Lesson: Probability of Hypergeometric Distribution

Definitions:

Hypergeometric Distribution:

- involves a series of dependent trials, each with success or failure as the only possible outcomes
- outcomes doesn't involve success or failure probability rate .
- probability of success changes as each trial is made
- random variable is the number of successful trials in an experiment
- probability involves Combination

$$P(x) = \frac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}} \qquad E(X) = \frac{ra}{n}$$

a: # of success available; x: # of success you need; $n-a$: # of failure available; $r-x$: # of failure you need; n: sample space, # of total trials; r: # of dependent trials needed

$E(X)$: r: # of dependent trials needed; a: # of success available; n: sample space

$a =$ *number of successful outcomes*

$x =$ *number of successes needed*

$n =$ *sample space*

$r =$ *number of dependent trials needed*

Example from Textbook Page 404 #1:

Which of these random variables have a hypergeometric distribution?

Random Variables	Hypergeometric
a) the number of clubs dealt from a deck give one card at a time — dependent —	Hyper.
b) the number of attempts before rolling a six with a die fixed # of "n" is missing	Not Hyper
c) the number of 3s produced by a random number generator - Independent	Binomial.
d) the number of defective screws in a random sample of 20 taken from a production line that has a 2% defect rate — independent — p=success n=20	Binomial.
e) the number of male names on a page selected at random from a telephone book — dependent —	Hyper.
f) the number of left-handed people in a group selected from the generate population — dependent —	Hyper
g) the number of left-handed people selected from a group comprised ~~equally of left-handed and right-handed people~~	Hyper

Example:

$r=6$ $n-a=8$ failure $\boxed{n=8+10=18}$ $a=10$

$\boxed{\text{6 Jurors}}$ are needed from a pool of $\boxed{\text{8 men}}$ and $\boxed{\text{10 women.}}$ success = women, failure = men; hyper.

a) Determine the probability distribution for the number of women on a civil-count jury selected.

b) What is the expected number of women on the jury?

(# of women selected)

$P(x)=\dfrac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}}$

x	$P(x)$
0	$\dfrac{\binom{10}{0}\binom{8}{6}}{\binom{18}{6}}$
1	$\dfrac{\binom{10}{1}\binom{8}{5}}{\binom{18}{6}}$
2	$\dfrac{\binom{10}{2}\binom{8}{4}}{\binom{18}{6}}$
3	$\dfrac{\binom{10}{3}\binom{8}{3}}{\binom{18}{6}}$
4	$\dfrac{\binom{10}{4}\binom{8}{2}}{\binom{18}{6}}$
5	$\dfrac{\binom{10}{5}\binom{8}{1}}{\binom{18}{6}}$
6	$\dfrac{\binom{10}{6}\binom{8}{0}}{\binom{18}{6}}$

# of success available; # of success we need; # failure available; # failure we need; $=r$; sample space; # of dependent trials need

$E(x)=\dfrac{ra}{n}$

$=\dfrac{\cancel{6}(10)}{\cancel{18}\ 3}$

$=\dfrac{10}{3}$

$\doteq 3.333$

Example from Textbook Page 401 Example 3:

A box contain seven yellow, three green, five purple, and six red candies jumbled together.

a) what is the expected number of red candies among five candies poured from the box?

$E(\text{red})=\dfrac{ra}{n}$

$r=5$

$a=6$

$n=7+3+5+6$

$=21$

$E(x)=\dfrac{5(\cancel{6})\,2}{\cancel{21}\ 7}$

$=\dfrac{10}{7}$

$=1.429$

Example from Textbook Page 402 Example 4:

$a = 500$

In the spring, the Ministry of the Environment caught and tagged $\boxed{\text{500 raccoons}}$ in a widerness area. The raccoons were released after being vaccinated against rabies. To estimate the raccoon population in the area, the ministry caught $\boxed{\text{40 raccoons}}$ during the summer. Of these $\boxed{\text{15 had tags.}}$ $E(X)$

Estimate $\underbrace{\text{the raccoon population}}_{n=?}$ in the widerness area.

$r = 40$

$$\boxed{E(X) = \frac{ra}{n}}$$

$$15 = \frac{40(500)}{n}$$

$$n = 1333.33$$

Example:

A hat contains 20 names, 12 of which are female. If five names are drawn from the hat,

a) what is the probability that there is exactly one female name is drawn?

$$P(1 \text{ female}) = \frac{\binom{12}{1}\binom{8}{4}}{\binom{20}{5}} = 0.0542$$

b) What is the expected number of female names?

$$E(X) = \frac{12(5)}{20}$$
$$= 3$$

Example:

What is the probability that a card game of 13 cards in hand continas six spades, four hearts, two diamonds, and one club?

$$P(X) = \frac{\binom{13}{6}\binom{13}{4}\binom{13}{2}\binom{13}{1}}{\binom{52}{13}}$$