6.5 Solving Inequalities of Combined Functions

Recall: An **inequality** is a mathematical statement that contains one of the following symbols:

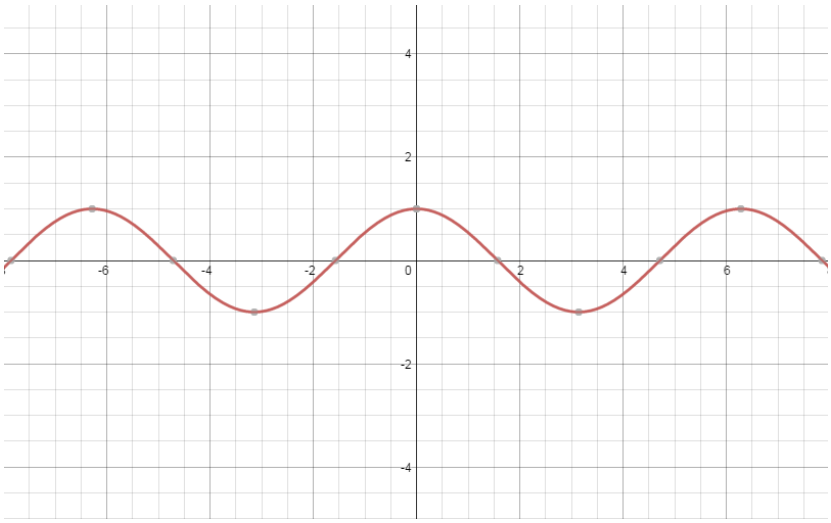
$$>, \geq, <, \leq, \neq$$

There are two main strategies used to solve an inequality involving $f(x)$ and $g(x)$

1. Inequality related to **Zeros**
   - The **chart method**
     - Used whenever you can factor $f(x) - g(x)$ or $g(x) - f(x)$
2. Inequality related to **Points of Intersection**
   - The **graphing method**
     - Used whenever you cannot factor the above but you can still graph $f(x)$ and $g(x)$

Ex.1: Given $f(x) = x^3 - 2x^2 + 5x + 20$ and $g(x) = 2x^2 + 14x - 16$. Solve $f(x) \geq g(x)$.

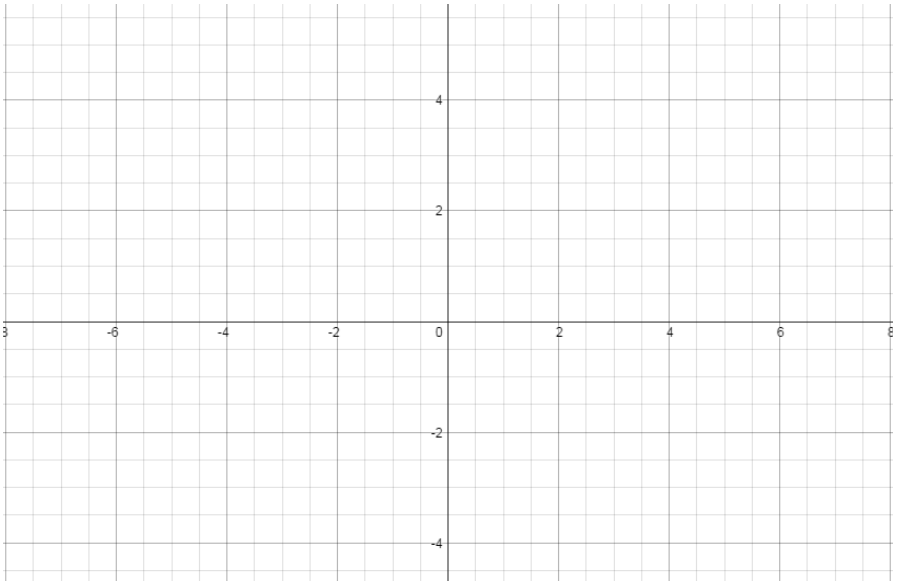
Ex.2: Given $f(x) = \cos(x)$ and $g(x) = x$. Determine all values of x such that $f(x) > g(x)$.



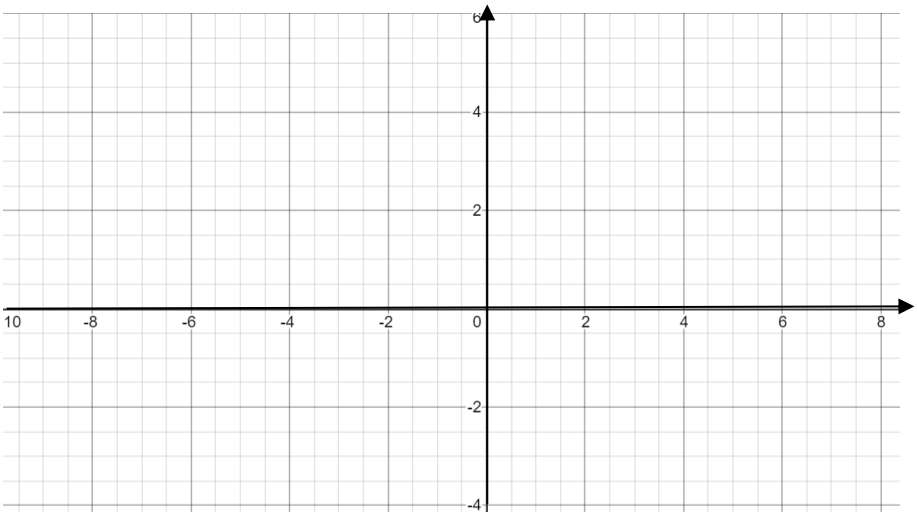
Ex.3: Given $f(x)=\frac{3x}{x+8}$ and $g(x)=\frac{x}{x-6}$, determine all values of x such that $f(x) < g(x)$.

Ex.4: Given $f(x) = x^2 + 4x - 5$ and $g(x) = x + 1$. Determine all values of x such that $f(x) > g(x)$.

Ex.5: Given $f(x) = \log_2(x^2)$ and $g(x)=\frac{1}{x^2}$ determine all values of x such that $f(x) > g(x)$.



Ex.6: Given $f(x) = \sqrt{2-x}$ and $g(x) = 2\sqrt{x} - 1$, sketch the graphs of f and g on the same set of axes. Using the graphs of these functions, solve $f(x) > g(x)$.



Group work

Multiple Choice: Write the CAPITAL LETTER corresponding to the most correct answer on the line provided.

1. If $\left(\frac{m}{n}\right)(x)=\frac{x+3}{2x+1}$ and $n(x)=2x^2-5x-3$, the function that defines $m(x)$ is: _____

   A) $m(x)=x^2+6x-9$ B) $m(x)=x+3$ C) $m(x)=x^2-9$
   D) $m(x)=x-3$ E) None of the above

2. If $D_f=\{x\mid x\in\mathbb{R}\}$ and $D_g=\{x\mid x\geq 0, x\in\mathbb{R}\}$, then which of the following is correct? _____

   A) $D_{f+g}=\{x\mid x\in\mathbb{R}\}$ B) $D_{f+g}=\{x\mid x\geq 0, x\in\mathbb{R}\}$ C) $D_{fg}=\{x\mid x\geq 0, x\in\mathbb{R}\}$ D) $D_{f-g}=\{x\mid x\in\mathbb{R}\}$ E) Both B and C

3. If $f(x)=2x-2$, $g(x)=x+4$ and $h(x)=f(x)-g(x)$ then which of the following statements is true? _____

   A) The y-intercept of $h(x)$ is 2 B) $f(x)$, $g(x)$ and $h(x)$ all intersect at the same point.
   C) $h(x)=f\left(f^{-1}(x)\right)-g(-10)$ D) $h(3)=-3$ E) Both C and D

4. Given $f(x)=3x^2$ and $g(x)=3x^4-x^2-5$, which of the following statements are true? _____

   A) $\left(\frac{g}{f}\right)(x)$ is quadratic B) $(f-g)(x)$ is quadratic C) $(f+g)(x)$ is cubic
   D) $(fog)(x)$ is an even function E) $(f-g)(x)$ is an odd function

Short Answer

5. Given $f(x)=11x^5$ and $g(x)=22x^2\log(x)$ determine :

   a) the domain of $\left(\frac{f}{g}\right)(x)$ b) equation of $\left(\frac{f}{g}\right)(x)$

6. Given the following functions, determine the following in simplest form:
   $f(x)=\{(1,2),(2,3),(3,5),(9,7)\}$ $g(x)=\{(1,4),(3,7),(5,9),(9,2)\}$
   $h(x)=\sqrt{x-4}$ $m(x)=\sqrt{x-3}$

   a) $(f+g)(x)$ b) $(fg)(3)$

   c) D_{h+m} d) $D_{\frac{h}{m}}$

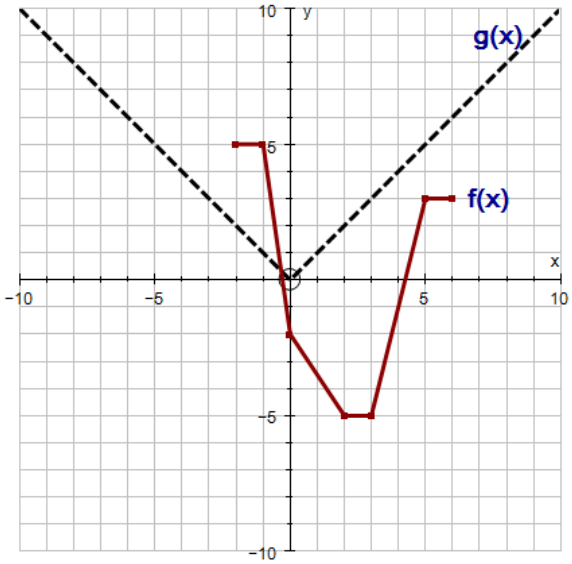
7. Given the functions $f(x)=2x^2-4x+3$ and $g(x)=3x^2-6x-4$ then determine the range of $(f-g)(x)$.

8. Determine two functions, f and g, whose product would result in $(fg)(x)=8\sin^3 x+27$.

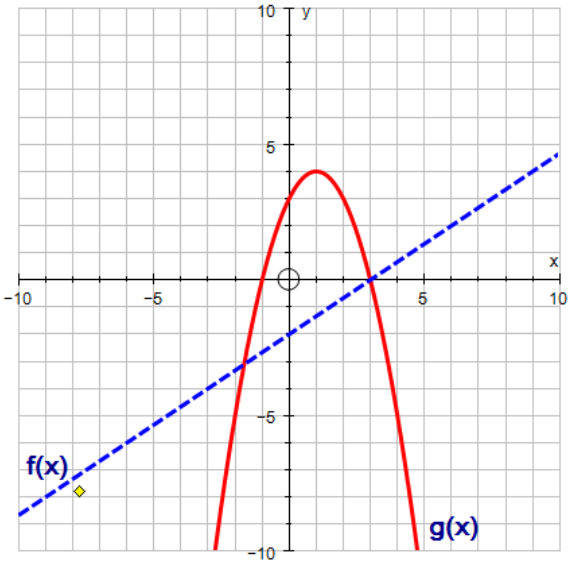
9. Given $f(x)=9x^4+6x^3+270$ and $g(x)=9x^4+51x^2+57x$ solve $f(x)\geq g(x)$.

10. Given the graphs below, sketch each of the following on the same set of axes provided.

a. $(f+g)(x)$



b. $\left(\frac{g}{f}\right)(x)$



11. If $f(x)=2x+3$ and $k(x)=2x^2+6x+9$, find $g(x)$ so that $(f\circ g)(x)=k(x)$.

12. Complete the table below.

Point on f(x)	Point on g(x)	Point on $(f\circ g)(x)$
	(3,4)	(3,1)
(1,5)	(2,1)	
(0,-1)	(1,__)	
(__,5)	(__,1)	(0,__)
(3,__)		(-1,3)

13. Using the functions listed below sketch the graph of $R(x)=\left(\frac{f}{g}\cdot h\cdot P\right)(x)$.

$f(x)=x^2+3x-40$ $\quad$ $g(x)=-3x^2+12x+15$

$h(x)=3x^3+3x^2$ $\quad$ $K(x)=2x$

$M(x)=4x+4$ $\quad$ $P(x)=(M\circ K)(x)$

14. Given the information below determine the range of $(f+g)(x)$.

$f(x)=2(x+7)^2-4$ $\quad$ $D_f=\{x|x\leq -9, x\in\mathbb{R}\}$

$g(x)=x+5$ $\quad$ $D_g=\{x|x\geq -12, x\in\mathbb{Z}\}$