6.2 Binomial Distributions

A **binomial distribution** looks at the distribution of the outcomes of several trials of a probability experiment in which:

1) 2 outcomes → success or failure
2) each trial is identical & independent
3) probability of success / failure

Example 1: A bag contains 3 yellow marbles and 4 blue marbles. Three marbles are selected at random from a bag, one at a time, with replacement.

a) Construct a probability distribution table for the number of blue marbles in the sample.

let x rep # of blue marbles

x	$P(X=x)$
0	$\left(\frac{3}{7}\right)^3 = 0.079$
1	${}_3C_1\left(\frac{4}{7}\right)\left(\frac{3}{7}\right)^2 = 0.315$
2	${}_3C_2\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right) = 0.419$
3	$(4/7)^3 = 0.187$

b) How many marbles would you **expect** to be blue?

$E(x) = 0(0.079) + 1(0.315) + 2(0.419) + 3(0.187)$

$= 1.7$

OR $E(x) = \left(\frac{4}{7}\right)(3) = 1.7$

Probability in a Binomial Distribution

$P(X=x) = {}_nC_x\, p^x q^{n-x}$

n = # of trials
x = # of successes
p = probability of success
q = probability of failure

Expectation for a Binomial Distribution

$E(x) = np$

Example 2: TJ is getting a new cell phone number. The first three digits of the number will be 446 and all combinations for the four remaining digits are equally likely. TJ's favourite numbers are the prime numbers 2, 3, 5 and 7.

a) Construct the probability distribution for the number of these prime digits in TJ's new number.

Let X rep the # of prime #'s.

$n = 4$
$p = 0.4$
$q = 0.6$

x	$P(X=x)$
0	$(0.6)^4 = 0.13$
1	$4C_1(0.6)^3(0.4) = 0.35$
2	$4C_2(0.6)^2(0.4)^2 = 0.35$
3	$4C_3(0.6)(0.4)^3 = 0.15$
4	$(0.4)^4 = 0.03$

b) What is the expected number of these prime digits in TJ's new cell phone number?

$E(x) = 4(0.4) = 1.6$ ∴ we expect 1.6 prime numbers.

Example 3: A jar contains 12 red balls and 8 green balls. Six balls are removed without replacement.

a) Explain why the binomial distribution is not a suitable model for this problem.

The probability of success / failure change after each trial "without replacement"

$n = 6$
$p = 0.6$
$q = 0.4$

b) If the balls are withdrawn one after another with replacement, each time, determine the probability that at least 3 balls are red. Let X rep # of red balls

Method ①: Indirect

$1 - P(X=0) - P(X=1) - P(X=2)$
$= 1 - (0.4)^6 - 6C_1(0.6)(0.4)^5 - 6C_2(0.6)^2(0.4)^4$
$= 1 - 0.0041 - 0.037 - 0.14$
$= 0.819$
or 81.9%

Method ②: Direct

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$
$= 6C_3(0.6)^3(0.4)^3 + 6C_4(0.6)^4(0.4)^2 + 6C_5(0.6)^5(0.4) + (0.6)^6$