3.3 Problem Solving with Combinations

Example 1: A tray of candies contains 1 red, 1 blue, 1 yellow, 1 green and 1 purple. In how many ways can you take some candy?

$\frac{2}{R} \times \frac{2}{B} \times \frac{2}{Y} \times \frac{2}{G} \times \frac{2}{P} = 2^5 - 1$ ← null set, nothing taken

$= 31$ ways

we understand "some" to be "all" → as a possibility.

> The total # of combinations containing **at least one item** chosen from a group of ***n*** distinct items is:
>
> $2^n - 1$ ← if you must take at least something (1 item)
> ← cannot have nothing.

Example 2: In how many ways can a committee with at least one member be appointed from a board with 12 members?

$2^{12} - 1 = 4095$ ways

Example 3: In how many ways can a committee with 8 members or less be appointed from a board with 12 members?

① Direct

$_{12}C_8 + {}_{12}C_7 + {}_{12}C_6 + {}_{12}C_5 + {}_{12}C_4 + {}_{12}C_3 + {}_{12}C_1$

$= 3796$ ways

② Indirect

Total − 9 or more

$= 2^{12} - 1 - {}_{12}C_9 - {}_{12}C_{10} - {}_{12}C_{11} - {}_{12}C_{12}$

$= 3796$ ways

* How do we know these aren't permutations? Language used?
* How could we change question to be a permutation?

Example 4: Mr. Subway has 8 possible toppings for their sandwiches. In how many ways could you build a sandwich.

$2^8 = 256$ ways

Example 5: A basket of fruit contains 3 apples, 2 mangoes, 5 plums and 9 bananas. In how many ways can you select some fruit to eat?

$\underset{A}{4} \times \underset{M}{3} \times \underset{P}{6} \times \underset{B}{10} - 1 = 719$ ways.

If **some items are alike**, then the total number of selections from ***p*** items of one kind, ***q*** items of another kind, and so on is:

$(p+1) \times (q+1) \times \ldots$

BUT ... don't forget to think whether NOT picking any items makes sense. If **at least one** item is chosen, then the total number of selections is:

$(p+1) \times (q+1) \times \cdots - 1$

Example 6: Lauren wants to plant a garden. She has 4 red, 3 yellow and 5 pink tulip bulbs. In how many ways can Lauren select some bulbs to plant her garden?

$(4+1) \times (3+1) \times (5+1) - 1$
$= (5)(4)(6) - 1$
$= 119$ ways.

* How can you change question to permutation?