W2 – 4.4 Compound Angle Formulas
MHF4U

SOLUTIONS

1) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}+\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{12}\right)$

$= \sin\left(\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3}\right)$

b) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}-\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{4}-\frac{\pi}{12}\right)$

$= \sin\left(\frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{6}\right)$

c) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3}\right)$

d) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}+\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{4}-\frac{\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{6}\right)$

$\cos\frac{2\pi}{9}\cos\frac{5\pi}{18}-\sin\frac{2\pi}{9}\sin\frac{5\pi}{18}$

$= \cos\left(\frac{2\pi}{9}+\frac{5\pi}{18}\right)$

$= \cos\left(\frac{9\pi}{18}\right)$

$= \cos\left(\frac{\pi}{2}\right)$

f) $\cos\frac{10\pi}{9}\cos\frac{5\pi}{18}+\sin\frac{10\pi}{9}\sin\frac{5\pi}{18}$

$= \cos\left(\frac{10\pi}{9}-\frac{5\pi}{18}\right)$

$= \cos\left(\frac{15\pi}{18}\right)$

$= \cos\left(\frac{5\pi}{6}\right)$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4}+\cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

b) $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)-\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

c) $\cos\left(\frac{2\pi}{3}-\frac{\pi}{4}\right) = \cos\frac{2\pi}{3}\cos\frac{\pi}{4}+\sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)+\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1+\sqrt{3}}{2\sqrt{2}}$

d) $\sin\left(\frac{2\pi}{3}-\frac{\pi}{4}\right) = \sin\frac{2\pi}{3}\cos\frac{\pi}{4}-\cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4}-\left(-\cos\frac{\pi}{3}\right)\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

e) $\tan\left(\frac{\pi}{4}+\pi\right) = \dfrac{\tan\frac{\pi}{4}+\tan\pi}{1-\tan\frac{\pi}{4}\tan\pi}$

$= \dfrac{1+0}{1-1(0)}$

$= \dfrac{1}{1}$

$= 1$

f) $\tan\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$

$= \dfrac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)}$

$= \dfrac{\frac{3}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{1+1}$

$= \dfrac{\left(\frac{2}{\sqrt{3}}\right)}{2}$

$= \dfrac{1}{\sqrt{3}}\dfrac{(\sqrt{3})}{(\sqrt{3})}$

$= \dfrac{\sqrt{3}}{3}$

4) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3}+\cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}}$

b) $\sin\frac{5\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6}+\cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

c) $\cos\frac{11\pi}{12}$

$= \cos\left(\frac{3\pi}{12}+\frac{8\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4}+\frac{2\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{2\pi}{3}-\sin\frac{\pi}{4}\sin\frac{2\pi}{3}$

$= \frac{1}{\sqrt{2}}\left(\frac{-1}{2}\right)-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{-1-\sqrt{3}}{2\sqrt{2}}$

d) $\cos\frac{5\pi}{12} = \cos\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)-\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

e) $\sin\frac{13\pi}{12} = \sin\left(\frac{4\pi}{12}+\frac{9\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{3\pi}{4}+\cos\frac{\pi}{3}\sin\frac{3\pi}{4}$

$= \frac{\sqrt{3}}{2}\left(-\frac{1}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \dfrac{-\sqrt{3}+1}{2\sqrt{2}}$

f) $\cos\frac{17\pi}{12} = \cos\left(\frac{9\pi}{12}+\frac{8\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4}+\frac{2\pi}{3}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{2\pi}{3}-\sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$

$= \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{1-\sqrt{3}}{2\sqrt{2}}$

g) $\sin\frac{19\pi}{12}$ $= \sin\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right)$

$= \sin\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$

$= \sin\frac{5\pi}{6}\cos\frac{3\pi}{4} + \cos\frac{5\pi}{6}\sin\frac{3\pi}{4}$

$= \frac{1}{2}\left(\frac{-1}{\sqrt{2}}\right) + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$

h) $\cos\frac{23\pi}{12}$ $= \cos\left(\frac{8\pi}{12} + \frac{15\pi}{12}\right)$

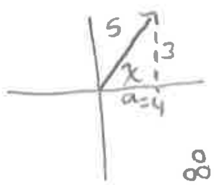
$= \cos\left(\frac{2\pi}{3} + \frac{5\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{5\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{5\pi}{4}$

$= \left(\frac{-1}{2}\right)\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{\sqrt{2}}\right)$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

5) Angles x and y are located in the first quadrant such that $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$. Determine exact values for $\cos x$ and $\sin y$.

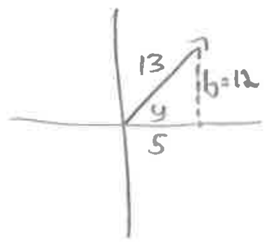


$a^2 + 3^2 = 5^2$

$a^2 = 16$

$a = 4$

$\therefore \cos x = \frac{4}{5}$



$5^2 + b^2 = 13^2$

$b^2 = 144$

$b = 12$

$\therefore \sin y = \frac{12}{13}$

6) Refer to the previous question. Determine an exact value for each of the following.

a) $\sin(x + y)$

$= \sin x\cos y + \cos x\sin y$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{3}{13} + \frac{48}{65}$

$= \frac{63}{65}$

b) $\sin(x - y)$ $= \sin x\cos y - \cos x\sin y$

$= \frac{3}{13} - \frac{48}{65}$

$= \frac{-33}{65}$

c) $\cos(x + y)$

$= \cos x\cos y - \sin x\sin y$

$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \frac{3}{5}\left(\frac{12}{13}\right)$

$= \frac{4}{13} - \frac{36}{65}$

$= \frac{-16}{65}$

d) $\cos(x - y)$

$= \cos x\cos y + \sin x\sin y$

$= \frac{4}{13} + \frac{36}{65}$

$= \frac{56}{65}$

7) Use a compound angle formula to show that $\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(2x) = \cos(x + x)$

$= \cos x \cdot \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$