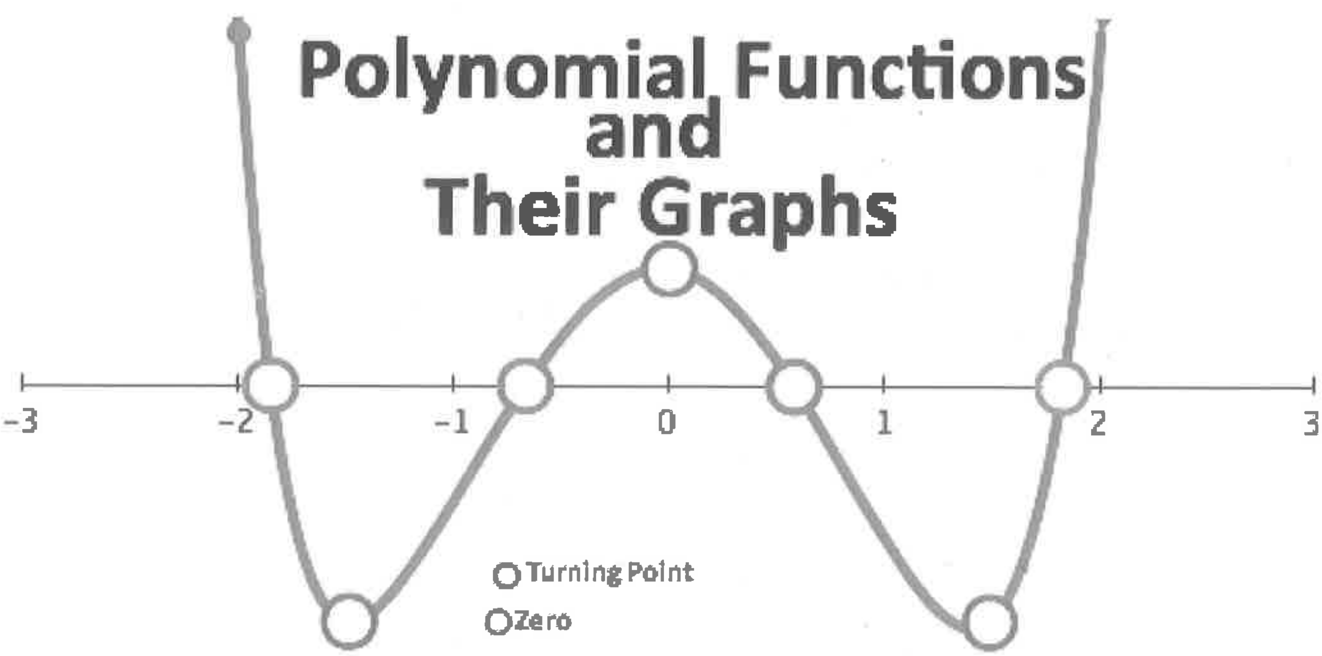
UNIT 1

Chapter 1- Polynomial Functions

WORKBOOK

SOLUTION MANUAL

MHF4U



W1 – 1.1 – Power Functions
MHF4U

ANSWERS

1) Identify which of the following are polynomial functions:

a) $p(x) = \cos x$

b) $h(x) = -7x$

c) $f(x) = 2x^4$

d) $y = 3x^5 - 2x^3 + x^2 - 1$

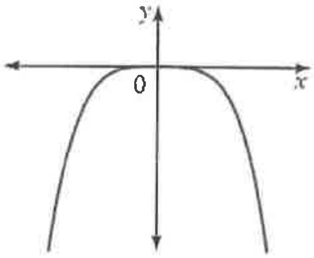
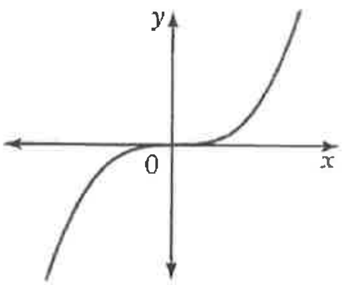
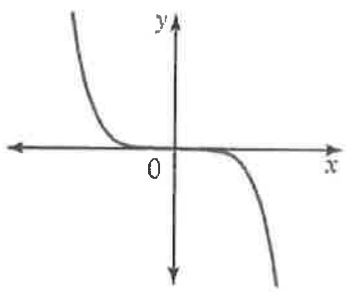
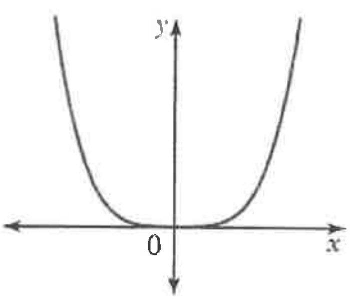
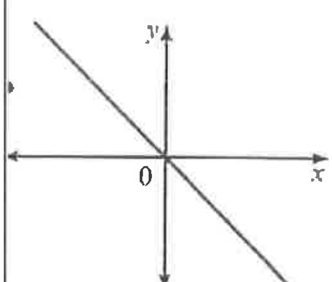
e) $k(x) = 8^x$

f) $y = x^{-3}$

2) State the degree and the leading coefficient of each polynomial

Polynomial	Degree	Leading Coefficient
$y = 5x^4 - 3x^3 + 4$	4	5
$y = -x + 2$	1	-1
$y = 8x^2$	2	8
$y = -\frac{x^3}{4} + 4x - 3$	3	$-\frac{1}{4}$
$y = -5$	0	-5
$y = x^2 - 3x$	2	1

3) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	Even	−	D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid y \leq 0\}$	Line	Q3 to Q4
	Odd	+	D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R}\}$	Point	Q3 to Q1
	Odd	−	D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R}\}$	Point	Q2 to Q4
	Even	+	D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid y \geq 0\}$	Line	Q2 to Q1
	Odd	−	D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R}\}$	Point	Q2 to Q4

4) Match each function to its end behavior

$y = -x^3$ $\quad y = \frac{3}{7}x^2$ $\quad y = 5x$

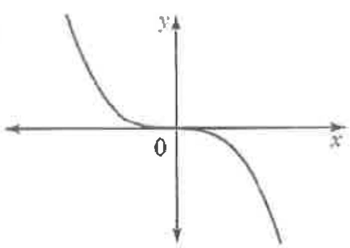
$y = 4x^5$ $\quad y = -x^6$ $\quad y = -0.1x^{11}$

$y = 2x^4$ $\quad y = -9x^{10}$

End Behaviour	Functions
Q3 to Q1	$y=4x^5$, $y = 5x$
Q2 to Q4	$y=-x^3$, $y = -0.1x^{11}$
Q2 to Q1	$y = 2x^4$, $y = \frac{3}{7}x^2$
Q3 to Q4	$y=-x^6$, $y = -9x^{10}$

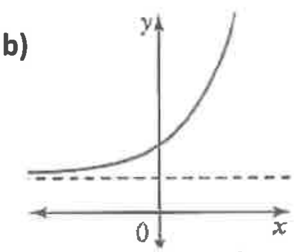
5) Determine whether each graph represents a power function, exponential function, a periodic function, or none of these.

a)



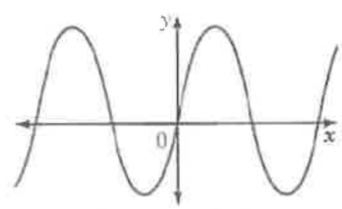
Power

b)



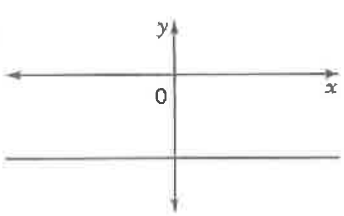
Exponential

c)



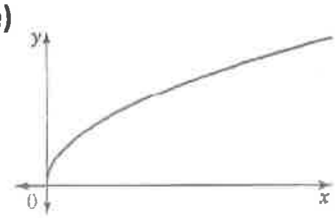
Periodic

d)



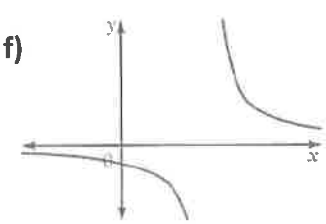
Power

e)



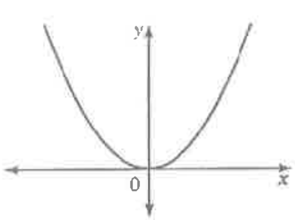
None
(square root)

f)



None
(rational)

g)



Power

Answer Key

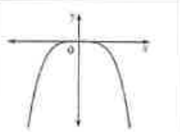
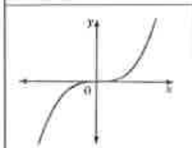
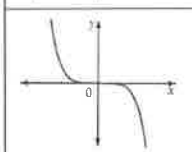
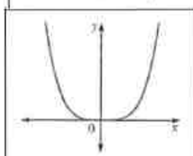
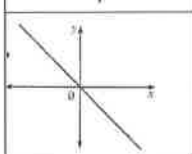
<u>W1</u>

1) a) No **b)** Yes **c)** Yes **d)** Yes **e)** No **f)** No

2)

Polynomial	Degree	Leading Coefficient
$y = 5x^4 - 3x^3 + 4$	4	5
$y = -x + 2$	1	-1
$y = 8x^2$	2	8
$y = -\frac{x^3}{4} + 4x - 3$	3	$-\frac{1}{4}$
$y = -5$	0	-5
$y = x^2 - 3x$	2	1

3)

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	EVEN	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} \mid y \leq 0\}$	Line	Q3 to Q4
	ODD	POSITIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q3 to Q1
	ODD	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q2 to Q4
	EVEN	POSITIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} \mid y \geq 0\}$	Line	Q2 to Q1
	ODD	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q2 to Q4

4)

End Behaviour	Functions
Q3 to Q1	$y = 4x^5, y = 5x$
Q2 to Q4	$y = -x^3, y = -0.1x^{11}$
Q2 to Q1	$y = 2x^4, y = \frac{3}{7}x^2$
Q3 to Q4	$y = -x^6, y = -9x^{10}$

5) a) power **b)** exponential **c)** periodic **d)** power **e)** none (square root) **f)** none (rational) **g)** power

W2 – 1.2 – Characteristics of Polynomial Functions
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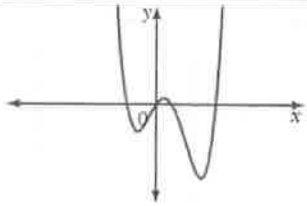
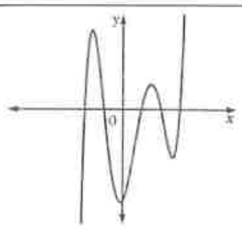
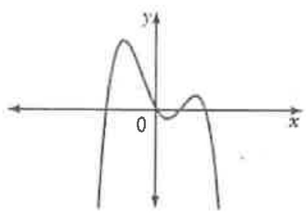
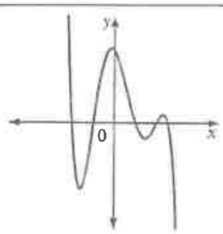
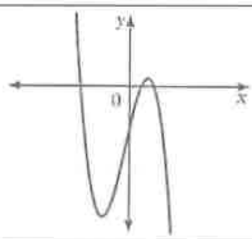
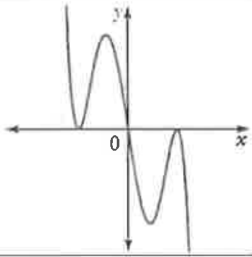
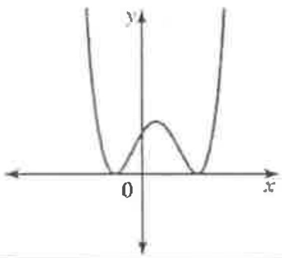
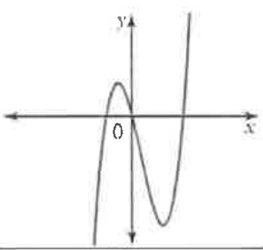
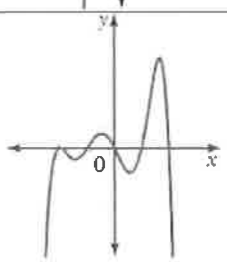
ANSWERS

Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q2 to Q1	None	3	4	4
	+	Odd	Q3 to Q1	None	4	5	5
	–	Even	Q3 to Q4	None	3	4	4
	–	Odd	Q2 to Q4	None	4	5	5
	–	Odd	Q2 to Q4	Point	2	3	3

2) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	−	Odd	Q2 to Q4	Point	4	3	5
	+	Even	Q2 to Q1	Line	3	2	4
	+	Odd	Q3 to Q1	Point	2	3	3
	−	Even	Q3 to Q4	None	5	5	6

3) Complete the following table

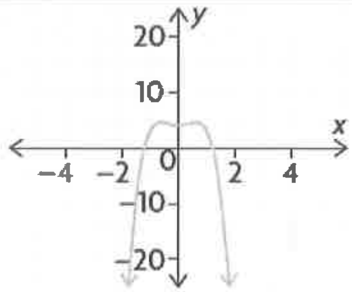
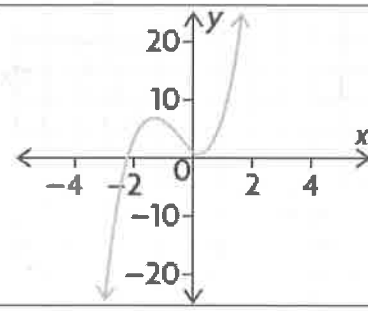
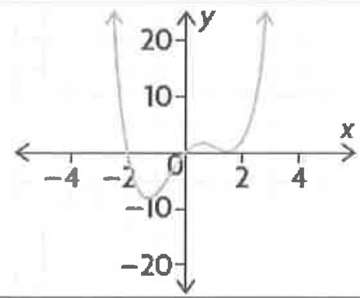
Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	−	Even	Q3 to Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	+	Odd	Q3 to Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	−	Odd	Q2 to Q4	2, 0	3, 2, 1
$h(x) = 2x(x-5)(3x+2)(4x-3)$	4	+	Even	Q2 to Q1	3, 1	4, 3, 2, 1, 0

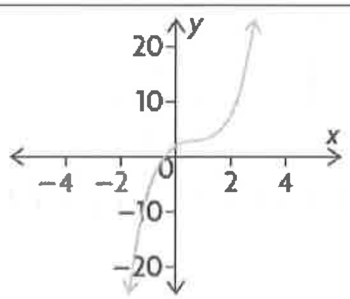
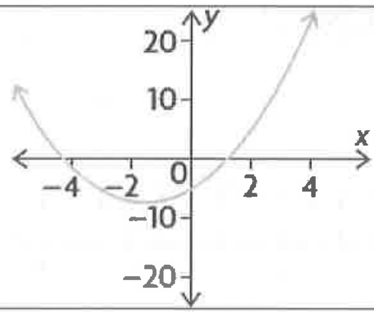
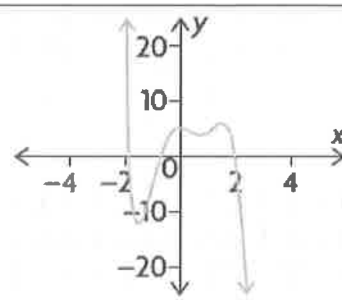
4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

A) $y = 2x^3 - 4x^2 + 3x + 2$ **B)** $y = -4x^4 + 3x^2 + 4$ **C)** $y = x^2 + 3x - 5$

D) $y = x^4 - x^3 - 4x^2 + 5x$ **E)** $y = -2x^5 + 3x^4 + 6x^3 - 10x^2 + 2x + 5$

F) $y = 3x^3 + 5x^2 - 3x + 1$

		
B	A	D

		
F	C	E

5) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

Degree 2

a) second differences = -8

$-8 = a(2!)$

$-8 = 2a$

$\boxed{-4 = a}$

Degree 4

b) fourth differences = 24

$24 = a(4!)$

$24 = 24a$

$\boxed{1 = a}$

6) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
−3	−45
−2	−16
−1	−3
0	0
1	−1
2	0
3	9
4	32

1st: 29, 13, 3, −1, 1, 9, 23

2nd: −16, −10, −4, 2, 8, 14

3rd: 6, 6, 6, 6, 6

Degree = 3

$6 = a(3!)$

$6 = 6a$

$\boxed{1 = a}$

b)

x	y
−2	−40
−1	12
0	20
1	26
2	48
3	80
4	92
5	30

1st: 52, 8, 6, 22, 32, 12, −62

2nd: −44, −2, 16, 10, −20, −74

3rd: 42, 18, −6, −30, −54

4th: −24, −24, −24, −24

Degree = 4

$-24 = a(4!)$

$-24 = 24a$

$\boxed{-1 = a}$

7) By analyzing the impact of growing economic conditions, a demographer establishes that the predicted population, P, of a town t years from now can be modelled by the function
$P(t) = 6t^4 - 5t^3 + 200t + 12000$

a) What is the value of the constant finite differences

$$\text{Finite Differences} = a(n!)$$
$$= 6(4!)$$
$$= 144$$

b) What is the current population of the town $P(0) = 6(0)^4 - 5(0)^3 + 200(0) + 12\,000$

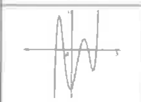
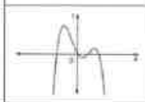
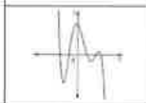
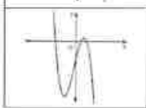
$$= 12\,000$$

c) What will the population of the town be 10 years from now

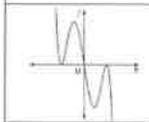
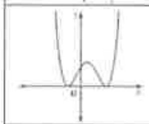
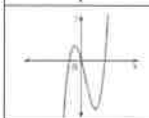
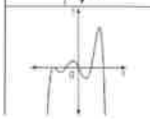
$$P(10) = 6(10)^4 - 5(10)^3 + 200(10) + 12\,000$$
$$= 69\,000$$

ANSWER KEY

1)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	POS	EVEN	Q2 to Q1	NONE	3	4	4
	POS	ODD	Q3 to Q1	NONE	4	5	5
	NEG	EVEN	Q3 to Q4	NONE	3	4	4
	NEG	ODD	Q2 to Q4	NONE	4	5	5
	NEG	ODD	Q2 to Q4	POINT	2	3	3

2)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	NEG	ODD	Q2 to Q4	Point	4	3	5
	POS	EVEN	Q2 to Q1	Line	3	2	4
	POS	ODD	Q3 to Q1	Point	2	3	3
	NEG	EVEN	Q3 to Q4	None	5	5	6

3)

Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	NEG	EVEN	Q3 → Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	POS	ODD	Q3 → Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	NEG	ODD	Q2 → Q4	2, 0	3, 2, 1
$h(x) = 2x(x-5)(3x+2)(4x-3)$	4	POS	EVEN	Q2 → Q1	3, 1	4, 3, 2, 1, 0

4) B F D
A C E

5) a) degree 2, $a = -4$ **b)** degree 4, $a = 1$

6) a) degree 3, $a = 1$ **b)** degree 4, $a = -1$

7) a) 144 **b)** 12 000 **c)** 69 000

W3 – 1.3 – Factored Form Polynomial Functions
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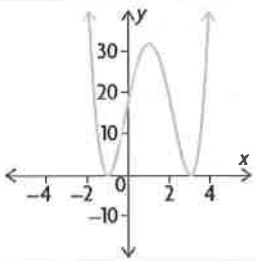
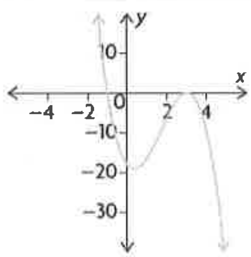
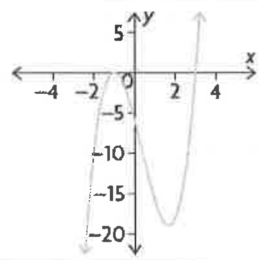
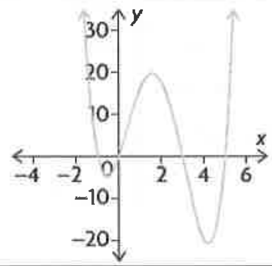
ANSWERS

1) Match each equation with the most suitable graph. Write the letter of the equation beneath the matching graph.

A) $f(x) = 2(x+1)^2(x-3)$ **B)** $f(x) = (x+1)^2(x-3)^2$

C) $f(x) = -2(x+1)(x-3)^2$ **D)** $f(x) = x(x+1)(x-3)(x-5)$

			
B	C	A	D

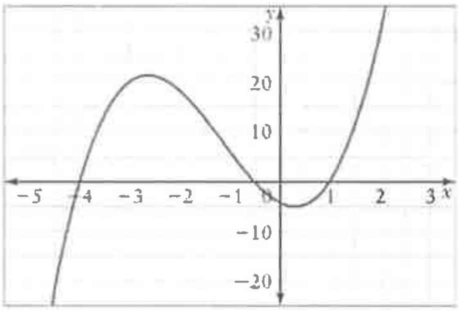
2) Complete the table

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x-4)(x+3)(2x-1)$	$(x)(x)(x)$ $= x^3$ 3	$(1)(1)(2)$ $= 2$	Q3 to Q1	$(4,0)$ $(-3,0)$ $(\frac{1}{2},0)$
$g(x) = -2(x+2)(x-2)(1+x)(x-1)$	$(x)(x)(x)(x)$ $= x^4$ 4	$-2(1)(1)(1)(1)$ $= -2$	Q3 to Q4	$(-2,0)$ $(-1,0)$ $(2,0)$ $(1,0)$
$h(x) = (3x+2)^2(x-4)(x+1)(2x-3)$	$(x^2)(x)(x)(x)$ $= x^5$ 5	$(3^2)(1)(1)(2)$ $= 18$	Q3 to Q1	$(-\frac{2}{3},0)$ $(\frac{3}{2},0)$ $(4,0)$ $(-1,0)$
$p(x) = -(x+5)^3(x-5)^3$	$(x^3)(x^3)$ $= x^6$ 6	$-1(1^3)(1^3)$ $= -1$	Q3 to Q4	$(-5,0)$ $(5,0)$

3) For each graph, state…

- **i)** the least possible degree and the sign of the leading coefficient
- **ii)** the x-intercepts (specify order of zero) and the factors of the function
- **iii)** the intervals where the function is positive/negative

a)



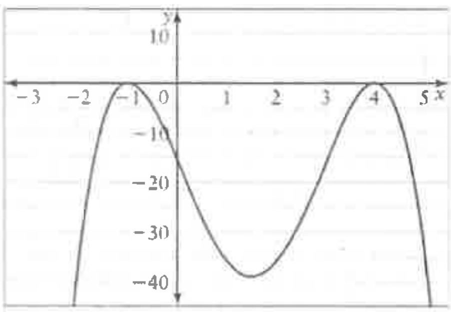
i) degree: 3
leading coefficient: Positive

ii) x-intercepts: $-4, -\frac{1}{2}, 1$
factors: $(x+4), (2x+1), (x-1)$

iii)

Interval	$(-\infty, -4)$	$(-4, -\frac{1}{2})$	$(-\frac{1}{2}, 1)$	$(1, \infty)$
Sign	−	+	−	+

b)



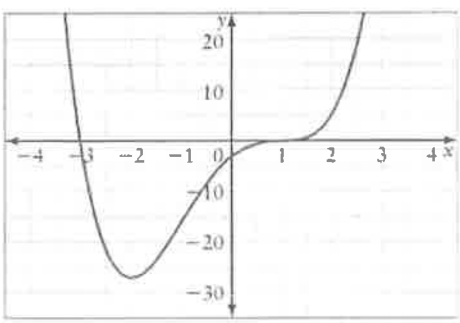
i) degree: 4
leading coefficient: Negative

ii) x-intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2, (x-4)^2$

iii)

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	−	−	−

c)



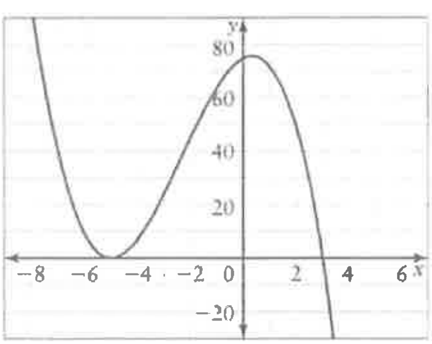
i) degree: 4
leading coefficient: Positive

ii) x-intercepts: -3, 1 (order 3)
factors: $(x+3), (x-1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	−	+

d)



i) degree: 3
leading coefficient: Negative

ii) x-intercepts: -5 (order 2), 3
factors: $(x+5)^2, (x-3)$

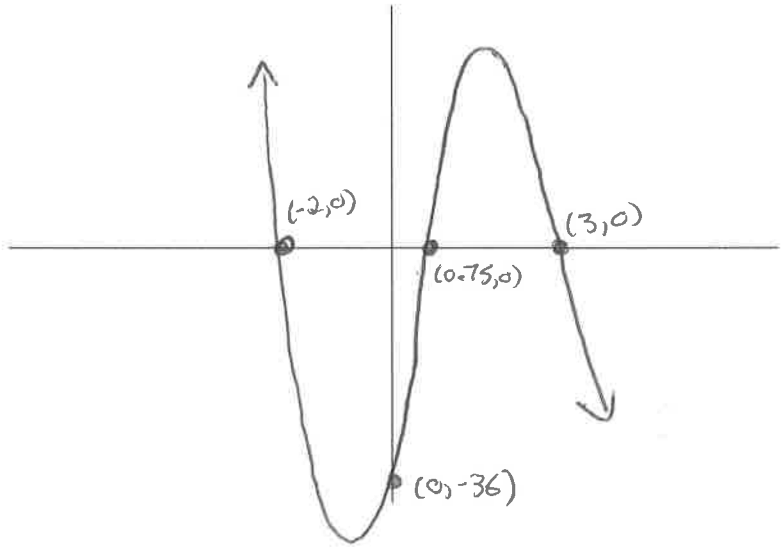
iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	−

4) For each function, complete the chart and sketch a possible graph of the function labelling key points.

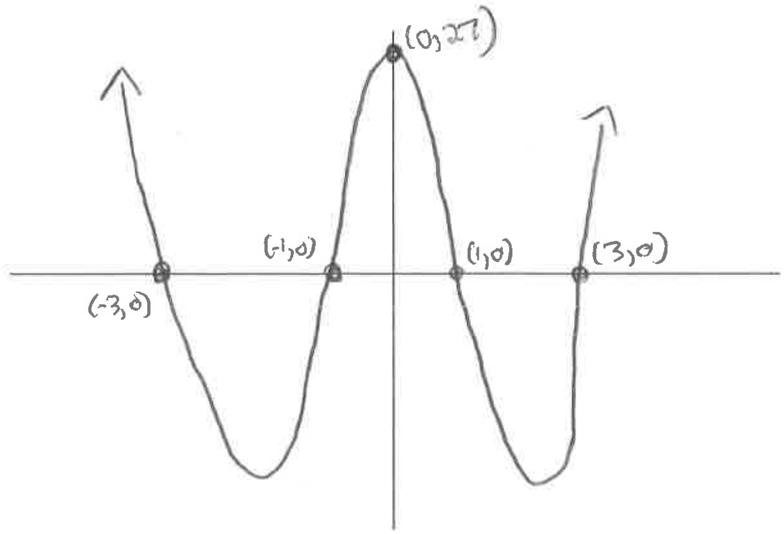
a) $f(x) = -2(x-3)(x+2)(4x-3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$-2(1)(1)(4)$ $= -8$	Q2 to Q4	$(3,0)$ $(-2,0)$ $(3/4,0)$	$f(0) = -2(-3)(2)(-3)$ $= -36$ $(0,-36)$



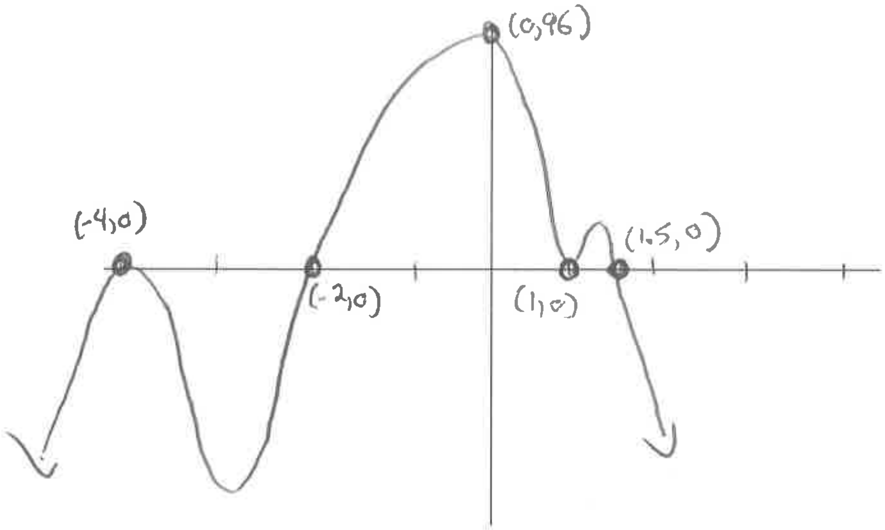
b) $g(x) = (x-1)(x+3)(1+x)(3x-9)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)(x)$ $= x^4$ Degree 4	$(1)(1)(1)(3)$ $= 3$	Q2 to Q1	$(1,0)$ $(3,0)$ $(-3,0)$ $(-1,0)$	$g(0) = (-1)(3)(1)(-9)$ $= 27$



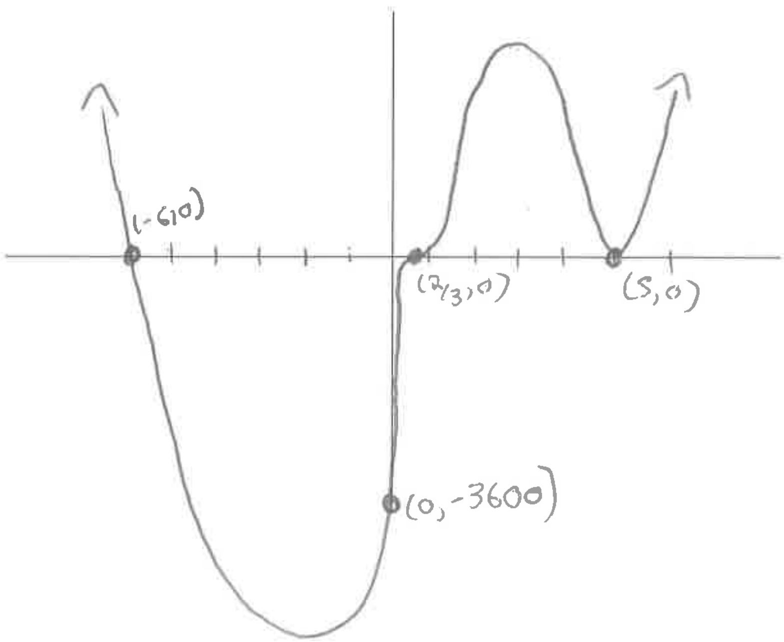
c) $h(x) = -(x+4)^2(x-1)^2(x+2)(2x-3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x^2)(x^2)(x)(x)$ $= x^6$ Degree 6	$-1(1^2)(1^2)(1)(2)$ $= -2$	Q3 to Q4	$(-4,0)$ order 2 $(1,0)$ order 2 $(-2,0)$ $(3/2,0)$	$h(0) = -1(4)^2(-1)^2(2)(-3)$ $= 96$



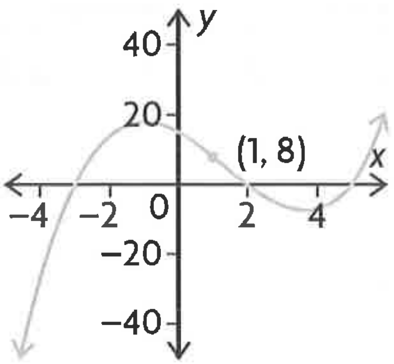
d) $p(x) = 3(x+6)(x-5)^2(3x-2)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x^2)(x^3)$ $= x^6$ Degree 6	$3(1)(1)^2(3)^3$ $= 81$	Q2 to Q1	$(-6,0)$ $(5,0)$ order 2 $(2/3,0)$ order 3	$p(0) = 3(6)(-5)^2(-2)^3$ $= -3600$



5) Write the equation of each function

a)



$f(x) = k(x+3)(x-2)(x-5)$

$8 = k(1+3)(1-2)(1-5)$

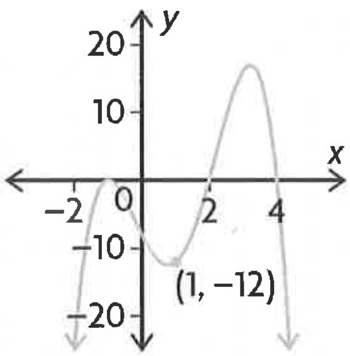
$8 = k(4)(-1)(-4)$

$8 = 16k$

$k = 1/2$

$\boxed{f(x) = \frac{1}{2}(x+3)(x-2)(x-5)}$

b)



$g(x) = k(x+1)^2(x-2)(x-4)$

$-12 = k(1+1)^2(1-2)(1-4)$

$-12 = k(4)(-1)(-3)$

$-12 = 12k$

$k = -1$

$\boxed{g(x) = -(x+1)^2(x-2)(x-4)}$

6) Determine an equation for a quintic function with zeros -1 (order 3) and 3 (order 2) that passes through the point (-2, 50)

$h(x) = k(x+1)^3(x-3)^2$

$50 = k(-2+1)^3(-2-3)^2$

$50 = k(-1)^3(-5)^2$

$50 = k(-1)(25)$

$50 = -25k$

$k = -2$

$\boxed{h(x) = -2(x+1)^3(x-3)^2}$

7) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$

$2x^2 - x - 1$ P: -2 S: -1 (-2 and 1)

$= 2x^2 - 2x + 1x - 1$

$= 2x(x-1) + 1(x-1)$

$= (x-1)(2x+1)$

$x^2 - 3x - 4$ P: -4 S: -3 (-4 and 1)

$= (x-4)(x+1)$

$f(x) = (x-1)(2x+1)(x-4)(x+1)$

$0 = (x-1)(2x+1)(x-4)(x+1)$

$x_1 = 1$ $x_2 = -1/2$ $x_3 = 4$ $x_4 = -1$

Answer Key

1) B C A D

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x-4)(x+3)(2x-1)$	3	2	Q3 → Q1	(4, 0) (-3, 0) $\left(\frac{1}{2}, 0\right)$
$g(x) = -2(x+2)(x-2)(1+x)(x-1)$	4	-2	Q3 → Q4	(-2, 0) (-1, 0) (1, 0) (2, 0)
$h(x) = (3x+2)^2(x-4)(x+1)(2x-3)$	5	18	Q3 → Q1	(4, 0) (-1, 0) $\left(-\frac{2}{3}, 0\right)$ $\left(\frac{3}{2}, 0\right)$
$p(x) = -(x+5)^3(x-5)^3$	6	-1	Q3 → Q4	(-5, 0) (5, 0)

3) a) **i)** degree: 3
leading coefficient: positive

ii) x-intercepts: -4, -0.5, 1
factors: $(x+4)$, $(2x+1)$, and $(x-1)$

iii)

Interval	$(-\infty,-4)$	$(-4,-0.5)$	$(-0.5,1)$	$(1,\infty)$
Sign	-	+	-	+

b) **i)** degree: 4
leading coefficient: negative

ii) x-intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2$, and $(x-4)^2$

iii)

Interval	$(-\infty,-1)$	$(-1,4)$	$(4,\infty)$
Sign	-	-	-

c) **i)** degree: 4
leading coefficient: positive

ii) x-intercepts: -3, 1 (order 3)
factors: $(x+3)$, and $(x-1)^3$

iii)

Interval	$(-\infty,-3)$	$(-3,1)$	$(1,\infty)$
Sign	+	-	+

d) **i)** degree: 3
leading coefficient: negative

ii) x-intercepts: -5 (order 2), 3
factors: $(x+5)^2$, and $(x-3)$

iii)

Interval	$(-\infty,-5)$	$(-5,3)$	$(3,\infty)$
Sign	+	+	-

4) a)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
3	-8	Q2 → Q4	(3, 0) (-2, 0) $\left(\frac{3}{4}, 0\right)$	(0, -36)

b)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
4	3	Q2 → Q1	(1, 0) (-3, 0) (-1, 0) (3,0)	(0, 27)

c)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	-2	Q3 → Q4	(-4, 0) order 2 (1, 0) order 2 (-2, 0) (1.5,0)	(0, 96)

d)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	81	Q2 → Q1	(-6, 0) (5, 0) order 2 $\left(\frac{2}{3}, 0\right)$ order 3	(0, -3600)

5) a) $y = 0.5(x+3)(x-2)(x-5)$ **b)** $y = -(x+1)^2(x-2)(x-4)$

$y = -2(x+1)^3(x-3)^2$

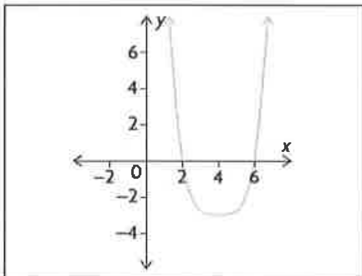
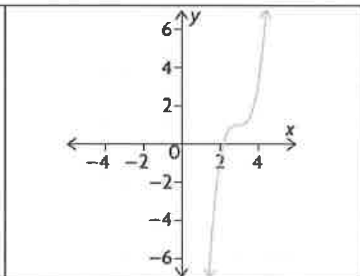
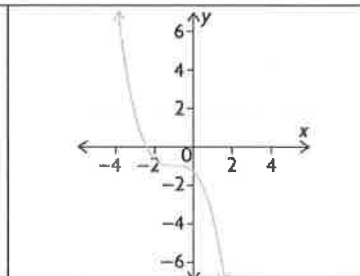
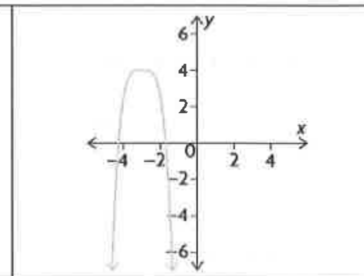
7) 4, 1, -1, and -0.5

W4 – 1.4 – Transformations
MHF4U

ANSWERS

1) Match each graph with the corresponding function.

A) $y = 2(x-3)^3 + 1$ **B)** $y = -\frac{1}{3}(x+1)^3 - 1$ **C)** $y = 0.2(x-4)^4 - 3$ **D)** $y = -1.5(x+3)^4 + 4$

			
C	A	B	D

2) List a good set of key points for the following parent functions:

$f(x) = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

$f(x) = x^5$	
x	y
-2	-32
-1	-1
0	0
1	1
2	32

3) Identify the a, k, d and c values and explain what transformation is occurring to the parent function:

a) $f(x) = -2(x-1)^2$

$a = -2$; vertical reflection
vertical stretch bafo 2 $(-2y)$

$d = 1$; shift 1 unit RIGHT $(x+1)$

b) $g(x) = [-\frac{1}{3}(x+5)]^4 - 1$

$k = -\frac{1}{3}$; horizontal reflection
horizontal stretch bafo 3 $(-3x)$

$d = -5$; shift 5 units LEFT $(x-5)$

$c = -1$; shift down 1 unit $(y-1)$

4) Write the full equation given the parent function and the transforming function:

a) $f(x) = x^5$, $g(x) = -3f[2(x+5)] - 1$

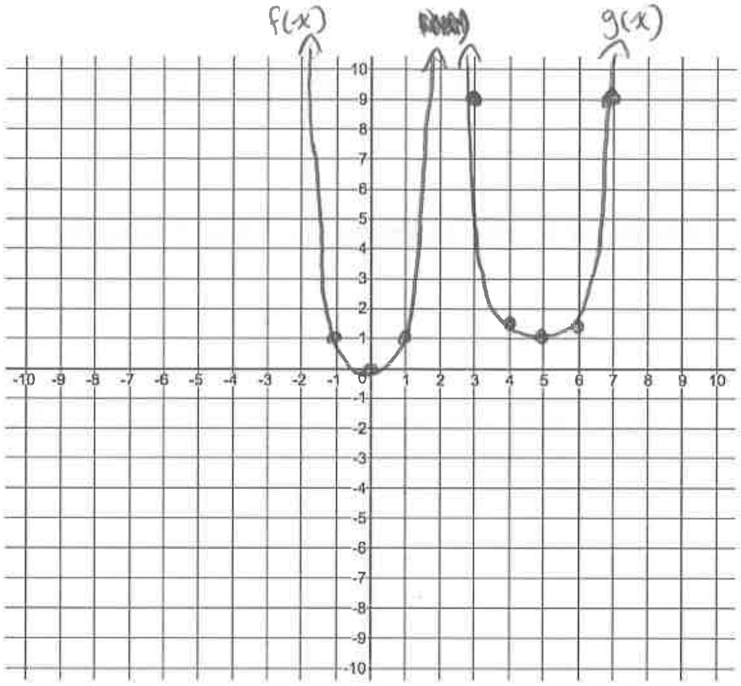
$g(x) = -3[2(x+5)]^5 - 1$

b) $f(x) = x^3$, $g(x) = \frac{1}{2}f\left[-\frac{1}{4}(x-4)\right] + 7$

$g(x) = \frac{1}{2}[-\frac{1}{4}(x-4)]^3 + 7$

5) For the following questions, use the key points of the parent function to perform transformations. Graph the parent and transformed function. Write the equation of the transformed function.

a) $f(x) = x^4$ $\qquad g(x) = \frac{1}{2}f[-(x-5)] + 1$



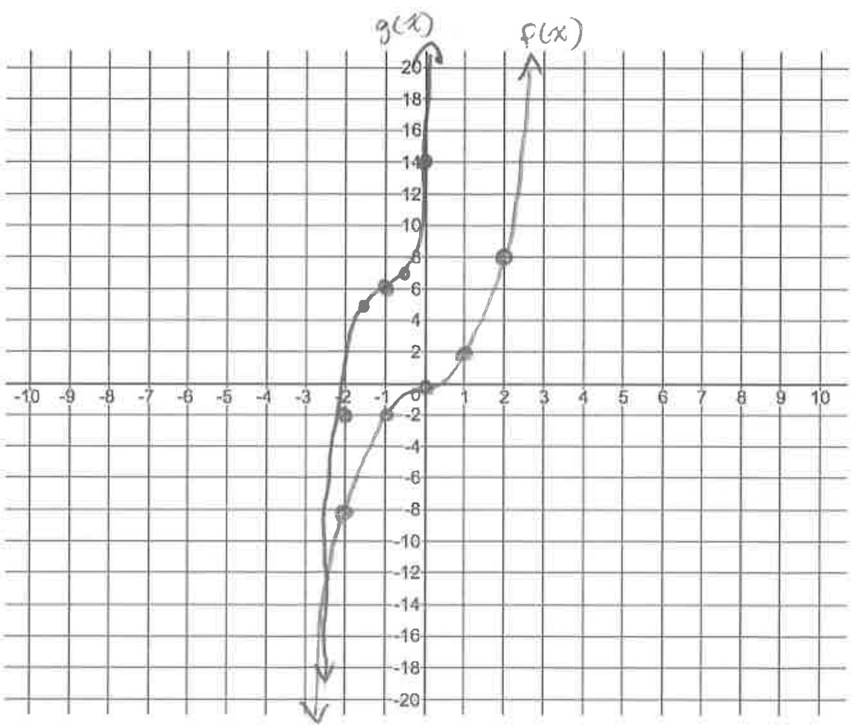
$f(x) = x^4$

x	y
-2	16
-1	1
0	0
1	1
2	16

$g(x) = \frac{1}{2}[-(x-5)]^4 + 1$

$-x+5$	$\frac{y}{2}+1$
7	9
6	1.5
5	1
4	1.5
3	9

b) $f(x) = x^3$ $\qquad g(x) = -f[-2(x+1)] + 6$



$f(x) = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

$g(x) = -[-2(x+1)]^3 + 6$

$\frac{x}{-2} - 1$	$-y+6$
0	14
-0.5	7
-1	6
-1.5	5
-2	-2

6) Write an equation for the function that results from the given transformations.

$d = -2$ $c = 3$

a) The function $f(x) = x^4$ is translated 2 units to the left and 3 units up.

$g(x) = (x+2)^4 + 3$

$k = 1/5$ $d = -12$

b) The function $f(x) = x^5$ is stretched horizontally by a factor of 5 and translated 12 units to the left.

$g(x) = \left[\frac{1}{5}(x+12)\right]^5$

$a = -3$

c) The function $f(x) = x^4$ is stretched vertically by a factor of 3, reflected vertically in the x-axis, and translated 6 units down and 1 unit to the left.

$c = -6$ $d = -1$

$g(x) = -3(x+1)^4 - 6$

$a = -1$ $k = \frac{-1}{5}$

d) The function $f(x) = x^6$ is reflected vertically in the x-axis, stretched horizontally by a factor of 5, reflected horizontally in the y-axis, and translated 3 units down and 1 unit to the right. $c = -3$, $d = 1$

$g(x) = -\left[-\frac{1}{5}(x-1)\right]^6 - 3$

ANSWER KEY

1) C A B D

2)

$f(x) = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

$f(x) = x^5$	
x	y
-2	-32
-1	-1
0	0
1	1
2	32

3) a) $a = -2$; vertical reflection and vertical stretch by a factor of 2 $(-2y)$
$d = 1$; shift right 1 unit $(x+1)$

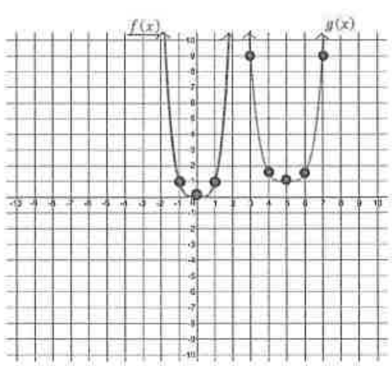
b) $k = -\frac{1}{3}$; horizontal reflection and horizontal stretch by a factor of 3 $(-3x)$
$d = -5$; shift left 5 units $(x-5)$
$c = -1$; shift down 1 unit $(y-1)$

4) a) $g(x) = -3[2(x+5)]^5 - 1$ **b)** $g(x) = \frac{1}{2}[-\frac{1}{4}(x-4)]^3 + 7$

5) a)

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

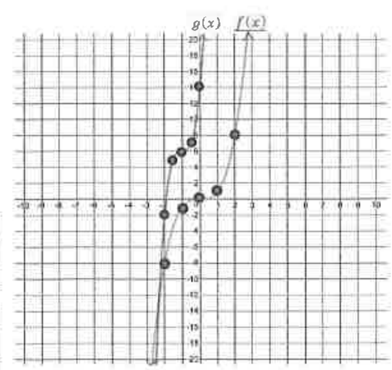
$g(x) = \frac{1}{2}[-(x-5)]^4 + 1$	
$-x+5$	$\frac{y}{2}+1$
7	9
6	1.5
5	1
4	1.5
3	9



b)

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$g(x) = -[-2(x+1)]^3 + 6$	
$\frac{x}{-2} - 1$	$-y+6$
0	14
-0.5	7
-1	6
-1.5	5
-2	-2



6) a) $g(x) = (x+2)^4 + 3$ **b)** $g(x) = \left[\frac{1}{5}(x+12)\right]^5$ **c)** $g(x) = -3(x+1)^4 - 6$ **d)** $g(x) = -\left[-\frac{1}{5}(x-1)\right]^6 - 3$

W5 – 1.3 – Symmetry in Polynomial Functions
MHF4U

ANSWERS

1) Determine whether each function is even, odd, or neither. Does it have line symmetry about the y-axis, point symmetry about the origin, or neither?

a) $y = x^4 - x^2$

Even

Line symmetry about y-axis

b) $y = -2x^3 + 5x$

Odd

Point symmetry about origin

c) $y = -4x^5 + 2x^2$

Neither

d) $y = x(2x+1)^2(x-4)$

$= x(4x^2+4x+1)(x-4)$

$= (4x^3+4x^2+x)(x-4)$

$= 4x^4 - 16x^3 + 4x^3 - 16x^2 + x^2 - 4x$

$= 4x^4 - 12x^3 - 15x^2 - 4x$

$\boxed{\text{Neither}}$

e) $y = -2x^6 + x^4 + 8$

Even

Line symmetry about the y-axis

2) State whether each function is even or odd. Verify algebraically.

a) $f(x) = x^4 - 13x^2 + 36$ Even

$f(-x) = (-x)^4 - 13(-x)^2 + 36$

$= (-1)^4(x)^4 - 13(-1)^2(x)^2 + 36$

$= x^4 - 13x^2 + 36$

$\therefore f(x) = f(-x)$

b) $g(x) = 6x^5 - 7x^3 - 3x$ ODD

$-g(x) = -1[6x^5 - 7x^3 - 3x]$

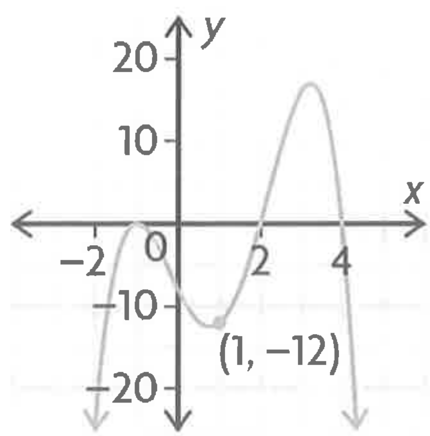
$= -6x^5 + 7x^3 + 3x$ $\therefore -g(x) = g(-x)$

$g(-x) = 6(-x)^5 - 7(-x)^3 - 3(-x)$

$= 6(-1)^5(x)^5 - 7(-1)^3(x)^3 - 3(-1)(x)$

$= -6x^5 + 7x^3 + 3x$

3) Use the given graph to state:



a) x-intercepts −1 (order 2), 2, and 4

b) number of turning points 3

c) least possible degree 4

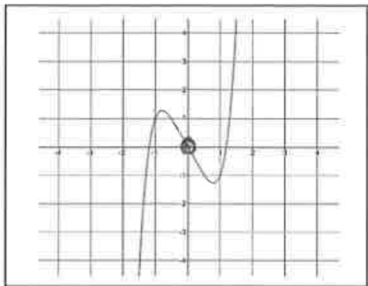
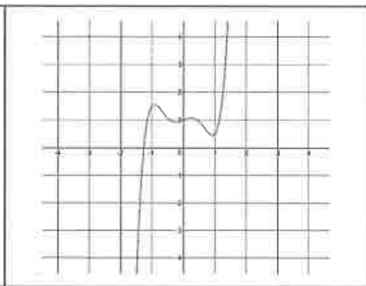
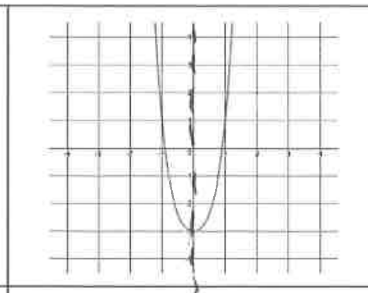
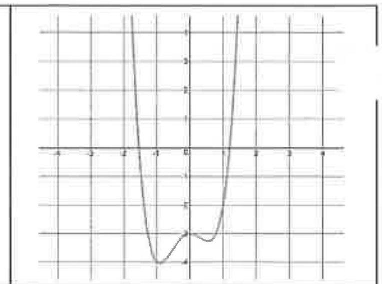
d) any symmetry present; even or odd function?

None, Neither

the intervals where $f(x) < 0$

$f(x) < 0$ when $x \in (-\infty, -1) \cup (-1, 2) \cup (4, \infty)$

4) Label each function as even, odd, or neither

			
ODD	NEITHER	EVEN	NEITHER

ANSWER KEY

1) a) even, line symmetry about y-axis **b)** odd, point symmetry about origin **c)** neither

d) neither **e)** even, line symmetry about y-axis

2) a) even, $f(-x) = f(x)$ **b)** odd, $f(-x) = -f(x)$

3) a) -1 (order 2), 2, and 4 **b)** 3 **c)** 4 **d)** no symmetry, neither **e)** $X \in (-\infty, -1) \cup (-1,2) \cup (4, \infty)$

4) odd, neither, even, neither