**W8 –The Natural Logarithm**
**MHF4U**

SOLUTIONS

**1)** Use a calculator to approximate each to the nearest thousandth

**a)** $\ln 6.2$

$\simeq 1.825$

**b)** $\ln 2.1$

$\simeq 0.742$

**c)** $\ln e$

$= 1$

**d)** $e^5$

$\simeq 148.413$

**2)** Expand each logarithm

**a)** $\ln x^2$

$= 2\ln x$

**b)** $\ln \sqrt[3]{x}$

$= \ln(x)^{1/3}$

$= \frac{1}{3}\ln x$

**c)** $\ln \frac{u^3}{wv^4}$

$= \ln u^3 - (\ln w + \ln v^4)$

$= 3\ln u - \ln w - 4\ln v$

**3)** Condense each expression to a single logarithm

**a)** $4\ln 2$

$= \ln(2)^4$

$= \ln 16$

**b)** $\ln 10 - 5\ln 7$

$= \ln 10 - \ln(7)^5$

$= \ln 10 - \ln 16807$

$= \ln\left(\frac{10}{16807}\right)$

**c)** $3\ln x + 3\ln y$

$= \ln(x)^3 + \ln(y)^3$

$= \ln(x^3 y^3)$

**4)** Solve each equation. Round your answer to 4 decimal places if necessary.

**a)** $e^x = 2$

$\ln(e)^x = \ln 2$

$x \ln e = \ln 2$

$x(1) = \ln 2$

$x \simeq 0.6931$

**b)** $e^{-3n} = 83$

$\ln(e)^{-3n} = \ln 83$

$-3n \ln e = \ln 83$

$-3n(1) = \ln 83$

$n = \frac{\ln 83}{-3}$

$n \simeq -1.4729$

**c)** $e^{k+7} = 26$

$\ln(e)^{k+7} = \ln 26$

$(k+7)\ln(e) = \ln 26$

$(k+7)(1) = \ln 26$

$k+7 = \ln 26$

$k = \ln(26) - 7$

$k \simeq -3.7419$

**d)** $9e^{1.4p-10} - 10 = 17$

$9e^{1.4p-10} = 27$

$e^{1.4p-10} = 3$

$\ln(e)^{1.4p-10} = \ln 3$

$(1.4p-10)\ln(e) = \ln 3$

$(1.4p-10)(1) = \ln 3$

$1.4p - 10 = \ln(3)$

$p = \frac{\ln(3)+10}{1.4}$

$p \simeq 7.9276$

**e)** $\ln x = -5$

$e^{-5} = x$

$x \simeq 0.0067$

**f)** $7.316 = e^{\ln(2x)}$

$\ln(7.316) = \ln(e)^{\ln(2x)}$

$\ln(7.316) = \ln(2x)\ln(e)$

$e^{\ln(7.316)} = 2x$

$x = \frac{e^{\ln(7.316)}}{2}$

$x = 3.658$

**g)** $\ln(-m) = \ln(m + 10)$

$-m = m + 10$

$-10 = 2m$

$m = -5$

**h)** $\ln(9x + 1) = \ln(x^2 + 9)$

$9x + 1 = x^2 + 9$

$0 = x^2 - 9x + 8$

$0 = (x - 8)(x - 1)$

$x_2 = 8 \quad x_2 = 1$

**i)** $\ln(1 - 8x) - 10 = -7$

$\ln(1 - 8x) = 3$

$e^3 = 1 - 8x$

$\frac{e^3 - 1}{-8} = x$

$x \simeq -2.3857$

**j)** $\ln(5 - 2x^2) + \ln 9 = \ln 43$

$\ln[(5 - 2x^2)(9)] = \ln(43)$

$9(5 - 2x^2) = 43$

$45 - 18x^2 = 43$

$-18x^2 = -2$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$