

W3 – 1.3 – Factored Form Polynomial Functions

MHF4U

ANSWERS

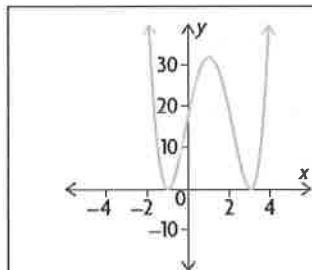
1) Match each equation with the most suitable graph. Write the letter of the equation beneath the matching graph.

A) $f(x) = 2(x + 1)^2(x - 3)$

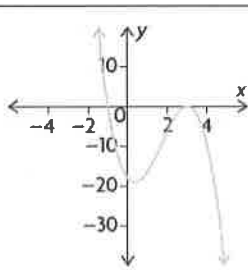
B) $f(x) = (x + 1)^2(x - 3)^2$

C) $f(x) = -2(x + 1)(x - 3)^2$

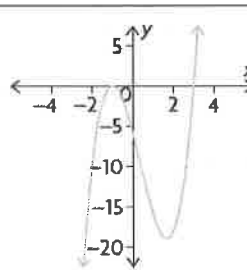
D) $f(x) = x(x + 1)(x - 3)(x - 5)$



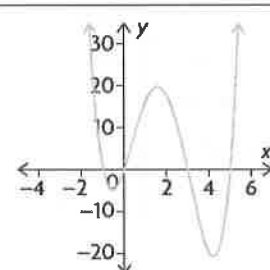
B



C



A



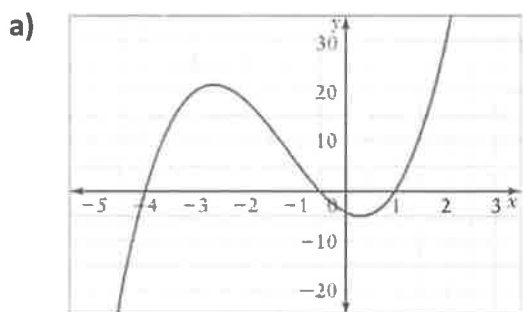
D

2) Complete the table

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x - 4)(x + 3)(2x - 1)$	$(x)(x)(x) = x^3$ ③	$(1)(1)(2) = 2$	Q3 to Q1	$(4, 0)$ $(-3, 0)$ $(\frac{1}{2}, 0)$
$g(x) = -2(x + 2)(x - 2)(1 + x)(x - 1)$	$(x)(x)(x)(x) = x^4$ ④	$-2(1)(1)(1)(1) = -2$	Q3 to Q4	$(-2, 0)$ $(-1, 0)$ $(2, 0)$ $(1, 0)$
$h(x) = (3x + 2)^2(x - 4)(x + 1)(2x - 3)$	$(x^2)(x)(x)(x) = x^5$ ⑤	$(3^2)(1)(1)(2) = 18$	Q3 to Q1	$(-\frac{2}{3}, 0)$ $(\frac{3}{2}, 0)$ $(4, 0)$ $(-1, 0)$
$p(x) = -(x + 5)^3(x - 5)^3$	$(x^3)(x^3) = x^6$ ⑥	$-1(1^3)(1^3) = -1$	Q3 to Q4	$(-5, 0)$ $(5, 0)$

3) For each graph, state...

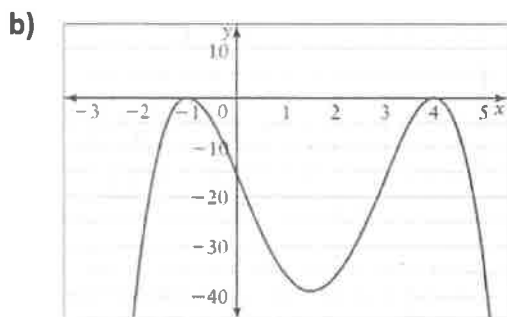
- i) the least possible degree and the sign of the leading coefficient
- ii) the x -intercepts (specify order of zero) and the factors of the function
- iii) the intervals where the function is positive/negative



- i) degree: 3
leading coefficient: Positive
- ii) x -intercepts: $-4, -\frac{1}{2}, 1$
factors: $(x+4), (2x+1), (x-1)$

iii)

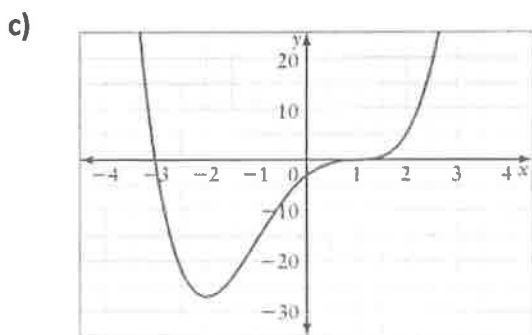
Interval	$(-\infty, -4)$	$(-4, -\frac{1}{2})$	$(-\frac{1}{2}, 1)$	$(1, \infty)$
Sign	-	+	-	+



- i) degree: 4
leading coefficient: Negative
- ii) x -intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2, (x-4)^2$

iii)

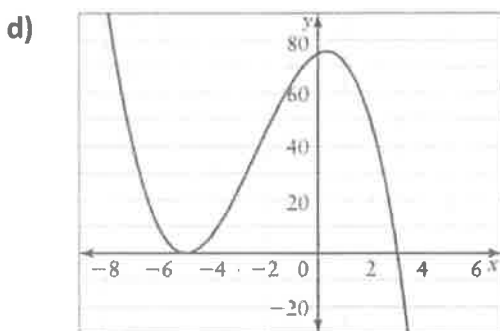
Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	-	-	-



- i) degree: 4
leading coefficient: Positive
- ii) x -intercepts: $-3, 1$ (order 3)
factors: $(x+3), (x-1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	-	+



- i) degree: 3
leading coefficient: Negative
- ii) x -intercepts: -5 (order 2), 3
factors: $(x+5)^2, (x-3)$

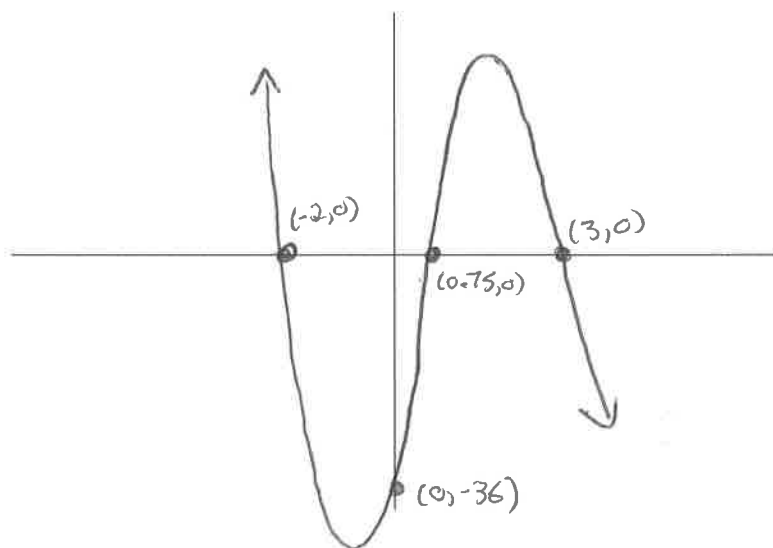
iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	-

4) For each function, complete the chart and sketch a possible graph of the function labelling key points.

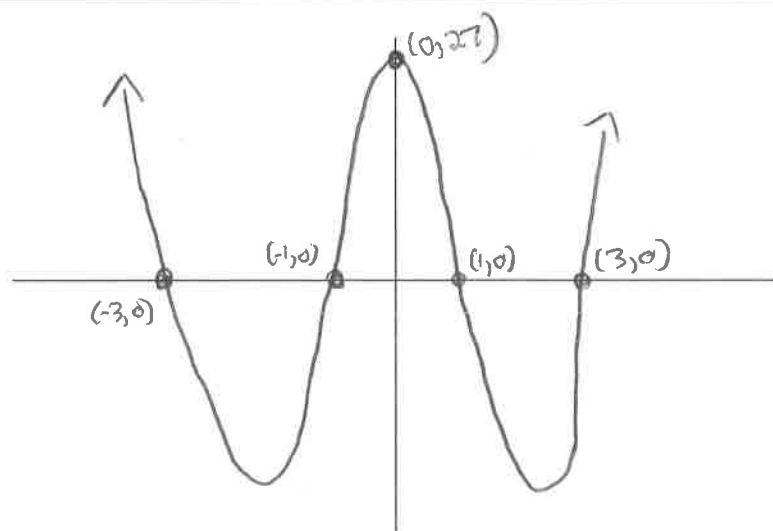
a) $f(x) = -2(x - 3)(x + 2)(4x - 3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$-2(1)(1)(4)$ $= -8$	Q2 to Q4	$(3, 0)$ $(-2, 0)$ $(\frac{3}{4}, 0)$	$f(0) = -2(-3)(2)(-3)$ $= -36$ $(0, -36)$



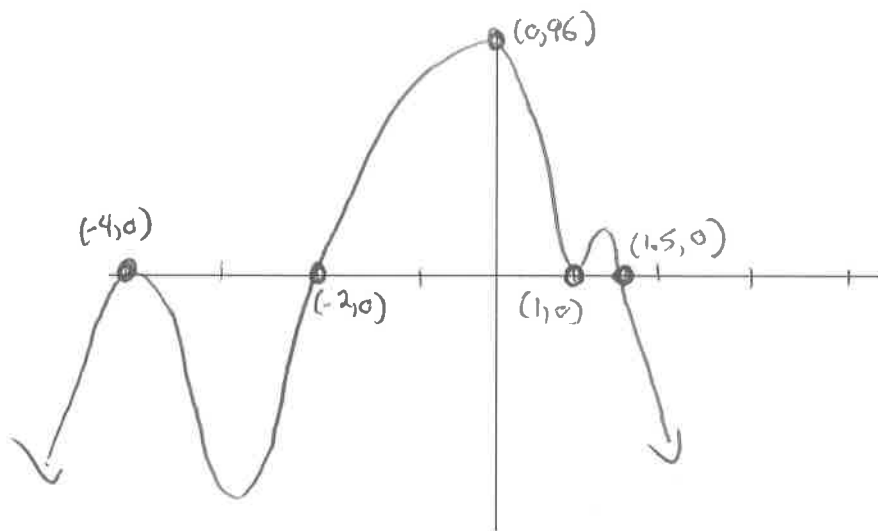
b) $g(x) = (x - 1)(x + 3)(1 + x)(3x - 9)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)(x)$ $= x^4$ Degree 4	$(1)(1)(1)(3)$ $= 3$	Q2 to Q1	$(1, 0)$ $(3, 0)$ $(-3, 0)$ $(-1, 0)$	$g(0) = (-1)(3)(1)(-9)$ $= 27$



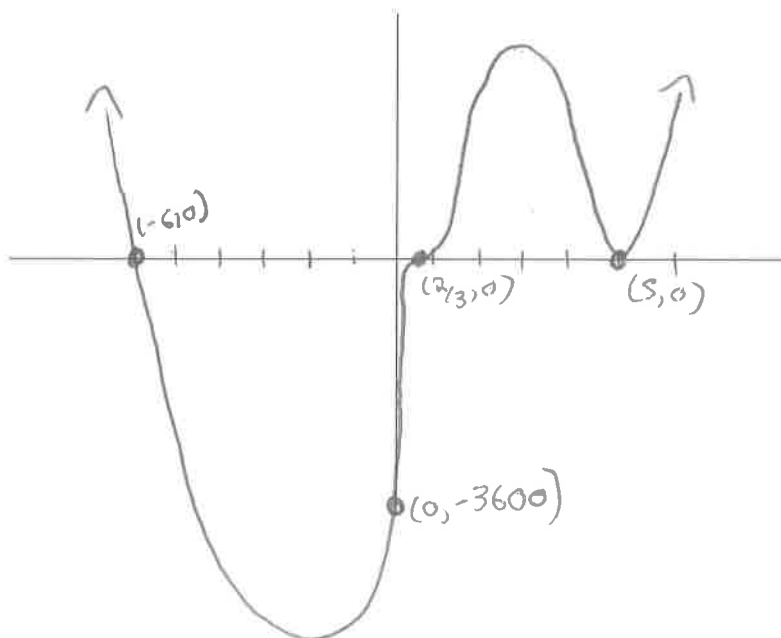
c) $h(x) = -(x+4)^2(x-1)^2(x+2)(2x-3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x^2)(x^2)(x)(x)$ $= x^6$ Degree 6	$-1(1^2)(1^2)(1)(2)$ $= -2$	Q3 to Q4	$(-4,0)$ order 2 $(1,0)$ order 2 $(-2,0)$ $(3/2,0)$	$h(0) = -1(4)^2(-1)^2(2)(-3)$ $= 96$



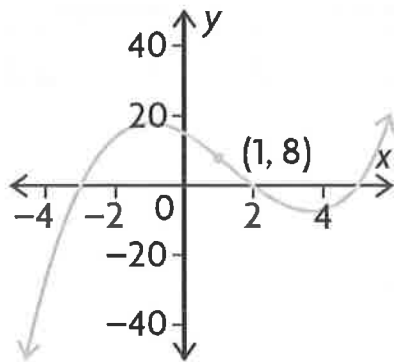
d) $p(x) = 3(x+6)(x-5)^2(3x-2)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x^2)(x^3)$ $= x^6$ Degree 6	$3(1)(1)^2(3)^3$ $= 81$	Q2 to Q1	$(-6,0)$ $(5,0)$ order 2 $(2/3,0)$ order 3	$p(0) = 3(6)(-5)^2(-2)^3$ $= -3600$



5) Write the equation of each function

a)



$$f(x) = k(x+3)(x-2)(x-5)$$

$$8 = k(1+3)(1-2)(1-5)$$

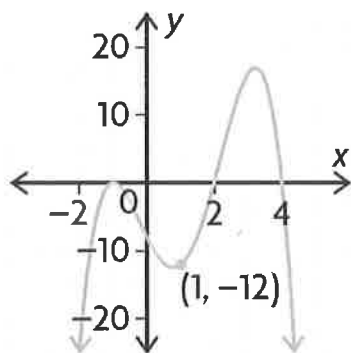
$$8 = k(4)(-1)(-4)$$

$$8 = 16k$$

$$k = \frac{1}{2}$$

$$f(x) = \frac{1}{2}(x+3)(x-2)(x-5)$$

b)



$$g(x) = k(x+1)^2(x-2)(x-4)$$

$$-12 = k(1+1)^2(1-2)(1-4)$$

$$-12 = k(4)(-1)(-3)$$

$$-12 = 12k$$

$$k = -1$$

$$g(x) = -(x+1)^2(x-2)(x-4)$$

6) Determine an equation for a quintic function with zeros -1 (order 3) and 3 (order 2) that passes through the point (-2, 50)

$$h(x) = k(x+1)^3(x-3)^2$$

$$50 = k(-2+1)^3(-2-3)^2$$

$$50 = k(-1)^3(-5)^2$$

$$50 = k(-1)(25)$$

$$50 = -25k$$

$$k = -2$$

$$h(x) = -2(x+1)^3(x-3)^2$$

7) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$

$$2x^2 - x - 1 \quad p: -2 \quad q: -1 \quad (-2 \text{ and } 1)$$

$$= 2x^2 - 2x + 1x - 1$$

$$= 2x(x-1) + 1(x-1)$$

$$= (x-1)(2x+1)$$

$$x^2 - 3x - 4 \quad p: -4 \quad q: -1 \quad (-4 \text{ and } 1)$$

$$= (x-4)(x+1)$$

$$f(x) = (x-1)(2x+1)(x-4)(x+1)$$

$$0 = (x-1)(2x+1)(x-4)(x+1)$$

$$x_1 = 1 \quad x_2 = -\frac{1}{2} \quad x_3 = 4 \quad x_4 = -1$$

Answer Key

1) B C A D

Equation	Degree	Leading Coefficient	End Behaviour	x-Intercepts
$f(x) = (x-4)(x+3)(2x-1)$	3	2	Q3 \rightarrow Q1	$(4, 0)$ $(-3, 0)$ $(\frac{1}{2}, 0)$
$g(x) = -2(x+2)(x-2)(1+x)(x-1)$	4	-2	Q3 \rightarrow Q4	$(-2, 0)$ $(-1, 0)$ $(1, 0)$ $(2, 0)$
$h(x) = (3x+2)^2(x-4)(x+1)(2x-3)$	5	18	Q3 \rightarrow Q1	$(4, 0)$ $(-1, 0)$ $(-\frac{2}{3}, 0)$ $(\frac{3}{2}, 0)$
$p(x) = -(x+5)^3(x-5)^2$	6	-1	Q3 \rightarrow Q4	$(-5, 0)$ $(5, 0)$

3) a) i) degree: 3
leading coefficient: positive

ii) x-intercepts: -4, -0.5, 1
factors: $(x+4)$, $(2x+1)$, and $(x-1)$

iii)

Interval	$(-\infty, -4)$	$(-4, -0.5)$	$(-0.5, 1)$	$(1, \infty)$
Sign	-	+	-	+

b) i) degree: 4
leading coefficient: negative

ii) x-intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2$, and $(x-4)^2$

iii)

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	-	-	-

c) i) degree: 4
leading coefficient: positive

ii) x-intercepts: -3, 1 (order 3)
factors: $(x+3)$, and $(x-1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	-	+

d) i) degree: 3
leading coefficient: negative

ii) x-intercepts: -5 (order 2), 3
factors: $(x+5)^2$, and $(x-3)$

iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	-

4) a)

Degree	3	Leading Coefficient	-8	End Behaviour	Q2 \rightarrow Q4	x-Intercepts	$(3, 0)$ $(-2, 0)$ $(\frac{3}{4}, 0)$	y-Intercept	$(0, -36)$
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b)

Degree	4	Leading Coefficient	3	End Behaviour	Q2 \rightarrow Q1	x-Intercepts	$(1, 0)$ $(-3, 0)$ $(-1, 0)$ $(3, 0)$	y-Intercept	$(0, 27)$
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c)

Degree	6	Leading Coefficient	-2	End Behaviour	Q3 \rightarrow Q4	x-Intercepts	$(-4, 0)$ order 2 $(1, 0)$ order 2 $(-2, 0)$ $(1.5, 0)$	y-Intercept	$(0, 96)$
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d)

Degree	6	Leading Coefficient	81	End Behaviour	Q2 \rightarrow Q1	x-Intercepts	$(-6, 0)$ $(5, 0)$ order 2 $(\frac{2}{3}, 0)$ order 3	y-Intercept	$(0, -3600)$
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5) a) $y = 0.5(x+3)(x-2)(x-5)$ b) $y = -(x+1)^2(x-2)(x-4)$

$$y = -2(x+1)^3(x-3)^2$$

7) 4, 1, -1, and -0.5