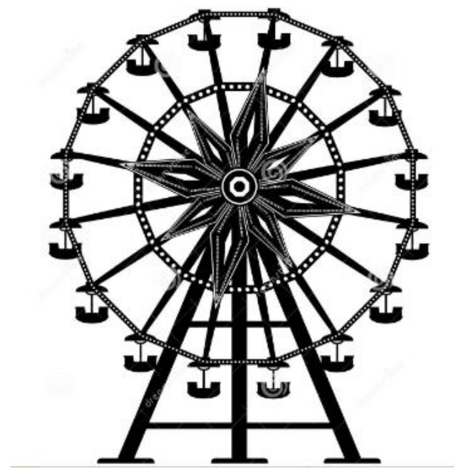
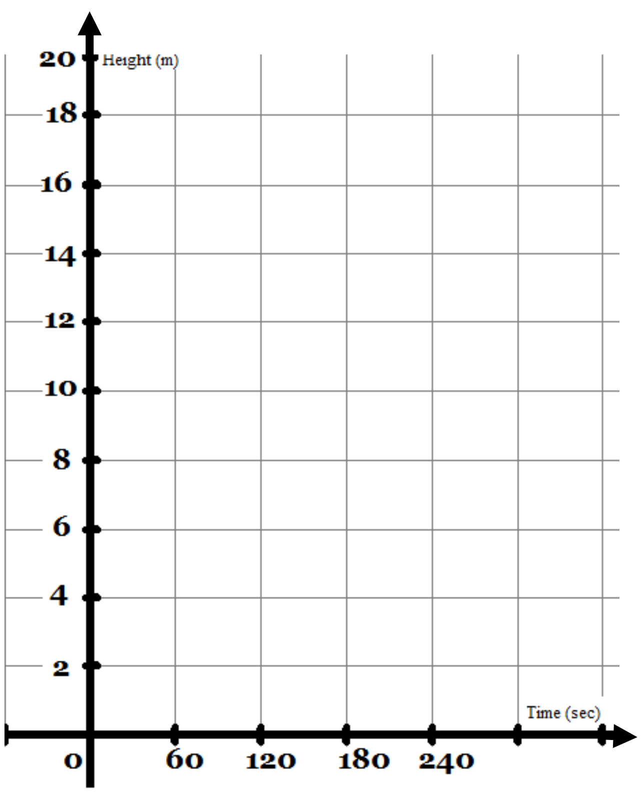
# 4.3 Modeling Sinusoidal Functions

1. At a county fair, the Ferris wheel has a diameter of 16 meters, and its lowest point is 2 meters above the ground. The wheel completes one complete revolution every 4 minutes. Riders begin a ride at the lowest position on the wheel.
   (a) Draw a sketch of the trip.
   (b) Determine the equation of this periodic function using both sine and cosine function.
   (c) Find the height of passengers at 2 minutes and 35 seconds.
   (d) Find the time(s) when the passengers are at a height of 7 m.

| Time (sec) | Height (m) |
|---|---|
| 0 sec | |
| 60 sec | |
| 120 sec | |
| 180 sec | |
| 240 sec | |

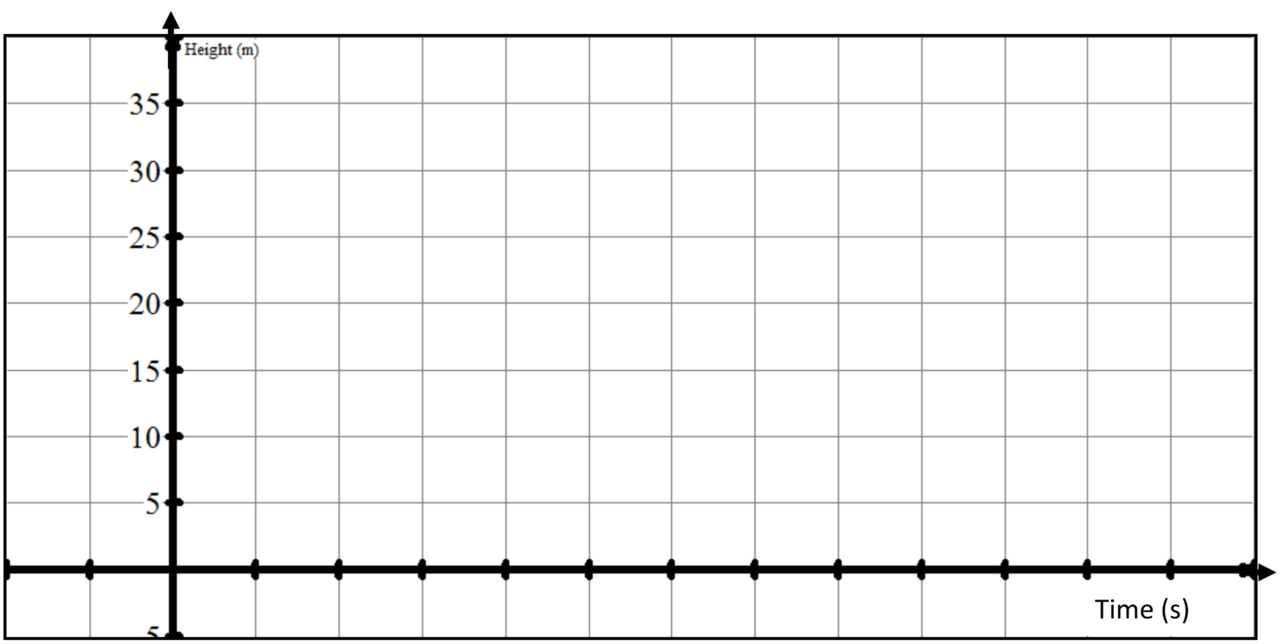




2. The SkyWheel is a Ferris wheel in Niagara Falls. It has a diameter of 30 metres and the ride lasts for 12 minutes for a total of 6 revolutions. It has a total of 42 gondolas that can each hold 6 passengers. Assuming that the height of the gondola follows a sinusoidal model, if you enter the gondola at a height of 2 m above the ground, what is your altitude at 11 minutes and 21 seconds? At what time(s) is your height 21 m high?

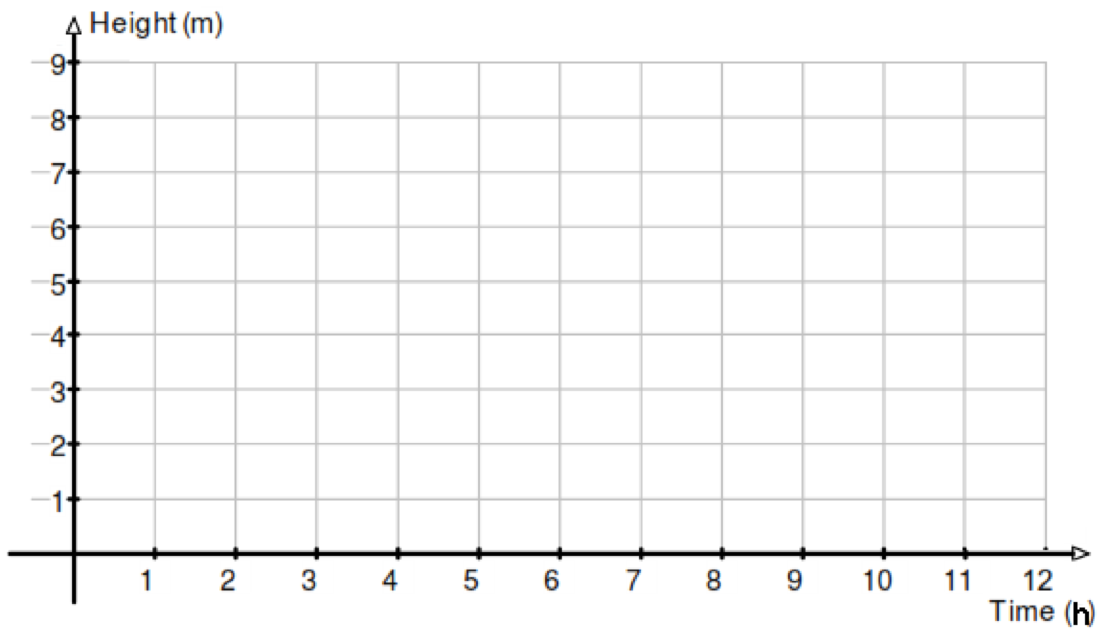
   (a) Draw a sketch of the trip.
   (b) Determine the equation of this periodic function using both sine and cosine function.
   (c) Find the height of your gondola at 11 minutes and 21 seconds.
   (d) Find the time(s) when your gondola is at a height of 21 m.

| Time (sec) | Height (m) |
|---|---|
| | |
| | |
| | |
| | |
| | |



3. The water at a local beach has an average depth of 1 meter at low tide. The average depth of the water at high tide is 8 m. If one cycle takes 12 hours:
   (a) Determine the equation of this periodic function using cosine as the base function where 0 time is the beginning of high tide.
   (b) What is the depth of the water at 2 am?
   (c) Many people dive into the beach from the nearby dock. If the water must be at least 3 m deep to dive safely, between what daylight hours should people dive?

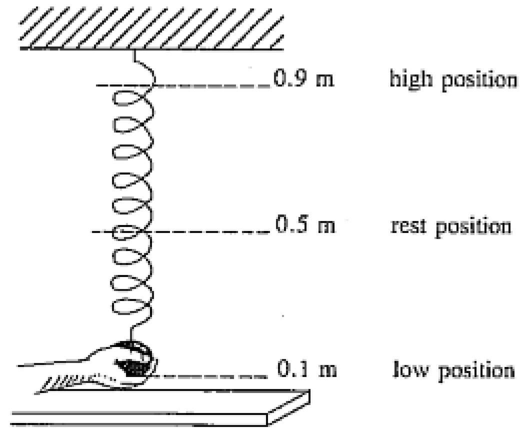
| Time (h) | Height (m) |
|---|---|
| | |
| | |
| | |
| | |
| | |

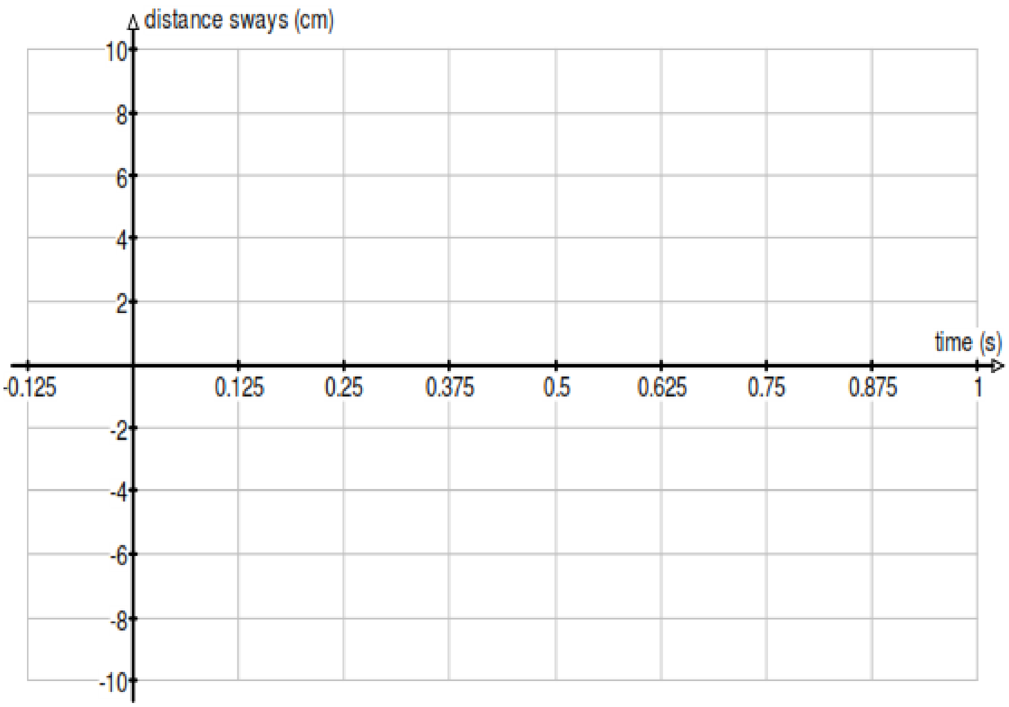


4. A certain mass is supported by a spring so that it is at rest 0.5 m above a table top. The mass is pulled down 0.4 m to its lowest position and released at time t=0, creating a periodic up and down motion, called simple harmonic motion. It takes 1.2 s for the mass to reach the highest position of 0.9 m and return to the lowest position each time.

a) Draw a graphing showing the height of the mass above the table top as a function for the first 2.0s.

b) Write an equation for this function.



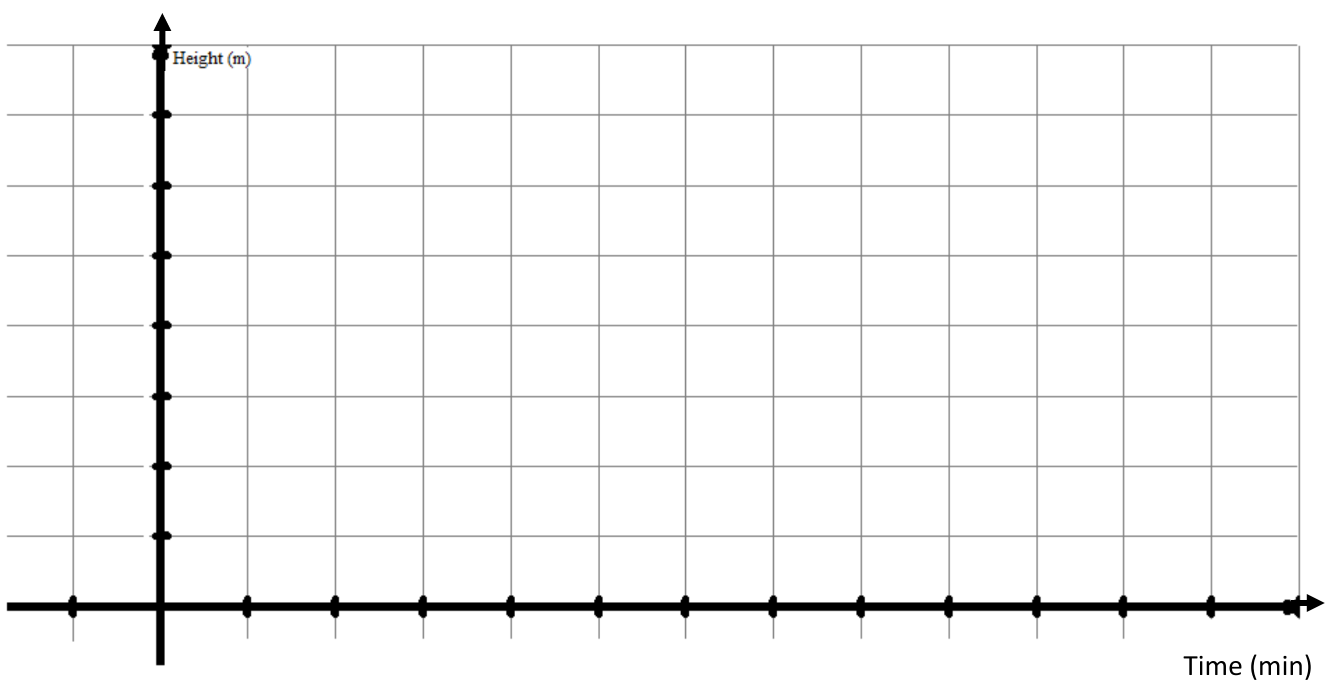



## EXIT CARD

A Ferris wheel with a radius of 9.5 m rotates once every 10 min. The bottom of the wheel is 1.2 m above the ground. Draw a graph to show how a person's height above the ground varies with time for two complete revolutions, starting when the person gets onto the Ferris wheel at its lowest point.

(a) Determine an equation for the graph based on a sine function.
(b) Determine an equation for the graph based on a cosine function.
(c) Find the time(s) when the passengers are at a height of 8.6 m.

| Time (min) | Height (m) |
|---|---|
| | |
| | |
| | |
| | |
| | |



# 4.3 Applications of Sinusoidal Functions-HW

1. Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time $t = 0$. A minimum number, 200 foxes, occurred when $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t = 5.1$ years.

a) Sketch a graph of this sinusoid.
b) Write an equation, using the sine function, to express the number of foxes as a function of time, $t$.
c) Predict the population when $t = 7$.
d) Foxes are declared to be an endangered species when their population drops below 300. Between what two non-negative values of $t$ were foxes first endangered?

2. A Ferris wheel ride reaches a maximum height of 30m and a minimum height of 2m above the ground. The Ferris wheel rotates at a speed of $0.8\pi$ m/s. You start the ride at the lowest point of the wheel and travel counterclockwise. $[\text{Hint}: k = \dfrac{speed}{radius}]$

a) Sketch a graph of this sinusoid.
b) Write an equation, using the cosine function, to express your height above the ground as a function of time, $t$.
c) How high will you be after 30s on the ride?

3. A group of students decided to study the sinusoidal nature of tides. Values for the depth of the water level were recorded at various times. At t=2 hours low tide was recorded at a depth of 1.8 m. At t=8 hours, high tide was recorded at a depth of 3.6 m.

a) Sketch the graph of this function
b) Write an equation expressing distance in terms of time
c) Give the depth of water at t=21 hours.

4. A city averages 14 hours of daylight in June, 10 in December, and 12 in both March and September. Assume that the number of hours of daylight varies sinusoidally over a period of one year.

a) Sketch the graph of this relationship.
b) Write an equation for n, the number of hours of daylight, as a cosine function of t.
c) (Let t be in months and t = 0 correspond to the month of January)
d) How many hours of daylight are there in the month of July?

5. At a certain ocean bay, the maximum height of the water is 4 m above mean sea level at 8:00 a.m. The height is at a maximum again at 8:24 p.m. Assuming that the relationship between the height, $h$, in meters, and the time, $t$, in hours, is sinusoidal, determine the height of the water above mean sea level at 10:00 a.m.

ANSWERS

1. b) $p(t) = 300\sin[\frac{5\pi}{11}(t-4)]+500$ or $p(t) = -300\cos[\frac{5\pi}{11}(t-2.9)]+500$

   c) 227 foxes

   d) 2.31years, 3.49years

2. b) $h(t) = 14\cos[\frac{2\pi}{35}\ (t-17.5)]+16$ or $h(t) = -14\cos\frac{2\pi}{35}t + 16$

   c) 7.27m

3. b) $h(t) = 0.4\sin[\frac{5\pi}{3}(t-0.6)]+0.5$ or $h(t) = -0.4\cos\left(\frac{5\pi}{3}t\right)+0.5$

4. b) $n(t) = 2\cos[\frac{\pi}{6}(t-5)]+12$

   c) 13.7h

5. a) $h(t) = 4\sin\frac{5\pi}{31}(t+3.1)$ or $h(t) = 4\cos\left(\frac{5\pi}{31}t\right)$

   b) $h(2) = 2.1\text{m}$

**Warm up**

Oceanic tides also display periodic behavior, where the high tide corresponds to the peak and the low tide corresponds to the trough of a sinusoidal curve. On a certain day, the depth of water at Sydney Harbour Bridge at high tide was 16m. After 6 hours (at low tide) the depth was 6m.

(a) How long is the tidal cycle.

(b) Write an equation for the depth of the water (in meters), in terms of time ( in hours).

(c) Using your equation, find the depth of the water 4 hours and 15 minutes *after the low tide*.

(d) When will the tide first reach a height of 14 m?