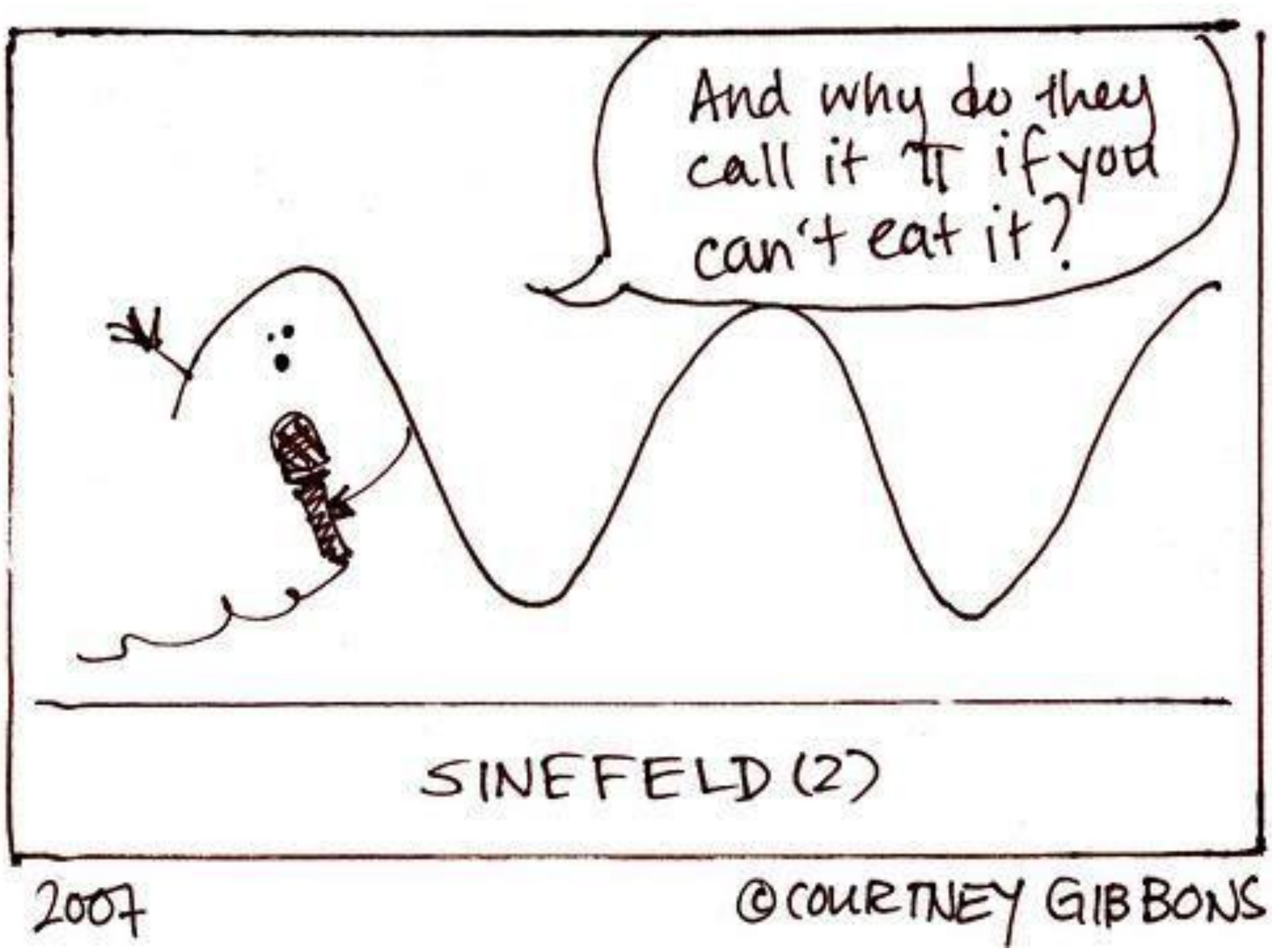
UNIT 4

Chapter 4/5 Part 1- Trigonometry in Radians

WORKBOOK

MHF4U



W1 – 4.1 Radian Measure
MHF4U

1) Determine mentally the exact radian measure for each angle, given that $30°$ is exactly $\frac{\pi}{6}$ radians.

a) $60°$ **b)** $90°$ **c)** $120°$ **d)** $150°$

2) Determine mentally the exact radian measure for each angle, given that $30°$ is exactly $\frac{\pi}{6}$ radians.

a) $15°$ **b)** $10°$ **c)** $7.5°$ **d)** $5°$

3) Determine mentally the exact radian measure for each angle, given that $45°$ is exactly $\frac{\pi}{4}$ radians.

a) $90°$ **b)** $135°$ **c)** $180°$ **d)** $225°$

4) Determine mentally the exact radian measure for each angle, given that $45°$ is exactly $\frac{\pi}{4}$ radians.

a) $22.5°$ **b)** $15°$ **c)** $9°$ **d)** $3°$

5) Determine the EXACT radian measure for each angle

a) $40°$ **b)** $10°$ **c)** $315°$

d) $210°$ **e)** $300°$ **f)** $75°$

6) Determine the APPROXIMATE radian measure, the nearest hundredth, for each angle.

a) $23°$ **b)** $51°$ **c)** $82°$

d) $128°$ **e)** $240°$ **f)** $330°$

7) Determine the EXACT degree measure for each angle.

a) $\frac{\pi}{5}$ **b)** $\frac{\pi}{9}$ **c)** $\frac{5\pi}{12}$

d) $\frac{5\pi}{18}$ **e)** $\frac{3\pi}{4}$ **f)** $\frac{3\pi}{2}$

8) Determine the APPROXIMATE degree measure, to the nearest tenth, for each angle.

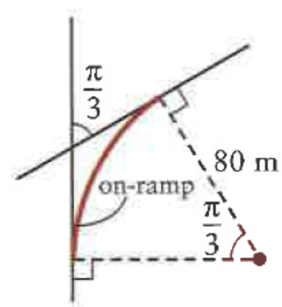
a) 2.34 **b)** 3.14 **c)** 5.27

d) 7.53 **e)** 0.68 **f)** 1.72

9) A circle of radius 25 cm has a central angle of 4.75 radians. Determine the length of the arc that subtends this angle.

10) Two highways meet at an angle measuring $\frac{\pi}{3}$ radians, as shown. An on-ramp in the shape of a circular arc is to be built such that the arc has a radius of 80 m.

a) Determine an EXACT expression for the length of the on-ramp.



b) Determine the length of the on-ramp, to the nearest tenth of a meter.

11) David made a swing for his niece Sarah using ropes 2.4 m long, so that Sarah swings through an arc length of 1.2 meters. Determine the angle through which Sarah swings, in both radians and degrees.

W2 – 4.2 Trig Ratios and Special Angles
MHF4U

1) Draw both special triangles using radian measures.

2) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

a) $\cos 3.43$ **b)** sin 2.92 **c)** tan 5.61

d) csc 1.27 **e)** cot 4.53 **f)** sec 0.98

3) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

a) cot $\frac{3\pi}{7}$ **b)** sec $\frac{16\pi}{3}$ **c)** csc $\frac{5\pi}{11}$

4) Use the unit circle and the cast rule to find exact expressions for each ratio

a) $\sin\frac{2\pi}{3}$ **b)** $\tan\frac{\pi}{6}$

c) $\cos\frac{5\pi}{4}$ **d)** $\tan\frac{7\pi}{4}$

5) Use the unit circle and cast rule to determine exact values of the primary trig ratios for each angle.

a) $\frac{2\pi}{3}$

b) $\frac{5\pi}{6}$

c) $\frac{3\pi}{2}$

d) $\frac{7\pi}{4}$

6) Use the special triangles determine exact values for the six trigonometric ratios for $\frac{11\pi}{6}$.

7) Lynda is flying her kite at the end of a 40-m string. The string makes an angle of $\frac{\pi}{4}$ with the ground. The wind speed increases, and the kite flies higher until the string makes and angle of $\frac{\pi}{3}$ with the ground.

a) Determine an exact expression for the horizontal distance that the kite moves between the two positions.

b) Determine and exact expression for the vertical distance that the kite moves between the two positions.

8) Determine an exact value for each expression

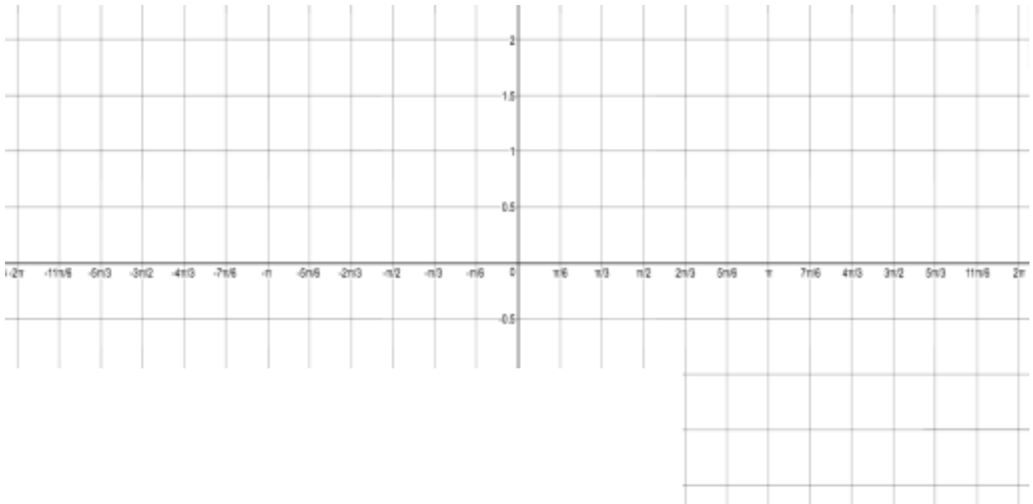
a) $\frac{\sin\frac{\pi}{3}\tan\frac{\pi}{6}}{\cos\frac{\pi}{4}}$

b) $\cot\frac{5\pi}{4} + \tan\frac{11\pi}{6}\tan\frac{5\pi}{3}$

W3 – 5.1/5.2 Graphing Trig Functions
MHF4U
Jensen

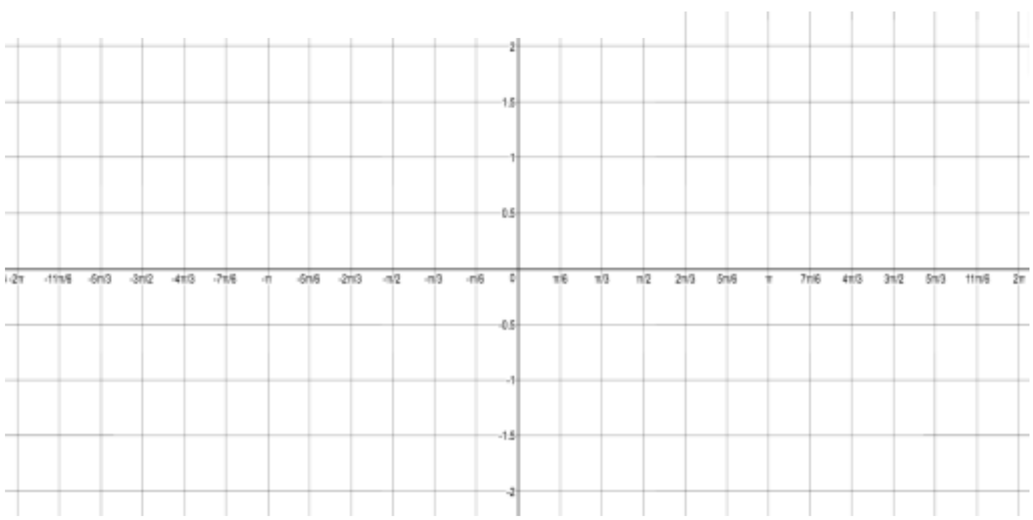
1) Complete the following table of values for the function $f(x) = \sin(x)$ and $g(x) = \csc(x)$. Use special triangles, the unit circle, or a calculator to find values for the function. Then graph both functions on the same grid. Draw asymptotes where necessary.

x	$f(x)$	$g(x)$
0		
$\frac{\pi}{6}$		
$\frac{2\pi}{6} = \frac{\pi}{3}$		
$\frac{3\pi}{6} = \frac{\pi}{2}$		
$\frac{4\pi}{6} = \frac{2\pi}{3}$		
$\frac{5\pi}{6}$		
$\frac{6\pi}{6} = \pi$		
$\frac{7\pi}{6}$		
$\frac{8\pi}{6} = \frac{4\pi}{3}$		
$\frac{9\pi}{6} = \frac{3\pi}{2}$		
$\frac{10\pi}{6} = \frac{5\pi}{3}$		
$\frac{11\pi}{6}$		
$\frac{12\pi}{6} = 2\pi$		



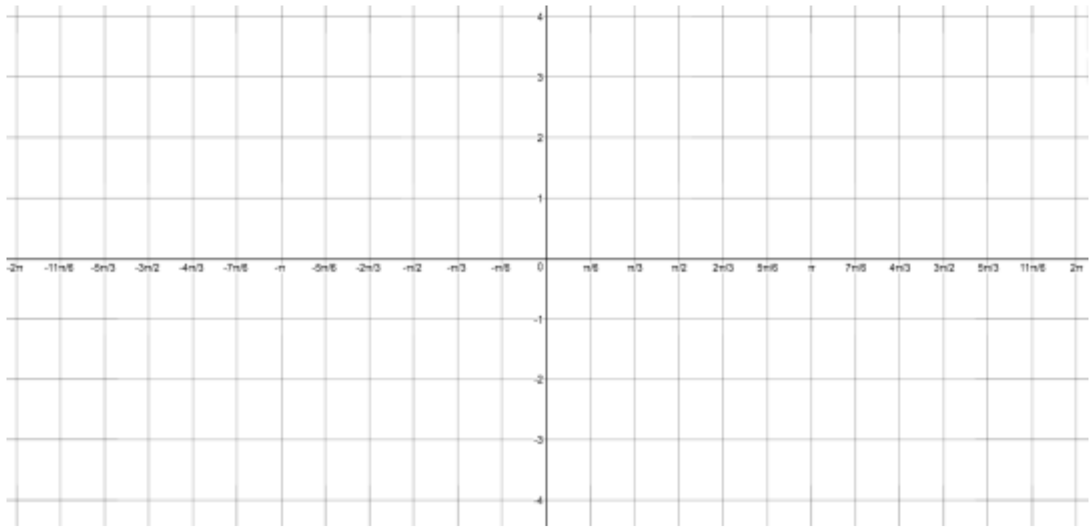
2) Complete the followi[n] ... $\sec(x)$. Use special
triangles, the unit circle, ... functions on the same
grid. Draw asymptotes v...

x	$f(x)$	
0		
$\frac{\pi}{6}$		
$\frac{2\pi}{6} = \frac{\pi}{3}$		
$\frac{3\pi}{6} = \frac{\pi}{2}$		
$\frac{4\pi}{6} = \frac{2\pi}{3}$		
$\frac{5\pi}{6}$		
$\frac{6\pi}{6} = \pi$		
$\frac{7\pi}{6}$		
$\frac{8\pi}{6} = \frac{4\pi}{3}$		
$\frac{9\pi}{6} = \frac{3\pi}{2}$		
$\frac{10\pi}{6} = \frac{5\pi}{3}$		
$\frac{11\pi}{6}$		
$\frac{12\pi}{6} = 2\pi$		



3) Complete the following table of values for the function $f(x) = \tan(x)$. Use the quotient identity to find y-values.

x	$f(x)$
0	
$\frac{\pi}{6}$	
$\frac{2\pi}{6} = \frac{\pi}{3}$	
$\frac{3\pi}{6} = \frac{\pi}{2}$	
$\frac{4\pi}{6} = \frac{2\pi}{3}$	
$\frac{5\pi}{6}$	
$\frac{6\pi}{6} = \pi$	
$\frac{7\pi}{6}$	
$\frac{8\pi}{6} = \frac{4\pi}{3}$	
$\frac{9\pi}{6} = \frac{3\pi}{2}$	
$\frac{10\pi}{6} = \frac{5\pi}{3}$	
$\frac{11\pi}{6}$	
$\frac{12\pi}{6} = 2\pi$	



4) A boat is in the water 150 meters from a straight shoreline. There is a rotating beam on the boat.

a) Determine a reciprocal trigonometric relation for the distance, d, from the boat to where the light hits the shoreline in terms of the angle of rotation x.

b) Determine an exact expression for the distance when $x = \frac{\pi}{6}$

c) Determine an approximate value, to the nearest tenth of a meter, for the distance.

5) A variant on the carousel at a theme park is the swing ride. Swings are suspended from a rotating platform and move outward to form an angle x with the vertical as the ride rotates. The angle is related to the radial distance, r, in meters, from the center of rotation; the acceleration, $g = 9.8$ m/s^2, due to gravity; and the speed, v, in meters per second, of the swing, according to the formula

$$\cot x = \frac{rg}{v^2}$$

Determine the angle x for a swing located 3.5 meters from the center of rotations and moving at 5.4 m/s, to the nearest hundredth of a radian.

6) Explain the difference between $\csc\frac{1}{\sqrt{2}}$ and $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

W4 – 5.3 Transformations of Trig Functions
MHF4U

1) For each function, fill in the table of information and then graph two cycles of the transformed function using transformations of the parent function. Choose an appropriate scale.

a) $y = 5\sin(3x)$

Amplitude:	Period:
Phase shift:	Vertical shift:
Max:	Min:

x	y

b) $y = -3\cos\left(\frac{3}{4}x\right)$

Amplitude:	Period:
Phase shift:	Vertical shift:
Max:	Min:

x	y

c) $y = 4\sin\left[3\left(x - \frac{\pi}{3}\right)\right] - 2$

Amplitude:	Period:
Phase shift:	Vertical shift:
Max:	Min:

x	***y***

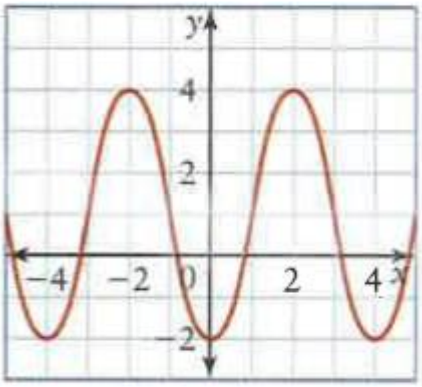
d) $y = 2\sin\left[\frac{1}{2}\left(x + \frac{5\pi}{6}\right)\right] + 4$

Amplitude:	Period:
Phase shift:	Vertical shift:
Max:	Min:

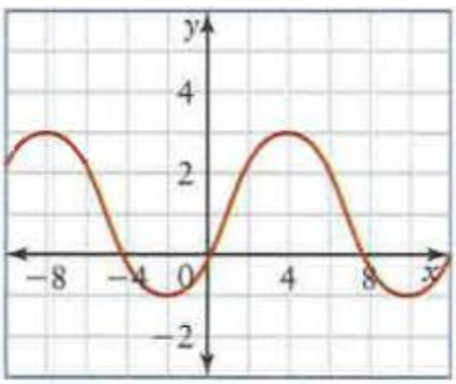
x	y

2) Model each graph shown as a sine and cosine function.

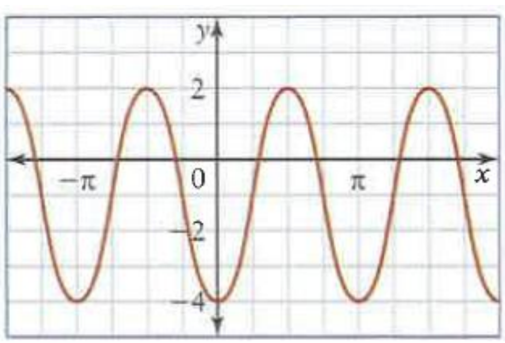
a)



b)



c)



3) A sine function has a maximum value of 7, a minimum value of -1, a phase shift of $\frac{3\pi}{4}$ radians to the left, and a period of $\frac{\pi}{2}$.

a) Write an equation for this function.

b) Write an equivalent cosine equation for this function.

4) A cosine function has a maximum value of 1, a minimum value of -5, a phase shift of 2 radians to the right, and a period of 3.

a) Write an equation for this function.

b) Write an equivalent sine equation for this function.

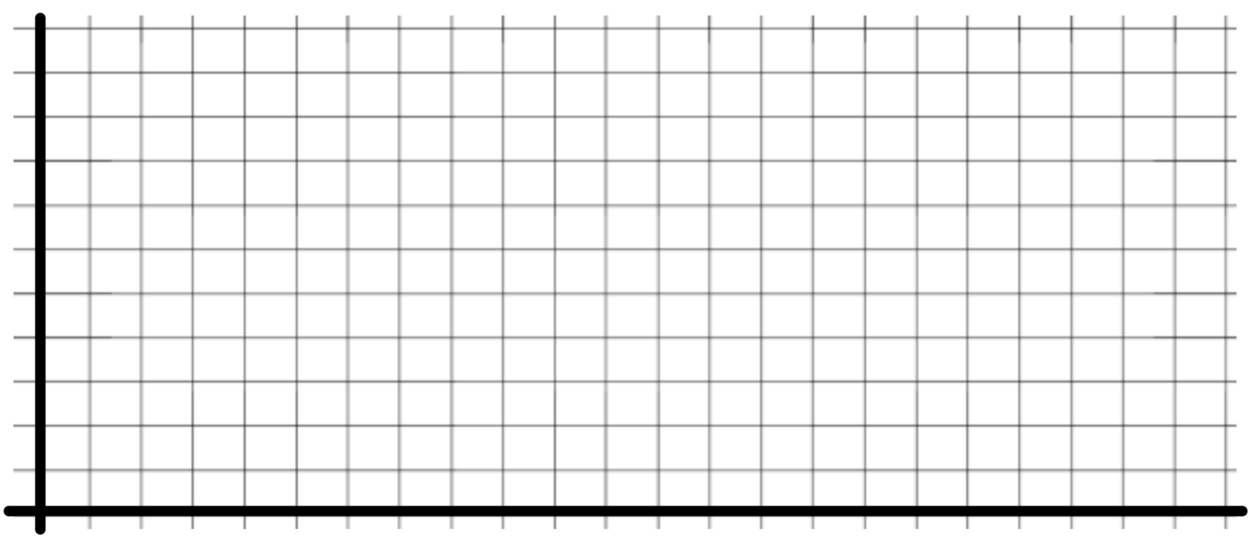
W5 – 5.3 Trig Applications
MHF4U

1) Consider a Ferris wheel with a diameter of 20 meters. A rider must step up 1 meter to enter her seat, and a complete turn of the wheel takes 60 seconds.

a) Write a cosine function that represents the height, h, of a rider t seconds after the ride starts. Start by deciding on the max and min height of the rider.

b) How high will a rider be after 10 seconds? After 160 seconds?

c) Sketch a graph for this function.



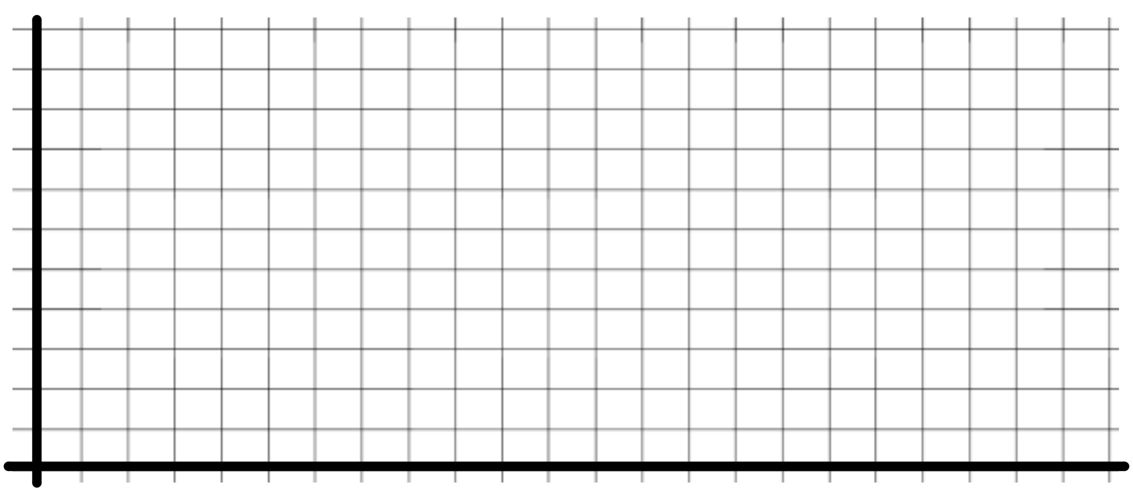
2) The equation $T = 40\cos\left[\frac{\pi}{10}(x - 10)\right] + 160$ describes how an element in a furnace heats and cools cyclically, where x is the time in minutes and T is the temperature in $°C$.

a) What is the period of the heating of this element? What does it mean?

b) At what point during the cycle of heating does the element reach its maximum temperature? What is the maximum temperature? What is the minimum?

c) Use this equation to calculate the temperature of the element 4 minutes and 12 minutes into the cycle.

d) Sketch a graph for this function.



3) During a 12-hour period, the tides in one area of the Bay of Fundy cause the water level to rise to 6 m above average sea level and to fall 6 m below average sea level. The depth of the water at low tide is 2 m.

a) What is the depth of the water at average sea level?

b) What is the depth of the water at high tide?

c) When do high tide and low tide occur if the water is at average sea level at midnight and the tide is coming in?

d) Create a sine equation to represent the scenario. Let y represent depth of the water in meters and let x represent hours after midnight.

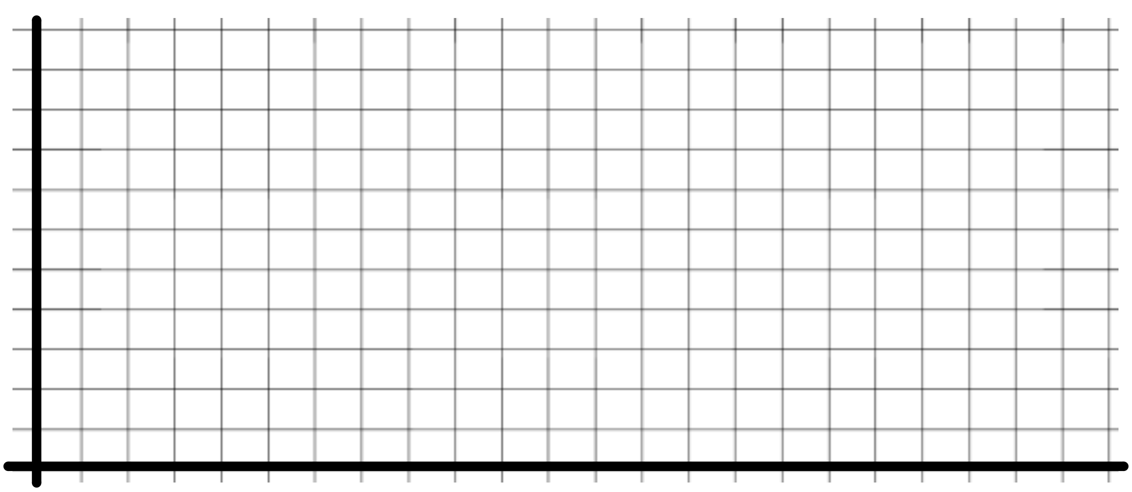
4) A robotic arm drives in 10 spikes per minute on an assembly line. It rises to a height of 28 cm, drops to a height of 4 cm as it strikes the spike, and repeats. The arm rests (or starts) at the maximum height.

a) What is the period of the motion of the robotic arm? In other words, how long does it take to go from maximum height, down to the spike and back up again to the maximum height if it does this 10 times in one minute?

b) Write a cosine function that represents the height, h, of the robotic arm t seconds after the process starts.

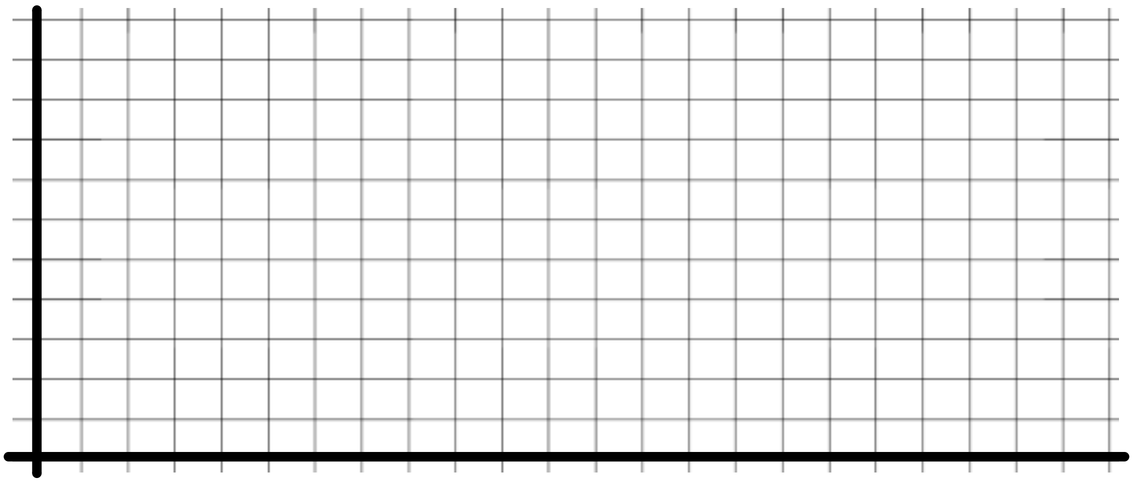
c) What will be the height of the arm after 10 seconds? After 50 seconds?

d) Sketch a graph for this function.



5) A rung on a hamster wheel, with a radius of 25 cm, is travelling at a constant speed. It makes one complete revolution in 3 seconds. The axle of the hamster wheel is 27 cm above the ground.

a) Sketch a graph of the height of the rung above the ground during two complete revolutions, beginning when the rung is closest to the ground.



b) Describe the transformations necessary to transform $y = \cos x$ into the function you graphed in part a)

c) Write the equation that models this situation

6) Each person's blood pressure is different, but there is a range of blood pressure values that is considered healthy. The function $P(t) = -20\cos\left(\frac{5\pi}{3}t\right) + 100$ models the blood pressure, p, in millimetres of mercury, at time t, in seconds, of a person at rest.

a) What is the period of the function? What does the period represent for an individual?

b) How many times does this person's heart beat each minute?

c) Sketch a graph of the function