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Tutorial 3

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Land Acknowledgment

I wish to acknoledge this land on which the University of Toronto operates. For thousands of years it has been the traditional land of the Huron-Wendat, the Seneca, and most recently the Mississaugas of the Credit River. Today, this meeting place is still the home of many Indigenous people from across Turtle Island and we are grateful to have the opportunity to work on this land.

Health Resources

- My Student Support Program (MySSP)
  - mentalhealth.utoronto.ca/my-student-support-program
  - Call phone number or download app
  - Call or chat in over 35 languages
  - Get advice about:
    - Being successful at school
    - Practical issues while studying
    - Relationships with friends and family
    - Language and cultural barriers
    - Stress, sadness and loneliness
    - Balancing work and school
    - Difficulty adjusting to life in Canada

Anouncements

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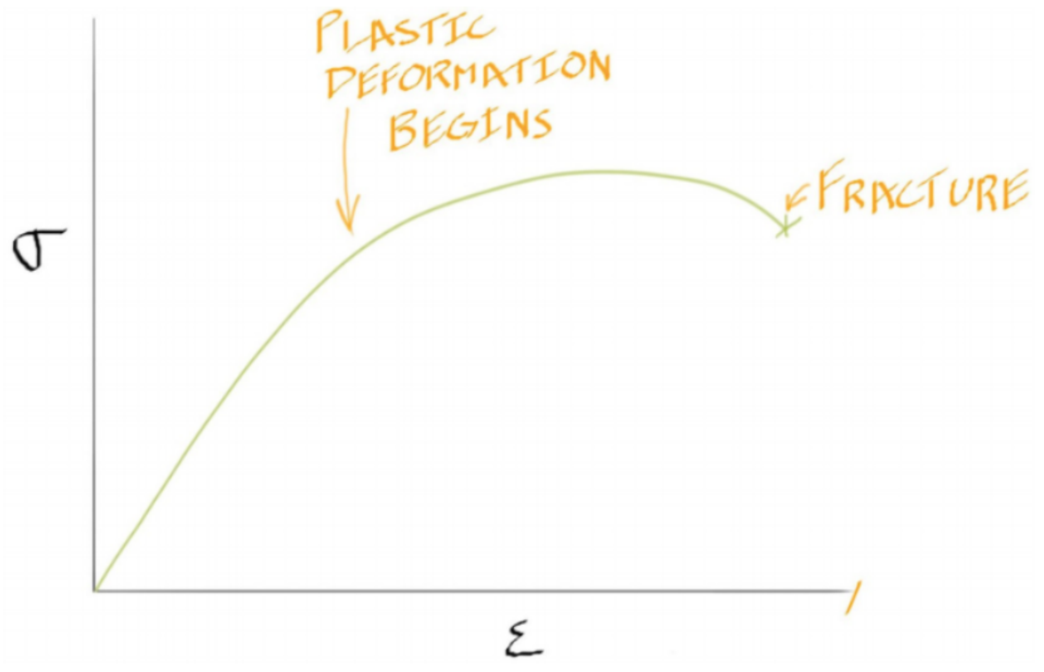
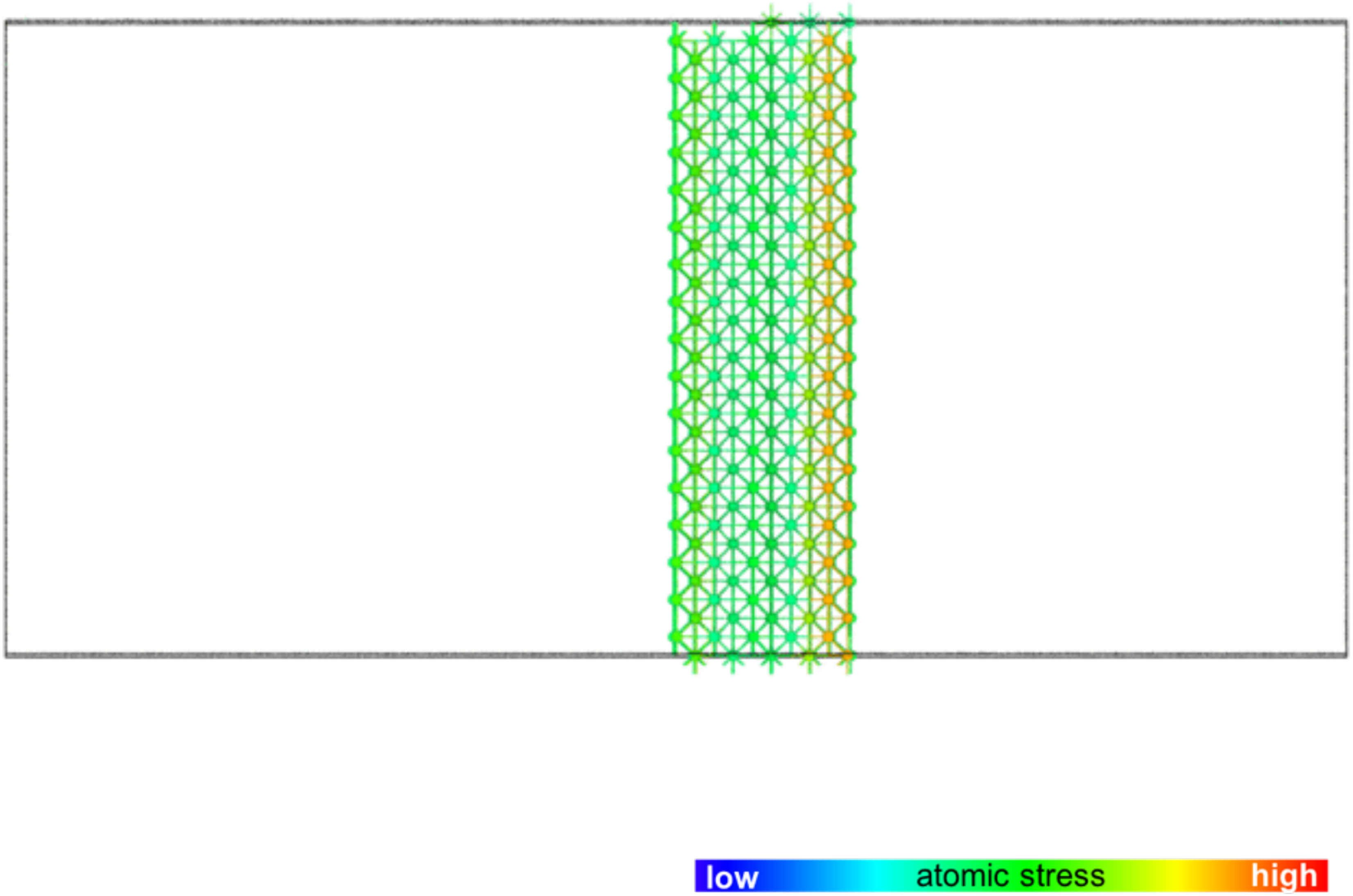
- Quiz will be available for all on Friday, from midnight to 11:59PM. If there are any questions during the test period about the quiz, you can email Desmond (symen.vandenberg@mail.utoronto.ca).
- Problem Set is also posted or will be shortly.
- For what will be covered on the Quiz and the Problem Set: check Learning Objectives spreadsheet.

- Elastic Deformation *vs* Plastic Deformation
  - Macroscopic: is the deformation recoverable?
  - Atomistic: upon unload, do the atoms go back to their original position?
  - The word *plastic* comes from *plastikos*, meaning *to sculpt* or *to shape*
    - Happens in polymers, but not only!



Theory Review

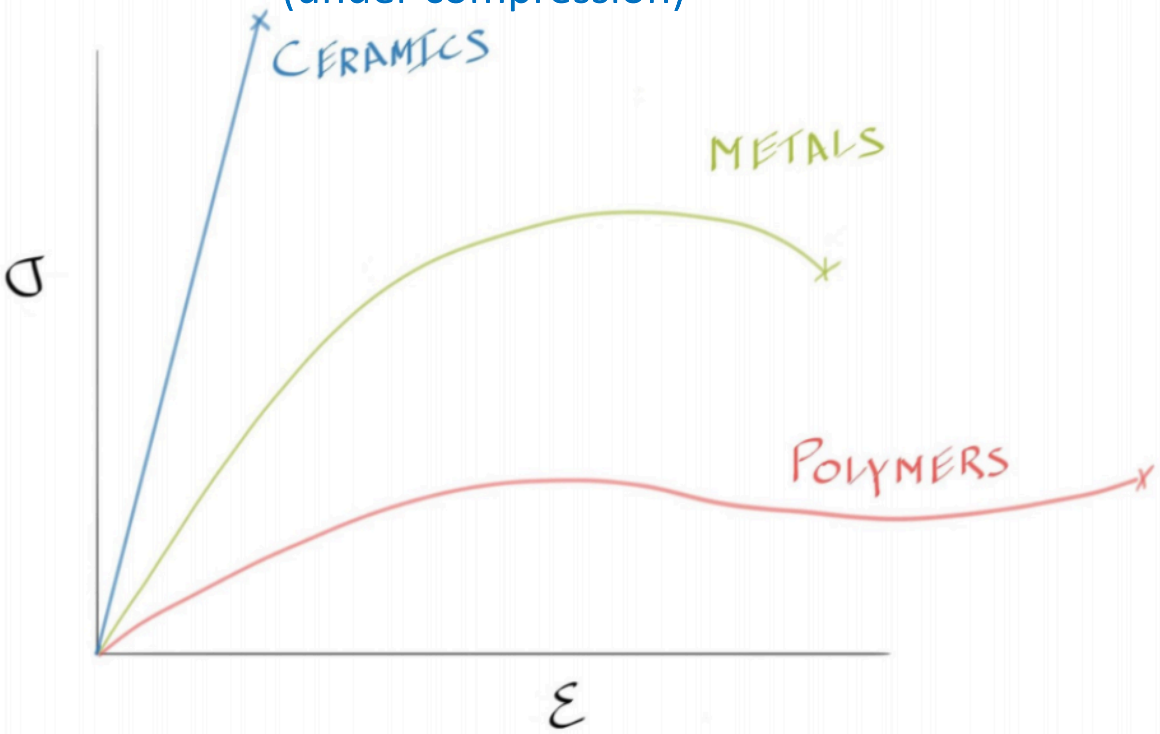
- Elastic *vs* Plastic
  - Molecular Dynamics simulation of a Fe Nanowire during uniaxial tensile test
  - Note: this is a perfect crystalline nanowire: monocrystalline, no defects



Theory Review

- Stress-Strain Curves of Ceramics, Metals and Polymers
  - Consider the trends for:
    - Young’s modulus
    - Plastic deformation
      - Brittle *vs* Ductile
    - Strain to failure
    - Strength
  - Possible applications for each?

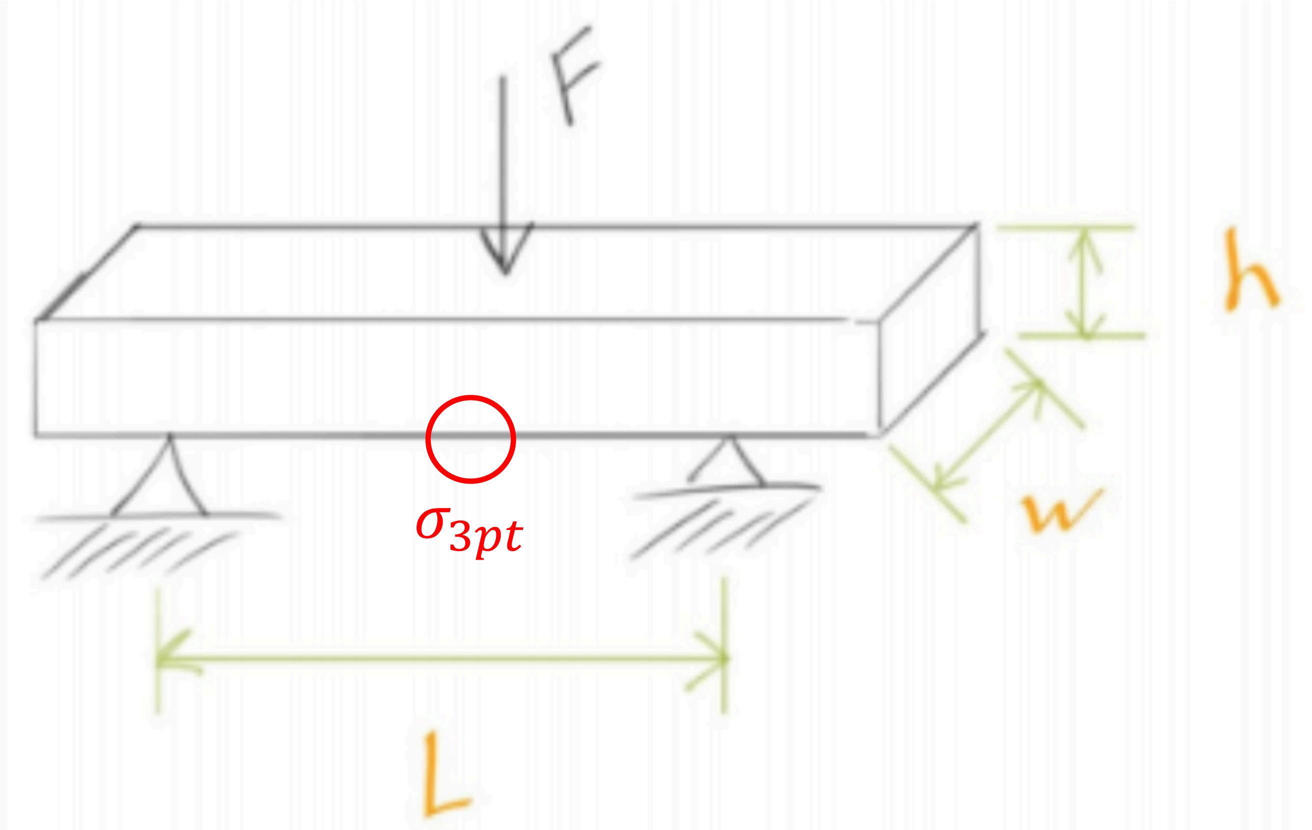
(under compression)



Theory Review

- 3-Point Bend Test
  - For ceramics, because tensile-testing them is hard
    - What are the three reasons?
  - Peak stress:

$$\sigma_{3pt} = \frac{3FL}{2wh^2}$$



Theory Review

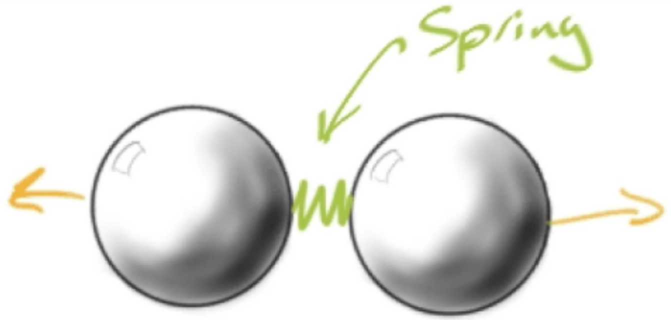
- Tempered Glass
  - How tempering produces residual stresses
  - How residual stresses make the material stronger
    - Outer layers in compression
    - Inner layer in tension
    - Why is this a good thing?
  - Upon fracture, stored energy is released → small pieces → safe

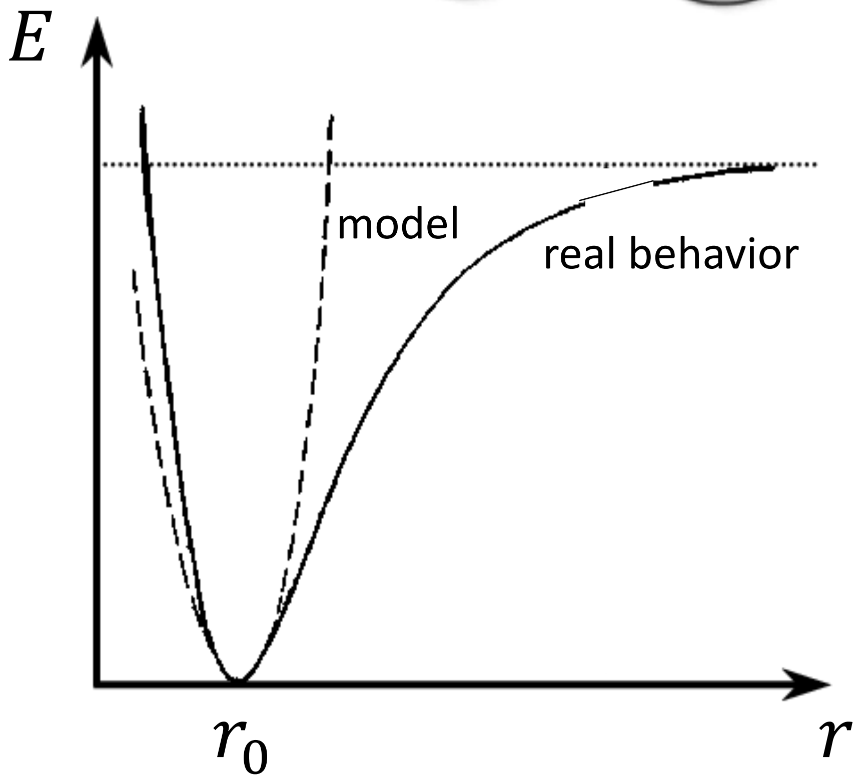
Theory Review

- Scientific models
  - Chemical bonds as springs?!
    - “Spring” means *quadratic potential*:

      $$E = E_0(r - r_0)^2$$

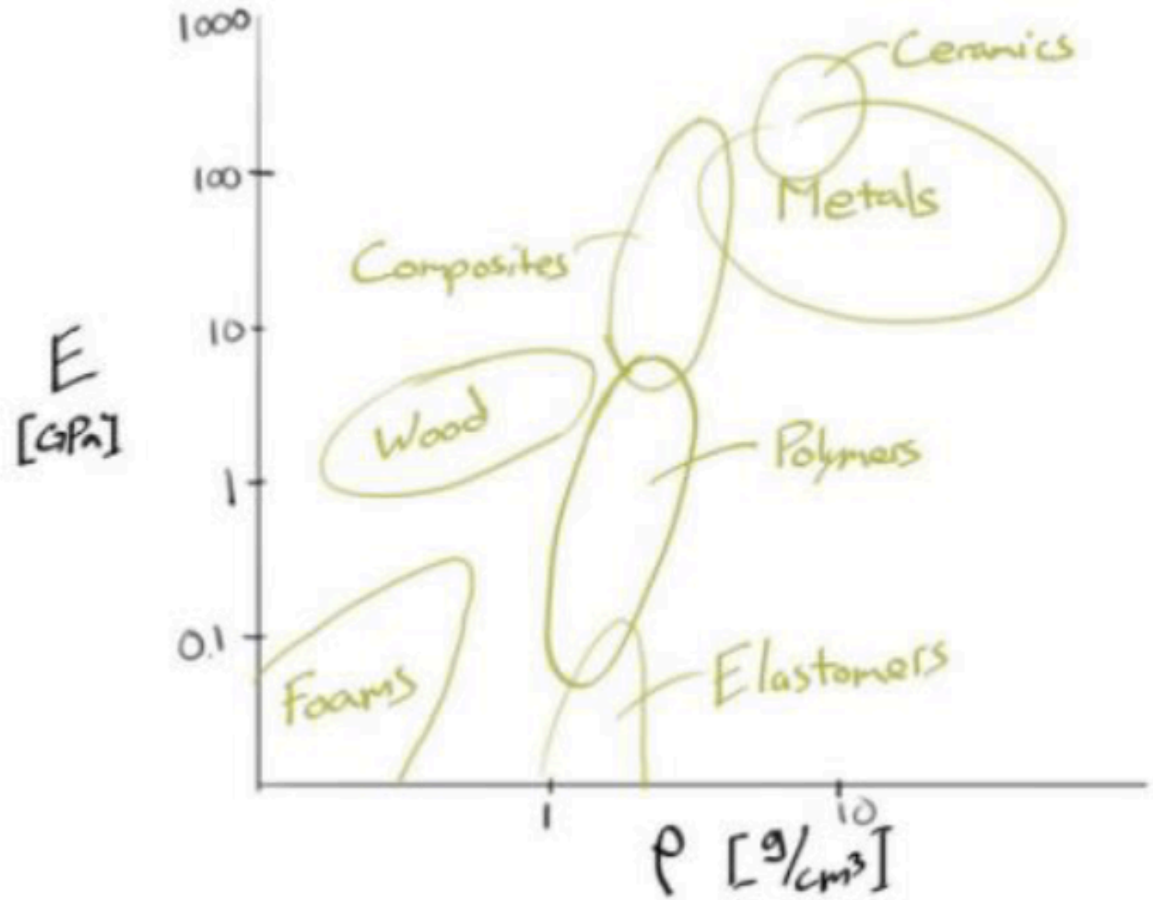
    - Works around r_0
    - Model used in Molecular Dynamics simulations at low temperatures (~300K)





Theory Review

- Young’s modulus as a function of Density
  - Where do we want to be?
  - Varies with application?



Sample Problems

1. A bar of steel is loaded by 960 N in three-point bending. The bar has a length of 70cm, a width of 4cm, and a height of 1.5cm. This sample has a yield strength of 515 MPa.

a. Will this bar plastically deform?

b. At what force would this bar plastically deform?

a. Load = F = 960 N

Length = L = 70 cm

Width = w = 4 cm

Height = h = 1.5 cm

Yield Strength = σ_Y = 515 MPa

$$\sigma_{3pt} = ? = \frac{3FL}{2wh^2} = 112\ MPa < \sigma_Y$$ ∴ No plastic deformation

b. $\sigma_{3pt} = \sigma_Y$

F = ?

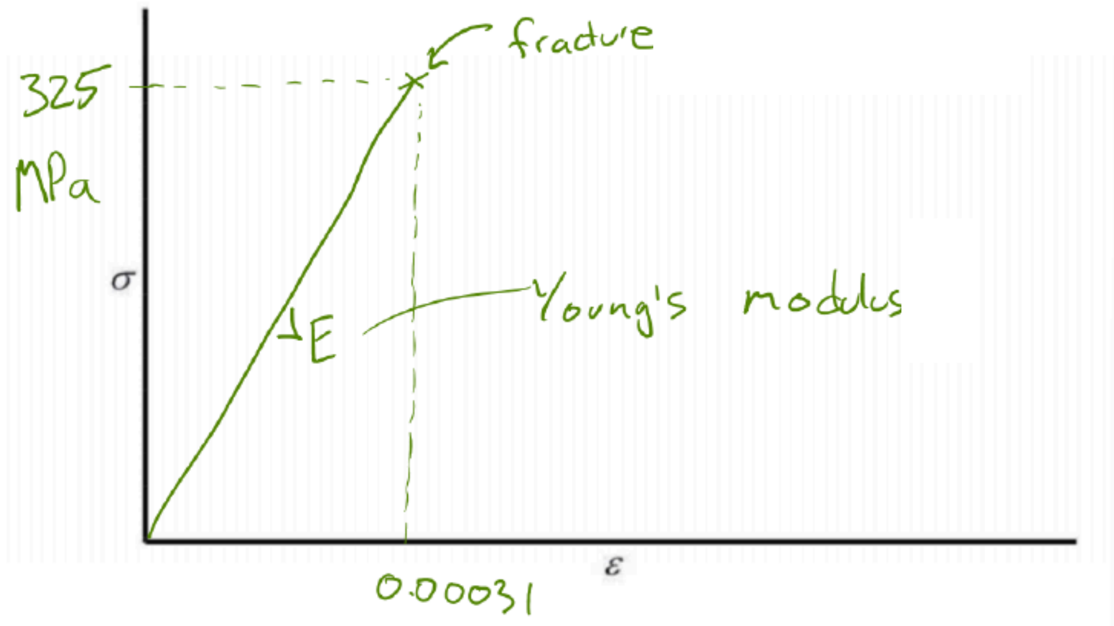
$$\sigma_{3pt} = \frac{3FL}{2wh^2} \Leftrightarrow F = \frac{2\sigma_{3pt}wh^2}{3L} = 4.4\ kN$$

Sample Problems

2. A rectangular cross-section beam of a hypothetical ceramic is loaded in 3pt bending and is found to have a fracture strength of 325 MPa when the strain in the mid-span of the lower surface of the beam is 0.00031, and the distance between the lower supports is 137 mm.

a. Sketch a stress-strain curve that you would expect for this sample, and label all the important points
b. Calculate the Young's modulus for this ceramic
c. If another beam of the same ceramic material was manufactured having a height of one-half of that in part (a), what would the maximum span (distance between lower supports) be that this beam could support without fracturing?

a.



b.

$$E = \frac{\sigma}{\epsilon} = \frac{325\ MPa}{0.00031} = 1050\ GPa$$

c.

$h_2 = ½\ h_1$

$L_2 = ?$

Assume $F_1 = F_2$

$$\sigma_{3pt} = \frac{3FL}{2wh^2}$$

$$L_2 = \frac{L_1}{4} = \frac{137\ \text{mm}}{4} = 34.3\ \text{mm}$$

Quizz (not graded)

1. You are designing a cylindrical tie to support a ceiling light fixture. The distance from the light to the ceiling must be 30cm, and the total supported weight is 1000N. To keep the costs as low as possible, you will be using a 1020 steel alloy with a modulus of elasticity of 207 GPa and a yield strength of 295 MPa.
    a. What is the minimum diameter that you can design this tie such that it will not undergo plastic deformation?
    b. If another tie was designed with a diameter of 2.3cm, how long would this tie be just prior to permanently deforming?

2. A load F (in N) is to be suspended from a cylindrical length of line of radius R (in m) and unloaded length lO (in m) and modulus E (in Pa) without breaking or plastically deforming the line. Develop an equation in terms of these variables for the change in length that the line will undergo when a weight is added to the end.

3. A hypothetical ceramic was tested utilizing three-point bending. The sample has a cross-section with a height of 7mm and a width of 10mm. This sample broke when a load of 5000 N was applied to it with a distance between the support points of 50 mm.
    a. Another test is to be performed on a sample made of the same ceramic material with a cross-section with a height of 5mm and a width of 7.5mm. If the span between the support points is maintained at 50mm, at what load would this second sample fracture?
    b. Discuss reasons why ceramics are tested using 3-point bending as opposed to a tensile test.

Quizz (not graded)

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4. Which general class of material has a Young's modulus that is roughly one hundred times less than that of metals?

   a. Polymers
   b. Ceramics
   c. Composites
   d. None of the above

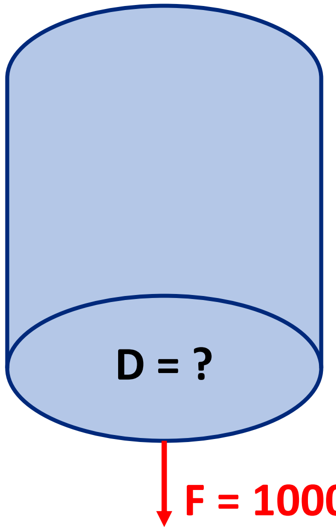
5. Two rectangular cross-section beams are prepared from the same ceramic material. The first beam has a length of 78 mm, height of 9.1 mm, and a width of 18 mm while the second beam has length 51 mm, height 9.4 mm, and width 12 mm. The first beam is found to fracture when the applied load, in 3 pt bending, is 2520 N. What load will the second beam fracture at?

   a. 1735.4 N
   b. 1195.1 N
   c. 1156.9 N
   d. 2741.6 N

Quizz (not graded) – Solutions

1. You are designing a cylindrical tie to support a ceiling light fixture. The distance from the light to the ceiling must be 30cm, and the total supported weight is 1000N. To keep the costs as low as possible, you will be using a 1020 steel alloy with a modulus of elasticity of 207 GPa and a yield strength of 295 MPa.

- **a. What is the minimum diameter that you can design this tie such that it will not undergo plastic deformation?**
- **b. If another tie was designed with a diameter of 2.3cm, how long would this tie be just prior to permanently deforming?**



$\boldsymbol{l_0 = 30\ cm}$

F = 1000 N

a. $\sigma_Y = \frac{F}{A} = \frac{F}{\pi D^2/4} \therefore D = 2.1\ mm$

b. $\epsilon_Y = \frac{\sigma_Y}{E} \approx 0.001425$

$$l_f = l_0(1 + \epsilon) \approx 30.0428\ cm$$

Quizz (not graded) – Solutions

2. A load F (in N) is to be suspended from a cylindrical length of line of radius R (in m) and unloaded length l_0 (in m) and modulus E (in Pa) without breaking or plastically deforming the line. Develop an equation in terms of these variables for the change in length that the line will undergo when a weight is added to the end.

$$\varepsilon = \frac{\Delta l}{l_0} \rightarrow \Delta l = \varepsilon \cdot l_0$$

$$\sigma = E\varepsilon \rightarrow \varepsilon = \frac{\sigma}{E}$$

$$\sigma = \frac{F}{A} \quad \text{where } A = \pi R^2$$

$$\Delta l = \varepsilon l_0 = \frac{\sigma}{E} l_0 = \frac{F l_0}{A_0 E} = \frac{F L_0}{\pi R^2 E}$$

Quizz (not graded) – Solutions

3. A hypothetical ceramic was tested utilizing three-point bending. The sample has a cross-section with a height of 7mm and a width of 10mm. This sample broke when a load of 5000 N was applied to it with a distance between the support points of 50 mm.

a. Another test is to be performed on a sample made of the same ceramic material with a cross-section with a height of 5mm and a width of 7.5mm. If the span between the support points is maintained at 50mm, at what load would this second sample fracture?

b. Discuss reasons why ceramics are tested using 3-point bending as opposed to a tensile test.

a.

$$\sigma_f = \frac{3FL}{2wh^2} = \frac{3 \cdot 5000 \cdot 50\times10^{-3}}{2 \cdot (10\times10^{-3}) \cdot (7\times10^{-3})^2}$$

$$\therefore \sigma_f = 765 \text{ MPa}$$

for sample 2 $\quad F = \frac{2\sigma_f w h^2}{3L} \simeq 1.91 \text{ kN}$

b.

1. Ceramics can crumble in grip
2. Ceramics difficult to machine/form into complex forms.
3. Ceramics difficult to align in a standard tensile tests (which rely on this alignment).

Quizz (not graded) – Solutions

4. Which general class of material has a Young's modulus that is roughly one hundred times less than that of metals?

a. **Polymers**
b. **Ceramics**
c. **Composites**
d. **None of the above**

Check the plot of Young's Modulus *versus* Density (slide 11)

Quizz (not graded) – Solutions

5. Two rectangular cross-section beams are prepared from the same ceramic material. The first beam has a length of 78 mm, height of 9.1 mm, and a width of 18 mm while the second beam has length 51 mm, height 9.4 mm, and width 12 mm. The first beam is found to fracture when the applied load, in 3 pt bending, is 2520 N. What load will the second beam fracture at?

a. 1735.4 N

b. 1195.1 N

c. 1156.9 N

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Same logic as Question 3:

1. Calculate fracture strenght (σ_{3pt}) from the fracture point of the first beam
2. Calculate the load (F) needed to break the second beam, given that it is the same material and therefore has the same fracture strenght