MHF4U EXAM REVIEW SOLUTIONS

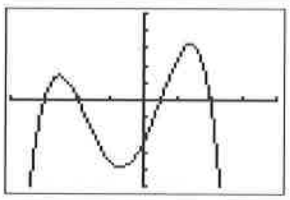
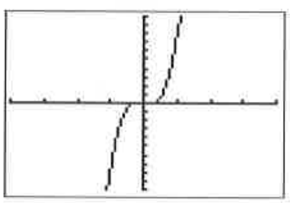
Unit 1: Polynomial Functions

1) Match each function to its end behavior

$y = -3x^2$ $\quad$ $y = 5x^4$ $\quad$ $y = 0.5x^3$ $\quad$ $y = -\frac{1}{3}x^5$

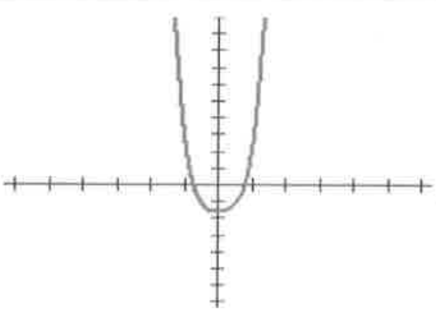
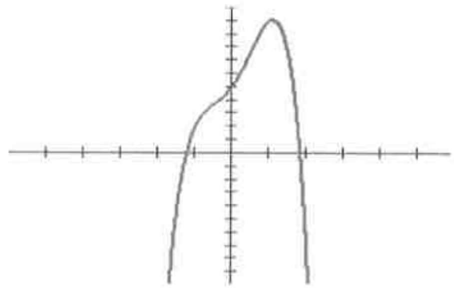
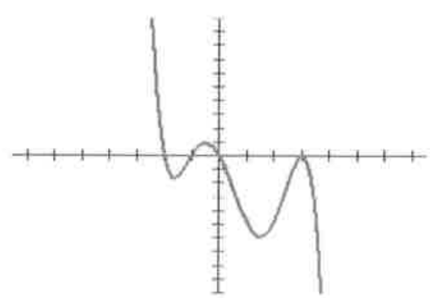
End Behaviour	Function
Q3 to Q1	$y = 0.5x^3$
Q2 to Q4	$y = -\frac{1}{3}x^5$
Q2 to Q1	$y = 5x^4$
Q3 to Q4	$y = -3x^2$

2) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	EVEN	−	D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid y \leq 3\}$	NONE	Q3 → Q4
	ODD	+	D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R}\}$	Point at origin	Q3 → Q1

3) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

A) $g(x) = -2x^4 + 3x^2 + 4x + 5$ **B)** $h(x) = -x^5 + 3x^4 + 7x^3 - 15x^2 - 18x$ **C)** $p(x) = x^6 + 5x^4 + 2x^2 - 3$

		
C	A	B

4) Use the polynomial, $P(x) = 2x^3 + 6x^2 - 8$ to answer the following

a) Complete the chart

Degree	Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts
3	2	Q3→Q1	2, 0	3, 2, 1

b) Is this an even function, odd function, or neither? Explain.

Neither ; mixture of even and odd degree terms.

c) Is there any type of symmetry present? Explain.

Point symmetry ; all cubics have point symmetry

5) Use the polynomial, $P(x) = -6x^4 + 2x^2 - 1$ to answer the following

a) Complete the chart

Degree	Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts
4	-6	Q3→Q4	3, 1	4, 3, 2, 1, 0

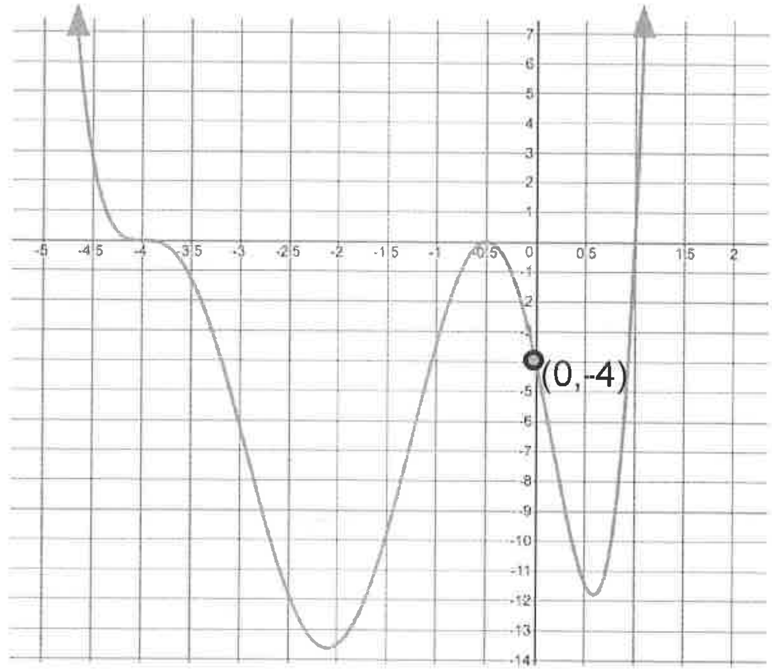
b) Is this an even function, odd function, or neither? Explain.

Even ; all terms are even degree

c) Is there any type of symmetry present? Explain.

Line symmetry about the y-axis ; all even functions have this.

Given the graph below:



a) Complete the following table

Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Number of turning points	x-intercepts with their order	Least Possible Degree
+	EVEN	Q2 → Q1	3	(-4,0) order 3 (-0.5,0) order 2 (1,0) order 1	6

b) Find the equation in factored form

$$f(x) = k(x+4)^3(2x+1)^2(x-1)$$

$$-4 = k(0+4)^3[2(0)+1]^2(0-1)$$

$$-4 = k(64)(1)(-1)$$

$$-4 = -64k$$

$$k = \frac{1}{16}$$

$$\boxed{f(x) = \frac{1}{16}(x+4)^3(2x+1)^2(x-1)}$$

6) Find the equation of a quartic function that has zeros at -4, 1, and 3 (order 2) and passes through the point (2, 6)

$f(x) = k(x+4)(x-1)(x-3)^2$

$6 = k(2+4)(2-1)(2-3)^2$

$6 = k(6)(1)(-1)^2$

$6 = 6k$

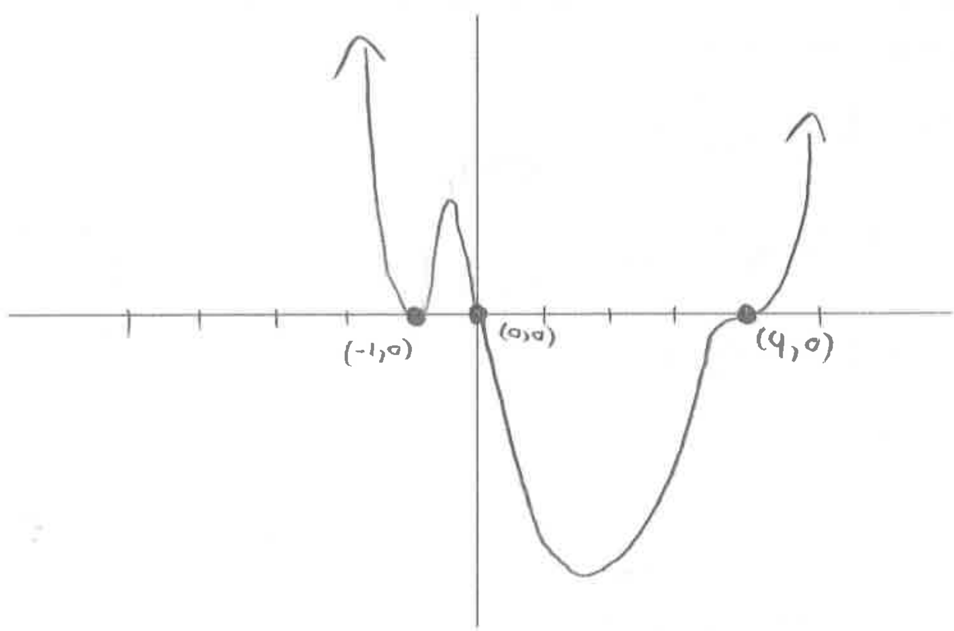
$k = 1$

$$\boxed{f(x) = (x+4)(x-1)(x-3)^2}$$

7) Complete the chart and sketch a possible graph of the function labelling key points.

$$f(x) = \frac{1}{2}x(x-4)^3(x+1)^2$$

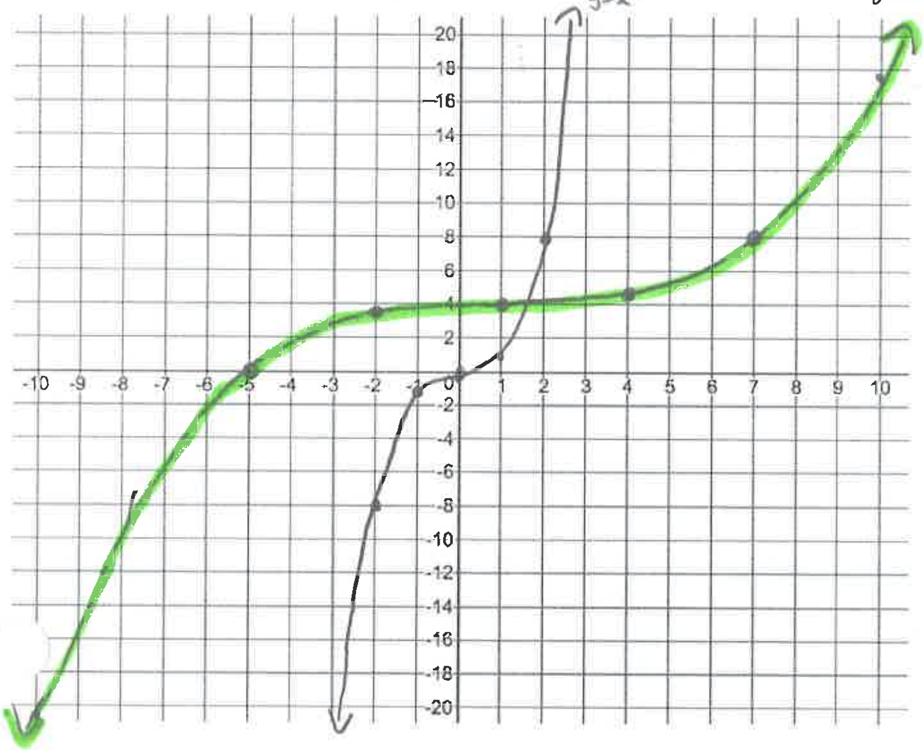
Degree	Leading Coefficient	End Behaviour	x-intercepts with orders	y-intercept
$(x)(x^3)(x^2)$ $= x^6$ Degree 6	$\frac{1}{2}(1)^3(1)^2$ $= \frac{1}{2}$	Q2 → Q1	(0,0) order 1 (4,0) order 3 (-1,0) order 2	$f(0) = \frac{1}{2}(0)(0-4)^3(0+1)^2$ $f(0) = 0$ (0, 0)



8) What is the parent function of, $g(x) = \frac{1}{2}\left[\frac{1}{3}(x-1)\right]^3 + 4$?

$y = x^3$

b) Use transformations to graph both the parent function and transformed function below. Clearly label each function. Show your calculations and tables of values.



$y = x^3$

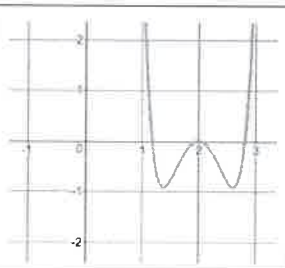
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$g(x)$

$3x+1$	$\frac{y}{2}+4$
-5	0
-2	3.5
1	4
4	4.5
7	8

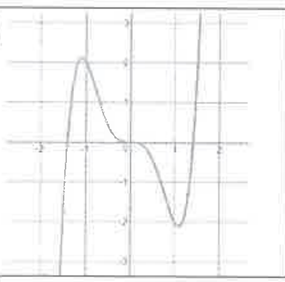
9) Circle all that apply for each function

a)



No symmetry
Even function
Odd function
Line Symmetry
Point Symmetry

b)



No symmetry
Even function
Odd function
Line Symmetry
Point Symmetry

c) $f(x) = 3x^4 + 2x + 1$

No symmetry
Even function
Odd function
Line Symmetry
Point Symmetry

d) $f(x) = 2x^4 + 4x^2 - 5$

No symmetry
Even function
Odd function
Line Symmetry
Point Symmetry

Unit 2: Factor Theorem

10) Divide $3x^3 - x^2 - 1$ by $x + 2$. Show using long and synthetic division. Express the result in quotient form.

$$\begin{array}{r} 3x^2 - 7x + 14 \\ x+2 \overline{\big) 3x^3 - 1x^2 + 0x - 1} \\ 3x^3 + 6x^2 \qquad\qquad \\ \hline -7x^2 + 0x \qquad \\ -7x^2 - 14x \qquad \\ \hline 14x - 1 \\ 14x + 28 \\ \hline R = -29 \end{array}$$

$$\boxed{\frac{3x^3 - x^2 - 1}{x+2} = 3x^2 - 7x + 14 - \frac{29}{x+2}}$$

$$\begin{array}{r|rrrr} -2 & 3 & -1 & 0 & -1 \\ & \downarrow & -6 & 14 & -28 \quad + \\ \hline \times & 3 & -7 & 14 & -29 \\ & x^2 & x & \# & R \end{array}$$

$$\boxed{\frac{3x^3 - x^2 - 1}{x+2} = 3x^2 - 7x + 14 - \frac{29}{x+2}}$$

11)a) What would be the remainder if you divided $p(x) = x^3 + 3x^2 + 4x + 7$ by $x + 3$? Use the remainder theorem, do not divide!

$p(-3) = (-3)^3 + 3(-3)^2 + 4(-3) + 7$

$p(-3) = -5$

$\boxed{\text{The remainder is } -5}$

b) Use synthetic division to verify your answer. Express your answer using the multiplication statement that can be used to check the division.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 4 & 7 \\ & \downarrow & -3 & 0 & -12 \\ \hline \times & 1 & 0 & 4 & -5 \\ & x^2 & x & \# & R \end{array}$$

$$\boxed{x^3 + 3x^2 + 4x + 7 = (x+3)(x^2+4) - 5}$$

12) Perform each division using the most appropriate method. Express your answer using the multiplication that can used to check the division.

) $(4x^3 + 6x^2 - 4x + 2) \div (2x - 1)$

$$
\begin{array}{r}
2x^2+4x+0 \\
2x-1 \,\big)\, 4x^3+6x^2-4x+2 \\
4x^3-2x^2 \downarrow \\
\hline
8x^2-4x \\
8x^2-4x \\
\hline
0x+2 \\
0x+0 \\
\hline
R=2
\end{array}
$$

$\boxed{4x^3+6x^2-4x+2=(2x-1)(2x^2+4x)+2}$

b) $(2x^3 - 4x + 8) \div (x - 2)$

$$
\begin{array}{c|cccc}
2 & 2 & 0 & -4 & 8 \\
 & \downarrow & 4 & 8 & 8 \quad + \\
\hline
\times & 2 & 4 & 4 & 16 \\
 & x^2 & x & \# & R
\end{array}
$$

$\boxed{2x^3-4x+8=(x-2)(2x^2+4x+4)+16}$

) Determine the value of k such that when $f(x) = 3x^5 - 4x^3 + kx^2 - 1$ is divided by $x + 2$, the remainder is -5.

$f(-2)=3(-2)^5-4(-2)^3+k(-2)^2-1$

$-5=-96+32+4k-1$

$-5=4k-65$

$60=4k$

$\boxed{k=15}$

14) Factor the polynomial $P(x) = x^3 + x^2 - 10x + 8$. I am looking to see a FULL list of possible zeros, your test of the zero, and your polynomial division.

Possible Zeros: $x = \pm 1, \pm 2, \pm 4, \pm 8$

Test(s): $f(1) = (1)^3 + (1)^2 - 10(1) + 8$

$f(1) = 0$

$\therefore$ $x-1$ is a factor

1	1	1	-10	8	
	↓	1	2	-8	+
x	1	2	-8	0	
	x^2	x	#	R	

$P(x) = x^3 + x^2 - 10x + 8$

$P(x) = (x-1)(x^2 + 2x - 8)$

$P(x) = (x-1)(x+4)(x-2)$

Factored Form:

$P(x) = (x-1)(x+4)(x-2)$

15) Factor the polynomial $P(x) = 3x^4 + x^3 - 14x^2 - 4x + 8$. I am looking to see a FULL list of possible zeros, your test of the zero, and your polynomial division.

Possible Zeros: $x = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 8, \pm \frac{8}{3}$

Test(s): $f(-1) = 3(-1)^4 + (-1)^3 - 14(-1)^2 - 4(-1) + 8$

$f(-1) = 0$

$\therefore$ $x+1$ is a factor

-1	3	1	-14	-4	8	
	↓	-3	2	12	-8	+
x	3	-2	-12	8	0	
	x^3	x^2	x	#	R	

$P(x) = 3x^4 + x^3 - 14x^2 - 4x + 8$

$P(x) = (x+1)(3x^3 - 2x^2 - 12x + 8)$

$P(x) = (x+1)\left[x^2(3x-2) - 4(3x-2)\right]$

$P(x) = (x+1)(3x-2)(x^2-4)$

$P(x) = (x+1)(3x-2)(x-2)(x+2)$

Factored Form:

$P(x) = (x+1)(3x-2)(x-2)(x+2)$

16) Create an equation to represent a **family** of ~~quartic~~ cubic polynomials with zeros at -3 and at $1 \pm \sqrt{6}$. You may leave your answer in factored form (you do not have to expand in to standard form).

factors:

$x = -3$ $\qquad$ $x = 1 \pm \sqrt{6}$

$x+3 = 0$ $\qquad$ $x - 1 = \pm\sqrt{6}$

$(x-1)^2 = 6$

$x^2 - 2x + 1 = 6$

$x^2 - 2x - 5 = 0$

Equation for Family:

$y = k(x+3)(x^2 - 2x - 5)$

17) Solve the following equations OR inequalities by first factoring completely. Use **any** factoring techniques. For inequalities, show a sketch of the polynomial or a factor table to support your solution.

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

a) $y = 27x^3 - 64$

$0 = (3x-4)[(3x)^2 + 12x + (4)^2]$

$0 = (3x-4)(9x^2 + 12x + 16)$

$3x - 4 = 0$

$\boxed{x = \frac{4}{3}}$

$b^2 - 4ac = 12^2 - 4(9)(16)$

$= -432$

∴ no solutions

b) $f(x) = x^3 - 2x^2 + 16x - 32$

$0 = x^2(x-2) + 16(x-2)$

$0 = (x-2)(x^2+16)$

$x - 2 = 0$

$\boxed{x = 2}$

$x^2 + 16 = 0$

$x^2 = -16$

$x = \pm\sqrt{-16}$

∴ No solutions

c) $g(x) = x^4 - 29x^2 + 100$ $\qquad$ $g(2) = 0$; ∴ $x-2$ is a factor

2	1	0	-29	0	100
	↓	2	4	-50	-100 +
×	1	2	-25	-50	0
	x^3	x^2	x	#	R

$0 = (x-2)(x^3 + 2x^2 - 25x - 50)$

$0 = (x-2)[x^2(x+2) - 25(x+2)]$

$0 = (x-2)(x+2)(x^2-25)$

$0 = (x-2)(x+2)(x-5)(x+5)$

$\boxed{x_1 = 2}$ $\boxed{x_2 = -2}$ $\boxed{x_3 = 5}$ $\boxed{x_4 = -5}$

$h(x) = x^3 - x^2 - 10x - 8$

d) [redacted]

$h(-1) = 0$

∴ $x+1$ is a factor

-1	1	-1	-10	-8
	↓	-1	2	8 +
×	1	-2	-8	0
	x^2	x	#	R

$0 = (x+1)(x^2 - 2x - 8)$

$0 = (x+1)(x-4)(x+2)$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 4}$ $\boxed{x_3 = -2}$

e) $2x^2 + x - 6 < 0$

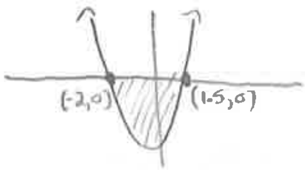
-12

$\frac{2}{1}\ \frac{4}{2}\ \frac{-3}{2}$

$(x+2)(2x-3) < 0$

x-int at $x = -2, 1.5$

$\left.\begin{array}{l} +\text{L.C.} \\ \text{Even Degree} \end{array}\right\}$ Q2 → Q1



Solution:

when $-2 < x < 1.5$

OR

when $x \in (-2, 1.5)$

f) $x^3 - 2x^2 - 13x \leq 10$

f(-1) = 0

∴ x+1 is a factor

$x^3 - 2x^2 - 13x - 10 \leq 0$

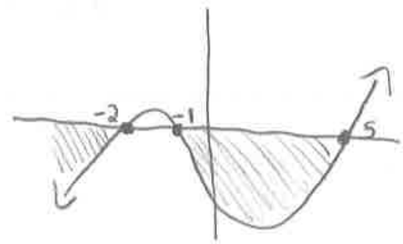
-1	1	-2	-13	-10
	↓	-1	3	10
×	1	-3	-10	0
	x^2	x	#	R

$(x+1)(x^2 - 3x - 10) \leq 0$

$(x+1)(x-5)(x+2) \leq 0$

x-int at $x = -2, -1, 5$

$\left.\begin{array}{l} +\text{L.C.} \\ \text{odd degree} \end{array}\right\}$ Q3 → Q1



Solution:

when $x \leq -2$ or $-1 \leq x \leq 5$

when $x \in (-\infty, -2] \cup [-1, 5]$

Unit 3: Exponential and Logarithmic Functions

18) Rewrite each equation in logarithmic form.

a) $5^4 = 625$

$\log_5(625) = 4$

b) $4^x = 12$

$\log_4(12) = x$

c) $y = 12^3$

$\log_{12}(y) = 3$

19) Rewrite each equation in exponential form.

a) $x = \log 8$

$10^x = 8$

b) $4 = \log_5 x$

$5^4 = x$

c) $7 = \log_b 200$

$b^7 = 200$

20) Evaluate without using a calculator.

a) $\log_2 256$

$= \log_2(2)^8$

$= 8$

b) $\log_4 64$

$= \log_4(4)^3$

$= 3$

c) $\log_2 0.25$

$= \log_2(2)^{-2}$

$= -2$

d) $2^{\log 1000}$

$= 2^3$

$= 8$

e) $\log\left(\frac{1}{1000}\right)$

$= \log(10)^{-3}$

$= -3$

f) $\log_6 6$

$= 1$

21) Solve each exponential equation. Use and show appropriate methods. Round to 3 decimal places where necessary. Make sure to check for extraneous routes where necessary.

a) $3^x = 12$

$\log_3(12) = x$

$\frac{\log(12)}{\log(3)} = x$

$\boxed{x \simeq 2.26}$

b) $4^{2x+5} = 32^{4-x}$

$(2^2)^{2x+5} = (2^5)^{4-x}$

$2^{4x+10} = 2^{20-5x}$

$4x+10 = 20-5x$

$9x = 10$

$\boxed{x = \frac{10}{9}}$

c) $27^{2-3x} = \left(\frac{1}{9}\right)^{2x}$

$(3^3)^{2-3x} = (3^{-2})^{2x}$

$3^{6-9x} = 3^{-4x}$

$6-9x = -4x$

$6 = 5x$

$\boxed{x = \frac{6}{5}}$

d) $4^{2x-5} = 7^x$

$\log(4)^{2x-5} = \log(7)^x$

$(2x-5)\log(4) = x\log(7)$

$2x\log(4) - 5\log(4) = x\log(7)$

$2x\log(4) - x\log(7) = 5\log(4)$

$x(2\log(4) - \log(7)) = 5\log(4)$

$x = \frac{5\log(4)}{2\log(4) - \log(7)}$

$\boxed{x \simeq 8.385}$

e) $5^{x+4} = 2^{4-5x}$

$\log(5)^{x+4} = \log(2)^{4-5x}$

$(x+4)\log(5) = (4-5x)\log(2)$

$x\log(5) + 4\log(5) = 4\log(2) - 5x\log(2)$

$x\log(5) + 5x\log(2) = \log(16) - \log(625)$

$x(\log(5) + 5\log(2)) = \log(16) - \log(625)$

$x = \frac{\log(16) - \log(625)}{\log(5) + 5\log(2)}$

$\boxed{x \simeq -0.722}$

f) $3^{2x} - 4(3)^x + 3 = 0$

Let $k = 3^x$

$k^2 - 4k + 3 = 0$

$(k-3)(k-1) = 0$

$k = 3$ or $k = 1$

CASE 1:

$3^x = 3$

$\boxed{x = 1}$

Case 2:

$3^x = 1$

$\log_3(1) = x$

$\frac{\log(1)}{\log(3)} = x$

$\boxed{x = 0}$

22) Write as a single logarithm and then evaluate. Round to 2 decimal places if necessary.

a) $\log_4 12 - \log_4 3$

$= \log_4\left(\frac{12}{3}\right)$

$= \log_4(4)$

$= \underline{1}$

b) $3\log 6 + 2\log 5 - \log 54$

$= \log(6)^3 + \log(5)^2 - \log(54)$

$= \log(216) + \log(25) - \log(54)$

$= \log\left(\frac{216\times 25}{54}\right)$

$= \log 100$

$= 2$

23) Simplify.

a) $\log(x^2 - 4x - 12) - \log(3x - 18)$

$= \log\left(\frac{x^2-4x-12}{3x-18}\right)$

$= \log\left[\frac{\cancel{(x-6)}(x+2)}{3\cancel{(x-6)}}\right]$

$= \log\left(\frac{x+2}{3}\right)$

b) $\log(x^3 - 27) - \log(x - 3)$

$= \log\left(\frac{x^3-27}{x-3}\right)$

$= \log\left[\frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}}\right]$

$= \log(x^2+3x+9)$

24) Solve the following logarithmic equations. Use and show appropriate methods. Round to 3 decimal places where necessary. Make sure to check for extraneous routes where necessary.

a) $\log_3(3x + 7) = 2$

$3^2 = 3x+7$

$9 = 3x+7$

$2 = 3x$

$\boxed{x = \frac{2}{3}}$

b) $\log_5(2x + 1) = 1 - \log_5(x + 2)$

$\log_5(2x+1) + \log_5(x+2) = 1$

$\log_5[(2x+1)(x+2)] = 1$

$5^1 = 2x^2+5x+2$

$0 = 2x^2+5x-3$

$0 = 2x^2-1x+6x-3$

$0 = x(2x-1)+3(2x-1)$

$0 = (2x-1)(x+3)$

$\boxed{x_1 = \frac{1}{2}}$ $\cancel{x_2 = -3}$

extraneous root

c) $\log_4 x = \log_4 15 - \log_4 3$

$\log_4(x) = \log_4\left(\frac{15}{3}\right)$

$\log_4(x) = \log_4(5)$

$\boxed{x = 5}$

d) $\log_4(x^2 - 9x + 18) - \log_4(x - 3) = 2$

$\log_4\left(\frac{x^2-9x+18}{x-3}\right) = 2$

$\log_4\left[\frac{\cancel{(x-3)}(x-6)}{\cancel{x-3}}\right] = 2$

$\log_4(x-6) = 2$

$4^2 = x-6$

$16 = x-6$

$\boxed{x = 22}$

Exponential Formulas		
$A(t) = A_0(1+i)^t$ where i is percent growth (+) or decay (-)	$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$ where H is the half-life period	$A(t) = A_0(2)^{\frac{t}{D}}$ where D is the doubling period
Logarithmic Formulas		
$pH = -\log[H^+]$ where pH is acidity and $[H^+]$ is concentration of hydronium ions in mol/L	$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$ where β is the loudness in dB and I is the intensity of sound in W/m^2	$M = \log\left(\frac{I}{I_0}\right)$ Where M is the magnitude measured by richters and I is intensity.

25) If the pH of a solution is 4.2, what is the concentration of the hydronium ions?

$4.2 = -\log[H^+]$

$-4.2 = \log[H^+]$

$10^{-4.2} = [H^+]$

$[H^+] \simeq 0.0000630957$ OR 6.31×10^{-5}

26) An investment earns interest compounded annually for 12 years. In that time, its value grows from $2500 to $7100. What was the interest rate, to the nearest tenth of a percent?

$7100 = 2500(1+i)^{12}$

$2.84 = (1+i)^{12}$

$(2.84)^{1/12} = 1+i$

$(2.84)^{1/12} - 1 = i$

$i \simeq 0.091$

∴ The interest rate was about 9.1 %

27) A 30-mg sample of a radioactive isotope decays to 27 mg in 12.5 h.

a) Calculate its half-life, to two decimal places.

$27 = 30\left(\frac{1}{2}\right)^{12.5/H}$

$0.9 = \left(\frac{1}{2}\right)^{12.5/H}$

$\log_{0.5}(0.9) = \frac{12.5}{H}$

$H = \frac{12.5}{\log_{0.5}(0.9)}$

$\boxed{H \simeq 82.24 \text{ hours}}$

b) How long will it take (to the nearest hour) until only 5 mg of the sample remain?

$5 = 30\left(\frac{1}{2}\right)^{t/82.24}$

$\frac{1}{6} = \left(\frac{1}{2}\right)^{t/82.24}$

$\log_{0.5}\left(\frac{1}{6}\right) = \frac{t}{82.24}$

$t = 82.24\log_{0.5}\left(\frac{1}{6}\right)$

$\boxed{t \simeq 212.59 \text{ hours}}$

28) The population of a species of animal in a nature reserve grows by 12.2% each year. Initially, there are 200 of that species.

a) Write an equation for the population of the species as a function of time, in years.

$$P(t) = 200(1.122)^t$$

b) What will the population be after 20 years?

$$P(20) = 200(1.122)^{20}$$

$$\boxed{P(20) \simeq 1999}$$

c) How long does it take the population to double?

$$400 = 200(1.122)^t$$

$$2 = 1.122^t$$

$$\log 2 = \log 1.122^t$$

$$\log 2 = t \log 1.122$$

$$t = \frac{\log 2}{\log 1.122}$$

$$\boxed{t \simeq 6.02 \text{ years}}$$

29) a) The intensity of a sound at the threshold of hearing (0 dB) is 10^{-12} W/m^2. What is the intensity of a 50 dB sound?

β_1 $\quad I_1$

$$\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$$

β_2

$$50 - 0 = 10 \log\left(\frac{I_2}{10^{-12}}\right)$$

$$5 = \log\left(\frac{I_2}{10^{-12}}\right)$$

$$10^5 = \frac{I_2}{10^{-12}}$$

$$I_2 = (10^5)(10^{-12})$$

$$I_2 = 10^{-7}$$

$$\boxed{I_2 = 10^{-7} \text{ W/m}^2}$$

b) How many times as intense is an 85 dB sound than a 50 dB sound?

β_2 $\quad \beta_1$

$$85 - 50 = 10 \log\left(\frac{I_2}{I_1}\right)$$

$$3.5 = \log\left(\frac{I_2}{I_1}\right)$$

$$10^{3.5} = \frac{I_2}{I_1}$$

$$\frac{I_2}{I_1} \simeq 3162.28$$

About 3162 times as intense

c) A noise is 400 times as intense as a 60 dB sound. What is the decibel rating of this noise?

$\frac{I_2}{I_1}$ $\quad \beta_1$

$$\beta_2 - 60 = 10 \log(400)$$

$$\beta_2 = 10 \log(400) + 60$$

$$\boxed{\beta_2 \simeq 86.02 \text{ dB}}$$

$M = \log\left(\frac{I}{I_0}\right)$

30)a) The magnitudes of two earthquakes are 4.7 and 7.1. How many times as intense was the stronger earthquake than the less severe one?

$7.1 - 4.7 = \log\left(\frac{I}{I_0}\right)$

$10^{2.4} = \frac{I}{I_0}$

$\frac{I}{I_0} \simeq 251.2$

About 251.2 times as intense

b) An earthquake is detected that is 450 times as intense as an earthquake with a magnitude of 5.2. What is the magnitude of the new earthquake?

$M - 5.2 = \log(450)$

$M = \log(450) + 5.2$

$M \simeq 7.85$

Unit 4: Trig in Radians

31) Determine the approximate degree measure, to the nearest tenth, for each angle.

3.62 $\times \frac{180}{\pi}$

$\simeq 207.4°$

b) 0.54 $\times \frac{180}{\pi}$

$\simeq 30.9°$

32) Determine the exact radian measure of each angle.

a) 125° $\times \frac{\pi}{180}$

$= \frac{25\pi}{36}$ radians

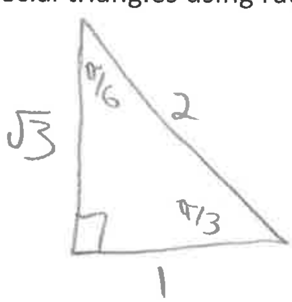
b) 80° $\times \frac{\pi}{180}$

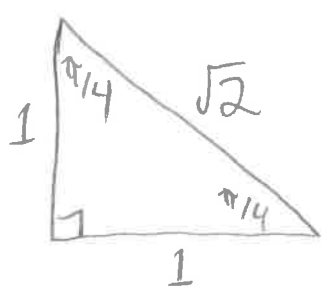
$= \frac{4\pi}{9}$ radians

c) 16° $\times \frac{\pi}{180}$

$= \frac{4\pi}{45}$ radians

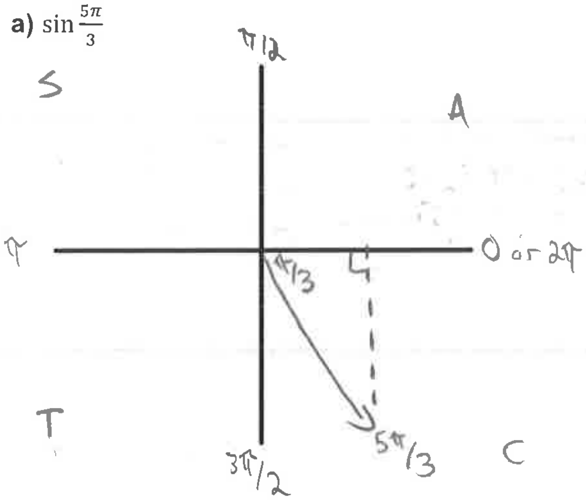
33) Draw both special triangles using radian measures.





34) Find the exact value of the expressions below. Use special triangles and the CAST rule to find your answer.

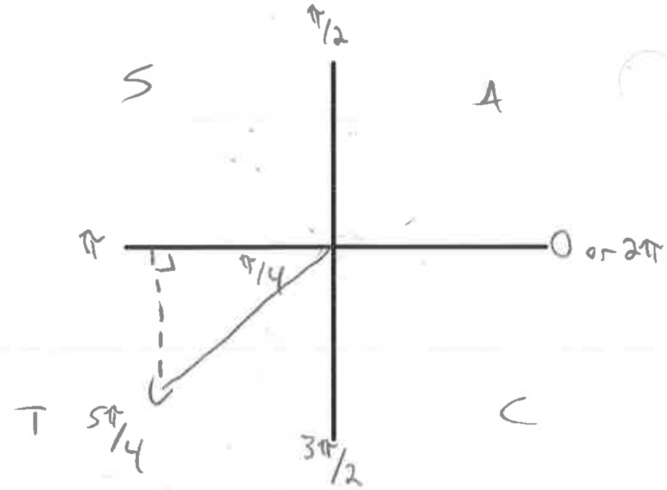
a) $\sin\frac{5\pi}{3}$



$\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2}$

b) $\cot\frac{5\pi}{4}$



$\cot\left(\frac{5\pi}{4}\right) = \frac{1}{\tan\left(\frac{5\pi}{4}\right)}$

$= \frac{1}{\tan\left(\frac{\pi}{4}\right)}$

$= \frac{1}{(1)}$

$= \underline{1}$

35) Determine an exact value for each expression.

a) $\dfrac{\sin\frac{\pi}{6}\tan\frac{\pi}{3}}{\csc\frac{\pi}{4}} = \dfrac{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{1}\right)}{\left(\frac{\sqrt{2}}{1}\right)}$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{\sqrt{6}}{4}$

b) $\sec\frac{4\pi}{3}\cot\frac{5\pi}{6} - \tan\frac{3\pi}{4}$

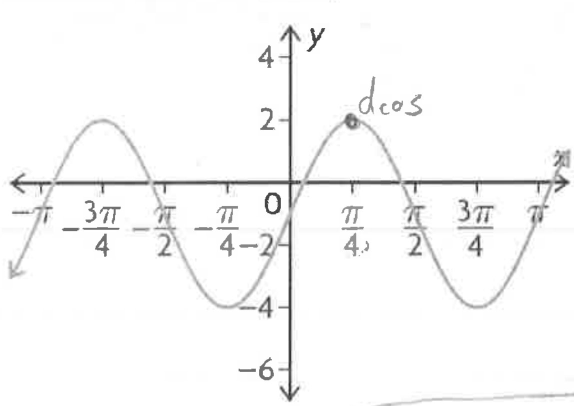
$= \frac{1}{\cos\left(\frac{4\pi}{3}\right)} \times \frac{1}{\tan\left(\frac{5\pi}{6}\right)} - \tan\left(\frac{3\pi}{4}\right)$

$= \frac{1}{-\cos\left(\frac{\pi}{3}\right)} \times \frac{1}{-\tan\left(\frac{\pi}{6}\right)} - \left(-\tan\frac{\pi}{4}\right)$

$= -2 \times -\sqrt{3} + 1$

$= 2\sqrt{3} + \underline{1}$

36) Find two equations (one sine and one cosine) to represent the function on the graph below. Show your calculations for full marks.



$a = \frac{\max - \min}{2} = \frac{2-(-4)}{2} = 3$

$K = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$

$c = \max - |a| = 2 - 3 = -1$

$d_{cos} = \frac{\pi}{4}$

$d_{sin} = d_{cos} - \frac{\pi}{2k} = \frac{\pi}{4} - \frac{\pi}{2(2)} = 0$

$y = 3\cos\left[2\left(x - \frac{\pi}{4}\right)\right] - 1$

$y = 3\sin(2x) - 1$

37) A sine function has a maximum value of 6, a minimum value of -2, a period of $\frac{\pi}{2}$, and a phase shift of $\frac{\pi}{6}$ radians to the right.

Write an equation for the function

$a = \frac{6-(-2)}{2} = 4$ $\quad d_{sin} = \frac{\pi}{6}$

$k = \frac{2\pi}{(\frac{\pi}{2})} = 4$

$c = max - |a| = 6 - 4 = 2$

$\boxed{y = 4\sin\left[4\left(x - \frac{\pi}{6}\right)\right] + 2}$

b) Write an equivalent cosine equation for the function

$d_{cos} = d_{sin} + \frac{\pi}{2k} = \frac{\pi}{6} + \frac{\pi}{2(4)} = \frac{4\pi}{24} + \frac{3\pi}{24} = \frac{7\pi}{24}$

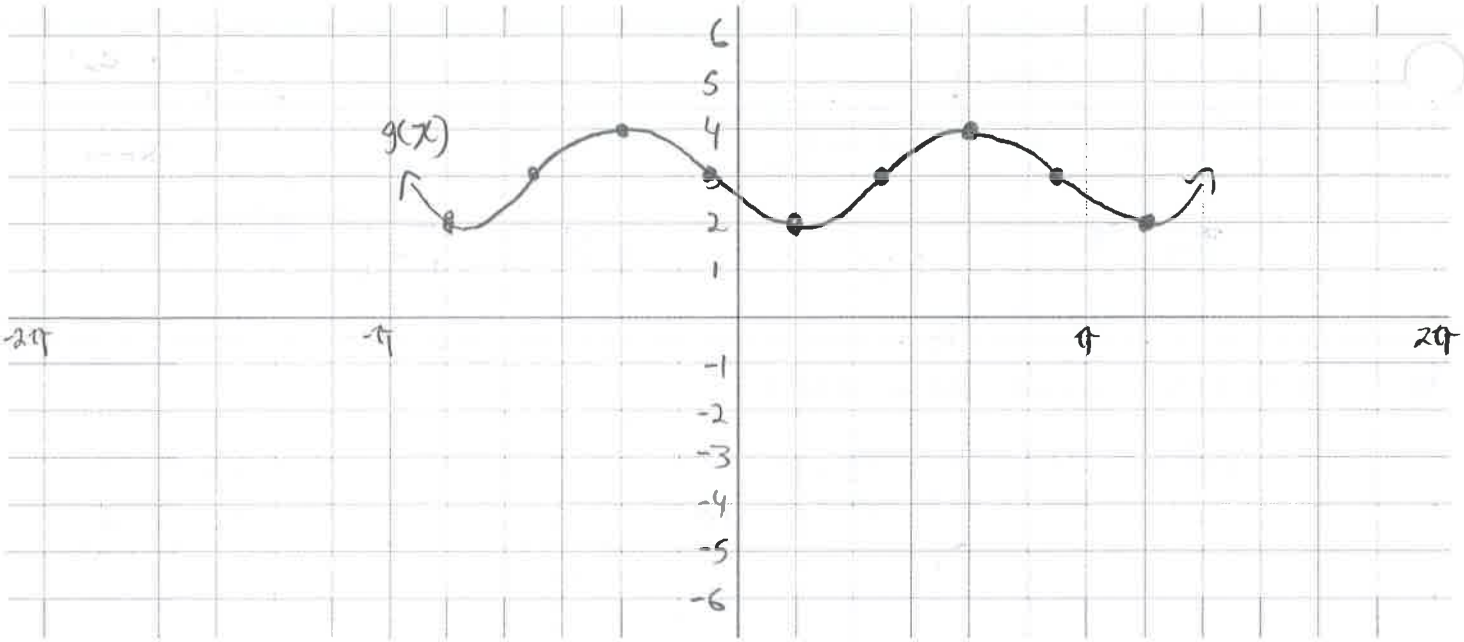
$\boxed{y = 4\cos\left[4\left(x - \frac{7\pi}{24}\right)\right] + 2}$

38) For the function $g(x) = -\cos\left[2\left(x - \frac{\pi}{6}\right)\right] + 3$, fill in the table of information and then graph two cycles of the transformed function using transformations of the parent function. Choose an appropriate scale. [7]

Amplitude: $= \lvert a \rvert = 1$	Period: $= \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$ radians
Phase shift: $\frac{\pi}{6}$ RIGHT	Vertical shift: 3 UP
Max: $= c + \lvert a \rvert = 3 + 1 = 4$	Min: $= c - \lvert a \rvert = 3 - 1 = 2$

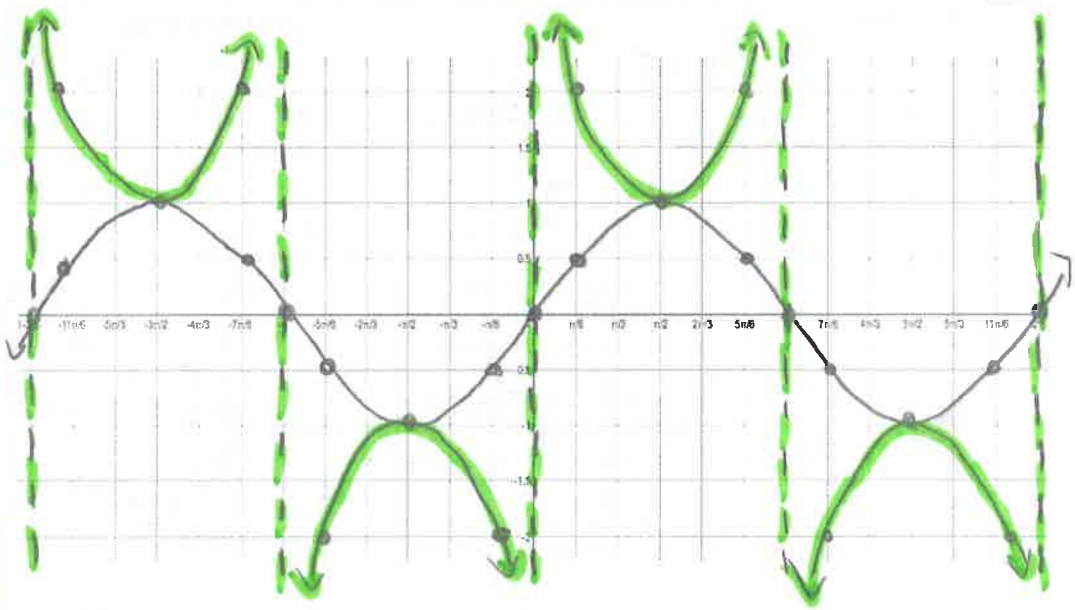
$y = \cos x$	
x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$g(x)$	
$\frac{x}{2} + \frac{\pi}{6}$	$-y + 3$
$\pi/6$	2
$5\pi/12 = \frac{2.5\pi}{6}$	3
$4\pi/6$	4
$11\pi/12 = \frac{5.5\pi}{6}$	3
$7\pi/6$	2



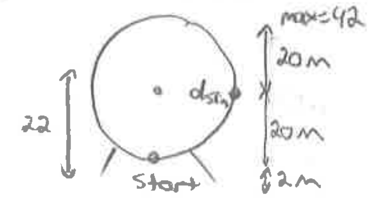
39) Complete the following table of values for the function $f(x) = \sin(x)$ and $g(x) = \csc(x)$. Use special triangles, the unit circle, or a calculator to find values for the function. Then graph both functions on the same grid. Draw asymptotes where necessary.

x	$f(x)$	$g(x)$
0	0	und
$\frac{\pi}{6}$	1/2	2
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2} = 0.87$	$\frac{2}{\sqrt{3}} = 1.15$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	0.87	1.15
$\frac{5\pi}{6}$	1/2	2
$\frac{6\pi}{6} = \pi$	0	und
$\frac{7\pi}{6}$	-1/2	-2
$\frac{8\pi}{6} = \frac{4\pi}{3}$	-0.87	-1.15
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	-0.87	-1.15
$\frac{11\pi}{6}$	-1/2	-2
$\frac{12\pi}{6} = 2\pi$	0	und



40) A Ferris wheel at an amusement park completes one revolution every 60 seconds. The wheel has a radius of 20 meters and its center is 22 meters above the ground. Assume the rider starts at the bottom.

a) Model the rider's height above the ground with a sine function

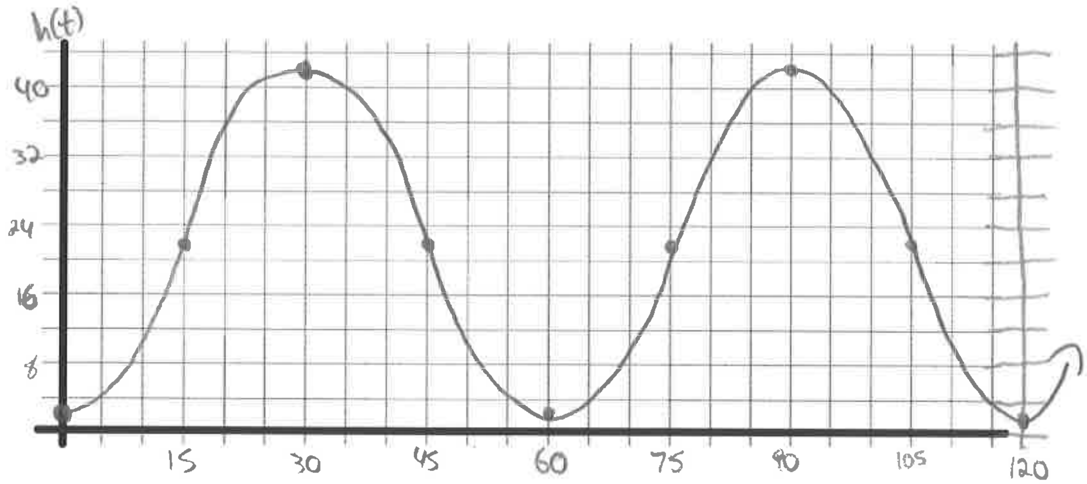


$a = \frac{42-2}{2} = 20$ $\quad d_{sm} = \frac{60}{4} = 15$

$k = \frac{2\pi}{60} = \frac{\pi}{30}$

$c = 42 - 20 = 22$ $\quad \boxed{h(t) = 20\sin\left[\frac{\pi}{30}(t-15)\right] + 22}$

b) Sketch a graph of the rider's height above the ground for 2 cycles.



Unit 5: Trig Identities and Equations

41) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each.

a) $\sin\pi\cos\frac{\pi}{2} + \cos\pi\sin\frac{\pi}{2}$

$= \sin\left(\pi + \frac{\pi}{2}\right)$

$= \sin\left(\frac{3\pi}{2}\right)$

$= -1$

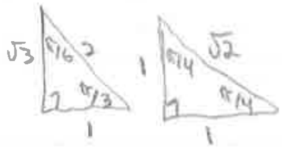
b) $\cos\pi\cos\frac{\pi}{2} + \sin\pi\sin\frac{\pi}{2}$

$= \cos\left(\pi - \frac{\pi}{2}\right)$

$= \cos\left(\frac{\pi}{2}\right)$

$= 0$

42) Use an appropriate compound angle formula to determine an exact value for each.



a) $\cos\frac{\pi}{12}$

$= \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

b) $\sin\frac{11\pi}{12}$

$= \sin\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$= \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \left(-\cos\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

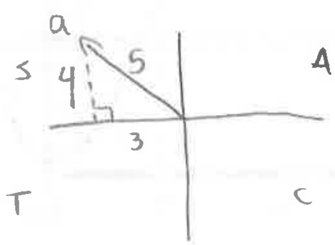
$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

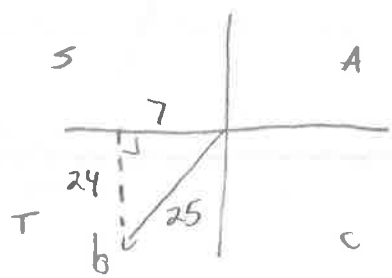
43) Angle a lies in the second quadrant and angle b lies in the third quadrant such that $\cos a = -\frac{3}{5}$ and $\tan b = \frac{24}{7}$. Determine an exact value for

2,214297436

4,428594

diagrams:





a) $\cos(a+b)$

$= \cos a\cos b - \sin a\sin b$

$= \left(\frac{-3}{5}\right)\left(\frac{-7}{25}\right) - \left(\frac{4}{5}\right)\left(\frac{-24}{25}\right)$

$= \frac{21+96}{125}$

$= \frac{117}{125}$

b) $\sin(a-b)$

$= \sin a\cos b - \cos a\sin b$

$= \left(\frac{4}{5}\right)\left(\frac{-7}{25}\right) - \left(\frac{-3}{5}\right)\left(\frac{-24}{25}\right)$

$= \frac{-28-72}{125}$

$= \frac{-100}{125}$

$= -\frac{4}{5}$

c) $\sin(2a)$

$= 2\sin a\cos a$

$= 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)$

$= \frac{-24}{25}$

d) $\cos(2b)$

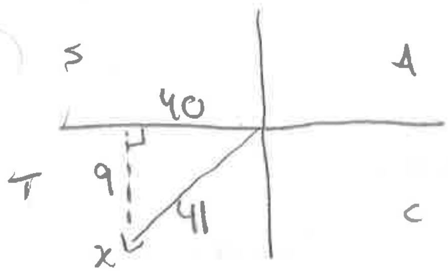
$= \cos^2 b - \sin^2 b$

$= \left(\frac{-7}{25}\right)^2 - \left(\frac{-24}{25}\right)^2$

$= \frac{49-576}{625}$

$= \frac{-527}{625}$

44) Angle x lies in the third quadrant, and $\tan x = \frac{9}{40}$. Determine an exact value for $\cos(2x)$.



$$\cos(2x) = \cos^2 x - \sin^2 x$$
$$= \left(\frac{-40}{41}\right)^2 - \left(\frac{-9}{41}\right)^2$$
$$= \frac{1600 - 81}{1681}$$
$$= \frac{1519}{1681}$$

45) Prove the following identities. Use a separate piece of paper.

a) $\sec x - \tan x = \frac{1-\sin x}{\cos x}$

b) $(\csc x - \cot x)^2 = \frac{1-\cos x}{1+\cos x}$

c) $\sin(2a) = \frac{2\tan a}{\sec^2 a}$

d) $\cos(x+y)\cos(x-y) = \cos^2 x + \cos^2 y - 1$

e) $\frac{\cos(2x)}{1-\sin(2x)} = \frac{\cos x + \sin x}{\cos x - \sin x}$

f) $\sin(2x) = 2\sin x \cos x$

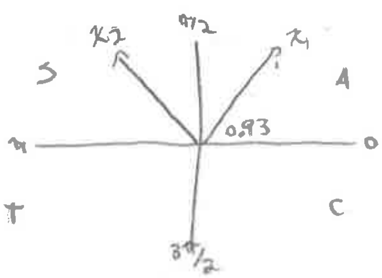
g) $\frac{\tan x - \tan y}{\cot x - \cot y} = -\tan x \tan y$

h) $\frac{2\tan x}{1+\tan^2 x} = \sin(2x)$

) Determine solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible.

a) $\sin x - 0.8 = 0$

$\sin x = 0.8$



$x_1 = \sin^{-1}(0.8)$

$\boxed{x_1 \simeq 0.93}$

$x_2 = \pi - 0.93$

$\boxed{x_2 \simeq 2.21}$

b) $\tan x - \frac{3}{4} = 0$

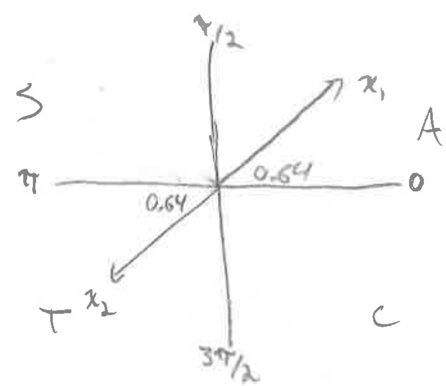
$\tan x = \frac{3}{4}$

$x_1 = \tan^{-1}\left(\frac{3}{4}\right)$

$\boxed{x_1 \simeq 0.64}$

$x_2 = \pi + 0.64$

$\boxed{x_2 \simeq 3.78}$



c) $2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

From Δ; $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

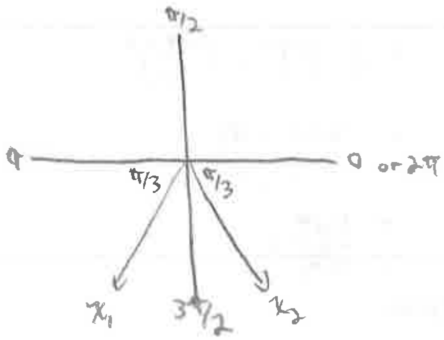
place in Q3 + Q4

$x_1 = \pi + \frac{\pi}{3}$

$\boxed{x_1 = \frac{4\pi}{3}}$

$x_2 = 2\pi - \frac{\pi}{3}$

$\boxed{x_2 = \frac{5\pi}{3}}$



d) $2\sin x\cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0$

From unit circle:

$\boxed{x_1 = \frac{\pi}{2}}$

$\boxed{x_2 = \frac{3\pi}{2}}$

$2\sin x - 1 = 0$

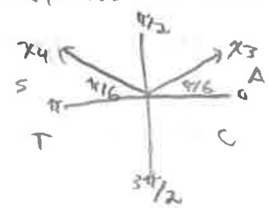
$\sin x = \frac{1}{2}$

From Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

place in Q1 + Q2

$\boxed{x_3 = \frac{\pi}{6}}$

$\boxed{x_4 = \frac{5\pi}{6}}$



e) $\csc^2 x = 2 + \csc x$

$\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$

from part d)

$\boxed{x_1 = \frac{\pi}{6}}$

$\boxed{x_2 = \frac{5\pi}{6}}$

$\csc x = -1$

$\sin x = -1$

from unit circle:

$\boxed{x_3 = \frac{3\pi}{2}}$

g) $64\sin^2 x - 25 = 0$

$\sin^2 x = \frac{25}{64}$

$\sin x = \pm\sqrt{\frac{25}{64}}$

$\sin x = \pm\frac{5}{8}$

$\sin x = \frac{5}{8}$

$\boxed{x_1 \simeq 0.675}$

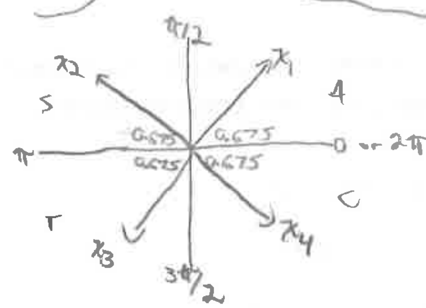
$\boxed{x_2 \simeq 2.47}$

$\sin x = -\frac{5}{8}$

place 0.675 in Q3+Q4

$\boxed{x_3 \simeq 3.82}$

$\boxed{x_4 \simeq 5.61}$



h) $2\csc^2 x - 9\csc x + 10 = 0$

20

$\frac{-2}{1} = \frac{-4}{2}$ $\quad \frac{-5}{2}$

-9

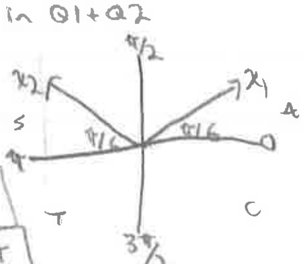
$(\csc x - 2)(2\csc x - 5) = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$

From Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

place in Q1 + Q2



$\boxed{x_1 = \frac{\pi}{6}}$

$\boxed{x_2 = \frac{5\pi}{6}}$

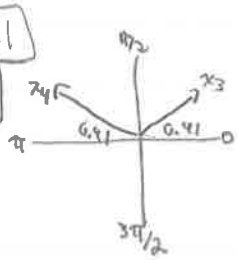
$\csc x = \frac{5}{2}$

$\sin x = \frac{2}{5}$

$x_3 = \sin^{-1}\left(\frac{2}{5}\right)$

$\boxed{x_3 \simeq 0.41}$

$\boxed{x_4 \simeq 2.73}$



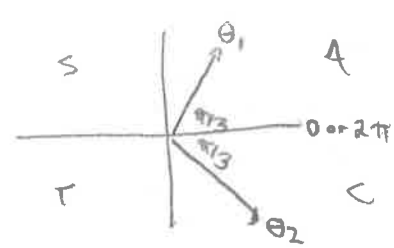
i) $2\cos(2x) = 1$

$\cos(2x) = \frac{1}{2}$ let $\theta = 2x$

$\cos\theta = \frac{1}{2}$

From Δ; $\cos\frac{\pi}{3} = \frac{1}{2}$

Place in Q1 + Q4



$\theta_1 = \frac{\pi}{3}$ $\quad \theta_2 = \frac{5\pi}{3}$

$2x = \frac{\pi}{3}$ $\quad 2x = \frac{5\pi}{3}$

$\boxed{x_1 = \frac{\pi}{6}}$ $\quad \boxed{x_2 = \frac{5\pi}{6}}$

$x_3 = x_1 + \pi$ $\quad x_4 = x_2 + \pi$

$\boxed{x_3 = \frac{7\pi}{6}}$ $\quad \boxed{x_4 = \frac{11\pi}{6}}$

* Add period of π to find other answers

Unit 6: Rational Equations/Inequalities and Rates of Change

47) Solve each equation

a) $\frac{x-2}{3x-1} = \frac{2}{x+1}$

$(x+1)(x-2) = 2(3x-1)$

$x^2 - x - 2 = 6x - 2$

$x^2 - 7x = 0$

$x(x-7) = 0$

$\boxed{x_1 = 0}$ $\quad x - 7 = 0$ $\quad \boxed{x_2 = 7}$

b) $3 = \frac{6}{2x^2-x-4}$

$3(2x^2 - x - 4) = 6$

$6x^2 - 3x - 12 = 6$

$6x^2 - 3x - 18 = 0$

$3(2x^2 - x - 6) = 0$

$(2x^2 - 4x + 3x - 6) = 0$

$[2x(x-2) + 3(x-2)] = 0$

$(x-2)(2x+3) = 0$

$x - 2 = 0$ $\quad 2x + 3 = 0$

$\boxed{x_1 = 2}$ $\quad \boxed{x_2 = -\frac{3}{2}}$

48) Solve each inequality

a) $\frac{2x-3}{x+5} > \frac{2x+7}{x-3}$

$\frac{2x-3}{x+5} - \frac{2x+7}{x-3} > 0$

$\frac{(x-3)(2x-3) - (2x+7)(x+5)}{(x+5)(x-3)} > 0$

$\frac{2x^2 - 3x - 6x + 9 - (2x^2 + 10x + 7x + 35)}{(x+5)(x-3)} > 0$

$\frac{\cancel{2x^2} - 9x + 9 - \cancel{2x^2} - 17x - 35}{(x+5)(x-3)} > 0$

$\frac{-26x - 26}{(x+5)(x-3)} > 0$

$\frac{-26(x+1)}{(x+5)(x-3)} > 0$

x-int: $x = -1$

restrictions: $x \neq -5, 3$

	$-\infty$ to -5: -6	-5 to -1: -2	-1 to 3: 0	3 to ∞: 4
-26	−	−	−	−
$x+1$	−	−	+	+
$x+5$	−	+	+	+
$x-3$	−	−	−	+
overall	+	−	+	−

Solution:

$x < -5$ or $-1 < x < 3$

$x \in (-\infty, -5) \cup (-1, 3)$

b) $\frac{x^2-8x+15}{x^2+5x+4} \leq 0$

$\frac{(x-3)(x-5)}{(x+4)(x+1)} \leq 0$

x-int: $x = 3, 5$

restrictions: $x \neq -4, -1$

	$-\infty$ to -4: -5	-4 to -1: -2	-1 to 3: 0	3 to 5: 4	5 to ∞: 6
$x-3$	−	−	−	+	+
$x-5$	−	−	−	−	+
$x+4$	−	+	+	+	+
$x+1$	−	−	+	+	+
overall	+	−	+	−	+

Solution:

$-4 < x < -1$ or $3 \leq x \leq 5$

$x \in (-4, -1) \cup [3, 5]$

49) The population of a small town, p, is modelled by the function $p(t) = 10\,050 + 225t - 20t^2$, where t is the time in years from now. Determine the average rate of change of the population from

a) year 0 to year 5

$m = \frac{\Delta y}{x - x}$

$= \frac{p(5) - p(0)}{5 - 0}$

$= \frac{10\,675 - 10\,050}{5}$

$\boxed{= 125 \text{ ppl / year}}$

b) year 5 to year 8

$m = \frac{p(8) - p(5)}{8 - 5}$

$= \frac{10570 - 10678}{3}$

$\boxed{= -36 \text{ ppl / year}}$

50) A soccer ball is kicked into the air. The following table of values shows the height of the ball above the ground at various times during its flight:

Time (seconds)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Height (meters)	0.5	11.78	20.6	26.98	30.9	32.38	31.4	27.98	22.1	13.78	3.0

a) Find the average rate of change in the height of the ball over the first 3 seconds.

$m = \frac{h(3) - h(0)}{3 - 0}$

$= \frac{31.4 - 0.5}{3}$

$\boxed{= 10.3 \text{ m / sec}}$

b) Estimate the ***instantaneous*** rate of change of the height at 4 seconds.

Method 1: surrounding

$\left.\frac{dh}{dt}\right|_{t=4} \simeq \frac{h(4.5) - h(3.5)}{4.5 - 3.5}$

$= \frac{13.78 - 27.98}{1}$

$\boxed{= -14.2 \text{ m/sec.}}$

Method 2: Average preceding and following

m for $3.5 \le t \le 4$

$m = \frac{22.1 - 27.98}{4 - 3.5}$

$= -11.76$

m for $4 \le t \le 4.5$

$m = \frac{13.78 - 22.1}{4.5 - 4}$

$= -16.64$

$\left.\frac{dh}{dt}\right|_{t=4} \simeq \frac{-11.76 + (-16.64)}{2}$

$\boxed{= -14.2 \text{ m/sec.}}$

51) Use the chart below to calculate several average rates of change to help you estimate the instantaneous rate of change for the function $f(x) = -2x^2 + x$ at $x = 1$. Have 4-decimal place accuracy in the $\frac{\Delta y}{\Delta x}$ column.

Interval	Change in y = Δy	Δx	$\frac{\Delta y}{\Delta x}$ = Avg. Rate of Change
$0 \le x \le 1$	$f(1) - f(0)$ $= -1 - 0$ $= -1$	$1 - 0$ $= 1$	$= \frac{-1}{1}$ $= -1$
$0.5 \le x \le 1$	$f(1) - f(0.5)$ $= -1 - 0$ $= -1$	$1 - 0.5$ $= 0.5$	$= \frac{-1}{0.5}$ $= -2$
$0.9 \le x \le 1$	$f(1) - f(0.9)$ $= -1 + 0.72$ $= -0.28$	$1 - 0.9$ $= 0.1$	$= \frac{-0.28}{0.1}$ $= -2.8$
$0.99 \le x \le 1$	$f(1) - f(0.99)$ $= -1 + 0.9702$ $= -0.0298$	$1 - 0.99$ $= 0.01$	$= \frac{-0.0298}{0.01}$ $= -2.98$

$$\left.\frac{dy}{dx}\right|_{x=1} \simeq -3$$

) Use the Newton Quotient to find the equation of the derivative for each of the following functions. Also, find the instantaneous rate of change for the function when $x = 4$.

a) $f(x) = x^2 + 6$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 6 - (x^2+6)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 6 - x^2 - 6}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$f'(x) = 2x + 0$$

$$\boxed{f'(x) = 2x}$$

$$f'(4) = 2(4)$$

$$\boxed{f'(4) = 8}$$

b) $f(x) = 2x^2 + 3x - 4$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + 3(x+h) - 4 - (2x^2 + 3x - 4)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + 3x + 3h - 4 - 2x^2 - 3x + 4}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h \cancel{-4} \cancel{-2x^2} \cancel{-3x} \cancel{+4}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{4xh + 2h^2 + 3h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 3)}{\cancel{h}}$$

$$f'(x) = 4x + 2(0) + 3$$

$$\boxed{f'(x) = 4x + 3}$$

$$f'(4) = 4(4) + 3$$

$$\boxed{f'(4) = 19}$$

53) Determine the equation of the tangent line at $x = 3$ for the function $f(x) = 5x^2 - 10x - 7$

$f'(x) = \lim_{h \to 0} \frac{5(x+h)^2 - 10(x+h) - 7 - (5x^2 - 10x - 7)}{h}$

$f'(x) = \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{10x} - 10h - \cancel{7} - \cancel{5x^2} + \cancel{10x} + \cancel{7}}{h}$

$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(10x + 5h - 10)}{\cancel{h}}$

$f'(x) = 10x + 5(0) - 10$

$f'(x) = 10x - 10$

$f'(3) = 10(3) - 10$
$= 20$

∴ slope of tangent is 20.

$f(3) = 5(3)^2 - 10(3) - 7$
$= 8$

∴ (3, 8) is on tangent line

$y = mx + b$
$8 = 20(3) + b$
$b = -52$

$\boxed{y = 20x - 52}$

54) Calculate the following limits:

a) $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$

$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}}$

$= 5 + 5$

$\boxed{= 10}$

b) $\lim_{x \to 3} \frac{2x^2 - 5x - 3}{x - 5}$

$= \frac{2(3)^2 - 5(3) - 3}{3 - 5}$

$= \frac{0}{-2}$

$\boxed{= 0}$

55) Calculate the following limits using the graph provided.

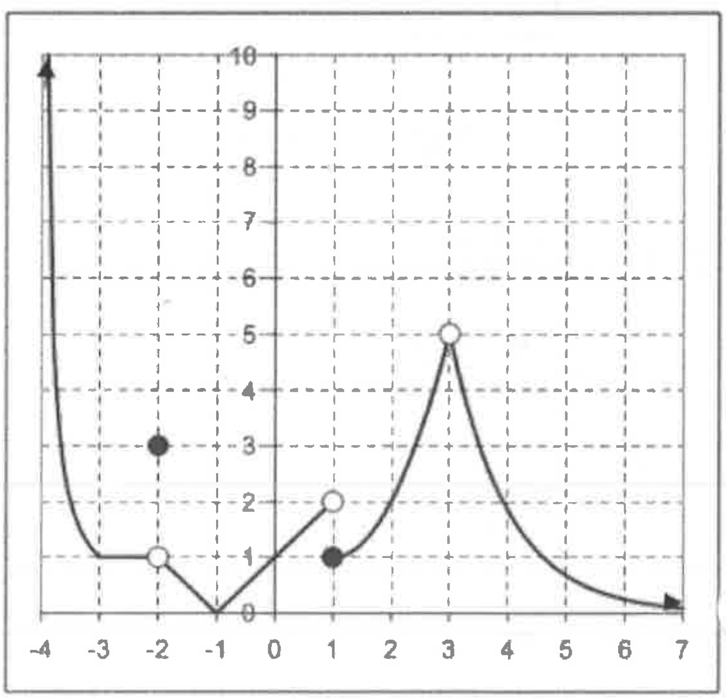
a) $\lim_{x \to \infty} f(x) = 0$

b) $\lim_{x \to -4^+} f(x) = \infty$

c) $\lim_{x \to 1^+} f(x) = 1$

d) $\lim_{x \to 1^-} f(x) = 2$

e) $\lim_{x \to 1} f(x)$ Does Not Exist



Unit 7: Graphing

56) Graph $f(x)=\dfrac{4}{x-2}$ by first graphing the denominator, then graphing it's reciprocal. Don't forget about the -on top! Label any asymptotes

HA: $y=0$

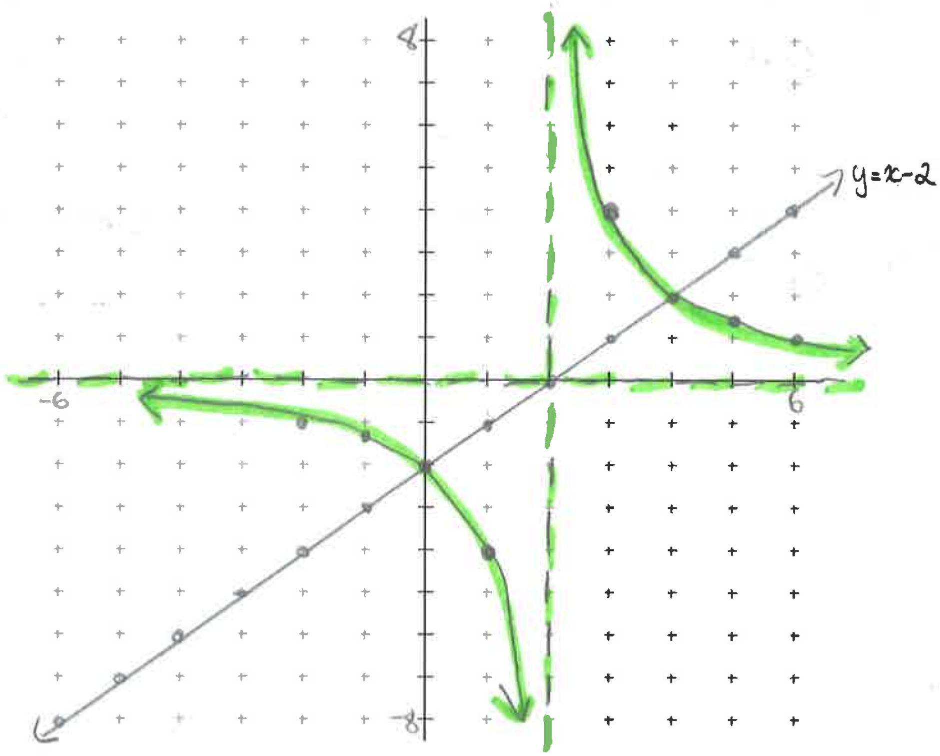
VA: $x=2$

$y=x-2$

x	y
-1	-3
0	-2
1	-1
2	0
3	1
4	2
5	3

$f(x)=\dfrac{4}{x-2}$

x	$\frac{4}{y}$
-1	-1.33
0	-2
1	-4
2	undefined
3	4
4	2
5	1.33



57) Graph $g(x)=\frac{1}{x^2-4}$ by first graphing the denominator, then graph it's reciprocal. Label any asymptotes.

$g(x)=\dfrac{1}{(x-2)(x+2)}$

HA: $y=0$

VA: $x=-2$ and $x=2$

$y=x^2-4$

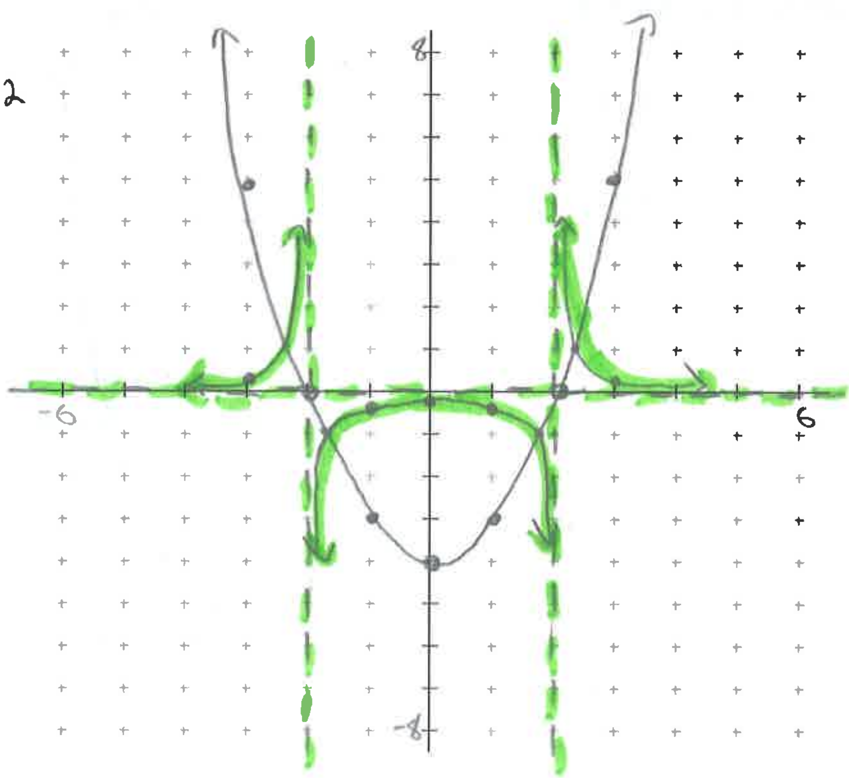
vertex at $(0,-4)$

x-int at $(-2,0)$ and $(2,0)$

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

$g(x)=\dfrac{1}{x^2-4}$

x	$\frac{1}{y}$
-3	0.2
-2	und
-1	-0.33
0	-0.25
1	-0.33
2	und
3	0.2



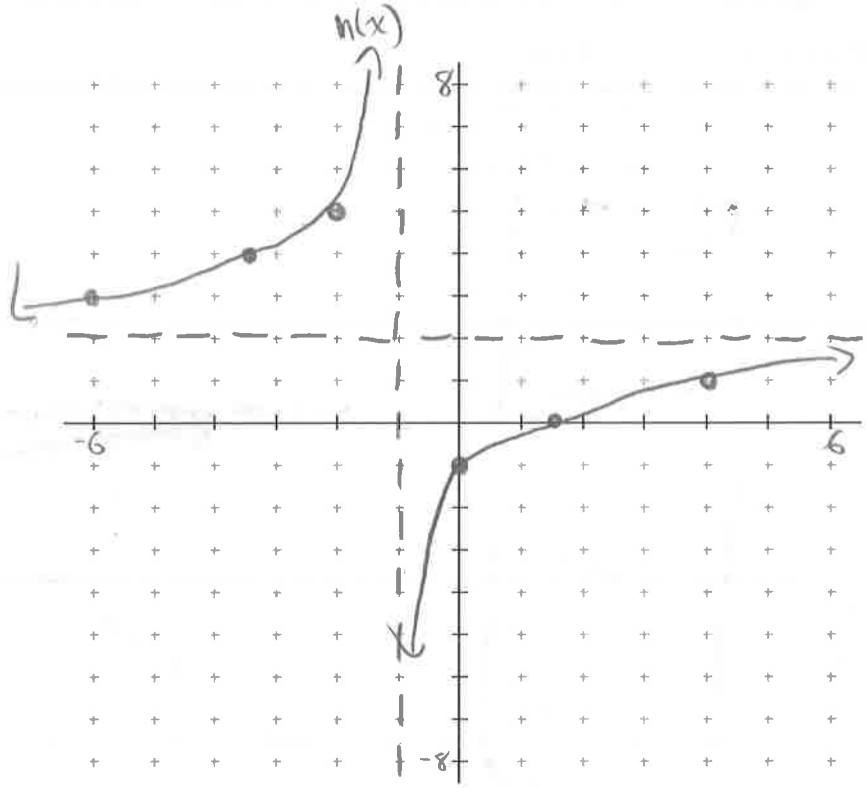
58) Graph the function $h(x) = \frac{2x-3}{x+1}$. Clearly state the key features of this graph including asymptotes and intercepts.

VA : $x = -1$
HA : $y = 2$

x-int: $0 = 2x - 3$
$x = 1.5$
$(1.5, 0)$

y-int: $h(0) = \frac{-3}{1}$
$(0, -3)$

other point:
$h(4) = \frac{2(4)-3}{4+1}$
$= 1$
$(4, 1)$



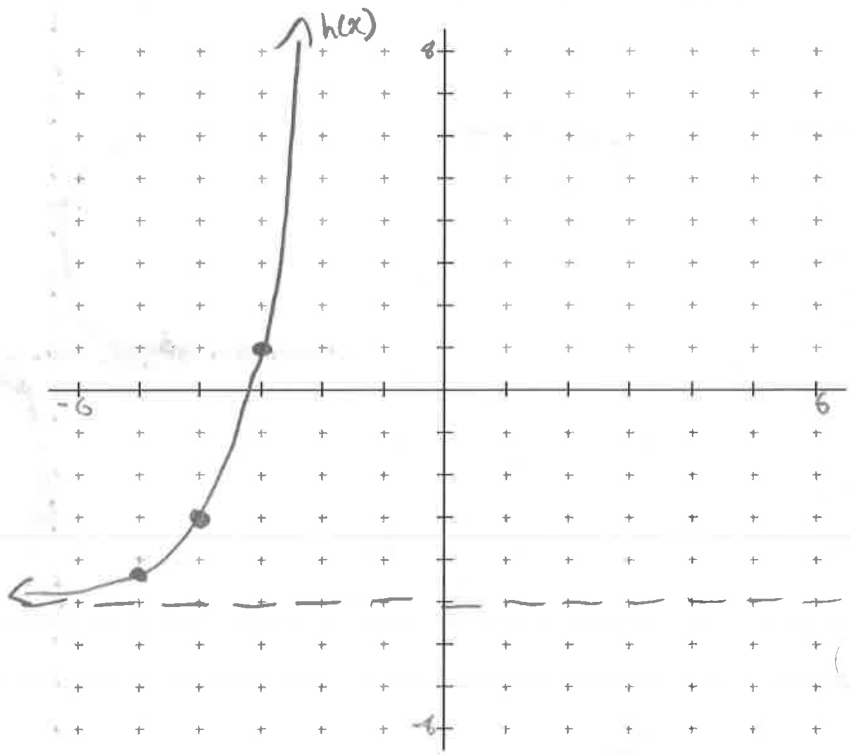
59) Graph the transformed function $h(x) = 2(3)^{x+4} - 5$ using transformations.

$y = 3^x$

x	y
-1	0.33
0	1
1	3
HA	$y=0$

$h(x)$

$x-4$	$2y-5$
-5	-4.33
-4	-3
-3	1
HA	$y=-5$



60) Evaluate each of the following, correct to 3 decimal places.

a) e^5

$\simeq 148.413$

b) $\ln 200$

$\simeq 5.298$

61) Simplify the following expression $\ln e^{2x}$

$= 2x \ln(e)$

$= 2x\ (1)$

$= 2x$

62) Solve the following equations, correct to 3 decimal places.

a) $e^x = 5$

$x = \ln(5)$

$x \simeq 1.609$

b) $1000 = 20e^{\frac{x}{4}}$

$50 = e^{x/4}$

$\frac{x}{4} = \ln(50)$

$x = 4\ln(50) \simeq 15.648$

63) Given that $g(x) = x + 5$ and $f(x) = x^2 + 4x - 1$, find a formula for the following…

a) $f(g(x))$

$= f(x+5)$

$= (x+5)^2 + 4(x+5) - 1$

$= x^2 + 10x + 25 + 4x + 20 - 1$

$= x^2 + 14x + 44$

b) $g(f(x))$

$= g(x^2 + 4x - 1)$

$= x^2 + 4x - 1 + 5$

$= x^2 + 4x + 4$

c) $g^{-1}(f(x))$

$g^{-1}(x)$

$x = y + 5$

$y = x - 5$

$g^{-1}(x) = x - 5$

$g^{-1}(f(x))$

$= g^{-1}(x^2 + 4x - 1)$

$= x^2 + 4x - 1 - 5$

$= x^2 + 4x - 6$

d) $f(x) - g(x)$

$= x^2 + 4x - 1 - (x + 5)$

$= x^2 + 4x - 1 - x - 5$

$= x^2 + 3x - 6$

45) a)

LS

$= \sec x - \tan x$

$= \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$= \frac{1 - \sin x}{\cos x}$

RS

$= \frac{1 - \sin x}{\cos x}$

LS = RS

b)

LS

$= (\csc x - \cot x)^2$

$= \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)^2$

$= \left(\frac{1 - \cos x}{\sin x}\right)^2$

$= \frac{(1 - \cos x)^2}{\sin^2 x}$

RS

$= \frac{1 - \cos x}{1 + \cos x} \frac{(1 - \cos x)}{(1 - \cos x)}$

$= \frac{(1 - \cos x)^2}{1 - \cos^2 x}$

$= \frac{(1 - \cos x)^2}{\sin^2 x}$

LS = RS

c)

LS

$= \sin(2a)$

$= 2 \sin a \cos a$

RS

$= \frac{2 \tan a}{\sec^2 a}$

$= \frac{\left(\frac{2 \sin a}{\cos a}\right)}{\left(\frac{1}{\cos^2 a}\right)}$

$= \frac{2 \sin a}{\cancel{\cos a}} \times \frac{\cos^{\cancel{2}} a}{1}$

$= 2 \sin a \cos a$

LS = RS

d)

LS | RS

$= \cos(x+y)\cos(x-y)$

$= (\cos x\cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= (\cos x \cos y)^2 - (\sin x \sin y)^2$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x \cdot \cos^2 y - (1-\cos^2 x)(1-\cos^2 y)$

$= \cos^2 x \cos^2 y - (1 - \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y)$

$= \cos^2 x \cos^2 y - 1 + \cos^2 y + \cos^2 x - \cos^2 x \cos^2 y$

$= \cos^2 x + \cos^2 y - 1$

RS:

$= \cos^2 x + \cos^2 y - 1$

LS = RS

e)

LS	RS
$= \dfrac{\cos(2x)}{1-\sin(2x)}$	$= \dfrac{\cos x + \sin x}{\cos x - \sin x} \dfrac{(\cos x - \sin x)}{(\cos x - \sin x)}$
$= \dfrac{\cos^2 x - \sin^2 x}{1 - 2\sin x \cos x}$	$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x - 2\cos x \sin x + \sin^2 x}$
	$= \dfrac{\cos^2 x - \sin^2 x}{1 - 2\cos x \sin x}$

LS = RS

p)

LS	RS
$= \sin(2x)$	$= 2\sin x \cos x$
$= \sin(x+x)$	
$= \sin x \cos x + \cos x \sin x$	
$= 2\sin x \cos x$	

LS = RS

9) LS | RS

LS:

$$= \frac{\tan x - \tan y}{\cot x - \cot y}$$

$$= \frac{\tan x - \tan y}{\left(\frac{1}{\tan x} - \frac{1}{\tan y}\right)}$$

$$= \frac{\tan x - \tan y}{\left(\frac{\tan y}{\tan x \tan y} - \frac{\tan x}{\tan x \tan y}\right)}$$

$$= \frac{\tan x - \tan y}{\left(\frac{\tan y - \tan x}{\tan x \tan y}\right)}$$

$$= \frac{\tan x - \tan y}{1} \times \frac{\tan x \tan y \ (-1)}{\tan y - \tan x \ (-1)}$$

$$= \frac{\cancel{\tan x - \tan y}}{1} \times \frac{-\tan x \tan y}{\cancel{\tan x - \tan y}}$$

$$= -\tan x \tan y$$

RS:

$$= -\tan x \tan y$$

LS = RS

h)

LS

$$= \frac{2\tan x}{1+\tan^2 x}$$

$$= \frac{2\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}\right)}$$

$$= \frac{\left(\frac{2\sin x}{\cos x}\right)}{\left(\frac{1}{\cos^2 x}\right)}$$

$$= \frac{2\sin x}{\cancel{\cos x}} \times \frac{\cos^{\cancel{2}\,1} x}{1}$$

$$= 2\sin x\cos x$$

RS

$$= \sin(2x)$$

$$= 2\sin x\cos x$$

LS=RS