L2 – 3.3 Quotient of Linear Functions
MHF4U

Part 1: Key Features of the Quotient of Linear Functions

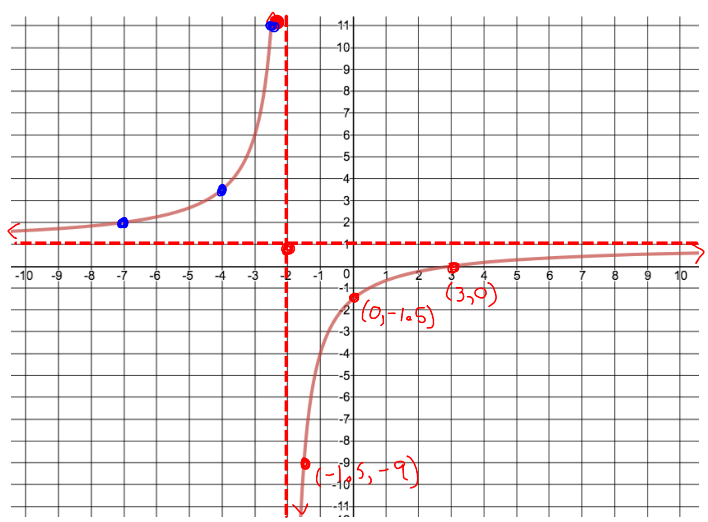
Features of $f(x) = \frac{ax+b}{cx+d}$

- If an x value is a zero of the denominator ONLY, this results in a vertical asymptote
  - Equation of vertical asymptote is $x = \frac{-d}{c}$
- If an x value is a zero of the numerator AND denominator, this results in a **hole** in the graph NOT a vertical asymptote
- There is a horizontal asymptote at the ratio of the leading coefficients
  - Equation of horizontal asymptote is $y = \frac{a}{c}$
- Forms a **Hyperbola**: the two branches of the graph of the function are equidistant from the point of intersection of the vertical and horizontal asymptotes
  - Once you know the shape of one branch, you can translate the points to graph the other branch
- You can find the x-intercept by setting $y = 0$ and solving for x
  - This results in $\left(\frac{-b}{a}, 0\right)$
- You can find the y-intercept by setting $x = 0$ and solving for y
  - This results in $\left(0, \frac{b}{d}\right)$

Part 2: Graphing a Quotient of Linear Functions

Example 1: Graph each of the following functions

a) $f(x) = \frac{x-3}{x+2}$



VA: $x+2=0$
$x=-2$

HA: $\frac{1}{1}=1$
$y=1$

x-int: $0 = \frac{x-3}{x+2}$
$0 = x-3$
$x = 3$
$(3,0)$

y-int: $f(0) = \frac{0-3}{0+2}$
$= -\frac{3}{2}$
$(0,-1.5)$

Another point:
$f(-1.5) = \frac{-1.5-3}{-1.5+2}$
$= \frac{-4.5}{0.5}$
$= -9$

b) $g(x) = \frac{2x-3}{x-1}$

VA: $x - 1 = 0$

$x = 1$

HA: $\frac{2}{1} = 2$

$y = 2$

x-int: $0 = \frac{2x-3}{x-1}$

$0 = 2x - 3$

$x = \frac{3}{2}$

$(1.5, 0)$

y-int: $f(0) = \frac{2(0)-3}{0-1}$

$= 3$

$(0, 3)$

Other points:

$f(2) = \frac{2(2)-3}{2-1} = \frac{1}{1} = 1$

$(2, 1)$

$f(3) = \frac{2(3)-3}{3-1} = \frac{3}{2}$

$(3, 1.5)$