L4 – 8.3 Composite Functions
MHF4U

Two functions, f and g can be combined using a process called composition, which can be represented by:

$$f(g(x)) \text{ OR } (f \circ g)(x)$$

This is read as "f composite g"

Part 1: Determine the Composition of Two Functions

To determine an equation for a composite function, substitute the second function into the first.

To determine $f(g(x))$, substitute $g(x)$ in for x in to $f(x)$

Example 1: If $f(x) = x^2$ and $g(x) = x + 3$, determine an equation for each composite function and then graph the function.

a) $(f \circ g)(x)$

$= f(g(x))$

$= f(x+3)$

$= (x+3)^2$

b) $(g \circ f)(x)$

$= g(f(x))$

$= g(x^2)$

$= x^2 + 3$

c) $g^{-1}(g(x))$

Start by finding $g^{-1}(x)$

$y = x + 3$

$x = y + 3$

$x - 3 = y$

$g^{-1}(x) = x - 3$

Now find $g^{-1}(g(x))$

$= g^{-1}(x+3)$

$= (x+3) - 3$

$= x$

a) $(f \circ g)(x) = (x+3)^2$

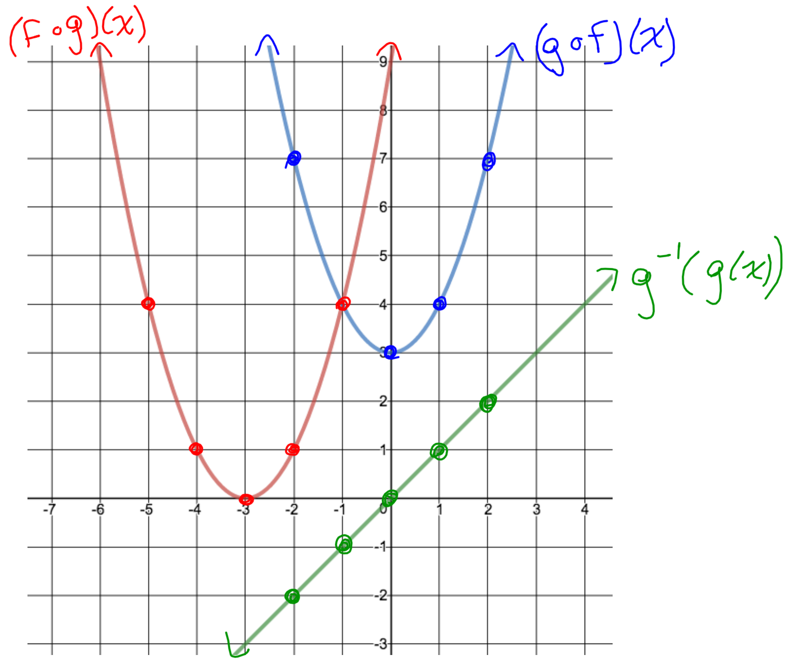
x	y
-5	4
-4	1
-3	0
-2	1
-1	4

b) $(g \circ f)(x) = x^2 + 3$

x	y
-2	7
-1	4
0	3
1	4
2	7

c) $g^{-1}(g(x)) = x$

x	y
-2	-2
-1	-1
0	0
1	1
2	2



Part 2: Evaluate a Composite Function

To evaluate a composite function $f(g(x))$ at a specific value, evaluate $g(x)$ at the specific value and then substitute the result in to $f(x)$.

Example 2: If $u(x) = x^2 + 3x + 2$ and $w(x) = \frac{1}{x-1}$

a) Evaluate $(u \circ w)(2)$

$= u(w(2))$

$= u(1)$

$= (1)^2 + 3(1) + 2$

$\boxed{= 6}$

$w(2) = \frac{1}{2-1}$

$= \frac{1}{1}$

$= 1$

b) Evaluate $w(u(-3))$

$w(u(-3)) = w(2)$

$= \frac{1}{2-1}$

$\boxed{= 1}$

$u(-3) = (-3)^2 + 3(-3) + 2$

$= 2$

Part 3: Application

Example 3: The number of rabbits, R, in a wildlife reserve as a function of time, t, in years can be modelled by the function $R(t) = 50\cos(t) + 100$. The number of wolves, W, in the same reserve can be modelled by the function $W(t) = 0.2[R(t-2)]$. Find the full equation for $W(t)$

$R(t-2) = 50\cos(t-2) + 100$

$W(t) = 0.2[R(t-2)]$

$W(t) = 0.2[50\cos(t-2) + 100]$

$W(t) = 10\cos(t-2) + 20$