

## L8 – The Natural Logarithm

MHF4U

### Part 1: What is 'e'?

**Example 1:** Suppose you invest \$1 at 100% interest for 1 year at various compounding levels. What is the highest amount of money you can have after 1 year?

Note: the formula used for compound interest of \$1 at 100% interest annually compounded  $n$  times during the year is:

$$A = 1 \left( 1 + \frac{1}{n} \right)^n$$

| Compounding Level, $n$             | Amount, $A$ in dollars |
|------------------------------------|------------------------|
| Annually (once a year)             |                        |
| Semi-annually (2-times)            |                        |
| Quarterly (4-times)                |                        |
| Monthly (12-times)                 |                        |
| Daily (365-times)                  |                        |
| Secondly (31 536 000-times)        |                        |
| Continuously (1 000 000 000-times) |                        |

### Properties of $e$ :

- $e = \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n$  \_\_\_\_\_
- $e$  is an \_\_\_\_\_ number, similar to  $\pi$ . They are non-terminating and non-repeating.
- $\log_e x$  is known as the \_\_\_\_\_ and can be written as \_\_\_\_\_
- Many naturally occurring phenomena can be modelled using base- $e$  exponential and logarithmic functions.
- $\log_e e = \ln e =$  \_\_\_\_\_

## Part 2: Reminder of Log Rules

|                            |   |
|----------------------------|---|
| Power Law of Logarithms    | $\log_b x^n = n \log_b x$ for $b > 0, b \neq 1, x > 0$                                      |
| Product Law of Logarithms  | $\log_b(mn) = \log_b m + \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$                      |
| Quotient Law of Logarithms | $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$ |
| Change of Base Formula     | $\log_b m = \frac{\log m}{\log b}, m > 0, b > 0, b \neq 1$                                  |
| Exponential to Logarithmic | $y = b^x \rightarrow x = \log_b y$  |
| Logarithmic to Exponential | $y = \log_b x \rightarrow x = b^y$  |
| Other useful tips          | $\log_a(a^b) = b$ $\log a = \log_{10} a$ $\log_b b = 1$                                     |

## Part 2: Solving Problems Involving $e$

**Example 2:** Evaluate each of the following

a)  $e^3$

b)  $\ln 10$

c)  $\ln e$

**Example 3:** Solve each of the following equations

a)  $20 = 3e^x$

b)  $e^{1-2x} = 55$

c)  $2 \ln(x - 3) - 7 = 3$

d)  $\ln(4e^x) = 2$

**Part 3: Graphing Functions Involving  $e$**

**Example 4:** Graph the functions  $y = e^x$  and  $y = \ln x$

| $y = e^x$ |     |
|-----------|-----|
| $x$       | $y$ |
|           |     |
|           |     |
|           |     |
|           |     |

| $y = \ln x$ |     |
|-------------|-----|
| $x$         | $y$ |
|             |     |
|             |     |
|             |     |
|             |     |

**Note:**  $y = \ln x$  is the inverse of  $y = e^x$

