4.4 Solving Trigonometric Equations

What is a trig equation?

- A trig equation is an equation that contains one or more trigonometric functions.

How is it similar to a trig identity?

- A trig equation *can* be but *does not have to* be a trig identity
  - A trig identity is an equation that is true for ***all values*** of the variable for which expressions of both sides of the equation are defined
- A trig equation that is not an identity is only true for ***certain values*** of the variable.

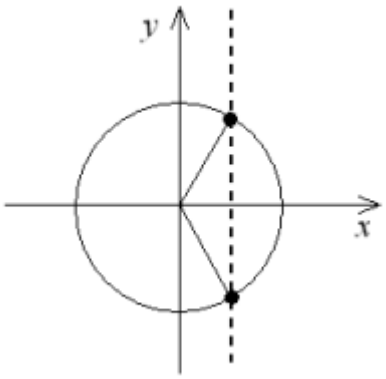
What does it mean to **SOLVE** a trig equation?

- Much like solving a linear equation, we are looking for all of the values of the variable that makes the equation true.

General Solutions

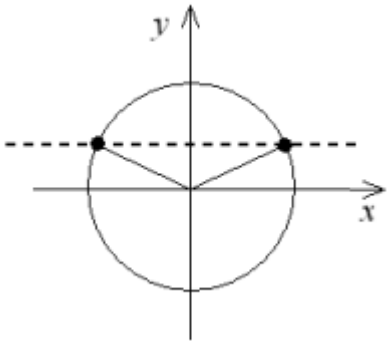
i) If cos(x)=cos(α) , then x = 2πk ± α , kϵ Z

Ex.1) Find the exact solutions for $2\cos(x)+\sqrt{3}=0$ for $x\in[0,2\pi]$.

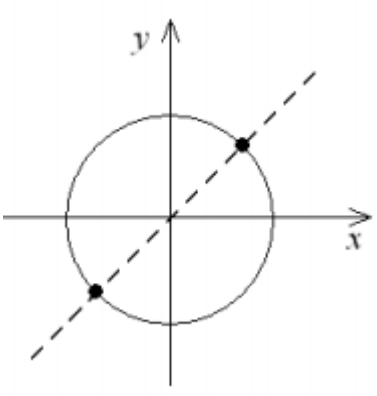


ii) If sin(x)=sin(α) , then x = 2πk + α or x = 2πk + (π-α) , kϵ Z

Ex.2) Find the exact solutions for $2\sin(2x)=\sqrt{3}$ for $x\in[0,2\pi]$.



iii) If tan(x)=tan(α) , then x = kπ + α , kϵZ



Ex.3) Find the exact solutions for $3\tan^2(\theta) = 1$ for $\theta \in [0, 2\pi]$.

Ex.4) Find the exact solutions for $\tan^2(\theta) + 3\sec(\theta) + 3 = 0$, $0 \le \theta \le 2\pi$

Ex.5) Find the solutions for $3\cos^2(\theta) - \sin(\theta) - 1 = 0\ , 0 \leq \theta \leq 2\pi$

Ex.6) Solve the equation $\sin(4x) - \cos(2x) = 0\ , x \in [0, 2\pi]$

Ex.7) Solve $\sin(2x) - \cos(2x) = 0, \quad 0 \le x \le 2\pi$

Ex.8) Solve $\tan(\theta)(\csc(\theta) + 2) = 0 \ , 0 \le \theta \le 2\pi$

Exit Card!

Solve $4\sin^4(\theta)+3\sin^2(\theta)-1=0\ ,0\le\theta\le2\pi$

Practice

1. Solve for x on the interval $[0, 2\pi]$.

a) $6\sin^2(x) - \sin(x) - 1 = 0$

b) $\cot(x)\cos^2(x) = \cot(x)$

c) $4\tan(x) - \sec^2(x) = 0$

d) $2\sin^2(x) - \sin(x) - 1 = 0$

e) $4\sin^3(x) + 2\sin^2(x) - 2\sin(x) - 1 = 0$

f) $\sin(x) + \sqrt{2} = -\sin(x)$

g) $2\cos(3x - 1) = 0$

h) $\sin(2x)\cos(x) - \cos(2x)\sin(x) = 0$

i) $\sec^2(x) - 2\tan(x) = 4$

j) $3\tan\left(\frac{x}{2}\right) + 3 = 0$

k) $-6\sin(2x)\cos(x) + 8\cos(2x) + 3\sin(x) + 4 = 0$

l) $\cot(x)\cos^2(x) = 2\cot(x)$

m) $\frac{1+\sin(x)}{\cos(x)} + \frac{\cos(x)}{1+\sin(x)} = 4$

n) $2\sin^2(x) + 3\cos(x) - 3 = 0$

o) $2\sin(x)\tan(x) - \tan(x) - 2\sin(x) + 1 = 0$

p) $\cos(x)\tan(x) - 1 + \tan(x) - \cos(x) = 0$

2. A weight hanging from a spring is set in motion moving up and down. Its distance, d, (in cm) above or below its "rest" position is described by $d(t) = 5(\sin(6t) - 4\cos(6t))$. At what times during the first 2 seconds is the weight at the rest position (d=0).

Warm up

Solve the following equations for $x \epsilon [0, 2\pi]$.

a) $3\sec^2(x) - 7 = -1$

b) $1 + \sin(x) = \sqrt{3}\cos(x)$

c) $\cos^2(2x) = 3\sin^2(x) - 2$

d) $\dfrac{\cos(2x)}{\cos(x) + \sin(x)} = 1$