① normal distribution ② discrete

Lesson: Normal Approximation for Binomial Distribution

① success = p, failure = q ② independent event

- The Normal distribution can be used to approximate Binomial probabilities when n is large and p is close to 0.5. In answer to the question
- the approximation should only be used when both $np > 5$ and $nq > 5$.
- $(mean) \quad \mu \text{ or } = np$
- $(standard\ deviation) \quad \sigma = \sqrt{npq}$

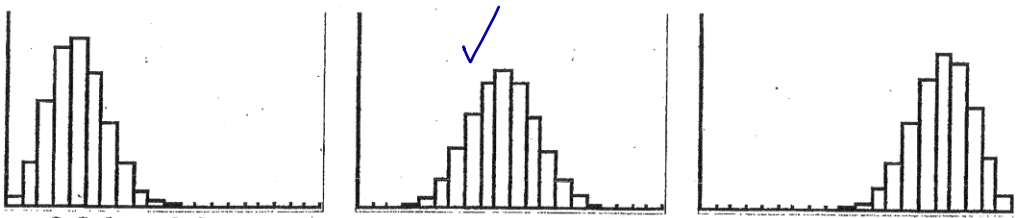
Example #1:

discrete normal distribution answering each question independent

Bill is preparing for a test with 20 questions. He wants to determine his chances of passing if the certainty of answering each question is:

i) 20% p= ✓ ii) 45% ✓p= iii) 75% ✓ p=

The distribution for the number of questions Bill would get correct is binomial. For all three situations, n = 20 but the values of p and q change. The following graphs are the probability distributions for each case:



p=0.2 & q=0.8 p=0.45 & q=0.55 p=0.75 & q=0.25

a) Which graph appears to have a normal distribution?

second one.

b) Which case meets the conditions required for normal distribution?

n=20
p=0.45
q=0.55

np = 20 (0.45) = 9 > 5 ✓

nq = 20(0.55) = 11 > 5 ✓

c) Calculate the mean and the standard deviation for normal approximation.

$\mu = np = 20(0.45) = 9$

$\sigma = \sqrt{npq} = \sqrt{20(0.45)(0.55)} \approx 2.22$

d) Determine the probability that the person passes the test using normal approximation.

n=20

Answer at least 10 questions correctly.

$P(10 \le X \le 20)$ apply continuity correction for discrete

$= P(9.5 < X < 20.5)$

$= P(X > 9.5)$

$P(X > 9.5)$

$= P\left(Z > \frac{9.5 - 20(0.45)}{\sqrt{20(0.45)(0.55)}}\right)$

$= P(Z > 0.22\ldots)$

$= 1 - 0.5871$

$= 0.4129$

Example #2:

The bottle of Cepsi know that they have 42% of the market. At a booth, 80 people take the Cepsi Challenge.

a) What type of distribution is the number of people who choose Cepsi?

Binomial Distribution
- independent
- discrete
- $p = 42\%$ $q = 58\%$

b) Can a normal approximation be made?

$np > 5$ $nq > 5$ $n = 80$

$= 80(0.42)$ $= 80(0.58)$

$= 33.6 > 5$ ✓ $= 46.4 > 5$ ✓

∴ Yes.

c) What is the mean and standard deviation of the approximate normal distribution?

mean: np

$= 80(0.42)$

$= 33.6$

standard deviation. $\sqrt{npq}$.

$= \sqrt{80(0.42)(0.58)}$

$\doteq 4.41$

d) What is the probability that the number of people that will choose Cepsi is between 25 and 40?

Apply Continuity Correction

exclusive. $(25 < x < 40.)$

$P(25.5 < x < 39.5)$

$$= P\left(\frac{25.5 - 80(0.42)}{\sqrt{80(0.42)(0.58)}} < Z < \frac{39.5 - 80(0.42)}{\sqrt{80(0.42)(0.58)}}\right)$$

$= P(-1.83 < Z < 1.34)$

$= 0.9099 - 0.0336$

$= 0.8763$