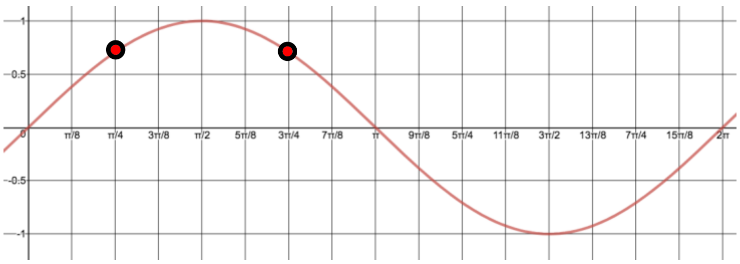
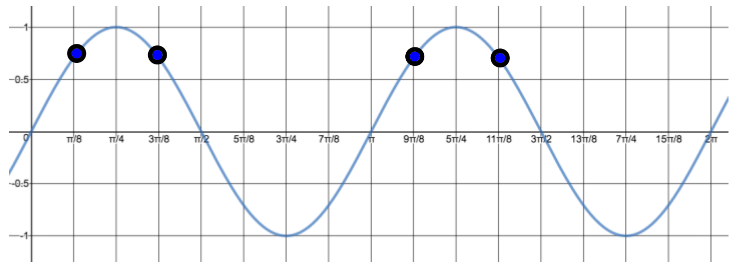
L6 – 5.4 Solve Double Angle Trigonometric Equations
MHF4U

Part 1: Investigation

$\boldsymbol{y = \sin x}$



$\boldsymbol{y = \sin(2x)}$



a) What is the period of both of the functions above? How many cycles between 0 and 2π radians?

For $y = \sin x$ → $period = 2\pi$

For $y = \sin(2x)$ → $period = \frac{2\pi}{2} = \pi$

b) Looking at the graph of $y = \sin x$, how many solutions are there for $\sin x = \frac{1}{\sqrt{2}} \approx 0.71$?

2 solutions

$$\sin\frac{\pi}{4} = \sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$$

c) Looking at the graph of $y = \sin(2x)$, how many solutions are there for $\sin(2x) = \frac{1}{\sqrt{2}} \approx 0.71$?

4 solutions

$$\sin\left[2\left(\frac{\pi}{8}\right)\right] = \sin\left[2\left(\frac{3\pi}{8}\right)\right] = \sin\left[2\left(\frac{9\pi}{8}\right)\right] = \sin\left[2\left(\frac{11\pi}{8}\right)\right] = \frac{1}{\sqrt{2}}$$

d) When the period of a function is cut in half, what does that do to the number of solutions between 0 and 2π radians?

Doubles the number of solutions

Part 2: Solve Linear Trigonometric Equations that Involve Double Angles

Example 1: $\sin(2\theta) = \frac{\sqrt{3}}{2}$ where $0 \leq \theta \leq 2\pi$

Let $2\theta = x$

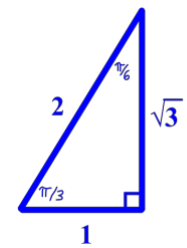
$\sin x = \frac{\sqrt{3}}{2}$

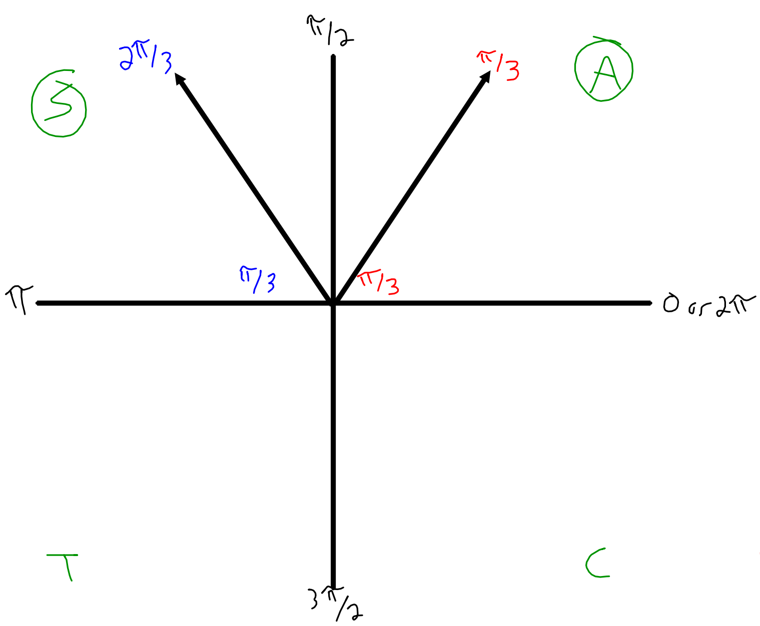
Graph → NO

△ → yes

$\sin \pi/3 = \frac{\sqrt{3}}{2}$

Put reference angle in Q1 + Q2 where sine is (+)





$x_1 = \pi/3$

$x_2 = \pi - \pi/3$

$x_2 = 2\pi/3$

$2\theta = x$

$2\theta = \pi/3$

$\theta_1 = \pi/6$

$2\theta = 2\pi/3$

$\theta_2 = 2\pi/6$

$\theta_2 = \pi/3$

$y = \sin(2\theta)$ has a period of π; add π to θ_1 and θ_2 to find other angles $0 \leq \theta \leq 2\pi$ that have equivalent ratios

$\theta_3 = \theta_1 + \pi$

$= \pi/6 + \pi$

$= \frac{7\pi}{6}$

$\theta_4 = \theta_2 + \pi$

$\theta_4 = \pi/3 + \pi$

$\theta_4 = \frac{4\pi}{3}$

$$\boxed{\sin\left[2\left(\tfrac{\pi}{6}\right)\right] = \sin\left[2\left(\tfrac{\pi}{3}\right)\right] = \sin\left[2\left(\tfrac{7\pi}{6}\right)\right] = \sin\left[2\left(\tfrac{4\pi}{3}\right)\right] = \frac{\sqrt{3}}{2}}$$

Example 2: $\cos(2\theta) = -\frac{1}{2}$ where $0 \le \theta \le 2\pi$

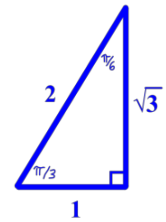
Let $2\theta = x$

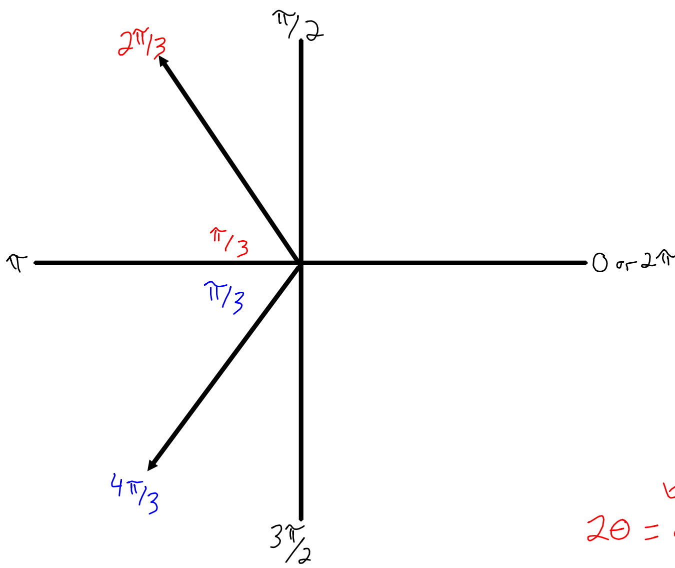
$\cos x = -\frac{1}{2} \rightarrow$ Graph $\rightarrow$ Δ

NO yes

$\cos \pi/3 = \frac{1}{2}$

Put reference angle in Q2 + Q3 where cosine is $\ominus$





$x_1 = \pi - \pi/3$

$x_1 = \frac{2\pi}{3}$

$x_2 = \pi + \pi/3$

$x_2 = \frac{4\pi}{3}$

$2\theta = x$

$2\theta = \frac{2\pi}{3}$

$\theta_1 = \frac{2\pi}{6}$

$\theta_1 = \frac{\pi}{3}$

$2\theta = \frac{4\pi}{3}$

$\theta_2 = \frac{4\pi}{6}$

$\theta_2 = \frac{2\pi}{3}$

Remember that $\cos(2\theta)$ has a period of π; add π to θ_1 and θ_2 to find other solutions $0 \le \theta \le 2\pi$

$\theta_3 = \theta_1 + \pi$

$\theta_3 = \frac{\pi}{3} + \pi$

$\theta_3 = \frac{4\pi}{3}$

$\theta_4 = \theta_2 + \pi$

$\theta_4 = \frac{2\pi}{3} + \pi$

$\theta_4 = \frac{5\pi}{3}$

$$\boxed{\cos\left[2\left(\frac{\pi}{3}\right)\right] = \cos\left[2\left(\frac{2\pi}{3}\right)\right] = \cos\left[2\left(\frac{4\pi}{3}\right)\right] = \cos\left[2\left(\frac{5\pi}{3}\right)\right] = -\frac{1}{2}}$$

Example 3: $\tan(2\theta) = 1$ where $0 \le \theta \le 2\pi$

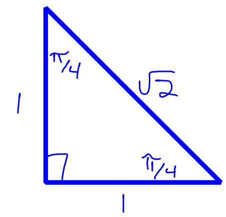
Let $2\theta = x$

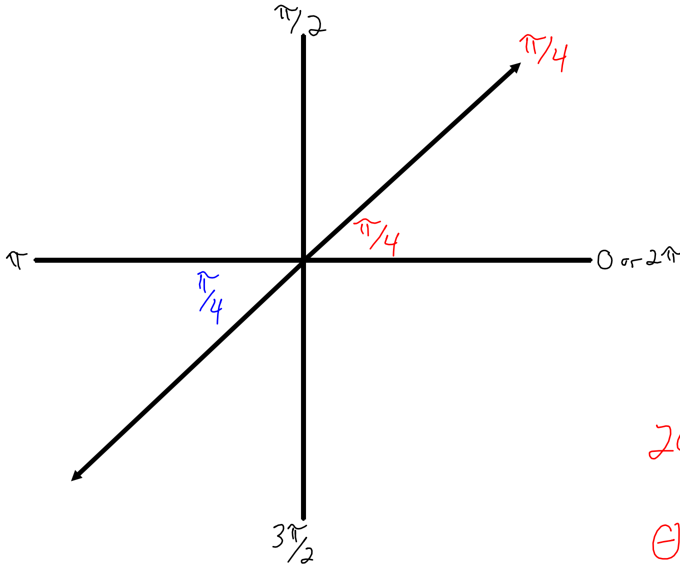
$\tan x = 1$ → Graph → Δ

NO yes

$\tan\frac{\pi}{4} = 1$

Put reference angle in Q1 + Q3 where tangent is (+)





$x_1 = \frac{\pi}{4}$

$x_2 = \pi + \frac{\pi}{4}$

$x_2 = \frac{5\pi}{4}$

$2\theta = x$

$2\theta = \frac{\pi}{4}$ $\qquad$ $2\theta = \frac{5\pi}{4}$

$\theta_1 = \frac{\pi}{8}$ $\qquad$ $\theta_2 = \frac{5\pi}{8}$

Remember that $\tan(2\theta)$ has a period of $\frac{\pi}{2}$; add $\frac{\pi}{2}$ to θ_1 and θ_2 to find other solutions $0 \le \theta \le 2\pi$

$$\theta_3 = \theta_2 + \frac{\pi}{2} = \frac{5\pi}{8} + \frac{4\pi}{8} = \frac{9\pi}{8}$$

$$\theta_4 = \theta_3 + \frac{\pi}{2} = \frac{9\pi}{8} + \frac{4\pi}{8} = \frac{13\pi}{8}$$

$$\boxed{\tan\left[2\left(\frac{\pi}{8}\right)\right] + \tan\left[2\left(\frac{5\pi}{8}\right)\right] + \tan\left[2\left(\frac{9\pi}{8}\right)\right] + \tan\left[2\left(\frac{13\pi}{8}\right)\right] = 1}$$