

## MDM4U1 - Final Exam Review Packages

### Exam Review Package 1: Unit 2 Organized Counting

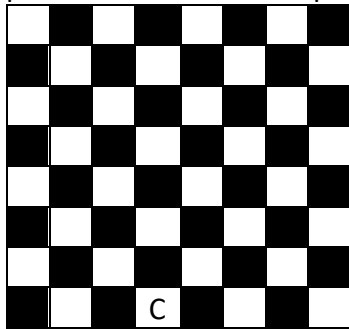
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1. Evaluate each of the following expressions:
  - a)  $3! P(5,2)$
  - b)  $6! + \binom{6}{4} - P(5,0)$
  - c)  $(9 - 9)! + \binom{4}{4} + P(5,5)$
  - d)  $\binom{9}{8} + P(7,6)$
  - e)  $\binom{11}{7} - \binom{11}{4}$
  - f)  $4! P(9,4)$
  - g)  $\binom{7}{4} \binom{3}{2}$
  - h)  $P(9,6) + 6! \binom{9}{6}$
  - i)  $\binom{12}{5} \binom{7}{3} \binom{4}{4}$
2. Express each of the following using factorial notation.
  - a) The number of seven-digit numbers in which no digit appears more than once.
  - b) The number of "words" that can be formed using all the letters of PROBLEMS.
  - c) The number of choices of three witch costumes from a wardrobe of eight such costumes.
  - d) The number of inspection teams of two mechanics and three supervisors chosen from seven mechanics and five supervisors.
3. Carmencita and her four sisters compete in a race at the family reunion picnic. In how many ways can the first and second prizes be awarded for the race?
4. There are eight runways at the regional airport. Three planes are coming in for a landing at approximately the same time. In how many ways can air-traffic control assign the planes to different runways?
5. Six candidates run for class representative to the student council. If two reps are to be chosen, how many possible results are there?
6. In how many ways can nine people be seated around a circular table?
7. How many even five-digit numbers can be formed using only numbers 1 through 7 if repetition is not allowed?
8. There are seven seats in the family's new mini-van. If four of the seven family members can drive, how many possible seating arrangements are there for a family trip in the van?
9. The prime factorization of 2520 is  $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$ . Find the number of even divisors of 2520.
10. How many different routes can Timothy take to the neighbourhood convenience store which is four blocks west and three blocks south of his house if he is always travelling toward his destination?
11. Find the number of ways in which at least one piece of fruit could be chosen from a basket containing four apples, five bananas, two cantaloupes, and three pears.
12. How many five-digit numbers greater than 60,000 are divisible by 5?
13. The basketball team carries 14 players: three first-year players, five second-players, and six experienced third-year players.
  - a) In how many ways can the coach choose a starting lineup (five players) with at least one first-year players?
  - b) In how many ways can he set up a starting lineup with two second-year and three third-year players or the reverse (two third-year and three second-year)?
14. If  $n(A \cup B) = n(A) + n(B)$ , what relationship exists between the sets A and B?
15. To what other expressions in the form  $\binom{n}{r}$  is each of the following equal?
  - a)  $\binom{16}{7}$
  - b)  $\binom{21}{9} + \binom{21}{10}$
  - c)  $\binom{n-3}{k+1} + \binom{n-3}{k+2}$
  - d)  $\binom{8}{3} + \binom{8}{4} + \binom{9}{5} + \binom{10}{6}$

16. Use the properties of Pascal's Triangle to evaluate the following.

- a)  $\binom{7}{0} + \binom{7}{1} + \binom{7}{2} + \binom{7}{3} + \binom{7}{4} + \binom{7}{5} + \binom{7}{6} + \binom{7}{7}$   
 b)  $\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8}$   
 c)  $\binom{5}{0}^2 + \binom{5}{1}^2 + \binom{5}{2}^2 + \binom{5}{3}^2 + \binom{5}{4}^2 + \binom{5}{5}^2$

17. An eight-square by eight-square checkerboard is given. A checker, indicated by C, is positioned in the fourth square of the bottom row. The checker is allowed to move one square at a time, diagonally left or right, to the row above. After seven moves the checker will be in the top row. How many different paths will lead to the top row?



1. In the arrangement of the letters given, starting from the top, proceed to the row below by moving diagonally to the immediate right or left. How many different paths will spell ARITHMETICO?

										A										
										R										
										I										
										T										
										H										
M	H	T	I	R	A	R	I	T	H	M	H	T	I	R	A	R	I	T	H	M
	E	M	H	T	I	R	A	R	I	T	H	M	H	T	I	R	A	R	I	T
		T	I	R	A	R	I	T	H	M	H	T	I	R	A	R	I	T	H	M
			I	R	A	R	I	T	H	M	H	T	I	R	A	R	I	T	H	M
				C	O	C														

## ANSWERS

- |           |                                |          |                        |                        |            |
|-----------|--------------------------------|----------|------------------------|------------------------|------------|
| 1. a) 120 | 2. a) $9 \times \frac{9!}{3!}$ | 4. 336   | 11. 359                | 15. a) $\binom{16}{9}$ | 16. a) 128 |
| b) 734    | b) $8!$                        | 5. 15    | 12. 7999               | b) $\binom{22}{10}$    | b) 256     |
| c) 122    | c) $\frac{8!}{3!5!}$           | 6. 40320 | 13. a) 1540            | c) $\binom{n-2}{k+2}$  | c) 252     |
| d) 5049   | d) $\frac{7!}{2!3!2!}$         | 7. 1080  | b) 350                 |                        | 17. 103    |
| e) 0      | 3. 20                          | 8. 2880  | 14. Mutually exclusive | $\binom{11}{6}$        | 18. 252    |
| f) 72576  |                                | 9. 36    |                        |                        |            |
| g) 105    |                                | 10. 35   |                        |                        |            |
| h) 120960 |                                |          |                        |                        |            |
| i) 27720  |                                |          |                        |                        |            |

## Exam Review Package 2: Unit 4: Probability and Unit 5 Probability Distribution

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1. A committee is to be formed to study the Canadian immigration policy. The committee is to be chosen randomly from five recent immigrants and five second-generation citizens. If the committee is to consist of seven members, what is the probability that
  - a) There are exactly three recent immigrants on the committee?
  - b) There are more recent immigrants than second-generation citizen on the committee?
2. The game of bingo has 15 numbers associated with each letter of the word bingo. For example, the letter "B" has the numbers 1 through 15; "I" has the number 16 through 30, and so on. If your bingo card has five different numbers under the "B", what is the probability that
  - a) On the first draw, one of your numbers under the "B" is drawn?
  - b) On the second draw, one of your numbers under the "B" was successfully drawn given that one of your numbers under the "B" is drawn from the first draw?
3. What is the probability that at least two of the 12 councillors elected in the next municipal election will have birthdays on the same day?
4. That odds against winning in craps are 507 to 493. What is your expected return on a \$20.00 wager?
5. Recent estimates indicate that the foreign car manufacturer Honda has 15 percent of the car market. If a random sample of 20 automobiles is conducted, what is the probability that
  - a) Exactly one of the cars was made by Honda?
  - b) At least two of the cars were made by Honda?
6. A basketball game in a municipal league between the Sharks and the Jets was scheduled for Thursday night. Only four players from the Sharks showed up and so the Jets, who had six players, won by default. So the evening would not be spoiled, the players that arrived for the game decided to have a pick-up scrimmage, and picked the two teams by drawing names from a hat. What is the probability that the four Sharks end up on the same team?
7. The Canadian government is concerned about the impact of arctic development on the wildlife there. To study the effects of development, it is necessary to estimate the population sizes of the various species of animals. One year 100 polar bears were tagged. The following year 15 of 65 polar bears that were sighted were noted to have tags. What estimate could be made of the bear population?
- ~~8. At the outset of a billiard game, it is estimated that the probability of a professional billiard player sinking a ball in a pocket is 0.85, and it remains the same on each subsequent shot until the end of his turn.
  - a) What is the probability that he will miss on his third shot?
  - b) What is the expected number of shots that he will have in turn?~~
9. In a certain manufacturing process the lengths of sides of silicon computer chips are distributed normally with mean 1.325 mm and standard deviation 0.005 mm. Find the expected proportion of the chips that will be outside the range 1.317 mm to 1.333 mm.

## ANSWERS

- |                   |                   |
|-------------------|-------------------|
| 1.                | 5.                |
| a) $\frac{5}{12}$ | a) 0.137          |
| b) $\frac{1}{2}$  | b) 0.824          |
| 2.                | 6. $\frac{1}{42}$ |
| a) $\frac{1}{15}$ | 7. 433            |
| b) $\frac{2}{37}$ | 8.                |
| 3. 0.167          | a) 0.108          |
|                   | b) 6              |
| 4. \$20.57        | 9. 10.96%         |

### Exam Review Package 3 "It's Probably Probability"

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1. Mixed Question
  - a) A die is rolled. What is the probability that the number rolled is greater than 2 and even?
  - b) From a standard deck of cards, one card is drawn. What is the probability that the card is black and a jack?
  - c) A standard deck of cards is shuffled and one card is drawn. Find the probability that the card is a queen or an ace.
  - d) What is the probability of drawing a card that is red or a jack?
  - e) A piggybank contains 2 toonies, 3 loonies, 4 nickels and 5 pennies. One coin is removed. What is the probability it is a loonie or a nickel?
  - f) A standard deck of cards is shuffled and one card is drawn. Find the probability that the card is a queen given that it is a red card?
  - g) What are the odds in favour of drawing a face card from a standard deck of cards?
2. A test has four true/false questions.
  - a) Draw a tree diagram to represent the test.
  - b) What is the probability that the student will guess all four questions correct?
3. At the start of flu season, Dr. Sickie McSick examines 50 patients over two days. Of those 50 patients, 30 have a headache, 24 have a cold, and 12 have neither symptoms. Draw a Venn diagram for this scenario and find the probability that a patient selected at random has:
  - a) Just a headache
  - b) Has a headache or a cold
  - c) Does not have either symptom
4. In a math class, there are 12 boys and 14 girls. Five of the boys and eight of the girls wear glasses. A person is chosen at random to be stuffed into a cannon and fired through a flaming loop into a pool or tiger and crocodiles (the math class was at a circus school). What is the probability that:
  - a) The person wears glasses?
  - b) The person is a boy or the person wears glasses?
  - c) The person is a boy and the person wears glasses?
  - d) The person is a boy given the fact that they wear glasses?
5. A bag contains 7 red, 5 blue, 4 green, 5 white and 9 yellow marbles. Five marbles are drawn. What is the probability that:
  - a) 3 red and 2 blue marbles are drawn and the marbles are replaced.
  - b) 3 red and 2 blue marbles are drawn and the marbles are not replaced.
6. Sir Guess A. Lot is writing the next Data Management test and is not very sure of his answers. The probability of him answering the first question correctly is  $\frac{1}{3}$ . If he answers correctly, he gets very excited and the probability the next question is correct is  $\frac{4}{5}$ . However, if he answers a question incorrectly, he becomes depressed and the probability of getting the next one correct is only  $\frac{2}{3}$ .
  - a) Draw a tree diagram to illustrate Sir Guess A. Lot answering the first 3 questions of the test.
  - b) Determine his probability of answering at least 2 of the first 3 questions correctly.
7. A paper bag has 3 types of candy. More specifically it has 10 chocolate bars, 7 gum balls, and 3 packages to toffee. However, you are not just allowed to reach in to take a candy. You must play a game where you pick out a piece of candy, then put it back and then you select a second candy. You may keep the candy if you select the same type of candy both selections in a row. What is the probability of each following scenario:
  - a) You are able to keep a chocolate bar
  - b) You are able to keep any candy
  - c) You are NOT able to keep any candy
8. Postal codes in Canada are 6 characters long in a pattern LDL DLD (L is letter and D is digit). If Toronto postal codes start with an M, what is the probability that a randomly generated postal code will be a Toronto one?

### ANSWERS

- 1a)  $\frac{1}{3}$  b)  $\frac{1}{26}$  c)  $\frac{2}{13}$  d)  $\frac{7}{13}$  e)  $\frac{1}{2}$  f)  $\frac{1}{13}$  g) 3:10
- 2a) 16 outcomes b)  $\frac{1}{16}$
- 3) a)  $\frac{7}{25}$  b)  $\frac{19}{25}$  c)  $\frac{6}{25}$
- 4) a)  $\frac{1}{2}$  b)  $\frac{10}{13}$  c)  $\frac{5}{26}$  d)  $\frac{5}{13}$
- 5) a) 343/97200 b) without replacement 25/10179
- 6) a) 8 outcomes b)  $\frac{2}{3}$
- 7) a)  $\frac{1}{4}$  b)  $\frac{79}{200}$  c)  $\frac{121}{200}$
- 8)  $\frac{1}{26}$

### Exam Review Package 4: Pascal's Triangle

1. In Ziggy's neighbourhood, the streets are on a grid as shown. Ziggy wants to take a different path to school every day. Will he be forced to repeat a path before the year is up?

Home


School

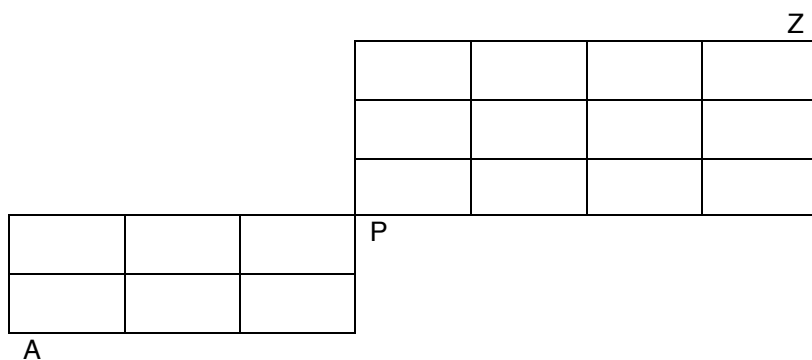
2. In the neighbourhood from question #1, there's a park (see below)
  - a) What percentage of the path from Ziggy's house to the school pass through the park?
  - b) If he does not go through the park, how many ways are there to get to school? (He may still pass along the edge of the park.)

Home

			Park			

School

3. What if he does go through the park, how many ways are there to get to school? (i.e. he walks inside the park, not just along the edge) If possible, find a direct method to confirm the result of the indirect method.
4. How many paths are there from A to Z?



### ANSWERS

1. no, he won't be forced to repeat a path before the year is up because there are 792 ways
- 2.a) 60.1%
- 2.b) 316 ways
3. If he goes through the park and alongside doesn't count:  $792 - 316 = 476$  ways
4. 350 ways

**Exam Review Package 5: Extra Practice**

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1. A game is played by tossing two coins. Each toss cost \$2.00. The player wins when both coins turn up the same and loses when they differ. The payouts are given in the table in the margin.

Outcome	Winnings (\$)
HH	5
HT	0
TH	0
TT	2

- a) Create a probability distribution for this game.  
b) What can a player expect to win playing this game?  
c) Is this a fair game? Explain.
2. Four teachers and five students have volunteered to serve on the school's fundraising committee, which is chaired by the principal. The principal wishes to select the committee using a random draw. He places each person's name on a slip of paper and draws four names.  
a) What is the probability that there is at least one teacher on the committee?  
b) What is the expected number of students on the committee?
3. Simplify  
a)  $\frac{\binom{74}{21}}{\binom{73}{20}}$   
b)  $\binom{20}{15} + \binom{20}{16}$
4. Find the probability that in 10 tosses of a fair coin,  
a) There will be exactly 5 heads  
b) There will be exactly 8 heads  
c) There will be exactly 2 heads  
d) There will be at least 2 heads
5. A student is selling chocolates to raise money for school athletics. She has found that she is able to sell to one in every three people she calls on. Determine the probability that she will have at least two sales in an evening in which she calls on four homes.
6. A multiple-choice test has 10 questions, each of which has 4 possible answers. If a student guesses the answers, determine the probability of getting exactly  
a) 5 answers correct  
b) 6 answers correct  
c) 1 answer correct  
d) All answers correct
7. A pair dice is rolled 30 times. Determine the probability that  
a) Double 3s are rolled exactly five times  
b) Double 3s are rolled at least once

8. In a manufacturing process, 0.2% of the brake pads that come off the production line are considered defective.
  - a) Determine the probability that at least one brake pads is defective in an order of 200 pads.
  - b) What is the expected number of defective brake pads in an order of 200?
9. In a bag, there are 10 white marbles and 6 red ones. Four are selected at random.
  - a) Determine the probability that there are exactly 3 white marbles in the sample.
  - b) Can this situation be represented by a binomial distribution? Explain.
  - c) If the discrete random variable,  $X$ , is defined as the number of white marbles in a sample of 4, determine the probability distribution.
  - d) Determine the expected number of white marbles in the sample of 4.
10. You roll a die 10 times and 6 times it comes up showing 5. What is the probability of this occurring?

#### ANSWERS

1. Let  $X$  be the winnings (\$)  
 a)
 

$X$	0	2	5
$P(X)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
- b)  $E(X) = -\$0.25$
- c) No, since in a fair game, the expected winnings would be the same as the cost of playing.
2. A) 0.96 B) 2.22
3. A)  $74/21$  B) 20,349
4. A) 0.246 B) 0.044 C) 0.044 D) 0.989
5. 0.407 or  $11/27$
6. A) 0.058 B) 0.016 C) 0.188 D) 0.000001
7. A) 0.001 B) 0.570
8. A) 0.330 B) 0.4
9. A) 0.396 B) No, because the probabilities change every time a marble is drawn

C)

0	0.008
1	0.110
2	0.371
3	0.396
4	0.115

D) 2.5

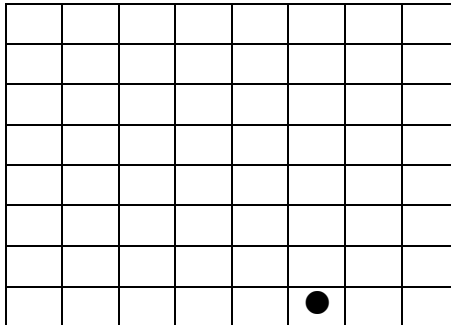
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**Exam Review Package 6: Mixed Questions**

1. At several branches of the Trillium Bank, transaction times with tellers during a two-hour period were recorded. Time, in the table, has been rounded to the nearest minute.

Time (min)	1	2	3	4	5	6 or more
Frequency	20	12	9	5	3	1

- a) Define the random variable,  $X$ .  
b) Create a probability distribution from these data  
c) Determine the expected transaction time at the Trillium Bank.
2. A baseball player has a batting average of 0.275. In most games, a player has four at-bats. Determine the probability that the player gets
- a) Exactly three hits in a game  
b) At least one hit in a game  
c) The expected number of hits in a game
3. A penny is placed in the bottom row of an eight-by-eight grid, as shown. If the penny can be moved one square at a time to the row above, either diagonally or straight ahead, how many paths will lead to the square in the top left-hand corner?

**ANSWERS**

1. A) times in minutes if transaction times with tellers during a 2-hour period.

B)

$X$	1	2	3	4	5	6
$P(X)$	$2/5$	$6/25$	$9/50$	$1/10$	$3/50$	$1/50$

C) 2.24mins

2. A) 0.06 B) 0.724 C) 1.1
3. 27



## Exam Review Package 7: Mixed Questions

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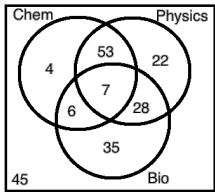
1. Compute the value of each of the following expressions.
  - a)  $P(4, 4)$
  - b)  $11 - 0!$
  - c)  $\frac{20 \times 19!}{20!}$
  - d)  $P(5, 0)$
  - e)  $C(5, 5)$
  - f)  $\binom{18}{2}$
2. Two hundred students have enrolled in at least one Grade 12 university math course this year.
  - 68 are in Calculus and Vectors
  - 78 are in Data Management
  - 60 are in Calculus and Vectors and Advanced Functions
  - 35 are in Advanced Functions and Data Management
  - 13 are in Calculus and Vectors and Data Management
  - 10 take all three
  - a) Draw a Venn diagram to represent this situation.
  - b) Use the Venn diagram to determine the total number of students who take Advanced Functions.
3. A six-sided die is rolled and then a coin is tossed.
  - a) Draw a tree diagram that represents all possible result of the two actions taken together.
  - b) Determine the number of outcomes in which the roll of the die is an odd number.
  - c) Determine the total number of outcomes in which the roll of the die odd and the coin toss is head.
4. Determine the probability of each of the following situations.
  - a) A red card is drawn from a standard deck of 52 playing cards.
  - b) Two even numbers are rolled on two consecutive rolls of a die
  - c) At least one 3 turns up when three dice are rolled
  - d) A five-card poker hand dealt from a standard deck of 52 cards result in a full house (three of a kind and two of a kind)
  - e) Two face cards are drawn in a row (without replacement) from a standard deck of 52 cards given that the first card drawn is a king
  - f) A committee of six people randomly chosen from seven males and eight females is either all male or all female
  - g) In a six-person sprint, Jesse finishes first, Marnie second, and Raul last
5. Use the appropriate counting techniques to answer each of the following.
  - a) Twenty books are to be placed on a shelf. Determine the number of ways the first five books can be placed on the shelf.
  - b) In how many ways can nine people place themselves in nine seats in a row?
  - c) Out of 15 different stores, how many ways can a salesperson visit 10 of the stores once each?
  - d) A team consisting of 3 members is to be chosen from a group of 12 people. How many different teams are possible if there must be a chairperson, a secretary, and a treasurer?
  - e) A class has 12 students. In how many different ways can the students be put into lab groups consisting of 3 students in each group?
  - f) In how many permutations of the digits 123456789 are the numbers 1 and 2 beside each other?
  - g) A school has 48 girls and 52 boys. How many committees of 5 members can be formed if there must be at least 1 boy on each committee?
  - h) How many groups consisting of at least 2 people can be chosen from a group of 10 people?

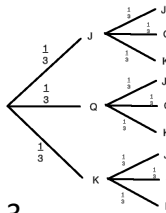


## Exam Review Package 8: Mixed Questions

- Two hundred students have enrolled in at least one Grade 12 university course this year.
  - 70 are in Chemistry
  - 110 are in Physics
  - 76 are in Biology
  - 60 are in Chemistry and Physics
  - 35 are in Physics and Biology
  - 13 are in Biology and Chemistry
  - 7 are in all three
  - Draw a Venn diagram to represent this situation.
  - Use the Venn diagram to determine the number of students who take Grade 12 courses but no Grade 12 university science courses.
  - Determine the probability that a student selected at random from the group is taking Biology.
  - Determine the probability that a student selected at random from the group is taking Chemistry or Physics.
- Three playing cards, a jack, a queen, and a king, are placed in a box. A card is drawn from the box, its value is recorded, and then the card is put back into the box. The process is then repeated a second time.
  - Draw a tree diagram for all possible outcomes of the two draws.
  - Determine the number of outcomes in which a king is the first card drawn from the box.
  - Find the probability that the first and second cards drawn are the same.
- A five-card poker hand is dealt and the last card is turned face up. Determine the probability that you have been dealt four aces, given that the card turned over is an ace.
- In how many ways can a group of 10 people be chosen from 6 adults and 8 children if the group must contain at least 2 adults?
- Nine horses are entered in a horse race. If you "box" three horses (three are chosen and they can finish in any of the first 3 positions in the race), determine the probability that you will hold the winning ticket.
- A drawer contains four red socks and five blue socks.
  - Three socks are drawn one at a time and then put back before the next selection. Determine the probability that...
    - Exactly two red socks are selected
    - At least two red socks are selected
  - Repeat part (a) without replacement
- There are 25 students in a Data Management class. Determine the probability that at least two of them share the same birthday (assume that every year has 365 days)

## ANSWERS

- |    |    |       |          |        |
|----|----|-------|----------|--------|
| 1. | a. | b. 45 | c. 19/50 | d. 3/5 |
|----|----|-------|----------|--------|
- 
- 
- |           |                          |         |
|-----------|--------------------------|---------|
| 3.        | 1/20,825                 | 4. 1001 |
| 5. 1/84   | 6.                       |         |
|           | a) i) 80/243 ii) 304/729 |         |
|           | b) i) 5/14 ii) 17/42     |         |
| 7. 0.5687 |                          |         |

- |    |    |
|----|----|
| 2. | a. |
|----|----|
- 
- |        |
|--------|
| b. 3   |
| c. 1/3 |

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} \qquad \sigma = \sqrt{\frac{\sum f(x_i - \mu)^2}{N}} \qquad z = \frac{x - \bar{x}}{s} \qquad \bar{x} = \frac{\sum x_i w_i}{\sum w_i} \qquad \bar{x} = \frac{\sum x_i}{n}$$

$$IQR = Q_3 - Q_1 \qquad P(A) = \frac{n(A)}{n(S)} \qquad P(A') = 1 - P(A) \qquad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) \times P(B) \qquad P(A \cup B) = P(A) + P(B)$$

$$C(n,r) = \frac{n!}{r!(n-r)!} \qquad P(n,r) = \frac{n!}{(n-r)!} \qquad n! = n(n-1)(n-2)(n-3) \ldots \times 2 \times 1$$

$$E(X) = \sum x_i P(X = x) \qquad \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$P(x) = \binom{n}{x} p^x (1 - p)^{n-x} \qquad E(X) = np \qquad P(x) = \frac{\binom{a}{x} \binom{n-a}{r-x}}{\binom{n}{r}} \qquad E(X) = \frac{ra}{n}$$

$$z = \frac{x - \bar{\mu}}{\sigma} \qquad \mu = np \qquad \sigma = \sqrt{npq} \qquad \text{odds in favour of } A = \frac{h}{k}, P(A) \\ = \frac{h}{h+k}$$