L3 – 8.1/8.2 Sum/Difference and Product/Quotient of Functions
MHF4U

Part 1: Sum and Difference of Functions

When two functions $f(x)$ and $g(x)$ are combined to form the function $(f+g)(x)$ or $(f-g)(x)$, the new function is called the sum or difference of f and g.

The graph of $f+g$ or $f-g$ can be obtained by adding or subtracting corresponding y-coordinates. This is called the s***uperposition principle***.

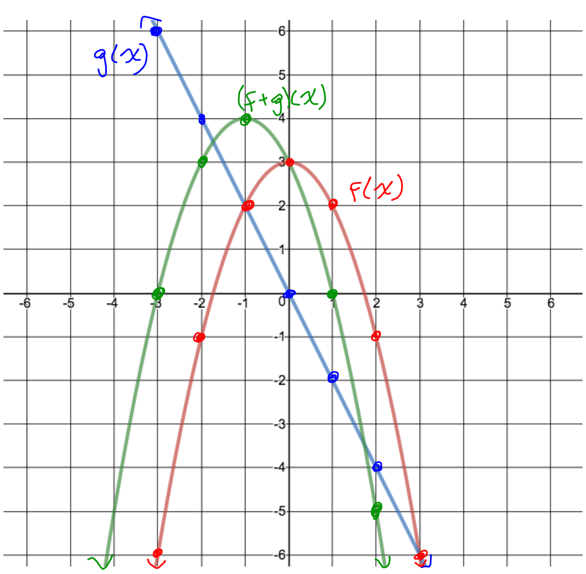
$$(f+g)(x) = f(x) + g(x)$$

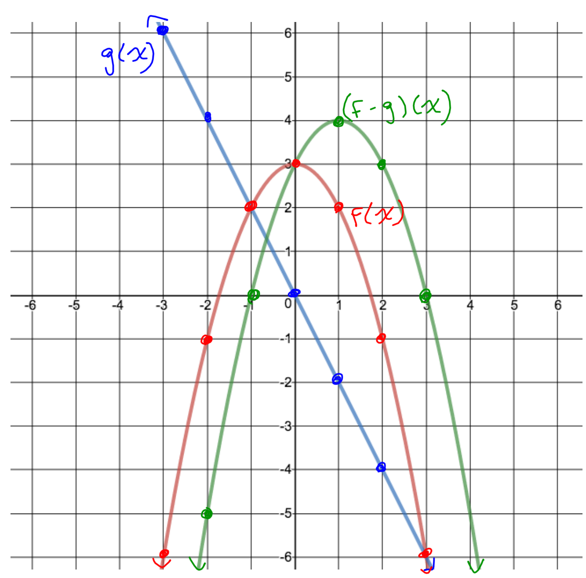
$$(f-g)(x) = f(x) - g(x)$$

Example 1: Given $f(x) = -x^2 + 3$ and $g(x) = -2x$ determine the graphs of $(f+g)(x)$ and $(f-g)(x)$.

Method 1: Graphically

x	$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)-g(x)$
-3	-6	6	0	-12
-2	-1	4	3	-5
-1	2	2	4	0
0	3	0	3	3
1	2	-2	0	4
2	-1	-4	-5	3
3	-6	-6	-12	0





Method 2: Algebraically

$(f+g)(x) = f(x) + g(x)$
$(f+g)(x) = (-x^2+3) + (-2x)$
$(f+g)(x) = -x^2 - 2x + 3$

Complete Square to find Vertex

$(f+g)(x) = -(x^2+2x) + 3$
$(f+g)(x) = -(x^2+2x+1-1) + 3$
$(f+g)(x) = -(x^2+2x+1) + 1 + 3$
$(f+g)(x) = -(x+1)^2 + 4$

vertex is (-1, 4)

x	$(f+g)(x)$
-4	-5
-3	0
-2	3
-1	4
0	3
1	0
2	-5

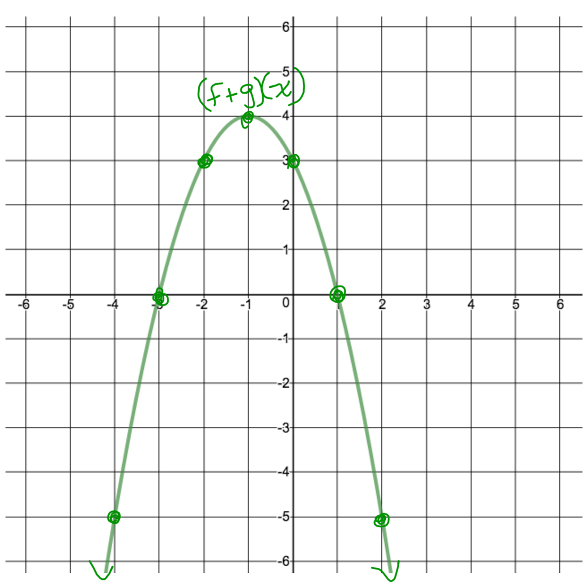
$(f-g)(x) = f(x) - g(x)$
$(f+g)(x) = (-x^2+3) - (-2x)$
$(f+g)(x) = -x^2 + 2x + 3$

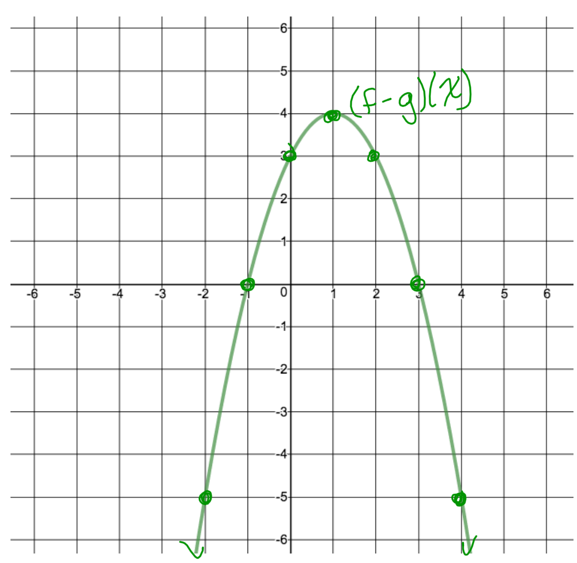
Complete Square to find Vertex

$(f+g)(x) = -(x^2-2x) + 3$
$(f+g)(x) = -(x^2-2x+1-1) + 3$
$(f+g)(x) = -(x^2-2x+1) + 1 + 3$
$(f+g)(x) = -(x-1)^2 + 4$

vertex is (1, 4)

x	$(f-g)(x)$
-2	-5
-1	0
0	3
1	4
2	3
3	0
4	-5





$(f+g)(x)$

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R} | y \leq 4\}$

$(f-g)(x)$

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R} | y \leq 4\}$

Note: *The domain of the sum or difference of functions is the intersection of the domains of f and g*

Part 2: Product and Quotient of Functions

When two functions $f(x)$ and $g(x)$ are combined to form the function $(f \cdot g)(x)$ or $(f \div g)(x)$, the new function is called the product or quotient of f and g.

The graph of $f \cdot g$ or $f \div g$ can be obtained by multiplying or dividing corresponding y-coordinates.

$$(f \times g)(x) = f(x) \times g(x)$$

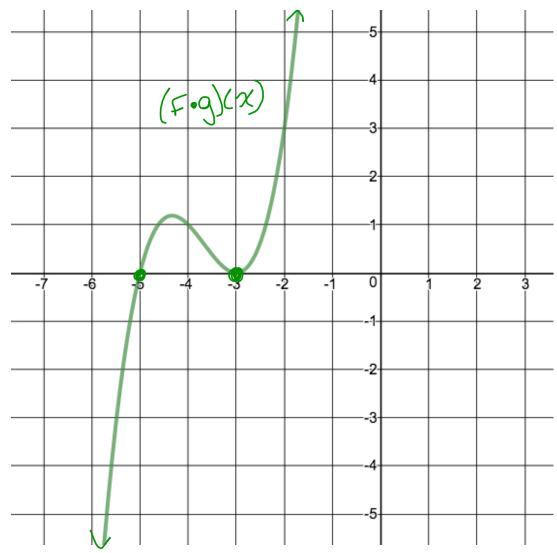
$$(f \div g)(x) = f(x) \div g(x)$$

Example 2: Let $f(x) = x + 3$ and $g(x) = x^2 + 8x + 15$. Determine an equation and graph for

a) $(f \times g)(x)$

$(f \times g)(x) = f(x)g(x)$
$(f \times g)(x) = (x + 3)(x^2 + 8x + 15)$
$(f \times g)(x) = (x + 3)(x + 3)(x + 5)$
$(f \times g)(x) = (x + 3)^2(x + 5)$

x –intercepts at -3 (order 2) and -5 (order 1)
Extends from Q1 to Q3



b) $(f \div g)(x)$

$(f \div g)(x) = \frac{f(x)}{g(x)}$

$(f \div g)(x) = \frac{x+3}{(x+3)(x+5)}$

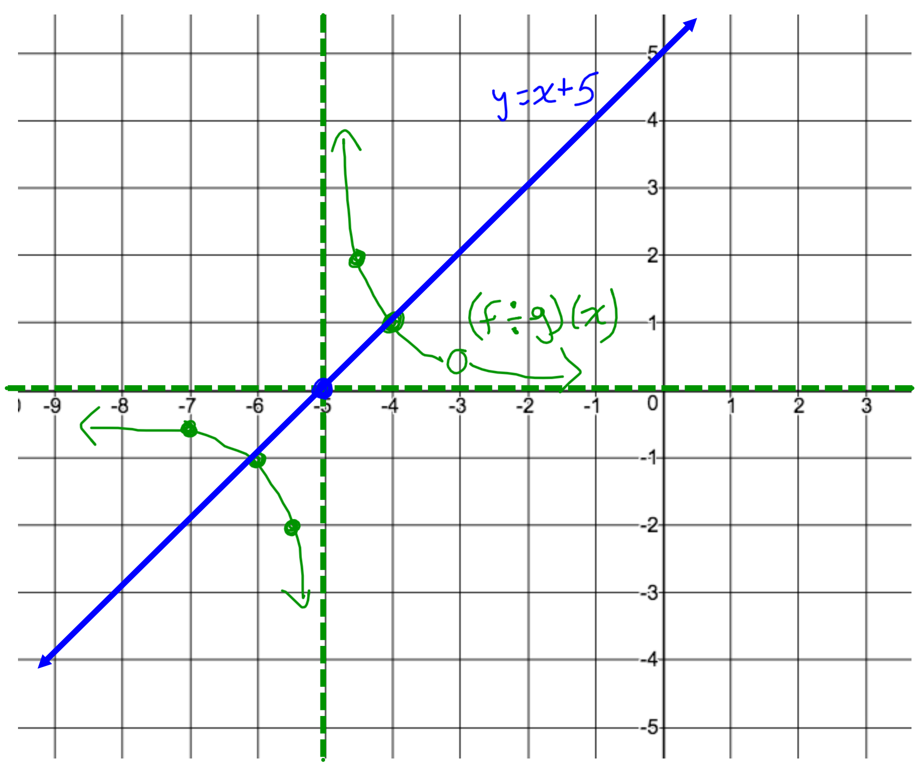
$(f \div g)(x) = \frac{1}{x+5}; x \neq -5, -3$

VA: $x = -5$
HA: $y = 0$
Hole at $x = -3$

Note: always a HA at $y = 0$ when denominator is higher degree than numerator

$y = x + 5$	
x	y
-7	-2
-6	-1
-5.5	-0.5
-5	0
-4.5	0.5
-4	1
-3	2

$y = \frac{1}{x+5}$	
x	$\frac{1}{y}$
-7	-0.5
-6	-1
-5.5	-2
-5	Und
-4.5	2
-4	1
-3	Und



c) State the domain and range of both functions

$(f \times g)(x)$

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R}\}$

$(f \div g)(x)$

D: $\{X \in \mathbb{R} | x \neq -5, -3\}$

R: $\{Y \in \mathbb{R} | y \neq 0, 0.5\}$