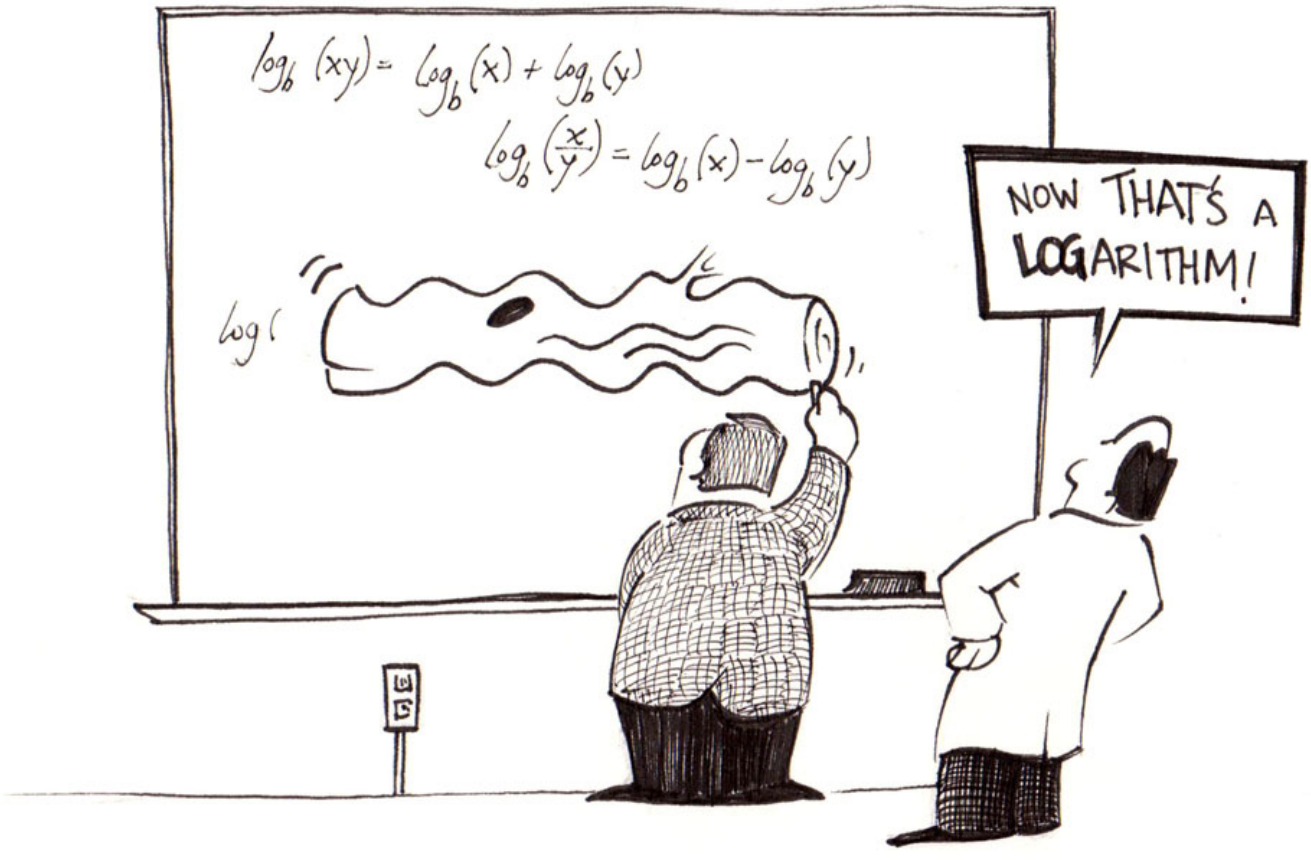
UNIT 3

Chapter 6/7- Logarithmic and Exponential Functions

WORKBOOK

MHF4U



W1 – 6.1/6.2 – Intro to Logarithms and Review of Exponentials
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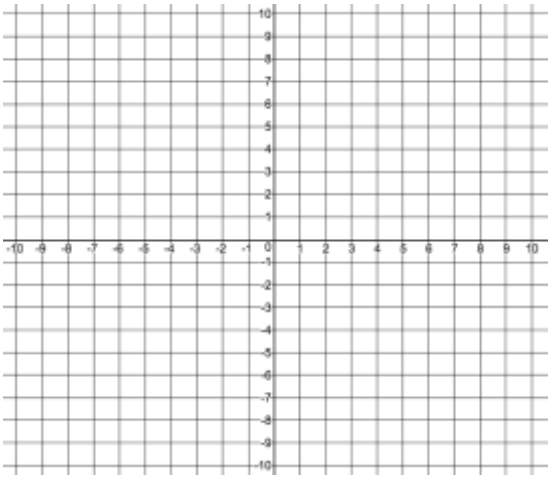
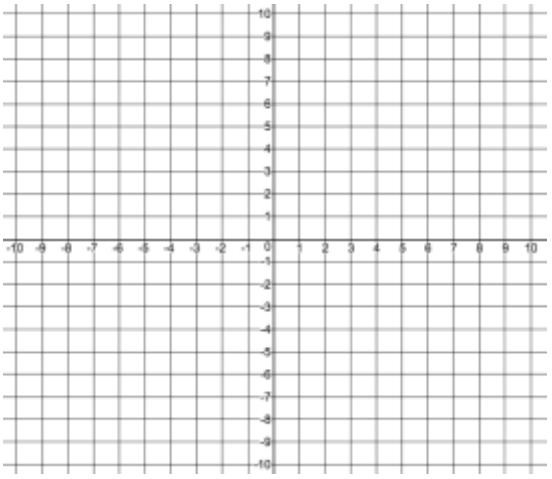
1) Sketch a graph of each function. Then, sketch a graph of the inverse of each function. Label each graph with its equation.

a) $y = 2^x$

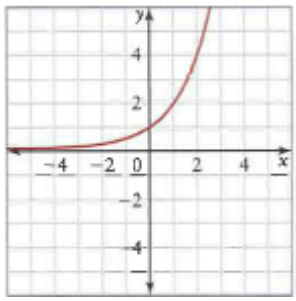
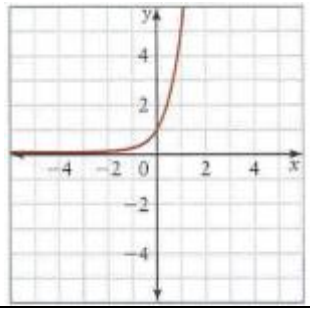
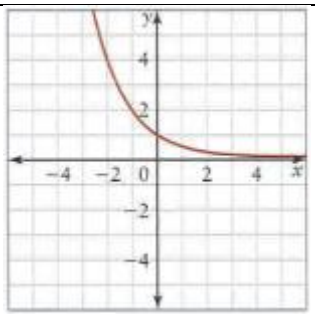
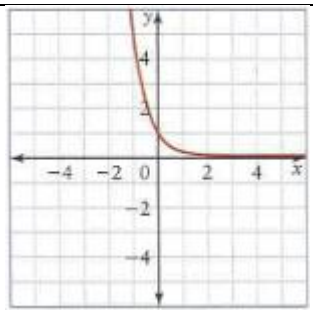


b) $y = 4^x$



2) Match each equation to its corresponding graph.

A) $y = 5^x$ **B)** $y = \left(\frac{1}{2}\right)^x$ **C)** $y = 2^x$ **D)** $y = \left(\frac{1}{5}\right)^x$

3) An influenza virus is spreading according to the function $N = 10(2)^t$, where N is the number of people infected and t is the time, in days.

a) How many people have the virus at each time?

i) initially, when $t = 0$

ii) after 1 day

iii) after 2 days

iv) after 3 days

b) After how many days will 40960 people be infected?

4) Rewrite each equation in logarithmic form

a) $4^3 = 64$

b) $128 = 2^7$

c) $5^{-2} = \frac{1}{25}$

d) $\left(\frac{1}{2}\right)^2 = 0.25$

e) $6^x = y$

f) $10^5 = 100\,000$

g) $\frac{1}{27} = 3^{-3}$

5) Evaluate each logarithm

a) $\log_2 64$ **b)** $\log_3 27$ **c)** $\log_2 \left(\frac{1}{4}\right)$

d) $\log_4 \left(\frac{1}{64}\right)$ **e)** $\log_5 125$ **f)** $\log_2 1024$

6) Evaluate each common logarithm

a) $\log 1000$ **b)** $\log \left(\frac{1}{10}\right)$ **c)** $\log 1$

d) $\log 0.001$ **e)** $\log 10^{-4}$ **f)** $\log 1\ 000\ 000$

7) Rewrite in exponential form

a) $\log_7 49 = 2$ **b)** $5 = \log_2 32$ **c)** $\log 10\ 000 = 4$

d) $w = \log_b z$ **e)** $\log_2 8 = 3$ **f)** $-2 = \log \left(\frac{1}{100}\right)$

W2 – 6.4 – Power Law of Logarithms
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1) Evaluate.

a) $\log_2 16^3$ **b)** $\log_4 8^2$ **c)** $\log 100^{-4}$ **d)** $\log 0.1^{\frac{1}{2}}$

e) $\log_2 \sqrt{8}$ **f)** $\log_3 \left(\sqrt[3]{81}\right)^6$

2) Solve for t to two decimal places.

a) $10 = 4^t$ **b)** $5^t = 250$ **c)** $2 = 1.08^t$ **d)** $500 = 100(1.06)^t$

3) An investment earns 7% interest, compounded annually. The amount, A, that the investment is worth as a function of time, t, in years, is given by $A(t) = 500(1.07)^t$.

a) Use the equation to determine the value of the investment after 4 years.

b) How long will it take for the investment to double in value?

4) Use the change of base formula to evaluate each of the following. Round to 3 decimal places.

a) $\log_3 23$ **b)** $\log_6 20$ **c)** $-\log_{12} 4$ **d)** $\log_{\frac{1}{2}} 30$

5) Write each as a single logarithm

a) $\frac{\log 8}{\log 5}$ **b)** $\frac{\log 17}{\log 9}$ **c)** $\frac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{2}{3}\right)}$ **d)** $\frac{\log(x+1)}{\log(x-1)}$

6)a) Evaluate $\log_2 8^5$ without using the power law of logarithms.

b) Evaluate the same expression by applying the power law of logarithms.

c) Which method do you prefer?

7) Solve for x, correct to 3 decimal places.

a) $2 = \log 3^x$ **b)** $100 = 10 \log 1000^x$ **c)** $4 = \log_3 15^x$

W3 – 7.3 – Product and Quotient Laws of Logarithms
MHF4U

1) Simplify using laws of logarithms and then evaluate.

a) $\log 9 + \log 6$ **b)** $\log 48 - \log 6$ **c)** $\log_3 7 + \log_3 3$

2) Simplify each algebraic expression.

a) $\log x + \log y + \log(2z)$ **b)** $\log_2 a + \log_2(3b) - \log_2(2c)$ **c)** $2\log m + 3\log n - 4\log y$

3) Evaluate using the product law of logarithms.

a) $\log_6 18 + \log_6 2$ **b)** $\log 40 + \log 2.5$ **c)** $\log_{12} 8 + \log_{12} 2 + \log_{12} 9$

4) Evaluate using the quotient law of logarithms.

a) $\log_3 54 - \log_3 2$ **b)** $\log 50\,000 - \log 5$ **c)** $\log_4 320 - \log_4 5$

5) Evaluate, using the laws of logarithms

a) $3\log_{16} 2 + 2\log_{16} 8 - \log_{16} 2$

b) $\log 20 + \log 2 + \frac{1}{3}\log 125$

6) Write as a sum or difference of logarithms. Simplify, if possible.

a) $\log_7(cd)$ **b)** $\log_3\left(\frac{m}{n}\right)$ **c)** $\log(uv^3)$ **d)** $\log\left(\frac{a\sqrt{b}}{c^2}\right)$ **e)** $\log_2 10$

7) Simplify

a) $\log\left(\frac{x^2}{\sqrt{x}}\right)$

b) $\log\sqrt{k} + \log\left(\sqrt{k}\right)^3 + \log\sqrt[3]{k^2}$

c) $\log(x^2 - 4) - \log(x - 2)$

d) $\log(x^2 - x - 6) - \log(2x - 6)$

W4 – 7.1/7.2 – Solving Exponential Equations
MHF4U

1) Write each expression with base 2.

a) 4^6 **b)** 8^3 **c)** $\left(\frac{1}{8}\right)^2$ **d)** 14

2) Write each expression as a power of 4.

a) $\left(\sqrt{16}\right)^3$ **b)** $\sqrt[3]{16}$ **c)** $\sqrt{64} \times \left(\sqrt[4]{128}\right)^3$

3) Solve each equation

a) $2^{4x} = 4^{x+3}$ **b)** $3^{w+1} = 9^{w-1}$ **c)** $4^{3x} = 8^{x-3}$ **d)** $125^{2y-1} = 25^{y+4}$

4) Consider the equation $10^{2x} = 100^{2x-5}$

a) Solve this equation by expressing both sides as powers of a common base.

b) Solve the same equation by taking the common logarithm of both sides.

5) Solve $2^{3x} > 4^{x+1}$

6) Solve for t. Round answers to 2 decimal places.

a) $2 = 1.07^t$ **b)** $100 = 10(1.04)^t$ **c)** $15 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

7) Solve each equation. Round answers to 3 decimal places.

a) $2^x = 3^{x-1}$ **b)** $5^{x-2} = 4^x$ **c)** $7^{2x+1} = 4^{x-2}$

8) Solve $2^{2x} + 2^x - 6 = 0$ using the quadratic formula (or by factoring). Clearly identify any extraneous roots.

9) Solve $8^{2x} - 2(8^x) - 5 = 0$ using the quadratic formula. Clearly identify any extraneous roots.

10) Use the decay equation for polonium-218, $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{3.1}}$, A is the amount remaining after t minutes and A_0 is the initial amount.

a) How much will remain after 90 seconds from an initial sample of 50 mg?

b) How long will it take for this sample to decay to 10% of its initial amount of 50 mg?

11) A 20-mg sample of thorium-233 decays to 17 mg after 5 minutes.

a) What is the half-life of thorium-233?

b) How long will it take this sample to decay to 1 mg?

W5 – 7.4 – Solving Logarithmic Equations
MHF4U

1) Find the roots of each equation

a) $2 = \log(x + 25)$ **b)** $1 - \log(w - 7) = 0$ **c)** $6 - 3\log(2n) = 0$

2) Solve

a) $5 = \log_2(2x - 10)$ **b)** $9 = \log_5(x + 100) + 6$ **c)** $\log_3(n^2 - 3n + 5) = 2$

3) Solve. Make sure to reject any extraneous roots.

a) $\log x + \log(x - 4) = 1$ **b)** $\log x^3 - \log 2 = \log(2x^2)$

c) $\log(v-1) = 2 + \log(v-16)$

d) $\log(k+2) + \log(k-1) = 1$

4) Solve. Check for extraneous roots.

a) $\log\sqrt{x^2-3x} = \frac{1}{2}$

b) $\log\sqrt{x^2+48x} = 1$

5) Solve. Check for extraneous roots.

a) $\log_2(x+5) - \log_2(2x) = 8$

b) $\log(2k+4) = 1 + \log k$

W6 – 6.5 – Applications of Logarithms AND Exponentials in Physical Sciences
MHF4U

Exponential Formulas

$A(t) = A_0(1+i)^t$

general, where i is percent growth(+) or decay(-)

$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$

half-life, H is the half-life period

$A(t) = A_0(2)^{\frac{t}{D}}$

doubling, D is the doubling period

Logarithmic Formulas

$pH = -\log[H^+]$

Where pH is acidity and [H+] is concentration of hydronium ions mol/L

$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$

Where β is loudness in dB and I is intensity of sound in W/m^2

$M = \log(\frac{I}{I_0})$

Where M is magnitude measure by richters, I is intensity

1) The half-life of a radioactive form of tritium is about 2 years. How much of a 5-kg sample of this material would remain after …

a) 8 years

b) 12 months

2) The population of Littleton is currently (2014) 23000, and is increasing exponentially with a growth rate of 2% per year. Estimate when Littleton will have a population of 30000.

3) The population of purple martins in Algonquin park was estimated to be 35000 in 1992. Ten years later, in 2002, the population had risen to 44400.

a) What is the annual growth rate for the purple martin population?

b) Estimate the population for 2010 to the nearest hundred.

4) After an accident at a nuclear plant, the radiation level in the plant was 950 R (roentgens). Five hours later the level was 800 R. How long will it take before safe levels of radiation are reached, which is less than 0.01 R?

5) The value of a new minivan drops 40% after the first year, and then decreases exponentially at a rate of 12% per year after that. When will a minivan that cost $35000 new be worth less than $10000?

6) A crab fossil contains 38.6% of its original Carbon14 isotope, which has a half life of 5370 years. Approximately how old is the crab fossil?

7) A Trimark mutual fund has track record of 4.2% growth per year. What is the doubling period for this investment?

8) A treatment to help a patient stop smoking involves chewing nicotine gum. Each gum introduces 1.5 mg of nicotine into the patient's system. Nicotine has a half life of 3 hours. The patient will feel the urge to smoke when the level of nicotine drops below 0.45 mg in her system. If she chewed a gum at 8:00 a.m., and another at 10:00 a.m., at what time will she next feel the urge to smoke?

9) Determine the pH of a solution with hydronium ion concentration:
a) 0.01 **b)** 1.5×10^{-10}

10) Determine the hydronium concentration, in moles per litre, of a solution with pH:
a) 8.5 **b)** 3

11) How many times as intense is the sound of a shout as the sound of a whisper?

12) A loud car stereo has a decibel level of 110 dB. How many times as intense as the sound of a loud car stereo is the sound of a rock concert speaker?

13) The sound intensity of a pin drop is about 1/30 000 of the sound intensity of a normal conversation. What is the decibel level of a pin drop?

14) On September 26, 2001, an earthquake in North Bay, Ontario, occurred that was 10 000 times as intense as I_0. What was the measure of this earthquake on the Richter scale?

15) On February 10, 2000, an earthquake happen in Welland, Ontario, that measured 2.3 on the Richter scale.

a) How many times as intense was this as a standard low-level earthquake?

b) On July 22, 2001, an earthquake in St. Catharines measured 1.1 on the Richter scale. How many times as intense as the St.Catharines earthquake was the Welland earthquake?

16) The stellar magnitude scale compares the brightness of stars using the equation $m_2 - m_1 = \log(\frac{b_1}{b_2})$, where $m_2\ and\ m_1$ are the apparent magnitude of the two stars being compared (how bright they appear in the sky) and b_1 and b_2 are their brightness (how much light they actually emit). This relationship does not factor in how far from Earth the stars are.

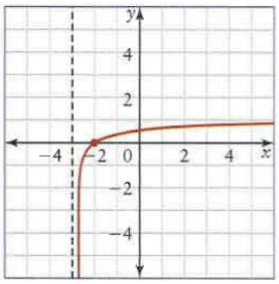
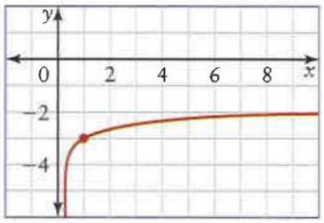
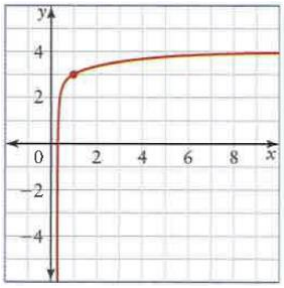
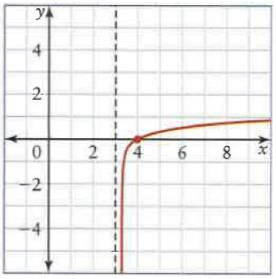
a) Sirius is the brightest-appearing star in our sky, with an apparent magnitude of -1.5. How much brighter does Sirius appear than Betelgeuse, whose apparent magnitude in 0.12?

b) The Sun appears about 1.3×10^{10} times as brightly in our sky as does Sirius. What is the apparent magnitude of the Sun?

W7 – 6.3 Transformations of Exponential and Logarithmic Functions
MHF4U

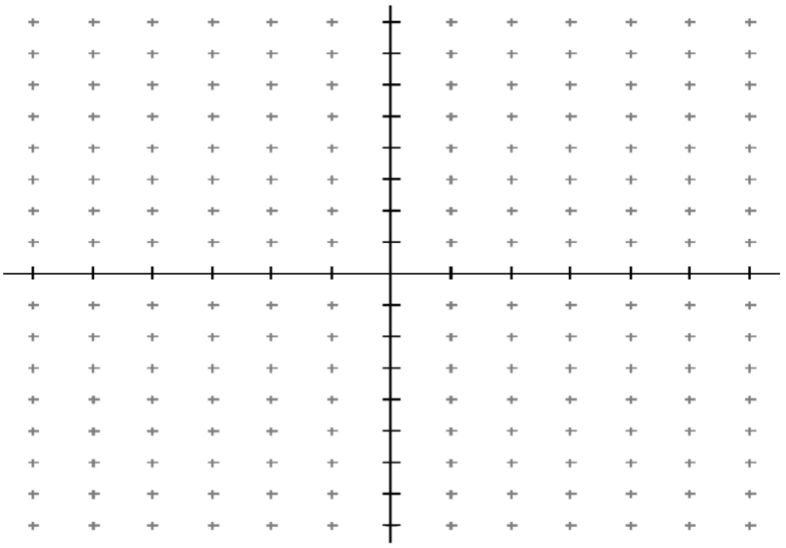
1) Write the letter of the equation under the corresponding graph

A) $y = \log(x - 3)$ **B)** $y = \log x - 3$ **C)** $y = \log(x + 3)$ **D)** $y = \log x + 3$

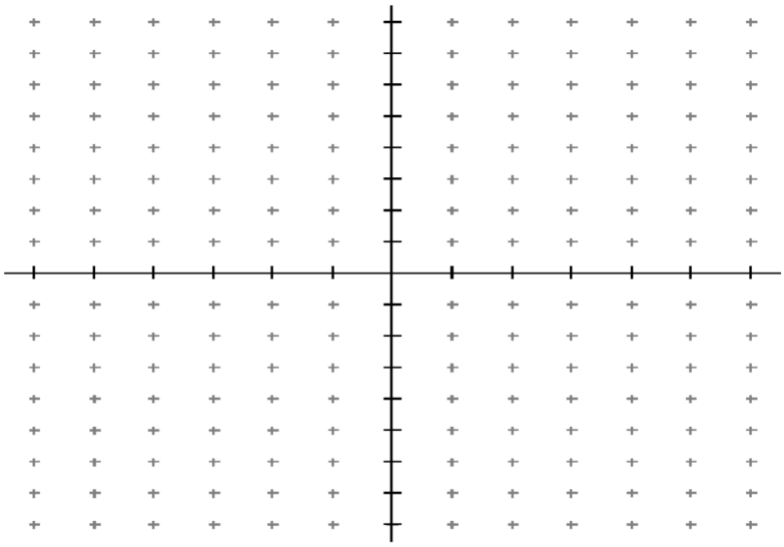
			

2) Sketch a graph of each of the following logarithmic functions by applying transformations to the parent function. Make sure to identify key points such as asymptotes and x-intercepts.

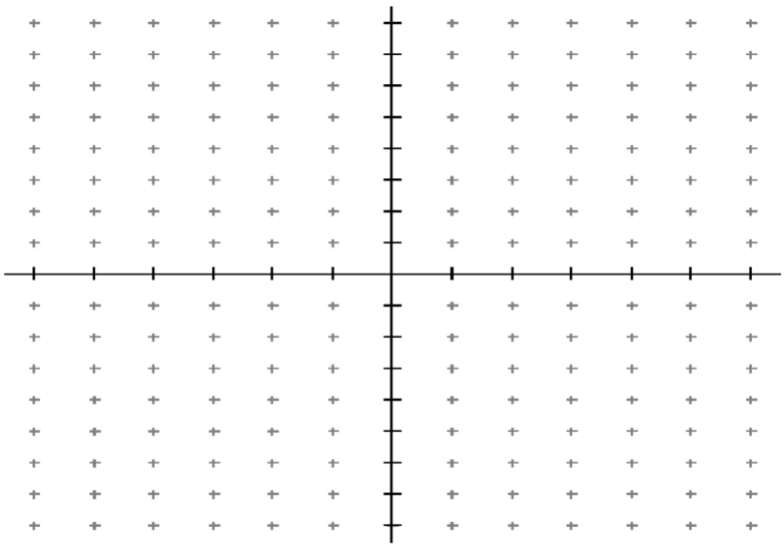
a) $f(x) = -2\log_2 x - 1$



b) $g(x) = \log_4(x - 1) + 4$

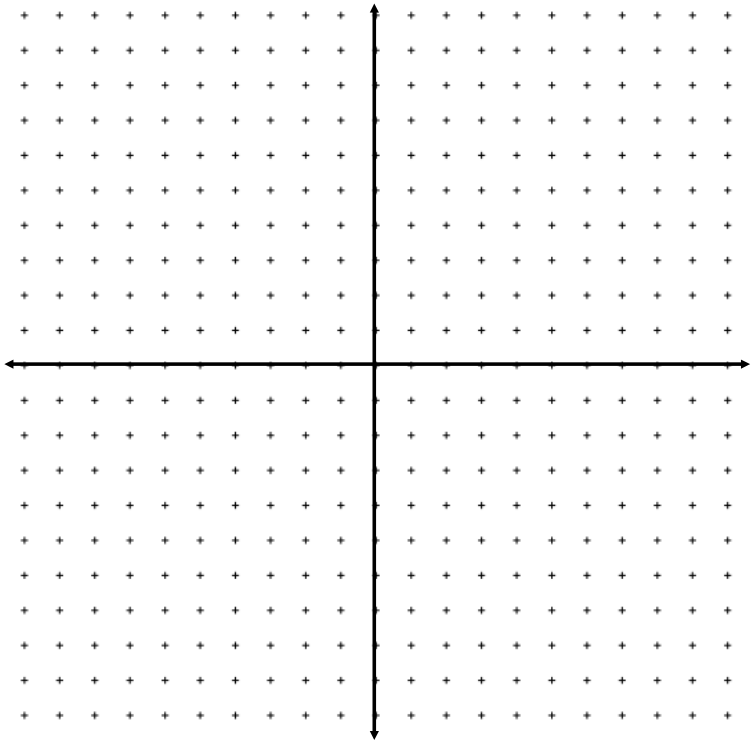


c) $h(x) = 4\log_3\left[\frac{1}{2}(x + 2)\right] - 3$

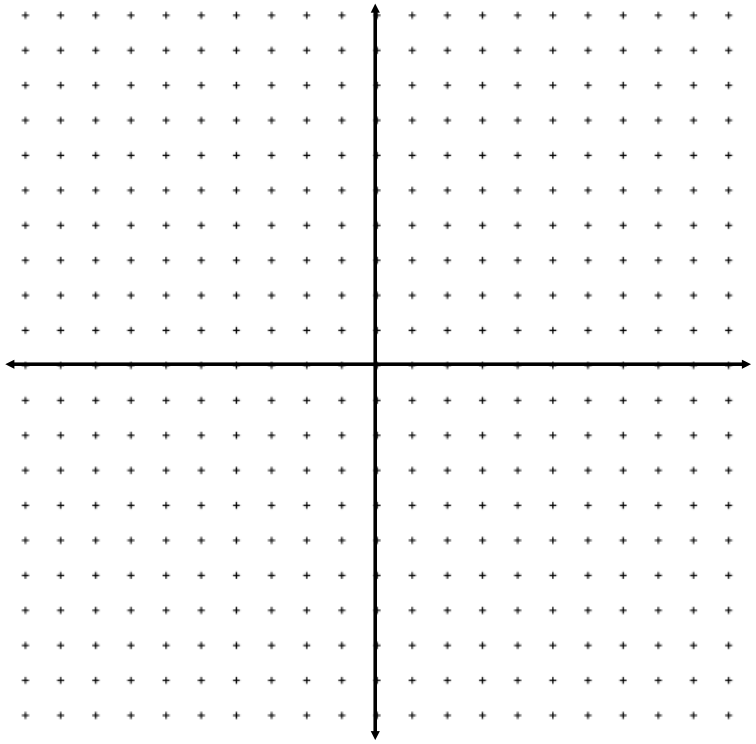


3) Sketch a graph of each of the following exponential functions by applying transformations to the parent function. Make sure to identify key points such as asymptotes and y-intercepts.

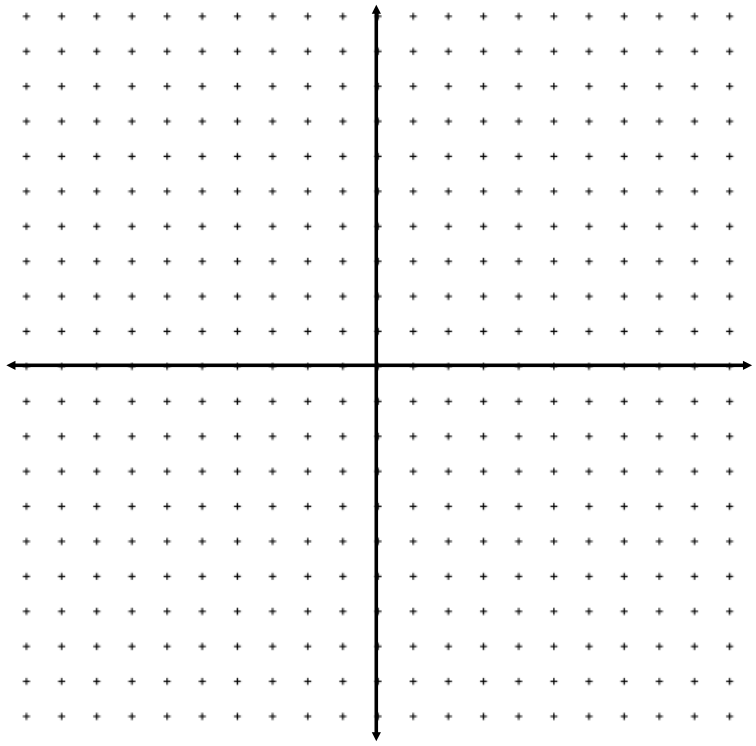
a) $f(x) = -3(2)^x + 1$



b) $g(x) = 3^{x-3} - 4$



c) $h(x) = 2(4)^{\frac{1}{2}(x+1)} - 3$



W8 – The Natural Logarithm
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1) Use a calculator to approximate each to the nearest thousandth

a) $\ln 6.2$ **b)** $\ln 2.1$ **c)** $\ln e$ **d)** e^5

2) Expand each logarithm

a) $\ln x^2$ **b)** $\ln \sqrt[3]{x}$ **c)** $\ln \frac{u^3}{wv^4}$

3) Condense each expression to a single logarithm

a) $4 \ln 2$ **b)** $\ln 10 - 5 \ln 7$ **c)** $3 \ln x + 3 \ln y$

4) Solve each equation. Round your answer to 4 decimal places if necessary.

a) $e^x = 2$ **b)** $e^{-3n} = 83$

c) $e^{k+7} = 26$ **d)** $9e^{1.4p-10} - 10 = 17$

e) $\ln x = -5$

f) $7.316 = e^{\ln(2x)}$

g) $\ln(-m) = \ln(m + 10)$

h) $\ln(9x + 1) = \ln(x^2 + 9)$

i) $\ln(1 - 8x) - 10 = -7$

j) $\ln(5 - 2x^2) + \ln 9 = \ln 43$