L1 – 6.1/6.2 – Intro to Logarithms and Review of Exponentials
MHF4U

In this section you will learn about how a logarithmic function is the inverse of an exponential function. You will also learn how to express exponential equations in logarithmic form.

Part 1: Review of Exponential Functions

Equation: $\boldsymbol{y = a(b)^x}$

$a =$ initial amount

$b =$ growth $(b > 1)$ or decay $(0 < b < 1)$ factor

$y =$ future amount

$x =$ number of times a has increased or decreased

To calculate x, use the equation: $x = \frac{total\ time}{time\ it\ takes\ for\ one\ growth\ or\ decay\ period}$

Example 1: An insect colony has a current population of 50 insects. Its population doubles every 3 days.

a) What is the population after 12 days?

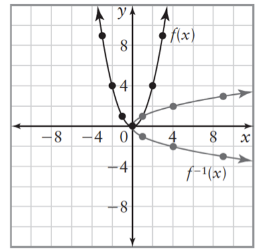
b) How long until the population reaches 25 600?

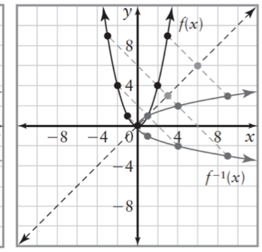
Part 2: Review of Inverse Functions

Inverse of a function:

· The inverse of a function *f* is denoted as f^{-1}
· The function and its inverse have the property that if f(*a*) = *b*, then $f^{-1}(b) = a$
· So if $f(5) = 13$, then $f^{-1}(13) = 5$
· More simply put: The inverse of a function has all the same points as the original function, except that the *x*'s and *y*'s have been reversed.

The **graph** of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$. This is true for all functions and their inverses.





Example 2: Determine the equation of the inverse of the function $f(x) = 3(x-5)^2 + 1$

Algebraic Method for finding the inverse:

1. Replace $f(x)$ with "y"
2. Switch the x and y variables
3. Isolate for y
4. replace y with $f^{-1}(x)$

Equation of inverse:

Part 3: Review of Exponent Laws

Name	Rule
Product Rule	$x^a \cdot x^b =$
Quotient Rule	$\frac{x^a}{x^b} =$
Power of a Power Rule	$(x^a)^b =$
Negative Exponent Rule	$x^{-a} =$
Exponent of Zero	$x^0 =$

Part 4: Inverse of an Exponential Function

Example 3:

a) Find the equation of the inverse of $f(x) = 2^x$.

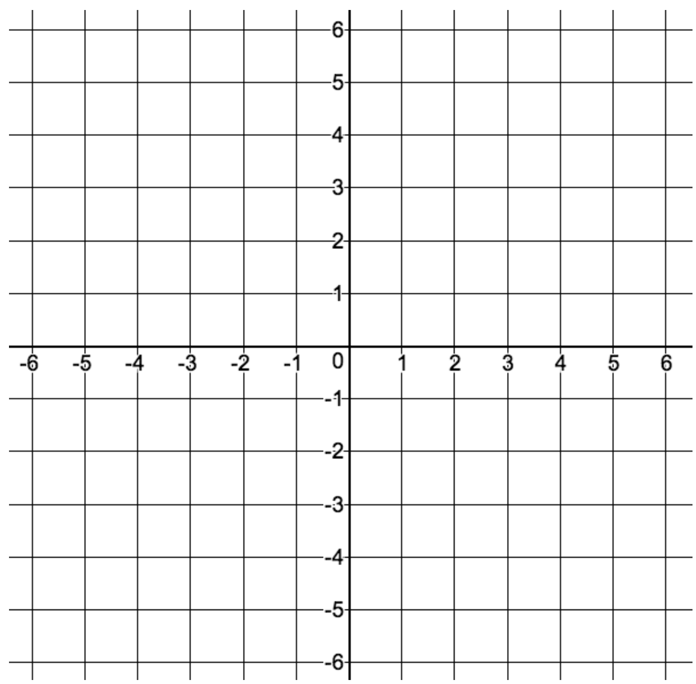
This step uses the 'change of base' formula that we will cover later in the unit.

$\log_b m = \frac{\log m}{\log b}$

b) Graph the both $f(x)$ and $f^{-1}(x)$.

$f(x) = 2^x$	
x	y

$f^{-1}(x) = log_2 x$	
x	y



Note: just swap x and y coordinates to get key points for the inverse of a function. The graph should appear to be a reflection across the line $y = x$.

c) Complete the chart of key properties for both functions

$y = 2^x$	$y = \log_2 x$
x-int:	x-int:
y-int:	y-int:
Domain:	Domain:
Range:	Range:
Asymptote:	Asymptote:

Part 5: What is a Logarithmic Function?

The logarithmic function is the _______________ of the exponential function with the same base.

> The **logarithmic function** is defined as $y = \log_b x$, or y equals the logarithm of x to the base b.
>
> The function is defined only for ____________________
>
> In this notation, ____ is the exponent to which the base, ____, must be raised to give the value of ____.

In other words, the solution to a logarithm is always an _______________.

The logarithmic function is most useful for solving for unknown ________________

_________________________________ are logarithms with a base of 10. It is not necessary to write the base for common logarithms: $\log x$ means the same as $\log_{10} x$

Part 6: Writing Equivalent Exponential and Logarithmic Expressions

Exponential equations can be written in logarithmic form, and vice versa

$y = b^x$ →

$y = \log_b x$ →

Example 4: Rewrite each equation in logarithmic form

a) $16 = 2^4$ **b)** $m = n^3$ **c)** $3^{-2} = \frac{1}{9}$

Example 5: Write each logarithmic equation in exponential form

a) $\log_4 64 = 3$ **b)** $y = \log x$

Note: because there is no base written, this is understood to be the common logarithm of x.

Part 7: Evaluate a Logarithm

Example 6: Evaluate each logarithm without a calculator

Rule: if $x^a = x^b$, then $a = b$

a) $y = \log_3 81$

Rule: $\log_a(a^b) = b$

a) $y = \log_4 64$

Note: *either of the rules presented above are appropriate to use for evaluating logarithmic expressions*

b) $y = \log\left(\frac{1}{100}\right)$

c) $y = \log_2\left(\frac{1}{8}\right)$