Lesson: Measures of Central Tendency (One-variable statistics)

When you analyze data that has been collected, the first measure to find is the "average" result for the collection.

There are three measures of central tendency that we can calculate.

- Mean – add all the data and divide by the number of data items.
- The symbol used for sample mean is $\bar{x}$.
- The symbol used for population mean is μ.

  The formula used to calculate the sample mean is $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, where n represents the total number of values and x represents the observed values.

- Median – the middle piece of data when the items are ranked from least to greatest

  To calculate the median, arrange the data values for least to greatest. Calculate $\frac{n+1}{2}$, where n represents the number of values, to determine the location of the middle value. If the data presents an even number of values, average the two values in the middle position.

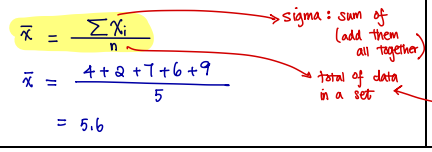
- Mode – the most frequencyly occurring data value

  To find the mode, determine the most frequent value. It is possible for there to exist more than one mode.

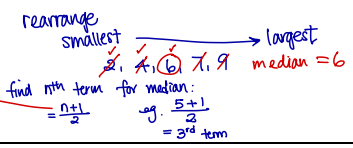
When given a set of numbers:

1. Given a set of numbers: 4, 2, 7, 6, 9

   a. Find the mean.

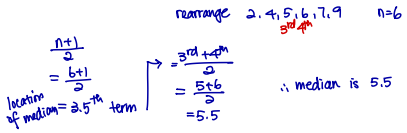


   b. Find the median.



2. Given a set of numbers: 5, 2, 7, 4, 9, 6

   a. Find the median.



3. Given a set of number:
   1, 1, 2, 3, 4, 5
   Find the mode. Unimodal

   1

4. Given a set of number:
   1, 1, 2, 2, 3, 3, 4, 4
   Find the mode.

   no mode

5. Given a set of number:
   1, 1, 2, 2, 3, 4, 5
   Find the mode.

   1 and 2 Bimodal

When given a set of numbers involving frequencies:

6. Given a set of numbers involving frequencies:

	①	③	②	④
x	2	5	4	9
frequencies	3	2	1	4

2, 2, 2, 5, 5, 4, 9, 9, 9, 9

a. Find the mean.

$\bar{x} = \frac{\sum f x_i}{n}$ ← frequency

$= \frac{2(3)+5(2)+4(1)+9(4)}{10}$

$= 5.6$

b. Find the median.

location $= \frac{n+1}{2}$

$= \frac{10+1}{2}$

$= 5.5^{th}$ term

$= \frac{5^{th}+6^{th}}{2}$

$= \frac{5+5}{2}$ ∴ median is 5

$= 5$

c. Find the mode.

mode = 9 ✓

When given a group of distribution:

7. Given a group distribution:

x	0-9	10-19	20-29	30-39
midpoint	4.5	14.5	24.5	34.5
frequencies	3	2	1	4

a. Find the mean.

$\bar{x} = \frac{\sum f x_i}{n}$

$= \frac{4.5(3)+14.5(2)+24.5(1)+34.5(4)}{10}$

$= 20.5$

b. Find the median.

$= \frac{n+1}{2}$

$= \frac{10+1}{2}$

$= 5.5^{th}$ term ∴ median is 19.5

$= \frac{5^{th}+6^{th}}{2}$

$= \frac{14.5+24.5}{2}$

$= 19.5$

c. Find the mode.

mode = 34.5 ✓

When given a set of continuous data:

8. Given a set of continuous data:
1.2111, 1.1212, 1.1213, 2.1111, 2.1212, 3.1314, 4.1516
Find the mode.

intervals	frequencies
$1 \leq x < 2$	3
$2 \leq x < 3$	2
$3 \leq x < 4$	1
$4 \leq x < 5$	1

modal interval = $1 \leq x < 2$

Practice Measure of Central Tendency:

Knowledge Questions:

1) Given the following set of data:

4,9,4,5,3,8,10,12

a) Find the mean of the given set of data. [6.875]

b) Find the median of the given set of data. [6.5]

c) Find the mode of the given set of data. [4]

a) $\bar{x} = \frac{4+9+4+5+3+8+10+12}{8}$

$= 6.875$

b) 3, 4, 4, 5, 8, 9, 10, 12

$n = 8$

location: $\frac{8+1}{2} = 4.5^{th}$ term $= \frac{4^{th}+5^{th}}{2} = \frac{5+8}{2} = 6.5$

c) mode = 4

2) Determine the interval size if 8 groups are intervals are needed: [6]

12,23,25,27,27,32,34,39,40,41,41,45,47,49,50,51,52,56,59,60,66,70

$\frac{70-12}{8}$

$= 7.25$

$\doteq 8$

Application Questions:

3) A class consists of 50 students, out of which 30 are girls. The mean of marks scored by girls in a test is 73 out of 100, and that of boys is 71 out of 100. Determine the mean score of the whole class. [72.2]

Let m be the total marks of all girls
n be the total marks of all boys
$\bar{x}$ be the mean score of the whole class

$\frac{m}{30} = 73$ | $\frac{n}{20} = 71$ | $\bar{x} = \frac{m+n}{50}$

$m = 73(30)$ | $n = 71(20)$ | $= \frac{2190+1420}{50}$

$m = 2190$ | $n = 1420$ | $= 72.2$

∴ The mean score of the class is 72.2 out of 100.

4) The mean of the following distribution is 50. Find the value of a and the frequencies of 30 and 70. [5, 28,24]

x	$frequency$
10	17
30	$5a+3$
50	32
70	$7a-11$
90	19

$\frac{10(17)+30(5a+3)+50(32)+70(7a-11)+90(19)}{17+(5a+3)+32+(7a-11)+19} = 50$

$\frac{170+150a+90+1600+490a-770+1710}{60+12a} = 50$

$2800+640a = 50(60+12a)$

$2800-3000 = 600a-640a$

$200 = 40a$

$a = 5$

For $x=30$: $f = 5a+3 = 5(5)+3 = 25+3 = 28$

For $x=70$: $f = 7a-11 = 7(5)-11 = 35-11 = 24$

Challenging Question:

5) Given three positive integers, a, b, and c. Their average is 20, where $a \le b \le c$. If the median is $(a+11)$, what is the least possible value of c? [25]

$\frac{a+b+c}{3} = 20$

median is b

$b = a+11$

least possible value of c could be:

① if $b = c$

$\frac{a+(a+11)+(a+11)}{3} = 20$

$3a+22 = 60$

$3a = 38$

$a = 12.67$

Since a is not an integer, ∴ $b \neq c$

② if $c = b+1$

$\frac{a+(a+11)+(a+12)}{3} = 20$

$3a+23 = 60$

$3a = 37$

$a = 12.3$

Since a is not an integer, ∴ $c \neq b+1$

③ if $c = b+2$

$\frac{a+(a+11)+(a+13)}{3} = 20$

$3a+24 = 60$

$3a = 36$

$a = 12$

Since a is an integer, ∴ the least possible value of c is $a+13 = 25$.

Lesson: Measures of Central Tendency (One-variable statistics)

Weighted Mean: A weighted mean is calculated when some of the data items in an average are worth more than others.

The formula used to calculate a weighted mean is $\overline{x_w} = \frac{\sum w_i x_i}{\sum w_i}$

Example: The overall performance of a student in Data Management was recorded as follows:

CATEGORY	WEIGHT	STUDENT MARK %
Knowledge and Understanding	24%	75%
Application	18%	70%
Thinking	14%	60%
Communication	14%	80%
Culminating Task (ISP)	15%	70%
Final Exam	15%	55%

a) Calculate the student's term mark. (The term mark does not include the culminating task or the final exam.)

$$\frac{75(0.24) + 70(0.18) + 60(0.14) + 80(0.14)}{0.70}$$

$\doteq 71.71$

∴ The student's term mark is 72%

b) Calculate the student's final course mark.

$75(0.24) + 70(0.18) + 60(0.14) + 80(0.14) + 70(0.15) + 55(0.15)$

$= 68.95$

∴ The student's final mark is 69%.

Time to determine your own updated mark in this course from TeachAssist:

Categories	Knowledge (22%)	Application (28%)	Thinking (10%)	Communication (10%)
Unit 1 Assignment				
Unit 2 Day 1 Test				
Unit 2 Day 2 Test				
Total				