5.4 Probability Involving Counting Techniques (Day 2)

Example 1: To win the grand prize in a fundraising draw, you need to match seven numbers from 1 to 15, without regard to order. combinations

a) What is the probability of winning the grand prize?

A = all 7 numbers correct
$n(A) = {}_7C_7 = 1$
$n(S) = {}_{15}C_7 = 6435$
$P(A) = \frac{1}{6435}$ or 0.016%

b) What is the probability of winning second prize, which requires matching six of the seven winning numbers?

B = 6 of 7 correct
$n(B) = {}_7C_6 \times {}_8C_1 = 56$
$n(S) = 6435$
$P(B) = \frac{56}{6435}$ or 0.9%

Example 2: A shipment of 12 calculators contains 3 that are defective. A customer buys 5 of the calculators. What is the probability that:

a) All 5 are working?

A = 5 working
$n(A) = {}_9C_5 = 126$
$n(S) = {}_{12}C_5 = 792$
$P(A) = \frac{126}{792} = \frac{7}{44}$ or 15.9%

b) 3 are working but 2 are defective?

B = 3 working, 2 defective
$n(B) = {}_9C_3 \times {}_3C_2 = 252$
$P(B) = \frac{252}{792} = \frac{7}{22}$ or 31.8%

Example 3: Four people are needed to help at a party. Determine the probability that you and 2 of your 3 friends will be chosen for this job if the four people are randomly selected from a group of 12.

A = you and 2 friends
$n(A) = {}_3C_2 \times {}_8C_1 \times {}_1C_1 = 24$
(3 friends, 8 non-friends, you)
$n(S) = {}_{12}C_4 = 495$
$P(A) = \frac{24}{495} = \frac{8}{165}$
or 4.8%

Example 4: Ten juniors and twelve seniors decide to rent a 16-passenger van and a 6-passenger car to drive to a concert in a nearby city. If the group is distributed randomly between the vehicles, what is the probability that:

a) There are no juniors in the car?

A = no juniors in car

$n(A) = {}_{12}C_6 \times {}_{16}C_{16} = 924$

$n(S) = {}_{22}C_6 \times {}_{10}C_{16} = 74613$

$P(A) = \frac{924}{74613} = \frac{4}{323}$

or 1.2%

b) There are exactly 5 juniors in the car?

B = 5 juniors in car

$n(B) = {}_{10}C_5 \times {}_{12}C_1 \times {}_{16}C_{16}$

$= 3024$

$P(B) = \frac{3024}{74613} = \frac{144}{3553}$ or

$= 4.1\%$

c) Steve (junior) and Sally (senior) are both in the van?

C = Steve and Sally in van

$n(C) = {}_{20}C_{14} \times {}_{6}C_6 = 38760$

$P(C) = \frac{38760}{74613}$

$= \frac{40}{77}$

$=$ or 51.9%

d) Either Steve (junior) or Sally (senior) are in the van (but not both)

D = either Steve or Sally in van

$n(D) = ({}_{20}C_{15} \times {}_{5}C_5) \times 2 = 31008$

$P(D) = \frac{31008}{74613} = \frac{32}{77}$ or 41.6%

Example 5: What is the probability that in a class of 25 students, two (or more) students will share the same birthday?

(i.e., what is P(will not have a unique birthday)?)

$$P(\text{all different}) = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \cdots \times \frac{340}{365}$$

365 possibilities for first person's birthday

365 days in a year

no repeats of a birthday

$$= \frac{{}_{365}P_{25}}{365^{25}}$$

$$P(\text{at least 2 the same}) = 1 - P(\text{all different})$$

$$= 1 - \frac{{}_{365}P_{25}}{365^{25}}$$

$$\doteq 1 - 0.4313$$

$$\doteq 0.5687$$

∴ the probability that at least two students will have the same birthday is 56.87%.