Using your sheet of all identities learned this unit, prove each of the following:

Example 1: Prove $\frac{\sin(2x)}{1+\cos(2x)} = \tan x$

$$\underline{\text{LS}}$$

$$= \frac{\sin(2x)}{1+\cos(2x)}$$

$$= \frac{\sin(2x)}{1+2\cos^2 x - 1}$$

$$= \frac{2\sin x \cos x}{2\cos^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$$\underline{\text{RS}}$$

$$= \tan x$$

$$= \frac{\sin x}{\cos x}$$

$$LS = RS$$

Example 2: Prove $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$$\underline{\text{LS}}$$

$$= \cos\left(\frac{\pi}{2} + x\right)$$

$$= \cos\left(\frac{\pi}{2}\right)\cos x - \sin\left(\frac{\pi}{2}\right)\sin x$$

$$= 0(\cos x) - 1\sin x$$

$$= -\sin x$$

$$\underline{\text{RS}}$$

$$= -\sin x$$

$$LS = RS$$

Example 3: Prove $\csc(2x) = \dfrac{\csc x}{2\cos x}$

$$\underline{\text{LS}}$$

$$= \csc(2x)$$

$$= \dfrac{1}{\sin(2x)}$$

$$= \dfrac{1}{2\sin x \cos x}$$

$$\underline{\text{RS}}$$

$$= \dfrac{\csc x}{2\cos x}$$

$$= \csc x \cdot \dfrac{1}{2\cos x}$$

$$= \dfrac{1}{\sin x} \cdot \dfrac{1}{2\cos x}$$

$$= \dfrac{1}{2\sin x \cos x}$$

$$LS = RS$$

Example 4: Prove $\cos x = \dfrac{1}{\cos x} - \sin x \tan x$

$$\underline{\text{LS}}$$

$$= \cos x$$

$$\underline{\text{RS}}$$

$$= \dfrac{1}{\cos x} - \sin x \tan x \quad \Big\} \text{QI}$$

$$= \dfrac{1}{\cos x} - \sin x \left(\dfrac{\sin x}{\cos x} \right)$$

$$= \dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$$

$$= \dfrac{\cos^2 x}{\cos x} \quad \Big/ \text{PI}$$

$$= \cos x$$

$$LS = RS$$

Example 5: Prove $\tan(2x) - 2\tan(2x)\sin^2 x = \sin 2x$

<div align="center">

LS	RS

</div>

LS:

$= \tan(2x) - 2\tan(2x)\sin^2 x$

) factor

$= \tan(2x)\left[1 - 2\sin^2 x\right]$

) DA

$= \tan(2x)\cos(2x)$

) QI

$= \dfrac{\sin(2x)}{\cos(2x)} \cdot \cos(2x)$

$= \sin(2x)$

RS:

$= \sin(2x)$

$$\therefore LS = RS$$

Example 6: Prove $\dfrac{\cos(x-y)}{\cos(x+y)} = \dfrac{1+\tan x \tan y}{1-\tan x \tan y}$

<div align="center">

LS	RS

</div>

LS:

$= \dfrac{\cos(x-y)}{\cos(x+y)}$

$= \dfrac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$

RS:

$= \dfrac{1+\tan x \tan y}{1-\tan x \tan y}$

$= \dfrac{1+\left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)}{1-\left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)} \times \dfrac{\cos x \cos y}{\cos x \cos y}$

$= \dfrac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$