L3 – 7.3 – Product and Quotient Laws of Logarithms
MHF4U

Part 1: Proof of Product Law of Logarithms

Let $x = \log_b m$ and $y = \log_b n$

Written in exponential form:

$b^x = m$ and $b^y = n$

$mn = b^x b^y$

$mn = b^{x+y}$

$\log_b(mn) = x + y$

$\log_b(mn) = \log_b m + \log_b n$

Part 2: Summary of Log Rules

Power Law of Logarithms	$\boldsymbol{\log_b x^n = n \log_b x}$ for $b > 0, b \neq 1, x > 0$
Product Law of Logarithms	$\boldsymbol{\log_b(mn) = \log_b m + \log_b n}$ for $b > 0, b \neq 1, m > 0, n > 0$
Quotient Law of Logarithms	$\boldsymbol{\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n}$ for $b > 0, b \neq 1, m > 0, n > 0$
Change of Base Formula	$\boldsymbol{\log_b m = \frac{\log m}{\log b}}$, $m > 0, b > 0, b \neq 1$
Exponential to Logarithmic	$y = b^x \rightarrow x = \log_b y$
Logarithmic to Exponential	$y = \log_b x \rightarrow x = b^y$
Other useful tips	$\log_a(a^b) = b$ $\quad$ $\log a = \log_{10} a$ $\quad$ $\log_b b = 1$

Part 3: Practice Using Log Rules

Example 1: Write as a single logarithm

a) $\log_5 6 + \log_5 8 - \log_5 16$

$= \log_5\left(\frac{6\times 8}{16}\right)$

$= \log_5 3$

b) $\log x + \log y + \log(3x) - \log y$

$= \log x + \log(3x)$

Started by collecting like terms. Must have same base and argument.

$= \log[(x)(3x)]$

$= \log(3x^2)$

Can't use power law because the exponent 2 applies only to x, not to $3x$.

c) $\frac{\log_2 7}{\log_2 5}$

$= \log_5 7$

Used change of base formula.

d) $\log 12 - 3\log 2 + 2\log 3$

$= \log 12 - \log 2^3 + \log 3^2$

$= \log 12 - \log 8 + \log 9$

$= \log\left(\frac{12\times 9}{8}\right)$

$= \log\left(\frac{27}{2}\right)$

Example 2: Write as a single logarithm and then evaluate

a) $\log_8 4 + \log_8 16$

$= \log_8(4\times 16)$

$= \log_8 64$

$= \frac{\log 64}{\log 8}$

$= 2$

b) $\log_3 405 - \log_3 5$

$= \log_3\left(\frac{405}{5}\right)$

$= \log_3 81$

$= \frac{\log 81}{\log 3}$

$= 4$

c) $2\log 5 + \frac{1}{2}\log 16$

$= \log 5^2 + \log\sqrt{16}$

$= \log 25 + \log 4$

$= \log(25\times 4)$

$= \log 100$

$= 2$

Example 3: Write the Logarithm as a Sum or Difference of Logarithms

a) $\log_3(xy)$

$= \log_3 x + \log_3 y$

b) $\log 20$

$= \log 4 + \log 5$

c) $\log(ab^2c)$

$= \log a + \log b^2 + \log c$

$= \log a + 2\log b + \log c$

Example 4: Simplify the following algebraic expressions

a) $\log\left(\frac{\sqrt{x}}{x^2}\right)$

$= \log\left(\frac{x^{\frac{1}{2}}}{x^{\frac{4}{2}}}\right)$

$= \log x^{-\frac{3}{2}}$

$= -\frac{3}{2}\log x$

b) $\log\left(\sqrt{x}\right)^3 + \log x^2 - \log\sqrt{x}$

$= \log x^{\frac{3}{2}} + \log x^2 - \log x^{\frac{1}{2}}$

$= \frac{3}{2}\log x + 2\log x - \frac{1}{2}\log x$

$= \frac{3}{2}\log x + \frac{4}{2}\log x - \frac{1}{2}\log x$

$= 3\log x$

c) $\log(2x-2) - \log(x^2-1)$

$= \log\left(\frac{2x-2}{x^2-1}\right)$

$= \log\left[\frac{2(x-1)}{(x-1)(x+1)}\right]$

$= \log\frac{2}{x+1}$