W1 – 2.1 – Long Division of Polynomials and The Remainder Theorem
MHF4U

1) Use the remainder theorem to determine the remainder when $2x^3 + 7x^2 - 8x + 3$ is divided by each binomial.

a) $x + 1$

$P(-1) = 2(-1)^3 + 7(-1)^2 - 8(-1) + 3$
$= 16$

b) $x - 2$

$P(2) = 2(2)^3 + 7(2)^2 - 8(2) + 3$
$= 31$

c) $x + 3$

$P(-3) = 2(-3)^3 + 7(-3)^2 - 8(-3) + 3$
$= 36$

2)a) Divide $x^3 + 3x^2 - 2x + 5$ by $x + 1$. Express the result in quotient form.

$$\begin{array}{r|l} & x^2 + 2x - 4 \\ x+1 & x^3 + 3x^2 - 2x + 5 \\ & x^3 + x^2 \\ & \quad 2x^2 - 2x \\ & \quad 2x^2 + 2x \\ & \qquad -4x + 5 \\ & \qquad -4x - 4 \\ & \qquad R = 9 \end{array}$$

$$\boxed{\frac{x^3 + 3x^2 - 2x + 5}{x+1} = x^2 + 2x - 4 + \frac{9}{x+1}}$$

b) Write the corresponding statement that can be used to check the division.

$$x^3 + 3x^2 - 2x + 5 = (x+1)(x^2 + 2x - 4) + 9$$

3) Divide $3x^4 - 4x^3 - 6x^2 + 17x - 8$ by $3x - 4$. Express the result in quotient form.

$$\begin{array}{r|l} & x^3 + 0x^2 - 2x + 3 \\ 3x-4 & 3x^4 - 4x^3 - 6x^2 + 17x - 8 \\ & 3x^4 - 4x^3 \\ & \quad 0x^3 - 6x^2 \\ & \quad 0x^3 - 0x^2 \\ & \qquad -6x^2 + 17x \\ & \qquad -6x^2 + 8x \\ & \qquad\quad 9x - 8 \\ & \qquad\quad 9x - 12 \\ & \qquad\quad R = 4 \end{array}$$

$$\boxed{\frac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{3x-4} = x^3 - 2x + 3 + \frac{4}{3x-4}}$$

b) Write the corresponding statement that can be used to check the division.

$$3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x-4)(x^3 - 2x + 3) + 4$$

4) Perform each division. Express the result in quotient form.

a) $x^3 + 7x^2 - 3x + 4$ divided by $x + 2$

$$\begin{array}{r|l} & x^2+5x-13 \\ x+2 & x^3+7x^2-3x+4 \\ & x^3+2x^2 \\ & 5x^2-3x \\ & 5x^2+10x \\ & -13x+4 \\ & -13x-26 \\ & R=30 \end{array}$$

$$\boxed{\frac{x^3+7x^2-3x+4}{x+2} = x^2+5x-13+\frac{30}{x+2}}$$

b) $6x^3 + x^2 - 14x - 6$ divided by $3x + 2$

$$\begin{array}{r|l} & 2x^2-1x-4 \\ 3x+2 & 6x^3+x^2-14x-6 \\ & 6x^3+4x^2 \\ & -3x^2-14x \\ & -3x^2-2x \\ & -12x-6 \\ & -12x-8 \\ & R=2 \end{array}$$

$$\boxed{\frac{6x^3+x^2-14x-6}{3x+2} = 2x^2-x-4+\frac{2}{3x+2}}$$

c) $10x^3 + 11 - 9x^2 - 8x$ divided by $5x - 2$

$$\begin{array}{r|l} & 2x^2-1x-2 \\ 5x-2 & 10x^3-9x^2-8x+11 \\ & 10x^3-4x^2 \\ & -5x^2-8x \\ & -5x^2+2x \\ & -10x+11 \\ & -10x+4 \\ & R=7 \end{array}$$

$$\boxed{\frac{10x^3-9x^2-8x+11}{5x-2} = 2x^2-x-2+\frac{7}{5x-2}}$$

d) $11x - 4x^4 - 7$ divided by $x - 3$

$$\begin{array}{r|l} & -4x^3-12x^2-36x-97 \\ x-3 & -4x^4+0x^3+0x^2+11x-7 \\ & -4x^4+12x^3 \\ & -12x^3+0x^2 \\ & -12x^3+36x^2 \\ & -36x^2+11x \\ & -36x^2+108x \\ & -97x-7 \\ & -97x+291 \\ & R=-298 \end{array}$$

$$\boxed{\frac{-4x^4+11x-7}{x-3} = -4x^3-12x^2-36x-97-\frac{298}{x-3}}$$

e) $6x^3 + x^2 + 7x + 3$ divided by $3x + 2$

$$\begin{array}{r|l} & 2x^2-1x+3 \\ 3x+2 & 6x^3+x^2+7x+3 \\ & 6x^3+4x^2 \\ & -3x^2+7x \\ & -3x^2-2x \\ & 9x+3 \\ & 9x+6 \\ & R=-3 \end{array}$$

$$\boxed{\frac{6x^3+x^2+7x+3}{3x+2} = 2x^2-1x+3-\frac{3}{3x+2}}$$

f) $8x^3 + 4x^2 - 31$ divided by $2x - 3$

$$\begin{array}{r|l} & 4x^2+8x+12 \\ 2x-3 & 8x^3+4x^2+0x-31 \\ & 8x^3-12x^2 \\ & 16x^2+0x \\ & 16x^2-24x \\ & 24x-31 \\ & 24x-36 \\ & R=5 \end{array}$$

$$\boxed{\frac{8x^3+4x^2-31}{2x-3} = 4x^2+8x+12+\frac{5}{2x-3}}$$

g) $6x^2 - 6 + 8x^3$ divided by $4x - 3$

$$\begin{array}{r|l} & 2x^2+3x+\frac{9}{4} \\ 4x-3 & 8x^3+6x^2+0x-6 \\ & 8x^3-6x^2 \\ & 12x^2+0x \\ & 12x^2-9x \\ & 9x-6 \\ & 9x-\frac{27}{4} \\ & R=3/4 \end{array}$$

$$\boxed{\frac{8x^3+6x^2-6}{4x-3} = 2x^2+3x+\frac{9}{4}+\frac{3}{4(4x-3)}}$$

5) The volume, in cubic cm, of a rectangular box can be modelled by the polynomial expression $2x^3 + 17x^2 + 38x + 15$. Determine possible dimensions of the box if the height, in cm, is given by $x + 5$.

$$
\begin{array}{r|l}
 & 2x^2 + 7x + 3 \\
\hline
x+5 & 2x^3 + 17x^2 + 38x + 15 \\
 & 2x^3 + 10x^2 \\
\hline
 & 7x^2 + 38x \\
 & 7x^2 + 35x \\
\hline
 & 3x + 15 \\
 & 3x + 15 \\
\hline
 & R = 0
\end{array}
$$

$2x^3 + 17x^2 + 38x + 15 = (x+5)(2x^2 + 7x + 3)$

Factor

$= (x+5)(x+3)(2x+1)$

height length width

P 6, S 7: $\frac{3}{1}$, $\frac{6}{2}$, $\frac{1}{2}$

6) Determine the value of k such that when $P(x) = kx^3 + 5x^2 - 2x + 3$ is divided by $x + 1$, the remainder is 7.

$P(-1) = k(-1)^3 + 5(-1)^2 - 2(-1) + 3$

$7 = -1k + 5 + 2 + 3$

$7 = -1k + 10$

$-3 = -1k$

$\boxed{k = 3}$

ANSWER KEY

1) a) 16 **b)** 31 **c)** 36

2)a) $\frac{x^3+3x^2-2x+5}{x+1} = x^2 + 2x - 4 + \frac{9}{x+1}$ **b)** $x^3 + 3x^2 - 2x + 5 = (x + 1)(x^2 + 2x - 4) + 9$

3)a) $\frac{3x^4-4x^3-6x^2+17x-8}{3x-4} = x^3 - 2x + 3 + \frac{4}{3x-4}$ **b)** $3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x - 4)(x^3 - 2x + 3) + 4$

4)a) $\frac{x^3+7x^2-3x+4}{x+2} = x^2 + 5x - 13 + \frac{30}{x+2}$ **b)** $\frac{6x^3+x^2-14x-6}{3x+2} = 2x^2 - x - 4 + \frac{2}{3x+2}$

c) $\frac{10x^3-9x^2-8x+11}{5x-2} = 2x^2 - x - 2 + \frac{7}{5x-2}$ **d)** $\frac{-4x^4+11x-7}{x-3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x-3}$

e) $\frac{6x^3+x^2+7x+3}{3x+2} = 2x^2 - x + 3 - \frac{3}{3x+2}$ **f)** $\frac{8x^3+4x^2-31}{2x-3} = 4x^2 + 8x + 12 + \frac{5}{2x-3}$

g) $\frac{6x^2-6+8x^3}{4x-3} = 2x^2 + 3x + \frac{9}{4} + \frac{3}{4(4x-3)}$

5) $2x^3 + 17x^2 + 38x + 15 = (x + 5)(x + 3)(2x + 1)$

6) $k = 3$