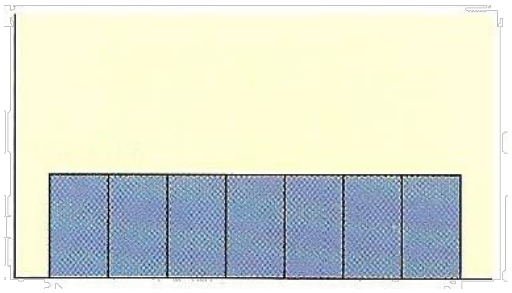
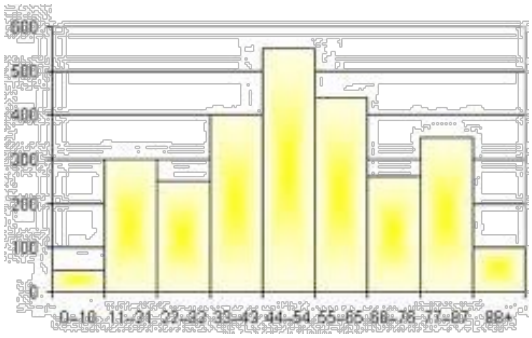
Lesson: Expectation / Fair Game

Definitions:

Probability in a Discrete Uniform Distribution:

$P(x) = \frac{1}{n}$, where n is the number of possible outcomes in the experiment

Uniform Distribution	Non-Uniform Distribution
	

Fair Game

- The expected outcome in a fair game is ZERO
- A game in which each player is not more likely to win than another

Fair Game	Not a Fair Game
$E(X) = 0$	$E(X) \neq 0$

Question Page 374 #2:

Explain whether each of the following experiments has a uniform probability distribution:

Experiment	Uniform/Non-Uniform Distribution
a) Selecting the winning number for a lottery	U
b) Selecting three people to attend a conference	N
c) Flipping a coin	U
d) Generating a random number between 1 and 20 with a calculator	U
e) Guessing a person's age	N
f) Cutting a card from a well-shuffled deck	U
g) Rolling a number (sum of the two dice) with two dice	N

Example "GAMES"

Ex: You are playing a game with a deck of cards. You randomly chose a card. The following are the results. If it is a face card, you win $10. If it is an even card, you lose $3. If it is an odd card, you lose $5. What is the expectation for this game? (Assume Ace = 1 = odd)

	Face	Odd	Even
X	+10	−5	−3
P(X)	$\frac{12}{52}$	$\frac{20}{52}$	$\frac{20}{52}$

$E(X) = 10\left(\frac{12}{52}\right) + (-5)\left(\frac{20}{52}\right) + (-3)\left(\frac{20}{52}\right) = \frac{-10}{13}$ or -0.77

∴ The expectation is losing an average of $0.77 in each game.

Example "FAIR GAMES"

Ex: A die game cost $3 to play. If you roll an even number, you win double the value. If you roll an odd number, you lose the value. Is this a fair game?

(Remember: If a game costs money upfront, then to be fair, the expectation has to have the same amount as the cost of the game.)

Roll a die	Random Variable, X		Probability, P(x)
	method	①	②
1	−1	−1−3 = −4	$\frac{1}{6}$
2	4	4−3 = 1	$\frac{1}{6}$
3	−3	−6	$\frac{1}{6}$
4	8	5	$\frac{1}{6}$
5	−5	−8	$\frac{1}{6}$
6	12	9	$\frac{1}{6}$

$E(X) = (-1)\left(\frac{1}{6}\right) + (4)\left(\frac{1}{6}\right) + (-3)\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) + (-5)\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right) - 3$

$= 2.5 - 3$

$= -0.5$

∴ This is not a fair game.

$E(X) = \frac{1}{6}(-4+1-6+5-8+9)$

$= -0$