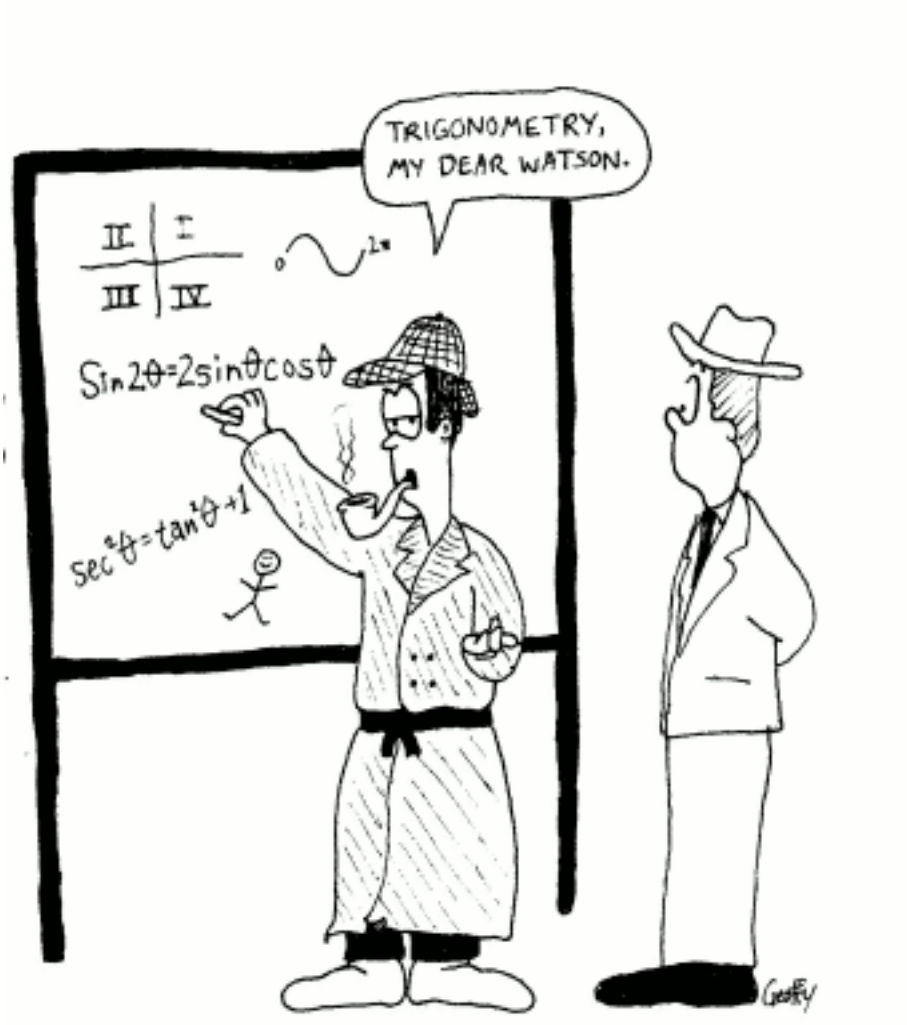
Chapter 4/5 Part 2- Trig Identities and Equations

WORKBOOK

MHF4U



W1 – 4.3 Co-function Identities
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1) Simplify.

a) $\sin x \left(\frac{1}{\cos x}\right)$ **b)** $(\cos x)(\sec x)$ **c)** $1 - \cos^2 x$ **d)** $1 - \sin^2 x$

e) $\frac{\tan x}{\sin x}$ **f)** $(1 - \sin x)(1 + \sin x)$ **g)** $\left(\frac{1}{\tan x}\right)\sin x$ **h)** $\frac{1+\tan^2 x}{\tan^2 x}$

i) $\frac{\sin x \cos x}{1-\sin^2 x}$ **j)** $\frac{1-\cos^2 x}{\sin x \cos x}$

2) Prove the following identities.

a) $\sin^2 x\,(1 + \cot^2 x) = 1$ **b)** $1 - \cos^2 x = \tan x \cos x \sin x$

c) $\cos x \tan^3 x = \sin x \tan^2 x$

d) $1 - 2\cos^2 \theta = \sin^4 \theta - \cos^4 \theta$

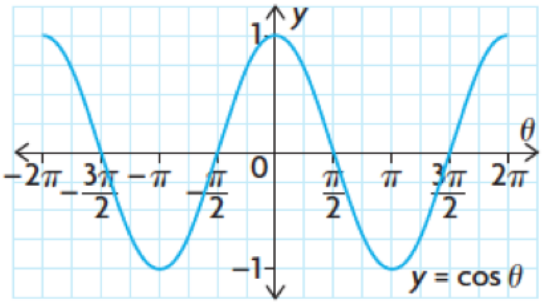
e) $\cot x + \frac{\sin x}{1+\cos x} = \csc x$

f) $\frac{\sec x}{\sin x} + \frac{\csc x}{\cos x} = \frac{2}{\sin x \cos x}$

g) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

h) $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

3)a) Use transformations and the cosine function to write three equivalent expressions for the following graph:



b) Transform your 3 equations from part a) to write the equation of 3 sine functions that represent the graph.

4) Use the co-function identities to write an expression that is equivalent to each of the following expressions.

a) $\sin\frac{\pi}{6}$ **b)** $\cos\frac{5\pi}{12}$ **c)** $\cos\frac{5\pi}{16}$

5) Write an expression that is equivalent to each of the following expressions, using the related acute angle.

a) $\sin\frac{7\pi}{8}$ **b)** $\cos\frac{13\pi}{12}$ **c)** $\cos\frac{11\pi}{6}$

6) Given that $\sin\frac{\pi}{6}=\frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{\pi}{3}=\frac{1}{2}$

7) Given that $\sin\frac{\pi}{6}=\frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{2\pi}{3}=-\frac{1}{2}$

8) Given that $\csc\frac{\pi}{4}=\sqrt{2}$, use an equivalent trigonometric expression to show that $\sec\frac{3\pi}{4}=-\sqrt{2}$

9) Given that $\cos\frac{3\pi}{11}\sim 0.6549$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $\sin\frac{5\pi}{22}$

b) $\sin\frac{17\pi}{22}$

W2 – 4.4 Compound Angle Formulas
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1) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}+\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

b) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}-\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

c) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

d) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}+\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

e) $\cos\frac{2\pi}{9}\cos\frac{5\pi}{18}-\sin\frac{2\pi}{9}\sin\frac{5\pi}{18}$

f) $\cos\frac{10\pi}{9}\cos\frac{5\pi}{18}+\sin\frac{10\pi}{9}\sin\frac{5\pi}{18}$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

b) $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

c) $\cos\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$

d) $\sin\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$

e) $\tan\left(\frac{\pi}{4}+\pi\right)$

f) $\tan\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$

4) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin\frac{7\pi}{12}$

b) $\sin\frac{5\pi}{12}$

c) $\cos\frac{11\pi}{12}$

d) $\cos\frac{5\pi}{12}$

e) $\sin\frac{13\pi}{12}$

f) $\cos\frac{17\pi}{12}$

g) $\sin\frac{19\pi}{12}$

h) $\cos\frac{23\pi}{12}$

5) Angles x and y are located in the first quadrant such that $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$. Determine exact values for $\cos x$ and $\sin y$.

6) Refer to the previous question. Determine an exact value for each of the following.

a) $\sin(x+y)$

b) $\sin(x-y)$

c) $\cos(x+y)$

d) $\cos(x-y)$

7) Use a compound angle formula to show that $\cos(2x) = \cos^2 x - \sin^2 x$

W3 – 4.5 Double Angle Formulas
MHF4U

1) Express each of the following as a single trig ratio.

a) $2\sin(5x)\cos(5x)$ **b)** $\cos^2\theta - \sin^2\theta$ **c)** $1 - 2\sin^2(3x)$

d) $\frac{2\tan(4x)}{1-\tan^2(4x)}$ **e)** $4\sin\theta\cos\theta$ **f)** $2\cos^2\frac{\theta}{2} - 1$

2) Express each of the following as a single trig ratio and then evaluate

a) $2\sin 45°\cos 45°$ **b)** $\cos^2 30° - \sin^2 30°$ **c)** $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

d) $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$ **e)** $1 - 2\sin^2\frac{3\pi}{8}$ **f)** $2\tan 60°\cos^2 60°$

3) Use a double angle formula to rewrite each trig ratio

a) $\sin(4\theta)$ **b)** $\cos(3x)$ **c)** $\tan x$

d) $\cos(6\theta)$ **e)** $\sin x$ **f)** $\tan(5\theta)$

4) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos\theta = \frac{3}{5}$ and $0 \leq \theta \leq \frac{\pi}{2}$

5) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\tan\theta = -\frac{7}{24}$ and $\frac{\pi}{2} \leq \theta \leq \pi$

6) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\sin\theta = -\frac{12}{13}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$

7) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos\theta = -\frac{4}{5}$ and $\frac{\pi}{2} \leq \theta \leq \pi$

8) Determine the value of a in the equation $2\tan x - \tan(2x) + 2a = 1 - \tan(2x)\tan^2 x$

W4 – 4.5 Prove Trig Identities
MHF4U

Prove each identity using the space on the following pages.

a) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

b) $\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$

c) $\sin(2x) = 2\sin x \cos x$

d) $\cos(2x) = \cos^2 x - \sin^2 x$

e) $\cot\theta - \tan\theta = 2\cot(2\theta)$

f) $\frac{\sin(2\theta)}{1-\cos(2\theta)} = \cot\theta$

g) $\sin x \sec x = \tan x$

h) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

i) $\frac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

j) $\frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x} = \sec x \csc x$

k) $\frac{1-\sin^2 x \cos^2 x}{\cos^4 x} = \tan^4 x + \tan^2 x + 1$

l) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

m) $\cot\theta - \tan\theta = 2\cot(2\theta)$

n) $(\sin x + \cos x)^2 = 1 + \sin(2x)$

o) $\frac{2\tan x}{1+\tan^2 x} = \sin(2x)$

p) $\sin\left(\frac{\pi}{4}+x\right) + \sin\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos x$

q) $\cos^4 x - \sin^4 x = \cos(2x)$

r) $\csc(2x) + \cot(2x) = \cot x$

s) $\cos(2x) = 2\cos^2 x - 1$

t) $\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$

u) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

v) $\cot x + \tan x = 2\csc(2x)$

W5 – 5.4 Solve Linear Trigonometric Equations
MHF4U

1) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\sin x - \frac{1}{4} = 0$

b) $\cos x + 0.75 = 0$

c) $\tan x - 5 = 0$

d) $\sec x - 4 = 0$

e) $3 \cot x + 2 = 0$

f) $2 \csc x + 5 = 0$

2) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin x + \frac{\sqrt{3}}{2} = 0$

b) $\cos x - 0.5 = 0$

c) $\tan x - 1 = 0$

d) $\cot x + 1 = 0$

3) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\sin^2 x - 0.64 = 0$

b) $\cos^2 x - \frac{4}{9} = 0$

c) $\tan^2 x - 1.44 = 0$

d) $\sec^2 x - 2.5 = 0$

4) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin^2 x - \frac{1}{4} = 0$

b) $\cos^2 x - \frac{3}{4} = 0$

c) $\tan^2 x - 3 = 0$

d) $3\csc^2 x - 4 = 0$

5) Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $3\sin x = \sin x + 1$

b) $5\cos x - \sqrt{3} = 3\cos x$

c) $7 \sec x = 7$

d) $2 \csc x + 17 = 15 + \csc x$

W6 – 5.4 Solve Double Angle Trigonometric Equations
MHF4U

Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible.

a) $\sin(2x) - 0.8 = 0$

b) $5\sin(2x) - 3 = 0$

c) $-4\sin(2x) + 3 = 0$

d) $\sin(2x) = \frac{1}{\sqrt{2}}$

e) $\sin(4x) = \frac{1}{2}$

f) $\sin(3x) = -\frac{\sqrt{3}}{2}$

g) $\cos(4x) = -\frac{1}{\sqrt{2}}$

h) $\cos(2x) = -\frac{1}{2}$

W7 – 5.4 Solve Quadratic Trigonometric Equations
MHF4U

1) Solve $\sin^2 x - 2\sin x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

2) Solve $\csc^2 x - \csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

3) Solve $2\sec^2 x - \sec x - 1 = 0$ on the interval $0 \leq x \leq 2\pi$

4) Solve $\tan^2 x - \tan x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$. Round answers to the nearest hundredth of a radian.

5) Solve $6\cos^2 x + 5\cos x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$

6) Solve $3\csc^2 x - 5\csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

7) Solve $2\tan^2 x - 5\tan x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

8) Solve $\cot x \csc^2 x = 2\cot x$ on the interval $0 \leq x \leq 2\pi$

9) Solve for θ to the nearest hundredth, where $0 \leq \theta \leq 2\pi$

a) $3\tan^2 \theta - 2\tan \theta = 1$

b) $12\sin^2\theta + \sin\theta - 6 = 0$

c) $5\cos(2\theta) - \cos\theta + 3 = 0$