## 6.5 Normal Approximation to the Binomial Distribution

***RECALL*** A binomial distribution is a discrete probability distribution in which:

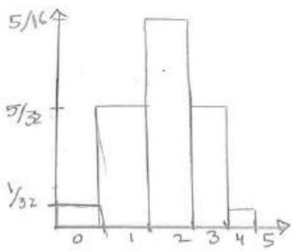
- There are only two outcomes – success and failure
- Each trial is identical and independent
- The probability of success or failure is unchanged

If a data set is reasonably large, and the data fall into a symmetrical bell shape, we can approximate discrete data with a normal distribution model. The normal model can then be used to make predictions!

Example 1*:* A coin is tossed 5 times. Construct a probability distribution table and probability histogram. for the number of tails.

Let X rep. # of tails.

| $x$ | $P(X=x)$ | |
|---|---|---|
| 0 | $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$ | |
| 1 | ${}_5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4$ | $=\frac{5}{32}$ |
| 2 | ${}_5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3$ | $=\frac{5}{16}$ |
| 3 | ${}_5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$ | $=5/16$ |
| 4 | ${}_5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1$ | $=5/32$ |
| 5 | ${}_5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$ | $=\frac{1}{32}$ |




Notice that the graph resembles a normal distribution!

Example 2*:* Lana tosses a fair coin 50 times. Estimate the probability that she will get heads less than 20 times.

Let X rep the number of heads

$$P(X<20) = P(X=0) + P(X=1) + \dots + P(X=19)$$

$$= \left(\frac{1}{2}\right)^{50} + {}_{50}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{49} + \dots + {}_{50}C_{19}\left(\frac{1}{2}\right)^{19}\left(\frac{1}{2}\right)^{31}$$

Too many calculations!







To avoid this tedious task, we use a *NORMAL APPROXIMATION.*

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***Rules for Normal Approximations***

1. Must be a binomial distribution
2. $np > 5$, $nq > 5$
3. $\mu = np$ (mean), $\sigma = \sqrt{npq}$ (standard deviation)
4. Discrete values must be converted to continuous intervals (continuity correction)
5.

Now, try using a normal approximately to solve Example 2.

Step 1: Test
$n = 50$
$p = 1/2$
$q = 1/2$
$np = 50(\frac{1}{2}) = 25 > 5$ ✓
$nq = 50(\frac{1}{2}) = 25 > 5$ ✓

Step 2: Mean / St dev
$\mu = np = 25$
$\sigma = \sqrt{npq} = \sqrt{50(\frac{1}{2})(\frac{1}{2})} = \sqrt{12.5} \doteq 3.5$

Step 3: Continuity correction
$P(X < 20) = P(X < 19.5)$

Step 4: $P(X < 20.5) = P\left(Z < \frac{19.5 - 25}{3.5}\right)$
$= P(Z < -1.57)$
$= 0.0582$
$= 5.8\%$





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$= 0.0582$

$= 5.8\%$





Example 3: The probability that a car part is defective is 8%. Out of 100 parts that are sampled, what is the probability that:

a) 10 or more ~~More than 10~~ will be defective?

Let X rep the number of defective parts

① $n = 100$ $\quad np = 100(0.08) = 8 > 5$ ✓ $\quad$ ② $\mu = np = 8$

$p = 0.08$ $\quad \sigma = \sqrt{100(0.08)(0.92)}$

$q = 0.92$ $\quad nq = 100(0.92) = 92 > 5$ ✓ $\quad = \sqrt{7.36} \doteq 2.7$

③ $P(X > 10)$
$= P(X > 9.5)$

④ $P\left(Z > \frac{9.5-8}{2.7}\right)$
$= P(Z > 0.56)$

$P = 1 - 0.7123$
$= 0.2877$ or $28.8\%$

b) Exactly 15 will be defective?

$P(X = 15) = {}_{100}C_{15}\,(0.08)^{15}(0.92)^{85}$

$= 0.0074$ or $0.7\%$

**Summary – Binomial or Normal?**

1. When we want to find the probability of a range of successes (a lot), we use the normal approximation (eg. $P(X>15)$)

2. When we want to find the probability of an exact number of successes, use the Binomial Distribution, eg $P(X=15)$.

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