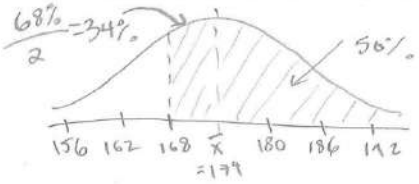
6.4 Introduction to Normal Distributions

In a **normal distribution:**

- 68% of the data lies within one standard deviation of the mean
- 95% of the data lies within two standard deviations of the mean
- 99.7% of the data lies within three standard deviations of the means

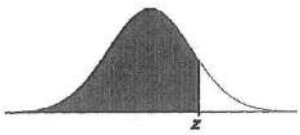
Example 1: Giselle is 168 cm tall. In her high school, boys' heights are normally distributed with a mean of 174 cm and a standard deviation of 6 cm. What is the probability that the first boy Giselle sees at school tomorrow will be taller than her?



Probability is 34% + 50%
= 84%

Example 1 is an example in which the area under the normal curve was taken from a standard deviation line, in which cases the percentages listed above could be used to find the probability. If the area under the normal curve has to be taken when the value **does not** fall on a distribution line, then we need to find the z-score(s) of the value(s) and use the z-score chart to find the corresponding probability. You can find the chart on **page 480 – 481** in your textbook.

Note that the area listed in the chart with respect to each z-score is the area under the normal curve taken from the very left edge of the curve to the value.



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a) $P(z < -0.78)$

$= 0.2177$

or 21.8%

b) $P(z \geq 1.53)$

$= 1 - P(z < 1.53)$

$= 1 - 0.9370$

$= 0.063$

c) $P(-1.00 < z < 1.50)$

$P(z < 1.50) - P(z < -1$

$= 0.9332 - 0.1587$

$= 0.7745$

The z-score table is only useful to use for $N(0, 1^2)$. Data with any other mean and standard deviation needs to be converted to a mean of 0 and a standard deviation of 1 using a standardization process.

$$z = \frac{x - \mu}{\sigma}$$

z: z-score; x: data value; μ: mean; σ: standard deviation

Example 3: Find the probability that a first year student picked at random at a university got between 70 and 80 as an overall average if the mean of the averages was 72 and the standard deviation was 5, given a normal distribution.

$z = \frac{x - \mu}{\sigma}$

$= \frac{70 - 72}{5}$

$= -0.4$

$z = \frac{x - \mu}{\sigma}$

$= \frac{80 - 72}{5}$

$= 1.6$

$P(70 < x < 80)$

$= P(-0.4 < z < 1.6)$

$= P(z < 1.6) - P(z < 0.4)$

$= 0.9452 - 0.3446$

$= 0.6006$

∴ there is a 60.06% probability that the student will get between 70% - 80%.