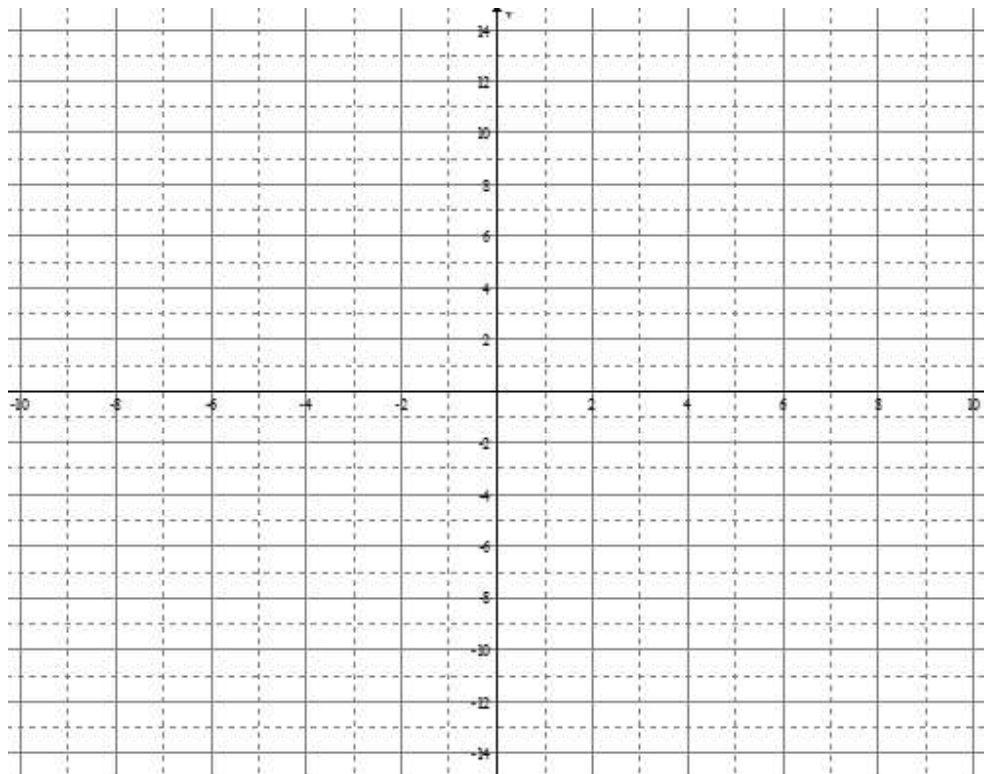


Putting it all together

Ex1. Given the following functions, use the characteristics of polynomials and rational functions to describe its behavior and sketch the function

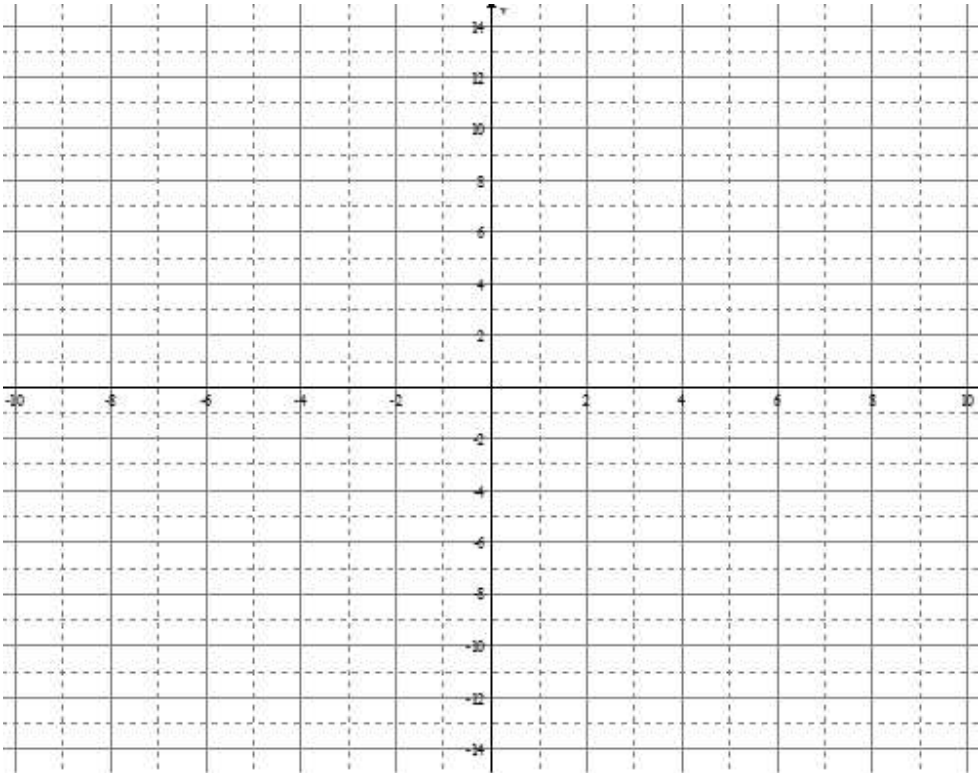
a) $f(x) = \frac{(x+1)^2(x-3)}{(x+3)^2(x-2)}$

As $x \rightarrow$	$f(x) \rightarrow$
-3^+	
-3^-	
2^+	
2^-	
$+\infty$	
$-\infty$	



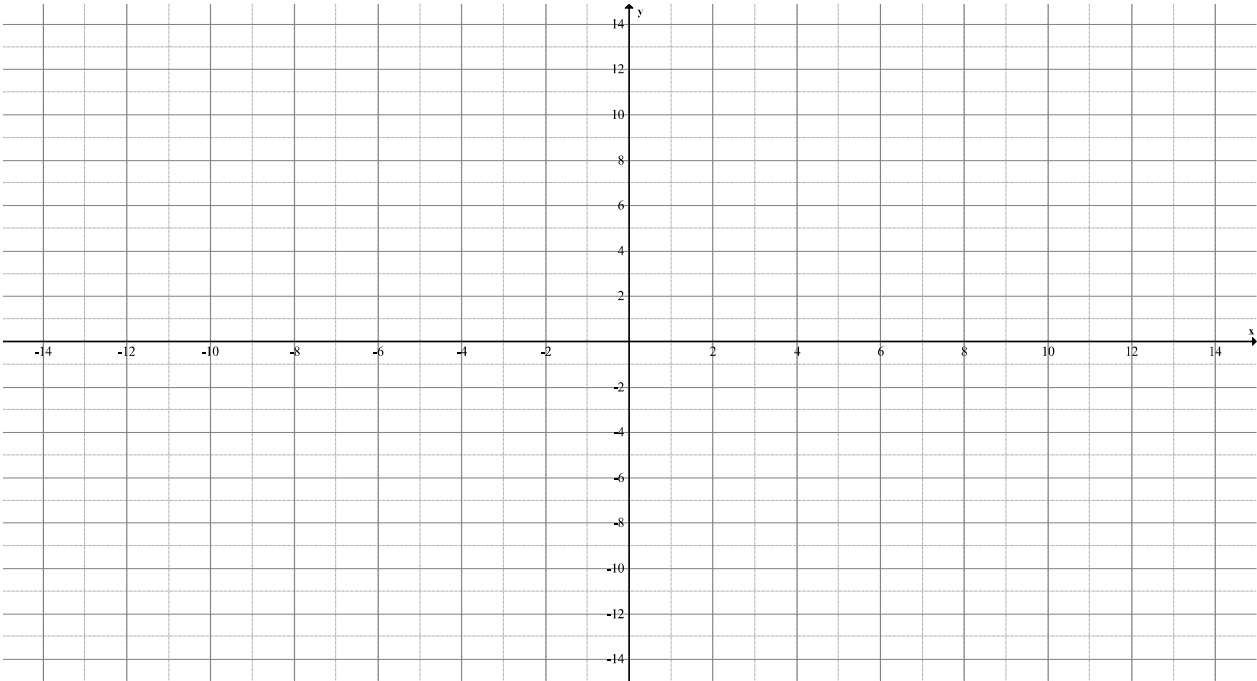
b) $f(x) = \frac{(x+2)(x-3)}{(x+1)^2(x-2)}$

As $x \rightarrow$	$f(x) \rightarrow$
-1^+	
-1^-	
2^+	
2^-	
$+\infty$	
$-\infty$	

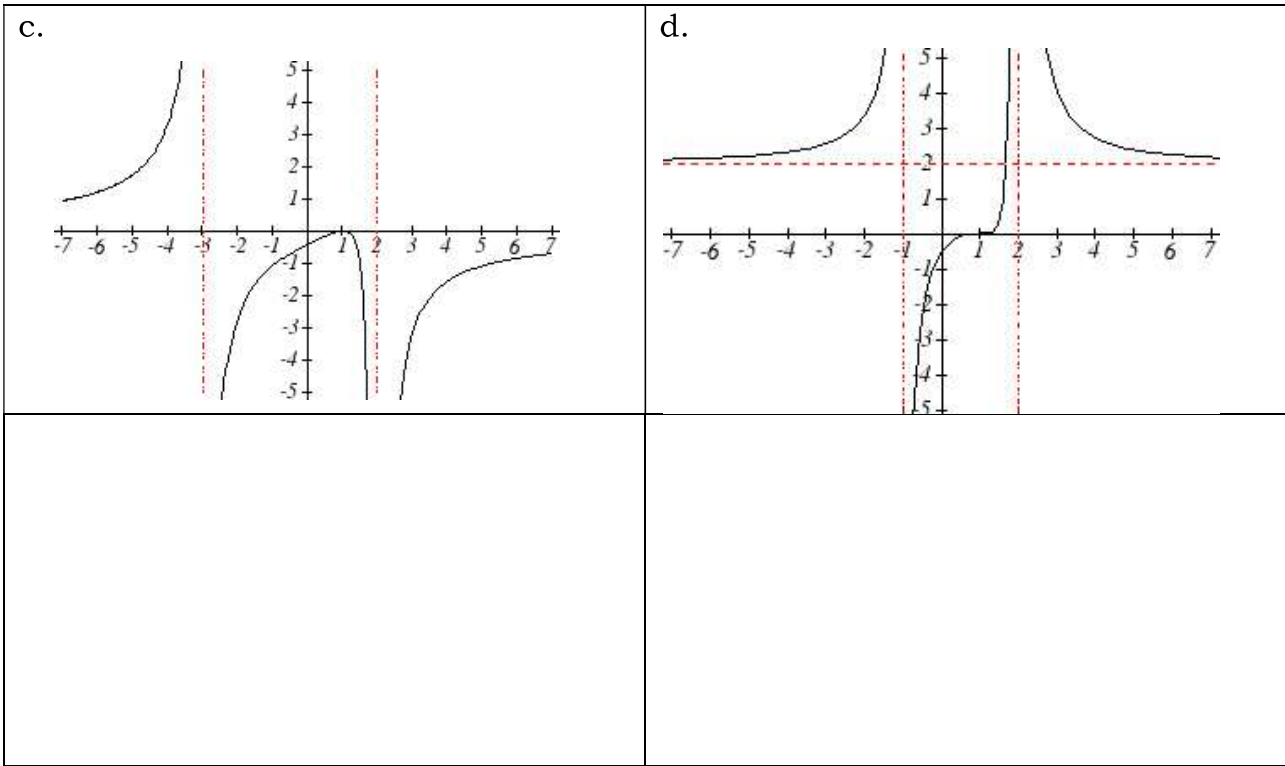
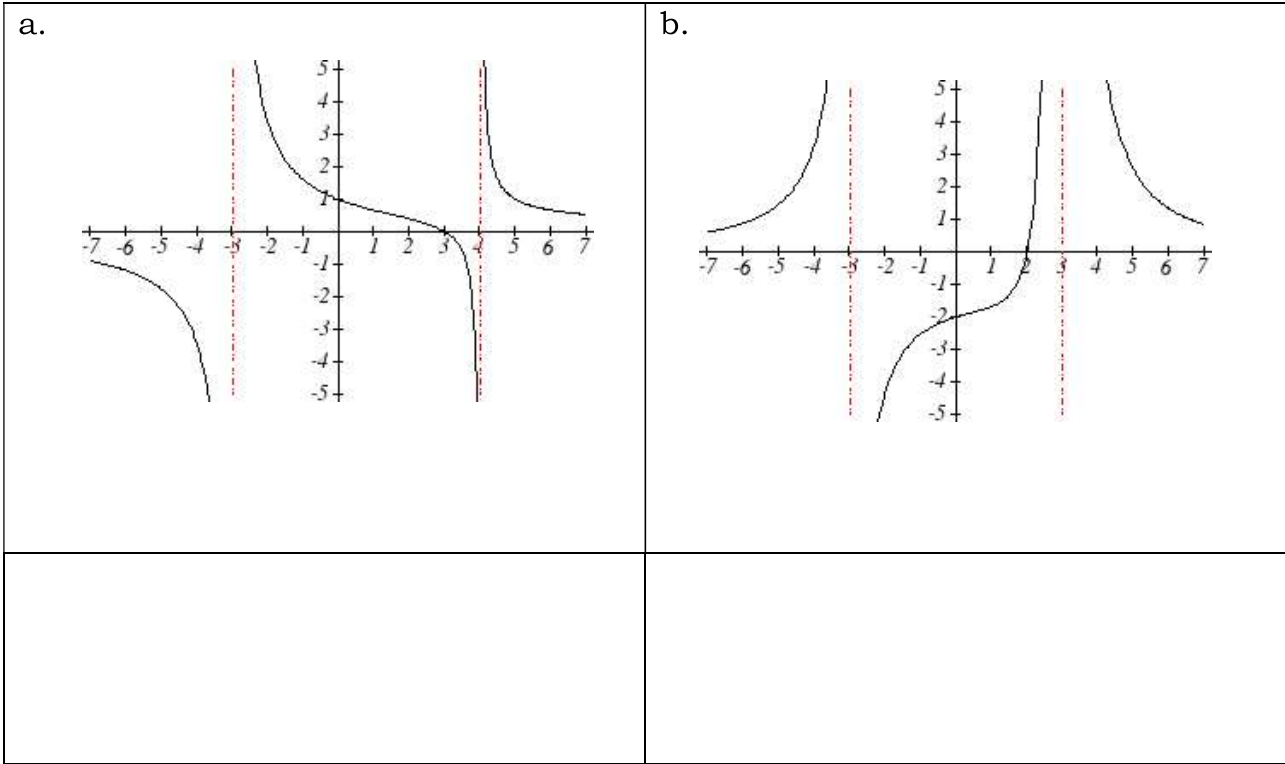


c) $f(x) = \frac{x^2 - 4x + 4}{x^2 + x - 20}$

As $x \rightarrow$	$f(x) \rightarrow$
$+\infty$	
$-\infty$	

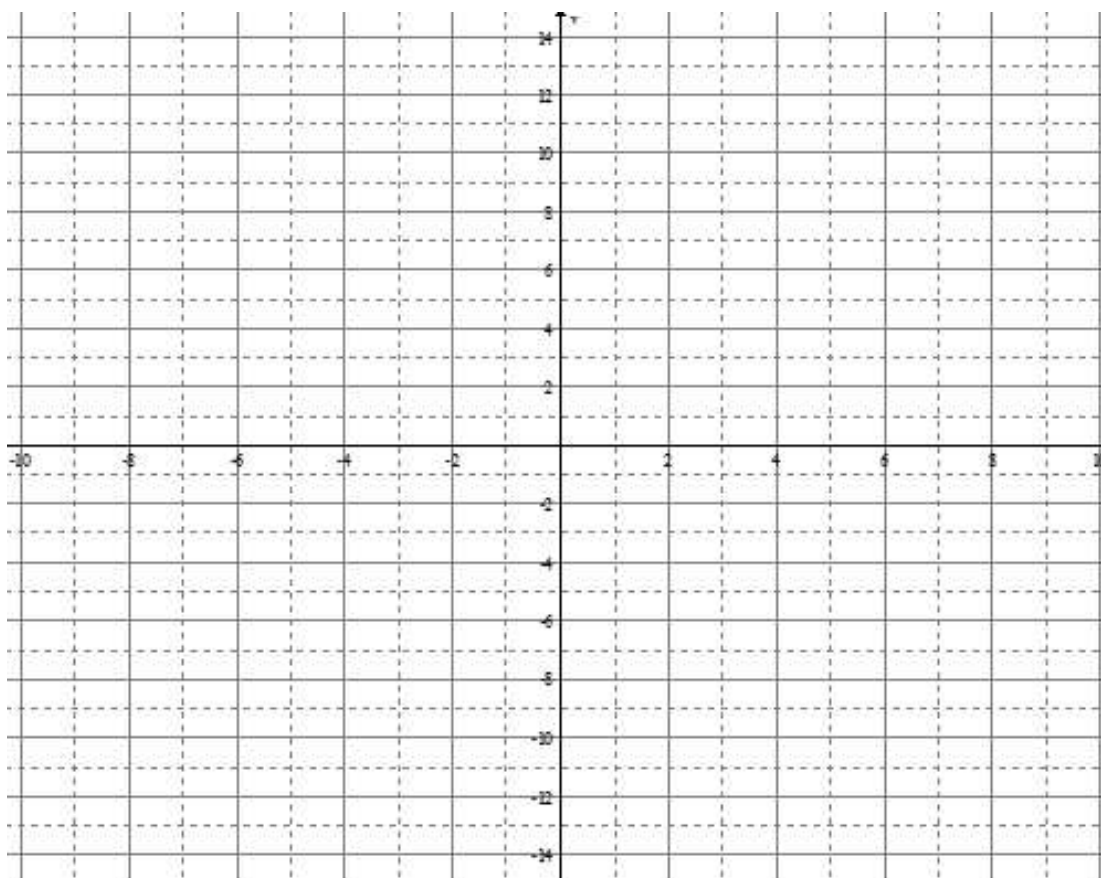


Ex.2 Write an equation for the function graphed



Ex. 3 Use the information below to sketch the function.

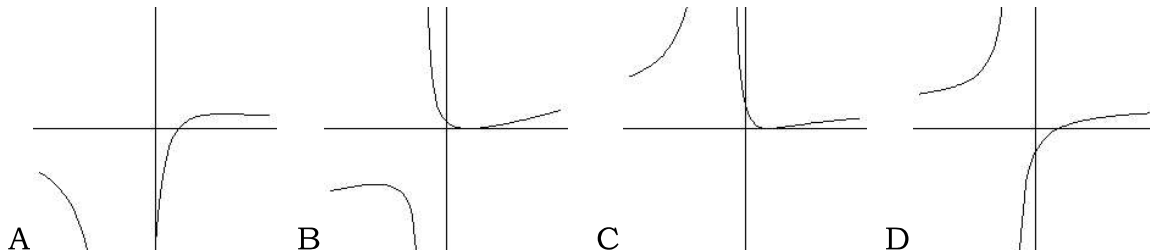
- There is a horizontal asymptote at $f(x) = 0$.
- The Domain of the function is $D: \{x \neq 4, x \neq 3, x \in \mathbb{R}\}$
- $f(2) = \frac{3}{2}$
- $f(1) = f(5) = 0, f(3) = \text{undefined}$
- $f(x) < 0$ when $x < 1, 3 < x < 4, 4 < x < 5$
- $f(x) > 0$ when $1 < x < 3, x > 5$



2.6 Practice

Match each equation form with one of the graphs

1. $f(x) = \frac{x-A}{x-B}$ 2. $g(x) = \frac{(x-A)^2}{x-B}$ 3. $h(x) = \frac{x-A}{(x-B)^2}$ 4. $k(x) = \frac{(x-A)^2}{(x-B)^2}$



Answer questions 5-12 in separate sheet of paper.

For each function, find the intercepts, the asymptotes and end behaviour of function. Use that information to sketch a graph.

5. $p(x) = \frac{2x-3}{x+4}$

6. $s(x) = \frac{4}{(x-2)^2}$

7. $f(x) = \frac{3x^2 - 14x - 5}{3x^2 + 8x - 16}$

8. $a(x) = \frac{x^2 + 2x - 3}{x^2 - 1}$

9. $h(x) = \frac{2x^2 + x - 1}{x - 4}$

10. $n(x) = \frac{3x^2 + 4x - 4}{x^3 - 4x^2}$

11. $w(x) = \frac{(x-1)(x+3)(x-5)}{(x+2)^2(x-4)}$

12. $m(x) = \frac{5-x}{2x^2 + 7x + 3}$

Write an equation for a rational function with the given characteristics.

13. Vertical asymptotes at $x=5$ and $x=-5$

x -intercepts at $(2,0)$ and $(-1,0)$

y -intercept at $(0,4)$

hole at $(1, 25/6)$

14. Vertical asymptotes at $x=-4$ and $x=-5$

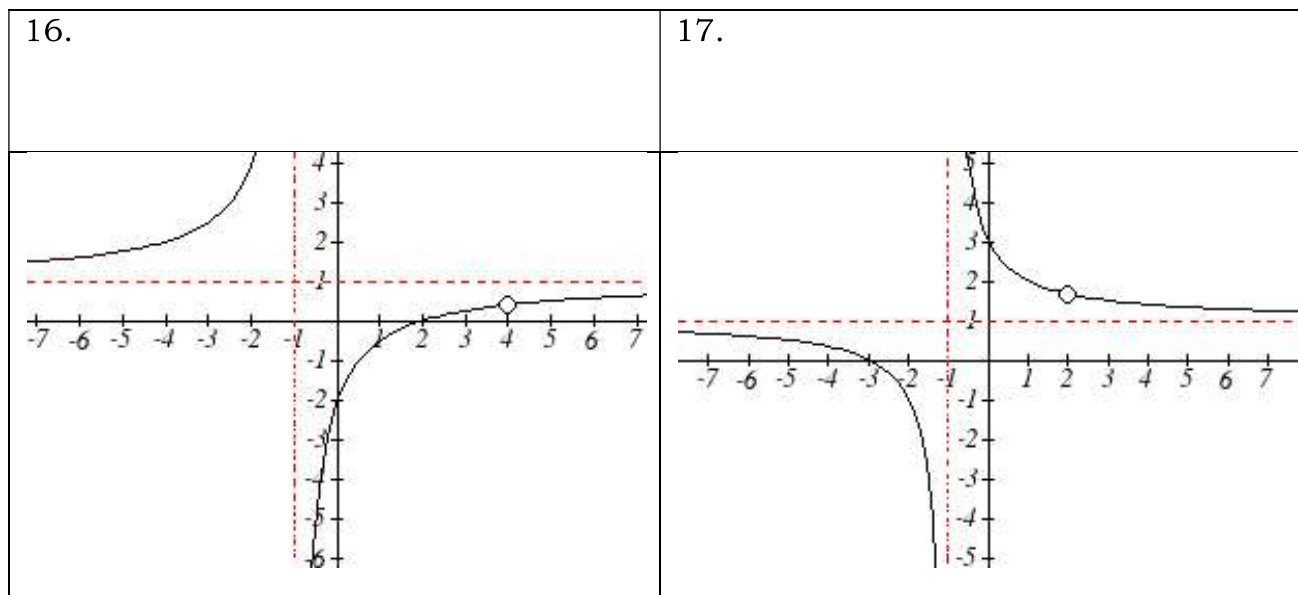
x -intercepts at $(4,0)$ and $(-6,0)$

Horizontal asymptote at $y=7$

15. Vertical asymptote at $x=-1$ Oblique asymptote : $y=x-5$

Double zero at $x=2$

y -intercept $(0,4)$



18. Estimate the slope of the tangent to the graph of $f(x) = \frac{x}{x^2 - 4}$ at the point where $x=3$. Explain why there cannot be a tangent line at $x=2$.

19. The concentration, C , of a drug in the bloodstream t hours after the drug was taken orally is given by $C(t) = \frac{5t}{7+t^2}$ where c is measured in milligrams per litre.

- (a) Calculate the average rate of change in the drug's concentration during the first 2 h since ingestion.
- (b) Estimate the rate at which the concentration of the drug is changing after exactly 3 h.

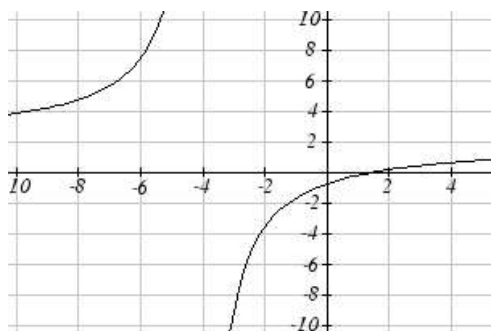
Answer to Practice Questions

1. D 2. B 3. A 4. C

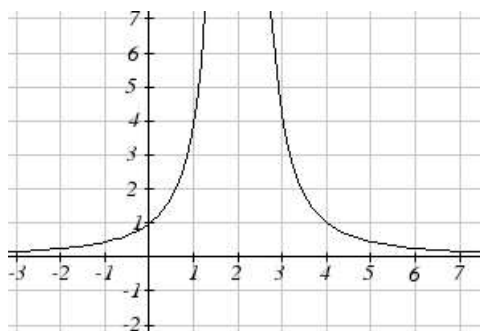
	Vertical Asymptotes	Horizontal Asymptote	y -Intercept	x -intercept
5.	$x = -4$	$y = 2$	$(0, -3/4)$	$(3/2, 0)$
6.	$x = 2$	$y = 0$	$(0, 1)$	DNE
7.	$x = -4, 1\frac{1}{3}$	$y = 1$	$(0, 5/16)$	$(-1/3, 0), (5, 0)$
8.	$x = -1$, hole at $x = 1$	$y = 1$	$(0, 3)$	$(-3, 0)$
9.	$x = 4$	none	$(0, 1/4)$	$(-1, 0), (1/2, 0)$

		$y=2x$ (oblique)		
10.	$x = 0, 4$	$y = 0$	DNE	$(-2, 0), (2/3, 0)$
11.	$x = -2, 4$	$y = 1$	$(0, 15/16)$	$(1, 0), (-3, 0), (5, 0)$
12.	$x = -3, \frac{-1}{2}$	$y = 0$	$(0, 5/3)$	$(5, 0)$

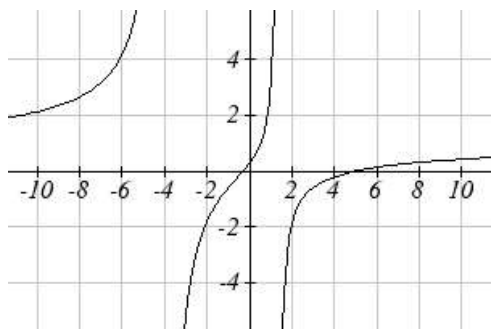
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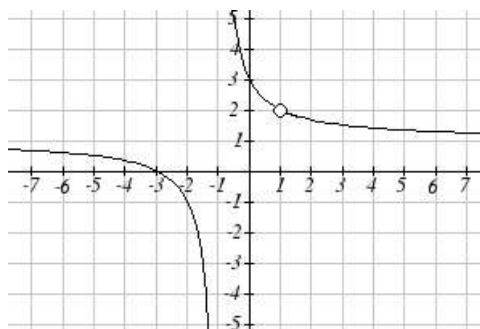
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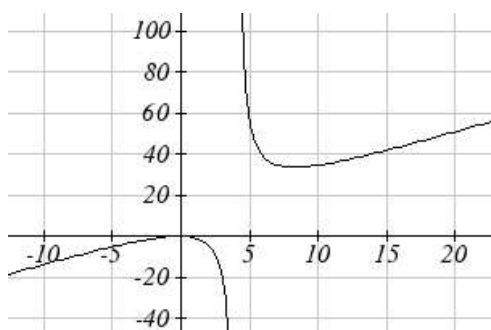
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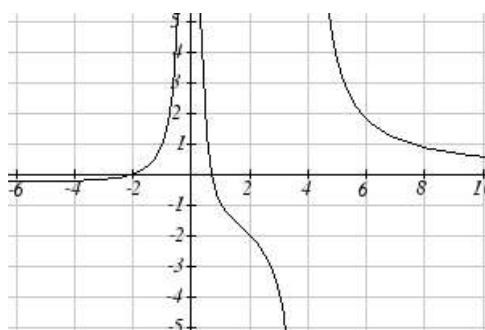
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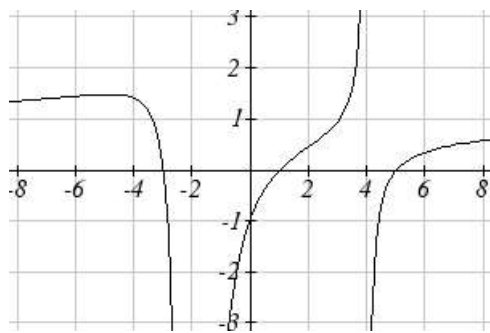
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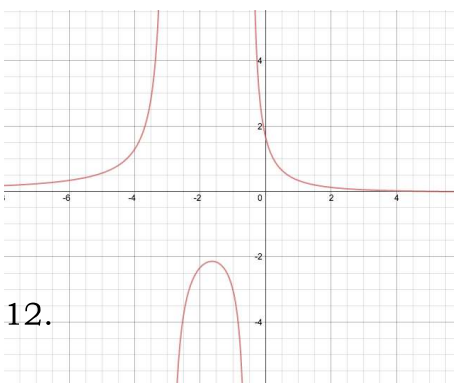
10.



11.



12.



$$13. \quad y = \frac{50(x-2)(x+1)(x-1)}{(x+5)(x-5)(x-1)}$$

$$14. \quad y = \frac{7(x-4)(x+6)}{(x+4)(x+5)}$$

$$15. \quad y = \frac{(x-2)^2}{x+1}$$

$$16. \quad y = \frac{(x-4)(x-2)}{(x-4)(x+1)}$$

$$17. \quad y = \frac{(x+3)(x-2)}{(x+1)(x-2)}$$

18 . slope of tangent at $x=3$ is -0.52 , slope of tangent at $x=2$ is undefined.

19. a) AROC when $t \in [0,2]$ is $5/11$.

b) IROC when $t=3$ is approximately -0.04 .

Warm up

1. Solve for $x \in \mathbb{R}$.

a) $\frac{x^2 + 2x - 15}{x^2 + 7x} > 0$

b) $\frac{x^2 + 5x - 10}{1 - x^2} \leq 2$

2. A closed-topped cylindrical tin can is to be made with a volume of $25\pi \text{ cm}^3$. Determine the values of the radius that will produce a surface area no more than the area of a letter sized piece of paper measuring $190\pi \text{ cm}^2$.

Unit 2-Review

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided.

1. Given the function $f(x) = \frac{x^a + k}{x^b + m}$, a linear oblique asymptote will occur when: _____
 A) $a \geq b$ B) $b > a$ C) $a - b = 1$ D) $a - b = 2$ E) none of the above

2. Which of the following statements is **true** if $f(x)$ is the reciprocal of a quadratic with x intercepts at $x = \pm 4$ and a vertex of $(0, 8)$? _____
 A) $f(x)$ has two vertical asymptotes B) $\frac{1}{f(x)}$ has a local maximum at $(0, \frac{1}{8})$
 C) $\frac{1}{f(x)}$ has a local minimum at $(0, \frac{-1}{8})$ D) $\frac{1}{f(x)} > 0$ when $x \in (-\infty, \infty)$

3. Given $f(x) = \frac{(6-2x)}{(x^2-4)(x-3)}$ which of the following is **true**? _____
 A) $f(x)$ crosses at least one of its asymptotes B) $f(x)$ has a hole at $(3, \frac{-2}{5})$
 C) $f(x)$ has a horizontal asymptote at $y = -2$ D) $f(x)$ has 3 vertical asymptotes

4. Which of the following functions has an asymptote that passes through the origin? _____
 A) $f(x) = \frac{x(x-4)(x-9)}{(x-8)(x-3)^2}$ B) $f(x) = \frac{(x^2-4)(2x-9)}{(x)(2x-3)}$, $x \neq -1$
 C) $f(x) = \frac{(x-6)(x+4)}{x^3-8}$ D) Both B and C

5. Which of the following function(s) cross at least one of their asymptotes? _____
 A) $f(x) = \frac{(x+1)(x+5)(2x-9)}{2x^2-7x-9}$ B) $f(x) = \frac{1}{x^2+16}$
 C) $f(x) = \frac{x(x-4)(x-9)}{x^3-14x^2+57x-72}$ D) All of the above

6. Complete the table below given the following function $f(x) = \frac{-x(x-3)(x-4)}{(2x-8)(x+2)(x+5)}$

x-intercept(s), if any.	
y-intercept, if any.	
Equation of vertical asymptote(s), if any.	
Equation of horizontal or oblique asymptote, if any.	

7. Determine the equation of the oblique asymptote given $f(x) = \frac{2x^2+9x-12}{x+4}$

8. Create the equation of a function $g(x)$ with the following properties:

B) x-intercept of $\frac{1}{4}$, y-intercept of $-\frac{1}{2}$, vertical asymptote of $x = \frac{-2}{3}$ and horizontal asymptote of $y = \frac{4}{3}$.

9. For the function, $g(x) = \frac{mx-3}{4-nx}$, find the values of m and n such that $g(x)$ has a vertical asymptote when $x = 6$ and a horizontal asymptote at $y = -3$.

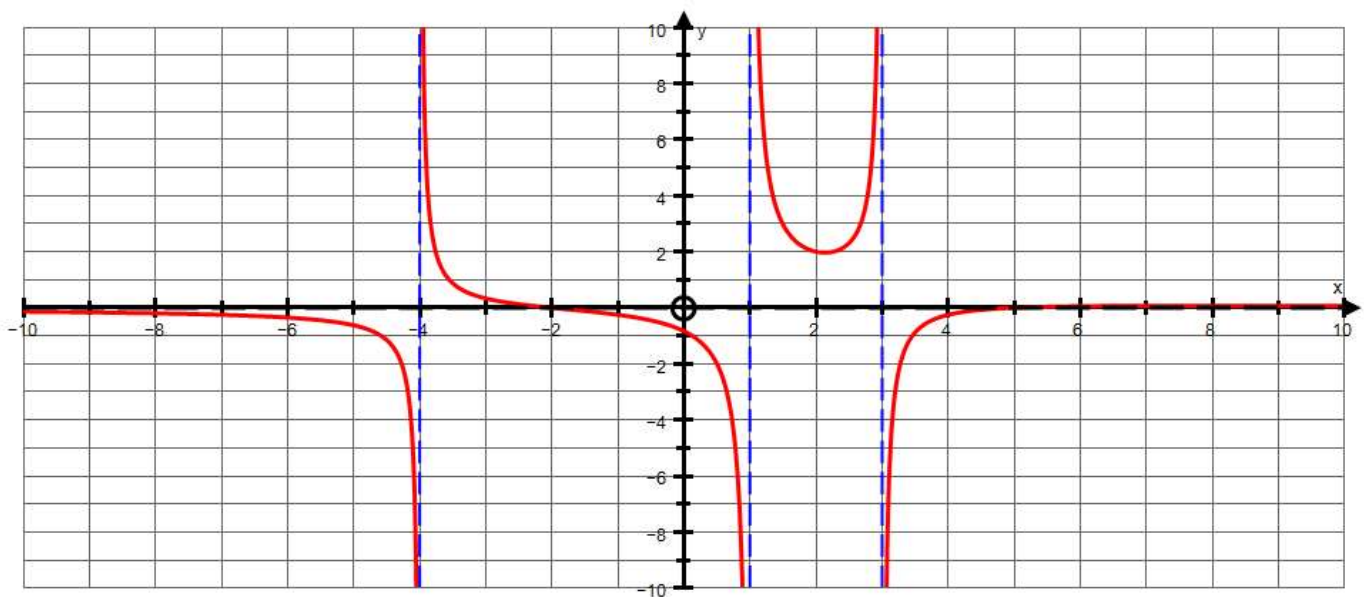
10. Sketch the graph of $f(x) = \frac{x^3 - x^2 - 4x + 4}{x^2 + x - 20}$

11. Determine any points of intersection for the function $f(x) = -x^2 - 5x - 6$ and its reciprocal function. Leave answer(s) in exact form where necessary.

12. Given the graph of the rational function $f(x)$ below solve $\frac{4x^2-20x}{f(x)} \geq 0$

Please note two things regarding $f(x)$:

- The y intercept is $(0, \frac{-5}{6})$
- $f(x)$ crosses its horizontal asymptote at $x = -2$ and $x = 5$
- The degree of the numerator of $f(x)$ is 2 and the degree of the denominator of $f(x)$ is 3.



13. Solve

$$a) \frac{x^3 + 7x^2 + 12x}{x^2 + 9x + 20} \geq \frac{x+1}{x^2 - 3x - 4} - \frac{2x+5}{2x^2 - 3x - 20}$$

$$b) \frac{x^3 + 2x^2 - 46x - 125}{(x^2 + 5x - 6)(x+3)} \geq \frac{-1}{(x+6)(x+3)}$$

14. Determine the perimeter of the quadrilateral, rounded to the nearest tenth of a unit, created by the intersections of $h(x)$ and its reciprocal graph $f(x)$. Given: $h(x) = -2(x+4)(x-6)$ and

$$f(x) = \frac{1}{-2x^2 + 4x + 48}.$$

15. Describe what is known about the equation of a rational function with vertical asymptotes at $x = 5$ and

$x = -3$ and a horizontal asymptote of $y = 0$.

16. The concentration of a toxic chemical in a spring-fed lake is given by the equation $C(x) = \frac{60x}{x^2 + 3x + 6}$, where C is given in grams per litre and x is the time in days. Find the instantaneous rate of change at 4 days.

17. Sketch the graph of $y = \frac{x^3 - 6x^2 + 32}{x^3 - x^2 - 4x + 4}$.

18. Use the information below to sketch the function.

- There is a horizontal asymptote at $f(x) = -2$.
- The Domain of the function is $D: \left\{ x \neq -3, x \neq 4, x \in \mathbb{R} \right\}$
- The range of the function is $R: \left\{ y > -8, y \in \mathbb{R} \right\}$
- $f(0) = \frac{28}{81}$
- There is a hole at $(4, \frac{9}{40})$
- One of the factors of the numerator is $(x^2 + 5x - 6)$
- $f(x) > 0$ when $-6 < x < -3$, $-3 < x < 1$, $1 < x < 4$, $4 < x < 7$
- $f(7) = 0$