

# W4 – 5.3 Transformations of Trig Functions

MHF4U

SOLUTIONS

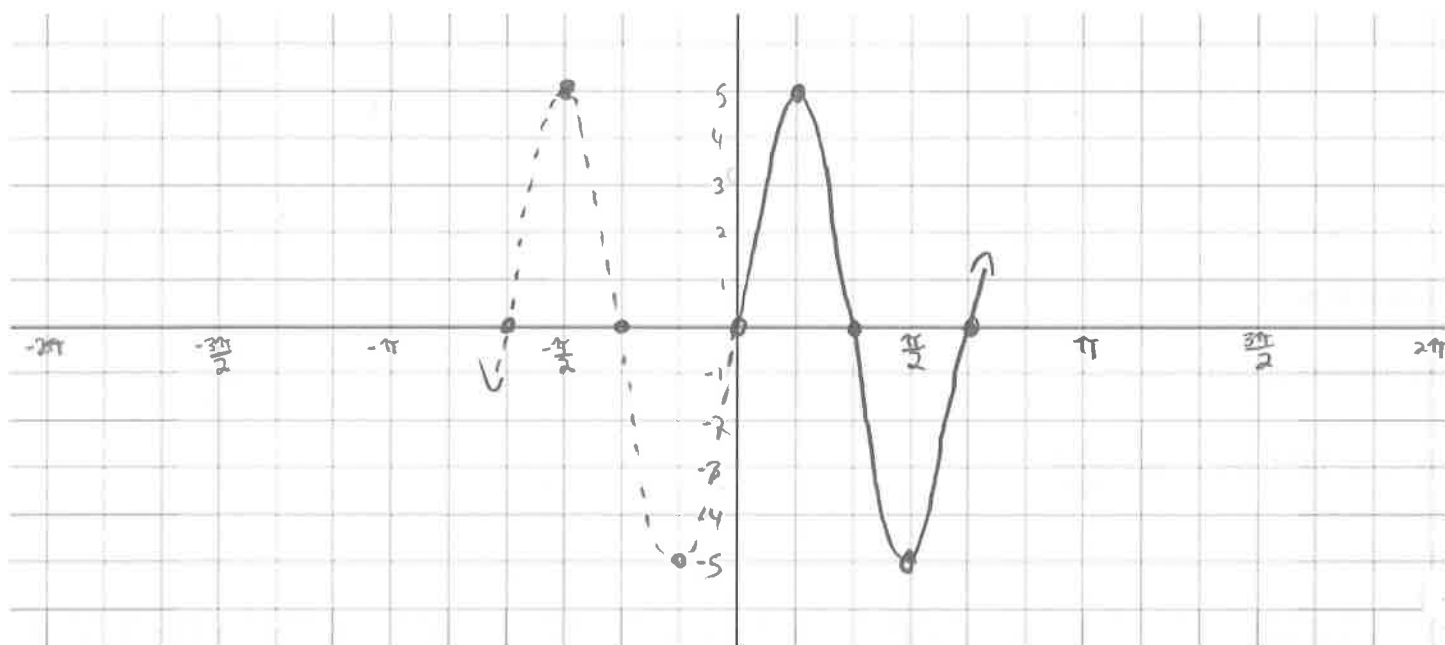
1) For each function, fill in the table of information and then graph two cycles of the transformed function using transformations of the parent function. Choose an appropriate scale.

a)  $y = 5 \sin(3x)$

Amplitude: $=  a  = 5$	Period: $\frac{2\pi}{ k } = \frac{2\pi}{3}$
Phase shift: $\text{none}$	Vertical shift: $\text{none}$
Max: $C +  a  = 0 + 5 = 5$	Min: $C -  a  = 0 - 5 = -5$

$y = \sin x$	
$x$	$y$
0	0
$\pi/2$	1
$\pi$	0
$3\pi/2$	-1
$2\pi$	0

$y = 5 \sin(3x)$	
$\frac{x}{3}$	$5y$
0	0
$\pi/6$	5
$\pi/3$	0
$3\pi/6 = \pi/2$	-5
$2\pi/3$	0

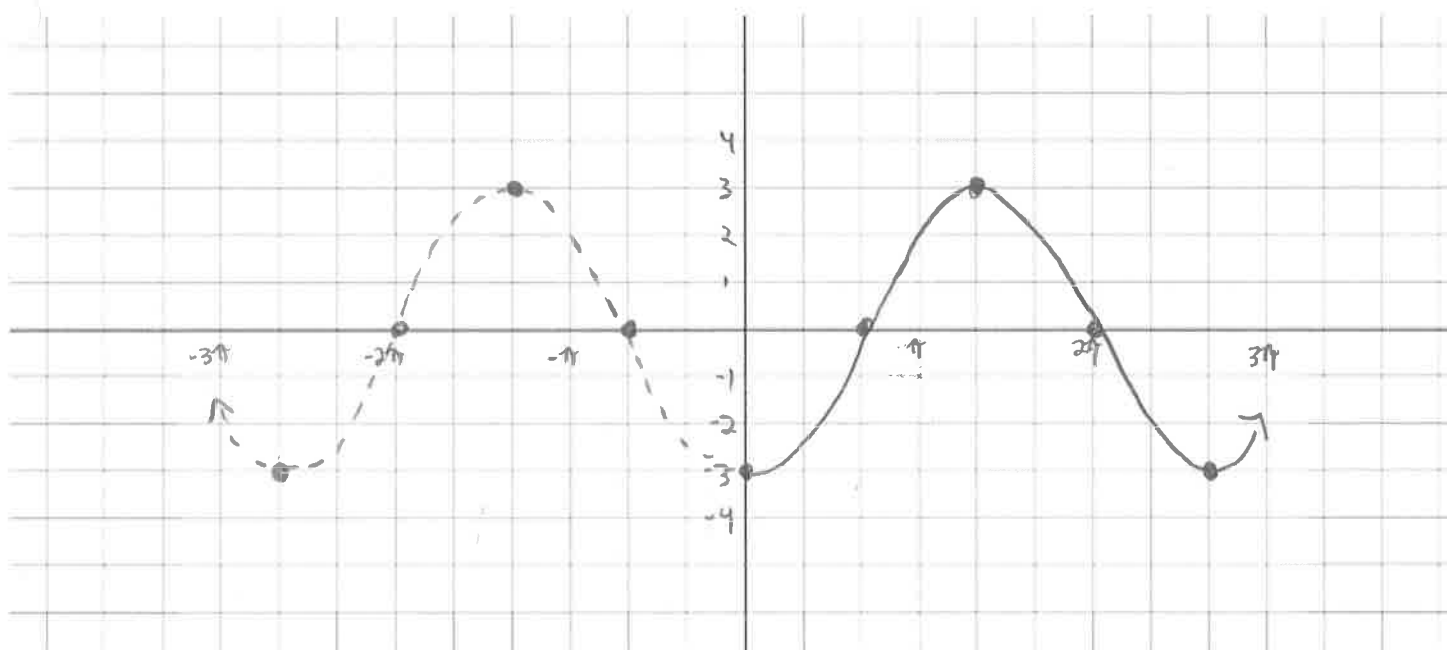


b)  $y = -3 \cos\left(\frac{3}{4}x\right)$

Amplitude: $=  a  =  -3  = 3$	Period: $= \frac{2\pi}{ k } = \frac{2\pi}{(\frac{3}{4})} = \frac{8\pi}{3}$
Phase shift: $NONE$	Vertical shift: $NONE$
Max: $= c +  a  = 0 + 3 = 3$	Min: $= c -  a  = 0 - 3 = -3$

$y = \cos x$	
$x$	$y$
0	1
$\pi/2$	0
$\pi$	-1
$3\pi/2$	0
$2\pi$	1

$y = -3 \cos\left(\frac{3}{4}x\right)$	
$\frac{4}{3}x$	$-3y$
0	-3
$\frac{4\pi}{6} = \frac{2\pi}{3}$	0
$\frac{4\pi}{3}$	3
$\frac{6\pi}{3} = 2\pi$	0
$\frac{8\pi}{3}$	-3

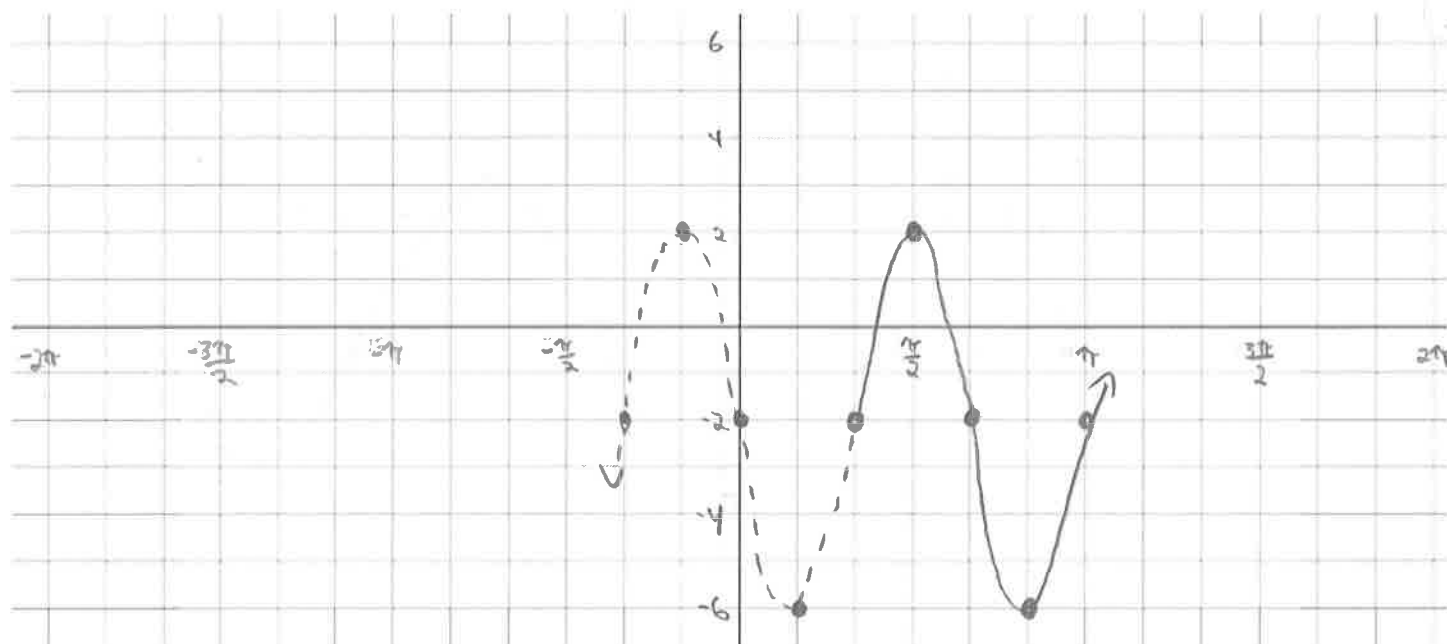


c)  $y = 4 \sin \left[ 3 \left( x - \frac{\pi}{3} \right) \right] - 2$

Amplitude: $=  a  = 4$	Period: $\frac{2\pi}{ k } = \frac{2\pi}{3}$
Phase shift: $\frac{\pi}{3}$ to the RIGHT	Vertical shift: 2 DOWN
Max: $C +  a  = -2 + 4 = 2$	Min: $C -  a  = -2 - 4 = -6$

$y = \sin x$	
$x$	$y$
0	0
$\pi/2$	1
$\pi$	0
$3\pi/2$	-1
$2\pi$	0

$y = 4 \sin \left[ 3 \left( x - \frac{\pi}{3} \right) \right] - 2$	
$\frac{x}{3} + \frac{\pi}{3}$	$4y - 2$
$\pi/3$	-2
$\frac{3\pi}{6} = \pi/2$	2
$2\pi/3$	-2
$5\pi/6$	-6
$3\pi/3 = \pi$	-2

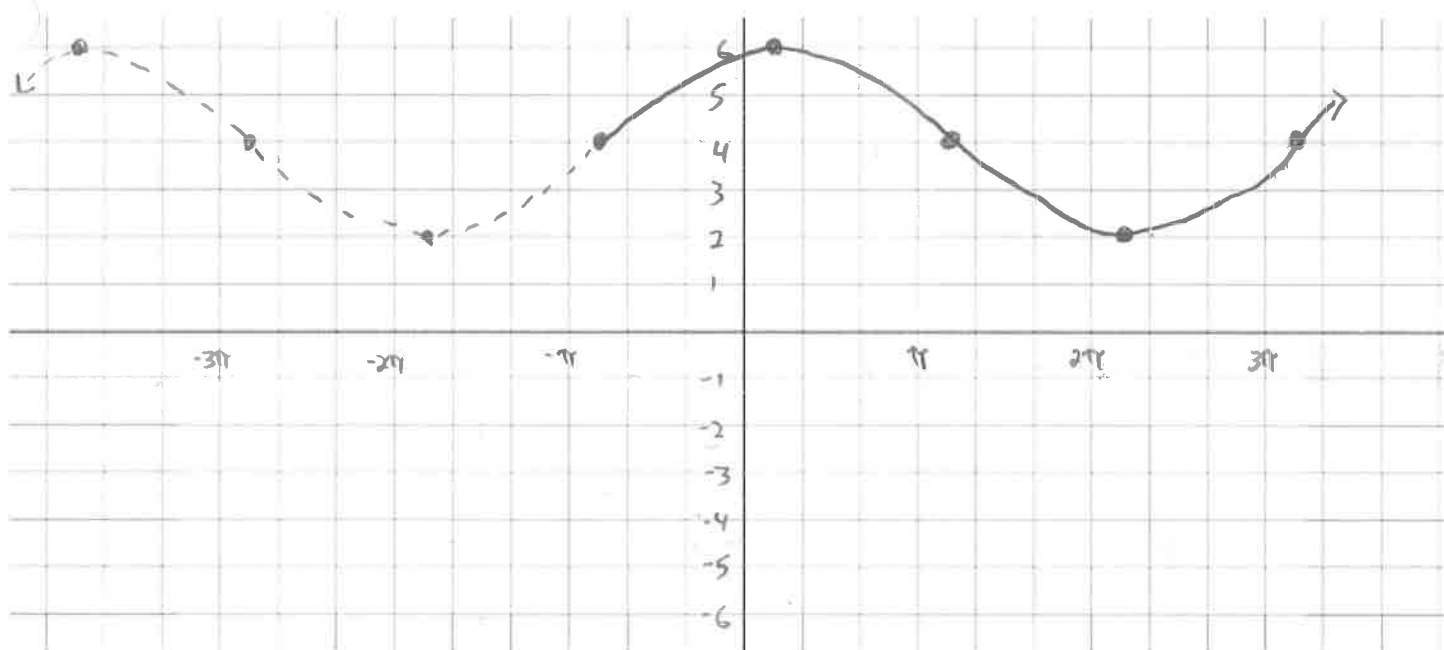


d)  $y = 2 \sin \left[ \frac{1}{2} \left( x + \frac{5\pi}{6} \right) \right] + 4$

Amplitude: $=  a  = 2$	Period: $= \frac{2\pi}{ k } = \frac{2\pi}{(\frac{1}{2})} = 4\pi$
Phase shift: $\frac{5\pi}{6}$ Left	Vertical shift: 4 up
Max: $c +  a  = 4 + 2 = 6$	Min: $c -  a  = 4 - 2 = 2$

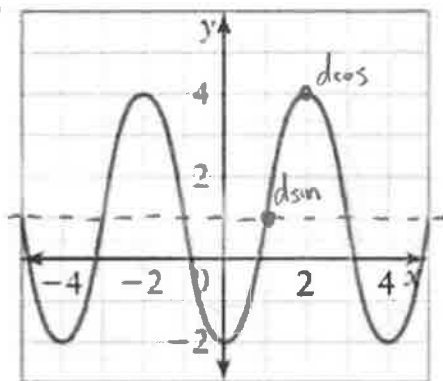
$y = \sin x$	
$x$	$y$
0	0
$\pi/2$	1
$\pi$	0
$3\pi/2$	-1
$2\pi$	0

$y = 2 \sin \left[ \frac{1}{2} \left( x + \frac{5\pi}{6} \right) \right] + 4$	
$2x - \frac{5\pi}{6}$	$2y + 4$
$-\frac{5\pi}{6}$	4
$\pi/6$	6
$7\pi/6$	4
$13\pi/6$	2
$19\pi/6$	4



2) Model each graph shown as a sine and cosine function.

a)



$$a = \frac{\max - \min}{2} = \frac{4 - (-2)}{2} = 3$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$c = \max - |a| = 4 - 3 = 1$$

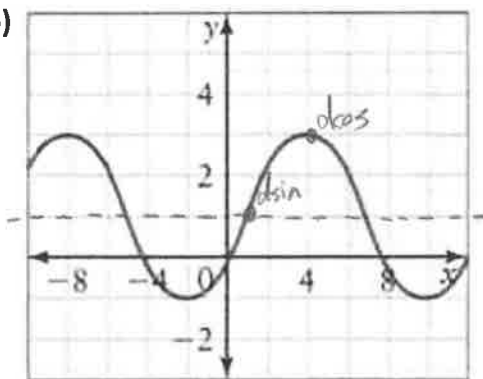
$$d \sin = 1$$

$$d \cos = 2$$

$$y = 3 \sin \left[ \frac{\pi}{2}(x-1) \right] + 1$$

$$y = 3 \cos \left[ \frac{\pi}{2}(x-2) \right] + 1$$

b)



$$a = \frac{\max - \min}{2} = \frac{3 - (-1)}{2} = 2$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$c = \max - |a| = 3 - 2 = 1$$

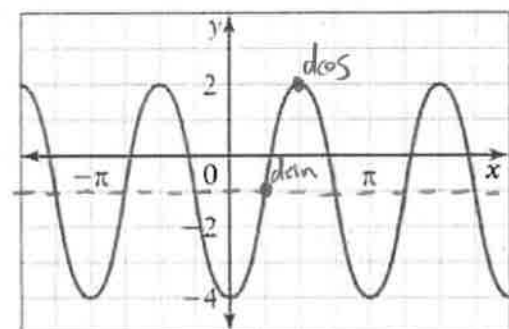
$$d \cos = 4$$

$$d \sin = d \cos - \frac{\pi}{2k} = 4 - \frac{\pi}{\left(\frac{2\pi}{6}\right)} = 4 - \pi \left(\frac{6}{2\pi}\right) = 4 - 3 = 1$$

$$y = 2 \sin \left[ \frac{\pi}{6}(x-1) \right] + 1$$

$$y = 2 \cos \left[ \frac{\pi}{6}(x-4) \right] + 1$$

c)



$$a = \frac{\max - \min}{2} = \frac{2 - (-4)}{2} = 3$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$

$$c = \max - |a| = 2 - 3 = -1$$

$$d \cos = \frac{\pi}{2}$$

$$d \sin = \frac{\pi}{4}$$

$$y = 3 \sin \left[ 2 \left( x - \frac{\pi}{4} \right) \right] - 1$$

$$y = 3 \cos \left[ 2 \left( x - \frac{\pi}{2} \right) \right] - 1$$

3) A sine function has a maximum value of 7, a minimum value of -1, a phase shift of  $\frac{3\pi}{4}$  radians to the left, and a period of  $\frac{\pi}{2}$ .

Write an equation for this function.

$$a = \frac{\max - \min}{2} = \frac{7 - (-1)}{2} = 4$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{(\frac{\pi}{2})} = 2\pi \left(\frac{2}{\pi}\right) = 4$$

$$d = -\frac{3\pi}{4}$$

$$c = \max - |a| = 7 - 4 = 3$$

$$y = 4 \sin\left[4\left(x + \frac{3\pi}{4}\right)\right] + 3$$

b) Write an equivalent cosine equation for this function.

$$d_{\cos} = d_{\sin} + \frac{\pi}{2k} = -\frac{3\pi}{4} + \frac{\pi}{2(4)} = -\frac{6\pi}{8} + \frac{\pi}{8} = -\frac{5\pi}{8}$$

$$y = 4 \cos\left[4\left(x + \frac{5\pi}{8}\right)\right] + 3$$

4) A cosine function has a maximum value of 1, a minimum value of -5, a phase shift of 2 radians to the right, and a period of 3.

a) Write an equation for this function.

$$a = \frac{\max - \min}{2} = \frac{1 - (-5)}{2} = 3$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{3}$$

$$d = 2$$

$$c = \max - |a| = 1 - 3 = -2$$

$$y = 3 \cos\left[\frac{2\pi}{3}(x - 2)\right] - 2$$

b) Write an equivalent sine equation for this function.

$$d_{\sin} = d_{\cos} - \frac{\pi}{2k} = 2 - \frac{\pi}{\left(\frac{4\pi}{3}\right)} = 2 - \pi\left(\frac{3}{4\pi}\right) = \frac{8}{4} - \frac{3}{4} = \frac{5}{4}$$

$$y = 3 \sin\left[\frac{2\pi}{3}\left(x - \frac{5}{4}\right)\right] - 2$$