SOLUTIONS

Section 1: 1.1 Power Functions

1) State the degree and the leading coefficient of each polynomial

Polynomial	Degree	Leading Coefficient
$y = 2x^3 + 3x - 1$	3	2
$y = 5x - 6$	1	5
$y = x^3 - 2x^2 - 5x^4 + 3$	4	-5
$y = -3x^5 + 2x^3 - x - 1$	5	-3
$y = 21 - 2x + 4x^2 - 6x^3$	3	-6

2) Match each function to its end behavior
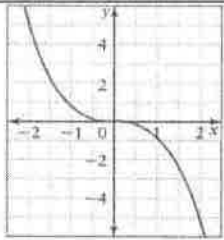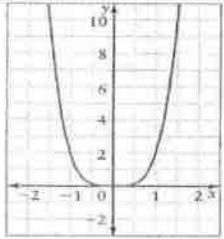
$$y = 3x^7 \qquad y = -\frac{1}{2}x^3 \qquad y = 2x^4 \qquad y = -0.25x^6$$

End Behaviour	Functions
Q3 to Q1	$y = 3x^7$
Q2 to Q4	$y = -\frac{1}{2}x^3$
Q2 to Q1	$y = 2x^4$
Q3 to Q4	$y = -0.25x^6$

3) Complete the following table

| Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	Odd	−	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$	Point about the origin	Q2 → Q4
	Even	+	D: $(-\infty, \infty)$ R: $[0, \infty)$	Line about the y-axis	Q2 → Q1

Section 2: 1.2 Characteristics of Polynomial Functions

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

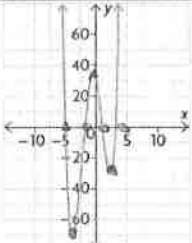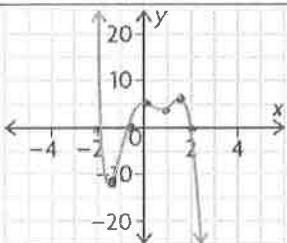**A)** $g(x) = 0.5x^4 - 3x^2 + 5x$ **B)** $h(x) = x^5 - 7x^3 + 2x - 3$ **C)** $p(x) = -x^6 + 5x^3 + 4$

B   C   A
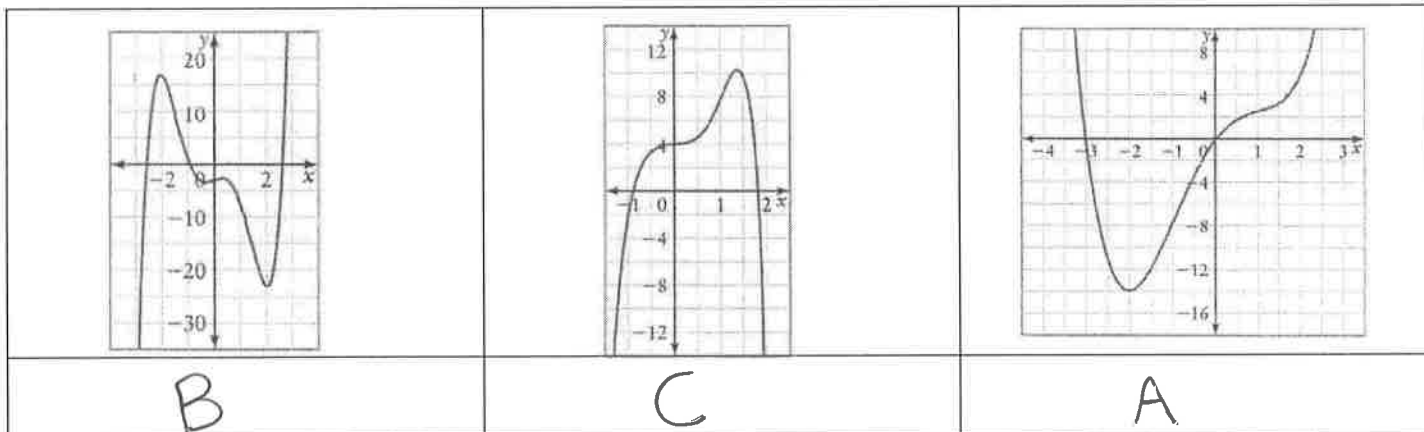
5) Complete the following table

Equation	Degree	Sign of Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = 6x^3 + 2x$	3	+	Q3→Q1	2, 0	3,2,1
$g(x) = -20x^6 - 5x^3 + x^2 - 17$	6	−	Q3→Q4	5,3,1	6,5,4,3,2,1,0
$p(x) = 22x^4 - 4x^3 + 3x^2 - 2x + 2$	4	+	Q2→Q1	3,1	4,3,2,1,0
$h(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$	5	−	Q2→Q4	4,2,0	5,4,3,2,1

6) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q2→Q1	None	3	4	4
	−	Odd	Q2→Q4	None	4	3	5

7) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) fifth differences = -60   Degree 5

$$-60 = a(5!)$$
$$-60 = 120a$$
$$\boxed{a = -\frac{1}{2}}$$

b) third differences = 42   Degree 3

$$42 = a(3!)$$
$$42 = 6a$$
$$\boxed{a = 7}$$

8) For each function, find the value of the constant finite differences.

a) $g(x) = 0.5x^4 - 3x^2 + 5x$

finite Differences $= 0.5(4!)$
$$= 0.5(24)$$
$$\boxed{= 12}$$

b) $h(x) = x^5 - 7x^3 + 2x - 3$

Finite Differences $= 1(5!)$
$$\boxed{= 120}$$

9) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
-3	124
-2	41
-1	8
0	1
1	-4
2	-31
3	-104
4	-247

1st: -83, -33, -7, -5, -27, -73, -143
2nd: 50, 26, 2, -22, -46, -70
3rd: -24, -24, -24, -24, -24

Degree = 3

$$-24 = a(3!)$$
$$-24 = 6a$$
$$\boxed{a = -4}$$

b)

x	y
-2	-229
-1	-5
0	3
1	-7
2	-53
3	-129
4	35
5	1213

1st: 224, 8, -10, -46, -76, 164, 1178
2nd: -216, -18, -36, -30, 240, 1014
3rd: 198, -18, 6, 270, 774
4th: -216, 24, 264, 504
5th: 240, 240, 240, 240

Degree = 5

$$240 = a(5!)$$
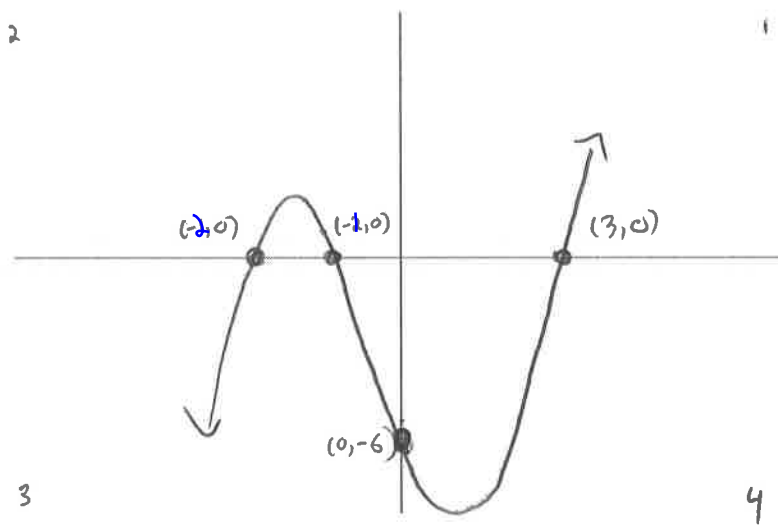$$240 = 120a$$
$$\boxed{a = 2}$$

Section 3: 1.3 Factored Form Polynomial Functions

10) For each function, complete the chart and sketch a possible graph of the function labelling key points.
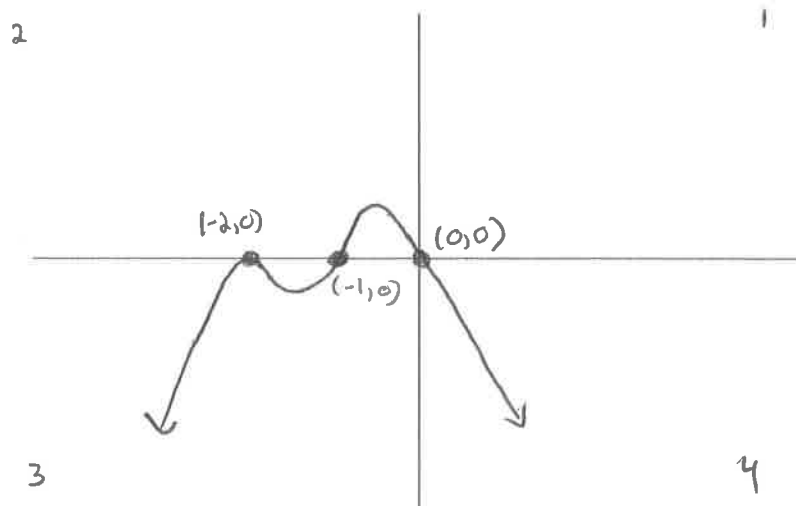
a) $f(x) = (x+1)(x-3)(x+2)$

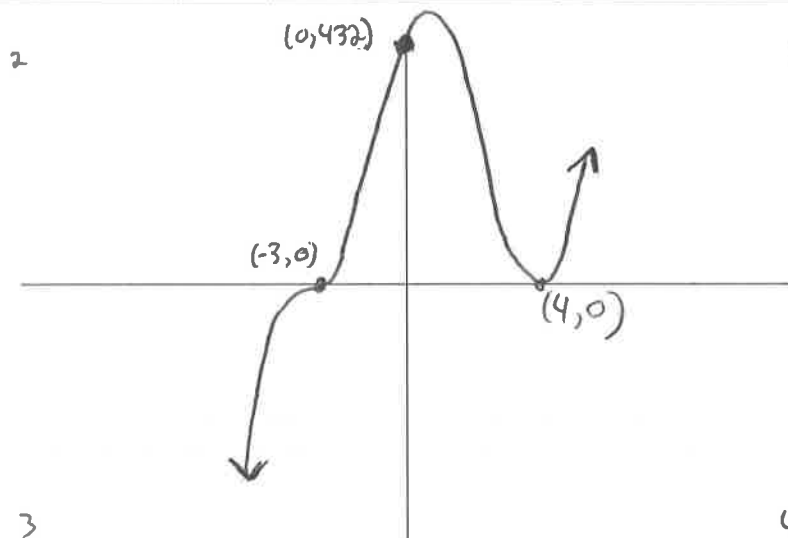| Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$(1)(1)(1)$ $= 1$	Q3 → Q1	$(-1,0)$ $(3,0)$ $(-2,0)$	$f(0) = (0+1)(0-3)(0+2)$ $= (1)(-3)(2)$ $= -6$

2

$(-2,0)$ $(-1,0)$ $(3,0)$

$(0,-6)$

3   4

b) $g(x) = -x(x+1)(x+2)^2$

| Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x^2)$ $= x^4$ Degree 4	$-1(1)(1)^2$ $= -1$	Q3 → Q4	$(0,0)$ $(-1,0)$ $(-2,0)$ order 2	$g(0) = -(0)(0+1)(0+2)^2$ $= -(0)(1)(4)$ $= 0$

2

$(-2,0)$ $(0,0)$

$(-1,0)$

3   4

c) $h(x) = (x-4)^2(x+3)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x^2)(x^3)$ $= x^5$ Degree 5	$(1)^2(1)^3$ $= 1$	Q3 → Q1	$(4,0)$ order 2 $(-3,0)$ order 3	$h(0) = (0-4)^2(0+3)^3$ $= (16)(27)$ $= 432$

2

$(0,432)$

$(-3,0)$

$(4,0)$

3 4

d) $p(x) = -4(2x+5)(x-2)(x+4)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$-4(2)(1)(1)$ $= -8$	Q2 → Q4	$(-\frac{5}{2},0)$ $(2,0)$ $(-4,0)$	$p(0) = -4[2(0)+5](0-2)(0+4)$ $= -4(5)(-2)(4)$ $= 160$

2

$(0,160)$

$(-2.5,0)$ $(2,0)$

$(-4,0)$

3 4
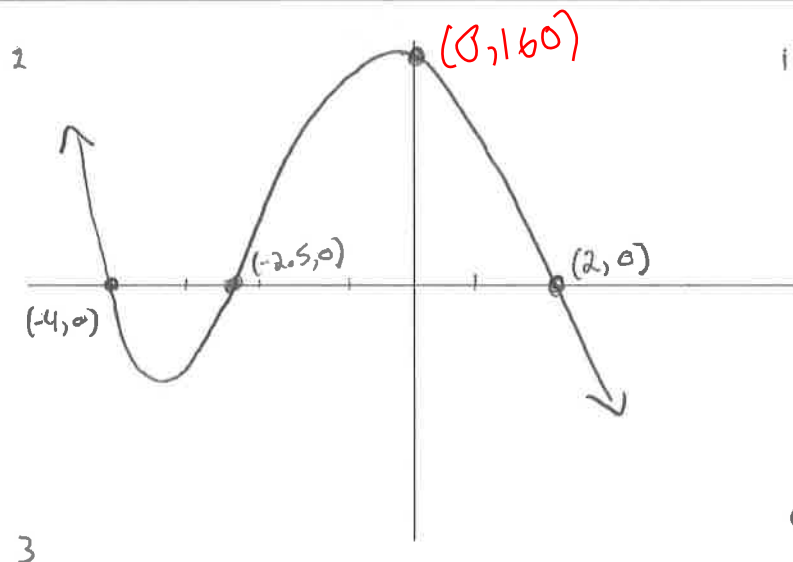
11) For each graph, state...

    i)   the least possible degree and the sign of the leading coefficient

    ii)   the x-intercepts (specify order of zero) and the factors of the function

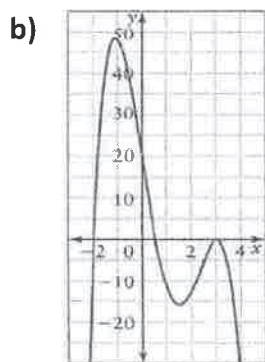   **iii)**   the intervals where the function is positive/negative

a)
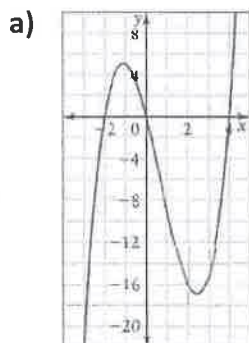
i) degree: 3

    leading coefficient: POSITIVE

ii) x-intercepts: $-2, 0, 4$

    factors: $(x+2)$, x, $(x-4)$

iii)

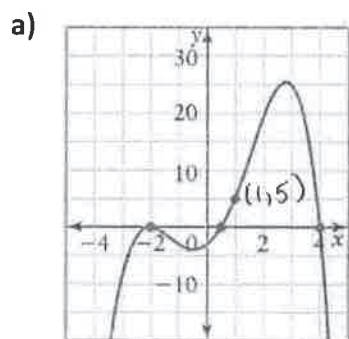| Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 4)$	$(4, \infty)$
Sign	$-$	$+$	$-$	$+$

b)

i) degree: 4

    leading coefficient: NEGATIVE

ii) x-intercepts: -2, $\frac{1}{2}$, 3 (order 2)

    factors: $(x+2)$, $(2x-1)$, $(x-3)^2$

iii)

Interval	$(-\infty, -2)$	$(-2, 0.5)$	$(0.5, 3)$	$(3, \infty)$
Sign	$-$	$+$	$-$	$-$

12) Write the equation of each of the following functions:

a)

$(1,5)$

$$f(x) = K(x+2)^2(2x-1)(x-4)$$
$$5 = K(1+2)^2[2(1)-1](1-4)$$
$$5 = K(9)(1)(-3)$$
$$5 = -27K$$
$$\text{zeros} \quad K = \frac{-5}{27}$$

$$\boxed{f(x) = -\frac{5}{27}(x+2)^2(2x-1)(x-4)}$$

b) The quartic function has at -3, -1, and 2 (order 2) and passes through the point (1, 4)

$$g(x) = K(x+3)(x+1)(x-2)^2$$
$$4 = K(1+3)(1+1)(1-2)^2$$
$$4 = K(4)(2)(1)$$
$$4 = 8K$$
$$K = \frac{1}{2}$$

$$\boxed{g(x) = \frac{1}{2}(x+3)(x+1)(x-2)^2}$$

Section 4: 1.4 Transformations of Polynomial Functions

12) Write an equation for the function that results from the given transformations.

a) The function $f(x) = x^4$ is compressed vertically by a factor of $\frac{3}{5}$, stretched horizontally by a factor of 2, reflected horizontally in the y-axis, and translated 1 unit up and 4 units to the left.

$$g(x) = \frac{3}{5}\left[-\frac{1}{2}(x+4)\right]^4 + 1$$

b) The function $f(x) = x^3$ is compressed horizontally by a factor of $\frac{1}{4}$, stretched vertically by a factor of 5, reflected vertically in the x-axis, and translated 2 units to the left and 7 units up.

$$g(x) = -5\left[4(x+2)\right]^3 + 7$$

13) Identify the a, k, d and c values and explain what transformation is occurring to the parent function for $g(x) = 2[-4(x+7)]^4 - 1$
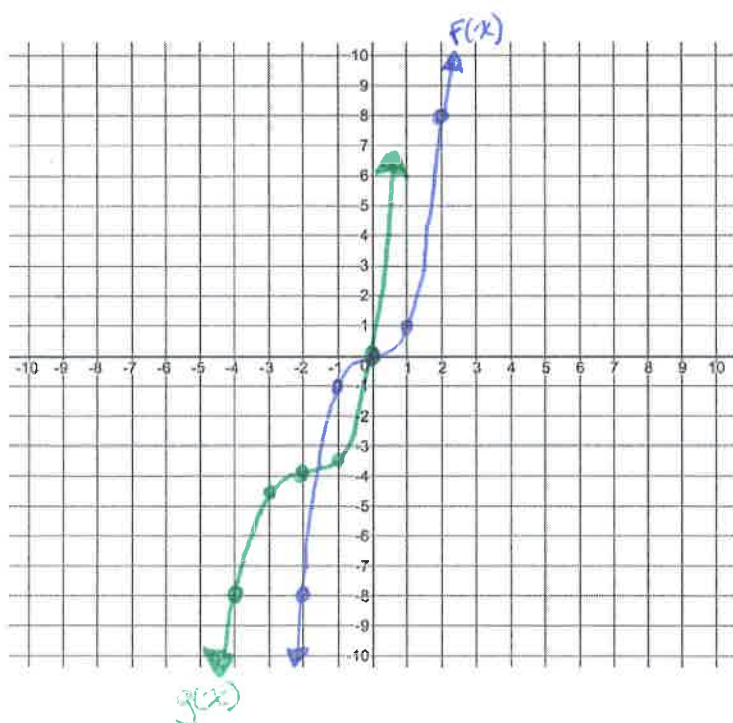
$a = 2$; vertical stretch by a factor of 2 $(2y)$

$k = -4$; horizontal reflection and horizontal compression bafo $\frac{1}{4}$ $\left(\frac{x}{-4}\right)$

$d = -7$; shift left 7 units $(x-7)$

$c = -1$; shift down 1 unit. $(y-1)$

14) For the following questions, use the key points of the parent function to perform transformations. Graph the parent and transformed function. Write the equation of the transformed function.

a) $f(x) = x^3 \qquad g(x) = \frac{1}{2}f(x+2) - 4$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

$g(x)$

$x-2$	$\frac{y}{2}-4$
-4	-8
-3	-4.5
-2	-4
-1	-3.5
0	0

$Eq^n : g(x) = \frac{1}{2}(x+2)^3 - 4$

b) $f(x) = x^4$ $\qquad$ $g(x) = -f\left[\frac{1}{2}(x-1)\right] + 7$

$f(x)$

$g(x)$
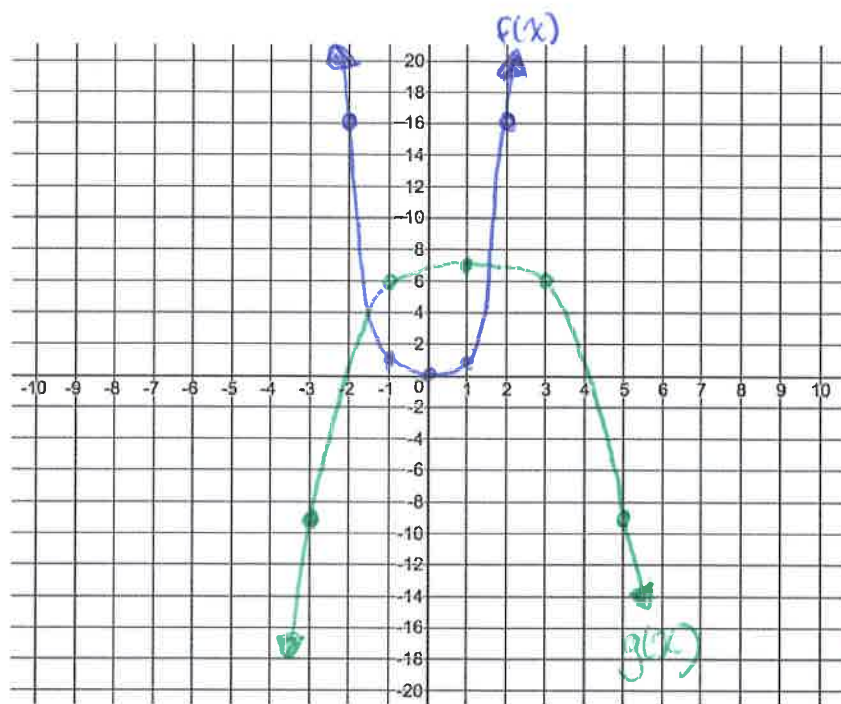
$F(x)$

x	y
-2	16
-1	1
0	0
1	1
2	16

$g(x)$

$2x+1$	$-y+7$
-3	-9
-1	6
1	7
3	6
5	-9

$Eq^n: -\left[\frac{1}{2}(x-1)\right]^4 + 7$

Section 5: 1.5 Symmetry

`5) Circle all that apply for each function

a)

No symmetry
Even function
Odd function
~~Line Symmetry~~ (circled)
Point Symmetry

d)

$f(x) = 3x^6 + 2x^2 - 5$

No symmetry
Even function (circled)
Odd function
Line Symmetry (circled)
Point Symmetry

b)

No symmetry
Even function
Odd function
Line Symmetry
Point Symmetry (circled)

e)

$f(x) = x^3 - 4x^2 + 1$

No symmetry
Even function
Odd function
Line Symmetry
Point Symmetry (circled)

All cubics have point symmetry.

c)

No symmetry
Even function
Odd function
Line Symmetry
Point Symmetry (circled)

f)

$f(x) = x^4 + 5x$

No symmetry (circled)
Even function
Odd function
Line Symmetry
Point Symmetry

16) Consider the polynomial function $f(x) = -3x^4 + 6x^2 - 10$

a) Show algebraically whether f is even, odd or neither.

$f(-x) = -3(-x)^4 + 6(-x)^2 - 10$

$f(-x) = -3(-1)^4(x)^4 + 6(-1)^2(x)^2 - 10$

$f(-x) = -3x^4 + 6x^2 - 10$

Since $f(-x) = f(x)$, it is an even function.

$\therefore f(-x) = f(x)$

b) For what finite difference will f give a constant value, and what will that constant value be?

It is degree 4, so the 4th differences will be constant.

finite differences $= -3(4!)$

$= -72$

c) What are the maximum and minimum number of zeros the above polynomial could have?

Min zeros $= 0$

Max zeros $= 4$

Possible zeros for degree 4 are 4,3,2,1, or 0.

17) Use the given graph to state:

a) x-intercepts -2 (order 2), and 1

b) number of turning points 2

c) least possible degree 3

d) any symmetry present; even or odd function?

Point symmetry. Not an even or odd function.

e) the intervals where $f(x) < 0$

$f(x) < 0$ when: $x < -2$ or $-2 < x < 1$

$f(x) < 0$ when $x \in (-\infty, -2) \cup (-2, 1)$