



MDM4U Unit 6: Probability Distributions

6.3 Hypergeometric Distributions

A **hypergeometric distribution** has a specific number of dependent trials with two possible outcomes, success or failure; in other words, we are sampling without replacement. So, the probability of success changes as each trial is made since each selection reduces the number of items that could be selected in the next trials.

Example 1: Three cards are drawn at random from a shuffled deck of regular playing cards (without replacement). Construct a probability distribution table for the number of spades in the hand.

Let X rep. the # of spades

x	$P(X=x)$
0	$\frac{{}^{39}C_3}{{}^{52}C_3} = 0.41$
1	$\frac{{}^{13}C_1 \times {}^{39}C_2}{{}^{52}C_3} = 0.44$
2	$\frac{{}^{13}C_2 \times {}^{39}C_1}{{}^{52}C_3} = 0.14$
3	$\frac{{}^{13}C_3}{{}^{52}C_3} = 0.01$

Probability in a Hypergeometric Distribution

$$P(X=x) = \frac{{}_a C_x \cdot {}_{n-a} C_{r-x}}{{}_n C_r}$$

a = # of desired items available
 x = # of desired items chosen
 n = # of items available
 r = # of items chosen

Expectation for a Hypergeometric Distribution

In a hypergeometric distribution, the probability of success (or failure) in a sample should be proportion to the population.

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Example 2: A group of 6 people is to be selected from a group of 8 adults and 10 teens. Determine whether this situation can be modeled with a hypergeometric distribution.

= trials are dependent (not replacing)





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a) Construct a probability distribution table for the number of women in the jury.

Let X rep the # of women

x	$P(X=x)$
0	${}^8C_6 / {}^{18}C_6 = 0.002$
1	${}^{10}C_1 \times {}^8C_5 / {}^{18}C_6 = 0.003$
2	${}^{10}C_2 \times {}^8C_4 / {}^{18}C_6 = 0.17$
3	${}^{10}C_3 \times {}^8C_3 / {}^{18}C_6 = 0.36$
4	${}^{10}C_4 \times {}^8C_2 / {}^{18}C_6 = 0.32$
5	${}^{10}C_5 \times {}^8C_1 / {}^{18}C_6 = 0.11$
6	${}^{10}C_6 / {}^{18}C_6 = 0.01$

b) How many teens would you expect to be in the group?

$$\frac{10}{18} = \frac{x}{6}$$

$$18x = 60$$

$$x = 3.3$$

∴ we would expect there to be 3.3 women.

Example 3: In the spring, the Ministry of the Environment caught and tagged 500 raccoons in a wilderness area. The raccoons were released after being vaccinated against rabies and tagged. To estimate the raccoon population in the area, the ministry caught 40 raccoons later in the summer and 15 of them had tags. Estimate the raccoon population in the wilderness area.

Let X rep total raccoon population

tagged
total

$$\frac{15}{40} = \frac{500}{x}$$

$$15x = 20000$$

$$x = 1333.3$$

∴ the estimated raccoon pop is 1333.3

