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UNIT 1 Dynamics

OVERALL EXPECTATIONS

- analyze technological devices that apply the principles of the dynamics of motion, and assess the technologies' social and environmental impact
- investigate, in qualitative and quantitative terms, forces involved in uniform circular motion and motion in a plane, and solve related problems
- demonstrate an understanding of the forces involved in uniform circular motion and motion in a plane

BIG IDEAS

- Forces affect motion in predictable and quantifiable ways.
- Forces acting on an object will determine the motion of that object.
- Many technologies that utilize the principles of dynamics have societal and environmental implications.

UNIT TASK **PREVIEW**

In the Unit Task, you will apply some of the principles of physics that are used in sports and games. The Unit Task is described in detail on page 146. As you work through the unit, look for Unit Task Bookmarks to see how information in the section relates to the Unit Task.

APPLYING THE DYNAMICS OF MOTION

One of the most exciting and dangerous winter sports is the one-person luge (sled), where athletes race down icy tracks with high banked curves on top of a small luge, as seen in the image on the facing page.

The thrill of the luge comes from the high levels of speed the athlete can reach. To help reduce air resistance and reach high speeds, athletes try to be as aerodynamic as possible by lying down and keeping their heads down and toes pointed. They also wear specially designed aerodynamic racing helmets and suits and use lightweight luges. In fact, many racers spend hours in wind tunnels designed to help them find the ideal body position to minimize drag.

The terrain and the mass of the luge also affect the speed. The steeper the hill, the faster the luge goes. However, if the luge crashes, the impact will be greater as well. The smoother the track, the less friction between the ice and the sled, and the faster the luge will go. The only brakes are the athlete's feet. Their shoes are covered with treads, like tire treads, to help protect them from the tremendous amount of friction created by braking.

The most dangerous points on a luge run are the turns and turn combinations, where circular motion and high velocities buffet the athlete. To maintain speed, the athlete must find just the right spot on the luge to perfectly balance the opposing forces.

Think about all the different kinds of motion and forces that occur as the luger speeds down the track. Which forces, if any, do you think have no direct effect on the motion of the luge? Which forces speed it up or slow it down? Gravity pulls the luger down the track, but some of this force is balanced by the other forces acting on the luge. For example, friction between the sled and the track works against gravity.

Questions

1. Which features of the luge and the athlete's technique help decrease the time of the run?
2. Why are the turns banked? Explain what you know about the forces acting on the athlete in the turns.
3. What are the main forces that cause the luge to speed up? When do the largest accelerations occur?
4. In what direction is the net force on the luge and the athlete when going around a banked curve? Explain your reasoning.
5. The sport of luging is extremely dangerous. In your opinion, should more stringent conditions or rules be implemented for luge courses? Explain your reasoning.

UNIT 1 ARE YOU READY?

CONCEPTS

- kinematics
- Newton's laws of motion
- vectors and scalars
- friction
- Pythagorean theorem
- trigonometric ratios
- sine law and cosine law

SKILLS

- solving for unknown lengths and angles using trigonometry
- communicating scientific information clearly and accurately
- analyzing graphs
- solving motion problems using kinematics equations
- drawing free-body diagrams and force diagrams
- determining vector components and net force

Concepts Review

1. Two students travel to school by different means. One takes the bus, and his path includes many turns and stops. The other student rides her bicycle and travels directly to school without any stops and turns. Describe each student's displacement. K/U C
2. Which object has more acceleration, a jet cruising with a constant speed of 600 km/h or a baseball player just after he hits the ball and starts running from rest? K/U
3. Describe a specific case in which an object's velocity and acceleration vectors point in opposite directions. K/U C A
4. (a) Some people dry their hands by flicking the water off (moving the hands rapidly and then stopping them suddenly). Explain how this action illustrates one of Newton's laws of motion.
   (b) Explain, using Newton's laws, how this knowledge can help you get ketchup out of a glass bottle in the most efficient way.
   (c) Explain, using Newton's laws, why hitting the bottom of a ketchup bottle is not the most effective way to get the ketchup out. Then explain why it works at all. K/U T/I C A
5. With some effort, a man can push his car to a nearby service station, even though the car is much more massive than he is (**Figure 1**). Describe the forces between the man and the car. Discuss where friction helps him and where friction hinders him in this situation. K/U A

Figure 1

6. A large football player collides with a smaller football player during a game. They exert forces on each other when they collide. K/U A
   (a) How do the magnitude and direction of the forces compare?
   (b) Which player is more likely to experience a greater acceleration? Explain your reasoning.
   (c) Explain why football players wear protective equipment even though it slows them down.
7. You push on a large heavy box with a horizontal and gradually increasing force. At first, the box does not move, but eventually it begins to accelerate. K/U T/I C
   (a) Which force keeps the box at rest when you start to push? Describe the magnitude and direction of this force.
   (b) Which forces act on the box when it is moving? Draw a free-body diagram of the box when it is moving.
   (c) Sketch a simple graph of the force of friction acting on the box (vertical axis) as a function of the applied force on the box (horizontal axis). Explain your reasoning.
8. You slide a dynamics cart up an incline. The cart moves directly up the ramp and then back down to the bottom, where you catch it. Sketch the three motion graphs for the cart when it is moving freely on the ramp without you exerting an applied force on it. Justify your reasoning. K/U T/I C

Skills Review

9. A hockey puck slides across the ice, eventually coming to rest a long distance from where it was hit. K/U T/I C
   (a) Draw a system diagram of the side view of the puck as it is sliding to the right.
   (b) Draw a free-body diagram of the puck.

10. For many centuries, people believed Aristotle's theory of free fall, which said two things: (a) objects immediately reach a constant velocity after they are released, and (b) the constant falling velocity depends on the mass of the object. Describe an investigation that you could conduct to test the validity of Aristotle's claims. K/U T/I C

11. A launcher applies a constant force to several different masses. An observer measures the acceleration during the launch for each mass. Which of the graphs in **Figure 2** shows the correct plot of acceleration versus mass? K/U T/I

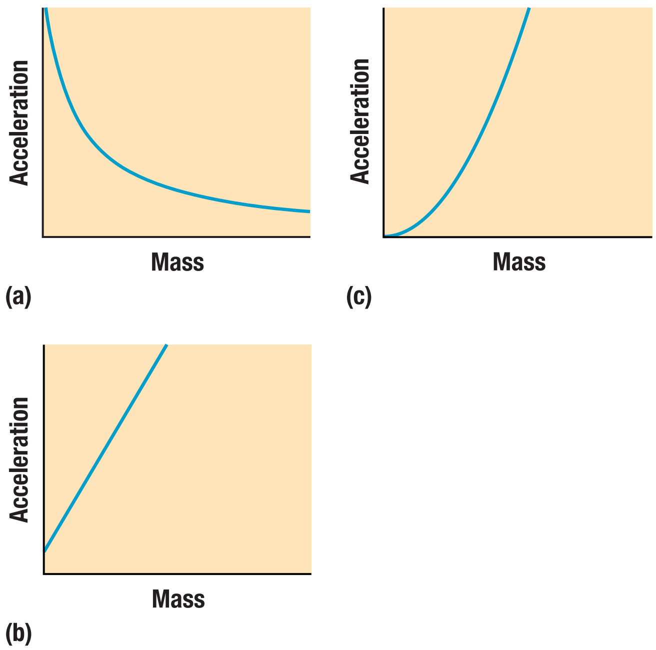


Figure 2

12. A storm front moving in approximately a straight line reaches Toronto at 3:45 p.m. and Peterborough at 4:30 p.m. the same day. If the storm continues moving at the same rate, when will it reach Ottawa (nearly in a line with the other cities)? Peterborough is 90 km from Toronto, and Ottawa is 220 km from Peterborough. K/U T/I A

13. A speed boat cruises with a velocity of 41.0 km/h [N]. K/U T/I
    (a) How far will the boat travel in 15 min?
    (b) Determine the net force acting on the boat.
    (c) How does the total of all the frictional forces acting on the boat compare to the applied force of the water on the boat?

14. A 12 kg mass is pushed by two forces, A and B. Force A is 55 N [W], and force B is 82 N [E]. K/U T/I
    (a) Calculate the net force due to forces A and B on the mass.
    (b) Calculate the acceleration of the mass.

15. Calculate the magnitude of all the forces acting on each mass below. K/U T/I
    (a) A 14 kg mass sits at rest on top of a desk.
    (b) A 3.2 kg mass is pulled horizontally across the floor at a constant velocity with a force of 4.5 N.
    (c) A 4.7 kg mass is pushed horizontally forward by an 8.6 N force, and the mass accelerates at 1.1 m/s^2 [forward].

16. A 15 kg mass sits on top of a scale calibrated in newtons. You push straight down on the mass with a force of 22 N, and the reading on the scale goes up. K/U T/I
    (a) What was the reading on the scale before you pushed down?
    (b) What was the reading on the scale after you pushed down?
    (c) The reading on the scale provides the magnitude of one of the forces acting on the mass. Which force is it? Explain your reasoning.

17. Solve for the unknown lengths, *a*, *b*, *c*, and *d*, in the right-angled triangles in **Figure 3**. T/I

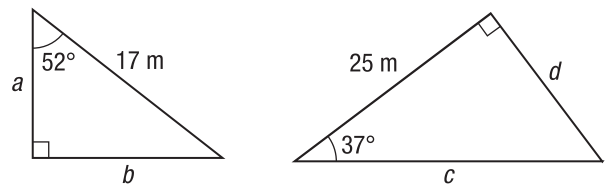


Figure 3

18. Solve for the unknown length, *c*, and the unknown angles, *x* and *y*, in the scalene triangle in **Figure 4**. T/I

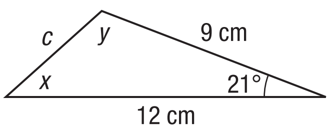


Figure 4

CAREER PATHWAYS PREVIEW

Throughout this unit, you will see Career Links. Go to the Nelson Science website to find information about careers related to Dynamics. On the Chapter Summary page at the end of each chapter, you will find a Career Pathways feature that shows you the educational requirements of the careers. There are also some career-related questions for you to research.

CHAPTER 1

Kinematics

KEY CONCEPTS

After completing this chapter you will be able to

- solve one-dimensional and two-dimensional motion problems involving average speed and average velocity
- calculate average acceleration using vector subtraction in two dimensions
- solve projectile motion problems using components
- solve relative motion problems in two dimensions
- conduct an inquiry to observe projectile motion

How Can Two-Dimensional Motion Be Analyzed?

An acrobat launched from a cannon at a target, such as the one on the facing page, is a familiar sight at circuses and fairs around the world. How you see an acrobat's trajectory (path) depends on your point of view. Suppose you are watching the launch outside on a sunny day. As the acrobat leaves the cannon, passing across your field of view, you notice her shadow on the ground. How does it appear to move across the ground when the Sun is directly overhead? Now suppose you are watching from behind the cannon. How would the motion of the acrobat appear from this point of view?

You may not see an acrobat launched from a cannon very often, but you do encounter the same type of projectile motion in everyday life. Anything hurled, pitched, tossed, or otherwise projected with force becomes a projectile. A football spiralling toward the receiver is a projectile, as are fireworks shot into the air, water squirting from a fountain, and an athlete hurtling over a high-jump bar.

Although it may seem simple, the study of projectile motion involves many factors. The acrobat's landing on the target requires several calculations before launching from the cannon. She needs to be fired at a specific speed and at a particular angle. Since gravity will pull her to the ground, she needs to know how far the cannon will propel her forward and at what point she will begin her descent so that the target is set up in the right place.

A strong understanding of the physics of motion is necessary in other situations as well, such as designing ski equipment, hot-air balloons, and GPS and weather satellites. The physics of motion also plays an integral role in the everyday world of elevators and traffic, the fun areas of entertainment and sports, and the serious endeavours of technological innovation and space exploration. It is as necessary in implementing safety measures as it is in achieving new horizons in exploration.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. In what direction is the acceleration of the human cannonball when
   (a) inside the cannon during firing?
   (b) moving through the air?
2. What kind of trajectory will the acrobat follow after leaving the cannon?
3. Which motion values are constant, and which values change, when the acrobat is moving through the air?
4. How do you think the motion of the shadow across the ground would look when the Sun is directly overhead?
5. Describe what you think you would see if you watched the acrobat fired out of the cannon when you were directly behind the cannon.

Mini Investigation

Launching Projectiles

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this activity, you will use a projectile launcher to model and observe projectile motion.

Equipment and Materials: eye protection; projectile launcher; ball of modelling clay or other soft material

1. Set the projectile launcher to fire at an angle of 45° (**Figure 1**).

Direct the launcher away from observers into an obstacle-free area. Obtain your teacher's permission before using the launcher.

2. Put on your eye protection, and launch the ball.
3. Watch the path of the ball from different points of view as it goes up and then falls to the ground.
4. Observe the ball's trajectory, maximum height, horizontal distance, and time of flight.

Figure 1

5. Repeat the activity by launching the ball at several different angles, including 0°, 30°, 60°, and 90°.

A. What happens to the maximum height of the ball as the launch angle increases? T/I A

B. How does the angle affect the horizontal distance and the time of flight? T/I A

C. How would you describe the motion of the ball in
(a) the horizontal direction?
(b) the vertical direction? T/I A

1.1 Motion and Motion Graphs

Cars drive by on the street, people walk and cycle past us, and garbage cans blow in a high wind. From quiet suburbs to busy highways, different kinds of motion happen all the time during a normal day (**Figure 1**). We often take this motion for granted, but we react to it instinctively: We dodge out of the way of objects flying or swerving toward us. We change our own motion to avoid hitting objects in our way or to get to school on time.

Kinematics is the study of motion without considering the forces that produce the motion. **Dynamics**, which is the topic of study in Chapters 2 and 3, is the study of the causes of motion. An understanding of kinematics and dynamics is essential in understanding motion.

Figure 1 A highway is a good example of the physics of motion in action.

kinematics the study of motion without considering the forces that produce the motion

dynamics the study of the causes of motion

scalar a quantity that has magnitude (size) but no direction

vector a quantity that has both magnitude (size) and direction

Kinematics Terminology

In physics, the terms we use to describe motion—displacement, distance, speed, velocity, and acceleration—all have specific definitions and equations that connect them. We can divide the mathematical quantities we use to describe the motion of objects into two categories: scalars and vectors. **Scalars** are quantities that have only a magnitude, or numerical value. **Vectors** have both a magnitude and a direction.

Position and Displacement

To start, consider motion along a straight line—one-dimensional motion—such as a hockey puck sliding on a horizontal icy surface. **Figure 2(a)** shows what a multiple-exposure image of a hockey puck in motion might look like. The dots represent evenly spaced intervals of time. The puck is moving the same distance in each interval, so the puck is moving at a constant speed. Here, we measure **position**—the distance and direction of an object from a reference point—as the distance from the origin on the horizontal axis to the centre of the hockey puck. For one-dimensional motion, distance specifies the position of the object. We can use the information in Figure 2(a) to construct a graph of the puck's position as a function of time, as shown in **Figure 2(b)**. Notice in Figure 2(b) that the distance axis is now vertical as we plot the position, $\vec{d}$, as a function of time, t. In such a position–time plot, it is conventional to plot time along the horizontal axis. The change in the puck's position, in one direction, is its **displacement**. Mathematically, for one-dimensional motion, displacement is written as

position ($\vec{d}$) the straight-line distance and direction of an object from a reference point

displacement ($\Delta\vec{d}$) the change in position of an object

$$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$$

where $\vec{d}_1$ is the object's initial position and $\vec{d}_2$ is the object's final position. While displacement tells us how far an object moves, it does not tell us how fast it moved.

Vectors provide two pieces of information, so we need a specific way of presenting this information clearly and unambiguously. For example, a displacement of 15 m [E] clearly identifies a magnitude of 15 and a direction of east.

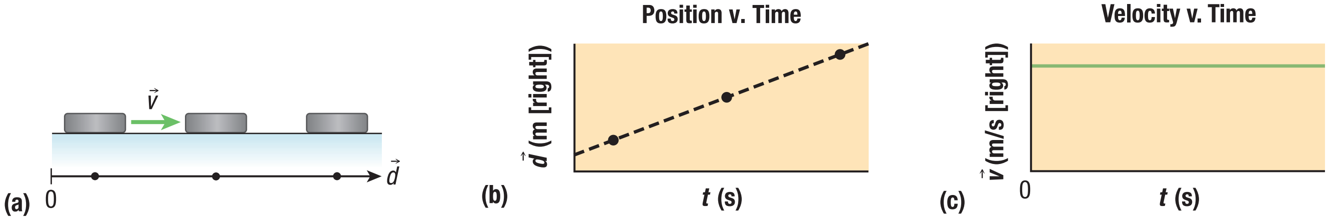


Figure 2 (a) If we took multiple exposures of a hockey puck travelling across an icy surface at a constant speed, the photo might look like this. (b) Each dot in the graph corresponds to a position of the puck in (a). (c) The velocity of the puck versus time is a straight line in this case.

Speed and Velocity

Speed and velocity are related quantities, but they are not the same. Many people use them interchangeably in everyday language, but strictly speaking this is incorrect in scientific terms. Speed tells how fast an object is moving, and speed is always a positive quantity or zero. The distance between adjacent dots in Figure 2(a) shows how far the puck has moved during each time interval. The **average speed**, v_{av}, of an object is the total distance travelled divided by the total time to travel that distance. Speed is a scalar quantity; it does not have a direction. The SI unit for speed is metres per second (m/s). You can calculate the average speed using the equation

average speed (v_{av}) the total distance travelled divided by the total time to travel that distance

$$v_{av} = \frac{\Delta d}{\Delta t}$$

Velocity, the change in position divided by the time interval, is a vector quantity, so it is written with the vector arrow, $\vec{v}$. The direction of $\vec{v}$ indicates the direction of the motion. In the puck example, the direction of the velocity is to the right. But it could have been negative (motion to the left, toward smaller or more negative values of $\vec{d}$) or even zero. So, velocity can be positive, negative, or zero.

velocity ($\vec{v}$) the change in position divided by the time interval

How does an object's velocity relate to its position? For a particular time interval that begins at time t_1 and ends at time t_2, the time interval is $\Delta t = t_2 - t_1$. You can then calculate the average velocity using the equation

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

The hockey puck in Figure 2(a) is sliding at a constant speed, so its velocity also has a constant value, as shown in the velocity–time graph in **Figure 2(c)**. In this case, the velocity is positive, which means that the direction of motion is toward increasing values of $\vec{d}$ (that is, toward the right).

When an object moves with a constant speed, the **average velocity**, $\vec{v}_{av}$—the displacement divided by the time interval for that change—is constant throughout the motion, and the position–time graph has a constant slope, as in Figure 2(b). For more general cases, the average velocity is the slope of the line segment that connects the positions at the beginning and end of the time interval, called the **secant**. This is illustrated in the hypothetical position–time graph in **Figure 3**. In Tutorial 1, you will solve two simple problems related to average velocity and average speed.

average velocity ($\vec{v}_{av}$) the displacement divided by the time interval for that change; the slope of a secant on a position–time graph

secant a straight line connecting two separate points on a curve

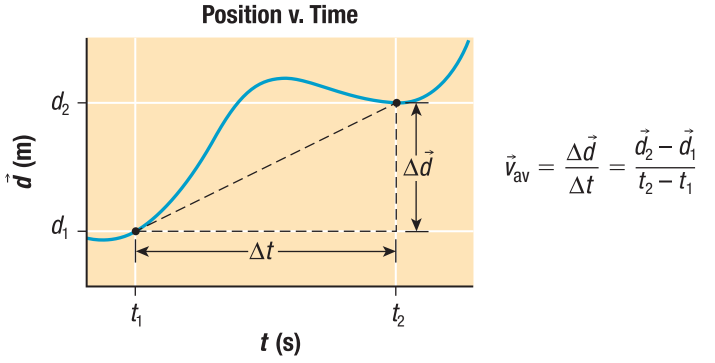


Figure 3 The average velocity during the time interval from t_1 to t_2 is the slope of the dashed line connecting the two corresponding points on the curve.

Tutorial 1 Distinguishing between Average Speed and Average Velocity

The following Sample Problem reviews how to calculate average speed and average velocity.

Sample Problem 1: Calculating Average Velocity and Average Speed

A jogger takes 25.1 s to run a total distance of 165 m by running 140 m [E] and then 25 m [W]. The displacement is 115 m [E].

(a) Calculate the jogger's average velocity.

(b) Calculate the jogger's average speed.

Solution

(a) **Given:** $\Delta\vec{d} = 115$ m [E]; $\Delta t = 25.1$ s

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$

Solution:
$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{115\ \text{m [E]}}{25.1\ \text{s}}$$
$$\vec{v}_{av} = 4.58\ \text{m/s [E]}$$

Statement: The jogger's average velocity is 4.58 m/s [E].

(b) **Given:** $\Delta d = 165$ m; $\Delta t = 25.1$ s

Required: v_{av}

Analysis: $v_{av} = \dfrac{\Delta d}{\Delta t}$

Solution:
$$v_{av} = \frac{\Delta d}{\Delta t} = \frac{165\ \text{m}}{25.1\ \text{s}}$$
$$v_{av} = 6.57\ \text{m/s}$$

Statement: The jogger's average speed is 6.57 m/s.

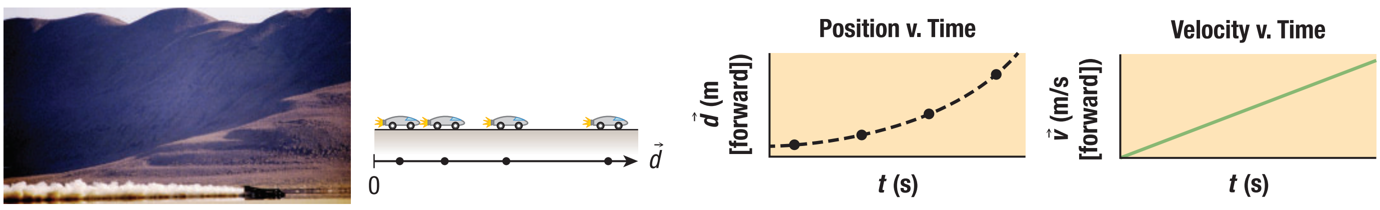
Practice

1. A woman leaves her house to walk her dog. They stop a few times along a straight path. They walk a distance of 1.2 km [E] from their house in 24 min. In another 24 min, they turn around and take the same path home. Give your answers to the following questions in kilometres per hour. T/I A
   (a) Determine the average speed of the woman and her dog for the entire route. [ans: 3.0 km/h]
   (b) Calculate the average velocity from their house to the farthest position from the house. [ans: 3.0 km/h [E]]
   (c) Calculate the average velocity for the entire route. [ans: 0.0 km/h]
   (d) Are your answers for (b) and (c) different? Explain why or why not.
2. A bus driver begins a descent down a steep hill and suddenly sees a deer about to cross the road. He applies the brakes. During the bus driver's 0.32 s reaction time, the bus maintains a constant velocity of 27 m/s [forward]. Determine the displacement of the bus during the time the driver takes to react. T/I [ans: 8.6 m [forward]]
3. Drivers at the Daytona 500 Speedway in Florida must complete 200 laps of a track that is 4.02 km long. Calculate the average speed, in kilometres per hour, of a driver who completes 200 laps in 6.69 h. T/I [ans: 1.20×10^2 km/h]
4. A student in a mall walks 140 m [E] in 55 s to go to his favourite store. The store is not open yet, so he walks 45 m [W] in 21 s to go to another store. Calculate his
   (a) average speed [ans: 2.4 m/s]
   (b) average velocity T/I [ans: 1.2 m/s [E]]
5. A delivery truck heads directly south for 62 km, stopping for an insignificant amount of time, and then travels 78 km directly north. The average speed for the entire trip is 55 km/h. K/U T/I C
   (a) Determine the average velocity for the entire trip in kilometres per hour. [ans: 6.3 km/h [N]]
   (b) Why is the average velocity of the truck so much smaller than the average speed?

Graphical Interpretation of Velocity

Consider another example of one-dimensional motion: a rocket-powered car travelling on a straight, flat road (**Figure 4(a)**). Assume the car is initially at rest; "initially" means that the car is not moving when the clock reads zero. At $t = 0$, the driver turns on the rocket engine and the car begins to move forward in a straight-line path. **Figure 4(b)** is a motion diagram showing the position of the car at evenly spaced instants in time. **Figure 4(c)** shows the corresponding position–time graph for the car, where again we use dots to mark the car's position at evenly spaced time intervals. Notice that the dots are not equally spaced along the position axis. Instead, their spacing increases as the car travels. This means that the car moves farther during each successive and equal time interval, so the velocity of the car increases with time.

In this example, the car moves toward increasing values of position, so the velocity is again positive and increases smoothly with time, as shown in **Figure 4(d)**.

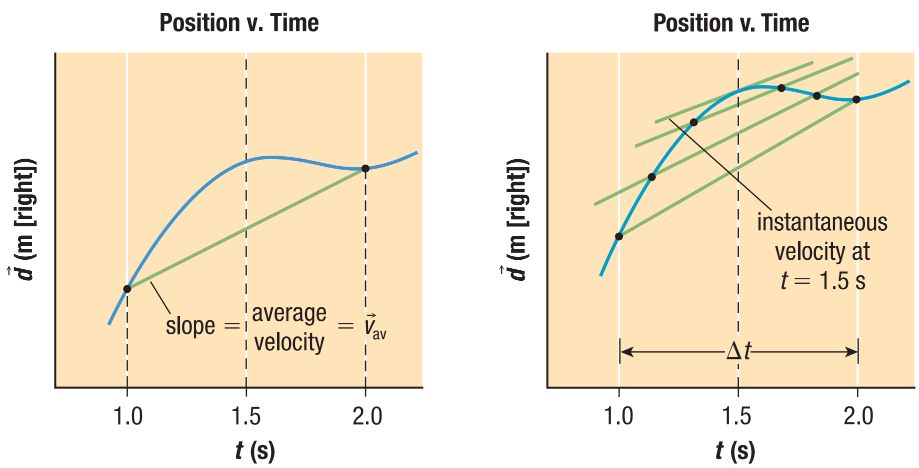


(a) (b) (c) (d)

Figure 4 (a) In 1997, the rocket-powered Thrust SSC became the first car to break the sound barrier by travelling at a speed of over 1200 km/h. (b) A time-lapse image of a rocket-propelled car travelling along a horizontal road might look like this. (c) Position versus time for the rocket-powered car. (d) The velocity–time graph indicates that the car's velocity is not constant.

Instantaneous Velocity

Figure 5 is a position–time graph for an object moving a certain displacement over a short time period. Again, we use dots to mark the position at the beginning and end of a particular time interval, which starts at $t = 1.0$ s and ends at $t = 2.0$ s. The average velocity during this time interval is just the displacement during the interval divided by the length of the time interval. Figure 5(a) shows that this average velocity is the slope of the secant connecting the two points on the curve.



(a) (b)

Figure 5 (a) The average velocity during a particular time interval is the slope of the line connecting the start of the interval to the end of the interval. (b) The instantaneous velocity at a particular time is the slope of the position–time curve at that time. The instantaneous velocity in the middle of a time interval is not necessarily equal to the average velocity during the interval.

With this approach, though, we lose the details about what happens in the middle of the interval. In Figure 5(a), the slope of the position–time curve varies considerably as we move through the interval from $t = 1.0$ s to $t = 2.0$ s. If we want to get a more accurate description of the object's motion at a particular instant within this time interval, say at $t = 1.5$ s, it is better to use a smaller interval. How small an interval should we use? In Figure 5(b), we consider slopes over a succession of smaller time intervals. Intuitively, we expect that using a smaller interval will give a better measure of the motion at a particular instant in time.

From Figure 5(b), we see that as we take smaller and smaller time intervals, we are actually approximating the slope of the position–time curve at the point of interest ($t = 1.5$ s) using a tangent. A **tangent** is a straight line that intersects a curve at a point and has the same slope as the curve at the point of intersection. The slope of a tangent to a position–time curve is called the **instantaneous velocity**, $\vec{v}$, which is the velocity of an object at a certain instant of time.

tangent a straight line that intersects a curve at a point and has the same slope as the curve at the point of intersection

instantaneous velocity ($\vec{v}$) the velocity of an object at a particular instant; the slope of the tangent to a position–time graph

instantaneous speed (v) the speed of an object at a particular instant; the magnitude of the slope of the tangent to a position–time graph

In some cases, a moving object changes its speed during its motion, so we need to clarify the difference between average speed and instantaneous speed: As mentioned earlier in this section, average speed is the total distance divided by the total time. **Instantaneous speed**, v, refers to the speed of an object at any given instant in time and is defined as the magnitude of the slope of the tangent to a position–time graph.

Moving objects do not always travel with changing speeds. Objects often move at a steady rate with a constant speed. The difference between the average and instantaneous values can be understood using the analogy of a car's speedometer. The speedometer reading gives your instantaneous speed, which is the magnitude of your instantaneous velocity at a particular moment in time. If you are taking a long drive, your average speed will generally be different because the average value will include periods when you are stopped in traffic, accelerating to pass other cars, and so on. In many cases, such as in discussions with a police officer, the instantaneous value will be of greater interest. CAREER LINK

The instantaneous velocity gives a mathematically precise measure of how the position is changing at a particular moment, making it much more useful than the average velocity. For this reason, from now on in this textbook we refer to the instantaneous velocity as simply the "velocity," and we denote it by $\vec{v}$.

In Tutorial 2, you will analyze a position–time graph, use the graph to determine average velocity, and then create a velocity–time graph.

Tutorial 2 Working with Motion Graphs

In the following Sample Problem, we will calculate average velocity from a position–time graph. Then, we will analyze a position–time graph and use the data to make a velocity–time graph.

Sample Problem 1: Calculating Average Velocity and Sketching a Velocity–Time Graph

The position–time graph in **Figure 6** shows the details of how an object moved.

(a) Calculate the average velocity during the time interval $t_1 = 1.0$ s to $t_2 = 2.5$ s.

(b) Analyze the position–time graph in Figure 6. Use your analysis to sketch a qualitative velocity–time graph of the object's motion.

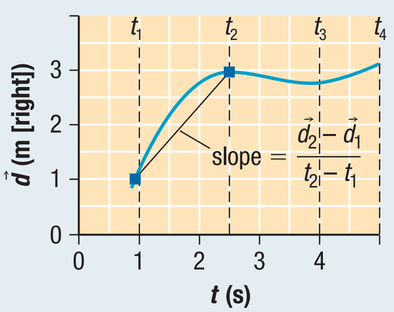


Figure 6

Solution

(a) **Given:** $t_1 = 1.0$ s; $t_2 = 2.5$ s

Required: $\vec{v}_{av}$ over the interval $t_1 = 1.0$ s to $t_2 = 2.5$ s

Analysis: To determine $\vec{v}_{av}$ between t_1 and t_2, calculate the slope of the line between t_1 and t_2.

Solution: Reading the values from the graph,

$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}\\ &= \frac{3.0\text{ m [right]} - 1.0\text{ m [right]}}{2.5\text{ s} - 1.0\text{ s}}\\ \vec{v}_{av} &= 1.3\text{ m/s [right]}\end{aligned}$$

Statement: The average velocity during the time interval $t_1 = 1.0$ s to $t_2 = 2.5$ s is 1.3 m/s [right].

(b) **Step 1.** To generate the data for the velocity–time graph, analyze the motion of the object in Figure 6.

This object is initially moving to the right. The object reverses direction near 2.5 s (t_2) and 4.0 s (t_3).

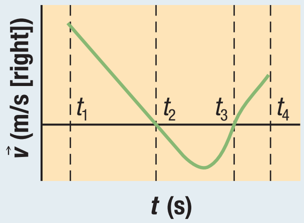
At t_1, the slope is large and positive, so v is large and positive at t_1.

At t_2, the slope is approximately zero, so v is near zero.

Between t_2 and t_3, the object is moving toward smaller values of position, so the slope of the position–time curve and, hence, the velocity, are negative. At t_3, the slope is zero, so the velocity is zero.

Finally, at t_4, the object is again moving to the right because d is increasing with time. So, v is again positive.

Step 2. After estimating the position–time slope at these places, we can construct a qualitative velocity–time graph.



Practice

1. Examine the position–time graphs in **Figure 7**. K/U A
   (a) In which graph(s) does the velocity increase with time? [ans: (c)]
   (b) In which graph(s) does the velocity decrease with time? [ans: (b)]

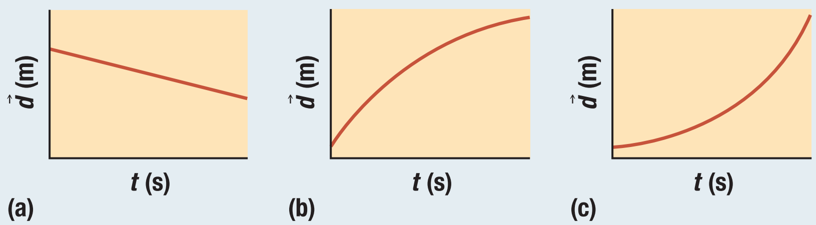


Figure 7

2. Analyze the graphs in **Figure 8**. Create a corresponding velocity–time graph for each graph. T/I C

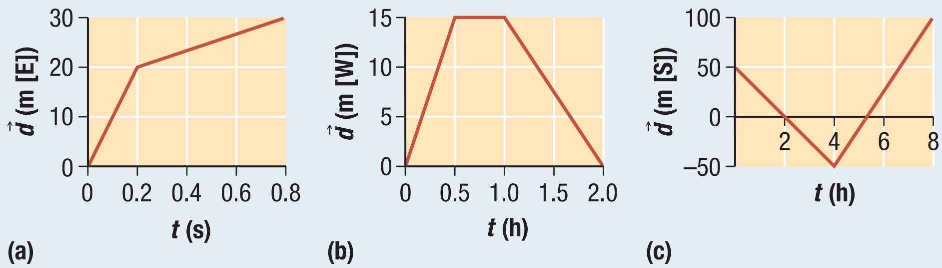


Figure 8

The average velocity is the slope of a secant on a position–time graph. To analyze the velocity, we take approximate values by drawing lines tangent to the position–time curve at several places and calculating their slopes. The instantaneous velocity at a particular time is always equal to the slope of the position–time curve at that time.

Acceleration

Acceleration is a measure of how velocity changes with time. The SI unit for acceleration is metres per second squared (m/s^2). Sometimes objects move at constant velocity, but usually the velocities we observe are changing. When an object's velocity is changing, that object is accelerating.

We can study acceleration using velocity–time graphs. These graphs display time values on the horizontal axis and velocity on the vertical axis. Velocity–time graphs can be useful when studying objects moving with uniform (constant) velocity (zero acceleration) or uniform acceleration (velocity changing, but at a constant rate). The velocity–time graphs for both uniform velocity and uniform acceleration are always straight lines. By contrast, the position–time graph of an accelerated motion is curved.

average acceleration ($\vec{a}_{av}$) the change in velocity divided by the time interval for that change

When an object's velocity changes by $\Delta\vec{v}$ over time Δt, the **average acceleration**, $\vec{a}_{av}$, or the change in velocity divided by the time interval for that change, during this interval is

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

instantaneous acceleration ($\vec{a}$) the acceleration at a particular instant in time

As with velocity, we often need to know the acceleration at a particular instant in time, or the **instantaneous acceleration**, $\vec{a}$. The instantaneous acceleration equals the slope of the velocity–time graph at a particular instant in time. If the velocity–time graph is straight during a time interval, then the acceleration is constant. This means that the instantaneous acceleration is equal to the average acceleration, and we can omit the subscript "av" in the equation above. You can now apply these concepts by completing Tutorial 3.

Tutorial 3 Working with Motion Graphs

In the following Sample Problem, we will create an acceleration–time graph from a velocity–time graph and analyze the graph to determine the maximum acceleration.

Sample Problem 1: Creating an Acceleration–Time Graph and Calculating the Maximum Acceleration

Suppose the sprinter in **Figure 9(a)** is running a 100 m dash. The sprinter's time and distance data have been recorded and used to make the velocity–time graph in **Figure 9(b)**.

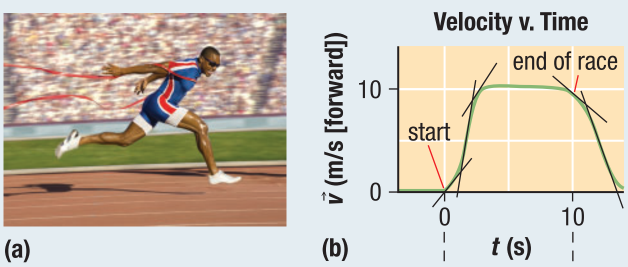


Figure 9

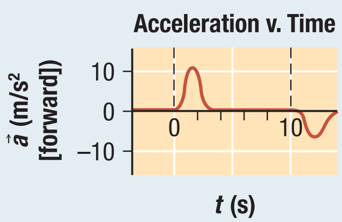
(a) Analyze the velocity–time graph in Figure 9(b). Use your analysis to make a sketch of an acceleration–time graph of the sprinter's motion.

(b) From your acceleration–time graph, determine the maximum acceleration and the time at which it occurs.

Solution

(a) **Step 1.** Acceleration is the slope of the velocity–time graph, so first we must estimate this slope at several different values of t to be able to graph the sprinter's acceleration as a function of time. Figure 9(b) shows several lines tangent to the velocity–time curve at various times. The slopes of these tangent lines give the acceleration. Estimate the slopes.

Step 2. Use the slope estimations to sketch the acceleration–time graph. The resulting acceleration–time graph is only qualitative (approximate). More accurate results would be possible if we had started with a more detailed graph of the velocity.



(b) The largest value of the acceleration is approximately 11 m/s^2. This occurs near the start of the race, around $t = 1.5$ s, when the sprinter is gaining speed. Note that this is where the slope of the velocity–time graph is largest. At the end of the race, as the runner crosses the finish line, he slows down and eventually comes to a stop with $v = 0$.

Practice

1. **Figure 10** shows a graph of the motion of a car along a straight road. K/U T/I C

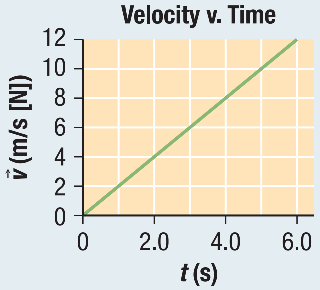


Figure 10

   (a) How can you tell from the graph that the car has a constant acceleration?
   (b) Describe the motion of the car.
   (c) Determine the acceleration of the car. [ans: 2.0 m/s^2 [N]]

2. Examine the velocity–time graph in **Figure 11**. K/U C A

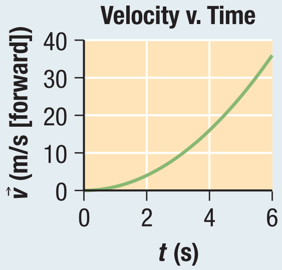


Figure 11

   (a) Determine the average acceleration for the entire trip. [ans: 6 m/s^2 [forward]]
   (b) Determine the instantaneous acceleration at 3 s and at 5 s. [ans: 6 m/s^2 [forward]; 10 m/s^2 [forward]]
   (c) Draw a reasonable acceleration–time graph of the motion.

3. Examine the velocity–time graph in **Figure 12**. T/I C

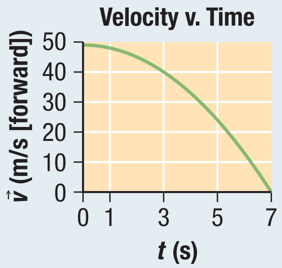


Figure 12

   (a) Determine the average acceleration for the entire trip. [ans: 7 m/s^2 [backward]]
   (b) Determine the instantaneous acceleration at 2 s, 4 s, and 6 s. [ans: 4 m/s^2 [backward]; 8 m/s^2 [backward]; 12 m/s^2 [backward]]
   (c) Draw a reasonable acceleration–time graph of the motion.

Acceleration is the slope of a velocity–time graph. Therefore, given a velocity–time graph, we can describe the behaviour of an object's acceleration. An interesting feature of the graphs in Sample Problem 1, on page 14, is that the maximum velocity and the maximum acceleration do not occur at the same time. It is tempting to think that if the "motion" is large, both v and a will be large, but this notion is incorrect. Acceleration is the slope—the rate of change—of the velocity versus time. The time at which the rate of change in velocity is greatest may not be the time at which the velocity itself is greatest.

1.1 Review

Summary

- The equation for average speed is $v_{av} = \dfrac{\Delta d}{\Delta t}$, and the equation for average velocity is $\vec{v}_{av} = \dfrac{\Delta \vec{d}}{\Delta t}$. The slope of an object's position–time graph gives the velocity of the object.
- Acceleration describes how quickly an object's velocity changes over time. The equation for average acceleration is $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$. The slope of an object's velocity–time graph gives the object's acceleration.

Questions

1. A cardinal flies east for 2.9 s in a horizontal plane for a distance of 22 m from a fence post to a bush. It then flies north another 11 m to a bird feeder for 1.5 s. T/I
   (a) Calculate the total distance travelled.
   (b) Calculate the cardinal's average speed.
   (c) Calculate the cardinal's average velocity.
2. At a sled race practice field in North Bay, Ontario, a dogsled team covers a single-lap distance of 2.90 km at an average speed of 15.0 km/h. T/I
   (a) Calculate the average speed in metres per second.
   (b) Calculate the time, in seconds, needed to complete the lap.
3. A skater travels straight across a circular pond with a diameter of 16 m. It takes her 2.1 s. T/I
   (a) Determine the skater's average speed.
   (b) How long would it take the skater to skate around the edge of the pond at the same average speed?
4. An airplane flies 450 km at a compass heading of 85° for 45 min. T/I
   (a) Calculate the airplane's average speed.
   (b) Calculate the airplane's average velocity.
5. A model rocket accelerates from rest to 96 km/h [W] in 4.1 s. Determine the average acceleration of the rocket. T/I
6. A batter hits a baseball in a batting-practice cage. The ball undergoes an average acceleration of 1.37×10^3 m/s^2 [W] in 3.12×10^{-2} s before it hits the cage wall. Calculate the velocity of the baseball when it hits the wall. T/I A
7. A track runner begins running at the starting whistle and reaches a velocity of 9.3 m/s [forward] in 3.9 s. Calculate the runner's average acceleration. T/I
8. The position–time graph in **Figure 13** represents the motion of a race car moving along a straight road. K/U T/I C

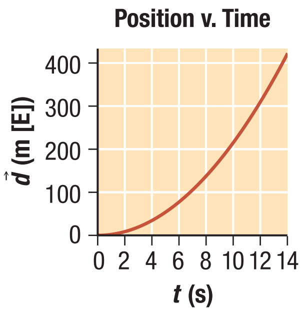


Figure 13

   (a) Determine the average velocity for the entire trip.
   (b) Determine the average velocity for the last 10 s of the motion. Why are the two average velocities different?
   (c) Determine the instantaneous velocity at 4.0 s, 8.0 s, and 12.0 s.
   (d) Sketch a qualitative velocity–time graph for the motion of the car.
9. Study the graph in **Figure 14**. K/U T/I C

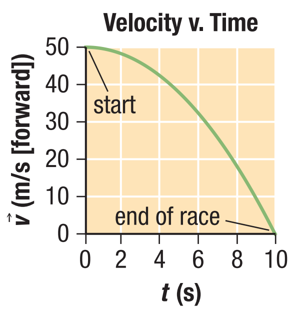


Figure 14

   (a) Determine the average acceleration for the entire trip.
   (b) Determine the instantaneous acceleration at 3.0 s, 6.0 s, and 9.0 s.
   (c) Sketch a qualitative acceleration–time graph of the motion.

Equations of Motion

You are driving a car along a road at a constant speed. Suddenly, you see a deer crossing the road ahead of you (**Figure 1**). You apply your brakes, slowing the car down. If you slow down at a constant rate, how far will you travel before you stop? Will you hit the animal?

To solve this kind of problem, you can use motion graphs. Using these graphs, you can calculate the distance you would travel while applying the brakes, which is the braking distance. You can also determine your braking time. The braking time is how long it takes to stop while applying the brakes.

There is another way to solve the problem, though: using motion equations. Motion equations relate different variables such as velocity, acceleration, and displacement. In most cases, it is easier and faster to solve problems using motion equations than by drawing motion graphs.

For motion with constant acceleration, motion equations relate five variables: the object's initial velocity, its final velocity, its acceleration, its displacement, and the time interval.

Figure 1 Understanding the physics of motion can help you prevent accidents.

One-Dimensional Motion with Constant Acceleration

The defining equation for average acceleration is

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

In this section and for all our equations of motion, the acceleration is constant, so we will use the symbol $\vec{a}$ for $\vec{a}_{av}$.

Figure 2 shows a velocity–time graph for an object with constant acceleration. The object starts with initial velocity $\vec{v}_i$, accelerates for a time Δt with acceleration $\vec{a}$, and ends with final velocity $\vec{v}_f$. The displacement is equal to the area under the line. The shape under the line is a trapezoid, and its area is

$$\Delta \vec{d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$$

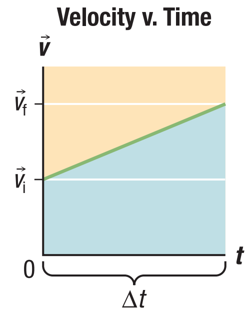


Figure 2 The displacement of an object is equal to the area under the line on a velocity–time graph. For constant acceleration, this shape is a trapezoid.

Notice that the above equation does not use the variable $\vec{a}$. This means we can combine it with the defining equation for average acceleration to derive other useful equations. There are five variables (Δt, $\Delta \vec{d}$, $\vec{v}_i$, $\vec{v}_f$, and $\vec{a}$), and we can derive equations that link any four of them.

For example, to eliminate $\vec{v}_f$, we can combine our equations. We can rearrange the defining equation for average acceleration to get $\vec{v}_f$:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

Substituting the above equation for $\vec{v}_f$ into the equation for displacement gives

$$\Delta \vec{d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$$

$$= \frac{1}{2}((\vec{v}_i + \vec{a}\Delta t) + \vec{v}_i)\Delta t$$

$$\Delta \vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

We can use similar substitutions to derive two more equations that eliminate $\vec{v}_i$ and Δt. The equations of motion for constant acceleration are shown in **Table 1**. In the following Tutorial, you will solve motion problems using these kinematics equations.

UNIT TASK BOOKMARK

You can use some of the equations in Table 1 when you complete the Unit Task on page 146.

Table 1 The Five Key Equations for Uniformly Accelerated Motion

	Equation	Variables found in equation	Variable not in equation
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$	$\Delta\vec{d}$, Δt, $\vec{v}_f$, $\vec{v}_i$	$\vec{a}$
Equation 2	$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$	$\vec{v}_f$, $\vec{v}_i$, $\vec{a}$, Δt	$\Delta\vec{d}$
Equation 3	$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$	$\Delta\vec{d}$, $\vec{v}_i$, Δt, $\vec{a}$	$\vec{v}_f$
Equation 4	$v_f^2 = v_i^2 + 2a\Delta d$	v_f, v_i, a, Δd	Δt
Equation 5	$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}\Delta t^2$	$\Delta\vec{d}$, $\vec{v}_f$, Δt, $\vec{a}$	$\vec{v}_i$

Tutorial 1 Using the Equations of Motion to Solve for Accelerated Motion

In the following Sample Problems, we will use the kinematics equations to analyze motion.

Sample Problem 1: Solving for Time and Final Velocity

Two cars are at rest on a straight road. Car A starts 120 m ahead of car B, and both begin moving in the same direction at the same time. Car A moves at a constant velocity of 7.0 m/s [forward]. Car B moves at a constant acceleration of 2.0 m/s^2 [forward]. Calculate how long it will take for car B to catch up with car A, and calculate the velocities of the two cars when they meet.

Given: We have two sets of variables, one for each car. We will label them with subscripts A and B: $\vec{v}_{Ai}$; $\vec{v}_{Bi}$; $\vec{a}_A$; $\vec{a}_B$; $\vec{v}_{Ai} = 7.0$ m/s [forward]; $\vec{a}_A = 0$ m/s^2; $\vec{v}_{Bi} = 0$ m/s; $\vec{a}_B = 2.0$ m/s^2 [forward]

Required: Δt; $\vec{v}_{Af}$; $\vec{v}_{Bf}$

Analysis: Using $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$, express the displacement of each car using forward as positive, and then relate the two displacements.

Solution: The displacement of car A from its starting position is

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$\Delta\vec{d}_A = (7.0 \text{ m/s [forward]})\Delta t + \left(\frac{1}{2}(0)\right)\Delta t^2$$

$$\Delta\vec{d}_A = (7.0 \text{ m/s [forward]})\Delta t$$

The displacement of car B from its starting position is

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$\Delta\vec{d}_B = (0)\Delta t + \frac{1}{2}(2.0 \text{ m/s}^2 \text{ [forward]})\Delta t^2$$

$$\Delta\vec{d}_B = (1.0 \text{ m/s}^2 \text{ [forward]})\Delta t^2$$

We want to solve for the time at which car B has caught up with car A. That is when $\Delta\vec{d}_B = \Delta\vec{d}_A + 120$ m [forward] because, for car B to catch up, it has to cover the initial 120 m separation as well as the displacement of car A.

Using our expressions for $\Delta\vec{d}_A$ and $\Delta\vec{d}_B$, we get the following quadratic equation:

$$\Delta\vec{d}_B = \Delta\vec{d}_A + 120 \text{ m}$$

$$\Delta t^2 = 7.0\Delta t + 120$$

Note that we are omitting the units here to keep the equation uncluttered. Rearranging gives the equation below.

$$\Delta t^2 - 7.0\Delta t - 120 = 0$$

$$(\Delta t - 15)(\Delta t + 8.0) = 0$$

There are two solutions: $\Delta t = 15$ and $\Delta t = -8.0$. We want the positive time value, which is $\Delta t = 15$ s.

Since car A is not accelerating, the velocity of car A is unchanged: $\vec{v}_{Ai} = 7.0$ m/s [forward].

We can determine the final velocity of car B:

$$\vec{v}_{Bf} = \vec{v}_{Bi} + \vec{a}_B\Delta t$$

$$= 0 + (2.0 \text{ m/s}^2 \text{ [forward]})(15 \text{ s})$$

$$\vec{v}_{Bf} = 3.0 \times 10^1 \text{ m/s [forward]}$$

Statement: Car B catches up with car A after 15 s. When they meet, the velocity of car A is 7.0 m/s [forward], and the velocity of car B is 3.0 × 10^1 m/s [forward].

Sample Problem 2: Solving for Time and Displacement

A motorcyclist drives along a straight road with a velocity of 30.0 m/s [forward]. The driver applies the brakes and slows down at 5.0 m/s^2 [backward].

(a) Calculate the braking time.

(b) Determine the braking distance (displacement).

Solution

(a) **Given:** $\vec{v}_i = 30.0$ m/s [forward]; $\vec{a} = 5.0$ m/s^2 [backward]; $\vec{v}_f = 0$

Required: Δt

Analysis: $a = \dfrac{v_f - v_i}{\Delta t}$; the motorcycle is slowing down, so make the acceleration negative and the initial velocity positive.

Solution:

$$a = \frac{v_f - v_i}{\Delta t}$$

$$\begin{aligned}\Delta t &= \frac{v_f - v_i}{a} \\ &= \frac{0\text{ m/s} - 30.0\text{ m/s}}{-5.0\text{ m/s}^2} \\ \Delta t &= 6.0\text{ s}\end{aligned}$$

Statement: The braking time is 6.0 s.

(b) **Given:** $\vec{v}_i = 30.0$ m/s [forward]; $\vec{a} = -5.0$ m/s^2 [forward]; $\Delta t = 6.0$ s

Required: $\Delta \vec{d}$

Analysis: $\Delta \vec{d} = \vec{v}_i \Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$

Solution:

$$\begin{aligned}\Delta \vec{d} &= \vec{v}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2 \\ &= (30.0\text{ m/s [forward]})(6.0\text{ s}) + \\ &\qquad \frac{1}{2}(-5.0\text{ m/s}^2\text{ [forward]})(6.0\text{ s})^2 \\ &= 180.0\text{ m [forward]} - 90.0\text{ m [forward]} \\ \Delta \vec{d} &= 90.0\text{ m [forward]}\end{aligned}$$

Statement: The braking distance is 90.0 m [forward].

Practice

1. A motorcyclist is travelling at 15.0 m/s [forward] and applies the brakes. The motorcycle slows down at 5.0 m/s^2 [backward]. T/I A
   (a) Determine the motorcycle's braking distance. [ans: 22 m [forward]]
   (b) Compare your answer to (a) to the answer for Sample Problem 2. What do these two problems indicate about speeding and traffic safety?
2. A man starts at rest and then runs north with a constant acceleration. He travels 120 m in 15 s. Calculate his acceleration. T/I [ans: 1.1 m/s^2 [N]]
3. A bus is moving at 22 m/s [E] for 12 s. Then the bus driver slows down at 1.2 m/s^2 [W] until the bus stops. Determine the total displacement of the bus. T/I [ans: 4.7×10^2 m]
4. In a 100.0 m sprint, a runner starts from rest and accelerates to 9.6 m/s [W] in 4.2 s. T/I
   (a) Calculate the acceleration of the runner. [ans: 2.3 m/s^2 [W]]
   (b) Calculate the displacement of the runner. [ans: 2.0×10^1 m [W]]
   (c) The runner runs at a constant velocity for the rest of the race. What is the total time? [ans: 13 s]
5. Two football players separated by 42 m run directly toward each other. Football player 1 starts from rest and accelerates at 2.4 m/s^2 [right], and football player 2 moves uniformly at 5.4 m/s [left]. K/U T/I A
   (a) How long does it take for the players to collide? [ans: 4.1 s]
   (b) How far does each player move? [ans: player 1: 20 m; player 2: 22 m]
   (c) How fast is football player 1 moving when the players collide? [ans: 9.8 m/s]
6. A jet lands on a runway at 110 m/s [forward]. When stopping, the jet can accelerate at 6.2 m/s^2 [backward]. K/U T/I A
   (a) Calculate the minimum time for the jet to stop. [ans: 18 s]
   (b) What is the minimum safe length for this runway? [ans: 9.8×10^2 m]
   (c) Explain why the runway should be much longer than the minimum safe length.

Freely Falling Objects

When you let go of a ball, it will fall down because Earth's gravity pulls it down. When you throw a ball upward, it will move upward for a time, stop, change direction, and then move downward. In both cases, the ball is moving under the influence of gravity. An object that is only moving under the influence of gravity is said to be in **free fall**. The ball is in free fall when you drop it, and the ball is in free fall when you throw it upward. Gravity moves objects downward with a constant acceleration of 9.8 m/s^2. This important value is called the acceleration due to gravity, g.

On Earth, other factors affect moving objects, such as air resistance. You will read more about air resistance in Chapter 2.

free fall the motion of a falling object where the only force acting on the object is gravity

Tutorial 2 Using the Equations of Motion for Accelerated Motion Due to Gravity

Sample Problem 1: Calculating Time and Velocity

A ball is thrown from a height of 52 m from the top of a building with a velocity of 24 m/s straight up.

(a) Determine the velocity of the ball at ground level.

(b) How long does it take for the ball to reach the ground?

Solution

(a) **Given:** $\vec{v}_i = 24$ m/s [up]; $\vec{g} = 9.8$ m/s^2 [down]; $\Delta\vec{d} = 52$ m [down]

Required: $\vec{v}_f$

Analysis: Use $v_f^2 = v_i^2 + 2a\Delta d$ to calculate the final velocity. Use up as positive and down as negative.

Solution:
$$v_f^2 = v_i^2 + 2a\Delta d$$
$$v_f^2 = (+24\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(-52\text{ m})$$
$$v_f = \pm 39.9\text{ m/s (one extra digit carried)}$$

Since the ball is moving down, use the negative value.

Statement: The velocity of the ball at ground level is -4.0×10^1 m/s, or 4.0×10^1 m/s [down].

(b) **Given:** $\vec{v}_i = +24$ m/s; $\vec{v}_f = -39.9$ m/s; $\vec{g} = -9.8$ m/s^2

Required: Δt

Analysis: $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$\Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$$

Solution: $\Delta t = \dfrac{\vec{v}_f - \vec{v}_i}{\vec{a}}$

$$= \frac{(-39.9\text{ m/s}) - (+24\text{ m/s})}{-9.8\text{ m/s}^2}$$
$$\Delta t = 6.5\text{ s}$$

Statement: It takes 6.5 s for the ball to reach the ground.

Practice

1. An apple is thrown upward from ground level at 22 m/s. Determine the maximum height of the apple. T/I [ans: 25 m]
2. Calculate the length of time it would take an object to fall 10.0 m if g were one-sixth the value of Earth's g (the acceleration due to gravity on the Moon). T/I [ans: 3.5 s]
3. A ball is thrown straight down at 12 m/s toward the ground from a height of 45 m. T/I
   (a) How long does it take to land? [ans: 2.0 s]
   (b) How fast is the ball moving when it lands? [ans: 32 m/s]
   (c) How does your answer to (b) change if the ball is initially thrown at 12 m/s [upward]?
4. In a physics experiment, a ball is released from rest, and it falls toward the ground. The timer was not paying attention but estimates that it took 1.5 s for the ball to fall the last 32 m. T/I
   (a) Calculate the velocity of the ball when it is 32 m above the ground. [ans: 14 m/s [downward]]
   (b) Calculate the total displacement of the ball. [ans: 42 m [downward]]
5. Matt tosses a set of keys straight up to Sanjit, who catches them from a window 1.1 s later at a height of 14 m above the point of release. T/I
   (a) Determine the initial velocity of the keys. [ans: 18 m/s [up]]
   (b) Determine the velocity of the keys when they were caught. [ans: 7.3 m/s [up]]

1.2 Review

Summary

- The five key equations for uniformly accelerated motion, listed in Table 1 on page 18, use the following variables: displacement, initial velocity, final velocity, acceleration, and time interval.
- Free fall is the motion of an object when it is moving only under the influence of gravity.
- The acceleration due to gravity on Earth is 9.8 m/s^2 [down].

Questions

1. Upon leaving the starting gate, a racehorse accelerates at a constant 4.1 m/s^2 [forward] for 5.2 s. Determine the horse's
   (a) displacement
   (b) final velocity T/I A
2. An electron travelling at 7.72×10^6 m/s [E] enters a force field that reduces its velocity to 2.46×10^6 m/s [E]. The acceleration is constant. The displacement during the acceleration is 0.478 m [E]. T/I A
   (a) Determine the electron's acceleration.
   (b) Determine the time interval over which the acceleration occurs.
3. A police officer at rest at the side of the highway notices a speeder moving at 62 km/h along a straight level road near an elementary school. When the speeder passes, the officer accelerates at 3.0 m/s^2 in pursuit. The speeder does not notice until the police officer catches up. T/I A
   (a) How long will it take for the officer to catch the speeder?
   (b) How far will they move from the position where the officer was at rest?
   (c) Calculate the speed of the police car when the officer catches the speeder. Is this reasonable?
   (d) Now assume that the police officer accelerates until the police car is moving 10.0 km/h faster than the speeder and then moves at a constant velocity until the police officer catches up. How long will it take to catch the speeder? Is this scenario more reasonable than the scenario in (c)? Explain your answer.
4. A cliff rises straight up from the Mediterranean Sea. Explain how you could calculate the height of the cliff above the sea using a stopwatch and a small stone. What assumption must you make? K/U T/I C A
5. A baseball player throws a ball straight up and then catches it 2.4 s later at the same height from which he threw it. Determine
   (a) the initial velocity
   (b) the maximum height of the ball T/I A
6. You throw a ball straight up into the air at 18 m/s from a height of 32 m above the ground. T/I
   (a) Calculate the time the ball takes to hit the ground.
   (b) Calculate the velocity of the ball when it hits the ground.
   (c) Calculate the maximum height of the ball.
   (d) Explain why you cannot use half the time for (a) to answer (c).
7. A toy rocket starts from rest on the ground and then accelerates at 39.2 m/s^2 [up] for 5.0 s. T/I
   (a) Calculate the velocity of the rocket when the engines stop firing after 5.0 s.
   (b) Calculate the maximum height of the rocket.
   (c) How long will it take the rocket to hit the ground from rest at the maximum height and initial rest?
8. When a driver is forced to make a panic stop by pressing down on the brake as hard as possible, the car will undergo a large acceleration to stop. K/U T/I A
   (a) Copy **Table 2** into your notebook. Calculate the braking distances.
   (b) Compare the reaction times, and discuss the implications, to the driver and to speeding, of using cellphones while driving.

Table 2

	Acceleration (m/s^2)	Reaction time (s)	Speed (km/h)	Braking distance (m)
(i)	9.5	0.80	60.0	
(ii)	9.5	0.80	120.0	
(iii)	9.5	2.0	60.0	

1.3 Displacement in Two Dimensions

So far, you have learned about motion in one dimension. This is adequate for learning basic principles of kinematics, but it is not enough to describe the motions of objects in real life. Cars and buses do not always move in a straight line because streets do not always follow straight lines (**Figure 1**). Even train tracks change directions, and airplanes have both vertical and horizontal displacements.

Figure 1 Streets typically change direction, as this image shows. The position of a vehicle on such streets requires coordinates in more than one dimension.

In this section, you will learn how to combine vectors to describe the position of an object in two dimensions. From this, you will be able to determine the object's two-dimensional displacement using different methods. This basic but important skill will prepare you for describing two-dimensional velocity and acceleration using vector addition. CAREER LINK

Displacement Vectors and Their Properties

In Sections 1.1 and 1.2, you reviewed how some quantities, such as speed, are described solely in terms of magnitude (scalars), and other quantities, such as displacement and velocity, are described in terms of both magnitude and direction (vectors).

As with equations, in diagrams and figures an arrow represents a vector quantity. The arrow's length indicates the magnitude of the vector (for example, how fast a car is moving), and the direction of the arrow indicates the direction of the vector relative to a chosen coordinate system (for example, which way a car is moving). In most cases, we use reference coordinates that are perpendicular to each other (such as x and y, or north and east) and then describe the vector in two dimensions with respect to the coordinate system.

In Sections 1.1 and 1.2, our vector notation described situations in one dimension. Now that we are describing situations in two dimensions, we need to slightly modify the notation. For example, suppose you walk 15 m toward the west. Your displacement will be 15 m [W]. Now suppose you turn and walk 15 m in a direction that is west 35° north. You express this displacement as 15 m [W 35° N] (**Figure 2**). The direction [W 35° N] can be read as "point west, and then turn 35° toward north."

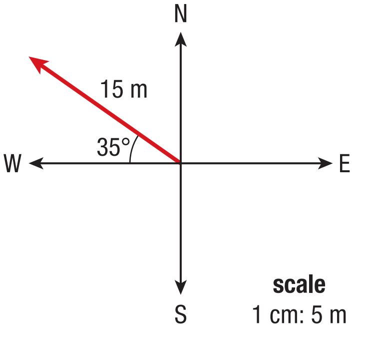


Figure 2 This scale diagram represents a displacement of 15 m [W 35° N].

Determining Total Displacement

Often, we will want to determine the total displacement of an object that has changed direction during its motion. In such cases, we will treat the object's linear displacement in each direction as a separate vector. Below are three different methods to determine the total displacement, $\Delta\vec{d}_T$: the scale diagram method, the cosine and sine laws method, and the perpendicular components of a vector method.

The Scale Diagram Method

Representing vectors as arrows is a convenient way to add them together and determine the total displacement. Drawing a scale diagram is the most direct way of doing this. Simply draw the vectors, making sure that you draw the magnitudes to scale with respect to each other, and orient their directions correctly with respect to the coordinate system using a protractor. This approach to solving the problem lacks accuracy, but it makes it easier to visualize the vector addition.

Figure 3 shows two displacements, $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$, drawn to scale and added together. Each vector has a magnitude and a direction, and both the lengths and the directions of the arrows are important. When you draw each vector, you can choose where you want to locate the arrow, just as long as it has the same length and the same direction. If the arrow keeps its properties of magnitude and direction, it is still the same vector. You can write this addition of vectors as

$$\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$$

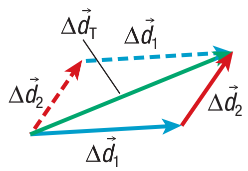


Figure 3 Drawing the tail of $\Delta\vec{d}_2$ at the tip of $\Delta\vec{d}_1$ is a convenient way to add vectors. The total displacement $\Delta\vec{d}_T$ equals $\Delta\vec{d}_1 + \Delta\vec{d}_2$.

Note that the addition of $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$ and the total displacement $\Delta\vec{d}_T$ form a triangle, with $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$ directed one way around the triangle (counterclockwise in this case) and $\Delta\vec{d}_T$ directed the other way. The triangle concept can be difficult to understand: Think of walking around a triangle on the ground. The two individual displacements, $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$, indicate one way you could walk around the triangle. The total displacement vector, $\Delta\vec{d}_T$, indicates the other way you could go. Also note that the order in which you add the vectors does not matter; that is, the addition is commutative. As long as you scale the arrows properly, orient them in the right directions, and add the tail of one vector to the tip of the other, the total displacement arrow will extend from the tail of the first displacement to the tip of the second displacement. This is true of all vectors, not just displacements. You must also remember to convert the answer you obtain from the scale diagram back into the actual answer using the scale.

The Cosine and Sine Laws Method

Another method to determine the sum of two vectors is to use the cosine and sine laws. These trigonometric relations allow you to calculate the length of the total displacement vector and its angle of orientation with respect to the coordinate system. This trigonometric method only works when adding two vectors at a time, but the result is more accurate than that of a scale diagram. In Tutorial 1, you will practise adding displacement vectors to determine total displacement by drawing scale diagrams and by using trigonometry.

Tutorial 1 Adding Displacement Vectors

This Tutorial models two methods of adding two displacement vectors to determine the total displacement. In Sample Problem 1, you will use the scale diagram method. In Sample Problem 2, you will use the cosine and sine laws method.

Sample Problem 1: Vector Addition by Scale Diagram

Suppose you walk to a friend's house, taking a shortcut across an open field. Your first displacement is 140 m [E 35° N] across the field. Then you walk 200.0 m [E] along the sidewalk. Determine your total displacement using a scale diagram.

Given: $\Delta\vec{d}_1 = 140$ m [E 35° N]; $\Delta\vec{d}_2 = 200.0$ m [E]

Required: $\Delta\vec{d}_T$; the angle for $\Delta\vec{d}_T$, θ

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Decide on a scale so that you can draw each vector to scale with the correct direction with respect to the coordinate axes.

Solution:

Step 1. Choose a suitable scale. In this case, set the scale so that 1 cm : 40 m. Then, determine the lengths of the arrows for $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$.

$$\Delta\vec{d}_1\text{: } \frac{140\ \cancel{\text{m}}}{40\ \cancel{\text{m}}/\text{cm}} = 3.5\text{ cm};\ \Delta\vec{d}_2\text{: } \frac{200.0\ \cancel{\text{m}}}{40\ \cancel{\text{m}}/\text{cm}} = 5.0\text{ cm}$$

Step 2. Using a ruler and a protractor, draw the two vectors, placing the tail of $\Delta\vec{d}_2$ at the tip of $\Delta\vec{d}_1$, as shown in **Figure 4**.

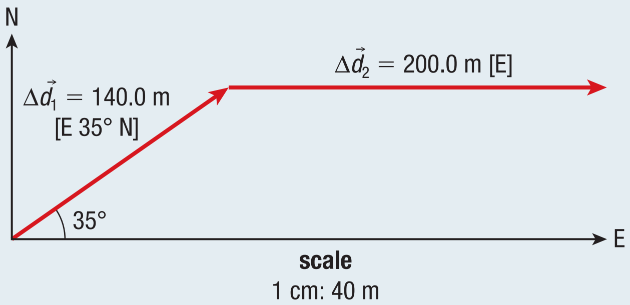


Figure 4 Set the scale, and draw vectors $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$.

Step 3. Draw the total displacement vector $\Delta\vec{d}_T$ from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$, measure the length of the vector, and measure the angle the displacement vector makes to the horizontal, as shown in **Figure 5**. The measured length of the total displacement vector is 8.1 cm. Convert to metres:

$$8.1\ \cancel{\text{cm}} \times 40\frac{\text{m}}{\cancel{\text{cm}}} = 324\text{ m}$$

To two significant digits, the total displacement is 320 m. The measured angle between east and the total displacement vector is about 14°.

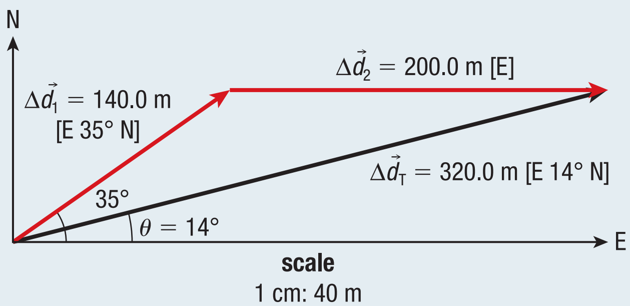


Figure 5 Draw the total displacement vector, $\Delta\vec{d}_T$.

Statement: The total displacement $\Delta\vec{d}_T$ is approximately 320 m [E 14° N].

Sample Problem 2: Vector Addition Using the Cosine and Sine Laws

Using your solution diagram for Sample Problem 1 (Figure 5), determine the total displacement using the cosine and sine laws.

Given: $\Delta\vec{d}_1 = 140$ m [E 35° N]; $\Delta\vec{d}_2 = 200.0$ m [E]

Required: $\Delta\vec{d}_T$; the angle for $\Delta\vec{d}_T$, θ

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$. To determine the magnitude of the displacement, use the cosine law. To calculate the angle of $\Delta\vec{d}_T$ with respect to the horizontal axis E, use the sine law.

Solution: The second displacement is parallel to the E axis, and $\theta_2 + 35° = 180°$, so the angle θ_2 is 145° (**Figure 6**).

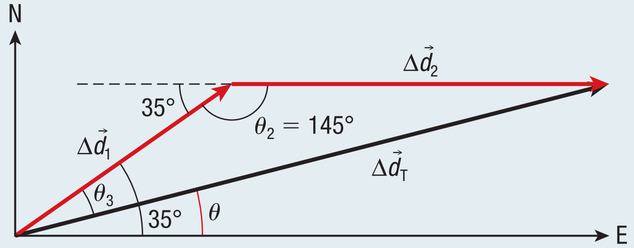


Figure 6

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$|\Delta\vec{d}_T|^2 = |\Delta\vec{d}_1|^2 + |\Delta\vec{d}_2|^2 - 2|\Delta\vec{d}_1||\Delta\vec{d}_2|\cos\theta_2$$

$$= (140\text{ m})^2 + (200.0\text{ m})^2 - 2(140\text{ m})(200.0\text{ m})(\cos 145°)$$

$$|\Delta\vec{d}_T|^2 = 105\,473\text{ m}^2$$

$$|\Delta\vec{d}_T| = 324.8\text{ m (two extra digits carried)}$$

$$\frac{\sin\theta_3}{|\Delta\vec{d}_2|} = \frac{\sin\theta_2}{|\Delta\vec{d}_T|}$$

$$\sin\theta_3 = \frac{|\Delta\vec{d}_2|\sin\theta_2}{|\Delta\vec{d}_T|}$$

$$= \frac{(200.0\text{ m})(\sin 145°)}{324.8\text{ m}}$$

$$\theta_3 = 20.7°\text{ (one extra digit carried)}$$

The angle θ between east and the total displacement is therefore $35° - 20.7° = 14.3°$.

Statement: Using the cosine and sine laws, the displacement is $\Delta\vec{d}_T = 320\text{ m [E 14° N]}$.

Practice

1. A car starts from a parking lot and travels 1.2 km south and then 3.1 km in a direction 53° north of east. Relative to the parking lot, what is the car's total displacement? Solve the problem using the scale diagram method, and express your answer in terms of distance and an angle. K/U T/I A [ans: 2.3 km [E 34° N]]
2. A boater travels across a river from one point on the western shore to a point 95.0 m south on the eastern shore. The river is 77.0 m wide as measured directly from west to east. Calculate the boater's total displacement. K/U T/I A [ans: 122 m [E 51.0° S]]
3. A helicopter flies 65 km [N 32° E] and then 42 km [E 21° N]. Determine the total displacement of the helicopter. K/U T/I A [ans: 1.0×10^2 km [E 44° N]]

The Perpendicular Components of a Vector Method

A more straightforward way of adding two or more displacement vectors is to resolve, or separate, each vector into perpendicular components. For two dimensions, the perpendicular **components of a vector** are the parts of the vector that lie along either the x-axis or the y-axis. This makes it easy to add the components of several vectors that are all parallel. We obtain the components of the total displacement vector by adding the parallel displacement components.

component of a vector in two dimensions, either of the x-vector and y-vector that are combined into an overall vector

To do this, we need to be familiar with obtaining the components of a vector, which requires trigonometry. For example, suppose you walked a distance of 5.0 km in a direction that is east 37° toward north, or $\Delta\vec{d} = 5.0$ km [E 37° N], as shown in **Figure 7**. In this example, we will use the convention that east and north are positive. To draw the x-component, you position the tail of the Δd_x vector at the origin and make a vector going east. You draw this one first because it is indicated as the first direction in [E 37° N]. From the tip of the x-component you draw the y-component, Δd_y, directly north and stop at the tip of $\Delta\vec{d}$, as shown in Figure 7. You draw the y-component second because it is the second direction listed in [E 37° N]. Then the components of this vector can be determined using the sine and cosine trigonometric ratios:

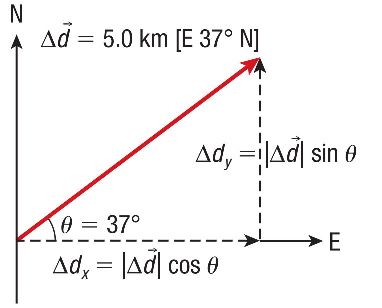


Figure 7 The displacement vector can be broken down into its perpendicular x- and y-components.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\Delta d_x}{|\Delta\vec{d}|} \qquad \sin\theta = \frac{\Delta d_y}{|\Delta\vec{d}|}$$

$$\Delta d_x = |\Delta\vec{d}|\cos\theta \qquad \Delta d_y = |\Delta\vec{d}|\sin\theta$$

For the situation described in Figure 7, the components are

$$\begin{aligned} \Delta d_x &= |\Delta \vec{d}| \cos\theta & \Delta d_y &= |\Delta \vec{d}| \sin\theta \\ &= (5.0 \text{ km})(\cos 37°) & &= (5.0 \text{ km})(\sin 37°) \\ &= 4.0 \text{ km} & &= 3.0 \text{ km} \\ \Delta d_x &= 4.0 \text{ km [E]} & \Delta d_y &= 3.0 \text{ km [N]} \end{aligned}$$

When drawing these displacement components, be sure to place the tail of the vertical displacement (Δd_y) at the tip of the horizontal component (Δd_x). Keep in mind that θ will not always be the angle between the x-axis and the displacement. Sometimes, θ will be situated between the y-axis and the displacement. For this reason, always consider which component is opposite θ and which one is adjacent to θ to determine the components. To add and subtract vectors using components, you must first become adept at determining the components of vectors. Tutorial 2 models how to do this.

Tutorial 2 Determining the Components of a Displacement Vector

This Tutorial shows how to use trigonometry to determine the components of a displacement vector.

Sample Problem 1: Determining Vector Components Using Trigonometry

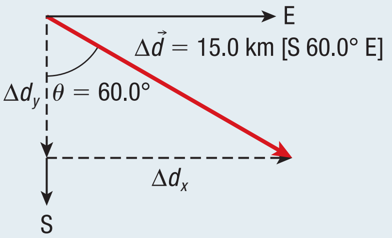
A polar bear walks toward Churchill, Manitoba. The polar bear's displacement is 15.0 km [S 60.0° E]. Determine the components of the displacement.

Given: $\Delta \vec{d} = 15.0$ km [S 60.0° E]

Required: Δd_x; Δd_y

Analysis: Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.

Solution:



$$\begin{aligned} \Delta d_x &= +|\Delta \vec{d}| \sin\theta \text{ (positive because the component points east)} \\ &= (15.0 \text{ km})(\sin 60.0°) \\ \Delta d_x &= 12.99 \text{ km (one extra digit carried)} \\ \Delta d_y &= -|\Delta \vec{d}| \cos\theta \text{ (negative because the component points south)} \\ &= -(15.0 \text{ km})(\cos 60.0°) \\ &= -7.50 \text{ km} \\ \Delta d_y &= 7.50 \text{ km [S]} \end{aligned}$$

To check these results, use the magnitudes of the components to determine the magnitude of the total displacement, $|\Delta \vec{d}| = \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}$, and then calculate the angle using the equation $\theta = \tan^{-1}\left(\frac{|\Delta d_x|}{|\Delta d_y|}\right)$.

$$\begin{aligned} |\Delta \vec{d}| &= \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2} \\ &= \sqrt{(12.99 \text{ km})^2 + (7.50 \text{ km})^2} \\ |\Delta \vec{d}| &= 15.0 \text{ km} \\ \theta &= \tan^{-1}\left(\frac{|\Delta d_x|}{|\Delta d_y|}\right) \\ &= \tan^{-1}\left(\frac{12.99 \text{ km}}{7.50 \text{ km}}\right) \\ \theta &= 60.0° \end{aligned}$$

The magnitude of the displacement and the angle are the same as those given in the Sample Problem.

Statement: The components of the polar bear's displacement are $\Delta d_x = 13.0$ km [E] and $\Delta d_y = 7.50$ km [S].

Practice

1. Determine the perpendicular vector components for each of the following displacements. T/I A
   (a) 25.0 km [E 45.0° N] [ans: $\Delta d_x = 17.7$ km [E]; $\Delta d_y = 17.7$ km [N]]
   (b) 355 km [N 42.0° W] [ans: $\Delta d_x = 238$ km [W]; $\Delta d_y = 264$ km [N]]
   (c) 32.3 m [E 27.5° S] [ans: $\Delta d_x = 28.7$ m [E]; $\Delta d_y = 14.9$ m [S]]
   (d) 125 km [S 31.2° W] [ans: $\Delta d_x = 64.8$ km [W]; $\Delta d_y = 107$ km [S]]

Adding Vectors Algebraically

Now that you know how to separate a vector into its perpendicular components, you can add two or more displacement vectors by separating each vector into its horizontal and vertical components. Once you have these components, add all the horizontal components as one-dimensional vectors ($\Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x} + \cdots$). Then do the same with the vertical displacement components ($\Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y} + \cdots$). In both cases, you obtain a total component, Δd_x and Δd_y, in each direction (**Figure 8**). These components represent the horizontal and vertical components of the total vector $\Delta \vec{d}_T$.

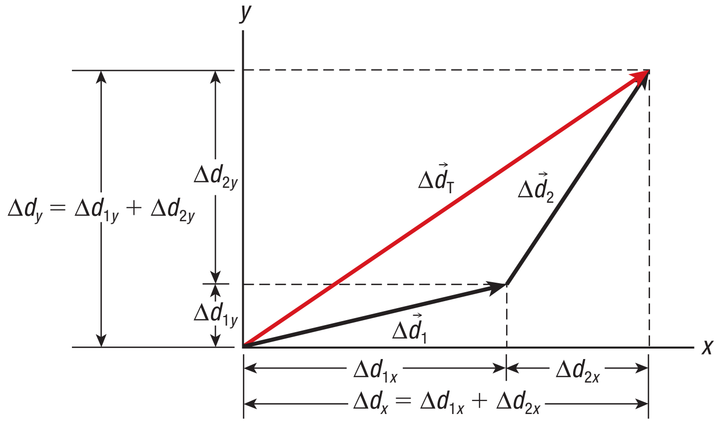


Figure 8 To calculate the sum of two vectors, add their components. Here, the x-components of $\Delta \vec{d}_1$ and $\Delta \vec{d}_2$ are added to get the total x-component Δd_x. The same procedure with the y-components yields Δd_y.

The magnitude of $\Delta \vec{d}_T$ is given by the Pythagorean theorem,

$$|\Delta \vec{d}_T| = \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}$$

and the angle between the total vector and the positive horizontal axis is given by

$$\theta = \tan^{-1}\left(\frac{|\Delta d_y|}{|\Delta d_x|}\right)$$

In Tutorial 3, you will practise adding vectors algebraically.

Tutorial 3 Adding Vectors Algebraically

This Tutorial models how to add vectors algebraically to determine the total displacement.

Sample Problem 1: Determining Displacement by Adding Vectors Algebraically

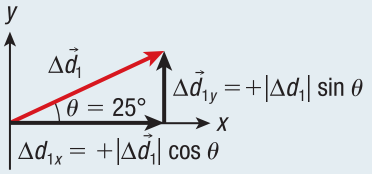
An airplane flies 250 km [E 25° N], and then flies 280 km [S 13° W]. Using components, calculate the airplane's total displacement.

Given: $\Delta \vec{d}_1 = 250$ km [E 25° N]; $\Delta \vec{d}_2 = 280$ km [S 13° W]

Required: $\Delta \vec{d}_T$

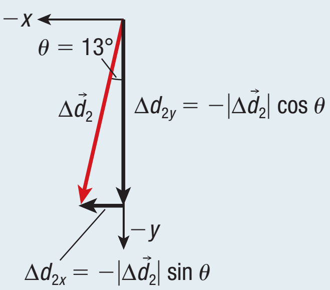
Analysis: Draw a simple scale vector diagram of each vector, and determine the corresponding components. Add the x-components and then the y-components to determine the total displacement in these directions. Use north and east as positive, and south and west as negative. Then use the Pythagorean theorem and the tangent ratio to determine the total displacement.

Solution: For the first vector:



$$\Delta d_{1x} = |\Delta d_1| \cos\theta$$
$$= (250 \text{ km})(\cos 25°)$$
$$\Delta d_{1x} = 226.6 \text{ km (two extra digits carried)}$$
$$\Delta d_{1y} = |\Delta\vec{d}_1| \sin\theta$$
$$= (250 \text{ km})(\sin 25°)$$
$$\Delta d_{1y} = +105.7 \text{ km (two extra digits carried)}$$

For the second vector:



$$\Delta d_{2x} = -|\Delta\vec{d}_2| \sin\theta$$
$$= -(280 \text{ km})(\sin 13°)$$
$$\Delta d_{2x} = -62.99 \text{ km (two extra digits carried)}$$
$$\Delta d_{2y} = -|\Delta\vec{d}_2| \cos\theta$$
$$= -(280 \text{ km})(\cos 13°)$$
$$\Delta d_{2y} = -272.8 \text{ km (two extra digits carried)}$$

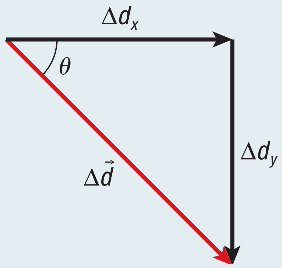
Add the horizontal components:

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$$
$$= 226.6 \text{ km} + (-62.99 \text{ km})$$
$$\Delta d_x = 163.6 \text{ km [E]}$$

Add the vertical components:

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$$
$$= 105.7 \text{ km} + (-272.8 \text{ km}) \text{ [N]}$$
$$= -167.1 \text{ km [N]}$$
$$\Delta d_y = 167 \text{ km [S]}$$

Combine the total displacement components to determine the total displacement.



$$|\Delta\vec{d}| = \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}$$
$$= \sqrt{(163.6 \text{ km})^2 + (167.1 \text{ km})^2}$$
$$|\Delta\vec{d}| = 233.9 \text{ km}$$
$$\theta = \tan^{-1}\left(\frac{|\Delta\vec{d}_y|}{|\Delta\vec{d}_x|}\right)$$
$$= \tan^{-1}\left(\frac{167.1 \text{ km}}{163.6 \text{ km}}\right)$$
$$\theta = 46°$$

Statement: The airplane's total displacement is 230 km [E 46° S].

Practice

1. An airplane flies 276.9 km [W 76.70° S] from Edmonton to Calgary and then continues 675.1 km [W 11.45° S] from Calgary to Vancouver. Using components, calculate the plane's total displacement. T/I A [ans: 830.0 km [W 29.09° S]]
2. A person drives 120 km [N 32° W] in a car to a friend's place to visit and then drives 150 km [W 24° N] to visit family. Determine the total displacement of the trip. T/I [ans: 260 km [W 39° N]]
3. In a helicopter ride, the pilot first flies 12 km [N], then 14 km [N 22° E], and then 11 km [E]. Determine the total displacement. [ans: 3.0×10^1 km [E 57° N]]

When adding displacement vectors, you can choose the method that you find convenient. The scale diagram method ("tip-to-tail") gives you a visual sense of how the displacements relate, but it is not the most accurate way of determining the total displacement. The trigonometric method using the sine and cosine laws is accurate, but you can only combine two vectors at a time. The algebraic component method is less familiar but is accurate, and you can use it for any number of displacement vectors. In time, you will find it easier to use as well. For these reasons, it is the preferred approach in physics for determining the total displacement or adding a number of vectors of any type.

1.3 Review

Summary

- To add displacement vectors using the scale diagram method, draw the vectors tip to tail and to the proper scale on a graph. Then determine the total displacement by measuring the length of the displacement vector and its angle. Finally, rescale the answer to the appropriate units.
- To calculate the sum of two vectors using the cosine and sine laws, first draw the two vectors tip to tail. Then apply the cosine and sine laws.
- Two-dimensional displacement vectors can be resolved into two perpendicular component vectors, one parallel to the x-axis and one parallel to the y-axis.
- You can use the component method to determine two perpendicular components using the sine and cosine ratios. The x-component of the total displacement equals the sum of the individual x-displacements. The y-component of the total displacement equals the sum of the individual y-displacements.
- Using the perpendicular vector component method, you can add two or more displacement vectors by separating each vector into its components and adding the components to obtain a total component in each direction. The total component represents a component of the total displacement vector $\Delta\vec{d}_T$.

Questions

1. Consider the displacements $\Delta\vec{d}_1 = 7.81$ km [E 50° N] and $\Delta\vec{d}_2 = 5.10$ km [W 11° N]. T/I C
   (a) Draw a scale diagram of these two vectors, and determine the total displacement using the scale diagram method.
   (b) Determine the total displacement mathematically using a different method.
   (c) Compare the two answers by determining the percent difference.
2. Consider the following three vectors: $\Delta\vec{d}_1 = 5.0$ cm [E 30.0° N], $\Delta\vec{d}_2 = 7.5$ cm [E], and $\Delta\vec{d}_3 = 15.0$ cm [E 10.0° S]. Add these vectors using the scale diagram method to determine the total displacement. T/I C
3. Determine the components of the vector 2.50 m [N 38.0° W]. T/I
4. A student walks 25.0 m [E 30.0° N]. Determine the components of the student's displacement. T/I
5. A vector has the following components: $\Delta d_x = 54$ m [E] and $\Delta d_y = 24$ m [N]. T/I
   (a) Calculate the length of the total displacement vector.
   (b) Determine the vector's direction.
6. A driver travels 15.0 km to the west and then turns and drives 45.0 km to the south. Finally, she travels 32 km [N 25° W]. Determine the driver's total displacement. T/I A
7. Using components, determine the total displacement from the following individual displacements: $\Delta\vec{d}_1 = 2.5$ m [W 30.0° S], $\Delta\vec{d}_2 = 3.6$ m [S], and $\Delta\vec{d}_3 = 4.9$ m [E 38.0° S]. K/U T/I
8. A boat travelling on the St. Lawrence River moves 2.70 km [E 25.0° N] and then changes direction and moves 4.80 km [E 45.0° S]. Determine (a) the components of the total displacement and (b) the total displacement of the boat. T/I
9. An airplane flies 1512.0 km [W 19.30° N] from Toronto to Winnipeg, continues 571.0 km [W 4.35° N] from Winnipeg to Regina, and then changes direction again and flies 253.1 km [W 39.39° N] from Regina to Saskatoon. Determine the total displacement of the plane. T/I A
10. A car takes a trip consisting of two displacements. The first displacement is 25 km [N], and the total displacement is 62 km [N 38° W]. Determine the second displacement. T/I A
11. A plane flies 450 km [W] and then another 220 km in an unknown direction. Determine the maximum and minimum displacement of the plane. Explain your reasoning. K/U T/I A
12. Figure 3 on page 23 shows, among other things, that vector addition is commutative. What does this mean? Make a similar diagram (or two separate diagrams) that shows that vector addition is commutative when using vector components. K/U T/I C

1.4 Velocity and Acceleration in Two Dimensions

Figure 1 An object's velocity changes whenever there is a change in the velocity's magnitude (speed) or direction, such as when these cars turn with the track.

Suppose you are driving south along a straight side road that has a speed limit of 60 km/h. You stay on this road for 1 h and then reach a highway that has a speed limit of 100 km/h. You turn and travel southwest on the highway for 2 h.

This simple example gives you an idea of why you must carefully consider how to determine average velocity in a two-dimensional situation. Velocity is a vector, and like displacement, can be described in more than one dimension. A change in velocity occurs when there is a change in the velocity's magnitude (speed) or direction, such as the race cars taking a curve in **Figure 1**. Acceleration depends on the change in velocity, so acceleration in two dimensions also depends on a change in the velocity's magnitude, direction, or both.

Now that you are familiar with the component method for adding vectors, you can use this method to calculate two-dimensional average velocity and average acceleration. First, we look at velocity and speed in two dimensions and then subtracting vectors.

Velocity and Speed in Two Dimensions

In general, average velocity is the change in total displacement over time and is described by the equation

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

If displacement is in two dimensions, then you must first determine the total displacement using components (or a similar method) before determining the average velocity.

The notation for describing a velocity vector is the same as that for displacement, except that average velocity has units of length divided by time (for example, metres per second). Suppose a car has a displacement of 200 m [E 30° N], and travels this distance and in this direction in 10 s. The average velocity is therefore

$$\vec{v}_{av} = \frac{200\ \text{m}\ [\text{E}\ 30°\ \text{N}]}{10\ \text{s}}$$

$$\vec{v}_{av} = 20\ \text{m/s}\ [\text{E}\ 30°\ \text{N}]$$

What happens when there are several displacements, each with a different direction? The average velocity for the entire trip is based on the total displacement. Therefore, this average velocity will always have the same direction as the total displacement. In the equation

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t},$$

$\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3 + \cdots$. To calculate total displacement, add the horizontal and vertical components of the individual displacements, and combine them to obtain the magnitude and direction of the total displacement as in Section 1.3.

Average speed, on the other hand, is a scalar property based on the length of time travelled and the total distance travelled, regardless of the direction. Therefore, when an object returns to its starting point, the distance it has travelled is the sum of all displacement magnitudes, and is thus not zero. Average speed is simply this total distance divided by the time of travel and is greater than zero. In the following Tutorial, you will review how to calculate average velocity and average speed in two dimensions.

Tutorial 1 Calculating Average Velocity and Average Speed in Two Dimensions

This Tutorial reviews how to calculate average velocity and average speed in two dimensions.

Sample Problem 1: Calculating Average Velocity and Average Speed

A family drives from Saint John, New Brunswick, to Moncton. Assuming a straight highway, this part of the drive has a displacement of 135.7 km [E 32.1° N]. From Moncton, they drive to Amherst, Nova Scotia. The second displacement is 51.9 km [E 25.9° S]. The total drive takes 2.5 h to complete.

(a) Calculate the average velocity of the family's vehicle.

(b) Calculate the average speed of the family's vehicle.

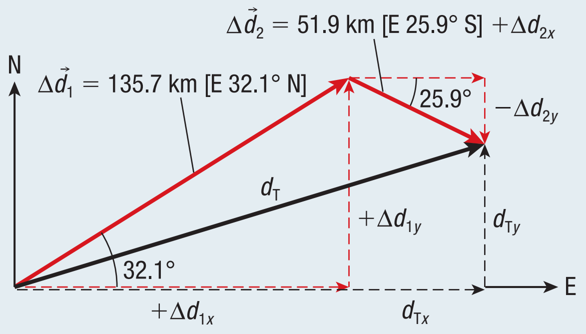
Solution

(a) **Given:** $\Delta\vec{d}_1 = 135.7$ km [E 32.1° N]; $\Delta\vec{d}_2 = 51.9$ km [E 25.9° S]; $\Delta t = 2.5$ h

Required: $\vec{v}_{av}$

Analysis: Make a scale diagram to show the situation. Determine the components of the two vectors. Then determine the total horizontal displacement, $\Delta d_{Tx} = \Delta d_{1x} + \Delta d_{2x}$, and the total vertical displacement, $\Delta d_{Ty} = \Delta d_{1y} + \Delta d_{2y}$. Calculate the magnitude of the total displacement using the Pythagorean theorem, and use the inverse tangent equation to calculate the angle of orientation for the total displacement. The average velocity is then the total displacement divided by the time of travel.

Solution:



$$\begin{aligned}
\Delta d_{Tx} &= \Delta d_{1x} + \Delta d_{2x} \\
&= (+\Delta\vec{d}_1 \cos\theta_1) + (+\Delta\vec{d}_2 \cos\theta_2) \\
&= (135.7 \text{ km})(\cos 32.1°) + (51.9 \text{ km})(\cos 25.9°) \\
&= 161.6 \text{ km} \\
\Delta d_{Tx} &= 161.6 \text{ km [E] (two extra digits carried)} \\
\Delta d_{Ty} &= \Delta d_{1y} + \Delta d_{2y} \\
&= (+\Delta\vec{d}_1 \sin\theta_1) + (-\Delta\vec{d}_2 \sin\theta_2) \\
&= (135.7 \text{ km})(\sin 32.1°) - (51.9 \text{ km})(\sin 25.9°) \\
&= 49.44 \text{ km} \\
\Delta d_{Ty} &= 49.44 \text{ km [N] (two extra digits carried)}
\end{aligned}$$

$$\begin{aligned}
|\Delta\vec{d}_T| &= \sqrt{(\Delta d_{Tx})^2 + (\Delta d_{Ty})^2} \\
&= \sqrt{(161.6 \text{ km})^2 + (49.44 \text{ km})^2} \\
|\Delta\vec{d}_T| &= 169.0 \text{ km (two extra digits carried)} \\
\theta_T &= \tan^{-1}\left(\frac{|\Delta d_{Ty}|}{|\Delta d_{Tx}|}\right) \\
&= \tan^{-1}\left(\frac{49.44 \text{ km}}{161.6 \text{ km}}\right) \\
\theta_T &= 17°
\end{aligned}$$

The total displacement is 170 km [E 17° N]. The average velocity is therefore

$$\begin{aligned}
\vec{v}_{av} &= \frac{\Delta\vec{d}_T}{\Delta t} \\
&= \frac{169.0 \text{ km [E 17° N]}}{2.5 \text{ h}} \\
\vec{v}_{av} &= 68 \text{ km/h [E 17° N]}
\end{aligned}$$

Statement: The average velocity of the vehicle is 68 km/h [E 17° N].

(b) **Given:** $\Delta\vec{d}_1 = 135.7$ km [E 32.1° N]; $\Delta\vec{d}_2 = 51.9$ km [E 25.9° S]; $\Delta t = 2.5$ h

Required: v_{av}

Analysis: To calculate the average speed of the vehicle, determine the total distance travelled. Distance is not a vector sum, but a scalar addition of the separate displacement magnitudes. Therefore,

$$\Delta d_T = |\Delta\vec{d}_1| + |\Delta\vec{d}_2|$$

and $v_{av} = \dfrac{\Delta d_T}{\Delta t}$.

Solution:
$$\begin{aligned}
d_T &= |\Delta\vec{d}_1| + |\Delta\vec{d}_2| \\
&= 135.7 \text{ km} + 51.9 \text{ km} \\
d_T &= 187.6 \text{ km} \\
v_{av} &= \frac{\Delta d_T}{\Delta t} \\
&= \frac{187.6 \text{ km}}{2.5 \text{ h}} \\
v_{av} &= 75 \text{ km/h}
\end{aligned}$$

Statement: The average speed of the vehicle is 75 km/h.

Practice

1. A plane makes the following displacements: $\Delta\vec{d}_1 = 72.0$ km [W 30.0° S], $\Delta\vec{d}_2 = 48.0$ km [S], and $\Delta\vec{d}_3 = 150.0$ km [W]. The entire flight takes 2.5 h. T/I
   (a) Calculate the total displacement of the plane. [ans: 230 km [W 22° S]]
   (b) Calculate the average velocity of the plane. [ans: 91 km/h [W 22° S]]
   (c) Calculate the average speed of the plane. [ans: 110 km/h]
2. An elk walks 25.0 km [E 53.13° N], then walks 20.0 km [S], and then runs 15.0 km [W]. The journey takes 12 h. T/I
   (a) Calculate the elk's average velocity. [ans: 0 km/h]
   (b) Calculate the elk's average speed. [ans: 5.0 km/h]
   (c) Explain the difference in the two answers for the elk's average velocity and average speed.

Subtracting Vectors in Two Dimensions

Up to now, you have been working with vector addition. In some cases, though, you need to multiply vectors by scalars and subtract vectors. Before dealing with vector subtraction, first consider the multiplication of a vector by a scalar. Scalars are simply numbers, such as 2 and 5.7. Multiplication of a vector by a scalar changes the vector's length, or magnitude. If a scalar k is greater than 1 ($k > 1$), then the product of k and vector $\vec{A}$ is longer than $\vec{A}$. Similarly, if $0 < k < 1$, then the product of k and vector $\vec{A}$ is shorter than $\vec{A}$.

Now suppose the scalar k is negative ($k < 0$). If you multiply vector $\vec{B}$ by k, then $\vec{B}$ and $k\vec{B}$ point in opposite directions. Now you can see how vector subtraction arises from scalar multiplication and vector addition. Subtracting vector $\vec{B}$ from vector $\vec{A}$ is equivalent to adding the vectors $\vec{A}$ and $-\vec{B}$, for $k = -1$. See **Figure 2**. Expressing this as a vector equation yields

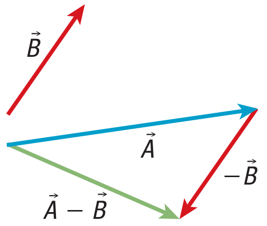


Figure 2 Vector subtraction is equivalent to adding a positive vector and a negative vector.

$$\begin{aligned}\vec{A} + (-1)(\vec{B}) &= \vec{A} + (-\vec{B}) \\ &= \vec{A} - \vec{B}\end{aligned}$$

You can use this same approach with the components of vectors. By multiplying one component by an appropriate negative scalar, you can subtract two vector components in one dimension by adding the positive component of one vector and the negative component of the other vector.

What does subtracting vectors mean physically? When a vector changes over an interval of time, the physical quantity of interest is the measure of the change, or the difference between the vectors in that time interval. For example, consider a car following a curve on a level road (**Figure 3**). Even if the driver keeps the speed of the car constant, the direction of the car changes. This change equals the difference between the velocity at one point in time and the velocity at any earlier point in time. In other words, the change in velocity is the subtraction of the final and initial velocities:

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

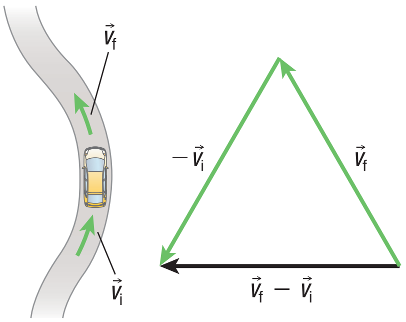


Figure 3 A change in a vector from $\vec{v}_i$ to $\vec{v}_f$ can be determined by vector subtraction, $\vec{v}_f - \vec{v}_i$.

Acceleration in Two Dimensions

We can now apply the principle of vector subtraction in two dimensions to determine the average acceleration in two dimensions. Recall that acceleration in one dimension is the change in velocity with time:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Average acceleration occurs when the velocity vector of an object changes in magnitude, direction, or both.

As with two-dimensional displacement vectors, you can break down the two velocity vectors into horizontal and vertical components. By subtracting each dimension's components, you obtain the net horizontal and vertical components:

$$\Delta v_x = v_{fx} - v_{ix}$$

$$\Delta v_y = v_{fy} - v_{iy}$$

From these, you can calculate the magnitude and direction of the net velocity using the Pythagorean theorem and the inverse tangent equation, respectively:

$$|\Delta\vec{v}| = \sqrt{\Delta v_x^2 + \Delta v_y^2}$$

$$\theta = \tan^{-1}\left(\frac{|\Delta v_y|}{|\Delta v_x|}\right)$$

Finally, the change in velocity divided by the time interval yields the average acceleration.

In Tutorial 2, you will learn how to calculate acceleration in two dimensions by vector subtraction of velocity components.

UNIT TASK BOOKMARK

Calculate the acceleration during your extreme sport for the Unit Task on page 146.

Tutorial 2 Calculating Acceleration in Two Dimensions

The following Sample Problem models how to determine acceleration in two dimensions by vector subtraction of velocity components.

Sample Problem 1: Calculating Acceleration in Two Dimensions

A car turns from a road into a parking lot and into an available parking space. The car's initial velocity is 4.0 m/s [E 45.0° N]. The car's velocity just before the driver decreases speed is 4.0 m/s [E 10.0° N]. The turn takes 3.0 s. Calculate the average acceleration of the car during the turn.

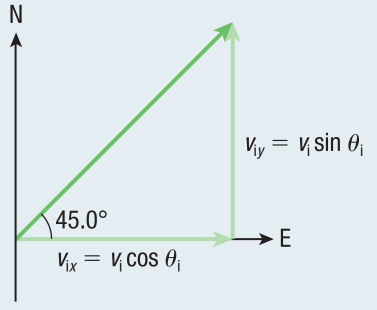
Given: $\vec{v}_i$ = 4.0 m/s [E 45.0° N]; $\vec{v}_f$ = 4.0 m/s [E 10.0° N]; Δt = 3.0 s

Required: $\vec{a}_{av}$

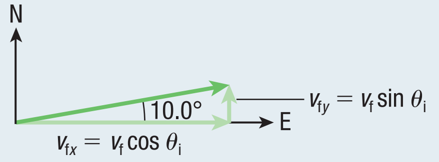
Analysis: Draw a vector diagram of the situation. Determine the components for each velocity vector, and then subtract the initial vector components from the final vector components, $\Delta v_x = \Delta v_{fx} - \Delta v_{ix}$ and $\Delta v_y = \Delta v_{fy} - \Delta v_{iy}$. Calculate the magnitude of the change in velocity using the Pythagorean theorem, and use the inverse tangent equation to calculate the angle of orientation for the net velocity. The average acceleration is then the change in velocity divided by the time interval.

Solution:

Components for the initial velocity vector:



Components for the final velocity vector:



For the initial vector:

$$v_{ix} = \vec{v}_i \cos\theta_i$$
$$= (4.0 \text{ m/s [E]})(\cos 45.0°)$$
$$v_{ix} = 2.83 \text{ m/s [E]}$$
$$v_{iy} = \vec{v}_i \sin\theta_i$$
$$= (4.0 \text{ m/s [N]})(\sin 45.0°)$$
$$v_{iy} = 2.83 \text{ m/s [N]}$$

For the final vector:

$$v_{fx} = \vec{v}_f \cos\theta_f$$
$$= (4.0 \text{ m/s [E]})(\cos 10.0°)$$
$$v_{fx} = 3.94 \text{ m/s [E]}$$
$$v_{fy} = \vec{v}_f \sin\theta_f$$
$$= (4.0 \text{ m/s [N]})(\sin 10.0°)$$
$$v_{fy} = 0.695 \text{ m/s [N]}$$

Subtract the horizontal components:

$$\Delta v_x = v_{fx} - v_{ix}$$
$$= 3.94 \text{ m/s [E]} - 2.83 \text{ m/s [E]}$$
$$\Delta v_x = 1.11 \text{ m/s [E]}$$

Subtract the vertical components:

$$\Delta v_y = v_{fy} - v_{iy}$$
$$= 0.695 \text{ m/s [N]} - 2.83 \text{ m/s [N]}$$
$$= -2.14 \text{ m/s [N]}$$
$$\Delta v_y = 2.14 \text{ m/s [S]}$$

Combine the net velocity components to determine the change in velocity:

$$|\Delta\vec{v}| = \sqrt{\Delta v_x^2 + \Delta v_y^2}$$
$$= \sqrt{(1.11 \text{ m/s})^2 + (-2.14 \text{ m/s})^2}$$
$$|\Delta\vec{v}| = 2.4 \text{ m/s}$$
$$\theta = \tan^{-1}\left(\frac{|\Delta v_y|}{|\Delta v_x|}\right)$$
$$= \tan^{-1}\left(\frac{2.14 \text{ m/s}}{1.11 \text{ m/s}}\right)$$
$$\theta = 63°$$

The change in velocity is 2.4 m/s [E 63° S]. The average acceleration is therefore

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{2.4 \text{ m/s [E 63° S]}}{3.0 \text{ s}}$$
$$\vec{a}_{av} = 0.80 \text{ m/s}^2 \text{ [E 63° S]}$$

Statement: The car's average acceleration is 0.80 m/s^2 [E 63° S].

Practice

1. A car heading east turns right at a corner. The car turns at a constant speed of 20.0 m/s. After 12 s, the car completes the turn, so that it is heading due south at 20.0 m/s. Calculate the car's average acceleration. T/I A [ans: 2.4 m/s^2 [W 45° S]]
2. Over a 15.0 min period, a truck travels on a road with many turns. The truck's initial velocity is 50.0 km/h [W 60.0° N]. The truck's final velocity is 80.0 km/h [E 60.0° N]. Calculate the truck's average acceleration, in kilometres per hour squared. T/I [ans: 2.80×10^2 km/h^2 [E 21.8° N]]
3. A bird flies from Lesser Slave Lake in northern Alberta to Dore Lake in northern Saskatchewan. The bird's displacement is 800.0 km [E 7.5° S]. The bird then flies from Dore Lake to Big Quill Lake, Saskatchewan. This displacement is 400.0 km [E 51° S]. The total time of flight is 18.0 h. Determine the bird's
   (a) total distance travelled [ans: 1.2×10^3 km]
   (b) total displacement [ans: 1.1×10^3 km [E 22° S]]
   (c) average speed [ans: 62 km/h]
   (d) average velocity T/I [ans: 62 km/h [E 22° S]]

1.4 Review

Summary

- Average velocity, in two dimensions, is the total displacement in two dimensions divided by the time interval during which the displacement occurs: $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$.
- You can determine the change in velocity in two dimensions by separating the velocity vectors into components and subtracting them using the vector property $\vec{v}_f - \vec{v}_i = \vec{v}_f + (-\vec{v}_i)$.
- Average acceleration in two dimensions is the change in velocity divided by the time interval between the two velocities: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$.

Questions

1. Explain why the average speed is always greater than or equal to the magnitude of the average velocity for an object moving in two dimensions. Give an example in your answer. K/U T/I C
2. During 4.0 min on a lake, a loon moves 25.0 m [E 30.0° N] and then 75.0 m [E 45.0° S]. Determine the loon's
   (a) total distance travelled
   (b) total displacement
   (c) average speed
   (d) average velocity T/I
3. A car driver in northern Ontario makes the following displacements:
   $\Delta\vec{d}_1 = 15.0$ km [W 30.0° N],
   $\Delta\vec{d}_2 = 10.0$ km [W 75.0° N],
   and $\Delta\vec{d}_3 = 10.0$ km [E 70.0° N].
   The trip takes 0.50 h. Calculate the average velocity of the car and driver. T/I
4. Explain how there can be average acceleration when there is no change in speed. K/U T/I C
5. In your own words, explain how to subtract vectors in two dimensions. K/U C
6. A pilot in a seaplane flies for a total of 3.0 h with an average velocity of 130 km/h [N 32° E]. In the first part of the trip, the pilot flies for 1.0 h through a displacement of 150 km [E 12° N]. She then flies directly to her final destination. Determine the displacement for the second part of the flight. T/I A
7. A student goes for a jog at an average speed of 3.5 m/s. Starting from home, he first runs 1.8 km [E] and then runs 2.6 km [N 35° E]. Then he heads directly home. How long will the entire trip take? T/I A
8. In **Figure 4**, a bird changes direction in 3.8 s while flying from point 1 to point 2. Determine the bird's average acceleration. T/I

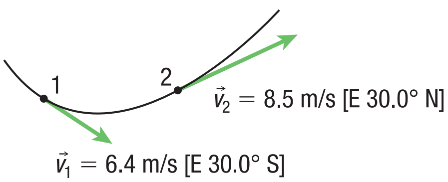


Figure 4

9. A helicopter travelling horizontally at 50.0 m/s [W] turns steadily, so that after 45.0 s, its velocity is 35.0 m/s [S]. Calculate the average acceleration of the helicopter. T/I A
10. A ball on a pool table bounces off the rail (side), as shown in **Figure 5**. The ball is in contact with the rail for 3.2 ms. Determine the average acceleration of the ball. T/I A

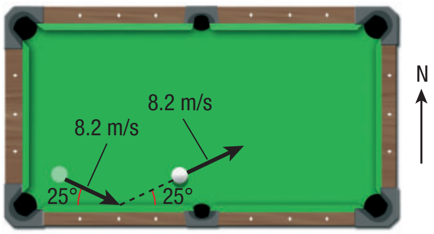


Figure 5

11. A speed boat is moving at 6.4 m/s [W 35° N] when it starts accelerating at 2.2 m/s^2 [S] for 4.0 s. Calculate the final velocity of the boat. T/I A
12. An airplane turns slowly for 9.2 s horizontally. The final velocity of the plane is 3.6×10^2 km/h [N]; the average acceleration during the turn is 5.0 m/s^2 [W]. What was the initial velocity of the plane? T/I A

1.5 Projectile Motion

In sports in which a player kicks, throws, or hits a ball across a field or court, the player's initial contact with the ball propels the ball upward at an angle. The ball rises to a certain point, and gravity eventually curves the path of the ball downward. If you ignore the effects of air resistance and Earth's rotation, the curved path, or trajectory, of the ball under the influence of Earth's gravity follows the curve of a parabola, as **Figure 1** shows. The ball acts like a **projectile**, which is an object that is moving through the air and accelerating due to gravity. The *x*-direction is horizontal and positive to the right, and the *y*-direction is vertical and positive upward.

projectile an object that is launched through the air along a parabolic trajectory and accelerates due to gravity

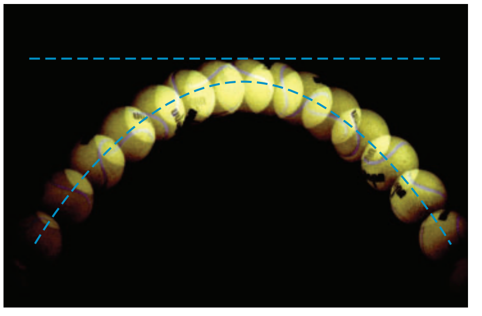


Figure 1 The path of a projectile follows the curve of a parabola.

The ball in Figure 1 was hit with a tennis racquet. If you draw an imaginary line through the ball images, you can trace the parabola from where the ball made contact with the racquet to the other end. Another imaginary line shows the uppermost point of the trajectory (at the top of the highest ball). After the ball leaves the racquet, its path curves upward to this highest point and then curves downward. You can see the symmetry of the ball's path because the shape of the upward-bound curve exactly matches the shape of the downward-bound curve. Anyone who has tossed any kind of object into the air has observed this parabolic trajectory called projectile motion. Before we formally define projectile motion, we will look at its properties.

Properties of Projectile Motion

Suppose you drop a soccer ball from the roof of a one-storey building while your friend stands next to you and kicks another soccer ball horizontally at the same instant. Will they both land at the same time? Some people are surprised to learn that the answer is yes.

Figure 2, on the next page, shows a strobe image of two balls released simultaneously, one with a horizontal projection, as your friend's soccer ball had. The horizontal lines represent equal time intervals—the time interval between the camera's strobe flashes is constant. The vertical components of the displacement increase by the same amount for each ball. The horizontal displacement of the projectile—in this instance, the ball—is called the horizontal **range**, Δd_x. The horizontal motion is also constant. The trajectory forms from the combination of the independent horizontal and vertical motions.

range (Δd_x) the horizontal displacement of a projectile

We observe the following properties about the motion of a projectile:

- The horizontal motion of a projectile is constant.
- The horizontal component of acceleration of a projectile is zero.
- The vertical acceleration of a projectile is constant because of gravity.
- The horizontal and vertical motions of a projectile are independent, but they share the same time.

Combining these properties helps us define projectile motion: **projectile motion** is the motion of an object such that the horizontal component of the velocity is constant and the vertical motion has a constant acceleration due to gravity.

projectile motion the motion of a projectile such that the horizontal component of the velocity is constant, and the vertical motion has a constant acceleration due to gravity

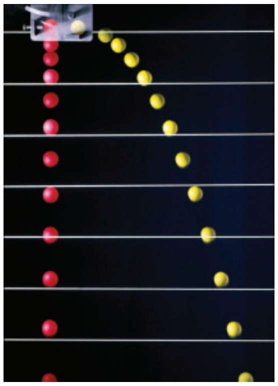


Figure 2 The two balls reach the lowest position at the same instant, even though one ball was dropped and the other was given an initial horizontal velocity.

The most important property of projectile motion in two dimensions is that the horizontal and vertical motions are completely independent of each other. This means that motion in one direction has no effect on motion in the other direction. This allows us to separate a complex two-dimensional projectile motion problem into two separate simple problems: one that involves horizontal, uniform motion and one that involves vertical, uniform acceleration down. **Figure 3** shows a baseball player hitting a fly ball and the path it follows. You can see that the horizontal velocity v_x is independent of the vertical velocity v_y.

Investigation **1.5.1**

Investigating Projectile Motion (page 50)
In this investigation, you will use an air table to investigate projectile motion.

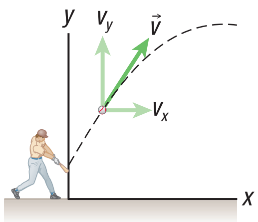


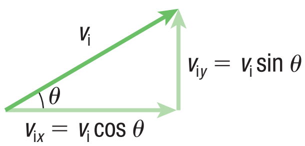
Figure 3 The horizontal and vertical components of velocity are independent of each other.

You can also see this result in the strobe images in Figure 2. The ball on the left simply drops, but the ball on the right has an initial horizontal velocity. The ball on the left falls straight down, while the ball on the right follows a parabolic path typical of projectile motion. The balls have quite different horizontal velocities at each "flashpoint" in the image. Nonetheless, they are at identical heights at each point. This shows that their displacements and velocities along the y-direction are the same. The image confirms that the motion along the vertical direction does not depend on the motion along the horizontal direction. WEB LINK

Analyzing Projectile Motion

In Section 1.2, you reviewed the equations that describe motion in one dimension. You can use these same equations to analyze the motion of a projectile in two dimensions. You simply have to apply the equations to the x- and y-motions separately. Assume that at $t = 0$ the projectile leaves the origin with an initial velocity v_i. If the velocity vector makes an angle θ with the horizontal, where θ is the projection angle, then from the definitions of the cosine and sine functions,

$$v_{ix} = v_i \cos\theta$$
$$v_{iy} = v_i \sin\theta$$



where v_{ix} is the initial velocity (at $t = 0$) in the x-direction, and v_{iy} is the initial velocity in the y-direction.

Table 1 summarizes the kinematics equations you can use with both horizontal and vertical components.

Table 1 Kinematics Equations with Horizontal and Vertical Components

Direction of motion	Description	Equations of motion
horizontal motion (x)	constant-velocity equation for the x-component only	$v_{ix} = v_i \cos\theta$ $v_{ix} = \text{constant}$ $\Delta d_x = v_{ix}\Delta t$ $\Delta d_x = (v_i \cos\theta)\Delta t$
vertical motion (y)	constant-acceleration equations for the y-component; constant acceleration has a magnitude of $\lvert \vec{g} \rvert = g = 9.8 \text{ m/s}^2$	$v_{fy} = v_i \sin\theta - g\Delta t$ $\Delta d_y = (v_i \sin\theta)\Delta t - \frac{1}{2}g\Delta t^2$ $v_{fy}^2 = (v_i \sin\theta)^2 - 2g\Delta d_y$

Mini Investigation

Analyzing the Range of a Projectile

Skills: Performing, Analyzing, Communicating

SKILLS HANDBOOK A2.2

You can calculate the horizontal range of a projectile by applying the kinematics equations step by step. In this activity, you will complete a table showing launch angle, time of flight, maximum height, and range.

Equipment and Materials: paper and pencil; calculator

1. Set up a table like the one in **Table 2**, either on paper or electronically.

Table 2

Launch angle (θ)	Time of flight (s)	Maximum height (m)	Range (m)
5			
15			
25			
85			

2. List several launch angles in increments of 10°, from 5° to 85°.
3. Complete the table for a projectile that has an initial velocity of magnitude 25 m/s and lands at the same level from which it was launched. Use two significant digits in your calculations.

A. What conclusion can you draw from the data about the relationship between the horizontal component of velocity and maximum height? K/U T/I

B. What conclusion can you draw from the data about how you can maximize the height of an object in projectile motion? K/U T/I

C. What conclusion can you draw from the data about how you can maximize the range of an object in projectile motion? K/U T/I

D. The sum of complementary angles is 90°. Identify pairs of complementary angles. Look at the range for each pair of complementary angles in your data. Write a statement that summarizes the relationship between complementary initial angles for projectile motion. K/U T/I C

In the following Tutorial, you will apply the projectile motion equations to Sample Problems in which an object launches horizontally and an object launches at an angle above the horizontal.

Tutorial 1 Solving Simple Projectile Motion Problems

This Tutorial demonstrates how to solve two-dimensional projectile motion problems. In Sample Problem 1, an object launches horizontally so that it has an initial horizontal velocity but no initial vertical velocity. In Sample Problem 2, an object launches at an upward angle so that it has both initial horizontal and vertical velocity components.

Sample Problem 1: Solving Projectile Motion Problems with No Initial Vertical Velocity

An airplane carries relief supplies to a motorist stranded in a snowstorm. The pilot cannot safely land, so he has to drop the package of supplies as he flies horizontally at a height of 350 m over the highway. The speed of the airplane is a constant 52 m/s. **Figure 4** shows the package (a) as it leaves the airplane, (b) in mid-drop, and (c) when it lands on the highway.

(a) Calculate how long it takes for the package to reach the highway.

(b) Determine the range of the package.

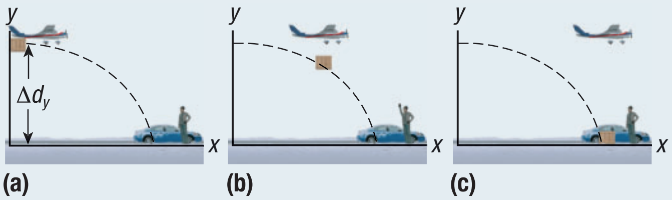


Figure 4

Solution

(a) **Given:** $\Delta d_y = -350$ m; $v_i = 52$ m/s

Required: Δt

Analysis: Set $d_i = 0$ as the altitude at which the plane is flying. Therefore, $\Delta d_y = -350$ m. Calculate Δt from the formula for the displacement along y:

$$\Delta d_y = (v_i \sin\theta)\Delta t - \frac{1}{2}g\Delta t^2$$

Solution:
$$\begin{aligned}\Delta d_y &= (v_i \sin\theta)\Delta t - \frac{1}{2}g\Delta t^2\\ &= (52\text{ m/s})(\sin 0^\circ)\Delta t - \frac{1}{2}g\Delta t^2\\ &= (0)\Delta t - \frac{1}{2}g\Delta t^2\\ \Delta d_y &= -\frac{1}{2}g\Delta t^2\\ \Delta t &= \sqrt{\frac{-2\Delta d_y}{g}}\\ &= \sqrt{\frac{-2(-350\text{ m})}{9.8\text{ m/s}^2}}\\ \Delta t &= 8.45\text{ s (one extra digit carried)}\end{aligned}$$

Statement: The package takes 8.5 s to reach the highway.

(b) **Given:** $\Delta d_y = -350$ m; $v_i = 52$ m/s; $\Delta t = 8.45$ s

Required: Δd_x

Analysis: Calculate Δd_x using the definition of cosine: $\Delta d_x = (v_i \cos\theta)\Delta t$

Solution:
$$\begin{aligned}\Delta d_x &= (v_i \cos\theta)\Delta t\\ &= (52\text{ m/s})(\cos 0^\circ)(8.45\text{ s})\\ \Delta d_x &= 4.4 \times 10^2\text{ m}\end{aligned}$$

Statement: The range of the package is 4.4×10^2 m.

Sample Problem 2: Solving Projectile Motion Problems with an Initial Vertical Velocity

A golfer hits a golf ball with an initial velocity of 25 m/s at an angle of 30.0° above the horizontal. The golfer is at an initial height of 14 m above the point where the ball lands (**Figure 5**).

(a) Calculate the maximum height of the ball.

(b) Determine the ball's velocity on landing.

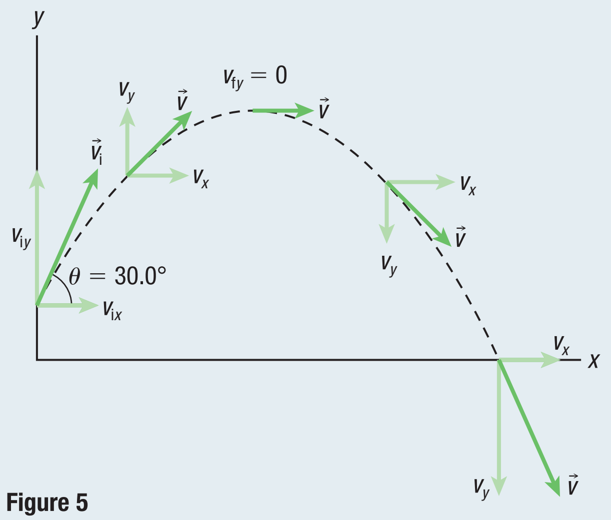


Figure 5

Solution

(a) **Given:** $v_i = 25$ m/s; $\theta = 30.0°$

Required: $\Delta d_{y\,max}$

Analysis: When the golf ball reaches its maximum height, the y-component of the ball's velocity is zero. So $v_{fy} = 0$. Set the formula for vertical velocity, $v_{fy} = v_i \sin\theta - g\Delta t$, equal to zero, and then determine the time at which the ball reaches this point. Then, determine the maximum height using the formula for vertical displacement,

$$\Delta d_y = (v_i \sin\theta)\Delta t - \frac{1}{2}g\Delta t^2.$$

Solution:

$$v_{fy} = v_i \sin\theta - g\Delta t$$
$$0 = v_i \sin\theta - g\Delta t$$
$$\Delta t = \frac{v_i \sin\theta}{g}$$
$$= \frac{(25\text{ m/s})(\sin 30.0°)}{9.8\text{ m/s}^2}$$
$$\Delta t = 1.28\text{ s (one extra digit carried)}$$
$$\Delta d_{y\,max} = (v_i \sin\theta)\Delta t - \frac{1}{2}g\Delta t^2$$
$$= (25\text{ m/}\cancel{\text{s}})(\sin 30.0°)(1.28\ \cancel{\text{s}}) - \frac{1}{2}(9.8\text{ m/}\cancel{\text{s}^2})(1.28\ \cancel{\text{s}})^2$$
$$\Delta d_{y\,max} = 8.0\text{ m}$$

Statement: The maximum height of the ball is 8.0 m.

(b) **Given:** $v_i = 25$ m/s; $\Delta d_y = 14$ m; $\theta = 30.0°$

Required: v_f

Analysis: Set $d_i = 0$ as the point at which the golfer strikes the golf ball. Therefore, $\Delta d_y = -14$ m. Use the equation $v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$ to calculate the final vertical velocity of the ball before it hits the ground. Then, calculate the velocity when the ball lands using $v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$ and the inverse tangent ratio.

Solution:

$$v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$$
$$= (v_i \sin\theta)^2 - 2g\Delta d_y$$
$$v_{fy} = \pm\sqrt{((25\text{ m/s})(\sin 30.0°))^2 - 2(9.8\text{ m/s}^2)(-14\text{ m})}$$
$$= \pm 20.8\text{ m/s}$$
$$v_{fy} = -20.8\text{ m/s (negative because the object is moving down)}$$
$$v_x = v_i \cos\theta$$
$$= 25(\cos 30.0°)$$
$$v_x = 21.7\text{ m/s}$$
$$v_f = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(21.7\text{ m/s})^2 + (-20.8\text{ m/s})^2}$$
$$v_f = 30.1\text{ m/s}$$
$$\theta = \tan^{-1}\left(\frac{|v_y|}{|v_x|}\right)$$
$$= \tan^{-1}\left(\frac{20.8}{21.7}\right)$$
$$\theta = 44°$$

Statement: The velocity of the ball when it lands is 30.1 m/s [44° below the horizontal].

Practice

1. A marble rolls off a table with a horizontal velocity of 1.93 m/s and onto the floor. The tabletop is 76.5 cm above the floor. Air resistance is negligible. K/U T/I A
   (a) Determine how long the marble is in the air. [ans: 0.40 s]
   (b) Calculate the range of the marble. [ans: 76 cm]
   (c) Calculate the velocity of the marble when it hits the floor. [ans: 4.3 m/s [64° below the horizontal]]
2. A baseball pitcher throws a ball horizontally. The ball falls 83 cm while travelling 18.4 m to home plate. Calculate the initial horizontal speed of the baseball. Air resistance is negligible. K/U T/I A [ans: 45 m/s]
3. In a children's story, a princess trapped in a castle wraps a message around a rock and throws it from the top of the castle. Right next to the castle is a moat. The initial velocity of the rock is 12 m/s [42° above the horizontal]. The rock lands on the other side of the moat, at a level 9.5 m below the initial level. Air resistance is negligible. K/U T/I A
   (a) Calculate the rock's time of flight. [ans: 2.4 s]
   (b) Calculate the width of the moat. [ans: 22 m]
   (c) Determine the rock's velocity on impact with the ground. [ans: 18 m/s [61° below the horizontal]]
4. A friend tosses a baseball out of his second-floor window with an initial velocity of 4.3 m/s [42° below the horizontal]. The ball starts from a height of 3.9 m, and you catch the ball 1.4 m above the ground. K/U T/I A
   (a) Calculate the time the ball is in the air. [ans: 0.48 s]
   (b) Determine your horizontal distance from the window. [ans: 1.5 m]
   (c) Calculate the speed of the ball as you catch it. [ans: 8.2 m/s]

The Range Equation

When you know the initial velocity and the launch angle of a projectile, you can calculate the projectile's range (Δd_x). Now suppose we launch a projectile that lands at the same height it started from, as shown in **Figure 6**. In this case, $\Delta d_y = 0$, and we can use this fact to significantly simplify the equations of motion. We can calculate the range using the equation $\Delta d_x = v_{ix}\Delta t$ if we know the initial velocity and launch angle.

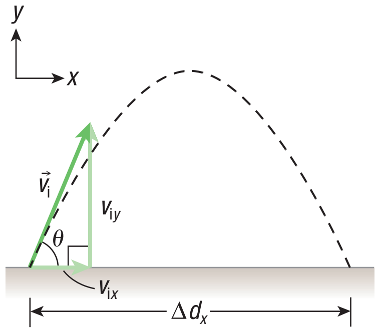


Figure 6 The projectile lands at the same height from which it was launched.

To determine the value of Δt, use the equation for vertical motion and $v_{iy} = v_i \sin\theta$:

$$\Delta d_y = (v_i \sin\theta)\Delta t - \frac{1}{2}g\Delta t^2$$

The final level is the same as the initial level, so $\Delta d_y = 0$. Substituting values in the vertical motion equation gives

$$0 = (v_i \sin\theta)\Delta t - \frac{1}{2}g\Delta t^2$$

$$0 = \Delta t\left(v_i \sin\theta - \frac{1}{2}g\Delta t\right)$$

Therefore, either $\Delta t = 0$ on takeoff or $v_i \sin\theta - \frac{1}{2}g\Delta t = 0$ on landing. Solving the latter equation for Δt gives the following equation:

$$\Delta t = \frac{2v_i \sin\theta}{g}$$

Now, we return to the equation for range, $\Delta d_x = v_i\Delta t$. Substituting Δt and the initial velocity in the x-direction, $v_{ix} = v_i \cos\theta$, gives

$$\Delta d_x = v_{ix}\Delta t$$

$$= v_i \cos\theta\left(\frac{2v_i \sin\theta}{g}\right)$$

$$\Delta d_x = \frac{v_i^2}{g}2\sin\theta\cos\theta$$

Substituting the trigonometry identity $2\sin\theta\cos\theta = \sin 2\theta$ into the above equation, we get the following for the range of a projectile:

$$\Delta d_x = \frac{v_i^2}{g}\sin 2\theta$$

where v_i is the magnitude of the initial velocity of a projectile launched at an angle θ to the horizontal. Note that this equation applies only when $\Delta d_y = 0$, that is, only when the projectile lands at the same height from which it was launched. The largest value of the range is when $\sin 2\theta = 1$ because the sine function has a maximum value of 1. This maximum value occurs when the angle is 90°. Since $2\theta = 90°$, then $\theta = 45°$, so the largest value the range can have occurs when $\theta = 45°$.

UNIT TASK **BOOKMARK**

If your extreme sport uses projectile motion, you can use the ideas in this section to complete the Unit Task on page 146.

All the previous discussion and examples of projectile motion have assumed that air resistance is negligible. This is close to the true situation in cases involving relatively dense objects moving at low speeds, such as a shot used in a shot put competition. However, for many situations you cannot ignore air resistance. When you consider air resistance, the analysis of projectile motion becomes more complex and is beyond the scope of this text.

In Tutorial 2, you will use the kinematics equations to calculate the maximum height and the range for a projectile that lands at the launching height.

Tutorial 2 Solving Projectile Motion Problems

Some projectile motion problems involve an object that starts and ends at the same height and is propelled at an angle above the horizontal. This Tutorial models how to solve projectile motion problems of this type.

Sample Problem 1: Solving Projectile Motion Problems in Which the Object Lands at the Same Height as the Launching Height

Suppose you kick a soccer ball at 28 m/s toward the goal at a launch angle of 21°.

(a) How long does the soccer ball stay in the air?

(b) Determine the distance the soccer ball would need to cover to score a goal (the range).

Solution

(a) **Given:** $v_i = 28$ m/s; $\theta = 21°$

Required: Δt

Analysis: $\Delta t = \dfrac{2v_i \sin\theta}{g}$

Solution:
$$\begin{aligned}\Delta t &= \frac{2v_i \sin\theta}{g}\\ &= \frac{2(28\ \text{m/s})\sin 21°}{9.8\ \text{m/s}^2}\\ \Delta t &= 2.0\ \text{s}\end{aligned}$$

Statement: The soccer ball stays in the air for 2.0 s.

(b) **Given:** $v_i = 28$ m/s; $\theta = 21°$

Required: Δd_x

Analysis: $\Delta d_x = \dfrac{v_i^2}{g}\sin 2\theta$

Solution:
$$\begin{aligned}\Delta d_x &= \frac{v_i^2}{g}\sin 2\theta\\ &= \frac{(28\ \text{m/s})^2}{9.8\ \text{m/s}^2}\sin(2(21°))\\ \Delta d_x &= 54\ \text{m}\end{aligned}$$

Statement: The range of the soccer ball is 54 m.

Practice

1. A projectile launcher is set at an angle of 45° above the horizontal and fires an object with a speed of 2.2×10^2 m/s. The object lands at the same height from which it was launched. Air resistance is negligible. Calculate the object's
   (a) time of flight [ans: 32 s]
   (b) horizontal range [ans: 4.9×10^3 m]
   (c) maximum height [ans: 1.2×10^3 m]
   (d) velocity at impact with the ground K/U T/I A [ans: 2.2×10^2 m/s [45° below the horizontal]]
2. A projectile is launched with an initial speed of 14.5 m/s at an angle of 35.0° above the horizontal. The object lands at the same height from which it was launched. Air resistance is negligible. Determine
   (a) the projectile's maximum height [ans: 3.5 m]
   (b) the projectile's horizontal displacement when it hits the ground [ans: 2.0×10^1 m]
   (c) how long the projectile takes to reach its maximum height K/U T/I A [ans: 0.85 s]
3. What happens to each of the following when the initial velocity of a projectile is doubled? Assume the projectile lands at the same height from which it was launched. K/U T/I A
   (a) the time of flight
   (b) the range
   (c) the maximum height

1.5 Review

Summary

- A projectile is an object that moves along a trajectory through the air, with only the force of gravity acting on it.
- An object moving with projectile motion has a constant horizontal velocity and a constant vertical acceleration.
- The time that a projectile moves in the horizontal direction is the same as the time that it moves in the vertical direction.
- When an object lands at the same height from which it was launched, use the range equation to determine the horizontal range: $\Delta d_x = \frac{v_i^2}{g} \sin 2\theta$.

Questions

1. A rock kicked horizontally off a cliff moves 8.3 m horizontally while falling 1.5 m vertically. Calculate the rock's initial speed. K/U T/I A
2. A projectile launcher sends an object with an initial velocity of 1.1×10^3 m/s [45° above the horizontal] into the air. The launch level is at the same level as the landing level. K/U T/I A
   (a) Calculate how long the object is airborne.
   (b) Determine its maximum range.
   (c) Determine the maximum height of the object.
3. In a physics demonstration, a volleyball is tossed from a window at 6.0 m/s [32° below the horizontal], and it lands 3.4 s later. Calculate (a) the height of the window and (b) the velocity of the volleyball at ground level. K/U T/I A
4. A person kicks a soccer ball with an initial velocity directed 53° above the horizontal. The ball lands on a roof 7.2 m high. The wall of the building is 25 m away, and it takes the ball 2.1 s to pass directly over the wall. K/U T/I A
   (a) Calculate the initial velocity of the ball.
   (b) Determine the horizontal range of the ball.
   (c) By what vertical distance does the ball clear the wall of the building?
5. A small asteroid strikes the surface of Mars and causes a rock to fly upward with a velocity of 26 m/s [52° above the horizontal]. The rock rises to a maximum height and then lands on the side of a hill 12 m above its initial position. The acceleration due to gravity on the surface of Mars is 3.7 m/s^2. K/U T/I A
   (a) Calculate the maximum height of the rock.
   (b) Determine the time that the rock is in flight.
   (c) What is the range of the rock?
6. A rock is thrown at an angle of 65° above the horizontal at 16 m/s up a hill that makes an angle of 30° with the horizontal. How far up the hill will the rock go before hitting the ground? K/U T/I A
7. A projectile launcher launches a snowball at 45 m/s from the top of building 1 in **Figure 7**. Does the snowball land on top of building 2? Support your answer with calculations. T/I

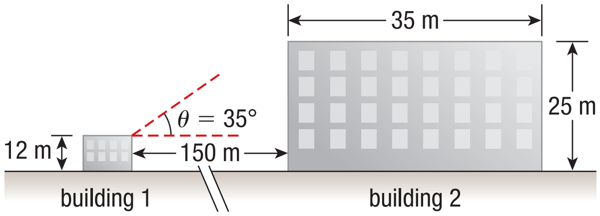


Figure 7

8. In a physics demonstration, a projectile launcher on the floor is aimed directly at a target hanging from the ceiling on the other side of the room (**Figure 8**). When the projectile is launched, the target is released at exactly the same time and the projectile hits the target. Explain why the projectile will always hit the target as long as it reaches the target before they strike the floor. K/U T/I C A

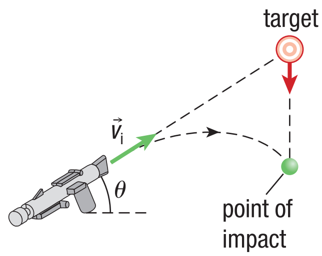


Figure 8

9. A football is thrown from the edge of a cliff from a height of 22 m at a velocity of 18 m/s [39° above the horizontal]. A player at the bottom of the cliff is 12 m away from the base of the cliff and runs at a maximum speed of 6.0 m/s to catch the ball. Is it possible for the player to catch the ball? Support your answer with calculations. K/U T/I A

1.6 Relative Motion

Suppose that you are flying in an airplane at a constant velocity south. Your view from the window might be similar to the view in **Figure 1(a)**. How would you describe the view? At first, the answer might seem simple and you would describe the clouds and the ground. After a little thought, you would realize that you can see much more. From the point of view of the plane, the ground and the clouds appear to be moving north and the plane appears to be stationary. However, from the point of view of an observer on the ground (**Figure 1(b)**), the plane is moving south, the ground is stationary, and the clouds are moving with the wind. CAREER LINK

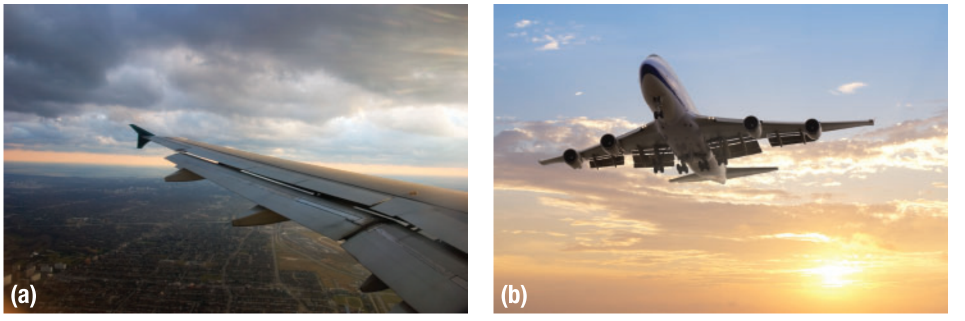


Figure 1 An airplane in flight is an excellent example of relative motion. (a) The view from an airplane window provides one perspective of an airplane's motion. (b) The view from the ground provides a completely different perspective of an airplane's motion.

Relative Velocity

frame of reference a coordinate system relative to which motion is described or observed

relative velocity the velocity of an object relative to a specific frame of reference

The airplane scenario above is an example of relative motion. The pilot and passengers in the plane are in one frame of reference (or have one point of view), and the observer on the ground is in another frame of reference. A **frame of reference** is a coordinate system relative to which motion is described or observed. The velocity of an object relative to a specific frame of reference is called the **relative velocity**.

When analyzing relative velocity problems, we will use the vector symbol, $\vec{v}$, with two subscripts in capital letters. The first subscript represents the moving object, and the second subscript represents the frame of reference. For example, suppose an airplane (P) is travelling at 450 km/h [N] relative to the frame of reference from Earth (E). Then $\vec{v}_{PE}$ is the velocity of the airplane relative to Earth, and we write the relative velocity like this: $\vec{v}_{PE} = 450$ km/h [N].

Now suppose we analyze the scenario further to be more realistic: At the altitudes at which airplanes fly, the air (A) often moves very fast relative to the ground. So, we must also consider the velocity of the plane relative to the air, $\vec{v}_{PA}$, and the velocity of the air relative to the ground, $\vec{v}_{AE}$, in addition to the velocity of the plane relative to Earth, $\vec{v}_{PE}$. The relationship that connects these three relative velocities is

$$\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$$

This equation applies whether the motion is in one, two, or three dimensions. In one dimension, solving the equation is straightforward. For example, if the plane is moving relative to the air at $\vec{v}_{PA} = 450$ km/h [N], but the air velocity relative to Earth is $\vec{v}_{AE} = 40$ km/h [N] (a tailwind), then the velocity of the plane relative to Earth is

$$\begin{aligned}\vec{v}_{PE} &= \vec{v}_{PA} + \vec{v}_{AE}\\ &= 450\text{ km/h }[\text{N}] + 40\text{ km/h }[\text{N}]\\ \vec{v}_{PE} &= 490\text{ km/h }[\text{N}]\end{aligned}$$

So, the ground speed increases with a tailwind, which makes sense. What would happen if the wind were a headwind? The airplane's ground speed would decrease to 410 km/h [N] if the magnitude of the wind's velocity were the same. This also makes sense because airplanes slow down relative to Earth because of headwinds.

In two dimensions, for example, when there is a crosswind, then the solution is also straightforward but requires more steps. You will work through relative velocity problems in one and two dimensions in Tutorial 1. But before you start the Tutorial, make sure you understand the patterns of the subscripts in the equation for relative velocity. In general, the equation takes the form

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

In the above equation, note that the outside subscripts on the right side of the equation (A and C) are in the same order as the subscripts on the left side of the equation, and the inside subscripts on the right side of the equation are the same (B).

If we add another frame of reference, the equation becomes

$$\vec{v}_{AD} = \vec{v}_{AB} + \vec{v}_{BC} + \vec{v}_{CD}$$

Tutorial 1 Solving Relative Motion Problems

A variety of situations involve relative motion. This Tutorial models a few examples in both one and two dimensions.

Sample Problem 1: Relative Motion in One Dimension

Passengers on a cruise ship are playing shuffleboard (**Figure 2**). The shuffleboard disc's velocity relative to the ship is 4.2 m/s [forward], and the ship is travelling in the same direction as the disc at 4.6 km/h relative to Earth when the water is stationary.

Figure 2

(a) Determine the disc's velocity relative to Earth.

(b) Determine the disc's velocity relative to Earth when the disc is moving in a direction opposite to that of the ship.

(c) Determine the disc's velocity relative to Earth when the water is moving at 1.1 m/s [forward].

Solution

(a) **Given:** Use the subscripts D for the disc, S for the ship, and E for Earth. $\vec{v}_{DS} = 4.2$ m/s [forward]; $\vec{v}_{SE} = 4.6$ km/h [forward]

Required: $\vec{v}_{DE}$

Analysis: Use the equation for relative velocity, $\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$, but first convert the ship's velocity to metres per second.

Solution:

$$\vec{v}_{SE} = \left(4.6\,\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\,[\text{forward}]\right)\left(\frac{1\,\cancel{\text{h}}}{60\,\cancel{\text{min}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\text{ s}}\right)\left(\frac{1000\text{ m}}{1\,\cancel{\text{km}}}\right)$$

$$= \frac{4.6 \times 1000\text{ m [forward]}}{60 \times 60\text{ s}}$$

$$\vec{v}_{SE} = 1.278\text{ m/s [forward] (two extra digits carried)}$$

$$\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$$

$$= 4.2\text{ m/s [forward]} + 1.278\text{ m/s [forward]}$$

$$\vec{v}_{DE} = 5.5\text{ m/s [forward]}$$

Statement: The velocity of the disc relative to Earth is 5.5 m/s [forward].

(b) **Given:** Use the subscripts D for the disc, S for the ship, and E for Earth. $\vec{v}_{DS} = 4.2$ m/s [backward]; $\vec{v}_{SE} = 4.6$ km/h [forward] = 1.278 m/s [forward]

Required: $\vec{v}_{DE}$

Analysis: Use the equation for relative velocity, $\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$. For the ship's velocity, use the value from part (a) after converting to metres per second.

Solution:

$$\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$$

$$= 4.2\text{ m/s [backward]} + 1.278\text{ m/s [forward]}$$

$$= 4.2\text{ m/s [backward]} - 1.278\text{ m/s [backward]}$$

$$\vec{v}_{DE} = 2.9\text{ m/s [backward]}$$

Statement: The velocity of the disc relative to Earth is 2.9 m/s [backward].

(c) **Given:** Use the subscripts D for the disc, S for the ship, W for water, and E for Earth. $\vec{v}_{DS} = 4.2$ m/s [forward]; $\vec{v}_{SE} = 4.6$ km/h [forward] $= 1.278$ m/s [forward]; $\vec{v}_{WE} = 1.1$ m/s [forward]

Required: $\vec{v}_{DE}$

Analysis: Use the equation for relative velocity, $\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SW} + \vec{v}_{WE}$. For the ship's velocity, use the value from part (a) after converting to metres per second.

Solution:

$$\begin{aligned}\vec{v}_{DE} &= \vec{v}_{DS} + \vec{v}_{SW} + \vec{v}_{WE}\\ &= 4.2\text{ m/s }[\text{forward}] + 1.278\text{ m/s }[\text{forward}]\\ &\qquad + 1.1\text{ m/s }[\text{forward}]\\ \vec{v}_{DE} &= 6.6\text{ m/s }[\text{forward}]\end{aligned}$$

Statement: The velocity of the disc relative to Earth is 6.6 m/s [forward].

Sample Problem 2: Relative Motion in Two Dimensions at Right Angles

The boat in **Figure 3** is heading due north as it crosses a wide river. The velocity of the boat is 10.0 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the boat's velocity relative to an observer on the riverbank.

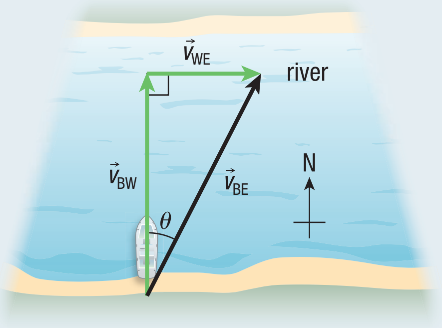


Figure 3

Given: Use the subscripts B for boat, W for water, and E for Earth. $\vec{v}_{BW} = 10.0$ km/h [N]; $\vec{v}_{WE} = 5.00$ km/h [E]

Required: $\vec{v}_{BE}$

Analysis: $\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$. This problem involves vectors in two dimensions, so we will use components to solve it. The vectors form a right-angled triangle, so the solution is straightforward. To determine the magnitude of the velocity of the boat relative to the ground, we can use the Pythagorean theorem. Then we can use the inverse tangent ratio to determine the direction.

Solution:

$$\begin{aligned}|\vec{v}_{BE}| &= \sqrt{|\vec{v}_{BW}|^2 + |\vec{v}_{WE}|^2}\\ &= \sqrt{(10.0\text{ km/h})^2 + (5.00\text{ km/h})^2}\\ |\vec{v}_{BE}| &= 11.2\text{ km/h}\end{aligned}$$

$$\begin{aligned}\tan\theta &= \frac{|\vec{v}_{WE}|}{|\vec{v}_{BW}|}\\ &= \frac{5.00\text{ km/h}}{10.0\text{ km/h}}\\ \theta &= 26.6^\circ\end{aligned}$$

Statement: The velocity of the boat relative to an observer on the riverbank is 11.2 km/h [N 26.6° E].

Sample Problem 3: Relative Motion in Two Dimensions

The driver of the boat in Sample Problem 2 moves with the same speed of 10.0 km/h relative to the water but now wants to arrive across the water at a location that is due north of his present location. The river is flowing east at 5.00 km/h.
In which direction should he head? What is the speed of the boat, according to an observer on the shore?

Given: Use the subscripts B for boat, W for water, and E for Earth. $\vec{v}_{BW} = 10.0$ km/h [?]; $\vec{v}_{WE} = 5.00$ km/h [E]; $\vec{v}_{BE} = ?$ [N]

Required: $|\vec{v}_{BE}|$; the heading of the boat, θ

Analysis: $\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$. This problem involves vectors in two dimensions, but we do not know two complete vectors. So, first we will draw the vector triangle and then resolve the triangle: draw $\vec{v}_{BW}$ in a northwest direction as shown in **Figure 4**, and then add $\vec{v}_{WE}$ head to tail. The sum of these two vectors, $\vec{v}_{BE}$, must be directed north as shown. Determine the heading of the boat using the definition of sine. To calculate the magnitude of the velocity of the boat relative to an observer on shore, use the Pythagorean theorem.

Solution:

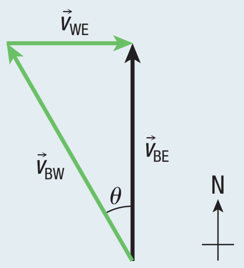


Figure 4

Determine the heading of the boat:

$$\begin{aligned}\sin\theta &= \frac{|\vec{v}_{WE}|}{|\vec{v}_{BW}|}\\ &= \frac{5.00\text{ km/h}}{10.0\text{ km/h}}\\ \theta &= 30.0^\circ\end{aligned}$$

$$|\vec{v}_{BE}| = \sqrt{|\vec{v}_{BW}|^2 - |\vec{v}_{WE}|^2}$$
$$= \sqrt{(10.0\ \text{km/h})^2 - (5.00\ \text{km/h})^2}$$
$$|\vec{v}_{BE}| = 8.66\ \text{km/h}$$

Statement: The heading of the boat is N 30.0° W, and the speed of the boat relative to the shore is 8.66 km/h.

Sample Problem 4: Using Trigonometry with Relative Motion

The air velocity of a small plane is 230 km/h [N 35° E] when the wind is blowing at 75 km/h [W 25° S]. Determine the velocity of the plane relative to the ground.

Given: Use the subscripts P for plane, A for air, and E for Earth. $\vec{v}_{PA} = 230$ km/h [N 35° E]; $\vec{v}_{AE} = 75$ km/h [W 25° S]

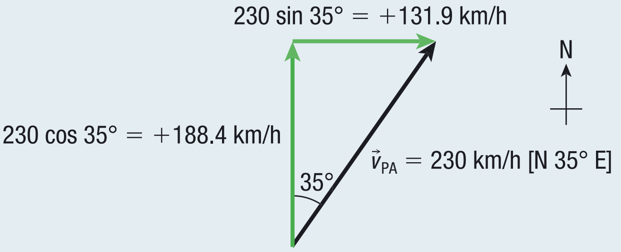
Required: $\vec{v}_{PE}$

Analysis: $\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$. This problem involves vectors in two dimensions, so we will use components to solve it.

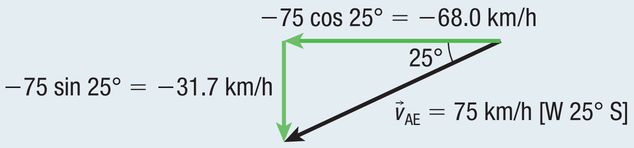
First, determine the components of each vector—the vector for the airplane and the vector for the wind—using the $+y$-direction as north and the $+x$-direction as east. Then use the Pythagorean theorem to calculate the speed of the plane and the inverse tangent ratio to calculate the direction of the plane.

Solution:

Airplane components:



Wind components:



Determine the vertical components, where the $+y$-direction is north:

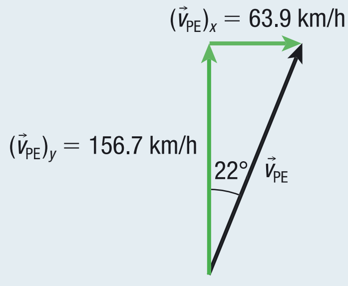
$$(\vec{v}_{PE})_y = (\vec{v}_{PA})_y + (\vec{v}_{AE})_y$$
$$= 188.4\ \text{km/h} + (-31.7\ \text{km/h})$$
$$(\vec{v}_{PE})_y = 156.7\ \text{km/h}$$

Determine the horizontal components, where the $+x$-direction is east:

$$(\vec{v}_{PE})_x = (\vec{v}_{PA})_x + (\vec{v}_{AE})_x$$
$$= 131.9\ \text{km/h} + (-68.0\ \text{km/h})$$
$$(\vec{v}_{PE})_x = 63.9\ \text{km/h}$$

Calculate the magnitude of the velocity of the plane relative to Earth and then the direction of the plane:

$$|\vec{v}_{PE}| = \sqrt{|(\vec{v}_{PE})_y|^2 + |(\vec{v}_{PE})_x|^2}$$
$$= \sqrt{(156.7\ \text{km/h})^2 + (63.9\ \text{km/h})^2}$$
$$|\vec{v}_{PE}| = 170\ \text{km/h}$$



$$\tan\theta = \frac{|(\vec{v}_{PE})_x|}{|(\vec{v}_{PE})_y|}$$
$$= \frac{63.9\ \text{km/h}}{156.7\ \text{km/h}}$$
$$\theta = 22°$$

Statement: The velocity of the plane relative to the ground is 170 km/h [N 22° E].

Practice

1. A group of teenagers on a ferry boat walk on the deck with a velocity of 1.1 m/s relative to the deck. The ship is moving forward with a velocity of 2.8 m/s relative to the water. K/U T/I A
   (a) Determine the velocity of the teenagers relative to the water when they are walking to the bow (front). [ans: 3.9 m/s [forward]]
   (b) Determine the velocity of the teenagers relative to the water when they are walking to the stern (rear). [ans: 1.7 m/s [forward]]

2. An airplane flies due north over Sudbury with a velocity relative to the air of 235 km/h and with a wind velocity of 65 km/h [NE]. Calculate the speed and direction of the airplane. K/U T/I A [ans: 280 km/h [E 81° N]]
3. A helicopter flies with an air speed of 175 km/h, heading south. The wind is blowing at 85 km/h to the east relative to the ground. Calculate the speed and direction of the helicopter. K/U T/I A [ans: 190 km/h [E 64° S]]
4. Suppose you are the pilot of a small plane flying due south between northern Ontario and Barrie. You want to reach the airport in Barrie in 3.0 h. The airport is 450 km away, and the wind is blowing from the west at 50.0 km/h. Determine the heading and air speed you should use to reach your destination on time. K/U T/I A [ans: 160 km/h [S 18° W]]
5. A large ferry boat is moving north at 4.0 m/s [N] with respect to the shore, while a child is running on the deck at a speed of 3.0 m/s. Determine the velocity of the child relative to Earth when the child is running in the following directions with respect to the deck of the boat:
   (a) north [ans: 7.0 m/s [N]]
   (b) south [ans: 1.0 m/s [N]]
   (c) east K/U T/I A [ans: 5.0 m/s [N 37° E]]
6. A plane is travelling with a velocity relative to the air of 3.5×10^2 km/h [N 35° W] as it passes over Hamilton. The wind velocity is 62 km/h [S]. K/U T/I A
   (a) Determine the velocity of the plane relative to the ground. [ans: 3.0×10^2 km/h [N 42° W]]
   (b) Determine the displacement of the plane after 1.2 h. [ans: 3.6×10^2 km [N 42° W]]
7. A person decides to swim across a river 84 m wide that has a current moving with a velocity of 0.40 m/s [E]. The person swims at 0.70 m/s [N] relative to the water. K/U T/I A
   (a) What is the velocity of the person with respect to Earth? [ans: 0.81 m/s [N 30° E]]
   (b) How long will it take to cross? [ans: 1.2×10^2 s]
   (c) How far downstream will the person land? [ans: 48 m]
   (d) In what direction should she swim if she lands at a point directly north of her starting position? [ans: [N 35° W]]
8. Two canoeists paddle with the same speed relative to the water, but one moves upstream at −1.2 m/s and the other moves downstream at +2.9 m/s, both relative to Earth. K/U T/I A
   (a) Determine the speed of the water relative to Earth. [ans: 0.85 m/s]
   (b) Determine the speed of each canoe relative to the water. [ans: 2.0 m/s]
9. An airplane maintains a velocity of 630 km/h [N] relative to the air as it makes a trip to a city 750 km away to the north. K/U T/I A
   (a) How long will the trip take when the wind velocity is 35 km/h [S]? [ans: 1.3 h]
   (b) How long will the same trip take when there is a tailwind of 35 km/h [N] instead? Why does the answer change? [ans: 1.1 h]
   (c) What will the pilot do if the wind velocity is 35 km/h [E] instead? How long will the trip take in this case? [ans: 1.2 h]

1.6 Review

Summary

- Relative motion is motion observed from a specific perspective or frame of reference. Each frame of reference has its own coordinate system. Relative velocity is the velocity of an object observed from a specific frame of reference.
- The relative velocity equation is $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$, where A is the object moving relative to the frame of reference C, which is moving relative to the frame of reference B.

Questions

1. A river has a steady current of 0.50 m/s [E]. A person can swim at 1.2 m/s in still water. The person swims upstream 1.0 km and then back to the starting point. K/U T/I A
   (a) How long does the trip take?
   (b) Will the time change if he swims downstream 1.0 km and then back instead? Explain your reasoning.
   (c) How much time is required to complete the same trip in still water? Why does the trip take longer when there is a current?
2. An airplane has an air velocity of 200 m/s [W]. The wind velocity relative to the ground is 60 m/s [N]. K/U T/I A
   (a) Determine the velocity of the airplane relative to the ground.
   (b) The airplane now faces a headwind of 60 m/s [E]. Calculate how long it takes the airplane to fly between two cities 300 km apart.
3. A helicopter travels at a velocity of 62 m/s [N] with respect to the air. Calculate the velocity of the helicopter with respect to Earth when the wind velocity is as follows: K/U T/I A
   (a) 18 m/s [N]
   (b) 18 m/s [S]
   (c) 18 m/s [W]
   (d) 18 m/s [N 42° W]
4. A person can swim 0.65 m/s in still water. She heads directly south across a river 130 m wide and lands at a point 88 m [W] downstream. K/U T/I A
   (a) Determine the velocity of the water relative to the ground.
   (b) Determine the swimmer's velocity relative to Earth.
   (c) Determine the direction she should swim to land at a point directly south of the starting point.
5. A pilot is required to fly directly from London, United Kingdom, to Rome, Italy, in 3.4 h. The displacement is 1.4×10^3 km [S 43° E]. The wind velocity reported from the ground is 55 km/h [S]. Determine the required velocity of the plane relative to the air. K/U T/I A
6. A pilot is flying to a destination 220 km [N] of her present position. An air traffic controller on the ground tells her the wind velocity is 42 km/h [N 36° E]. She knows her plane cruises at a speed of 230 km/h relative to the air. K/U T/I A
   (a) Determine the heading of the plane.
   (b) How long will the trip take?
7. An airplane flies 5.0×10^3 km from Boston to San Francisco at an air speed of 250 m/s. On the way to San Francisco, the airplane faces a headwind of 50.0 m/s blowing from west to east, and a tailwind of the same speed on the way back. K/U T/I A
   (a) Calculate the average speed of the airplane relative to the ground on the way west.
   (b) Calculate the average speed of the airplane relative to the ground on the way east.
8. A group of people on vacation on a cruise ship decide to go up to the top floor. Some decide to take an elevator, which moves at 2.0 m/s, while others climb the stairs at 2.0 m/s. The stairs are at an angle of elevation of 38° up from the east direction. The boat is cruising at a velocity of 3.2 m/s [E] relative to the water. K/U T/I A
   (a) Calculate the velocity of the people in the elevator relative to the water.
   (b) Calculate the velocity of the people taking the stairs relative to the water.
9. A car travels due east with a speed of 60.0 km/h relative to the ground. Raindrops are falling at a constant speed vertically relative to Earth. The traces of the rain on the side windows of the car make an angle of 70.0° with the vertical. Calculate the velocity of the rain relative to (a) the car and (b) Earth. K/U T/I A
10. A plane must reach a destination N 30.0° W of its present position. The wind velocity is 48 km/h [W], and the plane moves at 260 km/h relative to the air. Determine (a) the heading of the plane and (b) the speed of the plane relative to the ground. K/U T/I A

Investigation 1.5.1 OBSERVATIONAL STUDY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Investigating Projectile Motion

A straightforward way to observe and analyze two-dimensional projectile motion is to use an air table to launch a puck (**Figure 1**). The air table reduces the friction between the puck and the surface. If you change the angle of the air table and launch a puck with a velocity parallel to the inclined surface, the puck accelerates, undergoing projectile motion.

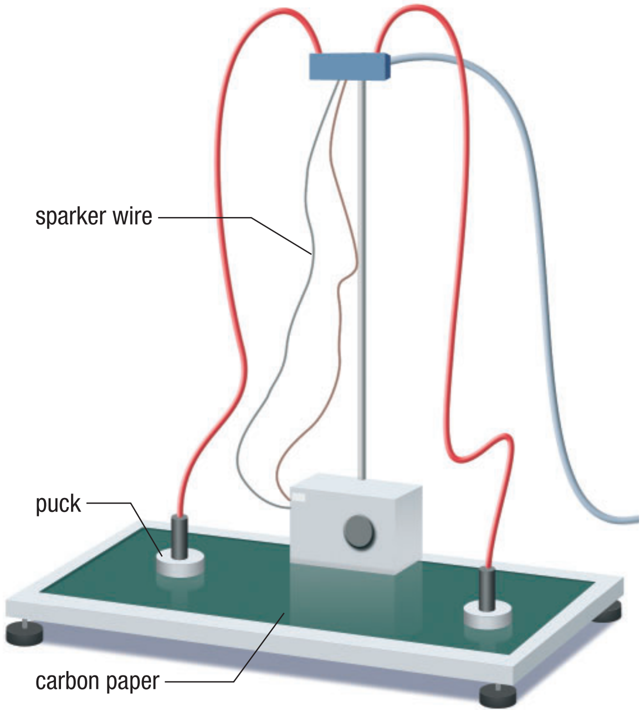


Figure 1 Air table with pucks

Purpose

SKILLS HANDBOOK A2.4

To analyze two-dimensional projectile motion using an air table

Equipment and Materials

- air table with sparker puck
- material to support one end of the air table, such as bricks
- metric ruler
- protractor
- construction paper

Do not touch the surface of the air table when the spark generator is on—you will get a shock. Keep both pucks in contact with the carbon paper on the air table when the generator is on. Keep the angle of elevation small.

Procedure

1. Raise one end of the air table, and use trigonometry or a protractor to determine the angle of incline as accurately as possible. Keep the angle low.
2. Turn the air table on but **not the sparker**. Have one person be ready to catch the puck before it hits the edge of the table.
3. Starting with the puck close to the top corner of the table, practise each of the following motions:

   motion A: $v_{ix} = 0$; $v_{iy} = 0$

   motion B: $v_{ix} > 0$; $v_{iy} = 0$

   motion C: $v_{ix} > 0$; $v_{iy} > 0$
4. Turn on the sparker—use a low frequency, such as 10 Hz—and create motions A, B, and C from Step 3. Use a separate sheet of construction paper for each motion. Label each motion, including the period and frequency of the sparker.
5. Draw between 6 and 10 velocity vectors, $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$, to represent the linear motion of motion A (**Figure 2**). (To determine the velocity vectors, draw displacement vectors and divide each one by the time interval for the displacement.)

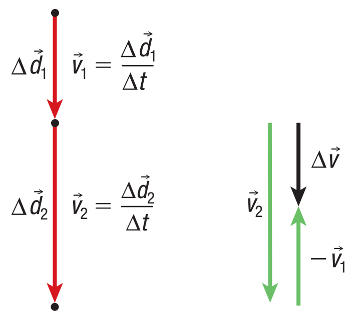


Figure 2 Motion A

6. Calculate the corresponding $\Delta\vec{v}$ vectors using vector subtraction.
7. Calculate the average acceleration for each $\Delta\vec{v}$ vector using the equation

$$\vec{a}_{\text{av},n} = \frac{\vec{v}_{n+1} - \vec{v}_n}{\Delta t}$$

where Δt is the time interval between $\vec{v}_n$ and $\vec{v}_{n+1}$. Finally, calculate the average acceleration of all $\vec{a}_{\text{av},n}$ values.
8. Repeat Steps 5, 6, and 7 for motion B (**Figure 3**). (Ignore sparker dots created when the puck comes near the edge of the table or is in contact with the pushing force.)

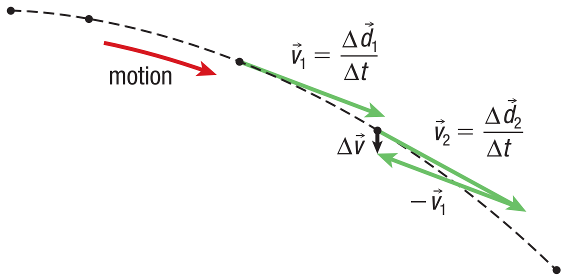


Figure 3 Motion B

9. Repeat Steps 5, 6, and 7 for motion C (**Figure 4**).

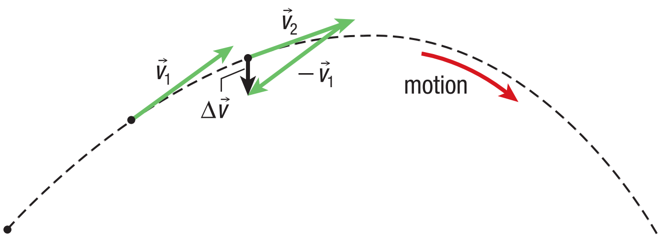


Figure 4 Motion C

Analyze and Evaluate

SKILLS HANDBOOK A5

(a) Compare the magnitude and direction of acceleration of the three motions you tested. T/I

(b) Determine the magnitude of acceleration down the inclined plane. Use the equation $a = g \sin\theta$, where g is the acceleration due to gravity (9.8 m/s^2). T/I

(c) Calculate the percent difference between your answer in (b) and the other average accelerations. T/I

(d) Why do you think you calculated the percent difference rather than the percent error in (c)? Explain your answer. T/I C

(e) What is the direction of the acceleration of a projectile on an inclined plane? T/I

(f) Describe random and systematic sources of error in this investigation. How could you minimize these sources of error? T/I

Apply and Extend

(g) Explain why the vertical component of projectile motion on an inclined plane is independent of the horizontal component. K/U

(h) Use a simulation to observe the motion of various projectiles. Manipulate the variables in the simulation, and observe how changing these variables affects the motion of the different objects. Summarize your results in a few paragraphs or a graphic organizer. K/U T/I C A

WEB LINK

CHAPTER 1 SUMMARY

Summary Questions

1. Create a study guide for this chapter based on the Key Concepts on page 6. For each point, create three or four subpoints that provide further information, relevant examples, explanatory diagrams, or general equations.
2. Look back at the Starting Points questions on page 6. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers that you wrote at the beginning of the chapter. Note how your answers have changed.
3. Draw a diagram showing the path of a ball undergoing projectile motion. Show the magnitude and direction of the horizontal and vertical components. Make separate vector diagrams for displacement, velocity, and acceleration. Describe the possible effects of air resistance on these quantities.

Vocabulary

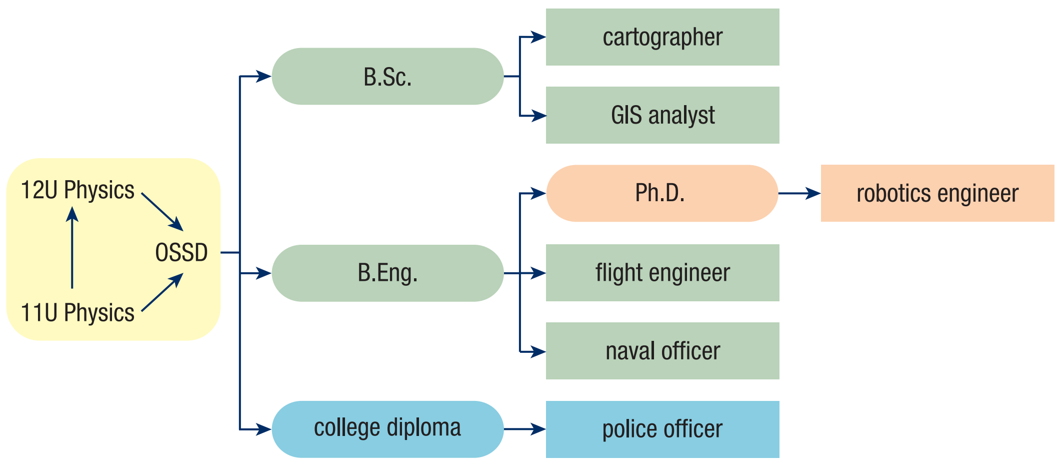
kinematics (p. 8)
dynamics (p. 8)
scalar (p. 8)
vector (p. 8)
position (p. 8)
displacement (p. 8)
average speed (p. 9)
velocity (p. 9)
average velocity (p. 9)
secant (p. 9)
tangent (p. 12)
instantaneous velocity (p. 12)
instantaneous speed (p. 12)
average acceleration (p. 14)
instantaneous acceleration (p. 14)
free fall (p. 20)
component of a vector (p. 25)
projectile (p. 36)
range (p. 36)
projectile motion (p. 36)
frame of reference (p. 44)
relative velocity (p. 44)

CAREER **PATHWAYS**

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. The graphic organizer below shows a few pathways to careers mentioned in this chapter.

1. Select an interesting career from the graphic organizer below or another career that relates to the study of kinematics and interests you. Research the educational pathway you would need to follow to pursue this career. Summarize your findings and share them with a classmate.



CAREER LINK

For each question, select the best answer from the four alternatives.

1. Under what condition is the average velocity equal to the instantaneous velocity? (1.1) K/U
   (a) always
   (b) when an object is moving with constant velocity
   (c) when an object is moving with constant acceleration
   (d) never
2. The space shuttle accelerates to 28 162 km/h in 8.5 min during a launch. What is the average acceleration? (1.1) K/U T/I
   (a) 7.5 m/s^2
   (b) 13 m/s^2
   (c) 14 m/s^2
   (d) 15 m/s^2
3. An object moving with initial speed v_i starts to slow down with an acceleration of magnitude a. How far does the object travel before stopping? (1.2) K/U T/I
   (a) $\frac{-v_i^2}{2a}$
   (b) $\frac{v_i^2}{2a}$
   (c) $\frac{v_i}{2a}$
   (d) $\frac{3v_i^2}{2a}$
4. When adding multiple two-dimensional displacement vectors, which of the following methods is most appropriate to accurately determine the total displacement? (1.3) K/U
   (a) scale diagram method
   (b) trigonometric method
   (c) algebraic component method
   (d) magnitude adding method
5. For a car moving forward and then to the right, how does the average speed compare to the average velocity? (1.4) K/U
   (a) The average speed is larger because the distance is greater than the magnitude of the displacement.
   (b) The average velocity is larger because the magnitude of the displacement is greater than the distance.
   (c) They are equal because the time is the same for both.
   (d) The average speed is larger because the magnitude of the displacement is larger than the distance.
6. A batter hits the ball in the air. The time the ball takes before it hits the ground depends on which of the following? (1.5) K/U
   (a) only the angle at which the ball is hit
   (b) the material from which the ball is made
   (c) only the initial speed with which the ball is hit
   (d) both the angle and the initial speed with which the ball is hit
7. A person is swimming with the flow of a stream. The swimmer's speed relative to the stream is 1.5 km/h, and the stream's speed relative to the bank is 1.0 km/h. What is the speed of the swimmer relative to the bank? (1.6) K/U T/I
   (a) 0.5 km/h
   (b) 1.0 km/h
   (c) 1.5 km/h
   (d) 2.5 km/h

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. The instantaneous velocity at a particular time is the slope of the displacement–time curve at that position. (1.1) K/U
9. An object can be in free fall after it is dropped or after it is thrown upward. (1.2) K/U
10. The addition of two displacement vectors depends on the order in which they are added. (1.3) K/U
11. If the velocity vector of an object changes only in direction, the average acceleration is zero. (1.4) K/U
12. For a ball thrown in a parabolic path, the y-component of the velocity at the highest point in its trajectory is equal to zero. (1.5) K/U
13. A stone projected horizontally from a cliff will reach the ground faster than a stone dropped vertically down from the same cliff. (1.5) K/U
14. The velocity of two cyclists relative to each other, if they are moving in the same direction with equal speed of 20 m/s, is zero. (1.6) K/U
15. If $\vec{v}_{AB} = 18.3$ m/s [S], then $\vec{v}_{BA} = -18.3$ m/s [N]. (1.6) K/U

Go to Nelson Science for an online self-quiz.

WEB LINK

Knowledge

For each question, select the best answer from the four alternatives.

1. Which is true for both uniform velocity and uniform acceleration as depicted by velocity–time graphs? (1.1) K/U
   (a) The velocity–time graphs for both uniform velocity and uniform acceleration are always straight lines.
   (b) The velocity–time graphs for both uniform velocity and uniform acceleration are always parallel to the *x*-axis.
   (c) The velocity–time graphs for both uniform velocity and uniform acceleration are always parallel to the *y*-axis.
   (d) The velocity–time graphs for both uniform velocity and uniform acceleration are always perpendicular to the *y*-axis.
2. You are standing at the origin of a set of coordinate axes. You walk 4.0 m [E] and then 4.0 m [N]. What is your displacement? (1.3) K/U T/I A
   (a) 5.7 m [NW]
   (b) 5.7 m [NE]
   (c) 8.0 m [NW]
   (d) 8.0 m [NE]
3. You walk 10 m [E 30° N]. What are the horizontal and vertical components of your displacement, respectively? (1.3) K/U T/I A
   (a) 9 m, 5 m
   (b) 5 m, 9 m
   (c) 5 m, 5 m
   (d) 9 m, 9 m
4. The speed of an object moving in a straight line increases from 10 m/s to 20 m/s in 2 s. What is the average acceleration? (1.4) K/U T/I A
   (a) 5 m/s^2 in the direction of motion of the object
   (b) 5 m/s^2 in the direction perpendicular to the motion of the object
   (c) 5 m/s^2 in the direction opposite to the motion of the object
   (d) $\sqrt{5}$ m/s^2 in the direction of motion of the object
5. The motion of a projectile is described in a coordinate system. At a particular instant, the magnitude of the horizontal component of velocity is 5 m/s and the magnitude of the vertical component of velocity is 8 m/s. Which is correct about the object? (1.5) K/U A
   (a) The projectile is at its maximum height.
   (b) The projectile is about to hit the ground.
   (c) The projectile is ascending.
   (d) The projectile has hit the ground.
6. A river flows with velocity 8 m/s [N] relative to the bank. A boat travels with a velocity of 6 m/s [E] relative to the river. What is the magnitude of the velocity of the boat relative to the bank? (1.6) K/U T/I A
   (a) 8 m/s
   (b) 10 m/s
   (c) 12 m/s
   (d) 14 m/s

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

7. Speed is always a positive quantity or zero, but never negative. (1.1) K/U
8. Average speed is the sum of all instantaneous speeds. (1.1) K/U
9. Velocity is the slope of the position–time graph, and acceleration is the slope of the velocity–time graph. (1.1) K/U
10. When a moving object starts to slow down on a straight track, the average acceleration of the object at any time interval after it starts slowing down is positive. (1.2) K/U
11. When two displacement vectors of equal magnitude are aligned opposite to each other, the resultant displacement is zero. (1.3) K/U
12. The average velocity of an object is always greater than or equal to the average speed. (1.4) K/U
13. A ball thrown horizontally from a cliff is an example of projectile motion. (1.5) K/U
14. Two students running toward each other with the same speed have the same velocity vector relative to each other. (1.6) K/U

Write a short answer to each question.

15. A toy car is moving on a straight track. (1.1) K/U C A
    (a) Can the toy car have a constant velocity but a varying speed? Explain.
    (b) Is the numerical ratio of speed to velocity of the toy car equal to one? Explain.
16. Is it possible for an object to have constant speed and variable velocity? Explain your answer. (1.1) K/U T/I C
17. Can two displacement vectors of the same length have a vector sum of zero? (1.3) K/U
18. Why can a sprinting football player not stop instantly? (1.4) K/U
19. A skier jumps off a ramp. In this case, air resistance is not negligible. How will air resistance affect the range and the speed with which she lands on the ground? (1.5) K/U
20. Explain what relative motion is using an example not mentioned in this section. (1.6) K/U C
21. In your own words, define relative velocity. (1.6) K/U C

Understanding

22. Discuss whether an object can have acceleration without speeding up or slowing down. (1.1) K/U T/I C
23. **Table 1** shows the combinations of values and corresponding signs for the velocity and the acceleration of an object in one dimension. Give an example of each situation in the table. (1.1) K/U A

Table 1

Velocity	Acceleration
(a) positive	positive
(b) positive	negative
(c) positive	zero

24. Compare the position–time graph and velocity–time graph for an object in uniform motion. Include a simple diagram of each. (1.1) K/U C
25. You note the odometer and the speedometer readings of your car at equal intervals of time over a long trip. What information about the motion of the car can you get from these readings? (1.1) K/U C
26. You throw a ball vertically upward, and it falls back to your hand. Identify the points where the instantaneous velocity is the same as the average velocity for the entire motion. (1.1) K/U T/I
27. Discuss what conditions are needed for three displacement vectors to have a vector sum of zero. (1.3) K/U C A
28. Provide an example in which an object moves in two dimensions but has acceleration in one dimension. (1.3) K/U A
29. A ball is thrown vertically upward from the roof of a building and lands back on the roof. Compare the displacement of the ball and the ball's velocity as seen from the roof and as seen by a person on the ground. (1.3) K/U
30. Give an example of why velocity and not acceleration should be taken into account when predicting the direction of motion of an object. (1.4) K/U C A
31. For a long jump event, describe the factors that affect the distance an athlete jumps. (1.5) K/U T/I A
32. A ball is dropped from the window of a moving car. Will the time it takes to fall to the ground be the same, more, or less than the time it takes to fall if the car is stationary? Explain your answer. (1.5) K/U C A
33. An object is at rest as well as in motion at the same time. Explain how this can be. (1.6) K/U C
34. You are piloting a fishing boat directly across a fast-moving river to reach a pier directly opposite your starting point. Explain how you would navigate the boat in terms of your velocity relative to the water. (1.6) K/U C

Analysis and Application

35. Describe the motion of an object in segments P, Q, R, and S in the position–time graph in **Figure 1**. (1.1) K/U T/I A

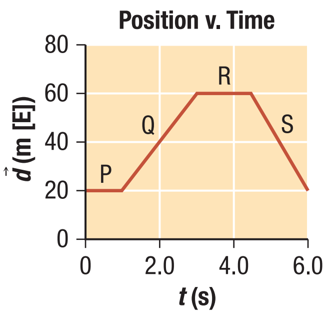


Figure 1

36. You start 1.5 m from a reference point, walk at a constant speed for 5 s, stay at this position for 1 s, and finally walk back with the same speed as earlier for the next 3 s. Draw a position–time graph of your movement. (1.1) K/U T/I C
37. Use concepts from this chapter to explain how a juggler is able to juggle balls with perfect timing. (1.2) K/U T/I A

38. A squirrel drops a nut from the top of a tree, and the nut falls to the ground. It takes 2.0 s for the nut to reach the ground. Calculate the height of the squirrel above the ground. (1.2) K/U T/I A

39. An athlete is running at a constant speed of 9 m/s. He takes 3 s to come to a stop after he crosses the finish line. Calculate his average acceleration from when he crosses the finish line to when he stops. (1.2) T/I A

40. At the start of a 100.0 m race, a sprinter increases her speed to 9.0 m/s in 2.0 s. (1.2) K/U T/I A
    (a) What is the acceleration of the sprinter during the first 2.0 s?
    (b) From this point, she runs the rest of the race with the same speed. Calculate the time to reach the finish line.

41. A race car reduces its speed from 20.0 m/s and comes to a complete stop after 50.0 m. (1.2) K/U T/I A
    (a) Determine the acceleration of the race car.
    (b) Calculate the time taken by the race car to come to a complete stop.

42. A bowler releases a ball at a bowling alley with a speed of 5.0 m/s. The ball covers the distance of 10.0 m to the pins in 2.2 s. Calculate the acceleration of the ball. (1.2) K/U T/I A

43. One stone is dropped from the top of a tall cliff, and a second stone with the same mass is thrown vertically from the same cliff with a velocity of 10.0 m/s [down], 0.50 s after the first. Calculate the distance below the top of the cliff at which the second stone overtakes the first. (1.2) K/U T/I A

44. Suppose the acceleration due to gravity on a certain planet is 2.0 m/s^2. (1.2) K/U T/I A
    (a) Will the height a high jumper can jump on this planet increase or decrease compared to a high jumper on Earth?
    (b) How high could you throw a baseball with an initial speed of 5 m/s on this planet?

45. A small aircraft is flying in a strong wind. The plane moves in a direction 60° west of south with a speed of 60 m/s. Determine the component of its velocity directed due west. (1.2) K/U T/I A

46. At the instant the traffic light turns green, a car starts from rest with a constant acceleration of 2 m/s^2. At that instant, a truck travelling with a constant speed of 10 m/s overtakes and passes the car. (1.2) K/U T/I A
    (a) How far beyond its starting point will the car overtake the truck?
    (b) Calculate how fast the car will be travelling.

47. A car slows down from 100.0 km/h to 0 km/h in 5.2 s. Determine the braking distance needed for the vehicle to come to a complete stop. (1.2) T/I A

48. A ball is thrown vertically upward from the ground with a velocity of 30.0 m/s. (1.2) K/U T/I A
    (a) How long will the ball take to rise to its highest point?
    (b) How high does the ball rise?
    (c) How long after the throw will the ball have a velocity of 10.0 m/s [upward]?
    (d) How long after the throw will the ball have a velocity of 10.0 m/s [downward]?
    (e) At what time is the displacement of the ball zero?

49. **Figure 2** shows the velocity of an object plotted as a function of time. (1.2) T/I A

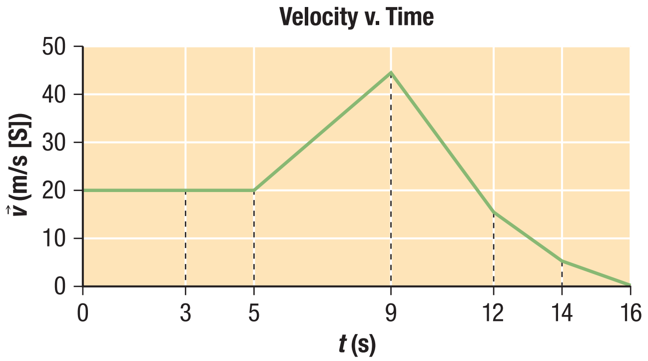


Figure 2

    (a) Calculate the instantaneous acceleration at $t = 3$ s, $t = 10$ s, and $t = 13$ s.
    (b) What is the average acceleration for the complete motion?

50. A ball thrown vertically upward passes the same height, h, at 2 s and 10 s on its way up and down, respectively. Calculate h. (1.2) K/U T/I A

51. Equation 3 in Table 1, page 18, is the equation of motion from which v_f has been eliminated. Show that Equation 3 is dimensionally correct. (1.2) K/U T/I

52. Refer to Table 1, page 18. (1.2) K/U T/I
    (a) Use Equations 1 and 2 to derive Equation 4.
    (b) Use Equations 1 and 2 to derive Equation 5.

53. Design an experimental procedure to determine the acceleration of a ball rolling down a slope. Describe your design in a few sentences. Which variables will you measure, and how will you calculate the acceleration? If possible, perform the activity. (1.2) T/I C A

54. The velocity–time graph in **Figure 3** describes the motion of an object. (1.2) K/U T/I A

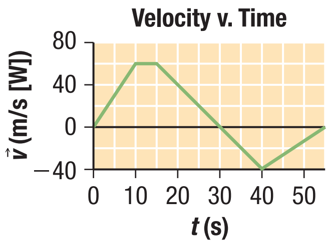


Figure 3

(a) At which intervals is the acceleration of the object positive, at which intervals is it negative, and at which intervals is it zero?

(b) Determine the average acceleration of the object for the complete motion.

(c) Determine the time(s) when the object changes its direction.

(d) How does the displacement between times 0 s and 10 s compare with the displacement between 10 s and 15 s?

55. You throw a dart horizontally with a speed of 10.0 m/s. The dart hits the board 0.49 m below the height from which it was thrown. Calculate your distance from the board. (1.3) K/U T/I A

56. Vector $\vec{A}$ of length 5.0 units makes an angle of 45° to another vector, $\vec{B}$, of length 5.0 units along the positive x-axis. Determine the components of $\vec{A} - \vec{B}$. (1.3) T/I A

57. A car travels 20.0 km due north and then 25.0 km in a direction 60.0° west of north. Determine the magnitude and direction of the car's resultant displacement. (1.3) K/U T/I A

58. In **Figure 4**, vectors $\vec{d}_1$ and $\vec{d}_2$ represent two displacements of a student. (1.3) K/U T/I A

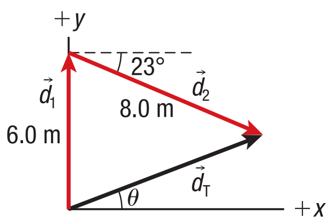


Figure 4

(a) Determine the components of the resultant displacement, $\vec{d}_T$.

(b) Determine the total displacement of the student.

59. A car is moving with a velocity of 15 m/s [E]. It makes a turn steadily in 5.0 s so that the velocity is 12 m/s [E 25° N]. Determine the average acceleration of the car. (1.4) K/U T/I A

60. A golf ball is hit from the ground, and it goes into a parabolic trajectory. What is the average acceleration in the x-direction? (1.4) K/U T/I A

61. The velocity–time graph in **Figure 5** shows the motion of a ball. (1.4) K/U T/I C A

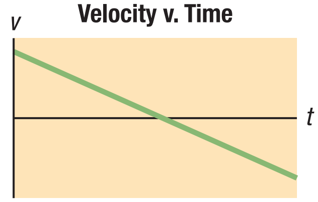


Figure 5

(a) Sketch, qualitatively, the corresponding position–time graph.

(b) Sketch, qualitatively, the corresponding acceleration–time graph.

62. A cyclist moves with a constant acceleration, covering the distance between two points in 6.0 s. The distance between these two points is 60.0 m. Her speed at the second point is 15 m/s. Calculate her acceleration and the speed at the first point. (1.4) K/U T/I A

63. A puma can jump to a height of 3.7 m when its initial velocity is at an angle of 45° to the horizontal. Calculate the initial speed of the puma. (1.5) K/U T/I A

64. Two footballs are kicked from the ground with equal initial speeds. Ball A is launched at a greater angle above the horizontal than ball B. (1.5) K/U T/I A

(a) Determine which ball reaches a higher elevation.

(b) Determine which ball stays in the air longer.

(c) Is it possible to calculate which ball travels farther?

65. A baseball player hits a 200.0 m home run. The ball travels at an angle of 45° with the horizontal just after being hit. Determine the initial speed with which the ball left the bat. Assume that air resistance is negligible and that the ball lands at approximately the same height from which it was hit. (1.5) K/U T/I A

66. A basketball player is standing 9.5 m from the basket, which is at a height of 3.1 m. She throws the ball from an initial height of 2.0 m at an angle of 35° above the horizontal. The ball goes straight through the basket. Determine the initial speed of the ball. (1.5) K/U T/I A

67. A batter hits a ball, which flies at an angle of 45° with the horizontal. The ball's speed after being hit by the bat is 30.0 m/s. Calculate the time the ball stays in the air. The ball lands at the same height at which it was hit. Air resistance is negligible. (1.5) K/U T/I A

68. A firefighter aims a hose at an angle of 60.0° with the horizontal. The water comes out of the hose with a speed of 60.0 m/s. (1.5) K/U T/I A
    (a) Calculate the maximum height the water can reach.
    (b) Determine the horizontal distance the water travels from the hose.
69. A dolphin leaps out of the water at an angle of 60.0° above the horizontal. The horizontal component of the dolphin's velocity is 8.0 m/s. Calculate the magnitude of the vertical component of its velocity. (1.5) K/U T/I A
70. A tennis ball is struck such that it leaves the racquet with a horizontal speed of 28.0 m/s. The ball hits the top of the net, and the player loses the point. What could she have done to avoid losing the point? (1.5) K/U T/I A
71. An athlete in a long jump trial leaves the ground at a certain angle and covers a horizontal distance of 8.7 m. The speed with which he can jump remains constant. What should he do to increase the distance of his jump? (1.5) K/U T/I A
72. In a snowball fight, a person throws one snowball at 26 m/s at an angle of 75° above the horizontal. While the target (his friend) is watching the snowball, he throws another at a smaller angle and the same speed as the first person, and both snowballs hit the friend at the same spot at the same time. Assume that the snowballs land at the same level as the initial throw. (1.5) K/U T/I A
    (a) What is the range of the first snowball?
    (b) At what angle was the second snowball thrown?
    (c) How long was the second snowball thrown after the first?
73. A soccer player kicks the ball in a parabolic path. The ball leaves the player's foot with a speed of 27 m/s, making an angle of 20.0° with the horizontal. (1.5) K/U T/I A
    (a) Calculate the maximum height of its trajectory.
    (b) Determine its speed as it hits the ground again. Air resistance is negligible.
74. In a practice session, a volleyball player hits a ball horizontally with a speed of 27 m/s from a height of 2.4 m. The ball travels until it hits the ground. (1.5) K/U T/I A
    (a) Determine the time the ball is in the air.
    (b) Determine the horizontal distance travelled by the ball.
    (c) Calculate the ball's speed as it hits the ground.
75. A boat is heading due north across a river with a speed of 12.0 km/h relative to the water. The water in the river has a uniform velocity of 6.00 km/h due east relative to the ground. Determine the velocity of the boat relative to an observer standing on either bank. (1.6) K/U T/I A
76. A person on a raft is drifting downstream with the current. Suddenly he dives off the raft and swims upstream for a quarter of an hour. He then swims downstream at the same velocity with respect to the water and catches back up to the raft at a position 1.0 km downstream from where he started. What is the speed of the current, in kilometres per hour? (1.6) K/U T/I A
77. A swimmer swims across a river at a velocity of 0.45 m/s [N] with respect to the water. The current is 2.5 m/s [W]. She crosses the river in 200.0 s. Determine the width of the river. (1.6) K/U T/I A
78. The current in a 35 m–wide river flows at a speed of 0.25 m/s. A student rows a boat directly across the river. The boat takes exactly 4.0 min to cross the river. Calculate the velocity of the boat relative to the water. (1.6) K/U T/I A
79. A plane is flying at 290 km/h [E 42° S] relative to the air when the wind velocity is 65 km/h [E 25° N]. Calculate the velocity of the plane relative to the ground. (1.6) K/U C

Evaluation

80. Two balls of different masses but the same surface area are dropped from the same height. Using equations of motion, prove that the time taken for both the balls to reach the ground is the same. (1.2) T/I A
81. Give an example of a scientific activity where the concepts of vectors and vector addition can be helpful. (1.4) K/U A
82. A javelin thrower argues with her coach that if her throw can keep the javelin in the air for a longer time, it will always travel a greater distance. Is the argument correct? Explain why or why not. (1.5) K/U T/I A
83. Using the concepts in the chapter, explain why an archer should aim at a point higher than the bull's-eye. (1.5) K/U T/I C A
84. Under non-windy conditions, a golfer can hit 200 m when the angle of flight of the ball is 12°. On a particular day, the wind is blowing from behind the golfer. Evaluate and explain how he should change the angle of flight of the ball such that it reaches a distance greater than 200 m. (1.5) K/U T/I A

85. The value for g on planet A is greater than the value for g on Earth, and the value for g on planet B is less than the value for g on Earth. An object is launched from planet A, and an identical object is launched from planet B. Both objects travel in a parabolic path. Speculate on how the equations and values for the time of flight, horizontal range, and maximum height compare to those on Earth. (1.5) T/I A

86. Using a projectile launcher, you launch a snowball at an angle of 35° from the roof of a building that is 45 m tall. The initial speed of the snowball is 29 m/s. The snowball lands on the ground. Your friend says the horizontal range of the snowball is 81 m. Is your friend correct? Explain why or why not. (1.5) K/U T/I A

Reflect on Your Learning

87. What did you find different from your preconceptions and intuitive understanding of the motion of objects? Which concept did you find most difficult to understand? Why was it difficult, and what helped clarify it? T/I C

88. You learned about different ways the motion of an object is measured and depicted and how various other parameters are calculated. Can you explain various physical phenomena where these concepts can be applied? Are there any exceptions to these concepts and theories in the real world? C A

89. Many people struggle to understand why the vertical acceleration of a projectile is constant. What helped to clarify this concept for you? T/I C

90. How can you apply the concepts in this chapter to enhance your performance in the sports you play? C A

Research

WEB LINK

91. Research how various scientists throughout history, such as Aristotle, Galileo, and Newton, have studied the motion of objects. Research the various experiments they conducted to test different laws. Create a timeline of advancements made by various scientists on the theory of motion and projectile motion. T/I C A

92. Research dirt bike tracks, and choose one. Research the design of the dirt bike track, with specific reference to the ramps and turns and how their design aids and affects the bikers' performance. Refer to velocity, acceleration, and trajectory. Identify the theories of kinematics that are used. T/I C A

93. Select a sport, and research how the theories of motion are involved in various aspects of the sport. Formulate a plan using the theories of motion to help an athlete perform better in the chosen sport. Your plan should contain concepts of trajectories and velocities. Describe the motion of any equipment used in the sport. T/I C A

94. In a 100 m race at the Olympics or any other prestigious event, runners start from rest and complete the event so quickly that it is difficult to see what is actually happening. Research the motion that occurs during one such event by looking up the split times for a particular runner. Use the terminology from this chapter to describe in detail how the runner moved to complete the race. Identify where the acceleration was largest and if at any time the velocity was basically uniform. Does the runner slow down at any time during the race? To help with your explanation, draw simple sketches of the motion graphs. T/I C A

95. A car driver's reaction time is the average time required for a driver to apply the brakes after seeing an emergency. The average reaction times for a car driver under the influence of alcohol are shown in **Figure 6**. T/I C A

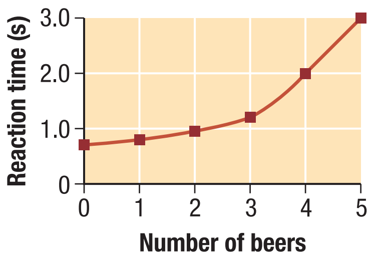


Figure 6

(a) Use Figure 6 to complete **Table 2**.

Table 2

Speed	Reaction distance (m)		
	No alcohol	4 bottles of beer	5 bottles of beer
17 m/s (60 km/h)			
25 m/s (90 km/h)			
33 m/s (120 km/h)			

(b) Using the Internet and other sources, research the effects of alcohol on the average reaction time. Search for direct evidence on how drivers are impaired when under the influence of alcohol. Prepare a brief summary of your findings.

(c) Some say there is no safe level of alcohol that can be consumed by drivers. Discuss the validity of this statement using examples from your research.

CHAPTER 2

Dynamics

KEY CONCEPTS

After completing this chapter you will be able to

- demonstrate an understanding of how forces affect the motion of an object
- solve two-dimensional motion and force problems
- predict the motion and forces acting on a system of objects and the forces involved
- explain the advantages and disadvantages of friction in situations involving various planes
- conduct investigations into the motion of a system of objects
- analyze a technological device that applies the principles of linear motion and assess the social and environmental impact of the device

How Does Our Understanding of Forces Affect the Design and Use of Technology?

Race car designers understand the forces that lead to speeding up and slowing down. The car's engine turns the wheels, which propel the car forward using the force of friction between the tires and the road. The larger the force of static friction between the tires and the road, the greater the acceleration. If the driver tries to accelerate too quickly, the wheels overcome the force of static friction and slip, spinning in place. When the tires slip on the road, the force of kinetic friction causes the burning rubber that you see in the image on the facing page. Once the car is moving, more frictional forces, such as air resistance, act to slow the car down.

The shape of the car is also important. Shape affects the air resistance as well as the force that the air exerts on the car. With an efficient body design, there is an increased downward force from the air. An increased downward force increases the normal force on the car, which affects the static friction between the tires and the road. As a result, the car can accelerate more without slipping.

Decreasing the amount of friction between the moving parts inside the car (the engine, the transmission, and other parts) is also important for maximizing speed. Motor oils help decrease the friction between the gears and other moving parts, increasing the total force the tires can apply backward to the road, propelling the car forward.

If the race car crashes, what happens to the driver? Newton's laws tell us that a sudden slowing down in the case of a crash means a large force on the driver. The car's safety features slow the driver in a way that decreases the force and, therefore, the effect of the force. New designs can also help make more efficient use of the materials required to make the car safe and reduce the environmental impact of production of not only race cars but everyday cars as well.

In this chapter, you will apply Newton's laws of motion to learn how forces affect the motion of an object in two dimensions.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. What forces are acting on the race car in the image on the facing page? What effect, if any, does each force have on the motion of the car?
2. What features of the car help reduce resistance due to the air and internal friction?
3. What safety features are visible in the car shown in the image?
4. What dangers should a race car driver be aware of?
5. What design features in a race car can be used to improve the safety or operation of a typical family car? If the features are not used in a family car, explain why.

Mini Investigation

Describing Motion Using Newton's Laws

Skills: Predicting, Performing, Observing, Analyzing, Communicating

At three different stations, you will model situations that demonstrate Newton's laws of motion. The first law describes what happens to an object that has a total force of zero acting on it. The second law describes the relationship between force and acceleration. The third law describes how an object reacts to a force exerted on it.

Equipment and Materials: pulley; 2 carts of equal mass, one spring-loaded; 50 g mass; 200 g mass; string

Station 1

1. Place the 50 g mass on top of the cart. Slowly push the cart toward the wall. Observe what happens to the mass when the cart hits the wall.

Take care when moving the mass and cart. Do not wear open-toed shoes. Do not allow the cart or the masses to fall on your hands or feet.

A. What happens to the mass when the cart hits the wall? K/U

B. Which of Newton's laws describes what happens to the motion of the mass? Explain your answer. K/U

Station 2

2. Use a string to attach the 50 g mass to the cart. Run the string over the pulley at the edge of the table so that when you release the mass it pulls the cart. Repeat the process with the 200 g mass.

C. Describe the speed of the cart as the mass pulls it. K/U

D. Describe how the motion of the cart changes with the different masses. K/U

E. Which of Newton's laws describes the motion of the cart when pulled by the mass? Explain your answer. K/U

Station 3

3. Place the two carts end to end with the loaded spring between them. Allow the spring to suddenly release.

F. What happens to each cart when the spring expands? K/U

G. Which of Newton's laws describes this motion? Explain your answer. K/U

2.1 Forces and Free-Body Diagrams

Figure 1 A trebuchet converts the force of gravity downward on the counterweight into the motion of the projectile.

Understanding forces is essential for designing and developing technologies, both ancient and modern. For example, the machine in **Figure 1** is a model of an ancient weapon called a trebuchet. Trebuchets were used for hundreds of years, before gunpowder was available, to launch projectiles into cities under siege. In a trebuchet, the force of gravity pulls downward on a counterweight. That force causes the motion of the projectile, which can travel quite quickly. WEB LINK

A modern example of converting force into motion is the linear actuator, which can be used to reduce the strain of repetitive motion in the workplace. A motor drives a series of gears or screws, which convert the motor's power into the force of the actuator. You will learn more about linear actuators in Section 2.5.

Common Forces

force ($\vec{F}$) a push or a pull

newton the SI unit of force; symbol N

contact force a force that acts between two objects when they touch each other

non-contact force a force that acts between two objects without the objects touching; also called action-at-a-distance force

force of gravity ($\vec{F}_g$) the force of attraction between all objects due to mass

A **force** is a push or a pull. The measure of force in the SI system of units is called the **newton** (N). You encounter different kinds of forces every day. A force can even stabilize an object by counteracting another force on that object. Physicists classify forces as **contact forces**, where one object exerts a force on another object when they touch each other, and **non-contact forces**, such as gravity, where the two objects need not touch to exert a force on each other. Non-contact forces are also called action-at-a-distance forces.

Earth's **force of gravity** is responsible for everything from keeping your textbooks on your desk to keeping satellites in orbit around Earth. The force of gravity is an *attractive* force: all objects have mass and therefore attract each other. This attraction is quite weak when the objects are small or far apart. Earth exerts a relatively large attractive force on everything around you compared to other masses because Earth is so massive compared to other masses on Earth, such as buildings and bridges. For example, the gravitational attraction between a 30.0 kg desk and a 1.0 kg textbook 0.10 m apart is only 2.0×10^{-7} N, but Earth's force on the same textbook is 9.8 N.

normal force ($\vec{F}_N$) a force perpendicular to the surface between objects in contact

If Earth's force of gravity pulls downward on the textbook on your desk, why does the book remain stationary? There must be a force pushing up on the book perpendicular to the surface to balance the force of gravity. This balancing force is called the **normal force**, which is a force perpendicular to the surface between objects in contact. In **Figure 2**, the normal force points upward because the contact surface is parallel to the ground.

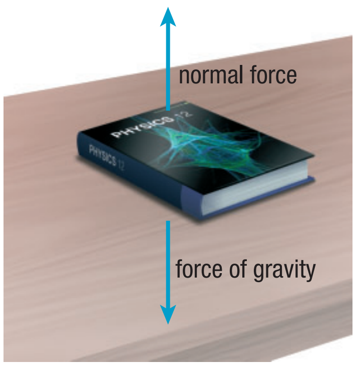


Figure 2 For a stationary object such as this textbook resting on a desk, Earth's gravity pulls downward while the normal force of the desk pushes upward, so the book does not move.

tension ($\vec{F}_T$) a force exerted by objects that can be stretched

Another common force is **tension**, which is a pulling force exerted by objects such as strings and ropes. **Figure 3(a)**, on the next page, shows how to measure tension using a spring scale. The more you stretch the spring in the spring scale, the more difficult it becomes to pull. The degree of difficulty indicates the amount of tension. Even when the direction of the force changes, such as when a string passes over a pulley (**Figure 3(b)**), the amount of tension stays uniform.

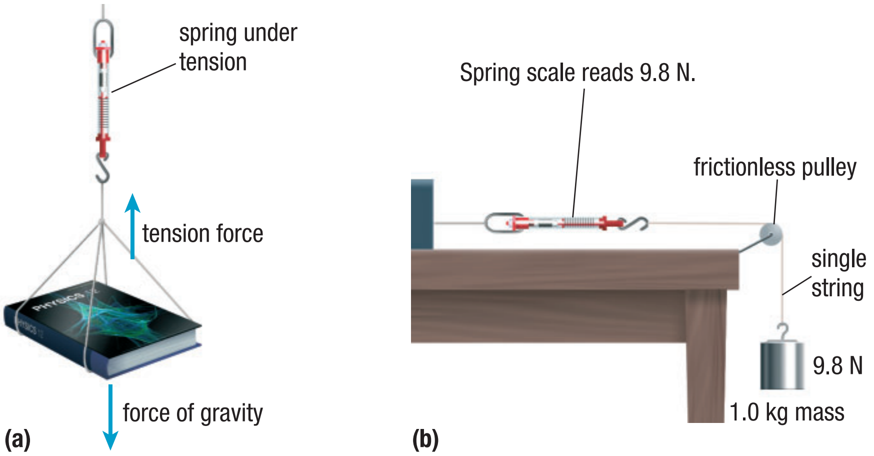


Figure 3 (a) The larger the stretch in the spring, the greater the tension. (b) The tension in the string is the same all along its length. The string pulls up on the 1.0 kg mass below the pulley with the same force as it pulls horizontally on the spring to the left of the pulley.

The force of **friction** exists between objects and always resists the sliding motion or attempted sliding motion between objects. Suppose a heavy box of books is on the floor. You try to push the box across the floor with a horizontal force, but the box does not move. You push harder, and the box starts to move. You have overcome the force of static friction. **Static friction** is a force that resists attempted motion between two surfaces—it keeps the stationary box of books from moving across the floor. **Kinetic friction** is a force exerted on a moving object by the surface in a direction opposite to the motion of the object. Pushing a box across the floor is made more difficult because kinetic friction acts in the direction opposite to motion. WEB LINK

Another important type of kinetic friction is air resistance. **Air resistance** is the friction between an object and the air around it. Air resistance is more noticeable for lightweight objects, such as a piece of paper falling through the air, and objects moving at high speeds, such as an airplane flying through the air. Air resistance can be neglected in most problem-solving situations unless it is logically required. CAREER LINK

Finally, an **applied force** is a force due to one object coming into contact with another object, such that a push or a pull results. When pushing on the box mentioned above with your hands, you are applying a force.

friction ($\vec{F}_f$) a force that opposes the sliding of two surfaces across one another; acts opposite to motion or attempted motion

static friction ($\vec{F}_S$) a force that resists attempted motion between two surfaces in contact

kinetic friction ($\vec{F}_K$) a force exerted on a moving object by a surface in the direction of motion opposite to the motion of the object

air resistance ($\vec{F}_{air}$) the friction between objects and the air around them

applied force ($\vec{F}_a$) a force due to one object pushing or pulling on another

Free-Body Diagrams

When solving physics problems, it is sometimes difficult to visualize all the forces acting on an object. One way to visualize all the different forces acting on an object is with a diagram. A **free-body diagram** (FBD) is a simple line drawing of an object that shows all the forces acting on the object at one moment in time. Arrows represent the approximate direction and magnitude of each force. A dot in the centre represents the object. The underlying assumption is that we are modelling the object as a point particle, so the dot makes this assumption visually apparent. Some people draw FBDs with a dot and a rectangle, but in this textbook the FBDs just have the dot. All forces point outward from the dot. Sometimes, you may need to sketch a system diagram showing all objects involved in a situation first before drawing an FBD (**Figure 4**).

free-body diagram a simple line drawing that shows all the forces acting on an object

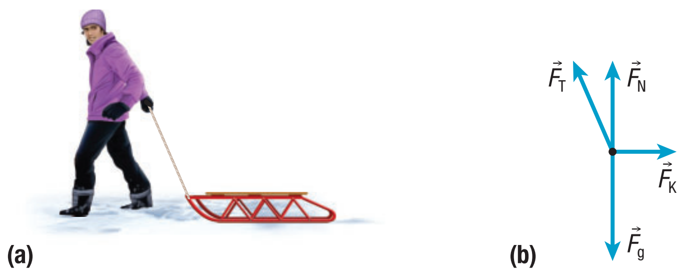


Figure 4 (a) A system diagram of a person pulling a sled. (b) An FBD shows the forces acting on the sled.

In Tutorial 1, you will practise drawing both an FBD and a system diagram for different forces.

Tutorial 1 Drawing Free-Body and System Diagrams

This Tutorial demonstrates how to draw free-body and system diagrams used to study forces.

Sample Problem 1: Applying a Horizontal Force

You are pushing with a horizontal force to the right against a large printer on a table. The printer remains stationary. Draw a system diagram and an FBD of the forces acting on the printer.

Solution

Step 1. Identify the objects in the scenario.

The objects in the scenario are two hands, a printer, a table, and Earth.

Step 2. Draw a simple system diagram.

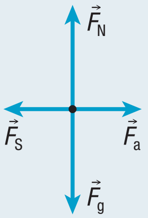
Step 3. Identify the forces acting on the printer.

The forces acting on the printer are the force of gravity, the normal force, and an applied force. The printer is not moving, so the force of static friction is also acting on the printer.

Step 4. Determine the direction of each force.

The normal force exerted by the desk pushes upward on the printer, and the force of gravity is pulling downward on the printer. The applied force is acting to the right. The force of static friction is acting on the printer to the left.

Step 5. Draw an FBD by drawing a dot to represent the printer. Draw individual arrows to represent each force and its direction. The lengths of the arrows should reflect the magnitudes of the forces. If two forces have the same magnitude, then the lengths of the arrows will be the same. Label each arrow with the appropriate force symbol.



Sample Problem 2: Applying a Non-horizontal Force

A rope pulls a skier up a hill to the right at a constant velocity. Draw a system diagram and an FBD of the forces acting on the skier.

Solution

Step 1. Identify the objects in the scenario.

The objects in the scenario are a skier, a rope, and an incline.

Step 2. Draw a simple system diagram.

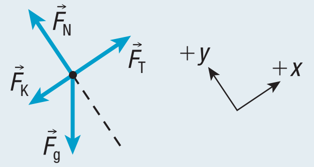
Step 3. Identify the forces acting on the skier.

The forces acting on the skier are gravity, the normal force, the force of tension in the rope, and the force of kinetic friction.

Step 4. Determine the direction of each force.

Gravity acts in the downward vertical direction. The normal force acts perpendicular to the slope of the hill. The tension of the rope acts on the skier to the upper right, and the kinetic friction between the skis and the snow acts on the skis in the opposite direction of the force of tension.

Step 5. Draw an FBD by drawing a dot to represent the skier. Draw individual arrows to represent each force and its direction. Indicate the magnitudes of the forces by the lengths of the arrows. Label each arrow with the appropriate force symbol.



Practice

1. Draw a simple system diagram and an FBD for each of the following objects. K/U C A
   (a) a pen sitting on a table
   (b) a rope connected to a crane raising a piano vertically upward at a constant speed
   (c) a lamp that has just begun falling from a table to the floor; air resistance is negligible
   (d) a dresser that is being pulled to the right up a ramp into a delivery truck by a cable parallel to the ramp; the ramp is at an angle of 14° above the horizontal
2. You throw a ball vertically upward. Air resistance is negligible. Draw an FBD of the ball
   (a) just after it leaves your hand
   (b) at the top of its motion
   (c) as it is falling back down K/U C A
3. A skydiver whose parachute is open can see his instantaneous height above ground level on an electronic screen. The skydiver has reached terminal speed. (Recall from earlier studies that when an object falls at terminal speed it is falling at a constant velocity.) For this question, assume the skydiver and the parachute together act as one body. Draw a system diagram and an FBD for the situation. K/U C A

Determining Net Force in Two Dimensions

Once you identify all of the forces acting on an object, you can calculate the **net force** on the object, or the sum of all the forces acting on an object. The term *net force* and the symbol $\Sigma\vec{F}$ are used to represent this sum, but the terms *total force* and *resultant force* are also used. The symbol for net force uses the Greek letter Σ (sigma) in front of the $\vec{F}$. In mathematics, sigma indicates a sum of several different terms or numbers. Sigma is used here to remind you to add up (or sum) all forces acting on a single object at one moment in time to calculate the net force.

net force ($\Sigma\vec{F}$) the sum of all the forces acting on an object

When several forces are acting on an object, those forces are not always parallel or perpendicular to each other. This can make determining the sum of the forces more difficult. In these cases, it is often convenient to think about the components of the forces in the *x*- and *y*-directions. We will use the symbols ΣF_x and ΣF_y for these components. In addition, FBDs are helpful to visualize the forces. In Tutorial 2, you will calculate the net force on an object in different contexts.

Tutorial 2 Determining Net Force

This Tutorial models how to determine the net force acting on objects when the individual forces are not all parallel or perpendicular to each other.

Sample Problem 1: Net Force above the Horizontal

A baseball player lightly bunts a baseball with an average force of 14 N at 29° above the horizontal (**Figure 5**). The force of gravity on the baseball is 1.4 N. Calculate the net force on the ball at the moment of contact, assuming that air resistance is negligible.

Figure 5

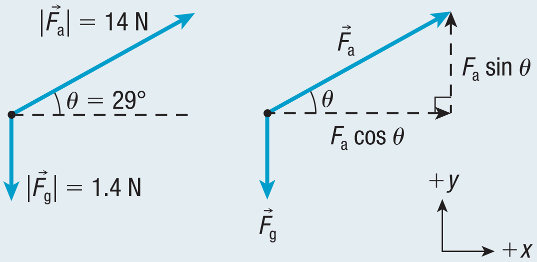
Given: $\vec{F}_{\mathrm{a}} = 14$ N; $\theta = 29°$ above the horizontal; $F_{gx} = 0.0$ N; $F_{gy} = 1.4$ N

Required: ΣF_x (net force); θ (direction)

Analysis: Draw FBDs to show the force on the baseball and the components of the force. Use $F_{ax} = F\cos\theta$ and $F_{ay} = F\sin\theta$ to determine the components of the force on the baseball. Add the components to the components of the force of gravity, and use $|\Sigma\vec{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$ to calculate the net force.

Use $\phi = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$ to determine the direction.

Solution: The FBDs are shown in **Figure 6** and **Figure 7**.



(a) **(b)**

Figure 6 (a) The FBD shows forces acting on the baseball. (b) Components of the forces in the vertical and horizontal directions.

$$F_{ax} = F\cos\theta$$
$$= (14\text{ N})\cos 29°$$
$$F_{ax} = 12.2\text{ N}$$

$$\Sigma F_x = F_{ax} + F_{gx}$$
$$= 12.2\text{ N} + 0.0\text{ N}$$
$$\Sigma F_x = 12.2\text{ N}$$

$$F_{ay} = F\sin\theta$$
$$= (14\text{ N})\sin 29°$$
$$F_{ay} = 6.79\text{ N (one extra digit carried)}$$

$$\Sigma F_y = F_{ay} + (-F_{gy})$$
$$= 6.79\text{ N} - 1.4\text{ N}$$
$$\Sigma F_y = 5.39\text{ N (one extra digit carried)}$$

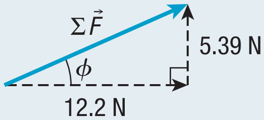


Figure 7 Net force on the baseball

$$|\Sigma\vec{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(12.2\text{ N})^2 + (5.39\text{ N})^2}$$
$$|\Sigma\vec{F}| = 13\text{ N}$$

$$\phi = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$
$$= \tan^{-1}\left(\frac{5.39\text{ N}}{12.2\text{ N}}\right)$$
$$\phi = 24°$$

Statement: The net force on the baseball is 13 N at 24° above the horizontal.

Sample Problem 2: Total Force of Friction on a Towed Object

A go-cart is being towed north by a car along a road with a net force of zero. The go-cart is attached to the car by two ropes. The tension in the ropes is the same, 31 N. The ropes make 22° angles to the direction of motion, one on the west side and the other on the east. Determine the force of friction on the go-cart. **Figure 8** shows a top view of the go-cart. The figure does not show the normal force and gravity because they are perpendicular to the page and cancel each other.

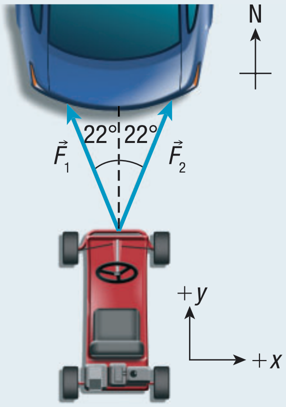


Figure 8

Given: $\Sigma\vec{F} = 0$; $\vec{F}_1 = 31$ N [N 22° W]; $\vec{F}_2 = 31$ N [N 22° E]

Required: force of friction on go-cart, $\vec{F}_K$

Analysis: Draw an FBD. There is no net force on the go-cart, so ΣF_x equals zero and ΣF_y equals zero. The normal force and the force of gravity cancel each other. Use $F_x = F\cos\theta$ and $F_y = F\sin\theta$ to determine the components of the tension forces on the go-cart. To calculate the force of friction on the go-cart, use $\Sigma F_x = F_{1x} + F_{2x} + F_{fx}$ and $\Sigma F_y = F_{1y} + F_{2y} + F_{fy}$. Use north and east as the positive directions when determining components.

Solution: The FBD is shown in **Figure 9**.

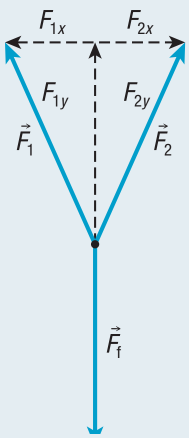


Figure 9

For the x-component of the tension force,

$$\begin{aligned} F_{1x} &= -F_1\cos\theta \\ &= -(31\text{ N})\cos 22° \\ F_{1x} &= -28.7\text{ N (one extra digit carried)} \end{aligned}$$

$$\begin{aligned} F_{2x} &= F_2\cos\theta \\ &= (31\text{ N})\cos 22° \\ F_{2x} &= 28.7\text{ N (one extra digit carried)} \end{aligned}$$

$$\begin{aligned} \Sigma F_x &= F_{1x} + F_{2x} + F_{fx} \\ F_{fx} &= \Sigma F_x - (F_{1x} + F_{2x}) \\ &= 0 - (-28.7\text{ N} + 28.7\text{ N}) \\ F_{fx} &= 0\text{ N} \end{aligned}$$

For the y-component of the tension force,

$$\begin{aligned} F_{1y} &= F_1\sin\theta \\ &= (31\text{ N})\sin 22° \\ F_{1y} &= 11.6\text{ N} \end{aligned}$$

$$\begin{aligned} F_{2y} &= F_2\sin\theta \\ &= (31\text{ N})\sin 22° \\ F_{2y} &= 11.6\text{ N} \end{aligned}$$

$$\begin{aligned} F_{fy} &= \Sigma F_y - (F_{1y} + F_{2y}) \\ &= 0 - 11.6\text{ N} - 11.6\text{ N} \\ F_{fy} &= -23\text{ N} \end{aligned}$$

Statement: The forces of the ropes in the x-direction cancel, so there is no force of friction in that direction. The force of friction is 23 N [S].

Practice

1. Determine the net force acting on each of the following objects. In each case assume that all forces acting on the object are given. K/U T/I
   (a) At an instant when a soccer ball is slightly off the ground, a player kicks it, exerting a force of 25 N at 40.0° above the horizontal. The force of gravity acting on the ball is 4.2 N [down]. [ans: 23 N [32° above the horizontal]]
   (b) Two children pull a sled across the ice. One child pulls with a force of 15 N [N 35° E], and the other pulls with a force of 25 N [N 54° W]. There is negligible friction acting on the sled. [ans: 29 N [N 23° W]]
   (c) In a circus act, a performer with a force of gravity of 4.4×10^2 N on her is lifted by two different ropes at the same time. One rope exerts a tension of 4.3×10^2 N [up and 35° left], and the other rope exerts a force of 2.8×10^2 N [up]. [ans: 3.1×10^2 N [up 38° left]]
2. Two tractors pull a large rock east through a construction site with a net force of zero on the rock. Tractor 1 exerts a force of 1.2×10^4 N [E 12° N] on the rock, and tractor 2 exerts a force of 1.2×10^4 N [E 12° S]. K/U T/I
   (a) Calculate the force of friction acting on the rock. [ans: 2.3×10^4 N [W]]
   (b) Discuss two ways someone could spot that the force of friction on the rock must be to the west before solving the problem.
3. **Figure 10** shows three masses connected by wires and hung vertically. Draw an FBD for each mass, and determine the tensions in the three wires. K/U T/I A [ans: top wire: 3.4×10^2 N; middle wire: 2.0×10^2 N; bottom wire: 1.3×10^2 N]

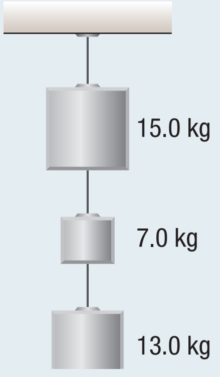


Figure 10

4. At one moment during its flight, a thrown baseball experiences a gravitational force of 1.5 N [down] and a force from air resistance of 0.40 N [32° above the horizontal]. Calculate the magnitude and direction of the net force on the ball. K/U T/I A [ans: 1.3 N [75° below the horizontal]]
5. The force of gravity on a basketball is 16 N [down]. K/U T/I A
   (a) What is the net force on the ball while held stationary in your hand? [ans: 0 N]
   (b) Neglecting air resistance, calculate the net force acting on the ball if you suddenly remove your hand. [ans: 16 N [down]]
   (c) You push the ball with a force of 12 N [right]. Calculate the net force on the ball. [ans: 20 N [right 53° down]]
   (d) You push the ball with a force of 26 N [up 45° right]. Calculate the net force on the ball. [ans: 19 N [right 7.4° up]]

Summary

- Examples of common forces that you encounter every day are Earth's gravity, the normal force, tension, friction, applied forces, and air resistance.
- Static friction prevents a stationary object from moving, and kinetic friction opposes the motion of an object. Air resistance opposes the motion of an object through air.
- A free-body diagram (FBD) is a simple drawing of an object that shows all forces acting on the object. FBDs can help you visualize the forces, determine the components, and calculate the net force.
- The net force, $\Sigma\vec{F}$, is the sum of all of the forces acting on an object.

Questions

1. Summarize the common forces in a table with the following headings: Name, Symbol, Contact/Non-contact, Direction, Example in daily life. K/U C
2. Study the traction system shown in **Figure 11**. The tension in the vertical cord above the mass is 22 N. A student claims that the tension in the vertical cord above the leg must be more than 22 N. Discuss the validity of this statement. K/U T/I C

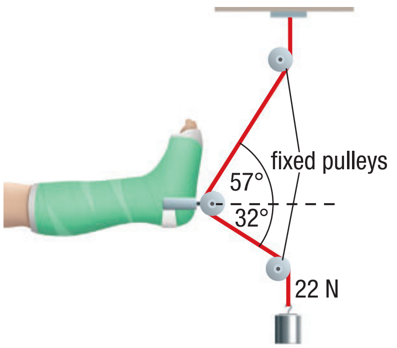


Figure 11

3. Explain why ropes can only pull and never push. K/U
4. You push your textbook at a constant velocity to the right across the table by applying a force at an angle of 23° below the horizontal. K/U T/I C A
   (a) List the forces acting on the textbook.
   (b) Draw an FBD of the textbook.
5. Given the forces $\vec{F}_A = 2.3$ N [S 35° W], $\vec{F}_B = 3.6$ N [N 14° W], and $\vec{F}_C = 4.2$ N [S 24° E], calculate the following:
   (a) $\vec{F}_A + \vec{F}_B + \vec{F}_C$
   (b) $\vec{F}_B - \vec{F}_C$ T/I
6. Given $\vec{F}_A = 33$ N [E 22° N] and $\vec{F}_B = 42$ N [S 15° E], calculate the force $\vec{F}_C$ needed so that $\vec{F}_A + \vec{F}_B + \vec{F}_C$ is zero. T/I
7. At the beach, three children pull on a floating toy. Child 1 pulls with a force of 15 N [N 24° E], child 2 pulls south, and child 3 pulls west. The net force on the toy is zero. Assume that there are no other significant forces acting on the toy. T/I
   (a) Calculate the magnitude of the forces exerted by child 2 and child 3 on the toy.
   (b) Child 2 lets go, and the other children maintain the same force. Calculate the net force on the toy.
   (c) What force must child 3 exert on the toy to cancel the force of child 1 on her own?
8. During a competition for charity, two students push horizontally on a heavy cart during a race. The net force on the cart is 180 N [E]. One student pushes with a force of 120 N [E 14° S]. Calculate the force that the second student exerts on the cart. T/I
9. You exert a force of 55 N on a heavy sled as shown in **Figure 12**. The force of gravity acting on the sled is 120 N [down]. The sled does not move across the rough horizontal surface, and the net force is zero. T/I

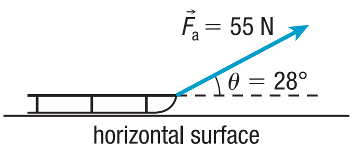


Figure 12

   (a) Draw an FBD of the sled.
   (b) Determine the normal force acting on the sled. Why is the magnitude of the normal force less than the magnitude of the force of gravity? Explain your answer.
   (c) Calculate the force of static friction acting on the sled.

2.2 Newton's Laws of Motion

Newton's laws of motion are three separate statements that explain how and why objects move or stay at rest. When a player hits a puck on an air hockey table, as in **Figure 1**, Newton's laws describe what happens to the puck before, during, and after the collision with the paddle. Newton's laws also describe the forces the objects exert on each other. You can use these three laws to explain the motion of many types of objects experiencing and exerting different types of forces.

Figure 1 Newton's laws of motion describe what happens to a puck on an air hockey table.

Newton's First Law of Motion

If you watch a puck move across an air hockey table, you will notice that it moves at a relatively constant velocity. This is because the puck floats on a cushion of air and moves with very little friction acting on it. In fact, the net force on the puck is virtually zero. In addition, when the puck is at rest and no one hits it, the puck will remain at rest. This example demonstrates **Newton's first law of motion**.

Newton's First Law of Motion

If the external net force on an object is zero, the object will remain at rest or continue to move at a constant velocity.

Some important implications of Newton's first law are the following:

- A net force is not required for an object to maintain a constant velocity.
- A net force is required to change the velocity of an object in magnitude, direction, or both.
- External forces are required to change the motion of an object. Internal forces have no effect on an object's motion.

Inertia and Mass

Galileo introduced the concept of inertia, and Newton used this concept to develop his first law of motion. **Inertia** is the property of matter that causes an object to resist any changes in motion. This means that an object at rest will stay at rest unless a net force acts on it. In addition, if an object is in motion, it will maintain a constant velocity unless a net force acts on it. The concept of inertia is closely related to Newton's first law.

inertia a measure of an object's resistance to change in velocity

Some objects have more inertia than others. In fact, objects that have more mass have more inertia. The degree to which an object resists a change in motion depends on the magnitude of the object's mass. **Mass** is a measure of the amount of matter in an object. The SI unit for mass is the kilogram (kg). Objects that contain a small amount of matter have a smaller mass and less inertia than objects that contain a large amount of matter. For example, a basketball has a mass of approximately 0.62 kg, and a volleyball has a mass of approximately 0.28 kg. If you hit each one with an equal force, the heavier ball (basketball) changes its motion less than the lighter ball (volleyball). Inertia is directly proportional to the mass of the object.

mass a measure of the amount of matter in an object

Newton's Second Law of Motion

Newton's second law of motion explains the relationship between mass, acceleration, and net force.

Newton's Second Law of Motion

If the net external force on an object is not zero, the object will accelerate in the direction of the net force. The magnitude of the acceleration is directly proportional to the magnitude of the net force and inversely proportional to the object's mass.

The acceleration of an object with mass m is then given by

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

Newton's second law is often written in the equivalent form

$$\Sigma\vec{F} = m\vec{a}$$

Recall from Section 2.1 that the net force, or total force, on an object is the sum of all the individual external forces, $\vec{F}_{net} = \vec{F}_{total} = \Sigma\vec{F}$. In most cases, more than one force acts on an object at any given time. To determine the total force, you add all these forces together. That resulting vector sum is the force $\Sigma\vec{F}$ in Newton's second law, as shown in **Figure 2**. Newton's second law, $\Sigma\vec{F} = m\vec{a}$, indicates that the acceleration of an object, $\vec{a}$, is always parallel to the net force, $\Sigma\vec{F}$, acting on the object (Figure 2). However, since acceleration and velocity might be in different directions, velocity and net force need not be in the same direction. For example, you can be moving forward in a car while applying the brakes. You and the car are still moving forward, but you are accelerating backward.

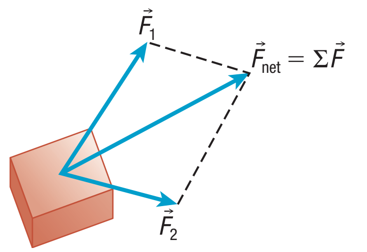


Figure 2 When several forces act on an object, the vector sum of these forces determines the acceleration, according to Newton's second law.

THE NEWTON

You can write the newton in terms of the SI units for mass (kilograms) and acceleration (metres per second squared):

$$\Sigma\vec{F} = m\vec{a}$$
$$\text{N} = \text{kg}\cdot\text{m/s}^2$$

The value of the newton as a unit of force is therefore

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$$

In the following Tutorial, you will use the equation for Newton's second law of motion to predict how several different forces act on an object.

Tutorial 1 Solving Two-Dimensional Problems Using Newton's Second Law

The following Sample Problem models how to calculate the acceleration of an object.

Sample Problem 1: Calculating Acceleration and Direction

Two tugboats are pulling a 4.2×10^3 kg barge into a harbour (**Figure 3**). The first tugboat exerts a constant force of 1.8×10^3 N [E 37° N]. The second tugboat exerts a constant force of 1.3×10^3 N [E 12° S]. Calculate the acceleration (magnitude and direction) of the barge. Assume there is no friction acting on the barge.

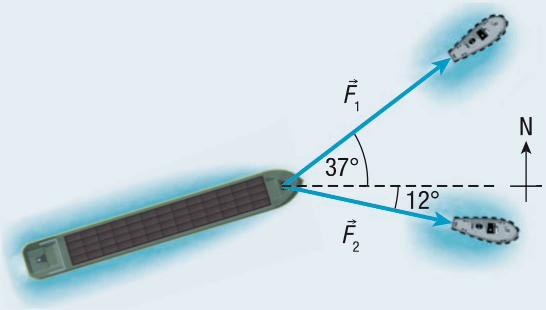


Figure 3

Given: $m = 4.2 \times 10^3$ kg; $F_1 = 1.8 \times 10^3$ N; $F_2 = 1.3 \times 10^3$ N; $\theta_1 =$ [E 37° N]; $\theta_2 =$ [E 12° S]

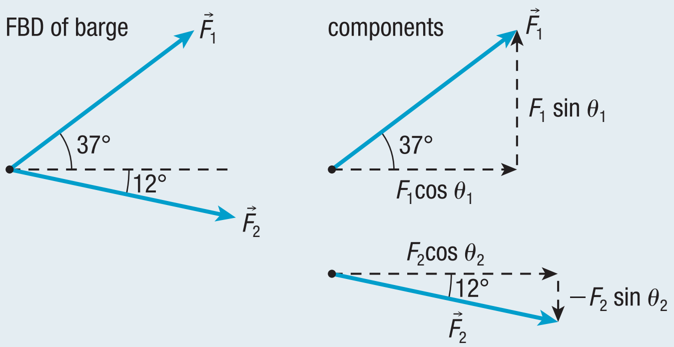
Required: the acceleration of the barge, a; the angle at which the barge moves, θ

Analysis: Use north and east as positive for components. Break the force into its components using $\Sigma F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2$ and $\Sigma F_y = F_1 \sin\theta_1 + (-F_2 \sin\theta_2)$. Draw an FBD.

Use $\theta = \tan^{-1}\left(\dfrac{\Sigma F_y}{\Sigma F_x}\right)$ to determine the direction of the force.

Use $|\Sigma \vec{F}| = \sqrt{(F_x)^2 + (F_y)^2}$ to calculate the magnitude of the force. Use $\Sigma \vec{F} = m\vec{a}$ to calculate the acceleration.

Solution:



$$\begin{aligned}
\Sigma F_x &= F_1 \cos\theta_1 + F_2 \cos\theta_2 \\
&= (1.8 \times 10^3\ \text{N})\cos 37^\circ + (1.3 \times 10^3\ \text{N})\cos 12^\circ \\
\Sigma F_x &= 2.7 \times 10^3\ \text{N}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_y &= F_1 \sin\theta_1 + (-F_2 \sin\theta_2) \\
&= (1.8 \times 10^3\ \text{N})\sin 37^\circ + (-(1.3 \times 10^3\ \text{N})\sin 12^\circ) \\
\Sigma F_y &= 8.13 \times 10^2\ \text{N (one extra digit carried)}
\end{aligned}$$

The direction of the force is

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) \\
&= \tan^{-1}\left(\frac{8.13 \times 10^2\ \text{N}}{2.7 \times 10^3\ \text{N}}\right) \\
\theta &= 17^\circ
\end{aligned}$$

The magnitude is

$$\begin{aligned}
|\Sigma \vec{F}| &= \sqrt{(F_x)^2 + (F_y)^2} \\
&= \sqrt{(2.7 \times 10^3\ \text{N})^2 + (8.13 \times 10^2\ \text{N})^2} \\
&= 2.82 \times 10^3\ \text{N} \\
|\Sigma \vec{F}| &= 2.8 \times 10^3\ \text{N}
\end{aligned}$$

The acceleration of the barge is

$$\begin{aligned}
\Sigma \vec{F} &= m\vec{a} \\
\vec{a} &= \frac{\Sigma \vec{F}}{m} \\
&= \frac{2.82 \times 10^3\ \text{N}\ [\text{E } 17^\circ\ \text{N}]}{4.2 \times 10^3\ \text{kg}} \\
\vec{a} &= 0.67\ \text{m/s}^2\ [\text{E } 17^\circ\ \text{N}]
\end{aligned}$$

Statement: The acceleration of the barge is 0.67 m/s^2 [E 17° N].

Practice

1. For each of the following, determine the acceleration of the mass. Assume no other forces act on the object other than the ones given. T/I
   (a) a mass of 1.2×10^2 kg with a force of 1.5×10^2 N [N] and a force of 2.2×10^2 N [W] acting on it [ans: 2.2 m/s^2 [N 56° W]]
   (b) a mass of 26 kg with a force of 38 N [N 24° E] and a force of 52 N [N 36° E] acting on it [ans: 3.4 m/s^2 [N 31° E]]

2. Two students push horizontally on a large, 65 kg trunk. The trunk moves east with an acceleration of 2.0 m/s^2. One student pushes with a force of 2.2×10^2 N [E 42° S]. The force of friction acting on the trunk is 1.9×10^2 N [W]. Determine the force that the other student applies to the trunk. T/I [ans: 2.1×10^2 N [E 43° N]]
3. Two ropes are used to lift a 1.5×10^2 kg beam with a force of gravity of 1.47×10^3 N [down] acting on it. One rope exerts a force of tension of 1.8×10^3 N [up 30.0° left] on the beam, and the other rope exerts a force of tension of 1.8×10^3 N [up 30.0° right] on the beam. Calculate the acceleration of the beam. T/I [ans: 11 m/s^2 [up]]

Newton's Third Law of Motion

Newton's first two laws of motion deal with a single object and the forces acting on it. **Newton's third law of motion** deals with the forces that two objects exert on each other.

Newton's Third Law of Motion

For every action force, there exists a simultaneous reaction force that is equal in magnitude but opposite in direction.

Newton's third law is also known as the action–reaction principle. For example, if you push east on a wall, the wall exerts a simultaneous force west on you, causing you to move away from the wall. Newton's third law states that action–reaction forces always come in pairs. According to Newton's third law, these two equal and opposite forces must always act on different objects. In Tutorial 2, you will use the equation for Newton's second law of motion to demonstrate Newton's third law of motion.

Tutorial 2 Solving Problems Related to Newton's Third Law

The following Sample Problem shows how to use Newton's second law of motion to calculate the acceleration of two objects in an action–reaction pair.

Sample Problem 1: Calculating Acceleration Due to Newton's Third Law

The person on roller blades in **Figure 4** is pushing on a refrigerator that sits on a level floor. Assume no force of friction exists on either the person or the refrigerator. The person has a mass of 60.0 kg, and the refrigerator has a mass of 1.2×10^2 kg. The force exerted by the person on the refrigerator is 1.8×10^2 N [forward]. Calculate the refrigerator's acceleration and the person's acceleration.

Figure 4

Given: $m_{person} = 60.0$ kg; $m_{refrigerator} = 1.2 \times 10^2$ kg; $\vec{F}_a = 1.8 \times 10^2$ N [forward]

Required: $\vec{a}_{refrigerator}$; $\vec{a}_{person}$

Analysis: The forces acting on the refrigerator are gravity, the normal force, and the force applied by the person on the refrigerator. The floor is level, so gravity and the normal force cancel each other. Use Newton's second law to determine the refrigerator's acceleration, $\vec{a} = \frac{\Sigma\vec{F}}{m}$. Use the same equation to determine the person's acceleration. The action force of the person on the refrigerator is equal to the reaction force of the refrigerator on the person in magnitude but opposite in direction.

Solution:

$$\vec{a}_{refrigerator} = \frac{\Sigma\vec{F}}{m} = \frac{1.8 \times 10^2 \text{ N [forward]}}{1.2 \times 10^2 \text{ kg}}$$

$$\vec{a}_{refrigerator} = 1.5 \text{ m/s}^2 \text{ [forward]}$$

$$\vec{a}_{person} = \frac{\Sigma\vec{F}}{m} = \frac{1.8 \times 10^2 \text{ N [backward]}}{60.0 \text{ kg}}$$

$$\vec{a}_{person} = 3.0 \text{ m/s}^2 \text{ [backward]}$$

Statement: The acceleration of the refrigerator is 1.5 m/s^2 [forward], and the acceleration of the person is 3.0 m/s^2 [backward].

Practice

1. Use Newton's third law to explain the motion of each of the following objects. Identify the action and reaction forces and their directions. K/U T/I A
   (a) a rocket leaving a launch pad
   (b) an airplane flying at a constant velocity
   (c) a runner's foot pushing straight down on the ground
2. A swimmer with a mass of 56 kg pushes horizontally against the pool wall toward the east for 0.75 s with a constant force. Having started from rest, the swimmer glides to a maximum speed of 75 cm/s. Neglecting friction, determine the magnitude of
   (a) the (constant) acceleration [ans: 1.0 m/s^2 [W]]
   (b) the force exerted by the swimmer on the wall [ans: 56 N [E]]
   (c) the force exerted by the wall on the swimmer [ans: 56 N [W]]
   (d) the displacement of the swimmer from the wall after 1.50 s K/U T/I A [ans: 0.84 m [W]]
3. A boy is floating on an air mattress in a swimming pool. The mass of the boy is 32.5 kg, and the mass of the mattress is 2.50 kg. T/I
   (a) Calculate the upward force of the water on the mattress. [ans: 3.4×10^2 N]
   (b) Calculate the force that the boy exerts on the mattress. [ans: 3.2×10^2 N]
   (c) Calculate the upward force of the mattress on the boy. [ans: 3.2×10^2 N]
4. A projectile launcher fires a projectile horizontally from a platform, which rests on a flat, icy, frictionless surface. Just after the projectile is fired and while it is moving through the launcher, the projectile has an acceleration of 25 m/s^2. At the same time, the launcher has an acceleration of 0.25 m/s^2. The mass of the projectile is 0.20 kg. Calculate the mass of the launcher. K/U T/I A [ans: 20 kg]

The Gravitational Force

Earth's force of gravity is something you are quite familiar with from everyday life. However, Newton realized that gravity is an attractive force between all objects, including the motion of the planets and stars. As mentioned in Section 2.1, the gravitational force is weak when the objects are far apart and/or small. However, Earth is so much more massive than any other nearby objects that the strongest gravitational attraction on objects around you is toward Earth.

You know that if you allow an object to fall, it will accelerate downward (in the absence of air resistance). The acceleration due to gravity near Earth's surface varies depending on the distance from Earth's surface. At or near Earth's surface, the acceleration due to gravity is $\vec{g} = 9.8 \text{ m/s}^2$ [down] (to two significant digits) when the object is in free fall. When an object is in free fall, it is moving toward Earth with only the force of gravity acting on it. In other words, we are assuming that air resistance is negligible.

Weight and the Normal Force

According to Newton's second law, when an object is in free fall, $\Sigma\vec{F} = m\vec{a}$, where $\vec{a} = \vec{g}$. Since the only force acting on the object is gravity, then $\Sigma\vec{F} = \vec{F}_g$. Therefore, the force of gravity is given by $\vec{F}_g = m\vec{g}$.

Another name for the gravitational force is **weight**. The weight, $\vec{F}_g$, of an object is a force and is therefore measured in newtons (N). It is often convenient to indicate a force that is directed downward by making the force negative, so when solving problems you will often use $F_g = -mg$. The negative sign here indicates that the force is in the negative y-direction, or down, toward the centre of Earth.

weight the gravitational force exerted by Earth on an object

As you can see in **Figure 5**, in addition to the gravitational force, another force acting on the person is the normal force exerted by the floor on her feet. The person in Figure 5 is at rest with an acceleration of zero. Using Newton's second law for components of the force and acceleration along y, you see that

$$\begin{aligned} \Sigma F &= -mg + F_N \\ ma &= -mg + F_N \\ 0 &= -mg + F_N \\ F_N &= mg \end{aligned}$$

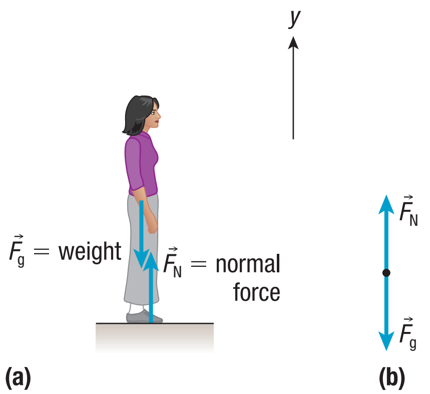


Figure 5 (a) The person standing still has two forces acting on her. (b) The FBD of the person in (a) shows all the forces acting on her.

In this case, the normal force is equal in magnitude and opposite in direction to the person's weight. This raises two common misconceptions in physics, which need to be cleared up at this point.

First, the normal force is not the reaction force to gravity. The reaction force to the force of gravity is another force of gravity. The normal force is the reaction force to the object applying a force to the surface.

Second, since the normal force is not the reaction force to gravity, the normal force is not always equal in magnitude and opposite in direction to the force of gravity. If someone pushed down on the shoulders of the person in Figure 5, the normal force would increase, but the force of gravity on the person remains the same. If the person in Figure 5 were standing on a ramp, the normal force would be perpendicular to the ramp and no longer vertical, but gravity would still point straight down.

UNIT TASK **BOOKMARK**

You can apply what you have learned about Newton's laws of motion to the Unit Task on page 146.

2.2 Review

Summary

- Newton's first law states that when the external net force on an object is zero, the object will remain at rest or continue moving at a constant velocity.
- Inertia causes matter to resist changes in motion.
- Newton's second law states that when the net external force on an object is not zero, the object will accelerate in the direction of the net force. The magnitude of the acceleration is directly proportional to the magnitude of the net force and inversely proportional to the mass: $\vec{a} = \frac{\Sigma\vec{F}}{m}$; also $\Sigma\vec{F} = m\vec{a}$.
- Newton's third law states that for every action force, there exists a simultaneous reaction force that is equal in magnitude and opposite in direction.
- Earth's force of gravity on an object is the object's weight. The force of gravity at Earth's surface is determined using the equation $\vec{F}_g = m\vec{g}$.

Questions

1. A snowboarder is sliding downhill when she suddenly encounters a rough patch. Use Newton's first law of motion to describe and explain what will likely happen to the snowboarder. K/U T/I
2. You are sitting on a bus moving at 50 km/h [E] when you toss a ball in front of you and straight up into the air. The ball reaches a height close to your eyes. Will the ball hit you in the face? Explain. K/U
3. A child is sliding across the ice on a sled with an initial velocity of 4.2 m/s [E]. The combined mass of the child and the sled is 41 kg. There is a constant force of friction between the ice and the sled of 25 N. K/U A
   (a) Calculate the child's acceleration across the ice.
   (b) How long will it take the child to stop?
4. An object moves with an acceleration of magnitude 12 m/s^2 while it is subjected to a force of magnitude 2.2×10^2 N. Determine the mass of the object. K/U T/I A
5. Your friend's car has broken down, so you volunteer to push it with your own car to the nearest repair shop, which is 2.0 km away. You carefully move your car so that the bumpers of the two cars are in contact. You then slowly accelerate to a speed of 2.5 m/s over the course of 1.0 min. The mass of your friend's car is 1.2×10^3 kg. K/U T/I A
   (a) Calculate the normal force between the two bumpers.
   (b) You then maintain the speed of 2.5 m/s. How long does it take you to reach the repair shop?
6. Two forces act on a 250 kg mass, 150 N [E] and 350 N [S 45° W]. Calculate the acceleration of the mass. T/I
7. In each of the examples below, identify an action–reaction pair of forces. K/U T/I A
   (a) A tennis racquet hits a tennis ball, exerting a force on the ball.
   (b) A car is moving at high speed and runs into a tree, exerting a force on the tree.
   (c) Two cars are moving in opposite directions and collide head-on.
   (d) A person leans on a wall, exerting a force on the wall.
   (e) A mass hangs by a string attached to the ceiling, and the string exerts a force on the mass.
   (f) A bird sits on a telephone pole, exerting a force on the pole.
8. Two 5.2 kg masses are suspended as shown in **Figure 6**. K/U T/I

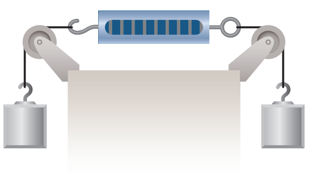


Figure 6

   (a) Determine the tension in each string.
   (b) Determine the reading on the spring scale.
   (c) How would your answers to (a) and (b) change if you replaced one mass with your hand and held everything at rest? Explain your answer.
9. An athlete with a mass of 62 kg jumps and lands on the ground on his feet. The ground exerts a total force of 1.1×10^3 N [backward 55° up] on his feet. Calculate the acceleration of the athlete. T/I

Applying Newton's Laws of Motion

2.3

As you read in Section 2.2, Newton's laws of motion describe how objects move as a result of different forces. In this section, you will apply Newton's laws to objects subjected to various forces in two dimensions, as well as objects that are accelerating.

For example, **Figure 1** shows a skier moving downhill. You can draw an FBD of all the forces acting on the skier. Earth's gravity acts directly downward and has components parallel and perpendicular to the slope. The normal force acts perpendicular to the hill and cancels the component of gravity perpendicular to the hill. Finally, friction acts parallel to the hill, opposing the skier's motion. You can use the sum of these forces and Newton's laws to learn about the motion of the skier.

Figure 1 The forces on this skier are gravity, the normal force, and friction. Compared to the other forces acting on the skier, air resistance is negligible here. These forces can be broken into components parallel and perpendicular to the hillside to analyze the motion of the skier.

Objects in Equilibrium

When the net force on an object is zero, that object is said to be in **equilibrium**. As discussed in Section 2.2, an object with no net force acting on it will not accelerate. So, an object in equilibrium will remain at rest or remain moving at a constant velocity until a force acts on it. Mathematically, an object is in equilibrium when $\Sigma\vec{F} = 0$, or, when you break the forces down into their components, both $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

When solving problems involving objects in equilibrium, you can set the positive x-axis in any direction, but you should draw the FBD first and then pick the most convenient direction. By "convenient" we mean the direction that will give you the fewest components.

The Tutorial on the next page shows you how to solve problems when an object is in equilibrium.

equilibrium a state in which an object has no net force acting on it

Investigation **2.3.1**

Static Equilibrium of Forces (page 95)
In this investigation, you will analyze the conditions for equilibrium using vector components.

Tutorial 1 Solving Problems for Objects That Are in Equilibrium

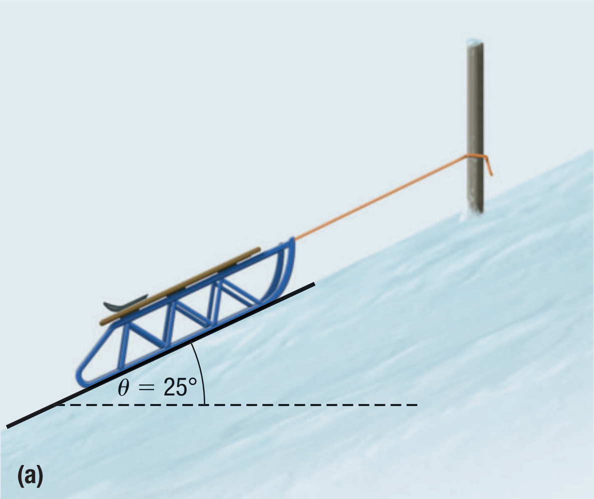
This Tutorial shows how to solve problems for objects in equilibrium when acted on by two-dimensional forces.

Sample Problem 1: Calculating Tension and the Normal Force

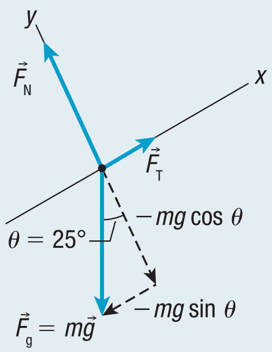
A sled has a mass of 14 kg and is on a hill that is inclined 25° to the horizontal, as shown in **Figure 2(a)**. The hill is very icy (negligible friction), and the sled is held at rest by a rope attached to a post. The rope is parallel to the hill as shown. **Figure 2(b)** shows the FBD.

(a) Calculate the magnitude of the tension in the rope.

(b) Calculate the magnitude of the normal force acting on the sled.



(a)



(b)

Figure 2

Solution

(a) **Given:** $m = 14$ kg; $\theta = 25°$

Required: F_T

Analysis: Tension is parallel to the hillside, and the normal force is perpendicular to the hillside. So let the positive x-axis point up the hillside as shown in Figure 2(b). Determine the components of gravity, and resolve the forces in the x-direction to solve for tension. The sled is in equilibrium, so the net force is zero.

Solution:

$$\begin{aligned}\Sigma F_x &= F_T + (-F_{gx})\\ \Sigma F_x &= F_T + (-mg\sin\theta)\\ 0 &= F_T - mg\sin\theta\\ F_T &= mg\sin\theta\\ &= (14\text{ kg})(9.8\text{ m/s}^2)\sin 25°\\ F_T &= 58\text{ N}\end{aligned}$$

Statement: The magnitude of the tension in the rope is 58 N.

(b) **Given:** $m = 14$ kg; $\theta = 25°$

Required: F_N

Analysis: Resolve the forces in the y-direction to solve for the normal force. The sled is in equilibrium, so the net force is zero.

Solution:

$$\begin{aligned}\Sigma F_y &= F_N + (-F_{gy})\\ \Sigma F_y &= F_N + (-mg\cos\theta)\\ 0 &= F_N - mg\cos\theta\\ F_N &= mg\cos\theta\\ &= (14\text{ kg})(9.8\text{ m/s}^2)\cos 25°\\ F_N &= 1.2 \times 10^2\text{ N}\end{aligned}$$

Statement: The magnitude of the normal force on the sled is 1.2×10^2 N.

Sample Problem 2: Force Applied at an Angle

Your car is stuck in the mud, and you ask a friend to help you pull it free using a rope. You tie one end of the rope to your car and then pull on the other end with a force of 10^3 N. Unfortunately, the car does not move. Your friend then suggests that you make a knot in the middle of the rope, tie the other end of the rope to a tree, and then pull on the knot. Although you are skeptical that your friend's idea will help, you try it anyway. You make a knot in the middle of the rope. You leave one end of the rope attached to the car and tie the other end to a tree at an angle $\theta = 10°$. Then you and your friend pull on the knot in the direction indicated by $\vec{F}_a$ in **Figure 3(a)**. **Figure 3(b)** shows the FBD with the forces acting on the knot at point O. You discover that when a 10^3 N force is applied to the knot in the middle of the rope in the direction shown in Figure 3(a), you are just able to free the car at a slow constant velocity. Why does this work?

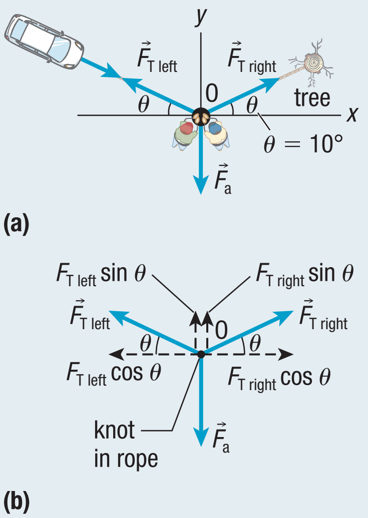


Figure 3

Given: $F_a = 10^3$ N; angle, θ, of the rope to the x-axis is 10°

Required: F_T

Analysis: Calculate the magnitude of the tension in the rope given the 10^3 N force exerted by you and your friend at the point where the car has just started to move at a slow constant velocity.

Since the car is just on the verge of moving, you can apply the conditions for equilibrium to this situation. Consider the forces acting on the rope at point 0 (the point at which you and your friend exert your force) to determine the tension in terms of the applied force.

Apply the conditions for static equilibrium to the rope at point 0. Use the x–y coordinate system to calculate the components of the three forces along x and y, and then apply the condition for equilibrium along y. The rope is continuous and the angles on the two sides are equal, so the tensions in the left and right portions of the rope are the same, $F_{T\ right} = F_{T\ left} = F_T$.

Solution: $\Sigma F_y = +F_{T\ right} \sin\theta + F_{T\ left} \sin\theta - F$

$$0 = +F_T \sin\theta + F_T \sin\theta - F$$
$$F = 2F_T \sin\theta$$
$$F_T = \frac{F}{2\sin\theta} = \frac{10^3\ \text{N}}{2\sin 10°}$$
$$F_T = 3 \times 10^3\ \text{N}$$

Statement: This arrangement multiplies the applied force. The tension in the rope is able to pull the car out because it is 3 times the applied force ($3F_a$).

Practice

1. The static friction on one block is holding another block up, as shown in **Figure 4**. Block A has a weight of 6.5 N, sits on a table, and is connected to a wall by a string. Block B has a weight of 2.8 N, is attached to a string, and is connected to block A's string at point P. The string from block A to point P is horizontal. The magnitude of the force of friction on block A is 1.4 N. K/U T/I C A
   (a) Draw an FBD for block B. Determine the magnitude of the tension in the vertical rope. [ans: 2.8 N]
   (b) Draw an FBD for block A. Determine the magnitude of the tension in the horizontal rope and the magnitude of the normal force acting on block A. [ans: 1.4 N; 6.5 N]
   (c) Draw an FBD of point P. Calculate the tension (the magnitude and the angle θ) in the third rope. [ans: 3.1 N [right 63° up]]

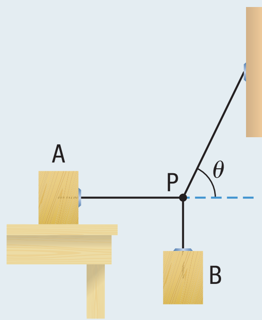


Figure 4

2. A 62 kg rock climber is attached to a rope that is allowing him to hang horizontally with his feet against the wall. The tension in the rope is 7.1×10^2 N, and the rope makes an angle of 32° with the horizontal. Determine the force exerted by the wall on the climber's feet. K/U T/I A [ans: 6.5×10^2 N [left 21° up]]
3. The three forces shown in **Figure 5** act on an object. The object is in equilibrium. Calculate the magnitude of the force F_3 and the angle θ_3. T/I [ans: 78 N; W 9.8° S]

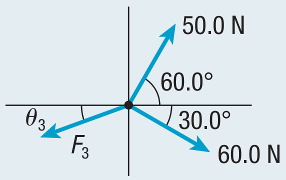


Figure 5

UNIT TASK BOOKMARK

You can apply what you have learned about forces and acceleration to the Unit Task on page 146.

Accelerating Objects

If an object is not in equilibrium, then it is accelerating in some direction. You can use Newton's second law, $\Sigma\vec{F} = m\vec{a}$, to determine the acceleration from the net force on the object, $\Sigma\vec{F}$.

When solving problems that involve accelerating objects, set the positive x-axis in the direction of the net force (acceleration). This will ensure that the net force has no additional y-component, which will simplify the solution. If you do not know the direction of the net force, then just set the positive x-axis in the direction that is the most convenient to solve the problem.

In the following Tutorial, you will use Newton's second law of motion to calculate velocity, acceleration, and tension for objects acted on by two-dimensional forces.

Tutorial 2 Solving Problems for Objects That Are Accelerating

The Sample Problems model how to calculate velocity, acceleration, and tension for objects that are accelerating when acted on by two-dimensional forces.

Sample Problem 1: Velocity Due to Acceleration

A sled is at the top of a hill, which makes an angle of 18° with the horizontal, as shown in **Figure 6(a)**. **Figure 6(b)** shows the FBD for the sled. The height of the hill is 25 m. Calculate the speed of the sled as it reaches the bottom of the hill. Assume that no friction acts on the sled.

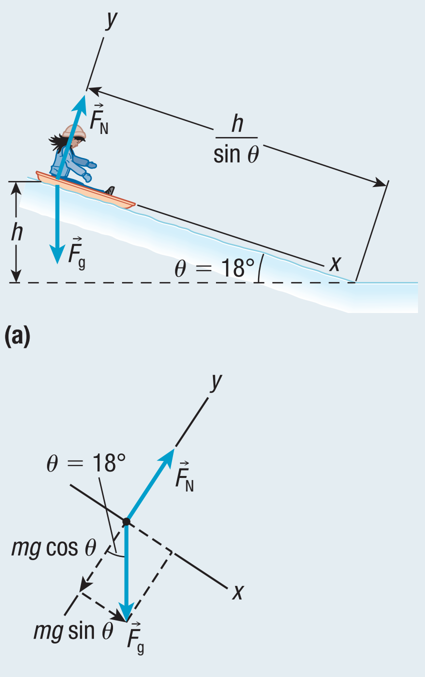


Figure 6

Given: $\Delta d_y = 25$ m; $\theta = 18°$, $v_i = 0$

Required: v_f

Analysis: The sled will accelerate down the hill, so the net force is down the hill according to Newton's second law. Therefore, make the positive x-axis down the hill. This means that the normal force is in the direction of the positive y-axis. Determine the components of the force of gravity in the x- and y-directions as defined by the coordinate axes in Figure 6(b). Use Newton's second law of motion to determine the acceleration along x; then apply $v_{fx}^2 = v_{ix}^2 + 2a_x\Delta d_x$ to calculate the final speed.

Solution:

$$\Sigma F_x = mg\sin\theta$$
$$\cancel{m}a_x = \cancel{m}g\sin\theta$$
$$a_x = g\sin\theta$$
$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta d_x$$
$$\Delta d_x = \frac{h}{\sin\theta}$$
$$v_{fx}^2 = 0^2 + 2(g\cancel{\sin\theta})\left(\frac{h}{\cancel{\sin\theta}}\right)$$
$$v_{fx} = \sqrt{2gh}$$
$$= \sqrt{2(9.8\ \text{m/s}^2)(25\ \text{m})}$$
$$v_{fx} = 22\ \text{m/s}$$

Statement: The speed of the sled at the bottom of the hill is 22 m/s.

Sample Problem 2: Acceleration and Tension

A crate with a mass of 32.5 kg sits on a frictionless surface and is connected to a second crate by a string that passes over a pulley, as shown in **Figure 7(a)**. The second crate has a mass of 40.0 kg. The pulley is frictionless and has no mass. The string also has no mass. FBDs are shown in **Figure 7(b)**. Determine the acceleration of the system of crates and the magnitude of the tension in the string.

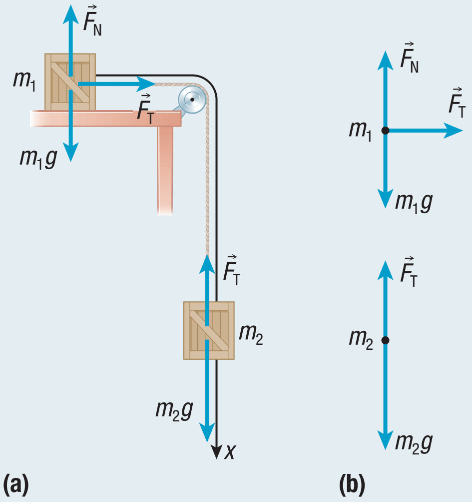


Figure 7

Given: $m_1 = 32.5$ kg; $m_2 = 40.0$ kg

Required: the acceleration of the crates, a; the magnitude of the tension in the string, F_T

Analysis: Apply Newton's laws to determine the acceleration of the crates, considering all the forces acting on them. The FBDs in Figure 7(b) show all these forces. The positive x-direction for each FBD is determined by the direction of the acceleration of each mass: right for mass 1 and down for mass 2. Write Newton's second law for each crate, and solve for the unknown values. To determine the magnitude of the tension, use the FBD for the crate on the surface. The accelerations of both masses are equal because they are tied together and the string does not stretch.

Solution:

$$\Sigma F_x = +F_T \quad \text{(For crate 1)}$$
$$m_1 a = F_T \quad \text{(Equation 1)}$$
$$\Sigma F_x = m_2 g - F_T \quad \text{(For crate 2)}$$
$$m_2 a = m_2 g - F_T \quad \text{(Equation 2)}$$
$$m_1 a + m_2 a = +F_T + m_2 g - F_T \quad \text{(Equation 1 + Equation 2)}$$
$$m_1 a + m_2 a = m_2 g$$
$$a = \frac{m_2 g}{m_1 + m_2}$$
$$= \frac{40.0\ \cancel{\text{kg}}(9.8\ \text{m/s}^2)}{32.5\ \cancel{\text{kg}} + 40.0\ \cancel{\text{kg}}}$$
$$a = 5.41\ \text{m/s}^2 \text{ (one extra digit carried)}$$
$$\Sigma F_x = +F_T$$
$$m_1 a = F_T$$
$$F_T = (32.5\ \text{kg})(5.41\ \text{m/s}^2)$$
$$F_T = 1.8 \times 10^2\ \text{N}$$

Statement: The acceleration of the crates is 5.4 m/s^2, and the magnitude of the tension in the string is 1.8×10^2 N.

Practice

1. Two blocks are fastened onto strings inside an elevator, as shown in **Figure 8**. The mass of the top block is 1.2 kg, and the mass of the bottom block is 1.8 kg. The elevator is accelerating up at 1.2 m/s^2. K/U T/I A

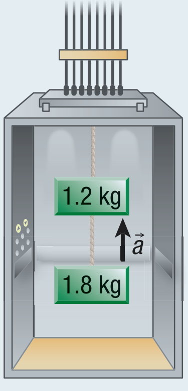


Figure 8

(a) Calculate the tension in each string. [ans: top string: 33 N; bottom string: 20 N]

(b) The maximum tension the strings can withstand is 38 N. Determine the maximum acceleration of the elevator that will not break the strings. [ans: 2.9 m/s^2 [up]]

2. A skier with a mass of 63 kg glides with negligible friction down a hill covered with hard-packed snow. The hill is inclined at an angle of 14° above the horizontal. K/U T/I A
   (a) Determine the magnitude of the normal force on the skier. [ans: 6.0×10^2 N]
   (b) Determine the magnitude of the skier's acceleration. (Hint: Remember to choose the $+x$-direction as the direction of the acceleration, parallel to the hillside.) [ans: 2.4 m/s^2]
3. A child on a toboggan slides down a hill with an acceleration of magnitude 1.9 m/s^2. Friction is negligible. Determine the angle between the hill and the horizontal. K/U T/I A [ans: 11°]
4. You pull a desk across a horizontal floor by exerting a force of 82 N, at an angle of 17° above the horizontal. The normal force exerted by the floor on the desk is 213 N. The acceleration of the desk across the floor is 0.15 m/s^2. K/U T/I A
   (a) Determine the mass of the desk. [ans: 24 kg]
   (b) Determine the magnitude of the friction force on the desk. [ans: 75 N]
5. A store clerk pulls three loaded shopping carts connected with two horizontal cords to help customers load their cars (**Figure 9**). Cart 1 has a mass of 9.1 kg, cart 2 has a mass of 12 kg, and cart 3 has a mass of 8.7 kg. Friction is negligible. A third cord, which pulls on cart 1 and is at an angle of 23° above the horizontal, has a tension of magnitude 29 N. K/U T/I A

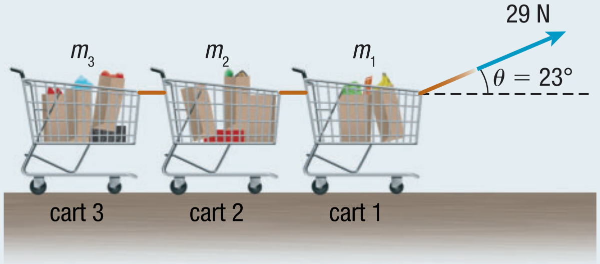


Figure 9

   (a) Determine the magnitude of the acceleration of the carts. [ans: 0.90 m/s^2]
   (b) Determine the magnitude of the tension in the cord between m_3 and m_2. [ans: 7.8 N]
   (c) Determine the magnitude of the tension in the cord between m_2 and m_1. [ans: 19 N]
6. Block A, with a mass of 4.2 kg, is suspended from a vertical string as shown in **Figure 10**. The string passes over a pulley and is attached to block B. The mass of block B is 1.8 kg. The pulley and the surface of the ramp are essentially frictionless. Calculate (a) the acceleration of the blocks and (b) the tension in the string. T/I [ans: (a) 5.3 m/s^2; (b) 19 N]

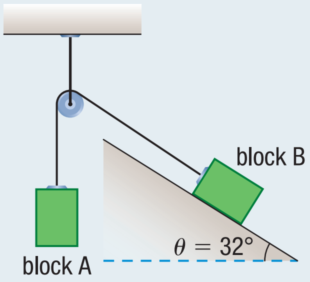


Figure 10

2.3 Review

Summary

- An object is in equilibrium when the net force on it is zero.
- For objects experiencing forces in two dimensions, break the motion into perpendicular components, which can be analyzed independently.
- Once you have determined the net force using components, use Newton's second law to determine the acceleration.

Questions

1. In **Figure 11**, two ropes are pulling on a skater, and they exert forces on her as shown in the figure. Calculate the magnitude and direction of the total force exerted by the ropes on the skater. T/I

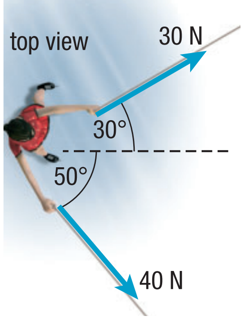


Figure 11

2. Determine the tensions in all three cables in **Figure 12**. T/I

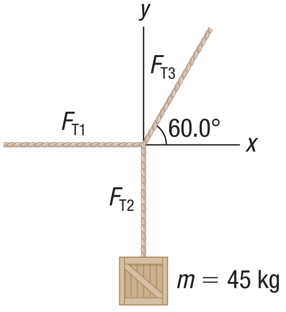


Figure 12

3. A flag of mass 2.5 kg is supported by a single rope as shown in **Figure 13**. A strong horizontal wind exerts a force of 12 N on the flag. Calculate the tension in the rope and the angle, θ, the rope makes with the horizontal. T/I

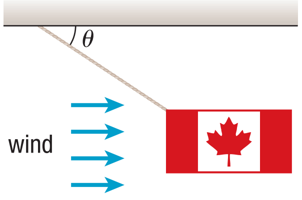


Figure 13

4. A car is parked on a slippery hill (**Figure 14**). The hill is at an angle of 15° to the horizontal. To keep it from sliding down the hill, the owner attaches a cable at the back of the car and to a post. The mass of the car is 1.41×10^3 kg. K/U T/I C

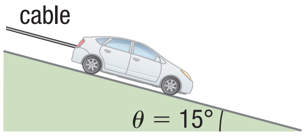


Figure 14

   (a) Draw an FBD showing the forces on the car.
   (b) Write the equations for the conditions for static equilibrium along the horizontal and vertical directions.
   (c) Calculate the tension in the cable. Assume there is no friction between the road and the tires.

5. A student pushes on a lawn mower from rest parallel to the handle of the mower. The student pushes with a force of magnitude 42 N. The handle makes an angle of 35° to the horizontal. The mower accelerates across a level driveway with negligible friction on the mower toward the lawn, 5.0 m away. The mass of the lawn mower is 18 kg. K/U T/I C
   (a) Draw the FBD of the mower.
   (b) Calculate the acceleration of the mower.
   (c) Calculate the normal force acting on the mower.
   (d) Calculate the velocity of the mower when it reaches the lawn.

6. In a physics experiment, a 1.3 kg dynamics cart is placed on a ramp inclined at 25° to the horizontal. The cart is initially at rest but is then pulled up the ramp with a force sensor. The force sensor exerts a force on the cart parallel to the ramp. Negligible friction acts on the cart. T/I
   (a) What force is required to pull the cart up the ramp at a constant velocity?
   (b) What force is required to pull the cart up the ramp at an acceleration of 2.2 m/s²?

2.4 Forces of Friction

Friction may seem like it always makes movement more difficult because it always opposes motion. However, friction is actually essential for much of the motion that we rely on. Have you ever tried getting around on ice? The reason that walking, driving, or riding a bicycle on ice is difficult is the lack of friction.

When you take a step, you push backward on the ground. The static friction of the ground opposes the attempted motion of your foot and exerts a simultaneous and opposite force that propels you forward. A sprinter, such as the one in **Figure 1**, tries to maximize the forward force of the ground. This means wearing shoes that have a large force of static friction with the running surface. It also means pushing on the ground with a force that has a large component parallel to the ground. However, if the sprinter pushes on the ground at an angle that is too shallow, his feet will overcome the static friction and slip on the track.

Figure 1 The sprinter is using the static friction between his shoes, $\vec{F}_s$, and the running surface to accelerate.

Types of Friction: Kinetic and Static

Figure 2(a) shows a hockey puck sliding across the ice. Although this surface is quite slippery, there is still a small amount of friction. Eventually, the force of friction stops the puck. **Figure 2(b)** is an FBD of the hockey puck. The puck moves horizontally to the right. Only one force acts along the horizontal, the force of kinetic friction. The puck's velocity is to the right, so the force of friction opposes this motion to the left. Two forces act in the vertical direction: the weight of the puck (the force of gravity) and the normal force exerted by the icy surface acting on the puck. (You read about these forces in Section 2.2.)

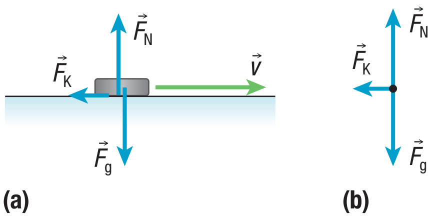


Figure 2 (a) The hockey puck experiences only one force in the horizontal direction, a small force of friction, which slows and eventually stops the puck. (b) The FBD shows all the forces acting on the puck.

Kinetic Friction

The force of gravity and the normal force cancel each other in the y-direction, and the puck's motion is entirely in the x-direction. So why do you think you need to know about the forces along y if they cancel each other? Actually, the normal force is closely connected to the force of friction. For a sliding object, the magnitudes of these two forces are related by $F_K = \mu_K F_N$. The number μ_K is the **coefficient of kinetic friction**, the number that relates the force of kinetic friction between two surfaces in contact with the normal force where they meet. The frictional force occurs when two surfaces are in motion (slipping) with respect to each other. As a coefficient, μ_K is a number without dimensions or units, and its value depends on the surface properties.

coefficient of kinetic friction (μ_K) the ratio of kinetic friction to the normal force

For a hockey puck on an icy surface, μ_K is relatively small, so the frictional force is similarly small. The value of μ_K depends on the smoothness of both the ice and the hockey puck, and might typically be 0.005. The coefficient of kinetic friction for two rough surfaces is larger. For example, the surface of wood is much rougher than the surface of ice, and the surface of wet snow is rougher than that of ice. For wood slipping on wet snow, the coefficient of kinetic friction is 0.10.

Table 1 lists coefficients of kinetic and static friction (discussed below) for some common materials. These values of μ_K show that the frictional force depends on the properties of the surfaces that are in contact. Note that the coefficient of kinetic friction is less than or equal to the coefficient of static friction. Note also the value for synovial joints in humans. Biomedical research into friction in joints is an advancing field. CAREER LINK

Table 1 Typical Values for the Coefficients of Kinetic Friction and Static Friction for Some Common Materials

Surface	μ_K	μ_S	Surface	μ_K	μ_S
rubber on dry concrete	0.6–0.85		steel on ice	0.01	0.1
rubber on wet concrete	0.45–0.75		rubber on ice	0.005	
rubber on dry asphalt	0.5–0.80		wood on dry snow	0.18	0.22
rubber on wet asphalt	0.25–0.75		wood on wet snow	0.10	0.14
steel on dry steel	0.42	0.78	Teflon on Teflon	0.04	0.04
steel on greasy steel	0.029–0.12	0.05–0.11	near-frictionless carbon	0.001	
leather on oak	0.52	0.61	synovial joints in humans	0.003	0.01
ice on ice	0.03	0.1			

Note: The values for μ_S for rubber and concrete are not normally provided because there are no reliable methods to determine them. In addition, the range depends on a variety of conditions.

Investigation 2.4.1

Inclined Plane and Friction (page 96)

The coefficients of friction can be determined experimentally by exerting a horizontal force and using measuring equipment. In this investigation, you will estimate these coefficients using objects on an inclined plane.

Remember, too, that the magnitude of the frictional force depends on the normal force. If you increase the normal force—perhaps by adding an additional mass to the top of the hockey puck—the frictional force increases. However, this relationship is not a "law" of nature. It is simply an approximation that works well in a wide variety of cases. To derive a fundamental law or theory of friction, we need to consider in detail the atomic interactions that occur when two surfaces are in contact. This problem is quite complicated and is currently a topic of much research.

Static Friction

The example of the sliding hockey puck illustrates surfaces that are moving (slipping) against each other. What about when two surfaces are in contact but not slipping? Such cases involve static friction.

Investigation 2.4.2

Motion and Pulleys (page 97)

You will design your own investigation in which you will predict the acceleration of a mass and then measure the acceleration. Then you will evaluate your results by comparing the measured value with the calculated value.

In **Figure 3(a)**, a worker is trying to push a refrigerator, but the refrigerator is not moving. This example demonstrates static friction. Unlike the hockey puck example, here, the person pushing on the refrigerator adds an additional force. The FBD in **Figure 3(b)** shows the additional force of the push working against the force of static friction. Intuitively, you know that when the force exerted by the person is small, the refrigerator will not move. Since the acceleration is then zero, the force of static friction and the force of the push cancel each other:

$$\begin{aligned} ma &= \Sigma F_x \\ &= F_a + (-F_S) \\ ma &= 0 \end{aligned}$$

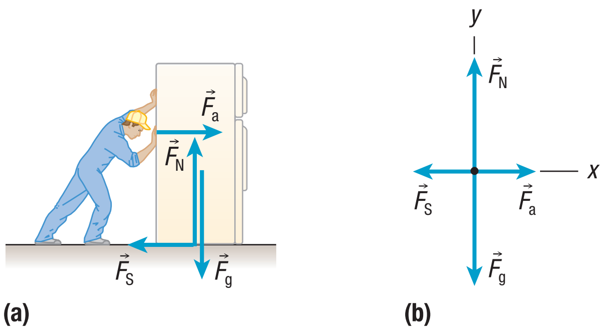


Figure 3 (a) Static friction opposes the attempted motion of the refrigerator. (b) FBD for the refrigerator.

The force of static friction is sufficiently strong that no relative motion (no slipping) occurs. In terms of magnitudes, $|\vec{F}_a| = |\vec{F}_S|$. However, the value of the applied force, F_a, can vary, and the refrigerator will still remain at rest. That is, the worker can push harder, a little, or not at all without the refrigerator moving. In all these cases, the frictional force exactly cancels F_a. The only way this can happen is when the magnitude of the frictional force varies depending on the value of F_a, as described by $|F_a| \leq \mu_S F_N$, where μ_S is the coefficient of static friction. The **coefficient of static friction** is the number that relates the force of static friction between two surfaces to the normal force where they meet.

coefficient of static friction (μ_S) the ratio of the maximum force of static friction to the normal force

The term *static* means that the two surfaces—the floor and the bottom of the refrigerator—are not moving relative to each other. The magnitude of the force of static friction can take any value up to a maximum of $\mu_S F_N$. If F_a in Figure 3 is small, the force of static friction is small and will cancel F_a so that the total horizontal force is zero. If F_a is increased, the force of static friction increases but again cancels F_a. However, the magnitude of the force of static friction has an upper limit of $\mu_S F_N$. If F_a is greater than this upper limit, the worker will succeed in moving the refrigerator.

UNIT TASK **BOOKMARK**

You can apply what you have learned about friction to the Unit Task on page 146.

Mini **Investigation**

Light from Friction

Skills: Observing, Analyzing, Communicating — SKILLS HANDBOOK A2.1

In this investigation, you will observe the production of light from friction. This is called triboluminescence (from the Greek *tribein*, meaning "to rub") and means that light is generated from the friction of materials rubbing together.

Equipment and Materials: eye protection; pliers; wintergreen mints

1. Extinguish the room lights and close the blinds to make the room as dark as possible. Alternatively, enter a dark closet for the investigation. Wait until your eyes adapt to the dark.
2. Put on your eye protection. Place the mints in the pliers and crush them. Observe the result. If nothing happens, try repeating the process.

Use caution when working in the dark. Wear eye protection when crushing the candy. Never eat or taste anything while in the science laboratory.

A. What happened when you crushed the mints? T/I
B. What materials rubbed together to create the friction that produced the light? K/U T/I

In the following Tutorial, you will explore friction problems and calculate acceleration, angle, and mass.

Tutorial 1 Solving Friction Problems

The Sample Problems in this Tutorial model how to use the coefficients of kinetic and static friction to calculate other unknowns, such as acceleration, angle, and mass.

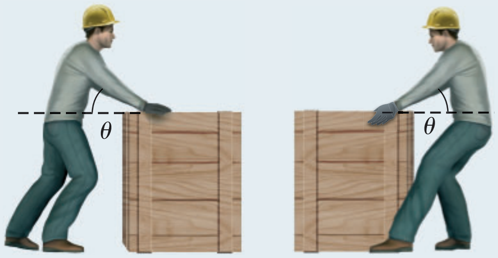
Sample Problem 1: Comparing Pushing and Pulling

A worker must move a crate that can be either pushed or pulled, as shown in **Figure 4**. The worker can exert a force of 3.6×10^2 N. The crate has a mass of 45 kg. The worker can push or pull the crate at an angle of 25°, and the coefficient of kinetic friction between the floor and the crate is 0.36. The worker wants to move the crate as quickly as possible, but he does not know whether it is better to push or pull.

(a) Calculate the acceleration when pushing the crate.

(b) Calculate the acceleration when pulling the crate.

(c) Evaluate your answers to (a) and (b). Does it matter whether the worker pushes or pulls the crate? Explain your answer.



(a) **(b)**

Figure 4

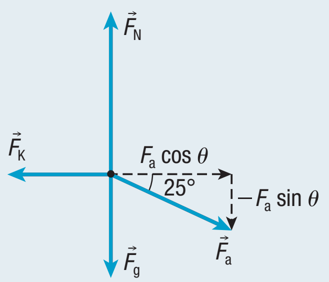
Solution

(a) **Given:** $F = 3.6 \times 10^2$ N; $m = 45$ kg; $\theta = 25°$; $\mu_K = 0.36$

Required: the acceleration when pushing on the crate, a_1

Analysis: Draw an FBD for the push. The force in the y-direction must be zero because the crate is not accelerating upward or downward. Calculate the normal force on the crate as well as the y-components of all the forces on the crate. The worker is pushing, so make the y-component downward. Use the horizontal components to calculate the acceleration.

Solution:



$$\begin{aligned}
\Sigma F_y &= +F_N + (-F_g) + (-F_a \sin\theta)\\
0 &= F_N - F_g - F_a \sin\theta\\
F_N &= F_g + F_a \sin\theta\\
&= mg + F_a \sin\theta\\
&= (45\ \text{kg})(9.8\ \text{m/s}^2) + (3.6 \times 10^2\ \text{N}) \sin 25°\\
F_N &= 593.1\ \text{N}\ \ \text{(two extra digits carried)}\\
\Sigma F_x &= F\cos\theta + (-\mu_K F_N)\\
&= (3.6 \times 10^2\ \text{N})\cos 25° - (0.36)(593.1\ \text{N})\\
\Sigma F_x &= 112.8\ \text{N}\ \ \text{(two extra digits carried)}\\
a_1 &= \frac{\Sigma F_x}{m}\\
&= \frac{112.8\ \text{N}}{45\ \text{kg}}\\
a_1 &= 2.5\ \text{m/s}^2
\end{aligned}$$

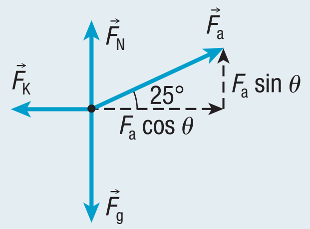
Statement: The acceleration when pushing the crate is 2.5 m/s^2.

(b) **Given:** $F = 3.6 \times 10^2$ N; $m = 45$ kg; $\theta = 25°$; $\mu_K = 0.36$

Required: the acceleration when pulling the crate, a_2

Analysis: In this case, the x-component of the force is the same, but the worker is now pulling so the y-component of the worker's force changes: it is now upward instead of downward. Draw an FBD for the pull.

Solution:



$$\begin{aligned}
\Sigma F_y &= +F_N + (-F_g) + (+F_a \sin\theta)\\
0 &= F_N - F_g + F_a \sin\theta\\
F_N &= F_g - F_a \sin\theta\\
&= mg - F_a \sin\theta\\
&= (45\ \text{kg})(9.8\ \text{m/s}^2) - (3.6 \times 10^2\ \text{N}) \sin 25°\\
F_N &= 288.9\ \text{N}\ \ \text{(two extra digits carried)}\\
\Sigma F_x &= F_a\cos\theta + (-\mu_K F_N)\\
&= (3.6 \times 10^2\ \text{N})\cos 25° - (0.36)(288.9\ \text{N})\\
\Sigma F_x &= 222.3\ \text{N}\ \ \text{(two extra digits carried)}
\end{aligned}$$

$$a_2 = \frac{\Sigma F_x}{m}$$
$$= \frac{222.3\ \text{N}}{45\ \text{kg}}$$
$$a_2 = 4.9\ \text{m/s}^2$$

Statement: The acceleration when pulling the crate is 4.9 m/s^2.

(c) **Statement:** The worker should pull the crate because pulling decreases the normal force on the crate. Consequently, the force of friction is less, which produces a greater acceleration.

Sample Problem 2: Overcoming Static Friction

A crate is placed on an inclined board as shown in **Figure 5**. One end of the board is hinged so that the angle θ is adjustable. The coefficient of static friction between the crate and the board is 0.30. Determine the angle at which the crate just begins to slip.

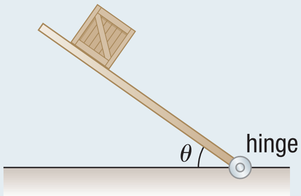


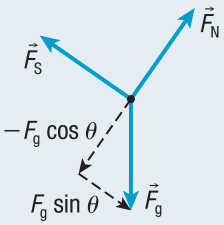
Figure 5

Given: $\mu_S = 0.30$

Required: θ

Analysis: The force of static friction on the crate can be as large as $F_S = \mu_S F_N$, where F_N is the normal force. First draw the FBD of the crate. Then calculate the normal force using the y-components. Then calculate the angle θ using the x-components and the fact that the object is in equilibrium.

Solution:



$$\Sigma F_y = +F_N + (-F_g \cos\theta)$$
$$0 = F_N - F_g \cos\theta$$
$$F_N = F_g \cos\theta$$

$$F_S = \mu_S F_N$$
$$= \mu_S (F_g \cos\theta)$$
$$F_S = \mu_S F_g \cos\theta$$

$$\Sigma F_x = +F_g \sin\theta - F_S$$
$$0 = F_g \sin\theta - F_S$$
$$F_g \sin\theta = F_S$$
$$\cancel{F_g} \sin\theta = \mu_S \cancel{F_g} \cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = \mu_S$$
$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\tan\theta = \mu_S$$
$$\theta = \tan^{-1}\mu_S$$
$$= \tan^{-1}(0.30)$$
$$\theta = 17^\circ$$

Statement: The angle at which the crate just begins to slip is 17°.

Sample Problem 3: Calculating Mass in Friction Problems

Figure 6 shows two blocks joined with a rope that runs over a pulley. The mass of m_2 is 5.0 kg, and the incline is 35°. The coefficient of static friction between m_1 and the inclined plane is 0.25. Determine the largest mass for m_1 such that both blocks remain at rest.

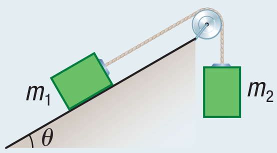


Figure 6

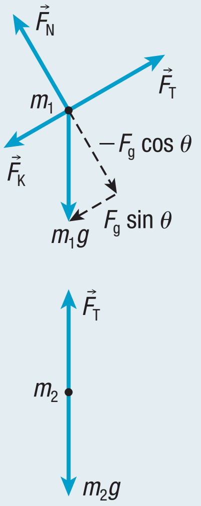
Given: $m_2 = 5.0$ kg; $\theta = 35°$; $\mu_S = 0.25$

Required: m_1

Analysis: Draw an FBD of the situation. As long as the blocks are at rest, the tension in the rope is equal to the force of gravity on m_2, $F_T = F_{g2} = m_2 g$.

Now consider m_1. To remain at rest, the net force must also be zero. First, use the y-components to determine the normal force; then use the x-components to calculate m_2.

Solution:



$$\begin{aligned}
\Sigma F_y &= +F_N + (-F_{g1}\cos\theta) \\
0 &= F_N - F_{g1}\cos\theta \\
F_N &= F_{g1}\cos\theta \\
F_S &= \mu_S F_N \\
&= \mu_S(F_{g1}\cos\theta) \\
&= \mu_S F_{g1}\cos\theta \\
F_S &= \mu_S m_1 g\cos\theta \\
\Sigma F_x &= +F_g\sin\theta + (-F_S) + (-F_T) \\
0 &= m_1\cancel{g}\sin\theta - \mu_S m_1\cancel{g}\cos\theta - m_2\cancel{g} \\
-m_1\sin\theta + \mu_S m_1\cos\theta &= -m_2 \\
m_1(\mu_S\cos\theta - \sin\theta) &= -m_2 \\
m_1 &= \frac{-m_2}{\mu_S\cos\theta - \sin\theta} \\
&= \frac{-(5.0\text{ kg})}{(0.25)\cos 35^\circ - \sin 35^\circ} \\
m_1 &= 14\text{ kg}
\end{aligned}$$

Statement: The largest mass for m_1 for the blocks to remain at rest is 14 kg.

Practice

1. A small textbook is resting on a larger textbook on a horizontal desktop. You apply a horizontal force to the bottom book and both books accelerate together. The top book does not slip on the lower book. K/U T/I C A
   (a) Draw an FBD of the top book during its acceleration.
   (b) What force causes the top book to accelerate horizontally?
2. A stack of dinner plates on a kitchen counter is accelerating horizontally at 2.7 m/s^2. Determine the smallest coefficient of static friction between the dinner plates that will prevent slippage. K/U T/I [ans: 0.28]
3. A rope exerts a force of magnitude 28 N, at an angle 29° above the horizontal, on a box at rest on a horizontal floor. The coefficients of friction between the box and the floor are $\mu_S = 0.45$ and $\mu_K = 0.41$. The box remains at rest. Determine the smallest possible mass of the box. K/U T/I [ans: 6.9 kg]
4. A sled takes off from the top of a hill inclined at 6.0° to the horizontal. The sled's initial speed is 12 m/s. The coefficient of kinetic friction between the sled and the snow is 0.14. Determine how far the sled will slide before coming to rest. K/U T/I A [ans: 2.1×10^2 m]
5. You are pulling a 39 kg box on a level floor by a rope attached to the box. The rope makes an angle of 21° with the horizontal. The coefficient of kinetic friction between the box and the floor is 0.23. Calculate the magnitude of the tension in the rope needed to keep the box moving at a constant velocity. (Hint: The normal force is not equal in magnitude to the force of gravity.) K/U T/I A [ans: 87 N]
6. A 24 kg box is tied to a 14 kg box with a horizontal rope. The coefficient of friction between the boxes and the floor is 0.32. You pull the larger box forward with a force of 1.8×10^2 N at an angle of 25° above the horizontal. Calculate (a) the acceleration of the boxes and (b) the tension in the rope. K/U T/I [ans: (a) 1.8 m/s^2 [forward]; (b) 59 N]
7. The coefficient of kinetic friction between a refrigerator and the floor is 0.20. The mass of the refrigerator is 100.0 kg, and the coefficient of static friction is 0.25. Determine the acceleration when you apply the minimum force needed to get the refrigerator to move. K/U T/I A [ans: 0.49 m/s^2]
8. You are given the job of moving a stage prop with a mass of 110 kg across a horizontal floor. The coefficient of static friction between the stage prop and the floor is 0.25. Calculate the minimum force required to just set the stage prop into motion. T/I [ans: 2.7×10^2 N [horizontal]]

2.4 Review

Summary

- The coefficients of static friction and kinetic friction relate the force of friction between two objects to the normal force acting at the surfaces of the objects. These coefficients have no units and depend on the nature of the surfaces.
- Frictional force increases as the normal force increases.
- The force of static friction, $F_S \leq \mu_S F_N$, opposes the force applied to an object, increasing as the applied force increases, until the maximum static friction is reached. At that instant, the object begins to move and kinetic friction, $F_K = \mu_K F_N$, opposes the motion.

Questions

1. A car is moving with a speed of 20 m/s when the brakes are applied. The wheels lock (stop spinning). After travelling 40 m, the car stops. Determine the coefficient of kinetic friction between the tires and the road. K/U T/I A
2. A hockey puck slides with an initial speed of 50.0 m/s on a large frozen lake. The coefficient of kinetic friction between the puck and the ice is 0.030. Determine the speed of the puck after 10.0 s. K/U T/I A
3. You are trying to slide a sofa across a horizontal floor. The mass of the sofa is 2.0×10^2 kg, and you need to exert a force of 3.5×10^2 N to make it just begin to move. K/U T/I A
   (a) Calculate the coefficient of static friction between the floor and the sofa.
   (b) After it starts moving, the sofa reaches a speed of 2.0 m/s after 5.0 s. Calculate the coefficient of kinetic friction between the sofa and the floor.
4. A crate is placed on an adjustable, inclined board. The coefficient of static friction between the crate and the board is 0.29. T/I
   (a) Calculate the value of θ at which the crate just begins to slip.
   (b) Determine the acceleration of the crate down the incline at this angle when the coefficient of kinetic friction is 0.26.
5. Friction can be helpful in some situations but cause problems in other situations. K/U T/I A
   (a) Describe two situations in which friction is helpful for an object moving on a horizontal surface.
   (b) Describe two situations in which it would be ideal if there were no friction when an object moves across a horizontal surface.
6. Two blocks are connected by a massless string that passes over a frictionless pulley (**Figure 7**). The coefficient of static friction between m_1 and the table is 0.45. The coefficient of kinetic friction is 0.35. Mass m_1 is 45 kg, and m_2 is 12 kg. K/U T/I

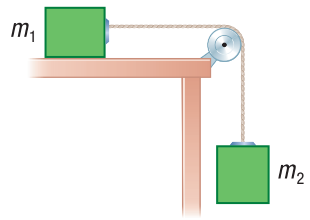


Figure 7

   (a) Is this system in static equilibrium? Explain.
   (b) Determine the tension in the string.
   (c) A mass of 20.0 kg is added to m_2. Calculate the acceleration.
7. A block of rubber is placed on an adjustable inclined plane and released from rest. The angle of the incline is gradually increased. T/I
   (a) The block does not move until the incline makes an angle of 42° to the horizontal. Calculate the coefficient of static friction.
   (b) The block stops accelerating when the incline is at an angle of 35° to the horizontal. Determine the coefficient of kinetic friction.
8. Two masses, connected by a massless string, hang over a pulley that connects two inclines (**Figure 8**). Mass m_1 is 8.0 kg, and mass m_2 is 12 kg. The coefficient of kinetic friction for both inclines is 0.21. Determine the acceleration of the two masses. T/I

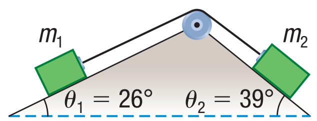


Figure 8

2.5 Explore an Application in Dynamics

Linear Actuators

SKILLS MENU

- Researching
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating
- Identifying Alternatives

A growing trend in dynamics is the field of ergonomics. Ergonomics is the study of the design and efficiency of different working environments, particularly how the working environment affects the health and safety of workers. A **linear actuator** is a device that converts energy into linear motion. Linear actuators can be used in many different working environments to prevent injuries in muscles, joints, and nerves resulting from repetitive motion and strain. They can also be used in rehabilitation centres to help everyone from infants to the elderly. CAREER LINK

linear actuator a device that converts energy into linear motion

Simply put, linear actuators convert energy into motion to turn a gear, which turns a screw, which pushes on a plunger (**Figure 1**). The plunger then applies a linear (constant) force. This force may lower the counter for a cashier, open or close power windows in cars, raise a workstation for an extremely tall worker, lift a patient into a harness and onto a stretcher for transportation, or tighten the screws fastening the dashboard to a vehicle in an automobile assembly line. Linear actuators consistently apply the same force every time. Due to this reliability, more and more industries and businesses are discovering new applications for this innovative technology.

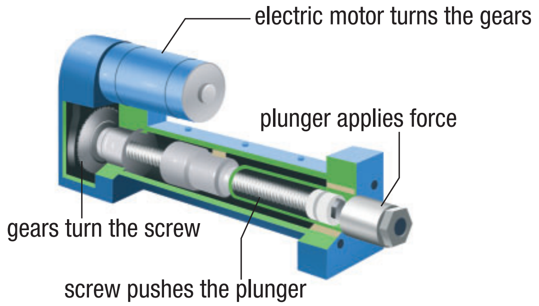


Figure 1 The linear actuator shown here uses an electric motor to turn a gear to push a plunger.

Linear actuators are classified by type; the total distance the plunger can move—called the stroke; the power of the motor; and speed. Different types of actuators use different types of energy, such as mechanical energy, electrical energy, and potential energy stored in compressed liquids or gases.

Actuators are also classified based on their energy source: electromechanical, mechanical, hydraulic (potential energy in compressed liquid, **Figure 2**), and pneumatic (potential energy in compressed gas). Each type has advantages and disadvantages.

Figure 2 Hydraulic actuators are used in lifts like the one shown here. Hydraulic actuators can exert large forces to lift heavy objects.

Mechanical actuators are typically inexpensive and do not require an external power source. That means, however, that they are not automated at all and are only manually operated. Electromechanical actuators are also typically inexpensive and can be automated. However, they have many moving parts that can wear out. Hydraulic and pneumatic actuators are useful for exerting large forces, but they are not as precise and repeatable as mechanical and electromechanical actuators.

The Application

You are considering getting a co-op placement. The co-op placement could be at any number of locations, such as a warehouse, a manufacturing facility, a rehabilitation centre, or an engineering department. However, each placement requires a working knowledge of linear actuators. You want to learn more about linear actuators and how they are used so you can make a good impression in the interview.

Your Goal

To learn how a specific linear actuator works, how it is used to make a task easier and safer for workers or patients, and how it affects society and the environment

Research

SKILLS HANDBOOK A4.1

Research the different types of linear actuators and how they are used. Then pick one type of linear actuator with a specific application. Once you have chosen an application, use the following points to guide your research:

- the application and how it works
- the advantages of the device to the job and to society
- any disadvantages of the device to society and the environment
- any new tasks on the horizon for the device WEB LINK

Summarize

Summarize your research and conclusions. Use the following questions as a guide:

- What type of linear actuator did you choose?
- How does the linear actuator work?
- What task does the linear actuator perform?
- What advantages does the device have in performing the task over other methods, such as manual labour?
- How does the device make the work environment more ergonomic?
- Does the device help reduce workdays lost to strain and injury?
- Are there disadvantages to the application?
- Summarize how the linear actuator compares with other methods of doing a task.
- Assess the impact of the device on society and the environment.

Communicate

Summarize your research in a format that you can review for your co-op interview:

- web page
- blog
- email to a friend
- electronic slide presentation
- written consumer report
- poster
- video
- oral presentation
- other format of your choosing

The Physics of Downhill Skiing

SKILLS HANDBOOK A3

ABSTRACT

Downhill skiing involves forces in a variety of different ways. Skiers race down the mountain as the force of Earth's gravity pulls them toward the bottom of the slope, while air resistance and kinetic friction resist the motion. The skier's stance and equipment help the skier reach the bottom of the slope as quickly as possible by reducing the air resistance on the skier as well as the friction between the skier and the snow. The skier must also maintain control while going down the slope by taking advantage of the friction between the skis and the snow. Finally, the design of the skier's safety equipment must take into account the forces on the skier during a crash.

The Forces Acting on a Skier

What are the forces acting on a downhill skier? Gravity acts to accelerate the skier down the hill, while various frictional forces oppose the skier's motion (**Figure 1**).

Figure 1 Downhill skiing is all about maximizing the forces acting down the slope and minimizing the forces that oppose the skier's motion.

Reducing friction is a significant element of downhill skiing. Wax on the bottom of the skis helps reduce the kinetic friction between the skis and the snow. This directly increases the acceleration of a skier because any reduction in the coefficient of kinetic friction between the skis and the snow will decrease the frictional force accordingly.

Body position is also important for reducing friction in the form of air resistance. The air resistance of an object is proportional to the area of the object. By making herself as small as possible, a skier can reduce the force of air resistance. This is why skiers go into a crouching position, called a tuck, as much as possible.

Having a large mass will not necessarily cause the skier to go faster. The mass of an object does not affect its acceleration due to gravity, but when air resistance becomes important, that can change. The equation for the acceleration of a skier with a mass of m at an angle θ (the slope of the mountain) incorporates the coefficient of kinetic friction, μ_K, and the force of air resistance, F_{air}. To show this, we draw an FBD and we have the positive x-axis pointing downhill, as shown in **Figure 2**.

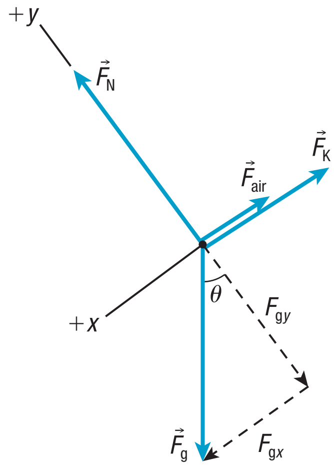


Figure 2

Therefore, the forces of kinetic friction and air resistance both point uphill and are negative. Taking components parallel to the incline gives the following:

$$\Sigma F_x = F_{gx} + (-F_K) + (-F_{air})$$

$$ma = mg\sin\theta - \mu_K mg\cos\theta - F_{air}$$

Divide both sides of the equation by the mass, m:

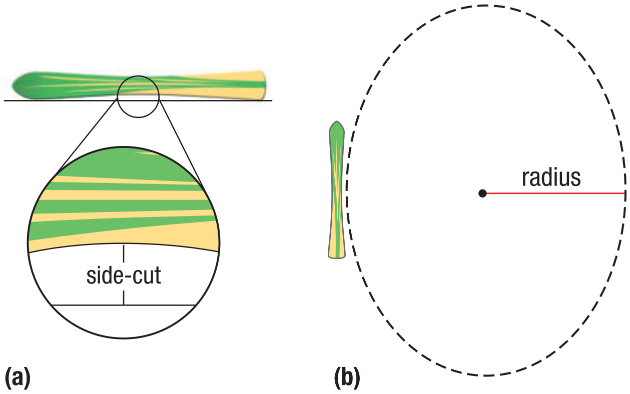
$$a = g\sin\theta - \mu_K g\cos\theta - \frac{F_{air}}{m}$$

Unlike the forces of gravity and kinetic friction, the force of air resistance does not depend on the mass of an object. So according to the above equation, a more massive skier should have a slightly larger acceleration as the skier's speed increases. However, the ability to make sharp turns is also important for a skier, and a heavier skier might have more trouble making such turns.

Another important aspect of downhill skiing is maintaining control going down the slope. This often requires making many sharp turns during the descent. A skier turns by using the friction between the skis and the snow to slow down and to help make turns. When turning, the skier has to angle the skis to dig into the snow, making use of the normal force (**Figure 3**). CAREER LINK

Figure 3 By angling his skis, a skier can change the relative values of the components of friction in different directions, causing him to turn.

The radius of curvature of these turns is important for overall speed. The tighter the curve, the shorter the overall distance the skier has to go and the faster he reaches the bottom. Skis that are shorter and side-cut can significantly reduce the radius of curvature (**Figure 4**).



(a) (b)

Figure 4 (a) The side-cut is the amount of curving at the sides of a ski. (b) The side-cut radius is an imaginary oval that you could draw if you followed the side-cut in the ski. The side-cut radius affects the skier's turning radius.

Leg strength is also important for making sharp, controlled turns that increase overall speed down the slope. Skiers train the muscles that allow them to make the purest possible carves (turns).

Skiers can also use poles to give a boost of extra force when they start from rest. When a skier pushes on the slope with the poles, the slope exerts a force on the skier according to Newton's third law. This force will have components both parallel and perpendicular to the slope (surface of the snow). The parallel force will directly increase acceleration down the slope. The perpendicular force will reduce the normal force of the slope on the skier and thus reduce the kinetic friction.

Finally, safety is a major issue in downhill skiing. During a crash, a skier's speed changes from a high speed to zero almost instantly. Newton's second law explains that this large change causes a large force to act on the skier. Safety equipment is intended to reduce the effect of the sudden slowing felt by the skier and, thus, the force on the skier. Reducing the force on the skier's head is particularly important. A helmet provides a cushion that allows the skier's head to take more time to slow down from full speed to zero during a crash.

UNIT TASK **BOOKMARK**

You can apply what you have learned about the physics of skiing to the Unit Task on page 146.

Further Reading

Lind, D., & Sanders, S.P. (2004). *The physics of skiing* (2nd ed.). New York: Springer.

The physics of skiing. (2011). *Real-world Physics Problems*.

Swinson, D.B. (1992). Physics and skiing. *Physics Teacher*, *30* (8), 458–63.

Weinstock, M. (2004, February). The physics of … skiing. *Discover Magazine*.

WEB LINK

2.6 Questions

1. List four forces that act on a downhill skier. K/U
2. How does equipment used by downhill skiers reduce friction and resistance? K/U
3. Does a large mass necessarily cause a skier to go faster? Explain your answer. K/U
4. Why is a helmet important for a downhill skier? K/U
5. Research improvements in skiing equipment, technology, and ski suits (clothing). Use search terms such as side-cut (or parabolic) skis and anti-drag suits. K/U T/I A
   (a) What is the relationship between improvements in skiing technology and safety?
   (b) Is there any evidence that supports the use of the technology?
   (c) Why do skiers wear special clothing?

WEB LINK

Investigation 2.3.1 OBSERVATIONAL STUDY

Static Equilibrium of Forces

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

In Section 2.3, you learned about using the components of force vectors to analyze the conditions required for static equilibrium. In this investigation, you will set up and evaluate the conditions for static equilibrium. This will require using the components of force vectors in two dimensions.

Purpose

SKILLS HANDBOOK A2.4

To analyze how forces of friction and other forces affect a system in equilibrium

Equipment and Materials

- eye protection
- 3 small pulleys
- padding, such as a towel or blanket
- circular protractor
- vertical force board (or a support structure)
- 3 hangers with masses (100 g and two 200 g)
- string

Procedure

1. Place padding at the base of the force board in case a string breaks and a mass falls.
2. Put on your eye protection. Hang the three different masses (m_1, m_2, and m_3) from strings tied together at a common point, with two of the strings hanging over pulleys as shown in **Figure 1(a)**. Make sure the masses are all at rest. Line up the origin of the protractor with the common point of the strings.

Do not use masses that are larger than 500 g. Do not wear open-toed shoes. Take care not to allow the masses to fall on your hands or feet.

3. Measure the angles a, b, c, and d (see Figure 1(a)).
4. Draw a system diagram for the setup. Label all angles and masses. Draw an FBD for each mass and for the common point of the strings.
5. Using the values of the masses and the angles of the strings, calculate the vertical components of the tensions in the strings attached to m_1 and m_3. Use up as positive. Calculate the vertical force produced by m_2. Compare the sum of the vertical components of the tensions in the strings attached to m_1 and m_3 to the vertical force produced by m_2 by calculating the percent difference.
6. Using the values of the masses and the angles of the strings, calculate the horizontal components of the tensions in the strings attached to m_1 and m_3 and compare them by calculating the percent difference. Use right as positive.
7. Change the angles of the strings connecting m_1 and m_3, while keeping the string supporting m_2 vertical, and repeat Steps 3 to 6.
8. Use the third pulley to offset the string holding m_2 so it is no longer vertical (see **Figure 1(b)**). Determine the vertical components of the tensions in the three strings and compare them. Determine the horizontal components of the tensions in the three strings and compare them.
9. Repeat Steps 7 and 8 for different force values and angles.

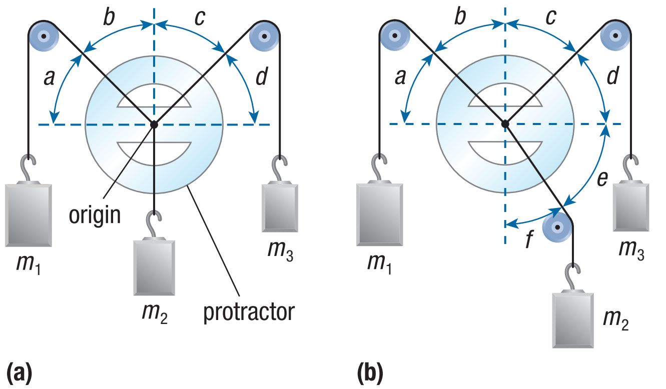


Figure 1 (a) Original setup of masses, as described in Step 1. (b) Add the third pulley, as described in Step 8.

Analyze and Evaluate

(a) What is the condition for static equilibrium? K/U

(b) Describe how friction between the strings and the pulleys affects the results of this investigation. T/I A

(c) How could you improve the accuracy of your measurements in this investigation? K/U T/I A

Apply and Extend

(d) If you have access to a force sensor, replace m_2 in Figure 1(a) with the sensor. Start with the strings attached to m_1 and m_3 as horizontal as possible, and then gradually pull down with the force sensor until the angles b and c decrease significantly. T/I A

(i) What happens to the reading on the force sensor?

(ii) What implications does this trend have?

(e) Imagine pulling down slightly on m_2 so that the point where the strings tie together moves, and then letting go. Predict what would happen. If the equipment is still available, try it. Describe the motion of the masses after you let go. Assume that you do not pull down far enough to pull any of the masses over their pulleys, and that the pulleys and strings are frictionless. Remember that after you let go of the mass, the net force goes back to zero. Was your prediction correct? T/I A

Investigation 2.4.1 CONTROLLED EXPERIMENT

Inclined Plane and Friction

In Section 2.4, you solved problems using the formulas for determining the coefficients of friction. In this investigation, you will approximate a reliable estimate of these coefficients using objects on an inclined plane.

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Testable Questions

SKILLS HANDBOOK A2.2

(a) How do the coefficients of static and kinetic friction compare for an object on a ramp?

(b) How do the coefficients of friction for different objects compare to each other when placed on the same ramp?

Hypothesis

After reading through the Experimental Design and Procedure, formulate hypotheses for the Testable Questions. Explain the reasoning for your hypotheses.

Variables

Identify the controlled, dependent, and independent variables in your investigation.

Experimental Design

You will test the effects of different objects (such as the sole of a running shoe or a friction block with rubber backing) on the coefficients of static friction and kinetic friction using an inclined plane. You will need to raise the inclined plane until each object begins to slide to determine the coefficient of static friction. You will need to lower the inclined plane until the object moves down the plane at a constant speed to determine the coefficient of kinetic friction. How much do the coefficients vary? Are they ever the same? These and similar questions should form the basis of your investigation.

Equipment and Materials

- metre stick
- inclined plane
- test objects (for example, running shoe, textbook, plastic block, piece of wood)
- protractor (optional)

Procedure

1. Place the first object on the inclined plane. Determine the angle at which the object just starts to slide down the incline. Explain how the mass of the object relates to the coefficient of static friction.

Do not wear open-toed shoes. Take care not to allow the test objects to fall on your hands or feet.

2. Use the angle from Step 1 to determine the coefficient of static friction.
3. Determine the angle that will allow the object to slide down the incline at a constant speed. You can do this by lowering the angle slightly once the object has started to move.
4. Use the angle from Step 3 to calculate the coefficient of kinetic friction.
5. Repeat Steps 1 to 4 for different test objects.

Analyze and Evaluate

(a) What variables were measured, recorded, and manipulated in this investigation? What type of relationship was being tested? K/U T/I

(b) Why are the coefficients of friction different for the different objects that you measured? K/U T/I

(c) Compare the coefficients of static and kinetic friction for shoe soles, wood, and plastic. Which object has the highest coefficient of friction? K/U T/I

(d) What actions could you take to improve the accuracy of your measurements in this activity? K/U T/I

Apply and Extend

(e) Describe an experimental procedure that shows that the coefficients of friction for two materials are independent of the mass of an object. T/I A

(f) Describe another experimental procedure that can determine the coefficient of kinetic friction using an inclined plane. What new equipment, if any, would this method require? T/I A

Investigation 2.4.2 OBSERVATIONAL STUDY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Motion and Pulleys

In Investigation 2.4.1, you determined the coefficients of friction of objects on an inclined plane. In this investigation, you will use the inclined plane, an object, and its coefficient of friction from Investigation 2.4.1. You will then calculate the acceleration of the object on the inclined plane. You will design your own investigation in which you will measure the acceleration of a mass and compare it to your calculated value.

Purpose

To design your own investigation to measure the acceleration of a mass and compare the measured value to the actual calculated value of the acceleration

Equipment and Materials

- eye protection
- ticker tape timer, motion sensor, or video camera
- metre stick
- protractor
- stopwatch
- masses (100 g, 200 g, 500 g)
- pulley
- string
- one object from Investigation 2.4.1
- inclined plane from Investigation 2.4.1

Procedure

1. Design an investigation to calculate and then measure the acceleration of a moving mass. Include all applicable safety precautions. Decide what you are going to measure and how you can determine the acceleration from the information that you have. Suggestion: Suspend one mass from a string that goes over a pulley, which is then tied to the object from Investigation 2.4.1 on the inclined plane.

Ensure the pulley is attached to a secure support. Do not wear open-toed shoes. Take care not to allow the test objects to fall on your hands or feet.

2. Before you conduct your investigation, draw an FBD of your setup.
3. Calculate the result that you expect for the acceleration using the coefficient of friction determined in Investigation 2.4.1.
4. Have your design approved by your teacher.
5. Conduct your investigation.
6. Repeat the investigation using a different mass.

Analyze and Evaluate

(a) Compare your measured values for acceleration with your predicted values. Account for any differences between the two sets of values. K/U T/I A

(b) Calculate the percent error. T/I

(c) Determine how you can improve the accuracy of your measurement. K/U T/I

Apply and Extend

(d) List some other values that you could measure using a similar setup. K/U T/I A

(e) Careful design of an experiment can help lead to accurate and precise results. Describe some common mistakes that students and researchers make when designing experiments. Explain how these mistakes affect the experimental results. T/I A

Summary Questions

1. One of the Key Concepts at the beginning of the chapter was to analyze a technological device that applies the principles of motion and forces and assess the social and environmental impact. Provide some relevant examples of such devices along with explanatory diagrams and equations.
2. Look back at the Starting Points questions on page 60 about the race car burning rubber. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers you wrote at the beginning of the chapter. Note how your answers have changed.

Vocabulary

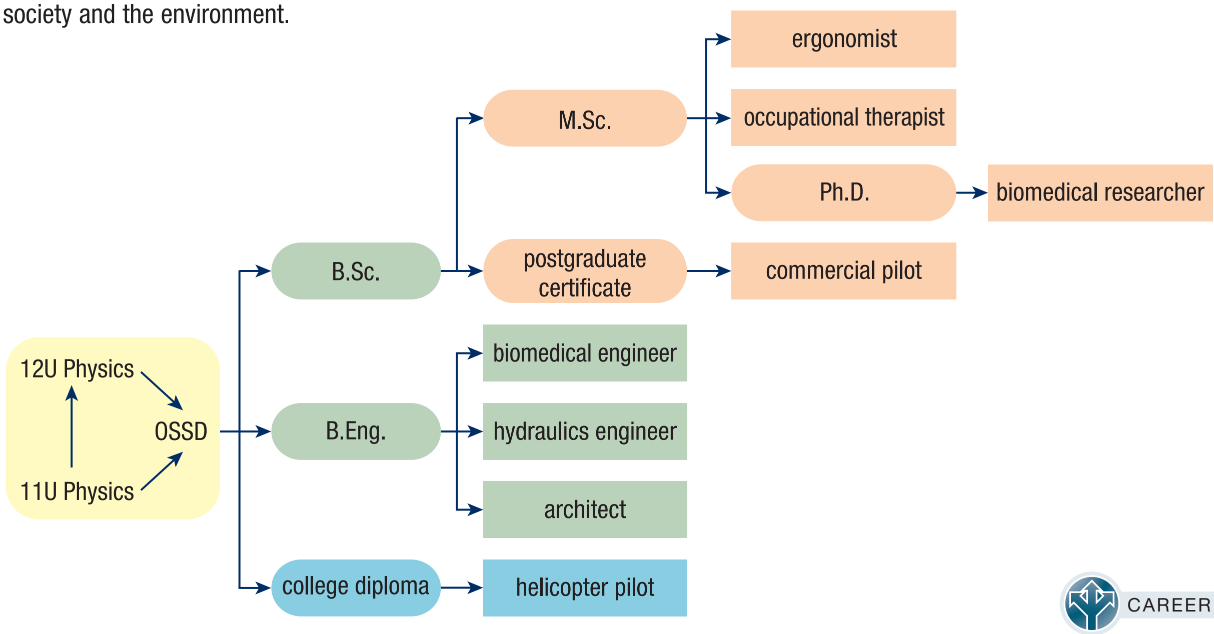
force (p. 62)
newton (p. 62)
contact force (p. 62)
non-contact force (p. 62)
force of gravity (p. 62)
normal force (p. 62)
tension (p. 62)
friction (p. 63)
static friction (p. 63)
kinetic friction (p. 63)
air resistance (p. 63)
applied force (p. 63)
free-body diagram (p. 63)
net force (p. 65)
Newton's first law of motion (p. 70)
inertia (p. 70)
mass (p. 71)
Newton's second law of motion (p. 71)
Newton's third law of motion (p. 73)
weight (p. 75)
equilibrium (p. 77)
coefficient of kinetic friction (p. 85)
coefficient of static friction (p. 86)
linear actuator (p. 91)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers mentioned in this chapter.

1. Select two careers related to Dynamics that you find interesting. Research the educational pathways that you would need to follow to pursue these careers. What is involved in the required educational programs? Prepare a brief report of your findings.
2. For one of the two careers that you chose above, describe the career, main duties and responsibilities, and working conditions. Also, outline how the career benefits society and the environment.



CAREER LINK

K/U Knowledge/Understanding   T/I Thinking/Investigation   C Communication   A Application

For each question, select the best answer from the four alternatives.

1. A pitcher throws a fastball. After the ball has left the pitcher's hand and is moving through the air, which forces are acting on the baseball? (2.1) K/U
   (a) the force from the throw and the downward force of gravity
   (b) the force from the throw, a force exerted by the air, and the downward force of gravity
   (c) a force exerted by the air and a force from the throw
   (d) a force exerted by the air and the downward force of gravity
2. A person with a mass of 62 kg is in an elevator moving at a constant velocity of 2.3 m/s [up]. What is the magnitude of the net force acting on the person? (2.1) K/U
   (a) 0 N
   (b) 610 N
   (c) 620 N
   (d) 6100 N
3. When the net force acting on an object is doubled, the effect on the acceleration of the object will be
   (a) tripled
   (b) doubled
   (c) quartered
   (d) constant (2.2) K/U
4. A force of 9.0 N exerted by a rope pulls a block with a mass of 4.5 kg. The block is resting on a smooth surface. What is the force of reaction exerted by the block on the rope? (2.2) T/I
   (a) 4.5 N
   (b) 9.0 N
   (c) 41 N
   (d) 44 N
5. An object is suspended from a spring balance in an elevator. The reading is 240 N when the elevator is at rest. The spring balance reading changes to 220 N. Which of the following describes how the elevator is moving? (2.3) K/U
   (a) downward with constant speed
   (b) downward with decreasing speed
   (c) downward with increasing speed
   (d) upward with increasing speed
6. A child with mass m is sliding down a slide that is inclined at an angle of θ above the horizontal. The magnitude of the normal force on the child is
   (a) $mg \tan \theta$
   (b) $mg \sin \theta$
   (c) $mg \cos \theta$
   (d) mg (2.3) T/I A
7. A horizontal force of 95.0 N is applied to a 60.0 kg crate on a rough, level surface. The crate accelerates at 1.20 m/s^2. What is the magnitude of the force of kinetic friction acting on the crate? (2.4) T/I
   (a) 16.0 N
   (b) 23.0 N
   (c) 33.0 N
   (d) 45.0 N
8. When the normal force is doubled, the coefficient of friction is
   (a) halved
   (b) doubled
   (c) quadrupled
   (d) unchanged (2.4) K/U

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

9. When you jump up in the air, the net force on you is equal to the force of air resistance. (2.1) K/U
10. A golf ball on the Moon has the same inertia as it has on Earth. (2.2) K/U
11. According to Newton's third law of motion, the forces of action and reaction always act on the same body and balance each other. (2.2) K/U
12. In a tug-of-war between two athletes, each pulls on the rope with a force of 300 N. The tension in the rope is 600 N. (2.3) K/U
13. A block with a mass of 0.10 kg is held against a wall by applying a horizontal force of 5.0 N on the block. The magnitude of the frictional force acting on the block is 0.98 N. (2.4) T/I
14. On a rainy day, it can be dangerous to drive a car at high speed, because the rain on the road surface increases the coefficients of friction. (2.4) K/U T/I
15. Downhill skiers go into a crouching position to increase air resistance and decrease speed. (2.6) K/U

Go to Nelson Science for an online self-quiz.
WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. When a body is at rest, the net force acting on it is
   (a) maximum
   (b) zero
   (c) minimum
   (d) constant (2.1) K/U
2. The inertia of an object is directly proportional to its
   (a) mass
   (b) velocity
   (c) acceleration
   (d) speed (2.2) K/U
3. When a constant net force acts on an object, which quantity remains constant? (2.2) K/U
   (a) velocity
   (b) displacement
   (c) acceleration
   (d) momentum
4. Action and reaction forces
   (a) act on two different objects
   (b) have the same direction
   (c) have unequal magnitude
   (d) always cancel each other (2.2) K/U
5. A 5.0 kg object undergoes an acceleration of 2.0 m/s^2. What is the magnitude of the resultant force acting on the object? (2.2) T/I
   (a) 0 N
   (b) 2.0 N
   (c) 5.0 N
   (d) 1.0×10^1 N
6. The tension in a cable supporting a beam at a construction site is less than the weight of the beam. The beam may be
   (a) going up or down with non-uniform velocity
   (b) going up with increasing velocity
   (c) going down with decreasing velocity
   (d) going down with increasing acceleration (2.3) K/U
7. A 2.0 kg mass sits at rest on a plane inclined at 30.0° from the horizontal. The coefficient of static friction is 0.70. What is the frictional force on the mass? (2.4) T/I
   (a) 5.9 N
   (b) 6.9 N
   (c) 8.5 N
   (d) 9.8 N
8. A bicyclist brakes suddenly, and the wheels skid across the ground. The force of friction between the wheels and the ground acts
   (a) backward on the front wheels and forward on the rear wheels
   (b) forward on the front wheels and backward on the rear wheels
   (c) backward on both wheels
   (d) forward on both wheels (2.4) K/U
9. A snowboarder is sliding down a frictionless hill inclined at an angle of θ to the horizontal. What is the acceleration of the skier? (2.4) K/U
   (a) $g \tan\theta$
   (b) $g \sin\theta$
   (c) $g \cos\theta$
   (d) g

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

10. If only three forces of equal magnitude act on an object, the object must have a non-zero net force. (2.1) K/U
11. When a non-zero net force acts on an object, the object will speed up in the direction of the net force. (2.2) K/U
12. When you pull up on an object at rest on a horizontal surface, you decrease the normal force on the object but not the weight of the object. (2.2) K/U
13. When a skater bumps into the boards in an arena, first the skater exerts a force on the boards, and then the boards exert an equal and opposite reaction force on the skater. (2.2) K/U
14. In an elevator with an acceleration of $0.20g$ up, the force exerted on the floor by a passenger of mass m is $1.2mg$. (2.3) K/U
15. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of the person's motion. (2.3) K/U
16. When you are sliding down a hill on a snowboard, the normal force on you is larger in magnitude than the force of gravity. (2.3) K/U
17. When two objects slip over each other, the force of friction between them is called static friction. (2.4) K/U
18. The magnitude of the frictional force depends on the nature of the two surfaces in contact. (2.4) K/U
19. One advantage of a linear actuator over manual labour is that a linear actuator can apply the same force each time. (2.5) K/U

Understanding

20. A 75 kg man stands in an elevator. Calculate the force that the floor exerts on him when the elevator starts moving upward with an acceleration of $2.0\ m/s^2$. (2.1) K/U T/I

21. The airline pilot in **Figure 1** is pulling a suitcase at a constant speed with force $\vec{F}$ applied to the handle at an angle of θ above the horizontal. A small force of friction resists the motion. (2.1) K/U T/I C A

Figure 1

(a) Draw an FBD of the suitcase.
(b) Determine the components of the net force. Choose the direction of motion to be the $+x$-direction.
(c) Determine the components of the forces. Choose the $+x$-direction as the direction in which the handle is pointing.
(d) Which choice of $+x$ is more convenient? Explain your answer. (Hint: Did you show the components of all the forces in your diagrams?)

22. Express the magnitude and direction of the net force acting on the following. (2.1) T/I A
(a) a drop of rain falling with a constant speed
(b) a cork with a mass of 10 g floating on still water
(c) a stone with a mass of 0.1 kg just after it is dropped from the window of a stationary train (Neglect air resistance.)
(d) the same stone at rest on the floor of a train, which is accelerating at $1.0\ m/s^2$

23. Draw an FBD for each of the following objects. (2.1) K/U T/I C A
(a) a saucepan hanging from a hook
(b) a person standing at rest on the floor
(c) a puck sliding in a straight line on the ice to the right
(d) a toboggan pulled by a rope at an angle above the horizontal to the right with significant friction on the toboggan

24. An astronaut is separated from his small spacecraft accelerating in interstellar space at a constant rate of $100\ m/s^2$. Determine the acceleration of the astronaut the instant he is outside the spaceship. (Assume that there are no nearby stars to exert a gravitational force on him.) (2.2) T/I A

25. Explain why action and reaction forces cannot cancel each other, even though they are equal and opposite. (2.2) K/U C

26. A constant horizontal force of magnitude 20.0 N is applied to block A with a mass of 4.0 kg, which pushes against block B with a mass of 6.0 kg. The blocks slide eastward over a frictionless surface. Calculate the acceleration of the blocks. (2.2) T/I

27. A student holding a spring balance in his hand suspends from it an object with a mass of 1.0 kg. The balance slips from his hands and falls down. What is the reading of the balance while it is in the air? (2.3) K/U T/I A

28. Is it always necessary for the coefficient of friction to be less than one? Explain your answer. (2.4) K/U C A

29. Provide one advantage and one disadvantage each of static and kinetic friction in situations involving inclined planes. Explain your reasoning. (2.4) K/U C

30. How are linear actuators used to make the workplace more ergonomic, reducing workdays lost to strain and injury? (2.5) K/U

Analysis and Application

31. Describe how a trebuchet applies the principles of linear motion. (2.1) K/U T/I

32. A rope with a mass of 0.53 kg is pulling a block with a mass of 8.6 kg with a force of 31.5 N. Calculate the force of reaction exerted by the block on the rope when the block is resting on a smooth horizontal surface. (2.1) T/I A

33. A helicopter with a mass of 1.5×10^3 kg rises with a vertical acceleration of $12\ m/s^2$. The crew and the passengers have a total mass of 4.2×10^2 kg. Express the magnitude and direction of the force on the floor by the crew and the passengers. (2.1) T/I A

34. Two forces are acting on a 23 kg object. One force has a magnitude of 45 N and is directed east. The other force has a magnitude of 29 N and is directed north. Determine the object's acceleration. (2.1) T/I A

35. Three forces act on an object: a 47 N force at 31° north of east, a 58 N force at 46° north of east, and a force of magnitude F at an angle of θ south of west. The object is in equilibrium. Calculate F and θ. (2.1) T/I A

36. Two blocks are connected by a string passing over a frictionless pulley, as shown in **Figure 2**. When the blocks are in motion, block A experiences a force of kinetic friction of magnitude 5.4 N. The mass of m_A is 2.3 kg, and the mass of m_B is 3.5 kg. (2.1, 2.2) T/I A

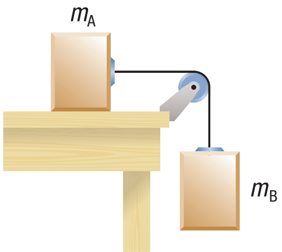


Figure 2

(a) Calculate the magnitude of the acceleration of the blocks.
(b) Calculate the magnitude of the tension in the string.

37. A man is floating on an air mattress in a swimming pool. (2.1) K/U T/I C A
(a) Draw two FBDs: one for the man and one for the mattress.
(b) Identify the reaction forces for all of the forces in your FBDs in (a).
(c) The mass of the man is 1.1×10^2 kg, and the mass of the mattress is 7.0 kg. Determine the normal force of the water on the mattress.
(d) Determine the normal force of the mattress on the man.

38. Describe how you could use this textbook, a piece of paper, and a desk to demonstrate Newton's first law of motion. (2.2) K/U T/I C A

39. The engines of an airplane exert a force of 1.2×10^2 kN [E] during takeoff. The mass of the airplane is 42 t (1 t = 10^3 kg). (2.2) T/I A
(a) Calculate the acceleration produced by the engines.
(b) Calculate the minimum length of runway needed if the speed required for takeoff on this runway is 71 m/s.

40. Two masses, 1.3 kg and 2.4 kg, are tied together with a string. The string has negligible mass and does not stretch. The masses are pulled on a frictionless horizontal surface with a force of 8.6 N [W]. (2.2) T/I A
(a) Determine the acceleration of the masses.
(b) Calculate the force acting on the 1.3 kg mass.

41. A horizontal force of 5.3×10^2 N pulls two masses, 11 kg and 19 kg, which are at rest on a frictionless table and connected by a light string. Calculate the tension in the string
(a) when the applied force pulls directly on the 11 kg mass
(b) when the applied force pulls directly on the 19 kg mass (2.3) T/I A

42. A sign with a mass of 2.5 kg is supported by a single rope, as shown in **Figure 3**. A strong horizontal wind exerts a force of 12 N on the sign. Calculate
(a) the tension in the rope
(b) the angle, θ, the rope makes with the horizontal (2.3) T/I A

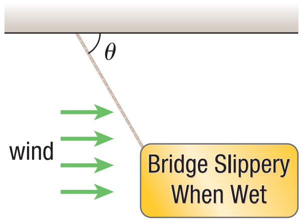


Figure 3

43. A circus performer with a mass of 54 kg hangs from some ropes, as shown in **Figure 4**. (2.3) T/I A

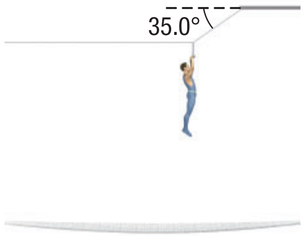


Figure 4

(a) Calculate the tension in each rope.
(b) What would happen to the tension in each rope if the horizontal rope were slightly longer and attached to the wall at a higher point? Explain your reasoning.

44. A water skier with a mass of 65 kg is pulled behind a boat that is moving with a constant speed of 25 m/s. The tension in the horizontal rope is 1.2×10^3 N [S]. Calculate the magnitude and direction of the force that the water exerts on the skier's ski. (2.3) T/I A

45. A hockey stick is in contact with a puck for 0.011 s. The speed of the puck when it leaves the stick is 32 m/s. The hockey puck has a mass of 160 g. Calculate the magnitude of the force applied to the puck. (Assume that this force is constant while the stick and puck are in contact.) (2.3) T/I A

46. A spacecraft requires a force of 1.7×10^4 N to accelerate in deep space at 6.9 m/s^2. How much force is required for the same spacecraft to accelerate at the same rate upward from Earth? (2.3) T/I A

47. Three masses are connected by strings, as shown in **Figure 5**. Assume that the masses of the strings are negligible. Calculate the acceleration of each mass and the tension in the string. (2.3) T/I A

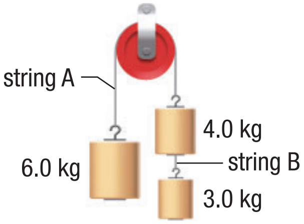


Figure 5

48. The tightrope walker in **Figure 6** gets tired and decides to stop for a rest. During this rest period, she is in equilibrium. She stops at the middle of the rope and notices that both sides of the rope make an angle of 15° below the horizontal. Calculate the tension in the rope on both sides of the tightrope walker. The mass of the tightrope walker is 60.0 kg. (2.3) K/U T/I C A

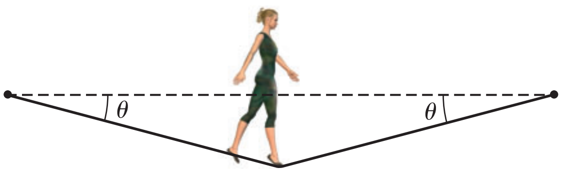


Figure 6

49. Your moving company runs out of rope, so you are forced to push two crates along the floor, one in front of the other. The crates have masses of 45 kg and 22 kg, and you push on the 45 kg crate. The crates are moving at constant speed, and the coefficient of kinetic friction between both crates and the floor is 0.35. Calculate the magnitude of the normal force between the two crates. (2.4) T/I A

50. A person sits on an office chair with wheels on it. A co-worker exerts a force of 2.2×10^2 N [right 35° down] on the chair. The person and the chair accelerate at 0.62 m/s^2 [right]. The force of friction acting on the chair is 1.4×10^2 N [left]. (2.4) K/U T/I
    (a) Determine the total mass of the chair and person.
    (b) Calculate the normal force acting on the chair.

51. You are given the job of moving a refrigerator with a mass of 1.3×10^2 kg across a horizontal floor. The coefficient of static friction between the refrigerator and the floor is 0.25. Calculate the magnitude of the minimum force that is required just to set the refrigerator into motion. (2.4) T/I A

52. You are trying to slide a heavy trunk across a horizontal floor. The mass of the trunk is 85 kg, and you need to exert a force of 3.3×10^2 N to make it just begin to move. (2.4) T/I A
    (a) Determine the coefficient of static friction between the floor and the trunk.
    (b) After the trunk starts moving, you continue to push with this force. The trunk reaches a speed of 2.0 m/s after 5.0 s. Calculate the coefficient of kinetic friction.

53. A race car driver discovers that she can accelerate at 4.0 m/s^2 without spinning her tires, but if she tries to accelerate more rapidly, she always "burns rubber." Determine the coefficient of static friction between the driver's tires and the road. (2.4) T/I A

54. A hockey puck slides along a rough, icy surface. It has an initial speed of 35 m/s and slides to a stop after travelling a distance of 95 m. Calculate the coefficient of kinetic friction between the puck and the ice. (2.4) T/I A

55. A piece of wood with a mass of 2.4 kg is held in a vise sandwiched between two wooden jaws, as shown in **Figure 7**. A blow from a hammer drives a nail that exerts a force of 450 N on the wood. The coefficient of static friction between the wood surfaces is 0.67. Calculate the magnitude of the minimum normal force that each jaw of the vise exerts on the wood block to hold the block in place. (2.4) T/I A

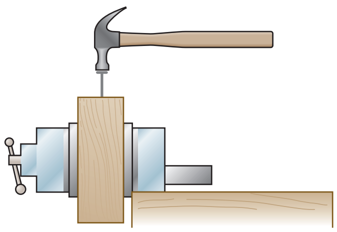


Figure 7

56. Two crates with masses of 15 kg and 35 kg are stacked on the back of a truck, with the lighter crate on top of the heavier. The frictional forces are strong enough that the crates do not slide off the truck. The truck is accelerating at 1.7 m/s^2. Draw FBDs for both crates, and determine the values of all forces in your diagrams. Indicate the direction of the truck's motion in your diagrams. (2.1, 2.4) K/U T/I C A

57. A block with a mass of 2.0 kg is placed on a plane inclined at 32° to the horizontal. The coefficient of friction between the block and the plane is 0.70. (2.4) T/I A
    (a) Calculate the force of friction acting on the block.
    (b) Is the force of friction static or kinetic? Explain.

58. A chest of drawers with a mass of 66 kg rests on the floor. The coefficient of static friction between the chest and the floor is 0.45. Calculate the magnitude of the minimum horizontal force that a person must apply to start the chest moving. (2.4) T/I A

59. A block resting at the top of a plane inclined at 26° with the horizontal slides down with an acceleration of $\frac{g}{5}$ [down the plane]. Calculate the coefficient of kinetic friction. (2.4) T/I A

60. A person is standing without slipping in a train that is accelerating forward. The coefficient of static friction between the passenger and the train floor is 0.43. (2.4) T/I C A
    (a) Draw an FBD of the person.
    (b) Determine the maximum acceleration of the train before the person starts to slip.
    (c) Describe what the person can do, without changing any of his clothing or grabbing on to anything, to keep from slipping when the acceleration exceeds this value.

61. Determine the maximum acceleration of a train in which a box lying on its floor will remain stationary, given that the coefficient of static friction between the box and the train floor is 0.16. (2.4) T/I A

62. A block slides down an incline of angle 26° with an acceleration of 2.5 m/s^2. Determine the coefficient of kinetic friction between the block and the incline. (2.4) T/I A

63. A metal block with a mass of 2.2 kg is resting on a table. The coefficient of kinetic friction between the block and the table is 0.41. Calculate the acceleration of the block when a force of 18 N [E] is applied on it. (2.4) T/I A

64. An object sliding on a rough horizontal plane slows down at 6.6 m/s^2. Calculate the coefficient of kinetic friction between the object and the plane. (2.4) T/I A

65. A box with a mass of 2.2 kg sits on top of another box with a mass of 3.8 kg. The coefficient of friction between the two boxes is 0.25, and the coefficient of kinetic friction between the larger box and the horizontal surface is 0.32. Determine the largest horizontal force that can be applied to the larger box so that the smaller box does not slip off. (2.4) T/I A

Evaluation

66. A student says, "If action–reaction forces cancel, then the net force must always be zero. How can anyone ever accelerate?" Discuss the validity of this statement. (2.2) K/U T/I C

67. You tie a rope to a large crate and try to pull it across a horizontal surface toward a fence, but it will not move. Suggest a way of moving the crate without getting any help from another person. Discuss any limitations of the method. (2.3) T/I C A

68. A person with a mass of 59.2 kg sits on a light seat attached to a rope that runs over a pulley, as shown in **Figure 8**. The person pulls down on the rope to move herself up at a constant velocity. (2.3) T/I A

Figure 8

    (a) Explain why this setup provides an advantage over climbing up a rope.
    (b) What magnitude of force does the person exert on the rope?
    (c) What assumptions are you making about the setup when calculating the force exerted by the person?

69. The following steps summarize the strategy for solving two-dimensional force problems that require Newton's second law of motion. Place the steps in the correct order. (2.3) K/U T/I C
    (a) Solve the problem using Newton's second law of motion.
    (b) Determine the x- and y-components of each force, and write the necessary equations.
    (c) Identify the given variables and the required variables.
    (d) Choose a coordinate system, and draw an FBD. Include a label for each force.
    (e) Identify the object on which the forces act.
    (f) Read the problem before trying to solve it.

70. A student claims the following about cross-country skiing: "I wish there was no friction when I'm sliding down those small hills, but I'm sure glad there is a little friction when I'm trying to go uphill." Why do you think the student would say this? Explain your reasoning. (2.4) K/U T/I A

71. The coefficient of static friction between your running shoes and dry pavement is 0.81. When the pavement is wet, the coefficient of static friction is 0.58. (2.4) T/I
   (a) Determine the maximum acceleration you can achieve in both cases.
   (b) How quickly could you run 100 m if you could sustain these accelerations? Are these times reasonable?

72. Two workers move a 52 kg crate by sliding it across the floor. Worker 1 can exert a force of 3.4×10^2 N, and worker 2 can exert a force of 1.7×10^2 N. One worker must push on the crate below the horizontal and the other must pull at the same angle above the horizontal (**Figure 9**). Determine the acceleration of the crate. Assume that the coefficient of kinetic friction is 0.52 and $\theta = 25°$. (2.4) T/I A

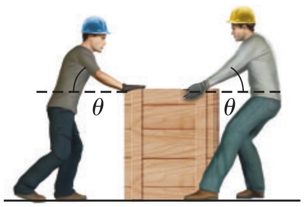


Figure 9

73. A driver makes an emergency stop and inadvertently locks up the brakes of the car, which skids to a stop on dry concrete. Consider the effect of heavy rain on this scenario. Using the values in **Table 1**, determine how much farther the car would skid (expressed as a percentage of the dry-weather skid) if the concrete were wet instead. What does this question imply about driving safely? (2.4) T/I A

Table 1 Coefficients of Kinetic Friction for Rubber on Concrete

Surface	μ_K
rubber on dry concrete	0.85
rubber on wet concrete	0.45

74. Two skiers are racing down a hill that is inclined at 23° to the horizontal. Skier 1 has a mass of 59 kg, and skier 2 has a mass of 73 kg. The coefficient of kinetic friction between the skis and the hill is 0.10. Use the equation below to answer the following questions. (2.6) T/I A

$$a = g\sin\theta - \mu_K g\cos\theta - \frac{F_{air}}{m}$$

   (a) Compare the accelerations of the skiers if air resistance is negligible. Explain your answer.
   (b) The air resistance on race day is 82 N. Compare the accelerations of the skiers.
   (c) Evaluate the difference in accelerations. Does the difference affect the race?

Reflect on Your Learning

75. How would you explain the common forces experienced in everyday life to a class if you were the teacher and your students had not taken physics? How would you explain FBDs to your class? T/I C A

76. In what areas of your daily experience do you now see the physics concepts that were explored in this chapter? T/I C

77. How did the information you learned in this chapter affect your thinking about frictional forces? T/I C A

78. Identify a situation you have experienced that involves concepts from this chapter. Write a question about it, and share it with a classmate. A

79. What was the most surprising thing you learned in this chapter? C

80. Was there any example in this chapter that you found particularly relevant to your daily life? C

Research

WEB LINK

81. Research biomechanics. How are the principles of biomechanics used by athletes? Identify the various fields that make use of a force platform and discuss its use. K/U T/I C A

82. Research automobile seat belts. Explain why seat belts are equipped with pre-tensioners and web clamps. Prepare a short oral presentation on automatic belt systems to give to your class. K/U T/I C A

83. Research the effect of belt friction. Describe the various techniques involved in friction management for climbing operations. K/U T/I C A

84. Research the physics of archery. What aspects of the principles of motion are applied in archery (**Figure 10**)? K/U C A

Figure 10

CHAPTER 3

Uniform Circular Motion

KEY CONCEPTS

After completing this chapter you will be able to

- distinguish between inertial and non-inertial frames of reference
- analyze uniform circular motion qualitatively and quantitatively
- derive the equations for uniform circular motion
- investigate the forces and acceleration experienced by an object in uniform circular motion
- conduct inquiries into uniform circular motion
- analyze a technological device that applies the principles of uniform circular motion and assess the social and environmental impact

What Conditions Are Necessary for an Object to Move in a Circle at a Constant Speed?

One of the thrills of professional auto racing is the danger of the sport. Drivers must drive at high speeds, even on curves, like the one on the facing page. Yet curves are where it is easiest to skid out of control. If drivers slow down to maintain control of the car, they fall behind other racers. How are they able to maintain high speeds at all times and do so safely? The design of the race cars themselves, along with the skills of the drivers, helps the drivers maintain high speeds along the track. The design of the racetrack is also extremely important.

Consider Newton's first law of motion: an object will keep the same velocity unless acted on by an external force. As a car enters a turn, in the absence of any external forces the car's tendency is to continue going straight. The driver steering the car and the friction between the tires and the pavement provide some of the force for changing the direction of the car. Yet these may not be enough, depending on the car's speed and the radius of the curve.

One simple way to keep the car on the track during the curve is to bank the track. A banked track has an incline, so that the inner side of the track has a lower elevation than the outer side. When a car moves on a banked track, gravity pulls the car straight down—not toward the lower inner side—and is unchanged in magnitude. When the other forces acting on the car are not quite large enough to keep it in a curved path, how does the banked curve help?

The idea of banked roads is just one example of how to make moving objects change direction, in particular, along circular paths. In this chapter, you will learn how accelerations and forces appear in different frames of reference, and how these accelerations and forces cause circular motion at constant speed. You will then apply these principles in specific and familiar situations.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. What forces act on a car speeding along a racetrack?
2. How do you think increasing the banking angle affects the maximum speed of the car? Explain your answer.
3. Why do you think that banked curves are a common safety feature of exit ramps on highways and other roads?
4. Why do you think city streets are not built with banked curves?

Mini Investigation

Observing Circular Motion

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this Mini Investigation you will explore circular motion at constant speed.

Equipment and Materials: eye protection; small eye screw; small rubber stopper; plastic tube; three small 50 g masses; alligator clip; string

Be sure the area around you is clear from material hanging from the ceiling that you may accidentally hit while swinging the stopper. Ensure no one can be hit by the swinging stopper.

1. Place the eye screw into the rubber stopper. Ensure the screw is secure.
2. Tie the string through the eye of the eye screw. Make sure that the knot is tight and will not slip during the activity. Place the string through the tube, and hang one mass on the other end of the string. Place the alligator clip on the string between the mass and the tube.
3. Put on the eye protection.
4. Swing the rubber stopper in a horizontal circle when holding the tube in your hand above your head. Note the force exerted on your hand.
5. Repeat Step 4, but spin the stopper at both faster and slower speeds using a different number of masses.
6. Shorten the radius of the stopper's circular path by putting the alligator clips at different spots. Repeat Steps 4 and 5.

A. Did you notice the force on your hand when you swung the stopper in a circle? How do you think the tension in the string is related to the circular motion of the stopper? T/I A

B. As the speed of the stopper increases, what happens to the stopper and the angle that the string makes with respect to the vertical? T/I A

C. How did the tension in the string change with the speed of the stopper? T/I A

D. How did the tension in the string change with the length of the string? T/I A

3.1 Inertial and Non-inertial Frames of Reference

Think about riding on a bus. When the bus moves at a steady speed without changing direction, you could close your eyes and possibly not even be aware that the bus is moving. If you place a ball in the aisle of the bus, the ball does not move relative to the bus (**Figure 1**). When the bus slows down, however, you will see the ball roll forward. When the bus speeds up again, the ball will roll backward. When the bus turns, the ball will roll to the side. What forces cause the ball to accelerate? The fact is, no forces act on the ball to accelerate it. WEB LINK

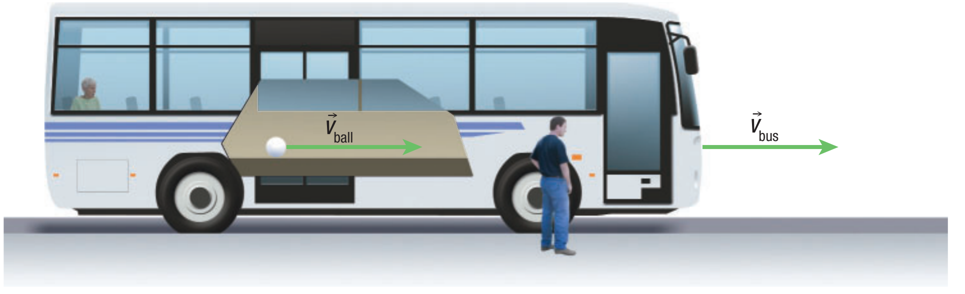


Figure 1 The bus and the ball placed in the aisle of the bus are moving at the same velocity with respect to the ground, although the ball is not moving relative to the bus.

Defining Frames of Reference

frame of reference a coordinate system relative to which motion is described or observed

inertial frame of reference a frame of reference that moves at a zero or constant velocity; a frame in which the law of inertia holds

non-inertial frame of reference a frame of reference that accelerates with respect to an inertial frame; the law of inertia does not hold

A person on the bus will view all motion from the point of view of the bus, so to her, the moving bus in Figure 1 is a moving frame of reference. A **frame of reference** is an observer's choice of coordinate system, including an origin, for describing motion. A person standing on the sidewalk is in the stationary frame of reference of the ground. In many cases, people use the ground as a frame of reference, especially when they are not moving with respect to the ground. When you are at rest in a frame of reference that has a constant velocity, whether that frame is itself at rest ($\vec{v} = 0$) or moving with a non-zero velocity, your velocity relative to that frame is zero. When the bus is moving at a constant velocity, no net force acts on it. Therefore, no net force acts on you inside it. Newton's first law of motion (the law of inertia) states that any object moving at a constant velocity (including $\vec{v} = 0$) remains at that velocity if no net force acts on it. The bus in this case is an **inertial frame of reference**, which is a frame of reference in which the law of inertia is valid.

Now think about what happens when the bus slows down and the ball begins to roll forward inside the bus. What causes the ball to move forward? As the bus slows, it accelerates (that is, changes velocity) in a direction that is opposite to the direction of motion. Therefore, the bus is no longer an inertial frame of reference. It becomes a **non-inertial frame of reference**, which is a frame of reference in which the law of inertia is no longer valid. The reason there appears to be a net force on the ball is that the ball continues to move with the same velocity it had before. Obeying the law of inertia, the ball does not immediately slow down with the bus (**Figure 2**). Within the accelerating bus, or non-inertial frame of reference, this looks as if a force is pushing the ball forward.

Similarly, when the bus starts from rest and speeds up, the ball tends to stay at rest. To you and other passengers in the non-inertial frame, it appears as if there was a push on the ball toward the back of the bus. There also appears to be a force on the ball when the bus turns a corner. The ball continues moving in the same direction as the bus before the turn. From your point of view in the bus, this would be like a force pushing the ball in the direction opposite to that in which the bus was turning.

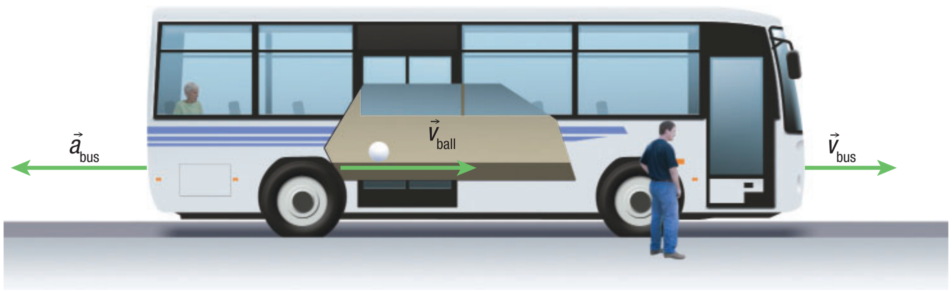


Figure 2 As the bus slows, the ball continues to move forward. In the bus, it appears as if a force has been applied to the ball.

To explain these various motions, we invent the idea of fictitious forces. **Fictitious forces** are apparent but non-existent forces that explain the motion in accelerating (non-inertial) frames of reference. In the bus example, a fictitious force pushed the ball in the direction opposite the acceleration of the bus. As you continue learning about mechanics, you will encounter other fictitious forces. Fictitious forces simply explain non-accelerated motion in accelerated frames of reference. Tutorial 1 shows how to solve problems involving an object placed in a non-inertial frame of reference.

fictitious force an apparent but non-existent force invented to explain the motion of objects within an accelerating (non-inertial) frame of reference

Tutorial 1 Solving Problems Related to Objects in a Non-inertial Frame of Reference

This Tutorial models how to solve problems involving objects placed in a non-inertial frame of reference.

Sample Problem 1: Calculating the Acceleration of a Non-inertial Frame of Reference

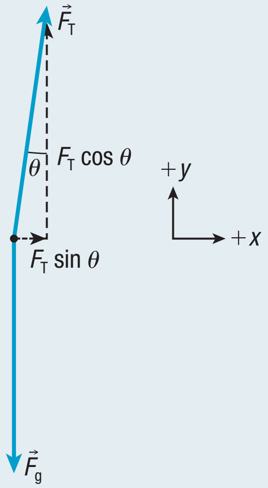
A teacher suspends a small cork ball from the ceiling of a bus. When the bus accelerates at a constant rate forward, the string suspending the ball makes an angle of 10.0° with the vertical. Calculate the magnitude of the acceleration of the bus.

Given: $\theta = 10.0°$; $g = 9.8\ \text{m/s}^2$

Required: a_x

Analysis: Look at the situation from an Earth (inertial) frame of reference. Draw an FBD to show the forces acting on the cork ball. The horizontal component of the tension $\vec{F}_T$ balances the acceleration, so express the components of the tension in terms of the horizontal and vertical applied forces. Then calculate the magnitude of the acceleration.

Solution:



Vertical components of force:

$$\Sigma F_y = 0$$
$$F_T \cos\theta - mg = 0$$
$$F_T = \frac{mg}{\cos\theta}$$

Horizontal components of force:

$$\Sigma F_x = ma$$
$$F_T \sin\theta = ma$$

Substitute $F_T = \dfrac{mg}{\cos\theta}$

$$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}a$$
$$g\left(\frac{\sin\theta}{\cos\theta}\right) = a$$

Substitute $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$$a = g\tan\theta$$
$$= (9.8\ \text{m/s}^2)(\tan 10.0°)$$
$$a = 1.7\ \text{m/s}^2$$

Statement: The magnitude of the bus's acceleration is 1.7 m/s².

Practice

1. You are in a car moving with a constant velocity of 14 m/s [E]. A baseball lies on the floor at your feet. K/U T/I C
   (a) Describe the motion of the ball from your point of view. How is it different from when the car is at rest?
   (b) How would an observer on the sidewalk describe the motion of the ball?
   (c) Now the car accelerates forward. Describe the ball's motion from your point of view.
   (d) Draw two FBDs showing the ball's motion in (c) from the frame of reference of the car and the frame of reference of the sidewalk. Which frame of reference is non-inertial? In which frame do you observe a fictitious force?
2. You use a string to suspend a cork ball with a mass of 22.0 g from the ceiling of a moving speedboat. The ball and string hang at an angle of 32.5° from the vertical. K/U T/I C A
   (a) Calculate the magnitude of the speedboat's acceleration. Do you need to know the mass of the ball to make this calculation? Why or why not? [ans: 6.2 m/s^2]
   (b) Determine the magnitude of the tension in the string. Do you need to know the mass of the ball to make this calculation? Why or why not? [ans: 0.26 N]
3. A person is standing in a subway train holding a strap that is attached to a piece of luggage on wheels (**Figure 3**). The mass of the luggage is 14 kg. The strap makes an angle of 35° to the vertical. Assume there is no friction between the luggage wheels and the floor. K/U T/I A

Figure 3

   (a) Determine the tension in the strap when the subway is moving at a constant velocity. [ans: 0 N]
   (b) Determine the tension in the strap when the subway is accelerating forward at 1.4 m/s^2. [ans: 34 N]
4. A passenger stands in a train that is accelerating forward. The passenger is able to stay in place because of the force of static friction between his shoes and the floor. The coefficient of static friction between the shoes and the floor is 0.42. Determine the maximum amount the train, relative to the track, can accelerate before the passenger begins to slip along the floor. K/U T/I A [ans: 4.1 m/s^2]

Apparent Weight

The study of vertical acceleration can also introduce fictitious forces. To understand this, consider what happens when you stand on a bathroom scale (**Figure 4(a)**). As in all cases when you stand up, you feel a force pushing upward against the soles of your feet. This is the normal force, and it is equal and opposite to the weight (mg) of your body when you stand on level ground.

Now suppose that you stand on the same scale inside an elevator. When the elevator is at rest, the normal force is again the same as your weight. This is also true when the elevator is moving at a constant non-zero speed upward or downward. However, what happens when the elevator accelerates? When the elevator accelerates downward, the normal force decreases, so that the magnitude of the reading on the scale is less than your weight, mg (**Figure 4(b)**). Similarly, when the elevator accelerates upward, the normal force increases, resulting in a greater reading on the scale.

As with the bus at the beginning of this section, the elevator can be an inertial frame of reference when it has a constant velocity going up or down. It becomes a non-inertial frame of reference when it accelerates, resulting in a normal force that is either greater or less than your weight. The magnitude of this normal force in a non-inertial frame of reference is called the **apparent weight**.

apparent weight the magnitude of the normal force acting on an object in an accelerated (non-inertial) frame of reference

Other non-inertial frames of reference produce other values for apparent weight. On a free-fall ride at an amusement park, the acceleration of the ride is equal to g, and the normal force is zero (**Figure 4(c)**). Thus, the scale will read an apparent weight of zero. Similarly, an astronaut aboard the International Space Station is constantly in free fall. Therefore, the normal force acting on the astronaut is zero (**Figure 4(d)**).

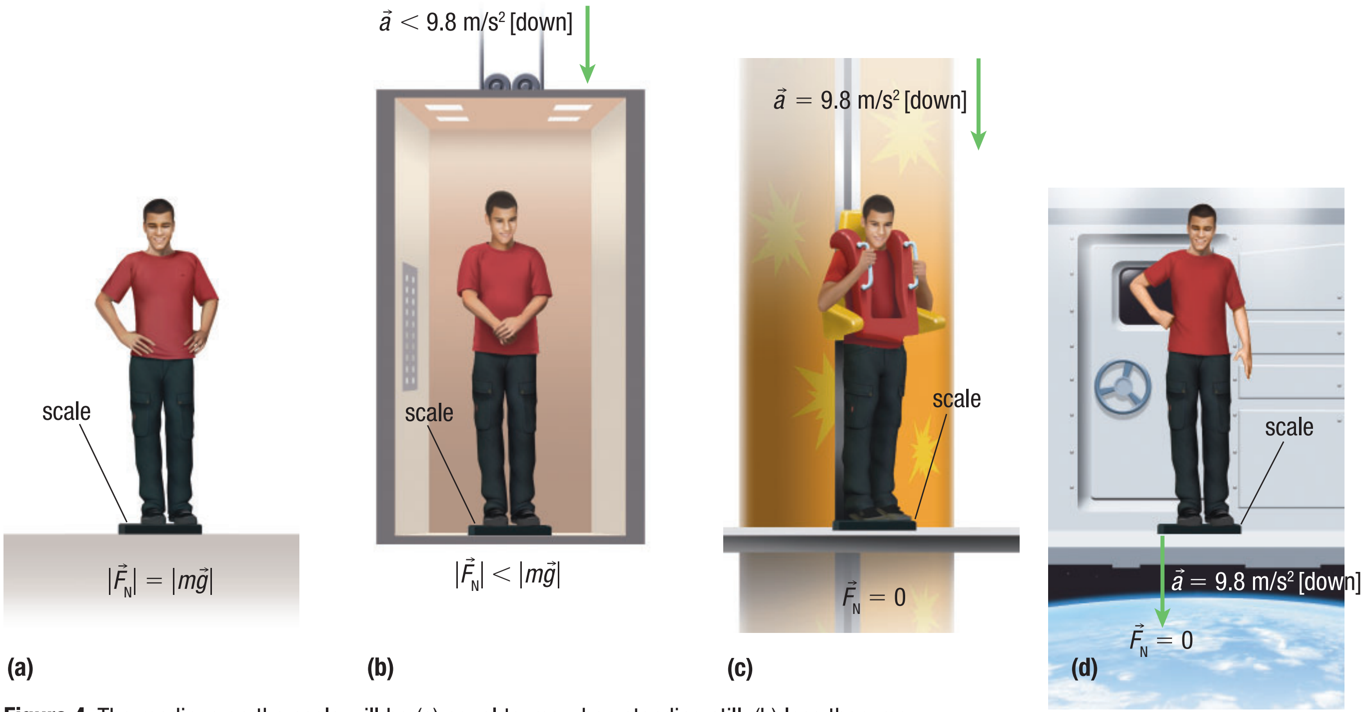


Figure 4 The readings on the scale will be (a) equal to mg when standing still, (b) less than mg when the elevator accelerates downward, (c) equal to zero in vertical free fall on an amusement park ride, and (d) equal to zero in free fall during orbit.

In the following Tutorial, you will learn how to solve problems that involve the apparent weight of an object in a non-inertial frame of reference.

Tutorial 2 Solving Problems Related to Apparent Weight in a Non-inertial Frame of Reference

Sample Problem 1: Apparent Weight in an Accelerating Elevator

An elevator accelerates upward with an acceleration of magnitude 1.5 m/s^2, after which it moves with a constant velocity. As the elevator approaches its stopping point, it undergoes a downward acceleration of magnitude 0.9 m/s^2. Calculate the apparent weight of a passenger with a mass of 75 kg when

(a) the elevator undergoes positive acceleration

(b) the elevator moves at constant velocity

(c) the elevator undergoes negative acceleration

Solution

(a) **Given:** $a = 1.5$ m/s^2 [up]; $m = 75$ kg; $g = 9.8$ m/s^2 [down]

Required: F_N

Analysis: Draw an FBD of the passenger, and solve for the normal force. Use up as positive.

Solution:

$$
\begin{aligned}
+F_N + (-mg) &= ma \\
F_N &= mg + ma \\
&= m(g + a) \\
&= (75 \text{ kg})(9.8 \text{ m/s}^2 + 1.5 \text{ m/s}^2) \\
F_N &= 8.5 \times 10^2 \text{ N}
\end{aligned}
$$

Statement: The apparent weight of the passenger when the elevator undergoes positive acceleration of 1.5 m/s^2 is 8.5×10^2 N.

(b) **Given:** $a = 0$ m/s^2; $m = 75$ kg; $g = 9.8$ m/s^2 [down]

Required: F_N

Analysis: In an inertial frame, there is no acceleration, so the apparent weight of the passenger is mg; $F_N = mg + 0 = mg$.

Solution:
$$
\begin{aligned}
F_N &= mg \\
&= (75 \text{ kg})(9.8 \text{ m/s}^2) \\
F_N &= 7.4 \times 10^2 \text{ N}
\end{aligned}
$$

Statement: The apparent weight of the passenger when the elevator moves at constant velocity is 7.4×10^2 N.

(c) **Given:** $a = 0.9$ m/s^2 [down]; $m = 75$ kg; $g = 9.8$ m/s^2 [down]

Required: F_N

Analysis: Draw an FBD of the passenger, and solve for the normal force. Use down as positive.

Solution:

$$
\begin{aligned}
-F_N + (+mg) &= ma \\
F_N &= mg - ma \\
&= m(g - a) \\
&= (75 \text{ kg})(9.8 \text{ m/s}^2 - 0.9 \text{ m/s}^2) \\
F_N &= 6.7 \times 10^2 \text{ N}
\end{aligned}
$$

Statement: The apparent weight of the passenger when the elevator undergoes negative acceleration of 0.9 m/s^2 is 6.7×10^2 N.

Practice

1. A student with a mass of 55 kg stands in an elevator that accelerates (a) upward at 2.9 m/s^2 and then (b) downward at 2.9 m/s^2. Determine the student's apparent weight during each acceleration. K/U T/I A [ans: (a) 7.0×10^2 N; (b) 3.8×10^2 N]
2. Two boxes of books are stacked and placed on the floor of an elevator. The masses of the bottom and top boxes are 9.5 kg and 2.5 kg, respectively. The normal force between the floor and the bottom box is 70.0 N. K/U T/I A
   (a) Determine the magnitude and direction of the elevator's acceleration. [ans: 4.0 m/s^2 [down]]
   (b) Determine the force that the larger box exerts on the smaller box. [ans: 15 N [up]]
3. Rope A is tied to block 1, and rope B is attached to both block 1 and block 2 (**Figure 5**). The mass of block 1 is 4.2 kg, and the mass of block 2 is 2.6 kg. You lift both blocks straight up. Calculate the tension in the ropes when the blocks (a) move at a constant velocity of 1.5 m/s [up] and (b) accelerate at 1.2 m/s^2 [up]. K/U T/I A
   [ans: (a) rope A: 67 N; rope B: 25 N; (b) rope A: 75 N; rope B: 29 N]
4. The Taipei 101 Tower, in Taipei, has 101 floors. Although the elevators in the Taipei 101 Tower are the fastest in the world, they have an acceleration of only 0.98 m/s^2. Calculate the apparent weight of a passenger with a mass of 61 kg when one of these elevators is accelerating downward. K/U T/I A [ans: 5.4×10^2 N]

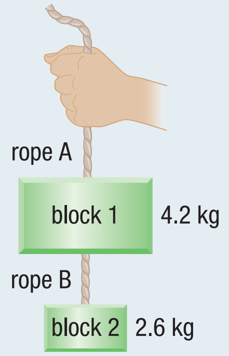


Figure 5

3.1 Review

Summary

- A frame of reference is a coordinate system relative to which motion is described or observed.
- An inertial frame of reference is one that moves at a constant velocity or is at rest. The law of inertia holds.
- A non-inertial frame of reference is one that undergoes acceleration because of an external force. The law of inertia does not hold.
- Fictitious forces help explain motion in a non-inertial frame of reference.
- Apparent weight is the magnitude of the normal force acting on an object in a non-inertial frame of reference.

Questions

1. Suppose you are on a train moving with a constant velocity. Another train on parallel tracks is moving with the same velocity. A passenger in the other train is tossing a ball vertically in the air. T/I C A
   (a) Describe how the path of the ball would look to you.
   (b) Describe how the path of the ball would look if the trains moved in opposite directions.
2. A mass on a string is suspended from the ceiling of an airplane. Calculate the angle that the mass makes when the airplane has a horizontal acceleration of magnitude 1.5 m/s^2. K/U T/I A
3. A jet reaches a takeoff speed of 255 km/h in 10.0 s. This jet has a ball-on-a-string accelerometer hanging from the ceiling of the cabin. Assume the jet accelerates uniformly during takeoff. Calculate the angle of the string during takeoff. K/U T/I A
4. A student constructs an accelerometer by attaching cork balls to strings anchored to the bottom of a fish tank. When the student fills the tank with water, the balls float to the surface. When the tank is at rest, the strings align in the vertical direction. While riding in a car and holding the tank level, the student notices that the strings make an angle of 16° with respect to the vertical. Calculate the magnitude of the car's acceleration. K/U T/I A
5. The passenger elevators at the Brookfield Place towers in Toronto reach a top speed of about 6.0 m/s upward. Suppose one of the elevators reaches this speed in 10.0 s. Calculate the apparent weight of a passenger whose mass is 64 kg. T/I
6. A student on a free-fall ride at an amusement park brings a scale to check her apparent weight during the ride. At one point, she notices a reading of 255 N. The student's mass is 52 kg. Calculate the acceleration of the ride at the time of the reading. T/I A
7. A vintage sports car accelerates down a hill at an angle of 17° to the ground, as shown in **Figure 6**. The driver notices that the string of ornamental fuzzy dice hanging from his rear-view mirror is perpendicular to the roof of the car. K/U T/I C A

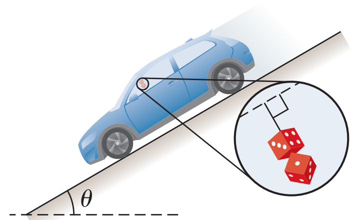


Figure 6

   (a) Draw an FBD of the dice from the frame of reference of the level ground, both when the car is at rest on the hill and when it is accelerating. Explain how the two FBDs differ.
   (b) Calculate the car's acceleration.
8. In **Figure 7**, mass 1 does not slide with respect to the surface when the horizontal force shown is applied. Determine the magnitude of the horizontal force in both Figure 7(a) and Figure 7(b). Assume there is no friction. K/U T/I A

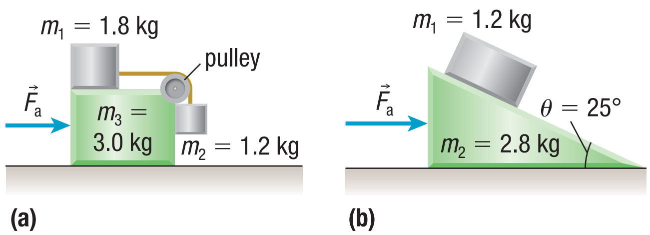


Figure 7

3.2 Centripetal Acceleration

The hammer throw is a track-and-field event in which an athlete throws a "hammer"—a heavy metal ball attached to a wire and handle—the farthest distance possible (**Figure 1**). To do this, the athlete first swings the ball in a circular path. After giving it a maximum circular speed with three or four turns, the athlete releases the hammer, which then flies down the field. Just before the hammer is released, it has **uniform circular motion**, which is motion in a circular path at a constant speed.

uniform circular motion the motion of an object with a constant speed along a circular path of constant radius

Figure 1 To give the hammer enough speed to travel a long distance down the field, the athlete must move it rapidly in a circular path.

By moving the ball in a circle, the athlete introduces the force of tension in the wire. This tension keeps the ball in a circular path. The greater the tension, the greater the acceleration toward the centre of the circle and the faster the ball moves in a circular path. When the tension is very large, so is the speed of the ball. When the athlete releases the hammer, the ball travels far down the field.

You may not always realize it, but objects moving in circular paths are all around you. Clothes in a washing machine during the spin cycle, the drum of a clothes dryer, the hands of certain electric clocks, and the spinning blades of a blender and a lawn mower: all of these objects move with uniform circular motion. What you may not have considered is that these are all among the most common non-inertial frames of reference. These objects move in a circular path, so their velocity constantly changes direction. Therefore they are accelerating. Acceleration that is directed toward the centre of a circular path is called **centripetal acceleration**, $\vec{a}_c$.

centripetal acceleration ($\vec{a}_c$) the instantaneous acceleration that is directed toward the centre of a circular path

Equations for Centripetal Acceleration

Recall that the average acceleration, $\vec{a}_{av}$, of an object equals the change in velocity, $\Delta\vec{v}$, during an interval of time, Δt: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$. For an object moving with uniform circular motion, the velocity changes direction continuously with time, so $\Delta\vec{v}$ is definitely not zero. Therefore, centripetal acceleration is not zero.

To calculate centripetal acceleration, we now consider the example of a runner moving at a constant speed along a circular track. The velocity of the runner changes with time and is always tangential to the circular path, as shown in **Figure 2**. Figure 2(a) shows a runner's velocity vectors at two nearby positions: A′ and A. Figure 2(b) shows the corresponding change in velocity, $\Delta\vec{v}$, over a short time interval, Δt. Recall from Section 1.4 that the difference in velocity vectors is the same as adding one vector to the negative of the other vector. First, shift vector $\vec{v}_2$ down so that its head is at point A (Figure 2(b)). Then reverse $\vec{v}_1$ so $\vec{v}_1$ becomes $-\vec{v}_1$. Place the tail of vector $-\vec{v}_1$ at the head of vector $\vec{v}_2$, so that the sum of the vectors is $\Delta\vec{v}$, which points toward the centre of the circle. It should be noted that we actually need to decrease the size of the time interval until it is very small for $\Delta\vec{v}$ to point directly toward the centre. For the sake of discussion and illustrating the concept, however, our model in Figure 2 is satisfactory.

As you can see in Figure 2(a), the individual velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are both tangent to the circle, perpendicular to the circle's radius, and equal in length (magnitude). This is true for all of the runner's velocity vectors along the circular path. The acceleration vector has the same direction as $\Delta\vec{v}$, so it follows that the centripetal acceleration of an object must always point toward the centre of the circular path.

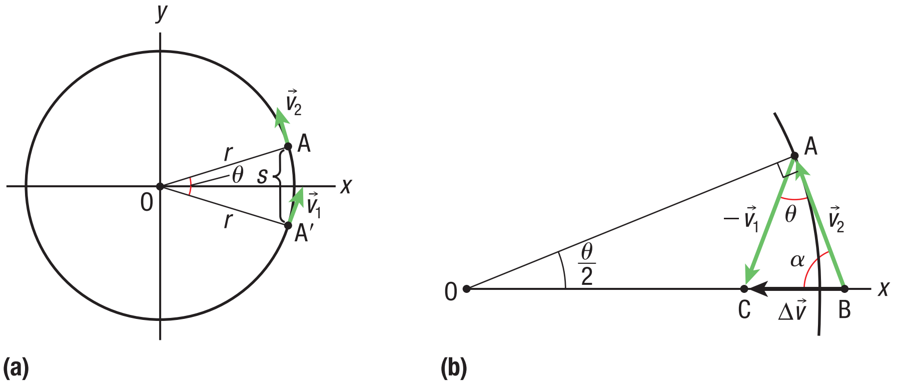


Figure 2 (a) The velocity $\vec{v}$ of an object (in this case, a runner) moving with uniform circular motion is shown as $\vec{v}_1$ and $\vec{v}_2$ at two different locations along the circular path. The distance travelled in going from point A′ to point A is *s*. (b) The difference in the velocity vectors, $\Delta\vec{v}$, is directed toward the centre of the circle when the time interval is very small.

Now use the triangle BAC in Figure 2(b) to calculate the magnitude of the acceleration. This triangle has two sides with equal lengths, $|\vec{v}_1|$ and $|\vec{v}_2|$. In general, the velocity magnitude is the same at any point around the circle, so $|\vec{v}_1| = |\vec{v}_2| = |\vec{v}|$, or simply v. The third side of this triangle is the vector $\Delta\vec{v}$, which has a length of $|\Delta\vec{v}|$.

The triangle BAC has the same interior angles as triangle AOA′ in Figure 2(a), so these triangles are similar. You can check this by using the relations below to show that the angle θ is the same for both triangles. Note in Figure 2(b) that the angle at point A between $\vec{v}_2$ and the radius is 90°, so

$$\frac{\theta}{2} + \alpha = 90^\circ$$
$$\theta + 2\alpha = 180^\circ$$
$$\theta = 180^\circ - 2\alpha$$

When you add the two equal angles, α, within triangle BAC to the third angle, they equal 180°, so the third angle must equal

$$180 - 2\alpha = \theta$$

Two sides of triangle AOA′ are along the radius of the circle, so they have length r, while the other side (between points A′ and A) has length s. When Δt is small, the arc length between points A′ and A approaches a straight-line length that connects A′ and A. Therefore, the distance the runner travels from A′ to A is approximately equal to the distance given by $s \approx v\Delta t$. The triangles BAC and AOA′ are similar, so the ratios of their corresponding sides are equal. Substituting v for the magnitude of either $\vec{v}_1$ or $\vec{v}_2$, and assuming a very small Δt:

$$\frac{|\Delta\vec{v}|}{v} = \frac{s}{r}$$
$$\frac{|\Delta\vec{v}|}{v} = \frac{v\Delta t}{r}$$
$$|\Delta\vec{v}| = \frac{v^2\Delta t}{r}$$

The magnitude of the average acceleration equals the magnitude of the difference in the velocities ($|\Delta\vec{v}|$) divided by Δt:

$$a_{\mathrm{av}} = \frac{|\Delta\vec{v}|}{\Delta t}$$

$$= \frac{\frac{v^2 \cancel{\Delta t}}{r}}{\cancel{\Delta t}}$$

$$a_{\mathrm{av}} = \frac{v^2}{r}$$

In the above equation, $a_{\mathrm{av}} = a_{\mathrm{c}}$ when the time interval is very small:

$$\boxed{a_{\mathrm{c}} = \frac{v^2}{r}}$$

where a_{c} is the magnitude of the centripetal acceleration, v is the speed of the object moving along the circular path, and r is the radius of the circular path. Note that, although this derivation started with the definition of average acceleration, the result becomes exact for a very small time interval Δt, so the centripetal acceleration in this case is an instantaneous quantity directed toward the centre of the circle.

The equation for centripetal acceleration indicates that, when the speed of an object moving with uniform circular motion is large for a constant radius, such as in the case of the hammer in the hammer throw, the direction of the velocity changes more rapidly than it would for a smaller speed. This means that, to produce these rapid changes in velocity, you need a larger acceleration. When the radius is larger for a constant speed, the direction of the velocity changes more slowly, so the object has a smaller acceleration.

Sometimes you may not know the speed of an object moving with uniform circular motion. However, you may be able to measure the time it takes for the object to move once around the circle, or the **period**, T. Then you can calculate the speed. Remember that the speed is constant, and that it equals the length of the path the object travels (the circumference of the circle, or $2\pi r$) divided by the period, T:

period (*T*) the time required for a rotating, revolving, or vibrating object to complete one cycle

$$v = \frac{\Delta d}{\Delta t}$$

$\Delta d = 2\pi r$ and $\Delta t = T$, so

$$v = \frac{2\pi r}{T}$$

Substitute the above expression for v into the above equation for centripetal acceleration to obtain the acceleration in terms of the period and the radius:

$$a_{\mathrm{c}} = \frac{v^2}{r}$$

$$= \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$= \frac{\frac{4\pi^2 r^{\cancel{2}}}{T^2}}{\cancel{r}}$$

$$\boxed{a_{\mathrm{c}} = \frac{4\pi^2 r}{T^2}}$$

For high rotational speeds, frequency is the preferred quantity of measurement. The **frequency**, f, equals the number of revolutions per unit of time, or

$$f = \frac{1}{T}$$

The unit of frequency is hertz (Hz), or cycles per second.

In terms of frequency and radius, the equation for centripetal acceleration takes the form

$$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ &= \frac{4\pi^2 r}{\left(\frac{1}{f}\right)^2} \\ &= \frac{4\pi^2 r}{\frac{1}{f^2}} \end{aligned}$$

$$a_c = 4\pi^2 r f^2$$

frequency (f) the number of rotations, revolutions, or vibrations of an object per unit of time; the inverse of period; SI unit Hz

You now have three equations for determining the magnitude of the centripetal acceleration. When dealing with the vector of this acceleration, remember that centripetal acceleration always points toward the centre of the circle. The following Tutorial models how to solve problems involving centripetal acceleration.

UNIT TASK **BOOKMARK**

You can use some of the equations for centripetal acceleration when you complete the Unit Task on page 146.

Tutorial 1 Solving Problems with Objects Moving with Centripetal Acceleration

This Tutorial shows how to calculate the centripetal acceleration for an object undergoing uniform circular motion using the different equations for the magnitude of centripetal acceleration.

Sample Problem 1: Calculating the Magnitude of Centripetal Acceleration

A child rides a carousel with a radius of 5.1 m that rotates with a constant speed of 2.2 m/s. Calculate the magnitude of the centripetal acceleration of the child.

Given: $r = 5.1$ m; $v = 2.2$ m/s

Required: a_c

Analysis: $a_c = \frac{v^2}{r}$

Solution:

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(2.2\ \text{m/s})^2}{5.1\ \text{m}} \\ a_c &= 0.95\ \text{m/s}^2 \end{aligned}$$

Statement: The magnitude of the centripetal acceleration of the child is 0.95 m/s^2.

Sample Problem 2: Calculating the Magnitude and Direction of Centripetal Acceleration

A salad spinner with a radius of 9.7 cm rotates clockwise with a frequency of 12 Hz. At a given instant, the lettuce in the spinner moves in the westward direction (**Figure 3**). Determine the magnitude and direction of the centripetal acceleration of the piece of lettuce in the salad spinner at the moment shown in Figure 3.

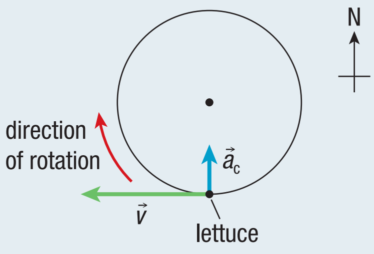


Figure 3

Given: $r = 9.7$ cm $= 0.097$ m; $f = 12$ Hz

Required: $\vec{a}_c$

Analysis: First, determine the direction of the acceleration from Figure 3. Then calculate the magnitude of the acceleration using the equation $a_c = 4\pi^2 r f^2$.

Solution: The westward velocity vector is at the south end of the spinner, as Figure 3 indicates. The direction of the centripetal acceleration is north.

$$a_c = 4\pi^2 r f^2$$
$$= 4\pi^2 (0.097 \text{ m})(12 \text{ Hz})^2$$
$$a_c = 5.5 \times 10^2 \text{ m/s}^2$$

Statement: The centripetal acceleration of the lettuce at the moment shown in Figure 3 is 5.5×10^2 m/s^2 [N].

Sample Problem 3: Calculating Frequency and Period of Rotation for a Spinning Object

The centripetal acceleration at the end of an electric fan blade has a magnitude of 1.75×10^3 m/s^2. The distance between the tip of the fan blade and the centre is 12 cm. Calculate the frequency and the period of rotation of the fan.

Given: $a_c = 1.75 \times 10^3$ m/s^2; $r = 12$ cm $= 0.12$ m

Required: f; T

Analysis: Use the equation for centripetal acceleration that includes frequency and radius: $a_c = 4\pi^2 r f^2$; rearrange and solve for f. Then use the equation relating frequency and period to calculate the period of rotation: $T = \frac{1}{f}$.

$$a_c = 4\pi^2 r f^2$$
$$\frac{a_c}{4\pi^2 r} = f^2$$
$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$

Solution: $f = \sqrt{\dfrac{a_c}{4\pi^2 r}}$

$$= \sqrt{\frac{1.75 \times 10^3 \text{ m/s}^2}{4\pi^2 (0.12 \text{ m})}}$$
$$= \pm 19.2 \text{ Hz}$$

Choose the positive root because frequency cannot be negative.

$$f = 19.2 \text{ Hz} \quad \text{(one extra digit carried)}$$
$$T = \frac{1}{f}$$
$$= \frac{1}{19.2 \text{ Hz}}$$
$$T = 5.2 \times 10^{-2} \text{ s}$$

Statement: The frequency of the fan is 19 Hz, and the period of rotation is 5.2×10^{-2} s.

Practice

1. At a distance of 25 km from the eye (centre) of a hurricane, the wind moves at nearly 50.0 m/s. Assume that the wind moves in a circular path. Calculate the magnitude of the centripetal acceleration of the particles in the wind at this distance. T/I A [ans: 0.10 m/s^2]
2. An athlete in a hammer throw competition swings the hammer with uniform circular motion clockwise as viewed from above at a speed of 4.24 m/s and a distance of 1.2 m from the centre of the circle. At a given instant, the hammer's velocity is directed southward. Determine the centripetal acceleration at this instant. T/I [ans: 15 m/s^2 [W]]
3. A ball on a string moves in a horizontal circle of radius 1.4 m. The centripetal acceleration of the ball has a magnitude of 12 m/s^2. Calculate the speed of the ball. T/I A [ans: 4.1 m/s]
4. The planet Venus moves in a nearly circular orbit around the Sun. The average radius of its orbit is 1.08×10^{11} m. The centripetal acceleration of Venus has a magnitude of 1.12×10^{-2} m/s^2. Calculate Venus's period of revolution around the Sun (a) in seconds and (b) in Earth days. T/I A [ans: (a) 1.95×10^7 s; (b) 226 days]
5. Suppose a satellite revolves around Earth in a circular orbit. The speed of the satellite is 7.27×10^3 m/s, and the radius of its orbit, with respect to Earth's centre, is 7.54×10^6 m. Calculate the magnitude of the satellite's centripetal acceleration. T/I A [ans: 7.01 m/s^2]
6. A research apparatus called a centrifuge undergoes centripetal acceleration with a magnitude of 3.3×10^6 m/s^2. The centrifuge has a radius of 8.4 cm. Calculate the frequency of the centrifuge (a) in hertz and (b) in revolutions per minute (rpm). T/I A [ans: (a) 1.0×10^4 Hz; (b) 6.0×10^5 rpm]

3.2 Review

Summary

- Uniform circular motion is the motion of any body that follows a circular path at a constant speed.
- Centripetal acceleration is the instantaneous acceleration of an object toward the centre of a circular path.
- There are three equations to determine centripetal acceleration: $a_c = \frac{v^2}{r}$, $a_c = \frac{4\pi^2 r}{T^2}$, and $a_c = 4\pi^2 r f^2$.

Questions

1. You have a puck on a string, and you twirl the puck with uniform circular motion in a horizontal circle along virtually frictionless ice. K/U T/I A
   (a) What causes the centripetal acceleration of the puck?
   (b) How does doubling the radius of the circle and leaving the speed unchanged affect the centripetal acceleration?
   (c) How does doubling the speed and leaving the radius unchanged affect the centripetal acceleration?
2. Two athletes compete in the hammer throw. One athlete can spin the hammer twice as fast as the second athlete. Compare the magnitudes of the two centripetal accelerations for the two hammer throws. Explain your answer. T/I C A
3. In a rodeo, a performer twirls a lasso (rope) at a constant speed, and the lasso turns in a circle of radius 0.42 m. The lasso has a period of rotation of 1.5 s. Calculate the magnitude of the centripetal acceleration of the lasso. T/I A
4. A motorcyclist maintains a constant speed of 28 m/s while racing on a circular track with a constant radius of 135 m. Calculate the magnitude of the centripetal acceleration of the motorcyclist. T/I A
5. The centripetal acceleration of an object at Earth's equator results from the daily rotation of Earth. Calculate the object's centripetal acceleration, given that the radius of Earth at the equator is 6.38×10^6 m. T/I A
6. An amusement park ride consists of a rotating cylinder with a coarse fabric on the walls, for friction. Participants on this ride stand against the wall as the cylinder rotates. After the cylinder reaches a constant speed, the floor of the ride drops away beneath the occupants. They remain against the wall because of the centripetal acceleration, which must be greater than about 25 m/s^2. This ride has a radius of 2.0 m. Determine the minimum frequency of rotation of the cylinder. T/I A
7. The centripetal acceleration of a car moving around a circular curve at a constant speed of 22 m/s has a magnitude of 7.8 m/s^2. Calculate the radius of the curve. T/I A
8. A jogger is running around a circular track that has a circumference of 478 m. The magnitude of the centripetal acceleration of the jogger is 0.146 m/s^2. Calculate the jogger's speed in kilometres per hour. T/I A
9. A bicycle wheel with a radius of 0.300 m is spinning clockwise at a rate of 60.0 rpm. T/I A
   (a) Calculate the period of the wheel's motion.
   (b) Calculate the centripetal acceleration of a point on the edge of the wheel if at that instant it moves westward.
10. The Moon's period of revolution is 27.3 days, and the magnitude of its centripetal acceleration is about 2.7×10^{-3} m/s^2. T/I A
    (a) Calculate the distance between the centre of the Moon and the centre of Earth. Assume that the orbit of the Moon is circular and that its speed is constant.
    (b) Compare your answer with the value provided in Appendix B. If different, suggest reasons why.
11. The record distance for the hammer throw is about 87 m. To achieve this distance, an athlete must produce a centripetal acceleration of nearly 711 m/s^2. K/U T/I A
    (a) Given a radius of 1.21 m, calculate the speed of the ball when it is released.
    (b) The athlete lets go when the ball is 2.0 m above the ground and moving at an angle of 42° above the horizontal. Determine the range. Ignore any air friction.

3.3

Centripetal Force

Think of a time when you were a passenger in a car going around a sharp curve at high speed (**Figure 1**). If the car were going fast enough, you might feel the side of the car door pushing on your side. If you look closely at Figure 1, you can see that the road is banked. In this case, the bank assists the force of friction to help make the car and passengers move in a circle more safely for a given speed. A force is also required on objects when you play crack the whip when ice skating, or when you swing a yo-yo on a string around your head. If you were to let go of the crack-the-whip line, you would no longer feel the force and would continue in a straight line in the direction you were moving. The implications of the forces causing circular motion is one reason highway exit ramps often have banked curves like the one in Figure 1. CAREER LINK

Figure 1 Passengers in a car that is going around a sharp curve at high speed will experience a strong force pushing them toward the centre of the curve.

Forces That Cause Centripetal Acceleration

As you learned in Section 3.2, any object moving with uniform circular motion has a centripetal acceleration of magnitude

$$a_c = \frac{v^2}{r}$$

From Newton's second law, we know that forces cause accelerations. So, for an object moving with uniform circular motion, we have

$$\Sigma F = ma_c$$

$$F_c = \frac{mv^2}{r}$$

where F_c is the magnitude of the net force required to make an object of mass m travel with a constant speed v in a circle of radius r. Centripetal acceleration is directed toward the centre of the circle, so the centripetal force must also be directed toward the centre of the circle according to Newton's second law.

To further analyze the forces involved in uniform circular motion, suppose a person is twirling a yo-yo on a string so that the yo-yo moves in a circle. To keep the situation simple, assume this demonstration is being performed by an astronaut in deep space, where gravitational forces are negligible (**Figure 2(a)**).

Since gravitational forces are negligible, the only force on the yo-yo comes from the string. According to Newton's second law and the equation

$$F_c = \frac{mv^2}{r}$$

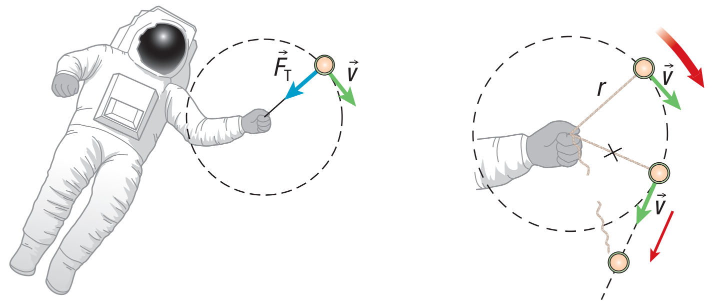
a force of magnitude $\frac{mv^2}{r}$ causes this acceleration. And since the force comes from the tension F_T in the string,

$$F_T = \frac{mv^2}{r}$$

Investigation 3.3.1

Simulating Uniform Circular Motion (page 135)

You have learned about the principles related to objects in uniform circular motion. In this investigation, you will use a simulation program to verify the relationship between frequency and variables such as force, mass, speed, radius, and period.



(a) (b)

Figure 2 (a) An astronaut in deep space twirls a yo-yo on a string. In deep space all gravitational forces are negligible, so the only force on the yo-yo is due to the tension in the string. (b) When $F_T = \frac{mv^2}{r}$, the yo-yo will move with uniform circular motion. If the string breaks, the yo-yo will move along a straight line, obeying Newton's first law.

Investigation 3.3.2

Analyzing Uniform Circular Motion (page 136)
An object in circular motion at a constant speed is constantly undergoing centripetal acceleration directed toward the centre of the circle. This investigation will give you an opportunity to observe an object in uniform circular motion and collect data to describe relationships between the object, its mass, and the radius of its path.

For the yo-yo to travel in a circle, the tension must have this value. In other situations, the force might be due to gravity, friction, or some other source. The net force that causes centripetal acceleration is called the **centripetal force**, F_c. Without such a force, the object cannot move in uniform circular motion.

centripetal force (F_c) the net force that causes centripetal acceleration

What happens if the string in Figure 2 suddenly breaks? After the string breaks (**Figure 2(b)**), the force on the yo-yo is zero. According to Newton's first law, the yo-yo will then move away in a straight-line path with a constant velocity. The yo-yo does not move radially outward, nor does it "remember" its circular trajectory. The only way the yo-yo can move in a circle is when there is a force that makes it do so. Before the string broke, the string provided that force. The following Tutorial models how to solve problems that involve different centripetal forces.

Tutorial 1 Solving Problems Related to Centripetal Force

In this Tutorial, you will solve for different variables in situations in which an object is moving with uniform circular motion.

Sample Problem 1: Determining Centripetal Acceleration and Identifying the Centripetal Force

Suppose a bug is sitting on the edge of a horizontal DVD. The bug has a mass of 5.0 g, and the DVD has a radius of 6.0 cm. The DVD is spinning such that the bug travels around its circular path three times per second. Calculate the centripetal acceleration of the bug and the net force on the bug. Also, identify the force or forces responsible for the centripetal force.

Given: $m = 5.0\text{ g} = 0.0050\text{ kg}$; $r = 6.0\text{ cm} = 0.060\text{ m}$; $t = 1.0\text{ s}$

Required: a_c; F_c; origin of F_c

Analysis: Draw an FBD to determine the force or forces responsible for the centripetal force. Calculate the speed of the DVD using $v = \frac{\Delta d}{\Delta t}$. Then use the speed in the equation for centripetal acceleration, $a_c = \frac{v^2}{r}$, and calculate the centripetal force, $\Sigma F_c = ma_c$; circumference of a circle $= 2\pi r$.

Solution: The FBD is shown in **Figure 3**. The bug's acceleration is in the horizontal plane, so the bug's acceleration in the vertical plane is zero. Therefore, the normal force, F_N, and the force of gravity, mg, must cancel: $F_N = mg$. The bug is moving with uniform circular motion, so we know a third force must provide the force required to produce the centripetal acceleration, a_c. This force keeps the bug from slipping relative to the DVD, so the force is the force of static friction. To make the bug move with uniform circular motion, the force of static friction must be directed toward the centre of the circle.

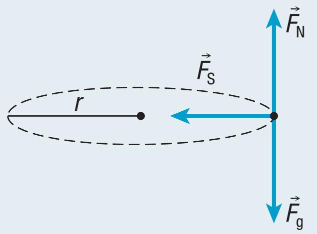


Figure 3

The bug travels around the circle three times in 1.0 s, so it travels a distance equal to three times the circumference each second:

$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{(3)(2\pi r)}{\Delta t}$$
$$= \frac{(6\pi)(0.060\ \text{m})}{1.0\ \text{s}}$$
$$v = 1.13\ \text{m/s (one extra digit carried)}$$
$$a_c = \frac{v^2}{r}$$
$$= \frac{(1.13\ \cancel{\text{m}}/\text{s})^2}{0.060\ \cancel{\text{m}}}$$
$$a_c = 21.3\ \text{m/s}^2\ \text{(one extra digit carried)}$$

The force is

$$\Sigma F_c = ma_c$$
$$= (0.0050\ \text{kg})(21.3\ \text{m/s}^2)$$
$$\Sigma F_c = 0.11\ \text{N}$$

Statement: The centripetal acceleration of the bug is 21 m/s^2, and the total force on the bug is 0.11 N. The centripetal force on the bug is static friction.

Sample Problem 2: Calculating Speed Using Apparent Weight

A roller coaster car is at the lowest point on its circular track. The radius of curvature is 22 m. The apparent weight of one of the passengers in the roller coaster car is 3.0 times her true weight. Determine the speed of the roller coaster.

Given: $r = 22$ m; $F_N = 3.0mg$

Required: v

Analysis: Draw an FBD for the scenario. The uniform circular motion is in the vertical plane in this problem. At the lowest point on the circular track, the forces on the person are gravity and the normal force. The normal force is the apparent weight. Since the roller coaster is at the low point of the track, the normal force is directed toward the centre of the circular arc defined by the track (up), and gravity is downward, away from the centre. Apply Newton's second law to relate the normal force to the speed of the roller coaster. Then apply the equation for circular motion, $F_c = \frac{mv^2}{r}$.

Solution: Figure 4 shows the FBD.

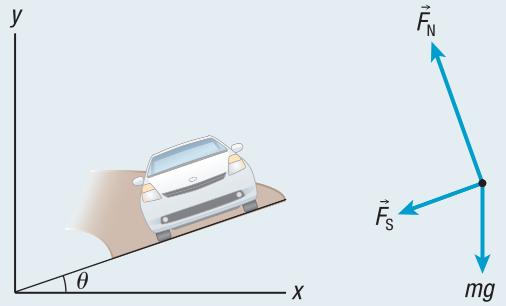
$$\Sigma F = +F_N + (-mg)$$
$$F_c = F_N - mg$$
$$\frac{mv^2}{r} = 3.0mg - mg$$
$$\frac{mv^2}{r} = 2.0mg$$
$$\frac{\cancel{m}v^2}{r} = 2.0\cancel{m}g$$
$$v = \sqrt{2.0rg}$$
$$= \sqrt{(2.0)(22\ \text{m})(9.8\ \text{m/s}^2)}$$
$$v = 21\ \text{m/s}$$

Figure 4

Statement: The speed of the roller coaster is 21 m/s.

Sample Problem 3: Calculating Speed on a Banked Turn

A car making a turn on a dry, banked highway ramp is experiencing friction (**Figure 5**). The coefficient of static friction between the tires and the road is 0.60. Determine the maximum speed at which the car can safely negotiate a turn of radius 2.0×10^2 m with a banking angle of 20.0°.



(a) (b)

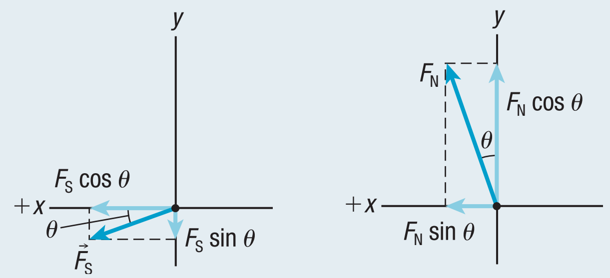
Figure 5

Given: $\mu_S = 0.60$; $r = 2.0 \times 10^2$ m; $\theta = 20.0°$

Required: v

Analysis: In two separate diagrams, draw the vector components of the frictional force and the vector components of the normal force. The car is moving with uniform circular motion, so the centripetal force on the car is $F_c = \frac{mv^2}{r}$ directed toward the centre of the circular path. In Figure 5, the centre is 2.0×10^2 m to the left of the car. Use the direction toward the centre as positive for the x-direction, and use up as positive for the y-direction. We have two unknown quantities: the speed of the car and the normal force. We can get two equations by applying Newton's second law along the vertical and horizontal directions. We can then solve for F_N and v; $F_S = \mu_S F_N$

Solution: The vector component diagrams are shown in **Figure 6**.



(a) **(b)**

Figure 6 (a) The vector components for the force of static friction (b) The vector components for the normal force

Vertical components of force:

The car is not slipping up or down the incline, so the acceleration along y is zero. The total force along y must be zero. From Figure 6(a) and 6(b),

$$\Sigma F_y = 0$$
$$\Sigma F_y = +F_N \cos\theta - F_S \sin\theta - mg$$
$$+F_N \cos\theta - F_S \sin\theta - mg = 0$$
$$F_S = \mu_S F_N$$
$$F_N \cos\theta - \mu_S F_N \sin\theta - mg = 0$$
$$F_N(\cos\theta - \mu_S \sin\theta) = mg$$
$$F_N = \frac{mg}{\cos\theta - \mu_S \sin\theta}$$

Horizontal components of force:

The total force along the horizontal direction provides the centripetal acceleration. From Figure 6(a) and 6(b),

$$\Sigma F_x = F_N \sin\theta + F_S \cos\theta$$
$$F_S = \mu_S F_N$$
$$\Sigma F_x = F_N \sin\theta + \mu_S F_N \cos\theta$$
$$ma_c = F_N \sin\theta + \mu_S F_N \cos\theta$$
$$\frac{mv^2}{r} = F_N(\sin\theta + \mu_S \cos\theta)$$

Solving for v then gives

$$v = \sqrt{\frac{F_N r(\sin\theta + \mu_S \cos\theta)}{m}}$$

Insert the result for the normal force:

$$v = \sqrt{\left(\frac{\cancel{m}g}{\cos\theta - \mu_S \sin\theta}\right)\left(\frac{r(\sin\theta + \mu_S \cos\theta)}{\cancel{m}}\right)}$$
$$= \sqrt{gr\left(\frac{\sin\theta + \mu_S \cos\theta}{\cos\theta - \mu_S \sin\theta}\right)}$$
$$= \sqrt{(9.8\ \text{m/s}^2)(2.0 \times 10^2\ \text{m})\left(\frac{\sin 20.0° + (0.60)\cos 20.0°}{\cos 20.0° - (0.60)\sin 20.0°}\right)}$$
$$v = 49\ \text{m/s}$$

Statement: The maximum speed at which the car can safely negotiate a turn with a radius of 2.0×10^2 m and with a banking angle of 20.0° is 49 m/s. It is really the horizontal components of the normal force and the force of friction that contribute to the net force and the acceleration. (Note: This is the maximum speed the car can go but not the speed the car should go.)

Practice

1. A model airplane with a mass of 0.211 kg pulls out of a dive. The bottom of the dive is a circular arc with a radius of 25.6 m. At the bottom of the arc, the plane's speed is a constant 21.7 m/s. Determine the magnitude of the upward lift on the plane's wings at the bottom of the arc. K/U T/I A [ans: 5.9 N]
2. A curved road with a radius of 450 m in the horizontal plane is banked so that the cars can safely navigate the curve. Calculate the banking angle for the road that will allow a car travelling at 97 km/h to make it safely around the curve when the road is covered with black ice. (Assume no friction.) K/U T/I A [ans: 9.3°]
3. A 2.00 kg mass is spinning horizontally in a circle on a virtually frictionless surface. It completes 5.00 revolutions in 2.00 s. The mass is attached to a string 4.00 m long. Calculate the magnitude of the tension in the string. Air resistance is negligible. K/U T/I A [ans: 2.0×10^3 N]
4. A barn swallow chasing a moth is flying in a vertical loop of radius 150 m. At the top of the loop, the vertical force exerted by the air on the bird is zero. At what speed is the swallow flying at this point? K/U T/I A [ans: 38 m/s]
5. The highway ramp in Sample Problem 3 was dry. Now suppose the highway is wet or covered in ice. Predict how the maximum speed will change. Test your prediction by using the value 0.25 for the coefficient of static friction and determining the maximum speed. K/U T/I A [ans: 36 m/s]

3.3 Review

Summary

- An object moving with uniform circular motion experiences a net force directed toward the centre of the object's circular path.
- The net force that causes uniform circular motion is the centripetal force, which may comprise one or more other forces such as gravity, the normal force, or tension.
- Combine the equation for Newton's second law with the equations for centripetal acceleration to calculate the magnitude of the net force: $\Sigma F = ma_c$; $F_c = \dfrac{mv^2}{r}$.

Questions

1. The track near the top of a roller coaster has a circular shape with a diameter of 24 m forming a hill. When you are at the top, you feel as if your weight is only one-third your true weight. Calculate the speed of the roller coaster as it rolls over the top of the hill. T/I A
2. A car with a mass of 1000.0 kg is travelling over the top of a hill, as shown in **Figure 7**. The hill's curvature has a radius of 40.0 m, and the car is travelling at 15 m/s. T/I C A

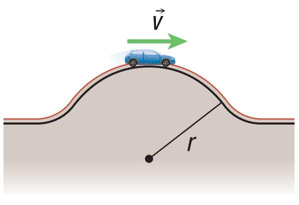


Figure 7

   (a) Draw an FBD.
   (b) Determine the magnitude of the normal force between the hill and the car at the top of the hill.
   (c) Determine the speed required to make the driver feel weightless at the top of the hill.
3. A civil engineer is designing a banked curve on a highway. The banked curve is designed to allow the cars to move safely in a horizontal circle. What will happen to the maximum speed of a car on the curve when the following changes are made? Explain your reasoning, considering each change separately. K/U T/I A
   (a) The banking angle between the road and the horizontal is increased.
   (b) The coefficient of friction between the tires and the road is larger.
   (c) A heavier car is used.
4. A car moves in a horizontal circle on a test track with a radius of 1.2×10^2 m. The coefficient of static friction between the tires and the road is 0.72. Draw an FBD, and calculate the maximum speed of the car. T/I C A
5. Consider a banked curve on an exit ramp for a highway in the middle of winter when the road surface is covered with very slippery ice. K/U T/I A
   (a) How does the banking angle of the road help drivers make it safely around the curve? What force (or component of force) is responsible? Explain your reasoning.
   (b) Explain why drivers must go much more slowly under these circumstances.
   (c) A student claims, "If banking angles help drivers safely navigate curved sections of road, why not make the banking angles significantly larger?" Identify one problem that might occur if this suggestion were used. Justify your answer.
6. An air puck with a mass of 0.26 kg is tied to a string and moves at a constant speed in a circle of radius 1.2 m. The other end of the string goes through a hole in the air table and straight down to a suspended mass of 0.68 kg, which hangs at rest (**Figure 8**). Calculate the speed of the air puck. K/U T/I A

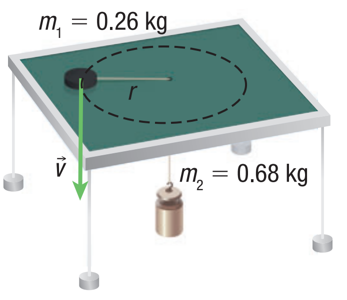


Figure 8

Rotating Frames of Reference

3.4

The force of gravity acting on any object is due strictly to the other masses in the space around it. On Earth, the gravity we experience is mainly due to Earth itself because of its large mass and the fact that we are on it. There is no device that can make or change gravity. So how can we simulate gravity? The answer is uniform circular motion. Incorporating the principles of uniform circular motion in technology has led to advances in many fields, including medicine, industry, and the space program. For example, while in training, astronauts and jet pilots lie in the compartment at the end of a large centrifuge like the one in **Figure 1**. A **centrifuge** is a device that spins rapidly. The arm of the centrifuge in Figure 1 spins around the centre, and the astronauts and pilots experience large forces that feel like a larger force of gravity pulling on them. Experiencing such forces allows us to better understand how the human body reacts during launches and in space. CAREER LINK

centrifuge a rapidly rotating device used to separate substances and simulate the effects of gravity

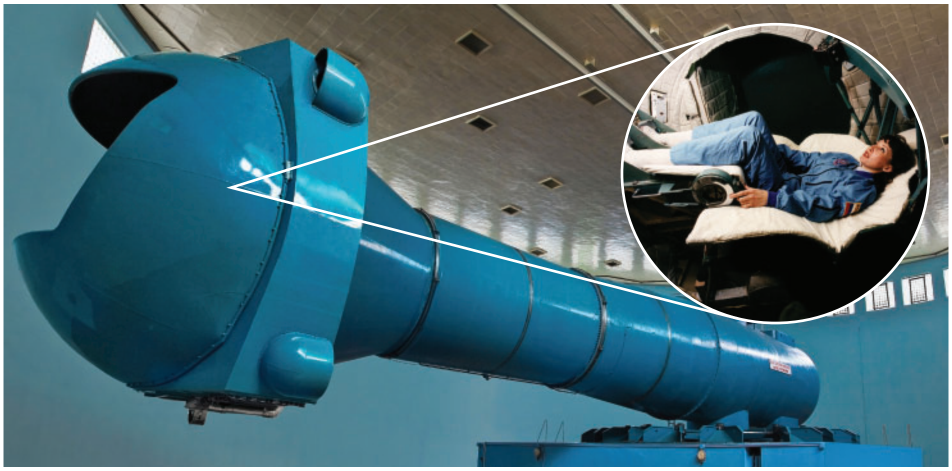


Figure 1 Russian cosmonauts (astronauts) lie in a large centrifuge such as this one as part of their training for space missions.

Centrifugal Force and Rotating Frames of Reference

Before we discuss how circular motion can simulate gravity, we need to look more closely at frames of reference and uniform circular motion. Merry-go-rounds and other rides in which people move in a circle are popular, so we will start with a merry-go-round. When you watch a merry-go-round from your vantage point on the ground, you are observing the motion of the merry-go-round relative to your reference frame on the ground. But the riders sitting on the merry-go-round observe you from a rotating frame of reference.

Now imagine that you are one of the riders sitting on the merry-go-round, leaning against a handrail as it spins. You feel as if the rail is pushing against your body. From Earth's frame of reference (the inertial frame), Newton's first law of motion explains the force that you feel when you tend to maintain your initial velocity in both magnitude and direction. When the merry-go-round turns left, you tend to go straight, but the rail prevents you from going straight. The rail pushes on you toward the centre of the ride and causes you to go in a circular path along with the merry-go-round (**Figure 2** on the next page). The centripetal force acting on your body in this situation is the push from the rail.

centrifugal force the fictitious force in a rotating (accelerating or non-inertial) frame of reference

Consider the same situation from the accelerating frame of reference of the merry-go-round. As it spins, you feel as if you are being pushed to the right toward the outside of the merry-go-round's circle. This force in a rotating frame of reference, acting away from the centre, is a fictitious force called the **centrifugal force** (**Figure 3**).

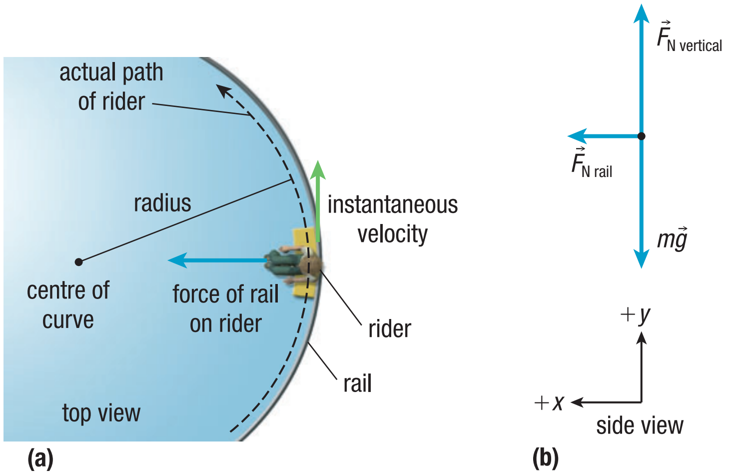


Figure 2 (a) The top view of a rider on a merry-go-round from Earth's frame of reference as the ride turns to the left. (b) The side-view FBD of the rider shows the forces acting on the rider.

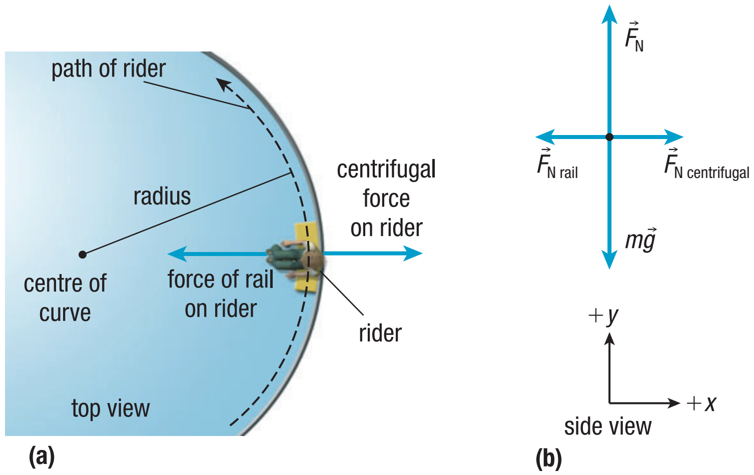


Figure 3 (a) The top view of a rider on a merry-go-round from the merry-go-round's frame of reference as the ride turns to the left. (b) The side-view FBD of the rider shows the (fictitious) centrifugal force in the non-inertial frame of reference.

Centrifugal Force and Centrifuges

Centrifuges are frequently used in medical laboratories to separate blood samples. The centrifuge rotates the test tubes containing blood samples at high speeds. Red blood cells are the densest components of blood. If the red blood cells are near the top of a test tube as the centrifuge starts spinning, centrifugal force will move the cells toward the bottom of the tube. The red blood cells settle on the bottom due to the spinning motion of the centrifuge (**Figure 4**). CAREER LINK

Figure 4 A centrifuge rotates at an extremely high rate, producing a large centripetal acceleration for the contents of the test tubes. As a result, the higher-density cells move to the outer end of the tube and separate.

To further clarify how a centrifuge works, consider the single, dense particle in the test tube at A in **Figure 5**. Note that A is near the top of the test tube. To keep the situation as simple as possible, we will disregard the fluid friction acting on this particle. As the centrifuge spins, the particle continues to move at a constant velocity because the net force acting on it is zero. This velocity will carry the particle along in a straight line toward B, near the bottom of the test tube. In the rotating frame of reference of the test tube, the fictitious centrifugal force appears to move the particle toward B. Relative to Earth's frame of reference, the particle moves according to Newton's first law of motion because it is moving in a straight line at a constant velocity while the test tube and the contents accelerate toward the centre of the centrifuge.

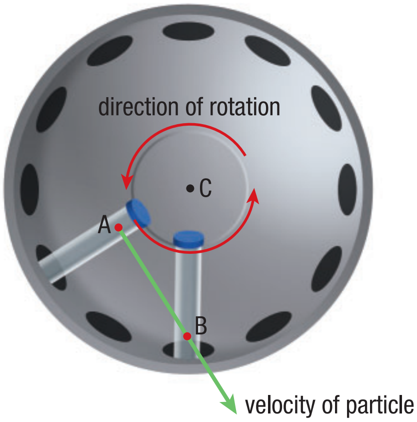


Figure 5 The particle at position A moves according to Newton's first law of motion as the centrifuge spins.

Centrifugal Force and Earth's Surface

Earth's surface is another example of a rotating and, therefore, non-inertial frame of reference. Objects near the surface of Earth are pulled down by gravity toward the centre of Earth by a centripetal force. The rotation of Earth on its axis creates a centrifugal force on objects at Earth's surface, but the effects are very small. If you stand at the equator and drop a rock, the force of gravity pulls the rock straight toward Earth's centre. There is also a centrifugal force directed away from Earth's centre relative to Earth's rotating frame of reference. The net force on the rock you dropped in Earth's rotating frame is less than the force of gravity in a non-rotating frame of reference, as shown in the FBD in **Figure 6**. The rock's acceleration at the equator is about 0.34 % less than the acceleration by gravity alone. At the equator, the magnitude of the centrifugal force is at a maximum. As you move toward the north or the south, the magnitude of the centrifugal force decreases, eventually reaching zero when you reach the poles.

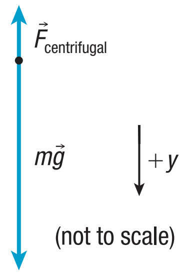


Figure 6 A falling rock at the equator experiences a small centrifugal force as well as gravity.

THE CORIOLIS FORCE

When studying the physics of the motion of objects in Earth's rotating frame of reference more closely, we discover another fictitious force. This fictitious force, called the **Coriolis force**, is perpendicular to the velocity of an object in the rotating frame of reference. The Coriolis force, named after the French mathematician Gaspard-Gustave de Coriolis, acts on objects that are in motion relative to the rotating Earth.

Coriolis force a fictitious force that acts perpendicular to the velocity of an object in a rotating frame of reference

The effect of the Coriolis force is not very noticeable on objects moving on Earth's surface. The effect is more noticeable for objects that move for a very long time above Earth's surface. Weather patterns are one example. On the television news, you may see a weather map detailing low-pressure systems that rotate counterclockwise in the northern hemisphere and clockwise in the southern hemisphere. The Coriolis force is responsible for this rotation. WEB LINK

Mini Investigation

Foucault Pendulum

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

It is difficult to envision Earth as a rotating frame of reference because, standing on its surface, you cannot see Earth move. In 1851, French physicist Jean Foucault designed an experiment to prove that Earth rotates—he strung a weight on a wire over 60 m long above Earth's surface. In this activity, you will work in a group and use a smaller pendulum and a globe to model Foucault's demonstration.

Equipment and Materials: eye protection; globe (or large ball); 50 g mass; wooden splints or straws; string; tape

1. Put on your eye protection. Make a pendulum and attach it to the globe, in a setup similar to that in **Figure 7**.

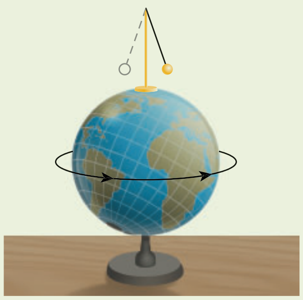


Figure 7

2. Rotate the globe slowly, and observe what happens to the mass.

Use caution when spinning the globe to ensure that the pendulum remains securely attached.

3. Rotate the globe more quickly, and observe what happens.

A. How does the rotating globe affect the behaviour of the pendulum mass? T/I

B. How does the period of rotation affect the behaviour of the pendulum mass? What does this imply about the effect of the rotation of Earth on a Foucault pendulum? T/I

C. How would the observed behaviour of a Foucault pendulum at the equator differ from the observed behaviour at your latitude? T/I A

Artificial Gravity

Now that we have had a closer look at frames of reference and uniform circular motion, we are ready to discuss how circular motion can simulate gravity. Have you ever wondered why astronauts and other objects in orbiting spacecraft look as if they are floating? The spacecraft and everything in it are in free fall, and that makes the apparent weight of the spacecraft and all the objects zero.

Over the past several decades, researchers have investigated the effects of extended free fall on the human body. We know that the absence of forces against the human body causes the muscles to become smaller and the bones to lose calcium and become brittle. The heart and blood vessels swell from the buildup of excess body fluids in the upper body. This imbalance of fluids causes the kidneys to release excess urine.

Astronaut-training programs include vigorous exercise programs on space flights to help astronauts reduce these negative effects on their bodies. The problems caused by extended free fall would still be catastrophic if humans travelled in space over the long periods needed to reach Mars and other parts of the solar system. To combat this problem, scientists and engineers are designing interplanetary spacecraft that have **artificial gravity**, which is a situation in which the value of gravity has been changed artificially.

artificial gravity a situation in which the value of gravity has been changed artificially to more closely match Earth's gravity

Making a spacecraft rotate constantly can simulate gravity. And, if the spacecraft rotates at the appropriate frequency, the simulated gravity can equal Earth's gravity, making the astronauts' apparent weight equal to their weight on Earth. The following Tutorial illustrates the variables needed to simulate Earth's gravity in space.

Tutorial 1 Simulating Gravity

This Tutorial models how to solve problems in which an object moving with uniform circular motion simulates the effects of gravity.

Sample Problem 1: Designing a Space Station

Consider a rotating space station similar to the one in **Figure 8**. The radius of the station is 40.0 m. How many times per minute must the space station rotate to produce a force due to artificial gravity equal to 30.0 % of Earth's gravity?

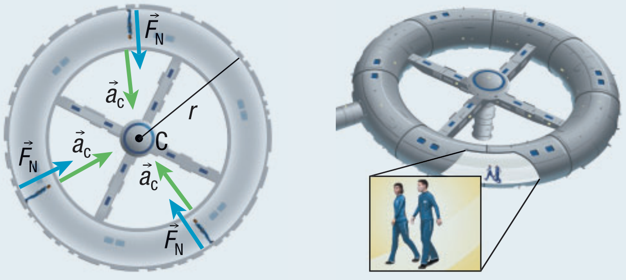


Figure 8

Given: $F_N = 0.30mg$; $r = 40.0$ m; $g = 9.8$ m/s^2

Required: f (revolutions per minute, rpm)

Analysis: The only force acting on the astronauts in Figure 8 is the normal force, and it is directed toward the centre of the station. Therefore, $\Sigma F_c = F_N = \dfrac{mv^2}{r}$. We can use this equation to determine the speed of the space station. Once we have the speed, we can determine the period of rotation, $T = \dfrac{d}{v}$. Then we can use the period, T, to determine the frequency (in revolutions per minute), $f = \dfrac{60 \text{ s/min}}{T}$; circumference of a circle $= 2\pi r$.

Solution:
$$F_N = \frac{mv^2}{r}$$
$$0.30\cancel{m}g = \frac{\cancel{m}v^2}{r}$$
$$v = \sqrt{0.30gr}$$
$$= \sqrt{(0.30)(9.8 \text{ m/s}^2)(40.0 \text{ m})}$$
$$v = 10.8 \text{ m/s (one extra digit carried)}$$
$$T = \frac{d}{v}$$
$$= \frac{2\pi r}{v}$$
$$= \frac{2\pi(40.0 \cancel{\text{m}})}{10.8 \cancel{\text{m}}/\text{s}}$$
$$T = 23.3 \text{ s (one extra digit carried)}$$
$$f = \frac{60 \text{ s/min}}{T}$$
$$= \frac{60 \cancel{\text{s}}/\text{min}}{23.3 \cancel{\text{s}}}$$
$$f = 2.6 \text{ rpm}$$

Statement: The space station must rotate 2.6 times per minute to produce a force due to artificial gravity equal to 30.0 % of Earth's gravity.

Practice

1. A spacecraft travelling to Mars has an interior diameter of 324 m. The craft rotates around its axis at the rate required to give astronauts along the interior wall an apparent weight equal in magnitude to their weight on Earth. K/U T/I A
   (a) Calculate the speed of the astronauts relative to the centre of the spacecraft. [ans: 39.8 m/s]
   (b) Determine the period of rotation of the spacecraft. [ans: 26 s]
2. Suppose there are two astronauts on the space station in Sample Problem 1. One has a mass of 45 kg, and the other has a mass of 65 kg. Would each astronaut experience artificial gravity equal to about 30.0 % of Earth's gravity? Explain your answer. K/U T/I A
3. Imagine another planet with an acceleration of 10.00 m/s^2 at its equator when ignoring the rotation of the planet. The radius of the planet is 6.2×10^6 m. An object dropped at the equator yields an acceleration of 9.70 m/s^2. Determine the length of one day on this planet. K/U T/I A [ans: 7.9 h]
4. A 56 kg astronaut stands on a bathroom scale inside a rotating circular space station. The radius of the space station is 250 m. The bathroom scale reads 42 kg. At what speed does the space station floor rotate? K/U T/I A [ans: 43 m/s]
5. In theory, if a car went fast enough it could fly off Earth's surface. This is because, at a fast enough speed, Earth's gravity is not strong enough to pull the car in a circle with a radius equal to the radius of Earth. Approximately how fast would a car have to move for this to happen? Refer to Appendix B for Earth's radius. K/U T/I A [ans: 7.9×10^3 m/s]

3.4 Review

Summary

- A centrifuge is a device that spins rapidly and is used to separate substances by density, as well as simulate the effects of gravity. A spinning centrifuge applies a centrifugal force to the objects it contains.
- A rotating frame of reference is the frame of reference of any object moving in a circle.
- Centrifugal force is a fictitious force used to explain the outward force observed in a rotating frame of reference.
- Making a spacecraft rotate at the appropriate frequency can simulate gravity equal to Earth's gravity.

Questions

1. When you swing a bucket full of water in a vertical circle at just the right speed, the water stays inside. Explain why. K/U C A
2. Explain how the spin cycle of a washing machine uses circular motion to remove water from clothes. K/U C A
3. You are standing 2.7 m from the centre of a spinning merry-go-round holding one end of a string tied to a 120 g mass. The merry-go-round has a period of 3.9 s. K/U T/I C A
   (a) Draw a system diagram of the situation.
   (b) Draw an FBD of the mass in Earth's frame of reference.
   (c) Draw an FBD of the mass in the merry-go-round's rotating frame of reference.
   (d) What angle does the string make with the vertical?
   (e) Determine the magnitude of the tension in the string.
4. Show that the acceleration of an object dropped at the equator is about 0.34 % less than the acceleration due to gravity alone. K/U T/I A
5. In a science fiction movie, a spacecraft has a rotating section to provide artificial gravity for the long voyage. A physicist watches a scene filmed from the interior of the spacecraft and notices that the diameter of the rotating section of the craft is about five times the height of an astronaut walking in that section (or about 10 m). Later, in a scene showing the spacecraft from the exterior, she notices that the living quarters of the ship rotate with a period of about 30 s. Did the movie get the physics right? Compare the centripetal acceleration of a 1.7 m–tall astronaut at his feet to that at his head. Compare these accelerations to g. T/I C A
6. A space station has a radius of 100 m. K/U T/I A
   (a) What period of rotation is needed to provide an artificial gravity of g at the rim?
   (b) At what speed is the rim moving?
   (c) What is your apparent weight if you run along the rim at 4.2 m/s opposite the rotation direction?
   (d) What is your apparent weight if you instead run in the direction of rotation?
   (e) In which direction would you run to get the best workout, with or against the rotation? Or does it matter?
7. An astronaut with a mass of 65 kg is in a rotating space station with a radius of 150 m. She stands on a scale, and the reading is 540 N. K/U T/I A
   (a) At what acceleration do objects fall when dropped near the floor of the space station?
   (b) Calculate the speed of rotation of the outer rim of the space station.
   (c) Calculate the period of rotation of the space station.
8. A centrifuge spins with a frequency of 1.1×10^3 Hz. A particle in a test tube is positioned 3.4 cm from the centre of the centrifuge. K/U T/I A
   (a) Determine the acceleration of the particle at this position from Earth's frame of reference.
   (b) Why do you think centrifuges need such a high frequency?
   (c) Why do you think medical researchers want to separate particles at all?
9. Research large-scale centrifuges. In a format of your choosing, describe how large-scale centrifuges are used in wastewater treatment. K/U T/I C A

WEB LINK

The Physics of Roller Coasters

SKILLS HANDBOOK A3

ABSTRACT

The earliest known ride resembling a roller coaster appeared in seventeenth-century Russia in the form of a large ice slide built on top of a wooden structure. Over the centuries, roller coasters have become more sophisticated in design and structure. The first roller coaster design that had a loop appeared in the early twentieth century. The roller coaster car had to move fast enough that it could complete the circle without falling. However, the speed required to accomplish this is too fast, and many people were injured on the ride. Today, roller coaster loops are in a shape called a clothoid.

Roller Coaster Designs

The earliest rides classified as roller coasters date from the early seventeenth century, in St. Petersburg, Russia. Builders constructed 21 m–tall wooden structures and covered them with sheets of ice. Riders climbed stairs at the back of the structure, sat on a sled, and coasted down slopes hundreds of metres long. Later, grooved tracks were added and the sleds were fitted with wheels.

Amusement park designers constructed the first looping roller coaster in the early twentieth century. It had one circular loop. The roller coaster car had to be fast enough that it could complete the circle without falling, but many people were injured on the ride because of the high speed required. Designers soon abandoned this dangerous design in favour of a safer one. Today, looping roller coasters have a much different design. They curve with a radius that is longer at the bottom of the loop and shorter at the top of the loop. This shape is called a clothoid loop (**Figure 1**).

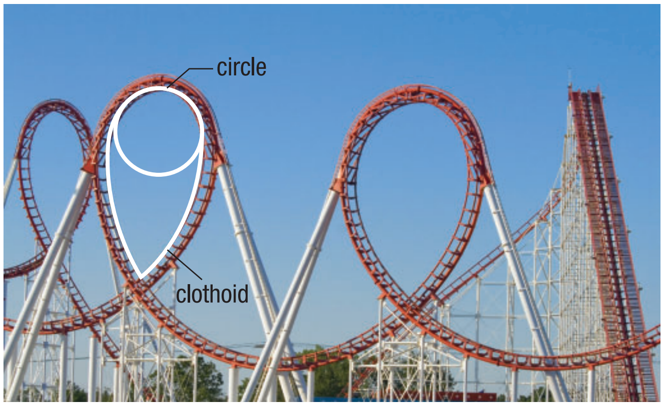


Figure 1 Modern roller coasters have clothoid loops.

Comparing the Two Designs

Using our understanding of circular motion, we can compare the old, circular roller coaster design to the clothoid design and see why the old design is dangerous. To accomplish this, first assume we have two riders, one on each roller coaster design. Next, assume that the heights of both designs are the same but the radius of the circular design is twice the radius of the clothoid design at the top (**Figure 2**).

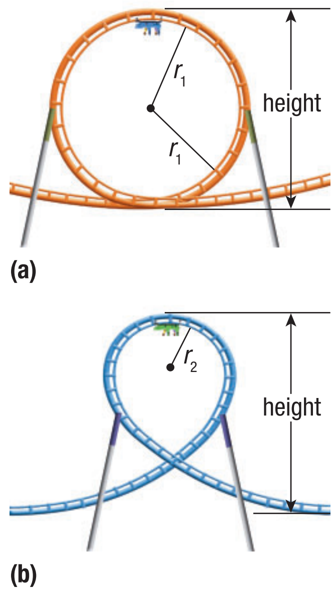


Figure 2 (a) The circular design (b) The clothoid design

Assuming the roller coaster car is not attached to the track in any way, the car would need a minimum speed at the top of either type of loop or it would simply fall off at some point. We can calculate the minimum speed for each type of loop and compare them. For simplicity, assume the radius of the circular loop is 15 m and the radius of the clothoid at the top is 7.5 m.

For the circular loop:

$$\Sigma F = ma_c$$

$$F_N + mg = \frac{mv^2}{r}$$

Set $F_N = 0$ to calculate the minimum speed.

$$0 + \cancel{m}g + = \frac{\cancel{m}v^2}{r}$$

$$g = \frac{v^2}{r}$$

$$v^2 = gr$$

$$v = \sqrt{gr} \quad \text{Take the positive root.}$$

$$= \sqrt{(9.8\ \text{m/s}^2)(15\ \text{m})}$$

$$v = 12\ \text{m/s}$$

For the clothoid loop:

$$v = \sqrt{gr}$$
$$= \sqrt{(9.8 \text{ m/s}^2)(7.5 \text{ m})}$$
$$v = 8.6 \text{ m/s}$$

The minimum speed of the old design roller coasters had to be much faster than the minimum speed of the clothoid design roller coaster to clear the loop, even though the heights of both loops are equal (Figure 2). Changing the radius of the loop made the roller coaster safer.

If you were moving at 8.6 m/s at the top of a clothoid loop of this design, you would feel weightless for an instant. This results because the normal force drops down to zero at the top and your apparent weight drops to zero. Keep in mind that gravity still acts on you at this point to keep you moving in a circle, but you lose sense of it because you are in free fall.

Roller Coasters and Apparent Weight

Typically, the ride moves much faster than the minimum speed required, and the riders experience normal forces much larger than those in everyday life. This is part of the thrill, of course, but if your apparent weight (normal force) becomes too large or suddenly increases, it can be dangerous. In fact, this is another reason for using the clothoid design.

Now consider an old roller coaster that has a horizontal section of track leading into a circular loop. Once the ride enters the loop, the riders will suddenly experience a centripetal force directed up toward the centre of the loop. The normal force must suddenly increase, not only to overcome gravity but to produce this large centripetal force. This sudden increase in apparent weight can be dangerous to riders.

How does a clothoid design solve this problem? One of the features of a clothoid loop is that the radius of curvature of a clothoid gradually decreases from top to bottom. Even though the radius of curvature of the clothoid at the top is much smaller than that of a circular loop, the same clothoid has a much larger radius of curvature at the bottom. This larger radius of curvature decreases the centripetal force experienced by riders at the bottom of the loop according to the equation $F_c = \frac{mv^2}{r}$. Since the radius (in the denominator) is larger, the centripetal force required is smaller. This means the normal force required at the start of the loop is reduced and riders do not experience a large sudden increase in apparent weight. In addition, since the radius gradually decreases, the apparent weight is gradually increased, allowing the rider time to adjust without taking away from the excitement of the ride. CAREER LINK

Further Reading

Alcorn, Steve. (2007). *Theme park design: Behind the scenes with an engineer*. Seattle: CreateSpace.

Baine, Celeste. (2007). *The fantastical engineer: A thrillseeker's guide to careers in theme park engineering*. Springfield, OR: Engineering Education Service Center. Print and e-book.

The Imagineers. (2010). *Walt Disney imagineering: A behind the dreams look at making more magic real*. New York: Disney editions. 1st ed.

WEB LINK

3.5 Questions

1. You are on a clothoid roller coaster upside down at the top of a loop with an accelerometer like the one in **Figure 3**. The radius of the loop's curvature is 18 m. You experience a force of 2.0 times your normal weight from your seat. K/U T/I A

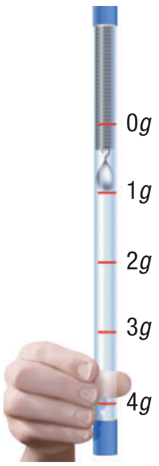


Figure 3 When this accelerometer is stationary, the reading is 1*g*, as shown here.

   (a) Calculate your speed at the top of the loop.
   (b) Identify the forces acting on the accelerometer.
   (c) What reading will you observe at the top of the loop if the accelerometer is calibrated as in Figure 3?
   (d) If you actually tried to use an accelerometer like the one in Figure 3 on a roller coaster, what likely sources of error would you expect?

2. A clothoid loop in a roller coaster has the same height as a circular loop but half the radius at the top. A rider at the top of either loop typically experiences a net force of 1.5*mg*. What is the ratio of the speed at the top of the circular loop to the speed at the top of the clothoid loop? K/U T/I A

3. Explain in your own words why the normal force must be set to zero to calculate the minimum possible speed a rider can have at the top of a loop. K/U T/I C A

4. A 62 kg rider is moving at a speed of 22 m/s at the bottom of a loop that has a radius of 35 m at that point. Determine the normal force acting on the rider due to the seat. K/U T/I

3.6 Explore an Issue in Dynamics

Improvements in Athletic Technology

SKILLS MENU

- Defining the Issue
- Researching
- Identifying Alternatives
- Analyzing
- Defending a Decision
- Communicating
- Evaluating

Athletic performance is often a function of innate physical abilities and technical equipment. Winners in early tennis competitions played with speed, agility, and coordination. They also wore specialized shoes and clothing, and used specialized racquets. The quality of play in those early tennis matches was different in many ways from that of the tennis matches we watch today. Over the decades, clothing and equipment improvements have been made in every sport: track and field, hockey, cycling, tennis, baseball, and snowboarding, to name a few (**Figure 1**).

Figure 1 (a) Some early snowboards were made of plywood. (b) Modern snowboards are lighter weight and faster because of the new materials and construction.

Thanks to innovations in the field, athletes today run faster, jump higher, and hit harder. The technological innovations have also expanded to include sports nutritionists, physicians, trainers, and psychologists. Everyone seems determined to push the boundaries of human ability, and engineers and scientists are designing the equipment that keeps moving that boundary forward. CAREER LINK

The list of athletic items subject to technological innovation for enhanced performance is staggering. We tend to think of bats, balls, and clothes, but engineers are also redesigning playing surfaces, such as field turf and court flooring. Sports physicians are developing more precise surgeries, and nutritionists are creating recipes for muscle-recovery drinks. CAREER LINK

With professional sports teams in Canada generating roughly $1.5 billion in revenue per year, sports is a serious business. In 2010, Americans spent in excess of $414 billion on their sports industry. Innovations in athletic technology will continue to generate profits, and interest in sports will continue to grow.

The Issue

SKILLS HANDBOOK A4

Sports technology is a rapidly growing industry. With thousands of new products on the market claiming to improve performance, athletes, consumers, and event organizers need to consider which products live up to their claims and which products do not. Is it fair to compare the results of previous sporting events with those aided by improved technology and equipment?

You are a member of an athletic committee for the Olympics or a professional league that is considering the fairness of comparing previous accomplishments and records to current results. The committee has asked you to investigate the issue and report back. Your teacher and classmates will represent the members of the committee.

Goal

To research the issue, take a stand on the issue, and then report your findings and make recommendations to the committee

Research

Choose a sport. Research specific details related to the sport you have chosen. You may wish to consider the following ideas in your research:

- Serious research and engineering goes into the design of sports products, such as shoes, swimwear, balls, javelins, and clothing. Some of these innovations have stemmed from NASA research. What are some recent developments in sports products? How have they affected performance or even the nature of the sport?
- Is there any evidence to suggest that changing from wood to aluminum hockey sticks improves hockey shots?
- Is there any evidence to suggest that the change in tennis racquets over the years corresponds to improved serves or performance?
- Have any new developments in cycling ever produced a measurable improvement in the performance of an athlete during the Tour de France?
- At the turn of the twentieth century, 18 professional football players in the United States died as a result of the poorly padded equipment they wore. What is the origin of protective football gear?
- In the sport of jai alai, players use a scoop made of old-fashioned wicker to catch and throw a ball at speeds of around 300 km/h. The equipment has not changed over the years. Are modern wood resins and metal alloys better materials?
- Are there any disadvantages to society and the environment in improving sporting equipment and related technologies? WEB LINK

UNIT TASK BOOKMARK

You can use some of the sports concepts in this section when you complete the Unit Task on page 146.

Possible Solutions

You may wish to consider the following questions to help you form an opinion:

- Summarize any evidence you found in your research related to improved athletic ability and modern equipment and clothing related to the sport you chose.
- Are manufacturers contributing to sports in a positive way, or are they simply in the market to improve profits?
- Have any controversies developed over the use of new sports technology?
- What are the implications for professional athletes as sports technology continues to advance?
- What are the advantages and disadvantages of improved sports technology to society and the environment?

Decision

Decide whether you think that the advances in sports technologies have improved sports in a quantitative way. Is the change desirable or controversial? Identify and consider all stakeholders before making a final decision.

Communicate

- Prepare a slide show or multimedia presentation for your committee. Your presentation should help you convince the committee of your decision.
- The presentation should highlight the changes in sports technology over the years.
- Include performance statistics and comparisons to similar products, using charts, tables, flow charts, or any other suitable format.
- Be prepared to answer questions from the members of the committee.

Plan for Action

Prepare a letter to be submitted to the international governing body for your sport. Your letter should include a summary of the findings that led to your decision and a recommendation for the governing body to implement.

Investigation 3.3.1 OBSERVATIONAL STUDY

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Simulating Uniform Circular Motion

You have learned about the principles related to objects in uniform circular motion. In this investigation, you will use a simulation program to verify the relationship between centripetal force and variables such as mass, speed, radius, frequency, and period for a mass attached to a rope swinging in a circle with uniform circular motion.

Purpose

SKILLS HANDBOOK A2.4

To verify the relationship between centripetal force and variables such as mass, speed, radius, frequency, and period in uniform circular motion

Equipment and Materials

- access to a computer with an Internet connection
- graph paper or graphing software

Procedure

1. Go to the Nelson Science website and start the simulation.
2. Display the vectors for force, acceleration, and velocity in your simulation. Determine net force (centripetal force) for different values of the radius, keeping the mass and speed constant. Record all information in a table, and use at least five different values of the radius.
3. Graph the centripetal force versus the radius and the centripetal force versus $\frac{1}{\text{radius}}$ separately.
4. Repeat Steps 1 and 2, but change the mass and keep the speed and radius constant. Graph the centripetal force versus the mass.
5. Repeat Steps 1 and 2, but change the speed and keep the radius and mass constant. Graph the centripetal force versus the speed and the centripetal force versus the square of the speed.
6. Determine the period and frequency for each speed used in Step 5, and record the results in your table.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) Examine all your graphs. Calculate the slope of each graph that produces a straight line. How is the slope of each graph related to mass, speed, and/or radius? Explain your reasoning. K/U T/I C

(b) What is the effect of increasing the following on the centripetal force? Explain your reasoning. T/I

(i) radius

(ii) mass

(iii) speed

(c) What would a graph of centripetal force versus $\frac{mv^2}{r}$ look like? What would the slope of this graph equal? Explain your reasoning. K/U T/I C A

(d) How is the speed of the mass related to the frequency and the period? T/I

(e) What happens to the centripetal force when the frequency increases? When the period increases? T/I

Apply and Extend

(f) How would you use this simulation to describe the motion of a satellite in orbit? T/I C A

WEB LINK

Investigation 3.3.2 CONTROLLED EXPERIMENT

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Analyzing Uniform Circular Motion

You have learned that while an object is in uniform circular motion at a constant speed, its velocity is also constantly changing direction. This causes the object to accelerate toward the centre of its circular path. In this investigation, you will build an apparatus to observe a small object in uniform circular motion and collect data to describe relationships between the object, its mass, and the radius of its path.

Testable Question

SKILLS HANDBOOK A2.2

How do the magnitude of the force, the radius of a circular path, and an object's mass affect the frequency of the revolution of an object in uniform circular motion?

Prediction

Predict the relationship between the frequency of revolution and each variable in the Testable Question. Explain your reasoning.

Variables

Read the Testable Question, Experimental Design, and Procedure, and identify the dependent, independent, and controlled variables.

Experimental Design

Figure 1 shows a simple setup that can be used to perform this investigation. You will hold a hollow tube vertically in your hand while you twirl the rubber stopper around in a horizontal circle. A string is tied to the rubber stopper and then passed down through the hollow tube, where it is tied to the mass. The force of gravity provides the tension required to make the rubber stopper move in a circle.

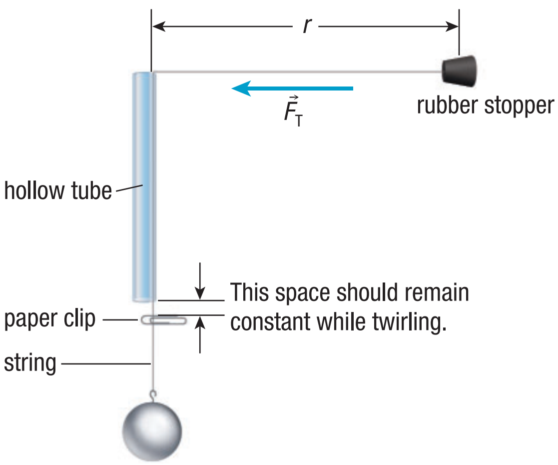


Figure 1

Equipment and Materials

- eye protection
- electronic balance or scale
- 3 small rubber stoppers with centre holes
- hollow tube
- 50 g, 100 g, 200 g, 250 g masses
- metre stick
- 1.5 m string or fishing line
- paper clip or masking tape
- graph paper or graphing software

Procedure

1. Create a data table for each of the following observations: three sets of values for changing the force of tension, three sets for changing the radius, and three sets for changing the mass.
2. Measure and record the mass, in kilograms, of each rubber stopper.
3. Tie one rubber stopper tightly to one end of the string.
4. Thread the string through the tube, hang the 200 g mass on the other end (Figure 1), and put on your eye protection.

Wear eye protection. Be sure the area around you is clear from material hanging from the ceiling that you may accidentally hit while swinging the stopper. Be careful not to drop the masses on your feet. Do not wear open-toed shoes. Ensure no one can be hit by the stopper.

5. Practise swinging the stopper around your head with a constant speed and constant radius. Make sure you are comfortable and proficient with this step before proceeding to the next step.
6. Lay the equipment on the floor, and measure a 75 cm distance from the centre of the stopper to the top of the hollow tube. Fix this radius by placing the paper clip at the bottom of the tube (Figure 1).
7. Swing the stopper at a constant speed at a radius of 75 cm. Try to keep the paper clip slightly below the bottom of the tube to ensure that it is not pushing on the tube and increasing the tension.

8. Complete 20 cycles, and record the time in your table. To obtain a reasonable average value, you will need to repeat this step several times.
9. Calculate the frequency of revolution, and record it in your table.
10. Repeat Steps 6 to 9 using a different tension force by changing the mass at the end of the string to 150 g and then to 100 g.
11. Measure the time for 20 complete cycles when the radius is 60 cm and when the radius is 45 cm, with the same mass of stopper and the tension force due to the 100 g mass. Calculate all the frequencies and record them in your table.
12. Add another stopper and measure the time for 20 complete cycles at a constant radius of 75 cm and a constant tension force due to the 100 g mass.
13. Repeat the process with a third stopper and the other masses. Calculate the frequencies and record them in your table.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) What variables were measured and/or manipulated in this investigation? What type of relationship was being tested? T/I

(b) Graph the relationships between the frequency of revolution and each of the following:
- the magnitude of the tension force
- the radius of the circle
- the mass of the object in motion T/I C

(c) Derive an equation for the frequency in terms of the tension, the radius, and the mass by combining your results from (b). T/I

(d) The equation $F_c = 4\pi^2 mrf^2$ gives the magnitude of the net force causing the acceleration of an object in uniform circular motion. T/I A
   (i) Manipulate the equation to solve for frequency. How does this compare with your results from (c)?
   (ii) What are the most likely causes for any discrepancies?

(e) To obtain the best accuracy, the tension force acting on the stopper should be horizontal. What happens to the accuracy as the frequency of revolution of the stopper increases? T/I A

(f) What sources of error did you encounter and how did you minimize them? T/I A

Apply and Extend

(g) Explain how this investigation illustrates all three of Newton's laws of motion. T/I C

(h) How might you apply your findings to a sports activity that involves circular motion? T/I A

(i) If you look carefully at the swinging stopper, you will notice that the string tied to the stopper is not completely horizontal. K/U T/I C
   (i) What effect, if any, will this have on the accuracy of your results?
   (ii) What happens to the orientation of the string as the speed of the stopper increases? Explain your reasoning.

Summary Questions

1. Create a study guide for this chapter based on the Key Concepts on page 106. For each point, create three or four subpoints that provide further information, relevant examples, explanatory diagrams, or general equations.
2. Look back at the Starting Points questions on page 106. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers you wrote at the beginning of the chapter. Note how your answers have changed.
3. Design a one-page graphic organizer that summarizes and connects the concepts in this chapter.

Vocabulary

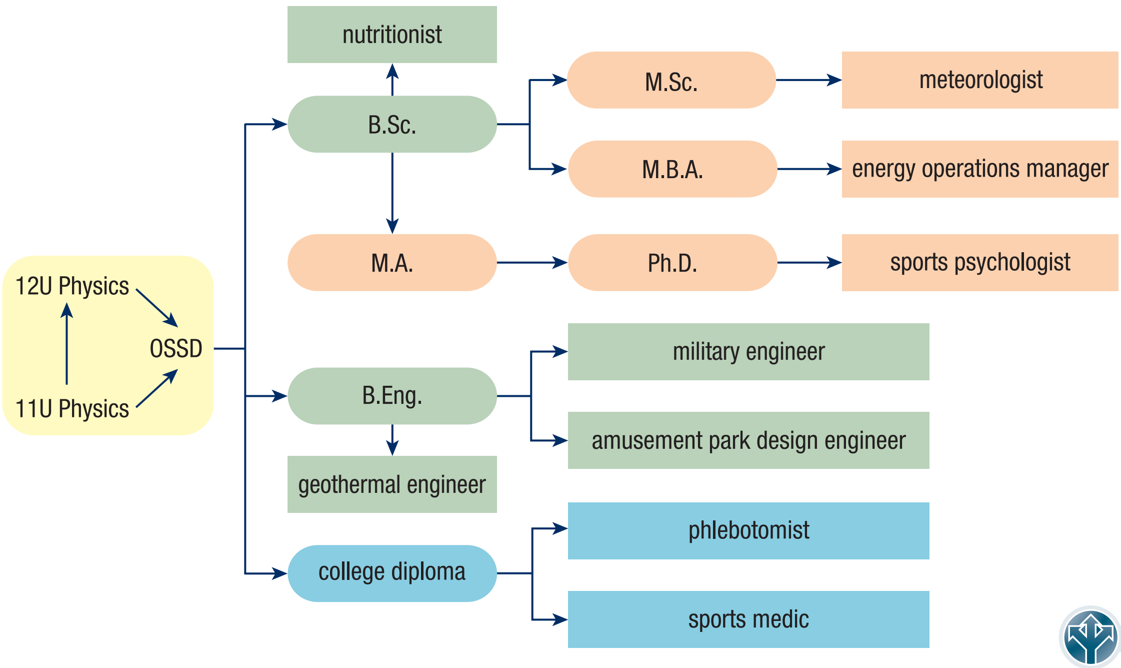
frame of reference (p. 108)
inertial frame of reference (p. 108)
non-inertial frame of reference (p. 108)
fictitious force (p. 109)
apparent weight (p. 111)
uniform circular motion (p. 114)
centripetal acceleration (p. 114)
period (p. 116)
frequency (p. 117)
centripetal force (p. 121)
centrifuge (p. 125)
centrifugal force (p. 126)
Coriolis force (p. 127)
artificial gravity (p. 128)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers mentioned in this chapter.

1. Select an interesting career that involves the use of centrifuges. Research the educational pathway you would need to follow to pursue this career.
2. What is involved in becoming an amusement park design engineer? Research at least two pathways that could lead to this career, and present your findings in the form of a graphic organizer similar to the one shown here.



CAREER LINK

CHAPTER 3 SELF-QUIZ

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. How does the acceleration of a non-inertial frame of reference cause objects in the frame to move? (3.1) K/U
   (a) in a straight line
   (b) at a constant velocity
   (c) as if a force were acting on them
   (d) as if they remained at rest
2. In addition to the radius of an object's path, what variable do you need in order to calculate an object's centripetal acceleration? (3.2) K/U
   (a) the object's direction
   (b) the object's mass
   (c) the object's period
   (d) the object's circumference
3. An object in uniform circular motion with speed v experiences a centripetal force, F_c. What value of F_c is needed if the radius of the path is halved and the velocity is kept the same? (3.3) K/U T/I A
   (a) $\frac{F_c}{4}$
   (b) $\frac{F_c}{2}$
   (c) $2F_c$
   (d) $4F_c$
4. Which particle experiences the largest centripetal force in a centrifuge? (3.3) K/U T/I
   (a) a 0.05 g particle at a distance of 2 cm from the centre
   (b) a 0.05 g particle at a distance of 5 cm from the centre
   (c) a 0.1 g particle at a distance of 2 cm from the centre
   (d) a 0.1 g particle at a distance of 5 cm from the centre
5. A 30.0 kg mass is attached to the end of a light wooden stick of length 1.0 m and swung around in a vertical circle at a constant speed of 12 m/s. What is the maximum tension in the stick? (3.3) K/U T/I A
   (a) 2.9×10^2 N
   (b) 4.0×10^3 N
   (c) 4.3×10^3 N
   (d) 4.6×10^3 N
6. A car moves around a banked curve at a constant speed over black ice with virtually no friction. What causes the car to accelerate? (3.3) K/U A
   (a) the normal force
   (b) gravity
   (c) the horizontal component of the normal force
   (d) the vertical component of the normal force
7. When will a bathroom scale with nothing on it read zero? (3.4) K/U A
   (a) on the ground
   (b) in an elevator moving up
   (c) in an elevator moving down
   (d) in space
8. A cart on a roller coaster is upside down at the top of a clothoid loop. At this point, the riders feel weightless. The radius of the loop at the top is 10.0 m. What is the speed of the cart? (3.5) K/U T/I A
   (a) 7.0 m/s
   (b) 9.9 m/s
   (c) 14 m/s
   (d) 98 m/s

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

9. During free fall, the apparent weight is zero because there is no force of gravity acting on the objects. (3.1) K/U
10. For two objects moving in the same circular path with different speeds, the faster object experiences less centripetal acceleration. (3.2) K/U
11. When two objects move in the same circular path with the same speed, the heavier object requires a greater centripetal force. (3.3) K/U
12. Earth's surface is an example of a rotating non-inertial frame of reference. (3.4) K/U
13. The effect of the Coriolis force is very noticeable on everyday objects moving along Earth's surface. (3.4) K/U

Go to Nelson Science for an online self-quiz.
WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. Which of the following describes an inertial frame of reference? (3.1) K/U
   (a) one in which Newton's first law of motion holds true
   (b) one in which Newton's first law of motion does not apply
   (c) one in which Newton's second law of motion no longer applies
   (d) one in which Newton's third law of motion no longer applies
2. Which of the following is an example of a non-inertial frame of reference? (3.1) K/U
   (a) a spinning centrifuge
   (b) a digital clock on a moving bus
   (c) an airplane moving with a constant velocity
   (d) a stationary DVD
3. Which of the following describes an object that follows a circular path at a constant speed? (3.2) K/U
   (a) inertial motion
   (b) uniform circular motion
   (c) motion with constant acceleration
   (d) motion with constant velocity
4. Which of the following would result if a tetherball on a rope came off the rope midway through its path around the pole? (3.3) K/U A
   (a) The ball would continue its circular path around the pole, eventually dropping with the force of gravity.
   (b) The ball would fly away from the pole in the straight-line direction it was travelling at the moment it came off the rope.
   (c) The ball would drop to the ground at the moment it came off the rope.
   (d) The ball would continue to move in its circular path around the pole but with a decreasing radius.
5. In which of the following directions is the centripetal force acting on an object undergoing circular motion? (3.3) K/U
   (a) in a straight line away from the centre of the object's path
   (b) in a straight line away from the object at a 90° angle
   (c) toward the centre of the circular path
   (d) along the object's path
6. Which of the following causes merry-go-round riders to feel as if they are being pushed away from the centre of the ride? (3.4) K/U
   (a) being in an inertial reference frame
   (b) the Coriolis force
   (c) centripetal acceleration
   (d) centrifugal force

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

7. An amusement park ride moving down with a constant velocity is an example of a non-inertial frame of reference. (3.1) K/U
8. The law of inertia does not hold in a non-inertial frame of reference. (3.1) K/U
9. The direction of centripetal acceleration for a car on a banked curve is always down the incline parallel to the road surface. (3.2) K/U
10. The magnitude of an object's centripetal acceleration increases with the mass, the radius of the circular path, and the velocity of the object. (3.2) K/U
11. An observer looking down on a passenger in a car driving around a sharp curve would see that the passenger is being pushed by the car in the direction of the curve. (3.3) K/U A
12. The Moon is not an example of an object in uniform circular motion. (3.4) K/U A
13. Objects moving in a rotating frame of reference experience a force parallel to the velocity of the object in the rotating frame. (3.4) K/U
14. A Foucault pendulum demonstrates that Earth is not a rotating frame of reference. (3.4) K/U
15. A roller coaster car in free fall has no apparent weight. (3.4) K/U A

Write a short answer to each question.

16. You are swinging your keys at the end of a lanyard in a horizontal circle around your head. What is the effect on the magnitude of the centripetal acceleration of the keys in each case? (3.2) K/U
    (a) You keep the radius of the circle constant but double the speed.
    (b) The speed of the keys stays the same, but you double the radius of the circle.
17. Two cars with the same mass are driving around a curved road at different velocities. Which car will experience a greater centripetal force, the one moving with the faster velocity or the one moving with the slower velocity? (3.3) K/U A

18. How are centrifuges used in blood analysis? (3.4) K/U C

19. Identify the force that is causing the centripetal force in each situation. (3.3, 3.4) K/U
    (a) the Moon orbiting Earth
    (b) a car turning a corner
    (c) a rock twirled on the end of a string

Understanding

20. While riding in a car heading east, you hold an accelerometer in your hand, like the one in **Figure 1**. The angle of the bead changes with the acceleration of the car. (3.1) K/U T/I C

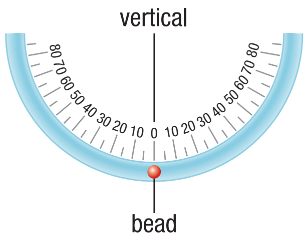


Figure 1

    (a) How must you hold the accelerometer so that it correctly measures acceleration? Explain your answer.
    (b) Describe what happens to the bead when the vehicle is at rest.
    (c) Describe what happens to the bead when the vehicle is accelerating toward the east.
    (d) Describe what happens to the bead when the vehicle is moving with a constant velocity.
    (e) Describe what happens to the bead when the vehicle begins to slow down while moving toward the east.
    (f) The bead is at an angle of 13° from the vertical. Calculate the magnitude of the car's acceleration.

21. Determine the magnitude of the centripetal acceleration in each scenario. (3.2) K/U T/I A
    (a) A penny is 13 cm from the centre of a vinyl record. The record is playing on a turntable at 33.5 rpm.
    (b) A rodeo performer is twirling his lasso with uniform circular motion. One complete revolution of the rope takes 1.2 s. The distance from the end of his rope to the centre of the circle is 4.3 m.
    (c) An electron is travelling around a nucleus at 2.18×10^6 m/s. The diameter of the electron's orbit is 1.06×10^{-10} m.

22. You are operating a remote-controlled car around a circular path in an open field. The car is undergoing centripetal acceleration of 33.8 m/s^2. The radius of the car's path is 125 m. Calculate the car's speed. (3.2) K/U T/I A

23. WindSeeker, a 30-storey swing ride at Canada's Wonderland, ascends 91.7 m, spreads its metal arms, and swings riders at speeds up to 50.0 km/h. Calculate the ride's centripetal acceleration when the ride operates at maximum speed and at full swing with a diameter of 33.5 m. (3.2, 3.3) K/U T/I A

24. The track near the top of your favourite roller coaster is looped with a diameter of 20 m. When you are at the top, you feel as if you weigh one-third of your true weight. How fast is the roller coaster moving? (3.3) K/U T/I A

25. A locomotive engine of mass $3m$, pulling an empty cargo car of mass m, is making a turn on a track. Assuming that the engine and cargo car are moving at the same speed, compare the centripetal forces acting on each. Explain your answer. (3.3) K/U T/I A

26. You are riding on Air Gliders, a thrill ride at Calaway Park, Calgary, that swings riders around in a circle while metal arms move the cars up and down. (3.3) K/U T/I A
    (a) What is the centripetal force experienced by a 90 kg rider swinging around at 20 m/s in a circle with a 16 m radius?
    (b) Calculate the force when the ride's arms close to a radius of 10 m.
    (c) Calculate the force when the ride slows to 5 m/s, keeping the radius at 10 m.

27. A discus thrower at a track meet hurls a 2.0 kg discus. She exerts a horizontal force of 2.8×10^2 N on it as she spins. She rotates the disc, with her arm outstretched, in uniform circular motion, with a radius of 1.00 m. How fast will the discus travel when released? (3.4) K/U T/I A

28. A 2.0 kg jewellery box is sitting at the edge of a rotating shelf in a mechanical display case. The radius of the rotating shelf is 0.50 m. Calculate the centripetal force when
    (a) the shelf is rotating at 1.0 rpm
    (b) the shelf frequency increases to 5.0 rpm
    (c) the shelf frequency decreases to 0.50 rpm (3.3, 3.4) K/U T/I A

29. On the Drop Tower at Canada's Wonderland, riders free-fall 23 storeys at speeds close to 100 km/h. At some point during the ride, a person experiences a force equivalent to 2g and the ride's seat is pushing up with a force of 1.1×10^3 N. What is the person's weight at this point? (3.4) K/U T/I A

Analysis and Application

30. The blades of a blender of radius 0.030 m are spinning at a rate of 60 rpm. What is the centripetal acceleration of a single point on the edge of one of the blades? (3.2) K/U T/I A

31. The rock in **Figure 2** is moving with uniform circular motion in a horizontal circle on a frictionless surface. The string is old and can only exert a maximum force of 25 N on the rock. Determine the minimum speed the rock can have without breaking the string. (3.3) K/U T/I A

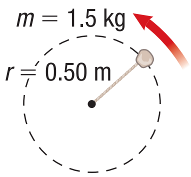


Figure 2

32. A roller coaster car is near the bottom of its track, as shown in **Figure 3**. At this point, the normal force on the roller coaster is 3.5 times its weight. The speed of the roller coaster is 26 m/s. Determine the radius of the track's curvature. (3.3) T/I A

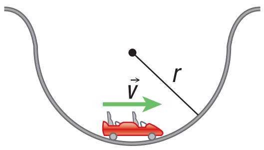


Figure 3

33. A 35 kg child sits on a Ferris wheel that has a diameter of 22 m. The wheel rotates 3.5 times per minute. (3.3) T/I C A
    (a) What force does the seat exert on the child at the top of the ride?
    (b) What force does the seat exert on the child at the bottom of the ride?

34. A rock with a mass of 1.5 kg attached to a light rod with a length of 2.0 m twirls in a vertical circle as shown in **Figure 4**. The speed v of the rock is constant; that is, it is the same at the top and at the bottom of the circle. The tension in the rod is zero when the rock is at its highest point. Calculate the tension when the rock is at the bottom. (3.3) T/I A

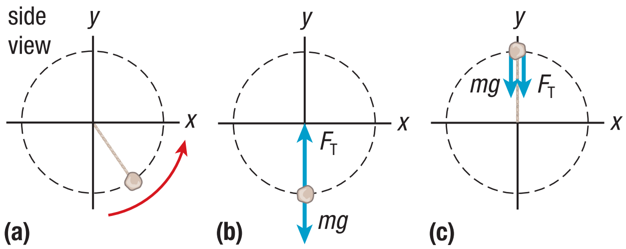


Figure 4

35. A rock tied to a string spins in a circle of radius 1.5 m, as shown in **Figure 5**. The speed of the rock is 10.0 m/s. (3.3) K/U T/I C A

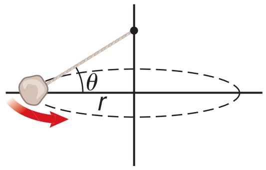


Figure 5

    (a) Draw two simple diagrams: one that shows a top view and one that shows a side view of the motion of the rock.
    (b) Draw an FBD for the rock.
    (c) Determine the total force on the rock directed toward the centre of its circular path. Express your answer in terms of the (unknown) tension in the string, F_T.
    (d) Apply Newton's second law along the vertical and the horizontal directions to calculate the angle the string makes with the horizontal.

36. A car with a mass of 1.7×10^3 kg is travelling without slipping on a flat, curved road with a radius of curvature of 35 m. The speed of the car is 12 m/s. Calculate the frictional force between the road and the tires. (3.3) K/U T/I A

37. A stone with a mass of 0.30 kg is tied to a string with a length of 0.75 m and is swung in a horizontal circle with speed v. The string has a breaking-point force of 50.0 N. What is the largest value v can have without the string breaking? Ignore any effects due to gravity. (3.3) K/U T/I A

38. A hammer thrower is swinging a ball on a rope. The mass of the ball is 70.0 kg, and it is swinging at 2.0 m/s in a circle of radius 1.0 m. Calculate the centripetal force. (3.3) K/U T/I A

39. A 30.0 kg child is riding a bicycle around a circular driveway with a diameter of 20.0 m. He is experiencing 32 N of centripetal force. How fast is the child cycling? (3.3) K/U T/I A

40. Roller coaster cars are travelling around a clothoid loop in the track at 55 m/s. The cars have a mass of 125 kg, and the loop has a radius of 25 m. Calculate the centripetal force. (3.3, 3.5) K/U T/I A

41. A child is operating a remote-controlled boat around the edge of a pond with a radius of 2 m. The boat is moving with a speed of 2 m/s. The centripetal force is 16 N. (3.3) K/U T/I A
    (a) Determine the mass of the boat.
    (b) In order to decrease the centripetal force to 4 N, how fast should the boat go?

42. **Figure 6** shows a car travelling around a curve in the road. (3.3.) K/U T/I A

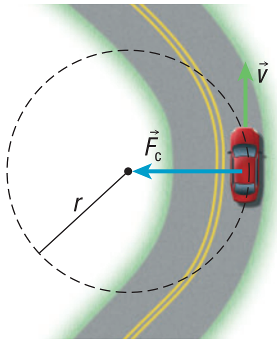


Figure 6

(a) If the car doubles its speed, how much of an increase in centripetal force from friction is needed to keep the car in a circular path?

(b) What would happen to the car's path if the road was covered in ice and there was no friction?

43. Determine the centripetal force needed to keep a 105 kg motorboat moving in a circular path on a lake at 7.0 m/s. The radius of the path's curve is 15 m. (3.3) K/U T/I A

44. Two masses are tied together by strings as shown in **Figure 7** and swung around in a horizontal circle with a period of 2.00 s on a frictionless surface. Mass 1 is 3.00 kg, and mass 2 is 5.00 kg. Determine the tension in each string. (3.3) K/U T/I

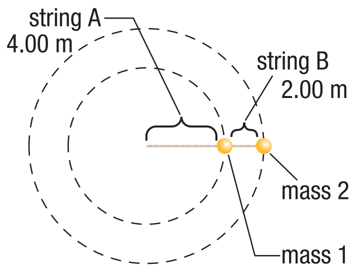


Figure 7

45. Mass 1 (2.0 kg) sits on top of mass 2 (5.0 kg), which rests on a frictionless surface (**Figure 8**). The coefficient of static friction between mass 1 and mass 2 is 0.30. A string of length 5.0 m is tied to mass 2, and both masses are swung around in a horizontal circle. Calculate (a) the maximum speed of the masses and (b) the tension in the string. (3.3) K/U T/I

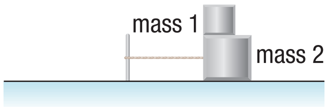


Figure 8

46. In an amusement park ride, a motor rotates two platforms with a period of 4.0 s in a vertical circle (**Figure 9**). The mass of platform 1 is 1200 kg, and the mass of platform 2 is 1800 kg. Calculate the tension in each support when the platforms are at the bottom as shown in the figure. (3.3, 3.4) K/U T/I A

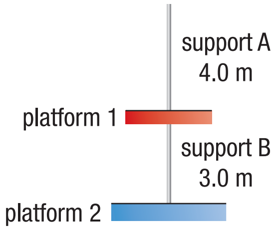


Figure 9

47. The amusement park ride shown in **Figure 10** is a large, rapidly spinning cylindrical room with a radius of 3.0 m. The riders stand up against the wall, and the room starts to spin. Once the room is spinning fast enough, the riders stick to the wall. Then the floor slowly lowers, but the riders do not slide down the wall. Assume the coefficient of friction between the wall and the riders is 0.40. (3.3, 3.4) K/U T/I C A

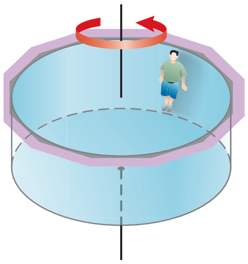


Figure 10

(a) Draw an FBD of a person on the ride. What force or forces cause the net force on the rider?

(b) Calculate the minimum speed of the rider required to keep the person stuck to the wall when lowering the floor.

48. A 6.0 kg object is attached to two 5.0 m–long strings (**Figure 11**) and swung around in a circle at 12 m/s. Determine the tension in the two strings, and explain why the tensions are not the same. (3.3, 3.4) K/U T/I A

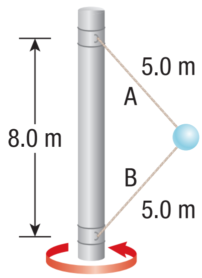


Figure 11

49. A race car driver wants to complete two laps in 1 min around a circular track with a 30.0 m radius. The combined mass of her body and her car equals 9.8×10^2 kg. What is the magnitude of centrifugal force she will feel? (3.4) K/U T/I A
50. A top-loading washing machine with 2.0 kg of clothes inside is on spin cycle. The tub, with a radius of 0.35 m, is rotating at 50.0 rpm. Determine the centripetal force acting on the clothes. (3.4) K/U T/I A
51. A coin is resting on a vinyl record. The coin slips off the record when the rotation rate is 0.30 rps (rotations per second). Determine the coefficient of static friction between the coin and the record. The radius of the record is 15 cm. (3.4) K/U T/I A
52. A roller coaster car is at the lowest point on its track, where the radius of curvature is 20.0 m. At this point, the apparent weight of a passenger on the roller coaster is 3.00 times her true weight. What is the speed of the roller coaster? (3.4, 3.5) K/U T/I A
53. A space station is rotating at 12 m/s. The artificial gravity is equal to 50.0 % of that found on Earth. What is the radius of the station? (3.4) K/U T/I A
54. A bucket of water is attached to a rope and is being swung around in a vertical circle. (3.4) K/U T/I A
    (a) What force is responsible for keeping the bucket moving in a circle?
    (b) Identify the source of the force in (a).
    (c) The water-filled bucket has a mass of 15 kg and is swinging at a velocity of 2 m/s in a circle with a radius of 2 m. Calculate the magnitude of the force.
55. A popular circus act features a daredevil motorcycle rider encased in a spherical metal cage, as shown in **Figure 12**. The diameter of the cage is 4 m. (3.4) K/U T/I A

Figure 12

    (a) A 65 kg performer on a 95 kg motorcycle rides horizontally around the middle of the cage. He completes 22 loops in one minute. Calculate the coefficient of friction he needs between his tires and the cage to keep him in place.
    (b) How many loops will the rider make per second?
    (c) If the performer rides around the cage in vertical loops at 6 m/s, what force is needed at the top and bottom of the cage to support his mass?
56. Two skaters are performing on ice. One skater is gripping the other's hand and spinning her in an arc around his body. The distance between the skaters' grip and the outer edge of the arc is 3.0 m. The skater is being swung around at 2.0 rpm and has a mass of 54 kg. Calculate the centripetal force. (3.4) K/U T/I A
57. A horse trainer is leading a 450 kg horse on a long lead rope around a training pen, making one rotation around the ring per minute. The centripetal force on the horse is 48 N. Determine the length of the lead rope. (3.4) K/U T/I A
58. Consider the performer in **Figure 13**. How fast must the horse go around a circus ring with a radius of 25 m in order to maintain constant centripetal acceleration of 1.0*g*? Give your answer in kilometres per hour. (3.5) K/U T/I A

Figure 13

Evaluation

59. Using your knowledge of forces, explain the following in a format of your choice. (3.1, 3.2, 3.3) T/I C
    (a) centrifugal force
    (b) Coriolis force
    (c) fictitious forces, and why they are called that
60. Describe the effects on a person in each of the following frames of reference. (3.1) T/I C A
    (a) riding the elevator to the top of the CN Tower in Toronto
    (b) free falling in a skydive from an airplane
61. Create a three-column table, either electronically or on paper. (3.2) K/U T/I C A
    (a) In the first column, list the three equations for centripetal acceleration. In the second column, identify the variables found in each equation. In the third column, identify the variables not found in each equation. Give your table a title.
    (b) In your own words, briefly describe how each equation was derived.

62. A rodeo performer spins a lasso above her head. (3.2) T/I C A
    (a) Explain the purpose of twirling the rope before throwing it.
    (b) Describe how she could maximize the distance the rope can be thrown.
    (c) Describe the path the rope will take once she releases it.
63. Explain how the principles of centripetal force are used to make safer driving conditions. (3.3) T/I C A
64. How would you explain the concepts of artificial gravity to a fellow student who has not taken physics? (3.4) T/I C A

Reflect on Your Learning

65. What did you learn in this chapter that was surprising? Explain your answer. T/I C
66. In this chapter, you learned how to solve some types of centripetal force problems. What questions do you still have about solving centripetal force problems? T/I C A
67. Prepare a Know–Want to Know–What You Learned (K-W-L) chart on the topic of artificial gravity or another topic from this chapter. T/I C A
68. How has your understanding of uniform circular motion changed? Did you learn anything particularly relevant to you on this topic? T/I C A
69. Consider the different topics you have studied in this chapter. Choose one that you feel has an important impact on your life. Write a one-page report about the topic, explaining why it is important to you. What else would you like to know about this topic? How could you go about learning this? T/I C A

Research

WEB LINK

70. Research the history of roller coasters, showing how the designs have changed over the centuries. Present your findings in a timeline, on paper, as a Wiki page, as an electronic slide presentation, or in another format of your choosing. T/I C A
71. Research the effects of the Coriolis force in meteorology. In your own words, describe the effect using the movement of a hurricane as an example. C A
72. Research the effects of uniform circular motion on growing plants. What effects would a continuously spinning pot of soil have on the grass seed planted in it? How would the grass grow differently? T/I C A
73. Gas centrifuge technology is an emerging technology. The technology enriches mined uranium to levels at which it can be used to generate nuclear power. The use of centrifuges increases the concentration of the isotope uranium-235 in the uranium. Research the various applications of gas centrifuge technology. How has it affected the efficiency of energy production? T/I C A
74. Astronauts undergo rigorous physical training to be able to function in the altered environments in space. Research astronaut training. How has astronaut training changed from the first piloted space mission to today's missions? What are the health risks associated with space flight and travel? What technologies are in development to help astronauts prepare for longer space travel than has ever been attempted? T/I C A
75. The centrifuge is an integral piece of machinery in many industries, from oil production to laundry applications to the dairy industry. T/I C A
    (a) Choose an industry, and trace the use of centrifuges in the industry over the past century. How have centrifuges contributed to advances in the industry?
    (b) List two major implications of the use of centrifuges on society.
76. Research the track layout and dimensions of the Behemoth, a ride at Canada's Wonderland. Prepare a concept map on all the possible forces riders will experience at each new twist in the track. T/I C A
77. Research windmills and wind turbines, how they work, and their effect on the environment. T/I C A
    (a) How do windmills and wind turbines use the principles of dynamics and circular motion to generate power? Include a simple diagram in your answer.
    (b) What is the environmental impact of wind power and wind farms?
78. Using an online resource, design your own roller coaster. List each design feature you have included and explain your reasoning. Explain how you have kept the ride exciting while keeping it safe for customers. Decide on a theme for your roller coaster, and try to include the theme in your design. K/U T/I C A

A New Extreme Sport

Sports use physics in some of the most original and extreme ways. In speed sports such as luge (**Figure 1**), motorcycle racing, skateboarding, and bobsledding, the goal is to reach extreme speeds while navigating courses filled with twists and turns.

Figure 1 In luge competitions, the athlete rides on a small sled at high speeds (140 km/h) along an iced track. The design of the track must keep the competitor on the track while allowing for extreme speeds.

These sports are particularly dangerous, so designing tracks for them requires an understanding of several laws and principles of physics in order to make each sport as safe as possible. The extreme physics of the tracks requires the use of banked and pitched sides, so that centripetal forces are large enough to keep the participants and vehicles on the track. Minimizing friction and drag helps participants achieve the highest speeds possible.

In addition, sports such as snowboarding, skateboarding, and ski jumping require an understanding of projectile motion, so that the participants land in the right places. These sports also rely on the ability of the participant, as well as the proper design of the equipment, machinery, and course on which they take place.

The Task

In this Unit Task, you will design a new extreme sport and apply the physics principles in Unit 1 to ensure that anyone can participate in the sport reliably and safely. You will then build a model of your sport that you can then analyze in terms of the physics principles that you have studied so far.

Read through the questions at the end of the Unit Task to help you plan your procedure and presentation.

Extreme Sport Design

Brainstorm ideas in a group. Your task is to come up with a new extreme sport and then design and build a model track, a series of obstacles, or a device that the participant will travel across or use safely. Here are some suggestions:

- an extreme snowboarding track
- an extreme bicycle motocross or skate track
- an extreme race car track

Note how your sport uses the track, obstacles, or device to reach high speeds and large changes in velocity while still permitting the participant to safely complete the course.

Once you have selected your sport and have discussed the features it will require, you will design those features, using the principles of physics that you have learned so far. Before you begin construction, research the topic of sports physics using print sources and the Internet to understand how certain principles (acceleration, friction, centripetal force) affect different features and challenges of a given sport. Then have your teacher approve your sport. WEB LINK

Once you have collected the essential information for your design, write a proposal describing your sport; the design of the track, course, device, or obstacles involved; and what the participants must achieve to successfully and safely perform the sport. Be sure that your teacher approves your proposal for completeness and accuracy.

Have your teacher approve your design before you begin constructing your model. Obtain permission before using any tools in the lab. Review safety and design rules before you begin. Use all tools safely, and test your design in a safe and controlled manner. Wear eye protection.

Using your approved proposal as a guide, construct a working model based on your design. Use materials that are easy to obtain and require readily available tools. You could use some of the following materials:

- eye protection
- marbles
- wheels
- steel ball bearings
- wire coat hangers
- cardboard tubes
- rubber tubing
- ice blocks
- elastic bands
- paper
- tape
- staples
- glue
- wood

Construction and Demonstration

During the construction process, test your model to be sure it works properly, and modify it if it does not. Upon completion of the model, test it again, and modify it as needed until the task is successfully completed. Keep a log or blog during this process to monitor your progress.

At this point, you are ready to analyze your model to see how well it performs. Collect and record data (for example, measurements of time or angle), and apply the equations for kinematics, force, and uniform circular motion. Determine how well your model performs. Assess your results. Use them to improve the model.

Demonstrate your model to the class. Describe the ideas behind the extreme sport, and share the data and the analysis of your process. Describe how you applied the principles of kinematics, linear dynamics, and circular motion in your design. Mention any setbacks and changes that you needed to make to the design and why you needed to make them.

Analyze and Evaluate

(a) Which physics principles did you use to design your model? K/U A

(b) Briefly describe your testing procedure. T/I C A

(c) Take measurements and calculate the various velocities and accelerations of the object tested in your model. Estimate how friction, gravity, and centripetal acceleration affect the object's motion. If your model uses projectile motion, determine the inclination angle, maximum height, and speed of the object. T/I A

(d) What measures did you take to ensure that the object travelled safely from start to finish? How did these measures change during the design and testing of your model? How would these safety measures relate to a full-scale version of your model? T/I C A

(e) Describe any changes you made to the design after testing your model. Which parts of your model worked well, and which did not? T/I C A

(f) List problems that you encountered while designing the model. Describe how you overcame them. T/I C A

(g) Create a flow chart or other graphic organizer of the process you followed to design and build your model. Did your final design meet your original expectations? If you had to do this task again, how would you change the process? What changes would you make to your design? T/I C A

Apply and Extend

(h) Assess the environmental impact of your design. What could you do to reduce the impact of your device on the environment? T/I C A

ASSESSMENT CHECKLIST

Your completed Unit Task will be assessed according to these criteria:

Knowledge/Understanding

- ✓ Demonstrate knowledge and understanding of velocity and acceleration.
- ✓ Demonstrate knowledge and understanding of forces.
- ✓ Demonstrate understanding of two-dimensional motion.
- ✓ Demonstrate knowledge and understanding of uniform circular motion, projectile motion, centripetal acceleration, and centripetal force.
- ✓ Demonstrate safety skills in the laboratory.

Thinking/Investigation

- ✓ Investigate the relationships between velocity and acceleration in real-life situations.
- ✓ Analyze velocities and accelerations in your model.
- ✓ Incorporate safety features in your model corresponding to the safety features in a full-scale version of the model.
- ✓ Improve the design of your model based on testing.
- ✓ Construct a working model of an extreme-sport design.
- ✓ Evaluate the success of your model.
- ✓ Evaluate and improve your design process.
- ✓ Identify and locate relevant research sources.
- ✓ Use appropriate terminology related to dynamics.
- ✓ Express the results of any calculations involving data accurately and precisely.

Communication

- ✓ Communicate design, procedure, and modifications in the form of a flow chart or other graphic organizer.
- ✓ Demonstrate and explain the purpose, design, and functionality of the model in a report and/or presentation.
- ✓ Explain the design process and various modifications of the design in a format determined by your teacher.
- ✓ Communicate your results clearly and concisely.

Application

- ✓ Assess the cost and environmental impacts of your design. Take into account how your design may require changes to terrain, and how this may affect the environment.

UNIT 1 SELF-QUIZ

K/U Knowledge/Understanding   T/I Thinking/Investigation   C Communication   A Application

For each question, select the best answer from the four alternatives.

1. The position–time graph shown in **Figure 1** depicts which of the following situations? (1.1) K/U

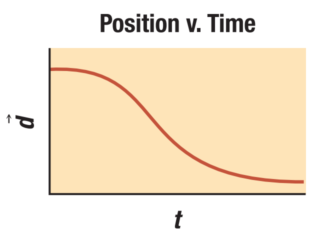


Figure 1

(a) positive velocity, positive acceleration, negative acceleration
(b) zero velocity, negative acceleration, positive acceleration
(c) zero velocity, positive acceleration, negative acceleration
(d) negative velocity, zero acceleration, positive motion

2. An ice hockey player set a record speed for a slap shot: 177.58 km/h. Suppose the hockey player accelerated the hockey puck through a distance of 1.25 m. What was the magnitude of the acceleration? (1.2) K/U T/I A
(a) 19.7 m/s^2
(b) 71.0 m/s^2
(c) 973 m/s^2
(d) 1260 m/s^2

3. A ship sails 150.0 km [E 60° N] into Hudson Bay from Fort Severn. The ship then changes course and travels 350.0 km [N]. What is the total displacement of the ship? (1.3) K/U T/I A
(a) $\Delta\vec{d}_T$ = 485.7 km [E 81.12° N]
(b) $\Delta\vec{d}_T$ = 485.7 km [E 8.712° N]
(c) $\Delta\vec{d}_T$ = 444.4 km [E 73.00° N]
(d) $\Delta\vec{d}_T$ = 444.4 km [E 17.01° N]

4. To cross a river with a current in the least amount of time, how should a boat point? (1.3) K/U T/I A
(a) somewhat downstream
(b) somewhat upstream
(c) directly at the opposite shore
(d) in a direction that will take the boat directly across

5. A helicopter moves in an arc so that its velocity changes from 77 km/h [S] to 77 km/h [E] in 15 s. What is the acceleration of the helicopter? (1.4) K/U T/I A
(a) 1.4 m/s^2 [W 45° S]
(b) 2.0 m/s^2 [E 45° N]
(c) 5.1 m/s^2 [E 45° S]
(d) 5.1 m/s^2 [E 45° N]

6. A ball kicked with a speed of 12.0 m/s at an angle of θ above the horizontal lands at a distance of 14.47 m away from the kicker. The total vertical displacement of the ball is zero. What is the value of θ? (1.5) K/U T/I A
(a) 30.0°
(b) 35.0°
(c) 40.0°
(d) 45.0°

7. A ship travelling at 39.0 km/h [N] across the St. Lawrence Seaway encounters a current of 13.0 km/h [E 50.0° N]. What is the velocity of the ship with respect to the shore? (1.6) K/U T/I A
(a) 48.4 km/h [E 78.1° N]
(b) 48.4 km/h [E 11.9° N]
(c) 49.7 km/h [E 80.3° N]
(d) 49.7 km/h [E 9.7° N]

8. The block in **Figure 2** remains motionless against the wall because of an applied force and the force of static friction between the block and the wall. The coefficient of static friction is μ_S. Which equation correctly describes the magnitude of the frictional force between the wall and the block? (2.1) K/U A

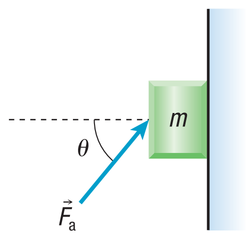


Figure 2

(a) $|\vec{F}_S| = (|m\vec{g}|\cos\theta)\mu_S$
(b) $|\vec{F}_S| = (|\vec{F}_a|\sin\theta - mg)\mu_S$
(c) $|\vec{F}_S| = (|\vec{F}_a|\cos\theta - mg)\mu_S$
(d) $|\vec{F}_S| = (|\vec{F}_a|\cos\theta)\mu_S$

9. A car and driver with a combined mass of 1.5×10^3 kg experience a forward force of 7.67×10^3 N and forces of friction and drag of 7.7×10^2 N. What is the acceleration of the car and driver? (2.2) K/U T/I A
   (a) 4.1 m/s^2
   (b) 4.6 m/s^2
   (c) 5.1 m/s^2
   (d) 5.6 m/s^2
10. An object is pushed horizontally at a constant velocity. What is true about the forces acting on the object? (2.2) K/U T/I
   (a) The force or forces acting forward are greater than the force or forces acting backward.
   (b) The sum of all forces is directed forward.
   (c) The forces acting on the object can be said to be "unbalanced."
   (d) The sum of all forces is zero.
11. A winch pulls a mass of 1.75×10^3 kg up a 24° slope. Friction is negligible. What is the tension in the winch cable? (2.3) K/U T/I A
   (a) 7.0×10^3 N
   (b) 9.0×10^3 N
   (c) 1.6×10^4 N
   (d) 1.7×10^4 N
12. A force of 1.0×10^3 N moves a heavy box up a ramp with a 21° incline. The weight of the box is 1.69×10^3 N, and it moves up the ramp at a constant speed. What is the coefficient of kinetic friction between the box and the ramp? (2.4) K/U T/I A
   (a) 0.02
   (b) 0.25
   (c) 0.48
   (d) 0.71
13. An object is pulled across a rough horizontal surface with an applied force of 300 N [40° below the horizontal]. The applied force is slowly rotated up toward the horizontal, decreasing the 40° angle. What will happen to the force of friction acting on the object and the acceleration? (2.4) K/U T/I
   (a) Both increase.
   (b) Both decrease.
   (c) Friction decreases and acceleration increases.
   (d) Friction increases and acceleration decreases.
14. For a frame of reference to be inertial, it must
   (a) undergo positive acceleration
   (b) undergo negative acceleration
   (c) change direction without changing speed
   (d) move in one direction with constant speed (3.1) K/U
15. An object moving in a circular path with radius 0.25 m experiences a centripetal acceleration of 2.5 m/s^2. What is the frequency? (3.2) K/U T/I A
   (a) 0.016 Hz
   (b) 0.13 Hz
   (c) 0.25 Hz
   (d) 0.50 Hz
16. You are spinning two identical balls attached to strings in uniform circular motion. Ball 2 has a string that is twice as long as the string with ball 1, and the rotational speed of ball 2 is three times the rotational speed of ball 1. What is the ratio of the centripetal force of ball 2 to that of ball 1? (3.3) K/U
   (a) $\frac{3}{4}$
   (b) $\frac{3}{2}$
   (c) $\frac{9}{4}$
   (d) $\frac{9}{2}$

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

17. A straight line on a position–time graph indicates that the acceleration is zero. (1.1) K/U
18. The variables in the kinematics equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$ are $\Delta\vec{d}$, $\vec{v}_i$, and $\vec{a}$. (1.2) K/U
19. The magnitude of the average velocity is always greater than or equal to the average speed in two dimensions. (1.4) K/U
20. The time of flight for a projectile fired horizontally from a given height equals the time it takes for the same projectile to fall vertically from the same height. (1.5) K/U
21. An FBD shows only the forces acting on an object or a group of objects. (2.1) K/U
22. According to Newton's third law, the action force occurs first, causing an equal and opposite reaction force. (2.2) K/U
23. For any two materials, the coefficient of static friction is greater than or equal to the coefficient of kinetic friction. (2.4) K/U
24. For the apparent weight of a passenger in an elevator to equal zero, the elevator must accelerate upward at 9.8 m/s^2. (3.1) K/U
25. The tension acting on a horizontal string swinging a ball around in a horizontal circle on a frictionless surface is $\vec{F}_T = m\frac{v^2}{r}$ [toward the centre of the circle]. (3.2, 3.3) K/U
26. In a centrifuge, the walls of the test tubes provide the centripetal force. (3.4) K/U

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. Which of the position–time graphs in **Figure 1** depicts motion due to a constant negative acceleration? (1.1) K/U T/I

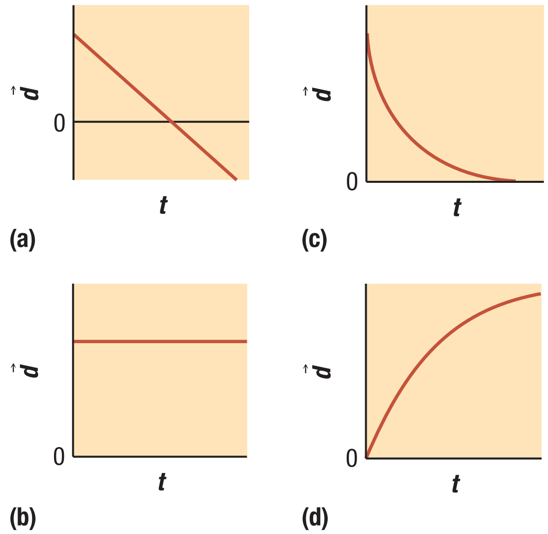


Figure 1

2. You drop a stone from a cliff. Which equation describes the displacement of the stone? (1.2) K/U
   (a) $\Delta d_y = -\frac{1}{2}g(\Delta t)^2$
   (b) $\Delta d_y = \frac{1}{2}g(\Delta t)^2 + v_{iy}\Delta t$
   (c) $\Delta d_y = v_{iy}\Delta t$
   (d) $\Delta d_y = \frac{v_{iy}^2}{2g}$
3. A student walks 50.0 m [W 60.0° N]. Which pair of displacements equals the east–west and north–south displacements? (1.3) K/U T/I A
   (a) 30.0 m [W], 40.0 m [N]
   (b) 40.0 m [W], 30.0 m [N]
   (c) 25.0 m [W], 43.3 m [N]
   (d) 43.3 m [W], 25.0 m [N]
4. A train approaches a rail yard with a velocity of 18 m/s [W]. After 95 s, the train has a velocity of 7.0 m/s [W]. What is the average acceleration of the train? (1.4) T/I A
   (a) 0.12 m/s^2 [W]
   (b) 0.12 m/s^2 [E]
   (c) 0.26 m/s^2 [W]
   (d) 0.26 m/s^2 [E]
5. A plane travelling at 192 km/h [S] is blown eastward by a wind with a speed of 56.0 km/h. What is the velocity of the plane with respect to the ground? (1.6) K/U T/I A
   (a) 1.36×10^2 km/h [E 16.3° S]
   (b) 2.00×10^2 km/h [E 73.7° S]
   (c) 2.00×10^2 km/h [N 73.7° E]
   (d) 2.48×10^2 km/h [E 73.7° S]
6. A mover attempts to pull a washing machine away from the wall. A force of static friction acts in the opposite direction of the applied force. The applied force is parallel to the level ground. Identify the FBD in **Figure 2** that correctly depicts the forces acting on the machine. (2.1) K/U T/I A

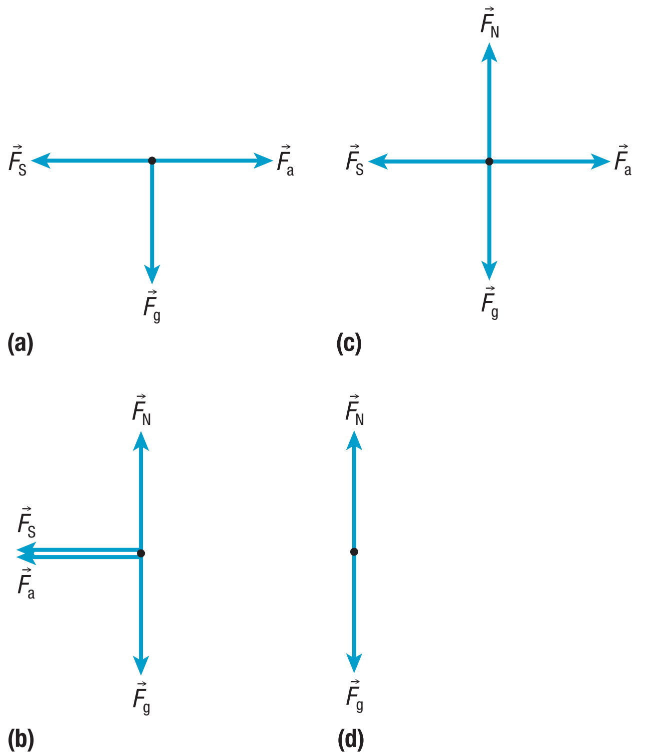


Figure 2

7. Which of the following describes a situation explained by Newton's third law? (2.2) K/U
   (a) a person pushes left on a wall while the wall pushes right on a person
   (b) clothes in a washing machine moving to the edge of the drum during the spin cycle
   (c) a spacecraft moving far from any massive bodies
   (d) a car accelerating forward by an amount equal to the ratio of the net forward force and the mass of the car

8. A skier with mass m slides down a slope that makes an incline of θ with the horizontal. Which expression describes the component of the force of gravity parallel to the slope? (2.3) K/U T/I A
   (a) mg
   (b) $mg\sin\theta$
   (c) $mg\cos\theta$
   (d) $mg\tan\theta$
9. The coefficient of static friction between a heavy box and a ramp is 0.45. The weight of the box is 1.2×10^3 N, and the ramp has an incline of 16°. What minimum force is needed for the box to overcome the force of static friction? (2.4) K/U T/I A
   (a) 1.9×10^2 N
   (b) 3.8×10^2 N
   (c) 5.2×10^2 N
   (d) 7.4×10^2 N
10. Which of the following best describes a fictitious force? (3.1) K/U
   (a) an apparent force used to explain the motion of objects within an inertial frame of reference
   (b) an apparent force used to explain the motion of objects within a non-inertial frame of reference
   (c) a real force that appears to act in the opposite direction because of the motion of the frame of reference
   (d) a force that is imaginary when observed in a non-inertial frame of reference but real when observed in an inertial frame of reference
11. What provides the centripetal force for a car moving around in a circle on a banked curve covered with very slippery ice? (3.3) K/U A
   (a) friction between the tires and banked curve
   (b) the horizontal component of the normal force
   (c) the vertical component of the normal force
   (d) the weight of the car

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

12. Acceleration always occurs in the same direction as the motion of the object. (1.1) K/U
13. Use the equation $v_f^2 = v_i^2 + 2a\Delta d$ when you know the changes in distance and speed, and acceleration is constant. (1.2) K/U
14. The force of gravity affects only the vertical component of the velocity of a projectile and not the horizontal component of the velocity. (1.5) K/U
15. To cross a river as quickly as possible, the velocity of the boat relative to the water should be directed upstream. (1.6) K/U
16. A boat travels with velocity $\vec{v}_{BR}$ with respect to a river. The river moves with velocity $\vec{v}_{RS}$ with respect to the shore. The equation that describes the boat's velocity with respect to the shore is therefore $\vec{v}_{BS} = \vec{v}_{BR} - \vec{v}_{RS}$. (1.6) K/U
17. In the process of determining the net force on an object, you can first separate all the forces acting on the object into x- and y-components, and then add components as vectors to obtain the x- and y-components of the net force. (2.1) K/U
18. An object is in a state of equilibrium when the net force acting on it is zero. (2.3) K/U
19. Static friction resists the motion of an object as long as the applied force is larger than the force of static friction. (2.4) K/U
20. For an object kept in a circular path, the centripetal acceleration increases with the period of revolution. (3.2) K/U
21. Objects move in uniform circular motion because they are acted on by a force that points toward the centre of the circular motion. (3.3) K/U

Match each term on the left with the most appropriate description on the right.

22. (a) acceleration
   (b) inertia
   (c) kinetic friction
   (d) static friction
   (e) equilibrium
   (f) inertial frame of reference
   (g) non-inertial frame of reference
   (h) centrifugal force
   (i) linear actuator
   (j) centripetal acceleration

   (i) at rest or moving with constant velocity
   (ii) fictitious force
   (iii) undergoes acceleration
   (iv) directed toward the centre of a circular path
   (v) converts energy into linear motion
   (vi) the difference between two velocities divided by an interval of time
   (vii) resists the motion of a sliding mass
   (viii) proportional to an object's mass
   (ix) net force is zero
   (x) keeps objects from slipping on surfaces

   (1.3, 1.4, 2.2, 2.4, 2.5, 3.1, 3.4) K/U

Write a short answer to each question.

23. What must be true about the average velocity of an object when the position–time graph for the object is a straight line with a negative slope? (1.1) K/U C
24. What must be true about the average acceleration of an object when the position–time graph for the object is a straight line with a positive slope? (1.1) K/U C
25. Describe how the position of a freely falling object changes with time. (1.1, 1.2) K/U C
26. Determine the acceleration of a drag racer who starts at rest and reaches a speed of 39.6 m/s in 1.2 s. (1.2) K/U T/I A
27. Identify the condition that must be satisfied when using the kinematics equations. (1.2) K/U
28. When solving problems related to vector addition, what are the advantages and disadvantages of using the cosine and sine laws compared to the component method of vector addition? (1.3) K/U
29. Write the expression for the magnitude of a vector displacement in terms of its horizontal and vertical components. (1.3) K/U
30. Explain why average speed can be larger than average velocity. (1.4) K/U C
31. How would you modify the equation for projectile motion with a non-horizontal initial velocity to obtain the equation for projectile motion with a horizontal initial velocity? (1.5) K/U
32. One student kicks a ball from 30.0 cm above the ground with an initial speed of 8.0 m/s, at an angle of 55° above the horizontal. Shortly after, another student catches the ball at 30.0 cm above the ground. Write the equation for the horizontal range the ball has travelled. (1.5) K/U T/I A
33. A plane drops a crate of parts for use by technicians on the ground. Unfortunately, the parachute fails to open. (1.6) K/U C A
    (a) Describe where the plane is with respect to the crate at the moment it lands.
    (b) Describe the path of the crate as observed from the cargo hatch of the plane.
34. A plane travels into a headwind. In terms of vectors, describe how to determine the relative velocity of the plane with respect to the ground. (1.6) K/U C A
35. A train passes by a platform. To a passenger at rest on the train, objects on the platform appear to be moving south at 72 km/h. What is the velocity of the train? (1.6) K/U A
36. The car in **Figure 3** is in a state of equilibrium on a frictionless surface. The cable is parallel to the incline. (2.1) K/U A

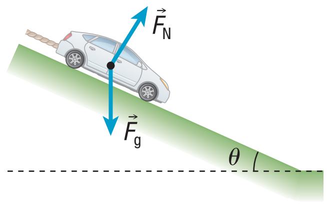


Figure 3

    (a) Write an expression for the tension in the cable.
    (b) Write an expression for the normal force acting on the car.
    (c) What will happen to each force acting on the car if the steepness of the incline is gradually decreased? Explain your reasoning.
37. Define inertia. (2.2) K/U
38. When you push against a heavy object, what evidence is there of Newton's third law? (2.2) K/U
39. Explain what must be true for the forces on an object to be in a state of equilibrium. (2.3) K/U
40. Suppose you attach a wooden ball with a weight of 1.0 N to a string. A wind exerts a horizontal force of 4.0 N on the ball, producing a tension in the string. Determine the magnitude of the tension. (2.3) K/U T/I A
41. Explain why a car jack (**Figure 4**) is a mechanical linear actuator. (2.5) K/U A

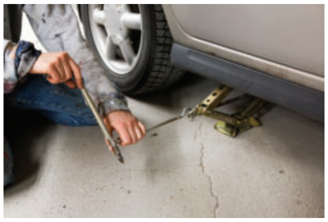


Figure 4

42. Identify three things in **Figure 5** that help make the skier complete the race faster. (2.6) K/U

Figure 5

43. In a Venn diagram, compare inertial and non-inertial frames of reference. (3.1) K/U C
44. Describe the conditions under which you can observe fictitious forces. (3.1) K/U

45. Describe how centripetal acceleration varies with distance from the centre of the circular path. (3.2) K/U

46. Describe two examples of centripetal force that hold a car on a curved stretch of road. (3.3) K/U C

47. A car moves in a circular track with an initial speed of 7.5 m/s. The car accelerates to a speed of 15 m/s. By how much does the centripetal force that keeps the car in the circular path change? (3.3) K/U C A

48. A disc-shaped space station rotates to produce artificial gravity. Write the equation that relates the radius of the station to the speed of rotation needed to produce the same gravitational force as on Earth's surface. (3.4) K/U T/I C

Understanding

49. Provide an example for each condition below. (1.1) K/U A
    (a) negative velocity with positive acceleration
    (b) negative displacement with zero acceleration
    (c) negative displacement with positive acceleration
    (d) positive velocity with negative acceleration

50. You walk 1.3 km to the store and then return home. The entire trip takes 40.0 min, ignoring the time spent shopping. (1.1) K/U T/I A
    (a) Determine your total displacement, and explain how you obtained this result.
    (b) Determine your average speed, in kilometres per hour, and explain how you obtained this result.
    (c) Determine your average velocity, in kilometres per hour, and explain how you obtained this result.

51. A rock falls off a cliff on the Moon. (1.2) K/U A
    (a) Identify how the motion of the rock differs from the motion of a rock falling from a cliff on Earth.
    (b) Identify how the motion of the rock is similar to the motion of a rock falling from a cliff on Earth.

52. Describe the method for algebraically determining the sum of two vectors. (1.3) K/U C A

53. Suppose that you have the vector displacement information for the two-dimensional motion of a train. (1.4) K/U C A
    (a) Explain how you would calculate the average velocity of the train.
    (b) Explain how you would calculate the average speed of the train.

54. A moon orbits a planet in a circular orbit so that its velocity along the circular path changes continuously with time. (1.4, 3.2) K/U C A
    (a) Does the force of gravity act perpendicular or parallel to the moon's velocity? Explain.
    (b) Explain how the force of gravity changes the moon's velocity.

55. Write the projectile motion equation for the magnitude of vertical displacement in each situation. (1.5) K/U A
    (a) A rock thrown upward from the edge of a high cliff with a speed of v_i and at an angle θ from the horizontal falls toward a canyon floor.
    (b) A rock kicked horizontally with a speed of v_i from the edge of a high cliff falls toward a canyon floor.
    (c) A rock dropped from the edge of a high cliff falls toward a canyon floor.

56. If you row a small boat or raft horizontally across a river, the current of the river displaces you downstream by a certain distance. Indicate how the various sides of a right triangle relate to the vectors of relative motion in this situation. (1.6) K/U A

57. A plane flies due north into a wind that is directed 28° south of east. The speed of the plane is 3.6×10^2 km/h with respect to the air, and the speed of the wind is 75 km/h with respect to the ground. Describe how to express the relative motion of the plane with respect to the ground. Then show the equations. (1.6) K/U T/I C A

58. A student is pushing a heavy box up a ramp with a 15° incline. The box is speeding up as it moves up the ramp. A small force of friction opposes the force applied by the student. (2.1) K/U C A
    (a) Draw an FBD for the box.
    (b) Identify the direction of the net force acting on the box.

59. Describe the procedure for determining the net force on an object, starting with drawing an FBD. (2.1) K/U C

60. Workers lift a piano from street level to the top floor of a building using cables and pulleys. At one point, the piano is at rest. Summarize how to calculate the tension in the cables at this point using vectors and Newton's first and second laws. (2.3) K/U C A

61. Distinguish between the two kinds of frictional forces. (2.4) K/U

62. An elevator rises from the ground floor of a tall building to the top floor. Describe how the upward acceleration of the elevator affects the apparent weight of the passengers. (3.1) K/U A

63. You are riding on a bus. During your trip, you place a heavy book on the floor. When the bus turns right at a constant speed, the book slides to the left side of the bus. (3.1) K/U C A
    (a) Describe how the book's motion provides an example of a fictitious force in a non-inertial frame of reference.
    (b) Suppose the book did not slide but remained in its original position on the floor. How would this demonstrate the existence of a centripetal force?

64. For an object moving in a circular path, distinguish between the behaviour of centripetal acceleration and acceleration along the direction of motion. (3.2) K/U C

65. List three examples in which centripetal forces affect the motion of an object. (3.3) K/U

66. A spacecraft moves with constant speed in a circular orbit around a planet. Then the spacecraft accelerates in the direction it is moving. Arrange the statements below in the correct order to explain why the distance of the spacecraft from the planet must increase in order to remain in a circular orbit. (3.3) K/U C A
    (a) The centripetal force is provided by gravity, which depends on mass and distance.
    (b) To offset the increase in speed, the distance must also increase.
    (c) An acceleration in the direction the spacecraft is moving means that the spacecraft is increasing in speed.
    (d) The masses of both the spacecraft and the planet are constant, so the distance from the planet must change.

67. A cylindrical spacecraft rotates around its axis. (3.4) K/U C A
    (a) Explain how rotating a spacecraft produces artificial gravity.
    (b) Explain how the spacecraft could be used to simulate the gravity an astronaut would experience on Earth.

Analysis and Application

68. The position–time graph in **Figure 6** describes the westward motion of a car. (1.1) K/U T/I A

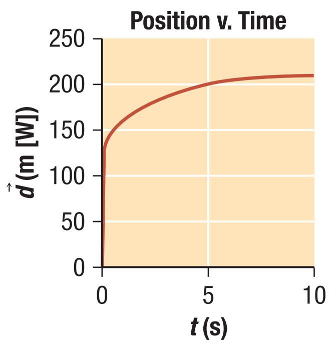


Figure 6

    (a) From the graph, estimate the total displacement of the car.
    (b) From the graph, estimate the average velocity of the car.

69. The velocity–time graph in **Figure 7** shows the change in velocity for a truck. (1.1) K/U T/I A

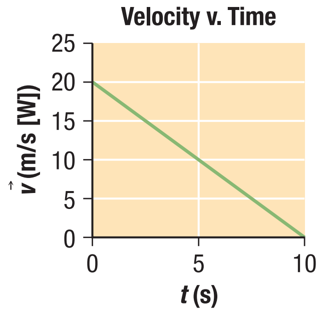


Figure 7

    (a) At what point is the instantaneous velocity of the truck equal to the average velocity?
    (b) Determine the magnitude of the acceleration of the truck from the graph in Figure 7.

70. A race car begins accelerating at a constant rate. In 25 s, the race car travels 7.8×10^2 m. Determine the magnitude of the acceleration. (1.2) K/U T/I A

71. A sprinter accelerates at a constant rate of 4.8 m/s^2 during the first 10.0 m of a 100.0 m event. Determine the runner's speed at the end of the first 10.0 m of the race. (1.2) K/U T/I A

72. A parachutist falling at a terminal velocity of 50.0 m/s [down] opens her parachute. At that point a net acceleration of about 12.5 m/s^2 [up] reduces her velocity to about 10.0 m/s [down]. Calculate the time over which this acceleration takes place. (1.2) K/U T/I A

73. A student throws a ball vertically upward. It takes the ball 0.50 s to come to a stop and another 0.50 s to return to the student. The maximum height of the ball is 1.225 m. Calculate the initial speed with which the student threw the ball. (1.2) K/U T/I A

74. A student walks across a field with a displacement of 125 m [E 60.0° N]. Determine the components of the displacement. (1.3) K/U T/I A

75. A sailboat first sails 125 km [E], then 85.0 km [E 45.0° N], and finally 94.0 km [S]. The voyage takes place in 6.75 h. (1.3) K/U T/I A
    (a) Use addition of vector components to determine the total displacement of the sailboat.
    (b) Calculate the average velocity of the sailboat.
    (c) Calculate the average speed of the sailboat.

76. An airplane with an initial velocity of 65 m/s [S] turns until its velocity after 15 s is 75 m/s [E]. Calculate the average acceleration of the airplane. (1.4) K/U T/I A

77. The Cap aux Diamants is a high bluff overlooking the St. Lawrence River at Québec City, rising about 98 m above the river. Suppose you launch a projectile horizontally over the river from the bluff with an initial speed of 2.9×10^2 m/s. (1.5) K/U T/I A
   (a) Calculate the time it takes for the projectile to land.
   (b) Calculate the range of the projectile, in kilometres.
78. For the same bluff as in Question 77, make the following assumptions: the terrain south and southeast of the Cap aux Diamants has the same level as the river, the initial speed of the projectile is 290 m/s, and the projectile launcher points up at an angle of 24° above the horizontal. (1.5) K/U T/I C A
   (a) Calculate the time of flight for the projectile.
   (b) Calculate the range of the projectile, in kilometres.
   (c) What would the range be if you set the angle of the projectile launcher at 45° above the horizontal?
   (d) Consider the range equation when the net vertical displacement is zero. Explain why your answer to (c) equals the greatest possible range for the given initial speed of the projectile.
79. Consider an airplane that travels at 800.0 km/h [E] against a wind with a velocity of 75.0 km/h [W 30.0° S]. (1.6) K/U T/I A
   (a) Determine the speed of the plane with respect to the ground.
   (b) Calculate how many kilometres south of its due east course the plane would be displaced after 3.0 h of flight.
80. A student paddles a canoe with a speed of 5.0 m/s relative to the river. The river has a current that moves south with a speed of 3.0 m/s. Determine the time it takes the student to travel 4.0 km upstream and then 5.0 km downstream. (1.6) K/U T/I A
81. An applied force pulls a car up an inclined plane that has an angle of 25°. The weight of the car is 6.8×10^3 N. (2.1) K/U T/I C A
   (a) Draw an FBD of the car.
   (b) Calculate the magnitude of the minimum applied force required to pull the car up the incline.
82. A space probe far from any planets and stars requires a force of 1.2×10^3 N to accelerate at 2.4 m/s^2. Determine the force needed to accelerate the probe at the same rate away from Earth's surface. (2.2) K/U T/I A
83. Two ice skaters, one with a mass of 57 kg and the other with a mass of 75 kg, stand facing each other on the surface of a frozen pond. One of the skaters exerts a 135 N force against the other. Calculate the magnitude of the acceleration of each skater. (2.2) K/U T/I A
84. Two heavy crates, m_1 and m_2, lie on different inclines with angles 18.0° and 33.0°, respectively (**Figure 8**). A cable, which runs over a pulley, connects the crates. The masses of the cable and the pulley are negligible. Assume that each incline is frictionless and that the system is in equilibrium. The mass of m_1 is 4.26×10^2 kg. (2.3) K/U T/I A

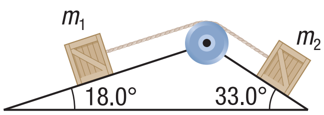


Figure 8

   (a) Determine the tension in the cable.
   (b) Calculate the mass of m_2 needed to keep the system in a state of equilibrium.
   (c) Now suppose that the mass of m_1 is 4.26×10^2 kg, and the mass of m_2 is 1.95×10^2 kg. The crates move to the left. The angles of the inclines and all other conditions are the same. Determine the magnitude of the acceleration of the crates.
85. You are helping a friend move, and you need to load a 2.65×10^2 kg box of books. You slide the box up a ramp, which has an incline of 30.0° and a coefficient of static friction of 0.45. You apply the force on the box at an angle of 39.0° with respect to the ramp. (2.4) K/U T/I A
   (a) Calculate the minimum force needed to slide the box up the ramp.
   (b) Now you want to stop halfway up the ramp. What coefficient of static friction must exist between the box and the ramp for the box to stay in place?
86. An elevator accelerates upward in 3.4 s to a final speed of 7.4 m/s. Determine the apparent weight of a passenger with a mass of 56 kg. (3.1) K/U T/I A
87. Suppose Earth turned with a greater rotation speed. Determine what the period of rotation, in hours, must be for the centripetal acceleration to equal *g*. The radius at Earth's equator is 6.378×10^3 km. (3.2) K/U T/I A
88. A skilled skateboarder can do a loop-the-loop. A concrete vertical-loop track has a radius of 6.53 m. Determine the minimum speed needed for the skateboarder to remain on the track when upside down. (3.2) K/U T/I A
89. A cylindrical space station has a radius of 57 m. The rotation produces artificial gravity equal to 90.0 % of the gravity on Earth's surface. (3.2, 3.4) K/U T/I A
   (a) Calculate the centripetal acceleration along the wall of the station.
   (b) Determine the period of rotation of the station.

90. A way to test the tensile strength of a wire is to place a mass at the end of it and spin the mass with uniform circular motion. The speed at which the wire breaks is a measure of its strength. The maximum frequency of rotation before a certain wire breaks is 22.5 Hz. The wire is 51.5 cm long and has a mass of 0.656 kg. (3.3) K/U T/I A
    (a) Determine the speed with which the mass moves.
    (b) Calculate the centripetal force acting on the wire.
91. Humans can endure accelerations over 45 times as great as *g*. One of the main ways of testing human endurance of these accelerations was the "human centrifuge," the first of which was built by the U.S. Navy in 1950 and operated by them from 1950 through 1996. This device consisted of a large metal sphere in which the subject sat. This sphere connected to the end of a 15.2 m metal arm, which rotated rapidly to create accelerations as great as 40*g*. Calculate the maximum rotational speed of the centrifuge. (3.4) K/U T/I A
92. A clothoid loop of a roller coaster is 40.0 m high with a radius of curvature of 10.0 m at the top. Assume that a cart rolls around the inside of the loop with nothing holding it onto the track. (3.5) K/U T/I A
    (a) What is the minimum speed of the cart at the top of the clothoid loop?
    (b) What is the minimum speed of the cart on a circular loop of the same height?
    (c) Explain why modern roller coasters often have clothoid loops instead of circular loops.

Evaluation

93. As straightforward as the algebraic component method is for adding vectors, it can become very involved if you have more than three vectors to add. Propose how you could program a computer, an electronic spreadsheet, or a calculator to determine the total displacement for a set of 20 ordered displacement vectors. Identify what each step of the program would do. (1.3) K/U T/I C
94. Traction between tires and the road is critical for automobile safety and efficiency. Suppose a road surface provides, by means of static friction, an average acceleration of 6.37 m/s^2. A car travelling at 35.0 m/s around a circular segment of highway changes its direction by 45°. (1.4) K/U T/I A
    (a) Using the equation that defines average acceleration and noting that only the direction changes, calculate the minimum time it takes for the driver to follow the curve without skidding.
    (b) Suppose the road is a complete circular track. Calculate the minimum time in which the driver could complete the circle once without skidding.
    (c) From your answer to (b), determine the radius of the circular track.
95. There are a number of things to consider when designing a building. List three forces that you need to anticipate before designing a building. Draw an FBD of a tall apartment building, showing as many of these forces as possible acting on the structure. (2.1) K/U T/I C A
96. Before they were decommissioned, the NASA space shuttles required two solid rocket boosters (SRBs) to launch the shuttle from Earth's surface. Both SRBs produced 2.5×10^7 N at liftoff. The combined mass of a shuttle and rocket boosters was about 2.0×10^6 kg. (2.2) K/U T/I A
    (a) Calculate the net acceleration of a space shuttle and rockets at the time of liftoff.
    (b) Calculate, in kilometres per hour, the speed of the shuttle and rockets after 5.0 s.
    (c) Calculate, in kilometres per hour, the speed of the shuttle and rockets after 15 s. (Note that the speed is actually greater because of the increase in thrust shortly after takeoff and reduced mass due to spent fuel.)
97. Pick one everyday activity that involves motion and forces, for example, cycling, running, or driving. (2.2, 2.3) K/U T/I C A
    (a) Describe the fundamentals of the activity using Newton's laws.
    (b) Using examples, describe how Newton's laws have changed the way you think about this activity.
98. A block and tackle is a combination of pulleys and rope that makes it easier to lift heavy objects (**Figure 9**). As the number of pulleys in the block and tackle increases, the force needed to raise a given weight is reduced. However, the length of rope needed to raise the object increases in proportion to the amount that the applied force decreases. Suppose a worker lifts a 2.5×10^3 N crate 22 m to the top of a building. The worker pulls 88 m of rope to raise the crate completely. (2.3) K/U T/I A

Figure 9

    (a) Determine the force the worker exerts on the rope.
    (b) What is the tension in each segment of rope in the block and tackle?

99. The alloy aluminum magnesium boride (called BAM) has one of the lowest coefficients of friction: 0.02. A ramp with a 30.0° incline has a BAM coating. A box with a mass of 122 kg slides down the ramp. (2.4) K/U T/I A
   (a) Calculate the acceleration of the box.
   (b) How does this value compare to the acceleration of the box if the ramp were truly frictionless?
100. While it is fairly easy to imagine a non-inertial frame of reference, it is much more difficult to visualize a true inertial frame. Think about the requirement for a frame of reference to obey the law of inertia. Then describe how objects would appear and behave in that frame of reference. (3.1) K/U T/I C A
101. In a graphic organizer, organize the following statements to justify why launching rockets near the equator is more advantageous than launching them at latitudes far from the equator. (3.3) K/U T/I C A
   - An object's apparent weight is less at the equator than at the poles (where it is greatest).
   - The maximum rotational motion at the equator gives rockets a boost if they are launched in an orbit that moves with Earth (that is, eastward).
   - Gravity must provide more of the centripetal force at the equator than anywhere else on Earth's surface.
   - At Earth's equator, the velocity of uniform circular motion is greatest.
   - The lower apparent weight makes it easier to rise against Earth's gravity.

Reflect on Your Learning

102. How did the information you learned in this unit affect your thinking about the directions of the vectors for displacement, velocity, and acceleration? Describe in your own words how these three properties of motion can point in different directions. K/U T/I C
103. How did your study of forces help you understand how objects move or do not move? For the various objects around you—your computer, a table or desk, anything that you can move from one place to another—ask yourself, "What forces act on that?" Try to explain these forces in terms of Newton's three laws of motion. In particular, think of how the various forces interact according to Newton's third law. K/U T/I C
104. What new perspective on the motion of objects did your studies of relative motion and projectile motion give you? Consider why relative motion is important when launching rockets and probes for space exploration. Why does the fact that planets and other bodies in the solar system move with respect to each other make it necessary to apply concepts of relative motion? K/U T/I C A
105. After completing this unit, how does your understanding of dynamics relate to your understanding of how technologies use dynamics? K/U T/I C
106. How did your learning in this unit connect to your prior studies in dynamics? K/U T/I C

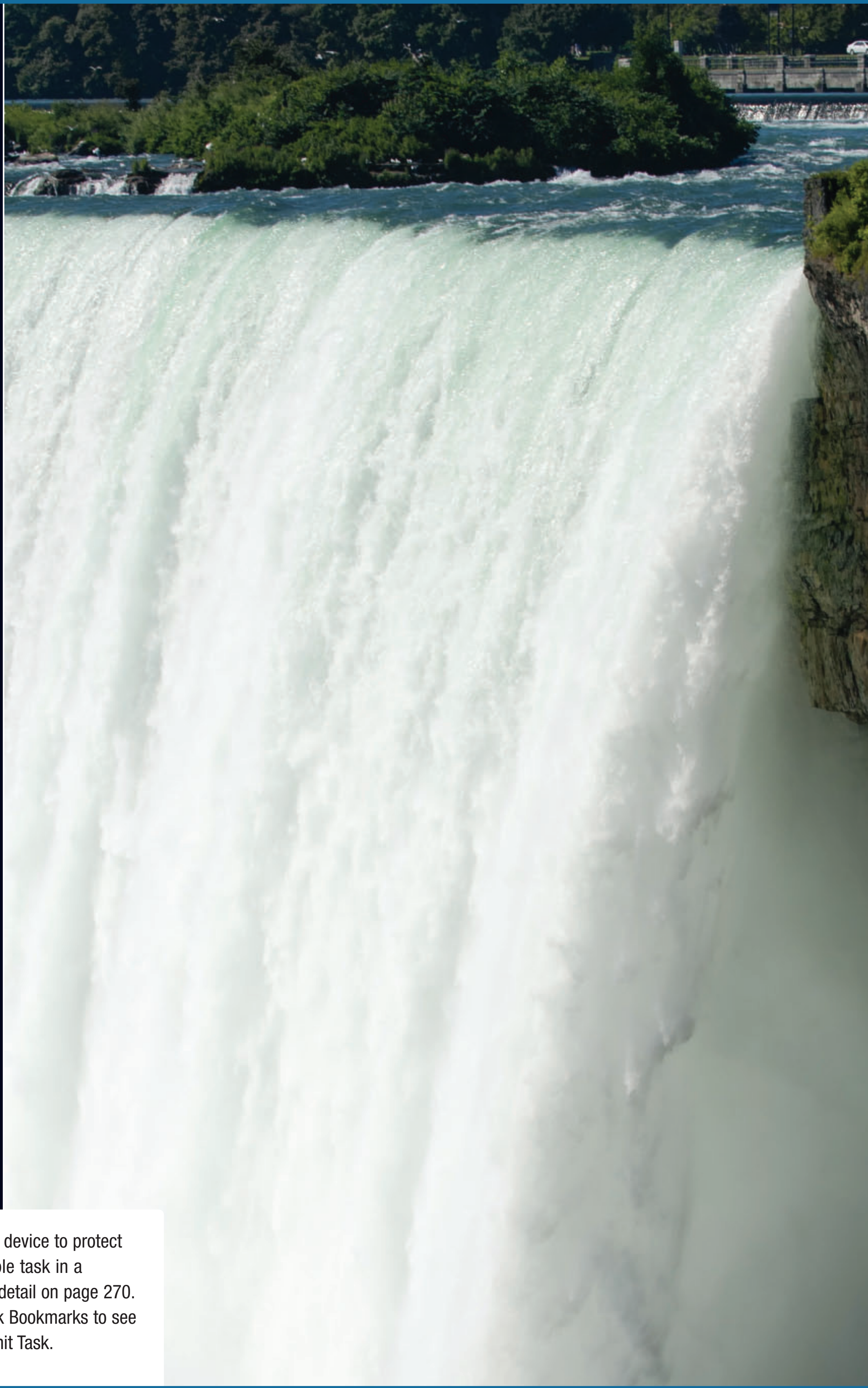
Research

WEB LINK

107. Research the manoeuvres, or "tricks," used by skateboarders and how they relate to the physics of motion and forces. See how some of the basic tricks, such as an Ollie, a frontside 180, or a 360 flip, are done and how skateboarders use forces to make them happen. Note also how skateboard design makes tricks easier or possible. Compile your results and present them to your class as an oral report or visual presentation. K/U T/I C A
108. Research drag racing, noting how the performance of drag racers differs from the performance of other racers and sports cars. Find out what features of drag racers affect their ability to accelerate, and learn how changes over the past several decades have led to greater accelerations. Examine the physics of drag racing in terms of kinematics, Newton's second law, the weight and normal force of the racer, and the coefficients of friction between pavement and different types of tires. Present your findings to your class orally. K/U T/I C A
109. Research the substance aluminum magnesium boride (BAM). Learn about the properties of this ceramic, in particular, its low coefficient of friction for both static and kinetic cases and its extreme hardness. Examine the results of early tests on this material, and evaluate applications of BAM to moving parts subjected to extreme mechanical forces and wear from friction. Choose a format to present your findings. K/U T/I C A
110. Research "pop bottle" rockets. How do you make one? How does a pop bottle rocket use the principles of motion? K/U C
111. Research Elizabeth MacGill, and describe some of her contributions. K/U T/I C
112. Oscar Pistorius, also known as the blade runner, is a double-leg amputee. He was the first amputee to win an able-bodied world track medal in 2011. K/U T/I C
   (a) Research the athletic accomplishments of Oscar Pistorius.
   (b) Why is there some controversy surrounding his artificial legs?
   (c) What possible ethical questions does it raise to allow a person with artificial limbs to compete against those without artificial limbs?
   (d) In your opinion, should people with artificial limbs be allowed to compete against those without? Write a paragraph defending your position.

UNIT 2

Energy and Momentum



OVERALL EXPECTATIONS

- analyze, and propose ways to improve, technologies or procedures that apply principles related to energy and momentum, and assess the social and environmental impact of these technologies or procedures
- investigate, in qualitative and quantitative terms, through laboratory inquiry or computer simulation, the relationship between the laws of conservation of energy and conservation of momentum, and solve related problems
- demonstrate an understanding of work, energy, momentum, and the laws of conservation of energy and conservation of momentum, in one and two dimensions

BIG IDEAS

- Energy and momentum are conserved in all interactions.
- Interactions involving the laws of conservation of energy and conservation of momentum can be analyzed mathematically.
- Technological applications that involve energy and momentum can affect society and the environment in positive and negative ways.

UNIT TASK **PREVIEW**

In this Unit Task, you will design either a safety device to protect a fragile object or a machine that does a simple task in a complicated way. The Unit Task is described in detail on page 270. As you work through the unit, look for Unit Task Bookmarks to see how information in the section relates to the Unit Task.

FOCUS ON **STSE**

INNOVATIONS IN ENERGY TECHNOLOGY

Society uses an enormous amount of energy to do its work. Power companies and other commercial energy providers give us easy access to needed energy. They extract this energy from many different sources, including natural sources such as falling water, wind, and sunlight, and induced sources such as burning coal and nuclear fission.

Energy providers do not create energy. They convert certain forms of energy into other more useful forms. The image on these pages shows Niagara Falls, over which almost 2 000 000 L of water flows every second. The water drops over 52 m, or almost the height of a 20-storey building. Hydroelectric power plants on the Niagara River use some of the energy of the flowing water to supply one-quarter of Ontario's power needs.

For many years, our society has used fossil fuels to meet most of our energy needs. We now face problems as these non-renewable resources dwindle and the environmental impact of their use becomes clear. Coal-fired power plants, for example, use the thermal energy of combusting coal to create electricity. Coal mining, however, produces poisonous chemicals such as sulfuric acid, while coal combustion releases greenhouse gases such as sulfur dioxide. As a result of these concerns, scientists and engineers have begun to study alternative energy sources more seriously to find clean, inexpensive, and renewable forms of energy.

Humans have used hydro power since ancient times. In 1881, the world's first hydroelectric power plant opened at Niagara Falls. Current technology, however, allows us to extract more energy than ever before from a growing assortment of water sources. Developing technology will also allow us to extract more energy while having fewer negative effects on the natural environment of the sources. For instance, new innovations in hydroelectric power allow us to harness energy from ocean tides, currents, and waves, and even from the difference in temperature between deep and surface ocean water.

Work, energy, and the physics of collisions are important concepts related to energy production. Scientists and engineers must understand these concepts to make innovations in energy technology. In this unit, you will learn about these concepts and the role they play in society's production and use of energy.

Questions

1. What forms of energy are present in Niagara Falls?
2. Is Niagara Falls a source of renewable energy? Explain your answer.
3. Do you think Niagara Falls is a sustainable energy source? Explain your thinking.
4. How does hydroelectricity compare with other sources of energy used by society?

UNIT 2 ARE YOU READY?

CONCEPTS

- energy transformations
- Newton's laws of motion
- acceleration
- forces
- friction
- vector and scalar quantities

SKILLS

- drawing and interpreting graphs
- drawing and interpreting free-body diagrams
- solving problems using algebraic equations
- identifying and analyzing social and environmental issues related to the use of energy

Concepts Review

1. Describe an energy transformation that can be used to produce electrical energy. K/U C
2. Describe the energy transformations that occur when an airplane takes off. K/U C
3. Briefly state Newton's three laws of motion. Then, give a specific example that illustrates each law. K/U C
4. A worker moves a box with mass *m* along a warehouse floor (**Figure 1**). What variables determine the amount of work done on the box by the worker? Be specific in your answer. K/U

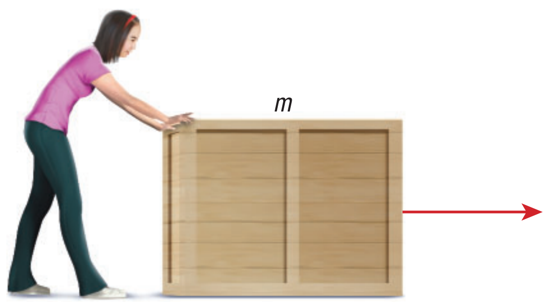


Figure 1

5. Explain why it is important to consider the effect of friction when examining the forces that act on an object. K/U C
6. Airbags are a type of safety device installed in vehicles to protect passengers from injury in a collision. T/I C A
   (a) Explain how energy transformations are used in airbags to protect passengers from injury during a collision.
   (b) What are some of the limitations of airbags as a safety device?

Skills Review

7. **Figure 2** is a velocity–time graph of a falling brick. Determine the acceleration of the brick at $t = 3.0$ s. K/U T/I

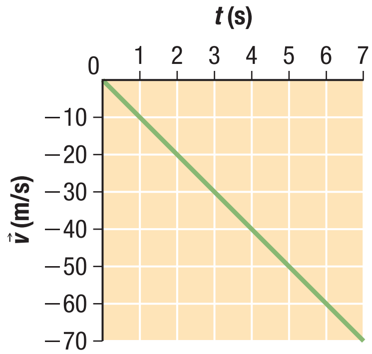


Figure 2

8. Draw a free-body diagram for each object in italics in the sentences below. K/U T/I C
   (a) A *cellphone* lies at rest on a countertop.
   (b) A *skydiver* whose parachute has opened falls toward the ground with constant speed. Assume there is no wind.
   (c) A *box of books* is pushed up a rough inclined ramp.
   (d) A *sports car* travelling at a constant speed makes a banked turn on an icy (frictionless) highway exit ramp.
9. You push a small cup of water across a table with a force of 5.5 N. The force of friction on the cup is 4.0 N. K/U T/I C
   (a) Draw a free-body diagram of the cup.
   (b) Calculate the net force on the cup.
10. Explain the difference between vector and scalar quantities. K/U C
11. Solve the equation $3x - 17 = x^2 - 27$ for x. T/I

12. Write the proportionality statement relating the variables that are defined in italics, and sketch the corresponding graph for each case. K/U T/I
    (a) The *gravitational potential energy* of a person walking up a flight of stairs doubles when the *height* of the stairs doubles.
    (b) The *kinetic energy* of a vehicle depends on the square of its *speed*.
    (c) The *acceleration* of a particle triples as the *applied force* increases by a factor of three.
13. Calculate the magnitude and sign of the *x*-component and *y*-component of the force shown in each part of **Figure 3**. K/U T/I

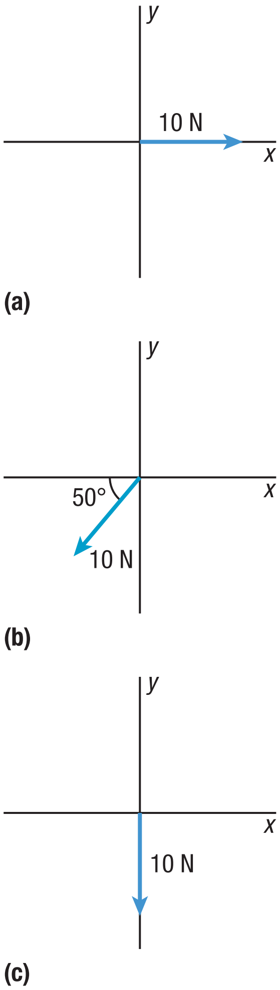


Figure 3

14. Your class is investigating how the speed of a mass when it hits the floor depends on the distance from which it is dropped. What safety precautions would you follow for this investigation? T/I C A
15. Compare the costs and benefits of operating a laptop computer using its battery versus using an electrical outlet. Include at least one social impact, one environmental impact, and one economic impact in your response. T/I C A
16. A digital balance scale is often used to measure the mass of an object (**Figure 4**). K/U T/I C
    (a) Describe how to calibrate the scale using a set of reference masses.
    (b) Explain why you should check that the scale is calibrated before measuring the mass of any object in an investigation.
    (c) What other sources of error should you minimize when attempting to determine an object's mass?

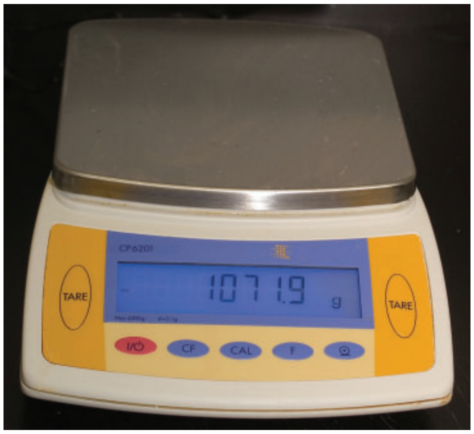


Figure 4

CAREER PATHWAYS **PREVIEW**

Throughout this unit, you will see Career Links. Go to the Nelson Science website to find information about careers related to Energy and Momentum. On the Chapter Summary page at the end of each chapter, you will find a Career Pathways feature that shows you the educational requirements of the careers. There are also some career-related questions for you to research.

CHAPTER 4

Work and Energy

KEY CONCEPTS

After completing this chapter you will be able to

- explain the concepts of work, energy, friction, and the work–energy theorem
- explain the concepts of gravitational potential energy, conservation of energy, Hooke's law, elastic potential energy, and simple harmonic motion and solve related problems in one and two dimensions
- describe technological applications involving energy and explain how they can affect society and the environment
- analyze the relationship between the work–energy theorem and the law of conservation of energy and solve related problems in one and two dimensions
- conduct an inquiry to test the law of conservation of energy

How Can Understanding Work and Energy Improve Our Quality of Life?

Energy is such a central part of our lives that we tend to use it without thinking about it. Energy allows us to travel long distances in a short time, communicate with others around the world, enjoy movies and games, and stay comfortable during the changing seasons. Life would be quite different without the scientific and technological advances that apply energy to our advantage. To understand the role of energy in our everyday lives, it is important to grasp basic energy concepts.

Energy takes many forms and can transform between forms. A tree converts radiant energy from the Sun into chemical energy through photosynthesis. The chemical energy transforms to thermal energy if you burn wood from the tree. When wood burns in a steam engine, the thermal energy transforms into mechanical energy.

Sports and other activities revolve around exchanges between forms of energy. Bungee jumping, downhill skiing, and roller coasters are all examples. During a bungee jump, the type of energy is different at the top of the jump than it is in the middle of the fall or at the bottom.

All sports and activities must balance excitement and safety. Successful safety equipment protects us from extreme exchanges of energy. Hockey pads protect the player by absorbing the kinetic energy of a check and transforming it into another form of energy. An athlete's physical condition also helps. Peak health means an athlete can generate and absorb energy efficiently. In this chapter, you will learn about work and energy and how their applications affect society and the environment.

STARTING POINTS

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. What forces are acting on a bungee jumper before the fall? During the fall?
2. What type of energy do you think is present at the top of the bungee jump? In the middle? At the bottom?
3. Describe the nature of a bungee jumper's motion.
4. What do you think a graph of the bungee jumper's height versus time would look like?
5. At what point(s) would the bungee jumper's speed be at a maximum? At what point(s) would the bungee jumper's speed be at a minimum?
6. Would the bungee jumper's speed ever be zero? Explain.

Mini **Investigation**

On the Rebound

Skills: Predicting, Performing, Observing, Analyzing, Evaluating

SKILLS HANDBOOK A2.1, A5.5

In this activity, you will investigate the energy changes of a bouncing ball.

Equipment and Materials: computer with graphing software; motion sensor; ball; metre stick

1. Determine the height from which you will hold the ball before you drop it. Record the height of the ball using the height at the bottom of the ball as your measurement.
2. Predict how a graph of ball height versus time will look. Sketch a rough graph of height versus time to show your prediction. Your graph should show the height of the ball on the vertical axis and time on the horizontal axis.
3. Place the motion sensor directly above the ball. Start the sensor, and then drop the ball. Let it bounce several times.
4. Make a graph of your data from the motion sensor.

A. Compare your prediction with the actual result. Account for any discrepancies. K/U T/I

B. Describe the form of energy of the ball before it is dropped. K/U T/I

C. Describe the form the energy of the ball takes as it hits the floor and as it continues to bounce. K/U T/I

D. At what point did the ball have the greatest amount of energy? Explain. K/U T/I

E. What happens to the total energy of the ball over time? Why does this happen? K/U T/I

4.1 Work Done by a Constant Force

Studying can feel like a lot of work. Imagine studying several hours for a difficult test or spending all afternoon writing a report for class. While this is a significant amount of hard work, in the scientific sense of the word, you have done no work at all. In physics, **work** is the energy that a force gives to an object when the force moves the object. When you read your notes, you do no work because you do not exert a force on an object to move it.

work the product of the magnitude of an object's displacement and the component of the applied force in the direction of the displacement

Your everyday life, however, is filled with examples of work in the scientific sense. You do work on a backpack when you lift it to your shoulders. You do work on the classroom door when you push it open or pull it closed. You do work on a basketball by bouncing or throwing it. In these cases, a force does work on an object to move it.

You may notice that you do more work lifting a backpack filled with heavy books than lifting an empty one, and more work lifting a backpack from the floor than from a desk because the distance from the floor to your shoulder is greater than the distance from a desk to your shoulder. The scientific definition of work is consistent with these experiences.

Work

Work depends on the magnitude of the force applied to an object and the distance the object moves. In fact, work depends only on the object's displacement in the direction of the applied force and not displacement perpendicular to the force. In other words, the work depends only on the component of force in the direction of motion and not the force perpendicular to the motion.

Figure 1 The force of the shovel does work on the snow. Only the horizontal force component contributes to the work.

Suppose, for example, you are shovelling snow. You might push on the snow at an angle θ to the ground, but the snow moves horizontally (**Figure 1**). The component of force directed into the ground does not help move the snow across the ground. Only the component of force along the ground moves the snow.

In any situation, only the force in the direction of an object's displacement does work on the object. The equation for calculating the work, W, that a constant force, $\vec{F}$, does to cause the displacement, $\Delta\vec{d}$, of an object is

$$W = F\Delta d \cos\theta$$

where F is the magnitude of the force, Δd is the magnitude of the object's displacement, and θ is the angle between the force and the displacement. The component of the force in the direction of the motion is $F\cos\theta$ (**Figure 2**).

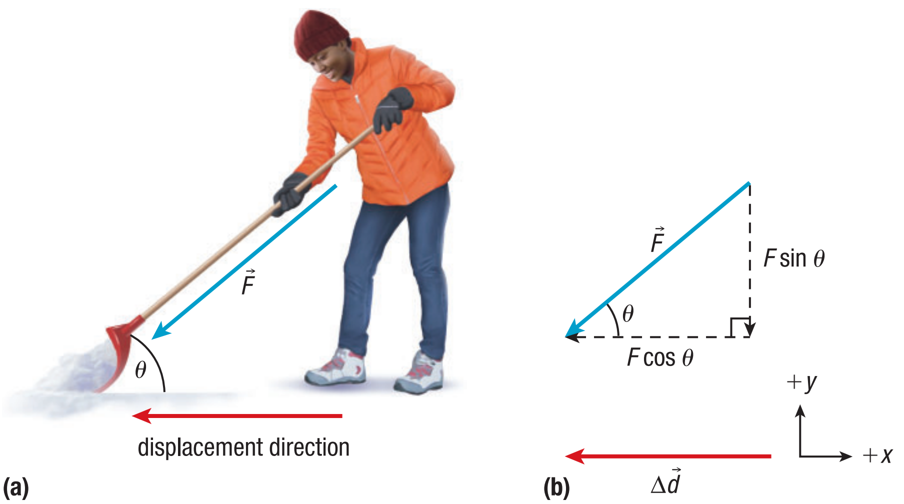


Figure 2 (a) A force acts at an angle to the displacement. (b) Only the component parallel to the displacement does work.

The work done by a force depends on two vectors: the applied force and the resulting displacement. The work done on an object is proportional to both force, $\vec{F}$, and displacement, $\Delta\vec{d}$. However, the amount of work done is a scalar quantity, not a vector.

The SI units of work are newton-metres (N·m), or kilograms times metres squared per second squared (kg·m^2/s^2). This unit is called a **joule** (J). In the following Tutorial, you will explore how to use the work equation to determine the work done in different situations.

joule the SI unit of work and energy; a force of 1 N acting over a displacement of 1 m does 1 J of work; symbol J

Tutorial 1 Calculating Work Done

You can determine the amount of work done on an object if you know the applied force, the displacement of the object, and the angle between the force and the displacement.

Sample Problem 1: Calculating the Work Done When the Force Is Parallel to the Displacement

The displacement of an object is often in the same direction as the applied force. In this case, the equation for calculating the work done by the force is simplified. Suppose a hockey player slides a puck along the ice with a constant force of 85 N in the forward direction (**Figure 3**). The puck moves a horizontal distance of 0.20 m while in contact with the hockey stick. Calculate the amount of work done on the puck by the stick.

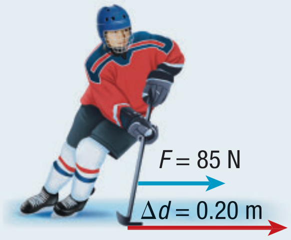


Figure 3

Given: $F = 85$ N; $\Delta d = 0.20$ m

Required: W

Analysis: Use the equation for work, $W = F\Delta d \cos\theta$. F and Δd are in the same direction, so the angle between them is zero, $\theta = 0$.

Solution:
$$\begin{aligned} W &= F\Delta d\cos\theta \\ &= (85\text{ N})(0.20\text{ m})\cos 0° \\ &= (17\text{ N·m})(1) \\ &= 17\text{ N·m} \\ W &= 17\text{ J} \end{aligned}$$

Statement: The hockey stick does 17 J of work on the puck. Note that the stick only does work on the puck while the stick is in contact with the puck, applying force.

Sample Problem 2: Calculating the Work Done When the Force Is at an Angle to the Displacement

The applied force and the displacement can be at an angle to each other. For example, a student pushes a lawnmower forward with a constant force of 48 N for a distance of 7.5 m (**Figure 4**). The angle between the force and the displacement of the lawnmower is 32°. Calculate how much work is done on the lawnmower by the student.

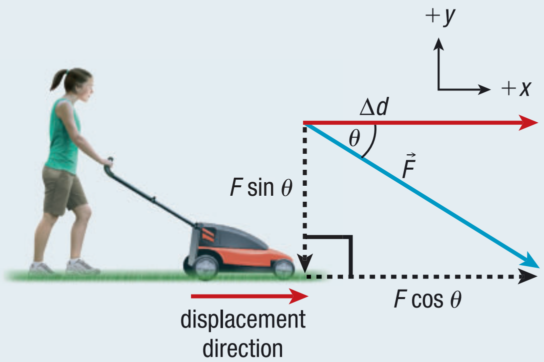


Figure 4

Given: $F = 48$ N; $\Delta d = 7.5$ m; $\theta = 32°$

Required: W

Analysis: The work done on the lawnmower by the student depends only on the component of force in the direction of the mower's displacement. Use the equation for work, $W = F\Delta d\cos\theta$. F and Δd are at an angle of 32° to each other.

Solution:
$$\begin{aligned} W &= F\Delta d\cos\theta \\ &= (48\text{ N})(7.5\text{ m})\cos 32° \\ &= (360\text{ N·m})(0.848) \\ W &= 3.1 \times 10^2\text{ J} \end{aligned}$$

Statement: The student does 3.1×10^2 J of work on the lawnmower.

Sample Problem 3: Calculating the Work Done When the Force Is Perpendicular to the Displacement

Suppose you carry books with a weight of 22 N a distance of 3.8 m across a room (**Figure 5**). Determine the work done on the books.

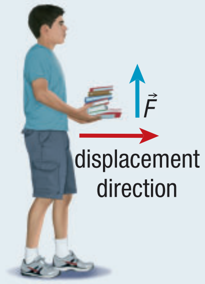


Figure 5

Given: $F = 22$ N; $\Delta d = 3.8$ m; $\theta = 90°$

Required: W

Analysis: By Newton's third law, the upward force you exert on the books is equal to the weight of the books, 22 N. Use the equation for work, $W = F\Delta d \cos\theta$. F and Δd are at an angle of 90° to each other.

Solution:

$$\begin{aligned} W &= F\Delta d \cos\theta \\ &= (22\text{ N})(3.8\text{ m})\cos 90° \\ &= (22\text{ N})(3.8\text{ m})(0) \\ &= 0\text{ N}\cdot\text{m} \\ W &= 0\text{ J} \end{aligned}$$

Statement: No work is done on the books.

Practice

1. A weightlifter uses a force of 275 N to lift weights directly upward through a distance of 0.65 m. Determine the work done on the weights by the weightlifter. T/I A [ans: 1.8×10^2 J]
2. Calculate the work done on a wall if you push on it with a constant force of 9.4 N and the wall does not move. T/I A [ans: 0 J]
3. A pool cue stick strikes a ball with a constant force of 0.73 N, causing the ball to move 0.65 m in the direction of the force. The ball moves 0.080 m while the cue stick is in contact with it. Calculate the work done on the ball by the cue stick. T/I A [ans: 0.058 J]
4. A tow truck uses a winch with a rope attached to pull a car that is stuck in a ditch. The rope exerts a force of 9.9×10^3 N on the car body, and the angle between the rope and the direction the car moves is 12°. Determine the amount of work done on the car by the tow truck to move the car 4.3 m. T/I A [ans: 4.2×10^4 J]

Positive and Negative Work

The force of the hockey stick on the puck in Sample Problem 1 resulted in positive work that caused the puck's speed to increase. What if the puck slides to rest? The displacement of the puck and the force on the puck by friction are in opposite directions. When an object moves in a direction opposite to an applied force, the force does negative work. Negative work will cause a loss of kinetic energy.

Forces that cause negative work are exerted at an angle between 90° and 180°, opposite to the object's direction. As you read in Chapter 2, friction is the force resisting the motion of objects moving against each other. Tutorial 2 gives you an opportunity to calculate the negative work that occurs when a force, such as friction, and displacement are in opposite directions.

Tutorial 2 Calculating Negative Work

Sample Problem 1: Calculating Negative Work in One Dimension

Suppose a car is moving along a straight road when the driver suddenly applies the brakes. The force of friction between the ground and the car tires is opposite to the car's direction of motion and decreases the car's speed. Calculate the work done by a constant frictional force of 1.4 kN over a distance of 1.2×10^2 m.

Given: $F = 1.4\text{ kN} = 1.4 \times 10^3$ N; $\Delta d = 1.2 \times 10^2$ m; $\theta = 180°$

Required: W

Analysis: $W = F\Delta d \cos\theta$

Solution: $W = F\Delta d \cos\theta$
$= (1.4 \times 10^3 \text{ N})(1.2 \times 10^2 \text{ m}) \cos 180°$
$= (1.4 \times 10^3 \text{ N})(1.2 \times 10^2 \text{ m})(-1)$
$= -1.70 \times 10^5 \text{ N·m}$
$W = -1.70 \times 10^5 \text{ J}$

Statement: Friction does -1.7×10^5 J of work on the car to slow it down. The negative sign indicates that the force is opposing the motion.

Sample Problem 2: Calculating Negative Work in Two Dimensions

An ice skater slides to a stop by pushing her blades against the ice (**Figure 6**). The ice exerts a constant force of 95 N on the skater, and the skater stops in 1.2 m. The angle between the force and the skater's direction of motion is 140°. Calculate the work done on the skater by the ice.

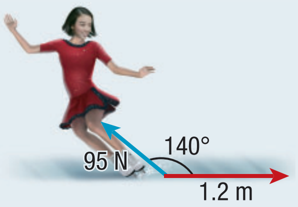


Figure 6

Given: $F = 95$ N; $\Delta d = 1.2$ m; $\theta = 140°$

Required: W

Analysis: Use the work equation, $W = F\Delta d \cos\theta$. Remember that $\cos\theta$ is negative for angles between 90° and 180°.

Solution: $W = F\Delta d \cos\theta$
$= (95 \text{ N})(1.2 \text{ m}) \cos 140°$
$= -87 \text{ N·m}$
$W = -87 \text{ J}$

Statement: The ice does –87 J of work on the skater.

Practice

1. A drop tower ride lifts riders at a constant speed to a height of 78 m and suddenly drops them. T/I A
   (a) Determine the work done on a 56 kg rider by the machine as she is lifted to the top of the ride. [ans: 4.3×10^4 J]
   (b) Determine the work done on the rider by gravity as she is lifted to the top of the ride. [ans: -4.3×10^4 J]
2. As a passenger airplane touches down it skids across the runway to a stop. Friction between the ground and the plane's wheels applies a constant force of 5.21 kN as the plane slides a distance of 355 m. T/I A
   (a) Calculate the work done on the airplane by friction. [ans: -1.85×10^6 J]
   (b) Determine the distance the plane would slide if friction applied the same force but did -1.52×10^6 J of work. [ans: 292 m]
3. A skier slides down a snowy hill and then stops by pressing his skis at an angle to the snow. The snow exerts a constant force of 5.9 N on the skier at an angle of 150° to the skier's displacement. The skier moves a distance of 3.5 m. Calculate the work done on the skier by the snow. [ans: -18 J]

You observed in Tutorial 1 that a force does positive work on an object when the object's displacement is in the same direction as the force. The work is also positive when the direction of the object's displacement is at an angle between 0° and 90° to the applied force. Similarly, Tutorial 2 showed that a force does negative work on an object when the object's displacement is opposite to the direction of the force, at an angle between 90° and 180° to the applied force. Recall that zero work is done on an object when the object's displacement is exactly 90° to the applied force. Are there other situations involving an applied force in which zero work is done?

Zero Work

When the direction of an object's displacement is exactly 90° to the applied force, the force does no work on the object. Look back at Sample Problem 3 in Tutorial 1. As the student walks across the room, he pushes up on the books, but the books move to the right. The angle between the force and the displacement is 90°. Since $W = F\Delta d \cos\theta$ and $\cos 90° = 0$, the student does no work on the books.

The work also equals zero when the force on an object is zero. Consider a probe travelling in space far from any gravitational forces. The probe's motion is due to inertia, but the force on it is zero. As a result, zero work is done on the probe.

The third variable in the work equation is the displacement of the object. Have you ever tried to twist the lid off a jar, but the lid was stuck? Regardless of how much force you exert on the lid, you do zero work unless the lid moves. The work on an object is zero when any of the force, the displacement, or the cosine of the angle between the force and the displacement is zero. In the following Tutorial, we will examine another example of a force that does zero work.

Tutorial 3 Work and Centripetal Acceleration

When a centripetal force acts on an object moving along a circular path, the direction of the force is perpendicular to the direction of the object's motion.

Sample Problem 1: Calculating the Work Done on an Object Moving in a Circular Path

A student attaches one end of a string to a rubber stopper. She then holds the other end of the string and twirls the rubber stopper in a horizontal circle around her head. The string exerts a tension force on the stopper directed toward the centre of the circle (**Figure 7**). Determine the work done on the stopper by the string during a revolution.

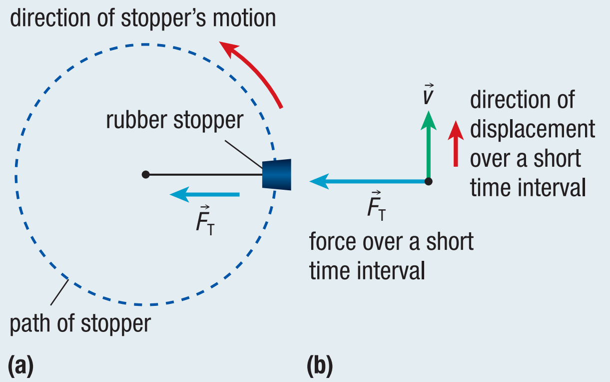


Figure 7 (a) The circular path of the rubber stopper, seen from above. (b) During a very short time interval, the displacement of the stopper is perpendicular to the tension force.

Given: $\theta = 90°$

Required: W

Analysis: The tension force, $\vec{F}_T$, causes the stopper's centripetal acceleration. At each moment, the stopper's instantaneous velocity is at an angle of 90° to the tension force. During a very short time interval, the very small displacement of the stopper is also at an angle of 90° to the tension force. We can break one loop around into a series of many small displacements, each occurring during a very short time interval. During each time interval, the tension and the displacement are perpendicular. The total work done during one loop around will equal the sum of work done during each small displacement. For each small displacement, use the work equation, $W = F\Delta d \cos\theta$, with $\theta = 90°$.

Solution:
$$\begin{aligned} W &= F\Delta d \cos\theta \\ &= F\Delta d \cos 90° \\ &= F\Delta d\,(0) \\ W &= 0\ \text{J} \end{aligned}$$

Statement: Summing the work done during all the small displacements of the loop gives a total of $W = 0$ J during each revolution. The tension force exerted by the string does zero work on the stopper during the revolution.

Practice

1. Earth exerts a gravitational pull that causes the Moon to experience a centripetal acceleration during its orbit. Assume that the Moon's orbit around Earth is circular. Determine the work done by Earth's gravitational pull on the Moon. As part of your solution, include a diagram that illustrates Earth, the Moon, the direction of travel, and the force at one instant in time. K/U T/I C [ans: 0 J]

In Tutorial 3, the work does not depend on the magnitude of the tension force or the total distance the stopper moves. We treated the stopper's orbit as a series of tiny displacements perpendicular to the tension force. For each tiny displacement, the tension force does zero work on the stopper. We conclude that the work done by the centripetal force acting on an object in circular motion is zero.

Work Done by Multiple Forces

Almost all real-world examples of work involve friction plus other forces. Tutorial 4 explores how the presence of multiple forces affects the work done on an object.

Tutorial 4 Calculating Work Done by Multiple Forces

Sample Problem 1: Calculating the Work Done by Multiple Forces on a Dragged Object

Figure 8 shows a long-distance hiker pulling a sled across a snowy field.

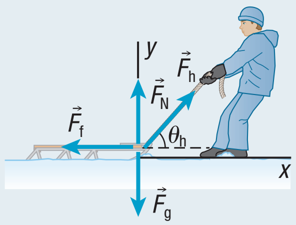


Figure 8

The hiker exerts a constant force of 135 N on the sled at a 48.0° angle to the sled's displacement. At the same time, a constant 67.0 N frictional force on the sled from the snow opposes the motion. The sled also experiences the force from gravity and the normal force from the snow, but these forces do not contribute to the work. Calculate the work done by the hiker (h), the work done by friction (f), and the total (T) work done on the sled when the hiker pulls the sled 345 m over the snow.

Given: $F_h = 135$ N; $F_f = 67.0$ N; $\Delta d = 345$ m; $\theta_h = 48.0°$; $\theta_f = 180°$

Required: W_h; W_f; W_T

Analysis: $W = F\Delta d \cos\theta$. The total work done is the sum of the work done by the individual forces.

Solution:

$$\begin{aligned} W_h &= F_h \Delta d \cos\theta_h \\ &= (135\ \text{N})(345\ \text{m})\cos 48.0° \\ &= 3.116 \times 10^4\ \text{N·m} \\ W_h &= 3.116 \times 10^4\ \text{J (one extra digit carried)} \end{aligned}$$

$$\begin{aligned} W_f &= F_f \Delta d \cos\theta_f \\ &= (67.0\ \text{N})(345\ \text{m})\cos 180° \\ &= -2.312 \times 10^4\ \text{N·m} \\ W_f &= -2.312 \times 10^4\ \text{J (one extra digit carried)} \end{aligned}$$

$$\begin{aligned} W_T &= W_h + W_f \\ &= 3.116 \times 10^4\ \text{J} - 2.312 \times 10^4\ \text{J} \\ W_T &= 8.04 \times 10^3\ \text{J} \end{aligned}$$

Statement: The hiker does 3.12×10^4 J of work on the sled. Friction does -2.31×10^4 J of work on the sled. The total work done on the sled is 8.04×10^3 J.

Practice

1. A hiker pulls a sled a distance of 223 m with a constant force of 122 N exerted at an angle of 37°. Friction acts on the sled with a constant force of 72.3 N. Calculate the work done on the sled by the hiker and by friction, and the total work done on the sled. T/I A
   [ans: 2.2×10^4 J; -1.6×10^4 J; 5.6×10^3 J]
2. If together the hiker and friction do 2.42×10^4 J of total work on the sled in Figure 8, how far did the hiker pull the sled? T/I A [ans: 963 m]

A frictional force is present in many parts of the hiker and sled problem in Tutorial 4. Friction acts, for example, between the hiker's hands and the rope, between the rope and the sled, and between the hiker's boots and the snow.

A frictional force acts and does work on any surfaces that slide past each other. In each case, the frictional force transfers energy to surfaces, increasing their kinetic energy. The energy does not disappear. The increase in temperature occurs because of the motion of atoms at the surfaces. All substances are composed of particles in constant motion. When friction heats two surfaces, the vibrations of the particles become larger, and the particles move more quickly. The faster-moving particles have more energy, and the energy stored in the chemical bonds increases.

In summary, when analyzing the total work done on an object, all forces that are present, including friction, must be considered. The net effect of these forces can result in either positive, negative, or zero total work done on the object.

4.1 Review

Summary

- Work occurs when a force $\vec{F}$ is applied to move an object a displacement $\Delta\vec{d}$.
- The work done on an object is given by $W = F\Delta d \cos\theta$, where θ is the angle between $\vec{F}$ and $\vec{d}$. The SI unit of work is the joule (J).
- When an object moves at an angle to an applied force, only the component of the force in the direction of the displacement does work on the object.
- When an object moves in a direction opposite to an applied force, the force does negative work on the object.
- A force does zero work on an object when the angle between the force and the object's displacement is 90°. Zero work is also done when either the force or the displacement is zero.

Questions

1. Two ropes pull on a crate toward the left with forces of equal magnitude, F, causing the crate to move horizontally (**Figure 9**). Which rope does more work on the crate? Explain. K/U

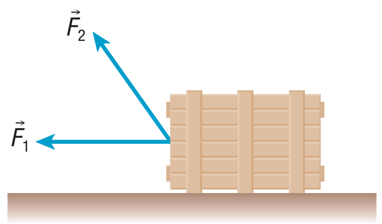


Figure 9

2. A toy consists of a small plastic tube connected at the centre to one end of a long string. A girl holds the other end of the string and swings the toy in a horizontal, circular path above her head. Is work done on the toy by the string during each revolution? Explain your reasoning. K/U
3. A shopper pushes a loaded grocery cart with a force of 12.6 N. The force makes an angle of 21.8° above the horizontal. Determine the work done on the cart by the shopper as he pushes the cart 14.2 m. T/I A
4. The horse in **Figure 10** pulls a rider on a sleigh across a snowy horizontal field. The force of the rope is 22.8 N, and the horse does 9.53×10^2 J of work pulling the sleigh a distance of 52.6 m. Calculate the angle between the rope and the horizontal. T/I A

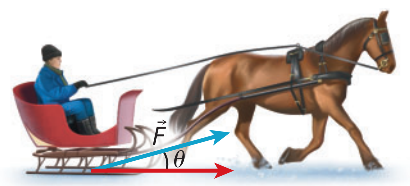


Figure 10

5. A warehouse worker pushes a crate of mass 24 kg up a ramp (**Figure 11**). Assume that the friction between the crate and ramp can be ignored. K/U T/I

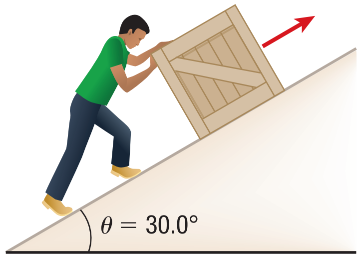


Figure 11

   (a) Determine the component of gravitational force directed along the ramp's surface.
   (b) Calculate the force required to move the crate at a constant speed up the ramp.
   (c) Calculate the work done in pushing the crate 23 m as measured along the ramp. Assume the crate moves at a constant velocity.
   (d) Assume the coefficient of kinetic friction between the crate and the ramp is $\mu_K = 0.25$. Calculate the work done on the crate by the worker and by friction, and calculate the total work done as the worker pushes the crate 16 m up the ramp.
6. A boy and a girl pull and push a crate along an icy horizontal surface, moving it 13 m at a constant speed. The boy exerts 75 N of force at an angle of 32° above the horizontal, and the girl exerts a force of 75 N at an angle of 22° above the horizontal. Calculate the total work done by the boy and girl together. T/I

Kinetic Energy and the Work–Energy Theorem

4.2

Imagine the energy that a jet, like the one in **Figure 1**, needs to climb into the air. The jet has a mass of hundreds of thousands of kilograms, yet during takeoff, it seems to rise easily above the ground.

How can the jet have so much energy? You read in the previous section that the work a force does on an object is proportional to the distance the object moves. The work is also proportional to the component of the force in the direction of the object's displacement. The jet engines cause a force that pushes the jet forward. As it moves along the runway, air rushing past the wings exerts an upward force on the wings. The engines help maintain this effect. The forces of the engines and the air do work on the jet as it takes off. In this section, you will learn about the relationship between work done on an object and energy transferred to the object. The jet engines and air are the sources of energy that the jet uses to fly.

Figure 1 Work done by jet engines and the surrounding air gives a jet the kinetic energy needed to take off.

Kinetic Energy

Kinetic energy, E_k, is the energy an object has due to its motion. An object's kinetic energy is directly related to its mass and the square of its speed, according to the following relationship:

kinetic energy (E_k) the energy an object has because of its motion

$$E_k = \frac{1}{2}mv^2$$

where m is the mass of the object and v is its speed.

Consider how this relationship affects the kinetic energy of the jet in Figure 1. The jet's kinetic energy increases during takeoff because its speed increases. If the speed doubles, for example, the kinetic energy increases by a factor of 4. The jet has a tremendous amount of kinetic energy because its mass is so great.

Notice that kinetic energy is a scalar quantity. The equation above defines the magnitude of kinetic energy, but kinetic energy does not have a direction associated with it. The mass and speed of the airplane, and not its direction, determine its kinetic energy.

You can use dimensional analysis of the above equation to identify the units of kinetic energy. Expressing the mass in kilograms (kg) and the speed in metres per second (m/s) shows that the units of kinetic energy are joules (J):

$$\begin{aligned}
[E_k] &= [\text{kg}]\left[\frac{\text{m}}{\text{s}}\right]^2 \\
&= [\text{kg}]\frac{[\text{m}]^2}{[\text{s}]^2} \\
&= \frac{[\text{kg}][\text{m}]}{[\text{s}]^2}[\text{m}] \\
&= [\text{N}]\cdot[\text{m}] \\
[E_k] &= [\text{J}]
\end{aligned}$$

Notice that the units of kinetic energy are the same as the units of work. We will explore this important result further after Tutorial 1, which shows how to determine the kinetic energy of a moving object when you know its mass and its speed.

Tutorial 1 Solving Problems Involving Kinetic Energy

Sample Problem 1: Calculating Kinetic Energy

A car has a mass of 1.50×10^3 kg and is travelling at a speed of 85.0 km/h. Calculate the car's kinetic energy.

Given: $m = 1.50 \times 10^3$ kg; $v = 85.0$ km/h

Required: E_k

Analysis: Use the equation for kinetic energy, $E_k = \frac{1}{2}mv^2$. First, convert speed into units of metres per second.

Solution: $v = 85.0 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}}$

$$v = 23.6\ \text{m/s}$$

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(1.50 \times 10^3\ \text{kg})(23.6\ \text{m/s})^2$$

$$E_k = 4.18 \times 10^5\ \text{J}$$

Statement: The car's kinetic energy is 4.18×10^5 J.

Sample Problem 2: Using Kinetic Energy to Determine the Speed of an Object

When fleeing a predator, a 1.4 kg rabbit has a kinetic energy of 96 J. Calculate the speed of the rabbit.

Given: $m = 1.4$ kg; $E_k = 96$ J

Required: v

Analysis: Rearrange the kinetic energy equation, $E_k = \frac{1}{2}mv^2$, to isolate the unknown variable, v.

Solution: $E_k = \frac{1}{2}mv^2$

Multiply both sides of the equation by 2. Then, divide both sides by m to isolate v on one side of the equation.

$$2E_k = \cancel{2}\left(\frac{1}{\cancel{2}}mv^2\right)$$

$$2E_k = mv^2$$

$$\frac{2E_k}{m} = \frac{\cancel{m}v^2}{\cancel{m}}$$

$$\frac{2E_k}{m} = v^2$$

Take the square root of both sides.

$$v = \sqrt{\frac{2E_k}{m}}$$

Substitute known values into the equation and solve.

$$v = \sqrt{\frac{2(96\ \text{J})}{1.4\ \text{kg}}}$$

$$v = 12\ \text{m/s}$$

Statement: The rabbit's speed is 12 m/s.

Practice

1. By what factor does a car's kinetic energy increase when the car's speed (a) doubles [ans: 4] (b) triples [ans: 9] (c) increases by 26 % T/I [ans: 1.6]
2. If a bowling ball with mass 8.0 kg travels down the lane at 2.0 m/s, what is its kinetic energy? T/I [ans: 16 J]
3. Calculate the mass of a blue jay moving at 15 km/h with 0.83 J of kinetic energy. T/I [ans: 0.095 kg]

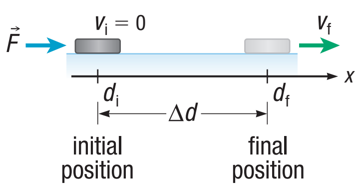


Figure 2 When a force, F, acts on this hockey puck, the puck accelerates. The force does work on the puck, and the puck's kinetic energy changes.

Kinetic Energy and the Work–Energy Theorem

Newton's second law of motion tells us that when an object is subject to a net external force, it accelerates in the same direction as the force. This motion results in work being done on the object. When the object's speed changes from this acceleration, then its kinetic energy also changes (**Figure 2**).

You have seen that for the simplest case of one-dimensional motion, with the force directed parallel to the displacement, the work done by a force on an object is equal to the magnitude of the force multiplied by the object's displacement: $W = F\Delta d$. Using Newton's second law ($F_T = ma$ for this one-dimensional case), the equation becomes

$$W = F_T \Delta d$$

$$W = ma\Delta d$$

Assume that the force is constant so that the acceleration is also constant. Recall the following kinematics equation for motion with constant acceleration:

$$v^2 = v_i^2 + 2a\Delta d$$

Notice that the subscript i indicates the *initial* velocity and position and the subscript f indicates the *final* velocity. The displacement is just $\Delta d = (d_f - d_i)$, so

$$v^2 = v_i^2 + 2a(d_f - d_i) \quad \text{or} \quad a(d_f - d_i) = \frac{v_f^2 - v_i^2}{2}$$

To calculate the work done on the object as it moves from the initial position d_i to the final position d_f, combine the previous equations:

$$\begin{aligned} W &= ma(d_f - d_i) \\ &= m\frac{v_f^2 - v_i^2}{2} \\ W &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \end{aligned}$$

The equation shows that doing work on an object changes its kinetic energy. The following shows this relationship explicitly:

$$\begin{aligned} W &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= E_{k\,f} - E_{k\,i} \\ W &= \Delta E_k \end{aligned}$$

This equation is the **work–energy theorem**: the total work done on an object by an external force equals the change in its kinetic energy. This relation tells us how work, force, and displacement connect to the kinetic energy of an object. Notice that this theorem is also consistent with the result discovered earlier, that work and kinetic energy are both measured in the same units, joules.

work–energy theorem the total work done on an object equals the change in its kinetic energy

Tutorial 2 Applying the Work–Energy Theorem

You can use the work–energy theorem to solve problems involving the kinetic energy transferred when a force does work on an object.

Sample Problem 1: Using the Work–Energy Theorem to Calculate Work Done

A blue whale with a mass of 1.5×10^5 kg is swimming with a speed of 6.1 m/s. A nearby boat startles the whale, and the whale increases its speed to 12.8 m/s. Calculate the work done on the whale by the water.

Given: $m = 1.5 \times 10^5$ kg; $v_i = 6.1$ m/s; $v_f = 12.8$ m/s

Required: W

Analysis: As the whale swims, it exerts a backward force on the water. By Newton's third law, the water exerts an equal and opposite forward force on the whale, causing it to accelerate and gain kinetic energy. Use the work–energy theorem, $W = \Delta E_k$, to calculate the positive work done by the water on the whale.

Solution: First, determine the initial and final kinetic energies.

$$\begin{aligned} E_{k\,i} &= \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(1.5 \times 10^5\text{ kg})(6.1\text{ m/s})^2 \\ E_{k\,i} &= 2.79 \times 10^6\text{ J} \\ E_{k\,f} &= \frac{1}{2}mv_f^2 \\ &= \frac{1}{2}(1.5 \times 10^5\text{ kg})(12.8\text{ m/s})^2 \\ E_{k\,f} &= 1.23 \times 10^7\text{ J} \end{aligned}$$

Apply the work–energy theorem.

$$\begin{aligned} W &= \Delta E_k \\ &= E_{k\,f} - E_{k\,i} \\ &= 1.23 \times 10^7\text{ J} - 2.79 \times 10^6\text{ J} \\ W &= 9.5 \times 10^6\text{ J} \end{aligned}$$

Statement: The water does 9.5×10^6 J of work on the whale.

Sample Problem 2: Applying the Work–Energy Theorem in the Presence of Friction

A shuffleboard player wants to slide a 430 g disc a distance of precisely 12 m. If the coefficient of kinetic friction between the disc and the playing surface is 0.62, calculate the initial speed at which the player must release the disc.

Given: $m = 430 \text{ g} = 0.43 \text{ kg}$; $\mu_K = 0.62$; $\Delta d = 12 \text{ m}$

Required: v

Analysis: The force due to friction is $\vec{F}_f = \mu_K \vec{F}_N$, where $\vec{F}_N$ is the normal force. Calculate the force due to friction, then the work done by friction. The work done by friction is $W = F_f \Delta d \cos\theta$.

The work–energy theorem is $W = \Delta E_k$, and $E_k = \frac{1}{2}mv^2$.

Solution: The force due to friction is

$$\begin{aligned}\vec{F}_f &= \mu_K \vec{F}_N \\ &= \mu_K mg \\ &= (0.62)(0.43 \text{ kg})(9.8 \text{ m/s}^2) \\ \vec{F}_f &= 2.61 \text{ N}\end{aligned}$$

Friction opposes the motion of the disc, so θ is 180°, and $\cos\theta$ is −1.

The work done by friction is

$$\begin{aligned}W &= F\Delta d \cos\theta \\ &= (2.61 \text{ N})(12 \text{ m})(\cos 180°) \\ W &= -31.3 \text{ J}\end{aligned}$$

The work–energy theorem tells us that the change in kinetic energy will equal the work done, or −31.3 J. The final velocity is zero, so the final kinetic energy is zero, and the initial kinetic energy is the negative of the work done.

$$\begin{aligned}E_k &= E_f - E_i \\ &= 0 - (-31.3 \text{ J}) \\ E_k &= 31.3 \text{ J}\end{aligned}$$

We can solve for the initial speed:

$$\begin{aligned}E_k &= \frac{1}{2}mv^2 \\ \frac{2E_k}{m} &= v^2 \\ v &= \sqrt{\left(\frac{2E_k}{m}\right)} \\ &= \sqrt{\left(\frac{2(31.3 \text{ J})}{0.43 \text{ kg}}\right)} \\ v &= 12 \text{ m/s}\end{aligned}$$

Statement: The initial speed of the disc must be 12 m/s.

Sample Problem 3: Applying the Work–Energy Theorem to Calculate Initial Speed

A police car of mass 2.4×10^3 kg is travelling on the highway when the officers receive an emergency call. They increase the speed of the car to 33 m/s. The increase in speed results in 3.1×10^5 J of work done on the car. Determine the initial speed of the police car in kilometres per hour.

Given: $m = 2.4 \times 10^3 \text{ kg}$; $v_f = 33 \text{ m/s}$; $W = 3.1 \times 10^5 \text{ J}$

Required: v_i

Analysis: Rearrange the work–energy equation to solve for the initial speed of the car. First, calculate the final kinetic energy of the car; then, subtract the work done to determine the initial kinetic energy.

Solution: First, determine the final and initial kinetic energies.

$$\begin{aligned}E_{k\,f} &= \frac{1}{2}mv_f^2 \\ &= \frac{1}{2}(2.4 \times 10^3 \text{ kg})(33 \text{ m/s})^2 \\ E_{k\,f} &= 1.31 \times 10^6 \text{ N}\end{aligned}$$

$$\begin{aligned}W &= \Delta E_k \\ &= E_{k\,f} - E_{k\,i} \\ E_{k\,i} &= E_{k\,f} - W \\ &= 1.31 \times 10^6 - 3.1 \times 10^5 \text{ J} \\ E_{k\,i} &= 1.00 \times 10^6 \text{ J}\end{aligned}$$

Then, solve for v_i.

$$\begin{aligned}E_{k\,i} &= \frac{1}{2}mv_i^2 \\ \left(\frac{2}{m}\right)E_{k\,i} &= \left(\frac{\cancel{2}}{\cancel{m}}\right)\frac{1}{\cancel{2}}\cancel{m}v_i^2 \\ v_i^2 &= \left(\frac{2}{m}\right)E_{k\,i} \\ v_i &= \sqrt{\left(\frac{2}{m}\right)E_{k\,i}} \\ &= \sqrt{\left(\frac{2}{2.4 \times 10^3 \text{ kg}}\right)1.00 \times 10^6 \text{ J}} \\ v_i &= 28.9 \text{ m/s}\end{aligned}$$

The initial speed is 29 m/s. To determine the speed in kilometres per hour,

$$\begin{aligned}v_i &= 29\,\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\,\frac{1 \text{ km}}{1000\,\cancel{\text{m}}}\,\frac{3600\,\cancel{\text{s}}}{1 \text{ h}} \\ v_i &= 1.0 \times 10^2 \text{ km/h}\end{aligned}$$

Statement: The initial speed of the police car was 1.0×10^2 km/h.

Practice

1. An archer pulls back her bowstring (**Figure 3**) loaded with a 22 g arrow and then releases the string. The arrow's speed as it leaves the bowstring is 220 km/h. Calculate the work done on the arrow by the bowstring. T/I [ans: 41 J]

Figure 3

2. A space probe travels far out in the galaxy to a point where the force of gravity is very weak. The probe has a mass of 3.8×10^4 kg and an initial speed of 1.5×10^4 m/s. The probe's engines exert a force of 2.2×10^5 N in the original direction of motion as the probe travels a distance of 2.8×10^6 m. Calculate the final speed of the probe. K/U T/I [ans: 1.6×10^4 m/s]
3. A skater moves across the ice a distance of 12 m before a constant frictional force of 15 N causes him to stop. His initial speed is 2.2 m/s. Calculate the skater's mass. K/U T/I [ans: 74 kg]

Underlying Assumptions Related to the Work–Energy Theorem

You can use the work–energy theorem to solve several types of physics problems. However, you cannot control all of the variables in the real world as easily as you can in a physics experiment. The work–energy theorem is only true if no energy losses occur. In many real-world situations, energy will seem to disappear in the form of light, sound, heat, or changes in the shape of an object. For instance, in a car collision, energy goes into the sounds of the crash and the bending of materials in the car. The work done on a given car does not equal the change in the kinetic energy of that car.

This discussion assumes that the applied force is constant. The derivation of the work–energy theorem for a varying force requires calculus, but the result is the same. The work–energy theorem holds true, even when the applied force is not constant.

Investigation 4.2.1

The Work–Energy Theorem **(page 209)**
Now that you have learned about the work–energy theorem, you are ready to complete an investigation to test the theorem. This Controlled Experiment will give you an opportunity to calculate the force of friction.

4.2 Review

Summary

- Kinetic energy, E_k, is the energy an object has due to its motion. It is a scalar quantity because no direction is associated with it. The units of kinetic energy are joules (J).
- An object's kinetic energy is related to its mass, m, and its speed, v, by the equation $E_k = \frac{1}{2}mv^2$.
- According to the work–energy theorem, the total work done on an object is equal to the change in the object's kinetic energy: $W = \Delta E_k$.

Questions

1. Could an elephant walking slowly across a field have more kinetic energy than a cheetah chasing its prey? Explain your answer. K/U T/I
2. A cat with a mass of 5.0 kg is chasing a mouse with a mass of 35 g. The mouse is running away from the cat at a constant speed in a straight line. K/U T/I
   (a) The cat's kinetic energy is 100 times the mouse's kinetic energy. Will the cat be able to catch up with the mouse? Explain your answer.
   (b) What is the minimum kinetic energy that the cat must have to keep up with the mouse?
3. A car of mass 1.5×10^3 kg is initially travelling at a speed of 11 m/s. The driver then accelerates to a speed of 25 m/s over a distance of 0.20 km. Calculate the work done on the car. K/U T/I
4. A truck of mass 9.1×10^3 kg is travelling along a level road at an initial speed of 98 km/h and then slows to a final speed of 27 km/h. Determine the total work done on the truck. T/I
5. Two objects have the same kinetic energy. One has a speed that is 2.5 times the speed of the other. Determine the ratio of their masses. K/U T/I
6. Consider a small car of mass 1.2×10^3 kg and a large sport utility vehicle (SUV) of mass 4.1×10^3 kg. The car is travelling at 99 km/h. The car and the SUV have the same kinetic energy. Calculate the speed of the SUV. K/U T/I
7. An archer is able to shoot an arrow with a mass of 0.020 kg at a speed of 250 km/h. If a baseball of mass 0.14 kg is given the same kinetic energy, determine its speed. T/I
8. A hockey player shoots a puck at a speed of 150 km/h. The mass of the puck is 0.16 kg, and the player's stick is in contact with it over a distance of 0.25 m. Calculate the average force exerted on the puck by the player. T/I
9. At room temperature, an oxygen molecule with mass 5.31×10^{-26} kg has kinetic energy of about 6.25×10^{-21} J. Determine the speed of the molecule. T/I
10. A horizontal force of 15 N pulls a block of mass 3.9 kg across a level floor. The coefficient of kinetic friction between the block and the floor is $\mu_K = 0.25$. If the block begins with a speed of 0.0 m/s and is pulled for a distance of 12 m, determine the final speed of the block. T/I
11. Centripetal forces do non-zero work on objects in non-circular orbits. Satellites in non-circular orbits around Earth have different speeds at different positions in their orbit. The change in kinetic energy comes from work done on the satellites by gravity. A satellite of mass 5.55×10^3 kg has a speed of 2.81 km/s at one point in its orbit and a speed of 3.24 km/s at a second point. Calculate the work done on the satellite by gravity as the satellite moves from the first point to the second point. T/I A
12. **Figure 4** shows the kinetic energy of a robot as a function of its speed. K/U T/I

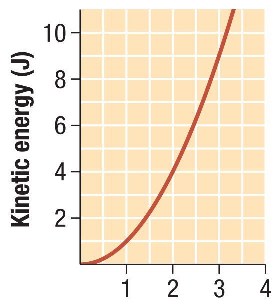


Figure 4

   (a) What type of function is this?
   (b) Why does the graph pass through the origin?
   (c) Determine the mass of the robot.
   (d) Write an equation that relates kinetic energy as a function of speed for the robot.

Gravitational Potential Energy

4.3

The extreme sport of heli-skiing starts with a helicopter ride to the top of a remote mountain. A skier like the one in **Figure 1** then jumps out of the helicopter onto the steep slopes. How do you know that the skier in Figure 1 has energy before jumping from the helicopter? How can this energy be transformed into another form? In this section, you will learn the answers to these questions.

Figure 1 The skier's kinetic energy will increase as she falls. Her height gives her potential energy.

Potential Energy

You can sense that the skier in Figure 1 has energy because of her height above the ground. The force of gravity will do work on the skier as soon as she steps out of the helicopter. As she falls, and then skis down the mountain, her kinetic energy will increase. Her height above the ground means that she has the potential to pick up kinetic energy from the force of gravity. This potential to increase kinetic energy is a form of stored energy. The stored energy that an object has that can be released into another form of energy is **potential energy**.

potential energy the stored energy an object has that can be converted into another form of energy

Many forms of potential energy exist. A long-jumper poised to jump has potential energy stored in his muscles. The flexed muscles have stored biomechanical energy that is released as the jumper springs into the air. A stretched elastic band also has elastic potential energy. When you let an elastic band go, the potential energy becomes kinetic energy.

Kinetic energy and potential energy have a close relationship, and one form can transform into the other as work is done on or by an object. The sum of the kinetic energy and potential energy of an object is called the **mechanical energy** of the object. You will read more about properties of mechanical energy in Section 4.5. This section will focus on the potential energy an object has due to gravity.

mechanical energy the sum of an object's kinetic and potential energies

Gravitational Potential Energy

An object near Earth's surface has a potential energy that depends on the object's mass, m, and height, h. The **gravitational potential energy**, E_g, is stored energy as a result of the gravitational force between the object and Earth. Although we will often speak of the gravitational potential energy of an object, note that E_g is actually a property of the object and Earth together. Potential energy is always a property of a *system* of objects, as we will discuss in Section 4.5.

gravitational potential energy (E_g) stored energy an object has because of its position and the applied gravitational force

Analyzing gravitational potential energy mathematically can help clarify how to apply it when solving problems. Suppose a worker lifts a crate onto the back of a pickup truck (**Figure 2**).

Figure 2 A worker applies a force F_a to a crate, doing work as he lifts the crate from the ground up to the truck bed. The change in height of the crate is Δy.

Let Δy be the change in the elevation of the crate. The force the worker applies to the crate to raise it is in the same direction as the crate's displacement. The work done on the crate by the force is

$$\begin{aligned} W &= F\Delta d \cos\theta \\ &= F\Delta y \cos\theta \\ &= mg\Delta y\,(\cos 0°) \end{aligned}$$

$$W = mg\Delta y$$

Note that we replaced Δd with Δy because we often use y to represent vertical displacements. When the bottom of the crate reaches the height of the truck's bed, the crate has gravitational potential energy relative to the ground. Suppose the worker decides instead to place crates on top of each other. You could then describe a crate's gravitational potential energy relative to the ground, or describe its gravitational potential energy relative to the truck bed. Since gravitational potential energy depends on position *relative* to an object, it is a relative quantity. Its value depends on the height of the object above some point of reference that you choose. You can choose the level that is most convenient for solving any given problem. The work done to increase the elevation of an object relates to the *change* in the elevation instead of a specific height:

$$\Delta E_g = mg\Delta y$$

where ΔE_g is the change in gravitational potential energy, m is the mass, g is the gravitational acceleration, and Δy is the vertical component of the displacement.

The equation for gravitational potential energy presents a few important points to consider. First, the equation determines the *change* in gravitational potential energy for a given change in elevation. It does not assign a fixed value of potential energy to a given elevation, but only the difference in potential energy between different elevations, Δy. You can freely choose a reference point to act as a zero gravitational potential elevation. When solving a problem, choose the elevation that is most convenient for that problem. Earth's surface is often a convenient choice, but it is not the only choice, or necessarily the best choice. Choose a reference point that will result in the easiest calculations. Often, choosing the surface of Earth for this point is most convenient because the gravitational potential energy will then be either zero or positive.

Second, Δy does not depend on any changes in the horizontal position or on the path the object took to reach its new height. When the object increases its height, Δy is positive; when the object falls to a lower height, Δy is negative.

Finally, you will read in Chapter 6 that the acceleration due to gravity, g, actually varies slightly by height above Earth's surface. The equation above is only accurate when the change in height is small enough that you can ignore the change in g.

An object thrown upward will begin with kinetic energy, but the amount of kinetic energy decreases as the object slows down because of the gravitational force. At the same time, the object gains potential energy as its height increases. Whenever an object falls, the force of gravity will do work on the object, giving it kinetic energy according to the work–energy theorem. At the same time, it loses gravitational potential energy.

Suppose that you hold a physics book above your desk. If you drop the book, its initial velocity is zero, but it has gravitational potential energy relative to the desktop. As it falls, the force of gravity does work on the book and converts its gravitational potential energy to kinetic energy. Before it hits the desktop, all of the initial potential energy relative to the desktop has converted into kinetic energy. As the book strikes the desktop, kinetic energy is converted into sound energy, thermal energy, and other forms of energy.

UNIT TASK **BOOKMARK**

You can apply what you have learned about work, kinetic energy, and gravitational potential energy to the Unit Task on page 270.

Units of Gravitational Potential Energy

Gravitational potential energy and kinetic energy are different manifestations of the same quantity—energy. They both have the same units—joules—and both are scalar quantities. In the equations below, you can see how to calculate the units of gravitational potential energy, $\Delta E_g = mg\Delta y$. The units of m are kilograms. The units of g are metres per second squared. The units of Δy are metres. Putting these facts together gives

$$\begin{aligned}[E_g] &= [\text{kg}]\frac{[\text{m}]}{[\text{s}^2]}[\text{m}] \\ &= [\text{N}]\cdot[\text{m}] \\ [E_g] &= [\text{J}]\end{aligned}$$

Similar to kinetic energy, no direction is associated with gravitational potential energy. The following Tutorial examines how to use gravitational potential energy in problems involving rising and falling objects.

Tutorial 1 Applications Involving Gravitational Potential Energy

Sample Problem 1: Calculating Gravitational Potential Energy

A hiker stands near the edge of a cliff and accidentally drops a rock of mass 1.2 kg to the ground at the base of the cliff 28 m below (**Figure 3**). Calculate the potential energy of the rock relative to the ground just before the hiker drops it.

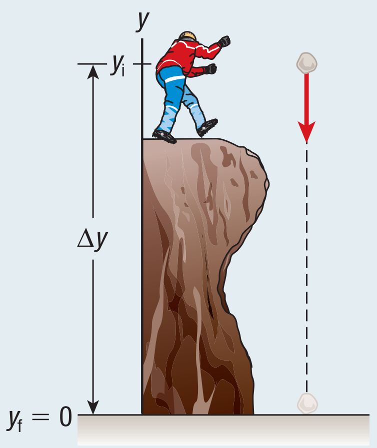


Figure 3

Given: $m = 1.2$ kg; $g = 9.8$ m/s^2; $\Delta y = 28$ m

Required: ΔE_g

Analysis: Use the gravitational potential energy equation, $\Delta E_g = mg\Delta y$. Let the positive y-direction be upward. Let the reference point, $y_f = 0$, be the base of the cliff.

Solution:
$$\begin{aligned}\Delta E_g &= mg\Delta y \\ &= (1.2\text{ kg})(9.8\text{ m/s}^2)(28\text{ m}) \\ \Delta E_g &= 3.3 \times 10^2\text{ J}\end{aligned}$$

Statement: The rock's potential energy just before the hiker drops it is 3.3×10^2 J.

Sample Problem 2: Applying Gravitational Potential Energy to Determine Mass

A weightlifter raises a loaded barbell 2.2 m. The lift increases the gravitational potential energy of the barbell by 490 J. Determine the mass of the loaded barbell.

Given: $\Delta y = 2.2$ m; $g = 9.8$ m/s^2; $\Delta E_g = 490$ J

Required: m

Analysis: Rearrange the gravitational potential energy equation, $\Delta E_g = mg\Delta y$, to solve for m.

Solution: $\Delta E_g = mg\Delta y$

Divide both sides of the equation by $g\Delta y$.

$$m = \frac{\Delta E_g}{g\Delta y}$$
$$= \frac{490\ \text{J}}{(9.8\ \text{m/s}^2)(2.2\ \text{m})}$$
$$m = 23\ \text{kg}$$

Statement: The mass of the loaded barbell is 23 kg.

Sample Problem 3: Gravitational Potential Energy and the Work–Energy Theorem

A physics textbook is 3.6 cm thick and has a mass of 1.6 kg. A student stacks 10 of the books in a single pile on a table. Each book starts from table level before the student lifts it to the top of the stack.

(a) Calculate the gravitational potential energy of the stack of books with respect to the table.
(b) Determine the total work done by the student to make the stack of books.
(c) Calculate the work done by the student to move the books from the stack and set each on the desk.

Given: height of each book, $h = 3.6$ cm $= 0.036$ m; number of books $= 10$

Required: W

Analysis: $\Delta E_g = mg\Delta y$

(a) Each book has a separate gravitational potential energy. Since the first book does not move, no energy is expended. The second book is lifted 3.6 cm, the third is lifted 2×3.6 cm, and so on. The gravitational potential energy of the stack is the sum of the gravitational potential energies for the individual books.
(b) Since work and energy use the same units, W is equal to the gravitational potential energy of the stack.
(c) The work done to move the books from the stack is equal in magnitude to W but opposite in sign.

Solution: $\Delta E_g = mg\Delta y$

$$= (1.6\ \text{kg})(9.8\ \text{m/s}^2)[0(0.036\ \text{m}) + 1(0.036\ \text{m}) + 2(0.036\ \text{m}) + \cdots + 9(0.036\ \text{m})]$$
$$= (1.6\ \text{kg})(9.8\ \text{m/s}^2)(0.036\ \text{m})(0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)$$
$$= (0.56\ \text{J})(45)$$
$$\Delta E_g = 25\ \text{J}$$
$$W = 25\ \text{J}$$

Statement:

(a) The potential energy of the stack relative to the table is 25 J.
(b) The work done by the student to create the stack is 25 J.
(c) The work done to move the books from the stack is -25 J.

Practice

1. A grey squirrel drops a 0.02 kg walnut from a branch that is 8.0 m high. Determine the change in potential energy of the walnut between the branch and the ground. T/I [ans: 1.6 J]
2. The weightlifter in Sample Problem 2 increases the mass of the weights on the barbell and lifts it one more time. The new lift increases the potential energy of the bar by 660 J. Calculate the new mass of the barbell. T/I [ans: 31 kg]
3. Calculate the work required by the student in Sample Problem 3 to stack 2 more physics books on the original stack of 10. T/I [ans: 12 J]

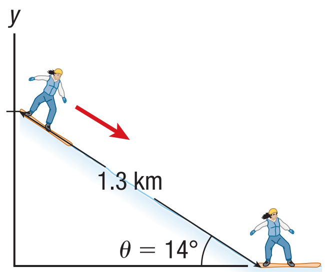
4.3 Review

Summary

- Potential energy is the stored energy an object has that can be released into another form of energy.
- Mechanical energy is the sum of the kinetic and potential energies.
- Gravitational potential energy is the energy that an object has due to its height above a reference point. It is a scalar quantity and is measured in joules (J).
- When solving problems related to gravitational potential energy, choose a reference point, $y = 0$, from which to measure the gravitational potential energy.
- The gravitational potential energy of an object near Earth's surface depends on the object's mass, m; the acceleration due to gravity, g; and the object's change in height as measured from a reference height, Δy: $\Delta E_g = mg\Delta y$.

Questions

1. A 2.5 kg piece of wood falls onto a carpenter's table from a height of 2.0 m above the table. T/I
   (a) Calculate the kinetic energy of the wood as it hits the table.
   (b) Calculate the speed of the wood as it hits the table.
2. Calculate the gravitational potential energy relative to the ground of a 5.0 kg Canada goose flying at a height of 553 m above the ground. T/I
3. A hockey referee drops a 175 g hockey puck from rest vertically downward from a height of 1.05 m above the ice surface. K/U T/I
   (a) Determine the gravitational potential energy of the puck relative to the ice before the referee drops it.
   (b) Calculate the change in gravitational potential energy of the puck as it drops from the referee's hand to the ice surface.
   (c) Calculate the work done on the puck by gravity as the puck travels from the referee's hand to the ice surface.
4. You lift your pet cat vertically by 2.0 m, and then you lower it vertically by 2.0 m. During this exercise, is the total work done by gravity positive, negative, or zero? Explain your answer. K/U C
5. A pole vaulter clears the bar at a height of 5.4 m, and then falls to the safety mat. The change in the pole vaulter's gravitational potential energy from the bar to the mat is -3.1×10^3 J. Calculate the pole vaulter's mass. T/I
6. A 0.46 kg golf ball on a tee is struck by a golf club. The golf ball reaches a maximum height where its gravitational potential energy has increased by 155 J from the tee. Determine the ball's maximum height above the tee. T/I
7. A 59 kg snowboarder descends a 1.3 km ski hill from the top of a mountain to the base (**Figure 4**). The slope is at an angle of 14° to the horizontal. Determine the snowboarder's gravitational potential energy relative to the mountain base when she is at the top. T/I



Drawing not to scale

Figure 4

8. Suppose that you have N identical boxes in your room, each with mass m and height Δy. You stack the boxes in a vertical pile. K/U T/I
   (a) Determine the work done to raise the last box to the top of the pile. Express your answer in terms of the variables m, g, N, and Δy.
   (b) Determine the gravitational potential energy, in terms of m, g, N, and Δy, that is stored in the entire pile.
9. A gallon of gas contains about 1.3×10^8 J of chemical potential energy. Determine how many joules of chemical potential energy are stored in each litre of gas (3.79 L = 1 gallon). Calculate the height that the amount of chemical potential energy in 1 L of gas could raise all the students in your class if it was all converted to gravitational potential energy. You will have to make assumptions about the mass of the students. State your assumptions and show your calculations. T/I A

4.4 Explore an Issue in Energy Generation

SKILLS MENU

- Defining the Issue
- Researching
- Identifying Alternatives
- Analyzing
- Defending a Decision
- Communicating
- Evaluating

Gravitational Potential Energy and Hydroelectricity

Although new technology and innovative methods for generating electricity are being developed, traditional methods remain important. One of these methods is hydroelectric power, which uses the kinetic energy of moving water to produce electrical energy (**Figure 1**). Hydroelectric power relies on a renewable energy source and is much cleaner to produce than power generated by burning fossil fuels.

Figure 1 The water moving through the Oldman hydroelectric dam in Alberta contains a tremendous amount of kinetic energy.

As with any large-scale energy production method, however, hydroelectric power has drawbacks. Many hydroelectric power plants use large dams and reservoirs, which are expensive to build and can destroy surrounding ecosystems. Problems with sediment buildup, evaporation of water from reservoirs, and the impact of climate change all influence the production of hydroelectricity.

Delivering the electricity to consumers also poses problems. Placing the large power lines required to deliver energy from a hydroelectric source to the people who need it might not be feasible for economic and environmental reasons. Electricity can be generated using water from a number of sources, but all of the methods used present distinct benefits and challenges. Harnessing ocean wave and tidal energy, for instance, is difficult without adversely affecting coastal areas and interrupting shipping lanes. Useful ocean currents may run far off the coast, making it expensive to transfer the electricity generated to inland locations.

The Issue

In Canada, most electricity is generated at hydroelectric power plants. This technology has not expanded, however, since the 1970s. Some Canadians believe that the country should increase its use of hydroelectricity. Other Canadians, however, feel that the negative aspects of hydroelectricity are too great. Your role is to be a resident of a community that must decide whether to fund a new hydroelectric power project. Representatives of both sides will present their arguments at a local forum. You must then vote on the project at the forum's conclusion. You will prepare for this forum by gathering information about the topics that are relevant to your community and the members of various communities across Canada.

Goal

To decide whether Canada should expand its use of hydroelectricity and present evidence that supports your decision

Research

SKILLS HANDBOOK A4.1

Contact your local providers of electric power to determine where the electricity used by your community is generated. Research the environmental concerns about power generation in your area, and the ways in which those concerns were resolved. Prepare a summary of your findings, highlighting any issues or strategies that you believe would be relevant if new hydroelectric plants were proposed for your area.

Choose one new technology related to modern hydroelectric power generation, and research the costs and benefits associated with it, including possible environmental and social impacts. Your research might include

- run-of-the-river generating stations, such as the Beauharnois Power Plant near Montréal
- tidal power plants, such as the Annapolis Tidal Station in Nova Scotia
- technologies used in different locations around the world WEB LINK

Possible Solutions

Consider your perspective on the issues raised. What are the options available for hydroelectric power in the future? What environmental and social outcomes would result from each energy strategy? What improvements can you think of to increase the capacity and efficiency of hydroelectric power? Think about

- advances in environmentally friendly construction methods that might be used during plant construction
- new materials or methods that might minimize cost concerns
- increasing concerns related to traditional electricity generation methods

Decision

SKILLS HANDBOOK A4.2

Which hydroelectric options would you support? Explain your decision, using your summary and research findings.

Communicate

- Design a chart, electronic slide presentation, web page, blog post, or other visual presentation that explains the pros and cons of each method.
- Prepare a map of Ontario or all of Canada that shows the current locations of hydroelectric power plants. Select areas that you believe, based on your research, might be good choices for development of new plants.
- Choose one location, and create a list of specific concerns that might trouble people living in the area surrounding the site you have chosen.

Plan for **Action**

Take part in a debate on this issue with your classmates, as you might in a community forum. Prepare five questions for each representative in the forum. At the end of the forum, vote on whether to build the plant in your community. Write a summary of your decision to present to people or groups outside your community who might be interested in your findings, including news agencies, government entities, surrounding communities, non-government energy groups, and environmental groups.

4.5 The Law of Conservation of Energy

Imagine the thrill of the riders as the roller coaster in **Figure 1** moves up and around the loops, over and over again. What do riders feel as they near the top of each loop? What do they feel as they move down toward the ground again? What makes them feel these sensations?

In this section, you will explore some of the physics related to roller coasters, sports activities, and other movements you experience every day. You will read about the exchange between gravitational potential energy and kinetic energy that occurs when objects move.

Figure 1 Roller coasters offer both thrills and a chance to learn about physics.

Energy Transformations

As a diver climbs the steps to the top of a diving platform, her gravitational potential energy increases relative to the water surface. As she dives toward the water, her gravitational potential energy decreases. At the same time, her kinetic energy increases as her downward speed builds. When she hits the water, her gravitational potential energy relative to the water surface is zero, and her kinetic energy is at a maximum.

Although her gravitational potential energy decreases, that energy does not just disappear. As her kinetic energy increases, it too does not just appear from nowhere. The potential energy transforms into kinetic energy as the diver falls. Energy is neither created nor destroyed; it simply changes form. In fact, the total mechanical energy—the sum of kinetic and potential energy—remains constant. This important law of nature is called the **law of conservation of energy:**

Law of Conservation of Energy

Energy is neither created nor destroyed. It can only change form.

The law of conservation of energy is one of the fundamental principles of physics. To take into account apparent energy losses due to friction and other effects, the statement above will be refined in the next section. You will use the law of conservation of energy as a tool for solving many problems.

Mini **Investigation**

Various Energies of a Roller Coaster

Skills: Predicting, Analyzing, Evaluating, Communicating

SKILLS HANDBOOK A5.5

In this activity, you will analyze differences in energy at various heights of a roller coaster car. A roller coaster car starts from rest at point A, moves down the hill, and up and around the loop, to point F (**Figure 2**). Assume that energy losses due to friction are negligible and can be ignored. In this scenario, the height of the roller coaster is the independent variable, and the different types of energy are the dependent variables.

1. Create a table with the headings Height, Gravitational Potential Energy, Kinetic Energy, and Total Energy.
2. Record the height values for the six labelled points in Figure 2.
3. Calculate the potential, kinetic, and total mechanical energy values for each of the six points, and record the values in your table. Assume the mass of the roller coaster car is 875 kg.

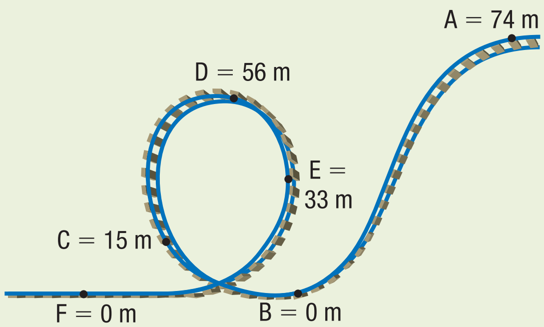


Figure 2

4. Sketch a graph of energy versus height for the roller coaster. Show each of the three energies (gravitational potential energy, kinetic energy, and total energy) on the same graph, but use different colours or line styles for each type.

A. Describe and explain the shape of the total energy graph. K/U T/I A

B. Compare the shapes of the graphs for gravitational potential energy and kinetic energy. How do they relate to the total energy graph? T/I A

C. Explain why it was necessary to know the height of point A. How would the graph change if the height of point A were greater? T/I A

D. Discuss how your graph would change if the mass of the roller coaster car were greater. K/U T/I A

WEB LINK

The total mechanical energy of a moving roller coaster car is the same at every point. That is, the sum of the gravitational energy and the kinetic energy remains constant. The total initial energy, $E_{\text{T i}}$, equals the total final energy, $E_{\text{T f}}$:

$$E_{\text{T i}} = E_{\text{T f}}$$

For situations involving only gravitational potential energy and kinetic energy, the equation can be written as

$$E_{\text{g i}} + E_{\text{k i}} = E_{\text{g f}} + E_{\text{k f}}$$

Knowing the energy of a roller coaster car at the start of the ride enables you to determine the gravitational potential energy and kinetic energy at other points. The following Tutorial examines how to use conservation of energy to solve problems involving motion. WEB LINK

Tutorial 1 Applying the Law of Conservation of Energy

Sample Problem 1: Making Connections between Gravitational Potential Energy and Kinetic Energy

A 67 kg snowboarder starts at the top of an icy (frictionless) hill of vertical height 22 m with an initial speed of 15 m/s (**Figure 3**).

(a) Calculate the snowboarder's mechanical energy at the top of the hill.

(b) Calculate the snowboarder's speed at the midway point and at the bottom of the hill.

(c) Describe the energy transformation that occurs as the snowboarder moves down the hill.

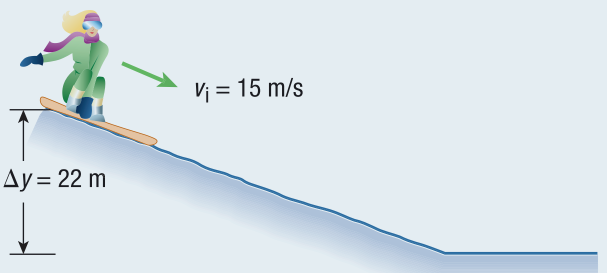


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Figure 3

Solution

(a) **Given:** $m = 67$ kg; $\Delta y = 22$ m; $v_i = 15$ m/s

Required: E_T

Analysis: Choose the bottom of the hill as the $h = 0$ reference point. Then, set the gravitational potential energy at the top of the hill equal to this amount.

$$\Delta E_g = mg\Delta y;\ E_k = \frac{1}{2}mv^2;\ E_T = E_g + E_k$$

Solution: The change in gravitational potential energy from the bottom of the hill to the top of the hill is

$$\begin{aligned}\Delta E_g &= mg\Delta y \\ &= (67\text{ kg})(9.8\text{ m/s}^2)(22\text{ m}) \\ \Delta E_g &= 1.444 \times 10^4\text{ J (two extra digits carried)}\end{aligned}$$

The kinetic energy at the top of the hill is

$$\begin{aligned}E_k &= \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(67\text{ kg})(15\text{ m/s})^2 \\ E_k &= 7.537 \times 10^3\text{ J (two extra digits carried)}\end{aligned}$$

The total mechanical energy at the top of the hill is then

$$\begin{aligned} E_T &= E_g + E_k \\ &= 1.444 \times 10^4 \text{ J} + 7.537 \times 10^3 \text{ J} \\ E_T &= 2.198 \times 10^4 \text{ J (two extra digits carried)} \end{aligned}$$

Statement: The snowboarder's mechanical energy at the top of the hill, relative to the bottom of the hill, is 2.2×10^4 J.

(b) **Given:** $m = 67$ kg; $\Delta y_{mid} = 11$ m and $\Delta y_{bottom} = 0$ m; $E_{T\,i} = 2.198 \times 10^4$ J

Required: v_{mid}; v_{bottom}

Analysis: Use the law of conservation of energy to relate the initial and final energies:

$$E_{g\,i} + E_{k\,i} = E_{g\,f} + E_{k\,f}$$

In each case,

$$\Delta E_g = mg\Delta y;\ E_k = \frac{1}{2}mv^2$$

Solution: The total mechanical energy at the midpoint and the bottom of the hill will equal the total energy at the top of the hill, 2.2×10^4 J. Since the hill bottom is the $h = 0$ reference point, the gravitational potential energy at the midpoint is

$$\begin{aligned} E_g &= mg\Delta y \\ &= (67 \text{ kg})(9.8 \text{ m/s}^2)(11 \text{ m}) \\ E_g &= 7.223 \times 10^3 \text{ J (two extra digits carried)} \end{aligned}$$

The kinetic energy is

$$\begin{aligned} E_k &= E_T - E_g \\ &= 2.198 \times 10^4 \text{ J} - 7.223 \times 10^3 \text{ J} \\ E_k &= 1.476 \times 10^4 \text{ J} \end{aligned}$$

Then, calculate the speed at the midpoint.

$$\begin{aligned} E_{k\,mid} &= \frac{1}{2}mv^2 \\ \frac{2E_{k\,mid}}{m} &= v^2 \\ \sqrt{\frac{2E_{k\,mid}}{m}} &= v \\ v &= \sqrt{\frac{2(1.476 \times 10^4 \text{ J})}{67 \text{ kg}}} \\ v &= 21 \text{ m/s} \end{aligned}$$

Next, perform the same calculations for the bottom of the hill. Note that at the bottom of the hill the mechanical energy is all kinetic energy.

$$\begin{aligned} E_{g\,bottom} &= mg\Delta y \\ &= (67 \text{ kg})(9.8 \text{ m/s}^2)(0 \text{ m}) \\ E_{g\,bottom} &= 0 \text{ J} \\ E_{k\,bottom} &= E_T - E_{g\,bottom} \\ &= 2.198 \times 10^4 \text{ J} - 0 \text{ J} \\ E_{k\,bottom} &= 2.198 \times 10^4 \text{ J (two extra digits carried)} \end{aligned}$$

Now calculate her speed at the bottom of the hill:

$$\begin{aligned} v_{bottom} &= \sqrt{\frac{2E_{k\,bottom}}{m}} \\ &= \sqrt{\frac{2(2.198 \times 10^4 \text{ J})}{67 \text{ kg}}} \\ v_{bottom} &= 26 \text{ m/s} \end{aligned}$$

Statement: Halfway down the hill, the snowboarder's speed is 21 m/s. Her speed at the bottom of the hill is 26 m/s.

(c) As the snowboarder moves down the hill, her height above the reference point decreases, reducing her gravitational potential energy. Her speed, however, increases. The gravitational potential energy continuously transforms into kinetic energy until she reaches the bottom.

Sample Problem 2: Determining Maximum Height and Speed Using the Law of Conservation of Energy

Figure 4 shows a 0.45 kg bullfrog jumping with an initial speed of 6.2 m/s at an angle of 49° above the horizontal. Assume that energy losses due to air resistance are negligible and can be ignored.

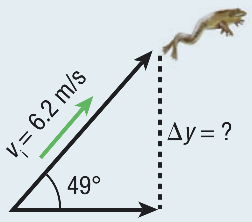


Figure 4

(a) Calculate the maximum height of the bullfrog's jump.

(b) Calculate the components of the bullfrog's velocity when it first reaches a height of 0.82 m.

Solution

(a) **Given:** $m = 0.45$ kg; $h_i = 0$; $v_i = 6.2$ m/s; $\theta = 49°$

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$; $E_k = \frac{1}{2}mv^2$; $E_T = E_g + E_k$;

$v_x = v_i \cos\theta$

The bullfrog's velocity in the horizontal direction does not change, because no horizontal force acts on the frog. Since the change in the horizontal velocity is zero, it has no effect on the change in kinetic energy of the bullfrog. The kinetic energy from the bullfrog's vertical component of velocity decreases and transforms into gravitational potential energy. You can solve the problem by comparing just the vertical parts of the kinetic energy and the gravitational potential energy.

Solution: The initial speed is 6.2 m/s. The initial horizontal velocity is

$$\begin{aligned} v_x &= v_i \cos\theta \\ &= (6.2 \text{ m/s}) \cos 49° \\ v_x &= 4.07 \text{ m/s} \end{aligned}$$

At the high point of the jump, the vertical component of the velocity is zero, but the horizontal component is still 4.07 m/s. The speed at the high point is then 4.07 m/s. Using conservation of energy,

$$\begin{aligned} E_{k\,i} + E_{g\,i} &= E_{k\,f} + E_{g\,f} \\ E_{g\,i} - E_{g\,f} &= E_{k\,f} - E_{k\,i} \\ -\Delta E_g &= \Delta E_k \end{aligned}$$

Use this equation to solve for the change in height.

$$\begin{aligned} -\cancel{m}g\Delta y &= \frac{1}{2}\cancel{m}(v_f^2 - v_i^2) \\ \Delta y &= \left(\frac{v_i^2 - v_f^2}{2g}\right) \\ &= \left(\frac{(6.2 \text{ m/s})^2 - (4.07 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)}\right) \\ \Delta y &= 1.1 \text{ m} \end{aligned}$$

Statement: The bullfrog reaches a maximum height of 1.1 m above the ground.

(b) **Given:** $y_i = 0$; $m = 0.45$ kg; $v_i = 6.2$ m/s; $y_f = 0.82$ m

Required: $v_{x\,f}$; $v_{y\,f}$

Analysis: Use the law of conservation of energy to relate the initial and final energies.

Solution: The change in the gravitational potential energy is

$$\begin{aligned} \Delta E_g &= mg\Delta y \\ &= (0.45 \text{ kg})(9.8 \text{ m/s}^2)(0.82 \text{ m}) \\ \Delta E_g &= 3.6 \text{ J} \end{aligned}$$

As in part (a), the horizontal speed of the bullfrog does not change, so the only change to its kinetic energy comes from a change in its vertical speed.

$$\begin{aligned} \Delta E_k &= \frac{1}{2}m(v_{x\,f}^2 + v_{y\,f}^2) - \frac{1}{2}m(v_{x\,i}^2 + v_{y\,i}^2) \\ &= \frac{1}{2}m(\cancel{v_{x\,f}^2} + v_{y\,f}^2 - \cancel{v_{x\,i}^2} - v_{y\,i}^2) \\ \Delta E_k &= \frac{1}{2}m(v_{y\,f}^2 - v_{y\,i}^2) \end{aligned}$$

The initial vertical speed is

$$\begin{aligned} v_{y\,i} &= v_i \sin\theta \\ &= (6.2 \text{ m/s}) \sin 49° \\ v_{y\,i} &= 4.68 \text{ m/s} \end{aligned}$$

Now use conservation of energy:

$$\begin{aligned} \Delta E_k &= -\Delta E_g \\ \frac{1}{2}mv_{y\,f}^2 &= \frac{1}{2}mv_{y\,i}^2 - \Delta E_g \end{aligned}$$

Multiply both sides of the equation by 2, and divide both sides by m.

$$\begin{aligned} v_{y\,f}^2 &= v_{y\,i}^2 - \frac{2\Delta E_g}{m} \\ v_{y\,f} &= \sqrt{v_{y\,i}^2 - \frac{2\Delta E_g}{m}} \\ &= \sqrt{(4.68 \text{ m/s})^2 - \frac{2(3.6 \text{ J})}{0.45 \text{ kg}}} \\ v_{y\,f} &= 2.4 \text{ m/s} \end{aligned}$$

Statement: The bullfrog's speed at a height of 0.82 m is 2.4 m/s.

Practice

1. A soccer player kicks a 0.43 kg soccer ball down a smooth (frictionless) hill 18 m high with an initial speed of 7.4 m/s (**Figure 5**). T/I
   (a) Calculate the ball's speed as it reaches the bottom of the hill. [ans: 2.0×10^1 m/s]
   (b) The soccer player stands at the same point on the hill and gives the ball a kick up the hill at 4.2 m/s. The ball moves up the hill, comes to rest, and rolls back down the hill. Determine the ball's speed as it reaches the bottom of the hill. T/I [ans: 19 m/s]
2. A tennis player begins a serve by tossing a 57 g tennis ball straight up. T/I
   (a) After leaving the player's hand, the ball rises another 1.8 m. Calculate the speed of the ball as it leaves the player's hand. [ans: 5.9 m/s]
   (b) On the next serve, the tennis player tosses the ball with $\frac{1}{4}$ the speed in (a). Determine the ratio of the maximum rise of the ball after leaving the player's hand after this toss to the maximum rise in (a). [ans: 1:16]

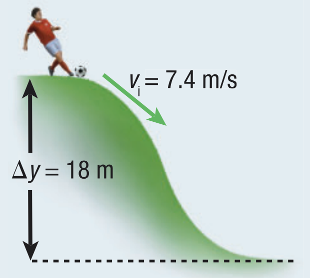


Figure 5

Investigation **4.5.1**

Energy and Pulleys (page 210)
The law of conservation of energy applies to real-world mechanical systems. In this controlled experiment you will investigate this law.

Isolated and Open Systems

In the previous section, you learned about the conservation of mechanical energy. Mechanical energy includes gravitational potential energy and kinetic energy. However, the conservation of energy includes other forms of energy, such as thermal, elastic, electrical, chemical, light, and sound. To understand transformations between these forms of energy, it is important to know the distinction between an isolated and an open system.

isolated system a system that cannot interact or exchange energy with external systems; also called a closed system

An **isolated system** is a system that cannot interact with any other system or exchange energy with its surroundings. An object in an isolated system might exchange energy with other objects within the system, but energy never moves into or out of the system. The parts of the system are isolated from any influences outside the system.

In reality, the universe itself is the only completely isolated system, but you can define a system that has minor influence from its surroundings as an isolated system. For example, you may consider a diver to be an isolated system if you ignore influences such as air friction and the energy of vibrations in the diving board.

open system a system that can interact with another external system

An **open system** is a system that can interact with another external system. An open system can exchange energy with its surroundings. In reality, a diver is an open system that exchanges energy with the air, the diving board, the water, and other real-world systems. Physicists sometimes refer to an isolated system as a *closed system* to contrast it with an open system. Many systems analyzed in this book can be modelled as isolated systems. The concepts of isolated and open systems allow us to state the **law of conservation of energy** more formally:

Law of Conservation of Energy

Energy is neither created nor destroyed in an *isolated system*. It can only change form.

Biological Energy Transformations

biochemical energy a type of chemical potential energy stored in the cells and other basic structures of biological organisms

It is difficult to imagine a second of the day without observing some type of energy transformation. Each time you turn on a light, walk to class, or listen to your favourite song, energy changes from one form to another. Energy conservation involves transformations between many types of energy. Many biologically important energy transformations involve chemical energy. **Biochemical energy** is the energy stored in the cells of organisms and is used to perform all life processes. Green plants produce biochemical energy during photosynthesis. Every time you blink your eyes, raise your arm, or move muscles, you transform biochemical energy stored in your body to mechanical energy that enables you to move.

Biochemical energy can also change to forms other than mechanical energy. The bioluminescent jellyfish in **Figure 6** uses the transformation of biochemical energy to light energy as a defence mechanism against predators. Electric eels transform biochemical energy to electrical energy to stun their prey. During intense exercise, the change of biochemical energy to thermal energy in your body causes you to feel warm. In each of these transformations, total energy is conserved.

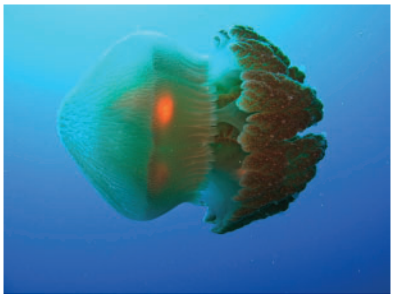


Figure 6 Some jellyfish transform biochemical energy into light energy.

Power

You learned that hydroelectric stations generate millions of kilowatts of power. The terms *power* and *kilowatt* are probably familiar to you, but what do they actually mean? Time enters into work–energy ideas through the concept of **power**, which relates the rate of change in energy of a system over time. A man using a rope to lift a crate does work on the crate as it moves from the floor to a height h (**Figure 7**). The displacement changes the potential energy of the crate by an amount $mg\Delta y$. This energy comes from the man as he pulls on the other end of the rope and does an amount of work $W = mg\Delta y$ on the rope.

power the rate of work done by a force over time, or the rate at which the energy of an open system changes

If this work is expended during a time t, then the power P exerted by the man is defined as

$$P = \frac{W}{t}$$

The SI unit of power is the watt (W), named after James Watt (1736–1819), a developer of the steam engine. One watt is equal to one joule per second. Note that the symbol for watt is not italicized (W), but the variable used for work is an italic W.

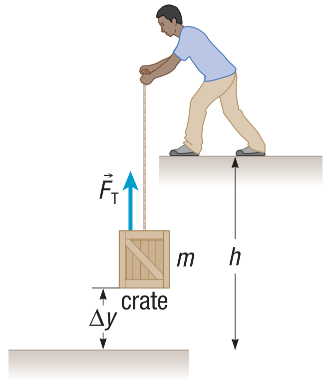


Figure 7 The man produces power as he lifts the crate.

Notice that power is the *rate* at which work is done. Work is a way to transfer energy from one system to another over time. As a result, power is also equal to the energy output of a device per unit time.

In addition to mechanical energy, you can consider how other types of energy, including chemical energy and electrical energy, are involved in conservation of energy situations. The concept of power also applies to chemical and electrical processes and devices. There is a power output or input associated with many of the electrical devices in your home. For example, a typical compact fluorescent lamp is rated at 23 W, or 23 J/s. This rating means that the lamp consumes 23 J of electrical energy for every second it is turned on. Likewise, electronic devices such as computers and DVD players also have a power rating. **Table 1** lists the power output and consumption of a number of common devices.

Table 1 Typical Values for the Power Output or Consumption of Some Common Appliances and Devices

Device	Power output or consumption (W)
portable DVD player	20
laptop computer	40
desktop computer	125
elite bicycle racer	400
automobile engine (small car)	7.5×10^4
automobile engine (race car)	5.2×10^5

The distinction between power consumption and power output is important. For example, a fluorescent lamp consumes a certain amount of electrical energy (for which you pay the utility company), and it outputs a certain amount of energy in the form of visible light along with a certain amount of thermal energy.

The following Tutorial illustrates the definition of power as change in energy over time.

Tutorial 2 Calculating Power

Sample Problem 1: Power as a Rate of Change in Kinetic Energy

A car accelerates from rest to a speed of 27.8 m/s in 7.7 s. The mass of the car is 1.1×10^3 kg. Ignoring friction, determine how much power the car requires.

Given: $m = 1.1 \times 10^3$ kg; $v_i = 0$ m/s; $v_f = 27.8$ m/s; $t = 7.7$ s

Required: P

Analysis: Use the power equation, $P = \frac{W}{t}$. Use the work–energy theorem to relate W to the change in kinetic energy ΔE_k.

Solution: The work done on the car equals its change in kinetic energy. The car starts from rest, so the change in kinetic energy equals the final kinetic energy:

$$\begin{aligned} W &= \Delta E_k \\ &= E_{k\,f} \\ &= \frac{1}{2}mv_f^2 \\ &= \frac{1}{2}(1.1 \times 10^3\ \text{kg})(27.8\ \text{m/s})^2 \\ W &= 4.251 \times 10^5\ \text{J (two extra digits carried)} \end{aligned}$$

Therefore,

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{4.251 \times 10^5\ \text{J}}{7.7\ \text{s}} \\ P &= 5.5 \times 10^4\ \text{W} \end{aligned}$$

Statement: The car requires 55 kW of power.

Practice

1. A firefighter climbs a ladder at a speed of 1.4 m/s. The ladder is 5.0 m long, and the firefighter weighs 65 kg. T/I A
   (a) Determine the firefighter's power output while climbing the ladder. [ans: 890 W]
   (b) How long does it take her to climb the ladder? [ans: 3.6 s]
2. A Grand Prix race car accelerates to twice the speed of the car in Sample Problem 1, in the same amount of time. Calculate the ratio of the power needed by the Grand Prix car to the power needed by the car in Sample Problem 1. K/U T/I A [ans: 4:1]
3. Every year, the Calgary Tower hosts a foot race to the top of the tower. The vertical distance travelled up the 802 steps is about 190 m, and a champion racer can make the climb in less than 5.0 min. If a 62 kg racer completes the climb in 4 min 50 s, determine his average power output during the race. T/I A [ans: 0.40 kW]

UNIT TASK **BOOKMARK**

You can apply what you have learned about energy transformations and the conservation of energy to the Unit Task on page 270.

4.5 Review

Summary

- The law of conservation of energy states that energy can neither be created nor destroyed in an isolated system; it can only change form. This law is useful for solving many physics problems involving motion.
- An isolated, or closed, system cannot exchange energy with its surroundings. However, an open system can exchange energy with its surroundings.
- Power is the rate of work done during a time interval, or the rate at which the energy of a system changes.

Questions

1. A child tosses a tennis ball straight up into the air with an initial speed of 11 m/s. Ignore the height of the child, and assume that air resistance is negligible. K/U T/I C
   (a) Determine the maximum height that the ball will reach.
   (b) Sketch a graph of potential energy versus time for the ball during its time in the air. Explain why the graph has the shape that it does. Label the minimum and maximum potential energy values.
   (c) On the same set of axes, sketch a graph of total energy versus time and a graph of kinetic energy versus time. Explain why they are shaped the way that they are. Identify any maximum or minimum values.
2. An apple falls from a branch to the ground below. K/U
   (a) At what moment is the kinetic energy of the apple greatest?
   (b) At what moment is the gravitational potential energy greatest?
3. A hockey puck slides along a level surface, eventually coming to rest. K/U T/I
   (a) Is the energy of the hockey puck conserved? Explain your answer.
   (b) Discuss what happens to the initial kinetic energy of the puck.
4. (a) A skier of mass 110 kg travels down a frictionless ski trail with a top elevation of 210 m. Calculate the work done on the skier by gravity as the skier travels from the top of the trail to the bottom.
   (b) Calculate the speed of the skier when he reaches the bottom of the ski trail. Assume he starts from rest. T/I A
5. A 62 kg snowboarder is moving across a horizontal ledge at 8.1 m/s when she encounters a drop-off and becomes airborne. Ignore air resistance. The snowboarder lands 3.7 m below the drop-off. Calculate her speed at the moment she hits the ground. T/I A
6. A dolphin is trying to jump through a hoop that is fixed at a height of 3.5 m above the surface of her pool. The dolphin leaves the water at an angle of inclination of 40°. Determine the minimum speed the dolphin will need when leaving the water in order to reach the height of the hoop. T/I
7. A roller coaster car with mass 640 kg moves along the track shown in **Figure 8**. Assume all friction is negligible. K/U T/I A

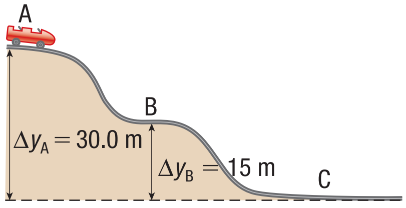


Figure 8

   (a) Is the mechanical energy of the roller coaster conserved? Explain your answer.
   (b) If the roller coaster starts from rest at point A, what is its total mechanical energy at point A?
   (c) What is the total mechanical energy at point B?
   (d) Calculate the speed of the roller coaster when it reaches points B and C.
   (e) If the car starts with a speed of 12 m/s at point A, calculate the speed of the roller coaster when it reaches points B and C.
8. A 52 kg woman jogs up a hill in 24 s. Calculate the power the woman exerts if the hill is 18 m high. T/I A

4.6 Elastic Potential Energy and Simple Harmonic Motion

Figure 1 Jumping on a trampoline requires transformations between kinetic energy, gravitational potential energy, and elastic potential energy.

Jumping on a trampoline is fun, but is it also work? Each time the student in **Figure 1** moves downward, she has to bend her knees and push hard against the trampoline. According to Newton's third law of motion, the trampoline also exerts an equal force in the opposite direction, pushing her upward.

What type of energy does the student jumping on the trampoline have? You know that she has gravitational potential energy relative to the ground when she is in the air above the trampoline. She also has kinetic energy because she is moving. Does she have other types of energy?

As the girl pushes down on the trampoline, she stretches the elastic fabric and springs of the trampoline. The downward force of her feet does work on the trampoline, transferring energy to it. This energy temporarily becomes stored energy in the fabric and the springs. We will explore the nature of this type of stored energy in this section.

Spring Forces

One important type of potential energy is associated with springs and other elastic objects. You are probably familiar with a simple spring, such as the tight coil of wire shown in **Figure 2**. In its relaxed state, with no force applied to its end, the spring is at rest, as shown in Figure 2(a). Suppose you pull on the spring with a force F_{pull}, causing the spring to stretch to the right, as shown in Figure 2(b). When stretched, the spring exerts a force F_{spring} to the left. Likewise, if you push on the spring with a force F_{push}, it compresses to the position shown in Figure 2(c). When compressed, the spring exerts a force F_{spring} to the right. In both cases, F_{spring} is called the restorative force because it tends to restore the spring to its natural length.

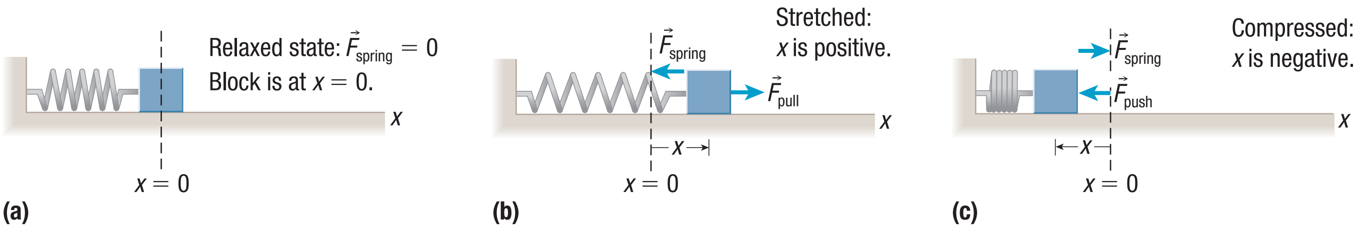


Figure 2 The force exerted by a spring is always opposite to the displacement of the end of the spring. (a) When a spring is unstretched and uncompressed, the force exerted by the spring is zero. (b) The spring is stretched by pulling it to the right, so the force exerted by the spring is to the left. (c) The spring is compressed, and the force exerted by the spring is to the right.

Hooke's law the amount of force exerted by a spring is directly proportional to the displacement of the spring

The amount of force exerted by a spring is proportional to the spring's displacement. This is **Hooke's law,** named after Robert Hooke (1635–1703), who discovered the relationship in 1678. Hooke's law for the force exerted by the spring is

$$\vec{F}_x = -k\Delta\vec{x}$$

spring constant (k) the constant of variation between the force exerted by an ideal spring and the spring's displacement

In this equation, $\vec{F}_x$ is the force exerted by the spring on whatever stretches it, and $\Delta\vec{x}$ is the displacement of the spring from its unstretched, equilibrium position. The constant of proportionality k is called the **spring constant** of the spring, and it corresponds to the stiffness of the spring. Springs that are stiff have a larger value for k and require a larger force to extend or compress them. Springs that stretch easily have smaller values of k. An essential feature of Hooke's law is that the direction of the spring force is opposite to the direction of displacement from equilibrium. If $\Delta\vec{x}$ is upward, then $\vec{F}_x$ is downward. If $\Delta\vec{x}$ is downward, then $\vec{F}_x$ is upward.

Mini Investigation

Spring Force

Skills: Performing, Observing, Analyzing, Evaluating, Communicating

SKILLS HANDBOOK A5.5

In this activity, you will explore the force exerted by a stretched spring.

Equipment and Materials: eye protection; support stand with desk clamp; extension spring; clamp for extension spring; mass set (masses from 50 g to 200 g for a sensitive spring; masses from 500 g to 2000 g for a stiff spring); ruler; soft material

1. Put on your eye protection. Set up the equipment as shown in **Figure 3**, and place soft material below the spring.

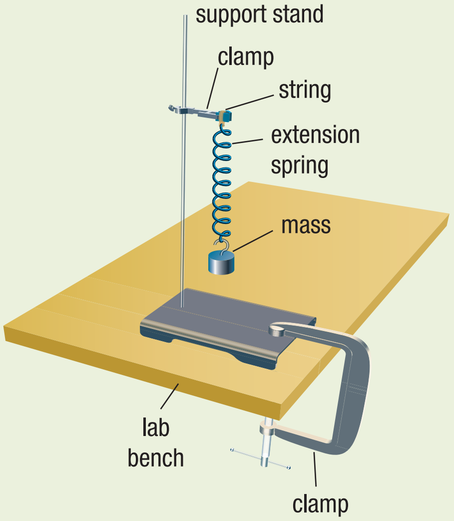


Figure 3

2. Attach a mass to the spring. Measure the displacement Δx, and record your measurement in a data table.

Be careful not to let the masses fall on your hands or feet. Do not let the springs overstretch.

3. Repeat Step 2 for the other masses. Remember to record the masses you use.
4. For each measurement, calculate and record the force using the equation $F_g = mg$.
5. Create a graph of F_g versus Δx. Draw a line of best fit.

A. Describe the relationship between F_g and Δx. K/U T/I

B. Calculate the slope of the line of best fit. What does this slope represent? T/I A

C. Write the equation $F = k\Delta x$ for the equipment you used in this investigation, where k is the slope of the line of best fit. K/U T/I

A spring that obeys Hooke's law exactly is called an **ideal spring**, and no internal or external friction acts on it. Although we have only discussed springs so far, Hooke's law applies to many elastic devices. In Tutorial 1, you will use Hooke's law to predict the effect of an applied force on a spring.

ideal spring any spring that obeys Hooke's law; it does not experience any internal or external friction

Tutorial 1 Applying Hooke's Law

The following Sample Problem examines how to determine the spring constant of a spring and how to use the spring constant to predict the stretch of the spring when a mass is attached to it.

Sample Problem 1: Determining and Applying the Spring Constant

A spring hangs at rest from a support. If you suspend a 0.46 kg mass from the spring, its deflection is 7.9 cm (**Figure 4,** next page).

(a) Determine the spring constant.

(b) Calculate the displacement, in centimetres, of the same spring when a 0.75 kg mass hangs from it instead.

(c) Suppose the 0.75 kg mass is pushed upward, so that it rises past the spring's unstretched position, compressing the spring. Calculate the net force on the mass when the spring is compressed 5.3 cm. Include a free-body diagram.

(d) Determine the acceleration of the mass at the position given in (c) once it is released.

Solution

(a) **Given:** $m = 0.46$ kg; $\Delta x = 7.9$ cm $= 0.079$ m

Required: k

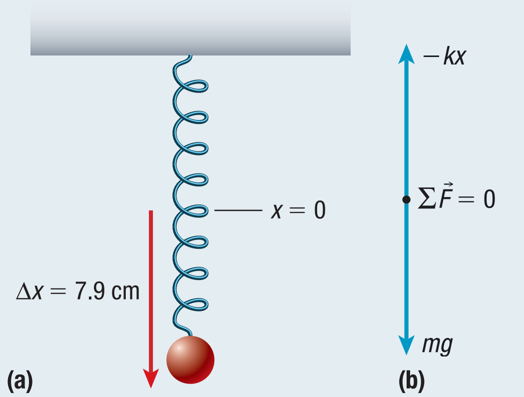


Figure 4 (a) Mass suspended from a vertical spring (b) FBD

Analysis: The force of gravity on the mass points down. The restorative spring force on the mass points up because the spring is stretched down. To calculate the total force, subtract the magnitudes:

$$\vec{F}_g = mg\,[\text{down}] = -mg\,[\text{up}];\ \vec{F}_x = -k\Delta\vec{x} = k\Delta x\,[\text{up}]$$

Since the mass is not accelerating, $\Sigma\vec{F} = 0$ according to Newton's second law.

Solution: $\Sigma\vec{F} = 0$

$$k\Delta x - mg = 0$$

$$k = \frac{mg}{\Delta x}$$

$$= \frac{(0.46\text{ kg})(9.8\text{ m/s}^2)}{(0.079\text{ m})}$$

$$k = 57.1\text{ N/m (one extra digit carried)}$$

Statement: The spring constant is 57 N/m.

(b) **Given:** $m = 0.75$ kg; $k = 57.1$ N/m

Required: $\Delta\vec{x}$

Analysis: The force of gravity on the mass points down. The spring force on the mass points up because the spring is displaced down.

$$\vec{F}_g = mg\,[\text{down}] = -mg\,[\text{up}];\ \vec{F}_x = -k\Delta\vec{x} = k\Delta x\,[\text{up}]$$

Since the mass is not accelerating, $\Sigma\vec{F} = 0$.

Solution: $\Sigma\vec{F} = 0$

$$k\Delta x - mg = 0$$

$$\Delta x = \frac{mg}{k}$$

$$= \frac{(0.75\text{ kg})(9.8\text{ m/s}^2)}{57.1\text{ N/m}}$$

$$\Delta x = 0.13\text{ m}$$

The displacement is in the downward direction.

Statement: The displacement of the spring is 13 cm [down].

(c) **Given:** $m = 0.75$ kg; $k = 57.1$ N/m; $\Delta x = 5.3$ cm $= 0.053$ m

Required: $\vec{F}_{\text{net}}$

Analysis: The free-body diagram for the mass is shown in **Figure 5**.

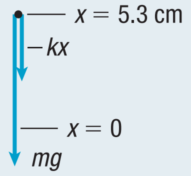


Figure 5

The force of gravity on the mass points down. The spring force on the mass points down because the spring is compressed upward.

$$\vec{F}_g = mg\,[\text{down}],\ \vec{F}_x = -k\Delta\vec{x} = k\Delta x\,[\text{down}]$$

$$\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_x$$

Solution: $\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_x$

$$= mg\,[\text{down}] + k\Delta x\,[\text{down}]$$

$$= (0.75\text{ kg})(9.8\text{ m/s}^2)\,[\text{down}]$$

$$+ (57.1\text{ N/m})(0.053\text{ m})\,[\text{down}]$$

$$\vec{F}_{\text{net}} = 10.4\text{ N [down]}$$

Statement: The net force is 10 N [down] when the spring has been compressed by 0.053 m.

(d) **Given:** $\vec{F}_{\text{net}} = 10.4$ N [down]; $m = 0.75$ kg

Required: $\vec{a}$

Analysis: $\vec{F}_{\text{net}} = m\vec{a}$

Solution: $\vec{F}_{\text{net}} = m\vec{a}$

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

$$= \frac{10.4\text{ N}}{0.75\text{ kg}}\,[\text{down}]$$

$$\vec{a} = 14\text{ m/s}^2\,[\text{down}]$$

Statement: The acceleration is 14 m/s² [down] when the spring is compressed 0.053 m. If the mass is moving upward, the downward acceleration means that it is slowing down due to the force of gravity and the elastic force of the spring.

Practice

1. (a) A 0.65 kg mass hangs at rest from a spring. The spring is stretched 0.44 m from its equilibrium position. Determine the spring constant. [ans: 14 N/m]
   (b) You remove the mass from the spring and attach a new mass to the spring. The new mass stretches the spring 0.74 m from its equilibrium position. Determine the new mass. T/I [ans: 1.1 kg]
2. A 5.3 kg mass hangs vertically from a spring with spring constant 720 N/m. The mass is lifted upward and released. Calculate the force and acceleration on the mass when the spring is compressed by 0.36 m. T/I [ans: 310 N [down]; 59 m/s^2 [down]]

Elastic Potential Energy

The student jumping on the trampoline in Figure 1 does work on the trampoline every time she pushes down on it. As the student comes down from a jump and hits the trampoline surface, she has kinetic energy. As the trampoline fabric and springs stretch, she transfers her kinetic energy into potential energy stored in the trampoline. Energy that is stored in objects that are compressed or stretched is called **elastic potential energy**. This stored energy in the trampoline can be transferred back to the student, giving her the kinetic energy she needs for her next upward jump.

elastic potential energy the potential energy due to the stretching or compressing of an elastic material

Unlike gravitational potential energy, elastic potential energy does not depend on an object's elevation. Instead, it depends on the amount of compression or stretching. To determine the potential energy, we can calculate the change in kinetic energy of a mass attached to a spring as the spring is compressed or stretched. The change in kinetic energy equals the work done by the spring force. The work done by the spring force can be determined from a graph of applied force versus displacement.

The area under a force–displacement graph for an ideal spring has the shape of a triangle. The area of this triangle equals the work done on the spring by the applied force. This applied force is equal but opposite to the spring force on an attached object. Therefore, the work done on the spring to displace it will equal the negative of the work done by the spring on the object as it is displaced.

Figure 6 shows a graph of the spring force on a mass attached to a spring as the spring is stretched or compressed. The slope of the line equals $-k$, following Hooke's law: the direction of the spring force is always opposite to the displacement. We can interpret the area between the F versus x line and the x-axis for a given x value as the total work done on the spring as the spring stretches or compresses by Δx. This work is the negative of the total work done by the spring.

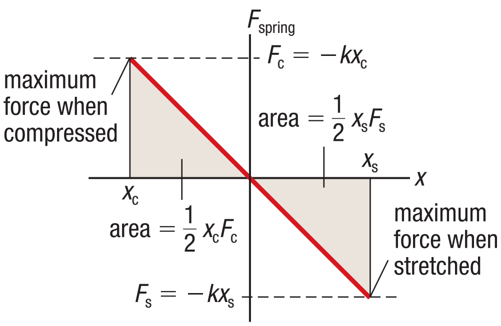


Figure 6 The work done by a variable force is equal to the area under the force–displacement graph. This figure shows the force exerted by a spring for a given displacement, which is the negative of the force applied to the spring to stretch it. For a spring, this area is a triangle. When the spring is stretched, $x = x_s$ and W is the area of the triangle at the lower right. When the spring is compressed, $x = x_c$ and W is the area of the triangle at the upper left. The work done by the spring is negative, so the change in potential energy of an attached object is positive.

Since the area of a triangle equals one-half the base length times the height, the work, W, done on a spring with a spring constant k is

$$W = \frac{1}{2}\Delta x(k\Delta x)$$

$$W = \frac{1}{2}k(\Delta x)^2$$

The work done by the spring force is the negative of this amount, and is also the negative of the change in potential energy. This means that the work done stretching or compressing the spring is transformed into elastic potential energy. Remember that work is a scalar quantity and thus directions can be ignored. We can equivalently write the equation as

$$E_e = \frac{1}{2}k(\Delta x)^2$$

where E_e is the elastic potential energy.

As with all types of energy, elastic potential energy can be transformed to kinetic energy, or to other types of potential energy. When the student jumps on the trampoline, some of the energy transforms into kinetic energy and gravitational potential energy. Some also transforms into the vibrational energy of the trampoline, sound energy, and thermal energy. The following Tutorial examines elastic potential energy.

Tutorial 2 Calculating and Applying Elastic Potential Energy

The following Sample Problem shows how elastic potential energy can be calculated and applied in simple situations.

Sample Problem 1: Calculate Elastic Potential Energy

A 42 kg teenager balances briefly on a pogo stick, causing the spring in the stick to compress downward by 0.18 m. Determine the elastic potential energy of the teenager.

Given: $m = 42$ kg; $\Delta x = 0.18$ m

Required: E_e

Analysis: The force of gravity on the teenager points down. The spring force on the teenager points up because the spring is compressed down.

$$\vec{F}_g = mg\,[\text{down}]$$

$$\vec{F}_x = -k\Delta\vec{x}$$

$$\vec{F}_x = k\Delta x\,[\text{up}]$$

Since the teenager is not accelerating, $\Sigma\vec{F} = 0$.

Solution: Determine the spring constant:

$$\begin{aligned}
\Sigma\vec{F} &= 0 \\
k\Delta x\,[\text{up}] - mg\,[\text{down}] &= 0 \\
k &= \frac{mg}{\Delta x} \\
&= \frac{(42\text{ kg})(9.8\text{ m/s}^2)}{(0.18\text{ m})} \\
k &= 2.29 \times 10^3\text{ N/m}
\end{aligned}$$

Use the spring constant to determine the elastic potential energy:

$$\begin{aligned}
E_e &= \frac{1}{2}k(\Delta x)^2 \\
&= \frac{1}{2}(2.29 \times 10^3\text{ N/m})(0.18\text{ m})^2 \\
E_e &= 37\text{ J}
\end{aligned}$$

Statement: The teenager on the pogo stick has 37 J of elastic potential energy.

Practice

1. The teenager from Sample Problem 1 has a brother twice her mass. Calculate the ratio of his elastic potential energy when balancing on the pogo stick to his sister's. T/I [ans: 4:1]
2. A spring-loaded toy uses a compressed spring to fire a marble out of a tube. A force of 220 N compresses the spring by 0.14 m. Calculate the elastic potential energy of the toy. T/I [ans: 15 J]

Periodic Motion

A person usually jumps more than one time on a trampoline or a pogo stick. Often a person will jump up and down, over and over again. The motion usually changes slightly with each jump, but suppose a jumper has regular motion so that the height and time for each jump are always the same. Motion that repeats in this way is called periodic motion.

Simple Harmonic Motion

Suppose a block is connected to a spring and both are resting on a frictionless surface. The block is at equilibrium when it is resting at its initial position, $x = 0$, as shown in **Figure 7(a)**. In **Figure 7(b)**, the spring is stretched to its maximum limit, or **amplitude**, A; displacement, $x = +A$, is maximized and the block stops momentarily. The block's motion is then reversed as the spring pulls it back toward the equilibrium point (**Figure 7(c)**). The block continues to move past the equilibrium point and stops when the spring is fully compressed and negative displacement, $x = -A$, is maximized (**Figure 7(d)**). The restorative force of the spring moves the system toward equilibrium, and the cycle continues.

amplitude (A) the maximum displacement of a wave

Notice that the force exerted by the spring is not constant. As the spring approaches its equilibrium point, the displacement, Δx, decreases. Since F_x is proportional to the magnitude of this displacement, F_x also decreases during this time.

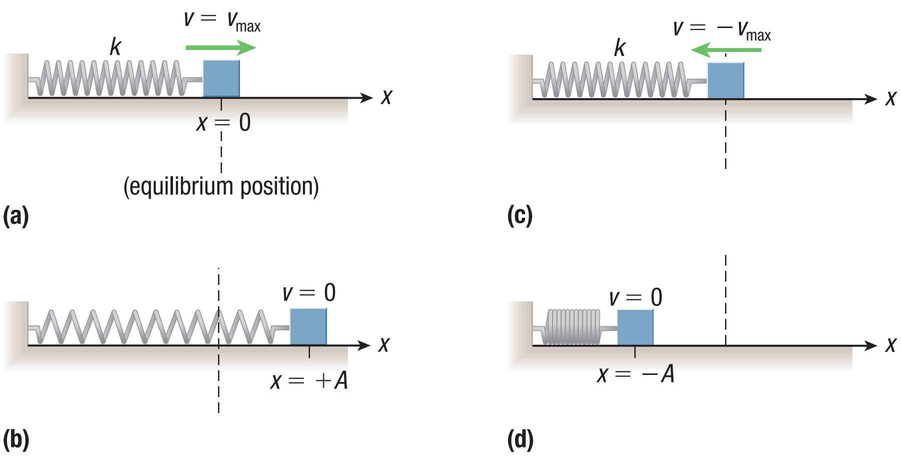


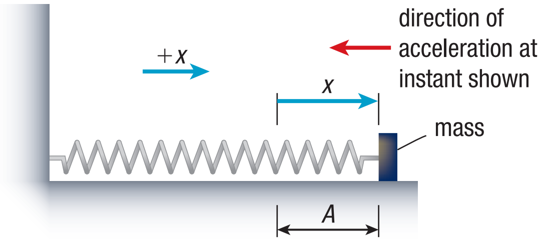
Figure 7 A mass–spring system undergoing simple harmonic motion

The mass in Figure 7 will continue to move right and left between A and $-A$ if both the spring and surface are frictionless. This back-and-forth motion is an example of simple harmonic motion. **Simple harmonic motion** (SHM) is back-and-forth, or periodic, motion in which the moving object experiences a force that is proportional and opposite to the displacement. An object undergoing SHM is often referred to as a simple harmonic oscillator. WEB LINK

simple harmonic motion periodic motion in which the acceleration of the moving object is proportional to its displacement

You can visualize back-and-forth motion that is similar to SHM by thinking about a tennis ball hit from one side of the net to the other, over and over again. If the players stand still in the same positions, and if they apply the same force to the ball with each hit, the ball will continually have the same back-and-forth motion.

When describing SHM mathematically, picture a reference circle, like the CD shown in **Figure 8** on the next page. The mass attached to the spring in the figure vibrates left and right with SHM. At the same time, the point shown on the CD rotates with uniform circular motion. Suppose that the amplitude of the SHM equals the reference point's radius of revolution. What if the period of vibration for the SHM exactly equals the period of rotation for the CD? Then, the x-coordinates of the mass and the point on the CD will remain equal at all times. This means that the acceleration of the mass is the same as the acceleration of the x-coordinate of the reference point at all times.



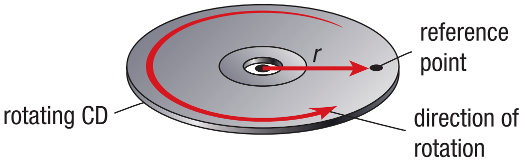


Figure 8 The CD's rotation period is the same as the mass's oscillation period. The mass and the reference point on the CD have the same x-coordinate at all times. The acceleration of the mass and the acceleration in the x-direction of the reference point are also the same at all times.

When the reference point and the mass are at the point of maximum stretch in the spring, the centripetal acceleration of the reference point a_c is directed toward negative x. The acceleration of the x-coordinate at this moment is a_c, so the acceleration of the mass at this moment is also a_c. This fact will allow us to calculate the period of motion of the mass.

For an object with radius r in uniform circular motion with period T, the centripetal acceleration is

$$a_c = \frac{4\pi^2 r}{T^2}$$

We can rewrite this equation to solve for T:

$$T^2 = \frac{4\pi^2 r}{a_c} \quad \text{or} \quad T = 2\pi\sqrt{\frac{r}{a_c}}$$

Applying this result to SHM, let $r = A$ (amplitude) for the reference circle. We then have

$$T = 2\pi\sqrt{\frac{A}{a_c}}$$

Next, we can use Hooke's law and Newton's second law to calculate the acceleration of the mass. The equation

$$\vec{F}_x = -k\Delta\vec{x} = m\vec{a}_x$$

means that

$$\vec{a}_x = -\frac{k\Delta\vec{x}}{m}$$

The ratio of the magnitude of the displacement to the magnitude of the acceleration is

$$\frac{\Delta x}{a_x} = \frac{m}{k}$$

At the point of maximum stretch,

$$\frac{\Delta x}{a_x} = \frac{A}{a_c}$$

which means

$$\frac{m}{k} = \frac{A}{a_c}$$

If you substitute this result into the equation for the period, we obtain the equation of the period of a mass on a spring:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The period, T, of SHM is the amount of time for one cycle of motion. Period is measured in seconds. The inverse of T equals the number of cycles per second, or the frequency, f. The units of frequency are called hertz, and 1 Hz = 1 cycle/s. WEB LINK

In Tutorial 3, you will use the equations for simple harmonic motion, period, and frequency to solve problems.

Tutorial 3 Application of Period and Frequency

The following Sample Problem shows how to calculate the period and frequency of a diver standing on a diving board.

Sample Problem 1: Calculating Period and Frequency

When a diving board bends, the restoring force due to the board's elastic properties obeys Hooke's law for an elastic material. Suppose a diver of mass 85 kg stands on a diving board with spring constant 8.1×10^3 N/m. The mass of the board is much smaller than the diver's mass. Calculate the period and frequency at which the board vibrates.

Given: $m = 85$ kg; $k = 8.1 \times 10^3$ N/m

Required: f, T

Analysis: Use the equations for simple harmonic motion period and frequency:

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ and } f = \frac{1}{T}$$

Solution: $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{85 \text{ kg}}{8.1 \times 10^3 \text{ N/m}}}$$

$$T = 0.64 \text{ s}$$

$$f = \frac{1}{T}$$

$$= \frac{1}{0.64 \text{ s}}$$

$$f = 1.6 \text{ Hz}$$

Statement: The diving board will vibrate with a period of 0.64 s and a frequency of 1.6 Hz.

Practice

1. A 105 kg swimmer stands on a diving board with a spring constant of 7.6×10^3 N/m. Determine the period and frequency of the board vibrations. T/I [ans: 0.74 s; 1.4 Hz]
2. A car mounted on the springs in its suspension acts like a mass on a spring. How will the frequency of oscillations change if passengers are added to the car? Will the frequency increase, decrease, or stay the same? Explain your answer. K/U T/I A

Notice that the calculations in Tutorial 3 did not require you to identify how far the diving board bent. The period and frequency depend only on the mass that exerts the force and the spring constant of the elastic material.

You can identify numerous examples of SHM in everyday life (if you ignore friction). A child on a playground swing may swing back and forth at a regular rate. Strings on guitars and violins vibrate when plucked. The planets move in periodic orbits around the Sun. Automobiles use springs to provide a cushioning effect. However, you do not experience long periods of SHM when riding in a car because shock absorbers provide friction on the springs. We will explore the behaviour of shock absorbers further in Section 4.7.

4.6 Review

Summary

- Hooke's law states that the force exerted by a spring (or, equivalently, the force applied to a spring) is directly proportional to the spring's displacement from its rest equilibrium position, $\vec{F} = -k\Delta\vec{x}$.
- The force exerted by a spring is a restorative force. It acts in the opposite direction of the displacement to return the spring to its natural length.
- The constant of proportionality in Hooke's law is the spring constant, k. The spring constant is large when a spring is stiff and small when a spring is loose. The spring constant is measured in newtons per metre.
- The energy stored in an object that is stretched, compressed, twisted, or bent is called elastic potential energy, $E_e = \frac{1}{2}k(\Delta x)^2$.
- Simple harmonic motion (SHM) is periodic motion in which an object moves in response to a force that is directly proportional and opposite to its displacement.

Questions

1. Spring A has a spring constant of 70 N/m, and spring B has a spring constant of 50 N/m. Explain which spring is more difficult to stretch. K/U
2. A force of 5 N is applied to a block attached to the free end of a spring stretched from its relaxed length by 10 mm. Determine the spring constant. T/I
3. Is the elastic potential energy stored in a spring greater when the spring is stretched by 1.5 cm or when it is compressed by 1.5 cm? Explain your answer. K/U C
4. Calculate the elastic potential energy stored in a spring with a spring constant of 5.5×10^3 N/m when it
   (a) stretches 2.0 cm
   (b) compresses 3.0 cm K/U T/I
5. A 0.63 kg mass rests on top of a vertical spring with spring constant 65 N/m. T/I
   (a) When the mass sits at rest, determine the distance that the spring is compressed from its equilibrium position.
   (b) The mass is held at the unstretched position of the spring and released. Calculate the acceleration of the mass after it falls 4.1 cm.
6. A 5.2 kg mass hung from a spring vibrates with a period of 1.2 s. Calculate the spring constant. T/I
7. A spring has a spring constant of 1.5×10^3 N/m. Determine the length that the spring should be stretched to store 80.0 J of energy. T/I
8. Calculate the work done by a spring force acting on a spring attached to a box, stretched from its relaxed length by 15 mm. The spring constant of the spring is 400.0 N/m. T/I
9. A mass–spring system undergoes SHM. The elastic potential energy at maximum stretch is 7.50 J, the mass is 0.20 kg, and the spring constant is 240 N/m. Calculate the frequency and amplitude of oscillation. T/I
10. The springs in the suspension of a car with worn-out shock absorbers will undergo SHM after hitting a bump in the road. Suppose that a car with worn-out shock absorbers has two identical rear axle springs that each support 5.5×10^2 kg. After hitting a large pothole, the rear end of the car vibrates through six cycles in 4.4 s. Calculate the spring constant of either spring. T/I A
11. Pyon pyon "flying shoes" were invented by Yoshiro Nakamatsu of Japan (**Figure 9**). Research this unique invention. Draw a diagram showing the forces at work when the wearer takes a step. How do you think the shoe's designer incorporated Hooke's law into the shoe design? K/U A

Figure 9

WEB LINK

Springs and Conservation of Energy

4.7

Most drivers try to avoid collisions, but not at a demolition derby like the one shown in **Figure 1**. The point of a demolition derby is to crash your car into as many other cars as possible. Each car tries to damage the other cars so much that they will stop working. The harder the crash, the more damage you are likely to do. The last car running is the winner.

Figure 1 In a demolition derby, the cars can be crashed, but the drivers must remain safe.

How can the drivers of demolition cars avoid serious injury? What types of safety equipment do they use? Like most drivers, they wear seat belts to hold themselves securely in their seat. They have shoulder straps to prevent lurching forward. Padding inside the driver's-side door might provide cushioning from side impacts. Most cars on the road, however, have safety features that are missing or unimportant in demolition cars. Cars you ride in probably have airbags to cushion the passengers during a crash. They may have anti-lock brakes or other computer-controlled systems that act during emergency situations. In this section, you will explore the physics behind safety equipment and other systems in which energy is stored and transformed. CAREER LINK

Conservation of Mechanical Energy

Systems that make a car safe use either springs or elastic materials, so they have elastic potential energy. You have read about the conservation of energy in an isolated system. The law of conservation of energy includes elastic potential energy: Energy is neither created nor destroyed in an isolated system, but it can be transformed between kinetic energy, gravitational potential energy, elastic potential energy, and other forms of energy. In this section, we will explore interactions of systems in which mechanical energy is conserved; that is, the total amount of kinetic, gravitational potential, and elastic potential energy remains constant. Energy losses due to effects such as friction, air resistance, thermal energy, and sound can be ignored as negligible.

UNIT TASK **BOOKMARK**

You can apply what you learn about springs and conservation of energy to the Unit Task on page 270.

Suppose, for example, a student jumps up from a diving board. Assuming air friction is negligible, the mechanical energy of the diver will be conserved. Use the diving board as the reference point, $y = 0$, for measuring the gravitational potential energy. At the maximum height h above the diving board, the diver has only gravitational potential energy equal to $mg\Delta y$. He then starts to fall toward the board, gaining kinetic energy because of his motion.

The diver's distance above the reference point is decreasing, so his gravitational potential energy is decreasing. His total mechanical energy does not change. Halfway to the board, his gravitational potential energy has decreased by half, so it exactly equals his kinetic energy. At the moment just before the diver hits the diving board, the gravitational potential energy is zero, and he has kinetic energy that equals his starting gravitational potential energy.

Conservation of mechanical energy still applies after the diver hits the diving board. He applies a downward force on the board, displacing it a distance x. This work transfers the diver's kinetic energy to the board. The energy is stored in the board as elastic potential energy. As the board dips down, the diver drops below $y = 0$, so his gravitational potential energy becomes negative. The total elastic potential energy increases to offset the decrease in gravitational potential energy. In reality, some energy is lost as friction, sound, and vibrations of the diving board. If we ignore these effects, mechanical energy is conserved. You will apply the conservation of mechanical energy in the following Tutorial.

Tutorial 1 Applying the Law of Conservation of Energy

In this Tutorial, we will analyze the transformations of gravitational potential, kinetic, and elastic potential energies of various systems.

Sample Problem 1: Analyzing Energy Transformations

In this problem, you will model a collision and apply conservation of energy to analyze the outcome. A model car of mass 5.0 kg slides down a frictionless ramp into a spring with spring constant $k = 4.9$ kN/m (**Figure 2**).

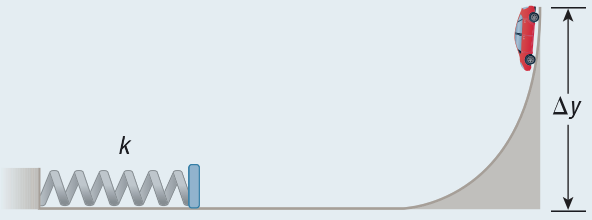


Figure 2

(a) The spring experiences a maximum compression of 22 cm. Determine the height of the initial release point.

(b) Calculate the speed of the model car when the spring has been compressed 15 cm.

(c) Determine the maximum acceleration of the car after it hits the spring.

Solution

(a) **Given:** $m = 5.0$ kg; $k = 4.9$ kN/m $= 4.9 \times 10^3$ N/m; $\Delta x = 22$ cm $= 0.22$ m

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$; $E_e = \frac{1}{2}k(\Delta x)^2$

Since energy is conserved, the change in potential energy of the model car must equal the change in elastic potential energy when the spring is compressed.

Solution: If we choose the bottom of the ramp to be the $y = 0$ reference point, the car will have no gravitational potential energy at the bottom of the ramp. The initial gravitational potential energy has been converted into kinetic energy. When the spring is fully compressed, the kinetic energy has been converted to elastic potential energy. Therefore, the spring's initial gravitational potential energy must equal its final elastic potential energy:

$$E_g = E_e$$

$$mg\Delta y = \frac{1}{2}k(\Delta x)^2$$

$$\Delta y = \frac{k(\Delta x)^2}{2\,mg}$$

$$= \frac{(4.9 \times 10^3\ \text{N/m})(0.22\ \text{m})^2}{2(5.0\ \text{kg})(9.8\ \text{m/s}^2)}$$

$$\Delta y = 2.42\ \text{m (one extra digit carried)}$$

Statement: The initial height of the model car is 2.4 m.

(b) **Given:** $m = 5.0$ kg; $k = 4.9$ kN/m $= 4.9 \times 10^3$ N/m; $\Delta x = 15$ cm $= 0.15$ m; $\Delta y = 2.42$ m

Required: v

Analysis: $\Delta E_g = mg\Delta y$; $E_e = \frac{1}{2}k(\Delta x)^2$; $E_k = \frac{1}{2}mv^2$

Since energy is conserved, the sum of the kinetic energy and the elastic potential energy when the spring is compressed must equal the initial gravitational potential energy.

Solution: The initial gravitational potential energy is

$$E_g = mg\Delta y$$
$$= (5.0 \text{ kg})(9.8 \text{ m/s}^2)(2.42 \text{ m})$$
$$E_g = 119 \text{ J}$$

When the spring is compressed to $\Delta x = 15$ cm, the elastic potential energy is

$$E_e = \frac{1}{2}k(\Delta x)^2$$
$$= \frac{1}{2}(4.9 \times 10^3 \text{ N/m})(0.15 \text{ m})^2$$
$$E_e = 55.1 \text{ J}$$

The kinetic energy when $x = 15$ cm must equal the difference between the initial gravitational potential energy and the final elastic potential energy:

$$E_k = E_g - E_e$$
$$= 119 \text{ J} - 55.1 \text{ J}$$
$$E_k = 63.9 \text{ J}$$

Finally, use E_k to solve for v:

$$\frac{1}{2}mv^2 = E_k$$
$$v^2 = \frac{2E_k}{m}$$
$$v = \sqrt{\frac{2E_k}{m}}$$
$$= \sqrt{\frac{2(63.9 \text{ J})}{5.0 \text{ kg}}}$$
$$v = 5.1 \text{ m/s}$$

Statement: The speed of the model car when the spring is compressed 15 cm is 5.1 m/s.

(c) **Given:** $m = 5.0$ kg; $k = 4.9$ kN/m $= 4.9 \times 10^3$ N/m; $\Delta x = 22$ cm $= 0.22$ m

Required: $\vec{a}$

Analysis: $\vec{F}_e = -k\Delta\vec{x}$; $\vec{F}_{net} = m\vec{a}$

The maximum acceleration occurs when the maximum force is acting, and this occurs when the spring is at the maximum compression of 22 cm.

Solution: Combining Hooke's law and Newton's second law gives

$$\vec{F}_{net} = \vec{F}_x$$
$$m\vec{a} = -k\Delta\vec{x}$$
$$\vec{a} = -\frac{k\Delta\vec{x}}{m}$$
$$= \frac{(4.9 \times 10^3 \text{ N/m})(-0.22 \text{ m})}{5.0 \text{ kg}}$$
$$\vec{a} = 2.2 \times 10^2 \text{ m/s}^2 \text{ [toward ramp]}$$

Statement: The maximum acceleration of the model car is 2.2×10^2 m/s^2 directed toward the ramp.

Sample Problem 2: Using Elastic Potential, Kinetic, and Gravitational Potential Energies

A 48 kg child bounces on a pogo stick. At the lowest point of one bounce, the compressed spring in the stick has 120 J of elastic potential energy as it compresses 0.19 m. Assume that the pogo stick is light enough that we can ignore its mass.

(a) Determine the child's maximum height during the jump following the bounce.

(b) Determine the child's maximum speed during the jump.

Solution

(a) **Given:** $m = 48$ kg; $E_e = 120$ J

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$

Solution: Choose the lowest point of the bounce as the $y = 0$ reference point. At the maximum height, Δy, all elastic potential energy has converted to gravitational potential energy.

$$E_g = E_e$$
$$mg\Delta y = E_e$$
$$\Delta y = \frac{E_e}{mg}$$
$$= \frac{120 \text{ J}}{(48 \text{ kg})(9.8 \text{ m/s}^2)}$$
$$\Delta y = 0.26 \text{ m}$$

Statement: The child rises 0.26 m from the lowest point of the bounce.

(b) **Given:** $m = 48.5$ kg; $E_e = 120$ J; $h = 0.19$ m

Required: v

Analysis: The point of maximum speed is the point at which the spring is at its equilibrium position. At this point, all of the elastic potential energy has been converted to gravitational potential energy and kinetic energy.

$$\Delta E_g = mg\Delta y;\ E_k = \frac{1}{2}mv^2$$

Solution: If we choose the lowest part of the bounce as the $y = 0$ reference point, then at the equilibrium position,

$$\begin{aligned} E_g &= mg\Delta y \\ &= (48\ \text{kg})(9.8\ \text{m/s}^2)(0.19\ \text{m}) \\ E_g &= 89.4\ \text{J (one extra digit carried)} \end{aligned}$$

The kinetic energy is the difference between the initial elastic potential energy and the gravitational potential energy:

$$\begin{aligned} E_k &= E_e - E_g \\ &= 120\ \text{J} - 89.4\ \text{J} \\ E_k &= 30.6\ \text{J (one extra digit carried)} \end{aligned}$$

Now solve for v:

$$\begin{aligned} \frac{1}{2}mv^2 &= E_k \\ v &= \sqrt{\frac{2E_k}{m}} \\ &= \sqrt{\frac{2(30.6\ \text{J})}{48\ \text{kg}}} \\ v &= 1.1\ \text{m/s} \end{aligned}$$

Statement: The child's maximum speed is 1.1 m/s.

Sample Problem 3: A Block Pushed Up a Frictionless Ramp by a Spring

A block with a mass of 2.0 kg is held against a spring with spring constant 250 N/m. The block compresses the spring 22 cm from its equilibrium position. After the block is released, it travels along a frictionless surface and then up a frictionless ramp. The ramp's angle of inclination is 30.0°, as shown in **Figure 3**.

(a) Determine the elastic potential energy stored in the spring before the mass is released.

(b) Calculate the speed of the block as it travels along the horizontal surface.

(c) Determine how far along the ramp the block will travel before it stops.

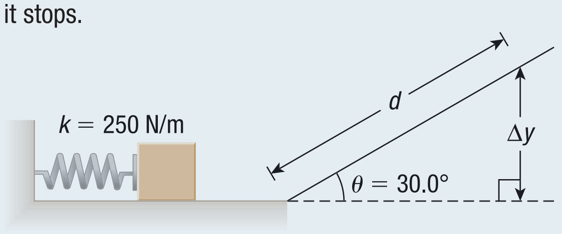


Figure 3

(a) **Given:** $k = 250$ N/m; $x = 22$ cm $= 0.22$ m

Required: E_e

Analysis: Before the block is released, the entire mechanical energy of the block–spring system is in the form of elastic potential energy stored in the compressed spring. We can use the given information to determine the amount of stored energy, $E_e = \frac{1}{2}k(\Delta x)^2$.

Solution:

$$\begin{aligned} E_e &= \frac{1}{2}k(\Delta x)^2 \\ &= \frac{1}{2}(250\ \text{N/m})(0.22\ \text{m})^2 \\ E_e &= 6.05\ \text{J (one extra digit carried)} \end{aligned}$$

Statement: The elastic potential energy stored in the spring before the mass is released is 6.0 J.

(b) **Given:** $E_k = 6.05$ J; $m = 2.0$ kg

Required: v

Analysis: As the block travels along the flat surface, all of the elastic potential energy is converted to kinetic energy. Use this to determine the speed of the block using the equation for kinetic energy: $E = \frac{1}{2}mv^2$.

Solution:

$$\begin{aligned} E &= \frac{1}{2}mv^2 \\ \frac{2E}{m} &= v^2 \\ v &= \sqrt{\frac{2E}{m}} \\ &= \sqrt{\frac{2(6.05\ \text{J})}{(2.0\ \text{kg})}} \\ v &= 2.46\ \text{m/s} \end{aligned}$$

Statement: The block will travel at a constant speed of 2.5 m/s along the frictionless horizontal surface.

(c) **Given:** $E_g = 6.05$ J; $m = 2.0$ kg; $g = 9.8$ m/s^2

Required: Δy, d

Analysis: As the block travels up the ramp, kinetic energy is gradually converted to gravitational potential energy. When the block reaches its maximum height, all energy will be in potential form. Use this to determine the vertical height attained, Δy, and then use trigonometry to calculate the distance travelled along the ramp, d, as shown in Figure 3.

$$E_g = mg\Delta y;\ \frac{\Delta y}{d} = \sin\theta$$

Solution: Mechanical energy is conserved throughout this problem because there are no energy losses due to friction.

The total potential energy at the top of the block's path is therefore 6.05 J.

$$E_g = mg\Delta y$$

$$\Delta y = \frac{E_g}{mg}$$

$$= \frac{6.05\ \text{J}}{(2.0\ \text{kg})(9.8\ \text{m/s}^2)}$$

$$\Delta y = 0.308\ \text{m}$$

Now use the sine ratio to determine how far along the ramp the block travels, *d*.

$$\frac{\Delta y}{d} = \sin\theta$$

Rearrange this equation to express *d* in terms of Δy and θ.

$$\Delta y = d\sin\theta$$

$$d = \frac{\Delta y}{\sin\theta}$$

$$= \frac{0.308\ \text{m}}{\sin 30.0°}$$

$$d = 0.62\ \text{m}$$

Statement: The block will travel a distance of 0.62 m, or 62 cm, along the ramp.

Practice

1. A block slides down a ramp from a fixed height and collides with a spring, compressing the spring until the block comes to rest. Compare the amount of compression in the case that the ramp is frictionless to the case where the ramp is not frictionless. Explain your answer. K/U T/I
2. A 3.5 kg mass slides from a height of 2.7 m down a frictionless ramp into a spring. The spring compresses 26 cm. Calculate the spring constant. T/I [ans: 2.7×10^3 N/m]
3. A 43 kg student jumps on a pogo stick with spring constant 3.7 kN/m. On one bounce, he compresses the stick's spring by 37 cm. Calculate the maximum height he reaches on the following jump. T/I [ans: 0.60 m above the compressed point]
4. A 0.35 kg branch falls from a tree onto a trampoline. If the branch was initially 2.6 m above the trampoline, and the trampoline compresses 0.14 m, calculate the spring constant of the trampoline. T/I [ans: 9.6×10^2 N/m]
5. Consider the block in Sample Problem 3. Suppose that the mass of the block is doubled at the top of its path of motion before returning down the frictionless ramp. K/U T/I
   (a) Determine the speed of the block as it returns along the horizontal surface. [ans: 2.5 m/s]
   (b) Does the block have the same kinetic energy as before along the horizontal surface? Explain your answer.
   (c) Will the block compress the spring twice as far as it did before? Explain your answer. If your answer is no, determine the new value for *x*.
   (d) Suppose the coefficient of friction of the ramp is 0.15. Does your answer to (c) change? Explain your answer. If your answer is yes, determine the new value for *x*.

Perpetual Motion Machines

An ideal spring would never lose energy and would continue with SHM forever, or as long as you did not disturb it. A machine that can continue to operate for an unlimited amount of time without outside help is a **perpetual motion machine**. To be a true perpetual motion machine, the machine must be able to run forever without restarting or refuelling. A grandfather clock, for example, is not a perpetual motion machine, since you must wind it up every now and then.

perpetual motion machine a machine that can operate forever without restarting or refuelling

Figure 4 shows a device called Newton's cradle. The leftmost ball has gravitational potential energy with respect to the other balls. Once released, the ball will interact with the other balls in such a way as to imitate perpetual motion. You will have the opportunity to explore the physics behind Newton's cradle in Chapter 5.

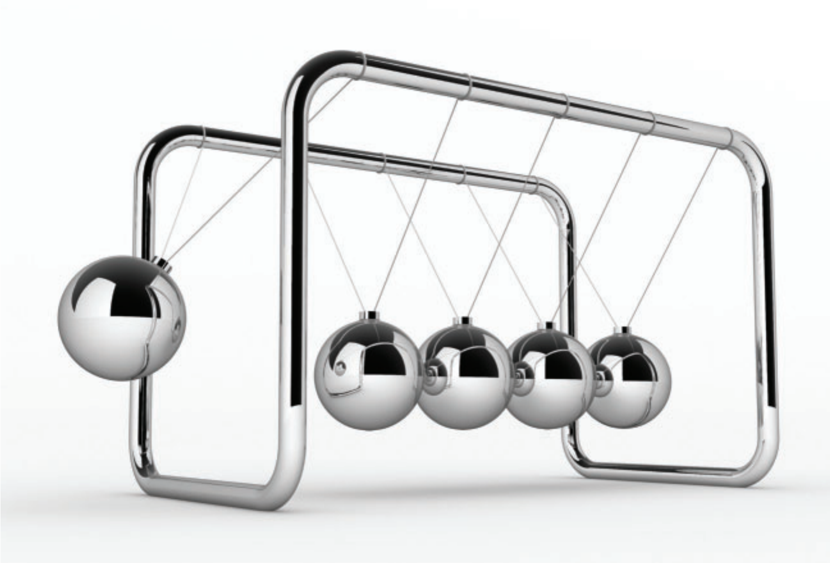


Figure 4 An ideal version of Newton's cradle would be a perpetual motion machine because it would never lose energy.

Investigation 4.7.1

Energy and Springs (page 211)
You have learned about the spring constant and how the movement of springs is related to the conservation of energy. Now you are ready to conduct an investigation to observe the conservation of energy.

An ideal version of Newton's cradle would never lose energy, and the cycle of falling, colliding, and rising would continue forever. It would then be a perpetual motion machine. Can you build such a machine?

The answer is no. In real-world machines, some mechanical energy will always be lost from the system as thermal energy, sound energy, or other forms of energy. This loss of energy can be useful. For example, the purpose of shock absorbers in cars, which we mentioned in Section 4.6, is to use friction to stop the SHM of the car's springs.

Research This

Perpetual Motion Machines

Skills: Researching, Communicating SKILLS HANDBOOK A4.1

Hobbyists and serious researchers alike have attempted to design perpetual motion machines. They have not been successful, but some of their ideas have useful applications.

1. Choose one machine, such as an analog watch, a metronome, a flywheel, or a child's swing, that relies on ongoing, consistent motion to work properly.
2. Research the design principles that have been incorporated into modern versions of the machine to make it work more efficiently.

A. What scientific principles explain how the machine operates? A
B. How has the design of the machine been improved over time? K/U
C. Have improvements been the result of the development of new materials, new technology, or new scientific discoveries? T/I
D. Prepare a short presentation that summarizes your findings. C

WEB LINK

Damped Harmonic Motion

So far, we have ignored the effect of friction on the motion of a simple harmonic oscillator. The friction in a real periodic system is referred to as damping, and the harmonic motion of a system affected by friction is called **damped harmonic motion**. The presence of friction means that the mechanical energy of the system will be transformed into thermal energy, and the system's motion will not continue perpetually. We can classify damped motion into three categories: underdamped, overdamped, and critically damped (**Figure 5**).

damped harmonic motion periodic motion affected by friction

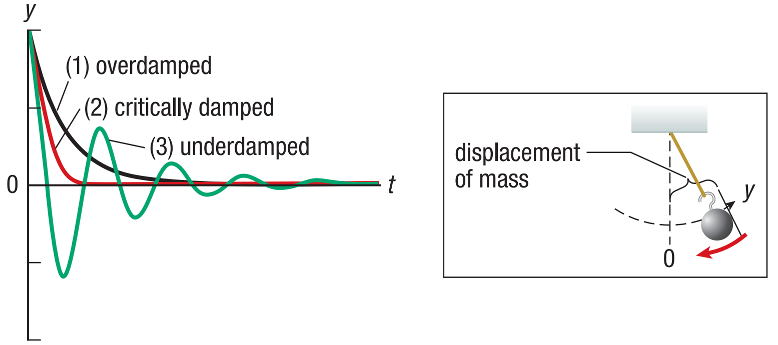


Figure 5 When a damped oscillator is given a non-zero displacement at $t = 0$ and then released, it can exhibit three different types of behaviour: (1) overdamped, (2) critically damped, and (3) underdamped.

Consider a pendulum. Curve 3 in Figure 5 shows how the displacement varies with time when the damping is weak, that is, when there is only a small amount of friction. This curve applies to any weakly damped harmonic oscillator. It describes the back-and-forth swinging of a pendulum or the motion of a mass attached to a spring on a horizontal surface when the surface is quite slippery. The system still oscillates because the displacement alternates between positive and negative values, but the amplitude of the oscillation gradually decreases with time. The amplitude eventually goes to zero, but the system undergoes many oscillations before damping brings it to rest. This type of motion is an underdamped oscillation.

When quite a bit of friction exists, the oscillator is overdamped. The resulting displacement as a function of time in this case is shown as curve 1 in Figure 5. This type of motion happens when the mass moves through a very thick fluid, like the hydraulic fluid inside the closing mechanism on many doors. If you pull the mass of an overdamped oscillator to one side and then release it, the mass moves extremely slowly back to the equilibrium.

Critically damped motion falls in between the two extreme cases. In underdamped motion, displacement always passes through zero—the equilibrium point—at least once, and usually many times, before the system comes to rest. In contrast, an overdamped system released from rest moves just to the equilibrium point, but not beyond. In critically damped motion, displacement falls to zero as quickly as possible without moving past the equilibrium position. Displacement as a function of time for the critically damped case is illustrated by curve 2 in Figure 5.

These different categories of damping have different applications. For example, a car's shock absorbers provide damping for springs that support the car's body (**Figure 6**).

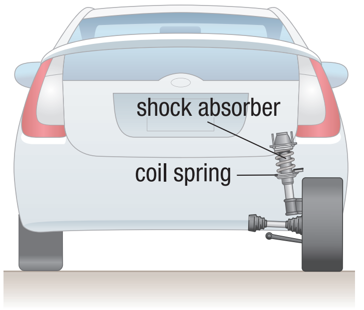


Figure 6 The shock absorbers on a car serve to dampen its coil springs. The goal is usually to have a car respond to bumps in the road as a critically damped oscillator.

Shock absorbers enable the tires to move up and down over bumps in the road without directly passing vibrations to the car's body or passengers. When the car hits a bump, the springs compress. To make the ride as comfortable as possible, the shock absorbers critically damp the motion of the springs. The critically damped motion means the body of the car returns to its original height as quickly as possible. Worn-out shock absorbers lead to underdamped motion, and the car bounces up and down more. Overdamped shock absorbers give a soft "spongy" ride with poor steering response and handling. CAREER LINK

4.7 Review

Summary

- For an isolated mass–spring system, the total mechanical energy—kinetic energy, elastic potential energy, and gravitational potential energy—remains constant.
- A perpetual motion machine is a machine that can operate forever without restarting or refuelling.
- Damped harmonic motion is periodic motion in which friction causes a decrease in the amplitude of motion and the total mechanical energy.

Questions

1. A mass hangs from a vertical spring and is initially at rest. A person then pulls down on the mass, stretching the spring. Does the total mechanical energy of this system (the mass plus the spring) increase, decrease, or stay the same? Explain. K/U
2. A mass rests against a spring on a horizontal, frictionless table. The spring constant is 520 N/m, and the mass is 4.5 kg. The mass is pushed against the spring so that the spring is compressed by 0.35 m, and then it is released. Determine the velocity of the mass when it leaves the spring. T/I
3. A toy airplane ejects its 8.4 g pilot using a spring with a spring constant of 5.2×10^2 N/m. The spring is initially compressed 5.2 cm. T/I
   (a) Calculate the elastic potential energy of the compressed spring.
   (b) Calculate the speed of the pilot as it ejects upward from the airplane.
   (c) Determine the maximum height that the pilot will reach.
4. In a pinball game, a compressed spring with spring constant 1.2×10^2 N/m fires an 82 g pinball. The pinball first travels horizontally and then travels up an inclined plane in the machine before coming to rest. The ball rises up the ramp through a vertical height of 3.4 cm. Determine the distance of the spring's compression. T/I
5. A bungee jumper of mass 75 kg is standing on a platform 53 m above a river. The length of the unstretched bungee cord is 11 m. The spring constant of the cord is 65.5 N/m. Calculate the jumper's speed at 19 m below the bridge on the first fall. T/I
6. A spring with a spring constant of 5.0 N/m has a 0.25 kg box attached to one end such that the box is hanging down from the string at rest. The box is then pulled down another 14 cm from its rest position. Calculate the maximum height, the maximum speed, and the maximum acceleration of the box. T/I
7. A 0.22 kg block is dropped on a vertical spring that has a spring constant of 280 N/m. The block attached to the spring compresses it by 11 cm before momentarily stopping. Determine the height from which the block was dropped. T/I
8. A block of 1.0 kg with speed 1.0 m/s hits a spring placed horizontally, as shown in **Figure 7**. The spring constant is 1000.0 N/m. T/I
   (a) Calculate the maximum compression of the spring.
   (b) How far will the block travel before coming to rest? Assume that the surfaces are frictionless.

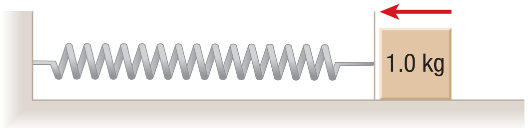


Figure 7

9. A wooden box of mass 6.0 kg slides on a frictionless tabletop with a speed of 3.0 m/s. It is brought to rest by a compressing spring. The spring constant is 1250 N/m. T/I
   (a) Calculate the maximum distance the spring is compressed.
   (b) Determine the speed and acceleration of the block when the spring is compressed a distance of 14 cm.
10. A tennis coach uses a machine to help with tennis practice. The machine uses a compressed spring to launch tennis balls. The spring constant is 440 N/m, and the spring is initially compressed 45 cm. A 57 g tennis ball leaves the machine horizontally at a height of 1.2 m. Calculate the horizontal distance that the tennis ball can travel before hitting the ground. T/I

Investigation 4.2.1 CONTROLLED EXPERIMENT

The Work–Energy Theorem

The work–energy theorem states that the change in kinetic energy of a system equals the work done on the system. In this investigation, you will test the validity of the work–energy theorem using a block pulled along a bench.

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Testable Question

SKILLS HANDBOOK A2.2

Does the work–energy theorem hold true as the applied force is varied for a block being pulled along a surface?

Hypothesis

Formulate a hypothesis based on the Testable Question.

Variables

Preview the Experimental Design and Procedure, and identify the independent, dependent, and controlled variables.

Experimental Design

You will make measurements to determine the change in energy of a block-and-mass system as gravity and friction do work on it. Then, you will use your data to test the validity of the work–energy theorem.

Equipment and Materials

- eye protection
- motion sensor
- bench or table
- block
- scale
- pulley
- mass set with hanger
- metre stick
- string
- box containing soft material

Procedure

1. Read through the Procedure for this investigation and create a data table to record your results.
2. Record the mass of the block.
3. Put on your eye protection. Set up the investigation as shown in **Figure 1**. Place a box containing soft material below the hanging mass.
4. Choose initial and final points along the path of the block. Measure and record the distance between the points. This is the distance the mass will fall when the block moves from the initial position to the final position.

Take care that the mass does not fall on your hands or feet.

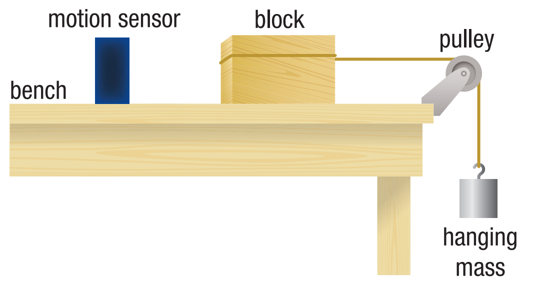


Figure 1

5. Place a 100 g mass on the hanger and let it drop. Use the measurements from the motion sensor to determine the acceleration of the block. Adjust the amount of the loaded mass until the motion sensor indicates that the block does not accelerate as it slides across the bench. It may be difficult to achieve zero acceleration with your set of masses, but come as close as you can. You may have to add mass to the block and then record the new mass of the block.
6. Draw a free-body diagram of the sliding block.
7. Solve for the force of friction between the block and the bench in the case where the block does not accelerate. Use your diagram as a guide.
8. Calculate the coefficient of kinetic friction, μ_K, between the block and the bench using the equation for the force of friction, your measurement from Step 5, and your result from Step 7.
9. Add mass to the hanger so that you have more mass than the special value you calculated in Step 5. Hold the block at the initial position and let the masses drop. Use the motion sensor to determine the speed of the block at the final position.
10. Repeat Step 9 using three different masses. Record your observations.

Analyze and Evaluate

(a) What type of relationship did you test in this investigation? T/I

(b) What is the work done by gravity on just the block as it moves from the initial position to the final position? T/I

(c) What is the work done by gravity on the mass for a general value of the mass m? T/I

(d) What is the work done by gravity on the total system? T/I

(e) Calculate the work done by gravity on the total system for the actual values of m that you used, using your answer from Question (d). T/I

(f) Calculate the work done by friction on the block as it moves from its initial position to its final position using the definition of work and your value of μ_K. T/I

(g) What is the total work done on the system by gravity and friction for each value of m that you used? Organize your results in a table. T/I C

(h) Use your measurements of the block's speed and mass and your values of m to calculate the change in kinetic energy of the system for each value of m. T/I

(i) Answer the Testable Question. How does the work done on the system compare to the change in kinetic energy for each value of m? T/I

(j) Suggest possible sources of error and recommend ways to improve the accuracy of your results. T/I

Apply and Extend

(k) Imagine repeating a similar experiment but with one major change. Instead of varying the value of m, you hold m fixed and vary the distance between the block's initial and final positions. How do you think the work done on the system and the change in kinetic energy would compare in this experiment? Explain your reasoning. T/I

Investigation 4.5.1 CONTROLLED EXPERIMENT

Energy and Pulleys

Conservation of energy allows you to make predictions about the motion of different objects. In this investigation, you will explore how energy changes from one form to another as a falling mass pulls a cart along a ramp.

Testable Question

Is the mechanical energy of a real system conserved as its initial potential energy changes?

Hypothesis

Use the law of conservation of energy to formulate a hypothesis based on the Testable Question.

Variables

Preview the Experimental Design and Procedure. Identify the independent, dependent, and controlled variables.

Experimental Design

You will use a motion sensor to measure the speed of a cart while a falling mass pulls the cart along a horizontal plane and down an inclined plane. You will use your measurements to determine whether energy is conserved.

Equipment and Materials

- eye protection
- horizontal surface
- motion sensor
- cart
- scale
- pulley
- mass
- metre stick
- blocks or books
- barrier for the cart
- box containing soft material
- string

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Procedure

Part A: Horizontal Plane

1. Record the mass of the cart.
2. Put on your eye protection. Set up the equipment as shown in **Figure 1**. Place a barrier between the cart and the edge of the horizontal surface to prevent the cart from falling off the table. Set a box containing soft material below the mass.

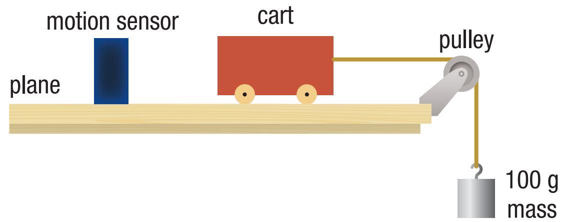


Figure 1

3. Choose initial and final positions for the cart along the horizontal surface. Measure and record the distance between the positions.
4. Let the 100 g mass drop. Use the motion sensor to measure the speed of the cart at the final position.

Take care that the mass does not fall on your hands or feet.

Part B: Inclined Plane

5. Use blocks or books to raise the height of the end of the surface with the pulley 10 cm to 20 cm. Measure the change in height of the cart between the initial and final positions.

6. Let the 100 g mass drop. Use the motion sensor to measure the speed of the cart at the final position.
7. Raise the height of the end of the surface another 10 cm. Measure the new change in height of the cart between the initial and final positions. Repeat Step 6.

Analyze and Evaluate

(a) What type of relationship between the variables did you test in this investigation? T/I

(b) Use your data to calculate the initial and final kinetic and potential energies of the cart and the mass for all three trials. How do the initial and final total energies compare in each case? T/I C

(c) If the initial and final values are not the same, calculate the percent difference and explain what might have happened to the missing energy. T/I

(d) Answer the Testable Question. T/I

Apply and Extend

(e) Imagine doing the investigation with a block with no wheels instead of a cart. Do you expect the difference between the initial and final energy to increase, decrease, or stay about the same? Explain. T/I

Investigation 4.7.1 CONTROLLED EXPERIMENT

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Energy and Springs

In this investigation, you will use a mass hanging from a spring to explore the conservation of energy.

Testable Question

To what extent is mechanical energy conserved during cycles of simple harmonic motion (SHM) in a real mass–spring system?

Hypothesis

Formulate a hypothesis based on the Testable Question.

Variables

Preview the experimental design and procedure. Identify the independent, dependent, and controlled variables.

Experimental Design

You will use a motion sensor to measure the motion of a mass–spring system as it moves through several cycles of harmonic motion. You will calculate the total energy after each cycle and determine whether energy is conserved.

Equipment and Materials

- eye protection
- motion sensor
- support stand with desk clamp
- extension spring
- clamp for extension spring
- mass set with hanger
- metre stick
- box containing soft material

Procedure

1. Determine the spring constant of your spring.
2. Set up the support stand and hang the spring from it. Attach a mass to the other end of the spring. Position the motion sensor so that it measures the displacement of the mass as it goes through SHM. Record the mass used.
3. Use the metre stick to measure the height of the mass at the equilibrium position.
4. Let the mass fall from rest at the natural unstretched position of the spring. Use the motion sensor to measure the maximum displacement for each of 10 cycles. Record your results.

Take care that the mass does not fall on your hands or feet.

5. Repeat the previous step and record the measurements for another 10 cycles.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) Use your measurement from Step 3 and the two sets of data from the motion sensor to measure the maximum displacement and the mass height at the top and bottom of each cycle. Calculate the energy of the system at the top and bottom of each cycle. T/I

(b) Explain what happens to the total energy of the mass–spring system over time. T/I

(c) Make a graph of the total energy versus the time. Show both sets of data on the same graph. T/I C

(d) Identify sources of error, and suggest a method for improving the accuracy of your measurements. T/I

Apply and Extend

(e) Use your results above to estimate the speed of the mass at the midpoint of each cycle. T/I

CHAPTER 4 SUMMARY

Summary Questions

1. Create a study guide for this chapter based on the Key Concepts on page 162. For each point, create three or four subpoints that provide further information, relevant examples, explanatory diagrams, or general equations.
2. Review the Starting Points questions on page 162. Answer these questions using what you have learned in this chapter. Compare your latest answers with those that you wrote at the beginning of the chapter, and note how your answers have changed.

Vocabulary

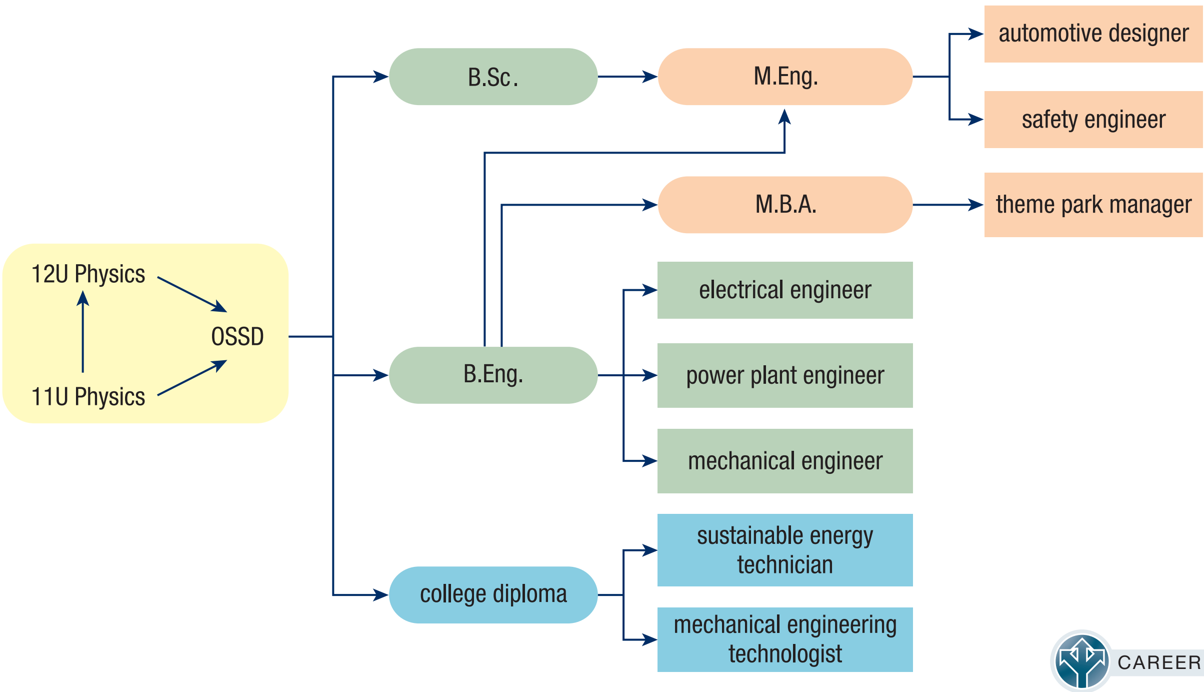
work (p. 164)
joule (p. 165)
kinetic energy (p. 171)
work–energy theorem (p. 173)
potential energy (p. 177)
mechanical energy (p. 177)
gravitational potential energy (p. 177)
law of conservation of energy (pp. 184, 188)
isolated system (p. 188)
open system (p. 188)
biochemical energy (p. 188)
power (p. 189)
Hooke's law (p. 192)
spring constant (p. 192)
ideal spring (p. 193)
elastic potential energy (p. 195)
amplitude (p. 197)
simple harmonic motion (p. 197)
perpetual motion machine (p. 205)
damped harmonic motion (p. 207)

CAREER PATHWAYS

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers related to topics covered in this chapter.

SKILLS HANDBOOK A6

1. Select two careers related to Work and Energy that you find interesting. Research the educational pathways you would need to follow to pursue these careers. What is involved in the required educational programs? Prepare a brief report of your findings.
2. For one of the two careers that you chose above, describe the career, main duties and responsibilities, working conditions, and setting. Also, outline how the career benefits society and the environment.



CAREER LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. A force, F, is applied to an object with a displacement, Δd. When does the equation $W = F\Delta d$ equal the work done by the force on the object? (4.1) K/U
   (a) always
   (b) when the force is in the same direction as the displacement
   (c) when the force is perpendicular to the displacement
   (d) when the force is at an angle of 45° to the displacement
2. Suppose that a spacecraft of mass 5.4×10^3 kg at rest in space fires its rockets to achieve a speed of 8.2×10^2 m/s. How much work has the fuel done on the spacecraft? (4.2) K/U T/I A
   (a) 2.2×10^6 J
   (b) 1.8×10^9 J
   (c) 3.6×10^9 J
   (d) 9.8×10^{12} J
3. Which of the following statements correctly describes the relationship between an object's gravitational potential energy and its height above the ground? (4.3) K/U
   (a) proportional to the square of the object's height above the ground
   (b) directly proportional to the object's height above the ground
   (c) inversely proportional to the object's height above the ground
   (d) proportional to the square root of the object's height above the ground
4. What happens to the kinetic energy of a hockey puck as it moves across the ice and is stopped by a hockey stick? (4.5) K/U T/I
   (a) The kinetic energy is dissipated due to friction with the ice.
   (b) The kinetic energy is dissipated due to air resistance.
   (c) The kinetic energy is absorbed in the collision with the hockey stick.
   (d) All of the above are true.
5. At a construction site, a constant force lifts a stack of wooden boards, which has a mass of 565 kg, to a height of 4.5 m in 13 s. The stack rises at a steady pace. How much power is needed to move the stack to this height? (4.5) K/U
   (a) 1.9×10^2 W
   (b) 1.6×10^3 W
   (c) 1.9×10^3 W
   (d) 1.6×10^4 W
6. If a mass of 0.65 kg attached to a vertical spring stretches the spring 4.0 cm from its original equilibrium position, what is the spring constant? (4.6) K/U
   (a) 0.27 N/m
   (b) 16 N/m
   (c) 60 N/m
   (d) 160 N/m
7. Which of the following does not operate using the conversion of elastic potential energy into other forms of energy? (4.7) K/U A
   (a) a slingshot
   (b) a guitar
   (c) a hinge
   (d) a spring toy

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. Work is a vector quantity. (4.1) K/U
9. Work is always a positive quantity. (4.1) K/U
10. The kinetic energy of an object at rest is always zero. (4.2) K/U
11. The amount of gravitational potential energy depends on where the reference height $y = 0$ is set. (4.3) K/U
12. The production of hydroelectricity provides clean energy and creates no environmental concerns. (4.4) K/U
13. There are many examples of isolated systems in the real world. (4.5) K/U
14. Gravitational potential energy increases as a pendulum's amplitude increases. (4.6) K/U
15. True perpetual motion is not possible due to the damping effect of friction and air resistance. (4.7) K/U

Go to Nelson Science for an online self-quiz.
WEB LINK

Knowledge

For each question, select the best answer from the four alternatives.

1. A race car brakes and skids to a stop on the road. Which statement best describes what happens? (4.1) K/U
   (a) The race car does work on the road.
   (b) The friction of the road does negative work on the race car.
   (c) The race car and the road do equal work on each other.
   (d) Neither does work on the other.
2. A mover pushes a sofa across the floor of a van. The mover applies 475 N of horizontal force to the sofa and pushes it 1.2 m. The work done on the sofa by the mover is
   (a) 285 J
   (b) 396 J
   (c) 570 J
   (d) Not enough information is given to answer the question. (4.1) K/U
3. Two identical cars are racing against each other, as shown in **Figure 1**. Neither car is able to pass the other. Which of the following is true? (4.1) K/U

Figure 1

   (a) Each car is limited by the motion of the other.
   (b) The cars are doing no work on each other.
   (c) The cars are doing equal and non-zero work on each other.
   (d) The cars are doing non-equal work on each other.
4. Two identical marbles are dropped in a classroom. Marble A is dropped from 1.00 m, and marble B is dropped from 0.50 m. Compare the kinetic energies of the two marbles just before they strike the ground. (4.2) K/U
   (a) Marble A has the same kinetic energy as marble B.
   (b) Marble A has 1.4 times as much kinetic energy as marble B.
   (c) Marble A has 2.0 times as much kinetic energy as marble B.
   (d) Marble A has 4.0 times as much kinetic energy as marble B.
5. A 0.30 kg soccer ball is released from the top of a 10 m building. The ball strikes the ground with a speed of 12 m/s. Use the conservation of energy to determine the energy lost due to the work done by air resistance. (4.3) K/U A
   (a) 7.8 J
   (b) 13.2 J
   (c) 21.6 J
   (d) 29.4 J
6. Which of the following prevents Earth and the Moon from being an isolated system? (4.5) K/U
   (a) the gravitational attraction of the Sun
   (b) the gravitational attraction of Saturn
   (c) the gravitational attraction of Pluto
   (d) all of the above
7. Which of the following *cannot* be described using simple harmonic motion? (4.6) K/U
   (a) a playground swing swinging through a small angle
   (b) a DVD spinning in a DVD player
   (c) the pendulum of a grandfather clock
   (d) a guitar string vibrating

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. All moving objects have kinetic energy. (4.2) K/U
9. If you push as hard as you can on a brick wall for 1 h, and the wall does not move, you have done no work on the wall. (4.2) K/U
10. The gravitational potential energy of an object 5 m above the ground in Ontario is the same as an identical object 5 m above the ground on the Moon. (4.3) K/U
11. The joule (J) is the SI unit for three quantities: work, energy, and power. (4.3) K/U

12. A marble is shot from a slingshot on a planet with no atmosphere. At any given moment, before the marble hits the ground, the sum of the kinetic energy and the gravitational potential energy is constant. (4.5) K/U
13. The farther you pull a spring beyond its equilibrium point, the more work you do on it. (4.6) K/U
14. In an oscillating spring, the elastic potential energy when the spring is completely compressed is equal to the kinetic energy when the spring is fully extended (**Figure 2**). (4.7) K/U

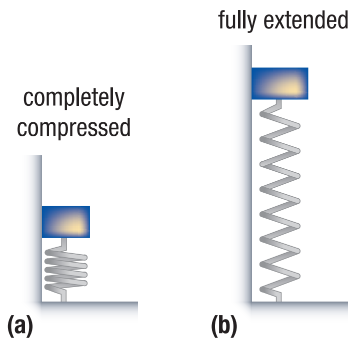


Figure 2

Understanding

15. A car stuck in a snow bank is spinning its wheels and unable to move either forward or backward. Discuss whether work is being performed by
    (a) the car
    (b) the snow bank (4.1) K/U
16. Give two examples in which a non-zero force acts on an object, but the total work done by that force is zero. (4.1) K/U
17. Is it possible to do work on an object if the object does not move? (4.1) K/U
18. A car is parked on a hill. The gravitational force on the car is 9.31×10^3 N straight downward, and the angle of the hill is 4.00° from the horizontal (**Figure 3**). The car's brakes fail, and the car slides 30.0 m downhill. (4.1) K/U

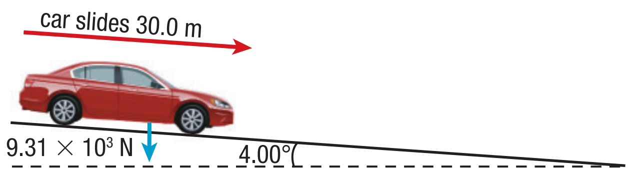


Figure 3

    (a) Calculate the component of the gravitational force that acts parallel to the car's motion.
    (b) Calculate the work done on the car by gravity as the car slides.

19. A spy, code-named 001, uses a pulley to lower another spy, code-named 002, on a harness down from the roof of a building at a constant speed. The mass of spy 002 is 65 kg, and the building is 100.0 m high. (4.1, 4.2, 4.3) K/U
    (a) Calculate the work done on spy 002 by spy 001.
    (b) Calculate the work done on spy 002 by the force of gravity.
    (c) Calculate spy 002's kinetic energy when she is lowered at a speed of 2.5 m/s down the side of the building.
    (d) Determine the change in spy 002's gravitational potential energy when she reaches the ground.
20. Explain, in your own words, how work represents a relationship between forces and energy. (4.2) K/U
21. A sprinter with a mass of 68 kg is running at a speed of 5.8 m/s. In a burst of speed to win the race, she increases her speed to 6.9 m/s. Determine the work that the sprinter does to increase her speed. (4.2) K/U
22. In a curling match, a 20.0 kg stone (**Figure 4**) with an initial speed of 2.0 m/s glides to a stop after 30.0 m. Determine the work done on the stone by friction. (4.2) K/U

Figure 4

23. A diver with a mass of 60.0 kg dives from a board 10.0 m above the surface of the pool. Calculate the diver's change in gravitational potential energy during his dive. (4.3) K/U
24. A grocery store employee lifts a case of cereal from the floor to a shelf 1.2 m high. The gravitational potential energy of the case increases by 5.8 J. Calculate the mass of the case. (4.3) K/U
25. Explain why, when choosing a site for a hydroelectric plant, both the height of the waterfall and the volume of water are important. (4.4) T/I

26. An extension that increases the height of the top of a hydroelectric dam is constructed (**Figure 5**). How does this extension affect the amount of energy produced by the plant? Explain your answer. (4.4) T/I A

Figure 5

27. In a graphic organizer of your choosing, compare and contrast work and power. Include characteristics and examples of each. (4.5) K/U C
28. An egg rolls off a countertop that is 1.2 m high. The egg has a mass of 55 g. Calculate the gravitational potential energy (relative to the floor) of the egg before it falls. (4.5) K/U
29. Every second at Niagara Falls, 5.7×10^5 kg of water falls an average distance of 21 m. Determine the power generated by this process. (4.5) K/U A
30. Describe an everyday object that converts between three different types of energy. (4.5) K/U A
31. Consider two diving boards made of the same material, one long and one short. Which do you think has a larger spring constant? Explain your reasoning. (4.6) T/I A
32. Interpret, in your own words, the meaning of the spring constant k in Hooke's law. (4.6) C
33. Compare the simple harmonic motion of two identical masses oscillating up and down on springs with different spring constants, k. (4.6) K/U C
34. Consider two different masses oscillating on springs with the same spring constant. Describe how the simple harmonic motion of the masses will differ. (4.6) T/I
35. To give an arrow maximum speed, explain why an archer should release it when the bowstring is pulled back as far as possible. (4.7) T/I
36. For maximum speed, why does it make sense for an arrow to be as light as possible? (4.7) T/I A
37. A table tennis ball is launched horizontally from a compressed spring. Use algebraic reasoning to express the table tennis ball's initial speed, v, in terms of the compression distance of the spring, Δx; the spring constant, k; and the mass of the ball, m. (4.7) T/I
38. When a force is exerted on a stationary object, such as a wall, and the object's kinetic energy does not change, what can you conclude about the work done on the object by the force? Explain your answer. (4.2) T/I A

Analysis and Application

39. A rope at an angle of 18.5° above the horizontal provides a tension force of 11.8 N to pull a toboggan along a smooth, horizontal surface. The rope does 214 J of work. Calculate how far the toboggan moves. (4.1) T/I A
40. Suppose you are on a merry-go-round and you let go, and then fly off. Has centripetal force done work on you? Explain why or why not. (4.1) T/I A
41. Periodic comets travel in elliptical orbits around the Sun (**Figure 6**). Copy Figure 6 into your notebook. (4.1) T/I C A

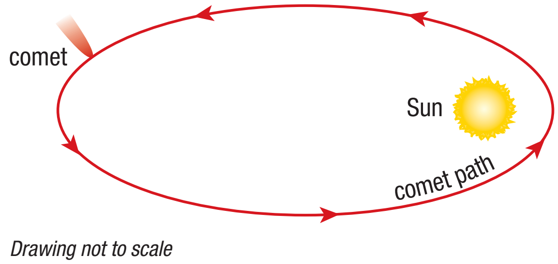


Figure 6

(a) On your drawing, show the points at which the Sun does positive, negative, or zero work on the comet.
(b) Describe what happens as the comet approaches the Sun.
(c) Describe what happens as the comet moves away from the Sun.

42. Using the definition of work done by a constant force in the same direction as displacement, $W = F\Delta d$, use dimensional analysis to verify that the units match those of kinetic energy, $E_k = \frac{1}{2}mv^2$. (4.2) K/U T/I A

43. A motorcycle has broken down (**Figure 7**), and to get the motor to start, you must push the motorcycle with a constant force so that it attains a speed of 10 km/h over a distance of 10 m. You try to stand directly behind the motorcycle and push, but it is difficult to keep your arms exactly parallel to the ground. Draw a graph of how much force you must use as the angle your arms make with the ground increases to 0°, 15°, 30°, 45°, 60°, and 75°. What happens as the angle increases? (4.2) T/I C A

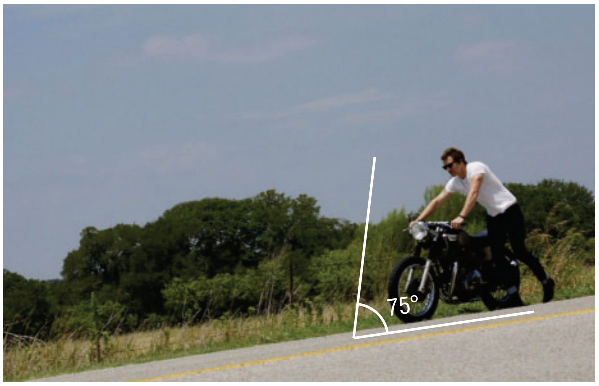


Figure 7

44. A spacecraft needs to tow four loads from one side of a space station to the other. The masses of the loads are 1000 kg, 2000 kg, 3000 kg, and 4000 kg. Each load will be moved with a constant force at a speed of 10 m/s for a distance of 1 km. Draw a scatter plot graph showing how the force the spacecraft must apply varies with the mass of the load it is towing. (4.2) T/I C A

45. For the spacecraft described in Question 44, suppose it is towing four new loads, each with a mass of 2000 kg. Each new load is to be towed the 1 km distance, but it will tow each load at a different speed. The towing speeds are 10 m/s, 15 m/s, 20 m/s, and 25 m/s. Sketch a graph showing how the force the spacecraft applies will vary depending on the speed it must achieve. (4.2) T/I C A

46. A frictionless merry-go-round is given an initial push and allowed to spin. A student stands at the edge, holding tight to a railing. For each of the following, state whether the variable is constant or changing, and explain your answer. (4.2) T/I A
    (a) student's velocity
    (b) student's speed
    (c) student's kinetic energy

47. In the expression $E_g = mg\Delta y$, we assume that we are dealing with objects close to Earth's surface. When we move farther from Earth's surface, what happens to the value of g? (4.3) T/I A

48. (a) The same object weighs less on the Moon than on Earth. How can you change the expression for gravitational potential energy, $E_g = mg\Delta y$, for an object near the Moon's surface instead of Earth's surface?
    (b) Suppose that you are able to conduct investigations on the Moon. List the steps required to use known masses to determine the acceleration due to gravity on the Moon. (4.3) T/I A

49. One hydroelectric plant, X, has an elevated reservoir twice as high as that of another plant, Y. (4.5) T/I A
    (a) If they both release water at the same rate, how many times as much power does the plant with the higher reservoir generate?
    (b) If plant Y now releases water twice as quickly as plant X does, how do the two plants compare in terms of power production?

50. At a post office, a package of mass 1.3 kg is pushed down a slanted chute with an initial speed of 1.8 m/s. The upper end of the chute is 4.0 m above the floor. When it reaches the floor, the package has a speed of 0.9 m/s. Determine the energy lost through air resistance and friction with the chute. (4.5) T/I A

51. Two water balloons of the same mass are dropped: a red balloon from height h and a blue balloon from one-quarter that height. Neglecting air resistance, compare the speeds of the two balloons when they reach the ground. (4.5) K/U A

52. Explain why it is just as important for a pole vaulter (**Figure 8**) to improve his speed, as it is to improve his arm strength and vaulting form. (4.5) T/I A

Figure 8

53. A snowboarder with a mass of 57 kg starts from rest at the top of a frictionless slope at a height of 45 m. She follows the frictionless path shown in **Figure 9**. Calculate her speed at the second peak. (4.5) T/I A

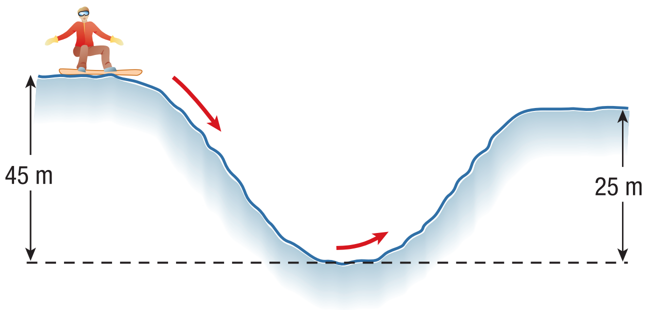


Figure 9

54. An amusement park uses large compressible springs to stop cars at the end of a ride. Assume the springs are ideal, with no weight, mass, or damping losses. The combined mass of the car and passengers averages 450 kg, and the cars make contact with the spring at a speed of 3.5 m/s. Determine the minimum spring constant to bring the car and its riders to a stop in 2.0 m. (4.5) T/I A

55. A soccer player kicks the ball in a parabolic arc to the opposite goal. The ball leaves the player's foot at a speed of 27 m/s, making an angle of 20.0° above the horizontal. The mass of the ball is 0.43 kg. (4.5) T/I A
    (a) Determine the maximum height of the ball's trajectory.
    (b) Determine the ball's speed as it hits the ground again. (Neglect air resistance.)

56. A 55 kg student bounces up from a trampoline with a speed of 5.4 m/s. (4.5) K/U A
    (a) Determine the work done on the student by the force of gravity when she is 1.3 m above the trampoline.
    (b) Determine her speed at 1.3 m above the trampoline.
    (c) Has she reached her maximum height? Explain your answer.

57. Suppose you set a spring with spring constant 4.5 N/m into damped harmonic motion at noon, measuring its maximum displacement from equilibrium to be 0.75 m. When you return 15 min later, the spring is still oscillating, but its maximum displacement has decreased to 0.5 m. (4.7) T/I
    (a) Determine how much energy the system has lost.
    (b) What is the power loss of the system?

58. A ball is attached to a vertical spring with a spring constant of 6.0 N/m. It is held at the equilibrium position of the spring and then released. It falls 0.40 m and then bounces back up again. Calculate the mass of the ball. (4.7) T/I

59. A ball of mass 0.50 kg is attached to a horizontal spring. The spring is compressed 0.25 m from its equilibrium and then released. The ball undergoes simple harmonic motion, achieving a maximum speed of 1.5 m/s. (4.7) T/I A
    (a) Determine the spring constant.
    (b) Calculate the speed of the ball when the spring is halfway to its equilibrium point.
    (c) When the ball is halfway to its equilibrium point, what fraction of its energy has been converted from elastic potential energy to kinetic energy?

60. Two objects of different masses are suspended from two springs that have the same spring constant. The heavier of the two objects will extend its spring farther beyond the equilibrium point. Why? (4.7) T/I A

61. A bungee jumper leaps from a high bridge. Draw a sketch of the jumper as he falls and as he bounces back up to show when each of the following three quantities is increasing or decreasing:
    (a) gravitational potential energy
    (b) elastic potential energy
    (c) kinetic energy (4.7) T/I C A

62. Name a type of energy other than kinetic, gravitational, or elastic potential energy. Give an example of a transformation involving it and one of the three given energy forms. (4.5) T/I A

63. To demonstrate automotive safety, a group of engineers measures how long it takes to stop a car with increasing initial speeds. A constant braking force is used throughout. Their data are shown in **Table 1**. (4.5) T/I C A

Table 1 Data for Automotive Safety

Initial speed (km/h)	Stopping distance (m)
15	5
30	18
45	40
60	68

    (a) Plot the data in Table 1 in a graph of stopping distance versus initial speed.
    (b) Use the concepts of work and energy to explain why the graph is non-linear.
    (c) Discuss how these data can help people drive more safely.

Evaluation

64. Give an example of a situation where more than one force is exerted on an object. For each force, specify whether or not it does work on the object. (4.1) T/I A
65. Using your knowledge of physics principles, speculate how Earth's life forms might be different if Earth were
    (a) less massive (the force of gravity would be less)
    (b) more massive (the force of gravity would be more) (4.3) T/I A
66. Analyze, in terms of various types of energy, how a spring toy is able to step down the stairs (**Figure 10**). (4.6) T/I A

Figure 10

67. Use concepts from this chapter to explain why roller coasters always start with a very high hill. (4.5) T/I A
68. All cars have a hard shell. Older cars, however, often have harder frames and are less likely to crumple in a collision than cars built today. Use concepts from this chapter to argue why a car that is more likely to crumple is safer for passengers. (4.7) T/I A

Reflect on Your Learning

69. What did you find most surprising in this chapter? What did you find the most difficult to understand? How can you learn more about the surprising or difficult topics? C
70. You learned about different sources of commercial energy, such as hydroelectric power. Do you have a better understanding of how we use different energy sources in our daily lives, and the pros and cons of each? Explain why or why not. C
71. What topics in this chapter are you still unsure of? Describe two ways you can improve your understanding of these topics. C
72. How can you apply the concepts you learned in this chapter to aspects of your daily life? C

Research

WEB LINK

73. You can use a pendulum to measure the force of gravity at a specific location. This force varies by as much as 5 % in various places on Earth. Pendulums have been used by scientists (and clockmakers) for centuries. Research how pendulums have been used by various scientists throughout history to study gravity and the orbital motion of the planets. Prepare a report or brochure that describes your research and its relevance to today's understanding of gravity. T/I C A
74. Choose two hydroelectric plants in Canada and research their history, operation, and efficiency. In a graphic organizer of your choice, compare their advantages and disadvantages. T/I C A
75. Select one common amusement park ride, other than a roller coaster. Research the design of the ride and create a presentation, such as a poster, display board, or slide show, that shows the forces and energies associated with the motion of the ride. Your presentation should include a detailed technical drawing or labelled photograph showing the forces acting on the riders at a minimum of three points in the ride. T/I C A

CHAPTER 5

Momentum and Collisions

KEY CONCEPTS

After completing this chapter you will be able to

- define momentum and impulse
- solve problems involving collisions and explosions using conservation of momentum in one and two dimensions
- distinguish between elastic and inelastic collisions
- analyze the results of head-on and glancing collisions using conservation of momentum
- identify and describe technological applications of momentum and conservation of momentum, and analyze their impact on society and the environment

How Is Momentum Related to Sports Safety?

High-energy sports are exciting, and they also provide a great opportunity to see physics in action. Checks in hockey or lacrosse, tackles in football, and kicks and blocks in karate all demonstrate the properties of collisions. Two objects that collide exchange energy, whether they are stick and puck or ball, player and turf, or foot and pad.

The development of better equipment means athletes can move faster, jump higher, and hit harder than ever before. However, more excitement means more safety risk. Fortunately, new technology also means engineers can build better safety equipment.

Athletes in all sports must be protected against injuries such as concussions. A hit to the body that causes an athlete's head to jerk fast enough can damage the brain. Studies show that receiving a concussion at a young age puts an athlete at risk for other health issues later in life. Studies also show that receiving one concussion puts an athlete at higher risk for receiving another.

Advances in sports technology help protect athletes by allowing engineers to design better helmets and pads, and can also allow the athletes to train in new ways that help them avoid injuries.

As you study this chapter, think about the following questions. What sports or exciting activities interest you? What safety risks do these activities have? By studying momentum and collisions, you can learn about how to minimize risk and increase safety in these and many other activities.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. Have you ever heard the term *momentum* used in conversation? Was it referring to a physical concept or an abstract one, such as "the momentum of the game shifted"? Based on your prior knowledge, what do you think momentum is?
2. Why might a test car colliding with a wall produce a greater average net force than a tennis ball colliding with the same wall at twice the speed of the car?
3. Suppose you are watching a football tackle in slow motion. Describe what you see, step by step, as the tackle proceeds from initial contact to its finish.

Mini Investigation

Keep On Rolling

Skills: Predicting, Controlling Variables, Performing, Observing, Analyzing

SKILLS HANDBOOK A2.1

In this Mini Investigation, you will explore how adding or subtracting mass affects the speed of a dynamics cart moving down an inclined plane.

Equipment and Materials: eye protection; inclined plane with barrier; dynamics cart; stopwatch; ruler or metre stick; mass

1. Set up a slightly inclined plane so that the slope of the plane is just enough to overcome the frictional forces on the dynamics cart after it starts moving. The cart should move at a relatively constant speed after you nudge it down the ramp.
2. Put on your eye protection. Determine the base speed by timing the cart's descent down the ramp for three trials. Average the result. This is the base speed.
3. Predict what will happen to the cart's speed after you carefully add a mass to the moving cart midway down the ramp. Perform this step and record your observations.
4. Repeat Step 3 two more times. Record your observations, and then average the results of your three trials.
5. Add a mass to the dynamics cart at the top of the inclined plane. Predict what will happen to the cart's speed after you carefully remove the mass from the moving cart midway down the ramp. Perform this step three times and record your observations.

A. Describe the change in speed of the cart in Steps 3 to 5. K/U

B. Could the change in the normal force and its effect on friction between the cart and the plane be enough to account for the changes in speed? Explain. K/U T/I A

C. What is the primary reason for the changes in speed that you observed? K/U T/I A

5.1 Momentum and Impulse

Objects in motion are a big part of everyday life. It is important that we understand how to put objects in motion, how to change their direction, and how to bring them to a stop. These changes in motion often occur as a result of collisions between two or more objects. A collision between a baseball and a bat, for example, brings about a sudden change in velocity of the ball but also has an effect on the bat (**Figure 1**). A collision between a car and a tree can have negative impacts for both; however, a well-designed collision between the driver and an airbag can save a life.

Why does a puck propelled by a slap shot travel faster than a puck propelled by a wrist shot? How do modern tennis racquets allow today's players to hit the ball with much greater speed than was possible with older wooden racquets? Why do golf courses have to be lengthened from time to time to remain challenging? The concepts of momentum and impulse will help you understand the science of collisions and answer these questions. CAREER LINK

Figure 1 When you hit a ball with a bat, the resulting collision has an effect on both the ball and the bat.

Momentum

Two variables, velocity and acceleration, describe the motion of a single object. An additional quantity, linear momentum, is useful for dealing with a collection of objects. The **linear momentum**, $\vec{p}$, of a single object of mass m moving with velocity $\vec{v}$ is

linear momentum ($\vec{p}$) a quantity that describes the motion of an object travelling in a straight line as the product of its mass and velocity

$$\vec{p} = m\vec{v}$$

Note that for the rest of this section, the term *momentum* refers to linear momentum. The momentum $\vec{p}$ of an object is directly proportional to the object's velocity, so the momentum vector is along the same direction as the velocity. Note also that $\vec{p}$ is proportional to the mass of the object. In the following Tutorial, you will learn more about how to calculate momentum.

Tutorial 1 Calculating Momentum

The following Sample Problem shows you how to use the equation $\vec{p} = m\vec{v}$ to calculate the momentum of an object.

Sample Problem 1: The Vector Nature of Momentum

(a) Calculate the momentum of a 2.5 kg rabbit travelling with a velocity of 2.0 m/s [E].

(b) Calculate the momentum of a 5.0 kg groundhog travelling with a velocity of 1.0 m/s [S].

(c) Compare the momentum and kinetic energies of the rabbit and the groundhog.

Given: $m_{\text{rabbit}} = 2.5$ kg; $\vec{v}_{\text{rabbit}} = 2.0$ m/s [E]; $m_{\text{groundhog}} = 5.0$ kg; $\vec{v}_{\text{groundhog}} = 1.0$ m/s [S]

Required: $\vec{p}_{\text{rabbit}}$; $E_{\text{k rabbit}}$; $\vec{p}_{\text{groundhog}}$; $E_{\text{k groundhog}}$

Analysis: $\vec{p} = m\vec{v}$; $E_{\text{k}} = \frac{1}{2}mv^2$

Solution:

(a)
$$\vec{p} = m\vec{v}$$
$$\vec{p}_{\text{rabbit}} = (2.5 \text{ kg})(2.0 \text{ m/s [E]})$$
$$\vec{p}_{\text{rabbit}} = 5.0 \text{ kg·m/s [E]}$$

(b)
$$\vec{p} = m\vec{v}$$
$$\vec{p}_{\text{groundhog}} = (5.0 \text{ kg})(1.0 \text{ m/s [S]})$$
$$\vec{p}_{\text{groundhog}} = 5.0 \text{ kg·m/s [S]}$$

(c) For the rabbit:

$$E_{\text{k}} = \frac{1}{2}mv^2$$
$$E_{\text{k rabbit}} = \frac{1}{2}(2.5 \text{ kg})(2.0 \text{ m/s})^2$$
$$E_{\text{k rabbit}} = 5.0 \text{ J}$$

For the groundhog:

$$E_{\text{k groundhog}} = \frac{1}{2}(5.0 \text{ kg})(1.0 \text{ m/s})^2$$
$$E_{\text{k groundhog}} = 2.5 \text{ J}$$

Statement:

(a) The momentum of the rabbit is 5.0 kg·m/s [E].

(b) The momentum of the groundhog is 5.0 kg·m/s [S].

(c) The momenta of the two animals are equal in magnitude but in different directions. Although the momenta of the rabbit and the groundhog are of the same magnitude, the kinetic energy, E_k, of the rabbit is twice that of the groundhog. Note that, unlike momentum, kinetic energy is a scalar quantity and does not have a direction.

Practice

1. Calculate the momentum and kinetic energy of a hockey puck with a mass of 160 g travelling with a velocity of 40.0 m/s [E]. K/U T/I A [ans: 6.4 kg·m/s; 130 J]
2. Compare the momentum of a bowling ball with a mass of 6.2 kg travelling with a velocity of 1.6 m/s [E] to that of a hockey puck with a mass of 160 g travelling with a velocity of 40.0 m/s [E]. What is the difference in their momenta? T/I A [ans: 3.5 kg·m/s]

Impulse

A golf ball resting on a tee has mass but zero velocity. Its momentum, therefore, is zero. Although the golfer may not think of it in these terms, the goal is to use the golf club to change the momentum of the ball. If the golfer is successful, the golf ball will fly through the air with considerable momentum an instant after colliding with the golf club. What happens during this transition?

Newton's first law states that the velocity of an object is constant unless acted on by an external force. So, in the absence of an external force, an object with constant mass must also have constant linear momentum. If a net force is applied to the object, its velocity will change and, therefore, its momentum will also change. Consider a force, $\vec{F}$, acting on a golf ball with a mass of 45 g. You can use linear momentum and Newton's second law, $\vec{F} = m\vec{a}$, to calculate the change in momentum.

The object's acceleration, $\vec{a}$, is related to the change in the velocity according to $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$. Suppose that the object has an initial velocity $\vec{v}_i$ just before the force is applied, and a final velocity $\vec{v}_f$ after a time Δt.

$$\begin{aligned}\vec{F} &= m\vec{a}\\ &= m\frac{\Delta\vec{v}}{\Delta t}\\ \vec{F} &= m\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\end{aligned}$$

From the definition of momentum, the initial momentum of the object is $\vec{p}_i = m\vec{v}_i$ and the final momentum is $\vec{p}_f = m\vec{v}_f$. Rearrange the previous equation and then substitute these expressions.

$$\begin{aligned}\vec{F}\Delta t &= m(\vec{v}_f - \vec{v}_i)\\ &= m\vec{v}_f - m\vec{v}_i\\ &= \vec{p}_f - \vec{p}_i\\ \vec{F}\Delta t &= \Delta\vec{p}\end{aligned}$$

The product $\vec{F}\Delta t$ is called the **impulse** and is the change in the momentum of an object:

$$\vec{F}\Delta t = \Delta\vec{p}$$

impulse the product of force and time that acts on an object to produce a change in momentum

Impulse is a vector; its direction is the same as the direction of the total force on the object. You can see from this equation that applying a large force for a short time could produce the same change in momentum as applying a smaller force for a longer time. Dimensional analysis shows that the SI units for impulse (newton seconds, or N·s) are the same as the units for momentum (kg·m/s):

$$[\mathrm{N \cdot s}] = \left[\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}\right][\mathrm{s}]$$

$$[\mathrm{N \cdot s}] = \left[\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^{\cancel{2}}}\right][\cancel{\mathrm{s}}]$$

$$[\mathrm{N \cdot s}] = \left[\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}\right]$$

Impulse in Sports

Figure 2 A modern tennis racquet can transfer a greater impulse to a tennis ball than older wooden racquets.

How do modern tennis racquets allow players to hit the ball with much greater speed than was possible with older wooden racquets (**Figure 2**)? You can investigate this using the equations for momentum and impulse. Assume the mass of the tennis ball is still the same; if its velocity is greater, its momentum must also be greater. So the tennis player is providing a greater impulse, or change in momentum, to the ball. Assuming also that the force the player uses does not change, then the change in impulse can only come from a change in Δt. For the most part, the greater speeds are the result of new materials and designs that allow more contact time between the ball and the racquet.

Impulse and Force–Time Graphs

In situations where the force applied to an object varies over time, you can use a force–time graph to estimate the impulse. A force–time graph shows force as a function of time during a collision within a time interval Δt. **Figure 3** shows the force–time graph of a struck tennis ball. The area under the force–time graph is equal to the impulse, because this area represents the product of force and time over the course of the collision.

UNIT TASK BOOKMARK

You can apply what you learn about momentum and impulse to the Unit Task on page 270.

One way to estimate impulse from the force–time graph is to count the number of squares and partial squares under the variable force–time graph, as shown in Figure 3(a). The total will represent the area under the curve, and therefore the impulse due to the applied force. Another way is to consider the average force exerted on the ball over the duration of the collision. A constant average force will produce a horizontal straight-line graph, as shown in Figure 3(b). Note that the rectangular area under this graph is approximately equal to the area under the variable force–time graph found by counting squares. In many situations it is easier to estimate the impulse produced by a variable force by assuming a constant average force acting over the same time interval.

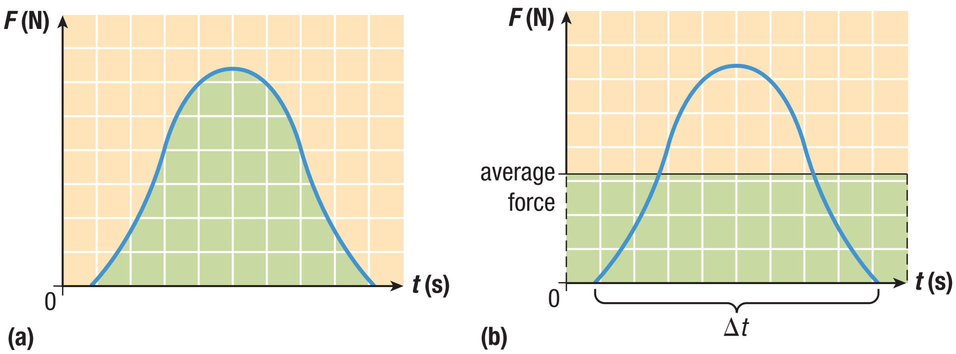


Figure 3 (a) A variable force–time graph and (b) its corresponding constant average force–time graph. The area under the curve is equal to approximately 30 square units, which corresponds to 30 N·s.

In the following Tutorial, you will learn how to calculate impulse and use force–time graphs.

Tutorial 2 Calculating Impulse

The Sample Problems in this Tutorial demonstrate various ways to calculate impulse.

Sample Problem 1: Impulse as a Change in Momentum

A 0.160 kg puck is travelling at 5.0 m/s [N]. A slapshot produces a collision that lasts for 0.0020 s and gives the puck a velocity of 40.0 m/s [S].

(a) Calculate the impulse imparted by the hockey stick.

(b) Determine the average force applied by the stick to the puck.

Solution

(a) **Given:** $m = 0.160$ kg; $\vec{v}_i = 5.0$ m/s [N]; $\vec{v}_f = 40.0$ m/s [S]

Required: $\Delta\vec{p}$

Analysis: $\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$

Solution:
$$\begin{aligned}\Delta\vec{p} &= m(\vec{v}_f - \vec{v}_i)\\ &= 0.160 \text{ kg } [40 \text{ m/s } [\text{S}] - 5 \text{ m/s } [\text{N}]]\\ &= 0.160 \text{ kg } [40 \text{ m/s } [\text{S}] - (-5 \text{ m/s } [\text{S}])]\\ &= 0.160 \text{ kg } (40 \text{ m/s } [\text{S}] + 5 \text{ m/s } [\text{S}])\\ &= 0.160 \text{ kg } (45 \text{ m/s } [\text{S}])\\ &= 7.2 \text{ kg}\cdot\text{m/s } [\text{S}]\\ \Delta\vec{p} &= 7.2 \text{ N}\cdot\text{s } [\text{S}]\end{aligned}$$

Statement: The impulse imparted by the hockey stick is 7.2 N·s [S].

(b) **Given:** $\Delta t = 0.0020$ s; $\Delta\vec{p} = 7.2$ N·s [S]

Required: $\vec{F}$

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

Solution:
$$\begin{aligned}\vec{F} &= \frac{\Delta\vec{p}}{\Delta t}\\ &= \frac{7.2 \text{ N}\cdot\cancel{\text{s}} \; [\text{S}]}{0.0020 \; \cancel{\text{s}}}\\ \vec{F} &= 3600 \text{ N } [\text{S}]\end{aligned}$$

Statement: The average force applied by the stick to the puck is 3.6×10^3 N [S].

Sample Problem 2: Impulse as the Product of Force and Time

A volleyball player starts a serve by throwing the ball vertically upward. The 260 g volleyball comes to rest at its maximum height. The server then hits it and exerts an average horizontal force of magnitude 6.5 N on the ball.

(a) Determine the speed of the ball after the player hits it if the average force is exerted on the ball for 615 ms.

(b) On the next serve, the volleyball player hits the ball with the same amount of horizontal force, but the time interval is 875 ms. Determine the speed of the ball.

Solution

(a) **Given:** $m = 260$ g $= 0.260$ kg; $\vec{F} = 6.5$ N; $\Delta t_a = 615$ ms $= 0.615$ s

Required: $\vec{v}_f$

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$

$$\begin{aligned}\Delta\vec{p} &= m(\vec{v}_f - \vec{v}_i)\\ \vec{F}\Delta t &= m(\vec{v}_f - \vec{v}_i)\\ \vec{v}_f &= \frac{\vec{F}\Delta t}{m} + \vec{v}_i\end{aligned}$$

Solution:
$$\begin{aligned}\vec{v}_f &= \frac{\vec{F}\Delta t}{m} + \vec{v}_i\\ &= \frac{6.5 \; \cancel{\text{kg}}\cdot\dfrac{\text{m}}{\text{s}^{\cancel{2}}}(0.615 \; \cancel{\text{s}})}{0.260 \; \cancel{\text{kg}}} + 0 \text{ m/s}\\ \vec{v}_f &= 15 \text{ m/s}\end{aligned}$$

Statement: The speed of the volleyball when the force is exerted for 615 ms is 15 m/s.

(b) **Given:** $m = 260$ g $= 0.260$ kg; $\vec{F} = 6.5$ N; $\Delta t_b = 875$ ms $= 0.875$ s

Required: $\vec{v}_f$

Analysis: Use the same equation for $\vec{v}_f$ we derived in (a), substituting the different time interval:

$$\vec{v}_f = \frac{\vec{F}\Delta t}{m} + \vec{v}_i$$

Solution:
$$\begin{aligned}\vec{v}_f &= \frac{\vec{F}\Delta t}{m} + \vec{v}_i\\ &= \frac{6.5 \; \cancel{\text{kg}}\cdot\dfrac{\text{m}}{\text{s}^{\cancel{2}}}(0.875 \; \cancel{\text{s}})}{0.260 \; \cancel{\text{kg}}} + 0 \text{ m/s}\\ \vec{v}_f &= 22 \text{ m/s}\end{aligned}$$

Statement: The speed of the volleyball when the force is exerted for 875 ms is 22 m/s. (Note: By following through on the serve, the player increases the time interval of the applied force, resulting in a faster serve.)

Sample Problem 3: Impulse as Area under a Force–Time Curve

Two figure skaters approach each other in a straight line. They meet hand to hand and then push off in opposite directions. The increase and decrease of force are both linear, which produces a force–time curve that is in the shape of a triangle. The force–time curve for this interaction is shown in **Figure 4**. Determine the impulse for this interaction.

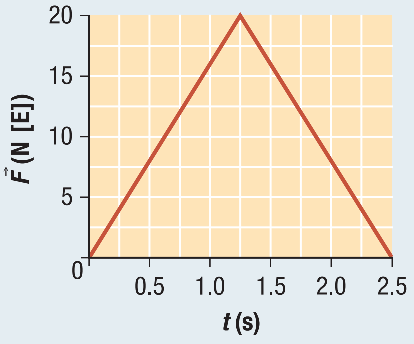


Figure 4

Given: Force–time graph of the skaters' interaction

Required: $\vec{F}\Delta t$

Analysis: Determine the impulse by calculating the area under the force–time curve of the collision. Use the equation for the area of a triangle: $A = \frac{1}{2}bh$; $A = \vec{F}\Delta t$.

Solution:
$$\begin{aligned}\vec{F}\Delta t &= \frac{1}{2}bh \\ &= \frac{1}{2}(2.5\text{ s})(20.0\text{ N}) \\ \vec{F}\Delta t &= 25\text{ N}\cdot\text{s}\end{aligned}$$

Statement: The impulse of the interaction of the two skaters is 25 N·s away from each other.

Practice

1. A hockey player passes a puck with an average force of 250 N. The hockey stick is in contact with the puck for 0.0030 s, and the mass of the puck is 180 g. The puck is not moving before the player passes it. T/I A
   (a) Determine the impulse imparted by the hockey stick. [ans: 0.75 kg·m/s [forward]]
   (b) Calculate the velocity of the puck as a result of this collision. [ans: 4.2 m/s [forward]]
2. A hockey player collides with a wall, and then pushes away from it. The collision occurs over 2.9 s and the average force applied by the player in the collision is 468 N. Draw a force–time graph similar to the one in Figure 3(b) and use it to determine the impulse of the collision. T/I C [ans: 1400 N·s [away from the wall]]

As you have seen, the concepts of linear momentum and impulse are relevant to an understanding of motion in sports and in the design of improved sports gear. You can also apply the concepts of momentum and impulse in many other areas, ranging from the analysis of motor vehicle collisions to the motion of rockets. You will learn more about applications of momentum and impulse later in the chapter. CAREER LINK

5.1 Review

Summary

- Linear momentum is the product of an object's mass and its velocity, expressed in units of kilograms times metres per second (kg·m/s): $\vec{p} = m\vec{v}$.
- Impulse is the change in momentum caused by the application of a force over a time interval, expressed in units of newton seconds (N·s): $\vec{F}\Delta t = \Delta\vec{p}$.
- The magnitude of an impulse can be found by measuring the area under a force–time curve.

Questions

1. Calculate the momentum of each of the following: K/U
   (a) a male moose of mass 4.25×10^2 kg running at 6.9 m/s [N]
   (b) a city bus of mass 9.97×10^3 kg moving at 5 km/h [forward]
   (c) a flying squirrel of mass 995 g gliding at 16 m/s [S]
2. In your own words, describe what impulse is. K/U C
3. A bicycle and rider have a combined mass of 79.3 kg and a momentum of 2.16×10^3 kg·m/s [W]. Determine the velocity of the bicycle. K/U
4. A projectile travelling at 9.0×10^2 m/s [W] has a momentum of 4.5 kg·m/s [W]. What is the mass of the projectile? K/U
5. A downhill skier travelling at a constant velocity of 29.5 m/s [forward] has a momentum of 2.31×10^3 kg·m/s [forward]. Determine the mass of the skier. T/I
6. Explain how increasing the time interval over which a force is applied can affect performance in sports. Use a sport not discussed in this section in your answer. K/U A
7. A teacher drops a tennis ball and a basketball from the same height onto the floor. The force from the floor produces an impulse on each ball. If the basketball is heavier than the tennis ball, which impulse is larger? Explain your answer. T/I C A
8. A hockey player passes a puck that is initially at rest. The force exerted by the stick on the puck is 1100.0 N [forward], and the stick is in contact with the puck for 5.0 ms. T/I
   (a) Determine the impulse imparted by the stick to the puck.
   (b) If the puck has a mass of 0.12 kg, calculate the speed of the puck just after it leaves the hockey stick.
9. You accidentally drop a cellphone, which has a mass of 225 g, from a height of 74 cm. T/I A
   (a) Calculate the cellphone's momentum at the moment of impact with the sidewalk.
   (b) If the cellphone lands on a grassy lawn, is its momentum less, the same, or greater? Explain your answer.
10. A rubber ball with a mass of 0.25 kg is dropped from a height of 1.5 m onto the floor. Just after bouncing from the floor, the ball has a velocity of 4.0 m/s [up]. T/I
    (a) Determine the impulse imparted by the floor to the ball.
    (b) If the average force of the floor on the ball is 18 N [up], for how long is the ball in contact with the floor?
11. An archer shoots an arrow with a mass of 0.030 kg. The arrow leaves the bow with a horizontal velocity of 88 m/s. T/I
    (a) Determine the impulse imparted to the arrow.
    (b) If the arrow is in contact with the bowstring for 0.015 s after the archer releases, what is the approximate average force of the bowstring on the arrow?
12. A tennis player hits a serve at a speed of 63 m/s [W], and the opponent returns the 0.057 kg tennis ball to the server with a speed of 41 m/s [E]. T/I
    (a) Calculate the magnitude of the impulse imparted to the ball by the opponent.
    (b) Calculate the approximate average force on the ball if the opponent's racquet is in contact with the ball for 0.023 s.

5.2 Conservation of Momentum in One Dimension

Figure 1 Curling requires a firm understanding of momentum and impulse to control the movement of the stones.

Success in the sport of curling relies on momentum and impulse. A player must accelerate a curling stone to a precise velocity to collide with an opponent's stone so that both end up in the desired location (**Figure 1**). The same is true in billiards. You must not only control where the target ball will go, but also where the cue ball will travel. Likewise, a football player receiving a pass, a tennis player delivering a serve, and a lacrosse player attempting a pass must all be in control of momentum and impulse.

In this section, you will examine what happens when two bodies interact such that momentum is exchanged. You will also examine a new aspect of momentum by considering what happens when an impulse is generated from within a single object, such as a rocket taking off, to create the motion of two independent masses, each with its own momentum.

The Law of Conservation of Momentum

collision the impact of one body with another

You may recall that the concept of a system plays an important role in discussions of energy and the law of conservation of energy. You can also apply ideas about momentum and impulse to the motion of a system of objects by examining a **collision**, where two or more objects come together. When two objects collide, they create an associated collision force. In the case of two hockey pucks or two billiard balls, this collision force is a normal (contact) force.

Consider a system of two colliding objects, as shown in **Figure 2**. WEB LINK

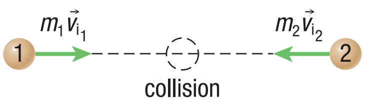


Figure 2 When two objects collide, the total momentum just after the collision is equal to the total momentum just before the collision.

Let $\vec{F}_{21}$ be the force exerted by object 2 on object 1 and $\vec{F}_{12}$ be the force exerted by object 1 on object 2. These forces are an action–reaction pair, so according to Newton's third law, they must be equal in magnitude and opposite in direction: $\vec{F}_{21} = -\vec{F}_{12}$. If this collision takes place over a time interval Δt, the impulse of object 1 is $\vec{F}_{21}\Delta t$. Since the impulse equals the change in momentum of object 1,

$$\vec{F}_{21}\Delta t = \Delta\vec{p}_1$$

and

$$\Delta\vec{p}_1 = \vec{p}_{f_1} - \vec{p}_{i_1}$$

Substituting gives

$$\vec{F}_{21}\Delta t = \vec{p}_{f_1} - \vec{p}_{i_1}$$

Here, $\vec{p}_{i_1}$ is the initial momentum of object 1 just before the collision and $\vec{p}_{f_1}$ is its final momentum just after the collision. Likewise, the impulse imparted to object 2 is $\vec{F}_{12}\Delta t$, which equals the change in the momentum of object 2:

$$\vec{F}_{12}\Delta t = \Delta\vec{p}_2$$
$$\Delta\vec{p}_2 = \vec{p}_{f_2} - \vec{p}_{i_2}$$
$$\vec{F}_{12}\Delta t = \vec{p}_{f_2} - \vec{p}_{i_2}$$

Since $\vec{F}_{21} = -\vec{F}_{12}$ and the interaction times Δt are the same, the impulse imparted to object 1 is equal in magnitude but opposite in sign to the impulse imparted to object 2:

$$\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$$
$$m_1\vec{a}_1 = -m_2\vec{a}_2$$
$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$
$$m_1(\vec{v}_{f_1} - \vec{v}_{i_1}) = -m_2(\vec{v}_{f_2} - \vec{v}_{i_2})$$
$$m_1\vec{v}_{f_1} - m_1\vec{v}_{i_1} = -m_2\vec{v}_{f_2} + m_2\vec{v}_{i_2}$$

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

This equation summarizes the **law of conservation of momentum** for two colliding objects:

Law of Conservation of Momentum

When two objects collide in an isolated system, the collision does not change the *total* momentum of the two objects. Whatever momentum is lost by one object in the collision is gained by the other. The total momentum of the system is conserved.

The law of conservation of momentum applies not only to isolated systems of two objects, but also to complex systems. For example, you can easily see how the momentum of the cue ball in a billiards game is transferred to the other balls in the system during the opening break. Momentum is always conserved, whether the colliding objects bounce off one another, as with billiard balls, or remain together, as in the case of a football player who catches a pass.

Investigation 5.2.1

Conservation of Momentum in One Dimension (page 258)
Now that you understand conservation of momentum, perform Investigation 5.2.1 to see how a collision between two objects in one dimension affects the momentum of each object.

Interactions within a System

You have explored the momentum of an object as the product of its mass, m, and its velocity, $\vec{v}$. You have also examined how the application of a force over a specified period of time, an impulse, can change the momentum of an object. You have seen how to derive the equation describing the conservation of motion in an isolated system. In the remainder of this section, you will read more about the concepts of momentum and impulse within an isolated system. Two general categories of interactions exist within a system: collisions and explosions.

COLLISIONS

Momentum is conserved in a system when two or more objects come together in a collision. Examples of a collision are a cue ball hitting another billiard ball, a car rear-ending another car, and one lacrosse player delivering a body check to another.

EXPLOSIONS

Momentum is also conserved in systems when an object or a collection of objects breaks apart in an **explosion**. Fireworks provide vivid images of explosions that give us a feel for the masses and velocities of the many individual objects involved. Accounting for the masses and velocities of all the elements is a challenging task.

explosion a situation in which a single object or group of objects breaks apart

Other examples of explosions may be less obvious. A force used to send an arrow flying affects the momentum of the arrow and the momentum of the bow and archer. Similarly, a squid gains momentum by ejecting water that possesses its own momentum. The same principle is used to put spacecraft into orbit. The following Tutorial illustrates how the law of conservation of momentum can be used to predict the outcome of a collision or explosion. CAREER LINK

Tutorial 1 Applying the Law of Conservation of Momentum

The Sample Problems in this Tutorial apply the law of conservation of momentum to problems involving collisions or explosions in one dimension.

Sample Problem 1: Collision Analysis

A hockey player of mass 97 kg skating with a velocity of 9.2 m/s [S] collides head-on with a defence player of mass 105 kg travelling with a velocity of 6.5 m/s [N]. An instant after impact, the two skate together in the same direction. Calculate the final velocity of the two hockey players.

Given: $m_1 = 97$ kg; $\vec{v}_1 = 9.2$ m/s [S]; $m_2 = 105$ kg; $\vec{v}_2 = 6.5$ m/s [N]

Required: $\vec{v}_f$

Analysis: The players stick together after the collision, so they can be treated as a single object having mass $m_1 + m_2$, velocity v_f, and momentum p_f.

$$\vec{p}_f = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{\vec{p}_f}{(m_1 + m_2)}$$

By conservation of momentum,

$$\vec{p}_f = \vec{p}_1 + \vec{p}_2$$

First determine the total momentum before the collision and then use this result to calculate the velocity of the two players after the collision.

Solution:
$$\begin{aligned}\vec{p}_f &= \vec{p}_1 + \vec{p}_2\\ &= (97\text{ kg})(9.2\text{ m/s [S]}) + (105\text{ kg})(6.5\text{ m/s [N]})\\ &= (97\text{ kg})(9.2\text{ m/s [S]}) - (105\text{ kg})(6.5\text{ m/s [S]})\\ &= 892\text{ kg}\cdot\text{m/s [S]} - 682\text{ kg}\cdot\text{m/s [S]}\\ \vec{p}_f &= 210\text{ kg}\cdot\text{m/s [S]}\\ \vec{v}_f &= \frac{\vec{p}_f}{(m_1 + m_2)}\\ &= \frac{210\ \cancel{\text{kg}}\cdot\text{m/s [S]}}{(202\ \cancel{\text{kg}})}\\ \vec{v}_f &= 1.0\text{ m/s [S]}\end{aligned}$$

Statement: After the collision, the two skaters will be travelling at 1.0 m/s [S].

Sample Problem 2: Explosion Analysis

In a science fiction novel, a large asteroid is approaching Earth. Scientists decide to use explosive devices to blow the asteroid into two equal halves before impact with Earth. The asteroid has a mass of 2.4×10^9 kg. For each half to safely miss Earth, an explosion must cause each to travel a minimum of 8.0×10^6 m at a right angle away from Earth within 24 h (**Figure 3**). Assume this is a one-dimensional problem. The magnitude of the impulse applied to each fragment is the same, but the halves are directed in opposite directions.

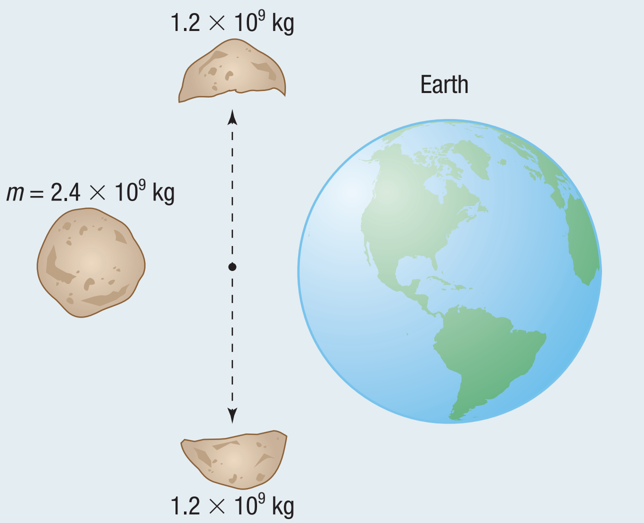


Figure 3

(a) Calculate the momentum of each part of the asteroid after the explosion.

(b) Determine the impulse delivered to each part by the explosion.

Solution

(a) **Given:** $m = 2.4 \times 10^9$ kg; $v = 8.0 \times 10^6$ m/24 h

Required: $\vec{p}$

Analysis: $\vec{p} = m\vec{v}$. Since the total mass of the asteroid is 2.4×10^9 kg, the mass of each half is 1.2×10^9 kg. Use this mass to calculate the momentum. First, convert the speed to metres per second.

$$\frac{8.0 \times 10^6\text{ m}}{24\text{ h}} = \frac{8.0 \times 10^6\text{ m}}{24\ \cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}}$$
$$= 92.6\text{ m/s (one extra digit carried)}$$

Solution:
$$\begin{aligned}\vec{p} &= m\vec{v}\\ &= (1.2 \times 10^9\text{ kg})(92.6\text{ m/s})\\ \vec{p} &= 1.111 \times 10^{11}\text{ kg}\cdot\text{m/s (two extra digits carried)}\end{aligned}$$

Statement: The momentum of each part of the asteroid is 1.1×10^{11} kg·m/s.

(b) **Given:** $m = 2.4 \times 10^9$ kg; $v = 8.0 \times 10^6$ m/24 h = 92.6 m/s; $\vec{p} = 1.111 \times 10^{11}$ kg·m/s

Required: $\Delta\vec{p}$

Analysis: The original momentum of each half of the asteroid is zero. So the change in momentum for each half is

$$\Delta\vec{p} = 1.1 \times 10^{11} \text{ kg·m/s}$$

Solution: $\Delta\vec{p} = 1.1 \times 10^{11}$ kg·m/s

$$\Delta\vec{p} = 1.1 \times 10^{11} \text{ N·s}$$

The impulse for the other fragment is 1.1×10^{11} N·s in the opposite direction.

Statement: The explosion will deliver an impulse of magnitude 1.1×10^{11} N·s to each fragment in the directions shown in Figure 3.

Practice

1. A 1350 kg car travelling at 72 km/h [S] collides with a slow-moving car of mass 1650 kg, also initially travelling south. After the collision, the velocity of the two cars together is 24 km/h [S]. Determine the initial velocity at which the second car was travelling. T/I [ans: 15 km/h [N]]
2. After shooting a 28 g arrow with an initial velocity of 92 m/s [forward], an archer standing on a frictionless surface travels in the opposite direction at a speed of 0.039 m/s. Calculate the combined mass of the archer and the bow. T/I A [ans: 6.6×10^1 kg]

You can demonstrate conservation of momentum experimentally by creating systems that minimize the influence of external forces, such as friction. For example, air pucks interacting on a cushion of air near motion sensors provide a means for carefully measuring the initial and final velocities of two objects after a collision.

Keep in mind that while momentum is always conserved in an isolated system, real-life systems are subject to many outside influences, such as friction and other complicated forces, which can make detecting conservation of momentum difficult. For example, if you jump on Earth, it does move, even if you cannot sense the movement. Momentum is conserved when you jump, but because of the huge mass of Earth, the change in its velocity is quite small.

UNIT TASK BOOKMARK

You can apply what you have learned about conservation of momentum to the Unit Task on page 270.

Rocket Propulsion

Momentum is conserved when a rocket engine burns fuel and expels a continuous stream of gases at an extremely high velocity (**Figure 4**). In this explosion, the expanding gases act against the rocket, propelling the rocket forward. In the vacuum of deep space, where gravity is negligible, it is possible to achieve a nearly perfectly isolated system, free from the influences of friction and gravity, providing ideal conditions for the study of momentum. The study of rocket propulsion is complicated, however, because the mass of the rocket changes continuously as the rocket burns fuel. For this reason, a detailed study of rocket propulsion requires knowledge of calculus, so we will not discuss it here. However, it may be something you wish to explore if you have already studied calculus or plan to do so in the future. CAREER LINK

Figure 4 The momentum of combusted gases ejected from the rocket is balanced by the forward momentum of the rocket.

5.2 Review

Summary

- For any interaction involving a system that experiences no external forces, the total momentum before the interaction is equal to the total momentum after the interaction.
- During an interaction between two objects in a system that experiences no external forces, the change in momentum of one object is equal in magnitude but opposite in direction to the change in momentum of the other object: $m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$.
- Interactions within a system can be categorized as collisions, in which two or more objects come together, or explosions, in which a single object or collection of objects separates.

Questions

1. Identify the conditions required for the total momentum of a system to be conserved. K/U
2. A 55 kg student stands on a 4.6 kg surfboard moving at 2.0 m/s [E]. The student then walks with a velocity of 1.9 m/s [E] relative to the surfboard. Determine the resulting velocity of the surfboard, relative to the water. Neglect friction. T/I
3. Two stationary hockey players push each other so that they move in opposite directions. One player has a mass of 35.6 kg and a speed of 2.42 m/s. What is the mass of the other player if her speed is 3.25 m/s? Neglect friction. T/I
4. A baseball pitcher with a mass of 80 kg is initially standing at rest on extremely slippery artificial turf. He then throws a baseball with a mass of 0.14 kg with a horizontal velocity of 50 m/s. Determine the recoil velocity of the pitcher. T/I
5. Consider a collision in one dimension that involves two objects of masses 4.5 kg and 6.2 kg. The larger mass is initially at rest, and the smaller mass has an initial velocity of 16 m/s [E]. The final velocity of the larger object is 10.0 m/s [E]. Calculate the final velocity of the smaller object after the collision. T/I
6. Two objects of masses m and $3m$ undergo a collision in one dimension. The lighter object is moving at three times the speed of the heavier object. Describe what happens to their speeds after the collision. Explain your reasoning. Assume that the lighter mass is moving to the right. K/U T/I C A
7. An object of mass $m_1 = 2.5$ kg has a one-dimensional collision with another object of mass $m_2 = 7.5$ kg, as shown in **Figure 5**. Their initial speeds along x are $v_1 = +6.0$ m/s and $v_2 = -15$ m/s. The two objects stick together after the collision. Calculate the velocity after the collision. T/I

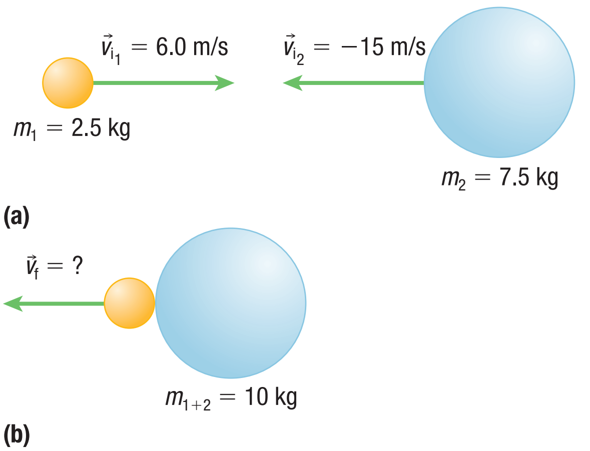


Figure 5

8. An astronaut on a spacewalk outside the International Space Station (ISS) has a safety equipment failure that leaves her floating in space just out of reach of the station airlock. Fortunately, she is still holding a tool bag. Explain how she can use the tool bag and conservation of momentum to return safely to the ISS. K/U T/I A

Collisions

If you have ever been to an amusement park, chances are you have ridden in the bumper cars, where the objective is for you and your friends to crash into one another (**Figure 1**). In Section 5.2, you learned about two types of interactions within a system—collisions and explosions. In this section, you will learn more about different types of collisions, as well as what happens to the energy of systems when a collision occurs. How can you use what you have learned about linear momentum and kinetic energy to understand what happens when two bumper cars collide and bounce apart, and what happens when they collide and stay together?

Figure 1 Momentum and kinetic energy can help explain what happens to the directions and speeds of objects when they collide with one another.

Elastic and Inelastic Collisions

In Section 5.2, you learned how to analyze a system of two objects. The total momentum of the objects before a collision is equal to the total momentum of the objects after the collision. In this section, you will apply the law of conservation of momentum to analyze several different types of collisions. In general, a collision changes the velocities of the objects involved. The final velocities (those found just after the collision) are different from the initial velocities (from just before the collision). Since the kinetic energy of an object depends on its speed, the kinetic energy of the object also changes as a result of the collision. Collisions fall into two general types, depending on what happens to the total kinetic energy of the entire system: elastic and inelastic collisions.

Elastic Collisions

In an **elastic collision**, the system's kinetic energy is conserved. That is, the total kinetic energy of the two objects after the collision is equal to the total kinetic energy of the two objects before the collision. This is called **conservation of kinetic energy**. The term *elastic* can help you understand how and why a collision affects the kinetic energy. For example, an extremely elastic ball (such as a rubber ball) is compressed during a collision, and this compression stores energy in the ball just as energy is stored in a compressed spring. In an ideal rubber ball, all of this potential energy is turned back into kinetic energy when the ball decompresses (springs back) at the end of the collision.

elastic collision a collision in which momentum and kinetic energy are conserved

conservation of kinetic energy the total kinetic energy of two objects before a collision is equal to the total kinetic energy of the two objects after the collision

Figure 2 shows two rubber balls before and after an elastic collision.

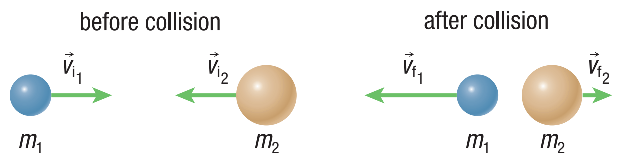


Figure 2 In elastic collisions, both momentum and kinetic energy are conserved.

Inelastic Collisions

inelastic collision a collision in which momentum is conserved, but kinetic energy is not conserved

In contrast to elastic collisions, **inelastic collisions** are collisions in which some kinetic energy is lost. The kinetic energy is transformed into other forms, such as thermal energy or sound energy. Consider a collision involving a ball composed of soft putty or clay. The ball will not spring back at the end of the collision. Energy is absorbed, causing the kinetic energy after the collision to be less than the kinetic energy before the collision. The collision, therefore, is inelastic.

In summary,

- In an elastic collision, both momentum and kinetic energy are conserved.
- In an inelastic collision, momentum is conserved, but kinetic energy is not conserved.

Note that in both types of collisions, momentum is conserved. The total energy is also conserved in both cases, even if the kinetic energy is not. WEB LINK

Mini Investigation

Newton's Cradle

Skills: Performing, Observing, Analyzing, Communicating — SKILLS HANDBOOK A2.1

In this investigation, you will explore the concept of conservation of momentum in collisions using a Newton's cradle (**Figure 3**).

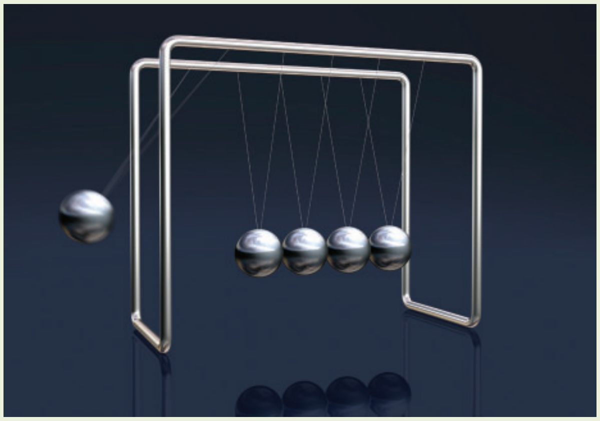


Figure 3

Equipment and Materials: Newton's cradle

1. Set up the Newton's cradle. Make sure all of the metal spheres are correctly aligned.
2. Pull back one of the spheres at the end and release. Observe how momentum is transferred from one end of the cradle to the other.
3. Explore what happens when you change the initial setup. For example, try changing the number of active spheres by holding up one of the end spheres during the investigation. You can also try moving one of the middle spheres out of the way of the collisions.

A. What happened during Step 2? How did your results change when you modified the setup in Step 3? T/I A

B. Do the collisions appear to conserve momentum? Explain your answer. K/U C

C. Do the collisions appear to conserve kinetic energy? Explain your answer. K/U C

D. Does the device as a whole appear to conserve mechanical energy? If not, identify some reasons for the energy loss. K/U T/I C A

Perfectly Elastic Collisions and Perfectly Inelastic Collisions

A **perfectly elastic collision** is an idealized situation where friction and other external forces are negligible, and therefore momentum and kinetic energy are perfectly conserved. On the other hand, a **perfectly inelastic collision** is one in which the two objects in a collision stick together after the collision so that the objects have the same final velocity. Perfectly elastic and perfectly inelastic collisions occur in isolated systems in which the effects of friction and other external forces are negligible. Perfectly elastic and perfectly inelastic collisions are extremely rare in the world around us, and represent idealized cases. Most real collisions fall somewhere between these two extreme situations. However, it is useful to consider perfectly elastic and perfectly inelastic collisions as ideal examples of Newton's laws. As you do this, be mindful of external forces that may additionally affect the systems.

perfectly elastic collision an ideal collision in which external forces are minimized to the point where momentum and kinetic energy are perfectly conserved

perfectly inelastic collision an ideal collision in which two objects stick together perfectly so they have the same final velocity; in this situation, momentum is perfectly conserved, but kinetic energy is not conserved

Perfectly Elastic Collisions

By applying some basic assumptions, you can use collisions with billiard balls, bumper cars, and asteroid–planet systems as reasonable examples of perfectly elastic collisions.

In perfectly elastic collisions, both momentum and kinetic energy are conserved:

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

In one-dimensional collisions, each vector can point in only one of two ways. Designate directions in a way that is convenient for solving a particular problem. For example, you may choose to assign right as positive and left as negative. Then you can express the vectors using only their magnitudes, understanding that a negative value implies a left direction. In Tutorial 1, you will use these equations to explore perfectly elastic collisions in one dimension.

Tutorial 1 Perfectly Elastic Collisions in One Dimension

In the following Sample Problem, you will use conservation of momentum and kinetic energy to analyze a perfectly elastic collision.

Sample Problem 1: Analyzing Perfectly Elastic Collisions

Suppose you have two balls with different masses involved in a perfectly elastic collision. Ball 1, with mass $m_1 = 0.26$ kg travelling at a velocity $v_1 = 1.3$ m/s [right], collides head-on with stationary ball 2, which has a mass of $m_2 = 0.15$ kg. Determine the final velocities of both balls after the collision.

Given: $m_1 = 0.26$ kg; $\vec{v}_{i_1} = 1.3$ m/s [right]; $m_2 = 0.15$ kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$, $\vec{v}_{f_2}$

Analysis: The collision is perfectly elastic, which means that kinetic energy is conserved. Apply conservation of momentum and conservation of kinetic energy to construct and solve a linear-quadratic system of two equations in two unknowns. First use the conservation of momentum equation to express the final velocity of ball 1 in terms of the final velocity of ball 2. Then substitute the resulting equation into the conservation of kinetic energy equation to solve for the final velocity of ball 2. Use the result to solve for the final velocity of ball 1. This is a one-dimensional problem, so omit the vector notation for velocities, recognizing that positive values indicate motion to the right and negative values indicate motion to the left.

Solution: First use conservation of momentum to solve for $\vec{v}_{f_1}$ in terms of $\vec{v}_{f_2}$:

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$$

Isolate the term containing $\vec{v}_{f_1}$ on the left side of the equation. Since $v_{i_2} = 0$, the equation becomes

$$m_1v_{f_1} = m_1v_{i_1} - m_2v_{f_2}$$

Divide both sides by m_1:

$$v_{f_1} = v_{i_1} - \frac{m_2}{m_1}v_{f_2}$$
$$= 1.3 \text{ m/s} - \frac{0.15 \text{ kg}}{0.26 \text{ kg}}v_{f_2}$$
$$v_{f_1} = 1.3 \text{ m/s} - 0.58v_{f_2} \qquad \text{(Equation 1)}$$

The conservation of kinetic energy equation can be simplified by multiplying both sides of the equation by 2 and noting that $v_{i_2} = 0$:

$$\cancel{2}\left(\frac{1}{\cancel{2}}m_1 v_{i_1}^2 + \frac{1}{\cancel{2}}m_2 v_{i_2}^2\right) = \cancel{2}\left(\frac{1}{\cancel{2}}m_1 v_{f_1}^2 + \frac{1}{\cancel{2}}m_2 v_{f_2}^2\right)$$

$$m_1 v_{i_1}^2 = m_1 v_{f_1}^2 + m_2 v_{f_2}^2 \qquad \text{(Equation 2)}$$

Substitute Equation 1 and the given values into Equation 2:

$$(0.26 \text{ kg})(1.3 \text{ m/s})^2 = (0.26 \text{ kg})(1.3 \text{ m/s} - 0.58v_{f_2})^2 + (0.15 \text{ kg})v_{f_2}^2$$

Expand and simplify both sides of the equation:

$$0.439 \text{ kg}\cdot\text{m}^2/\text{s}^2 = (0.26 \text{ kg})(1.69 \text{ m}^2/\text{s}^2 - 1.51 \text{ m/s } v_{f_2} + 0.34v_{f_2}^2) + (0.15 \text{ kg})v_{f_2}^2$$
$$0.439 \text{ kg}\cdot\text{m}^2/\text{s}^2 = 0.439 \text{ kg}\cdot\text{m}^2/\text{s}^2 - 0.39 \text{ kg}\cdot\text{m/s } v_{f_2} + 0.088 \text{ kg } v_{f_2}^2 + (0.15 \text{ kg})v_{f_2}^2$$
$$0.439 \cancel{\text{kg}}\cdot\text{m}^2/\text{s}^2 = 0.439 \cancel{\text{kg}}\cdot\text{m}^2/\text{s}^2 - 0.39 \cancel{\text{kg}}\cdot\text{m/s } v_{f_2} + 0.24 \cancel{\text{kg}} \, v_{f_2}^2$$
$$0 = -0.39 \text{ m/s } v_{f_2} + 0.24v_{f_2}^2$$

Express the quadratic equation in standard form:

$$0 = -0.39 \text{ m/s } v_{f_2} + 0.24v_{f_2}^2$$

Solve by common factoring:

$$0 = (-0.39 \text{ m/s} + 0.24v_{f_2})v_{f_2}$$

The factor of v_{f_2} means the equation has a solution $v_{f_2} = 0$ m/s. This solution describes the system before the collision. The equation has a second solution describing the system after the collision:

$$0.24v_{f_2} - 0.39 \text{ m/s} = 0$$
$$v_{f_2} = \frac{0.39 \text{ m/s}}{0.24}$$
$$v_{f_2} = 1.63 \text{ m/s (one extra digit carried)}$$

Substitute this value into Equation 1 and calculate the final velocity of ball 1:

$$v_{f_1} = 1.3 \text{ m/s} - 0.58v_{f_2}$$
$$= 1.3 \text{ m/s} - (0.58)(1.63 \text{ m/s})$$
$$v_{f_1} = 0.35 \text{ m/s}$$

Statement: The second ball has a final velocity of 1.6 m/s [right] and the first ball has a final velocity of 0.35 m/s [right].

Practice

1. Two balls collide in a perfectly elastic collision. Ball 1 has mass 3.5 kg and is initially travelling at a velocity of 5.4 m/s [right]. It collides head-on with stationary ball 2 with mass 4.8 kg. Determine the final velocity of ball 2. T/I [ans: 4.6 m/s [right]]
2. A curling stone with initial speed v_1 collides head-on with a second, stationary stone of identical mass m. Calculate the final speeds of the two curling stones. K/U T/I A [ans: $v_{f_1} = 0$; $v_{f_2} = v_1$]

The analysis and solution used above apply to the special case of a perfectly elastic collision where one body is initially at rest. A more general case of perfectly elastic collisions involves two bodies that are already in motion before the collision. Collisions of this type will be explored in greater depth later in this chapter.

Perfectly Inelastic Collisions

The simplest type of inelastic collision is a perfectly inelastic collision, in which two objects stick together after the collision so that the objects have the same final velocity. If the colliding objects bounce, it is not a perfectly inelastic collision. A good example of a perfectly inelastic collision is one in which two cars lock bumpers. **Figure 4** on the next page shows two cars coasting on a straight road in one dimension with velocities $\vec{v}_{i_1}$ and $\vec{v}_{i_2}$. The cars collide and lock bumpers, and they have the same velocity $\vec{v}_f$ after the collision. The cars are moving in a horizontal direction, x, and if they are coasting, there are no external forces on the cars in this direction. Therefore, the total momentum along x is conserved.

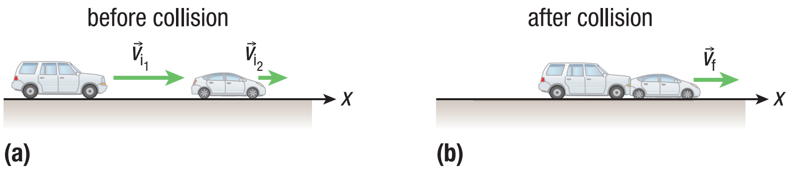


Figure 4 In an inelastic collision, only momentum is conserved. (a) Velocities just before a one-dimensional collision. (b) After the collision the cars travel with $\vec{v}_f$.

The condition for conservation of momentum along *x* is

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

In this case, there is only one unknown, the final velocity $\vec{v}_f$, since the two velocities are the same when the two objects stick together. Solving the above equation for $\vec{v}_f$ gives

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$$

This equation relates the final velocity of two objects in a perfectly inelastic collision to their masses and their initial velocities. You will apply this equation in Tutorial 2.

UNIT TASK BOOKMARK

You can apply what you learn about collisions to the Unit Task on page 270.

Tutorial 2 Perfectly Inelastic Collisions in One Dimension

In these Sample Problems, you will calculate the final velocities of two vehicles after a one-dimensional perfectly inelastic collision.

Sample Problem 1: Applying the Conservation of Momentum to a One-Dimensional Perfectly Inelastic Collision

Two cars, a sports utility vehicle (SUV) of mass 2500 kg and a compact model of mass 1200 kg, are coasting at a constant velocity along a straight road. Their initial velocities are 40.0 m/s [W] and 10.0 m/s [W], respectively, so the SUV is catching up to the compact car. When they collide, the cars lock bumpers. Assume no external forces exist along the direction of travel, and the total momentum along *x* is conserved. Determine the velocity of the cars just after the collision.

Given: $m_1 = 2500$ kg; $\vec{v}_{i_1} = 40.0$ m/s [W]; $m_2 = 1200$ kg; $\vec{v}_{i_2} = 10.0$ m/s [W]

Required: $\vec{v}_{f_1}$

Analysis: The cars stick together after the collision, so this is an example of a perfectly inelastic collision. In this system, both vehicles are moving in the same direction (west).

Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$ to calculate the final velocity.

Solution:

$$\vec{v}_f = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$$

$$= \frac{(2500\ \text{kg})(40.0\ \text{m/s [W]}) + (1200\ \text{kg})(10.0\ \text{m/s [W]})}{2500\ \text{kg} + 1200\ \text{kg}}$$

$$= \frac{(100\,000\ \cancel{\text{kg}}\cdot\text{m/s [W]}) + (12\,000\ \cancel{\text{kg}}\cdot\text{m/s [W]})}{3700\ \cancel{\text{kg}}}$$

$$\vec{v}_f = 3.0 \times 10^1\ \text{m/s [W]}$$

Statement: The final velocity of the cars is 3.0×10^1 m/s [W].

Sample Problem 2: Applying Conservation of Momentum and Energy in a One-Dimensional Perfectly Inelastic Collision

A child with a mass of 22 kg runs at a horizontal velocity of 4.2 m/s [forward] and jumps onto a stationary rope swing of mass 2.6 kg. The child "sticks" on the rope swing and swings forward.

(a) Determine the horizontal velocity of the child plus the swing just after impact.

(b) How high do the child and swing rise?

Solution

(a) **Given:** Let m_1 and v_{i_1} represent the mass and initial velocity of the child, and m_2 and v_{i_2} represent the mass and initial velocity of the swing. $m_1 = 22$ kg; $\vec{v}_{i_1} = 4.2$ m/s [forward]; $m_2 = 2.6$ kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_f$

Analysis: Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$ to determine the final velocity just after the child collides with the swing.

Solution:

$$\begin{aligned}
\vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2} \\
&= \frac{(22\text{ kg})(4.2\text{ m/s}) + (2.6\text{ kg})(0\text{ m/s})}{22\text{ kg} + 2.6\text{ kg}} \\
&= \frac{(92.4\ \cancel{\text{kg}}\cdot\text{m/s}) + (0\ \cancel{\text{kg}}\cdot\text{m/s})}{24.6\ \cancel{\text{kg}}} \\
\vec{v}_f &= 3.76\text{ m/s [forward] (one extra digit carried)}
\end{aligned}$$

Statement: The child and rope swing have a final velocity of 3.8 m/s [forward] just after the collision.

(b) **Given:** $m_1 = 22$ kg; $m_2 = 2.6$ kg; $\vec{v}_f = 3.76$ m/s

Required: Δy

Analysis: Conservation of energy requires that the initial kinetic energy of the child and swing transform to gravitational potential energy at the highest point of the swing: $E_k = mg\Delta y$.

Solution:

$$\begin{aligned}
E_k &= mg\Delta y \\
\frac{1}{2}\cancel{(m_1 + m_2)}(\vec{v}_f)^2 &= \cancel{(m_1 + m_2)}g\Delta y \\
\Delta y &= \frac{(\vec{v}_f)^2}{2g} \\
&= \frac{(3.76\text{ m/s})^2}{2(9.8\text{ m/s}^2)} \\
&= \frac{(3.76)^2\text{ m}^{\cancel{2}}/\cancel{\text{s}^2}}{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}^2})} \\
\Delta y &= 0.72\text{ m}
\end{aligned}$$

Statement: The child and rope swing will rise to 0.72 m above the initial height.

Practice

1. A child rolls a 4.0 kg ball with a speed of 6.0 m/s toward a 2.0 kg ball that is stationary. The two balls stick together after the collision. What is their velocity immediately after the perfectly inelastic collision? K/U A [ans: 4.0 m/s [forward]]
2. In a scene in an action film, a car with a mass of 2200 kg, travelling at 60.0 km/h [E], collides with a car of mass 1300 kg that is travelling at 30.0 km/h [E], and the two cars lock bumpers. T/I A
   (a) Calculate the velocity of the vehicles immediately after the perfectly inelastic collision. [ans: 14 m/s [E]]
   (b) Calculate the total momentum of the two cars before and after the collision. [ans: 4.8×10^4 kg·m/s]
   (c) Determine the decrease in kinetic energy during the collision. [ans: 2.8×10^4 J]
3. A 66 kg snowboarder slides down a hill 25 m high and has a perfectly inelastic collision with an initially stationary 72 kg skier at the bottom of the hill. Friction is negligible. Calculate the speed of each person after the collision. K/U A [ans: 11 m/s]

5.3 Review

Summary

- In an isolated system, the total momentum of the system is conserved for all elastic, inelastic, perfectly elastic, and perfectly inelastic collisions:

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

- In a perfectly elastic collision, total kinetic energy is conserved:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

- In an inelastic collision, the total kinetic energy is not conserved, although total energy is always conserved.

Questions

1. Two boxes are on an icy, frictionless, horizontal surface. You push one box, which collides with the other. If the boxes stick together after the collision, which of the following is conserved in the collision? Explain your reasoning. K/U
   (a) momentum
   (b) kinetic energy
2. An 85 kg skateboarder takes a running jump onto his skateboard, which has a mass of 8.0 kg and is initially at rest. After he lands on the skateboard, the speed of the board plus the skateboarder is 3.0 m/s. Determine the speed of the skateboarder just before he landed on the skateboard. T/I
3. A student puts two dynamics carts with a speed bumper between them on a track and presses them together. The total mass of the carts is 3.0 kg. Once the student releases them, the carts spring apart and roll away from each other. One cart has a mass of 2.0 kg and a final velocity of 2.5 m/s [S]. Calculate the final velocity of the other cart. T/I
4. Two people are riding inner tubes on an ice-covered (frictionless) lake. The first person has a mass of 85 kg and is travelling with a speed of 6.5 m/s. He collides head-on with the second person with a mass of 120 kg who is initially at rest. They bounce apart after the perfectly elastic collision. The final velocity of the first person is 1.1 m/s in the opposite direction to his initial direction. K/U T/I C A
   (a) Are momentum and kinetic energy conserved for this system? Explain your answer.
   (b) Determine the final velocity of the second person.
5. Two skaters are studying collisions on an ice-covered (frictionless) lake. Skater 1 has a mass of 95 kg and is initially travelling with a speed of 5.0 m/s, and skater 2 has a mass of 130 kg and is initially at rest. Skater 1 then collides with skater 2, and they lock arms and travel away together. K/U T/I
   (a) Does the system undergo an elastic collision or an inelastic collision? Explain your answer.
   (b) Solve for the final velocity of the two skaters.
6. Two cars of equal mass (1250 kg) collide head-on in a perfectly inelastic collision. Just before the collision, one car is travelling with a velocity of 12 m/s [E] and the other at 12 m/s [W]. Determine the velocity of each car after the collision. A T/I
7. A moving object collides with a stationary object. If the collision is perfectly elastic, is it possible for both objects to be at rest after the collision? Explain your answer. K/U C A
8. A truck of mass 1.3×10^4 kg, travelling at 9.0×10^1 km/h [N], collides with a car of mass 1.1×10^3 kg, travelling at 3.0×10^1 km/h [N]. The collision is perfectly inelastic. T/I A
   (a) Calculate the magnitude and direction of the velocity of the vehicles immediately after the collision.
   (b) Calculate the total kinetic energy before and after the collision described in (a).
   (c) Determine the decrease in kinetic energy during the collision.

5.4 Head-on Elastic Collisions

In previous sections, you read about systems involving elastic and inelastic collisions. You applied the law of conservation of momentum and the law of conservation of kinetic energy to solve problems. You also read about the ideal cases of the perfectly elastic and perfectly inelastic collisions, and learned that in both cases momentum is conserved. You discovered that kinetic energy is conserved in the case of perfectly elastic collisions, but not inelastic collisions. In this section, you will examine in greater detail situations involving perfectly elastic collisions in one dimension. You will calculate the final velocities of two objects after an elastic collision in one dimension with equations for special cases. Once again, we consider near-perfectly elastic collisions in one dimension (**Figure 1**).

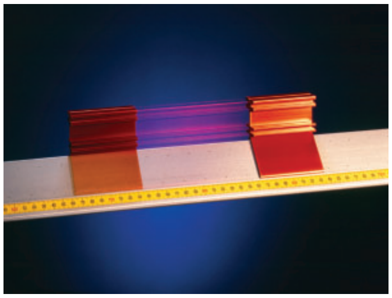


Figure 1 Gliders on an air track can undergo elastic collisions in one dimension.

Perfectly Elastic Head-on Collisions in One Dimension

head-on elastic collision an impact in which two objects approach each other from opposite directions; momentum and kinetic energy are conserved after the collision

In a one-dimensional **head-on elastic collision**, two objects approach each other from opposite directions and collide. In such collisions, both momentum and kinetic energy are conserved. You can derive expressions for the final velocities of two objects in a head-on collision in terms of the initial velocities and the objects' masses.

Suppose an object of mass m_1 travels with initial velocity v_{i_1} and collides head-on with an object of mass m_2 travelling at velocity v_{i_2}. If we assume a one-dimensional collision, we can omit the vector notation for velocities, and instead use positive and negative values to identify motion in one direction or the opposite direction. We begin the analysis with the conservation of momentum:

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2} \qquad \text{(Equation 1)}$$

Rewrite Equation 1 by bringing all the terms with m_1 to one side and all the terms with m_2 to the other side, and common factoring the m coefficients:

$$m_1v_{i_1} - m_1v_{f_1} = m_2v_{f_2} - m_2v_{i_2}$$

$$m_1(v_{i_1} - v_{f_1}) = m_2(v_{f_2} - v_{i_2}) \qquad \text{(Equation 2)}$$

Since this is an elastic collision, conservation of total kinetic energy can be applied:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

Multiply both sides of the equation by 2 to clear the fractions:

$$\cancel{2}\left(\frac{\cancel{1}}{\cancel{2}}m_1v_{i_1}^2 + \frac{\cancel{1}}{\cancel{2}}m_2v_{i_2}^2\right) = \cancel{2}\left(\frac{\cancel{1}}{\cancel{2}}m_1v_{f_1}^2 + \frac{\cancel{1}}{\cancel{2}}m_2v_{f_2}^2\right)$$

$$m_1v_{i_1}^2 + m_2v_{i_2}^2 = m_1v_{f_1}^2 + m_2v_{f_2}^2$$

Collect m_1 terms on the left side and m_2 terms on the right side and divide out the common factors.

$$m_1v_{i_1}^2 - m_1v_{f_1}^2 = m_2v_{f_2}^2 - m_2v_{i_2}^2$$

$$m_1(v_{i_1}^2 - v_{f_1}^2) = m_2(v_{f_2}^2 - v_{i_2}^2)$$

Factor both sides using the difference of squares:

$$m_1(v_{i_1} - v_{f_1})(v_{i_1} + v_{f_1}) = m_2(v_{f_2} - v_{i_2})(v_{f_2} + v_{i_2}) \qquad \text{(Equation 3)}$$

Divide Equation 3 by Equation 2:

$$\frac{\cancel{m_1(v_{i_1} - v_{f_1})}(v_{i_1} + v_{f_1})}{\cancel{m_1(v_{i_1} - v_{f_1})}} = \frac{\cancel{m_2(v_{f_2} - v_{i_2})}(v_{f_2} + v_{i_2})}{\cancel{m_2(v_{f_2} - v_{i_2})}}$$

$$v_{i_1} + v_{f_1} = v_{f_2} + v_{i_2} \qquad \text{(Equation 4)}$$

Rearranging Equation 4 to isolate v_{f_2} on the left gives

$$v_{f_2} = v_{i_1} + v_{f_1} - v_{i_2} \qquad \text{(Equation 5)}$$

Substitute Equation 5 into Equation 1 to express v_{f_1} in terms of the masses and their initial speeds:

$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2(v_{i_1} + v_{f_1} - v_{i_2})$$
$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{i_1} + m_2 v_{f_1} - m_2 v_{i_2}$$

Collect the v_{f_1} terms on the right side of the equation and terms involving initial velocities on the left side, then collect like terms and divide out common m coefficients:

$$m_1 v_{i_1} - m_2 v_{i_1} + m_2 v_{i_2} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{f_1}$$
$$(m_1 - m_2)v_{i_1} + 2m_2 v_{i_2} = (m_1 + m_2)v_{f_1}$$

Divide both sides by $m_1 + m_2$ to isolate v_{f_1}:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

This equation expresses the final velocity of the first object in terms of the masses and initial velocities of the two objects. Similarly, we can rearrange Equation 4 to isolate v_{f_1} on the left:

$$v_{f_1} = v_{f_2} + v_{i_2} - v_{i_1} \qquad \text{(Equation 6)}$$

To derive a similar equation for v_{f_2}, follow the above steps for v_{f_1}, starting with substituting Equation 6 into Equation 1, and ending by dividing both sides by $m_1 + m_2$ and isolating v_{f_2} on the left side:

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

This equation expresses the final velocity of the second object in terms of the masses and initial velocities of the two objects. Note that the equation for $\vec{v}_{f_2}$ is the same as the equation for $\vec{v}_{f_1}$ if you interchange all the 1 and 2 subscripts. It is important to note that these equations hold true only for perfectly elastic collisions in one dimension.

In some cases, one of the objects is initially at rest. For instance, if v_2 is initially zero, the equations above simplify to

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$$
$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

In Tutorial 1, you will use these velocity relationships as an alternative way of analyzing some head-on elastic collisions. As you do Tutorial 1, compare these methods to the methods used in Section 5.3. WEB LINK

Tutorial 1 Analyzing Head-on Elastic Collisions

In these Sample Problems, you will use the relative velocity relationships derived above to solve problems related to head-on elastic collisions in one dimension.

Sample Problem 1: Head-on Elastic Collision with One Object at Rest in One Dimension

Consider an elastic head-on collision between two balls of different masses, as shown in **Figure 2**. The mass of ball 1 is 1.2 kg, and its velocity is 7.2 m/s [W]. The mass of ball 2 is 3.6 kg, and ball 2 is initially at rest. Determine the final velocity of each ball after the collision.

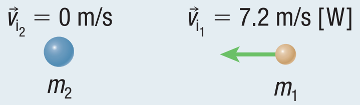


Figure 2

Given: $m_1 = 1.2$ kg; $\vec{v}_{i_1} = 7.2$ m/s [W]; $m_2 = 3.6$ kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Since one of the objects is initially at rest in the head-on elastic collision, use the simplified equations:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution: Let the negative *x*-direction represent west.

For ball 1,

$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&= \left(\frac{1.2\text{ kg} - 3.6\text{ kg}}{1.2\text{ kg} + 3.6\text{ kg}}\right)(-7.2\text{ m/s}) \\
&= 3.6\text{ m/s} \\
\vec{v}_{f_1} &= 3.6\text{ m/s [E]}
\end{aligned}$$

For ball 2,

$$\begin{aligned}
\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&= \left(\frac{2(1.2\text{ kg})}{1.2\text{ kg} + 3.6\text{ kg}}\right)(-7.2\text{ m/s}) \\
&= -3.6\text{ m/s} \\
\vec{v}_{f_2} &= 3.6\text{ m/s [W]}
\end{aligned}$$

Statement: The final velocity of ball 1 is 3.6 m/s [E]. The final velocity of ball 2 is 3.6 m/s [W].

Sample Problem 2: Head-on Elastic Collision with Both Objects Moving in One Dimension

In a bumper car ride, bumper car 1 has a total mass of 350 kg and is initially moving at 4.0 m/s [E]. In a head-on completely elastic collision, bumper car 1 hits bumper car 2. The total mass of bumper car 2 is 250 kg, and it is moving at 2.0 m/s [W]. Calculate the final velocity of each bumper car immediately after the collision.

Given: Let east be positive and west be negative; $m_1 = 350$ kg; $\vec{v}_{i_1} = 4.0$ m/s [E] $= +4.0$ m/s; $m_2 = 250$ kg; $\vec{v}_{i_2} = 2.0$ m/s [W] $= -2.0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution:

$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2} \\
&= \left(\frac{350\text{ kg} - 250\text{ kg}}{350\text{ kg} + 250\text{ kg}}\right)(4.0\text{ m/s}) \\
&\quad + \left(\frac{2(250\text{ kg})}{350\text{ kg} + 250\text{ kg}}\right)(-2.0\text{ m/s}) \\
&= -1.0\text{ m/s} \\
\vec{v}_{f_1} &= 1.0\text{ m/s [W]} \\
\vec{v}_{f_2} &= \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&= \left(\frac{250\text{ kg} - 350\text{ kg}}{350\text{ kg} + 250\text{ kg}}\right)(-2.0\text{ m/s}) \\
&\quad + \left(\frac{2(350\text{ kg})}{350\text{ kg} + 250\text{ kg}}\right)(4.0\text{ m/s}) \\
&= 5.0\text{ m/s} \\
\vec{v}_{f_2} &= 5.0\text{ m/s [E]}
\end{aligned}$$

Statement: The final velocity of bumper car 1 is 1.0 m/s [W]. The final velocity of bumper car 2 is 5.0 m/s [E].

Practice

1. A ball of mass 80.0 g is moving at 7.0 m/s [W] when it undergoes a head-on elastic collision with a stationary ball of mass 60.0 g. Assume the collision is one-dimensional. Calculate the velocity of each ball after the collision. T/I [ans: 1.0 m/s [W]; 8.0 m/s [W]]
2. Cart 1 has a mass of 1.5 kg and is moving on a track at 36.5 cm/s [E] toward cart 2. The mass of cart 2 is 5 kg, and it is moving toward cart 1 at 42.8 cm/s [W]. The carts collide. The collision is cushioned by a Hooke's law spring, making it an elastic head-on collision. Calculate the final velocity of each cart after the collision. T/I [ans: cart 1: 90 cm/s [W]; cart 2: 6 cm/s [W]]

Special Cases

Using these new equations for head-on elastic collisions in one dimension, special cases of collisions, such as objects of equal mass, produce some interesting results.

CASE 1: OBJECTS HAVE THE SAME MASS

The first case we consider is when the objects that are colliding have the same mass, so let $m_1 = m_2 = m$.

$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m - m}{m + m}\right)\vec{v}_{i_1} + \left(\frac{2m}{m + m}\right)\vec{v}_{i_2} \\
&= \left(\frac{0}{2m}\right)\vec{v}_{i_1} + \left(\frac{2m}{2m}\right)\vec{v}_{i_2} \\
&= \left(\frac{2m}{2m}\right)\vec{v}_{i_2} \\
\vec{v}_{f_1} &= \vec{v}_{i_2} \\
\vec{v}_{f_2} &= \left(\frac{m - m}{m + m}\right)\vec{v}_{i_2} + \left(\frac{2m}{m + m}\right)\vec{v}_{i_1} \\
&= \left(\frac{0}{2m}\right)\vec{v}_{i_2} + \left(\frac{2m}{2m}\right)\vec{v}_{i_1} \\
&= \left(\frac{2m}{2m}\right)\vec{v}_{i_1} \\
\vec{v}_{f_2} &= \vec{v}_{i_1}
\end{aligned}$$

In other words, when two objects with the same mass undergo a head-on elastic collision in one dimension, they exchange velocities almost as if they pass through each other.

CASE 2: A LIGHTER OBJECT COLLIDING WITH A MUCH HEAVIER, STATIONARY OBJECT

Our second case deals with situations in which the mass of one of the objects is much greater than the mass of the other object, and the heavier object is stationary. For example, if object 2 is stationary and has a much greater mass, then since m_2 is much greater than m_1, you can consider m_1 to be approximately zero, or negligible. So

$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&\approx \left(\frac{0 - m_2}{0 + m_2}\right)\vec{v}_{i_1} \\
\vec{v}_{f_1} &\approx -\vec{v}_{i_1} \\
\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&\approx \left(\frac{2(0)}{0 + m_2}\right)\vec{v}_{i_1} \\
\vec{v}_{f_2} &\approx 0
\end{aligned}$$

Investigation 5.4.1

Head-on Elastic Collisions (page 259)

Now that you have an understanding of how head-on elastic collisions work, perform Investigation 5.4.1 to study these types of collisions in greater detail.

In other words, if an object collides with a stationary, much heavier object, the velocity of the light object is reversed, and the heavier object stays at rest. To put this scenario into perspective, consider a collision between a table tennis ball and a stationary transport truck: the transport truck will not move and the table tennis ball will bounce back with the same speed. You will explore more special cases in the questions at the end of Section 5.4.

Conservation of Mechanical Energy

You have discovered what happens to momentum in head-on elastic collisions. What do you suppose happens to the conservation of total mechanical energy during elastic collisions? One of the two gliders in **Figure 3** has been fitted with a spring bumper. When the two gliders collide head-on in an elastic collision, the bounce is not immediate. If you viewed the collision in slow motion, you would see the bumper compress initially and then spring back to its original shape. During the compression, some of the kinetic energy of the moving gliders is converted into elastic potential energy. This potential energy is converted back into kinetic energy during the rebound.

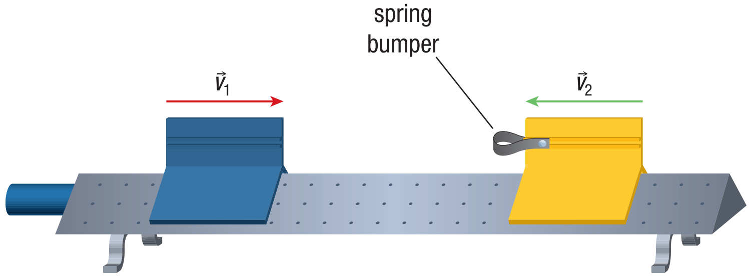


Figure 3 When the two gliders collide, the duration of the collision is greater than it would be without the spring bumper on one of the gliders.

If the compression of the spring bumper during the collision is x, then the law of conservation of energy states:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2 + \frac{1}{2}kx^2$$

This equation and the graph in **Figure 4** both show that as the spring compresses, the elastic energy increases and the total kinetic energy of the two carts decreases. The total mechanical energy, however, stays constant. As the compression decreases, the elastic energy decreases and the total kinetic energy increases. The total mechanical energy still remains constant.

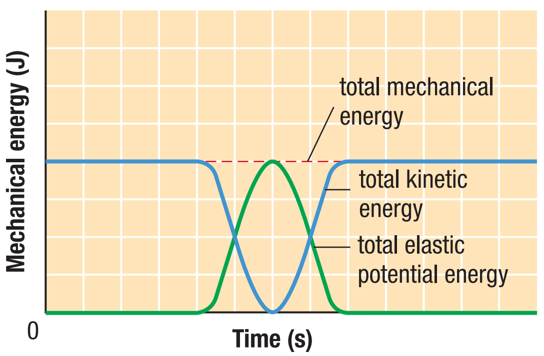


Figure 4 In this graph of total mechanical energy versus time, you can see how the total mechanical energy, the total kinetic energy, and the total elastic potential energy relate to each other throughout the collision.

To determine the maximum compression of the spring during the collision, use the fact that when the two gliders collide they have the same velocity at that point. If they did not have the same velocity at maximum compression, then one would be catching up to the other or pulling away from the other. Therefore, at maximum compression (closest approach), the two objects must have the same velocity, v_f. The equation above then reduces to the following:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}(m_1 + m_2)v_f^2 + \frac{1}{2}kx^2$$

In Tutorial 2, you will apply the conservation of mechanical energy to problems involving the physics of spring carts.

Tutorial 2 Applying Conservation of Mechanical Energy

In the following Sample Problem, you will apply the conservation of mechanical energy to solve collision problems.

Sample Problem 1: Two-Cart Spring System

Dynamics cart 1 has a mass of 1.8 kg and is moving with a velocity of 4.0 m/s [right] along a frictionless track. Dynamics cart 2 has a mass of 2.2 kg and is moving at 6.0 m/s [left]. The carts collide in a head-on elastic collision cushioned by a spring with spring constant $k = 8.0 \times 10^4$ N/m (**Figure 5**).

(a) Determine the compression of the spring, in centimetres, during the collision when cart 2 is moving at 4.0 m/s [left].

(b) Calculate the maximum compression of the spring, in centimetres.

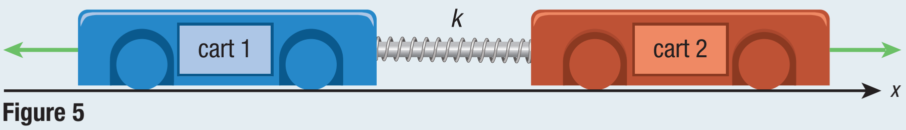


Figure 5

Solution

(a) **Given:** $m_1 = 1.8$ kg; $\vec{v}_{i_1} = 4.0$ m/s [right]; $m_2 = 2.2$ kg; $\vec{v}_{i_2} = 6.0$ m/s [left]; $\vec{v}_{f_2} = 4.0$ m/s [left]; $k = 8.0 \times 10^4$ N/m

Required: x

Analysis: Use the conservation of momentum to determine the velocity of cart 1 during the collision, when cart 2 is moving 4.0 m/s [left]. Then apply the conservation of mechanical energy to determine the compression of the spring at this particular moment during the collision. Consider right to be positive and left to be negative, and omit the vector notation.

Solution: Begin with the conservation of momentum equation.

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$$

Rearrange this equation to express the final velocity of cart 1 in terms of the other given values.

$$m_1v_{i_1} + m_2v_{i_2} - m_2v_{f_2} = m_1v_{f_1}$$

$$\frac{m_1v_{i_1} + m_2v_{i_2} - m_2v_{f_2}}{m_1} = v_{f_1}$$

Substitute the given values and solve.

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2} - m_2 v_{f_2}}{m_1}$$

$$= \frac{(1.8\ \text{kg})(4.0\ \text{m/s}) + (2.2\ \text{kg})(-6.0\ \text{m/s}) - (2.2\ \text{kg})(-4.0\ \text{m/s})}{1.8\ \text{kg}}$$

$$v_{f_1} = 1.56\ \text{m/s (one extra digit carried)}$$

Cart 1 is moving 1.6 m/s [right] when cart 2 is moving 4.0 m/s [left].

Now use the conservation of mechanical energy to determine the compression of the spring, x.

$$\frac{1}{2}m_1 v_{i_1}^2 + \frac{1}{2}m_2 v_{i_2}^2 = \frac{1}{2}m_1 v_{f_1}^2 + \frac{1}{2}m_2 v_{f_2}^2 + \frac{1}{2}kx^2$$

Multiply both sides of the equation by 2 to clear the fractions, and then isolate the term containing x on one side of the equation.

$$\cancel{2}\left(\frac{1}{\cancel{2}}m_1 v_i^2 + \frac{1}{\cancel{2}}m_2 v_i^2 - \frac{1}{\cancel{2}}m_1 v_f^2 - \frac{1}{\cancel{2}}m_2 v_f^2\right) = \cancel{2}\left(\frac{1}{\cancel{2}}kx\right)$$

Divide both sides by k and then take the square root of both sides.

$$\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - m_1 v_{f_1}^2 - m_2 v_{f_2}^2}{k} = \frac{\cancel{k}x^2}{\cancel{k}}$$

$$\sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - m_1 v_{f_1}^2 - m_2 v_{f_2}^2}{k}} = x$$

Substitute the known values to determine the compression of the spring when cart 2 is moving 4.0 m/s [left].

$$x = \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - m_1 v_{f_1}^2 - m_2 v_{f_2}^2}{k}}$$

$$x = \sqrt{\frac{(1.8\ \text{kg})(4.0\ \text{m/s})^2 + (2.2\ \text{kg})(-6.0\ \text{m/s})^2 - (1.8\ \text{kg})(1.56\ \text{m/s})^2 - (2.2\ \text{kg})(4.0\ \text{m/s})^2}{8.0 \times 10^4\ \text{N/m}}}$$

$$x = 2.9 \times 10^{-2}\ \text{m}$$

Statement: The compression of the spring is 2.9 cm during the collision, when cart 2 is moving 4.0 m/s [left].

(b) **Given:** $m_1 = 1.8$ kg; $\vec{v}_{i_1} = 4.0$ m/s [right]; $m_2 = 2.2$ kg; $\vec{v}_{i_2} = 6.0$ m/s [left]; $k = 8.0 \times 10^4$ N/m

Required: x

Analysis: At the beginning of the collision, as the carts come together and the spring is being compressed, cart 1 is moving faster than cart 2. Toward the end of the collision, as the carts separate and the spring is being released, cart 2 will be moving faster than cart 1. At the point of maximum compression of the spring, the two carts will have the same velocity, v_f. Use the conservation of momentum equation to determine this velocity. Then apply the conservation of mechanical energy to calculate the maximum compression of the spring.

Solution: $m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_f + m_2 v_f$

Factor out the common factor v_f.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2)v_f$$

Divide both sides by $m_1 + m_2$ to isolate v_f.

$$\frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2} = v_f$$

Substitute the given values to calculate the velocity of both carts at maximum compression.

$$v_f = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

$$= \frac{(1.8\text{ kg})(+4.0\text{ m/s}) + (2.2\text{ kg})(-6.0\text{ m/s})}{1.8\text{ kg} + 2.2\text{ kg}}$$

$$v_f = -1.5\text{ m/s}$$

Now use the law of conservation of mechanical energy to determine the maximum compression of the spring. Clear the fractions first, and then isolate x.

$$\cancel{2}\left(\frac{1}{\cancel{2}}m_1 v_{i_1}^2 + \frac{1}{\cancel{2}}m_2 v_{i_2}^2\right) = \cancel{2}\left(\frac{1}{\cancel{2}}(m_1 + m_2)v_f^2 + \frac{1}{\cancel{2}}kx^2\right)$$

$$m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2)v_f^2 = kx^2$$

$$\sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2)v_f^2}{k}} = x$$

Substitute the known values and solve for the maximum compression of the spring.

$$x = \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2)v_f^2}{k}}$$

$$= \sqrt{\frac{(1.8\text{ kg})(4.0\text{ m/s})^2 + (2.2\text{ kg})(-6.0\text{ m/s})^2 - (1.8\text{ kg} + 2.2\text{ kg})(-1.5\text{ m/s})^2}{8.0 \times 10^4\text{ N/m}}}$$

$$x = 3.5 \times 10^{-2}\text{ m}$$

Statement: The maximum compression of the spring is 3.5 cm.

Practice

1. A 1.2 kg glider moving at 3.0 m/s [right] undergoes an elastic head-on collision with a glider of equal mass moving at 3.0 m/s [left]. The collision is cushioned by a spring whose spring constant, k, is 6.0×10^4 N/m. T/I A
   (a) Determine the compression in the spring when the second glider is moving at 1.5 m/s [right]. [ans: 1.6 cm]
   (b) Calculate the maximum compression of the spring. [ans: 1.9 cm]
2. A student designs a new amusement park ride that involves a type of bumper car that has a spring on the front to cushion collisions. To test the bumper, the student attaches one spring to the front of a single car. In a collision, car 1 with total mass 4.4×10^2 kg is moving at 3.0 m/s [E] toward car 2 with total mass 4.0×10^2 kg moving at 3.3 m/s [W]. During the collision, the spring compresses a maximum of 44 cm. Determine the spring constant. T/I [ans: 4.3×10^4 N/m]

UNIT TASK **BOOKMARK**

You can apply what you learn about head-on elastic collisions and conservation of mechanical energy to the Unit Task on page 270.

5.4 Review

Summary

- In a perfectly elastic head-on collision in one dimension, momentum and kinetic energy are conserved.
- Using the law of conservation of momentum and the law of conservation of kinetic energy, we can derive equations to determine the final velocities of two objects in a perfectly elastic head-on collision in one dimension: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$ and $\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$.
- In cases where $\vec{v}_2$ is initially zero, $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$ and $\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$.
- In cases where the masses of the colliding objects are identical, $\vec{v}_{f_1} = \vec{v}_{i_2}$ and $\vec{v}_{f_2} = \vec{v}_{i_1}$.
- In cases in which one mass is significantly larger than the other mass, and the larger mass is stationary, $\vec{v}_{f_1} \approx -\vec{v}_{i_1}$ and $\vec{v}_{f_2} \approx 0$.
- During a head-on collision in one dimension, the kinetic energy of the moving masses is converted into elastic potential energy, and then back into kinetic energy during the rebound. Total mechanical energy is conserved throughout the collision.

Questions

1. Is it possible for two moving masses to undergo an elastic head-on collision and both be at rest immediately after the collision? Is it possible for an inelastic collision? Explain your reasoning. K/U
2. In curling, you will often see one curling stone hit another and come to rest while the stationary stone moves away from the one-dimensional collision. Explain how this can happen. K/U
3. The particles in **Figure 6** undergo an elastic collision in one dimension. Particle 1 has mass 1.5 g and particle 2 has mass 3.5 g. Their velocities before the collision are $\vec{v}_{i_1} = 12$ m/s [right] and $\vec{v}_{i_2} = 7.5$ m/s [left]. Determine the velocity of the two particles after the collision. K/U T/I A

Figure 6

4. Two chunks of space debris collide head-on in an elastic collision. One piece of debris has a mass of 2.67 kg. The other chunk has a mass of 5.83 kg. After the collision, both chunks move in the direction of the second chunk's initial velocity with speeds of 185 m/s for the smaller chunk and 172 m/s for the larger. What are the initial velocities of the two chunks? K/U T/I
5. Dynamics cart 1 has a mass of 0.84 kg and is initially moving at 4.2 m/s [right]. Cart 1 undergoes an elastic head-on collision with dynamics cart 2. The mass of cart 2 is 0.48 kg, and cart 2 is initially moving at 2.4 m/s [left]. The collision is cushioned by a spring with spring constant 8.0×10^3 N/m. T/I
   (a) Calculate the final velocity of each cart after they completely separate.
   (b) Determine the compression of the spring during the collision at the moment when cart 1 is moving at 3.0 m/s [right].
   (c) Determine the maximum compression of the spring.
6. Ball 1 has a mass of 2.0 kg and is suspended with a 3.0 m rope from a post so that the ball is stationary. Ball 2 has a mass of 4.0 kg and is tied to another rope. The second rope also measures 3.0 m but is held at a 60.0° angle, as shown in **Figure 7**. When ball 2 is released, it collides, head-on, with ball 1 in an elastic collision. T/I
   (a) Calculate the speed of each ball immediately after the first collision.
   (b) Calculate the maximum height of each ball after the first collision.

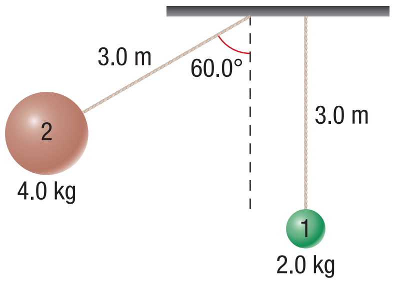


Figure 7

Collisions in Two Dimensions: Glancing Collisions

5.5

So far, you have read about collisions in one dimension. In this section, you will examine collisions in two dimensions. In **Figure 1**, the player is lining up the shot so that the cue ball (the white ball) will hit another billiard ball at an angle, directing it toward the corner pocket. What component of the cue ball's momentum will be transferred to the target ball if the shot is successful?

The laws of conservation of momentum and conservation of kinetic energy will apply just as they did for one-dimensional interactions. However, to calculate momentum for two-dimensional problems, consider the x-components and y-components of force and motion independently.

Figure 1 Billiards requires players to master the use of glancing collisions.

Mini Investigation

Glancing Collisions

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this investigation, you will model glancing collisions to observe and analyze how they work.

Equipment and Materials: eye protection; air table; 2 pucks; marbles; billiard balls

1. Put on your eye protection. Set up an air table with two pucks.

When you unplug the air table, pull the plug and not the cord. Wear closed-toe shoes to protect your feet in case the puck flies off the table. Push the objects lightly and cautiously.

2. Push the first puck toward the second to cause a gentle head-on collision. Observe the changes in the speeds of the pucks after the collision.
3. Repeat this process, but vary the angles of collision and the initial speed of the first puck. Observe the changes in the speeds of the pucks and the directions of their final velocities.
4. Try the same investigation using different objects, such as marbles or billiard balls.

A. How does the speed of the second puck compare to the initial speed of the first puck as you vary the angle of collision? K/U T/I A

B. How did using marbles or billiard balls affect the changes in direction and velocity? How was this different from using the pucks? Briefly summarize your observations. T/I A

Components of Momentum

Dealing with collisions in two dimensions involves the same basic ideas as dealing with collisions in one dimension. Now, however, the final velocity of each object involves two unknowns: the two components of the velocity vector. For objects in motion in two dimensions, the change in momentum for each component can be considered independently:

$$\Sigma\vec{F}_x\Delta t = \Delta\vec{p}_x$$

$$\Sigma\vec{F}_y\Delta t = \Delta\vec{p}_y$$

Similarly, the conservation of momentum equation can be expressed in terms of horizontal and vertical components:

$$\vec{p}_{\mathrm{i}_{1x}} + \vec{p}_{\mathrm{i}_{2x}} = \vec{p}_{f_{1x}} + \vec{p}_{\mathrm{f}_{2x}}$$
$$\vec{p}_{\mathrm{i}_{1y}} + \vec{p}_{\mathrm{i}_{2y}} = \vec{p}_{\mathrm{f}_{1y}} + \vec{p}_{\mathrm{f}_{2y}}$$

Consider the collision of two billiard balls shown in **Figure 2**. In this shot, the cue ball (1) will collide with the target ball (2), initially at rest, sending it at an angle ϕ toward the corner pocket, and the cue ball will continue travelling at an angle θ from its original direction of travel. Both objects are travelling at an angle to the directions of their original courses. This type of collision is called a **glancing collision**. In the following Tutorial, we use components to analyze the physics of a glancing collision.

glancing collision a collision in which the first object, after an impact with the second object, travels at an angle to the direction it was originally travelling

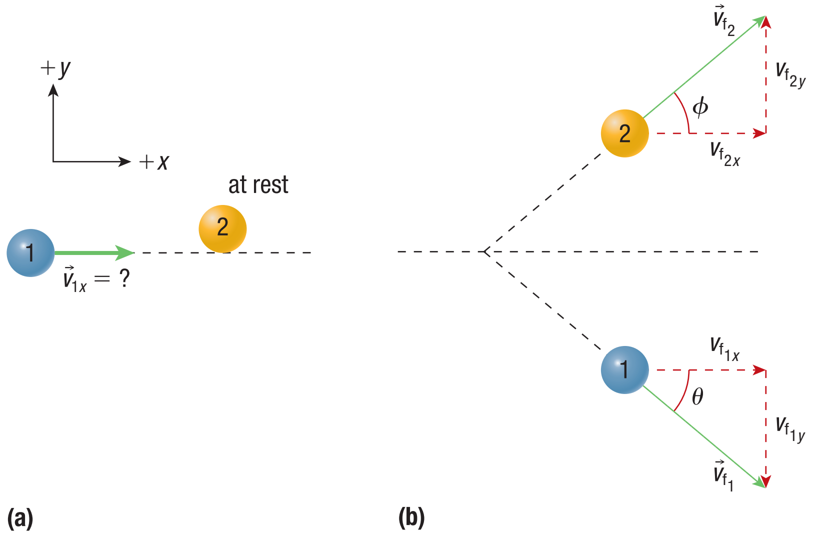


Figure 2 A cue ball striking another ball at an angle causes both balls to change direction.

Investigation 5.5.1

Conservation of Momentum in Two Directions (page 260)
After learning about the physics of glancing collisions, perform Investigation 5.5.1 to explore how momentum is conserved in collisions that occur in two dimensions.

Tutorial 1 Analysis of Glancing Collisions

In these Sample Problems, you will apply conservation of momentum in two dimensions.

Sample Problem 1: Analysis of a Glancing Collision

In a game of curling, a collision occurs between two stones of equal mass. The object stone is initially at rest. After the collision, the stone that is thrown has a speed of 0.56 m/s in some direction, represented by θ in **Figure 3.**

The object stone acquires a velocity $\vec{v}_{f_2} = 0.42$ m/s at an angle of $\phi = 30.0°$ from the original direction of motion of the thrown stone. Determine the initial velocity of the thrown stone.

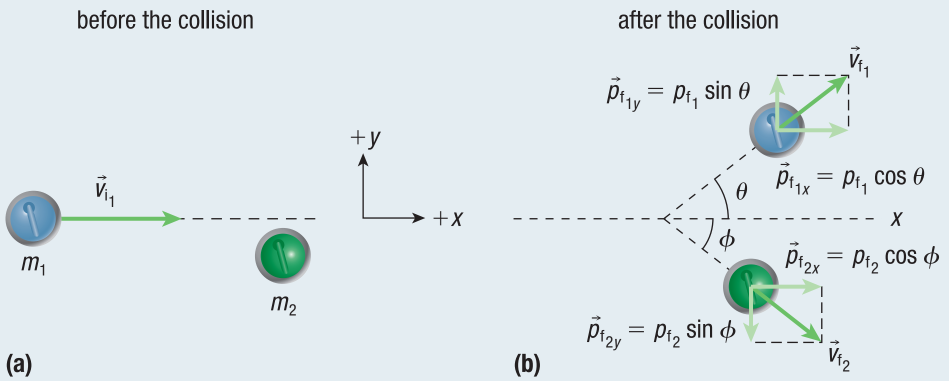


Figure 3 (a) The curling stone collides with the object stone in a glancing collision. (b) Both curling stones move off in different directions. We can analyze their final velocities to determine the initial velocity of the thrown curling stone.

Given: $m_1 = m_2$; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_{f_1} = 0.56$ m/s; $\vec{v}_{f_2} = 0.42$ m/s; $\phi = 30.0°$

Required: $\vec{v}_{i_1}$

Analysis: Choose a coordinate system to identify directions: let positive x be to the right and negative x be to the left. Let positive y be up and negative y be down.

Apply conservation of momentum independently in the x-direction and the y-direction. Begin by applying conservation of momentum in the y-direction to determine the direction of the final velocity of the thrown stone. Then apply conservation of momentum in the x-direction to calculate the initial velocity of the thrown stone.

Solution: In the y-direction, the total momentum before and after the collision is zero.

$$p_{T_{iy}} = p_{T_{fy}} = 0$$

Therefore, after the collision:

$$mv_{f_{1y}} + mv_{f_{2y}} = 0$$

Divide both sides by m and substitute the vertical component of each velocity vector. Note that the vertical component of the first stone's velocity is directed up, so its value is positive, whereas the vertical component of the second stone's velocity is directed down, so its value is negative.

$$\frac{\cancel{m}(v_{f_{1y}} + v_{f_{2y}})}{\cancel{m}} = \frac{0}{m}$$

$$v_{f_1} \sin\theta - v_{f_2} \sin\phi = 0$$

Rearrange this equation to isolate $\sin\theta$.

$$v_{f_1} \sin\theta = v_{f_2} \sin\phi$$

$$\sin\theta = \frac{v_{f_2} \sin\phi}{v_{f_1}}$$

Substitute the given values and solve for $\sin\theta$.

$$\sin\theta = \frac{(0.42\ \cancel{\text{m/s}})(\sin 30°)}{(0.56\ \cancel{\text{m/s}})}$$

$$\sin\theta = 0.375$$

Apply the inverse sine to both sides to solve for θ.

$$\theta = \sin^{-1}0.375$$
$$\theta = 22.0° \text{ (one extra digit carried)}$$

The first stone is travelling at an angle of 22° above the horizontal after the collision.

Now use conservation of momentum in the x-direction to solve for the initial speed of the thrown stone.

$$p_{T_{ix}} = p_{T_{fx}}$$

Note that the object stone is at rest before the collision, so its initial momentum is zero.

$$mv_{i_{1x}} + \cancel{mv_{i_{2x}}} = mv_{f_{1x}} + mv_{f_{2x}}$$

Divide both sides of the equation by m.

$$\frac{\cancel{m}v_{i_{1x}}}{\cancel{m}} = \frac{\cancel{m}(v_{f_{1x}} + v_{f_{2x}})}{\cancel{m}}$$

$$v_{i_{1x}} = v_{f_{1x}} + v_{f_{2x}}$$

Substitute the known values for the final horizontal velocity components of the stones. Note that all vectors are directed to the right, so all velocities are positive.

$$\begin{aligned} v_{i_{1x}} &= v_{f_1}\cos\theta + v_{f_2}\cos\phi \\ &= (0.56\cos 22.0°)\ \text{m/s} + (0.42\cos 30°)\ \text{m/s} \\ &= 0.519\ \text{m/s} + 0.364\ \text{m/s (one extra digit carried)} \\ v_{i_{1x}} &= 0.88\ \text{m/s} \end{aligned}$$

Statement: The initial velocity of the thrown stone is 0.88 m/s [right].

Sample Problem 2: Inelastic Glancing Collisions

Two cross-country skiers are skiing to a crossing of horizontal trails in the woods as shown in **Figure 4**. Skier 1 is travelling east and has a mass of 84 kg. Skier 2 is travelling north and has a mass of 72 kg. Both skiers are travelling with an initial speed of 5.1 m/s. One of the skiers forgets to look, resulting in a right-angle collision with the skis locked together after the collision. Calculate the final velocity of the two skiers.

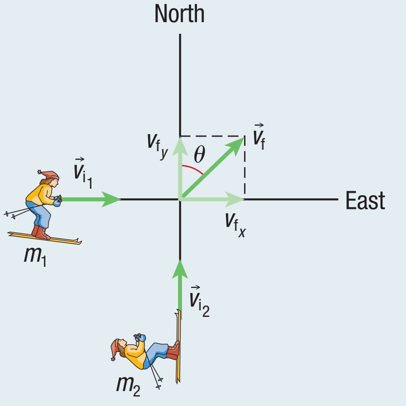


Figure 4

Given: inelastic collision; $m_1 = 84$ kg; $m_2 = 72$ kg; $\vec{v}_{i_1} = 5.1$ m/s [E]; $\vec{v}_{i_2} = 5.1$ m/s [N]

Required: $\vec{v}_f$

Analysis: According to the law of conservation of momentum, $\vec{p}_{T_i} = \vec{p}_{T_f}$. Since the initial velocities are at right angles to each other, as shown in **Figure 5**, you can calculate the total velocity and momentum using the Pythagorean theorem and trigonometry:

$$p^2 = p_1^2 + p_2^2, \text{ and } \tan\theta = \left(\frac{p_1}{p_2}\right)$$

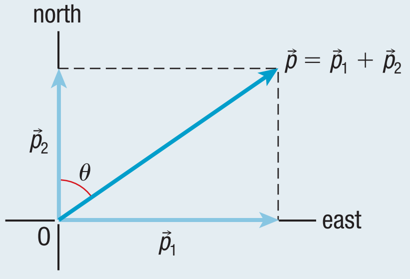


Figure 5

Solution: The first skier's momentum is

$$\vec{p}_1 = m_1\vec{v}_1$$
$$= (84\text{ kg})\left(5.1\ \frac{\text{m}}{\text{s}}\right)[\text{E}]$$
$$\vec{p}_1 = 428\text{ kg·m/s }[\text{E}] \text{ (one extra digit carried)}$$

The second skier's momentum is

$$\vec{p}_2 = m_2\vec{v}_2$$
$$= (72\text{ kg})\left(5.1\ \frac{\text{m}}{\text{s}}\right)[\text{N}]$$
$$\vec{p}_2 = 367\text{ kg·m/s }[\text{N}] \text{ (one extra digit carried)}$$

The magnitude of the total momentum can be calculated by applying the Pythagorean theorem:

$$p^2 = p_1^2 + p_2^2$$
$$p = \sqrt{p_1^2 + p_2^2}$$
$$= \sqrt{\left(428\ \frac{\text{kg·m}}{\text{s}}\right)^2 + \left(367\ \frac{\text{kg·m}}{\text{s}}\right)^2}$$
$$p = 564\text{ kg·m/s (one extra digit carried)}$$

The direction can be determined by applying the tangent ratio:

$$\tan\theta = \left(\frac{p_1}{p_2}\right)$$
$$\theta = \tan^{-1}\left(\frac{p_1}{p_2}\right)$$
$$= \tan^{-1}\left(\frac{428\ \frac{\text{kg·m}}{\text{s}}}{367\ \frac{\text{kg·m}}{\text{s}}}\right)$$
$$\theta = 49^\circ$$

The direction of the two skiers is [N 49° E].

Conservation of momentum tells us that the final total momentum of the skiers must equal this initial momentum. Since the collision is perfectly inelastic, both skiers have the same final velocity:

$$\vec{p}_f = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$
$$= (m_1 + m_2)\vec{v}_f$$
$$\vec{v}_f = \frac{\vec{p}_f}{m_1 + m_2}$$
$$= \frac{564\ \frac{\cancel{\text{kg}}\text{·m}}{\text{s}}}{84\ \cancel{\text{kg}} + 72\ \cancel{\text{kg}}}[\text{N } 49^\circ\text{ E}]$$
$$\vec{v}_f = 3.6\text{ m/s }[\text{N } 49^\circ\text{ E}]$$

Statement: After the collision, the skiers are travelling together with a velocity of 3.6 m/s [N 49° E].

Practice

1. Two freight trains have a completely inelastic collision at a track crossing. Engine 1 has a mass of 1.4×10^4 kg and is initially travelling at 45 km/h [N]. Engine 2 has a mass of 1.5×10^4 kg and is initially travelling at 53 km/h [W]. Calculate the final velocity. T/I A
   [ans: 9.7 m/s [N 52° W]]
2. A star of mass 2×10^{30} kg moving with a velocity of 2×10^4 m/s [E] collides with a second star of mass 5×10^{30} kg moving with a velocity of 3×10^4 m/s at a right angle to the path of the first star. If the two join together, what is their common velocity? T/I A [ans: 2×10^4 m/s [15° to the initial path of the second star]]

5.5 Review

Summary

- The laws of conservation of momentum and conservation of kinetic energy for collisions in two dimensions are the same as they are for one-dimensional collisions.
- Momentum is conserved for elastic and inelastic collisions.
- Kinetic energy is conserved only in elastic collisions.
- The fact that momentum is a vector quantity means that problems involving two-dimensional collisions can be solved by independently analyzing the x-components and y-components.

Questions

1. Two balls of equal mass undergo a collision (see **Figure 6**). Ball 1 is initially travelling horizontally with a speed of 10.0 m/s, and ball 2 is initially at rest. After the collision, ball 1 moves away with a velocity of 4.7 m/s at an angle of $\theta = 60.0°$ from its original path and ball 2 moves away at an unknown angle ϕ. Determine the magnitude and direction of velocity of ball 2 after the collision. K/U T/I

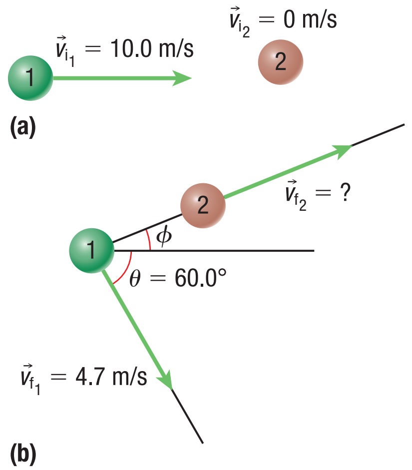


Figure 6

2. A hockey puck of mass 0.16 kg, sliding on a nearly frictionless surface of ice with a velocity of 2.0 m/s [E], strikes a second puck at rest with a mass of 0.17 kg. The first puck has a velocity of 1.5 m/s [N 31° E] after the collision. Determine the velocity of the second puck after the collision. T/I A
3. Two hockey pucks of equal mass approach each other. Puck 1 has an initial velocity of 20.0 m/s [S 45° E], and puck 2 has an initial velocity of 15 m/s [S 45° W]. After the collision, the first puck is moving with a velocity of 10.0 m/s [S 45° W]. K/U T/I C
   (a) Determine the final velocity of the second puck.
   (b) Is this collision elastic, perfectly inelastic, or (non-perfectly) inelastic? Explain your reasoning.
4. An automobile collides with a truck at an intersection. The car, of mass 1.4×10^3 kg, is travelling at 32 km/h [S]; the truck has a mass of 2.6×10^4 kg and is travelling at 48 km/h [E]. The collision is perfectly inelastic. Determine their velocity just after the collision. T/I
5. Two balls of equal mass m undergo a collision. One ball is initially stationary. After the collision, the velocities of the balls make angles of 25.5° and −45.9° relative to the original direction of motion of the moving ball. T/I C
   (a) Draw and label a diagram to show the balls before and after the collision. Label the angles θ and ϕ.
   (b) Calculate the final speeds of the balls if the initial ball had a speed of 3.63 m/s.
6. A carbon-14 nucleus, initially at rest, undergoes a nuclear reaction known as beta decay. The nucleus emits two particles horizontally: one with momentum 7.8×10^{-21} kg·m/s [E] and another with momentum 3.5×10^{-21} kg·m/s [S]. T/I A
   (a) Calculate the direction of the motion of the nucleus immediately following the reaction.
   (b) Determine the final momentum of the nucleus.
   (c) The mass of the residual carbon-14 nucleus is 2.3×10^{-26} kg. Determine its final velocity.
7. A neutron of mass 1.7×10^{-27} kg, travelling at 2.2 km/s, hits a stationary helium nucleus of mass 6.6×10^{-27} kg. After the collision, the velocity of the helium nucleus is 0.53 km/s at 52° to the original direction of motion of the neutron. Determine the final velocity of the neutron. T/I
8. Your classmate makes the following statement: "For a head-on elastic collision between two objects of equal mass, the after-collision velocities of the objects are at right angles to each other." Evaluate the accuracy of this statement. K/U T/I A

5.6 Explore Applications of Momentum

SKILLS MENU

- Researching
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating
- Identifying Alternatives

Staying Safe at Every Speed

According to Transport Canada, almost 3000 people per year die in traffic accidents in Canada. This number is high, but it is only half the rate of traffic fatalities that occurred in the 1970s. Many factors have cut the number of fatalities, but advances in vehicle safety devices have played a huge role. In fact, Transport Canada reports that seat belts save 1000 Canadian lives per year.

Motor vehicle safety devices either help prevent accidents or help protect us in an accident. Anti-lock brakes give the driver more control over the vehicle when stopping suddenly, helping to avoid surprises on the road. Active head restraints cushion a passenger's head in a rear-end collision, avoiding damage to the neck (**Figure 1**).

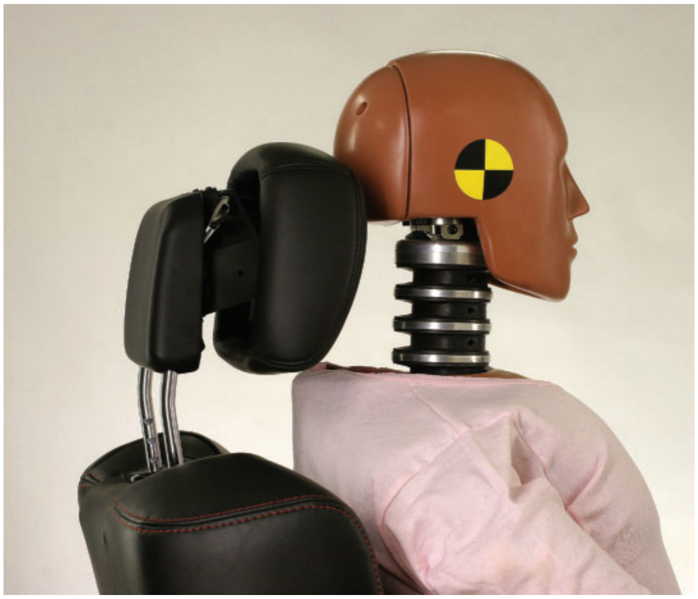


Figure 1 Active head restraints protect a passenger's head and neck during low-impact rear-end collisions.

Scientists and engineers design safety devices by analyzing the transfer of energy and momentum in collisions. Years of collision data provide information on how long it takes to stop a car and how much force a human body can tolerate. We know more about how the parts of a vehicle will bend or break during a collision, and how a passenger's body will respond. Armed with data, technicians can try to improve devices and build new ones to efficiently and safely absorb energy and momentum.

Seat belts, for instance, work by holding the passenger to the seat. This simple action means that the passenger will slow down with the car, reducing the chance of injury. Active head restraints work by narrowing the space between the passenger's head and the headrest, reducing the chance of whiplash, an injury caused by jerking the neck back quickly.

The Application

SKILLS HANDBOOK A4

Transport Canada requires motor vehicles to have seat belts. Vehicle makers add many other devices as standard features, such as airbags. Other devices are only optional, such as electronic stability control. Drivers can choose to have these features installed at an additional cost.

Although we would always like to drive the safest car possible, sometimes the cost of a device does not seem worth paying for. Consumers have to make careful decisions on safety when purchasing a new car or an old car without modern devices. Understanding more about how safety devices work and how they keep us safe can help us make a good decision.

Your Goal

To communicate information about an automobile safety device to your family using the concepts of energy and momentum

Research

Suppose that a family friend plans to buy a new car. Choose one motor vehicle safety device (other than seat belts). Prepare a presentation to help your friend decide whether to have this device installed in the new car. Conduct library or Internet research to learn about your chosen safety device. Be sure to investigate the following:

- how the device works
- how the device applies the concepts of energy and momentum to help prevent a collision or to protect you in a collision
- the costs of installing the device
- statistics or estimates of accidents prevented or lives saved by the device
- limitations or problems with the device
- continuing research to improve the device WEB LINK

Summarize

Summarize your research:

- How does the device keep you safe?
- What technology does the device use?
- How does the device use principles of energy and momentum?
- What research is being undertaken to improve the device? CAREER LINK

Communicate

Prepare a presentation of your findings that will help the average person decide whether to install the device in a vehicle. Present your findings in a slide presentation, video, poster, blog, website, or other format of your choice.

Plan for Action

Many schools and school districts purchase new vehicles, including school buses. Plan a presentation of your findings to the school board or parent–teacher association that will convince them to install or not install the device on future vehicles. Be sure to compare the cost of installing the device to the cost of damage to vehicles and harm to passengers.

Momentum and the Neutrino

SKILLS HANDBOOK A3

ABSTRACT

Neutrinos, which are subatomic particles that do not carry an electric charge, were first predicted by Wolfgang Pauli in 1930 to account for missing energy and momentum in nuclear reactions. This theory began a search for the elusive particle, which could not be detected at the time. Neutrinos are produced in nuclear reactions in the Sun and on Earth. Predictions about the number of neutrinos produced by the Sun helped to explain how the Sun works and revealed secrets about particle physics. Most modern neutrino detectors lie deep underground to avoid noise from cosmic radiation.

First Hints

Wolfgang Pauli first predicted the existence of neutrinos in 1930. Pauli needed a way to account for missing energy and momentum from certain radioactive decays of atomic nuclei, called beta decay. Experiments at the time seemed to show that beta decay did not conserve energy or momentum. So Pauli suggested that a new particle, which had so far escaped detection, was responsible for carrying off the missing energy and momentum.

Pauli originally called his new particle the neutron, but the same name was being used by other scientists to describe the much more massive particles found in the nuclei of atoms. The name of the still-unproven particle eventually changed to neutrino, or "little neutral one," after the suggestion of Italian physicist Enrico Fermi. Fermi developed Pauli's suggestion of a missing particle into a full theory of beta decay in 1933.

The prediction of the neutrino solved the problem of conservation of energy and momentum during beta decay. However, it proved difficult to actually detect a neutrino and measure its properties. Physicists did not know whether neutrinos had mass or not, but it was obvious that these particles rarely interacted with matter. How could they detect a neutrino if it rarely interacted with other charges or magnets? Over time, researchers found sophisticated methods for detecting neutrinos. Today, several experiments around the world test the neutrino's properties.

Missing Neutrinos

Calculations of the number of neutrinos produced by nuclear reactions in the Sun indicated that billions of neutrinos should be passing through each square centimetre of Earth's surface each second. Despite this fact, the first neutrinos were not actually detected in particle experiments until 1956. The first naturally produced neutrinos were discovered in 1965 in experiments in gold mines in Africa and India.

However, as neutrino detectors improved, scientists detected far fewer neutrinos from the Sun than calculations had predicted. The problem of the missing neutrinos was a challenge for models of the Sun's interior and the accepted theories of particle physics.

In the early 2000s, researchers at the Sudbury Neutrino Observatory (SNO), 2 km below ground near Sudbury, Ontario, found the missing neutrinos (**Figure 1**). The neutrino originally hypothesized by Pauli to explain beta decay was only one of three types of neutrinos. These are now called electron neutrinos, muon neutrinos, and tau neutrinos. The Sun produces the first type, electron neutrinos, the only type that the older experiments could detect.

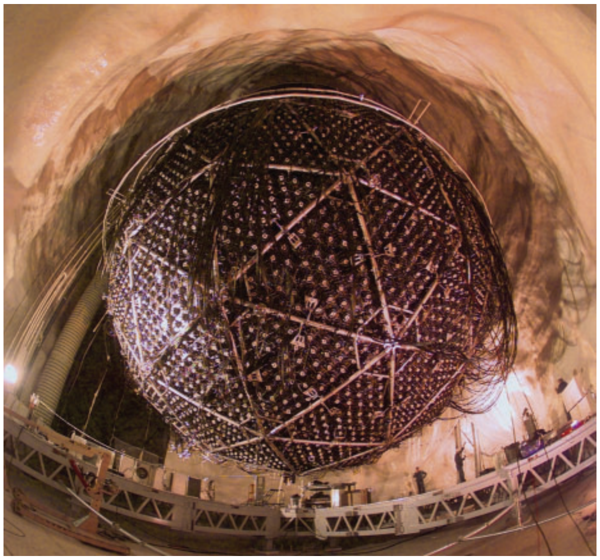


Figure 1 The Sudbury Neutrino Observatory detector sits 2 km underground near Sudbury, Ontario.

SNO, however, could detect all three types, and the total number of all neutrinos detected equalled the predicted total number produced by the Sun. This discovery verified the theory of *neutrino oscillations*, which predicted that a neutrino of one type can transform into either of the other two types and back again. During their journey from the Sun to Earth, the electron neutrinos produced in solar nuclear reactions transform into muon and tau neutrinos. The earlier detectors did not detect the transformed neutrinos, but SNO did.

This discovery supported the theory that neutrinos have mass. Fermi's original theory of beta decay assumed that neutrinos did not have mass, but the theory of neutrino oscillations predicts that the oscillations between types of neutrino can only happen if they do have mass. The theory also predicts that the rate at which the oscillations between types occurs depends on the mass, so researchers can measure the neutrino mass by measuring the oscillation rate. The quest to pin down the mass of the neutrino continues today.

Modern Neutrino Observatories

Today, laboratories used to measure neutrinos are often built deep underground, for example, SNO, Super-Kamiokande in caverns on the island of Honshu, Japan, and IceCube under the Antarctic ice (**Figure 2**). The surrounding material shields detectors from the cosmic rays—high-energy particles from space—that would hide the neutrino signal.

Even in its subterranean location, the SNO detector used heavy water to increase the likelihood of neutrino interaction. Heavy water has extra neutrons that interact with the neutrinos in a process similar to the reverse of beta decay. (Although hydrogen has no neutrons, heavy water is composed of oxygen and deuterium, which has one neutron.) A very small percentage of neutrinos interact with the heavy water, but they were enough to help researchers detect the missing solar neutrinos. Although the SNO detector no longer operates, neutrino research continues in the Sudbury mine at SNOLAB.

Figure 2 The IceCube Neutrino Observatory experiment consists of an array of over 5000 small detectors each buried over 2 km deep under the frozen Antarctic ice. In this photo, one of the small detectors is lowered down a narrow channel in the ice to its final position.

Further Reading

Bahcall, J. (2004). Solving the mystery of the missing neutrinos. Nobel Foundation.

Learned, J., and S. Pakvasa (2005). A neutrino timeline. Department of Physics & Astronomy, University of Hawaii at Manoa.

Siegel, E. (2010). Starts with a bang! The story of the neutrino. *ScienceBlogs*.

WEB LINK

5.7 Questions

1. What is a neutrino? What scientific observations led to its prediction? K/U
2. Studying neutrinos helped to explain how our Sun works but led to changes in theories of particle physics. How is this process consistent with the scientific process? How can details about a theory be adjusted without undermining other discoveries made through the theory's predictions? T/I C
3. Explain how the theory of neutrino oscillations helps to explain the missing electron neutrinos from the Sun. K/U
4. Building large detectors underground can have an impact on the environment around the detector site. Do you think the knowledge we gain from studying neutrinos is worth the environmental impact? Explain your reasoning. C

Investigation 5.2.1 CONTROLLED EXPERIMENT

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Conservation of Momentum in One Dimension

In this controlled experiment, you will explore inelastic collisions in order to test the law of conservation of momentum in one dimension. You will use two dynamics carts, each loaded with various masses, configured to collide in a nearly perfect inelastic manner.

Testable Question

SKILLS HANDBOOK A2.2

How does a collision between two objects in one dimension in an isolated system affect the momentum of each object?

Hypothesis

Read through the experimental design and the procedure for this investigation. Then formulate a hypothesis based on the Testable Question. Your hypothesis should include predictions and reasons for your predictions.

Variables

Consider which factor you will control and which factors will change in response. Then identify all dependent (responding) and independent (manipulated) variables.

Experimental Design

This will be a controlled experiment. You will explore inelastic collisions to test the conservation of momentum in one dimension, using dynamics carts and motion sensors or tickertape timers.

Equipment and Materials

- eye protection
- 2 dynamics carts with magnets and Velcro tabs
- 1 cart track
- masses of varying sizes from 2 g to 500 g
- motion sensors or tickertape timers

Procedure

1. Create a data table in which to record your observations. Put on your eye protection. Set up your track on a level surface. Place a cart at rest on the track to be sure it does not move. Make adjustments to the levelling feet of the track if necessary.

Wear closed-toe shoes to protect your feet from falling masses. To unplug the sensors or timers, pull the plug and not the cord. Use caution when pushing the carts.

2. Place one cart in the middle of the track and the other cart at the far end. The magnets of the carts should face each other so that the carts stick together when they collide. Place the same amount of mass in each cart. Secure the masses to the carts.
3. If you are using a dynamics track, place the motion sensors at the ends of the track so that each will record the speed of one of the carts. You will need to take one measurement for the initial velocity of the first cart, and another for the carts once they are attached.
4. Start the motion sensors. Give the cart at the end of the track an initial velocity toward the cart in the middle using a gentle push.
5. Record the velocities of each cart from the motion sensor in a data table. Be sure to select a direction for positive and negative, and record the velocities properly according to the direction you chose. Also record the total masses of the loaded carts. Use the mass and velocity to calculate the momentum in each case.
6. Repeat Steps 2 through 5 at least once, changing the size of the masses in the carts but keeping them equal. Complete a separate table for the new trial.
7. Repeat Steps 2 through 5 at least twice, using unequal masses in the carts each time. Complete a separate table for each trial.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) According to your data:
  (i) What was the total momentum before and after each collision?
  (ii) How did the velocity change in each trial?
  (iii) How did this change in velocity affect the change in momentum?
  (iv) Explain why you need to compare changes in momentum rather than changes in velocity during a collision. T/I C A

(b) Did you observe any trends in the momentum? Was momentum conserved better in certain types of interactions? Explain. T/I A

(c) To what extent was momentum conserved in your investigation? Express your answer as percentage losses. Explain how some momentum might have been lost. T/I A

(d) How might your results have been affected if you had not balanced the dynamics track carefully before starting? T/I

Apply and Extend

(e) Design a method to measure an unknown mass placed on top of one of the carts. Use your method to measure the masses of several objects, and check your results with an accurate scale. Describe the accuracy of your results. T/I A

(f) Consider a situation in which you are the driver of a car stopped at a red light and you see a car of similar mass approaching rapidly from behind. Use the results of your experiment to discuss possible strategies for reducing the impact of the impending collision. For example, should you take your foot off the brake? Should you accelerate forward? T/I A

(g) Suppose you are riding a skateboard along a narrow path and realize that you are about to have a head-on collision with another skateboarder of similar mass approaching from the opposite end of the path. Use the results of your experiment to describe your best strategy for minimizing injuries from the collision. Assume that jumping off the skateboard is not an option. T/I A

Investigation 5.4.1 CONTROLLED EXPERIMENT

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Head-on Elastic Collisions

In this investigation you will explore head-on elastic collisions using dynamics carts in order to test the law of conservation of momentum and the conservation of kinetic energy. You will investigate cases in which one object is initially at rest and cases in which both objects are in motion. You will also investigate special cases in which one mass is much greater than the other. Alternatively, you can use a computer simulation to conduct this experiment (see Procedure: Part B).

Testable Questions

SKILLS HANDBOOK A2.2

- Is momentum conserved in an elastic collision if one of the objects is at rest before the collision?
- Is momentum conserved in an elastic collision if both of the objects are moving before the collision?
- What effect do mass and velocity have on the momentum of objects in head-on elastic collisions?
- What effect do mass and velocity have on the kinetic energy of objects in head-on elastic collisions?

Hypothesis

Read through the Procedure for this investigation. Formulate a hypothesis based on the Testable Questions. Consider each of the cases in which the speed and the mass vary.

Variables

Identify and record all dependent and independent variables.

Experimental Design

In this controlled experiment, you will explore head-on elastic collisions to test the conservation of momentum in one dimension and the conservation of kinetic energy.

Equipment and Materials

- eye protection
- 2 dynamics carts
- 1 cart track
- masses of varying sizes from 2 g to 500 g
- motion sensors or tickertape timers

Wear closed-toe shoes to protect your feet from falling masses. To unplug the sensors or timers, pull the plug and not the cord. Use caution when pushing the carts.

Procedure

Part A: Dynamics Cart Experiment

1. Read through the procedure and create a data table in which to record your observations.
2. Put on your eye protection. Set up your track on a level surface. Place a cart at rest on the track to be sure it does not move. Make adjustments to the track if necessary.
3. Place one cart in the middle of the track and the other cart at the far end. Place the same amount of mass in each cart.

4. If you are using a dynamics track, place the motion sensors at the ends of the track. The sensors must record the velocity of each cart before and after the collision.
5. Predict whether momentum and kinetic energy will be conserved if a cart in motion strikes a cart at rest.
6. Start the motion sensors. Give the cart at the end of the track an initial velocity toward the cart in the middle.
7. Record the velocities of each cart in your data table. Choose a direction for positive and negative velocity. Also record the total mass of each cart. Use the mass and velocity to calculate the momentum and kinetic energy in each case.
8. Repeat Steps 3 through 7, but this time start both carts at the ends of the track, and give both an initial velocity toward each other. Before you start, predict whether momentum and kinetic energy will be conserved.
9. Repeat Steps 3 through 7, with the mass for the first cart much larger than the mass for the second cart.
10. Repeat Steps 3 through 7, with the mass for the first cart much smaller than the mass for the second cart.

Part B: Simulation

1. Go to the Nelson Science website.
2. Run the simulation. Follow the Procedure in Part A.
3. Record the velocities of each cart in a data table. Choose a direction for positive and negative. Also record the masses you added to each cart. Use the mass and velocity to calculate the momentum and kinetic energy in each case.

WEB LINK

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) Based on this investigation, what can you conclude about the following cases? In each case, explain how your data support your conclusion. T/I
 (i) Is momentum conserved when one of the carts is at rest?
 (ii) Is momentum conserved when both of the carts are moving?
 (iii) Is kinetic energy conserved when one of the carts is at rest?
 (iv) Is kinetic energy conserved when both of the carts are moving?
 (v) Are momentum and kinetic energy conserved when one mass is much larger than the other?

(b) Were the predictions you made before each of your measurements correct? Explain. T/I C

(c) How do you think your results would have differed if the carts had stuck together after the collision? T/I

(d) Do you think the data collected in this investigation might have been affected by the experimental setup or the method of measurement? Explain. T/I C

Apply and Extend

(e) Section 5.4 showed the derivation of the following velocity formulas for a head-on elastic collision in which one object is initially at rest:

$$v_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{i_1} \qquad v_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{i_1}$$

Insert the data you collected for each trial into these equations to analyze your results. You may wish to use a spreadsheet or graphing calculator to carry out the calculations. T/I A

(f) Describe how one or more of the results you obtained in this investigation apply to a situation in everyday life. Address the topics of mass, velocity, momentum, and kinetic energy in your answer. T/I A

Investigation 5.5.1 CONTROLLED EXPERIMENT

Conservation of Momentum in Two Dimensions

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

You read about the dynamics of collisions in two dimensions in Section 5.5. In this controlled experiment, you will design an experiment to explore conservation of momentum in two-dimensional collisions. Alternatively, you can use a computer simulation to conduct this experiment (see Procedure: Part B).

Testable Questions

- Is momentum conserved in all collisions in two dimensions?
- How can conservation of momentum be demonstrated in a glancing collision?

Hypothesis

Read the Testable Questions, Experimental Design, and Procedure for this investigation. Formulate a hypothesis about the testable questions that addresses the concept of conservation of momentum in two dimensions. Your hypothesis should include a prediction and a reason for the prediction.

Variables

Identify and record all dependent and independent variables. You might need to change these variables as you are designing your experiment.

Experimental Design

You will work in teams to design an experiment to test the conservation of momentum in two dimensions. Include in your design some method of making and analyzing a video of the motion of objects during a two-dimensional collision. Consider which objects will move, and how they will collide. How will you record and analyze the motion? One possibility is to use a flip camera to record the motion and use a computer media player to analyze the motion you recorded. You might instead choose to use an online simulation of the motion.

Equipment and Materials

Possible equipment and materials:

- eye protection
- air table
- strobe lights
- digital camera
- 2 pucks
- 2 launchers

Wear closed-toe shoes to protect your feet from falling pucks. Make sure others do not crowd the air table. To unplug the strobes or air table, pull the plug and not the cord. Do not look straight into the strobe lights. Strobe lighting can trigger seizures in people with certain medical conditions.

Procedure

Part A: Air Table Experiment

1. Decide how many trials (at least three) you will include in your investigation to test whether momentum is conserved in two-dimensional collisions. Create one or more data tables based on the dependent and independent variables you identified for your investigation. Predict how the angle at which objects collide and the mass of the objects will vary in the trials. Write a description of your procedures and draw a diagram of your design setup. Include all necessary safety precautions. Ask your teacher to approve your plans.
2. Put on your eye protection. Set up the design for your experiment. If you are using a strobe light, conduct the experiment in the dark. Make sure everyone in the classroom is ready before your teacher dims the lights.
3. Practise launching the objects and operating the camera before conducting your experiments. Be careful not to launch the objects so hard that they leave the table.
4. If you are using a strobe light, the strobes should be set to flash 10 times per second (600 rpm).
5. Conduct your experiment and record your data.

Part B: Simulation

1. Go to the Nelson Science website.
2. Decide how many trials (at least three) you will include in your investigation to test whether momentum is conserved in two-dimensional collisions.
3. Create one or more data tables based on the dependent and independent variables you identified for your investigation. Write a description of your procedures and draw a diagram of the simulation. Ask your teacher to approve your plans.
4. Predict how the angle at which objects collide and the mass of the objects will vary in the trials.
5. Run the simulation and record your data.

WEB LINK

Analyze and Evaluate

(a) What type of relationship was being tested in this investigation? K/U T/I

(b) How can you use your data to determine to what extent momentum is conserved in a two-dimensional interaction? In your answer, explain why you must consider the perpendicular vector components of the collisions. T/I

(c) What effect did changing the angle of the collision have on your results? T/I

(d) Is it possible to predict the path of each object after a collision if you know only the initial masses and velocities? If not, what additional information do you need to make these predictions? T/I

Apply and Extend

(e) How can you apply the results of this investigation to improve your chances of causing a billiard ball at rest to move into a pocket of the billiards table if the cue ball strikes the ball at rest with a glancing collision? T/I A

Summary Questions

1. Read the Key Concepts on page 220. For each point, create a list of related key terms and equations. Create a one-page learning aid that summarizes one of the Key Concepts, using images and important definitions.
2. Look back at the Starting Points questions on page 220. Working in pairs, review the answers you gave for each question. Discuss the answers that you would change and explain your reasoning. As you work, create a graphic organizer that summarizes what you learned. Share your results with the class. Compare these results to those of other groups. In what areas did you have misconceptions before studying the concepts presented in the chapter?

Vocabulary

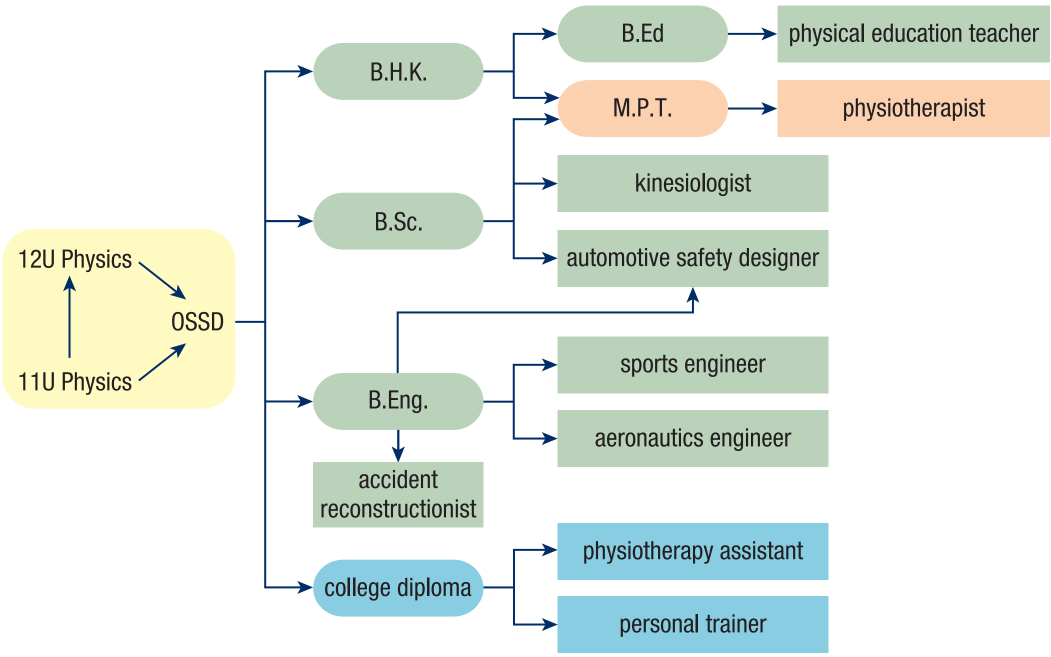
linear momentum (p. 222)
impulse (p. 223)
collision (p. 228)
law of conservation of momentum (p. 229)
explosion (p. 229)
elastic collision (p. 233)
conservation of kinetic energy (p. 233)
inelastic collision (p. 234)
perfectly elastic collision (p. 235)
perfectly inelastic collision (p. 235)
head-on elastic collision (p. 240)
glancing collision (p. 250)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers mentioned in this chapter.

1. Select two careers related to momentum and collisions that you find interesting. Research the educational pathways you would need to follow to pursue these careers. What is involved in the required educational programs? Prepare a brief report of your findings.
2. For one of the two careers that you chose, describe the career, main duties and responsibilities, working conditions, and setting. Also outline how the career benefits society and the environment.



CAREER LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. Which of the following has the greatest magnitude of momentum? (5.1) K/U T/I
   (a) a 60 kg skier travelling at 80 km/h
   (b) a jumbo jet with a mass of 408 233 kg taxiing on the runway at 3 km/h
   (c) a 1 kg object ejected from an airplane with a speed of 960 km/h relative to the ground
   (d) a proton with a mass of 1.67×10^{-27} kg travelling at 99.995 % of the speed of light (3.0×10^{8} m/s)
2. Suppose two billiard balls, A and B, with the same mass undergo a head-on elastic collision. Ball A was initially stationary. Which of the following outcomes is possible following the collision? (5.2) K/U
   (a) One ball is moving and one is stationary.
   (b) Both balls are moving.
   (c) Both balls are stationary.
   (d) All of the above outcomes are possible.
3. Which of the following uses conservation of momentum to move? (5.2) K/U T/I
   (a) a rocket being launched out of Earth's atmosphere
   (b) a squid taking in water and expelling it in one direction
   (c) a balloon deflating and flying around the room
   (d) all of the above
4. A cannon with a mass of 346 kg shoots a 12 kg cannonball at a speed of 126 m/s. At what speed does the machine recoil? (5.2) K/U T/I
   (a) 2.2 m/s
   (b) 4.4 m/s
   (c) 8.7 m/s
   (d) 1.5×10^{2} m/s
5. Which of the following collisions can be treated as most elastic? (5.3) K/U T/I A
   (a) A chef throws a piece of spaghetti against the wall to test whether it is done.
   (b) In a game of marbles, the shooter marble strikes two smaller marbles.
   (c) An egg rolls off the counter and hits the kitchen floor.
   (d) A meteorite strikes the Moon and creates a crater.
6. In a perfectly elastic collision, a small marble collides with a stationary marble three times its mass. What percentage of the small marble's kinetic energy is transferred to the stationary marble after the collision? (5.4) K/U
   (a) 25 %
   (b) 50 %
   (c) 75 %
   (d) 100 %
7. An asteroid moves through deep space and suddenly breaks into two pieces of equal mass. The two pieces fly off at a right angle to each other. What can you conclude? (5.5) K/U
   (a) The pieces have equal final speed.
   (b) The pieces both travel at 45° to the original direction of the asteroid.
   (c) The pieces have equal kinetic energy.
   (d) All of the above are true.

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. Momentum is a scalar quantity. (5.1) K/U
9. When a collision occurs between two objects, the vector sum of the momenta changes. (5.1) K/U
10. All collisions conserve momentum and can be distinguished on the basis of whether they also conserve kinetic energy. (5.2) K/U
11. During a hockey game, a stationary goalie stops a puck. If we know his kinetic energy after the save, we can determine the initial kinetic energy of the puck. (5.2) K/U T/I
12. When two bodies travelling toward each other at the same speed collide, the resultant velocity of each body is different. (5.2) K/U
13. In reality, collisions between heavy objects can only be approximately elastic. (5.3) K/U
14. A curling stone hits a wall at a right angle and rebounds with a final speed that is nearly equal to its initial speed. Its final path will almost be the mirror image of its initial path. (5.5) K/U T/I A

Go to Nelson Science for an online self-quiz.
WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. Suppose object A has greater momentum than object B. Which of the following can you conclude? (5.1) K/U
   (a) Object A has a greater mass than object B.
   (b) Object A has a greater velocity than object B.
   (c) Object A has greater kinetic energy than object B.
   (d) None of the above is necessarily true.
2. A 61 kg gymnast falls vertically from a jump onto a trampoline. Her speed as she hits the trampoline is 5.2 m/s, and she comes to a stop in 0.20 s. What is the average magnitude of the force exerted on the gymnast by the trampoline? (5.1) K/U
   (a) 6.1 N
   (b) 305 N
   (c) 610 N
   (d) 1600 N
3. A ball with a mass of 0.5 kg, initially at rest, is struck with a bat and acquires a velocity of 4.0 m/s. What is the magnitude of the change in momentum of the ball? (5.2) K/U
   (a) 0.5 kg·m/s
   (b) 2.0 kg·m/s
   (c) 2.5 kg·m/s
   (d) 10.0 kg·m/s
4. Two tennis balls undergo a head-on elastic collision. Under which of these initial conditions is it impossible for both balls to be moving in the same direction after the collision? (5.4) K/U T/I
   (a) The lighter ball is stationary and the heavier ball is in motion.
   (b) The two balls have the same mass and are initially moving in the same direction, and they collide because the faster-moving ball overtakes the slower-moving one.
   (c) The two balls have the same mass and only one is moving.
   (d) none of the above

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

5. You can determine the average force exerted on an object during a collision if you know only the object's momentum before and after the collision. (5.1) K/U
6. When an acorn falls and hits the ground, Earth's response is imperceptible. (5.2) K/U A
7. In an inelastic collision only momentum is conserved. (5.3) K/U
8. When two objects undergo a perfectly elastic head-on collision, each object will always have a final velocity equal to the initial velocity of the other object. (5.4) K/U T/I
9. A head-on collision is a collision in which the initial and the final velocities of colliding masses lie in the same line. (5.4) K/U
10. In glancing collisions in two dimensions, momentum is no longer conserved. (5.5) K/U
11. Scientists can detect neutrinos using conservation of momentum. (5.7) K/U

Understanding

12. Verify, using the definition of momentum, that the units for momentum are the same as those for force multiplied by time. (5.1) K/U
13. Two friends in a hurry to go picnicking decide to stand outside a window, holding a cloth to catch a watermelon tossed out the window by a third friend. Explain why a stretchy cloth is less likely to tear than an inflexible cloth when the watermelon hits it. (5.1) K/U T/I A
14. Two construction workers use different hammers to pound in nails. Both swing their hammers with the same speed, and the duration of both hammers' collisions with the nails is equal. However, one worker seems to achieve more force than the other. Offer a possible explanation. (5.1) K/U T/I A
15. A 57 g tennis ball approaches a player horizontally at a speed of 6.0 m/s. The player hits the ball with a racquet in a collision that lasts 4.0 ms. To return the ball with a horizontal speed of 7.0 m/s, how much average force must the player apply? (5.1) K/U A
16. A car with a mass of 1100 kg is travelling at a speed of 33 m/s. Determine the magnitude of the total momentum. (5.1) T/I
17. When a meteor enters Earth's atmosphere, it slows down. Explain why this does not violate conservation of momentum. (5.2) K/U T/I A

18. In each of the following situations, explain why conservation of momentum appears to fail. These are not isolated systems. (5.2) K/U A
    (a) On a stretch of gravel, two race cars collide and come to a stop.
    (b) In a flowing river, a stick repeatedly bumps the shore.
    (c) A bus slows down, picks up a passenger, and speeds back up again.
19. A cup is sliding over a frictionless table while a waiter pours water into it from above. Describe what will happen to the cup's speed. (5.2) K/U T/I A
20. A bobsled team rides a sled across a horizontal runway of ice. Describe what happens to the sled's speed as the sledders jump off the sled. (5.2) K/U T/I A
21. Two tennis balls of equal mass are moving in directions opposite to each other. The tennis balls are travelling with equal speed when they collide head-on. You can assume that this collision is perfectly elastic. Describe in your own words what happens after the tennis balls collide. (5.2) T/I C A
22. Using conservation of momentum, explain whether the following situation is possible: Two objects collide head-on with equal and opposite velocities. When they rebound, the velocity of each object is doubled. (5.2) T/I
23. Two soccer players collide head-on and are stopped. If the mass of one player is 1.2 times the mass of the other player, what can you conclude about their initial speeds? (5.3) K/U
24. Classify the following collisions as elastic, inelastic, or perfectly inelastic. (5.3) K/U T/I A
    (a) A child throws a lump of modelling clay against a refrigerator. It rebounds, but a part of it sticks.
    (b) Two electrons collide in a cyclotron.
    (c) A ball is thrown into jelly.
    (d) Two marbles bounce off each other after colliding.
25. The data in **Table 1** represent the given information for a head-on elastic collision in one dimension. Determine the final velocities for each row. (5.4) T/I

Table 1

(a)	$m_1 = 25$ kg	$v_1 = 6.0$ m/s [E]	$m_2 = 15$ kg	$v_2 = 0$ m/s
(b)	$m_1 = 12$ kg	$v_1 = 8.0$ m/s [E]	$m_2 = 22$ kg	$v_2 = 2.0$ m/s [E]
(c)	$m_1 = 150$ kg	$v_1 = 2.0$ m/s [N]	$m_2 = 240$ kg	$v_2 = 3.0$ m/s [S]

26. Hockey player 1 is travelling at a velocity of 12 m/s [N] and hockey player 2 is travelling at a velocity of 18 m/s [S] when they collide head-on. After colliding, the hockey players hang on to each other and slide along the ice with a velocity of 4.0 m/s [S]. If hockey player 1 weighs 120 kg, calculate how much hockey player 2 weighs. (5.4) K/U T/I
27. In a demonstration in physics class, a 1.2 kg dynamics cart starts from rest at the top of a ramp. The ramp is 2.4 m above the ground. The cart then rolls down to the bottom of the ramp, where it collides with a stationary 1.4 kg dynamics cart. Assume that an elastic head-on collision occurs. Calculate the speed of each cart just after the collision. (5.4) T/I
28. A 1.2 kg cart slides eastward down a frictionless ramp from a height of 1.8 m and then onto a horizontal surface where it has a head-on elastic collision with a stationary 2.0 kg cart cushioned by an ideal Hooke's law spring. The maximum compression of the spring during the collision is 2.0 cm. (5.4) T/I
    (a) Determine the spring constant.
    (b) Calculate the velocity of each cart just after the collision.
    (c) After the collision, the 1.2 kg cart rebounds up the ramp. Determine the maximum height reached by the cart.
29. Two people on inner tubes collide head-on on a frictionless surface of ice. The first inner tube and its rider have a total mass of 81 kg, and the second inner tube with rider has a total mass of 93 kg. The final velocities of the two inner tubes, including the riders, are 1.7 m/s [N] and 1.1 m/s [S], respectively. (5.4) T/I A
    (a) Determine the initial velocities of the inner tubes.
    (b) Determine the total kinetic energy of the inner tubes and riders.
    (c) Determine the total momentum of the inner tubes and riders.
30. A sailboat with a mass of 240 kg glides at a speed of 4.3 m/s on frictionless ice, runs aground on mud, and comes to a stop after 3 s. Determine the average force of friction exerted by the mud on the boat. (5.1) K/U A
31. Two balls of different masses and equal speeds undergo a head-on elastic collision. If the balls are moving in opposite directions after the collision, how can you determine from the outcome of the collision which object has a greater mass? (5.4) K/U

32. A curling stone travelling at 5.0 m/s collides with a stationary stone of the same mass. Following the collision, the two stones travel at angles of 17° and 38° in opposite directions with respect to the initial motion of the first stone. (5.5) K/U T/I C
   (a) Draw a diagram of the stones' motion.
   (b) Calculate the speed of each stone after the collision.
33. During a spacewalk, three astronauts wearing jetpacks approach each other at equal speeds along lines equally spaced by an angle of 120° (**Figure 1**). As the astronauts approach each other, they take each other's hands. If the astronauts come to rest after colliding, what conclusion can you draw? (5.5) T/I

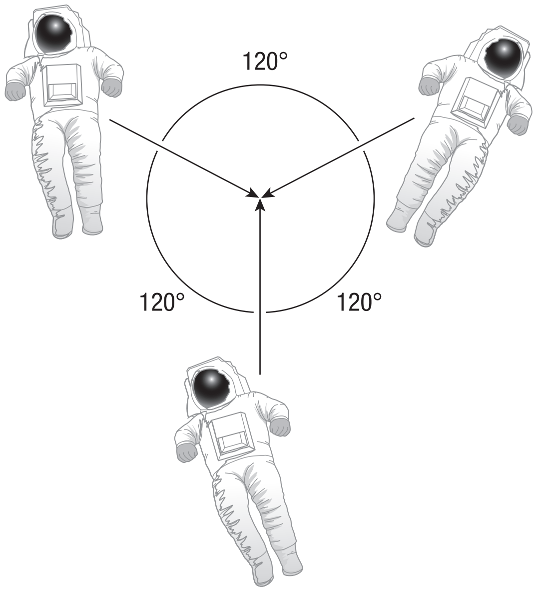


Figure 1

Analysis and Application

34. (a) If two objects with non-zero mass and non-zero velocity have equal momentum and equal kinetic energy, what can you conclude about their velocities?
   (b) Can you draw any conclusion about their masses? Explain your answer. (5.1) K/U T/I A
35. Draw three different graphs of force applied to an object over a time interval so that in each graph, the impulse is the same. (5.1) K/U T/I C
36. For each of the following collisions, calculate the force exerted on the object. (5.1) K/U T/I C A
   (a) A baseball with a mass of 0.152 kg hits a cement wall. Immediately before the collision, the baseball is travelling horizontally at 35 m/s. The collision lasts 1.6 ms. Immediately after the collision, the baseball is travelling horizontally away from the wall at 29 m/s.
   (b) A squash ball with a mass of 0.125 kg collides horizontally with a cement wall at a speed of 25 m/s. The collision lasts for 0.25 s. Immediately after the collision, the squash ball travels horizontally away from the wall at 23 m/s.
   (c) In a forensics test, a metal projectile with a mass of 0.06 kg collides horizontally with a cement wall at a speed of 340 m/s. The collision lasts 0.1 ms. Immediately after the collision, the projectile travels horizontally away from the wall at 3 m/s.
37. Ball 1 of mass 0.1 kg makes an elastic head-on collision with ball 2 of unknown mass that is initially at rest. If ball 1 rebounds at one-third of its original speed, determine the mass of ball 2. (5.1) K/U C A
38. At the circus, a human cannon is used to convert the potential energy of the performer to kinetic energy (**Figure 2**). To achieve maximum height, the organizers are debating whether to use a lighter performer with higher speed, or a heavier performer with less speed. Explain which they should choose. (5.1) K/U T/I C

Figure 2

39. Suppose a watermelon with a mass of 2.0 kg undergoes a head-on elastic collision on a frictionless counter with a grapefruit with a mass of 0.8 kg. If the total kinetic energy of the system is 10.5 J and the total momentum is 7.5 kg·m/s, determine the possible initial and final velocities for the watermelon and the grapefruit. (5.4) K/U A

40. A team of four 63 kg bobsledders push their sled, which has a mass of 210 kg when empty. They start to push their sled over a flat frictionless surface at an initial speed of 3.0 m/s. One by one, at intervals of 2.0 s, each bobsledder sprints forward at a speed of 2.0 m/s faster than the sled's current speed and then jumps in. (5.2) K/U T/I A
   (a) Determine the sled's final speed once all of the sledders are in.
   (b) Determine the sled's final momentum once all of the sledders are in.

41. A force acts on a 2.4 kg object with the magnitude shown in **Figure 3** as a function of time. (5.1) K/U C A

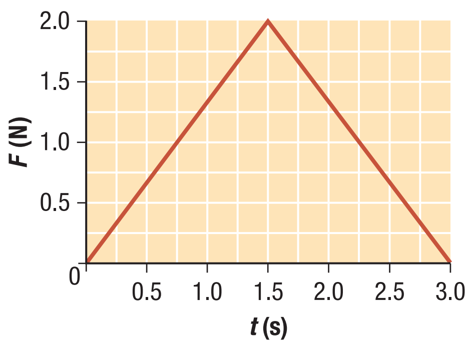


Figure 3

   (a) Determine the impulse imparted by the force on the object.
   (b) Determine the final velocity of the object if it had an initial velocity of 14 m/s in the negative direction.

42. A frog leaps at a constant horizontal speed from a lily pad to an adjacent lily pad. The lily pads have the same mass and are initially stationary on a frictionless surface. When the frog has completed the leap, both lily pads are moving. (5.2) K/U T/I
   (a) What are the directions of the lily pads' motion?
   (b) Which lily pad has a higher speed? Explain why.

43. A rocket in deep space with an initial velocity of 12 km/s [forward] sheds its rear stages, which represent two-thirds of its mass. The rear stages travel with a velocity of 10.0 km/s [backward]. The remaining rocket continues in the initial direction of motion. Calculate the rocket's velocity after shedding the rear stages. (5.2) K/U T/I A

44. Two 0.3 kg gliders collide elastically on a frictionless track. Prior to the collision, their total kinetic energy is 0.52 J and their total momentum is 0.12 kg·m/s [left] along the track. Calculate the final velocities of the gliders. (5.2) K/U T/I A

45. A boy is at rest on a sheet of flat frictionless ice. He throws a snowball of mass 0.02 kg at a speed of 18 m/s in a horizontal direction. If his mass is 75 kg, how fast will the recoil make him drift on the ice? (5.2) K/U A

46. (a) In Question 45, what is the total kinetic energy
      (i) before the boy throws the snowball?
      (ii) after the boy throws the snowball?
   (b) Why are your answers in (a) different?
   (c) Where did the difference in kinetic energy come from? (5.3) T/I A

47. Suppose a circus selects a performer of mass 78 kg to be shot from the human cannon with a kinetic energy of 1.2×10^5 J. On his way out of the cannon, the performer holds his arm ahead of him to punch a stationary beach ball of mass 40.0 g balanced on a post. The beach ball flies out in the direction of the performer's motion, and the performer's speed is reduced by 0.1 m/s. (5.3) K/U T/I A
   (a) What is the speed of the beach ball after the collision?
   (b) If the collision of the performer's fist with the beach ball lasts for 5 ms, how much average force did he exert on the ball?
   (c) What is the performer's final speed?
   (d) Could you have answered (a) and (b) without determining the performer's final speed? Explain.

48. On a frictionless sheet of ice, an 810 kg adult moose skids toward a stationary baby moose at a speed of 5.2 m/s, and they collide and continue together in the same direction. The final velocity of the adult moose and baby moose system is 4.85 m/s. Determine the mass of the baby moose. (5.3) K/U A

49. A block of ice of mass 50.0 g slides along a frictionless, frozen lake at a speed of 0.30 m/s. It collides with a 100.0 g block of ice that is sliding in the same direction at 0.25 m/s (**Figure 4**). The two blocks stick together. (5.3) K/U A
   (a) How fast are the two blocks moving after the collision?
   (b) How much kinetic energy is lost?

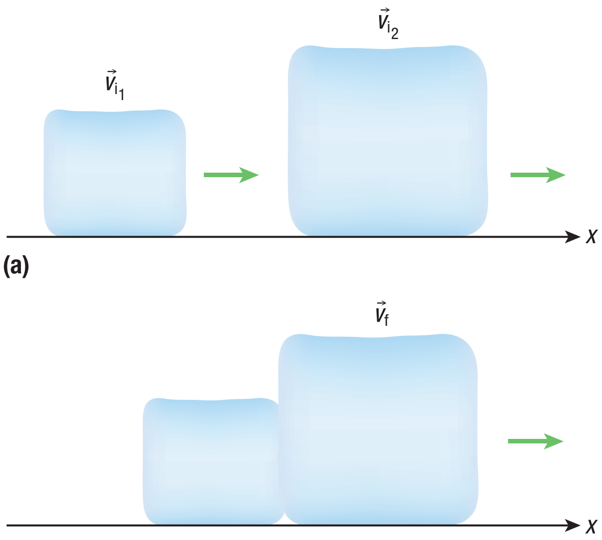


Figure 4

50. Using your answer from Question 48, suppose that the adult moose again approaches at 5.2 m/s. This time, after the two collide, the baby moose's final velocity is 8.0 m/s in the direction of the adult moose's original motion. What is the adult moose's final velocity? (5.4) K/U A
51. Two equal-mass hockey pucks undergo a glancing collision. Puck 1 is initially at rest and is struck by puck 2 travelling at a velocity of 13 m/s [E]. Puck 1 travels at an angle of [E 18° N] after the collision. Puck 2 travels at an angle of [E 4° S]. Determine the final velocity of each puck. (5.5) K/U A
52. In a 1500 kg car, 1.3 m of its front section is designed to crumple in an accident, protecting the driver and passengers. Suppose the car is travelling at 32 m/s and comes to a stop while uniformly slowing down over 1.3 m. (5.6) T/I A
   (a) What is the duration of the collision?
   (b) What is the average force exerted on the car?

Evaluation

53. Give two examples of collisions in open systems. For each example, define the open system and explain how to expand an open system to be closed. (5.1) T/I C A
54. Consider a perfectly elastic head-on collision between two objects of equal mass m. (5.2) T/I C A
   (a) Use algebraic reasoning to prove that conservation of kinetic energy can be expressed as $v_1^2 + v_2^2 = C$, where C is a constant and v_1 and v_2 represent the speeds of the objects at any time other than the instant of the collision.
   (b) Similarly, show that conservation of momentum can be expressed as $v_1 + v_2 = C'$, where C' is another constant.
   (c) Graph these two equations on the same set of axes for v_1 versus v_2.
   (d) At how many points do the graphs intersect? What do the intersections represent?
55. Suppose that a bowling ball collides elastically with a row of stationary bowling balls all of the same mass. All the bowling balls are confined to move only along the gutter beside the lane in a bowling alley. Prove that after the collision only one ball can be in motion. Use the laws of conservation of momentum and conservation of energy (kinetic energy) to support your answer. (5.2) T/I C A
56. Two objects undergo an elastic head-on collision in one dimension, with one object initially at rest and the other moving at 12 m/s [E]. Make a prediction for each scenario below, explaining your reasoning. Then calculate the velocity of each object after the collision for each situation. (5.4) K/U T/I A
   (a) The moving object is twice the mass of the stationary object.
   (b) The stationary object is twice the mass of the moving object.
   (c) The moving object is 106 times the mass of the stationary object.
   (d) The stationary object is 106 times the mass of the moving object.
57. Suppose that two objects undergo a perfectly elastic collision. The first object, with an initial velocity of v_i, is much more massive than the second object, which is initially at rest. (5.4) T/I C
   (a) Predict what the final velocities will be after the collision.
   (b) Use the final velocity equations to determine the approximate final velocities of both masses.
   (c) Compare these results with your predictions.

Reflect on Your Learning

58. What did you find most surprising in this chapter, and what did you find most interesting? T/I C
59. (a) Do you feel that you could explain momentum to a fellow student who has not taken physics? Discuss your answer with a classmate.
    (b) Does anything about momentum and conservation of momentum still confuse you? T/I C

Research

WEB LINK

60. Research Newton's cradle, which was named after Sir Isaac Newton. Explore how the device works and observe what factors are being conserved. T/I C A
61. A Galilean cannon demonstrates the conservation of momentum. Research the Galilean cannon. In a short oral presentation, describe how you could use a basketball and a tennis ball to demonstrate the principle behind this device. T/I C A
62. While landing on the ground, skydivers and paratroopers always keep their knees bent (**Figure 5**). Research and write a report on how the various laws of conservation work in this case. If possible, speak with a professional in this field and highlight the safety measures that skydivers and paratroopers should take while landing. T/I C A

Figure 5

63. Research the standard spacing and height of 10-pin bowling pins and the standard radius of bowling balls. Discuss in a one-page report how the game would be easier or more difficult if these standards were changed. Would it be possible to knock down all the pins with one shot? At what point would it become inevitable? T/I C A
64. Research the length of crumple zones in cars. How should the length of the crumple zone vary with the mass of the car? Research what parameters are varied in crash tests and what information is contained in crash test ratings. Write up your findings in a short report. K/U T/I C A
65. Principles of momentum play a part in the safe demolition of buildings. As modern buildings become larger and taller and urban areas become more densely populated, methods of safe building demolition must improve. Research methods of building demolition and how the methods have evolved. Prepare a multimedia report that includes video examples of demolition techniques. Include your thoughts on practical ways to improve demolition technology. K/U T/I C A
66. Use the concept of momentum to explain how child car seats help protect children riding in motor vehicles. Describe some ways that the design of standard child car seats or the materials used in them might be improved. K/U T/I C A
67. Research the history of crash test dummies and summarize the impact their use has had on motor vehicle safety research. Why are crash test dummies used? What impact have innovations in crash test dummy design and use had on traffic accident injuries? K/U T/I C A
68. In the Unit Task on page 270, you may choose to design a Rube Goldberg machine. Research Rube Goldberg machines and write a short summary of your findings. Include the following information in your summary: T/I C A
    (a) What was Rube Goldberg's educational background and career pathway? Did his education and the jobs he held have an impact on his designs?
    (b) Describe two of Rube Goldberg's designs and identify any physics principles at work.
    (c) Discuss how Rube Goldberg's machines have had an impact on society.

Applications of Energy and Momentum in Engineering Design

When engineers and technicians develop new technologies, they must carefully analyze how their devices use energy and momentum. For example, safety technologies such as airbags and bicycle helmets must protect us from the forces in collisions. To do that, the safety devices need to transform energy and absorb momentum so that we do not.

Machines work by transforming energy and redirecting momentum. For example, a solar-powered car first transforms solar energy into electrical energy in its solar cells, and then transforms electrical energy into mechanical energy in its motor to run the car. Engineers design crumple zones, seat belts, and airbags to redirect momentum in collisions and protect passengers. CAREER LINK

The Task

SKILLS HANDBOOK A1.2

In this Unit Task, you will perform one of two tasks. You will either design a new safety device that transforms energy and absorbs momentum to protect a fragile object, such as an egg, or you will design a Rube Goldberg machine, which is a machine that completes a simple task in a complicated way (**Figure 1**). Consider how your device uses the concepts you have studied in this unit. Read through the questions at the end of the Task to help you plan your procedure and presentation.

Review safety and design rules before you begin. You must use all tools safely, and you must test your design in a safe and controlled manner.

Use equipment you are comfortable with. Have your teacher approve your design plans before you execute them, and seek permission before you use any tools in the lab. Consider all necessary safety precautions before building.

Option 1: Egg-Drop Protector Device

Your task is to design and build a container that will protect an egg from breaking when it is dropped from a height of 2 m, or another height as determined by your teacher. For this project, you will need an egg and simple materials for construction, such as

- paper
- tape
- packing supplies
- polystyrene foam
- tissue
- cotton balls

Analyze how your design uses principles of energy and momentum to complete the task successfully. Test your design and make improvements based on the results.

Finally, demonstrate the functionality of your design to the class. Prepare a presentation to describe the usefulness of your final design and the process you followed to create it.

Option 2: Rube Goldberg Machine

Your task is to design and build a Rube Goldberg machine. Rube Goldberg was a cartoonist in the early twentieth century who used his engineering talents to point out, in a humorous way, how people often like to do things the hard way. You can physically construct your machine or use simulation software to design it. If you build your machine consider all necessary safety precautions. If you use a simulation it must be of real (not imaginary) equipment and materials. Materials you may wish to use could include:

- wood
- nails
- cardboard
- recycled plastic or metal containers
- tape
- string
- wire
- eye protection

Figure 1 Rube Goldberg machines are complicated devices built to perform simple tasks.

Analyze how the machine's operation relies on the principles of energy and momentum. Prepare a presentation to describe the purpose of your design and the process you followed to create it.

Analyze and Evaluate

Option 1

(a) Which physics principles did you use to design your container? K/U A

(b) Calculate the energy and momentum of your egg when its container hits the ground. T/I A

(c) Describe what happened to the momentum and energy of the egg during its fall. Calculate the relative gravitational potential energy, kinetic energy, elastic potential energy, and momentum at both the top and the bottom of the drop. K/U T/I C A

(d) What safety measures did you follow while designing and testing your container? C A

(e) Describe your testing procedure. T/I C A

(f) Describe the changes you made to the design after testing your container. T/I C

(g) List problems that you encountered while designing the container, and describe how you overcame them. T/I C A

(h) Compare your design to the designs of other students. What types of features did the successful designs have in common? T/I C A

(i) Create a flow chart or other graphic organizer of the process you followed to design and build your container. Did your final design successfully protect the egg? If you had to do this task again, how would you change the process? How would you change the design? T/I C A

Option 2

(j) Which physics principles did you use to design your machine? K/U A

(k) Describe the purpose of your machine. C A

(l) Estimate the cost of building a permanent, fully functional version of your machine. You may have to do some research. T/I

(m) List the sequence of energy transformations in the operation of your machine. K/U T/I A

(n) Analyze the exchange of momentum for one to three key collisions that occurred in your machine. Estimate the gravitational potential energy, kinetic energy, elastic potential energy, and momentum at each of these key points. K/U T/I A

(o) Compare your design to the design of other students. Which machines were the most successful at accomplishing their tasks? Why do you think they were successful? T/I C A

(p) Describe the design process that you followed. Did your machine work? Which parts worked well, and which did not? If you had to do this task again, how would you change the process? T/I C A

Apply and Extend

Option 1

(q) Describe how to extend your design to protect larger objects, including parts of the human body, from damage during a collision. C A

(r) Assess the environmental impact of your design. Explain what you could do to reduce the negative impact of your device. T/I C A

Option 2

(s) Consider the costs that you estimated in Step (l). Based on your research, how might you reduce some of these costs? C A

(t) Describe how to adapt your machine for a second use by making the fewest changes possible. C A

(u) Assess the environmental impact of your design. Explain what you could do to reduce the negative impact of your device. T/I C A

ASSESSMENT **CHECKLIST**

Your completed Unit Task will be assessed according to the following criteria:

Knowledge/Understanding

- ✓ Demonstrate knowledge of concepts of work, energy, and momentum.
- ✓ Demonstrate knowledge of conservation of energy and momentum.
- ✓ Demonstrate safety skills in the laboratory.

Thinking/Investigation

- ✓ Investigate relationships between conservation of energy and momentum in real-life and imagined collisions.
- ✓ Analyze conservation of energy and momentum in interactions and collisions.
- ✓ Develop a plan for designing a safety device or complicated machine.
- ✓ Improve the design of an egg-drop container or Rube Goldberg machine.
- ✓ Construct a successful egg-drop container or Rube Goldberg machine.
- ✓ Evaluate the success of your design.
- ✓ Evaluate and improve your design process.

Communication

- ✓ Communicate your design, procedure, and modifications in the form of a flow chart or other graphic organizer.
- ✓ Demonstrate and explain the functionality and design of the device in a presentation.
- ✓ Communicate the results of your design clearly and concisely.

Application

- ✓ Propose alternative uses for your device and describe the impact on society of your device.
- ✓ Assess the cost and environmental impact of your design.

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. A weightlifter lifts a 200 kg mass over his head, and then sets it back on the floor where it started. Which of the following statements is true? (4.1) K/U
   (a) The weightlifter did positive work on the mass.
   (b) The weightlifter did negative work on the mass.
   (c) The weightlifter did zero net work on the mass.
   (d) The mass did work on the weightlifter.
2. The units of energy can be written as
   (a) J
   (b) N·m
   (c) kg·m/s^2
   (d) kg·m/s (4.1) K/U
3. A large water tank is filled by pumping water from a reservoir below using electric pumps. About 3.2×10^6 kg of the water is lifted 79 m in 12 min. How much work is done by the pumps? (4.1) K/U T/I A
   (a) 6.4×10^5 J
   (b) 3.2×10^6 J
   (c) 3.0×10^7 J
   (d) 2.4×10^9 J
4. How much power is produced by the pumps in Question 3? (4.5) K/U T/I A
   (a) 6.0×10^5 W
   (b) 3.4×10^6 W
   (c) 2.9×10^7 W
   (d) 2.3×10^9 W
5. A 10.0 kg toy airplane starts from rest and speeds up to 5.0 m/s in 3.0 s. What work is done by the airplane's motor, assuming the work done by friction is negligible? (4.2) K/U
   (a) 0 J
   (b) 25 J
   (c) 42 J
   (d) 130 J
6. A girl can produce 710 W of power over small intervals of time. If the girl's mass is 42 kg, how many seconds will it take her to climb a flight of stairs that is 12 m high? (4.3) K/U A
   (a) 7.0 s
   (b) 1.3 s
   (c) 4.5 s
   (d) 5.2 s
7. The roller coaster car in **Figure 1** is at point A. At which point is its potential energy with respect to the ground greater than it is at point A? (4.3) K/U A
   (a) B
   (b) C
   (c) D
   (d) E

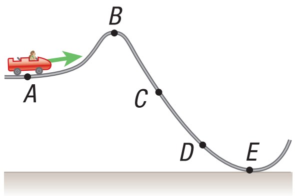


Figure 1

8. In Question 7, if the total of the kinetic energy and potential energy at point A is 100 000 J, then what is the total energy at point C (assuming no friction forces)? (4.5) K/U T/I A
   (a) more than 100 000 J
   (b) exactly 100 000 J
   (c) less than 100 000 J
   (d) none of the above
9. A 5900 kg airplane flies at 220 m/s at an altitude of 1600 m. What is the total gravitational potential energy of the airplane with respect to the ground? (4.5) K/U T/I A
   (a) 3.0×10^7 J
   (b) 5.9×10^7 J
   (c) 6.0×10^8 J
   (d) 9.3×10^7 J
10. If a boy throws a 2.0 kg stone straight up into the air with a speed of 12 m/s just over the edge of a cliff that looms 55 m above the ocean, at what speed does the stone hit the water? (4.5) K/U T/I A
    (a) 35 m/s
    (b) 98 m/s
    (c) 130 m/s
    (d) 210 m/s
11. A student is bouncing on a trampoline. When does she have maximum speed? (4.7) K/U T/I A
    (a) at the top of her motion, above the trampoline
    (b) halfway down
    (c) as she is depressing the trampoline surface
    (d) as she just leaves the flat trampoline surface

12. Which of the following is an application of simple harmonic motion? (4.7) K/U T/I A
    (a) a rubber ball bouncing on the ground
    (b) the pendulum on a grandfather clock
    (c) a person jumping on a trampoline
    (d) a swimmer swimming laps
13. A hockey player taps the hockey puck during a game. The puck does not go very fast because
    (a) the force was large and the time of contact was long
    (b) the force was large and the time of contact was short
    (c) the force was small and the time of contact was short
    (d) the force was small and the time of contact was long (5.1) K/U T/I A
14. An egg falling on a wood floor will break, but an egg falling from the same height onto a pillow will not break. What is the difference in these situations? (5.1) T/I A
    (a) The pillow delivers a small force over a long time to stop the egg.
    (b) The pillow delivers less impulse in stopping the egg.
    (c) The wood floor delivers a small force over a long time.
    (d) The pillow delivers more impulse in stopping the egg.
15. Tripling the speed of an object has what result on its momentum? (5.1) T/I A
    (a) multiplying it by 2
    (b) multiplying it by 3
    (c) multiplying it by 4
    (d) multiplying it by 9
16. An 82 kg hockey forward carries the puck at 2.5 m/s [N]. A 110 kg defender moves at 1.2 m/s [S] and delivers a check to the forward. The players slide across the ice together after the check. Calculate their final velocity. (5.3) T/I A
    (a) 0.13 m/s [S]
    (b) 0.23 m/s [S]
    (c) 0.38 m/s [N]
    (d) 0.32 m/s [N]
17. In the grocery store, Marie stops her cart in an aisle. Gerard gives his cart, twice the mass of Marie's cart, a push and the cart heads toward Marie's, at a speed of 3 m/s. The carts collide and move off together with a common speed. What is the final speed of the carts, and in which direction do the carts move with respect to the original direction of Gerard's cart? (5. 3) K/U T/I
    (a) 2 m/s, in the opposite direction to Gerard's cart
    (b) 3 m/s, in the opposite direction to Gerard's cart
    (c) 3 m/s, in the same direction as Gerard's cart
    (d) 2 m/s, in the same direction as Gerard's cart
18. Two marbles of the same mass collide head-on. The first marble moves at 11 cm/s to the right. The second marble moves at 18 cm/s to the left. After the collision, the first marble moves at 16 cm/s to the left. What is the velocity of the second marble? (5.4) T/I A
    (a) 9.0 cm/s [right]
    (b) 16 cm/s [left]
    (c) 23 cm/s [right]
    (d) 11 cm/s [left]
19. A 71 kg boy and a 43 kg girl, both wearing skates, face each other at rest on a skating rink. The boy pushes the girl eastward with a speed of 4.6 m/s. Ignoring friction, determine the velocity of the boy. (5.4) T/I A
    (a) 2.8 m/s [W]
    (b) 2.8 m/s [E]
    (c) 3.3 m/s [W]
    (d) 3.3 m/s [E]

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

20. Energy is a scalar quantity. (4.1) K/U
21. The unit of both work and energy is the joule. (4.1) K/U
22. Excluding any energy losses due to friction, the net work done on an object is equal to the change in kinetic energy of that object. (4.1) K/U
23. Friction does positive work on a baseball player sliding into home plate. (4.1) K/U T/I A
24. The energy contained in gasoline is referred to as kinetic energy. (4.3) K/U
25. *Energy* and *power* are different words that have the same meaning. (4.5) K/U
26. A diver has 800 J of kinetic energy as he hits the water if he has 500 J of potential energy and 300 J of kinetic energy at one instant in his dive. (4.5) T/I A
27. Baseball players follow through on their swings because it increases the impulse delivered to the ball. (5.1) T/I
28. In any collision, momentum is conserved. (5.2) K/U
29. When two objects collide in a perfectly elastic collision, kinetic energy is conserved but momentum is not. (5.3) K/U
30. The total momentum of two bumper cars before a collision is the same as the momentum after the collision. (5.3) K/U T/I A
31. In a perfectly elastic two-body head-on collision, the objects collide and travel in the reverse direction with twice their original speed. (5.4) K/U
32. If two billiard balls of equal mass travelling at equal but opposite initial velocities collide in a glancing collision, their final velocities will also be equal but opposite. (5.5) K/U

Knowledge

For each question, select the best answer from the four alternatives.

1. A worker lifts a box upward from the floor and then carries it across the warehouse. When is he doing work? (4.1) K/U
   (a) while lifting the box from the floor
   (b) while carrying the box across the warehouse
   (c) while standing in place with the box
   (d) at no time during the process
2. The equivalent to 1 J is
   (a) 1 kg·m/s^2
   (b) 1 kg·m^2/s
   (c) 1 kg·m^2/s^2
   (d) 1 kg·m/s (4.1) K/U
3. A car of mass 1.0×10^3 kg travels forward at 12 m/s. How much distance is required to completely stop the car using only a soft braking force of 720 N? (4.1) T/I
   (a) 50 m
   (b) 100 m
   (c) 200 m
   (d) 500 m
4. The car in Question 3 doubles its speed. What happens to its kinetic energy? (4.2) K/U
   (a) The kinetic energy stays the same.
   (b) The kinetic energy doubles.
   (c) The kinetic energy triples.
   (d) The kinetic energy quadruples.
5. A grocer is stocking items on a store shelf. She lifts a box of detergent from the floor and stacks it on top of another box of detergent that is resting on the shelf. At which point is the second box of detergent's gravitational potential energy greatest relative to the ground? (4.3) K/U
   (a) when the box is at rest on the ground
   (b) when the top of the box reaches the shelf height
   (c) when the bottom of the box reaches the shelf height
   (d) when the bottom of the box is set on top of the box resting on the shelf
6. A cart at the farmer's market is loaded with potatoes and pulled at constant speed up a ramp to the top of a hill. If the mass of the loaded cart is 3.0 kg and the top of the hill has a height of 0.45 m, then what is the potential energy of the loaded cart at the top of the hill? (4.3) K/U
   (a) 1.3 J
   (b) 0.13 J
   (c) 13 J
   (d) 130 J
7. A cheetah cub is resting on a tree branch 20.0 m above the ground. The cub has a mass of 6.0 kg. How much gravitational potential energy does the cheetah cub have relative to the ground? (4.3) K/U
   (a) 12 J
   (b) 120 J
   (c) 1200 J
   (d) 12 000 J
8. A baseball player drops the ball from his glove. At what moment is the ball's kinetic energy the greatest? (4.3) K/U
   (a) when the baseball player is holding the ball
   (b) just before the ball hits the ground
   (c) at the ball's highest point before beginning to fall
   (d) the moment the ball leaves the baseball player's glove
9. Which of the following will increase the kinetic energy of a hammer as it strikes the head of a nail? (4.5) K/U
   (a) using a hammer with greater mass
   (b) swinging the hammer with a greater downward velocity
   (c) lifting the hammer to a greater height before swinging it down
   (d) all of the above
10. Which of the following represents a way to calculate power? (4.5) K/U
   (a) Power equals work divided by time.
   (b) Power equals work divided by kinetic energy.
   (c) Power equals time divided by work.
   (d) Power equals mass divided by time.

11. The driver of a car applies her brakes. What happens to the car's kinetic energy as it comes to rest? (4.5) K/U
    (a) It is transformed into potential energy.
    (b) It is transformed into thermal energy and sound energy.
    (c) It is completely lost.
    (d) It becomes power.
12. A skier maintains a constant speed as she descends a mountain slope. Which of the following best describes the energy transfer during this process? (4.5) K/U
    (a) Kinetic energy is transformed into potential energy.
    (b) Thermal energy is transformed into kinetic energy.
    (c) Potential energy is transformed into thermal energy.
    (d) Thermal energy is transformed into potential energy.
13. Two students of different masses run a 50 m sprint. They finish in the same amount of time. Which student produces more power during the sprint? (4.5) K/U
    (a) the student with less mass
    (b) the student with more mass
    (c) the student who took longer strides
    (d) Both students produce the same power.
14. A hand-held candy dispenser ejects small candy pieces using a spring. The candy is placed in the dispenser and the spring is compressed. When a button is pushed, the candy flies forward. Which of the following best accounts for the candy's motion? (4.6) K/U
    (a) The potential energy stored in the spring is converted to work, which then becomes the kinetic energy of the candy.
    (b) The kinetic energy of the compressed spring is converted to the potential energy of the candy.
    (c) The simple harmonic motion of the spring becomes the kinetic energy of the candy.
    (d) The simple harmonic motion of the spring transfers potential energy to the candy.
15. A mass attached to an ideal spring oscillates on a horizontal frictionless surface. The velocity of the mass is greatest when
    (a) the displacement is at a maximum
    (b) the kinetic energy is at a minimum
    (c) the mass is at the equilibrium position
    (d) the spring force is at a maximum (4.6) K/U
16. What shape is the force–displacement graph for an ideal spring? (4.6) K/U
    (a) parabolic
    (b) linear
    (c) exponential
    (d) none of the above
17. For a spring-powered candy dispenser, which of the following correctly expresses the relationship of the candy's speed to the compression distance of the spring? (4.7) K/U
    (a) The speed is proportional to the square root of the compression distance of the spring.
    (b) The speed is directly proportional to the compression distance of the spring.
    (c) The speed is inversely proportional to the compression distance of the spring.
    (d) The speed is proportional to the square of the compression distance of the spring.
18. An archer stores 150 J of potential energy in a bow as he pulls back the string. The 0.30 kg arrow leaves the bow with an initial speed of 30 m/s. How much energy was lost to vibrations and deformation of the bow? (4.7) K/U
    (a) 0 J
    (b) 15 J
    (c) 30 J
    (d) 50 J
19. The system comprising a block of mass m and a spring with spring constant k forms a damped oscillator. As energy is transferred to thermal energy, the mechanical energy of the system
    (a) increases
    (b) decreases
    (c) does not change
    (d) oscillates in value (4.7) K/U
20. In a high school hockey game, two players of the same mass are at rest. One player pushes the other away. What is true about their velocities? (5.3) K/U
    (a) They are equal and in the same direction.
    (b) They are equal but in opposite directions.
    (c) The player who pushes has twice the speed of the other player, but in the opposite direction.
    (d) The player who got pushed has twice the speed of the other player, but in the opposite direction.

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

21. Gravity does positive work on a mountaineer as she climbs up the side of a cliff. (4.1) K/U
22. If an object has no velocity, it has no kinetic energy. (4.2) K/U
23. Power is the rate at which work is done. (4.4) K/U
24. At a hydroelectric power plant, the gravitational potential energy of water can be transformed into kinetic energy. (4.5) K/U

25. A diver jumps from a diving board. If the work done by air resistance is negligible, at any given moment until she enters the water the sum of her kinetic energy and her gravitational potential energy is not constant. (4.5) K/U
26. In a classic gravity-driven roller coaster, the initial hilltop must be the highest. (4.5) K/U A
27. As you expand a spring, the force necessary to continue pulling it apart decreases. (4.6) K/U
28. As a particle moves in circular motion about the origin, the *x*-component of the particle's displacement is an example of simple harmonic motion. (4.6) K/U
29. The spring constant, *k*, is measured in joules. (4.6) K/U
30. A child jumping on a trampoline causes energy transformations between kinetic energy, gravitational potential energy, and elastic potential energy. (4.6) K/U
31. The minus sign in the equation for Hooke's law, $F = -kx$, indicates that the spring force is always directed opposite to the spring's displacement. (4.6) K/U
32. A ball bouncing up and down due to gravity is in simple harmonic motion. (4.6) K/U A
33. Both gravitational potential energy and elastic potential energy depend on an object's elevation. (4.6) K/U
34. A mass oscillating vertically on a spring can have three types of energy: kinetic, gravitational potential, and elastic potential. (4.6) K/U A
35. The amplitude in simple harmonic motion is the magnitude of the displacement of the particle in one direction only. (4.6) K/U A
36. A larger spring constant means a weaker pull or push of the spring for a given displacement. (4.7) K/U A

Understanding

37. Would you do the same work to lift a 2 kg box vertically through 1.5 m on the Moon as you would to lift it on Earth? Explain. (4.1) K/U T/I
38. A box slides down an inclined plane, increasing in speed. Does the normal force do any work on the box? Explain. (4.1) K/U T/I
39. A push is applied to an object, and the object undergoes a displacement. However, the object's speed does not increase. What can you conclude about the system? (4.1) K/U T/I
40. A horizontal force of 50 N is needed to push a 500 kg piano across a floor. How much work is done by the force in displacing the piano 20 m? (4.1) K/U
41. To gain height on a playground swing, a child raises and lowers his legs at just the right moment. Discuss the energy transformations as the child swings gradually higher and higher. (4.2) T/I C A
42. A weightlifter is able to lift 250 kg through 2.1 m in 2.4 s. Determine his power output. (4.2) K/U
43. A swimmer experiences a horizontal reaction force from the blocks to her feet at the start of a race. What will the work done on the blocks and change in energy be? (4.2) K/U T/I
44. A 41 kg block is uniformly slowed by a friction force of 5.0 N. The block travels 29 m before coming to rest. (4.2) K/U
    (a) Calculate the work done by friction.
    (b) Calculate the initial speed of the block.
45. A compact car has a fuel economy of 12 km/L. A mid-sized car has a fuel economy of 7 km/L. Explain, using energy concepts, why there is a difference in fuel economy. (4.2) K/U T/I
46. (a) A 150 kg wrecking ball is lifted to a height of 25 m. Determine the gravitational potential energy of the wrecking ball with respect to the ground. (4.3)
    (b) If the wrecking ball is dropped to the ground, neglecting air resistance, what is the kinetic energy of the ball on impact? (4.5) K/U T/I
47. A 91 000 kg airplane is flying at 980 km/h at a height of 12 km. Determine its total energy (kinetic plus gravitational potential). Assume $g = 9.8 \text{ m/s}^2$. (4.3) K/U
48. From what vertical height should a 3.6 g marble be dropped so that it hits a rubber mat with a speed of 6.5 m/s? (4.3) T/I
49. A rock climber with a mass of 110 kg slips and falls 12 m before safely reaching the end of his rope. (4.3) T/I
    (a) Determine the change in his potential energy during the fall.
    (b) Determine the climber's speed when the rope stops his fall. What assumption do you need to make?
50. A rubber ball is thrown upward. As it rises, gravity does negative work on it. Describe what happens to the kinetic energy of the ball. (4.5) K/U T/I
51. Consider two springs: one is a suspension spring on a car (**Figure 1**); the other is a spring on the screen door of a house. Which do you think has a larger spring constant? Explain your answer. (4.6) T/I A

Figure 1 Car suspension spring

52. A spring is used to project a 0.021 kg ball into the air. If the spring constant is 160 N/m and if the spring is compressed 0.13 m, determine the height to which the ball rises. (4.7) K/U T/I

53. A soccer player intends to use his chest to control a ball kicked by another player. (5.1) T/I
   (a) He first uses his chest to redirect the kicked ball directly upward to a maximum height of 3.0 m above his chest. The mass of the ball is 0.43 kg. Calculate the momentum of the ball when it strikes the player's chest after falling from this height.
   (b) The player then uses his chest to bounce the ball straight upward again. If the impact of the ball with the player's chest lasts for 0.2 s, and he imparts an average force of 31 N [up], calculate the speed of the ball after this second collision.
54. A bungee jumper with a mass of 78 kg, tied to a 39 m cord, jumps off the Sault Ste. Marie bridge from a height of 69 m. She falls to 6.0 m above the ground before the cord brings her momentarily to rest. Calculate the impulse exerted on the jumper by the cord as it stretches. (5.1) T/I
55. **Figure 2** shows the graph of the force of a toy car crashing into a brick wall. Determine the impulse. (5.1) T/I A

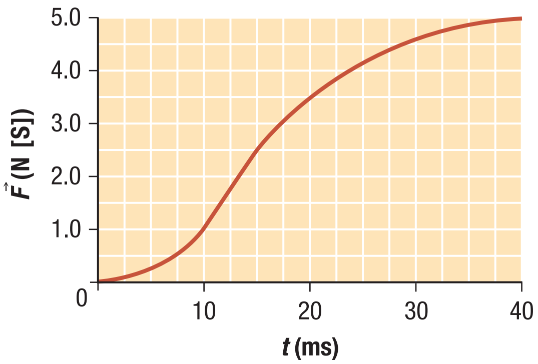


Figure 2

56. How did concepts of energy and momentum play a part in the discovery of the neutrino? (5.7) K/U A
57. A grapefruit is tossed across a room. Ignoring air friction, describe what happens to the horizontal component of the linear momentum. (5.2) K/U
58. A wet snowball collides with a stationary parked car. Is this an example of an inelastic collision? Why or why not? (5.3) K/U
59. (a) Give an example of a collision in everyday life where one of the objects is at rest after the collision.
   (b) Give an example of a collision where both objects are at rest after the collision. (5.3) K/U
60. In a movie stunt, two cars collide head on and lock bumpers on an icy, frictionless road. Car 1 has a mass of 1850 kg and an initial velocity of 26 m/s [E]. Car 2 has a mass of 1200 kg. The velocity of the cars after the collision is 6.5 m/s [E]. Determine the initial velocity of car 2. (5.3) T/I
61. A 46 kg hockey player accelerates from rest at 3.4 m/s^2 for 2.7 s and then has a perfectly inelastic collision with a stationary 56 kg player. Friction is negligible. What is the speed of each hockey player immediately after the collision? (5.3) T/I
62. Two objects of masses $m_1 = 1.5$ kg and $m_2 = 3.5$ kg undergo a one-dimensional head-on collision as shown in **Figure 3**. Their initial velocities along x are $v_{i_1} = 12$ m/s and $v_{i_2} = -7.5$ m/s. The two objects stick together after the perfectly inelastic collision. (5.3) T/I A

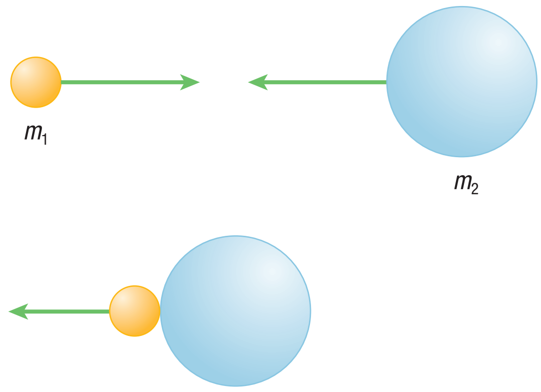


Figure 3

   (a) Calculate the velocity after the collision.
   (b) Determine how much kinetic energy is lost in the collision.

Analysis and Application

63. A child pulls a sled through the snow a distance of 12 m by applying a force of 480 N at an angle of 21° with the horizontal. Calculate the work he does, assuming no friction. (4.1) T/I A
64. A father pushes a stroller 200.0 m through the park with a force of 0.25 N. Ignoring friction, how much work has he done on the stroller? (4.1) T/I A
65. A gardener pushes a lawnmower with a force of magnitude 9.3 N. If this force does 87 J of work on the lawnmower while pushing it 11 m across level ground, determine the angle between the applied force and the horizontal. (4.1) K/U T/I
66. A 1200 kg car travels at 25 m/s. The brakes are applied and the car slows down at −8.0 m/s^2. How far does the car travel before it stops? (4.1) T/I A
67. A force of 52 N acts on a wooden block at an angle of 13° from the horizontal. The block moves a horizontal distance of 3.8 m. Calculate the work done by the applied force. (4.1) T/I A

68. Calculate the work done by an applied force to lift an 18 kg box 5.1 m vertically at a constant speed. (4.1) K/U T/I A

69. A 73 kg skier moving at 4.2 m/s encounters a hill inclined at 10° to the horizontal. She coasts up the hill until she comes to rest. Friction is negligible. Determine the distance up the hill (not vertical distance) the skier slides before stopping. (4.2) T/I

70. A 2.0 kg stone is dropped from 10.0 m above the ground. Ignoring air resistance, determine the stone's speed when it hits the ground. (4.3) T/I

71. A pole vaulter uses a pole to jump to a height of 4.0 m at a high school track event. At this height, the pole vaulter's potential energy is 2.7 kJ and he has zero velocity. Calculate the mass of the pole vaulter. (4.3) T/I

72. Is it possible for a rubber ball to be dropped from shoulder height and then bounce to a height above your head? Explain your answer. (4.3) K/U T/I A

73. A baseball is thrown from a cliff 41 m high with an initial velocity of 22 m/s at an angle of 37° above the horizontal. (4.3) K/U T/I
    (a) Determine the speed of the ball just before it hits the ground.
    (b) How does your answer change if the angle of elevation changes to 60°?

74. A 55 kg snowboarder practises in a hemispherical half-pipe with radius 4.0 m (**Figure 4**). Assume the half-pipe sides are frictionless. Determine
    (a) the potential energy of the snowboarder at the top of the half-pipe
    (b) the kinetic energy of the snowboarder at the bottom of the half-pipe
    (c) the potential and kinetic energy of the snowboarder at a point C if point C is 2.0 m above the half-pipe bottom (4.5) T/I A

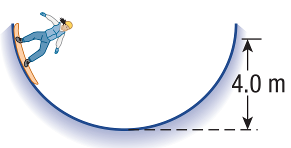


Figure 4

75. When a meteorite collides with the Moon, surface material at the impact site melts. Explain why. (4.5) T/I C A

76. A fire hose directs a stream of water at a rate of 20 kg/s and with a speed of 30 m/s against a flat plate of metal. Calculate the force required to hold the plate in place. (4.5) T/I

77. A girl and her bicycle have a total mass of 39 kg. At the top of a hill, the girl's speed is 3.4 m/s. The hill is 11 m high and 11 m long horizontally (not along the slope). If the force of friction as she rides down the hill is 22 N, determine her speed at the bottom. (4.5) K/U T/I

78. The water in the Niagara River at the top of Niagara Falls is held by a dam (**Figure 5**). Describe the transformation of the water's energy as it drops over the falls. (4.5) K/U A

Figure 5

79. A 60 g golf ball is dropped from a height of 2.0 m and rebounds to 1.5 m. Determine how much energy is lost. (4.5) K/U T/I A

80. A skier starts at the top of a frictionless slope and pushes off with a speed of 2.0 m/s. The elevation of the slope is 45 m. She skis down the slope to a valley with elevation 0.0 m and then glides to the peak of an adjacent slope that is at an elevation of 31 m. Calculate her speed at the second peak. (4.5) T/I A

81. A baseball player bats a baseball so that it travels in a parabolic arc. The player bats the ball near ground level and the ball leaves the bat at a speed of 45 m/s, making an angle of 35° above the horizontal. The mass of the baseball is 0.25 kg. (4.5) T/I A
    (a) Calculate the maximum height of its trajectory.
    (b) What is its speed as it hits the ground again? (Neglect air resistance.)

82. A 1990 kg rocket sled on horizontal frictionless rails is loaded with 102 kg of propellant. It exhausts the propellant in a burn of 25 s. The rocket starts at rest; it moves with a speed of 240 m/s after the burn. (4.5) T/I A
    (a) Determine the impulse experienced by the rocket sled.
    (b) Calculate the average force experienced by the rocket during the burn.
83. A baseball with a mass of 0.2 kg reaches home plate at a speed of 40 m/s and is batted straight back to the pitcher at a speed of 60 m/s. (4.5) T/I
    (a) Determine the impulse experienced by the ball.
    (b) Calculate the magnitude of the change in the ball's momentum.
84. A 100.0 kg hockey player moves east at a speed of 5.0 m/s and collides with a 130 kg opponent moving west at 3.0 m/s. The collision is perfectly inelastic. Find the velocity of the two players immediately after the collision. (5.4) T/I A
85. A neutron in a nuclear reactor moves at 1.0×10^6 m/s and makes an elastic head-on collision with a carbon atom initially at rest. The mass of the carbon atom is 12 times the mass of the neutron. Determine the fraction of the neutron's kinetic energy transferred to the carbon atom. (5.3) T/I A
86. A paintball gun launches a paintball off a cliff at an angle of elevation of 45°. The cliff is 165 m high. The paintball is initially moving at 180 m/s. Calculate the speed of the paintball as it hits the ground. (4.5) T/I
87. **Figure 6** shows a roller coaster car at rest on the track at the top of a hill that is 15 m high. The car has a mass of 10.0 kg. It begins to move and reaches a speed of 10.0 m/s at the top of the second hill, point A, on the track. Calculate the height of hill A. (4.5) T/I

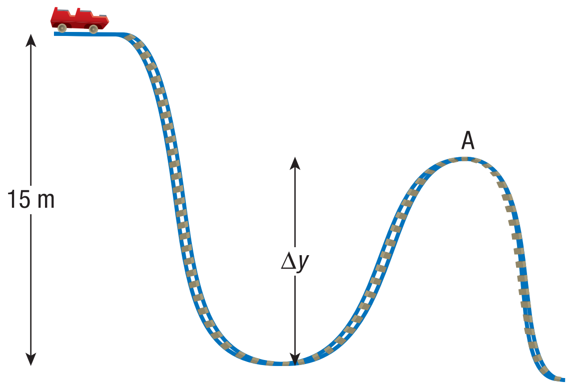


Figure 6

88. A spring is stretched 0.62 m from equilibrium by a force of 199 N. (4.6) T/I
    (a) Determine the spring constant.
    (b) Calculate the magnitude of the force required to stretch the spring 0.25 m from equilibrium.
    (c) Determine the work done on the spring to stretch it 0.25 m from equilibrium.
    (d) Determine the work done on the spring to compress it 0.50 m from equilibrium.
89. Give one example in which the damping of vibrations is
    (a) useful
    (b) not useful (4.7) C A
90. A 9.1 g ball is hit into a 98 g block of clay at rest on a level surface. After impact, the block slides 8.0 m before coming to rest. If the coefficient of friction is 0.60, determine the speed of the ball before impact. (4.7) T/I
91. Engineers perform a crash test with a minivan and a compact car. The mass of the minivan is 8.0×10^2 kg, and the mass of the compact car is 560 kg. The minivan was moving north, and the compact car was moving east. After the collision, the two cars crumpled together and moved at 15 m/s [N 30° E]. Determine the initial velocity of each vehicle. (5.5) T/I
92. You drop a ball toward Earth. (5.2) K/U T/I A
    (a) What is the direction of the gravitational force exerted by Earth on the ball?
    (b) What is the direction of the gravitational force exerted by the ball on Earth?
    (c) Which exerts the greater force?
    (d) If Earth and the ball are initially stationary, how does Earth move after the ball drops?
    (e) If somebody in China drops an identical ball at exactly the same instant from the same height, how does that modify your answers? State your assumptions.
93. While clearing off the surface of a frozen lake for an ice rink, a worker strikes an ice chunk with a shovel, causing the chunk to shatter into two fragments. The fragments slide across the frozen surface in opposite directions. The speed of the first fragment, with a mass of 2.2 kg, is 1.2 m/s. The mass of the second fragment is 3.3 kg. Determine the speed of the second fragment. (5.2) T/I A

Evaluation

94. A 25.6 kg child pulls a 4.81 kg toboggan up a hill inclined at 25.7° to the horizontal. The vertical height of the hill is 27.3 m. Friction is negligible. (4.1) T/I A
    (a) Determine how much work the child must do on the toboggan to pull it at a constant velocity up the hill.
    (b) Now suppose that the hill is inclined at an angle of 19.6° but the vertical height is still 27.3 m. What conclusion can you make?
95. If during a physical process the only forces acting on a moving object are friction and a normal force, what must be assumed about the object's kinetic energy? (4.5) T/I
96. Summarize the design and operation of a roller coaster in terms of conservation of mechanical energy. Include answers to the following questions. (4.5) K/U T/I A
    (a) What happens to the speed of the car as it reaches the top of a hill?
    (b) What happens to the speed of the car as it goes down the hill? Explain.
97. (a) Earth travels in an elliptical orbit around the Sun. When closer to the Sun, Earth moves more quickly. In terms of Earth's gravitational potential and kinetic energy in relation to the Sun, explain why Earth moves faster when closer to the Sun.
    (b) Does Earth have more total energy in relation to the Sun when it is closer or farther away? Explain. (4.5) K/U T/I C A
98. A gymnast with a mass of 105 kg stands on the edge of a springboard. The spring constant is $k = 7600$ N/m. Calculate the period and frequency of the board's vibrations when she jumps. (4.6) T/I A
99. A student measures the amount of stretch of an elastic band under increasing applied forces (**Table 1**). (4.7) T/I C A
    (a) Graph these data on a force–stretch-distance graph.
    (b) Use the graph to determine the average spring constant of the elastic band in newtons per metre.

Table 1 Stretch and Force of an Elastic Band

Stretch (cm)	Force (N)
5	0.7
10	1.6
15	2.5
20	3.4
25	4

100. Explain how knowledge of vibrations and damping is useful to an automotive engineer when designing cars. (4.7) K/U T/I C A
101. During a tennis match, a pro player hits a serve at 53 m/s. Determine the work done on the 61 g ball to achieve that speed. (4.7) K/U T/I A
102. When a ball collides with a stationary clay block, and the block moves off in the direction of the ball, momentum and total energy are conserved, but mechanical energy is not conserved. Explain how this is possible. (5.1) K/U T/I C A

Reflect on Your Learning

103. What did you find most surprising in this unit? What did you find most difficult to understand? How can you learn more about the surprising or difficult topics? C
104. Describe the connections between force, work, energy, and momentum. K/U C
105. (a) How can you relate Hooke's law to everyday applications?
     (b) Describe the limit to which something elastic can stretch. K/U C
106. What are the practical applications of understanding the law of conservation of energy? K/U C

Research

WEB LINK

107. An important task in automobile accident reconstruction is the analysis of skid marks. The data on a car's tires and the road surface can help a reconstruction engineer make a good estimate of a car's speed just before the driver hit the brakes. Research accident reconstruction engineering and discuss how the simple idea of friction is used to get a basic idea about what happened in an accident. Alternatively, perform Internet research to determine the skills required to become an accident reconstruction engineer. Briefly summarize your findings in a format of your choosing. K/U T/I C A

108. Swings and trapezes (**Figure 7**) are examples of one of the classic problems of physics: the pendulum. Hanging a mass and making it swing may be very simple, but the pendulum is an example of a complicated system called a non-linear oscillator. The application of conservation of energy allows us to obtain some important details about the non-linear oscillator. Research the pendulum and write a brief report that summarizes the speeds at various points in the cycle, including at the bottom and at the extreme ends. K/U T/I C A

Figure 7

109. Research photoelectric solar energy. Discuss the energy output of solar panels. How large an area of land must be covered in solar panels to provide all the electrical needs of Canada? K/U T/I C A

110. Research reflective solar energy as used in the power station in the French Pyrenees and the research station in California (**Figure 8**). How much energy is produced at these stations? How many cities could these plants supply with electrical energy? K/U T/I C A

Figure 8

111. Research wind power from modern windmills. Compare the energy production with solar and other energy sources. T/I C A

112. A torsion test measures the strength of any material against maximum bending. It is an extremely common test used in material mechanics to measure how much bend a certain material can withstand before cracking. Research and write a brief report on how Hooke's law plays a part in conducting this test. K/U T/I C A

113. Research Newton's original statement of his second law of motion, in terms of momentum and force. Give the equation expressing this relationship. K/U T/I C A

114. Research airbags. Describe the beneficial effects of airbags used to cushion passengers from collision with the dashboard or steering wheel of a car during an accident. In your description, use the concepts of impulse and momentum. Determine the projected number of lives saved in Canada every year by these safety devices. K/U T/I C A

115. Research the "impulse engine" for use in space travel. Discuss with a classmate the principle of how it works and summarize your findings. K/U T/I C A

116. Compare the transformation of energy in Canada's hydroelectric power plants to that of power plants using fossil fuels. Create a list of pros and cons associated with each system. Based on your research, which generation method is more energy efficient? Should Ontario invest in hydroelectric power plants or fossil fuel generation? Support your reasoning with evidence. K/U C A

117. Research bioluminescent organisms. What advances in human technology have been made possible through the study of bioluminescence in microorganisms, plants, fungi (**Figure 9**), and insects? K/U C A

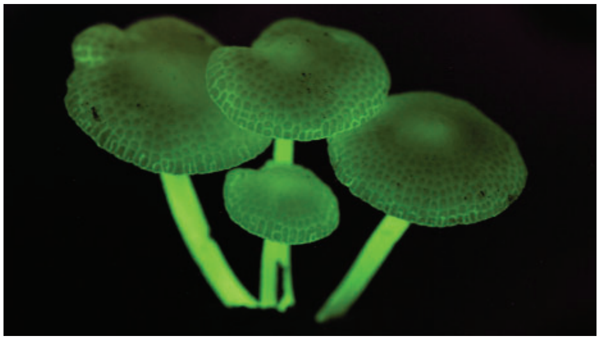


Figure 9

UNIT 3 Gravitational, Electric, and Magnetic Fields

OVERALL EXPECTATIONS

- analyze the operation of technologies that use gravitational, electric, or magnetic fields, and assess the technologies' social and environmental impact
- investigate, in qualitative and quantitative terms, gravitational, electric, and magnetic fields, and solve related problems
- demonstrate an understanding of the concepts, properties, principles, and laws related to gravitational, electric, and magnetic fields and their interactions with matter

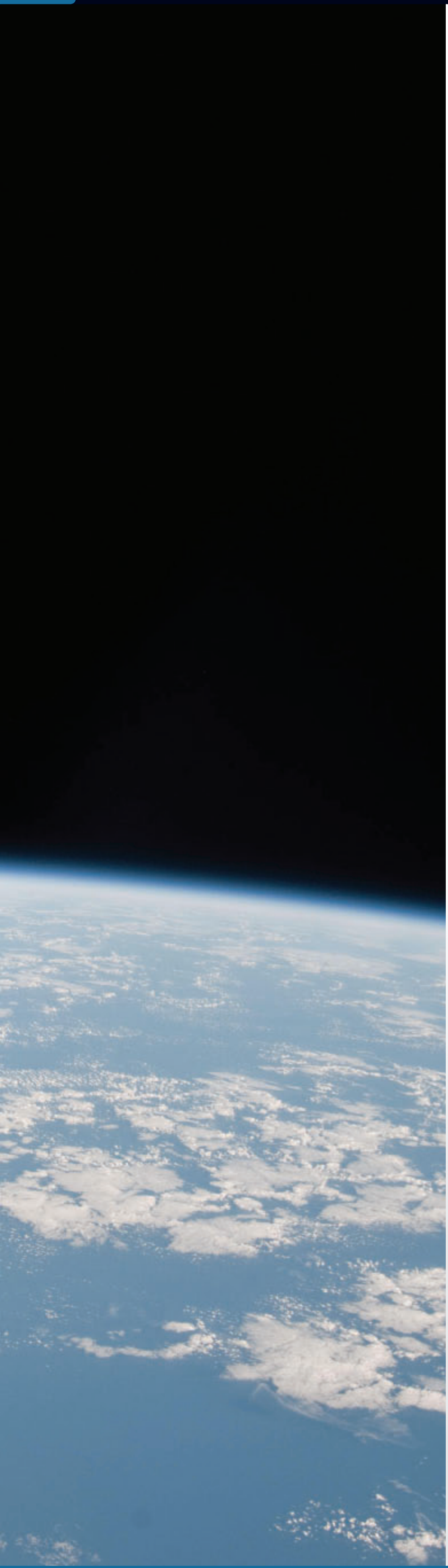
BIG IDEAS

- Gravitational, electric, and magnetic forces act on matter from a distance.
- Gravitational, electric, and magnetic fields share many similar properties.
- The behaviour of matter in gravitational, electric, and magnetic fields can be described mathematically.
- Technological systems that involve gravitational, electric, and magnetic fields can have an effect on society and the environment.

UNIT TASK **PREVIEW**

In this Unit Task, you will analyze a technological system, such as an underwater vehicle navigation system, that uses one or more types of fields. You will examine the effects of gravitational, electric, and magnetic fields on the system. You will assess the impact of this system on society and on the environment.

The Unit Task is described in detail on page 422. As you work through the unit, look for Unit Task Bookmarks to see how information in the section relates to the Unit Task.



FOCUS ON **STSE**

THE INTERNATIONAL SPACE STATION

The weather reports you rely on to plan events depend on artificial satellites. So do cellphones, Global Positioning System (GPS) units, and many other electronic devices that are common to modern living. Scientists must calculate where to put these satellites so that they remain in orbit around Earth. These calculations require an understanding of gravitational fields. The satellite must achieve the correct velocity so that Earth's gravitational pull keeps it in orbit at a specific height above Earth's surface. The precise velocity depends on the altitude above Earth where the satellite will orbit. Scientists and engineers use their knowledge of gravity to calculate a flight plan for the satellite's launch.

Once the satellite is in orbit, the engineers and scientists on the ground (ground control) adjust its motion to make the orbit as circular as possible. Ground control uses radio signals to communicate with the satellite. Radio waves are a type of electromagnetic radiation, which means that an understanding of electric and magnetic fields is also essential to the successful launching and operation of satellites.

The International Space Station (ISS) is a research satellite built through the cooperative efforts of many countries. Earth's gravitational field keeps the ISS in orbit. In addition, much of the technology in the ISS involves electric and magnetic fields. The robotic arms, computers, circuitry, and measurement instruments use both electric and magnetic fields to operate; the station is shielded from the solar wind by Earth's own magnetic field. Onboard the ISS, scientists conduct experiments on the effects of gravitational, electric, and magnetic fields and investigate scientific ideas that may prove useful for various applications, from pharmaceuticals to space exploration itself. Some of these experiments might involve testing new technologies for future satellites. For example, scientists are developing groups of microsatellites the size of volleyballs that fly in formation. Due to their small size, these microsatellites are efficient to launch and versatile in design.

ISS scientists have also conducted studies in biotechnology. Perhaps you have read about salmonella outbreaks in the news. Identifying how salmonella bacteria function in space and studying proteins that regulate the genes involved in their reproduction have helped researchers develop a vaccine to curb this sometimes deadly source of food poisoning.

Questions

1. How do satellites directly affect your life?
2. What do you think happens to the force of gravity as the astronauts move from Earth up to the ISS?
3. How is Earth's magnetic field helpful to the ISS?
4. How is the communication between the ground and the ISS a good example of using both electric and magnetic fields?
5. Suppose an astronaut brings a compass into space. Where do you think the needle will point as the astronaut moves farther and farther from Earth? Will the compass stop working at some point?

UNIT 3 ARE YOU READY?

CONCEPTS

- force terminology
- principles of electromagnetism
- the electrical nature of matter
- different methods of charging
- law of electric charges
- law of magnetic poles
- magnetic and gravitational fields

SKILLS

- communicating in writing and diagrams
- designing an experimental procedure
- applying scientific theories and models
- calculating the effects of gravitational fields
- applying the right-hand rule for a straight conductor

Concepts Review

1. Can you make a magnet that can be turned on and off? Explain your reasoning. K/U A
2. Why do electronic devices (computers, radios, televisions) get dusty more rapidly than wooden furniture? T/I A
3. Give an example of a medical technology that uses magnetic fields to take images of the human body. K/U A
4. What is a force field? K/U
5. (a) Explain in your own words how an object can become electrically charged.
   (b) Compare and contrast positively charged objects, negatively charged objects, and neutral objects.
   (c) What would you observe if a negatively charged object was placed near a positively charged object?
   (d) What would you observe if two negatively charged objects were brought near each other? K/U
6. Explain in your own words the difference between conductors and insulators. K/U
7. (a) Give an example of how the Bohr–Rutherford atom is similar to the solar system.
   (b) Give an example of how the Bohr–Rutherford atom is different from the solar system. K/U A
8. In a short paragraph, summarize what you know about magnetic resonance imaging (MRI) and how it works. Include in your paragraph a description of the positive impacts of MRI and some of the issues surrounding its use in Canada. K/U C
9. Give an example of an everyday device whose operation can be disrupted by the presence of electromagnetic fields. K/U A
10. (a) Use what you know about magnetic fields to explain how magnetic levitation trains (**Figure 1**) can hover above the tracks.
    (b) Coils along the sides of the tracks for magnetic levitation trains constantly alternate polarity. Use what you know about magnets to explain how this can cause the train to move. K/U C A

Figure 1

11. Summarize the right-hand rule for
    (a) a straight conductor
    (b) a solenoid or coiled conductor
    (c) force on a current-carrying conductor in an external magnetic field K/U C
12. List three properties of magnetic field lines. K/U
13. Computer hard drives use electromagnetism to operate. How would placing the following devices near the hard drive affect the data stored on it? K/U A
    (a) a battery not connected to anything
    (b) a battery connected to a solenoid
    (c) a permanent magnet
    (d) a piece of iron

14. (a) In your own words, define gravitational field strength.
(b) What happens to the strength of Earth's gravitational field as you move away from the surface of Earth?
(c) Is the gravitational field strength the same at Earth's equator and the poles? Explain. K/U
15. What is the difference between mass and weight? K/U A
16. Would you weigh less on the Moon than on Earth? Explain your answer. K/U A
17. A student says that the force of gravity is zero for astronauts on board the International Space Station and that this explains why they appear weightless. Do you agree with this student? Explain your answer. K/U
18. What makes more sense for an astronaut to use on the ISS—a pencil or a ballpoint pen? Why? K/U A
19. If a friend told you about a tourist attraction in another province where the force of gravity was repulsive instead of attractive, would you believe him or her? Why or why not? T/I
20. (a) In which direction is the acceleration due to gravity?
(b) Is the direction always the same, or does it depend on whether the object is moving up or down? Explain your answer. K/U

Skills Review

21. Describe how to determine the direction of current in a wire given the direction of the magnetic field. K/U
22. (a) Describe an experimental procedure that you could use to give an object an electrical charge.
(b) Describe an experiment that tests the interaction of two charged objects.
(c) Describe using words or diagrams how you could use a positively charged object to attract a neutral object. K/U
23. (a) Draw a diagram of the magnetic field surrounding a wire with a current in it.
(b) How would the magnetic field change if the current in the wire were increased?
(c) Suppose that the wire is now bent into a loop, as in **Figure 2**. Draw a diagram of its magnetic field.

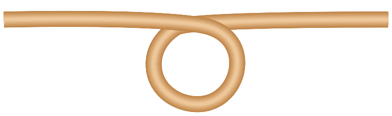


Figure 2

(d) Explain why your diagrams from (a) and (c) describe the construction of an electromagnet. K/U T/I C

24. **Figure 3** shows two opposite magnetic poles. What happens when you try to move them together? K/U T/I

Figure 3

25. **Figure 4** shows conductors with the current moving into and out of the page. Copy the diagrams in Figure 4 into your notebook, and draw the direction of the magnetic field lines. K/U C

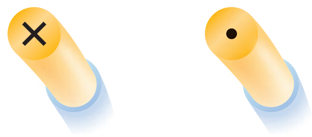


Figure 4

26. (a) What is the acceleration due to gravity of an apple falling from a tree on Earth (neglecting air resistance)?
(b) What is the acceleration due to gravity of a barbell falling from an exercise machine on Earth (neglecting air resistance)?
(c) What effect does mass have on the acceleration due to gravity?
(d) What is the relationship between air resistance and the speed of a falling object? K/U T/I
27. (a) Draw the field lines of Earth's magnetic field.
(b) At various points along Earth's magnetic field lines, draw arrows indicating the direction of the force a compass needle would experience. K/U C A
28. (a) Draw a simple diagram of Earth, showing arrows indicating the magnitude and direction of the force any object would experience due to Earth's gravitational field.
(b) How are the magnetic field lines different from the gravitational field lines? K/U T/I C

CAREER PATHWAYS PREVIEW

Throughout this unit, you will see Career Links. Go to the Nelson Science website to find information about careers related to Gravitational, Electric, and Magnetic Fields. On the Chapter Summary page at the end of each chapter, you will find a Career Pathways feature that shows you the educational requirements of the careers. There are also some career-related questions for you to research.

CHAPTER

6 Gravitational Fields

KEY CONCEPTS

After completing this chapter you will be able to

- describe key properties of fields
- describe specific properties of Earth's gravitational field
- solve problems related to circular motion and universal gravitation
- analyze planetary orbits and solve problems based on orbiting bodies
- assess the impact on society and the environment of technologies such as satellites that use gravitational fields
- compare gravity as understood through Newtonian mechanics to gravity as understood through general relativity
- describe black holes and dark matter
- model the trajectory of a rocket at varying altitudes and use the data to derive *G*, the gravitational constant
- use simulation software to explore orbital properties of multi-body systems and to design a solar system

What Effect Does Gravity Have on Objects That Are Not Near the Surface of Earth?

Your everyday experience confirms that gravity affects objects here on Earth. However, gravitational fields exist anywhere and everywhere, whether near Earth's surface or very far away from Earth. Wherever there is mass, there will be a gravitational field. Moreover, multiple masses within the same system create complex gravitational effects. Our solar system is a perfect example of this.

In 2004, the MESSENGER spacecraft was launched from Earth, heading for its destination in orbit around Mercury. It arrived seven years later, in 2011. MESSENGER owed its journey to the various gravitational forces it encountered along the way. Scientists designed its path so the spacecraft could pick up energy and shift direction through its interactions with planetary gravitational fields. MESSENGER's path looped once by Earth, twice by Venus, and three times by Mercury before settling into orbit.

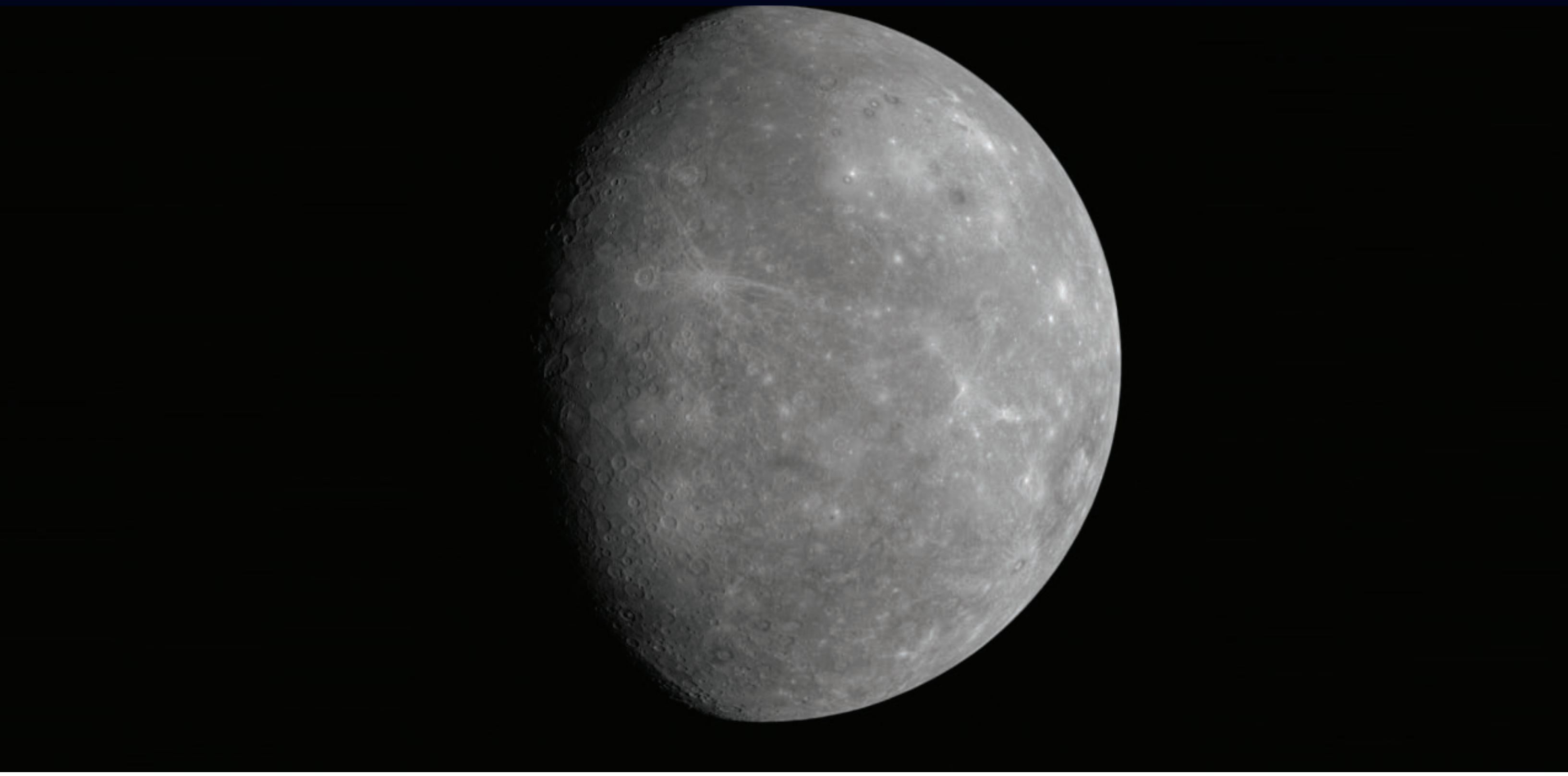
MESSENGER's route minimized the amount of energy it had to transform using rocket propellant. To follow a direct line to Mercury, MESSENGER would have needed to overcome strong forces that would have pulled it off course. Although longer, MESSENGER's path was more efficient because gravitational forces accelerated MESSENGER at just the right time to position it for the next phase of its trip.

The planets' orbits—and MESSENGER's path across them—represent an extreme example of gravitational fields and forces. In this chapter, you will explore gravity and gravitational effects through some fundamental models and systems, including gravitational fields, gravitational forces between two masses, and masses orbiting around larger bodies.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. What role do gravitational fields play in spacecraft launches?
2. What happens to the strength of the gravitational field acting on an object as the object gets farther from Earth?
3. What happens to the gravitational field strength on the object when it is very far from Earth?
4. How do gravitational fields affect the orbits of planets, moons, and other bodies?



Mini Investigation

Artificial Gravity

Skills: Planning, Observing, Analyzing, Communicating

SKILLS HANDBOOK A5.2

Astronauts experience the forces of simulated gravity inside spaceships and space stations. In this investigation, you will place a ball in a bucket and then move the bucket in uniform circular motion to simulate the forces felt by an astronaut in a rotating spaceship.

Perform this investigation outdoors and away from windows. While swinging the bucket, be sure to be a safe distance away from other people.

Equipment and Materials: plastic bucket with a strong handle; tennis ball; metre stick; stopwatch

1. Measure the distance from the base of the bucket to the shoulder of the person who will be swinging the bucket. The person's shoulder is the centre of revolution.
2. Place the tennis ball in the bucket.
3. Have the person swing the bucket back and forth like a pendulum, increasing the distance of the swing each time.
4. After the bucket has been swung in an approximately 180° arc, swing the bucket in a complete vertical circle at a high, constant speed.
5. While the bucket is being swung in a complete circle, have another group member, standing a safe distance away, use the stopwatch to determine the time for five complete revolutions of the bucket moving at a constant speed.
6. Slow the swing down to the minimum speed needed to keep the ball inside the bucket.
7. Record the time for five complete revolutions of the bucket while it is moving at the slowest speed possible.

A. Draw a system diagram and a free-body diagram for the ball at the top of its loop in Steps 4 and 5. T/I C A

B. Assuming that the bucket is moving in completely uniform circular motion, calculate the speed of the bucket at the top of the loop in Steps 4 and 5. Use this speed to calculate the magnitude of the centripetal acceleration of the ball at the top of the loop. T/I A

C. Determine the ratio of the apparent weight of the ball at the top of the loop to its weight on Earth. T/I A

D. Repeat A, B, and C for the values you recorded in Steps 6 and 7. T/I A

E. Describe the strengths and weaknesses of this model of artificial gravity. T/I C

6.1 Newtonian Gravitation

Early in the formation of our galaxy, tiny gravitational effects between particles began to draw matter together into slightly denser configurations. Those, in turn, exerted even greater gravitational forces, resulting in more mass joining the newly forming structures. Eventually, those repetitive and continuous gravitational effects formed and shaped our Milky Way galaxy, as depicted in **Figure 1**. The same process of gravitational attraction—on different scales—accounts for the overall structure of the entire universe, despite being the weakest of the four fundamental forces.

Gravity accounts for how the planets in our solar system move and orbit around the Sun. By the late 1700s, scientists had identified all the inner terrestrial planets as well as the gas giants, Jupiter and Saturn. Then, British astronomer William Herschel (1738–1822) used observations of the relative movements of the stars to determine that a presumed "star" was actually an additional planet. The new planet was Uranus. Scientists then observed that Uranus's path was anomalous. It seemed to respond to the pull of another distant but unknown body. Using mathematical analysis, scientists predicted where the unknown body would have to be and began searching for it. In 1846, scientists discovered the planet Neptune (**Figure 2**). CAREER LINK

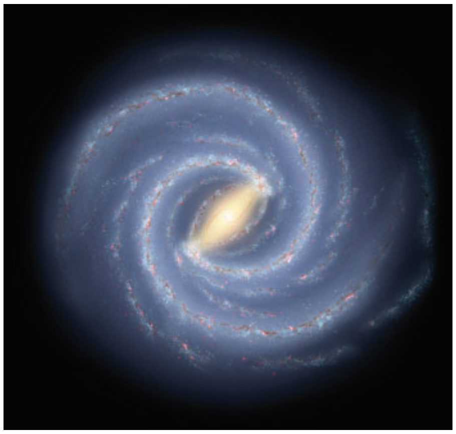


Figure 1 Our galaxy, the Milky Way, was shaped by gravitational forces, depicted here in an artist's conception.

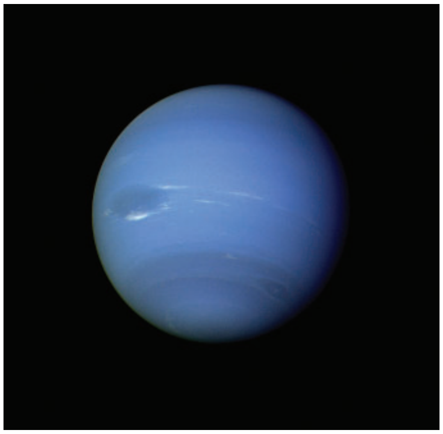


Figure 2 Scientists discovered Neptune by observing its gravitational effects on Uranus.

Universal Gravitation

The force that causes Uranus to wobble slightly in its orbit is gravity—the same force that causes Earth and the other planets to revolve around the Sun. Sir Isaac Newton, whose laws of motion provide the foundation of our study of mechanics, used known data about the solar system to describe the system of physical laws that govern the movement of celestial bodies around the Sun. Through this inquiry, he formulated the **universal law of gravitation**.

Universal Law of Gravitation

There is a gravitational attraction between *any* two objects. If the objects have masses m_1 and m_2 and their centres are separated by a distance r, the magnitude of the gravitational force on either object is directly proportional to the product of m_1 and m_2 and inversely proportional to the square of r:

$$F_g = \frac{Gm_1m_2}{r^2}$$

G is a constant of nature called the **gravitational constant**, which is equal to $6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

gravitational constant a constant that appears in the universal law of gravitation; the constant is written as G and has a value of $6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

Newton's law of gravitation plays a key role in physics for two reasons. First, his work showed for the first time that the laws of physics apply to all objects. The same force that causes a leaf to fall from a tree also keeps planets in orbit around the Sun. This fact had a profound effect on how people viewed the universe. Second, the law provided us with an equation to calculate and understand the motions of a wide variety of celestial objects, including planets, moons, and comets.

The gravitational force is always attractive (**Figure 3**). Every mass attracts every other mass. Therefore, the direction of the force of gravity on one mass (mass 1) due to a second mass (mass 2) points from the centre of mass 1 toward the centre of mass 2.

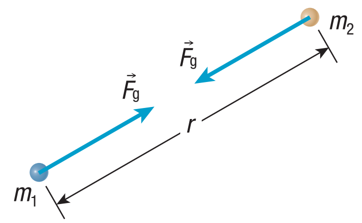


Figure 3 The gravitational force between two point masses m_1 and m_2 that are separated by a distance r is given in the universal law of gravitation.

The magnitude of the gravitational force exerted by mass 1 on mass 2 is equal to the magnitude of the gravitational force exerted by mass 2 on mass 1. Since the forces are both attractive, this result is precisely what we would expect from Newton's third law ($\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$). The two gravitational forces are an action–reaction pair because they are equal in magnitude and opposite in direction and they act on different members of the pair of objects.

Another important feature of the universal law of gravitation is that the force follows the inverse-square law. The **inverse-square law** is a mathematical relationship between variables in which one variable is proportional to the inverse of the square of the other variable. When applied to gravitational forces, this relationship means that the force is inversely proportional to the square of the distance between the mass centres, or $F \propto \frac{1}{r^2}$. In other words, the force of attraction drops quickly as the two objects move farther apart. No matter how large the distance between the mass centres, however, they will still experience a gravitational force. Every massive object in the universe exerts a force of attraction on every other massive object.

inverse-square law a mathematical relationship in which one variable is proportional to the inverse of the square of another variable; the law applies to gravitational forces and other phenomena, such as electric field strength and sound intensity

If both objects have a small mass compared to the distance between their centres, they will experience a small gravitational force. For the force to be noticeable, at least one of the objects must have a large mass relative to the distance between the object centres.

The Value of *g*

On Earth, we can calculate the acceleration due to the force of gravity, g, from the universal law of gravitation. Near Earth's surface, g has an approximate value of 9.8 m/s^2. The precise value of g, however, decreases with increasing height above Earth's surface based on the inverse-square law (**Table 1**). The value of g also varies on the surface of Earth because the surface varies in distance from the centre of Earth.

Table 1 Gravity versus Distance from Earth's Surface

Altitude (km)	g (m/s^2)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13

Calculating the Force of Gravity

The first measurement of the gravitational constant G was carried out in 1798 in a famous experiment by Henry Cavendish (1731–1810). Cavendish wanted to measure the gravitational force between two objects on Earth using two large lead spheres. He needed to use spheres with a very small distance between them or he would have found the gravitational force nearly impossible to observe. Cavendish arranged two large spheres of mass m_1 in a dumbbell configuration and suspended them from their centre point by a thin wire fibre, as shown in **Figure 4** on the next page. He placed another pair of large spheres with mass m_2 close to the suspended masses.

The gravitational forces between the pairs of masses, m_1 and m_2, caused the dumbbell to rotate. As the fibre twisted, tension in the fibre caused a force resisting the twist that increased as the rotation increased. At a certain angle, this twisting force balanced the gravitational force. By carefully measuring the angle θ, Cavendish could determine the force on the dumbbell, as well as the separation of the spheres. By also measuring the masses of the spheres and inserting the values into the universal law of gravitation, Cavendish could measure G.

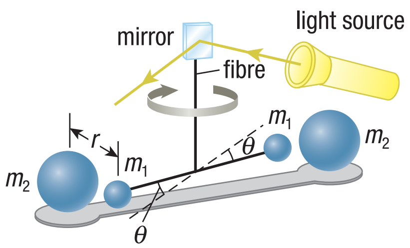


Figure 4 Cavendish used an apparatus like this one to measure the force of gravity between terrestrial objects. The amount that the light is deflected from its original path gives an indication of the angle of rotation θ.

Through this method, Cavendish measured the value of G: 6.67×10^{-11} N·m²/kg². The constant G has this combination of units because it gives a gravitational force in newtons. When you multiply the units of G by two masses in kilograms and divide by the square of a distance in metres, you will be left with newtons. Following his calculation of G, Cavendish was able to calculate the mass of Earth. In fact, all masses of planetary bodies can be determined by using the universal law of gravitation. Although Cavendish conducted his experiment more than 200 years ago, his design still forms the basis for experimental studies of gravitation today. Tutorial 1 shows how to calculate the force of gravity between two spherical masses, as well as how to calculate the mass of a celestial body.

Investigation 6.1.1

Universal Gravitation (page 308)
You have learned the basic information about the universal law of gravitation. This investigation will give you an opportunity to verify this law through an observational study.

Tutorial 1 Calculating the Force of Gravity

The following Sample Problems involve the force of gravity.

Sample Problem 1: Calculating the Force of Gravity between Two Ordinary Masses

The centres of two uniformly dense spheres are separated by 50.0 cm. Each sphere has a mass of 2.00 kg.

(a) Calculate the magnitude of the gravitational force of attraction between the two spheres.

(b) How much of an effect will this force have on the two spheres?

Solution

(a) **Given:** $G = 6.67 \times 10^{-11}$ N·m²/kg²; $r = 50.0$ cm $= 0.500$ m; $m_1 = m_2 = 2.00$ kg

Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution:
$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(2.00\ \cancel{\text{kg}})(2.00\ \cancel{\text{kg}})}{(0.500\ \cancel{\text{m}})^2}$$
$$F_g = 1.07 \times 10^{-9}\ \text{N}$$

Statement: The magnitude of the gravitational force of attraction between the two spheres is 1.07×10^{-9} N.

(b) The gravitational force of attraction between the two spheres (1.07×10^{-9} N) is too small to have any noticeable effect on the motion of these two spheres under normal circumstances.

Sample Problem 2: Calculating the Force of Gravity and Solving for Mass

Eris, a dwarf planet, is the ninth most massive body orbiting the Sun. It is more massive than Pluto and three times farther away from the Sun. Eris is estimated to have a radius of approximately 1200 km. Acceleration due to gravity on Eris differs from the value of g on Earth. In this three-part problem, you will explore the force of gravity on the surface of Eris.

(a) Suppose that an astronaut stands on Eris and drops a rock from a height of 0.30 m. The rock takes 0.87 s to reach the surface. Calculate the value of g on Eris.

(b) Calculate the mass of Eris.

(c) Suppose that an astronaut stands on Eris and drops a rock from a height of 2.50 m. Calculate how long it would take the rock to reach the surface.

Solution

(a) **Given:** $\Delta d = 0.30$ m; $v_i = 0$ m/s; $\Delta t = 0.87$ s

Required: g_{Eris}

Analysis: Near the surface of Eris, the gravitational acceleration will be approximately constant, like it is near the surface of Earth. We can determine the value of g_{Eris} by using the equation for the motion of an object falling under constant acceleration: $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$. Since $v_i = 0$, the equation simplifies to $\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$.

For this problem, $a = g_{Eris}$. Choose up as positive, so down is negative.

Solution:

$$\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$$

$$\vec{a} = \frac{2\Delta\vec{d}}{\Delta t^2}$$

$$\vec{g}_{Eris} = \frac{2(-0.30\text{ m})}{(0.87\text{ s})^2}$$

$$\vec{g}_{Eris} = -0.7927\text{ m/s}^2 \text{ (two extra digits carried)}$$

Statement: The value of g on Eris is 0.79 m/s^2.

(b) **Given:** $g_{Eris} = 0.7927$ m/s^2; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2; $r = 1200$ km $= 1.2 \times 10^6$ m

Required: m_{Eris}

Analysis: Use the equations for the force of gravity, $F_g = mg$, and the universal law of gravitation, $F_g = \frac{Gm_1m_2}{r^2}$.

Solution: Equate these two expressions for the gravitational force on the rock, $F_g = m_{rock}g_{Eris}$ and $F_g = \frac{Gm_{rock}m_{Eris}}{r^2}$, and use 1 N = 1 kg·m/s^2.

$$F_g = F_g$$

$$m_{rock}g_{Eris} = \frac{Gm_{rock}m_{Eris}}{r^2}$$

$$m_{Eris} = \frac{g_{Eris}r^2}{G}$$

$$= \frac{\left(0.7927\,\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)(1.2 \times 10^6\,\cancel{\text{m}})^2}{6.67 \times 10^{-11}\,\frac{\cancel{\text{kg}}\cdot\cancel{\text{m}}}{\cancel{\text{s}^2}} \cdot \frac{\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}}$$

$$m_{Eris} = 1.7 \times 10^{22}\text{ kg}$$

Statement: The mass of Eris is 1.7×10^{22} kg.

(c) **Given:** $\Delta d = 2.50$ m; $v_i = 0$ m/s; $a = g_{Eris} = 0.7927$ m/s^2

Required: Δt

Analysis: Since the value of g_{Eris} is known, we can use the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ to determine the time it would take the rock to reach the surface. Since $v_i = 0$, the equation simplifies to $\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$. Choose up as positive, so down is negative.

Solution:

$$\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$$

$$\Delta t^2 = \frac{2\Delta\vec{d}}{\vec{a}}$$

$$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$$

$$= \sqrt{\frac{2(-2.50\,\cancel{\text{m}})}{-0.7927\,\frac{\cancel{\text{m}}}{\text{s}^2}}}$$

$$\Delta t = 2.5\text{ s}$$

Statement: It would take the rock 2.5 s to fall 2.50 m.

Sample Problem 3: Calculating the Force of Gravity in a Three-Body System

Figure 5 shows three large, spherical asteroids in space, which are arranged at the corners of a right triangle ABC. Asteroid A has a mass of 1.0×10^{20} kg. Asteroid B has a mass of 2.0×10^{20} kg and is 50 million kilometres (5.0×10^{10} m) from asteroid A. Asteroid C has a mass of 4.0×10^{20} kg and is 25 million kilometres (2.5×10^{10} m) away from asteroid A along the other side of the triangle.

(a) Determine the net force on asteroid A from asteroids B and C.

(b) Determine the net force on asteroid B from asteroid C.

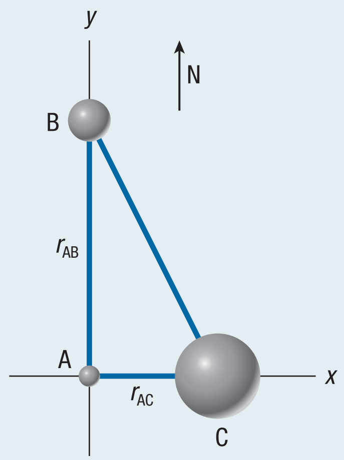


Figure 5

Solution

(a) **Given:** $m_A = 1.0 \times 10^{20}$ kg; $m_B = 2.0 \times 10^{20}$ kg; $m_C = 4.0 \times 10^{20}$ kg; $r_{AB} = 5.0 \times 10^{10}$ m; $r_{AC} = 2.5 \times 10^{10}$ m

Required: $\vec{F}_{\text{net A}}$

Analysis: The force of gravity on mass m_1 due to mass m_2 is $F_g = \frac{Gm_1m_2}{r^2}$ directed from the centre of m_1 toward the centre of m_2. The force on asteroid A from asteroid B will be along side AB, and the force on asteroid A from asteroid C will be along side AC of triangle ABC. So, use the Pythagorean theorem to determine the magnitude of the net force on asteroid A from asteroids B and C, $F_{\text{net A}} = \sqrt{F_{AB}^2 + F_{AC}^2}$, and use trigonometry to calculate the angle that the net force makes with side AC: $\theta = \tan^{-1}\left(\frac{F_{AB}}{F_{AC}}\right)$.

Solution: Calculate the force on asteroid A due to asteroid B, F_{AB}.

$$F_{AB} = \frac{Gm_Am_B}{r_{AB}^2} = \frac{\left(6.67 \times 10^{-11}\,\frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(1.0 \times 10^{20}\,\cancel{\text{kg}})(2.0 \times 10^{20}\,\cancel{\text{kg}})}{(5.0 \times 10^{10}\,\cancel{\text{m}})^2}$$

$F_{AB} = 5.336 \times 10^8$ N (two extra digits carried)

Calculate the force on asteroid A due to asteroid C, F_{AC}.

$$F_{AC} = \frac{Gm_Am_C}{r_{AC}^2} = \frac{\left(6.67 \times 10^{-11}\,\frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(1.0 \times 10^{20}\,\cancel{\text{kg}})(4.0 \times 10^{20}\,\cancel{\text{kg}})}{(2.5 \times 10^{10}\,\cancel{\text{m}})^2}$$

$F_{AC} = 4.269 \times 10^9$ N (two extra digits carried)

$\vec{F}_{\text{net A}}$ is the vector sum of $\vec{F}_{AB}$ and $\vec{F}_{AC}$. Since the force vectors lie along the sides of a right triangle, we can use the Pythagorean theorem to calculate the magnitude of the net force on asteroid A, $F_{\text{net A}}$.

$$F_{\text{net A}} = \sqrt{F_{AB}^2 + F_{AC}^2} = \sqrt{(5.336 \times 10^8\ \text{N})^2 + (4.269 \times 10^9\ \text{N})^2}$$

$F_{\text{net A}} = 4.3 \times 10^9$ N

Now calculate the angle that the net force on asteroid A makes with side AC.

$$\theta = \tan^{-1}\left(\frac{F_{AB}}{F_{AC}}\right) = \tan^{-1}\left(\frac{5.336 \times 10^8\,\cancel{\text{N}}}{4.269 \times 10^9\,\cancel{\text{N}}}\right)$$

$\theta = 7.1°$

Statement: The net force on asteroid A from asteroids B and C is 4.3×10^9 N [E 7.1° N], directed toward asteroid A.

(b) **Given:** $m_B = 2.0 \times 10^{20}$ kg; $m_C = 4.0 \times 10^{20}$ kg; $r_{AB} = 5.0 \times 10^{10}$ m; $r_{AC} = 2.5 \times 10^{10}$ m

Required: $\vec{F}_{BC}$

Analysis: Use the Pythagorean theorem to determine the distance between asteroid B and asteroid C, $r_{BC} = \sqrt{r_{AB}^2 + r_{AC}^2}$. Then use the universal law of gravitation, $F_g = \frac{Gm_1m_2}{r^2}$, directed from the centre of m_1 toward the centre of m_2. The force will act along the hypotenuse of triangle ABC, so the angle that the force makes with side AB is given by $\theta = \tan^{-1}\left(\frac{r_{AC}}{r_{AB}}\right)$.

Solution: Calculate the distance between asteroid B and asteroid C.

$$r_{BC} = \sqrt{r_{AB}^2 + r_{AC}^2} = \sqrt{(5.0 \times 10^{10}\ \text{m})^2 + (2.5 \times 10^{10}\ \text{m})^2}$$

$r_{BC} = 5.590 \times 10^{10}$ m (two extra digits carried)

Calculate the magnitude of the gravitational force between asteroid B and asteroid C.

$$F_{BC} = \frac{Gm_Bm_C}{r_{BC}^2} = \frac{\left(6.67 \times 10^{-11}\,\frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(2.0 \times 10^{20}\,\cancel{\text{kg}})(4.0 \times 10^{20}\,\cancel{\text{kg}})}{(5.590 \times 10^{10}\,\cancel{\text{m}})^2}$$

$F_{BC} = 1.7 \times 10^9$ N

As Figure 5 indicates, the net force on asteroid B due to asteroid C acts along side BC. Calculate the angle that this force makes with side AB.

$$\theta = \tan^{-1}\left(\frac{r_{AC}}{r_{AB}}\right) = \tan^{-1}\left(\frac{2.5 \times 10^{10}\,\cancel{\text{m}}}{5.0 \times 10^{10}\,\cancel{\text{m}}}\right)$$

$\theta = 27°$

Statement: The force on asteroid B due to asteroid C is 1.7×10^9 N [S 27° E], directed toward asteroid B.

Practice

1. Two spherical asteroids have masses as follows: $m_1 = 1.0 \times 10^{20}$ kg and $m_2 = 3.0 \times 10^{20}$ kg. The magnitude of the force of attraction between the two asteroids is 2.2×10^9 N. Calculate the distance between the two asteroids. T/I [ans: 3.0×10^{10} m]
2. Jupiter has a mass of 1.9×10^{27} kg and a mean radius at the equator of 7.0×10^7 m. Calculate the magnitude of g on Jupiter, if it were a perfect sphere with that radius. T/I [ans: 26 m/s^2]
3. Uniform spheres A, B, and C have the following masses and centre-to-centre distances: $m_A = 40.0$ kg, $m_B = 60.0$ kg, and $m_C = 80.0$ kg; $r_{AB} = 0.50$ m and $r_{BC} = 0.75$ m. If the only forces acting on B are the gravitational forces due to A and C, determine the net force acting on B with the spheres arranged as in **Figures 6(a)** and **(b)**. T/I [ans: (a) 7.1×10^{-8} N [left]; (b) 8.6×10^{-7} N [W 42° S]]

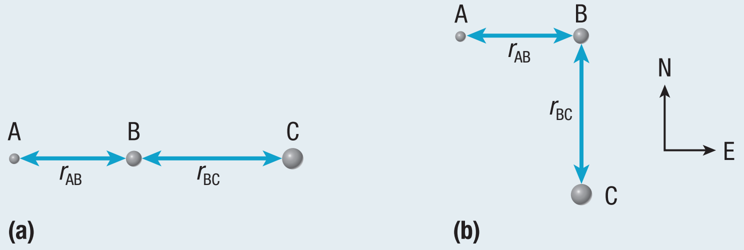


Figure 6

Gravitational Fields

The universal law of gravitation tells us that at any point in space surrounding a massive object, such as Earth, we can calculate the gravitational force on a second object sitting at that point in space. Earth has a mean radius of approximately 6380 km. So an object that is 10 km above Earth's surface, or 6390 km from Earth's centre, will have the same gravitational attraction to Earth no matter which land mass or ocean it is positioned above. A vector exists at every point in space surrounding the central object, pointing toward it and depending on the object's mass and the distance from its centre (**Figure 7**). The **gravitational field** of the central object can be represented by this collection of vectors. A gravitational field exerts forces on objects with mass. The **gravitational field strength** is the force of attraction per unit mass of an object placed in a gravitational field, and it equals the gravitational force on the object divided by the object's mass. On Earth, the gravitational field strength is approximately 9.8 N/kg. Notice that this has the same magnitude as the acceleration due to gravity on Earth's surface, and thus has the same symbol, g.

gravitational field a collection of vectors, one at each point in space, that determines the magnitude and direction of the gravitational force

gravitational field strength the magnitude of the gravitational field vector at a point in space

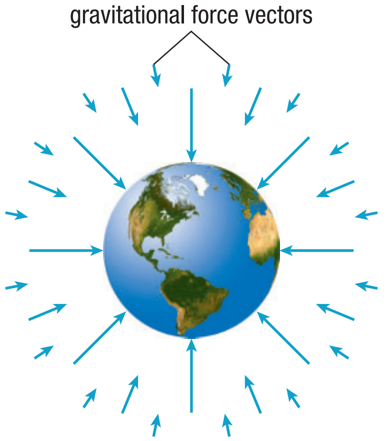


Figure 7 Earth's gravitational field strength diminishes with increasing distance from the planet's centre.

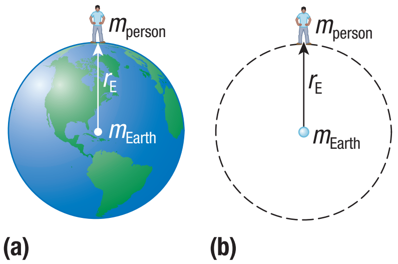


Figure 8 If we approximate Earth as a sphere, we can assume that the gravitational force that Earth exerts on a person or an object is equal to the force experienced if all the mass were located at Earth's centre.

For spherical objects, the strength of the gravitational field at a distance from the surface is the same whether the mass actually fills its volume or sits at a point in the centre. We can therefore use the gravitational force equation as though all of the object's mass were located at its centre; this is why we measure centre-to-centre distances (**Figure 8**).

To calculate the gravitational field strength as a function of a central spherical mass, we combine the universal law of gravitation with Newton's second law. Let us calculate the acceleration due to gravity g on a small mass m_{object} near the surface of a spherical planet of mass m_{planet} and radius r.

$$F_g = F_g$$

$$m_{object}g = \frac{Gm_{planet}m_{object}}{r^2}$$

$$g = \frac{Gm_{planet}}{r^2}$$

We can apply this formula to other planets and stars by substituting the appropriate values for m_{planet} and r. The value of g depends on the mass of the central body, the distance from that body's centre, and the gravitational constant, G. An object's acceleration due to gravity does not depend on its own mass. Tutorial 2 shows how to calculate the gravitational field strength on other planets.

Tutorial 2 Solving Problems Related to Gravitational Field Strength

In the following Sample Problem, you will learn how to calculate the gravitational field strength.

Sample Problem 1: Determining the Gravitational Field Strength

(a) Calculate the magnitude of the gravitational field strength on the surface of Saturn, assuming that it is perfectly spherical with a radius of 6.03×10^7 m. The mass of Saturn is 5.69×10^{26} kg.

(b) Determine the ratio of Saturn's gravitational field strength to Earth's gravitational field strength (9.8 N/kg).

Solution

(a) **Given:** $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$; $m_{Saturn} = 5.69 \times 10^{26}$ kg; $r = 6.03 \times 10^7$ m

Required: g_{Saturn}

Analysis: $g_{Saturn} = \frac{Gm_{Saturn}}{r^2}$

Solution:

$$g_{Saturn} = \frac{Gm_{Saturn}}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(5.69 \times 10^{26}\ \cancel{\text{kg}})}{(6.03 \times 10^7\ \cancel{\text{m}})^2}$$

$g_{Saturn} = 10.438$ N/kg (two extra digits carried)

Statement: The gravitational field strength on the surface of Saturn is 10.4 N/kg.

(b) **Given:** $g_{Saturn} = 10.438$ N/kg; $g_{Earth} = 9.8$ N/kg

Required: $g_{Saturn} : g_{Earth}$

Analysis:

Calculate $\frac{g_{Saturn}}{g_{Earth}}$.

Solution:

$$\frac{g_{Saturn}}{g_{Earth}} = \frac{10.438 \frac{\cancel{\text{N}}}{\cancel{\text{kg}}}}{9.8 \frac{\cancel{\text{N}}}{\cancel{\text{kg}}}}$$

$$\frac{g_{Saturn}}{g_{Earth}} = 1.1$$

Statement: The ratio of Saturn's gravitational field strength to Earth's gravitational field strength is 1.1:1.

Practice

1. The radius of a typical white dwarf star is just a little larger than the radius of Earth, but a typical white dwarf has a mass that is similar to the Sun's mass. Calculate the surface gravitational field strength of a white dwarf with a radius of 7.0×10^6 m and a mass of 1.2×10^{30} kg. Compare this to the surface gravitational field strength of Earth. T/I A [ans: 1.6×10^6 N/kg]
2. Suppose that Saturn expanded until its radius doubled, while its mass stayed the same. Determine the gravitational field strength on the new surface relative to the old surface. T/I [ans: $\frac{1}{4} g_{Saturn}$]

Tutorial 2 demonstrates that the gravitational field strength on the surface of Saturn is only slightly greater than the gravitational field strength on the surface of Earth. Each planet in our solar system has a different gravitational field strength that depends on the radius and the mass of the planet. **Table 2** lists the relative values for all the planets in our solar system.

Table 2 Surface Gravitational Field Strength of the Planets in the Solar System

Planet	Value of g_{planet} relative to Earth	Value of g (N/kg)
Mercury	0.38	3.7
Venus	0.90	8.8
Earth	1.00	9.8
Mars	0.38	3.7
Jupiter	2.53	24.8
Saturn	1.06	10.4
Uranus	0.90	8.8
Neptune	1.14	11.2

Research This

Gravitational Field Maps and Unmanned Underwater Vehicles

Skills: Researching, Analyzing, Communicating SKILLS HANDBOOK A4.1

Unmanned underwater vehicles (UUVs) navigate underwater without a human driver to conduct searches, collect images, or recover submerged materials. Some UUVs are operated remotely by a human pilot, and others operate independently. To ensure that UUVs stay on course, gravitational field maps are used to correct errors in UUV navigation systems.

1. Research UUVs (also called autonomous underwater vehicles, or AUVs), and locate several examples of both human-operated and independent UUVs.
2. Research gravitational field maps and how they work. Find one example of a visualization of gravitational field data.
3. Research the factors that affect gravitational fields and why and how the fields can vary.
4. Determine how gravitational field maps are used to correct UUV navigation systems.

A. Explain in your own words how a gravitational field map works, how it is created, and how it is used. T/I A

B. Draw a diagram highlighting the design features and functions of two UUV examples. K/U C

C. Create a one-page report or short presentation outlining how gravitational field maps are used to correct UUV navigation systems. K/U C A

WEB LINK

6.1 Review

Summary

- The universal law of gravitation states that the force of gravitational attraction between any two objects is directly proportional to the product of the masses of the objects and inversely proportional to the square of the distance between their centres: $F_g = \frac{Gm_1m_2}{r^2}$.
- The gravitational constant, $G = 6.67 \times 10^{-11}$ N·m²/kg², was first determined experimentally by Henry Cavendish in 1798.
- The gravitational field can be represented by a vector at each point in space. A gravitational field exerts forces on objects with mass. The gravitational field strength at a distance r from a body of mass m equals the magnitude of gravitational acceleration at that distance: $g = \frac{Gm}{r^2}$.

Questions

1. At what altitude above Earth would your weight be one-half your weight on the surface? Use Earth's radius, r_E, as the unit. T/I A
2. In a hydrogen atom, a proton and an electron are 5.3×10^{-11} m apart. Calculate the magnitude of the gravitational attraction between the proton and the electron. The mass of a proton is 1.67×10^{-27} kg, and the mass of an electron is 9.11×10^{-31} kg. T/I A
3. Two objects are a distance r apart. The distance r increases by a factor of 4. K/U
   (a) Does the gravitational force between the objects increase or decrease? Explain your answer.
   (b) By what factor does the gravitational force between the objects change?
4. A satellite of mass 225 kg is located 8.62×10^6 m above Earth's surface. T/I A
   (a) Determine the magnitude and direction of the gravitational force. (Hint: The values for Earth's mass and radius can be found in Appendix B.)
   (b) Determine the magnitude and direction of the resulting acceleration of the satellite.
5. On the surface of Titan, a moon of Saturn, the gravitational field strength has a magnitude of 1.3 N/kg. Titan's mass is 1.3×10^{23} kg. What is Titan's radius? T/I A
6. Earth's gravitational field strength at the surface is 9.80 N/kg. Determine the distance, as a multiple of Earth's radius, r_E, above Earth's surface at which the magnitude of the acceleration due to gravity is 3.20 N/kg. T/I A
7. Calculate the gravitational field strength of the Sun at a distance of 1.5×10^{11} m from its centre (Earth's distance). T/I
8. The gravitational field strength between two objects is the sum of two vectors pointing in opposite directions. Somewhere between the objects, the vectors will cancel, and the total force will be zero. Determine the location of zero force as a fraction of the distance r between the centres of two objects of mass m_1 and m_2. T/I A
9. A 537 kg satellite orbits Earth with a speed of 4.3 km/s at a distance of 2.5×10^7 m from Earth's centre. K/U T/I
   (a) Calculate the acceleration of the satellite.
   (b) Calculate the gravitational force on the satellite.
10. Calculate the value of Mercury's surface gravitational field strength, and compare your answer to the value provided in Table 2 on page 295. K/U
11. The gravitational field strength is 5.3 N/kg at the location of a 620 kg satellite in orbit around Earth. K/U T/I
    (a) Calculate the satellite's altitude. (Hint: The values for Earth's mass and radius can be found in Appendix B.)
    (b) Determine the gravitational force on the satellite.
12. Through experimentation, Henry Cavendish was able to determine the value of the gravitational constant. Explain how to use his result together with astronomical data on the motion of the Moon to determine the mass of Earth. T/I C A
13. Determine the location between two objects with masses equal to Earth's mass and the Moon's mass where you could place a third mass so that it would experience a net gravitational force of zero. T/I A

Orbits

6.2

RADARSAT-1 and RADARSAT-2 are Earth-observation satellites designed and commissioned by the Canadian Space Agency. These "eyes in the skies" peer down from orbit, capturing images and data that help scientists monitor environmental changes and the planet's natural resources. Examples of satellite monitoring include detecting oil spills, tracking ice movements, identifying ships at sea, and monitoring natural disasters. **Figure 1** shows an image of RADARSAT-2.

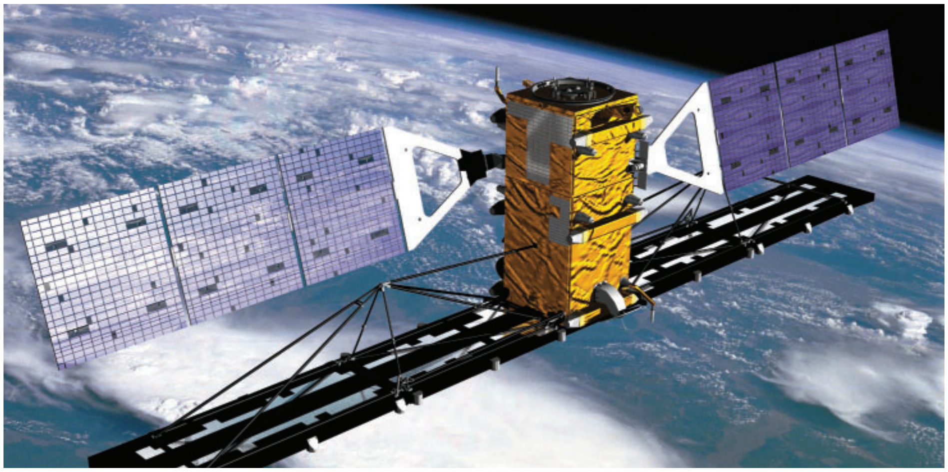


Figure 1 RADARSAT-2 uses sophisticated microwave-based radar to collect images of Earth day and night, even through cloud cover.

Satellites and Space Stations

A **satellite** is an object or a body that revolves around another body that usually has much more mass than the satellite. For example, the planets are natural satellites revolving around the Sun. Planetary moons, including Earth's moon, are natural satellites, too. **Artificial satellites**, on the other hand, are human-made objects that orbit Earth or other bodies in the solar system. CAREER LINK

RADARSAT-1 and RADARSAT-2 are examples of artificial satellites. Another well-known example of artificial satellites is the network of 24 satellites that make up the Global Positioning System (GPS). By coordinating several signals at once, as shown in **Figure 2**, the system can locate an object on Earth's surface to within 15 m of its actual position.

satellite an object or a body that revolves around another body due to gravitational attraction

artificial satellite an object that has been intentionally placed by humans into orbit around Earth or another body; referred to as "artificial" to distinguish from natural satellites such as the Moon

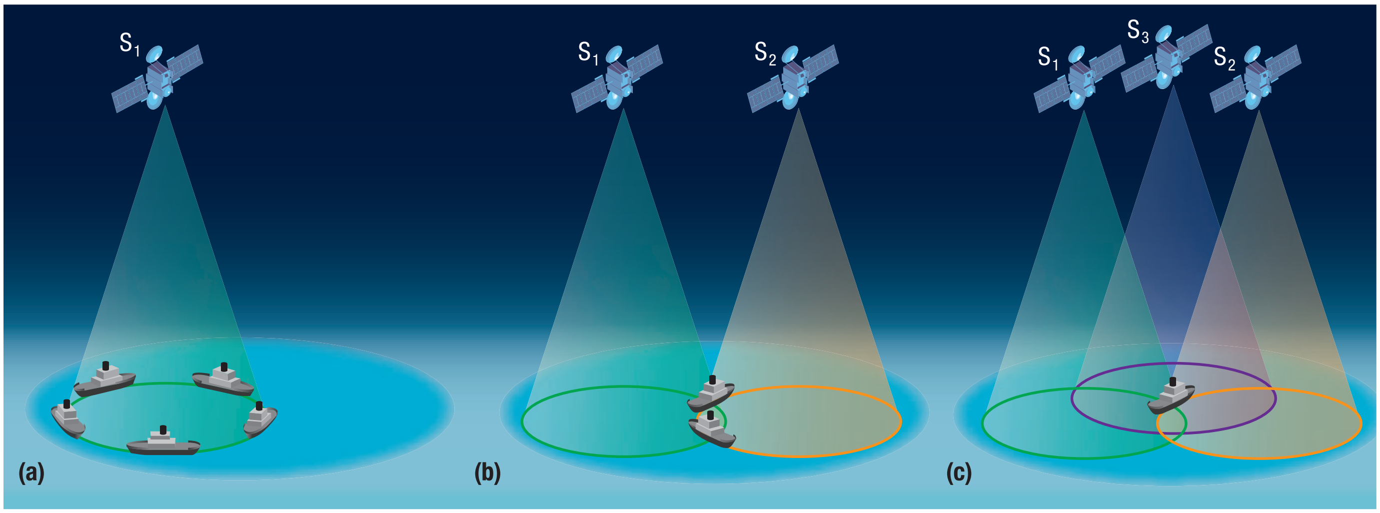


Figure 2 GPS satellites can determine the location of an object, in this case a boat. (a) The data from one satellite will show that the location is somewhere along the circumference of a circle. (b) Two satellites consulted simultaneously will refine the location to one of two intersection spots. (c) With three satellites consulted simultaneously, the intersection of three circles will give the location of the boat to within 15 m of its actual position.

The boat shown in Figure 2 on the previous page has a computer-controlled GPS receiver that detects signals from three satellites simultaneously. The system calculates distances based on signal speeds and transmission times. A single satellite can identify the boat's location somewhere along the circumference of a circle. Two simultaneous satellite signals can pinpoint the location at one of two intersecting spots where two circles intersect. With a third satellite—and therefore three intersecting circles—the boat's location can be pinpointed. This is referred to as triangulation.

space station a spacecraft in which people live and work

Another example of an artificial satellite in Earth orbit is a **space station**, a spacecraft in which people live and work. An example is the International Space Station (ISS), shown in **Figure 3**. The ISS is a permanent orbiting laboratory that supports many different research projects. In the process, scientists are also able to study human responses to space travel and "zero" gravity or, more accurately, microgravity: 1×10^{-6} times the value of g. A microgravity environment is present when any object is in free fall. So when you dive from a dive tower into a swimming pool, you are in microgravity until you hit the water. Similarly, astronauts aboard the ISS are in a constant state of free fall and are thus in a microgravity environment. It is important to note the difference between microgravity and the gravitational field strength, since the value of g at the altitude of the ISS is approximately 8.7 N/kg. Clearly, there is still a significant gravitational force at that altitude and it is incorrect to say that the astronauts are in zero gravity. A gravitational force of approximately zero would only occur if you were extremely far away from any mass.

Figure 3 The International Space Station is an orbiting spacecraft in which astronauts live and work in space.

The knowledge gained from research by orbiting space stations enables scientists to design spacecraft that can safely transport people through space and perform experiments in microgravity environments. These microgravity experiments can lead to breakthroughs in medicine and chemistry. CAREER LINK

Mini Investigation

Exploring Gravity and Orbits

Skills: Performing, Observing, Analyzing, Communicating

In this investigation, you will use a simulation to create and explore different configurations of orbiting bodies. Move the planets, moons, or the Sun to see how the orbital paths change. Change the sizes of the objects and the distances between them. Explore the variations that occur as the force of gravity is changed or when gravity is removed from the model.

Equipment and Materials: computer with Internet access

1. Go to the Nelson Science website.
2. Load the supporting software, if necessary.
3. Select the option to view the Sun, Earth, and Moon and the options to show Gravity Force and the Path.
4. Play the simulation and allow Earth to complete one full revolution around the Sun.
5. Pause the simulation.
6. Using the slider bar, increase the size of the Sun and start the simulation again. Observe the motion of Earth and the Moon around the Sun for one full revolution.
7. Pause the simulation again. Return the Sun to its original size and then increase the size of Earth.
8. Start the simulation again and observe Earth's revolution.

A. What happens to the orbit of Earth when you increase the size of the Sun? T/I A

B. What happens to the Moon when you increase the size of the Sun? What happens when you increase the size of Earth? T/I A

C. What happens to Earth's orbit when you increase the size of Earth? T/I A

D. The MESSENGER probe mentioned at the beginning of this chapter made use of several gravity assists to reach Mercury without using too much of its own energy. This method is also known as a gravitational slingshot. It works by using the gravity of a celestial body to accelerate, slow down, or redirect the path of a spacecraft. Gravity assists can save fuel, time, and expense. Try to design a system of orbiting elements within the simulation that demonstrates this effect. K/U T/I C A

WEB LINK

Satellites in Circular Orbits

When Newton developed the idea of universal gravitation, he also theorized that the same force that pulls objects to Earth also keeps the Moon in its orbit. One difference, of course, is that the Moon does not hit Earth's surface. The Moon orbits Earth at a distance from Earth's centre—called the **orbital radius**. The orbit of the Moon about Earth is another example of centripetal motion, which you studied in Chapter 3. The force of gravity on the Moon due to Earth is a centripetal force that pulls the Moon toward Earth's centre. As the Moon orbits Earth, the Moon has velocity perpendicular to the radius vector. Without gravity, the Moon would fly off in a straight line. Without its orbital velocity, however, the force of gravity would pull the Moon straight to Earth's surface. The orbital motion of the Moon depends on both the centripetal force due to gravity and the Moon's orbital velocity.

orbital radius the distance between the centre of a satellite and the centre of its parent body

The Moon's orbit, similar to the orbits of the planets around the Sun, is actually elliptical. We can closely approximate the orbits, however, by assuming that they are circular. This approximation is useful for most problem-solving purposes. To analyze the motion of a satellite in uniform circular motion, combine Newton's law of universal gravitation with the mathematical expression describing centripetal acceleration. Using the universal law of gravitation from Section 6.1, we can say that the gravitational field strength of Earth with mass m_E at the location of a satellite at height r above Earth's centre is

$$g = \frac{Gm_E}{r^2}$$

Recall from Chapter 3 that the formula for centripetal acceleration based on the orbiting object's speed v is

$$a_c = \frac{v^2}{r}$$

For a satellite in a circular orbit, the gravitational force provides the centripetal force. Combining the above two equations gives

$$a_c = g$$
$$\frac{v^2}{r} = \frac{Gm_E}{r^2}$$

UNIT TASK BOOKMARK

You can apply what you have learned about orbits and satellites to the Unit Task on page 422.

Solving for the speed of the satellite and using only the positive square root gives

$$\boxed{v = \sqrt{\frac{Gm_E}{r}}}$$

This equation holds for an orbiting body in a central gravitational field. If a satellite orbits around any other large body with mass m, we can replace the mass of Earth in this equation and generalize it to

$$\boxed{v = \sqrt{\frac{Gm}{r}}}$$

This equation indicates that the speed of a satellite depends on its orbital radius and is independent of the satellite's own mass. For a satellite to maintain an orbit of radius r, its speed v must be constant.

geosynchronous orbit the orbit around Earth of an object with an orbital speed matching the rate of Earth's rotation; the period of such an orbit is exactly one Earth day

A communications satellite in **geosynchronous orbit**—that is, a satellite orbiting Earth with a speed matching that of Earth's own rotation—is an example of an artificial satellite with a constant orbital radius (**Figure 4**). The orbital period, represented by the symbol T, is the time it takes an object to complete one orbit around another object. A geosynchronous satellite's orbital speed leads to an orbital period that exactly matches Earth's rotational period. To an observer on Earth, the satellite will appear to travel through the same point in the sky every 24 h. A geostationary orbit is a type of geosynchronous orbit in which the satellite orbits directly over the equator. To an observer on Earth, a geostationary satellite will appear to remain fixed in the same point in the sky at all times. CAREER LINK

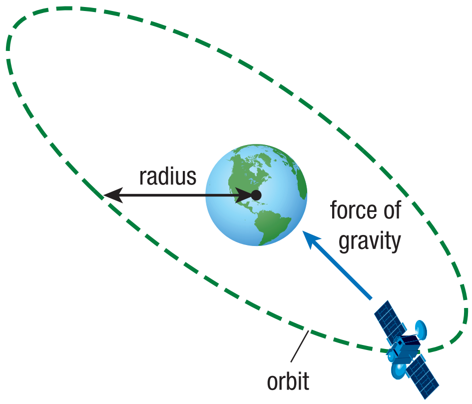


Figure 4 A satellite with a geosynchronous orbit travels at the same speed as Earth's rotation. Its orbital period is one Earth day.

In the following Tutorial you will explore how you can use the equation for orbital speed in problem solving.

Tutorial 1 Solving Problems Relating to Circular Orbits

The Sample Problems in this Tutorial show how to determine the properties of an object in a circular orbit within a gravitational field around a larger object.

Sample Problem 1: Calculating the Speed and Orbital Period of a Satellite

The International Space Station (ISS) orbits Earth at an altitude of about 350 km above Earth's surface.

(a) Determine the speed needed by the ISS to maintain its orbit.

(b) Determine the orbital period of the ISS in minutes.

Solution

(a) **Given:** $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$; $m_\text{E} = 5.98 \times 10^{24}$ kg; $r_\text{E} = 6.38 \times 10^6$ m; $h_\text{ISS} = 350\ \text{km} = 3.5 \times 10^5$ m

Required: v

Analysis: $v = \sqrt{\dfrac{Gm_\text{E}}{r}}$; $r = r_\text{E} + h_\text{ISS} = 6.73 \times 10^6$ m

Solution: $v = \sqrt{\dfrac{Gm_\text{E}}{r}}$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})}{6.73 \times 10^6\ \cancel{\text{m}}}}$$

$v = 7.698 \times 10^3$ m/s (two extra digits carried)

Statement: The ISS requires a speed of 7.7×10^3 m/s to maintain its orbit.

(b) **Given:** $v = 7.698 \times 10^3$ m/s; $r = 6.73 \times 10^6$ m

Required: T

Analysis: The distance travelled in one period is $2\pi r$. The orbital period, T, is the time it takes for the space station to travel this distance, so it is the distance divided by the speed, $T = \dfrac{2\pi r}{v}$.

Solution:

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi(6.73 \times 10^6\ \cancel{\text{m}})}{7.698 \times 10^3\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}} \times \frac{1\ \text{min}}{60\ \cancel{\text{s}}}$$

$$T = 92\ \text{min}$$

Statement: The ISS has an orbital period of 92 min.

Sample Problem 2: Calculating the Speeds of Planets around the Sun

Determine the speeds of Venus and Earth as they orbit the Sun. The Sun's mass is 1.99×10^{30} kg. Venus has an orbital radius of 1.08×10^{11} m, and Earth has an orbital radius of 1.49×10^{11} m.

Given: $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$; $m_S = 1.99 \times 10^{30}$ kg; $r_V = 1.08 \times 10^{11}$ m; $r_E = 1.49 \times 10^{11}$ m

Required: v_V; v_E

Analysis: $v = \sqrt{\dfrac{Gm}{r}}$

Solution:

$$v_V = \sqrt{\frac{Gm_S}{r_V}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(1.99 \times 10^{30}\ \cancel{\text{kg}})}{1.08 \times 10^{11}\ \cancel{\text{m}}}}$$

$$v_V = 3.51 \times 10^4\ \text{m/s}$$

$$v_E = \sqrt{\frac{Gm_S}{r_E}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(1.99 \times 10^{30}\ \cancel{\text{kg}})}{1.49 \times 10^{11}\ \cancel{\text{m}}}}$$

$$v_E = 2.98 \times 10^4\ \text{m/s}$$

Statement: Venus orbits the Sun at a speed of 3.51×10^4 m/s, and Earth orbits the Sun at a speed of 2.98×10^4 m/s.

Practice

1. Astronomers have determined that a black hole sits at the centre of galaxy M87 (**Figure 5**). Observations show matter at a distance of 5.34×10^{17} m from the black hole and travelling at speeds of 7.5×10^5 m/s. Calculate the mass of the black hole, assuming the matter being observed moves in a circular orbit around it. T/I A [ans: 4.5×10^{39} kg]

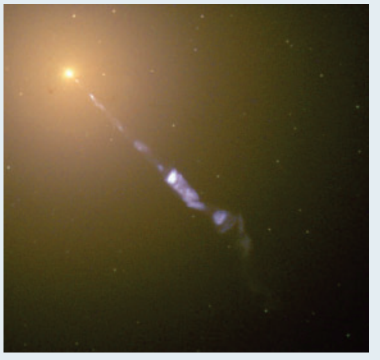


Figure 5

2. Mars orbits the Sun in a nearly circular orbit of radius 2.28×10^{11} m. The mass of Mars is 6.42×10^{23} kg. Mars experiences a gravitational force from the Sun of magnitude 1.63×10^{21} N. Calculate the speed of Mars and the period of revolution for Mars in terms of Earth years. T/I A [ans: 2.41×10^4 m/s; 1.90 Earth years]
3. Calculate the speed of a satellite in a circular orbit 600.0 km above Earth's surface. Determine the orbital period of the satellite to two significant digits. T/I A [ans: 7.56×10^3 m/s; 97 min]
4. Satellites can orbit the Moon very close to the Moon's surface because the Moon has no atmosphere to slow the satellite through air resistance. Determine the speed of a satellite that orbits the Moon just 25 m above the surface. (Hint: Refer to Appendix B for radius and mass data for the Moon.) T/I A [ans: 1.7×10^3 m/s]

Investigation 6.2.1

Design a Solar System (page 309)
With what you have learned about orbits and the movement of planetary bodies, you are ready to take the next step. This investigation will give you an opportunity to create your own solar system with a sun, several planets, and moons.

Research This

Space Junk

Skills: Researching, Analyzing, Communicating

SKILLS HANDBOOK A4.1

Space junk is debris from artificial objects orbiting Earth. It is just one example of how beneficial technology can have unwanted environmental effects. In this activity, you will research space junk and discover how an orbiting body can go from being a functioning satellite to being space junk.

1. Research the mechanisms that satellites have to maintain speed and orbital radius.
2. Research methods of dealing with different forms of space junk.
3. Explore one story of space junk that catches your interest.

A. Review this chapter's formulas pertaining to the relationship between orbital speed and orbital radius. Describe effects that could make a satellite slow down in its orbit and slip into a lower orbit. C A

B. Describe what happens when a satellite drifts so low that it enters Earth's atmosphere. C A

C. Are there any ways to avoid creating space junk? T/I C

D. Are there any effective ways to get rid of existing space junk? C A

E. Compose an email to a friend describing what space junk is. Include the interesting example you researched in Step 3. C

WEB LINK

6.2 Review

Summary

- Satellites can be natural, such as moons around planets, or artificial, such as the RADARSAT satellites and the International Space Station.
- The speed, v, of a satellite in uniform circular motion around a central body depends on the mass of the central body, m, and the radius of the orbit, r:

$$v = \sqrt{\frac{Gm}{r}}$$

- For a given orbital radius, a satellite in circular orbit has a constant speed.

Questions

1. What is the difference between natural and artificial satellites? Give an example of each. K/U
2. Explain what microgravity is. K/U
3. Explain in your own words how GPS satellites work. K/U
4. (a) What is a geosynchronous orbit?
   (b) How does a satellite in geosynchronous orbit appear to an observer on Earth?
   (c) How does a satellite in geostationary orbit appear to an observer on Earth? K/U C
5. Calculate the orbital radius of a satellite in geosynchronous orbit. K/U T/I A
6. Neptune orbits the Sun in 164.5 Earth years in an approximately circular orbit at a radius of 4.5×10^9 km. T/I A
   (a) Determine the orbital speed of Neptune.
   (b) Determine the mass of the Sun.
7. Saturn makes one complete orbit of the Sun every 29 Earth years with a speed of 9.69 km/s. Calculate the radius of the orbit of Saturn. Assume a circular orbit. T/I A
8. The region of the solar system between Mars and Jupiter, called the Asteroid Belt, contains many asteroids that orbit the Sun. Consider an asteroid in a circular orbit of radius 5.03×10^{11} m. T/I A
   (a) Calculate the speed of the asteroid around the Sun.
   (b) Calculate the period of the orbit in years.
9. In recent years, astronomers have discovered that a number of nearby stars have planets of their own, called exoplanets. A newly discovered exoplanet orbits a star with our Sun's mass (1.99×10^{30} kg) in a circular orbit with an orbital radius of 4.05×10^{12} m. What is the orbital speed of the exoplanet in kilometres per hour? T/I A
10. The orbital radius of one exoplanet is 4.03×10^{11} m, with a period of 1100 Earth days. Calculate the mass of the star around which the exoplanet revolves. T/I A
11. Phobos (**Figure 6**), one of Mars's moons, has an elliptical orbit around Mars with an orbital radius that varies between 9200 km and 9500 km. Calculate the orbital period of Phobos in Earth days, assuming a circular orbit of radius 9.38×10^6 m. The mass of Mars is 6.42×10^{23} kg. T/I A

Figure 6

12. Determine the speed of a satellite, in kilometres per hour, that is in a geosynchronous orbit about Earth. (Hint: Use the equation for the speed of an object in circular motion and equate that to the speed of a satellite in orbit around a central body. Rearrange the equation to solve for the radius. Use the radius to calculate the speed.) K/U T/I A
13. (a) Calculate the orbital speeds of the planets Mercury, Venus, Earth, and Mars using the solar system data in Appendix B.
    (b) What can you conclude about the speed of the planets in orbit farther from the Sun? T/I A
14. Scientists wish to place a geosynchronous satellite near a moon at an altitude of 410 km. The mass of the moon is 7.36×10^{22} kg and it has a radius of 1.74×10^6 m. Calculate the velocity and the period of the satellite. T/I A

6.3 Explore Applications of Gravitational Fields

SKILLS MENU

- Researching
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating
- Identifying Alternatives

Satellites

Orbiting high above Earth, satellites peer down, collecting data about the oceans, ice, land, the environment, and the atmosphere. One of the primary and most well-known functions of satellites is Earth imaging. This includes environmental monitoring, geological observations, mapping missions, forestry observations, and coastal and ocean tracking. Satellites also serve in communications systems and for the Global Positioning System.

Other satellites peer up rather than down. The Hubble Space Telescope is in orbit around Earth, but its job is to look outward toward the universe rather than down to the planet.

At any given time, thousands of satellites orbit Earth. Their orbital altitude determines their orbital speed. A satellite's orbital speed relative to Earth's rotational period will determine its ground coverage pattern and the frequency of its passing over a particular region.

The Application

Nearly 70 % of Earth's fresh water is in the Antarctic region, so environmental and climate changes there have a strong effect on worldwide sea levels and water systems. RADARSAT-1 has been collecting data and images from Antarctica since 1997, through the Antarctic Mapping Mission project (**Figure 1**). The goals of the mission include testing for effects of global warming and monitoring human impacts on Antarctica.

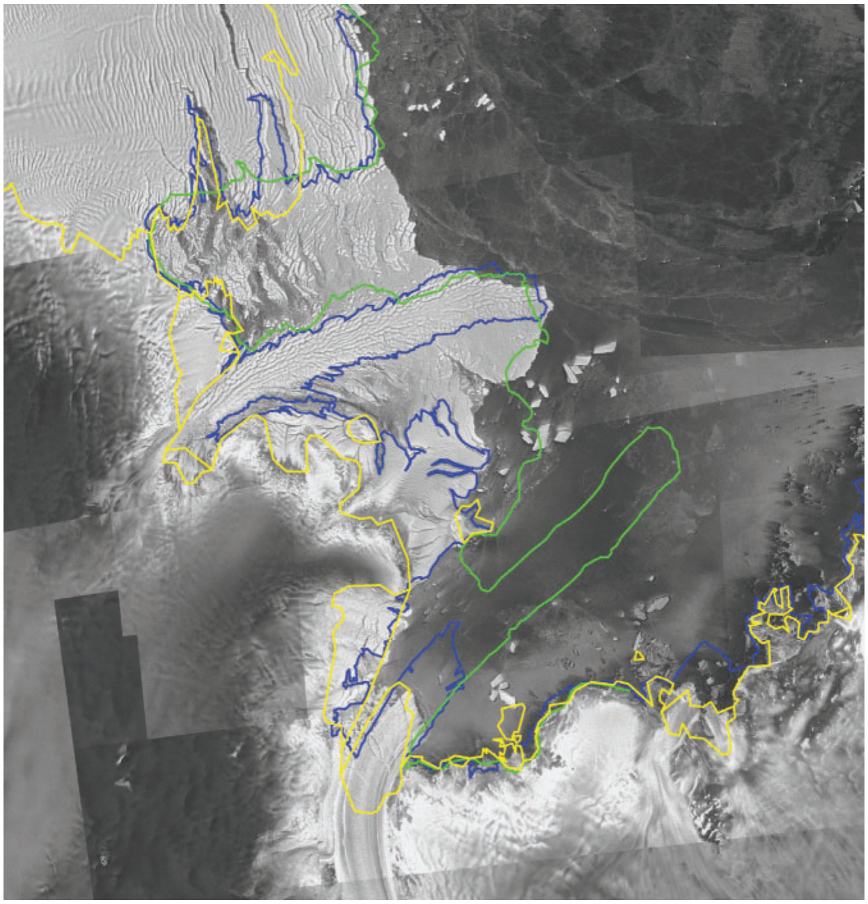


Figure 1 This RADARSAT image, compiled from satellite imaging, shows how the Shirase Glacier in Antarctica has changed over the decades. The blue represents the 1997 coastline, the yellow represents the coastline in the mid-1970s, and the green represents the coastline in 1962.

Climate change research is just one of many applications of satellites. In addition, Earth-imaging satellites are used for weather tracking and naval support, search and rescue, media broadcasting, and scientific studies. Satellites are crucial for our navigation and communications systems, and they also have military applications.

Your Goal

To become informed about satellite usage and to communicate this information to your peers

Research

Choose one application of satellites that interests you. Explore your chosen topic using the following points as a guide:

- review data or imagery captured by the satellite
- explore the purpose of the satellite
- investigate the way in which this particular satellite uses Earth's gravitational field to obtain the placement and frequency of orbit required to do its job correctly
- identify how the satellite collects, uses, and shares data and images
- examine how the data the satellite collects affects society and the environment
- include a summary of research findings and any open questions raised by the data the satellite makes available
- include both the positive and negative effects of the functions that the satellite performs WEB LINK

Summarize

Summarize your research on your chosen topic. Outline some related questions scientists are currently studying, and suggest a new one. Use these questions to summarize your research:

- How is Earth's gravitational field used to obtain the desired orbit of the satellite?
- How does the satellite collect, use, or share data?
- In what ways does the satellite's operation affect society and the environment?
- Has the satellite provided any new or useful information to researchers?
- Has information collected by this satellite raised any new questions for researchers?

Communicate

Prepare a presentation that includes a summary of your research. You may wish to include images of the satellite you are presenting and some images or data the satellite provides. Identify the sources you have used. Take questions from your audience after your presentation, and be prepared to encourage discussion.

UNIT TASK BOOKMARK

You can apply what you have learned about satellites to the Unit Task on page 422.

6.4 Physics JOURNAL

General Relativity

SKILLS HANDBOOK A3

ABSTRACT

According to Newtonian physics, gravity is an attractive force between two objects. This conception of gravity is powerful and effective: it accurately describes and predicts physical effects and applies not only to objects on Earth, but also to the Moon, the motions of the planets, and more.

General relativity explains falling bodies and orbiting masses, too, but through a very different perspective. The theory of general relativity explains gravity in terms of the geometry of space and time. This way of seeing the universe has led to breakthrough ideas such as black holes, gravitational lenses, and other mysterious phenomena.

Einstein's Mental Laboratory

Albert Einstein (1879–1955), a theoretical physicist often regarded as the father of modern physics, developed the general theory of relativity. The general theory of relativity explains gravitational effects through an advanced form of geometry. In 1905, while working as a patent clerk in Switzerland, Einstein completed his doctoral degree and published four highly influential research papers. Einstein's laboratory was mostly in his mind. He developed and shaped his theories through "what-if" style mental exercises called thought experiments. One of Einstein's most-quoted lines is, "Imagination is more important than knowledge." Due to the success of Einstein's methods, thought experiments are considered valid scientific studies today.

From Newtonian Gravitation to General Relativity

Einstein's breakthrough idea that led to the theory of general relativity was that there is no experiment that observers can perform to distinguish whether acceleration occurs because of a gravitational force or because their reference frame is accelerating, as shown in **Figure 1**.

For example, there would be no way to test (by, say, dropping or tossing balls or any other experiment that involves applications of Newton's laws) whether an observer is standing on Earth and therefore under the influence of Earth's gravitational field or whether the observer is standing on a spaceship accelerating at 9.8 m/s^2. In both cases, the observer experiences the same effects. Einstein called this relation the "principle of equivalence."

Einstein created brilliant thought experiments to study the principle of equivalence. His results led him to create a theory of gravity based on an advanced version of geometry. This theory, now called general relativity, deals with the curvature of space-time in the universe (**Figure 2**). It has several differences from Newton's theory of gravity.

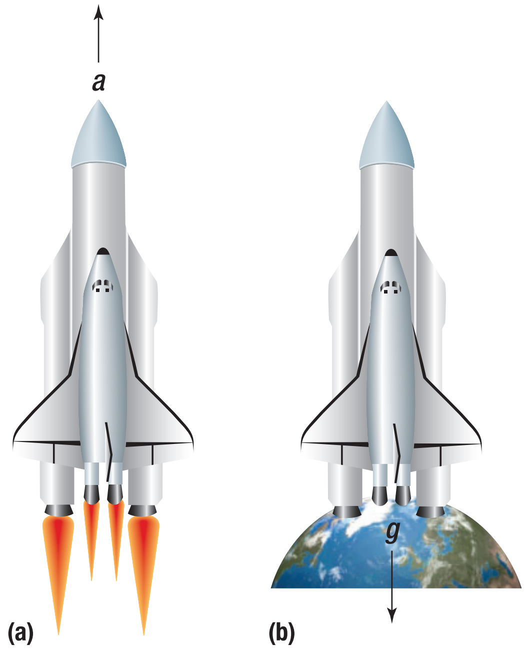


Figure 1 Einstein's theory states that there is no physical difference between (a) an accelerating frame of reference and (b) a frame of reference in a gravitational field.

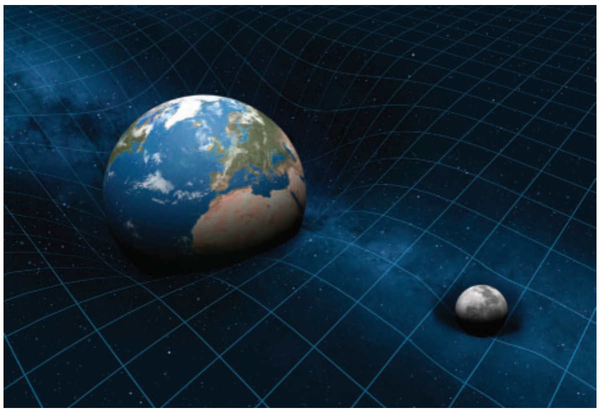


Figure 2 Albert Einstein's theory of general relativity deals with the curvature and other geometric features of space-time.

One important difference is that gravity has a "speed limit" in general relativity. In Newton's theory, a change in the position of a mass in one part of the universe instantly changes its gravitational field in all other parts of the universe. In general relativity, changes in the gravitational field travel at the speed of light but no faster.

These changes can travel across the universe as gravitational waves. A pair of stars locked in orbit around each other lose energy by emitting gravitational waves. The loss of energy means that the two stars fall toward each other and eventually collide or tear each other apart. A pair of stars, or a binary system, in orbit in Newton's theory would continue to circle each other with no changes. Measurements of changes in orbits of binary systems currently provide the most accurate experimental tests of general relativity.

Another important difference is that general relativity predicts that gravity affects light. Light does not have mass, so Newton's theory predicts that light experiences no force and exerts no force. In general relativity, the gravitational field can bend the path of light. For example, light travelling from a distant galaxy past a very massive object, such as a galaxy cluster, will bend and deflect around the object. This effect, called gravitational lensing (**Figure 3**), causes an observer on the other side of the massive object to see multiple, distorted images of the original light source. Gravitational lensing can create two or more images: a bright ring called an Einstein ring, partial rings, or other patterns.

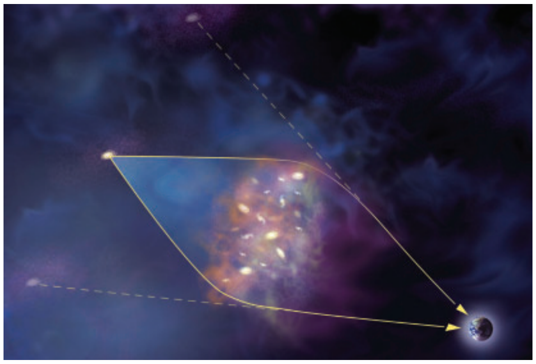


Figure 3 A massive object, such as a galaxy cluster, acts as a gravitational lens. Light from a distant star passes on both sides of the galaxy. An observer sees two separate images of the star.

One of the most mysterious predictions made by general relativity is the existence of black holes. Black holes are regions in space where the gravitational field is so strong that nothing, including light, can escape from the region after travelling into it. Black holes form as one possible product of the end of a star's life. We cannot directly see black holes, since no light can escape from one. Scientists, however, can detect black holes by studying the behaviour of objects near the suspected black hole. As material gets pulled into a black hole, the material emits X-rays and other particles that can be detected and analyzed on Earth. Black holes are so mysterious, in fact, that not even general relativity completely explains what happens to the material after it travels into the black hole.

What Is Next?

Although general relativity answers many questions about our universe, it currently faces a tough challenge. Very large objects in the universe, such as galaxies and galaxy clusters, and the universe as a whole do not behave exactly the way general relativity predicts. The orbital speed of objects at the edges of galaxies, for instance, should depend on the mass collected in the inner regions of the galaxy. Astronomers have found that the actual speeds of stars and dust in many galaxies are much faster than they would expect from the amount of mass that they can observe in the galaxies.

One solution to this challenge is that some exotic form of matter that we have never before detected exists in the universe. Physicists refer to this unknown matter as *dark matter*, since we do not directly see it. Another solution is that we have to modify the theory of general relativity so that the theory gives the correct description of the universe's behaviour. Either solution will change the way we think about gravity and the universe.

Further Reading

Guéron, E. (2009). Surprises from general relativity: "Swimming" in spacetime. *Scientific American*, August 2009, p. 34.

Einstein, A. (1952). *The principle of relativity.* New York, NY; Dover.

Schutz, B. (2004). *Gravity from the ground up.* New York, NY: Cambridge University Press.

WEB LINK

6.4 Questions

1. Explain what Einstein meant by his principle of equivalence. K/U C
2. Describe some of the differences between Einstein's and Newton's theories about gravity. K/U C
3. A person standing on Earth drops a ball. At the same time, a person standing at the bottom of a spaceship accelerating at 9.8 m/s^2, in the absence of any significant field in deep space, drops a ball. What would these people observe? T/I A
4. Explain why black holes are so difficult for scientists to study. K/U C A
5. Research general relativity and problems with the theory. Identify two possible solutions to problems with general relativity and the behaviour of galaxies. Do the solutions seem plausible to you? Why or why not? T/I C A

WEB LINK

Investigation 6.1.1 OBSERVATIONAL STUDY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Universal Gravitation

Using models is an important aspect of scientists' work in addition to traditional experiments. In this investigation, you will model the trajectory of a rocket at varying altitudes. You will build a set of data points, plot a curve, and determine the slope of the curve. Finally, you will use your data and analysis to derive *G*, the gravitational constant.

Purpose

To determine the value of the gravitational constant, *G*

Equipment and Materials

- calculator
- graph paper and/or graphing calculator or software

Procedure

SKILLS HANDBOOK A5.5

1. A rocket with a mass of 2.75×10^6 kg rises from Earth's surface. **Table 1** shows its height and the gravitational force on the rocket at various points along its trajectory. Use the data in the table to create a graph of gravitational force as a function of distance from the centre of Earth. (Hint: The rocket's height above the surface is not the same as its distance from Earth's centre. Use the empty column in the data table to determine the rocket's height including Earth's radius.) Note that distance should go on the *x*-axis of the graph.
2. Briefly describe the graph's shape.
3. Create a data table showing gravitational force versus $\frac{1}{r^2}$, where *r* is Earth's radius, 6.38×10^6 m, plus the height of the rocket. Earth's mass is 5.98×10^{24} kg. If graphing software is available, do an inverse-square curve fit. Record the constants involved.
4. Determine the slope of the graph you created in Step 3.

Table 1 The Gravitational Force for a Sampling of Rocket Heights

Gravitational force (N)	Height above Earth's surface (m)	Height including Earth's radius (m)
2.70×10^7	0	
2.32×10^7	500 000	
2.02×10^7	1 000 000	
1.56×10^7	2 000 000	
1.25×10^7	3 000 000	
1.02×10^7	4 000 000	
8.47×10^6	5 000 000	
7.16×10^6	6 000 000	
6.13×10^6	7 000 000	
5.30×10^6	8 000 000	
4.64×10^6	9 000 000	
4.09×10^6	10 000 000	
1.57×10^6	20 000 000	
8.28×10^5	30 000 000	
5.09×10^5	40 000 000	
3.45×10^5	50 000 000	
2.49×10^5	60 000 000	
1.88×10^5	70 000 000	
1.47×10^5	80 000 000	
1.18×10^5	90 000 000	
9.68×10^4	100 000 000	

Analyze and Evaluate

(a) How did your first graph differ from your second graph? T/I A

(b) To what does the slope of the second graph correspond? T/I A

(c) Describe in your own words how you used the data in the table to determine *G*. T/I C A

(d) What answer did you calculate for *G*? The current accepted value for *G* is 6.67×10^{-11} N·m²/kg². Evaluate the procedure you used to calculate *G*. Were you able to come up with the correct value? T/I C

Apply and Extend

(e) Jupiter is much larger and more massive than Earth. Explain how you think your graphs would appear if you conducted this study on Jupiter. What would happen to the value of *G*? Explain your answer. T/I C A

Investigation 6.2.1 OBSERVATIONAL STUDY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Design a Solar System

Solar systems share many characteristics. For example, most of the mass in a solar system is located in the star. The planets that orbit it are relatively small in comparison. You can determine the speed on a moon by first considering the planet at rest and then applying what you learned about relative motion from Chapter 1.

Simulation software allows you to create a wide range of orbital scenarios, observe the gravitational interactions, reset the parameters, and repeat the process over and over. In this investigation, you will use simulation software to explore orbital properties of multi-body systems and then design your own solar system. By applying the concepts and formulas you learned in this chapter, you will describe and quantify the orbits you have created.

Purpose

SKILLS HANDBOOK A2.4

To understand the dynamics of a solar system by analyzing orbital scenarios

Equipment and Materials

- access to Internet resources

Procedure

1. Go the Nelson Science website.
2. Consult the simulation software for this investigation and begin exploring possible scenarios, including options for the number of masses and the position and velocity of each.
3. Try at least 20 different scenarios, varying the starting conditions and keeping track of the results in a data table. Be sure to explore all of the preset examples to see a range of possible results.
4. Try a scenario with a sun and one planet, and change the magnitude of the velocity. At some point along the changing velocity scale, observe if there is a shift in how the two objects interact. Experiment with different velocity magnitudes and directions. Record your observations.
5. Design a solar system that includes a sun and several planets. At least one planet should have an orbiting moon.
6. For the solar system you designed, provide and solve the equations that describe the orbital path for each element, based on the mass around which each object is orbiting.

Analyze and Evaluate

(a) What happens in the simulation when you have a sun and a planet, but the planet has no velocity? T/I

(b) What happens when there is a sun and one planet, and the planet's velocity vector points away from the sun? T/I A

(c) Explain why you can ignore the masses of other planets when considering a planet's speed, but you cannot ignore those effects when calculating the speed of a planet's moon. T/I C

(d) How can you set up a stable three-body orbit? K/U T/I

(e) Experiment with complex configurations of three, four, or more objects to determine whether you can find one that combines short-period and long-period orbits. Describe your results. T/I C A

(f) Use the values in **Table 1** as your initial setup for a four-body orbital model on the simulation site. Describe your observations. T/I C A

Table 1 Orbit Simulation Settings for a Four-Body Configuration

		Position		Velocity	
Object	**Mass**	***x***	***y***	***x***	***y***
1	200	0	0	0	−1
2	10	142	0	0	140
3	0.001	166	0	0	74
4	0.001	−84	0	0	−133

Apply and Extend

(g) How do you think simulations like these can lead to new understanding of complex systems? Why do you think scientists use simulations to run virtual experiments? Explain your answer. T/I A

(h) In the virtual environment, why do you think you might have to choose between accuracy and speed of modelling? Explain your reasoning. T/I A

(i) If you were creating a new simulation environment to test gravitational effects, what would you add, change, or enhance? Explain your reasoning. T/I A

WEB LINK

CHAPTER 6 SUMMARY

Summary Questions

1. Create a concept map or other graphic organizer for this chapter based on the Key Concepts found on page 286. For each point, create three or four subpoints that provide further information, relevant examples, explanatory diagrams, or general equations.
2. Look back at the Starting Points questions on page 286. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers that you wrote at the beginning of the chapter. Note how your answers may have changed.
3. Design a graphic organizer or create a storyboard mapping the relationship between the universal law of gravitation, circular orbits, and centripetal acceleration. Include a description or a storyboard frame about how our conception of gravity was expanded by general relativity.

Vocabulary

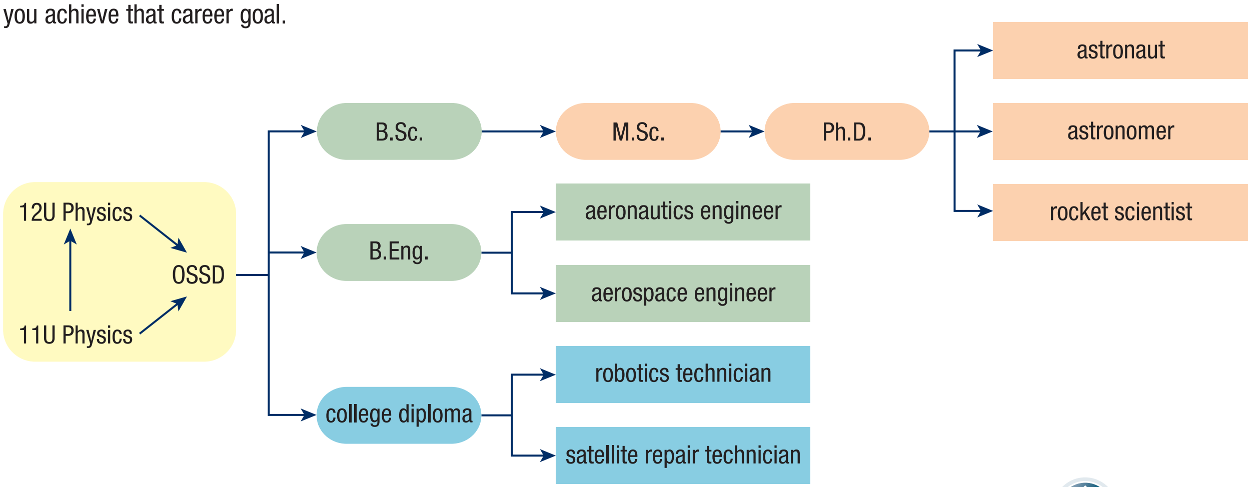
universal law of gravitation (p. 288)
gravitational constant (p. 288)
inverse-square law (p. 289)
gravitational field (p. 293)
gravitational field strength (p. 293)
satellite (p. 297)
artificial satellite (p. 297)
space station (p. 298)
orbital radius (p. 299)
geosynchronous orbit (p. 300)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers related to topics covered in this chapter.

1. Identify at least three careers related to understanding gravitational fields; launching objects through gravitational fields; solving equations of orbital mechanics; designing, maintaining, and using satellite systems; and understanding and modelling large-scale universal gravitation.
2. Choose one of these careers and create a graphic organizer similar to the one below mapping two possible education and career pathways that would help you achieve that career goal.



CAREER LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. The gravitational force between two spherical masses is F_g. Which of the following would increase the gravitational force between the objects to $16F_g$? (6.1) K/U
   (a) increasing the distance by a factor of 4
   (b) increasing the distance by a factor of 16
   (c) increasing the mass of one object by a factor of 4
   (d) increasing the mass of both objects by a factor of 4
2. For a given mass, *m*, and a given distance, *d*, separating you and the object, which of the following would exert a stronger gravitational force on you? (6.1) K/U
   (a) an object of mass 7*m* a distance 15*d* away
   (b) an object of mass 9*m* a distance 5*d* away
   (c) an object of mass 8*m* a distance 3*d* away
   (d) an object of mass 12*m* a distance 4*d* away
3. What is the gravitational force between an object of mass 3.6×10^2 kg and a second object of mass 4.3×10^3 kg when the distance between their centres is 53 m? (6.1) T/I
   (a) 3.7×10^{-8} N
   (b) 1.9×10^{-6} N
   (c) 5.4×10^{-3} N
   (d) 2.9×10^{-1} N
4. Suppose a planet has half the mass of Earth but the same radius. An astronaut stands on the surface of the planet and drops a rock from a height of 2.4 m. How much longer does it take the rock to hit the ground on that planet than it would on Earth? (6.1) T/I A
   (a) 1.2 times longer
   (b) $\sqrt{2}$ times longer
   (c) 2 times longer
   (d) 4.8 times longer
5. A satellite is in a circular orbit around Earth at an altitude of 650 km. What orbital speed must the satellite maintain to stay in orbit at this altitude? (6.2) T/I
   (a) 5.7×10^3 m/s
   (b) 7.5×10^3 m/s
   (c) 2.5×10^4 m/s
   (d) 7.9×10^4 m/s
6. Mars has a mass of 6.42×10^{23} kg and a radius of 3.4×10^6 m. What orbital speed must a satellite maintain to stay in a circular orbit at an altitude of 3.80×10^5 m above the surface of Mars? (6.2) T/I
   (a) 3.0×10^3 m/s
   (b) 3.4×10^3 m/s
   (c) 8.3×10^3 m/s
   (d) 1.1×10^4 m/s
7. Which of the following is true about both dark matter and black holes? (6.4) K/U
   (a) They are massless.
   (b) They do not have a gravitational field.
   (c) They cannot be seen directly.
   (d) They exist only outside our galaxy.

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. The strength of the gravitational field at any given distance around a 2 kg object is twice the strength of the field around a 1 kg object at the same distance. (6.1) K/U
9. The strength of Earth's gravitational field is inversely proportional to the distance from Earth's centre. (6.1) K/U
10. The gravitational constant *G* is the same for all objects. (6.1) K/U
11. The gravitational force between two 500 kg objects is twice the gravitational force between two 250 kg objects. (6.1) T/I
12. If a satellite is moving in a circular orbit, its period is directly proportional to its speed. (6.2) K/U
13. If the speed of a satellite in a circular orbit doubles, its orbital radius decreases by one half. (6.2) K/U
14. Satellites have only positive impacts on society and the environment. (6.3) K/U
15. Both Newton's and Einstein's theories of gravity predict that gravity has a speed limit. (6.4) K/U

Go to Nelson Science for an online self-quiz.
WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. Which of the following best describes the strength of the gravitational force on Earth due to your mass? (6.1) K/U
   (a) It is zero as long as you are standing on Earth's surface.
   (b) It is much greater than Earth's gravitational force on you because Earth's mass is so great.
   (c) It is equal to the magnitude of the force that Earth exerts on you.
   (d) It is negligible compared to the force of gravity on you because your mass is so small when compared to Earth's mass.
2. The gravitational force on a small rock sitting on a 20 m–high cliff on Earth is F_g. How does the gravitational force on the rock change if a hiker picks up the rock and carries it to a 200 m–high cliff? (6.1) K/U
   (a) It will decrease by an insignificant amount.
   (b) It will decrease by about one-tenth.
   (c) It will decrease by about one-fourth.
   (d) It will decrease by about one-half.
3. What major obstacle did Henry Cavendish face when measuring the gravitational force between two objects on Earth? (6.1) K/U
   (a) The masses required had to be very small.
   (b) The gravitational force between the two masses was very small.
   (c) The distance between the two masses had to be extremely large.
   (d) The gravitational force between the two masses was extremely large.
4. Ball A, with mass *m*, is a distance *d* from ball B, which has a mass of 3*m*. At which of the following distances is the gravitational attraction of the balls on each other equal? (6.1) K/U
   (a) $\frac{d}{9}$
   (b) $\frac{d}{3}$
   (c) $\frac{2d}{3}$
   (d) any separation distance
5. Spherical planet A has mass *m* and radius *r*. Spherical planet B has mass $\frac{m}{2}$ and radius 2*r*. How does the gravitational field strength at the surface of planet B compare to the gravitational field strength at the surface of planet A? (6.1) K/U
   (a) It is the same as planet A.
   (b) It is twice that of planet A.
   (c) It is half that of planet A.
   (d) It is one-eighth that of planet A.
6. Two satellites are orbiting a planet at the same height above its surface. The mass of satellite A is *m*, and the mass of satellite B is 2*m*. What can you conclude about the planet's gravitational force on the satellites? (6.2) K/U
   (a) The planet's gravitational force on both satellites is the same.
   (b) The planet's gravitational force on satellite B is half the gravitational force on satellite A.
   (c) The planet's gravitational force on satellite B is twice the gravitational force on satellite A.
   (d) The planet's gravitational force on satellite B is four times the gravitational force on satellite A.
7. How does a planet's gravity help keep a satellite in a circular orbit? (6.2) K/U
   (a) It pulls the satellite in the same direction as its motion.
   (b) It pulls the satellite at an angle of 30° to its direction of motion.
   (c) It pulls the satellite at an angle of 60° to its direction of motion.
   (d) It pulls the satellite at an angle of 90° to its direction of motion.
8. To pinpoint the location of your vehicle within 15 m using a Global Positioning System (GPS), how many satellites' signals must interact? (6.2) K/U
   (a) 1
   (b) 2
   (c) 3
   (d) 4
9. Which of the following conditions are necessary to place a satellite in geosynchronous orbit? (6.2) K/U
   (a) a varying orbital velocity so that it maintains a constant position
   (b) a varying orbital radius so that it maintains a constant height above Earth
   (c) a constant period that is equal to the orbital speed of Earth about the Sun
   (d) a constant period that matches the revolution rate of Earth about its axis

10. The period of a satellite is independent of
    (a) its own mass
    (b) the mass of the planet it orbits
    (c) the value of the gravitational constant
    (d) the orbital radius (6.2) K/U

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

11. The gravitational constant, G, near the Moon is different than G near Earth. (6.1) K/U
12. The gravitational field around Earth at a fixed distance from its centre would be the same if Earth had half the radius but the same mass. (6.1) K/U
13. A book is surrounded by its own gravitational field. (6.1) K/U
14. Unlike most satellites, a geosynchronous satellite has a fixed position and does not orbit Earth. (6.2) K/U
15. In order for a satellite to stay in a uniform circular orbit, its speed must be constant. (6.2) K/U
16. The velocity of a satellite in uniform circular motion depends on the satellite's mass. (6.2) K/U
17. Satellites are useful for communication, astronomical observations, and atmospheric studies. (6.3) K/U
18. According to the theory of general relativity, gravity has no effect on light because light has no mass. (6.4) K/U
19. Gravitational lensing occurs when the gravitational field changes the direction of motion of a massive object. (6.4) T/I

Match each term on the left with the most appropriate description on the right.

20. (a) satellite — (i) a spacecraft in which people live and work
    (b) artificial satellite — (ii) an object or body that revolves around another body
    (c) space station — (iii) an object that has been intentionally placed by humans into orbit around Earth or another body (6.2) K/U

Understanding

21. The gravitational force is inversely proportional to the square of the separation of two masses: $F_g \propto \frac{1}{r^2}$. Earth's gravitational pull on an object is defined as the object's weight. Explain why the weight of any object on Earth is not infinite, even though the distance between the object and Earth is zero. (6.1) T/I A
22. An Internet site states that the value of g on Earth is 9.806 65 N/kg. Is this figure accurate for all places on Earth? Why or why not? (6.1) K/U T/I C
23. Relate the universal law of gravitation to Newton's third law of motion. (6.1) K/U C
24. Henry Cavendish conducted experiments to measure the force of gravity between two objects on Earth. In a short paragraph, summarize Cavendish's experimental setup and results. (6.1) K/U C
25. Explain why you do not feel the gravitational force between you and a car 5 m away, even though a car's mass is so great that you cannot lift it. (6.1) T/I
26. Every object is surrounded by a gravitational field. (6.1) K/U C
    (a) The unit of the gravitational field strength g is newtons per kilogram (N/kg). Explain how the unit of gravitational field strength relates to the unit of force.
    (b) How does g vary with distance? How does it vary with the object's mass?
    (c) Describe the direction of the gravitational field around a spherical object.
27. The universal law of gravitation describes the force of gravity between two bodies. What does it say about the strength of the gravitational field? How does the size of the object affect the use of the gravitational force equation? (6.1) K/U
28. Rank the following from least to greatest gravitational attraction on you. (6.1) T/I
    (a) a mass of $4m$ a distance of $2d$ away
    (b) a mass of $6m$ a distance of $5d$ away
    (c) a mass of $2m$ a distance of $3d$ away
    (d) a mass of m a distance of $2d$ away
29. A 68 kg spherical boulder is sitting 1.5 m from a 27 kg spherical rock. What is the gravitational force between the boulder and the rock? (6.1) K/U
30. A traffic officer is standing 4.5 m from a 1200 kg pickup truck. The gravitational force between the officer and the truck is 1.7×10^{-7} N. What is the officer's mass? (6.1) T/I A
31. In 2005, the space probe Deep Impact launched a 370 kg projectile into Comet Tempel 1. Observing the collision helped scientists learn about the comet's characteristics. The comet is estimated to have a mass of about 9.0×10^{13} kg. (6.1) K/U T/I
    (a) Assuming the estimated mass of the comet at that time was correct, at what distance from the comet's centre was the gravitational force between the comet and the projectile 32 N?
    (b) What was the magnitude of the gravitational force between the comet and the projectile at a distance of 350 m?
    (c) Deep Impact also released a probe to fly by the comet and record images of the collision. Determine the strength of the comet's gravitational field at the probe's distance of 5.0×10^3 km from the comet.

32. Which is a more important factor in order for a satellite to remain in orbit at a certain distance above Earth's surface: the speed or the mass of the satellite? Explain your answer. (6.2) T/I A
33. Two identical satellites are orbiting different planets at the same orbital radius, but one planet has twice the mass of the other planet. How do the satellite's orbital speeds compare with each other? (6.2) K/U T/I
34. Two identical satellites are orbiting different planets at the same orbital radius, but one satellite's orbital speed is twice as fast as the other's. What can you conclude about the masses of the planets the satellites are orbiting? (6.2) K/U T/I A
35. The RADARSAT-1 and RADARSAT-2 satellites were placed in orbit at an altitude of approximately 800 km and have a mass of about 2750 kg each. RADARSAT Constellation satellites orbit at approximately 600 km and have masses of about 1300 kg. (6.3) K/U A
    (a) Which set of satellites has greater speeds?
    (b) What effect does the mass have on the speeds?
36. Why is the concept of dark matter sometimes referred to as the missing mass problem? (6.4) K/U C

Analysis and Application

37. Three balls are sitting on the ground, as shown in **Figure 1**. The centre of each ball is an equal distance from you. Ball A has mass m and radius r. Ball B has mass $2m$ and radius r. Ball C has mass m and radius $2r$. Compare the gravitational force of each ball on you. Explain your answer. (6.1) T/I A

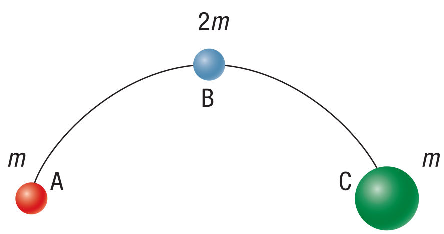


Figure 1

38. Two people are standing 1.0 m apart (centre to centre). Assume that each person has a mass of 45 kg. Calculate the gravitational force between the two people. (6.1) T/I
39. Two small balls of mass 22 kg and 25 kg are a distance of 1.2 m apart. (6.1) T/I
    (a) Calculate the gravitational force between the balls.
    (b) How far apart would two balls of mass 16 kg and 21 kg have to be to have this same gravitational force between them?
40. Henry Cavendish used freely moving balls to measure the gravitational force between two masses on Earth's surface. Suppose a scientist repeated the measurement using masses $m_1 = 0.032$ kg and $m_2 = 5500$ kg. What is the gravitational force between the masses when the distance between their centres is $r = 0.75$ m? (6.1) T/I
41. Calculate the strength of the gravitational field of a 6520 kg elephant at a distance of 5.75 m. (6.1) T/I
42. The world's largest ball of twine was made by one man in Minnesota in the United States. A basketball sitting 55.0 m (measured from centre to centre) from the ball of twine would experience a gravitational field of 1.74×10^{-10} N/kg from the ball of twine. Calculate the ball of twine's mass. (6.1) T/I
43. The highest peak in Canada is Mount Logan (**Figure 2**), which has an altitude of 5959 m above sea level. Assume that sea level defines the height of Earth's surface. (6.1) T/I

Figure 2

    (a) Calculate the strength of Earth's gravitational field at the altitude of Mount Logan.
    (b) What is the ratio of the strength of Earth's gravitational field at the top of Mount Logan to the strength at Earth's surface?
44. Neptune, the most distant planet in our solar system, is at an average distance of 4.5×10^9 km from the Sun. Its mass is 1.03×10^{26} kg. (6.1) T/I A
    (a) Calculate the strength of the Sun's gravitational field at Neptune's location.
    (b) Calculate the strength of Neptune's gravitational field at the Sun's location.
    (c) Calculate the gravitational force between the Sun and Neptune.

45. The gravitational force due to the Sun on the planets in our solar system decreases as the planetary distance from the Sun increases. In your notebook, draw a larger version of **Figure 3**, and complete it for the force of gravity on an imaginary Earth–mass planet if its distance were between the Sun's radius, r_S, and $100r_S$. (6.1) K/U T/I C

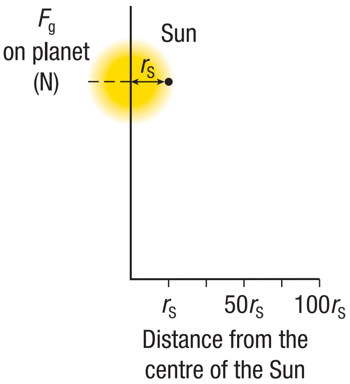


Figure 3

46. How does the *weight* of a Mars lander change as it travels from Earth to Mars? Does the weight ever equal zero? Does the mass of the lander change? Explain your answers. (6.1) K/U T/I C

47. Ceres is a dwarf planet located in the asteroid belt between the orbits of Mars and Jupiter. The radius of Ceres is 4.76×10^5 m. Suppose an astronaut stands on the surface of Ceres and drops a 0.85 kg hammer from a height of 1.25 m. The hammer takes 3.0 s to reach the ground. (6.1) T/I A
   (a) Determine the gravitational field strength of Ceres at this height.
   (b) Calculate the mass of Ceres.
   (c) Determine the gravitational field strength of Ceres at an altitude of 150 km above its surface.

48. Three balls of mass $m_1 = 13$ kg, $m_2 = 17$ kg, and $m_3 = 12$ kg are arranged in a straight line. Mass m_1 is in the middle, 6 m from both mass m_2 and mass m_3. Calculate the total gravitational force exerted by balls 2 and 3 on ball 1. State both the magnitude and the direction of the force in your answer. (6.1) T/I

49. Titan, one of the moons of Saturn, has a radius of 2.57×10^6 m and a mass of 1.35×10^{23} kg. (6.1) T/I A
   (a) Determine the gravitational field strength on the surface of Titan.
   (b) What is the ratio of Titan's gravitational field strength at its surface to the gravitational field strength on the surface of Earth?

50. The International Space Station (ISS) orbits Earth at a height of approximately 375 km. (6.1) T/I A
   (a) Calculate the gravitational field strength on the ISS.
   (b) Are astronauts truly weightless?
   (c) Why do astronauts and other objects on the ISS appear to float?

51. Consider what you have learned about the inverse-square law. Would it be possible for the force of gravity between two very heavy supertankers to cause them to float toward each other and collide? Explain your reasoning. (6.1) T/I C A

52. A satellite in orbit above Earth's equator is travelling at an orbital speed of 7.45 km/s. (6.2) T/I
   (a) Determine the altitude of the satellite.
   (b) Determine the satellite's period.

53. Saturn rotates once in 645 min (just under 11 h) and has a mass of 5.69×10^{26} kg. Suppose that scientists have placed a satellite in orbit around Saturn that has the same period as Saturn. (6.2) T/I C
   (a) Calculate the radius at which the satellite must orbit.
   (b) In a few sentences, compare this radius to Saturn's equatorial radius of 6.03×10^7 m, and compare the ratio of these two numbers to the same ratio for a satellite in geostationary orbit (around Earth).

54. Neptune has an orbital radius from the Sun of 4.5×10^9 km. (6.2) T/I A
   (a) Assume the orbit is circular. Calculate the orbital speed of Neptune. Express your answer in metres per second and in kilometres per hour.
   (b) Calculate Neptune's orbital period in Earth years.

55. Two satellites are placed in their desired orbit by releasing them from the International Space Station using the Canadarm2. Satellite A is released to an orbital radius of r. Satellite B is released to an orbital radius of $\frac{9}{10}r$. How does the velocity of satellite B compare to the velocity of satellite A? (6.2) K/U T/I

56. The microsatellite MOST (Microvariability and Oscillations of STars) has a mass of just 52 kg. It travels in an almost circular orbit at an average altitude of 820 km above Earth's surface. (6.2) T/I A
   (a) Calculate the gravitational force between Earth and the MOST satellite at this altitude.
   (b) What speed does the MOST satellite need to maintain its altitude? Express the speed in metres per second and in kilometres per hour.
   (c) Determine the orbital period of MOST.

57. Determine the ratio of the speed of a satellite in orbit around Earth to the speed of a similar satellite in orbit around the Moon, assuming the satellites have equal orbital radii. The Moon's mass is 1.23 % of Earth's mass and its radius is 27.2 % of Earth's radius. (6.2) T/I A

58. A space vehicle is in circular orbit at a height of 390 km above Earth's surface. Explain how the orbital speed of the vehicle would have to change in order for its altitude above Earth to decrease by 75 km. (6.2) T/I

59. The Canadian Telesat communications satellite Anik F2 has a mass of 5900 kg and orbits 35 000 km above the equator. (6.2, 6.3) T/I A
    (a) Determine the gravitational field of Earth at this altitude.
    (b) Determine the gravitational force between the satellite and Earth.
    (c) Calculate the speed needed by Anik F2 to maintain its orbit. Express the speed in metres per second and in kilometres per hour.
    (d) Calculate the orbital period of Anik F2.

60. The black hole at the centre of the Milky Way galaxy is called Sagittarius A* (**Figure 4**). Determining its mass is difficult, but a typical value calculated for the mass is 4.3×10^6 times the mass of the Sun. The mass of the Sun is 1.99×10^{30} kg. (6.4) T/I

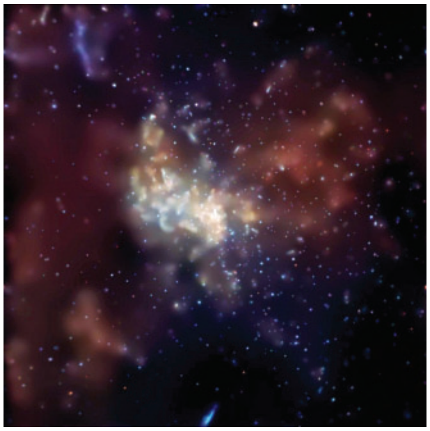


Figure 4

    (a) If this value for the mass of Sagittarius A* is correct, how would the black hole's gravitational force on a 1 kg object compare with the gravitational force on a 1 kg object the same distance from the Sun?
    (b) Suppose an 8.5 kg space probe is a distance of 4.5×10^{12} m from the black hole's centre. (This is about the distance from Neptune to the Sun.) What gravitational force does the black hole exert on the probe?

Evaluation

61. A magazine article claims that people are influenced by the movement of the planets. Use the following steps to evaluate this claim. (6.1) T/I A
    (a) The planet closest to Earth is Venus. It has a mass of 4.85×10^{24} kg, and its distance to Earth is 1.5×10^{10} m. Calculate the gravitational force of Venus on an 85 kg person.
    (b) Calculate the gravitational pull on an 85 kg person by a 10 000 kg school bus a distance of 0.5 m away.
    (c) What is the ratio of the gravitational pull of the bus to the gravitational pull of Venus?
    (d) Interpret your findings.

62. Isaac Newton developed the equation for universal gravitation several decades before Henry Cavendish did his experiment. It was not until he did his experiment that calculations using Newton's equation could produce data from observations. Cavendish's experiment yielded a value for G that is slightly higher than today's accepted value of $6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. Some more recent measurements have shown the value to be $6.69 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. What would be the effect of changing the value of the constant? (6.1) T/I A

63. In 1970, a NASA spacecraft called *Apollo 13* experienced an explosion which crippled the spacecraft. Engineers and scientists evaluated whether they should turn the spacecraft around immediately and use rockets to get the astronauts aboard the spacecraft home or use the Moon's gravitational field to get back. They opted for the use of the Moon's gravitational field. Suggest some reasons for this decision. (6.2) T/I A

64. A satellite is in orbit with velocity v at a distance d above Earth's surface. A student says that the satellite's velocity would not change if it were in orbit at the same distance d around a planet with twice the mass and twice the radius. (6.2) T/I C A
    (a) Use the equation $v = \sqrt{\frac{Gm}{r}}$ to determine whether or not the student is correct.
    (b) Would the satellite's velocity around the more massive planet be higher or lower? Defend your answer.

65. Earth's orbit around the Sun is almost but not quite circular. We can approximate, however, a small piece of the orbit as though it is part of a perfectly circular orbit of the same radius. Earth's orbital speed is slightly greater during the winter in the northern hemisphere than during the summer in the northern hemisphere. (6.2) K/U T/I C A
   (a) In which season, winter or summer, is Earth closest to the Sun?
   (b) Does your answer to (a) explain why summer in the northern hemisphere is so much warmer than in the winter? Why or why not?
66. Consider a specific type of artificial satellite and assess the impact of that satellite technology on society or the environment. (6.3) T/I C A
67. Canada first used satellites in the early 1960s for atmospheric observations. In the 1970s, however, the use shifted to communications satellites. Satellites are also used in Canada for weather and environmental observations. Make a poster explaining the ways that satellites affect your everyday life. Evaluate how your life would be different without this type of technology. (6.3) T/I C A
68. Communication satellites have made talking anywhere in the world on a cellphone possible. These communication satellites are difficult to service if anything goes wrong. If a satellite has stopped working completely, it is often left up in space to orbit. As more and more satellites end up in orbit, they will create clutter and possibly space junk. How will this clutter affect future space travel? (6.3) T/I A

Reflect on Your Learning

69. When studying this chapter, you first read about universal gravitation and then about gravitational fields. Write a short paragraph explaining why it was helpful to learn about these topics in this order instead of the reverse order. T/I C
70. Look back at the diagrams and images in this chapter. Create a slide show presentation that shows how they helped you understand universal gravitation and orbits. Be sure to include specific examples in your presentation. T/I C
71. Consider the different topics you have studied in this chapter. Choose one that you feel has an important impact on your life. Formulate your thoughts on paper and then express your thoughts to a parent or sibling, explaining about the topic and why it is important to you. What else would you like to know about this topic? How could you go about learning this? T/I C

Research

WEB LINK

72. View some actual radar images of Earth. Choose a current meteorological or environmental event, explore the information available on the Internet, and then report your findings in the form of a brief news release. T/I C
73. Application of gravitational concepts has enabled great advancements in astronomical research and understanding. Gravity explains how stars are bound together in galaxies, how galaxies are bound together in groups, and how groups of stars and galaxies are bound together in clusters. Knowledge of gravity helps scientists develop theories about black holes and dark matter. Research and prepare a report, two pages or longer, about how the application of gravitational concepts has helped astronomers. T/I C
74. The Lagrange points, labelled numerically as shown in **Figure 5**, are positions in space where satellites can be placed in stationary orbits relative to two larger objects, such as Earth and the Sun. Research Lagrange points. T/I C A

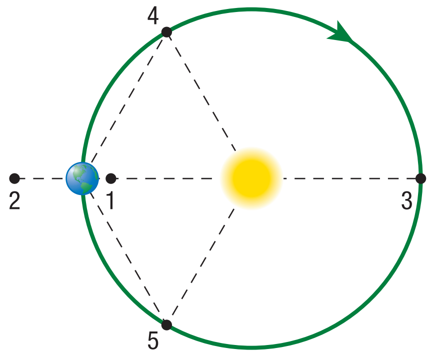


Figure 5

   (a) In an email to a peer, explain why there are five Lagrange points in the Earth–Sun system.
   (b) What do the designations 1, 2, and so on, mean?
   (c) How do scientists use these points when choosing the placement of satellites in orbits?
   (d) What satellites are currently in orbit at different Lagrange points and why?
75. A geostationary satellite is a geosynchronous satellite in orbit directly above the equator. In a few sentences, describe why a satellite must orbit above the equator to be geostationary and not just geosynchronous. K/U T/I
76. Technology now allows researchers to map Earth's gravitational field and to use the map to study the material making up Earth's interior. Research gravity surveys and how gravitational fields are used to search for mineral deposits. C A

CHAPTER

7 Electric Fields

What Effect Does the Electric Force Have on the Motion of Charges?

KEY CONCEPTS

After completing this chapter you will be able to

- describe properties of electric charges and the electric force, and describe how electric forces affect the motion of charges
- describe properties of electric fields
- solve problems related to electric force and electric fields
- analyze the operation of technologies that use electric fields and assess their social and environmental impact
- solve problems related to electric potential and electric potential energy
- conduct laboratory inquiries to examine the behaviour of particles in a field

Although physicists have answered many questions about our world, many puzzles still remain. For one thing, researchers still do not fully understand the phenomenon of lightning. We do know that lightning is an electrical effect in which an imbalance of electric charge forms in storm clouds and suddenly causes a discharge to try to cancel the imbalance. The violent energy of the storm somehow causes this charge imbalance, but researchers do not yet understand the exact cause. They also do not yet know how to predict when and where lightning will strike.

Researchers do know that when the charge imbalance becomes great enough, a current runs from cloud to ground, cloud to cloud, or even ground to cloud. In fact, a rare form of lightning called ball lightning forms as a sphere of charge that can travel along the ground with a ghostly, dangerous glow. A bolt of lightning can provide up to a million times the voltage of a normal wall receptacle. The flow of charge can heat the air around a lightning bolt to seven times the temperature of the surface of the Sun. This sudden immense heating causes the air to expand rapidly, leading to the sound wave we know as thunder.

Astrophysicists have recently discovered another surprising phenomenon. The imbalance of electric charge in storm clouds can cause a burst of antimatter! Antimatter is the opposite of regular matter in some specific properties such as charge. For example, the antimatter version of an electron is called the positron. A positron has all of the properties of an electron except that it is positively charged. The storm releases electrons upward from the top of the clouds, where the electrons bump into air molecules. The collision leads to bursts of particles high above the clouds, including positrons. This phenomenon is depicted in an artist's rendering on the facing page. When antimatter and regular matter collide, they annihilate each other to produce a significant amount of energy. You will learn more about antimatter in Unit 5. To understand this effect and other puzzles of lightning, we must understand the electric force and how it affects electrons and other charged particles. You will be learning about the properties of electric charges and electric fields in this chapter.

STARTING POINTS

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. What might cause an imbalance of charges inside an object?
2. Can the path of an electron be directed?
3. How do electric fields compare to gravitational fields?
4. Is it possible to do work with electric charges?
5. What are some applications of electric fields?

Mini Investigation

The Van de Graaff Generator

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

Your teacher will first demonstrate how a Van de Graaff generator works. You will then have a chance to touch the generator and observe its effect on your hair.

The Van de Graaff generator produces a very high voltage and can give you a dangerous shock, especially if you have a heart condition. Remove metal jewellery. Follow your teacher's instructions.

Equipment and Materials: Van de Graaff generator; grounding electrode; tall wooden stool

1. Observe as your teacher turns on the Van de Graaff generator and slowly brings the grounding electrode close to the generator sphere. Record your observations.
2. Your teacher will discharge the generator before having you touch it and then turning it on. Sit on the stool, which is insulated from the ground, and touch the generator with both hands. Do not let go when the generator is turned on. Have your classmates observe what happens to your hair during the demonstration. Note any sensations that you feel when you touch the generator. The generator must be discharged before you let go. Observe what happens when other students touch the generator. Record your observations.

A. Describe what happened when your teacher brought the grounding electrode near the Van de Graaff generator. K/U

B. Suppose the spark represented a lightning strike. Explain how to apply what you observed to protecting buildings from lightning damage. T/I A

C. Describe what you observed in Step 2 when you were touching the generator. Describe what you observed in Step 2 while watching your classmates. Did you note any differences in the effect on students with long hair compared to the effect on students with short hair? K/U C A

7.1 Properties of Electric Charge

A visit to a science museum can be, literally, a hair-raising experience. In **Figure 1**, the device that the child is touching is a Van de Graaff generator, which produces a large amount of electric charge. When you touch the generator, the electric charge spreads along your skin and onto each individual hair. The charges repel each other, so the hairs spread out and stand on end. How do electric charges move from one place to another and charge an object? In this section, you will learn the answers to this and other questions related to the properties of electric charges.

Figure 1 Electric charges on each strand of hair exert a repulsive force on the other strands, causing the hair to rise and spread out.

Electric Charge

About 2500 years ago, Greek scholars first reported that rubbing amber with a piece of animal fur caused the amber to attract dust particles. You can demonstrate this effect with modern materials, such as a plastic rod and small bits of paper (**Figure 2**). Note that neither the fur nor the rod attracts the paper pieces under normal conditions, but rubbing the two materials together seems to create an attractive force on each object. An even more remarkable feature is that the rod attracts the pieces of paper without making direct contact with them.

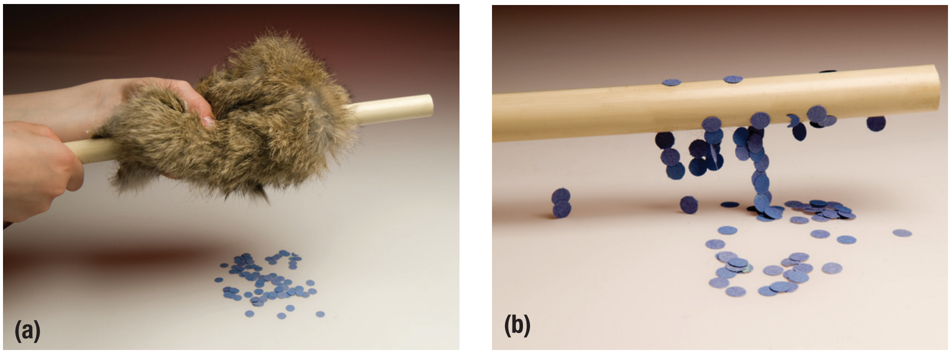


Figure 2 When a plastic rod is rubbed with fur (a), the rod acquires an electric charge. (b) The charged rod attracts small bits of paper and other objects.

By the early 1900s, physicists had identified the subatomic particles called the electron and the proton as the basic units of charge. All protons carry the same amount of positive charge, *e*, and all electrons carry an equal but opposite charge, $-e$. Charges interact with each other in very specific ways governed by the **law of electric charges** (**Figure 3**).

Law of Electric Charges

Like charges repel each other; unlike charges attract.

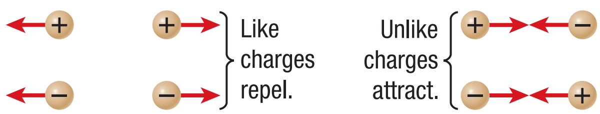


Figure 3 The electric force between two like charges (charges with the same sign) is repulsive, whereas the electric force between unlike charges (charges with opposite signs) is attractive.

An atom or a molecule is considered to have a charge of zero because the number of protons is equal to the number of electrons. Atoms and molecules can become charged to form ions. Ions can be positively or negatively charged. In a positive ion, or a cation, the number of protons must be greater than the number of electrons. Electrons are removed from an atom or a molecule to become a cation. Conversely, in a negative ion, or an anion, the number of electrons must be greater than the number of protons. Electrons are added to an atom or a molecule to make an anion.

When looking at objects larger than atoms and molecules, we consider the total charge. The total charge on an object is the sum of all the charges in that object and can be positive, negative, or zero, just as in the case for atoms and molecules. A deficit of electrons means that the object is positively charged, and an excess of electrons means that the object is negatively charged. An object is said to have a total charge of zero when the number of negative charges equals the number of positive charges. An object with a charge of zero is said to be neutral.

In addition, we know that charge can move from place to place, and from one object to another, but the total charge of the universe does not change. This is stated clearly in the **law of conservation of charge**.

Law of Conservation of Charge

Charge can be transferred from one object to another, but the total charge of a closed system remains constant.

What Is Electric Charge?

In the SI system, the basic unit of charge is called the **coulomb** (C), in honour of the French physicist Charles de Coulomb (1736–1806). The charge of a single electron, $-e$, is -1.60×10^{-19} C, and the charge of a single proton, $+e$, is $+1.60 \times 10^{-19}$ C. The symbol e often denotes the magnitude of the charge on an electron or a proton. In this text, e will have the positive quantity ($+1.60 \times 10^{-19}$ C).

coulomb the SI unit of electric charge; symbol C

The symbol q denotes the amount of charge, such as the total charge on a small piece of paper. To say that a particle has a charge q or that it carries a charge q is simply a way of saying that the total charge of the particle is q. For example, an alpha particle is a helium nucleus (a helium atom with no electrons). The helium nucleus has two protons, so its charge is $2e$, or 3.20×10^{-19} C ($2 \times 1.60 \times 10^{-19}$ C).

Conductors and Insulators

To understand how researchers observe electric forces, you need to understand the various ways that charge can be transferred from one material to another.

Conductors

In most metals, each individual atom is electrically neutral, with equal numbers of protons and electrons. When these neutral atoms come together to form a large piece of metal, one or more electrons from each atom are able to escape from the parent atom and move freely through the entire piece of metal. These electrons are called conduction electrons, and the metal is a conductor. A **conductor** is a substance in which electrons can move easily among atoms. Copper is a good example of a conductor. The conduction electrons leave behind positively charged ions, which are bound in place and do not move. By adding or removing electrons from a conductor, the conductor can acquire a net negative or positive charge.

conductor any substance in which electrons are able to move easily from one atom to another

Insulators

On the other hand, electrons in **insulators** cannot move freely from atom to atom or escape from the molecules, so no conduction electrons are available to carry charge through the solid. Extra electrons placed on the surface of an insulator do not move about freely. Instead, these added electrons stay where they were initially placed. Examples of insulators are amber, plastic, and quartz.

insulator any substance in which electrons are not free to move easily from one atom to another

Conductivity of Liquids and Gases

In the case of liquids, the atoms and molecules are free to move throughout the substance. Certain substances, such as table salt (NaCl), dissolve in water to form separately charged atoms, or ions. The sodium ion (Na^+) and the chloride ion (Cl^-) provide an electric charge in the salt–water solution. This property of salt water makes it a good conductor, which is especially important for marine animals that rely on bioelectricity for survival.

The conductivity of gases is similar to that of liquids. The constituent atoms and molecules of a gas are mostly neutral, but a few are present as ions and are able to carry charge from place to place. Some free electrons are usually present in a gas as well.

Placing Charge on an Insulator

To understand the electrical behaviour of an object, you must understand what happens when you add or remove charge from the object. Take, for instance, the case of an insulator such as quartz, when it is dry and surrounded by dry air. If you place a few electrons at a particular location on this insulator's surface, these electrons will stay in that location (**Figure 4(a)**). This is because there are no free electrons in an insulator, and the insulator does not allow the extra electrons to move about easily.

In reality, excess charge will not stay on an insulator indefinitely. If an insulator contains some excess electrons, they will attract the stray positive ions that are usually part of the surrounding air. These stray ions will combine with the electrons on the insulator and cause the net charge on the insulator to become zero.

Placing Charge on a Conductor

Metals are excellent conductors because electrons can move easily through a metal. We have to refer to the law of electric charges to understand what happens when an excess of electrons is present in a conductor. Electrons naturally repel each other. In a metal object, the electrons are free to move from atom to atom, but they will never end up moving toward other electrons. The electrons do not concentrate at the centre of the piece of metal; rather, they move as far away from one another as possible. Also, the electrons do not spontaneously leave the metal to get away from each other. There are attractive forces from the protons in the atoms preventing the electrons from leaving. Since the electrons must repel each other while staying within the metal, excess electrons on a piece of metal must all spread evenly on the metal's surface (**Figure 4(b)**).

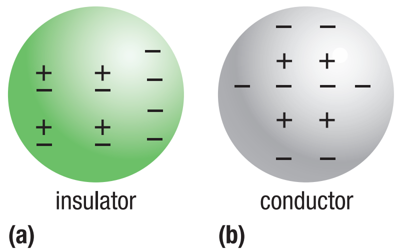


Figure 4 (a) When excess electrons are placed on an insulator, they generally stay where they are placed for a period of time. (b) Excess electrons placed on a conductor, however, spread out on the surface of the material immediately.

This does not mean that there are no charges within the metal. The interior of a neutral, uncharged metal contains equal numbers of positive and negative charges. These charges are present inside the metal at all times. Only the excess charge resides at the metal's surface. In Figure 4(b), excess charge on the metal surface distributes as negative charge, by adding electrons to the conductor, or as positive charge, by removing electrons from the conductor.

Charging an Object by Friction

Electricity was discovered when a piece of amber was rubbed with fur. The act of rubbing caused electrons to move from the fur to the amber. The amber thus acquired a net negative charge (an excess of electrons), whereas the fur was left with a net positive charge (an excess of positive ions). This process is called *charging by friction*. You can also charge a glass rod by rubbing it with a silk cloth, in which case electrons leave the glass and move to the silk, so the glass acquires a net positive charge. Through investigation, scientists have found that some materials have a stronger hold on their electrons than others. For example, glass has a weaker hold on its electrons than does silk. If you were to let glass and silk touch each other, some electrons would leave the glass and travel to the silk. Rubbing allows more surface to come into contact, and the friction it produces rips more electrons off the glass to go to the silk. **Figure 5** shows the electrostatic series, which indicates the relative hold on electrons that different materials have when being charged by friction.

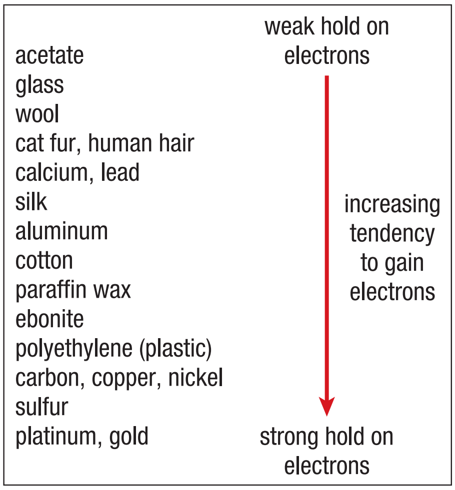


Figure 5 The electrostatic series

Charging an Object by Induced Charge Separation

If you look back at the example of using a charged plastic rod to attract pieces of paper, you may notice that something seems wrong. You now know that rubbing the rod with fur charges the rod. However, the pieces of paper start out neutral with a total charge of zero. How is the neutral paper attracted to the charged rod?

Although the paper is electrically neutral, the presence of the rod nearby causes some slight movement of the charges in the paper. The negatively charged rod repels electrons in the paper, which are then redistributed throughout the material. The electrons thus move a short distance away from the rod, as shown in **Figure 6**. A net positive charge remains on the portion of the paper nearest the rod, and a net negative charge stays on the paper opposite the rod. This process is called charging by *induced charge separation*.

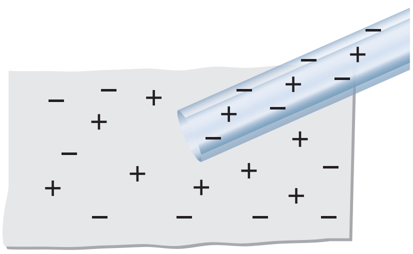


Figure 6 The negative charges in the paper have been repelled by the negative charges in the rod, leaving the area of the paper nearest to the rod positively charged and causing attraction between the paper and the rod.

Unlike charges attract, and the positive side of the paper is closer than the negative side of the paper to the negatively charged rod. This makes the paper attractive, overall, to the charged rod. In this way, an electric force pulls on an electrically neutral object once the object has undergone induced charge separation.

Another example of induced charge separation is shown in **Figure 7**. Ordinarily, the stream of water flows straight downward due to the force of gravity. However, if you place a charged balloon near the stream, it exerts a deflecting force on the water. The charged balloon induces a charge separation in the water molecules, producing an attractive force similar to that produced by the charged rod on the pieces of paper.

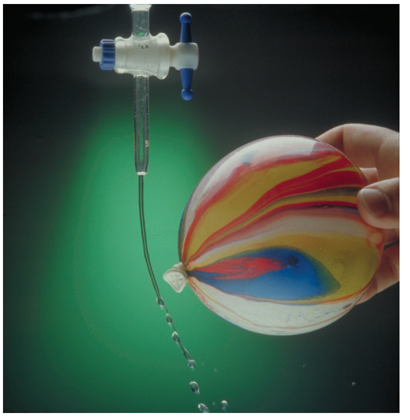


Figure 7 A stream of water is deflected by a nearby charged balloon.

Grounding

Suppose you place a charged rubber rod on a table. If you watch carefully with sensitive electronic measuring equipment, you will find that the excess electrons originally on the rod move to the table and then to other adjacent objects. Grounding occurs whenever any charge imbalance is cancelled out by either sending excess electrons into the ground or moving excess electrons from the ground into the object. Grounding works because Earth is so large that any extra electrons going into or out of the ground have an insignificant effect on Earth. The concept of grounding is used by electrical engineers to ensure that buildings and appliances are safe. Ground wires direct excess charges away from users, protecting them from electric shock. CAREER LINK

Charging by Contact and by Induction

How would you transfer some of the electrons on a negatively charged rubber rod to a piece of metal? You could do this by simply touching the rubber rod to the metal. The rod contains an excess of electrons, and some of these electrons will move to the metal upon contact (**Figure 8**). This is an example of *charging by contact*, or *charging by conduction*.

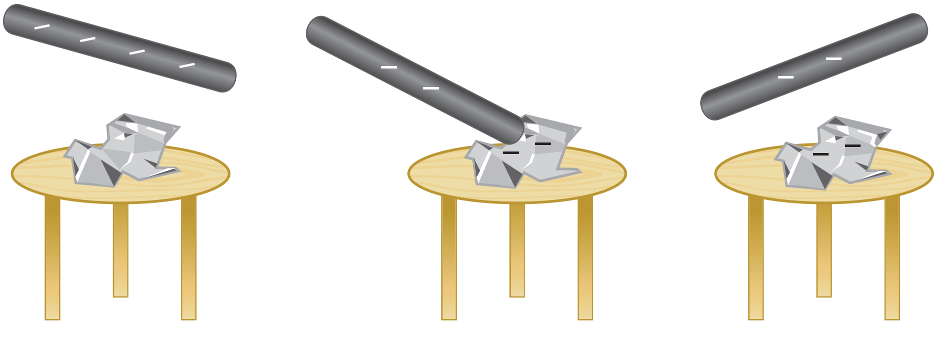


Figure 8 If one charged object touches a second object, the second object will usually acquire some of the excess charge. Hence, the second object is charged by contact. The stand in this illustration is an insulating stand.

Now suppose you want to give the metal a net positive charge using the same negatively charged rubber rod. This might seem an impossible task, but an approach called *charging by induction* will accomplish it. This approach uses the properties of induced charge separation and an electrical ground.

First, bring the negatively charged rod near the metal. This polarizes the conductor by moving conduction electrons to the side opposite the rod and leaving the side of the metal near the rod with a net positive charge (**Figure 9(a)**). Now, connect the negatively charged portion of the metal to an electrical ground using a metal wire. This allows electrons to move even farther from the charged rod by travelling into the electrical ground region (**Figure 9(b)**). Finally, after removing the grounding wire, the original piece of metal has a net positive charge (**Figure 9(c)**). Notice that no positive charges are transferred to the metal; instead, electrons are removed and the charge left on the metal is positive.

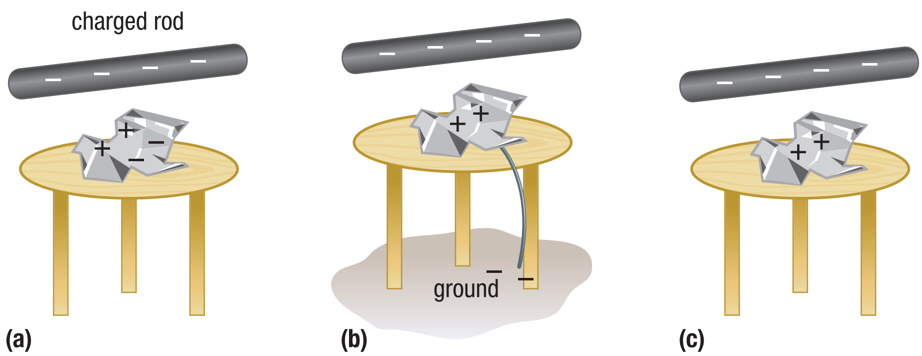


Figure 9 An object can be charged by the process of induction. (a) The object is first brought near a charged rod, separating the charges on the object. (b) The object is then connected to a ground; some electrons flow between it and the ground. (c) The object is left with an opposite excess charge when it is disconnected from the ground.

Mini **Investigation**

Observing Electric Charge

Skills: Controlling Variables, Performing, Observing, Analyzing, Communicating   SKILLS HANDBOOK A2.1

In this investigation, you will explore how different objects become charged. You will draw conclusions based on how these charged objects affect small pieces of paper.

Equipment and Materials: plastic pen; pencil; glass stirring rod; tissue paper; fabric

1. Tear a piece of tissue paper into several small pieces and gather them into a pile.
2. Charge the plastic pen by rubbing it on a piece of fabric, such as part of an old flannel shirt.
3. Bring the charged object near the pieces of paper.
4. Observe how the pieces of paper behave relative to the charged object used, and record your observations.
5. Gather the pieces of paper into a pile again, and repeat the steps, first using the pencil, and then using the glass rod. Record your observations.

A. Why does each charged object attract the pieces of paper differently? K/U T/I

B. Why do some pieces of paper fall off your charged objects after a short while? K/U T/I

C. If you used a metal sphere with a large charge, the pieces of paper would jump off instead of falling. Explain why this occurs. K/U T/I

7.1 Review

Summary

- Electric charges are either positive or negative. Objects that have more negative charge than positive charge are negatively charged. Objects that have more positive charge than negative charge are positively charged. Objects that have an equal number of positive and negative charges are neutral and have a total charge of zero.
- Electrons are the only subatomic particle capable of being transferred from one object to another. Protons are bound to the atomic nucleus.
- The law of electric charges states the following: Unlike, or opposite, electric charges attract each other. Like, or similar, electric charges repel each other.
- The law of conservation of charge states that charge can be created or destroyed, but the total charge of a closed system remains constant.
- Objects can be charged by friction, by induced charge separation, by contact, and by induction.

Questions

1. When you rub a glass rod with silk, the rod becomes positively charged; when you rub a plastic rod with fur, the rod becomes negatively charged. Suppose you have a charged object but do not know whether it carries a positive or a negative charge. Explain how you could use a glass rod and a piece of silk to determine the sign of the charge on the unknown object. K/U T/I
2. Explain how two objects attract one another due to an electric force, when one object has zero net charge. K/U C
3. Suppose children at a party rub balloons on their hair and then place the balloons on the wall. If the rubbing process puts excess electrons on the balloons, how do the balloons stay attached to the wall? K/U A
4. The end of a charged rubber rod attracts small pellets of foam plastic that, having made contact with the rod, quickly move away from it. Explain why this happens. T/I C
5. When two objects, such as a glass rod and a silk cloth, are rubbed together, electrons move from one object to the other. Can protons also move from one object to the other? Explain why or why not. K/U T/I
6. If you walk across a thick carpet on a cold, dry day, electrons will move from the carpet to your body. K/U
   (a) How does the charge on your body compare to the charge on the rug?
   (b) Which of the three methods of charging is demonstrated in this example?
7. A student upgrades the memory in her computer by replacing the memory chip. Before handling the chip, she first touches the metal casing of the computer. Why is this a good precaution? K/U A
8. Provide an example of how to give a neutral object a positive charge using only a negatively charged object. K/U C
9. In the following examples, describe the change in charge on each rod and charging material or object in terms of the movement of electrons. K/U
   (a) A glass rod is rubbed with a wool rag.
   (b) A plastic rod is rubbed with a silk scarf.
   (c) A platinum rod with a negative charge is touched with a similar rod with a positive charge.
   (d) A small metal rod touches a large positively charged metal sphere.
10. A student shakes hands with his father, who is wearing a wool sweater. As soon as they shake hands, a spark jumps between their hands. Explain what caused the spark. K/U
11. Fabric softener sheets are supposed to reduce the static cling between the clothes in a dryer. Research fabric softener sheets, and answer the following questions. K/U T/I
    (a) Why do clothes cling to each other when removed from a dryer?
    (b) How does a fabric softener sheet reduce the static cling among clothes?

WEB LINK

Coulomb's Law

7.2

Recall that charged objects attract some objects and repel others at a distance, without making any contact with those objects. **Electric force**, F_E, or the force acting between two charged objects, is somewhat similar to gravity. Both are non-contact forces. Similar to the force of gravity, the electric force becomes weaker as the distance, r, between the charged objects increases (**Figure 1**). Electric force becomes stronger as the amount of charge on either object increases, in the same way that the force of gravity becomes stronger with an increase in mass of either object. In this section, you will learn more about how the electric force depends on charge and distance. You will also learn how to solve problems related to the electric force.

electric force (F_E) a force with magnitude and direction that acts between two charged particles

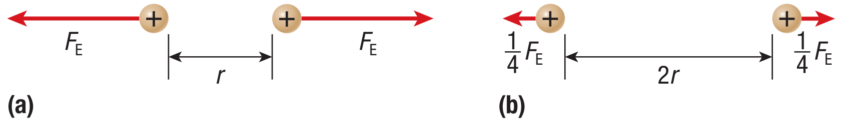


Figure 1 The electric force of repulsion between two identical charges decreases as the separation increases.

The Electric Force

If you drop a tennis ball, the force of gravity is responsible for its fall. The tennis ball will take approximately 1 s to fall from a height of 5 m. How long do you think the tennis ball will take to stop as it hits the ground? It will take a lot less time to stop than it took to fall. What force is responsible for making the tennis ball stop? The electric force of repulsion between the protons in the tennis ball and the protons in the ground stop it. This electric force must be significantly stronger than gravity.

Consider two charged objects so tiny that you can model them as point particles. These objects have charges q_1 and q_2 and are separated by a distance r (**Figure 2**). The magnitude of the electric force between q_1 and q_2 is expressed by the equation

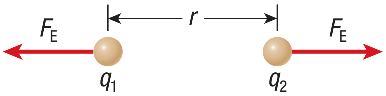


Figure 2 The electric force between two point charges q_1 and q_2 is given by Coulomb's law.

$$F_E = k\frac{q_1 q_2}{r^2}$$

This equation represents **Coulomb's law**.

Coulomb's Law

The force between two point charges is inversely proportional to the square of the distance between the charges and directly proportional to the product of the charges.

The constant k, which is sometimes called **Coulomb's constant**, has the value $k = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. Do not confuse this constant k with the spring constant in Hooke's law. Most physicists use the same symbol for both.

Coulomb's constant (k) the proportionality constant in Coulomb's law; $k = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$

The direction of the electric force on each of the charges is along the line that connects the two charges, as illustrated in Figure 2 for the case of two like charges. As mentioned earlier, this force is repulsive for two charges with the same type of charge. Strictly speaking, the value of F_E in the equation for Coulomb's law applies only to two point charges. However, it is a good approximation whenever the sizes of the particles are much smaller than their distance of separation.

Investigation 7.2.1

Coulomb's Law (page 366)

Now that you have learned how to calculate the electric force between two charges with a given separation, perform Investigation 7.2.1 to determine how electric force varies with distance r and $\frac{1}{r^2}$. You will use this information with two different charge values to determine Coulomb's law.

Coulomb's law has several important properties:

- You have already seen that the electric force is repulsive for like charges and attractive for unlike charges. Mathematically, this property results from the product q_1q_2 in the numerator of the Coulomb's law equation. The magnitude of the electric force, F_E, is always positive. In physics, it is convenient to use a negative sign to show direction. Including the negative sign of a negatively charged object would imply a direction. You never include the sign of the charge when solving problems related to Coulomb's law.
- The electric force is inversely proportional to the square of the distance between the two particles.
- The magnitude of the electric force is the magnitude of the force exerted on each of the particles. That is, a force of magnitude F_E is exerted on charge q_1 by charge q_2, and a force of equal magnitude and opposite direction is exerted on q_2 by q_1. This pairing of equal but opposite forces is described by Newton's third law, the action–reaction principle.

Using the equation for Coulomb's law, we can show that electric forces can be extremely large. Suppose you have two boxes of electrons, each with a total charge of $q_T = -1.8 \times 10^8$ C separated by a distance r of 1.0 m. For simplicity, assume that each box is so small that it can be modelled as a point particle. The magnitude of the total electric force is

$$\begin{aligned}F_E &= k\frac{q_1q_2}{r^2}\\ &= \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)\frac{(1.8 \times 10^8 \ \cancel{\text{C}})(1.8 \times 10^8 \ \cancel{\text{C}})}{(1.0 \ \cancel{\text{m}})^2}\\ F_E &= 2.9 \times 10^{26} \ \text{N}\end{aligned}$$

This is an extremely large force, all from just two small containers of electrons. Note that the negative sign for charge was not included.

Why, then, does the electric force not dominate everyday life? The fact is that it is essentially impossible to obtain a box containing only electrons. Recall from earlier science studies that a neutral atom contains equal numbers of electrons and protons. If our two point-like boxes had contained equal numbers of electrons and protons, their total charges, q_T, would have been zero, and so would the force calculated using Coulomb's law.

Ordinary matter consists of equal, or nearly equal, numbers of electrons and protons. The total charge is therefore either zero or very close to zero. At the atomic and molecular scales, however, it is common to have the positive and negative charges (nuclei and electrons) separated by a small distance. In this case, the electric force is not zero, and these electric forces hold matter together.

Comparing Coulomb's Law and Universal Gravitation

The equation for Coulomb's law may seem familiar to you, even though you have only just learned it. This is because it is similar to the universal law of gravitation, which you learned in Chapter 6. Both of these laws describe the force between two objects. In addition, both depend on certain properties of the objects involved. For gravitation, this property is mass. For the electric force, this property is electric charge. Both mass and charge can be considered to be concentrated at a central point. If you think of the mass or charge as a solid sphere, you can treat this same mass or charge as if it were a point at the centre of that sphere.

Another similarity between the two laws is that both the gravitational force and the electric force decrease as the distance between the two interacting objects increases. If the distance between the objects is great (compared to the size of either object), the actual size and shape of the objects involved become mathematically irrelevant. The theoretical mass and charge at the centre point of each object are then used in calculations.

Although there are similarities between electric and gravitational forces, there are also important differences. Gravitational forces are always attractive. The direction of electric forces depends on the types of charge: unlike charges attract, and like charges repel. Also, the magnitude of the electric force is much greater than the magnitude of the gravitational force over the same distance.

The Superposition Principle

So far, you have learned about the electric force between two point particles. Now consider how to use Coulomb's law with more than two charges. Suppose you have three particles. One particle has a charge of q_1. The second particle has a charge of $2q_1$ and is a distance $2r$ from the first particle, as shown in **Figure 3**. What is the electric force on a third charge, q_2, placed midway between these two charges?

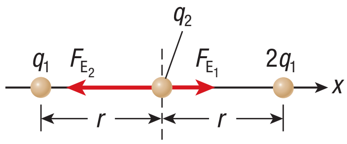


Figure 3 The total force on q_2 equals the sum of the forces from charges q_1 and $2q_1$.

Solve this problem by first using Coulomb's law to calculate the force exerted by charge q_1 on q_2. Then use Coulomb's law a second time to calculate the force exerted by the charge $2q_1$ on q_2. The total force on q_2 equals the vector sum of these two separate contributions. This combining of two forces is an example of the superposition principle. The **superposition principle** says that the total force acting on q_2 is the sum, or superposition, of the individual forces exerted on q_2 by all the other charges in the problem. Remember that force is a vector, so be careful to add these forces as vectors. This means that, for charges not on a straight line, the solution requires some trigonometry involving triangles or the use of vector components.

superposition principle the resultant, or net, vector acting at a given point equals the sum of the individual vectors from all sources, each calculated at the given point

When all charges lie in a straight line, the superposition principle simplifies so that you can add or subtract the various individual forces to or from one another to obtain the resultant, or net, force. As shown in Figure 3, the separation between q_2 and q_1 is r. Coulomb's law for this pair of charges is therefore

$$F_{E_1} = \frac{kq_1q_2}{r^2}$$

Using the coordinate system in Figure 3, this corresponds to a force on q_2 by q_1 in the positive x-direction because like charges repel. So, F_{E_1} is the component of the force along the positive x-axis. The charges line up along the x-axis, so the component of the force along the y-axis is zero.

Now consider the force on q_2 by the charge $2q_1$ in a similar way. The separation of the charges is again r, so, using the equation for Coulomb's law, you get

$$F_{E_2} = k\frac{(2q_1)q_2}{r^2}$$

$$F_{E_2} = \frac{2kq_1q_2}{r^2}$$

From Figure 3 you can see that this corresponds to the force on q_2 by $2q_1$ in the negative x-direction, again because like charges repel. This is because the electric force acts along the line connecting the two charges, which in this case is the x-axis. For this reason, the component of the force along the y-axis is again zero. So, the force F_{E_2} exerted by $2q_1$ on q_2 is in the negative x-direction, or to the left. The total force on q_2 is the sum of the two electric forces, F_{E_1} and F_{E_2}. Be careful to take direction into account for the calculation. In this case, the sign of each force indicates its direction.

$$\begin{aligned}\vec{F}_{E_{net}} &= \vec{F}_{E_1} + \vec{F}_{E_2} \\ &= +\frac{kq_1q_2}{r^2} - \frac{2kq_1q_2}{r^2} \\ \vec{F}_{E_{net}} &= -\frac{kq_1q_2}{r^2}\end{aligned}$$

The negative sign means that, for the positive product q_1q_2, the force exerted on q_2 is along the negative x-direction.

The equation for Coulomb's law is more accurately represented by $|\vec{F}_E| = \frac{k|q_1||q_2|}{r^2}$. Here, the vertical bars surrounding the electric force vector, $|\vec{F}_E|$, represent the magnitude of the electric force, and the vertical bars surrounding the charges q_1 and q_2, $|q_1|$ and $|q_2|$, represent the absolute values of the electric charges. This representation is more accurate because the magnitude of the electric force is a strictly positive quantity. If you just use the equation $F_E = \frac{kq_1q_2}{r^2}$, you will get a negative value for the magnitude of the electric force when one of the charges is negative. The absolute value bars keep the values of the charges positive. However, many people find the more accurate version of the equation cumbersome and confusing and prefer to use $F_E = \frac{kq_1q_2}{r^2}$. This is what we will do when solving problems. Always remember to drop the negative signs on charges when using this equation.

The following Tutorial will give you some practice in solving problems using Coulomb's law.

Tutorial 1 Using Coulomb's Law

This Tutorial shows how to use Coulomb's law to solve charge distribution problems in both one and two dimensions.

Sample Problem 1: Applying Coulomb's Law in One Dimension

An early model of the hydrogen atom depicted the electron revolving around the proton, much like Earth revolving around the Sun. The proton and electron both have mass, so they exert a gravitational force upon each other. They also have charge, so they exert an electric force on each other.

(a) The distance between the electron and the proton in a hydrogen atom is 5.3×10^{-11} m, the charge of each is 1.6×10^{-19} C, the mass of the electron is 9.11×10^{-31} kg, and the mass of the proton is 1.67×10^{-27} kg. Calculate the ratio of the electric force F_E to the gravitational force F_g.

(b) Determine the accelerations of the electron caused by both the electric force and the gravitational force.

Solution

(a) **Given:** $r = 5.3 \times 10^{-11}$ m; $q = 1.6 \times 10^{-19}$ C; $m_e = 9.11 \times 10^{-31}$ kg; $m_p = 1.67 \times 10^{-27}$ kg; $k = 8.99 \times 10^9$ N·m²/C²; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: $\frac{F_E}{F_g}$

Analysis: Use Coulomb's law to calculate F_E, and use the equation for universal gravitation to calculate F_g.

$F_g = \frac{Gm_em_p}{r^2}$ and $F_E = \frac{kq_eq_p}{r^2}$, where $q_e = q_p$.

From these results, calculate $\frac{F_E}{F_g}$.

Solution:
$$F_E = \frac{kq_eq_p}{r^2}$$
$$= \frac{\left(8.99 \times 10^9\ \text{N}\cdot\frac{\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(1.6 \times 10^{-19}\ \cancel{\text{C}})^2}{(5.3 \times 10^{-11}\ \cancel{\text{m}})^2}$$
$$F_E = 8.193 \times 10^{-8}\ \text{N (two extra digits carried)}$$

$$F_g = \frac{Gm_em_p}{r^2}$$
$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(9.11 \times 10^{-31}\ \cancel{\text{kg}})(1.67 \times 10^{-27}\ \cancel{\text{kg}})}{(5.3 \times 10^{-11}\ \cancel{\text{m}})^2}$$
$$F_g = 3.613 \times 10^{-47}\ \text{N (two extra digits carried)}$$
$$\frac{F_E}{F_g} = \frac{8.193 \times 10^{-8}\ \cancel{\text{N}}}{3.613 \times 10^{-47}\ \cancel{\text{N}}}$$
$$\frac{F_E}{F_g} = 2.3 \times 10^{39}$$

Statement: The electric force F_E between the electron and the proton of a hydrogen atom is 2.3×10^{39} times the gravitational force F_g between these same particles.

(b) **Given:** $m_e = 9.11 \times 10^{-31}$ kg; $F_g = 3.613 \times 10^{-47}$ N; $F_E = 8.193 \times 10^{-8}$ N

Required: a_E; a_g

Analysis: Use the equation for electric force to calculate a_E using F_E and m_e. Likewise, use the equation for gravitational force to calculate a_g using the values of F_g and m_e.

$a_E = \frac{F_E}{m_e}$ and $a_g = \frac{F_g}{m_e}$. Use 1 N = 1 kg·m/s².

Solution:
$$a_E = \frac{F_E}{m_e}$$
$$= \frac{8.193 \times 10^{-8}\ \cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^2}}{9.11 \times 10^{-31}\ \cancel{\text{kg}}}$$
$$a_E = 9.0 \times 10^{22}\ \text{m/s}^2$$

$$a_g = \frac{F_g}{m_e}$$

$$= \frac{3.613 \times 10^{-47}\ \cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^2}}{9.11 \times 10^{-31}\ \cancel{\text{kg}}}$$

$$a_g = 4.0 \times 10^{-17}\ \text{m/s}^2$$

Statement: The acceleration of the electron caused by the electric force of the proton is 9.0×10^{22} m/s^2, and the acceleration of the electron caused by the gravitational force is 4.0×10^{-17} m/s^2.

Sample Problem 2: Determining Electrostatic Equilibrium

Two charges, $q_1 = -2.00 \times 10^{-6}$ C and $q_2 = -1.80 \times 10^{-5}$ C, are separated by a distance, L, of 4.00 m. A third charge, $q_3 = +1.50 \times 10^{-6}$ C, is placed somewhere between q_1 and q_2, as shown in **Figure 4**, where the net force exerted on q_3 by the other two charges is zero. Determine the location of q_3.

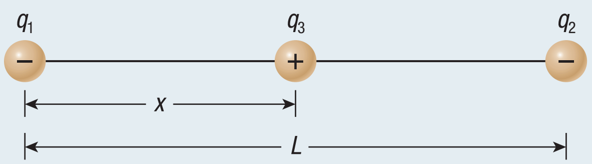


Figure 4

Given: $L = 4.00$ m; $q_1 = -2.00 \times 10^{-6}$ C; $q_2 = -1.80 \times 10^{-5}$ C; $q_3 = +1.50 \times 10^{-6}$ C; $k = 8.99 \times 10^9$ N·m^2/C^2

Required: x, the location of q_3

Analysis: Use the superposition principle, and use the equation for Coulomb's law to calculate the forces F_{E_1} exerted on q_3 by q_1 and F_{E_2} exerted on q_3 by q_2.

The force equations are $F_{E_1} = \frac{kq_1q_3}{x^2}$ and $F_{E_2} = \frac{kq_2q_3}{(L-x)^2}$.

Since the net force on q_3 is zero, $F_{E_1} - F_{E_2} = 0$, or $F_{E_1} = F_{E_2}$.

Solution:

$$F_{E_1} = F_{E_2}$$

$$\frac{kq_1q_3}{x^2} = \frac{kq_2q_3}{(L-x)^2}$$

Divide both sides of the equation by the common terms k and q_3, and then simplify.

$$\frac{q_1}{x^2} = \frac{q_2}{(L-x)^2}$$

$$q_1(L-x)^2 = q_2x^2$$

$$q_1(L^2 - 2Lx + x^2) = q_2x^2$$

$$q_1L^2 - 2q_1Lx + q_1x^2 = q_2x^2$$

$$(q_2 - q_1)x^2 + 2q_1Lx - q_1L^2 = 0$$

$$\frac{(q_2 - q_1)}{q_1}x^2 + 2Lx - L^2 = 0$$

Substitute the values for q_1, q_2, and L into the equation, noting that x is in metres with three significant digits. Solve for x.

$$\frac{(1.80 \times 10^{-5}\ \text{C} - 2.00 \times 10^{-6}\ \text{C})}{2.00 \times 10^{-6}\ \text{C}}x^2 + 2(4.00)x - (4.00)^2 = 0$$

$$\left(\frac{1.60 \times 10^{-5}\ \cancel{\text{C}}}{2.00 \times 10^{-6}\ \cancel{\text{C}}}\right)x^2 + (8.00)x - 16.0 = 0$$

$$8.00x^2 + (8.00)x - 16.0 = 0$$

$$x^2 + x - 2 = 0$$

$$(x - 1)(x + 2) = 0$$

$$x = 1 \text{ or } x = -2$$

Distance is always a positive quantity, so $x = 1$.

Statement: The location of q_3, such that the electric forces from q_1 and q_2 cancel, is $x = 1.00$ m, or 1.00 m to the right of q_1.

Sample Problem 3: Applying Coulomb's Law in Two Dimensions

Two point particles have equal but opposite charges of $+q_1$ and $-q_1$. The particles are arranged as shown in **Figure 5**. Suppose a charge q_2 is placed on the x-axis as shown. $q_1 = 5.0 \times 10^{-6}$ C, $q_2 = 1.0 \times 10^{-6}$ C, and the distance between $+q_1$ and $-q_1$ is 8.0 m measured vertically along the y-axis. Calculate the magnitude and the direction of the net electric force on q_2.

Given: $q_1 = 5.0 \times 10^{-6}$ C; $-q_1 = -5.0 \times 10^{-6}$ C; $q_2 = 1.0 \times 10^{-6}$ C; $k = 8.99 \times 10^9$ N·m^2/C^2

Required: net electric force on q_2

Analysis: As shown in Figure 5, there are symmetrical right triangles above and below the x-axis, so we can use the Pythagorean theorem to calculate r (the distance separating $+q_1$ and q_2). Then use r to calculate the electric force between $+q_1$ and q_2 and that between $-q_1$ and q_2. Use trigonometry to calculate the horizontal and vertical components of these two forces to determine the magnitude and direction of the net electric force. Use Coulomb's law to calculate the force between the charges.

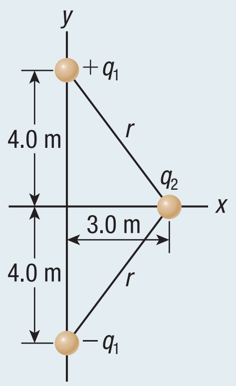


Figure 5

Solution: First, calculate the magnitudes of the electric forces along the horizontal and vertical directions. Both of the forces along the x-axis act on the left side of the charge q_2, so the net horizontal force is the sum of the two horizontal force components.

Use the Pythagorean theorem to calculate r, the distance between $+q_1$ and q_2, and that between $-q_1$ and q_2.

$$r = \sqrt{(4.0\text{ m})^2 + (3.0\text{ m})^2}$$

$$r = 5.0\text{ m}$$

Calculate the force of $+q_1$ on q_2, F_{E_1}.

$$F_{E_1} = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(5.0 \times 10^{-6}\,\cancel{\text{C}})(1.0 \times 10^{-6}\,\cancel{\text{C}})}{(5.0\,\cancel{\text{m}})^2}$$

$$F_{E_1} = 1.798 \times 10^{-3}\text{ N (two extra digits carried)}$$

The force between a positive charge and another positive charge is repulsion, so this force is directed toward q_2 along the line connecting $+q_1$ and q_2.

Calculate the force of $-q_1$ on q_2, F_{E_2}.
Since the magnitude of $-q_1$ is the same as that of $+q_1$, the magnitude of F_{E_2} is the same as that of F_{E_1}.

$$F_{E_2} = F_{E_1}$$

$$F_{E_2} = 1.798 \times 10^{-3}\text{ N (two extra digits carried)}$$

The force between a negative charge and a positive charge is attraction, so this force is directed away from q_2 along the line connecting $-q_1$ and q_2.

Now calculate the x-component of F_{E_1}, F_{Ex_1}.
Let θ be the angle that F_{E_1} makes with the x-axis.

$$\frac{F_{Ex_1}}{F_{E_1}} = \cos\theta$$

$$F_{Ex_1} = F_{E_1}\cos\theta$$

$$= (1.798 \times 10^{-3}\text{ N})\left(\frac{3.0\,\cancel{\text{m}}}{5.0\,\cancel{\text{m}}}\right)$$

$$F_{Ex_1} = 1.079 \times 10^{-3}\text{ N (two extra digits carried)}$$

F_{E_1} is directed toward q_2, so F_{Ex_1} is directed toward q_2 along the positive x-direction.

By symmetry, the x-component of F_{E_2}, F_{Ex_2}, must have the same magnitude as the x-component of F_{E_1} and is directed away from q_2 along the negative x-direction.

$$F_{Ex_2} = F_{Ex_1}$$

$$F_{Ex_2} = 1.079 \times 10^{-3}\text{ N (two extra digits carried)}$$

Determine the vector sum of the two horizontal force components.

$$\vec{F}_{Ex_{net}} = \vec{F}_{Ex_1} + \vec{F}_{Ex_2}$$

$$= +1.079 \times 10^{-3}\text{ N} - 1.079 \times 10^{-3}\text{ N}$$

$$\vec{F}_{Ex_{net}} = 0\text{ N}$$

There is no net force on q_2 along the x-direction.

Now calculate the y-component of F_{E_1}, F_{Ey_1}.

$$\frac{F_{Ey_1}}{F_{E_1}} = \sin\theta$$

$$F_{Ey_1} = F_{E_1}\sin\theta$$

$$= (1.798 \times 10^{-3}\text{ N})\left(\frac{4.0\,\cancel{\text{m}}}{5.0\,\cancel{\text{m}}}\right)$$

$$F_{Ey_1} = 1.438 \times 10^{-3}\text{ N (two extra digits carried)}$$

F_{E_1} is directed toward q_2 from $+q_1$, so F_{Ey_1} is directed toward q_2 downward.
Since $F_{E_2} = F_{E_1}$, the y-component of F_{E_2}, F_{Ey_2}, has the same magnitude as the y-component of F_{E_1}, F_{Ey_1}.

$$F_{Ey_2} = F_{Ey_1}$$

$$F_{Ey_2} = 1.438 \times 10^{-3}\text{ N (two extra digits carried)}$$

F_{E_2} is directed away from q_2 from $-q_1$, so F_{Ey_2} is also directed downward.
Determine the vector sum of the two vertical force components.
Choose upward as positive, so downward is negative.

$$\vec{F}_{Ey_{net}} = \vec{F}_{Ey_1} + \vec{F}_{Ey_2}$$

$$= -1.438 \times 10^{-3}\text{ N} + (-1.438 \times 10^{-3}\text{ N})$$

$$\vec{F}_{Ey_{net}} = -2.9 \times 10^{-3}\text{ N}$$

Statement: The net electric force acting on q_2 is 2.9×10^{-3} N [down].

Practice

1. Determine the magnitude of the electric force between two charges of 1.00×10^{-4} C and 1.00×10^{-5} C that are separated by a distance of 2.00 m. T/I [ans: 2.25 N]
2. Two particles of charge q and $-2q$ (with q positive) are located as shown in **Figure 6**. A third charge q_3 is placed on the x-axis to the left of q. The total electric force on q_3 is zero. The charges are separated by 1.000 m. Determine where q_3 must be located. K/U T/I [ans: 2.414 m [left of q]]

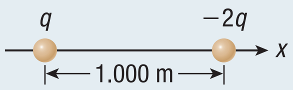


Figure 6

3. Three point charges are placed at the following points on the x-axis: $+2.0\ \mu$C at $x = 0$, $-3.0\ \mu$C at $x = 40.0$ cm, and $-5.0\ \mu$C at $x = 120.0$ cm. Determine the force on the $-3.0\ \mu$C charge. T/I [ans: 0.55 N toward the negative x-direction, or 0.55 N [left]]

7.2 Review

Summary

- According to Coulomb's law, the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the charges, given as $F_E = \frac{kq_1q_2}{r^2}$.
- Coulomb's constant, k, is equal to 8.99×10^9 N·m^2/C^2.
- Coulomb's law applies to point charges and to charges that can be concentrated equivalently in points located at the centre, when the sizes of the charges are much smaller than their distance of separation.
- There are similarities between the electric force and the gravitational force.
- The superposition principle states that the total force acting on a charge q is the vector sum, or superposition, of the individual forces exerted on q by all the other charges in the problem.

Questions

1. Two charged objects have a repulsive force of 0.080 N. The distance separating the two objects is tripled. Determine the new force. T/I
2. Two charged objects have an attractive force of 0.080 N. Suppose that the charge of one of the objects is tripled and the distance separating the objects is tripled. Calculate the new force. T/I
3. Determine the magnitude of the electric force between two electrons separated by a distance of 0.10 nm (approximately the diameter of an atom). K/U T/I
4. Two point charges are separated by a distance r. Determine the factor by which the electric force between them changes when the separation is reduced by a factor of 1.50. K/U T/I
5. Determine the distance of separation required for two 1.00 μC charges to be positioned so that the repulsive force between them is equivalent to the weight (on Earth) of a 1.00 kg mass. T/I
6. Consider an electron and a proton separated by a distance of 1.0 nm. K/U T/I
   (a) Calculate the magnitude of the gravitational force between them.
   (b) Calculate the magnitude of the electric force between them.
   (c) Explain how the ratio of these gravitational and electric forces would change if the distance were increased to 1.0 m.
7. Particles of charge q and $3q$ are placed on the x-axis at $x = -40$ and $x = 50$, respectively. A third particle of charge q is placed on the x-axis, and the total electric force on this particle is zero. Determine the position of the particle. K/U T/I
8. Two charges of 2.0×10^{-6} C and -1.0×10^{-6} C are placed at a separation of 10 cm. Determine where a third charge should be placed on the line connecting the two charges so that it experiences no net force due to these two charges. K/U T/I A
9. Three charges with $q = +7.5 \times 10^{-6}$ C are located as shown in **Figure 7**, with $L = 25$ cm. Determine the magnitude and direction of the total electric force on each particle listed below. T/I

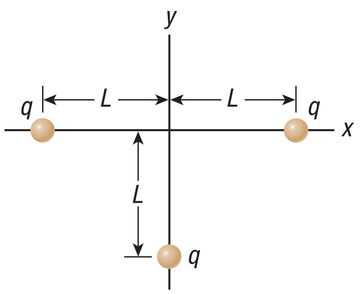


Figure 7

   (a) the charge at the bottom
   (b) the charge on the right
   (c) an electron placed at the origin
10. Two pith balls, each with a mass of 5.00 g, are attached to non-conducting threads and suspended from the same point on the ceiling. Each thread has a length of 1.00 m. The balls are then given an identical charge, which causes them to separate. At the point that the electric and gravitational forces balance, the threads are separated by an angle of 30.0°. Calculate the charge on each pith ball. K/U A

7.3 Electric Fields

If you have typed a letter on a computer, heard musical tones from a cellphone, or even just pressed a floor button in an elevator, then you have applied an electric force. In fact, almost everywhere you go you will find a device that, in one way or another, uses electric fields. An electric field is what causes the electric force. At the dentist's office, the X-ray machine uses an electric field to accelerate electrons as part of the process for producing X-rays. At coal-burning power plants, electric fields in smokestack scrubbers remove soot and other pollutants before gases are released into the air. Even when you are speaking into a telephone or listening to the other person on the line, electric fields help convert sound to electricity and back to sound again.

The liquid crystal display (LCD) is a device that uses electric fields. Nearly all computer monitors, digital cameras, and smart phones use LCDs for their visual components (**Figure 1**). LCDs consist of a liquid crystal between two transparent sheets of glass or plastic, with a thin conducting material on the outside of the sheets. An electric field across the crystal causes its molecular arrangement to change, so that light passing through it is made either lighter or darker, depending on the design of the device. In this way, small changes in the electric field can make nearly any pattern appear on the LCD.

Figure 1 LCDs use changing electric fields to alter the crystal's optical properties, creating images.

Properties of Electric Fields

electric field ($\vec{\varepsilon}$) the region in which a force is exerted on an electric charge; the electric force per unit positive charge; unit is N/C

We can describe the electric force between a pair of charges using Coulomb's law, but there is another way to describe electric forces. Suppose there is a single isolated point charge that is far from any other charges. This charge produces an **electric field**, a region in which a force is exerted on an electric charge. The electric field is similar to the gravitational field near an isolated mass, as discussed in Chapter 6. Were another charge to enter this field, the electric field would exert a force on it, much as a gravitational field exerts a force on a mass. The electric field has both magnitude and direction, so electric field is a vector denoted by $\vec{\varepsilon}$.

Consider a particular point in space where there is a uniform electric field $\vec{\varepsilon}$ (**Figure 2**). A point charge or an arrangement of several charges may have produced this field. A charge q at this location in the field will be affected by the electric field and experience an electric force given by

$$\vec{F}_E = q\vec{\varepsilon}$$

The electric force, $\vec{F}_E$, is thus parallel to $\vec{\varepsilon}$, with direction depending on whether the charge is positive or negative.

The charge q_1, or the charge affected by the field, in Figure 2 is called a test charge. As a convention, physicists use a positive test charge to determine direction. By measuring the force on a positive test charge, you can determine the magnitude and direction of the electric field at the location of the test charge. Since we are working with a positive test charge, q_1, the electric field points in the same direction as the force that the test charge experiences. If q_1 happens to be a negative charge, then the direction of the electric field is in the opposite direction of the force that the negative charge experiences. The units for the electric field can be determined from the equation relating electric force to electric field. Force is measured in newtons (N), and charge is measured in coulombs (C), so the electric field is expressed in newtons per coulomb (N/C).

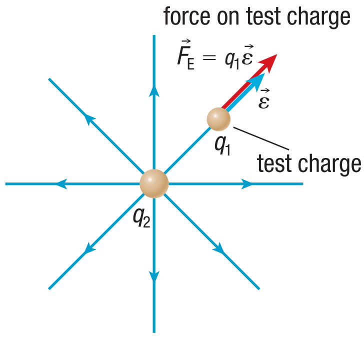


Figure 2 The electric field at a particular point in space is related to the electric force on a test charge q_1 at that location.

The electric field $\vec{\varepsilon}$ also relates to Coulomb's law for electric force. Coulomb's law for electric force allows you to calculate the electric field using the amount of charge that produces the field, q_2, and the distance of the field from the charge. As an example, calculate the magnitude of the electric field at a distance r from a charge q_2. The test charge is, once again, q_1. According to Coulomb's law, the electric force exerted on q_1 has a magnitude of

$$F_E = \frac{kq_1q_2}{r^2}$$

$$F_E = q_1\frac{kq_2}{r^2}$$

Inserting this expression into the equation relating electric force to the electric field gives the result

$$q\varepsilon = F_E$$

$$q\varepsilon = q_1\frac{kq_2}{r^2}$$

In this equation $q = q_1$, so we get

$$\boxed{\varepsilon = \frac{kq_2}{r^2}}$$

This is the magnitude of the electric field at a distance r from a point charge q_2. The direction of $\vec{\varepsilon}$ lies along the line that connects the charge producing the field, q_2, to the point where the field is measured. The direction of the electric field is determined by a positive test charge. If q_2 is positive, then a positive test charge will be repelled away from it, and thus the electric field points in a direction away from a positive charge. If q_2 is negative, then a positive test charge will be attracted toward it and thus the electric field points in a direction toward a negative charge. As a result of the direction of the electric field depending on the type of charge and the location in relation to the charge, we do not include the signs of charges in the equations to avoid implying a direction.

In Tutorial 1, you will solve problems related to the electric field in both one and two dimensions. When using the equation for electric field, the symbol q really means the absolute value of q, just like it does for Coulomb's law.

Tutorial 1 Determining Electric Fields

This Tutorial shows how to determine the electric field due to charge distribution at a point some distance from the charge.

Sample Problem 1: Electric Field Due to Two Point Charges in One Dimension

Two point charges are 45 cm apart (**Figure 3**). The charge on q_1 is 3.3×10^{-9} C, and the charge on q_2 is -1.00×10^{-8} C.

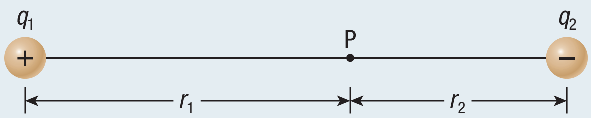


Figure 3

(a) Calculate the net electric field at point P, 27 cm from the positive charge, on the line connecting the charges.

(b) A new charge of $+2.0 \times 10^{-12}$ C is placed at P. Determine the electric force on this new charge.

Solution

(a) **Given:** $r_{12} = 45$ cm; $r_1 = 27$ cm; $q_1 = 3.3 \times 10^{-9}$ C; $q_2 = -1.00 \times 10^{-8}$ C; $k = 8.99 \times 10^9\ \text{N·m}^2/\text{C}^2$

Required: net electric field $\vec{\varepsilon}_{\text{net}}$ at point P

Analysis: The net electric field at point P equals the vector sum of the electric fields from the charges producing the fields. Use the equation $\varepsilon = \dfrac{kq}{r^2}$ to calculate ε for each of the charges q_1 and q_2 at point P. Then determine the direction of each field based on the signs of the charges. Combine the two vector quantities to calculate $\vec{\varepsilon}_{\text{net}}$.

$r_2 = r_{12} - r_1 = 45\text{ cm} - 27\text{ cm} = 18\text{ cm} = 0.18\text{ m}$

Solution: Calculate the magnitude of the electric field at a distance r_1 from charge q_1 and determine the field's direction.

$$\varepsilon_1 = \frac{kq_1}{r_1^2} = \frac{\left(8.99 \times 10^9\ \frac{\text{N·}\cancel{\text{m}^2}}{\text{C}^{\cancel{2}}}\right)(3.3 \times 10^{-9}\ \cancel{\text{C}})}{(0.27\ \cancel{\text{m}})^2}$$

$\varepsilon_1 = 4.070 \times 10^2$ N/C (two extra digits carried)

Since q_1 is a positive charge, the electric field on a positive test charge at P will be directed away from q_1.

$\vec{\varepsilon}_1 = 4.070 \times 10^2$ N/C [right]

Calculate the magnitude of the electric field at a distance r_2 from charge q_2 and determine the field's direction.

$$\varepsilon_2 = \frac{kq_2}{r_2^2} = \frac{\left(8.99 \times 10^9\ \frac{\text{N·}\cancel{\text{m}^2}}{\text{C}^{\cancel{2}}}\right)(1.00 \times 10^{-8}\ \cancel{\text{C}})}{(0.18\ \cancel{\text{m}})^2}$$

$\varepsilon_2 = 2.775 \times 10^3$ N/C (two extra digits carried)

Since q_2 is a negative charge, the electric field on a positive test charge at P will be directed toward q_2.

$\vec{\varepsilon}_2 = 2.775 \times 10^3$ N/C [right]

Now determine the vector sum of the two electric fields. Choose right as positive, so left is negative.

$$\begin{aligned}\vec{\varepsilon}_{\text{net}} &= \vec{\varepsilon}_1 + \vec{\varepsilon}_2 \\ &= +4.070 \times 10^2\text{ N/C} + 2.775 \times 10^3\text{ N/C} \\ &= +3.182 \times 10^3\text{ N/C (two extra digits carried)}\end{aligned}$$

$\vec{\varepsilon}_{\text{net}} = 3.2 \times 10^3$ N/C [right]

Statement: The net electric field is 3.2×10^3 N/C to the right of point P.

(b) **Given:** $\vec{\varepsilon}_{\text{net}} = 3.182 \times 10^3$ N/C, directed to the right of point P; $q = +2.0 \times 10^{-12}$ C

Required: net electric force $\vec{F}_{\text{E}_{\text{net}}}$

Analysis: Use the equation $\vec{F}_{\text{E}} = q\vec{\varepsilon}$ to calculate the net electric force for the test charge q at point P.

Solution: Use the sign of the charge to determine the direction of the electric field. Since the charge is positive, the electric force is directed to the right. Choose right as positive, so left is negative.

$$\begin{aligned}\vec{F}_{\text{E}_{\text{net}}} &= q\vec{\varepsilon}_{\text{net}} \\ &= (+2.0 \times 10^{-12}\text{ C})(+3.182 \times 10^3\text{ N/C}) \\ &= +6.4 \times 10^{-9}\text{ N}\end{aligned}$$

$\vec{F}_{\text{E}_{\text{net}}} = 6.4 \times 10^{-9}$ N [right]

Statement: The electric force acting on the new charge at point P is 6.4×10^{-9} N to the right of point P.

Sample Problem 2: Electric Field Due to Two Point Charges in Two Dimensions

Two point charges are arranged as shown in **Figure 4**. $q_1 = 4.0 \times 10^{-6}$ C, $q_2 = -2.0 \times 10^{-6}$ C, and $r = 3.0$ cm. Calculate the magnitude of the electric field at the origin.

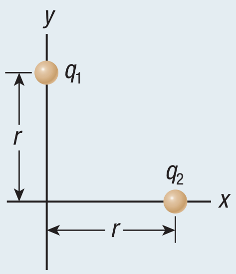


Figure 4

Given: $r = 3.0 \text{ cm} = 0.030$ m; $q_1 = 4.0 \times 10^{-6}$ C; $q_2 = -2.0 \times 10^{-6}$ C; $k = 8.99 \times 10^9 \text{ N·m}^2/\text{C}^2$

Required: magnitude of net electric field at the origin, ε_{net}

Analysis: The electric field at the origin results from the electric fields of charges q_1 and q_2. Use the equation $\varepsilon = \frac{kq}{r^2}$ for a point charge q to calculate the magnitudes of the net electric fields at the origin along the horizontal and vertical directions due to the charges q_1 and q_2. Then use the equation $\varepsilon_{\text{net}} = \sqrt{\varepsilon_{x_{\text{net}}}^2 + \varepsilon_{y_{\text{net}}}^2}$ to calculate the magnitude of the net electric field at the origin.

Solution: First, calculate the magnitudes of the electric fields along the horizontal and vertical directions. The electric field of q_2 has only an x-component.

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x \\
&= \frac{kq_2}{r^2} \\
&= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{C}^{\cancel{2}}}\right)(2.0 \times 10^{-6}\ \cancel{\text{C}})}{(0.030\ \cancel{\text{m}})^2}
\end{aligned}$$

$$\varepsilon_x = 1.998 \times 10^7 \text{ N/C (two extra digits carried)}$$

The electric field of q_1 has only a y-component.

$$\begin{aligned}
\varepsilon_{y_{\text{net}}} &= \varepsilon_y \\
&= \frac{kq_1}{r^2} \\
&= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{C}^{\cancel{2}}}\right)(4.0 \times 10^{-6}\ \cancel{\text{C}})}{(0.030\ \cancel{\text{m}})^2}
\end{aligned}$$

$$\varepsilon_{y_{\text{net}}} = 3.996 \times 10^7 \text{ N/C (two extra digits carried)}$$

Then calculate the magnitude of the net electric field at the origin.

$$\begin{aligned}
\varepsilon_{\text{net}} &= \sqrt{\varepsilon_{x_{\text{net}}}^2 + \varepsilon_{y_{\text{net}}}^2} \\
&= \sqrt{(1.998 \times 10^7 \text{ N/C})^2 + (3.996 \times 10^7 \text{ N/C})^2}
\end{aligned}$$

$$\varepsilon_{\text{net}} = 4.5 \times 10^7 \text{ N/C}$$

Statement: The magnitude of the net electric field at the origin is 4.5×10^7 N/C.

Practice

1. An electric force with a magnitude of 2.5 N, directed to the left, acts on a negative charge of -5.0 C. T/I
   (a) Determine the electric field in which the charge is located. [ans: 0.50 N/C [toward the right]]
   (b) Calculate the electric field when the force is the same but the charge is -0.75 C. [ans: 3.3 N/C [toward the right]]
2. Calculate the magnitude and direction of the electric field at a point 2.50 m to the right of a positive point charge $q = 6.25 \times 10^{-6}$ C. T/I [ans: 8.99×10^3 N/C [toward the right]]
3. Calculate the electric field at point Z in **Figure 5**, due to the point charges $q_1 = 5.56 \times 10^{-9}$ C at point X and $q_2 = -1.23 \times 10^{-9}$ C at point Y. T/I [ans: -50.3 N/C, or 50.3 N/C [toward the left]]

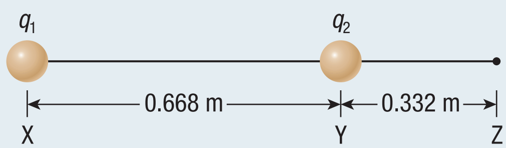


Figure 5

Electric Field Lines

electric field lines the continuous lines of force around charges that show the direction of the electric force at all points in the electric field

An electric field exists in a region around a charge. The field and its properties can be represented with **electric field lines**. These electric field lines can help us determine the direction of the force on a nearby test charge.

As was stated earlier, electric fields point away from positive charges and toward negative charges. This convention is based on using a positive test charge to determine direction. To show what the electric field would look like around a positive point charge, we draw electric field lines that extend radially outward from the charge, as shown in **Figure 6(a)**. For a negative charge, the field lines are directed inward, toward the charge (**Figure 6(b)**). As you may expect, the electric field lines are parallel to $\vec{\varepsilon}$, and the density of the field lines is proportional to the magnitude of $\vec{\varepsilon}$. In both parts of Figure 6, the field lines are densest near the charges. In both cases, the magnitude of $\vec{\varepsilon}$ increases as the distance to the charge decreases.

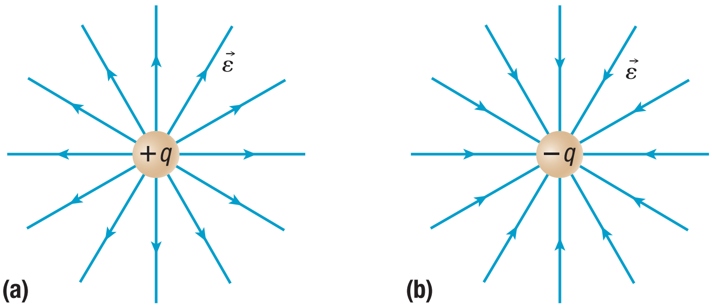


Figure 6 Electric field lines near a point charge placed at the origin. (a) If the charge is positive, the electric field lines are directed outward, away from the charge. (b) If the charge is negative, the electric field lines are directed inward, toward the charge.

According to Coulomb's law, the force between two point charges varies as $\frac{1}{r^2}$, where r is the separation between the two charges. In a similar way, the electric field produced by a point charge also varies as $\frac{1}{r^2}$. The electric field thus obeys an inverse-square law, just as the gravitational force does (**Figure 7(a)**).

The magnitude of the electric force is proportional to the density of field lines—that is, the number of field lines per unit area of space—at some distance r from the charge or charges producing the field. For a point charge q (**Figure 7(b)**), the density of field lines decreases farther away from the charge. The field lines from q spread out through a surface area. Therefore, the number of field lines per unit area (A), and thus the respective strengths of the electric field and electric force, decreases as r^2 increases.

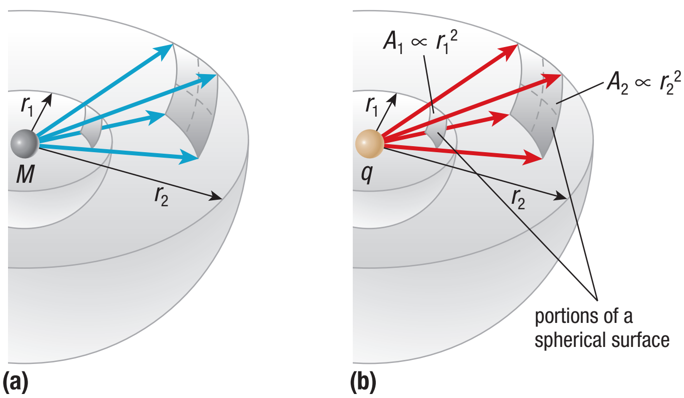


Figure 7 (a) The gravitational force lines from a mass M spread out as r increases in all directions. (b) The electric field lines from a charge q also spread out in all directions. Both types of field lines pass through larger spherical surface areas at greater distances. The surface areas (A) over which the fields act increase as r^2 increases, so the fields themselves exert inverse-square forces.

Another interesting aspect of the electric force relates to the question of action at a distance. How do two point charges that interact through Coulomb's law "know" about each other? In other words, how does one charge transmit electric force to another charge? In terms of the electric field lines, every charge generates (or carries with it) an electric field, through which the electric force is transmitted.

Electric Dipoles

Consider the two point particles with equal but opposite charge in **Figure 8**. Charges $-q$ and $+q$, where q is the positive magnitude of the charge, are separated by a small distance r. This charge configuration is called an **electric dipole**.

electric dipole a pair of equal and opposite electric charges with centres separated by a small distance

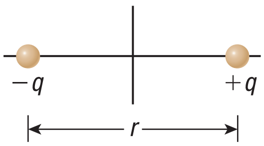


Figure 8 Two opposite charges separated by a distance r form an electric dipole.

The two charges in an electric dipole give rise to a more complicated electric field than the one associated with a single electric charge. This is because the electric fields around the individual charges interact most strongly with each other at close distances, such as those that are similar in size to the dipole separation. Initially, the fields at the negative charge radiate inward toward the charge, and the fields at the positive charge radiate outward from the charge (**Figure 9(a)**). As the fields extend into the space around the other charge, they interact with each other, producing field lines that bend toward the other charge (**Figure 9(b)**).

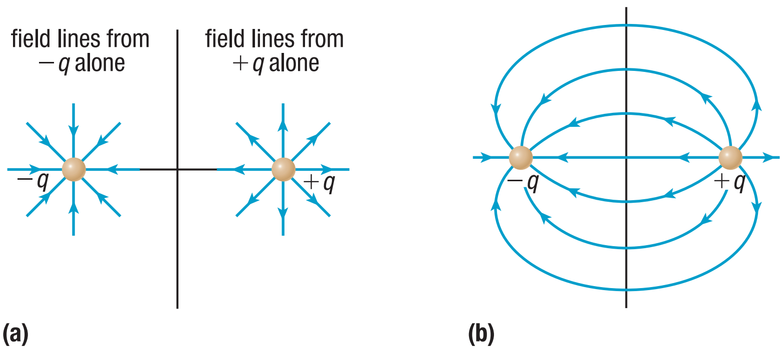


Figure 9 The electric field lines around each individual charge of an electric dipole (a) are affected by the field lines from the other charge, causing them to bend (b). It is important to note that the field lines extend in three dimensions around the charges, and that the view depicted here is of the field lines in a plane perpendicular to the line of sight.

Notice that, along the vertical axis midway between the two charges, the electric field is parallel to the line connecting the two charges. This remains true along the vertical axis at all distances from the dipole, although the magnitude of the electric field decreases at distances that are farther from the dipole. The direction of this electric field always points from the positive charge to the negative charge.

Although the field lines of a dipole merge at the midpoint, field lines do not cross. Instead, the cumulative effect, or the vector sum, of the electric fields from both charges produces a net electric field. That net electric field is represented by the electric field line.

Now consider an arrangement of charges slightly different from an electric dipole. In this case, a positive charge, $+q$, replaces the negative charge, $-q$, so that the two charges are equal and alike. Now the electric field lines extend outward from both charges. Instead of the field lines from a positive charge merging with the lines from a negative charge, the lines from similar charges do not connect at any point. This arrangement produces a disk-shaped region of zero electric field everywhere around the midpoint between the two charges (**Figure 10**).

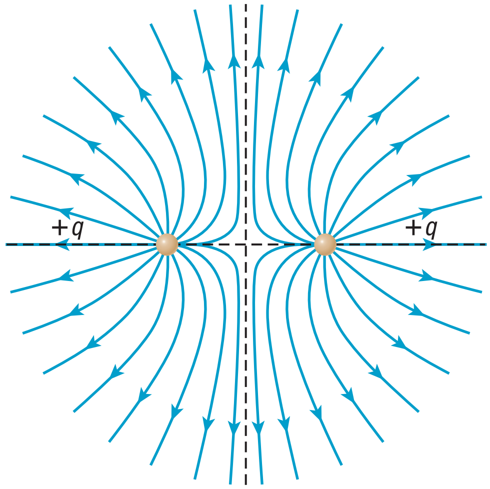


Figure 10 Two identical charges separated by a small distance produce this electric field pattern. Again, the field lines occupy all the space around the charges. This illustration shows how they appear in a plane. Note that the electric field is zero along the line that bisects the line connecting the charges.

Notice that, farther away from the charges, the field behaviour starts to resemble that of a single charge. That is, the field lines appear to be radiating from a single point charge. This makes sense because, at a large distance from the two charges, the separation between them is not noticeable, and both charges have the same sign. Midway between the two charges, there is a gap where there are no field lines. This is expected because the vector sum of the two electric fields from both charges is zero at the midpoint.

Note that this electric field pattern would be the same if the two charges were both negative. The difference would be in the direction in which the field lines were pointing but not the shape of the combined fields.

Finally, consider a dipole-like arrangement of two charges that have different magnitudes and signs. If the positive charge $+q$ is replaced with a charge $+2q$, the symmetry of the dipole field is altered (**Figure 11** on the next page). This is because the number of field lines for each charge is proportional to the magnitude of that charge. The number of field lines leaving the positive charge is therefore twice the number of field lines meeting at the negative charge. Half of these lines converge on the negative charge, while the other half emanate outward, as if there were only one charge ($+2q$). At large distances, where r is much greater than the charge separation, the electric field radiates outward as it would for a charge of $+2q$.

The field-line pattern for unequal and opposite charges includes regions near the charges where the density of field lines becomes very high. Note that this does not mean that the electric field is stronger in these regions. The electric field is still strongest along the line connecting the charges. Electric field patterns can become very complex, but many simulations exist to show what the electric field would look like in a multi-charge system. WEB LINK

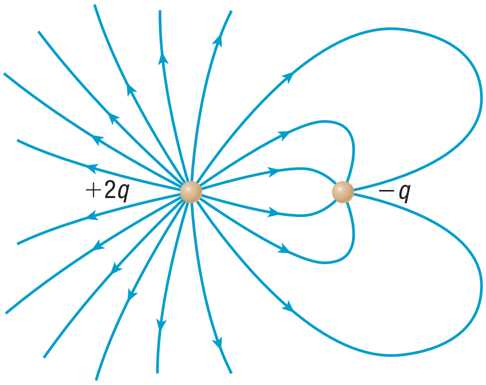


Figure 11 The electric field lines around a dipole consisting of charges $-q$ and $+2q$. The number of lines from the larger charge is twice the number of lines from the smaller charge. Half of these lines extend from the positive charge to the negative charge. The other half extend outward, as if there were only one charge, $+2q$.

Uniform Electric Fields

So far, you have learned about electric fields that vary with distance from a charge. In the case of a dipole, for instance, the strength of the electric field varies with the number of charges, their placement, and the distance from the charges.

A different electric field arises from a different type of dipole. Instead of point charges, suppose you have two parallel planes of charge. As with the dipole, one plane has a positive charge and the other plane has a negative charge. In both cases, the charge spreads uniformly along each plane.

Just as the electric field along the line connecting two unlike charges extends straight from the positive to the negative charge, the electric field between the planes of charge extends from the positive plane of charge to the negative plane and is uniform. These field lines are straight, parallel to each other, and perpendicular to the planes of charge. At any location between the planes, the electric field has the same magnitude and direction. Outside the planes, the vector sum of the electric fields from all the individual charges in the two parallel planes yields a value of zero.

This description of planes of charges involves "infinite" parallel planes carrying "infinite" amounts of charge and thus does not exist in real-world scenarios. However, you can create a close approximation by using two large conducting plates charged by dry cells. These plates are parallel and carry equal and opposite charges. As long as the separation between the plates is much smaller than their surface area, the electric field between the plates remains uniform (**Figure 12**). In fact, except near the edges of the plates, the magnitude of the electric field depends only on the amount of charge, the area of the plates, and the material between the plates.

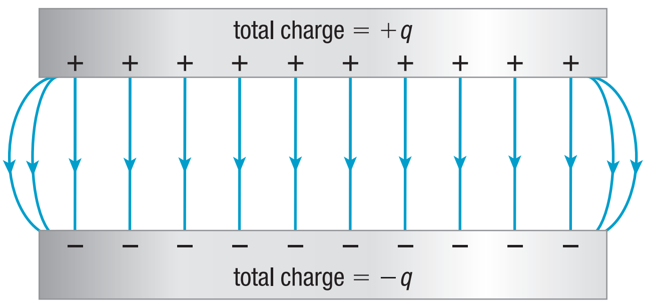


Figure 12 The electric field between two parallel conducting plates is uniform in direction and magnitude.

Earth's Electric Field

Energy from the Sun bombards Earth's upper atmosphere. Some of this energy strips electrons from atoms, leaving a region of positively charged ions and free electrons. Some of the electrons recombine with the ions, but others travel into space and other regions of the atmosphere. This region, appropriately called the ionosphere, therefore has a positive charge and is able to conduct electricity.

In contrast to the ionosphere, Earth's surface is more negatively charged. Both areas tend to stay charged, so that a permanent electric field exists throughout the atmosphere. Near Earth's surface, when the sky is clear of storms, the electric field has an average magnitude of about 120 N/C.

This electric field varies seasonally, but the greatest and most sudden change occurs during thunderstorms, when the atmospheric electric field can reverse direction. Cloud-to-ground lightning, which often accompanies these storms, is both very common and potentially very destructive, so it is essential that scientists understand how Earth's electric field changes during storms, and what field conditions are likely to result in a lightning strike.

One device, called an electric field mill, or just field mill, is widely used to measure Earth's electric field. A field mill makes use of the uniform electric field between two parallel conducting plates and detects changes in the field strength at a given location.

The design of a field mill incorporates two circular conducting plates (**Figure 13**). The conducting plate at the front of the mill is the sensor plate. This part of the mill is exposed to the atmospheric electric field. A set of motor-driven, rotating shutter blades (the mill) exposes the sensor plate for a short time and then blocks the plate from the electric field for an identical length of time. During exposure, the plate becomes charged. During non-exposure, an analyzer circuit measures the electric field between the charged sensor plate and the uncharged detector plate. This process repeats continuously, providing information about variations in the field strength of Earth's electric field over a given time interval and in a particular region of the atmosphere. CAREER LINK

UNIT TASK BOOKMARK

You can apply what you have learned about electric fields to the Unit Task on page 422.

Field mills have fairly simple designs and generally perform reliably. Although often set up in permanent positions on the ground, field mills located in balloons and aircraft measure field changes at different elevations. Field mills are used in cases where lightning could do extreme damage, such as to spacecraft before launch.

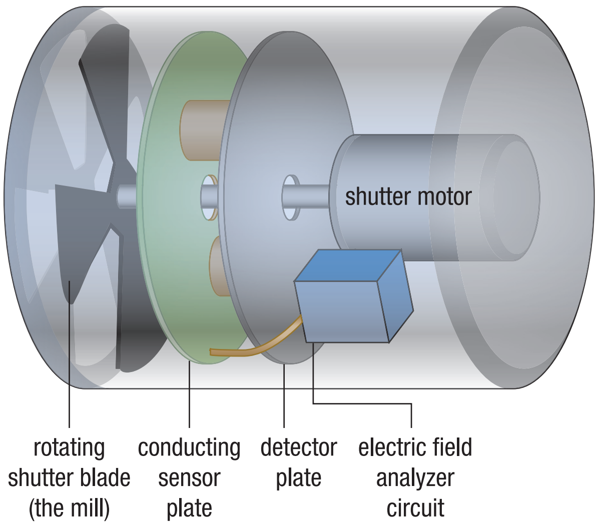


Figure 13 A field mill measures the electric field between two parallel conducting plates to determine the changes in Earth's electric field.

Electrostatic Precipitators

During the Industrial Age, heavily polluted air resulted from the smoke pouring out of chimneys and smokestacks. Most of these emissions, called flue gases because they passed through the flues of chimneys, consisted of gases such as nitrogen and carbon dioxide, both of which are clear substances. However, tiny particles of carbon, sulfur compounds, and dust produced by various chemical processes and combustion combined with the gases. These particles gave the air its smoky appearance.

Today, industrial processes continue to pollute our air. Air containing polluting gases leads to many environmental concerns such as climate change and acid precipitation. Acid precipitation is known to harm both plant and aquatic life. The polluting gases also affect the respiratory health of not only the people who live nearby but also those who live where the prevailing winds tend to push the gases.

In recent years, devices called electrostatic precipitators have reduced the numbers of these particles released into the atmosphere. Electrostatic precipitators use electric fields to remove extremely small particles of soot, dust, and ash from flue gases and other emissions produced by combustion, smelting, and refining.

The exact arrangements of different electrostatic precipitators vary, but the basic principle is the same in all of them. In the design shown in **Figure 14**, the flue gas and particles it contains pass between a grid of negatively charged conducting wires and a conducting plate carrying a positive charge. The wires transfer electrons to the various particles that come into contact with the wires, making the particles negatively charged. The electric field between the wires and the plates is about 1×10^6 N/C, so the force drawing these negatively charged particles toward the positively charged plates is very large. Shaking the plates from time to time loosens the particles that accumulate on them. A storage (collection) hopper below the precipitator collects this refuse. Repeating the procedure of passing the flue gas through several series of plates and wires removes about 99 % of the various particles from the gas.

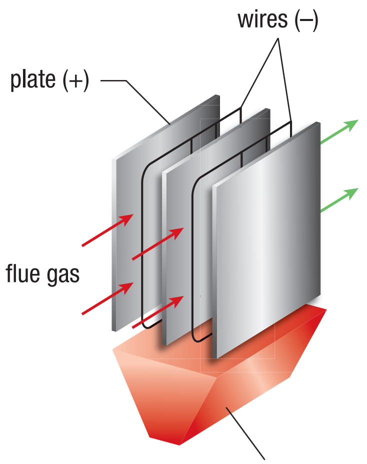


Figure 14 An electrostatic precipitator uses electric fields to remove particles from flue gases.

Some of the devices used for home air purification use the basic principles of electrostatic precipitators. However, the results from these air cleaners have been mixed, partly because of the comparatively low levels of particles in household air, and partly because the electric fields are not as strong as in industrial precipitators.

Electric Fields in Nature

Electric fields are also produced by animals. These fields are often weak and produced by ordinary actions, such as motion in the muscles. Some animals have organs that detect and respond to these weak electric fields. Hammerhead sharks, for instance, detect fields as low as 6 N/C in fish that hide beneath the sand or in tunnelled shelters along shallow ocean bottoms.

The hammerhead shark swims close to the sandy ocean floor (**Figure 15**). It preys on goby, small fish that hide in sand-covered holes. A goby produces electric fields from muscular movements of its fins or gills. Although the shark cannot see the goby, it can detect these electric fields up to 25 cm above the sand. Having detected the goby, the hammerhead shark swims in a figure eight until it pinpoints the location of greatest electric field strength and then catches and consumes the goby.

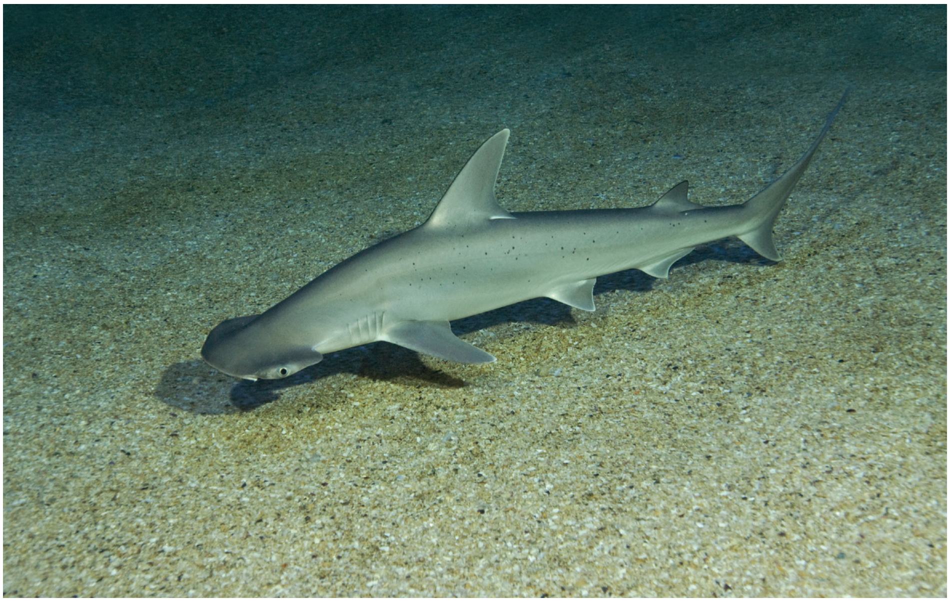


Figure 15 A hammerhead shark can detect the electric fields produced by the movements of its prey.

Research This

Fish and Electric Fields

Skills: Researching, Communicating SKILLS HANDBOOK A4.1

Many fish use electric fields to detect or stun their prey, or to ward off predators. Some examples are electric eels, electric catfish, elephant fish, Nile knifefish, and torpedo fish.

1. Research one of these fish on the Internet, and determine how it detects or uses electric fields.
2. Compare this fish's abilities with those of the hammerhead shark.
3. Write a brief report of your findings that includes answers to the following questions.

A. What types of behaviours that are related to electric fields are typical for the organism you chose? T/I

B. Why are fish that stun prey with electric fields typically freshwater species? T/I

C. Many fish are able to detect weak electric fields from prey that live in rivers with large amounts of silt and soil suspended in the water. Why would an adaptation such as electric-field detection be beneficial for these fish? T/I

WEB LINK

7.3 Review

Summary

- An electric field exists in a region of space when a test charge placed at any point in the region has a force exerted upon it.
- The electric field is a vector and is denoted by $\vec{\varepsilon}$. A test charge q will experience an electric force given by $\vec{F}_E = q\vec{\varepsilon}$. The directions of the electric force and electric field are determined by a positive test charge.
- For a point charge q_2, the magnitude of the electric field at a distance r from the charge is $\varepsilon = \dfrac{kq_2}{r^2}$.
- Electric field lines are continuous lines of force that show the direction of electric force at all points in the electric field around a charge or charges.
- An electric dipole consists of two equal but opposite charges separated by some small distance.
- The electric field between two parallel plates of charge is uniform and perpendicular to the plates. The electric field outside the parallel plates is zero.
- One application of electric fields is in electrostatic precipitators, which use electric fields to remove extremely small particles of soot, dust, and ash from flue gases.
- Some organisms can detect the weak electric fields produced by the movement of other organisms.

Questions

1. A proton and an electron are placed in a uniform electric field. Comment on the magnitudes of the forces experienced by both. K/U
2. Calculate the magnitude of the electric field at a distance of 1.5 m from a point charge with $q = 3.5$ C. K/U
3. A point particle of charge $q_1 = 4.5 \times 10^{-6}$ C is placed on the x-axis at $x = -10$ cm. A second particle of charge q_2 is placed on the x-axis at $x = +25$ cm. The electric field at the origin is zero. Determine the charge q_2. K/U T/I
4. A ring with a radius of 25 cm and total charge 5.00×10^{-4} C is centred at the origin as shown in **Figure 16**. The charge is distributed uniformly around the ring. Calculate the electric field at the origin. (Hint: Think of applying symmetry.) K/U T/I

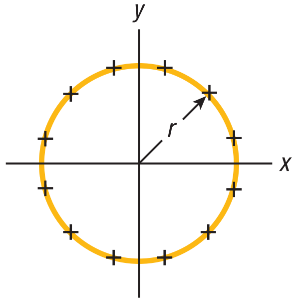


Figure 16

5. When drawing electric field lines, what determines the number of lines originating from a charge? K/U
6. Two point particles with charges q_1 and q_2 are separated by a distance L, as shown in **Figure 17**. The electric field is zero at point A, which is a distance $\frac{L}{4}$ from q_1. Determine the ratio q_1:q_2. K/U T/I

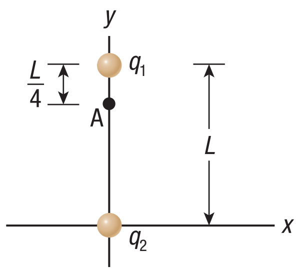


Figure 17

7. Five point charges, all with $q = 7.5$ C, are spaced equally along a semicircle with a radius of 2.3 m, as shown in **Figure 18**. Calculate the electric field at the origin. K/U T/I

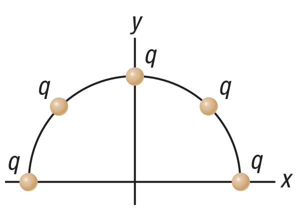


Figure 18

8. Draw the electric field lines between the wires and plates of an electrostatic precipitator. T/I C

7.4 Potential Difference and Electric Potential

In the previous section, you learned how two parallel charged surfaces produce a uniform electric field. From the definition of an electric field as a force acting on a charge, it follows that, for a given uniform electric field, charge, and particle mass, the particle undergoes a uniform acceleration. Another way of thinking about the physics of this situation is that the electric field does work on the charged particle. This view works well if the charge stays constant, but in reality the work done by the field varies with charge. What if, instead of describing the electric field in terms of force per charge, you expressed it in terms of energy per charge? As it turns out, we can describe the field in this way, as you will learn in this section.

No matter how you choose to describe the uniform electric field, its ability to accelerate charged particles with known conditions has proven useful to physicists and engineers. Devices such as particle accelerators, which can accelerate particles to speeds near the speed of light, can only work if the particles are moving in the first place. Electric fields cause the initial motion of these particles by accelerating them. Particle acceleration is important in some everyday devices as well. Inkjet printers accelerate charged ink particles toward specific parts of the paper (**Figure 1**). Old television sets and computer monitors have cathode-ray tubes. These tubes accelerate electrons toward a phosphor screen. Variations in the deflection of the accelerated electrons determine the brightness and colour of the screen.

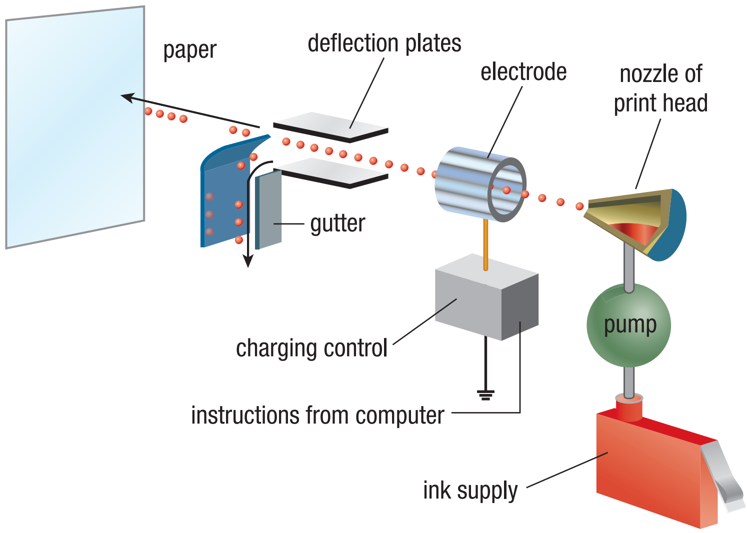


Figure 1 Ink droplets from the print head are either charged or uncharged. The uncharged droplets move to the paper undeflected, forming the letters. Charged droplets are deflected into the gutter, leaving those parts of the paper blank.

Work and Electric Potential Difference

Recall that in a region of space where the electric field $\vec{\varepsilon}$ is constant, $\vec{\varepsilon}$ has the same magnitude and direction at all points. A point charge q in this region experiences an electric force

$$\vec{F}_E = q\vec{\varepsilon}$$

The force is parallel to $\vec{\varepsilon}$, as shown in **Figure 2**.

Suppose this charge moves a certain distance Δd, starting at point A and ending at point B. For simplicity, assume this displacement is parallel to the electric force $\vec{F}_E$. According to the definition of work (W), the work done by the electric force on the charge is

$$W = F_E \Delta d$$

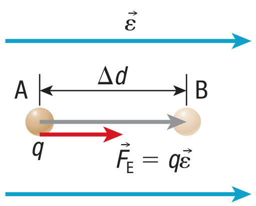


Figure 2 The electric field gives rise to an electric force, which moves in the same direction as the charge if the charge is positive.

The electric force does work on the charge and is independent of the path it takes from A to B. We can now define the **electric potential energy**, E_E, which is the energy stored in the system that can do work W on a positively charged particle. From your studies of work and energy you know that the change in the potential energy associated with this type of force is equal to $-W$, where W is the work done by that force. A more detailed explanation of the meaning of the negative sign will follow. So, if the electric force does an amount of work W on a charged particle, the change in the electric potential energy is

electric potential energy (E_E) the energy stored in a system of two charges a distance Δd apart, or the energy stored in an electric field that can do work on a positively charged particle

$$\Delta E_E = -W$$
$$\Delta E_E = -F_E \Delta d$$

Combining this equation with the equation relating force to the electric field, the change in electric potential energy when the charged particle moves from A to B in Figure 2 is

$$\begin{aligned}\Delta E_E &= -W \\ &= -F_E \Delta d\end{aligned}$$

$$\boxed{\Delta E_E = -q\varepsilon \Delta d}$$

This equation gives the *change* in the potential energy as the charge moves through a displacement Δd, in a region where the electric field is parallel to the displacement. Note that the change in the potential energy depends on the starting and ending locations but not on the path taken. In Figure 2, the displacement is along a line, but the charge may move from A to B along many other paths without affecting ΔE_E.

Electric potential energy is stored through the potential effect of the electric field on an electric charge. This effect is illustrated in **Figure 3(a)**, where the charge q is moved from point B to point A by an external force $\vec{F}_a$ directed to the left.

Force $\vec{F}_a$ results from some external agent, which could be your hand. The electric force on q points to the right, assuming that q is positive, so the displacement is opposite to the direction of the electric force. The work done by the electric field on the particle is thus negative. Therefore, according to the electric potential energy equation, the change in the electric potential energy must be positive. In other words, a positive amount of energy has now been stored in the system composed of the charge q and the electric field. That energy came from the positive amount of work done by the external force $\vec{F}_a$ that moved the charge from B to A.

As the charge moves from A back to B, the process is reversed (**Figure 3(b)**). Now the electric field does a positive amount of work on the particle because the electric force and the particle's displacement are parallel and the change in the electric potential energy is negative. Energy stored in the electric field and particle system is now taken out of the system. This energy can appear as an increase in the kinetic energy of the particle when it reaches B. Potential energy becomes kinetic energy.

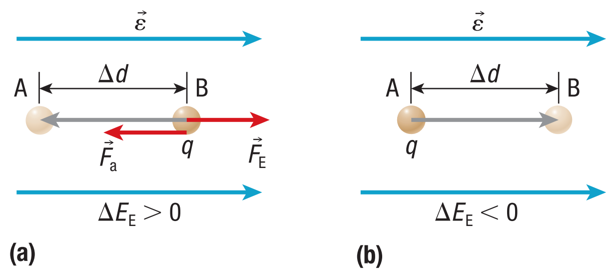


Figure 3 (a) To move q from B to A, the external force acting on a charged particle works against the electric field and produces a positive change in ΔE_E of the field and particle system. (b) The charged particle takes energy stored in the electric field and converts it to kinetic energy. This produces a negative change in ΔE_E of the field and particle system.

In the following Tutorial, you will learn more about how to solve problems that involve electric potential energy.

Tutorial 1 Solving Problems Involving Electric Potential Energy

This Tutorial explains how to determine the change in electric potential energy for a charge in a uniform electric field, given the position and magnitude of that field.

Sample Problem 1: Potential Energy Difference in an Electric Field

A charged particle moves from rest in a uniform electric field.

(a) For a proton, calculate the change in electric potential energy when the magnitude of the electric field is 250 N/C, the starting position is 2.4 m from the origin, and the final position is 3.9 m from the origin.

(b) Calculate the change in electric potential energy for an electron in the same field and with the same displacement.

(c) Calculate the change in electric potential energy for an electron accelerated in an electric field with the same magnitude but opposite direction as in (a) and (b), and with a starting position of 2.4 m from the origin and a final position of 5.0 m from the origin.

Solution

(a) **Given:** $d_i = 2.4$ m; $d_f = 3.9$ m; $q = +1.60 \times 10^{-19}$ C; $\varepsilon = 250$ N/C

Required: ΔE_E

Analysis: Use the equation for electric potential energy in terms of q, ε, and Δd, where $\Delta d = d_f - d_i$: $\Delta E_E = -q\varepsilon\Delta d = -q\varepsilon(d_f - d_i)$. Note that ΔE_E is negative for a positive charge travelling in the same direction as ε. Thus, the sign of q takes care of whether there is a gain (for negative q) or loss (for positive q) of electric potential energy. Note that we include the sign of the charge because we are dealing with energy, which is not a vector quantity and does not have a direction.

Solution:

$$\begin{aligned}\Delta E_E &= -q\varepsilon\Delta d \\ &= -q\varepsilon(d_f - d_i) \\ &= -(+1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\ \frac{\text{N}}{\cancel{\text{C}}}\right)(3.9\text{ m} - 2.4\text{ m}) \\ &= -6.0 \times 10^{-17}\ \text{N}\cdot\text{m}\end{aligned}$$

$$\Delta E_E = -6.0 \times 10^{-17}\ \text{J}$$

Statement: The change in electric potential energy due to the movement of the proton in the uniform electric field is -6.0×10^{-17} J. The negative sign indicates that the electric field loses potential energy by doing work on the proton.

(b) **Given:** $d_i = 2.4$ m; $d_f = 3.9$ m; $q = -1.60 \times 10^{-19}$ C; $\varepsilon = 250$ N/C

Required: ΔE_E

Analysis: Use the same equation for electric potential energy: $\Delta E_E = -q\varepsilon\Delta d = -q\varepsilon(d_f - d_i)$. Note that we include the sign of the charge to determine whether there is a gain or loss of electric potential energy.

Solution:

$$\begin{aligned}\Delta E_E &= -q\varepsilon\Delta d \\ &= -q\varepsilon(d_f - d_i) \\ &= -(-1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\ \frac{\text{N}}{\cancel{\text{C}}}\right)(3.9\text{ m} - 2.4\text{ m}) \\ &= 6.0 \times 10^{-17}\ \text{N}\cdot\text{m}\end{aligned}$$

$$\Delta E_E = 6.0 \times 10^{-17}\ \text{J}$$

Statement: The change in electric potential energy due to the movement of the electron in the uniform electric field is 6.0×10^{-17} J. The positive value indicates that the electric field gains potential energy as the electron is accelerated in the field.

(c) **Given:** $d_i = 2.4$ m; $d_f = 5.0$ m; $q = -1.60 \times 10^{-19}$ C; $\varepsilon = -250$ N/C

Required: ΔE_E

Analysis: Note that the sign for the electric field is negative because the electron is accelerating in a direction opposite to ε. Use the same equation for electric potential energy: $\Delta E_E = -q\varepsilon\Delta d = -q\varepsilon(d_f - d_i)$.

Solution:

$$\begin{aligned}\Delta E_E &= -q\varepsilon\Delta d \\ &= -q\varepsilon(d_f - d_i) \\ &= -(-1.6 \times 10^{-19}\ \cancel{\text{C}})\left(-250\ \frac{\text{N}}{\cancel{\text{C}}}\right)(5.0\text{ m} - 2.4\text{ m}) \\ &= -1.0 \times 10^{-16}\ \text{N}\cdot\text{m}\end{aligned}$$

$$\Delta E_E = -1.0 \times 10^{-16}\ \text{J}$$

Statement: When the direction of the electric field is reversed, the change in the electric potential energy is -1.0×10^{-16} J. The negative value indicates that the electric field loses potential energy by doing work on the electron.

Sample Problem 2: Dynamics of Charged Particles

(a) Using the law of conservation of energy, calculate the speed of the proton in part (a) of Sample Problem 1 for the given displacement. Assume that the proton starts from rest.

(b) Determine the initial speed of the electron in part (b) of Sample Problem 1, assuming its speed has decreased to half of its initial speed after the same displacement, Δd.

Solution

(a) **Given:** $d_i = 2.4$ m; $d_f = 3.9$ m; $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg; $\varepsilon = 250$ N/C

Required: v

Analysis: The law of conservation of energy states that the total change in potential energy of the field and particle system and the change in kinetic energy of the particle equals zero: $\Delta E_E + \Delta E_k = 0$. The kinetic energy of the particle is related to its speed v by the equation $\Delta E_k = \frac{1}{2}mv^2$. Use this equation and the equation $\Delta E_E = -q\varepsilon\Delta d$ for the change in potential energy of the field and proton system. Note that $\Delta d = 3.9 \text{ m} - 2.4 \text{ m} = 1.5$ m and $1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$.

Solution: By the law of conservation of energy,

$$\Delta E_E + \Delta E_k = 0$$

$$-q\varepsilon\Delta d + \frac{1}{2}mv^2 = 0$$

$$\frac{1}{2}mv^2 = q\varepsilon\Delta d$$

$$v = \sqrt{\frac{2q\varepsilon\Delta d}{m}}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\dfrac{\cancel{\text{kg}}\cdot\dfrac{\text{m}}{\text{s}^2}}{\cancel{\text{C}}}\right)(1.5\text{ m})}{1.67 \times 10^{-27}\ \cancel{\text{kg}}}}$$

$$v = 2.7 \times 10^5 \text{ m/s}$$

Statement: The speed of the proton accelerated for a distance of 1.5 m by an electric field of 250 N/C is 2.7×10^5 m/s.

(b) **Given:** $\Delta d = 1.5$ m; $q = -1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $\varepsilon = 250$ N/C; $v_f = 0.5v_i$

Required: v_i

Analysis: For situations that do not involve an object at rest, the change in kinetic energy is given by the equation $\Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. Use this equation and the equation $\Delta E_E = -q\varepsilon\Delta d$ for the change in potential energy of the field and electron system.

Solution: By the law of conservation of energy,

$$\Delta E_E + \Delta E_k = 0$$

$$-q\varepsilon\Delta d + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = 0$$

$$\frac{1}{2}m\left(\frac{1}{2}v_i\right)^2 - \frac{1}{2}mv_i^2 = q\varepsilon\Delta d$$

$$-\frac{3}{8}mv_i^2 = q\varepsilon\Delta d$$

$$v_i = \sqrt{-\frac{8q\varepsilon\Delta d}{3m}}$$

$$v_i = \sqrt{-\frac{8(-1.6 \times 10^{-19}\text{ C})\left(250\dfrac{\text{N}}{\text{C}}\right)(1.5\text{ m})}{3(9.11 \times 10^{-31}\text{ kg})}}$$

$$= \sqrt{\frac{8(1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\dfrac{\cancel{\text{kg}}\cdot\dfrac{\text{m}}{\text{s}^2}}{\cancel{\text{C}}}\right)(1.5\text{ m})}{3(9.11 \times 10^{-31}\ \cancel{\text{kg}})}}$$

$$v_i = 1.3 \times 10^7 \text{ m/s}$$

Statement: The initial speed of the electron before entering the electric field is 1.3×10^7 m/s.

Practice

1. An electron enters a uniform electric field of 145 N/C pointed toward the right. The point of entry is 1.5 m to the right of a given mark, and the point where the electron leaves the field is 4.6 m to the right of that mark. T/I A
   (a) Determine the change in the electric potential energy of the electron. [ans: 7.2×10^{-17} J]
   (b) The initial speed of the electron was 1.7×10^7 m/s when it entered the electric field. Determine its final speed. [ans: 1.1×10^7 m/s]
2. Calculate the work done in moving a proton 0.75 m in the same direction as the electric field with a strength of 23 N/C. T/I A [ans: 2.8×10^{-18} J]
3. An electron experiences a change in kinetic energy of $+4.2 \times 10^{-16}$ J. Calculate the magnitude and direction of the electric field when the electron travels 0.18 m toward the right. T/I A [ans: 1.5×10^4 N/C [toward the left]]

Electric Potential

electric potential (V) the value, in volts, of potential energy per unit positive charge for a given point in an electric field; 1 V = 1 J/C

Electric potential energy is a property of a system of charges or of a point charge in an electric field, where the field is created by other charges. In either case, this electric potential energy is not the property of a single charge alone. For this reason, the potential energy depends on the values of the charges and the electric field involved in the interaction. This leads to a new quantity called **electric potential**, V, which is a measure of how much electric potential energy is associated with a specific quantity of charge at a particular location in an electric field. Based on this definition,

$$V = \frac{E_E}{q}$$

Electric potential, or just potential, is a convenient measure because it is independent of the amount of charge at a particular location in the field. It depends only on the electric field strength at that location. For example, if you had 1 C of electrons at a particular location in a uniform electric field, you would possess a certain amount of electric potential energy. If you doubled the amount of electrons to 2 C at the same location in the electric field, you would have double the electric potential energy. In both these situations, you would have the same electric potential.

The SI unit of electric potential is the volt (V), named in honour of physicist Alessandro Volta (1745–1827). The volt relates to other SI units in the following equation:

$$1 \text{ V} = 1 \text{ J/C} = 1 \text{ N·m/C}$$

The volt is used in the measurement of many electrical quantities. Note in particular that the units of volts per metre are equivalent to units of newtons per coulomb, the units for electric field strength. These equivalent units give you several different ways to express the units of the electric field. The ones most commonly used are 1 V/m = 1 N/C.

electric potential difference (ΔV) the amount of work required per unit charge to move a positive charge from one point to another in the presence of an electric field

Another convenient definition relating to electric potential energy is **electric potential difference**, ΔV. We return to the concept of the change in potential difference and the displacement of the particle: You can define the change in the potential, or potential difference, for a charge q that moves between two points:

$$\Delta V = \frac{\Delta E_E}{q}$$

For the case of a uniform electric field, the equation for electric potential difference becomes

$$\begin{aligned}\Delta V &= \frac{\Delta E_E}{q} \\ &= \frac{-W}{q} \\ &= \frac{-q\varepsilon\Delta d}{q} \\ \Delta V &= -\varepsilon\Delta d\end{aligned}$$

This relationship shows how a non-uniform electric field varies with the change in electric potential (that is, electric potential difference) and the change in position in the field:

$$\varepsilon = -\frac{\Delta V}{\Delta d}$$

This equation states that the magnitude of the electric field is largest in regions where V is large and changes rapidly with small changes in displacement. Conversely, the electric field is zero in regions where V is constant. Notice that because of the negative sign in the equation, the electric field points from regions of high potential to regions of low potential (**Figure 4**). If we consider a circuit in which a battery is the source of electrical energy, a positive test charge will naturally move from the positive terminal where a high potential exists to the negative terminal where a low potential exists. Since the electric field points from positive to negative, the positive test charge will also move in the same direction as the field. Conversely, electrons will naturally travel from a region of low potential to a region of high potential, in a direction opposite to the direction of the electric field.

UNIT TASK **BOOKMARK**

You can apply what you have learned about electric potential to the Unit Task on page 422.

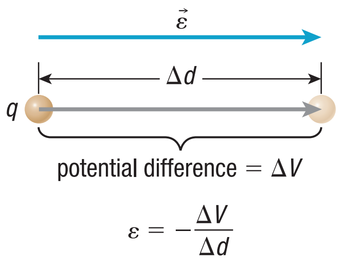


Figure 4 The magnitude and direction of the electric field are related to how the electric potential V changes with position.

The relation between the electric field and the changes in the potential and position involves the component of the electric field that is in the direction parallel to the displacement Δd. The electric field is a vector, so if you want to determine $\vec{\varepsilon}$ in a particular direction, you must consider how the potential V changes for a test charge moving along that direction.

With this equation and knowledge of $\vec{\varepsilon}$, you can, in principle at least, calculate how the electric potential changes as you move from place to place within the find. Strictly speaking, this relation only holds for small steps Δd in a constant field. You can, however, combine the potential changes ΔV from many such small steps to determine the change in the potential difference over a large distance. The following Tutorial will demonstrate how to solve problems involving electric potential.

Tutorial **2** Solving Problems Related to Electric Potential

This Tutorial shows how to use electric potential to solve problems related to a charge in a uniform electric field.

Sample Problem 1: TV Tubes and Particle Accelerators

The cathode-ray tubes in old television sets and computer monitors work in a way that is similar to certain parts in particle accelerators. Both devices accelerate particles in a similar way, using the uniform electric field between conducting plates. Looked at another way, the particles accelerate as they move through an electric potential difference.

(a) An electron leaves the negative plate of a cathode-ray tube and travels toward the positive plate. The electric potential difference between the plates is 1.5×10^4 V. Using the law of conservation of energy and the definition of electric potential difference, calculate the speed of an electron as it reaches the positive plate in a cathode-ray tube. Assume that the electron is initially at rest. The mass of an electron is 9.11×10^{-31} kg.

(b) Calculate the magnitude of the electric field at a distance of 15 cm, which is at the end of the cathode-ray tube.

(a) **Given:** $q = -1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $\Delta V = 1.5 \times 10^4$ V; $v_i = 0$ m/s

Required: v_f

Analysis: Use the equation for the law of conservation of energy, $\Delta E_E + \Delta E_k = 0$, and the equation for the electric potential difference, $\Delta E_E = q\Delta V$. The kinetic energy of the particle initially at rest is related to its final speed v_f by the equation $\Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. Since $v_i = 0$, $\Delta E_k = \frac{1}{2}mv_f^2$. Note that 1 V = 1 N·m/C = 1 kg·m/s²·m/C.

Solution: By the law of conservation of energy,

$$\Delta E_E + \Delta E_k = 0$$
$$q\Delta V + \frac{1}{2}mv_f^2 = 0$$
$$\frac{1}{2}mv_f^2 = -q\Delta V$$
$$v_f^2 = \frac{-2q\Delta V}{m}$$
$$v_f = \sqrt{-\frac{2q\Delta V}{m}}$$

Calculate the final speed v_f of the electron as it reaches the positive plate.

$$v_f = \sqrt{-\frac{2(-1.6 \times 10^{-19}\ \text{C})(1.5 \times 10^4\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}}$$
$$= \sqrt{\frac{2(1.6 \times 10^{-19}\ \cancel{\text{C}})\left(1.5 \times 10^4\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2} \cdot \frac{\text{m}}{\cancel{\text{C}}}\right)}{9.11 \times 10^{-31}\ \cancel{\text{kg}}}}$$
$$v_f = 7.3 \times 10^7\ \text{m/s}$$

Statement: The speed of the electron as it reaches the positive plate is 7.3×10^7 m/s.

(b) **Given:** $\Delta d = 15$ cm $= 0.15$ m; $\Delta V = 1.5 \times 10^4$ V

Required: ε

Analysis: Use the equation for the electric field in terms of potential difference and displacement:

$$\varepsilon = -\frac{\Delta V}{\Delta d}$$

Solution: $\varepsilon = -\dfrac{\Delta V}{\Delta d}$

$$= -\frac{1.5 \times 10^4\ \text{V}}{0.15\ \text{m}}$$
$$\varepsilon = -1.0 \times 10^5\ \text{V/m}$$

Statement: The electric field has a magnitude of 1.0×10^5 V/m. The negative sign indicates that the field extends from high to low potential. The electric field vector always points from regions of high V to regions of low V because $\vec{\varepsilon}$ is parallel to the direction that a positive test charge would move if it were placed at that location.

Sample Problem 2: Calculating the Speed of a Deflected Charged Particle

An electron moves horizontally with a speed of 1.6×10^6 m/s between two horizontal parallel plates. The plates have a length of 12.5 cm, and a plate separation that allows a charged particle to escape even after being deflected (**Figure 5**). The magnitude of the electric field within the plates is 150 N/C. Calculate the final velocity of an electron as it leaves the plates.

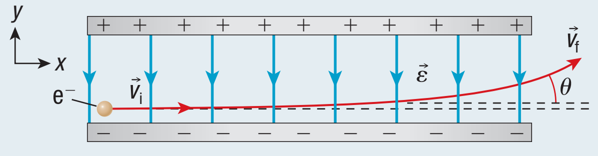


Figure 5

Given: $v_i = 1.6 \times 10^6$ m/s; $L = 12.5$ cm $= 0.125$ m; $q = -1.60 \times 10^{-19}$ C; $\varepsilon = -150$ N/C

Required: $\vec{v}_f$

Analysis: The electric field is uniform and directed downward between the plates, so the electric force acting on a proton is also constant and directed downward. A constant downward force means a constant downward acceleration on a positive charge. However, because the charge is negative, the acceleration must be upward. To calculate the magnitude of the final velocity, solve for each component of the velocity: the constant horizontal component and the accelerated upward component. Then use the equation $v_f = \sqrt{v_{x_f}^2 + v_{y_f}^2}$ for the magnitude of the final velocity.

The components of the velocity are given by $v_{x_f} = v_i = \dfrac{L}{\Delta t}$ (Δt can be calculated from v_i and L) and $v_{y_f} = v_{y_i} + a_y\Delta t = a_y\Delta t$ (since $v_{y_i} = 0$).

By combining the equations for Newton's second law, $F_{net} = ma$, and the electric force on a charge in an electric field ε, $F_E = q\varepsilon$, we can determine the vertical acceleration $a_y = \dfrac{F_{net}}{m} = \dfrac{F_E}{m} = \dfrac{q\varepsilon}{m}$.

Solution: Calculate a_y.

$$a_y = \frac{q\varepsilon}{m}$$
$$= \frac{(-1.6 \times 10^{-19}\ \cancel{\text{C}})\left(-150\dfrac{\cancel{\text{kg}} \cdot \dfrac{\text{m}}{\text{s}^2}}{\cancel{\text{C}}}\right)}{9.11 \times 10^{-31}\ \cancel{\text{kg}}}$$
$$a_y = 2.634 \times 10^{13}\ \text{m/s}^2\ \text{(two extra digits carried)}$$

Calculate Δt.

$$v_i = \frac{L}{\Delta t}$$

$$\Delta t = \frac{L}{v_i}$$

$$= \frac{0.125\ \cancel{m}}{1.6 \times 10^6\ \frac{\cancel{m}}{s}}$$

$\Delta t = 7.812 \times 10^{-8}$ s (two extra digits carried)

Calculate v_{y_f}.

$$v_{y_f} = a_y \Delta t$$

$$= \left(2.634 \times 10^{13}\ \frac{m}{s^{\cancel{2}}}\right)(7.812 \times 10^{-8}\ \cancel{s})$$

$v_{y_f} = 2.058 \times 10^6$ m/s (two extra digits carried)

Now calculate the magnitude of the net velocity.

$$v_f = \sqrt{v_{x_f}^2 + v_{y_f}^2}$$

$$= \sqrt{(1.6 \times 10^6\ \text{m/s})^2 + (2.058 \times 10^6\ \text{m/s})^2}$$

$$v_f = 2.6 \times 10^6\ \text{m/s}$$

Calculate the angle θ to determine the direction of the electron.

$$\theta = \tan^{-1}\left(\frac{v_{y_f}}{v_{x_f}}\right)$$

$$= \tan^{-1}\left(\frac{2.058 \times 10^6\ \frac{\cancel{m}}{\cancel{s}}}{1.6 \times 10^6\ \frac{\cancel{m}}{\cancel{s}}}\right)$$

$$\theta = 52°$$

Statement: The final velocity of the electron is 2.6×10^6 m/s [E 52° N].

Practice

1. An old television cathode-ray tube creates a potential difference of 1.6×10^4 V across the parallel accelerating plates. These plates accelerate a beam of electrons toward the target phosphor screen. The separation between the plates is 12 cm. K/U T/I
   (a) Using the principle of energy conservation and the definition of electric potential difference, calculate the speed at which the electrons strike the screen. [ans: 7.5×10^7 m/s]
   (b) Calculate the magnitude of the electric field. [ans: 1.3×10^5 N/C]
2. Four parallel plates are connected in a vacuum as shown in **Figure 6**. An electron at rest in the hole of plate X is accelerated to the right. The electron passes through holes at W and Y with no acceleration at all. It then passes through the hole at Y and slows down as it heads to plate Z. T/I

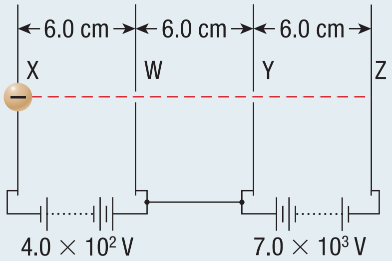


Figure 6

   (a) Calculate the speed of the electron at hole W. [ans: 1.2×10^7 m/s]
   (b) Calculate the distance, in centimetres, from plate Z to the point at which the electron changes direction. [ans: 5.7 cm [to the left of Z]]
3. An electron enters a parallel plate apparatus that is 8.0 cm long and 4.0 cm wide, as shown in **Figure 7**. The electron has a horizontal speed of 6.0×10^7 m/s. The potential difference between the plates is 6.0×10^2 V. Calculate the electron's velocity as it leaves the plates. K/U A [ans: 6.0×10^7 m/s [E 3.3° N]]

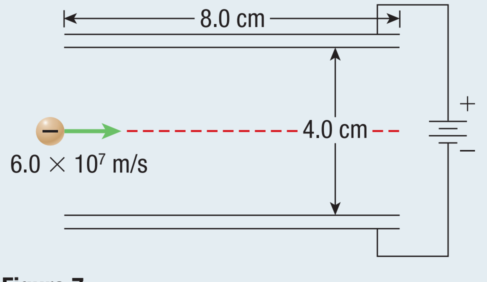


Figure 7

7.4 Review

Summary

- The change in electric potential energy depends on the electric field, the charge being moved, and the charge's displacement: $\Delta E_{\mathrm{E}} = -q\varepsilon\Delta d$.
- For $\Delta E_{\mathrm{E}} > 0$, work is done against the electric field $(-W)$, resulting in energy stored in the field. For $\Delta E_{\mathrm{E}} < 0$, work is done by the electric field $(+W)$ on a particle moving in the field, which typically increases the kinetic energy of the particle.
- The electric potential is the electric potential energy per unit charge at a given point in an electric field: $V = \dfrac{E_{\mathrm{E}}}{q}$.
- The magnitude of an electric field varies with the electric potential difference and the change in position in the field: $\varepsilon = -\dfrac{\Delta V}{\Delta d}$.

Questions

1. An electron moves from an initial location between parallel plates where the electric potential is $V_{\mathrm{i}} = 30$ V to a final location where $V_{\mathrm{f}} = 150$ V. K/U T/I
   (a) Determine the change in the electron's potential energy.
   (b) Determine the average electric field along a 10 cm–long line segment that connects the initial and final locations of the electron. Be sure to give both the magnitude and the direction of $\vec{\varepsilon}$.
2. The electric potential difference between two parallel metal plates is ΔV. The plates are separated by a distance of 3.0 mm and the electric field between the plates is $\varepsilon = 250$ V/m. Calculate ΔV. K/U T/I
3. A proton of mass 1.67×10^{-27} kg moves from a location where $V_{\mathrm{i}} = 75.0$ V to a spot where $V_{\mathrm{f}} = -20.0$ V. K/U T/I
   (a) Calculate the change in the proton's kinetic energy.
   (b) Replace the proton with an electron, and determine its change in kinetic energy.
4. An electron moves from a region of low potential to a region of higher potential where the potential change is $+45$ V. K/U T/I
   (a) Calculate the work, in joules, required to push the electron.
   (b) What is doing the work?
5. The electrons in an old TV picture tube are accelerated through a potential difference of 2.5×10^4 V. K/U T/I A
   (a) Do the electrons move from a region of high potential to a region of low potential, or vice versa?
   (b) Calculate the change in the kinetic energy of one of the electrons.
   (c) Calculate the final speed of an electron when the initial speed is zero.
6. A pair of parallel plates has an electric field of 2.26×10^5 N/C. Determine the change in the electric potential between points that are 2.55 m (initial) and 4.55 m (final) from the plates. K/U T/I A
7. An electron with a horizontal speed of 4.0×10^6 m/s and no vertical component of velocity passes through two horizontal parallel plates, as shown in **Figure 8**. The magnitude of the electric field between the plates is 150 N/C. The plates are 6.0 cm long. K/U T/I

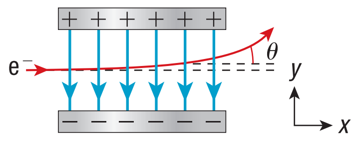


Figure 8

   (a) Calculate the vertical component of the electron's final velocity.
   (b) Calculate the final velocity of the electron.
8. An electric field of 20 N/C exists along the x-axis in space. Calculate the potential difference $\Delta V = V_{\mathrm{B}} - V_{\mathrm{A}}$, where the points A and B are given by
   (a) A = 0 m; B = 4 m
   (b) A = 4 m; B = 6 m T/I A
9. Points A and B are at the same potential. Determine the net work done in moving a charge from point A to point B. K/U T/I
10. The potential at a point is 20 V. Calculate the work done in bringing a charge of 0.5 C to this point. K/U A

Electric Potential and Electric Potential Energy Due to Point Charges

7.5

In the first section of this chapter, you saw how a Van de Graaff generator in a science museum causes the hair of anyone in contact with the device to stand on end. At that point, the discussion dealt simply with the properties of electric charge, and how the like charges (electrons) on individual hairs caused the hairs to repel each other and spread out in all directions.

We can look at this phenomenon in another way: the charge on the Van de Graaff generator creates a high electric potential on the conducting sphere of the generator. This potential provides an electric field that exerts a force on charged particles near the generator. This force, in turn, accelerates these particles. In fact, when the Van de Graaff generator was invented, it was an early type of particle accelerator. As a particle accelerator, however, the Van de Graaff generator has limitations. For one thing, it can only accelerate charges like those on the sphere. Also, if the potential on the sphere becomes too high with respect to the generator's surroundings, the air around the generator will ionize. This causes large sparks to jump between the sphere and objects at lower electric potentials, resulting in artificial lightning (**Figure 1**).

At a distance, the conducting sphere of a Van de Graaff generator resembles a point charge. In this section, you will learn about electric potentials, electric potential energies, and electric potential differences for point charges and their surroundings.

Figure 1 The charge on the Van de Graaff generator has a large electric potential difference with its surroundings. This can cause large sparks to appear between the generator and its surroundings.

Electric Potential Due to a Point Charge

Earlier you learned about the electric properties of a point charge. This simple example is useful in many situations. In Section 7.4, we defined electric potential V in terms of a test charge q and the electric potential energy E_E. The electric potential energy, in turn, depends on the position d of the test charge in an electric field $\vec{\varepsilon}$. If the charge producing the electric field has the same sign as the test charge q, the work done by the electric field on q is negative. This means that $\Delta E_E > 0$ for $\Delta d > 0$, and the sign of potential energy (and thus potential) is reversed from that of the uniform electric field.

For a point charge q, the magnitude of the electric field is

$$\varepsilon = \frac{kq}{r^2}$$

Setting the location in the field equal to the radius of the field ($\Delta d = r$), the **electric potential due to a point charge** is inversely proportional to the distance from the charge and directly proportional to the amount of charge. So, for a point charge q producing an electric field, at a distance r from the charge, the electric potential is

electric potential due to a point charge the electric potential is inversely proportional to the distance from the charge and proportional to the amount of charge producing the field

$$V = \frac{kq\Delta d}{r^2}$$

$$V = \frac{kq}{r}$$

These results reinforce what you learned about potential energy and the behaviour of like and unlike charges.

Suppose you place a source charge ($q > 0$) at the origin of the coordinate system in **Figure 2**. The charge produces a potential given by $\frac{kq}{r}$. Then you place a positive test charge nearby. Both the source and the test charge are positive, so they repel one another, and the test charge experiences a force that carries it "downhill" and away from the origin along the blue potential curve in Figure 2. When the source charge is negative, the potential is described by the lower, red curve in Figure 2. The source charge is now negative, so the attractive force carries a positive test charge along the red potential curve toward the origin.

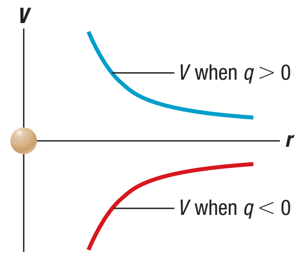


Figure 2 Electric potential near a point charge. The blue curve is for a positive point charge, and the red curve is for a negative point charge.

Only changes in potential energy are important, so you must always be clear about choosing the reference point for potential energy. Electric potential is proportional to electric potential energy, so you must also pay attention to the reference point for electric potential. For example, when dealing with a point charge, follow the standard convention by choosing $V = 0$ at an infinite distance from the source charge. Infinite distance represents a distance so far away that any potential produced by a charge is negligible and for all intents and purposes equal to zero. In many other problems, choose Earth as $V = 0$ because Earth conducts charge well. Put another way, the electric ground is the point where $V = 0$.

For two or more charges, use the superposition principle. Remember that electric potentials of negative charges are negative, and electric potentials of positive charges are positive. However, because electric potential is not a vector, the total electric potential of several point charges equals the algebraic sum of the electric potentials resulting from each of the individual charges and their specific distances from the chosen location.

Electric Potential Energy of Two Point Charges

Now consider the electric potential energy of a pair of charges. The electric potential energy of a pair of charges is the potential energy possessed by each charge in the pair because of its position. For a pair of charges, this potential energy is inversely proportional to the distance between the charges and directly proportional to the product of the two charges. For a pair of charges q_1 and q_2 separated by a distance r, use the equation for the electric field for the point charge q_2 and the definition of electric potential energy for the point charge q_1, in terms of V, to determine the potential energy for the two charges:

$$\begin{aligned} E_E &= q_1 V \\ &= q_1 \frac{kq_2}{r} \\ E_E &= \frac{kq_1q_2}{r} \end{aligned}$$

The electric potential energy of a pair of charges is then given by

$$\boxed{E_E = \frac{kq_1q_2}{r}}$$

This is qualitatively similar to the equation for Coulomb's law, except that E_E varies as $\frac{1}{r}$, not $\frac{1}{r^2}$. If the charges are initially separated by a distance r_i and then brought together to a final separation r_f, the change in the potential energy is

$$\Delta E_E = E_{Ef} - E_{Ei}$$
$$\Delta E_E = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

Note that only changes in potential energy matter, and that E_E approaches zero when the two charges are very far apart. The blue curve in **Figure 3** shows the general behaviour of the electric potential energy for two like charges. In this case, E_E is positive and increases as the charges are brought together. If the charges have opposite signs, the electric force is attractive (negative), and the potential energy is then also negative, as shown by the red curve in Figure 3. The following Tutorial will demonstrate how to solve problems involving electric potential and electric potential energy.

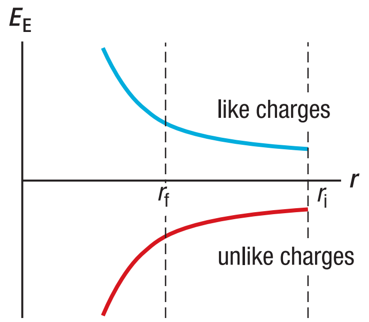


Figure 3 Electric potential energy as a function of the separation r between two charges q_1 and q_2. The blue curve is for a repulsive force, and the red curve is for an attractive force.

Tutorial 1 Electric Potential and Electric Potential Energy

Sample Problem 1 models how to calculate the total electric potential of two point charges and the work required to move a third charge into this potential. In Sample Problem 2, we will use the electric potential energy and conservation of energy to calculate the speed of a charge at a given location. Finally, in Sample Problem 3, we show how to calculate the minimum particle separation by using electric potential energy, conservation of energy, and conservation of momentum.

Sample Problem 1: Calculating the Electric Potential

A point charge with a charge of 4.00×10^{-8} C is 4.00 m due west from a second point charge with a charge of -1.00×10^{-7} C.

(a) Calculate the total electric potential due to these charges at a point P, 4.00 m due north of the first charge.

(b) Calculate the work required to bring a third point charge with a charge of 2.0×10^{-9} C from infinity to point P.

Solution

(a) **Given:** $q_1 = 4.00 \times 10^{-8}$ C; $q_2 = -1.00 \times 10^{-7}$ C; $r_{12} = 4.00$ m; $r_{1P} = 4.00$ m; $k = 8.99 \times 10^9$ N·m²/C²

Required: V_T

Analysis: The total potential at P is the sum of the potentials due to the two individual charges q_1 and q_2. We can use the equation $V = \frac{kq}{r}$ to calculate the potential due to each charge and the Pythagorean theorem to calculate the distance of point P from q_2: $r_{2P} = \sqrt{r_{12}^2 + r_{1P}^2}$.

Solution: V_T is the sum of the potentials due to the individual charges q_1 and q_2.
Calculate the potential V_1 at P due to q_1.

$$
\begin{aligned}
V_1 &= \frac{kq_1}{r_{1P}} \\
&= \frac{\left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\text{C}^{\cancel{2}}}\right)(+4.00 \times 10^{-8}\ \cancel{\text{C}})}{4.00\ \cancel{\text{m}}} \\
&= 8.99 \times 10^1\ \text{N}\cdot\text{m/C} \\
V_1 &= 8.99 \times 10^1\ \text{J/C}
\end{aligned}
$$

To calculate the potential V_2 at P due to q_2, first calculate the distance r_{2P} between P and q_2.

$$
\begin{aligned}
r_{2P} &= \sqrt{r_{12}^2 + r_{1P}^2} \\
&= \sqrt{(4.00\ \text{m})^2 + (4.00\ \text{m})^2} \\
r_{2P} &= 5.6569\ \text{m (two extra digits carried)}
\end{aligned}
$$

Then calculate V_2.

$$
\begin{aligned}
V_2 &= \frac{kq_2}{r_{2P}} \\
&= \frac{\left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\text{C}^{\cancel{2}}}\right)(-1.00 \times 10^{-7}\ \cancel{\text{C}})}{5.6569\ \cancel{\text{m}}} \\
&= -1.5892 \times 10^2\ \text{N}\cdot\text{m/C} \\
V_2 &= -1.5892 \times 10^2\ \text{J/C (two extra digits carried)}
\end{aligned}
$$

Now calculate the total potential V_T.

$$
\begin{aligned}
V_T &= V_1 + V_2 \\
&= 8.99 \times 10^1\ \text{J/C} + (-1.5892 \times 10^2\ \text{J/C}) \\
V_T &= -69.02\ \text{J/C (one extra digit carried)}
\end{aligned}
$$

Statement: The total electric potential at point P is −69.0 J/C.

(b) **Given:** $V_T = -69.02$ J/C; $q_3 = 2.0 \times 10^{-9}$ C

Required: work required to move q_3 from $r = \infty$ to P

Analysis: Use the definition of electric potential energy. The work done on q_3 is $W = q_3(V_\infty - V_T)$. At infinity, the electric potential is zero, so $W = -q_3V_T$.

Solution:

$$
\begin{aligned}
W &= -q_3V_T \\
&= -(2.0 \times 10^{-9}\ \cancel{\text{C}})\left(-69.02\ \frac{\text{J}}{\cancel{\text{C}}}\right) \\
W &= 1.4 \times 10^{-7}\ \text{J}
\end{aligned}
$$

Statement: The work done on the third point charge as it is brought from infinity to point P is 1.4×10^{-7} J. The positive value of W indicates that the electric field does work on the charge to increase the charge's kinetic energy as it approaches P.

Sample Problem 2: Electric Potential Energy and Dynamics

A point charge q_1 with charge 2.0×10^{-6} C is initially at rest at a distance of 0.25 m from a second charge q_2 with charge 8.0×10^{-6} C and mass 4.0×10^{-9} kg (**Figure 4**). Both charges are positive. Charge q_1 remains fixed at the origin, whereas q_2 travels to the right upon release. Determine the speed of charge q_2 when it reaches a distance of 0.50 m from q_1.

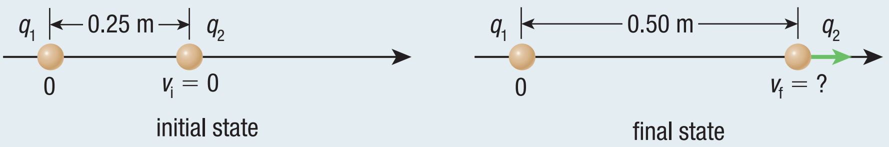


Figure 4

Given: $q_1 = 2.0 \times 10^{-6}$ C; $q_2 = 8.0 \times 10^{-6}$ C; $m = 4.0 \times 10^{-9}$ kg; $r_1 = 0.25$ m; $r_2 = 0.50$ m

Required: v_f

Analysis: No external forces are applied to the system, so energy must be conserved. We can apply the law of conservation of energy, using electric potential energy: $E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$. Charge q_1 remains fixed at all times, so E_k equals the kinetic energy of charge q_2, and E_E equals the electric potential energy of the system of two charges. Charge q_2 is initially at rest, so the initial kinetic energy of q_2 is zero. Note that r_1 and r_2 are related by $r_2 = 2r_1$. $E_{E_i} = \frac{kq_1q_2}{r_1}$; $E_{E_f} = \frac{kq_1q_2}{2r_1}$; $E_{k_i} = \frac{1}{2}mv_i^2 = 0$; $E_{k_f} = \frac{1}{2}mv_f^2$

Solution:

$$E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$$

$$\frac{kq_1q_2}{r_1} + 0 = \frac{kq_1q_2}{2r_1} + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv_f^2 = \frac{kq_1q_2}{2r_1}$$

$$v_f = \sqrt{\frac{kq_1q_2}{mr_1}}$$

$$= \sqrt{\frac{\left(8.99 \times 10^9 \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{C}^2}}\right)(2.0 \times 10^{-6}\ \cancel{\text{C}})(8.0 \times 10^{-6}\ \cancel{\text{C}})}{(4.0 \times 10^{-9}\ \cancel{\text{kg}})(0.25\ \cancel{\text{m}})}}$$

$$v_f = 1.2 \times 10^4 \text{ m/s}$$

Statement: The speed of charge q_2 when it reaches a distance of 0.50 m from charge q_1 is 1.2×10^4 m/s.

Sample Problem 3: A Head-on "Collision"

Two particles, a proton with charge 1.60×10^{-19} C and mass 1.67×10^{-27} kg and an alpha particle (helium-4 nucleus) with charge 3.20×10^{-19} C and mass 6.64×10^{-27} kg, are separated by an extremely large distance. They approach each other along a straight line with initial speeds of 3.00×10^6 m/s each. Calculate the separation between the particles when they are closest to each other.

Given: $q_1 = 1.60 \times 10^{-19}$ C; $q_2 = 3.20 \times 10^{-19}$ C; $m_1 = 1.67 \times 10^{-27}$ kg; $m_2 = 6.64 \times 10^{-27}$ kg; $v_{i_1} = v_{i_2} = 3.00 \times 10^6$ m/s; $r_i \rightarrow \infty$ m; $k = 8.99 \times 10^9$ N·m²/C²

Required: minimum separation of charges, r_f

Analysis: We can use the conservation of energy to determine the separation of the charges at the moment when both particles have converted their kinetic energy into electric potential energy: $E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$. At an extremely large distance, $r_i \rightarrow \infty$, the electric potential energy of the pair of charges will be zero. At minimum separation, the kinetic energy of the charges will be zero, so

$$E_{E_i} = \frac{kq_1q_2}{r_i} = 0;\ E_{E_f} = \frac{kq_1q_2}{r_f};\ E_{k_i} = \frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2;\ E_{k_f} = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2 = 0$$

Solution:

$$E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$$

$$0 + \frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{kq_1q_2}{r_f} + 0$$

$$r_f = \frac{2kq_1q_2}{m_1v_{i_1}^2 + m_2v_{i_2}^2}$$

$$= \frac{2\left(8.99 \times 10^9 \frac{\cancel{\text{kg}} \cdot \frac{\text{m}}{\cancel{\text{s}^2}} \cdot \cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(1.60 \times 10^{-19}\ \cancel{\text{C}})(3.20 \times 10^{-19}\ \cancel{\text{C}})}{(1.67 \times 10^{-27}\ \cancel{\text{kg}})\left(3.00 \times 10^6 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2 + (6.64 \times 10^{-27}\ \cancel{\text{kg}})\left(3.00 \times 10^6 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}$$

$$r_f = 1.23 \times 10^{-14}\ \text{m}$$

Statement: The minimum separation of the proton and the alpha particle is 1.23×10^{-14} m.

Practice

1. Three charges, $q_1 = +6.0 \times 10^{-6}$ C, $q_2 = -3.0 \times 10^{-6}$ C, $q_3 = -3.0 \times 10^{-6}$ C, are located at the vertices of an equilateral triangle (**Figure 5**). K/U T/I
   (a) Calculate the electric potential at the midpoint of each side of the triangle. [ans: between q_1 and q_2 is $V_1 = 7.6 \times 10^3$ J/C; between q_1 and q_3 is $V_2 = 7.6 \times 10^3$ J/C; between q_2 and q_3 is $V_3 = -1.5 \times 10^4$ J/C]
   (b) Calculate the total electric potential energy of the group of charges. [ans: -8.1×10^{-2} J]
2. Four point charges, each with $q = 4.5 \times 10^{-6}$ C, are arranged at the corners of a square of side length 1.5 m. Determine the electric potential at the centre of the square. K/U T/I [ans: 1.5×10^5 J/C]
3. Two electrons start at rest with a separation of 5.0×10^{-12} m. Once released, the electrons accelerate away from each other. Calculate the speed of each electron when they are a very large distance apart. K/U T/I [ans: 7.1×10^6 m/s]
4. Two protons move toward each other. They start at infinite separation. One has an initial speed of 2.3×10^6 m/s, and the other has an initial speed of 1.2×10^6 m/s. Calculate the separation when the protons are closest to each other. K/U T/I [ans: 4.1×10^{-14} m]

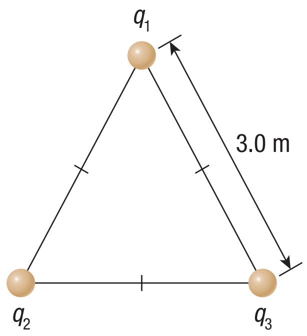


Figure 5

The Van de Graaff Generator

The Van de Graaff generator, discussed earlier, produces an electric potential on the conducting sphere, or dome, by separating charge at one place, and moving the charge to the conductor. As you can see in **Figure 6**, a moving cloth or rubber conveyor-style belt receives electrons from a lower conducting comb at point A. The negatively charged part of the belt moves upward toward the conducting sphere and gives up the negative charge to the upper comb at point B. The charge travels to the conducting sphere and gives the dome a net negative charge. As the generator continues to run, the amount of charge on the sphere builds. When the charge reaches high enough levels, the electrons jump from the sphere through the air to another conductor brought close to the sphere, producing the artificial lightning seen in Figure 1.

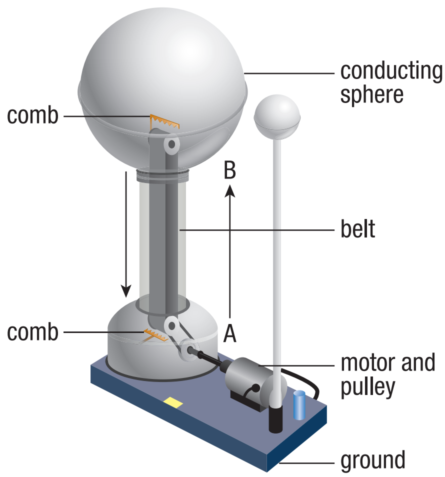


Figure 6 In a Van de Graaff generator, charge is carried to the dome by a moving belt. The charge is deposited on the belt at point A and conveyed to the dome at point B.

7.5 Review

Summary

- The electric potential at a distance r from a point charge q is $V = \frac{kq}{r}$.
- The total electric potential at a point P for a system of charges equals the algebraic sum of the potentials at P due to each individual charge at a distance that separates the charge and P.
- For an electric potential from a point source charge q_1, the work done moving a test charge q_2 from an initial separation r_i to a final separation r_f equals the change in the electric potential energy:

$$W = E_{E_f} - E_{E_i}$$

$$W = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

- For a system of two charges, q_1 and q_2, separated by a distance r, the electric potential energy is $E_E = \frac{kq_1q_2}{r}$.
- The potential energy difference of two point charges, q_1 and q_2, as the separation between them changes from r_i to r_f is

$$\Delta E_E = E_{E_f} - E_{E_i}$$

$$\Delta E_E = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

- When the amount of charge on the dome of a Van de Graaff generator is high enough, the electrons may jump from the sphere, producing artificial lightning.

Questions

1. A charged particle is released from rest and begins to move as a result of the electric force from a nearby proton. For the following particles, answer these questions. Does the particle move to a region of higher or lower potential energy? Does the particle move to a region of higher or lower electric potential? K/U T/I
   (a) The particle is a proton.
   (b) The particle is an electron.
2. Two particles are at locations where the electric potential is the same. Do these particles have the same electric potential energy? Explain. K/U T/I
3. How much work is required to move a charge from one spot to another with the same electric potential? Explain your answer. K/U T/I
4. Two point particles with charges $q_1 = 4.5 \times 10^{-5}$ C and $q_2 = 8.5 \times 10^{-5}$ C have a potential energy of 40.0 J. Calculate the distance between the charges, in centimetres. K/U T/I
5. Two point particles with charges $q_1 = 4.5 \times 10^{-5}$ C and $q_2 = 8.5 \times 10^{-5}$ C are initially separated by a distance of 2.5 m. They are then brought closer together so that the final separation is 1.5 m. Determine the change in the electric potential energy. K/U T/I
6. Two point charges with charges $q_1 = 3.5 \times 10^{-6}$ C and $q_2 = 7.5 \times 10^{-6}$ C are initially very far apart. They are then brought together, with a final separation of 2.5 m. Calculate how much work it takes to bring them together. K/U T/I
7. A simple model of a hydrogen atom shows the electron and proton as point charges separated by a distance of 5.00×10^{-11} m. Calculate how much work is required to break apart these two charges and to separate them by a very large distance when the electron is initially at rest. T/I A
8. A Van de Graaff generator used for classroom demonstrations has a conducting spherical dome with a radius of 15 cm. The electric potential produced by the charged generator is -8.5×10^4 V near its surface. The sphere has a uniform distribution of charge, so you may assume that all charge on the sphere is concentrated at the sphere's centre. Assume that the electric potential at a large distance and at Earth's surface is zero. K/U T/I A
   (a) Calculate the charge on the sphere.
   (b) Calculate the magnitude of the electric field near the surface of the sphere.
   (c) In which direction does the electric field point?

7.6 The Millikan Oil Drop Experiment

Physicists now know that each fundamental particle has a characteristic electric charge that does not change. In fact, the amount of charge is part of the definition of the kind of particle. At the start of the twentieth century, though, physicists still had many questions about the charge of fundamental particles. A big question was, is there a smallest unit of charge that nature will allow, and if so, what is the value of this charge? In this section, you will read about a brilliant experiment aimed at answering this question that resulted in the first measurement of the charge of the electron.

Figure 1 Robert Millikan

fundamental physical constant a measurable natural value that never varies and can be determined by experimentation

elementary charge (*e*) the magnitude of the electric charge carried by a proton, equal to the absolute value of the electric charge of an electron

Millikan's Experiment

Physicist Robert Millikan (**Figure 1**) set out in 1909 to examine the existence of fundamental charge using a series of experiments. Millikan's work demonstrated that the electron is a fundamental particle with a unique charge. This electric charge is considered one of a few **fundamental physical constants**—measurable values that can be determined by demonstration and do not vary—that define natural laws.

Millikan hypothesized that an **elementary charge**, *e*, the smallest unit of charge in nature, did exist, and that the charge of the electron equalled this elementary charge. To measure the charge, Millikan used a fine mist of oil droplets sprayed from an atomizer similar to what you may find on perfume bottles. The droplets picked up electric charges due to friction when sprayed from the atomizer. Millikan further hypothesized that the amount of charge any one drop picked up would be a whole-number multiple of the fundamental charge.

To measure the charge on a drop, Millikan used a device called an electrical microbalance. He allowed the oil drops to fall into a region between two oppositely charged parallel plates. The charges on the plates mean that there is an electric field in the space between the plates, which creates a potential difference between the top plate and the bottom plate. Millikan connected the plates to a series of adjustable batteries so that he could adjust the magnitude of the electric field and, therefore, the electric force on the droplets. By adjusting the electric force to balance the downward gravitational force, Millikan could bring a charged drop of oil to rest in the region between the plates (**Figure 2**).

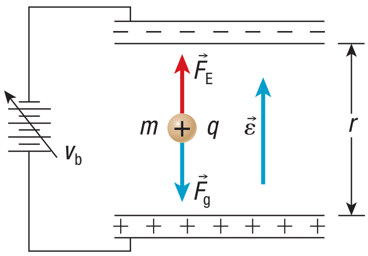


Figure 2 A charged oil drop will be suspended between two charged plates when the electric force on the drop balances the gravitational force.

We can understand how Millikan used the electrical microbalance to determine the charge on an oil drop by comparing the electric and gravitational forces. For a drop of charge q, the electric force from the field $\vec{\varepsilon}$ is

$$\vec{F}_E = q\vec{\varepsilon}$$

If we assume that the charge is positive, then we can charge the plates so that the electric field points upward and gives an upward force to the drop. We can then carefully adjust the potential difference so that the falling drop comes to rest between the plates. When this happens, the electric force balances the gravitational force, and the magnitudes will be equal:

$$F_E = F_g$$

$$q\varepsilon = mg$$

In Section 7.4, you learned that the electric field between two charged plates depends on the potential difference ΔV as

$$\varepsilon = \frac{\Delta V}{\Delta d}$$

where Δd is the plate separation distance. Therefore, we can solve for the electric charge of the drop as

$$q = \frac{mg}{\varepsilon}$$

$$q = \frac{mg\Delta d}{\Delta V_b}$$

where ΔV_b is the special value of potential difference that balances the drop.

If we assume that we can measure the potential difference of our batteries and the plate separation, then we will know the charge of the drop if we can measure the drop's mass.

To measure the mass of a drop, Millikan simply switched off the electric field and observed the final speed of the drop as it fell onto the bottom plate. From the final speed, he could calculate the mass of the drop if he accounted for both the gravitational force and the force due to air friction. With this information, he could determine the charge on the drop.

Millikan repeated his experiment many times, balancing a drop, measuring the voltage, letting the drop fall, and measuring its final speed. When he analyzed the data, he discovered the hypothesized pattern. The values of the charges he measured were whole-number multiples of some smallest value, and no drops had less charge than this value. Millikan concluded that this charge value equalled the elementary charge of the electron. In fact, we now think of this positive number as the charge of the proton, but the absolute value is the same for electrons.

Later experiments by other researchers confirmed Millikan's results and improved the accuracy of his findings. The current accepted value of the elementary charge e to four significant digits is

$$e = 1.602 \times 10^{-19}\ \text{C}$$

With this value, we can connect the charge of an object and the difference in the number of electrons versus protons in the object. If an object has N more protons than electrons, it has a charge q given by

$$q = Ne$$

The following Tutorial examines problems involving the elementary charge.

Investigation 7.6.1

The Millikan Experiment (page 367)
In Investigation 7.6.1, you will recreate Millikan's famous experiment using hands-on and online models.

Tutorial 1 Solving Problems Related to the Elementary Charge

Sample Problem 1: Calculating the Charge on an Object

Calculate the charge on a small sphere with an excess of 3.2×10^{14} electrons.

Given: $N = 3.2 \times 10^{14}$ electrons

Required: q

Analysis: $q = Ne$; note that in this case $e = -1.602 \times 10^{-19}$ C because we are dealing with electrons.

Solution:
$$\begin{aligned} q &= Ne \\ &= (3.2 \times 10^{14})(-1.602 \times 10^{-19}\ \text{C}) \\ q &= -5.1 \times 10^{-5}\ \text{C} \end{aligned}$$

Statement: The charge on the sphere is -5.1×10^{-5} C.

Sample Problem 2: The Elementary Charge and an Oil Drop

In a Millikan-type experiment, two horizontal plates maintained at a potential difference of 360 V are separated by 2.5 cm. A latex sphere with a mass of 1.41×10^{-15} kg hangs between the plates, the upper plate of which is positive.

(a) Is the sphere negatively or positively charged?

(b) Calculate the magnitude of the charge on the latex sphere.

(c) Determine the number of excess or deficit particles on the sphere.

Solution

(a) The electric field lines run from positive charges to negative charges, so the field between the plates points downward. We know that protons will move in the same direction as an electric field and that electrons will move in the opposite direction to the electric field. The electric force on the sphere must point upward to balance the gravitational force, so the sphere's charge must be negative.

(b) **Given:** $r = 2.5\ \text{cm} = 2.5 \times 10^{-2}$ m; $m = 1.41 \times 10^{-15}$ kg; $\Delta V_b = 360$ V

Required: q

Analysis: When the sphere is balanced, the electric force balances the gravitational force. So, we can use the equation for solving for the electric charge of the oil drop used in Millikan's oil drop experiment: $q = \dfrac{mg\Delta d}{\Delta V_b}$.

Note that 1 V = 1 N·m/C and 1 N = 1 kg·m/s^2.

Solution: $q = \dfrac{mg\Delta d}{\Delta V_b}$

$$= \frac{(1.41 \times 10^{-15}\ \cancel{\text{kg}})\left(9.8\ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)(2.5 \times 10^{-2}\ \cancel{\text{m}})}{360\ \cancel{\text{kg}} \cdot \dfrac{\cancel{\text{m}}}{\cancel{\text{s}^2}} \cdot \dfrac{\cancel{\text{m}}}{\text{C}}}$$

$q = 9.596 \times 10^{-19}$ C (two extra digits carried)

The actual charge is negative. This value gives the magnitude of the charge.

Statement: The magnitude of the charge on the latex sphere is 9.6×10^{-19} C.

(c) **Given:** $q = 9.596 \times 10^{-19}$ C

Required: N

Analysis: Use the equation that connects the charge of an object with the number of excess protons in the object: $q = Ne$.

Solution: $q = Ne$

$$N = \frac{q}{e}$$

$$= \frac{9.596 \times 10^{-19}\ \cancel{\text{C}}}{1.602 \times 10^{-19}\ \cancel{\text{C}}}$$

$$N = 6$$

Since the sphere has a negative charge, the excess charges are electrons.

Statement: There are 6 excess electrons on the sphere.

Practice

1. Calculate the force of repulsion between two plastic spheres placed 110 cm apart. Each sphere has a deficit of 1.2×10^8 electrons. T/I [ans: 2.7×10^{-12} N]
2. An oil drop with a mass of 2.48×10^{-15} kg is balanced between two parallel, horizontal plates 1.7 cm apart, maintained at a potential difference of 260 V. The upper plate is positive. Calculate the charge on the drop in coulombs and as a multiple of the elementary charge. Determine whether there is an excess or a deficit of electrons. T/I [ans: -1.6×10^{-18} C; $-10e$; 10 excess electrons]
3. Due to the positive charge of Earth's ionosphere, Earth's surface is surrounded by an electric field similar to the field surrounding a negatively charged sphere. The magnitude of this field is approximately 1.0×10^2 N/C. What charge would an oil drop with a mass of 2.4×10^{-15} kg need in order to remain suspended by Earth's electric field? Give your answer in both coulombs and as a multiple of the elementary charge. T/I [ans: -2.4×10^{-16} C; $-1.5 \times 10^3 e$]

Charge of a Proton

The elementary charge is the electric charge of a proton, which is equal in magnitude to the absolute value of the electric charge of the electron. Careful experimentation has consistently shown that the two particles have charges that are equal in magnitude. This result is actually a surprise, because the electron and proton have very little else in common, including their masses and the roles they play in the structure of matter.

Furthermore, physicists think of the electron as a fundamental particle with no inner workings, but they now view the proton as a combination of more fundamental particles called quarks. The proton consists of three quarks, all of which have charges that are either exactly one-third or two-thirds of the elementary charge. Despite having fractional charges, though, no experiment has detected any combination of quarks in nature that have a total charge whose magnitude is less than *e*. For this reason, physicists still refer to *e* as the elementary charge.

In fact, every subatomic particle that researchers have so far detected has a charge whose magnitude is equal to a whole-number multiple of *e*. Researchers also believe that the amount of charge in an isolated system is conserved like energy. Unlike energy, electric charge does not come in different types and cannot change from one form to another. No interaction can destroy or create electric charge, and the total electric charge of the universe remains constant.

7.6 Review

Summary

- Robert Millikan used his oil drop experiments to determine the magnitude of an electron's charge.
- The elementary charge, *e*, is equal to 1.602×10^{-19} C and represents the electric charge carried by a proton. This value is a fundamental physical constant. It is also the absolute value of the electric charge carried by an electron.
- The value of the charge on an electron is -1.602×10^{-19} C.
- Every subatomic particle so far detected has a charge whose magnitude is equal to a whole-number multiple of the elementary charge.

Questions

1. Calculate the number of electrons that must be removed from an uncharged object to give it a positive charge of 3.8×10^{-14} C. K/U
2. Calculate the magnitude of the electric field and the electric potential at a distance of 0.35 m from an object with an excess of 6.1×10^{6} electrons. T/I
3. Consider two parallel plates 2.00 mm apart with a potential difference of 240 V and a positively charged upper plate. A charged oil droplet with a mass of 5.88×10^{-10} kg is suspended between the plates. Determine the sign and magnitude of the electric charge on the oil droplet, and calculate the electron deficiency or excess. T/I
4. A 3.3×10^{-7} kg drop of water is suspended by a uniform 8.4×10^{3} N/C electric field directed upward. K/U T/I
   (a) Is the charge on the drop positive or negative? Explain your answer.
   (b) Calculate the electron excess or deficit on the drop.
5. A 5.2×10^{-15} kg oil drop hangs between two parallel plates (**Figure 3**). K/U T/I

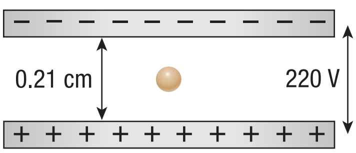


Figure 3

   (a) Calculate the oil drop's charge.
   (b) Calculate the electron excess or deficit on the drop.
6. Sphere A has a mass of 4.2×10^{-2} kg and is tethered to a wall by a thin thread. Sphere A has an excess of 1.2×10^{12} electrons. Sphere B has a deficit of 3.5×10^{12} electrons and is 0.23 m from sphere A (**Figure 4**). T/I

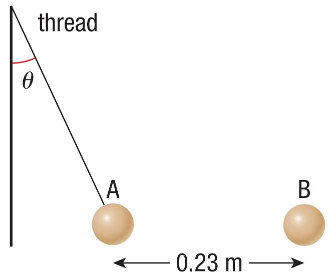


Figure 4

   (a) Determine the angle between the thread and the wall.
   (b) Determine the tension in the thread.
7. Earth has an electric field on its surface with an approximately constant magnitude of 1.0×10^{2} N/C directed toward the centre. K/U T/I A
   (a) Compare Earth's electric field and gravitational field in terms of the fields' direction and shape, and how they change as the altitude increases.
   (b) Calculate the mass of a particle that can be suspended by Earth's electric field if the particle has the elementary charge on it.
8. In a beam of sunlight coming through a window, you can often observe tiny dust particles floating in still air. Explain the cause of this effect and describe a procedure to test your answer. T/I C A

Investigation 7.2.1 OBSERVATIONAL STUDY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Coulomb's Law

The force between point electric charges can be determined using Coulomb's law. Coulomb's law states that the force between two point charges is inversely proportional to the square of the distance between the charges and directly proportional to the product of the charges.

In this investigation, you will examine the nature of forces between charges.

Purpose

SKILLS HANDBOOK A2.4

Part A: To examine the force between charges at varying distances from each other

Part B: To investigate the force between charges at different distances or different charges at one distance

Equipment and Materials

- ruler
- calculator
- computer with Internet access
- computer with graphing software (optional)
- graph paper

Procedure: Part A

1. **Table 1** shows data for the force between two equally charged particles separated by a distance *r*. Using the data in Table 1, graph force versus $\frac{1}{r^2}$.
2. Draw a line of best fit for the plotted data, and determine the slope of the line.

Procedure: Part B

3. Go to the Nelson Science website and locate the simulation for this investigation.
4. Design an experiment that will allow you to test Coulomb's law by varying the two test charges while holding their distance fixed, or by varying the distance while holding the charges fixed.

WEB LINK

Analyze and Evaluate: Part A

(a) Based on the slope of the line you created in Step 2, determine the charge of each particle. Use the value of Coulomb's constant as part of this calculation. T/I

Table 1 Experimental Data

r (m)	F_E (N)
0.1×10^{-6}	9.21×10^{-14}
0.2×10^{-6}	2.30×10^{-14}
0.3×10^{-6}	1.02×10^{-14}
0.4×10^{-6}	5.75×10^{-15}
0.5×10^{-6}	3.68×10^{-15}
0.6×10^{-6}	2.56×10^{-15}
0.7×10^{-6}	1.88×10^{-15}
0.8×10^{-6}	1.44×10^{-15}
0.9×10^{-6}	1.14×10^{-15}
1.0×10^{-6}	9.21×10^{-16}
1.1×10^{-6}	7.61×10^{-16}
1.2×10^{-6}	6.39×10^{-16}
1.3×10^{-6}	5.45×10^{-16}
1.4×10^{-6}	4.70×10^{-16}
1.5×10^{-6}	4.09×10^{-16}
1.6×10^{-6}	3.60×10^{-16}
1.7×10^{-6}	3.19×10^{-16}
1.8×10^{-6}	2.84×10^{-16}
1.9×10^{-6}	2.55×10^{-16}
2.0×10^{-6}	2.30×10^{-16}

Analyze and Evaluate: Part B

(b) Explain whether or not your results are consistent with Coulomb's law. If you note any discrepancy, explain the possible reasons for it. C

Apply and Extend

(c) Describe advantages and disadvantages of using the computer simulation to test a scientific law. How might the advantages that you described be applied to scientific research? K/U C A

(d) Design a real-world experiment to test Coulomb's law. T/I A

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

The Millikan Experiment

Millikan was able to determine the charge of an electron without actually observing one. Physicists use models to demonstrate physics principles and to explore experiments that are impossible or too expensive to run in the real world. In Part A of this investigation, you will model the Millikan oil drop experiment. In Part B, you will use given data to test your predictions from Part A.

Purpose

SKILLS HANDBOOK A2.4

Part A: To model Millikan's oil drop experiment

Part B: To use given data to simulate Millikan's experiment and test your results from Part A

Equipment and Materials

- 10 opaque bags with varying numbers of marbles
- digital scale
- graph paper or graphing software

Procedure

Part A

1. Read through the Procedure and create a data table in which to record your observations.
2. Measure the mass of each bag. (Remember to subtract the mass of the bag to obtain the mass of the marbles.)
3. Calculate the differences between the masses of the bags, and record the differences in your table.
4. Calculate the smallest (non-zero) difference.
5. Divide each of the masses by the smallest difference. If any differences are not whole numbers, round the difference to obtain a whole number. You will call this the "whole difference." Record the data in your table.
6. Graph the mass differences against the numbers calculated in Step 5.
7. Draw a line of best fit for the points on your graph, and determine the slope of the line.
8. Formulate predictions about the mass of a single marble. If each marble represents a single electron, make a prediction about the charge on a single electron.

Part B

9. Use the data in **Table 1** to determine the charge of each oil drop. Assume a plate separation distance of 0.50 cm. Record your observations in your notebook.

Observations

Table 1 Experimental Data

Mass of oil drop (kg)	Electric potential difference (V)	Charge (C)
1.9×10^{-15}	290.9	
2.1×10^{-15}	160.8	
2.2×10^{-15}	673.8	
2.3×10^{-15}	176.1	
2.4×10^{-15}	147.0	
2.5×10^{-15}	382.8	
2.8×10^{-15}	428.8	
3.1×10^{-15}	237.3	
3.2×10^{-15}	140.0	
3.4×10^{-15}	173.5	
3.5×10^{-15}	214.4	
3.7×10^{-15}	566.6	
3.9×10^{-15}	597.2	
4.2×10^{-15}	214.4	
4.3×10^{-15}	263.4	

Analyze and Evaluate

(a) Using your data from Part A, determine the mass of a single marble. T/I

(b) Determine the elementary charge of an electron using the data in Table 1. K/U T/I

(c) How are the methods used in Part A to model the Millikan oil drop experiment similar to the analysis you did in Part B? What are the differences? What are the limitations of the model? A

(d) What must be true about individual elementary charges for your method of determining their values to be valid? A

(e) Why must a large number of values be used to get a reliable answer? What error might result if only a few oil drops were used, or if all the oil drops contained an even number of charges? A

Apply and Extend

(f) Millikan's experiment did not determine the precise value of the elementary charge. Explain why this is the case. List possible sources of error for this type of investigation. T/I A

CHAPTER 7 SUMMARY

Summary Questions

1. Create a concept map for this chapter based on the Key Concepts on page 318. Show how electric charge, electric potential, electric fields, and electric force relate to each other, and describe the properties of each. Provide further information, relevant examples, explanatory diagrams, or general equations under each of these concepts.

2. Look back at the Starting Points questions on page 318. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers you wrote at the beginning of the chapter. Note how your answers have changed.

Vocabulary

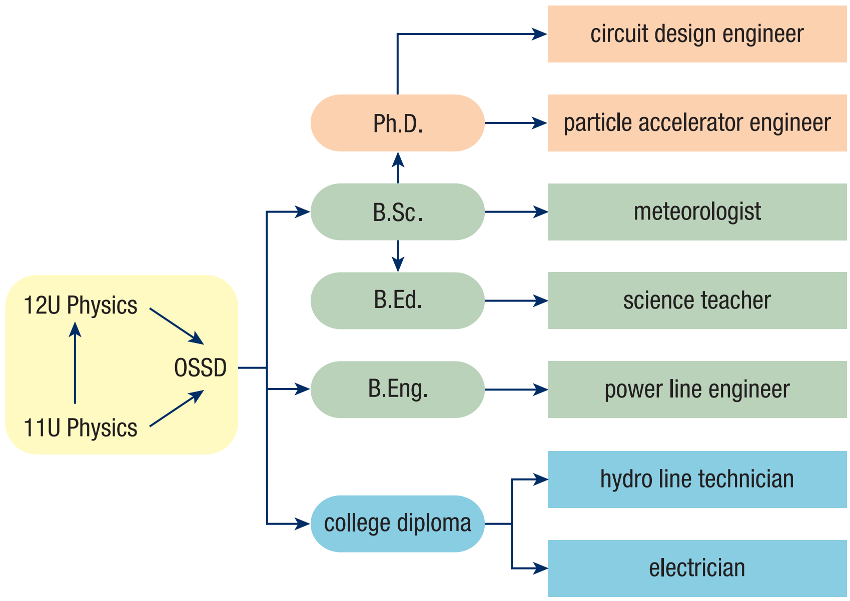
law of electric charges (p. 320)
law of conservation of charge (p. 321)
coulomb (p. 321)
conductor (p. 321)
insulator (p. 321)
electric force (p. 327)
Coulomb's law (p. 327)
Coulomb's constant (p. 327)
superposition principle (p. 329)
electric field (p. 334)
electric field lines (p. 338)
electric dipole (p. 339)
electric potential energy (p. 347)
electric potential (p. 350)
electric potential difference (p. 350)
electric potential due to a point charge (p. 355)
fundamental physical constant (p. 362)
elementary charge (p. 362)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers related to topics covered in this chapter.

1. Select two careers related to electric fields that you find interesting. Research the educational pathways that you would need to follow to pursue these careers. What is involved in the required educational programs? Prepare a brief report of your findings.
2. For one of the two careers that you chose above, describe the career, main duties and responsibilities, working conditions, and setting. Also outline how the career benefits society and the environment.



CAREER LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. The SI unit of electrostatic charge is the
   (a) electron
   (b) ion
   (c) coulomb
   (d) newton (7.1) K/U
2. A negatively charged plastic rod comes close to, but does not touch, a neutral metal sphere. While the rod is still close, someone momentarily touches the opposite side of the sphere with her finger. The charge on the metal sphere is
   (a) negative
   (b) positive
   (c) neutral
   (d) cannot be determined (7.1) K/U
3. When a positively charged metal sphere comes in contact with an electrical ground,
   (a) protons flow from the object into the ground
   (b) electrons flow from the object into the ground
   (c) protons flow from the ground into the object
   (d) electrons flow from the ground into the object (7.1) K/U
4. Two charged metal spheres, separated by 20 cm, attract each other with a 12 N force. Determine the resulting force when the charge on each sphere is reduced by half. (7.2) T/I
   (a) 3 N
   (b) 6 N
   (c) 24 N
   (d) 48 N
5. Two plastic spheres are placed on a number line. Both spheres have a negative charge of equal magnitude. Determine the region where the net electric field is zero. (7.2) K/U
   (a) to right of the rightmost sphere
   (b) to the left of the leftmost sphere
   (c) in between the spheres on the number line
   (d) below the spheres
6. The electric field at a distance of 1.0 m from a charged sphere is 100 N/C. At what distance from the sphere will the electric field be 50 N/C? (7.3) T/I
   (a) 1.1 m
   (b) 1.4 m
   (c) 2.0 m
   (d) 4.0 m
7. How much kinetic energy is obtained by a 5 C charge as it moves from a position of 12 V electric potential to a position of 4 V? (7.4) T/I
   (a) 20 J
   (b) 30 J
   (c) 40 J
   (d) 60 J
8. A particle of dust obtains a charge of -4.8×10^{-19} C. Determine how many excess electrons the dust obtained. (7.6) T/I
   (a) 2
   (b) 3
   (c) 4
   (d) 5

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

9. Two identical balloons are each rubbed with a wool cloth. When held close together, the balloons will repel each other. (7.1) K/U
10. Electrons and protons are attracted to each other because they have the same magnitude of electric charge. (7.1) K/U
11. For large objects, such as moons and planets, electrostatic forces are stronger than gravitational forces. (7.2) K/U
12. Liquid crystal displays (LCDs) use electric fields to display pictures and messages on cellphones. (7.3) K/U
13. An electrostatic precipitator first imparts a negative charge to a dust or a flue gas particle. Then the precipitator captures that particle through electrostatic attraction. (7.3) A
14. The amount of work done by the electric field in moving a charged object from point A to point B within the field does not depend on the path taken between the points. (7.4) K/U
15. A negative electric charge is moved from infinity to a point some distance from a positive charge. The electric potential will increase. (7.5) K/U
16. Every subatomic particle detected so far by researchers has an electric charge equal to a positive or negative whole-number multiple of the fundamental charge *e*. (7.6) K/U

Go to Nelson Science for an online self-quiz.
WEB LINK

Knowledge

For each question, select the best answer from the four alternatives.

1. The main difference between an insulator and a conductor is that
   (a) an insulator cannot obtain a net electric charge
   (b) an insulator will not allow charges to flow freely
   (c) a conductor will obtain only net negative charges
   (d) a conductor will not emit an electric field (7.1) K/U
2. Objects A and B attract each other. Objects B and C attract each other. Objects A and C will most likely
   (a) attract each other
   (b) repel each other
   (c) not interact with each other
   (d) first repel, then attract each other (7.1) T/I
3. Which one of the following magnitudes is *not* a possible magnitude of charge for a real-world particle to obtain? (7.1) T/I
   (a) 0.9×10^{-19} C
   (b) 1.6×10^{-19} C
   (c) 3.2×10^{-19} C
   (d) 4.8×10^{-19} C
4. When a woollen carpet is rubbed with rubber shoes, the woollen carpet
   (a) acquires positive charge
   (b) acquires negative charge
   (c) becomes grounded
   (d) remains neutral (7.1) K/U
5. A student negatively charges a rubber balloon, and then leaves the balloon hanging by a string in the classroom overnight. Select the most likely explanation why the balloon does not display an electric charge when the student returns the following morning. (7.1) K/U
   (a) The electrons were absorbed by the balloon's protons.
   (b) Someone must have grounded the balloon in the middle of the night.
   (c) Stray ions in the surrounding air absorbed the excess electrons that were on the balloon.
   (d) The balloon's excess electrons lost their charge overnight.
6. The individual chloride ion (Cl^-) has one more electron than it has protons. Determine the magnitude of charge on this chloride ion. (7.1) K/U
   (a) 1.0 C
   (b) -1.0 C
   (c) 1.6×10^{-19} C
   (d) -1.6×10^{-19} C
7. After successfully charging a plastic rod, a student tries to charge a metal rod while holding it and rubbing it with rabbit fur. Select the statement that best predicts what will happen. (7.1) K/U
   (a) Any excess charge on the metal rod will flow into the student's body.
   (b) The metal rod will hold a charge similar to the way the plastic rod held a charge.
   (c) The metal will take on the same charge as the rabbit fur.
   (d) The metal rod will polarize when wrapped in rabbit fur.
8. To determine the magnitude and direction of the electrostatic force between two charged particles, one must know
   (a) the sign of each charged particle
   (b) the distance between the particles
   (c) the magnitude of charge on each particle
   (d) the sign of the charge on each particle, the magnitude of the charges, and the distance between the particles (7.2) K/U
9. One correct comparison between the electrostatic force and the gravitational force is that
   (a) the electrostatic force increases with increasing distance
   (b) the electrostatic force is weaker than gravity on the atomic scale
   (c) the gravitational force diminishes with the inverse cube of distance
   (d) the gravitational force is only attractive (7.2) K/U
10. Two small charged spheres a distance r apart exert an electrostatic force on each other of magnitude F. What is the new force when the distance between the spheres is reduced to $\frac{r}{3}$? (7.2) T/I
    (a) $9F$
    (b) $3F$
    (c) $\frac{F}{3}$
    (d) $\frac{F}{9}$
11. Two positively charged particles attract each other with a force of 20 N. Determine the resulting force when the distance between the particles is doubled. (7.2) T/I
    (a) 5 N
    (b) 10 N
    (c) 40 N
    (d) 80 N

12. Determine the electrostatic force on a 0.06 C charged object when it is placed in an electric field of magnitude 1500 N/C. (7.3) T/I
    (a) 90 N
    (b) 1950 N
    (c) 4500 N
    (d) 25 000 N
13. Which of the following makes use of electric fields? (7.3) K/U
    (a) an LCD screen showing a movie
    (b) an electrostatic precipitator removing pollutants from the air
    (c) a shark hunting for prey beneath the sand
    (d) All of the above examples make use of electric fields.
14. An electron moving at high speed toward the right enters a uniform electric field directed straight upward. What is the resulting path of the electron while it is in the electric field? (7.4) T/I
    (a) a parabola curving downward
    (b) a parabola curving upward
    (c) a diagonal line pointing down and to the right
    (d) a diagonal line pointing up and to the right
15. A small, positively charged sphere is released from rest and moves directly away from a larger, positively charged sphere. During this process, the electrostatic force
    (a) does positive work and increases the kinetic energy of the small sphere
    (b) does negative work and increases the kinetic energy of the small sphere
    (c) does positive work and decreases the kinetic energy of the small sphere
    (d) does negative work and decreases the kinetic energy of the small sphere (7.4) K/U
16. The electric potential due to a point charge at a point depends on
    (a) the direction of the electric field
    (b) the distance from the point charge
    (c) the velocity of the point charge
    (d) the mass of the point charge (7.5) K/U
17. The charge of an electron compared to the charge of a proton is
    (a) equal in sign and equal in magnitude
    (b) different in sign but equal in magnitude
    (c) equal in sign but different in magnitude
    (d) different in sign and different in magnitude (7.6) K/U

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

18. Only insulators can hold a net positive charge. (7.1) K/U
19. A balloon will attract a thin stream of water only if the balloon is positively charged. (7.1) T/I
20. You can increase the conductivity of a glass of water by stirring in table salt. (7.1) K/U
21. A positively charged particle will create an electric field that repels all other charged particles. (7.1) K/U
22. In a large object, such as a person, the total number of protons is essentially equal to the total number of electrons. (7.1) K/U
23. The total charge on an object depends only on the sum of the individual charges of the particles in the object. (7.1) K/U
24. Grounding occurs whenever any charge imbalance is cancelled out. (7.1) K/U
25. When a metal plate is given a negative charge, the excess electrons accumulate in the centre of the plate. (7.1) K/U
26. In Coulomb's law, the denominator r represents the radius of the larger charge. (7.2) K/U
27. The net force on a charged object surrounded by other charged objects can be calculated through vector addition. (7.2) K/U
28. The value of the proportionality constant k in Coulomb's law depends upon the distance between charges. (7.2) K/U
29. The magnitude of the force on charge A due to charges B and C equals the magnitude of the force due to just B plus the magnitude of the force due to just C. (7.2) K/U
30. An electric dipole consists of two point particles with equal but opposite charge located a small distance apart. (7.3) K/U
31. It is impossible for an electric field to exist in empty space. (7.3) K/U
32. The density of electric field lines is proportional to the magnitude of the electric field. (7.3) K/U
33. At least two charges are necessary to create an electric field. (7.3) K/U
34. The electric field inside any conductor is always zero. (Hint: Think of applying symmetry.) (7.3) K/U
35. An electron will experience a force in the same direction as an electric field line. (7.3) K/U
36. On average, Earth has an electric field of magnitude 120 N/C that is directed toward the ground. (7.3) K/U
37. The electric field between two parallel plates is stronger near the positive plate. (7.3) K/U

38. Volts per metre and newtons per coulomb are two ways to measure electric potential. (7.4) K/U
39. If the electric field at a certain point is zero, then the electric potential at that same point is also zero. (7.4) K/U
40. The electric field is a vector quantity; the electric potential is a scalar quantity. (7.4) K/U
41. The electric potential due to a point charge is directly proportional to the distance from the charge and inversely proportional to the amount of charge. (7.5) K/U
42. Millikan measured the charge on an electron by suspending individual electrons with an electric field. (7.6) K/U
43. Every subatomic particle so far detected has a charge whose magnitude is equal to a whole-number multiple of the elementary charge. (7.6) K/U

Understanding

44. Explain why a person's hair sometimes stands on end when the person's hand is on a continuously charged conductor, such as a Van de Graaff generator. (7.1) K/U
45. Explain why excess charge can never stay on an insulator. (7.1) K/U
46. Explain how you can use a negatively charged plastic rod to place a positive charge on an isolated metal sphere. (7.1) K/U
47. A student rubs a balloon against her hair. She then places the side of the balloon that is rubbed against a wall, and the balloon sticks to the wall. (7.1) K/U A
    (a) Explain why the balloon sticks to the wall.
    (b) If the balloon is turned around so the other side touches the wall, will it still stick to the wall? Justify your answer.
48. Describe the similarities and differences between electrostatic and gravitational forces. (7.2) K/U
49. Verify that the unit V/m is equivalent to the unit N/C. (7.3) T/I
50. Compare the electric potential experienced by a 5 C charge and a 50 C charge when they are placed (one at a time) at 1.0 m from a −1 C point charge. (7.4) K/U
51. A 1.0 C charge moves in a surrounding electric field. Determine the work needed to move the charge along a path perpendicular to the electric field lines. Calculate the change in electric potential along such a path. (7.4) K/U T/I
52. Calculate the electric potential at 0.25 m from a 1.2 C point charge. (7.5) K/U
53. Summarize the operation of a Van de Graaff generator. (7.5) K/U C
54. Identify the different forces that play a part in Millikan's oil drop experiment and the role each force plays. (7.6) K/U C

Analysis and Application

55. An oxygen ion consists of 8 protons, 8 neutrons, and 10 electrons. Determine the net charge on this oxygen ion in coulombs. (7.1) T/I A
56. A graphite-coated pith ball with a charge of −6.8 nC contacts an identical uncharged pith ball such that half of the charge gets transferred to the uncharged pith ball. Determine the new charge on each pith ball after contact is made. (7.1) T/I A
57. Two identical metal spheres sit on top of insulating stands. Sphere A has a charge of +6 μC. Sphere B has a charge of −10 μC. The spheres then come into contact with each other, giving each sphere an equal charge (**Figure 1**). (7.1) T/I

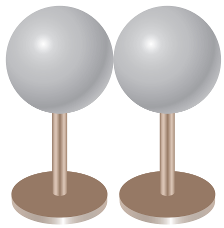


Figure 1

    (a) Determine the charge on each sphere after contact is made.
    (b) Sphere B is then touched to the ground. Determine the final charge on sphere B.
58. Two oppositely charged particles are separated at increasing distances along a number line. Draw a graph of the electrostatic force versus the separation distance. (7.2) T/I C A
59. Two metal spheres are separated a distance of 25 cm. Each sphere has a charge of $+2.3 \times 10^{-10}$ C. Determine the force between the spheres. (7.2) T/I
60. Two particles, each with charge +4.2 μC, exhibit a repulsive force of 0.25 N. Calculate the distance that separates the particles. (7.2) T/I A
61. Two charged particles separated by 60 cm attract each other with a force of 1.8 N. One particle has a charge of −83 μC. What are the sign and magnitude of the other charge? (7.2) T/I A

62. Ernest Rutherford used alpha particles (charge $+2e$) and thin gold foil to probe the structure of the atom. Given that the nucleus of a gold atom has 79 protons, calculate the electrostatic repulsive force between the alpha particle and the gold nucleus when the separation distance between the two is 6.2×10^{-14} m. (7.2) T/I A

63. Three charged particles are situated on the x-axis. A charge of q is fixed at the origin, a charge of $3q$ is fixed at $x = 12$ cm, and a third charge, $+q$, is placed between q and $3q$. Determine the position at which $+q$ experiences a net force of zero. (7.2) T/I A

64. Three identical charged particles sit at the corners of an equilateral triangle of side length 30 cm (**Figure 2**). Each particle has a charge of -8.5 μC. Determine the magnitude and direction of the net force on each particle. (7.2) T/I A

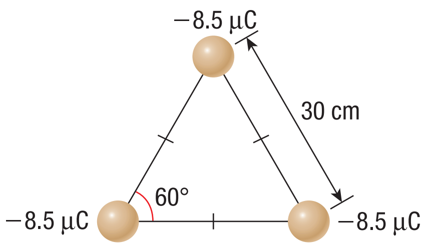


Figure 2

65. Three charged particles sit along the x-axis: $q_1 = +41$ nC at $x = 0.0$ cm, $q_2 = -19$ nC at $x = 6.0$ cm, and $q_3 = -28$ nC at $x = -4.0$ cm. Calculate the magnitude and direction of the net force on q_1, q_2, and q_3. (7.2) T/I A

66. Explain why many tall buildings have a sharp piece of copper sticking up from the highest point on their roof. (7.3) K/U A

67. Two charged particles of $+76$ μC are fixed to opposite corners of a square with side length 15 cm. A third charged particle of -24 μC is fixed to one of the other corners of the square. Determine the magnitude of the electric field at the empty corner of the square. (7.3) T/I A

68. Determine the electric field 0.040 m away from a particle with a charge of 1.2×10^{-8} C. (7.3) T/I A

69. A small metal sphere has charge -56 nC. Calculate the distance from the centre of the sphere where the electric field is
    (a) 3000 N/C
    (b) 1500 N/C
    (c) 400 N/C (7.3) T/I

70. Two charges are fixed on the x-axis. Charge q_1 is $+4.0$ μC and is fixed at $x = 0.0$ cm. Charge q_2 is fixed at $x = 18$ cm, but the value of its charge is unknown. Measurements indicate that the electric field is zero at $x = 36$ cm. Determine the magnitude and sign of charge q_2. (7.3) T/I A

71. Determine the electrostatic force on a particle that has 1.8 μC of charge and is in an electric field of magnitude 6700 N/C. (7.3) T/I

72. A small charged particle experiences 0.063 N of force when placed in an electric field of magnitude 4200 N/C. (7.3) T/I A
    (a) Calculate the magnitude of charge on the particle in microcoulombs.
    (b) If the force is opposite the direction of the electric field, what is the sign of the charge?

73. A soot particle enters an electrostatic precipitator and experiences 2.3×10^{-5} N of force pulling it toward an accumulator plate. The soot particle has a charge of 4.5×10^{-11} C. Determine the electric field strength of the precipitator. (7.3) T/I

74. A nitrogen ion with a mass of 2.4×10^{-26} kg has a $-2e$ charge and is situated in an atmospheric electric field of 200.0 N/C. Calculate the magnitude of the initial acceleration of the ion. (7.4) T/I A

75. A particle with charge 0.6 C requires 3.0 J of work to move it from point A to point B in an electric field. Determine the electrical potential difference between points A and B. (7.4) T/I A

76. A particle of unknown charge obtains 0.042 J of kinetic energy as it moves from point A to point B. Point A has an electric potential of 700.0 V; and point B has an electric potential of 200.0 V. Determine the magnitude and sign of the charge. (7.4) T/I A

77. A particle accelerator uses electric fields to increase the energy of charged particles. Consider a proton with a mass of 1.67×10^{-27} kg that starts at rest and crosses an electrical potential difference of 5.3×10^5 V. (7.4) T/I A
    (a) Calculate the kinetic energy gained by the proton.
    (b) Calculate the final speed of the proton.

78. An electron with a mass of 9.11×10^{-31} kg is accelerated from rest across a set of parallel plates that have a potential difference of 150 V and are separated by 0.80 cm. (7.4) T/I A
    (a) Determine the kinetic energy of the electron after it crosses between the plates.
    (b) Determine the final speed of the electron.
    (c) Determine the acceleration of the electron while it is between the plates.
    (d) Determine the time required for the electron to travel across the plates.

79. A spark will jump between two conductors separated by air when the electric field between the two conductors is approximately 3.0×10^4 V/cm. This is known as the dielectric breakdown, and the exact value can vary depending on the conditions of the air. Determine the potential difference between two conductors in the following situations. (7.4) T/I A
   (a) Two students send a 6 mm spark between their fingertips.
   (b) A Van de Graaff generator and a spherical grounding wand receive a 12 cm spark.
   (c) A thundercloud and the top of a mountain receive a 950 m lightning bolt.
80. Two parallel plates are separated by 4.9 cm and have a potential difference of 85 V. Calculate the electric field between the plates. (7.4) T/I
81. A scientist needs to create an electric field of 5400 N/C and has a set of parallel plates that are 2.5 cm apart. Determine the potential difference to which the plates should be set so they reach the desired electric field strength. (7.4) T/I A
82. Determine the point charge for which the electric potential at 12 cm from it is 98 V. (7.4) T/I A
83. Determine the work done on a 1.0 C charge as it moves in a circle of radius 85 cm around a −1.0 C charge. (7.5) T/I A
84. Two protons, each of charge $+1.6 \times 10^{-19}$ C, are kept 3.0×10^{-15} m apart. Determine their mutual potential energy. (7.5) T/I A
85. A 1.3×10^{-6} C point charge is moved from 1.4 m to 0.45 m away from a 3.3×10^{-3} C charged sphere. How much work was required to move the point charge? (7.5) K/U T/I
86. A small conducting sphere has an excess of 1.5×10^{12} electrons. Determine the electric potential and the magnitude of the electric field at a point 1.2 m from the sphere. (7.5) K/U T/I
87. A metal-coated table tennis ball with a mass of 2.1 g is suspended by an insulating thread. A similar table tennis ball touches the first, giving each ball an equal negative charge. When the two balls are separated horizontally by 5.0 cm, the string hangs at 15° with respect to the vertical (**Figure 3**). (7.2, 7.6) T/I C A

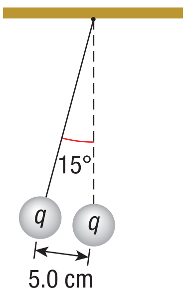


Figure 3

   (a) Draw an FBD showing the forces acting on the table tennis ball that is hung by the thread.
   (b) Calculate the charge on each table tennis ball in coulombs.
   (c) Determine approximately how many excess electrons are on each table tennis ball.
88. An oil droplet is sprayed into a uniform electric field of adjustable magnitude. The 0.04 g droplet hovers motionless (gravity force equalling electrostatic force) when the electric field is set to 370 N/C and directed downward. (7.6) T/I A
   (a) Determine the sign and magnitude of the charge on the oil droplet.
   (b) Determine the approximate number of excess electrons that are on the oil droplet.
89. A plastic rod is rubbed with fur and obtains a net charge of −0.50 μC. Determine the number of excess electrons on the rod. (7.6) T/I A
90. Determine the magnitude of charge (expressed in nanocoulombs) of 7.6×10^9 electrons. (7.6) T/I A

Evaluation

91. Design an investigation to determine the average charge, in coulombs, held on a small rubber balloon after the balloon has been rubbed with another type of material. What materials would be best to use? What results do you predict? What factors might affect those results? (7.2) T/I A
92. A charged particle is positioned at the origin on a number grid. Voltage readings are taken along the x-axis at 1 cm intervals. From $x = 1$ cm to $x = 10$ cm, the voltages are measured as follows: 800 V, 400 V, 267 V, 200 V, 160 V, 133 V, 114 V, 100 V, 89 V, 80 V. (7.4) T/I C
   (a) Graph the voltage versus distance for all the points given.
   (b) Approximate the electric field (in terms of volts per metre) at each of the following positions: $x = 2.5$ cm, $x = 5.5$ cm, and $x = 8.5$ cm.
   (c) Describe a real-world situation in which information on electric field strength is important. How would the graph need to be modified to fit the situation?

93. Two parallel plates are brought close together to make a uniform electric field pointing downward. The electric potential difference between the plates is 2400 V, with the top plate positive and the bottom plate negative. A positively charged particle, $+q$, is free to move between the plates. Point A is near the top positive plate. Point B is exactly halfway between the plates. Point C is near the bottom negative plate. (7.4) T/I C A
    (a) Construct a diagram of the situation described.
    (b) At what point does $+q$ have the greatest electric potential energy? Mark the point on your sketch.
    (c) At what point does $+q$ have the least electric potential energy? Mark the point on your sketch.
    (d) At what point(s) is the force on $+q$ the greatest? Mark the point on your sketch.
    (e) Does your diagram adequately illustrate the concept described? What other media could you use to demonstrate the concept more effectively?
94. A charged particle $+q$ is located at $x = 10$ cm, and a charged particle $-q$ is located at $x = -10$ cm. A charge $-q$ is initially placed on the y-axis at $y = 10$ cm (**Figure 4**). (7.5) T/I C A

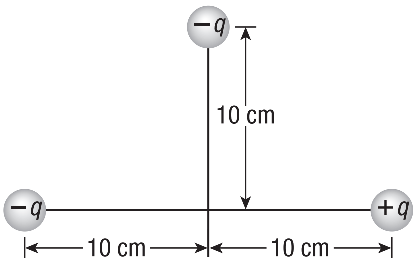


Figure 4

    (a) Draw and label a pathway for $-q$ that would increase its electric potential energy.
    (b) Draw and label a pathway for $-q$ that would decrease its electric potential energy.
    (c) Draw and label a pathway for $-q$ that would not change its electric potential energy.
    (d) Design a three-dimensional model that you could use to illustrate this concept to a fellow student.
95. Millikan assumed that the smallest charge value of any oil drop he had measured was the smallest possible charge value of any object. In this way, he deduced the charge on an electron without directly measuring the charge. Criticize or defend Millikan's method. (7.6) K/U T/I C A

Reflect on Your Learning

96. What did you find most interesting in this chapter? Summarize what you learned in this chapter in a format of your choosing, such as a graphic organizer, a poster, a pamphlet, or an electronic presentation. K/U C
97. How would you explain the idea of the electric field to a friend who has not taken physics? K/U C
98. Consider the different topics you have studied in this chapter. Choose one that you feel has an important impact on your life. Write a one-page report about the topic and why it is important to you. What else would you like to know about this topic? How could you go about learning this? K/U C A

Research

WEB LINK

99. Research the frequency and causes of a lightning strike. Describe the charge separation in a thundercloud. Identify safety measures to practise regarding lightning. T/I C A
100. Research how sharks use electric fields to hunt for prey. T/I C A
    (a) Explain why electroreception is better suited to sea creatures than to land creatures.
    (b) Define the term *ampullae*.
    (c) Analyze scientists' methods for measuring the sensitivity of sharks' electroreception.
101. Atmospheric scientists divide Earth's atmosphere into five layers with varying electrical properties. Identify the various layers of the atmosphere and tabulate their electrical properties. Identify a technological system that makes use of the particular electrical properties of one of the atmosphere's layers, and analyze the system's operation. Summarize your findings in a one-page report. K/U T/I C A
102. High-voltage lines transmit needed power from energy providers to homes and businesses, but exposure to the electric fields from power lines may have undesired effects. Research and assess the effects of exposure to high-voltage lines on human health. C A
103. The electrocardiogram (EKG or ECG) uses electric potential sensors to take a picture of the heart. Research ECG technology and analyze its operation. How does the ECG monitor the heart's electrical activity without direct contact? K/U C A
104. Research photocopiers, and analyze how a photocopier uses electrostatics to create images on paper. Explain the purpose of the selenium drum. Describe how the toner particles position themselves correctly on the page. T/I C A

CHAPTER 8

Magnetic Fields

KEY CONCEPTS

After completing this chapter you will be able to

- describe the properties of magnetic fields
- analyze the operation of technologies that use magnetic fields and assess the social and environmental impact
- solve problems related to magnetic force on a moving charge and a current-carrying conductor
- solve problems related to circular motion and a moving charge in a magnetic field
- conduct a laboratory inquiry or use a computer simulation to examine the behaviour of a particle in a field

What Effect Does a Uniform Magnetic Field Have on the Motion of a Moving Charge?

Magnetic fields literally surround us. Our planet is one large magnet, complete with a north and a south magnetic pole. We use magnetism in many aspects of our lives. Without magnetism we could not start our cars, talk on cellphones, run computer programs, or listen to music through our headphones. Even more important, we might not be able to diagnose some serious medical conditions.

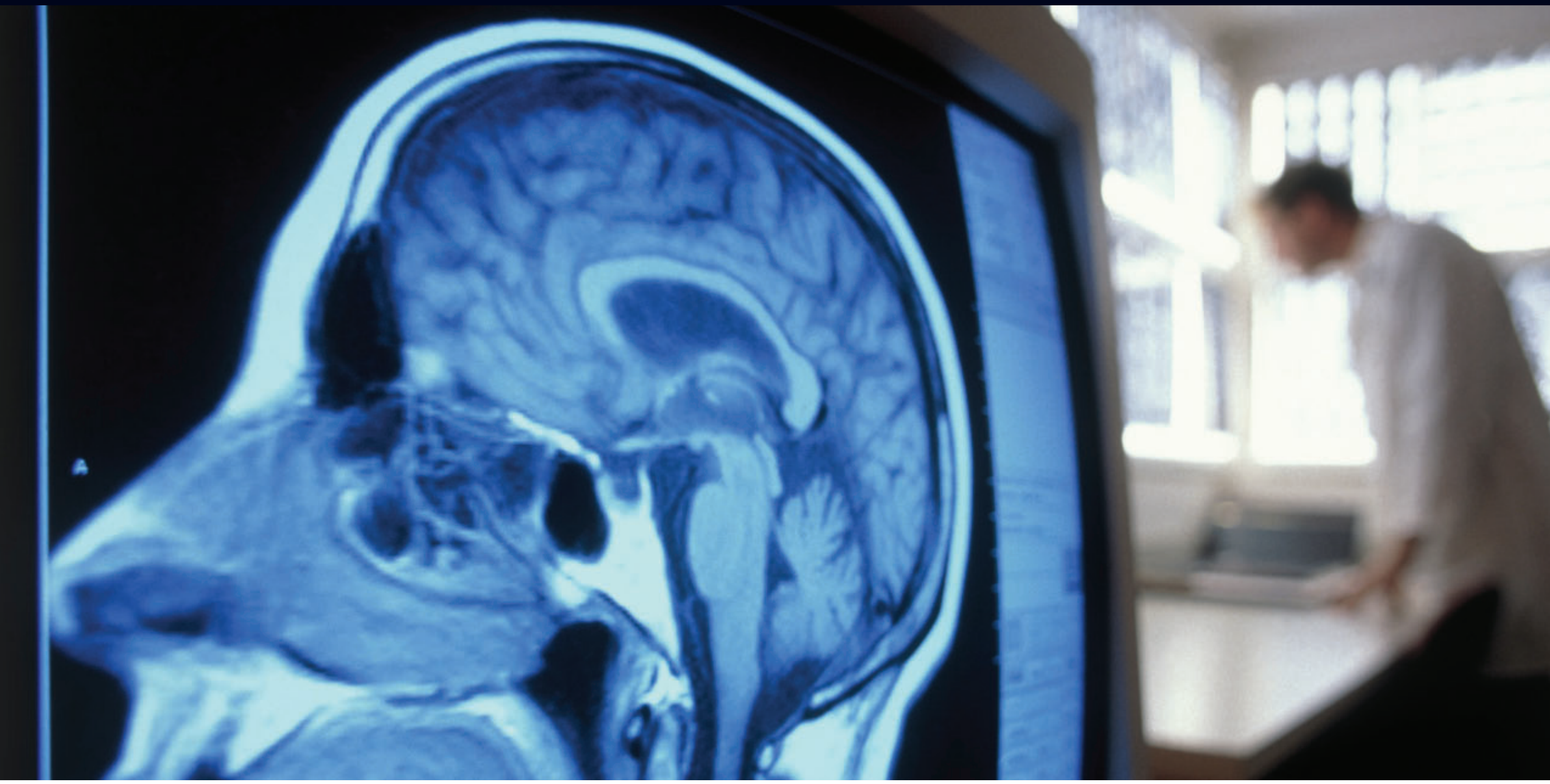
Magnetic resonance imaging (MRI) technology uses superconducting magnets to create a magnetic field around the body. This field is typically more than 100 000 times as strong as Earth's magnetic field. The magnetic field applied by the MRI alters the magnetization of the body's atoms as the MRI unit scans soft tissues in the brain, heart, eyes, and other organs. Doctors then use the properties of magnetic fields to detect heart disease, tumours, internal bleeding, and other problems.

To use MRI technology, we need to understand properties of magnetic fields and how they interact with other magnetic fields and electric currents. This knowledge helps us use the magnetic force to our advantage. In this chapter, you will learn how understanding the properties of magnetic fields can lead to innovations in medical technology and other areas. You will also learn how to solve problems related to magnetic force and electric charges.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. Describe what you know about magnetic fields.
2. Explain what it means when an object is magnetic.
3. How is a magnetic field created inside an MRI machine?



Mini Investigation

Building a Magnetic Dipping Needle

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

A magnetic compass rotates horizontally toward Earth's magnetic north pole. Earth's magnetic field is not perfectly horizontal or parallel to Earth's surface. We can use a magnetic dipping needle (or magnetic inclinometer) that rotates vertically to observe the angle between Earth's horizontal surface and the orientation of Earth's magnetic field. In this investigation, you will make your own magnetic dipping needle and use it to observe the magnetic inclination, or magnetic dip, at your location.

Equipment and Materials: bar magnet; compass; 10 cm piece of wire; string; sewing needle; 1 cm^3 foam block; toothpick

1. Bend the wire into a stirrup, as shown in **Figure 1**.

Figure 1

2. Tie the string to the stirrup, and suspend the stirrup so that it can freely rotate.
3. Push the sewing needle carefully into the foam block, and push the toothpick into the foam block perpendicular to the needle as shown in Figure 1. Push the needle only far enough so that it is balanced horizontally.
4. Magnetize the needle by rubbing it against the north pole of the bar magnet several times. Then place the foam block with the needle and toothpick onto the stirrup, and allow the system to settle into position.
5. Use the compass to identify the direction of Earth's north magnetic pole. Be sure to place the magnet used in Step 4 far away from where you are observing to eliminate any effects from the magnet.
6. Point the dipping needle toward north. Observe the angle between the horizontal and the needle.

A. Describe how the dipping needle works. K/U

B. Estimate the angle of the needle's inclination, or dip, relative to the ground. T/I

C. Explain how the observations of both compasses help you understand the nature of Earth's magnetic field. K/U

D. What orientation of the dipping needle do you think would make it align with a "magnetic equator"? A

8.1 Magnets and Electromagnets

You probably became familiar with magnets by playing with them when you were a child. You learned that magnets are attracted to some materials but not to others. In this section, you will learn about magnetic materials and how some of these materials deep inside Earth produce Earth's magnetic field. You will also learn how a magnetic field surrounds all magnets and how moving electric charges produce a magnetic field.

Auroras

Imagine going camping in northern Canada, and on your first night you look up and see bright greenish-white ribbons of light stretching across the night sky. The glowing lights, shown in **Figure 1**, are in motion, rippling up and down. What causes these lights to appear? Why do they move and swirl?

Figure 1 The aurora borealis flashes green light over the northern skyline.

The swirls of lights that you see in the sky are the aurora borealis. The same phenomenon in the southern hemisphere is called the aurora australis. One reason these auroras appear is because Earth exhibits the properties of a large magnet. A magnetic field surrounds Earth as if Earth were a bar magnet. Recall from Grade 11 that magnetic fields are strongest at the poles and that magnetic field lines fan out from Earth's south pole and converge at Earth's north pole. A stream of charged particles called the solar wind escapes the Sun's gravity and flows past Earth. Charged particles entering Earth's magnetic field travel in spiral paths along the magnetic field lines toward the poles, where they spiral down the field lines toward Earth's surface. In the upper atmosphere, the charged particles collide with oxygen and nitrogen atoms. These collisions energize the gas atoms. The atoms then release their extra energy as light that we see as the auroras. When you see the lights of the aurora borealis, you are really watching the interplay of electricity and magnetism. WEB LINK

Permanent Magnets

The first observations of magnetic fields involved materials that can easily be magnetized, called permanent magnets. Permanent magnets are used in numerous devices, such as compass needles, refrigerator magnets, speakers, and some motors. **Figure 2(a)** shows a permanent magnet made in the shape of a bar, with small iron filings sprinkled around the bar magnet. The magnetic field of the bar magnet, indicated by $\vec{B}$ in the figure, is not visible, but the iron filings follow imaginary lines, called **magnetic field lines**, corresponding to the magnetic field's strength and direction. **Figures 2(b)** and **(c)** show how the magnetic fields of two magnets interact when the magnets are close together. The poles of a magnet are analogous to positive and negative electric charges.

magnetic field line one of a set of lines drawn to indicate the strength and direction of a magnetic field

Just as opposite electric charges attract, opposite magnetic poles attract. The north pole (N) of one magnet attracts the south pole (S) of another magnet. The effect is different, however, when like poles approach each other. Two north poles repel each other, and two south poles repel each other. This is analogous to the way two positive electric charges or two negative electric charges repel each other, except that the magnetic poles are electrically neutral.

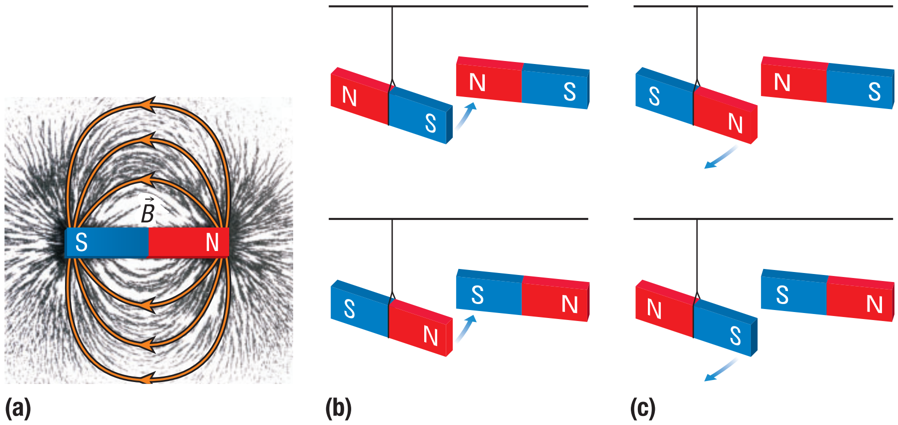


Figure 2 (a) The iron filings around the bar magnet show that the magnetic field $\vec{B}$ extends from the north pole of a magnet to the south pole. (b) The interaction of the magnetic fields of two magnets causes unlike poles to attract each other. (c) The interaction of the magnetic fields causes like poles to repel each other.

You can see in Figure 2(a) that the iron filings are crowded together near each pole. This crowding indicates that the magnetic field is strongest at the poles. Magnetic field lines move outward from the north pole of a magnet and inward toward the south pole. Opposite poles attract each other because the magnetic fields are oriented in opposite directions. Like poles repel each other because the magnetic fields are oriented in the same direction.

The attraction and repulsion of magnetic poles explain the alignment of the iron filings with the magnetic field lines of the bar magnet in Figure 2(a). Iron is a magnetic material; it produces a magnetic field in response to an applied magnetic field. Each magnetized iron filing has its own north and south poles. The north pole of each iron filing is attracted to the south pole of the large bar magnet and is repelled from the north pole. At the same time, the south pole of an iron filing is attracted to the north pole of the large bar magnet and is repelled from the south pole. These magnetic forces cause the iron filing to align parallel to the magnetic field lines and hence parallel to $\vec{B}$. This is similar to the situation with electric dipoles and electric fields (**Figure 3**), with an important difference. An electric field can result from a single charge. For example, the electric field of a single positive charge radiates outward from the charge. A magnetic field, however, will always result from a magnetic dipole. There will always be a north pole and a south pole producing the magnetic field. You can never have only a south pole or only a north pole.

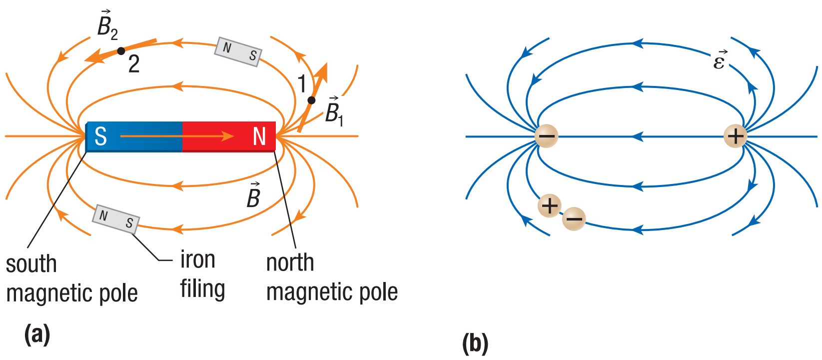


Figure 3 The magnetic field lines of a bar magnet are similar to the electric field lines of an electric dipole. (a) The iron filings align with $\vec{B}$. (b) The electric dipoles align with $\vec{\varepsilon}$.

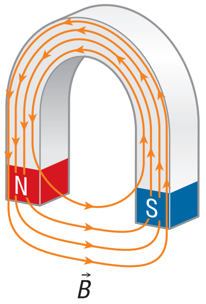


Figure 4 The magnetic field lines of a horseshoe magnet form closed loops pointing from the north pole to the south pole, just as they do with a bar magnet.

The magnetic field lines of a bar magnet extend from the north pole to the south pole outside the magnet and from south to north inside the magnet, forming a closed loop. Magnetic field lines always form closed loops. You can make a horseshoe magnet by simply bending a bar magnet (**Figure 4**). One end of the horseshoe magnet has a north pole, and the other end has a south pole. The field lines form closed loops as they circulate through the horseshoe. The field lines extend across the gap between the ends of the magnet so the direction of the field is from the north pole toward the south pole, just as it is with a bar magnet. The magnetic field strength is greatest in the gap between the poles. Magnets with horseshoe shapes have many applications, including in motors and generators.

Permanent magnets come in many shapes and sizes. Regardless of the shape of a magnet, the magnetic field lines form closed loops, the field lines point from the north pole toward the south pole, and the field is strongest at the poles.

Earth's Magnetic Field

The largest magnet on Earth is Earth itself. A compass needle is a small bar magnet mounted so that it can swivel freely about its centre (**Figure 5(a)**). Earth's magnetic field exerts a torque (twisting force) on this magnet. From experiments done with compass needles, William Gilbert, a sixteenth-century English scientist, reasoned that Earth acts as a very large, permanent magnet oriented as shown in **Figure 5(b)**.

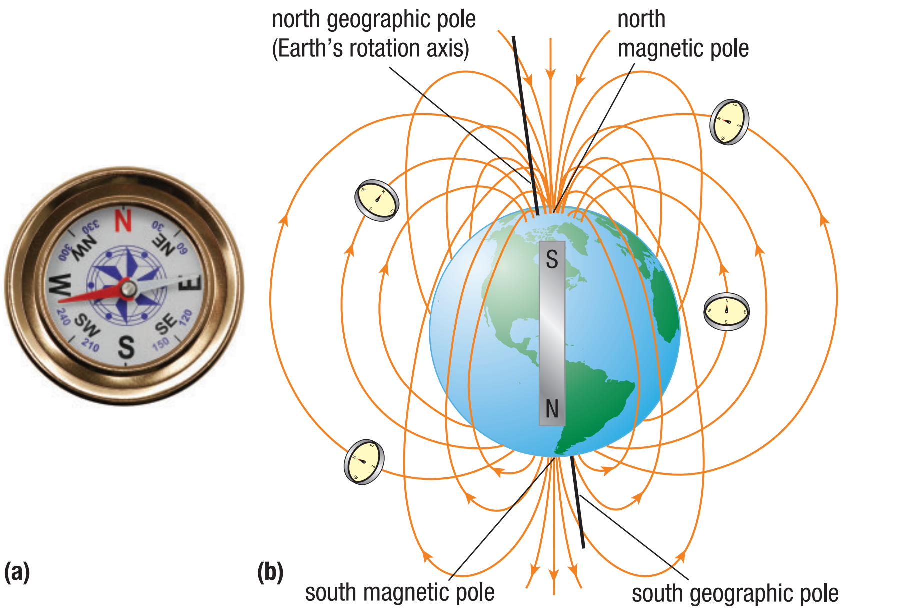


Figure 5 (a) A compass. (b) If we were to place compasses at different spots in Earth's magnetic field, each compass needle would be aligned parallel to the field.

Earth has two geographic poles, the north pole and the south pole, where Earth's axis of rotation meets Earth's surface. Every magnet, including Earth, has two magnetic poles, a north and a south pole. A compass needle aligns itself to point to Earth's magnetic poles. Earth's geographic poles are not exactly in the same location as its magnetic poles, but they are close enough that we can say that the north magnetic pole of a compass needle points approximately toward Earth's north geographic pole. This behaviour is why the poles of a bar magnet were given the names "north" and "south." Note: the north magnetic pole of one magnet (the compass needle) points to the south magnetic pole of a second magnet, so Earth's north geographic pole is actually a south magnetic pole. However, it is convention to refer to the north magnetic pole as Earth's north magnetic pole, even though it is a south pole of a magnet.

Our knowledge of what causes Earth's magnetic field is incomplete, but several clues point to an explanation. First, we know that Earth's magnetic poles do not quite coincide with its geographic poles. In fact, Earth's magnetic poles move slowly from day to day and year to year. For centuries Earth's magnetic north pole was in northern Canada (**Figure 6**). In 2011, the north magnetic pole was located at 84.7° N, 129.1° W, well within the Arctic Ocean, and was moving toward Russia at approximately 60 km per year. Second, geological studies show that Earth's magnetic field has completely reversed direction many times during the planet's history. The last reversal occurred about 780 000 years ago. In the years just before that time, the north pole of a compass needle would have pointed toward Earth's south magnetic pole.

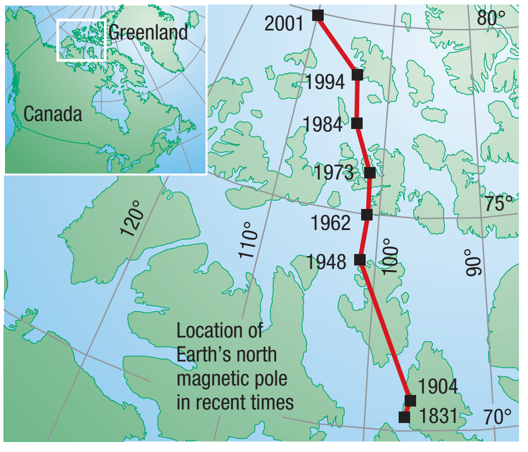


Figure 6 Earth's magnetic poles move from year to year. The north magnetic pole was near latitude 70° in northern Canada less than 200 years ago.

Electric currents in Earth's core probably cause this behaviour of the magnetic field. Earth's core is made of liquid metal. This liquid conducts electricity, and the spin of Earth about its axis causes the liquid to circulate much like the current in a conducting loop. The circulating current causes a magnetic field. Scientists believe that circulation within Earth's core has a complicated flow pattern that varies with time. These variations cause changes in the magnetic field, resulting in the movement of Earth's magnetic poles. Scientists still do not have a complete understanding of these phenomena, however. As is the case with any phenomenon that is not fully understood, more studies will take place and scientists' working theories will continue to be modified.

We usually think of the regions far above Earth's atmosphere between Earth, the Moon, and the Sun as empty. These regions actually contain many charged particles, including electrons and protons, that come from the Sun or from outside our solar system. Such particles are called cosmic rays. Earth's magnetic field affects their motion because they have electric charge. **Figure 7** shows two positively charged cosmic ray particles that approach Earth's equator.

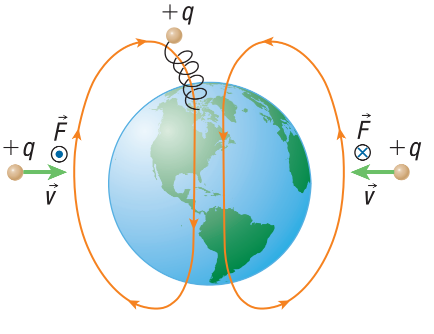


Figure 7 Earth's magnetic field affects the motion of cosmic rays. Cosmic ray particles at the poles spiral in along the field lines, while cosmic rays at the equator are deflected away from Earth. The circled x on the right of the figure indicates that the force is directed into the page for that particle. The circled dot on the left indicates the force is directed out of the page for that particle.

A magnetic field exerts a force on a charged particle that is perpendicular to both the field and the particle's velocity. Consider the particle on the right in Figure 7 on the previous page. At this location, the magnetic field runs parallel to Earth's surface in a geographic south-to-north direction. The magnetic force on the charged particle points perpendicular to the plane of the drawing and parallel to Earth's surface in a west-to-east direction. Earth's magnetic field deflects charged cosmic rays near the equator so that they tend to move away from the surface. At the poles, the magnetic field is perpendicular to the surface and much stronger. Cosmic rays spiral along magnetic field lines, and at the poles they spiral downward. The interaction of these cosmic rays with atmospheric gases causes the aurora borealis that you read about at the start of this chapter. You will learn more about the circular motion of charged particles in magnetic fields in Section 8.4.

Electromagnetism

In 1820, Danish physicist Hans Christian Oersted was demonstrating how a wire becomes warmer when electric charge flows through it. In the course of his demonstration, he noticed that the needle in a nearby compass moved each time he switched on the electricity. This strange event led Oersted to conclude that a magnetic field surrounds moving electric charges. This idea is now known as the **principle of electromagnetism**.

Principle of Electromagnetism

Moving electric charges produce a magnetic field.

LEARNING **TIP**

Current Direction

The direction of conventional current is defined as the direction of flow of positive charge. The direction in which electrons flow (negative charge) in a wire is opposite to the direction of conventional current. When you apply the right-hand rule, consider the direction of conventional current.

Magnetic Field of a Straight Conductor

Moving charges, like those in an electric current, produce a magnetic field. Current in a straight wire or other long, straight conductor creates a magnetic field whose lines look like circles centred on the wire (**Figure 8(a)**).

You can determine the direction of the magnetic field lines around a straight wire by using the **right-hand rule for a straight conductor** (**Figure 8(b)**). If you reverse the direction of the conventional current, the magnetic field lines also reverse.

Right-Hand Rule for a Straight Conductor

If your right thumb is pointing in the direction of conventional current, and you curl your fingers forward, your curled fingers point in the direction of the magnetic field lines.

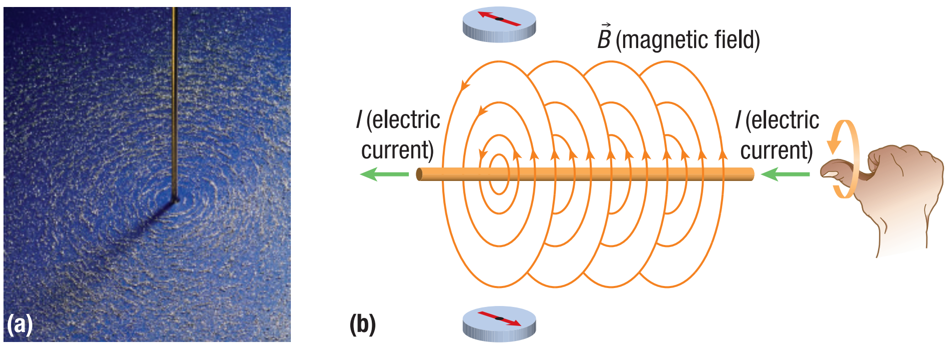


Figure 8 (a) Iron filings indicate the circular magnetic field around a conducting wire. (b) The right-hand rule for a straight conductor indicates the direction of the magnetic field.

Magnetic Field of a Current Loop

If you make a circular loop from a straight wire and run a current through the wire, the magnetic field will circle around each segment of the loop. The field lines inside the loop create a stronger magnetic field than those on the outside because they are closer together. You can still use the right-hand rule for a straight conductor to determine the direction of the magnetic field for a single loop (**Figure 9**).

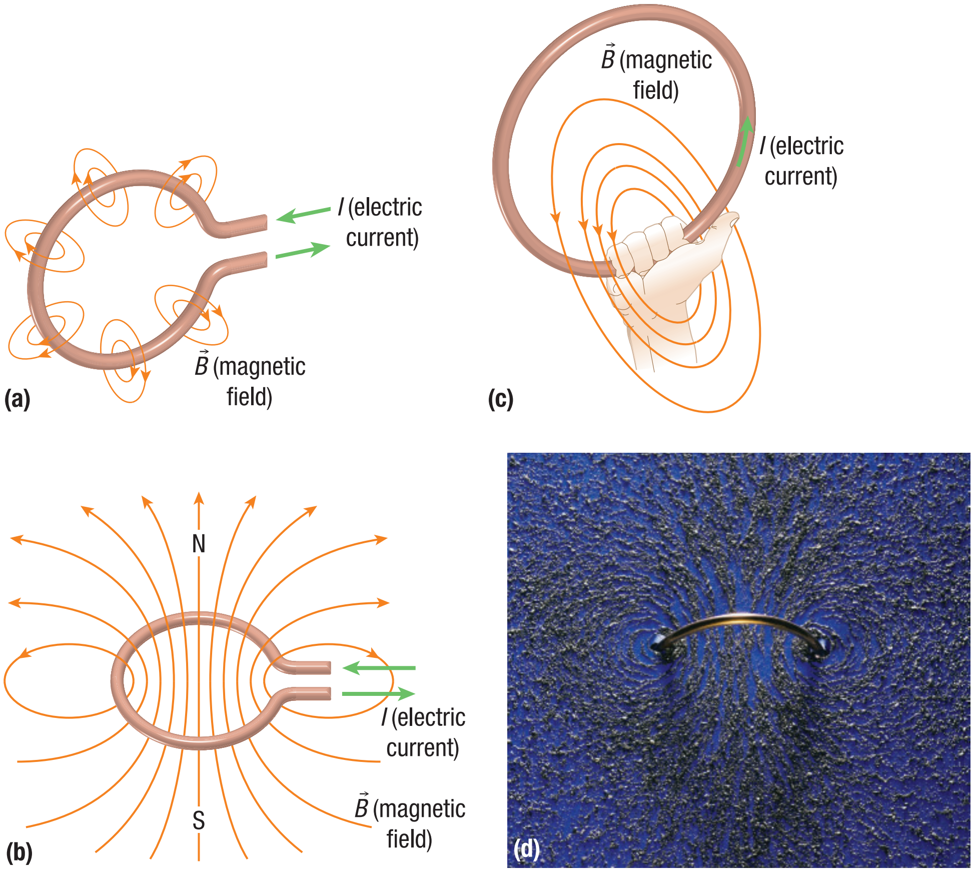


Figure 9 (a) Each segment of a current loop produces a magnetic field, $\vec{B}$, similar to that of a straight conductor. (b) The fields of each segment combine to produce a field similar to that of a bar magnet. (c) The right-hand rule for a straight conductor indicates the magnetic field direction for a single loop. (d) Iron filings show the magnetic field circling the loop.

Magnetic Field of a Coil or Solenoid

A solenoid is a conducting wire wound into a coil. The magnetic field of a solenoid is composed of the combined fields of all its loops. The field is strongest inside the coil because the field lines are closer together. The more tightly you wind the coil, the straighter and closer the field lines become. When the solenoid is loosely wound, field lines within the coil are curved.

To determine the direction of the magnetic field in coiled wire, you must use the **right-hand rule for a solenoid**.

Right-Hand Rule for a Solenoid

If you coil the fingers of your right hand around a solenoid in the direction of the conventional current, your thumb points in the direction of the magnetic field lines in the centre of the coil.

The magnetic field lines of a solenoid look like the field lines of a bar magnet (**Figure 10**). This occurs because the strengths of the fields can be added together, much like the net electric field is the vector sum of all the electric fields. Similarly, in a solenoid, the magnetic field around each segment of the loops corresponds to a straight conductor for that segment. A circular magnetic field forms around the wire at that point of the segment, as you can see from Figures 10(b) and (c). Adding all the magnetic fields together gives the resulting magnetic field of a solenoid. The magnetic field lines extend through the centre of the coil and then loop around the outside. The solenoid has the useful feature that we can switch the current in the wire on or off. Turning the current on and off enables us to control the magnetic field.

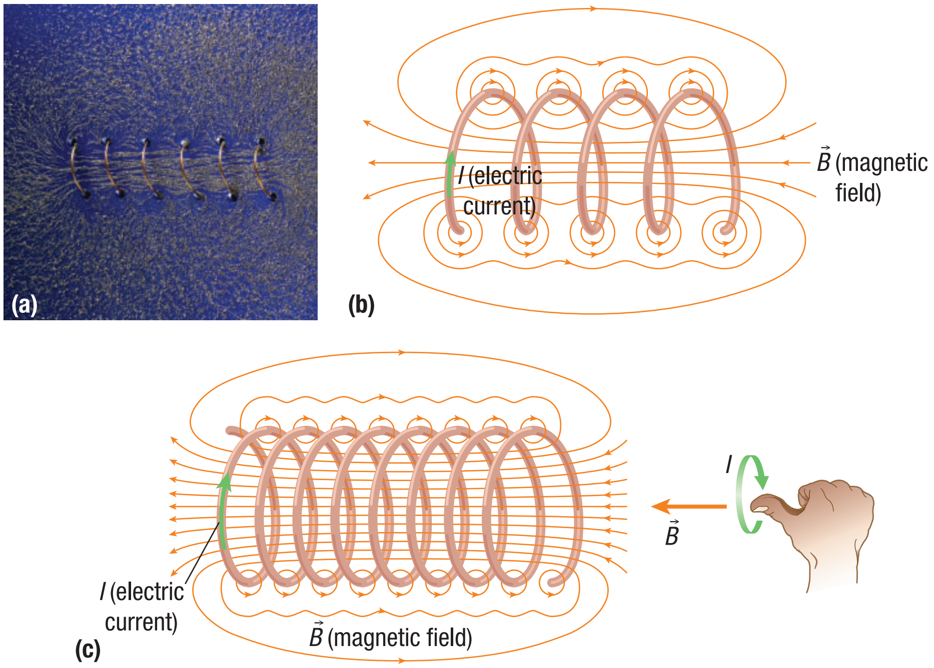


Figure 10 Magnetic field, $\vec{B}$, of a solenoid. (a) The iron filings indicate the direction of the solenoid's magnetic field. (b) The field lines are curved when the coil of the solenoid is loosely wound. (c) The field lines are straight for a tightly wound solenoid. The right-hand rule indicates the direction of the magnetic field through the solenoid.

Applying a current through a solenoid as described above causes the solenoid to become an electromagnet. Stronger electromagnets can be made using a solenoid with a magnetic material, such as iron, nickel, or cobalt, within the coil. The effect of this core material is to increase the strength of the magnetic field by aligning the electrons within the core material in such a way as to enhance the magnetic field.

Electromagnets have many applications:

- They are used in scrap-metal yards to pick up and drop large metal objects such as cars.
- They are used in washing machines and dishwashers to regulate the flow of water.
- They are used in doorbells to pull a lever against a bell and release it.
- Electromagnets also form the central piece of the MRI unit you read about at the start of this chapter.

UNIT TASK **BOOKMARK**

You can apply what you have learned about magnets and electromagnets to the Unit Task on page 422.

8.1 Review

Summary

- All magnets have magnetic poles. Opposite magnetic poles attract one another, and like magnetic poles repel one another.
- A magnetic field surrounds all magnets and goes from north to south outside the magnet and from south to north inside the magnet.
- Earth's magnetic field resembles that of a bar magnet. Earth's magnetic field changes orientation over time and is able to direct the motion of charged particles from space.
- The principle of electromagnetism states that moving electric charges produce a magnetic field.
- The right-hand rule for a straight conductor states that if your thumb points in the direction of conventional current, your curled fingers indicate the direction of the magnetic field lines around the straight conductor.
- The right-hand rule for a solenoid states that if the fingers of your right hand curl in the direction of the conventional current, your thumb points in the direction of the magnetic field lines in the centre of the coil.

Questions

1. What is a permanent magnet? K/U
2. Earth is surrounded by a magnetic field. K/U T/I A
   (a) Where are Earth's magnetic poles?
   (b) Explain how Earth's magnetic field contributes to the aurora borealis.
   (c) How can a hiker use Earth's magnetic field to identify directions?
3. Describe how you would use the right-hand rule to determine the direction of the magnetic field around
   (a) a long, straight conductor with a steady current
   (b) a loop of wire with a steady current
   (c) a long coil of wire with a steady current K/U
4. Look at the magnetic field lines of a tightly wound solenoid, as shown in Figure 10 on page 384. Describe the difference in the magnetic field lines inside and outside the coils. K/U
5. Examine the diagram of the interior of a doorbell shown in **Figure 11**. Use key terms from this section (which include *magnetic field line*, *principle of electromagnetism*, *right-hand rule for a straight conductor*, and *right-hand rule for a solenoid*) to explain how it works. K/U T/I C A

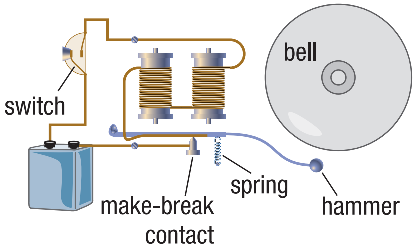


Figure 11

6. Design an experimental procedure to demonstrate the principle of electromagnetism. List the steps you would follow. T/I A
7. Scientists have discovered many living creatures that use Earth's magnetic field in different ways. Magnetotactic bacteria, honey bees, homing pigeons, and dolphins all rely on Earth's magnetic field in some way. **Figure 12** shows a micrograph of a magnetotactic spirilla cell. The dark, round dots inside the cell are magnetite crystals. Write a paragraph explaining how you think the bacterium might be using the crystals. Afterwards, research and prepare a presentation about the bacterium and other animals that use magnetism. T/I C A

Figure 12

WEB LINK

8.2 Magnetic Force on Moving Charges

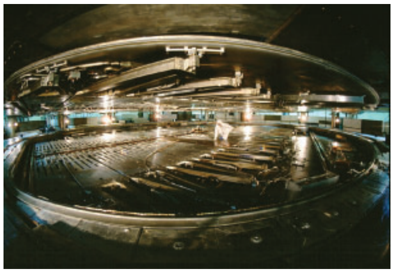


Figure 1 The 18 m cyclotron at TRIUMF is the largest in the world. The magnetic force of electromagnets guides protons along an expanding spiral path.

tesla the SI unit of measure for describing the strength of a magnetic field; $1\ \text{T} = 1\ \frac{\text{kg}}{\text{C·s}}$

Each second, the cyclotron particle accelerator at TRIUMF national laboratory in Vancouver (**Figure 1**) accelerates trillions of protons to speeds of 224 000 km/s. At this speed, a proton travelling in a straight line would reach the Moon in less than 2 s. The magnetic force of a series of large electromagnets directs the protons in the cyclotron along a spiral path with an increasing radius. These powerful magnets produce a magnetic field over 10 000 times as strong as Earth's magnetic field. WEB LINK

The unit of magnetic field strength in the SI system is the **tesla** (T). A tesla is defined in terms of other SI units by this conversion:

$$1\ \text{T} = 1\ \frac{\text{kg}}{\text{C·s}}$$

The magnetic field close to a refrigerator magnet has a magnitude of about 0.001 T. The magnetic field strength near Earth's surface is approximately 50 μT (5×10^{-5} T). An MRI unit, which you read about at the beginning of this chapter, can have a magnetic field strength of 7 T.

We can observe a magnetic field exerting a force on moving charges using a cathode ray tube and a magnet. In **Figure 2(a)**, a beam of electrons moves across the cathode ray tube. Bringing a magnet near the cathode ray tube (**Figure 2(b)**) shows how a magnetic force deflects the electron beam.

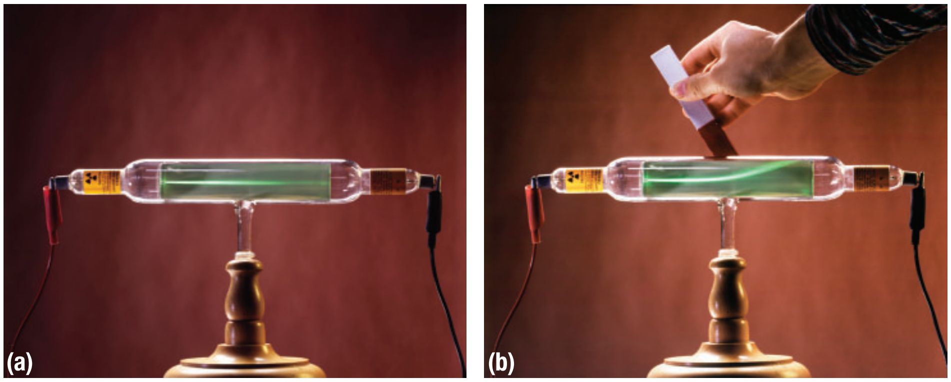


Figure 2 (a) Electrons move straight through the cathode ray tube. (b) When a bar magnet is placed near the electrons, the magnetic force deflects their path.

A Charge in a Magnetic Field

The eighteenth-century French scientist André-Marie Ampère demonstrated that two current-carrying parallel wires could be made to attract or repel one another depending on the direction of the current in each wire. He also showed that a magnetic field could be made to force a current-carrying conductor to move. Ampère's discovery applies to more than just electrons flowing in a wire. Magnetic forces act on all types of electric charges, such as electrons, protons, and ions. An important property of the magnetic force is that it depends on the velocity of the charge. The magnetic force is non-zero only if a charge is in motion. This property of the magnetic force is not shared by the gravitational force and the electric force. Those forces are both independent of velocity.

Figure 3 shows a particle with positive charge q moving with velocity $\vec{v}$. The direction of $\vec{v}$ is south, the magnetic field $\vec{B}$ points east, and the magnetic force on the charge is acting vertically upward. Given a magnetic field $\vec{B}$ and a charge q moving with velocity $\vec{v}$, the magnetic force exerted by $\vec{B}$ on q is

$$F_M = qvB\sin\theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$. In Figure 3, this angle is 90°, so in this case, $\sin\theta$ is equal to 1.

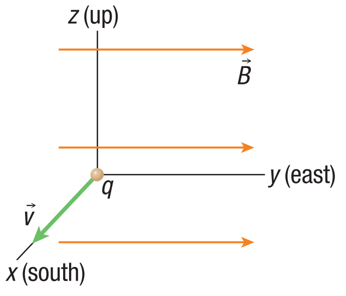


Figure 3

The magnetic force, just like any other force, is a vector quantity. Notice that the equation provides only the magnitude of the magnetic force. **Figure 4** shows how we can also determine the direction of the force using the **right-hand rule for a moving charge in a magnetic field.**

Right-Hand Rule for a Moving Charge in a Magnetic Field

If you point your right thumb in the direction of the velocity of the charge ($\vec{v}$), and your straight fingers in the direction of the magnetic field ($\vec{B}$), then your palm will point in the direction of the resulting magnetic force ($\vec{F}_M$).

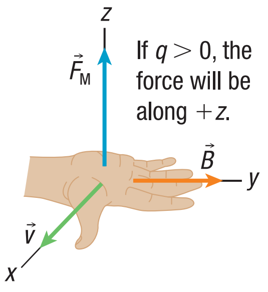


Figure 4 You can use the right-hand rule to determine the direction of the magnetic force on a moving charged particle.

Applying the right-hand rule to Figure 3, the direction of the magnetic force is vertically upward along the $+z$-axis. Using the right-hand rule gives the direction of the force when q is positive in the equation for magnetic force. The magnetic force is up for a positive charge ($q > 0$) with $\vec{v}$ and $\vec{B}$ directed as shown in Figure 4. The force for a negative charge, such as an electron, is down.

Investigation 8.2.1

Observing the Magnetic Force on a Moving Charge (page 412)

You have learned how the magnetic force on a moving electric charge relates to the magnitude of the charge, its velocity, and the magnetic field. Now perform Investigation 8.2.1 to apply this relationship.

The magnetic force has several distinctive characteristics. One already mentioned is that $\vec{F}_M$ depends on velocity. Another characteristic different from gravitational and electric forces is that the direction of $\vec{F}_M$ is always perpendicular to both the magnetic field and the particle's velocity. The direction in which the force acts and the direction of the field are always parallel for electric and gravitational fields.

In the following Tutorial, you will calculate the magnitude of a magnetic force and use the right-hand rule to determine the direction of the force.

Tutorial 1 Calculating the Magnetic Force on Moving Charges

A charged particle moving through a magnetic field experiences a magnetic force. This Tutorial shows how to calculate the magnetic force for a negative charge and for a positive charge, and determine the direction of deflection of a charge by Earth's magnetic field.

Sample Problem 1: Magnetic Force on a Negative Charge in Motion

The electron in **Figure 5** moves at a speed of 54 m/s through a magnetic field with a strength of 1.2 T. The angle between the electron's velocity vector and the magnetic field is 90°. Assume a value for the electron's charge of $q = -e = -1.60 \times 10^{-19}$ C.

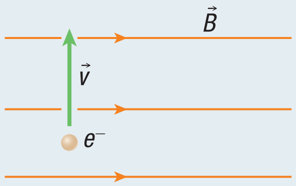


Figure 5

(a) What is the magnitude of the magnetic force on the electron? Express your answer in newtons (N), using $1 \text{ T} = 1 \frac{\text{kg}}{\text{C}\cdot\text{s}}$.

(b) Use the right-hand rule to determine the direction of the magnetic force.

(c) Calculate the gravitational force on the electron. The mass of an electron is 9.11×10^{-31} kg.

(d) What is the ratio of the gravitational force on the electron to the magnetic force on the electron?

Solution

(a) **Given:** $q = 1.60 \times 10^{-19}$ C; $v = 54$ m/s; $B = 1.2$ T; $\theta = 90°$

Required: F_M

Analysis: $F_M = qvB\sin\theta$; use a positive value for q since we are calculating a magnitude.

Solution:

$$F_M = qvB\sin\theta$$

$$= (1.60 \times 10^{-19} \cancel{\text{C}})\left(54 \frac{\text{m}}{\text{s}}\right)\left(1.2 \frac{\text{kg}}{\cancel{\text{C}}\cdot\text{s}}\right)(\sin 90°)$$

$$= 1.037 \times 10^{-17} \text{ kg}\cdot\text{m/s}^2$$

$$F_M = 1.037 \times 10^{-17} \text{ N (two extra digits carried)}$$

Statement: The magnitude of the magnetic force on the electron is 1.0×10^{-17} N.

(b) Using the right-hand rule for a positive charge, point the fingers of your right hand in the direction of the magnetic field. Then point your thumb in the direction of the velocity of the particle. Your palm points in the direction of the magnetic force for a positive charge. In this case, the magnetic force for a positive charge points into the page, but the magnetic force for an electron is in the opposite direction of the magnetic force for a positive charge and points out of the page.

(c) **Given:** $m_e = 9.11 \times 10^{-31}$ kg

Required: F_g

Analysis: The gravitational force on the electron is $m_e g$, the product of the mass of the electron and the gravitational acceleration.

Solution:

$$F_g = m_e g$$

$$= (9.11 \times 10^{-31} \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right)$$

$$F_g = 8.928 \times 10^{-30} \text{ N (two extra digits carried)}$$

Statement: The gravitational force on the electron is 8.9×10^{-30} N.

(d) **Given:** $F_g = 8.928 \times 10^{-30}$ N; $F_M = 1.037 \times 10^{-17}$ N

Required: $\frac{F_g}{F_M}$

Analysis: Divide the gravitational force by the magnetic force.

Solution:

$$\frac{F_g}{F_M} = \frac{8.928 \times 10^{-30} \cancel{\text{N}}}{1.037 \times 10^{-17} \cancel{\text{N}}}$$

$$\frac{F_g}{F_M} = 8.6 \times 10^{-13}$$

Statement: The ratio of the gravitational force to the magnetic force on the electron is $8.6 \times 10^{-13} : 1$.

Sample Problem 2: Magnetic Force on a Positive Charge in Motion

A proton is moving along the x-axis at a speed of 78 m/s. It enters a magnetic field of strength 2.7 T. The angle between the proton's velocity vector and the magnetic field is 38° (**Figure 6**). The mass of a proton is 1.67×10^{-27} kg.

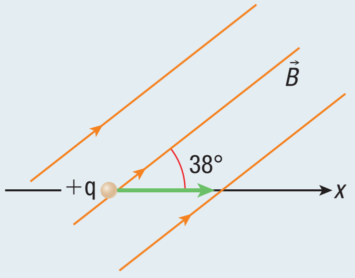


Figure 6

(a) Calculate the initial magnitude and the direction of the magnetic force on the proton.

(b) Determine the proton's initial acceleration.

Solution

(a) **Given:** $q = 1.60 \times 10^{-19}$ C; $v = 78$ m/s; $B = 2.7$ T; $\theta = 38°$

Required: magnitude and direction of magnetic force, F_M

Analysis: Use $F_M = qvB\sin\theta$ to calculate the magnitude of the force. Use the right-hand rule to determine the direction of the force.

Solution:

$$F_M = qvB\sin\theta$$
$$= (1.60 \times 10^{-19}\ \cancel{C})\left(78\ \frac{m}{s}\right)\left(2.7\ \frac{kg}{\cancel{C}\cdot s}\right)(\sin 38°)$$
$$= 2.075 \times 10^{-17}\ kg\cdot m/s^2$$
$$F_M = 2.075 \times 10^{-17}\ \text{N (two extra digits carried)}$$

Point the fingers of your right hand in the direction of the magnetic field, and direct your thumb along the velocity vector of the proton. Your palm is facing in the direction of the magnetic force—out of the page.

Statement: The magnitude of the magnetic force on the proton is 2.1×10^{-17} N. The direction of the magnetic force is out of the page.

(b) **Given:** $F_M = 2.075 \times 10^{-17}$ N; $m = 1.67 \times 10^{-27}$ kg

Required: acceleration, a

Analysis: $a = \frac{F_M}{m}$

Solution:

$$a = \frac{F_M}{m}$$
$$= \frac{2.075 \times 10^{-17}\ \cancel{kg}\frac{m}{s^2}}{1.67 \times 10^{-27}\ \cancel{kg}}$$
$$a = 1.2 \times 10^{10}\ m/s^2$$

Statement: The proton's initial acceleration is 1.2×10^{10} m/s^2.

Sample Problem 3: Determining the Effect of Earth's Magnetic Field on the Direction of Lightning Strikes

During a thunderstorm, positive charge accumulates near the top of a cloud, and negative charge accumulates near the bottom of a cloud (**Figure 7**). When the charge buildup is strong enough, negative charge moves rapidly from the cloud to the ground as a lightning strike. Assume the charge velocity vector is perpendicular to the ground, and Earth's magnetic field is horizontal, directed north. Determine the direction of the deflection of this charge by Earth's magnetic field.

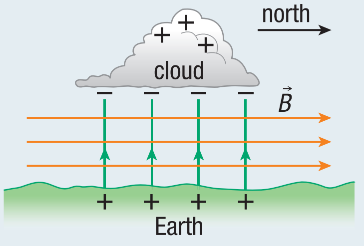


Figure 7

Given: Negative electric charge moves downward. Earth's magnetic field is directed to the north.

Required: direction of deflection of charge

Analysis: The deflection is in the direction of the magnetic force due to Earth's magnetic field. Apply the right-hand rule for a moving charge in a magnetic field to determine the direction of the magnetic force.

Solution: Point the fingers of your right hand in the direction of the velocity of the positive charge, from the ground toward the cloud. Next, point your fingers in the direction of the magnetic field, north. Your palm points in the direction of the resulting magnetic force, into the page, which is west.

Statement: When lightning strikes, Earth's magnetic field deflects the charge toward the west.

Practice

1. A proton with a mass of 1.67×10^{-27} kg is moving horizontally eastward at 9.4×10^4 m/s as it enters a magnetic field of strength 1.8 T. The field is directed vertically upward (**Figure 8**). T/I

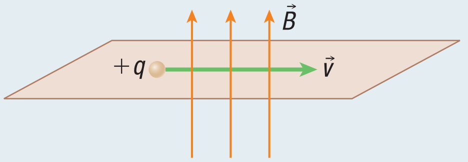


Figure 8

(a) Calculate the magnitude and direction of the magnetic force on the proton. [ans: 2.7×10^{-14} N [S]]

(b) Determine the gravitational force on the proton. [ans: 1.6×10^{-26} N]

(c) Determine the ratio of the gravitational force to the magnetic force on the proton. [ans: 6.0×10^{-13} : 1]

2. Determine the magnitude and direction of the magnetic field for an electron moving upward through a uniform magnetic field with a speed of 3.5×10^5 m/s. The particle experiences a maximum magnetic force of 7.5×10^{-14} N [right]. T/I [ans: 1.3 T [horizontal, into the page]]

3. The cyclotron at TRIUMF accelerates protons from a central injection point outward along an increasing spiral path (**Figure 9**). The protons reach a speed of 2.24×10^8 m/s. Magnets then direct the protons out of the spiral along a linear path toward a target. T/I A

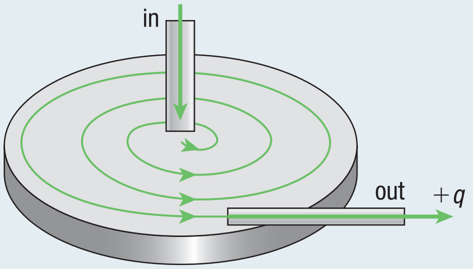


Figure 9

(a) Suppose a proton passes through a magnetic field that has strength 0.56 T directed vertically upward at the moment just *before* the proton leaves the spiral. What are the magnitude and direction of the magnetic force on the proton at this moment? [ans: 2.0×10^{-11} N [horizontally outward from the spiral]]

(b) What is the magnitude of the magnetic force acting on the proton at the moment just *after* it leaves the spiral if the only magnetic field around it is Earth's field of 5.5×10^{-5} T? [ans: 2.0×10^{-15} N]

4. An electron moving at a velocity of 6.7×10^6 m/s [E] enters a magnetic field with a magnitude of 2.3 T directed at an angle of 47° to the direction of motion and upward in the vertical plane. T/I

(a) What are the magnitude and direction of the magnetic force on the electron? [ans: 1.8×10^{-12} N [N]]

(b) What is the electron's acceleration? Its mass is 9.11×10^{-31} kg. [ans: 2.0×10^{18} m/s^2]

(c) What would the acceleration be if the charged particle were a proton of mass 1.67×10^{-27} kg? [ans: 1.1×10^{15} m/s^2]

8.2 Review

Summary

- The formula for calculating the magnitude of the magnetic force on a moving charge is $F_M = qvB \sin \theta$, where θ is the angle between $\vec{v}$ and $\vec{B}$.
- You can use the right-hand rule to determine the direction of the magnetic force on a moving particle. First, point your right thumb in the direction of $\vec{v}$. Then, point your fingers in the direction of $\vec{B}$. If the charge is positive, your palm indicates the direction of a force on the charge. The direction of force is in the opposite direction for a negative charge.

Questions

1. You have now learned three forms of the right-hand rule. Explain the use of each. K/U
2. The charged particle in **Figure 10** moves east. It experiences a force that acts out of the page. Does the particle have a positive charge or a negative charge? K/U

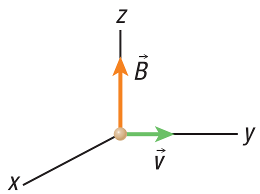


Figure 10

3. A charged particle moving along the $+y$-axis passes through a uniform magnetic field oriented in the $+z$ direction. A magnetic force acts on the particle in the $-x$ direction. Does the particle have positive charge or negative charge? How would the force change if the charge of the particle were tripled but the velocity were halved? K/U
4. A proton moves at a speed of 1.4×10^3 m/s in a direction perpendicular to a magnetic field with a magnitude of 0.85 T. T/I
   (a) Calculate the magnitude of the magnetic force on the proton.
   (b) What would the magnitude of the magnetic force be if the particle in (a) were an electron?
5. An electron with speed 235 m/s moves through a magnetic field of 2.8 T. The magnitude of the force on the electron is 5.7×10^{-17} N. What angle does the electron's velocity vector make with $\vec{B}$? T/I
6. The particle in **Figure 11** has a positive charge. Its initial velocity is to the left when it passes into a magnetic field directed into the plane of the page. In which direction is the particle deflected? K/U

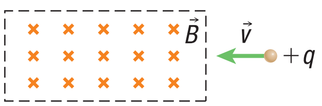


Figure 11

7. A particle of charge $q = 6.4\ \mu$C is moving at an angle of 27° with respect to the y-axis and a speed of 170 m/s. It passes through a uniform magnetic field of magnitude 0.85 T parallel to the y-axis in the x–y plane. T/I
   (a) Calculate the magnitude of the magnetic force on the particle.
   (b) Determine the direction of the magnetic force.
   (c) What would the magnetic force be on an identical particle travelling at the same speed along the y-axis? Explain.
8. The particle in **Figure 12** has a negative charge. Its velocity vector lies in the x–y plane and makes an angle of 75° with the y-axis. If the magnetic field is along the $+x$ direction, what is the direction of the magnetic force on the particle? T/I

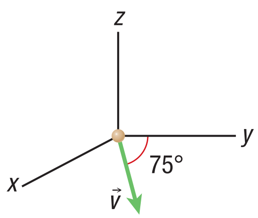


Figure 12

9. A particle of charge $q = -7.9\ \mu$C has a speed of 580 m/s. The particle lies in the x–y plane and travels at an angle of 55° with respect to the x-axis. There is a uniform magnetic field of magnitude 1.3 T parallel to the y-axis. What are the magnitude and direction of the magnetic force on the particle? T/I
10. An alpha particle is a helium nucleus that consists of two protons bound with two neutrons; its mass is 6.644×10^{-27} kg. An alpha particle moving through a perpendicular magnetic field of 1.4 T experiences an acceleration of 2.4×10^3 m/s^2. T/I A
    (a) What is the force on the alpha particle?
    (b) What is the alpha particle's speed?

8.3 Magnetic Force on a Current-Carrying Conductor

Other than the musicians, probably the most important parts of a concert are the speakers. You might have seen the large boxes that transmit sound to both the performers and the audience (**Figure 1**). Speakers are so common that we rarely think about the impact this invention has had on our culture. People use speakers at music concerts, in cars, and in portable devices. Telephones, computers, and televisions all rely on speakers to transmit sound. Intercom systems in buildings, megaphones used by firefighters and police officers, and even electronic readers for sight-impaired people are all possible because of the speaker.

Figure 1 Musicians rely on the magnetic force in speakers to transmit sound during a performance.

How does a speaker work? Inside a speaker is an electromagnet as well as a permanent magnet. The magnetic field of the permanent magnet exerts a force on the current in the coil of the electromagnet. The speaker uses this force to produce sound waves. Simply put, a speaker works because a current-carrying conductor experiences a force in a magnetic field.

Magnetic Force and Current

An electric current consists of a collection of moving charges. We can therefore use the formula for the magnetic force on a single moving charge to determine the magnetic force on a current-carrying wire. This force is important in many applications, including electric motors. Consider a current-carrying wire placed in an external magnetic field as shown in **Figure 2**. The magnetic field $\vec{B}$ is uniform and perpendicular to the wire. An external force, not the current in the wire, produces this field. We can calculate the magnetic force on the wire due to this external field by adding the magnetic forces on all the moving charges in the wire.

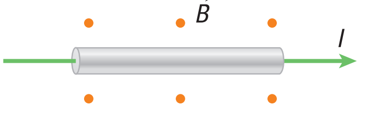


Figure 2 This current-carrying wire is in an external magnetic field. The magnetic field is perpendicular to the wire and is directed out of the page as indicated by the dots.

A segment of the wire of length L is shown in **Figure 3**. The current, I, in this segment is

$$I = \frac{q}{\Delta t}$$

where q is the electric charge that passes by one end of the wire segment in a time interval Δt.

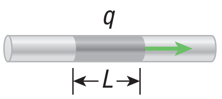


Figure 3 In our analysis to determine the magnetic force on a current-carrying wire, we examine a section of the wire and call it length *L*.

This equation is the relationship between charge and current you have studied before. The magnetic force on this moving charge is given by

$$F_M = qvB\sin\theta$$

The speed of the charge is just

$$v = \frac{L}{\Delta t}$$

Substituting the speed, v, in the magnetic force equation gives

$$\begin{aligned} F_M &= qvB\sin\theta \\ &= q\frac{L}{\Delta t}B\sin\theta \\ F_M &= \frac{q}{\Delta t}LB\sin\theta \end{aligned}$$

Using the relationship between current and charge, $I = \frac{q}{\Delta t}$, we get

$$F_M = ILB\sin\theta$$

The magnetic force on the moving charge is really a force on the wire. We can rewrite the equation to express this relationship explicitly:

$$\boxed{F_{\text{on wire}} = ILB\sin\theta}$$

where θ represents the angle between I and $\vec{B}$.

Figure 4 shows how to use the right-hand rule to determine the direction of $\vec{F}_{\text{on wire}}$. Begin with the fingers of your right hand in the direction of the magnetic field and point your thumb in the direction of the current. Your palm then points in the direction of the force on the wire. The external magnetic field pulls this wire downward as long as the current is directed to the right. The magnetic force on a current is due to the force on a moving charge, so the right-hand rule used here is similar to other right-hand rules you have used before. The direction of the current replaces the direction of $\vec{v}$ for a moving charge.

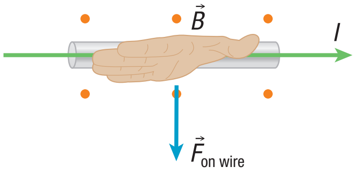


Figure 4 You can use the right-hand rule to determine the direction of the magnetic force on the wire.

In the following Tutorial, you will calculate the magnetic force on a current-carrying wire in a magnetic field.

Tutorial 1 Calculating the Magnetic Force on a Wire

Sample Problem 1: Calculating the Magnitude of the Magnetic Force on a Wire in a Uniform Magnetic Field

A piece of wire 45.2 cm long has a current of 12 A (**Figure 5**). The wire moves through a uniform magnetic field with a strength of 0.30 T. Calculate the magnitude of the magnetic force on the wire when the angle between the magnetic field and the wire is (a) 0°, (b) 45°, and (c) 90°.

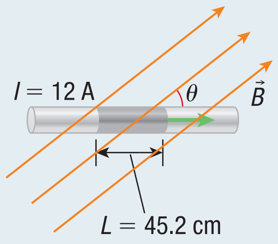


Figure 5

Given: $I = 12$ A; $L = 45.2$ cm $= 0.452$ m; $B = 0.30$ T

Required: $F_{\text{on wire}}$

Analysis: Use the equation for the magnitude of the magnetic force on a current-carrying wire: $F_{\text{on wire}} = ILB\sin\theta$. Note that 1 A = 1 C/s.

Solution:

$$\begin{aligned} F_{\text{on wire}} &= ILB\sin\theta \\ &= \left(12\,\frac{\cancel{\text{C}}}{\text{s}}\right)(0.452\text{ m})\left(0.30\,\frac{\text{kg}}{\cancel{\text{C}}\cdot\text{s}}\right)\sin\theta \\ &= (1.627\text{ kg}\cdot\text{m/s}^2)\sin\theta \\ F_{\text{on wire}} &= (1.627\text{ N})\sin\theta \text{ (two extra digits carried)} \end{aligned}$$

(a) When $\theta = 0°$, then $\sin\theta = 0$, so

$$\begin{aligned} F_{\text{on wire}} &= (1.627\text{ N})(0) \\ F_{\text{on wire}} &= 0\text{ N} \end{aligned}$$

(b) When $\theta = 45°$, then $\sin\theta = 0.707$, so

$$\begin{aligned} F_{\text{on wire}} &= (1.627\text{ N})(0.707) \\ F_{\text{on wire}} &= 1.2\text{ N} \end{aligned}$$

(c) When $\theta = 90°$, then $\sin\theta = 1$, so

$$\begin{aligned} F_{\text{on wire}} &= (1.627\text{ N})(1) \\ F_{\text{on wire}} &= 1.6\text{ N} \end{aligned}$$

Statement: The magnitude of the force on the wire is 0 N when $\theta = 0°$, 1.2 N when $\theta = 45°$, and 1.6 N when $\theta = 90°$.

Sample Problem 2: Determining Magnetic Force on a Segment of a Current-Carrying Wire in Earth's Magnetic Field

Two electrical poles support a current-carrying wire. The mass of a 2.5 m segment of the wire is 0.44 kg. A 15 A current travels through the wire. The conventional current is oriented due east, horizontal to Earth's surface. The strength of Earth's magnetic field at the location is 57 μT and is oriented due north, horizontal to Earth's surface (**Figure 6**).

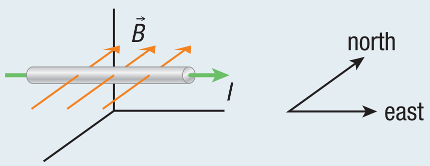


Figure 6

(a) Determine the magnitude and the direction of the magnetic force on the 2.5 m segment of wire.

(b) Calculate the gravitational force on the 2.5 m segment of wire.

Solution

(a) **Given:** $B = 57\ \mu\text{T} = 5.7 \times 10^{-5}$ T; $I = 15$ A; $L = 2.5$ m; $\theta = 90°$

Required: $F_{\text{on wire}}$

Analysis: Use the equation $F_{\text{on wire}} = ILB\sin\theta$ to determine the magnitude of the magnetic force; then use the right-hand rule for a current-carrying wire in a magnetic field to determine the direction.

Solution:

$$\begin{aligned} F_{\text{on wire}} &= ILB\sin\theta \\ &= \left(15\,\frac{\cancel{\text{C}}}{\text{s}}\right)(2.5\text{ m})\left(5.7 \times 10^{-5}\,\frac{\text{kg}}{\cancel{\text{C}}\cdot\text{s}}\right)(\sin 90°) \\ &= 2.1 \times 10^{-3}\text{ kg}\cdot\text{m/s}^2 \\ F_{\text{on wire}} &= 2.1 \times 10^{-3}\text{ N} \end{aligned}$$

Using the right-hand rule, point the fingers of your right hand in the direction of the current, east. Next, curl your fingers in the direction of the magnetic field, north. Your thumb points in the direction of the resulting magnetic force, upward.

Statement: The magnitude of the magnetic force on the 2.5 m segment of wire is 2.1×10^{-3} N. The force is directed upward.

(b) **Given:** $m = 0.44$ kg

Required: F_g

Analysis: $F_g = mg$

Solution:

$$\begin{aligned} F_g &= mg \\ &= (0.44\text{ kg})(9.8\text{ m/s}^2) \\ F_g &= 4.3\text{ N} \end{aligned}$$

Statement: The gravitational force on the 2.5 m segment of wire is 4.3 N.

Practice

1. A 155 mm part of a wire has a mass of 0.27 kg and carries an electric current of 3.2 A. The conventional current passes through a uniform magnetic field of 1.8 T. The direction of the wire and the magnetic field are shown in **Figure 7**. T/I
   (a) What is the magnitude of the magnetic force on the wire? [ans: 0.89 N]
   (b) Use the right-hand rule to determine the direction of the magnetic force. [ans: in the direction of the *x*-axis]
2. The magnitude of the force exerted on a length of wire in an electric motor is 0.75 N. The 15 A current in the wire passes at a 90° angle to a uniform magnetic field of 0.20 T. Calculate the length of the wire, in centimetres. T/I [ans: 25 cm]
3. Earth's magnetic field exerts a force of 1.4×10^{-5} N on a 0.045 m segment of wire in a truck motor. The motor wire is positioned at an 18° angle to Earth's magnetic field, which has a magnitude of 5.3×10^{-5} T at the truck's location. Calculate the current in the wire. T/I A [ans: 19 A]
4. An electrical cord in a lamp carries a 1.5 A current. A 5.7 cm segment of the cord is tilted at a right angle to Earth's magnetic field. This segment experiences a 5.7×10^{-6} N magnetic force due to Earth's magnetic field. Calculate the magnitude of Earth's magnetic field around the lamp. T/I [ans: 6.7×10^{-5} T]

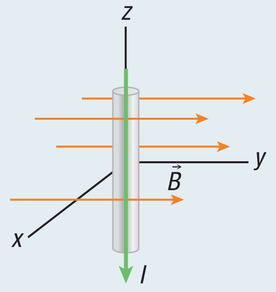


Figure 7

Loudspeakers

Figure 8 shows how a loudspeaker uses a magnetic force on a current to produce sound. The wire coil inside the speaker is part of an electromagnet. Electrical signals corresponding to sounds produce a changing current in the coil. The changing current produces a changing magnetic field around the coil. The permanent magnet also has a magnetic field. This field exerts a force on the current-carrying wire. Variations in the current produce variations in the force on the wires in the coil. The coil moves back and forth in response. The vibrating coil causes the cone to vibrate, pushing sound waves through the air and into your ears.

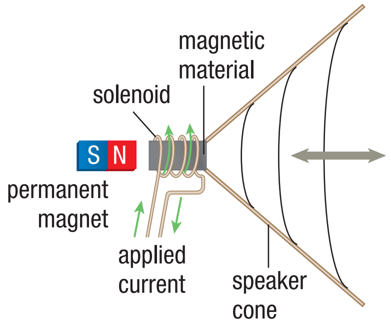


Figure 8 A loudspeaker changes electrical signals to sound using a permanent magnet and an electromagnet.

Electromagnetic Pumps

An understanding of the magnetic force on a current enabled medical researchers to devise electromagnetic pumps to move fluids in kidneys and artificial hearts. Traditional mechanical pumps can cause damage to blood cells. The use of magnetic fields eliminates this problem. Scientists are able to keep the blood flowing to the heart during kidney dialysis, for example, by creating a magnetic field over tubes of blood containing an electric current. A magnetic force acting on the charged particles keeps the blood in motion. WEB LINK

UNIT TASK BOOKMARK

You can apply what you have learned about magnetic force and current to the Unit Task on page 422.

8.3 Review

Summary

- The magnetic force on a current-carrying wire is equivalent to the sum of the magnetic forces on all of the moving charges in the wire.
- A straight, current-carrying conductor in a uniform external magnetic field $\vec{B}$ experiences a magnetic force due to the field. The magnitude of the force is $F_{\text{on wire}} = ILB \sin \theta$, where I is the current in the conductor, L is the length of the conductor, and θ is the angle between the direction of the current and the direction of the magnetic field.
- A magnetic field does not exert a force on a current moving parallel to the direction of the magnetic field. The magnetic force on the current is greatest when the current moves perpendicular to the direction of the magnetic field.
- You can use the right-hand rule for a moving charge in a magnetic field to determine the direction of the magnetic force on the conductor. Point the fingers of your right hand in the direction of the magnetic field. Your right thumb points in the direction of the conventional current in the conductor. The palm of your right hand points in the direction the wire will be forced.
- Many technologies exist that use magnetic forces acting on wires.

Questions

1. A current is carried by a straight conductor in a magnetic field of 1.4 T. The conductor is perpendicular to the magnetic field. The conductor is 2.3 m long, and the magnetic field exerts a 1.8 N force on it. T/I
   (a) Calculate the current in the conductor.
   (b) What is the angle between the conductor and the magnetic field when the magnetic force is at a maximum? Explain your reasoning.
2. A 120 mm segment of wire has a mass of 0.026 kg. The segment of wire is oriented at a 45° angle to a uniform magnetic field. The strength of the magnetic field is 0.40 T, and the current in the wire segment is 2.3 A. T/I
   (a) Calculate the magnitude of the magnetic force on the wire segment.
   (b) What is the direction of the magnetic force if the direction of the conventional current is due south and both the wire and the magnetic field are horizontal?
3. A 2.6 m wire carries a current of 2.5 A. The direction of the conventional current is due west. The strength of Earth's magnetic field at the location of the wire is 5.0×10^{-5} T and the orientation of the field is north. Both the wire and Earth's magnetic field are horizontal. T/I
   (a) What are the magnitude and direction of Earth's magnetic force on the wire?
   (b) What will the force on the wire be if the wire is rotated to an angle of 72° with the magnetic field?
4. A long, straight wire of length 1.4 m carries a current of $I = 3.5$ A. A magnetic field of magnitude $B = 1.5$ T is directed perpendicular to the wire. Calculate the magnitude of the force on the wire. T/I
5. The current loop in **Figure 9** forms a right-angled triangle. The loop carries a current I. A uniform magnetic field is directed perpendicular to one edge of the loop. The angle θ in Figure 9 is 39°, L has a length of 1.2 m, and the magnetic field strength has a magnitude of 1.6 T. T/I

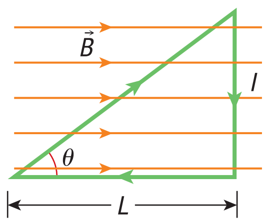


Figure 9

   (a) What is the force on the part of the loop parallel to the magnetic field?
   (b) What is the sum of the forces on the remaining two segments of the loop? (Hint: Write the length of each segment in terms of θ and L, and then use trigonometry.)
   (c) What can you conclude about the magnetic force on a closed loop in a uniform magnetic field?

Motion of Charged Particles in Magnetic Fields

8.4

Atoms and molecules are particles that are the building blocks of our universe. How do scientists study the nature of these small particles? The mass spectrometer shown in **Figure 1** is an instrument scientists use to study atoms and molecules. Mass spectrometers are used to define the elemental composition of a sample or molecule, to determine masses of particles, and to reveal the chemical structures of molecules.

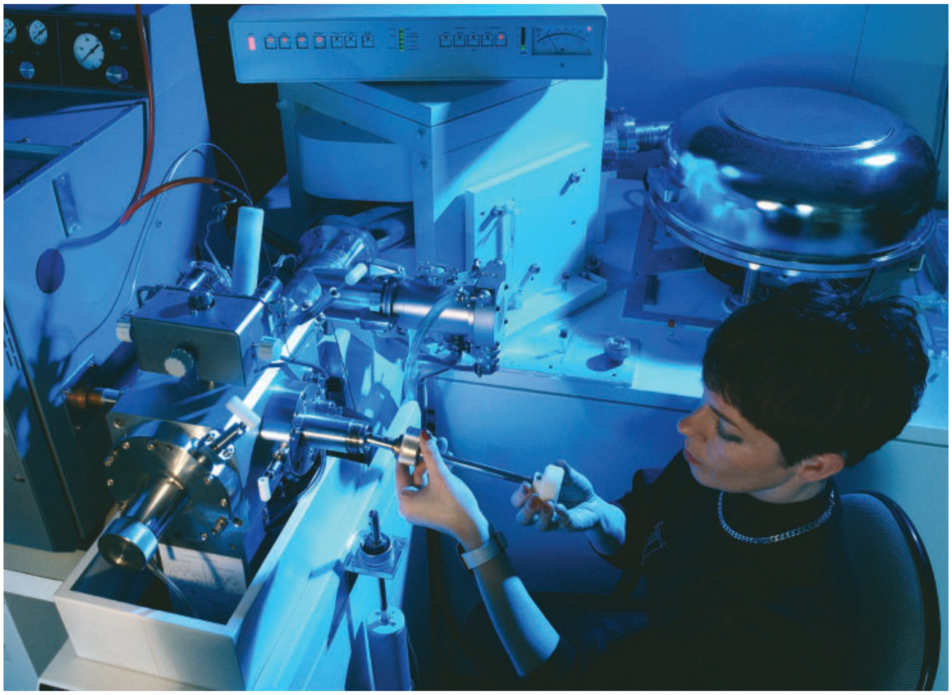


Figure 1 A scientist inserts a sample into a mass spectrometer.

How does a mass spectrometer work? Imagine a billiard table with a billiard ball rolling across the table from your left to your right. If you hit the ball with a sideways force, the ball will move away from you. Now suppose a bowling ball rolls across the table in the same direction. If you apply the same sideways force on the bowling ball, it will also move away from you but not as far. The masses of the billiard ball and bowling ball determine the distance they will be deflected by the force. If you know the amount of force, the speeds of the balls, and the curve of their paths, you can calculate the mass of each ball. The less deflection there is, the heavier the ball must be.

In a similar way, a mass spectrometer uses a magnetic field to deflect electrically charged particles. Atoms are converted into ions and then accelerated into a finely focused beam. Different ions are then deflected by the magnetic field by different amounts, depending on the mass of the ion and its charge. Lighter ions are deflected more than heavier ones. Ions with more positive charges are deflected more than ions with fewer positive charges. Only some ions make it all the way through the machine to the ion detector, where they are detected electrically. If you vary the magnetic field, different types of ions will reach the detector.

Scientists use the mass spectrometer to identify unknown compounds, to determine the structure of a compound, and to understand the isotopic makeup of molecular elements. The mass spectrometer has applications in the medical field, the food industry, genetics, carbon dating, forensics, and space exploration. CAREER LINK

Charges and Uniform Circular Motion

To understand how a mass spectrometer works, we first need to understand how a directional force affects the motion of an object—in this case, a charged particle. Consider the direction of a magnetic force $\vec{F}_M$ and how this force affects the motion of a charged particle. We know $F_M = qvB \sin \theta$. For simplicity, we assume the magnetic field, $\vec{B}$, is uniform, so the magnitude and direction of $\vec{B}$ are the same everywhere. **Figure 2(a)** shows a charged particle, $+q$, moving at velocity $\vec{v}$ parallel to the direction of $\vec{B}$. In this case, the angle θ between $\vec{v}$ and $\vec{B}$ is zero. The factor $\sin \theta$ in $F_M = qvB \sin \theta$ is then zero, so the magnetic force in this case is also zero. If a charged particle has a velocity parallel to $\vec{B}$, the magnetic force on the particle is zero.

Figure 2(b) shows a charged particle moving perpendicular to $\vec{B}$. Now we have $\theta = 90°$, and $\sin \theta = 1$. The magnitude of the magnetic force is thus $F_M = qvB$, and the force is perpendicular to the velocity.

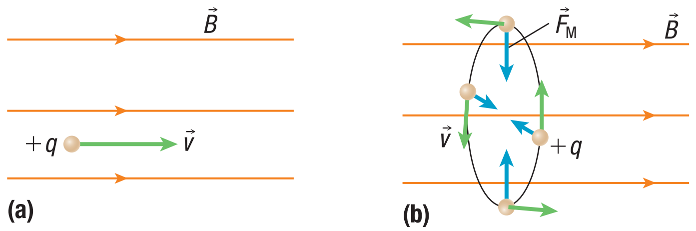


Figure 2 (a) When the velocity of a charged particle is parallel to the magnetic field $\vec{B}$, the magnetic force on the particle is zero. (b) When $\vec{v}$ makes a right angle with $\vec{B}$ ($\theta = 90°$), the charged particle moves in a circle that lies in a plane perpendicular to $\vec{B}$.

Recall that when a particle experiences a force of constant magnitude perpendicular to its velocity, the result is circular motion, as shown in Figure 2(b). Hence, if a charged particle is moving perpendicular to a uniform magnetic field, the particle will move in a circle. This circle lies in the plane perpendicular to the field lines.

The radius of the circle can be determined from Newton's second law and centripetal acceleration. Recall that for a particle to move in a circle of radius r, there must be a force of magnitude $\frac{mv^2}{r}$ directed toward the centre of the circle. Here, the force producing circular motion is the magnetic force, so we have

$$F_M = \frac{mv^2}{r}$$

The magnetic force is perpendicular to the velocity and $\sin 90° = 1$, so we can insert $F_M = qvB$:

$$qvB = \frac{mv^2}{r}$$

Solving for r gives

$$\boxed{r = \frac{mv}{qB}}$$

Now calculate the value of r for an electron that has a speed of 5.5×10^6 m/s moving in a magnetic field of strength 5.0×10^{-4} T. Inserting these values into the equation $r = \frac{mv}{qB}$ and using $1\ \text{T} = 1 \frac{\text{kg}}{\text{C}\cdot\text{s}}$, we get

$$r = \frac{mv}{qB}$$

$$= \frac{(9.11 \times 10^{-31}\ \cancel{\text{kg}})\left(5.5 \times 10^6\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{(1.60 \times 10^{-19}\ \cancel{\text{C}})\left(5.0 \times 10^{-4}\ \frac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\cancel{\text{s}}}\right)}$$

$$r = 6.3 \times 10^{-2}\ \text{m}$$

This calculation shows that we can determine the radius of a particle's deflection if we know the mass of the particle, its velocity, its charge, and the strength of the magnetic field through which it moves.

Tutorial 1 Solving Problems Related to Charged Particles in Circular Motion in Magnetic Fields

Sample Problem 1: An Electron in a Magnetic Field

An electron starts from rest. A horizontally directed electric field accelerates the electron through a potential difference of 37 V. The electron then leaves the electric field and moves into a magnetic field. The magnetic field strength is 0.26 T, directed into the page (**Figure 3**), and the mass of the electron is 9.11×10^{-31} kg.

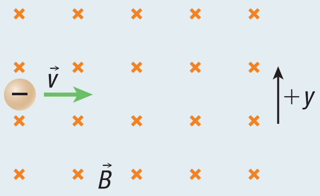


Figure 3

(a) Determine the speed of the electron at the moment it enters the magnetic field.

(b) Determine the magnitude and direction of the magnetic force on the electron.

(c) Determine the radius of the electron's circular path.

Solution

(a) **Given:** $\Delta V = 37$ V; $m_e = 9.11 \times 10^{-31}$ kg; $q = 1.60 \times 10^{-19}$ C

Required: v_i

Analysis: The decrease in the electron's electric potential energy equals the increase in its kinetic energy, $-\Delta E_E = \Delta E_k$, where $-\Delta E_E = q\Delta V$ and $E_k = \frac{1}{2}mv^2$.

The speed of the electron before it enters the electric field is zero. Therefore, $\Delta E_k = E_{k_f}$. Use $1\ \text{V} = 1\frac{\text{J}}{\text{C}}$ and $1\ \text{J} = 1\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$.

Solution: $-\Delta E_E = \Delta E_k$

$$q\Delta V = \frac{1}{2}mv_i^2$$

$$v_i = \sqrt{\frac{2q\Delta V}{m}}$$

$$= \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})(37\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$= \sqrt{\frac{2(1.60 \times 10^{-19}\ \cancel{\text{C}})\left(37\ \frac{\cancel{\text{kg}}\cdot\text{m}^2}{\cancel{\text{C}}\cdot\text{s}^2}\right)}{9.11 \times 10^{-31}\ \cancel{\text{kg}}}}$$

$$v_i = 3.605 \times 10^6\ \text{m/s (two extra digits carried)}$$

Statement: The initial speed of the electron at the moment it enters the magnetic field is 3.6×10^6 m/s.

(b) **Given:** $B = 0.26$ T; $q = 1.60 \times 10^{-19}$ C; $\theta = 90°$; $v_i = 3.605 \times 10^6$ m/s

Required: F_M and its direction

Analysis: Use $F_M = qvB\sin\theta$ to determine the magnitude of the force. Then use the right-hand rule to determine the direction. The magnetic force will be the opposite of this direction because the charge is negative.

Solution:

$$F_M = qvB\sin\theta$$

$$= (1.60 \times 10^{-19}\ \cancel{\text{C}})\left(3.605 \times 10^6\ \frac{\text{m}}{\text{s}}\right)\left(0.26\ \frac{\text{kg}}{\cancel{\text{C}}\cdot\text{s}}\right)(\sin 90°)$$

$$F_M = 1.5 \times 10^{-13}\ \text{N}$$

Apply the right-hand rule for an electric charge moving through a magnetic field: point the fingers of your right hand in the direction of the external magnetic field, into the page. Point your right thumb in the direction that the charge is moving, to the right. Your palm points in the direction of the magnetic force for a positive charge, up the page. The charge is negative, so the magnetic force is down the page.

Statement: The magnitude of the magnetic force on the electron is 1.5×10^{-13} N down the page.

(c) **Given:** $m_e = 9.11 \times 10^{-31}$ kg; $B = 0.26$ T; $q = 1.60 \times 10^{-19}$ C; $v = 3.605 \times 10^6$ m/s

Required: r

Analysis: The only force acting on the electron is the magnetic force. This force is perpendicular to the electron's velocity, causing it to move in uniform circular motion.

The magnetic force is the centripetal force, $F_c = \frac{mv^2}{r}$.

Solution: $F_M = F_c$

$$qvB = \frac{mv^2}{r}\ \text{(because } \sin 90° = 1\text{)}$$

$$r = \frac{mv}{qB}$$

$$= \frac{(9.11 \times 10^{-31}\ \cancel{\text{kg}})\left(3.605 \times 10^6\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{(1.60 \times 10^{-19}\ \cancel{\text{C}})\left(0.26\ \frac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\cancel{\text{s}}}\right)}$$

$$r = 7.9 \times 10^{-5}\ \text{m}$$

Statement: The radius of the electron's circular path is 7.9×10^{-5} m.

Sample Problem 2: The Mass Spectrometer: Identifying Particles

A researcher using a mass spectrometer observes a particle travelling at 1.6×10^6 m/s in a circular path of radius 8.2 cm. The spectrometer's magnetic field is perpendicular to the particle's path and has a magnitude of 0.41 T.

(a) Calculate the mass-to-charge ratio of the particle. (In 1910, Robert Millikan accurately determined the charge carried by an electron. His finding allowed researchers to calculate the mass of charged particles using the mass-to-charge ratio.)

(b) Identify the particle using **Table 1**.

Table 1

Isotope	***m*** **(kg)**	***q*** **(C)**	$\frac{m}{q}$ **(kg/C)**
hydrogen	1.67×10^{-27}	1.60×10^{-19}	1.04×10^{-8}
deuterium	3.35×10^{-27}	1.60×10^{-19}	2.09×10^{-8}
tritium	5.01×10^{-27}	1.60×10^{-19}	3.13×10^{-8}

Solution

(a) **Given:** $v = 1.6 \times 10^6$ m/s; $r = 8.2$ cm $= 0.082$ m; $\theta = 90°$; $B = 0.41$ T

Required: $\frac{m}{q}$

Analysis: The only force acting on the electron is the magnetic force, $F_M = qvB\sin\theta$. This force is perpendicular to the electron's velocity, causing it to move in uniform circular motion. The magnetic force is the centripetal force, $F_c = \frac{mv^2}{r}$.

Solution:

$$F_M = F_c$$

$$qvB = \frac{mv^2}{r} \text{ (because } \sin 90° = 1)$$

$$\frac{m}{q} = \frac{rB}{v}$$

$$= \frac{(0.082\ \cancel{\text{m}})\left(0.41\ \frac{\text{kg}}{\text{C}\cdot\cancel{\text{s}}}\right)}{1.6 \times 10^6\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}$$

$$\frac{m}{q} = 2.1 \times 10^{-8} \text{ kg/C}$$

Statement: The mass-to-charge ratio of the particle is 2.1×10^{-8} kg/C.

(b) According to Table 1, the particle is the isotope deuterium.

Sample Problem 3: The Mass Spectrometer: Separating Isotopes

A researcher uses a mass spectrometer in a carbon dating experiment (**Figure 4**). The incoming ions are a mixture of $^{12}C^+$ and $^{14}C^+$, and they have speed $v = 1.0 \times 10^5$ m/s. The strength of the magnetic field is 0.10 T. The mass of the electron is 9.11×10^{-31} kg. The mass of the proton and the mass of the neutron are both 1.67×10^{-27} kg.

The researcher first positions the ion detector to determine the value of r for $^{12}C^+$ and then moves it to determine the value of r for $^{14}C^+$. How far must the detector move between detecting $^{12}C^+$ and $^{14}C^+$?

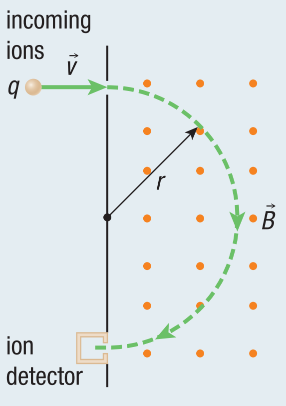


Figure 4

Given: $q = 1.60 \times 10^{-19}$ C; $m_e = 9.11 \times 10^{-31}$ kg; $m_p = m_n = 1.67 \times 10^{-27}$ kg; $v = 1.0 \times 10^5$ m/s; $B = 0.10$ T

Required: Δd

Analysis: Use the mass of the proton and the mass of the neutron to determine the mass of each isotope. Then use the equation for the radius of curvature for a particle deflected in a magnetic field, $r = \frac{mv}{qB}$. The detector will have to move a distance equal to twice the difference between the two radii.

Solution: Determine the mass of each isotope.

$$m_{C12} = 6m_p + 6m_n + 5m_e$$

$$= 6(1.67\times10^{-27}\text{ kg}) + 6(1.67\times10^{-27}\text{ kg}) + 5(9.11\times10^{-31}\text{ kg})$$

$$m_{C12} = 2.004 \times 10^{-26} \text{ kg (two extra digits carried)}$$

$$m_{C14} = 6m_p + 8m_n + 5m_e$$

$$= 6(1.67\times10^{-27}\text{ kg}) + 8(1.67\times10^{-27}\text{ kg}) + 5(9.11\times10^{-31}\text{ kg})$$

$$m_{C14} = 2.338 \times 10^{-26} \text{ kg (two extra digits carried)}$$

Calculate the radius of curvature of each particle.

$$r_{C12} = \frac{m_{C12}v}{qB}$$

$$= \frac{(2.004 \times 10^{-26}\ \cancel{\text{kg}})\left(1.0 \times 10^5\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{(1.60 \times 10^{-19}\ \cancel{\text{C}})\left(0.10\ \frac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\cancel{\text{s}}}\right)}$$

$$r_{C12} = 0.1252 \text{ m (two extra digits carried)}$$

$$r_{C14} = \frac{m_{C14}v}{qB}$$

$$= \frac{(2.338 \times 10^{-26}\ \cancel{\text{kg}})\left(1.0 \times 10^5\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{(1.60 \times 10^{-19}\ \cancel{\text{C}})\left(0.10\ \frac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\cancel{\text{s}}}\right)}$$

$r_{C14} = 0.1461$ m (two extra digits carried)

Calculate how far the detector must move.

$$\Delta d = 2(r_{C14} - r_{C12})$$
$$= 2(0.1461\ \text{m} - 0.1252\ \text{m})$$
$$\Delta d = 0.04\ \text{m}$$

Statement: The ion detector must move a distance equal to the difference in the diameters of the circular trajectories, so it must move a distance of 0.04 m.

Practice

1. A helium 2+ ion with charge 3.2×10^{-19} C and mass 6.7×10^{-27} kg enters a uniform 2.4 T magnetic field at a velocity of 1.5×10^7 m/s, at right angles to the field. Calculate the radius of the ion's path. T/I [ans: 0.13 m]
2. A proton with mass 1.67×10^{-27} kg moves in a plane perpendicular to a uniform 1.5 T magnetic field in a circle of radius 8.0 cm. Calculate the proton's speed. T/I [ans: 1.1×10^7 m/s]
3. Consider a mass spectrometer used to separate the two isotopes hydrogen and deuterium. The isotope hydrogen has a proton, and deuterium has a proton and a neutron. Assume both ions have a 1+ charge and they enter the magnetic field region with a speed of 6.0×10^5 m/s. Calculate the magnitude of the magnetic field that is required to give a detector placement difference of 1.5 mm as measured from the initial entry point into the spectrometer compared to when the ions leave the spectrometer. T/I A [ans: 8.4 T]
4. The Bainbridge-type mass spectrometer uses a velocity selector to select only those ions with the proper velocity. The selector has two charged parallel plates to create an electric field pointing up, and copper coils to create a magnetic field. Positive ions pass through the selector, with velocity directed to the right (**Figure 5**). T/I A
   (a) In which direction should the magnetic field point in order to balance the electric force against the magnetic force?
   (b) If the electric field has magnitude ε, the magnetic field has magnitude B, and the ion has charge q, determine the proper velocity for the ions to pass through the selector without deflection.
   (c) Predict the paths of ions that have too great, and too small, a velocity. Justify your answers.

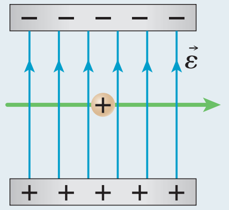


Figure 5

Mini Investigation

Simulating a Mass Spectrometer

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

Equipment and Materials: eye protection; 50 cm wooden or plastic ramp; bar magnet; 2 small steel ball bearings of different masses

Wear closed-toed shoes for this activity.

1. Set up the ramp at approximately a 45° angle.
2. Place the bar magnet on the level surface at the ramp's base. One pole of the magnet should be facing the bottom of the ramp.
3. Put on your eye protection. Roll a steel ball bearing down the ramp, but not directly at the magnet.
4. Observe the path of the ball.
5. Draw its trajectory on a piece of paper.
6. Repeat Steps 3 to 5 using a ball bearing with a different mass.
7. Draw the new trajectory next to the first one and note any differences.

A. Compare this activity to the function of a mass spectrometer. How is it similar? K/U C

B. How is this activity different from the function of a mass spectrometer? K/U C

Earth's Magnetic Field

Charged particles travelling parallel to a magnetic field do not experience a magnetic force and continue moving along the field direction. Charged particles travelling perpendicular to a magnetic field experience a force that keeps them moving in a circular path. Charged particles with velocity components that are both parallel and perpendicular to a magnetic field experience a combination of these effects. The result is a spiral path that resembles the shape of a coil of wire. The particle travels with a looping motion along the direction of the field (**Figure 6**).

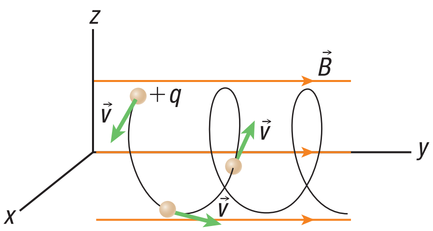


Figure 6 When the velocity of a charged particle has non-zero components parallel and perpendicular to the magnetic field, the particle will move along a spiral path.

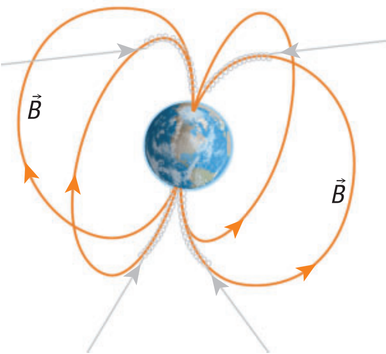


Figure 7 Earth's magnetic field deflects charged particles from outside the atmosphere. The particles travel in spiral paths along the field lines toward the magnetic poles.

Charged particles entering Earth's magnetic field are deflected in this way. Since they are charged particles with a component of the velocity perpendicular to the magnetic field, they will spiral along the field lines toward the magnetic poles. This motion results in a concentration of charged particles at Earth's north and south magnetic poles (**Figure 7**).

Collisions between the charged particles and atoms in the atmosphere release light that causes the glow of the aurora borealis in the northern hemisphere and the aurora australis in the southern hemisphere (**Figure 8**).

Figure 8 The aurora australis. The glow of the auroras occurs when charged particles spiral along Earth's magnetic field lines and collide with molecules in the atmosphere above the polar regions.

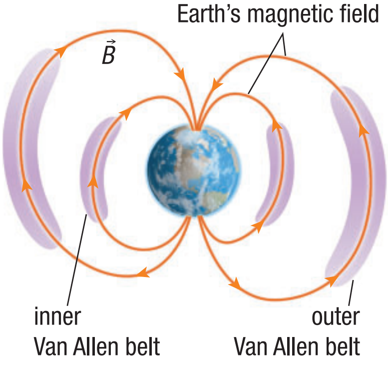


Figure 9 The Van Allen belts are regions of charged particles and radiation trapped by Earth's magnetic field.

At high altitudes in Earth's magnetic field are zones of highly energetic charged particles called the Van Allen radiation belts (**Figure 9**). James A. Van Allen, an American physicist, discovered the toroidal (doughnut-shaped) zones of intense radiation while studying data from a satellite he built in 1958. Van Allen was able to show that charged particles from cosmic rays were trapped in Earth's magnetic field.

Most intense over the equator, the Van Allen belts are almost absent over Earth's poles and consist of an inner region and an outer region. The outer Van Allen belt contains charged particles from the atmosphere and the Sun, mostly ions from the solar wind. The inner Van Allen belt is a ring of highly energetic protons. The concentration of charged particles and radiation can easily damage electronic equipment, so researchers program the paths and trajectories of satellites and spacecraft to avoid the belts.

Field Theory

We associate the term *force* with a physical action of one object on another. When we talk about the force of a bat against a baseball, our minds use a concept of contact between the objects, which transmits the force. To develop a more accurate concept of force, we need to talk about it in terms of fields.

We know that all objects are made of atoms interacting without actually touching each other. There are spatial gaps between the atoms in a bat and a baseball, so the idea that the bat makes contact with the ball is deceptive. In reality, electromagnetic forces affect the interacting atoms in each object.

How do we create an understanding of the gravitational, electric, and magnetic forces? We need a scientific model that describes different types of forces that exist at different points in space, and field theory does that. **Field theory** is a scientific model that describes forces in terms of entities, called *fields*, that exist at every point in space. The general idea of fields links different kinds of forces once thought of as separate. Field theory states that if an object experiences a specific type of force over a continuous range of positions in an area, then a field exists in that area. Field theory can be applied in explaining the minute interactions of subatomic particles as well as describing motions of galaxies throughout the universe.

field theory a scientific model that describes forces in terms of entities that exist at every point in space

Studying gravitational, electric, and magnetic forces has revealed differences and similarities between these forces and their respective fields. The electric and magnetic fields have a stronger effect on the motion of subatomic particles, such as protons and electrons, but the gravitational field has a stronger effect on large objects, such as planets, galaxies, and clusters of galaxies (**Figure 10**).

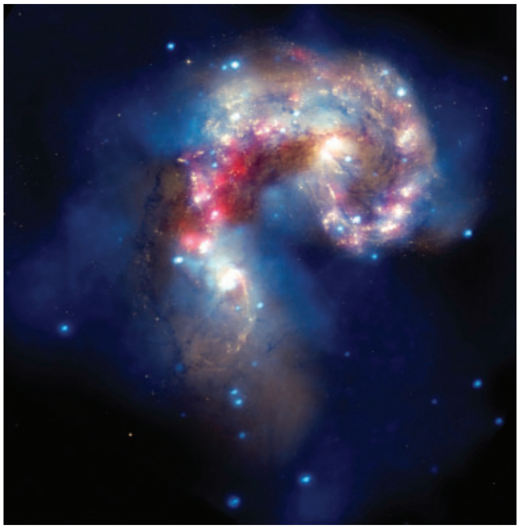


Figure 10 Gravity controls the collision of two clusters of galaxies, while electricity and magnetism affect the release of radiation during the collision. (Colours have been added to the image to enhance the visual representation.)

The electric and gravitational forces resemble each other in that the force on an object depends on the location of the object. The magnetic force, however, depends on a charged object's motion. The direction of electric and gravitational forces points from the object toward the charge or mass source. The direction of the magnetic force depends on the motion of charged particles with respect to the magnetic field.

Despite these similarities and differences, field theory states that electric and magnetic fields are more closely related to one another than they are to the gravitational field. In fact, the electric and magnetic fields are thought to be different aspects of a single field, the electromagnetic field. They are used in conjunction with one another in a multitude of innovative technologies ranging from particle accelerators to artificial hearts.

8.4 Review

Summary

- If a charged particle moves in a uniform magnetic field so that its initial velocity is parallel to the field, it will not experience a magnetic force. If it moves so that its initial velocity is perpendicular to the field, it will move in a circular path in a plane perpendicular to the magnetic field.
- If a charged particle moves in a uniform magnetic field with a velocity that is neither parallel nor perpendicular to the field, it will move in a spiral path along the field lines.
- The radius r of the circular path a charged particle takes in a uniform magnetic field can be determined from Newton's second law and centripetal acceleration and is given by $r = \frac{mv}{qB}$, where m is the mass of the particle, q is its charge, v is its speed, and B is the magnitude of the magnetic field.
- Charged particles entering Earth's magnetic field are deflected and spiral along the field lines toward the magnetic poles. This motion results in a concentration of charged particles at Earth's north and south magnetic poles.
- Field theory states that if an object experiences a specific type of force over a continuous range of positions in an area, then a field exists in that area.

Questions

1. Explain how a mass spectrometer works. Include a sketch as part of your answer. K/U C
2. Consider a mass spectrometer used to separate the two isotopes of uranium, $^{238}U^{3+}$ (3.952×10^{-25} kg) and $^{235}U^{3+}$ (3.903×10^{-25} kg). Assume the ions enter the magnetic field region of strength 9.5 T with identical speeds and leave the spectrometer with a separation of 2.2 mm (as measured from the entry point) after completing a half-circle turn. Calculate the initial speed of the ions. T/I
3. An electron moves in a circular path perpendicular to a magnetic field of magnitude 0.424 T. The kinetic energy of the electron is 2.203×10^{-19} J. Calculate the radius of the electron's path. Refer to Appendix B for the mass of the electron. T/I
4. A particle carries a charge of 4×10^{-9} C. When it moves with velocity 3×10^3 m/s [E 45° N], a uniform magnetic field exerts a force directly upward. When the particle moves with a velocity of 2×10^4 m/s directly upward, there is a force of 4×10^{-5} N [W] exerted on it. What are the magnitude and direction of the magnetic field? T/I
5. An electron, after being accelerated through a potential difference of 100.0 V, enters a uniform magnetic field of 0.0400 T perpendicular to its direction of motion. Calculate the radius of the path described by the electron. T/I
6. A velocity selector is a device that can choose the velocity of a charged particle moving through a region in which the electric field is perpendicular to the magnetic field, and with both fields perpendicular to the initial velocity of the particle (**Figure 11**). To make the charged particle travel straight through the parallel plates, the downward deflection due to the electric field must equal the upward deflection due to the magnetic field. Suppose you want to design a velocity selector that will allow protons to pass through, undeflected, only if they have a speed of 5.0×10^2 m/s. T/I A

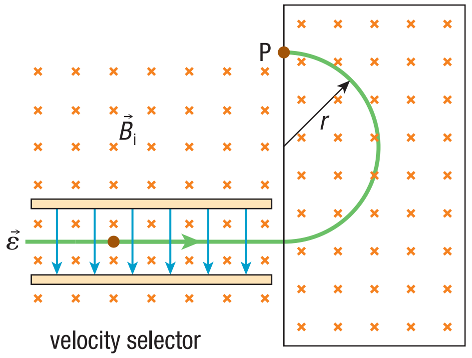


Figure 11

(a) The magnetic field is $B = 0.050$ T. Calculate the electric field you need.
(b) What is the radius of the path the proton takes to get to point P? Refer to Appendix B for the mass of the proton.

Applications of Electric and Magnetic Fields

8.5

If you have a pet dog or cat, you probably have tags on it that give information on how to contact you if the pet ever gets lost. Unfortunately, these tags can fall off. A more secure way to identify, or ID, your pet is to have your veterinarian implant an ID microchip under the skin. The microchip stores information electronically and can be read by a scanner.

Microchips have many uses. Credit card companies and banks use them to help prevent theft of account information and fraud. Microchips are more secure than magnetic stripes, less easily damaged, and very convenient. With contactless chip cards, which are read using a magnetic field, you do not even have to swipe or insert your card; you just wave the card over the microchip reader. These advances in technology are due to our understanding of fields.

RFID Chips

Radio-frequency identification technology (RFID) is a tracking technology that uses microchips less than a millimetre in size (**Figure 1**). The microchips act as transmitters and responders (transponders) to communicate data by radio waves. This technology uses electromagnetic waves, which are a combination of electric and magnetic fields. The tag detects a specific radio signal sent by an RFID reader. When the transponder receives the radio signal, it transmits a unique numerical identification code back to the transceiver. Every tag is encoded with a unique set of numbers for the purpose of identifying and tracking items. RFID was invented in 1969 but has only recently become widely available in commercial applications. RFID tags have uses in product tracking, transportation and logistics, animal and plant identification, and payment systems.

radio-frequency identification technology (RFID) a technology that uses microchips that act as transmitters and responders to communicate data by radio waves

Figure 1 RFID chips come in all shapes and sizes, ranging from pill-shaped capsules to flat tags that can be embedded in credit cards, smart phones, clothing, and even pets.

RFID tags have many technological advantages over bar codes. RFID tags can be read inside containers and through materials such as water and body tissue. They can be embedded into any item not made of metal. They are used in wooden shipping pallets (to identify the products the pallets contain), plastic key fobs, hotel keys, credit cards, gas cards, and driver's licences. While bar codes can only be read one at a time, hundreds of RFID tags can be read simultaneously. Imagine if all the items in a store were enabled with RFID tags; when you pushed your cart through the checkout scanner, the RFID reader and tags could instantly calculate the prices of all the items in the cart (**Figure 2**).

Figure 2 A shopping cart equipped with an RFID scanner and display

Research This

Privacy Concerns Associated with RFID Technology

Skills: Researching, Analyzing, Evaluating, Communicating

SKILLS HANDBOOK A4.1

RFID technology is used in many ways that benefit society. This technology reduces costs for retailers and increases efficiency for consumers in stores by tracking individual products through unique identification codes. This same technology can be used in more controversial ways that may have a negative impact on society by allowing unauthorized, free access to personal information.

1. Research RFID technology, and choose a current product or service that uses RFID tags.
2. Describe how the tags are being used, as well as the advantages of using the RFID technology in this product or service.
3. Research RFID privacy concerns.

A. List privacy concerns associated with this product or service that are currently compromised by the RFID. Discuss possible future privacy issues that might arise with products or services if RFID tags are implemented. K/U T/I C

B. Describe steps that could be taken to reduce privacy issues. T/I C

C. How are RFID tags currently being used? K/U T/I

D. What are some privacy risks associated with RFID tags? K/U T/I

E. What actions should businesses take to ensure the privacy of personal information? K/U T/I

WEB LINK

MR Fluid Dampers

magnetorheological fluid a fluid containing suspended iron particles that, when subjected to a magnetic field, changes to a solid

Can you imagine constructing a building with a material that changes from a solid to a liquid and back to a solid again? Architects and engineers have long known that for a structure to withstand the seismic waves of an earthquake, the structure must be flexible, not rigid. **Magnetorheological fluid** (MR fluid) is a material that can change state from solid to liquid and back to solid again using a magnetic field (**Figure 3**).

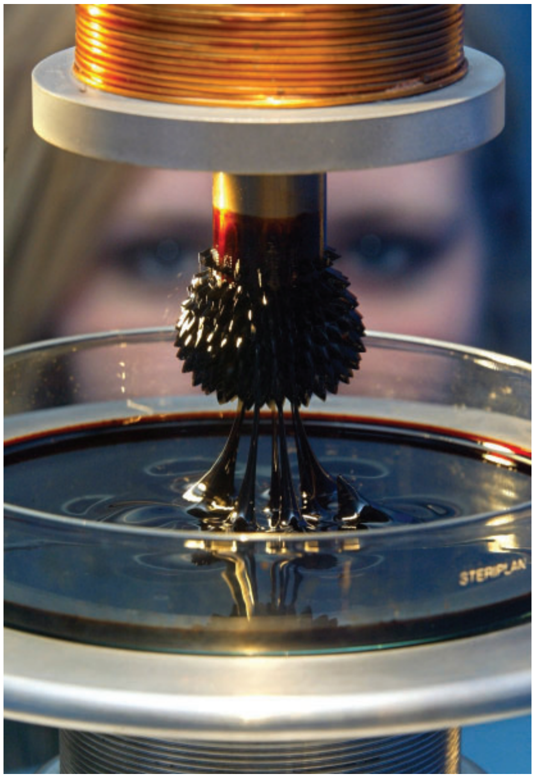


Figure 3 MR fluid reacts to a magnetic field.

Although MR fluids may seem like an idea from a science fiction novel, they are now being used in the construction materials of buildings in earthquake-prone regions. Under normal conditions, an MR fluid is solid, but it changes to a liquid in response to sensors placed in strategic locations that control a magnetic field during an earthquake. This semi-liquid state of certain building components allows a building to absorb shockwaves and reduces potential damage. Buildings constructed with MR fluids are called smart structures. CAREER LINK

The liquid portion of MR fluid material is usually a type of high-viscosity (thick) oil that keeps small iron particles suspended in it. The iron particles are the key to changing the fluid into a solid and vice versa. When a magnetic field is activated near the MR fluid, the fluid greatly increases its viscosity.

Although MR fluid was introduced in the 1940s, the technology required to control the force of the magnetic field, and thus the strength of the fluid, is a recent development. Due to advancements in the technology, MR fluids are now being used in car shock absorbers, washing machines, prosthetics, and exercise equipment.

High-Voltage Power Lines

Every day, we come into contact with invisible lines of force from electric and magnetic fields. They surround any transmission or use of electricity, from high-voltage transmission and power lines to the wiring and lighting in our homes. Electromagnetic fields are also found near household appliances, electronic equipment such as cellphones, and electric motors.

In the 1980s, people began to worry that exposure to the intense electromagnetic fields around high-voltage wires posed serious health risks. Some early studies showed a link between magnetic field strength and an increased risk of cancer. People, particularly young children, living under or near large, high-voltage transformers were thought to be at high risk for developing leukemia. Since that time, however, scientists from Health Canada, the Federal-Provincial-Territorial Radiation Protection Committee (FPTRPC), and the U.S. National Institute of Environmental Health Sciences have independently reviewed over two decades of research involving adults and children exposed to electric and magnetic fields. To date, they have not found clear evidence linking high exposures with the adult cancers studied (breast cancer, brain cancer, and adult leukemia). In addition, they concluded that the studies showed only a weak association between exposure to electric and magnetic fields and childhood leukemia.

The case is not closed, however. Studies involving electromagnetic fields are difficult to perform because they are not controlled investigations. In a controlled investigation, scientists can manipulate one variable and see the outcome on a responding variable while keeping all other conditions constant. Studies related to exposure to electromagnetic fields are often correlational. A correlational study looks for relationships or patterns between measured variables and may depend on many variables affecting the outcome. Sometimes a correlation is weak but is still reported as a result. For example, a neighbourhood may have a high incidence of cancer and be located next to a high-voltage line. A result may be reported saying that high-voltage lines have been linked to cancer. However, the neighbourhood may also be located near a factory that emits a carcinogenic pollutant. The difficulty for scientists is to determine what caused the effect: was it the factory, the high-voltage lines, or both, or neither? It is always important to consider the type of study when interpreting the conclusions. WEB LINK

Medical Applications

magnetic resonance imaging (MRI) a process in which magnetic fields interact with atoms in the human body, producing images that doctors can use to diagnose injuries and diseases

Water, composed of hydrogen and oxygen atoms, is a part of all cells. The human body is approximately two-thirds water by mass and contains billions of hydrogen atoms. In a **magnetic resonance imaging (MRI)** device, magnetic fields interact with these hydrogen atoms, producing images that doctors can use to diagnose injuries and disease. The MRI uses a superconducting magnet to create a large, stable magnetic field of approximately 2.0 T. The large magnetic field is needed to produce precise images of the soft tissues inside the human body.

For an MRI machine to obtain images, a patient must lie on a movable bed that slides into a tube in the centre of the magnetic field. Hydrogen atoms in the body can behave like atomic-sized compasses whose north and south poles normally point in random, changing directions. When the body enters the magnetic field, the body's hydrogen atoms align their poles either in the direction of the field or opposite to the direction of the field. The number of atoms aligned with the field will almost equal the number aligned opposite, but there will be a small difference in the numbers (about one in a million). This difference in the number of atoms aligned versus anti-aligned depends on the particular material that the atoms are part of (such as skin, bone, or organs) and whether the material is normal and healthy or abnormal and diseased.

Next, a radio-frequency pulse is directed toward the area of the body to be examined. The pulse will cause the anti-aligned atoms to spin and align with the magnetic field. When the pulse ends, those atoms spin around again, but emit energy they absorbed from the pulse. The MRI device sends a regularly repeating radio-frequency pulse, which causes the atoms to emit a regular energy signal that can be detected by receivers.

While this is happening, three gradient magnets are activated, quickly turning on and off in a particular pattern. The gradient magnets are much smaller than the primary magnet, but they allow for precise alteration of the magnetic field. By altering the gradient magnets, the magnetic field can be specifically focused on a selected part of the body. The MRI device sends signal information to a computer, which converts the data into an image (**Figure 4**). Manipulating the gradient magnets in the MRI allows doctors to obtain three-dimensional pictures of specific body areas without moving the patient's body.

UNIT TASK **BOOKMARK**

You can apply what you have learned about applications of electric and magnetic fields to the Unit Task on page 422.

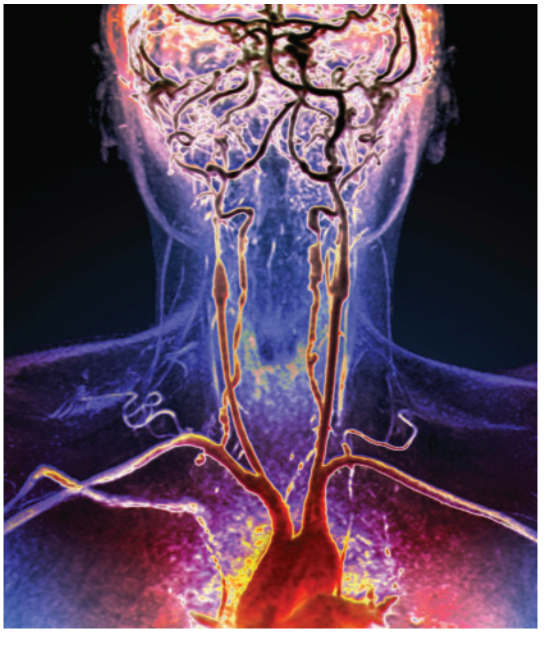


Figure 4 MRI imagery, combined with a contrast medium, has revealed aneurysm swellings in the neck and brain arteries in this 37-year-old patient. An aneurysm is a swelling caused by weakened blood vessel walls. If an aneurysm ruptures, it can cause a stroke.

8.5 Review

Summary

- Radio-frequency identification (RFID) technology is possible through the use of electromagnetic waves, which are a combination of electric and magnetic fields.
- Magnetorheological (MR) fluid is a fluid of iron particles suspended in a thick liquid that can change state from solid to liquid and back again when subjected to magnetic force.
- Current research indicates that exposure to high-voltage electric fields does not increase the risk of developing certain types of cancers, but research is ongoing.
- Magnetic resonance imaging (MRI) uses magnetic fields to produce three-dimensional images of internal body systems that provide doctors with clear and precise information about the patient's condition.

Questions

1. Describe RFID technology in your own words. K/U
2. Suppose your local grocery store uses RFID technology to keep track of all its products. T/I A
   (a) Describe the effect this technology will have on each of the following and explain your reasoning:
      (i) the number of cashiers required
      (ii) the amount of shoplifting
      (iii) the convenience for the shopper
      (iv) inventory tracking
   (b) Suppose all grocery stores started using RFID technology. List possible negative effects.
3. Highlight and analyze the major challenges ahead in the utilization of RFID technology. T/I C A
4. Explain the significance of magnetism to magnetorheological fluid. K/U
5. (a) Identify three uses for magnetorheological fluid.
   (b) Choose one of these applications, research it, and describe how the MR fluid is used in this application. K/U T/I C
6. What is a smart structure? K/U
7. Describe our current understanding of the relationship between high-voltage electric fields and human health. K/U
8. Outline the basic principle involved in magnetic resonance imaging devices. K/U
9. (a) Explain the function of magnetic fields in an MRI.
   (b) How do doctors see the differences between healthy and damaged tissue? K/U
10. An emerging technology, magnetophoresis, involves studying the motion of dispersed magnetic particles in a fluid influenced by a magnetic field (**Figure 5**). The movement of magnetic particles can be used to detect or isolate specific components in the fluid. This has implications in medicine and biotechnology. Research the various applications of magnetophoretic technology. C A

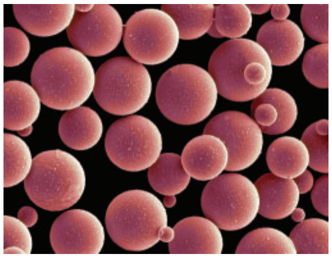


Figure 5 One application of magnetophoresis is magnetic flow sorting, where tiny magnetic balls are used to select cells.

    (a) How have they improved medical treatments?
    (b) In what ways have they advanced biotechnology?
11. Research and then write a brief article about superconductivity. What are the tools and conditions needed to achieve superconductivity? Discuss some of the many technologies made possible by the immense magnetic fields generated by superconducting wires. C A
12. Research geomagnetism and migratory animals. What species are especially sensitive to magnetic fields (**Figure 6**)? How do researchers measure this sensitivity, and how do they track the animals? Write a short report to share with the class. C A

Figure 6 The loggerhead sea turtle uses Earth's magnetic field for navigating.

WEB LINK

8.6 Explore an Application in Electromagnetic Fields

SKILLS MENU

- Researching
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating
- Identifying Alternatives

Particle Accelerators

Particle acceleration started with the idea that smashing atoms together could reduce them to their most fundamental parts. This led to the discovery of subatomic particles other than electrons, protons, and neutrons, as well as their anti-particles. From the first smaller-scale atom-smashing accelerators in the early nineteenth century to the Large Hadron Collider (LHC) at CERN, Switzerland (**Figure 1**)—which actually propels subatomic particles—particle accelerator engineering has changed a great deal.

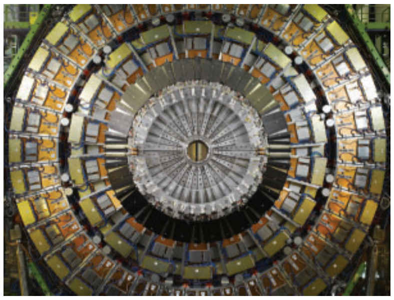


Figure 1 Large Hadron Collider (LHC) at CERN, Switzerland

Figure 2 Stanford Linear Accelerator at Stanford University

Particles can be accelerated in a linear or circular manner. Linear particle accelerators, such as the Stanford Linear Accelerator (SLAC) (**Figure 2**), move charged particles in a straight path. The particles are confined in a narrow beam by electromagnets and move through a series of long vacuum "drift" tubes. The particles are accelerated as they are attracted to alternate charges in the tubes and continue to accelerate until they are smashed against a target. Linear accelerators can range in size from a cathode ray tube a few centimetres long to the 3.2 km–long SLAC.

Circular accelerators, such as the TRIUMF cyclotron at the University of British Columbia (**Figure 3**), use a circular path and a series of superconducting magnets to accelerate particles. Charged particles are propelled around the track and continually accelerated by the magnetic fields. Particle collisions are controlled at specific points along the path.

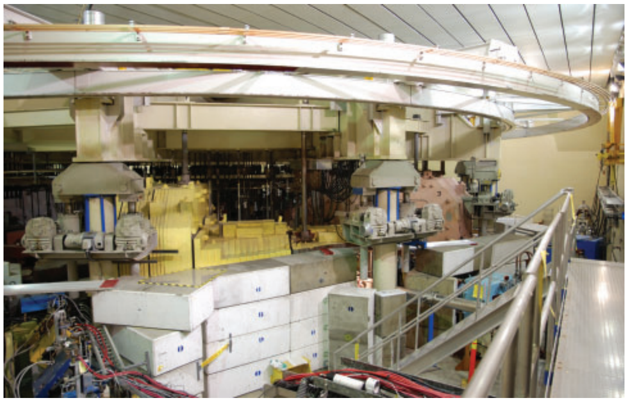


Figure 3 TRIUMF cyclotron at the University of British Columbia

Particle accelerator technology has everyday applications. A simple form of accelerator can be found in the ordinary cathode ray tube found in older televisions and computer monitors. Aside from the large machines primarily used for research, particle accelerators are used in radiotherapy, ion implantation, industrial processes and research, and biomedical and other low-energy research.

The Application

There are more than 70 large-scale, research-oriented particle accelerators around the globe, perhaps the most famous of which is the LHC. In addition, there are more than 26 000 accelerators worldwide being used for research and in industrial or commercial applications.

Your Goal

To analyze the operation of particle accelerators and assess their impact on society and the environment

Research

SKILLS HANDBOOK A5.1

Working in pairs or small groups, research particle accelerators. Be sure that each group member is responsible for an area of the research. Some suggested areas of research are as follows:

- Particle accelerators in general: Define the different types of accelerators currently in use. Why are there different types? What typical speeds do the particles reach in each type? What happens to the particles after they are accelerated into the target, and how do scientists observe them?
- Compare accelerators used in pure research to those used in industrial applications. What types of research have been conducted with each type of particle accelerator, and what were the outcomes?
- Research the TRIUMF cyclotron at the University of British Columbia. How large is it? For what research projects is it currently being used?
- Some particle accelerators provide high-energy particles that can be used for medical imaging. Research how these particles are used. What are some other uses for particle accelerators?
- The cost of creating a large-scale particle accelerator is high, which is often an argument against them. Research this aspect of particle accelerators.
- What jobs, locally and internationally, are created through the construction and operation of an accelerator? How might the information learned from research done with a large-scale particle accelerator benefit society?
- Large-scale particle accelerators require a great deal of land to be built. Research the impact these have had on surrounding communities. Consider the impact on both people and the environment. WEB LINK

Summarize

Use the following questions to summarize your group's findings about particle accelerators:

- How has pure research using particle accelerators benefited society in the past?
- What are the differences between scientific labs that function for pure research and those that conduct research leading to the development of products? How does each affect society locally and globally?
- What effects do particle accelerators have on the surrounding community? Consider both positive and negative effects, such as job opportunities, and environmental factors, such as pollution and effects on wildlife and habitat.

Communicate

Communicate your findings in a short speech to your class. You and your partner or group can include visual aids, such as T-charts, flow charts, video clips, or any other suitable format. If you are not presenting individually, be sure that each group member has an individual role during the presentation.

Investigation 8.2.1 CONTROLLED EXPERIMENT

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Observing the Magnetic Force on a Moving Charge

You have learned that the strength of a magnetic field varies and that a magnetic field can affect charged particles as they pass through the field. You have also learned about technology that uses these principles. This controlled experiment will give you an opportunity to demonstrate and analyze the factors that affect magnetic force on moving charges.

This investigation is sometimes done using a cathode ray tube (**Figure 1**), but you will use simulation software that allows you to send charged particles through magnetic fields.

Figure 1

Testable Question

How does a magnetic field affect moving charges?

Prediction

Based on what you have learned in this chapter, predict the effect of

(i) the direction of the magnetic field on the charged particle

(ii) the strength of the magnetic field on the charged particle

(iii) the speed of the particle on its path through the magnetic field

(iv) the amount of charge on the particle on the motion of the particle

(v) the sign of the charge on the particle on the motion of the particle

Variables

Identify the controlled, independent, and dependent variables.

Experimental Design

Simulation software will be used to represent charges being fired into a magnetic field. The amount of charge on the particles as well as the sign of the charge will be changed and the effect of the path of the particles observed.

Equipment and Materials

- computer with simulation software

Procedure

Part A: Magnetic Field

1. Create a table to record your observations.
2. Go to the Nelson Science website. Using the simulation software, fire a charged particle into the magnetic field; then change the value of the magnetic field strength and direction, and record and sketch your observations.

Part B: Speed of Charges

3. Using the simulation software, adjust the initial speed of the particle. Record your observations, taking care to note the effect on the magnetic force by sketching the paths of the charged particles at different velocities. Be as quantitative as possible in recording the effect of the force on the charges.

Part C: Charge on the Particles

4. Using the simulation software, change the amount of charge on the particles. Record the effect of the force on the charge.

Part D: Sign of the Charge on the Particle

5. Repeat Step 4, using charges of opposite sign, and record any changes in the path of the particles.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) What variables were manipulated in this investigation? What type of relationship was being tested? T/I

(b) How did the direction of the magnetic field impact the moving charges? What evidence did you collect to support your answer? T/I

(c) Construct a graph of the data you collected in Part A to communicate the effects of strong and weak magnetic fields on a beam of electrons. T/I C

(d) Analyze your graph from (c). What can you conclude? T/I C

(e) Summarize the effects of increasing the initial speed of the particles on the particles' path through the magnetic field. T/I C

(f) What information would you need to be able to calculate the magnitude of the force on the electrons? K/U T/I

(g) Summarize the effects of increasing the amount of charge on the particles. T/I C

(h) What effect did changing the sign of the charge have on the particles' path? T/I

(i) Consider the predictions you made before this experiment. Did your predictions match what you observed? Why or why not? T/I C

Apply and Extend

(j) Use the results of your experiment to answer the following questions: K/U T/I A

  (i) Explain why moving a speaker close to a television screen distorts the image. Assume both devices are turned on.

  (ii) Explain to a patient who is about to undergo an MRI test why he should remove metal jewellery.

(k) What additional tests or simulations might improve the results of this experiment? T/I

WEB LINK

CHAPTER 8 SUMMARY

Summary Questions

1. Create a study guide or graphic organizer for all equations used in determining the direction of a magnetic field and for calculating magnetic force. Create a sample problem for each formula.
2. Look back at the Starting Points questions on page 376. Consider how you first answered them in comparison with what you have learned. Write a summary of the changes in your thinking on each question.
3. List two technologies that use the principles of magnetism. Describe the principle(s) at work in each technology.

Vocabulary

magnetic field line (p. 378)

principle of electromagnetism (p. 382)

right-hand rule for a straight conductor (p. 382)

right-hand rule for a solenoid (p. 383)

tesla (p. 386)

right-hand rule for a moving charge in a magnetic field (p. 387)

field theory (p. 403)

radio-frequency identification technology (RFID) (p. 405)

magnetorheological fluid (p. 406)

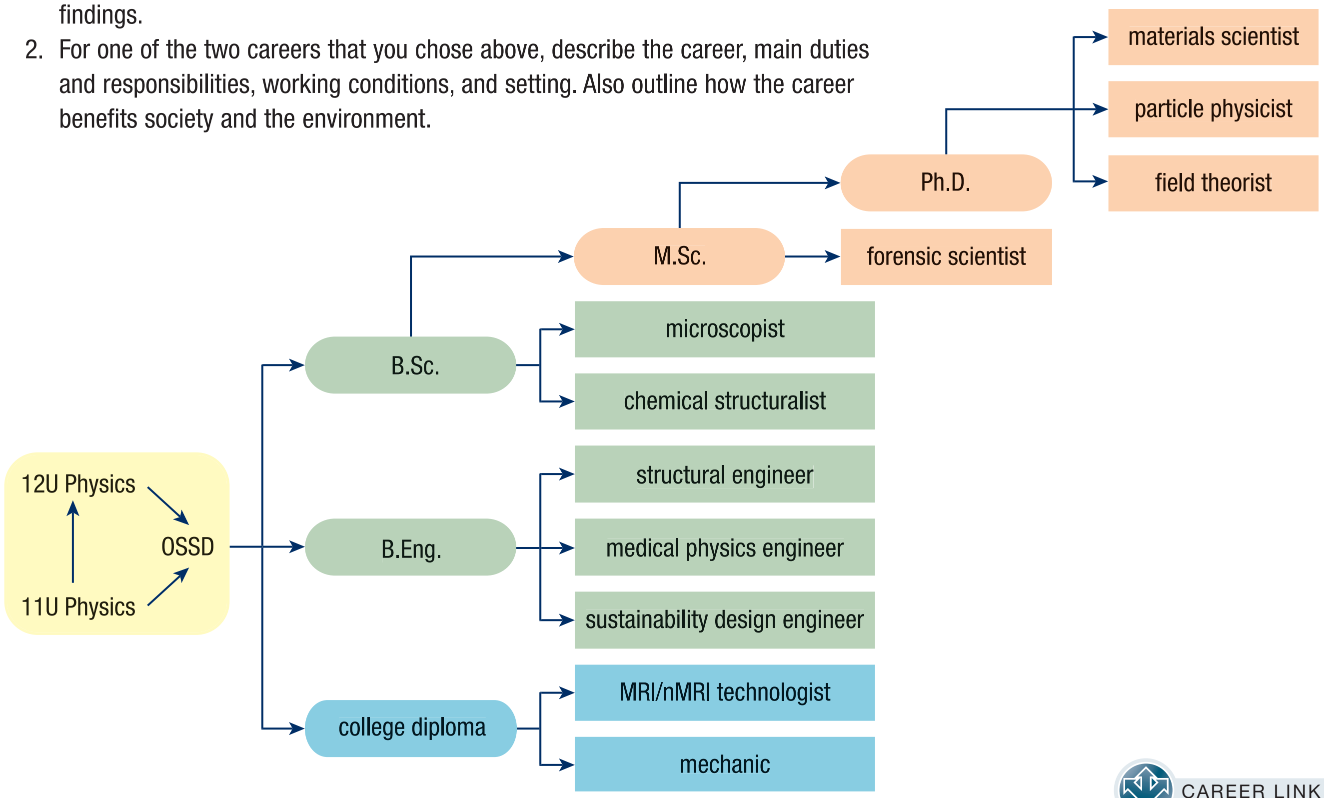
magnetic resonance imaging (MRI) (p. 408)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers related to topics covered in this chapter.

1. Select two careers related to magnetic fields that you find interesting. Research the educational pathways that you would need to follow to pursue these careers. What are the requirements for the educational programs? Prepare a brief report of your findings.
2. For one of the two careers that you chose above, describe the career, main duties and responsibilities, working conditions, and setting. Also outline how the career benefits society and the environment.



CAREER LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. The SI unit of magnetic field strength is the
   (a) watt
   (b) tesla
   (c) coulomb
   (d) newton (8.1) K/U
2. The magnetic lines of force around a straight current-carrying conductor are
   (a) parallel to the conductor
   (b) opposite to the direction of the current
   (c) in the same direction as the current
   (d) circular (8.1) K/U
3. Parallel and equidistant lines of forces indicate that the magnetic field is
   (a) very high
   (b) very low
   (c) uniform
   (d) non-uniform (8.1) K/U
4. A current flows in a conductor from east to west. The direction of the magnetic field at a point above the conductor is
   (a) toward the north
   (b) toward the south
   (c) toward the east
   (d) toward the west (8.1) T/I
5. The strength of Earth's magnetic field has an average value of about
   (a) 5×10^{-5} T
   (b) 2×10^{-6} T
   (c) 3×10^{-4} T
   (d) 4×10^{-2} T (8.2) K/U
6. A power line lies along the east–west direction and carries a current of 10 A. The force acting on a 1 m segment due to Earth's magnetic field of 10^{-5} T is
   (a) 10^{-2} N
   (b) 10^{-4} N
   (c) 10^{-3} N
   (d) 10^{-5} N (8.3) T/I
7. When a conductor is carrying a current, there will be no force acting on it when the conductor is placed in a magnetic field
   (a) parallel to it
   (b) perpendicular to it
   (c) at an angle of 90° to it
   (d) at an angle of 60° to it (8.3) K/U
8. An electric charge, q, moves with a constant velocity, v, parallel to the lines of force of a uniform magnetic field, B. The force experienced by the charge is
   (a) Bqv
   (b) $\frac{qv}{B}$
   (c) zero
   (d) $\frac{q}{Bv}$ (8.3) K/U
9. An MRI machine uses this to align atoms. (8.5) K/U
   (a) a powerful electric field
   (b) a weak electric field
   (c) a powerful magnetic field
   (d) a weak magnetic field

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

10. The points of weakest magnetism are near the poles. (8.1) K/U
11. A long wire is called a solenoid. (8.1) K/U
12. Using an iron core inside a solenoid makes the solenoid's magnetic field stronger. (8.1) K/U
13. Magnetic fields exert a force on a charged particle whether it is moving or at rest. (8.1) K/U
14. Moving electrically charged particles, such as a current-carrying conductor, cannot create magnetic fields. (8.1) K/U
15. The source of a magnetic field is analogous to the source of an electric field. (8.1) K/U
16. When an electron is moving parallel to the direction of a magnetic field, the motion of the electron will not be affected. (8.2) K/U
17. Speakers in sound systems operate on the principle of a magnetic force acting on a current-carrying wire in a magnetic field. (8.3) K/U
18. A current-carrying wire experiences a sideways force in the presence of a magnetic field. (8.3) K/U
19. A mass spectrometer measures the mass-to-charge ratio of a charged particle. (8.4) K/U
20. Auroras are shaped by the atmosphere. (8.4) K/U
21. Transponders are used to communicate data via radio waves in RFID technology. (8.5) K/U A
22. A particle accelerator uses electromagnetic fields to propel charged particles to high speeds. (8.6) A

Go to Nelson Science for an online self-quiz.
WEB LINK

Knowledge

For each question, select the best answer from the four alternatives.

1. Which of the following magnetic properties is exhibited by Earth? (8.1) K/U
   (a) Earth has a magnetic north and a magnetic south pole.
   (b) Earth has magnetic field lines that are uniformly spaced.
   (c) Earth attracts and repels magnetic compass needles.
   (d) Earth repels all cosmic rays.
2. Which of the following results in an auroral display? (8.1) K/U
   (a) protons and neutrons sliding along Earth's magnetic field lines, creating an electric field
   (b) electrically charged particles colliding in Earth's atmosphere, releasing energy in the form of light
   (c) increased solar wind creating flares in space visible on Earth
   (d) Earth's magnetic field increasing in intensity and releasing charged particles as visible light
3. Which of the following is a type of magnetic material? (8.1) K/U
   (a) cadmium
   (b) aluminum
   (c) magnesium
   (d) cobalt
4. Which of the following principles is attributed to Hans Oersted? (8.1) K/U
   (a) the principle of magnetism
   (b) the principle of electricity
   (c) the principle of electromagnetism
   (d) the principle of magnetic fields
5. Which of the following requires the use of the right-hand rule? (8.1) K/U
   (a) determining the direction of the magnetic field around a straight conductor
   (b) determining the speed of the current in a wire
   (c) determining the direction of the electric field in a conductor
   (d) determining the mass of a particle in a magnetic field
6. A charged particle moves with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The magnetic force experienced by the particle is
   (a) always zero
   (b) never zero
   (c) zero if $\vec{B}$ and $\vec{v}$ are perpendicular
   (d) zero if $\vec{B}$ and $\vec{v}$ are parallel (8.2) K/U
7. Which of the following factors are needed to calculate the magnitude of a magnetic force on a moving charge in a magnetic field? (8.2) K/U
   (a) velocity of the charge, mass of the charged particle, density of the magnetic field
   (b) amplitude of the electric current, distance the charge is travelling, strength of the magnetic field
   (c) length of the magnetic field, number of moving charges, velocity of the charge
   (d) velocity and charge of the moving charge, angle between the velocity and the magnetic field, strength of the magnetic field

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. When lightning strikes, Earth's magnetic field deflects the negatively charged particles. (8.2) K/U
9. A right-hand rule gives the direction of the magnetic force on a current-carrying conductor in an external magnetic field. To determine this, point your fingers in the direction of the current and wrap your fingers in the direction of the magnetic field. (8.3) K/U
10. Scientists use a mass spectrometer to identify unknown compounds, to determine the structure of a compound, and to understand the isotopic makeup of molecular elements. (8.4) K/U
11. Auroras are formed when charged particles collide with atoms and molecules in the upper atmosphere. (8.4) K/U
12. Field theory includes the study of the principles of magnetic fields, electric fields, spectral fields, and gravitational fields. (8.4) K/U
13. Electric fields and magnetic fields are more closely related to each other than they are to gravitational fields. (8.4) K/U
14. Exposure to high-voltage electric fields increases the risk of developing cancers. (8.5) K/U
15. Magnetic resonance imaging devices are used to create three-dimensional images of the body's soft tissues. (8.5) K/U

Understanding

16. **Figure 1** shows a bar magnet surrounded by iron filings. (8.1) T/I

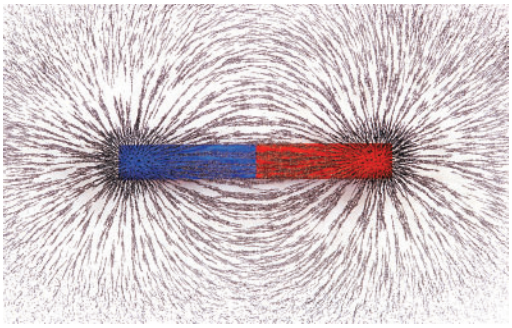


Figure 1

(a) In which direction do the iron filings align with the bar magnet?
(b) In which direction do the magnetic field lines point?

17. **Figure 2** shows a horseshoe magnet. (8.1) T/I C

Figure 2

(a) In your notebook, draw the magnetic field lines, showing their direction.
(b) In which area is the magnetic field strongest?

18. In your school lab, you place iron filings in a glass tube. Describe the effect on iron filings in the glass tube if the tube is placed near a strong magnetic force. (8.1) K/U
19. A compass needle placed on a level surface points to the letter N. What does this tell the compass reader? (8.1) K/U
20. Earth's magnetic north and south poles move from year to year. What is the probable cause of this phenomenon? (8.1) K/U
21. An electric cord from a lamp is placed near a compass. Explain what happens to the compass when the lamp is turned on. (8.1) K/U
22. Which solenoid in **Figure 3** has a stronger magnetic field? (8.1) T/I

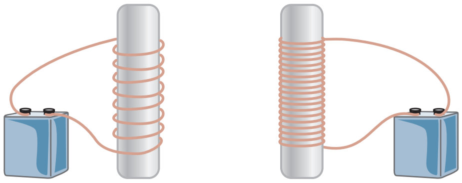


Figure 3

23. What can be added to a solenoid to create a stronger magnetic field? (8.1) K/U T/I
24. For the solenoid in **Figure 4**, the direction of the current is shown. Copy the figure into your notebook. Indicate the direction of the magnetic field inside the solenoid. (8.1) T/I

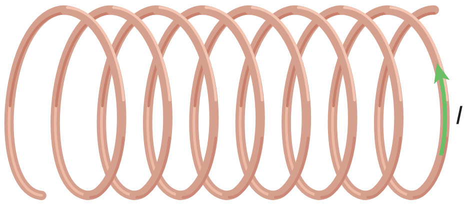


Figure 4

25. The magnitude of the magnetic force, F_M, on a charge, q, moving with velocity $\vec{v}$ perpendicular to a magnetic field, $\vec{B}$, is given by $F_M = qvB$. What is the direction of $\vec{F}_M$ relative to $\vec{v}$ and $\vec{B}$? (8.2) T/I
26. A uniform magnetic field with a magnitude of 1.2×10^{-3} T points vertically upward through a laboratory chamber. A proton with a velocity of 3.2×10^7 m/s enters the chamber moving horizontally from south to north. (8.2) T/I
    (a) What is the magnitude of the magnetic force on the proton?
    (b) What is the direction of the magnetic force on the proton?
27. An electron moving with speed v enters a uniform magnetic field. Describe the path of the electron through the field for the following conditions. (8.2) T/I
    (a) It enters parallel to the field direction.
    (b) It enters perpendicular to the field direction.
    (c) It enters at some other angle to the field direction.
28. An electron is travelling near Earth's equator. In which direction does it tend to deflect if its velocity is directed
    (a) vertically downward?
    (b) horizontally westward?
    (c) horizontally southward? (8.2) T/I

29. (a) Determine the magnitude of the maximum magnetic force that could be exerted on a proton of mass 1.67×10^{-27} kg moving through a magnetic field of magnitude 1.50 T at a speed of 6.00×10^{6} m/s.
(b) What is the magnitude of the maximum acceleration of the proton? (8.2) T/I

30. A shock absorber used in building a structure to withstand earthquakes uses the properties of tiny iron particles suspended in oil. In the presence of a magnetic field, the particles align and make the liquid nearly solid. The liquid is activated by a magnetic field with a magnitude of 0.080 T. A particle with a charge of 2.0×10^{-11} C moves perpendicular to the magnetic field with a speed of 4.8 cm/s. Determine the magnitude of the force that the particle will experience. (8.2) T/I

31. **Figure 5** shows a straight conductor between the poles of a permanent magnet. If a current is moving in the direction indicated, what is the direction of the force on the conductor due to the magnetic field? (8.1, 8.3) T/I

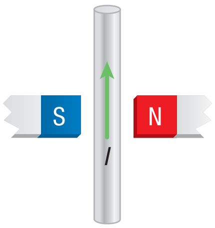


Figure 5

32. In early televisions, pictures were created when a beam of electrons streamed from the back of the set and struck the screen. If a television were positioned horizontal to Earth's magnetic field, in which direction(s) should it be oriented so that the beam undergoes the largest deflection? (8.3) T/I

33. A current of 10.0 A flows through a wire at a 30.0° angle to the direction of a 0.300 T magnetic field. Determine the magnetic force on a 5.00 m length of that wire. (8.3) T/I

34. Calculate the minimum magnitude of the magnetic field that would be needed to make a proton move in a circular path around the equator of Earth with a speed of 1.0×10^{7} m/s. Earth's radius is 6.4×10^{6} m, and the mass of the proton is 1.67×10^{-27} kg. (8.4) T/I

35. A mass spectrometer separates ions according to their charges and masses. One simple design for such a device is shown in Figure 4 on page 400. Ions of mass m, charge q, and speed v enter a region in which the magnetic field B is uniform and perpendicular to the plane. The ions then travel in a circular arc and leave the spectrometer a distance $L = 2r$ from their entry point. Consider a hypothetical mass spectrometer used to study the isotopes of hydrogen. Calculate r for H^+, $^2H^+$ (deuterium), and $^3H^+$ (tritium). (8.4) K/U

36. A charged particle is moving through a velocity selector (a region in which the electric field is perpendicular to the magnetic field, with both fields perpendicular to the initial velocity of the particle) (**Figure 6**). Consider a case with an electric field strength of 510 V/m and a magnetic field strength of 0.025 T. (8.4) K/U T/I

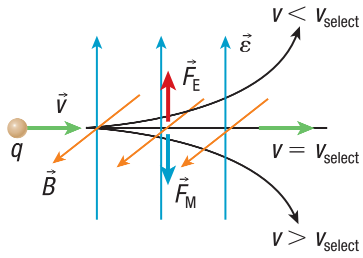


Figure 6

(a) Calculate the speed selected for protons.
(b) Calculate the speed selected for Ca^{2+} ions.
(c) Describe the change necessary to Figure 6 if you were to use your selector to select all incoming F^- ions with speeds less than a selected value.

Analysis and Application

37. **Figure 7** shows the magnetic field around a current-carrying wire. The arrows indicate the direction of the magnetic field. (8.1) T/I

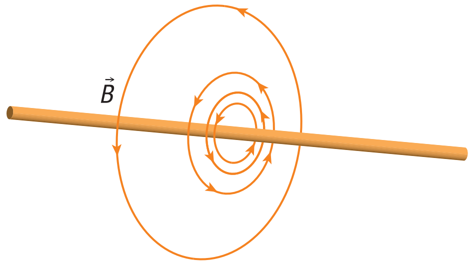


Figure 7

(a) In which direction is the current?
(b) In which direction should the current move to reverse the magnetic field direction?

38. In **Figure 8**, a wire is shown to carry a conventional current in three different directions. Determine the direction of the magnetic field that will produce the magnetic force in each case. (8.2) T/I

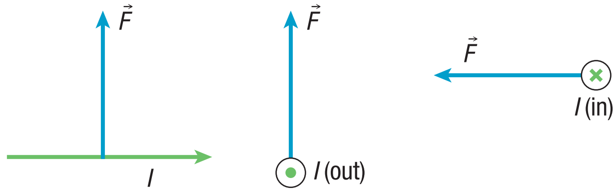


Figure 8

39. The magnetic field in **Figure 9** has a magnitude of 3.0 T. Determine the magnetic force experienced by

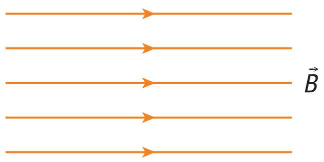


Figure 9

(a) an electron at rest
(b) an electron moving upward at 2.0 m/s
(c) an electron moving to the left at 2.0 m/s (8.2) T/I

40. In **Figure 10**, an electron is travelling to the left and moves into a magnetic field directed out of the page. Describe the path of the particle through the field. (8.2) T/I

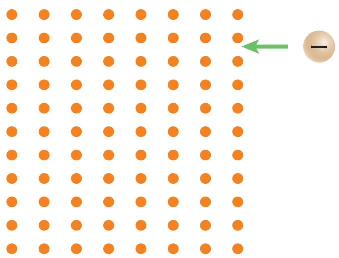


Figure 10

41. A magnetic field has strength 0.3 T. A particle with a positive charge of 0.006 C and speed 400 m/s is moving perpendicular to the magnetic field. Determine the magnitude of the magnetic force exerted on the particle. (8.2) T/I

42. A proton moves into a magnetic field of 5.4×10^{-2} T in a direction perpendicular to the field at a speed of 4.8×10^5 m/s. Determine the force on the particle. (8.2) T/I

43. An electron moves with a speed of 6.9×10^3 m/s in a direction perpendicular to a magnetic field directed south to north and of strength 1.3×10^{-2} T. What are the magnitude and direction of the force on the electron if the electron is moving from east to west? (8.2) T/I

44. A small particle with a charge of 5.0×10^{-16} C enters a magnetic field of strength 2.4×10^{-2} T at right angles with a speed of 4.9×10^5 m/s. Determine the magnitude of the magnetic force on the particle. (8.2) T/I C

45. An electron moves at 3.0×10^3 m/s perpendicularly into a magnetic field of intensity 2.4×10^{-2} T. Determine the force on the particle. (8.2) T/I

46. An electron travelling at right angles to a magnetic field with a speed of 8.0×10^4 m/s experiences a force of 3.8×10^{-18} N. Determine the magnetic field strength. (8.2) T/I

47. An oxygen ion, O^{2-}, with a mass of 2.7×10^{-26} kg is moving with a speed of 310 m/s in a direction perpendicular to a magnetic field of magnitude B. The acceleration of the O^{2-} ion is 1.5×10^9 m/s^2. What is B? (8.2) T/I A

48. A nitrogen ion, N^+, is travelling through the atmosphere at a speed of 520 m/s. What is the approximate force of this ion on Earth's magnetic field when the ion is moving perpendicular to Earth's magnetic field? Assume a value of 5.5×10^{-5} T for Earth's magnetic field. (8.2) T/I A

49. A proton travels with a velocity of 1.00×10^5 m/s [E]. The proton enters a research building that is using a strong magnetic field of 55.0 T. When the proton moves eastward, the magnetic force acting on it is directed straight upward, and when it moves northward, no magnetic force acts on it. (8.2) T/I
    (a) Determine the direction of the magnetic field.
    (b) What is the magnitude of the magnetic force when the proton moves eastward?

50. Assume an electron is moving at a speed of 2.5×10^5 m/s due west in a 55.0 T magnetic field directed north to south. (8.2) T/I
    (a) What is the magnitude of the magnetic force on the electron?
    (b) What is the direction of the magnetic force?

51. A particle is travelling with a speed of 400 m/s parallel to a magnetic field of strength 0.3 T. The particle has a positive charge of 0.006 C. Determine the magnitude of the magnetic force exerted on the particle. (8.2) T/I

52. How should a current-carrying conductor be placed relative to a magnetic field to maximize the force on the conductor? (8.3) K/U

53. If a current-carrying conductor is placed along the x-axis in a magnetic field along the z-axis, what will be the direction of the magnetic force? (8.3) K/U

54. If a current-carrying conductor is placed parallel to the direction of the magnetic field, what is the magnitude of the force acting on the conductor? (8.3) K/U

55. A wire carries a conventional current along the $+z$-direction and experiences a force along the $-y$-direction. If the magnetic field is perpendicular to the wire, what is the direction of the magnetic field? (8.3) T/I

56. A straight wire of length 0.65 m carries a current of $I = 1.7$ A. The strength of the magnetic field in the region is 1.6 T. If the force on the wire is 1.1 N, determine the angle between the field and the wire. (8.3) T/I

57. In **Figure 11**, a negative charge is moving in a downward direction near an electric current passing through a straight conductor. Determine the direction of the magnetic force on the charge. (8.3) T/I

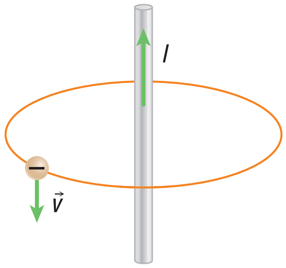


Figure 11

58. A 10 cm–long segment of straight wire is perpendicular to a magnetic field of 1 T. If the wire experiences a force of 2.5 N, determine the current through the wire. (8.3) T/I

59. If the charge of a particle moving in a magnetic field is changed from positive to negative, what happens to the direction of the magnetic force on the charge? (8.3) T/I

60. A conductor 25 cm long carries a current of 50.0 A. It is placed in a magnetic field of strength 49 T. Determine the force exerted on the conductor when it makes each of the following angles with the magnetic lines of force. (8.3) T/I
    (a) 0°
    (b) 45°
    (c) 90°

61. A 5.0 cm segment of wire carrying 2.5 A inside a solenoid is perpendicular to a magnetic field of 25 T. Determine the magnitude of the magnetic force on the segment. (8.3) T/I

62. A 36.0 m length of electrical wire carries a current of 22.0 A from a transformer to a building in a west-to-east direction. The magnetic field of Earth at the transformer location is horizontal and directed from south to north with a magnitude of 5.00×10^{-5} T. Determine the magnitude and direction of the magnetic force on the wire. (8.3) T/I

63. A 30 cm length of appliance cord carries a current of 4.0 A. It is oriented at right angles to a magnetic field of strength 0.3 T. Determine the magnitude of the magnetic force on this segment of the cord. (8.3) T/I

64. A straight conductor is carrying a current of 5.0 A moving toward the east. A magnetic field of 0.2 T points straight up across 1.5 m of the conductor. What is the force acting on the conductor? (8.3) T/I

65. In **Figure 12**, a length of wire is strung between the poles of a magnet. The magnetic field is designated by the orange crosses. (8.4) T/I C
    (a) In which direction is the magnetic field moving?
    (b) In which direction must the current be moving to cause the deflection of the wire shown in each diagram?

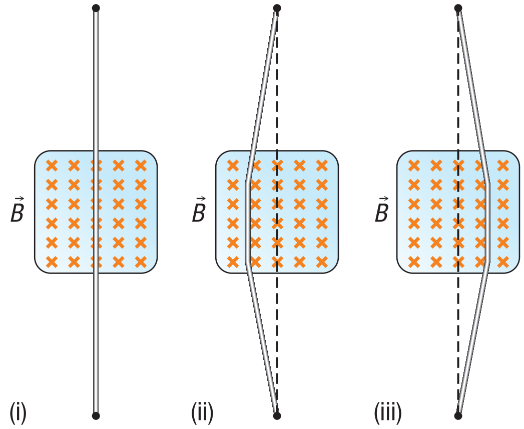


Figure 12

66. A bubble chamber is a device used to study the trajectories of elementary particles such as electrons and protons. **Figure 13** shows the trajectories of some typical particles, one of which is indicated by a white arrow. Suppose this particle is moving in the direction indicated by the green arrow; the particle follows a curved path due to the presence of a magnetic field perpendicular to the plane of the photo. (8.4) K/U T/I A

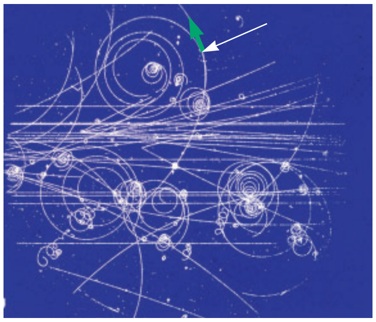


Figure 13

    (a) For this question, we assume that the magnetic field is directed into the page. Does this particle have a positive charge or a negative charge? Explain your answer.
    (b) Describe how the trajectory changes when the charge on the particle is increased by a factor of 3.
    (c) Describe how the trajectory changes when the mass is decreased by a factor of 10.

Evaluation

67. An electrical wire is strung horizontally between two utility poles. The current is travelling toward the east. (8.2) T/I C
    (a) State the rule used to determine the direction of the force on the wire due to Earth's magnetic field.
    (b) Give the formula for calculating the magnitude of the magnetic force on the wire due to Earth's magnetic field.
    (c) Predict what would happen to the magnetic field if the current were to flow toward the west.
68. Summarize how the principles of electricity and magnetic force are applied to make a speaker work. (8.3) T/I C
    (a) What role does electric current play?
    (b) How is sound produced?
69. A vivid and intense aurora is seen over northern Canada but is not visible south of Manitoba. Using what you know about auroras and Earth's magnetic field, answer the following questions. (8.1, 8.4) T/I C
    (a) What accounts for the intensity of the light display?
    (b) How can you explain the range of visibility?
    (c) What is the relationship of the Van Allen belts to this phenomenon?
70. Summarize the properties of the gravitational, electric, and magnetic fields by drawing a table with three columns, labelled Gravitational, Electric, and Magnetic. Label the rows with Affected Particles, Factors Determining Magnitude of Force, Factors Determining Direction of Force, and Relative Strength. Complete the table. (8.4) K/U C
71. Assess and justify how Ampère's advances on the principle of electromagnetism changed scientific thinking. (8.2, 8.5) T/I C

Reflect on Your Learning

72. What did you find most interesting in this chapter? Explain why a particular topic interested you. T/I C
73. Draw the magnetic field lines of a bar magnet and the electric field lines for a current-carrying wire. Compare the two types of lines of force and summarize your result in the form of a short practical demonstration. T/I C
74. Identify and make a list of a few situations where the right-hand rule is used. T/I C
75. Find an analogy of electricity in magnetism. Prepare a short report and discuss with your class. T/I C

Research

WEB LINK

76. Before the advent of the "chip and PIN" system, on the back of most credit cards, transit cards, and bank cards, you would see a magnetic strip that provided information when the card was swiped through a card reader. Research and prepare a one-page summary of how magnetic strip cards work. T/I C A
77. Research and answer the following questions about reversals in Earth's magnetic field. T/I C A
    (a) How did scientists discover field reversals?
    (b) Explain why the magnetic field reverses.
    (c) How often does the magnetic field reverse?
    (d) How may a reversal affect life on Earth?
78. Insulated wires carrying currents in opposite directions are often twisted together in electric circuits. C A
    (a) What is the advantage of doing this?
    (b) What are the implications of other configurations found in electrical wiring plans?
79. Wireless power charging for electric cars is in development. C A
    (a) What are some advantages of this technology?
    (b) What are the challenges in implementing this technology?
80. High-voltage power lines were once thought to pose serious health risks to children and adults who lived near them. Long-term research cast doubt on this idea. New studies are emerging that suggest cellphone use may lead to health risks, including cancer. C A
    (a) What published research can you find that supports this risk?
    (b) What studies are published that dispute it?
81. Magnetic resonance imaging has been a successful instrument in modern medicine and has recently changed due to advances in design technology. Trace the development of magnetic resonance imaging. C A
    (a) How does MRI create a magnetic field inside it?
    (b) What were some limitations of early MRIs?
    (c) How have recent advances improved medical diagnosis and treatment?
82. Space land rovers are instrumental in collecting and delivering information from planets in our solar system. They are possible through scientific collaboration and the application of field theory. Research the relationship among gravitational, electric, and magnetic field science involved in the operation of the Mars land rover. C A

Field Technology in Use

Scientific research in field theory continues in many different areas, including gravity, electricity, and magnetism. Researchers from all of these disciplines collaborate regularly. This collaboration has led to advances in medicine, transportation, space exploration, computer science, and communications networks.

One advance in field technology that has benefited the medical community is the Navigation Brain Stimulation (NBS) **(Figure 1)**. NBS is a safe and non-invasive device that uses short magnetic pulses to monitor the activity of the human brain and central nervous system.

Figure 1 The Navigated Brain Stimulation (NBS) is a technology that uses magnetic fields to safely monitor the workings of the human brain.

How does NBS work? Magnetic pulses from the device stimulate specific areas of the brain. NBS then monitors and records the brain's localized and general reactions to the stimulation. The brains of people who have diseases or a history of trauma will react differently from those of healthy people. NBS is therefore a very useful tool for diagnosing certain disorders of the brain and central nervous system. Researchers also think that specific pulses from NBS may one day be useful in treating depression and chronic pain.

The principles of gravitational, electric, and magnetic fields are used in a broad range of technologies. Field theory principles are often used together to design and build more complex technology. In order to launch communications satellites and keep them in orbit, for example, you need a deep understanding of all three fields (**Figure 2**). To design a magnetic resonance imaging (MRI) machine requires a strong knowledge of both electric and magnetic fields.

Figure 2 Launching and monitoring communications satellites requires an understanding of the principles of gravitational, electric, and magnetic fields.

In this research task, you will use knowledge you acquired from this unit to demonstrate your understanding of field theory, its applications in technology, and its societal impacts.

The Task

Your task is to research and analyze an existing technology that uses a combination of two or more fields. Consider how the different fields are used together, and how this combination affects the design of the technology. Consider the technology's impact on society, the economy, and the environment. You will prepare and present a full technical report of your analysis and evaluations.

Procedure

SKILLS HANDBOOK A4.1

1. Choose resource tools such as the Internet, field experts, and magazine and journal articles to research a technology that operates using applied field theory. Examples include medical devices, electric cars (**Figure 3**), space land rovers (**Figure 4**), and communications devices. WEB LINK

Figure 3 Electric car designs incorporate advances in field theory.

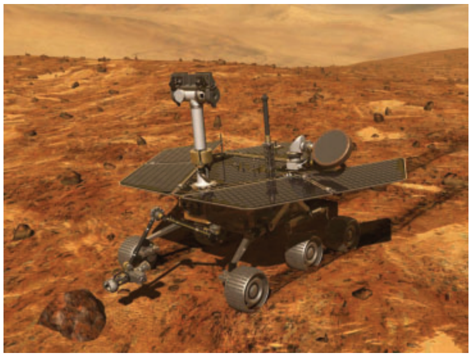


Figure 4 Space land rovers are another type of technology that uses field theory.

2. Once you have chosen a technology to research, analyze and describe the interactions of the fields. Explain how their interactions influenced the design of the technology.
3. Develop an evaluation tool to assess the social, economic, and environmental impacts of your chosen technology. How will you determine the effects? Consider and discuss both positive and negative impacts.

   Consider the following in your assessment:
   - What is the developmental timeline of the technology you chose?
   - What physics principles are present in the design of the technology?
   - What mathematical and physics formulas may have been applied in the technology?
   - What are some advantages and disadvantages of the technology?
   - How does the technology impact the economy, the environment, and society?
   - What careers are associated with the technology?
   - What improvements are being made to the design and use of the chosen technology?
   - Create your own list of questions to analyze this technology.

Communicate

Prepare a professional communication that summarizes your analysis and evaluation of your chosen technology and its impacts on society. Your presentation may be a video documentary; a written report including statistics, images, and graphics; an audiovisual presentation; or another format of your choice.

ASSESSMENT CHECKLIST

Your completed Unit Task will be assessed according to these criteria:

Knowledge/Understanding

- ✓ Demonstrate knowledge of gravitational, electric, and magnetic fields.
- ✓ Demonstrate knowledge of technologies that apply principles of and concepts related to field theory.
- ✓ Describe the economic impact of a technology that uses field theory.
- ✓ Describe social and environmental issues related to field technology.
- ✓ Summarize field theory and its applications.
- ✓ Use symbols, terms, and equations correctly.

Thinking/Investigation

- ✓ Research technologies that combine gravitational, electric, and magnetic fields.
- ✓ Determine mathematical and physics formulas applied in technology that uses field theory.
- ✓ Analyze the impact of field theory on the design of technology.
- ✓ Predict future societal impacts from the use of field theory technology.

Communication

- ✓ Synthesize findings in the form of a written report.
- ✓ Communicate findings in an audiovisual format.

Application

- ✓ Apply knowledge to an unfamiliar context.

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. What is the direction of the net force on a satellite moving in a circular orbit around Earth? (6.1) K/U
   (a) tangentially, in the same direction as the satellite's motion
   (b) radially inward, toward the centre of Earth
   (c) radially outward, away from the centre of Earth
   (d) tangentially, in the opposite direction to the satellite's motion
2. RADARSAT-1 and RADARSAT-2 are Earth observation satellites designed and commissioned by the Canadian Space Agency. Examples of satellite monitoring include
   (a) identifying ships at sea
   (b) determining the tracking of storms and ice flows
   (c) detecting oil spills
   (d) all of the above (6.2) K/U
3. Two satellites of different masses complete perfectly circular orbits around Earth at the same altitude. Choose the most correct statement regarding the satellites. (6.2) K/U
   (a) The satellite with the larger mass must have a larger tangential velocity.
   (b) The satellite with the larger mass experiences a greater force of attraction from Earth.
   (c) The satellite with the larger mass has a longer period of rotation.
   (d) The satellite with the smaller mass must have a larger tangential velocity.
4. Earth imaging is a function of satellite monitoring. Earth imaging does not include
   (a) mapping missions
   (b) environmental monitoring
   (c) identifying coordinates of ships at sea
   (d) coastal and ocean tracking (6.3) K/U
5. A solid copper sphere is given an electric charge. How does the charge distribute itself across the copper sphere? (7.1) K/U
   (a) All the charge is concentrated in the centre of the sphere.
   (b) The charge is most concentrated in the centre and becomes less concentrated at the outer surface.
   (c) All the charge sinks to the lower half of the sphere.
   (d) All the charge is distributed around the outer surface of the sphere.
6. When a piece of amber is rubbed with animal fur (both initially neutral), the amber gains a net negative charge. Determine the change in the atomic level during this process. (7.1) T/I
   (a) The amber gained protons.
   (b) The animal fur lost protons.
   (c) The amber gained electrons.
   (d) The animal fur gained electrons.
7. Two protons attract each other through the force of gravity, but repel each other through the force of electricity. Which force is stronger when the protons are separated by a small distance? (7.2) K/U
   (a) the gravitational force
   (b) the electrostatic force
   (c) both forces are always equal in magnitude
   (d) the answer depends on the energy of the protons
8. Two charged objects in space are separated by a distance r and attract each other with an electric force of magnitude F. Which of the following represents the magnitude of force if the separation distance decreases to $\frac{r}{4}$? (7.2) K/U T/I
   (a) $4F$
   (b) $8F$
   (c) $16F$
   (d) $32F$
9. The electric field due to two oppositely charged point particles (a dipole) has its greatest magnitude
   (a) closest to either of the charges
   (b) exactly between the charges
   (c) anywhere on the line connecting the two charges
   (d) very far away from the charges (7.3) K/U
10. Particle A has positive charge $+q$ and is fixed in place. Particle B has negative charge $-q$ and is fixed in place 0.5 m directly north of particle A. Determine the direction in which the electric field points midway on the line connecting the two particles. (7.3) K/U
    (a) north
    (b) east
    (c) west
    (d) south
11. An aluminum key, a copper hinge, and a steel bolt are all brought close to a permanent magnet. Which of the following object(s) will be attracted to the magnet? (8.1) K/U
    (a) the aluminum key
    (b) the copper hinge
    (c) the steel bolt
    (d) all of the above

12. A charged particle enters a magnetic field with a speed v and experiences a force of magnitude F. Determine the force on the charged particle if it enters the same magnetic field in the same direction, but with a speed $3v$. (8.2) K/U
   (a) F
   (b) $3F$
   (c) $9F$
   (d) $18F$
13. A straight copper wire placed horizontally on a table carries a strong electric current. When a traditional, flat compass is placed beside the wire (at the same height), the needle shows very little response. Choose the best explanation as to why the needle does not show a large deflection. (8.2) K/U
   (a) The magnetic field at the compass points vertically, and the needle only reacts to horizontal magnetic fields.
   (b) The compass will only react to a changing current; the wire must be carrying a constant current.
   (c) The needle is being attracted to the copper in the wire because copper is a strong ferromagnetic material.
   (d) The magnetic field at the compass points horizontally, and the needle reacts only to vertical magnetic fields.
14. Which of the following explanations best describes why the magnetic force cannot do work on a moving charged particle? (8.2) K/U
   (a) The magnetic force is not strong enough to push the particle in a new direction.
   (b) The magnetic force is always parallel to the particle's velocity.
   (c) The magnetic force is always perpendicular to the particle's velocity.
   (d) The magnetic force is always less than the electric force.
15. The magnetic force on a current-carrying wire is perpendicular to
   (a) both the wire and the magnetic field
   (b) only the wire
   (c) only the magnetic field
   (d) neither the wire nor the magnetic field (8.3) K/U

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

16. Henry Cavendish performed the first precise test of the universal law of gravitation for objects in the solar system. (6.1) K/U
17. As a satellite moves in an elliptical orbit, it will move more slowly when it is closer to Earth. (6.2) K/U T/I
18. The altitude of the International Space Station's orbit is approximately $\frac{1}{20}$ of Earth's radius. (6.2) K/U
19. If you measure the orbital period of a satellite, then you can calculate the mass of the satellite. (6.2) T/I
20. As a planet orbits the Sun, the Sun also moves slightly because of the gravitational force from the planet. (6.2) K/U
21. Gravitational forces do not exist between protons, neutrons, and electrons. (7.1) K/U
22. If you rub a glass rod with a silk scarf, the scarf becomes negatively charged, but if you rub a plastic rod with a silk scarf, the scarf becomes positively charged. (7.1) K/U T/I
23. The gravitational force and the electric force both obey inverse-square laws. (7.2) K/U
24. The value of k in Coulomb's law is different for each type of charge. (7.2) K/U
25. At least two charged particles are necessary to create an electric field. (7.3) K/U
26. As an electron is released from rest in an electric field, the electron will move in the opposite direction to the electric field lines. (7.3) K/U
27. One joule per coulomb is equivalent to an electric potential of one volt. (7.4) K/U
28. Positive work done by an outside force is required to move a proton from a point of electric potential 1.0×10^1 V to a point of electric potential 5.0×10^1 V. (7.4) K/U T/I
29. Aluminum is a ferromagnetic material. (8.1) K/U
30. The principle of electromagnetism states that moving charges produce an electromagnetic field. (8.1) K/U
31. An electron moving with velocity $\vec{v}$ in a uniform magnetic field experiences the opposite force as a proton moving with velocity $-\vec{v}$ in the same field. (8.2) T/I
32. Stereo speakers rely on changing currents and permanent magnets to create sound. (8.3) K/U
33. An electron and a proton enter a uniform magnetic field with the same velocity, which is perpendicular to the field. Both particles will move in a circular path, but the proton's path will have a smaller radius. (8.4) K/U
34. Magnetic resonance imaging devices are used to explore the human body with better resolution and greater safety than X-rays. (8.5) A
35. Modern particle experiments use electric and magnetic fields to accelerate particles to high speeds. (8.6) K/U

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. A planet outside our solar system has eight times the mass of Earth and twice Earth's radius. The acceleration due to gravity on Earth's surface is approximately 10 m/s^2. What is the acceleration due to gravity on the surface of the other planet? (6.1) T/I
   (a) 5 m/s^2
   (b) 10 m/s^2
   (c) 20 m/s^2
   (d) 80 m/s^2
2. If the distance between two point masses is doubled, the gravitational attraction between them
   (a) is doubled
   (b) is increased by a factor of four
   (c) is reduced to half
   (d) is reduced to a quarter (6.1) K/U
3. Why do satellites orbit Earth and not crash down to Earth's surface? (6.2) K/U
   (a) There is no gravity above Earth's atmosphere.
   (b) The satellite has the necessary orbital velocity to keep it orbiting Earth.
   (c) The gravitational pull of the Moon and other planets on the satellite is just the right amount to keep it from falling into Earth.
   (d) All of the above are true.
4. Which one of the following is an example of a natural satellite? (6.2) K/U
   (a) the International Space Station
   (b) RADARSAT-1
   (c) a planetary moon
   (d) a satellite used in the Global Positioning System
5. Which of the following quantities does the orbital period of a satellite depend on? (6.2) K/U
   (a) the mass of the satellite
   (b) the orbital radius of the satellite
   (c) the amount of time the satellite has been in orbit
   (d) all of the above
6. The minimum value of charge on any charged body is
   (a) 1.6×10^{-19} C
   (b) 1 C
   (c) 1 μC
   (d) 4.8×10^{-12} C (6.2) K/U
7. When a negatively charged plastic rod is brought near bits of confetti, the confetti is attracted to the rod. Select the best explanation as to why this happens. (7.1) K/U
   (a) The confetti has a net positive charge, and it is attracted to the negative plastic rod.
   (b) The confetti has a net negative charge, and it is attracted to the negative plastic rod.
   (c) The confetti is plastic, and it is attracted to other plastic items.
   (d) The confetti is neutral, but it is attracted to the rod through induced charge separation.
8. Select the true statement concerning charging by induction. (7.1) K/U
   (a) In charging by induction, the initially charged object comes close to, but does not touch, the object being charged.
   (b) Transfer of charge to or from an electric ground is necessary to permanently charge an object by induction.
   (c) In charging by induction, the initially charged object and the object being charged will end up with charges of opposite signs.
   (d) All of the above are true.
9. What fundamental force is responsible for keeping electrons attracted to their host nucleus in an atom? (7.2) K/U
   (a) the gravitational force
   (b) the electric force
   (c) the magnetic force
   (d) the strong nuclear force
10. A charged particle experiences a force of 100 N when placed in an electric field of magnitude ε. If the same charged particle is placed in an electric field of magnitude 3ε, determine the resulting force. (7.3) K/U
    (a) 300 N
    (b) 150 N
    (c) 100 N
    (d) 33.3 N
11. A long, straight wire wound into a coil is called a
    (a) dipping needle
    (b) magnetic field
    (c) solenoid
    (d) galvanometer (8.1) K/U

12. As a charged particle enters a magnetic field, what pathways are possible for the particle to follow? (8.2) K/U
 (a) circle or helical path only
 (b) straight line, circle, or helical path
 (c) circle or parabola only
 (d) straight line only
13. A beam of electrons travels vertically upward from the ground. How will this beam initially be deflected by Earth's magnetic field if Earth's magnetic field points directly north? (8.2) K/U T/I
 (a) westward
 (b) southward
 (c) eastward
 (d) northward
14. Two parallel current-carrying wires will attract each other magnetically
 (a) if the currents run in the same direction
 (b) if the currents run in the opposite direction
 (c) if the currents have the same magnitude
 (d) never (8.3) K/U T/I

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

15. The gravitational constant G has the same value throughout the universe and is a scalar quantity. (6.1) K/U
16. All satellites orbit Earth in circular pathways. (6.2) K/U
17. As the altitude of a satellite increases, its orbital speed decreases. (6.2) K/U
18. The orbit of Earth around the Sun would become more elliptical if you could increase its mass. (6.2) K/U
19. The electric force is much stronger than the gravitational force. (7.1) K/U
20. Ordinary muscular actions in most animals will create weak electric fields. Some predators have developed senses to detect these electric fields. (7.3) K/U
21. Two positive charges are placed along the x-axis. Other than at points infinitely far away, there are two places where the electric field is equal to zero. (7.3) K/U T/I
22. As an electron moves away from a positively charged plate toward a negatively charged plate, the potential energy of the system decreases. (7.4) K/U
23. The north end of a bar magnet is attracted to and sticks to a refrigerator door. If someone removes the magnet and brings the south end near the refrigerator door, the magnet will be repelled. (8.1) K/U
24. The aurora borealis results from charged particles emitting energy in the form of light as they spiral down through Earth's magnetic field. (8.1) K/U
25. The magnetic force applied on a moving charged particle will always be in the same direction as the particle's velocity. (8.2) K/U
26. Giant horseshoe magnets are used to create the 2.0 T magnetic fields inside an MRI machine. (8.5) K/U

Understanding

27. Is the value of acceleration due to gravity, g, the same everywhere on the surface of Earth? Justify your answer. (6.1) K/U A
28. Two artificial satellites orbit Earth. If you know that the satellites have the same orbital radius but different masses, what can you say about their speeds? (6.2) K/U T/I
29. Explain why it is not possible to keep a military intelligence satellite in geosynchronous, or fixed, orbit over Antarctica. (6.2) K/U A
30. A wooden metre stick is carefully balanced with its centre on the corner of a table. When a negatively charged plastic rod is brought near one side of the metre stick, the stick shows a slight attraction to the rod. (7.1) K/U C
 (a) Explain why there is a slight attraction between the neutral metre stick and the negatively charged plastic rod.
 (b) Predict what would happen if the negatively charged rod were brought near the other end of the metre stick.
 (c) Predict what would happen if the plastic rod were positively charged.
31. Identify each of the following as a conductor or an insulator. (7.1) K/U
 (a) aluminum
 (b) rubber
 (c) paper
 (d) copper
 (e) salt water
 (f) glass
32. Two particles with electric charges q and $-3q$ are separated by a distance of 1.2 m. (7.2) K/U T/I
 (a) Determine the magnitude of the electric force between the two particles with $q = 4.5$ C.
 (b) If $q = -4.5$ C, how does the answer change?
33. Determine the separation required between two equal charges of 1.0 C so that the force acting between them equals the weight of a 50 kg person on Earth. (7.2) K/U T/I
34. Two charged particles are placed at a distance of 1.0 cm apart. Calculate the minimum possible magnitude of the electric force acting on each charge. (7.2) K/U T/I

35. Determine the electric force between two protons separated by a distance of 10^{-15} m. The protons in a nucleus remain at a separation of this order. (7.2) K/U T/I

36. Two oppositely charged particles, q_1 and q_2, are fixed on the x-axis. Point P is a small distance above the x-axis and midway between the charges. A proton at point P experiences a net force directed up and to the right, as shown in **Figure 1**. (7.3) K/U T/I

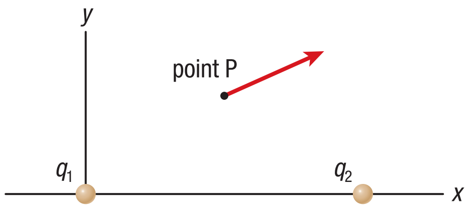


Figure 1

(a) Which particle is positively charged, and which is negatively charged?

(b) Which particle has the greater magnitude of charge? Or, do they have the same magnitude of charge? Justify your answer.

37. **Figure 2** shows a negatively charged ion moving to the right as it enters a uniform electric field formed by two oppositely charged parallel plates. The ion deflects along an upward parabolic pathway. (7.4) K/U C A

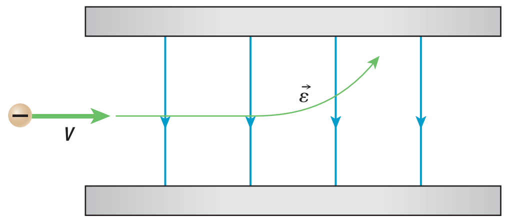


Figure 2

(a) What is the charge on each plate? Justify your answer.

(b) Sketch the predicted pathway of a doubly charged negative ion that enters the field at the same location with the same velocity.

38. Explain how birds can sit on a non-insulated high-voltage power line without being electrocuted. (7.4) K/U C A

39. Explain how a solenoid can create a magnetic field that is similar to that of a bar magnet. Sketch the magnetic field of a solenoid. (8.1) K/U C

40. Describe how a highly sensitive dipping needle would behave in each of the following locations. (8.1) K/U A

(a) Edmonton, Canada

(b) Quito, Ecuador

(c) Santiago, Chile

41. A road worker putting up street signs uses a sledgehammer to drive in a vertical steel post. Afterwards, the worker notices that the top of the steel post shows strong attraction to one end of a compass needle. (In Canada, Earth's magnetic field has a large vertical component.) (8.1) T/I C

(a) Explain what happened to the steel post.

(b) Predict the behaviour of the compass needle when it is brought close to the bottom of the steel post.

42. A student finds a bar magnet with poles marked W and X. The student places a compass near the magnet as shown in **Figure 3**. Sketch the entire magnetic field of the bar magnet, and properly assign the north and south poles. (8.1) K/U C A

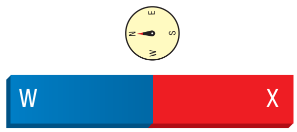


Figure 3

43. Discuss the differences in degrees between the geographic north pole and the magnetic north pole. Explain how they vary across Canada. (8.1) K/U A

44. A positively charged particle is moving upward in the plane of the page. Explain how you can use a magnetic field to direct this particle toward the right. (8.2) T/I

45. A rectangular loop of current-carrying wire sits above a long, straight, current-carrying wire. The rectangular loop has a current running clockwise, and the long, straight wire has a current running to the left, as shown in **Figure 4**. (8.3) T/I C

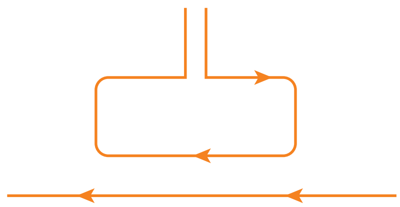


Figure 4

(a) Determine the direction of the magnetic force on each of the four sides of the rectangular loop.

(b) Determine the direction of the net force on the rectangular loop of wire. Justify your answer.

46. The energy of a charged particle moving in a uniform magnetic field does not change. Explain. (8.3) K/U

Analysis and Application

47. A sphere of mass 40 kg is attracted by a second sphere of mass 15 kg, when their centres are 63 cm apart, with a force of 1.0×10^{-7} N. Calculate the value of the gravitational constant. (6.1) T/I

48. A rocket is fired from Earth toward the Moon. At what distance from the Moon is the gravitational force on the rocket equal to zero? The mass of Earth is 6.0×10^{24} kg. The mass of the Moon is 7.4×10^{22} kg, and the orbital radius of the Moon is 3.8×10^{8} m. Neglect the effect of the Sun and other planets. (6.1) T/I

49. An apple of mass 0.25 kg falls from a tree. Determine the acceleration of the apple toward Earth. Also calculate the acceleration of Earth toward the apple. (6.1) T/I

50. If a planet existed whose mass and radius were both half those of Earth, what would be the value of acceleration due to gravity on its surface as compared to what it is on the surface of Earth? (6.1) T/I

51. Calculate the gravitational field strength of a planet on which the weight of a 60 kg astronaut is 300 N. (6.1) T/I

52. All masses attract gravitationally. The Sun should therefore attract us away from Earth when the Sun is overhead. The Sun has a mass of 2.0×10^{30} kg and is 1.5×10^{11} m away from Earth. (6.1) T/I
    (a) Calculate the force that the Sun exerts on a 50 kg person standing on Earth's surface.
    (b) Determine the ratio of the Sun's gravitational force to Earth's gravitational force on the same 50 kg person.

53. A certain satellite orbits 7.00×10^{6} m from Earth's centre. (6.2) T/I
    (a) Determine the orbital speed of the satellite.
    (b) Determine the orbital period of the satellite in minutes.

54. An extra-solar planet orbits the distant star Pegasi 51. The planet has an orbital velocity of 2.3×10^{5} m/s and an orbital radius of 6.9×10^{9} m from the centre of the star. Determine the mass of the star. (6.2) T/I

55. An observational satellite is designed to orbit Mars at an altitude of 5.2×10^{5} m. Mars has a mass of 6.4×10^{23} kg and an average radius of 3.4×10^{6} m. Calculate the necessary orbital velocity of the satellite, and its orbital period in minutes. (6.2) T/I

56. When Sputnik 1 was launched by the U.S.S.R. in October 1957, American scientists wanted to know as much as possible about this new artificial satellite. If Sputnik orbited Earth once every 96 min, calculate its orbital velocity and altitude. (6.2) T/I

57. The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge −0.8 μC has magnitude 0.2 N. (7.2) T/I
    (a) Calculate the distance between the two spheres.
    (b) Identify the nature of the force on the second sphere due to the first.

58. Two identically charged particles separated by 10 cm repel each other with a force of 80 mN. (7.2) T/I
    (a) Determine the repulsive force if the separation distance changes to 20 cm.
    (b) Determine the repulsive force if the separation distance changes to 2 cm.
    (c) Determine the charge on each particle.

59. An electron is released from rest in an electric field of magnitude 300 N/C. (7.3) T/I
    (a) Determine the magnitude of the electric force on the electron.
    (b) Determine the magnitude of the acceleration of the electron.
    (c) Determine the electron's final speed after it travels a distance of 1.5 cm.

60. An electron is travelling through a region of space in which the electric field lies along the positive x-direction and has a magnitude of 2.0×10^{3} N/C. (7.3) K/U T/I A
    (a) Determine the magnitude and direction of the acceleration of the electron.
    (b) Using the acceleration you calculated in (a) and assuming that the electron's initial speed is negligible, what is the speed of the electron after 1.0 min of travel in the electric field? Do you think that this speed is reasonable?

61. A plastic sphere of mass $m = 100.0$ g is attached to a string as shown in **Figure 5**. An electric field with a magnitude of 100.0 N/C is directed along the x-direction. The string makes an angle of $\theta = 30.0°$ with the y-axis. Calculate the charge on the sphere. (7.3) K/U T/I A

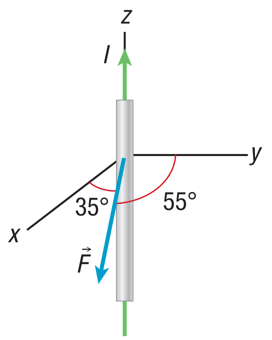


Figure 5

62. Two positively charged particles, each of magnitude 5.0×10^{-8} C, are fixed on the x-axis. One charge is placed at the origin and the other at $x = 0.10$ m. (7.3) T/I C A
    (a) Sketch the combined electric field created by these charges.
    (b) Determine the magnitude and direction of the electric field at each of the following points:
        (i) $x = 0.05$ m
        (ii) $x = 0.07$ m

63. A lightweight aluminum sphere ($m = 15.1$ g) hangs by an insulating thread in a uniform electric field of 585 N/C directed upward as shown in **Figure 6**. The tension in the sphere is 0.167 N. (7.3) T/I C A

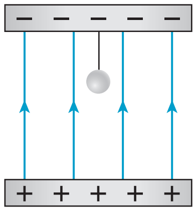


Figure 6

(a) Sketch all the forces acting on the sphere.
(b) Determine the magnitude and direction of the electric force acting on the sphere.
(c) Calculate the magnitude and sign of the charge on the sphere.

64. A proton starts at rest and travels through a potential difference of 8000 V. (7.4) T/I
(a) Determine the final kinetic energy of the proton.
(b) Determine the final speed of the proton.

65. Consider a point, P, and a charge of 4.0×10^{-7} C located 9.0 cm away from point P. (7.5) T/I
(a) Calculate the potential at point P.
(b) Determine the work done in bringing a charge of 2.0×10^{-9} C from infinity to point P.
(c) Does the work done depend on the path along which the charge is brought from infinity?

66. Three point charges, $q_1 = 2.5 \times 10^{-6}$ C, $q_2 = 4.5 \times 10^{-6}$ C, and $q_3 = -3.5 \times 10^{-6}$ C, are arranged as shown in **Figure 7**. Calculate the total electric potential energy of this system. (7.5) K/U T/I

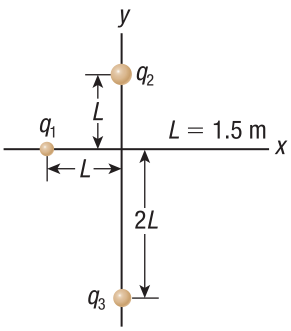


Figure 7

67. Consider again the three charges in Figure 7 with $q_1 = 2.5 \times 10^{-6}$ C, $q_2 = 4.5 \times 10^{-6}$ C, and $q_3 = -3.5 \times 10^{-6}$ C. A fourth charge, $q_4 = -5.0 \times 10^{-6}$ C, is brought from very far away and placed at the origin. (7.5) K/U T/I
(a) How much work is required in this process?
(b) Determine how much work is required to move q_4 from the origin to a distance that is very far away.

68. An electron and a proton are 7.5×10^{-9} m from each other. Calculate how much energy is required to increase their separation by a factor of 2. (7.5) K/U T/I

69. An oxygen ion, O^{2-} (charge of 3.20×10^{-19} C), is at a distance of 5.0×10^{-10} m from an H^+ ion (charge 1.60×10^{-19} C). Determine how much energy is required to separate them completely. Treat both ions as point charges. (7.5) T/I A

70. Which is larger, 1 C or the charge on an electron? Determine the number of electrons with a total charge equal to 1 C. (7.6) T/I

71. In Millikan's experiment, a potential difference of 2.0 kV is applied between the two plates, which are 8 mm apart, to balance an oil drop of mass 4.9×10^{-14} kg. Calculate the number of elementary charges on the drop ($g = 10$ m/s^2). (7.6) T/I

72. A proton travelling east at 5.9×10^6 m/s enters a northward-directed magnetic field of strength 0.800 T. (8.2) T/I
(a) Determine the magnitude of the magnetic force exerted on the proton.
(b) Describe the path taken by the proton as it stays in the magnetic field.

73. A 15 cm length of copper wire carries 5.1 A of current through a horseshoe magnet so that the wire is perpendicular to the magnetic field lines. The copper wire experiences a force of 0.05 N. Determine the magnitude of the magnetic field created by the horseshoe magnet. (8.3) T/I

74. The circuit shown in **Figure 8** includes a copper bar held horizontally between the poles of a horseshoe magnet. (8.3) K/U T/I A

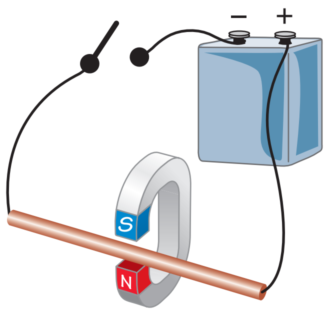


Figure 8

(a) Explain what will happen when the switch is closed and conventional current goes through the copper bar.
(b) What variable(s) could be changed to increase the magnitude of the force on the bar?
(c) Describe the effect on the magnetic force if the bar were tilted down slightly instead of being horizontal.

75. A wire carries a current along the $+z$ direction and experiences a force that lies in the x–y plane (**Figure 9**). What might be the direction of the magnetic field? (8.3) T/I

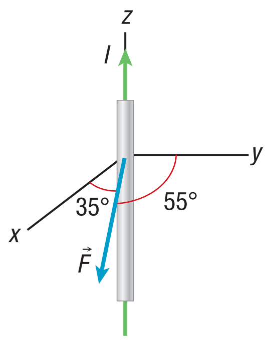


Figure 9

76. In a mass spectrometer, a beam of singly charged ions of mass 2.0×10^{-26} kg is accelerated to a final speed of 6.5×10^5 m/s. A magnetic field then steers the ions in a circle of radius 35 cm. Determine the necessary strength of the magnetic field in the mass spectrometer. (8.4) T/I

77. A proton and an electron move perpendicular to a uniform magnetic field with the same speed. Determine the ratio of the radii of the circular paths of the proton and electron if $m_p = 1840\ m_e$. (8.4) T/I

78. Cosmic rays undergo a spiral trajectory as they travel to Earth. Consider a cosmic ray that is an iron ion, Fe^+, with a mass of 9.3×10^{-26} kg, travelling at 1.2×10^7 m/s. What is the approximate minimum radius of the spiral when the ion is near Earth's surface? Assume a value of 5.5×10^{-5} T for Earth's magnetic field. (Hint: Use Newton's second law of motion for circular motion, $F = ma_c$, where a_c is the centripetal acceleration, equal to $\frac{v^2}{r}$.) (8.4) T/I

Evaluation

79. An astronaut on the Moon measures the acceleration due to gravity to be 1.7 m/s^2. He knows that the radius of the Moon is 0.27 times that of Earth. Predict his estimate of the ratio of the mass of Earth to that of the Moon. (6.1) T/I A

80. A group of students research satellite data from the Canadian Space Agency. They chart the orbital period and orbital radius of five different satellites (**Table 1**). Orbital radius is measured from the centre of Earth. (6.2) T/I A

Table 1

Orbital radius (km)	Orbital speed (km/s)
6900	7.6
7500	7.3
8800	6.7
9300	6.5
9700	6.4

(a) Plot the orbital speed versus orbital radius on a graph.
(b) Draw a smooth curve through all the data points. From your graph, extrapolate to predict the following.
  (i) the orbital speed for a satellite with orbital radius 10 300 km
  (ii) the orbital radius for a satellite travelling at 7.9 km/s

81. Astronomers observed Jupiter's moons and carefully noted the orbital period and orbital radius for each moon (**Table 2**). (6.2) T/I A

Table 2

Moon	Orbital period (days)	Orbital radius (km)	Orbital speed (km/s)
Io	1.8	422 000	
Europa	3.5	670 000	
Ganymede	7.5	1 070 000	
Callisto	17	1 880 000	

(a) Determine the orbital speed of each moon.
(b) Use these data to calculate an average value for the mass of Jupiter. (Hint: Convert to metres per second when calculating with G.)

82. An ebonite rod can be charged by holding it in your hand and rubbing it with a piece of cloth. A copper rod cannot be charged like this. Why? (7.1) T/I A

83. Consider an electron and a proton separated by a distance of 1.0 nm. (7.2) T/I A
(a) Calculate the magnitude of the gravitational force between them.
(b) Calculate the magnitude of the electric force between them.
(c) How would the ratio of these gravitational and electric forces change if the distance were increased to 1.0 m?

84. A charge of 1 C is placed on top of your school and another equal charge at the top of your house. Assume the separation between the two charges equals the distance between your house and your school. (7.3) K/U T/I A
   (a) Determine the force exerted by the charges on each other.
   (b) How many times your weight is this force?
85. Three equal charges of 2.0×10^{-6} C each are fixed at the three corners of an equilateral triangle with side length 5.0 cm. Determine the net force experienced by one of the charges due to the other two charges. (7.3) K/U T/I A
86. When more than one charged object is present in an area, how can the total electric force on one of the charged objects be determined? (7.3) K/U T/I
87. An electron and a proton are situated in an electric field. Will the electric force on them be equal? Will their acceleration be equal? Justify your answer. (7.3) K/U T/I A
88. When we move a charge through an electric field, we do work. (7.4) K/U T/I A
   (a) Is the work done in moving a charge between two points in an electric field independent of the path followed?
   (b) Is the electrostatic potential necessarily zero at a point where the electric field strength is zero? Illustrate your answer with an example.
89. **Table 3** shows data from a researcher who has followed Millikan's procedure. (7.6) T/I C A

Table 3

Oil drop	Charge (C)
1	9.60×10^{-19}
2	6.70×10^{-19}
3	1.62×10^{-19}
4	5.16×10^{-19}
5	3.24×10^{-19}

   (a) Without using any prior knowledge about the elementary charge, describe a procedure that could be used to find the value of the elementary charge from these data.
   (b) Use your procedure to determine the elementary charge.
   (c) Compare your results with the known value of *e* and discuss any problems.
90. Compare and contrast the similarities and differences between electric fields and magnetic fields using a Venn diagram. (8.1) K/U C
91. A large constant current runs upward through a horizontal plane (**Figure 10**). Describe the magnetic field in the plane. Show the direction of the needles for the eight compasses. (8.1) K/U A

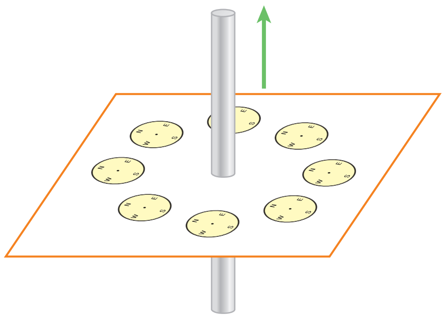


Figure 10

92. A proton is moving through a magnetic field. (8.2) K/U T/I A
   (a) When does the proton experience a maximum magnetic force?
   (b) When does the proton experience a minimum magnetic force?
93. A long, straight wire carries a current of $I = 125$ A in a region where the magnetic field has magnitude $B = 7.3$ T, but the force on the wire is zero. Explain how that can be. (8.3) T/I
94. A long current-carrying wire is parallel to the y-axis with the conventional current $I = 2.5$ A along the $+y$-direction (**Figure 11**). A magnetic field of magnitude $B = 1.4$ T lies in the y–z plane, directed 25° away from the y-axis. (8.3) T/I

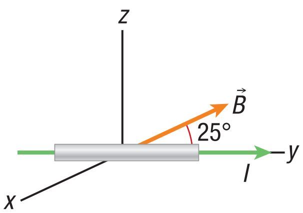


Figure 11

   (a) What is the direction of the magnetic force on the wire?
   (b) What is the magnitude of the magnetic force on a 1.5 m–long section of the wire?

95. The velocities of two alpha particles, A and B, entering a uniform magnetic field are in the ratio 4:1. On entering the field, they move in different circular paths. Determine the ratio of the radii of curvature of the paths of the particles. (8.4) T/I

96. Two students use a current-carrying wire, a compass, and a metre stick to determine the strength of Earth's magnetic field. The wire carries a current vertically upward. The compass sits directly south of the wire with its centre a distance *r* away from the centre of the wire (**Figure 12**).

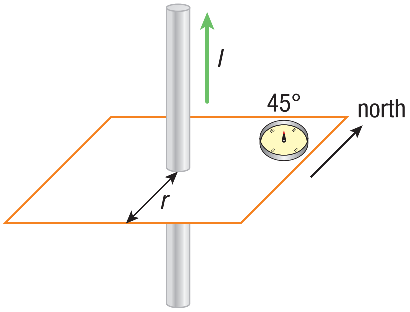


Figure 12

The students set the compass at various distances from the wire. For each distance, the students increase the current until the compass needle deflects 45° away from north. The result of each trial is recorded in **Table 4**. When the compass deflects 45°, then the magnetic field of Earth is equal to the magnetic field created by the wire. (8.4) T/I A

Table 4

Wire-to-compass distance (cm)	Current in wire (A)
4.0	10.4
8.0	21.3
12.0	32.1

(a) Use the trials to compute an average value for Earth's magnetic field. Calculate the strength of the magnetic field with the following equation: $B = \frac{(1.3 \times 10^{-6})I}{2\pi r}$

(b) After comparing their experimental value to accepted values of Earth's magnetic field, the students in Canada find that their value is too low. Explain why this experiment gives a low value for Earth's magnetic field.

(c) Predict the behaviour of the compass if the experiment was repeated, but this time the compass was placed directly north of the wire.

97. A uniform magnetic field acts at right angles to the velocity of a group of electrons. As a result, the electrons move in a circular path of radius 2 cm. Determine the radius of their circular path if the speed of the electrons is doubled. (8.4) T/I A

Reflect on Your Learning

98. What did you find most surprising in this unit? What did you find hardest to understand? How can you learn more about the surprising or difficult topics? C

99. How would you describe the similarities between gravitational forces and electric forces to a friend who has not taken physics? C

100. List some of the important technologies that make use of the physics ideas you learned about in this unit. C

Research

WEB LINK

101. Research the efforts astronomers are making to find planets outside our solar system (exoplanets). How many planets have scientists discovered to date? Write a blog entry that describes the techniques and equipment used for making these discoveries. T/I C A

102. Research the variations in gravity around Earth. Create a poster that illustrates how the factors of altitude, latitude, and local geology can change the strength of gravity. Where is gravity the weakest? Where is gravity the strongest? C A

103. When airplanes fly, charges can build up on the wings during flight. To address this issue, long, thin strips called static wicks are made using a conducting material and attached to the airplane's wings. Research this technology and answer the following questions. K/U T/I A

(a) Why does a charge build up on the airplane's wings during flight?

(b) What problems can a charge buildup cause?

(c) How does the shape and material of the wicks help remove the charge?

104. Research the motion of a charged particle in an electric field of your own creation. Go to Nelson Science for websites related to particle motion. Create a presentation, including drawings or models, in which you describe how the aspects of motion (force, acceleration, inertia) successfully control particle motion. C A

105. Create a poster or brochure that explains the physics of loudspeakers. List the necessary components of a speaker, and explain how an electrical signal is transformed into a sound wave. C A

UNIT 4

The Wave Nature of Light

OVERALL EXPECTATIONS

- analyze technologies that use the wave nature of light, and assess their impact on society and the environment
- investigate, in qualitative and quantitative terms, the properties of waves and light, and solve related problems
- demonstrate an understanding of the properties of waves and light in relation to diffraction, refraction, interference, and polarization

BIG IDEAS

- Light has properties that are similar to the properties of mechanical waves.
- The behaviour of light as a wave can be described mathematically.
- Technologies that use the principles of the wave nature of light can have societal and environmental implications.

UNIT TASK PREVIEW

The goal of the Unit Task is to use the knowledge gained in this unit to analyze digital media. The Unit Task is described in detail on page 556. As you work through the unit, look for Unit Task Bookmarks to see how information in the section relates to the Unit Task.

FOCUS ON **STSE**

LIGHT AND INTERFERENCE

A hummingbird is one of the most colourful animals in nature. If you have ever watched a hummingbird, you might have noticed how, when viewed from different angles, the hummingbird's colour seems to change. Hummingbird feathers have specialized cells, or platelets, on their top layers that act as prisms. These prisms split white light into many different colours. This phenomenon is called iridescence. What causes a hummingbird's iridescent coloration? It is partly caused by light passing through the platelets. The birds typically have 8 to 10 top layers of these feathers, stacked on top of each other. This stacking tends to intensify and purify the resulting colour.

An iridescent object appears to change colour as your angle of view of the object changes. Many other insects and animals have iridescent features, such as the wings of dragonflies, the shells of beetles, and the eyes of many nocturnal animals. The eerie glow of a house cat's eyes in a photograph occurs because of iridescence due to structures that improve the cat's night vision. For hummingbirds, and other birds such as peacocks, the iridescent feathers play a part during courting and mating. The male hummingbirds display their colourful throats in an attempt to attract female hummingbirds, which do not display as much iridescence.

Iridescence is not found in insects and animals only—you can observe it in everyday situations. Think about the soap bubbles created when washing dishes or blowing soap bubbles using a bubble wand. As you blow bubbles, you can observe the colours on the bubble's surface change as the bubbles change in shape and size. The iridescent properties of the bubbles, like the brilliant colours of the hummingbird feathers, are a result of the light reflecting from the bubble's surface. Different surfaces, viewing angles, and thicknesses result in different colours.

We are able to see different colours because different surfaces reflect, refract, and absorb white light in different ways. For example, when an object absorbs all wavelengths of light, you perceive this object as black. No light is reflected. The colour of an object is not a part of the object, but rather depends on which wavelengths of light that object reflects and absorbs. A green grape is not made up of the colour green, but rather reflects a wavelength of light that appears green to the observer, and all other visible wavelengths of light are absorbed. The wave properties of light affect how we see colours, shapes, and textures.

Questions

1. What do you think might happen to the appearance of a hummingbird's feathers if you viewed the feathers under a fluorescent light instead of natural sunlight?
2. Why do you think the angle from which a hummingbird is viewed affects the colour that you see?
3. Can you think of other everyday scenarios where you might see a colourful pattern similar to the examples discussed?
4. What role do you think is played by (a) refraction, (b) reflection, and (c) interference in the phenomenon of iridescence?

CONCEPTS

- energy transfer through waves
- wave characteristics
- the universal wave equation
- index of refraction
- total internal reflection
- electromagnetic spectrum
- interference

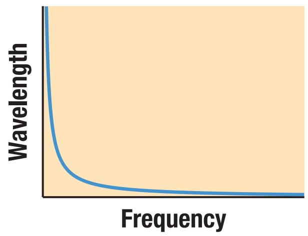
SKILLS

- planning investigations
- solving algebraic equations for one or two unknowns
- interpreting graphs
- communicating scientific information clearly and accurately through words and diagrams

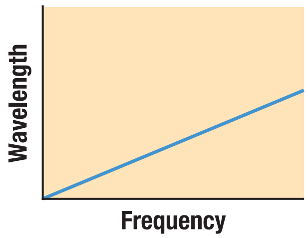
Concepts Review

1. Use words and a diagram to explain interference. Explain why only waves, and not particles, can interfere. K/U C
2. (a) Describe total internal reflection.
   (b) What are the conditions under which total internal reflection takes place?
   (c) Discuss two cases that illustrate this phenomenon. K/U C A
3. Explain why an empty test tube dipped into water in a beaker appears silvery when viewed from a certain direction. T/I A
4. List two everyday devices that use electromagnetic waves. (Hint: Which technologies rely on transmission towers?) K/U C
5. Theories and experiments are two very different methods used in scientific research. K/U C
   (a) Explain the difference between theory and experiment in science.
   (b) Discuss why both theory and experiment are useful in researching the wave nature of light.
6. Which statement is true of a transverse wave travelling along a string toward an end that is free to move? K/U
   (a) The wave will be reflected with a phase change of half a wavelength.
   (b) The wave will not be reflected.
   (c) The wave will be reflected with no change of phase.
   (d) The wave will be reflected with a phase change of a quarter wavelength.
7. Describe the direction in which a ray of light bends as it travels
   (a) from air into diamond
   (b) from water into air
   (c) normal to the interface of two media K/U
8. Which graph shows the relationship between wavelength and frequency for sound waves that travel at constant speed? K/U

(a)



(b)



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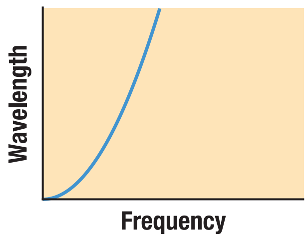


Skills Review

9. Identify the parts of the wave labelled in **Figure 1**. K/U

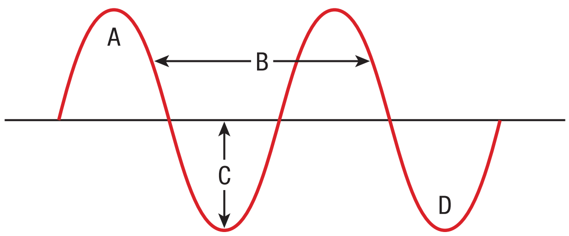


Figure 1

10. Explain how you would measure the wavelength and amplitude of the wave in **Figure 2**. If you could watch the wave over time, explain how you would measure its period and frequency. K/U T/I C

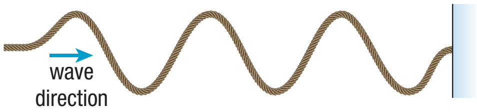


Figure 2

11. A swimming pool pump generates 10 water waves every 15 s. T/I C
    (a) Determine the frequency of the waves.
    (b) When the wave crests are separated by 2.0 m, determine the speed of the waves.
    (c) Predict what will happen as two wave crests interfere near the middle of the pool.
12. **Figure 3** shows a ray of light travelling from air into a glass block. Copy the figure into your notebook. K/U C
    (a) Draw the normal and the reflected light ray.
    (b) Draw the general direction of the refracted light ray. Is it toward the normal, away from the normal, or along the normal?

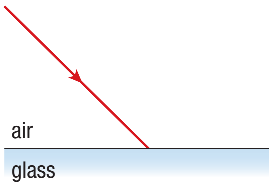


Figure 3

13. Green light has a frequency of 5.70×10^{14} Hz. Use the universal wave equation to calculate the wavelength, in nanometres, of green light. The speed of light in a vacuum is 3.0×10^8 m/s. T/I
14. Solve each equation for the variable x. T/I
    (a) $\left(x - \frac{1}{2}\right)\left(\frac{4}{3}\right) = 1$
    (b) $\left(8 - \frac{1}{2}\right)\left(\frac{x}{14}\right) = 1$
    (c) $1.1 \sin 60° = 1.66 \sin x$
    (d) $\frac{1.47}{1.33} = \frac{478}{x}$
15. Solve for θ in the following equations. K/U T/I
    (a) $\sin\theta = \left(n - \frac{1}{2}\right)\frac{x}{y}$,
    where $n = 2$, $x = 5.1 \times 10^{-7}$, and $y = 8.0 \times 10^{-6}$
    (b) $\frac{\sin\theta_1}{\sin\theta} = \frac{n_2}{n_1}$,
    where $\theta_1 = 27.0°$, $n_1 = 1.00$, and $n_2 = 1.35$
16. Determine the minimum and maximum wavelengths of sound in water that is within the audible range 20 Hz–20 000 Hz for an average human ear. The speed of sound in water is approximately 1496 m/s. T/I
17. Calculate the frequency of radio waves transmitted by a radio station when the wavelength of the waves is 300 m. The speed of radio waves in air is 3.0×10^8 m/s. T/I
18. A source is producing 15 wavelengths in 3.0 s. The distance between a crest and a trough is 10 cm. Calculate
    (a) the frequency
    (b) the wavelength
    (c) the speed of the wave T/I
19. The vibrating end of the standing wave in **Figure 4** makes 25 complete vibrations in one minute. Calculate (a) the wavelength of the waves and (b) the speed of the waves. T/I

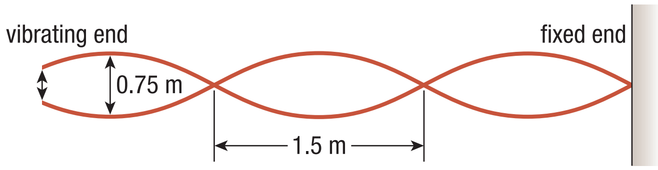


Figure 4

20. The index of refraction for glass is 1.5. Calculate the speed of light in glass. T/I
21. Solve each system of linear equations for the variables x and y. T/I
    (a) $3x - y = -2; -5x + 2y = 6$
    (b) $2x + 3y = -1; -3x - 5y = 4$
22. List two safe laboratory practices to observe when performing investigations that involve light. K/U
23. Suppose your teacher has given you a ripple tank, a metre stick, and a marble. Describe how you could use these items to determine the period, frequency, speed, and wavelength of a water wave. K/U T/I

CAREER PATHWAYS **PREVIEW**

Throughout this unit, you will see Career Links. Go to the Nelson Science website to find information about careers related to The Wave Nature of Light. On the Chapter Summary page at the end of each chapter, you will find a Career Pathways feature that shows you the educational requirements of the careers. There are also some career-related questions for you to research.

CHAPTER 9

Waves and Light

KEY CONCEPTS

After completing this chapter you will be able to

- distinguish between reflection, refraction, and the interference of light
- describe various properties of light using wave diagrams
- discuss the historical development of theories about the wave–particle nature of light
- communicate and explain the key concepts of wave interference and wavelength
- analyze the two-point interference pattern produced in ripple tanks using diagrams
- analyze the interference of light using diagrams
- describe the properties of light used in medicine, communication, astronomy, and security

How Can We Use Properties of Light to Create Technologies That Enhance Our Lives?

Visible light, a type of electromagnetic radiation, includes all the colours of the rainbow. The warmth you feel when you stand near a hot stove—thermal energy—is also a form of electromagnetic radiation. These examples are only a small part of a spectrum that includes all electromagnetic energy, such as radio waves, X-rays, and gamma rays.

For centuries, scientists have studied the properties of light and used their findings to create new technologies. Even today, we are still discovering new aspects of light that we can use to enhance our lives.

Photocells, which affect electric current in a circuit based on the amount of light detected by a sensor, have many uses, including automatic door openers and security systems. Infrared cameras that detect the thermal energy emitted by hot objects became popular as night-vision goggles, but they also help with energy efficiency by allowing inspectors to locate areas in a room where thermal energy is entering or escaping. Compact fluorescent lamps (CFLs) and light-emitting diodes (LEDs), two important energy-saving technologies, resulted from a better understanding of how to produce light.

Today, we use light in many ways other than illuminating objects. If a light beam is varied, the light can carry information. Your computer's Internet connection may transmit data by means of light passing through fibre optic cables like the fibre optic cables on the facing page. Laser light shining on a CD, DVD, or Blu-ray disc decodes information stored on the disc. The digital information from the disc is interpreted as music, a movie, or computer data. In this chapter, you will learn about the properties of waves and light, and how these properties can be used in a variety of fields, including medicine, communications, astronomy, and security.

STARTING POINTS

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. How does the composition of a medium affect how light travels in it?
2. Does light exhibit any wave-like properties?
3. Does light exhibit any particle-like properties?
4. What properties of light can be used to create useful applications for society?

Mini Investigation

Does Light Have Wave Properties?

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this Mini Investigation, you will preview some of the experimental evidence that supports the theory that light has wave-like properties.

Equipment and Materials: light box or flashlight; double-slit plate with holder; red and green filters

1. Shine the light, without filters, through the double-slit plate, and observe the pattern on a wall. The lights need to be off and the room darkened for this to work. Record your observations. Note the width of the bands and the spacing between bands.

To unplug the light, pull on the plug, not the cord. Ensure that no one can trip on the electric cord and that the lamp is properly housed.

2. Place the red filter in front of the light box. Shine the red light through the slits and observe the pattern on the wall. Record your observations, again noting the width of the bands and the spacing between them.
3. Move the light and double-slit plate closer to the wall, 10 cm at a time. Observe the band pattern, and record your observations each time.
4. Repeat the process with the green filter.

A. What pattern did you see on the wall when you shone the light through the plate in Step 1? Why do you think this pattern appeared? T/I

B. What pattern did you see on the wall when you used the red filter? The green filter? Why do you think the patterns differed from each other, and from those in Step 1? T/I

C. What happened to the pattern as you moved the light, each coloured filter, and the double-slit plate closer to the wall? Why do you think this happened? T/I

9.1 Properties of Waves and Light

If you have ever gone surfing at a water park, or in a lake or an ocean, or watched a news report about a tsunami, you know that water waves can transmit a large amount of energy (**Figure 1**).

Figure 1 Water waves can transmit a large amount of energy.

Other examples of waves are mechanical waves, such as the wave on a vibrating string or the surface of a ringing bell, sound waves such as AM and FM waves, and seismic waves produced by earthquakes. All waves share the same basic properties.

Properties of Waves

periodic wave a wave with a repeated pattern over time or distance

A wave is a moving disturbance that transports energy from one place to another but does not necessarily transport matter. The simplest wave is a periodic wave. A **periodic wave** is a wave that repeats itself at regular intervals.

Figure 2 illustrates one way to generate a mechanical wave. In Figure 2(a), the person's hand exerts a vertical force on the string, creating a wave pulse. Since the force and the resulting pulse act along the length of the string, the force does work on the string. This means that the wave pulse carries energy along the string, from the hand to the wall. In Figure 2(b), as the hand continues to move, the wave becomes periodic. Notice that the points on the strings (matter) do not move horizontally.

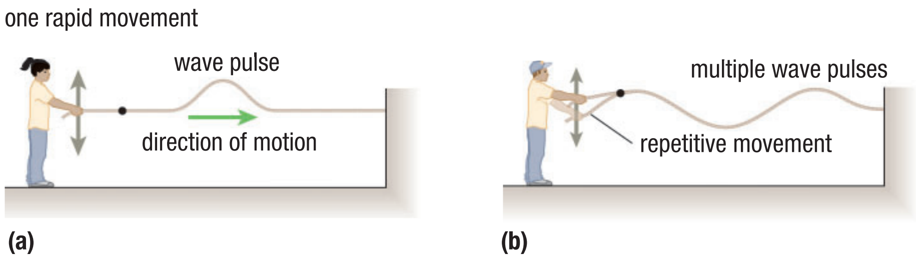


Figure 2 (a) Wave pulse and (b) periodic waves along a string

Suppose that the person is shaking the string such that the string is moving in a periodic fashion in the vertical direction. **Figure 3** shows a snapshot of the wave as it begins to move to the right.

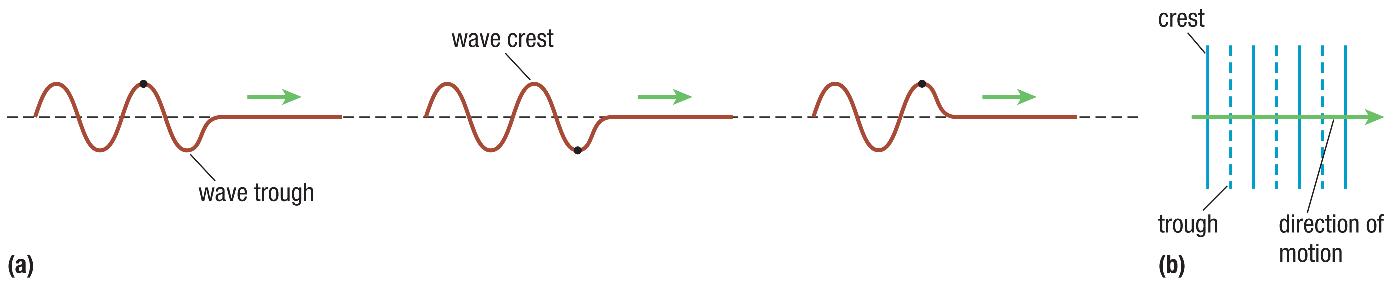


Figure 3 (a) The wave moves to the right. (b) The crests and troughs are shown as waves when viewed from above.

The front edge of a wave is called the **wave front**. The **crest** of the wave is the upper half of the wave. The **trough** of the wave is the lower half of the wave. One complete crest and one complete trough represent one cycle of the wave. The amplitude, A, is the maximum or minimum value of the wave (**Figure 4**). The **wavelength**, λ, of the wave is the distance from one positive amplitude to the next positive amplitude (or one negative amplitude to the next negative amplitude). Two points on a wave that are at the same place in a wave cycle (for example, two successive crests) are said to have the same **phase**. Generally, the phase is the offset of the wave from a reference point. The period, T, of a wave is the time required for one wave cycle to pass a particular point, and the frequency, f, is the number of wave cycles that pass a particular point per unit of time. The SI unit of frequency is the hertz, Hz. One hertz is equal to one cycle per second.

wave front the continuous line or surface at the start of a wave as it travels in time

crest the upper part of a wave

trough the lower part of a wave

wavelength (λ) the distance between one positive amplitude and the next

phase the offset of the wave from a reference point

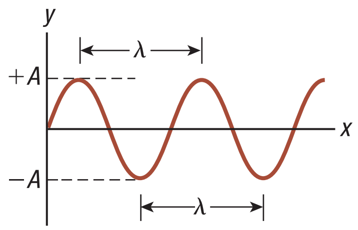


Figure 4 The amplitude is the maximum displacement of a wave. The wavelength is the distance between one positive amplitude, or one negative amplitude, and the next.

How can you use these properties to measure the speed of a wave? Figure 4 shows that the wave moves a distance of $\Delta x = \lambda$ during a period of motion ($\Delta t = T$). The following equation determines the speed of the wave:

$$v = \frac{\Delta x}{\Delta t}$$

$$v = \frac{\lambda}{T}$$

Frequency, f, and period, T, are related according to the following relationships:

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$

Substituting the equation for period into the equation for v, you can see that

$$v = \frac{\lambda}{\left(\frac{1}{f}\right)}$$

or

$$\boxed{v = f\lambda}$$

This is called the universal wave equation, and it shows that the speed of a periodic wave is related to its frequency and wavelength. A unit analysis of this equation shows that frequency in hertz corresponds to speed in metres per second:

$$
\begin{aligned}
v &= f\lambda \\
\frac{[\mathrm{m}]}{[\mathrm{s}]} &= [\mathrm{s}]^{-1}[\mathrm{m}] \\
\frac{[\mathrm{m}]}{[\mathrm{s}]} &= \frac{[\mathrm{m}]}{[\mathrm{s}]}
\end{aligned}
$$

Reflection

ray approximation treating the propagation of light waves as though they move in straight lines called rays

reflection a change in direction of a light ray when it meets an obstacle where the incoming ray and the outgoing ray are on the same side of the obstacle

In many instances, light travels in straight lines. This model, which treats the propagation of light waves as though they move in straight lines, is called **ray approximation**. **Figure 5(a)** shows the reflection of light modelled by rays. **Reflection** is a change in direction after meeting an obstacle, for example, a mirror, where the original ray and the reflected ray are on the same side of the obstacle, or mirror. **Figure 5(b)** shows the angle of incidence, θ_i, and angle of reflection, θ_r, of one of the rays.

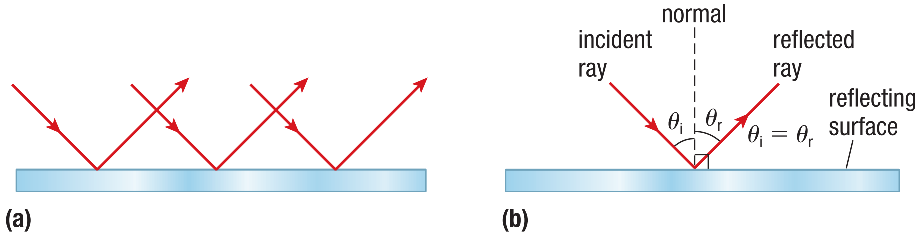


Figure 5 (a) Rays reflecting from a flat surface. Note how the reflected rays are parallel. (b) The angle of incidence is equal to the angle of reflection.

normal the line drawn at a right angle to the boundary at the point where an incident ray strikes the boundary

angle of incidence the angle between the incident ray and the normal

angle of reflection the angle between the reflected ray and the normal

Consider the reflection of light from a flat mirror. The **normal** line is a reference line drawn at a right angle to the surface of the mirror. The **angle of incidence** is the angle between the incoming, or incident, ray and the normal. The **angle of reflection** is the angle between the outgoing, or reflected, ray and the normal. Light, then, behaves according to the **law of reflection**.

Law of Reflection

For reflection from a flat surface, the angle of incidence is always equal to the angle of reflection.

specular reflection the reflection of light from a surface where all the reflected rays are in the same direction

diffuse reflection the reflection of light from a surface where all the reflected rays are directed in many different directions

Reflections from a flat surface, such as the reflecting surface in Figure 5, are called **specular reflections**. If, however, the reflecting surface is rough, as shown in **Figure 6**, then the reflected rays are directed in many different directions as light strikes the different parts of the surface. This is called a **diffuse reflection**.

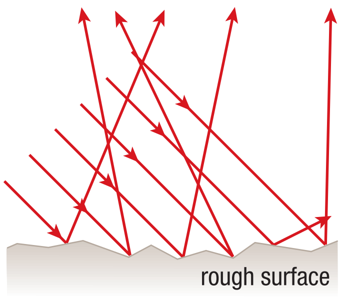


Figure 6 When parallel incident rays reflect from a rough surface, the resulting reflected rays are not parallel. Compare with Figure 5(a).

9.1 Review

Summary

- A wave is a moving disturbance that transports energy from one place to another but does not transport matter.
- The speed of a periodic wave is related to its wavelength and frequency by the universal wave equation, $v = f\lambda$.
- For reflection, we measure the angle between the incident ray and the normal to the reflecting surface, and the reflected ray and the normal to the reflecting surface.
- The law of reflection states that the angle of incidence is equal to the angle of reflection, $\theta_i = \theta_r$.

Questions

1. Explain what determines the frequency of a wave. K/U
2. Explain what determines the speed of a wave. K/U
3. Explain what determines the amplitude of a wave. K/U
4. Explain what determines the wavelength of a wave. K/U
5. A light ray strikes a flat surface making an angle of 10° with the surface. T/I C
   (a) Determine the angle of incidence.
   (b) Calculate the angle of reflection.
   (c) Sketch the path for both the incident and the reflected rays.
6. **Figure 7** shows a wave. The frequency of this wave is 40 Hz. What is the approximate speed of the wave? T/I

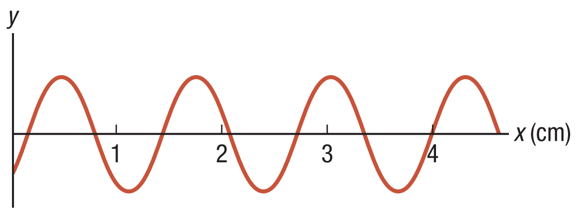


Figure 7

7. A wave travels 0.3 m in 3.5 s and has a frequency of 4.6 Hz. Calculate the wavelength. T/I
8. Determine the frequency of a wave with a period of 0.05 s. T/I A
9. All light waves have a speed of 3.0×10^8 m/s. Calculate the wavelength of light that has a frequency of 5.0×10^{14} Hz. T/I
10. Calculate the frequency of red light waves that have a wavelength of 750 nm. T/I
11. Calculate the wavelength of a violet light with frequency 6.0×10^{14} Hz. T/I
12. A light ray from a source on one wall strikes a flat mirror on the opposite wall. The distance between the walls is 2.5 m. The reflected ray hits the original wall at a point that is 1.2 m below the light source. Determine the angle of incidence, θ_i. T/I
13. Sound waves in water travel at approximately 1.5×10^3 m/s. Calculate the wavelength of a sound wave that has a frequency of 4.4×10^2 Hz. T/I
14. A wave completes one cycle as it moves a distance of 2.0 m at a speed of 20.0 m/s. Calculate the frequency of the wave. T/I
15. A wave has a frequency of 3.1 kHz and a wavelength of 0.13 m. Calculate the speed of the wave. T/I
16. One wavelength of visible radiation has a frequency of 7.9×10^{14} Hz. This radiation has a speed of 3.0×10^8 m/s in air. Determine the wavelength of the radiation. T/I
17. A microwave oven emits microwaves with a frequency of 310 MHz. The speed of microwaves is 3.0×10^8 m/s. Calculate the wavelength of the microwaves. T/I
18. Explain why mirrors can reflect images. K/U
19. Two waves with equal speeds have frequencies that differ by a factor of three. What is the ratio of their wavelengths? K/U A
20. A wave on a string has a frequency of 0.83 Hz and a wavelength of 0.56 m. Determine the wavelength when a new wave of frequency 0.45 Hz is established on this string and the wave speed does not change. T/I A
21. Identify whether the following surfaces cause specular or diffuse reflection, and justify your answer. T/I A
    (a) a flat mirror
    (b) a piece of notebook paper
    (c) the surface of a puddle on a calm day
    (d) the surface of a lake on a windy day

9.2 Refraction and Total Internal Reflection

When a light wave strikes a transparent material such as glass or water, some of the light is reflected from the surface (as described in Section 9.1). The rest of the light passes through (transmits) the material. **Figure 1** shows a ray that has entered a glass block that has two parallel sides. The part of the original ray that travels into the glass is called the refracted ray, and the part of the original ray that is reflected is called the reflected ray.

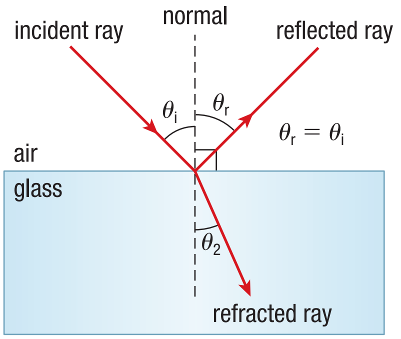


Figure 1 A light ray that strikes a glass surface is both reflected and refracted.

Refracted and reflected rays of light account for many things that we encounter in our everyday lives. For example, the water in a pool can look shallower than it really is. A stick can look as if it bends at the point where it enters the water. On a hot day, the road ahead can appear to have a puddle of water, which turns out to be a mirage. These effects are all caused by the refraction and reflection of light.

Refraction

refraction the bending of light as it travels at an angle from one medium to another

optical density the property of a material that determines how light behaves when it travels through the material

principle of reversibility a light ray will follow exactly the same path if its direction of travel is reversed

index of refraction the ratio of the speed of light in a vacuum to the speed of light in another medium

The direction of the refracted ray is different from the direction of the incident ray, an effect called **refraction**. As with reflection, you can measure the direction of the refracted ray using the angle that it makes with the normal. In Figure 1, this angle is labelled θ_2. The size of this angle depends on the incident angle (which is shown as θ_i in Figure 1) as well as the optical densities of the two media. The **optical density** of a medium is a measure of its tendency to absorb the energy of an electromagnetic wave, which is different from the material's mass density. The more optically dense a material is, the slower a wave will move through it. When light travels from a less optically dense medium such as air to a more optically dense medium such as glass, light is refracted toward the normal, as shown in Figure 1. If we extend the refracted ray in Figure 1 to show the light leaving the glass and entering air again, you would see that the light is refracted away from the normal and emerges parallel to the incident ray. This is shown in **Figure 2** on the next page. It is an example of the **principle of reversibility**, which states that the path a light ray follows remains the same if its direction of travel is reversed.

Consider the speed of light in various media. The speed of light in a vacuum is 3.0×10^8 m/s. In most cases, this value is a good approximation of the speed of light in air. However, when light enters a different medium, it interacts with the atoms and its average speed through the medium is changed. The ratio of the speed of light in a vacuum to the speed of light in another medium, $\frac{c}{v}$, is called the **index of refraction**, n. **Table 1**, on the next page, shows the speed of light and indices of refraction in different media.

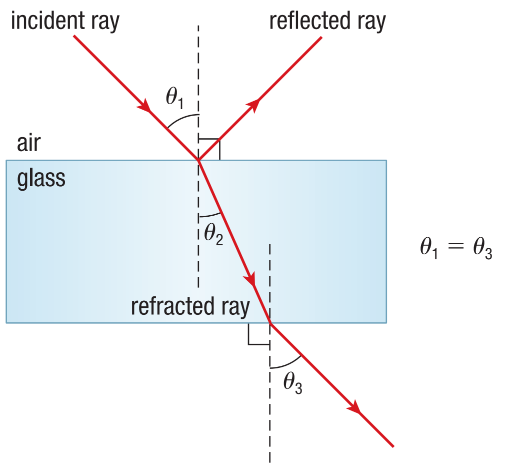


Figure 2 The refracted ray that emerges from the bottom of the glass is parallel to the incident ray.

Table 1 The Speed of Light and Indices of Refraction in Different Media

Medium	Index of refraction	Speed of light (m/s)	Medium	Index of refraction	Speed of light (m/s)
vacuum	1.00	2.9979×10^8	lens of human eye	1.41	2.1262×10^8
air	1.0003	2.9970×10^8	quartz crystal	1.46	2.0534×10^8
ice	1.30	2.3061×10^8	Pyrex glass	1.47	2.0394×10^8
liquid water	1.33	2.2541×10^8	Plexiglas (plastic)	1.51	1.9854×10^8
aqueous humour (liquid between the lens and cornea)	1.33	2.2541×10^8	benzene	1.50	1.9986×10^8
cornea of human eye	1.38	2.1724×10^8	zircon	1.92	1.5601×10^8
vitreous humour (liquid between the lens and retina)	1.38	2.1724×10^8	diamond	2.42	1.2388×10^8

Notice in Table 1 that as the index of refraction increases, the speed of light in various media decreases. **Figure 3** shows how the change in the speed of light from a vacuum to glass affects the orientation of the wave fronts and the direction of the rays inside the glass, causing refraction. Each incident ray is at a right angle to its respective wave front. The incident rays arrive at an angle of θ_1, and the refracted rays are at an angle of θ_2.

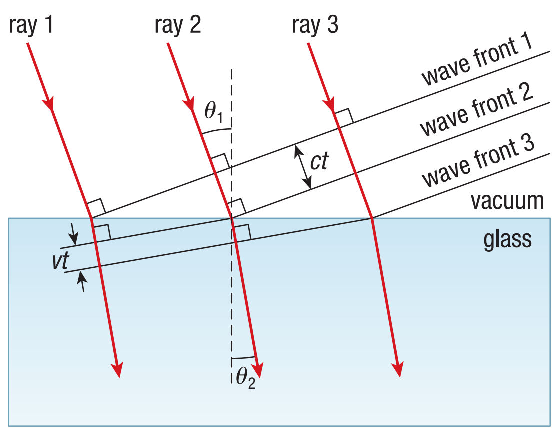


Figure 3 Refraction is caused by the difference in wave speed in two media. Here the distance between the wave fronts inside the glass is less than outside the glass. The speed of light has decreased; however, the frequency of the light is the same.

Figure 3 also shows the wave fronts at equally spaced moments in time. The speed of light in the vacuum is c, so the wave travels a distance of ct through the vacuum in time t. In the glass, the speed of light is reduced to v, so the wave travels a distance of vt in time t. Since the speed of light in a vacuum (c) is greater than the speed of light in the glass (v), the distance travelled in time t in the glass (vt) is less than the distance travelled in the vacuum (ct). This effect causes the wave fronts to bend at the point where they enter the glass, which means that the light rays bend toward the normal as they enter the glass. The result is that θ_2 is less than θ_1. The angle θ_2 between the refracted light ray and the normal is called the **angle of refraction** and is often written as θ_R. **Figure 4** shows a more detailed view of the relationship between the angles θ_1 and θ_2 as the wave fronts enter the glass.

angle of refraction the angle that a light ray makes with respect to the normal to the surface when it has entered a different medium

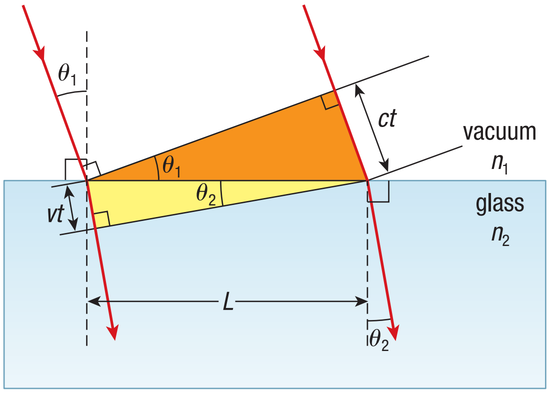


Figure 4 You can use the geometry of the wave fronts to examine the relationship between the incident and refracted angles.

The orange triangle on the glass edge in Figure 4 has sides of length ct and L, with the angle adjacent to L being θ_1. The following trigonometric relationship is true for this triangle:

$$\sin\theta_1 = \frac{ct}{L}$$

The corresponding yellow triangle has sides of length vt and L, with the angle adjacent to L being θ_2. For this triangle, the same relationship holds:

$$\sin\theta_2 = \frac{vt}{L}$$

Rearrange these equations to solve for L:

$$L = \frac{ct}{\sin\theta_1} \quad \text{and} \quad L = \frac{vt}{\sin\theta_2}$$

Therefore,

$$\frac{ct}{\sin\theta_1} = \frac{vt}{\sin\theta_2}$$

Since t cannot equal zero, we can divide each side of the equation by t:

$$\frac{c}{\sin\theta_1} = \frac{v}{\sin\theta_2}$$

or

$$\sin\theta_1 = \frac{c\sin\theta_2}{v}$$

Recall that the ratio $\frac{c}{v}$ is a dimensionless number called the index of refraction and is given the symbol n:

$$n = \frac{c}{v}$$

We can rewrite the above equation as

$$\sin\theta_1 = n_2\sin\theta_2$$

where n_2 is the index of refraction of the glass.

The preceding derivation describes the refraction of light from a vacuum to glass. If you consider the common situation where light passes between two substances, such as air and glass or air and water, then the following relationship, called **Snell's law**, is true:

Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where n_1 and θ_1 are, respectively, the index of refraction in the incident medium and the angle of incidence, and n_2 and θ_2 are, respectively, the index of refraction for the medium being studied and the angle of refraction.

Using the relationship between speed, frequency, and wavelength, as well as the fact that the frequency of light does not change when passing from one substance to another, the following equation can be derived:

$$n = \frac{c}{v}$$

$$= \frac{f\lambda_1}{f\lambda_2}$$

$$n = \frac{\lambda_1}{\lambda_2}$$

where λ_1 is the wavelength of light in a vacuum, and λ_2 is the wavelength of light in the substance. When the first medium is not a vacuum and has index of refraction n_1, this equation becomes

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$$

In **Figure 5**, incident light hits the block at angle θ_1 on side 1. It is then refracted at an angle θ_2, which can be calculated using Snell's law if we have enough information, such as the indices of refraction of the media. When the incident light hits the boundary on the opposite side, at an angle θ_2, the light is refracted at an angle equal to θ_1 because the surfaces are parallel.

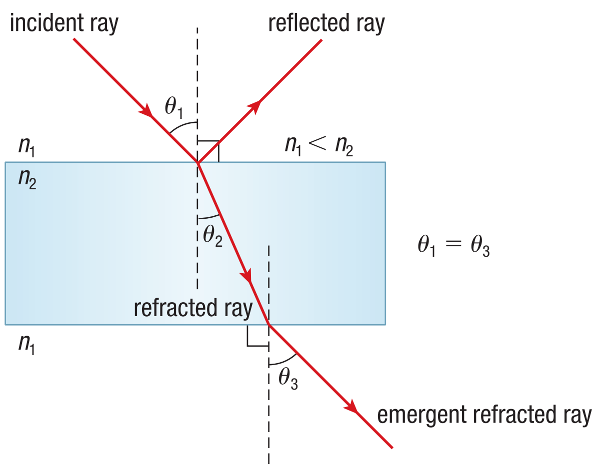


Figure 5 Snell's law can be used to calculate the angles of refraction.

In Tutorial 1, you will use Snell's law to calculate the angle of refraction as well as the speed, wavelength, and frequency for refracted light.

Tutorial 1 Solving Problems by Applying Snell's Law

You can calculate angles of incidence and refraction using Snell's law, given information about the index of refraction. In the following Sample Problem, we will use Snell's law to solve for the angle of refraction and the speed, wavelength, and frequency of light.

Sample Problem 1: Angle of Refraction for Light from a Vacuum into Glass

(a) Calculate the angle of refraction for light moving from a vacuum into a plate of glass with index of refraction 1.47. The angle of incidence is 40.0°.

(b) The light continues through the glass and emerges back into a vacuum. Calculate the angle of refraction when the light exits the glass.

(c) Suppose the light exits into water instead of a vacuum. Calculate the angle of refraction for the light moving from glass into water. The index of refraction for water is 1.33.

Solution

(a) **Given:** $n_2 = 1.47$; $\theta_1 = 40.0°$; $n_1 = 1.0$

Required: θ_2

Analysis:

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}$$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

Solution:

$$\begin{aligned}\sin\theta_2 &= \frac{n_1 \sin\theta_1}{n_2}\\ &= \frac{(1.0)\sin 40.0°}{1.47}\\ \theta_2 &= 25.9° \text{ (one extra digit carried)}\end{aligned}$$

Statement: The angle of refraction for light moving from the vacuum into glass is 26°.

(b) **Given:** $n_3 = 1.0$; $\theta_2 = 25.9°$

Required: θ_3

Analysis:

$$\frac{\sin\theta_3}{\sin\theta_2} = \frac{n_2}{n_3}$$

$$\sin\theta_3 = \frac{n_2 \sin\theta_2}{n_3}$$

Solution:

$$\begin{aligned}\sin\theta_3 &= \frac{n_2 \sin\theta_2}{n_3}\\ \sin\theta_3 &= \left(\frac{1.47}{1}\right)(\sin 25.9°)\\ &= 0.64\\ \theta_3 &= 40°\end{aligned}$$

Statement: The angle of refraction for light moving from the glass back into the vacuum is 40°. This is the same as the angle of incidence of the ray entering the glass.

(c) **Given:** $n_3 = 1.33$; $\theta_2 = 25.9°$; $n_2 = 1.47$

Required: θ_3

Analysis:

$$\frac{\sin\theta_3}{\sin\theta_2} = \frac{n_2}{n_3}$$

$$\sin\theta_3 = \frac{n_2 \sin\theta_2}{n_3}$$

Solution:

$$\begin{aligned}\sin\theta &= \frac{n_2 \sin\theta_2}{n_3}\\ &= \frac{(1.47)\sin 25.9°}{1.33}\\ \theta_3 &= 29°\end{aligned}$$

Statement: The angle of refraction for light moving from glass into water is 29°.

Sample Problem 2: Refraction and Lasers

Light travels at 3.0×10^8 m/s. Laser light with a wavelength of 520 nm enters a sheet of plastic. The index of refraction for the plastic is 1.49.

(a) Calculate the speed of the laser light in the plastic.

(b) Calculate the wavelength of the laser light in the plastic.

(c) Calculate the frequency of the laser light in the plastic.

Solution

(a) **Given:** $n = 1.49$; $c = 3.0 \times 10^8$ m/s

Required: v

Analysis:

$$n = \frac{c}{v}$$

$$v = \frac{c}{n}$$

Solution:

$$v = \frac{c}{n}$$

$$= \frac{3.0 \times 10^8 \text{ m/s}}{1.49}$$

$$v = 2.01 \times 10^8 \text{ m/s (one extra digit carried)}$$

Statement: The speed of the laser light in the plastic is 2.0×10^8 m/s.

(b) **Given:** $n = 1.49$; $\lambda_1 = 520 \text{ nm} = 5.2 \times 10^{-7}$ m

Required: λ_2

Analysis:

$$n = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \frac{\lambda_1}{n}$$

Solution:

$$\lambda_2 = \frac{\lambda_1}{n}$$

$$= \frac{5.2 \times 10^{-7} \text{ m}}{1.49}$$

$$\lambda_2 = 3.49 \times 10^{-7} \text{ m (one extra digit carried)}$$

Statement: The wavelength of the laser light in the plastic is 3.5×10^{-7} m.

(c) **Given:** $\lambda_2 = 3.49 \times 10^{-7}$ m; $v = 2.01 \times 10^8$ m/s

Required: f

Analysis:

$$f = \frac{v}{\lambda_2}$$

Solution:

$$f = \frac{v}{\lambda_2}$$

$$= \frac{2.01 \times 10^8 \, \cancel{\text{m}}/\text{s}}{3.49 \times 10^{-7} \, \cancel{\text{m}}}$$

$$f = 5.77 \times 10^{14} \text{ Hz}$$

Statement: The frequency of the laser light in the plastic is 5.8×10^{14} Hz.

Practice

1. In an experiment, light shines on a flat mirror with an angle of incidence of 65°. The experiment takes place underwater. What is the angle of incidence? The index of refraction for water is 1.33. K/U [ans: 65°]
2. A light ray enters an unknown medium from air. Some reflects at an angle of 47.5°, and the rest of the light is refracted at an angle of 34.0°. According to Table 1 on page 445, what is the unknown medium? T/I [ans: water]
3. Calculate the index of refraction of water when a light ray that hits the top of a glass of water at an angle of incidence of 35° is refracted at an angle of 25°. T/I [ans: 1.36]
4. Determine the speed of light in a diamond. Use 3.0×10^8 m/s for the speed of light in air. Refer to Table 1 on page 445 for the index of refraction for diamond. T/I [ans: 1.2×10^8 m/s]
5. Calculate the wavelength of light in quartz if the wavelength in a vacuum is 5.6×10^{-7} m and the index of refraction is 1.46. T/I [ans: 3.8×10^{-7} m]
6. Light with a wavelength of 450 nm in a vacuum enters a sample of glass. The index of refraction of the glass is 1.45. Determine the frequency of the light inside the glass. T/I [ans: 6.7×10^{14} Hz]

Dispersion and Prisms

Visible light contains waves with a spectrum of different wavelengths. In a vacuum, all these waves travel at the same speed, 3.0×10^8 m/s. When visible light travels from one medium to another, the speeds of the various waves change.

The speed of the wave depends on its frequency and wavelength. When the wave enters the new medium, its frequency does not change but its wavelength does. This means that the speed of the refracted light depends on its wavelength. This dependence of the speed of light on wavelength causes light waves to separate in a phenomenon called **dispersion**. **Figure 6** shows the variation of the speed of visible light with wavelength for three materials.

dispersion the separation of a wave into its component parts according to a given characteristic, such as frequency or wavelength

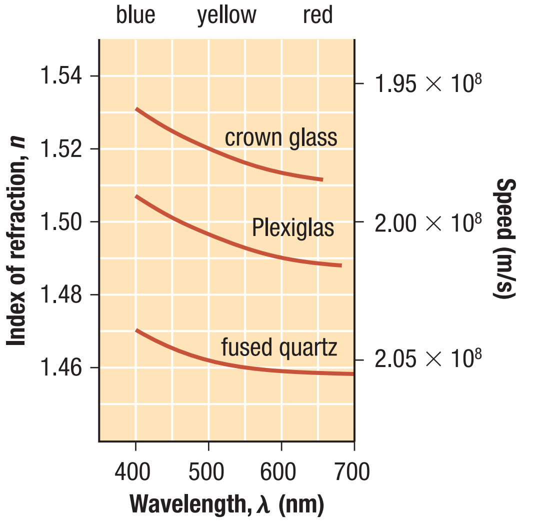


Figure 6 The speed of visible light varies in different materials.

The index of refraction for a material depends on the wavelength of light. The index of refraction for red light (at one end of the visible spectrum) and the index of refraction for blue light (at the opposite end of the spectrum) are slightly different for the same material. For example, Figure 6 shows that, in quartz, the index of refraction for red light is approximately 1.46 and the index of refraction for blue light is approximately 1.47. The difference between these values is 0.01, which is much smaller than the difference between the (average) indices of refraction for quartz and water ($1.46 - 1.33 = 0.13$). However, the difference is large enough to lead to a difference in the angle of refraction for different colours in quartz. This is true in other materials as well. **Figure 7** shows how this effect is used in a prism to disperse (separate) an incident beam of white light into its component colours.

UNIT TASK **BOOKMARK**

You can apply what you have learned about dispersion and prisms to the Unit Task on page 556.

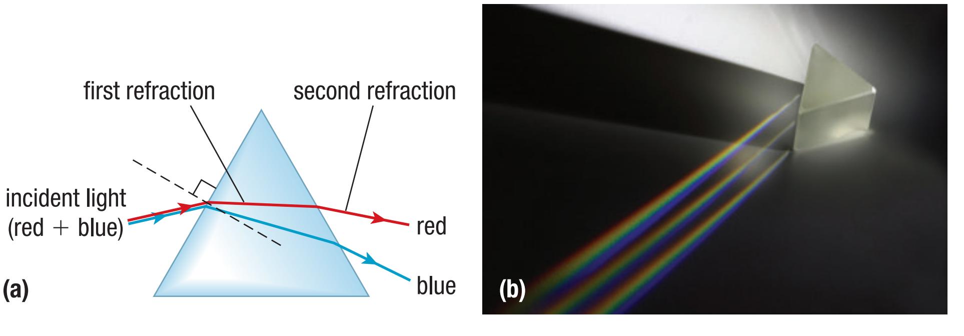


Figure 7 (a) A simplified comparison between red and blue light being dispersed by a prism. Note that the drawing is not to scale. (b) In a real prism white light is dispersed into its component colours.

Prisms are often composed of glass, and are triangular in shape, as shown in Figure 7(b). In Figure 7(a), the beam of light that is incident on the prism is taken, for simplicity, to be composed of two colours (red and blue), represented as two rays. Each ray is refracted once when it enters the left-hand face of the prism, and again when it leaves the prism at the right-hand face. Note that the light does not curve within the material but only changes direction at the boundaries.

The incident blue light has a slightly larger index of refraction than red light, so its angle of refraction is less than the angle of refraction of the incident red light. Inside the glass, the blue and red light components travel along slightly different paths. This path difference is increased by the second refraction when the light exits the prism. The outgoing light appears slightly bluish at one side and slightly reddish at the other side of the beam.

Another effect of the triangular shape of a prism is that the light ray leaving the prism is not parallel to the light ray that enters the prism. The angle between the incident ray and the final outgoing ray is called the **angle of deviation**. The following Tutorial examines how you can use Snell's law to solve dispersion problems involving prisms.

angle of deviation the angle between the incident ray and the final outgoing ray after reflection or refraction

Tutorial 2 Solving Dispersion Problems Using Snell's Law

In this Tutorial, you will use Snell's law to calculate the angles at which a prism disperses different wavelengths of light.

Sample Problem 1: Angle of Refraction through a Prism

Figure 8 shows a ray of light incident on a glass triangular prism. It is refracted as it passes through the first face and is then refracted again through the second face.

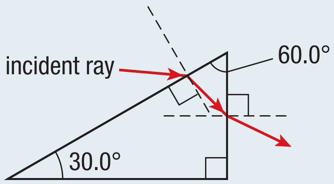


Figure 8

(a) Calculate the angles of refraction for blue light and for red light travelling from air into the left boundary of the prism. The angle of incidence at the left boundary of the prism is 40.0°. The index of refraction for blue light in the prism is 1.47, and the index of refraction for red light is 1.46. The index of refraction for air is 1.0003.

(b) Calculate the angle of incidence for blue and red light at the right boundary of the prism.

(c) Calculate the angle of refraction for blue and red light as the rays exit the prism.

Solution

(a) **Given:** $n_{2\text{ blue}} = 1.47$; $n_{2\text{ red}} = 1.46$; $n_{\text{air}} = 1.0003$; $\theta_1 = 40.0°$

Required: $\theta_{2\text{ blue}}$; $\theta_{2\text{ red}}$

Analysis: $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{n_{\text{air}}}{n_{\text{prism}}}$

Solution: For blue light:

$$\frac{\sin\theta_{2\text{ blue}}}{\sin\theta_1} = \frac{n_1}{n_{2\text{ blue}}}$$

$$\sin\theta_{2\text{ blue}} = \frac{n_1\sin\theta_1}{n_{2\text{ blue}}}$$

$$= \frac{(1.0003)\sin 40.0°}{1.47}$$

$$\theta_{2\text{ blue}} = 25.94° \text{ (one extra digit carried)}$$

For red light:

$$\theta_{2\text{ red}} = \sin^{-1}\left(\frac{(1.0003)\sin 40.0°}{1.46}\right)$$

$$\theta_{2\text{ red}} = 26.13° \text{ (one extra digit carried)}$$

Statement: The angle of refraction at the left boundary of the prism of blue light is 25.9°. The angle of refraction of red light is 26.1°.

(b) **Given:** $\theta_{2\text{ blue}} = 25.94°$; $\theta_{2\text{ red}} = 26.13°$

Required: $\theta_{3\text{ blue}}$; $\theta_{3\text{ red}}$

Analysis: If θ_2 is the angle of refraction at the left boundary and θ_3 is the angle of incidence at the right boundary, then from the geometry of the triangular prism we have $\theta_2 + \theta_3 = 60.0°$ (which is the angle in the top right of the triangle).

Solution: $\theta_3 = 60.0° - \theta_2$

For blue light: $60.0° - 25.94° = 34.06°$

For red light: $60.0° - 26.13° = 33.87°$

Statement: The angle of incidence at the right boundary of the prism of blue light is 34.1°. The angle of incidence of red light is 33.9°.

(c) **Given:** $\theta_{3\text{ blue}} = 34.06$; $\theta_{3\text{ red}} = 33.87$; $n_{2\text{ blue}} = 1.47$; $n_{4\text{ blue}} = 1.46$; $n_{\text{air}} = 1.0003$

Required: $\theta_{4\text{ blue}}$; $\theta_{4\text{ red}}$

Analysis: $\dfrac{\sin\theta_4}{\sin\theta_3} = \dfrac{n_{\text{prism}}}{n_{\text{air}}}$

Solution: For blue light:

$$\frac{\sin\theta_{4\text{ blue}}}{\sin\theta_{3\text{ blue}}} = \frac{n_{2\text{ blue}}}{n_1}$$

$$\sin\theta_{4\text{ blue}} = \frac{n_{2\text{ blue}}\sin\theta_{3\text{ blue}}}{n_1}$$

$$= \frac{(1.47)(\sin 34.06°)}{1.0003}$$

$$\theta_{4\text{ blue}} = 55.4°$$

For red light:

$$\theta_{4\text{ red}} = \sin^{-1}\frac{(1.46)(\sin 33.87°)}{1.0003}$$

$$\theta_{4\text{ red}} = 54.4°$$

Statement: The angle of refraction for the blue light as it exits the prism is 55.4°. The angle of refraction for the red light as it exits the prism is 54.4°.

Practice

1. Consider Figure 8 on page 451, but this time only one colour of light strikes the prism. The index of refraction for this colour in the prism is 1.465, and its angle of incidence is 40.0°. T/I
   (a) Determine the angle of refraction for this light at the left boundary of the prism. [ans: 26.0°]
   (b) Determine the angle of refraction for this light as it exits the prism. [ans: 54.9°]
2. In Sample Problem 1(c), why is the final angle of refraction not equal to the angle of incidence? K/U C
3. Monochromatic light is incident on the prism in **Figure 9**. The index of refraction of the prism is 1.60. The angle of incidence is 55°. What angle does the outgoing ray make with the horizontal? K/U T/I A [ans: 21° [below the horizontal]]

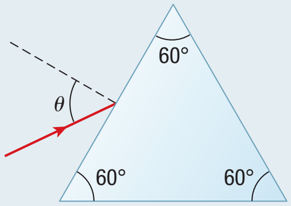


Figure 9

Research This

Using Spectroscopy to Determine Whether Extra-Solar Planets Can Support Life

Skills: Questioning, Researching, Analyzing, Communicating — SKILLS HANDBOOK A4.1

Research spectroscopy. Find out how scientists use star spectra (**Figure 10**) to look for planets that contain substances necessary for life, such as carbon dioxide, methane, oxygen, and water, in their atmospheres. Your research should aim to answer the following questions.

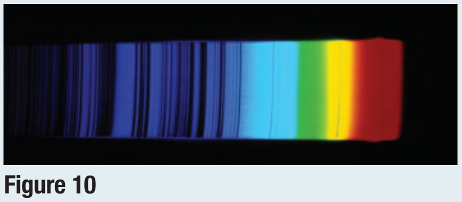


Figure 10

A. How are dispersion and spectroscopy used to determine which gases are present on other planets? T/I A

B. Name some of the key chemical compounds that astrophysicists and astrobiologists look for, and explain why these compounds are so important. T/I

C. When scientists use this technique, do they consider light to be a wave, a particle, or both? Explain. A

WEB LINK

Total Internal Reflection

The angle of refraction depends on the angle of incidence; however, for some values of the angle of incidence, there is no refracted ray. Consider a light ray that travels in medium 1 and meets the boundary of medium 2, as shown in **Figure 11**. If the index of refraction n_1 is greater than n_2, the angle of refraction θ_2 is greater than θ_1. Light will refract away from the normal. As the incident angle θ_1 increases, the refracted angle θ_2 also increases.

When θ_2 reaches 90°, the refracted ray will travel parallel to the surface (Figure 11(b)). The value of the angle of incidence at which this occurs is called the **critical angle**, θ_c. If the angle of incidence θ_1 increases beyond the critical angle, there is no refracted ray, and the incident light ray is totally reflected at the boundary (Figure 11(c)). This effect, called **total internal reflection**, can occur when light encounters a boundary between an initial medium with a higher index of refraction and a second medium with a lower index of refraction.

critical angle the smallest angle of incidence at which a light ray passing from one medium to another less refractive medium can be totally reflected from the boundary between the two

total internal reflection an effect that occurs when light encounters a boundary between a medium with a higher index of refraction and one with a lower index of refraction

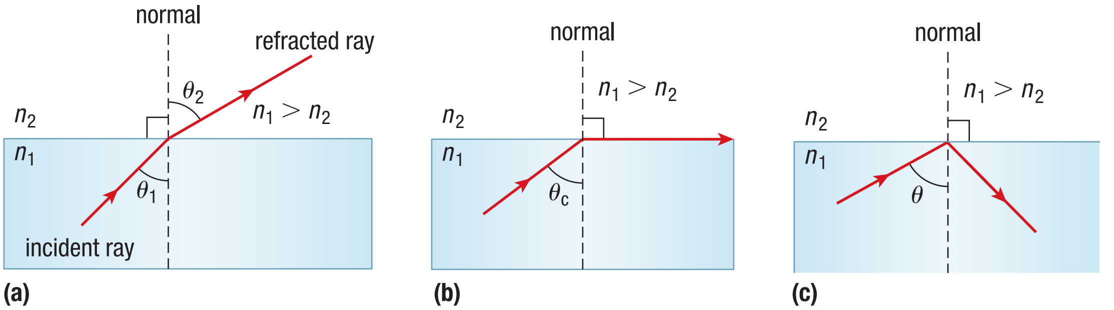


Figure 11 (a) Different angles of incidence cause different angles of refraction. (b) At the critical angle, the angle of refraction is 90°. (c) When the angle of incidence is greater than the critical angle, no light exits medium 1.

Figure 12 shows an example of total internal reflection. Here, light from a laser enters the side of a tank of water and travels through it. At the air–water boundary the angle of incidence exceeds the critical angle, so there is no refracted ray. The ray is reflected as though it had hit a surface like a mirror. The incident and reflected rays obey the law of reflection: $\theta_i = \theta_r$.

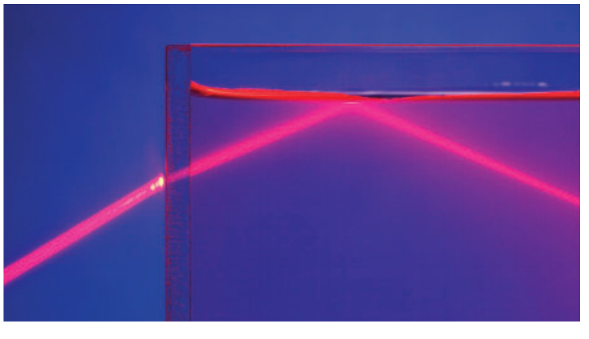


Figure 12 At $\theta_i > \theta_c$, the laser light totally reflects within the water.

By applying Snell's law, you can determine the critical angle since $\theta_1 = \theta_c$ and $\theta_2 = 90°$. From Snell's law,

$$n_1 \sin \theta_c = n_2 \sin 90° = n_2$$

Therefore,

$$\sin \theta_c = \frac{n_2}{n_1}$$

Since the sine of an angle can never be greater than 1, there can be no critical angle unless n_1 is greater than n_2. This means that total internal reflection only occurs when light travels through a medium and encounters a boundary of another medium with a lower index of refraction, $n_1 > n_2$.

UNIT TASK **BOOKMARK**

You can apply what you have learned about total internal reflection and Snell's law to the Unit Task on page 556.

Total internal reflection can occur even when the media appear to be otherwise transparent. If you look carefully at a glass of water, you can observe that at certain angles you do not see the table on which the glass sits. Instead, you see a reflection of the inside of the glass. However, if you pick up the glass you will immediately see your fingers.

Total internal reflection has many practical applications. When combined with glass or prisms, total internal reflection can change the direction of a light ray, as shown in **Figure 13**.

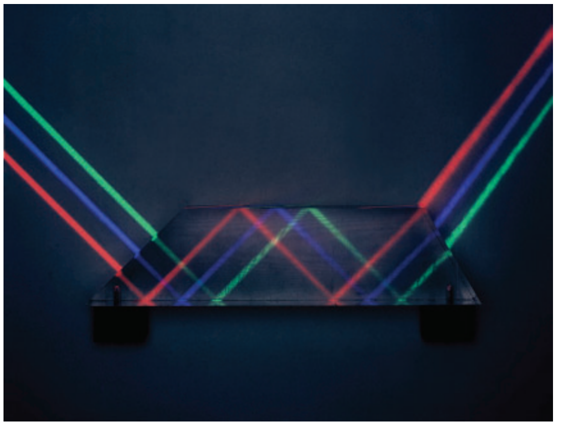


Figure 13 Total internal reflection can be used to make devices that change the direction of a light beam.

Prisms that are isosceles right-angled triangles are often used to control the direction of light. As **Figure 14(a)** shows, light enters from a side, is turned 90°, and exits out the other side. **Figure 14(b)** shows how to use the two short sides of a prism to make the light entering the base take two 90° turns before exiting back out the base. **Figure 14(c)** shows an application in which two prisms are used together to alter the path of a ray of light, as in the periscope of a submarine.

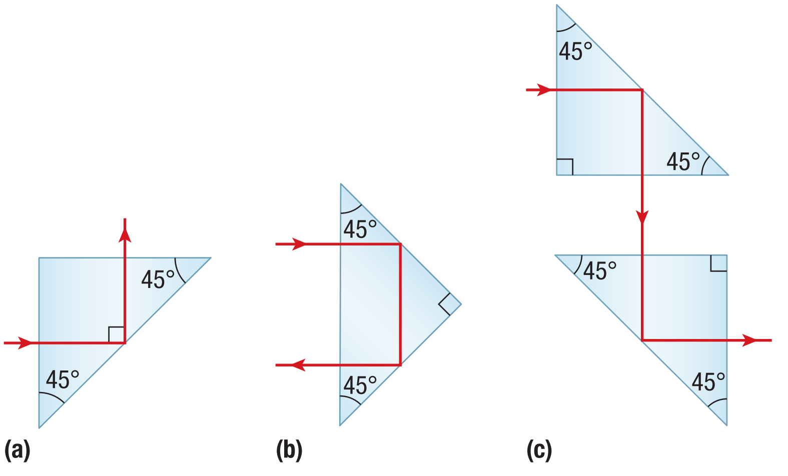


Figure 14 Various configurations of prisms can control the direction of light rays.

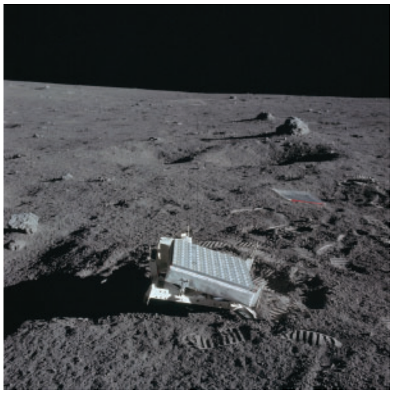


Figure 15 Retro-reflectors left on the Moon in a series of Apollo missions reflect laser light back toward Earth.

Retro-reflectors left on the surface of the Moon by astronauts during some of the Apollo missions are an interesting application of the reflection of light. These retro-reflectors, shown in **Figure 15**, contain hundreds of prisms and are oriented to reflect laser light back to Earth. Much like the reflector on a bicycle, these retro-reflectors have the property of always reflecting an incoming light ray back in the direction it came from. When scientists aim a laser at the retro-reflector on the Moon, the reflection enables them to measure the Earth–Moon distance with a precision of about 15 cm.

You can also make use of total internal reflection and dispersion to achieve a filter-like effect using a semicircular piece of glass, as shown in **Figure 16**. If a ray of light travels toward the centre of the semi-circle, at point P, it will strike the round edge along the normal and not refract. Imagine now rotating the piece of glass to make larger angles of incidence. Because the index of refraction depends on the wavelength of light, the critical angle for the glass–air interface also depends on the wavelength of light. In fact, the critical angle for blue light is less than the critical angle for red light. Therefore, as the angle increases, the light emerging along the flat edge of the glass becomes increasingly red in colour as other wavelengths totally reflect.

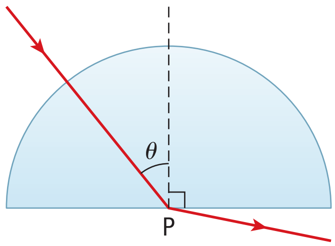


Figure 16 Total internal reflection combined with dispersion can turn a semicircular piece of glass into a red filter.

Fibre Optics

fibre optics a technology that uses glass or plastic wire (fibre) through which data are transmitted using internally reflected light impulses

Fibre optics is a technology that uses flexible strands of glass, called optical fibres, to conduct and transmit light. Recall that light undergoes partial reflection and refraction when it encounters an interface between materials with different indices of refraction. Although an optical fibre is cylindrical, its diameter is large compared to the wavelength of light. Consequently, the reflection of light within the optical fibre is similar to reflection from a flat interface. Even if the fibre is curved, the light remains inside due to total internal reflection (**Figure 17**).

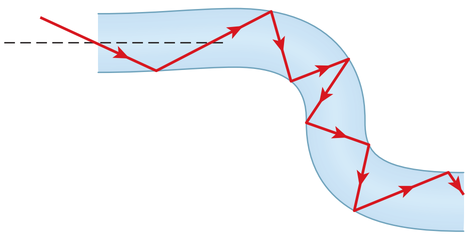


Figure 17 Light moves through the fibre optic cable by reflection.

The optical fibre in Figure 17 is a simplified illustration. All practical optical fibres consist of at least two different types of glass, as shown in **Figure 18**. The central core is surrounded by an outer layer called the cladding. The core and the cladding are both made of glass, but with different compositions and different indices of refraction. The index of refraction of the cladding is less than the core's index of refraction, enabling total internal reflection of light within the core.

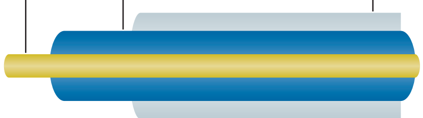


Figure 18 Fibre optic cable is surrounded by a layer of glass cladding as well as a protective outer sheath.

In an application, light from a laser enters the fibre at one end, such that the angle of incidence permits the light to always undergo total internal reflection at the boundary between the core and the cladding. Signals, such as a telephone signal, are carried from one end of the fibre to the other by pulses of laser light within the core. These fibres can be very long—many kilometres—and a single fibre can carry many more simultaneous signals than is possible with a conventional metal wire. Optical fibres can be quite flexible, and an optical transmission line can be constructed by bundling parallel fibres together. CAREER LINK

The range of angles at which light can enter the fibre is called the acceptance cone (**Figure 19**). If incoming light is at an angle within this cone, then it will be transmitted through the fibre by total internal reflection. If incoming light is at an angle outside this cone, then it can exit out of the fibre core. The size of the acceptance cone can be adjusted. It would become larger if the critical angle for the core were made smaller. Using a material with a smaller index of refraction for the cladding material makes the difference between the indices of refraction for the core and cladding materials greater.

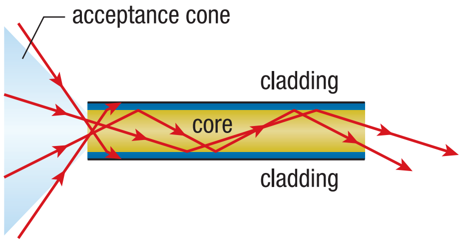


Figure 19 Light from within the acceptance cone travels inside the fibre optic core. Light from outside the acceptance cone can exit outside of the fibre core.

If a fibre optic cable bends too much at one point, an incident light ray can hit the side at an angle less than θ_c. The possibility of total internal reflection is lost, and the light can exit the core fibre. If a light ray travelling inside the fibre meets the side wall of the fibre with angle of incidence θ_1, there is a reflected ray with an angle of reflection equal to θ_1. If, however, the fibre is bent to the point that θ_1 is less than the critical angle θ_c, then total internal reflection is lost, the refracted ray in the air travels at an angle θ_2, and there is a loss of light within the fibre.

Fibre optic cables can be made very small and flexible, so they have many uses for transporting signals and light through small or dangerous places. Doctors can study a patient's internal tissues using an instrument called an endoscope (**Figure 20**) which uses fibre optics. With an endoscope, a doctor can check a patient's digestive system for internal blockages, damaged tissues, stomach ulcers, and other medical issues without performing major surgery.

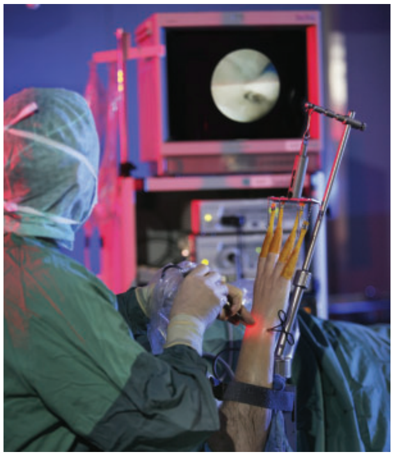


Figure 20 An endoscope uses fibre optic cables to allow doctors to explore this patient's wrist without invasive surgery.

Fibre optic cables have transformed communications by allowing signals to travel quickly (at the speed of light) across long distances with relatively little loss of signal strength. Silica fibres, which are commonly used, have a loss rate of approximately 5 % to 10 % of the signal intensity per kilometre. There are some cables with losses of less than 5 %. However, over large distances technicians have to boost the signal along the way to take care of the signal intensity losses. Many communications companies in Ontario and across Canada depend on fibre optic technology for Internet and telephone services. Even transatlantic cables, which were once made of metal, now consist of fibre optic cables, including the CANTAT-3, which runs from Nova Scotia on the western end to Iceland, the Faroe Islands, England, Denmark, and Germany on the eastern end. CAREER LINK

Tutorial 3 Solving Problems Related to Total Internal Reflection

The Sample Problem in this Tutorial models how to solve problems related to total internal reflection and the critical angle.

Sample Problem 1: The Critical Angle for a Water–Air Boundary

(a) Calculate the critical angle for light passing through water (index of refraction 1.33) into air (index of refraction 1.0003).

(b) Describe what an underwater swimmer sees if she looks toward the surface at angles of 40°, θ_c from (a), and 60° relative to the normal.

Solution

(a) **Given:** $n_2 = 1.0003$ (index of refraction for air); $n_1 = 1.33$ (index of refraction for water)

Required: θ_c

Analysis: $\sin\theta_c = \dfrac{n_2}{n_1}$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Solution: $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

$$= \sin^{-1}\left(\frac{1.0003}{1.33}\right)$$

$$\theta_c = 48.8°$$

Statement: The critical angle for light passing through water into air is 48.8°.

(b) The path of a light ray is reversible, so a light ray that reaches the swimmer's eye must travel the same path as a light ray that leaves the swimmer's eye. The 40° angle is less than the critical angle. A light ray directed from the swimmer's eye to the surface at an angle less than the critical angle will pass through the surface, so the swimmer sees light from outside the water when she looks at an angle of 40°.

A light ray directed from her eye to the surface at the critical angle will travel along the surface, so the swimmer sees light travelling along the surface when she looks at θ_c.

The 60° angle is greater than the critical angle. A light ray directed from her eye to the surface at an angle greater than the critical angle will reflect from the surface back into the water, so she sees light from underwater when she looks at 60°.

Practice

1. Replace the water in Sample Problem 1(a) with a transparent fluid that has a lower index of refraction. Describe the change to the critical angle at the liquid–air boundary. K/U C
2. Calculate the critical angle for light travelling through a layer of benzene floating on water at the benzene–water boundary. Use the indices of refraction in Table 1 on page 445. T/I [ans: 62.5°]
3. Calculate the critical angle for light passing through a thin rod made from glass with an index of refraction of 1.40 when the rod is surrounded by air (index of refraction 1.0003). T/I [ans: 45.6°]
4. Diamonds, with a high index of refraction of 2.42, are known for their attractive sparkle under lights. Similar-looking materials, such as crown glass (index of refraction 1.52) and zircon (index of refraction 1.92), do not have the same sparkle (**Figure 21**). Calculate the critical angle between each material and air, and express how the result explains the sparkle. T/I C A [ans: $\theta_{c,d} = 24.4°$; $\theta_{c,g} = 41.2°$; $\theta_{c,z} = 31.4°$]

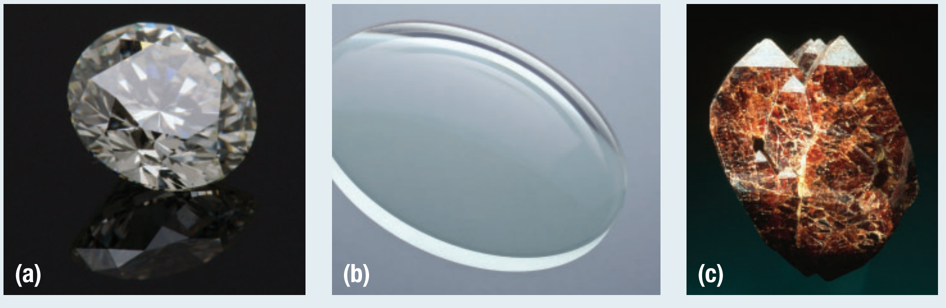


Figure 21 (a) Diamond has an index of refraction of 2.42. (b) Crown glass has an index of refraction of 1.52. (c) Zircon has an index of refraction of 1.92.

9.2 Review

Summary

- Snell's law describes the relationship between the incident and refracted angles of a light ray and the indices of refraction of two media: $n_1 \sin\theta_1 = n_2 \sin\theta_2$.
- The index of refraction, n, of a medium is the ratio of the speed of light in a vacuum, c, to the speed of light in the medium, v: $n = \frac{c}{v}$.
- The index of refraction, n, of a medium is equal to the ratio of the wavelength of light in a vacuum, λ_1, to the wavelength of light in the medium, λ_2: $n = \frac{\lambda_1}{\lambda_2}$. For two different media, $\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$. The frequency of light is unchanged.
- Dispersion is the separation of a wave into its component parts according to a given characteristic, such as wavelength.
- When light passes from one medium to another, partial reflection and partial refraction can occur, and the wavelength changes based upon the index of refraction for the second medium.
- Total internal reflection occurs when light is completely reflected at a boundary between two media. Two conditions must be met: (1) the incident light must originate in the more optically dense medium, and (2) the angle of incidence must be greater than the critical angle.
- The critical angle can be calculated using $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$.

Questions

1. Why is a beam of light said to be "bent" when it undergoes refraction? K/U
2. Define "angle of deviation" in your own words. K/U C
3. Light from a laser has a wavelength of 630 nm in a vacuum. Calculate its wavelength in water (index of refraction is 1.33). T/I
4. The speed of light in a medium is measured to be 3.0×10^8 m/s. Calculate the index of refraction of the medium. T/I
5. A piece of glass ($n = 1.47$) is coated with a thin film of water, with light incident from below. Calculate the angle of refraction of the final outgoing ray when the angle of incidence is 30.0°. K/U
6. Light travels through an optical fibre ($n = 1.44$) to air. The angle of incidence of light in the fibre is 30.0°. Calculate the angle of refraction outside the fibre. T/I
7. Suppose that the angle of incidence of a laser beam in water aimed at the surface is 50.0°. Use Snell's law to calculate the angle of refraction. T/I
8. (a) Calculate the critical angle for light travelling through glass ($n = 1.65$) and water ($n = 1.33$).
   (b) Does the light start in the glass or in the water? K/U T/I
9. Light travels from air into a transparent material that has an index of refraction of 1.30. The angle of refraction is 45°. T/I
   (a) Calculate the angle of incidence.
   (b) Determine the critical angle for total internal reflection to occur in the transparent material.
10. A ray of light passes from water ($n = 1.33$) into carbon disulfide ($n = 1.63$). T/I
    (a) Calculate the angle of refraction when the angle of incidence is 30.0°.
    (b) Is it possible for total internal reflection to occur in this case? Explain your answer.
11. Give an example where total internal reflection is used in medicine. A
12. Research transatlantic fibre optic cables. T/I C
    (a) What plans for new submarine cables are currently being developed?
    (b) Discuss some of the pros and cons of transatlantic submarine cables in a short paragraph.
13. Over long distances, the intensity of the signal carried by light in a fibre optic cable is reduced. Research what causes the signal to be reduced. Provide one reason for why this happens. K/U T/I A

WEB LINK

Diffraction and Interference of Water Waves

9.3

Have you ever noticed how people relaxing at the seashore spend so much of their time watching the ocean waves moving over the water, as they break repeatedly and roll onto the shore? Water waves behave similarly to other kinds of waves in many ways. **Figure 1** illustrates one characteristic behaviour of waves when a part of the wave enters through a narrow opening. The section of the wave that gets through acts as a source of new waves that spread out on the other side of the opening, and the waves from the two sources combine in specific ways. You will learn about these characteristic behaviours of waves in this section.

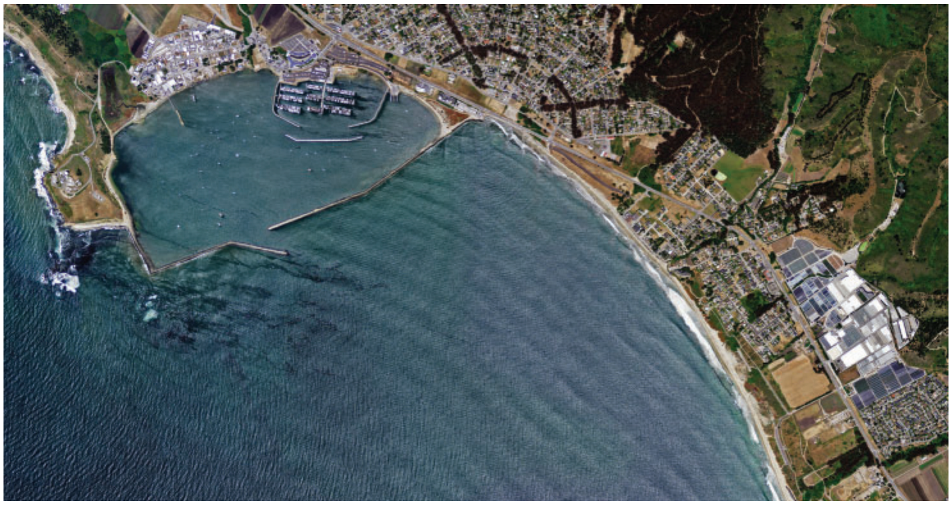


Figure 1 When part of a wave enters a narrow opening, such as this bay, new waves are created and the two waves combine in predictable ways.

Diffraction

If you observe straight wave fronts in a ripple tank, you can see that they travel in a straight line if the water depth is constant and no obstacles are in the way. If, however, the waves pass by an edge of an obstacle or through a small opening, the waves spread out. **Diffraction** is the bending of a wave as the wave passes through an opening or by an obstacle. The amount of diffraction depends on the wavelength of the waves and the size of the opening. In **Figure 2(a)**, an obstacle diffracts shorter wavelengths slightly. In **Figure 2(b)**, the same obstacle diffracts longer wavelengths more.

diffraction the bending and spreading of a wave when it passes through an opening; dependent on the size of the opening and the wavelength of the wave

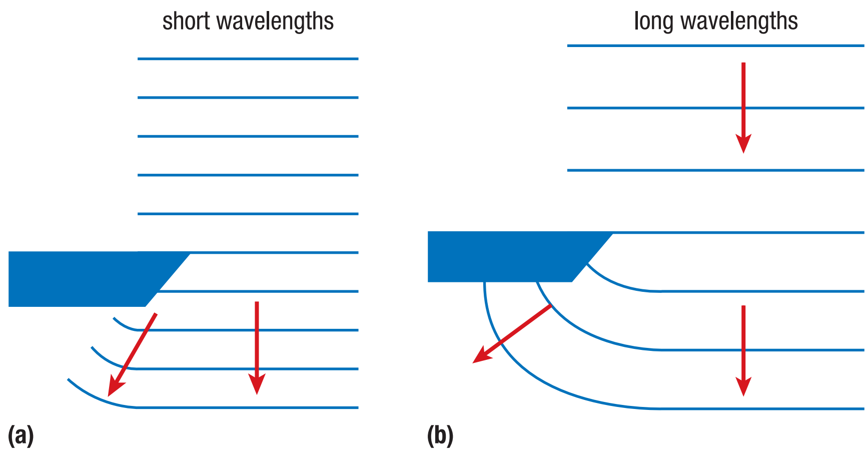


Figure 2 When waves travel by an edge, (a) shorter wavelengths diffract less than (b) longer wavelengths.

Although diffraction also occurs for sound and light waves, we will study the phenomenon first for water waves because their long wavelength allows for easier observation of the effects of diffraction.

In **Figure 3(a)**, you can see a wave encountering a slit with a certain width, w. In **Figure 3(b)** and **Figure 3(c)**, the width of the opening is the same but the wavelength of the incident waves has changed. Notice how the amount of diffraction changes. Therefore, as the wavelength increases but the width of the slit does not, the diffraction also increases. In Figure 3(a), the wavelength is approximately one-third of w. Notice that only the part of the wave front that passes through the slit creates the series of circular wave fronts on the other side. In Figure 3(b), the wavelength is about half of w, which means that significantly more diffraction occurs, but areas to the edge exist where no waves are diffracted. In Figure 3(c), the wavelength is approximately two-thirds of w, and the sections of the wave front that pass through the slit are almost all converted to circular wave fronts.

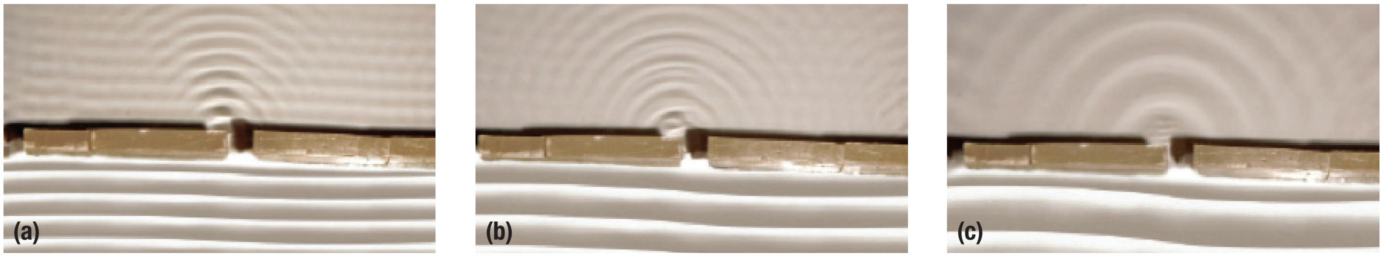


Figure 3 As the wavelength increases, the amount of diffraction increases.

What happens if you change the width of the slit but keep the wavelength fixed? As **Figure 4(a)** and **Figure 4(b)** show, as the size of the slit decreases, the amount of diffraction increases. If waves are to undergo more noticeable diffraction, the wavelength must be comparable to or greater than the slit width ($\lambda \geq w$). For small wavelengths (such as those of visible light), you need to have narrow slits to observe diffraction. **Figure 4(c)** shows what happens to the diffraction pattern when the wavelength decreases but the size of the slit does not change.

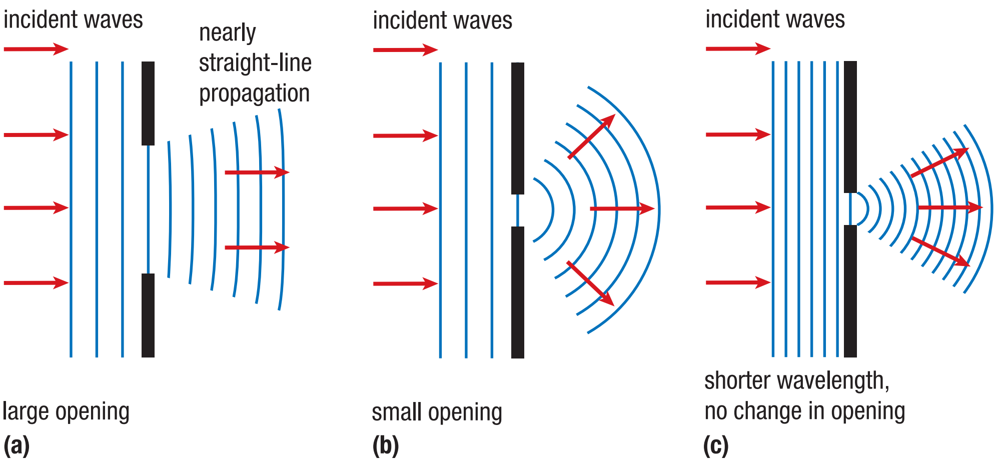


Figure 4 (a) and (b) As the size of the slit (aperture) decreases, diffraction increases. (c) With a shorter wavelength and no change in the size of the opening, there is less diffraction.

Investigation 9.3.1

Properties of Water Waves (page 487)
You have learned how observing water waves helps in understanding the properties of waves in general. This investigation will give you an opportunity to test conditions for diffraction to occur.

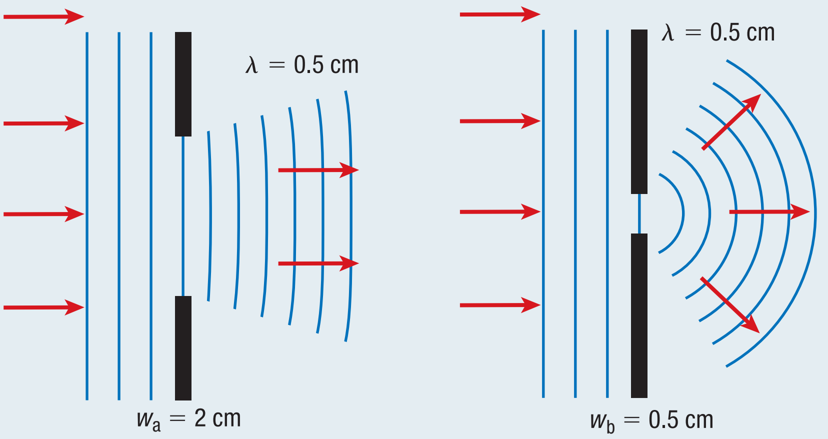
The relationship between wavelength, width of the slit, and extent of diffraction is perhaps familiar for sound waves. You can hear sound through an open door, even if you cannot see what is making the sound. The primary reason that sound waves diffract around the corner of the door is that they have long wavelengths compared to the width of the doorway. Low frequencies (the longer wavelengths of the sound) diffract more than high frequencies (the shorter wavelengths of the sound). So if a sound system is in the room next door, you are more likely to hear the lower frequencies (the bass). In the following Tutorial, you will learn how to solve problems relating to diffraction.

Tutorial 1 Diffraction

The following Sample Problem provides an example and sample calculations for the concepts of diffraction.

Sample Problem 1: Diffraction through a Slit

Determine and explain the difference between the diffractions observed in **Figure 5(a)** and **Figure 5(b)**.



(a) **(b)**

Figure 5

Given: $\lambda = 0.5$ cm; $w_a = 2$ cm; $w_b = 0.5$ cm

Required: diffraction analysis

Analysis: $\frac{\lambda}{w} \geq 1$

Solution:

For Figure 5(a),

$$\frac{\lambda}{w_a} = \frac{0.5\text{ cm}}{2\text{ cm}}$$
$$= 0.25$$
$$\frac{\lambda}{w_a} < 1$$

For Figure 5(b),

$$\frac{\lambda}{w_b} = \frac{0.5\text{ cm}}{0.5\text{ cm}}$$
$$\frac{\lambda}{w_b} = 1$$

Statement: Since the ratio in Figure 5(a) is less than 1, little diffraction occurs. Since the ratio in Figure 5(b) is 1, more noticeable diffraction occurs.

Practice

1. Determine whether diffraction will be noticeable when water waves of wavelength 1.0 m pass through a 0.5 m opening between two rocks. T/I [ans: yes]
2. A laser shines red light with a wavelength of 630 nm onto an adjustable slit. Determine the maximum slit width that will cause significant diffraction of the light. T/I [ans: 6.3×10^{-7} m]

Interference

interference the phenomenon that occurs when two waves in the same medium interact

constructive interference the phenomenon that occurs when two interfering waves have displacement in the same direction where they superimpose

When two waves cross paths and become superimposed, they interact in different ways. This interaction between waves in the same medium is called **interference**. You experience interference when you listen to music and other types of sound. For example, sound waves from two speakers may reach the listener's ears at the same time, as illustrated in **Figure 6**. If the crest of one wave coincides with the crest of the other, then the waves are in phase and combine to create a resultant wave with an amplitude that is greater than the amplitude of either individual wave—resulting in a louder sound. This phenomenon is called **constructive interference**.

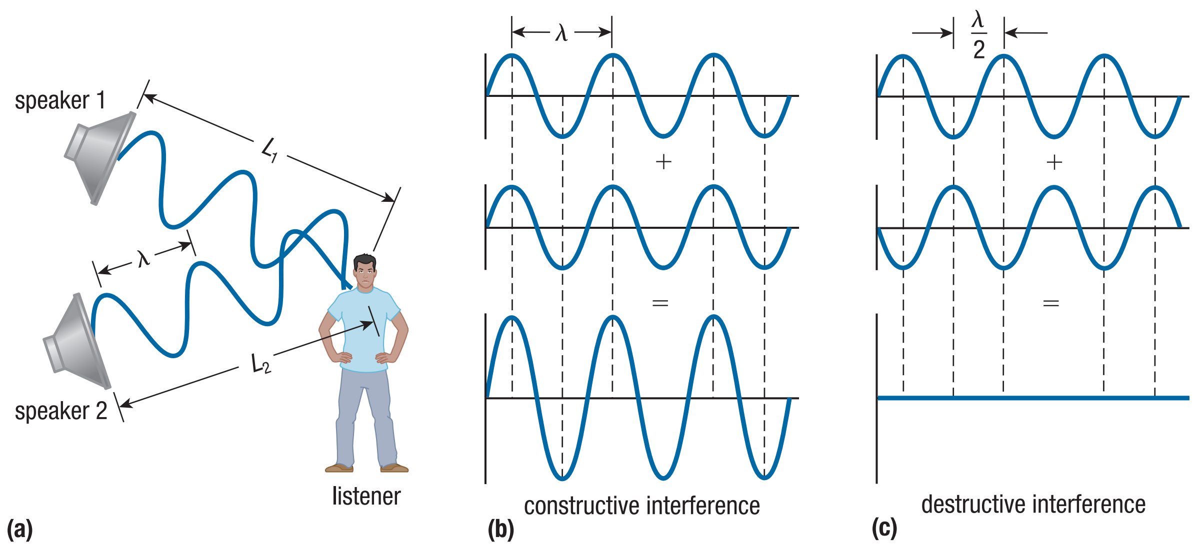


Figure 6 (a) If L_1 and L_2 are the same, the waves arrive at the listener in phase. (b) Waves interfere constructively if they arrive in phase. (c) If L_1 and L_2 differ by, say, $\frac{\lambda}{2}$, the waves arrive 180° out of phase, and they interfere destructively.

destructive interference the phenomenon that occurs when two interfering waves have displacement in opposite directions where they superimpose

For two waves that differ in phase by $\frac{\lambda}{2}$, shown in Figure 6(c), the crest of one wave coincides with the trough of the other wave. This corresponds to a phase difference of 180°. The two waves combine and produce a resulting wave with an amplitude that is smaller than the amplitude of either of the two individual waves. This phenomenon is called **destructive interference**.

The following conditions must be met for interference to occur:

1. Two or more waves are moving through different regions of space over at least some of their way from the source to the point of interest.
2. The waves come together at a common point.
3. The waves must have the same frequency and must have a fixed relationship between their phases such that over a given distance or time the phase difference between the waves is constant. Waves that meet this condition are called **coherent**.

coherent composed of waves having the same frequency and fixed phases

Figure 7 shows water waves interfering.

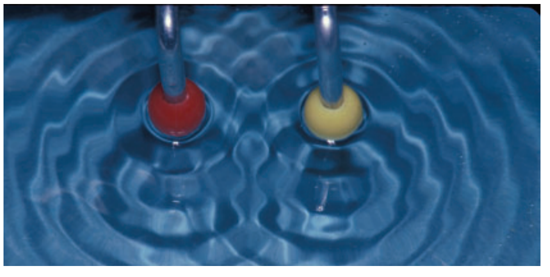


Figure 7 Interference of water waves in a ripple tank

In Figure 7, two point sources have identical frequencies. The sources produce waves that are in phase and have the same amplitudes. Successive wave fronts travel out from the two sources and interfere with each other. Constructive interference occurs when a crest meets a crest or when a trough meets a trough. Destructive interference occurs when a crest meets a trough (resulting in zero amplitude.)

Symmetrical patterns spread out from the sources, producing line locations where constructive and destructive interference occur. A **node** is a place where destructive interference occurs, resulting in zero amplitude (a net displacement of zero). As shown in **Figure 8**, the interference pattern includes lines of maximum displacement, caused by constructive interference, separated by lines of zero displacement, caused by destructive interference. The lines of zero displacement are called **nodal lines**.

node a point along a standing wave where the wave produces zero displacement

nodal line a line or curve along which destructive interference results in zero displacement

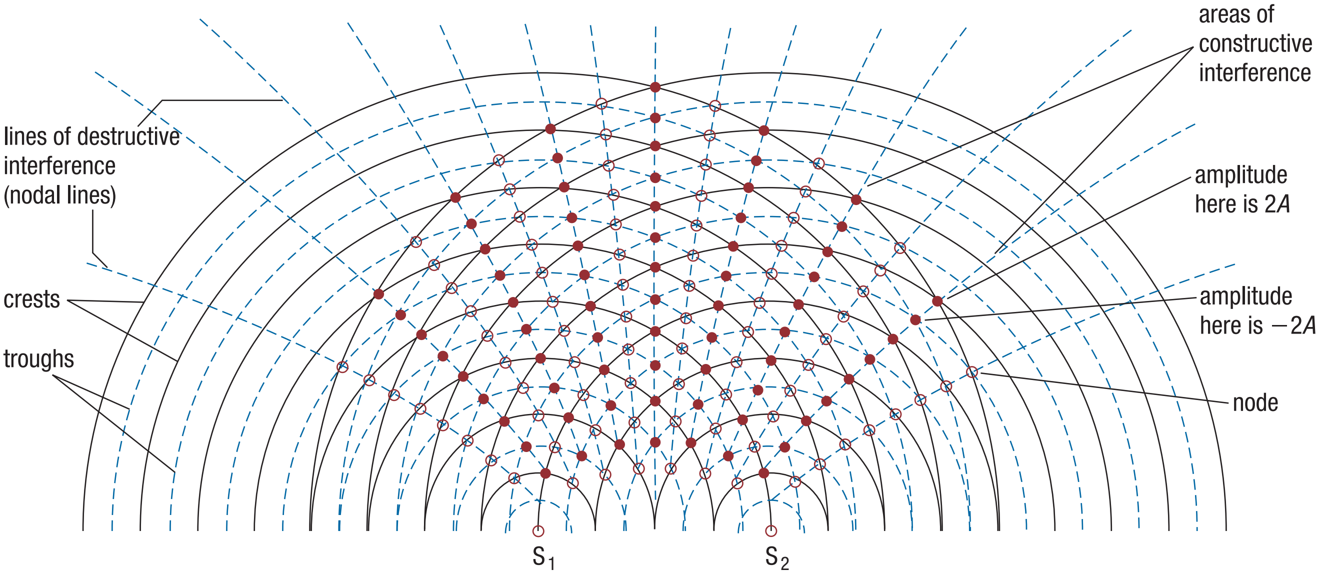


Figure 8 Interference pattern between two identical sources each with positive amplitude *A*

When the frequency of the two sources increases, the wavelength decreases, which means that the nodal lines come closer together and the number of nodal lines increases. When the distance between the two sources increases, the number of nodal lines also increases. The symmetry of the pattern does not change when these two factors are changed. However, if the relative phase of the two sources changes, then the pattern shifts, as shown in **Figure 9**, but the number of nodal lines stays the same. In Figure 9(a), the sources are in phase, but in Figure 9(b), there is a phase difference of 180°.

Investigation 9.3.2

Interference of Waves in Two Dimensions (page 488)

So far, you have read about two-point-source interference in theory, with diagrams and photographs. This investigation gives you an opportunity to see and measure interference for yourself.

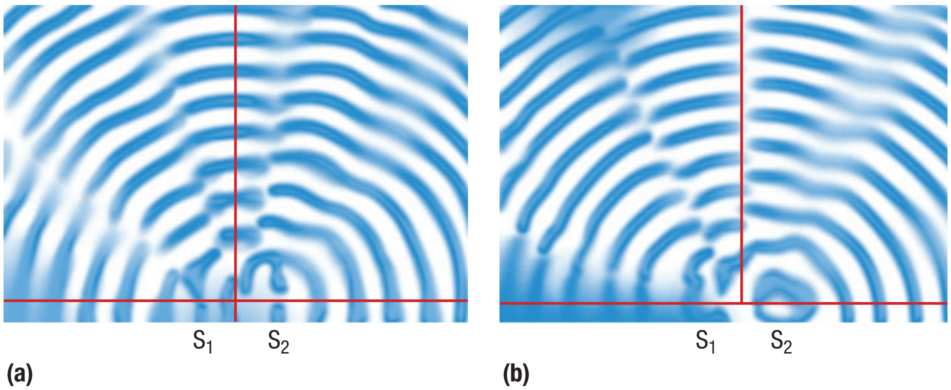


Figure 9 Effect of phase change on interference pattern for (a) zero phase difference and (b) 180° phase difference

Mini Investigation

Interference from Two Speakers

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

At the start of the discussion on interference, you considered sound waves coming from two speakers. In this investigation, your teacher will set up two speakers in the classroom, which will emit a single-frequency sound. You will examine how the sound intensity varies from place to place because of interference.

Equipment and Materials: speakers; plan of classroom

1. Before you begin, decide how to identify areas of constructive and destructive interference. How will these manifest themselves with sound?
2. Mark the location of the speakers on the classroom plan.
3. Your teacher will turn on the speakers.
4. Walk around the different areas of the classroom and mark on your plan where constructive and destructive interference occur.

A. How should your distances to the two speakers differ to get constructive interference? K/U

B. How should your distances to the two speakers differ to get destructive interference? K/U

C. Use your results marked on your class plan to identify two locations for either constructive or destructive interference, estimate their distances from the speakers, and use these estimates to estimate the wavelength of the sound. A

WEB LINK

Mathematics of Two-Point-Source Interference

You can measure wavelength using the interference pattern produced by two point sources and develop some mathematical relationships for studying the interference of other waves. **Figure 10** shows an interference pattern produced by two point sources in a ripple tank.

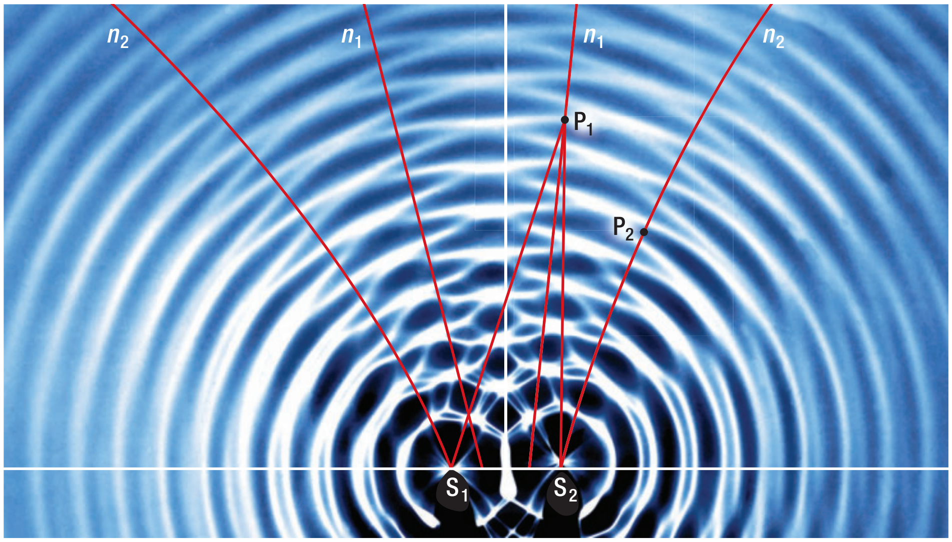


Figure 10 Ripple tank interference patterns can be used to develop relationships to study interference.

The two sources, S_1 and S_2, separated by a distance of three wavelengths, are vibrating in phase. The bisector of the pattern is shown as a white line perpendicular to the line that joins the two sources. You can see that each side of the bisector has an equal number of nodal lines, which are labelled n_1 and n_2 on each side. A point on the first nodal line (n_1) on the right side is labelled P_1, and a point on the second nodal line (n_2) on the right side is labelled P_2. The distances P_1S_1, P_1S_2, P_2S_1, and P_2S_2 are called **path lengths**.

path length the distance from point to point along a nodal line

If you measure wavelengths, you will find that $P_1S_1 = 4\lambda$ and $P_1S_2 = \frac{7}{2}\lambda$. The **path length difference**, Δs_1, on the first nodal line is equal to

path length difference the difference between path lengths, or distances

$$\Delta s_1 = |P_1S_1 - P_1S_2|$$
$$= \left|4\lambda - \frac{7}{2}\lambda\right|$$
$$\Delta s_1 = \frac{1}{2}\lambda$$

This equation applies for any point on the first nodal line. We use the absolute value because only the size of the difference in the two path lengths matters, not which one is greater than the other. A node can be found symmetrically on either side of the perpendicular bisector. The path length difference, Δs_2, on the second nodal line is equal to

$$\Delta s_2 = |P_2S_1 - P_2S_2|$$
$$\Delta s_2 = \frac{3}{2}\lambda$$

Extending this to the nth nodal line, the equation becomes

$$\Delta s_n = |P_nS_1 - P_nS_2|$$
$$\Delta s_n = \left(n - \frac{1}{2}\right)\lambda$$

where P_n is any point on the nth nodal line. Thus, by identifying a specific point on a nodal line and measuring the path lengths, you can use this relationship to determine the wavelength of the interfering waves.

This technique will not work if the wavelengths are too small or the point P is too far away from the sources, because the path length difference is too small to measure accurately. If either or both of those conditions apply, you need to use another method to calculate the path length difference.

For any point P_n, the path length difference is AS_1 (**Figure 11(a)**):

$$|P_nS_1 - P_nS_2| = AS_1$$

When P_n is very far away compared to the separation of the two sources, d, then the lines P_nS_1 and P_nS_2 are nearly parallel (**Figure 11(b)**).

UNIT TASK **BOOKMARK**

You can apply what you have learned about interference to the Unit Task on page 556.

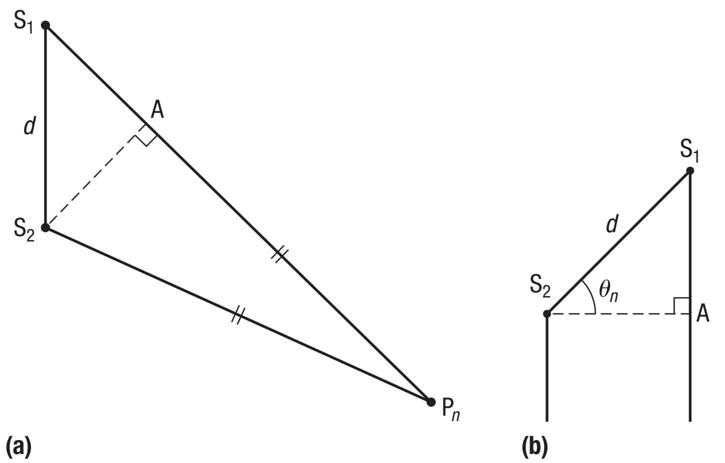


Figure 11 When developing the equations to calculate path length difference, it is important to consider the distance to P_n to be far enough away that P_nS_1 and P_nS_2 can be considered parallel.

In Figure 11(b), the lines AS_2, S_1S_2, and AS_1 form a right-angled triangle, which means that the difference in path length can be written in terms of the angle θ_n, since

$$\sin\theta_n = \frac{AS_1}{d}$$

Rearranging,

$$d\sin\theta_1 = AS_1$$

However, on the previous page we established that

$$AS_1 = |P_nS_1 - P_nS_2|$$

$$AS_1 = \left(n - \frac{1}{2}\right)\lambda$$

Combining this with the expression for AS_1 in terms of $\sin\theta_1$ leads to

$$\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

The angle for the *n*th nodal line is θ_n, the wavelength is λ, and the distance between the sources is d.

Using this equation, you can approximate the wavelength for an interference pattern. Since $\sin\theta_n$ cannot be greater than 1, it follows that $\left(n - \frac{1}{2}\right)\frac{\lambda}{d}$ cannot be greater than 1. The number of nodal lines on the right side of the pattern is the maximum number of lines that satisfies this condition. By counting this number and measuring d, you can determine an approximate value for the wavelength. For example, if d is 2.0 cm and n is 4 for a particular pattern, then

$$\left(n - \frac{1}{2}\right)\frac{\lambda}{d} \approx 1$$

$$\left(4 - \frac{1}{2}\right)\frac{\lambda}{2.0\text{ cm}} \approx 1$$

$$1.75\text{ cm} \approx \frac{1}{\lambda}$$

$$\lambda \approx 0.57\text{ cm}$$

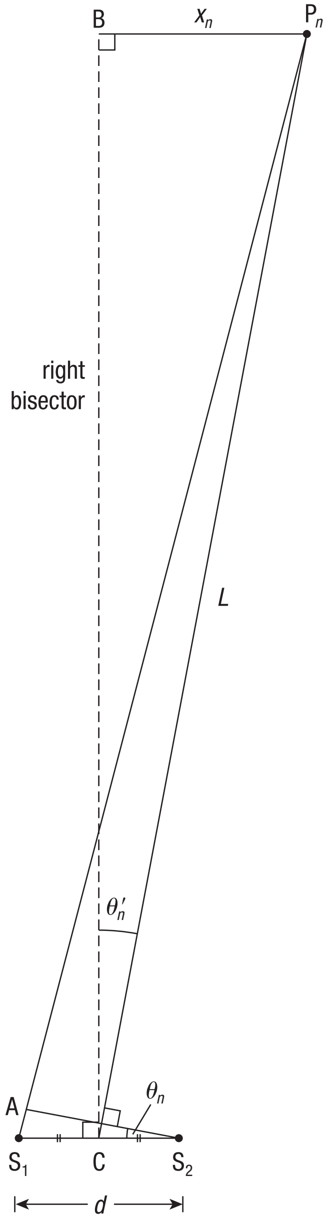


Figure 12 Use the triangle P_nBC to measure $\sin\theta'_n$.

Although it is relatively easy to measure θ_n for water waves in a ripple tank, for light waves, the measurement is not as straightforward. The wavelength of light is very small and so is the distance between the sources. Therefore, the nodal lines are close together. You need to be able to measure $\sin\theta_n$ without having to measure the angle directly. Assuming that a pair of point sources is vibrating in phase, you can use the following derivation.

For a point P_n that is on a nodal line and is distant from the two sources, the line from P_n to the midpoint of the two sources P_nC can be considered to be parallel to P_nS_1. This line is also at right angles to AS_2. The triangle P_nBC in **Figure 12** is used to determine $\sin\theta'_n$. In this derivation, d is the distance between the sources, x_n is the perpendicular distance from the right bisector to the point P_n, L is the distance from P_n to the midpoint between the two sources, and n is the number of the nodal line.

$$\sin\theta'_n = \frac{x_n}{L}$$

Figure 13 is an enlarged version of the triangle with base CS_2 in Figure 12. Since

$$\sin\theta_n = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

and, since the right bisector, CB, is perpendicular to S_1S_2, you can see in Figure 13 that $\theta'_n = \theta_n$. So we can say that

$$\frac{x_n}{L} = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

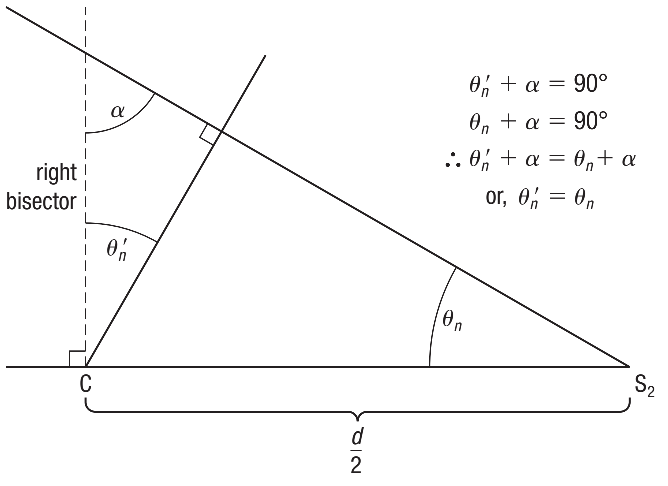


Figure 13 Enlargement of the base triangle from Figure 12

The following Tutorial will demonstrate how to solve problems involving two-dimensional interference.

Tutorial 2 Interference in Two Dimensions

This Tutorial provides examples and sample calculations for the concepts of two-point-source interference.

Sample Problem 1: Interference in Two Dimensions

The distance from the right bisector to a point P on the second nodal line in a two-point interference pattern is 4.0 cm. The distance from the midpoint between the two sources, which are 0.5 cm apart, to point P is 14 cm.

(a) Calculate the angle θ_2 for the second nodal line.

(b) Calculate the wavelength.

Solution

(a) **Given:** $n = 2$; $x_2 = 4.0$ cm; $L = 14$ cm

Required: $\sin\theta_2$

Analysis: $\sin\theta_n = \frac{x_n}{L}$

$$\theta_2 = \sin^{-1}\frac{x_2}{L}$$

Solution:

$$\theta_2 = \sin^{-1}\frac{x_2}{L}$$

$$= \sin^{-1}\left(\frac{4.0\text{ cm}}{14\text{ cm}}\right)$$

$$\theta_2 = 17°$$

Statement: The angle of the second nodal line is 17°.

(b) **Given:** $n = 2$; $x_2 = 4.0$ cm; $L = 14$; $d = 0.5$ cm

Required: λ

Analysis: $$\frac{x_n}{L} = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

$$\lambda = \frac{x_n d}{\left(n - \frac{1}{2}\right)L}$$

Solution: $$\lambda = \frac{x_2 d}{\left(n - \frac{1}{2}\right)L}$$

$$\lambda = \frac{(4.0\text{ cm})(0.5\text{ cm})}{\left(\frac{3}{2}\right)(14\text{ cm})}$$

$$\lambda = 0.095\text{ cm}$$

Statement: The wavelength is 9.5×10^{-2} cm.

Sample Problem 2: Determining Wave Pattern Properties Using Differences in Path Length

Two identical point sources are 5.0 cm apart, in phase, and vibrating at a frequency of 12 Hz. They produce an interference pattern. A point on the first nodal line is 5 cm from one source and 5.5 cm from the other.

(a) Determine the wavelength.

(b) Determine the speed of the waves.

Solution

(a) **Given:** $n = 1$; $P_1S_1 = 5.5$ cm; $P_1S_2 = 5.0$ cm; $d = 5.0$ cm

Required: λ

Analysis: $|P_nS_1 - P_nS_2| = \left(n - \frac{1}{2}\right)\lambda$

Solution: $$|P_nS_1 - P_nS_2| = \left(n - \frac{1}{2}\right)\lambda$$

$$|P_1S_1 - P_1S_2| = \left(1 - \frac{1}{2}\right)\lambda$$

$$|5.5\text{ cm} - 5.0\text{ cm}| = \left(1 - \frac{1}{2}\right)\lambda$$

$$0.5\text{ cm} = \left(\frac{1}{2}\right)\lambda$$

$$\lambda = 1.0\text{ cm}$$

Statement: The wavelength is 1.0 cm.

(b) **Given:** $f = 12$ Hz; $\lambda = 1.0$ cm

Required: v

Analysis: $v = f\lambda$

Solution: $$v = f\lambda$$
$$= (12\text{ Hz})(1.0\text{ cm})$$
$$v = 12\text{ cm/s}$$

Statement: The speed of the waves is 12 cm/s.

Practice

1. Two point sources, S_1 and S_2, are vibrating in phase and produce waves with a wavelength of 2.5 m. The two waves overlap at a nodal point. Calculate the smallest corresponding difference in path length for this point. T/I A [ans: 1.2 m]
2. A point on the third nodal line from the centre of an interference pattern is 35 cm from one source and 42 cm from the other. The sources are 11.2 cm apart and vibrate in phase at 10.5 Hz. T/I
   (a) Calculate the wavelength of the waves [ans: 2.8 cm]
   (b) Calculate the speed of the waves [ans: 29 cm/s]
3. Two point sources vibrate in phase at the same frequency. They set up an interference pattern in which a point on the second nodal line is 29.5 cm from one source and 25.0 cm from the other. The speed of the waves is 7.5 cm/s. T/I
   (a) Calculate the wavelength of the waves. [ans: 3.0 cm]
   (b) Calculate the frequency at which the sources are vibrating. [ans: 2.5 Hz]

9.3 Review

Summary

- Diffraction is the bending and spreading of a wave whose wavelength is comparable to or greater than the slit width, or where $\lambda \geq w$.
- Interference occurs when two waves in the same medium meet.
- Constructive interference occurs when the crest of one wave meets the crest of another wave. The resulting wave has an amplitude greater than that of each individual wave.
- Destructive interference occurs when the crest of one wave meets the trough of another wave. The resulting wave has an amplitude less than that of each individual wave.
- Waves with shorter wavelengths diffract less than waves with longer wavelengths.
- A pair of identical point sources that are in phase produce a symmetrical pattern of constructive interference areas and nodal lines.
- The number of nodal lines in a given region will increase when the frequency of vibration of the sources increases or when the wavelength decreases.
- When the separation of the sources increases, the number of nodal lines also increases.
- The relationship that can be used to solve for an unknown variable in a two-point-source interference pattern is $|P_nS_1 - P_nS_2| = \left(n - \frac{1}{2}\right)\lambda$.

Questions

1. Under what condition is the diffraction of waves through a slit maximized? K/U T/I
2. Two loudspeakers are 1.5 m apart, and they vibrate in phase at the same frequency to produce sound with a wavelength of 1.3 m. Both sound waves reach your friend in phase where he is standing off to one side. You realize that one of the speakers was connected incorrectly, so you switch the wires and change its phase by 180°. How does this affect the sound volume that your friend hears? K/U
3. (a) Determine the maximum slit width that will produce noticeable diffraction for waves of wavelength 6.3×10^{-4} m.
   (b) If the slit is wider than the width you calculated in (a), will the waves diffract? Explain your answer. K/U T/I
4. Two speakers are 1.0 m apart and vibrate in phase to produce waves of wavelength 0.25 m. Determine the angle of the first nodal line. T/I
5. What conditions are necessary for the interference pattern from a two-point source to be stable? K/U T/I
6. Two identical point sources are 5.0 cm apart. A metre stick is parallel to the line joining the two sources. The first nodal line intersects the metre stick at the 35 cm and 55 cm marks. Each crossing point is 50 cm away from the middle of the line joining the two sources. K/U T/I C A
   (a) Draw a diagram illustrating this.
   (b) The sources vibrate at a frequency of 6.0 Hz. Calculate the wavelength of the waves.
   (c) Calculate the speed of the waves if the frequency of the sources is the same as in part (a).
7. A student takes the following data from a ripple tank experiment where two point sources are in phase: $n = 3$, $x_3 = 35$ cm, $L = 77$ cm, $d = 6.0$ cm, $\theta_3 = 25°$, and the distance between identical points on 5 crests is 4.2 cm. From these data, you can work out the wavelength in three different ways. T/I
   (a) Carry out the relevant calculations to determine the wavelength.
   (b) Which piece of data do you think has been incorrectly recorded?

9.4 Light: Wave or Particle?

The period from 1650 to about 1710 was significant in the development of theories of light. By the end of the seventeenth century two opposing camps of physicists had emerged—one group followed the wave theory of light, and the other group followed the particle theory of light. The debate continued for over three hundred years.

In 1665, Francesco Grimaldi, who was the first scientist to use the term *diffraction*, suggested that observable diffraction took place when light passed through a narrow slit, creating rays of coloured light, thus showing that light was wave-like in nature.

Also in 1665, Robert Hooke developed his wave theory of light. Christiaan Huygens, in his *Treatise on Light* (1678), further developed Hooke's theory that light behaved as a wave. Huygens formulated the wave principle, called Huygens' principle. **Huygens' principle** states that all points on a wave front can be thought of as new sources of spherical waves. Huygens also claimed that light required an invisible medium in which to travel called the *ether*. Huygens' theory helped explain the concepts of reflection, refraction, and diffraction using wave concepts for light. However, a major objection to Huygens' ideas about light was that waves spread out in all directions.

Huygens' principle every point on a wave front can be considered as a point source of tiny secondary wavelets that spread out in front of the wave at the same speed as the wave itself

Isaac Newton, in contrast, thought of light as travelling in particles that he called corpuscles ("little particles"). The particles travel in straight lines with maximum velocity, and have kinetic energy. Newton's corpuscular, or particle, theory does not need a medium for light to travel in. This theory accounted for the **rectilinear propagation** of light, which means that light travels in a straight line. It also explained some other properties of light, such as reflection, and had some similarities with the much later theory of light. By 1700, Newton's theory still prevailed. It would be many years before direct experimental evidence countered Newton's views.

rectilinear propagation light travelling in straight lines

In the early nineteenth century, Thomas Young seemed to disprove Newton's theory. First, he used a ripple tank to demonstrate the idea of interference of water waves. Then, he recreated Grimaldi's experiment, demonstrating the interference of light using a double-slit experiment. (You experimented with this at the beginning of this chapter, and in Section 9.5 it is described in more detail.) Young described how he placed a narrow card with a single slit in a beam of light and saw fringes of colour in the shadow and to the sides of the card. When he put another card before or after the narrow card to stop the beam of light from striking one of the edges of the card, the fringes disappeared. This result supported the theory that light is composed of waves, results that Newton's theory could not explain.

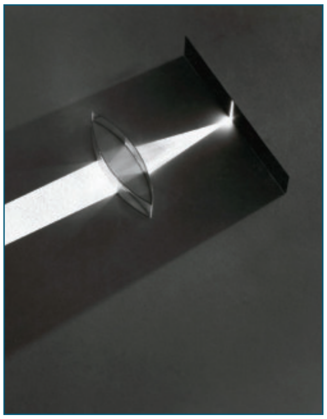
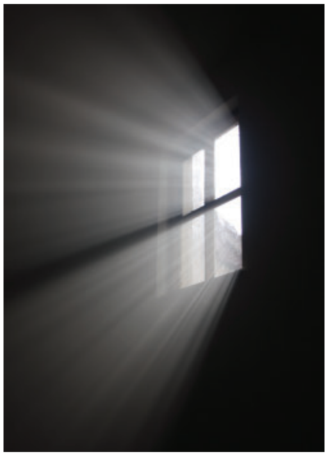
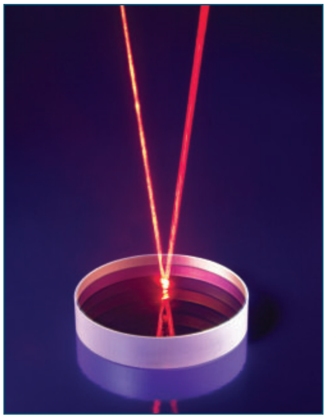
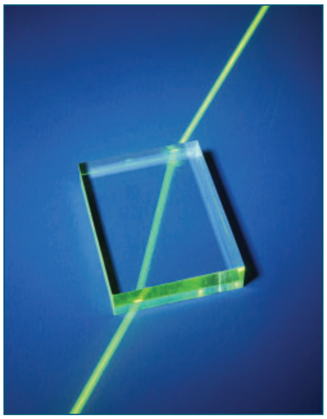
So, is light a wave or a particle? In some instances, light behaves as a wave, such as during interference. Other times, light behaves as a particle, such as light shining on metal. In Chapter 12, you will learn more about light behaving as a particle when shone on metal.

Newton's Particle Theory of Light

Newton's fascination with the ability of prisms to produce colours from white light led to his development of the particle theory of light. He stated that light corpuscles (little particles) travel in straight lines (rectilinear propagation) with a maximum velocity and therefore have kinetic energy. Newton's theory does not require a medium for the light to travel in. Furthermore, he was able to explain the properties of reflection and refraction using his theory. However, his explanation of diffraction showed the shortcomings of his theory.

Table 1, on the next page, summarizes Newton's explanations for these properties.

Table 1 Newton's Particle Theory Applied to Properties of Light

Phenomenon	Explanation	Example
rectilinear propagation	Newton argued that since light does not appear to curve, but travels in a straight line, light must consist of particles with extremely high speeds. In addition, since he did not notice any pressure from light, the mass of the particles must be quite low.	
diffraction	Newton argued that light cannot travel around corners as waves do. He argued that Grimaldi's observations were a result of collisions between light particles at the edges of the slit, rather than from light waves spreading out.	
reflection	Newton showed that, if light particles undergo perfectly elastic collisions, the law of reflection follows from the laws of motion. Horizontal velocity does not change, but vertical velocity is reversed, causing the particles to bounce. The magnitude of the velocity does not change.	
refraction	Newton claimed that particles will bend toward the normal if their speed increases. Particles accelerate at the boundary as they pass from one medium to another and the speed in the medium is greater than in air. This is the opposite of what actually happens.	

Huygens' Principle and the Wave Theory of Light

Huygens' principle leads to a geometric construction that determines the position of a new wave front based on knowledge of the previous wave front.

In **Figure 1(a)**, the straight wave front AB is moving toward the right. The straight wave front AB in Figure 1(a) corresponds to the circular wave front AB in **Figure 1(b)**. Huygens' viewpoint is that all the dots on the straight wave front can be considered as new sources of spherical waves. On the circular wave front AB, the dots represent the centres of the new wavelets—the small arcs of circles in Figure 1(b). The wave front A′B′ is the new position of the wave front a short time later. Notice how A′B′ is the tangent common to all the wavelets.

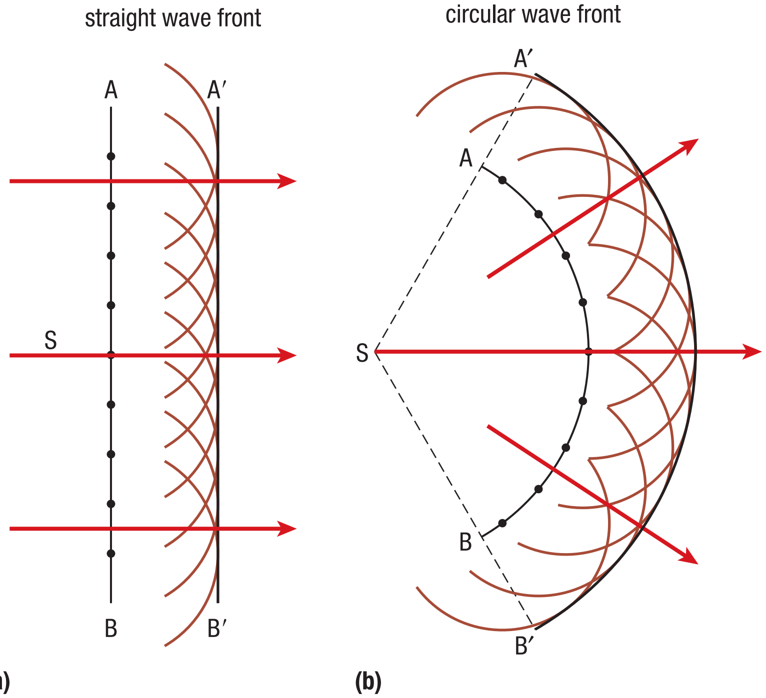


Figure 1 (a) In Huygens' construction of a straight wave front, the wave front is a straight line even though it is defined by circular waves. (b) In Huygens' construction of a spherical wave, the new wave front is drawn tangent to the circular wavelets radiating from the point sources on the original wave front.

Huygens' Principle and Reflection

Huygens' principle shows the derivation of the laws of reflection and refraction. **Figure 2** shows simplified wave fronts striking a surface. At the point where each wave front contacts the reflecting surface, a wave reflects. **Figure 3**, on the next page, simplifies reflection even further by showing one incident ray and one reflected ray, obeying the law of reflection.

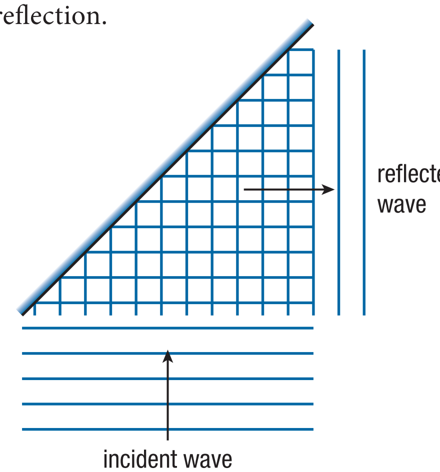


Figure 2 At the points where each wave touches the surface, the wave reflects.

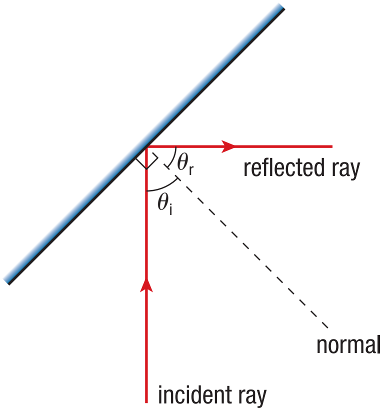


Figure 3 A single ray strikes the surface and reflects according to the law of reflection.

Huygens' Principle and Refraction

Huygens' principle can also be used to derive Snell's law of refraction, as illustrated in **Figure 4**. In time Δt, the portion of a wave front moving in the first medium covers a distance $v_1\Delta t$. The portion moving in the second medium covers a distance $v_2\Delta t$. If the second medium is a slower medium, the portion of the wave front in the second medium moves a shorter distance. As a result, the wave front and the ray deviate from the original path in medium 1. Since the waves in medium 2 do not travel as quickly, it will take longer to move away from the normal line. As a result, the refracted light ray is closer to the normal. This difference in path length results in the law of refraction. Applying trigonometric relations to Figure 4 to take account of the difference in path lengths results in a derivation of the law of refraction.

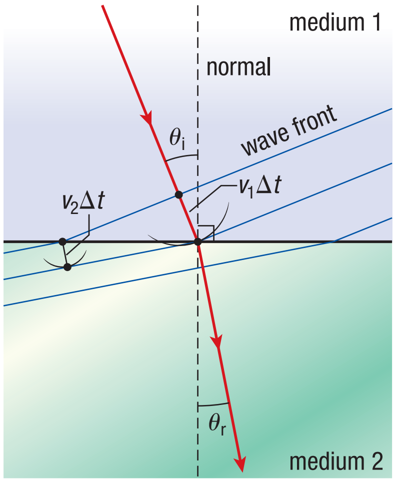


Figure 4 In Huygens' explanation of the law of refraction, the wave fronts bend due to differences in the speed of light in the two media. In this figure, the wave front is travelling faster in medium 1 than in medium 2.

Table 2, on the next page, summarizes the application of Huygens' principle to the properties of light.

Table 2 Huygens' Principle Applied to Properties of Light

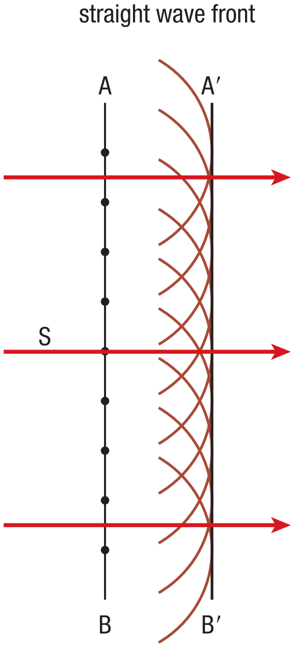
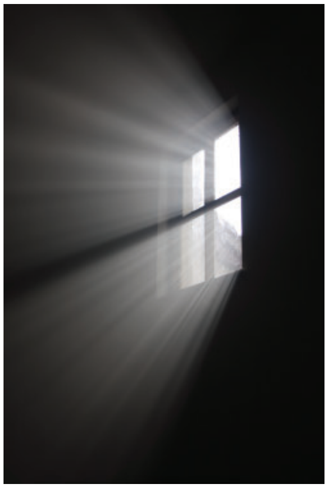
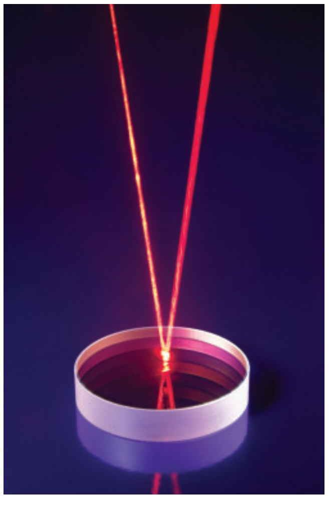
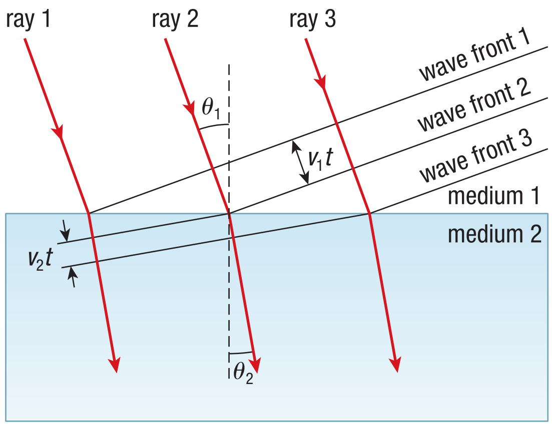
Phenomenon	Explanation	Example
rectilinear propagation	Each point on the wave front acts as a point source for a new spherical wavelet, and the wave propagates away from the source.	
diffraction	Each point passing through an opening acts as a point source for new spherical wavelets. Huygens' principle is consistent with large slit widths, edges of obstacles, and slits of the same magnitude as the wavelength.	
reflection	The incident rays hit points on the reflecting surfaces, which then act as point sources for spherical wavelets.	

Table 2 Huygens' Principle Applied to Properties of Light (*Continued*)

Phenomenon	Explanation	Example
refraction	The difference in the speed of wave fronts travelling in the two media causes the waves to bend toward or away from the normal. When the speed of the wave front decreases, the waves refract toward the normal.	

Huygens' wave theory explained many of the properties of light, including reflection, refraction, partial reflection, partial refraction, diffraction, dispersion, and rectilinear propagation. Although the wave theory made a strong case, Newton's particle theory dominated for much of the following century. This dominance was due largely to Newton's undoubted successes in other scientific areas, including studies of gravity, which gave him immense prestige within the scientific community. Newton's theory went into decline after Young's double-slit experiment demonstrated interference of light waves, which you will read about in Section 9.5.

Research This

Very Long Baseline Interferometry

Skills: Researching, Observing, Communicating SKILLS HANDBOOK A4.1

Very Long Baseline Interferometry (VLBI) is a technique used by radio astronomers that allows them to combine observations of the same object made at the same time by many telescopes. The technique measures the time differences between the arrivals of radio waves at separate receivers and is best known for producing images of distant cosmic radio sources, tracking spacecraft, and making measurements of astronomical distances. Studying Earth's rotation and accurately mapping movements of tectonic plates are other uses of VLBI.

1. Research VLBI and answer the following questions.

A. Which radio observatory in Canada uses this technique? T/I

B. How are the data recorded at each of the telescopes? T/I

C. How is the signal from the antenna sampled? T/I

D. How are the data sent in VLBI? T/I

E. When the data are played back, how are the data from the different telescopes synchronized? T/I

F. How do the wave-like properties of radio waves enable the compilation of images? T/I

WEB LINK

UNIT TASK BOOKMARK

You can apply what you have learned about the properties of light to the Unit Task on page 556.

9.4 Review

Summary

- Newton proposed the particle theory of light to explain reflection, refraction, and the rectilinear propagation of light. However, Newton's theory could not adequately explain diffraction.
- Huygens' principle states that every point on a wave front acts as a point source for secondary wavelets, which then spread out in front of the initial wave at the same speed as the initial wave. The new wave front appears as a line tangent to all the wavelets.
- The wave theory proposed by Huygens and embodied in Huygens' principle explains reflection, refraction, and diffraction.

Questions

1. Draw diagrams to show how light behaves like a wave in the following situations. K/U T/I C
   (a) rectilinear propagation of light
   (b) reflection
   (c) refraction
2. Which model of light, a wave model or a particle model, best explains known information about light? Explain. K/U C
3. State one piece of experimental evidence that light is a wave. K/U
4. Explain how you know that the speed of light does not change when it is reflected. K/U A
5. Why did Newton think that the mass of a light particle is very low? K/U
6. Why was Newton's theory of light the dominant theory for so long? K/U
7. Does Huygens' principle apply to water and sound waves? K/U
8. Determine whether a wave model or a particle model is best to use in each of the following situations. K/U T/I
   (a) light travelling from the Sun to Earth
   (b) energy travelling for TV, radio, X-rays, and so on
9. You shine laser light through an open window. The window is like a slit, but the laser light does not diffract at all as it passes through the window. Instead, it travels in a straight line. K/U T/I
   (a) What does your observation imply about the relative magnitude of the laser's wavelength and the width of the window?
   (b) How must you change this experiment so that the electromagnetic radiation (light) does diffract through the window?
10. Thomas Young showed that light passing through two parallel narrow slits produces a pattern of light and dark fringes. Did this support or contradict Newton's corpuscular theory of light? Explain your answer. K/U C
11. When light strikes a metal surface, waves of electrons travel along the surface. Researchers call these waves surface plasmon polaritons (SPPs), or surface plasmons for short (**Figure 5**). Research SPPs, and summarize answers to the following in a brief report. T/I C

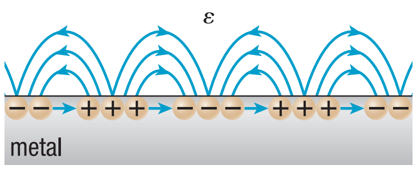


Figure 5

   (a) How do SPPs make use of the wave nature of light?
   (b) What applications do SPPs have for technology and society?
12. Research Newton's contributions to light theory. How else did Newton contribute to the study of light? In what way did Grimaldi's work influence Newton? Organize your findings in a format of your choice. T/I C
13. In 1665, Robert Hooke proposed that light travels as a wave. Research Hooke and his wave theory, and find out how he described this phenomenon. T/I C

WEB LINK

Interference of Light Waves: Young's Double-Slit Experiment

At the end of the 1600s and into the 1700s, the debate over the nature of light was in full swing. Newton's theory of light particles faced challenges from leading scientists such as Christiaan Huygens. Huygens' writings on the wave theory of light described what we now call Huygens' principle and proposed that light travelled in waves through an omnipresent ether, like sound travelling through air. Mathematician and scientist Leonard Euler developed his own wave theory of light and used the particle theory's poor description of diffraction as a key point in an argument against Newton's theory.

During this time, many researchers focused on the question of interference. If light travels as a wave, then it should interfere like a wave; experiments looking for interference, however, could not detect it. We now know that the experiments failed because of the extremely small wavelengths of light. Interference patterns in water waves in ripple tanks are easy to observe, because the wavelengths are large and the frequencies of the sources are relatively small. These properties make the distance between adjacent nodal lines visible to the eye. Most experiments with light before the 1800s involved the placement of two sources of light close to each other, with screens positioned near the sources. The scientists closely observed the screens to try to observe interference patterns. They never succeeded using this method, partly because the separation between the nodal lines was too small to be observable. However, these issues did not account for all of the problems in attempting to repeat the ripple tank experiment with light waves.

Atoms, including those in the Sun and in light bulbs, emit most visible light. You might expect the light waves emitted by two atoms in the same source to be identical and have the same frequency so that they could exhibit interference. However, collisions among the atoms can reset the phase of the emitted light wave. Experiments show that these phase jumps in typical light waves occur about every 10^{-8} s. This means the light from different atoms of a given source is **incoherent** because the waves have no fixed phase relationship to each other.

incoherent composed of waves that have no fixed phase relationship to each other and different frequencies

monochromatic composed of only one colour; light with one wavelength

If you direct light from a **monochromatic**, or single-wavelength, source through a narrow slit, however, the slit acts as a single point source, and the light travels from it as a coherent wave according to Huygens' principle. If you then direct the light from the single slit to a double slit (a closely spaced pair of slits), the double slit acts as a pair of sources of coherent, monochromatic light (**Figure 1**). Any change that occurs in the original source will occur in the two beams at the same time, which allows you to observe interference effects.

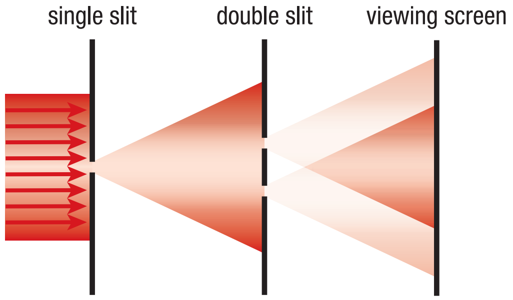


Figure 1 Monochromatic light passes through the single slit first, then through the double slit. According to Huygens' principle, both the single slit and the double slit act as a source of coherent light.

Young's Double-Slit Experiment

Figure 2 Thomas Young

At the very end of the 1700s, Thomas Young (**Figure 2**) performed a series of experiments to determine what happens when light passes through two closely spaced slits. We now refer to his basic setup as Young's double-slit experiment, and it demonstrated conclusively that light behaves as a wave. The experiment also provided a method to measure wavelength. **Figure 3(a)** shows the experimental setup, and **Figure 3(b)** shows an image produced with such a setup using white light.

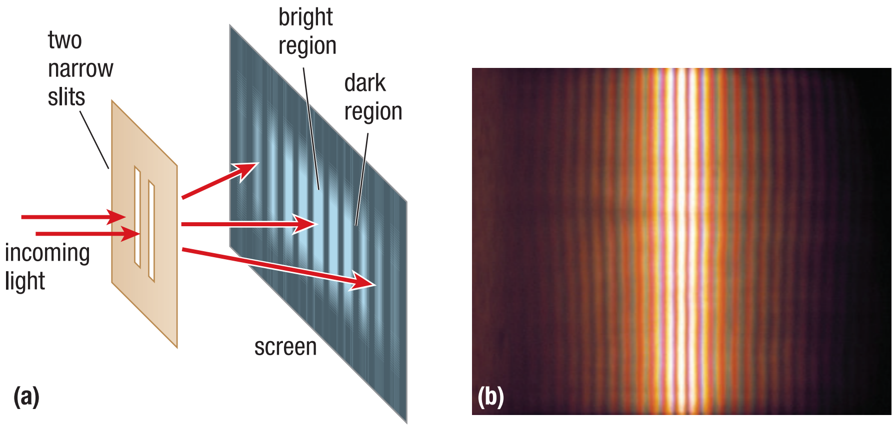


Figure 3 (a) Young used a similar setup in his light experiments in 1800. (b) When the white light passed through two small slits, an interference pattern of alternating bright and dark fringes was produced.

When monochromatic light, such as laser light, passes through the slits of an experiment setup similar to the one in Figure 3(a), the laser light hits the screen on the right and produces an interference pattern. This arrangement satisfies the general conditions required to create wave interference:

- The interfering waves travel through different regions of space (in this case they travel through two different slits).
- The waves come together at a common point where they interfere (in this case the screen).
- The waves are coherent (in this case they come from the same monochromatic source).

When the two slits in the double-slit setup are both very narrow, each slit acts as a simple point source of new light waves, and the outgoing waves from each slit are like the simple spherical waves in **Figure 4**.

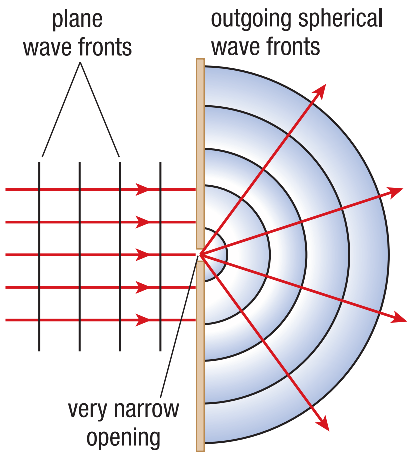


Figure 4 We can see the diffraction effects of light passing through two narrow slits by considering just one of the narrow slits. The same thing happens in the second narrow slit.

Investigation 9.5.1

Young's Double-Slit Experiment (page 490)
You have learned about Young's famous double-slit experiment. This investigation will give you an opportunity to recreate that experiment for yourself using a monochromatic light source and a series of different double-slit plates.

Interference will determine the intensity of light at any point on the screen. If one of the slits is covered, the screen shows a bright, wide centre line with closely spaced, dim, alternating dark and light bands, or **interference fringes**, on either side. You can read more about this single-slit pattern in Chapter 10. However, if both slits are open, then the screen shows a pattern of nearly uniform bright and dark fringes (**Figure 5**). The ideal double-slit pattern would show no dimming, but a real-world pattern does appear dimmer as you move away from the centre. This dimming occurs because the width of the slits causes each to act like a single slit, and the non-uniform single-slit pattern combines with the ideal double-slit pattern.

interference fringe one of a series of alternating light and dark regions that result from the interference of waves

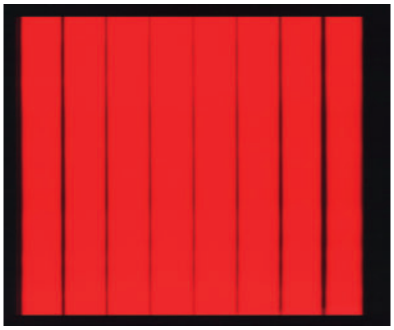


Figure 5 A display of Young's double-slit experiment using red light

The bright and dark fringes are alternate regions of constructive and destructive interference, respectively. To analyze the interference, you need to determine the path length difference between each slit and the screen. **Figure 6** illustrates the double-slit interference conditions.

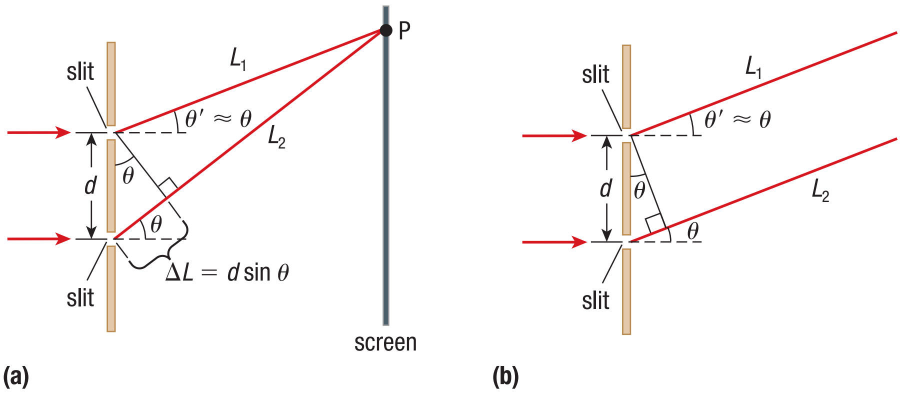


Figure 6 (a) Values of θ give the location of the bright fringes on the screen. (b) When the point P is very far away compared to the separation of the two sources, the lines L_1 and L_2 are nearly parallel.

To simplify the analysis in Figure 6, we will assume that the screen is a long way from the slits, so L is very large compared to d. The path lengths for waves from each of the slits to the point P on the screen are L_1 and L_2 in Figure 6. The distance L_2 is greater than the distance L_1, and, since L is large, the angles that specify the directions from the slits to point P are approximately equal, so both are shown as θ in Figure 6. Finally, we assume that the wavelength λ is much smaller than d, the spacing between the slits.

Since the slits are separated by a distance d, the path length difference between L_2 and L_1 is given by

$$\Delta L = d \sin\theta$$

For the two waves to be in phase when they reach the screen, and thus for constructive interference to occur, this path length difference needs to be a whole number of wavelengths. The condition for constructive interference and a bright interference fringe is therefore

$$d \sin\theta = m\lambda$$
$$m = 0, 1, 2, 3, \ldots$$
constructive interference

The light fringes, or **maxima**, are called zero-order maximum, first-order maximum, and so forth, for $m = 0, 1, 2, 3, \ldots$. In this notation, $m = 0$ denotes the maximum in the centre of the screen. Successive values of m correspond to successive maxima moving away in either direction from the centre of the screen.

maxima points of brightness, or maximum intensity, in an interference pattern

For the two waves to be out of phase when they reach the screen, and thus for destructive interference to occur, the path length difference needs to be $\left(n - \frac{1}{2}\right)\lambda$. Thus, the condition for destructive interference and a dark fringe in the interference pattern is

$$d \sin\theta = \left(n - \frac{1}{2}\right)\lambda$$

$$n = 1, 2, 3, \ldots$$

destructive interference

minima points of darkness, or minimum intensity, in an interference pattern

These dark lines, or **minima**, are called first-order minimum, second-order minimum, and so forth, for $n = 1, 2, 3, \ldots$.

Figure 7 shows a much-enlarged intensity pattern from a double-slit experiment. Notice that as the value of m changes, the angle θ also changes. At certain values of θ, the conditions for constructive interference are satisfied and a maximum in the intensity is produced, corresponding to a bright fringe. The fringe at the centre of the screen is the zero-order maximum and has the value 0 for m in the equation for constructive interference. The next bright fringe moving toward the top of the screen has the value $m = 1$, and so on.

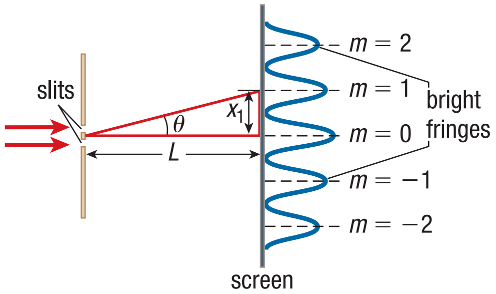


Figure 7 In this double-slit interference pattern, values of θ give the location of fringes on the screen.

How does the interference pattern change when the distance between the slits changes, or when the distance to the screen changes? First, calculate the separation between the central bright fringe ($m = 0$) and the next bright fringe ($m = 1$). The equation for constructive interference tells us that

$$\sin\theta = \frac{\lambda}{d}$$

To determine the separation between the bright fringes on the screen, you can use the right-angled triangle outlined in red in Figure 7. It has sides of lengths L and x_1, where x_1 is the distance between the two fringes corresponding to $m = 0$ and $m = 1$. These lengths are related by the trigonometric relation

$$x_1 = L \tan\theta$$

The wavelength is much smaller than the spacing between the slits, so $\frac{\lambda}{d}$ is very small. Therefore, θ is also very small, and the approximation $\sin\theta \approx \tan\theta$ holds:

$$x_1 = L \sin\theta$$

$$x_1 = \frac{L\lambda}{d}$$

For the mth-order bright fringe, this relation becomes

$$x_m = \frac{mL\lambda}{d}$$

and the separation between any two adjacent fringes is

$$\Delta x = \frac{L\lambda}{d}$$

LEARNING **TIP**

Small-Angle Approximation

When L is much larger than y and the angle θ is small, $\sin\theta \approx \tan\theta$, so the tangent function is a good approximation to the sine function. You can verify this yourself by checking values on your calculator.

Suppose the slit separation is 0.1 mm, the wavelength of light from a red laser is 630 nm, and the screen is 0.5 m from the slits. Substitute the corresponding values into this equation to determine the separation of the fringes for the red laser light:

$$\begin{aligned}\Delta x &= \frac{L\lambda}{d} \\ &= \frac{(0.5\text{ m})(6.3 \times 10^{-7}\text{ m})}{0.1 \times 10^{-3}\text{ m}} \\ &= 3.15 \times 10^{-3}\text{ m} \\ \Delta x &= 3.2\text{ mm}\end{aligned}$$

This separation is large enough to be seen by the unaided eye.

This analysis shows that, provided the slits are close enough together, the bright and dark fringes in the interference pattern are observable. A slit separation of 0.1 mm or smaller works well. Experiments have shown that when the spacing is larger than a few millimetres, the fringes are close together and difficult to see.

Using the equations for destructive interference, you can calculate the distance of each dark fringe from the centre of the screen. The result is

$$x_n = \left(n - \frac{1}{2}\right)\frac{L\lambda}{d} \qquad n = 1, 2, 3, \ldots$$

For example, the distances of the first, second, and third minima from the centre of the screen are

$$\begin{aligned}x_1 &= \left(1 - \frac{1}{2}\right)\frac{L\lambda}{d} = \frac{L\lambda}{2d} \\ x_2 &= \left(2 - \frac{1}{2}\right)\frac{L\lambda}{d} = \frac{3L\lambda}{2d} \\ x_3 &= \left(3 - \frac{1}{2}\right)\frac{L\lambda}{d} = \frac{5L\lambda}{2d}\end{aligned}$$

Although L actually has different values for each nodal line, in this case L is so large compared to d and the values of L for the various nodal lines are so similar, that we can treat L as a constant, being essentially equal to the perpendicular distance from the slits to the screen.

Modern interference experiments often inject a cloud of smoke between the slits and the viewing screen. Some of the light reflects off the particles in the cloud, so the viewer can easily see the path of the light. Will interference patterns appear in the cloudy region between the slits and the screen? The equations for constructive and destructive interference depend on the distance, L, from the slits, which can be any value considerably greater than d. The interference patterns will still occur in both places. In the cloud region, you will see bright lines directed toward the bright areas of the screen and dark lines directed toward the dark areas of the screen.

The following Tutorial will demonstrate how to calculate physical quantities in double-slit interference patterns.

UNIT TASK BOOKMARK

You can apply what you have learned about interference and the double-slit experiment to the Unit Task on page 556.

Tutorial 1 Calculating Physical Quantities in Double-Slit Interference Patterns

The following Sample Problems show how to measure the wavelength of light and slit separation in a double-slit experiment.

Sample Problem 1: Determining the Wavelength of a Light Source

A double-slit experiment is carried out with slit spacing $d = 0.41$ mm. The screen is at a distance of 1.5 m. The bright fringes at the centre of the screen are separated by a distance $\Delta x = 1.5$ mm. Calculate the wavelength of the light.

Given: $d = 0.41\text{ mm} = 4.1 \times 10^{-4}\text{ m}$; $L = 1.5\text{ m}$; $\Delta x = 1.5\text{ mm} = 1.5 \times 10^{-3}\text{ m}$

Required: λ

Analysis: In a double-slit experiment, the bright fringes occur when light waves from the two slits interfere constructively. This happens when the path difference, ΔL, is equal to a whole number of wavelengths:

$$\Delta x = \frac{L\lambda}{d}$$

$$\lambda = \frac{\Delta x d}{L}$$

Solution:

$$\lambda = \frac{\Delta x d}{L}$$

$$= \frac{(1.5 \times 10^{-3}\ \text{m})(4.1 \times 10^{-4}\ \text{m})}{1.5\ \text{m}}$$

$$\lambda = 4.1 \times 10^{-7}\ \text{m}$$

Statement: The wavelength of the light is 4.1×10^{-7} m.

Sample Problem 2: Determining Slit Separation

The third-order dark fringe of 660 nm light is observed at an angle of 20.0° when the light falls on two narrow slits. Determine the slit distance.

Given: $n = 3$; $\theta_3 = 20.0°$; $\lambda = 660\ \text{nm} = 6.6 \times 10^{-7}\ \text{m}$

Required: d

Analysis: $$\sin\theta_n = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

$$d = \frac{\left(n - \frac{1}{2}\right)\lambda}{\sin\theta_n}$$

Solution: $$d = \frac{\left(n - \frac{1}{2}\right)\lambda}{\sin\theta_n}$$

$$= \frac{\left(3 - \frac{1}{2}\right)(6.6 \times 10^{-7}\ \text{m})}{\sin 20.0°}$$

$$d = 4.8 \times 10^{-6}\ \text{m}$$

Statement: The slit separation is 4.8×10^{-6} m.

Practice

1. Calculate the wavelength of the monochromatic light that produces a fifth-order dark fringe at an angle of 3.8° with a slit separation of 0.042 mm. T/I [ans: 6.2×10^{-7} m]
2. Two slits are separated by a distance of 0.050 mm. A monochromatic beam of light with a wavelength of 656 nm falls on the slits and produces an interference pattern on a screen that is 2.6 m from the slits. Calculate the fringe separation at the centre of the pattern. T/I [ans: 3.4 cm]
3. The third-order dark fringe of light with a wavelength of 652 nm is observed when light passes through two double slits onto a screen. The slit distance is 6.3×10^{-6} m. At what angle is the fringe observed? T/I [ans: 15°]

More Developments in the Theory of Light

Young's evidence for the wave nature of light was not accepted by the scientific community until 1818, when Augustin Fresnel proposed his own wave theory, complete with the mathematics. A mathematician named Simon Poisson showed how Fresnel's equations predicted a unique diffraction pattern when light is projected past a small solid object, as shown in **Figure 8**.

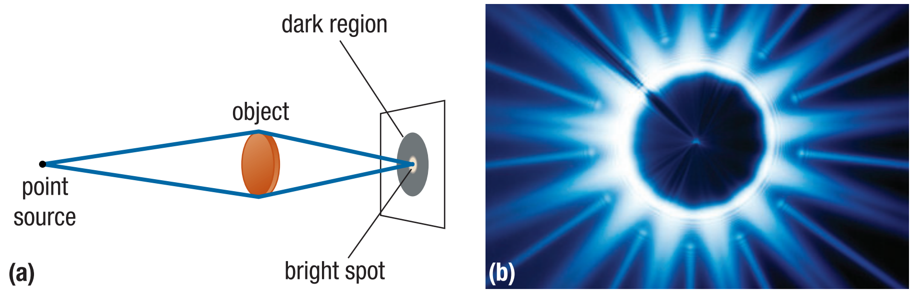


Figure 8 (a) The prediction of Fresnel and Poisson, confirmed by Arago, says that diffraction will cause a small bright spot to appear at the centre of the shadow of a small solid object. (b) A small disc refracts light and shows Poisson's bright spot.

Poisson's argument was that, if light behaved as a wave, then the light diffracting around the edges of the disc should interfere constructively to produce a bright spot at the centre of the diffraction pattern. This was impossible according to the particle theory of light. In 1818, Poisson's prediction was tested experimentally by Dominique Arago, and, to many people's surprise, he successfully observed the bright spot.

Arago's experiment demonstrated what is now called "Poisson's bright spot." The wave theory had been generally accepted by 1850, although it could not explain the movement of light through a vacuum such as space. Therefore, as mentioned in Section 9.4, scientists developed a theory that a fluid called ether filled all regions of space. Despite many experiments devised to try to detect this ether, it was never found.

Mini Investigation

Wavelengths of Light

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this investigation, you will answer the question, "What are some of the wavelengths that constitute white light?"

Equipment and Materials: clear showcase lamp; rotor stand and clamp; white screen; ruler; paper sliders; red and green transparent filters; double-slit plate; metre stick; elastic bands

1. Set up the apparatus as shown in **Figure 9**. View the lamp through the double slit and describe the pattern seen on either side of it.

If a lamp is used, unplug it by pulling on the plug, not the cord.

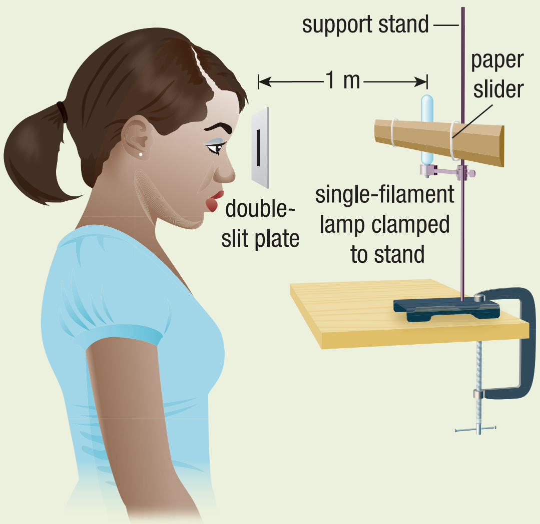


Figure 9 Setup for the wavelength experiment

2. Cover the upper half of the bulb with the green filter and the lower half with the red filter, securing each filter with elastic bands. Compare the interference patterns for green and red light.
3. Cover the lamp completely with the red filter. Use the ruler mounted in front of the light to count the number of nodal lines you can see in a fixed distance. Use the paper sliders to mark the distance on the ruler.
4. Predict the relative wavelengths of red and green light.
5. Leaving the paper sliders in place, repeat Step 3 with the green filter.
6. Record the number of bright fringes between the markers and the distance between the paper sliders. Calculate the average separation, Δx, of the nodal lines.
7. Use the relationship $\frac{\Delta x}{L} = \frac{\lambda}{d}$ to determine the wavelength of that colour of light (d should be recorded on your slit plate or should be available from your teacher).
8. Compare your results with those of other students.

A. Describe the interference patterns you saw for each filter. T/I C A

B. How many nodal lines did you see at a fixed distance for each filter? T/I

C. Calculate the wavelength of the colour of light for each filter. T/I

9.5 Review

Summary

- Earlier experiments that attempted to show the interference of light using two sources of light some distance apart were unsuccessful because the sources of light were out of phase and the wavelength of light is so small.
- Thomas Young's successful experiment used diffraction through a double slit to create two sources of light from a single coherent source of light so that the sources were close together and in phase.
- Young's experiment produced a series of light and dark fringes on a screen placed in the path of the light, with a pattern that resembled the results of interference of water waves in a ripple tank.
- For a setup with slit separation d and light of wavelength λ, the locations of bright fringes are given by $d \sin\theta = m\lambda$, for $m = 0, 1, 2, 3, \ldots$, and the locations of dark fringes are given by $d \sin\theta = \left(n - \frac{1}{2}\right)\lambda$, for $n = 1, 2, 3, \ldots$.
- For a viewing screen at a distance L, the mth-order bright fringe is at location $x_m = \frac{mL\lambda}{d}$, and the nth-order bright fringe is at location $x_n = \left(n - \frac{1}{2}\right)\frac{L\lambda}{d}$.
- Young's diffraction experiment was evidence for the wave nature of light. The wave theory could explain all properties of light except its propagation through a vacuum.

Questions

1. How is the interference pattern produced by monochromatic red light, $\lambda = 650$ nm, different from the pattern produced by monochromatic blue light, $\lambda = 470$ nm, when all other factors are kept constant? K/U
2. When a monochromatic light source shines through a double slit with a slit separation of 0.20 mm onto a screen 3.5 m away, the first dark band in the pattern appears 4.6 mm from the centre of the bright band. Calculate the wavelength of the light. T/I
3. Suppose a double-slit experiment is carried out first in air, then completely underwater, with all factors being the same in each medium. How do the interference patterns differ? K/U T/I
4. Two slits are separated by 0.30 mm and produce an interference pattern. The fifth minimum is 1.28×10^{-1} m from the central maximum. The wavelength of the light used is 4.5×10^{-7} m. Determine the distance at which the screen is placed. T/I
5. In a double-slit experiment, the distance to the screen is 2.0 m, the slits are 0.15 mm apart, and the dark fringes are 0.56 cm apart. T/I
   (a) Determine the wavelength of the source.
   (b) Determine the spacing of the dark fringes with a source of wavelength 600 nm.
6. The second-order dark angle in a double-slit experiment is 5.4°. Calculate the ratio of the separation of the slits to the wavelength of the light. T/I
7. In a double-slit experiment, the slits are 0.80 mm apart and an interference pattern forms on a screen, which is 49 cm from the slits. The distance between adjacent dark fringes is 0.33 mm. T/I
   (a) Determine which wavelength of monochromatic light is used in this experiment.
   (b) Determine the distance between the fringes if the same light were used in an experiment with slits that were 0.60 mm apart.
8. Light with a wavelength of 5.1×10^{-7} m passes through a double slit and onto a screen that is 2.5 m from the slits. T/I
   (a) Determine the slit spacing if two adjacent bright fringes are 12 mm apart.
   (b) If the slit spacing is now reduced by a factor of three, determine the new distance between the bright fringes.
9. Summarize the development in the theory of light. Can light be considered a wave only and not a particle? Why or why not? K/U

9.6 Explore an Issue in Light-Based Technology

Should Governments Restrict Network Access?

SKILLS MENU

- Defining the Issue
- Researching
- Identifying Alternatives
- Analyzing
- Defending a Decision
- Communicating
- Evaluating

Fibre optic technology has revolutionized access to information. Optical fibres use light to transmit large volumes of data quickly across great distances, even from one side of the world to another. In many places, this technology has greatly enhanced communication by computer and telephone (**Figure 1**). Optical fibres can be run underground in many kinds of terrain and under the oceans. This has enabled the creation of a communications network that connects people who live all over the world. Due to technology like this, the world has truly become a "global village."

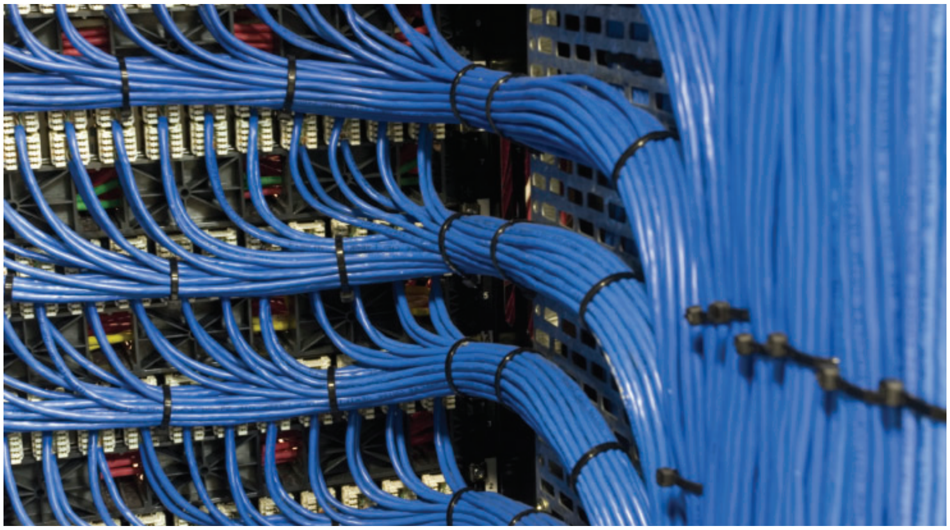


Figure 1 Communication technology is complex, but it allows people all over the world to share information.

As fibre optic technology has improved, controversy surrounding its use has also developed. Many people argue that unrestricted access to information helps to open up societies and improve human rights. Some governments, such as the government of the United Kingdom, are proposing that broadband access be made available to every home within its borders by a future target date.

Others, however, see problems with the new technology. Today's increased access to information creates security challenges and complicates privacy issues. Copyright issues surrounding the sharing of data, music, and other types of information have economic and legal implications. Some governments restrict Internet usage, and they give a variety of reasons for doing so.

The Issue

Should everyone be given open access to information, or should governments have regulations in place to control what type of information they release to their citizens? How does government control of access to the Internet affect your human rights as a Canadian citizen? What regulations are important for the protection of Canadian citizens?

Choose one of the following two roles:

- You are a campaigner for human rights, and your boss has asked you to prepare an argument opposed to government controls on the Internet, which will be presented at a public hearing.
- You are an intern for a member of the House of Commons, and your supervisor has asked you to prepare an argument in favour of government controls on the Internet, which will be presented at a public hearing.

Part of your task is to investigate whether restricting public access to the Internet has any effect on the control of terrorist activities or anti-government rioting, as well as how such restrictions might have an impact on human rights.

You will prepare a presentation to be given as part of a series of public hearings held on government controls for the Internet. Your audience will consist of public officials and the general public.

Goal

To persuade your audience to support or oppose government controls on the Internet

Research

SKILLS HANDBOOK A4

Research the pros and cons of government control of the Internet. You may wish to consider the following questions in your analysis.

- What are the implications of one nation's government controlling its citizens' access to a global entity such as the Internet?
- How might government controls on the Internet affect grassroots campaigns?
- How might government controls affect anti-government protests?
- How might Internet access affect public behaviour in countries with emerging democracies? What role might the Internet play in countries with more established democracies?
- Does limiting access to the Internet affect human rights? WEB LINK

Possible Solutions

- Should the Canadian government support controlled access to the Internet for the citizens of Canada?
- Can a compromise be reached between those who are opposed to government controls and those who support them?
- Should only certain information be limited? If so, what sort of information should be limited? If not, explain why.

Decision

SKILLS HANDBOOK A4.2

Based on your research, decide which solution you most support. Include reasons for your decision.

Communicate

Prepare a presentation for a mock public hearing. Include evidence that supports your opinion, including historical facts. Be respectful of differing opinions by addressing the pros and cons of government regulation in your presentation.

Plan for **Action**

As a class, create a brochure that highlights some of the issues that arise from human rights, information, and the Internet. Decide how you would circulate your brochure, and to whom. Be sure to present both sides of the issue.

Investigation 9.3.1 CONTROLLED EXPERIMENT

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

Properties of Water Waves

You have learned about how scientists have observed water waves in ripple tanks to study properties of waves in general. This investigation focuses on how the location and size of obstacles, such as slits (apertures) and edges, affect the diffraction of waves. This is analogous to the behaviour of a light wave as it travels through an opening.

Testable Question

SKILLS HANDBOOK A2.2

How do the size and placement of edges, openings, and other obstacles affect the amount of diffraction of a water wave with a fixed wavelength?

Prediction

Predict the conditions that will maximize diffraction around an object or through an opening.

Variables

Identify the independent, dependent, and controlled variables in the experiment.

Experimental Design

In Part A, you will use a computer simulation of water waves in a ripple tank. You will simulate how water waves behave around an obstacle, around an edge, and through an opening. In Part B, you will use a real ripple tank. You will fill the tank with water and use a wave generator to produce waves. You will design your own procedure to determine how the size, location, and position of obstacles relative to each other affect waves. You will be able to manipulate the size and shape of various obstacles and the openings between obstacles in the tank to observe the effect on the waves.

Equipment and Materials

- access to a computer with an Internet connection
- ripple tank and accessories
- wave generator with a straight source
- digital camera
- marked ruler
- wax blocks of various sizes

The light source and generators are electrical, so tape all wiring away from the water. The lab will be dark, so keep all bags, books, and other belongings out of the aisles and away from exits.

Procedure

Part A: Simulation

1. Go to the Nelson Science website.
2. Run the simulation.
3. Select one slit. Manipulate the slit width and barrier location to observe how water waves behave as they travel through an opening. Record your observations.
4. Select two slits. Manipulate the slit width and barrier location to observe how water waves behave as they travel through the openings. Record your observations.
5. Predict the behaviour of the waves through a single or double slit as you change the slit width and barrier location.

WEB LINK

Part B: Ripple Tank Experiment

6. Set up the ripple tank and wave generator, making sure the tank is level and filled with water to a depth of approximately 1 cm.

To unplug the water tank or the wave generator, pull on the plug, not the cord. When touching electrical cords, make sure that your hands are dry.

7. In small groups, discuss how adding obstacles to the tank will affect the water's movement. What details could you vary?
8. Design a series of obstacles to test the effect of obstacle and opening size on the waves. Use the wax blocks to implement your designs in the tank. Consider the following questions in your planning:
   - Will your wave fronts change speed?
   - How can you change your wave fronts from parallel to circular?
   - How will you alter the wavelength of your waves?
   - How will you use the wax blocks as barriers?
   - How will you create apertures in the barriers?
9. Use the digital camera and ruler to observe the results.
10. Using the image you create, trace out the nodal lines of the wave. Calculate wave speed and wavelength. How do they compare with the pattern that you predicted?

Analyze and Evaluate

(a) What variables were measured, recorded, or manipulated in this investigation? What type of relationship were you testing? K/U T/I

(b) What happened to the diffraction pattern behind the block when you increased the wavelength? T/I

(c) Does diffraction around an edge increase or decrease when the wavelength increases? K/U

(d) To keep diffraction small, how must the wavelength compare to the aperture width? T/I

(e) What value of $\frac{\lambda}{w}$ will produce noticeable diffraction? K/U

(f) Explain what you discovered about how waves behave around an obstacle, by an edge, and through an opening. C

(g) Was one of your designs more successful at producing examples of wave interference than the others, or did all of them produce similar results? C

(h) Using your digital image and your sketch, calculate wave speed and wavelength. T/I A

(i) Based on your predictions, what are the conditions for maximum diffraction through an opening? A

Apply and Extend

(j) In a bay where the water is calm, would you be more likely to see diffraction around smaller boats or larger boats? Explain your answer. A

(k) How would the observed pattern change if more than two sources were used and the sources were in phase? Out of phase by 180°? T/I A

(l) What changes to the interference pattern would result from changing the medium to another substance, such as motor oil or honey? T/I A

Investigation 9.3.2 CONTROLLED EXPERIMENT

Interference of Waves in Two Dimensions

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

As you learned in Section 9.3, interference is a behaviour unique to waves. Three factors affect interference: the frequency of the sources, the distance between the sources, and the relative phase of the sources. You have learned about the patterns of interference of waves in two dimensions. In this investigation, you will create an interference pattern and take some direct measurements. You will then test the theoretical analysis provided in the text and apply the mathematical equations on an actual two-point interference pattern.

Testable Question

How do the frequency of the waves, the relative phase of the waves, and the distance between identical wave sources affect interference?

Prediction

For Part A, predict the appearance of a wave interference pattern when the sources are in phase, and when they are out of phase by 180°. For Part B, predict how this interference pattern will change as the frequency of the sources increases, and as the distance between the sources increases.

Variables

Identify the independent, dependent, and controlled variables in this experiment.

Experimental Design

In Part A, you will use a computer simulation of wave sources in a ripple tank. This will allow you to view and interact with the simulation to observe how water waves behave when they meet while travelling through a medium.

In Part B, you will use real sources in a real tank. This will allow you to observe and analyze how a two-point interference pattern is affected by changes in the frequency of the sources, the distance between the sources, and the phase relationship between the sources. You will use the ripple tank with two wave sources to observe the interference pattern of the waves. You can adjust the phase of the sources, the distance between the sources, and the frequency of the waves that they produce.

Equipment and Materials

- ripple tank and accessories
- two point sources
- wave generator with adjustable wax blocks
- digital camera
- computer with Internet connection
- pencil
- ruler

The light source and generators are electrical, so tape all wiring away from the water. The lab will be dark, so keep all bags, books, and other belongings out of the aisles and away from exits.

Procedure

Part A: Simulation

1. Go to the Nelson Science website.
2. Open the simulation.
3. Click "Physics" in the menu in the left panel.
4. Click "Sound & Waves" in the menu in the left panel and click the "Wave Interference" icon. Run the simulation.
5. Select two slits and manipulate frequency and amplitude to observe patterns of interference.
6. Predict the two-point interference pattern produced using the diffraction of waves from a single wave source through two apertures.

WEB LINK

Part B: Ripple Tank Experiment

7. Set up the ripple tank, making sure it is level. Fill the tank with water to a depth of approximately 1 cm.

To unplug the water tank or the wave generator, pull on the plug, not the cord. When touching electrical cords, make sure that your hands are dry.

8. Connect the two point sources to the generator so that they are about 6 cm apart. Check to ensure that the two sources are in phase. (If your generator has a phase adjustment, set it at 0.)
9. Adjust the generator so that the two point sources are both dipping into the water at one end of the tank. Generate waves with a frequency of approximately 10 Hz. Identify the nodal lines and the lines of constructive interference on the screen. Sketch the nodal line pattern.
10. Adjust the frequency of the generator and record how the change in frequency affects the interference pattern.
11. While keeping the frequency constant, change the separation of the sources. Allow the interference pattern to stabilize between changes. Observe how the interference pattern is affected by changes in separation. Record your observations.
12. Change the frequency of the generator to produce a visible pattern on the screen. Place a pencil on the screen so that it lies along a nodal line. Turn the phase control of the generator from 0 to 1, 2, and so on (or 180°, and so on). Note any changes in the interference pattern.
13. Stop the generator and mark the positions of S_1 and S_2 on the screen. Draw a line between the two sources. Then, draw the right bisector of this line, extending it across the screen (see **Figure 1**).

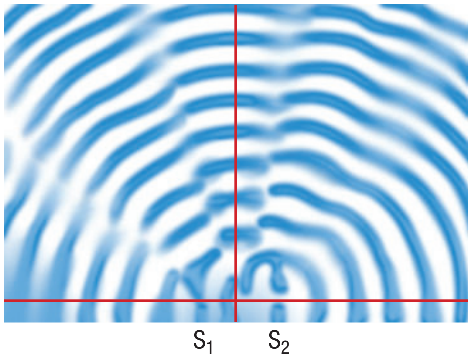


Figure 1

14. Adjust the frequency of the generator to produce three nodal lines on either side of the right bisector. Choose one of the nodal lines and mark three nodal points (labelled P_1, P_2, and P_3) on the screen. Make sure that one of these points is relatively close to S_1 and S_2 and that one is quite far away.
15. Calculate the wavelengths of the waves on the screen using appropriate measurements of the waves and your own calculations. Use two different equations.
16. Repeat Steps 14 and 15 for three points on a different nodal line, on the other side of the right bisector.
17. Determine an average calculated value for the wavelength of the waves.
18. Turn off the generator. Place the wax blocks in the ripple tank, creating two openings approximately 6 cm apart and 10 cm from the generator (**Figure 2**).

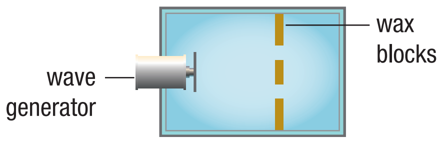


Figure 2 Setup for Step 18

19. Predict the interference pattern that will result, and draw a sketch that shows your prediction.
20. Turn on the wave generator. Use the ruler and digital camera to record the interference pattern.

Analyze and Evaluate

(a) What variables were measured, recorded, or manipulated in this investigation? What type of relationship was being tested? T/I

(b) How did the interference pattern change when the frequency increased? K/U

(c) How did the interference pattern change when the source separation increased? K/U

(d) How did the interference pattern change when the relative phase of your sources changed from 0° to 180°? K/U

(e) How did your prediction in the simulation compare with your results from Step 20? T/I

(f) How valid are the predicted wavelengths of the waves, based on mathematical wave interference relationships, when compared with direct measurements? T/I

(g) How successful were your procedure and experimental design in producing a two-point interference pattern? Include any changes to the procedure that you think would have produced better results. C A

Apply and Extend

(h) If the relative phase of two identical sources was constantly changing, how would this affect the interference pattern? A

(i) When two sources are used, why is the interference pattern less stable than it is when a single source is used? A

Investigation 9.5.1 CONTROLLED EXPERIMENT

Young's Double-Slit Experiment

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

In Section 9.5, you learned about Young's double-slit experiment. In Part A of this investigation, you will have an opportunity to recreate the experiment for yourself. While Young performed his experiment with two pinholes in a piece of paper and used the Sun as the light source, you will be using a monochromatic light source and a series of different double-slit plates. You will determine the wavelength of laser light.

In Part B, you will use a simulation of the double-slit experiment. The simulation allows you to manipulate the variables in the experiment. You will design your own procedure to show how interference patterns are affected by the distance between the slits and the distance from the slits to the screen, and verify your results.

Testable Question

How do the spacing between the slits and the distance of the double slit from the screen affect the interference pattern produced by waves?

Prediction

The wavelength of a certain source can be predicted using the mathematical relationship for a two-point interference pattern.

Variables

The variables in this controlled experiment are distance between the slits and the distance from the slits to the screen. Since single-slit interference patterns may be apparent, make sure you focus on double-slit interference patterns.

Equipment and Materials

- helium–neon laser (class 1 or 2 only)
- multiple-slit plate or red LED
- metre stick
- white screen
- support stand and clamp
- computer with Internet connection

Procedure

Part A: The Double-Slit Experiment

1. Position the screen so that it is 3.0 m from the slit plate (**Figure 1**, on the next page).

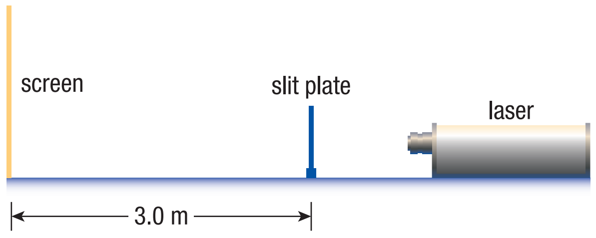


Figure 1

2. Direct the laser light through the double slit onto the screen so that the light lands on the screen.

Do not direct the laser light into anyone's eyes.

3. Measure the distance over at least seven bright points on the screen to determine Δx.
4. Calculate the wavelength of the helium–neon laser light using the relationship $\frac{\Delta x}{L} = \frac{\lambda}{d}$.
5. Repeat Steps 1 to 3 for two other pairs of slits that have different slit separations, d.
6. Obtain the wavelength of the light source from your teacher. Compare the average of your predictions in Steps 3 and 4 to the value obtained.

Part B: Simulation

7. Go to the Nelson Science website.
8. Run the simulation.
9. Select two slits and manipulate the frequency and amplitude to observe patterns of interference.
10. Predict the two-point interference pattern produced using the diffraction of waves from a single wave source through two apertures.
11. Work with a partner to design your own procedure for showing how interference patterns are affected by the distance between the two slits and the distance from the slits to the screen.
12. In small groups, discuss your design. Incorporate any ideas that might improve the design.
13. Use your design to test your prediction from Step 10.

WEB LINK

Analyze and Evaluate

SKILLS HANDBOOK A5.2

(a) What variables were measured, recorded, or manipulated in this investigation? What type of relationship was being tested? T/I
(b) Compare the average results for the wavelength of the light source with the known value, and calculate the percent uncertainty. T/I
(c) Did your observations in the investigation match your initial predictions?
(d) Compare your observations to the simulation. How were the results similar? How were they different?
(e) What are the factors that contribute to error in measurement of the wavelength of laser light? A
(f) What methods might you use to reduce error, given that the wavelength of the laser light is known to at least three significant digits? A
(g) How well did your design predict the wavelength of light when the spacing of the slits and the distance to the screen are known? T/I

Apply and Extend

(h) The wavelength of the light and the separation of the sources affect the number of nodal lines produced. Using your results from Steps 2 and 3, determine the effects of source separation on the nodal line structure for light. A
(i) Compare your results with the observations for water wave interference that you made in Investigation 9.3.2. T/I A
(j) Why does this investigation provide such strong support for the wave theory of light? A

CHAPTER 9 SUMMARY

Summary Questions

1. Create a study guide for this chapter based on the Key Concepts on page 438. For each point, create three or four subpoints that provide further information, relevant examples, explanatory diagrams, or general equations.
2. Look back at the Starting Points questions on page 438. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers that you wrote at the beginning of the chapter. Note how your answers have changed.

Vocabulary

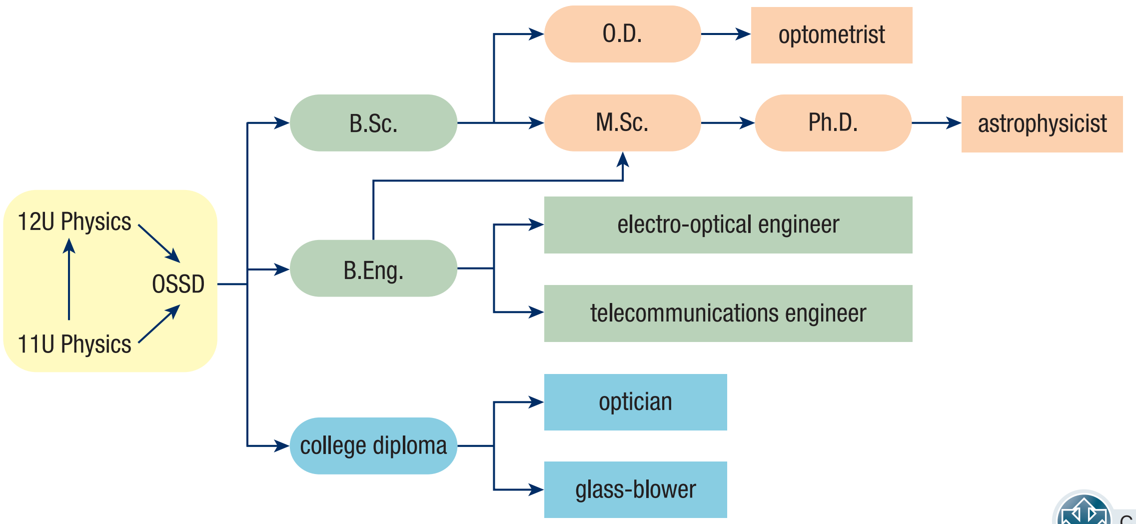
periodic wave (p. 440)
wave front (p. 441)
crest (p. 441)
trough (p. 441)
wavelength (p. 441)
phase (p. 441)
ray approximation (p. 442)
reflection (p. 442)
normal (p. 442)
angle of incidence (p. 442)
angle of reflection (p. 442)
law of reflection (p. 442)
specular reflection (p. 442)
diffuse reflection (p. 442)
refraction (p. 444)
optical density (p. 444)
principle of reversibility (p. 444)
index of refraction (p. 444)
angle of refraction (p. 446)
Snell's law (p. 447)
dispersion (p. 450)
angle of deviation (p. 451)
critical angle (p. 453)
total internal reflection (p. 453)
fibre optics (p. 455)
diffraction (p. 459)
interference (p. 462)
constructive interference (p. 462)
destructive interference (p. 462)
coherent (p. 462)
node (p. 463)
nodal line (p. 463)
path length (p. 464)
path length difference (p. 465)
Huygens' principle (p. 470)
rectilinear propagation (p. 470)
incoherent (p. 477)
monochromatic (p. 477)
interference fringe (p. 479)
maxima (p. 479)
minima (p. 480)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers mentioned in this chapter.

1. Select an interesting career that relates to the study of waves and light. Research the educational pathway that you would need to follow to pursue this career.
2. How might optics be related to work done through the Canadian Space Agency? Research at least two pathways that would involve in-depth study of light waves.



CAREER LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. What type of reflection is exhibited as light waves from an overhead projector reflect off a white projection screen and are scattered throughout a room? (9.1) K/U
   (a) diffuse reflection
   (b) specular reflection
   (c) incident reflection
   (d) periodic reflection
2. Which statement is true of the index of refraction? (9.2) K/U
   (a) All types of glass have the same index of refraction.
   (b) The index of refraction is the same for different wavelengths of light.
   (c) The index of refraction is a dimensionless number greater than or equal to 1.0.
   (d) The index of refraction is a dimensionless number less than 1.0.
3. Total internal reflection can occur only when
   (a) one of the two refractive media is air
   (b) light is travelling from a more optically dense medium into a less optically dense medium
   (c) a light of low intensity is used
   (d) a light of high intensity is used (9.2) K/U
4. As straight water waves pass through an opening and spread out in circular wave fronts, the waves undergo
   (a) reflection
   (b) refraction
   (c) dispersion
   (d) diffraction (9.3) K/U
5. Which frequencies of sound from an audio speaker will *best* diffract around corners and fill a room? (9.3) K/U
   (a) low frequencies
   (b) medium to high frequencies
   (c) high frequencies
   (d) Both low and high frequencies diffract the same amount.
6. Select the one statement upon which both Newton and Huygens would agree. (9.4) K/U
   (a) Light from the Sun is a shower of very small particles.
   (b) As light travels from air into glass, there is a change in the speed of light.
   (c) Rays of light are affected by the forces of gravity.
   (d) Various colours of light correspond to different wavelengths of light.
7. A group of students uses a double slit and a laser to produce bright fringes on a distant screen. To double the fringe spacing, the students should
   (a) decrease the slit spacing by half
   (b) decrease the laser wavelength by half
   (c) decrease the distance to the screen by half
   (d) increase the slit spacing (9.5) K/U

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. Specular reflection occurs when a light wave reflects from a smooth surface. (9.1) K/U
9. Fibre optic cables transmit information quickly and with relatively little signal loss by the process of total internal reflection. (9.2) K/U
10. The index of refraction has the same units as velocity. (9.2) K/U
11. Diffraction does not occur for sound waves. (9.3) K/U
12. Interference can take place only in transverse waves. (9.3) K/U
13. If the path length difference from two coherent wave sources to a single point is a whole-number multiple of the wavelength, then waves will interfere constructively at the point in question. (9.3) K/U
14. Newton argued that particles speed up as they travel from air into a dense, transparent medium, such as glass. (9.4) K/U
15. Light can behave as a wave or as a particle. (9.4) K/U
16. A light beam diffracting around a small solid disc will create a bright spot in the centre of the disc's shadow. (9.5) K/U

Go to Nelson Science for an online self-quiz.

WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. Water waves splash under a dock at regular intervals. The time that elapses between splashes is called the
   (a) incidence
   (b) frequency
   (c) specular
   (d) period (9.1) K/U
2. A water wave has a frequency of 0.20 Hz. How many wavelengths will have passed a fixed point after 1.5 min? (9.1) K/U
   (a) 12
   (b) 18
   (c) 90
   (d) 450
3. If a beam of light strikes a surface exactly perpendicular to the surface, then the angle of incidence is
   (a) 0°
   (b) 45°
   (c) 60°
   (d) 90° (9.1) K/U
4. In terms of light, which quantity remains constant regardless of the medium through which it travels? (9.2) K/U
   (a) wavelength
   (b) frequency
   (c) amplitude
   (d) speed
5. Visible light is one example of
   (a) an electromagnetic wave
   (b) a longitudinal wave
   (c) an acoustic wave
   (d) a compression wave (9.1) K/U
6. A ray of light refracts when travelling from medium 1 into medium 2. The angle of incidence is less than the angle of refraction. Select the true statement concerning this interaction. (9.2) K/U
   (a) Medium 1 must be a vacuum.
   (b) The speed of the light ray increases as it enters medium 2.
   (c) The frequency of the light ray decreases as it enters medium 2.
   (d) Medium 1 has a lower index of refraction.
7. Which interaction will exhibit the greatest amount of dispersion? (9.2) K/U
   (a) white light entering water at a small angle of incidence
   (b) white light entering water at a large angle of incidence
   (c) white light entering a diamond at a small angle of incidence
   (d) white light entering a diamond at a large angle of incidence
8. Predict what will happen as two water waves move toward each other. (9.3) K/U
   (a) The waves will bounce off each other and return in the opposite direction.
   (b) The waves will momentarily add together and then continue on in their original direction.
   (c) The waves will collide and cancel each other's energy.
   (d) The waves will scatter off each other, moving perpendicular to their original direction.
9. At any point on one of the nodal lines in the two-point-source interference pattern,
   (a) a wave crest is combining with a wave trough
   (b) a wave crest is combining with a wave crest
   (c) a wave crest is combining with a slightly smaller wave crest
   (d) a wave crest is reflecting off a wave crest (9.3) K/U
10. A teacher uses audio speakers to create a two-point-source interference pattern in the classroom. The speakers are placed at S_1 and S_2. A student stands at point P on the $n = 1$ nodal line. In terms of the wavelength, λ, what is the path length difference between PS_1 and PS_2? (9.3) K/U
   (a) There is no path length difference.
   (b) $\frac{1}{2}\lambda$
   (c) λ
   (d) $\frac{3}{2}\lambda$
11. Which action will decrease the angular separation of nodal lines in a two-point-source interference pattern? (9.3) K/U
   (a) moving the point sources farther apart
   (b) decreasing the frequency of the waves
   (c) increasing the wavelength of the waves
   (d) decreasing the screen distance

12. Newton's particle theory of light is unable to account for which phenomenon? (9.4) K/U
   (a) propagation
   (b) reflection
   (c) radiation pressure
   (d) diffraction

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

13. The angle of incidence is measured between the incoming ray and the reflecting surface. (9.1) K/U
14. The amplitude of a water wave is the vertical distance from the top of a crest to the bottom of a trough. (9.1) K/U
15. The critical angle is small for substances that have large indices of refraction. (9.2) K/U
16. The dependence of the speed of light on wavelength is called interference. (9.2) K/U
17. The index of refraction is the same for red and blue light in the same material. (9.2) K/U
18. In any situation, the angle of incidence is always greater than the angle of refraction. (9.2) K/U
19. Light moves faster in a quartz crystal than it does in a diamond. (9.2) K/U
20. As a light wave refracts from air into glass, the only quantity to remain constant is the light's wavelength. (9.2) K/U
21. The symbol *c* is used in reference to the speed of light in air at standard temperature and pressure. (9.2) K/U
22. Two sources that produce waves of the same frequency and phase are said to be coherent. (9.3) K/U
23. Diffraction of waves decreases as the width of the slit decreases. (9.3) K/U
24. Destructive interference occurs along a nodal line. (9.3) K/U
25. Light waves diffract a greater amount than sound waves. (9.3) K/U
26. Newton's corpuscular theory of light had no explanation for the small amount of diffraction displayed by visible light. (9.4) K/U
27. Huygens' wave principle states that all points on a wave front can be thought of as new sources of spherical waves. (9.4) K/U
28. Calculations with Young's experiment can be used to determine the wavelength of a light source or the slit spacing of a double slit. (9.5) K/U
29. For angles of less than 10°, the sine and the cosine of the angle are approximately equal. (9.5) K/U
30. An advantage of fibre optic technology is that large amounts of information can be transmitted over short distances. (9.6) K/U

Match each term on the left with the most appropriate description on the right.

31. (a) refraction
    (b) diffraction
    (c) interference
    (d) dispersion
    (e) critical angle
    (f) reflection
    (g) angle of refraction

    (i) two waves in the same medium interact
    (ii) the smallest angle of incidence at which a light ray can be totally reflected from the boundary between two media
    (iii) the angle that a light ray makes with respect to the normal to the surface when it has entered a different medium
    (iv) a change in direction of a light ray when it meets an obstacle
    (v) the bending of a wave when it passes through an opening
    (vi) separation of a wave into its component parts according to a given characteristic
    (vii) the bending of light as it travels from one medium to another

    (9.1, 9.2, 9.3) K/U

Understanding

32. Use unit analysis to show that the quantity $n_2 \sin\theta_2$ in Snell's law has no units. (9.1) K/U A
33. (a) Explain why a laser beam is invisible when it travels through the air, but can be seen as it strikes a white screen.
    (b) Describe how one could make the laser beam visible in the air, as in **Figure 1**. (9.1) K/U A

Figure 1

34. Explain using a diagram and words how absolutely still water can exhibit specular reflection while choppy water exhibits diffuse reflection. (9.1) T/I C A

35. Explain the separation of light into colours as light travels through a prism (**Figure 2**). (9.2) K/U

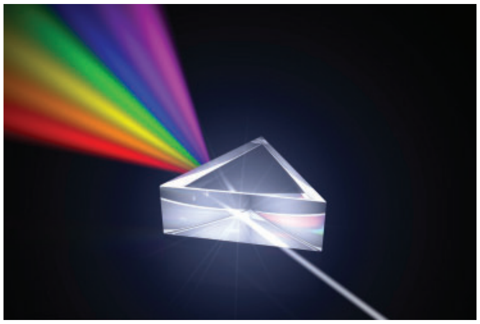


Figure 2

36. Summarize the key differences between Newton's particle theory and Huygen's principle. (9.4) K/U C

37. Explain how Young's double-slit experiment demonstrated that light has the properties of a wave. (9.5) C

38. List the benefits and drawbacks of fibre optic cables. (9.6) K/U C

Analysis and Application

39. The wavelength of red light is approximately 650 nm. How many wavelengths of red light will fit across the width of a 1.0 cm fingernail? (9.1) T/I

40. A certain radio station broadcasts with a frequency of 88.7 MHz. Radio waves travel at the speed of light (3.0×10^8 m/s). Determine the wavelength of these radio waves. (9.1) T/I

41. A light year is defined as the distance light travels in one year, and is useful for astronomical measurements. Alpha Centauri, the nearest star system to our own, is 4.4 light years away. Determine the distance to Alpha Centauri in metres. (9.1) T/I

42. A Pyrex test tube submerged in vegetable oil becomes "invisible" (**Figure 3**). Use refraction to explain this illusion. (9.2) T/I A

Figure 3

43. The Cassini Space Probe entered into orbit around Saturn in July 2004. On average, Saturn is about 1.1 billion kilometres away from Earth. How many hours does it take for a message travelling at the speed of light to reach Earth from Cassini? (9.1) K/U

44. A beam of light refracts from air into Plexiglas. The index of refraction for Plexiglas is 1.51. Determine the speed of the light in the Plexiglas. (9.2) K/U

45. The speed of light is reduced by 45 % as it refracts from a vacuum into an unknown transparent material. Calculate the index of refraction of the transparent material. (9.2) T/I

46. The beam of a green laser travels from air into Pyrex, then into water. Calculate the wavelength, in nanometres, of the green light in the Pyrex and in the water. The index of refraction for Pyrex is 1.47, the index of refraction for air is 1.0003, and the index of refraction for water is 1.33. The wavelength of the green laser is 5.30×10^{-7} m. (9.2) T/I

47. A ray of monochromatic light in water hits quartz crystal at an incident angle of 52.0°. Determine the angle of refraction. The index of refraction for quartz crystal is 1.46. (9.2) K/U

48. A ray of light in the air hits a block of transparent material at an incident angle of 62°. The angle of refraction is 44°. (9.2) T/I C
    (a) Sketch the situation, labelling the incident ray, the refracted ray, the reflected ray, and the normal.
    (b) Determine the index of refraction of the transparent block.
    (c) Determine the speed of light in the block.

49. A ray of monochromatic light travelling in air strikes the end of a 21 cm–wide block of Plexiglas at an incident angle of 65° (**Figure 4**). The index of refraction for Plexiglas is 1.51. (9.2) K/U T/I
    (a) Calculate the angle of refraction as the light enters the block.
    (b) The light travels to the opposite side of the block and refracts out into the air. The path of the refracted light ray is parallel to the original light ray, but displaced a distance *d*. Calculate the value of *d*.

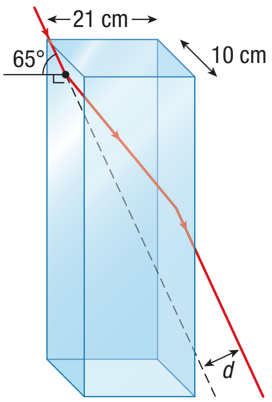


Figure 4

50. A right-triangular prism has edge lengths of 3.0 cm, 4.0 cm, and 5.0 cm. A ray of red light in air hits the 3.0 cm edge at an incident angle of 0.0° (**Figure 5**). The prism is made of flint glass, which has an index of refraction of 1.65 for red light. (9.2) T/I

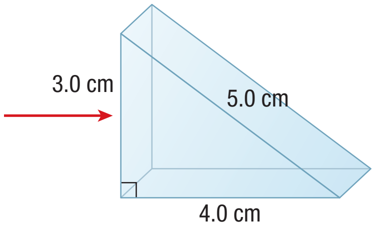


Figure 5

(a) Calculate the angle of deviation of the light beam from its original path after it leaves the prism. The angle of deviation is the angle between the incident ray and the final outgoing ray.
(b) Calculate the angle of deviation if a beam of violet light was used. Violet light has an index of refraction of 1.67 in flint glass.

51. Determine the critical angle of the following interfaces. The index of refraction for Pyrex is 1.47, for air it is 1.0003, and for water it is 1.33. (9.2) T/I
(a) Pyrex to air
(b) Pyrex to water
(c) water to air

52. A ray of light travels from air into an optical fibre with an index of refraction of 1.44. (9.2) K/U T/I
(a) In which direction does the light bend?
(b) The angle of incidence on the end of the fibre is 33°. Determine the angle of refraction inside the fibre.

53. A ray of light travels through a liquid. The speed of light in the liquid is 1.6×10^8 m/s, and the wavelength of the light ray in the liquid is 440 nm. Determine the following values. (9.2) T/I
(a) the index of refraction of the liquid
(b) the wavelength of the light ray in a vacuum

54. The index of refraction of ethyl alcohol is 1.36. Calculate the critical angle for a light ray travelling from ethyl alcohol into air. (9.2) K/U

55. Calculate the critical angle for a glycerin and water interface. The index of refraction for glycerin is 1.47 and for water is 1.33. (9.2) K/U

56. Calculate the critical angle for a diamond and crown glass interface. The index of refraction for diamond is 2.42, and for crown glass is 1.52. (9.2) K/U

57. A small light source shines upward from the bottom of a 35 cm–deep pond. When viewed from above, because of internal reflection, the light source makes a disc of light on the water's surface. Calculate the diameter of this disc. (9.2) T/I

58. A student uses a laser beam and a semicircular acrylic block to study refraction. Light is incident on the block at the following increasing angles, 15°, 30°, 45°, and 60°. The student measures the refracted angles as 11°, 21°, 29°, and 38°, respectively. (9.2) K/U C
(a) Use the data to plot a graph showing the sines of the refracted angles versus the sines of the incident angles.
(b) Determine the slope of the graph to two decimal places. Use the slope to determine the index of refraction of acrylic to two decimal places.

59. Light with a wavelength of 590 nm passes through double slits with a spacing of 0.085 mm to a screen located 1.10 m away. Calculate how far apart the bright fringes will be on the screen. (9.5) T/I

60. Discuss the impact of two technologies based on the wave theory of light. Focus on the economic and social impact of the technologies. (9.2, 9.6) K/U A

61. A student uses a ripple tank to create a two-point-source interference pattern. The student places a dot somewhere along the $n = 2$ nodal line. The distances from the dot to each source are 16.3 cm and 21.9 cm. Determine the wavelength of the waves. (9.3) T/I

62. Two coherent point sources 4.5 cm apart create an interference pattern in a ripple tank. There are 10 nodal lines in the entire interference pattern. Determine the wavelength of the water waves. (9.3) T/I

63. Water waves hit a straight barrier with two small openings separated by 14 m. Waves diffract from the openings and interfere with each other, creating a pattern containing a total of 12 nodal lines. Determine the wavelength of the water waves. (9.3) T/I

64. Two sources are vibrating at the same frequency in phase a distance apart in a ripple tank. How would you change each of the following to increase the number of nodal lines? (9.3) T/I
(a) frequency
(b) wavelength
(c) separation of the sources

65. Two sources are 7.2 cm apart and vibrate in phase at 7.0 Hz. A point on the third nodal line is 30.0 cm from one source and 37 cm from the other. (9.3) K/U T/I
(a) Calculate the wavelength of the waves.
(b) Calculate the speed of the waves.

66. Two towers of a radio station are 400 m apart along an east–west line. The towers act as point sources radiating at a frequency of 1.0×10^6 Hz. Radio waves travel at a speed of 3.0×10^8 m/s. Determine the first angle at which the radio signal strength is at a maximum for listeners who are on a line 20.0 km north of the station. (9.5) T/I A

67. Two coherent point sources 14 cm apart create an interference pattern in a ripple tank. The frequency of the waves is 3.1 Hz, and the waves travel 30.0 cm in 1.8 s. (9.3) T/I
   (a) Determine the wavelength of the waves.
   (b) Determine the total number of nodal lines in the entire interference pattern that may be observed.
68. List the successes and failures of the particle and wave models in accounting for the behaviour of light as follows: (9.4) K/U T/I
   (a) Name three optical phenomena adequately accounted for by both models.
   (b) Name two optical phenomena not adequately accounted for by the particle model.
   (c) Name one phenomenon not adequately accounted for by the wave model.
69. Describe and explain the experimental evidence collected up to the end of this chapter in support of the wave theory of light. (9.4) K/U T/I
70. Red laser light ($\lambda = 658$ nm) passes through a double slit of unknown slit spacing. The third bright fringe is observed at an angle of 2.8°. Determine the slit spacing, in millimetres. (9.5) T/I
71. Red laser light ($\lambda = 650$ nm) passes through a double slit, and the slits are spaced 2.1×10^{-4} m apart. Predict the distance between adjacent bright fringes when the light hits a screen positioned 5.0 m away from the slits. (9.5) T/I
72. Violet laser light of unknown wavelength passes through two slits that are spaced 7.3×10^{-4} m apart. A distance of 4.3 mm separates each bright fringe on a screen 6.5 m from the slits. Calculate the wavelength of the violet light, in nanometres. (9.5) T/I
73. A group of students uses an intense white light and a red filter to create bright fringes in a double-slit experiment. Describe the differences in the observed results when the students make the following changes, and explain the differences. (9.5) T/I A
   (a) Move the screen farther from the slits.
   (b) Temporarily block one of the slits.
   (c) Replace the red filter with a green filter.
   (d) Remove the filter and use white light.
74. In a double-slit experiment using a monochromatic source, the recorded distance between the first and seventh nodal lines is 6.0 cm. The slit separation is 2.2×10^{-4} m, and the screen is 3.0 m from the slits. Calculate the wavelength of the light. (9.5) T/I
75. Two slits produce an interference pattern. The perpendicular distance from the midpoint between the two slits to the screen is 7.7 m. The two third-order maxima are separated from each other by a distance of 3.29×10^{-1} m. The wavelength of the light is 4.9×10^{-7} m. Calculate the separation between the slits. (9.5) T/I
76. A double-slit experiment uses two slits 0.35 mm apart to produce an interference pattern on a screen 1.5 m from the slits. Determine the wavelength of the incident light. The distance between adjacent bright spots is 2.4 mm. (9.5) T/I
77. Light of wavelength 4.8×10^{-7} m shines on a slide containing two slits at a separation of 0.050 mm that is 1.0 m away from a screen. Determine the distance between two consecutive bright bands on an interference pattern. (9.5) T/I
78. Calculate the wavelength of the light that produces dark fringes 4.95×10^{-3} m apart on a screen 1.25 m away after passing through two slits that are spaced 0.100 mm apart. (9.5) T/I
79. A pattern of fringes appears on a screen 175 cm away, with a spacing of 7.7 mm between bright fringes. The wavelength of the light is 5.5×10^{-7} m. Determine the slit spacing. (9.5) T/I
80. Light with a wavelength of 530 nm shines on a double-slit apparatus. The bright fringes that appear on a distant screen have an angular separation of 2.1°. Determine the separation between the slits. (9.5) T/I
81. Two cellphone towers separated by 550 m transmit an identical 790 MHz signal. You discover that you have an optimal signal while standing 12 km from each of the towers. The towers lie along a north–south line, and as you walk directly north, the signal decreases. Calculate how far north you must walk until you have an optimal signal again. (9.5) T/I A
82. A light source shines light of wavelengths 490 nm and 560 nm onto a pair of slits separated by 0.44 mm. Calculate the angular location and the location in centimetres of the second-order dark fringes on a screen 1.4 m from the slits. (9.5) C T/I

Evaluation

83. Draw two flat mirrors at right angles to each other. Use the law of reflection to show that as a wave bounces off both surfaces, the wave undergoes a full 180° reversal of direction. (9.1) C A

84. Imagine that a friend tells you about his plans for a special room in a haunted house. Your friend says, "I could put a tank of water at the end of a hallway and hide underwater with a diving mask. If I choose a spot at the end of the tank, light rays travelling from me toward people in the hallway will reflect internally along the water surface. The people in the hallway will not see me, but I will see them and can jump out and scare them!" Assess the plan's chances of success. (9.2) T/I C A

85. Design a procedure to determine the wavelength of water waves that uses a ripple tank, two point sources, and other laboratory equipment. (9.3) T/I C A

86. Imagine that you are standing a few kilometres from a radio tower, with the wall of a tall building covered in metallic siding a few hundred metres directly behind you as you face the tower. (The siding will reflect radio waves.) A friend hands you a radio tuned to the tower's station with the radio's frequency display covered, and challenges you to guess the frequency. Devise a procedure to determine the station's frequency. (9.3) T/I C A

87. The debate over whether light is a particle or a wave lasted for many years. At any moment during that time, you could find scientists who claimed that light was a particle as well as scientists who claimed that light was a wave. Imagine a time before Young's experiment, and justify the viewpoints of scientists on both sides of the debate. (9.4) C A

88. Use the concepts of reflection and interference to propose why a car radio might lose reception of certain stations at certain places. (9.4) K/U T/I C A

89. A student tries to recreate Young's double-slit experiment by using two closely spaced miniature light bulbs as the light sources. The student does not observe the expected bright and dark fringes. Evaluate the student's experiment. (9.5) T/I A

Reflect on Your Learning

90. What did you find most surprising in this chapter, and what did you find most interesting? How can you learn more about these topics? K/U T/I C

91. How would you explain the concepts of total internal reflection, refraction, and diffraction to a fellow student who has not taken physics? K/U T/I C

92. In what areas of your daily experience do you now see the physics concepts that were explored in this chapter? K/U T/I C

93. What properties of waves and light are still confusing to you? How can you improve your understanding of the properties of waves and light? K/U T/I C

Research

WEB LINK

94. Research the working and use of fibre optic technology. Identify purposes, and illustrate the positions of the core, the cladding, and the buffer coating. List at least five advantages of a fibre optic communication system over a traditional metal wire system and discuss with a classmate. T/I C A

95. Night-vision goggles allow their user to see infrared light using technology known as thermal imaging. Research thermal imaging and its impact on society. Explain how this technology uses the wave nature of light. Identify other uses of thermal imaging besides night vision. Summarize your findings in a short report. C A

96. Modern double-slit experiments often make use of lasers to provide a coherent light source. Research current technology used for double-slit experiments and any applications of double-slit interference effects. Summarize your results in a short statement. C A

97. Light-based technologies can be used to analyze human skin for medical and cosmetic concerns. T/I C A
   (a) Investigate optics-based skin technologies and summarize how they work.
   (b) Identify the light-based technology used in optics-based skin analysis.
   (c) Discuss some of the benefits and drawbacks of this type of skin analysis.
   (d) List two careers associated with optics-based skin analysis and describe the pathways to each of these careers.

98. The colour theory of vision is based on the particle theory of light. Research this theory, and explain some of its successes and shortcomings. T/I A

99. Research heat mirages, and discuss how the refraction of light can cause mirages to appear in the distance on a paved road on a warm summer day or in a desert (**Figure 6**). K/U T/I C A

Figure 6

100. Research the work of Francesco Grimaldi (1619–1642) and his contributions to the theory of light. K/U T/I C A
   (a) Summarize Grimaldi's observations relating to light.
   (b) Using Grimaldi's work as an example, express how new theories change scientific thought.

CHAPTER

10 Applications of the Wave Nature of Light

KEY CONCEPTS

After completing this chapter you will be able to

- describe and explain thin-film interference in soap films and air wedges
- explain how single-slit diffraction creates interference patterns to increase the resolution of images
- explain how to use diffraction gratings in single-slit and double-slit investigations
- describe the electromagnetic spectrum, the uses of different wavelengths of light, and the production of electromagnetic radiation
- understand that polarization is a wave property of light, and explain how polarization can be used to control how light travels in various media
- describe some applications of the wave nature of light, and discuss their impact on society and the environment

How Can We Make Use of the Wave Nature of Light to Create Technologies?

Many technologies in several different industries use the wave properties of light. For example, you may have seen 3D movies in theatres. The technology involved in creating a 3D image relies on the wave properties of light. Movie projectors display two similar but slightly displaced images on a movie screen. The projector uses two sets of filters, one for each image, that give differing wave-like properties to the light for each image. The 3D viewing glasses use different filters for each eye that allow only light with one orientation to reach each eye. Your eyes and brain process these different images, producing the 3D effect. In this chapter, you will learn more about this property of light, which is called polarization.

The image on the facing page shows another application of polarization. Researchers use a computer to create a 3D image of a complicated molecule whose structure would be difficult to draw otherwise. Polarization effects are also used in sunglasses to reduce glare.

Some beautiful and sometimes surprising effects are the result of other wave properties of light. You may have seen colours reflected in soap bubbles and insect wings, and the iridescent colours of some birds and fish. These colours do not come from any pigment or dye, but from the interference of light waves reflected from different surfaces.

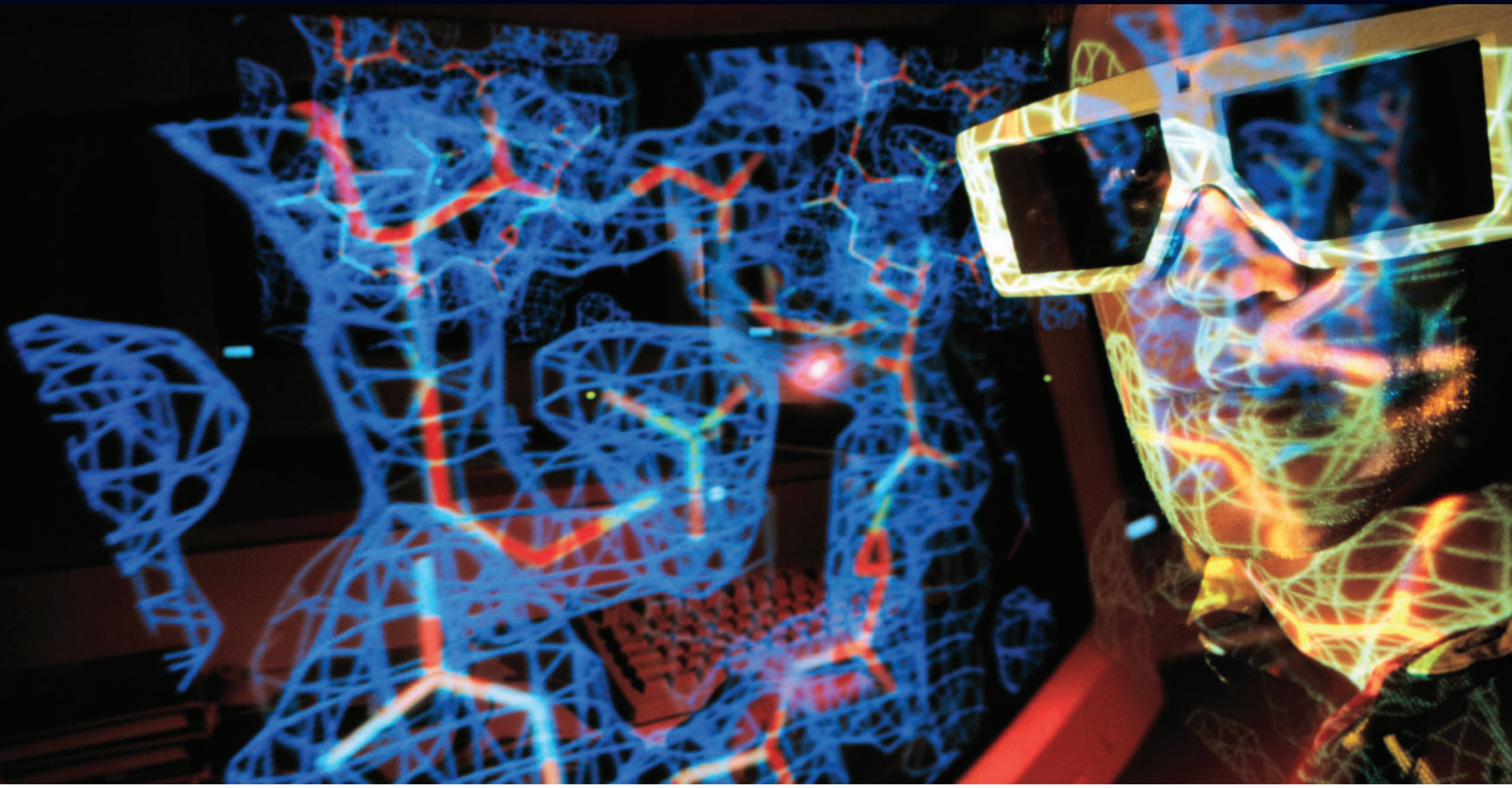
The colours you see in light reflected from a CD or DVD result from light waves being reflected off the grooves of the disc, which then interfere with each other. Another technology uses laser light to reflect off atoms in the atmosphere and back to the emitter, revealing information about the composition of the atmosphere at that location.

In this chapter, you will learn more about how the wave properties of light can be used in numerous practical applications.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. Why do you think the researcher in the image on the facing page needs to wear special eyeglasses to see the three-dimensional image?
2. What do you think causes soap bubbles to contain so many varying colours?
3. How can we use light to probe our natural world and further our understanding of the universe?
4. How do you think polarized sunglasses reduce glare?



Mini Investigation

Soap Bubbles

Skills: Performing, Observing, Analyzing, Communicating SKILLS HANDBOOK A2

In this activity, you will examine the interference effects of light passing through soap films (**Figure 1**).

Equipment and Materials: eye protection; showcase lamp or other bright lamp; plastic tray (such as a cafeteria tray); liquid soap (such as dish detergent)

1. Put on your eye protection. Pour a small amount of liquid soap onto a clean, plastic tray.
2. Dim the lights in the room.
3. Direct the light onto the soap on the tray.

To unplug the lamp, pull on the plug, not the cord. Do not touch the lamp because it will be hot after use. Use caution when working in a darkened room.

4. Record the different colours and areas of interference by making a sketch, writing a short description, or taking digital images.
5. Record how the pattern changes as you view the soap film.

Figure 1

A. What do you think the dark areas that you saw in the soap film represent? T/I
B. What do you think caused the colours that you saw? T/I
C. Why do you think the pattern kept changing as you viewed the soap film? T/I

10.1 Interference in Thin Films

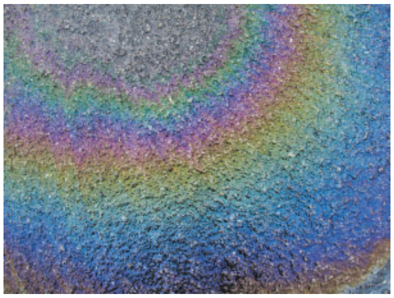


Figure 1 The colours in a thin film of oil on a road depend on the thickness of the oil layer.

thin film a very thin layer of a substance, usually on a supporting material

You may have noticed the colours of reflected light in a thin layer of oil in a puddle on a road, like the one in **Figure 1**. To understand why reflection from a layer of oil produces this colourful pattern, you need to understand the interference of light waves when they are reflected from or transmitted through a thin layer of material, often called a **thin film**.

Phase Change Due to Reflection

When light rays travelling from air meet the upper surface of an oil layer, some light waves are reflected. Similar behaviour occurs at the lower surface of the oil layer, with the result that light rays from both surfaces are reflected. These rays interfere either constructively or destructively, depending on their phase difference. For example, if the ray travels from one medium into a more optically dense medium, the reflected wave is inverted so it will not constructively interfere.

You can see this effect for yourself by attaching one end of a string tightly to a wall, holding the other end of the string in your hand, and moving your wrist sharply to create a wave. The incident wave travels quickly down the string and hits the wall. The wall pushes against the string with a force that is equal in magnitude and opposite in direction, according to Newton's third law of motion. This causes the wave to be inverted when it is reflected. Now suppose that the string is not fixed at one end or the incident wave encounters another medium in which the wave can travel faster. Then the reflected wave will not be inverted. It will have the same phase as the incident wave.

A wave that goes right through the medium, or is transmitted, does not reflect, so it is never inverted. You can verify this by tying a long metal wire to the loose end of a string. Hold the other end of the wire in your hand, and move your wrist sharply as before. The incident wave will travel quickly down the wire and encounter the string. Since the string is more flexible than the wire, the string will offer little resistance to the incident wave, and some of the wave's energy will carry forward through the string. Whatever wave reflects back to the wire will have the same phase as the incident wave.

Similar to the waves in the string, light waves will also invert when they encounter the surface of more optically dense media. A more optically dense medium has a higher index of refraction, so light waves are slowed down by the medium. As you can see in **Figure 2(a)**, when an incident wave reaches a fixed end or the boundary of a medium in which its speed will decrease, the reflected wave is inverted. On the other hand, if the wave reaches a free end or the boundary of a medium in which its speed will increase (**Figure 2(b)**), the reflected wave is not inverted and no phase change occurs.

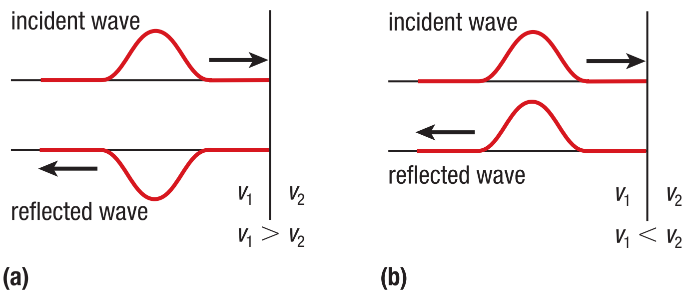


Figure 2 (a) A wave is inverted when it reflects from a fixed end or encounters a medium in which the speed of the wave decreases. (b) A wave is not inverted when it reflects from a free end or encounters a medium in which the speed of the wave increases.

Wavelength and the Index of Refraction

How does wavelength relate to the index of refraction? **Figure 3** shows a light wave striking a thin film of oil on a glass slide. The waves reflect from the top and bottom surfaces of the film of oil. Assume that the angle of incidence is zero. In the following figures, the incident ray is drawn with a larger angle of incidence so that the two reflected rays are easily visible.

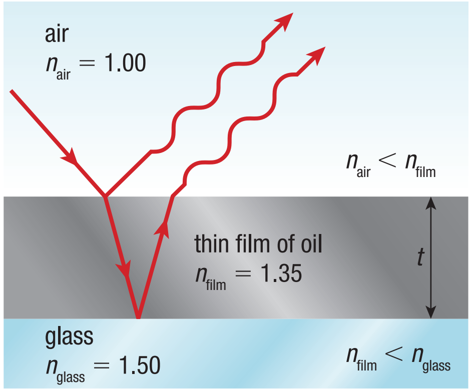


Figure 3 Light waves reflect from the top and bottom surfaces of a thin film of oil on glass. The indices of refraction are related as follows: $n_{air} < n_{film} < n_{glass}$.

For a film of thickness t, the distance travelled by the ray reflecting off the bottom layer is $2t$. This path difference will cause a phase difference between the rays reflected off the top and bottom layers of the film. To determine the total phase difference, however, you also need to account for the change in wavelength of the wave travelling through the film due to the index of refraction of the film.

The frequency, f, and wavelength, λ_{vac}, of a light wave in a vacuum are related by

$$c = \lambda_{vac} f$$

where $c = 3.0 \times 10^8$ m/s, the speed of light in a vacuum. When light travels through a substance with index of refraction n, its speed, v, is $\frac{c}{n}$. Since n for any substance is greater than 1, light travels more slowly in the substance than in a vacuum. The product of the wavelength in the film, λ_{film}, and the frequency of the light in the film, f_{film}, equals the wave speed. Thus,

$$\lambda_{film} f_{film} = \frac{c}{n_{film}}$$

Comparing the two previous equations shows that the wavelength, the frequency, or both must change when light travels from a vacuum into the thin film. **Figure 4** shows that as the light wave travels from one medium to the other, the wavelength changes but the frequency remains the same. If this were not the case, the waves just outside and just inside the film would not remain in phase, and the wave fronts would "pile up" at the boundary.

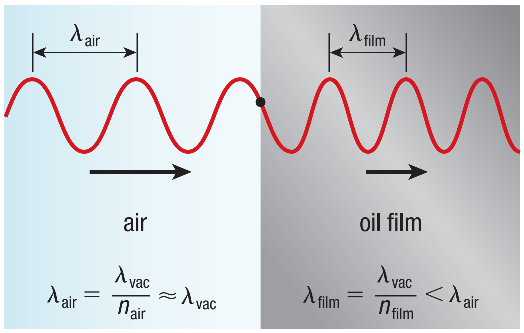


Figure 4 When light travels from air to a different medium, such as an oil film, the light waves are in phase. This can only happen if the frequency is the same on both sides of the boundary.

Although the frequency of light is unchanged as it enters the film, its wavelength changes. If we divide the previous equation by f, the wavelength inside the film becomes

$$\lambda_{\text{film}} = \frac{1}{f}\frac{c}{n_{\text{film}}}$$

Combining this with the relation between λ and f in a vacuum then leads to

$$\lambda_{\text{film}} = \frac{\lambda_{\text{vac}}}{n_{\text{film}}}$$

The wavelength of light inside the film is thus shorter by a factor of $\frac{1}{n_{\text{film}}}$. The same is true for light travelling in air, with $\lambda_{\text{air}} = \frac{\lambda_{\text{vac}}}{n_{\text{air}}}$. Given that n_{air} is very close to 1 ($n_{\text{air}} = 1.0003$), interference effects are not usually considered in air. Therefore, the wavelength of a light wave in air is close to its wavelength in a vacuum:

$$\lambda_{\text{air}} \approx \lambda_{\text{vac}}$$

The ray travelling through the oil film in Figure 3, on the previous page, travels the distance $2t$ inside the film. The extra number of wavelengths, N, this wave travels is thus

$$N = \frac{2t}{\lambda_{\text{film}}}$$

For example, if N has a value of 4, the ray will travel four complete, extra wavelengths compared with the ray reflecting off the top of the film.

To determine whether this combination of reflected rays produces constructive or destructive interference, you need to combine the phase change due to the path difference with phase changes due to reflection.

Thin Films and Interference

To understand what leads to the colours from a film such as the one in Figure 1, on page 502, consider a thin film between two glass slides. For simplicity's sake, assume that the light is monochromatic (single pure colour), so it has a single wavelength. When the light ray in **Figure 5** strikes the upper surface of the film, part of the light wave reflects and part of it transmits through the film. The transmitted part will then partially reflect from the boundary between the material under the film and the bottom surface of the film, and part of the wave will also transmit through the bottom surface. The intensity of transmitted and reflected light depends on the properties of the reflecting material. For a transparent material, much of the wave transmits and little reflects. For an opaque material, much of the wave reflects and little transmits.

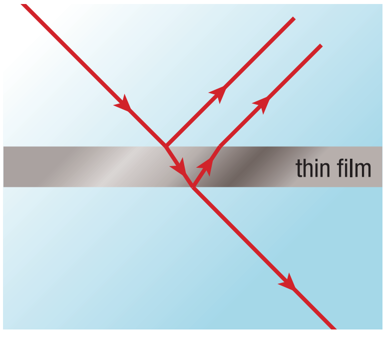


Figure 5 Both surfaces of a thin film reflect and transmit part of the incident light.

You will need to follow several steps to determine the relative phase of the interfering reflected waves. The difference in phase due to path difference depends on the difference in the lengths of the two paths and the wavelength of the light in the film. The phase change due to reflection depends on the relative sizes of the indices of refraction of the film and the surrounding substances.

Now suppose a thin film, for example a soap film, lies between air on one side and a substance with a higher index of refraction on the other, such as glass, so that $n_{\text{air}} < n_{\text{film}} < n_{\text{glass}}$ (**Figure 6**). Two phase changes occur when the index of refraction increases across each successive boundary, so both waves reflecting from the thin film invert upon reflection. Since they both change by the same amount, this change introduces no difference in phase between the two waves. If the number of extra wavelengths, *N*, is a whole number, light waves that travel along rays 2 and 3 in Figure 6 are in phase and interfere constructively with one another. If *N* is not a whole number, they are out of phase. For both waves having a phase change,

$$2t = \frac{n\lambda}{n_{\text{film}}} \text{ (constructive interference); } 2t = \frac{\left(m + \frac{1}{2}\right)\lambda}{n_{\text{film}}} \text{ (destructive interference)}$$

where $n = 1, 2, 3, \ldots,$ and $m = 0, 1, 2, \ldots.$

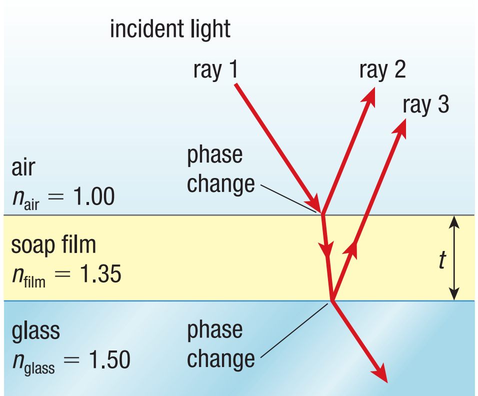


Figure 6 The incident light ray, ray 1, meets the boundary between the air and the soap film and is partially reflected (ray 2) and partially transmitted through the film. When it encounters the boundary between the soap film and the glass, it is partially reflected (ray 3) and partially transmitted.

Now we look at the case when only one of the two waves has a phase change on reflection, as, for example, in the case of a soap film with air on both sides (**Figure 7**). To simplify the analysis, we continue to consider the case where the incident and reflected rays are all approximately normal (perpendicular) to the film. This means that the rays that pass through the film travel a total distance of $2t$, where t is the thickness of the film. As before, the diagram does not show that the rays are perpendicular. The reason is that the rays would lie on top of each other, making the diagram difficult to understand and analyze.

The index of refraction of the soap film in Figure 7 is 1.35, so part of ray 1 reflects and becomes inverted (ray 2) when it encounters a medium with a higher index of refraction. Part of ray 1 transmits through the soap film, where it encounters the bottom layer of the film. It reflects and continues back through the soap film. There is no phase change when it encounters a medium with a lower index of refraction. The inverted phase change is equivalent to a shift of the wave by $\frac{\lambda}{2}$. Thus, when only one of the two waves undergoes an inverted phase change,

$$2t = \frac{\left(m + \frac{1}{2}\right)\lambda}{n_{\text{film}}} \text{ (constructive interference); } 2t = \frac{n\lambda}{n_{\text{film}}} \text{ (destructive interference)}$$

where $m = 0, 1, 2, \ldots,$ and $n = 1, 2, 3, \ldots.$

These conditions depend on the thickness of the film. In other words, the thickness of the film determines the colour of light that will be strongly reflected. The thicker the thin film, the longer the wavelength of light that will produce constructive interference. The oil film in Figure 1 varies in thickness, so different colours of light are reflected. The following Tutorial shows you how to calculate the interference effects in thin films such as soap bubbles.

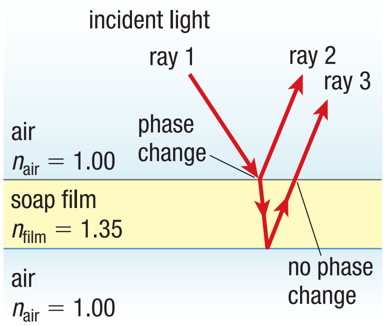


Figure 7 When ray 1 reflects at the air–film boundary, the reflected ray (ray 2) is inverted. The ray reflecting from the lower film–air boundary (ray 3) does not change phase at the upper film–air boundary.

Investigation 10.1.1

Investigating Interference Using Air Wedges (page 544)

Now that you have learned about thin films and interference, apply this knowledge to perform Investigation 10.1.1 to determine the thickness of a human hair.

Tutorial 1 Determining Interference Effects in Thin Films

In the following Sample Problems, you will determine the interference effects in thin films with one inverted reflection and two phase changes.

Sample Problem 1: Determining the Colour and Thickness of a Soap Film

Consider a soap film that is the thinnest film that will produce a bright blue light when illuminated with white light. The index of refraction of the soap film is 1.35, and the blue light is monochromatic with wavelength 411 nm (**Figure 8**).

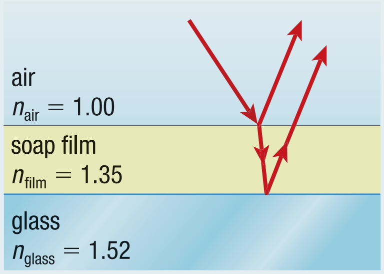


Figure 8 Interference from a soap film with air on one side depends on the substance on the other side of the film.

(a) Calculate the thickness of the film if the soap covers a piece of crown glass with index of refraction 1.52.

(b) Suppose the reflections occur instead from a soap film on water with index of refraction 1.33. Determine the thickness of the film on water that will produce the same blue colour of reflected light.

Solution

(a) **Given:** $n_{glass} = 1.52$; $n_{film} = 1.35$; $\lambda = 411 \text{ nm} = 4.11 \times 10^{-7} \text{ m}$

Required: t

Analysis: Both reflections involve inverted phases, since the index of refraction increases from layer to layer. Therefore, use the formula for constructive interference of the two waves when phase changes occur in both reflections. Let $n = 1$.

$$2t = \frac{n\lambda_{blue}}{n_{film}}$$

Solution:
$$2t = \frac{n\lambda_{blue}}{n_{film}}$$
$$t = \frac{n\lambda_{blue}}{2n_{film}}$$
$$= \frac{(1)4.11 \times 10^{-7} \text{ m}}{2(1.35)}$$
$$t = 1.52 \times 10^{-7} \text{ m}$$

Statement: The thickness of the film on the glass is 1.52×10^{-7} m.

(b) **Given:** $n_{water} = 1.33$; $n_{film} = 1.35$; $\lambda = 411 \text{ nm} = 4.11 \times 10^{-7} \text{ m}$

Required: t

Analysis: One ray is partially reflected from the upper surface and inverts because it is moving into a more optically dense medium. However, the second ray reflects from the bottom surface and does not change phase because it is moving into a less optically dense medium. Therefore, use the formula for constructive interference of two waves with one phase change. Set m equal to zero.

Solution:
$$2t = \frac{\left(m + \frac{1}{2}\right)\lambda_{blue}}{n_{film}}$$
$$2t = \frac{\lambda_{blue}}{2n_{film}}$$
$$t = \frac{\lambda_{blue}}{4n_{film}}$$
$$= \frac{4.11 \times 10^{-7} \text{ m}}{4(1.35)}$$
$$t = 7.61 \times 10^{-8} \text{ m}$$

Statement: The thickness of the film on water is 7.61×10^{-8} m.

Sample Problem 2: Making Solar Cells More Efficient

In solar cells, incoming light passes through an anti-reflective coating to increase the efficiency of the cell (**Figure 9**). Suppose the index of refraction of the coating is $n_1 = 1.45$ and the index of refraction of the material below the coating is $n_2 = 3.50$. In this case, you need to maximize the amount of transmitted light to minimize the reflected light. Determine the thickness of the anti-reflective coating that will minimize the reflection of light with a wavelength of 7.00×10^{-7} m.

Given: $n_1 = 1.45$; $n_2 = 3.50$; $\lambda = 7.00 \times 10^{-7}$ m

Required: t

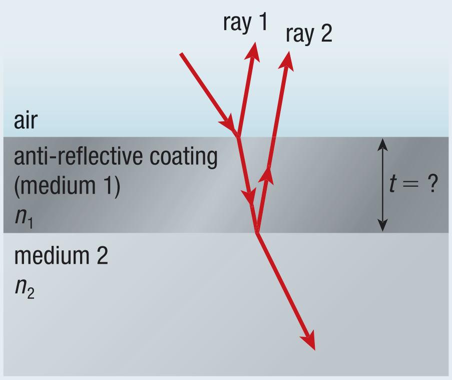


Figure 9

Analysis: An incident light ray moving from air into medium 1 will partially reflect at the upper surface of the coating. Since medium 1 is more optically dense than air, a light wave will be reflected and inverted at the upper surface. Similarly, at the lower surface of medium 1, a light wave will be reflected and inverted because medium 2 is more optically dense than medium 1. To maximize the amount of transmitted light through the coating, the two reflected rays from the upper and lower surface of the coating must undergo destructive interference. This means that the distance, $2t$, that ray 2 must travel compared to ray 1 is equal to $\frac{\lambda}{2}$. Therefore, use the formula for destructive interference when both waves change phase, which is the same as when both waves have no change of phase. Set m equal to zero.

$$2t = \frac{(m + \frac{1}{2})\lambda_{\text{blue}}}{n_{\text{film}}}$$

Solution:

$$2t = \frac{\lambda}{2n_{\text{film}}}$$

$$t = \frac{\lambda}{4n_{\text{film}}}$$

$$= \frac{7.00 \times 10^{-7}\ \text{m}}{4(1.45)}$$

$$t = 1.21 \times 10^{-7}\ \text{m}$$

Statement: The thickness of the anti-reflective coating is 1.21×10^{-7} m.

Practice

1. A soap film produces constructive interference of light of wavelength 500.0 nm, and a second film produces constructive interference of light with a wavelength of 600.0 nm. Determine which film is thicker. Explain your answer. K/U C
2. Calculate the smallest thickness of a soap film on glass capable of producing reflective destructive interference with a wavelength of 745 nm in air. Assume that the index of refraction for soapy water is the same as that for pure water, which is 1.33. T/I A [ans: 1.40×10^{-7} m]
3. A 510 nm wavelength of yellowish light is incident on an oil slick with an index of refraction of 1.50 on top of a pool of pure water. Calculate the thickness the oil slick needs to be so that you cannot see the yellowish light. T/I A [ans: 1.70×10^{-7} m]
4. Most camera lenses have an anti-reflective coating made of magnesium fluoride (MgF_2). The magnesium fluoride coating has an index of refraction of 1.38. Determine the thickness of the anti-reflective magnesium fluoride coating needed for red light with a wavelength of 610 nm. K/U A [ans: 1.1×10^{-7} m]

Mini **Investigation**

Observing a Thin Film on Water

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

When white light is incident on a thin film, each different wavelength of light should have a different interference pattern. In this investigation, you will examine and compare the patterns for white, red, and blue light, and analyze the patterns in terms of the conditions for constructive and destructive interference.

Equipment and Materials: bright light source; black cloth or construction paper; a flat piece of glass; liquid dropper; water; light machine oil; red and blue filters; digital camera (optional)

1. Spread the cloth or construction paper on a flat table or desk.
2. Place the glass on top of the black surface. Using a liquid dropper, cover the surface of the glass with a thin layer of water.
3. Place a few drops of light machine oil on the water.
4. Direct a bright white light source at the surface. Darken the room. Observe the interference pattern on the surface of the water, and record your observations.

To unplug the lamp, pull on the plug, not the cord. Do not touch the lamp because it will be hot after use. Use caution when working in a darkened room.

5. Place a red filter in front of the light. Observe the changes in the pattern, and record your observations in labelled sketches or as digital images, if a digital camera is available.
6. Repeat Step 5 using a blue filter.

A. What did the dark areas in the film represent? K/U
B. What caused the patterns that you saw? K/U
C. Why did the pattern change when the colour of the light changed? K/U

Newton's Rings and Air Wedges

If you place a glass disc with a convex surface in contact with a flat glass surface, as shown in **Figure 10(a)**, and illuminate it with a beam of light, a phenomenon called Newton's rings occurs (**Figure 10(b)**). Light reflects from both the upper surface of the flat glass and the lower surface of the curved glass. The light reflecting from the flat glass plate changes phase because the air between the two glass surfaces has a lower index of refraction than the glass. However, the light reflecting from the lower surface of the curved glass does not change phase. The difference in path length, t, increases with the distance from the point of contact of the two glass surfaces. Therefore, monochromatic light of wavelength λ has a bright fringe at each location where the separation t is half a whole-number multiple of λ. It has a dark fringe from destructive interference at each location where the separation t is a whole-number multiple of λ.

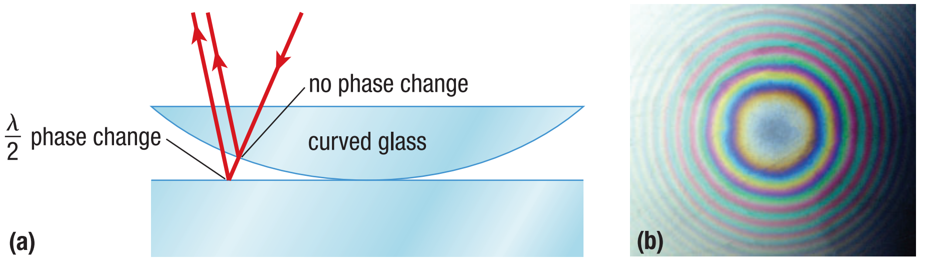


Figure 10 (a) Light reflects from both the upper surface of the flat glass and the lower surface of the curved glass. (b) Newton's rings, a series of concentric rings, form from light reflecting between a flat surface and an adjacent curved surface.

A practical application of Newton's rings is checking lenses for imperfections. Lenses are typically spherical in shape and, if shaped properly, will produce perfectly circular Newton's rings when illuminated with light (**Figure 11(a)**). However, if the lens is imperfectly shaped, it will produce a pattern that clearly indicates a defective lens shape, as shown in **Figure 11(b)**. CAREER LINK

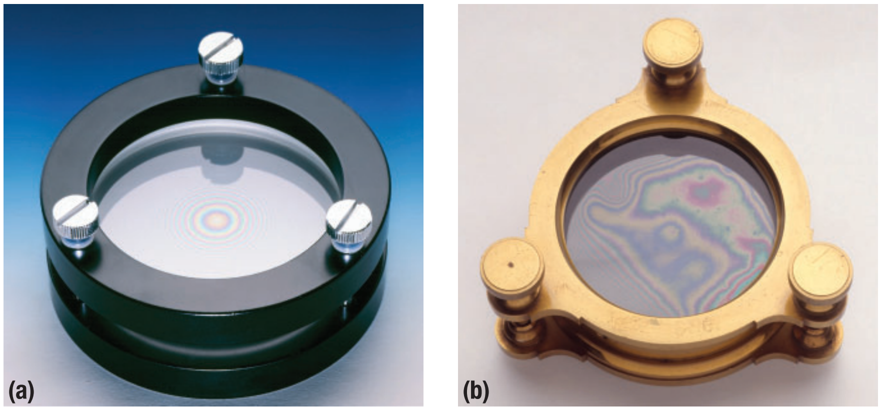


Figure 11 (a) A well-made lens has the shape of a section of the surface of a sphere. When illuminated with light, the interference patterns are circular Newton's rings. In this image, white light was used. (b) A defective lens produces non-circular Newton's rings that provide information on how to regrind the lens.

Air Wedges

air wedge the air between two sheets of flat glass angled to form a wedge

To create a measurable pattern of destructive and constructive interference, researchers often use an **air wedge**, which is a wedge of air between two sheets of flat glass that have been angled to form a wedge. The upper glass is slightly raised by a very small distance, t, and illuminated with monochromatic light, as shown in **Figure 12**. The interference patterns produced, such as the patterns in **Figure 13**, can be used to measure very small distances.

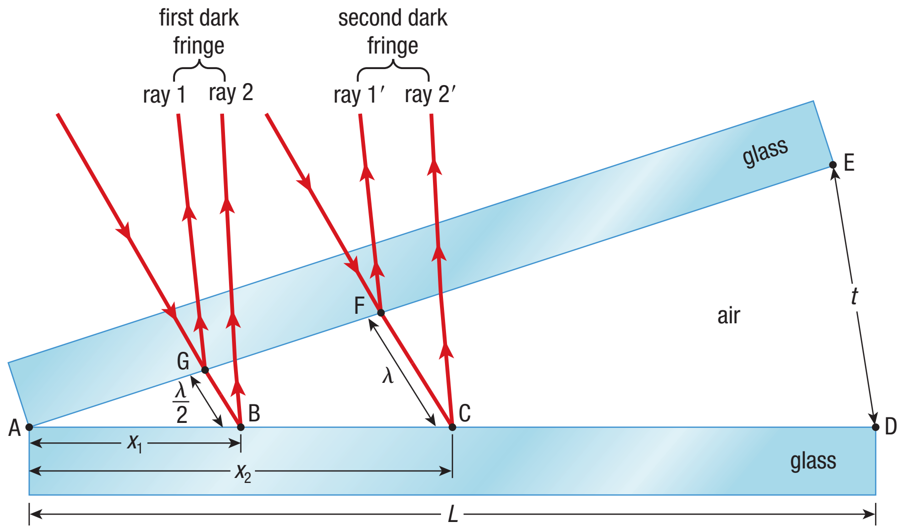


Figure 12 Two flat layers of glass separated at one end form an air wedge.

The measurable distance between fringes depends on the wavelength of light and the distance, t. Thus, if the distance is known, you can use the fringe pattern to determine the wavelength of light used. If you already know the wavelength, you can use the measurable pattern to determine the width of the very small object causing the separation of the layers of glass.

To calculate the fringe pattern from t, λ, and the length of the plate, L, you need to determine the relative phase of waves reflecting from the bottom surface of the upper glass plate (for example, point G in Figure 12) and the upper surface of the bottom glass plate (for example, point B). At point A, where the glass edges meet, the 180° phase change of just one reflected wave leads to destructive interference and a dark fringe. At point G, a light wave travelling through the first glass layer reaches a boundary where the index of refraction for air is less than that for glass. So light that reflects at point G has no phase change. The same reasoning applies at point F. The reflection of light at points B and C, however, undergoes a phase change of 180°. The reason is that light is travelling from air into glass, which is more optically dense than air. Thus, the dark fringes occur whenever the separation distance of the two plates at a particular location is a multiple of one wavelength.

To determine the first minimum, x_1, set the difference in path length to $\frac{\lambda}{2}$, which is the condition for destructive interference to occur. Triangles AGB and AFC in Figure 12 demonstrate that the first minimum meets the following condition:

$$\frac{x_1}{L} = \frac{\left(\frac{\lambda}{2}\right)}{t}$$

$$x_1 = \frac{L\lambda}{2t}$$

The next fringe, x_2, occurs when the difference in path length for the reflection from the upper and the lower plates is λ:

$$\frac{x_2}{L} = \frac{\lambda}{t}$$

$$x_2 = \frac{L\lambda}{t}$$

Thus, the separation between the two fringes is

$$\Delta x = x_2 - x_1$$

$$\Delta x = \frac{L\lambda}{t} - \frac{L\lambda}{2t}$$

$$\Delta x = \frac{L\lambda}{2t}$$

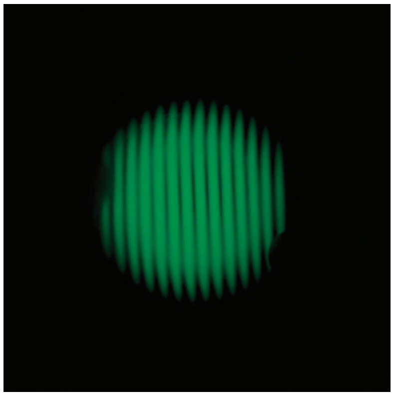


Figure 13 These dark fringes resulted from an air wedge experiment using $t = 546$ nm.

UNIT TASK BOOKMARK

You can apply what you have learned about interference in thin films to the Unit Task on page 556.

Tutorial 2 Calculating Interference Effects in an Air Wedge

This Sample Problem shows how to use interference effects to measure small distances.

Sample Problem 1: Interference Effects in an Air Wedge

Two glass plates are separated on one side by a human hair. The light shining on the plates has a wavelength of 6.00×10^{-7} m. The light intensity is zero at the point of contact of the two plates, followed by nine alternating bright and dark fringes (**Figure 14**). Estimate the thickness of the hair.

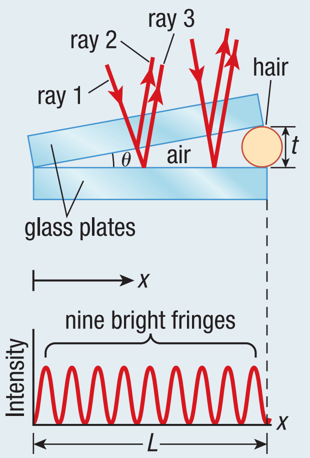


Figure 14

Given: $\lambda = 6.00 \times 10^{-7}$ m; nine complete cycles of passing from dark to light to dark areas along the wedge

Required: t

Analysis: Let $L = 9\Delta x$.

$$\Delta x = \frac{L\lambda}{2t}$$

$$2t = \frac{L\lambda}{\Delta x}$$

$$2t = \frac{9\cancel{\Delta x}\lambda}{\cancel{\Delta x}}$$

$$2t = 9\lambda$$

$$t = \frac{9\lambda}{2}$$

Solution: $t = \dfrac{9\lambda}{2}$

$$= \frac{9(6.00 \times 10^{-7}\ \text{m})}{2}$$

$$t = 2.70 \times 10^{-6}\ \text{m}$$

Statement: The estimated thickness of the hair is 2.70 μm.

Practice

1. A sheet of paper 0.012 cm thick separates two sheets of glass to form an air wedge 10.8 cm long. When the air wedge is illuminated with monochromatic light, the distance between the centres of the first and eighth dark bands is 2.4 mm. Determine the wavelength of the light. K/U T/I [ans: 7.6×10^{-7} m]
2. An air wedge 6.0 cm long is formed from two pieces of glass separated at one end by a piece of paper. When light of wavelength 730 nm is reflected from the wedge, interference fringes appear. Between the dark fringes at both ends of the wedge, 62 bright fringes appear. Calculate the thickness of the paper. K/U T/I [ans: 2.3×10^{-5} m]

Research This

Thin Films and Cellphones

Skills: Researching, Communicating SKILLS HANDBOOK A4.1

A new thin-film technology ("green glow") uses several layers of thin films to detect and amplify infrared light, while other layers, which are called organic light-emitting diodes (OLEDs), convert infrared to visible light. This thin-film technology enables researchers to turn infrared light into visible light to create night vision. Some practical applications of green glow are cellphone cameras, and windshields and eyeglasses for night vision.

1. Choose one type of application either listed above or discovered through your own research as you learn more about OLEDs and thin films.
2. Research the application.

A. Explain how your chosen technology works and how the interference properties of thin films play a role. T/I C

B. Determine the current state of development of the application that you chose. T/I

C. Determine from your research what further improvements are needed to make the technology more usable, perhaps for another application. Discuss any concerns, hazards, or disadvantages of the technology that would need to be addressed. T/I C A

D. Prepare a brief presentation or a one-page written summary of the application that you researched. C

WEB LINK

10.1 Review

Summary

- Light waves become inverted when they reflect from the boundary of a medium that has a higher index of refraction than the original medium. No phase change occurs when light waves reflect from the boundary of a medium that has a lower index of refraction than the original medium.
- Light waves that reflect from the two surfaces of a thin film produce interference fringes that depend on the different path lengths travelled by the two waves, the wavelength of the light, any phase changes that occur from reflection, and the indices of refraction for the materials involved.
- If only one wave has a phase change,

  $$2t = \frac{\left(m + \frac{1}{2}\right)\lambda}{n_{\text{film}}} \quad \text{(constructive interference); } m = 0, 1, 2, 3, \ldots$$

  $$2t = \frac{n\lambda}{n_{\text{film}}} \quad \text{(destructive interference); } n = 1, 2, 3, \ldots$$

  where t is the thickness of the film, λ is the wavelength, and n_{film} is the index of refraction for the film.
- If both waves have a phase change,

  $$2t = \frac{n\lambda}{n_{\text{film}}} \quad \text{(constructive interference); } n = 1, 2, 3, \ldots$$

  $$2t = \frac{\left(m + \frac{1}{2}\right)\lambda}{n_{\text{film}}} \quad \text{(destructive interference); } m = 0, 1, 2, 3, \ldots$$
- Newton's rings and fringes in air wedges result from light reflecting, transmitting, and interfering with surfaces that have different separations at different locations.

Questions

1. A coating, 177.4 nm thick, is applied to a lens to minimize reflections. The refractive indices of the coating and the lens material are 1.55 and 1.48, respectively. Calculate the wavelength in air that is minimally reflected for light rays that strike the lens along the normal. Include a diagram. K/U T/I C
2. A transparent film of $n = 1.29$ spills onto the surface of water, with $n = 1.33$, producing a maximum of reflection with normally incident orange light, with a wavelength of 7.00×10^{-7} m in air. Assuming that the maximum occurs in the first order, determine the thickness of the film. Include a diagram in your solution. K/U T/I C
3. An extremely thin film of soapy water of $n_{\text{film}} = 1.35$ sits on top of a flat glass plate of $n_{\text{glass}} = 1.50$. The soap film has a red colour when the incident light reflects perpendicularly off the surface of the water. Determine the thickness of the film when $\lambda_{\text{red}} = 6.00 \times 10^{-7}$ m. T/I
4. Change the index of refraction for the glass plate in Question 3 to 1.10. Determine the thickness of the film when $\lambda_{\text{red}} = 6.00 \times 10^{-7}$ m. T/I
5. **Figure 15** shows the bands of reflected colour produced by exposing a thin soap-bubble film to white light. What produces the colours? Is the bubble of uniform thickness? Explain. K/U T/I A

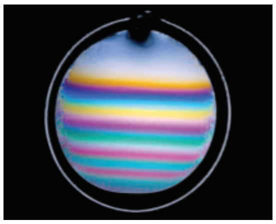


Figure 15

6. Use diagrams to explain why the top of a soap film appears bright from one side and dark from the other side when light is transmitted through it. K/U C
7. A very thin sheet of glass of $n_{\text{glass}} = 1.55$ floats on the surface of water of $n_{\text{water}} = 1.33$. When illuminated with white light at normal incidence, the reflected light consists predominantly of the wavelengths 5.60×10^{-7} m and 4.00×10^{-7} m. Determine the thickness of the glass. T/I

10.2 Single-Slit Diffraction

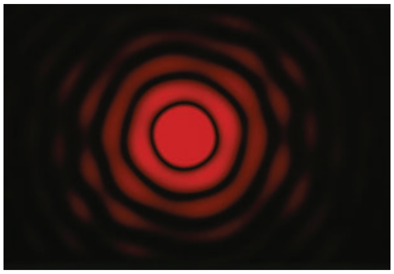


Figure 1 The interference of light passing through a single slit from a circular opening produces a pattern of concentric rings.

If you shine a beam of light through a wide-enough opening, you might expect the beam to pass through with very little diffraction. However, when light passes through a progressively narrower opening, the wave properties of light often produce unusual-looking patterns, such as the diffraction pattern shown in **Figure 1**. In this section, you will learn about the formation of such diffraction patterns.

Single-Slit Diffraction

To understand how a narrow opening (for example, 100 μm wide) can affect light passing through it, consider the behaviour of light passing through a single slit as shown in **Figure 2(a)**. While investigating this configuration, assume that the following conditions exist:

1. The width, *w*, of the slit is narrow enough that a substantial amount of diffraction can occur, but not so narrow that it acts as a single point source of waves.
2. The light source is monochromatic with wavelength λ.
3. The light source is far from the slit, so that light rays that reach the slit are all travelling approximately parallel to each other.

Fraunhofer diffraction an interference pattern that shows distinctive differences between the bright central fringe and darker flanking fringes

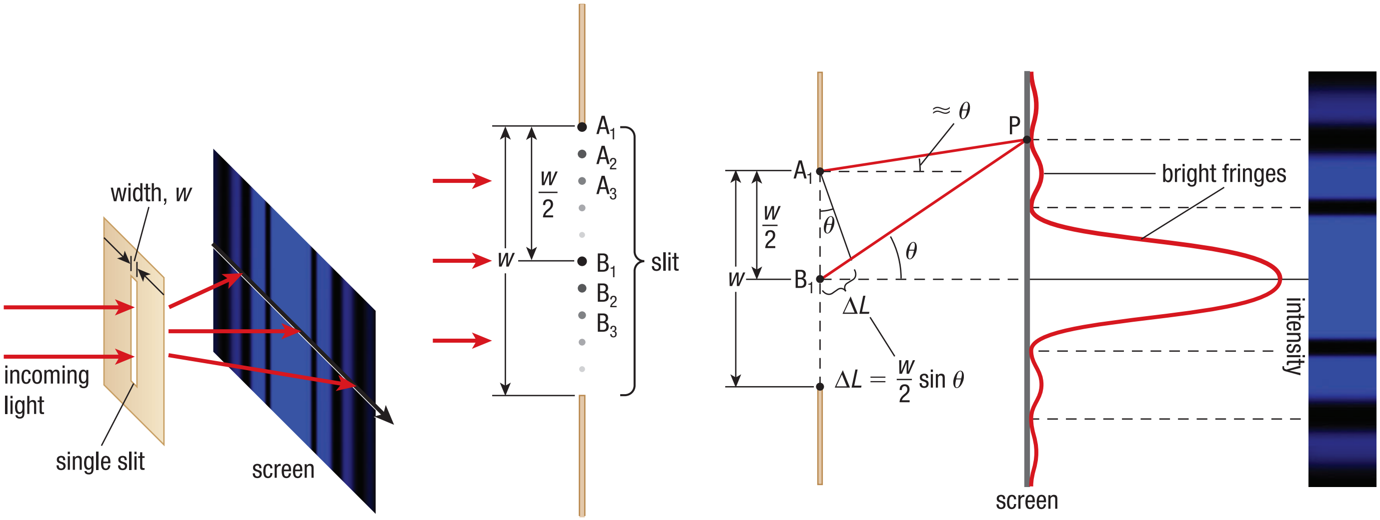
central maximum the bright central region in the interference pattern of light and dark lines produced in diffraction

secondary maxima the progressively less-intense bright areas, outside the central region, in an interference pattern

Such configurations produce **Fraunhofer diffraction**, which shows a bright central fringe called the **central maximum**. The central maximum is flanked by dark fringes, called minima, and less-intense bright fringes, called **secondary maxima**.

Why do these fringes occur, and how can you predict where bright fringes and dark fringes will appear on the screen? The key to answering these questions is Huygens' principle. Recall from Chapter 9 that Huygens' principle states that each point on a wave front acts as a new source of waves. **Figure 2(b)** is a magnified view of the slit looking down on it from above, showing its width. According to Huygens' principle, all points across the slit width in Figure 2(b) act as wave sources, and these different waves create an interference pattern at the screen.

When the slit is divided in half (Figure 2(b)), coherent waves that interfere with each other appear as the light passes through the slit. The points in one half are labelled A, and the points in the other half are labelled B. Each of these points is a source of Huygens waves (spherical waves), including point A_1 at one edge of the slit and the corresponding source point B_1 at the centre of the slit.



(a) (b) (c)

Figure 2 (a) Light passes through a single slit and hits a screen. (b) A view from above the slit looking down at its width. Each point in the slit acts as a source of spherical waves. (c) Looking down at the slit and screen, the single-slit pattern is apparent. Assume that point P is far away.

Figure 2(c) shows the distances from these points to a point P on the screen—the path lengths—and the path length difference ΔL. Although Figure 2 shows the screen close to the slit, assume that the screen is quite far away, so all the angles denoted by θ are approximately equal.

Suppose the waves from A_1 and B_1 interfere destructively when they reach a particular point P on the screen. The path length difference is $\Delta L = \frac{\lambda}{2}$. When this condition for destructive interference is met for points A_1 and B_1, the waves from A_2 and B_2 will also interfere destructively, as will the waves from points A_3 and B_3, and so on, for all similar pairs of points within the slit. Destructive interference produces a dark fringe on the screen. From Figure 2(c), you can see that

$$\Delta L = \frac{w}{2}\sin\theta_n$$

where w is the width of the slit.

Substituting $\frac{\lambda}{2}$ for ΔL leads to $\frac{w}{2}\sin\theta_n = \frac{\lambda}{2}$, which gives the angle of the first dark fringe on one side of the centre of the screen. On the other side, the corresponding value gives the angle of the first dark fringe to that side of the centre. The first dark fringes are thus at the angles

$w\sin\theta_n = \lambda$ single-slit diffraction: first dark fringes (destructive interference)

Figure 3 shows the method of determining the position of the second dark fringe. In Figure 3(a), the slit is divided into four regions. The points in these four regions are denoted by A, B, C, and D. Figure 3(b) shows the path lengths from the farthest point in each region (A_1, B_1, C_1, and D_1) to a point P on the screen. Suppose the path length difference for the Huygens waves from A_1 and B_1 is $\Delta L = \frac{\lambda}{2}$, so these waves interfere destructively. The path length difference for C_1 and D_1 has the same value, so these waves also interfere destructively. The path length difference is the same for waves from other pairs of points A_2 and B_2, C_2 and D_2, and so on, so these waves also interfere destructively, resulting in no light, or a total intensity of zero, on the screen. Thus, for the second dark fringe, $\Delta L = \frac{w}{4}\sin\theta$. Setting this expression equal to $\frac{\lambda}{2}$, the interference condition in this case is the following, where the angle again corresponds to fringes left and right of the centre of the screen:

$w\sin\theta_2 = 2\lambda$ single-slit diffraction: second dark fringes (destructive interference)

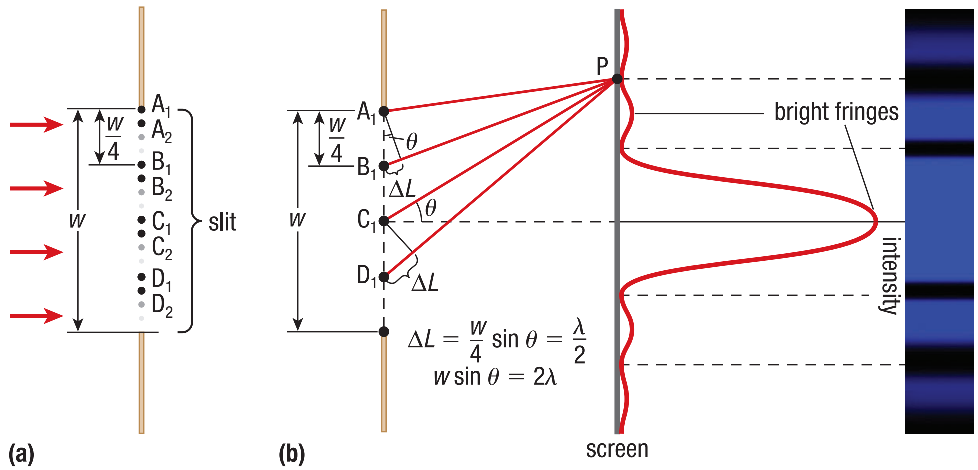


Figure 3 (a) To determine the locations of the second dark fringes, we divide the width into four regions, A, B, C, and D. (b) The locations of the second dark fringes can then be determined. In a scale drawing, the slit width would be much smaller than the width of the central diffraction peak.

The same method can predict the third and other dark fringes by dividing the slit into 6, 8, 10, or any even number of regions. The result for the nth dark fringe is

$$w \sin \theta_n = n\lambda \quad \text{single-slit diffraction: } n\text{th dark fringes (destructive interference)}$$

where $n = 1, 2, 3, \ldots$. The values of θ that satisfy this equation give the angles of all dark fringes.

To determine the angles for the maxima, we use a similar method of dividing up the slit. For maxima, however, we divide the slit into an odd number of regions. Light from all but one region will interfere destructively, as in the discussion for minima, and the light from the remaining region provides the maxima.

To calculate the first maximum after the central peak, divide the slit into three regions, for example. In the equation for destructive interference of two of the regions, the effective slit width for each region is $\frac{w}{3}$, and the path length difference is a half-wavelength as always. The location of the fringe is then

$$\frac{w}{3} \sin \theta = \frac{\lambda}{2}, \text{ or}$$

$$w \sin \theta = \frac{3}{2}\lambda \quad \text{single-slit diffraction: first bright fringes (constructive interference)}$$

You can calculate additional bright fringes by breaking the slit into 5, 7, 9, or any odd number of regions. The result is that maxima occur in general when

$$w \sin \theta_m = \left(m + \frac{1}{2}\right)\lambda \quad \text{single-slit diffraction: } m\text{th bright fringes (constructive interference)}$$

where $m = 1, 2, 3, \ldots$, and the values of θ that satisfy this equation give the angles of all bright fringes.

Figure 4 shows the intensity curve for single-slit diffraction. For each successive bright fringe, more of the slit acts as point sources of light that interferes destructively. The intensity of bright fringes therefore decreases away from the central peak.

Investigation 10.2.1

Diffraction of Light Using a Single Slit (page 545)

Now that you have learned about single-slit diffraction, perform Investigation 10.2.1 to study how changes in slit width, wavelength, and source distance affect the interference patterns.

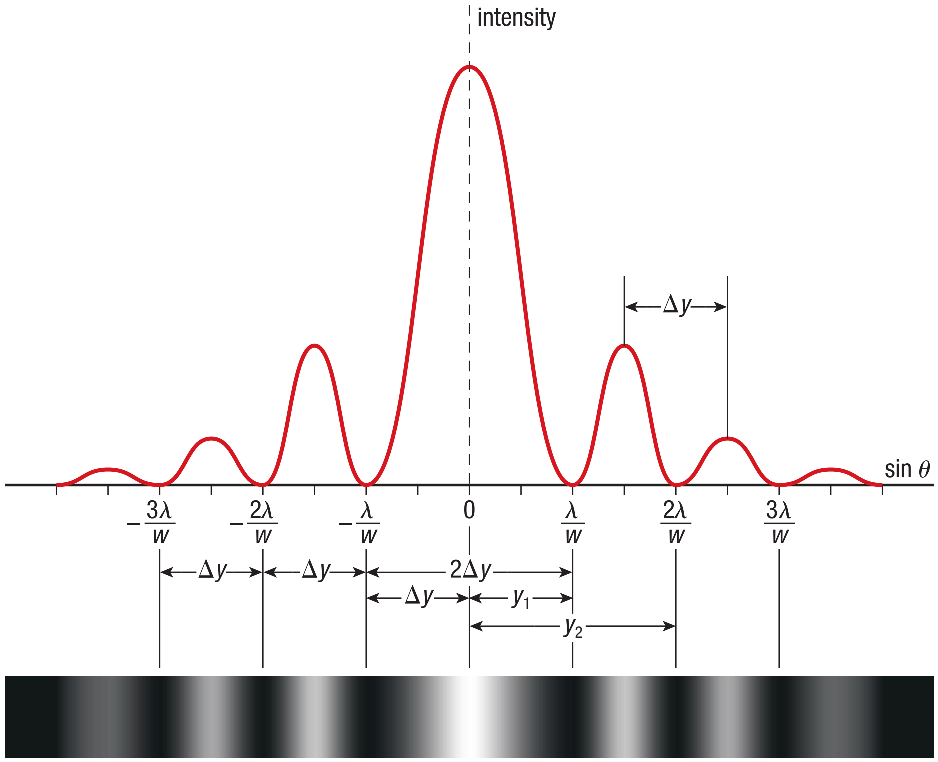


Figure 4 A full calculation of the intensity curve shows that the central bright fringe is about 20 times as intense as the bright fringes on either side.

You can approximate the width of the central bright fringe by using the angular separation of the first dark fringes on either side. You can calculate the angles for these first dark fringes by setting $n = 1$ to obtain

$$\sin\theta_1 = \frac{\lambda}{w}$$

Figure 5 shows the projection of the fringes on the screen. The distance y_1 of the first minimum from the central maximum (using $\sin\theta \approx \tan\theta$ for small angles) is then

$$y_1 = \frac{L\lambda}{w}$$

The width of the central maximum is the distance between first minima, $2y_1$, or

$$2\Delta y = 2\frac{L\lambda}{w}$$

The separation of the central maximum from the first minimum, $2\Delta y$, is half of this, or just y_1, which gives $\lambda = \dfrac{w\Delta y}{L}$. From the equation for the location of the minima, the distance from each minimum to the next one is also y_1. This is useful for predicting the positions of the fringes and the wavelength of light from the diffraction pattern. In particular, a longer wavelength or a smaller width produces a larger value of Δy and a wider diffraction pattern.

Figure 5 also shows the angles for the single-slit diffraction pattern. By determining the angle and distance from the screen, you can calculate the distances between fringes. The following Tutorial shows how to calculate the width of the central maximum of a single-slit diffraction pattern.

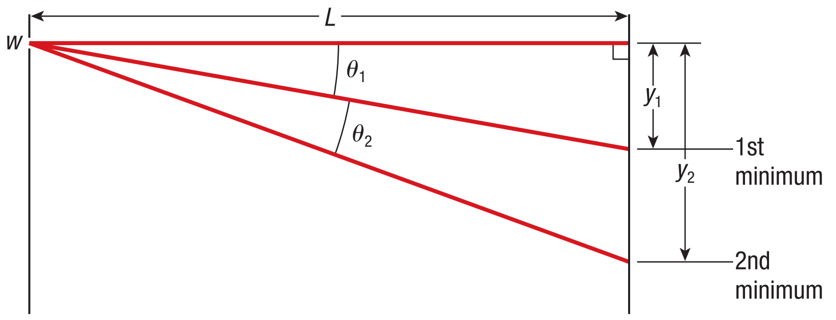


Figure 5 You can determine the value of $\sin\theta_1$ using the approximation $\sin\theta_1 \approx \tan\theta_1 = \dfrac{y_1}{L}$.

Tutorial 1 Analyzing Single-Slit Interference

In the following Sample Problems, you will analyze interference patterns produced during single-slit diffraction.

Sample Problem 1: Determining the Distance between Dark Fringes and the Central Maximum

Light with a wavelength of 5.40×10^2 nm is incident on a slit of width 11 μm and produces a diffraction pattern on a screen located 80.0 cm behind the slit. Calculate the distance of the first dark fringe from the central maximum on the screen.

Given: $\lambda = 5.40 \times 10^2\ \text{nm} = 5.40 \times 10^{-7}\ \text{m}$; $w = 11\ \mu\text{m} = 1.1 \times 10^{-5}\ \text{m}$; $L = 80.0\ \text{cm} = 0.800\ \text{m}$

Required: y_1

Analysis: The distance from the central maximum to the first minimum is $\Delta y = y_1$.

$$\lambda = \frac{w\Delta y}{L}$$

$$\Delta y = \frac{L\lambda}{w}$$

Solution: $\Delta y = \dfrac{L\lambda}{w}$

$$= \frac{(0.800\ \text{m})(5.40 \times 10^{-7}\ \cancel{\text{m}})}{1.1 \times 10^{-5}\ \cancel{\text{m}}}$$

$$\Delta y = 3.9 \times 10^{-2}\ \text{m}$$

Statement: The first dark fringe is 3.9 cm from the central maximum.

Sample Problem 2: Determining the Width of the Central Maximum

Light with a wavelength of 670 nm is incident on a slit of width 12 μm and produces a diffraction pattern on a screen that is 30.0 cm behind the slit.

(a) Calculate the angular width and the absolute width, in centimetres, of the central maximum.

(b) Calculate the distance between the first and second minima.

(c) Determine the distance between the second and third maxima. Compare this answer to your answer in (b). Does this answer make sense? Explain why or why not.

Solution

(a) **Given:** $n = 1$; $\lambda = 670\text{ nm} = 6.7 \times 10^{-7}\text{ m}$; $w = 12\ \mu\text{m} = 1.2 \times 10^{-5}\text{ m}$; $L = 30.0\text{ cm} = 0.300\text{ m}$

Required: θ_1; $2\Delta y$, the width of the central maximum

Analysis: The distance from the central maximum to the first minimum on each side is $\Delta y = y_1$. The angle for the first minimum is $\sin\theta_1 = \frac{\lambda}{w}$, where $n = 1$ and the angular width is $2\theta_1$. To calculate $2y_1$, use $\lambda = \frac{w\Delta y}{L}$.

Solution:
$$\begin{aligned}
\sin\theta_1 &= \frac{\lambda}{w} \\
&= \frac{6.7 \times 10^{-7}\ \cancel{\text{m}}}{1.2 \times 10^{-5}\ \cancel{\text{m}}} \\
&= 0.055\ 83 \\
\theta_1 &= 3.201^\circ \text{ (two extra digits carried)} \\
\lambda &= \frac{w\Delta y}{L} \\
2\Delta y &= 2\frac{L\lambda}{w} \\
&= \frac{2(0.300\text{ m})(6.7 \times 10^{-7}\ \cancel{\text{m}})}{1.2 \times 10^{-5}\ \cancel{\text{m}}} \\
2\Delta y &= 0.0335\text{ m (one extra digit carried)}
\end{aligned}$$

Statement: The angular width of the central maximum is 6.4°, corresponding to 3.4 cm on the screen.

(b) **Given:** $\lambda = 670\text{ nm} = 6.7 \times 10^{-7}\text{ m}$; $w = 12\ \mu\text{m} = 1.2 \times 10^{-5}\text{ m}$; $L = 30.0\text{ cm} = 0.300\text{ m}$

Required: distance between the first and second minima

Analysis: This is half the width of the central maximum in (a), since $\Delta y = y_1$
$$\begin{aligned}
2\Delta y &= 2y_1 \\
y_1 &= \frac{2\Delta y}{2}
\end{aligned}$$

Solution:
$$\begin{aligned}
y_1 &= \frac{2\Delta y}{2} \\
&= \frac{0.0335\text{ m}}{2} \\
y_1 &= 1.7\text{ cm}
\end{aligned}$$

Statement: The distance between the first and second minima is 1.7 cm.

(c) **Given:** $\lambda = 670\text{ nm} = 6.7 \times 10^{-7}\text{ m}$; $w = 12\ \mu\text{m} = 1.2 \times 10^{-5}\text{ m}$; $L = 30.0\text{ cm} = 0.300\text{ m}$

Required: distance between the second and third maxima

Analysis: This is half the width of the central maximum in (a);
$$y_1 = \frac{2\Delta y}{2}$$

Solution:
$$\begin{aligned}
y_1 &= \frac{2\Delta y}{2} \\
&= \frac{0.0335\text{ m}}{2} \\
y_1 &= 1.7\text{ cm}
\end{aligned}$$

Statement: The distance between the second and third maxima is 1.7 cm. Answers (b) and (c) are the same. This makes sense because the distance between adjacent maxima is identical to the distance between adjacent minima because the pattern is symmetrical.

Practice

1. Would the central maximum in Sample Problem 1 be wider or narrower if the slit and the screen were submerged in water? K/U
2. Helium–neon laser light, with a wavelength of 7.328×10^{-7} m, passes through a single slit with a width of 43 μm onto a screen 3.0 m from the slit. Calculate the separation of adjacent minima, other than those on either side of the central maximum T/I [ans: 5.1 cm]
3. Monochromatic light falls onto a slit 3.00×10^{-6} m wide. The angle between the first dark fringes on either side of the central maximum is 25.0°. Calculate the wavelength of the light. T/I [ans: 6.49×10^{-7} m]
4. The first dark fringe in a single-slit diffraction pattern is located at an angle of $\theta_a = 56^\circ$. With the same light, the first dark fringe formed with another single slit is located at $\theta_b = 34^\circ$. Calculate the ratio $\frac{w_a}{w_b}$. T/I [ans: 0.67]

Resolution

The **resolution** of an optical device is the device's ability to separate closely spaced objects into distinctly different images. Light passing through a small opening is diffracted, and smaller openings produce greater diffraction. When light from two objects passes simultaneously through the same opening, the light from both sources is diffracted. It produces overlapping diffraction patterns and the images appear fuzzy. If the opening is quite small and the objects are close to each other, distinguishing between the two images can be quite challenging (**Figure 6**).

resolution the ability of an optical device to separate close objects into distinct and sharp images

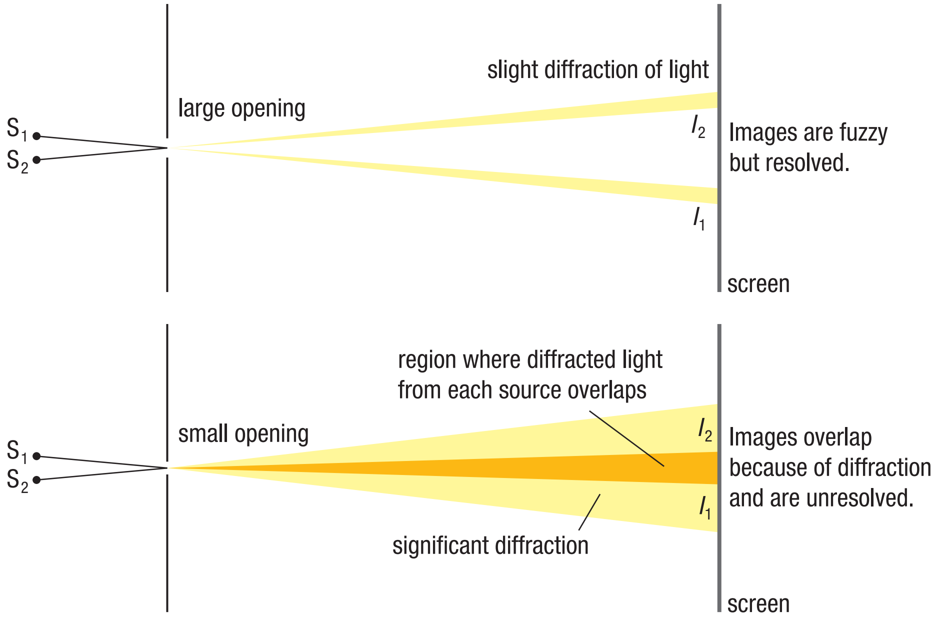


Figure 6 With a large opening, the images from two light sources are fuzzy but distinguishable. With a smaller opening, the images overlap and are difficult to distinguish.

In many optical devices, a small lens replaces the opening. The resolution of the device is then the ability of the lens to form two sharp images. When light passes through the lens, the diffraction leads to a single bright spot surrounded by concentric circles in a diffraction pattern called an Airy disc (**Figure 7**). Two small objects will each produce an image surrounded by diffraction fringes. If the bright parts of the diffraction pattern for the first image fall on the dark areas of the other image, the two images may be inseparable. Lenses and openings that are small relative to the wavelength of light usually produce lower-resolution images by increasing the effects of diffraction, as seen in **Figure 8** on the next page. But factors other than diffraction affect the resolution of lenses such as those on cameras.

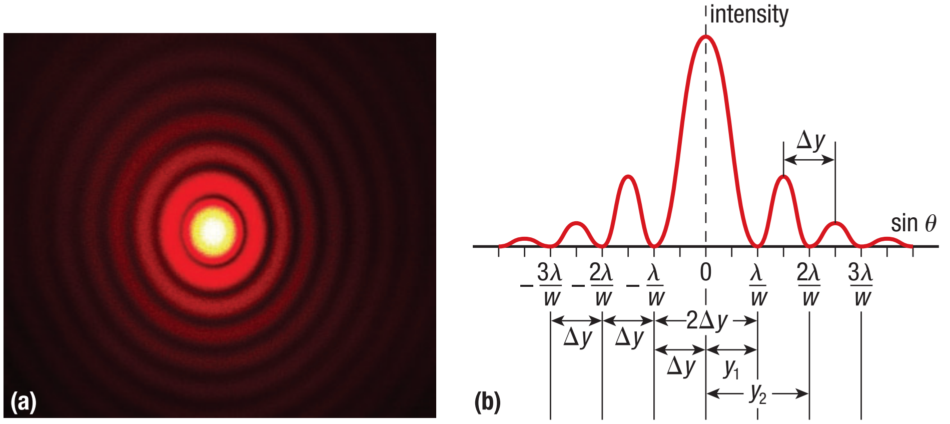


Figure 7 (a) The classical Airy disc is a pattern of concentric circles of light around a bright central spot. (b) The light intensity pattern of an Airy disc is similar to a single-slit diffraction pattern.

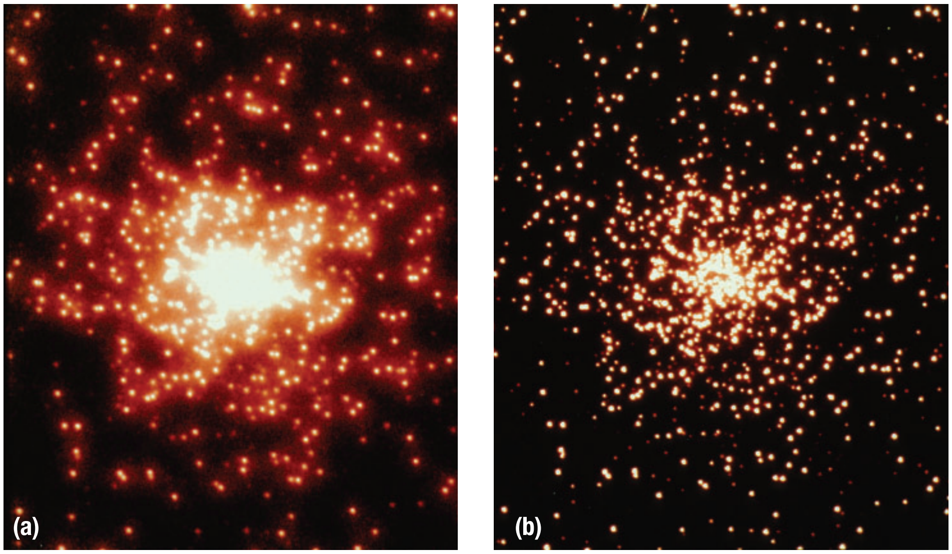


Figure 8 (a) When the resolution of an optical device is not sharp enough, stars that are in close proximity appear blurry. (b) When the resolution is sufficient, more individual stars can be clearly seen.

In an ideal set of optics, 84 % of the incident light is concentrated in the central maximum. If there are obstructions in the optical path, such as a mirror, more light will end up being spread into the rings, making the image less clear. This principle makes a refracting telescope the sharpest theoretical optical instrument for a given diameter. A refracting telescope uses a lens with no obstructions in the optical path. Reflecting telescopes use mirrors instead of lenses, but they often contain more than one mirror in the optical path.

Optical microscopes have small-diameter objective lenses. Due to diffraction, the best optical microscopes can typically achieve a magnification no greater than 1000 times. One way to improve resolution is to use shorter-wavelength light. Therefore, an ultraviolet microscope can achieve a much greater resolution than one that uses visible light.

Most observatory telescopes use curved mirrors (**Figure 9**). Astronomers use the telescopes with the largest mirrors to view the most distant objects, not only because they collect more light, but also because the large apertures reduce diffraction effects and offer greater resolution.

Figure 9 The telescope inside the Canada–France–Hawaii Observatory (left) detects visible light and infrared radiation.

10.2 Review

Summary

- Monochromatic light passing through a single slit produces a diffraction pattern that consists of a bright central region surrounded by alternating light and dark bands (maxima and minima) resulting from constructive and destructive interference.
- Fraunhofer diffraction is a special case of diffraction that shows distinctive differences between the central fringe and darker flanking fringes.
- In single-slit diffraction, the dark bands (minima) occur at angles θ_n that satisfy $\sin\theta_n = \frac{n\lambda}{w}$ $(n = 1, 2, \ldots)$
- In single-slit diffraction, the bright bands (maxima) occur at angles θ_m that satisfy $\sin\theta_m = \frac{(m + \frac{1}{2})\lambda}{w}$ $(m = 1, 2, \ldots)$
- The distance between successive minima, and successive maxima, is given by $\Delta y = \frac{\lambda L}{w}$, and the width of the central maximum is $2\Delta y$.
- The resolution of an optical instrument is its ability to separate closely spaced objects into distinctly different images limited by diffraction.

Questions

1. Light with a wavelength of 794 nm produces a single-slit diffraction pattern in which the ninth dark fringe lies at 6.48 cm from the direction of the central maximum. The distance to the screen is 1.0 m. Determine the width of the slit. T/I
2. Red light ($\lambda = 600.0$ nm) is diffracted by a single slit, and the first dark fringe occurs at $\theta_1 = 6.9°$. Determine the width of the slit. T/I
3. A slit of width 0.15 mm is located 10.0 m from a screen. Light with a wavelength of 450 nm passes through this slit. Determine the distance between the first and the third dark fringes on the screen. T/I
4. Light with a wavelength of 550 nm passes through a single slit and illuminates a screen 2.0 m away. The distance from the first dark fringe to the centre of the interference pattern is 5.5 mm. Determine the width of the slit. T/I
5. Light from a helium–neon laser ($\lambda = 630$ nm) passes through a single slit to a screen 3.0 m from the slit. The slit is 0.25 mm wide. What is the width of the central maximum on the screen? T/I
6. Yellow light from a vapour lamp passes through a single slit 0.0295 cm wide to a screen 60.0 cm away. A first-order dark fringe is 0.120 cm from the centre of the central maximum. Determine the wavelength of the yellow light. K/U T/I
7. Suppose monochromatic light incident on a single slit produces a diffraction pattern. K/U
   (a) How would the pattern differ if you doubled the wavelength of the light?
   (b) How would the pattern differ if you doubled both the wavelength and slit width at the same time?
8. Blue light passing through a single narrow slit forms an interference pattern. Determine how the spacing of the maxima would be different if you replaced blue light with green light. Explain your reasoning. K/U T/I C
9. A doorway acts as a single slit for light that passes through it, but a doorway is much larger than the wavelength of visible light, so the diffraction angles are very small. Calculate the angle of the tenth minimum for a typical doorway. K/U T/I
10. Digital images consist of pixels, or small coloured dots. If an image is 84 pixels wide by 62 pixels high, how could you achieve better resolution? (Hint: Consider aperture and distance.) K/U T/I A
11. The star Mizar is actually a double star that only appears as two distinct stars when resolved by a telescope. Explain why. K/U T/I C
12. Explain how a single-slit interference pattern differs from a double-slit interference pattern. K/U T/I C

10.3 The Diffraction Grating

diffraction grating a device with a large number of equally spaced parallel slits that produces interference patterns

It is difficult to measure the wavelength of light accurately using the interference pattern from either a double slit or a single slit. The interference pattern may be dull or the resolution fuzzy. To solve these problems, most researchers use a **diffraction grating**, which is a device that has an array of many parallel slits. White light is a mixture of light of different wavelengths, so as it passes through the slits in the diffraction grating, the waves originating from the slits interfere. The interference produces light of various wavelengths that travel along different paths (**Figure 1**). This creates an effect similar to passing white light through a prism.

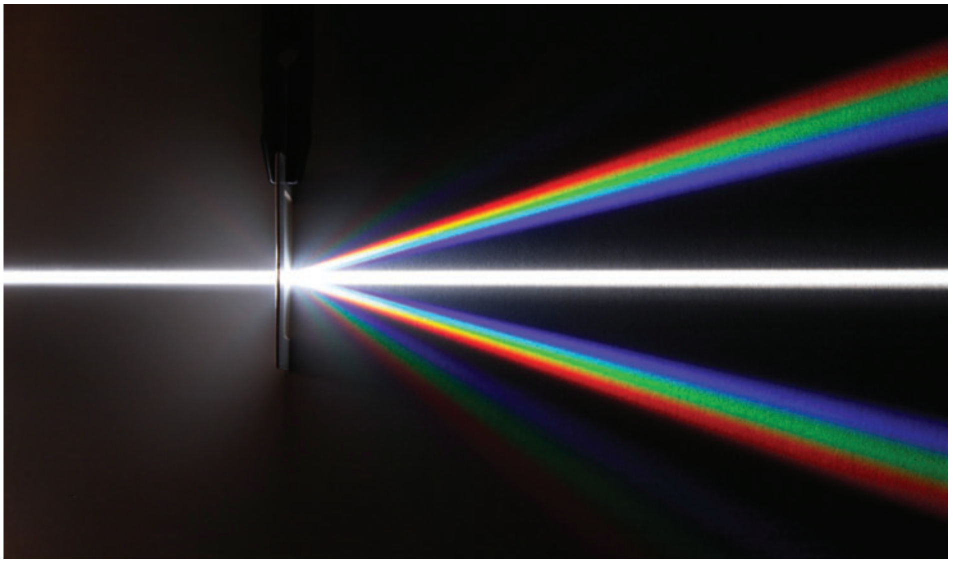


Figure 1 A diffraction grating splits white light into light of different wavelengths that travel along different paths. The effect shown here is due to interference, not dispersion.

Diffraction Gratings

There are two types of diffraction gratings: transmission gratings, which transmit light, and reflection gratings, which reflect light. CDs and DVDs are common examples of a reflection grating. When illuminated, they both produce iridescent reflections. A transmission grating, however, usually has an anti-reflection coating. Transmission gratings are typically used in spectroscopy. Most of the discussion here concerns transmission gratings, but the same concepts apply to both types.

The process of manufacturing an effective transmission grating involves precision machinery. One method uses a diamond tip to etch closely spaced parallel lines on the grating surface. The lines are opaque, and the transparent spaces between them serve as the slits. Newer photographic methods use interference from lasers to produce the pattern on photographic film. Then, the film is processed to produce the parallel lines. A typical diffraction grating might have 10 000 lines per centimetre. CAREER LINK

In Chapter 9, you learned that when coherent monochromatic light passes through a double slit, it produces a pattern of alternating bright and dark fringes on a screen located far from the slits. The bright fringes (maxima) occur in directions for which the path length from slits to screen are whole-number multiples of the wavelength of light used. In these directions, light waves arrive in phase and interfere constructively.

Three equally spaced slits would produce the same type of interference pattern for the same reasons. Consider what happens when light of wavelength λ passes through a large number, N, of equally spaced slits, as in a diffraction grating and as shown in **Figure 2**. The spacing between the slits is represented by w.

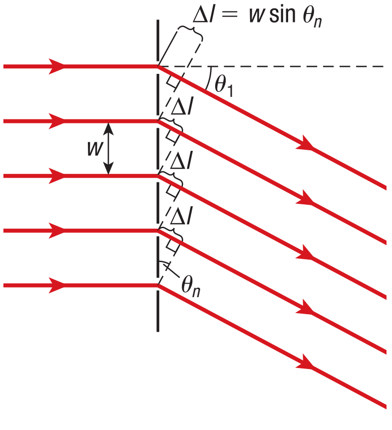


Figure 2 Light waves pass through a diffraction grating to produce a constructive interference pattern.

For an angle $\theta = 0°$, all of the waves arrive in phase, with a maximum in intensity. This is the **zero-order maximum**, which is the same for all wavelengths of light.

zero-order maximum the location of maximum intensity in the diffraction pattern at $\theta = 0°$

The next maximum occurs at an angle θ_1, where the path length difference between successive slits is exactly λ. The waves are again in phase when they reach the screen. If you apply trigonometry to Figure 2, you will see that the path length difference, Δl, is equal to $w \sin \theta_1$. The first maximum of intensity, called the **first-order maximum**, occurs when this path is equal to one wavelength, according to the equation

first-order maximum the first maximum of intensity on either side of the zero-order maximum in an interference pattern from a diffraction grating

$$\lambda = w \sin \theta_1$$

This condition is exactly the same as the condition for the first maximum for the double slit. The result is constructive interference at an angle θ from each slit and a bright maximum in that direction.

For each whole number m, the angle θ_m must satisfy the following condition:

$$m\lambda = w \sin \theta_m$$

where $m = 0, 1, \ldots.$

This corresponds to a path that differs by a whole-number multiple of a wavelength. Again, the waves arrive in phase at this angle and interfere constructively, resulting in a maximum called the mth-order maximum. Here, m is called the **order number**. At angles between maxima, the waves from each slit differ in phase and interfere to produce relatively wide dark areas on the screen. The overall result is a pattern of extremely narrow maxima.

order number the value of m for a given maximum in a diffraction-grating interference pattern; sequentially numbers the maxima on either side of the zero-order maximum

Figure 3 compares the interference patterns for different numbers of slits. The same equation describes where the maxima occur for a single given slit separation w, so the maxima are all at the same angle. As the number of slits increases, each maximum becomes narrower. Since a typical diffraction grating has thousands of slits, the maxima it produces are in precisely defined directions. In addition, since the separation between slits is typically quite small, the maxima are widely separated from each other.

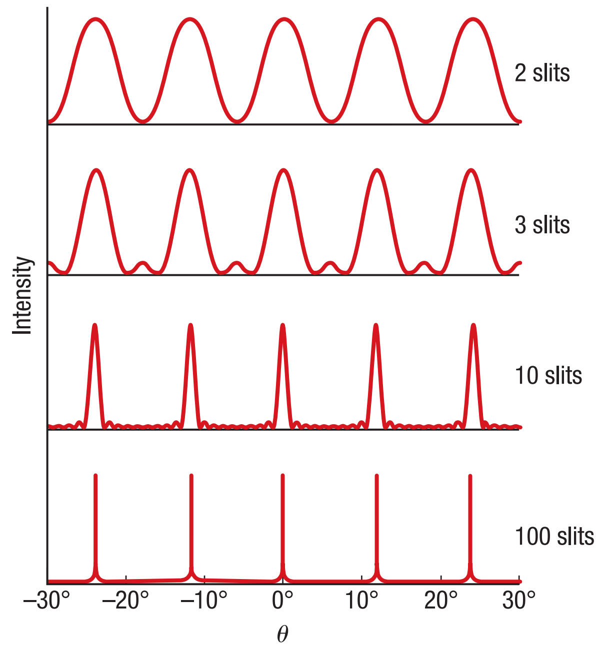


Figure 3 As the number of slits increases, the maxima become narrower and more sharply peaked. The resulting patterns of bright and dark lines are called diffraction fringes.

Using Gratings as a Spectrometer

When light of different wavelengths is incident on a diffraction grating, each wavelength produces diffraction peaks in different directions. This makes a diffraction grating a powerful tool for separating light of different wavelengths.

Figure 4 shows what happens when a mixture of green and red light passes through a diffraction grating and then onto a screen behind it. The resulting spots on the screen are circular because the deflected beam is circular (Figure 4(b)).

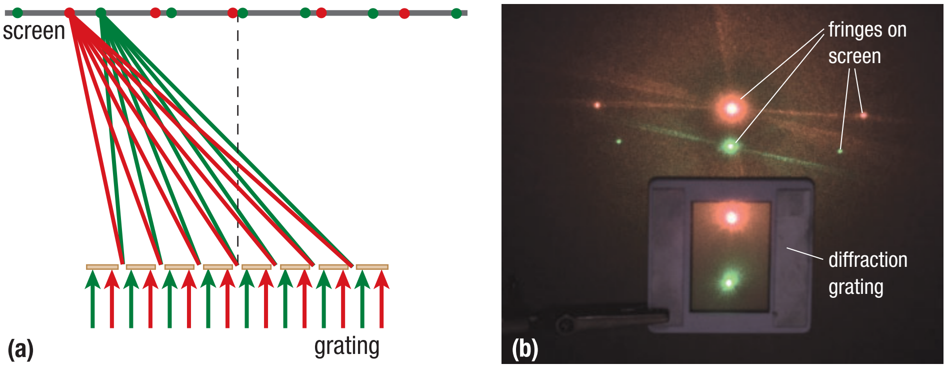


Figure 4 (a) Red and green light diffract at different angles. (b) The resulting interference pattern is seen on the screen.

Figure 4(a) also shows that the angles at which maxima occur have a simple and precise relation to the wavelength of the light. This behaviour of the diffraction grating makes it a powerful tool for precisely measuring wavelengths of light.

The spectrometer shown in **Figure 5** is a device for measuring wavelengths of light. Light from a source passes through the slit and into the collimator. A collimator is a system of mirrors or lenses that produces parallel wave fronts. The light then passes through a diffraction grating and onto a telescope. The telescope produces an image of the slit that appears as a line formed from the given wavelength of light. The observer positions the telescope so that the crosshairs mounted in it fall on the slit image. The observer then reads the angle from the scale below the telescope. Since the number of lines per centimetre and therefore the line spacing for the diffraction grating being used is known, the wavelength of light can be calculated from the measured angle. Astronomers use spectrometers to identify elements in space and on other planets. CAREER LINK

Investigation 10.3.1

CD and DVD Storage Capacity (page 547)
In Investigation 10.3.1, you will use a CD and a DVD as a diffraction grating and determine the groove spacing of each. You will use your data to assess which disc can hold more data.

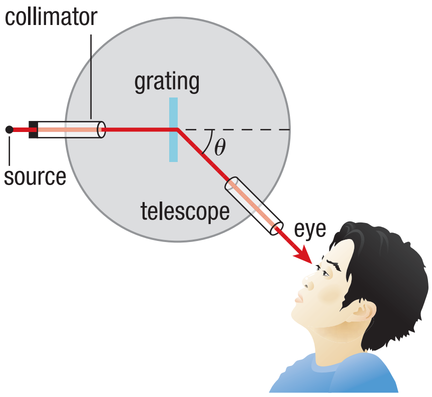


Figure 5 You can use a grating mounted in a spectrometer to measure the angles of interference maxima.

The Tutorial on the next page models how to locate and number the maxima produced by a diffraction grating.

Tutorial 1 Locating Maxima on a Diffraction Grating

The following Sample Problems show how to determine the location and number of maxima produced by a given diffraction grating.

Sample Problem 1: Determining the Maxima for a Diffraction Grating

Light with a wavelength of 540 nm is incident on a diffraction grating that has 8500 lines/cm. Calculate the angles of the maxima.

Given: $\lambda = 540 \text{ nm} = 5.4 \times 10^{-7}$ m; $N = 8500$ lines/cm

Required: the angle, θ_m, giving the locations of the mth-order maxima for $m = 1, 2, \ldots$, etc.

Analysis: The equation $w = \frac{1}{N}$ can be used to calculate the slit separation from the number of lines. Then use the equation $m\lambda = w \sin\theta_m$ to locate the maximum for each value of m.

Solution: $w = \frac{1}{N}$

$$= \frac{1}{8500 \text{ lines/cm}} \times \frac{1 \text{ m}}{100 \text{ cm}}$$

$$w = 1.176 \times 10^{-6} \text{ m (two extra digits carried)}$$

For the first-order maximum, $m = 1$:

$$m\lambda = w \sin\theta_1$$
$$(1)(5.4 \times 10^{-7} \text{ m}) = (1.176 \times 10^{-6} \text{ m}) \sin\theta_1$$
$$\sin\theta_1 = 0.4591$$
$$\theta_1 = 27^\circ$$

For the second-order maximum, $m = 2$:

$$m\lambda = w \sin\theta_2$$
$$(2)(5.4 \times 10^{-7} \text{ m}) = (1.176 \times 10^{-6} \text{ m}) \sin\theta_2$$
$$\sin\theta_2 = 0.9180$$
$$\theta_2 = 67^\circ$$

Performing the same calculation for $m = 3$ to determine the third-order maximum leads to 1.377 as the value required for $\sin\theta_3$. Since the sine of an angle can never be greater than 1, no third-order maximum exists.

Statement: The first-order maximum is at 27°, the second-order maximum is at 67°, and no other maximum is possible.

Sample Problem 2: Calculating Angles of Diffraction in a Diffraction Grating

Light emitted by a particular source is incident on a diffraction grating with 9000 lines/cm and produces a first-order maximum at 32.0°. Determine the wavelength of the light.

Given: $N = 9000$ lines/cm; $\theta_1 = 32.0^\circ$; $m = 1$

Required: λ

Analysis: Use $w = \frac{1}{N}$ to determine the slit separation, w. Then use the equation $m\lambda = w \sin\theta_m$ to determine the wavelength from w and the given angle, θ_1.

Solution: $w = \frac{1}{9000 \text{ lines/cm}} \times \frac{1 \text{ m}}{100 \text{ cm}}$

$$w = 1.111 \times 10^{-6} \text{ m (one extra digit carried)}$$

For the first-order maximum, $m = 1$:

$$\lambda = \frac{w \sin\theta_1}{m}$$
$$= (1.111 \times 10^{-6} \text{ m}) \sin 32.0^\circ$$
$$\lambda = 5.89 \times 10^{-7} \text{ m}$$

Statement: The light has a wavelength of 589 nm.

Practice

1. Consider two diffraction gratings, one with 10 000 lines/cm and one with 8500 lines/cm. Compare the separations between adjacent maxima for these two gratings. K/U T/I C
2. Calculate the angular separation of successive maxima of the same colour when light with a wavelength of 660 nm is incident on a diffraction grating with 8500 lines/cm. T/I A [ans: 34°]
3. A diffraction grating produces a third-order bright fringe at an angle of 22.0° for red light with a wavelength of 694.3 nm. Calculate the number of lines per centimetre on the grating. T/I [ans: 1798 lines/cm]

UNIT TASK **BOOKMARK**

You can apply what you have learned about diffraction gratings to the Unit Task on page 556.

CDs and DVDs as Diffraction Gratings

When you move a CD or a DVD under white light, you can see that the data side produces a spectrum of colours that change as you move it. The disc has a reflective surface and a long, microscopically thin track that spirals thousands of times around the disc from the centre to the edge. Light reflected from adjacent edges of the spiral track interferes in the same way as light interferes from slits in a reflection diffraction grating. Interference between waves from the track edges leads to a diffraction pattern. Different wavelengths contained in the white light have interference peaks in different directions, giving the colour pattern that you see.

CDs and DVDs take advantage of destructive interference to store data. The disc-manufacturing process uses a sharply focused laser beam that burns microscopic, quarter-wavelength-deep pits at precise intervals along the length of the spiral track. Then, a reflective coating is applied to the entire disc, including the track and its pits. Upon playback, the player rotates the disc and shines a sharply focused, low-power laser beam on the track. When the beam hits the leading or trailing edge of a pit, the light briefly reflects from both the pit and the undisturbed surface of the track (called the "land"), as shown in **Figure 6**.

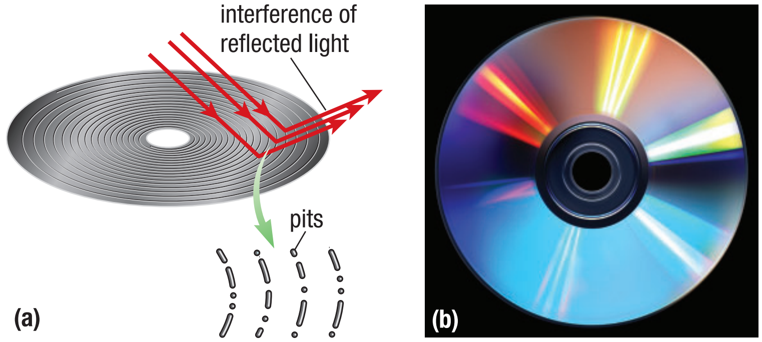


Figure 6 Interference of light reflected from nearby tracks in (a) produces the pattern of colours seen from the CD in (b).

Since the pit is a quarter-wavelength deeper than the land, the light wave that the pit reflects is half a wavelength out of phase with the wave that the adjacent land reflects. The two reflected waves interfere destructively, producing a momentary decrease in the intensity of the light reflecting from the track. A photodetector monitoring the reflected light detects these intensity changes, and the player converts them into usable data.

CDs use a near-infrared 780 nm laser, whereas DVDs use shorter wavelengths of 635 nm. The DVD's shorter wavelength allows for a smaller track separation, a smaller pit depth, and a smaller pit length. As a result, a DVD can store much more data than a CD on the same size disc. WEB LINK

Research This

Blu-ray Technology

Skills: Researching, Analyzing, Communicating — SKILLS HANDBOOK A4.1

DVD and CD players use the interference between two reflected beams to read CDs and DVDs. The interference depends on the relation between the wavelength and the difference in height between a pit and the land in the reflecting surfaces. A newer technology is called Blu-ray.

1. Research Blu-ray technology.

A. Why is the technology called Blu-ray? K/U

B. How does the technology work? K/U

C. What can a Blu-ray recording do that is superior to a CD or a DVD recording? K/U A

D. What enables Blu-ray to accomplish these improvements? K/U

E. Analyze, assess, and compare CD, DVD, and Blu-ray technologies in a visual format. Describe any disadvantages, hazards, and concerns associated with Blu-ray technologies. T/I C A

WEB LINK

10.3 Review

Summary

- A diffraction grating consists of a large number of closely spaced parallel slits.
- Diffraction gratings produce interference patterns that are similar to those from a double slit, but the maxima are far narrower and more intense.
- The angle, θ_m, for the mth-order maximum of a diffraction grating with slit spacing w and light wavelength λ is given by $m\lambda = w \sin \theta_m$.
- The colours that you can see in a CD or a DVD result from interference similar to the interference of light from a diffraction grating.

Questions

1. When a CD reflects white light, the result is a rainbow-like display of different colours. Explain what this indicates about the surface of the CD. K/U C A
2. A diffraction grating has 2800 lines/cm. Determine the distance between two lines in the grating. T/I A
3. Light incident on a diffraction grating with 10 000 lines/cm produces first-order, second-order, and third-order maxima at angles of 31.2°, 36.4°, and 47.5°, respectively. Determine the wavelength, in nanometres, of light that produces each maximum. T/I A
4. A square diffraction grating of width 2.0 cm contains 6000 slits. At what angle does blue light with a wavelength of 450 nm produce the first intensity maximum? T/I A
5. Red light with a wavelength of 600.0 nm is incident on a diffraction grating with a slit spacing of 25 μm. At what angle from $\theta = 0°$ is the first-order maximum in intensity? T/I A
6. Light with a wavelength of 780 nm from a laser pointer is incident on a diffraction grating with a screen located 10 m behind it. The maxima near $\theta = 0°$ are spaced 0.50 m apart. Determine the spacing between the lines in the diffraction grating. T/I A
7. In an experiment, light is reflected on a diffraction grating that has 300 lines/cm, and the diffraction grating is 0.84 m from a screen. The distance between the $m = 0$ and $m = 3$ bright fringe is 3.6 cm. Calculate the wavelength of the light. K/U T/I A
8. Determine the maximum order number possible in an interference pattern when light with a wavelength of 5.4×10^{-7} passes through a diffraction grating with 3000 lines/cm. T/I
9. The molecular planes in a crystal act as a diffraction grating when X-rays are incident on the crystal (**Figure 7**). The molecular planes in a crystal are 0.50 nm apart, and the X-rays have a wavelength of 0.050 nm. K/U T/I C A

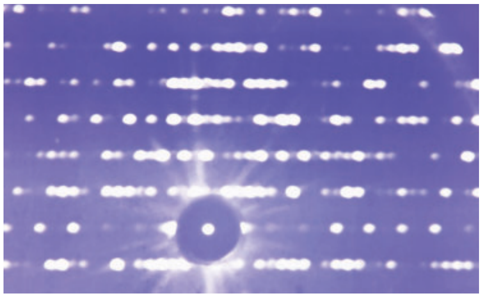


Figure 7 The white spots show the diffraction pattern as X-rays pass through a protein. From the pattern, scientists can determine the structure of the protein.

   (a) Assume that the maxima in the diffraction pattern are given by the same equation that applies for a grating with slits with the same value of w. Determine the angles for the first three maxima.
   (b) Assume that light with a wavelength of 600 nm is incident on the crystal instead of X-rays. At what angle is the first bright fringe?
   (c) What does this show about the prospects of using visible light for diffraction by crystals? Explain your answer.

10. Light with a wavelength of 5.00×10^2 nm produces a first-order maximum at an angle of 20.0° in a specific spectroscope. When the measurement is repeated with the same spectroscope on a distant star that is known to have a planet in orbit about it, the same light produces a first-order maximum at 18.0°. Determine the index of refraction of the atmosphere on the planet as it passes in front of its host star and the star's spectrum is analyzed. K/U T/I A

10.4 Electromagnetic Radiation

In the early decades of the nineteenth century, discoveries by scientists such as Hans Christian Oersted, André-Marie Ampère, and Michael Faraday established the basic relationship between electric and magnetic fields. Other scientists then worked on extending electromagnetic radiation theory to related phenomena. **Electromagnetic radiation** consists of interacting electric and magnetic fields that travel at the speed of light. Some examples are visible light, X-rays, infrared light, ultraviolet light, and gamma rays. One result of this work was the discovery of how electromagnetic waves could travel through space.

electromagnetic radiation radiation that consists of interacting electric and magnetic fields that travel at the speed of light

Faraday's law describes how a changing magnetic field by itself can produce an electric field. Many physicists wondered if the reverse were possible, whether an electric field by itself could produce a magnetic field. Scottish physicist and mathematician James Clerk Maxwell (**Figure 1**) hypothesized that this was possible. He proposed that a changing electric field produces a magnetic field. Maxwell expressed his theory of electromagnetism in four equations, generally called Maxwell's equations. The key ideas that his equations express are the following:

Figure 1 James Clerk Maxwell formulated the known laws of electromagnetism into a theory that led to the prediction of electromagnetic waves.

- Electric charges in space produce an electric field, and currents produce a magnetic field.
- Magnetic field lines form continuous closed loops that have neither a beginning nor an end. Electric field lines always begin and end on charges.
- A changing electric field produces a magnetic field.
- A changing magnetic field produces an electric field.

The symmetry of the relationship between electric and magnetic fields led Maxwell to an important conclusion. In 1864, he predicted the existence of repeated motions, or oscillations, between electric and magnetic fields. Moreover, he believed that these would take the form of electromagnetic waves travelling through space. Maxwell devised a series of equations that allowed him to calculate the speed of propagation of these oscillating electric and magnetic fields. From these equations, he was able to determine that the waves travelled at the speed of light. Heinrich Hertz confirmed these predictions in 1887 when he discovered radio waves and verified that they obey Maxwell's equations.

The changing fields of an electromagnetic wave can be produced by an accelerating electric charge. In practice, this could be from an oscillating current in a coil or from charged particles that change velocity in some other way. If periodic motion produces the changing fields, the frequency of the electromagnetic wave is the same as the frequency of the current producing it.

Faraday's law explains one important property of these waves. According to Faraday's law, the direction of the electric field, $\vec{\varepsilon}$, is perpendicular to the direction of the magnetic field, $\vec{B}$, that produced it. Similarly, the magnetic field produced by a changing electric field is perpendicular to the electric field that produced it. Therefore, the electric and magnetic fields in an electromagnetic wave are perpendicular to each other (**Figure 2**). In addition, the direction of travel, or propagation, of an electromagnetic wave is perpendicular to both $\vec{\varepsilon}$ and $\vec{B}$, and the two fields oscillate in phase.

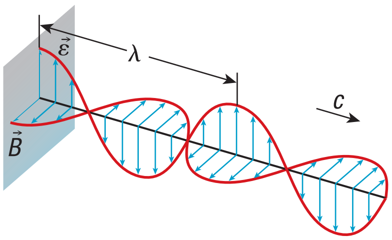


Figure 2 In an electromagnetic wave, the electric and magnetic fields are perpendicular to each other and to the direction of propagation. The fields oscillate in phase with each other.

The three central properties of electromagnetic waves are as follows:

- An electromagnetic wave has both an electric field and a magnetic field.
- The electric and magnetic fields are perpendicular to each other and oscillate in phase.
- The direction of propagation of the wave is perpendicular to both $\vec{\varepsilon}$ and $\vec{B}$.

The Electromagnetic Spectrum

All electromagnetic waves travel through a vacuum at the speed of light. Researchers have measured this speed, designated by the letter c, to be $2.997\ 924\ 58 \times 10^8$ m/s. The measurements are so accurate that, as of 1983, the speed of light is now used to define the length of the metre. In this textbook, though, we use the number 3.0×10^8 m/s.

Researchers classify electromagnetic waves according to their wavelength, λ, and frequency, f. The wavelength and frequency are related to each other through the universal wave equation, $v = f\lambda$. The universal wave equation applies to electromagnetic waves as well as to mechanical waves. However, for electromagnetic waves, we use c to represent speed instead of v. A longer wavelength corresponds to a lower frequency. **Figure 3** shows the frequencies and wavelengths of the parts of the **electromagnetic spectrum**, which is the range of all possible electromagnetic waves. The lowest and highest frequencies have no specific lower or upper limit. The assigned names in different frequency ranges reflect how the waves are typically generated and how they interact with matter.

electromagnetic spectrum the range of frequencies and wavelengths of all electromagnetic waves

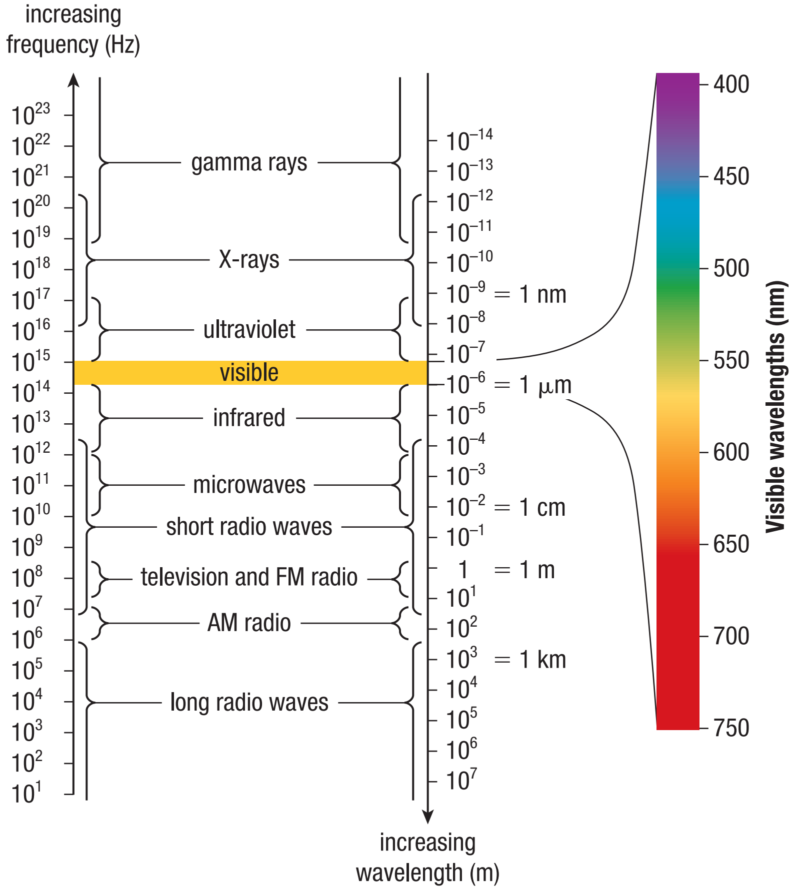


Figure 3 In this diagram of the electromagnetic spectrum, wavelength increases from top to bottom and frequency increases from bottom to top. However this is arbitrary. Some diagrams of the electromagnetic spectrum show the opposite.

Radio waves of all types have frequencies from as low as a few hertz to over 10^{12} Hz. The corresponding wavelengths vary from about 10^8 m to less than a millimetre. Radio waves are usually produced by an alternating-current (AC) circuit attached to an antenna. AC current in an antenna oscillates with time. The moving charges in the antenna accelerate as they oscillate. In general, when an electric charge accelerates, it produces electromagnetic radiation. The frequency of a radio wave generated in this way is equal to the frequency of the current.

Microwaves have frequencies between about 10^9 Hz and 10^{12} Hz. The corresponding wavelengths vary from a few centimetres to a few tenths of a millimetre. Microwave radiation interacts strongly with water molecules. Since most foods contain water, microwaves can be quite useful in the kitchen. Microwave ovens generate radiation with a frequency near 2.5×10^9 Hz. Water molecules readily absorb the energy transmitted by waves of this frequency. As the water molecules absorb this energy, they begin to vibrate. Molecular motion, including vibration, involves the transformation of potential energy within the molecule into thermal energy. That is the way in which microwaves heat food.

Infrared radiation falls in the frequency range from around 10^{12} Hz to about 4×10^{14} Hz (wavelengths from a few tenths of a millimetre to less than a micrometre). Satellite cameras that are sensitive to infrared radiation are useful for monitoring Earth's weather (**Figure 4**). Infrared radiation interacts strongly with molecules and is absorbed by most substances. Vibrating molecules can produce infrared radiation, as can electrons undergoing energy transitions within an atom.

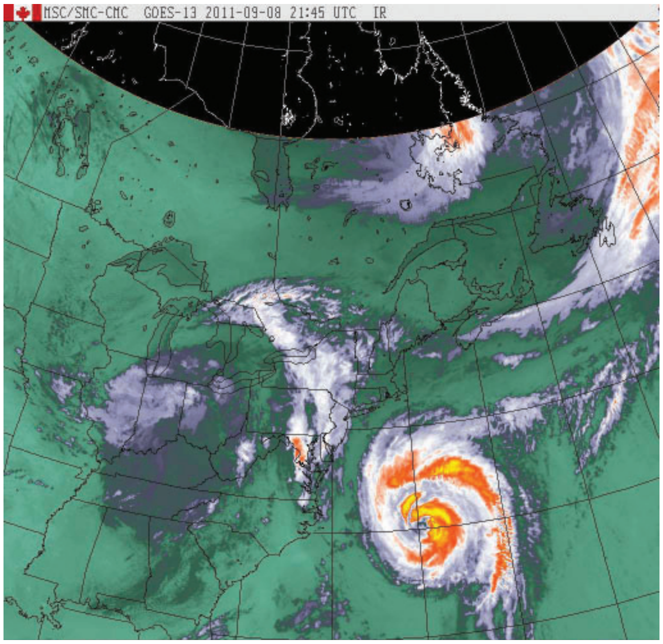


Figure 4 The computer-generated colours correspond to different temperatures. The red, for example, corresponds to higher temperatures.

Figure 5 Laser light, such as in this keychain laser, is monochromatic.

Visible light is the part of the electromagnetic spectrum that we can detect with our eyes. It has a narrow frequency range from about 4×10^{14} Hz to near 8×10^{14} Hz, corresponding to wavelengths from about 750 nm to 400 nm. The colour of visible light varies with its frequency. Light in the lowest frequency range (with the longest wavelengths) appears red. Increasing frequencies correspond in sequence to red, orange, yellow, green, blue, indigo, and violet. White light is a mixture of all these colours of light. Many sources of visible light, such as incandescent light bulbs, produce radiation with a range of frequencies. Some sources, like lasers, produce light of only a single wavelength (**Figure 5**).

Ultraviolet (UV) light has frequencies from about 8×10^{14} Hz to 10^{17} Hz, corresponding to wavelengths from about 3 nm to 400 nm. Ultraviolet radiation stimulates the production of vitamin D in the human body. Nearly all other biological effects of UV radiation are harmful. Excessive exposure to UV light can cause sunburn, skin cancer, and cataracts in the eyes. At shorter wavelengths, the damage to exposed human skin increases. The UV portion of the spectrum is commonly subdivided into several regions, including UV-A, UV-B, and UV-C. UV-C radiation has the highest frequency and the greatest potential for damaging tissue because it has the highest energy. Fortunately, ozone in Earth's atmosphere absorbs UV radiation more strongly at shorter wavelengths, so most UV-C radiation from the Sun does not reach Earth's surface.

X-rays have frequencies of approximately 10^{17} Hz to 10^{20} Hz, with wavelengths of about 0.01 nm to 10 nm. German physicist Wilhelm Röntgen discovered X-rays in 1895 and found that he could use them to form images from inside living tissue. X-rays passing through soft tissue blacken a photographic plate when the plate is processed, while the places where bones absorb the X-rays remain relatively clear. X-rays can show bone fractures, such as in **Figure 6**. All X-ray images used this method until the 1970s, when researchers developed a technique called computed axial tomography (CAT or CT). A CT scan takes many X-ray images at many different angles and then uses sophisticated computer analysis to combine these images into a three-dimensional representation of the object. This allows medical professionals to accurately measure the shape and size of bones, tumours, and so on within the body. Although X-ray imaging is valuable, excessive exposure to X-ray radiation can be harmful. At high exposures, X-rays can damage DNA molecules, which can then cause cancer and other serious diseases.

Gamma rays lie at the high-frequency end of the electromagnetic spectrum, with frequencies above about 10^{20} Hz and wavelengths less than about 10^{-12} m. No precise boundary exists between the X-ray and gamma ray parts of the spectrum. Energy transformations that occur within the nucleus of an atom, such as nuclear fission in nuclear power plants and nuclear fusion in the Sun, produce gamma rays. Gamma rays also reach Earth from sources outside our solar system (**Figure 7**).

Figure 6 Doctors use X-rays as an imaging tool to show exactly where bone fractures occur.

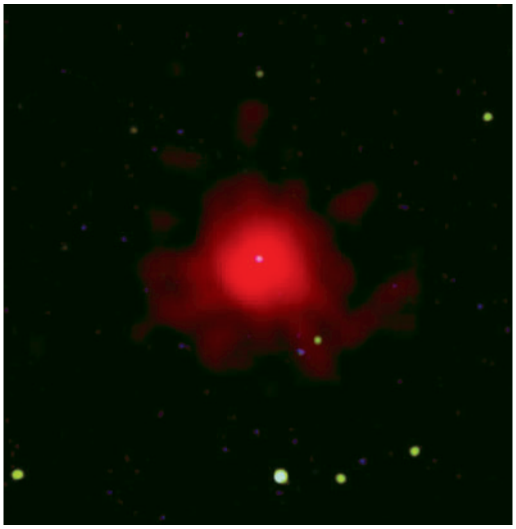


Figure 7 Supernovas are one source of gamma rays that reach Earth.

UNIT TASK **BOOKMARK**

You can apply what you have learned about electromagnetic radiation to the Unit Task on page 556.

The following Tutorial shows you how to calculate the frequency, wavelength, and energy of electromagnetic waves.

Tutorial 1 Determining Characteristics of Electromagnetic Waves

Sample Problem 1: Calculating the Frequency of Electromagnetic Waves

Microwaves with a wavelength of 1.5 cm carry television signals using a sequence of relay towers.

(a) Determine the frequency of the microwaves.

(b) How much time does it take for a microwave signal to travel 5.0×10^3 km across Canada from St. John's, Newfoundland, to Victoria, British Columbia?

Solution

(a) **Given:** $\lambda = 1.5 \text{ cm} = 1.5 \times 10^{-2}$ m

Required: f

Analysis: $\lambda f = c$

$$f = \frac{c}{\lambda}$$

Solution: $f = \dfrac{c}{\lambda}$

$$= \frac{3.0 \times 10^8 \cancel{\text{m}}/\text{s}}{1.5 \times 10^{-2} \cancel{\text{m}}}$$

$$f = 2.0 \times 10^{10} \text{ s}^{-1}$$

Statement: The frequency of the microwaves is 2.0×10^{10} Hz.

(b) **Given:** $d = 5.0 \times 10^3 \text{ km} = 5.0 \times 10^6$ m

Required: Δt

Analysis: $d = c\Delta t$

$$\Delta t = \frac{d}{c}$$

Solution: $\Delta t = \dfrac{d}{c}$

$$= \frac{5.0 \times 10^6 \cancel{\text{m}}}{3.0 \times 10^8 \cancel{\text{m}}/\text{s}}$$

$$\Delta t = 1.7 \times 10^{-2} \text{ s}$$

Statement: The time required for the microwave signal to travel from St. John's to Victoria is 1.7×10^{-2} s.

Sample Problem 2: Analyzing the Energy of Electromagnetic Waves

An electromagnetic wave in the form of an X-ray transfers its energy to an electron to change its state. The energy of the electromagnetic wave is proportional to its frequency. Suppose the X-ray has a wavelength of 0.025 nm. How does the energy of the electron transition compare with that of 540 nm visible light?

Given: $\lambda_{\text{visible}} = 540$ nm; $\lambda_{\text{X-ray}} = 0.025$ nm

Required: the ratio of energies, $\dfrac{E_{\text{X-ray}}}{E_{\text{visible}}}$

Analysis: The energy of the electromagnetic wave is proportional to its frequency:

$$\frac{E_{\text{X-ray}}}{E_{\text{visible}}} = \frac{f_{\text{X-ray}}}{f_{\text{visible}}}$$

Substitute the equation for wavelength and frequency into that for the ratio of energies:

$$\frac{E_{\text{X-ray}}}{E_{\text{visible}}} = \frac{\dfrac{c}{\lambda_{\text{X-ray}}}}{\dfrac{c}{\lambda_{\text{visible}}}}$$

$$\frac{E_{\text{X-ray}}}{E_{\text{visible}}} = \frac{\lambda_{\text{visible}}}{\lambda_{\text{X-ray}}}$$

Solution: $\dfrac{E_{\text{X-ray}}}{E_{\text{visible}}} = \dfrac{540 \text{ nm}}{0.025 \text{ nm}}$

$$\frac{E_{\text{X-ray}}}{E_{\text{visible}}} = 2.2 \times 10^4$$

Statement: The ratio of X-ray energy to visible light energy is 2.2×10^4 to 1 for the electron transition.

Practice

1. An FM station broadcasts at 107.1 MHz. Calculate the wavelength of its signal. T/I [ans: 2.8 m]
2. A particular X-ray machine produces X-rays with a frequency of 3.00×10^{17} Hz. Calculate the wavelength of the X-rays. T/I A [ans: 1.0×10^{-7} cm]
3. A helium–neon laser emits light with a wavelength of 638 nm. Calculate the period of the wave. T/I [ans: 2.1×10^{-15} s]
4. Determine how many wavelengths of radiation from a 60.0 Hz electrical transmission line would be needed to travel across North America (approximately 5.0×10^3 km). T/I A [ans: 1]

10.4 Review

Summary

- Maxwell's theory of electromagnetism predicts that oscillating electric and magnetic fields propagate through space at the speed of light.
- Hertz confirmed experimentally the existence of electromagnetic waves.
- Electromagnetic waves are produced by accelerating electric charges.
- Electromagnetic waves consist of magnetic and electric fields that are perpendicular to each other and to the direction of propagation, and oscillate in phase.
- The electromagnetic spectrum is the range of all possible electromagnetic radiation of various wavelengths and frequencies.
- The main categories of waves in the electromagnetic spectrum are radio waves, microwaves, visible light, ultraviolet light, infrared light, X-rays, and gamma rays.

Questions

1. The light used in a CD player has a frequency of about 5.0×10^{14} Hz. Determine its wavelength. T/I
2. The human eye is most sensitive to light with a wavelength of about 550 nm. Calculate this light's frequency. T/I
3. The dial on an FM radio contains numbers ranging from about 88 to about 108. These numbers correspond to the frequencies of the radio stations as measured in megahertz. Determine the corresponding range of wavelengths. T/I
4. X-rays are electromagnetic waves with very short wavelengths. Suppose an X-ray has a wavelength of 0.10 nm. Calculate its frequency. T/I
5. A microwave oven generates electromagnetic waves that have a wavelength of 12.24 cm. T/I A
   (a) Calculate the frequency of this radiation.
   (b) Explain why most microwave ovens contain rotating carousels.
6. Some cordless telephones use radio waves with a frequency near 2.4 GHz to transmit to their base station. Calculate the wavelength of these waves. T/I
7. The AM radio station 680 News in Toronto uses a frequency of 680 kHz. Determine the corresponding wavelength. T/I
8. Microwaves with a frequency of 7.5 GHz are incident on a slit 6.0 cm wide. Determine the angle from the central maximum to the first diffraction minimum. K/U T/I A
9. Use the Internet or print sources to identify why it is necessary to travel into deep space to detect all parts of the electromagnetic spectrum. Explain your answer. C A
10. When television correspondents are interviewed live in a distant part of the world, there is a delay before hearing their responses. Explain why. K/U A
11. Nearby mountains and airplanes can reflect radio waves. The reflections can interfere with the signal arriving directly from a station. K/U C A
    (a) What kind of interference results when an antenna receives 75 MHz television signals directly from a distant station and also receives waves reflected from an airplane 134 m directly above. (Assume that the phase changes by $\frac{\lambda}{2}$ on reflection.) Explain your answer.
    (b) Identify the type of interference that would result if the plane were 42 m closer to the antenna. Explain your answer.
12. Research how radio transmitters work (**Figure 8**). Write a short journal piece, with an abstract, that describes how a vibrating electric dipole produces electromagnetic radiation. Include an explanation of why antennas are different lengths. T/I C A

Figure 8

WEB LINK

10.5 Polarization of Light

Electromagnetic waves have electric and magnetic fields that are perpendicular to each other and to the direction of propagation. These fields can take many different directions and still be perpendicular to the direction of propagation. When you describe an electromagnetic wave, you need to include the direction of the electric field, as well as the amplitude, frequency, wavelength, and direction of propagation. These directions determine the polarization of the light, which is the subject of this section.

What Is Polarized Light?

A variety of sources are capable of producing electromagnetic radiation. Examples are the Sun, a microwave oven, an X-ray machine, a radio transmitter, and a light bulb. Each source produces electromagnetic waves that transmit energy. The polarization of electromagnetic waves describes the way in which the electric field is oriented relative to the direction of propagation. The light waves in **polarized light** vibrate in a single plane. A mixture of light with different waves and different directions of propagation is **unpolarized light**. In other words, a source of light creates oscillating electric and magnetic fields, perpendicular to each other, that move away from the source. Light can also be a mixture of polarized and unpolarized light, called partially polarized light. Polarization occurs when you block one of the axes of light oscillation with a polarizer (**Figure 1**). A **polarizer** is a filter or device that allows only light with an electric field along one direction to pass through. If you do not block any of the axes, then the light remains unpolarized.

polarized light light waves that vibrate in a single plane

unpolarized light light waves that vibrate in many different planes

polarizer a device that allows only light with an electric field along a single direction to pass through

Many applications make use of the polarization of electromagnetic waves. For example, sunglasses use filters to block glare reflected from horizontal surfaces such as the surface of a lake. Materials such as glass and plastic have different indices of refraction for different polarizations of light. The plastic instruments in **Figure 2** produce a colourful display when viewed through polarized sunglasses. This effect has practical uses in analyzing stresses to identify where structural failure might occur.

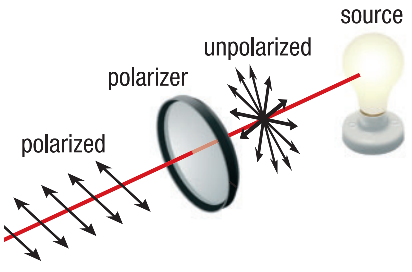


Figure 1 In this example, only light that vibrates in the horizontal plane passes through the polarizing filter. This light is said to be polarized along the horizontal direction.

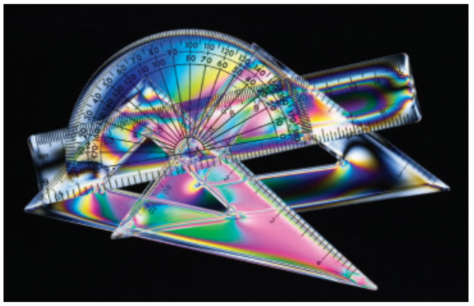


Figure 2 Areas of weakness within these plastic geometry instruments are visible under polarized light.

Generating Waves with Specific Polarizations

Radio stations can generate waves with specific polarizations by using various antenna configurations. Light that is entirely polarized in one direction that is perpendicular to the direction of propagation is said to be **linearly polarized**, or **plane polarized**.

linearly polarized (plane polarized) the quality of light waves that are polarized in one direction, perpendicular to the direction of propagation

Many natural phenomena, such as reflection, change unpolarized light to partially polarized light. The amount of light reflecting from a surface for each angle of incidence typically depends on whether the electric field of the light is along the surface or perpendicular to the surface. That is why light reflecting from road and water surfaces is typically partially polarized. Light from the Sun encounters small particles that scatter the waves. Shorter wavelengths are scattered more than longer wavelengths, which is why the sky appears blue. As a result, the sky often appears more vivid when viewed through a pair of polarized sunglasses (**Figure 3**). The degree of polarization depends on the relative orientation of the viewer, the source of the light, and the direction of the sky being viewed.

Figure 3 The colours in the image on the right are much more vivid because the photo was taken through a polarized filter.

Polarization by Selective Absorption

Ordinary, unpolarized light can act as a source of polarized light if you pass it through a polarizer. The direction of the electric field that the polarizer allows through is the **transmission axis** of the polarizer (**Figure 4**). Light that leaves the polarizer is always linearly polarized along the direction of the axis of the polarizer.

transmission axis the direction of the electric field that a polarizer allows through

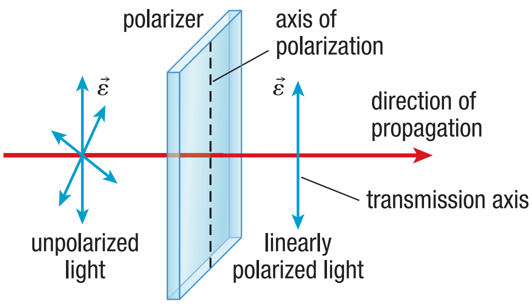


Figure 4 The polarizer allows light to pass through and become linearly polarized.

One example of a polarizer, called Polaroid, consists of a type of film used in instant cameras to generate developed film images. Polaroid film is a plastic sheet. It absorbs light when the electric field is perpendicular to the transmission axis and transmits light when the electric field is parallel to the axis of the polarizer. Polaroid film contains molecules (usually polyvinyl alcohol dyed with iodine) that selectively absorb light whose oscillating electric field is along the direction of the molecules. Light with an electric field that is perpendicular to the direction of the molecules passes through. The result is that the light passing through the film is linearly polarized, making detecting polarized light in nature easy.

How can you determine the strength, or intensity, of light passing through a polarizer? Recall that intensity is related to wavelength—the shorter the wavelength of light, the greater is its intensity. Consider what happens when the incident light is at an angle θ with respect to the polarization axis of the polarizer. The component of the electric field that is parallel to the polarizer is given by $\varepsilon_{in} \cos \theta$. The intensity, I, of the light is proportional to the square of the magnitude of its electric field vector. So, the transmitted intensity is related to the incident intensity by a relation called **Malus's law**:

Malus's Law

$$I_{out} = I_{in} \cos^2 \theta$$

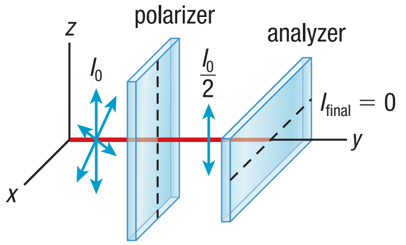


Figure 5 Light that is transmitted through the first polarizer passes through a second polarizer, called an analyzer, to verify that the light has indeed been polarized.

analyzer a second polarizer used to verify that the light from the first polarizer is polarized

The SI unit for intensity is the candela. To determine the proportion of unpolarized light that passes through a polarizer, you calculate average intensity using Malus's law over all possible incident polarization directions. As you might expect, only half the light passes through the polarizer:

$$I_{out} = \frac{1}{2} I_{in}$$

If you wanted to verify that the light from the first polarizer is indeed polarized, you could pass it through a second polarizer, called an **analyzer** (**Figure 5**). When the analyzer in Figure 5 is oriented at 90° to the polarizer, no light passes through. As the analyzer is rotated through 90° in either direction, the light intensity increases to its maximum value. The variation of intensity with rotation of an analyzer indicates the state of polarization of the light reaching it. You can use a similar arrangement to observe how some materials rotate the direction of polarization by observing the analyzer to determine the minimum transmission when the sample is between the polarizer and the analyzer.

Mini **Investigation**

Observing Polarization from Reflection

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

Light from the Sun and from most artificial light sources is unpolarized. Many familiar processes such as reflection can polarize light. In this activity, you will examine how a polarizer and analyzer work using Polaroid filters, and you will examine polarization effects in reflection.

Equipment and Materials: bright source of light; 2 Polaroid filters

1. Hold a Polaroid filter up to the light. Record your observations.

To unplug the lamp, pull on the plug, not the cord. Do not touch the lamp because it will be hot after use. Never look directly at the light source. Never look directly at the Sun, even through the Polaroid filter.

2. Hold two Polaroid filters up to the light, one above the other. Keeping one fixed, rotate the other 180°. Record your observations.
3. Create a glare on a surface using the light source. Observe the glare through a single Polaroid filter. Record your observations.
4. Hold the Polaroid filter against various regions of a clear sky away from the Sun. Rotate the filter.

A. Describe the effects of rotating the second filter. What happens to the light? T/I C

B. Explain why glare on a surface is reduced when you use a Polaroid filter. T/I C

C. Explain the variations in light seen by changing the location of a Polaroid filter against various regions of the sky. T/I C

Polarization by Reflection

In **Figure 6**, on the next page, an unpolarized light wave strikes the flat surface of a non-metallic material at an angle of less than 90° (not normal to the surface). Upon impact, the light's oscillating electric fields vibrate within the surface of the material. This, in turn, produces new outgoing light waves. Some of these new waves reflect from the surface and some refract into the material. The following discussion analyzes the mechanics of these interactions.

Like all electromagnetic waves, the electric field of the incoming light waves oscillates in all directions that are perpendicular to the direction of propagation. Thus, some of the electric field oscillates parallel to the surface of the material. Most of it oscillates in directions that are partially parallel and partially perpendicular to the surface. Light waves that penetrate the surface of the material transfer their energy to some of the electrons within the material. This, in turn, causes those electrons to begin to oscillate, which in turn produces electromagnetic, or light, waves. Some of these waves vibrate parallel to the surface, which means that these waves also vibrate perpendicular to the direction of reflection. These light waves encounter no obstacles when radiating in that direction and are free to produce a reflected wave.

However, light waves that vibrate partially perpendicular to the surface do not vibrate completely perpendicular to the direction of reflection. Since an electric field must oscillate perpendicular to the direction of reflection to propagate a wave in that direction, these light waves cannot radiate as effectively. The result is that some of the light from these waves propagates in the reflection direction, and some of it refracts into the material. The overall effect is that the light reflecting from the surface is partially polarized along the direction parallel to the surface. The relative intensity of the reflected waves depends on the angle of incidence, so the degree of polarization of the reflected light depends on the angle of incidence.

At some particular angle of incidence, called **Brewster's angle**, the direction of the reflected portion of the wave is perpendicular to the direction of the refracted portion of the wave (Figure 6). Maximum polarization of the reflected light occurs at Brewster's angle. You can calculate this angle using **Brewster's law**, which states that the tangent of Brewster's angle equals the ratio of the indices of refraction:

Brewster's Law

$$\tan \theta_B = \frac{n_2}{n_1}$$

At this angle, part of the electric field of the incident light oscillates along the direction of reflection. The photons that penetrate the material cannot radiate in this direction, so that portion of the incident light is not reflected at all. Meanwhile, the part of the light that oscillates parallel to the surface keeps reflecting. Thus, at an angle of incidence equal to Brewster's angle, the reflected light is 100 % polarized.

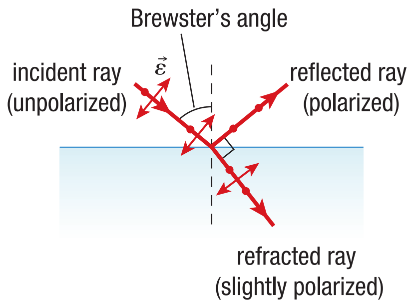


Figure 6 At Brewster's angle, the law of reflection and Snell's law of refraction require that the direction of the reflected ray is perpendicular to the direction of the refracted ray. Light waves that vibrate into the surface instead of along it, in response to the electric fields for incident polarization, cannot radiate in the reflected direction. Note that the red dots represent electric field lines that are perpendicular to the page.

Brewster's angle the angle at which the direction of the reflected portion of the wave is perpendicular to the direction of the refracted portion of the wave

Polarization by Scattering

Figure 7 shows that sunlight, which is unpolarized, can become polarized by **scattering**, that is, after it is scattered, or dispersed, after collisions between molecules in the atmosphere. Figure 7(a) shows an unpolarized wave of sunlight approaching a molecule in the atmosphere. When the wave hits the molecule, energy is transferred from the light wave to the molecule. This causes the positive and negative charges within the molecule to oscillate. Figures 7(b) and 7(c) show this motion for light polarized in two different directions. In Figure 7(b), the direction of polarization and the direction of the electric fields are vertical. Therefore, the molecular charges vibrate up and down and produce new outgoing waves that are also polarized vertically. These outgoing waves are called scattered waves. Figure 7(c) shows how the molecular charges move when the incoming light is polarized horizontally. The charges now oscillate in a horizontal direction and again produce scattered light waves. Since vibrating charges do not radiate light waves in the direction that the charges move, no scattered light reaches the observer looking along the vibration direction. Now combine the results in Figures 7(b) and 7(c) to understand how unpolarized sunlight is scattered. The part of this light that is polarized vertically scatters to the observer, but none of the horizontally polarized scattered light propagates in the observer's direction. Hence, the light that reaches the observer is said to be polarized by scattering.

scattering the change in direction of light waves as a result of collisions

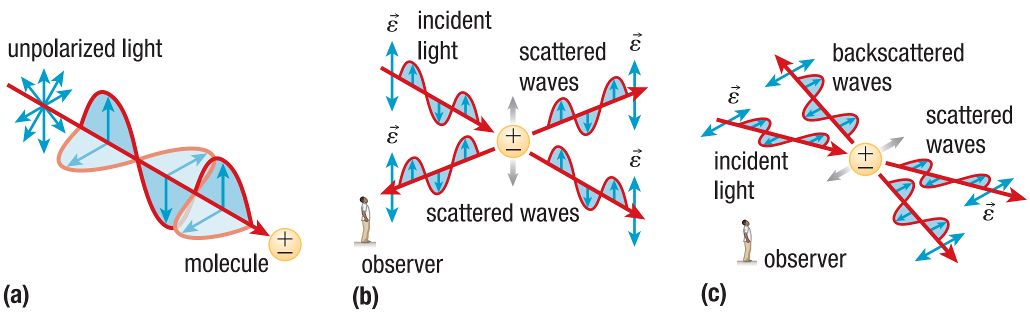


Figure 7 (a) Light from the Sun is unpolarized. (b) Light becomes polarized after scattering. (c) Different polarizations are scattered differently, which affects what an observer sees.

Optical Activity and Liquid Crystal Displays

optical activity the rotation of the direction of polarization when linearly polarized light interacts with certain molecules

liquid crystal display (LCD) a thin, flat display that makes use of polarizers and optical activity

When linearly polarized light passes through certain materials, the materials rotate the direction of the light's polarization (**Figure 8**) in an effect called **optical activity**. Materials that are optically active usually contain molecules with a screw-like or helical structure. Simple sugars, such as fructose and sucrose, are examples of optically active molecules. The thin, flat displays on wristwatches, cellphones, and many other devices, called **liquid crystal displays (LCDs)**, make use of polarizers to create optical activity. **Figure 9** shows how LCDs work. A polarizer linearly polarizes the incident light (Figure 9(a)). The polarized light then passes through a film that contains the optically active liquid crystal. The long, screw-shaped molecules in the liquid crystal rotate the plane of polarization by 90° so that the outgoing light can pass through an output polarizer. To turn off the display, a circuit applies a voltage to the liquid crystal, aligning the molecules in such a way that they no longer rotate the direction of polarization (Figure 9(b)). The light that comes out of the liquid crystal is then polarized at 90° relative to the output polarizer, so no light is transmitted and the display appears dark.

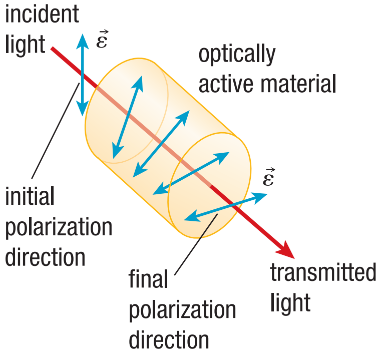


Figure 8 Polarized light passes through an optically active material, and the material changes the direction of polarization.

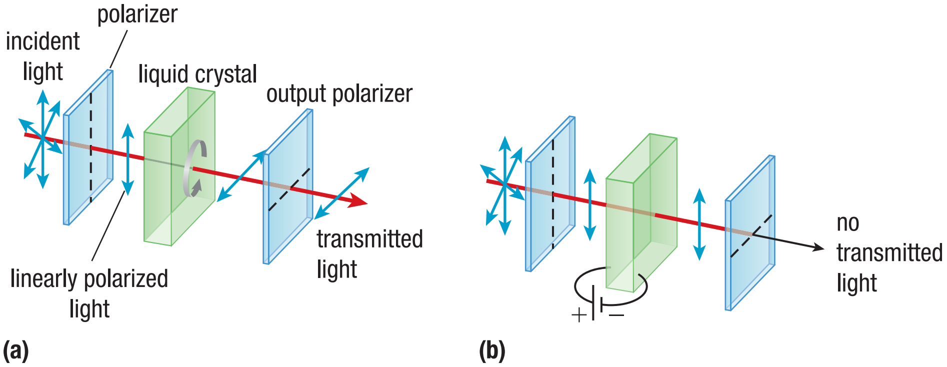


Figure 9 (a) When no voltage is applied, the liquid crystal linearly polarizes the light. (b) An applied voltage causes the liquid crystal to stop rotating its plane of polarization.

By applying different voltages to different areas of the liquid crystal, an LCD device can form various patterns of light and dark regions corresponding to letters or numbers. This is how the display of a wristwatch, a cellphone, and an MP3 player, as well as other similar displays, form the letters and numbers that you see. You can tell that the light from an LCD screen is polarized by placing a polarizer like your sunglasses in front of the screen.

Research This

Holography

Skills: Researching, Analyzing, Evaluating, Communicating SKILLS HANDBOOK A4.1

You may have seen a three-dimensional image produced by holography, which uses interference effects to reconstruct an image (**Figure 10**). Holography has many potential uses, and some newer techniques under development use polarization holography.

1. Research holography on the Internet.

A. What is holography, and how does it work? K/U

B. What equipment is needed to make a hologram? K/U

C. How does polarization affect holography? K/U

D. List two everyday applications of holography and two novel applications of this technology. K/U

E. Prepare a short presentation summarizing the results of your research. K/U C

Figure 10 This holographic image of a dinosaur skull shows a three-dimensional effect.

WEB LINK

10.5 Review

Summary

- Polarized light is produced using filters to selectively block the transmission of light waves.
- Linearly polarized, or plane polarized, light is entirely polarized in one direction that is perpendicular to the direction of propagation.
- Selective absorption, reflection, and scattering are three ways that polarized light can be produced from unpolarized light.
- Malus's law states how the transmitted intensity of light through a polarizer is related to the incident intensity of light: $I_{out} = I_{in} \cos^2 \theta$.
- Polarization filters have many uses, including stress analysis in materials, sunglasses, and the design and production of LCD displays.
- Simple sugars, such as fructose and sucrose, are optically active molecules that rotate in the direction of the polarization of light.
- By applying different voltages to different areas of a liquid crystal, an LCD device can form various patterns of light and dark regions corresponding to letters or numbers.

Questions

1. Describe the characteristics of unpolarized light and partially polarized light. K/U C
2. You are shopping for sunglasses and want to test whether a pair you like has polarizing lenses or simply darkened plastic lenses. Describe two ways you could quickly test them. K/U A
3. Briefly describe how selective absorption, reflection, and scattering polarize light. K/U C
4. A polarizer and an analyzer have perpendicular axes, so that light through them is blocked. Describe what will happen, and why, when you place between them a Polaroid sheet with its transmission axis
   (a) parallel to the transmission axis of the polarizer
   (b) parallel to the transmission axis of the analyzer
   (c) at a 45° angle to the two other axes K/U T/I A
5. The sky often looks very different when viewed through polarizing sunglasses. Explain what causes this effect. K/U C
6. The light reflected from the surface of a still pond is observed through a polarizer. How can you tell whether the reflected light is polarized? K/U
7. Using what you know about polarization, explain why the sky appears blue. K/U
8. You are viewing a selective polarization experiment through protective goggles. The original intensity of the incident polarized light is 250 candelas, and the resultant light is 17 % as intense. Determine the polarization angle of the incident light with respect to the polarization angle of the filter. T/I
9. A flashlight is directed at the flat side of a piece of quartz with an index of refraction of 1.54. The light travels through air. Calculate the angle of incidence that results in perfectly polarized light. What is the name of this angle? K/U
10. Explain in your own words how materials are able to reflect polarized light when the light source is unpolarized. K/U C
11. On a sunny day, the reflection of the sky in a window can make it difficult to see through the window to the inside of the building. Describe how polarized sunglasses can make it easier to see through the window. K/U C A
12. Is it possible for light intensity to be unaffected by a polarization filter? Explain your answer. K/U C A
13. One method of creating three-dimensional images in movies relies on polarization technology to create the optical illusion of a three-dimensional image. Research 3D technology and its use of polarization. Summarize your findings in a format of your choice. C A
14. Theatres showing movies that appear three-dimensional frequently make polarizing eyeglasses available to their patrons. Research the role that polarizing eyeglasses have in producing the 3D appearances of the images, and explain your findings orally to a fellow student. T/I C A

WEB LINK

10.6 Explore Applications of Light Technology

SKILLS MENU

- Researching
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating
- Identifying Alternatives

Electromagnetic Waves

Electromagnetic waves provide a means of expanding human knowledge. The applications of our knowledge of light have benefits for society and the environment.

One example is the use of photoelasticity. Some materials, such as glass and Lucite, have an index of refraction that is different for different polarizations of light. When subjected to stress, such as bending, stretching, or compression, these photoelastic materials refract light in a particular way. When you place Lucite between polarizing and analyzing filters, the strain patterns (and thus the stresses) in the material become visible (**Figure 1**). Engineers often begin their design process by building Lucite models of their designs. This enables them to analyze the stresses in the models before completing the design of the actual structures. Under mechanical stress, these Lucite models reveal areas where stress accumulates. This allows engineers to make any necessary design changes before constructing a particular device. CAREER LINK

Figure 1 Through a polarizer, you can see the stress patterns in this small piece of Lucite.

Another innovative light technology is the remote-sensing application of lidar, which uses electromagnetic radiation to examine the properties of objects or substances. The lidar user aims a laser beam at a remote object of interest and observes the resulting scattered light. The laser may be ultraviolet or near infrared. Scientists use lidar to examine rocks, rain, chemical compounds in the atmosphere, and clouds. Usually, the laser beam is pulsed so the timing between the pulse and detection of the scattered light can be measured. These measurements provide precise information about the distances between the laser and the object under observation. **Figure 2** shows a laser beam that measures air density, pressure, temperature, water vapour, and other constituents in the air such as oxygen, nitrogen, and carbon dioxide, providing scientists with information about the atmosphere at that location.

Figure 2 In this photo taken by Luc Girard, the University of Western Ontario's Purple Crow lidar sends laser light into the atmosphere. The laser light reflects off air molecules, revealing information about the atmosphere.

Allan Carswell, shown in **Figure 3**, on the next page, while a professor at York University in Toronto, began studying laser research. Later, he led a team that developed the first CO_2 laser in Canada and the first Canadian commercial helium–neon laser. In 1974, Carswell founded Optech, Inc. to develop practical applications of lidar systems. Since then, Optech has become a world leader in developing lidar applications.

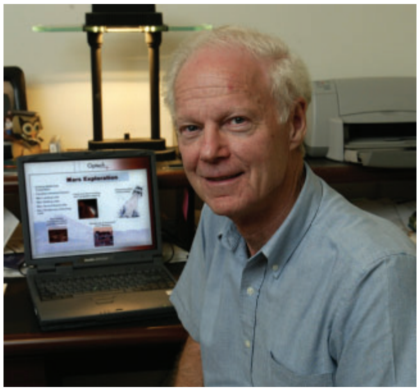


Figure 3 Allan Carswell is an internationally known Canadian physicist involved in lidar applications.

The Application

Lidar and photoelasticity tests are just two examples of the many applications of electromagnetic waves. Light technology is used in many diverse disciplines, including biology, chemistry, meteorology, medicine, engineering, and entertainment. Examples of applications of light technology are the study of human and animal vision, holography, laser eye surgery, mining, xenon lights, and stress analysis in materials. In this activity, you will choose one application of electromagnetic waves to investigate.

Your Goal

SKILLS HANDBOOK A4

To learn about the benefits, costs, and risks of a technology that uses the wave nature of light

Research

Select one of the applications of electromagnetic waves listed above, or another application of your choosing. Have your teacher approve your topic before you begin. Research the technology to learn how it works, why it is beneficial, and how it impacts society and the environment. WEB LINK

Summarize

Use these questions to summarize your research:

- How is this technology useful? Give examples of its applications.
- How does the application use electromagnetic waves?
- What portion of the electromagnetic spectrum is used?
- Who benefits from this application?
- What are the costs, risks, or concerns surrounding the application?
- Can this phenomenon work if another type of electromagnetic wave is used? Explain why or why not.

Communicate

Communicate your findings in an interesting and informative way to your peers. With your teacher's approval, choose one of these formats for your presentation:

- video
- video journal
- electronic slide presentation
- blog/website
- news article
- Wiki page
- written report
- poster
- another format of your choice

Physics JOURNAL

Light Nanotechnology and Counterfeit Prevention

SKILLS HANDBOOK A2

ABSTRACT

Clint Landrock, an engineer and inventor from British Columbia, developed a unique anti-counterfeiting method based on nanotechnology. His discovery may soon replace the holograms that are currently used to protect banknotes and other documents from forgery. Landrock's technology uses tiny "nanoscopic" perforations, each 1/1500 the size of a human hair, that reflect and refract light. These nanostructures produce a unique shimmering iridescence, and, unlike a hologram, they cannot be duplicated or copied.

Current Anti-Counterfeiting Technology

Since the late 1970s, many industries and governments have used holograms as a security measure to prevent counterfeiting. For instance, the shimmering circles of film on sports-league items, which prove they are authentic, are holograms (**Figure 1**). However, counterfeiters have discovered ways to duplicate holograms, so they are no longer effective security measures.

Figure 1 Holograms are used to validate the authenticity of many sports-league items.

Dennis Gabor invented holography in 1947. Holography is a way to produce a three-dimensional image on a single film. Gabor assumed that light waves reflecting from various points on an illuminated object interfere with each other. He hypothesized that photographing the interference patterns coming from an illuminated object would produce a three-dimensional image of the object. Using white light, Gabor demonstrated that his idea works. However, white light is incoherent because it contains a wide range of frequencies. As a result, the images Gabor produced were fuzzy.

Researchers were able to overcome this problem after the development of lasers in 1960. Emmett N. Leith and Juris Upatnieks split a laser beam and illuminated an object with one part of the beam and directed the other part of the beam to a photographic plate or film. When the light from both beams interfered on the surface of the film, the film recorded the pattern (**Figure 2**). The film appeared to contain grey smudges when viewed in white light. However, when Leith and Upatnieks illuminated the pattern with the same laser light, they saw a true three-dimensional image.

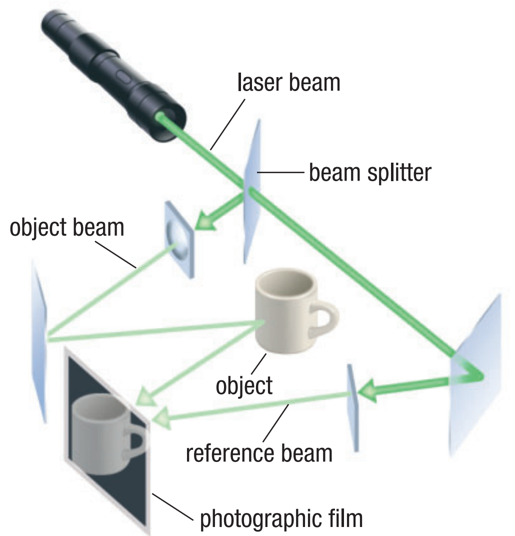


Figure 2 A laser beam is split into two beams. The resulting interference patterns are then recorded on film, creating a three-dimensional image.

Holograms have worked effectively for decades as security measures. In the past few years, the cost of holographic printers has decreased. As a result, counterfeiters have been able to purchase holographic printers and have become more skilled at duplicating security holograms.

Inspiration from Nature

During a trip to Costa Rica, Clint Landrock, an engineer and inventor from British Columbia, was inspired by the blue morpho butterfly (**Figure 3**). The blue morpho has tiny perforations in its wings that reflect and refract light to give the butterfly a shimmering, iridescent appearance (**Figure 4**).

Figure 3 The iridescent wings of a blue morpho butterfly show luminescence.

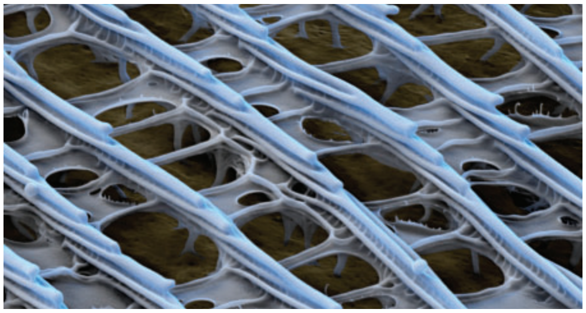


Figure 4 This scanning electron micrograph image shows the tiny perforations in the blue morpho's wings. It is these perforations, and not a pigment, that produce an iridescent effect.

A few years later, while studying solar cells at Simon Fraser University in British Columbia, Landrock remembered the blue morpho butterfly and how it reflected and refracted light. He noticed that thin metal sheets containing tiny perforations created iridescence, just as the wings of the blue morpho butterfly do. The iridescent light from the metal sheets reflects as a beam of pure colour, just as light from a blue morpho's wings reflects as a brilliant blue colour. Landrock and his colleagues realized that these unusual properties could be developed into a technology that prevents counterfeiting in banknotes and passports. They contacted a number of international banks, including the Bank of Canada, and the research team is in the process of marketing this new technology.

The Science behind the Technology

This new process is a combination of nanotechnology (the study of manipulating matter on an atomic and a molecular scale) and entomology (the study of insects). The technology is simple: drill numerous, tiny holes about 1/1500 the thickness of a human hair in thin pieces of metal and illuminate the metal. The resulting colour, like the iridescence of the blue morpho butterfly, comes from the reflection and refraction of the light, not from inks, dyes, or pigments. Since no colour pigment exists, the counterfeit mark cannot be copied or scanned. Furthermore, the minuscule size of the perforations makes them almost impossible to replicate. The technology is called nano-optic technology for enhanced security (NOtES).

Applications

According to the inventors of NOtES, preventing counterfeit banknotes and passports is just the beginning. There will be many other practical applications, from authenticating legal documents and stock certificates to preventing counterfeit pharmaceuticals. In fact, the inventors hope to replace holograms in a variety of merchandise, including sports brands, running shoes, and DVD software.

Further Reading

Drexler, E.K. (1996). *Engines of creation: The coming era of nanotechnology*. London: Fourth Estate.

Johnston, S.F. (2006). *Holographic visions: A history of new science*. Oxford: Oxford University Press.

WEB LINK

10.7 Questions

1. What concepts related to the wave nature of light apply to nanotechnology? Explain one of these concepts and how it applies to the technology. K/U T/I C A
2. Explain how butterfly wings reflect light to produce vivid colours. K/U C
3. Research the future implications of nanotechnology in optics. Summarize your findings in a short list. T/I C
4. In a graphic organizer, compare holography and nanotechnology in terms of the counterfeiting application. T/I C
5. How does this application of nanotechnology demonstrate the roles of creativity and chance in science? A
6. How does this application of nanotechnology demonstrate the link between science and technology? A

WEB LINK

10.8

Explore an Issue in Light Technology

SKILLS MENU

- Defining the Issue
- Researching
- Identifying Alternatives
- Analyzing
- Defending a Decision
- Communicating
- Evaluating

Global Positioning Systems

Many drivers rely on Global Positioning System (GPS) technology to help them find addresses in unfamiliar neighbourhoods. Many cars now come equipped with GPS devices (**Figure 1**). Others have dashboard- or window-mounted monitors. Are these devices a dangerous distraction? One British insurance company studied the effects of GPS use and discovered that drivers using a GPS were twice as likely to lose concentration as drivers who read a map.

Figure 1 How distracting is using a GPS system while driving?

Researchers originally developed GPS technology in the 1970s for military use, to track ships and submarines. Since then, GPS technology applications have expanded to construction, mining, transportation, farming, law enforcement, and leisure activities (such as the popular high-tech treasure hunting game of geocaching). Scientists also used GPS technology to measure the altitude of Mount Everest and to measure the effects of earthquakes. However, the use of GPS technology also has problems. For example, in construction, GPS surveying instruments can provide surveyors with precise measurements in any kind of weather. However, the data they provide may be inaccurate if obstructions, such as tall buildings and trees, interfere with the signal. In addition, some researchers have suggested that frequent use of GPS technology may affect memory.

The Issue

Apparent shortcomings exist in GPS technology, both in the device itself and in the effects of using the device. Are they severe enough to be regulated? Some insurance companies claim that GPS use has been the cause of driver distraction leading to accidents.

Your role as a member of a task force is to investigate the connection between traffic accidents and GPS use in Ontario. Is GPS use linked to traffic accidents? If so, what types of accidents occur when people use GPS systems? You also need to determine whether distracted drivers, incorrect GPS information, or both cause the accidents. You will present your findings to the government and citizens of your town in a series of town hall meetings.

Goal

To make a recommendation to the provincial government either to support or not to support regulations controlling GPS use in Ontario

Research

SKILLS HANDBOOK A4

Research this topic using the Internet and print resources. Ensure that your sources are credible. Do not rely on your personal opinion, and remember to take into account the different stakeholders that could be affected by your recommendation. Consider these questions in your research:

- How does a GPS device use electromagnetic waves?
- What type of electromagnetic radiation does GPS technology use?
- What are some of the benefits of GPS technology?
- What are some of the shortcomings with GPS use?
- Do GPS units ever provide incorrect information?
- What happens when the GPS is down?
- Do you think people rely on GPS devices too much for navigating? Explain your reasoning.
- How has GPS technology changed the world?
- Do distractions due to viewing or using GPS devices while driving contribute to traffic accidents? Provide evidence (statistical data, police reports, newspaper coverage) to support your claim.
- How might GPS technology be exploited? What are some privacy concerns with GPS devices in cellphones, cars, and so on? WEB LINK

Possible Solutions

- Does Ontario need a law to regulate GPS device use in automobiles?
- Can stakeholders reach a compromise?
- If so, what is the compromise and how will it be implemented?

Decision

Prepare an argument in favour of or against the regulation of GPS device use in automobiles in the province. Summarize the evidence that you have gathered, your reasoning, and your recommendations. Include in your summary some of the costs, benefits, and risks that you analyzed that led you to your decision.

Communicate

Present a summary of your findings to your classmates. You may wish to write a report, prepare a presentation, or create a poster. State your recommendations with your reasons. Remember that the members of your target audience are not engineers or scientists but consumers and government officials, and these people will make the final decision on whether or not to proceed with your recommendation.

Plan for **Action**

Speak to friends and family about GPS technologies and how government monitoring of GPS use would work. Then poll your family and friends to determine how many people support the decision and how many are against it. Write one or two paragraphs outlining some of the reasons for the decision. Share these results with the class.

WEB LINK

Investigation 10.1.1 CONTROLLED EXPERIMENT

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Investigating Interference Using Air Wedges

Light waves reflected from the two surfaces in an air wedge produce interference fringes. In Part A of this investigation, you will apply the concepts of interference, using an air wedge, to measure the thickness of a human hair. In Part B, you will use a more traditional method to measure the thickness of the same hair and compare the two results.

Testable Question

SKILLS HANDBOOK A2.2

How does the accuracy of measuring a small object using an air wedge compare to other methods of measuring?

Hypothesis

An air wedge is a more accurate means of measuring the thickness of a small object than direct measurement with either a micrometer or a microscope.

Variables

What variables will you use in your calculation? Which can you identify as the dependent (responding) variables and which as the independent (manipulated) variables?

Experimental Design

In this investigation, you will design two procedures to measure a human hair. One procedure will involve using an air wedge with a laser, and the other measurements will be done using a microscope or a micrometer. You will then compare the methods and analyze by how much the measurements vary.

Equipment and Materials

- class 1 or 2 helium–neon laser
- microscope or micrometer
- diverging lens ($f = -8$ mm)
- 2 glass microscope slides
- 2 elastic bands

If the laser uses 120 V electricity, to unplug it, pull on the plug, not the cord. Do not look into the laser light. Do not point the laser at anyone. Lower-power helium–neon laser light can damage your eyes.

Procedure
Part A: Using an Air Wedge

1. Design a procedure to investigate the thickness of a human hair using a helium–neon laser and an air wedge. Use the setup shown in **Figure 1**. Two microscope slides touch at one end and hold a human hair at the other end. Both ends must be secured with elastic bands. A helium–neon laser provides monochromatic light for the observation.

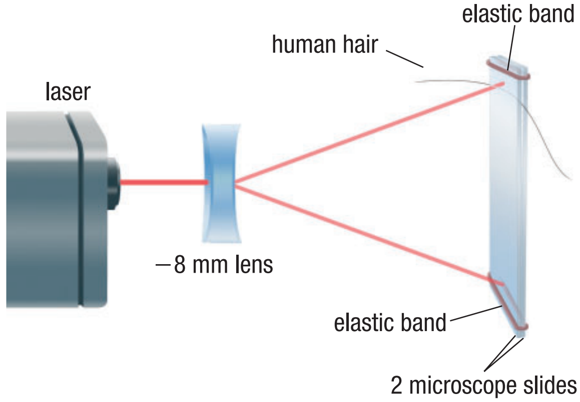


Figure 1

2. Get your teacher's approval for your design before conducting your investigation.
3. Determine the thickness of the human hair by measuring interference effects in the air wedge.

Procedure
Part B: Using a Microscope or Micrometer

4. Design a procedure to investigate the thickness of a human hair using a microscope or micrometer.
5. Obtain your teacher's approval for your design before conducting your experiment.
6. Determine the thickness of the human hair using the microscope or micrometer.

Analyze and Evaluate

(a) What variables were measured, recorded, and manipulated in this investigation? What type of relationship was being tested? T/I

(b) Compare the measurements of the hair from each method. K/U T/I

(c) How does the accuracy of measuring a small object using an air wedge compare to other ways of measuring the same object? C A

(d) Explain the sources of error and suggest ways to improve your procedures. K/U T/I

Apply and Extend

(e) Describe a different application of the concepts used in this lab, and explain how the methods of this experiment can be applied. C A

Investigation 10.2.1 CONTROLLED EXPERIMENT

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Diffraction of Light Using a Single Slit

In Part A of this investigation, you will make qualitative observations of diffraction from a single slit, and explore how the slit width and wavelength (colour) of the light affect the interference pattern. In Part B, you will make quantitative measurements of single-slit interference using a helium–neon laser or an LED. You will analyze the resulting data using the mathematical expressions in Section 10.2. Finally, in Part C, you will predict and then observe the interference pattern produced by shining a laser beam on a human hair. This investigation will prepare you for sections on diffraction gratings and other interference effects discussed in this chapter.

Testable Question

Do changes in slit width, wavelength, and source distance affect the measurement of interference fringe separation?

Hypothesis

If changes are made to the slit width, wavelength, or source distance, the interference fringe separation pattern will also change.

Variables

What variables will you use in your calculations? Which are the dependent (responding) variables, and which are the independent (manipulated) variables?

Experimental Design

In this investigation, you will use three procedures to observe and measure single-slit diffraction and interference patterns. What are the differences that you observe among the three procedures? How do slit width and wavelength affect the interference pattern? These questions should form the basis of your study.

Equipment and Materials

- long-filament showcase lamp or halogen–quartz bulb
- commercially prepared single-slit plate
- commercially prepared variable-slit plate
- red and green transparent filters
- white screen
- class 1 or 2 helium–neon laser or LED
- razor blade and plastic gloves
- diverging lens
- metric measuring tape or ruler
- photometer or computer interface (optional)
- elastic bands
- microscope slides

Procedure

Part A: Single-Slit Diffraction

SKILLS HANDBOOK A1.2

1. Arrange the lamp so that its filament is vertical.

The lamp bulb can get very hot. Do not touch it.

2. Using a commercially prepared variable-slit plate, look through the widest single slit at the light source. Now view the light with the rest of the series of single slits one at a time, so that you are observing the effect of changing slit width for the same wavelength. Record your observations with a simple sketch of what you see.
3. Cover the light source with the red filter and repeat Step 2.
4. Remove the red filter. View the light through a single slit. Slowly move the slit away from the light, and then back. Record any changes in the pattern that you observe.
5. Now, use the red and green filters. Cover the upper half of the light source with the red filter and the lower half with the green filter. Keep the distance between the source and the slit fixed. Determine the distance, y, from the first minimum to the central maximum for red and for green light. Compare these two values.

Part B: Diffraction with the Laser as the Light Source

6. Place the white screen about 1.0 m from the helium–neon laser. Aim the helium–neon laser to project its light on the white screen.

If the laser uses 120 V electricity, to unplug it, pull on the plug, not the cord. Do not look into the laser light. Do not point the laser at anyone. Lower-power helium–neon laser light can damage your eyes.

7. Use gloves to position the edge of the razor blade to one side of the laser beam. Record what you see on the screen.

Be extremely careful when handling the razor blade.

8. Pass the laser light through the variable-slit plate. Slowly narrow the slit. Record any changes that you see in the pattern on the screen.
9. Aim the laser beam through a commercially prepared single-slit plate. Enlarge the beam with a diverging lens. Record the pattern that you see on the screen.
10. With the screen still 1.0 m in front of the laser aperture, remove the single-slit plate. On the screen, mark the exact middle of the laser beam. Measure the distance, L, from the laser aperture to this point on the screen.
11. Place the single-slit plate right in front of the laser beam to produce a diffraction pattern on the screen. You may have to adjust the slit so that the centre of the central maximum is at the point that you marked on the screen in Step 10. Measure the distances from the centre of the central maximum on the screen to the first dark line. Calculate the average, y_1, of the measured distances on each side.
12. Record the width, w, of the single slit. Using the expression $\lambda = \dfrac{wy_1}{L}$, calculate λ.
13. Repeat Steps 11 and 12 for a different slit. Use the value for λ from Step 12 to calculate the width, w, of the slit.

Part C: Diffraction by a Human Hair

14. Estimate the size of your hair, or research the typical size of a human hair.
15. Carefully place your hair on a microscope slide.
16. Predict the interference pattern when you insert the slide into the laser beam. Draw a sketch of your prediction.
17. Mount the hair vertically in front of the laser beam using the elastic bands. Focus a clear image of the hair onto the screen with the diverging lens. Sketch the pattern that you observe.

Analyze and Evaluate

(a) What variables were measured, recorded, and manipulated in this investigation? What type of relationship was being tested? K/U

(b) What approximate slit width do you need to produce diffraction? What does the width indicate about the wavelength? Explain. C A

(c) According to your observations, which has the longer wavelength—red or green light? Why? K/U C A

(d) According to your observations of passing white light through the single slit, how do the respective wavelengths of the spectral colours differ? K/U C A

(e) Evaluate the Hypothesis. C A

For the next three questions, write your answers as proportions.

(f) How does the slit width, w, affect the distance, y_1, to the first dark line? K/U C

(g) What is the relationship between the distance, y_1, from the central line to the first minimum and the distance, L, from the source to the first minimum? K/U C

(h) What is the relationship between y and the wavelength, λ? K/U C

(i) Combine the proportionality statements from (f) to (h) into one proportionality statement between y, L, and w, with λ as the dependent variable. K/U T/I C

(j) Does the observed diffraction pattern for the human hair agree with your prediction? Explain. K/U T/I C

(k) How do your observations of intensity versus position compare with Figure 4 in Section 10.2? Compare your value for the laser wavelength with the accepted value of 6.328×10^{-7} m. State your experimental error. C

(l) Compare the slit width that you determined with the value provided by your teacher. State your experimental error. C

(m) Describe the most likely sources of error in determining the wavelength and slit width. T/I C

(n) Suggest ways to improve the accuracy of wavelength determination. T/I C

Apply and Extend

(o) If you have access to a photometer or a probe connected to a computer interface, you can measure the interference patterns. Position the single-slit plate as in Step 11. Position the diverging lens as in Step 9. Darken the room and move a sensitive photometer or a probe connected to a computer interface across the pattern produced on the screen, from one end to the other. Record the intensity at 5 mm intervals. Plot the light intensity versus position, defining the origin to be the centre of the pattern. T/I

Investigation 10.3.1 OBSERVATIONAL STUDY

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

CD and DVD Storage Capacity

If you look closely at the surface of a CD, you will notice a long spiral track circling from the centre of the disc out toward the rim. This track forms a reflection diffraction grating. The angular location of each maximum depends on the spacing between slits and on the wavelength of the light source. In this investigation, you will use a CD as a diffraction grating and record where its maxima fall. You will then use that information and the known wavelength of the laser light to determine the groove spacing of the CD. You will then repeat the investigation using a DVD to determine the difference between the two, and use this information to assess which disc can hold more data.

Purpose

To determine whether a CD or a DVD can hold more data

Equipment and Materials

- class 1 or 2 laser or LED
- scissors or other cutting device
- ruler or other measuring device
- clamps for mounting the screen
- thick cardboard or pegboard
- grid paper mounted on a piece of thick cardboard or pegboard
- glue
- one CD and one DVD

Procedure

1. Predict what you expect to observe regarding the relative spacing of the CD and DVD tracks.
2. Create a data table in which to record your observations.
3. Set up the laser and the screen as shown in **Figure 1**. Set up the cardboard to be perpendicular to the laser beam and 10 cm from the screen.
4. Glue a CD to the cardboard. Test that the CD is glued firmly in position.

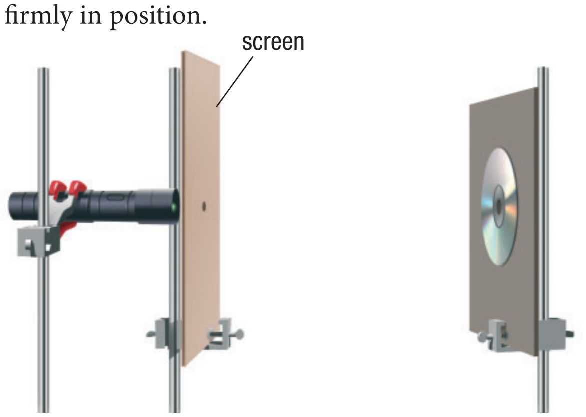


Figure 1 The CD is mounted perpendicular to the laser beam and parallel to the screen.

5. Direct the laser light through the hole in the screen onto the mounted CD. The beam should strike the CD toward the outer edge, not closer to the middle. The light is incident perpendicular to the CD. The grooves on the CD act as a diffraction grating to produce a reflection interference pattern on the screen.

If the laser uses 120 V electricity, to unplug it, pull on the plug, not the cord. Do not look into the laser light. Do not point the laser at anyone.

6. Get your teacher's approval for your setup. The interference pattern produced should clearly show the central maximum and secondary maxima.
7. Measure the distance from the central maximum to the first maximum (x_1), and from the central maximum to the second maximum (x_2). Record your data.
8. Use the equation

$$w = \frac{\lambda}{\dfrac{x_2}{\sqrt{(2L - x_2)^2 + x_2^2}} - \dfrac{x_1}{\sqrt{(2L - x_1)^2 + x_1^2}}}$$

and your measured data to determine the spacing of the grooves on the CD. Show your calculations.

9. Repeat Steps 5 to 8 using the DVD.
10. Record your qualitative observations of the differences between the groove spacings on the CD and the DVD.

Analyze and Evaluate

(a) Let w represent the unknown spacing between the CD or DVD grooves and θ_1 and θ_2 represent the angles of the first and second minima. Draw a diagram and label these quantities and the path difference between light from successive slits. C

(b) How many maxima did you observe on each side of the central maximum in the CD interference pattern? The DVD interference pattern? K/U T/I A

(c) Compare the DVD and CD measured track separation, and discuss what that comparison implies about which one stores more data. T/I C

Apply and Extend

(d) Explain how the spacing of the grooves on the CD needs to be different to observe more maxima. How would the wavelength need to be different instead? Is the same true for the DVD? C A

Summary Questions

1. Create a study guide for this chapter based on the Key Concepts on page 500. For each point, create three or four subpoints that provide further information, relevant examples, explanatory diagrams, or general equations.
2. Imagine you are a graduate student in nanotechnology, and your professor just won a grant. Write a research proposal based on the chapter concepts and describe how they apply.
3. Review the Starting Points questions on page 500. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers that you wrote at the beginning of the chapter. Note how your answers have changed.

Vocabulary

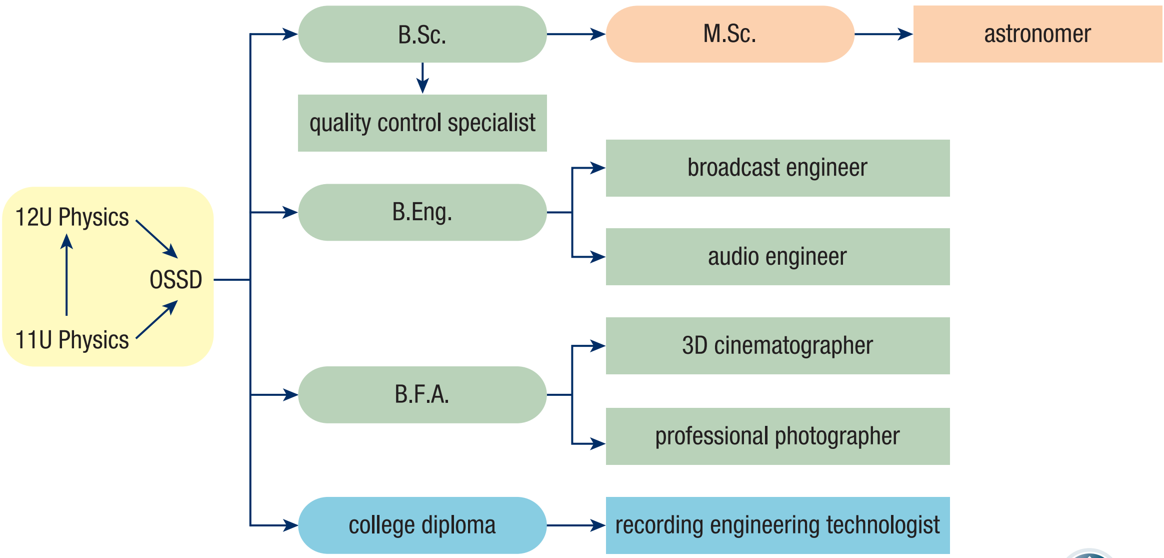
thin film (p. 502)
air wedge (p. 508)
Fraunhofer diffraction (p. 512)
central maximum (p. 512)
secondary maxima (p. 512)
resolution (p. 517)
diffraction grating (p. 520)
zero-order maximum (p. 521)
first-order maximum (p. 521)
order number (p. 521)
electromagnetic radiation (p. 526)
electromagnetic spectrum (p. 527)
polarized light (p. 532)
unpolarized light (p. 532)
polarizer (p. 532)
linearly polarized (plane polarized) (p. 532)
transmission axis (p. 533)
Malus's law (p. 533)
analyzer (p. 534)
Brewster's angle (p. 535)
Brewster's law (p. 535)
scattering (p. 535)
optical activity (p. 536)
liquid crystal display (LCD) (p. 536)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers mentioned in this chapter.

1. Select two careers related to the wave nature of light that you find interesting. Research the educational pathways that you would need to follow to pursue these careers. What is involved in the required educational programs? Prepare a brief report of your findings.
2. For one of the two careers that you chose, describe the career, main duties and responsibilities, working conditions, and setting.



CAREER LINK

CHAPTER 10 SELF-QUIZ

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. A beam of monochromatic light travels through air with wavelength λ. When the beam enters a thin film with index of refraction n_{film}, the new wavelength will be
   (a) λn_{film}
   (b) $\dfrac{\lambda}{cn_{film}}$
   (c) $\dfrac{\lambda}{n_{film}}$
   (d) λc (10.1) K/U
2. The colours in anti-reflective coatings on eyeglasses and solar cells, and the colours seen as sunlight shines on a soap bubble, can be explained by
   (a) light interfering as it reflects within a thin film
   (b) light diffracting within a thin film
   (c) light dispersing across a thick film
   (d) light polarizing inside a thin film (10.1) K/U
3. To increase the distance of the first dark fringe from the central maximum in a single-slit diffraction pattern, you should
   (a) use more intense light
   (b) use light of a higher frequency
   (c) use light of a longer wavelength
   (d) replace the slit with a wider opening (10.2) K/U
4. In the equation $m\lambda = w \sin \theta_m$, the variable w represents
   (a) the width of the central bright spot
   (b) the diameter of the light beam
   (c) the distance between adjacent slits
   (d) the width of a diffraction grating (10.3) K/U
5. An electromagnetic wave with frequency 1.0×10^7 Hz and wavelength 30 m should be classified as a
   (a) radio wave
   (b) microwave
   (c) visible light wave
   (d) gamma ray (10.4) K/U
6. Polarizing sunglasses work by
   (a) blocking the most energetic wavelengths of light
   (b) blocking light whose electric field is aligned in a certain direction
   (c) reducing the intensity of all incoming light waves by a factor of three
   (d) creating interference between incident and reflected waves (10.5) K/U
7. Light polarized in all directions with intensity I_{in} passes through a linear polarizer. What is the intensity of the light that comes out of the polarizer? (10.5) K/U
   (a) $\dfrac{1}{2}I_{in}$
   (b) $\dfrac{1}{4}I_{in}$
   (c) I_{in}
   (d) $2I_{in}$

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. A thin-film anti-reflective coating of constant thickness can block out all wavelengths of light equally well. (10.1) K/U
9. Optical resolution is limited by the effects of diffraction as light passes through small openings. (10.2) K/U
10. A diffraction grating will produce more intense maxima than a double slit. (10.3) K/U
11. A major difference between DVD technology and CD technology is that the light used to read DVDs has a longer wavelength than the light used to read CDs. (10.3) K/U
12. Earth's atmosphere absorbs radio waves most strongly at higher frequencies. (10.4) K/U
13. As light from the sky reflects off the road, most of the reflected light is polarized parallel to the road. (10.5) K/U
14. If a material can reorient the direction of linearly polarized light, then the material is said to be inactive. (10.5) K/U
15. Photoelasticity is a measure of how easily a material can bend a light ray. (10.6) K/U
16. Photographing the interference patterns coming from an illuminated object will produce a three-dimensional image of the object. (10.7) K/U
17. GPS devices have only positive impacts on society. (10.8) K/U

Go to Nelson Science for an online self-quiz.
WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. A light wave travels in a medium with a low index of refraction. A portion of this light is reflected off a medium with a higher index of refraction. The reflected light will experience a phase change of
   (a) 0°
   (b) 45°
   (c) 90°
   (d) 180° (10.1) K/U
2. Monochromatic light of wavelength λ strikes perpendicular to the surface of a soap bubble with uniform thickness. The light interferes constructively. Possible values for the thickness of the bubble are
   (a) $1\lambda, 5\lambda, 7\lambda$
   (b) $\frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$
   (c) $2\lambda, 4\lambda, 6\lambda$
   (d) $\frac{1}{4}\lambda, \frac{3}{4}\lambda, \frac{5}{4}\lambda$ (10.1) T/I
3. The wavelength of light in a certain thin film is 70 % of its wavelength in a vacuum. What is the index of refraction of the thin film? (10.1) K/U
   (a) 0.75
   (b) 1.00
   (c) 1.33
   (d) 1.43
4. **Figure 1** shows a light ray incident on a thin film of oil with an index of refraction of 1.41 spread across a block of glass with an index of refraction of 1.52. Which statement is true? (10.1) K/U
   (a) Only ray 2 undergoes a phase change.
   (b) Only ray 3 undergoes a phase change.
   (c) Rays 2 and 3 both undergo a phase change.
   (d) Rays 1, 2, and 3 undergo a phase change.

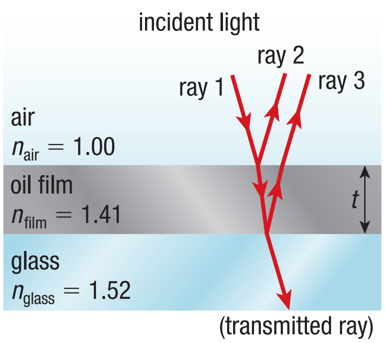


Figure 1

5. The incident light in Figure 1 has wavelength λ in air. The minimum thickness for the film to cause constructive interference is
   (a) $\frac{1}{2}\frac{\lambda}{n_{film}}$
   (b) $\frac{1}{4}\frac{\lambda}{n_{film}}$
   (c) λn_{film}
   (d) $2\lambda n_{film}$ (10.1) T/I
6. Monochromatic light of wavelength λ passes through a slit and creates a central maximum with diameter d on a screen, located a distance L away. What is the diameter of the slit? (10.2) T/I
   (a) $\frac{2\lambda L}{d}$
   (b) $\frac{2\lambda}{Ld}$
   (c) $\frac{Ld}{\lambda}$
   (d) $\frac{1}{2}\left(\frac{L}{\lambda d}\right)$
7. Optical resolution will increase by
   (a) increasing the frequency of light
   (b) decreasing the aperture size
   (c) decreasing the intensity of light
   (d) increasing the aperture size (10.2) K/U
8. Monochromatic light passes through a grating with slit spacing w. The fifth maximum falls at an angle θ. What is the wavelength of the light? (10.3) K/U T/I
   (a) $\frac{1}{5}w\sin\theta$
   (b) $\frac{5\sin\theta}{w}$
   (c) $\frac{5}{2}w\sin\theta$
   (d) $\frac{1}{5}\frac{\sin\theta}{w}$
9. A transmission grating has 11 000 lines/cm. Determine the distance between adjacent slits. (10.3) T/I
   (a) 9.1×10^{-7} m
   (b) 9.1×10^{-5} m
   (c) 1.3×10^{3} m
   (d) 1.3×10^{4} m

10. Electromagnetic waves consist of magnetic and electric fields that are
    (a) parallel to each other and to the direction of propagation, and oscillate in phase
    (b) perpendicular to each other and to the direction of propagation, and oscillate in phase
    (c) parallel to each other and to the direction of propagation, and oscillate out of phase
    (d) perpendicular to each other and to the direction of propagation, and oscillate out of phase (10.4) K/U
11. A microwave oven operates at a frequency of 2.45 GHz. What is the wavelength of the microwaves? (10.4) T/I
    (a) 0.122 mm
    (b) 0.122 cm
    (c) 0.122 m
    (d) 0.244 m
12. Malus's law relates
    (a) the refracted intensity to the transmitted intensity
    (b) the incident intensity to the refracted intensity
    (c) the transmitted intensity to the incident intensity
    (d) the incident intensity to the reflected intensity (10.5) K/U
13. Engineers often analyze the design of a large structure by constructing Lucite models. This process enables engineers to measure which property of the real structure? (10.6) K/U
    (a) the effect on atmospheric pressure waves
    (b) the interference with radio and cellphone signals
    (c) the reflection of light from the material's surface
    (d) the stresses within the structure
14. Dennis Gabor demonstrated holography by using
    (a) ultraviolet light
    (b) visible light
    (c) a laser
    (d) infrared light (10.7) K/U
15. Researchers have found that GPS systems may have direct harmful effects on which of the following? (10.8) K/U
    (a) brain
    (b) eyes
    (c) heart
    (d) ears

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

16. Newton's rings can be used to detect very small flaws in the shape of a convex lens. (10.1) K/U
17. When light travelling inside water reflects from a piece of crown glass, the reflected light wave will experience a 180° phase change. (10.1) K/U
18. A phase change of 180° corresponds to waves that are separated by one-half of a wavelength. (10.1) K/U
19. The particle theory of light best explains the bright and dark fringes surrounding the central maximum in a single-slit diffraction demonstration. (10.2) K/U
20. To achieve the best possible resolution of their images, astronomers try to maximize the effects of diffraction from distant stars. (10.2) K/U
21. Doubling the wavelength of light in a single-slit diffraction pattern will increase the diameter of the central maximum by four times. (10.2) K/U
22. Diffraction of sunlight through a single slit may not be noticed because sunlight has all the visible wavelengths, and the fringes of each wavelength overlap. (10.2) K/U
23. A CD is a good example of a transmission grating. (10.3) K/U
24. Increasing the number of lines per centimetre in a transmission grating will also increase the diffraction angle. (10.3) K/U
25. Diffraction gratings will display interference patterns only for visible light wavelengths if the slit spacing is appropriate. (10.3) K/U
26. Gamma rays travel through a vacuum at a higher speed than microwaves. (10.4) K/U
27. Microwave ovens work by radiating the food with waves of select frequencies that interact strongly with water and carbon dioxide molecules. (10.4) K/U
28. Light from the Sun is linearly polarized before reflecting off particles in the atmosphere. (10.5) K/U
29. When light reflects off a transparent surface at Brewster's angle, the reflected light is entirely polarized. (10.5) K/U
30. Lidar technology relies on visible light. (10.6) K/U
31. Photoelasticity is a property of certain materials that allows engineers to measure the stress distribution in a structure. (10.6) K/U
32. The technique of holography allows us to store, retrieve, and process information optically. (10.7) K/U
33. All technology is beneficial and has no shortcomings. (10.8) K/U

Understanding

34. Many camera lenses are manufactured with a thin film over the glass to minimize reflection. Using diagrams and words, explain how this anti-reflective coating works. Discuss how the following variations will affect the properties of the anti-reflective coating. (10.1) K/U C A
    (a) changing the wavelength of light
    (b) changing the angle of incidence of light

35. A light ray passes through the normal to the surface of a thin film of thickness t. One portion of the light ray reflects off the top of the film. Another portion transmits through the film, reflects off the bottom of the film, and returns to the top of the film. What is the path difference between these two reflected rays? (10.1) K/U

36. (a) Explain what is meant by a thin film.
    (b) Approximately how thin must a transparent film be to achieve optical interference? (10.1) K/U C

37. Explain why it is impossible to see interference effects in thick films. (10.1) K/U

38. Describe two factors that affect the angle of diffraction as monochromatic light passes through a small opening. (10.2) K/U

39. Do the maxima created by a diffraction grating all have the same intensity? (10.3) K/U

40. Sometimes, your car radio loses reception as you drive around mountains or over long distances where the curvature of Earth becomes a factor. Generally, FM waves are lost more readily than AM waves. Given that FM waves are measured in megahertz, and AM waves are measured in kilohertz, explain why the FM waves lose reception before the AM waves. (10.4) K/U T/I A

41. Your friend insists that he is "listening to" a radio wave. State the differences between a sound wave and a radio wave, and explain how information from the radio station reaches the listener's ear. (10.4) K/U C

42. When a radio transmitter uses a vertical antenna, the best reception occurs when the receiver also uses a vertical antenna. Explain why a horizontal antenna would give poorer reception. (10.4) K/U A

43. Explain the term "optically active," and describe two devices around your home that make use of optical activity. (10.5) K/U C A

44. Describe how you can tell whether or not a pair of sunglasses is polarized. (10.5) K/U C

Analysis and Application

45. A thin film of soapy water with an index of refraction of 1.35 has a thickness of 2.50×10^{-7} m at one point. This film is surrounded by air. (10.1) T/I
    (a) Determine the three longest wavelengths of light that will interfere constructively when reflecting from the soapy film.
    (b) Which of these wavelengths is visible?

46. A plastic film with a thickness of 250 nm appears to be green when light with a wavelength of 500.0 nm illuminates it. Calculate the index of refraction when viewed in reflection at normal incidence (**Figure 2**). (10.1) T/I

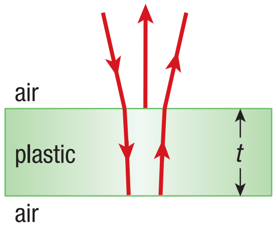


Figure 2

47. A thin film with an index of refraction of 1.45 is spread across flint glass with an index of refraction of 1.53. Green light with a wavelength of 560 nm, from the air, hits the thin film nearly perpendicular to the surface. (10.1) T/I
    (a) Calculate the wavelength of green light in the film.
    (b) Determine whether the waves experience a phase change when reflecting from the air–film surface and from the film–glass surface.
    (c) Calculate the minimum thickness of the film that will create constructive interference of the green light.

48. A group of students uses thin-film interference in an air wedge to calculate the thickness of a human hair. The hair is sandwiched at the ends of two microscope slides each of length 6.0 cm. The students count 25 evenly spaced interference fringes when red light with a wavelength of 6.50×10^{-7} m illuminates the slides. Calculate the thickness of the hair. (10.1) T/I

49. A thin-film coating with a refractive index of 1.39 is used on a camera lens that has a refractive index of 1.47 to cancel out reflected light with a wavelength of 5.40×10^{-7} m. (10.1) T/I
    (a) Calculate the minimum thickness for the coating that will accomplish this anti-reflection.
    (b) The manufacturers report that they cannot apply a coating this thin. Determine the next two possible thicknesses that will also accomplish this anti-reflection.

50. A CD uses the interference of reflected waves from the CD to encode information. Light from a red laser with a wavelength of 630 nm is reflected from small pits in the CD, and waves from the bottom of a pit and the adjacent top edge interfere. Determine the minimum pit depth that would cause the two light waves to interfere destructively. Assume that the pit is on the surface of the CD so that the region above the pit is air. (10.1) T/I A

51. The pits on a CD are covered with a layer of plastic so that surface scratches do not damage the pits. The index of refraction of the plastic coating is $n = 1.70$. Calculate the minimum pit depth needed to produce destructive interference using reflected light with $\lambda = 630$ nm (**Figure 3**). (10.1) T/I A

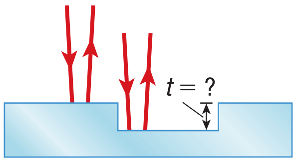


Figure 3

52. Two glass plates are arranged so that light reflected from the bottom surface of the top plate interferes with light reflected from the top surface of the bottom plate. Moving along the *x*-axis, the total reflected intensity alternates between bright and dark fringes. There are 25 bright fringes across the entire width of the plates. Determine the plate spacing at the right edge. Assume that the source of illumination is blue light with a wavelength of 420 nm. (10.1) T/I

53. A device called a Fabry–Perot interferometer contains two parallel mirrors as shown in **Figure 4**. Multiple reflections occur between the inner mirror surfaces, and the waves transmitted at the top can interfere. Assume that the light waves travel nearly perpendicular to the mirrors, the transmitted waves interfere constructively, and the spacing between the mirrors is exactly 3.5 μm. What is a possible value for the wavelength in the visible range between 600 nm and 700 nm? (10.1) T/I

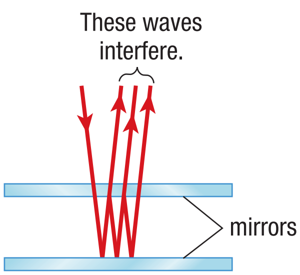


Figure 4

54. Light rays with a wavelength of 350 nm perpendicularly strike two flat glass surfaces separated by an air layer. Use a diagram to support your reasoning in both of the following questions. (10.1) T/I C
   (a) Calculate the thickness of the air layer needed to make the glass appear bright when the light is reflected.
   (b) Calculate the thickness of the air layer needed to make the glass appear opaque when the light is reflected.

55. Two glass plates are separated on one side by a strip of paper 1.92×10^{-3} cm thick. The plates are 9.8 cm long and are touching at the other end. The distance between the second and eighth dark bands is 1.23 cm. Calculate the wavelength of the light. (10.1) T/I

56. A beam of green light with a wavelength of 560 nm passes through a single slit and casts a diffraction pattern on a screen 6.3 m away. The central maximum measures 1.3 cm in diameter. Determine the width of the slit in micrometres. (10.2) K/U

57. Monochromatic light passes through a single slit with a width of 8.2×10^{-6} m. The third dark fringe (minimum) falls at an angle of 15° from the centre line. Determine the wavelength of the light in nanometres. (10.2) K/U

58. Two students use a violet laser to create a single-slit diffraction pattern on a screen 1.5 m away. The slit has a width of 5.6×10^{-5} m. The students measure the distance from the centre of the pattern to the fifth maximum to be 6.1 cm. (10.2) T/I
   (a) Determine the distance between successive maxima in centimetres.
   (b) Determine the wavelength of the violet light in nanometres.

59. Green light has a wavelength of approximately 510 nm. When green light is diffracted through a single slit with a width of 17 μm, the observed separation of the first dark fringe and the central maximum is 2.6 cm. Determine the distance between the slit and the screen for this observation to have been made. (10.2) T/I

60. Determine the angle of the first-order maximum as red light with a wavelength of 660 nm passes through a diffraction grating with 8000 lines/cm. (10.3) T/I

61. Predict the pattern that would appear if you set two diffraction gratings in front of a laser beam. Consider the following two cases. (10.3) T/I
   (a) The lines on one grating are perpendicular to the lines on the other grating.
   (b) The lines on one grating are parallel to the lines on the other grating.

62. (a) Use the equation $\lambda m = w \sin \theta_m$ to explain why visible light cannot exhibit diffraction when reflected off crystals that have a spacing of 1×10^{-8} m between the molecular planes within the crystal.
   (b) Discuss what type of wave will exhibit diffraction in this situation. (10.3) K/U T/I A

63. Violet light with a wavelength of 430 nm passes through a transmission grating in a spectroscope. The first-order maximum lies 16° from the centre line. (10.3) T/I
   (a) Determine the spacing between adjacent slits on the diffraction grating.
   (b) Determine the number of lines per centimetre on the diffraction grating.

64. A group of students examines the diffraction pattern created when red laser light with a wavelength of 650 nm is normal upon the surface of a CD. The students observe a first-order maximum 34° from the centre line on one side, and the other first-order maximum is 31° from the centre on the other side. (10.3) T/I A
   (a) Use both measurements to determine the average slit spacing on the CD in metres.
   (b) Calculate the number of lines per centimetre that are etched onto the CD.
   (c) The CD has an etched radius of 4.0 cm, and it plays for 50 min. Determine how many rotations the CD will make if one line is read each rotation.
   (d) Determine the average rotational speed of the CD in revolutions per minute.
65. A beam of light with a mix of wavelengths between 400.0 nm and 700.0 nm passes through a transmission grating that has 2000 lines/cm. Determine the width of the first-order rainbow spectrum that appears on a screen 4.0 m behind the grating. (10.3) T/I
66. A beam of red light with a wavelength of 660 nm passes through a transmission grating with 5000 lines/cm. Determine how many maxima are present on each side of the central maximum. (Hint: The last observable maximum will appear at an angle of 90°.) (10.3) T/I
67. A certain cellphone operates at a frequency of 2.0×10^9 Hz. Calculate the corresponding wavelength. (10.4) T/I
68. A common remote control uses an infrared diode that emits radiation of wavelength 940 nm. (10.4) T/I
   (a) Determine the frequency of this radiation.
   (b) The remote is used to operate a television set 2.5 m away. Calculate how long it takes for the signal to reach the television.
69. A satellite TV signal operates at 89 GHz. Determine the wavelength of this signal. (10.4) T/I
70. Unpolarized light passes through two polarizing filters. The axis of the second filter is rotated 60° relative to the axis of the first filter. Determine the intensity of the transmitted light when the incoming light has intensity I_{in}. (10.5) T/I
71. Unpolarized light passes through two polarizing filters. The transmitted light has one-tenth the intensity of the incoming light. Calculate the rotation angle of the second polarizer relative to the first. (10.5) T/I
72. Light incident at 61° in air strikes a transparent material and produces reflected light that is linearly polarized in just one direction, satisfying the condition of Brewster's angle. (10.5) K/U T/I
   (a) Determine the angle of reflection.
   (b) Determine the angle of refraction.
   (c) Determine the index of refraction of the material.
   (d) Calculate the angle of incidence that would yield completely polarized reflected light if the transparent material were water.
73. Most computer LCD projectors (**Figure 5**) emit polarized light of just three colours: red, green, and blue. A student projects the image of a white screen from an LCD projector. When the student holds a polarizing filter in front of the projector lens, the "shadow" cast by the filter is bright green. (10.5) T/I A

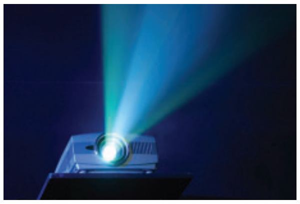


Figure 5

   (a) Explain why the shadow is green.
   (b) Predict what would happen if the student rotated the polarizing filter by 90°.
74. A small incandescent bulb glows above a laboratory table. As you look directly at the bulb through a polarizer, the light is dimmed by the same amount no matter how you change the angle of the polarizer. However, as you look through the polarizer at the glare of the light bulb reflected off the shiny laboratory table, you can effectively eliminate the glare by turning the polarizer to the correct angle. Explain why the polarizer will block out the light in the second situation, but not in the first situation. (10.5) T/I A
75. When unpolarized light shines through two polarizing filters set at 90° to each other, no light will pass through. However, when a third polarizing filter is inserted between the first two at an angle of 45° relative to the first, some light will pass through all three filters (**Figure 6**). The original light has intensity I_0. Determine the intensity of the light that passes through all three filters. (10.5) T/I

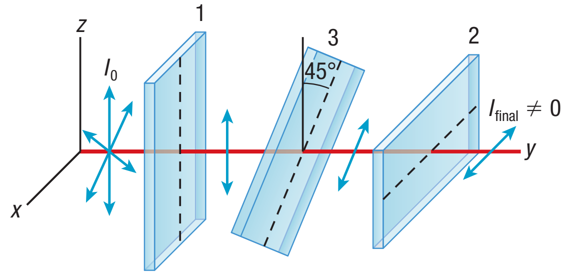


Figure 6

Evaluation

76. Assess the following sentence, then rewrite or reword it to make it more accurate: "The light intensity is black at the point of contact of the two plates, followed by seven bright fringes, followed by another dark fringe." (10.1) K/U T/I C
77. Violet light with a wavelength of 435 nm reflects from a diffraction grating. There are 6400 lines/cm on the diffraction grating. If you were asked to calculate how far the first-order maximum lies from the centre line, in degrees, how many significant digits would your answer have? Explain your answer. (10.3) K/U T/I
78. A classmate attempts to use an analogy to describe the polarization of light by a filter. Your classmate says, "Pass a rope between two pickets of a picket fence and then try to create waves on the rope. Only waves that vibrate along the direction of the pickets will pass through the pickets. That is exactly like light polarization." Assess the accuracy of your classmate's analogy, taking into account Malus's law. (10.5) C A
79. Assess the ways in which publishing magazines and newsletters on the Internet, rather than printing and distributing them, benefits the environment. (10.4, 10.6) A

Reflect on Your Learning

80. What did you find most surprising in this chapter, and what did you find most interesting? How can you learn more about these topics? K/U
81. How would you explain the concepts of thin-film interference and polarization to a fellow student who has not taken physics? K/U C
82. In what areas of your daily experience do you now see the physics concepts that were explored in this chapter? K/U A

Research

WEB LINK

83. Research the iridescence of colours in butterfly wings. Based on your research and findings, prepare a short oral presentation that discusses the following points: T/I C A
    (a) the differences between scales and cuticles.
    (b) the purposes of the brilliant colours in butterfly wings
84. Research the superior eyesight of a bald eagle. T/I C A
    (a) What is the advantage of binocular vision?
    (b) Describe how the fovea (a depression in the retina) is different in eagles than in humans.
    (c) Explain how the eyelids, eyebrows, and flexible lens also improve an eagle's vision.
85. **Figure 7** shows the Very Large Array (VLA), a radio astronomy observatory in New Mexico. Research the VLA, and answer the following questions. Discuss your findings with your classmates. T/I C A

Figure 7

    (a) At what elevation does the VLA stand?
    (b) How many independent antennas are present in the observatory?
    (c) What is the diameter of one of these circular dishes?
    (d) What do astronomers observe with the VLA?
    (e) What are the advantages of receiving radio waves instead of other types of waves?
86. You may have seen the new QR codes on many products and posters (**Figure 8**). Research QR codes. T/I A

Figure 8

    (a) What does QR stand for?
    (b) Where did the QR code originate?
    (c) Why are QR codes becoming so popular compared with barcodes?
    (d) Barcodes are read by lasers. QR codes are read with cellphone cameras, but cellphone cameras do not use lasers. Why does this work?
87. Research the work and contributions of Canadian physicist and engineer Reginald Fessenden, one of the pioneers in the field of radio. What were Fessenden's notable contributions to radio and the use of radio for long-distance communication? Discuss how Fessenden's work involved using the wave model for light. Connect his work in radio to modern cellphone technology. Discuss your findings with a classmate. T/I C A

Optical Pattern Analysis

The physics of light affects how we see the world around us. The concepts that you studied in this unit explain many of the properties of light waves that you interact with every day. A mirage above hot pavement on a summer day and the sparkle of a crystal can both be explained by the properties of light (**Figure 1**).

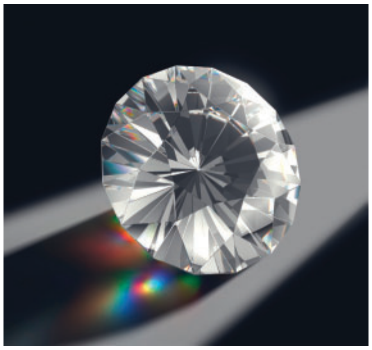


Figure 1 White light shining on a diamond will disperse into a rainbow of colours as it passes through the diamond.

Our eyes allow us to see some of the effects of wave properties of light around us. However, our eyes cannot detect and record all the particular features of light waves. Video recorders and even the simple cameras on many mobile devices can help us see many of the features that our eyes miss.

Innovative instruments and computer tools developed by researchers and engineers constantly improve both the hardware and the software for digital cameras. High-dynamic-range photography, for example, combines multiple images to capture more contrast in a single image, and image-editing software can apply visual effects to an image to change it in many different ways.

The Task

In this Unit Task, you will use photo and video technology to analyze examples of the wave-like behaviour of light that you studied in this unit. You will begin by collecting your own examples of the wave-like behaviour of light in various forms of digital media. Some examples are as follows:

- light interference
- thin films
- polarization
- dispersion

You can use digital photo or video-recording devices. You can take the images yourself, or you can use examples that you find online or in other sources.

Work in groups of two or three. Read through the rest of the task description before you begin collecting your examples so that you have an idea of what to look for. Spend some time thinking about the types of images that would help you complete the task, and discuss your plans with your group members. Some examples may be visually appealing but may not be easy to analyze. You will want to be certain that the phenomenon in the image can be described using the concepts that you learned about in this unit. Two examples of such images are shown in **Figure 2** and **Figure 3**.

Figure 2 A thin film of gasoline on asphalt illustrates the interference of light.

Figure 3 A thin soapy solution illustrates the colours resulting from the interference of light.

Part A: Qualitative Analysis of Images

After you have collected a few sample images, apply the concepts of physical optics that you have learned in this unit. For each image, describe the physics being illustrated in the image using the concepts that you have learned. Summarize your findings in a report or visual presentation that includes the images. If you use videos, you may design your report as a webpage or in another format.

Part B: Quantitative Analysis of Images

Use photo- or video-editing software to analyze your images quantitatively. Look for examples where you can apply equations from Unit 4 to determine the properties of the waves displayed in the images. You may have to make certain assumptions about the wavelength of light, distances from or number of sources, placement of slits, or other details. Prepare a summary of your findings that includes the equations that you were able to apply and the assumptions that you had to make.

Analyze and Evaluate

(a) Describe the wave-like effects of light that your images illustrate. K/U C

(b) List the quantitative data that you determined for each image, including your assumptions. K/U T/I C A

Apply and Extend

(c) Describe common technologies that use the properties displayed in your images. Do these technologies have an environmental impact? Explain your answer. K/U C A

(d) Identify some careers related to the physics of light. K/U C

(e) Create a piece of art with the images that you took. The piece of art could be in the form of a coloured print, or it could be electronic and interactive. Organize all the pieces of art from the class in an art gallery, displayed either on walls or desks or in an electronic art gallery. Invite others to view the gallery.

ASSESSMENT CHECKLIST

Your completed Unit Task will be assessed according to the following criteria:

Knowledge/Understanding

- ✓ Demonstrate an understanding of the properties of light waves in relation to diffraction, refraction, interference, and polarization.
- ✓ Demonstrate knowledge of how physics concepts apply to qualitative analysis of images.
- ✓ Demonstrate an ability to apply equations involving the wavelength of light, the distance from or number of sources, the index of refraction, the speeds of different colours of light, and the placement of slits.
- ✓ Demonstrate knowledge of the terminology associated with light waves.
- ✓ Describe the impact on the environment of some technologies that make use of the properties of light waves.

Thinking/Investigation

- ✓ Effectively use digital technologies and image-manipulation software.
- ✓ Investigate, qualitatively and quantitatively, the properties of light waves, and solve related problems.
- ✓ Identify and analyze properties of light waves, such as interference, diffraction, and refraction.
- ✓ Identify technologies that use the wave nature of light.

Communication

- ✓ Synthesize findings in the form of a written report.
- ✓ Communicate findings in a visual format using different multimedia tools.
- ✓ Use appropriate terminology related to the physics of light.

Application

- ✓ Assess the influence of technology that uses the wave-like nature of light on the environment.
- ✓ Identify careers associated with the physics of light.

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. A wave has an angle of incidence of 24°. What is the angle of reflection? (9.1) K/U
   (a) 24°
   (b) 48°
   (c) 76°
   (d) 90°
2. The distance between two consecutive positive amplitudes in a wave is 2 m. Two complete waves pass through any point per second. What is the speed of the wave? (9.1) K/U
   (a) 1 m/s
   (b) 2 m/s
   (c) 3 m/s
   (d) 4 m/s
3. The change of direction of a transmitted light wave at the boundary of two different media is called
   (a) diffraction
   (b) reflection
   (c) refraction
   (d) incidence (9.2) K/U
4. A ray of light travels through a sequence of different materials and is diffracted as shown in **Figure 1**. The respective speeds of light in the three materials are v_1, v_2, and v_3. Which of the following is true? (9.2) K/U T/I
   (a) $v_1 < v_2 < v_3$
   (b) $v_2 < v_1 < v_3$
   (c) $v_2 < v_3 < v_1$
   (d) $v_3 < v_2 < v_1$

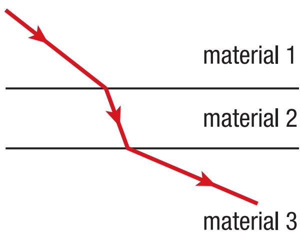


Figure 1

5. When a light wave travelling in air enters water, the physical quantity that remains unchanged is
   (a) direction
   (b) wavelength
   (c) frequency
   (d) speed (9.2) K/U
6. When two interfering waves have a displacement in the same direction, which of the following occurs? (9.3) K/U
   (a) constructive interference
   (b) destructive interference
   (c) refraction
   (d) diffraction
7. Which phenomenon is not explained by Newton's particle theory of light? (9.4) K/U
   (a) refraction
   (b) reflection
   (c) diffraction
   (d) rectilinear propagation
8. In Young's double-slit experiment, the wavelength of light being used is halved and the distance between the slits is doubled. The separation of the resulting fringes is
   (a) unchanged
   (b) zero
   (c) decreased
   (d) increased (9.5) K/U
9. When monochromatic light is used, the centre of Newton's rings is
   (a) dark
   (b) bright
   (c) neither bright nor dark
   (d) red (10.1) K/U
10. The ability of a lens to resolve two closely spaced images depends on which of the following properties? (10.2) K/U
    (a) the speed of the light passing through the lens
    (b) the thickness of the lens
    (c) the diameter of the lens
    (d) the distance between the lens and the images
11. A single-slit diffraction pattern is obtained using a beam of yellow light. If blue light is used instead, then
    (a) the diffraction pattern remains the same
    (b) the diffraction pattern fringes become narrower and closer together
    (c) the diffraction pattern disappears
    (d) the diffraction pattern becomes uniform with no maxima or minima (10.2) K/U

12. The spread of the diffraction pattern that forms when light waves pass through a single slit increases with
    (a) the slit width
    (b) the wavelength of the light
    (c) the speed of the light
    (d) the index of refraction (10.2) K/U
13. A beam of monochromatic light is incident normally on a diffraction grating with 6000 lines/cm. A first-order spectral line is found to be diffracted at an angle of 25°. What is the wavelength of the light? (10.3) K/U
    (a) 654 nm
    (b) 704 nm
    (c) 754 nm
    (d) 804 nm
14. In electromagnetic waves, the electric and magnetic fields oscillate
    (a) parallel to the direction of propagation
    (b) perpendicular to the direction of propagation
    (c) out of phase with each other
    (d) parallel to each other (10.4) K/U
15. Red light differs from blue light in its
    (a) speed
    (b) intensity
    (c) amplitude
    (d) frequency (10.4) K/U
16. A light wave that vibrates in one plane only is
    (a) refracted
    (b) polarized
    (c) diffracted
    (d) dispersed (10.5) K/U
17. The polarization of sunlight by Earth's atmosphere represents an example of which type of polarization? (10.5) K/U
    (a) polarization by reflection
    (b) polarization by absorption
    (c) polarization by scattering
    (d) polarization by optical activity

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

18. The angle of reflection of a light ray from a surface depends on the index of refraction of the surface material. (9.1) K/U
19. Light rays are always normal to the wave front. (9.1) K/U
20. The speed of a light wave in a material decreases as the index of refraction of the material increases. (9.2) K/U
21. Total internal reflection can take place when light travels from glass to water. (9.2) K/U
22. Two light waves from a single source that meet after a path difference of an odd number of wavelengths will interfere destructively. (9.3) K/U
23. A regular series of water waves pass through a vertical opening in a small dam. As the opening is closed, the waves will diffract more. (9.3) K/U
24. Poisson's bright spot is predicted by the wave model of light. (9.5) K/U
25. As the slits in a double-slit experiment are moved closer together, the fringes move closer together. (9.5) K/U
26. Light travelling from glass to water undergoes a half-wavelength phase change when it reflects from the water. (10.1) K/U T/I
27. Destructive interference in a thin film will occur if the light reflecting from the top and the bottom of the film has a phase difference of a whole number of wavelengths. (10.1) K/U
28. A CD player uses the interference of reflected waves from the CD to encode information. (10.1) K/U
29. Astronomers use telescopes with large aperture sizes to view distant objects because the large size reduces diffraction and increases resolution. (10.2) K/U
30. Double-slit and diffraction-grating interference patterns are similar, but the diffraction-grating maxima are narrower and more intense. (10.2) K/U
31. Increasing the number of slits per centimetre in a diffraction grating increases the width of each maximum. (10.3) K/U
32. The speed of an electromagnetic wave is always 3.0×10^8 m/s, regardless of the material through which it propagates. (10.4) K/U
33. Microwaves have less energy than X-rays, but more energy than radio waves. (10.4) K/U
34. Ultraviolet images are used for weather forecasting. (10.4) K/U
35. Sunlight will be partially polarized when it reflects from the window of a building. (10.5) K/U
36. The liquid crystal displays in many televisions, cellphones, and computer screens work due to the polarization of light. (10.5) K/U

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. You are at a beach and count the waves as they hit the shore. In 2 min you count 30 waves. What is the frequency of the waves? (9.1) K/U
   (a) 0 Hz
   (b) 0.25 Hz
   (c) 0.5 Hz
   (d) 0.9 Hz
2. A wave with high frequency has a
   (a) small period
   (b) large period
   (c) large amplitude
   (d) small amplitude (9.1) K/U
3. A light wave strikes a surface with an angle of incidence of 76°. What is the angle of reflection? (9.2) K/U
   (a) 0°
   (b) 24°
   (c) 48°
   (d) 76°
4. What is the change in direction of a light wave due to a change in its speed called? (9.2) K/U
   (a) diffraction
   (b) reflection
   (c) refraction
   (d) interference
5. The speed of light in a medium is 1.25×10^8 m/s. What is the medium's index of refraction? (9.2) K/U
   (a) 1.3
   (b) 1.4
   (c) 2.2
   (d) 2.4
6. Snell's law, a relationship between the indices of refraction of different media, can also be called the
   (a) law of reflection
   (b) law of refraction
   (c) law of cosines
   (d) law of diffraction (9.2) K/U
7. The interference of light waves differs from diffraction in that, unlike diffraction,
   (a) interference fringes are of varying intensity
   (b) interference cannot be observed with white light
   (c) interference minima may be perfectly dark
   (d) interference fringes are of unequal width (9.3) K/U
8. When two interfering waves have a displacement in the same direction,
   (a) constructive interference occurs
   (b) destructive interference occurs
   (c) refraction occurs
   (d) nothing happens (9.3) K/U
9. According to Huygens' principle, a wave front is
   (a) tangent to the circular waves
   (b) a concentric circle to the circular waves
   (c) normal to the circular waves
   (d) a plane wave (9.4) K/U
10. Huygens' principle of secondary waves is used to
    (a) obtain the new position of the wave front geometrically
    (b) explain interference phenomena
    (c) explain polarization
    (d) determine the speed of light (9.4) K/U
11. Lasers are a
    (a) monochromatic source of light
    (b) polychromatic source of light
    (c) maximum source of light
    (d) minimum source of light (9.5) K/U
12. Young's double-slit experiment proved that light
    (a) is monochromatic
    (b) is polychromatic
    (c) has wave properties
    (d) is sinusoidal (9.5) K/U
13. You can see many colours on a film of gasoline spread over a surface. The colour effect is due primarily to
    (a) diffraction
    (b) interference
    (c) absorption
    (d) transmission (10.1) K/U
14. A glass camera lens has an index of refraction greater than 1.2. The coating on the lens has an index of refraction of 1.2. What coating thickness is required to minimize the intensity of reflected light, where λ is the wavelength of light? (10.1) K/U
    (a) $\dfrac{\lambda}{2}$
    (b) $\dfrac{\lambda}{4(1.2)}$
    (c) $\dfrac{(1.2)\lambda}{4}$
    (d) $\dfrac{\lambda}{2(1.2)}$

15. In a single-slit diffraction experiment, the slit is exposed to white light. What colour is the fringe surrounding the central fringe? (10.2) K/U
   (a) red
   (b) yellow
   (c) violet
   (d) green
16. A ray of light strikes a glass plate at an angle of incidence of 60°. The reflected and refracted rays are perpendicular to each other. What is the index of refraction of the glass? (10.5) K/U
   (a) $\sqrt{5}$
   (b) $\sqrt{3}$
   (c) $\sqrt{2}$
   (d) 3
17. Which of the following statements is correct? (10.5) K/U
   (a) Brewster's angle is independent of the wavelength of light.
   (b) Brewster's angle is independent of the nature of the reflecting surface.
   (c) Brewster's angle is different for different wavelengths.
   (d) Brewster's angle depends on the wavelength but not on the nature of the reflecting surface.
18. When unpolarized light of intensity I is incident on a polarizing sheet, the intensity of the light that does not get transmitted is
   (a) $\frac{I}{2}$
   (b) $\frac{I}{4}$
   (c) 0
   (d) I (10.5) K/U

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

19. Radio waves go through more cycles per second than X-rays. (9.1, 10.4) K/U
20. Light waves with higher frequencies have longer wavelengths, and light waves with lower frequencies have shorter wavelengths. (9.1) K/U
21. In a ripple tank, a decrease in the frequency of the two vibrating sources causes an increase in the number of nodal lines. (9.3) K/U
22. When two light waves from the same source meet after a path difference of 2.5 wavelengths, they interfere destructively. (9.3) K/U
23. The wave model of light cannot explain all wave properties of light. (9.4) K/U
24. When light travels from air to glass, the wave theory predicts that the light will slow down but Newtonian particle theory predicts that the light will speed up. (9.4) K/U
25. When a Young's double-slit apparatus is operated in air and then submerged in water, the fringe pattern becomes less spread out. (9.5) K/U
26. Together, Fresnel's and Huygens' principles explain the rectilinear propagation of light and diffraction effects. (9.5) K/U
27. Increasing the width of the slit in a single-slit apparatus causes the distance between adjacent dark fringes to decrease. (10.2) K/U
28. For a diffraction grating, the angular separation of the maxima is generally small because the slit spacing is so small. (10.3) K/U
29. Polaroid is used in sunglasses to protect your eyes from glare. (10.5) K/U

Write a short answer to each question.

30. State the law of reflection. (9.1) K/U
31. Identify the factors that determine the frequency of a wave. (9.1) K/U
32. How does the speed of a light wave vary with the index of refraction? (9.2) K/U
33. Define the node and nodal line. (9.3) K/U
34. A dam with a small opening is placed in a ripple tank. The water waves have a wavelength of 4 cm. What size of opening will produce the largest diffraction: a 1 cm opening, an 8 cm opening, or a 20 cm opening? Explain your answer. (9.3) K/U
35. A wave encounters a barrier that has an opening that is similar in size to the wavelength. What happens to the wave? (9.3) K/U
36. Identify the quantities that determine the amount of diffraction of a light wave due to a single slit. (9.3) K/U
37. Describe what happens to the spacing of nodal lines in a ripple tank with two sources when the source separation increases. (9.3) K/U T/I
38. How did Newton explain refraction in his particle (corpuscular) theory of light? (9.4) K/U
39. Which model of light would you expect to better explain polarization: the corpuscular model or the wave model? Explain your answer. (9.4, 10.5) K/U
40. Summarize Huygens' principle in your own words. (9.4) K/U C
41. Identify two obstacles that challenged early attempts to show light interference. (9.5) K/U

42. How did Young solve the problem of phase difference in the light sources for his double-slit experiment? (9.5) K/U
43. Name a property of light that the wave theory of light, as corroborated by Young's experiment, could not explain. (9.5) K/U
44. How would the interference pattern in Young's experiment change if the two slits had a smaller separation? (9.5) K/U
45. The index of refraction of a piece of glass is 1.5. Compare the frequency of a light wave as it travels through the glass to the frequency of the light wave as it travels through a vacuum. (10.1) K/U
46. State the conditions that cause the formation of Newton's rings. (10.1) K/U
47. Describe the effect on resolution of increasing the wavelength of light passing through the aperture of an optical instrument. (10.2) T/I
48. Describe what happens to the distance between maxima and minima in single-slit diffraction as the slit width decreases. (10.2) K/U T/I
49. How does the spread of different wavelengths of light by a diffraction grating depend on the number of slits in the grating? (10.3) K/U
50. Determine the ratio of the speeds of light rays of wavelengths 400 nm and 800 nm in a vacuum. (10.4) K/U
51. Identify three methods of polarizing light. (10.5) K/U
52. State how the optical activity of molecules in a solution can be detected using two polarizers. (10.5) K/U
53. What is Brewster's angle? Describe the polarization of the reflected light when the angle of incidence is equal to Brewster's angle. (10.5) K/U

Understanding

54. In a graphic organizer of your choice, compare light and transverse water waves. (9.1) K/U
55. Use Snell's law to explain why total internal reflection only occurs for waves passing from a medium with a higher index of refraction into a medium with a lower index of refraction. (9.2) K/U
56. Explain why white light does not break down into colours when it passes through a flat piece of window glass, as it does with a prism. (9.2) K/U
57. A boat is on a pond opposite a straight line of shore. Suppose you placed two devices that generate mechanical waves in the water at two points on the shore. (9.3) K/U
    (a) Describe the behaviour of the boat at a point where constructive interference of the waves takes place.
    (b) Describe the behaviour of the boat at a point where destructive interference of the waves takes place.
58. Explain why we can hear sounds around corners, but we cannot see around corners, even though sound and light both travel as waves. (9.3) K/U
59. What assumption did Newton make about the refraction of light "corpuscles" that was later shown to be incorrect? (9.4) K/U
60. Examine the successes and failures of the particle and wave models of light. (9.4) K/U
    (a) Name three properties of light accounted for by both models.
    (b) Name three properties not accounted for by the particle model.
    (c) Name one property not accounted for by the wave model.
61. Assuming that you had a means of measuring the change in radio wave intensity, explain why performing the Young double-slit experiment with radio waves would be easier than performing it with visible light. (9.5) K/U
62. Explain why in Young's time the observation of double-slit interference was more convincing than the observation of diffraction as evidence for the wave theory of light. (9.5) K/U
63. Explain why the light from two incandescent bulbs does not form an interference pattern. (9.5) K/U
64. Explain why the constructive interference for a thin film that covers a piece of glass differs from that for a thin film surrounded by air. (10.1) K/U
65. Explain why the first-order maximum in a diffraction pattern is not as intense as the central maximum. (10.2) K/U
66. The central bright fringe in a diffraction pattern has a width that equals the distance between the screen and the slit. Determine the ratio of the wavelength of light to the width of the slit in the single-slit diffraction setup. (10.2) K/U T/I
67. Explain why increasing the aperture of an optical instrument affects the instrument's resolution. (10.2) K/U

68. Identify the advantages of a diffraction grating over a prism for use in spectral analysis experiments. (10.3) K/U C A
69. Describe what would happen to the distance between the bright fringes produced by a diffraction grating if you immersed the apparatus in water, including the light source, grating, and screen. (10.3) K/U T/I
70. Summarize how polarization validates the wave theory of light. (10.5) K/U
71. A beam of polarized light passes through a sheet of polarizing material. (10.5) K/U
   (a) Describe the intensity when the angle between the axis of the polarizer and the electric field of the light is 0°.
   (b) Describe the intensity when the angle between the axis of the polarizer and the electric field of the light is 90°.
   (c) Explain how Malus's law describes the change in intensity of polarized light.
72. Illustrate with a sketch how polarizing sunglasses reduce glare from reflected sunlight. (10.5) K/U T/I C

Analysis and Application

73. A quartz prism has an index of refraction for red light of 1.459 and an index of refraction for blue light of 1.467. The angle of incidence of white light is 25° on one side of the prism. Calculate the difference in the angles of refraction for red and blue light to the nearest tenth of a degree. (9.2) T/I
74. A fish tank is made of glass with an index of refraction of 1.50. A beam of light strikes one surface at an angle of 35.0° with respect to the normal. Determine the angle of refraction of the light through the glass. (9.2) K/U T/I
75. Calculate the angle of refraction for a ray of light that enters a bucket of water at an angle of incidence of 25.0°. The index of refraction of the water is 1.33. (9.2) T/I
76. Sunlight passes into a raindrop at an angle of 22.5° from the normal. The raindrop has an index of refraction of 1.33. What is the angle of refraction? (9.2) T/I
77. Determine the critical angle for light inside a diamond at the diamond–air boundary. The diamond has an index of refraction of 2.42. (9.2) T/I
78. The critical angle as light passes from acrylic to air is 42.4°. Calculate the index of refraction of the acrylic. (9.2) T/I
79. Silicate flint glass has an index of refraction of 1.60. Determine the wavelength of cyan light in the glass. The wavelength of cyan in air is 485 nm. (9.2) T/I
80. A nearly plane water wave with a wavelength of 1.4 m passes through two narrow openings within a concrete pier. The openings are 8.6 m apart. Calculate the angle of the third nodal line. (9.3) T/I A
81. Two point sources produce waves with the same phase and frequency and a speed of 7.5 m/s. The waves produce an interference pattern, and a point on the second nodal line is 23.0 cm from one source and 25.5 cm from the other source. Calculate
   (a) the wavelength of the waves
   (b) the frequency of the waves (9.3) T/I
82. You perform Young's double-slit experiment and observe a distance of 5.8 cm between the first and seventh bright fringes on a screen located 3.50 m from the slit plate. The slits have a separation of 2.2×10^{-4} m. Calculate the wavelength of the light that you used and identify its colour. (9.5) T/I
83. In a Young's double-slit experiment, the second dark fringe on either side of the central bright fringe is at an angle of 5.1°. Determine the ratio of the slit separation to the wavelength of the light. (9.5) T/I
84. Monochromatic light falls on two slits with a separation of 0.018 mm. The fifth-order dark fringe appears on a screen at an angle of 8.2°. Calculate the wavelength of the light. (9.5) T/I
85. Light with a wavelength of 638 nm falls on two narrow slits. The third-order bright fringe appears at an angle of 8.0°. Calculate the distance between the slits. (9.5) T/I
86. A helium–neon laser directs red light with a wavelength of 633 nm at a screen with two narrow slits separated by 0.100 mm. A fringe pattern appears on a screen 2.10 m away. Calculate the distance from the first dark fringe to the central maximum. (9.5) T/I
87. Monochromatic light falls on two narrow slits separated by 0.042 mm. The separation between minima on a screen 4.00 m away is 5.5 cm. Calculate
   (a) the wavelength of the light
   (b) the frequency of the light (9.5) T/I
88. Light with a wavelength of 639 nm falls on two narrow slits separated by 0.048 mm. The light produces an interference pattern on a screen 2.80 m away. Determine the distance from the first dark fringe to the central maximum. (9.5) T/I
89. In a Young's double-slit experiment, the two slits are 0.5 mm apart. The screen is placed 1.0 m from the slits. The distance between the eleventh fringe and the first fringe is 1.0 cm. Calculate the wavelength of light used. (9.5) T/I

90. In a Young's double-slit experiment, two slits are separated by 3.0 mm and illuminated by light with a wavelength of 480 nm. The screen is 2.0 m from the plane of the slits. Calculate the separation between the eighth bright fringe and the third dark fringe observed with respect to the central bright fringe. (9.5) T/I

91. Light with a wavelength of 6.5×10^{-7} m is incident on two slits 1.0 mm apart. A screen is placed at a distance of 1.0 m from the slits. Calculate the distance between the third dark fringe and the fifth bright fringe. (9.5) T/I

92. Light with a wavelength of 656 nm falls on two narrow slits, producing an interference pattern on a screen 1.50 m away. The fourth-order maxima are separated from the central maximum by 48.0 mm. Calculate the slit separation. (9.5) T/I

93. Light with a wavelength of 465 nm in a vacuum falls on two slits separated by 5.00×10^{-4} m and produces an interference pattern on a screen 50.0 cm away. The apparatus sits under water with an index of refraction of 1.33. Calculate the fringe separation distance. (9.5) T/I

94. Light illuminates an oil slick on water, which then appears blue at normal incidence. Calculate the minimum thickness of the oil slick, assuming that the blue light has a wavelength of 410 nm and that the oil has an index of refraction of 1.40. (10.1) T/I

95. The glass of a camera lens, with an index of refraction of 1.52, is covered by a non-reflective coating of magnesium fluoride. The index of refraction of the coating is 1.38. The coating prevents the reflection of yellow–green light at a wavelength of 582 nm. Determine the minimum thickness of the coating. (10.1) T/I

96. A piece of glass with an index of refraction of 1.52 has a transparent coating with an index of refraction of 1.62. The glass appears black when viewed in reflected monochromatic light with a wavelength of 587 nm. Calculate the two smallest possible non-zero values for the coating's thickness. (10.1) T/I

97. Anti-reflective coatings deposited on lenses reduce reflection and improve the transmission of light. This occurs by destructive interference at the coating's surface. The coating has a minimum thickness of 121 nm, and the light chosen has a wavelength of 566 nm. Calculate the index of refraction of the coating. (10.1) T/I A

98. A glass plate with an index of refraction of 1.52 is spread with a thin film of ethyl alcohol with an index of refraction of 1.36. The plate is illuminated with white light, and a region of the film reflects only green light with a wavelength of 524 nm. Determine the thickness of the alcohol film. (10.1) T/I

99. A soap bubble with an index of refraction of 1.33 and a thickness of 109 nm is illuminated by white light. Determine the wavelength and colour of the light that is most constructively reflected. (10.1) T/I

100. A thin plastic sheet separates one end of two pieces of flat glass, making an air wedge. You see 36 dark lines when you illuminate the glass with 628 nm light. Determine the thickness of the sheet. (10.1) T/I

101. You create an air wedge between two sheets of glass with a sheet of paper 7.62×10^{-5} m thick. Determine the number of bright fringes that you would see along the wedge when you illuminate it with 529 nm light. (10.1) T/I

102. You create an air wedge with two 15.4 cm glass plates and a fine metal foil inserted between the plates at one end. You illuminate the wedge with 532 nm light and observe that the average distance between two dark bands in the interference pattern is 1.4 mm. Calculate the thickness of the metal foil. (10.1) T/I

103. You dip a wire loop into a soap solution and then hold it in front of monochromatic yellow light. You notice at one instant that the reflected pattern looks similar to **Figure 1**. (10.1) T/I A

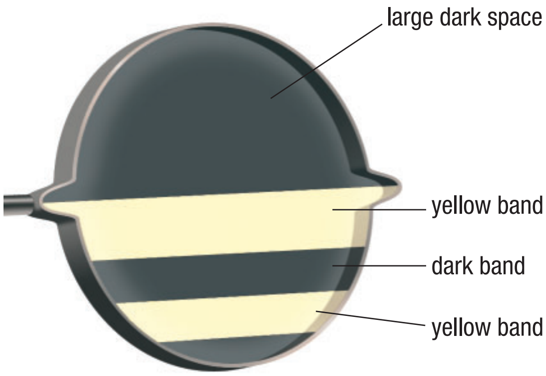


Figure 1

(a) Explain the large dark space that you see at the top of the soap film.
(b) Describe the changes that you will see in the pattern as time goes on. Explain your answer.
(c) Calculate the difference between the thicknesses of the soap film in adjacent bright bands. Express your answer in terms of the wavelength of the reflected light and the index of refraction of the film.
(d) The soap has an index of refraction of 1.33, and the yellow light has a wavelength of 587 nm. Calculate the thickness of the soap film in the lowest dark band.

104. Monochromatic blue light with a wavelength of 425 nm passes through a slit with width 0.35 mm. Calculate the width of the central maximum on a screen located 2.0 m from the slit. (10.2) T/I

105. Light shines through a single 5.60×10^{-4} m slit. A diffraction pattern forms on a screen 3.00 m away. The separation between the middle of the central maximum and the first dark fringe is 3.5 mm. Calculate the wavelength of the light. (10.2) T/I

106. Light with a wavelength of 589 nm falls on a 1.08×10^{-6} m slit and produces a diffraction pattern. (10.2) T/I
   (a) Determine the angle of the first minimum.
   (b) Does a second minimum appear? Explain your answer.

107. A helium–neon laser produces infrared light with a wavelength of 1.15×10^{-6} m. The laser illuminates a narrow single slit. The centre of the fourth dark band lies at an angle of 8.4° off the central axis. Determine the width of the slit. (10.2) T/I

108. A beam of 639 nm light shines through a single slit that is 4.2×10^{-4} m wide onto a screen 3.50 m away. Determine the separation between the maxima. (10.2) T/I

109. Telescopes are designed to observe the universe in different parts of the electromagnetic spectrum. Why do you think the telescopes used to study X-ray sources have excellent resolution, yet do not have large apertures? (10.2) K/U A

110. While performing a single-slit diffraction experiment, you observe the diffraction pattern in **Figure 2(a)**. You make some adjustments to the equipment and then observe that the pattern has changed, as in **Figure 2(b)**. Identify the changes that could have produced the result, and explain your answer. (10.2) K/U T/I A

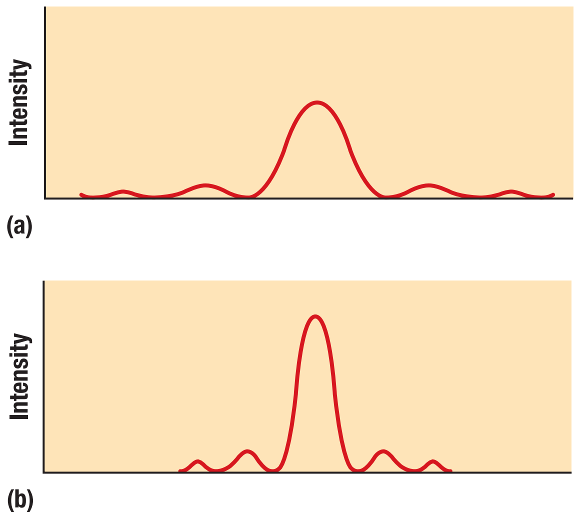


Figure 2

111. You perform a single-slit diffraction experiment with a microwave source emitting 22.5 GHz microwaves through a slit that is 1.9 cm wide onto a screen 0.45 m away. In **Figure 3**, the distance from the central maximum to the first dark fringe is labelled B, and the distance from the slit to the first dark band is labelled C. Predict the numerical values for the lengths of B and C. (10.2) T/I

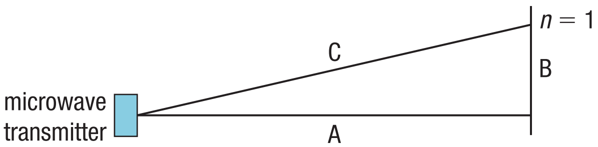


Figure 3

112. You have directed a parallel beam of microwaves at a metal screen with an opening 20.0 cm wide. As you move a microwave detector parallel to the plate, you locate a minimum at 36° from the central axis. Determine the wavelength of the microwaves. (10.2) T/I

113. Diffraction can limit the magnification power of a telescope. (10.2) K/U C
   (a) Describe how changing the aperture of the telescope affects the diffraction of the incoming light.
   (b) Due to diffraction, a point source of light will appear as an Airy disk. Describe the distinctive features of an Airy disk.
   (c) In an ideal optics arrangement, 84 % of the incoming light is concentrated in the central maximum of the Airy disk. If there are obstructions, like mirrors, in the optical path, then more light will be spread into the rings. Based on this, do you expect a refracting or a reflecting telescope to give the clearer image?

114. Describe how to design a diffraction grating that displays just first-order maxima. (10.3) T/I

115. A certain diffraction grating has slits separated by 2.1×10^{-6} m. You illuminate the grating with a mixture of light with wavelengths ranging from 411 nm to 664 nm and observe rainbow-like spectra on a screen 2.96 m away. Calculate the angle and location of the first-order spectrum. (10.3) T/I

116. Monochromatic light passes through a diffraction grating with 5100 lines/cm. A second maximum appears in the spectra at 33°. Determine the wavelength of the light. (10.3) T/I

117. You use an 8600 lines/cm spectroscope to observe a light source and measure two first-order spectral lines at angles, to one side of centre, of 26.6° and 41.1°. Calculate the wavelengths of the observed light. (10.3) T/I

118. (a) Prove that a 26 000 lines/cm diffraction grating cannot produce a maximum for visible light.
(b) Determine the longest wavelength for which this grating can produce a first-order maximum. (10.3) T/I

Evaluation

119. Describe in your own words the advantage of observing interference and diffraction of water waves over light waves. (9.3, 9.5) K/U C

120. Your cellphone provider adds a second tower in your neighbourhood that transmits signals in phase with its original tower. Are you guaranteed better reception? Justify your answer. (9.3) K/U T/I A

121. A radio wave transmitter tower and a receiver tower stand 62.0 m above the ground and 0.40 km apart. The receiver receives signals directly from the transmitter and indirectly from reflections off the ground. If a half-wavelength phase shift occurs on reflection, determine the longest possible wavelengths that interfere at the receiver
(a) constructively
(b) destructively (9.3) K/U T/I A

122. In **Figure 4**, narrow slit S_1 is positioned far from a screen. Between the slit and the screen is a plane mirror, which is perpendicular to the screen. A beam of monochromatic light is incident on S_1. Some of the light goes through the slit to the screen, and some reflects off the mirror. The image of the slit in the mirror provides a second source of light, which illuminates the screen. This is a classic experiment called Lloyd's mirror. (9.5) K/U T/I A

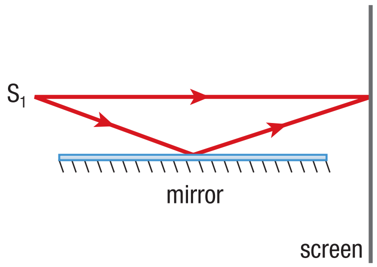


Figure 4

(a) Is the image of the slit in the mirror coherent with the slit itself? Explain.
(b) Does the screen show a double-slit interference pattern or a pair of single-slit diffraction patterns? Explain.
(c) If the pattern is an interference pattern, is the fringe closest to the mirror surface bright or dark? Explain.

123. Light with a wavelength of 583 nm shines on a convex lens that rests on a flat glass surface. A total of 32 dark Newton's rings appear, not counting the central dark spot. Determine the difference in thickness between the centre and edges of the lens. (10.1) K/U T/I

124. Light passing through the pupil of your eye undergoes diffraction similar to that of a single slit. Consider light with a wavelength of 685 nm. Assume that the width of the average pupil is 6.5 mm. Since the opening is circular, the equations for the diffraction pattern are different than those for slits. The angular location of the first minimum is given by the equation $\sin\theta_1 = \frac{1.22\lambda}{d}$, where d is the diameter of the aperture. What is the angle between the central and first dark fringes? (10.2) T/I A

125. Telescope A has an aperture diameter of 100 mm and telescope B has an aperture diameter of 50 mm. (10.2) T/I A
(a) Which telescope experiences more diffraction?
(b) A common rule says that the maximum magnification of a telescope is equal to twice the aperture diameter in millimetres. According to the rule, what are the maximum magnifications for telescopes A and B?
(c) A telescope advertisement describes a telescope with aperture 75 mm and maximum magnification of 625 times. Do you believe the advertisement? Why or why not?

126. A sonar transmitter under water produces 12 kHz waves with the intensity pattern shown in **Figure 5**. The solid lines represent regions of constant sound intensity. The intensity pattern resembles a single-slit interference pattern in optics with the transmitter acting as the slit. Calculate the width of the vibrating surface of the transmitter. The speed of sound in water is 1.40×10^3 m/s. (10.2) K/U T/I A

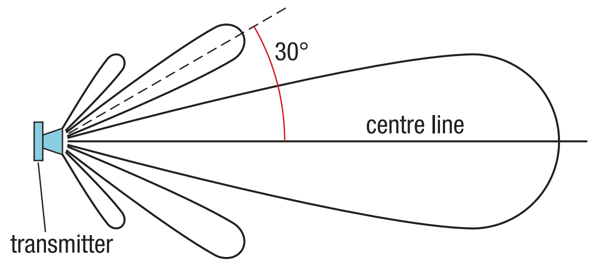


Figure 5

127. Radar systems use radio waves to "see" objects, analogous to the way our eyes use visible light. Explain whether or not it would be easier to resolve nearby objects using X-rays rather than radio waves. (10.2) K/U T/I A

128. White light shone through a certain diffraction grating produces just two full spectral orders on either side of the central maximum. Calculate the maximum possible number of lines per centimetre for this grating. (10.3) K/U T/I

129. White light reflects from a soap film with an index of refraction of 1.33 onto the slit of a spectrograph. The light shines at normal incidence through the slit onto a diffraction grating with 520 lines/mm. A dark band appears in the first-order spectrum at an angle of 17° to the normal. Calculate the minimum possible thickness of the soap film. (10.3) K/U T/I

130. A traditional radio receiver picks up pops and whistle sounds. These sounds result from lightning in storms. Explain how this happens. (10.4) K/U A

131. In a graphic organizer of your choice, compare counterfeiting technology based on using holograms and on using NOtES. Include the advantages and disadvantages of each technology. Which technology do you think is more effective? Explain your answer. Give an example of how your preferred technology can be used in a way not mentioned in this text. (10.7) T/I

132. Compare and evaluate the pros and cons of GPS technology in a graphic organizer. (10.8) T/I C

Reflect on Your Learning

133. How did interference and diffraction of water waves help you understand the same processes when they occurred with light waves? K/U

134. What further aspects would you like to know about fibre optics in telecommunications? K/U A

135. How did the information that you learned in this unit affect your thinking about diffraction gratings? T/I A

136. What would you like to know about astronomical observatories that use different regions of the electromagnetic spectrum? T/I C

137. GPS technology is becoming increasingly prevalent in our everyday lives. The technology is used in cellphones, cars, ships, and personal tracking devices. Based on what you now know about GPS technology, does it change the way you think about privacy? Explain your answer to a peer. T/I C

138. In Grade 10 science you also studied light waves. What is the difference between the unit on light in Grade 10 and the unit on light in Grade 12? K/U T/I A

Research

WEB LINK

139. Investigate the design and functioning of fibre optics technology. Research the Internet and other sources. In a few brief paragraphs, discuss the advantages and disadvantages of fibre optics technology. C A

140. Research the production of diffraction gratings. Summarize in a few paragraphs current methods for producing gratings with different spacings, including the smallest gratings currently made. C A

141. Research the polarization of rainbows. Describe in a format of your choice (oral presentation, diagrams, or a one-page report) the major causes of polarized light in rainbows. Describe how double rainbows are formed. C A

142. Image processing is very important in modern optical systems. Computers can be used to remove image distortions due to diffraction. Research the use of a point spread function. Investigate different techniques of image processing as used in astronomy. Summarize your findings in a format of your choice. C A

143. Laser surgery has become very common for a range of treatments, such as eye surgery, knee replacements, and treating varicose veins (**Figure 6**). Research laser surgery, and compare it with traditional surgery. A

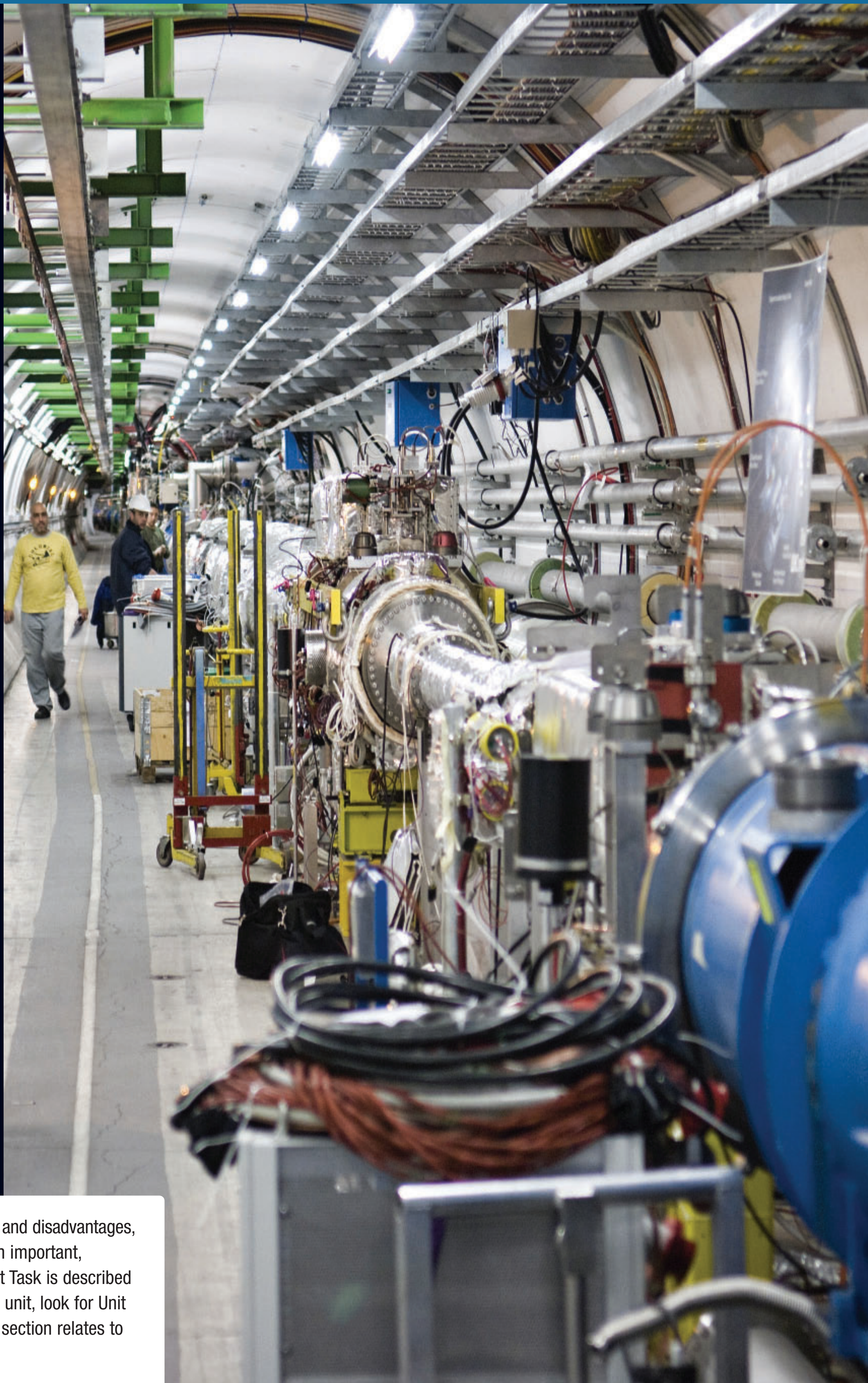
Figure 6 Varicose veins are dilated and bulging veins that can be uncomfortable and can interfere with circulation. The surgeon heats the vein using pulses of laser light through a catheter. The varicose vein disappears over time.

(a) Is the same type of laser used for all surgeries? Provide some examples.
(b) What are some advantages of laser surgery compared with traditional surgery?
(c) What are some disadvantages of laser surgery?

144. Research the production of X-rays and how X-rays are used in medicine. K/U T/I C A
(a) Describe how X-rays can be produced based on energy transmissions in an atom.
(b) Describe how an X-ray emitter works.
(c) List two risks of using X-rays to treat tumours.
(d) Describe an emerging nanotechnology that uses carbon nanotubes to produce X-rays. Compare and contrast the emerging technology to traditional X-ray technology.

UNIT 5

Revolutions in Modern Physics: Quantum Mechanics and Special Relativity



OVERALL EXPECTATIONS

- analyze, with reference to quantum mechanics and relativity, how the introduction of new conceptual models and theories can influence and/or change scientific thought and lead to the development of new technologies
- investigate special relativity and quantum mechanics, and solve related problems
- demonstrate an understanding of the evidence that supports the basic concepts of quantum mechanics and Einstein's theory of special relativity

BIG IDEAS

- Light can show particle-like and wave-like behaviour, and particles can show wave-like behaviour.
- The behaviour of light as a particle and the behaviour of particles as waves can be described mathematically.
- Time is relative to a person's frame of reference.
- The effects of relativistic motion can be described mathematically.
- New theories can change scientific thought and lead to the development of new technologies.

UNIT TASK **PREVIEW**

In the Unit Task you will analyze the advantages and disadvantages, including costs, of the Large Hadron Collider, an important, expensive modern physics experiment. The Unit Task is described in detail on page 666. As you work through the unit, look for Unit Task Bookmarks to see how information in the section relates to the Unit Task.

FOCUS ON **STSE**

PARTICLE ACCELERATION AND THE LARGE HADRON COLLIDER

The Large Hadron Collider (LHC), a particle accelerator at the European laboratory CERN, is the most complicated machine ever built. The LHC has a circumference of 27 km and lies in a tunnel under Switzerland and France. The experimental team involves over 10 000 scientists from over 100 countries. Construction took over a decade, and CERN plans to operate the LHC for the next several years. The amount of data produced in a single year could fill 100 000 DVDs. The final bill for construction and operation will be billions of dollars. This investment has caused controversy, because some people believe there are more pressing problems that the money and effort of scientists could be allocated to solving.

The LHC is able to create two beams of particles that travel in opposite directions inside the machine's circular accelerator. Each time the particles complete a lap, they gain more and more energy. Eventually, physicists cause the particles to collide, releasing an immense amount of energy that physicists use to try to recreate the conditions just after the big bang. The collider will allow researchers to test our current understanding of fundamental particles and other basic physics concepts. The current model of particle physics explains much of what we see in the universe, but not everything. One predicted fundamental particle, the Higgs boson, has never been detected. Researchers at the LHC expect to observe new physics and possibly signs of the Higgs boson.

Building and operating the LHC requires huge amounts of time, money, and effort, but the machine will produce some incredible results, both for science and for society. The experimental results could change the world, giving physicists profound insight into the true nature of reality itself. Their findings could revolutionize our current understanding of the universe, forcing scientists to no longer support some currently accepted ideas and enter into uncharted territory. The LHC has also already proven that countries from all over the globe can work together to increase our understanding of science. Furthermore, researchers and engineers have developed new technology to turn the original design of the LHC into a real, functioning machine. The new technologies have resulted in many spinoff technologies. Understanding the scientific issues of the LHC requires learning about modern physics, including relativity and the physics of subatomic particles.

Questions

1. Do you think scientific experiments will continue to grow in size and cost? What are the limits?
2. What do you hope scientists will discover with the LHC?
3. Do you think limits should be set on the resources allocated to "pure research"? If so, who should regulate it, and what guidelines should be used? Explain your reasoning.

CONCEPTS

- motion problems
- inertial and non-inertial reference frames
- field theory
- interference patterns of light
- standing waves and harmonics
- theory and experimentation
- conservation of energy and momentum

SKILLS

- interpreting images and patterns on graphs
- solving for unknowns in algebraic equations
- practising safe laboratory procedures
- designing experiments to test hypotheses
- communicating scientific information clearly and accurately

Concepts Review

1. A child stands in the aisle of a train moving eastward at 40.0 m/s. For a short time, the child runs forward in the same direction as the train is moving at a constant speed, covering 15 m in 3 s. K/U T/I
   (a) Calculate the velocity of the child relative to an observer on the train.
   (b) Calculate the velocity of the child relative to an observer on the ground.
   (c) A second train moves westward at a speed of 30.0 m/s. Calculate the velocity of the child relative to an observer on the second train.
2. Consider each of the following situations. For each situation, identify whether the person is in an inertial reference frame or a non-inertial reference frame. Explain your choices. K/U A
   (a) a taxi driver accelerating away from a stop light
   (b) a child riding around the outer edge of a carousel
   (c) a race car driver travelling at a constant 200 km/h down the straight section of a racetrack
   (d) a driver backing straight out of a garage at a slow and constant speed
   (e) a student riding swiftly down the first hill of a roller coaster
   (f) an airplane maintaining a fixed direction and constant velocity
3. State the law of conservation of energy. K/U
4. (a) What is electromagnetic radiation?
   (b) Give three examples of types of electromagnetic radiation.
   (c) List the three central properties of electromagnetic waves.
   (d) At what speed do electromagnetic waves travel through a vacuum? K/U
5. State the law of conservation of momentum. K/U
6. A student tries using two white LEDs as light sources in a double-slit interference experiment. Unfortunately, the student does not see the expected bright fringes. Explain why the two LEDs are not viable light sources for this experiment. T/I C
7. Red laser light shines through a double slit and creates equally spaced bright fringes 2.3 cm apart on a distant screen. Explain what would happen to the fringe spacing if
   (a) green laser light were used instead of red
   (b) the slits were moved closer together T/I A
8. A negatively charged plastic rod is brought close to, but does not touch, a metal sphere (**Figure 1**). Explain what happens inside the sphere, and explain why it happens. K/U C

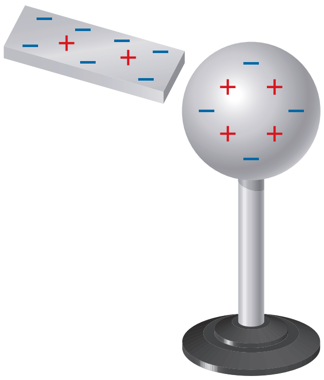


Figure 1

9. Science develops through a combination of theory and experiment. K/U C
   (a) Explain the conflict between Newton and Huygens regarding their theories of light.
   (b) Describe how Young's double-slit experiment changed scientific understanding of light.

10. (a) What is a diffraction grating?
    (b) What information can be obtained using a diffraction grating?
    (c) List some applications of spectroscopy. K/U
11. Explain how standing waves result when waves created at the loose end of a string reflect off the fixed end. K/U C
12. Match each term on the left with the most appropriate description on the right. K/U
    (a) hypothesis — (i) an explanation that has been tested and confirmed as a general principle to explain a natural phenomenon
    (b) postulate — (ii) a generally accepted formal statement about the occurrence of a natural phenomenon
    (c) scientific theory — (iii) a predicted answer to a testable question
    (d) scientific law — (iv) a statement assumed to be true from which a theory is developed
13. Describe the Bohr model of the atom. K/U

Skills Review

14. Use the wave equation to calculate the frequency of each of the wavelengths below. T/I
    (a) 640 nm
    (b) 510 nm
    (c) 320 nm
15. Which wavelength in Question 14 has the highest energy? K/U T/I A
16. When gas of a certain element is heated enough to emit light, the light shines with only certain wavelengths. Use Bohr's atomic theory to interpret these results. K/U
17. A 1.2 m–long string fixed at both ends vibrates at 51 Hz. Three antinodes appear on the string between the fixed ends. K/U T/I
    (a) Determine in what harmonic mode the string is vibrating.
    (b) Calculate the fundamental frequency.
    (c) Predict the frequency that would cause seven antinodes to appear on the fixed string.
18. Your classmate is about to perform an investigation involving an electric circuit. Write a list of safety precautions that your classmate must follow to ensure the investigation is completed safely. K/U C
19. Solve for x in each of the following equations. T/I
    (a) $\left(1 - \dfrac{x^2}{9 \times 10^{16}}\right)^{-\frac{1}{2}} = 8.7$
    (b) $6.3 = (2.4 \times 10^{-15})x - 1.7$
    (c) $x = \dfrac{5.23 \text{ units}}{\sqrt{1 - \dfrac{(2.0 \times 10^8)^2}{(3.0 \times 10^8)^2}}}$
20. (a) Plot the data in **Table 1** on a graph.

Table 1

x	y
0	100.0
20.0	150.0
40.0	200.0
60.0	250.0

    (b) Determine the y-intercept.
    (c) Determine the slope of the line.
    (d) Derive the equation that represents the data.
    (e) What can you determine about the relationship between x and y? T/I C
21. Look at the graph in **Figure 2**. K/U T/I

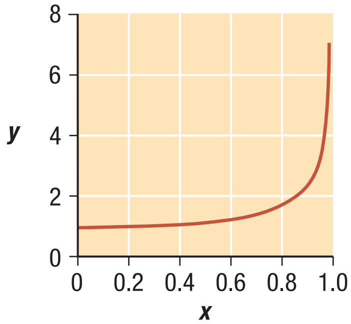


Figure 2

    (a) What type of curve is this?
    (b) What happens to the curve in Figure 2 as it approaches 1.0?

CAREER PATHWAYS PREVIEW

Throughout this unit, you will see Career Links. Go to the Nelson Science website to find information about careers related to Revolutions in Modern Physics. On the Chapter Summary page at the end of each chapter, you will find a Career Pathways feature that shows you the educational requirements of the careers. There are also some career-related questions for you to research.

CHAPTER 11 Relativity

KEY CONCEPTS

After completing this chapter you will be able to

- describe why Einstein questioned the scientific understanding of the late nineteenth century, and in doing so revolutionized our understanding of time and space
- understand Einstein's two postulates of special relativity
- describe some of the experimental evidence that supports the predictions of special relativity
- understand how developments in relativity theory can lead to new applications and technologies that impact society and the environment
- solve problems and analyze data for situations requiring special relativity, including time dilation, length contraction, simultaneity, and relativistic momentum
- understand the origins of Einstein's famous equation, $E = mc^2$, and how it relates to energy issues
- solve problems using the mass–energy equivalence equation

How Can an Understanding of Space and Time Help Solve Our Energy Problems?

Just over a century ago, a crisis occurred in physics. The models that had worked well in the 1800s could not explain the results of several experiments, including one that tried to prove the existence of the medium that electromagnetic waves were thought to travel through. These problems led Albert Einstein to consider how light behaves for observers moving at very high speeds. Einstein's conclusions have had a significant impact on our ideas about space and time.

A far-reaching part of Einstein's theory was the discovery of the connection between mass and energy. This vision led researchers to study how to convert the mass of certain atomic nuclei into other forms of energy. Out of this research came the discovery and application of nuclear fission, both for nuclear weapons and for power-generating nuclear reactors.

The equivalence of mass and energy, described by Einstein's theory, also explains how stars, including the Sun, produce energy. Would it be possible to create an artificial Sun here on Earth? Imagine if artificial suns that do not produce dangerous greenhouse gases or radioactive waste could supply the energy needs of the world.

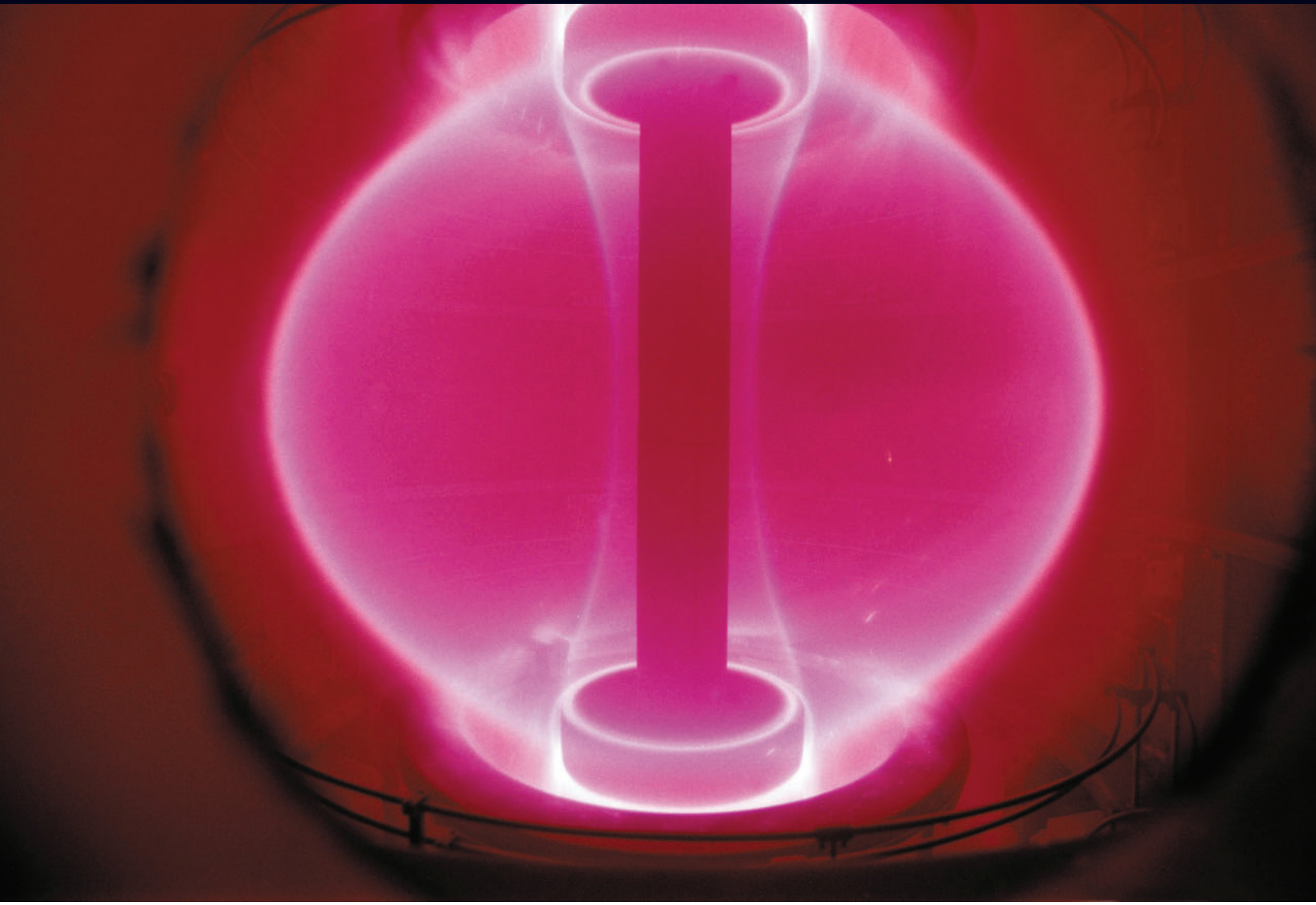
Researchers have tried to recreate the nuclear processes that occur in the Sun in the laboratory and are developing new technologies to improve methods of achieving nuclear fusion. The image on the facing page shows nuclear fusion research involving plasma. Fusion research teams include international collaborations such as ITER, a large-scale, international experiment related to harnessing nuclear fusion, and private companies such as Canada's General Fusion. This process has yet to become practical, but hope remains that some day nuclear fusion could provide energy from hydrogen for centuries to come.

In this chapter, you will learn about the theories behind our current understanding of space and time, and you will learn how this knowledge has the potential to solve our energy problems.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. Suppose you are in a car with no windows. The car moves at a constant speed in a straight line. Is there any way you can tell if you are moving? Why or why not?
2. Suppose you throw a rock into a pond exactly at noon. Do the waves reach the other shore at noon as well? Why or why not?
3. What do you think the relationship is between matter and energy?



Mini **Investigation**

Who Is Moving?

Skills: Planning, Performing, Observing, Communicating

SKILLS HANDBOOK A2.5

If you have ever stood near a moving train, you may have had the sensation that you were the one moving. This sensation is one way of understanding Einstein's theory about space and time. In this investigation, you will make a short video that deliberately confuses your viewing audience by making it impossible for them to tell whether the camera is moving or whether the object being recorded is moving.

Equipment and Materials: cellphone or digital camera that makes video recordings; object of your choice

1. Use the camera to show motion between the camera and an object of your choice. The motion should be such that no one viewing the video can tell whether the object or the camera is moving. (Hint: Keep in mind that the motion of background objects indicates whether an object is moving.)

A. What did you need to do to trick the audience? C A

B. Write two statements describing the apparent motion in the video. T/I C

11.1 The Special Theory of Relativity

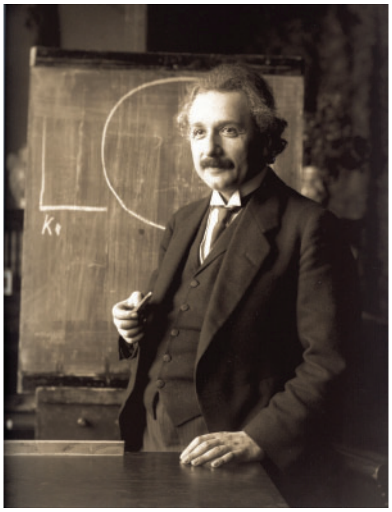


Figure 1 Albert Einstein's ideas in physics changed our perception of space and time.

At the turn of the twentieth century, most of the physics community enjoyed a sense of accomplishment. Newtonian mechanics (also called classical mechanics), based on Newton's laws of motion, provided the principles by which all matter behaved. Physicists expanded these principles to the atomic level and established the idea of energy conservation. Building on James Clerk Maxwell's mathematical description of electric and magnetic fields, physicists successfully unified the subjects of electricity, magnetism, and optics. Maxwell's ideas had an apparent answer to the true nature of light: light is a combination of oscillating electric and magnetic fields.

At that time, only a few questions remained. The nature of the atom was still unclear. Physicists had observed that light emitted from perfect radiators (called "blackbodies," which you will learn more about in Chapter 12) did not behave as predicted. This gave rise to the so-called blackbody problem. The phenomenon by which metals give up electrons under certain kinds of light also had physicists mystified. Moreover, contrary to Maxwell's assumption that electromagnetic waves, like all waves, needed a medium, there was no evidence to support this. Physicists expected to solve these problems within the framework of familiar physical models and principles. However, new models and principles needed to be developed in order to explain these problems, and these new ideas radically changed the way physicists understood their once-familiar universe. One of the main contributors to these new models was Albert Einstein (1879–1955) (**Figure 1**). CAREER LINK

The Principle of Relativity

Suppose you are inside a room that has no windows. A billiard table stands near the centre of the room. If you strike the cue ball with the cue stick, you will notice that the ball moves in the direction of the applied force. This is exactly what you would expect to happen, according to Newton's second law of motion ($\vec{F} = m\vec{a}$). Similarly, you observe that linear momentum is conserved when the cue ball collides with another ball. You will also note that mechanical energy is conserved if the collision is elastic. Everything that you observe is consistent with the principles and laws of Newtonian mechanics.

What you do not know is that the room is not simply a windowless room in a building—it is in a railway car that can move on a set of tracks. Now suppose that the railway car starts to move, and for a brief time accelerates from zero to a final constant velocity. You will notice during this time that the laws of motion will appear different in the room. The billiard balls on the table will roll backward on their own, as if acted upon by an unseen force. You may even have trouble walking because of the sensation of a force acting on you. All of this is occurring because everything in the room is accelerating along with the railway car.

By the time the car is moving along a straight, level stretch of smooth track at a constant speed, the conditions in the car are the same as when it was at rest. You cannot feel or hear the motion of the car. The railway car is no longer accelerating, and nothing inside it—not you, not the table, not the balls on the table—is accelerating either. What happens, then, if you hit the cue ball with the stick in exactly the same way as before?

The answer is that the ball moves in exactly the same way. Nothing about the motion of the balls, or any object inside the room, indicates that the room is moving. The reason is that all objects inside the railway car share the motion of the car. The *relative* motion between you and the ball is the same for all constant velocities of the railway car. This includes the special case of zero velocity when the railway car is at rest. In fact, you cannot tell from the motion of any object inside the railway car, including yourself, that the railway car is moving, as long as the railway car does not accelerate.

The railway car in this situation is an example of a frame of reference. Recall from earlier chapters that a **frame of reference** is a coordinate system that you can use to observe and describe motion. In this case, you could measure the motion of the cue ball with respect to the reference frame. In particular, no external force acts on the objects in the frame of reference because the frame (the railway car) does not accelerate. This is the condition described in Newton's first law of motion, or the law of inertia. Any frame of reference that is at rest or moves with a constant velocity is an **inertial frame of reference**.

frame of reference a coordinate system relative to which motion is described or observed

inertial frame of reference a frame of reference that moves at zero or a constant velocity; a frame in which the law of inertia holds

Within an inertial frame, moving with constant velocity $\vec{v}$ ($\vec{v} = 0$ is a special case of constant velocity), mechanical laws apply in the same way that they would if there were no movement. Would you expect these laws to remain unchanged for an observer in a different inertial frame? As it turns out, there is only one difference between two inertial reference frames. For an observer in one frame, the velocities of objects in the other frame must be added, by vector addition, to the velocity with which that frame moves away from the first observer. This changes the way in which the motion of the object appears to the first observer. An example is illustrated in **Figure 2**. Once corrected, however, by considering the velocities of the two reference frames, the physical results become the same for both reference frames. This situation provides the basis for the **principle of relativity**: for all inertial frames of reference, the laws of Newtonian mechanics are the same. The main idea of the principle of relativity has a long history, going back at least to the work of Galileo in the 1600s.

principle of relativity the laws of motion are the same in all inertial frames

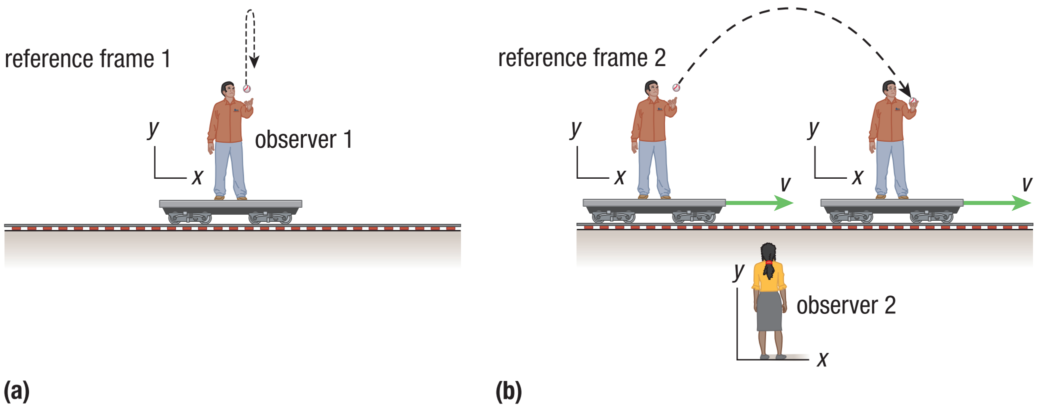


Figure 2 (a) When observer 1 throws a ball upward, he observes that the ball's motion is purely along the vertical (y) direction. (b) When the railway car for observer 1 has a speed v relative to observer 2, the ball appears to undergo projectile motion with a displacement along both x and y in reference frame 2. However, as viewed by observer 1 using his reference frame and coordinates x and y, the ball's motion is still purely vertical, as in (a).

Any two cars moving at different speeds on a straight road can each define a different inertial frame of reference. Accelerating objects, such as planets in orbit about their star and satellites in Earth orbit, are not inertial reference frames, but they are reference frames that move with respect to each other. Earth's surface is a non-inertial (accelerating) reference frame because it spins on its axis, rotates about the Sun, moves through the galaxy as a part of the solar system, and so on. However, because the accelerations involved in these motions are small, Earth's surface is very close to being an inertial reference frame. We will use Earth's surface to define an inertial frame of reference throughout this chapter.

Mini Investigation

Understanding Frames of Reference

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.5

In this activity, you will observe actions at slow, everyday speeds. You will then describe how the laws of Newtonian mechanics change for objects in different frames of reference that move with respect to each other. You will need to work in groups for this activity.

Equipment and Materials: small rubber or plastic ball

1. Have one member of the group stand still and toss the ball forward to a second group member. Observe how the speed of the ball depends on the speed with which the ball is tossed.
2. The student throwing the ball now walks slowly forward while tossing the ball to the second student, taking care to throw the ball with the same speed as in Step 1.
3. Repeat Step 2, but with the student walking quickly forward as the ball is tossed.
4. Repeat Step 2, but with the student walking slowly backwards as the ball is tossed.

A. Based on your observations in Step 2, infer how the speed of the ball depends on the speed with which the ball is tossed and the speed of the student tossing the ball. K/U T/I

B. Compare what happened in Step 3 with what happened in Step 2. What can you infer about how the speed of the ball depends on the speed with which the ball is tossed and the speed of the student tossing the ball? K/U T/I

C. Compare what happened in Step 4 with what happened in Step 2. What can you infer about how speed and direction affect the speed of the ball? K/U T/I

D. If you know the speed of an object in one inertial reference frame, can you determine its speed in another inertial reference frame? Explain your answer. T/I A

Is the Principle of Relativity Universal?

ether the proposed medium through which electromagnetic waves were once believed to propagate

Maxwell's mathematical description of electric and magnetic fields unified the subjects of electricity and magnetism but worked under the assumption that electromagnetic waves needed a medium through which to travel. The medium was called the luminiferous **ether**, or just ether. At the time of Maxwell's theory, physicists thought that ether filled the vacuum of space, had no mass, and had no drag effect on the motions of the planets.

For 200 years, physicists knew that Newton's laws of motion remained the same, or were invariant, in all inertial frames. However, electromagnetic waves seemed to be different. As it was then understood, Maxwell's theory stated that the speed of light with respect to the ether was always $c = 3.0 \times 10^8$ m/s. This meant that if you were at rest and observed a light source move relative to the ether, you would measure the speed of light to be different from c, depending on where you were standing (**Figure 3**). This is similar to the motion of waves in other media: an observer moving through the medium would measure the velocity of the waves to be different from what an observer at rest with respect to the medium would measure. WEB LINK

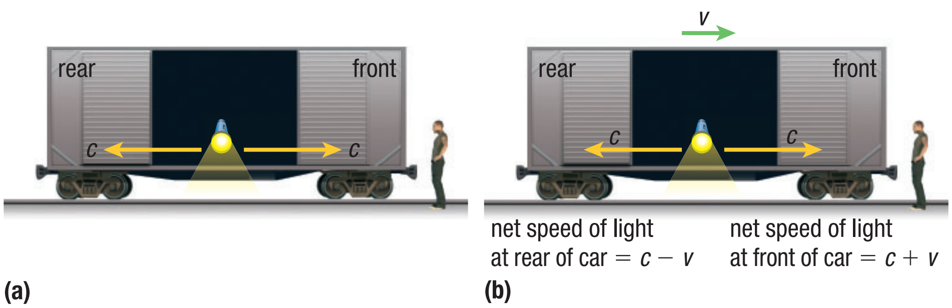


Figure 3 (a) When the railway car is at rest, the speed of light from the flashlight in both directions is the speed of light with respect to the ether. (b) When the railway car moves at speed v relative to the ether, we would expect a stationary observer in front of the car to measure $c + v$ for the speed of light because it takes less time for light to reach the observer. By similar reasoning, we would expect a stationary observer behind the car to measure $c - v$ for the speed of light because it takes longer for light to reach the observer.

The Development of Einstein's Postulates

Experimental evidence did not support the idea that the speed of light varied with the speed of the inertial frame. Experiments performed by Michelson and Morley in 1887, and by Trouton and Noble between 1901 and 1903, to try to detect the ether showed no change in the speed of light with the motion of Earth. Ultimately, the results of these and similar experiments suggested that electromagnetic waves do not require a medium in which to propagate, and that the existence of ether could not be proven experimentally.

What were the implications of the absence of ether? For one thing, the motion of the reference frame in which the speed of light was being measured did not seem to affect that measurement. What did these experimental results say about the laws of physics that govern the motion of material objects such as electrons and baseballs?

Einstein was not convinced that electromagnetic phenomena depended on the motion of the reference frame. He examined Maxwell's ideas as applied to a frame-of-reference experiment that required only a magnet and a closed coil of wire. Einstein used a method called a **thought experiment**, which is an experiment carried out in the imagination but not actually performed. A thought experiment examines the logic behind a hypothesis, theory, or principle by analyzing a virtual version of an experiment. The goal of a thought experiment is to illustrate or explore the consequences of a theory. Thought experiments are useful when the actual experiment may be difficult or impossible to perform, or when the researcher wants to emphasize certain details or unusual predictions of a theory. WEB LINK

thought experiment a mental exercise used to investigate the potential consequences of a hypothesis or postulate

In the magnet-and-coil thought experiment, Maxwell's theory predicts that when a magnet moves toward a coil of wire, an electric field forms near the moving magnet. This electric field moves charges within the coil, thus inducing an electric current. However, Maxwell's theory also predicts that if the coil moves and the magnet remains at rest, a current exists in the coil, not because an electric field forms, but because the magnetic field exerts a force on the charges in the moving coil.

To Einstein, it seemed illogical that selecting a frame of reference in which either the magnet or the coil is at rest would change the way we understand what is happening. Why would moving the magnet near a coil be different from moving the coil near a magnet? Nothing else in the universe described by physics behaved in this way.

Given this unsatisfactory consequence of classical electromagnetism, as well as the lack of evidence for the ether, Einstein decided to address the problem in the most fundamental and straightforward way possible. He began with the basic assumption that electromagnetism, like Newtonian mechanics, did not change when transformed between inertial frames. That is, the laws were the same in one inertial frame as in another. This meant that basic conservation principles—energy and momentum—held in all frames. Then he proposed two **postulates**, or conditions believed or known to be true at the time. From these, the special theory of relativity was developed.

postulate a statement assumed to be true from which a theory is developed

Postulate 1: The Principle of Relativity

The laws of physics are the same in all inertial frames of reference. No physics experiment can ever determine whether you are at rest or moving at a constant velocity.

Postulate 2: The Speed of Light Principle

There is at least one inertial frame of reference in which, for an observer at rest in this frame of reference, the speed of light, c, in a vacuum is independent of the motion of the source of the light.

Postulate 1 states that each inertial observer can consider himself or herself to be "at rest." If nature behaves in a certain way in one inertial frame, it should behave in the same way in all inertial frames. You should not be able to detect inertial motion inside a closed room using experiments of any kind. While not proven directly, this is a reasonable guess.

Postulate 2 is also reasonable. The speed of a wave is always independent of the speed of the source. Sound waves, water waves, and earthquake waves travel at a fixed speed relative to the medium through which they are travelling. In Postulate 2, Einstein assumed that there exists a frame of reference in which light waves behave in the same way as other waves, regardless of whether a medium to carry them actually exists.

Separately, the postulates were not extraordinary. When considered together, however, the two postulates led to an entirely new understanding about our universe. Postulate 1 implies that if Postulate 2 is true in one inertial frame of reference, it must be true in all frames of reference. Essentially, the property of light that is true for all models of waves cannot be true in one frame and false in another; otherwise, the laws of physics of light waves would differ in the two reference frames.

The consequence of this is that the speed of light must be constant and the same in all inertial frames of reference because the laws of physics do not prefer one frame of reference over another. Put another way, the speed of light is the same even when an observer is moving relative to the source of the light. If one observer sees a source of light in motion and another observer sees the source at rest, both measure the speed of the light as c. If you move very rapidly toward or away from a flashlight, the speed of light you measure is c, not $c + v$ or $c - v$. This result seems absurd, yet it is the logical result of Einstein's postulates.

Einstein published his work in 1905 in a paper titled "On the Electrodynamics of Moving Bodies." This paper was the basis for the **special theory of relativity**.

Special Theory of Relativity

All physical laws are the same in all inertial frames of reference, and the speed of light is independent of the motion of the light source or its observer in all inertial frames of reference.

The principle that the speed of light in a vacuum is the same in all frames of reference is an essential feature of the special theory of relativity. In fact, the laws of electromagnetism did not change at all as a result of special relativity, but scientists' understanding of the laws had to change. The laws of physics no longer made sense, and the understanding of space and time had to change. Einstein realized that the simplest and most natural way to develop relativistic physical laws would require a new understanding of space and time. This understanding was not specific to light—it would affect all natural phenomena.

We will consider a number of features of the special theory of relativity. First, the speed of light is much greater than the speeds of objects that you observe from day to day. As a result, in your everyday life, you cannot observe the unusual effects upon space and time hinted at in this section. Another important outcome is that, while advances in experimental techniques at the beginning of the twentieth century made measurements of the speed of light more accurate, it was still difficult to test the results of the special theory of relativity. Einstein used some of these experimental results to draw his conclusions and then relied on thought experiments to test the postulates and complete the theory. Throughout the twentieth century, numerous experiments have confirmed special relativity. For example, global positioning satellite systems would not work accurately if the technology did not take special relativity into account. You will learn more about this in Section 11.2.

11.1 Review

Summary

- A frame of reference is a coordinate system in which we can observe and measure the motion of an object. A frame of reference that moves with a constant velocity—in which the law of inertia holds—is an inertial frame of reference.
- The principle of relativity states that the laws of physics are the same in all inertial frames of reference.
- The ether is a hypothetical medium through which electromagnetic waves were thought to propagate. Tests such as the Michelson–Morley experiment failed to verify that there is an ether.
- Einstein's special theory of relativity is based on two postulates: The laws of physics are the same in all inertial frames of reference, and in at least one inertial frame of reference the speed of light is independent of the motion of the source of the light.
- One result of Einstein's postulates is that the speed of light is the same in all inertial frames of reference, regardless of their velocities.

Questions

1. A student on inline skates moves with a constant speed along the deck of a cruise ship, which is moving with constant velocity parallel to the shoreline. T/I C
   (a) Identify three distinct frames of reference from which to view the skater.
   (b) Describe the skater's motion in each reference frame.
2. Consider the properties of inertial and non-inertial frames of reference. K/U C
   (a) How does an inertial frame of reference differ from a non-inertial frame of reference?
   (b) Give two examples of each type of frame of reference.
3. Suppose an astronaut in a rocket moving at $0.5c$ along Earth's surface shines a light forward from the rocket. K/U T/I A
   (a) Calculate the speed of the light compared to that of the astronaut.
   (b) Calculate the speed of the light compared to that of Earth's surface.
4. Gabor stands on a cart that moves with a constant velocity. In his frame of reference, he tosses a ball vertically up and down. Lutaaq stands on the ground nearby. She also tosses a ball vertically up and down with respect to her frame of reference. T/I C A
   (a) Explain what the motion of Gabor's ball looks like from Lutaaq's frame of reference.
   (b) Explain what the motion of Lutaaq's ball looks like from Gabor's frame of reference.
5. What feature of Einstein's coil and magnet thought experiment did he find troubling? K/U
6. State the two postulates of the special theory of relativity. K/U C
7. What conclusion results from the combination of Einstein's two postulates? K/U C
8. Explain what a thought experiment is and give an example. K/U C
9. You are travelling in a spacecraft without windows. You are also far from any planets or stars. Describe an experiment that you could perform to determine whether you are in an inertial or a non-inertial frame of reference. K/U C
10. Suppose you are inside a windowless railway car. A billiard table is at the centre of the car. K/U C
    (a) While rolling a cue ball forward, you notice that the ball slows down suddenly, even though you have not applied a backward force on the ball. Explain the motion of the ball in terms of inertial or non-inertial frames of reference.
    (b) While rolling a cue ball forward, you notice that the ball rolls to the right (**Figure 4**), even though you have not applied a sideways force on the ball. Explain the motion of the ball in terms of inertial or non-inertial frames of reference.

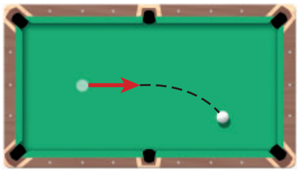


Figure 4

11.2 Time Dilation

Figure 1 This artist's depiction conveys the idea that time behaves very differently for an observer at rest compared to an observer moving at close to the speed of light.

time dilation the slowing down of time in one reference frame moving relative to an observer in another reference frame

The implications that arise from Einstein's postulates and the constant speed of light for all inertial frames are not obvious. While Einstein showed that physical laws in inertial frames behave in understandable and expected ways, he also showed, using another thought experiment, that time behaves in ways that are unexpected and counter-intuitive for a stationary observer watching another observer who is moving at a speed close to the speed of light. Although the distorted clock in **Figure 1** is art, special relativity's predictions about time in an inertial frame of reference are no less bizarre.

In this section, you will see the development of a model for the behaviour of time in different frames of reference. This model, called **time dilation**, explains the slowing down of time in one reference frame moving relative to an observer in another reference frame. This treatment will start in terms of a thought experiment and then expand through a simple algebraic derivation. You will then learn about the physical significance of time dilation, as well as the experimental evidence supporting the results predicted by special relativity.

Time Dilation

Einstein's two postulates seem straightforward. The first postulate—that the laws of physics must be the same in all inertial reference frames—agrees with Newton's laws, so it does not seem that this postulate can lead to anything new for mechanics. The second postulate concerns the speed of light, and it is not obvious what it will mean for objects other than light. Einstein, however, showed that these two postulates together lead to a surprising result concerning the very nature of time. He did so by carefully considering how time in inertial frames is measured.

Einstein analyzed the operation of the simple clock in **Figure 2** in a thought experiment. This clock keeps time in a frame that, for the purpose of this thought experiment, is at rest. The clock measures time using a pulse of light that travels back and forth between two mirrors. A distance d separates the mirrors, and light travels between them at speed c. The time required for a light pulse to make one round trip through the clock is thus $\frac{2d}{c}$. That is the time required for the clock to "tick" once.

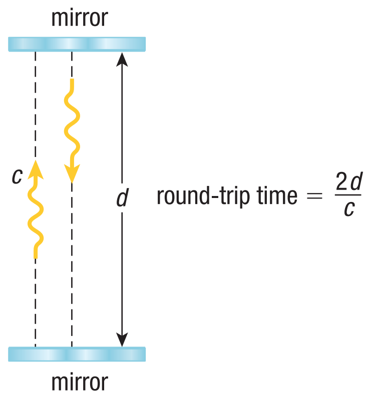


Figure 2 Each round trip of a light pulse between the mirrors corresponds to one tick of the light clock.

Now the light clock moves with constant horizontal speed v relative to a clock at rest. For this thought experiment, a railway car capable of moving at high speeds along a long, straight track provides this motion. How does this motion affect the operation of the clock? See **Figure 3**. For observer 1, who rides with the clock on the car, the return path of the light pulse, and thus one tick of the clock, appears as it does for any observer at rest with respect to the clock. The pulse simply travels up and down between the two mirrors. The motion of the car has no effect on the measurement of the speed of light for observer 1, in accordance with the postulates of special relativity. The separation of the mirrors is still d, so the round-trip time is still $\frac{2d}{c}$.

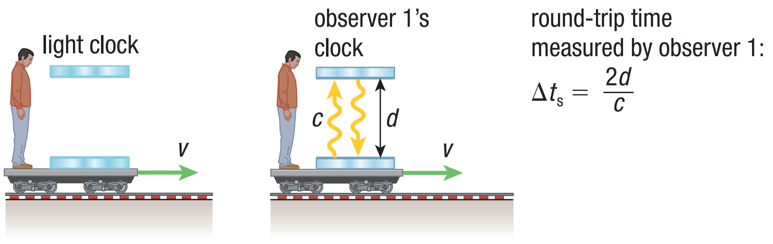


Figure 3 (a) A light clock is travelling with observer 1 on his railway car. (b) Light pulses travel back and forth in the clock. Each tick of the clock takes a time $\Delta t_s = \frac{2d}{c}$. According to observer 1, the operation of the clock is the same whether or not the railway car is moving.

The term Δt represents the time interval for one tick of the clock. When an observer is at rest (stationary) with respect to the clock, we write Δt_s for the time the observer measures for one tick. We write Δt_m for the time interval measured by an observer who sees the clock moving relative to her.

If Δt_s is the time required for the clock to make one tick as measured by observer 1 (who is stationary relative to the clock), then

$$\Delta t_s = \frac{2d}{c}$$

Observer 2 sees the clock moving while standing on the ground, as shown in **Figure 4**. She also measures the same speed of light as observer 1, but for her the light pulse moves a greater distance. This situation is analogous to the observer on a railway car tossing a ball vertically in the air (Figure 2 in Section 11.1). To the observer on the ground, the ball traces out a parabolic arc because of the two-dimensional motion provided by the horizontally moving car and the vertically displaced ball. In the case of the pulse in the light clock, observer 2 sees the light move at speed c, but along a path that has a horizontal deflection dependent on the speed, v, of the railway car.

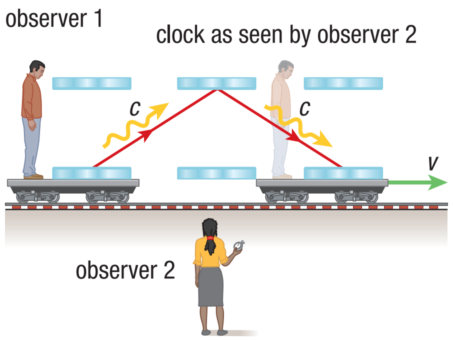


Figure 4 Observer 2, who is at rest on the ground, views the motion of the light pulses in the clock and sees the light pulse move a greater distance.

The distance that the light travels is longer for observer 2 than for observer 1, but the speed of light is the same for both observers. So, for observer 2, the time taken for the light to complete one tick (Δt_m) as it travels between the mirrors will be longer than for observer 1. The mathematical expression for Δt_m in terms of v, c, and d can be derived using geometry and the Pythagorean theorem.

In Figure 4, observer 2 sees the light pulse travel at speed c. As it travels, the light pulse covers a total vertical distance of $2d$ and a total horizontal distance of $v\Delta t_m$. The path of the light pulse forms the hypotenuse, z, of the two back-to-back right triangles in **Figure 5**.

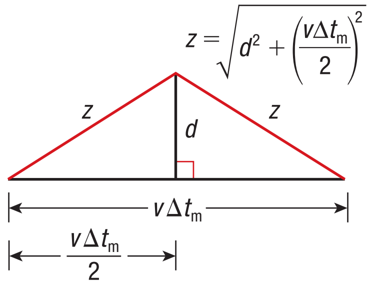


Figure 5 According to observer 2, the round-trip travel distance for a light pulse is $2z$, where $z = \sqrt{d^2 + \left(\frac{v\Delta t_m}{2}\right)^2}$, which is longer than the round-trip distance $2d$ seen by observer 1.

Applying the Pythagorean theorem to each triangle,

$$z^2 = d^2 + \left(\frac{v\Delta t_m}{2}\right)^2 \qquad \text{(Equation 1)}$$

We know that $z = c\Delta t_m$. Since z is half the total round-trip distance, and replacing v in Figure 5 with c, the speed of light, we get

$$z = \frac{c\Delta t_m}{2}$$

Squaring z,

$$z^2 = \frac{c^2(\Delta t_m)^2}{4}$$

Substitute z^2 into Equation 1:

$$\frac{c^2(\Delta t_m)^2}{4} = d^2 + \frac{v^2(\Delta t_m)^2}{4}$$

Now solve for Δt_m:

$$(\Delta t_m)^2 = \frac{4d^2}{c^2} + \frac{v^2}{c^2}(\Delta t_m)^2$$

$$(\Delta t_m)^2\left(1 - \frac{v^2}{c^2}\right) = \frac{4d^2}{c^2}$$

$$(\Delta t_m)^2 = \frac{\frac{4d^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)}$$

Taking the square root of both sides and expressing the equation in terms of Δt_s, where $\Delta t_s = \frac{2d}{c}$, leads to

$$\Delta t_m = \frac{\frac{2d}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Recall that Δt_m is the time interval measured by an observer who sees the clock moving relative to herself, and Δt_s is the time interval for an observer who is stationary with respect to the moving clock. In other words, the equation indicates that these times are different for each observer. These times are **relativistic times**, which means that time changes relative to an observer. The time interval required for the pulses of light to travel between the two mirrors depends on the relative motion between the observers. This is one of Einstein's key insights: time is not absolute.

relativistic time time that is not absolute, but changes relative to the observer

Now consider the implications of the last equation in more detail. The clock in Figures 3 and 4 is at rest relative to observer 1, and observer 1 measures a time Δt_s for each tick. The same clock is moving with speed v relative to observer 2, and according to the equation she measures a longer time Δt_m for each tick. This result is not limited to light clocks. Postulate 1 of special relativity states that all the laws of physics must be the same in all inertial reference frames. We could use a light clock to time any process in any reference frame. Since the equation holds for light clocks, it must therefore apply to any process, including biological processes.

Divide both sides of the previous equation by Δt_s. Then, we find that the ratio of Δt_m (the time measured by observer 2) to Δt_s (the time measured by observer 1) is

$$\boxed{\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

This equation describes the phenomenon of time dilation. Looking at the above equation, if v were greater than c, then the term under the square root sign would be negative. Since the square root of a negative number is undefined, v can never be greater than the speed of light, c. So the right side of the equation is always greater than 1. Hence, the ratio $\frac{\Delta t_m}{\Delta t_s}$ is greater than 1, which means that observer 2 measures a longer time for the clock than observer 1 does. In other words, according to observer 2, a moving clock will take longer for each tick. Therefore, special relativity predicts that moving clocks run more slowly from the point of view of an observer at rest.

This result seems very strange because your everyday experience tells you that a clock (such as your wristwatch) travelling in a car gives the same time as an identical clock at rest. If the equation for time dilation is true (and experiments have conclusively shown that it is), why have you not noticed time dilation before now? The graph of $\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ shown in **Figure 6** indicates the answer as a function of the speed, v, of the clock. At ordinary terrestrial speeds, v is much smaller than the speed of light c, and v^2 is even smaller than c^2. Therefore, the ratio $\frac{\Delta t_m}{\Delta t_s}$ is very close to 1 for speeds less than $0.1c$.

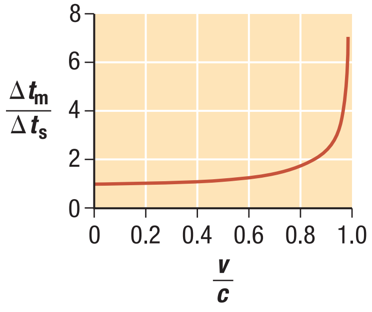


Figure 6 For typical terrestrial speeds, $\frac{v}{c}$ is very small and $\Delta t_m \approx \Delta t_s$.

For example, when $v = 50$ m/s, the ratio is

$$\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{(50 \text{ m/s})^2}{(3.0 \times 10^8 \text{ m/s})^2}}}$$

$$\frac{\Delta t_m}{\Delta t_s} = 1.000\ 000\ 000\ 000\ 014$$

The result is extremely close to 1, so for typical terrestrial speeds, the difference between Δt_s and Δt_m is negligible.

proper time (Δt_s) the time interval measured by an observer at rest with respect to a clock

The time interval for a particular clock (or process) as measured by an observer who is stationary relative to that clock is called the **proper time**, Δt_s. The word "proper" does not mean that measurements of time in other frames are incorrect. Proper time is always measured by an observer at rest relative to the clock or any observed process being studied. Therefore, while observer 1 is moving on his railway car in Figure 4, the clock is moving along with him. Therefore, he is at rest relative to this clock and he measures the clock's proper time. On the other hand, observer 2 sees the clock moving relative to her, so she does not measure the proper time. The time interval measured by an observer who is in relative motion with respect to a clock or process Δt_m is always longer than the proper time of that clock or process.

When an observer is at rest relative to a clock or process, the start and end of the process occur at the same location for this observer. For the light clock in Figure 4, observer 1 might be standing next to the bottom mirror, so from his viewpoint the light pulse starts and ends at the same location. By comparison, for observer 2 in Figure 4, the light pulse begins at the bottom mirror when the clock is at the left and returns to this mirror when the clock is in a different location relative to this observer. Observer 2 therefore measures a longer time interval, Δt_m. The proper time is always the shortest possible time that can be measured for a process, by any observer.

Time dilation is just one consequence of special relativity. In Section 11.3, you will learn that changes in time between frames of reference are accompanied by changes in length along the direction of motion. Therefore, it is not correct to think of time dilation as if it is an isolated effect. When you treat space and time as being interconnected, you will find it is easier to understand some of the contradictions of special relativity.

Tutorial 1 illustrates time dilation in a frame of reference moving close to the speed of light with respect to an observer in another frame of reference.

Tutorial 1 Determining Time Dilation

Sample Problem 1: Time Dilation for an Astronaut

On Earth, an astronaut has a pulse of 75.0 beats/min. He travels into space in a spacecraft capable of reaching very high speeds.

(a) Determine the astronaut's pulse with respect to a clock on Earth when the spacecraft travels at a speed of 0.10*c*.

(b) Determine the astronaut's pulse with respect to a clock on Earth when the spacecraft travels at a speed of 0.90*c*.

Solution

(a) **Given:** $\frac{1}{\Delta t_s} = 75.0$ beats/min; $v = 0.10c$

Required: $\frac{1}{\Delta t_m}$

Analysis: To calculate the time of one beat, take the reciprocal of the pulse rate. The process at rest in the moving reference frame is the time of one beat, which is the proper time, Δt_s. The time interval of the pulse as observed from Earth (Δt_m) must be longer than the proper time because the astronaut (who is basically the same as the stationary clock in his reference frame) moves with respect to Earth. Use $\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$ to calculate Δt_m.

Solution:

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\left(\frac{1}{75.0 \text{ beats/min}}\right)}{\sqrt{1 - \frac{(0.10c)^2}{c^2}}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{1 - 0.10^2}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{0.99}}$$

$\Delta t_m = 1.34 \times 10^{-2}$ min/beat (one extra digit carried)

Converting this result to a pulse rate yields

$$\text{pulse} = \frac{1}{\Delta t_m}$$

$$= \frac{1}{1.34 \times 10^{-2} \text{ min/beat}}$$

$$= 74.6 \text{ beats/min}$$

pulse = 75 beats/min

Statement: The observed pulse of the astronaut is 75 beats/min to two significant digits; however, the exact rate is slightly slower than his pulse in his own frame of reference (the spacecraft), or when he was on Earth and not moving with respect to Earth.

(b) **Given:** $\frac{1}{\Delta t_s} = 75.0$ beats/min; $v = 0.90c$

Required: $\frac{1}{\Delta t_m}$

Analysis: Follow the same steps as in (a).

Solution:

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\left(\frac{1}{75.0 \text{ beats/min}}\right)}{\sqrt{1 - \frac{(0.90c)^2}{c^2}}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{1 - 0.90^2}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{0.19}}$$

$\Delta t_m = 3.05 \times 10^{-2}$ min/beat (one extra digit carried)

Converting this result to a pulse rate yields

$$\text{pulse} = \frac{1}{\Delta t_m}$$

$$= \frac{1}{3.05 \times 10^{-2} \text{ min/beat}}$$

pulse = 33 beats/min

Statement: The observed pulse of the astronaut is 33 beats/min, which is much slower than his pulse in his own frame of reference (the spacecraft). This occurs because the speed of the spacecraft is close to the speed of light, which causes the time dilation to be large.

Practice

1. Determine how much longer a 1.00 s proper time interval appears to a stationary observer when a clock is moving with a speed of $0.60c$. T/I [ans: 0.25 s]
2. A beam of particles travels at a speed of 2.4×10^8 m/s. Scientists in the laboratory measure the average lifetime of the particle in the beam as 3.7×10^{-6} s. Calculate the average lifetime of the particles when they are at rest. K/U T/I [ans: 2.2×10^{-6} s]
3. An 8.0 s interval as measured on a moving spacecraft is measured as 10.0 s on Earth. Calculate how fast, relative to Earth, the spacecraft is moving. T/I A [ans: 1.8×10^8 m/s]
4. A spacecraft has a speed of $0.700c$ with respect to Earth. The crew of the spacecraft observes two events on Earth. According to the spacecraft's clocks, the time between the events is 30.0 h. K/U T/I A
   (a) Calculate the proper time, in hours, between the two events. [ans: 21.4 h]
   (b) What time interval does the crew measure when their craft travels at $0.950c$? [ans: 68.6 h]
5. Two astronauts travel to the Moon at a speed of 1.1×10^4 m/s. Their clock is accurate enough to detect time dilation. T/I C A
   (a) Determine the ratio of Δt_m to Δt_s to nine decimal places. [ans: 1.000 000 001]
   (b) What does your answer to (a) mean?

Investigation 11.2.1

Analyzing Relativistic Data (page 604)

Now that you have learned how to calculate time dilation, perform the part of Investigation 11.2.1 that uses the time dilation equation.

Verification of Time Dilation

In the early twentieth century, very few experiments could confirm, much less test directly, the predicted results of special relativity. However, since 1905, scientists have performed and repeated many experiments that have confirmed time dilation. Two of these experiments are discussed briefly here.

CLOCKS AND PASSENGER JETS

In the early 1970s, a series of experiments using atomic clocks took place. Atomic clocks use the vibrations of cesium atoms to measure time intervals very precisely. Four of these clocks were placed on separate jet aircraft and flown around the world twice. The purpose of this experiment, called the Hafele–Keating experiment after the two physicists who designed it, was to see if clocks moving at different speeds with respect to the centre of Earth run slower relative to a clock recording proper time. CAREER LINK

One complication posed by the experiment is that Earth rotates. Thus, a clock on Earth's surface is also a moving clock. To correct for this, one clock was placed on a plane moving westward against the rotation of Earth. This plane had the slowest speed and served as the clock to record the proper time for the system of clocks. A clock on Earth's surface was the next fastest-moving clock, and the clock on the plane flying east was moving fastest. In the final analysis of the data, the scientists made various corrections for gravitational effects as well as for the fact that none of the clocks were in a truly inertial frame.

The results showed that after two trips around the planet, the clock on Earth's surface ran 273 billionths of a second (or 273 ns) slower than the westbound clock (proper time), and that the eastbound clock ran 59 ns slower than the Earth clock, or 332 ns slower than the westbound clock. The error in these measurements was about 25 ns. Later repetitions of the experiment improved the accuracy, and all have been consistent with the predicted time dilation.

RELATIVITY AND GPS

Figure 7 Location-based games use GPS systems, which rely on relativistic corrections to operate with continuous high accuracy.

Location-based games are video games that use the player's location as part of the software that the game uses (**Figure 7**). A typical form of this game is a treasure hunt, called geo-caching, that does not use a map. Rather, it uses a GPS (global positioning satellite) system to indicate exactly where the player is and when the player is close to certain items, such as "buried gold." How is the computer able to know exactly where you are?

Satellites orbiting Earth send electromagnetic signals outward. A GPS computer on the ground determines from the signal the position of the satellite and the time it takes for the signal to arrive from the satellite. The computer gathers this data from three or more satellites and uses the speed of light (which for all satellites is the same) to determine its location on Earth. This simple procedure gives fairly good results. However, the speed of the signal is the speed of light, a very large number. So the error from just three measurements can be as much as a third of a kilometre and is usually only accurate to about 15 m. Adding a fourth satellite's signal as a time correction for all the GPS satellite signals improves accuracy to within 10 m or better. CAREER LINK

Relativity affects the long-term accuracy of the GPS system. Gravity affects the rate at which a clock runs (as described by the theory of general relativity) and must be corrected for. Another correction takes the fast relative motions of the satellites themselves into account. This matters because each satellite moves at nearly 3900 m/s with respect to Earth, causing time dilation. Without correction, these two kinds of relativistic effects cause the GPS system to lose accuracy by up to 11 km/day. WEB LINK

11.2 Review

Summary

- An observer in an inertial frame of reference will see the time in another inertial frame of reference as running slower.
- The equation for time dilation is $\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, where v is the relative speed of an object (or observer) with respect to an observer, c is the speed of light, and Δt_s is the proper time of the object, as measured by an observer at rest with respect to the object. For v greater than zero, $\Delta t_m > \Delta t_s$.
- The value $\sqrt{1 - \frac{v^2}{c^2}}$ is undefined for v greater than c. No object can move at a speed greater than or equal to c.
- Time dilation is a natural result of the two postulates of special relativity and the realization that the speed of light is the same for all observers.
- Numerous experiments have provided evidence of time dilation, including the Hafele–Keating experiment using passenger jets and atomic clocks.
- Time dilation, along with general relativity corrections, has to be taken into account to maintain the accuracy of GPS systems.

Questions

1. Refer to the thought experiment in Figure 4 on page 581. Explain what would have to be true about observer 2 for her to measure the same time on the light clock that observer 1 measures. K/U T/I
2. A process takes place in a given amount of time. K/U
   (a) Does the process seem to take longer for an observer moving relative to the process, or for an observer at rest with respect to the process?
   (b) Which observer measures the proper time of the process?
3. Two identical clocks are synchronized. One clock stays on Earth, and the other clock orbits Earth for one year, as measured by the clock on Earth. After the year elapses, the orbiting clock returns to Earth for comparison with the stationary clock. K/U
   (a) Do the clocks remain synchronized?
   (b) Will the clock that was in orbit run slower after it returns?
   (c) Will the clock that was in orbit have the same time as the clock that stayed on Earth or a different time?
   (d) Does the clock that stayed on Earth have the wrong time? Explain.
   (e) Does the clock that was in orbit have the wrong time? Explain.
4. Suppose an atomic clock is placed on a jet flying westward around Earth at a constant altitude. The jet lands at the same airport from which it departed 8.64×10^4 s earlier. A similar clock at the airport was synchronized with respect to the clock on the plane before the plane took off. Determine which clock ran slower (ignore the various forces in the non-inertial frames of the two clocks). Explain your answer. K/U T/I C
5. Why do you think the accuracy of a GPS system depends on correcting satellite clocks for special relativity? K/U T/I C A
6. Roger is travelling with a speed of 0.85c relative to Mia. Roger travels for 30 s as measured on his watch. K/U T/I A
   (a) Determine who measures the proper time for Roger's trip, Roger or Mia. Explain your answer.
   (b) Calculate the elapsed time on Mia's watch during this motion.
7. An astronaut travels at a speed of 0.95c away from Earth. The astronaut sends a light signal back to Earth every 1.0 s, as measured by her clock. An observer on Earth notes that these signals arrive at intervals equal to Δt_m. Calculate the value of Δt_m. T/I A

11.3 Length Contraction, Simultaneity, and Relativistic Momentum

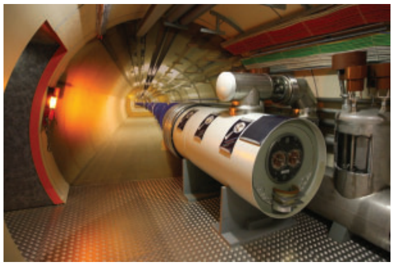


Figure 1 Particle accelerators, such as the Large Hadron Collider, increase particle speeds to nearly the speed of light. The resulting increase in the inertia of the particles means that strong magnetic fields are needed to bend the trajectories of the particles into a circle.

Time dilation is only one of the consequences of Einstein's postulates. There are also consequences that deal with space, such as the contraction, or compression, of length. In this section, we will explore this concept using a thought experiment.

As well as changing our understanding of time and length, special relativity also changes our understanding of momentum, energy, and mass. Despite the shortcomings of some of the older ways of thinking about these concepts, they are still useful in many situations. Science is a process; sometimes there are incremental changes, and sometimes new information forces a complete rethinking of what we know.

For example, you will read below that the momentum of a moving object in special relativity is different from the momentum in Newtonian mechanics. An understanding of both of the ways of thinking about momentum, however, is useful for solving different types of problems. The equations of special relativity also reveal that mass and energy are equivalent. This insight can lead to the discovery of new physical processes, such as nuclear fission, that can convert mass to mechanical energy. CAREER LINK

The effects of relativity become very important in particle accelerators (**Figure 1**). Particle accelerators accelerate subatomic particles to nearly the speed of light. To describe the particles correctly at such high speeds, we must use special relativity.

Length Contraction

In the past two sections, you learned how special relativity contradicts the concept of absolute time in Newtonian mechanics. Measurements of time intervals are relative, in that they can be different for different observers. However, time is just one aspect of a reference frame; reference frames also involve measurements of position and length. How are these measurements affected by relativity?

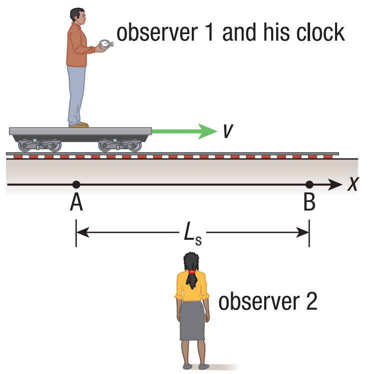


Figure 2 Observer 1 measures the distance between points A and B by using a clock to measure the time, Δt_s, it takes him to travel between the two points, together with his known speed, v.

proper length (L_s) the length of an object or distance between two points as measured by an observer who is stationary relative to the object or distance

Recall the example of observer 1 on the railway car and observer 2 on the ground near the tracks. Consider how observers 1 and 2 might each measure a particular length or distance (**Figure 2**). Suppose observer 2 marks two locations, A and B, on the ground. She then measures these locations to be a distance L_s apart on the x-axis. Observer 1 travels in the positive x-direction at a constant speed v, and as he passes point A he reads his clock. Observer 1 reads his clock again when he passes point B and calls the difference between the two readings Δt_s. This is the proper time interval because observer 1 measures the start and finish times at the same location (the centre of his railway car) with the same clock.

Like proper time, **proper length** is a measurement made by an observer who is stationary relative to the object being measured. Just as we denote the proper time by Δt_s, we will denote the proper length by L_s. An observer at rest relative to the object measures the length as proper length.

Recall from Section 11.2 that when observer 2 measures with her clock the time it takes for observer 1 to travel from A to B, the value she determines for Δt_m is given by the time dilation equation:

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Multiplying both sides of this equation by v gives

$$v\Delta t_m = \frac{v\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For observer 2, the time that observer 1 travels multiplied by v is simply the distance between A and B, or L_s:

$$v\Delta t_m = L_s$$

Similarly, the distance measured by observer 1 is the speed, v, times the proper time measured in his reference frame, so

$$v\Delta t_s = L_m$$

Substituting these last two equations into the time dilation equation gives the following result:

$$L_s = \frac{L_m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$$

Note that, because Δt_m is different from Δt_s due to time dilation, the lengths measured by the two observers will also be different. The length, L_m, measured by observer 1 is shorter than the length, L_s, measured by observer 2. This effect, called **length contraction**, or compression, is the shortening of distances in an inertial frame of reference moving relative to an observer in another inertial frame of reference. Contraction occurs along the direction of motion. Length contraction is the spatial counterpart to time dilation.

length contraction the shortening of length or distance in an inertial frame of reference moving relative to an observer in another inertial frame of reference

Consider the same situation, but now points A and B are the two ends of a metre stick. Observer 2 and the metre stick are in motion, while observer 1 is at rest (**Figure 3**). The metre stick is at rest relative to observer 2, so she measures the length of the metre stick and determines that $L_s = 1$ m, exactly. Observer 1 measures the length of the metre stick as it moves past him; he measures a length L_m that is shorter than L_s.

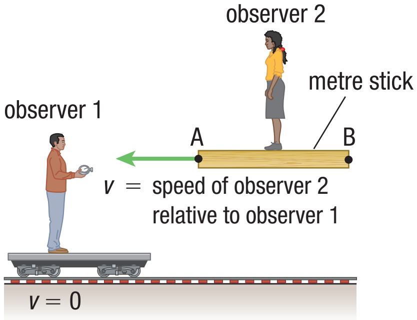


Figure 3 Observer 1 is at rest and observer 2, along with the metre stick, is in a reference frame moving with speed v relative to observer 1. Observer 1 observes that the moving metre stick is shorter than the length measured by observer 2.

Another way of saying this is that a metre stick moving relative to a stationary observer becomes shortened. The proper length, L_s, is the length measured by an observer at rest relative to the metre stick. It follows, then, that the length L_m, which is measured by the other observer and is always shorter than the proper length, is the **relativistic length**.

relativistic length (L_m) the length of an object or the distance between two points as measured by an observer moving with respect to the object or distance

Length contraction is described by the following equation:

$$\frac{L_m}{L_s} = \sqrt{1 - \frac{v^2}{c^2}}$$

The graph of $\frac{L_m}{L_s}$ versus the ratio $\frac{v}{c}$ in **Figure 4** shows that, for speeds that are small compared to c, the fraction $\frac{L_m}{L_s}$ is nearly 1. At speeds where v approaches c, the fraction $\frac{L_m}{L_s}$ approaches zero.

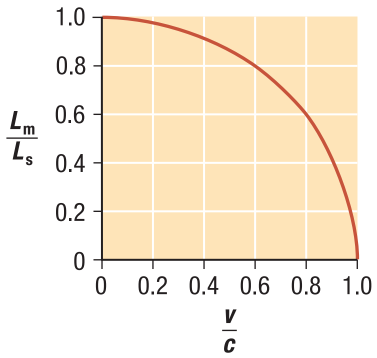


Figure 4 For typical terrestrial speeds, $\frac{v}{c}$ is very small and $L_m \approx L_s$.

In the previous example, the metre stick is in one frame of reference and the clock is in the other, so each observer must use a different relativistic property to obtain correct measurements. Observer 1 uses proper time to measure the length L_m, which he sees contracted. Observer 2 observes the proper length in her frame but must use time dilation for the time Δt_m observed in the frame of observer 1.

Length contraction occurs along the direction of motion (in these examples, along the x-axis). The following Tutorial models how to solve problems in which length is contracted.

Tutorial 1 Solving Problems Related to Length Contraction

The following Sample Problem illustrates how length is contracted for an observer in a moving frame of reference.

Sample Problem 1: Calculating Length Contraction

An observer on Earth measures the length of a spacecraft travelling at a speed of $0.700c$ to be 78.0 m long. Determine the proper length of the spacecraft.

Given: $L_m = 78.0$ m; $v = 0.700c$

Required: L_s

Analysis: $\frac{L_m}{L_s} = \sqrt{1 - \frac{v^2}{c^2}}$

$$L_s = \frac{L_m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Solution:

$$\begin{aligned} L_s &= \frac{L_m}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \frac{78.0 \text{ m}}{\sqrt{1 - \frac{(0.700c)^2}{c^2}}} \\ &= \frac{78.0 \text{ m}}{\sqrt{1 - 0.700^2}} \\ L_s &= 109 \text{ m} \end{aligned}$$

Statement: The proper length of the spacecraft is 109 m.

Practice

1. An object at rest is 5.0 m long, but when it drives past a stationary observer, the observer measures it to be only 4.5 m long. Determine how fast the object is moving. T/I [ans: 0.44c, or 1.3×10^8 m/s]
2. A spacecraft passes you at a speed of 0.80c. The proper length of the spacecraft is 120 m. Determine the length that you measure as it passes you. T/I [ans: 72 m]
3. (a) A car with proper length 2.5 m moves past you at speed v, and you measure its length to be 2.2 m. Determine the car's speed. [ans: 1.4×10^8 m/s]

   (b) A rocket with a proper length of 33 m moves past you at speed v, and you measure its length to be 26 m. Determine the rocket's speed. T/I [ans: 1.8×10^8 m/s]

Muons and Evidence for Length Contraction and Time Dilation

The decay of unstable elementary particles called muons demonstrates how length contraction and time dilation complement each other. Muons are particles that are about 207 times as massive as electrons, travel at speeds of about 0.99c, and decay in 2.2 ms for an observer at rest relative to the muons.

One source of muons is the cosmic radiation that collides with atoms in Earth's upper atmosphere. In Newtonian mechanics, most of these muons should decay after travelling about 660 m into the atmosphere. Yet experimental evidence shows that a large number of muons decay after travelling 4800 m—over seven times as far.

Why does this happen? The only known explanation comes from special relativity. Consider Earth as the stationary frame of reference. As observed from Earth, these muons undergo time dilation. They also undergo length contraction, but they are so small to begin with that this is a minor effect. Due to time dilation at very high speeds, the muons' "clocks" run slower relative to Earth clocks, so their lifetimes as measured on Earth increase by a factor of seven. This allows them to travel the greater distance.

What does this physical situation look like in the muon's frame of reference? An observer moving with the muon would notice a contracted Earth rushing toward the muon. More importantly, the distance from the upper atmosphere to Earth's surface would appear to be about one-seventh its normal thickness. Therefore, while the muons decay in their own frame of reference in just 2.2 ms, the 4800 m distance they must travel shortens in their frame of reference to 660 m.

With this example, you start to see the interrelationship between space and time through the complementary effects of length contraction in one reference frame and time dilation in the other. We will examine the inseparability of space and time later in this section.

Investigation 11.2.1

Analyzing Relativistic Data (page 604)

Now that you have learned how to calculate length contraction, perform the portion of Investigation 11.2.1 that uses the length contraction equation.

Relativity of Simultaneity

Suppose you are driving down the street, and you see two different traffic lights change colour at the same time. This is an example of **simultaneity**—the occurrence of two or more events at the same time. Your everyday experiences and intuition suggest that the notion of simultaneity is *absolute*; that is, two events are either simultaneous or not simultaneous for all observers. However, determining whether or not two events are simultaneous involves the measurement of time. Your study of time dilation in this chapter has already shown that different observers do not always agree on measurements involving clocks, time intervals, and lengths. What does special relativity imply for our perception of simultaneity?

simultaneity the occurrence of two or more events at the same time

If two events appear to be simultaneous to one observer, will other observers also find these events to be simultaneous? Consider the situation shown in **Figure 5**. Observer 1 is standing in the middle of his railway car, moving with a speed v relative to observer 2, when two lightning bolts strike the ends of the car. The lightning bolts leave burn marks on the ground (points A and B), which indicate the locations of the two ends of the car when the bolts struck. We now ask, "Did the two lightning bolts strike simultaneously?"

Observer 2 is midway between the burn marks at A and B. The light pulses from the lightning bolts reach her at the same time (Figure 5(c)). Observer 2 concludes that, because she is midway between points A and B and the light pulses reach her at the same time, the lightning bolts struck the railway car at the same time. Therefore, the bolts are simultaneous, as viewed by observer 2.

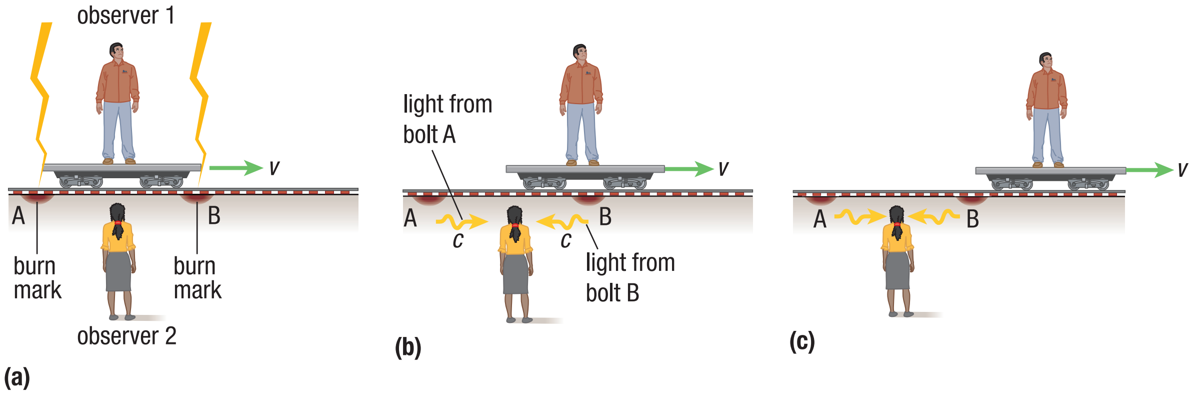


Figure 5 In a thought experiment to study simultaneity, (a) two lightning bolts strike the ends of the moving railway car, leaving burn marks on the ground. (b) According to observer 2, the lightning bolts are simultaneous. She comes to this conclusion because she is midway between the two burn marks. (c) The light pulses from the two bolts also reach observer 2 at the same time.

What does observer 1 see? He stands in the middle of his railway car, so, like observer 2, he is also midway between the places where the lightning bolts strike. Hence, if the two events are simultaneous as viewed by observer 1, the light pulses should reach him at the same time. Do they? Observer 2 can answer this question.

She observes that the railway car moves to the right, and because observer 1 is moving, the flash at B will reach him before the flash from A. Even with the distortions of time and space that arise from relativity, events do not occur out of sequence. So observer 1 will see the lightning strike at B before the lightning strike at A. The speeds of the light pulses from A and B are the same (a consequence of Einstein's postulates), and the distances that the pulses travel are the same. Therefore, observer 1 must conclude that the light pulses were not emitted at the same time.

The two lightning bolts in Figure 5 are therefore simultaneous for one observer (observer 2) but not for another observer (observer 1). Yet both observers are correct in their own reference frames, even if their observations are different. No reference frame is preferred. The observation of simultaneity can be different in different reference frames.

Relativistic analysis of simultaneity can help clarify apparent paradoxes. For example, in a simple time dilation experiment, one clock (C) travels between two clocks (A and B) that are stationary and synchronized with respect to each other. A stationary observer with respect to A and B notes that 10 s elapse on C as it moves from A to B, while 20 s elapse on A and B. A person holding C, however, will see A and B running slow by the same factor of $\frac{10}{20}$, or $\frac{1}{2}$. She will see only $\frac{1}{2}$ of 10 s, or 5 s, elapse on A and B as she moves from A to B.

How is this possible? Where did the "extra" 15 s go? The answer is that while clocks A and B are synchronized in their reference frame, they are not synchronized in the reference frame of C. At any instant of time for the moving observer, a snapshot of A and B will show that B is 15 s ahead of clock A. As the moving observer moves from A to B, A will go from 0 s to 5 s, and B will go from 15 s to 20 s.

Relativity of simultaneity is necessary to make sense of the reciprocity of time dilation, as well as the reciprocity of length contraction. Together, these concepts ensure that no reference frame is preferred. Relativity of simultaneity also works to ensure that the existence of a universal speed limit, c, does not lead to logical problems, as illustrated by the lightning example above. All of these concepts work together to ensure that special relativity makes sense.

If the previous sentences sound familiar, the reason is that they are a restatement of Einstein's postulates for relativity. Originally, Einstein wanted to call his model "the special theory of invariance" because it was of utmost importance to him that all laws of physics be invariant between inertial frames. Led by this principle, methodical logic, and early experimental evidence, he was able to conclude that the speed of light is invariant between and within inertial frames. To achieve this conclusion, he had to abandon the traditional views of time (in which time measurements were the same for all observers) and space (in which all spatial measurements were the same for all observers). Einstein's postulates reveal that the speed of light is independent of the speed not only of the source (just like all waves) but also of the observer.

The Twin Paradox

With our new understanding of space and time as properties that are both affected by motion comes the challenge of keeping track of what happens in each reference frame. Problems requiring special relativity become difficult to conceptualize without the classical notions of simultaneous events and absolute space and time. One of the most famous examples of this type of problem is the **twin paradox**—a thought experiment in which a traveller in one frame of reference returns from a voyage to learn that time has dilated in his reference frame, but not in the reference frame of his Earth-bound twin.

twin paradox a thought experiment in which a traveller in one frame of reference returns from a voyage to learn that time had passed more slowly in his spacecraft relative to the passage of time on Earth, despite the seemingly symmetric predictions of special relativity

Consider an astronaut who travels to the Sirius star system, which is 8.6 light years (ly) from Earth (**Figure 6**). His spacecraft is capable of a maximum speed of 0.90c, which means that he can reach the Sirius system in about 9.6 years. A round trip will take him just over 19 years. While on his mission, a crew of scientists on Earth, one of whom just happens to be the astronaut's twin sister, tracks the astronaut's health. The scientists' observations of the astronaut's biological and physical clocks indicate that he is aging more slowly than he would have done on Earth (although within his own frame of reference, he is, of course, unaware of any change in the flow of time). During the 19-year round trip, the crew notes that he ages only 8.3 years.

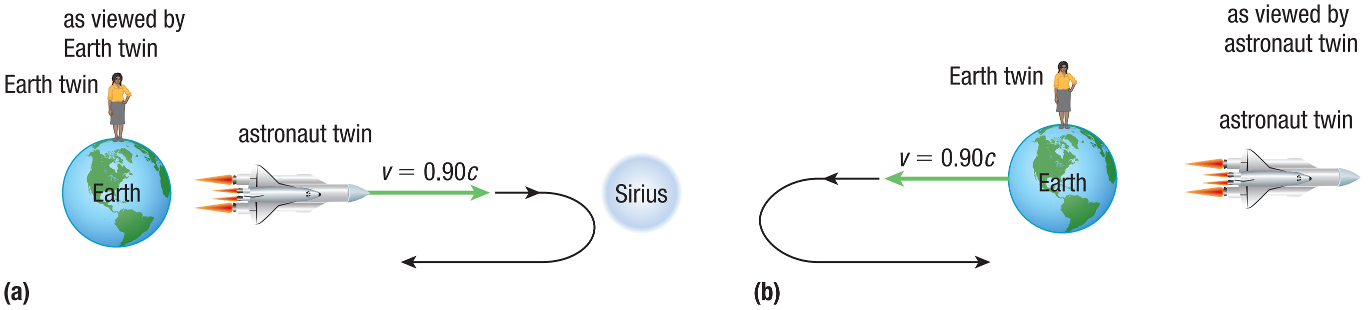


Figure 6 (a) An astronaut travels to the Sirius star system and then returns while his twin sister on Earth monitors his trip. (b) As viewed by the astronaut (in his reference frame), his Earth twin and planet Earth take a journey in the direction opposite to that in (a).

From the astronaut's frame of reference, Earth recedes from him at a rate of 0.90c. He therefore expects everyone on Earth, including his sister, to age only 8.3 years, while he ages 19 years. Imagine his surprise, then, when upon his return, his sister's analysis is correct and his analysis is wrong—she aged 19 years compared to his 8.3 years.

What happened here? Did the special theory of relativity fail, or is the error in the interpretation of relativistic effects? To understand the situation correctly, you need to consider that the astronaut moved in a frame of reference that was not truly inertial. So far, all the examples in this chapter have involved observers moving with constant velocities. In the case of the astronaut, however, he had to accelerate to change direction during the trip. The situation is, therefore, not symmetrical between the astronaut and his non-accelerating sister, and he cannot draw the same conclusions as his sister. Put another way, his frame of reference is not equivalent to that of his sister.

Nevertheless, there is a relativistic observation that the astronaut can use that completely reconciles the imagined paradox. Recall the interpretation of muon decay in the atmosphere and how the observations in each frame complement each other through the interconnection of space and time. **Space-time** is the four-dimensional coordinate system in which the three space coordinates (x, y, and z) are combined with a fourth dimension—time. From the astronaut's frame of reference, the universe undergoes length contraction. The distance that he must travel each way is, from his point of view, not 8.6 ly, but 3.7 ly. With a total distance of 7.4 ly and a speed that is most of the time $0.90c$, the astronaut sees the mission take only 8.3 years—the same amount by which his twin has determined he will age during the voyage. So he returns to find that his sister has aged 19 years while he has aged only 8.3 years.

space-time a four-dimensional coordinate system in which the three space coordinates are combined with time, a fourth dimension

Relativistic Momentum

As you learned in Chapter 5, an object with mass m moving with speed v has a momentum equal to the product of the two values. Momentum is conserved when there are no external forces acting on the object. This is an essential concept in physics. In fact, the conservation of momentum is one of the fundamental conservation rules in physics and is believed to be satisfied by all laws of physics, including the theory of special relativity.

When an object's speed is small compared with the speed of light, the object's momentum can be determined using the Newtonian formula for momentum, $p = mv$. However, as v approaches the speed of light, we have to take special relativity into account. Newtonian momentum gives a linear relation between p and v. By contrast, in special relativity the **relativistic momentum**—the momentum of objects moving at speeds near the speed of light—becomes extremely large as the object's speed approaches c. **Figure 7** graphically compares the momentum in Newtonian physics and special relativity.

relativistic momentum the momentum of objects moving at speeds near the speed of light

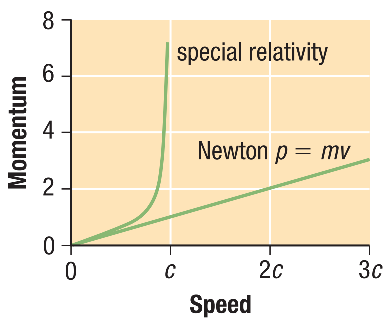


Figure 7 Newtonian mechanics predicts that momentum increases linearly with speed, while special relativity predicts that momentum approaches infinity at speeds close to c.

The effects of time dilation and length contraction are not included in the Newtonian momentum used in classical mechanics. To account for the relativistic effects on the momentum of objects moving near the speed of light, Einstein showed that proper time should be used to calculate momentum. This amounts to using a clock that travels along with the object. At the same time, an observer who watches the object move with speed v should take the measurement of length. The proper time is given by the expression $\Delta t_s = \Delta t_m\sqrt{1 - \frac{v^2}{c^2}}$, where Δt_m is the dilated time. Speed v is $\frac{\Delta x}{\Delta t}$, so when we replace t with Δt_s in the equation for Newtonian momentum, the resulting equation is

$$p = m\frac{\Delta x}{\Delta t_s}$$

$$p = m\frac{\Delta x}{\Delta t_m\sqrt{1 - \frac{v^2}{c^2}}}$$

where v is the speed of the object as viewed by the observer in the stationary reference frame. Since $v = \frac{\Delta x}{\Delta t_m}$, we get

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is the equation for relativistic momentum, which is used to calculate the momentum of objects moving at close to the speed of light. The momentum of an object differs noticeably from the predictions of Newtonian mechanics for speeds greater than about $0.1c$.

An important feature of the equation for relativistic momentum is the rest mass, m. **Rest mass** is the mass of the object as measured at rest with respect to the observer. This value is sometimes called the proper mass. The rest mass is an invariant value in special relativity; that is, it does not change at different speeds. However, at speeds close to the speed of light, the measured mass of an object will differ from the rest mass. This measured mass, or **relativistic mass**, is observed in a frame moving at speed v with respect to the observer.

rest mass the mass of an object measured at rest with respect to the observer; also called the proper mass

relativistic mass the mass of an object measured by an observer moving with speed v with respect to the object

The application of forces increases an object's momentum. So, after a large force is applied or a collision occurs, the object's momentum becomes very large. However, even when the momentum is very large, the object's speed never quite reaches the speed of light. In the following Tutorial, you will compare the values obtained using the classical momentum equation and the relativistic momentum equation.

Tutorial 2 Calculating Relativistic Momentum

The following Sample Problem illustrates the difference between classical and relativistic momentum.

Sample Problem 1: Comparing Classical and Relativistic Momentum

In experiments to study the properties of subatomic particles, physicists routinely accelerate electrons to speeds close to the speed of light. An electron has a mass of 9.11×10^{-31} kg and moves with a speed of $0.99c$.

(a) Calculate the electron's momentum using the non-relativistic equation.

(b) Calculate the electron's relativistic momentum. Compare the relativistic momentum and the non-relativistic momentum.

Solution

(a) **Given:** $m = 9.11 \times 10^{-31}$ kg; $v = 0.99c$; $c = 3.0 \times 10^8$ m/s

Required: $p_{classical}$

Analysis: $p_{classical} = mv$

Solution:

$$\begin{aligned} p_{classical} &= mv \\ &= m(0.99c) \\ &= (9.11 \times 10^{-31}\ \text{kg})(0.99)(3.0 \times 10^8\ \text{m/s}) \\ p_{classical} &= 2.7 \times 10^{-22}\ \text{kg·m/s} \end{aligned}$$

Statement: The non-relativistic momentum of the electron is 2.7×10^{-22} kg·m/s.

(b) **Given:** $m = 9.11 \times 10^{-31}$ kg; $v = 0.99c$; $c = 3.0 \times 10^8$ m/s

Required: $p_{\text{relativistic}}$

Analysis: $p_{\text{relativistic}} = \dfrac{mv}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Solution:

$$p_{\text{relativistic}} = \frac{mv}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$= \frac{m(0.99c)}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$= \frac{(9.11 \times 10^{-31}\ \text{kg})(0.99)(3.00 \times 10^8\ \text{m/s})}{\sqrt{1 - \dfrac{(0.99c)^2}{c^2}}}$$

$$p_{\text{relativistic}} = 1.9 \times 10^{-21}\ \text{kg}\cdot\text{m/s}$$

Statement: The relativistic momentum of the electron is 1.9×10^{-21} kg·m/s, which is about seven times as great as the momentum predicted by the classical definition.

Practice

1. A proton with a mass of 1.67×10^{-27} kg moves in a particle accelerator at $0.85c$. Calculate the proton's
   (a) non-relativistic momentum [ans: 4.3×10^{-19} kg·m/s]
   (b) relativistic momentum T/I [ans: 8.1×10^{-19} kg·m/s]
2. Suppose a 100.0 g projectile is launched with a speed $0.30c$ relative to Earth. Determine its relativistic momentum with respect to Earth. T/I [ans: 9.4×10^6 kg·m/s]
3. A proton moves at $0.750c$ relative to an inertial system in a lab. Given that the proton's mass is 1.67×10^{-27} kg, determine its relativistic momentum in the lab's frame of reference. T/I [ans: 5.68×10^{-19} kg·m/s]
4. A cube of iridium has the following dimensions: 0.100 m $\times$ 0.100 m $\times$ 0.100 m. Suppose the cube is moving at $0.950c$, in the direction of the y-axis. The density of iridium is 2.26×10^4 kg/m^3 when measured at rest. K/U T/I A
   (a) Which of the three directions, x, y, or z, is affected by the motion? [ans: y]
   (b) Calculate the relativistic volume of the cube. [ans: 3.12×10^{-4} m^3]
   (c) Calculate the relativistic momentum of the cube. [ans: 2.06×10^{10} kg·m/s]

Investigation 11.2.1

Analyzing Relativistic Data (page 604)

Now that you have learned about relativistic momentum, perform the portion of Investigation 11.2.1 that uses the relativistic momentum equation.

The Universal Speed Limit

Throughout this chapter, you have seen the development of the various components of special relativity—time dilation, length contraction, simultaneity, and relativistic momentum. Each of these concepts involves the expression $\sqrt{1 - \dfrac{v^2}{c^2}}$, which is a mathematical consequence of the postulates of special relativity. All physical laws remain invariant between inertial frames of reference with a relative velocity, and the speed of light remains the same in all frames of reference, regardless of whether the frame, light source, or observer is moving.

After using thought experiments to discover time dilation, length contraction, simultaneity, and relativistic momentum, Einstein also realized that, as a consequence of relativity, the speed c is a unique speed that plays a unique role in the universe. Although Einstein originally concentrated on the behaviour of light, researchers now understand that the speed c is special in its own right, independent of the properties of light waves. The universe truly does have an ultimate speed limit.

11.3 Review

Summary

- Proper length, L_s, is the length of an object as measured by an observer who is at rest with respect to the object. Relativistic length, L_m, is the length of the object as measured by an observer not at rest with respect to the object.
- The equation for length contraction is $L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$. For v greater than zero, $L_m < L_s$. Contraction occurs along the direction of motion.
- For two observers in motion relative to each other, events that appear simultaneous for one observer are not simultaneous for the other observer. However, in both cases, events appear to both observers in the order that they occur. The observers perceive the time between the two events differently.
- The twin paradox describes a thought experiment in which a moving observer ages more slowly than his or her "twin," despite the reciprocity of time dilation because the reference frame of the moving observer is not inertial.
- The equation for relativistic momentum is $p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$. Relativistic momentum increases as the speed increases and is limited by the speed of light.
- The rest mass of an object is the mass of the object as measured by an observer at rest with respect to the object.
- No object with a rest mass greater than zero can move as fast as, or faster than, the speed of light.

Questions

1. Spacecraft 1 passes spacecraft 2 at $0.755c$ relative to spacecraft 2. An astronaut on spacecraft 1 measures the length of spacecraft 2 to be 475 m. Calculate the proper length of spacecraft 2. T/I
2. An astronaut moving at $0.55c$ with respect to Earth measures the distance to a nearby star as 8.0 ly. Another astronaut makes the same voyage at $0.85c$ with respect to Earth. Calculate the distance the second astronaut measures between Earth and the star. T/I
3. François is travelling at a speed of $0.95c$ on a railway car, which François has measured as 25 m in length. Soledad, who is located on the ground near the railway tracks, arranges for two small explosions to occur on the ground next to the ends of the railway car. According to Soledad, the two explosions occur simultaneously, and she uses the burn marks on the ground to measure the length of François's railway car. According to François, do the two explosions occur simultaneously? If not, then according to François, which explosion occurs first? K/U T/I
4. In experiments, physicists routinely accelerate protons to speeds quite close to the speed of light. The mass of a proton is 1.67×10^{-27} kg, and the proton is moving with a speed of $0.99c$. T/I A
   (a) Calculate the proton's momentum, according to Newton's definition.
   (b) Calculate the proton's relativistic momentum.
   (c) Determine the ratio of the relativistic momentum to the Newtonian momentum.
5. The relativistic momentum of a particle of rest mass m and speed v is equal to $5mv$. Calculate the speed of the particle. T/I A
6. An electron with a speed of $0.999c$ has a momentum that is equal to the momentum of a ship with a mass of 4.38×10^7 kg moving at a certain speed. Determine the speed of the ship. T/I
7. If you were travelling on a spacecraft at $0.99c$ relative to Earth, would you feel compressed in the direction of travel? Explain your answer. K/U
8. Why do we not notice the effects of length contraction in our everyday lives? For example, why do cars not appear shorter when they drive past us at high speeds? K/U

11.4 Mass–Energy Equivalence

In Section 11.3, you learned how to adjust the definition of momentum using the mathematics of special relativity so that the observations between inertial frames of reference make sense at speeds near c. Now we will examine the concepts of work and energy conservation.

In the Newtonian definition of work, the mass of an object remains constant, and any energy transferred to increase its kinetic energy results in an increase in speed only. You would expect this relation to remain valid for a continuous net force. However, this does not happen, as scientists discovered in experiments performed in the early twentieth century. Only when special relativity was applied to the problem, so that acceleration and force were proportional with a factor of $\sqrt{1 - \frac{v^2}{c^2}}$, could the results of these experiments be reconciled with the laws of physics.

When you apply this result to the classical work–kinetic energy equation, an additional term results from special relativity. This additional term forms what is probably the most famous equation in physics: $E = mc^2$. Perhaps the most familiar application of this equation is in nuclear physics. During nuclear fusion, protons and neutrons combine in the interior of the Sun to form helium nuclei, as well as the nuclei of other elements (**Figure 1**). The difference in the masses of the reactants and products of fusion, multiplied by c^2, equals the released energy.

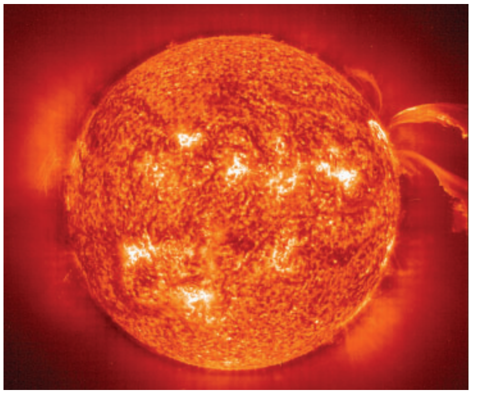


Figure 1 The Sun's energy comes from the fusion of hydrogen nuclei and neutrons to produce helium nuclei and energy. This process is a direct result of mass converting to energy, as predicted in the special theory of relativity.

Mass–Energy Equivalence

Recall from the discussion of relativistic momentum (Section 11.3) that the relativistic mass of an object varies with the inertial reference frames in which the mass and an observer are located. From the relativistic momentum equation, it follows that the equation for relativistic mass is

$$m_{\text{relativistic}} = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m is the rest mass of the object.

Using the equations of special relativity, Einstein concluded that the total energy, E_{total}, for an object with rest mass m moving with speed v is equal to

$$E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

When the object is at rest ($v = 0$), the equation simplifies to

$$E_{\text{rest}} = mc^2$$

This is Einstein's famous $E = mc^2$ equation. It means that the rest mass of an object and its energy are equivalent. This energy is called the object's rest energy. The **rest energy**, E_{rest}, is the amount of energy an object at rest has with respect to an observer, and it does not change.

rest energy (E_{rest}) the amount of energy an object at rest has with respect to an observer

When an object is in motion, its total energy is larger than its rest energy. The **relativistic kinetic energy**, E_k, is the difference between the total energy of the object and its rest energy:

relativistic kinetic energy (E_k) the energy of an object in excess of its rest energy

$$E_k = E_{\text{total}} - E_{\text{rest}}$$

$$E_k = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

Remember that, because of the relativistic treatment, kinetic energy can become very large, but the object's speed never quite reaches the speed of light (**Figure 2**). From the total energy expression, you can see that as the speed increases, the object responds as if it had a mass larger than its mass at low speeds.

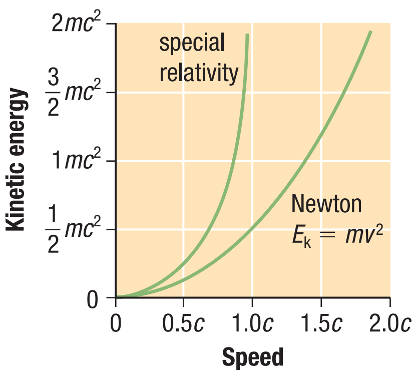


Figure 2 Even when an object has a very large kinetic energy, its speed never reaches the speed of light.

As in the case of relativistic momentum, the mass here is the object's rest mass (or proper mass). Unlike the various types of potential energy, where a force is acting on an object without moving it, rest energy is a property of matter itself. Just as theories about space and time shifted to a unified space-time theory, so too did theories about mass and energy shift to a unified mass–energy theory. Consequently, the conservation of energy principle is now the principle of **conservation of mass–energy**, which states that the rest energy is equal to rest mass times the speed of light squared.

conservation of mass–energy the principle that rest mass and energy are equivalent

Since the speed of light is a very large number, the magnitude of the rest energy can be very large, even when the rest mass is small. In fact, a rest mass of only 1 kg corresponds to a tremendous amount of rest energy:

$$\begin{aligned} E_{\text{rest}} &= mc^2 \\ &= (1\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2 \\ E_{\text{rest}} &= 9 \times 10^{16}\ \text{J} \end{aligned}$$

To put this number into perspective, the total daily energy consumption for Canada in 2008 was 2.4×10^{16} J, which is less than a third of the amount of energy available from a rest mass of 1 kg. You can now understand why scientists and engineers search for ways to convert mass into energy. The most efficient nuclear fission reactors convert less than 1 % of the rest mass of nuclear fuel to energy, yet their energy output is enormous. Since the late 1940s, physicists have looked at ways to produce controlled nuclear fusion reactions (**Figure 3**), similar to the reactions that produce energy in the Sun and other stars. This process also uses the conversion of mass to energy, but uses hydrogen (the most common element in the universe) as fuel. One of the advantages of the fusion process is fewer radioactive waste products. However, researchers have not yet overcome the challenge of confining hydrogen at the temperatures and pressures needed for fusion to take place.

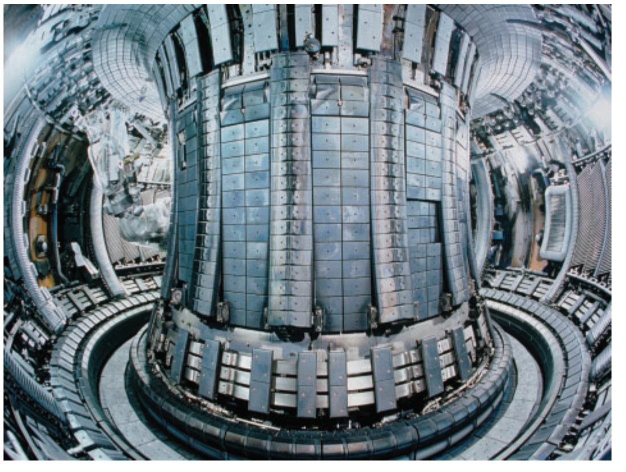


Figure 3 Nuclear fusion reactions have been studied and partially achieved in devices such as this tokamak, which uses magnetic fields to create conditions similar to those within the Sun.

UNIT TASK **BOOKMARK**

You can apply what you have learned about mass–energy conversion to the Unit Task on page 666.

While many scientists think that mass–energy conversion occurs in most processes that release energy, such as chemical reactions, the change in mass that occurs is extremely small. For instance, the energy released by the combustion of 1.0 kg of coal is equivalent to a mass of about 3.6×10^{-10} kg. This amount is too small to detect, even with the best electronic balance. It is only at the nuclear level or smaller that the mass–energy conversion becomes both significant and measurable. The Large Hadron Collider discussed at the beginning of this unit operates at energies where this mass–energy conversion is important. CAREER LINK

The following Tutorial will demonstrate how to solve problems involving mass–energy equivalence.

Tutorial 1 Solving Problems Related to Mass–Energy Equivalence

Sample Problem 1 involves mass–energy conversion. Sample Problem 2 models how to calculate total energy, kinetic energy, and rest energy using units of electron-volts instead of joules.

Sample Problem 1: Calculating Energy Equivalence

The average home in Canada uses 3.6×10^{10} J of energy per day. Imagine that a cabbage with a rest mass of 0.750 kg could be completely converted to another form of energy (although in a nuclear reaction only a fraction of the mass is converted into electrical energy).

(a) Calculate how much energy is released by the cabbage.

(b) Determine the number of days this cabbage could supply energy for an average home in Canada.

Solution

(a) **Given:** $m = 0.750$ kg; $c = 3.0 \times 10^8$ m/s

Required: E_{rest}

Analysis: $E_{rest} = mc^2$

Solution:
$$\begin{aligned} E_{rest} &= mc^2 \\ &= (0.750 \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 \\ E_{rest} &= 6.75 \times 10^{16} \text{ J (one extra digit carried)} \end{aligned}$$

Statement: The energy released by the conversion of the cabbage's rest mass would be 6.8×10^{16} J.

(b) **Given:** $E_{rest} = 6.75 \times 10^{16}$ J; $E_{home} = 3.6 \times 10^{10}$ J per day usage

Required: days of energy provided by cabbage, d

Analysis: $d = \dfrac{E_{rest}}{E_{home}}$

Solution:
$$d = \frac{6.75 \times 10^{16} \cancel{\text{J}}}{3.6 \times 10^{10} \dfrac{\cancel{\text{J}}}{\text{day}}}$$
$$d = 1.9 \times 10^6 \text{ days}$$

Statement: If a cabbage could be completely converted to energy, it would provide enough energy to power an average home in Canada for 1.9×10^6 days, or over 5000 years.

Sample Problem 2: Calculating an Electron's Energy

For subatomic particles with extremely small masses, the standard SI units of kilograms and joules are not convenient. Therefore, physicists use the electron-volt (eV) as the unit of energy. The electron-volt is defined as the work done on an electron by 1 V of electric potential. The conversion factor between electron-volts and joules is 1 eV $= 1.60 \times 10^{-19}$ J. Mass–energy equivalence allows us to express the mass of a subatomic particle in electron-volts.

An electron has a speed of $0.900c$ in a laboratory, and the rest mass of an electron is 9.11×10^{-31} kg. With respect to the laboratory's frame of reference, calculate the electron's rest energy, total energy, and kinetic energy in electron-volts.

Given: $m = 9.11 \times 10^{-31}$ kg; $v = 0.900c$; $c = 3.0 \times 10^8$ m/s; 1 eV $= 1.60 \times 10^{-19}$ J

Required: E_{rest}; E_{total}; E_k

Analysis: Use the equation $E_{rest} = mc^2$ to determine the rest energy of the electron. Include the conversion factor for electron-volts. Use the equation $E_{total} = \dfrac{mc^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ to calculate the total relativistic energy of the electron. To determine the kinetic energy of the electron, use the equation $E_k = E_{total} - E_{rest}$.

Solution:

$$E_{\text{rest}} = mc^2$$

$$= (9.11 \times 10^{-31}\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2 \left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right)$$

$$= 5.12 \times 10^5\ \text{eV}$$

$$E_{\text{rest}} = 0.512\ \text{MeV (one extra digit carried)}$$

$$E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{(9.11 \times 10^{-31}\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2}{\sqrt{1 - \frac{(0.900c)^2}{c^2}}} \left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right)$$

$$= 1.17 \times 10^6\ \text{eV}$$

$$E_{\text{toal}} = 1.17\ \text{MeV (one extra digit carried)}$$

$$E_{\text{k}} = E_{\text{total}} - E_{\text{rest}}$$

$$= 1.17\ \text{MeV} - 0.512\ \text{MeV}$$

$$E_{\text{k}} = 0.66\ \text{MeV}$$

Statement: The rest energy of an electron is 0.51 MeV. The total relativistic energy of an electron moving at 0.900*c* is 1.2 MeV. The kinetic energy of an electron moving at 0.900*c* is 0.66 MeV.

Practice

1. A cellphone has a rest energy of 2.25×10^{16} J. Calculate its rest mass. T/I A [ans: 0.25 kg]
2. A proton moves with a speed of 0.800*c* through a particle accelerator. In the accelerator's frame of reference, calculate (a) the total and (b) the kinetic energies of the proton in megaelectron-volts. T/I A [ans: (a) 1.57×10^3 MeV; (b) 6.26×10^2 MeV]
3. Nuclear power stations in Canada use a type of reactor invented in Canada called a Canada Deuterium Uranium (CANDU) reactor. A typical CANDU fuel bundle has a mass of 23 kg. T/I A
   (a) Calculate the amount of electrical energy produced by a single CANDU fuel bundle, assuming that the entire bundle is converted to electrical energy. [ans: 2.1×10^{18} J]
   (b) An average Canadian home uses 3.6×10^{10} J of energy per day. Determine the number of days that a single CANDU fuel bundle could theoretically provide energy to a home. [ans: 5.8×10^7 days]
4. An asteroid with a mass of 2500 kg has a relativistic kinetic energy of 1.5×10^{20} J. Calculate the speed of the asteroid. T/I A [ans: 0.80*c*]

The applications of mass–energy equivalence and other concepts from special relativity have had a deep effect on technology, society, and the environment. Conversion of mass into energy through nuclear fission, for example, led to nuclear power and nuclear weapons, and scientists hope that experiments on nuclear fusion will lead to a source of clean, efficient energy.

11.4 Review

Summary

- The rest energy of an object with mass m is the energy of an object that is not in motion. It is a measure of the energy that is intrinsically contained in the matter that makes up the object and is given by the equation $E_{\text{rest}} = mc^2$.
- The total relativistic energy of an object with rest mass m is $E_{\text{total}} = \dfrac{mc^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

  This expression is equal to the sum of the kinetic energy and the rest energy of the object: $E_{\text{total}} = E_{\text{rest}} + E_{\text{k}}$.
- Special relativity shows that mass and energy are equivalent and thus establishes the principle of conservation of mass–energy.
- The energy produced from nuclear reactions has many applications, including nuclear power and nuclear weapons.

Questions

1. A 1 kg object is travelling at 0.95c. Calculate the object's relativistic mass. K/U T/I
2. In your own words, explain how the equation $E_{\text{rest}} = mc^2$ leads to the conservation of mass–energy. K/U T/I C
3. The chemical energy released by 1.00 kg of the explosive TNT equals about 4.20×10^6 J. Determine the rest mass, when converted, that produces a rest energy identical to the amount of energy released by TNT. T/I A
4. The anti-proton is a type of antimatter and is the antimatter "cousin" of the proton. The two particles have the same rest mass, which is 1.67×10^{-27} kg. It is possible for a proton and an anti-proton to collide and annihilate each other, producing pure energy in the form of gamma radiation. Calculate the energy that a proton and an anti-proton release when they annihilate each other. Assume that the two particles are at rest just before the annihilation. T/I A
5. In a physics laboratory, a subatomic particle has a rest energy of 1.28 MeV and a total relativistic energy of 1.72 MeV with respect to the laboratory's frame of reference. T/I A
   (a) Calculate the particle's rest mass.
   (b) Calculate the particle's kinetic energy in the laboratory's frame.
   (c) Determine the particle's speed in the laboratory.
6. The Moon has a rest mass of 7.35×10^{22} kg and an average orbital speed of 1.02×10^3 m/s. Calculate the amount of mass that would need to be converted to energy to accelerate the Moon from rest to its final orbital speed. K/U A
7. Tritium is a heavy form of hydrogen, with a nucleus consisting of one proton and two neutrons. The rest energy of a tritium nucleus is 2809.4 MeV. The formation of a tritium nucleus releases energy in the forms of increased kinetic energy of the nucleus and gamma radiation. Determine the amount of energy released when a proton with rest energy 938.3 MeV combines with two neutrons, each with a rest energy of 939.6 MeV, to form a tritium nucleus. T/I A
8. The kinetic energy of a particle is equal to its rest energy. Calculate the speed of the particle. K/U T/I
9. A spacecraft with a mass of 2500 kg has a relativistic kinetic energy of 2.0×10^{19} J. Determine the speed of the spacecraft. K/U A
10. Using the measured speed and rest mass of the electron (Newtonian mechanics), an electron in a television picture tube has a classical kinetic energy of 30.0 keV. Determine the actual kinetic energy of the electron, in kiloelectron-volts, by using the total relativistic energy and the rest energy. K/U T/I A
11. A car requires 1.0×10^8 J of energy to drive 30.0 km. Calculate how many kilometres you could hypothetically drive using the energy contained in the rest mass of 100.0 mg of fuel. K/U
12. A piece of uranium with a mass of 100.000 kg is placed in a reactor. Four years later, a mass of 99.312 kg is left over. Determine the energy that was released during this time. For this question, use the speed of light to six significant digits, which is $2.997\ 99 \times 10^8$ m/s. T/I A

Investigation 11.2.1 ACTIVITY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Analyzing Relativistic Data

Parts A and B of this activity will show how time dilation, length contraction, and relativistic momentum depend on the speed at which an object is moving. You will determine the relativistic effects for a rocket moving at different speeds relative to a stationary observer. Since we do not have a rocket that can move near the speed of light, you will use a computer spreadsheet, calculations, and graphs to show how the time onboard the rocket, the length of the rocket, and its relativistic momentum (as viewed by a stationary observer) depend on the rocket's speed. In Part C, you will perform another thought experiment in which you speculate on how your life would change if the speed of light were much less than its actual value.

Purpose

To determine the relativistic effects for a rocket moving at different speeds relative to a stationary observer

Materials

- computer spreadsheet software

Procedure

Part A: Inputting the Data

1. Design a computer spreadsheet to record the data for your activity. You will use the equations of special relativity in your spreadsheet to calculate the rocket's time dilation, length, and relativistic momentum as measured by an Earth-bound (stationary) observer.
2. Create columns in your table for the following properties of the rocket: speed, time onboard, length, and relativistic momentum (all as measured by a stationary observer), as illustrated in **Table 1**.

Table 1 Rocket's Speed, Time, Length, and Relativistic Momentum (as measured by a stationary observer)

Speed (m/s)	Time (s)	Length (m)	Relativistic momentum (kg·m/s)
$0.001c$			

3. Express the rocket's speed as a fraction of the speed of light for values from $0.001c$ to $0.999c$. Choose increments for speed such that there are at least 100 values.
4. In the time column, calculate the time dilation experienced by the rocket relative to a stationary observer. Use the time dilation equation,

$$\Delta t_{\mathrm{m}} = \frac{\Delta t_{\mathrm{s}}}{\sqrt{1 - \dfrac{v^2}{c^2}}},$$

to calculate how long 1.0 s of proper time ($\Delta t_{\mathrm{s}} = 1.0$ s) on the rocket appears to a stationary observer.

5. In the length column, calculate the length of the rocket at each speed as it appears to a stationary observer watching it pass by. Assume that the length of the rocket when it is at rest is 55 m. Use the length contraction equation,

$$L_{\mathrm{m}} = L_{\mathrm{s}}\sqrt{1 - \frac{v^2}{c^2}},$$

where L_{m} is the length of the moving rocket as it appears to the stationary observer, and L_{s} is the length of the rocket at rest from the perspective of an astronaut on the rocket.

6. In the momentum column, calculate the magnitude of the relativistic momentum of the moving rocket as it appears to a stationary observer. Assume that the rest mass of the rocket is 2.1×10^4 kg. Use the relativistic momentum equation,

$$p = \frac{mv}{\sqrt{1 - \dfrac{v^2}{c^2}}},$$

and make sure that your speed is in the correct units.

7. Copy the equations that you have entered in each column to all cells in your table so that the time, length, and momentum values are calculated for all your speeds.

Procedure

Part B: Graphing the Data

8. Using the spreadsheet program and the data that you generated, create three scatter graphs: time versus speed, length versus speed, and momentum versus speed. For each graph, plot the speed on the *x*-axis. Give your graphs titles, and label the axes, including the SI units.

Procedure

Part C: Thought Experiment

9. Suppose that the speed of light suddenly decreased to 100 km/h. Describe in a few paragraphs how this change would affect your daily life.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) Study the three graphs from Step 8, and describe in a few sentences the shape of each curve. T/I C A

(b) Assume that you could only notice a 10 % or greater change in the length of the rocket. Determine the speed at which relativistic effects become noticeable. T/I

(c) Describe the change in the shape of the rocket as the speed increases, as viewed by an Earth-bound observer. T/I

(d) Describe the change in the shape of the rocket as the speed increases, as viewed by an astronaut onboard the rocket. T/I

(e) Describe the changes that you would expect to see in the curves if you extrapolated them to higher speeds. T/I

(f) State the important result of special relativity that your answer to (e) demonstrates. K/U

Apply and Extend

(g) Would the shape of the time dilation graph be different if you plotted the duration of 1 h on the rocket instead of 1 s? Explain your answer. K/U

(h) Determine the amount of energy required by the Large Hadron Collider to accelerate a proton from $0.955c$ to $0.999c$. T/I A

(i) Briefly explain why you think that thought experiments are important in physics research. T/I C A

CHAPTER 11 SUMMARY

Summary Questions

1. Create a concept map for this chapter based on the Key Concepts on page 572. Show how inertial frames of reference, the constant speed of light, time dilation, length contraction, relativistic momentum, rest mass, total energy, rest energy, and relativistic kinetic energy relate to each other, and describe the properties of each. Provide further information, relevant examples, explanatory diagrams, or general equations under each of these concepts.
2. Look back at the Starting Points questions on page 572. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers that you wrote at the beginning of the chapter. Note how your answers have changed.

Vocabulary

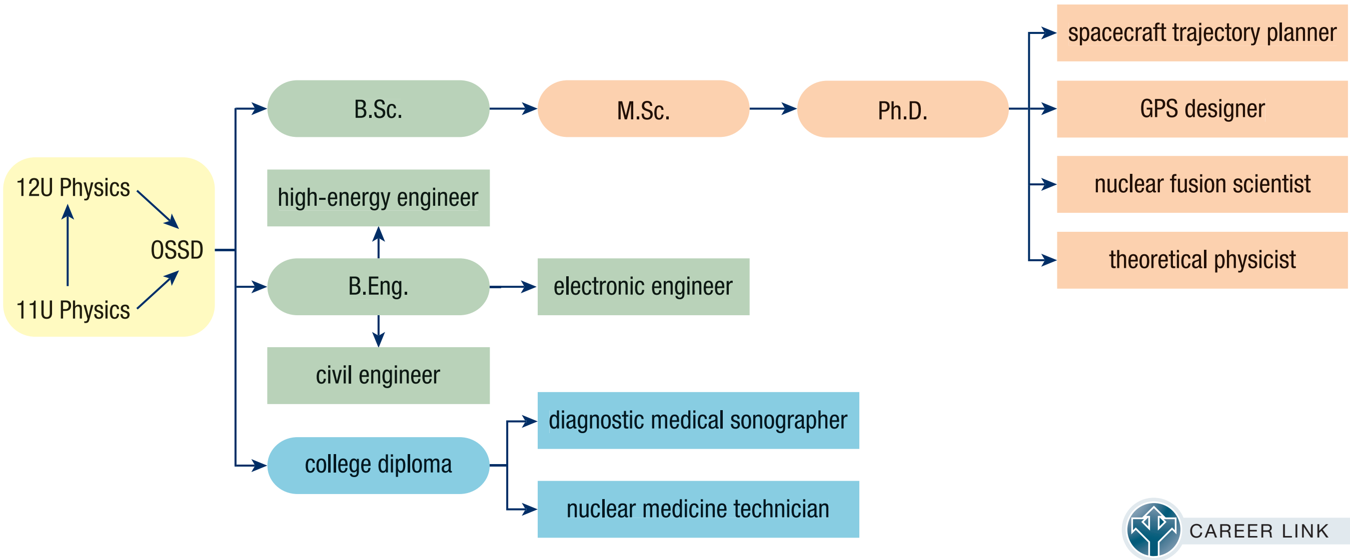
frame of reference (p. 575)
inertial frame of reference (p. 575)
principle of relativity (p. 575)
ether (p. 576)
thought experiment (p. 577)
postulate (p. 577)
special theory of relativity (p. 578)
time dilation (p. 580)
relativistic time (p. 583)
proper time (p. 584)
proper length (p. 588)
length contraction (p. 589)
relativistic length (p. 589)
simultaneity (p. 591)
twin paradox (p. 593)
space-time (p. 594)
relativistic momentum (p. 594)
rest mass (proper mass) (p. 595)
relativistic mass (p. 595)
rest energy (p. 599)
relativistic kinetic energy (p. 599)
conservation of mass–energy (p. 599)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma or a B.Sc. degree. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers mentioned in this chapter.

1. Select two careers related to relativity that you find interesting. Research the educational pathways that you would need to follow to pursue these careers. What is involved in the required educational programs? Prepare a brief report of your findings.
2. For one of the two careers that you chose above, describe the career, main duties and responsibilities, working conditions, and setting. Also outline how the career benefits society and the environment.



CAREER LINK

K/U Knowledge/Understanding   T/I Thinking/Investigation   C Communication   A Application

For each question, select the best answer from the four alternatives.

1. What was the direct source of inspiration for Einstein's theory of relativity? (11.1) K/U
   (a) dissatisfaction with the inconsistency of electromagnetism in moving frames
   (b) the positive results of the Michelson–Morley experiment
   (c) the inability of Newtonian mechanics to solve momentum problems
   (d) principles of thermodynamics
2. A mental activity often used to examine the flaws of a particular theory is called
   (a) hypothesizing
   (b) a thought experiment
   (c) theory dilation
   (d) reflection (11.2) K/U
3. Why do people not observe time dilation in their daily experiences? (11.2) K/U
   (a) Time dilation only occurs on railroad cars, not in automobiles.
   (b) People need to be travelling backward for time dilation to activate.
   (c) Typical terrestrial speeds are much too low to notice the effects of time dilation.
   (d) Typical terrestrial speeds are much too high to notice the effects of time dilation.
4. A spacecraft measures 70 m in length when stationary. As the spacecraft travels at a speed close to the speed of light, what length will be measured by a stationary observer? (11.3) K/U
   (a) slightly more than 70 m
   (b) slightly less than 70 m
   (c) exactly 70 m
   (d) less than 10 m
5. In a graph depicting the relativistic momentum of an object versus speed, which of the following best describes the shape of the line? (11.3) K/U
   (a) The line is a straight line with a positive slope.
   (b) The line is a straight line with a negative slope.
   (c) The line approaches infinity as the object approaches the speed of light.
   (d) The line approaches zero as the object approaches the speed of light.
6. Rajesh holds a metre stick while moving at a speed near the speed of light on a railroad car. Arun stands stationary on the ground and watches. Both Rajesh and Arun measure the length of the metre stick. Who will measure the proper length of the metre stick? (11.3) K/U
   (a) Rajesh
   (b) Arun
   (c) both Rajesh and Arun
   (d) Neither Rajesh nor Arun will be able to measure the proper length of the metre stick.
7. A constant force is applied to a proton. As the proton approaches the speed of light, what will happen to the proton? (11.4) K/U
   (a) The proton's relativistic mass will increase.
   (b) The proton's length will increase.
   (c) The proton's speed will increase until a maximum relative mass is achieved.
   (d) The proton's electric charge will increase.

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. It is convention to say that the Moon revolves around Earth, but we can also say that Earth revolves around the Moon. (11.1) K/U
9. The observed speed of light depends on the speed of the observer's frame of reference. (11.1) K/U
10. The proper time is time measured by an observer at rest, relative to the process being studied. (11.2) K/U
11. The relativistic length is always longer than the proper length. (11.3) K/U
12. Special relativity states that events that are simultaneous in one frame of reference are not simultaneous in all frames of reference. (11.3) K/U
13. The energy released by a nuclear weapon is an example of mass converting to energy as heavier nuclei undergo fission and form lighter nuclei. (11.4) K/U
14. The relativistic energy of an object is equal to the sum of the object's rest energy and kinetic energy. (11.4) K/U

Go to Nelson Science for an online self-quiz.

WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. During which period did physicists enjoy a great sense of accomplishment, just before they needed to radically alter their core ideas about the universe? (11.1) K/U
   (a) mid-1700s
   (b) late 1700s
   (c) mid-1800s
   (d) late 1800s
2. Sam can swim 6 km/h in still water. If Sam swims directly against a river's current that is moving at 4 km/h, what is Sam's velocity relative to the shoreline? (11.1) T/I
   (a) 2 km/h upstream
   (b) 2 km/h downstream
   (c) 10 km/h upstream
   (d) 10 km/h downstream
3. A person is in a railway car without any windows. What test can the person perform to determine whether the car is at rest or is moving at constant velocity? (11.1) K/U
   (a) toss a ball horizontally
   (b) drop a ball vertically
   (c) let two masses collide on the floor
   (d) There is no experiment the person can do to determine whether the car is at rest or moving at constant velocity.
4. Which of the following is an example of an inertial frame of reference? (11.1) K/U
   (a) a race car accelerating from the starting line
   (b) a child riding on the outside of a carousel moving around at constant speed
   (c) a subway train slowing down as it approaches a station
   (d) a woman driving on the highway at constant velocity
5. A stationary lantern emits light with speed c. A spacecraft moves away from the lantern with speed $0.25c$. Determine the speed of the lantern's light as observed by people onboard the spacecraft. (11.1) K/U
   (a) $0.25c$
   (b) $0.75c$
   (c) c
   (d) $1.25c$
6. Charles moves on a railway car at a high speed relative to Priya, who stands at rest. Both Charles and Priya measure time using a light clock with light pulses that bounce up and down. According to Priya, what is the outcome of this situation? (11.2) K/U
   (a) Charles's light pulses move faster than Priya's.
   (b) Charles's time passes faster than Priya's time.
   (c) Charles's time passes slower than Priya's time.
   (d) Charles's light pulses move slower than Priya's.
7. The shortest possible time measured for any process is called the
   (a) proper time
   (b) dilated time
   (c) inertial time
   (d) non-inertial time (11.2) K/U
8. In the equation for calculating time dilation, the denominator will always be
   (a) less than zero
   (b) greater than zero, but less than 1.0
   (c) greater than 1.0
   (d) any numerical value (11.2) K/U
9. Sherin rides on a spacecraft at a speed near the speed of light. Leo remains stationary on Earth. When Sherin leaves, her clock is synchronized with Leo's clock. After some time, Leo checks his clock and notices that 60 min have passed. How much time has passed on Sherin's clock with respect to Leo's clock? (11.2) K/U
   (a) slightly less than 60 min
   (b) slightly more than 60 min
   (c) exactly 120 min
   (d) exactly 3600 min
10. Gilbert and Herbert are identical twins. Herbert leaves on a long space voyage, while Gilbert stays on Earth. When the two twins reunite at home on Earth, which twin will have aged more? (11.3) K/U
    (a) Herbert
    (b) Gilbert
    (c) Both twins will age the same amount.
    (d) It is impossible to know which twin ages more.
11. A mass, m, has rest energy E_{rest}. How much mass is necessary to achieve a rest energy of $4E_{rest}$? (11.4) K/U
    (a) $4m$
    (b) $2m$
    (c) $1.4m$
    (d) $\frac{m}{4}$

12. Stationary nucleus X undergoes fission and splits into nucleus Y and nucleus Z. Energy is released during the process in the form of gamma rays (**Figure 1**). Nuclei Y and Z have high kinetic energies after the reaction. Which statement correctly compares the masses of all three nuclei? (11.4) K/U

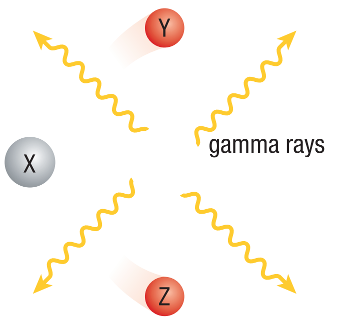


Figure 1

(a) Nucleus X is more massive than the sum of nuclei Y and Z.
(b) Nucleus X is less massive than the sum of nuclei Y and Z.
(c) Nucleus X has the same mass as the sum of nuclei Y and Z.
(d) The masses of all three nuclei are equal.

13. The Sun emits huge amounts of energy in the form of thermal energy and light. What is the source of this energy? (11.4) K/U
(a) chemical bonds being broken as carbon-based materials burn inside the Sun
(b) mass transforming into energy as lighter nuclei fuse into heavier nuclei inside the Sun
(c) electrons emitting radiation as they jump down in energy levels inside hydrogen atoms
(d) mass transforming into energy as heavier nuclei split into lighter nuclei inside the Sun

14. Why have engineers not yet designed a working nuclear fusion reactor? (11.4) K/U
(a) There are too many waste products involved.
(b) There is not a sufficient amount of starting fuel available to run the process.
(c) No containment materials can withstand the temperatures and pressures needed for fusion to occur.
(d) The hypothetical energy output is no higher than that from a nuclear fission reactor.

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

15. The principle of relativity states that for all inertial reference frames, the laws of mechanics are the same. (11.1) K/U
16. Two satellites orbiting Earth are examples of inertial reference frames. (11.1) K/U
17. Another way to state the second postulate of special relativity is to say that the speed of a light wave is independent of the speed of the source. (11.1) K/U
18. When considering problems of motion, no one frame of reference can be considered superior to another. (11.1) K/U
19. Electromagnetic waves, unlike sound waves, can travel through space without a medium. (11.1) K/U
20. Einstein's technique of thought experiments was especially useful when considering objects with speeds close to the speed of light. (11.2) K/U
21. Time dilation is still the result of a thought experiment and has no actual experimental verification. (11.2) K/U
22. For any process, the dilated time will always be less than the proper time. (11.2) K/U
23. Given a stationary observer and an observer who is moving near the speed of light, time will pass slower for the stationary observer. (11.2) K/U
24. The equation for time dilation predicts that to bring about relativistic effects, objects must be moving faster than the speed of light. (11.2) K/U
25. Calculations using global positioning satellites need to account for relativistic effects, such as time dilation. (11.2) K/U
26. Only electrons can be accelerated to speeds faster than the speed of light. (11.2) K/U
27. As an object moves at a speed close to the speed of light, length measured in the direction of the speed contracts. (11.3) K/U
28. Einstein originally wanted to call his model "the special theory of reference frames." (11.3) K/U
29. The mass of a particle travelling at nearly the speed of light increases beyond its rest mass. (11.3) K/U
30. The twin paradox is proof that special relativity is just an exercise for the mind and cannot really happen. (11.3) K/U
31. An object has rest mass m. If the object's relativistic momentum is $4mc$, then its speed must be $4c$. (11.3) K/U
32. When a particle doubles its speed from $0.45c$ to $0.90c$, the relativistic energy will increase by more than a factor of four. (11.4) K/U
33. When two protons and two neutrons fuse into a helium nucleus, energy is emitted and mass remains constant. (11.4) K/U
34. The rest energy of an object is always less than the classical kinetic energy of the same object. (11.4) K/U

Write a short answer to each question.

35. What fields of physics are unified in Maxwell's equations? (11.1) K/U
36. Two identical, very precise clocks are started at the same time. One clock is taken on a trip at a speed close to the speed of light, and the other is left at rest on Earth. When the travelling clock is returned to Earth, it shows that 1 h has passed. Describe the time that has passed on the Earth clock. (11.2) K/U
37. What quantity must be invariant between all inertial reference frames? (11.3) K/U
38. A spacecraft passes you at a speed near the speed of light. Inside the spacecraft is a metre stick aligned in the direction of motion of the spacecraft. Describe the appearance of the length of the metre stick from your perspective on Earth. (11.3) K/U
39. Identify which laws of conservation are applicable for objects moving at speeds near the speed of light. (11.4) K/U

Understanding

40. A driver hangs a small ornament by a string from his rear-view mirror. Describe the precise motion of the ornament as the driver speeds up, slows down, drives at constant velocity, and takes a turn. Use the term *inertial reference frame* where appropriate. (11.1) K/U C
41. A softball player runs at a constant speed over level ground. As she runs, she tosses a ball high in the air and catches it again without breaking her stride. (11.1) K/U
    (a) Describe the path of the softball relative to the softball player.
    (b) Describe the path of the softball relative to a stationary observer.
42. (a) What was the purpose of the ether in physics?
    (b) Why do physicists not need the ether today?
    (c) How did the Michelson–Morley experiment contribute to the demise of the ether theory? (11.1) K/U C
43. A railway car moves with speed v. A light flashes in the centre of the car and travels out in all directions with speed c. Observer Miguel stands at the front of the car. Observer Sofia stands at the back of the car. (11.1) K/U A
    (a) According to Newtonian theory, with what speed will Miguel and Sofia each measure the light beam?
    (b) According to special relativity, with what speed will Miguel and Sofia each measure the light beam?
44. Would an astronaut on a spacecraft moving at $0.99c$ relative to Earth feel as if she were living in slow motion? Explain. (11.2) K/U
45. Use the equation for time dilation to explain why an object cannot travel at, or exceed, the speed of light. (11.2) T/I C A
46. (a) Explain how highly accurate clocks and passenger jets were used to verify time dilation.
    (b) Discuss the purpose of the eastbound and westbound planes.
    (c) Describe quantitatively how much time was dilated for passengers onboard the jets. (11.2) K/U C
47. (a) How long is the average lifetime of a muon at rest?
    (b) How far could this muon travel before decaying without the aid of time dilation? (11.3) K/U C A
48. (a) Explain the twin paradox.
    (b) Why is this considered a paradox?
    (c) What is the resolution of the paradox? (11.3) K/U C A
49. Discuss the terms *rest mass*, *proper mass*, and *relativistic mass*. Which mass(es) can change and under what circumstances? (11.3) K/U C
50. A pilot of a spacecraft travelling at 90 % of the speed of light ($0.9c$) turns on its headlights just as it passes a stationary observer. Compare the measurement of the speed of light emitted by the headlights by the pilot and the observer. (11.3) K/U
51. A train car moves at a relativistic speed into a tunnel. Someone standing on the ground outside the tunnel activates two flashes of light that occur simultaneously at the front and rear of the tunnel. According to a passenger on the train, which flash of light is seen first? Justify your reasoning. (11.3) K/U T/I A
52. Explain how the speed of a particle must have a limit, but the momentum of a particle does not have a limit. (11.4) K/U C A
53. A reaction annihilates a particle and its anti-particle, and the energy released is E. Determine the mass of each particle. (11.4) K/U T/I

Analysis and Application

54. Two rugby players, Greg and Dave, are running north at 5 m/s. Greg starts 1 m behind Dave. (11.1) T/I
    (a) Determine Dave's velocity relative to Greg.
    (b) Greg then slows to 2 m/s, still going north. What is Dave's velocity relative to Greg?
    (c) Greg reverses direction and runs at 3 m/s to the south. What is Dave's velocity relative to Greg?

55. Some distant astronomical objects, such as quasars, move away from Earth with a relative speed of about half that of light. Determine the speed of light from these objects as measured from Earth. (11.1) T/I A
56. A spacecraft travels at 2.5×10^8 m/s relative to Earth. A process onboard the ship as measured by a stationary clock on Earth takes 95 min. How long would the same process take as measured by a clock onboard the ship? (11.2) T/I
57. An elementary particle called the pion has an average lifetime of 2.40×10^{-8} s in a stationary reference frame. Calculate the lifetime of the pion in a frame in which the pion is moving at 75 % of the speed of light. (11.2) T/I
58. An observer on a spacecraft measures a solar flare (**Figure 2**) to last for 17 min. A stationary observer on Earth measures the same flare to last 13 min. Determine the speed of the spacecraft relative to Earth. (11.2) T/I

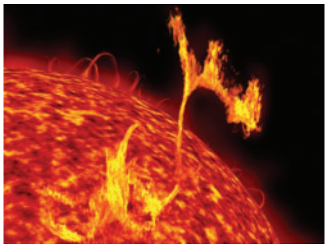


Figure 2

59. The average lifetime of high-speed muons is 320 μs, as measured in the muon's reference frame. The average lifetime of a muon at rest is 220 μs. Calculate the speed of the muons in Earth's reference frame. (11.2) T/I
60. The clock onboard a spacecraft travelling at 0.80*c* relative to the Moon records exactly 45 min for a process on the Moon's surface to occur. (11.2) T/I
    (a) Calculate the proper time for this process to occur.
    (b) Determine the reading on the clock onboard the spacecraft if this process occurs when the ship is travelling at 0.95*c*.
61. Relativistic effects need to be considered when actual time intervals differ from classically predicted time intervals by about 1 %. Assume an interval of 1 h. Determine the minimum speed an object must travel in order to include relativistic calculations. (11.2) T/I
62. A particle at rest takes 2.20×10^{-6} s to decay. Relative to the laboratory's frame of reference in which it moves, the particle decays in 4.40×10^{-6} s. Determine the speed of the particle. (11.2) T/I A
63. A spacecraft activates a warning light that flashes 45 times per minute as measured by people onboard the craft. The spacecraft is travelling at 2.4×10^8 m/s relative to a person standing on Earth. Calculate how many flashes per minute a person on Earth observes from the spacecraft. (11.2) T/I
64. A certain spacecraft measures 65 m in length when it is at rest. The spacecraft then travels at 80 % of the speed of light. Determine the length of the spacecraft
    (a) according to a passenger onboard the ship
    (b) according to an Earthbound observer (11.3) T/I
65. The Andromeda Galaxy is 2.5 million light years from Earth, as measured by astronomers on Earth. A spacecraft travels at 0.90*c* toward Andromeda. Determine the travel distance as measured by the spacecraft passengers. (11.3) T/I
66. A high-speed muon travels 5110 m in its lifetime, as measured by an observer on Earth. From the muon's reference frame, that distance measures 662 m. Calculate the speed of the muon. (11.3) T/I
67. A car with a proper length of 5.0 m passes by. Calculate the contracted length when the speed of the car is 0.99*c*. (11.3) T/I
68. In a thought experiment, a person boards a rocket on Earth and flies off at 0.88*c*, while her twin brother remains at home on Earth. Onboard the rocket, the first twin travels for 2.6×10^6 s (about 30 days), stops suddenly, and travels back to Earth at 0.88*c*, again taking 2.6×10^6 s. Determine how far into "the future" the first twin travels by calculating the difference in the times of the twins' clocks. Express your answer in seconds and days. Assume that the turnaround happens quickly enough that the rocket's velocity is approximately constant during each leg of the trip. (11.3) T/I
69. A particle with rest mass *m* travels with speed 0.50*c*. (11.3) T/I
    (a) Calculate the relativistic momentum of the particle, in terms of *m* and *c*, as measured by a stationary observer.
    (b) Determine how quickly, in terms of *c*, the particle must travel if it is to double its relativistic momentum from (a).
    (c) Determine how quickly, in terms of *c*, the particle must travel if it is to triple its initial relativistic momentum from (a).
70. A proton with a mass of 1.67×10^{-27} kg moves with a speed of 0.800*c*. Calculate the magnitude of the Newtonian momentum, and compare it with the proton's relativistic momentum. (11.3) T/I A
71. An object with rest mass 2.5 kg is travelling at 0.85*c*. Calculate the object's relativistic mass. (11.4) K/U T/I

72. As a certain nuclear process occurs, 4.6×10^{-9} kg of mass is converted to energy. Determine the energy released in
   (a) joules
   (b) megaelectron-volts (Hint: 1 eV = 1.6×10^{-19} J.) (11.4) T/I
73. The rest masses of an electron and a positron are both 9.11×10^{-31} kg. The two particles collide and annihilate each other and emit pure energy (**Figure 3**). Calculate the resulting energy in megaelectron-volts. Assume that the particles are at rest before the collision. (11.4) T/I

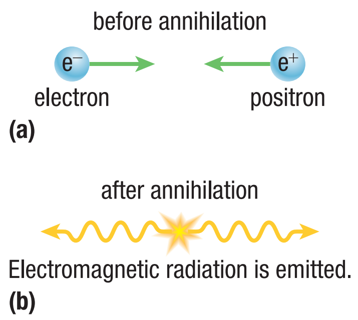


Figure 3

74. Calculate the total relativistic energy, in megaelectron-volts, of a proton moving at 94 % of the speed of light. The rest mass of a proton is 1.67×10^{-27} kg. (11.4) T/I
75. An electron with rest energy 0.511 MeV moves with speed 0.80*c*. Calculate its total relativistic energy and kinetic energy. (11.4) T/I
76. A proton starting at rest is accelerated through a potential difference of 1.4 GV. Calculate the final speed of the proton as a fraction of the speed of light. (11.4) T/I
77. The daily energy output of a certain power plant is 5.3×10^{13} J. Determine how much mass, in grams, must be converted to pure energy to produce an equivalent amount of energy. (11.4) T/I

Evaluation

78. The laws of physics are the same in all frames of reference at rest with respect to one another. What can you conclude about the laws of physics in frames of reference moving relative to one another at constant velocities? (11.1, 11.3) K/U C A
79. A spacecraft travelling toward Earth with speed v has a rod sticking out at right angles to its direction of travel. When a light at the spacecraft end of the rod flashes, the light pulse that travels along the rod is reflected back to the spacecraft by a mirror at the end of the rod. The returning light pulse activates a very fast mechanism that makes the light flash again. Let the frequency of flashes as determined from the spacecraft be f. Assume that time dilation is the only effect to take into account, and determine the period of the flash on Earth clocks. (11.2) K/U T/I A
80. Another one of Einstein's thought experiments was to look into a mirror and then race backward at speeds that increased toward and surpassed the speed of light. Discuss the outcome of this experiment. (11.3) C A
81. Two passengers are riding in a train, each sitting at one end of the same car. The train travels east at almost the speed of light. Within the train, these two passengers glance up at the same time. (11.3) T/I
   (a) Express the sequence of events in Earth's reference frame, as seen by an observer standing behind the train.
   (b) Imagine yourself in a similar, real-world situation, with a train moving at a typical terrestrial speed. Describe what you would actually expect to see in this situation. Evaluate your answer to (a) and justify any differences.
82. A friend of yours insists that time travel is possible according to the ideas put forth by Einstein with his theory of special relativity. Your friend believes that we will soon be visited by travellers from the future. Assess the realistic limits of "time travel" afforded by Einstein's theory. (11.3) K/U C
83. Evaluate the mass of an object as its speed approaches the speed of light. (11.4) K/U T/I

Reflect on Your Learning

84. What did you find most surprising in this chapter, and what did you find most interesting? How can you learn more about these topics? K/U T/I
85. How would you explain the postulates of special relativity, the consequences of time dilation, and the consequences of length contraction to a student who has not taken physics? K/U T/I C

86. Consider the different topics that you studied in this chapter. Choose one that you feel has an important impact on your life. Formulate your thoughts on paper, and then express them to a parent, sibling, or friend, explaining the topic and why it is important to you. What else would you like to know about this topic? How could you go about learning this? K/U T/I C

Research

WEB LINK

87. Research the Michelson–Morley experiment. T/I C A
    (a) When and where was this experiment first conducted, and what was its purpose?
    (b) What is an interferometer? Explain how the interferometer allowed for measurements of very small differences in distances. Copy **Figure 4** into your notebook, and add labels where indicated.
    (c) Describe the larger repercussions of this "failed" experiment.

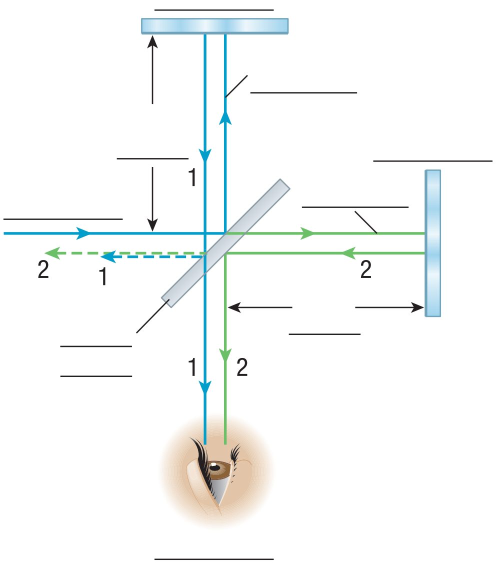


Figure 4

88. Research cold fusion. T/I C A
    (a) What are the promises offered by cold fusion?
    (b) Discuss the incompatibilities with conventional fusion.
    (c) Describe the mistakes made by Pons and Fleishmann in their announcement of cold fusion.
89. Research special relativity in science fiction. List three novels or movies that include special relativity as part of their plot. Do you think the novels or movies do an adequate job of presenting the essential ideas of special relativity? T/I C A
90. Physicists celebrated 2005 as the international year of physics because it marked the centenary celebration of Einstein's miracle year, 1905. T/I C A
    (a) What did Einstein accomplish during 1905?
    (b) What was Einstein doing for a living at this time?
    (c) What else did Einstein accomplish in his scientific and political careers?
91. Research Galileo and his principle of relativity. Describe Galileo's principle of relativity with an illustration. How does Galileo's theory help in the understanding of Einstein's special theory of relativity? T/I C A
92. Although we give Einstein a lot of credit for developing the special theory of relativity, the work of many other researchers led up to Einstein's work. Research the experiments and discoveries that led to the development of special relativity. Choose two examples and summarize in a brief report
    (a) the researchers responsible for the work
    (b) the technology behind the work
    (c) how the work relates to special relativity T/I C A
93. Although Einstein's special theory of relativity made many unusual claims about the world, many different experiments have confirmed its claims. Research experimental tests of special relativity and summarize your findings in a short report. Choose one experimental test, and analyze the technology used in the experiment. T/I C A
94. A laser ring gyroscope, or laser ring gyro (**Figure 5**), measures rotation using the constancy of the speed of light in any frame. Laser ring gyros have uses in navigation systems on aircraft and in detailed studies of Earth's rotation. Research this application of relativity, and summarize your findings in a brief report. Include a description of two applications of laser ring gyros and an analysis of the technology behind them. T/I C A

Figure 5

CHAPTER

12 Quantum Mechanics

KEY CONCEPTS

After completing this chapter you will be able to

- analyze the development of a major revolution in modern physics and assess how it changed scientific thought and impacted society and the environment
- assess the importance of quantum mechanics to the development of various technologies
- describe some properties of photons
- understand, analyze, and simulate the photoelectric effect
- describe blackbody radiation
- explain Planck's hypothesis
- calculate Planck's constant
- explain wave–particle duality
- describe the standard model of particle physics

How Is Our Understanding of the Subatomic World Connected to Technological Developments in Our Society?

Once scientific thinking shifted to the microscopic world and the behaviour of very small particles, researchers discovered that a different set of rules came into play. In the macroscopic world, the laws of Newtonian mechanics apply. However, in the microscopic world, these laws do not apply. Understanding the new rules of the microscopic world has led to new technologies and services. For example, electron microscopy has allowed us to observe a wide range of very small objects, from micro-organisms to molecules. The image of cabbage white butterfly eggs on the facing page was produced by an electron microscope.

Computers and digital technology have transformed our world. The Internet has changed how people share information and communicate. Information and images are instantly transmitted from almost anywhere in the world and even from space. These innovations rely on the behaviour of subatomic particles and microscopic systems.

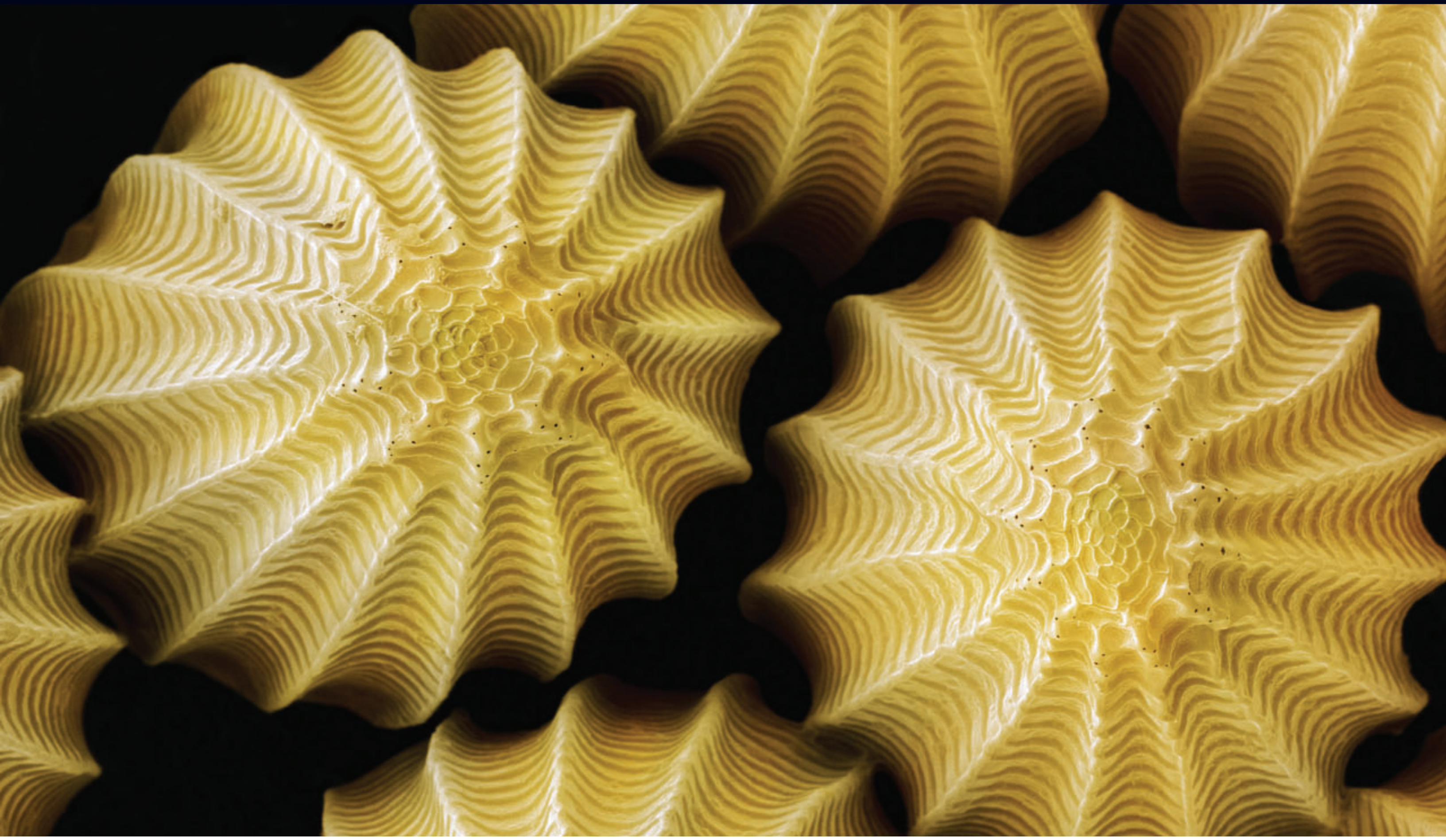
Lasers are another innovation that has developed tha-nks to our understanding of the microscopic world. Lasers allow us to scan a cereal box bar code at the grocery store checkout, play a DVD, print a document, have retinal laser surgery, and measure the distance to the Moon. Researchers even hope to use laser technology to generate safe, clean, sustainable energy through fusion, another application of an understanding of the microscopic world.

In the current time of intense information exchange, private information of individuals, corporations, and governments is continually at risk. Cryptography, the ability to code information and send it securely, has become necessary to prevent increasingly sophisticated methods of identity theft and fraud. New technologies using laws applicable in the microscopic world can detect sophisticated spying equipment and methods of eavesdropping. Future technologies may include a completely secure global satellite communication network. Perhaps a new method of computing will lead to an entirely new way of thinking about how we store, process, and transmit data. In this chapter, you will explore the laws of the microscopic world that make possible the new technologies that are changing society and the environment.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. How important are computers to society? How important are they in your daily life?
2. What are some applications that use lasers?
3. The word "quantum" appears in the title of this chapter. What do you think this word means? Have you heard this word used? If so, where?
4. What are some negative applications of the technologies described above?



Mini Investigation

Investigating Light with Glow-in-the-Dark Objects

Skills: Predicting, Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this activity, you will explore a subatomic effect of light using phosphorescent glow-in-the-dark stickers such as the stars in **Figure 1** or other glow-in-the-dark objects. You will need to prevent the stickers from being exposed to light before starting the investigation.

Equipment and Materials: glow-in-the-dark stickers; holiday LED lights of different colours (blue, white, red, yellow, violet); box or other object to cover the stickers.

1. Predict which colour of LEDs will cause the stickers to glow.
2. After your teacher turns off the lights in the classroom, uncover one of the stickers.
3. Shine the white LED on the sticker, and observe what happens.

Be sure to pull on the plug, and not the electric cord, when you unplug the holiday lights. Use caution when working in a dark room.

Figure 1

4. Repeat Step 3 with the other colours of LEDs, using a new sticker each time. Ensure that the stickers are kept in the dark until you are ready to use them, and do not reuse stickers that have been exposed to light.

A. Which LED colours caused the stickers to glow? T/I

B. Using what you know about light and energy, try to explain your observations. K/U T/I C

12.1 Introducing Quantum Theory

How does the microscopic world differ from the world as you know it? The laws of classical physics govern the motion of everyday objects, such as baseballs, automobiles, and planets. Do these laws apply to the microscopic world?

In classical physics, Newton's laws describing forces and Maxwell's work on electromagnetic radiation form the basis for the study of mechanics, electricity, and magnetism. However, by the late 1800s, experiments applying these laws and theories to the study of newly discovered subatomic particles were leading to some very strange results.

Physicists such as J.J. Thomson discovered that Newton's laws failed to explain the behaviour of electrons and atoms. Similarly, although Maxwell correctly described electromagnetic phenomena in the everyday world, his equations failed to describe the microscopic world. This microscopic world is called the quantum world, where **quantum** refers to a very small increment of energy. The study of the behaviour of these very small bundles of energy, called **quantum theory**, shook the classical foundations of physics in the early twentieth century. In this section, you will see how quantum behaviour is different from anything described before.

quantum the smallest amount of energy that a particle can emit or absorb; the plural is quanta

quantum theory the theoretical basis of modern physics that explains the nature and behaviour of matter and energy at the atomic and subatomic levels

Particles and Waves

According to Newton's laws and Maxwell's equations, energy can be carried from one point to another in two ways: by particles (such as hockey pucks and tennis balls) and by waves (such as sound and earthquake waves). We have an understanding of particles and waves through our everyday experiences, and it is natural to use this intuition when we consider the behaviour of all objects. However, we will see that this intuition is not accurate in the quantum world.

How Do Waves and Particles Differ?

Consider the double-slit experiment, which you learned about in Chapter 9. **Figure 1(a)** shows light incident on an opaque barrier that contains two extremely narrow openings. Recall from Chapter 9 that during a double-slit experiment, an interference pattern consisting of a series of alternating bright and dark fringes forms on the screen on the right. The bright fringes are produced by constructive interference between light waves that pass through the two slits, and the dark fringes are produced at locations where destructive interference occurs. Other types of waves, such as sound and water waves, produce similar results.

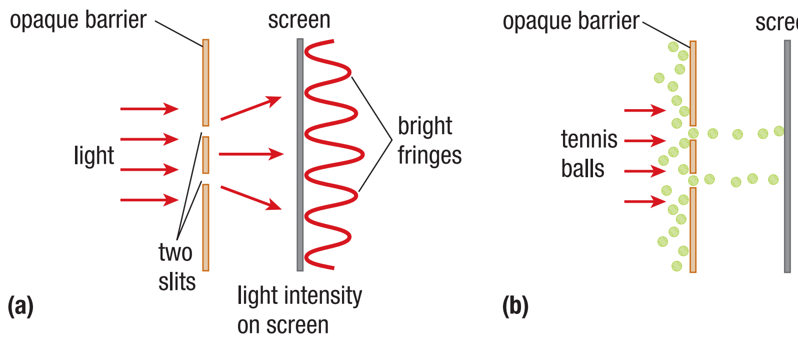


Figure 1 (a) When light passes through a double slit, constructive and destructive interference produces bright and dark fringes on the screen. (b) Classical objects such as tennis balls do not interfere when they pass through a double slit.

Figure 1(b) shows another double-slit experiment using tennis balls instead of light. You might predict that only tennis balls that pass through one or the other of the two slits will reach the screen on the right; the other tennis balls (the ones that strike the barrier) are stopped by the barrier. Your prediction would be correct. In addition, the pattern formed on the screen by the tennis balls that went through the slits will be quite different from the pattern formed by light waves. The tennis ball pattern corresponds only to the "shadows" of the slits. In this case, no constructive or destructive interference occurs.

The behaviour of the tennis balls in Figure 1 illustrates some important differences between particles and waves.

- Particles do not show interference effects.
- Waves do show interference effects.
- Particles deliver energy in discrete quantities, that is, separate, individual "parcels" of energy that transfer to the screen in the small area where the particle strikes. For example, when a tennis ball hits the screen, some energy instantly transfers to the screen at the spot where the tennis ball hit.
- Waves do not deliver energy in discrete quantities. Waves deliver their energy continuously over time and spread out over the screen.

The energy carried by a wave is described by its intensity, which equals the amount of energy the wave transports per unit time across a surface of unit area. For the light wave in Figure 1(a), the amount of energy absorbed by the screen depends on the intensity of the wave and the absorption time. The amount of absorbed energy can take on any non-negative value.

An Interference Experiment with Electrons

In classical physics, the experiment in Figure 1 shows the distinction between particles and waves. According to classical physics, only these two types of behaviour are possible: waves exhibit interference; particles do not. However, this separation of particles and waves is not found in the quantum world.

Figure 2 shows a double-slit experiment performed with electrons. (You will have a more detailed look at an actual electron double-slit experiment in Section 12.3.) A beam of electrons, all with the same speed, is incident from the left and passes through two slits. The electrons then travel to a screen on the right, which records where each electron strikes. Case 1 in Figure 2 shows the result after 20 electrons have passed through the slits. Each dot shows where a particular electron arrived at the screen—the arrival points seem to be distributed randomly. If you repeated this experiment with another 20 electrons, the precise arrival points would be different, but the general appearance would be the same.

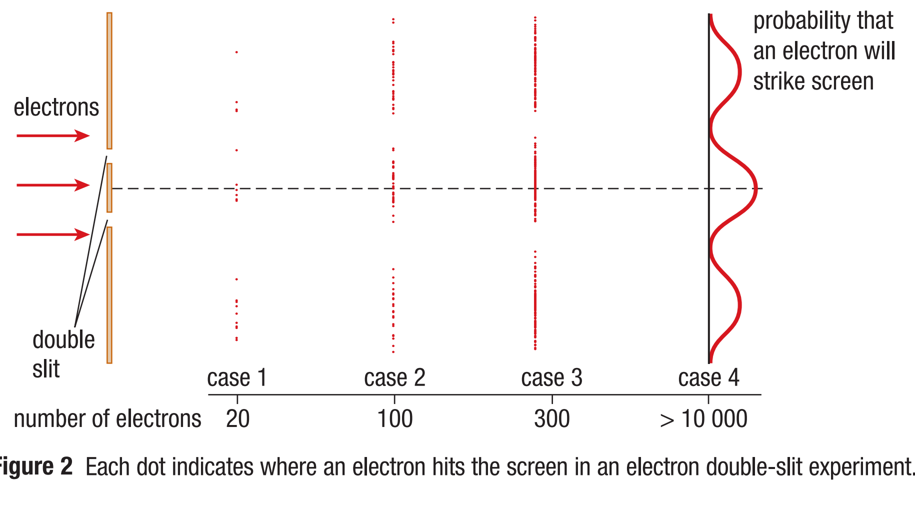


Figure 2 Each dot indicates where an electron hits the screen in an electron double-slit experiment.

If you proceed with the experiment and wait until 100 electrons have arrived, you will see the result shown in case 2. The arrival points are still spread out, but you can now see that the electrons are more likely to strike at certain points than at others. By the time 300 electrons have reached the screen in case 3, it is clear that electrons are much more likely to hit certain points on the screen.

In case 4, quite a large number of electrons have passed through the slits. This part shows the probability that electrons will arrive at different points. This probability curve has precisely the same form as the variation of light intensity in the double-slit interference experiment in Figure 1(a). The experiment shows that electrons constructively interfere at certain locations on the screen, giving a large probability for electrons to arrive at those locations. At other places, the electrons destructively interfere, and the probability for an electron to reach those locations is quite small or zero. **Figure 3** is an actual image taken during an electron double-slit experiment, and it shows how the electrons produce the same pattern as light in the double-slit experiment.

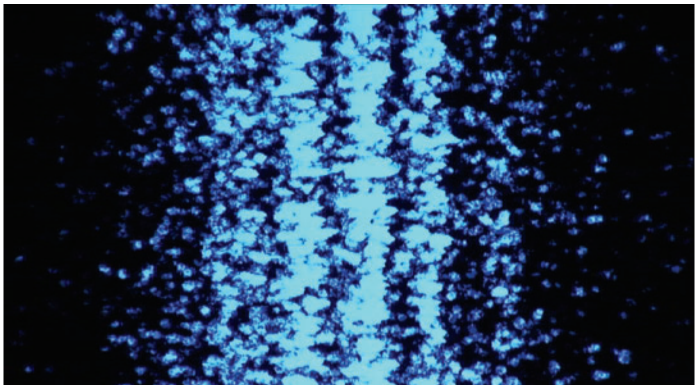


Figure 3 The electrons in the double-slit experiment produce the same pattern as light in a double-slit experiment.

The results in Figure 3 show that electrons can exhibit interference, a property that classical theory says is possible only for waves. This experiment also shows aspects of particle-like behaviour because the electrons arrive one at a time at the screen. Each dot in cases 1 through 3 in Figure 2 corresponds to the arrival of a single electron as it deposits its parcel of energy on the screen.

The behaviour in Figure 3 shows that electrons behave in some ways as both a classical particle and a classical wave. In fact, this behaviour is characteristic of the quantum world: Electrons, protons, atoms, and molecules all give the results shown in Figure 3. Even light and other electromagnetic radiation exhibit particle-like behaviour. The clear-cut distinction between particles and waves breaks down in the quantum world.

wave–particle duality the property of matter that defines its dual nature of displaying both wave-like and particle-like characteristics

The property of matter that defines its dual nature of exhibiting both wave-like and particle-like behaviours is sometimes called **wave–particle duality**, and includes the following properties:

- All quantum objects, including electromagnetic radiation and electrons, can exhibit interference.
- All quantum objects, including electromagnetic radiation and electrons, transfer energy in distinct, or discrete, amounts. These discrete “parcels” of energy are quanta.

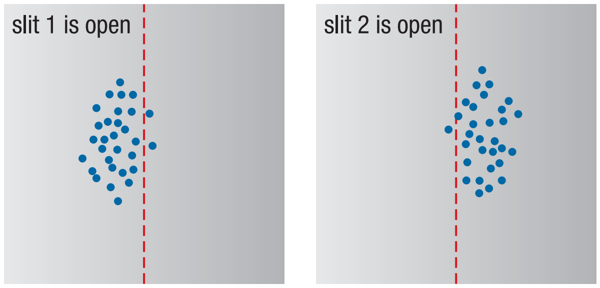
12.1 Review

Summary

- Classical physics refers to the everyday world of large objects where Newtonian mechanics and Maxwell's theory of electromagnetism apply.
- The quantum world is the world of microscopic particles and their behaviours.
- Many models in classical physics break down when applied to extremely small objects such as electrons.
- In classical physics, energy can be carried from one point to another by waves or by particles.
- In classical physics, waves exhibit interference; particles do not. Particles often deliver their energy in discrete amounts, but waves do not.
- In the quantum world, all objects, including electromagnetic radiation and electrons, can exhibit interference and transfer energy in discrete amounts called quanta.
- Wave–particle duality is the property of matter that defines its wave-like and particle-like characteristics.

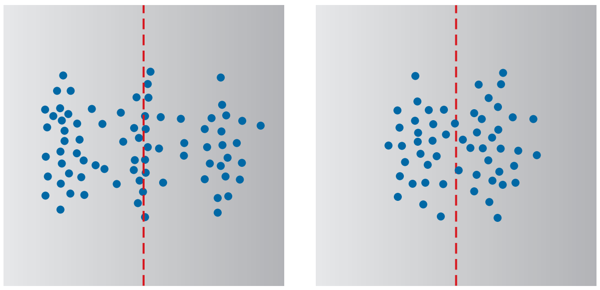
Questions

1. How is energy transferred according to classical physics? K/U
2. Describe the differences between the properties of classical particles and classical waves. K/U C
3. Describe what evidence the electron double-slit interference experiment provides that suggests that
   (a) electrons have particle properties
   (b) electrons have wave properties K/U T/I C
4. Explain why quantum theory revolutionized classical physics in the early twentieth century. T/I C
5. Describe the impact of the electron double-slit interference experiment on classical physics and the development of quantum theory. T/I C
6. Maxwell's theory describes the wave nature of light and all electromagnetic radiation. How is quantum theory more complete than Maxwell's theory? T/I C
7. You hit 50 golf balls at a barrier with 2 narrow slits in it. There is a wall directly behind the barrier. Draw a diagram that shows the distribution of golf balls hitting the wall. T/I C A
8. Imagine that you hit baseballs toward two slits, and that the balls leave marks on a wall after passing through the slits. **Figure 4** shows the distributions of the marks made on the wall when each slit is open. When slit 1 is open and slit 2 is closed, the baseballs hit the wall in the pattern shown in Figure 4(a). When slit 2 is open and slit 1 is closed, the baseballs hit the wall in the pattern shown in Figure 4(b). **Figure 5** shows sample distribution patterns. K/U T/I C A



(a) (b)

Figure 4



(a) (b)

Figure 5

   (a) Which distribution in Figure 5 best represents the distribution of baseballs when both slits are open at the same time?
   (b) Compare this pattern to the pattern that you would see if you used a beam of electrons instead of baseballs.
9. Explain why some of our intuitions about the macroscopic world do not apply to the quantum world. T/I C

12.2 Photons and the Quantum Theory of Light

Figure 1 Laser light shows at concerts are one application of the particle nature of light.

Lasers are used everywhere, from concert light shows to grocery store checkout lines to cutting-edge research labs (**Figure 1**). Although classical physics says that light behaves as a wave, the discovery that light also has particle properties led to the development of the laser and other breakthroughs in light technology. In this section, you will read about photons ("particles" of light) and the nature of light, as well as some key experiments and discoveries that led to a deeper understanding of the nature of electromagnetic radiation.

The Work Function

Around 1800, Thomas Young performed his double-slit interference experiment, which provided the first clear evidence that light is a wave. Maxwell worked out his theory of electromagnetic waves about 60 years later. Then, physicists developed a detailed theory of light as an electromagnetic wave and thought that the nature of light was well understood. In the 1880s, however, studies of what happens when light shines onto metal gave some very puzzling results that the wave theory of light could not explain.

Metal contains electrons that are free to move around within the metal. However, these electrons are still bound as a whole to the metal because of their attraction to the positive charges of the metal atom nuclei. Energy is required to remove electrons from the atoms.

work function (W) the minimum energy needed to remove an electron bound to a metal surface

The minimum energy required to remove a single electron from a piece of metal is called the **work function**, W. The work function has units of energy. For convenience, researchers often give the value of the work function in electron-volts (eV) rather than joules. One electron-volt is defined as the amount of energy given to an electron that accelerates through a potential difference of one volt. To convert electron-volts to joules, use the following conversion factor:

$$1\text{ eV} = 1.60 \times 10^{-19}\text{ J}$$

You can measure the work function of a metal by applying an electric potential. In **Figure 2**, when the electric potential energy of the electrons exceeds the work function, electrons are ejected from the top metal and move to the metal plate below. The smallest electric potential difference able to eject electrons gives the value of the work function, W:

$$W = e\Delta V$$

where ΔV is the electric potential difference at which electrons begin to jump across the vacuum gap in Figure 2, and e is the elementary charge, 1.6×10^{-19} C.

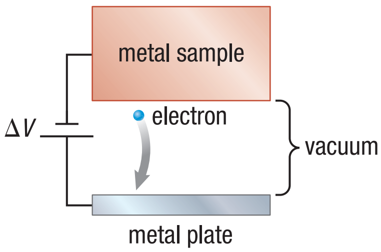


Figure 2 Electrons will leave a metal sample and will move into a metal plate when the applied voltage is above a certain level. That voltage times the elementary charge equals the work function.

Table 1 lists the values of the work functions for common materials in both joules and electron-volts.

Table 1 Work Functions of Several Metals

Metal	Work function (J)	Work function (eV)
aluminum (Al)	6.73×10^{-19}	4.20
calcium (Ca)	4.60×10^{-19}	2.87
cesium (Cs)	3.12×10^{-19}	1.95
copper (Cu)	8.17×10^{-19}	5.10
iron (Fe)	7.48×10^{-19}	4.67
lead (Pb)	6.81×10^{-19}	4.25
platinum (Pt)	9.04×10^{-19}	5.64
potassium (K)	3.67×10^{-19}	2.29
silver (Ag)	7.43×10^{-19}	4.64
sodium (Na)	3.78×10^{-19}	2.36

The Photoelectric Effect

Another way to extract electrons from a metal is by shining light onto it. Light striking a metal surface is absorbed by the electrons. If an electron absorbs an amount of light energy above the metal's work function, it ejects from the metal in a phenomenon called the **photoelectric effect**. **Figure 3(a)** shows the photoelectric effect. Experimental studies of the photoelectric effect carried out around 1900 revealed that no electrons are emitted unless the light's frequency is greater than the **threshold frequency**, f_0, which is the minimum frequency at which electrons are ejected from a material. When the frequency is above f_0, the kinetic energy of the emitted electrons varies linearly with frequency f, as shown in **Figure 3(b)**. Physicists tried to explain these results using the classical wave theory of light, but two difficulties existed with the classical explanations.

photoelectric effect the phenomenon of electrons being ejected from a material when exposed to electromagnetic radiation

threshold frequency (f_0) the minimum frequency at which electrons are ejected from a metal

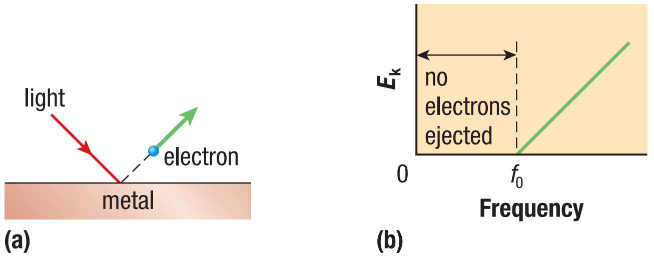


Figure 3 (a) In the photoelectric effect, electrons are ejected from a metal when light at or above a certain frequency strikes it. (b) Experiments show that the kinetic energy of the ejected electrons, E_k, depends on the frequency of the light. When the frequency is below the threshold frequency, no electrons are ejected.

First, experiments show that the threshold frequency is independent of the intensity of the light. Recall the discussion from Section 12.1 in which we compared the differences between particles and waves: According to classical wave theory, the energy carried by a light wave is proportional to the intensity of the light. It should always be possible to eject electrons by increasing the intensity to a sufficiently high value. Experiments found that when the frequency is below the threshold frequency, however, no electrons are ejected, no matter how great the light intensity.

Second, the kinetic energy of an ejected electron is independent of the light intensity. Classical theory predicts that increasing the intensity will increase the kinetic energy of the electrons, but experiments do not show this.

Many scientists refused to abandon the classical wave theory and continued to look for complicated ways to explain the photoelectric effect. Some scientists looked for interference from other sources of electromagnetic noise, and some considered errors from preparing the metallic samples. Quantum theory eventually prevailed as the accepted explanation.

Einstein's Quantum Theory of Light

photon a discrete bundle of energy carried by light

In 1905, Albert Einstein proposed that light should be thought of as a collection of particles, now called **photons**. Photons have two important properties that are quite different from classical particles. Photons do not have any mass, and they exhibit interference effects, as electrons do in double-slit interference experiments. Photons are unlike any other particle in classical physics.

According to Einstein, each photon carries a parcel of maximum kinetic energy (quantum) according to the following equation:

$$E_{\text{photon}} = hf$$

Planck's constant (*h*) a constant with the value 6.63×10^{-34} J·s; represents the ratio of the energy of a single quantum to its frequency

where f is the frequency of the light and h is a constant of nature called **Planck's constant**, which has the value 6.63×10^{-34} J·s. Planck's constant had been introduced a few years earlier by Max Planck to explain another unexpected property of electromagnetic radiation, the blackbody radiation spectrum, discussed later in this section.

Einstein's photon, or quantum, theory explains the two puzzles associated with photoelectric experiments. In fact, Einstein won the Nobel Prize for Physics in 1921 for his contributions regarding the photoelectric effect. First, the absorption of light by an electron is just like a collision between two particles, a photon and an electron. The photon carries energy hf, which the electron absorbs. When this energy is less than the work function, the electron is not able to escape from the metal. For monochromatic light, increasing the light intensity increases the number of photons that arrive each second. However, if the photon energy is less than the work function, even a high-intensity light will not eject electrons. The energy of a single photon—and hence the energy gained by any particular electron—depends on the frequency but not on the light intensity. Electrons can be ejected with varying speeds and, therefore, varying kinetic energies. However, in our discussion of the photoelectric effect, we are concerned with the electron's maximum kinetic energy.

Second, Einstein's quantum theory also explains why the kinetic energy of ejected electrons depends on light frequency but not intensity. The threshold frequency in the photoelectric effect (Figure 3(b)) corresponds to photons whose energy is equal to the work function, W:

$$hf_0 = W$$

Such a photon has barely enough energy to eject an electron from the metal, but the ejected electron then has no kinetic energy. If a photon has a higher frequency and thus a greater energy, the extra energy above the work function goes into the kinetic energy of the electron. So

$$E_{\text{k}} = hf - hf_0$$
$$E_{\text{k}} = hf - W$$

This is the equation of a straight line. Hence, the kinetic energy of an ejected electron should be linearly proportional to f. This linear behaviour is precisely what is found in experiments (**Figure 4** on the next page). The slope of this line is the factor that is multiplied by f, which is Planck's constant, h.

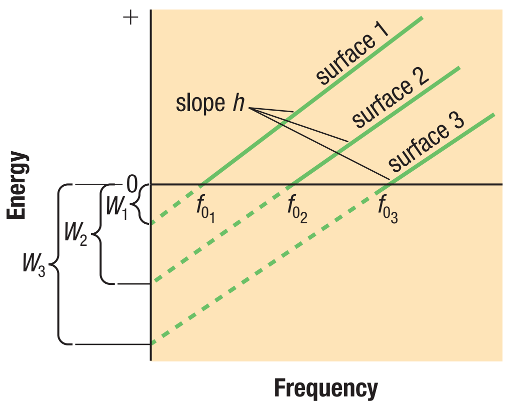


Figure 4 The slope of the energy of ejected electrons versus the frequency of incident light does not change for the three surfaces, even though they each have a different work function. The slope equals h.

Investigation 12.2.1

The Photoelectric Effect (page 654)
This investigation will give you an opportunity to simulate the photoelectric effect and work with the work function equation.

Photoelectric experiments give a way to measure h, and the values found agree with the value known prior to Einstein's quantum theory. Tutorial 1 models how to solve problems involving the photoelectric effect.

Tutorial 1 Solving Problems Involving the Photoelectric Effect

This Sample Problem models how to apply Einstein's theory of the photoelectric effect to determine the lowest photon energy that light must have to cause emission of electrons from a given metal.

Sample Problem 1: Determining Photon Energy

Aluminum is being used in a photoelectric effect experiment. According to Table 1 on page 621, the work function of aluminum is 6.73×10^{-19} J.

(a) Calculate the minimum photon energy and frequency needed to emit electrons.

(b) Incident blue light of wavelength 450 nm is used in the experiment. Determine whether any electrons are emitted, and if they are, determine their maximum kinetic energy.

Solution

(a) **Given:** $W = 6.73 \times 10^{-19}$ J; $h = 6.63 \times 10^{-34}$ J·s

Required: the lowest energy, E_{photon}, that photons must have to cause emission of electrons; frequency, f_0

Analysis: The lowest photon energy must satisfy the equation $E_{photon} = W$, and the frequency this corresponds to is f_0 in the equation $E_{photon} = hf_0$.

$$f_0 = \frac{E_{photon}}{h}$$

Solution: $E_{photon} = W$

$$E_{photon} = 6.73 \times 10^{-19}\ \text{J}$$

$$f_0 = \frac{E_{photon}}{h} = \frac{6.73 \times 10^{-19}\ \cancel{\text{J}}}{6.63 \times 10^{-34}\ \cancel{\text{J}}\cdot\text{s}}$$

$$f_0 = 1.02 \times 10^{15}\ \text{Hz}$$

Statement: The minimum energy needed to emit electrons is 6.73×10^{-19} J, corresponding to using incident light with a frequency of 1.02×10^{15} Hz.

(b) **Given:** $\lambda = 450$ nm $= 4.50 \times 10^{-7}$ m; $W = 6.73 \times 10^{-19}$ J; $h = 6.63 \times 10^{-34}$ J·s; $c = 3.0 \times 10^{8}$ m/s

Required: E_k

Analysis: The energy of the photon is $E_{photon} = hf$, and the frequency is related to the wavelength by $\lambda f = c$. Therefore,

$$\lambda f = c$$

$$f = \frac{c}{\lambda}$$

$$E_{photon} = hf$$

$$E_{photon} = \frac{hc}{\lambda}$$

If E_{photon} is greater than W, then electrons can be emitted by the incident photon. If that happens, then the maximum kinetic energy an ejected electron can have is the incident photon energy minus the work function W:

$$E_k = E_{photon} - W$$

Solution: $E_{photon} = \dfrac{hc}{\lambda}$

$$= \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\cancel{\text{s}})\left(3.0 \times 10^{8}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)}{4.50 \times 10^{-7}\ \cancel{\text{m}}}$$

$$E_{photon} = 4.4 \times 10^{-19}\ \text{J}$$

This energy is less than W. No electrons are ejected.

Statement: The frequency of the 450 nm wavelength light is too low and the photon energy is too low for electrons to be emitted from aluminum by means of the photoelectric effect.

Practice

1. A photoelectric effect experiment uses calcium instead of aluminum. Determine the lowest photon energy that can cause emission of electrons by means of the photoelectric effect. Refer to Table 1 on page 621. T/I A [ans: 4.60×10^{-19} J]
2. The wavelength of light incident on a clean metal surface is slowly decreased. The emission of electrons from the metal first occurs at a wavelength of 268 nm. Determine the work function of the metal. Then use Table 1 to determine whether the metal is silver or lead. T/I A [ans: 7.42×10^{-19} J; silver]

Photons Possess Energy and Momentum

Einstein's quantum theory states that light energy can only be absorbed or emitted in discrete parcels, that is, as single photons. Each photon carries an energy, E_{photon}, equal to hf. The classical theory of electromagnetic waves predicts that a light wave with energy E also carries a certain amount of momentum,

$$p = \frac{E}{c}$$

You can also derive this formula (as Einstein did) from the formulas for the relativistic energy and momentum for an object with zero mass.

Identifying the energy E with E_{photon}, Einstein's quantum theory predicts that the momentum of a single photon is

$$\boxed{p_{\text{photon}} = \frac{hf}{c}}$$

The wavelength of a light wave is related to its frequency as

$$f\lambda = c$$

So, substituting $f\lambda$ for c in the photon momentum equation gives

$$\begin{aligned} p_{\text{photon}} &= \frac{hf}{c} \\ &= \frac{h\cancel{f}}{\cancel{f}\lambda} \\ p_{\text{photon}} &= \frac{h}{\lambda} \end{aligned}$$

Evidence of Photon Momentum

In 1923, American physicist A.H. Compton (1892–1962) discovered a phenomenon that provided experimental evidence of the momentum carried by individual photons. Instead of using visible light, Compton directed a beam of high-energy X-ray photons at a thin metal foil. The foil ejected both electrons and lower-energy X-ray photons. This effect, in which incident X-ray photons lose energy and scatter off a metal foil along with free electrons, is called the **Compton effect**.

Compton effect the elastic scattering of photons by high-energy photons

Compton conducted a series of experiments using different metal foils and different beams of X-rays. X-rays have higher-energy photons, and the electron is able to absorb only some of this energy. In contrast, in a typical photoelectric experiment, lower-energy photons are used, and the electron is able to absorb all the energy, leaving none left over for a residual photon. Each test produced similar results that could not be explained using electromagnetic wave theory. The results suggested to Compton that each incident X-ray photon acts like a particle in an elastic collision with an electron in the metal. The photon emerges from the collision with lower energy and a different momentum. The electron deflects with the kinetic energy and momentum lost by the photon (**Figure 5**).

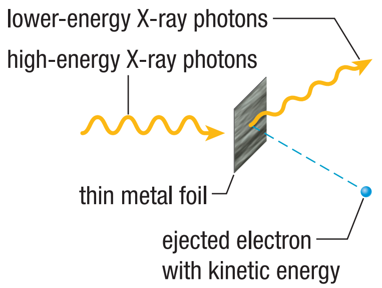


Figure 5 The Compton effect results when X-ray radiation scatters from a metal and ejects an electron. The effect can be explained as an elastic collision between a photon and an electron in the metal.

Compton's data indicated that the effect conserves both energy and momentum. Compton had to use equations of special relativity to analyze the collisions, including the equations for relativistic momentum that you learned about in Section 11.3. He would not have obtained the correct results without using special relativity. In this way, Einstein's ideas on relativity and the speed of light influenced work that confirmed Einstein's ideas about the behaviour and characteristics of photons.

Photon Energy

We can use the equation $E_{\text{photon}} = hf$ to calculate the energy carried by a single photon. For example, a green laser pointer with a wavelength of about 530 nm has a frequency of 5.7×10^{14} Hz. This frequency corresponds to an energy of 3.8×10^{-19} J, which is quite a small amount of energy. This is much smaller than what you might encounter in the everyday world. In most applications, you can detect the presence or absence of light (the presence or absence of one or more photons) through the energy carried by the light. If each photon of light has so little energy, you can infer that a laser beam must contain a huge number of photons.

In the following Tutorial, you will solve problems related to the momentum and energy of a photon.

Tutorial 2 Analyzing Photon Energy and Momentum

In the following Sample Problem, you will examine how the momentum and energy of a photon depend on the photon's frequency.

Sample Problem 1: Analyzing the Momentum and Energy of a Photon

A certain AM radio station has a frequency near 1.0 MHz, and a certain FM station has a frequency near 110 MHz. Radio waves are electromagnetic waves, so the radio waves produce photons.

(a) Compare the momentum of the photons from the AM station with the momentum of the photons from the FM station.

(b) Compare the energy of the photons from the AM and the FM stations.

Solution

(a) **Given:** $f_{\text{AM}} = 1.0\text{ MHz} = 1.0 \times 10^6\text{ Hz}$; $f_{\text{FM}} = 110\text{ MHz} = 1.10 \times 10^8\text{ Hz}$; $h = 6.63 \times 10^{-34}\text{ J}\cdot\text{s}$; $c = 3.0 \times 10^8\text{ m/s}$

Required: p_{photon}, the momentum of the photon at each given frequency

Analysis:

$$p_{\text{photon}} = \frac{h}{\lambda} = \frac{hf}{c}$$

Solution: For the AM station:

$$p_{\text{photon AM}} = \frac{hf}{c}$$

$$= \frac{(6.63 \times 10^{-34}\text{ J}\cdot\cancel{\text{s}})\left(1.0 \times 10^6\ \frac{1}{\cancel{\text{s}}}\right)}{3.0 \times 10^8\text{ m/s}}$$

$$p_{\text{photon AM}} = 2.2 \times 10^{-36}\text{ kg}\cdot\text{m/s}$$

For the FM station:

$$p_{\text{photon FM}} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\cancel{\text{s}})\left(1.10 \times 10^{8}\ \frac{1}{\cancel{\text{s}}}\right)}{3.0 \times 10^{8}\ \text{m/s}}$$

$$p_{\text{photon FM}} = 2.4 \times 10^{-34}\ \text{kg}\cdot\text{m/s}$$

Statement: The photons from the FM station have the greater momentum.

(b) **Given:** $f_{AM} = 1.0\ \text{MHz} = 1.0 \times 10^{6}\ \text{Hz}$; $f_{FM} = 110\ \text{MHz} = 1.10 \times 10^{8}\ \text{Hz}$; $h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$

Required: E_{photon}, the energy of the photon at each given frequency

Analysis: $E_{\text{photon}} = hf$

Solution: For the AM station:

$$E_{\text{photon AM}} = hf$$

$$= (6.63 \times 10^{-34}\ \text{J}\cdot\cancel{\text{s}})\left(1.0 \times 10^{6}\ \frac{1}{\cancel{\text{s}}}\right)$$

$$E_{\text{photon AM}} = 6.6 \times 10^{-28}\ \text{J}$$

For the FM station:

$$E_{\text{photon FM}} = (6.63 \times 10^{-34}\ \text{J}\cdot\cancel{\text{s}})\left(1.10 \times 10^{8}\ \frac{1}{\cancel{\text{s}}}\right)$$

$$E_{\text{photon FM}} = 7.3 \times 10^{-26}\ \text{J}$$

Statement: The higher-frequency FM photons have higher energy.

Practice

1. Calculate the momentum of a photon with a wavelength of 450 nm. T/I A [ans: 1.5×10^{-27} kg·m/s]
2. A hand-held laser pointer emits photons with a wavelength of 630 nm. Determine the energy of these photons. T/I A [ans: 3.2×10^{-19} J]
3. Suppose an atomic nucleus at rest emits a gamma ray with energy 140 keV (which is 2.2×10^{-14} J). Calculate the momentum of the gamma ray. T/I A [ans: 7.3×10^{-23} kg·m/s]

Photon Interactions

In both the photoelectric effect and the Compton effect, when a photon comes into contact with matter, an interaction takes place. Five main interactions can occur:

1. A photon may simply reflect, as when photons of visible light undergo perfectly elastic collisions with a mirror.
2. A photon may free an electron and be absorbed in the process, as in the photoelectric effect.
3. A photon may emerge with less energy and momentum after freeing an electron. After this interaction with matter, the photon still travels at the speed of light but with less energy and a lower frequency. This is the Compton effect.
4. A photon may be absorbed by an individual atom and elevate an electron to a higher energy level within the atom. (You will read more about energy levels in Section 12.6.) The electron remains within the atom but is in what is called an *excited* state.
5. A photon can undergo **pair creation**, where it becomes converted into two particles with mass. This process conserves energy and momentum because all the energy of the photon becomes converted into the kinetic energy of the new particles and their rest mass energy.

pair creation the transformation of a photon into two particles with mass

Blackbody Radiation

blackbody an object that absorbs all radiation reaching it

blackbody radiation radiation emitted by an ideal blackbody

In 1901, Max Planck was studying blackbodies and blackbody radiation. A **blackbody** is an object that absorbs all radiation reaching it, and **blackbody radiation** is radiation emitted by a blackbody. The specific problem that puzzled Planck is represented by the glowing oven in **Figure 6(a)**. This oven emits radiation over a range of wavelengths and frequencies, as shown in **Figure 6(b)**. To the eye, the colour of the oven is determined by the wavelength of the largest radiation intensity.

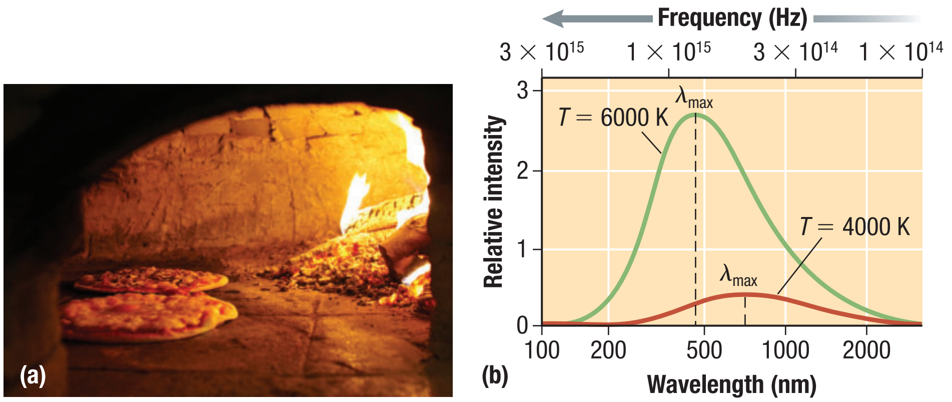


Figure 6 (a) This pizza oven is an approximate blackbody. (b) Light emitted from an ideal blackbody follows the blackbody spectrum shown here. The wavelength, λ_{max}, at which the radiation intensity is largest depends on the temperature of the blackbody.

Experiments prior to Planck's work showed that the intensity curve in Figure 6(b) has the same shape for a wide variety of objects. The blackbody intensity falls to zero at both long and short wavelengths, corresponding to low and high frequencies, respectively, with a peak in the middle. Planck tried to explain this behaviour.

At that time, physicists knew that electromagnetic waves form standing waves as they reflect back and forth inside an oven. These standing waves are just like the standing waves on a string. Standing waves on a string have frequencies that follow the pattern $f_n = nf_0$, where f_0 is the fundamental frequency and $n = 1, 2, 3, \ldots$ (**Figure 7**).

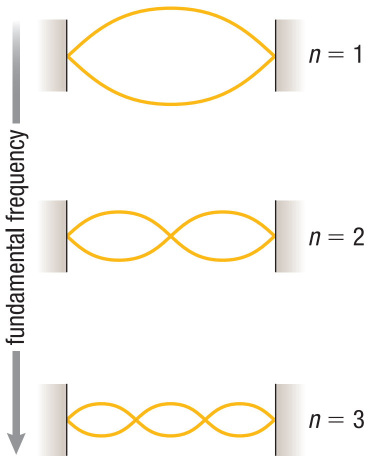


Figure 7 Standing waves form on a string fixed at both ends. The frequency increases as a multiple of the fundamental frequency, f_0.

Standing electromagnetic waves in an oven follow the same mathematical pattern. There is no limit to the value of n (as long as it is a whole number), so the frequency, f_n, of a standing electromagnetic wave in a blackbody can be infinitely large. According to classical physics, each of these standing waves carries energy, and as their frequency increases, so does the total energy. As a result, the classical theory predicts that the blackbody intensity should become infinite as the frequency approaches infinite values.

Investigation 12.2.2

Determining Planck's Constant (page 656)

In this investigation, you will measure Planck's constant in a simple circuit using LEDs.

LEARNING **TIP**

Kelvin Temperature Scale

The kelvin (K) temperature scale is an extension of the degree Celsius scale down to absolute zero, a hypothetical temperature characterized by a complete absence of thermal energy. To convert from a Celsius temperature to a kelvin temperature, simply add 273. Subtract 273 to convert from kelvins to degrees Celsius. For example, absolute zero is 0 K and –273 °C. Temperatures expressed in kelvins do not take the degree symbol.

Planck's Hypothesis

Planck and other physicists of the time thought that any theory that predicts that the intensity of radiation is infinite could not possibly be correct. Such a theory would also be in conflict with the experimental intensity curves in Figure 6(b) because the true intensity falls to zero at high frequencies. Furthermore, nearly all objects act as approximate blackbodies. It is quite difficult to imagine how all objects could emit an infinite amount of energy and still be consistent with our ideas about conservation of energy.

This disagreement between theory and experiment is called the ultraviolet catastrophe. It is called the ultraviolet catastrophe because the predicted infinite intensity is found at high frequencies, and high-frequency visible light falls at the ultraviolet end of the electromagnetic spectrum.

Despite much effort, physicists were not able to connect classical theory with the observed blackbody behaviour. Many researchers had different ideas about how they might solve the problem. Until Planck offered an explanation that fit all the details, multiple theories existed.

Planck resolved this disagreement by hypothesizing that the energy in a blackbody comes in discrete parcels (quanta). He believed that each parcel has energy equal to hf_n, where f_n is one of the standing-wave frequencies and h is a universal constant. Planck showed that this hypothesis supports the theory of blackbody radiation so that it correctly produces the blackbody spectrum in Figure 6(b). However, he could give no reason or justification for his assumption about standing-wave quanta.

His theory of blackbody radiation could fit the experiments perfectly, but no one (including Planck) knew why it worked. Einstein answered that question in part. The standing electromagnetic waves in a blackbody consist of photons. These photons have quantized energies given by Planck's expression, hf_n. While Einstein's photon theory supported Planck's result, physicists would still need several decades of study before they fully worked out and understood the photon concept.

Wien's Law

In Figure 6(b) on page 627, the wavelength at which the radiation intensity of a blackbody is largest is denoted by λ_{max} and is determined by the temperature, T, of the blackbody through an expression called Wien's law:

$$\lambda_{max} = \frac{2.90 \times 10^{-3}\,\text{m}\cdot\text{K}}{T}$$

Here, T is measured in the kelvin temperature scale. According to Wien's law, the value of λ_{max}, and hence the colour of the blackbody, depends on the temperature. A hotter object (higher T) has a smaller value of λ_{max}, and the entire blackbody curve in Figure 6(b) shifts to shorter wavelengths when the temperature is increased. A flame that appears blue (shorter wavelength) is therefore hotter than one that is red (longer wavelength).

One technology that applies Wien's law is an ear thermometer (**Figure 8**), which detects the radiation from inside your ear. According to Wien's law, the wavelength at which the radiation intensity is largest depends on temperature. This thermometer measures λ_{max} and then uses Wien's law to calculate your body temperature. In Tutorial 3, you will use Wien's law to solve blackbody problems.

Figure 8 Ear thermometers use blackbody radiation to calculate body temperature.

Tutorial 3 Solving Problems Related to Blackbody Radiation

This Tutorial models how to use Wien's law to solve problems related to blackbody radiation.

Sample Problem 1: Analyzing Blackbody Radiation

A blackbody would appear to our eye to have a colour determined by the wavelength at which the radiation is most intense. Use **Table 2** to determine the colour of a blackbody that radiates at 4143 K and a blackbody that radiates at 6444 K.

Table 2 Wavelengths of Visible Light

Colour	Wavelength range (m)
red	6.25×10^{-7} to 7.40×10^{-7}
orange	5.90×10^{-7} to 6.25×10^{-7}
yellow	5.65×10^{-7} to 5.90×10^{-7}
green	5.20×10^{-7} to 5.65×10^{-7}
cyan	5.00×10^{-7} to 5.20×10^{-7}
blue	4.35×10^{-7} to 5.00×10^{-7}
violet	3.80×10^{-7} to 4.35×10^{-7}

Given: $T_1 = 4143$ K; $T_2 = 6444$ K

Required: λ_1; λ_2

Analysis: $\lambda_{max} = \dfrac{2.90 \times 10^{-3}\ \text{m·K}}{T}$

Solution:

$$\lambda_{max} = \frac{2.90 \times 10^{-3}\ \text{m·K}}{T}$$

$$\lambda_1 = \frac{2.90 \times 10^{-3}\ \text{m·K}}{T_1}$$

$$= \frac{2.90 \times 10^{-3}\ \text{m·}\cancel{\text{K}}}{4143\ \cancel{\text{K}}}$$

$$\lambda_1 = 7.00 \times 10^{-7}\ \text{m}$$

$$\lambda_2 = \frac{2.90 \times 10^{-3}\ \text{m·K}}{T_2}$$

$$= \frac{2.90 \times 10^{-3}\ \text{m·}\cancel{\text{K}}}{6444\ \cancel{\text{K}}}$$

$$\lambda_2 = 4.50 \times 10^{-7}\ \text{m}$$

Statement: The 4143 K blackbody has a peak wavelength of 7.00×10^{-7} m, which is red light. The 6444 K blackbody has a peak wavelength of 4.50×10^{-7} m, which is blue light.

Practice

1. Determine the colour of a blackbody that radiates at 5100 K. Refer to Table 2. [ans: yellow] T/I
2. A certain blackbody appears to have an aqua colour. The wavelength of the emitted radiation is 510 nm. Determine the temperature of the blackbody. T/I A [ans: 5700 K]
3. The human body emits electromagnetic radiation according to the body's temperature (37 °C). Estimate λ_{max} for the human body. Can you see this electromagnetic radiation? Explain your answer. T/I A [ans: 9.4×10^{-6} m]

Wave–Particle Nature of Light

The photoelectric effect and blackbody radiation can only be understood in terms of the particle nature of light. While light has some properties like those of a classical particle, it also has wave properties such as interference.

In Section 12.1, you read about a double-slit experiment with electrons and observed how the electrons arrive one at a time at the screen. Light behaves in precisely the same way. Consider a double-slit experiment performed with light with a very low intensity. Suppose the screen responds to the arrival of individual photons by emitting light from the spot where the photon strikes. The results would then appear just as in **Figure 9** on the next page. The full interference pattern at the far right becomes visible only after many photons have reached the screen. As with electrons, this experiment shows both the particle nature of light (the arrival of individual photons) and the wave nature of light (interference) at the same time.

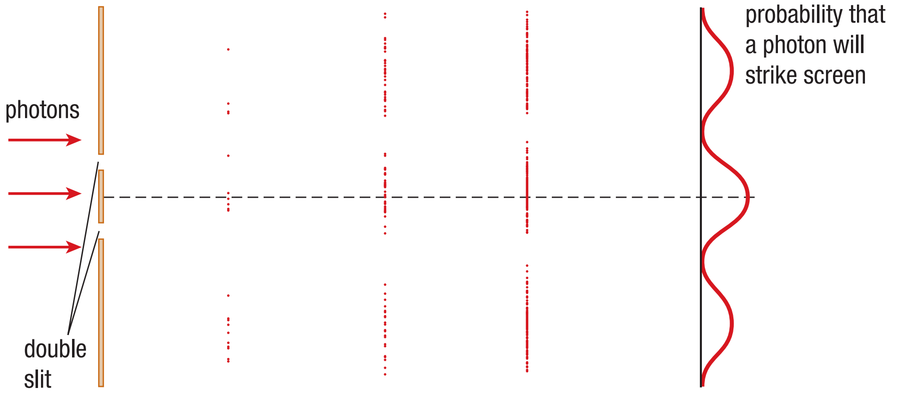


Figure 9 Light with a very low intensity produces an interference pattern of photons.

Research This

Exploring Photonics

Skills: Researching, Analyzing, Evaluating, Communicating

SKILLS HANDBOOK A4.1

The particle nature of light forms the basis for the development of photonics technology. Photonics technology uses light in different functions in various fields of electronics. In this activity, you will research a specific photonics technology that works because of the particle nature of light.

1. Choose a technology or process that uses the particle nature of light, such as lasers (light production), light sensors, photocells (**Figure 10**), or photosynthesis.

Figure 10 A photocell

2. Research your choice, and find out how the particle nature of light is applied.
3. Determine how quantum theory led to the development of the technology or process that you chose.
4. Investigate any new advances in the field of photonics related to your choice.
5. Research the economic, environmental, and social impacts of your chosen technology or process, if applicable. Include both positive and negative impacts in your research.
6. Research the Canadian Photonics Consortium.

A. How does the particle nature of light make the technology or process work? K/U A

B. How is the technology or concept made possible through the understanding of quantum theory? K/U T/I

C. What are some examples of other related emerging technologies? T/I A

D. How many Canadian companies are part of the Canadian Photonics Consortium? Approximately how many people are employed by photonics companies in the consortium?

WEB LINK

12.2 Review

Summary

- The work function, W, is the minimum energy needed to remove an electron bound to a metal surface. The work function equation is $W = eV$.
- A photon is a quantum of electromagnetic energy. The quantum theory of light says that photons have both energy and momentum.
- The energy of a photon is given by $E = hf$.
- The momentum of a photon is given by $p = \frac{hf}{c} = \frac{h}{\lambda}$.
- Planck's constant, h, is a universal constant with a value of 6.63×10^{-34} J·s.
- In the photoelectric effect, electrons are ejected when light with a certain minimum frequency strikes a metal. Energy is conserved during this process.
- In the Compton effect, electrons are ejected when X-rays strike a metal. Energy and momentum are conserved during this process.
- Planck explained the observed spectrum of blackbody radiation by hypothesizing that the energy in a blackbody comes in discrete parcels called quanta.
- The photoelectric effect and blackbody radiation demonstrate that photons exhibit both wave-like and particle-like properties.

Questions

1. The work function for a metal is 5.0 eV. Determine the minimum photon frequency that can just eject an electron from the metal. T/I
2. A photocell is a light sensor that affects the flow of current in a circuit based on the amount of light falling on the cell. When light strikes the cell, the photoelectric effect causes electrons to eject from a metal piece in the cell and flow through the circuit. You are designing a photocell to work with visible light and are considering the use of either aluminum or cesium. Aluminum has a work function of 4.20 eV, and cesium has a work function of 1.95 eV. K/U T/I A
   (a) Which one is the better choice? Explain your answer.
   (b) Calculate the lowest photon frequency that can be measured with your photocell.
   (c) Determine where this frequency falls in the electromagnetic spectrum. Refer to **Table 3**.

Table 3 Frequencies of Electromagnetic Waves

Wave	Frequency (Hz)
radio	$< 3.0 \times 10^9$
microwave	3.0×10^9 to 3.0×10^{12}
infrared	3.0×10^{12} to 4.3×10^{14}
visible light	4.3×10^{14} to 7.5×10^{14}
ultraviolet	7.5×10^{14} to 3.0×10^{17}
X-ray	3.0×10^{17} to 3.0×10^{19}
gamma	$> 3.0 \times 10^{19}$

3. A wavelength of dim red light ejects no electrons. Suppose you increase the intensity by a factor of 1000. Explain whether the red light can now eject electrons according to the photoelectric effect. T/I C
4. Calculate the photon energy and momentum for each of the following. T/I A
   (a) FM radio with a frequency of 100 MHz
   (b) red light with a wavelength of 633 nm
   (c) X-ray radiation with a wavelength of 0.070 nm
5. Determine which has greater energy, an ultraviolet photon or an X-ray photon. Refer to Table 3. Explain your answer. K/U C
6. The highest-energy photons emitted by a hydrogen atom have been measured to have an energy of 13.6 eV. K/U A
   (a) Express this energy in joules.
   (b) Calculate the frequency and wavelength of these photons. What type of electromagnetic radiation do they correspond to? (Use Table 3.)
7. The most common method for converting solar energy into electrical energy uses solar cells. Research the technology of solar cells, and describe to a classmate how they use quantum mechanics to produce electricity. K/U T/I C A

WEB LINK

12.3 Wave Properties of Classical Particles

Research into the quantum world has led to many discoveries in science and technology. In previous sections, you read about how photons have both wave and particle properties. The same is true of other particles. For example, electrons do not behave as particles only, but as both particles and waves. In this section, we will examine the results of one of the most important experiments in quantum physics: the electron double-slit experiment. This experiment produced the clearest evidence of how the properties of waves and particles are both present in the quantum world.

Wave-like Properties of Classical Particles

Earlier you read about the wave-like properties of electrons. The concept that the properties of both classical waves and classical particles are present at the same time—wave–particle duality—is essential for understanding the world of electrons, atoms, and molecules.

By the early 1920s, the photon theory of light was well established. However, physicists were still struggling with how to describe particles such as electrons in the quantum world. In 1924, Louis de Broglie first suggested that all classical particles have wave-like properties. At the time, experimental evidence of interference with electrons had not yet been discovered. That did not stop de Broglie, however, who developed his theory to agree with the behaviour of photons.

In Section 12.2, you read that a photon has a momentum given by

$$p_{\text{photon}} = \frac{h}{\lambda}$$

de Broglie turned this result around and hypothesized that a particle with momentum p has a wavelength of

$$\lambda = \frac{h}{p}$$

de Broglie wavelength the wavelength associated with the motion of a particle possessing momentum of magnitude p

matter wave the wave-like behaviour of particles with mass

This quantity—the wavelength associated with the motion of a particle with momentum p—is the **de Broglie wavelength**. If a particle has a wavelength, the particle should exhibit interference just as waves do. The test of de Broglie's hypothesis was to look for interference involving classical particles. Such an experiment is easiest if the wavelength is long, and according to the de Broglie wavelength equation, a long wavelength means that the value for momentum has to be small. For a classical particle, the momentum is mv. Therefore, a particle with a very small mass is required, and the lightest known particle at that time was the electron. As a result, the first observation of **matter waves**, the wave-like behaviour of massive particles, came from an experiment done with electrons.

The Electron Double-Slit Experiment

As you learned in Sections 9.5 and 12.1, a simple physics experiment to illustrate the wave nature of light is the double-slit experiment. In this experiment, a screen, called screen 1, with two slits is placed at a distance from a point source of light, and a second screen, called screen 2, is placed behind screen 1. When light is directed at screen 1, many interference fringes appear across screen 2, instead of just two bars of light directly in line with the light and the slits. Why does that happen? The slits diffract the light, and the fringes mark the interference of the light waves.

In 1927, physicists Clinton Davisson and Lester Germer performed an experiment in which they aimed a beam of electrons at a crystal target (**Figure 1**). The atoms in the target were spaced at regular intervals, acting as a series of slits for the electrons. Just as with the diffraction of light, the Davisson–Germer experiment exhibits interference when the wavelength of the electrons is similar to the spacing between the atoms in the crystal. The diffraction technique used in the Davisson–Germer experiment is still used today as a way to measure molecule spacing within a crystal. WEB LINK

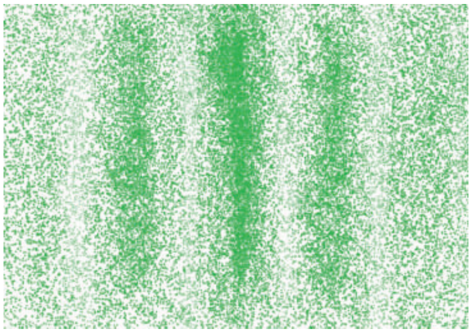


Figure 1 The atoms in a crystal act as slits when electrons or high-energy photons are directed at the crystal. When electrons exit the crystal, they form an interference pattern.

In the following Tutorial, you will quantitatively examine the wave-like properties of electrons.

Tutorial 1 Determining the Wavelength of an Electron

Davisson and Germer's demonstration of interference with electrons showed conclusively that electrons have wave-like properties. Careful measurements of the interference pattern allowed them to calculate the de Broglie wavelength of the electron. This Sample Problem shows how to calculate the de Broglie wavelength.

Sample Problem 1: Calculating the de Broglie Wavelength of an Electron

In their studies of interference with electrons, Davisson and Germer used electrons with a kinetic energy of approximately 50 eV, which equals 8.0×10^{-18} J. The mass of an electron is 9.11×10^{-31} kg.

(a) Calculate the de Broglie wavelength of these electrons in metres and nanometres.

(b) The spacing between atoms in a typical crystal is about 0.3 nm. How does this spacing compare with the wavelength of the electrons used by Davisson and Germer?

Solution

(a) **Given:** $m = 9.11 \times 10^{-31}$ kg; $E_k = 50$ eV $= 8.0 \times 10^{-18}$ J; $h = 6.63 \times 10^{-34}$ J·s

Required: λ

Analysis: Using the equation for kinetic energy, $E_k = \frac{1}{2}mv^2$, solve for the speed. Use the equation for momentum, $p = mv$, and the quantum relation $\lambda = \frac{h}{p}$ to calculate the de Broglie wavelength.

Solution:
$$E_k = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(8.0 \times 10^{-18}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}}$$
$$v = 4.19 \times 10^6\ \text{m/s}$$
$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J·s}}{(9.11 \times 10^{-31}\ \text{kg})(4.19 \times 10^6\ \text{m/s})}$$
$$\lambda = 1.74 \times 10^{-10}\ \text{m}$$

Statement: The wavelength of the electrons in the Davisson–Germer experiment was about 1.74×10^{-10} m, or 0.174 nm.

(b) This wavelength is less than, but similar to, the spacing between atoms in a crystal solid.

Practice

1. Calculate the de Broglie wavelength for an electron with momentum 1.8×10^{-25} kg·m/s. K/U T/I [ans: 3.7×10^{-9} m]
2. Calculate the de Broglie wavelength for a proton moving with a speed of 3.4×10^5 m/s. The mass of a proton is 1.7×10^{-27} kg. K/U T/I [ans: 1.1×10^{-12} m]
3. Calculate the de Broglie wavelength for a 140 g baseball moving at 140 km/h. T/I A [ans: 1.2×10^{-34} m]
4. Compare your answer to Question 3 with the diameter of a proton, which is about 10^{-15} m. What does the baseball's de Broglie wavelength mean? K/U T/I A

It is possible to determine the de Broglie wavelength of larger objects, such as baseballs. However, the momentum of large objects tends to be so large that it implies an incredibly small wavelength. That is why we are unable to see the interference of these objects.

Interpreting the Double-Slit Experiment

The results of double-slit experiments using photons, electrons, or any other small particle are surprising and difficult to understand. The picture of nature at very small scales differs greatly from the classical picture at large scales. Electrons arrive at a screen in single, particle-like amounts, but the spot on the screen where they arrive is determined by wave-like interference behaviour.

The equations of quantum mechanics make precise predictions about the results of a double-slit experiment. However, they do not give a clear description of what happens to photons, electrons, or other particles as they pass through the slits and travel to the screen. As a result of this uncertainty, researchers have proposed different interpretations of what happens in the quantum world. The debate still continues over which interpretation gives the best description. There are researchers, including some at Canada's Perimeter Institute for Theoretical Physics, who support each of these interpretations. CAREER LINK

COLLAPSE INTERPRETATION

In the collapse interpretation, the electron may behave sometimes as a wave and sometimes as a particle, but always one or the other. The laws that determine the motion of an electron differ in either case. The electron leaves its source behaving as a particle, but then it spreads out and travels as a wave until it is measured at the screen (**Figure 2**). When you measure the location of the electron, it somehow collapses back into a particle. It then arrives at one location on the screen, as a particle would. The collapse interpretation claims that an electron physically changes from a particle to a wave and back again. These two behaviours and the physical laws that go with them alternate in a way that is not predicted by quantum mechanics.

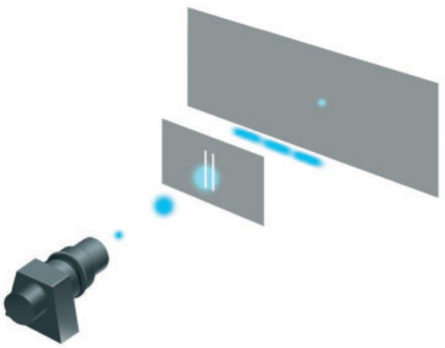


Figure 2 In the collapse interpretation, each electron in a double-slit experiment travels as a spread-out wave.

PILOT WAVE INTERPRETATION

In the pilot wave interpretation, the electron is just a simple particle whose motion is described by a single law. The pilot wave interpretation avoids any unexplained collapse, but the motion of the electron depends on a mysterious pilot wave (**Figure 3**). To obtain the interference pattern in the double-slit experiment, the behaviour of the pilot wave must depend on everything everywhere in the universe, including future events. For example, the pilot wave "knows" whether one or two slits are open, and whether or not a detector is turned on at the screen.

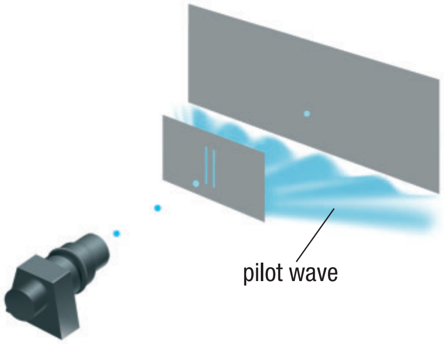


Figure 3 In the pilot wave interpretation, the electrons in a double-slit experiment are particles whose motion depends on a pilot wave.

MANY WORLDS INTERPRETATION

In the many worlds interpretation, electrons are simple particles, as in the pilot wave interpretation. The many worlds interpretation does not introduce a wave-like spreading or collapsing of the electron, though. Nor does it use a mysterious pilot wave. Instead, according to the many worlds interpretation, the universe constantly splits into many versions of itself. Each version exists as a separate parallel universe that cannot interact with any other version. A parallel universe exists for each of the electron's possible states. When the electron reaches the slits, the entire universe splits into two. In one version of the universe, the electron passes through the left slit, and in the other version it passes through the right slit. If the many worlds interpretation is true, then our universe consists of a vast number of parallel worlds, some holding versions of every one of us.

COPENHAGEN INTERPRETATION

The Copenhagen interpretation deals directly with the results of measurements made on physical objects. This interpretation of quantum mechanics was developed in the 1920s by Niels Bohr and his colleagues, primarily at the University of Copenhagen. The interpretation discusses only what you can actually do or observe. It does not use theoretical ideas such as undetectable collapses, invisible pilot waves, and multiple parallel universes. The Copenhagen view interprets the physical laws in terms of information about actual measurements made on a quantum-mechanical system. The Copenhagen interpretation basically says that certain questions do not have answers, such as what electrons are "doing" as they travel to the detection screen. You can only ask what the results will be if you do a certain experiment. This view was the dominant one for most of the twentieth century.

The debate over the different interpretations continues. Although the interpretations have some strange and useful features, they all make the same predictions for experimental results. Some researchers take the view that this fact means that the debate will never end, or that the debate does not need to be settled. They focus on using the predictions of quantum mechanics to explore new features of the quantum world and new technological uses.

The Wave Function: A Mathematical Description of Wave–Particle Duality

The equations of quantum physics describe wave–particle duality and the behaviour of photons, electrons, and other quantum objects with a mathematical tool called a wave function. A wave function gives the probability for a particle to take any possible path, or for the particle to show up at any possible location on the detection screen in the double-slit experiment. All the interpretations of quantum mechanics must include some discussion of the wave function. However, we will not include any wave functions in this discussion because they are beyond the scope of this book.

You can calculate the wave function using an equation developed by Erwin Schrödinger. He was one of the inventors of quantum theory and in 1933 received the Nobel Prize in Physics for this work. In quantum mechanics, researchers use the Schrödinger equation to determine the wave function and how it varies with time. This is in much the same way that physicists use Newton's laws of mechanics to determine the motion of an object.

In many situations, the solutions of the Schrödinger equation are mathematically similar to standing waves. As an example, consider an electron confined to a particular region of space, as shown in **Figure 4(a)**. In classical terms, you can think of this region of space as an extremely deep canyon or box from which the electron cannot escape. A classical particle moving around inside the box would simply travel back and forth, bouncing from one wall to another. The wave function for a particle–wave (such as an electron) inside this box is described by standing waves, similar to those you would see on a string.

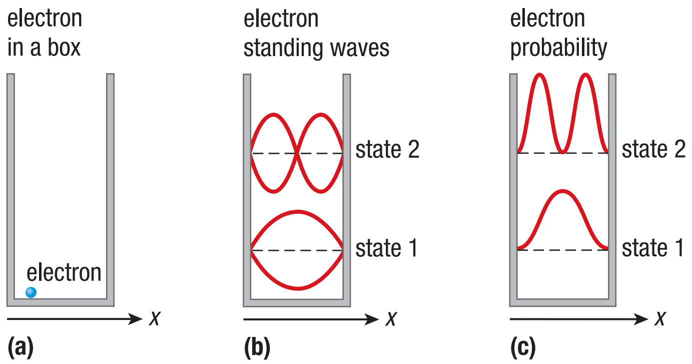


Figure 4 (a) A thought experiment of an electron that is trapped in a box. (b) The electron wave probability function forms a standing particle–wave similar to the standing waves on a string fastened to the walls of the box. The electron wavelength must therefore "fit" into the box as it would for a standing wave. (c) The quantum-mechanical probabilities of finding the electron at different locations in the box correspond to each of the two wave functions in (b).

Figure 4(b) shows two possible wave function solutions corresponding to electrons with different kinetic energies. The wavelengths of these standing waves are different, since the wavelength of an electron depends on its kinetic energy. In the case of mechanical waves in classical mechanics, these two solutions correspond to two standing waves with different wavelengths.

After determining the wave function for a particular situation, such as for the electron in Figure 4(b), you can try to calculate the position and speed of the electron. However, the results do not give a simple single value for x. Instead, the wave function allows the calculation of the probability of finding the electron at different locations in space. The probability for each of the wave functions in Figure 4(b) is shown in **Figure 4(c)** for the electron in a box. The probability of finding the electron at certain values of x is large in some regions and small in others. This corresponds to the anti-nodes and nodes of a standing wave. The probability distribution (how the probability varies with position within the box) is different for each wave function.

The Heisenberg Uncertainty Principle

German physicist Werner Heisenberg's early studies of the meaning of quantum mechanics led him to discover a limitation on measurements of quantum systems. The **Heisenberg uncertainty principle** says that there is a limit to how accurately simultaneous measurements of the position and momentum of a quantum object can be. If you measure the position of a quantum object with great accuracy, then you can only measure its momentum with little accuracy. In addition, the act of measuring the system itself disturbs the system. While this disturbance is not significant in the macroscopic world, the effects are obvious in the quantum world. The mathematical expression of the Heisenberg uncertainty principle is

Heisenberg uncertainty principle a mathematical statement that says that if Δx is the uncertainty in a particle's position, and Δp is the uncertainty in its momentum, then $\Delta x \, \Delta p \geq \frac{h}{4\pi}$, where h is Planck's constant

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

where h is Planck's constant.

Imagine that you set up a detector to measure electron diffraction through a single slit. The width of the slit equals the uncertainty in the position of the electron, Δx, as it passes through the first screen. If you decrease the width of the slit, you decrease the amount of uncertainty in the electron's position. As a consequence, according to Heisenberg's uncertainty principle, the uncertainty in the momentum of the electron, Δp, must increase. The direction in which the electron moves as it leaves the slit becomes more uncertain. As a result, the electron may hit the second screen in a wider range of locations, and the peaks of the diffraction pattern may become wider and less focused (**Figure 5**).

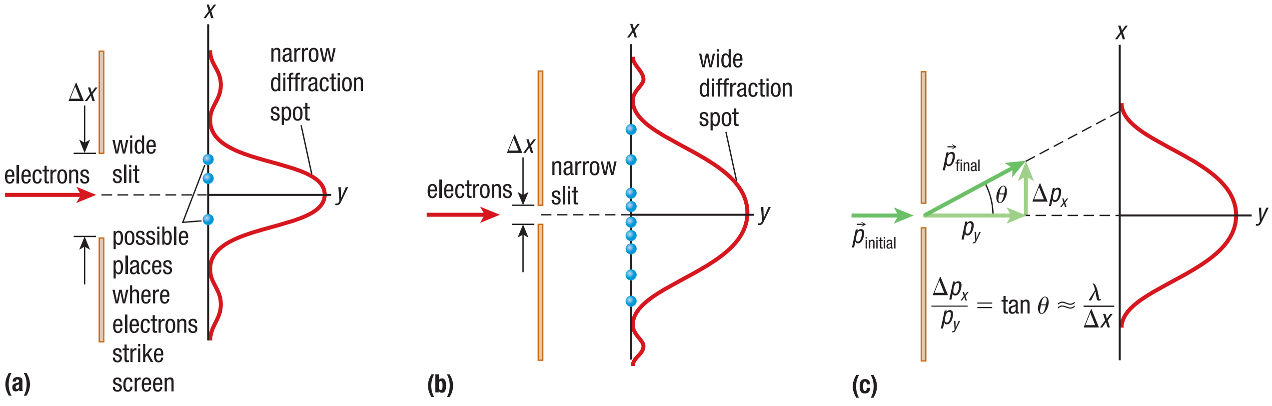


Figure 5 (a) When an electron passes through a wide slit, the diffraction spot on the screen is narrow. (b) When the slit is made narrower, the diffraction spot becomes wider. (c) The spreading of an electron wave as it passes through a slit gives a way to relate the uncertainty in the electron's position, Δx, to the uncertainty in its momentum, Δp.

Applications of the Quantum World

The double-slit interpretations and Heisenberg's ideas may leave you feeling that science cannot adequately explain quantum physics. The debate over the interpretations of what happens to electrons after they exit the double slits illustrates the idea that science does not always answer every question. Scientific understanding progresses tentatively and is always evolving and partly uncertain.

Quantum mechanics does, however, make quite accurate predictions about the statistics of observed results, such as the interference patterns made by many particles in a double-slit interference experiment. It simply says that the world is unpredictable for single events, such as a single electron passing through a double-slit setup. This unpredictability is in contrast to the fully predictable world of classical physics. That unpredictability has not slowed the advance of engineering designs and technologies using the principles of quantum physics.

UNIT TASK **BOOKMARK**

As you work on the Unit Task on page 666, apply what you have learned about technological applications of quantum theory.

Figure 6 This electron microscope image is a foot of a housefly. The white hairs under the claws allow the fly to cling to smooth surfaces.

Applications of wave–particle duality have led to the invention of LEDs, solar cells, and electron microscopes, and to advances in computers, the Internet, and nanotechnologies. In the case of the electron microscope, electron waves create high-resolution images of extremely small objects in a variety of fields (**Figure 6**). The resolution of a microscope depends on the wavelength of the waves used in its beam. In an electron microscope, a beam of electrons is aimed at an object using magnetic and electric fields. The wavelength of the beam is reduced to the de Broglie wavelength of the electrons, which is much smaller than the wavelengths in visible light.

Electronic devices that contain transistors are possible because engineers use the wave model of electrons to design the transistors. You are using the quantum nature of electrons when you talk on a cellphone, search the Internet on a computer, or listen to music on an MP3 player.

Medical imaging is another important application of quantum mechanics. Positron emission tomography (PET) scans allow doctors to produce images of biochemical processes in our bodies using the particle nature of light. Patients swallow a substance that emits positrons, or antimatter electrons, into the body. When positrons collide with an electron, two photons are released. The photons are detected by a device that can create highly detailed images. The use of photons in this way has enabled medical practitioners to observe brain function (**Figure 7**). CAREER LINK

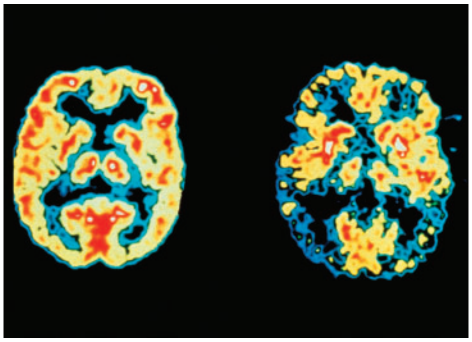


Figure 7 These false-colour PET scans show the brain of a normal patient on the left and the brain of a patient with Alzheimer's disease on the right. The red and yellow represent areas of high brain activity, and the blue and black represent areas of low brain activity.

Research This

Exploring Quantum Computers

Skills: Researching, Analyzing, Evaluating, Communicating — SKILLS HANDBOOK A4.1

A quantum computer is to a traditional digital computer what "a laser is to a light bulb," according to Seth Lloyd, a quantum engineer at the Massachusetts Institute of Technology. Unlike a digital computer, which uses bits and bytes, a quantum computer uses qubits and quantum properties to manipulate data. In this activity, you will research quantum computing and its potential applications.

1. Research quantum computers, and find out how they are designed.
2. Examine some of the problems facing scientists as they attempt to create workable quantum computers.
3. Identify some possible applications of quantum computing.

A. Describe the major differences in design between quantum computers and digital computers. T/I C

B. Describe some of the issues facing scientists attempting to create quantum computers. T/I C

C. Suggest some possible applications of quantum computing. T/I C

WEB LINK

12.3 Review

Summary

- Louis de Broglie hypothesized that electrons possess both particle-like and wave-like properties, including a wavelength related to their momentum as $\lambda = \frac{h}{p}$, where h is Planck's constant. The wavelength is called the de Broglie wavelength.
- Experiments by Davisson and Germer confirmed that electrons exhibit the wave-like property of interference with a wavelength given by de Broglie's wavelength.
- The equations of quantum mechanics do not explain what happens to a single electron during a double-slit experiment. Different interpretations of the equations attempt to address the question. The collapse interpretation says that the electron physically changes from a particle to a wave and back again. The pilot wave interpretation avoids any unexplained collapse, but states that the motion of the electron depends on a pilot wave. In the many worlds interpretation, the electron is a real particle that exists in several parallel universes. The Copenhagen interpretation states that we cannot comment on the nature of the electron between measurements.
- A wave function gives the probability for any quantum object to take any possible path, or for the object to be at any possible location on the detection screen in the double-slit experiment.
- The Heisenberg uncertainty principle says that we cannot simultaneously determine the position and momentum of a quantum object with great accuracy.
- Wave–particle duality has many technological applications, such as electron microscopy and PET scans.

Questions

1. An electron has a de Broglie wavelength of 150 nm. Determine its speed. K/U T/I
2. The mass of a proton is 1800 times the mass of an electron. An electron and a proton have the same de Broglie wavelength. Determine the ratio of their energies. K/U T/I
3. Determine the de Broglie wavelength of a 1000.0 kg car that (a) has a speed of 100.0 km/h, (b) has a speed of 10.0×10^3 km/h, and (c) is at rest. K/U T/I
4. Compare how particles are viewed in classical physics and in quantum mechanics. K/U C
5. List one example of experimental evidence for (a) wave-like properties of matter and (b) particle-like properties of electromagnetic radiation. K/U C A
6. Are wave functions real? Explain your answer. K/U T/I
7. Why do the different interpretations of quantum mechanics exist? Which interpretation do you think is most likely? Explain your answer. K/U T/I C A
8. Taking the Heisenberg uncertainty principle into account, explain whether it is possible to take exact measurements of an electron when it is at rest. K/U C
9. The late Canadian Nobel laureate Willard Boyle (**Figure 8**) developed an important application of quantum mechanics called a charge-coupled device (CCD). Research Boyle and CCDs, and prepare a summary that includes biographical information, the technology and the physics behind CCDs, the development of the technology, and how CCDs contribute to physics and society. Present your research in the form of a research paper, a website, or a short video documentary. T/I C A

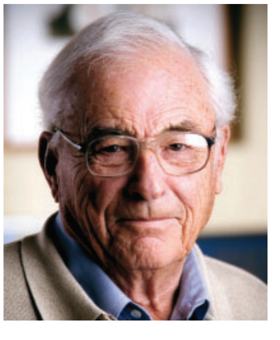


Figure 8

WEB LINK

12.4 Explore an Application in Quantum Mechanics

SKILLS MENU

- Researching
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating
- Identifying Alternatives

Medical Diagnostic Tools

The implications and applications of quantum theory are far-reaching and have affected society in many ways. Our understanding of quantum mechanics has been applied to the field of medicine for quite some time. The electron microscope is one example. Electron microscopy uses the wave-like nature of electrons to provide a detailed look at the microscopic world.

Another application is SQUID, a superconducting quantum interference device that interprets brain wave signals and converts them into operating instructions for artificial limbs. Lasik eye surgery is a technique that uses a laser to make incisions in the cornea for vision correction (**Figure 1**). Positron emission tomography (PET) is used to measure blood flow, oxygen use, and sugar metabolism to help doctors evaluate the health and function of organs and tissues. PET scans detect brain dysfunctions such as tumours, seizures, and memory disorders. Computed tomography (CT) can diagnose many different cancers, such as lung, liver, and pancreatic cancers. CAREER LINK

Figure 1 Laser surgery is commonly used to correct vision.

Many biological processes involve converting energy into forms that can be used in chemical transformations that are quantum mechanical in nature. They involve chemical reactions, light absorption, and the transfer of electrons and protons during photosynthesis and cellular respiration.

The Application

SKILLS HANDBOOK A4

The medical field has a long history of developing devices that enhance and assist in disease diagnosis and treatment. Sophisticated and well-funded medical research and development laboratories use the latest scientific discoveries to improve existing technology and design new tools. Suppose you are a graduate student in the physics department of a university. Your research professor has just received a grant to study how science can lead to technologies that benefit society.

Your Goal

To identify and describe a technological advance in medicine that uses applications of quantum mechanics

Research

Choose a technological medical application of quantum mechanics. Some examples are CT scans, PET scans, lasers for eye surgery, laser scalpels, UV light sterilizers, quantum dots used in cancer therapy, and SQUID.

Use the Internet and other sources to gather specific details related to how the technology operates due to our understanding of the quantum world. Use the following points as a guide:

- Trace the history of the development of the technology.
- Identify potential problems or negative impacts on the environment that might arise from this technology.
- Determine the advantages that the technology has over its predecessors.
- Identify the core science in the operation of the technology.
- Determine whether the technology is still being improved or tested.
- Determine the social and financial costs associated with the technology. Is it accessible to everyone? WEB LINK

Summarize

Use the following questions to summarize your research:

- How has quantum mechanics provided practical applications to medical technology?
- How has the technology you chose benefited the medical field?
- What are the major differences between the technology and its preceding models?
- What are the advantages that the technology has over its predecessors?
- What are the disadvantages of the technology?
- Summarize the impacts of the technology on society, the economy, and the environment.

Communicate

Prepare a multimedia presentation to present your research to the class. Your presentation should convey the advantages and disadvantages of the technology, the quantum nature of the technology, the connection between science and technology, and the larger effects—both positive and negative—of the technology on society and on the environment.

UNIT TASK BOOKMARK

As you work on the Unit Task on page 666, apply what you have learned about medical applications of quantum theory.

Raymond Laflamme and Quantum Information Theory

ABSTRACT
Raymond Laflamme is a leading thinker in the world of quantum mechanics, quantum computers, and the nature of our universe. He is a faculty member of the Perimeter Institute for Theoretical Physics in Waterloo, Ontario. Laflamme is enthusiastic about the current quantum research and the promises it holds for the future.

Introduction

Raymond Laflamme (**Figure 1**) grew up in Québec City, where he studied physics at Université Laval. He then worked as a doctoral student with Stephen Hawking, a world-renowned theoretical physicist best known for his work in cosmology and quantum gravity, at the University of Cambridge. While working there, Laflamme convinced Hawking that, in a contracting universe, time does not reverse. That story is retold in Hawking's book *A Brief History of Time*. In 2001, after working as a researcher at different laboratories around the world, Laflamme joined the Perimeter Institute for Theoretical Physics in Waterloo, Ontario. He is also director of the Institute for Quantum Computing at the University of Waterloo.

Figure 1 Laflamme is a Canadian pioneer in the study of quantum computing, specializing in error-correcting codes.

Quantum Information

As a research scientist at Los Alamos Research Laboratory, New Mexico, from 1992 to 2001, Laflamme became interested in quantum computing. Traditional computers operate through the manipulation of bits of data that exist in one of two states: 0s and 1s. Quantum computers, by contrast, are not limited to these two states. Instead, they encode data as quantum bits, or qubits, which can exist simultaneously as either a 0 or a 1 or as a superposition of the two. Physical qubits can be constructed from atoms, ions, photons, or electrons that work together as the computer memory and processor. A quantum computer can store and process information in multiple states simultaneously, so its computing power has the potential to surpass even the most powerful of traditional computers.

Quantum computing can open up a whole new domain for computers that may allow even personal computers to perform tasks never before dreamed of. A quantum computer, for example, can search collections of information, or databases, much more quickly than a classical computer. This ability would allow an increase in the speed of websites that use databases, including web browsers and social networking sites. A quantum computer can also encode information and decode encrypted information much more quickly than a classical computer.

Quantum information theory is Laflamme's passion. This field of study proposes that physical information about a system can be stored in a quantum state. This storage allows levels of information processing and encrypting that are not possible in the classical world. Quantum information theory leads to the type of quantum information processing that makes transmitting information completely secure.

At the Perimeter Institute, Laflamme collaborated in devising a mathematical framework for error-correcting codes in quantum computing. When information is stored or transmitted from one computer to another, errors in the information can arise due to accident, noise, and other problems. Error-correcting codes detect and correct these errors, so error correction is an important part of designing a reliable computer.

Quantum error correction prevents errors in quantum computers. Errors can arise from noise in the circuits and from interference from outside the computer. Noise comes from unwanted, meaningless data, often as an unwanted by-product of other transmissions. Quantum information can also be affected by a quantum effect called decoherence, which causes a quantum system to behave more like a classical system. Decoherence is a sort of "quantum noise" that reduces the quantum computer's advantage over current computers. Laflamme is working to develop methods for protecting quantum information against such noise. As of 2011, Laflamme's research group's 12-bit quantum computer is the world's largest quantum processor.

Developments in Quantum Cryptography

Cryptography is the science of encoding and decoding information in order to store and transmit it securely. Whenever you log in to a website, for instance, your login information will pass from your computer over the Internet in an encoded form to prevent anyone from stealing your information. Researchers see cryptography as an important application of quantum information science that will allow us to send information through public systems such as fibre optic networks without fear that the information could be intercepted.

Traditional cryptographic methods of encoding and decoding messages use mathematical codes and keys. For example, encryption software on a computer generates a key, which determines a random code and encrypts an email message with the code before sending it over the Internet. When the message is received, the key deciphers the message. Keys are very large numbers. The larger the key, the more number combinations are possible, making it more difficult to break the code. With standard encoding methods, it is impossible to know with certainty whether the key has been intercepted by an eavesdropper.

Quantum cryptography uses quantum mechanics. Recall from Section 10.5 that electromagnetic waves can be polarized. In the same way that electromagnetic waves can be polarized, so too can photons. Users of quantum cryptography will first exchange a key, which is a code connected to the polarization of the photons. Basically, the photons carrying the message are polarized in a certain direction, and the receiver must know the direction. The sender uses the key to lock the information, and the receiver uses the key to unlock the information.

An eavesdropper must know the direction of polarization to intercept the message. Measurement of a single photon's polarization, though, affects the polarization differently than the measurement of a classical wave, predicted by the Heisenberg uncertainty principle. So, unlike traditional cryptography methods, with quantum cryptography it is possible to detect whether or not the key has been intercepted. An eavesdropper will always be detected because it is impossible to measure (eavesdrop on) a quantum system without fundamentally changing it. Banks in Switzerland are already using quantum cryptography to encode information. **Figure 2** shows an example of some quantum cryptography equipment.

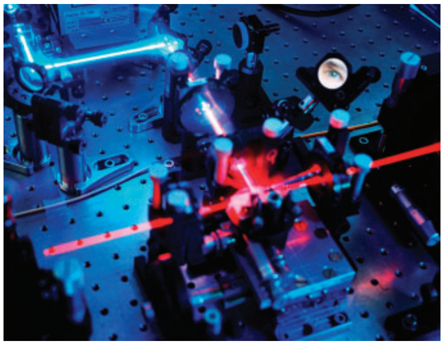


Figure 2 Quantum cryptography equipment uses lasers to produce streams of photons with particular polarizations.

Further Reading

Hawking, S. (1988). *A brief history of time*. Toronto, New York: Bantam Dell Publishing Group.

Kaye, P., Laflamme, R., Mosca, M. (2007). *An introduction to quantum computing*. Oxford University Press, USA.

WEB LINK

12.5 Questions

1. Explain some of Laflamme's contributions to quantum research. K/U C
2. Describe the significance of Laflamme's work in developing quantum error-correcting codes. C A
3. Describe the relationship between quantum information theory and its application in quantum cryptography. T/I C A
4. How do decoherence and noise affect information transmission? K/U C
5. Describe the impact of Laflamme's research on society. What impacts could this have on you personally? K/U T/I C A
6. How do quantum computers differ from traditional computers? K/U T/I
7. Research Laflamme and his current projects and areas of expertise. Describe one of the research projects to your classmates. K/U C

WEB LINK

12.6

The Standard Model of Elementary Particles

In science, some advances happen when old ideas generate new thinking. Other innovations occur because new ideas force the old ones to be discarded. In this section, you will learn why it was necessary to replace the Rutherford model of the atom with the Bohr model. You will discover how a deeper examination of the atom revealed that matter is made up of much more than just electrons, protons, and neutrons. You will learn about the concepts of matter and antimatter as described in one model of particle physics. Finally, the information in this section may cause you to wonder when it will be possible to describe our vast universe with a single unified theory of physics, or theory of everything.

Understanding the Atom

In 1909, physicist Ernest Rutherford and his students, Hans Geiger and Ernest Marsden, set out to learn more about the inner structure of atoms. During experiments, they aimed high-speed, positively charged particles at a thin sheet of gold foil. Rutherford and his team expected most of the particles to pass through the foil. Instead, they discovered that a small number of particles were deflected. Rutherford realized that this result meant that all the positive charge in an atom must be concentrated in a very small volume.

Rutherford suggested that the atom is like a miniature solar system, with electrons orbiting the nucleus just as planets orbit the Sun. This is called the planetary model of the atom. The electrons in the planetary model are not stationary. The electrons must move in orbits to avoid "falling" into the nucleus as a result of the electric force. The model also proposed that they could move in any orbit.

The charge carried by a proton is $+e$, and the charge carried by an electron is $-e$. Therefore, the total charge carried by a hydrogen atom, which contains one electron and a nucleus consisting of a single proton, is zero (**Figure 1**). The total charge of neutral atoms of all other elements is also zero. Therefore, the number of protons in the nucleus must be equal to the number of electrons in the neutral atom. This number, called the atomic number of the element, is denoted by Z.

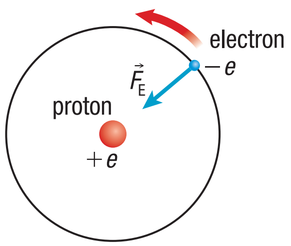


Figure 1 According to Rutherford's planetary model of the atom, electrons orbit the nucleus under the influence of the electric force. In a hydrogen atom, an electron with charge $-e$ orbits a proton with charge $+e$.

Nearly all atomic nuclei contain protons and neutrons. Protons are positively charged particles, so according to Coulomb's law, two or more protons collected in a tiny space the size of an atomic nucleus should repel each other and fly apart. Researchers now know that protons experience an additional force—what physicists call the strong force—that holds them together.

The neutron is a particle with a zero net electric charge. Protons are attracted to neutrons by this additional force. Since the neutrons do not have electric charge, they do not repel the protons or each other through the Coulomb force. Neutrons help make nuclei more stable by contributing to the strong force without a repulsive Coulomb force.

Problems with the Planetary Model

Calculations with the planetary model initially attempted to use Newtonian mechanics to describe the atom. However, some fundamental problems with this model were soon apparent. The biggest problem is the stability of an electron orbit.

Maxwell's classical theory of electromagnetism predicts that an electron emits electromagnetic radiation when it orbits a proton. The radiation carries away energy. If the electron in a hydrogen atom loses energy in this way, Newtonian mechanics predicts that it will spiral inward to the nucleus (**Figure 2**). According to classical physics, an atom in Rutherford's model is, by nature, unstable. If this model were correct, all atoms would collapse, which is not the case. Physicists were unable to modify the planetary model to make the atoms stable.

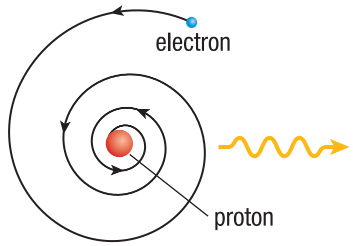


Figure 2 The problem with the planetary model of the atom is that the radiation emitted by an orbiting electron predicted by Maxwell's theory would result in the electron losing energy and spiralling into the nucleus.

The Bohr Model of the Atom

Shortly after Rutherford published his model of the atom, a young Danish physicist named Niels Bohr began to study the problems associated with the planetary model. Bohr proposed a quantum-mechanical approach to the motion of electrons within the atom. He was inspired by the Planck–Einstein introduction of quanta into the theory of electromagnetic radiation. As a result, he proposed a theory about the motion of electrons within atoms. Bohr's theory went against the well-established classical laws of mechanics and electromagnetism.

Investigation 12.6.1

Laser Simulation (page 657)
Try the simulation of how a laser works in Investigation 12.6.1 to get a better understanding of energy levels as well as how quantum mechanics is applied in technologies.

Bohr proposed that an electron in an atom can have only certain orbits with particular values for the radius of each orbit. This was in contrast to the Rutherford model, in which an electron could orbit at any radius. The Bohr model requires that a whole-number multiple of electron wavelengths equal the circumference of an orbit. If this condition is not met, then the Bohr model does not allow the orbit. The special values of the orbital radius meant that the electron could only have special values of potential energy and kinetic energy. The total energy could take on only certain discrete, quantized values. Each value of energy corresponds to what is now called an energy level.

According to Bohr, when an electron is in an allowed orbit, it does not radiate energy. An electron will emit a single photon when it moves from a higher energy level to a lower energy level. Bohr proposed that an atom could only absorb energy if that energy were equal to the energy difference between the lower state and a higher one.

Bohr's model was partially successful. It provided a physical model of the hydrogen atom. The model matched the internal energy levels to the levels observed in a hydrogen spectrum. At the same time, the model accounted for the stability of the hydrogen atom. Bohr's model, however, was incomplete. When applied to atoms with many electrons, the model broke down.

An explanation for the Bohr model came from de Broglie's matter waves, which were developed 10 years after Bohr's work. As mentioned above, the electrons in a given energy level have special values of kinetic energy. Therefore, those electrons have certain values of momentum. It follows that they have only certain wavelengths (Section 12.3).

Using de Broglie's model, the allowed electron orbits in hydrogen correspond exactly to those orbits in which electron waves form circular standing waves around the nucleus (**Figure 3**).

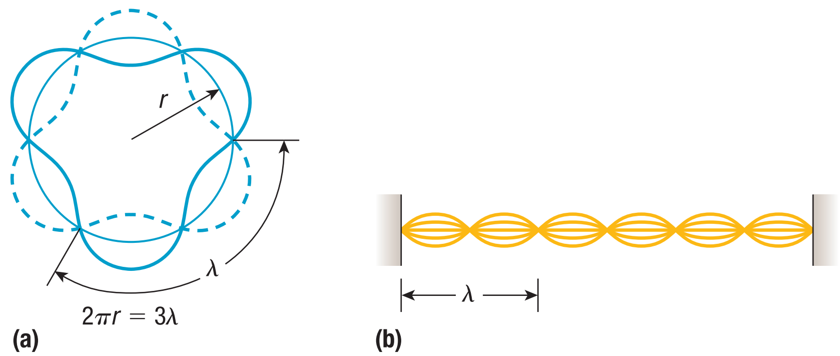


Figure 3 (a) The standing-wave pattern for an electron wave in a stable orbit of hydrogen requires that the orbital circumference equal a whole number of wavelengths. (b) The standing-wave pattern for a string fixed at both ends requires that the distance between supports equal a whole number of wavelengths.

Tutorial 1 examines the connection between electron standing waves and electron orbits in the Bohr model.

Tutorial 1 Standing Wave Orbits in the Bohr Model

Sample Problem 1: Electron Standing Waves around a Hydrogen Nucleus

The number of wavelengths in a standing wave represents the electron's energy level. **Figure 4(a)** shows two electron standing waves around a hydrogen nucleus. **Figure 4(b)** shows a single electron standing wave.

(a) Determine the energy level for each standing wave in Figure 4(a).

(b) Analyze Figure 4(b), and determine the energy level.

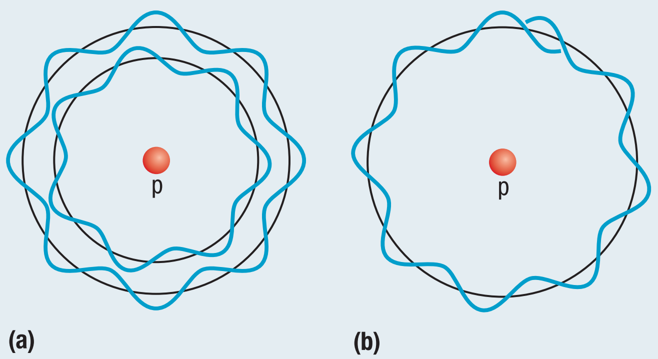


Figure 4

Solution

(a) The number of standing waves in each level represents the energy level. Count the number of wavelengths in each standing wave.

The inner energy level is 7, and the outer energy level is 8.

(b) The Bohr model requires that a whole-number multiple of electron wavelengths equal the orbital circumference.

In Figure 4(b), the standing wave does not join with itself for this particular wavelength. So, this wavelength is not allowed in the Bohr model. Therefore, no energy level exists.

Practice

1. **Figure 5** shows two allowed orbits of an electron in a hydrogen atom. Copy this diagram into your notebook. Draw all allowed orbits for energy levels between the two shown. K/U T/I C

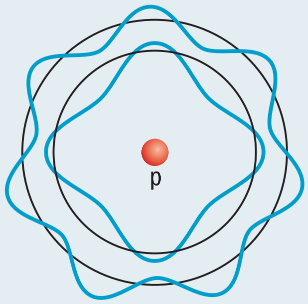


Figure 5

2. Draw the first three energy levels of a Bohr atom. K/U T/I C

Twenty-First-Century Physics and Antimatter

Rutherford's discovery of the atomic nucleus raised the question, "How is the nucleus itself put together?" The nucleus is not complicated: it is composed of just two different building blocks, protons and neutrons. The next level in the progression of understanding the atomic and subatomic world is the inner workings of the atomic nucleus and how protons, neutrons, and other subatomic particles are put together. CAREER LINK

Matter and Antimatter

One of the early developers of quantum physics was P.A.M. Dirac. He formulated a theory of quantum mechanics in the early 1930s. This theory combined the quantum theory of Schrödinger and Heisenberg with the postulates of special relativity. Dirac's theory predicted the existence of a completely new particle. This particle has the same mass and electric charge as the electron but the opposite sign of charge. This particle, called a positron, was later discovered in cosmic ray radiation, high-energy charged particles that strike Earth from all directions in space.

Positrons are written with the symbol e^+ to distinguish them from the electron, e^-. The electron carries a negative charge, and the positron carries a positive charge. The positron is an example of antimatter. **Antimatter** is any particle of matter that has the same mass and opposite charge as a corresponding particle of ordinary matter. Anti-protons and anti-neutrons are two more examples of antimatter.

antimatter a form of matter in which each particle has the same mass and an opposite charge as its counterpart in ordinary matter

Although the neutron and anti-neutron are both neutral, they are different particles. The reason, as you will read below, is that neutrons and anti-neutrons are actually made up of smaller particles called quarks that do have electric charges. The neutron is made up of a certain combination of quarks, and the anti-neutron is made up of the corresponding combination of anti-quarks.

Particles such as electrons, positrons, and protons undergo reactions, much like the reactions involving nuclei. When describing these reactions, we denote each of these particles by a symbol. Protons are denoted as p and neutrons as n; the corresponding anti-particles are denoted with the same letter but with an overbar. **Table 1** on the next page lists the mass and electric charge of the electron, proton, and neutron along with the values for their corresponding anti-particles.

Table 1 Some Properties of Electrons, Protons, Neutrons, and Their Anti-particles

Particle	Symbol	Mass (kg)	Mass (MeV/c^2)	Charge
electron	e^-	9.109×10^{-31}	0.511	-1
positron	e^+	9.109×10^{-31}	0.511	$+1$
proton	p	1.673×10^{-27}	938	$+1$
anti-proton	$\bar{p}$	1.673×10^{-27}	938	-1
neutron	n	1.675×10^{-27}	940	0
anti-neutron	$\bar{n}$	1.675×10^{-27}	940	0

Anti-particles give researchers a chance to see special relativity at work at the microscopic level. For example, when an electron encounters its anti-particle, the positron, the two undergo a reaction that destroys both particles. This reaction is written as

$$e^- + e^+ \rightarrow \gamma + \gamma$$

where the symbol γ (Greek letter gamma) represents one high-energy photon called a gamma ray. This process states that an electron plus a positron react to form two photons. The event needs to satisfy the law of conservation of energy. The total energy before the reaction of the original electron and positron must equal the final energy of the two photons. The total initial energy includes the kinetic energies of the electron and positron plus their rest energies. According to special relativity, the rest energy of an electron with rest mass m_e is $m_e c^2$. The positron also has rest mass m_e, so it has the same rest energy. When you know the initial kinetic energies of the particles, you can measure the energies of the two photons emitted in this event to determine the rest energies of the electron and positron and check the predictions of special relativity.

The Standard Model

Studies of nuclear and particle physics began in the early 1900s, when Rutherford and other physicists conducted atomic experiments. Various particles were aimed at atoms and nuclei, revealing that nuclei are composed of protons and neutrons. For a brief period, scientists thought that electrons, protons, and neutrons were the fundamental particles from which all matter is composed. That simple model did not last long. Many other particles were discovered in cosmic ray studies, in collision experiments, and in nuclear decay processes. Protons and neutrons are themselves composed of particles called **quarks**. According to current understanding, quarks are fundamental point particles whose charge can be either $\frac{2}{3}e$ or $-\frac{1}{3}e$. Particles composed of quarks—such as protons, neutrons, and their anti-particles—form a family of particles called **hadrons**. Not all particles are members of the hadron family, however. **Leptons**, another family of particles that includes electrons and positrons, are believed to be elementary, indivisible particles. The behaviour of hadrons and leptons is described by the standard model of elementary particles. The **standard model** is the current theory of fundamental particles and the forces that are present in their interactions.

quark an elementary particle that makes up protons, neutrons, and other hadrons

hadrons a class of particles that contains the neutron, the proton, and the pion; composed of combinations of quarks and anti-quarks

leptons a class of particles that includes the electron, the muon, the tauon, and the three types of neutrinos; not composed of smaller particles

standard model the modern theory of fundamental particles and their interactions

Quarks Bind Together to Form Hadrons

There are six different kinds—or flavours, as they are also referred to in particle physics—of quarks, called up (denoted by the symbol u), down (d), charm (c), strange (s), top (t), and bottom (b). These names do not refer to any physical properties of the quarks. Rather, they are whimsical names made up by physicists. Each of these quarks has a corresponding anti-quark, denoted by $\bar{u}$, $\bar{d}$, and so on. **Table 2** lists some properties of quarks.

Table 2 Types of Quarks and Their Properties

Type of quark (flavour)	Symbol	Quark charge (e)	Mass	Anti-quark	Anti-quark charge (e)
up	u	$+\frac{2}{3}$	1.7–3.1 MeV	$\bar{u}$	$-\frac{2}{3}$
down	d	$-\frac{1}{3}$	4.1–5.7 MeV	$\bar{d}$	$+\frac{1}{3}$
charm	c	$+\frac{2}{3}$	1.18–1.34 GeV	$\bar{c}$	$-\frac{2}{3}$
strange	s	$-\frac{1}{3}$	80–130 MeV	$\bar{s}$	$+\frac{1}{3}$
top	t	$+\frac{2}{3}$	172.9 GeV	$\bar{t}$	$-\frac{2}{3}$
bottom	b	$-\frac{1}{3}$	4.13–4.37 GeV	$\bar{b}$	$+\frac{1}{3}$

Quarks were first discovered in collision experiments involving protons. When a high-energy electron collides with a proton, the way that the electron scatters (that is, its outgoing direction and energy) gives information about how mass and charge are distributed inside the proton. This is similar to Rutherford's experiment, which showed how positively charged particles scattered from an atom, indicating that it has a massive nucleus at its centre.

Hadrons composed of three quarks are called baryons (**Figure 6**). Protons and neutrons are both baryons. Collision experiments with protons show that there are three point-like particles inside each proton. **Table 3** lists the quark composition of the proton: two up quarks and one down quark (uud). The total charge on the proton is the sum of the charges of the constituent quarks. Table 3 lists a few other baryons as well. Dozens of other baryons have been observed and their component quarks identified.

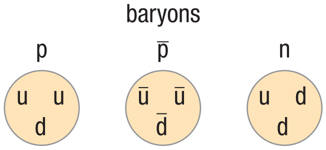


Figure 6 Baryons and anti-baryons are composed of three quarks. The three particles represented here are the proton (p), the anti-proton ($\bar{p}$), and the neutron (n).

Table 3 Properties of Some Baryons

Particle	Symbol	Constituent quarks	Lifetime (s)	Mass (MeV/c^2)
proton	p	uud	stable	938
neutron	n	udd	890	940
sigma plus	Σ^+	uus	0.8×10^{-10}	1189
sigma zero	Σ^0	uds	6.0×10^{-20}	1193
sigma minus	Σ^-	dds	1.5×10^{-10}	1197
xi minus	Ξ^-	dss	1.6×10^{-10}	1321

Note: There are many other baryons composed of other combinations of three quarks and anti-quarks.

A quark and an anti-quark can also combine to form a particle. Hadrons composed of just two quarks are called mesons (**Figure 7**), and a few are listed in **Table 4**.

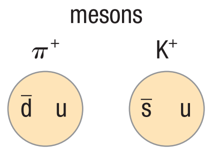


Figure 7 Mesons are composed of one quark and one anti-quark. These mesons are called pions (π^+) and kaons (K$^+$).

Table 4 Properties of Some Mesons

Particle	Symbol	Constituent quarks	Lifetime (s)	Mass (MeV/c^2)
pion (pi plus)	π^+	u$\bar{\text{d}}$	2.6×10^{-8}	140
pi zero*	π^0	d$\bar{\text{d}}$/u$\bar{\text{u}}$	8.4×10^{-17}	135
kaon (K plus)	K$^+$	u$\bar{\text{s}}$	1.2×10^{-8}	494
kaon (K minus)	K$^-$	s$\bar{\text{u}}$	1.2×10^{-8}	494
phi	ϕ	s$\bar{\text{s}}$	1.6×10^{-22}	1020

Note: There are many other mesons, which are composed of other combinations of quarks and anti-quarks.

* The π^0 is a quantum-mechanical combination of the d$\bar{\text{d}}$ and u$\bar{\text{u}}$ quark states.

All hadrons are composed of quarks, so the interactions between quarks determine the properties of hadrons and how they relate to one another. The two most important hadrons are the proton and the neutron, so the behaviour of quarks also determines the properties of nuclei. Quarks are charged, so they act on each other through the electric (Coulomb) force. They also interact through the strong force mentioned earlier. Quarks bind together to form protons and neutrons (nucleons), and the strong force is responsible for holding protons and neutrons together to make nuclei.

Leptons

Particle physicists think that leptons, like quarks, are not composed of any smaller particle. **Table 5** shows the six fundamental leptons plus their corresponding anti-particles. These particles group naturally into three pairs: the electron and the electron neutrino, the muon and the muon neutrino, and the tau and the tau neutrino. The muon and the tau are not stable. Electrons are stable, but the behaviour of neutrinos is more complicated. Recent experiments indicate that as neutrinos travel through space, they change from one to another of the three types of neutrinos listed in Table 5, an effect called neutrino oscillation.

Table 5 Leptons and Their Properties

Particle	Symbol	Lepton charge	Mass/c^2	Anti-lepton	Anti-lepton charge
electron	e$^-$	–1	0.511 MeV	e$^+$	1
electron neutrino	ν_e	0	0.05 eV < m < 2 eV	$\bar{\nu}_e$	0
muon	μ^-	–1	106 MeV	μ^+	1
muon neutrino	ν_μ	0	< 0.19 MeV	$\bar{\nu}_\mu$	0
tau	τ^-	–1	1780 MeV	τ^+	1
tau neutrino	ν_τ	0	< 18 MeV	$\bar{\nu}_\tau$	0

Another interesting property of neutrinos is that they have extremely small masses. Particle physicists have only approximations for the masses of neutrinos. However, to give you an idea of their mass, the best experiments to date give an approximate mass for the electron neutrino as less than 100 000 times the mass of an electron.

Leptons play important roles in certain reactions involving hadrons. For example, an isolated neutron (a neutron outside a nucleus) decays to form a proton, an electron, and an electron anti-neutrino:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Nuclear decay reactions also produce leptons. In fact, neutrinos were first observed in the 1950s in studies of the particles emitted inside a nuclear fission reactor. The nuclear fusion reactions that occur within the Sun provide a very large source of the neutrinos observed on Earth. Studying these solar neutrinos is one of the best ways to understand the nuclear reactions that take place inside the Sun.

Bosons: Force-Mediating Particles

The fundamental forces of nature are the ways in which individual particles interact with each other. Every interaction in the universe can be described using only three forces. (In general relativity, gravity is not a force.) These forces are the strong nuclear force, the weak nuclear force, and electromagnetism. The strong nuclear force holds the subatomic particles of the nucleus together. The weak nuclear force causes radioactive decay and starts the process of hydrogen fusion and other nuclear processes in stars. The electromagnetic force is responsible for the attraction and repulsion among electrical charges.

Quarks and leptons make up the building blocks of matter, and physicists classify these particles as **fermions**. In addition to fermions, the standard model includes another type of particle called a field particle, or boson. **Bosons** are responsible for transmitting the fundamental forces between the quarks and the leptons.

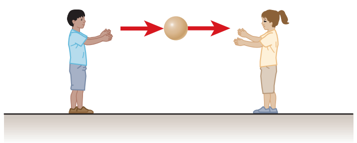
fermion a fundamental particle that forms matter

boson the particle responsible for transmitting electromagnetic, strong, and weak forces

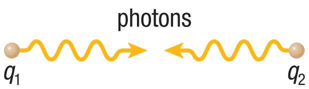
The standard model describes particle interactions involved in three of the fundamental forces: electromagnetism, the strong nuclear force, and the weak nuclear force. The electromagnetic force is "mediated" (see next paragraph) by the photon, which means that photons transmit the electromagnetic force acting on charged particles. The strong nuclear force is mediated between quarks by particles called **gluons**. Eight different types of gluons exist, and the force exerted between two quarks depends on the type of each quark. The weak nuclear force is mediated by a family of three particles called the W^+, W^-, and Z bosons. Unlike the photon and gluons, which have zero mass, the W^+, W^-, and Z bosons have mass.

gluon a particle that mediates the strong nuclear force

In the language of particle physicists, bosons mediate the interactions between other particles. **Figure 8(a)** gives a classical picture of how the exchange of particles can lead to a force between two objects. Here the "objects" are two children playing catch. The child throwing the ball experiences a recoil force during the throw while the child catching the ball experiences a recoil force when she catches it. The ball plays the role of a photon in the electromagnetic force, and both the ball and the photon mediate a force between two objects (**Figure 8(b)**).



(a)



(b)

Figure 8 (a) When two children play catch, the ball "carries" a force from one to the other. Physicists describe the ball as being an object that mediates a force between the two children. (b) The force carried by photons is similar to the situation of two children playing catch. In the quantum-mechanical model, photons mediate (or carry) the electric force from one charge to another.

Table 6 and **Table 7** summarize the particles of the standard model. Some scientists think that all the fundamental forces in nature are mediated by elementary particles.

Table 6 Fermions, the Building Blocks of Matter, in the Standard Model

Leptons		Quarks	
Name	**Charge**	**Name**	**Charge**
electron	–1	up	$+\frac{2}{3}$
electron neutrino	0	down	$-\frac{1}{3}$
muon	–1	charm	$+\frac{2}{3}$
muon neutrino	0	strange	$-\frac{1}{3}$
tau	–1	top	$+\frac{2}{3}$
tau neutrino	0	bottom	$-\frac{1}{3}$

Table 7 Bosons, Carriers of Forces, in the Standard Model

Name	Force
photon	electromagnetic force
W^+, W^-, and Z bosons	weak nuclear force
gluons (eight different types)	strong nuclear force

Higgs boson the theoretical particle thought to play a role in giving mass to other particles

According to the standard model, another boson, the Higgs boson, exists. The **Higgs boson** is a hypothetical massive elementary particle. It plays a part in the mechanism that gives other fundamental particles their mass. Rigorous tests of the standard model have resulted in the observation of all quarks, all leptons, and four bosons, but not the Higgs boson. Researchers working with the particle accelerator called the Large Hadron Collider at the European Organization for Nuclear Research (CERN) plan to collide protons with enough energy that a Higgs boson may be created.

A Theory of Everything

At a fundamental level, almost every concept that we understand about our universe comes from quantum mechanics or the theory of relativity, which includes special relativity (Chapter 11), and general relativity, the modern theory of gravity developed by Einstein in 1915.

Quantum mechanics describes the world at a subatomic scale, and general relativity describes the world at the macroscopic scale. However, neither theory sufficiently describes both. It is widely believed that these two ideas must somehow be combined into a **theory of everything**, which would combine the ideas of quantum mechanics and the ideas of general relativity. Such a theory would answer some of the most complex questions imaginable, such as "How is it possible for the universe to exist at all?"

theory of everything a theory that attempts to combine three fundamental forces (weak, strong, and electromagnetic) with gravity into a single theory

Discovering a theory of everything would be a great success for physics. The new theory would answer deep questions about the universe and reveal new secrets. It would also lead to new technologies that would impact society and our environment in major ways. Quantum mechanics itself began as a theory that mostly interested scientists, but it has now revolutionized our everyday life through computers, medical technology, nuclear power, and many more areas. It continues to change our lives through advances in quantum computing, quantum cryptography, and other new tools. We can only begin to imagine a theory of everything's potential impact on humanity.

12.6 Review

Summary

- Rutherford discovered the nucleus and proposed that electrons in an atom orbit the nucleus like a planetary system. Classical physics predicts that this system is not stable.
- The Bohr model of the atom proposes that electrons can only orbit the nucleus at certain allowed energy levels. These electrons transition between levels by emitting or absorbing photons whose energies are equal to the difference between the energy levels.
- Antimatter is a particle of matter that has the same mass and opposite charge as its corresponding particle of ordinary matter. The positron, for example, is the antimatter counterpart of the electron.
- All particles in the universe interact through the three fundamental forces of nature: the strong nuclear force, the weak nuclear force, and electromagnetism.
- The standard model is the current theory of particle physics, which predicts that nature consists of quarks, leptons, and bosons that interact through fundamental forces.
- Quarks combine to form hadrons, which include protons and neutrons. Leptons include electrons and neutrinos.
- Bosons mediate fundamental forces. The Higgs boson helps explain the origin of particle masses, but it has not yet been detected.
- A theory of everything attempts to explain and predict interactions in both the macroscopic and the quantum worlds by combining quantum mechanics and the theory of general relativity into one theory.

Questions

1. Explain why Rutherford's planetary model of the atom is inconsistent with classical physics and what we know about atoms. K/U C
2. In the progression of atomic models, describe the fundamental changes that were made between each new model. K/U T/I C
3. (a) Illustrate the constituent parts of a proton and a neutron.
   (b) Create a particle of your own using any combination of quarks.
   (c) Research your creation to see if it exists and what its properties are. T/I C A
4. Describe the role that bosons play in the standard model. K/U
5. Compare and contrast
   (a) fermions with bosons
   (b) mesons with baryons
   (c) leptons with hadrons K/U C
6. Describe the standard model of elementary particles in terms of the characteristics of quarks, leptons, and bosons. Explain the possible limitations of the standard model. K/U
7. What are the implications of the discovery of the Higgs boson? K/U
8. In a few sentences, explain why scientists think that there must be a theory of everything that is yet to be discovered. Describe what scientists are expecting to explain with a theory of everything. K/U T/I C
9. Newton's law of gravitation is one example of a unification of physics principles. In what way does his law of gravity unify the motion of celestial objects with the motion of earthly objects? K/U T/I C
10. The standard model developed through the work of many researchers. Research the history of the standard model, and choose a scientist who you feel made one of the most important contributions to the development of the standard model. Describe the experiments that the scientist conducted or organized, or the theory that the scientist formulated. Describe the discovery that the scientist made, and how the discovery built on earlier discoveries and influenced later discoveries. C A

WEB LINK

Investigation 12.2.1 OBSERVATIONAL STUDY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

The Photoelectric Effect

In Section 12.2, you read about the photoelectric effect and its relationship to the quantum theory of light. You also read about the work function equation. Recall that, in the Mini Investigation at the beginning of this chapter, you examined which frequencies of light from different-coloured holiday LED lights were able to light up the glow-in-the-dark stickers.

This investigation can be performed in two ways. In Part A, you will use a simulation of the photoelectric effect. In Part B, you will simulate the photoelectric effect and apply the work function equation by examining the data from a commercial phototube.

The photoelectric current will form any time light of sufficiently high frequency shines on a photoelectric surface. In this investigation, the collector will usually be negative, slowing the current in the phototube. At a sufficiently negative collector potential, V_0, the current will stop completely. You can measure the retarding potential difference, the work required to stop an electric charge between two points, using the formula $E_{\text{k max}} = qV_0$. Here, $E_{\text{k max}}$ is the maximum kinetic energy of the ejected photoelectrons, which is equal to the retarding potential difference.

Purpose

To simulate, visualize, and describe the photoelectric effect

Equipment and Materials

Part A

- computer with Internet access
- graph paper or graphing software

Part B

- graph paper or graphing software
- light source (for example, mercury vapour lamp)
- coloured filters (red, green, blue, violet)
- light collector
- variable resistor
- 6 V battery
- voltmeter
- microammeter
- connecting leads

Procedure

Part A: Simulation of the Photoelectric Effect

1. Go to the Nelson Science website.
2. Run the simulation.
3. Manipulate the intensity of the light source to observe the effect on the current and energy of the electrons as they move between two points.
4. Manipulate the wavelength of light to observe the effect on the current and energy of the electrons as they move between two points.
5. Manipulate the voltage of light to observe the effect on the current and energy of the electrons as they move between two points.
6. Change the material of the target, and observe the effect on the current and energy of the electrons as they move between the two points.
7. Determine the frequency of each colour using the equation $c = f\lambda$. Then plot the maximum kinetic energy of the ejected electrons versus the frequency of the photons. The energy axis should have a negative axis equal in magnitude to the positive axis, and the frequency should begin at zero.
8. Measure the slope of the energy-versus-frequency graph, and compare the value to Planck's constant.
9. In your graph from Step 7, the magnitude of the negative intercept on the energy axis represents the work function, W. Determine the value of W from your graph.

Part B: Analyzing the Photoelectric Effect

10. Use the data in **Table 1** to plot a graph of photocurrent versus retarding potential for each wavelength of light. Choose the scales of the axes so that all five graphs fit onto the same set of axes. Use electronic graphing software if available.
11. Set up the light collector and light source as shown in **Figure 1**. Have your teacher approve your setup.

When you unplug the light source or power supply, be sure to pull on the plug itself and not the electric cord. Do not shine the light in anyone's eyes.

Table 1 Colour Wavelengths and Photocurrents

Colour	Yellow	Green	Blue	Violet	
Wavelength	578 nm	546 nm	480 nm	410 nm (low intensity)	410 nm (high intensity)
Retarding potential (V)	Photocurrent (μA)				
0.00	3.2	10.4	11.2	8.5	14.8
0.05	2.3	8.7	10.1	8.1	14.0
0.10	1.3	7.1	9.0	7.6	13.3
0.15	0.6	5.5	8.0	7.2	12.6
0.20	0.2	4.0	7.0	6.7	11.9
0.25	0	2.4	6.0	6.2	11.1
0.30	0	1.0	4.9	5.7	10.4
0.35		0.2	3.8	5.3	9.6
0.40		0	2.8	4.8	8.9
0.45		0	1.7	4.4	8.2
0.50			1.2	3.9	7.0
0.55			0.7	3.4	6.7
0.60			0.4	3.0	6.0
0.65			0.1	2.5	5.2
0.70			0	2.0	4.5
0.75			0	1.6	3.7
0.80				1.1	3.0
0.85				0.9	2.3
0.90				0.7	1.7
0.95				0.5	1.2
1.00				0.3	0.8
1.05				0.2	0.4
1.10				0.1	0.1
1.15				0	0
1.20				0	0

12. Use the red filter to observe the effect on the current and energy of the electrons as they move between two points. Determine the stopping voltage (cut-off potential). Record your observations.
13. Change the intensity of the light, and record what happens to the current and energy of the electrons.
14. Repeat Steps 12 and 13 for each filter.
15. Determine the frequency of each colour using the equation $c = f\lambda$. Then plot the maximum kinetic energy of the ejected electrons, in electron-volts, versus the frequency of the photons, in hertz. The energy axis should have a negative axis equal in magnitude to the positive axis, and the frequency should begin at zero.

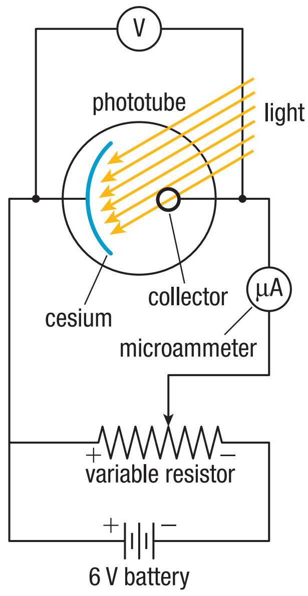


Figure 1 Light of different colours is directed at the photoelectric surface.

16. Measure the slope of the energy–frequency graph, and compare the value to Planck's constant.
17. Study your graph of kinetic energy versus frequency from Step 15. The magnitude of the negative intercept on the energy axis represents the work function, W. Determine the value of W from your graph.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) Determine the relationship between the intensity of the incident light and the maximum electric current. K/U T/I

(b) Identify the colour of light associated with the largest cut-off potential. K/U T/I

(c) The cut-off potential measures the maximum kinetic energy of the electrons ejected from the photoelectric surface. Determine the maximum kinetic energy of the photoelectrons, in both electron-volts and joules ($1 \text{ eV} = 1.6 \times 10^{-19}$ J). T/I A

(d) Which colour of light has the highest-energy photons? Explain your reasoning. T/I C

(e) How does the slope of your energy–frequency graph compare to Planck's constant? T/I A

(f) Using your energy–frequency graph, calculate the work function for cesium. T/I A

Apply and Extend

(g) What are some practical applications of the photoelectric effect? T/I A

(h) What is the relationship between the photoelectric effect and the quantum theory of light? T/I A

WEB LINK

Investigation 12.2.2 OBSERVATIONAL STUDY

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Determining Planck's Constant

In this investigation, developed at Canada's Perimeter Institute, you will observe that when a large enough potential difference is applied across a light-emitting diode (LED), all of the photons that are emitted have the same frequency. The energy of each electron that passes through the LED becomes converted into the energy of one photon. You will use the equipment to measure Planck's constant.

Purpose

To use an LED in a simple circuit to demonstrate the quantization of light and to measure Planck's constant, *h*

Equipment and Materials

- 1 kΩ potentiometer
- 5 connecting leads
- 6 V battery
- set of 5 LEDs (red, amber, yellow, green, blue)
- 330 Ω resistor
- voltmeter

Procedure

1. Set up the potentiometer so that its terminals point toward you. Turn the knob clockwise, and connect the battery's negative terminal to the left side of the potentiometer. Next, connect the battery's positive terminal to the terminal on the right side of the potentiometer (**Figure 1**).

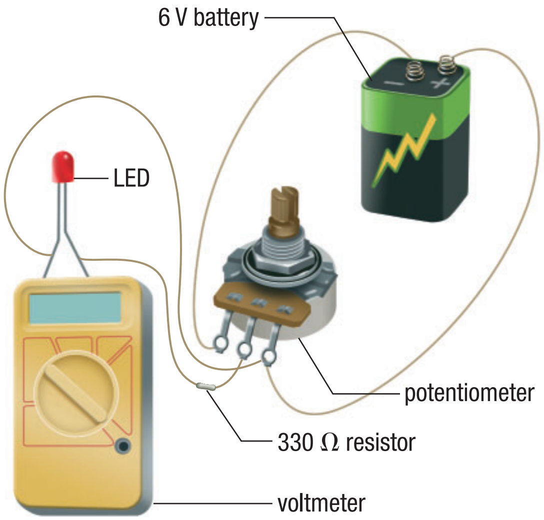


Figure 1 The longer LED wire attaches to the right-hand terminal.

2. Wire the first LED to the 330 Ω resistor. Then, connect the LED and the resistor to the central and right-hand terminals of the potentiometer (refer to Figure 1). Make sure that you attach the longer wire of the LED to the right-hand terminal.
3. Connect the voltmeter across the LED.
4. In your notebook, create a table similar to **Table 1** to summarize your measurements from Steps 5 and 6.

Table 1 Observations

Colour of LED	Red	Amber	Yellow	Green	Blue
Frequency ($\times 10^{14}$ Hz)	4.54	5.0	5.08	5.31	6.38
Potential difference (V)					

5. Slowly turn the potentiometer knob counterclockwise. This will increase the potential difference across the LED. Stop as soon as you see the LED beginning to glow. Record the potential difference on the potentiometer. (Hint: To get a more accurate reading, go backward and forward past the point at which the LED begins to glow. This will help you locate the exact potential difference.)

Do not stare directly at a lit LED, because it can damage your eyes.

6. Repeat Step 5 with the other LEDs. For the best results, turn the potentiometer knob fully clockwise before changing LEDs. This will ensure that the initial voltage across each LED is zero.
7. Plot the potential difference versus frequency on a graph and include a line of best fit.

Analyze and Evaluate

SKILLS HANDBOOK A5.5

(a) Determine the slope of the line from Step 7. T/I

(b) Calculate Planck's constant using this slope and the equation $e\Delta V = hf$, where e is the elementary charge, $+1.6 \times 10^{-19}$ C; ΔV is the potential difference across the LED; and f is the frequency of a photon emitted by the LED. T/I A

(c) Determine the percentage error for the value of Planck's constant that you calculated in (b). The standard accepted value is 6.63×10^{-34} J·s. T/I

(d) What are some possible sources of error in your investigation? T/I C

Apply and Extend

(e) A red laser pointer emits 700.0 nm light with a power rating of 1.00 mW. Calculate the number of photons emitted each second. T/I A

Investigation 12.6.1 OBSERVATIONAL STUDY

Laser Simulation

SKILLS MENU

• Questioning	• Planning	• Observing
• Researching	• Controlling Variables	• Analyzing
• Hypothesizing	• Performing	• Evaluating
• Predicting		• Communicating

The word *laser* stands for "light amplification by stimulated emission of radiation." In this investigation, you will run a simulation to visualize "stimulated emission of radiation" as well as the changes in energy levels as the photon interactions take place.

Purpose

To simulate, visualize, and describe how a laser works and the concept of energy levels

Equipment and Materials

- computer with Internet access

Procedure

1. Go to the Nelson Science website.
2. Run the simulation.
3. Set the intensity of the lamp so that single photons travel through the tube. The atom interacts with the photon in a process called *spontaneous emission*.
4. Increase the intensity of the red lamp. The process seen here is called *stimulated emission*.
5. Adjust the intensity of the red lamp to its lowest setting. In the "Energy Levels" area, click the "Three" button to increase the number of energy levels. Increase the intensity of the blue lamp that appears.
6. In the "Options" area, click on "Display photons emitted from upper energy state." Record your observations.
7. Increase the intensity of the red lamp. What adjustments can you make to increase the internal power to the highest level possible?
8. Click on "Enable mirrors," and increase the intensity of the blue light. Record your observations. Then decrease the "Mirror Reflectivity" to allow some photons to escape.

Analyze and Evaluate

(a) Describe what occurs when a single photon interacts with the atom. Refer to the energy-level diagram in the upper right-hand window. K/U C

(b) Describe the process of spontaneous emission. K/U C

(c) What did you observe when you increased the intensity of the red lamp in Step 4? How did the energy-level diagram change? K/U C

(d) Describe the process of stimulated emission. K/U C

(e) Describe your observations from Step 5. What colour are the photons that are emitted from the excited atom? K/U C

(f) What changes do you notice in the photons emitted from the excited atom in Step 6? K/U C

(g) What did you observe in Step 7? K/U C

(h) What happened when you enabled the mirrors? How does increasing intensity affect the internal power? K/U

(i) How does allowing photons to escape affect the internal and output power? K/U

Apply and Extend

(j) Click on the "Multiple Atoms (Lasing)" tab. In a stable state, electrons can theoretically remain forever. In an unstable state, an electron remains for about 10^{-8} s. After this time, the electron falls to the ground state and emits a photon. The energy (colour) of the emitted photon is equal to the energy lost by the electron. In a metastable state, an electron remains for 10^{2} s to 10^{-5} s before falling to the ground state and emitting a photon. How would you adjust the energy-level slide to represent an unstable or metastable state? K/U

(k) How can you adjust the "Lifetime" of each energy level to increase the internal power? Explain why. K/U

(l) How can you adjust the simulation to produce the highest possible output? Explain why. K/U

(m) Can you adjust the simulation so that the tube overheats and blows up? K/U

WEB LINK

Summary Questions

1. Create a study guide for this chapter based on the Key Concepts on page 614. For each point, create three or four subpoints that provide further information, relevant examples, explanatory diagrams, or general equations.

2. Look back at the Starting Points questions on page 614. Answer these questions using what you have learned in this chapter. Compare your latest answers with the answers that you wrote at the beginning of the chapter. Note how your answers have changed.

Vocabulary

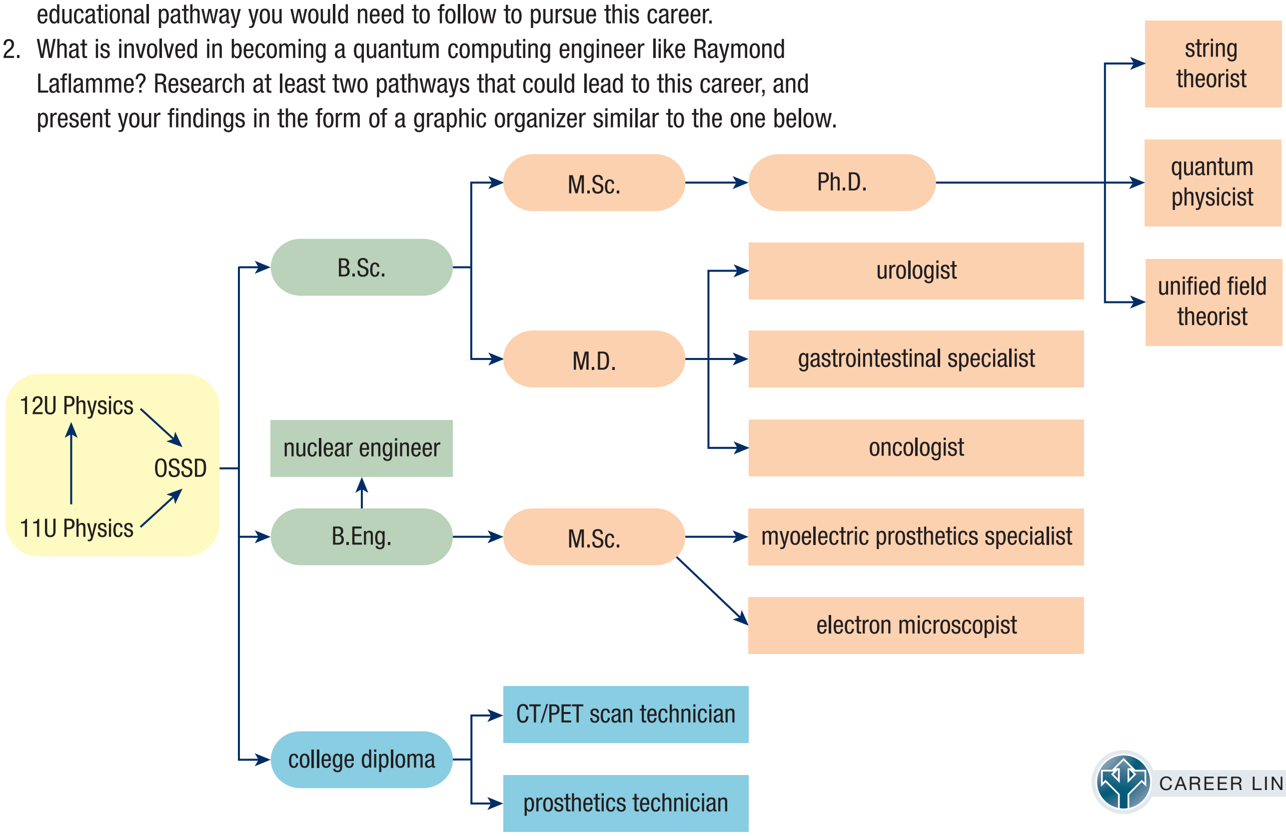
quantum (p. 616)
quantum theory (p. 616)
wave–particle duality (p. 618)
work function (p. 620)
photoelectric effect (p. 621)
threshold frequency (p. 621)
photon (p. 622)
Planck's constant (p. 622)
Compton effect (p. 624)
pair creation (p. 626)
blackbody (p. 626)
blackbody radiation (p. 626)
de Broglie wavelength (p. 632)
matter wave (p. 632)
Heisenberg uncertainty principle (p. 637)
antimatter (p. 647)
quark (p. 648)
hadrons (p. 648)
leptons (p. 648)
standard model (p. 648)
fermion (p. 651)
boson (p. 651)
gluon (p. 651)
Higgs boson (p. 652)
theory of everything (p. 652)

CAREER PATHWAYS

SKILLS HANDBOOK A6

Grade 12 Physics can lead to a wide range of careers. Some require a college diploma, a B.Sc. degree, or work experience. Others require specialized or postgraduate degrees. This graphic organizer shows a few pathways to careers mentioned in this chapter.

1. Select an interesting career that relates to quantum mechanics. Research the educational pathway you would need to follow to pursue this career.
2. What is involved in becoming a quantum computing engineer like Raymond Laflamme? Research at least two pathways that could lead to this career, and present your findings in the form of a graphic organizer similar to the one below.



CAREER LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. In quantum mechanics, what are individual bundles of energy called? (12.1) K/U
   (a) crests
   (b) rays
   (c) quanta
   (d) interferons
2. What does the word "discrete" mean? (12.1) K/U
   (a) continuous
   (b) careful
   (c) distinct
   (d) prudent
3. When electrons are used in a double-slit experiment, what pattern emerges on a far screen? (12.1) K/U
   (a) The electrons impact in only two places corresponding to the two slits.
   (b) The electrons impact in an interference pattern with alternating bright and dark fringes.
   (c) All the electrons impact the central area.
   (d) The electrons impact in only one place corresponding to a combination of the two slits.
4. What is the minimum amount of energy needed to remove an electron from a metal called? (12.2) K/U
   (a) double-slit threshold
   (b) quantum minimum
   (c) work function
   (d) zero-point energy
5. A particle with momentum p has a de Broglie wavelength of
   (a) $\frac{h}{p}$
   (b) hp
   (c) $\frac{1}{hp}$
   (d) $\left(\frac{h}{p}\right)^2$ (12.3) K/U
6. The Heisenberg uncertainty principle states that one can never know with exact accuracy
   (a) an electron's momentum and position
   (b) an electron's momentum and speed
   (c) an electron's mass and charge
   (d) an electron's mass and position (12.3) K/U
7. Which statement accurately describes Bohr's model of the atom? (12.6) K/U
   (a) Electrons may only exist in allowed orbits.
   (b) An atom's positive charge is spread throughout, and electrons are dotted about like plums in a pudding.
   (c) A dense nucleus contains most of the atom's mass.
   (d) An atom is a combination of quarks and gluons interacting with electrons.

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

8. Newton's laws work perfectly well on the atomic scale. (12.1) K/U
9. A quantum is a small and indivisible increment of energy. (12.1) K/U
10. The work function, W, is measured in hertz. (12.2) K/U
11. The wavelength of a light wave and the energy of the photons in that wave are inversely related. (12.2) K/U
12. One of the odd results of quantum mechanics is that a photon, although massless, can still have momentum. (12.2) K/U
13. Max Planck explained the problems of blackbody radiation by assuming that the energy in the cavity must be emitted in a continuous stream. (12.2) K/U
14. de Broglie suggested that if waves have momentum, then moving particles can have wavelength. (12.3) K/U
15. Classical physics cannot explain what happens to an electron as it passes through a double slit. (12.3) K/U
16. The collapse interpretation of the electron double-slit experiment states that the electron expands into a wave as it traverses both the double slits and then collapses back into a particle as it strikes one place on the screen. (12.3) K/U
17. When Rutherford first set up his gold foil experiment, he expected to see many alpha particles rebounding off the gold foil. (12.6) K/U
18. Electrons and positrons are examples of gluons. (12.6) K/U

Go to Nelson Science for an online self-quiz.

WEB LINK

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. The pillars of classical physics (mechanics and electromagnetism) are built on the laws of
   (a) Newton and Einstein
   (b) Einstein and Planck
   (c) Ampère and Ohm
   (d) Newton and Maxwell (12.1) K/U
2. According to the classical view, the amount of energy carried in a wave is described by the wave's
   (a) intensity
   (b) velocity
   (c) medium
   (d) wavelength (12.1) T/I
3. Innovations in computer technology are made possible through our understanding of which fundamental area of physics? (12.1) K/U
   (a) kinematics
   (b) thermodynamics
   (c) relativity theory
   (d) quantum mechanics
4. Light of a single wavelength shines on two sheets of metal with different work functions. Which metal will eject electrons with larger kinetic energies? (12.2) K/U T/I
   (a) the metal with the larger work function
   (b) the metal with the smaller work function
   (c) the metal with more electrons
   (d) the metal with fewer electrons
5. The wavelength at which an object emits maximum radiation intensity depends on the temperature of the object. Which of the colours that are radiated from an object indicate that the object is at the highest temperature? (12.2) K/U
   (a) blue
   (b) yellow
   (c) orange
   (d) red
6. On what does the energy of a photon depend? (12.2) K/U
   (a) only the frequency of the light
   (b) only the intensity of the light
   (c) both the frequency and the intensity of the light
   (d) the wavelength and intensity of the light
7. Light photons with energy 8.3 eV strike a metal surface and eject electrons with 4.9 eV of kinetic energy. What is the work function of the metal? (12.2) T/I
   (a) 3.4 eV
   (b) 6.8 eV
   (c) 13.2 eV
   (d) 26.4 eV
8. As the frequency of a light source increases, the momentum of photons coming from that source will
   (a) increase
   (b) decrease
   (c) remain constant
   (d) increase then decrease (12.2) K/U
9. A metal has work function W. What is the minimum frequency of a light photon that will eject an electron from this metal? (12.2) T/I
   (a) hW
   (b) $\frac{W}{h}$
   (c) $\frac{W^2}{h}$
   (d) $\frac{1}{Wh}$
10. In Compton's experiment, a high-energy photon strikes a thin metal foil. An electron and a low-energy photon are scattered after the collision. Which photon will have the longer wavelength? (12.2) K/U T/I
    (a) the incident photon
    (b) the scattered photon
    (c) it depends on the intensity of the light
    (d) the wavelength does not change
11. What is Einstein's quantum theory of light? (12.2) K/U
    (a) X-ray photons lose energy and scatter off metal foil.
    (b) A photon emerges with less energy and momentum after freeing an electron.
    (c) A metal's work function is given by $W = eV$.
    (d) A photon carries a quantum of energy according to $E = hf$.
12. Which experiment best demonstrates the particle-like nature of light? (12.2) K/U
    (a) a red laser beam refracting from air into a container of water
    (b) monochromatic light passing through a double slit and creating interference fringes on a distant screen
    (c) light striking a metal plate and ejecting electrons
    (d) light undergoing a phase change as it reflects off a medium of larger index of refraction

13. A scientist who deals only with the mathematics and measurements related to electron diffraction experiments would likely subscribe to which quantum-mechanical interpretation? (12.3) K/U
   (a) collapse
   (b) pilot wave
   (c) many worlds
   (d) Copenhagen

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

14. Electrons can behave like waves and like particles. (12.1) K/U
15. When two classical particles approach each other, one possible outcome is for the particles to pass through each other. (12.1) K/U
16. If the frequency of a light source doubles, then the energy of each photon from that light source will be halved. (12.2) K/U
17. The intensity of light is a measure of the number of photons per second emitted from the light source. (12.2) K/U
18. Planck's constant will be a different value depending on the metal that is used. (12.2) K/U
19. The photoelectric effect is an experiment that demonstrates the wave nature of light. (12.2) K/U
20. Planck's constant has units of energy per unit of time. (12.2) K/U
21. One of the observed results in the Compton effect is pair creation, where two non-zero mass particles are created out of the energy of the incident photon. (12.2) K/U
22. As the temperature of an object increases, the wavelength of maximum radiated intensity will also increase. (12.2) K/U
23. Interpretations of the electron diffraction experiment have led to theories that suggest multiple universes. (12.3) K/U
24. As a beam of electrons emerges from a narrow slit, the electrons will make a wide diffraction pattern. (12.3) K/U
25. Positron emission tomography interprets brain wave signals and converts them into instructions for operating artificial limbs. (12.4) K/U
26. The advantage of quantum computing over binary computing is that quantum computers are not limited to the two states 0 and 1. (12.5) K/U
27. Three up quarks and one down quark make up a proton. (12.6) K/U
28. Particles in the hadron family are composed of quarks. (12.6) K/U
29. When combined in the proper quantities, bosons can form any particle in the lepton family. (12.6) K/U

Match each term on the left with the most appropriate description on the right.

30. (a) photoelectric effect
    (b) matter waves
    (c) Higgs boson
    (d) collapse interpretation
    (e) pilot wave interpretation
    (f) quantum

    (i) the electron always behaves as a wave or a particle
    (ii) the smallest amount of energy a particle can emit or absorb
    (iii) the electron is just a simple particle whose motion is described by a single law
    (iv) the wave-like behaviour of massive particles
    (v) electrons are ejected from a material exposed to electromagnetic radiation
    (vi) thought to play a role in giving mass to other particles

    (12.1–12.3, 12.6) K/U

Write a short answer to each question.

31. Explain how the double-slit experiment is used to demonstrate the wave–particle duality of matter on the atomic scale. (12.1) K/U C
32. Discuss the photoelectric effect. (12.2) K/U C
    (a) Sketch the essential features of the photoelectric effect and how they are related: incident light, metal surface, and ejected electrons.
    (b) Explain why light with a higher intensity does not provide more kinetic energy to the electrons.
    (c) Explain why the kinetic energy of an ejected electron is not equal to the photon's energy.

Understanding

33. The words *quantum* and *quantized* can be difficult to understand. Answer the following questions with a classmate. (12.1) K/U C A
    (a) Describe how money is quantized.
    (b) Explain how the analogy of money can relate to quantum physics.
    (c) What other things in your life are quantized?
34. Explain the difference between classical mechanics and quantum mechanics. (12.1) K/U C

35. When the double-slit experiment is performed with subatomic particles such as electrons, there are some very surprising results. (12.1) K/U T/I A
    (a) What evidence collected from this experiment supports the theory that electrons act like particles?
    (b) What evidence collected from this experiment supports the theory that electrons act like waves?
    (c) In your own words, explain why this experiment shows that electrons must have a wave–particle duality.
36. Discuss the problems that physicists had in explaining blackbody radiation before the development of quantum mechanics. (12.2) K/U C
    (a) Sketch the relative intensity versus wavelength curves for two objects with different temperatures.
    (b) Discuss what the classical theory predicts should happen to the intensity as the frequency approaches infinite values.
    (c) Describe Max Planck's resolution to the problem.
37. In a graphic organizer of your choice, describe three pieces of experimental evidence that support a particle model of light. (12.2) K/U T/I C
38. Explain why it is the frequency, not the intensity, of the light source that determines whether photoemission will occur. (12.2) K/U C
39. In a table, compare and contrast four properties of a 1 eV electron and a 1 eV photon. (12.3) K/U C
40. Explain how an experiment involving electron diffraction from a single slit of various widths demonstrates the Heisenberg uncertainty principle. (12.3) K/U C
41. Describe the experimental evidence that supports a wave model of matter. (12.3) K/U T/I C
42. An electron microscope (**Figure 1**) can create high-resolution images with up to 10 million times magnification. Why are ordinary optical microscopes unable to achieve the same magnification? (12.3) K/U

Figure 1

43. Identify and describe the four interpretations of the electron double-slit experiment that were described in this chapter. (12.3) K/U C
44. Explain the difference between traditional cryptology and quantum cryptology. (12.5) K/U C A
45. What was the main theoretical problem with using the planetary model to describe electrons orbiting the nucleus? (12.6) K/U
46. Describe the development of our understanding of the atom through atomic models. Discuss the ideas of Rutherford and Bohr, and how theory and experiment led to changes through time. (12.6) K/U C A
47. In a table, list the three fundamental forces and the bosons that mediate these forces. (12.6) K/U
48. Describe the standard model of elementary particles in terms of the characteristics of quarks, hadrons, and bosons. (12.6) K/U C

Analysis and Application

49. Determine the temperature of a blackbody that appears orange. Its maximum intensity of radiation occurs at a wavelength of 625 nm. (12.2) T/I
50. Ear thermometers measure the blackbody emission of infrared light. A healthy human temperature is 37.0 °C. (12.2) T/I A
    (a) Convert 37.0 °C to degrees kelvin by adding 273 to the value in degrees Celsius.
    (b) Calculate the maximum wavelength.
51. The classical equation for momentum depends on an object's mass and speed, $p = mv$. However, a photon does not have mass. The equation for the momentum of a photon is $p_{\text{photon}} = \frac{h}{\lambda}$, which does not include mass. While studying the scattering of lower-energy photons by high-energy photons, Compton wondered about conservation momentum of massless particles. He solved the problem using Einstein's $E = mc^2$. Show how Compton derived the above equation for photon momentum. (12.2) T/I A
52. Light with wavelength λ ejects electrons from a metal with a kinetic energy of 1.0 eV. When the wavelength of the light source is switched to $\frac{\lambda}{2}$, the ejected electrons have a kinetic energy of 3.0 eV. What is the work function of the metal? (12.2) K/U T/I
53. Describe two possible procedures to determine the work function of a metal plate. One method uses a voltage source and a second plate. The other method uses a light source. (12.2) K/U T/I A

54. Light with a wavelength of 3.20×10^{-7} m is incident on the positive plate of the set of parallel plates shown in **Figure 2**. The light ejects electrons from the metal (cesium), which has a work function of 1.95 eV. (12.2) K/U T/I A

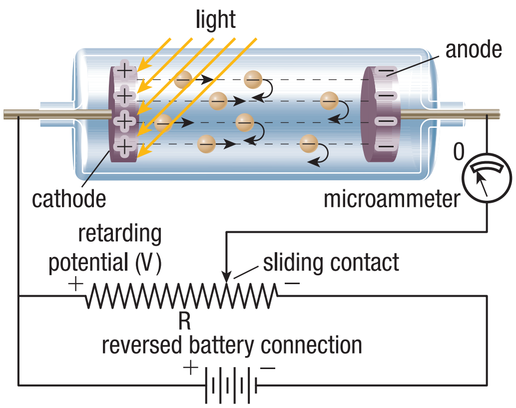


Figure 2

(a) Calculate the kinetic energy of an ejected electron.

(b) Calculate the potential difference across the two parallel plates required to stop the ejected electrons from reaching the opposite plate.

55. The maximum kinetic energy of electrons from a tungsten surface is 1.7 eV. The surface has a work function of 4.52 eV and it is illuminated by light. What is the wavelength of the light? (12.2) K/U T/I

56. Photosynthesis is a chemical reaction that occurs in green plants when exposed to light. **Figure 3** shows how the rate of photosynthesis depends on the wavelength of light. Answer the following questions using quantum theory and Wein's law. (12.2) K/U T/I A

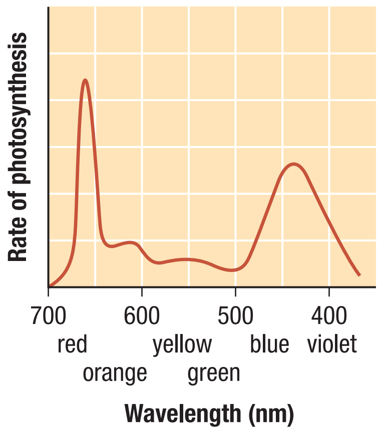


Figure 3

(a) Why do the plants appear green?

(b) Why does pure green light produce a very low reaction rate for photosynthesis?

(c) Why does an incandescent light operating at 5300 K produce a very low reaction rate?

57. A shiny silvery metal with an unknown work function is connected through a potential difference to another metal plate. The minimum potential that will eject electrons toward the bottom plate is 5.65 V. (12.2) K/U T/I A

(a) Determine the metal's work function in joules and in electron-volts.

(b) Using **Table 1**, determine the unknown metal.

Table 1 Work Functions of Several Metals

Metal	Work function (J)	Work function (eV)
aluminum (Al)	6.73×10^{-19}	4.20
copper (Cu)	8.17×10^{-19}	5.10
lead (Pb)	6.81×10^{-19}	4.25
molybdenum (Mo)	7.25×10^{-19}	4.53
platinum (Pt)	9.04×10^{-19}	5.64
silver (Ag)	7.43×10^{-19}	4.64

58. Create a graph of E versus f for copper and lead using the values from Table 1. (12.2) K/U T/I C A

(a) Determine the threshold frequency of each metal.

(b) Explain why the lines in your graph have the same slope.

59. Iron has a work function of 7.47×10^{-19} J, or 4.67 eV. (12.2) K/U T/I A

(a) Determine the minimum frequency, f_0, of a photon that will eject electrons from iron.

(b) X-ray light with a frequency of 7.9×10^{15} Hz now strikes the iron plate. Determine the kinetic energy of the ejected electrons. Provide your answer in both joules and electron-volts.

60. In a photoelectric experiment, light with unknown frequency and wavelength strikes a copper plate. Copper has a work function of 8.17×10^{-19} J, or 5.10 eV. Electrons are ejected with 2.1 eV of kinetic energy (**Figure 4**). (12.2) K/U T/I A

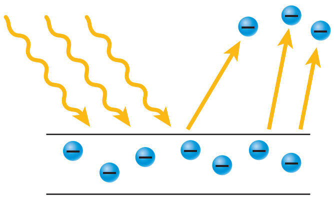


Figure 4

(a) Determine the frequency of the light.

(b) Determine whether the light is ultraviolet, visible, or infrared.

61. Calculate the momentum of a light photon with wavelength 580 nm. (12.2) K/U T/I A

62. An X-ray photon behaves like a particle with momentum equal to 2.03×10^{-24} kg·m/s. Calculate
  (a) the wavelength of the photon
  (b) the frequency of the photon (12.2) K/U T/I A

63. A certain photon has energy 6.40 eV. Determine the corresponding momentum of the photon. (12.2) K/U T/I

64. Explain how quantum mechanics applies to our everyday experiences. (12.3) K/U T/I C A

65. Suppose you want to perform a single-slit electron diffraction experiment to study the properties of de Broglie's matter waves. Determine the maximum slit width you need to notice matter wave effects for a 4 eV electron. Justify your answer. (12.3) T/I A

66. Determine the de Broglie wavelength of an electron that is travelling with a speed of 5.80×10^6 m/s. (12.3) K/U T/I A

67. To take advantage of the wave nature of an electron, a scientist needs to create a beam of electrons with a de Broglie wavelength of 0.220 nm. (12.3) K/U T/I A
  (a) Determine the speed of these electrons.
  (b) Determine the kinetic energy of these electrons in both joules and electron-volts.
  (c) Determine the voltage necessary to accelerate the electrons from rest.

68. Determine the ratio of the de Broglie wavelengths of a proton and of an electron both moving at 0.025*c*. (12.4) K/U T/I A

69. Scientific breakthroughs and technological advances have a close relationship. Describe an example of a technological advance that permitted a breakthrough in particle physics research. Describe an example of a technological application of particle physics research. (12.6) K/U C A

Evaluation

70. In your opinion, which scientist or scientists made the most important contributions to the development of quantum mechanics? In your answer, include information such as the experiments conducted and the discoveries made. (12.1–12.4) T/I C A

71. A very bright standard light bulb will not damage a person's skin, even at close distances, assuming the hot bulb does not touch the skin. However, ultraviolet light from the distant Sun can cause sunburn, even when the day is not particularly bright. Interpret these effects in terms of the energy of photons. (12.2) K/U A

72. A physics instructor conducts a photoelectric experiment with sodium as a target material. The instructor shines a decreasing range of wavelengths on the sodium sample and measures the kinetic energy of the ejected electrons (**Table 2**). (12.2) T/I C A

Table 2 Properties of Ejected Electrons

Wavelength (nm)	Frequency of light (Hz)	Kinetic energy of ejected electrons (J)
600		—
520		—
440		4.6×10^{-20}
360		1.4×10^{-19}
280		2.9×10^{-19}
200		5.7×10^{-19}

  (a) Copy and complete the table by calculating the frequency for each corresponding wavelength.
  (b) Plot all possible values on a kinetic energy versus frequency graph.
  (c) Determine the value of the *x*-intercept. Explain its meaning.
  (d) Use the data to calculate the slope (with units) of the graph.
  (e) Explain why some wavelengths do not produce any ejected electrons.

73. Particle physics researchers say that bosons "mediate" the fundamental forces. Justify the use of this word in this context. (12.6) K/U C A

Reflect on Your Learning

74. What did you find most surprising in this chapter, and what did you find most interesting? How can you learn more about these topics? T/I C

75. How would you explain the core ideas of quantum mechanics to a student who has not taken physics? T/I C

76. Do you need to consider the effects of quantum mechanics in your daily experiences? Discuss some technologies that you use regularly that rely on the effects of quantum mechanics. C A

Research

WEB LINK

77. Research the Davisson–Germer experiment and answer the following questions. T/I C A
  (a) How did this experiment confirm the electron's wave nature?
  (b) What materials did Davisson and Germer use in this experiment?

78. Cosmology is the study of the origin and future of the universe. Research how quantum mechanics has helped us better understand the origin of the universe. T/I C A
    (a) What observation did astronomer Edwin Hubble make in the 1920s about the galaxies in the universe?
    (b) What discovery did Arno Penzias and Robert Wilson make in 1965 when they were testing a microwave antenna for satellite communications?
    (c) How has the big bang theory explained these two experimental observations?
    (d) What particles do physicists think were present at the origin of the universe? How did the interaction of these particles change over time after the big bang occurred?
79. Einstein won the Nobel Prize for Physics in 1921 for his explanation of the photoelectric effect. T/I C A
    (a) Research the ideas and experiments that Einstein used when developing his theory.
    (b) Explain how Einstein's explanation of the photoelectric effect eventually led to the development of solar photovoltaic cells.
80. Einstein's general theory of relativity radically changed the way that physicists view the universe. However, he was unable to develop a theory that unified all fundamental forces with the force of gravity. T/I C A
    (a) Research why gravity posed a problem for Einstein when he developed his theory of general relativity.
    (b) What are some of the problems with gravity that physicists have encountered as they have tried to develop a theory of everything?
81. Physicists think of the quarks that make up baryons and mesons as "swimming in a sea of exchanged gluons." They also think that the force between quarks approaches zero when they are a very short distance apart (less than the size of a proton or a neutron). Research the unusual properties of the force between quarks. T/I C A
    (a) What happens when you try to separate the quarks that form a proton or a neutron? What particles do you end up with?
    (b) What happens when you increase the energy used to break apart a proton or neutron? What particles do you end up with?
    (c) Is it possible to separate two quarks given enough energy? Explain.
82. Physicist Richard Feynman contributed significantly to our understanding of how quarks interact, and he developed diagrams to help visualize these interactions. Research Feynman's contribution to particle physics. T/I C A
    (a) What projects did he work on that helped him develop our understanding of quantum mechanics?
    (b) Explain why physicists use Feynman diagrams to visualize quark interactions.
83. A charge-coupled device (CCD) will produce one conducting electron for each photon that strikes a pixel. K/U T/I A
    (a) Research the basic principles behind a CCD.
    (b) Why does this fact make CCDs useful for astronomers?
84. Research the technology of photovoltaic modules. T/I C A
    (a) When was the first photovoltaic module built?
    (b) What are the essential parts of a photovoltaic module, or a solar cell?
    (c) How is an array different from a module or a cell?
    (d) What other physics concepts that you have studied thus far do you see at work in a photovoltaic module (**Figure 5**)?

Figure 5

85. (a) Find examples of materials in nature that approach a "perfect" blackbody.
    (b) Discuss some of the applications of blackbody radiation. T/I C A
86. The Canadian Light Source (CLS) synchrotron, in Saskatchewan, produces intense beams of light at various wavelengths, particularly in the X-ray range. The extremely small wavelength of the light, coupled with its high intensity, makes the CLS an ideal probe with which to examine the most minute of structures. Research the CLS. T/I C A
    (a) What is synchrotron radiation?
    (b) Why is the electron the particle of choice for the CLS synchrotron?
    (c) Identify three uses for synchrotron light.

The Large Hadron Collider: The World's Biggest Microscope

The Large Hadron Collider (LHC) is a particle accelerator that lies underground along the Swiss–French border near Geneva, Switzerland (**Figure 1**). It allows scientists to "look" more closely at the makeup of subatomic particles by accelerating particles and smashing them together. With a circumference of 27 km, the LHC can be considered the world's biggest microscope. The LHC will enable scientists from around the world to make new discoveries about the structure of all matter, and it will provide further clues about the origin of the universe.

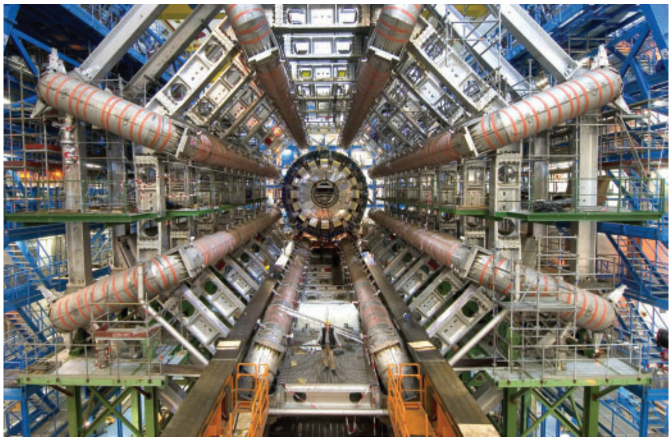


Figure 1 The Large Hadron Collider is the largest particle accelerator in the world.

Many of the modern-day technologies that we take for granted have resulted from particle physics research projects like the LHC. The Internet, MRI machines, medical radioactive tracers, and some cancer therapies can all be traced back to research in particle physics. This kind of research, especially the development of large, complicated machines such as the LHC, is very expensive. Operating the LHC requires enormous amounts of energy. There are also dangers related to technology based on particle physics. For example, exposure to radiation can have adverse effects on living things.

Your Goal

To gain an understanding and an awareness of the work and research taking place at the LHC, and to assess the impact that this work has on the scientific community and on the world in general

Research

You will investigate the research currently taking place at the LHC and similar research projects in subatomic physics that were performed in the past. Your investigation will look at the research from scientific, environmental, societal, and ethical standpoints. Consider the potential positive and negative impacts, and then prepare to discuss whether the advantages of such scientific research outweigh the disadvantages. You will be assigned one of the following three options to investigate.

OPTION 1

Research the work being conducted at the LHC from a scientific perspective. Investigate, in general terms, how the LHC is structured and how each section of the LHC works. Describe what the various detectors measure and how physicists use these measurements. How do assumptions from the theories of classical physics, special relativity, and quantum physics play a role at the LHC? Prepare a presentation to share with the rest of the class about the science behind the LHC. Gather any relevant pictures, charts, graphs, videos, and so on, that will help illustrate the presentation.

OPTION 2

Research the various spinoff technologies that have resulted from similar subatomic research projects in the past. Consider the positive and negative impacts that these technologies have had on society and the environment. Also, research potential technologies that scientists hope to develop based on results achieved at the LHC. Are there any lessons to be learned from the outcomes of spinoff technologies of the past? Prepare an electronic slide presentation to share your findings and analysis with the rest of the class.

OPTION 3

Research the broader societal and ethical factors associated with operating the LHC. For example, the LHC is not able to run at full power year round because of the increased demand it puts on the electrical generation capacity for the entire country of France. What impact might this have? How could this electrical need be addressed? How might potential solutions positively or negatively impact France and the other nearby countries? Consider the economic impact of the LHC, which is the most expensive and complicated machine ever built. Is the vast expense worth the potential payoff, even if it is only "knowledge for knowledge's sake"? Has the LHC directly or indirectly produced other economic benefits for the countries involved? How is the cost of this project divided among the participating countries? Investigate these and other societal issues, and share your results with the rest of the class in an oral presentation, using charts and graphs to support your findings.

Make a Decision

After each group has presented their findings, discuss whether the potential success of the scientific research that is taking place at the LHC is worth the economic costs, human resources, and potential impacts on society and the environment. You may be assigned to argue from a particular perspective, so be sure you understand the different sides of this task.

ASSESSMENT **CHECKLIST**

Your completed Unit Task will be assessed according to these criteria:

Knowledge/Understanding

- ✓ Demonstrate a basic understanding of how the LHC works.
- ✓ Demonstrate knowledge of the technologies that have developed out of past particle physics research.
- ✓ Demonstrate an understanding of the impacts, both positive and negative, that the LHC project is having on society and the environment.

Thinking/Investigation

- ✓ Research the purpose and function of the various components of the LHC.
- ✓ Research technologies that have emerged from particle physics research.
- ✓ Research the economic impact the LHC has had on participating countries.

Communication

- ✓ Synthesize findings in the form of an oral presentation.
- ✓ Communicate findings in an audiovisual format using a variety of multimedia tools.
- ✓ Communicate viewpoints and findings clearly and concisely.

Application

- ✓ Apply physics knowledge to an unfamiliar context.
- ✓ Justify a position on a physics-related issue through research.

For each question, select the best answer from the four alternatives.

1. Lupe is riding in a spacecraft moving away from Earth at a speed close to the speed of light. Richard is standing on Earth. As the spacecraft emits a flash of light, both observers will determine that the light
   (a) has the same frequency
   (b) has the same colour
   (c) moves at the same speed
   (d) oscillates with the same period (11.1) K/U
2. A property of an inertial frame of reference is that it is
   (a) not in motion relative to other inertial frames of reference
   (b) distinguishable from other inertial reference frames
   (c) a frame in which Newton's first law of motion holds
   (d) not in motion relative to Earth (11.1) K/U
3. Einstein used his thought experiment involving a light clock bouncing vertically between mirrors to prove that
   (a) light must move faster when emitted by reference frames in motion
   (b) light must move slower when emitted by reference frames in motion
   (c) moving clocks run slow compared to stationary clocks
   (d) moving clocks run fast compared to stationary clocks (11.2) K/U
4. A clock designed to tick once a second is in a spacecraft moving at a constant speed of $0.75c$ through an inertial frame of reference. Which is true if the clock is observed from Earth? (11.2) K/U
   (a) The clock is ticking once a second.
   (b) The clock is ticking at a rate that is faster than once a second.
   (c) The clock is ticking at a rate that is slower than once a second.
   (d) The clock is running backward.
5. As a particle with rest mass m is accelerated to a relativistic speed v, the particle's momentum
   (a) becomes less than mv
   (b) becomes greater than mv
   (c) becomes equal to mv
   (d) depends on whether or not the particle is moving faster than or slower than c (11.3) K/U
6. Muons decay after travelling 660 m (in the muon's frame of reference) into the atmosphere, yet they are detected on Earth's surface. This is evidence that
   (a) time contracts for objects in motion
   (b) length increases for objects in motion
   (c) length contraction and time dilation have complementary effects in different frames of reference
   (d) mass decreases for objects in motion (11.3) K/U
7. Kapil measures the length of a pipe to be 65 cm as he remains stationary on Earth. Kapil then takes the same pipe aboard a spacecraft and travels at relativistic speeds. While in motion, what will Kapil measure as the length of the pipe? (11.3) K/U
   (a) 65 cm
   (b) more than 65 cm
   (c) less than 65 cm
   (d) The answer depends on the direction of travel.
8. After a uranium nucleus at rest undergoes nuclear fission, the two daughter nuclei race apart from each other at high speeds. Where does this kinetic energy come from? (11.4) K/U
   (a) the gravitational potential energy of the system
   (b) the conversion of mass into energy
   (c) the collision between a third particle, heavier than uranium, and the original nucleus
   (d) the kinetic energy of moving neutrons
9. The fuel that produces a nuclear fusion reaction in the Sun is
   (a) uranium
   (b) hydrogen
   (c) plutonium
   (d) carbon (11.4) K/U
10. The concepts developed in quantum mechanics are only applicable for
    (a) very small objects, such as electrons
    (b) very large objects, such as stars
    (c) molecular phenomena, such as reactions
    (d) particle phenomena, such as billiard ball collisions (12.1) K/U

11. A sheet of aluminum will exhibit the photoelectric effect when
    (a) high-frequency light strikes the aluminum, causing electrons to be ejected
    (b) low-frequency light strikes the aluminum, causing electrons to be ejected
    (c) light strikes the metal and is reflected perpendicular to the surface
    (d) the metal vibrates enough to produce electromagnetic radiation (12.2) K/U
12. Three types of photons strike a metal plate. The photons have the following wavelengths: 370 nm, 530 nm, and 750 nm. Which photon is most likely to eject electrons from the metal plate? (12.2) K/U
    (a) the 370 nm photons
    (b) the 530 nm photons
    (c) the 750 nm photons
    (d) All three types of photons have an equal chance of ejecting electrons from the metal.
13. The Compton effect is evidence that
    (a) photons can have momentum and behave as particles
    (b) electrons can have wavelength and interfere with each other
    (c) energy is not conserved in the subatomic realm
    (d) momentum is not conserved in the subatomic realm (12.2) K/U
14. Which scientist first used the idea of quantized energy to explain blackbody radiation? (12.2) K/U
    (a) Albert Einstein
    (b) James Maxwell
    (c) Max Planck
    (d) Thomas Young
15. Consider a free electron. If the kinetic energy of this electron doubles, its de Broglie wavelength changes by a factor of
    (a) $\frac{1}{2}$
    (b) $\frac{1}{\sqrt{2}}$
    (c) 2
    (d) $\sqrt{2}$ (12.3) K/U

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

16. Maxwell's equations served as the starting point for Einstein's special theory of relativity. (11.1) K/U
17. Einstein often used thought experiments to test the predictions of traditional theories in extreme situations. (11.1) K/U
18. A spacecraft in space will have clocks running faster than a stationary clock by a factor of $\sqrt{1 - \frac{v^2}{c^2}}$. (11.2) K/U
19. An observer watching a moving object will see its length contract in the direction perpendicular to the direction of motion. (11.3) K/U
20. When two photons approach each other, their relative velocity is zero. (11.3) K/U
21. In the thought experiment involving the twin paradox, it is important to remember that only one twin is accelerating, and that the length of the universe will contract for only one twin. (11.3) K/U
22. Mass–energy conversion is detectable in any process that releases energy. (11.4) K/U
23. A beam of electrons passes through a double slit. All places on the distant screen will have the same probability of receiving an electron impact. (12.1) K/U
24. Wave–particle duality includes the property that electromagnetic radiation can exhibit interference. (12.1) K/U
25. If a stream of table tennis balls is launched through a double-slit setup, the impacts on a distant target will form an interference pattern. (12.2) K/U
26. According to Wien's law, the wavelength at which the blackbody radiation is most intense is inversely related to the temperature of the blackbody in degrees kelvin. (12.2) K/U
27. Techniques developed in the original Davisson–Germer experiment to show electron diffraction are still used today as a way of measuring molecular spacing. (12.3) K/U
28. The Heisenberg uncertainty principle implies that the more you know about the momentum of an electron, the less you can know about its position. (12.3) K/U
29. A superconducting quantum interference device interprets brainwave signals, detecting brain dysfunctions such as tumours and memory disorders. (12.4) K/U
30. Quantum cryptography can improve the security of information transmitted online. (12.5) K/U
31. Rutherford's model of the atom was unstable because orbiting electrons radiate energy. (12.6) K/U

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. Einstein developed his theory of relativity mainly because he
   (a) discovered errors in the work of Galileo and Newton
   (b) needed to explain the results of the Michelson–Morley experiment
   (c) grew dissatisfied with the inconsistency of electromagnetism in moving frames of reference
   (d) needed to explain Planck's theory of blackbody radiation (11.1) K/U
2. Thought experiments were important in the development of special relativity because, during Einstein's time,
   (a) scientists were not performing experiments about light
   (b) scientists did not think that experiments were necessary
   (c) the effects of relativity only become important at speeds close to the speed of light and scientists were unable to achieve these speeds
   (d) scientists would only believe thought experiments (11.1) K/U
3. Time dilation is the slowing down of time in an inertial reference frame moving relative to
   (a) an observer in the same inertial reference frame
   (b) an observer in another inertial reference frame
   (c) any observer
   (d) proper time (11.2) K/U
4. As a very small particle is accelerated to a relativistic speed, what will a stationary observer notice about the particle? (11.3) K/U
   (a) The particle's length increases.
   (b) The particle's length decreases.
   (c) The particle becomes hotter.
   (d) The particle becomes darker.
5. The length of a spacecraft at rest is 100 m. If the spacecraft travels at the appropriate speed, its length, as measured by a stationary observer, could be
   (a) 0 m
   (b) 80 m
   (c) 110 m
   (d) 200 m (11.3) K/U
6. Which statement about the twin paradox is true? (11.3) K/U
   (a) Only one of the twins is in an inertial frame of reference.
   (b) The twin paradox has no solution.
   (c) The twin paradox is only theoretical; in reality, the twins would be the same age.
   (d) The effects of quantum mechanics are needed to resolve the twin paradox.
7. Which example represents an inertial frame of reference for measuring all motion? (11.3) K/U
   (a) a railway car that moves uniformly at half the speed of light
   (b) the surface of Earth
   (c) the surface of the Sun
   (d) the frame of reference in which the distant stars are not moving
8. In Einstein's well-known formula, $E = mc^2$, m is the rest mass of a particle, c is the speed of light, and E is the
   (a) classical kinetic energy of the particle
   (b) relativistic kinetic energy of the particle
   (c) rest energy of the particle
   (d) potential energy of the particle (11.4) K/U
9. The word *quantum* in quantum mechanics means
   (a) fast-moving
   (b) a small amount of energy
   (c) a wave
   (d) a particle (12.1) K/U
10. The energy carried by a photon with frequency f is
   (a) hf
   (b) $\frac{hf}{c}$
   (c) $\frac{h}{f}$
   (d) $\frac{f}{h}$ (12.2) K/U
11. The energy of a photon is E. The energy of another photon is $2E$. What is the speed of the higher-energy photon? (12.2) K/U T/I
   (a) $\frac{c}{2}$
   (b) c
   (c) $\sqrt{2}c$
   (d) $2c$

12. A scientist shines green light on a copper plate in an attempt to demonstrate the photoelectric effect. However, no electrons are ejected. What type of light should the experimenter try next? (12.2) K/U
    (a) infrared
    (b) orange
    (c) yellow
    (d) blue
13. Which of these technologies is only possible due to our understanding of quantum mechanics? (12.2) K/U
    (a) a laser bar code scanner
    (b) radar
    (c) hydroelectric power
    (d) rocket engines
14. Two very hot objects emit light and thermal energy that is measured by researchers. Object A emits light with a wavelength of 490 nm. Object B emits light with a wavelength of 670 nm. Which object is hotter? (12.2) K/U
    (a) object A
    (b) object B
    (c) The answer depends on the volumes of the objects.
    (d) The answer depends on the compositions of the objects.
15. The difference in the photon energy and the kinetic energy of an ejected electron is equal to the
    (a) Copenhagen difference
    (b) work function
    (c) interference minimum
    (d) threshold frequency (12.2) K/U
16. According to de Broglie, the wavelength of a particle is inversely related to the particle's
    (a) period
    (b) momentum
    (c) energy
    (d) charge (12.3) K/U
17. The Heisenberg uncertainty principle describes the uncertainty in measurements due to
    (a) the poor accuracy of our measuring devices
    (b) a fundamental limit on the accuracy of measurements
    (c) human misunderstanding about very small objects
    (d) an inability to control very small objects in an experiment (12.3) K/U
18. Particles that are made from combinations of quarks are called
    (a) leptons
    (b) hadrons
    (c) combination quarks
    (d) bosons (12.6) K/U
19. Which scientist developed the first quantum model of the atom, suggesting that electrons could only occupy certain energy levels? (12.6) K/U
    (a) J.J. Thomson
    (b) Ernest Rutherford
    (c) Niels Bohr
    (d) Max Planck

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

20. At the end of the 1800s, many physicists thought that they had learned all there was to know in the areas of classical mechanics, thermodynamics, and electromagnetism. (11.1) K/U
21. Special relativity applies to objects moving at the speed of light. (11.1) K/U
22. The surface of Earth is not an inertial reference frame because Earth is rotating. (11.1) K/U
23. Visible light travels with speed *c*, but ultraviolet light travels with a speed slightly less than *c*. (11.1) K/U
24. Observers in different inertial frames measure the speed of light to be the same. (11.1) K/U
25. Special relativity affects only subatomic particles. (11.2) K/U
26. Muons are the only particles that can demonstrate time dilation and length contraction. (11.3) K/U
27. The proper mass of a particle depends on its speed. (11.3) K/U
28. An electric force does continual work on a proton as it approaches the speed of light, so eventually the proton's momentum will slow to zero. (11.4) K/U
29. Electrons behave like waves when they are near a light atom, such as hydrogen, but behave like particles when they are near a heavy atom, such as iron. (12.1) K/U
30. Both light and electrons exhibit wave-like and particle-like behaviour. (12.1) K/U
31. The value of Planck's constant depends on the velocity of the observer. (12.2) K/U
32. Classical physics predicts that increasing the intensity of light striking a metal surface increases the kinetic energy of the ejected electrons, but studies of the photoelectric effect did not show this result. (12.2) K/U
33. Matter waves demonstrate that particles with mass always behave as waves. (12.3) K/U

34. Electron microscopes can take high-resolution images of small objects because the de Broglie wavelength of electrons is very large. (12.3) K/U
35. The Heisenberg uncertainty principle is no longer valid because our modern tools of measurement have vastly improved since Heisenberg's time. (12.3) K/U
36. In the quantum model of an atom, electrons orbit the nucleus in a way similar to planets orbiting the Sun. (12.6) K/U
37. Antimatter was first predicted by theory and only later observed experimentally. (12.6) K/U
38. Antimatter is never produced in nature. (12.6) K/U
39. Electrons, protons, and neutrons are all fermions. (12.6) K/U
40. Quarks are the particles that carry the electromagnetic force. (12.6) K/U
41. There are eight types of quark: up, down, left, right, top, bottom, strange, and charm. (12.6) K/U
42. A proton is one type of hadron. (12.6) K/U
43. An electron is a lepton, and leptons are made up of quarks. (12.6) K/U
44. The standard model describes the fundamental particles and all their interactions. (12.6) K/U
45. Quarks have the same electric charge as electrons. (12.6) K/U
46. A theory of everything would combine quantum theory and general relativity into a single theory. (12.6) K/U

Understanding

47. You are in a sealed room with no windows or doors. You are asked whether the room is at rest or moving with a constant velocity. Are there any experiments that you could do to allow you to answer the question? Explain your reasoning. (11.1) K/U
48. Two observers are in separate frames of reference. The two frames of reference each move at different constant speeds. (11.1) K/U C
    (a) Can either observer know the speed of his or her own frame of reference? Explain your answer.
    (b) Can either observer know the speed of the other observer's frame of reference? Explain your answer.
    (c) Why will both observers measure the same relative speed for the two frames of reference?
    (d) What quantity will have the same value when measured by either observer?
49. Suppose you carried a mirror in your hand and ran at the speed of light. What, if anything, would you see in the mirror? Explain your reasoning. (11.1) K/U
50. In the 1970s, experiments that involved taking atomic clocks around the world on aircraft were done to test the effects of special relativity. List some of the factors that had to be corrected for when analyzing these experiments to test special relativity. (11.2) K/U
51. (a) Draw a simple diagram of Einstein's thought experiment with the light clock aboard a moving railway car.
    (b) Explain how this experiment, combined with a postulate of special relativity, proves that time is dilated.
    (c) Ali is aboard the railway car, and Jorge is standing outside at rest. Who will measure the correct time? (11.3) K/U C
52. A spacecraft travels at $0.86c$ past a stationary observer on another spacecraft. The stationary observer measures the spacecraft to be 58.0 m long. (11.3) T/I
    (a) Predict the length of the spacecraft according to the stationary observer when the spacecraft travels at $0.99c$.
    (b) Could the entire length of the spacecraft fit into a tunnel that is shorter than the proper length of the spacecraft? Justify your answer.
53. Two observers in inertial frames of reference, one on Earth and the other in a spacecraft, observe each other moving away from each other at a speed of $0.90c$. After a few years, they observe each other moving toward each other at the same speed. How will they know which of them was actually in motion? (11.3) T/I
54. According to the periodic table, the atomic weight of hydrogen is 1.007 94 atomic mass units, and the atomic weight of helium is 4.002 60 atomic mass units. During nuclear fusion, two protons and two neutrons fuse into one helium nucleus. Why is the atomic weight of helium less than four times that of hydrogen? (11.4) K/U A
55. A photon has energy E and wavelength λ. Calculate the wavelength of a photon that has five times the energy. (12.2) T/I
56. Before the invention of the electron microscope, many scientists stated that no one would ever "see" an atom. (12.2) K/U C
    (a) Explain why scientists felt confident in making this claim.
    (b) Describe another piece of technology or experiment discussed in Chapter 12 that changed scientists' minds about a subject.
57. One of the first successes of quantum theory was to explain the spectrum of a blackbody. (12.2) K/U
    (a) State the major problems with the classical theory of a blackbody spectrum.
    (b) Summarize Planck's suggestion that resolved the problem.

58. Describe the problems in trying to explain the photoelectric effect using the classical wave theory of light. (12.2) K/U
59. Electrons and photons can act as waves or as particles, depending on the situation. For each situation listed below, indicate whether the electrons or photons are acting as waves or as particles. Justify your answers. (12.3) K/U C
    (a) Young's double-slit experiment
    (b) the photoelectric effect
    (c) the Compton effect
    (d) electron diffraction experiment
60. Explain how astronomers can determine the temperature of a star just by measuring the electromagnetic radiation that is emitted by the star. (12.3) K/U C
61. Describe three applications of quantum mechanics in medicine. (12.4) K/U
62. Explain in your own words what quantum information theory is. (12.5) K/U
63. Summarize how quantum cryptography works and how it is used to protect information. (12.5) K/U
64. Which quark combination can make up (a) a proton and (b) a neutron (**Table 1**)? Explain your reasoning. (12.6) K/U

Table 1 Quarks

Type of quark	Symbol	Charge
up	u	$\frac{2}{3}$
down	d	$-\frac{1}{3}$

65. Complete **Table 2**. Do not include the antiparticles. (12.6) K/U

Table 2 The Standard Model

Fermions building blocks of matter				**Bosons** force mediators	
Leptons		**Quarks**		**Bosons**	
Particle	**Charge**	**Name**	**Charge**	**Particle**	**Force**

Analysis and Application

66. In special relativity, we often encounter the statement that "nothing can travel faster than the speed of light." When light travels through glass, its speed decreases. Can objects travelling through glass travel faster than the speed of light? Explain whether your answer conflicts with special relativity. (11.1) K/U T/I A
67. Two spacecraft move at half the speed of light relative to an observer on Earth. As the two spacecraft approach each other head-on, one sends a laser signal to the other. Determine the speed of the laser signal as measured from the second spacecraft. (11.1) T/I A
68. A car travels along a road at night. (11.1) T/I A
    (a) Determine the speed of light emitted from the headlights of the car relative to an observer on the roadside when the car is at rest.
    (b) Determine the speed of light relative to the roadside observer when the car moves forward at a speed of 40.0 m/s.
69. A spacecraft is travelling at 0.5*c* relative to Earth. An astronaut onboard shines a flashlight out ahead of the spacecraft. (11.1, 11.4) T/I
    (a) Determine the speed of the light from the flashlight as measured by the astronaut.
    (b) Determine the speed of the light from the flashlight as measured by an observer on Earth.

70. An astronaut travelling at $0.70c$ relative to Earth uses his watch to measure the time between two radio signals arriving from a distant satellite. He measures the time as 28 min. Calculate the time difference between the signals as measured by another observer at rest on Earth. (11.2) T/I

71. A subatomic particle decays in 2.63×10^{-10} s when measured at rest in a laboratory's frame of reference. Suppose the particle has a speed of $0.800c$ with respect to the lab. Calculate the measured time of decay of the particle. (11.2) T/I A

72. An astronaut travels at a speed of $0.90c$ from Earth to a distant star. Determine how much the astronaut has aged after 14 years have elapsed on Earth. (11.2) T/I A

73. A time interval measured on a moving spacecraft equals 8.0 s. This same time interval when measured on Earth equals 10.0 s. How fast, relative to Earth, is the spacecraft moving? (11.2) T/I A

74. Observer 1 is travelling on his railroad car with his light clock with a speed of $0.85c$ relative to observer 2. Observer 1 travels for 30.0 s as measured on his light clock (**Figure 1**). (11.2) K/U T/I A

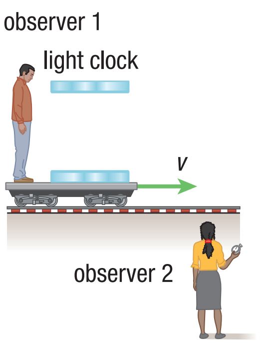


Figure 1

(a) Who measures the proper time, observer 1 or observer 2?

(b) How much time elapses on observer 2's watch during this motion?

75. Many relativity calculations use the quantity $\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$. (11.2) T/I

(a) Copy **Table 3** into your notebook. For each speed, calculate the value of this quantity.

Table 3

Speed	$\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$
$0.500c$	
$0.550c$	
$0.600c$	
$0.650c$	
$0.700c$	
$0.750c$	
$0.800c$	
$0.850c$	
$0.900c$	
$0.950c$	
$0.990c$	

(b) Write the time dilation, length contraction, relativistic momentum, and relativistic energy equations in terms of $\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

76. In the Hafele–Keating experiment, the atomic clocks placed on jets showed different times with respect to each other. They also differed from identical stationary clocks on Earth. **Table 4** shows the predicted and observed results with respect to the clocks on Earth. (The results have been corrected to take special relativity alone into account.) (11.2) K/U T/I A

Table 4

	Time differences (ns)	
	Clock moving east	**Clock moving west**
predicted differences	-184 ± 18	96 ± 10
observed differences	-193 ± 10	94 ± 10

(a) Which clock ran slower relative to the clock on Earth? Explain your answer.

(b) Why should the clock moving west gain time? (Recall the frame of reference used for proper time.)

77. Assess whether an astronaut on a spacecraft moving at $0.99c$ relative to Earth would feel as if she were living in slow motion. (11.2) T/I A

78. A spacecraft has a length of 49.0 m in its rest frame. The spacecraft travels with a constant speed relative to an outside observer, who measures the spacecraft's length as 42.0 m. Determine the spacecraft's speed relative to the observer. (11.3) T/I

79. The length of a spacecraft at rest is 100.0 m. (11.3) T/I
    (a) At what speed would the spacecraft have to be travelling to appear half as long to a stationary observer?
    (b) At what speed would the spacecraft have to be travelling to appear one-fourth as long to a stationary observer?
    (c) At what speed would the spacecraft have to be travelling to appear twice as long to a stationary observer?

80. An object with a rest mass of 1.2 kg is moving at a speed of $0.81c$. (11.3, 11.4) T/I
    (a) Determine the relativistic momentum of the object.
    (b) Determine the total relativistic energy, in joules, of the object according to a stationary observer.

81. An electron moving at a relativistic speed has a total energy of 1.7 MeV. (11.4) T/I
    (a) Determine the total energy of the electron in joules.
    (b) Determine the speed of the electron.

82. A proton is accelerated to a speed of 2.96×10^8 m/s. Determine
    (a) the relativistic momentum of the proton
    (b) the relativistic total energy of the proton in joules
    (c) the relativistic total energy of the proton in electron-volts (11.4) T/I

83. When uranium-235 undergoes fission, about 0.1 % of the original mass is released as energy. (11.4) T/I
    (a) Calculate the energy released when 1 kg of uranium-235 undergoes fission.
    (b) Calculate the amount of uranium-235 that must undergo fission per day in a nuclear reactor that provides energy to a 100 MW device.

84. Photons with frequency 9.7×10^{14} Hz strike a calcium plate with a work function of 4.5×10^{-19} J. Determine the kinetic energy of the ejected electrons in electron-volts. (12.2) T/I

85. At times it is helpful to write Planck's constant in units of electron-volts rather than joules. Write the numerical value of Planck's constant in electron-volts. (12.2) T/I

86. A photon has a wavelength of 210 nm. (12.2) T/I
    (a) Determine the frequency of the photon.
    (b) Determine the energy of the photon.
    (c) Determine the momentum of the photon.

87. A scientist arranges a flash of red light to have the same total energy as a flash of blue light. (12.2) T/I
    (a) Explain how this is possible.
    (b) Determine which colour of flash will have the greater intensity.

88. Light strikes a silver plate with a work function of 4.3 eV and ejects electrons with 3.9 eV of kinetic energy. Determine the wavelength of the light hitting the silver. (12.3) T/I

89. An electron exhibits wave-like behaviour that is consistent with a de Broglie wavelength of 0.105 nm. (12.3) T/I
    (a) Determine the momentum of the electron.
    (b) Determine the speed of the electron, assuming that relativistic effects do not play a role.
    (c) Calculate the relativistic momentum of an electron moving at the speed that you calculated in (b).
    (d) Explain why it is safe to assume that relativistic effects do not play a role in (b).

90. (a) Calculate the de Broglie wavelength of a 1000.0 kg car travelling at a speed of 20.0 m/s.
    (b) Is it possible to observe the wave-like properties of the car moving at this speed? (12.3) T/I

91. An electron microscope uses a beam of fast electrons focused by electric and magnetic fields to produce an enlarged image of a thin specimen on a screen or photographic plate. Determine the resolving power of an electron microscope that uses 15 keV electrons. Assume that this is equal to the electron wavelength. (12.3) T/I

92. A particle at rest of mass m decays into two particles of masses m_1 and m_2 with non-zero velocities. Calculate the ratio of the de Broglie wavelengths of the two particles. (12.3) T/I

93. Suppose that you annihilate 1.0 kg of matter and 1.0 kg of antimatter. Determine the total energy released. (12.6) T/I

94. Quarks are the building blocks of hadrons. (12.6) T/I
    (a) What is the total electric charge of two up quarks and one down quark?
    (b) Is this the same as the electric charge of a proton? Explain.

95. In neutron decay, a neutron changes into a proton, an electron, and an anti-electron neutrino. (12.6) T/I
(a) What is the electric charge of the neutron before the decay?
(b) What is the total charge of the three particles produced after the decay?
(c) The principle of conservation of charge states that the total charge before an interaction is the same as the total charge afterward. Does neutron decay obey this principle?

96. The decay of an atomic nucleus can produce particles with different kinetic energies. (12.6) K/U T/I A
(a) Can an electron produced in a highly energetic decay travel at the speed of light? Justify your answer.
(b) Can a photon produced in a decay travel at the speed of light? Justify your answer.

97. The introduction of new conceptual models and theories can influence scientific thought and lead to the development of new technologies. (12.1–12.6) K/U T/I C
(a) Create a graphic organizer showing the development of quantum theory based on the events and scientists mentioned in Chapter 12.
(b) How did Einstein's work on the photoelectric effect influence the development of quantum theory?
(c) Assess the importance of quantum mechanics to the development of various technologies.

Evaluation

98. In many science fiction stories, a character travels back in time to change the events of history. Does this example of time travel have a foundation in special relativity? Explain your answer. (11.2) K/U C A

99. The time dilation equation is $\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. (11.2) T/I C A
(a) When v gets closer to c, what can you say about the fraction on the right side of the equation?
(b) When v is greater than c, what can you say about the fraction on the right side of the equation?
(c) Evaluate what these mathematical properties indicate about the speed of light and the speed at which objects can travel in the universe.

100. At some time in the future, a person might attempt to retain his youth by travelling on a long space journey with a speed close to the speed of light. Eighty years might pass for people on Earth, but only eight years for the traveller. Assess the possibility of this scenario, and support your argument using principles of special relativity. (11.2) K/U C

101. Imagine a science fiction film where people travel in a spacecraft powered by annihilating matter and anti-matter that travel at speeds many times the speed of light. Assess which parts of this premise are allowed by the laws of physics and which are not. (11.4) K/U T/I A

102. (a) The metal sodium has a threshold frequency that corresponds to yellow light. In the case of the yellow light and sodium metal, describe what would happen in the following situations. (12.2) T/I C A
(i) You shine orange light on the sodium metal.
(ii) You shine very bright red light on the sodium metal.
(iii) You shine blue light on the sodium metal.
(b) Sketch a graph of the maximum kinetic energy of an electron emitted in the photoelectric effect versus the frequency of incident light for this metal.
(c) Evaluate the graph.

103. Show that when a positron and an electron both initially at rest are annihilated, creating two photons, both photons have the Compton wavelength $\lambda = \frac{h}{mc}$. (12.2) T/I A

104. After pair annihilation, two 1 MeV photons move off in opposite directions. Assess this situation, and determine the kinetic energy of the electron and the positron. (12.2) T/I A

105. Using an example, assess the reasons why the wave nature of matter is not observable in everyday situations. (12.3) T/I A

106. Identify which graph in **Figure 2** accurately represents the relationship between particle momentum and the associated de Broglie wavelength. (12.3) C A

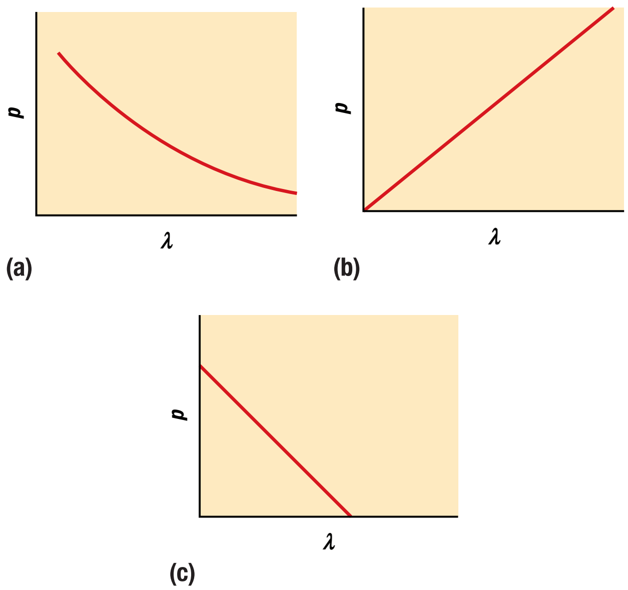


Figure 2

107. A hypothetical particle decay changes a proton into a neutron and an electron. (12.6) K/U T/I A
   (a) What is the total electric charge before the decay?
   (b) What is the total electric charge after the decay?
   (c) All decays have to satisfy the principle of conservation of charge. Assess whether this decay could actually occur.
108. The standard model answers the question, "What is an atom made of?" Describe what your answer to that question would be if you were a physicist in the year 1900. Hypothesize what your answer would be if you were a physicist in the year 2100. (12.6) K/U C A

Reflect on Your Learning

109. How has your understanding of relativity and atoms changed from studying this unit? K/U
110. Do you have a better understanding of quantum mechanics after completing this unit? Explain. K/U
111. In a graphic organizer, compare your understanding of relativity before you studied this unit with your new understanding of relativity. C
112. Was there any Sample Problem, equation, or illustration in this unit that you found particularly helpful? Explain the concept to a friend who has not taken physics. K/U
113. Is there a concept that you had difficulty with for which your understanding improved by the end of this unit? What did you do to improve your understanding? K/U

Research

WEB LINK

114. Research how special and general relativity concerns are involved with Global Positioning System (GPS) satellites. What are some predictions made by special and general relativity that are confirmed by a GPS? C A
115. In 2011, scientists at CERN, Switzerland, announced that they had accelerated neutrinos to faster than the speed of light. Research this experiment. Have the results been confirmed? What are the implications of this experiment if the results turn out to be true? K/U T/I A
116. Research space–time diagrams and answer the following questions. K/U C A
   (a) Who was Minkowski, and how did his work add to scientists' understanding of space and time?
   (b) What are some of the applications of space–time diagrams?
117. Research Max Planck (**Figure 3**). K/U C A

Figure 3

   (a) How did Planck start studying radiation?
   (b) What were some of the personal difficulties that Planck encountered?
   (c) What were some of his scientific discoveries before and after his quantum hypothesis?
118. The LHC particle accelerator at CERN contains four main particle experiments called ALICE, ATLAS, CMS, and LHCb. Research one of these experiments, and prepare a short report that analyzes the technology behind the experiment and summarizes the scientific questions that the experiment hopes to answer. K/U C A
119. Research the Higgs boson (**Figure 4**). K/U C A

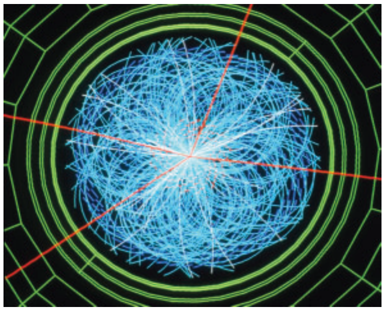


Figure 4 This image is a computer simulation of an event in which a Higgs boson decays and produces four muons (the red lines).

   (a) Why is the Higgs boson sometimes nicknamed the "god particle"?
   (b) What is the function of the Higgs boson in nature?
   (c) Explain how the whole science community is working together to investigate the Higgs boson.
120. Research the discovery of antimatter. In what year was antimatter predicted, and in what year was it discovered? Analyze the technology used to detect the first anti-particle. Describe a current use of antimatter-based technology in everyday society. K/U C A

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SKILLS HANDBOOK LINKS

Throughout this textbook you will see links to the Skills Handbook, where appropriate, in each activity. These links identify supporting material in the Skills Handbook that will help you with that activity.

A1 Safety

A1.1 Safety Conventions and Symbols

Although we make every effort to make the science experience a safe one, there are some inherent risks. These risks are generally associated with the materials and equipment used, and with disregard of safety instructions when conducting investigations. Most of these risks pose no more danger than we normally experience in everyday life. We can reduce these risks by doing the following: being aware of the possible hazards, knowing the rules, behaving appropriately, and using common sense.

Remember, you share the responsibility not only for your own safety, but also for the safety of those around you (**Figure 1**). Always alert your teacher in case of an accident. In this textbook, chemicals, equipment, and procedures that are hazardous are indicated by the appropriate Workplace Hazardous Materials Information System (WHMIS) symbol or by .

Figure 1 Taking the proper safety precautions will ensure a safe environment for you and your classmates.

WHMIS SYMBOLS AND HHPS

The Workplace Hazardous Materials Information System (WHMIS) provides workers and students with complete and accurate information about hazardous products. All chemical products supplied to schools, businesses, and industries must contain standardized labels and be accompanied by a Material Safety Data Sheet (MSDS). The MSDS provides detailed information about the product. Clear and standardized symbols are an important component of WHMIS (**Table 1**, page 680). These symbols must be present on the product's original container and shown on other containers if the product is transferred.

The *Canadian Hazardous Products Act* requires manufacturers of consumer products containing chemicals to include a symbol that specifies the nature of the hazard and whether it is the container or the contents that is dangerous. In addition, the label must state any secondary hazards, first-aid treatment, storage, and disposal. Household Hazardous Product Symbols (HHPS) are used to show the type of hazard. The shape of the frame around the symbol indicates whether the hazard is due to the contents or the container (**Figure 2**).

Symbol	Danger
	**Explosive** This container can explode if it is heated or punctured.
	**Corrosive** This product will burn skin or eyes on contact, or throat and stomach if swallowed.
	**Flammable** This product, or its fumes, will catch fire easily if exposed to heat, flames, or sparks.
	**Poisonous** Licking, eating, drinking, or sometimes smelling this product is likely to cause illness or death.

Figure 2 Household Hazardous Product Symbols (HHPS) appear on many products. A triangular frame indicates that the container is potentially dangerous. An octagonal frame indicates that the contents pose a hazard.

A1.2 Safety in the Laboratory

Safety in the laboratory is an attitude and a habit more than it is a set of rules. It is easier to prevent accidents than to deal with the consequences of an accident. Most of the following rules are common sense:

- Do not enter a laboratory unless a teacher or other supervisor is present, or you have permission to do so.
- Know your school's safety regulations.
- Tell your teacher about any allergies or medical problems you may have.
- Wear eye protection, a lab apron, and safety gloves when instructed by your teacher. Wear closed shoes.
- Tie back long hair and wear a protective lab coat over loose clothing. Remove any loose jewellery and finger rings.
- Keep yourself and your work area tidy and clean. Keep aisles clear.
- Never eat, drink, or chew gum in the laboratory.

(Section A1.2 continues on page 681.)

Table 1 The Workplace Hazardous Materials Information System (WHMIS)

Class and type of compounds	WHMIS symbol	Risks	Precautions
Class A: *Compressed Gas* Material that is normally gaseous and kept in a pressurized container		• could explode due to pressure • could explode if heated or dropped • possible hazard from both the force of explosion and the release of contents	• ensure container is always secured • store in designated areas • do not drop or allow to fall
Class B: *Flammable and Combustible Materials* Materials that will continue to burn after being exposed to a flame or another ignition source		• may ignite spontaneously • may release flammable products if allowed to degrade or when exposed to water	• store in designated areas • work in well-ventilated areas • avoid heating • avoid sparks and flames • ensure that electrical sources are safe
Class C: *Oxidizing Materials* Materials that can cause other materials to burn or support combustion		• can cause skin or eye burns • increase fire and explosion hazards • may cause combustibles to explode or react violently	• store away from combustibles • wear body, hand, face, and eye protection • store in container that will not rust or oxidize
Class D: *Toxic Materials Immediate and Severe* Poisons and potentially fatal materials that cause immediate and severe harm		• may be fatal if ingested or inhaled • may be absorbed through the skin • small volumes have a toxic effect	• avoid breathing dust and vapours • avoid contact with skin and eyes • wear protective clothing, and face and eye protection • work in well-ventilated areas and wear breathing protection
Class D: *Toxic Materials Long-Term Concealed* Materials that have a harmful effect after repeated exposures or over a long period		• may cause death or permanent injury • may cause birth defects or sterility • may cause cancer • may be sensitizers causing allergies	• wear appropriate personal protection • work in well-ventilated areas • store in appropriate designated areas • avoid direct contact • use hand, body, face, and eye protection • ensure respiratory and body protection is appropriate for the specific hazard
Class D: *Biohazardous Infectious Materials* Infectious agents or biological toxins that cause a serious disease or death		• may cause anaphylactic shock • include viruses, yeasts, moulds, bacteria, and parasites that affect humans • include fluids containing toxic products • include cellular components	• special training is required to handle materials • work in designated biological areas with appropriate engineering controls • avoid forming aerosols • avoid breathing vapours • avoid contamination of people and/or area • store in special designated areas
Class E: *Corrosive Materials* Materials that react with metals and living tissue		• eye and skin irritation on exposure • severe burns/tissue damage on longer exposure • lung damage if inhaled • may cause blindness if they contact eyes • environmental damage from fumes	• wear body, hand, face, and eye protection • use breathing apparatus • ensure protective equipment is appropriate • work in well-ventilated areas • avoid all direct body contact • use appropriate storage containers and ensure nonventing closures
Class F: *Dangerously Reactive Materials* Materials that may have unexpected reactions		• may react with water • may be chemically unstable • may explode if exposed to shock or heat • may release toxic or flammable vapours • may vigorously polymerize • may burn unexpectedly	• handle with care avoiding vibration, shocks, and sudden temperature changes • store in appropriate containers • ensure storage containers are sealed • store and work in designated areas

Section A1.2 continued

- Know the location of MSDS information, exits, and all safety equipment, such as the first-aid kit, fire blanket, fire extinguisher, and eyewash station, and be familiar with their contents and operation.
- Avoid moving suddenly or rapidly in the laboratory, especially near chemicals and sharp instruments.
- If you are not sure what to do, ask your teacher for directions.
- Never change anything or start an activity or investigation without your teacher's approval.
- Before you start an investigation that you have designed yourself, get your teacher's approval.
- Never attempt unauthorized experiments.
- Never work in a crowded area or alone in the laboratory.
- Always stand up when doing laboratory practical work. Do not sit down.
- Wash your hands with soap and warm water when you finish an investigation, and before you leave the laboratory.
- Use stands, clamps, and holders to secure any potentially dangerous or fragile equipment that could be tipped over.
- Do not taste any substance in a laboratory.
- Never smell chemicals unless specifically instructed to do so by your teacher. Do not inhale the vapours, or gas, directly from the container. Take a deep breath to fill your lungs with air, then waft or fan the vapours toward your nose.
- Report all accidents.
- Inform your teacher of any spills, and follow your teacher's instructions on how to clean up the spill. Clean up all spills, even water spills, immediately.
- Remember safety procedures when you leave the laboratory. Accidents can also occur outdoors, at home, and at work.

EYE, EAR, AND FACE SAFETY

- Always wear approved eye protection in a laboratory. Keep the safety glasses or goggles over your eyes, not on top of your head. For certain experiments, full face protection may be necessary.
- If you must wear contact lenses in the laboratory, be extra careful; whether or not you wear contact lenses, do not touch your eyes without first washing your hands. If you do wear contact lenses, make sure that your teacher is aware of it. Carry your lens case and a pair of glasses with you.
- If you wear prescription eyeglasses, you must still wear the appropriate eye protection on top of them.
- Do not stare directly at any bright source of light (for example, a burning magnesium ribbon, laser pointers, the Sun). You will not feel any pain if your retina is being damaged by intense radiation. You cannot rely on the sensation of pain to protect you.
- If a piece of glass or other foreign object enters your eye, seek immediate medical attention.
- Always wear ear protection when experimenting with loud sounds.

HANDLING GLASSWARE SAFELY

- Never use glassware that is broken, cracked, or chipped. Give such glassware to your teacher or dispose of it as directed. Do not put the item back into circulation.
- Never pick up broken glassware with your fingers. Use a broom and dustpan.
- Dispose of glass fragments in special containers marked "Broken Glass."
- Check with your teacher before heating any glassware. Heat glassware only if it is approved for heating.
- Be very careful when cleaning glassware. There is an increased risk of breakage from dropping when the glassware is wet and slippery.
- If you cut yourself, inform your teacher immediately and get appropriate first aid. Embedded glass or continued bleeding requires medical attention.

USING SHARP INSTRUMENTS SAFELY

- Make sure that your instruments are sharp. Dull cutting instruments require more pressure than sharp instruments and are, therefore, much more likely to slip.
- Select the appropriate instrument for the task. Never use a knife when scissors would work best.
- Always cut away from yourself and others.
- If you cut yourself, inform your teacher immediately and get appropriate first aid.

HEAT SAFETY

- Make sure that heating equipment, such as the burner, hot plate, or electric heater, is secure on the bench and clamped in place when necessary.
- Do not use a laboratory burner near wooden shelves, flammable liquids, or any other item that is combustible.
- Do not allow overheating if you are performing an experiment in a closed area. For example, if you are using a light source in a large cardboard box, be sure you have enough holes at the top of the box and on the sides to dissipate thermal energy.
- Always assume that hot plates and electric heaters are hot, and use protective gloves when handling.

- Do not touch a light source that has been on for some time. It may be hot and cause burns.
- In a laboratory where burners or hot plates are being used, never pick up a glass object without first checking the temperature by placing your hand near but not touching it. Glass and metal items that have been heated may not appear to be hot, but can cause burns.
- If you burn yourself, *immediately* run cold water gently over the burned area or immerse the burned area in cold water and inform your teacher.
- Never look down the barrel of a laboratory burner.
- Always pick up a burner by its base, never by its barrel.
- Never leave a lighted burner unattended.
- Any metal powder can be explosive. Do not put these in a flame.
- To heat a beaker, put it on the hot plate and secure with a ring support attached to a utility stand. (Placing a wire gauze under the beaker is optional.)
- Remember to include a cooling time in your experiment plan; do not put away hot equipment.

FIRE SAFETY

- Immediately inform your teacher of any fires. A very small fire in a container may be extinguished by covering the container with a wet paper towel or a ceramic square to cut off the supply of air. Alternatively, sand may be used to smother small fires. A bucket of sand with a scoop should be available in the laboratory.
- If anyone's clothes or hair catch fire, tell the person to drop to the floor and roll. Then use a fire blanket to smother the flames. Never wrap the blanket around a standing person on fire.
- For larger fires, immediately evacuate the area. Call the office or sound the fire alarm. Do not try to extinguish larger fires. As you leave the classroom, make sure that the windows and doors are closed.
- If you use a fire extinguisher, direct the extinguisher at the base of the fire and use a sweeping motion, moving the extinguisher nozzle back and forth across the front of the fire's base.

ELECTRICAL SAFETY

- Do not operate electrical equipment near running water or a large container of water. Water or wet hands should never be near electrical equipment such as a hot plate, a light source, or a microscope.
- Check the condition of electrical equipment. Do not use it if wires or plugs are damaged, or if the ground pin has been removed.
- If using a light source, check that the wires of the light fixture are not frayed, and that the bulb socket is in good shape and well secured to a stand.
- Make sure that electrical cords are not placed where someone could trip over them.
- When unplugging equipment, remove the plug gently from the socket. Do not pull on the cord.
- When using variable power supplies, start at low voltage and increase slowly.

WASTE DISPOSAL

Waste disposal at school, at home, and at work is a societal issue. Most laboratory waste can be washed down the drain or, if it is in solid form, placed in ordinary garbage containers. However, some waste must be treated more carefully. It is your responsibility to follow procedures and to dispose of waste in the safest possible manner according to your teacher's instructions.

FIRST AID

The following guidelines apply in case of an injury, such as a burn, cut, chemical spill, ingestion, inhalation, or splash in the eyes:

- Always inform your teacher immediately of any injury.
- If the injury is a minor cut or abrasion, wash the area thoroughly. Using a compress (for example, clean paper towels), apply pressure to the cut to stop the bleeding. When bleeding has stopped, replace the compress with a sterile bandage. If the cut is serious, apply pressure and seek medical attention immediately.
- If you get a solution in your eye, quickly use the eyewash or nearest running water. Continue to rinse the eye with water for at least 15 min. Unless you have a plumbed eyewash system, you will also need assistance in refilling the eyewash container. Have another student inform your teacher of the accident. The injured eye should be examined by a doctor.
- If the injury is a burn, immediately immerse the affected area in cold water, or run cold water gently over the burned area. This will reduce the temperature and prevent further tissue damage.
- In case of electric shock, unplug the appliance and do not touch it or the victim. Inform your teacher immediately.
- If a classmate's injury has rendered him/her unconscious, notify your teacher immediately. Your teacher will perform CPR if necessary. Do not administer CPR unless under specific instructions from your teacher. Call the school office and request emergency medical help.

A2 Scientific Inquiry

In our attempts to further our understanding of the natural world, we encounter questions, mysteries, or events that are not readily explainable. To develop explanations, we investigate using scientific inquiry. An important aspect of scientific inquiry is that science is only one of many ways people explore, explain, and come to know the world around them. Scientific inquiry is a multifaceted process that involves the following: identifying questions that can be answered through scientific investigations; using appropriate tools and techniques to gather, analyze, and interpret data; developing descriptions, explanations, predictions, and models using evidence; thinking critically and logically to make the relationships between evidence and explanations; recognizing and analyzing alternative explanations and predictions; and communicating scientific procedures and explanations.

The methods used in scientific inquiry depend, to a large degree, on the purpose of the inquiry. There are four common types of scientific inquiry: the controlled experiment, the correlational study, the observational study, and the activity. These types of scientific inquiry require specific skills. The skills are discussed below, followed by a detailed description of how they relate to each of the four types of scientific inquiry.

A2.1 Skills of Scientific Inquiry

Scientific inquiry requires certain skills that are important in the process of conducting an investigation. These skills can be organized into four categories: initiating and planning, performing and recording, analyzing and interpreting, and communicating.

INITIATING AND PLANNING

1. Questioning: Most scientific investigations begin with a question. It is important to ask the right questions. In certain types of scientific inquiry, the question must be testable. This means that it must ask about a possible cause-and-effect relationship. A cause-and-effect relationship is one in which a change in one variable (see #3) causes a change in another variable. A testable question might start in one of the following ways: What causes . . . ? How does . . . affect . . . ? If . . . , what happens to . . . ?
2. Researching: This is a skill that occurs across all four categories of scientific inquiry and includes preparing for research, accessing resources, processing information, and transferring learning. The process involves identifying the type of information that is required, using strategies to locate and access the information, recording the information, synthesizing findings, and formulating conclusions.
3. Identifying variables: Considering the variables involved in an investigation is an important step in designing an effective investigation. Variables are any factors that could affect the outcome of an investigation. There are three kinds of variables in a controlled experiment: the manipulated variable, the responding variable, and the controlled variables.
   - The manipulated variable (also known as the independent variable or cause variable) is the variable that is deliberately changed by the investigator.
   - The responding variable (also known as the dependent variable or effect variable) is the variable that the investigator believes will be affected by a change in the manipulated variable.
   - The controlled variables are variables that may affect the responding variable, but that are held constant so that they cannot affect the responding variable. A controlled experiment is a test of whether (and how) a manipulated variable affects a responding variable. To make the test fair, all other variables that may affect the responding variable are kept constant (unchanging).
4. Hypothesizing: A hypothesis is a predicted answer to the testable question. It proposes a possible explanation based on an already known scientific theory, law, or other generalization.

   A hypothesis may be written in the form of an "If . . . , then . . . because . . ." statement. If the manipulated (independent) variable is changed in a particular way, then we predict that the responding (dependent) variable will change in a particular way, and we provide a theoretical explanation for the prediction. You may create more than one hypothesis from the same testable question. For example,

   If an air-filled balloon is placed in a freezer and its temperature is decreased, then its volume will decrease because, according to the kinetic molecular theory, atoms and molecules slow down and occupy less space at lower temperatures.

   When you conduct an investigation, your observations do not always support the prediction in your hypothesis. When this happens, you may re-evaluate and modify your hypothesis and design a new experiment.
5. Planning: Planning an inquiry activity involves developing an experimental design, identifying variables, selecting necessary equipment and materials, addressing safety concerns, and writing a step-by-step procedure.

PERFORMING AND RECORDING

1. Conducting inquiry: As you perform an investigation, follow the steps in the procedure carefully and thoroughly. Check with your teacher if you find that you need to make significant alterations to your procedure. Use all equipment and materials safely, appropriately, and with precision.
2. Making observations: When you conduct an investigation, you should make accurate observations at regular intervals and record them carefully and accurately. Record exactly what you observe. Observations from an experiment may not always be what you expect them to be. Qualitative (descriptive) and quantitative (measured) observations may be made during an investigation. Some observations may also be provided for you during an investigation.

   Quantitative observations are based on measured quantities, such as temperature, volume, and mass. They are usually recorded in data tables.

   Qualitative observations describe characteristics that cannot be expressed in numbers, such as texture, smell, and taste. They can be recorded using words, pictures, tables, or labelled diagrams.
3. Collecting, organizing, and recording data: During an investigation you should collect and record all data and observations, and organize these into formats that are easily interpreted (such as tables and charts).

ANALYZING AND INTERPRETING

1. Analyzing: Analyzing involves looking for patterns and relationships that will help you explain your results and give you new information about the question you are investigating. Your analysis will tell you whether your observations support your hypothesis.
2. Evaluating: It is very important to evaluate the evidence that is obtained through observations and analysis. When evaluating the results of an investigation, here are some aspects you should consider:
   - Experimental design: Were there any problems with the way you planned your experiment? Did you control all the variables except for the manipulated variable?
   - Equipment and materials: Was the equipment adequate? Would other equipment have been better? Was something used incorrectly? Did you have difficulty with a piece of equipment?
   - Skills: Did you have the appropriate skills for the investigation? Did you have to use a skill that you were just beginning to learn?
   - Observations: Did you accurately record all the relevant observations?

COMMUNICATING

1. It is important to share both your process and your results. Other people may want to repeat your investigation, or they may want to use or apply your results in another situation. Your write-up or report should reflect the process of scientific inquiry that you used in your investigation.
2. At this stage, you should be prepared to extend insights and opinions from your findings, suggest areas for further investigation, and relate research findings to the world around you.

In the following sections, we will detail the components of the four types of investigations: controlled experiments, correlational studies, observational studies, and activities.

A2.2 Controlled Experiments

A controlled experiment is an example of scientific inquiry in which a manipulated variable is intentionally changed to determine its effect on a responding variable. All other variables are controlled (kept constant). Controlled experiments are performed when the purpose of the inquiry is to create, test, or use a scientific concept.

The common components of controlled experiments are outlined below. Note that there are normally many cycles through the steps during an actual experiment.

TESTABLE QUESTION

A testable question forms the basis for your controlled experiment. The investigation is designed to answer the question. Controlled experiments are about relationships among variables, so your question could be about the effects on variable A when variable B is changed.

VARIABLES

The primary purpose of a controlled experiment is to determine whether a change in a manipulated variable causes a noticeable change in a responding variable while all other variables remain constant. Therefore, you must identify all major variables that you will measure and/or control in your investigation. What is the manipulated (independent) variable? What is the responding (dependent) variable? What are the controlled variables?

When conducting a controlled experiment, change only one manipulated variable at a time, holding all the others (except the responding variable) constant. This way, you can assume that the results are caused by the manipulated variable and not by any of the other variables.

HYPOTHESIS/PREDICTION

When formulating a hypothesis, first read the testable question, the experimental design, and the procedure, if provided. Then, try to identify (and distinguish) the manipulated variable, the responding variable, and the controlled variables. Your hypothesis will be a predicted answer to the testable question accompanied by a theoretical explanation for your prediction.

EXPERIMENTAL DESIGN

The design of a controlled experiment shows how you plan to answer your question. The design outlines how you will change the manipulated variable, measure any variations in the responding variable, and control all the other variables. It is a summary of your plan for the experiment.

EQUIPMENT AND MATERIALS

Make a detailed list of all equipment and materials you will use, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection, lab apron, protective gloves, and tongs, where needed. Draw a diagram to show any complicated setup of apparatus.

PROCEDURE

Write a step-by-step description of how you will perform your investigation. It must be clear enough for someone else to follow, and it must explain how you will deal with each of the variables in your investigation. The first step in a procedure usually refers to any safety precautions that need to be addressed, and the last step relates to any cleanup that needs to be done.

OBSERVATIONS

There are many ways you can gather and record observations during your investigation. It is helpful to plan ahead and think about what data you will need to answer the question and how best to record them (for example, data tables, pictures, or labelled diagrams may be helpful). This helps to clarify your thinking about the question posed at the beginning, the variables, the number of trials, the procedure, the materials, and your skills. It will also help you organize your evidence for easier analysis.

ANALYZE AND EVALUATE

You will need to analyze and interpret your observations—this may include graphing your data and analyzing any patterns or trends that may be evident in your graphs. After thoroughly analyzing your observations, you may have sufficient and appropriate evidence to enable you to answer the testable question posed at the beginning of the investigation.

You must evaluate the processes that you followed to plan and perform the investigation. You will also evaluate the outcome of the investigation, which involves evaluating your hypothesis/prediction. You must identify and take into account any sources of error and uncertainty in your measurements.

Finally, compare your hypothesis/prediction with the evidence. Is your hypothesis supported by the evidence?

APPLY AND EXTEND

Reflect on how your investigation relates to the world around you: can you use what you have learned in everyday life?

REPORTING ON THE INVESTIGATION

Your lab report should describe your planning process and procedure clearly and in enough detail that the reader could repeat the experiment exactly as you performed it. You should present your observations, your analysis, and your evaluation of your experiment clearly, accurately, and honestly.

A2.3 Correlational Studies

When the purpose of scientific inquiry is to test a suspected relationship between two different variables, but a controlled experiment is not possible, a correlational study is conducted. In a correlational study, the investigator tries to determine whether one variable is affecting another without purposely changing or controlling any of the variables. Instead, variables are allowed to change naturally. It is often difficult to isolate cause and effect in correlational studies. A correlational study requires very large sample numbers and many replications to increase the certainty of the results.

A correlational study does not require experiments or fieldwork; for example, the investigator can use databases prepared by other researchers to find relationships between two or more variables. The investigator can, however, choose to make observations and measurements as well, through fieldwork, interviews, and surveys.

A hypothesis or prediction is not useful in a correlational study. Correlational studies are not intended to establish cause-and-effect relationships. However, the results of a correlational study can be used to formulate a hypothesis about the causal relationship between the variables.

The common components of a correlational study are outlined below. Even though the sequence is presented as linear, there are normally many cycles through the steps during an actual study.

PURPOSE

When planning a correlational study, it is important that you pose a question about a possible statistical relationship between variable A and variable B. Choose a topic that interests you. Determine whether you are going to replicate or revise a previous study, or create a new one. Indicate your decision in the statement of the purpose.

VARIABLES

In a correlational study you must determine whether two variables are related, without controlling any of the variables. You must identify all the major variables that will be measured and/or observed in your investigation.

STUDY DESIGN

When designing your correlational study you must identify the setting and the methods you will use in carrying out your investigation. You should describe the type(s) of data you plan to collect and its sources. Your design should address questions such as the following: Will you be conducting a survey? If so, where will the survey be conducted? Who will answer your questionnaire? When will the survey be conducted? How often will the survey be administered? If you are obtaining information from an existing database, then describe the source of the information and your plans for analyzing the information.

EQUIPMENT AND MATERIALS

Make a detailed list of all equipment and materials used, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection and ear protection, where needed. Draw a diagram to show any complicated setup of apparatus.

PROCEDURE

Write a step-by-step description identifying how you will gather data on the variables under study. You will also need to identify potential sources of data. There are two possible sources: observations made by you, the investigator; and existing data (databases, etc.).

OBSERVATIONS

If you are collecting your own data through observation, then you will need to plan ahead and think about what data you will need and how best to record them. There are many ways to gather and record your observations (such as data tables, pictures, or labelled diagrams). This is an important step because it helps to clarify your thinking about the question posed at the beginning, the variables, the procedure, the materials, and your skills. It will also help you organize your observations for easier analysis.

ANALYZE AND EVALUATE

You will need to analyze and interpret your observations or sourced data—this will usually include graphing the data and analyzing any patterns or trends that may be evident in your graphs. You will need to identify the relationship between your two variables. A positive correlation indicates a direct relationship between variables: an increase in one variable corresponds to an increase in another. A negative correlation indicates an inverse relationship: an increase in one variable corresponds to a decrease in the other variable. If there is no relationship between the variables, then there is no correlation.

After thoroughly analyzing your observations, you may have sufficient and appropriate evidence to enable you to answer the question you posed at the beginning of the investigation. Was there a relationship between variable A and variable B?

Evaluate the processes that you followed to plan and perform the investigation. Also evaluate the outcome of the investigation, which involves evaluating any prediction you made at the beginning of the investigation. You must identify and take into account any sources of error and uncertainty in your measurements.

APPLY AND EXTEND

Reflect on how your investigation relates to the world around you: how can you use what you have learned in your everyday life?

REPORTING ON THE INVESTIGATION

In preparing your report, your objectives should be to describe your design and procedure accurately and to report your observations, analyses, and evaluations accurately and honestly.

A2.4 Observational Studies

Often, the purpose of an inquiry is simply to study a natural phenomenon with the intention of gaining scientifically significant information to answer a question. Observational studies involve observing a subject or phenomenon in an unobtrusive or unstructured manner, usually with no specific hypothesis. The inquiry does not start off with a hypothesis, but a hypothesis may be generated as information is collected.

The stages and processes of scientific inquiry through observational studies are summarized below. Even though the sequence is presented as linear, there are normally many cycles through the steps during an actual study.

PURPOSE

When planning an observational study, it is important to pose a general question about the natural world. Choose a topic that interests you. Determine whether you are going to replicate or revise a previous study, or create a new one. Indicate your decision in a statement of the purpose.

EQUIPMENT AND MATERIALS

Make a detailed list of all equipment and materials used, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection, lab apron, protective gloves, and tongs, where needed. Draw a diagram to show any complicated setup of apparatus.

PROCEDURE

Write a step-by-step description of how you will make your observations. It must be clear enough for someone else to follow. The first step in a procedure usually refers to any safety precautions that need to be addressed, and the last step relates to any cleanup that needs to be done.

OBSERVATIONS

There are many ways that you can gather and record observations—including quantitative observations—during an observational study. All observations should be objective and unambiguous. Consider ways to organize your information for easier analysis.

ANALYZE AND EVALUATE

After thoroughly analyzing your observations, you may have sufficient and appropriate evidence to enable you to answer the question posed at the beginning of the investigation. You may also have enough observations and information to form a hypothesis for a controlled experiment.

At this stage of the investigation, you will evaluate the processes used to plan and perform the investigation. Evaluating the processes includes evaluating the materials, the design, the procedure, and your skills. The results of most such investigations will suggest further studies, perhaps controlled experiments or correlational studies, to explore tentative hypotheses you may have developed.

APPLY AND EXTEND

At this stage you should reflect on how your investigation relates to the world around you: how can you use what you have learned in your everyday life?

REPORTING ON THE INVESTIGATION

In your report, describe your design and procedure accurately, and report your observations accurately and honestly.

A2.5 Activities

An activity is a type of scientific inquiry that provides opportunities for you to demonstrate your knowledge and understanding of a specific concept. An activity might involve constructing a device, reasoning your way through a problem (a thought experiment), or using given information to complete a task. You might be asked to consider creative ways to present the concept, such as a performance, a digital platform, or a written report. The materials that are required for different activities can vary greatly, so materials may need special planning and attention.

Activities do not have structured stages and processes. However, to demonstrate a thorough understanding of the concept, you will need to communicate the key points. You will also need to analyze and evaluate your presentation, which may include looking at how your understanding of the concept has grown and applying what you have learned to other areas.

A2.6 Lab Reports

When carrying out investigations, it is important that scientists keep records of their plans and results, and share their findings. In order to have their investigations repeated (replicated) and accepted by the scientific community, scientists generally share their work by publishing reports in which details of their design, materials, procedure, evidence, analysis, and evaluation are provided.

Lab reports are prepared after an investigation is completed. To ensure that you can accurately describe the investigation, keep thorough and accurate records of your activities as you carry out the investigation. Your lab book or report should reflect the type of scientific inquiry that you used in the investigation (controlled experiment, correlational study, observational study, or activity) and should be based on the following headings, as appropriate:

TITLE

At the beginning of your report, write the number and title of your investigation. In this book, the title is always given, but if you are designing your own investigation, create a title that suggests what the investigation is about. Include the date the investigation was conducted and the names of all lab partners (if you worked as a team).

PURPOSE

State the purpose of the investigation. Why are you doing this investigation?

TESTABLE QUESTION

State the question that you attempted to answer in the investigation. Sometimes the question is provided for you; other times you are expected to formulate your own. If it is appropriate to do so, state the question in terms of manipulated and responding variables.

HYPOTHESIS/PREDICTION

For a controlled experiment you will usually have to compose a hypothesis or prediction. This will be a proposed answer to your testable question. When writing a hypothesis include both a prediction and a reason for the prediction, based on scientific theory, law, or other generalization. You may use the "If . . . , then . . . because . . ." form. A simple prediction may be written in the "If . . . , then . . ." form.

VARIABLES

Identify all major variables that you measured and/or controlled in the investigation. What is the manipulated variable? What is the responding variable? What are the major controlled variables?

EXPERIMENTAL DESIGN

Provide a brief general overview (one to three sentences) of what you did in your investigation. If your investigation involved manipulated, responding, and controlled variables, list them and indicate how they were changed, measured, or held constant. Identify any control or control group that was used in the investigation.

EQUIPMENT AND MATERIALS

Include a detailed list of all the equipment and materials that you used, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection, lab apron, protective gloves, and tongs, where needed. Draw a diagram to show any complicated setup of apparatus.

PROCEDURE

Describe, in detailed, numbered steps, the procedure you followed to carry out your investigation. Your teacher may specify which style you should use. Examples of three common writing styles are

1. third person past tense ("The test tubes were heated . . .")
2. first person plural past tense ("We heated the test tubes . . .")
3. second person imperative ("Heat the test tubes . . .")

Include steps to clean up and dispose of waste and all safety considerations.

OBSERVATIONS

Present your observations in a form that is easily understood. This includes all the qualitative and quantitative observations that you made. Be as precise as possible when describing quantitative observations. Include any unexpected observations and present your information clearly. If you have only a few observations, this could be a list; for controlled experiments and for many observations, a data table, labelled diagram, or written description would be more appropriate.

ANALYSIS

Complete the questions found in the Analyze and Evaluate section of the investigation. These questions will prompt you to analyze and interpret your observations, answer a testable question, draw conclusions, and evaluate both your experiment and your conclusions. You will also be prompted to graph your data and analyze these graphs where applicable.

If you are writing up an investigation for which there are no questions, write your own analysis. Interpret your observations and present the evidence in the form of titled tables, graphs, or illustrations, as appropriate. Include any calculations, the results of which can be shown in a table. Make statements about any patterns or trends you observed. Conclude the analysis with a statement based only on the evidence you have gathered, answering the question that initiated the investigation.

EVALUATION

The evaluation is your judgment about the quality of evidence obtained and about the validity of the prediction and hypothesis (if present). This section can be divided into two parts:

1. Evaluation of the experiment: Did your experiment provide reliable and valid evidence to enable you to answer the question? Consider the experimental design, the procedure, and your laboratory skills. Were they all adequate? Are you confident enough in the evidence to use it to evaluate any prediction and/or hypothesis you made?
2. Evaluation of the prediction: Was the prediction you made before the investigation supported or falsified by the evidence? Based on your evaluation of the evidence and prediction, is the hypothesis you used to make your prediction supported, or should it be rejected?

APPLY AND EXTEND

Answer any Apply and Extend questions in the investigation. Number your answers as they appear in the Apply and Extend section in the textbook.

A3 Scientific Publications

Communicating in Science

Advances in science and our understanding of the natural world are the result of scientists sharing ideas and information. Watson and Crick won the Nobel Prize for discovering the double helix structure of DNA, but they solved this structure with the help of X-ray photographs taken by Rosalind Franklin during her own research.

It is important for scientists to share ideas and research with other scientists. New research findings are shared at conferences and in journal publications. Research scientists take pride in being published. Being published confirms that the research adds to the knowledge base of the scientific community. Sharing information helps to spread knowledge, solve problems, and inspire other scientists.

The Scientific Journal

Scientific journals are publications that are used to present new research. There are thousands of different science journals published worldwide, and in many languages. A scientific journal may be specific to a subject (for example, *Canadian Journal of Physics*) or contain articles that cover a variety of subjects within a field (for example, *Nature*). These publications may be electronic (online) or in print and may be published weekly, monthly, bimonthly, or quarterly.

Peer Review

When an article is submitted to a journal, the research findings are critically reviewed by experts in the topic. This ensures that the research presents ideas that are supported by practices of good science. High-quality evidence and appropriate conclusions are necessary for the article to be accepted for publication. An article that is submitted to a reputable journal may take months to be approved for publication. The article may be returned to the author(s) for revision if necessary.

Scientists aim to be published in the most respected journals. The peer-review process contributes to the reputation of a journal. It helps to maintain standards and provide credibility. A scientific journal becomes reputable by ensuring that only articles of high quality are published. Very prestigious journals (such as *Nature, Science*, and *Physics Review Letters*) are known for publishing research backed by only the best practices of science. Reputable journals are widely read and considered reliable by the scientific community. Publication in a reputable journal brings immediate recognition for the author(s). Research that is not published in a peer-reviewed journal is often overlooked.

Format of Research Articles

Research articles have specific sections. Articles often include an abstract, introduction, methods, results, discussion, conclusions, and references. Go to the Nelson Science website to see an example of a real research article with a description of each section. WEB LINK

THE ABSTRACT

The abstract is a short summary of the article. It presents the purpose of the research, outlines the design of the experiment and methods used, and summarizes findings or conclusions. A well-written abstract is useful when looking for articles with specific information. Time may be saved by reading the abstract and then deciding if the article is going to be helpful in supporting research. The background material, methods used, results, main subject, and discussion are all summarized within the abstract.

CITING SOURCES AND GIVING CREDIT

Once a scientist has found useful sources and included them in an article or paper, information must be provided about the article so that someone else who is interested in learning more will be able to find it. This also shows the reader that information supporting the research is current.

More importantly, citing another scientist's work provides a measure of value of what he or she has published. It gives credit to the scientist. More citations often mean that the work is worthwhile and influential. An example of this is the frequency of citation of Kary Mullis's work that presented the polymerase chain reaction (PCR). Thousands of citations occurred within a few years of publication, confirming that his work was influential.

Further Reading

Day, R.A., & Gastel, B. (2006). *How to write and publish a scientific paper* (6th ed.). Cambridge: Cambridge University Press.

Frame, P.F., Hik, D.S., Cluff, H.D., Paquet, P.C. (June 2004). Long foraging movement of a denning tundra wolf. *Arctic*, 57 (2), 196–203.

WEB LINK

A4 Exploring Issues and Applications

Throughout this textbook you will have many opportunities to examine the connections between science, technology, society, and the environment (STSE) by exploring issues and applications.

An issue is a situation in which several points of view need to be considered in order to make a decision. There can be many positions on an issue, generally determined by the values that an individual or a society holds. Which solution is "best" is a matter of opinion; ideally, the solution that is implemented is the one that is most appropriate for society as a whole. Researching information about an issue will help you make an educated decision about it. All the skills listed in Section A4.1 may be useful in an activity that involves exploring an issue.

Scientific research produces knowledge or understanding of natural phenomena. Technologists and engineers look for ways to apply this knowledge in the development of practical products and processes. Technological inventions and innovations can have wide-ranging applications for, and impacts on, society and the environment. The purpose of exploring an application is to research a particular technological invention or innovation to determine how it works, how it is used, and how it may affect society and the environment. The skills of researching, communicating, and evaluating may be useful in an activity that involves exploring an application.

A4.1 Research Skills

The following skills are involved in many types of research. Some of these skills will help you research issues only, while some will help you research issues and applications. Refer to this section when you have questions about any of the following skills and processes.

DEFINING THE ISSUE

When exploring an issue, the first step in understanding the issue is to explain why it exists, the problems associated with it, and, if applicable, the individuals or groups, also known as stakeholders, that are involved in it. The issue includes information about the role a person takes when thinking about an issue as well as a description of who your audience will be. You could brainstorm questions involving Who? What? Where? When? Why? and How? Develop background information on the issue by clarifying facts and concepts, and identifying relevant attributes, features, or characteristics of the problem.

RESEARCHING

When beginning your research for both issues and applications, you need to formulate a research question that helps to limit, narrow, or define the scope of your research. You then need to develop a plan to find reliable and relevant sources of information. This includes outlining the stages of your research: gathering, sorting, evaluating, selecting, and integrating relevant information. You should gather information from a variety of sources if possible (for example, print, web, and personal interviews).

As you collect information, do your best to ensure that the information is reliable, accurate, and current. Avoid biased opinions, opinions that are not supported by, or that ignore, credible evidence. It is important to ensure that the information you have gathered addresses all aspects of the issue or application you are researching.

IDENTIFYING ALTERNATIVES

When exploring an issue, examine the situation and think of as many alternative solutions as you can. Be creative about combining the solutions. At this point, it does not matter whether or not the solutions seem unrealistic. To analyze the alternatives, you should examine the issue from a variety of perspectives. Stakeholders may bring different viewpoints to an issue, which may influence their position on the issue. Brainstorm or hypothesize how different stakeholders would feel about your alternatives.

ANALYZING THE ISSUE

An important part of exploring an issue is analyzing the issue. First, you should establish criteria for evaluating your information to determine its relevance and significance. You can then evaluate your sources, determine what assumptions may have been made, and assess whether you have enough information to make your decision.

To analyze an issue effectively,

- establish criteria for determining the relevance and significance of the data you have gathered
- evaluate the sources of information
- identify and determine what assumptions have been made
- challenge unsupported evidence
- evaluate the alternative solutions, possibly by conducting a risk–benefit analysis

Once the issue has been analyzed, you can begin to evaluate the alternative solutions. You may decide to carry out a risk–benefit analysis—a tool that enables you to look at each possible result of a proposed action and helps you make a decision. (See Section A4.2 for more information on risk–benefit analysis.)

DEFENDING A DECISION

After analyzing your information on your issue, you can answer your research question and take an informed position or draw a conclusion on the issue. If you are working as a group, this is the stage where everyone gets a chance to share ideas and information gathered about the issue. Then the group needs to evaluate all the possible alternatives and decide on their preferred solution based on the criteria.

Your position on the issue or conclusion must be justified using supporting information that you have researched. You should be able to defend your position to people with different perspectives. Ask yourself the following questions:

- Do I have supporting evidence from a variety of sources?
- Can I state my position clearly?
- Can I show why this issue is relevant and important to society?
- Do I have solid arguments (with solid evidence) supporting my position?
- Have I considered arguments against my position, and identified their faults?
- Have I analyzed the strong and weak points of each perspective?

COMMUNICATING

When exploring an issue, there are several things to consider when communicating your decision. You need to state your position clearly and take into consideration who your audience is. You should always support your decision with objective data and a persuasive argument if possible. Be prepared to defend your position against any opposition.

You should be able to defend your solution in an appropriate format—debate, class discussion, speech, position paper, multimedia presentation, brochure, poster, video, etc.

When exploring an application you should communicate the "need or want" for the application (why the application was developed in the first place), the "how" (how the application/technology actually works), and the risks and benefits to society, individuals, and the environment. You should conclude with your "assessment" of the application.

EVALUATING

The final phase of your decision making when exploring an issue includes evaluating the decision itself and the process used to reach the decision. After you have made a decision, carefully examine the thinking that led to your decision.

Some questions to guide your evaluation include:

- What was my initial perspective on the issue? How has my perspective changed since I first began to explore the issue?
- How did we make our decision? What process did we use? What steps did we follow?
- To what extent were my arguments factually accurate and persuasively made?
- In what ways does our decision resolve the issue?
- What are the likely short- and long-term effects of the decision?
- To what extent am I satisfied with the final decision?
- What reasons would I give to explain our decision?
- If we had to make this decision again, what would I do differently?

A4.2 Risk–Benefit Analysis Model

Risk-benefit analysis is a tool used to organize and analyze information gathered in research, especially when exploring a socio-scientific issue. A thorough analysis of the risks and benefits associated with each alternative solution can help you decide on the best alternative.

- Research as many aspects of the situation as possible. Look at it from different perspectives.
- Collect as much evidence as you can, including reasonable projections of likely outcomes if the proposal is adopted.
- Classify every individual potential result as being either a benefit or a risk.
- Quantify the size of the potential benefit or risk (perhaps as a dollar figure, or a number of lives affected, or on a scale of 1 to 5).
- Estimate the probability (percentage) of that event occurring.
- By multiplying the size of a benefit (or risk) by the probability of its happening, you can calculate a probability value for each potential result.
- Total the probability values of all the potential risks, and all the potential benefits.
- Compare the sums to help you decide whether to accept the proposed action.

A5 Math Skills

A5.1 Scientific Notation

It is difficult to work with very large or very small numbers when they are written in common decimal notation. Usually it is possible to accommodate such numbers by changing the SI prefix so that the number falls between 0.1 and 1000. For example, 237 000 000 mm can be expressed as 237 km, and 0.000 000 895 kg can be expressed as 0.895 mg. However, this prefix change is not always possible, either because an appropriate prefix does not exist or because it is essential to use a particular unit of measurement in a calculation. In these cases, the best method of dealing with very large and very small numbers is to write them using scientific notation. Scientific notation expresses a number by writing it in the form $a \times 10^n$, where $1 \leq |a| < 10$ and the digits in the coefficient a are all significant. **Table 1** shows situations where scientific notation would be used.

Table 1 Examples of Scientific Notation

Expression	Common decimal notation	Scientific notation
"124.5 million kilometres"	124 500 000 km	1.245×10^8 km
"154 thousand picometres"	154 000 pm	1.54×10^5 pm
"602 sextillion molecules"	602 000 000 000 000 000 000 000 molecules	6.02×10^{23} molecules

To multiply numbers in scientific notation, multiply the coefficients and add the exponents. To divide numbers in scientific notation, divide the coefficients and subtract the exponents. The answer is always expressed in scientific notation. Note that the coefficient should always be between 1 and 10. For example,

$$\begin{aligned}(4.73 \times 10^5\ \text{m})(5.82 \times 10^7\ \text{m}) &= 27.5 \times 10^{12}\ \text{m}^2 \\ &= 2.75 \times 10^{13}\ \text{m}^2\end{aligned}$$

$$\frac{(6.4 \times 10^6\ \text{m})}{(2.2 \times 10^3\ \text{s})} = 2.9 \times 10^3\ \text{m/s}$$

When evaluating exponents, the following rules apply:

$$x^a \cdot x^b = x^{a+b} \qquad (xy)^b = x^b y^b$$

$$\frac{x^a}{x^b} = x^{a-b} \qquad \left(\frac{x}{y}\right)^b = \frac{x^b}{y^b}$$

$$(x^a)^b = x^{ab}$$

SCIENTIFIC NOTATION WITH CALCULATORS

On many calculators, scientific notation is entered using a special key, labelled EXP or EE. This key includes "× 10" from the scientific notation; you need to enter only the exponent. For example, to enter

7.5×10^4 press 7.5 EXP 4

3.6×10^{-3} press 3.6 EXP +/− 3

Depending on the type of calculator you have, +/− may need to be entered after the relevant number.

A5.2 Uncertainty in Measurements

There are two types of quantities that are used in science: exact values and measurements. Exact values include defined quantities (1 m = 100 cm) and counted values (5 beakers or 10 trials). Measurements, however, are not exact because there is some uncertainty or error associated with every measurement.

SIGNIFICANT DIGITS

The certainty of any measurement is communicated by the number of significant digits in the measurement. In a measured or calculated value, significant digits are the digits that are known reliably, or for certain, and include the last digit that is estimated or uncertain. Significant digits include all digits correctly reported from a measurement.

Follow these rules to decide if a digit is significant:

1. All non-zero digits are significant.
2. If a decimal point is present, zeros to the left of other digits (leading zeros) are not significant.
3. If a decimal point is not present, zeros to the right of the last non-zero digit (trailing zeros) are not significant.
4. Zeros placed between other digits are always significant.
5. Zeros placed after other digits to the right of a decimal point are significant.
6. When a measurement is written in scientific notation, all digits in the coefficient are significant.
7. Counted and defined values have infinite significant digits.

Table 2, on the next page, shows examples of significant digits.

Table 2 Certainty in Significant Digits

Measurement	Number of significant digits
32.07 m	4
0.0041 g	2
5×10^5 kg	1
7002 N•m	4
6400 s	2
6.0000 A	5
204.0 cm	4
10.0 kJ	3
100 people (counted)	infinite

An answer obtained by multiplying and/or dividing measurements is rounded to the same number of significant digits as the measurement with the fewest significant digits, for example, using a calculator to solve the following equation:

$$77.8 \text{ km/h} \times 0.8967 \text{ h} = 69.763\ 26 \text{ km}$$

However, the certainty of the answer is limited to three significant digits, so the answer is rounded up to 69.8 km. The same applies to scientific notation. For example,

$$(5.5 \times 10^4) + (4.236 \times 10^4) = 9.7 \times 10^4$$

ROUNDING

When adding or subtracting measurements of different precisions, the answer is rounded to the same precision as the least precise measurement. For example, using a calculator,

$$11.7 \text{ cm} + 3.29 \text{ cm} + 0.542 \text{ cm} = 15.532 \text{ cm}$$

The answer must be rounded to 15.5 cm because the first measurement limits the precision to a tenth of a centimetre.

Follow these rules to round answers to calculations:

1. When the first digit to be dropped is 4 or less, the last digit retained should not be changed.
   3.141 326 rounded to 4 digits is 3.141
2. When the first digit to be dropped is greater than 5, or if it is a 5 followed by at least one digit other than zero, the last digit retained is increased by 1 unit.
   2.221 682 rounded to 5 digits is 2.2217
   4.168 501 rounded to 4 digits is 4.169
3. When the first digit discarded is 5 followed by only zeros, the last digit retained is increased by 1 if it is odd, but not changed if it is even.
   2.35 rounded to 2 digits is 2.4
   2.45 rounded to 2 digits is 2.4
   −6.35 rounded to 2 digits is −6.4

MEASUREMENT ERROR

There are two types of measurement errors: random error and systematic error. Random error results when an estimate is made to obtain the last significant digit for any measurement. The size of the random error is determined by the precision of the measuring instrument. For example, when measuring length with a measuring tape, it is necessary to estimate between the marks on the measuring tape. If these marks are 1 cm apart, the random error will be greater and the precision will be less than if the marks are 1 mm apart. Such errors can be reduced by taking the average of several readings.

Systematic error is associated with an inherent problem with the measuring system, such as the presence of an interfering substance, incorrect calibration, or room conditions. For example, if a balance is not zeroed at the beginning, all measurements will have a systematic error; using a slightly worn metre stick will also introduce a systematic error. Such errors are reduced by adding or subtracting the known error, calibrating the instrument, or performing a more complex investigation.

PRECISION AND ACCURACY

Precision and *accuracy* are two other terms used to describe how close a measurement is to a true value. The precision of a measurement depends upon the gradations of the measuring device. Precision is the place value of the last measurable digit. For example, a measurement of 12.74 cm is more precise than a measurement of 127.4 cm because the first value was measured to hundredths of a centimetre, whereas the latter was measured to tenths of a centimetre.

No matter how precise a measurement is, it still may not be accurate. Accuracy refers to how close a value is to its accepted value. An accurate measurement has a low uncertainty. **Figure 1** shows an analogy between precision and accuracy: the positions of golf balls on a golf course.

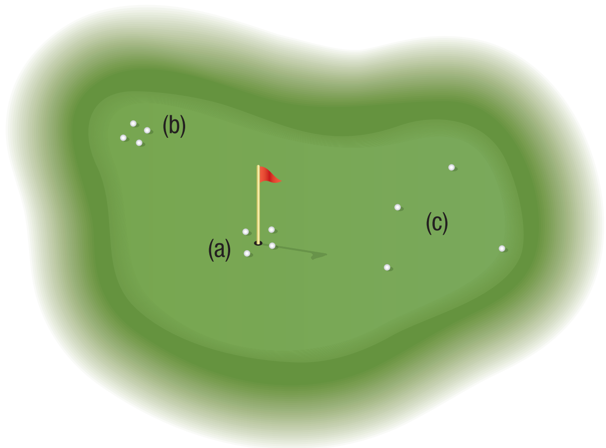


Figure 1 In (a) the results are precise and accurate, in (b) they are precise but not accurate, and in (c) they are neither precise nor accurate.

How certain you are about a measurement depends on two factors: the precision of the instrument used and the size of the measured quantity. More precise instruments give more certain values. For example, a mass measurement of 13 g is less precise than a measurement of 12.76 g; you are more certain about the second measurement than the first. Certainty also depends on the size of the measurement. For example, consider the measurements 0.4 cm and 15.9 cm; both have the same precision. However, if the measuring instrument is precise to $\pm$ 0.1 cm, the first measurement is 0.4 $\pm$ 0.1 cm (0.3 cm or 0.5 cm) for an error of 25 %, whereas the second measurement could be 15.9 $\pm$ 0.1 cm (15.8 cm or 16.0 cm) for an error of 0.6 %. For both factors—the precision of the instrument used and the value of the measured quantity—the more digits there are in a measurement, the more certain you are about the measurement.

REPORTING DATA INVOLVING MEASUREMENTS

A formal report of an experiment involving measurements should include an analysis of uncertainty, percentage uncertainty, and percentage error or percentage difference. Uncertainty is often assumed to be plus or minus half of the smallest division of the scale on the instrument; for example, the estimated uncertainty of 15.8 cm is $\pm$ 0.05 cm or $\pm$ 0.5 mm.

Whenever calculations involving addition or subtraction are performed, the uncertainties accumulate. Thus, to determine the total uncertainty, the individual uncertainties must be added. For example,

$$(34.7\,\text{cm} \pm 0.05\,\text{cm}) - (18.4\,\text{cm} \pm 0.05\,\text{cm}) = 16.3\,\text{cm} \pm 0.10\,\text{cm}$$

Percentage uncertainty is calculated by dividing the uncertainty by the measured quantity and multiplying by 100. Use your calculator to prove that 28.0 cm $\pm$ 0.05 cm has a percentage uncertainty of $\pm$ 0.18 %.

Whenever calculations involving multiplication or division are performed, the percentage uncertainties must be added. If desired, the total percentage uncertainty can be converted back to uncertainty. For example, consider the area of a certain rectangle:

$$\begin{aligned} A &= lw \\ &= (28.0\text{ cm} \pm 0.18\,\%)(21.5\text{ cm} \pm 0.23\,\%) \\ &= 602\text{ cm}^2 \pm 0.41\,\% \\ A &= 602\text{ cm}^2 \pm 2.5\text{ cm}^2 \end{aligned}$$

Percentage error can be determined only if it is possible to compare a measured value with the most commonly accepted value. The equation is

$$\%\text{ error} = \frac{\text{measured value} - \text{accepted value}}{\text{accepted value}} \times 100$$

Percentage difference is useful for comparing two measurements when the true measurement is not known or for comparing a measured value to a predicted value. The percentage difference is calculated as

$$\%\text{ difference} = \frac{\text{measured value} - \text{predicted value}}{\text{predicted value}} \times 100$$

A5.3 Use of Units

When solving problems in science, it is important to denote the units that go with a numerical value. The number 170 as an answer is unacceptable since there are no units. The quantity 170 g/mL denotes a density, 170 ºC denotes a temperature, 170 K denotes a temperature in kelvins, and 170 kPa denotes a pressure. Understanding and placing units with a value gives the proper context of the value.

You can also identify a formula by looking at the units. For instance, if a density of 9.01 g/cm^3 is given, you can note that the density units have grams (mass) divided by cm^3 (volume), so the formula for density is mass divided by volume or $D = \frac{M}{V}$.

A5.4 Mathematical Equations

Several mathematical equations involving geometry, algebra, and trigonometry can be applied in physics.

ABSOLUTE VALUE

Mathematicians tell us that the absolute value is the magnitude of a real number without regard to its sign. This means that, mathematically, if $a = 3$ and $b = -3$, both a and b have an absolute value of 3. This can be expressed as

$$|a| = |b| = 3$$

Notice the "absolute value bars" that are shown around the letters a and b. These bars indicate that we are taking the absolute value of each of these quantities.

"Magnitude" is a term frequently used by physicists. The magnitude of a quantity is the same as its absolute value. Magnitude is often referred to in problems involving vectors. In the previous example, we could have stated that a and b both have a magnitude of 3.

Linear vectors: Vectors have both magnitude and direction. Two displacement vectors are represented below:

$$\Delta\vec{d}_1 = 10\text{ m [E]};\ \Delta\vec{d}_2 = 10\text{ m [W]}$$

While these two vectors point in opposite directions, they have a common magnitude of 10 m. Algebraically, we can state that

$$|\Delta\vec{d}_1| = |\Delta\vec{d}_2| = 10\text{ m}$$

We can state this in words as "the magnitude of displacement one equals the magnitude of displacement two, which equals 10 m." Note that the magnitude of a vector does not include the use of a sign or direction.

Absolute value bars can also be used in vector equations where only the magnitude of a calculated vector is required. For example, we can write Coulomb's law, which describes the force between two charged particles, as

$$|\vec{F}_{\mathrm{E}}| = k\frac{|q_1||q_2|}{r^2}$$

This equation only indicates the magnitude of the electric force, not the direction associated with it.

Perpendicular vectors: Figure 2 shows two perpendicular displacement vectors joined tip to tail. A common method for adding two vectors algebraically involves using the Pythagorean theorem to determine the magnitude of the resultant vector.

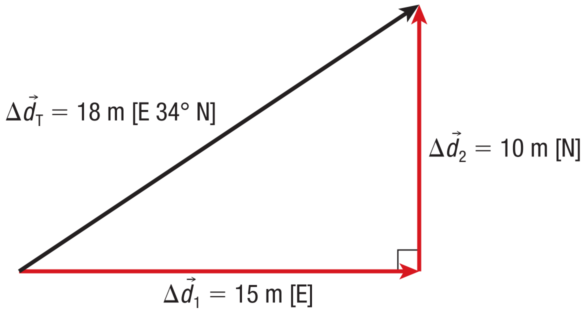


Figure 2 Use the Pythagorean theorem to determine the resultant vector when two perpendicular displacement vectors are added tip to tail.

The Pythagorean theorem is a scalar equation. We may use this equation for vector problems so long as we only use the magnitude of the given vectors in our calculation. For this example, the Pythagorean theorem can be expressed as

$$|\Delta\vec{d}_{\mathrm{T}}|^2 = |\Delta\vec{d}_1|^2 + |\Delta\vec{d}_2|^2$$

Note that it would be algebraically incorrect to write this equation as

$$\Delta\vec{d}_{\mathrm{T}}^{\,2} = \Delta\vec{d}_1^{\,2} + \Delta\vec{d}_2^{\,2}$$

This equation is meaningless because it requires us to square vectors. In this example, $\Delta\vec{d}_1^{\,2}$ could have an associated unit of (m [E])^2, which has no physical meaning.

As a result, we must use only the magnitude of each vector if our use of the Pythagorean theorem is to be valid.

Centripetal force: Figure 3 shows an object moving with uniform circular motion as it moves counterclockwise in a circle. The velocity and acceleration are shown at two points in the object's path. Since the object is moving with uniform circular motion, the magnitude of the velocity at each point is constant. From the diagram, it is clear that the direction of the velocity vector is different at the two points shown. Similarly, the centripetal acceleration is constant in magnitude at both points; however, its direction is different at each location. Centripetal acceleration always points toward the centre of the circle.

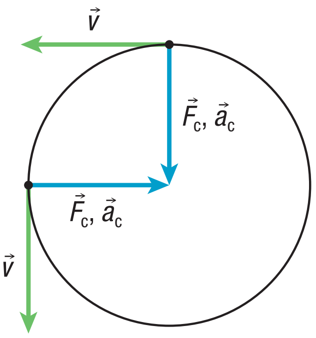


Figure 3 An object moving with uniform circular motion

The defining equation for centripetal force can be written as

$$|\vec{F}_{\mathrm{c}}| = \frac{m|\vec{v}|^2}{r}$$

Notice that absolute value bars have been placed around the centripetal force vector and the velocity vector. This is again because the velocity vector is squared, and squaring a vector has no physical meaning. Since the magnitude of the centripetal force and the magnitude of the object's velocity are constant throughout the object's motion, this use of absolute value bars is valid. Often this equation is simplified as $F_{\mathrm{c}} = \frac{mv^2}{r}$, where F_{c} denotes the magnitude of the centripetal force vector and v represents the magnitude of the velocity vector, or speed.

GEOMETRY

For a rectangle of length l and width w, the perimeter P and the area A are

$$P = 2l + 2w \quad \text{and} \quad A = lw$$

For a triangle of base b and altitude h, the area is

$$A = \frac{1}{2}bh$$

For a circle of radius r, the circumference C and the area A are

$$C = 2\pi r \quad \text{and} \quad A = \pi r^2$$

For a sphere of radius r, the area A and volume V are

$$A = 4\pi r^2 \quad \text{and} \quad V = \frac{4}{3}\pi r^3$$

For a right circular cylinder of height h and radius r, the area and volume are

$$A = 2\pi r^2 + 2\pi rh \quad \text{and} \quad V = \pi r^2 h$$

ALGEBRA

Quadratic formula: Given a quadratic equation in the form $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this equation, the discriminant $b^2 - 4ac$ indicates the number of real roots of the equation. When $b^2 - 4ac < 0$, the quadratic function has no real roots. When $b^2 - 4ac = 0$, the quadratic function has one real root. When $b^2 - 4ac > 0$, the quadratic function has two real roots.

Example 1

A ball is launched from the roof of a building that is 40.0 m tall. The ball is launched at an angle such that its vertical velocity is 10.0 m/s and its horizontal velocity is 17.3 m/s. Determine the time the ball takes to reach the ground.

Given: $\Delta\vec{d}_y = -40.0$ m; $\vec{v}_{iy} = 10.0$ m/s; $\vec{a}_y = -9.8$ m/s^2

Notice that in the given information, the vertical displacement and acceleration are listed as negative values. This indicates that both of these values are vectors that are pointing downward.

We can describe the vertical motion of the ball using the equation

$$\Delta\vec{d}_y = \vec{v}_{iy}\Delta t + \frac{1}{2}\vec{a}_y\Delta t^2$$

Required: Δt

Analysis: $\Delta\vec{d}_y = \vec{v}_{iy}\Delta t + \frac{1}{2}\vec{a}_y\Delta t^2$

Solution: $\Delta\vec{d}_y = \vec{v}_{iy}\Delta t + \frac{1}{2}\vec{a}_y\Delta t^2$

$$-40 \text{ m} = (10.0 \text{ m/s})\Delta t - \frac{1}{2}(9.8 \text{ m/s}^2)\Delta t^2$$

$$0 = -4.9\Delta t^2 + 10.0\Delta t + 40.0$$

This is a quadratic equation for the variable Δt. We can now substitute values into the quadratic formula to determine the correct values for time.

$$a = -4.9,\ b = 10.0, \text{ and } c = 40.0$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-10.0 \pm \sqrt{(10.0)^2 - 4(-4.9)(40.0)}}{2(-4.9)}$$

$$\Delta t = \frac{-10.0 \pm 29.7}{-9.8}$$

$$\Delta t = -2.0 \text{ s} \quad \text{or} \quad \Delta t = 4.1 \text{ s} \quad \text{(Choose the positive root.)}$$

Statement: It takes the ball 4.1 s to reach the ground.

TRIGONOMETRY

The first application of trigonometry was to solve right-angled triangles. Trigonometry derives from the fact that for similar triangles, the ratio of corresponding sides will be equal.

The right-angled triangle in **Figure 4** shows the three sides of the triangle labelled with reference to the indicated angle α (alpha). Notice that the opposite side is opposite α, the hypotenuse side is the longest side of the triangle, and the adjacent side is beside the angle α. Trigonometry provides us with three ratios that can be used for solving problems:

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\alpha = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\alpha = \frac{\text{opp}}{\text{adj}}$$

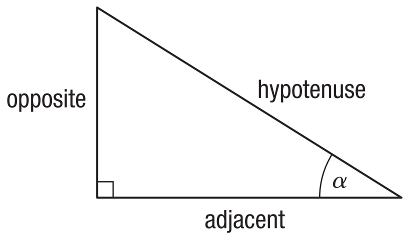


Figure 4

Figure 5 shows a right-angled triangle where one interior angle and the length of one side are given. We can use the trigonometric ratios shown above to determine the length of the two unknown sides.

$$\sin 54° = \frac{\text{opp}}{\text{hyp}} \qquad \cos 54° = \frac{\text{adj}}{\text{hyp}}$$

$$\sin 54° = \frac{Y}{2.3 \text{ cm}} \qquad \cos 54° = \frac{X}{2.3 \text{ cm}}$$

$$Y = (2.3 \text{ cm}) \sin 54° \qquad X = (2.3 \text{ cm}) \cos 54°$$

$$Y = 1.9 \text{ cm} \qquad X = 1.4 \text{ cm}$$

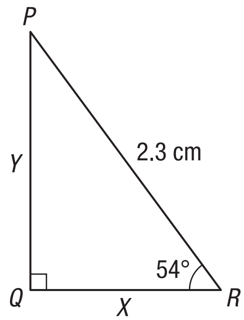


Figure 5

Pythagorean theorem: For the right-angled triangle in **Figure 6**, $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the other sides.

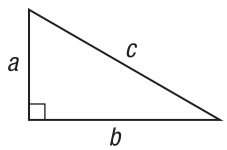


Figure 6 With a right-angled triangle, use the Pythagorean theorem.

Figure 7 shows a right-angled triangle with an unknown interior angle β (beta). We can use one of the trigonometric ratios to solve for this interior angle, since we are given the lengths of two sides of this triangle.

$$\tan\beta = \frac{\text{opp}}{\text{adj}}$$
$$\tan\beta = \frac{X}{Y}$$
$$\beta = \tan^{-1}\left(\frac{2.6\text{ cm}}{7.3\text{ cm}}\right)$$
$$\beta = 20°$$

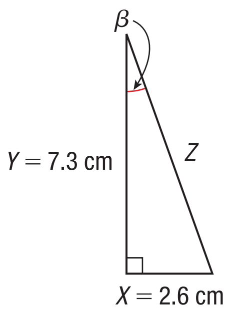


Figure 7

The length of the third side, Z, can be determined by using trigonometry, or by using the Pythagorean theorem. The Pythagorean theorem gives

$$Z^2 = X^2 + Y^2$$
$$Z = \sqrt{X^2 + Y^2}$$
$$Z = \sqrt{(2.6\text{ cm})^2 + (7.3\text{ cm})^2}$$
$$Z = 7.7\text{ cm}$$

For the obtuse triangle in **Figure 8** with angles A, B, and C, and opposite sides a, b, and c:

Sum of the angles: $A + B + C = 180°$

Sine law: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

To use the sine law, two sides and an opposite angle (SSA) or two angles and one side (AAS) must be known.

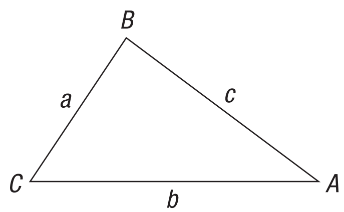


Figure 8 For an obtuse triangle, use the sine law.

The sine law can yield an acute angle rather than the correct obtuse angle when solving for an angle greater than 90°. This problem occurs because for an angle A between 0° and 90°, $\sin A = \sin(A + 90°)$. To avoid this problem, always check the validity of the angle opposite the largest side of a triangle.

Cosine law: $c^2 = a^2 + b^2 - 2ab\cos C$

To use the cosine law, three sides (SSS), or two sides and the contained angle (SAS) must be known. Notice in the cosine law that if $C = 90°$, the equation reduces to the Pythagorean theorem.

Trigonometric identities: You may find the following trigonometric identities useful:

$$\cos\theta = \sin(90° - \theta)$$
$$\sin\theta = \cos(90° - \theta)$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

A5.5 Analyzing Experimental Data

Controlled physics experiments are conducted to determine the relationship between variables. The experimental data can be analyzed in a variety of ways to determine how the dependent variable depends on the independent variable(s). Often the resulting derived relationship can be expressed as an equation.

PROPORTIONALITY STATEMENTS AND GRAPHING

The statement of how one quantity varies in relation to another is a proportionality statement, or a variation statement. Common proportionality statements are as follows:

$y \propto x$ (direct proportion)

$y \propto \frac{1}{x}$ (inverse proportion)

$y \propto x^2$ (square proportion)

$y \propto \frac{1}{x^2}$ (inverse square proportion)

A proportionality statement can be converted into an equation by replacing the proportionality sign with an equal sign and including a proportionality constant. Using k to represent this constant, the proportionality statements become the following equations:

$$y = kx$$
$$y = \frac{k}{x}$$
$$y = kx^2$$
$$y = \frac{k}{x^2}$$

The constant of proportionality can be determined by using graphing software or by applying regular graphing techniques as outlined in the following steps:

1. Plot a graph of the dependent variable as a function of the independent variable. If the resulting line of best fit is straight, the relationship is a direct variation. Proceed to Step 3.
2. If the line of best fit is curved, replot the graph to get a straight line as shown in **Figure 9**. If the first replotting results in a new curved line, draw yet another graph to obtain a straight line.
3. Determine the slope and y-intercept of the straight line on the graph. Substitute the values into the slope/y-intercept form of the equation that corresponds to the variables plotted on the graph with the straight line.
4. Check the equation by substituting original data points.
5. If required, use the equation (or the straight-line graph) to give examples of interpolation and extrapolation.

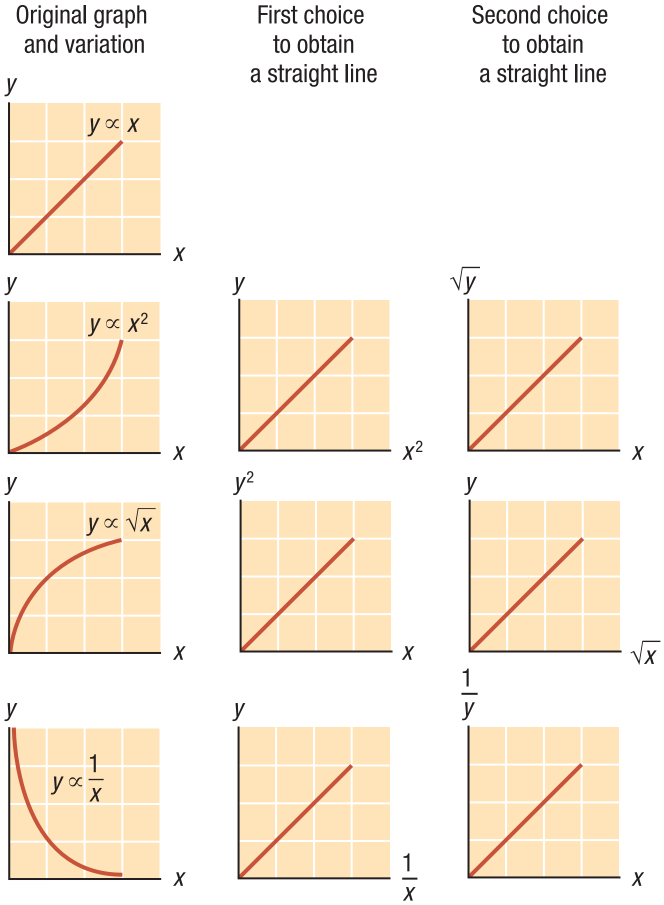


Figure 9 Replotting graphs to try to obtain a straight line

Example 2

Use regular graphing techniques to derive the equation relating the data given in **Table 3**.

Table 3 Velocity–Time Data

***t* (s)**	0.00	2.00	4.00	6.00
$\vec{v}$ (m/s [E])	10.0	15.0	20.0	25.0

Solution: Figure 10 is the graph that corresponds to the data in Table 3. The line is straight and has the following slope:

$$\text{slope} = \frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{25.0\text{ m/s [E]} - 10.0\text{ m/s [E]}}{6.00\text{ s} - 0.00\text{ s}}$$

$$\text{slope} = 2.50\text{ m/s}^2\text{ [E]}$$

The y-intercept is 10.0 m/s [E].
Using $y = mx + b$, the equation is

$$\vec{v} = 2.50\text{ m/s}^2\text{ [E]}\,(t) + 10.0\text{ m/s [E]}$$

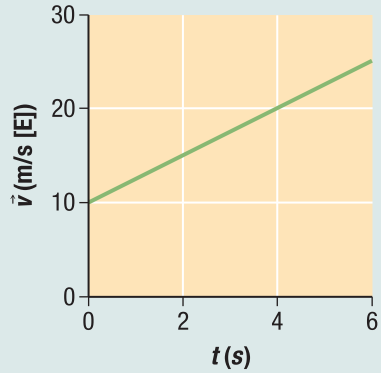


Figure 10 Velocity–time graph

Verify the equation by substituting $t = 4.00$ s:

$$\vec{v} = 2.50\text{ m/s}^2\text{ [E]}\,(4.00\text{ s}) + 10.0\text{ m/s [E]}$$

$$= 10.0\text{ m/s [E]} + 10.0\text{ m/s [E]}$$

$$\vec{v} = 20.0\text{ m/s [E]}$$

The equation is valid.
Using $t = 3.20$ s as an example of interpolation,

$$\vec{v} = 2.50\text{ m/s}^2\text{ [E]}\,(3.20\text{ s}) + 10.0\text{ m/s [E]}$$

$$= 8.00\text{ m/s [E]} + 10.0\text{ m/s [E]}$$

$$\vec{v} = 18.0\text{ m/s [E]}$$

Example 3

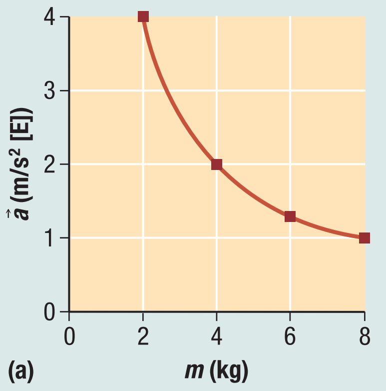
Use regular graphing techniques to derive the equation for the data in the first two rows of **Table 4**.

Table 4 Acceleration–Mass Data

m (kg)	2.0	4.0	6.0	8.0
$\vec{a}$ (m/s^2 [E])	4.0	2.0	1.3	1.0
$\frac{1}{m}$ (kg^{-1})*	0.50	0.25	0.167	0.125

* The third row is for the redrawn graph of the relationship.

Solution: Figure 11(a) is the graph of the data given in the first two rows of the table. **Figure 11(b)** shows the replotted graph with m replaced by $\frac{1}{m}$, which produces a straight line.



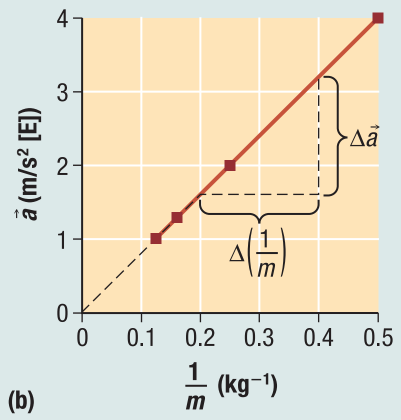


Figure 11 (a) Acceleration–mass graph. (b) The graph in (a) has been replotted with m replaced by $\frac{1}{m}$.

The slope of the straight line is

$$\text{slope} = \frac{\Delta\vec{a}}{\Delta\left(\frac{1}{m}\right)}$$

$$= \frac{3.2\ \text{m/s}^2\ [\text{E}] - 1.6\ \text{m/s}^2\ [\text{E}]}{0.40\ \text{kg}^{-1} - 0.20\ \text{kg}^{-1}}$$

$$\text{slope} = 8.0\ \text{kg}\cdot\text{m/s}^2\ [\text{E}]$$

The slope is 8.0 kg·m/s^2 [E], which can also be written 8.0 N [E].

The y-intercept is 0. Using $y = mx + b$, the equation is

$$\vec{a} = 8.0\ \text{kg}\cdot\text{m/s}^2\ [\text{E}] \times \frac{1}{m}$$

or $$\vec{a} = \frac{8.0\ \text{N}\ [\text{E}]}{m}$$

Verify the equation by substituting $m = 6.0$ kg:

$$\vec{a} = \frac{8.0\ \text{kg}\cdot\text{m/s}^2\ [\text{E}]}{6.0\ \text{kg}}$$

$$\vec{a} = 1.3\ \text{m/s}^2\ [\text{E}]$$

The equation is valid.

We can use this equation to illustrate extrapolation; for example, when the mass is 9.6 kg, the acceleration is

$$\vec{a} = \frac{8.0\ \text{kg}\cdot\text{m/s}^2\ [\text{E}]}{9.6\ \text{kg}}$$

$$\vec{a} = 0.83\ \text{m/s}^2\ [\text{E}]$$

The acceleration is 0.83 m/s^2 [E].

GRAPHING DATA ELECTRONICALLY

You can use a graphing calculator or a computer-graphing program for several purposes, including determining the roots of an equation and analyzing linear functions, quadratic functions, trigonometric functions, and conic functions. You can also create a graph of given or measured data, and determine the equation relating the variables plotted or solve two simultaneous equations with two unknowns.

Graphing calculators: Examples 4, 5, and 6 show different ways to program a TI-83 graphing calculator. If you have a different graphing calculator, refer to its instruction manual for detailed information about solving equations.

Example 4

A ball is tossed vertically upward with an initial speed of 9.0 m/s. At what times after its release will the ball pass a position 3.0 m above the position where it was released? (Neglect air resistance.)

Solution: Defining upward as the positive direction and using magnitudes only, the given quantities are $\Delta d = 3.0$ m, $v_i = 9.0$ m/s, and $a = -9.8$ m/s^2. The constant acceleration equation for displacement is

$$\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$3.0 = 9.0\Delta t - 4.9\Delta t^2$$

$$4.9\Delta t^2 - 9.0\Delta t + 3.0 = 0$$

To solve for Δt, we can use the quadratic formula and enter the data into the calculator. The equation is in the form $Ax^2 + Bx + C = 0$, where $A = 4.9$, $B = -9.0$, and $C = 3.0$.

1. Store the coefficients A and B, and the constant C in the calculator:
   - 4.9 $\boxed{\text{STO▸}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$
   - $\boxed{\text{ALPHA}}$ $\boxed{\text{:}}$
   - −9 $\boxed{\text{STO▸}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$
   - $\boxed{\text{ALPHA}}$ $\boxed{\text{:}}$
   - 3 $\boxed{\text{STO▸}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{C}}$
   - $\boxed{\text{ENTER}}$
2. Enter the expression for the quadratic formula:

   $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   - $\boxed{(}$ $\boxed{-}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ + $\boxed{\text{2nd}}$ $\boxed{\sqrt{}}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{x^2}$ $\boxed{-}$ 4 $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{C}}$ $\boxed{)}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 2 $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{)}$
3. Press $\boxed{\text{ENTER}}$ to determine one solution for the time. To determine the other solution, the negative must be used in front of the discriminant. The answers are 1.4 s and 0.44 s.

Example 5

Graph the function $y = \cos x$ for $0° \leq x \leq 360°$.

Solution:

1. Put the calculator in degree mode:
   - $\boxed{\text{MODE}}$ → Degree → $\boxed{\text{ENTER}}$.
2. Enter $y = \cos x$ into the equation editor:
   - Y = $\boxed{\text{COS}}$ $\boxed{\text{X,T,}\Theta,n}$ $\boxed{)}$.
3. Adjust the window so that it corresponds to the given domain:
   - $\boxed{\text{WINDOW}}$ → $X_{min} = 0$, $X_{max} = 360$, $X_{sel} = 90$ (for an interval of 90° on the x-axis), $Y_{min} = -1$, and $Y_{max} = 1$.
4. Graph the function using the ZoomFit:
   - $\boxed{\text{ZOOM}}$ $\boxed{0}$.

Consider an ellipse, which is important in physics because it is the shape of the orbits of planets and satellites. The standard form of the equation of an ellipse where the centre is the origin and the major axis is along the x-axis is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b.$$

The vertices of the ellipse are at $(a, 0)$ and $(-a, 0)$, as shown in **Figure 12**.

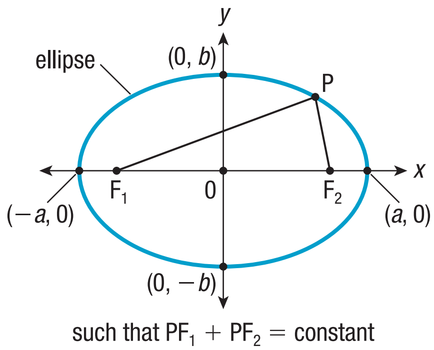


Figure 12 An ellipse

Example 6

Use the "Zap-a-Graph" feature to plot an ellipse centred on the origin of an x-y graph, and determine the effect of changing the parameters of the ellipse.

Solution:

1. Choose Ellipse from the Zap-a-Graph menu:
   - [DEFINE] → Ellipse.
2. Enter the parameters of the ellipse (e.g., $a = 6$ and $b = 4$), then plot the graph.
3. Alter the ellipse by choosing Scale from the Grid menu and entering different values.

Graphing on a spreadsheet: A spreadsheet is a computer program that can be used to create a table of data and a graph of the data. It is composed of cells indicated by a column letter (A, B, C, etc.) and a row number (1, 2, 3, etc.) In **Figure 13**, B1 and C3 are examples of cells. Each cell can hold a number, a label, or an equation.

	A	B	C	D	E
1	A1	B1	C1		
2	A2	B2			
3	A3				

Figure 13 Spreadsheet cells

To create a data table and plot the corresponding graph, you can follow the steps outlined in the next example.

Example 7

An object is moving with an initial velocity of 5.0 m/s [E] and a constant acceleration of 4.0 m/s^2 [E]. Set up a spreadsheet for the relationship $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$. Plot a graph of the data from $t = 0$ s to $t = 8.0$ s at intervals of 1.0 s.

Solution:

1. Access the spreadsheet, and label cell A1 the independent variable, in this case t, and cell B1 the dependent variable, in this case Δd.
2. Enter the values of t from 0 to 8.0 in cells A2 to A10. In cell B2 enter the right side of the equation in the following form: =5*A2+(1/2)*4*A2*A2 where "*" represents multiplication.
3. Use the cursor to select B2 down to B10 and choose the Fill Down command, or right drag cell B1 down to B10 to copy the equation to each cell.
4. Command the program to graph the values in the data table (e.g., choose Make Chart, depending on the program).

A spreadsheet can be used to solve a system of two simultaneous equations, which can occur, for example, when analyzing elastic collisions (discussed in Section 5.3). In a one-dimensional elastic collision, the two equations involved simultaneously are

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2' \quad \text{conservation of momentum}$$

and

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \quad \text{conservation of energy}$$

If the known quantities are m_1, m_2, v_1, and v_2, then the unknown quantities are v_1' and v_2'. To determine the solution to these unknowns, rewrite the equations so that one unknown is isolated and written in terms of all the other variables. In this example, enter v_2' in cell A1, enter the first v_1' based on the equation for the law of conservation of momentum in cell B1, and enter the second v_1' (call it v_1'') based on the equation for the law of conservation of energy in cell C1. Then proceed to enter the data according to the previous example, using reasonable values for the variables. Plot the data on a graph, and determine the intersection of the two resulting lines. That intersection is the solution to the two simultaneous equations.

A5.6 Unit Analysis

Unit analysis is a useful tool to confirm that a calculation has been performed correctly or to convert units. Three quantities that are commonly measured in physics are mass, m, distance, Δd, and time, Δt. Note that the units of these measurements are all base units—kilogram (kg), metre (m), and second (s), respectively. In unit analysis, all units are expressed as base units. Derived units can be written in terms of SI base units and, thus, base dimensions. For example, the newton has base units of kg·m/s^2, or dimensions of [M][L][T^{-2}]. Appendix B contains a list of derived units.

After a while, unit analysis will become second nature. Consider a situation where you solve an equation in which time Δt is the unknown, and your final answer is $\Delta t = 2.1$ kg. You know that something clearly has gone wrong, since time is measured in seconds (s). Check your calculation to see where the error was made. It is important to note that while unit analysis can clearly point out when you have made an error, units that work out correctly do not necessarily indicate that your answer is correct. We can use unit analysis to confirm that the equation $\Delta d = v_i t + \frac{1}{2}a\Delta t^2$ is mathematically valid.

To perform unit analysis, insert the appropriate units into the equation that you are analyzing. Place all units in square brackets, and ignore any fractions in the equation that you are analyzing.

$$\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$[\text{m}] = \frac{\text{m}}{\cancel{\text{s}}}[\cancel{\text{s}}] + \frac{\text{m}}{\cancel{\text{s}^2}}[\cancel{\text{s}^2}]$$

$$[\text{m}] = [\text{m}] + [\text{m}]$$

$$[\text{m}] = [\text{m}]$$

Since both sides of the equation have the same units, metres (m), unit analysis has shown that this equation is mathematically valid.

The reason we ignore fractions in dimensional analysis is because fractions have no units. They are "dimensionless quantities." Dimensionless quantities include

- all plain numbers (4, π, etc.) and counted quantities (12 people, 5 cars, etc.)
- angles (although angles have units)
- cycles
- trigonometric functions
- exponential functions
- logarithms

You can also use unit analysis to convert from one unit to another. For example, to convert 95 km/h to m/s, kilometres must be changed to metres, and hours must be changed to seconds. Using the conversions 1 km = 1000 m, 1 h = 60 min, and 1 min = 60 s, the following ratios can be written:

$$\frac{1000\text{ m}}{1\text{ km}} = 1; \quad \frac{1\text{ h}}{60\text{ min}} = 1; \quad \frac{1\text{ min}}{60\text{ s}} = 1$$

To convert from km/h to m/s we will use these three ratios strategically to cancel units that we do not want, and to keep units that we do want (m and s). These ratios can be inverted if necessary:

$$\frac{95\text{ km}}{\text{h}} = \frac{95\ \cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}}$$

Therefore,

$$\frac{95\text{ km}}{1\text{ h}} = \frac{95\ 000\text{ m}}{3600\text{ s}}$$
$$= 26.4\text{ m/s, or 26 m/s (to two significant digits)}$$

Example 8

A 2100 g object experiences a net force of magnitude 38.2 N. Determine the magnitude of the object's acceleration.

Given: $m = 2100$ g; $F_{\text{net}} = 38.2$ N

Required: a

Analysis: First, convert grams to kilograms. Then use the equation $F_{\text{net}} = ma$ to calculate acceleration.

Solution: $m = (2100\ \cancel{\text{g}})\left(\dfrac{1\text{ kg}}{1000\ \cancel{\text{g}}}\right)$

$$m = 2.1\text{ kg}$$
$$F_{\text{net}} = ma$$
$$a = \frac{F_{\text{net}}}{m}$$
$$= \frac{38.2\text{ N}}{2.1\text{ kg}}$$
$$a = 18\text{ N/kg}$$

It is more common to express acceleration using the SI units of metres per second squared, so we will use unit analysis to convert the units:

$$F_{\text{net}} = ma$$
$$[\text{N}] = \left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right]$$
$$a = \frac{18\text{ N}}{\text{kg}}$$
$$= \frac{\left[\dfrac{18\ \cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}\right]}{\cancel{\text{kg}}}$$
$$a = 18\text{ m/s}^2$$

Statement: The acceleration is 18 m/s^2.

A5.7 Vectors

Several quantities in physics are vector quantities—quantities that have both magnitude (size) and direction. Understanding and working with vectors is crucial in solving many physics problems.

VECTOR SYMBOLS

A vector is represented in a vector scale diagram by an arrow, or a directed line segment. The length of the arrow is proportional to the magnitude (size) of the vector, and the direction is the same as the direction of the vector. The tail of the vector is the initial point, and the tip of the vector is the end with the arrowhead. When the vector is drawn to scale, the scale should be indicated on the diagram, as should the direction north (**Figure 14**).

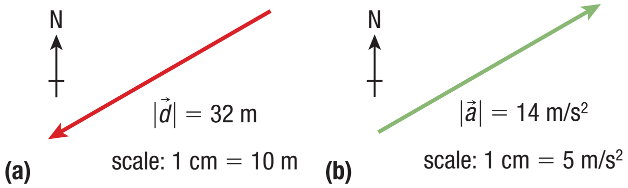


Figure 14 Examples of vector quantities: (a) displacement vector; (b) acceleration vector

In this text, a vector quantity is indicated by an arrow above the letter representing the vector (for example, $\vec{A}$, $\vec{a}$, $\vec{F}$, and $\vec{p}$). The magnitude of a vector, for example, $\vec{A}$, is indicated by the letter without the vector arrow, in this example, A. The magnitude is always positive.

DIRECTIONS OF VECTORS

The directions of vectors are indicated in square brackets following the magnitude and units of the measurement. The four compass directions—east, west, north, and south—are indicated as [E], [W], [N], and [S]. Other examples are [down], [forward], [11.5° below the horizontal], [toward Earth's centre], and [W 24° N].

Figure 15 shows a vector drawn in the northwest quadrant of a Cartesian coordinate system. We designate the direction of this vector as [W 24° N]. This can be read as, "the vector initially pointed west, and was then rotated 24° toward north." Since the complementary angle of 24° is 66°, this direction can also be expressed as [N 66° W]. Both of these directions are equally correct.

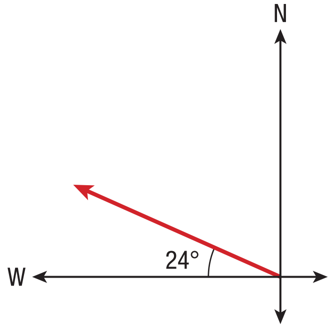


Figure 15 Locating the direction W 24° N. This represents a vector that was initially pointed west and was rotated 24° toward north.

COMPONENTS OF VECTORS

Any two-dimensional vector can be broken down into two perpendicular components. **Figure 16** shows a displacement vector $\Delta\vec{d} = 15$ m [E 30° N] drawn on a Cartesian coordinate system.

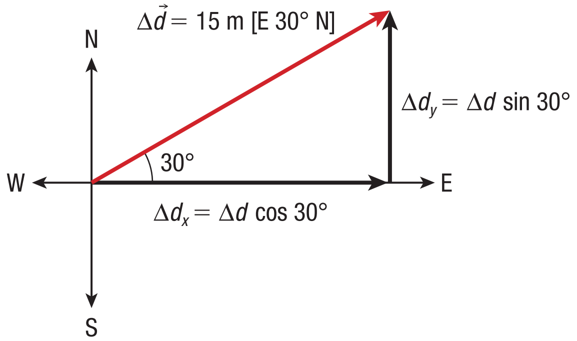


Figure 16 A vector broken down into two perpendicular components

This vector has been broken down into *x*- and *y*-components such that the *x*-component is drawn along the *x*-axis of the Cartesian coordinate system. When the two component vectors Δd_x and Δd_y are joined tip to tail as shown in the diagram, they will give the resultant vector $\Delta\vec{d} = 15$ m [E 30° N]. The directions of these two component vectors are clear from the diagram.

We can use trigonometry to determine the magnitude of each component vector: When possible, draw *x*-components along the *x*-axis; this will ensure that each *x*-component vector will have a cosine term. Similarly, each *y*-component vector will have a sine term. Depending on the problem being solved, θ will not always be the angle between the *x*-axis and the displacement. Sometimes, θ will be between the *y*-axis and the displacement. Therefore, always consider which component is opposite θ and which one is adjacent to θ to determine the components.

We can determine the magnitude and direction of our two component vectors as follows:

$$\cos 30° = \frac{\Delta d_x}{\Delta d} \qquad \sin 30° = \frac{\Delta d_y}{\Delta d}$$
$$\Delta d_x = \Delta d \cos 30° \qquad \Delta d_y = \Delta d \sin 30°$$
$$\Delta d_x = \Delta d \cos 30° \qquad \Delta d_y = \Delta d \cos 30°$$
$$\Delta d_x = (15 \text{ m}) \cos 30° \qquad \Delta d_y = (15 \text{ m}) \sin 30°$$
$$\Delta d_x = 13 \text{ m [E]} \qquad \Delta d_y = 7.5 \text{ m [N]}$$

VECTOR ADDITION

In arithmetic, 3 + 3 always equals 6. This is not always true when adding vector quantities. Vector addition must take into consideration the directions of the vectors. Here are some general guidelines regarding vector addition:

- Vector addition is commutative. The order of addition does not matter: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- Vector addition is associative. If more than two vectors are added, it does not matter how they are grouped: $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

In this text we model three methods to add vectors. Vector addition by scale diagram requires the use of a protractor and metric ruler to add vectors. The sine and cosine laws method only works when adding two vectors at a time, but the result is more accurate than that of a scale diagram. Vector addition by components is a purely algebraic method for adding vectors.

The Scale Diagram Method

To add vector quantities by scale diagram, the arrows representing the vectors are drawn to scale and are joined tip to tail. The resultant vector is drawn starting at the tail of the first vector and ending at the tip of the last vector added. Vector addition by scale diagram tends to be very simple for one-dimensional problems. We will solve a one-dimensional and a two-dimensional problem in the following examples.

Example 9

Add the following two displacements using the scale diagram method:

$$\Delta\vec{d}_1 = 3.0 \text{ m [E]}, \Delta\vec{d}_2 = 5.0 \text{ m [W]}$$

Note: When drawing a vector scale diagram, choose a scale that will produce a diagram that is approximately one-half to one full page in size. Be sure to indicate the scale and direction due north in your diagram.

Given: $\Delta\vec{d}_1 = 3.0 \text{ m [E]}; \Delta\vec{d}_2 = 5.0 \text{ m [W]}$

Required: $\Delta\vec{d}_T$

Analysis: Use the scale 1 cm = 1 m.

$\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$\Delta\vec{d}_T = 3.0 \text{ m [E]} + 5.0 \text{ m [W]}$

Figure 17 shows the total displacement, represented by the vector $\Delta\vec{d}_T$. From this diagram we can see that the total displacement is represented by a vector 2.0 cm long pointing due west. Applying our scale we get $\Delta\vec{d}_T = 2.0 \text{ m [W]}$.

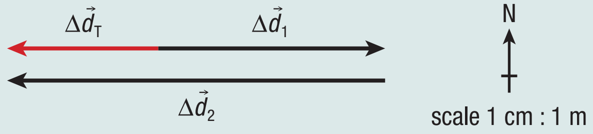


Figure 17

Statement: The total displacement is 2.0 m [W].

Example 10

Add the following two displacement vectors using the scale diagram method:

$$\Delta\vec{d}_1 = 4.0 \text{ m [E]}, \Delta\vec{d}_2 = 5.0 \text{ m [W 30° S]}$$

Solution: Use the scale 1 cm = 1 m. **Figure 18** shows these two vectors joined tip to tail in a scale diagram. The length of the total displacement is measured to be 4.6 cm. Applying our scale and using our protractor to measure the angle for the direction of this displacement vector, we get $\Delta\vec{d}_T = 4.6 \text{ m [E 71° S]}$.

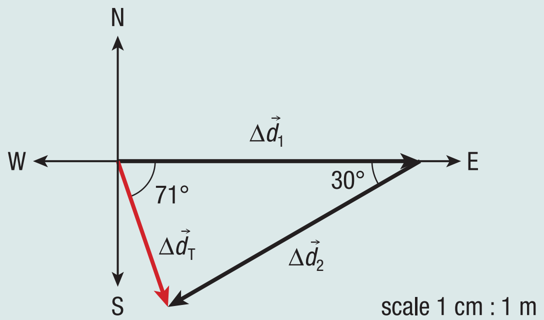


Figure 18

You can use the cosine and sine laws method with Example 10 and get the same results.

Adding Vectors Algebraically

While vector addition by scale diagram is effective, it is not terribly precise. It can also become quite complex when adding more than two vectors. A purely algebraic method of adding vectors is far more precise. This method uses trigonometry and the Pythagorean theorem.

Example 11

Consider two vector forces acting on a single object. Determine the vector sum of these two forces.

$\vec{F}_1 = 10.5 \text{ N [S]}, \vec{F}_2 = 14.0 \text{ N [W]}$

Given: $\vec{F}_1 = 10.5 \text{ N [S]}; \vec{F}_2 = 14.0 \text{ N [W]}$

Required: $\vec{F}_T$

Analysis: $\vec{F}_T = \vec{F}_1 + \vec{F}_2$

Solution: Figure 19 shows the two force vectors joined tip to tail. Note that this is only a sketch; it is not drawn to scale. We can determine the magnitude of the resultant vector $\vec{F}_T$ using the Pythagorean theorem.

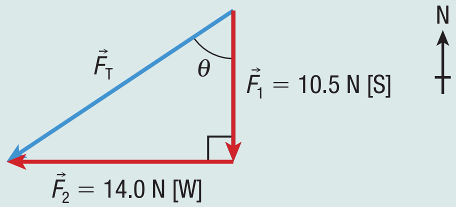


Figure 19 Determining the net force

$$\vec{F}_T = \vec{F}_1 + \vec{F}_2$$

$$F_T = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{(10.5 \text{ N})^2 + (14.0 \text{ N})^2}$$

$$F_T = 17.5 \text{ N}$$

To determine the direction of the resultant vector, use the tangent function:

$$\tan\theta = \frac{F_2}{F_1}$$

$$\theta = \tan^{-1}\left(\frac{14.0 \text{ N}}{10.5 \text{ N}}\right)$$

$$\theta = 53.1°$$

Statement: The total force acting on the object is 17.5 N [S 53.1° W].

Often we are required to add vectors that are not perpendicular to each other. We can still do this algebraically. The method is called vector addition by components. The goal is to take a question and convert it into the sample problem that we have just solved, that is, to take a general two-dimensional vector problem and convert it into a problem where we have two perpendicular vectors. To add any number of vectors by components, use the following steps:

1. Draw a Cartesian coordinate system.
2. Draw each vector to be added on the Cartesian coordinate system starting at the origin.
3. Break each vector down into *x*- and *y*-components such that the *x*-component is drawn along the *x*-axis.
4. Determine the sum of the *x*-components by adding all the individual *x*-components.
5. Determine the sum of the *y*-components by adding all the individual *y*-components.
6. Draw a sketch showing the overall *x*- and *y*-component vectors joined tip to tail.
7. Determine the magnitude and direction of the resultant vector by using the Pythagorean theorem and tangent function.

Example 12

Determine the vector sum of the two following displacements: $\Delta\vec{d}_1 = 25$ m [N 40.0° W], $\Delta\vec{d}_2 = 35$ m [S 20.0° E]

Given: $\Delta\vec{d}_1 = 25$ m [N 40.0° W]; $\Delta\vec{d}_2 = 35$ m [S 20.0° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Figure 20 shows our two displacement vectors drawn on a Cartesian coordinate system. Each vector is drawn from the origin and has an *x*-component, which is drawn along the *x*-axis.

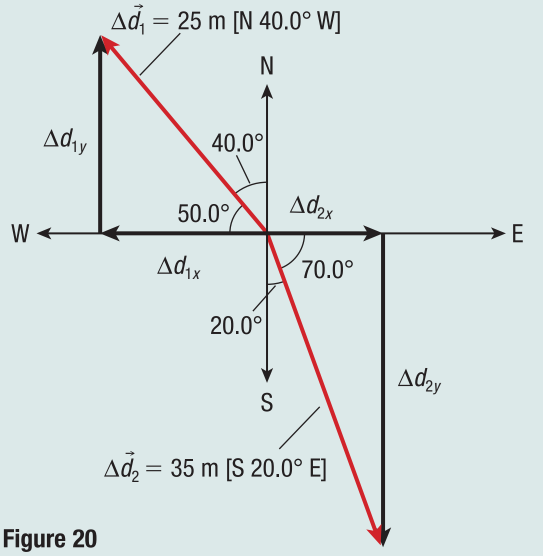


Figure 20

Calculate the vector sum of the *x*-components.

$$\begin{aligned}
\Delta d_{Tx} &= \Delta d_{1x} + \Delta d_{1x} \\
&= \Delta d_1 \cos\theta_1 + \Delta d_2 \cos\theta_2 \\
&= (25\text{ m})(\cos 50.0°)\ [\text{W}] + (35\text{ m})(\cos 70.0°)\ [\text{E}] \\
&= 16.1\text{ m [W]} + 12.0\text{ m [E]} \\
&= 16.1\text{ m [W]} - 12.0\text{ m [W]} \\
\Delta d_{Tx} &= 4.10\text{ m [W] (one extra digit carried)}
\end{aligned}$$

Calculate the vector sum of the *y*-components.

$$\begin{aligned}
\Delta d_{Ty} &= \Delta d_{1y} + \Delta d_{2y} \\
&= \Delta d_1 \sin\theta_1 + \Delta d_2 \sin\theta_2 \\
&= (25\text{ m})(\sin 50.0°)\ [\text{N}] + (35\text{ m})(\sin 70.0°)\ [\text{S}] \\
&= 19.2\text{ m [N]} + 32.9\text{ m [S]} \\
&= -19.2\text{ m [S]} + 32.9\text{ m [S]} \\
\Delta d_{Ty} &= 13.7\text{ m [S] (one extra digit carried)}
\end{aligned}$$

Figure 21 shows a sketch of the overall *x*- and *y*-component vectors joined tip to tail.

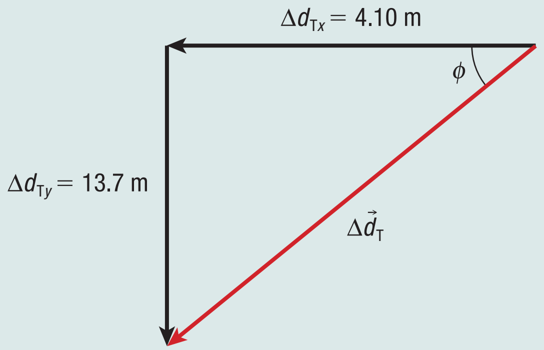


Figure 21

We can now use the Pythagorean theorem to determine the magnitude of the resultant vector and the tangent function to determine its direction.

$$\begin{aligned}
\Delta d_T^2 &= \Delta d_{Tx}^2 + \Delta d_{Ty}^2 \\
\Delta d_T &= \sqrt{\Delta d_{Tx}^2 + \Delta d_{Ty}^2} \\
\Delta d_T &= \sqrt{(4.10\text{ m})^2 + (13.7\text{ m})^2} \\
\Delta d_T &= 14\text{ m}
\end{aligned}$$

$$\begin{aligned}
\tan\phi &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}} \\
\phi &= \tan^{-1}\left(\frac{13.7\text{ m}}{4.10\text{ m}}\right) \\
\phi &= 73°
\end{aligned}$$

Statement: The vector sum of the displacements is 14 m [W 73° S].

THE SCALAR OR DOT PRODUCT OF TWO VECTORS

The scalar product of two vectors is equal to the product of their magnitudes and the cosine of the angle between the vectors. The scalar product is also called the dot product because a dot can be used to represent the product symbol. An example of a scalar product is the equation for the work W done by a net force $\vec{F}$ that causes an object to move by a displacement $\Delta\vec{d}$ (Section 4.1).

$$\begin{aligned} W &= \vec{F}\cdot\Delta\vec{d} \\ &= F\Delta d\cos\theta \end{aligned}$$

or $$W = (F\cos\theta)\Delta d$$

Thus, the defining equation of the scalar (or dot) product of vectors $\vec{A}$ and $\vec{B}$ is

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

where θ is the angle between $\vec{A}$ and $\vec{B}$, A is the magnitude of $\vec{A}$, and B is the magnitude of $\vec{B}$. Notice that $\vec{A}$ and $\vec{B}$ do not represent the same quantities.

A scalar product can be represented in a diagram as shown in **Figure 22** in which an applied force $\vec{F}_a$ is at an angle θ to the displacement of the object being pulled (with negligible friction).

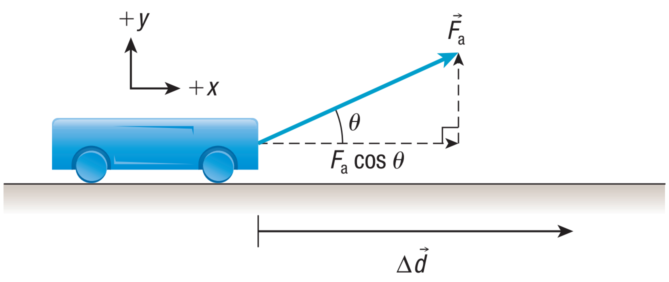


Figure 22 Assuming that there is negligible friction, the work done by $\vec{F}_a$ on the cart in moving it a displacement $\Delta\vec{d}$ is the scalar product, $F_a\cos\theta\Delta d$.

THE VECTOR OR CROSS PRODUCT OF TWO VECTORS

The vector product of two vectors has a magnitude equal to the product of the magnitudes of the two vectors and the sine of the angle between the vectors. The vector product is also called the cross product because a "×" is used to represent the product symbol. Thus, for vectors $\vec{A}$ and $\vec{B}$, the vector product $\vec{C}$ is defined by the following equation:

$$\vec{C} = \vec{A}\times\vec{B}$$

where the magnitude is given by $C = |\vec{C}| = |AB\sin\theta|$, and the direction is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. However, there are two distinct directions that are perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. To determine the correct direction, you can use the following rule, illustrated in **Figure 23**:

- Right-hand rule for the vector product: When the fingers of the right hand move from $\vec{A}$ toward $\vec{B}$, the outstretched thumb points in the direction of $\vec{C}$.

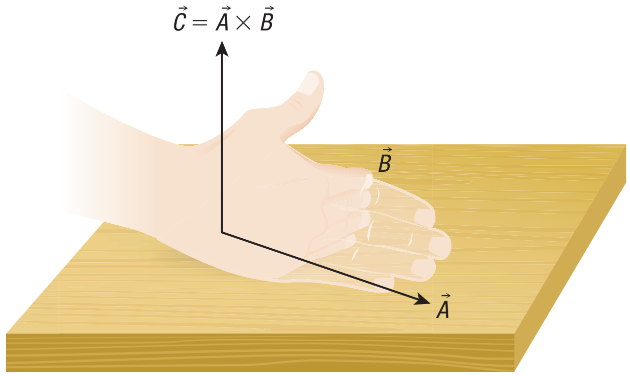


Figure 23 The right-hand rule to determine the direction of the vector resulting from the vector product $\vec{C} = \vec{A}\times\vec{B}$

The vector (or cross) product has the following properties:

- The order in which the vectors are multiplied matters because $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$. (Use the right-hand rule to verify this.)
- If $\vec{A}$ and $\vec{B}$ are parallel, $\theta = 0°$ or $180°$ and $\vec{A}\times\vec{B} = 0$ because $\sin 0° = \sin 180° = 0$. Thus, $\vec{A}\times\vec{A} = 0$.
- If $\vec{A}$ is perpendicular to $\vec{B}$ ($\theta = 90°$), then $|\vec{A}\times\vec{B}| = AB$ because $\sin 90° = 1$.
- The vector product obeys the distributive law; that is, $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$.

LEARNING TIP

Alternative Notation

In advanced physics textbooks, vectors are sometimes written in boldface rather than with an arrow above the quantity. Thus, you may determine the dot product and the cross product written as follows:

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

$\mathbf{A}\times\mathbf{B} = AB\sin\theta$

A6 Choosing Appropriate Career Pathways

Often, one of the most difficult tasks in high school is deciding what career path to follow after graduation. The science skills and concepts presented in this book will be of benefit to many careers, whether you are planning a career in scientific research (such as research geneticist or astrophysicist) or in areas related to science (such as environmental lawyer, pharmaceutical sales representative, or electrician). The strong critical-thinking and problem-solving skills that are emphasized in science programs are a valuable asset for any career.

Career Links and Pathways

Throughout this textbook you will have many opportunities to explore careers related to your studies in physics. The Career Links icons in the text indicate that you can learn more about a career related to the text by going to the Nelson Science website. At the end of each chapter you will also find a Career Pathways feature that illustrates sample educational pathways for some of the careers related to the chapter.

It is wise to begin researching academic requirements as early as possible. Understanding the options available to pursue a particular career will help you make decisions on whether to attend university or college, and which program of study you should take. In addition, understanding the terminology used by universities and colleges will play an integral role in planning your future.

University and College Programs

Undergraduate university programs generally lead to a three-year general bachelor degree or a four-year honours bachelor degree. These degree designations begin with a "B" followed by the area of specialization; for example, a B.Sc. (Hons.) indicates an Honours Bachelor of Science degree. These degrees can lead to employment or to further education in Masters or Doctorate postgraduate programs. The length of postgraduate degrees generally varies from one to four years.

College programs typically fall into three categories: one-year certificates, two-year diplomas, and three-year advanced diplomas. Certificates and diplomas can lead directly to employment opportunities or to graduate certificate programs. In some programs, there are "transfer agreements" with universities, which allow college graduates to enter university programs with advanced standing toward a university degree.

Pathways in Physics

The Career Pathways graphic organizer illustrates examples of pathways to follow after high school. Certain pathways lead to careers after university, and others may lead to careers after college. Look at **Figure 1** below to see three possible career pathways. Student A wishes to become an atmospheric scientist and must complete the Grade 11 and 12 University Physics courses (along with other prerequisites) and enter an undergraduate university program. Student A must obtain a Bachelor of Science degree, and then continue on to further education in Masters and Doctorate programs before becoming an atmospheric scientist.

Student B wishes to become a Geographic Information System (GIS) analyst and must complete the Grade 11 University Physics course (along with other prerequisites) and enter an undergraduate university program. Student B must obtain a Bachelor of Arts degree, for example in Geography, and then continue on to a college for a graduate certificate program in GIS before becoming a GIS analyst. Student C could become a GIS analyst by entering a college certificate program, then completing the same graduate certificate program as student B.

Planning for Your Future

Planning ahead for your educational and career paths will provide a rewarding future. You should consult your guidance counsellors for specific advice on career planning and which courses you should take in high school. Take the time to research university and college websites for specific program information because these sites will provide the prerequisite information and, most often, career planning advice. CAREER LINK

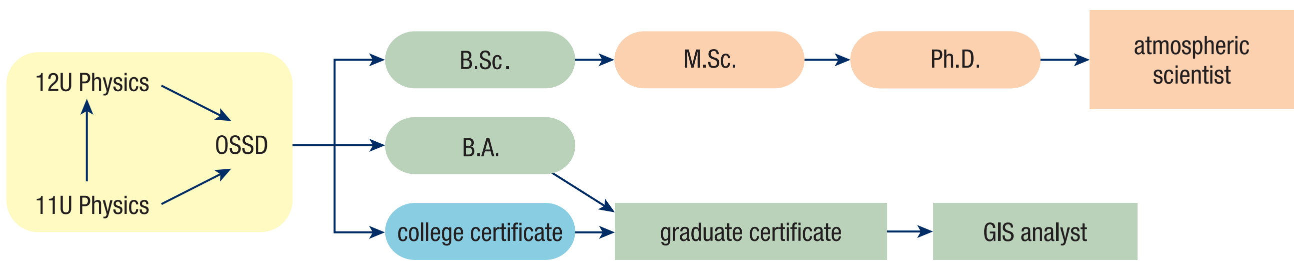


Figure 1 This graphic organizer shows three pathways to two careers that benefit from courses in physics.

Throughout *Nelson Physics 12* and in this reference section, we have attempted to be consistent in the presentation and usage of units. As much as possible, the text uses the International System of Units (SI). However, some other units have been included because of their practical importance, wide usage, or use in specialized fields.

Nelson Physics 12 has followed the most recent *Canadian Metric Practice Guide* (CAN/CSA-Z234.1-00), published in 2000 and updated in 2003 by the Canadian Standards Association.

Table 1 SI Base Units

Quantity	Symbol	Unit name	Symbol
amount of substance	n	mole	mol
electric current	I	ampere	A
length	L, l, h, d, w	metre	m
luminous intensity	I_v	candela	cd
mass	m	kilogram	kg
thermodynamic temperature	T	kelvin	K
time	t	second	s

Table 2 Some SI-Derived Units

Quantity	Symbol	Unit	Unit symbol	SI base unit
acceleration	$\vec{a}$	metre per second squared	m/s^2	m/s^2
area	A	square metre	m^2	m^2
Celsius temperature	t	Celsius	°C	K
density	ρ, D	kilogram per cubic metre	kg/m^3	kg/m^3
displacement	$\Delta\vec{d}$	metre	m	m
electric charge	Q, q	coulomb	C	A·s
electric field	$\vec{\varepsilon}$	volt per metre	V/m	$kg{\cdot}m/A{\cdot}s^3$
electric field intensity	ε	newton per coulomb (tesla)	N/C, T	$kg/A{\cdot}s^2$
electric potential	V	volt	V	$kg{\cdot}m^2/A{\cdot}s^3$
electric resistance	R	ohm	Ω	$kg{\cdot}m^2/A^2{\cdot}s^3$
energy	E, E_k, E_p	joule	J	$kg{\cdot}m^2/s^2$
force	F	newton	N	$kg{\cdot}m/s^2$
frequency	f	hertz	Hz	s^{-1}
gravitational field	$\vec{g}$	newton per kilogram	N/kg	m/s^2
heat	Q	joule	J	$kg{\cdot}m^2/s^2$
magnetic field	$\vec{B}$	weber per square metre (tesla)	T	$kg/A{\cdot}s^2$
momentum	$\vec{p}$	kilogram-metre per second	kg·m/s	kg·m/s
period	T	second	s	s
power	P	watt	W	$kg{\cdot}m^2/s^3$
pressure	p	newton per square metre	N/m^2	$kg/m{\cdot}s^2$
speed	v	metre per second	m/s	m/s
velocity	$\vec{v}$	metre per second	m/s	m/s
volume	V	cubic metre	m^3	m^3
wavelength	λ	metre	m	m
weight	W, w	newton	N	N, $kg{\cdot}m/s^2$
work	W	joule	J	$kg{\cdot}m^2/s^2$

Table 3 Numerical Prefixes

Powers and Subpowers of Ten		
Prefix	**Power**	**Symbol**
deca	10^1	da
hecto	10^2	h
kilo	10^3	k
mega	10^6	M
giga	10^9	G
tera	10^{12}	T
peta	10^{15}	P
exa	10^{18}	E
deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
atto	10^{-18}	a

Some Examples of Prefix Use

2000 metres	$= 2 \times 10^3$ metres	= 2 **kilo**metres or 2 km
0.27 metres	$= 27 \times 10^{-2}$ metres	= 27 **centi**metres or 27 cm
3 000 000 000 hertz	$= 3 \times 10^9$ hertz	= 3 **giga**hertz or 3 GHz

Table 4 The Greek Alphabet

Upper case	Lower case	Name
A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ε	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
O	o	omicron
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

LEARNING **TIP**

SI Prefixes

It can be difficult to remember the metric prefixes. A mnemonic is a saying that helps you remember something. "King Henry Doesn't Mind Drinking Chocolate Milk" is a mnemonic for kilo, hecto, deca, metre, deci, centi, and milli. Another helpful hint is that mega (M) represents a million ($\times 10^6$) and tera (T) represents a trillion ($\times 10^{12}$). The first letter of the prefix and the first letter of what it represents are the same.

Table 5 Physical Constants

Quantity	Symbol	Approximate value
speed of light in a vacuum	c	3.0×10^{8} m/s
gravitational constant	G	6.67×10^{-11} N·m^2/kg^2
Coulomb's constant	k	8.99×10^{9} N·m^2/C^2
charge on electron	$-e$	-1.60×10^{-19} C
charge on proton	e	1.60×10^{-19} C
electron mass	m_e	9.11×10^{-31} kg
proton mass	m_p	1.673×10^{-27} kg
neutron mass	m_n	1.675×10^{-27} kg
atomic mass unit	u	1.660×10^{-27} kg
Planck's constant	h	6.63×10^{-34} J·s
electron-volt	eV	1.60×10^{-19} J

Table 6 The Solar System

Object	Mass (kg)	Radius of object (m)	Period of rotation on axis (s)	Mean radius of orbit (m)	Period of revolution of orbit (s)	Orbital eccentricity
Sun	1.99×10^{30}	6.96×10^{8}	2.14×10^{6}	—	—	—
Mercury	3.28×10^{23}	2.44×10^{6}	5.05×10^{6}	5.79×10^{10}	7.60×10^{6}	0.206
Venus	4.85×10^{24}	6.05×10^{6}	2.1×10^{7}	1.08×10^{11}	1.94×10^{7}	0.007
Earth	5.98×10^{24}	6.38×10^{6}	8.64×10^{4}	1.49×10^{11}	3.16×10^{7}	0.017
Moon	7.35×10^{22}	1.74×10^{6}	2.36×10^{6}	3.84×10^{8}	2.36×10^{6}	0.055
Mars	6.42×10^{23}	3.40×10^{6}	8.86×10^{4}	2.28×10^{11}	5.94×10^{7}	0.093
Jupiter	1.90×10^{27}	7.15×10^{7}	3.58×10^{4}	7.78×10^{11}	3.75×10^{8}	0.048
Saturn	5.69×10^{26}	6.03×10^{7}	3.84×10^{4}	1.43×10^{12}	9.30×10^{8}	0.056
Uranus	8.80×10^{25}	2.56×10^{7}	6.20×10^{4}	2.87×10^{12}	2.65×10^{9}	0.046
Neptune	1.03×10^{26}	2.48×10^{7}	5.80×10^{6}	4.50×10^{12}	5.20×10^{9}	0.010
Pluto	1.3×10^{23}	1.15×10^{6}	5.51×10^{5}	5.91×10^{12}	7.82×10^{9}	0.248

Table 7 Atomic Masses of Selected Particles

Name	Symbol	Atomic mass (u)
neutron	n	1.008 665
proton	p	1.007 276
deuteron	d	2.013 553
alpha particle	α	4.002 602

Table 8 Quarks and Their Properties

Type of quark	Symbol	Quark charge (e)	Mass	Anti-quark	Anti-quark charge (e)
up	u	$+\frac{2}{3}$	1.7–3.1 MeV	$\bar{u}$	$-\frac{2}{3}$
down	d	$-\frac{1}{3}$	4.1–5.7 MeV	$\bar{d}$	$+\frac{1}{3}$
charm	c	$+\frac{2}{3}$	1.18–1.34 GeV	$\bar{c}$	$-\frac{2}{3}$
strange	s	$-\frac{1}{3}$	80–130 MeV	$\bar{s}$	$+\frac{1}{3}$
top	t	$+\frac{2}{3}$	172.9 GeV	$\bar{t}$	$-\frac{2}{3}$
bottom	b	$-\frac{1}{3}$	4.13–4.37 GeV	$\bar{b}$	$+\frac{1}{3}$

Table 9 Properties of Some Baryons

Particle	Symbol	Constituent quarks	Lifetime (s)	Mass (MeV/c^2)
proton	p	uud	stable	938
neutron	n	udd	890	940
sigma plus	Σ^+	uus	0.8×10^{-10}	1189
sigma zero	Σ^0	uds	6.0×10^{-20}	1193
sigma minus	Σ^-	dds	1.5×10^{-10}	1197
xi minus	Ξ^-	dss	1.6×10^{-10}	1321

Note: There are many other baryons composed of other combinations of three quarks and anti-quarks.

Table 10 Properties of Some Mesons

Particle	Symbol	Constituent quarks	Lifetime (s)	Mass (MeV/c^2)
pion (pi plus)	π^+	$u\bar{d}$	2.6×10^{-8}	140
pi zero*	π^0	$d\bar{d}/u\bar{u}$	8.4×10^{-17}	135
kaon (K plus)	K^+	$u\bar{s}$	1.2×10^{-8}	494
kaon (K minus)	K^-	$s\bar{u}$	1.2×10^{-8}	494
phi	ϕ	$s\bar{s}$	1.6×10^{-22}	1020

Note: There are many other mesons, which are composed of other combinations of quarks and anti-quarks.

* The π^0 is a quantum-mechanical combination of the $d\bar{d}$ and $u\bar{u}$ quark states.

Table 11 Leptons and Their Properties

Particle	Symbol	Lepton charge	Mass/c^2	Anti-lepton	Anti-lepton charge
electron	e^-	–1	0.511 MeV	e^+	1
electron neutrino	ν_e	0	0.05 eV < m < 2 eV	$\bar{\nu}_e$	0
muon	μ^-	–1	106 MeV	μ^+	1
muon neutrino	ν_μ	0	< 0.19 MeV	$\bar{\nu}_\mu$	0
tau	τ^-	–1	1780 MeV	τ^+	1
tau neutrino	ν_τ	0	< 18 MeV	$\bar{\nu}_\tau$	0

Nelson Physics 12

Key

atomic number → 26; 3+ ← most common ion charge; 2+ ← other ion charge

symbol of element (solids in black, liquids in blue, gases in red) → Fe

iron ← name of element

55.85 ↑ atomic mass (u)—based on C-12 atomic molar mass (g/mol)

	1	2	3	4	5	6	7	8	9
1	1 (1+, 1−) **H** hydrogen 1.01								
2	3 (1+) **Li** lithium 6.94	4 (2+) **Be** beryllium 9.01							
3	11 (1+) **Na** sodium 22.99	12 (2+) **Mg** magnesium 24.31							
4	19 (1+) **K** potassium 39.10	20 (2+) **Ca** calcium 40.08	21 (3+) **Sc** scandium 44.96	22 (4+, 3+) **Ti** titanium 47.87	23 (5+, 4+) **V** vanadium 50.94	24 (3+, 2+) **Cr** chromium 52.00	25 (2+, 4+) **Mn** manganese 54.94	26 (3+, 2+) **Fe** iron 55.85	27 (2+, 3+) **Co** cobalt 58.93
5	37 (1+) **Rb** rubidium 85.47	38 (2+) **Sr** strontium 87.62	39 (3+) **Y** yttrium 88.91	40 (4+) **Zr** zirconium 91.22	41 (5+, 3+) **Nb** niobium 92.91	42 (6+) **Mo** molybdenum 95.96	43 (7+) **Tc** technetium (98)	44 (3+, 4+) **Ru** ruthenium 101.07	45 (3+) **Rh** rhodium 102.91
6	55 (1+) **Cs** cesium 132.91	56 (2+) **Ba** barium 137.33	57 (3+, 2+) **La** lanthanum 138.91	72 (4+) **Hf** hafnium 178.49	73 (5+) **Ta** tantalum 180.95	74 (6+) **W** tungsten 183.84	75 (7+) **Re** rhenium 186.21	76 (4+) **Os** osmium 190.23	77 (4+) **Ir** iridium 192.22
7	87 (1+) **Fr** francium (223)	88 (2+) **Ra** radium (226)	89 (3+, 2+) **Ac** actinium (227)	104 **Rf** rutherfordium (261)	105 **Db** dubnium (262)	106 **Sg** seaborgium (266)	107 **Bh** bohrium (264)	108 **Hs** hassium (277)	109 **Mt** meitnerium (268)
	Alkali metals	Alkaline earth metals							

6	58 (3+) **Ce** cerium 140.12	59 (3+) **Pr** praseodymium 140.91	60 (3+) **Nd** neodymium 144.24	61 (3+) **Pm** promethium (145)	62 (3+, 2+) **Sm** samarium 150.36
7	90 (4+) **Th** thorium 232.04	91 (5+, 4+) **Pa** protactinium 231.04	92 (6+, 4+) **U** uranium 238.03	93 (5+) **Np** neptunium (237)	94 (4+, 6+) **Pu** plutonium (244)

- Metals
- Metalloids
- Non-metals
- Hydrogen

Periodic Table of the Elements

Measured values are subject to change as experimental techniques improve. Atomic molar mass values in this table are based on IUPAC website values (2011).

Period	10	11	12	13	14	15	16	17	18
1									2 — He helium 4.00
2				5 — B boron 10.81	6 — C carbon 12.01	7 3– N nitrogen 14.01	8 2– O oxygen 16.00	9 1– F fluorine 19.00	10 — Ne neon 20.18
3				13 3+ Al aluminum 26.98	14 — Si silicon 28.09	15 3– P phosphorus 30.97	16 2– S sulfur 32.07	17 1– Cl chlorine 35.45	18 — Ar argon 39.95
4	28 2+ 3+ Ni nickel 58.69	29 2+ 1+ Cu copper 63.55	30 2+ Zn zinc 65.38	31 3+ Ga gallium 69.72	32 4+ Ge germanium 72.64	33 3– As arsenic 74.92	34 2– Se selenium 78.96	35 1– Br bromine 79.90	36 — Kr krypton 83.80
5	46 2+ 3+ Pd palladium 106.42	47 1+ Ag silver 107.87	48 2+ Cd cadmium 112.41	49 3+ In indium 114.82	50 4+ 2+ Sn tin 118.71	51 3+ 5+ Sb antimony 121.76	52 2– Te tellurium 127.60	53 1– I iodine 126.90	54 — Xe xenon 131.29
6	78 4+ 2+ Pt platinum 195.08	79 3+ 1+ Au gold 196.97	80 2+ 1+ Hg mercury 200.59	81 1+ 3+ Tl thallium 204.38	82 2+ 4+ Pb lead 207.2	83 3+ 5+ Bi bismuth 208.98	84 2+ 4+ Po polonium (209)	85 1– At astatine (210)	86 — Rn radon (222)
7	110 Ds darmstadtium (281)	111 Rg roentgenium (272)	112 Cn copernicium (285)	113 Uut ununtrium (284)	114 Uuq ununquadium (289)	115 Uup ununpentium (288)	116 Uuh ununhexium (291)	117 Uus ununseptium (294)	118 Uuo ununoctium (294)
								Halogens	Noble gases

Period									
6	63 3+ 2+ Eu europium 151.96	64 3+ Gd gadolinium 157.25	65 3+ Tb terbium 158.93	66 3+ Dy dysprosium 162.50	67 3+ Ho holmium 164.93	68 3+ Er erbium 167.26	69 3+ Tm thulium 168.93	70 3+ 2+ Yb ytterbium 173.05	71 2+ Lu lutetium 174.97
7	95 3+ 4+ Am americium (243)	96 3+ Cm curium (247)	97 3+ 4+ Bk berkelium (247)	98 3+ Cf californium (251)	99 3+ Es einsteinium (252)	100 3+ Fm fermium (257)	101 2+ 3+ Md mendelevium (258)	102 2+ 3+ No nobelium (259)	103 3+ Lr lawrencium (262)

APPENDIX C ANSWERS

These pages include numerical and short answers to *Are You Ready?* questions, chapter section questions, and Chapter Self-Quiz, Chapter Review, Unit Self-Quiz, and Unit Review questions.

Unit 1

Are You Ready?, pp. 4–5

2. baseball player
7. (a) static friction
   (b) gravity, normal, applied, kinetic friction
11. (a)
12. 6:10 p.m.
13. (a) 1.0×10^1 km
    (b) 0 N
14. (a) 27 N [E]
    (b) 2.2 m/s^2 [E]
15. (a) 140 N; 140 N
    (b) 31 N; 31 N; 4.5 N; 4.5 N
    (c) 46 N; 46 N; 8.6 N; 3.4 N
16. (a) 150 N
    (b) 170 N
    (c) normal force of scale
17. $a = 10.5$ m; $b = 13.4$ m; $c = 31.3$ m; $d = 18.8$ m
18. $c = 4.8$ cm; $x = 42.2°$; $y = 116.8°$

1.1 Questions, p. 16

1. (a) 33 m
   (b) 7.5 m/s
   (c) 5.6 m/s [E 27° N]
2. (a) 4.17 m/s
   (b) 696 s
3. (a) 7.6 m/s
   (b) 6.6 s
4. (a) 170 m/s
   (b) 170 m/s [E 15° N]
5. 6.5 m/s^2 [W]
6. 42.7 m/s [E]
7. 2.4 m/s^2 [forward]
8. (a) 29 m/s [E]
   (b) 37 m/s [E]
   (c) 17 m/s [E]; 33 m/s [E]; 50 m/s [E]
9. (a) 5 m/s^2 [backward]
   (b) 3 m/s^2 [backward]
   6 m/s^2 [backward]
   9 m/s^2 [backward]

1.2 Questions, p. 21

1. (a) 55 m [forward]
   (b) 21 m/s [forward]
2. (a) 5.60×10^{13} m/s^2 [W]
   (b) 9.39×10^{-8} s
3. (a) 11 s
   (b) 2.0×10^2 m
   (c) 1.2×10^2 km/h
   (d) 24 s
5. (a) 12 m/s [up]
   (b) 7.1 m
6. (a) 5.0 s
   (b) 3.1×10 m/s [down]
   (c) 49 m
7. (a) 2.0×10^2 m/s [up]
   (b) 2.4×10^3 m
   (c) 47 s
8. (a) (i) 28 m
   (ii) 85 m
   (iii) 48 m

1.3 Questions, p. 29

1. (b) 7.0 km [N]
2. 27 cm [E]
3. 1.54 m [W]; 1.97 m [N]
4. 21.6 m [E]; 12.5 m [N]
5. (a) 59 m
   (b) [E 24° N]
6. 33 km [W 29° S]
7. 8.0 m [E 78° S]
8. (a) 5.84 km [E]; 2.25 km [S]
   (b) 6.26 km [E 21.1° S]
9. 2.30×10^3 km [W 17.8° N]
10. 45 km [W 32° N]
11. 670 km [W]; 230 km [W]

1.4 Questions, p. 35

2. (a) 1.0×10^2 m
   (b) 85.0 m [E 28.5° S]
   (c) 0.42 m/s
   (d) 0.35 m/s [E 28.5° S]
3. 58 km/h [W 65.4° N]
6. 3.1×10^2 km [N 11° E]
7. 2.4×10^3 s
8. 2.0 m/s^2 [E 76° N]
9. 1.36 m/s^2 [E 35.0° S]
10. 2.2×10^3 m/s^2 [N]
11. 7.3 m/s [W 44° S]
12. 1.1×10^2 m/s [E 65° N]

1.5 Questions, p. 43

1. 15 m/s
2. (a) 1.6×10^2 s
   (b) 1.2×10^2 km
   (c) 31 km
3. (a) 67 m
   (b) 37 m/s [82° below the horizontal]
4. (a) 2.0×10^1 m/s [53° above the horizontal]
   (b) 32 m
   (c) 4.4 m
5. (a) 57 m
   (b) 1.0×10^1 s
   (c) 1.7×10^2 m
6. 17 m

1.6 Questions, p. 49

1. (a) 34 min
   (b) no
   (c) 28 min
2. (a) 200 m/s [W 20° N]
   (b) 36 min
3. (a) 8.0×10^1 m/s [N]
   (b) 44 m/s [N]
   (c) 65 m/s [N 16° W]
   (d) 76 m/s [N 9.1° W]
4. (a) 0.44 m/s [W]
   (b) 0.78 m/s [S 34° W]
   (c) [S 43° E]
5. 3.7×10^2 km/h [S 49° E]
6. (a) [N 6.2° W]
   (b) 0.84 h
7. (a) 2.0×10^2 m/s
   (b) 3.0×10^2 m/s
8. (a) 3.8 m/s [E 32° up]
   (b) 4.9 m/s [E 14° up]
9. (a) 17.7 m/s [down 70.0° W]
   (b) 6.07 m/s [down]
10. (a) [N 21° W]
    (b) 280 km/h

Chapter 1 Self-Quiz, p. 53

1. (b)	6. (d)	11. F
2. (d)	7. (d)	12. T
3. (b)	8. F	13. F
4. (c)	9. T	14. T
5. (a)	10. F	15. F

Chapter 1 Review, pp. 54–59

1. (a)
2. (b)
3. (a)
4. (a)
5. (c)
6. (b)
7. T
8. F
9. T
10. F
11. T
12. F
13. T
14. F
15. (a) no
    (b) yes
16. yes
17. yes
19. both decrease
38. 2.0×10^1 m
39. 3 m/s^2 [backward]
40. (a) 4.5 m/s^2 [forward]
    (b) 12 s
41. (a) 4.00 m/s^2 [backward]
    (b) 5.00 s
42. 0.41 m/s^2 [backward]
43. 2.7 m
44. (a) increase
    (b) 6.2 m
45. 50 m/s [W]
46. (a) 100 m
    (b) 20 m/s
47. 72 m
48. (a) 3.1 s
    (b) 46 m
    (c) 2.0 s
    (d) 4.1 s
    (e) 6.1 s
49. (b) 1.2 m/s^2 [N]
50. 98 m
54. (b) 0 m/s^2
    (c) 30 s
    (d) both 300 m [W]
55. 3.2 m
56. x-component $= -1.5$ units
    y-component $= 3.5$ units
57. 39.1 km [N 33.7° W]
58. (a) x-component $= 7.4$ m
    y-component $= 2.9$ m
    (b) 7.9 m [21° above the x-axis]
59. 1.3 m/s^2 [W 51° N]
60. 0 m/s^2
62. 1.7 m/s^2 [forward]
    5.0 m/s
63. 12 m/s
64. (a) ball A
    (b) ball A
    (c) no
65. 44 m/s
66. 11 m/s
67. 4.3 s
68. (a) 138 m
    (b) 318 m
69. 14 m/s

72. (a) 34 m
    (b) 15°
    (c) 3.8 s
73. (a) 4.4 m
    (b) 27 m/s
74. (a) 0.70 s
    (b) 19 m
    (c) 28 m/s
75. 13.4 km/h [N 26.6° E]
76. 2.0 km/h
77. 9.0×10^1 m
78. 0.29 m/s [60° upstream]
79. 320 km/h [E 31° S]

2.1 Questions, p. 69

5. (a) 2.3 N [S 12° W]
   (b) 7.8 N [N 19° W]
6. 50 N [W 34° S]
7. (a) 14 N [S]; 6.1 N [W]
   (b) 14 N [N]
   (c) 15 N [S 24° W]
8. 70 N [E 25° N]
9. (b) 94 N [up]
   (c) 49 N [backward]

2.2 Questions, p. 76

3. (a) 0.61 m/s² [W]
   (b) 6.9 s
4. 18 kg
5. (a) 50 N
   (b) 7.7×10^2 s
6. 1.1 m/s² [S 22° W]
8. (a) 51 N
   (b) 51 N
9. 11 m/s² [backward 25° up]

2.3 Questions, p. 83

1. 54 N [E 17° S]
2. 250 N; 440 N; 510 N
3. 27 N; 64°
4. (c) 3.6×10^3 N
5. (b) 1.9 m/s² [forward]
   (c) 200 N [up]
   (d) 4.4 m/s [forward]
6. (a) 5.4 N [up ramp]
   (b) 8.2 N [up ramp]

2.4 Questions, p. 90

1. 0.5
2. 47 m/s
3. (a) 0.18
   (b) 0.14
4. (a) 16°
   (b) 0.28 m/s² [down incline]
6. (a) yes
   (b) 120 N
   (c) 2.1 m/s²
7. (a) 0.90
   (b) 0.70
8. 0.28 m/s² [clockwise]

Chapter 2 Self-Quiz, p. 99

1. (d)	6. (c)	11. F
2. (a)	7. (b)	12. F
3. (b)	8. (d)	13. T
4. (b)	9. F	14. F
5. (c)	10. T	15. F

Chapter 2 Review, pp. 100–105

1. (b)
2. (a)
3. (c)
4. (a)
5. (d)
6. (a)
7. (d)
8. (c)
9. (b)
10. F
11. F
12. T
13. F
14. T
15. F
16. F
17. F
18. T
19. T
20. 8.8×10^2 N [up]
24. 0 m/s²
26. 2.0 m/s² [E]
27. 0 N
28. no
32. 31.5 N
33. 9.2×10^3 N [downward]
34. 2.3 m/s² [E 33° N]
35. 1.0×10^2 N [W 39° S]
36. (a) 5.0 m/s²
    (b) 17 N
37. (c) 1.1×10^3 N
    (d) 1.1×10^3 N
39. (a) 2.9 m/s² [E]
    (b) 8.8×10^2 m
40. (a) 2.3 m/s² [W]
    (b) 3.0 N [W]
41. (a) 34 N
    (b) 19 N
42. (a) 27 N
    (b) 64°
43. (a) 760 N; 920 N
44. 1.4×10^3 N [N 28° up]
45. 4.7×10^2 N
46. 4.1×10^4 N
47. 0.75 m/s²; 63 N; 27 N
48. 1.1×10^3 N
49. 75 N
50. (a) 65 kg
    (b) 7.6×10^2 N
51. 3.2×10^2 N
52. (a) 0.40
    (b) 0.36
53. 0.41
54. 0.66
55. 3.5×10^2 N
57. (a) 10 N [up the plane]
    (b) static
58. 2.9×10^2 N
59. 0.27
60. (b) 4.2 m/s²
61. 1.6 m/s²
62. 0.20
63. 4.2 m/s² [E]
64. 0.67
65. 34 N
68. (b) 2.9×10^2 N
69. (f); (c); (e); (d); (b); (a)
71. (a) 7.9 m/s²; 5.7 m/s²
    (b) 5.0 s; 5.9 s; no
72. 4.5 m/s²
73. 90 %
74. (b) skier 1: 1.5 m/s²
    skier 2: 1.8 m/s²

3.1 Questions, p. 113

1. (a) straight up and down
   (b) parabola
2. 8.7°
3. 35.9°
4. 2.8 m/s²
5. 6.7×10^2 N
6. 4.9 m/s² [down]
7. (b) 2.9 m/s²
8. (a) 39 N
   (b) 17 N

3.2 Questions, p. 119

1. (b) $\frac{1}{2}a$
   (c) $4a$
2. 4:1
3. 7.4 m/s²
4. 5.8 m/s²
5. 3.37×10^{-2} m/s²
6. 0.56 Hz
7. 62 m
8. 12.0 km/h
9. (a) 1.00 s
   (b) 11.8 m/s² [N]
10. (a) 3.8×10^8 m
11. (a) 29.3 m/s
    (b) 85 m

3.3 Questions, p. 124

1. 8.9 m/s
2. (b) 1.5×10^4 N
   (c) 20 m/s
4. 29 m/s
6. 5.5 m/s

3.4 Questions, p. 138

3. (d) 36°
   (e) 0.96 N
5. no
6. (a) 2.0×10^1 s
   (b) 31 m/s
   (c) $7.3m$
   (d) $13m$
   (e) with the rotation
7. (a) 8.3 m/s² [down]
   (b) 35 m/s
   (c) 27 s
8. (a) 1.6×10^6 m/s²

3.5 Questions, p. 132

1. (a) 23 m/s
   (c) $3g$
2. 1.4:1
4. 1500 N [up]

Chapter 3 Self-Quiz, p. 139

1. (c)	6. (c)	11. T
2. (c)	7. (d)	12. T
3. (c)	8. (b)	13. F
4. (c)	9. F	
5. (d)	10. F	

Chapter 3 Review, pp. 140–145

1. (a)
2. (a)
3. (b)
4. (b)
5. (c)
6. (d)
7. F
8. T
9. F
10. F
11. T
12. F
13. F
14. F
15. T
16. (a) $4a$
    (b) $\frac{1}{2}a$
17. faster
19. (a) gravity
    (b) friction
    (c) tension
20. (f) 2.3 m/s²
21. (a) 1.6 m/s²
    (b) 1.2×10^2 m/s²
    (c) 8.97×10^{22} m/s²
22. 65 m/s
23. 11.5 m/s²
24. 8.1 m/s
25. $F_{engine} = 3F_{cargo\ car}$
26. (a) 2×10^3 N
    (b) 4×10^3 N
    (c) 2×10^2 N
27. 12 m/s
28. (a) 0.011 N
    (b) 0.27 N
    (c) 0.0027 N

29. 5.5×10^2 N
30. 1.2 m/s^2
31. 2.9 m/s
32. 28 m
33. (a) 2.9×10^2 N
    (b) 3.9×10^2 N
34. 29 N
35. (c) $F_T \cos\theta$
    (d) 8.4°
36. 7.0×10^3 N
37. 11 m/s
38. 2.8×10^2 N
39. 3.3 m/s
40. 1.5×10^4 N
41. (a) 8 kg
    (b) 1 m/s
42. (a) $4F$
43. 3.4×10^2 N
44. string A: 414 N
    string B: 296 N
45. (a) 3.8 m/s
    (b) 21 N
46. support A: 7.2×10^4 N
    support B: 4.9×10^4 N
47. (b) 8.6 m/s
48. string A: 2.8×10^2 N
    string B: 2.0×10^2 N
49. 1.3×10^3 N
50. 19 N
51. 0.053
52. 19.8 m/s
53. 29 m
54. (a) centrifugal force
    (b) tension
    (c) 30 N
55. (a) 0.92
    (b) 0.37 loops/s
    (c) top: 1.3×10^3 N
    bottom: 4.4×10^3 N
56. 7.1 N
57. 9.7 m
58. 56 km/h

Unit 1 Self-Quiz, pp. 148–149

1. (b)
2. (c)
3. (a)
4. (c)
5. (b)
6. (c)
7. (c)
8. (d)
9. (b)
10. (d)
11. (a)
12. (b)
13. (c)
14. (d)
15. (d)
16. (d)
17. T
18. F
19. F
20. T
21. T
22. F
23. T
24. F
25. T
26. T

Unit 1 Review, pp. 150–157

1. (d)
2. (a)
3. (c)
4. (b)
5. (b)
6. (c)
7. (a)
8. (b)
9. (c)
10. (b)
11. (a)
12. F
13. F
14. T
15. F
16. F
17. T
18. T
19. F
20. F
21. T
22. (a) (vi)
    (b) (viii)
    (c) (vii)
    (d) (x)
    (e) (ix)
    (f) (i)
    (g) (iii)
    (h) (ii)
    (i) (v)
    (j) (iv)
26. 33 m/s^2
27. constant acceleration
29. $|\Delta \vec{d}_T| = \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}$
32. $\Delta d_x = \dfrac{v_i^2}{g}\sin 2\theta$
33. (a) overhead
    (b) straight down
35. 72 km/h [N]
36. (a) $mg \sin\theta$
    (b) $mg \cos\theta$
39. $F_{net} = 0$
40. 4.1 N
47. $4a$
48. $v^2 = gr$
50. (a) 0 m
    (b) 3.9 km/h
    (c) 0 km/h
54. (a) perpendicular
55. (a) $\Delta d_y = v_i \sin\theta \Delta t - \dfrac{1}{2}g(\Delta t)^2$
    (b) $\Delta d_y = -\dfrac{1}{2}g(\Delta t)^2$
    (c) $\Delta d_y = -\dfrac{1}{2}g(\Delta t)^2$
57. $\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$
66. (c), (a), (d), (b)
68. (a) 220 m [W]
    (b) 22 m/s [W]
69. (a) 5 s
    (b) 2 m/s^2
70. 2.5 m/s^2
71. 9.8 m/s
72. 3.2 s
73. 4.9 m/s
74. 108 m [N], 62.5 m [E]
75. (a) 188 km [E 10.4° S]
    (b) 27.9 km/h [E 10.4° S]
    (c) 45.0 km/h
76. 6.6 m/s^2 [E 41° N]
77. (a) 4.5 s
    (b) 1.3 km
78. (a) 25 s
    (b) 6.6 km
    (c) 8.7 km
79. (a) 736 km/h [E 2.92° S]
    (b) 112 km
80. 2.6×10^3 s or 0.73 h
81. (b) 2.9×10^3 N
82. 6.1×10^3 N
83. 2.4 m/s^2; 1.8 m/s^2
84. (a) 1.29×10^3 N
    (b) 242 kg
    (c) 0.401 m/s^2
85. (a) 2.1×10^3 N
    (b) 0.577
86. 6.7×10^2 N
87. 1.408 h
88. 8.0 m/s
89. (a) 8.8 m/s^2
    (b) 16 s
90. (a) 72.8 m/s
    (b) 6.75×10^3 N
91. 77 m/s
92. (a) 9.90 m/s
    (b) 14.0 m/s
94. (a) 4.2 s
    (b) 34 s
    (c) 190 m
96. (a) 2.7 m/s^2
    (b) 49 km/h
    (c) 150 km/h
98. (a) 620 N
    (b) 620 N
99. (a) 4.7 m/s^2

Unit 2

Are You Ready?, pp. 160–161

7. -10 m/s^2
9. (b) 1.5 N [toward the cup]
11. $x = 5$ or $x = -2$
12. (a) $E_g \propto h$
    (b) $E_k \propto v^2$
    (c) $\vec{a} \propto \vec{F}_a$
13. (a) $F_x = +10$ N; $F_y = 0$ N
    (b) $F_x = -6.4$ N;
    $F_y = -7.7$ N
    (c) $F_x = 0$ N; $F_y = -10$ N

4.1 Questions, p. 170

3. 166 J
4. 37.4°
5. (a) 120 N [down the ramp]
   (b) 120 N [up the ramp]
   (c) 2.7×10^3 J
   (d) $W_{worker} = 2.7 \times 10^3$ J
   $W_{friction} = -8.1 \times 10^2$ J
   $W_{total} = 1.9 \times 10^3$ J
6. 1.7×10^3 J

4.2 Questions, p. 176

1. yes
2. (a) no
   (b) 140 times
3. 3.8×10^5 J
4. -3.1×10^6 J
5. 6.2:1
6. 54 km/h
7. 94 km/h
8. 560 N
9. 485 m/s
10. 5.8 m/s
11. 7.22×10^9 J
12. (a) quadratic
    (c) 2 kg
    (d) $E_k = v^2$

4.3 Questions, p. 181

1. (a) 49 J
   (b) 6.3 m/s
2. 2.7×10^4 J
3. (a) 1.8 J
   (b) -1.8 J
   (c) 1.8 J
4. 0 J
5. 59 kg
6. 34 m
7. 1.8×10^5 J
8. (a) $mg(N - 1)\Delta y$
   (b) $mgN(N - 1)\dfrac{\Delta y}{2}$
9. 3.4×10^7 J

4.5 Questions, p. 191

1. (a) 6.2 m
4. (a) 2.3×10^5 J
   (b) 64 m/s
5. 12 m/s
6. 13 m/s
7. (a) yes
   (b) 1.9×10^5 J
   (c) 1.9×10^5 J
   (d) 17 m/s at B; 24 m/s at C
   (e) 21 m/s at B; 27 m/s at C
8. 3.8×10^2 W

4.6 Questions, p. 200

1. spring A
2. 500 N/m
3. equal
4. (a) 1.1 J
   (b) 2.5 J
5. (a) 0.095 m
   (b) 5.6 m/s^2 [down]
6. 140 N/m
7. 0.33 m
8. 4.5×10^{-2} J
9. 5.5 Hz; 0.25 m
10. 4.0×10^4 N/m

4.7 Questions, p. 208

1. increases
2. 3.8 m/s [away from the spring]

3. (a) 0.70 J
   (b) 13 m/s
   (c) 8.5 m
4. 0.021 m
5. 18 m/s
6. 14 cm [above rest position]; 0.63 m/s; 2.8 m/s^2 [toward rest position]
7. 0.68 m
8. (a) 0.032 m
   (b) 0.032 m
9. (a) 0.21 m
   (b) 2.2 m/s; 29 m/s^2 [away from the spring]
10. 2.0×10^1 m

Chapter 4 Self-Quiz, p. 213

1. (b)	6. (d)	11. T
2. (b)	7. (c)	12. F
3. (b)	8. F	13. F
4. (d)	9. F	14. T
5. (c)	10. T	15. T

Chapter 4 Review, pp. 214–219

1. (b)
2. (c)
3. (b)
4. (c)
5. (a)
6. (d)
7. (b)
8. T
9. T
10. F
11. F
12. T
13. T
14. F
17. no
18. (a) 649 N
    (b) 1.95×10^4 J
19. (a) -6.4×10^4 J
    (b) 6.4×10^4 J
    (c) 2.0×10^2 J
    (d) -6.4×10^4 J
21. 4.7×10^2 J
22. -4.0×10^1 J
23. -5.9×10^3 J
24. 0.49 kg
28. 0.65 J
29. 1.2×10^8 W
37. $v = \left(\sqrt{\frac{k}{m}}\right)\Delta x$
39. 19.1 m
46. (a) changing
    (b) constant
    (c) constant
47. decreases
49. (a) 2 times
    (b) same
50. 53 J
51. $v_{blue} = \frac{1}{2}v_{red}$
53. 2.0×10^1 m/s
54. 1.4×10^3 N/m
55. (a) 4.4 m
    (b) 27 m/s
56. (a) -7.0×10^2 J
    (b) 1.9 m/s
    (c) no
57. (a) 0.7 J
    (b) 7.8×10^{-4} W
58. 0.12 kg
59. (a) 18 N/m
    (b) 1.3 m/s
    (c) $\frac{3}{4}$

5.1 Questions, p. 227

1. (a) 2.9×10^3 kg·m/s [N]
   (b) 1.4×10^4 kg·m/s [forward]
   (c) 16 N·s [S]
3. 27.2 m/s [W]
4. 5.0 g
5. 78.3 kg
8. (a) 5.5 N·s [forward]
   (b) 46 m/s
9. (a) 0.86 kg·m/s [down]
   (b) same
10. (a) 2.4 N·s [up]
    (b) 0.13 s
11. (a) 2.6 N·s [forward]
    (b) 1.8×10^2 N [forward]
12. (a) 5.9 N·s
    (b) 2.6×10^2 N [E]

5.2 Questions, p. 232

2. 0.25 m/s [E]
3. 26.5 kg
4. 9×10^{-2} m/s [backward]
5. 2.2 m/s [E]
7. 9.8 m/s [left]

5.3 Questions, p. 239

1. momentum
2. 3.3 m/s
3. 5.0 m/s [N]
4. (a) yes
   (b) 5.4 m/s [initial direction]
5. (a) inelastic
   (b) 2.1 m/s [initial direction]
6. 0 m/s; 0 m/s
7. no
8. (a) 85 km/h [N]
   (b) 4.1×10^6 J; 4.0×10^6 J
   (c) 1.4×10^5 J

5.4 Questions, p. 248

1. no; yes
3. 15 m/s [left]
   4.2 m/s [right]
4. 180 m/s [initial direction of 1st chunk]
5. (a) 0.60 m/s [left]
       6.0 m/s [right]
   (b) 3.5×10^{-2} m
   (c) 4.1×10^{-2} m
6. (a) 7.2 m/s; 1.8 m/s
   (b) 2.7 m; 0.17 m

5.5 Questions, p. 253

1. 8.7 m/s [28° to original path]
2. 1.8 m/s [S 44° E]
3. (a) 21 m/s [S 31° E]
   (b) inelastic
4. 47 km/h [S 88° E]
5. (b) 2.75 m/s; 1.65 m/s
6. (a) [W 24° N]
   (b) 8.5×10^{-21} kg·m/s [W 24° N]
   (c) 3.7×10^5 m/s [W 24° N]
7. 1.9 m/s [60° to original path]

Chapter 5 Self-Quiz, p. 263

1. (b)	6. (c)	11. T
2. (a)	7. (d)	12. F
3. (d)	8. F	13. T
4. (b)	9. F	14. T
5. (b)	10. T	

Chapter 5 Review, pp. 264–269

1. (d)
2. (d)
3. (b)
4. (c)
5. F
6. T
7. T
8. F
9. T
10. F
11. T
15. 1.9×10^2 N [opposite initial direction]
16. 3.6×10^4 kg·m/s
24. (a) inelastic
    (b) elastic
    (c) perfectly inelastic
    (d) elastic
25. (a) 1.5 m/s [E]; 7.5 m/s [E]
    (b) 0.24 m/s [E]
        6.2 m/s [E]
    (c) 4.2 m/s [S]
        0.85 m/s [N]
26. 140 kg
27. 0.53 m/s; 6.4 m/s
28. (a) 6.6×10^4 N/m
    (b) 1.5 m/s [W]
        4.5 m/s [E]
    (c) 0.11 m
29. (a) 1.3 m/s [S]; 1.5 m/s [N]
    (b) 1.7×10^2 J
    (c) 35 kg·m/s [N]
30. 3×10^2 N [backward]
32. (b) 3.8 m/s; 1.8 m/s
33. equal masses
34. (a) equal
    (b) equal
36. (a) 6.1×10^3 N [away from the wall]
    (b) 24 N [away from the wall]
    (c) 2×10^5 N [away from the wall]
37. 0.2 kg
39. watermelon: 3.0 m/s; 2.3 m/s
    grapefruit: 1.8 m/s; 3.6 m/s or interchange initial velocities
40. (a) 4.4 m/s
    (b) 2.0×10^3 kg·m/s [forward]
41. (a) 3.0 N·s
    (b) -13 m/s
42. (b) first lily pad
43. 56 km/s [forward]
44. 1.5 m/s [left]; 1.1 m/s [right]
45. 5×10^{-3} m/s
46. (a) 0 J; 3 J
47. (a) 2×10^2 m/s
    (b) 2000 N [forward]
    (c) 60 m/s
    (d) yes
48. 58 kg
49. (a) 0.27 m/s
    (b) 4×10^{-5} J
50. 4.6 m/s [original direction]
51. 2.4 m/s [E 18° N]
    11 m/s [E 4° S];
52. (a) 8.1×10^{-2} s
    (b) 5.9×10^5 N [backward]
54. (d) two
56. (a) 4.0 m/s [E]; 16 m/s [E]
    (b) 4.0 m/s [W];
        8.0 m/s [E]
    (c) 12 m/s [E]; 24 m/s [E]
    (d) 12 m/s [W];
        0.22 m/s [E]

Unit 2 Self-Quiz, pp. 272–273

1. (c)	9. (d)	17. (d)
2. (a)	10. (a)	18. (a)
3. (d)	11. (d)	19. (a)
4. (b)	12. (b)	20. T
5. (d)	13. (c)	21. T
6. (a)	14. (a)	22. T
7. (a)	15. (b)	23. F
8. (b)	16. (c)	24. F

25. F
26. T
27. T
28. T
29. F
30. T
31. F
32. T

Unit 2 Review, pp. 274–281

1. (a)
2. (c)
3. (b)
4. (d)
5. (d)
6. (c)
7. (c)
8. (b)
9. (d)
10. (a)
11. (b)
12. (c)
13. (b)
14. (a)
15. (c)
16. (b)
17. (b)
18. (b)
19. (b)
20. (b)
21. F
22. T
23. T
24. T
25. F
26. T
27. F
28. T
29. F
30. T
31. T
32. F
33. F
34. T
35. F
36. F
37. no
38. no
40. 1000 J
42. 2.1×10^3 W
44. (a) -1.4×10^2 J
    (b) 2.7 m/s
46. (a) 3.7×10^4 J
    (b) 3.7×10^4 J
47. 1.4×10^{10} J
48. 2.2 m
49. (a) 1.3×10^4 J
    (b) 15 m/s
50. decreases
51. suspension spring
52. 6.6 m
53. (a) 3.3 kg·m/s [down]
    (b) 6.8 m/s
54. 2.2×10^3 N·s [up]
55. 0.12 N·s
58. yes
60. 24 m/s [W]
61. 4.1 m/s
62. (a) 1.6 m/s [left]
    (b) 2.0×10^2 J
63. 5.4×10^3 J
64. 5.0×10^1 J
65. 32°
66. 39 m
67. 1.9×10^2 J
68. 9.0×10^2 J
69. 5.2 m
70. 14 m/s
71. 69 kg
72. no
73. (a) 36 m/s
74. (a) 2.2×10^3 J
    (b) 2.2×10^3 J
    (c) 1.1×10^3 J
76. 600 N [outward]
77. 14 m/s
79. 0.29 J
80. 17 m/s
81. (a) 34 m
    (b) 45 m/s
82. (a) 4.8×10^5 N·s
    (b) 1.9×10^4 N [forward]
83. (a) 20 N·s
    (b) 20 kg·m/s
84. 0.48 m/s [E]
85. 28 %
86. 1.9×10^2 m/s
87. 9.9 m
88. (a) 3.2×10^2 N/m
    (b) 8.0×10^1 N
    (c) 1.0×10^1 J
    (d) 4.0×10^1 J
90. 33 m/s
91. 22 m/s [N]; 18 m/s [E]
92. (a) down
    (b) up
    (c) same
    (d) up
93. 0.8 m/s
94. (a) 1.3×10^3 J
98. 0.74 s; 1.4 Hz
99. (b) 16 N/m
101. 86 J

Unit 3

Are You Ready?, pp. 284–285

1. yes
16. yes
18. pencil
20. (a) down
26. (a) 9.8 m/s^2
    (b) 9.8 m/s^2
    (c) none

6.1 Questions, p. 296

1. 0.41 r_E above Earth's surface
2. 3.6×10^{-47} N
3. (a) decrease
   (b) $\frac{1}{16}$
4. (a) 399 N [toward Earth's centre]
   (b) 1.77 m/s^2 [toward Earth's centre]
5. 2.6×10^6 m
6. $0.75r_E$
7. 5.9×10^{-3} N/kg
8. $\dfrac{r}{\left(1+\sqrt{\dfrac{m_2}{m_1}}\right)}$, where m_1 is the larger mass
9. (a) 0.64 m/s^2 [toward Earth's centre]
   (b) 340 N [toward Earth's centre]
10. 3.67 m/s^2, or 0.375g
11. (a) 2.3×10^6 m
    (b) 3.3×10^3 N [toward Earth's centre]
13. $\frac{r}{10}$ from the Moon's centre

6.2 Questions, p. 303

5. 4.2×10^7 m
6. (a) 5.5×10^3 m/s
   (b) 2.0×10^{30} kg
7. 1.4×10^{12} m
8. (a) 1.62×10^4 m/s
   (b) 6.17 years
9. 2.06×10^4 km/h
10. 4.3×10^{30} kg
11. 0.319 days
12. 1.11×10^4 km/h
13. (a) 4.79×10^4 m/s
    3.51×10^4 m/s
    2.98×10^4 m/s
    2.41×10^4 m/s
14. 1.5×10^3 m/s; 8.9×10^3 s

Chapter 6 Self-Quiz, p. 311

1. (d)
2. (c)
3. (a)
4. (b)
5. (b)
6. (b)
7. (c)
8. T
9. F
10. T
11. F
12. F
13. F
14. F
15. F

Chapter 6 Review, pp. 312–317

1. (c)
2. (a)
3. (b)
4. (d)
5. (d)
6. (c)
7. (d)
8. (c)
9. (d)
10. (a)
11. F
12. T
13. T
14. F
15. T
16. F
17. T
18. F
19. F
20. (a) (ii)
    (b) (iii)
    (c) (i)
28. (c) < (b) < (d) < (a)
29. 5.4×10^{-8} N
30. 43 kg
31. (a) 2.6×10^2 m
    (b) 18 N
    (c) 2.4×10^{-10} N/kg
32. speed
33. $\sqrt{2}$ times greater
35. (a) RADARSAT Constellation
    (b) none
37. $F_A = F_C$; $F_B = 2F_A$
38. 1.4×10^{-7} N
39. (a) 2.5×10^{-8} N
    (b) 0.94 m
40. 2.1×10^{-8} N
41. 1.32×10^{-8} N/kg
42. 7.89×10^3 kg
43. (a) 9.78 N/kg
    (b) 0.998
44. (a) 6.6×10^{-6} N/kg
    (b) 3.4×10^{-10} N/kg
    (c) 6.8×10^{20} N
47. (a) 0.28 N/kg
    (b) 9.4×10^{20} kg
    (c) 0.16 N/kg
48. 1.2×10^{-10} N [toward m_2]
49. (a) 1.36 N/kg
    (b) 0.139:1
50. (a) 8.74 N/kg
    (b) no
51. no
52. (a) 806 km
    (b) 1 h 41 min
53. (a) 1.13×10^8 m
54. (a) 5.4×10^3 m/s
    2.0×10^4 km/h
    (b) 1.7×10^2 Earth years
55. 1.05 times greater
56. (a) 4.0×10^2 N
    (b) 7.4×10^3 m/s
    2.7×10^4 km/h
    (c) 1.7 h
57. $v_E : v_M = 9.0{:}1$
58. increase by 43 m/s
59. (a) 0.23 N/kg
    (b) 1.4×10^3 N
    (c) 3.1×10^3 m/s
    1.1×10^4 km/h
    (d) 23 h

60. (a) 4.3×10^6 times greater
    (b) 2.4×10^2 N
61. (a) 1.2×10^{-4} N
    (b) 2.3×10^{-4} N
    (c) 1.9:1
64. (a) incorrect
    (b) higher
65. (a) winter
    (b) no

7.1 Questions, p. 326

5. no

7.2 Questions, p. 333

1. 8.9×10^{-3} N
2. 2.7×10^{-2} N
3. 2.3×10^{-8} N
4. 2.25
5. 0.030 m
6. (a) 1.0×10^{-49} N
   (b) 2.3×10^{-10} N
7. $x = -7$
8. 24 cm beyond the smaller charge
9. (a) 5.7 N [down]
   (b) 5.7 N [E 30° N]
   (c) 1.7×10^{-13} N [down]
10. 6.26×10^{-7} C

7.3 Questions, p. 345

2. 1.4×10^{10} N/C
3. 2.8×10^{-5} C
4. 0 N/C
6. 1:9
7. 3.1×10^{10} N/C [down]

7.4 Questions, p. 354

1. (a) 1.9×10^{-17} J
   (b) -1.2×10^3 N/C
2. 0.75 V
3. (a) 1.52×10^{-17} J
   (b) -1.52×10^{-17} J
4. (a) 7.2×10^{-18} J
   (b) electric field
5. (a) vice versa
   (b) 4.0×10^{-15} J
   (c) 9.4×10^7 m/s
6. 4.52×10^5 V
7. (a) 4.0×10^5 m/s [up]
   (b) 4.0×10^6 m/s [5.6° from the x-axis]
8. (a) -80 V
   (b) -40 V
9. 0 J
10. 10 J

7.5 Questions, p. 361

1. (a) lower; lower
   (b) lower; higher
2. no
3. 0 J
4. 86 cm
5. +9.2 J
6. 9.4×10^{-2} J
7. 4.6×10^{-18} J
8. (a) -1.4×10^{-6} C
   (b) -5.7×10^5 N/C
   (c) inward

7.6 Questions, p. 365

1. 2.4×10^5 electrons
2. 7.2×10^{-2} N/C; 2.5×10^{-2} V
3. -4.8×10^{-14} C; 3.0×10^5 excess
4. (a) positive
   (b) 2.4×10^9 deficit of electrons
5. (a) 4.9×10^{-19} C
   (b) deficit of 3 electrons
6. (a) 2.5°
   (b) 0.41 N [up the thread]
7. (b) 1.6×10^{-18} kg

Chapter 7 Self-Quiz, p. 369

1. (c)
2. (b)
3. (d)
4. (a)
5. (c)
6. (b)
7. (c)
8. (b)
9. T
10. F
11. F
12. T
13. T
14. T
15. F
16. T

Chapter 7 Review, pp. 370–375

1. (b)
2. (b)
3. (a)
4. (a)
5. (c)
6. (d)
7. (a)
8. (d)
9. (d)
10. (a)
11. (a)
12. (a)
13. (d)
14. (a)
15. (b)
16. (b)
17. (b)
18. F
19. F
20. T
21. F
22. T
23. T
24. T
25. F
26. F
27. T
28. F
29. F
30. T
31. F
32. T
33. F
34. T
35. F
36. T
37. F
38. F
39. F
40. T
41. F
42. F
43. T
47. (b) no
50. same
51. 0 J
52. 4.3×10^{10} V
55. -3.2×10^{-19} C
56. both -3.4 nC
57. (a) both -2 μC
    (b) 0 C
59. 7.6×10^{-9} N
60. 0.80 m
61. +87 μC
62. 9.5 N
63. 4.4 cm
64. 13 N [away from the centre]
65. q_1 : 4.5×10^{-3} N [left]
    q_2 : 1.5×10^{-3} N [left]
    q_3 : 6.0×10^{-3} N [right]
67. 3.8×10^7 N/C
68. 6.7×10^4 N/C
69. (a) 41 cm
    (b) 58 cm
    (c) 1.1 m
70. -1.0 μC
71. 0.012 N
72. (a) 15 μC
    (b) negative
73. 5.1×10^5 N/C
74. 2.7×10^9 m/s^2
75. 5 V, or 5 J/C
76. $+8.4 \times 10^{-5}$ C
77. (a) 8.5×10^{-14} J
    (b) 1.0×10^7 m/s
78. (a) 2.4×10^{-17} J
    (b) 7.3×10^6 m/s
    (c) 3.3×10^{15} m/s^2
    (d) 2.2×10^{-9} s
79. (a) 18 000 V
    (b) 3.6×10^5 V
    (c) 2.8×10^9 V
80. 1.7×10^3 V/m
81. 140 V
82. 1.3×10^{-9} C
83. 0 J
84. 7.7×10^{-14} J
85. 58 J
86. 1.8×10^3 V; 1.5×10^3 N/C
87. (b) both -3.9×10^{-8} C
    (c) 2.4×10^{11} excess
88. (a) -1.1 μC
    (b) 6.6×10^{12} excess
89. 3.1×10^{12} excess
90. 1.2 nC

8.2 Questions, p. 391

2. positive
3. negative
4. (a) 1.9×10^{-16} N
   (b) 1.9×10^{-16} N
5. 33°
6. downward
7. (a) 4.2×10^{-4} N
   (b) $-z$ direction
   (c) 0 N
8. $+z$ direction
9. 3.4×10^{-3} N [$-z$ direction]
10. (a) 1.6×10^{-23} N
    (b) 3.6×10^{-5} m/s

8.3 Questions, p. 396

1. (a) 0.56 A
   (b) 90°
2. (a) 7.8×10^{-2} N
   (b) upward
3. (a) 3.2×10^{-4} N [down]
   (b) 3.1×10^{-4} N
4. 7.4 N
5. (a) 0 N
   (b) 0 N

8.4 Questions, p. 404

2. 1.0×10^6 m/s
3. 9.34×10^{-6} m
4. 0.5 T [N]
5. 0.844 mm
6. (a) 25 N/C
   (b) 1.0×10^{-4} m

Chapter 8 Self-Quiz, p. 415

1. (b)
2. (d)
3. (c)
4. (a)
5. (a)
6. (b)
7. (a)
8. (c)
9. (c)
10. F
11. F
12. T
13. F
14. F
15. F
16. T
17. T
18. T
19. T
20. F
21. T
22. T

Chapter 8 Review, pp. 416–421

1. (a)
2. (b)
3. (d)
4. (c)
5. (a)
6. (d)
7. (d)
8. T
9. T

10. T
11. T
12. F
13. T
14. F
15. T
16. (a) parallel
    (b) south
17. (b) in the gap
22. right
25. perpendicular
26. (a) 6.1×10^{-15} N
    (b) east
28. (a) west
    (b) vertically upward
    (c) no deflection
29. (a) 1.44×10^{-12} N
    (b) 8.62×10^{14} m/s^2
30. 7.7×10^{-14} N
32. east or west
33. 7.50 N
34. 1.6×10^{-8} T
35. H^+: $r = (1.04 \times 10^{-8})\frac{v}{B}$
    deuterium:
    $r = (2.09 \times 10^{-8})\frac{v}{B}$
    tritium: $r = (3.13 \times 10^{-8})\frac{v}{B}$
36. (a) 2.0×10^4 m/s
    (b) 2.0×10^4 m/s
37. (a) right
    (b) left
38. (a) into the page
    (b) right
    (c) down
39. (a) 0 N
    (b) 9.6×10^{-19} N [out of the page]
    (c) 0 N
40. curved downward
41. 0.72 N
42. 4.1×10^{-15} N
43. 1.4×10^{-17} N [up]
44. 5.9×10^{-12} N
45. 1.2×10^{-17} N
46. 3.0×10^{-4} T
47. 0.41 T
48. 4.6×10^{-21} N
49. (a) south to north
    (b) 8.8×10^{-13} N
50. (a) 2.2×10^{-12} N
    (b) down
51. 0 N
52. perpendicular
53. y-axis
54. 0 N
55. $-x$
56. 38°
57. to the wire
58. 25 A
60. (a) 0 N
    (b) 4.3×10^2 N
    (c) 6.1×10^2 N
61. 3.1 N
62. 4.0×10^{-2} N [up]
63. 0.36 N
64. 1.5 N [S]
65. (a) into the page
    (b) (i) no current
    (ii) upward
    (iii) downward
66. (a) positive
    (b) $\frac{1}{3}r$
    (c) $\frac{1}{10}r$

Unit 3 Self-Quiz, pp. 424–425

1. (b)	13. (a)	25. F
2. (d)	14. (c)	26. T
3. (b)	15. (a)	27. T
4. (c)	16. F	28. T
5. (d)	17. F	29. F
6. (c)	18. T	30. T
7. (b)	19. F	31. F
8. (c)	20. T	32. T
9. (a)	21. F	33. F
10. (a)	22. T	34. T
11. (c)	23. T	35. T
12. (b)	24. F	

Unit 3 Review, pp. 426–433

1. (c)
2. (d)
3. (b)
4. (c)
5. (b)
6. (a)
7. (d)
8. (d)
9. (b)
10. (a)
11. (c)
12. (b)
13. (c)
14. (a)
15. T
16. F
17. T
18. F
19. T
20. T
21. F
22. F
23. F
24. T
25. F
26. F
27. no
31. (a) conductor
    (b) insulator
    (c) insulator
    (d) conductor
    (e) conductor
    (f) insulator
32. (a) 3.8×10^{11} N
    (b) no change
33. 4300 m
34. 2.3×10^{-24} N
35. 230 N
36. (a) q_1; q_2
    (b) q_1
37. (a) top positive; bottom negative
40. (a) point downward
    (b) lie level
    (c) point upward
45. (b) down
47. 6.6×10^{-11} N·m^2/kg^2
48. 3.8×10^7 m
49. 9.8 m/s^2; 4.1×10^{-25} m/s^2
50. $2g$
51. 5 N/kg
52. (a) 0.30 N
    (b) 6.1×10^{-4} : 1
53. (a) 7.6×10^3 m/s
    (b) 97 min
54. 5.5×10^{30} kg
55. 3.3×10^3 m/s; 124 min
56. 7.6×10^3 m/s; 5.7×10^5 m
57. (a) 0.12 m
    (b) attractive
58. (a) 20 mN
    (b) 2.0 N
    (c) both 3.0×10^{-7} C
59. (a) 4.8×10^{-17} N
    (b) 5.3×10^{13} m/s^2
    (c) 1.3×10^6 m/s
60. (a) 3.5×10^{14} m/s^2 [$-x$]
    (b) 2.1×10^{16} m/s; no
61. -5.66×10^{-3} C
62. (b) (i) 0 N/C
    (ii) 4.1×10^5 N/C [$-x$]
63. (b) 0.019 N [down]
    (c) -3.2×10^{-5} C
64. (a) 1.3×10^{-15} J
    (b) 1.2×10^6 m/s
65. (a) 4.0×10^4 V
    (b) 8.0×10^{-5} J
    (c) no
66. -7.2×10^{-3} J
67. (a) -0.16 J
    (b) 0.16 J
68. 1.5×10^{-20} J
69. 9.2×10^{-19} J
70. 1 C; 6.25×10^{18}
71. 12
72. (a) 7.6×10^{-13} N
73. 0.06 T
74. (b) current, magnetic field
    (c) decrease
75. 55° away from $+x$ in $-y$ direction
76. 0.23 T
77. 1840:1
78. 1.3×10^5 m
79. 79:1
81. (a) Io: 17 km/s;
    Europa: 14 km/s;
    Ganymede: 1.0×10^1 km/s;
    Callisto: 8.0 km/s
    (b) 1.8×10^{27} kg
83. (a) 1.0×10^{-49} N
    (b) 2.3×10^{-10} N
    (c) same
85. 25 N [from centre]
87. equal but opposite; no
88. (a) yes
    (b) no
94. (a) x
    (b) 2.2 N
95. 4:1
96. (a) approximately 5.5×10^{-5} T
    (c) point N 45° W
97. 4 cm

Unit 4

***Are You Ready?*, pp. 436–437**

6. (c)
8. (a)
11. (a) 0.67 Hz
    (b) 1.3 m/s
13. 530 nm
14. (a) 1.25
    (b) 1.87
    (c) 35°
    (d) 432
15. (a) 5.5°
    (b) 19.7°
16. 75 m; 7.5 cm
17. 1 MHz
18. (a) 5.0 Hz
    (b) 0.2 m
    (c) 1 m/s
19. (a) 3.0 m
    (b) 1.3 m/s
20. 2.0×10^8 m/s
21. (a) $x = 2$; $y = 8$
    (b) $x = 7$; $y = -5$

9.1 Questions, p. 443

5. (a) 80°
   (b) 80°
6. 0.5 m/s
7. 2×10^{-2} m
8. 20 Hz
9. 6.0×10^{-7} m
10. 4.0×10^{14} Hz
11. 5.0×10^{-7} m
12. 13°
13. 3.4 m
14. 10 Hz

15. 4.0×10^2 m/s
16. 3.8×10^{-7} m
17. 0.97 m
19. 3:1
20. 1.0 m
21. (a) specular
(b) diffuse
(c) specular
(d) diffuse

9.2 Questions, p. 458

3. 470 nm
4. 1.0
5. 47.3°
6. 46.0°
7. total internal reflection
8. (a) 53.7°
(b) glass
9. (a) 67°
(b) 50.3°
10. (a) 24.1°
(b) no

9.3 Questions, p. 469

3. (a) 6.3×10^{-4} m
4. 7.2°
6. (b) 2 cm
(c) 12 cm/s
7. (a) 1.0 cm

9.4 Questions, p. 476

7. yes

9.5 Questions, p. 484

2. 530 nm
4. 19 m
5. (a) 420 nm
(b) 0.8 cm
6. 16:1
7. (a) 540 nm
(b) 0.44 mm
8. (a) 0.11 mm
(b) 3.6 cm

Chapter 9 Self-Quiz, p. 493

1. (a)
2. (c)
3. (b)
4. (d)
5. (a)
6. (b)
7. (a)
8. T
9. T
10. F
11. F
12. F
13. T
14. T
15. T
16. T

Chapter 9 Review, pp. 494–499

1. (d)
2. (b)
3. (a)
4. (b)
5. (a)
6. (b)
7. (d)
8. (b)
9. (a)
10. (b)
11. (a)
12. (d)
13. F
14. F
15. T
16. F
17. F
18. F
19. T
20. F
21. F
22. T
23. F
24. T
25. F
26. F
27. T
28. T
29. F
30. F
31. (a) (vii)
(b) (v)
(c) (i)
(d) (vi)
(e) (ii)
(f) (iv)
(g) (iii)
39. 1.5×10^4 wavelengths
40. 3.4 m
41. 4.2×10^{16} m
43. 1.0 h
44. 2.0×10^8 m/s
45. 1.8
46. 361 nm; 398 nm
47. 45.9°
48. (b) 1.27
(c) 2.4×10^8 m/s
49. (a) 37°
(b) 12 cm
50. (a) 62°
(b) 62°
51. (a) 42.9°
(b) 64.8°
(c) 48.8°
52. (b) 22°
53. (a) 1.9
(b) 830 nm
54. 47.4°
55. 64.8°
56. 38.9°
57. 80 cm
58. (b) 0.71; 1.41
59. 7.6 mm
61. 3.73 cm
62. 1.0 cm
63. 2.5 m
64. (a) increase
(b) decrease
(c) increase
65. (a) 2.8 cm
(b) 2.0×10 cm/s
66. 49°
67. (a) 5.4 cm
(b) 6
70. 0.040 mm
71. 1.5 cm
72. 480 nm
74. 730 nm
75. 6.9×10^{-5} m
76. 560 nm
77. 9.6 mm
78. 396 nm
79. 0.12 mm
80. 1.4×10^{-5} m
81. 8.3 m
82. 0.096°; 0.23 cm; 0.11°; 0.27 cm

10.1 Questions, p. 511

1. 550 nm
2. 2.71×10^{-7} m
3. 2.22×10^{-7} m
4. 1.11×10^{-7} m
7. 4.52×10^{-7} m

10.2 Questions, p. 519

1. 1.1×10^{-4} m
2. 5.0×10^{-6} m
3. 6.0 cm
4. 0.20 mm
5. 1.5 cm
6. 5.90×10^{-7} m
9. $3.1 \times 10^{-4\circ}$

10.3 Questions, p. 525

2. 3.6×10^{-6} m
3. 518 nm; 297 nm; 246 nm
4. 7.8°
5. 1.4°
6. 1.6×10^{-5} m
7. 480 nm
8. 6th order
9. (a) 5.7°; 12°; 17°
(b) no first bright fringe
10. 1.11

10.4 Questions, p. 531

1. 6.0×10^2 nm
2. 5.5×10^{14} Hz
3. 3.4 m–2.8 m
4. 3.0×10^{18} Hz
5. (a) 2.5×10^9 Hz
6. 12 cm
7. 440 m
8. 42°
11. (a) constructive
(b) destructive

10.5 Questions, p. 537

8. 66°
9. 57°
12. yes

Chapter 10 Self-Quiz, p. 549

1. (c)
2. (a)
3. (c)
4. (c)
5. (a)
6. (b)
7. (a)
8. F
9. T
10. T
11. F
12. F
13. T
14. F
15. F
16. T
17. F

Chapter 10 Review, pp. 550–555

1. (d)
2. (d)
3. (d)
4. (c)
5. (a)
6. (a)
7. (d)
8. (a)
9. (a)
10. (b)
11. (c)
12. (c)
13. (d)
14. (b)
15. (a)
16. T
17. T
18. T
19. F
20. F
21. F
22. T
23. F
24. T
25. F
26. F
27. F
28. F
29. T
30. F
31. T
32. T
33. F
35. 2*t*
39. no
45. (a) 1.35×10^{-6} m; 4.50×10^{-7} m; 2.70×10^{-7} m
(b) 4.50×10^{-7} m
46. 1.50
47. (a) 390 nm
(b) yes; yes
(c) 190 nm
48. 8.1×10^{-6} m
49. (a) 9.71×10^{-8} m
(b) 2.91×10^{-7} m; 4.85×10^{-7} m
50. 160 nm
51. 93 nm
52. 5.3×10^{-6} m
53. 640 nm–700 nm

54. (a) 8.8×10^{-8} m
(b) 1.8×10^{-7} m
55. 8.03×10^{-7} m
56. 540 μm
57. 710 nm
58. (a) 1.1 cm
(b) 410 nm
59. 0.87 m
60. 32°
63. (a) 1.6×10^{-6} m
(b) 6400 lines/cm
64. (a) 1.2×10^{-6} m
(b) 8300 lines/cm
(c) 3.3×10^{4} rotations
(d) 660 rpm
65. 0.24 m
66. 3
67. 0.15 m
68. (a) 3.2×10^{14} Hz
(b) 8.3×10^{-9} s
69. 3.4×10^{-3} m
70. $\frac{1}{8}I_{in}$
71. 63°
72. (a) 61°
(b) 29°
(c) 1.8
(d) 53°
75. $\frac{1}{8}I_0$

Unit 4 Self-Quiz, pp. 558–559

1. (a)
2. (d)
3. (c)
4. (b)
5. (c)
6. (a)
7. (c)
8. (c)
9. (a)
10. (c)
11. (b)
12. (b)
13. (b)
14. (b)
15. (d)
16. (b)
17. (c)
18. F
19. T
20. T
21. T
22. F
23. T
24. T
25. F
26. F
27. F
28. T
29. T
30. T
31. F
32. F
33. T
34. F
35. T
36. T

Unit 4 Review, pp. 560–567

1. (b)
2. (a)
3. (d)
4. (c)
5. (d)
6. (b)
7. (c)
8. (a)
9. (a)
10. (a)
11. (a)
12. (c)
13. (b)
14. (d)
15. (a)
16. (b)
17. (c)
18. (a)
19. F
20. F
21. F
22. T
23. F
24. T
25. T
26. T
27. T
28. F
29. T
34. 1 cm opening
39. wave model
50. 1:1
66. 1:2
69. distance decreases
71. (a) maximum
(b) minimum
(c) $I_{out} = \cos^2\theta\, I_{in}$
73. 0.1°
74. 22.5°
75. 18.5°
76. 16.7°
77. 24.4°
78. 1.48
79. 303 nm
80. 24°
81. (a) 0.017 m
(b) 450 Hz
82. 610 nm
83. 17:1
84. 570 nm
85. 1.4×10^{-5} m
86. 6.65×10^{-3} m
87. (a) 580 nm
(b) 5.2×10^{14} Hz
88. 1.9×10^{-2} m
89. 500 nm
90. 1.8×10^{-3} m
91. 1.6×10^{-3} m
92. 8.20×10^{-5} m
93. 3.50×10^{-4} m
94. 7.3×10^{-8} m
95. 1.05×10^{-7} m
96. 1.81×10^{-7} m;
3.62×10^{-7} m
97. 1.17
98. 1.93×10^{-7} m
99. 5.80×10^{2} nm; yellow
100. 1.1×10^{-5} m
101. 288
102. 2.9×10^{-5} m
103. (c) $\frac{\lambda}{2n_{soap}}$
(d) 6.52×10^{-7} m
104. 4.9×10^{-3} m
105. 650 nm
106. (a) 33.0°
(b) no
107. 3.1×10^{-5} m
108. 5.3×10^{-3} m
111. 0.32 m; 0.55 m
112. 12 cm
115. 411 nm: 11°, 0.58 m
664 nm: 18°, 0.94 m
116. 530 nm
117. 5.2×10^{-7} m; 7.6×10^{-7} m
118. (b) 380 nm
121. (a) 38 m
(b) 19 m
122. (a) yes
(b) double slit
(c) bright
123. 9.3×10^{-6} m
124. 0.0074°
125. (a) telescope B
(b) 200 times; 100 times
(c) no
126. 0.35 m
128. 6700 lines/cm
129. 2.1×10^{-7} m

Unit 5

Are You Ready?, pp. 570–571

1. (a) 5 m/s [E]
(b) 45 m/s [E]
(c) 75 m/s [E]
2. (a) non-inertial
(b) non-inertial
(c) inertial
(d) inertial
(e) non-inertial
(f) inertial
4. (d) 3.0×10^{8} m/s
12. (a) (iii)
(b) (iv)
(c) (i)
(d) (ii)
14. (a) 4.7×10^{14} Hz
(b) 5.9×10^{14} Hz
(c) 9.4×10^{14} Hz
15. (c)
17. (a) 3rd harmonic
(b) 17 Hz
(c) 120 Hz
19. (a) $x = \pm 3 \times 10^{8}$
(b) $x = 3.3 \times 10^{15}$
(c) $x = 7.0$
20. (b) 100.0
(c) 2.50
(d) $y = 2.50x + 100.0$
(e) linear
21. (a) vertical and horizontal asymptote

11.1 Questions, p. 579

3. (a) c
(b) c

11.2 Questions, p. 587

3. (a) no
(b) no
(c) different
(d) no
(e) no
4. airport clock
6. (a) Roger
(b) 57 s
7. 3.2 s

11.3 Questions, p. 597

1. 724 m
2. 5.0 ly
3. no
4. (a) 5.0×10^{-19} kg·m/s
(b) 3.52×10^{-18} kg·m/s
(c) 7:1
5. $0.98c$
6. 1.39×10^{-28} m/s
7. no

11.4 Questions, p. 603

1. 3.2 kg
3. 4.67×10^{-11} kg
4. 3.01×10^{-10} J
5. (a) 2.28×10^{-30} kg
(b) 0.44 MeV
(c) 2.00×10^{8} m/s
6. 4.25×10^{11} kg
7. 8.1 MeV
8. 2.60×10^{8} m/s
9. $0.40c$
10. 32.9 keV
11. 2.7×10^{6} km
12. 6.1837×10^{16} J

Chapter 11 Self-Quiz, p. 607

1. (a)
2. (b)
3. (c)
4. (b)
5. (c)
6. (a)
7. (a)
8. T
9. F
10. T
11. F
12. T
13. T
14. T

Chapter 11 Review, pp. 608–613

1. (d)
2. (a)
3. (d)
4. (d)
5. (c)
6. (c)
7. (a)
8. (b)
9. (a)
10. (b)
11. (a)
12. (a)
13. (b)
14. (c)

15. T
16. F
17. T
18. T
19. T
20. T
21. F
22. F
23. F
24. F
25. T
26. F
27. T
28. F
29. T
30. F
31. F
32. T
33. F
34. F
37. c
43. (a) $c - v$; $c + v$
    (b) both c
44. no
47. (a) 2.2 ms
    (b) 660 m
50. both c
53. both $\frac{E}{2c^2}$
54. (a) 0 m/s [N]
    (b) 3 m/s [N]
    (c) 8 m/s [N]
55. c
56. 53 min
57. 3.6×10^{-8} s
58. 1.9×10^{8} m/s
59. 2.2×10^{8} m/s
60. (a) 27 min
    (b) 86 min
61. 4.2×10^{7} m/s
62. 2.6×10^{8} m/s
63. 27 flashes/min
64. (a) 65 m
    (b) 39 m
65. 1.1 Mly
66. 2.97×10^{8} m/s
67. 0.71 m
68. 2.7×10^{6} s; 31 days
69. (a) $0.58mc$
    (b) $0.76c$
    (c) $0.87c$
70. 4.01×10^{-19} kg·m/s
    6.68×10^{-19} kg·m/s
    3:5
71. 4.7 kg
72. (a) 4.1×10^{8} J
    (b) 2.6×10^{21} MeV
73. 1.0 MeV
74. 2.8×10^{3} MeV
75. 0.85 MeV; 0.34 MeV
76. $0.92c$
77. 0.59 g
79. $\dfrac{f}{\sqrt{1 - \left(\dfrac{v^2}{c^2}\right)}}$

12.2 Questions, p. 631

1. 1.2×10^{15} Hz
2. (a) cesium
   (b) 4.71×10^{14} Hz
   (c) visible light
4. (a) 7×10^{-26} J
   2×10^{-34} kg·m/s
   (b) 3.14×10^{-19} J
   1.05×10^{-27} kg·m/s
   (c) 2.8×10^{-15} J
   9.5×10^{-24} kg·m/s
5. X-ray photon
6. (a) 2.18×10^{-18} J
   (b) 3.28×10^{15} Hz
   9.14×10^{-8} m
   ultraviolet

12.3 Questions, p. 639

1. 4.9×10^{3} m/s
2. 1800:1
3. (a) 2.39×10^{-38} m
   (b) 2.39×10^{-40} m
   (c) undefined

Chapter 12 Self-Quiz, p. 659

1. (c)
2. (c)
3. (b)
4. (c)
5. (a)
6. (a)
7. (a)
8. F
9. T
10. F
11. T
12. T
13. F
14. T
15. T
16. T
17. F
18. F

Chapter 12 Review, pp. 660–665

1. (d)
2. (a)
3. (d)
4. (b)
5. (a)
6. (a)
7. (a)
8. (a)
9. (b)
10. (b)
11. (d)
12. (c)
13. (d)
14. T
15. F
16. F
17. T
18. F
19. F
20. F
21. T
22. F
23. T
24. T
25. F
26. T
27. F
28. T
29. F
30. (a) (v)
    (b) (iv)
    (c) (vi)
    (d) (i)
    (e) (iii)
    (f) (ii)
49. 4640 K
50. (a) 3.10×10^{2} K
    (b) 9.35×10^{-6} m
52. 1.0 eV
54. (a) 3.10×10^{-19} J
    (b) 1.93 V
55. 2.0×10^{-7} m
57. (a) 9.04×10^{-19} J; 5.65 eV
    (b) platinum
59. (a) 1.13×10^{15} Hz
    (b) 4.5×10^{-18} J; 28 eV
60. (a) 1.7×10^{15} Hz
    (b) ultraviolet
61. 1.1×10^{-27} kg·m/s
62. (a) 3.27×10^{-10} m
    (b) 9.19×10^{17} Hz
63. 3.41×10^{-27} kg·m/s
65. 0.6 nm
66. 1.25×10^{-10} m
67. (a) 3.31×10^{6} m/s
    (b) 4.98×10^{-18} J,
    or 31.2 eV
    (c) 31.2 V
68. 1:1800
72. (c) 6.1×10^{14} Hz
    (d) 6.4×10^{-34} J/Hz or J·s

Unit 5 Self-Quiz, pp. 668–669

1. (c)
2. (c)
3. (c)
4. (c)
5. (b)
6. (c)
7. (a)
8. (b)
9. (b)
10. (a)
11. (a)
12. (a)
13. (a)
14. (c)
15. (b)
16. T
17. T
18. F
19. F
20. F
21. T
22. F
23. F
24. T
25. F
26. T
27. T
28. T
29. T
30. T
31. T

Unit 5 Review, pp. 670–677

1. (c)
2. (c)
3. (b)
4. (b)
5. (b)
6. (a)
7. (a)
8. (c)
9. (b)
10. (a)
11. (b)
12. (d)
13. (a)
14. (a)
15. (b)
16. (b)
17. (b)
18. (b)
19. (c)
20. T
21. F
22. T
23. F
24. T
25. F
26. F
27. F
28. F
29. F
30. T
31. F
32. F
33. F
34. F
35. F
36. F
37. T
38. F
39. T
40. F
41. F
42. T
43. F
44. F
45. F
46. T
47. no
48. (a) no
    (b) no
52. (a) 16 m
    (b) yes
55. $\frac{\lambda}{5}$
64. (a) uud
    (b) udd
67. c
68. (a) c
    (b) c
69. (a) c
    (b) c
70. 2.0×10^{1} min
71. 4.38×10^{-10} s
72. 6.1 y
73. $0.60c$
74. (a) observer 1
    (b) 57 s
76. (a) east

78. 1.54×10^{8} m/s
79. (a) 2.60×10^{8} m/s
    (b) 2.90×10^{8} m/s
    (c) not possible
80. (a) 5.0×10^{8} kg·m/s
    (b) 1.8×10^{17} J
81. (a) 2.7×10^{-13} J
    (b) 2.9×10^{8} m/s
82. (a) 3.04×10^{-18} kg·m/s
    (b) 9.23×10^{-10} J
    (c) 5.77×10^{9} eV
83. (a) 9×10^{13} J
    (b) 96 g/day
84. 1.2 eV
85. 4.14×10^{-15} eV·s
86. (a) 1.4×10^{15} Hz
    (b) 9.5×10^{-19} J
    (c) 3.2×10^{-27} kg·m/s
87. (b) red
88. 1.5×10^{-7} m
89. (a) 6.31×10^{-24} kg·m/s
    (b) 6.93×10^{6} m/s
    (c) 6.31×10^{-24} kg·m/s
90. (a) 3.3×10^{-38} m
    (b) no
91. 1.0×10^{-11} m
92. 1:1
93. 1.8×10^{17} J
94. (a) $+e$
    (b) yes
95. (a) 0
    (b) 0
    (c) yes
96. (a) no
    (b) yes
104. both 0.489 MeV
106. (a)
107. (a) $+e$
    (b) $-e$
    (c) no

Glossary

A

air resistance ($\vec{F}_{air}$) the friction between objects and the air around them (p. 63)

air wedge the air between two sheets of flat glass angled to form a wedge (p. 508)

amplitude (A) the maximum displacement of a wave (p. 197)

analyzer a second polarizer used to verify that the light from the first polarizer is polarized (p. 534)

angle of deviation the angle between the incident ray and the final outgoing ray after reflection or refraction (p. 451)

angle of incidence the angle between the incident ray and the normal (p. 442)

angle of reflection the angle between the reflected ray and the normal (p. 442)

angle of refraction the angle that a light ray makes with respect to the normal to the surface when it has entered a different medium (p. 446)

antimatter a form of matter in which each particle has the same mass and an opposite charge as its counterpart in ordinary matter (p. 647)

apparent weight the magnitude of the normal force acting on an object in an accelerated (non-inertial) frame of reference (p. 111)

applied force ($\vec{F}_a$) a force due to one object pushing or pulling on another (p. 63)

artificial gravity a situation in which the value of gravity has been changed artificially to more closely match Earth's gravity (p. 128)

artificial satellite an object that has been intentionally placed by humans into orbit around Earth or another body; referred to as "artificial" to distinguish from natural satellites such as the Moon (p. 297)

average acceleration ($\vec{a}_{av}$) the change in velocity divided by the time interval for that change (p. 14)

average speed (v_{av}) the total distance travelled divided by the total time to travel that distance (p. 9)

average velocity ($\vec{v}_{av}$) the displacement divided by the time interval for that change; the slope of a secant on a position–time graph (p. 9)

B

biochemical energy a type of chemical potential energy stored in the cells and other basic structures of biological organisms (p. 188)

blackbody an object that absorbs all radiation reaching it (p. 626)

blackbody radiation radiation emitted by an ideal blackbody (p. 626)

boson the particle responsible for transmitting electromagnetic, strong, and weak forces (p. 651)

Brewster's angle the angle at which the direction of the reflected portion of the wave is perpendicular to the direction of the refracted portion of the wave (p. 535)

Brewster's law $\tan \theta_B = \frac{n_2}{n_1}$ (p. 535)

C

central maximum the bright central region in the interference pattern of light and dark lines produced in diffraction (p. 512)

centrifugal force the fictitious force in a rotating (accelerating or non-inertial) frame of reference (p. 126)

centrifuge a rapidly rotating device used to separate substances and simulate the effects of gravity (p. 125)

centripetal acceleration ($\vec{a}_c$) the instantaneous acceleration that is directed toward the centre of a circular path (p. 114)

centripetal force (F_c) the net force that causes centripetal acceleration (p. 121)

coefficient of kinetic friction (μ_K) the ratio of kinetic friction to the normal force (p. 85)

coefficient of static friction (μ_S) the ratio of the maximum force of static friction to the normal force (p. 86)

coherent composed of waves having the same frequency and fixed phases (p. 462)

collision the impact of one body with another (p. 228)

component of a vector in two dimensions, either of the x-vector and y-vector that are combined into an overall vector (p. 25)

Compton effect the elastic scattering of photons by high-energy photons (p. 624)

conductor any substance in which electrons are able to move easily from one atom to another (p. 321)

conservation of kinetic energy the total kinetic energy of two objects before a collision is equal to the total kinetic energy of the two objects after the collision (p. 233)

conservation of mass–energy the principle that rest mass and energy are equivalent (p. 599)

constructive interference the phenomenon that occurs when two interfering waves have displacement in the same direction where they superimpose (p. 462)

contact force a force that acts between two objects when they touch each other (p. 62)

Coriolis force a fictitious force that acts perpendicular to the velocity of an object in a rotating frame of reference (p. 127)

coulomb the SI unit of electric charge; symbol C (p. 321)

Coulomb's constant (k) the proportionality constant in Coulomb's law; $k = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$ (p. 327)

Coulomb's law the force between two point charges is inversely proportional to the square of the distance between the charges and directly proportional to the product of the charges (p. 327)

crest the upper part of a wave (p. 441)

critical angle the smallest angle of incidence at which a light ray passing from one medium to another less refractive medium can be totally reflected from the boundary between the two (p. 453)

D

damped harmonic motion periodic motion affected by friction (p. 207)

de Broglie wavelength the wavelength associated with the motion of a particle possessing momentum of magnitude p (p. 632)

destructive interference the phenomenon that occurs when two interfering waves have displacement in opposite directions where they superimpose (p. 462)

diffraction the bending and spreading of a wave when it passes through an opening; dependent on the size of the opening and the wavelength of the wave (p. 459)

diffraction grating a device with a large number of equally spaced parallel slits that produces interference patterns (p. 520)

diffuse reflection the reflection of light from a surface where all the reflected rays are directed in many different directions (p. 442)

dispersion the separation of a wave into its component parts according to a given characteristic, such as frequency or wavelength (p. 450)

displacement ($\Delta\vec{d}$) the change in position of an object (p. 8)

dynamics the study of the causes of motion (p. 8)

E

elastic collision a collision in which momentum and kinetic energy are conserved (p. 233)

elastic potential energy the potential energy due to the stretching or compressing of an elastic material (p. 195)

electric dipole a pair of equal and opposite electric charges with centres separated by a small distance (p. 339)

electric field ($\vec{\varepsilon}$) the region in which a force is exerted on an electric charge; the electric force per unit positive charge; unit is N/C (p. 334)

electric field lines the continuous lines of force around charges that show the direction of the electric force at all points in the electric field (p. 338)

electric force (F_E) a force with magnitude and direction that acts between two charged particles (p. 327)

electric potential (V) the value, in volts, of potential energy per unit positive charge for a given point in an electric field; 1 V = 1 J/C (p. 350)

electric potential difference (ΔV) the amount of work required per unit charge to move a positive charge from one point to another in the presence of an electric field (p. 350)

electric potential due to a point charge the electric potential is inversely proportional to the distance from the charge and proportional to the amount of charge producing the field (p. 355)

electric potential energy (E_E) the energy stored in a system of two charges a distance Δd apart, or the energy stored in an electric field that can do work on a positively charged particle (p. 347)

electromagnetic radiation radiation that consists of interacting electric and magnetic fields that travel at the speed of light (p. 526)

electromagnetic spectrum the range of frequencies and wavelengths of all electromagnetic waves (p. 527)

elementary charge (e) the magnitude of the electric charge carried by a proton, equal to the absolute value of the electric charge of an electron (p. 362)

equilibrium a state in which an object has no net force acting on it (p. 77)

ether the proposed medium through which electromagnetic waves were once believed to propagate (p. 576)

explosion a situation in which a single object or group of objects breaks apart (p. 229)

F

fermion a fundamental particle that forms matter (p. 651)

fibre optics a technology that uses glass or plastic wire (fibre) through which data are transmitted using internally reflected light impulses (p. 455)

fictitious force an apparent but non-existent force invented to explain the motion of objects within an accelerating (non-inertial) frame of reference (p. 109)

field theory a scientific model that describes forces in terms of entities that exist at every point in space (p. 403)

first-order maximum the first maximum of intensity on either side of the zero-order maximum in an interference pattern from a diffraction grating (p. 521)

force ($\vec{F}$) a push or a pull (p. 62)

force of gravity ($\vec{F}_g$) the force of attraction between all objects due to mass (p. 62)

frame of reference a coordinate system relative to which motion is described or observed (pp. 44, 108, 575)

Fraunhofer diffraction an interference pattern that shows distinctive differences between the bright central fringe and darker flanking fringes (p. 512)

free-body diagram a simple line drawing that shows all the forces acting on an object (p. 63)

free fall the motion of a falling object where the only force acting on the object is gravity (p. 20)

frequency (f) the number of rotations, revolutions, or vibrations of an object per unit of time; the inverse of period; SI unit Hz (p. 117)

friction ($\vec{F}_f$) a force that opposes the sliding of two surfaces across one another; acts opposite to motion or attempted motion (p. 63)

fundamental physical constant a measurable natural value that never varies and can be determined by experimentation (p. 362)

G

geosynchronous orbit the orbit around Earth of an object with an orbital speed matching the rate of Earth's rotation; the period of such an orbit is exactly one Earth day (p. 300)

glancing collision a collision in which the first object, after an impact with the second object, travels at an angle to the direction it was originally travelling (p. 250)

gluon a particle that mediates the strong nuclear force (p. 651)

gravitational constant a constant that appears in the universal law of gravitation; the constant is written as G and has a value of 6.67×10^{-11} N·m^2/kg^2 (p. 288)

gravitational field a collection of vectors, one at each point in space, that determines the magnitude and direction of the gravitational force (p. 293)

gravitational field strength the magnitude of the gravitational field vector at a point in space (p. 293)

gravitational potential energy (E_g) stored energy an object has because of its position and the applied gravitational force (p. 177)

H

hadrons a class of particles that contains the neutron, the proton, and the pion; composed of combinations of quarks and anti-quarks (p. 648)

head-on elastic collision an impact in which two objects approach each other from opposite directions; momentum and kinetic energy are conserved after the collision (p. 240)

Heisenberg uncertainty principle a mathematical statement that says that if Δx is the uncertainty in a particle's position, and Δp is the uncertainty in its momentum, then $\Delta x \Delta p \geq \frac{h}{4\pi}$, where h is Planck's constant (p. 637)

Higgs boson the theoretical particle thought to play a role in giving mass to other particles (p. 652)

Hooke's law the amount of force exerted by a spring is directly proportional to the displacement of the spring (p. 192)

Huygens' principle every point on a wave front can be considered as a point source of tiny secondary wavelets that spread out in front of the wave at the same speed as the wave itself (p. 470)

I

ideal spring any spring that obeys Hooke's law; it does not experience any internal or external friction (p. 193)

impulse the product of force and time that acts on an object to produce a change in momentum (p. 223)

incoherent composed of waves that have no fixed phase relationship to each other and different frequencies (p. 477)

index of refraction the ratio of the speed of light in a vacuum to the speed of light in another medium (p. 444)

inelastic collision a collision in which momentum is conserved, but kinetic energy is not conserved (p. 234)

inertia a measure of an object's resistance to change in velocity (p. 70)

inertial frame of reference a frame of reference that moves at a zero or constant velocity; a frame in which the law of inertia holds (pp. 108, 575)

instantaneous acceleration ($\vec{a}$) the acceleration at a particular instant in time (p. 14)

instantaneous speed (v) the speed of an object at a particular instant; the magnitude of the slope of the tangent to a position–time graph (p. 12)

instantaneous velocity ($\vec{v}$) the velocity of an object at a particular instant; the slope of the tangent to a position–time graph (p. 12)

insulator any substance in which electrons are not free to move easily from one atom to another (p. 321)

interference the phenomenon that occurs when two waves in the same medium interact (p. 462)

interference fringe one of a series of alternating light and dark regions that result from the interference of waves (p. 479)

inverse-square law a mathematical relationship in which one variable is proportional to the inverse of the square of another variable; the law applies to gravitational forces and other phenomena, such as electric field strength and sound intensity (p. 289)

isolated system a system that cannot interact or exchange energy with external systems; also called a closed system (p. 188)

J

joule the SI unit of work and energy; a force of 1 N acting over a displacement of 1 m does 1 J of work; symbol J (p. 165)

K

kinematics the study of motion without considering the forces that produce the motion (p. 8)

kinetic energy (E_k) the energy an object has because of its motion (p. 171)

kinetic friction ($\vec{F}_K$) a force exerted on a moving object by a surface in the direction of motion opposite to the motion of the object (p. 63)

L

law of conservation of charge charge can be transferred from one object to another, but the total charge of a closed system remains constant (p. 321)

law of conservation of energy energy is neither created nor destroyed in an isolated system; it can only change form (pp. 184, 188)

law of conservation of momentum when two objects collide in an isolated system, the collision does not change the total momentum of the two objects; whatever momentum is lost by one object in the collision is gained by the other; the total momentum of the system is conserved (p. 229)

law of electric charges like charges repel each other; unlike charges attract (p. 320)

law of reflection for reflection from a flat surface, the angle of incidence is always equal to the angle of reflection (p. 442)

length contraction the shortening of length or distance in an inertial frame of reference moving relative to an observer in another inertial frame of reference (p. 589)

leptons a class of particles that includes the electron, the muon, the tauon, and the three types of neutrinos; not composed of smaller particles (p. 648)

linear actuator a device that converts energy into linear motion (p. 91)

linear momentum ($\vec{p}$) a quantity that describes the motion of an object travelling in a straight line as the product of its mass and velocity (p. 222)

linearly polarized (plane polarized) the quality of light waves that are polarized in one direction, perpendicular to the direction of propagation (p. 532)

liquid crystal display (LCD) a thin, flat display that makes use of polarizers and optical activity (p. 536)

M

magnetic field line one of a set of lines drawn to indicate the strength and direction of a magnetic field (p. 378)

magnetic resonance imaging (MRI) a process in which magnetic fields interact with atoms in the human body, producing images that doctors can use to diagnose injuries and diseases (p. 408)

magnetorheological fluid a fluid containing suspended iron particles that, when subjected to a magnetic field, changes to a solid (p. 406)

Malus's law $I_{out} = I_{in}\cos^2\theta$ (p. 533)

mass a measure of the amount of matter in an object (p. 71)

matter wave the wave-like behaviour of particles with mass (p. 632)

maxima points of brightness, or maximum intensity, in an interference pattern (p. 479)

mechanical energy the sum of an object's kinetic and potential energies (p. 177)

minima points of darkness, or minimum intensity, in an interference pattern (p. 480)

monochromatic composed of only one colour; light with one wavelength (p. 477)

N

net force ($\Sigma\vec{F}$) the sum of all the forces acting on an object (p. 65)

newton the SI unit of force; symbol N (p. 62)

Newton's first law of motion if the external net force on an object is zero, the object will remain at rest or continue to move at a constant velocity (p. 70)

Newton's second law of motion if the net external force on an object is not zero, the object will accelerate in the direction of the net force; the magnitude of the acceleration is directly proportional to the magnitude of the net force and inversely proportional to the object's mass (p. 71)

Newton's third law of motion for every action force, there exists a simultaneous reaction force that is equal in magnitude but opposite in direction (p. 73)

nodal line a line or curve along which destructive interference results in zero displacement (p. 463)

node a point along a standing wave where the wave produces zero displacement (p. 463)

non-contact force a force that acts between two objects without the objects touching; also called action-at-a-distance force (p. 62)

non-inertial frame of reference a frame of reference that accelerates with respect to an inertial frame; the law of inertia does not hold (p. 108)

normal the line drawn at a right angle to the boundary at the point where an incident ray strikes the boundary (p. 442)

normal force ($\vec{F}_N$) a force perpendicular to the surface between objects in contact (p. 62)

O

open system a system that can interact with another external system (p. 188)

optical activity the rotation of the direction of polarization when linearly polarized light interacts with certain molecules (p. 536)

optical density the property of a material that determines how light behaves when it travels through the material (p. 444)

orbital radius the distance between the centre of a satellite and the centre of its parent body (p. 299)

order number the value of m for a given maximum in a diffraction-grating interference pattern; sequentially numbers the maxima on either side of the zero-order maximum (p. 521)

P

pair creation the transformation of a photon into two particles with mass (p. 626)

path length the distance from point to point along a nodal line (p. 464)

path length difference the difference between path lengths, or distances (p. 465)

perfectly elastic collision an ideal collision in which external forces are minimized to the point where momentum and kinetic energy are perfectly conserved (p. 235)

perfectly inelastic collision an ideal collision in which two objects stick together perfectly so they have the same final velocity; in this situation, momentum is perfectly conserved, but kinetic energy is not conserved (p. 235)

period (T) the time required for a rotating, revolving, or vibrating object to complete one cycle (p. 116)

periodic wave a wave with a repeated pattern over time or distance (p. 440)

perpetual motion machine a machine that can operate forever without restarting or refuelling (p. 205)

phase the offset of the wave from a reference point (p. 441)

photoelectric effect the phenomenon of electrons being ejected from a material when exposed to electromagnetic radiation (p. 621)

photon a discrete bundle of energy carried by light (p. 622)

Planck's constant (h) a constant with the value 6.63×10^{-34} J·s; represents the ratio of the energy of a single quantum to its frequency (p. 622)

polarized light light waves that vibrate in a single plane (p. 532)

polarizer a device that allows only light with an electric field along a single direction to pass through (p. 532)

position ($\vec{d}$) the straight-line distance and direction of an object from a reference point (p. 8)

postulate a statement assumed to be true from which a theory is developed (p. 577)

potential energy the stored energy an object has that can be converted into another form of energy (p. 177)

power the rate of work done by a force over time, or the rate at which the energy of an open system changes (p. 189)

principle of electromagnetism moving electric charges produce a magnetic field (p. 382)

principle of relativity the laws of motion are the same in all inertial frames (p. 575)

principle of reversibility a light ray will follow exactly the same path if its direction of travel is reversed (p. 444)

projectile an object that is launched through the air along a parabolic trajectory and accelerates due to gravity (p. 36)

projectile motion the motion of a projectile such that the horizontal component of the velocity is constant, and the vertical motion has a constant acceleration due to gravity (p. 36)

proper length (L_s) the length of an object or distance between two points as measured by an observer who is stationary relative to the object or distance (p. 588)

proper time (Δt_s) the time interval measured by an observer at rest with respect to a clock (p. 584)

Q

quantum the smallest amount of energy that a particle can emit or absorb; the plural is quanta (p. 616)

quantum theory the theoretical basis of modern physics that explains the nature and behaviour of matter and energy at the atomic and subatomic levels (p. 616)

quark an elementary particle that makes up protons, neutrons, and other hadrons (p. 648)

R

radio-frequency identification technology (RFID) a technology that uses microchips that act as transmitters and responders to communicate data by radio waves (p. 405)

range (Δd_x) the horizontal displacement of a projectile (p. 36)

ray approximation treating the propagation of light waves as though they move in straight lines called rays (p. 442)

rectilinear propagation light travelling in straight lines (p. 470)

reflection a change in direction of a light ray when it meets an obstacle where the incoming ray and the outgoing ray are on the same side of the obstacle (p. 442)

refraction the bending of light as it travels at an angle from one medium to another (p. 444)

relative velocity the velocity of an object relative to a specific frame of reference (p. 44)

relativistic kinetic energy (E_k) the energy of an object in excess of its rest energy (p. 599)

relativistic length (L_m) the length of an object or the distance between two points as measured by an observer moving with respect to the object or distance (p. 589)

relativistic mass the mass of an object measured by an observer moving with speed v with respect to the object (p. 595)

relativistic momentum the momentum of objects moving at speeds near the speed of light (p. 594)

relativistic time time that is not absolute, but changes relative to the observer (p. 583)

resolution the ability of an optical device to separate close objects into distinct and sharp images (p. 517)

rest energy (E_{rest}) the amount of energy an object at rest has with respect to an observer (p. 599)

rest mass the mass of an object measured at rest with respect to the observer; also called the proper mass (p. 595)

right-hand rule for a moving charge in a magnetic field if you point your right thumb in the direction of the velocity of the charge ($\vec{v}$), and your straight fingers in the direction of the magnetic field ($\vec{B}$), then your palm will point in the direction of the resulting magnetic force ($\vec{F}_M$) (p. 387)

right-hand rule for a solenoid if you coil the fingers of your right hand around a solenoid in the direction of the conventional current, your thumb points in the direction of the magnetic field lines in the centre of the coil (p. 383)

right-hand rule for a straight conductor if your right thumb is pointing in the direction of conventional current, and you curl your fingers forward, your curled fingers point in the direction of the magnetic field lines (p. 382)

S

satellite an object or a body that revolves around another body due to gravitational attraction (p. 297)

scalar a quantity that has magnitude (size) but no direction (p. 8)

scattering the change in direction of light waves as a result of collisions (p. 535)

secant a straight line connecting two separate points on a curve (p. 9)

secondary maxima the progressively less-intense bright areas, outside the central region, in an interference pattern (p. 512)

simple harmonic motion periodic motion in which the acceleration of the moving object is proportional to its displacement (p. 197)

simultaneity the occurrence of two or more events at the same time (p. 591)

Snell's law $n_1 \sin \theta_1 = n_2 \sin \theta_2$ (p. 447)

space station a spacecraft in which people live and work (p. 298)

space-time a four-dimensional coordinate system in which the three space coordinates are combined with time, a fourth dimension (p. 594)

special theory of relativity all physical laws are the same in all inertial frames of reference, and the speed of light is independent of the motion of the light source or its observer in all inertial frames of reference (p. 578)

specular reflection the reflection of light from a surface where all the reflected rays are in the same direction (p. 442)

spring constant (k) the constant of variation between the force exerted by an ideal spring and the spring's displacement (p. 192)

standard model the modern theory of fundamental particles and their interactions (p. 648)

static friction ($\vec{F}_S$) a force that resists attempted motion between two surfaces in contact (p. 63)

superposition principle the resultant, or net, vector acting at a given point equals the sum of the individual vectors from all sources, each calculated at the given point (p. 329)

T

tangent a straight line that intersects a curve at a point and has the same slope as the curve at the point of intersection (p. 12)

tension ($\vec{F}_T$) a force exerted by objects that can be stretched (p. 62)

tesla the SI unit of measure for describing the strength of a magnetic field; $1\ \text{T} = 1\ \dfrac{\text{kg}}{\text{C}\cdot\text{s}}$ (p. 386)

theory of everything a theory that attempts to combine three fundamental forces (weak, strong, and electromagnetic) with gravity into a single theory (p. 652)

thin film a very thin layer of a substance, usually on a supporting material (p. 502)

thought experiment a mental exercise used to investigate the potential consequences of a hypothesis or postulate (p. 577)

threshold frequency (f_0) the minimum frequency at which electrons are ejected from a metal (p. 621)

time dilation the slowing down of time in one reference frame moving relative to an observer in another reference frame (p. 580)

total internal reflection an effect that occurs when light encounters a boundary between a medium with a higher index of refraction and one with a lower index of refraction (p. 453)

transmission axis the direction of the electric field that a polarizer allows through (p. 533)

trough the lower part of a wave (p. 441)

twin paradox a thought experiment in which a traveller in one frame of reference returns from a voyage to learn that time had passed more slowly in his spacecraft relative to the passage of time on Earth, despite the seemingly symmetric predictions of special relativity (p. 593)

U

uniform circular motion the motion of an object with a constant speed along a circular path of constant radius (p. 114)

universal law of gravitation there is a gravitational attraction between any two objects; if the objects have masses m_1 and m_2 and their centres are separated by a distance r, the magnitude of the gravitational force on either object is directly proportional to the product of m_1 and m_2 and inversely proportional to the square of r (p. 288)

unpolarized light light waves that vibrate in many different planes (p. 532)

V

vector a quantity that has both magnitude (size) and direction (p. 8)

velocity ($\vec{v}$) the change in position divided by the time interval (p. 9)

W

wave front the continuous line or surface at the start of a wave as it travels in time (p. 441)

wavelength (λ) the distance between one positive amplitude and the next (p. 441)

wave–particle duality the property of matter that defines its dual nature of displaying both wave-like and particle-like characteristics (p. 618)

weight the gravitational force exerted by Earth on an object (p. 75)

work the product of the magnitude of an object's displacement and the component of the applied force in the direction of the displacement (p. 164)

work–energy theorem the total work done on an object equals the change in its kinetic energy (p. 173)

work function (W) the minimum energy needed to remove an electron bound to a metal surface (p. 620)

Z

zero-order maximum the location of maximum intensity in the diffraction pattern at $\theta = 0°$ (p. 521)

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