

W3 – 7.3 – Product and Quotient Laws of Logarithms**MHF4U***SOLUTIONS***1) Simplify using laws of logarithms and then evaluate.**

a) $\log 9 + \log 6$

$$\begin{aligned} &= \log(9 \times 6) \\ &= \log(54) \\ &= 1.732 \end{aligned}$$

b) $\log 48 - \log 6$

$$\begin{aligned} &= \log\left(\frac{48}{6}\right) \\ &= \log(8) \\ &= 0.903 \end{aligned}$$

c) $\log_3 7 + \log_3 3$

$$\begin{aligned} &= \log_3(7 \times 3) \\ &= \log_3(21) \\ &= \frac{\log(21)}{\log(3)} \\ &= 2.771 \end{aligned}$$

2) Simplify each algebraic expression.

a) $\log x + \log y + \log(2z)$

$$= \log(2xyz)$$

b) $\log_2 a + \log_2(3b) - \log_2(2c)$

$$= \log_2\left(\frac{3ab}{2c}\right)$$

c) $2 \log m + 3 \log n - 4 \log y$

$$\begin{aligned} &= \log m^2 + \log n^3 - \log y^4 \\ &= \log\left(\frac{m^2 n^3}{y^4}\right) \end{aligned}$$

3) Evaluate using the product law of logarithms.

a) $\log_6 18 + \log_6 2$

$$\begin{aligned} &= \log_6(18 \times 2) \\ &= \log_6(36) \\ &= \frac{\log 36}{\log 6} \\ &= 2 \end{aligned}$$

b) $\log 40 + \log 2.5$

$$\begin{aligned} &= \log(40 \times 2.5) \\ &= \log(100) \\ &= 2 \end{aligned}$$

c) $\log_{12} 8 + \log_{12} 2 + \log_{12} 9$

$$\begin{aligned} &= \log_{12}(8 \times 2 \times 9) \\ &= \log_{12}(144) \\ &= \frac{\log 144}{\log 12} \\ &= 2 \end{aligned}$$

4) Evaluate using the quotient law of logarithms.

a) $\log_3 54 - \log_3 2$

$$\begin{aligned} &= \log_3\left(\frac{54}{2}\right) \\ &= \log_3(27) \\ &= 3 \end{aligned}$$

b) $\log 50\,000 - \log 5$

$$\begin{aligned} &= \log\left(\frac{50\,000}{5}\right) \\ &= \log(10\,000) \\ &= 4 \end{aligned}$$

c) $\log_4 320 - \log_4 5$

$$\begin{aligned} &= \log_4\left(\frac{320}{5}\right) \\ &= \log_4(64) \\ &= 3 \end{aligned}$$

5) Evaluate, using the laws of logarithms

a) $3 \log_{16} 2 + 2 \log_{16} 8 - \log_{16} 2$

$$\begin{aligned} &= \log_{16}(2^3) + \log_{16}(8^2) - \log_{16}(2) \\ &= \log_{16}(8) + \log_{16}(64) - \log_{16}(2) \\ &= \log_{16}(256) \\ &= 2 \end{aligned}$$

b) $\log 20 + \log 2 + \frac{1}{3} \log 125$

$$\begin{aligned} &= \log(20 \times 2) + \log(125^{1/3}) \\ &= \log(40) + \log(5) \\ &= \log(40 \times 5) \\ &= \log(200) \\ &= 2.301 \end{aligned}$$

6) Write as a sum or difference of logarithms. Simplify, if possible.

a) $\log_7(cd)$

b) $\log_3\left(\frac{m}{n}\right)$

c) $\log(uv^3)$

d) $\log\left(\frac{a\sqrt{b}}{c^2}\right)$

e) $\log_2 10$

a) $\log_7(c) + \log_7(d)$

b) $\log_3(m) - \log_3(n)$

c) $\log(u) + \log(v^3)$
 $= \log(u) + 3\log(v)$

d) $\log a + \log(b^{1/2}) - \log(c^2)$
 $= \log a + \frac{1}{2} \log b - 2\log c$

e) $\log_2(2 \times 5)$
 $= \log_2(2) + \log_2(5)$
 $= 1 + \log_2(5)$

7) Simplify

$$\begin{aligned} \log\left(\frac{x^2}{\sqrt{x}}\right) &= \log x^2 - \log x^{1/2} \\ &= 2\log x - \frac{1}{2}\log x \\ &= \frac{3}{2}\log x \end{aligned}$$

b) $\log \sqrt{k} + \log(\sqrt{k})^3 + \log \sqrt[3]{k^2}$

$$\begin{aligned} &= \log(k^{1/2}) + \log(k^{3/2}) + \log(k^{2/3}) \\ &= \frac{1}{2}\log k + \frac{3}{2}\log k + \frac{2}{3}\log k \\ &= \frac{3}{6}\log k + \frac{9}{6}\log k + \frac{4}{6}\log k \\ &= \frac{8}{3}\log k \end{aligned}$$

c) $\log(x^2 - 4) - \log(x - 2)$

$$\begin{aligned} &= \log\left(\frac{x^2 - 4}{x - 2}\right) \\ &= \log\left[\frac{(x-2)(x+2)}{x-2}\right] \\ &= \log(x+2) \end{aligned}$$

d) $\log(x^2 - x - 6) - \log(2x - 6)$

$$\begin{aligned} &= \log\left(\frac{x^2 - x - 6}{2x - 6}\right) \\ &= \log\left[\frac{(x-3)(x+2)}{2(x-3)}\right] \\ &= \log\left(\frac{x+2}{2}\right) \end{aligned}$$

ANSWER KEY

1) $\log 54 = 1.732$ b) $\log 8 = 0.903$ c) $\log_3 21 = 2.771$

2) a) $\log(2xyz)$ b) $\log_2\left(\frac{3ab}{2c}\right)$ c) $\log\left(\frac{m^2n^3}{y^4}\right)$

3) a) 2 b) 2 c) 2

4) a) 3 b) 4 c) 3

5) a) 2 b) 2.301

6) a) $\log_7 c + \log_7 d$ b) $\log_3 m - \log_3 n$ c) $\log u + 3\log v$ d) $\log a + \frac{1}{2}\log b - 2\log c$ e) $1 + \log_2 5$

7) a) $\frac{3}{2}\log x$ b) $\frac{8}{3}\log k$ c) $\log(x+2)$ d) $\log\left(\frac{x+2}{2}\right)$