W4 – Limits
MHF4U

SOLUTIONS

1) Evaluate each limit

a) $\lim_{x\to 2} \frac{3x}{x^2+2}$

$= \frac{3(2)}{(2)^2+2}$

$= \frac{6}{6}$

$= 1$

b) $\lim_{x\to -1} (x^4 + x^3 + x^2)$

$= (-1)^4 + (-1)^3 + (-1)^2$

$= 1 - 1 + 1$

$= 1$

c) $\lim_{x\to 9} \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$= \left(\sqrt{9} + \frac{1}{\sqrt{9}}\right)^2$

$= \left(3 + \frac{1}{3}\right)^2$

$= \left(\frac{10}{3}\right)^2$

$= \frac{100}{9}$

2) Evaluate the limit of each

a) $\lim_{x\to 2} \frac{4-x^2}{2-x}$

$= \lim_{x\to 2} \frac{(2-x)(2+x)}{2-x}$

$= 2+2$

$= 4$

b) $\lim_{x\to -1} \frac{2x^2+5x+3}{x+1}$

P: 6, 5 — $\frac{1}{1} \neq \frac{2}{2} \times \frac{3}{3}$, 5

$= \lim_{x\to -1} \frac{(x+1)(2x+3)}{x+1}$

$= 2(-1)+3$

$= 1$

c) $\lim_{x\to 3} \frac{x^3-27}{x-3}$

$= \lim_{x\to 3} \frac{(x-3)(x^2+3x+9)}{x-3}$

$= (3)^2 + 3(3) + 9$

$= 27$

d) $\lim_{x\to 4} \frac{16-x^2}{x^3+64}$

$= \lim_{x\to 4} \frac{(4-x)(4+x)}{(x+4)(x^2-4x+16)}$

$= \frac{4-4}{(4)^2-4(4)+16}$

$= \frac{0}{16}$

$= 0$

e) $\lim_{x\to 4} \frac{x^2-16}{x^2-5x+6}$

$= \lim_{x\to 4} \frac{(x-4)(x+4)}{(x-2)(x-3)}$

$= \frac{(4-4)(4+4)}{(4-2)(4-3)}$

$= \frac{0(8)}{2(1)}$

$= \frac{0}{2}$

$= 0$

f) $\lim_{x\to -1} \frac{x^2+x}{x+1}$

$= \lim_{x\to -1} \frac{x(x+1)}{x+1}$

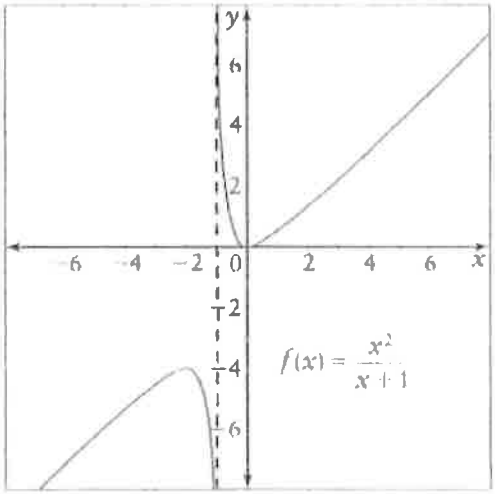
$= -1$

3) Complete the following table and use results to estimate $\lim_{x\to 2} \frac{x-2}{x^2-x-2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$\frac{x-2}{x^2-x-2}$	0.3448	0.3344	0.3334	0.3332	0.3322	0.3226

$\lim_{x\to 2} \frac{x-2}{x^2-x-2} \simeq \frac{1}{3}$

4) Use the graph to find the following limits:

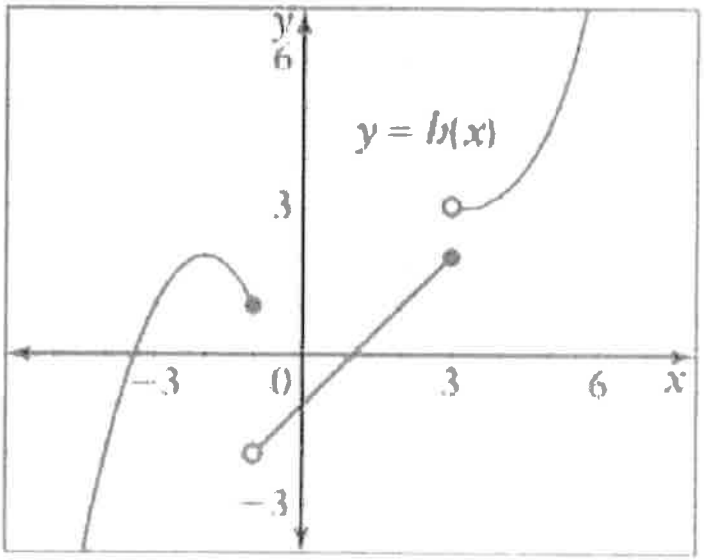


a) $\lim_{x\to -1^+} \frac{x^2}{x+1}$ $= \infty$

b) $\lim_{x\to -1^-} \frac{x^2}{x+1}$ $= -\infty$

c) $\lim_{x\to -1} \frac{x^2}{x+1}$ Does not exist

5) Use the graph to determine the following limits



a) $\lim_{x\to -1^+} h(x)$ $= -2$

b) $\lim_{x\to -1^-} h(x)$ $= 1$

c) $\lim_{x\to -1} h(x)$ Does not exist

d) $\lim_{x\to 3^+} h(x)$ $= 3$

e) $\lim_{x\to 3^-} h(x)$ $= 2$

f) $\lim_{x\to 3} h(x)$ Does not exist