Lesson: Probability of Binomial Distribution

Definitions:

Probability in a Binomial Distribution:

- Binomial probability distribution of the number of successes in a sequence of n independent experiment
- 2 possible outcomes only: "Success" or "Failure"
- "Success" $= p$ ← probability of success
- "Failure" $= q$ ← probability of failure.
- $p + q = 1$
- probability of each outcome remains constant throughout the trials (they are independent)

Probability in a Binomial Distribution:

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

# of total trials; probability of success.; probability of failure.; nCx Combination; # of success trials (you want)

Expectation for a Binomial Distribution:

$$E(X) = np$$

number of times rolling a "3" $P(x) = \binom{n}{x} p^x q^{n-x}$

X	$P(X)$
0	$\binom{10}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{10}$
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

$n=10$, $p=\frac{1}{6}$, $q=\frac{5}{6}$

Examples from Textbook Questions Page 385 #1:

Situations	Binomial or not Binomial
a) a child rolls a die ten times and counts the number of 3s (independent; n; success = 3 $\left(\frac{1}{6}\right)$; failure = not 3 $\left(\frac{5}{6}\right)$)	B
b) the first player in a free-throw basketball competition has a free-throw success rate of 88.4%. A second player takes over when the first player misses the basket. – dependent –	NB
c) A farmer gives 12 of the 200 cattle in a herd an antibiotic. The farmer then selects 10 cattle at random to test for infections to see if the antibiotic was effective. – dependent –	NB
d) A factory producing electric motors has a $\boxed{0.2\% \text{ defect rate}}$. A quality-control inspector needs to determine the expected number of motors that would fail in a day's production. (independent; n; p)	B

Textbook Example Page 379 Example #1:

A manufacturer of electronics components produces precision resistors designed to have a tolerance of $\pm 1\%$. From qualilty-control testing, the manufacturer knows that about one resistor in six is actually within just 0.3% of its normal value. A customer needs three of these precise resistors. What is the probability of finding exactly three such resistors among the first five tested?

Binomial = testing the normal value of each resistor. (independent)

Outcomes → fall within the normal value $(p) = \frac{1}{6}$
→ doesn't fall within the normal value. $(q) = \frac{5}{6}$

$n = 5$ $x = 3$

$$P(x) = \binom{n}{x} p^x q^{n-x}$$
$$= \binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2$$
$$\doteq 0.03215$$

∴ the probability of exactly three out of five meet the specification is approx. 3.2%.

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

Textbook Example Page 380 Example #2:

Tan's family moves to an area with a different telephone exchange, so they have to get a new telephone number. Telephone numbers in the new exchange start with 446, and all combinations for the four remaining digits are equally likely. Tan's favourite numbers are the prime numbers 2, 3, 5, and 7.

a) Calculate the probability distribution for the number of these prime digits in Tan's new telephone number.

$n=4$ $\boxed{x=?}$ $p=\frac{4}{10}$ $q=\frac{6}{10}$

(# of prime) 0-4

905 area code
884 exchange
4453

x	$P(X)$
0	$\binom{4}{0}\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^4 = 0.1296$
1	$\binom{4}{1}\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3 = 0.3456$
2	$\binom{4}{2}\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2 = 0.3456$
3	$\binom{4}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1 = 0.1536$
4	$\binom{4}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^0 = 0.0256$

b) What is the expected number of these prime digits in the new telephone number?

$$E(X) = np = 4\left(\frac{2}{5}\right) = 1.6$$

∴ on average, there will be 1.6 Tan's favourite digits in the phone number.

Textbook Example Page 384 Example 3:

The Choco-Latie Candies company makes candy-coated chocolates, 40% of which are red. The production line mixes the candies randomly and packages ten per box.

a) What is the probability that at least three candies in a given box are red?

Direct method: P(X=3)+P(X=4)+P(X=5)+P(X=6)….

Indirect method: 1-P(X=0)-P(X=1)-P(X=2)
= 0.8327

P(0)=(10C0)(0.4)^0(0.6)^10
P(1)=(10C1)(0.4)^1(0.6)^9
P(2)=(10C2)(0.4)^2(0.6)^8

b) What is the expected number of red candies in a box?

E(X)=np=10(0.4)=4
Therefore, on average, there will be 4 red in one box.