

Learning Goal: Independent and Dependent Events

Independent Events: <ul style="list-style-type: none"> - the occurrence of one event has no effect on the occurrence of another <p>Ex. $P(A)$=flipping a coin 1st time</p> <p>$P(B)$=flipping a coin 2nd time</p> $P(A \text{ and } B) = P(A) \times P(B)$	Dependent Events: <ul style="list-style-type: none"> - the probable outcome of an event, B, depends on the outcome of another event A. (B occurs, given that A has already occurred) $P(A \text{ and } B) = P(A) \times P(B A)$ <p style="text-align: right;">← depend on A (1st event)</p> $P(B A) = \frac{P(A \text{ and } B)}{P(A)}$
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Homework Question #1 from Textbook Page 334

Classify each pair of events as independent or dependent event:

	Event A	Event B	Independent (I) OR Dependent (D) Event
i)	Attending a rock concert on Tuesday night (until 5 am on Wednesday morning)	Passing a final examination the following Wednesday morning (at 9 am for two hours)	dependent
j)	Eating chocolate	Winning at checkers	independent
k)	Having blue eyes	Having poor hearing	independent
l)	Attending an employee training session	Improving personal productivity	dependent
m)	Graduating from university	Running a marathon	independent
n)	Going to a mall	Purchasing a new shirt (Trying on a shirt in a fitting room)	dependent

Textbook Page 328 Example#1:

- a) A coin is flipped and turns up heads. What is the probability that the second flip will turn up head?
Independent or Dependent?

$$P(\text{second flip is head}) = \frac{1}{2}$$

- b) A coin is flipped four times and turns up heads each time. What is the probability that the fifth trial will be head?
Independent or Dependent?

$$P(A) = \frac{1}{2}$$

- c) Find the probability of tossing five heads in a row.
Independent or Dependent?

$$P(A) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

Textbook Page 329 Example#2:

A coin is flipped while a die is rolled. What is the probability of flipping heads and rolling 5 in a single trial?
Independent or Dependent?

$$P(\text{head}) = \frac{1}{2} \quad P(\text{roll } 5) = \frac{1}{6}$$

$$P(\text{head} \cap \text{roll } 5) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Textbook Page 330 Example#3:

Soo-Ling travels the same route to work every day. She has determined that there is a 0.7 probability that she will wait for at least one red light and that there is a 0.4 probability that she will hear her favourite new song on her way to work.

- a) What is the probability that Soo-Ling will not have to wait at a red light and will hear her favourite song?
Independent or Dependent?

$$P(A^c) = 0.3 \\ P(B) = 0.4$$

$$P(A^c \cap B) = 0.3 \times 0.4 = 0.12 \quad \checkmark$$

- b) What are the odds in favour of Soo-Ling having to wait at a red light and not hearing her favourite song?

$$P(A) = 0.7 \\ P(B^c) = 0.6 \\ P(A \cap B^c) = 0.7 \times 0.6 = 0.42$$

$$\text{odds} = \frac{0.42}{1-0.42} = \frac{0.42}{0.58} = \frac{21}{29} \quad \checkmark$$

Textbook Page 331 Example#4:

A professional hockey team has eight wingers. Three of these wingers are 30-goal scorers, or "snipers." Every fall the team plays an exhibition match with the club's farm team. In order to make the match more interesting for the fans, the coaches agree to select two wingers at random from the pro team to play for the farm team. What is the probability that two snipers will play for the farm team?

Independent or Dependent?

Let A the event of first winger is a sniper
 B the event of second winger is a sniper, given the first winger is a sniper

$$\begin{aligned} P(A \text{ and } B) &= P(A) \times P(B|A) \\ &= \frac{\binom{3}{1}}{\binom{8}{1}} \times \frac{\binom{2}{1}}{\binom{7}{1}} \\ &= \frac{3}{28} \end{aligned}$$

Modified from Textbook Page 332 Example#5:

Sina's computer sometimes crashes while he is trying to use his e-mail program, Outlook. When Outlook "hangs" (stop responding to commands), Sina is usually able to close Outlook without a system crash. In a computer magazine, he reads that the probability of Outlook hanging in any 15-min period is 2.5%, while the chance of Outlook and the operating system failing together in any 15-min period is 1%. If Outlook is hanging, what is the probability that the operating system will crash?

Independent or Dependent?

Let A be the event of Outlook hangs
 B be the event of operating system fails, given Outlook is hanging

$$\begin{aligned} P(A) &= 0.025 \\ P(A \text{ and } B) &= 0.01 \\ P(B|A) &= \frac{0.01}{0.025} \\ &= 0.4 \end{aligned}$$

∴ There is 40% probability that the operating system crashes given Outlook is hanging.