Learning Goal: Introduction to Probability

3 Basic Types of Probability:

1. Subjective Probability
   - an estimate of likelihood based on intuition and experience
   - example: "I think...."
2. Empirical (Experimental) Probability
   - an estimate of likelihood based on an experiment by the number of trials
   - example: " 3 out of 5 times I tried rolling a die is 2."
3. Theoretical Probability
   - an estimate of likelihood based on analysis
   - example: "There's 50% chance of getting a head when flipping a coin."

Outcomes: possible results

Event: an occurrence

Trial: an examination/experiment

Sample Space: all possible outcomes of an event

Notation:

$$P(A) = \frac{n(A)}{n(S)}, \quad 0 \leq P(A) \leq 1$$

($n(A)$: number of elements of the event; $n(S)$: number of element of sample space)

$0 = impossible; 1 = occurs\ 100\%\ of\ the\ time$

Example#1: Event: Flipping a coin

P(head) = $\frac{1}{2}$

P(tail) = $\frac{1}{2}$

Example#2: Event: Flipping a coin if **both** sides are head.

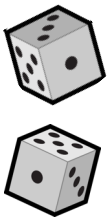
P(head) = $\frac{1}{1} = 1$

P(tail) = $\frac{0}{1} = 0$

Example#3:
Event: Rolling a die:

P(1) = $\frac{1}{6}$	P(4) = $\frac{1}{6}$
P(2) = $\frac{1}{6}$	P(5) = $\frac{1}{6}$
P(3) = $\frac{1}{6}$	P(6) = $\frac{1}{6}$

Example#4:
Event: Rolling **a pair** of standard dice to find **the sum**:



1st die \ 2nd die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

P(1) = $\frac{0}{36}$	P(evens) = $\frac{18}{36}=\frac{1}{2}$
P(2) = $\frac{1}{36}$	P(odds) = $\frac{1}{2}$
P(3) = $\frac{2}{36}=\frac{1}{18}$	P(doubles) = $\frac{6}{36}=\frac{1}{6}$
P(4) = $\frac{3}{36}=\frac{1}{12}$	P(prime) = $\frac{15}{36}=\frac{5}{12}$
P(7) = $\frac{6}{36}=\frac{1}{6}$	P(composite) = $\frac{7}{12}$
P(not 7) = $\frac{5}{6}$	

P(A')= Complement of event A = event A doesn't happen

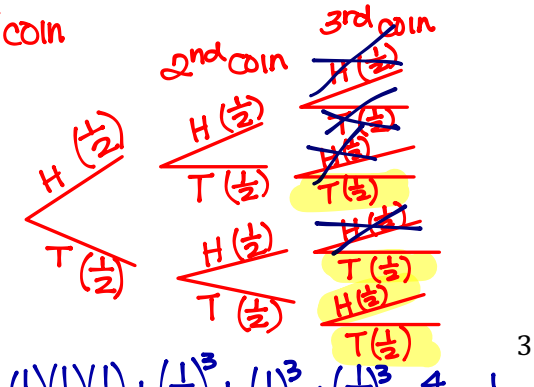
P(A)+P(A')=1

Example#5: Given the sample space {1,2,3,4,5,6,.....50}

P(evens) = $\frac{1}{2}$	P(odds) = $\frac{1}{2}$
P(prime) = $\frac{15}{50}=\frac{3}{10}$ {2,3,5,7,11,13,17,19,23,29,31,37,41,43,47}	P(composite) = $\frac{34}{50}=\frac{17}{25}$
P(perfect squares) = $\frac{7}{50}$	P(perfect cubes) = $\frac{3}{50}$
P(not perfect squares) = $\frac{43}{50}$	P(not perfect cubes) = $\frac{47}{50}$

Tree Diagram:

① Determine the probability of tossing at least 2 tails with 3 coins



$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^3=\frac{4}{8}=\frac{1}{2}$