MDM4U Unit 3: Combinations

3.4 Problem Solving with Combinations (Day 2)

Example 1: A student has been selected to choose the morning songs for this week. On his very limited iPod, he has 10 BTS songs, 5 Rihanna songs and 3 Justin Bieber songs. In how many ways can he select the five songs (order irrelevant) so there is at least *some* Bieber?

Direct:
${}_3C_1 \times {}_{15}C_4 + {}_3C_2 \times {}_{15}C_3 + {}_3C_3 \times {}_{15}C_2$
$= 5565$ ways

Indirect:
Total − no Bieber
$= {}_{18}C_5 - {}_{15}C_5$
$= 5565$

Example 2: The card game Euchre uses only 24 cards: 9s, 10s, jacks, queens, king and aces of each suit. Five card hands are dealt to the players. How many euchre hands contain:

a) Two or three hearts?
${}_6C_2 \times {}_{18}C_3 + {}_6C_3 \times {}_{18}C_2 = 15300$ ways
have, want

b) At least two hearts?
Total − no ♡ − 1 ♡
$= {}_{24}C_5 - {}_{18}C_5 - {}_{18}C_4 \times {}_6C_1 = 15576$ ways

c) A flush (all five cards the same suit?
${}_4C_1 \times {}_6C_5 = 24$ hands

Example 3: Molly has 8 pictures with her family and 5 pictures with her friends. Her wall shelf will only hold 5 pictures. Molly decides she wants to use 3 family and 2 friend pictures. In how many ways can Molly arrange the pictures of the shelf?

order matters

select photos: ${}_8C_3 \times {}_5C_2 = 560$

arrange: $560 \times 5! = 67200$ ways

3.4 Problem Solving with Combinations (Day 2)

Example 1: A student has been selected to choose the morning songs for this week. On his very limited iPod, he has 10 Drake songs, 5 Taylor Swift songs and 3 One Direction songs. In how many ways can he select the five songs (order irrelevant) so there is at least *some* One Direction?

Method ①: Direct

$_3C_1 \times {}_{15}C_4 + {}_3C_2 \times {}_{15}C_3 + {}_3C_3 \times {}_{15}C_2$

$= 5565$ ways

Method ②: Indirect

Total − no one Direction

$= {}_{18}C_5 - {}_{15}C_5$

$= 5565$

Example 2: The card game Euchre uses only 24 cards: 9s, 10s, jacks, queens, king and aces of each suit. Five card hands are dealt to the players. How many euchre hands contain:

a) Two or three hearts?

$_6C_2 \times {}_{18}C_3 + {}_6C_3 \times {}_{18}C_2 = 15,300$ ways

(2 hearts, 3 not hearts) (3 hearts, 2 not hearts)

b) At least two hearts?

Total − no ♡ − 1 ♡

$= {}_{24}C_5 - {}_{18}C_5 - {}_{18}C_4 \times {}_6C_1$

$= 15,576$ ways

c) A flush (all five cards the same suit)?

$_4C_1 \times {}_6C_5 = 24$ hands

Example 3: Molly has 8 pictures with her family and 5 pictures with her friends. Her wall shelf will only hold 5 pictures. Molly decides she wants to use 3 family and 2 friend pictures. In how many ways can Molly arrange the pictures on the shelf?

order

select photos: $_8C_3 \times {}_5C_2 = 560$

arrange → $560 \times 5! = 67,200$ ways

(give answer in ...)