## 6.1 Introduction to Probability Distributions (Day 1)

So far we have looked at the probability of **individual** outcomes for events. Next, we will consider models for distributions that show the probabilities of **all possible** outcomes that have equal or different likelihoods.

### Terminology Used:

**Random Variable**: A variable that can have any value within a range.

- **X** denotes the random variable while **x** represents all possible outcomes of **X**
- Example: let $x$ rep the numbers rolled on standard dice
  then $x = 1, 2, 3, 4, 5, 6$

**Discrete Variable:** A variable that can have only finite and distinct values within a range

- Example: The # of phone calls made by sales

**Continuous Variable**: A variable that takes on all real (infinite) values within a range

- Example: the length of time of each phone call made by sales

**Probability Distribution:** A function that gives the probability of all possible values of a random variable.

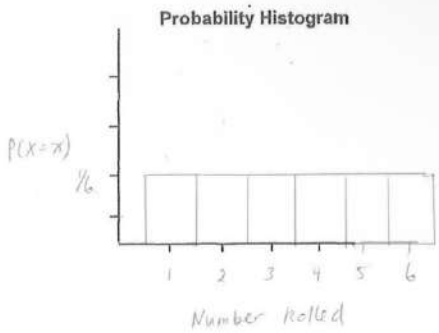
- Is denoted by $P(X=x)$ or $P(x)$

**Probability Histogram**: A graph of a probability distribution in which equal intervals are marked on the horizontal axis and the probabilities associated with these intervals are indicated by the areas of the bars.

Example 1: Consider an experiment that counts the outcomes of rolling one fair die. Construct a probability distribution table and probability histogram.

Let x rep # rolled

| $x$ | $P(X=x)$ |
|---|---|
| 1 | $\frac{1}{6}$ |
| 2 | $\frac{1}{6}$ |
| 3 | $\frac{1}{6}$ |
| 4 | $\frac{1}{6}$ |
| 5 | $\frac{1}{6}$ |
| 6 | $\frac{1}{6}$ |

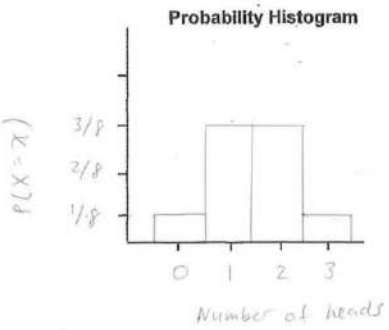



This distribution is called a uniform distribution because all of the probabilities are the same.

Example 2: Consider an experiment that counts the number of heads in 3 flips of a coin. Construct a probability distribution table and probability histogram.

Let x rep # of heads

| $x$ | $P(X=x)$ |
|---|---|
| 0 | $\frac{1}{8}$ |
| 1 | $\frac{3}{8}$ |
| 2 | $\frac{3}{8}$ |
| 3 | $\frac{1}{8}$ |



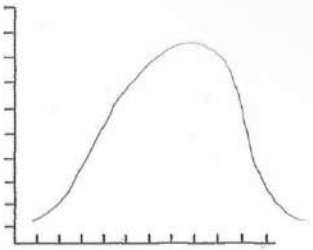

**NOTE:** The sum of the probabilities is always equal to 1.

Example 3: Consider an experiment involving the sum of the top faces of two fair dice. Construct a probability distribution table and probability histogram.

Let x rep the sum of the 2 dice.

| $x$ | $P(X=x)$ |
|---|---|
| 2 | $1/36$ |
| 3 | $2/36$ |
| 4 | $3/36$ |
| 5 | $4/36$ |
| 6 | $5/36$ |
| 7 | $6/36$ |
| 8 | $5/36$ |
| 9 | $4/36$ |
| 10 | $3/36$ |
| 11 | $2/36$ |
| 12 | $1/36$ |

Probability Histogram



An expectation or **expected value**, E(X), is the predicated average (weighted average) of all possible outcomes in an experiment.

$$E(x) = \sum_{i=1}^{n} (x_i \cdot P(x_i))$$

outcome, probability

Example 4: Calculate the expected value of the sum of the top faces of two fair die.

$E(x) = 2(1/36) + 3(2/36) + 4(3/36) + \cdots + 12(1/36)$

$= 7$

∴ You expect to roll a sum of 7.

Example 5: If you rolled a pair of dice 360 times, how many times would you expected to get 9?

$E(9) = \left(\frac{4}{36}\right)(360)$

$= 40$

∴ You expect to roll a 9, 40 times.

Example 6: A hospital is having a fundraising lottery to raise money for cancer research. A ticket costs $10 and 2 000 000 tickets are available. There are four levels of prizes, one $5 000 000 grand prize, three $100 000 second prizes, ten $1 000 prizes and 2000 free tickets for next year's lottery.

a) Construct a probability distribution for the possible earnings in this situation.

Let $x$ rep possible earnings

| $x$ | $P(X=x)$ |
|---|---|
| 5,000,000 | $\frac{1}{2,000,000}$ |
| 100,000 | $\frac{3}{2,000,000}$ |
| 1000 | $\frac{10}{2,000,000}$ |
| 10 | $\frac{2000}{2,000,000}$ |
| 0 | $\frac{1,997,986}{2,000,000}$ |

b) What are the expected **earnings** per ticket?

$E(x) = 5,000,000\left(\frac{1}{2,000,000}\right) + 100,000\left(\frac{3}{2,000,000}\right) + \cdots + 0\left(\frac{1,997,986}{2,000,000}\right)$

$= 2.67$

∴ you expect to earn $2.67

c) What is the expected **value** of each ticket?

$2.67 - 10$

$= -7.33$

∴ The expected value is a loss of $7.33