Chapter 4
Polynomial Equations and Inequalities

Getting Started, pp. 194–195

1. a) $5x - 7 = -3x + 17$
$5x + 3x = 17 + 7$
$8x = 24$
$x = 3$

b) $12x - 9 - 6x = 5 + 3x + 1$
$12x - 6x - 3x = 5 + 1 + 9$
$3x = 15$
$x = 5$

c) $2(3x - 5) = -4(3x - 2)$
$6x - 10 = -12x + 8$
$6x + 12x = 8 + 10$
$18x = 18$
$x = 1$

d) $\frac{2x + 5}{3} = 7 - \frac{x}{4}$

$12\left(\frac{2x + 5}{3} = 7 - \frac{x}{4}\right)$
$4(2x + 5) = 84 - 3x$
$8x + 20 = 84 - 3x$
$8x + 3x = 84 - 20$
$11x = 64$
$x = \frac{64}{11}$

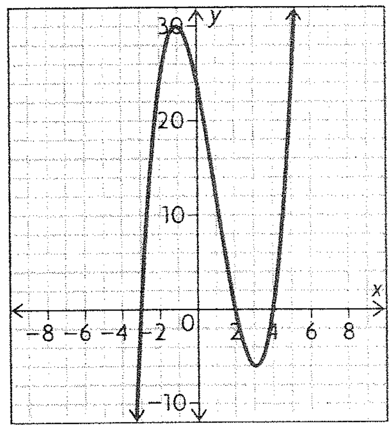
2. a) $x^3 + x^2 - 30x = x(x^2 + x - 30)$
$= x(x + 6)(x - 5)$

b) $x^3 - 64 = (x - 4)(x^2 + 4x + 16)$

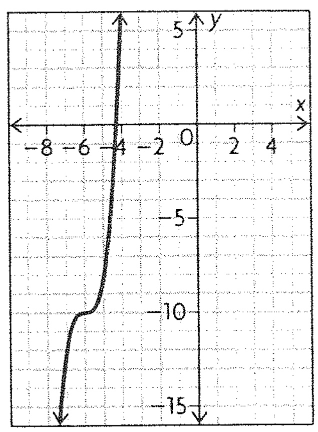
c) $24x^4 + 81x = 3x(8x^3 + 27)$
$= 3x(2x + 3)(4x^2 - 6x + 9)$

d) $2x^3 + 7x^2 - 18x - 63$
$= x^2(2x + 7) - 9(2x + 7)$
$= (x^2 - 9)(2x + 7)$
$= (x + 3)(x - 3)(2x + 7)$

3. a)



b)



4. The graph crosses the x-axis at $x = 2$ and at $x = 5$. The roots of the equation are 2 and 5.

5. a) $2x^2 = 18$
$x^2 = 9$
$x = \pm\sqrt{9}$
$x = \pm 3$

The roots are 3 and -3.

b) $(x^2 + 8x - 20) = 0$
$(x + 10)(x - 2) = 0$
$(x + 10) = 0$ and $(x - 2) = 0$
$x = -10, 2$

The roots are -10 and 2.

c) $6x^2 = 11x + 10$
$6x^2 - 11x - 10 = 0$
$6x^2 - 15x + 4x - 10 = 0$
$3x(2x - 5) + 2(2x - 5) = 0$
$(3x + 2)(2x - 5) = 0$

$(3x + 2) = 0$ and $(2x - 5) = 0$

$$x = -\frac{2}{3}, \frac{5}{2}$$

The roots are $-\frac{2}{3}$ and $\frac{5}{2}$.

d) $x(x + 3) = 3 - 5x - x^2$

$$x^2 + 3x = 3 - 5x - x^2$$

$$2x^2 + 8x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 2$, $b = 8$, and $c = -3$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{-8}{4} \pm \frac{\sqrt{64 - (-24)}}{4}$$

$$= -2 \pm \frac{\sqrt{88}}{4}$$

$$= -2 \pm \frac{2\sqrt{22}}{4}$$

$$= -2 \pm \frac{\sqrt{22}}{2}$$

$$\doteq 0.3452, -4.345$$

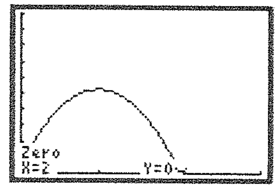
The roots are $0.3452, -4.345$.

6. a) (3, 7); Answers may vary. For example, the change in distance over time from $t = 3$ to $t = 7$ is greater than at other intervals of time.

b) From 0 seconds to 3 seconds, she walks a total of 1 metre for a rate of $\frac{1}{3}$ m/s. From 3 seconds to 7 seconds, she walks a total of 3 metres for a rate of $\frac{3}{4}$ m/s.

c) Answers may vary. For example, away; Erika's displacement, or distance from the sensor, is increasing.

7. a) Determine the value of t for which $h(t) = 0$. Use a graphing calculator.



The ball is in the air for 2 s.

b) $h(0) = 0.5$

$$h(1) = -5 + 9.75 + 0.5 = 5.25$$

$$\frac{5.25 - 0.5}{1 - 0} = 4.75 \text{ m/s}$$

c) Estimate the instantaneous rate of change by calculating the average rate of change from $t = 1.999$ s to $t = 2$ s.

$$\frac{h(2) - h(1.999)}{2 - 1.999} = \frac{0 - 0.010\,245}{2 - 1.999} = \frac{-0.010\,245}{0.001} = -10.245 \text{ m/s}$$

8. Answers given for justfication may vary. Sample answers are provided.

	Statement	Agree	Disagree	Justification
a)	The quadratic formula can only be used when solving a quadratic equation.		x	You could use the quadratic formula to solve $y = x^3 + 4x^2 + 3x$ because it equals $x(x^2 + 4x + 3)$.
b)	Cubic equations always have three real roots.		x	$y = (x + 3)^2(x - 2)$ is a cubic equation that will have two roots.
c)	The graph of a cubic function always passes through all four quadrants.		x	The equation $y = x^3$ will only pass through two quadrants.
d)	The graphs of all polynomial functions must pass through at least two quadrants.	x		All polynomials are continuous and all polynomials have a y-intercept.
e)	The expression $x^2 > 4$ is only true if $x > 2$.		x	$f(-3) = 9$
f)	If you know the instantaneous rates of change for a function at $x = 2$ and $x = 3$, you can predict fairly well what the function looks like in between.	x		The instantaneous rates of change will tell you whether the graph is increasing, decreasing, or not changing at those points.

4.1 Solving Polynomial Equations, pp. 204–206

1. a) $y = 2x(x - 1)(x + 2)(x - 2)$

$2x = 0$ and $x - 1 = 0$ and

$x + 2 = 0$ and $x - 2 = 0$

$x = 0, 1, -2, 2$

b) $y = 5(2x + 3)(4x - 5)(x + 7)$

$2x + 3 = 0$ and $4x - 5 = 0$ and $x + 7 = 0$

$x = -\frac{3}{2}, \frac{5}{4}, -7$

c) $y = 2(x - 3)^2(x + 5)(x - 4)$

$x - 3 = 0$ and $x + 5 = 0$ and $x - 4 = 0$

$x = 3, -5, 4$

d) $y = (x + 6)^3(2x - 5)$

$x + 6 = 0$ and $2x - 5 = 0$

$x = -6, \frac{5}{2}$

e) $y = -5x(x^2 - 9)$

$-5x = 0$ and $x^2 - 9 = 0$

$x = 0$ and $x^2 = 9$

$x = 0$ and $x = \pm 3$

$x = 0, -3, 3$

f) $y = (x + 5)(x^2 - 4x - 12)$

$y = (x + 5)(x + 2)(x - 6)$

$x + 5 = 0$ and $x + 2 = 0$ and $x - 6 = 0$

$x = -5, -2, 6$

2. a) $3x^3 = 27x$

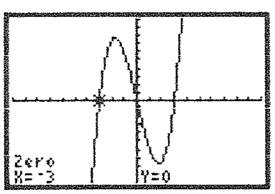
$3x^3 - 27x = 0$

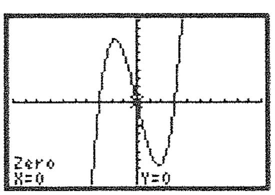
$3x(x^2 - 9) = 0$

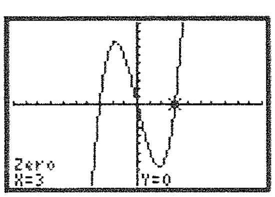
$3x(x + 3)(x - 3) = 0$

$3x = 0$ and $x + 3 = 0$ and $x - 3 = 0$

$x = 0, -3, 3$







b) $4x^4 = 24x^2 + 108$

$4x^4 - 24x^2 - 108 = 0$

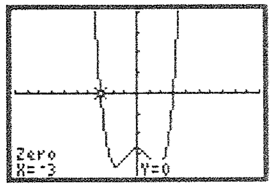
$x^4 - 6x^2 - 27 = 0$

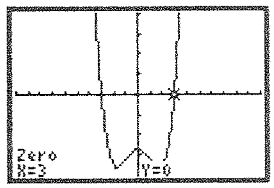
$(x^2 - 9)(x^2 + 3) = 0$

$x^2 - 9 = 0$ and $x^2 + 3 = 0$

$x^2 = 9$ and $x^2 = -3$

$x = \pm 3$ no real solutions





c) $3x^4 + 5x^3 - 12x^2 - 20x = 0$

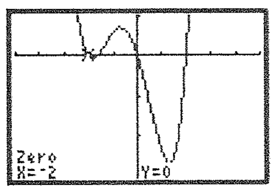
$x(3x^3 + 5x^2 - 12x - 20) = 0$

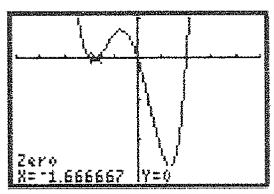
$x(x^2(3x + 5) - 4(3x + 5)) = 0$

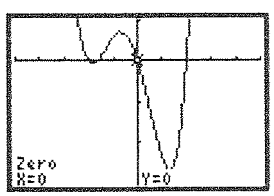
$x(x^2 - 4)(3x + 5) = 0$

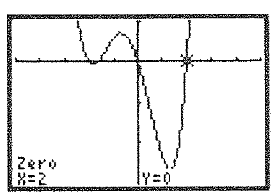
$x = 0$ and $x^2 - 4 = 0$ and $3x + 5 = 0$

$x = 0, 2, -2, -\frac{5}{3}$





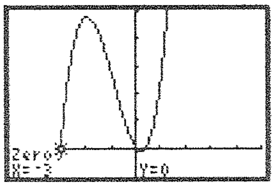


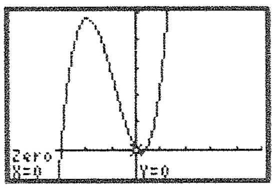


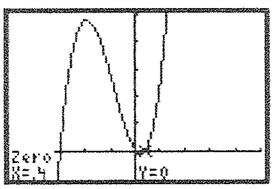
d) $$\begin{aligned} 10x^3 + 26x^2 - 12x &= 0 \\ 2x(5x^2 + 13x - 6) &= 0 \\ 2x(5x^2 + 15x - 2x - 6) &= 0 \\ 2x(5x(x + 3) - 2(x + 3)) &= 0 \\ 2x(5x - 2)(x + 3) &= 0 \end{aligned}$$

$2x = 0$ and $5x - 2 = 0$ and $x + 3 = 0$

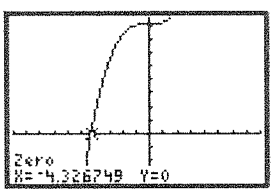
$x = 0, \frac{2}{5}, -3$







e) $$\begin{aligned} 2x^3 + 162 &= 0 \\ 2x^3 &= -162 \\ x^3 &= -81 \\ \sqrt[3]{x^3} &= \sqrt[3]{-81} \\ x &= -3\sqrt[3]{3} \end{aligned}$$

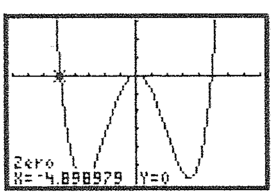


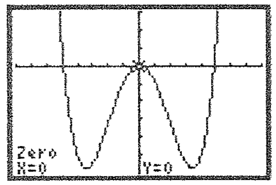
f) $$\begin{aligned} 2x^4 &= 48x^2 \\ 2x^4 - 48x^2 &= 0 \\ 2x^2(x^2 - 24) &= 0 \end{aligned}$$

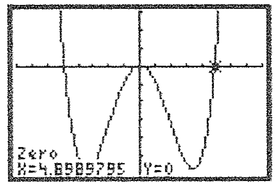
$2x^2 = 0$ and $x^2 - 24 = 0$

$x^2 = 0$ and $x^2 = 24$

$x = 0, \pm 2\sqrt{6}$







3. a) $y = 2x^3 - 17x^2 + 23x + 42$

$f(1) = 50$

$f(6) = 0$

$$\begin{array}{r|rrrr} 6 & 2 & -17 & 23 & 42 \\ & \downarrow & 12 & -30 & -42 \\ \hline & 2 & -5 & -7 & 0 \end{array}$$

$$\begin{aligned} (x - 6)(2x^2 - 5x - 7) &= 0 \\ (x - 6)(2x^2 - 7x + 2x - 7) &= 0 \\ (x - 6)(x(2x - 7) + 1(2x - 7)) &= 0 \\ (x - 6)(x + 1)(2x - 7) &= 0 \end{aligned}$$

$x - 6 = 0$ and $x + 1 = 0$ and $2x - 7 = 0$

$x = 6, -1, \frac{7}{2}$

b) From part a) $2x^3 - 17x^2 + 23x + 42 = 0$ or $(x - 6)(x + 1)(2x - 7) = 0$. The roots of either of these equations are the zeros of the function in part a).

4. Algebraically: Collect all terms on one side of the equation and solve by factoring.

$x^3 + 12x^2 + 21x - 4 = x^4 - 2x^3 - 13x^2 - 4$

$0 = x^4 - 3x^3 - 25x^2 - 21x$

$x(x^3 - 3x^2 - 25x - 21) = 0$

$x = 0$

$x^3 - 3x^2 - 25x - 21 = 0$

$f(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -25 & -21 \\ & \downarrow & -1 & 4 & 21 \\ \hline & 1 & -4 & -21 & 0 \end{array}$$

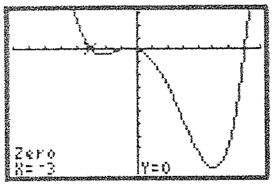
$(x + 1)(x^2 - 4x - 21) = 0$

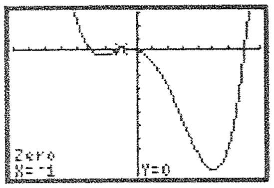
$(x + 1)(x + 3)(x - 7) = 0$

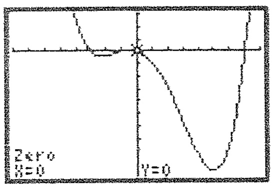
$x + 1 = 0$ and $x + 3 = 0$ and $x - 7 = 0$

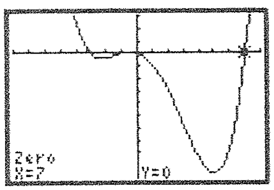
$x = -1, -3, 7, 0$

Graphically: Collect all terms on one side of the equation. Graph the resulting expression and use the calculator to determine the zeros.









$x = -3, -1, 0, 7$

5. $f(x) = 2x^4 - 11x^3 - 37x^2 + 156x$

$x(2x^3 - 11x^2 - 37x + 156) = 0$

$x = 0$

$2x^3 - 11x^2 - 37x + 156 = 0$

$f(1) = 110$

$f(2) = 54$

$f(3) = 0$

$$\begin{array}{r|rrrr} 3 & 2 & -11 & -37 & 156 \\ & \downarrow & 6 & -15 & -156 \\ \hline & 2 & -5 & -52 & 0 \end{array}$$

$(x - 3)(2x^2 - 5x - 52) = 0$

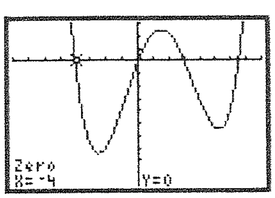
$(x - 3)(2x^2 - 13x + 8x - 52) = 0$

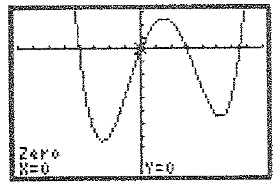
$(x - 3)(x(2x - 13) + 4(2x - 13)) = 0$

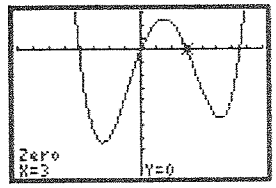
$(x - 3)(x + 4)(2x - 13) = 0$

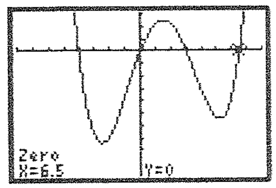
$x - 3 = 0$ and $x + 4 = 0$ and $2x - 13 = 0$

$x = 0, 3, -4, \frac{13}{2}$









6. a) $f(x) = x(x - 2)^2(x + 5)$

$x = 0$ and $(x - 2)^2 = 0$ and $x + 5 = 0$

$x = 0, 2, -5$

b) $f(x) = (x^3 + 1)(x - 17)$

$x^3 + 1 = 0$ and $x - 17 = 0$

$x = -1, 17$

c) $f(x) = (x^2 + 36)(8x - 16)$

$x^2 + 36 = 0$ and $8x - 16 = 0$

$x^2 = -36$ and $x = 2$

$x = 2$ (Since there is no number that, when squared, will equal -36.)

d) $f(x) = -3x^3(2x + 4)(x^2 - 25)$

$-3x^3 = 0$ and $2x + 4 = 0$ and $x^2 - 25 = 0$

$x = 0$ and $x = -2$ and $x = \pm 5$

$x = 0, -2, -5, 5$

e) $f(x) = (x^2 - x - 12)(3x)$

$x^2 - x - 12 = 0$ and $3x = 0$

$(x - 4)(x + 3) = 0$ and $x = 0$

$x - 4 = 0$ and $x + 3 = 0$

$x = 0, -3, 4$

f) $f(x) = (x + 1)(x^2 + 2x + 1)$

$x + 1 = 0$ and $x^2 + 2x + 1 = 0$

$x = -1$ and $(x + 1)(x + 1) = 0$

$x = -1$

7. a) $x^3 - 8x^2 - 3x + 90 = 0$

$f(-3) = 0$

$$\begin{array}{r|rrrr} -3 & 1 & -8 & -3 & 90 \\ & \downarrow & -3 & 33 & -90 \\ \hline & 1 & -11 & 30 & 0 \end{array}$$

$(x + 3)(x^2 - 11x + 30) = 0$

$(x + 3)(x - 6)(x - 5) = 0$

$x + 3 = 0$ and $x - 6 = 0$ and $x - 5 = 0$

$x = -3, 6, 5$

b) $x^4 + 9x^3 + 21x^2 - x - 30 = 0$

$f(1) = 0$

$$\begin{array}{r|rrrrr} 1 & 1 & 9 & 21 & -1 & -30 \\ & \downarrow & 1 & 10 & 31 & 30 \\ \hline & 1 & 10 & 31 & 30 & 0 \end{array}$$

$(x - 1)(x^3 + 10x^2 + 31x + 30) = 0$

For $x^3 + 10x^2 + 31x + 30 = 0$

$f(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & 10 & 31 & 30 \\ & \downarrow & -2 & -16 & -30 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$

$(x - 1)(x + 2)(x^2 + 8x + 15) = 0$

$(x - 1)(x + 2)(x + 3)(x + 5) = 0$

$x - 1 = 0$ and $x + 2 = 0$ and $x + 3 = 0$ and $x + 5 = 0$

$x = 1, -2, -3, -5$

c) $2x^3 - 5x^2 - 4x + 3 = 0$

$f(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & 3 \\ & \downarrow & -2 & 7 & -3 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$$\begin{aligned} (x + 1)(2x^2 - 7x + 3) &= 0 \\ (x + 1)(2x^2 - 6x - 1x + 3) &= 0 \\ (x + 1)(2x(x - 3) - 1(x - 3)) &= 0 \\ (x + 1)(2x - 1)(x - 3) &= 0 \end{aligned}$$

$x + 1 = 0$ and $2x - 1 = 0$ and $x - 3 = 0$

$x = -1, \frac{1}{2}, 3$

d) $2x^3 + 3x^2 = 5x + 6$

$2x^3 + 3x^2 - 5x - 6 = 0$

$f(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 2 & 3 & -5 & -6 \\ & \downarrow & -2 & -1 & 6 \\ \hline & 2 & 1 & -6 & 0 \end{array}$$

$$\begin{aligned} (x + 1)(2x^2 + x - 6) &= 0 \\ (x + 1)(2x^2 + 4x - 3x - 6) &= 0 \\ (x + 1)(2x(x + 2) - 3(x + 2)) &= 0 \\ (x + 1)(2x - 3)(x + 2) &= 0 \end{aligned}$$

$x + 1 = 0$ and $2x - 3 = 0$ and $x + 2 = 0$

$x = -1, \frac{3}{2}, -2$

e) $4x^4 - 4x^3 - 51x^2 + 106x = 40$

$4x^4 - 4x^3 - 51x^2 + 106x - 40 = 0$

$f(2) = 0$

$$\begin{array}{r|rrrrr} 2 & 4 & -4 & -51 & 106 & -40 \\ & \downarrow & 8 & 8 & -86 & 40 \\ \hline & 4 & 4 & -43 & 20 & 0 \end{array}$$

$(x - 2)(4x^3 + 4x^2 - 43x + 20) = 0$

For $4x^3 + 4x^2 - 43x + 20 = 0$,

$f(-4) = 0$

$$\begin{array}{r|rrrr} -4 & 4 & 4 & -43 & 20 \\ & \downarrow & -16 & 48 & -20 \\ \hline & 4 & -12 & 5 & 0 \end{array}$$

$$\begin{aligned} (x - 2)(x + 4)(4x^2 - 12x + 5) &= 0 \\ (x - 2)(x + 4)(4x^2 - 10x - 2x + 5) &= 0 \\ (x - 2)(x + 4)(2x(2x - 5) - 1(2x - 5)) &= 0 \\ (x - 2)(x + 4)(2x - 1)(2x - 5) &= 0 \end{aligned}$$

$x - 2 = 0$ and $x + 4 = 0$ and $2x - 1 = 0$ and $2x - 5 = 0$

$x = 2, -4, \frac{1}{2}, \frac{5}{2}$

f) $12x^3 - 44x^2 = -49x + 15$

$12x^3 - 44x^2 + 49x - 15 = 0$

$f\left(\frac{1}{2}\right) = 0$

$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & -44 & 49 & -15 \\ & \downarrow & 6 & -19 & 15 \\ \hline & 12 & -38 & 30 & 0 \end{array}$$

$\left(x - \frac{1}{2}\right)(12x^2 - 38x + 30) = 0$

For $12x^2 - 38x + 30 = 0$,

$f\left(\frac{5}{3}\right) = 0$

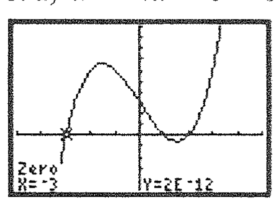
$$\begin{array}{r|rrr} \frac{5}{3} & 12 & -38 & 30 \\ & \downarrow & 20 & -30 \\ \hline & 12 & -18 & 0 \end{array}$$

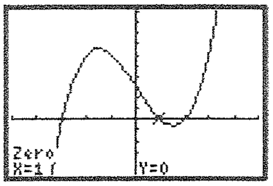
$\left(x - \frac{1}{2}\right)\left(x - \frac{5}{3}\right)(12x - 18) = 0$

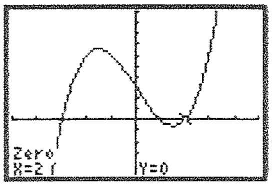
$x - \frac{1}{2} = 0$ and $x - \frac{5}{3} = 0$ and $12x - 18 = 0$

$x = \frac{1}{2}, \frac{5}{3}, \frac{3}{2}$

8. a) $x^3 - 7x + 6 = 0$

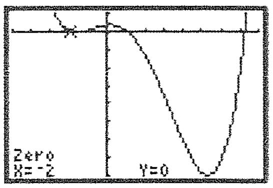


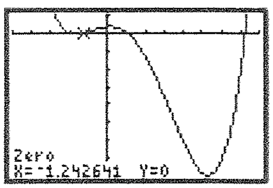


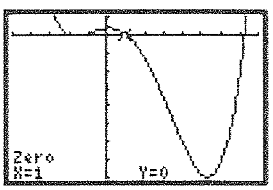


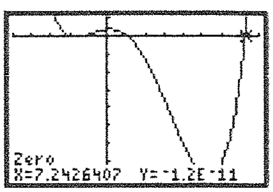
$x = -3, 1, 2$

b) $x^4 - 5x^3 - 17x^2 + 3x + 18 = 0$





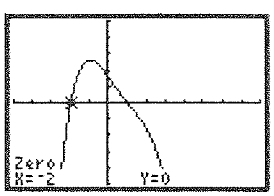


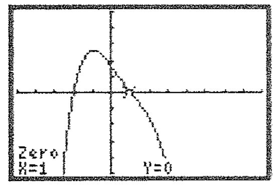


$x = -2, -1.24, 1, 7.24$

c) $3x^3 - 2x^2 + 16 = x^4 + 16x$

$-x^4 + 3x^3 - 2x^2 - 16x + 16 = 0$

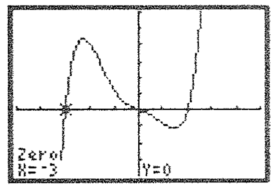


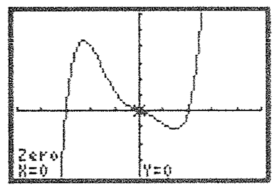


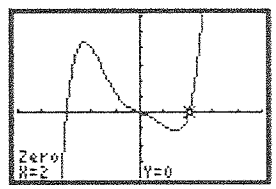
$x = -2, 1$

d) $x^5 + x^4 = 5x^3 - x^2 + 6x$

$x^5 + x^4 - 5x^3 + x^2 - 6x = 0$



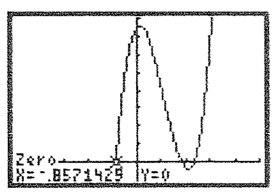


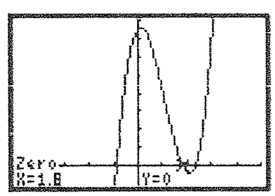


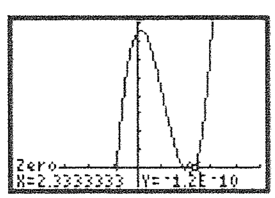
$x = -3, 0, 2$

e) $105x^3 = 344x^2 - 69x - 378$

$105x^3 - 344x^2 + 69x + 378 = 0$



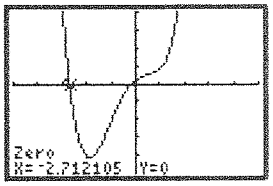


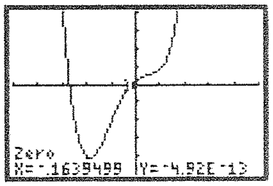


$x = -0.86, 1.8, 2.33$

f) $21x^3 - 58x^2 + 10 = -18x^4 - 51x$

$18x^4 + 21x^3 - 58x^2 + 51x + 10 = 0$





$x = -2.71, -0.16$

9. a) $x^3 - 6x^2 - x + 30 = 0$

$f(3) = 0$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & -1 & -30 \\ & \downarrow & 3 & -9 & 30 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$(x - 3)(x^2 - 3x - 10) = 0$

$(x - 3)(x + 2)(x - 5) = 0$

$x - 3 = 0$ and $x + 2 = 0$ and $x - 5 = 0$

$x = 3, -2, 5$

b) $9x^4 - 42x^3 + 64x^2 - 32x = 0$

$x(9x^3 - 42x^2 + 64x - 32) = 0$

For $9x^3 - 42x^2 + 64x - 32 = 0$

$f(2) = 0$

$$\begin{array}{r|rrrr} 2 & 9 & -42 & 64 & -32 \\ & \downarrow & 18 & -48 & 32 \\ \hline & 9 & -24 & 16 & 0 \end{array}$$

$x(x - 2)(9x^2 - 24x + 16) = 0$

$x(x - 2)(9x^2 - 12x - 12x + 16) = 0$

$x(x - 2)(3x(3x - 4) - 4(3x - 4)) = 0$

$x(x - 2)(3x - 4)(3x - 4) = 0$

$x = 0$ and $x - 2 = 0$ and $3x - 4 = 0$

$x = 0, 2, \frac{4}{3}$

c) $6x^4 - 13x^3 - 29x^2 + 52x = -20$

$6x^4 - 13x^3 - 29x^2 + 52x + 20 = 0$

$f(2) = 0$

$$\begin{array}{r|rrrrr} 2 & 6 & -13 & -29 & 52 & 20 \\ & \downarrow & 12 & -2 & -62 & -20 \\ \hline & 6 & -1 & -31 & -10 & 0 \end{array}$$

$(x - 2)(6x^3 - x^2 - 31x - 10) = 0$

For $6x^3 - x^2 - 31x - 10 = 0$

$f(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 6 & -1 & -31 & -10 \\ & \downarrow & -12 & 26 & 10 \\ \hline & 6 & -13 & -5 & 0 \end{array}$$

$(x - 2)(x + 2)(6x^2 - 13x - 5) = 0$

$(x - 2)(x + 2)(6x^2 - 15x + 2x - 5) = 0$

$(x - 2)(x + 2)(3x(2x - 5) + 1(2x - 5)) = 0$

$(x - 2)(x + 2)(3x + 1)(2x - 5) = 0$

$x - 2 = 0$ and $x + 2 = 0$ and $3x + 1 = 0$ and $2x - 5 = 0$

$x = 2, -2, -\frac{1}{3}, \frac{5}{2}$

d) $x^4 - 6x^3 + 10x^2 - 2x = x^2 - 2x$

$x^4 - 6x^3 + 9x^2 = 0$

$x^2(x^2 - 6x + 9) = 0$

$x^2(x - 3)(x - 3) = 0$

$x^2 = 0$ and $x - 3 = 0$

$x = 0, 3$

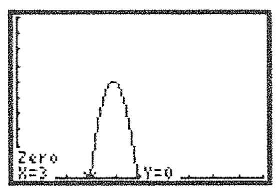
10. $V = lwh$

$(30 - 2x)(20 - 2x)(x) = 1008$

$(4x^2 - 100x + 600)(x) = 1008$

$4x^3 - 100x^2 + 600x - 1008 = 0$

$x^3 - 25x^2 + 150x - 252 = 0$



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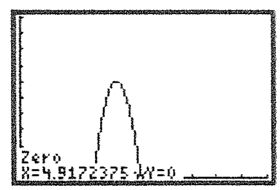


$x = 3, 4.92$

The dimensions of the cut square could be either 3 cm by 3 cm or 4.92 cm by 4.92 cm.

11. a) $x(x - 4)(x - 6) = 0$

$x = 0$ and $x - 4 = 0$ and $x - 6 = 0$

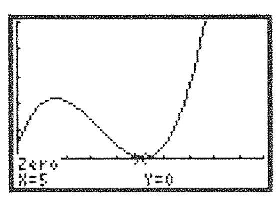
$x = 0, 4, 6$

Since there was not a game number 0, Maya's score was equal to zero after games 4 and 6.

b) $x(x - 4)(x - 6) = -5$

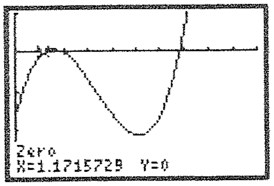
$x(x^2 - 10x + 24) = -5$

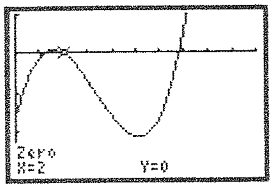
$x^3 - 10x^2 + 24x + 5 = 0$

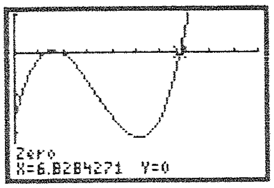


$x = 5$. Maya's score was -5 after game 5.

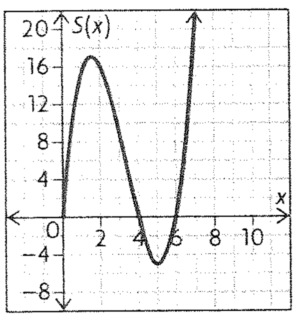
c) $x(x-4)(x-6)=16$
$x^3-10x^2+24x-16=0$







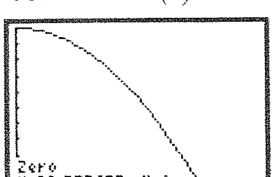
Since $x \in \mathbf{W}$, $x=2$. Maya's score was 16 after game 2.

d)


This is not a good model to represent Maya's score because the graph is shown for real numbers, but the number of games can only be a whole number.

12. $s(t)=-\frac{1}{2}(3.92)t^2+1000$
$=-1.96t^2+1000$

Use a graphing calculator to determine the value of t for which $s(t)=0$.



It takes the object about 22.59 s to hit the surface.

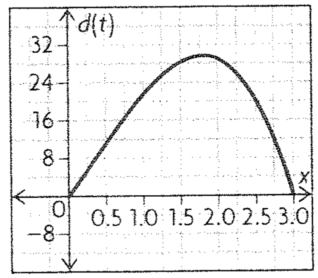
13. a) $d(t)=-3t^3+3t^2+18t$
$=-3t(t^2-t-6)$
$=-3t(t+2)(t-3)$

b) The ship is in the harbour when the distance equals 0.

$-3t(t+2)(t-3)=0$
$-3t=0$ and $t+2=0$ and $t-3=0$
$t=0,-2,3$

The ship returns to the harbour after 3 hours.

c) -2 is the other zero. It is not relevant because time cannot be negative.

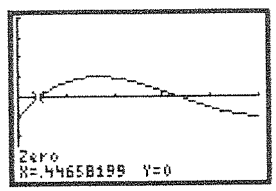
d)


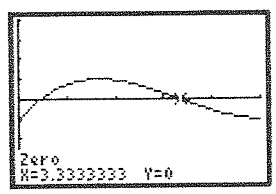
e) By examining the graph, it appears that the ship will begin its return trip to the harbour about 1.8 hours after departure.

14. a) $0 \le t \le 5$ because time cannot be negative and the model is only for a 5 s respiratory cycle.

b) Answers may vary. For example, because the function involves decimals, graphing technology would be the better strategy for answering the question.

c) $0.027t^3-0.27t^2+0.675t=0.25$
$0.027t^3-0.27t^2+0.675t-0.25=0$





At 0.45 s and 3.33 s, the person's lungs have a volume of air of 0.25 L.

15. All powers are even, which means every term is positive for all real numbers. Thus, the polynomial is always positive.

16. The zeros are 2, 3, and -5, so the polynomial function is of the form $f(x)=a(x-2)(x-3)(x+5)$. Since the graph of the function passes through (4, 36),

$36=a(4-2)(4-3)(4+5)$
$36=18a$
$2=a$

So $f(x)=2(x-2)(x-3)(x+5)$
$=2(x^2-5x+6)(x+5)$
$=2(x^3-19x+30)$

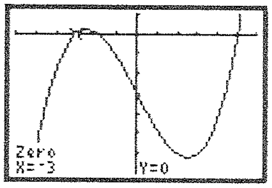
To determine the value of x for which $f(x) = 120$, set the polynomial equal to 120 and solve for x.

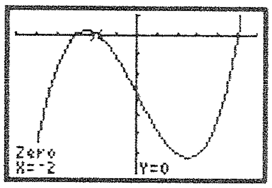
$2(x^3 - 19x + 30) = 120$

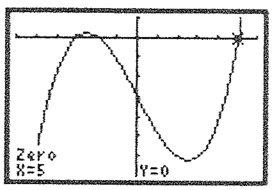
$x^3 - 19x + 30 = 60$

$x^3 - 19x - 30 = 0$

Graph $y = x^3 - 19x - 30$ on a graphing calculator and use the calculator to determine the zeros.







The fuction has a value of 120 at $x = -3$, $x = -2$, and $x = 5$.

17. Answers may vary. For example:

a) $x^3 + x^2 - x - 1 = 0$; $f(1) = 0$, so it is simple to solve using the factor theorem.

b) $x^2 - 2x = 0$; The common factor, x, can be factored out to solve the equation.

c) $x^3 - 2x^2 - 9x + 18$; An x can be factored out of the first two terms and a -2 out of the second two terms leaving you with the factors $(x - 2)(x^2 - 9)$.

d) $10x^2 - 7x + 1 = 0$; The roots are fractional which makes using the quadratic formula the most sensible approach.

e) $x^3 - 8 = 0$; This is the difference of two cubes.

f) $0.856x^3 - 2.74x^2 + 0.125x - 2.89 = 0$; The presence of decimals makes using graphing technology the most sensible strategy.

18. a) $0 = x^4 + 10$; x^4 is non-negative for all real x, so $x^4 + 10$ is always positive.

b) A degree 5 polynomial function $y = f(x)$ has opposite end behaviour, so somewhere in the middle it must cross the x-axis. This means its corresponding equation $0 = f(x)$ will have at least one real root.

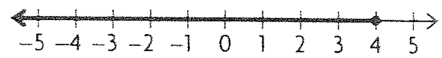
19. $y = x^5 + x + 1$; By the factor theorem, the only possible rational zeros are 1 and -1. Neither works. Because the degree is odd, the polynomial has opposite end behaviour, and hence must have at least one zero, which must be irrational.

4.2 Solving Linear Inequalities, pp. 213–215

1. a) $3x - 1 \leq 11$

$3x \leq 12$

$x \leq 4$

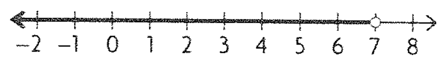


$\{x \in \mathbf{R} \mid x \leq 4\}$

b) $-x + 5 > -2$

$-x > -7$

$x < 7$

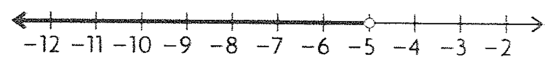


$\{x \in \mathbf{R} \mid x < 7\}$

c) $x - 2 > 3x + 8$

$-2x > 10$

$x < -5$



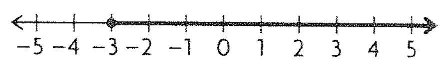
$\{x \in \mathbf{R} \mid x < -5\}$

d) $3(2x + 4) \geq 2x$

$6x + 12 \geq 2x$

$4x \geq -12$

$x \geq -3$



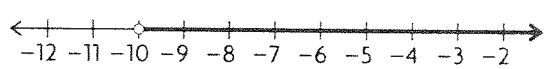
$\{x \in \mathbf{R} \mid x \geq -3\}$

e) $-2(1 - 2x) < 5x + 8$

$-2 + 4x < 5x + 8$

$-1x < 10$

$x > -10$



$\{x \in \mathbf{R} \mid x > -10\}$

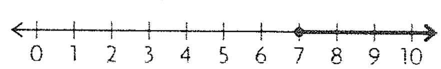
f) $\dfrac{6x + 8}{5} \leq 2x - 4$

$6x + 8 \leq 5(2x - 4)$

$6x + 8 \leq 10x - 20$

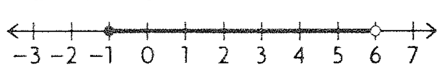
$-4x \leq -28$

$x \geq 7$

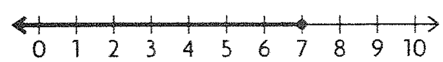


$\{x \in \mathbf{R} \mid x \geq 7\}$

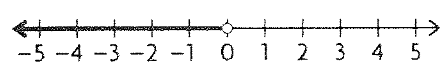
2. a) $2x - 5 \le 4x + 1$
$-2x \le 6$
$x \ge -3$
$x \in [-3, \infty)$
b) $2(x + 3) < -(x - 4)$
$2x + 6 < -x + 4$
$3x < -2$
$x < -\frac{2}{3}$
$x \in \left(-\infty, -\frac{2}{3}\right)$
c) $\frac{2x + 3}{3} \le x - 5$
$2x + 3 \le 3(x - 5)$
$2x + 3 \le 3x - 15$
$-x \le -18$
$x \ge 18$
$x \in [18, \infty)$
d) $2x + 1 \le 5x - 2$
$-3x \le -3$
$x \ge 1$
$x \in [1, \infty)$
e) $-x + 1 > x + 1$
$-2x > 0$
$x < 0$
$x \in (-\infty, 0)$
f) $\frac{x + 4}{2} \ge \frac{x - 2}{4}$
$4(x + 4) \ge 2(x - 2)$
$4x + 16 \ge 2x - 4$
$2x \ge -20$
$x \ge -10$
$x \in [-10, \infty)$
3. $3 \le 2x + 5 < 17$
$3 - 5 \le 2x + 5 - 5 < 17 - 5$
$-2 \le 2x < 12$
$-1 \le x < 6$



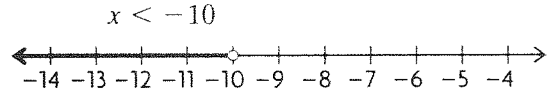
4. a) $x > -1$
$2 > -1$
Yes, $x = 2$ is contained in the solution set.
b) $5x - 4 > 3x + 2$
$5(2) - 4 > 3(2) + 2$
$10 - 4 > 6 + 2$
$6 > 8$
This is false. So, $x = 2$ is not contained in the solution set.
c) $4(3x - 5) \ge 6x$
$4(3(2) - 5) \ge 6(2)$
$4(6 - 5) \ge 12$
$4(1) \ge 12$
$4 \ge 12$
This is false. So, $x = 2$ is not contained in the solution set.
d) $5x + 3 \le -3x + 1$
$5(2) + 3 \le -3(2) + 1$
$10 + 3 \le -6 + 1$
$13 \le -5$
This is false. So, $x = 2$ is not contained in the solution set.
e) $x - 2 \le 3x + 4 \le x + 14$
$2 - 2 \le 3(2) + 4 \le 2 + 14$
$0 \le 6 + 4 \le 16$
$0 \le 10 \le 16$
Yes, $x = 2$ is contained in the solution set.
f) $33 < -10x + 3 < 54$
$33 < -10(2) + 3 < 54$
$33 < -20 + 3 < 54$
$33 < -17 < 54$
This is false. So, $x = 2$ is not contained in the solution set.
5. a) $2x - 1 \le 13$
$2x \le 14$
$x \le 7$



Check $x = 6$ to verify.
$2(6) - 1 \le 13$
$12 - 1 \le 13$
$11 \le 13$
b) $-2x - 1 > -1$
$-2x > 0$
$x < 0$



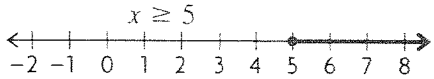
Check $x = -1$ to verify.
$-2(-1) - 1 > -1$
$2 - 1 > -1$
$1 > -1$
c) $2x - 8 > 4x + 12$
$-2x > 20$
$x < -10$



Check $x = -11$ to verify.
$2(-11) - 8 > 4(-11) + 12$

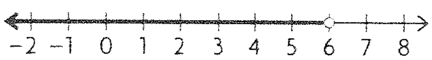
$$-22 - 8 > -44 + 12$$
$$-30 > -32$$

d) $5(x - 3) \geq 2x$
$$5x - 15 \geq 2x$$
$$3x \geq 15$$
$$x \geq 5$$



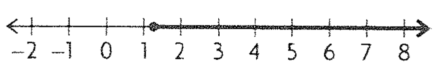
Check $x = 7$ to verify.
$$5(7 - 3) \geq 2(7)$$
$$5(4) \geq 14$$
$$20 \geq 14$$

e) $-4(5 - 3x) < 2(3x + 8)$
$$-20 + 12x < 6x + 16$$
$$6x < 36$$
$$x < 6$$



Check $x = 4$ to verify.
$$-4(5 - 3(4)) < 2(3(4) + 8)$$
$$-4(5 - 12) < 2(12 + 8)$$
$$-20 + 48 < 24 + 16$$
$$28 < 40$$

f) $\frac{x - 2}{3} \leq 2x - 3$
$$x - 2 \leq 3(2x - 3)$$
$$x - 2 \leq 6x - 9$$
$$-5x \leq -7$$
$$x \geq \frac{7}{5}$$



Check $x = 4$ to verify.
$$\frac{4 - 2}{3} \leq 2(4) - 3$$
$$\frac{2}{3} \leq 8 - 3$$
$$\frac{2}{3} \leq 5$$

6. a) $3x \leq 4x + 1$
$$3(0) \leq 4(0) + 1$$
$$0 \leq 0 + 1$$
$$0 \leq 1$$
Yes, 0 is contained in the solution set.

b) $-6x < x + 4 < 12$
$$-6(0) < 0 + 4 < 12$$
$$0 < 4 < 12$$
Yes, 0 is contained in the solution set.

c) $-x + 1 > x + 12$
$$-0 + 1 > 0 + 12$$
$$1 > 12$$
This is false. So, 0 is not contained in the solution set.

d) $3x \leq x + 1 \leq x - 1$
$$3(0) \leq 0 + 1 \leq 0 - 1$$
$$0 \leq 1 \leq -1$$
This is false. So, 0 is not contained in the solution set.

e) $x(2x - 1) \leq x + 7$
$$0(2(0) - 1) \leq 0 + 7$$
$$0 \leq 7$$
Yes, 0 is contained in the solution set.

f) $x + 6 < (x + 2)(5x + 3)$
$$0 + 6 < (0 + 2)(5(0) + 3)$$
$$6 < (2)(3)$$
$$6 < 6$$
This is false. So, 0 is not contained in the solution set.

7. a) $-5 < 2x + 7 < 11$
$$-5 - 7 < 2x + 7 - 7 < 11 - 7$$
$$-12 < 2x < 4$$
$$-6 < x < 2$$

b) $11 < 3x - 1 < 23$
$$11 + 1 < 3x - 1 + 1 < 23 + 1$$
$$12 < 3x < 24$$
$$4 < x < 8$$

c) $-1 \leq -x + 9 \leq 13$
$$-1 - 9 \leq -x + 9 - 9 \leq 13 - 9$$
$$-10 \leq -x \leq 4$$
$$-4 \leq x \leq 10$$

d) $0 \leq -2(x + 4) \leq 6$
$$0 \leq -2x - 8 \leq 6$$
$$0 + 8 \leq -2x - 8 + 8 \leq 6 + 8$$
$$8 \leq -2x \leq 14$$
$$-7 \leq x \leq -4$$

e) $59 < 7x + 10 < 73$
$$59 - 10 < 7x + 10 - 10 < 73 - 10$$
$$49 < 7x < 63$$
$$7 < x < 9$$

f) $18 \leq -12(x - 1) \leq 48$
$$18 \leq -12x + 12 \leq 48$$
$$18 - 12 \leq -12x + 12 - 12 \leq 48 - 12$$
$$6 \leq -12x \leq 36$$
$$-3 \leq x \leq -\frac{1}{2}$$

8. a) Answers may vary. For example:
$$3x + 1 > 9 + x$$
$$2x > 8$$
$$x > 4$$

b) Answers may vary. For example:

$$3x + 1 \le 4 + x$$
$$2x \le 3$$
$$x \le \frac{3}{2}$$

9. a) $\{x \in \mathbf{R} \mid -6 \le x \le 4\}$

b)
$$-13 \le 2x - 1 \le 7$$
$$-13 + 1 \le 2x - 1 + 1 \le 7 + 1$$
$$-12 \le 2x \le 8$$
$$-6 \le x \le 4$$

10. Attempting to solve $x - 3 < 3 - x < x - 5$ yields $3 > x > 4$, which has no solution. Solving $x - 3 > 3 - x > x - 5$ yields $3 < x < 4$.

11. a) $\frac{1}{2}x + 1 < 3$

b) $x < 4$

c)
$$\frac{1}{2}x + 1 < 3$$
$$\frac{1}{2}x < 2$$
$$x < 4$$

12. a) $18 \le \frac{5}{9}(F - 32) \le 22$

b)
$$18 \le \frac{5}{9}(F - 32) \le 22$$
$$9(18) \le 9\left(\frac{5}{9}(F - 32)\right) \le 9(22)$$
$$162 \le 5(F - 32) \le 198$$
$$\frac{162}{5} \le \frac{5(F - 32)}{5} \le \frac{198}{5}$$
$$32.4 \le F - 32 \le 39.6$$
$$32.4 + 32 \le F - 32 + 32 \le 39.6 + 32$$
$$64.4 \le F \le 71.6$$

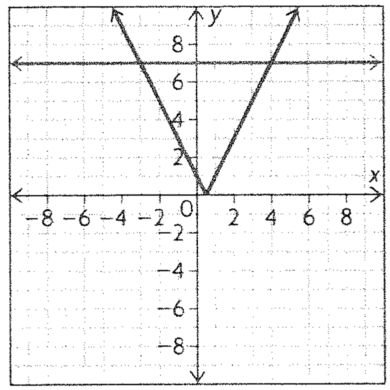
13.
$$0.50 + 0.10x \le 2.00$$
$$0.10x \le 1.50$$
$$x \le 15$$

The volunteers can talk for the initial 3 minutes plus an additional 15 minutes, or a total of 18 minutes.

14. a)
$$C = \frac{5}{9}(F - 32)$$
$$\frac{9}{5}C = F - 32$$
$$\frac{9}{5}C + 32 = F$$

b)
$$\frac{9}{5}C + 32 > C$$
$$\frac{4}{5}C > -32$$
$$C > -32\left(\frac{5}{4}\right)$$
$$C > -40$$

15. a)

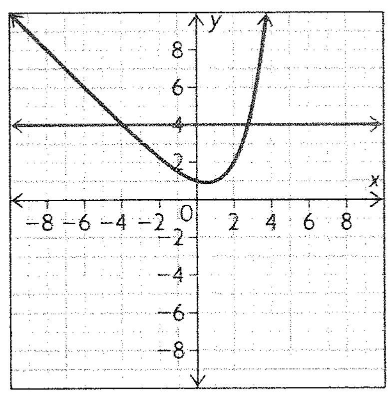


b) $-3 < x < 4$

16. The solution will always have an upper and lower bound due to the manner in which the inequality is solved. The only exception to this is when there is no solution set.

17. a) Isolating x is very hard.

b) A graphical approach as described in the lesson yields a solution of $x > 2.75$ (rounded to two places).



18. a) maintained

b) Maintained if both positive; switched if both negative; varies if one positive and one negative.

c) maintained

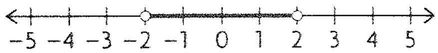
d) switched

e) Switched unless one is positive and the other is negative, in which case it is maintained. (If either side is zero, it becomes undefined.)

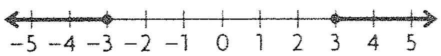
f) Maintained, except that $<$ and $>$ become $\le$ and $\ge$, respectively.

g) Maintained, but it is undefined for negative numbers.

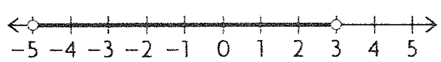
19. a) $x^2 < 4$
The solutions to this inequality are numbers that have a square less than 4.
The solution can be written as $\{x \in \mathbf{R} \mid -2 < x < 2\}$ or $(-2, 2)$.



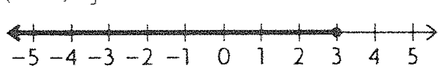
b) $4x^2 + 5 \geq 41$
$4x^2 \geq 36$
$x^2 \geq 9$
The solutions to this inequality are numbers that have a square greater than or equal to 9.
The solution can be written as $\{x \in \mathbf{R} \mid x \leq -3 \text{ or } x \geq 3\}$. In interval notation the solution is $(-\infty, -3]$ or $[3, \infty)$.



c) $|2x + 2| < 8$
Consider two cases.
If $2x + 2 \geq 0$, then $|2x + 2| = 2x + 2$.
$2x + 2 < 8$
$2x < 6$
$x < 3$
If $2x + 2 < 0$, the $|2x + 2| = -(2x + 2)$.
$-(2x + 2) < 8$
$2x + 2 > -8$
$2x > -10$
$x > -5$
The solution can be written as $\{x \in \mathbf{R} \mid -5 < x < 3\}$ or $(-5, 3)$.



d) $-3x^2 \geq 81$
Divide by -3 and switch the direction of the inequality.
$x^3 \leq 27$
$x \leq 3$
The solution can be written as $\{x \in \mathbf{R} \mid x \leq 3\}$ or $(-\infty, 3]$.



Mid-Chapter Review, p. 218

1. a) $0 = -2x^3(2x - 5)(x - 4)^2$
$-2x^3 = 0$ and $2x - 5 = 0$ and $(x - 4)^2 = 0$
$x = 0, \frac{5}{2}, 4$

b) $0 = (x^2 + 1)(2x + 4)(x + 2)$
$x^2 + 1 = 0$ and $2x + 4 = 0$ and $x + 2 = 0$
$x = -2$

c) $x^3 - 4x^2 = 7x - 10$
$x^3 - 4x^2 - 7x + 10 = 0$
$f(1) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & \downarrow & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$(x - 1)(x^2 - 3x - 10) = 0$
$(x - 1)(x + 2)(x - 5) = 0$
$x - 1 = 0$ and $x + 2 = 0$ and $x - 5 = 0$
$x = 1, -2, 5$

d) $0 = (x^2 - 2x - 24)(x^2 - 25)$
$x^2 - 2x - 24 = 0$ and $x^2 - 25 = 0$
$(x + 4)(x - 6) = 0$ and $\quad x^2 = 25$
$x + 4 = 0$ and $x - 6 = 0$ and $x = \pm\sqrt{25}$
$x = -4, 6, 5, -5$

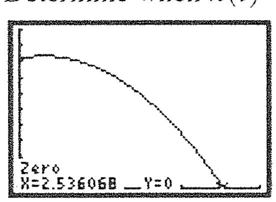
e) $0 = (x^3 + 2x^2)(x + 9)$
$0 = x^2(x + 2)(x + 9)$
$x^2 = 0$ and $x + 2 = 0$ and $x + 9 = 0$
$x = 0, -2, -9$

f) $-x^4 = -13x^2 + 36$
$0 = x^4 - 13x^2 + 36$
$(x^2 - 9)(x^2 - 4) = 0$
$x^2 - 9 = 0$ and $x^2 - 4 = 0$
$x^2 = 9$ and $x^2 = 4$
$x = \pm\sqrt{9}$ and $x = \pm\sqrt{4}$
$x = 3, -3, 2, -2$

2. a) $h(t) = -5(t - 0.3)^2 + 25$
$= -5(t - 0.3)(t - 0.3) + 25$
$= -5(t^2 - 0.6t + 0.09) + 25$
$= -5t^2 + 3t - 0.45 + 25$
$= -5t^2 + 3t + 24.55$

b) $h(0) = -5(0)^2 + 3(0) + 24.55$
$= 24.55$
The cliff is 24.55 metres high.

c) Graph $h(t)$ using a graphing calculator.
Determine when $h(t) = 0$.



Jude hits the water after about 2.5 s.

d) $t > 2.5$ seconds; Jude is below sea level (in the water)

3. $(30 - 2x)(30 - 2x)(x) = 1000$

$(900 - 120x + 4x^2)x = 1000$

$4x^3 - 120x^2 + 900x - 1000 = 0$

$f(10) = 0$

$$\begin{array}{r|rrrr} 10 & 4 & -120 & 900 & -1000 \\ & \downarrow & 40 & -800 & 1000 \\ \hline & 4 & -80 & 100 & 0 \end{array}$$

$(x - 10)(4x^2 - 80x + 100) = 0$

For $4x^2 - 80x + 100 = 0$

$$x = \frac{80 \pm \sqrt{(-80)^2 - 4(4)(100)}}{2(4)}$$

$$= \frac{80}{8} \pm \frac{\sqrt{6400 - 1600}}{8}$$

$$= 10 \pm \frac{\sqrt{4800}}{8}$$

$= 10 \pm 8.66$

$= 1.34, 18.66$

x cannot be 18.66 because $2x$ is longer than 30 cm.

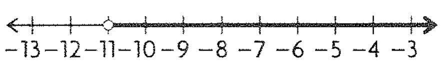
$x = 10, 1.34$

The dimensions of the squares are either 10 cm by 10 cm or 1.34 cm by 1.34 cm.

4. a) $2x - 4 < 3x + 7$

$-x < 11$

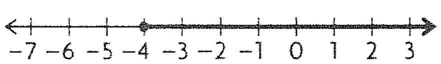
$x > -11$



b) $-x - 4 \le x + 4$

$-2x \le 8$

$x \ge -4$

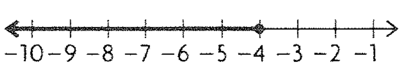


c) $-2(x - 4) \ge 16$

$-2x + 8 \ge 16$

$-2x \ge 8$

$x \le -4$

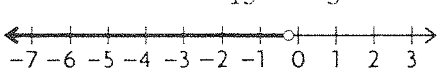


d) $2(3x - 7) > 3(7x - 3)$

$6x - 14 > 21x - 9$

$-15x > 5$

$$x < -\frac{5}{15} \text{ or } -\frac{1}{3}$$



5. $2x < \dfrac{3x + 6}{2} \le 4 + 2x$

$4x < 3x + 6 \le 8 + 4x$

$x < 6 \le x + 8$

Considering the two inequalities separately:

$x < 6$ and $6 \le x + 8$, or $-2 \le x$

So, the solution is $x \in [-2, 6)$.

6. a) Answers may vary. For example:

$2x + 1 > 15$

$2x > 14$

$x > 7$

b) Answers may vary. For example:

$4x - 1 < -33$

$4x < -32$

$x < -8$

$x \in (-\infty, -8)$

c) Answers may vary. For example:

$-3 \le 2x - 1 \le 13$

$-3 + 1 \le 2x - 1 + 1 \le 13 + 1$

$-2 \le 2x \le 14$

$-1 \le x \le 7$

d) Answers may vary. For example:

$x - 2 \le 3x - 8$

$x + 6 \le 3x$

$6 \le 2x$

$3 \le x$

7. a) $f(x) = -x + 1$; $g(x) = 2x - 5$

b) $x > 2$

c) $f(x) < g(x)$

$-x + 1 < 2x - 5$

$-3x < -6$

$x > 2$

8. a) $N(t) = 20 + 0.02t$; $M(t) = 15 + 0.03t$

b) $20 + 0.02t > 15 + 0.03t$

c) $20 + 0.02t > 15 + 0.03t$

$-0.01t > -5$

$t < 500$

Since time must be positive, $0 \le t < 500$.

d) Negative time has no meaning.

4.3 Solving Polynomial Inequalities, pp. 225–228

1. a) $(x + 2)(x - 3)(x + 1) \ge 0$

$x + 2 = 0$ and $x - 3 = 0$ and $x + 1 = 0$

$x = -2, -1, 3$

The roots are -2, 1, and 3. These numbers divide the real numbers into 4 intervals:

$x < -2$, $-2 < x < -1$, $-1 < x < 3$, $x > 3$

	$x < -2$	$-2 < x < -1$	$-1 < x < 3$	$x > 3$
$(x + 2)$	$-$	$+$	$+$	$+$
$(x - 3)$	$-$	$-$	$-$	$+$
$(x + 1)$	$-$	$-$	$+$	$+$
their product	$(-)(-)(-) = -$	$(+)(-)(-) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$

The intervals are $-2 \leq x \leq -1$ or $x \geq 3$.

b) $-2(x - 2)(x - 4)(x + 3) < 0$

$x - 2 = 0$ and $x - 4 = 0$ and $x + 3 = 0$

$x = 2, 4, -3$

The roots are 2, 4, and -3. These numbers divide the real numbers into 4 intervals:

$x < -3, -3 < x < 2, 2 < x < 4, x > 4$

	$x < -3$	$-3 < x < 2$	$2 < x < 4$	$x > 4$
$(x - 2)$	$-$	$-$	$+$	$+$
$(x - 4)$	$-$	$-$	$-$	$+$
$(x + 3)$	$-$	$+$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(-)(+) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$

The intervals are $-3 < x < 2$ or $x > 4$.

c) $(x - 3)(5x + 2)(4x - 3) < 0$

$x - 3 = 0$ and $5x + 2 = 0$ and $4x - 3 = 0$

$$x = 3, -\frac{2}{5}, \frac{3}{4}$$

The roots are 3, $-\frac{2}{5}$, and $\frac{3}{4}$. These numbers divide the real numbers into 4 intervals:

$$x < -\frac{2}{5}, -\frac{2}{5} < x < \frac{3}{4}, \frac{3}{4} < x < 3, x > 3$$

	$x < -\frac{2}{5}$	$-\frac{2}{5} < x < \frac{3}{4}$	$\frac{3}{4} < x < 3$	$x > 3$
$(x - 3)$	$-$	$-$	$-$	$+$
$(5x + 2)$	$-$	$+$	$+$	$+$
$(4x - 3)$	$-$	$-$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(+)(-) = +$	$(-)(+)(-) = -$	$(+)(+)(+) = +$

The intervals are $x < -\frac{2}{5}$ or $\frac{3}{4} < x < 3$.

d) $(x - 5)(4x + 1)(2x - 5) \geq 0$

$x - 5 = 0$ and $4x + 1 = 0$ and $2x - 5 = 0$

$$x = 5, -\frac{1}{4}, \frac{5}{2}$$

The roots are 5, $-\frac{1}{4}$, and $\frac{5}{2}$. These numbers divide the real numbers into 4 intervals:

$$x < -\frac{1}{4}, -\frac{1}{4} < x < \frac{5}{2}, \frac{5}{2} < x < 5, x > 5$$

	$x < -\frac{1}{4}$	$-\frac{1}{4} < x < \frac{5}{2}$	$\frac{5}{2} < x < 5$	$x > 5$
$(x - 5)$	$-$	$-$	$-$	$+$
$(4x + 1)$	$-$	$+$	$+$	$+$
$(2x - 5)$	$-$	$-$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(+)(-) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$

The intervals are $-\frac{1}{4} \leq x \leq \frac{5}{2}$ or $x \geq 5$.

2. a) $(-\infty, -5]$, $[-2, 0]$, and $[3, \infty)$

b) At $x = 1$

c) $[-7, -3]$ and $[0, 4]$

d) $(-\infty, -4]$ and $[2, 7]$

3. $f(x) > g(x)$

$2x^3 - x^2 + 3x + 10 > x^3 + 3x^2 + 2x + 4$

$x^3 - 4x^2 + x + 6 > 0$

$f(2) = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 1 & 6 \\ & \downarrow & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$(x - 2)(x^2 - 2x - 3) > 0$

$(x - 2)(x + 1)(x - 3) > 0$

If $x - 2 = 0$ and $x + 1 = 0$ and $x - 3 = 0$,

$x = 2, -1, 3$

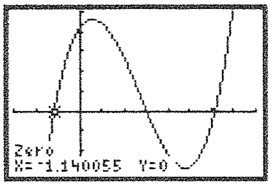
The roots are -1, 2, and 3. These numbers divide the real numbers into 4 intervals:

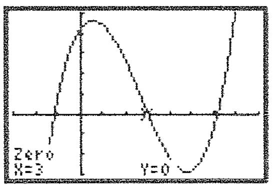
$x < -1, -1 < x < 2, 2 < x < 3, x > 3$

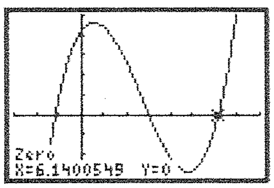
	$x < -1$	$-1 < x < 2$	$2 < x < 3$	$x > 3$
$(x - 2)$	$-$	$-$	$+$	$+$
$(x - 3)$	$-$	$-$	$-$	$+$
$(x + 1)$	$-$	$+$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(-)(+) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$

$2x^3 - x^2 + 3x + 10 > x^3 + 3x^2 + 2x + 4$ when $-1 < x < 2$ or $x > 3$.

4. To solve $x^3 - 7x^2 + 4x + 12 > x^2 - 4x - 9$, rewrite as an equivalent inequality with 0 on one side.
$x^3 - 8x^2 + 8x + 21 > 0$
Graph the polynomial on the left side of the inequality and determine the intervals for which the graph is above the x-axis.







The zeros are at $x = -1.14$, 3, and 6.14. The intervals for which $x^3 - 7x^2 + 4x + 12 > x^2 - 4x - 9$ is true are $-1.14 < x < 3$ and $x > 6.14$.

5. a) $(-1, 2)$, $(4, \infty)$
b) $(-2, 2)$, $(2, \infty)$
c) $(-\infty, -2)$, $(0, 1)$
d) $(-\infty, 2)$, $(2, \infty)$

6. a) $(x - 1)(x + 1) > 0$
$x - 1 = 0$ and $x + 1 = 0$
$x = 1, -1$
This divides the domain of real numbers into 3 intervals:
$x < -1$; $-1 < x < 1$; $x > 1$
Test for each interval:
$\boldsymbol{x < -1}$**:**
$f(-2) = (-2 - 1)(-2 + 1)$
$= (-3)(-1)$
$= 3 > 0$ Yes
$\boldsymbol{-1 < x < 1}$**:**
$f(0) = (0 - 1)(0 + 1)$
$= (-1)(1)$
$= -1 > 0$ No
$\boldsymbol{x > 1}$**:**
$f(2) = (2 - 1)(2 + 1)$
$= (1)(3)$
$= 3 > 0$ Yes
The intervals are $x < -1$ or $x > 1$.

b) $(x + 3)(x - 4) < 0$
$x + 3 = 0$ and $x - 4 = 0$
$x = -3, 4$
This divides the domain of real numbers into 3 intervals:
$x < -3$; $-3 < x < 4$; $x > 4$
Test for each interval:
$\boldsymbol{x < -3}$**:**
$f(-4) = (-4 + 3)(-4 - 4)$
$= (-1)(-8)$
$= 8 < 0$ No
$\boldsymbol{-3 < x < 4}$**:**
$f(0) = (0 + 3)(0 - 4)$
$= (3)(-4)$
$= -12 < 0$ Yes
$\boldsymbol{x > 4}$**:**
$f(5) = (5 + 3)(5 - 4)$
$= (8)(1)$
$= 8 < 0$ No
The interval is $-3 < x < 4$.

c) $(2x + 1)(x - 5) \geq 0$
$2x + 1 = 0$ and $x - 5 = 0$
$x = -\frac{1}{2}, 5$
This divides the domain of real numbers into 3 intervals:
$x \leq -\frac{1}{2}$; $-\frac{1}{2} \leq x \leq 5$; $x \geq 5$
Test for each interval:
$\boldsymbol{x \leq -\frac{1}{2}}$**:**
$f(-1) = (2(-1) + 1)(-1 - 5)$
$= (-1)(-6)$
$= 6 \geq 0$ Yes
$\boldsymbol{-\frac{1}{2} \leq x \leq 5}$**:**
$f(1) = (2(1) + 1)(1 - 5)$
$= (3)(-4)$
$= -12 \geq 0$ No
$\boldsymbol{x \geq 5}$**:**
$f(6) = (2(6) + 1)(6 - 5)$
$= (13)(1)$
$= 13 \geq 0$ Yes
The intervals are $x \leq -\frac{1}{2}$ or $x \geq 5$.

d) $-3x(x + 7)(x - 2) < 0$
$-3x = 0$ and $x + 7 = 0$ and $x - 2 = 0$
$x = 0, -7, 2$

This divides the domain of real numbers into 4 intervals:
$x < -7$; $-7 < x < 0$; $0 < x < 2$; $x > 2$
Test for each interval:
$x < -7$:
$$\begin{aligned} f(-8) &= (-3(-8))(-8 + 7)(-8 - 2) \\ &= (24)(-1)(-10) \\ &= 240 < 0 \end{aligned} \qquad \text{No}$$
$-7 < x < 0$:
$$\begin{aligned} f(-1) &= (-3(-1))(-1 + 7)(-1 - 2) \\ &= (3)(6)(-3) \\ &= -54 < 0 \end{aligned} \qquad \text{Yes}$$
$0 < x < 2$:
$$\begin{aligned} f(1) &= (-3(1))(1 + 7)(1 - 2) \\ &= (-3)(8)(-1) \\ &= 24 < 0 \end{aligned} \qquad \text{No}$$
$x > 2$:
$$\begin{aligned} f(3) &= (-3(3))(3 + 7)(3 - 2) \\ &= (-9)(10)(1) \\ &= -90 < 0 \end{aligned} \qquad \text{Yes}$$
The intervals are $-7 < x < 0$ or $x > 2$.

e) $(x - 3)(x + 1) + (x - 3)(x + 2) \geq 0$
$$\begin{aligned} (x^2 - 2x - 3) + (x^2 - x - 6) &\geq 0 \\ 2x^2 - 3x - 9 &\geq 0 \\ 2x^2 - 6x + 3x - 9 &\geq 0 \\ 2x(x - 3) + 3(x - 3) &\geq 0 \\ (2x + 3)(x - 3) &\geq 0 \end{aligned}$$
$2x + 3 = 0$ and $x - 3 = 0$
$$x = -\frac{3}{2}, 3$$
The roots are $-\frac{3}{2}$ and 3. These numbers divide the real numbers into 3 intervals:
$x < -\frac{3}{2}$, $-\frac{3}{2} < x < 3$, $x > 3$

	$x < -\frac{3}{2}$	$-\frac{3}{2} < x < 3$	$x > 3$
$(2x + 3)$	$-$	$+$	$+$
$(x - 3)$	$-$	$-$	$+$
their product	$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$

$(x - 3)(x + 1) + (x - 3)(x + 2) \geq 0$ when $x \leq -\frac{3}{2}$ or $x \geq 3$.

f) $2x(x + 4) - 3(x + 4) \leq 0$
$$(2x - 3)(x + 4) \leq 0$$
$2x - 3 = 0$ and $x + 4 = 0$
$$x = \frac{3}{2}, -4$$
The roots are $\frac{3}{2}$ and -4. These numbers divide the real numbers into 3 intervals:
$x < -4$, $-4 < x < \frac{3}{2}$, $x > \frac{3}{2}$

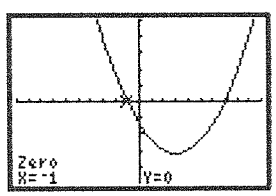
	$x < -4$	$-4 < x < \frac{3}{2}$	$x > \frac{3}{2}$
$(2x - 3)$	$-$	$-$	$+$
$(x + 4)$	$-$	$+$	$+$
their product	$(-)(-) = +$	$(-)(+) = -$	$(+)(+) = +$

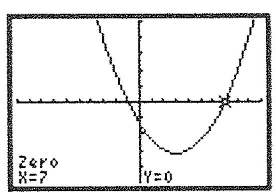
$2x(x + 4) - 3(x + 4) \leq 0$ when $-4 \leq x \leq \frac{3}{2}$.

7. a) $x^2 - 6x + 9 \geq 16$
$$\begin{aligned} x^2 - 6x - 7 &\geq 0 \\ (x - 7)(x + 1) &\geq 0 \end{aligned}$$
$x - 7 = 0$ and $x + 1 = 0$
$$x = 7, -1$$
The roots are -1 and 7. These numbers divide the real numbers into 3 intervals:
$x < -1$, $-1 < x < 7$, $x > 7$

	$x < -1$	$-1 < x < 7$	$x > 7$
$(x - 7)$	$-$	$-$	$+$
$(x + 1)$	$-$	$+$	$+$
their product	$(-)(-) = +$	$(-)(+) = -$	$(+)(+) = +$

$x^2 - 6x + 9 \geq 16$ when $x \leq -1$ or $x \geq 7$.



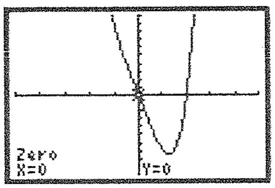


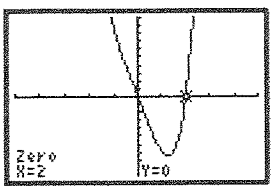
b) $x^4 - 8x < 0$
$x(x^3 - 8) < 0$
$x(x - 2)(x^2 + 2x + 4) < 0$

Since the discriminant of $x^2 + 2x + 4$ is -12, this factor has no real roots. The only roots are $x = 0$ and $x = 2$. These numbers divide the real numbers into 3 intervals: $x < 0, 0 < x < 2, x > 2$.

	$x < 0$	$0 < x < 2$	$x > 2$
x	$-$	$+$	$+$
$(x - 2)$	$-$	$-$	$+$
$(x^2 + 2x + 4)$	$+$	$+$	$+$
their product	$(-)(-)(+) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$

$x^4 - 8x < 0$ when $0 < x < 2$





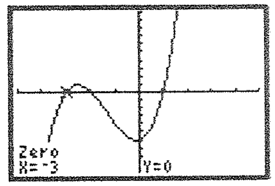
c) $x^3 + 4x^2 + x \leq 6$
$x^3 + 4x^2 + x - 6 \leq 0$
When $x = 1$, the left side is 0.

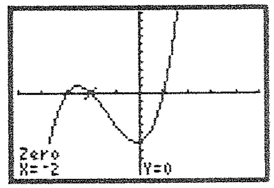
$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & \downarrow & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

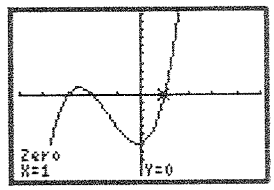
$(x - 1)(x^2 + 5x + 6) \leq 0$
$(x - 1)(x + 3)(x + 2) \leq 0$
$x - 1 = 0$ and $x + 3 = 0$ and $x + 2 = 0$
$x = 1, -3, -2$
The roots are $-3, -2$, and 1. These numbers divide the real numbers into 4 intervals:
$x < -3, -3 < x < -2, -2 < x < 1, x > 1$

	$x < -3$	$-3 < x < -2$	$-2 < x < 1$	$x > 1$
$(x - 1)$	$-$	$-$	$-$	$+$
$(x + 3)$	$-$	$+$	$+$	$+$
$(x + 2)$	$-$	$-$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(+)(-) = +$	$(-)(+)(+) = -$	$(+)(+)(+) = +$

$x^3 + 4x^2 + x \leq 6$ when $x \leq -3$ or $-2 \leq x \leq 1$.



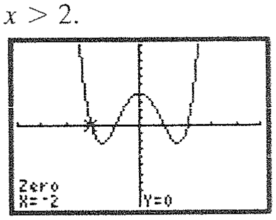


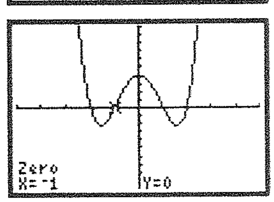


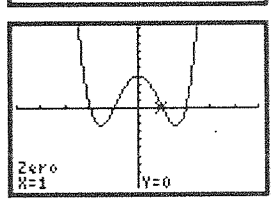
d) $x^4 - 5x^2 + 4 > 0$
$(x^2 - 4)(x^2 - 1) > 0$
$x^2 - 4 = 0$ and $x^2 - 1 = 0$
$x^2 = 4$ and $x^2 = 1$
$x = \pm 2$ and $x = \pm 1$
The roots are $-2, -1, 1$, and 2. These numbers divide the real numbers into 5 intervals:
$x < -2, \ -2 < x < -1, \ -1 < x < 1, 1 < x < 2,$
$x > 2$

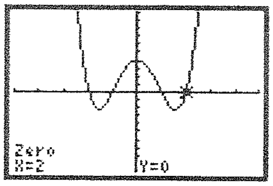
	$x < -2$	$-2 < x < 1$	$-1 < x < 1$	$1 < x < 2$	$x > 2$
$(x^2 - 1)$	$+$	$+$	$-$	$+$	$+$
$(x^2 - 4)$	$+$	$-$	$-$	$-$	$+$
their product	$(+)(+) = +$	$(+)(-) = -$	$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$

$x^4 - 5x^2 + 4 > 0$ when $x < -2, -1 < x < 1$, or $x > 2$.









e) $3x^3 - 3x^2 - 2x \le 2x^3 - x^2 + x$

$x^3 - 2x^2 - 3x \le 0$

$x(x^2 - 2x - 3) \le 0$

$x(x + 1)(x - 3) \le 0$

$x = 0$ and $x + 1 = 0$ and $x - 3 = 0$

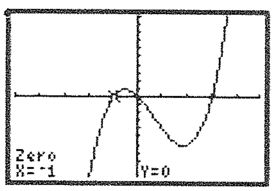
$x = 0, -1, 3$

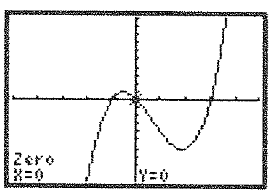
The roots are -1, 0, and 3. These numbers divide the real numbers into 4 intervals:

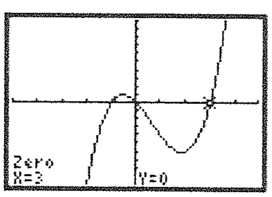
$x < -1, -1 < x < 0, 0 < x < 3, x > 3$

	$x < -1$	$-1 < x < 0$	$0 < x < 3$	$x > 3$
x	$-$	$-$	$+$	$+$
$(x - 3)$	$-$	$-$	$-$	$+$
$(x + 1)$	$-$	$+$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(-)(+) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$

$3x^3 - 3x^2 - 2x \le 2x^3 - x^2 + x$ when $x \le -1$ or $0 \le x \le 3$.







f) $x^3 - x^2 - 3x + 3 > -x^3 + 2x + 5$

$2x^3 - x^2 - 5x - 2 > 0$

$f(2) = 0$

$$\begin{array}{r|rrrr} 2 & 2 & -1 & -5 & -2 \\ & \downarrow & 4 & 6 & 2 \\ \hline & 2 & 3 & 1 & 0 \end{array}$$

$(x - 2)(2x^2 + 3x + 1) > 0$

$(x - 2)(2x^2 + 2x + 1x + 1) > 0$

$(x - 2)(2x(x + 1) + 1(x + 1)) > 0$

$(x - 2)(2x + 1)(x + 1) > 0$

$x - 2 = 0$ and $2x + 1 = 0$ and $x + 1 = 0$

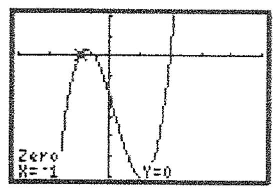
$x = 2, -\frac{1}{2}, -1$

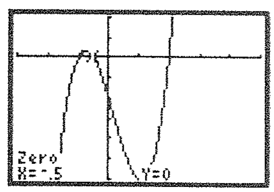
The roots are -1, $-\frac{1}{2}$, and 2. These numbers divide the real numbers into 4 intervals:

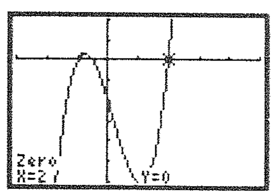
$x < -1, -1 < x < -\frac{1}{2}, -\frac{1}{2} < x < 2, x > 2$

	$x < -1$	$-1 < x < -\frac{1}{2}$	$-\frac{1}{2} < x < 2$	$x > 2$
$(x - 2)$	$-$	$-$	$-$	$+$
$(x + 1)$	$-$	$+$	$+$	$+$
$(2x + 1)$	$-$	$-$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(+)(-) = +$	$(-)(+)(+) = -$	$(+)(+)(+) = +$

$x^3 - x^2 - 3x + 3 > -x^3 + 2x + 5$ when $-1 < x < -\frac{1}{2}$ or $x > 2$.







8. $(-1, 1)$ and $(2, \infty)$

9. a) $x^3 + 11x^2 + 18x = 0$

b) Any values of x for which the graph of the corresponding function is above the x-axis ($y = 0$) are solutions to the original inequality.

c) $x^3 + 11x^2 + 18x + 10 > 10$

$x^3 + 11x^2 + 18x > 0$

$x(x^2 + 11x + 18) > 0$

$x(x + 9)(x + 2) > 0$

$x = 0$ and $x + 9 = 0$ and $x + 2 = 0$

$x = 0, -9, -2$

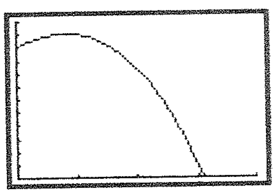
The roots are -9, -2, and 0. These numbers divide the real numbers into 4 intervals:
$x < -9$, $-9 < x < -2$, $-2 < x < 0$, $x > 0$

	$x < -9$	$-9 < x < -2$	$-2 < x < 0$	$x > 0$
x	$-$	$-$	$-$	$+$
$(x + 9)$	$-$	$+$	$+$	$+$
$(x + 2)$	$-$	$-$	$+$	$+$
their product	$(-)(-)(-) = -$	$(-)(+)(-) = +$	$(-)(+)(+) = -$	$(+)(+)(+) = +$

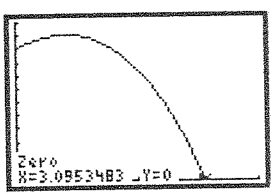
$x^3 + 11x^2 + 18x + 10 > 10$ when $-9 < x < -2$ or $x > 0$.

10. $f(x)$ has roots at -2, 1, and a double root at 3.
$f(x) = a(x + 2)(x - 1)(x - 3)^2$
Since $f(-1) = 96$,
$96 = a(-1 + 2)(-1 - 1)(-1 - 3)^2$
$96 = a(1)(-2)(16)$
$96 = -32a$
$-3 = a$
$f(x) = -3(x + 2)(x - 1)(x - 3)^2$

11. a)

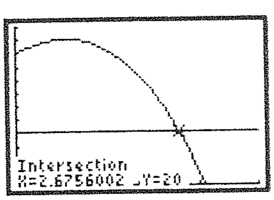


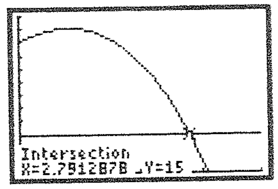
b) $-t^3 - 6t^2 + 12t + 50 > 0$



The zero is at approximately 3.095 348 3. So,
$3.095\,348\,3 \times 50\ °C \doteq 154.77\ °C$.
$v > 0$ for temperatures less than 154.77 °C.
$0 < v < 154.77$ °C

c) Use a graphing calculator to determine the value of t for which $v = 20$ and the value of t for which $v = 15$. These numbers will provide the temperature range for which $15 < v < 20$.

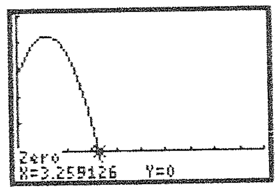




$2.675\,600\,2 \times 50\ °C = 133.78\ °C$.
$2.791\,287\,8 \times 50\ °C = 139.56\ °C$.
So, $15 < v < 20$ for temperatures between 133.78 °C and 139.56 °C

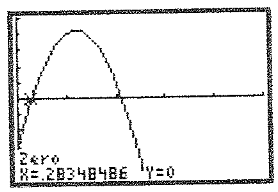
12. a) $h(0) = -5(0)^2 + 12(0) + 14 = 14$

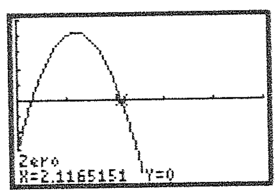
b) $-5t^2 + 12t + 14 > 0$



The rock is in the air for about 3.3 seconds.

c) $-5t^2 + 12t + 14 > 17$
$-5t^2 + 12t - 3 > 0$

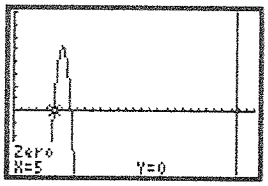


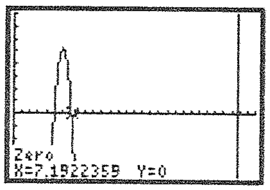


$0.3 < t < 2.1$; The rock is higher than 17 metres between 0.3 seconds and 2.1 seconds.

d) 2.1 s $-$ 0.3 s = 1.8 seconds

13. $V(x) = x(50 - 2x)(30 - 2x)$
First note that $0 < x < 15$, since $x = 0$ means that no squares are cut out and that there is no height. Also, if $x \geq 15$, an entire side is cut away and there is nothing to fold up.
$x(50 - 2x)(30 - 2x) > 4000$
$x(1500 - 160x + 4x^2) - 4000 > 0$
$4x^3 - 160x^2 + 1500x - 4000 > 0$
$x^3 - 40x^2 + 375x - 1000 > 0$





$5 < x < 7.19$

The volume of the box will be greater than 4000 cm^3 if the squares cut from the 4 corners measure between 5 cm and 7.19 cm.

14. a) Since all the powers are even and the coefficients are positive, the polynomial on the left is always positive.

b) Since all the powers are even and all the coefficients are negative (once all terms are brought to the left), the polynomial on the left is always negative.

15. You cannot divide by a variable expression because you do not know whether it is positive, negative, or zero.

$$(x + 1)(x - 2) > (x + 1)(-x + 6)$$
$$x^2 - x - 2 > -x^2 + 5x + 6$$
$$2x^2 - 6x - 8 > 0$$
$$2x^2 - 8x + 2x - 8 > 0$$
$$2x(x - 4) + 2(x - 4) > 0$$
$$(2x + 2)(x - 4) > 0$$

	$x < -1$	$-1 < x < 4$	$x > 4$
$(x + 1)$	$-$	$+$	$+$
$(x - 4)$	$-$	$-$	$+$
their product	$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$

$$2(x + 1)(x - 4) > 0$$

$x + 1 = 0$ and $x - 4 = 0$

$x = -1, 4$

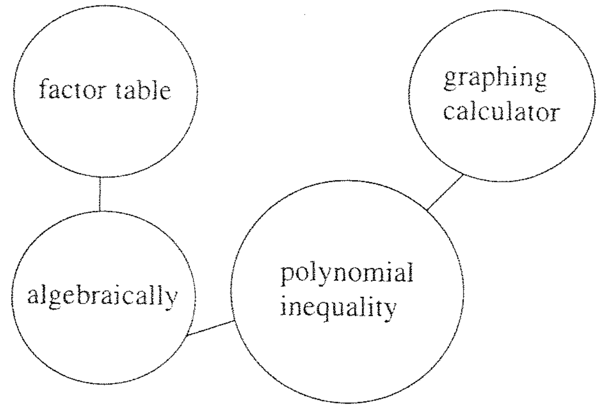
The roots are -1, and 4. These numbers divide the real numbers into 3 intervals:

$x < -1$, $-1 < x < 4$, $x > 4$

$(x + 1)(x - 2) > (x + 1)(-x + 6)$ when $x < -1$ or $x > 4$.

The correct solution is $x < -1$ or $x > 4$.

16. Answers may vary. For example:



17. a) $\dfrac{x^2 + x - 12}{x^2 + 5x + 6} < 0$

$$\frac{(x + 4)(x - 3)}{(x + 3)(x + 2)} < 0$$

The roots are -4 and 3 for the numerator, and -3, and -2 for the denominator. These numbers divide the real numbers into 5 intervals:

$x < -4$, $-4 < x < -3$, $-3 < x < -2$, $-2 < x < 3$, $x > 3$

	$x < -4$	$-4 < x < -3$	$-3 < x < -2$	$-2 < x < 3$	$x > 3$
$(x + 4)$	$-$	$+$	$+$	$+$	$+$
$(x - 3)$	$-$	$-$	$-$	$-$	$+$
$(x + 3)$	$-$	$-$	$+$	$+$	$+$
$(x + 2)$	$-$	$-$	$-$	$+$	$+$
their quotient	$\frac{(-)(-)}{(-)(-)} = +$	$\frac{(+)(-)}{(-)(-)} = -$	$\frac{(+)(-)}{(+)(-)} = +$	$\frac{(+)(-)}{(+)(+)} = -$	$\frac{(+)(+)}{(+)(+)} = +$

$\dfrac{x^2 + x - 12}{x^2 + 5x + 6} < 0$ when $-4 < x < -3$ or $-2 < x < 3$.

b) $\dfrac{x^2 - 25}{x^3 + 6x^2 + 5x} > 0$

$$\frac{(x + 5)(x - 5)}{x(x^2 + 6x + 5)} > 0$$

$$\frac{\cancel{(x + 5)}(x - 5)}{x(x + 1)\cancel{(x + 5)}} > 0$$

$$\frac{(x - 5)}{x(x + 1)} > 0$$

The roots are 5 for the numerator, and -1 and 0 for the denominator. These numbers divide the real numbers into 4 intervals:
$x < -1, -1 < x < 0, 0 < x < 5, x > 5$

	$x < -1$	$-1 < x < 0$	$0 < x < 5$	$x > 5$
$(x - 5)$	$-$	$-$	$-$	$+$
x	$-$	$-$	$+$	$+$
$(x + 1)$	$-$	$+$	$+$	$+$
their quotient	$\frac{(-)}{(-)(-)} = -$	$\frac{(-)}{(-)(+)} = +$	$\frac{(-)}{(+)(+)} = -$	$\frac{(+)}{(+)(+)} = +$

$\frac{x^2 - 25}{x^3 + 6x^2 + 5x} > 0$ when $-1 < x < 0$ or $x > 5$.

18. $(x + 1)(x - 2)(2^x) \ge 0$
2^x cannot equal zero. So, the roots must be found using $(x + 1)(x - 2)$.
$x + 1 = 0$ and $x - 2 = 0$
$x = -1, 2$
The roots are -1 and 2. These numbers divide the real numbers into 3 intervals:
$x < -1, -1 < x < 2, x > 2$

	$x < -1$	$-1 < x < 2$	$x > 2$
$(x + 1)$	$-$	$+$	$+$
$(x - 2)$	$-$	$-$	$+$
their product	$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$

Since 2^x is always positive, we only needed to check the sign of the product of the other factors.
$(x + 1)(x - 2)(2^x) \ge 0$ when $x \le -1$ or $x \ge 2$.

4.4 Rates of Change in Polynomial Functions, pp. 235–237

1. a) positive on (0, 1), (4, 7), (10, 15.5), (19, 20); negative on (1, 4), (7, 10), (15.5, 19); zero at $x = 1, 4, 7, 10, 15.5$, and 19
b) A positive slope means the cyclist's elevation is increasing (cyclist is going uphill), a negative slope means it is decreasing (cyclist is going downhill), and a zero slope means the cyclist's elevation is transitioning from increasing to decreasing or vice versa (at the top of a hill or bottom of a valley).

2. $f(x) = 3(x - 2)^2 - 2$
a) i) $2 \le x \le 4$

$$\text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(4) - f(2)}{4 - 2}$$
$$= \frac{(3(4 - 2)^2 - 2) - (3(2 - 2)^2 - 2)}{2}$$
$$= \frac{10 - (-2)}{2}$$
$$= 6$$

ii) $2 \le x \le 6$

$$\text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(6) - f(2)}{6 - 2}$$
$$= \frac{(3(6 - 2)^2 - 2) - (3(2 - 2)^2 - 2)}{4}$$
$$= \frac{46 - (-2)}{4}$$
$$= 12$$

iii) $4 \le x \le 6$

$$\text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(6) - f(4)}{6 - 4}$$
$$= \frac{(3(6 - 2)^2 - 2) - (3(4 - 2)^2 - 2)}{2}$$
$$= \frac{46 - 10}{2}$$
$$= 18$$

b) $x = 4$

$$\text{Slope} = \frac{f(a + h) - f(a)}{h}, \text{ where } a = 4; h = 0.001$$
$$= \frac{f(4.001) - f(4)}{0.001}$$
$$= \frac{(3(4.001 - 2)^2 - 2) - (3(4 - 2)^2 - 2)}{0.001}$$
$$= \frac{10.012\,003 - 10}{0.001}$$
$$\doteq 12$$

c) The graph is increasing on (2, 6).

d) $0 \le x \le 2$

$$\text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(2) - f(0)}{2 - 0}$$

$$= \frac{(3(2-2)^2 - 2) - (3(0-2)^2 - 2)}{2}$$

$$= \frac{-2 - 10}{2}$$

$$= -6$$

e) $(1, 1) \rightarrow x = 1$

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 1;\ h = 0.001$$

$$= \frac{f(1.001) - f(1)}{0.001}$$

$$= \frac{(3(1.001 - 2)^2 - 2) - (3(1-2)^2 - 2)}{0.001}$$

$$= \frac{0.994\,003 - 1}{0.001}$$

$$\doteq -6$$

3. $f(x) = x^3 - 4x^2 + 4x$

a) $x = 4$

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 2;\ h = 0.001$$

$$= \frac{f(2.001) - f(2)}{0.001}$$

$$= \frac{(2.001^3 - 4(2.001)^2 + 4(2.001))}{0.001}$$

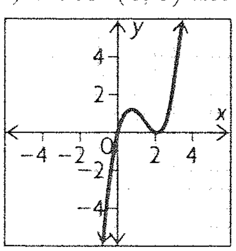
$$- \frac{(2^3 - 4(2)^2 + 4(2))}{0.001}$$

$$= \frac{-0.000\,002 - 0}{0.001}$$

$$\doteq 0$$

b) It indicates that $x = 2$ is a turning point in the graph.

c) Zeros: $(0, 0)$ and $(2, 0)$.



4. a) $f(4) = 0,\ f(5) = 7$

$$\text{Average rate of change} = \frac{f(5) - f(4)}{5 - 4}$$

$$= \frac{7 - 0}{1}$$

$$= 7$$

b) Answers may vary. For example, $(4.5, 3)$.

5. $x \in [2, 5]$

a) $f(x) = 3x + 1$

$$\text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(5) - f(2)}{5 - 2}$$

$$= \frac{(3(5) + 1) - (3(2) + 1)}{3}$$

$$= \frac{16 - 7}{3}$$

$$= 3$$

b) $t(x) = 3x^2 - 4x + 1$

$$= \frac{f(5) - f(2)}{5 - 2}$$

$$= \frac{(3(5)^2 - 4(5) + 1) - (3(2)^2 - 4(2) + 1)}{3}$$

$$= \frac{56 - 5}{3}$$

$$= 17$$

c) $g(x) = \frac{1}{x}$

$$= \frac{f(5) - f(2)}{5 - 2}$$

$$= \frac{\left(\frac{1}{5}\right) - \left(\frac{1}{2}\right)}{3}$$

$$= \frac{-\frac{3}{10}}{3}$$

$$= -\frac{1}{10}$$

d) $d(x) = -x^2 + 7$

$$= \frac{f(5) - f(2)}{5 - 2}$$

$$= \frac{(-(5)^2 + 7) - (-(2)^2 + 7)}{3}$$

$$= \frac{-18 - 3}{3}$$

$$= -7$$

e) $h(x) = 2^x$

$$= \frac{f(5) - f(2)}{5 - 2}$$

$$= \frac{(2^5) - (2^2)}{3}$$

$$= \frac{32 - 4}{3}$$

$$= \frac{28}{3}$$

f) $v(x) = 9$

Slope $= 0$

6. Instantaneous rate at $x = 3$

a) $f(x) = 3x + 1$

Slope $= \dfrac{f(a + h) - f(a)}{h}$, where $a = 3$; $h = 0.001$

$$= \frac{f(3.001) - f(3)}{0.001}$$
$$= \frac{(3(3.001) + 1) - (3(3) + 1)}{0.001}$$
$$= \frac{10.003 - 10}{0.001}$$
$$= 3$$

b) $t(x) = 3x^2 - 4x + 1$

$$= \frac{f(3.001) - f(3)}{0.001}$$
$$= \frac{(3(3.001)^2 - 4(3.001) + 1)}{0.001} - \frac{(3(3)^2 - 4(3) + 1)}{0.001}$$
$$= \frac{16.014\,003 - 16}{0.001}$$
$$\doteq 14$$

c) $g(x) = \dfrac{1}{x}$

$$= \frac{f(3.001) - f(3)}{0.001}$$
$$= \frac{\left(\frac{1}{3.001}\right) - \left(\frac{1}{3}\right)}{0.001}$$
$$\doteq -0.111$$
$$\doteq -\frac{1}{9}$$

d) $d(x) = -x^2 + 7$

$$= \frac{f(3.001) - f(3)}{0.001}$$
$$= \frac{(-(3.001)^2 + 7) - (-(3)^2 + 7)}{0.001}$$
$$= \frac{-2.006\,001 - (-2)}{0.001}$$
$$\doteq -6$$

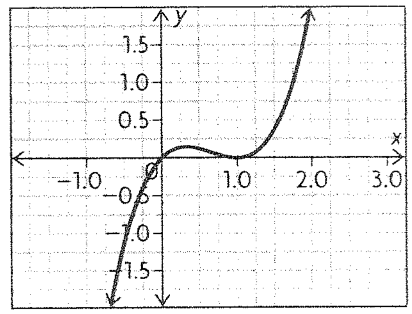
e) $h(x) = 2^x$

$$= \frac{f(3.001) - f(3)}{0.001}$$
$$= \frac{(2^{3.001}) - (2^3)}{0.001}$$
$$= \frac{8.005\,547\,1 - 8}{0.001}$$
$$\doteq 5.5$$

f) $v(x) = 9$

slope $= 0$

7.



Rate of change is positive on $(-\infty, \frac{1}{3})$ and $(1, \infty)$, negative on $(\frac{1}{3}, 1)$, and zero at $x = \frac{1}{3}$ and $x = 1$.

8. $s(t) = 320 - 5t^2, 0 \le t \le 8$

a) $3 \le t \le 8$

Average rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{f(8) - f(3)}{8 - 3}$$
$$= \frac{(320 - 5(8)^2) - (320 - 5(3)^2)}{5}$$
$$= \frac{0 - 275}{5}$$
$$= -55 \text{ m/s}$$

b) $t = 2$

Slope $= \dfrac{f(a + h) - f(a)}{h}$, where $a = 2$; $h = 0.001$

$$= \frac{f(2.001) - f(2)}{0.001}$$
$$= \frac{(320 - 5(2.001)^2) - (320 - 5(2)^2)}{0.001}$$
$$= \frac{299.979\,995 - 300}{0.001}$$
$$\doteq -20 \text{ m/s}$$

9. $f(x) = 3x^2 - 4x - 1$

a) $x = 1$

Slope $= \dfrac{f(a + h) - f(a)}{h}$, where $a = 1$; $h = 0.001$

$$= \frac{f(1.001) - f(1)}{0.001}$$
$$= \frac{(3(1.001)^2 - 4(1.001) - 1)}{0.001} - \frac{(3(1)^2 - 4(1) - 1)}{0.001}$$
$$= \frac{-1.997\,997 - (-2)}{0.001}$$
$$\doteq 2$$

b) $f(1) = 3(1)^2 - 4(1) - 1$
$= 3 - 4 - 1$
$= -2$

c) Slope = 2; point of tangency: $(1, -2)$

$$y = 2x + b$$
$$-2 = 2(1) + b$$
$$-4 = b$$
$$y = 2x - 4$$

10. $h(t) = -5t^2 + 50t$

a) $t = 4$

Slope $= \dfrac{f(a+h) - f(a)}{h}$, where $a = 4$; $h = 0.001$

$$= \frac{f(4.001) - f(4)}{0.001}$$
$$= \frac{(-5(4.001)^2 + 50(4.001))}{0.001} - \frac{(-5(4)^2 + 50(4))}{0.001}$$
$$= \frac{120.009\,995 - 120}{0.001}$$
$$\doteq 10 \text{ m/s.}$$

b) $t = 10$

Slope $= \dfrac{f(a+h) - f(a)}{h}$, where $a = 10$; $h = 0.001$

$$= \frac{f(10.001) - f(10)}{0.001}$$
$$= \frac{(-5(10.001)^2 + 50(10.001))}{0.001} - \frac{(-5(10)^2 + 50(10))}{0.001}$$
$$= \frac{-0.050\,005 - 0}{0.001}$$
$$\doteq -50 \text{ m/s}$$

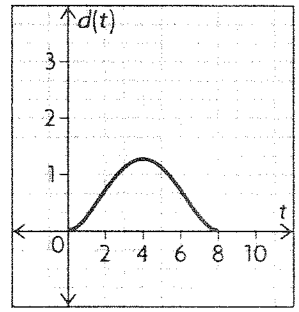
c) interval from $t = 0$ to $t = 10$

Average rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{f(10) - f(0)}{10 - 0}$$
$$= \frac{(-5(10)^2 + 50(10)) - (-5(0)^2 + 50(0))}{10}$$
$$= \frac{0 - 0}{10}$$
$$= 0 \text{ m/s}$$

11. $d(t) = \left(\dfrac{1}{200}\right)t^2(t - 8)^2$

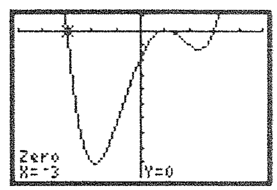
a)



The rate is positive for $t \in (0, 4)$, negative for $t \in (4, 8)$, and zero at $t = 0, 4$, and 8

b) When the rate of change is zero, the boat stops. When the rate of change is negative, the boat is headed back to the dock.

12. $y = x^4 - 2x^3 - 8x^2 + 18x - 9$

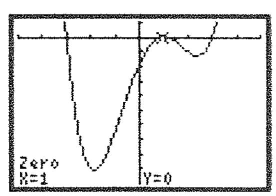


at $x = -3$

Slope $= \dfrac{f(a+h) - f(a)}{h}$, where $a = -3$;

$h = 0.001$

$$= \frac{f(-2.99) - f(-3)}{0.001}$$
$$= \frac{-0.095\,936\,014 - 0}{0.001}$$
$$\doteq -96$$

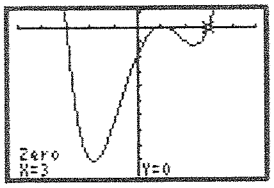


at $x = 1$

Slope $= \dfrac{f(a+h) - f(a)}{h}$, where $a = 1$; $h = 0.001$

$$= \frac{f(1.001) - f(1)}{0.001}$$

$$= \frac{-0.000\,007\,99 - 0}{0.001}$$
$$\doteq 0$$



at $x = 3$

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 3; h = 0.001$$
$$= \frac{f(3.001) - f(3)}{0.001}$$
$$= \frac{0.024\,028\,01 - 0}{0.001}$$
$$\doteq 24$$

13. $f(x) = x^2 + 3x - 5$

a) Instantaneous rate of change at $x = 1$,

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 1; h = 0.001$$
$$= \frac{f(1.001) - f(1)}{0.001}$$
$$= \frac{(1.001^2 + 3(1.001) - 5) - (1^2 + 3(1) - 5)}{0.001}$$
$$= \frac{-0.994\,999 - (-1)}{0.001}$$
$$\doteq 5$$

b) $\frac{f(x+h) - f(x)}{h}$

$$= \frac{((x+h)^2 + 3(x+h) - 5) - (x^2 + 3x - 5)}{h}$$
$$= \frac{(x^2 + 2xh + h^2 + 3x + 3h - 5)}{h} - \frac{(x^2 + 3x - 5)}{h}$$
$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h \cancel{- 5} \cancel{- x^2} \cancel{- 3x} \cancel{+ 5}}{h}$$
$$= \frac{2xh + 3h + h^2}{h}$$
$$= \frac{\cancel{h}(2x + 3 + h)}{\cancel{h}}$$
$$= 2x + 3 + h$$

c) It gets closer to $2x + 3$ as h becomes very close to 0.

d) The expression for the instantaneous rate of change is $2x + 3$. At $x = 1$: $2(1) + 3 = 5$

14. When the instantaneous rate of change is zero, the function potentially has a local maximum or a local minimum. If the rate is positive to the left and negative to the right, it has a local maximum. If the rate is negative to the left and positive to the right, it has a local minimum.

15. $f(x) = e^x$

a) Instantaneous rate of change at $x = 5$,

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 5; h = 0.001$$
$$= \frac{f(5.001) - f(5)}{0.001}$$
$$= \frac{e^{5.001} - e^5}{0.001}$$
$$= \frac{148.561\,646\,5 - 148.413\,159\,1}{0.001}$$
$$\doteq 148.4$$

$f(5) = e^5 \doteq 148.4$

b) Answers may vary. For example.

i) Instantaneous rate of change at $x = 1$,

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 1; h = 0.001$$
$$= \frac{f(1.001) - f(1)}{0.001}$$
$$= \frac{e^{1.001} - e^1}{0.001}$$
$$= \frac{2.721\,001\,47 - 2.718\,281\,828}{0.001}$$
$$\doteq 2.7$$

$f(1) = e^1 \doteq 2.7$

ii) Instantaneous rate of change at $x = 3$,

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 3; h = 0.001$$
$$= \frac{f(3.001) - f(3)}{0.001}$$
$$= \frac{e^{3.001} - e^3}{0.001}$$
$$= \frac{20.105\,632\,51 - 20.085\,536\,92}{0.001}$$
$$\doteq 20.1$$

$f(3) = e^3 \doteq 20.1$

iii) Instantaneous rate of change at $x = 4$,

$$\text{Slope} = \frac{f(a+h) - f(a)}{h}, \text{ where } a = 4; h = 0.001$$
$$= \frac{f(4.001) - f(4)}{0.001}$$

$$= \frac{e^{4.001} - e^4}{0.001}$$

$$= \frac{54.652\,775\,49 - 54.598\,150\,03}{0.001}$$

$$\doteq 54.6$$

$f(4) = e^4 \doteq 54.6$

c) The instantaneous rate of change of e^x for any value of x is e^x.

16. $f(x) = x^3 - 4x$

a) Instantaneous rate of change at $x = 1$.

Slope $= \dfrac{f(a+h) - f(a)}{h}$, where $a = 1$; $h = 0.001$

$$= \frac{f(1.001) - f(1)}{0.001}$$

$$= \frac{((1.001)^3 - 4(1.001)) - ((1)^3 - 4(1))}{0.001}$$

$$= \frac{-3.000\,996\,999 - (-3)}{0.001}$$

$$\doteq -1$$

b) $f(1) = 1^3 - 4(1) = -3$

Point of tangency: $(1, -3)$

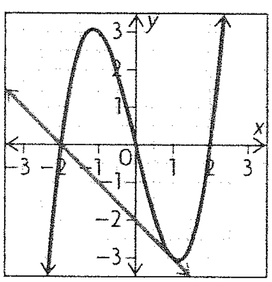
$y = -1x + b$

$-3 = -1(1) + b$

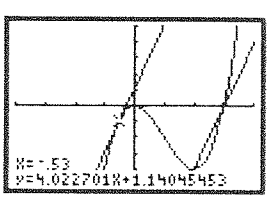
$-2 = b$

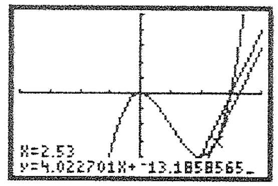
$y = -x - 2$

c) $(-2, 0)$



17. The slope of the secant line is $\frac{0 - (-4)}{3 - 2}$ or 4. Graph the curve and the secant using a graphing calculator. Estimate where the slope of the tangent to the curve is 4. Use the calculator to draw the tangents at various x-values until your estimate is accurate to two decimal places.





The slope of the tangent lines when $x = -0.53$ and $x = 2.53$ are the same as the slope of the given secant line.

Chapter Review, pp. 240–241

1. a) $x^4 - 16x^2 + 75 = 2x^2 - 6$

$x^4 - 18x^2 + 81 = 0$

$(x^2 - 9)(x^2 - 9) = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

b) $2x^2 + 4x - 1 = x + 1$

$2x^2 + 3x - 2 = 0$

$2x^2 + 4x - 1x - 2 = 0$

$2x(x + 2) - 1(x + 2) = 0$

$(2x - 1)(x + 2) = 0$

$2x - 1 = 0$ and $x + 2 = 0$

$x = \frac{1}{2}, -2$

c) $4x^3 - x^2 - 2x + 2 = 3x^3 - 2(x^2 - 1)$

$4x^3 - x^2 - 2x + 2 = 3x^3 - 2x^2 + 2$

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x + 2)(x - 1) = 0$

$x = 0$ and $x + 2 = 0$ and $x - 1 = 0$

$x = 0, -2, 1$

d) $-2x^2 + x - 6 = -x^3 + 2x - 8$

$x^3 - 2x^2 - x + 2 = 0$

$x^2(x - 2) - 1(x - 2) = 0$

$(x^2 - 1)(x - 2) = 0$

$x^2 - 1 = 0$ and $x - 2 = 0$

$x^2 = 1$ and $x = 2$

$x = \pm 1, 2$

2. $18x^4 - 53x^3 + 52x^2 - 14x - 8$

$= 3x^4 - x^3 + 2x - 8$

$15x^4 - 52x^3 + 52x^2 - 16x = 0$

$x(15x^3 - 52x^2 + 52x - 16) = 0$

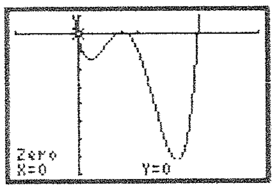
For $15x^3 - 52x^2 + 52x - 16 = 0, f(2) = 0$.

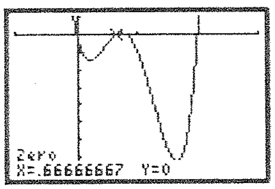
$$\begin{array}{r|rrrr} 2 & 15 & -52 & 52 & -16 \\ & \downarrow & 30 & -44 & 16 \\ \hline & 15 & -22 & 8 & 0 \end{array}$$

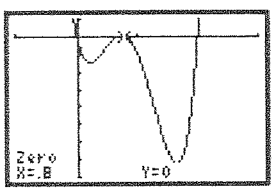
$$x(x-2)(15x^2 - 22x + 8) = 0$$
$$x(x-2)(15x^2 - 12x - 10x + 8) = 0$$
$$x(x-2)(3x(5x-4) - 2(5x-4)) = 0$$
$$x(x-2)(3x-2)(5x-4) = 0$$

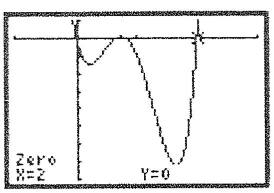
$x = 0$ and $x - 2 = 0$ and $3x - 2 = 0$ and $5x - 4 = 0$

$x = 0, 2, \frac{2}{3}, \frac{4}{5}$









3. a) $f(x) = a(x-1)(x-2)(x+1)(x+2)$

Since $f(x)$ has a y-intercept of 4, $f(0) = 4$.

$4 = a(-1)(-2)(1)(2)$

$4 = 4a$

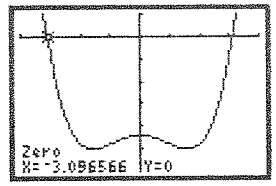
$1 = a$

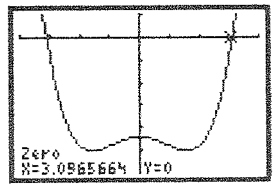
So, $f(x) = (x-1)(x-2)(x+1)(x+2)$ or $f(x) = x^4 - 5x^2 + 4$.

b) $x^4 - 5x^2 + 4 = 48$

$x^4 - 5x^2 - 44 = 0$

Graph $y = x^4 - 5x^2 - 44$ on a graphing calculator and use the calculator to determine the zeros.





The function has a value of 48 when $x \doteq -3.10 \doteq 3.10$.

4. $x(24 - 2x)(30 - 2x) = 1040$

$x(720 - 108x + 4x^2) = 1040$

$720x - 108x^2 + 4x^3 = 1040$

$4x^3 - 108x^2 + 720x - 1040 = 0$

$f(2) = 0$

$$\begin{array}{r|rrrr} 2 & 4 & -108 & 720 & -1040 \\ & \downarrow & 8 & -200 & 1040 \\ \hline & 4 & -100 & 520 & 0 \end{array}$$

$(x-2)(4x^2 - 100x + 520) = 0$

$x = 2$

For $4x^2 - 100x + 520 = 0$

$$x = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(4)(520)}}{2(4)}$$

$= \frac{25}{2} \pm \frac{\sqrt{1680}}{8}$

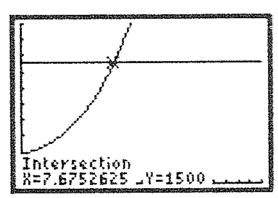
$= 7.4, 17.6$ (17.6 is too large)

2 cm by 2 cm or 7.4 cm by 7.4 cm

5. a) The given information states that the model is valid between 1985 and 1995, so it can be used for 1993, but not 2005.

b) Set $C(t) = 1500$ (since the units are in thousands) and solve using a graphing calculator.

c)



Sales reach 1.5 million in the 8th year after 1985, so in 1993.

6. a) Answers may vary. For example:

$2x + 1 > 17$

$2x > 16$

$x > 8$

b) Answers may vary. For example:

$3x - 4 \geq -16$

$3x \geq -12$

$x \geq -4$

c) Answers may vary. For example:

$2x + 3 \leq -21$

$2x \leq -24$

$x \leq -12$

d) Answers may vary. For example:

$-19 < 2x - 1 < -3$

$-18 < 2x < -2$

$-9 < x < -1$

7. a) $2(4x - 7) > 4(x + 9)$

$8x - 14 > 4x + 36$

$4x > 50$

$x > \frac{25}{2}$

$x \in \left(\frac{25}{2}, \infty\right)$

b) $\frac{x - 4}{5} \geq \frac{2x + 3}{2}$

$2(x - 4) \geq 5(2x + 3)$

$2x - 8 \geq 10x + 15$

$-8x \geq 23$

$x \leq -\frac{23}{8}$

$x \in \left[-\frac{23}{8}, \infty\right)$

c) $-x + 2 > x - 2$

$-2x > -4$

$x < 2$

$x \in (-\infty, 2)$

d) $5x - 7 \leq 2x + 2$

$3x \leq 9$

$x \leq 3$

$x \in (-\infty, 3]$

8. a) $-3 < 2x + 1 < 9$

$-3 - 1 < 2x + 1 - 1 < 9 - 1$

$-4 < 2x < 8$

$-2 < x < 4$

$\{x \in \mathbf{R} \mid -2 < x < 4\}$

b) $8 \leq -x + 8 \leq 9$

$8 - 8 \leq -x + 8 - 8 \leq 9 - 8$

$0 \leq -x \leq 1$

$-1 \leq x \leq 0$

$\{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$

c) $6 + 2x \geq 0 \geq -10 + 2x$

$6 + 2x - 2x \geq 0 - 2x \geq -10 + 2x - 2x$

$6 \geq -2x \geq -10$

$-3 \leq x \leq 5$

$\{x \in \mathbf{R} \mid -3 \leq x \leq 5\}$

d) $x + 1 < 2x + 7 < x + 5$

$x + 1 - x < 2x + 7 - x < x + 5 - x$

$1 < x + 7 < 5$

$-6 < x < -2$

$\{x \in \mathbf{R} \mid -6 < x < -2\}$

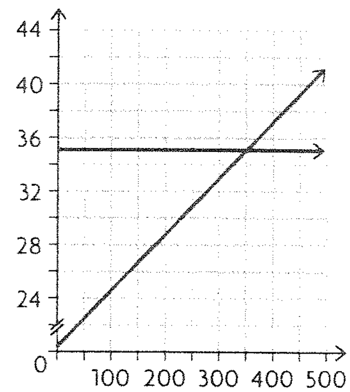
9. a) $0.04x + 20.95 > 34.95$

$0.04x > 14$

$x > 350$

The second plan is better if one calls more than 350 minutes per month.

b)



10. a) $(x + 1)(x - 2)(x + 3)^2 < 0$

$x + 1 = 0$ and $x - 2 = 0$ and $x + 3 = 0$

$x = -3, -1, 2$

This divides the domain of real numbers into 4 intervals:

$x \leq -3$; $-3 < x < -1$; $-1 < x < 2$; $x \geq 2$

Test for each interval:

$\boldsymbol{x \leq -3}$**:**

$f(-4) = (-4 + 1)(-4 - 2)(-4 + 3)^2 < 0$

$= (-3)(-6)(-1)^2 < 0$

$= 18 < 0$ No

$\boldsymbol{-3 < x < -1}$**:**

$f(-2) = (-2 + 1)(-2 - 2)(-2 + 3)^2 < 0$

$= (-1)(-4)(1)^2 < 0$

$= 4 < 0$ No

$\boldsymbol{-1 < x < 2}$**:**

$f(0) = (0 + 1)(0 - 2)(0 + 3)^2 < 0$

$= (1)(-2)(3)^2 < 0$

$= (-2)(9) < 0$

$= -18 < 0$ Yes

$\boldsymbol{x \geq 2}$**:**

$f(3) = (3 + 1)(3 - 2)(3 + 3)^2 < 0$

$= (4)(1)(6)^2 < 0$

$= (4)(36) < 0$

$= 144 < 0$ No

The interval is $-1 < x < 2$.

b) $\dfrac{(x - 4)(2x + 3)}{5} \geq \dfrac{2x + 3}{5}$

$(x - 4)(2x + 3) \geq 2x + 3$

$2x^2 - 5x - 12 \geq 2x + 3$

$2x^2 - 7x - 15 \geq 0$

$2x^2 - 10x + 3x - 15 \geq 0$

$2x(x - 5) + 3(x - 5) \geq 0$

$(2x + 3)(x - 5) \geq 0$

$2x + 3 = 0$ and $x - 5 = 0$

$x = -\dfrac{3}{2}, 5$

This divides the domain of real numbers into 3 intervals:

$x \leq -\dfrac{3}{2}; -\dfrac{3}{2} < x <; x \geq 5$

Test for each interval:

$\boldsymbol{x \leq -\dfrac{3}{2}}$**:**

$f(-2) = (2(-2) + 3)((-2) - 5) \geq 0$

$= (-1)(-7) \geq 0$

$= 7 \geq 0$ Yes

$\boldsymbol{-\dfrac{3}{2} < x < 5}$**:**

$f(0) = (2(0) + 3)((0) - 5) \geq 0$

$= (3)(-5) \geq 0$

$= -15 \geq 0$ No

$\boldsymbol{x \geq 5}$**:**

$f(6) = (2(6) + 3)((6) - 5) \geq 0$

$= (15)(1) \geq 0$

$= 15 \geq 0$ Yes

The intervals are $x \leq -\frac{3}{2}$ or $x \geq 5$.

c) $-2(x - 1)(2x + 5)(x - 7) > 0$

$x - 1 = 0$ and $2x + 5 = 0$ and $x - 7 = 0$

$x = -\dfrac{5}{2}, 1, 7$

This divides the domain of real numbers into 4 intervals:

$x < -\dfrac{5}{2}; -\dfrac{5}{2} < x < 1; 1 < x < 7; x > 7$

Test for each interval:

$\boldsymbol{x < -\dfrac{5}{2}}$**:**

$f(-3)$

$= -2((-3)-1)(2(-3) + 5)((-3)-7) > 0$

$= -2(-4)(-1)(-10) < 0$

$= 80 > 0$ Yes

$\boldsymbol{-\dfrac{5}{2} < x < 1}$**:**

$f(0) = -2((0) - 1)(2(0) + 5)((0) - 7)$

$= -2(-1)(5)(-7)$

$= -70 > 0$ No

$\boldsymbol{1 < x < 7}$**:**

$f(2) = -2((2) - 1)(2(2) + 5)((2) - 7)$

$= -2(1)(9)(-5)$

$= 90 > 0$ Yes

$\boldsymbol{x > 7}$**:**

$f(8) = (-2(8) - 1)(2(8) + 5)((8) - 7)$

$= -2(7)(21)(1)$

$= -294 > 0$ No

The intervals are $x < -\frac{5}{2}$ or $1 < x < 7$.

d) $x^3 + x^2 - 21x + 21 \leq 3x^2 - 2x + 1$

$x^3 - 2x^2 - 19x + 20 \leq 0$

$f(1) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -19 & 20 \\ & \downarrow & 1 & -1 & -20 \\ \hline & 1 & -1 & -20 & 0 \end{array}$$

$(x - 1)(x^2 - x - 20) \leq 0$

$(x - 1)(x - 5)(x + 4) \leq 0$

$x - 1 = 0$ and $x - 5 = 0$ and $x + 4 = 0$

$x = -4, 1, 5$

This divides the domain of real numbers into 4 intervals:

$x \leq -4; -4 < x < 1; 1 \leq x \leq 5; x > 5$

Test for each interval:

$\boldsymbol{x \leq -4}$**:**

$f(-5) = ((-5) - 1)((-5) - 5)((-5) + 4) \leq 0$

$= (-6)(-10)(-1) \leq 0$

$= -60 \leq 0$ Yes

$\boldsymbol{-4 < x < 1}$**:**

$f(0) = ((0) - 1)((0) - 5)((0) + 4) \leq 0$

$= (-1)(-5)(4) \leq 0$

$= 20 \leq 0$ No

$\boldsymbol{1 \leq x \leq 5}$**:**

$f(2) = ((2) - 1)((2) - 5)((2) + 4) \leq 0$

$= (1)(-3)(6) \leq 0$

$= -18 \leq 0$ Yes

$\boldsymbol{x > 5}$**:**

$f(6) = ((6) - 1)((6) - 5)((6) + 4) \leq 0$

$= (5)(1)(10) \leq 0$

$= 50 \leq 0$ No

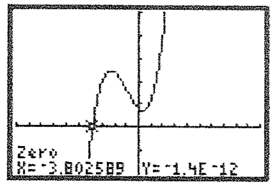
The intervals are $x \leq -4$ or $1 \leq x \leq 5$.

11. $f(x) = 2x^4 - 2x^3 - 32x^2 - 40x$
$x(2x^3 - 2x^2 - 32x - 40)$
For $2x^3 - 2x^2 - 32x - 40$, $f(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 2 & -2 & -32 & -40 \\ & \downarrow & -4 & 12 & 40 \\ \hline & 2 & -6 & -20 & 0 \end{array}$$

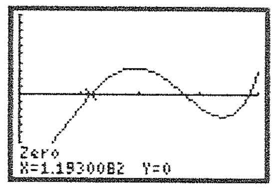
$x(x + 2)(2x^2 - 6x - 20)$
$x(x + 2)(2x^2 - 10x + 4x - 20)$
$x(x + 2)(2x(x - 5) + 4(x - 5))$
$x(x + 2)(2x + 4)(x - 5)$
Set each of these equal to zero to find the intervals and how each interval relates to zero (either positive or negative).
$x = 0$ and $x + 2 = 0$ and $2x + 4 = 0$ and $x - 5 = 0$
$x = -2, 0, 5$
This divides the domain of real numbers into 4 intervals:
$x \le -2$; $-2 < x < 0$; $0 \le x \le 5$; $x > 5$
Test for each interval:
$x \le -2$:
$f(-3) = -3((-3) + 2)(2(-3) + 4)((-3) - 5)$
$= -3(-1)(-2)(-8)$
$= 48$ Positive
$-2 < x < 0$:
$f(-1) = -1((-1) + 2)(2(-1) + 4)((-1) - 5)$
$= -1(1)(2)(-6)$
$= 12$ Positive
$0 \le x \le 5$:
$f(2) = 2((2) + 2)(2(2) + 4)((2) - 5)$
$= 2(4)(8)(-3)$
$= -192$ Negative
$x > 5$:
$f(6) = 6((6) + 2)(2(6) + 4)((6) - 5)$
$= 6(8)(16)(1)$
$= 768$ Positive
The intervals of the function are negative when $x \in (0, 5)$, positive when $x \in (-\infty, -2)$, $(-2, 0)$, $(5, \infty)$.

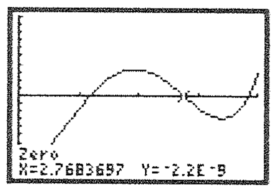
12. Rewrite as an equivalent inequality with 0 on one side; graph and determine the zeros of the polynomial function that is graphed.
$x^3 - 2x^2 + x - 3 \ge 2x^3 + x^2 - x + 1$
$0 \ge x^3 + 3x^2 - 2x + 4$

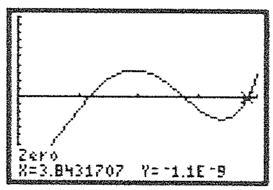


$x \le -3.81$

13. $f(x) = 1135x^4 - 8197x^3 + 15\,868x^2 - 2157x + 176\,608$, $0 \le x \le 4$
To determine in which years the harvest was less than 185 000 m^3, write an inequality, rewrite an equivalent inequality with 0 on one side, and use a graphing calculator to solve.
$1135x^4 - 8197x^3 + 15\,868x^2 - 2157x + 176\,608 \le 185\,000$
$1135x^4 - 8197x^3 + 15\,868x^2 - 2157x - 8392 \le 0$







Convert the zeros to years and months after January 1993. So the harvest is less than 185 000 m^3 between January 1993 and March 1994 and between October 1995 and October 1996.

14. a) $f(x) = x^2 - 2x + 3$
Average Rate (from $x = 2$ to $x = 7$):
Average rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$
$= \dfrac{f(7) - f(2)}{7 - 2}$
$= \dfrac{(7^2 - 2(7) + 3) - (2^2 - 2(2) + 3)}{5}$

$= \dfrac{38 - 3}{5}$
$= 7$

Instantaneous Rate (at $x = 5$).

$\text{Slope} = \dfrac{f(a + h) - f(a)}{h}$, where $a = 5$; $h = 0.001$

$= \dfrac{f(5.001) - f(5)}{0.001}$

$= \dfrac{(5.001^2 - 2(5.001) + 3) - (5^2 - 2(5) + 3)}{0.001}$

$= \dfrac{18.008\,001 - 18}{0.001}$

$\doteq 8$

b) $h(x) = (x - 3)(2x + 1)$

Average Rate (from $x = 2$ to $x = 7$):

$\text{Average rate of change} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(7) - f(2)}{7 - 2}$

$h = \dfrac{((7 - 3)(2(7) + 1))}{5} - \dfrac{((2 - 3)(2(2) + 1))}{5}$

$= \dfrac{60 - (-5)}{5}$

$= 13$

Instantaneous Rate (at $x = 5$).

$\text{Slope} = \dfrac{f(a + h) - f(a)}{h}$, where $a = 5$; $h = 0.001$

$= \dfrac{f(5.001) - f(5)}{0.001}$

$= \dfrac{((5.001 - 3)(2(5.001) + 1))}{0.001} - \dfrac{((5 - 3)(2(5) + 1))}{0.001}$

$= \dfrac{22.015\,002 - 22}{0.001}$

$\doteq 15$

c) $g(x) = 2x^3 - 5x$

Average Rate (from $x = 2$ to $x = 7$):

$\text{Average rate of change} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(7) - f(2)}{7 - 2}$

$= \dfrac{(2(7)^3 - 5(7)) - (2(2)^3 - 5(2))}{5}$

$= \dfrac{651 - 6}{5}$

$= 129$

Instantaneous Rate (at $x = 5$).

$\text{Slope} = \dfrac{f(a + h) - f(a)}{h}$, where $a = 5$; $h = 0.001$

$= \dfrac{f(5.001) - f(5)}{0.001}$

$= \dfrac{(2(5.001)^3 - 5(5.001)) - (2(5)^3 - 5(5))}{0.001}$

$= \dfrac{225.145\,03 - 225}{0.001}$

$\doteq 145$

d) $v(x) = -x^4 + 2x^2 - 5x + 1$

Average Rate (from $x = 2$ to $x = 7$):

$\text{Average rate of change} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(7) - f(2)}{7 - 2}$

$= \dfrac{(-(7)^4 + 2(7)^2 - 5(7) + 1)}{5} - \dfrac{(-(2)^4 + 2(2)^2 - 5(2) + 1)}{5}$

$= \dfrac{-2337 - (-17)}{5}$

$= -464$

Instantaneous Rate (at $x = 5$).

$\text{Slope} = \dfrac{f(a + h) - f(a)}{h}$, where $a = 5$; $h = 0.001$

$= \dfrac{f(5.001) - f(5)}{0.001}$

$= \dfrac{(-(5.001)^4 + 2(5.001)^2 - 5(5.001) + 1)}{0.001} - \dfrac{(-(5)^4 + 2(5)^2 - 5(5) + 1)}{0.001}$

$= \dfrac{-599.485\,148 - (-599)}{0.001}$

$\doteq -485$

15. By examining the graph, the instantaneous rate of change is positive when $-1 < x < 1$, negative when $x < -1$ or $x > 1$, and zero at $x = -1, 1$.

16. $h(t) = -5t^2 + 25$

a) $-5t^2 + 25 = 0$

$-5t^2 = -25$

$t^2 = 5$

$t \doteq 2.2$

The object hits the ground at 2.2 seconds.

b) Average rate from $t = 0$ to $t = 2.2$:

Average rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(2.2) - f(0)}{2.2 - 0}$

$= \dfrac{(-5(2.2)^2 + 25) - (-5(0)^2 + 25)}{2.2}$

$= \dfrac{0.8 - 25}{2.2}$

$= -11$ m/s

c) Instantaneous rate at $t = 2.2$

Slope $= \dfrac{f(a+h) - f(a)}{h}$, where $a = 2.2$;

$h = 0.001$

$= \dfrac{f(2.201) - f(2.2)}{0.001}$

$= \dfrac{(-5(2.201)^2 + 25) - (-5(2.2)^2 + 25)}{0.001}$

$= \dfrac{0.777\,995 - 0.8}{0.001}$

$\doteq -22$ m/s

17. $f(x) = 2x^3 + 3x - 1$

a) Average rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(3.0001) - f(3)}{3.001 - 3}$

$= \dfrac{(2(3.0001)^3 + 3(3.0001) - 1)}{0.0001}$

$- \dfrac{(2(3)^3 + 3(3) - 1)}{0.0001}$

$= \dfrac{62.005\,700\,18 - 62}{0.0001}$

$\doteq 57.002$

b) Average rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(3) - f(2.9999)}{3 - 2.9999}$

$= \dfrac{(2(3)^3 + (3) - 1) - (2(2.9999)^3}{0.0001}$

$+ \dfrac{3(2.9999) - 1)}{0.0001}$

$= \dfrac{62 - 61.994\,300\,18}{0.0001}$

$\doteq 56.988$

c) Both approximate the instantaneous rate of change at $x = 3$.

Instantaneous rate at $x = 3$

Slope $= \dfrac{f(a+h) - f(a)}{h}$, where $a = 3$; $h = 0.001$

$= \dfrac{f(3.001) - f(3)}{0.001}$

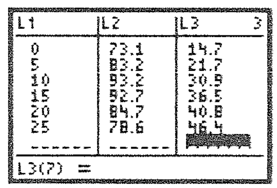
$= \dfrac{(2(3.001)^3 + 3(3.001) - 1)}{0.001}$

$- \dfrac{(2(3)^3 + 3(3) - 1)}{0.001}$

$= \dfrac{62.057018 - 62}{0.001}$

$\doteq 57$

18. a) Enter the data into a graphing calculator and then use the calculator to determine a cubic function for both sets of data. Let the independent variable represent the number of years since 1975, so $x = 0$ corresponds to 1975.

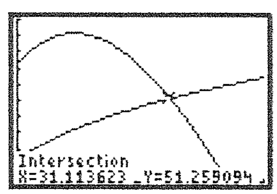


L1	L2	L3
0	73.1	14.7
5	83.2	21.7
10	93.2	30.9
15	92.7	36.5
20	84.7	40.8
25	78.6	46.4

L3(7) =

The cubic function for the male data is:
$f(x) = 0.001x^3 - 0.162x^2 + 3.394x + 72.365$.
The cubic function for the female data is:
$g(x) = 0.0002x^3 - 0.026x^2 + 1.801x + 14.369$.

b) Use a graphing calculator to determine when $g(x) > f(x)$.



The cubic functions intersect at approximately 31.11, so more females will have lung cancer in 2006.

c) The rate was changing faster for females, on average. Looking only at 1975 and 2000, the incidence among males increased only 5.5 per 100 000, while the incidence among females increased by 31.7.

d) Between 1995 and 2000, the incidence among males decreased by 6.1 while the incidence among females increased by 5.6. Since 1998 is about halfway between 1995 and 2000, an estimate for the instantaneous rate of change in 1998 is the average rate of change from 1995 to 2000. The two rates of change are about the same in magnitude, but the rate for females is positive, while the rate for males is negative.

Chapter Self-Test, p. 242

1. $3x^3 - 3x^2 - 7x + 5 = x^3 - 2x^2 - 1$

$2x^3 - x^2 - 7x + 6 = 0$

$f(1) = 0$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & -7 & 6 \\ & \downarrow & 2 & 1 & -6 \\ \hline & 2 & 1 & -6 & 0 \end{array}$$

$(x - 1)(2x^2 + x - 6) = 0$

$(x - 1)(2x^2 + 4x - 3x - 6) = 0$

$(x - 1)(2x(x + 2) - 3(x + 2)) = 0$

$(x - 1)(2x - 3)(x + 2) = 0$

$x - 1 = 0$ and $2x - 3 = 0$ and $x + 2 = 0$

$x = 1, \frac{3}{2}, -2$

2. a) By examining the graph, the function is positive when $x < -2$ and $0 < x < 2$, negative when $-2 < x < 0$, and $x > 2$ and zero at $x = -2, 0,$ and 2.

b) By examining the graph, the instantaneous rate of change is positive when $-1 < x < 1$, negative when $x < -1$ or $1 < x$, and zero at $x = -1, 1$.

$m = \frac{0 - 1}{2 - 1} = \frac{-1}{1} = -1$

3. a) Cost with card: $50 + 5n$;

Cost without card: $12n$

b) $12n > 50 + 5n$

$7n > 50$

$n > 7.14$

They must buy at least 8 pizzas to make the card worthwhile.

4. a) $4x - 5 < -2(x + 1)$

$4x - 5 < -2x - 2$

$6x < 3$

$x < \frac{1}{2}$

b) $-4 \le -(3x + 1) \le 5$

$-4 \le -3x - 1 \le 5$

$-4 + 1 \le -3x - 1 + 1 \le 5 + 1$

$-3 \le -3x \le 6$

$-2 \le x \le 1$

c) $(x + 1)(x - 5)(x + 2) > 0$

$(x + 1)(x - 5)(x + 2) = 0$

$x + 1 = 0$ and $x - 5 = 0$ and $x + 2 = 0$

$x = -2, -1, 5$

This divides the domain of real numbers into 4 intervals:

$x < -2; -2 < x < -1; -1 < x < 5; x > 5$

Test for each interval:

$x < -2$:

$f(-3) = (-3 + 1)(-3 - 5)(-3 + 2) > 0$

$= (-2)(-8)(-1) > 0$

$= -16 > 0$ No

$-2 < x < -1$:

$f(-1.5) = (-1.5 + 1)(-1.5 - 5)(-1.5 + 2) > 0$

$= (-0.5)(-6.5)(0.5) > 0$

$= 1.625 > 0$ Yes

$-1 < x < 5$:

$f(0) = (0 + 1)(0 - 5)(0 + 2) > 0$

$= (1)(-5)(2) > 0$

$= -10 > 0$ No

$x > 5$:

$f(6) = (6 + 1)(6 - 5)(6 + 2) > 0$

$= (7)(1)(8) > 0$

$= 56 > 0$ Yes

The intervals are $-2 < x < -1$ or $x > 5$.

d) $(2x - 4)^2(x + 3) \ge 0$

$2x - 4 = 0$ and $x + 3 = 0$

$x = -3, 2$

This divides the domain of real numbers into 3 intervals:

$x < -3; -3 \le x < 2; x \ge 2$

Test for each interval:

$x > -3$:

$f(-4) = (2(-4) - 4)^2((-4) + 3) \ge 0$

$= (-12)^2(-1) \ge 0$

$= -144 \ge 0$ No

$-3 \le x < 2$:

$f(0) = (2(0) - 4)^2((0) + 3) \ge 0$

$= (-4)^2(3) \ge 0$

$= 48 \ge 0$ Yes

$x \ge 2$:

$f(3) = (2(3) - 4)^2((3) + 3) \ge 0$

$= (2)^2(6) \ge 0$

$= 24 \ge 0$ Yes

The interval is $x \ge -3$.

5. $h(t) = -5t^2 + 20t + 15$

a) $h(0) = -5(0)^2 + 20(0) + 15 = 15$

15 metres

b) $-5t^2 + 20t + 15 = 0$

$t = \frac{-20 \pm \sqrt{(-20)^2 - 4(-5)(15)}}{2(-5)}$

$= \frac{-20}{-10} \pm \frac{\sqrt{400 - -300}}{-10}$

$= 2 \pm 2.6$

$= 4.6, -0.6$

Since time cannot be negative, it will take 4.6 seconds.

c) Average rate of change from $t = 0$ to $t = 4.6$

$$\text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(4.6) - f(0)}{4.6 - 0}$$
$$= \frac{(-5(4.6)^2 + 20(4.6) + 15)}{4.6} - \frac{(-5(0)^2 + 20(0) + 15)}{4.6}$$
$$= \frac{1.2 - 15}{4.6}$$
$$= -3 \text{ m/s}$$

6. $f(x) = x^3 + x^2 + 1$

a) The slope at $x = 1$:

$$\text{Slope} = \frac{f(a + h) - f(a)}{h}, \text{ where } a = 1; h = 0.001$$
$$= \frac{f(1.001) - f(1)}{0.001}$$
$$= \frac{(1.001^3 + 1.001^2 + 1) - (1^3 + 1^2 + 1)}{0.001}$$
$$= \frac{3.005\,004\,001 - 3}{0.001}$$
$$\doteq 5$$

b) $f(1) = 1^3 + 1^2 + 1 = 1 + 1 + 1 = 3$

The coordinates are $(1, 3)$.

c) $y = 5x + b$

$3 = 5(1) + b$

$-2 = b$

$y = 5x - 2$

7. Since all the exponents are even and all the coefficients are positive, all values of the function are positive and greater than or equal to 4 for all real numbers x.

8. a) $\{x \in \mathbf{R} \mid -2 < x < 7\}$

b) $-5 < 2x - 1 < 13$

$-5 + 1 < 2x - 1 + 1 < 13 + 1$

$-4 < 2x < 14$

$-2 < x < 7$

9. $(x)(x)(x + 13) = 60$

$x^2(x + 13) = 60$

$x^3 + 13x^2 - 60 = 0$

$f(2) = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 13 & 0 & -60 \\ & \downarrow & 2 & 30 & 60 \\ \hline & 1 & 15 & 30 & 0 \end{array}$$

$(x - 2)(x^2 + 15x + 30) = 0$

The roots of $x^2 + 15x + 30$ are both negative. Since the dimensions cannot be negative, $x - 2$ is the root that we use.

$x - 2 = 0$

$x = 2$

The dimensions are 2 cm by 2 cm by 15 cm.