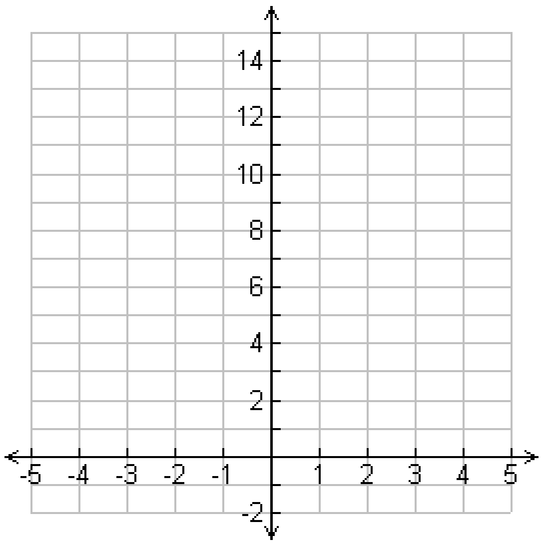
Unit 5: Exponential and Logarithmic Functions
5.1 The Exponential Function and its Inverse

In this section, you will be investigating the exponential function $f(x) = b^x$. Since you will be drawing several curves on each grid, remember to label each curve with its equation.

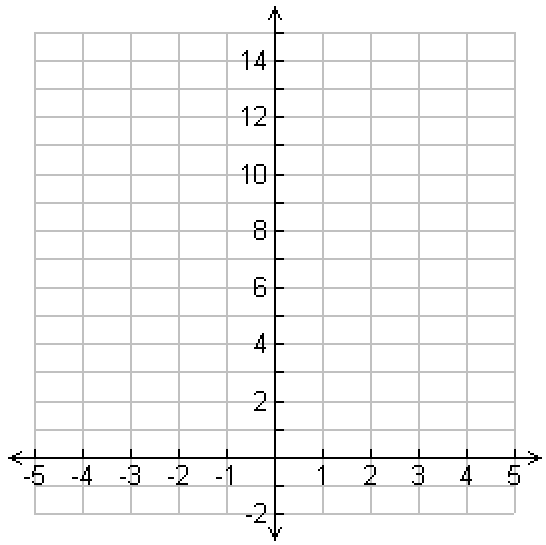
1. Use your graphing calculator to draw the graphs of $y = 2^x$, $y = 5^x$ and $y = 10^x$. Sketch the curves on the grid below. Be sure to label the y-intercept and any asymptotes.



	$y = 2^x$	$y = 5^x$	$y = 10^x$
Domain			
Range			
y-intercept			
Asymptotes			
Increasing/decreasing			

What are the common characteristics of these curves?

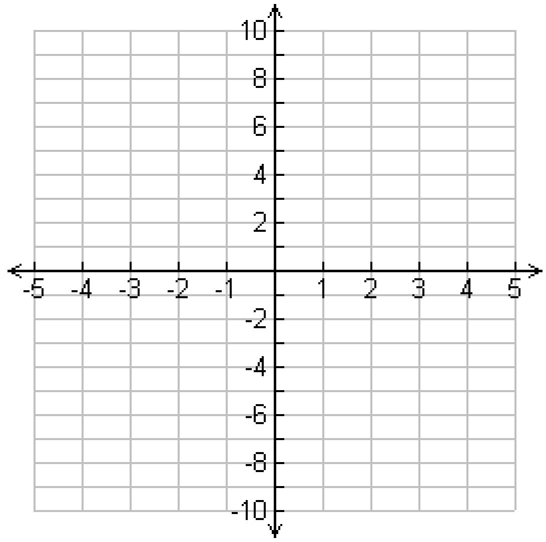
2. Use your graphing calculator to draw the graphs of $y=\left(\frac{1}{3}\right)^x$, $y=\left(\frac{1}{5}\right)^x$ and $y=\left(\frac{1}{10}\right)^x$. Note that we can express these functions as $y=3^{-x}$, $y=5^{-x}$, and $y=10^{-x}$. Sketch the curves on the grid below, labeling fully.



	$y=3^{-x}$	$y=5^{-x}$	$y=10^{-x}$
Domain			
Range			
y-intercept			
Asymptotes			
Increasing/decreasing			

What are the common characteristics of these curves?

3. Graph $y = 3^x$, $y = \left(\frac{1}{3}\right)^x$ and $y = -3^x$. Sketch, labeling the functions carefully.



(a) What transformation on $y = 3^x$ will give $y = \left(\frac{1}{3}\right)^x$ as its image?

(b) What transformation on $y = 3^x$ will give $y = -3^x$ as its image?

(c) The inverse of a function is obtained by ______________________________________.

(d) The inverse of $y = 2^x$ is _________________________.

(e) The graph of the inverse is obtained______________________________________.

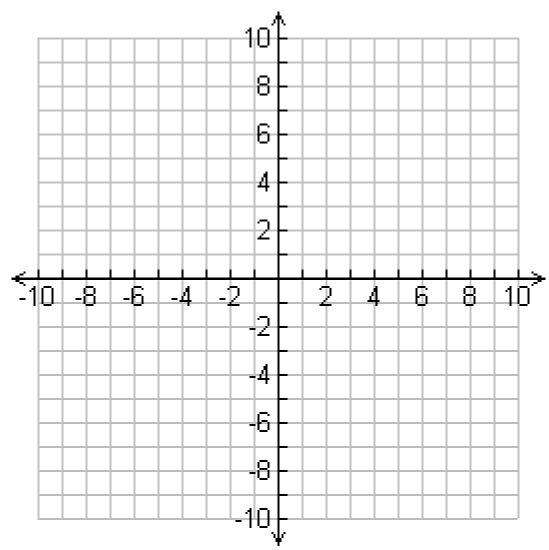
Ex. 1: Graph $y = 2^x$ and its inverse, on the same set of axes.

$y = 2^x$

x	y
-1	
0	
1	
2	
3	

inverse

x	y



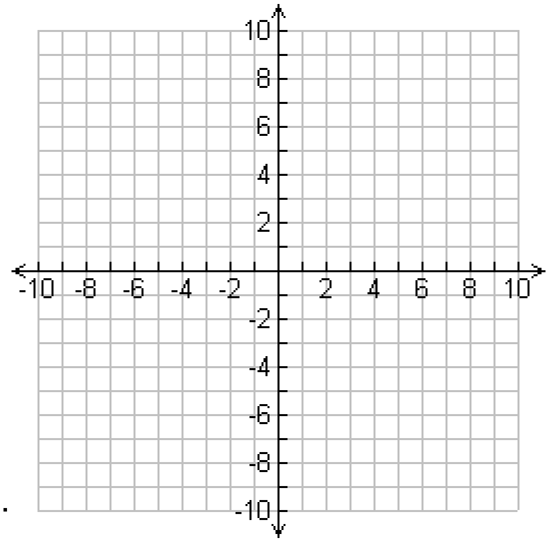
Ex. 2: Graph $y = \left(\frac{1}{2}\right)^x$ and its inverse, on the same set of axes.

$y = \left(\frac{1}{2}\right)^x$

x	y
-3	
-2	
-1	
0	
1	
2	

inverse

x	y



Ex. 3: Write an equation to fit the data in the table of values.

x	y			
-3	$\frac{1}{64}$			
-2	$\frac{1}{16}$			
-1	$\frac{1}{4}$			
0	1			
1	4			
2	16			
3	64			

Logarithmic Functions

Logarithms were first introduced by John Napier in the 17^{th} century for the purpose of simplifying calculations. This was accomplished with the development of logarithmic tables and, soon after, with logarithmic scales on a slide rule. With the introduction of the scientific calculator in the mid-1970s, this application of logarithms for computations became somewhat obsolete; however, logarithms are still used today in many areas such as

- scientific formulas and scales (the pH scale in chemistry and the Richter scale for measuring and comparing the intensity of earthquakes),
- astronomy (order of magnitude calculations comparing relative size of massive bodies),
- modelling and solving problems involving exponential growth and decay, and many areas of calculus

Introduction

We will begin our study of logarithms by introducing and exploring the **logarithmic function**. The logarithmic function is simply the inverse of the exponential function.

Exponential Form ←→ **Logarithmic Form**

$x = b^y$ $\quad$ $y = \log_b x$ $(b > 0$ and $b \neq 1)$

The logarithm of a number x with a given base is the exponent to which that base must be raised to yield x.

What is a Logarithm?

Logarithms can be set to any base. The LOG key on your calculator represents $\log_{10}$. Record the results in the space provide. The first example is done for you.

Logarithm	Value
$\log 100 = 2$	$10^2 = 100$
$\log 10$	
$\log 1000$	
$\log 0.01 =$	
$\log 0.0001 =$	
$\log \sqrt{10} =$	
$\log \sqrt{10000} =$	
$\log 0$	
$\log(-3)$	
$\log_3(81)$	
$\log_2(16)$	
$\log_6(216)$	
$\log_{25}\left(\frac{1}{625}\right)$	
$\log_4(64)$	

Ex. 1: Change to exponential form.

a. $\log_2 8 = 3$

b. $\log_2 32 = 5$

Ex. 2: Change to logarithmic form.

a. $4^3 = 64$

b. $\left(\frac{1}{2}\right)^{-4} = 16$

Ex. 3: Evaluate the following.

a. $\log_5 25$

b. $\log_3 27$

c. $\log_2\left(\frac{1}{4}\right)$

d. $\log_{\frac{1}{3}} 27$

e. $\log_2\left(\frac{1}{4}\right)+\log_{\frac{1}{2}} 4$

f. $\log_3\left(27\times\sqrt{27}\right)$

Ex. 4: For each function, *y*=*g*(*x*), determine the equation of $y=g^{-1}(x)$.

a. $g(x)=3^{2(x-1)}+5$

b. $g(x)=2\log_5(x+4)-1$

c. $g(x)=-\log_3\left(\frac{1}{2}(x-3)\right)+4$

Ex.5. Determine the point of intersection of $f(x)=2\log_2(x-2)$ and $g(x)=2-\log_3(x-3)$.

To conclude…

1. An exponential function of the form $y = b^x$, $b > 0$, $b \neq 1$, has
   - a repeating pattern of finite differences
   - a rate of change that is increasing proportional to the function for $b > 1$
   - a rate of change that is decreasing proportional to the function of $0 < b < 1$
   - has domain $\{x \mid x \in R\}$
   - has range $\{y \mid y > 0, y \in R\}$
   - has y-intercept 1
   - has horizontal asymptote with equation $y = 0$

2. The inverse of $y = b^x$ is a function that can be written as $x = b^y$. This function
   - has domain $\{x \mid x > 0, x \in R\}$
   - has range $\{y \mid y \in R\}$
   - has x-intercept 1
   - has vertical asymptote at $x = 0$
   - is a reflection of $y = b^x$ about the line $y = x$

Practice

Part A - Multiple Choice.

_____1. The range of the function $f(x) = 4(2)^x + 1$ is:

A. $y \in R$ B. $y > 4,\ y \in R$ C. $y < 1,\ y \in R$ D. $y > 1,\ y \in R$

_____2. Another way to write $2^{-3} = \frac{1}{8}$ is

A. $\log_2(-3) = 8$ B. $\log_2(-3) = \frac{1}{8}$ C. $\log_{\frac{1}{8}}(2) = -3$ D. $\log_2\left(\frac{1}{8}\right) = -3$

_____3. The domain of the logarithmic function is:

A. $x \in R$ B. $x < 0,\ x \in R$ C. $x > 0,\ x \in R$ D. $x > 1,\ x \in R$

_____ 4. The graph of $y = 5\log_2(3x + 12) - 5$ has a vertical asymptote at

A. $x = -5$ B. $x = -12$ C. $x = 4$ D. $x = -4$

Part B - Short Answer

1. Find the value(s) of x such that $\log_x(19x - 30) = 3$.

2. Find the inverse of the following functions:

a) $f(x) = \dfrac{2^x}{1 - 2^x}$

b) $f(x) = -2\log_3\left(\frac{2}{3}(x+1)\right) - 2$