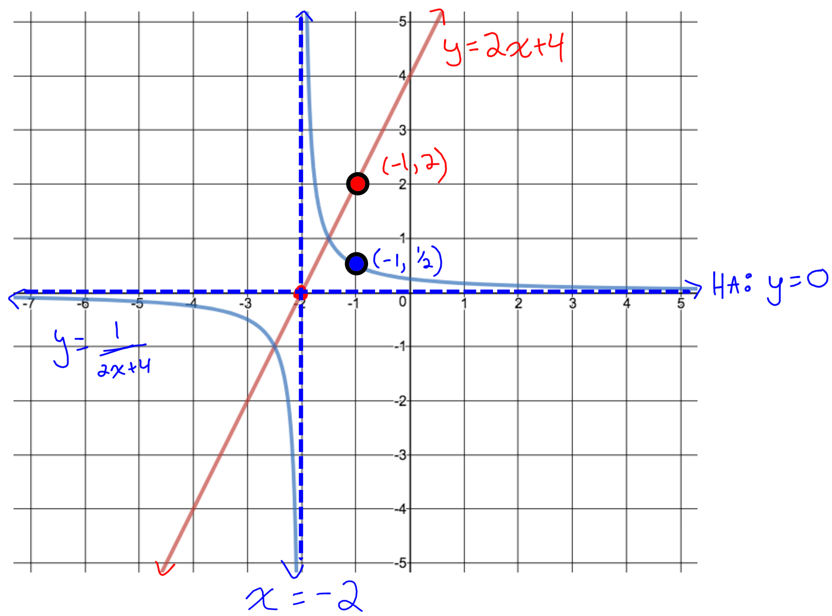
L1 – 3.1/3.2 Reciprocal of Linear and Quadratic Functions
MHF4U

Part 1: Analyze the Reciprocal of a Linear Function

Example 1:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

HA: $y = 0$
VA: $x = -2$

b) What graphical characteristic in the reciprocal function does the zero (x-int) of the original function correspond to?

The vertical asymptote of the reciprocal function passes through the x-intercept of the linear function.

c) When the original function is increasing, what is happening to the reciprocal function?

It is DECREASING

d) What are the y-coordinates of the points of intersection?

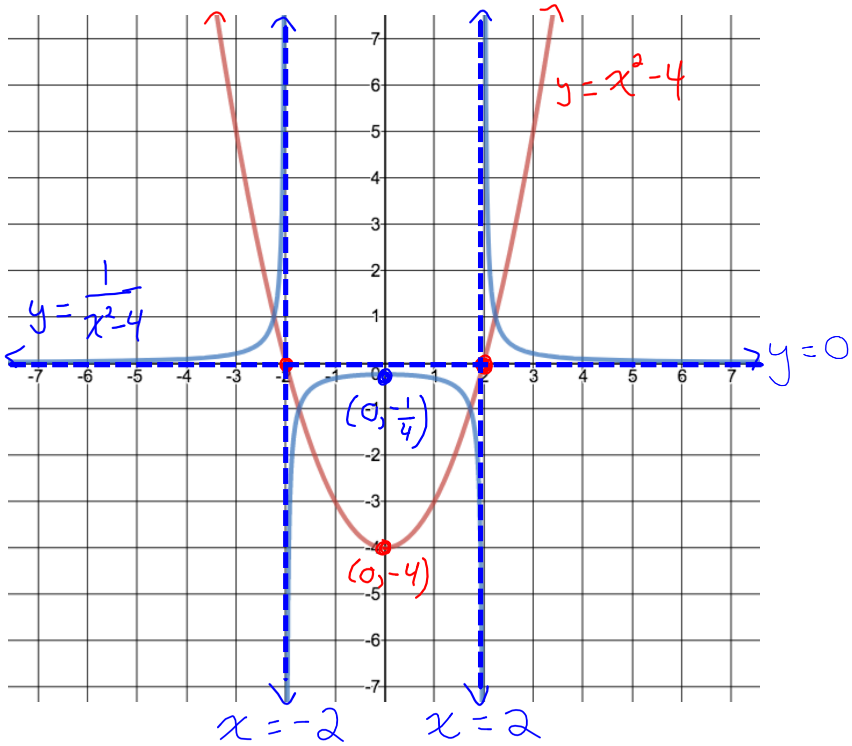
1 and -1. This is because the reciprocal of each of those does not change their value.

e) Label a point on the graph of both functions at $x = 2$. What do you notice about the y values of each point?

The y-value of the reciprocal function is the reciprocal of the y-value of the linear function

Part 2: Analyze the Reciprocal of a Quadratic Function

Example 2:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

VA: $x = 2$ and $x = -2$
HA: $y = 0$

b) What graphical characteristic in the reciprocal function do the zeros (x-int) of the original function correspond to?

The vertical asymptotes of the reciprocal function pass through the x-intercepts of the quadratic.

c) When the original function is decreasing, what is happening to the reciprocal function?

It is INCREASING

d) What are the y-coordinates of the points of intersection?

1 and -1

f) Label the local min or max point on each function. What do you notice about them?

They have the same x-coordinate which is exactly half way between the vertical asymptotes.

The quadratic has a local min but the reciprocal has a local max.

Properties of Reciprocal Functions

- All the y-coordinates of the reciprocal function are the reciprocals of the y-coordinates of the original function
- The graph of the reciprocal function has a vertical asymptote at the x-intercepts (zeros) of the original
  - This is because it makes the denominator of the reciprocal $= 0$
- $y = 0$ will always be a horizontal asymptote
- The reciprocal function has the same positive/negative intervals as the original function
- Intervals of increase on the original function are intervals of decrease on the reciprocal
- Intervals of decrease on the original function are intervals of increase on the reciprocal
- If 1 is in the range of the original function, this is where the functions will intersect
- If the original function has a local min point, the reciprocal will have a local max at the same x-value (and vice versa)

Part 3: Graphing Reciprocal Functions

Process:

- Find key features of the function in the denominator and graph it using a table of values
- Create a table of values for the reciprocal function by keeping the same x values but using the reciprocal of all y values
- Draw vertical asymptotes at any point that is a zero of the original linear/quadratic function
  - Reciprocal of 0 is undefined
- If the numerator is something other than 1, multiply the y-values by this stretch factor

Example 3: Graph each of the following reciprocal functions. Start by graphing the function in the denominator.

a) $y = \frac{1}{2x-1}$

$\boldsymbol{y = 2x - 1}$

$x - int: (\frac{1}{2}, 0)$

x	y
-2	-5
-1	-3
0	-1
0.5	0
1	1
2	3
3	5

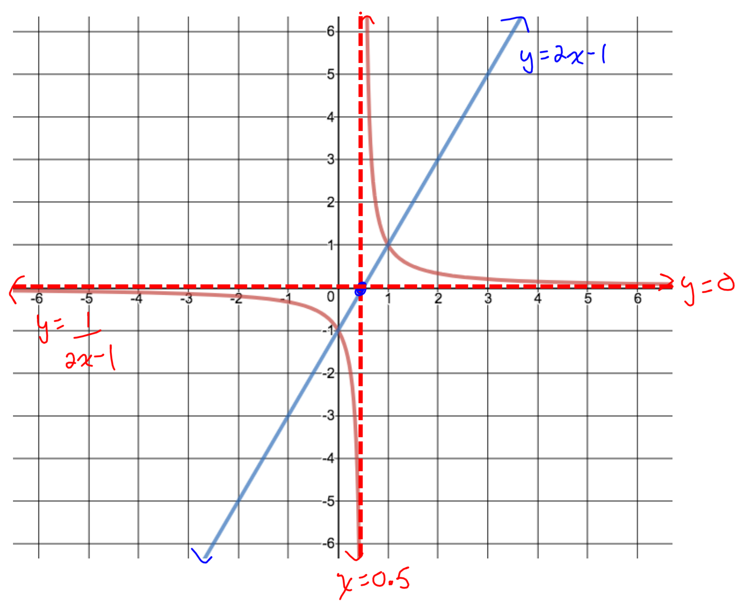
Center x-int

$\boldsymbol{y = \frac{1}{2x - 1}}$

$VA: x = \frac{1}{2}$

$HA: y = 0$

x	$\frac{1}{y}$
-2	-0.2
-1	-0.33
0	-1
0.5	$undefined$
1	1
2	0.33
3	0.2



b) $y = \frac{1}{x^2-2x-15} = \frac{1}{(x-5)(x+3)}$

$$\boldsymbol{y = x^2 - 2x - 15}$$

$x^2 - 2x - 15$

$= (x-5)(x+3)$

$x - int$: $(5,0)$ and $(-3,0)$

$x - vertex = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$

$$\boldsymbol{y = \frac{1}{x^2 - 2x - 15}}$$

$\frac{1}{x^2 - 2x - 15}$

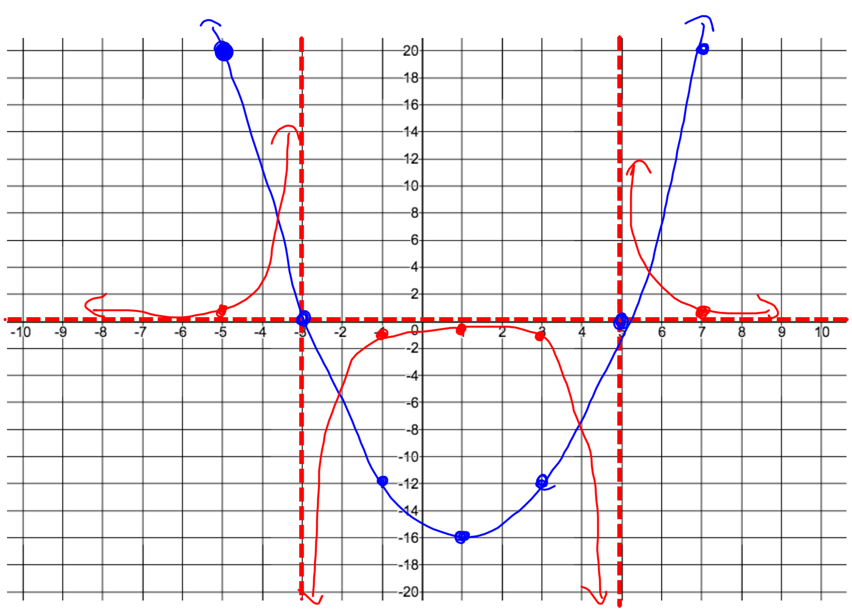
$= \frac{1}{(x-5)(x+3)}$

$VA: x = 5$ and $x = -3$

$HA: y = 0$

	x	y
	-5	20
Include x-int	-3	0
	-1	-12
Center the vertex	1	-16
	3	-12
Include x-int	5	0
	7	20

x	$\frac{1}{y}$
-5	0.05
-3	$undefined$
-1	-0.08
1	-0.0625
3	-0.08
5	$undefined$
7	0.05



c) $y = \frac{1}{x^2+4}$

$$\boldsymbol{y = x^2 + 4}$$

$\boldsymbol{x - int}$:

$x^2 + 4 \neq 0$
$\therefore no\ x - int$

$\boldsymbol{x - vert}$:

$$x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0$$

$$\boldsymbol{y = \frac{1}{x^2 + 4}}$$

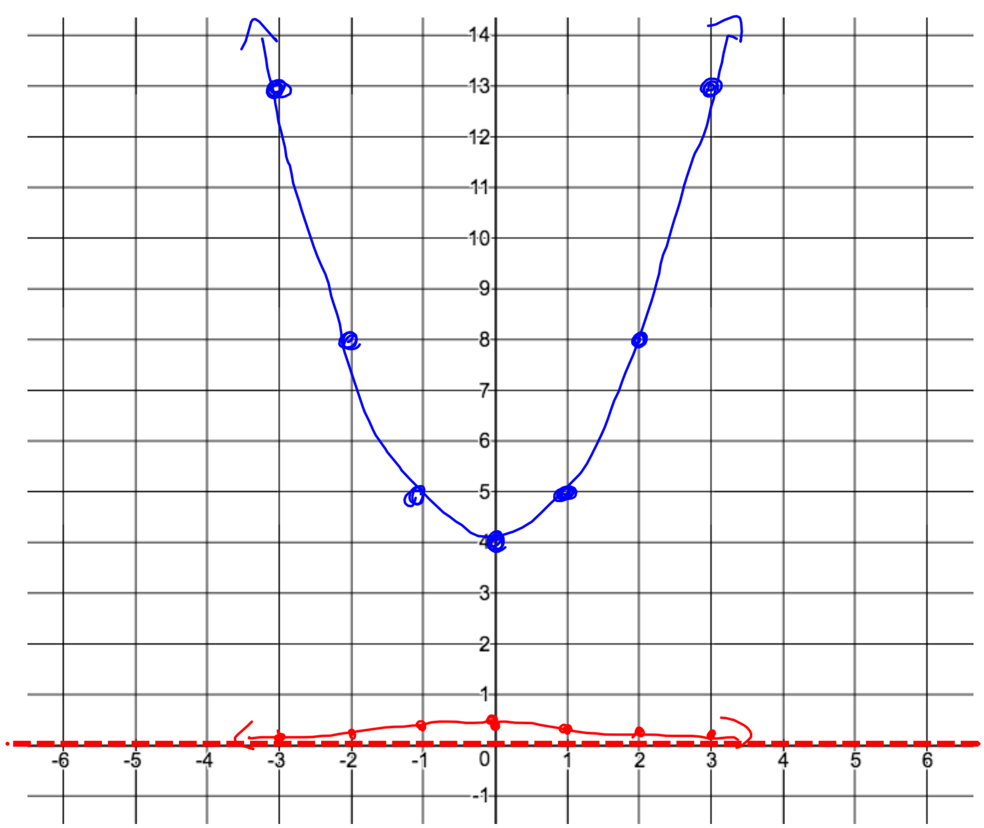
$VA: none$

$HA: y = 0$

Center the vertex

$\boldsymbol{x}$	$\boldsymbol{y}$
-3	13
-2	8
-1	5
0	4
1	5
2	8
3	13

$\boldsymbol{x}$	$\boldsymbol{\frac{1}{y}}$
-3	0.08
-2	0.125
-1	0.2
0	0.25
1	0.2
2	0.125
3	0.08



d) $y = \frac{2}{x^2-6x+9} = 2\left[\frac{1}{(x-3)^2}\right]$

$$\boldsymbol{y = x^2 - 6x + 9}$$

$x^2 - 6x + 9$

$= (x-3)^2$

$x - int: (3,0)$

$x - vertex = 3$

$$\boldsymbol{y = \frac{2}{x^2 - 6x + 9}}$$

$\frac{2}{x^2 - 6x + 9}$

$= 2\left[\frac{1}{(x-3)^2}\right]$

$VA: x = 3$

$HA: y = 0$

Center the vertex

$\boldsymbol{x}$	$\boldsymbol{y}$
0	9
1	4
2	1
3	0
4	1
5	4
6	9

$\boldsymbol{x}$	$\boldsymbol{\frac{2}{y}}$
0	0.22
1	0.5
2	2
3	$undefined$
4	2
5	0.5
6	0.22