6.2 The great beyond - Calculations beyond yielding

LEARNING GOALS

Learning Objectives

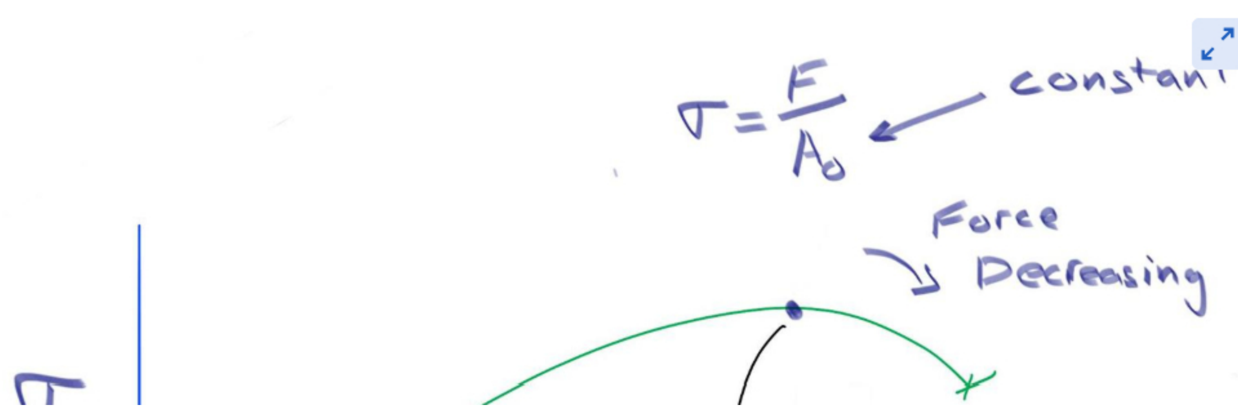
1. Distinguish between the engineering stress and strain and the true stress and strain
2. Evaluate the value of the true stress and strain
3. Utilize the strain hardening equation to relate true stress and true strain beyond yielding

You lied to us! We want the true stress!

I wanted to title this section using Jack Nicholson's famous line from A Few Good Men, "you can't handle the truth!" The only problem is, I know that you can handle it. True stress and true strain aren't that difficult to wrap your head around. In fact, many students find that it provides the necessary context to better mechanical deformation in general. So, here it is in a nutshell. The engineering stress is very useful for us and the majority of the time when we aren't considering plastic deformation we use the engineering stress. As you know, the engineering stress is the force divided by the initial cross-sectional area

$$\sigma = \frac{F}{A_0}$$

so it is clear that the denominator does not change in our calculation of engineering stress. The only thing that will change the engineering stress once a mechanical test begins is the force. This is why we see the engineering stress decreasing after the ultimate tensile strength.

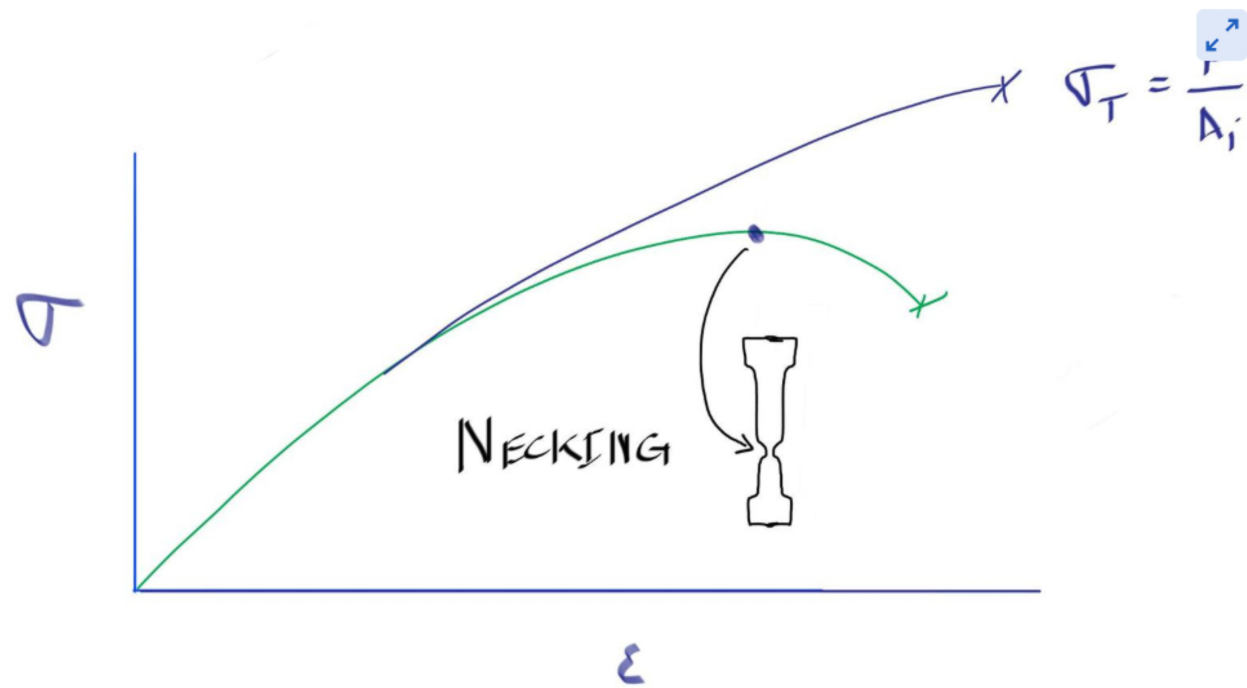


Engineering stress decreases after the UTS because the force required to continue elongating the sample goes down, because the actual area supporting the load decreases, but the engineering stress does not account for this decrease.

Even though the actual cross-sectional area supporting the load decreases the engineering stress doesn't give the sample any credit for that, if you will. The engineering stress says, "even though only a tiny area is supporting the load, I will only acknowledge the area that you started with!" This is where the true stress comes in. If we want to determine what the stress in the sample *actually* is we need to divide by the actual area, called the *instantaneous cross-sectional area* A_i. This is the so-called true stress and is denoted with a subscript capital T

$$\sigma_T = \frac{F}{A_i}$$

A plot of the true stress versus the true strain shows the true stress always increasing.



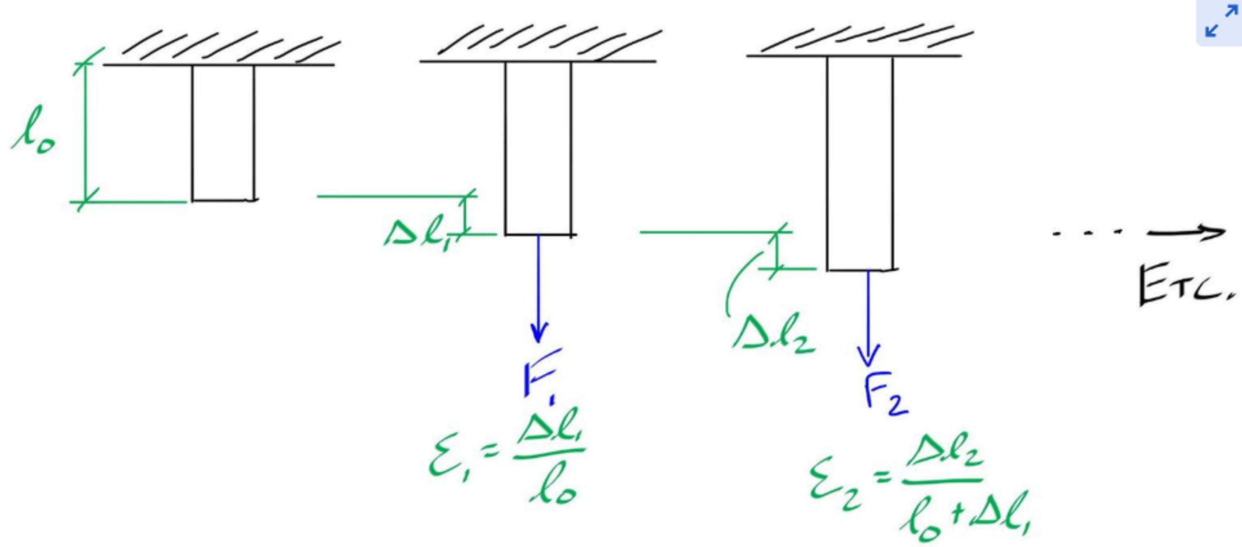
True stress always increases as it accounts for the reduction in the cross-sectional area that is supporting the load.

"Wait!" you say. It's all well and good to give the material proper credit for its impressively decreased cross-sectional area but why is that really important? The reason is simple. The true stress versus true strain behaviour for most metals can be fit very nicely with a simple equation known as the strain hardening equation

$$\sigma_T = K\epsilon_T^n$$

where K and n are the strain hardening coefficient and exponent, respectively, and are both material properties, specific to each metal. This is useful because it means that we can look up these values and then perform calculations relating stress and strain, beyond the linear elastic region.

Now is likely a good time to discuss what true strain is all about. So far we've discussed the engineering strain as the change in length over the initial length $\epsilon = \frac{\Delta_l}{l_0}$. The concept of the true strain is best understood by imagining that a small elongation occurs, followed by another small elongation, as shown in the figure below.



Considering a small elongation applied to a sample, the first strain is simple, just using the engineering strain equation. However, when the next little bit of elongation is applied, we realize that the next *initial length* over which this next little bit of elongation occurred was actually longer than the *original initial length*. To determine the true strain, we must sum up all of these little bits of elongation, which is the business of integration.

The first little bit of elongation Δ_l occurred over the initial length l_0 giving a strain calculated as before from the engineering strain. The second little bit of elongation gives us a little bit of trouble though (see what I did there, repeating "little bit" to get you thinking about integration? I'm a little bit sneaky.) You see, if we were to calculate the strain, at the second load, we would need to calculate the second strain, accounting for the fact that the sample was already a little longer than when it started. Finally, we'd have to sum up the little bits of strain to get the total strain. This is, of course, the business of integration, if we make the little bits of elongation so small as to be infinitesimally small, and so we can write the true strain as

$$\epsilon_T = \int_{l_0}^{l_i} \frac{dl}{l}$$

where l_i is the instantaneous length. As you likely know, the integral of $\frac{1}{l}$ is just $\ln l$ so we can write the true strain as

$$\epsilon_T = \ln \frac{l_i}{l_0}$$

The only problem with these equations for true stress and true strain is that they aren't that practically useful since we need to know the instantaneous area and length A_i and l_i. Thankfully, we can make a simplifying assumption that the volume of the sample stays the same, at least until necking.

$$V_0 = V_i$$
$$A_0 l_0 = A_i l_i$$
$$A_i = A_0 \frac{l_0}{l_i}$$

and we also know that the instantaneous length l_i is the initial length plus the change in length

$$l_i = l_0 + \Delta l$$

which we can substitute into the equation for A_i

$$A_i = A_0 \frac{l_0}{l_i}$$
$$A_i = A_0 \frac{l_0}{l_0 + \Delta l}$$

which we can substitute back into our equations for true stress and true strain

$$\sigma_T = \frac{F}{A_i}$$
$$\sigma_T = \frac{F}{A_0 \frac{l_0}{l_0 + \Delta l}}$$
$$\sigma_T = \frac{F}{A_0} \frac{l_0 + \Delta l}{l_0}$$
$$\sigma_T = \sigma(1 + \epsilon)$$

and

$$\epsilon_T = \ln(1 + \epsilon)$$

which are two useful equations, that we can apply prior to necking.

EXERCISE YOUR KNOWLEDGE