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Bayview Secondary School
Mathematics Department – Course Code: MDM4U1
Unit 5 Probability Distribution Assessment for Learning

Instruction:

- **Round all answers to 4 decimal places unless otherwise indicated.**
- **Show all necessary steps and work in a logical sequence to demonstrate the chain of thought to obtain full marks.**

KNOWLEDGE/UNDERSTANDING

Short answer questions. Please write your final answer in the table provided, you do not need to provide steps in this section. [Total 12 marks: 1 mark each]

Questions	Final Answers
1. Calculate the z-score of $x = 79$ for a population approximated with a mean of 81 and standard deviation of 36.	$\frac{79-81}{36} = -0.0556$
2. A coin is flipped 50 times. You win each time a tail appears. Calculate the probability of getting 26 wins.	$\binom{50}{26}\left(\frac{1}{2}\right)^{26}\left(\frac{1}{2}\right)^{24} = 0.1080$
3. 5 numbers, ranging from 1 to 5, are placed in a purple hat. You are to choose one number from the hat. Determine the expected value of the chosen number.	$\frac{1}{5}(1+2+3+4+5) = 3$
4. An ordinary die is thrown five times. If X represents the number of times a three is rolled,	
a. Determine the type of distribution modeled in this situation.	Binomial
b. What is the probability of success?	$\frac{1}{6}$
c. What is the expectation?	$5\left(\frac{1}{6}\right) = \frac{5}{6}$
d. What is P(5)?	$\binom{5}{5}\left(\frac{1}{6}\right)^{5}\left(\frac{5}{6}\right)^{0} = \frac{1}{7776} = 0.0001$
e. What is the probability that exactly two 3's turn up?	$\binom{5}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{3} = \frac{625}{3888} = 0.1608$
5. A box contains 6 white balls and 5 purple balls. 4 balls are randomly withdrawn without being replaced.	
a. What type of distribution is modeled in this situation?	Hypergeometric
b. Write an expression for the probability function $P(X)$ used to determine the probability of choosing x white marbles.	$\left(\frac{\binom{6}{x}\binom{5}{4-x}}{\binom{11}{4}}\right)$
c. What is the expected number of white balls selected?	$\left(\frac{4(6)}{11}\right) = 2.1818\ balls$
d. What is the probability that zero white balls will be chosen?	$\left(\frac{\binom{6}{0}\binom{5}{4}}{\binom{11}{4}}\right) = \frac{1}{66} = 0.0152$

APPLICATION

6. Suppose the time periods that people wait in line at a particular bank are normally distributed with a mean of 10 minutes and a standard deviation of 3 minutes. Calculate the probability that a person will wait:

* concluding statements are not required

a) less than 13 minutes ②

$$\begin{aligned} & P(x < 13) \\ &= P\left(Z < \frac{13-10}{3}\right) \\ &= P(Z < 1) \\ &= 0.8413 \end{aligned}$$

b) more than 15 minutes ②

$$\begin{aligned} & P(X > 15) \\ &= 1 - P(X < 15) \\ &= 1 - P\left(Z < \frac{15-10}{3}\right) \\ &= 1 - P(Z < 1.67) \\ &= 1 - 0.9525 \\ &= 0.0475 \end{aligned}$$

c) between 6 minutes and 11 minutes ②

$$\begin{aligned} & P(6 < X < 11) \\ &= P\left(\frac{6-10}{3} < Z < \frac{11-10}{3}\right) \\ &= P(-1.33 < Z < 0.33) \\ &= 0.6293 - 0.0918 \\ &= 0.5375 \end{aligned}$$

d) either more than 15 minutes or less than 13 minutes ①

$$\begin{aligned} & P(X > 15) + P(X < 13) \\ &= 0.0475 + 0.8413 \\ &= 0.8888 \end{aligned}$$

7. A single card is selected from a standard deck of 52 cards. The card is noted and replaced. If the process is repeated a total number of six times, calculate the probability that a face card is selected more than three times. ③

$$p = \frac{12}{52} \qquad q = \frac{40}{52} \qquad n = 6$$

$$P(4) + P(5) + P(6)$$
$$= \binom{6}{4}\left(\frac{12}{52}\right)^4\left(\frac{40}{52}\right)^2 + \binom{6}{5}\left(\frac{12}{52}\right)^5\left(\frac{40}{52}\right)^1 + \binom{6}{6}\left(\frac{12}{52}\right)^6\left(\frac{40}{52}\right)^0$$
$$= 0.0283$$

8. A random sample of 1000 elevators was tested. Calculate the probability of observing 27 or more defectives assuming that the elevators' average reliability is 0.98. ④

$$p = 0.02 \qquad q = 0.98$$
$$np = 1000(0.02) = 20 > 5$$
$$nq = 1000(0.98)980 > 5$$
normal approximation for binomial distribution can be used
continuity correction is needed
$$\sqrt{npq} = \sqrt{1000(0.02)(0.98)}$$
$$= \frac{7\sqrt{10}}{5}$$

$$P(X > 26.5)$$
$$= P\left(Z > \frac{26.5 - 20}{\frac{7\sqrt{10}}{5}}\right)$$
$$= 1 - P(Z < 1.47)$$
$$= 1 - 0.9292$$
$$= 0.0708$$

9. Slick decided to pick up some spare spending money using a game that he designed. The game is based on the roll of a six-sided fair die. The player rolls a "n". If n is a prime number, then player must pay Slick $\$2^n$, but if n is not a prime number, Slick must pay the player $\$n^2$. Construct a probability distribution table and determine the expected value of the game. Interpret this value. What should Slick charge to make this game fair? Explain. ④

Prime: 2, 3, 5. Composite: 1, 4, 6

Face value of the die	X=win $	P(X)	
1	1^2	$\frac{1}{6}$	$E(X) = \frac{1}{6}(1 - 4 - 8 + 16 - 32 + 36)$ $= \frac{1}{6}(9)$ $= 1.5$ *Therefore, Slick should charge* \$1.50 *each game to make this a fair game.*
2	-2^2	$\frac{1}{6}$	
3	-2^3	$\frac{1}{6}$	
4	4^2	$\frac{1}{6}$	
5	-2^5	$\frac{1}{6}$	
6	6^2	$\frac{1}{6}$	