

# MHF4U EXAM REVIEW

## Unit 1: Polynomial Functions

1) Match each function to its end behavior

$$y = -3x^2$$

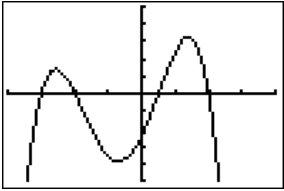
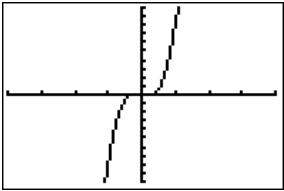
$$y = 5x^4$$

$$y = 0.5x^3$$

$$y = -\frac{1}{3}x^5$$

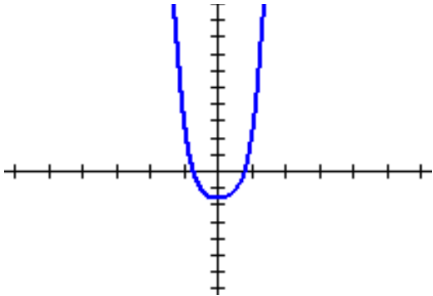
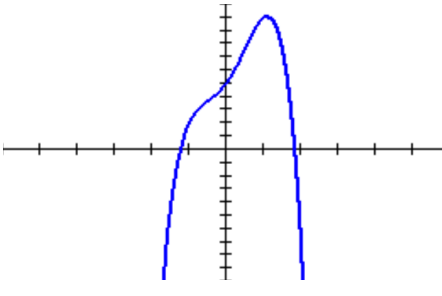
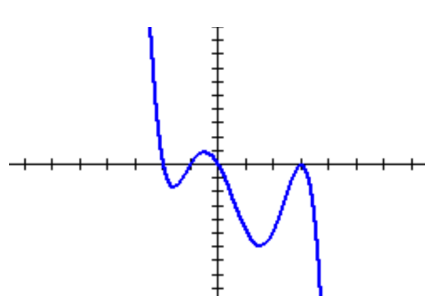
End Behaviour	Function
Q3 to Q1	
Q2 to Q4	
Q2 to Q1	
Q3 to Q4	

2) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
					
					

3) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

**A)**  $g(x) = -2x^4 + 3x^2 + 4x + 5$     **B)**  $h(x) = -x^5 + 3x^4 + 7x^3 - 15x^2 - 18x$     **C)**  $p(x) = x^6 + 5x^4 + 2x^2 - 3$

**4)** Use the polynomial,  $P(x) = 2x^3 + 6x^2 - 8$  to answer the following

**a)** Complete the chart

Degree	Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts

**b)** Is this an even function, odd function, or neither? Explain.

**c)** Is there any type of symmetry present? Explain.

**5)** Use the polynomial,  $P(x) = -6x^4 + 2x^2 - 1$  to answer the following

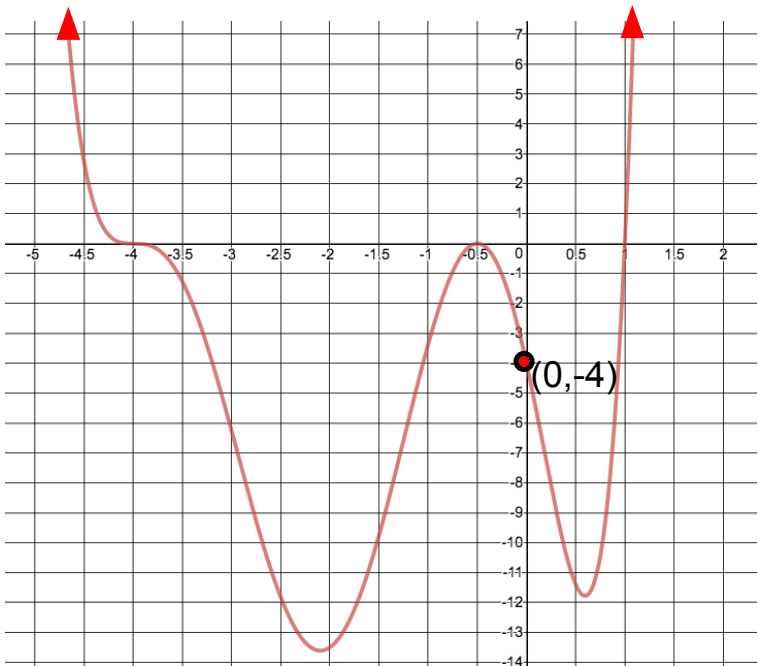
**a)** Complete the chart

Degree	Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts

**b)** Is this an even function, odd function, or neither? Explain.

**c)** Is there any type of symmetry present? Explain.

Given the graph below:



a) Complete the following table

Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Number of turning points	x-intercepts with their order	Least Possible Degree

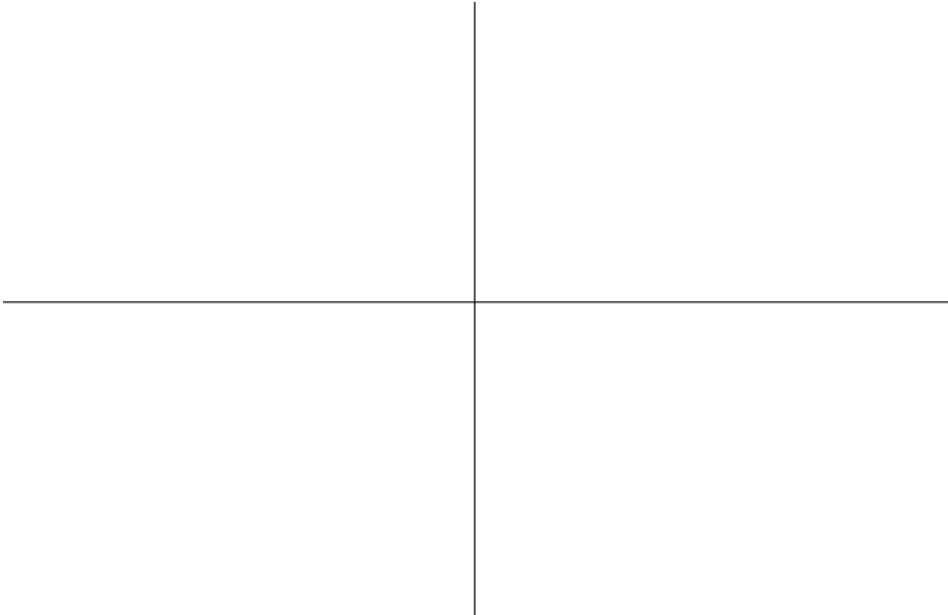
b) Find the equation in factored form

6) Find the equation of a quartic function that has zeros at -4, 1, and 3 (order 2) and passes through the point (2, 6)

7) Complete the chart and sketch a possible graph of the function labelling key points.

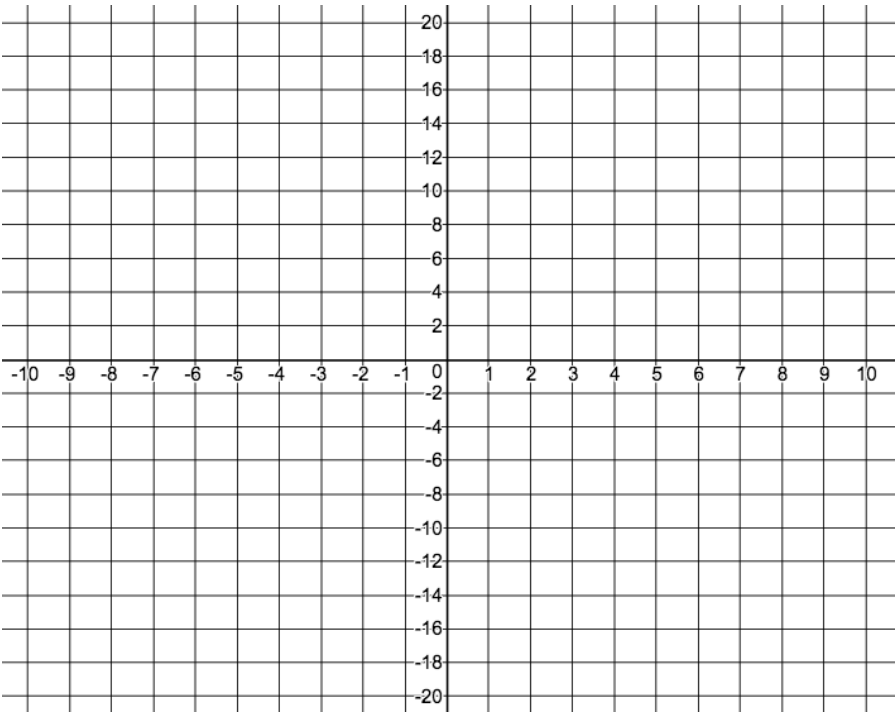
$$f(x) = \frac{1}{2}x(x - 4)^3(x + 1)^2$$

Degree	Leading Coefficient	End Behaviour	x-intercepts with orders	y-intercept



8) What is the parent function of,  $g(x) = \frac{1}{2}\left[\frac{1}{3}(x - 1)\right]^3 + 4$  ?

b) Use transformations to graph both the parent function and transformed function below. Clearly label each function. Show your calculations and tables of values.



9) Circle all that apply for each function

<div>a)</div>	No symmetry Even function Odd function Line Symmetry Point Symmetry	<div>c)</div> $f(x) = 3x^4 + 2x + 1$	No symmetry Even function Odd function Line Symmetry Point Symmetry
<div>b)</div>	No symmetry Even function Odd function Line Symmetry Point Symmetry	<div>d)</div> $f(x) = 2x^4 + 4x^2 - 5$	No symmetry Even function Odd function Line Symmetry Point Symmetry

## Unit 2: Factor Theorem

**10)** Divide  $3x^3 - x^2 - 1$  by  $x + 2$ . Show using long and synthetic division. Express the result in quotient form.



**11)a)** What would be the remainder if you divided  $p(x) = x^3 + 3x^2 + 4x + 7$  by  $x + 3$ ? Use the remainder theorem, do not divide!

**b)** Use synthetic division to verify your answer. Express your answer using the multiplication statement that can be used to check the division.

**12)** Perform each division using the most appropriate method. Express your answer using the multiplication that can be used to check the division.

**a)**  $(4x^3 + 6x^2 - 4x + 2) \div (2x - 1)$

**b)**  $(2x^3 - 4x + 8) \div (x - 2)$

**13)** Determine the value of  $k$  such that when  $f(x) = 3x^5 - 4x^3 + kx^2 - 1$  is divided by  $x + 2$ , the remainder is  $-5$ .

**14)** Factor the polynomial  $P(x) = x^3 + x^2 - 10x + 8$ . I am looking to see a FULL list of possible zeros, your test of the zero, and your polynomial division.

Possible Zeros:

Test(s):

Factored Form:

$P(x) =$

**15)** Factor the polynomial  $P(x) = 3x^4 + x^3 - 14x^2 - 4x + 8$ . I am looking to see a FULL list of possible zeros, your test of the zero, and your polynomial division.

Possible Zeros:

Test(s):

Factored Form:

$P(x) =$



**16)** Create an equation to represent a **family** of cubic polynomials with zeros at -3 and at  $1 \pm \sqrt{6}$ . You may leave your answer in factored form (you do not have to expand in to standard form).

Equation for Family:

**17) Solve** the following equations OR inequalities by first factoring completely. Use **any** factoring techniques. For inequalities, show a sketch of the polynomial or a factor table to support your solution.

**a)**  $0 = 27x^3 - 64$

**b)**  $0 = x^3 - 2x^2 + 16x - 32$

**c)**  $0 = x^4 - 29x^2 + 100$

**d)**  $0 = x^3 - x^2 - 10x - 8$

**e)**  $2x^2 + x - 6 < 0$

**f)**  $x^3 - 2x^2 - 13x \leq 10$

**Unit 3: Exponential and Logarithmic Functions**

**18)** Rewrite each equation in logarithmic form.

**a)**  $5^4 = 625$

**b)**  $4^x = 12$

**c)**  $y = 12^3$

**19)** Rewrite each equation in exponential form.

**a)**  $x = \log 8$

**b)**  $4 = \log_5 x$

**c)**  $7 = \log_b 200$

**20)** Evaluate without using a calculator.

**a)**  $\log_2 256$

**b)**  $\log_4 64$

**c)**  $\log_2 0.25$

d)  $2^{\log 1000}$

e)  $\log\left(\frac{1}{1000}\right)$

f)  $\log_6 6$

**21)** Solve each exponential equation. Use and show appropriate methods. Round to 3 decimal places where necessary. Make sure to check for extraneous routes where necessary.

a)  $3^x = 12$

b)  $4^{2x+5} = 32^{4-x}$

c)  $27^{2-3x} = \left(\frac{1}{9}\right)^{2x}$

d)  $4^{2x-5} = 7^x$

e)  $5^{x+4} = 2^{4-5x}$

f)  $3^{2x} - 4(3)^x + 3 = 0$

**22)** Write as a single logarithm and then evaluate. Round to 2 decimal places if necessary.

**a)**  $\log_4 12 - \log_4 3$

**b)**  $3 \log 6 + 2 \log 5 - \log 54$

**23)** Simplify.

**a)**  $\log(x^2 - 4x - 12) - \log(3x - 18)$

**b)**  $\log(x^3 - 27) - \log(x - 3)$

**24)** Solve the following logarithmic equations. Use and show appropriate methods. Round to 3 decimal places where necessary. Make sure to check for extraneous routes where necessary.

**a)**  $\log_3(3x + 7) = 2$

**b)**  $\log_5(2x + 1) = 1 - \log_5(x + 2)$

**c)**  $\log_4 x = \log_4 15 - \log_4 3$

**d)**  $\log_4(x^2 - 9x + 18) - \log_4(x - 3) = 2$

Exponential Formulas		
$A(t) = A_0(1 + i)^t$ where $i$ is percent growth (+) or decay (-)	$A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{H}}$ where $H$ is the half-life period	$A(t) = A_0(2)^{\frac{t}{D}}$ where $D$ is the doubling period
Logarithmic Formulas		
$pH = -\log[H^+]$ where $pH$ is acidity and $[H^+]$ is concentration of hydronium ions in mol/L	$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1}\right)$ where $\beta$ is the loudness in dB and $I$ is the intensity of sound in W/m <sup>2</sup>	$M = \log \left(\frac{I}{I_0}\right)$ Where $M$ is the magnitude measured by richters and $I$ is intensity.

**25)** If the pH of a solution is 4.2, what is the concentration of the hydronium ions?

**26)** An investment earns interest compounded annually for 12 years. In that time, its value grows from \$2500 to \$7100. What was the interest rate, to the nearest tenth of a percent?

**27)** A 30-mg sample of a radioactive isotope decays to 27 mg in 12.5 h.

**a)** Calculate its half-life, to two decimal places.

**b)** How long will it take (to the nearest hour) until only 5 mg of the sample remain?

**28)** The population of a species of animal in a nature reserve grows by 12.2% each year. Initially, there are 200 of that species.

**a)** Write an equation for the population of the species as a function of time, in years.

**b)** What will the population be after 20 years?

**c)** How long does it take the population to double?

**29) a)** The intensity of a sound at the threshold of hearing (0 dB) is  $10^{-12} \text{ W/m}^2$ . What is the intensity of a 50 dB sound?

**b)** How many times as intense is an 85 dB sound than a 50 dB sound?

**c)** A noise is 400 times as intense as a 60 dB sound. What is the decibel rating of this noise?

**30)a)** The magnitudes of two earthquakes are 4.7 and 7.1. How many times as intense was the stronger earthquake than the less severe one?

**b)** An earthquake is detected that is 450 times as intense as an earthquake with a magnitude of 5.2. What is the magnitude of the new earthquake?

#### **Unit 4: Trig in Radians**

**31)** Determine the approximate degree measure, to the nearest tenth, for each angle.

**a)** 3.62

**b)** 0.54

**32)** Determine the exact radian measure of each angle.

**a)**  $125^\circ$

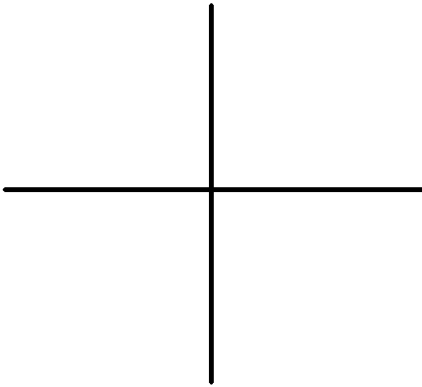
**b)**  $80^\circ$

**c)**  $16^\circ$

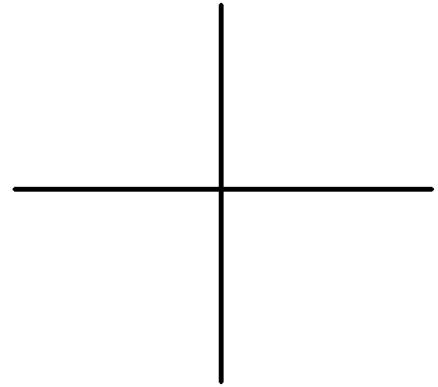
**33)** Draw both special triangles using radian measures.

**34)** Find the exact value of the expressions below. Use special triangles and the CAST rule to find your answer.

a)  $\sin \frac{5\pi}{3}$



b)  $\cot \frac{5\pi}{4}$

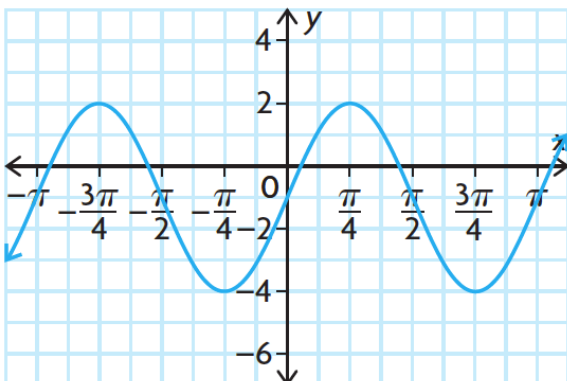


**35)** Determine an exact value for each expression.

a)  $\frac{\sin \frac{\pi}{6} \tan \frac{\pi}{3}}{\csc \frac{\pi}{4}}$

b)  $\sec \frac{4\pi}{3} \cot \frac{5\pi}{6} - \tan \frac{3\pi}{4}$

**36)** Find two equations (one sine and one cosine) to represent the function on the graph below. Show your calculations for full marks.





**37)** A sine function has a maximum value of 6, a minimum value of -2, a period of  $\frac{\pi}{2}$ , and a phase shift of  $\frac{\pi}{6}$  radians to the right.

**a)** Write an equation for the function

**b)** Write an equivalent cosine equation for the function

**38)** For the function  $g(x) = -\cos\left[2\left(x - \frac{\pi}{6}\right)\right] + 3$ , fill in the table of information and then graph two cycles of the transformed function using transformations of the parent function. Choose an appropriate scale. [7]

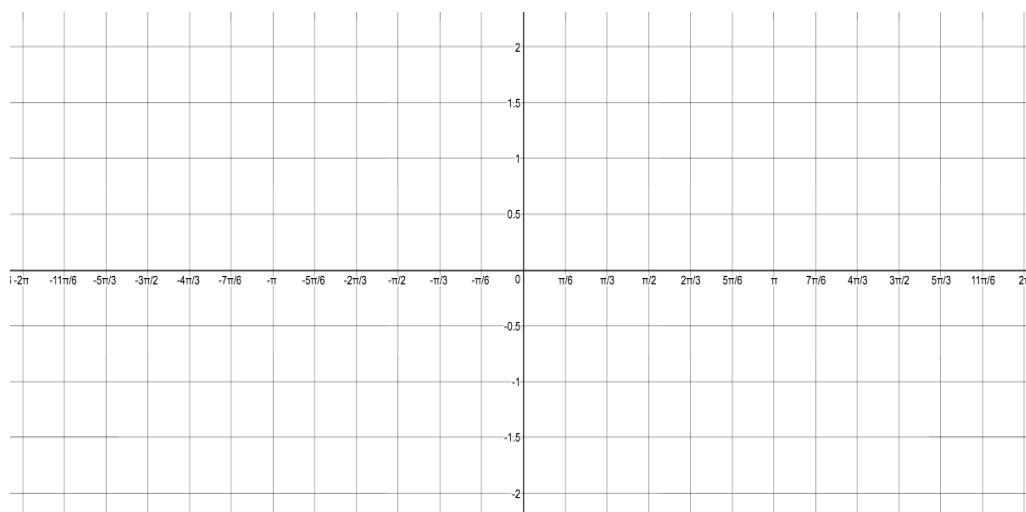
Amplitude:	Period:
Phase shift:	Vertical shift:
Max:	Min:

<i>x</i>	<i>y</i>




**39)** Complete the following table of values for the function  $f(x) = \sin(x)$  and  $g(x) = \csc(x)$ . Use special triangles, the unit circle, or a calculator to find values for the function. Then graph both functions on the same grid. Draw asymptotes where necessary.

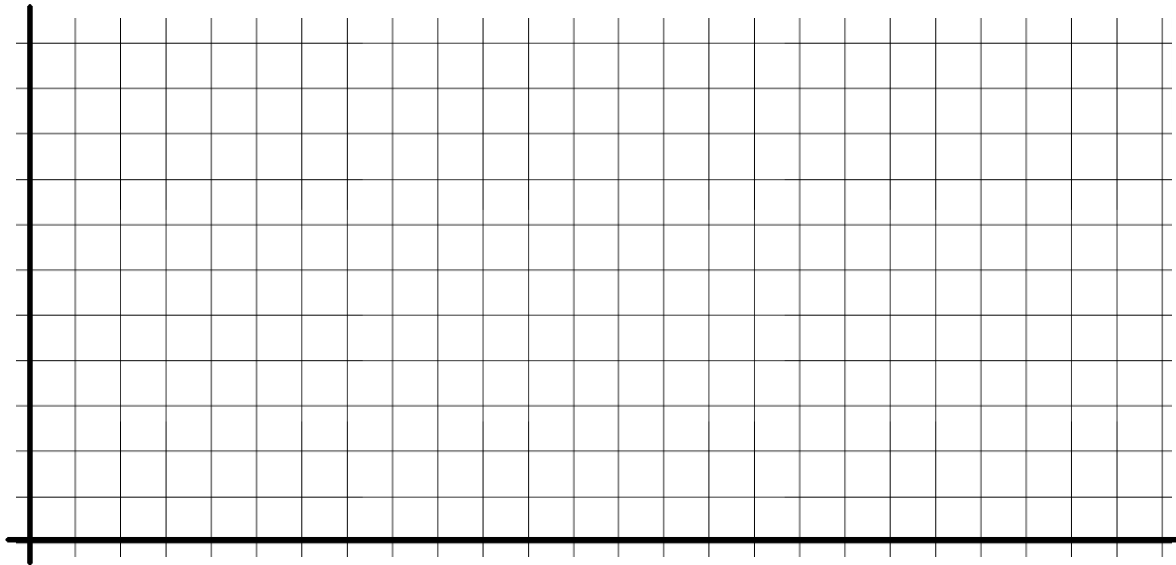
$x$	$f(x)$	$g(x)$
0		
$\frac{\pi}{6}$		
$\frac{2\pi}{6} = \frac{\pi}{3}$		
$\frac{3\pi}{6} = \frac{\pi}{2}$		
$\frac{4\pi}{6} = \frac{2\pi}{3}$		
$\frac{5\pi}{6}$		
$\frac{6\pi}{6} = \pi$		
$\frac{7\pi}{6}$		
$\frac{8\pi}{6} = \frac{4\pi}{3}$		
$\frac{9\pi}{6} = \frac{3\pi}{2}$		
$\frac{10\pi}{6} = \frac{5\pi}{3}$		
$\frac{11\pi}{6}$		
$\frac{12\pi}{6} = 2\pi$		



**40)** A Ferris wheel at an amusement park completes one revolution every 60 seconds. The wheel has a radius of 20 meters and its center is 22 meters above the ground.

**a)** Model the rider's height above the ground with a sine function

**b)** Sketch a graph of the rider's height above the ground for 2 cycles.



### Unit 5: Trig Identities and Equations

**41)** Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each.

**a)**  $\sin \pi \cos \frac{\pi}{2} + \cos \pi \sin \frac{\pi}{2}$

**b)**  $\cos \pi \cos \frac{\pi}{2} + \sin \pi \sin \frac{\pi}{2}$

**42)** Use an appropriate compound angle formula to determine an exact value for each.

**a)**  $\cos \frac{\pi}{12}$

**b)**  $\sin \frac{11\pi}{12}$

**43)** Angle  $a$  lies in the second quadrant and angle  $b$  lies in the third quadrant such that  $\cos a = -\frac{3}{5}$  and  $\tan b = \frac{24}{7}$ . Determine an exact value for

diagrams:

**a)**  $\cos(a + b)$

**b)**  $\sin(a - b)$

**c)**  $\sin(2a)$

**d)**  $\cos(2b)$

**44)** Angle  $x$  lies in the third quadrant, and  $\tan x = \frac{9}{40}$ . Determine an exact value for  $\cos(2x)$ .

**45)** Prove the following identities. Use a separate piece of paper.

**a)**  $\sec x - \tan x = \frac{1 - \sin x}{\cos x}$

**b)**  $(\csc x - \cot x)^2 = \frac{1 - \cos x}{1 + \cos x}$

**c)**  $\sin(2a) = \frac{2 \tan a}{\sec^2 a}$

**d)**  $\cos(x + y) \cos(x - y) = \cos^2 x + \cos^2 y - 1$

**e)**  $\frac{\cos(2x)}{1 - \sin(2x)} = \frac{\cos x + \sin x}{\cos x - \sin x}$

**f)**  $\sin(2x) = 2 \sin x \cos x$

**g)**  $\frac{\tan x - \tan y}{\cot x - \cot y} = -\tan x \tan y$

**h)**  $\frac{2 \tan x}{1 + \tan^2 x} = \sin(2x)$

**46)** Determine solutions for each equation in the interval  $0 \leq x \leq 2\pi$ , to the nearest hundredth of a radian. Give exact answers where possible.

**a)**  $\sin x - 0.8 = 0$

**b)**  $\tan x - \frac{3}{4} = 0$

**c)**  $2 \sin x = -\sqrt{3}$

**d)**  $2 \sin x \cos x - \cos x = 0$

**e)**  $\csc^2 x = 2 + \csc x$

**g)**  $64 \sin^2 x - 25 = 0$

**h)**  $2 \csc^2 x - 9 \csc x + 10 = 0$

**i)**  $2 \cos(2x) = 1$

## Unit 6: Rational Equations/Inequalities and Rates of Change

**47)** Solve each equation

a)  $\frac{x-2}{3x-1} = \frac{2}{x+1}$

b)  $3 = \frac{6}{2x^2-x-4}$

**48)** Solve each inequality

a)  $\frac{2x-3}{x+5} > \frac{2x+7}{x-3}$

b)  $\frac{x^2-8x+15}{x^2+5x+4} \leq 0$

**49)** The population of a small town,  $p$ , is modelled by the function  $p(t) = 10\,050 + 225t - 20t^2$ , where  $t$  is the time in years from now. Determine the average rate of change of the population from

**a)** year 0 to year 5

**b)** year 5 to year 8

**50)** A soccer ball is kicked into the air. The following table of values shows the height of the ball above the ground at various times during its flight:

<b>Time (seconds)</b>	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
<b>Height (meters)</b>	0.5	11.78	20.6	26.98	30.9	32.38	31.4	27.98	22.1	13.78	3.0

**a)** Find the average rate of change in the height of the ball over the first 3 seconds.

**b)** Estimate the *instantaneous* rate of change of the height at 4 seconds.



**51)** Use the chart below to calculate several average rates of change to help you estimate the instantaneous rate of change for the function  $f(x) = -2x^2 + x$  at  $x = 1$ . Have 4-decimal place accuracy in the  $\frac{\Delta y}{\Delta x}$  column.

Interval	Change in $y = \Delta y$	$\Delta x$	$\frac{\Delta y}{\Delta x} = \text{Avg. Rate of Change}$
$0 \leq x \leq 1$			
$0.5 \leq x \leq 1$			
$0.9 \leq x \leq 1$			
$0.99 \leq x \leq 1$			

**52)** Use the Newton Quotient to find the equation of the derivative for each of the following functions. Also, find the instantaneous rate of change for the function when  $x = 4$ .

**a)**  $f(x) = x^2 + 6$

**b)**  $f(x) = 2x^2 + 3x - 4$

**53)** Determine the equation of the tangent line at  $x = 3$  for the function  $f(x) = 5x^2 - 10x - 7$

**54)** Calculate the following limits:

a)  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

b)  $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x - 5}$

**55)** Calculate the following limits using the graph provided.

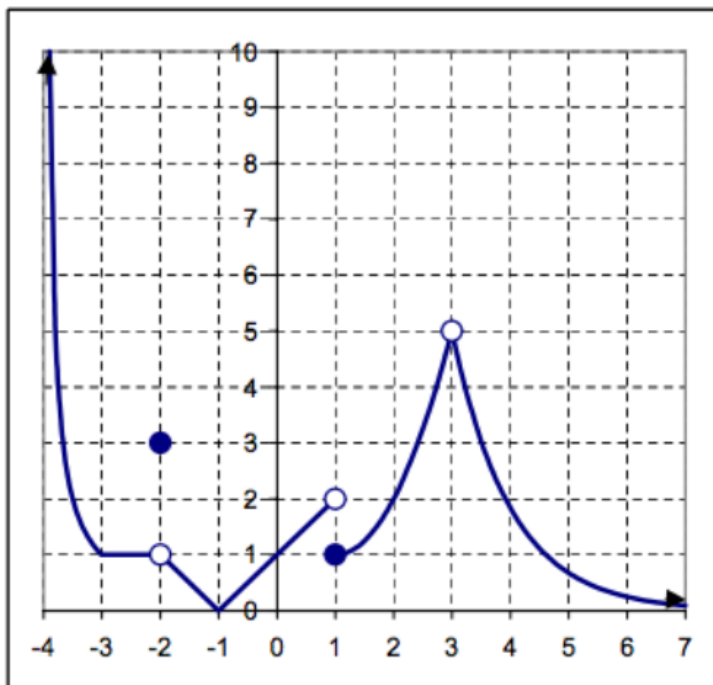
a)  $\lim_{x \rightarrow \infty} f(x)$

b)  $\lim_{x \rightarrow -4^+} f(x)$

c)  $\lim_{x \rightarrow 1^+} f(x)$

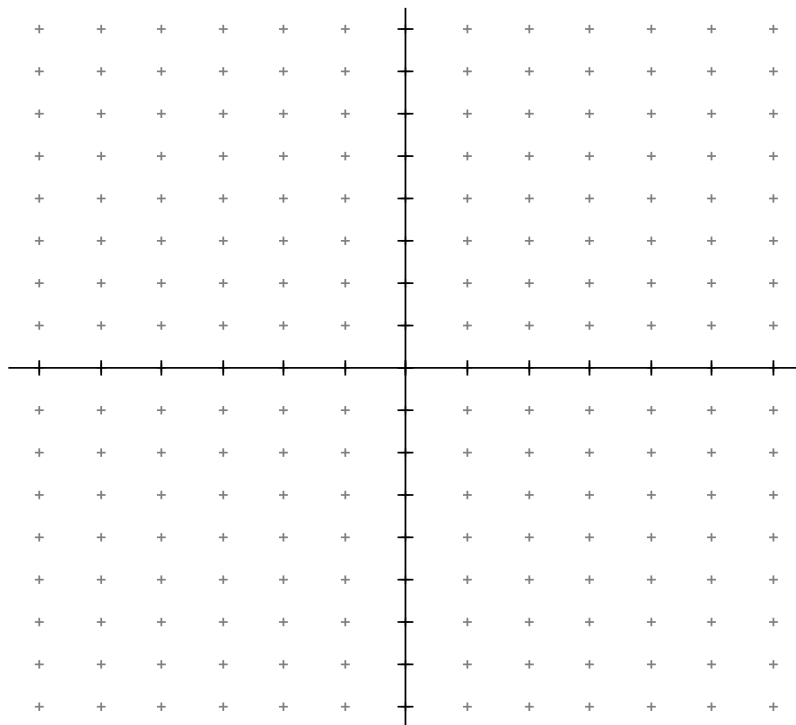
d)  $\lim_{x \rightarrow 1^-} f(x)$

e)  $\lim_{x \rightarrow 1} f(x)$

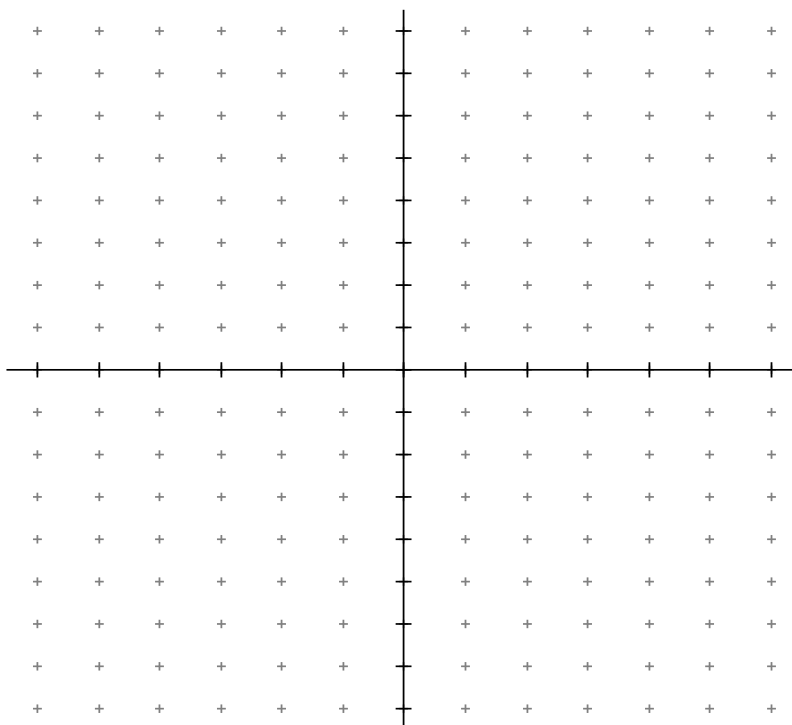


## Unit 7: Graphing

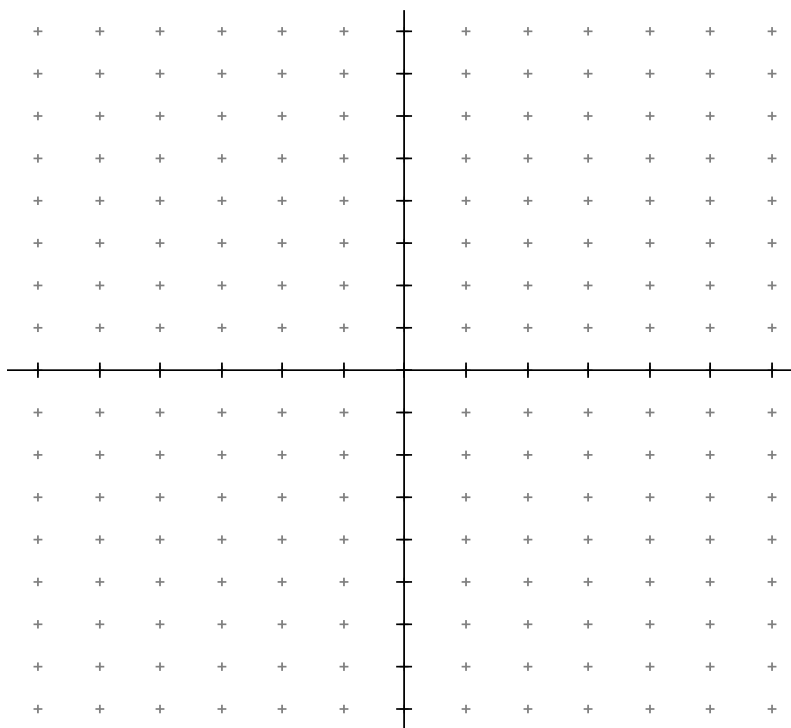
56) Graph  $f(x) = \frac{4}{x-2}$  by first graphing the denominator, then graphing it's reciprocal. Don't forget about the 4 on top! Label any asymptotes



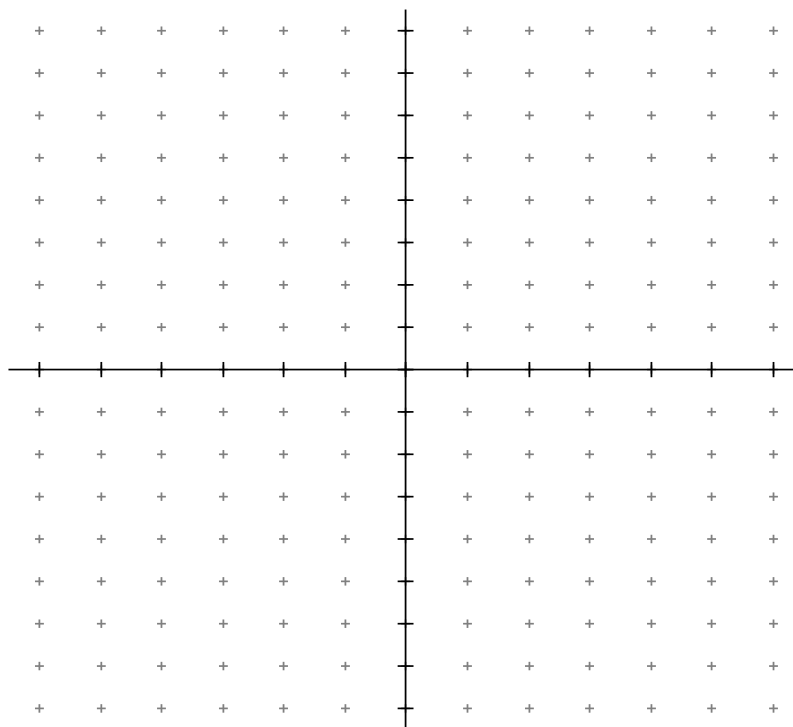
57) Graph  $g(x) = \frac{1}{x^2-4}$  by first graphing the denominator, then graph it's reciprocal. Label any asymptotes.



**58)** Graph the function  $h(x) = \frac{2x-3}{x+1}$ . Clearly state the key features of this graph including asymptotes and intercepts.



**59)** Graph the transformed function  $h(x) = 2(3)^{x+4} - 5$  using transformations.



**60)** Evaluate each of the following, correct to 3 decimal places.

**a)**  $e^5$

**b)**  $\ln 200$

**61)** Simplify the following expression  $\ln e^{2x}$

**62)** Solve the following equations, correct to 3 decimal places.

**a)**  $e^x = 5$

**b)**  $1000 = 20e^{\frac{x}{4}}$

**63)** Given that  $g(x) = x + 5$  and  $f(x) = x^2 + 4x - 1$ , find a formula for the following...

**a)**  $f(g(x))$

**b)**  $g(f(x))$

**c)**  $g^{-1}(f(x))$

**d)**  $f(x) - g(x)$