



## Google Drive

在“云端硬盘”App中打开

打开



scan\_kas ... 6-52.pdf



D

MDM4U

Unit 3: Combinations

### 3.7 Binomial Theorem

**Example 1:** Find a connection between the values in Pascal's triangle, the row/term number and combinations.

row 0                      1

row 1                      1      1

row 2                      1      2      1

row 3                      1      3      3      1

$2C_0$                        $1C_0$                        $1C_1$

$2C_0$                        $2C_1$                        $2C_2$

$3C_0$                        $3C_1$                        $3C_2$                        $3C_3$

\* row #  $C_{1 \text{ row}}$

### THE BINOMIAL THEOREM

Recall the a **binomial** is a polynomial with 2 terms, so it has the form  $a + b$ .

Expanding  $(a + b)^n$  is easy for small values of  $n$ , but becomes very laborious as  $n$  increases.

	Copy coefficients of terms here
For $n=0$ , $(a+b)^0 = 1$	1
For $n=1$ , $(a+b)^1 = a + b$	1 1
For $n=2$ , $(a+b)^2 = (a+b)(a+b)$ $= a^2 + ab + ab + b^2$ $= a^2 + 2ab + b^2$	1 2 1
For $n=3$ , $(a+b)^3 = (a+b)(a+b)^2$ $= (a+b)(a^2 + 2ab + b^2)$ $= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$ $= a^3 + 3a^2b + 3ab^2 + b^3$	1 3 3 1

(Pascal's  $\Delta$ )

Work: (value of  $t_{18,2}$  of Pascal's  $\Delta$ : 18C2)  
notation

Page 1 / 4

MDM4U

Unit 3: Combinations



drive.google.com





Google Drive  
在“云端硬盘”App中打开

打开



scan\_kas ... 6-52.pdf



MDM4U

Unit 3: Combinations

What do you notice about the coefficients of each expansion?

... follow the Pascal's  $\Delta$ .

What do you notice about the exponents of each expansion?

add up row #.

The formula for a binomial expansion is

$$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \dots + {}_nC_n a^0 b^n$$

Example 2: Expand  $(2x-1)^4$

$$\begin{aligned} \left. \begin{array}{l} n=4 \\ a=2x \\ b=-1 \end{array} \right\} &= {}_4C_0 (2x)^4 (-1)^0 + {}_4C_1 (2x)^3 (-1)^1 + {}_4C_2 (2x)^2 (-1)^2 + {}_4C_3 (2x)^1 (-1)^3 \\ &\quad + {}_4C_4 (2x)^0 (-1)^4 \\ &= (1)(16x^4) + (4)(8x^3)(-1) + (6)(4x^2)(1) + (4)(2x)(-1) + (1)(1) \\ &= 16x^4 - 32x^3 + 24x^2 - 8x + 1 \end{aligned}$$

Example 3: Expand  $(3x+2y)^3$

$$\begin{aligned} &= {}_3C_0 (3x)^3 (2y)^0 + {}_3C_1 (3x)^2 (2y)^1 + {}_3C_2 (3x)^1 (2y)^2 + {}_3C_3 (3x)^0 (2y)^3 \\ &= (1)(27x^3)(1) + (3)(9x^2)(2y) + (3)(3x)(4y^2) + (1)(8y^3) \\ &= 27x^3 + 54x^2y + 36xy^2 + 8y^3 \end{aligned}$$

2

Page 2 / 4



drive.google.com





Google Drive

在“云端硬盘”App中打开

打开



scan\_kas ... 6-52.pdf



D

MDM4U

3.7 (cont'd)

Unit 3: Combinations

Example 4: Write  $1 + 12x + 54x^2 + 108x^3 + 81x^4$  in the form  $(a+b)^n$ .

$$n = 4 \quad \checkmark \text{ (first term)}$$

$$a^n = 1$$

$$a^4 = 1$$

$$a = 1$$

$$\checkmark \text{ (last term)}$$

$$b^n = 81x^4$$

$$b^4 = 81x^4$$

$$b = \sqrt[4]{81x^4}$$

$$b = 3x$$

∴ The binomial is  $(1 + 3x)^4$

3

Page 3 / 4



drive.google.com

