

W3 - 4.5 Double Angle Formulas  
MHF4U

SOLUTIONS

1) Express each of the following as a single trig ratio.

a)  $2 \sin(5x) \cos(5x)$

$$= \sin[2(5x)]$$

$$= \sin(10x)$$

b)  $\cos^2 \theta - \sin^2 \theta$

$$= \cos(2\theta)$$

c)  $1 - 2 \sin^2(3x)$

$$= \cos[2(3x)]$$

$$= \cos(6x)$$

d)  $\frac{2 \tan(4x)}{1 - \tan^2(4x)}$

$$= \tan[2(4x)]$$

$$= \tan(8x)$$

e)  $4 \sin \theta \cos \theta$

$$= 2(2) \sin \theta \cos \theta$$

$$= 2 \sin(2\theta)$$

f)  $2 \cos^2 \frac{\theta}{2} - 1$

$$= \cos[2(\frac{\theta}{2})]$$

$$= \cos \theta$$

2) Express each of the following as a single trig ratio and then evaluate

a)  $2 \sin 45^\circ \cos 45^\circ$

$$= \sin(2 \times 45^\circ)$$

$$= \sin 90^\circ$$

b)  $\cos^2 30^\circ - \sin^2 30^\circ$

$$= \cos(2 \times 30^\circ)$$

$$= \cos 60^\circ$$

c)  $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$

$$= \sin[2(\frac{\pi}{12})]$$

$$= \sin(\frac{\pi}{6})$$

d)  $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$

$$= \cos[2(\frac{\pi}{12})]$$

$$= \cos(\frac{\pi}{6})$$

e)  $1 - 2 \sin^2 \frac{3\pi}{8}$

$$= \cos[2(\frac{3\pi}{8})]$$

$$= \cos(\frac{3\pi}{4})$$

f)  $2 \tan 60^\circ \cos^2 60^\circ$

$$= 2 \left( \frac{\sin 60^\circ}{\cos 60^\circ} \right) \cos^2 60^\circ$$

$$= 2 \sin 60^\circ \cos 60^\circ$$

$$= \sin[2(60^\circ)]$$

$$= \sin 120^\circ$$

3) Use a double angle formula to rewrite each trig ratio

a)  $\sin(4\theta) = \sin[2(2\theta)]$

$$= 2 \sin(2\theta) \cos(2\theta)$$

b)  $\cos(3x) = \cos[2(\frac{3x}{2})]$

$$= 2 \cos^2(\frac{3x}{2}) - 1$$

c)  $\tan x = \tan[2(\frac{x}{2})]$

$$= \frac{2 \tan(\frac{x}{2})}{1 - \tan^2(\frac{x}{2})}$$

d)  $\cos(6\theta) = \cos[2(3\theta)]$

$$= \cos^2(3\theta) - \sin^2(3\theta)$$

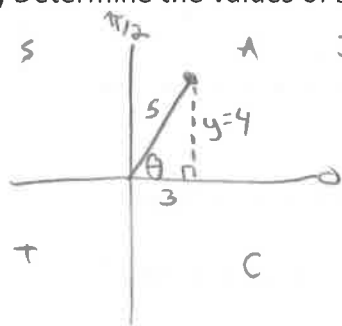
e)  $\sin x = \sin[2(\frac{x}{2})]$

$$= 2 \sin(\frac{x}{2}) \cos(\frac{x}{2})$$

f)  $\tan(5\theta) = \tan[2(\frac{5\theta}{2})]$

$$= \frac{2 \tan(\frac{5\theta}{2})}{1 - \tan^2(\frac{5\theta}{2})}$$

- 4) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\cos \theta = \frac{3}{5}$  and  $0 \leq \theta \leq \frac{\pi}{2}$



$$\begin{aligned} 3^2 + y^2 &= 5^2 \\ y^2 &= 16 \\ y &= 4 \end{aligned}$$

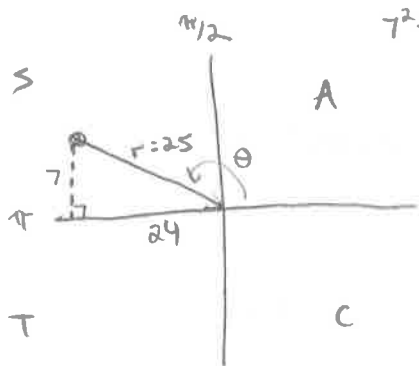
$$\begin{aligned} \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left( \frac{4}{5} \right) \left( \frac{3}{5} \right) \\ &= \frac{24}{25} \end{aligned}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= \left( \frac{3}{5} \right)^2 - \left( \frac{4}{5} \right)^2 \\ &= \frac{9}{25} - \frac{16}{25} \\ &= -\frac{7}{25} \end{aligned}$$

$$\begin{aligned} &= \frac{\left( \frac{24}{25} \right)}{\left( -\frac{7}{25} \right)} \\ &= -\frac{24}{7} \end{aligned}$$

- 5) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\tan \theta = -\frac{7}{24}$  and  $\frac{\pi}{2} \leq \theta \leq \pi$



$$\begin{aligned} 7^2 + 24^2 &= r^2 \\ 625 &= r^2 \\ r &= 25 \end{aligned}$$

$$\begin{aligned} \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left( \frac{7}{25} \right) \left( -\frac{24}{25} \right) \\ &= -\frac{336}{625} \end{aligned}$$

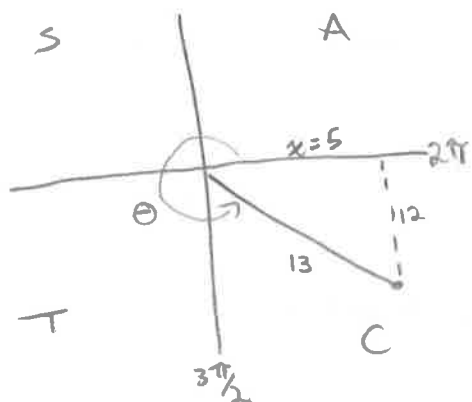
$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$\begin{aligned} &= \frac{\left( -\frac{336}{625} \right)}{\left( \frac{527}{625} \right)} \end{aligned}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= \left( -\frac{24}{25} \right)^2 - \left( \frac{7}{25} \right)^2 \\ &= \frac{576}{625} - \frac{49}{625} \\ &= \frac{527}{625} \end{aligned}$$

$$= -\frac{336}{527}$$

- 6) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\sin \theta = -\frac{12}{13}$  and  $\frac{3\pi}{2} \leq \theta \leq 2\pi$



$$\begin{aligned} x^2 + 12^2 &= 13^2 \\ x^2 &= 25 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left( -\frac{12}{13} \right) \left( \frac{5}{13} \right) \\ &= -\frac{120}{169} \end{aligned}$$

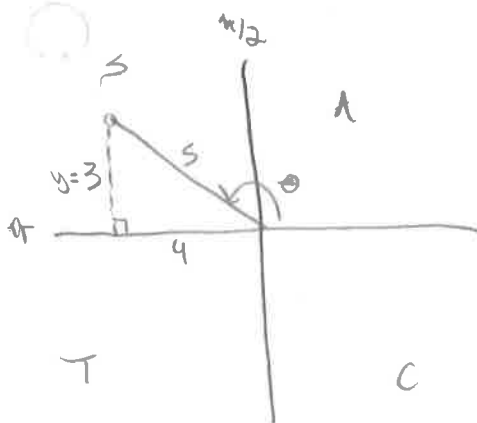
$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$\begin{aligned} &= \frac{\left( -\frac{120}{169} \right)}{\left( -\frac{119}{169} \right)} \end{aligned}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= \left( \frac{5}{13} \right)^2 - \left( -\frac{12}{13} \right)^2 \\ &= \frac{25}{169} - \frac{144}{169} \\ &= -\frac{119}{169} \end{aligned}$$

$$= \frac{120}{119}$$

7) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\cos \theta = -\frac{4}{5}$  and  $\frac{\pi}{2} \leq \theta \leq \pi$



$$x^2 + y^2 = 5^2$$

$$y^2 = 9$$

$$y = 3$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{3}{5} \right) \left( -\frac{4}{5} \right)$$

$$= -\frac{24}{25}$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= \left( -\frac{4}{5} \right)^2 - \left( \frac{3}{5} \right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$= \frac{\left( -\frac{24}{25} \right)}{\left( \frac{7}{25} \right)}$$

$$= -\frac{24}{7}$$

8) Determine the value of  $a$  in the equation  $2 \tan x - \tan(2x) + 2a = 1 - \tan(2x) \tan^2 x$

$$2 \tan x = \tan(2x) [1 - \tan^2 x] - 2a + 1$$

$$\frac{2 \tan x}{1 - \tan^2 x} = \frac{\tan(2x) [1 - \tan^2 x] - 2a + 1}{1 - \tan^2 x}$$

$$\tan(2x) = \tan(2x) + \frac{-2a + 1}{1 - \tan^2 x}$$

$$0 = -2a + 1$$

$$-1 = -2a$$

$$a = \frac{1}{2}$$