Bayview Secondary School
Math Department – MDM4U1-Semester 1 (2025)
Assessment of Organized Counting (Day 1)

Name: ______________________ Alpha # ________

K-19	T-5

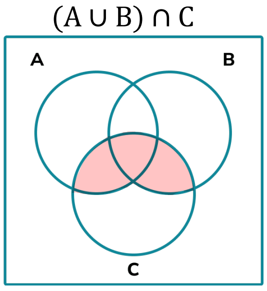
Instructions:

1) Show all work to obtain full marks for questions that are worth greater than 1 mark
2) 2 marks will be awarded for communication in total for both days of the assessment
3) The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.
4) Scientific calculators are allowed but cannot be shared.
5) Duration of the assessment on each day: 45 minutes

Knowledge and Understanding

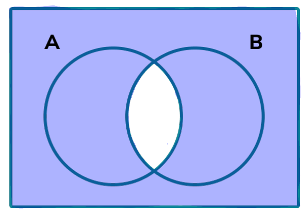
1) Shade the given region on the corresponding Venn Diagram. [1 mark]

$(A \cup B) \cap C$



2) Shade the given region on the corresponding Venn Diagram. [1 mark]

$A' \cup B'$



3) How many 5 letter arrangements can be made from the letters of the word POLICEMAN if all vowels must be used? [3 marks]

5C: P, L, C, M, N
4V: O, I, E, A

$\binom{5}{1}\left(\frac{4}{V}\times\frac{3}{V}\times\frac{2}{V}\times\frac{1}{V}\times\frac{5}{C}\right)$

$\binom{5}{1}\times 4! \times 5$

$= 600$ ways

4) A group of 10 summer camp students are to be assigned to three different cabins containing three beds, two beds and five beds respectively. In how many ways can the students be assigned to sleep in these three cabins? [2 marks]

$\binom{10}{3}\binom{7}{2}\binom{5}{5} = 2520$ ways

5) Given a standard deck of 52 cards and a card is randomly drawn, how many ways can you draw a heart or a face card? [2 marks]

$13 + 12 - 3$

$= 22$ ways

6) Given $A = \{3a-2, 5, 8\}$, $B = \{4, 2b+3, 9\}$, and $A \cap B = \{4, 5\}$, find the sum of a and b. [1 mark]

$4 = 3a - 2$
$6 = 3a$
$a = 2$

$5 = 2b + 3$
$2 = 2b$
$b = 1$

$a + b = 3$

7) Find the number of divisors of 260 other than 1. [2 marks]

2 | 260
13 | 130
2 | 10
5

$\frac{3}{2's} \times \frac{2}{5's} \times \frac{2}{13's} - 1$

$= 11$ ways

8) At a banquet, three couples are sitting at a round table, how many seating arrangements are possible for them if they must sit beside their spouse? [2 marks]

$(3-1)! \times 2! \times 2! \times 2! = 16$ ways

9) From a standard deck of 52 cards, find the number of ways with a hand of 5 cards where there are two different pairs and a random card that is not the same rank as the two pairs? [2 marks]

$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$

$= 123\ 552$ ways

10) Five different colour blocks are to be arranged, red, green, blue, yellow, and purple. In how many ways can the blocks be arranged if green and yellow cannot be adjacent to each other? [3 marks]

green & yellow together.

$5! - 2!4! = 72$ ways

Thinking

11) A box contains fifteen balls, with five balls of each color: red, yellow, and blue. Each set of five balls of the same color is labeled with a letter: A, B, C, D, or E. In how many ways can five balls be randomly chosen if the selection must include all three colors and all five letters? [5 marks]

Colour

R	Y	B	Letters.
3	1	1	$\binom{5}{3}\binom{2}{1}\binom{1}{1} = 20$
2	2	1	$\binom{5}{2}\binom{3}{2}\binom{1}{1} = 30$
2	1	2	$\binom{5}{2}\binom{3}{1}\binom{2}{2} = 30$
1	3	1	$\binom{5}{1}\binom{4}{3}\binom{1}{1} = 20$
1	2	2	$\binom{5}{1}\binom{4}{2}\binom{2}{2} = 30$
1	1	3	$\binom{5}{1}\binom{4}{1}\binom{3}{3} = 20$

✓ ✓ ✓

✓ $\therefore\ 20+30+30+20+30+20$

✓ $= 150$ ways

OR

① 3 of the same colour, 1 other, 1 other

letters → $\binom{5}{3}\binom{2}{1}\binom{1}{1} \times 3 = 60$ ✓

② 2 of the same for 2 colours, 1 other

$\binom{5}{2}\binom{3}{2}\binom{1}{1} \times 3 = 90$ ✓ ✓

$\therefore\ 60+90=150$ ways ✓

--- End of day 1 assessment ---

Bayview Secondary School
Math Department – MDM4U1-Semester 1 (2025)
Assessment of Organized Counting (Day 2)

Name: ______________________________ Alpha # ________

A-22	T-4	C-3

Instructions:

1) Show all work to obtain full marks for questions that are worth greater than 1 mark
2) 3 marks will be awarded for communication in total for both days of the assessment
3) The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.
4) Scientific calculators are allowed but cannot be shared.
5) Duration of the assessment on each day: 45 minutes

Application

1) Solve for $n, n \in \mathbb{N}$. [3 marks]

$$\frac{n!}{10} = P(n-1, n-3)$$

$$\frac{n!}{10} = \frac{(n-1)!}{[(n-1)-(n-3)]!}$$ ✓

$$\frac{n\cancel{(n-1)!}}{10} = \frac{\cancel{(n-1)!}}{(n-1-n+3)!}$$

$$\frac{n}{10} = \frac{1}{2!}$$ ✓

$$n = 10\left(\frac{1}{2}\right)$$

$$n = 5$$ ✓

2) A bookshelf has n fiction books and 6 non-fiction books. If there are 150 ways to choose two books of each type, how many fiction books are on the bookshelf? [3 marks]

$$\binom{n}{2}\binom{6}{2} = 150$$ ✓

$$\binom{n}{2}(15) = 150$$

$$\binom{n}{2} = 10$$

$$\frac{n!}{2!(n-2)!} = 10$$ ✓

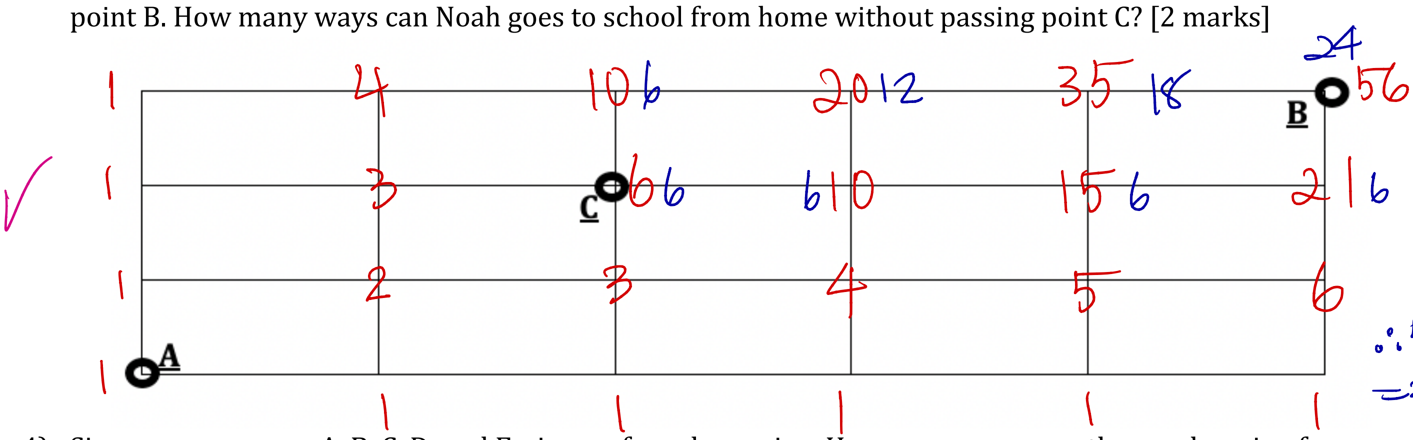
$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$$

$$n^2 - n - 20 = 0$$

$$(n-5)(n+4) = 0$$

$$n = 5 \text{ or } \cancel{n = -4}$$ ✓

3) The grid below shows the street in Noah's neighbourhood. Suppose Noah lives at point A and his school is at point B. How many ways can Noah goes to school from home without passing point C? [2 marks]



$\therefore 56 - 24$
$= 32$ ways ✓

4) Six runners, person A, B, C, D, and E, sign up for relay racing. How many ways can the coach assign four runners to participate for the 4×100 meter relay if person A cannot run the first sprint and person B cannot run the last sprint? [3 marks]

Indirect method.

All $-$ n(A 1st) $-$ n(B last) $+$ n(A 1st $\cap$ B last)

$${}_6P_4 - \left(\underset{A}{1} \times 5 \times 4 \times 3\right) - \left(5 \times 4 \times 3 \times \underset{B}{1}\right) + \left(\underset{A}{1} \times 4 \times 3 \times \underset{B}{1}\right)$$ ✓

$$= 360 - 60 - 60 + 12$$

$$= 252 \text{ ways}$$ ✓

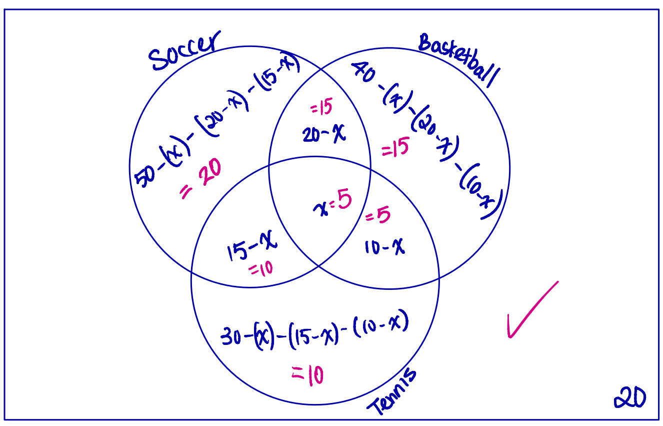
5) One hundred orders from Amazon.ca are about to ship to the customers. Three of the orders have the wrong shipping label on the box, how many ways can at least two of the "wrong" boxes be chosen if five boxes are chosen at random? [3 marks]

2 boxes + 3 boxes

$\binom{97}{3}\binom{3}{2} + \binom{97}{2}\binom{3}{3} = 446\,976$ ways

6) A survey was conducted among 100 students to determine their preferences for three different sports: Soccer, Basketball, and Tennis. The results were as follows:
- 50 students like soccer
- 40 students like basketball
- 30 students like tennis
- 20 students like both soccer and basketball
- 15 students like both soccer and tennis
- 10 students like both basketball and tennis
- 20 students like neither

How many students like all three sports? Include a Venn diagram as part of the solution to obtain full mark. [3 marks]



$100-20 = 50+40+30-20-15-10+x$

$80 = 75 + x$

$x = 5$

7) Eight people are to be seated in two rows: a front row and a back row. If person A and person B must be seated in the front row and person C must be seated in the back row, how many different seating arrangements are possible? [3 marks]

front row & back row

$\binom{2}{2}\binom{5}{2} \times 4!$ (A & B) $\times$ $\binom{1}{1}\binom{3}{3} \times 4!$ (C) $= 5760$ ways

8) How many 4-digit even numbers can be formed from the numbers 0-9 if repetition is not allowed? [2 marks]

① or ②

$\underline{9} \times \underline{8} \times \underline{7} \times \underline{1}_{0} + \underline{8}_{\not 0} \times \underline{8} \times \underline{7} \times \underline{4}_{2468}$

$= 504 + 1792$

$= 2296$ ways

Thinking

9) In how many ways can you pick any 3 of the numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 (without repetition) so that the sum of the three numbers is an even number and is greater or equal to 10? [4 marks]

evens $= \{0,2,4,6,8\}$, $n(\text{even})=5$, odds $= \{1,3,5,7,9\}$, $n(\text{odd})=5$

sum of 3 evens = even $\longrightarrow \binom{5}{3} = 10$

~~sum of 2 evens & 1 odd = odd~~

sum of 1 even & 2 odds = evens $\longrightarrow \binom{5}{1}\binom{5}{2} = 50$

$\therefore 10 + 50 - 9$

$= 51$ ways

sum of 3 evens < 10
0 2 4
0 2 6

sum of 1 even & 2odds < 10
0 1 3, 0 3 5, 2 1 3, 4 1 3
0 1 5, 2 1 5
0 1 7

$= 9$

3 marks for overall mathematical form throughout the test.

--- End of day 2 assessment ---