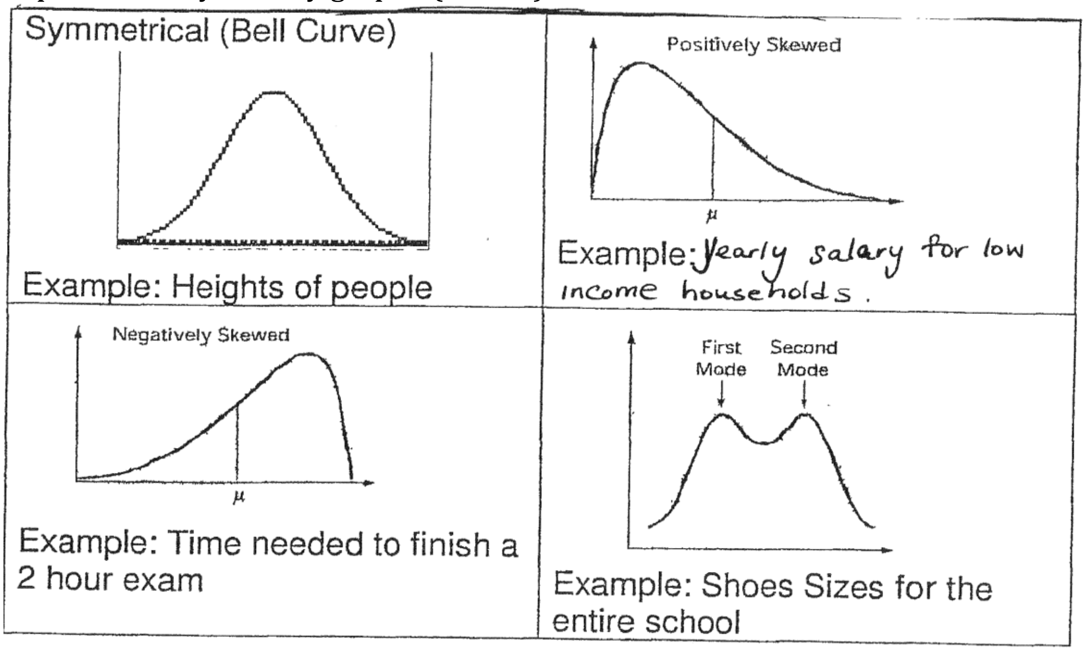
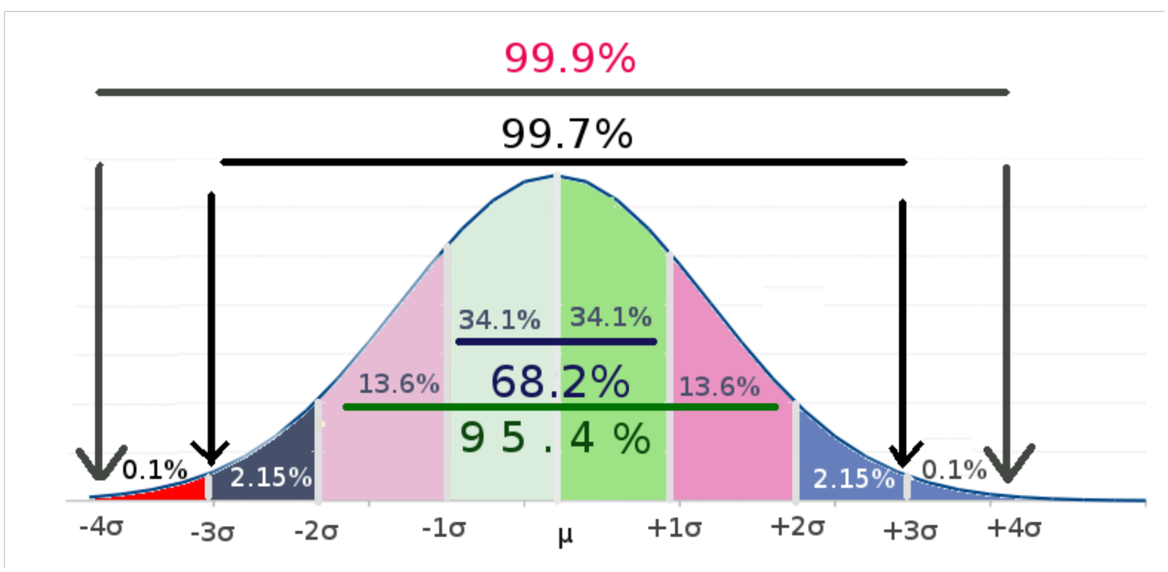
Lesson: Introduction to Continuous Probability Distribution

Continuous probability distributions:

- possible values of the random variable are any real numbers.
- often represented by density graphs (curves)



Characteristics of the Normal Distribution



- It is symmetrical about the mean (median and mode are the same value).
- It is bell-shaped.
- The total area under the density curve equals one
- When considering continuous distributions, we are most often interested in determining the probability that a variable falls within a particular range of values
- The probability may be determined using the area under the curve

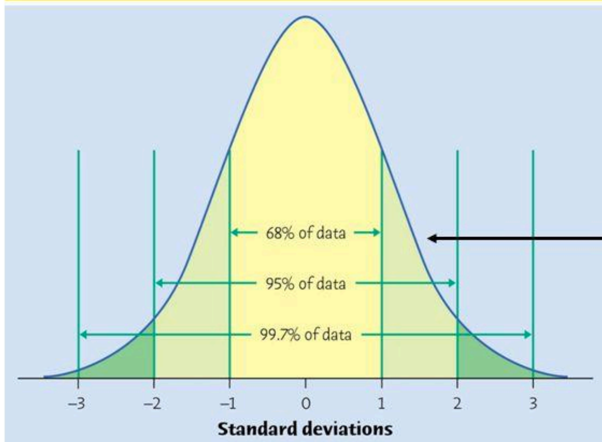
In a normal distribution, the proportions of the data set found within one, two and three standard deviations of the mean always were 68% (within one standard deviation of the mean), 95% (within two standard deviations of the mean), and 99.75% (within three standard deviations of the mean). That is the 68-95-99.75 rule.

Even though the mean and standard deviation affect the position and shape of the normal curve, changing their values does not distort the areas underneath the normal density function and as a result does not distort the proportions of observations found within these regions nor the probabilities determined for these regions. It is the relative position of the boundaries of these regions, measured in standard deviations from the mean, that are important and not the actual values themselves.

THE 68–95–99.7 RULE (aka – The Empirical rule)

In the Normal distribution with mean μ and standard deviation σ:

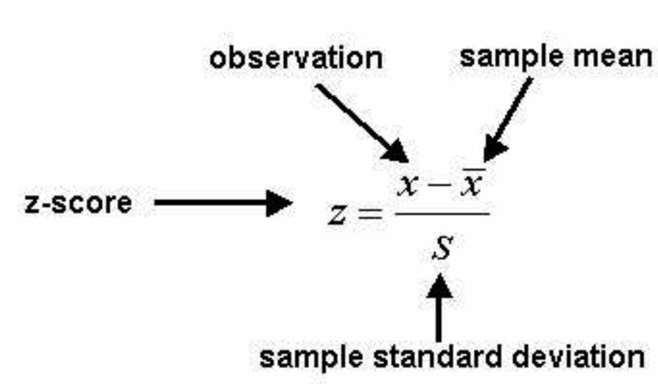
- Approximately **68%** of the observations fall within σ of the mean μ.
- Approximately **95%** of the observations fall within 2σ of μ.
- Approximately **99.7%** of the observations fall within 3σ of μ.



Normal distributions are abbreviated as ; N(μ,σ)

The Normal distribution with mean of 0 and standard deviation of 1 is called the standard Normal curve; N(0,1)

By calculating a Z-score, which "standardizes" our observations to the standard normal distribution, we are determining the number of standard deviations that our observation is from the mean and we can use a z-score table to measure the area under the curve.



Strictly speaking, the z-score is given by the following equation.

$$z = \frac{x - \mu}{\sigma}$$

where μ is the true mean and σ is the true standard deviation. Since we usually do not know μ and σ, we estimate these values using $\overline{x}$ and S

Example: Assuming that a normal distribution occurs in a situation, the mean is 14 and standard deviation is 4.5. Determine the probability for the random variable is less than 13.01

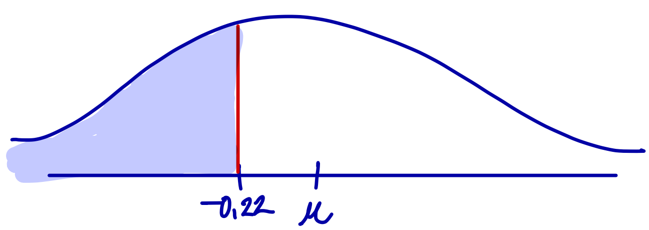
Step #1: Convert a particular normal distribution to the standard normal distribution by calculating the z-score

$$P(X < 13.01)$$

$$= P\left(Z < \frac{13.01 - 14}{4.5}\right)$$

$$= P(Z < -0.22)$$

Step #2: Identify the area under the bell curve that is needed for the probability



Step #3: Refer to the Normal Distribution Table for the probability of a z-score that is 0.4129

Solution: $P(X < 13.01)$

$$= P\left(Z < \frac{13.01 - 14}{4.5}\right)$$

$$= P(Z < -0.22)$$

$$\doteq 0.4129$$

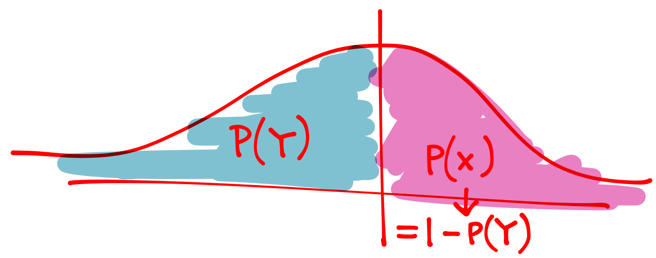
Example: Assuming that a normal distribution occurs in a situation, the mean is 14 and standard deviation is 4.5. Determine the probability for the random variable is greater than 17.

Step #1: Convert a particular normal distribution to the standard normal distribution by calculating the z-score

Step #2: Identify the area under the bell curve that is needed for the probability

$P(X > 17)$

$= P\left(Z > \frac{17-14}{4\ 5}\right)$



Step #3: Refer to the Normal Distribution Table for the probability of a z-score that is 74.86%

$= P(Z > 0\ 67)$

OR Solution :

$P(X>17)$

$= 1 - P(X<17)$

$= 1 - P\left(Z < \frac{17-14}{4.5}\right)$

$= 1 - 0.7486$

$= 0.2514$

Step #4: Determine the probability

$= 1 - 0\ 7486$

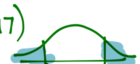
$= 0.2514$

Example: Assuming that a normal distribution occurs in a situation, the mean is 14 and standard deviation is 4.5. Determine the probability for the random variables between 10 to 17 inclusively.

"inclusively" means $P(10 \leq X \leq 17)$ ← but we cannot use "equal"

Therefore

$P(X<10)$ and $P(X>17)$ should be excluded



$1 - P(X<10) - P(X>17)$

$1 - P\left(Z < \frac{10-14}{4\ 5}\right) - P\left(Z > \frac{17-14}{4\ 5}\right)$

$= 0\ 5619$

Now, Let's do two scenarios "between 10 to 17 exclusively"

$P(10<X<17)$

$= P\left(\frac{10-14}{4.5} < Z < \frac{17-14}{4\ 5}\right)$

$= P(-0\ 89 < Z < 0\ 67)$

$= 0\ 7486 - 0\ 1867$

$= 0\ 5619$

Important Note***
if the calculated z-score value is a terminating decimal on the thousandth with a "5", you MUST calculate the mean of 2 z-score values from the table above and below

eg. $P\left(Z < \frac{108.89 - 110}{2}\right)$

$= P(Z < -0.555)$

$= \frac{P(Z<-0.55) + P(Z<-0.56)}{2}$

$= \frac{0.2912 + 0.2877}{2}$

$= 0.28945$

you will notice that the answers for both cases, $10 \leq X \leq 17$ and $10 < X < 17$, are the same, this indicates that the inclusivity will not make a difference in determining the probability.