Bayview Secondary School **Name:** _____
Mathematics Department – Course Code: MDM4U1
Unit 5 Probability Distribution Assessment of Learning

K/U	APPS	Comm
/18	/10	/2

Instructions:
- **Final answer expectations: Round to 4 decimal places for probabilities or in exact value; 2 decimal places for z-scores except the special case; 2 decimal places if expected value.**
- **Show all necessary steps and work in a logical sequence to demonstrate the chain of thought to obtain full marks if the question is worth more than 1 mark.**

KNOWLEDGE/UNDERSTANDING

Multiple Choice questions:
Place the CAPITAL Letter of the most appropriate choice in the box provided on your answer sheet on page 3. [12 marks – 1 mark each]

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
B	B	B	C	D	A	D	A	D	C	B	A

13. A wildlife researcher is studying a population of turtles in a protected wetland. She knows that 20 of the turtles in the area are caught and tagged from a previous study. During a routine check, she captures and examines 8 turtles at random and finds that the 3 turtles are tagged. Estimate the total number of turtles in the wetland. [2 marks]

$$3 = \frac{8(20)}{n}$$

$$n = 53.3333 \text{ turtles}$$

$\therefore$ 53 turtles.

14. A nature photographer takes photos of birds in a forest where the probability of capturing a photo of a rare bird in a single shot is 0.2. She takes 15 independent shots in one session. What is the probability that she captures exactly 3 photos of the rare bird? [2 marks]

$$P(X=3) = \binom{15}{3}(0.2)^3(0.8)^{12}$$

$$\doteq 0.2501$$

15. A jar contains 8 red marbles and 12 blue marbles. If 5 marbles are selected without replacement, what is the probability that exactly 3 red marbles are selected? [2 marks]

$$\frac{\binom{8}{3}\binom{12}{2}}{\binom{20}{5}}$$

$$= \frac{77}{323} \text{ or } 0.2384$$

Application

1. A company finds that 75% of customers who receive a promotional email open it. The company sends the email to 200 customers. What is the probability that at least 150 customers open the email? [3 marks]

$np = 200(0.75)$
$= 150 > 5$

$nq = 200(0.25)$
$= 50 > 5$

$\therefore$ Normal Approximation can be applied.

$P(X \geq 150)$
$= P(X > 149.5)$
$= P\left(z > \frac{149.5 - 150}{\sqrt{200(0.75)(0.25)}}\right)$
$= P(z > -0.08)$
$= 1 - 0.4681$
$= 0.5319$

2. In a city-wide math contest, student scores are normally distributed. However, the contest organizers did not release the mean or standard deviation of the scores.

They did reveal the following:

- 20% of students scored below 50.0%
- 10% of students scored above 80.0%

Assuming the scores follow a normal distribution, determine the mean and standard deviation of the students' scores on the contest. [4 marks]

$P(X < 50) = 0.2$

$\frac{50 - \mu}{\sigma} = -0.845$

$50 - \mu = -0.845\sigma$

$50 + 0.845\sigma = \mu$ ①

$P(X < 80) = 0.90$

$\frac{80 - \mu}{\sigma} = 1.285$

$80 - 1.285\sigma = \mu$ ②

let ① = ②

$50 + 0.845\sigma = 80 - 1.285\sigma$

$2.13\sigma = 30$

$\sigma = 14.0845\%$

$50 + 0.845\sigma = \mu$

$\mu = 61.9014\%$

3. A school club is running a fundraising game at a carnival. For $5, participants can spin a prize wheel. The wheel has 20 equal sections with the following prizes:

- 1 section wins $50
- 3 sections win $10
- 6 sections win $5
- 10 sections win nothing

The club wants to determine whether the game is profitable.

a) Create the probability distribution for the net profit per play from the club's perspective. [2 marks]

player's perspective

X	P(X)
50	$\frac{1}{20}$
10	$\frac{3}{20}$
5	$\frac{6}{20}$
0	$\frac{10}{20}$

club's perspective

X	P(X)
-45	$\frac{1}{20}$
-5	$\frac{3}{20}$
0	$\frac{6}{20}$
5	$\frac{10}{20}$

b) Calculate the expected profit per play for the club.
Include a concluding statement to interpret your calculated value. [Comm: 1 mark]

$-45\left(\frac{1}{20}\right) + (-5)\left(\frac{3}{20}\right) + 0\left(\frac{6}{20}\right) + 5\left(\frac{10}{20}\right)$

$= -0.5$

The expected profit per play for the club is -$0.50, meaning the club lose 50 cents on average per play.

Communication

One mark will be awarded for proper mathematical forms throughout the assessment. [1 mark]

--- End of Assessment ---