Unit 5: Exponential and Logarithmic Functions
5.5 Applications of Logarithmic Functions

The most common examples of logarithmic scales are the Richter, decibel, and pH scales. The Richter scale measures the magnitude of earthquakes. The decibel scale measures the loudness of sound. The pH scale measures the acidity of liquids.

The Richter Scale
The Richter scale was developed in 1935 by seismologist Charles F. Richter. It measures the magnitude of an earthquake by comparing the intensity of the earthquake to some reference earthquake. The formula developed by Richter is

***Richter Scale*:** to measure earthquakes

$$M = \log\left(\frac{I}{I_o}\right)$$

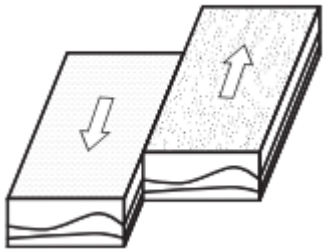
, where, M = magnitude of earthquake (measures on the Richter Scale)
I = intensity of the earthquake
I_o = intensity of a standard earthquake

Ex. 1: The San Francisco earthquake of 1989 measured 6.9 on the Richter scale. The Alaska earthquake of 1964 measured 8.5.

a) How many times as intense as the San Francisco earthquake was the Alaska earthquake?

b) Calculate the magnitude of an earthquake that is twice as intense as the 1989 San Francisco earthquake.

GENERALISING THE PROCEDURE



Earthquakes can be analyzed by the formula:

$$M_2 - M_1 = \log\left(\frac{I_2}{I_1}\right)$$

Where M_2 is the magnitude of the earthquake with intensity of I_2 and M_1 is the magnitude of the earthquake with intensity of I_1 .

Ex.2: How much more intense is an earthquake measuring 6.5 on the Richter scale than one measuring 6.4?

The Decibel Scale

The loudness of any sound is measured relative to the loudness of sound at the threshold of hearing. Sounds at this level are the softest that can still be heard.

Sound	Loudness (dB)
soft whisper	30
normal conversation	60
shouting	80
subway	90
screaming	100
rock concert	120
jet engine	140
space-shuttle launch	180

The formula used to compare sounds is

Decibels:

$$L = 10\log\left(\frac{I}{I_o}\right)$$

where, L = loudness measured in decibels (dB)
I = intensity of the sound (measured in W/m^2)
I_o = intensity of sound at the threshold of hearing. $\left(I_o = 10^{-12}\ W/m^2\right)$

Ex. 3: A sound is 1000 times more intense than a sound you can just hear. What is the measure of its loudness in decibels?

GENERALISING THE PROCEDURE

Sound intensity can be analyzed by the formula:

$$L_2 - L_1 = 10\log\left(\frac{I_2}{I_1}\right)$$

Where L_2 is the loudness of the sound with intensity of I_2 and L_1 is the loudness of the sound with intensity of I_1 .

Ex. 4: The loudness level of a heavy snore is 69 dB. The loudness level of a conversation is 60 dB. The loudness level of a whisper is 30 dB.

a) How many times as loud as a conversation is a heavy snore?

b) How many times as loud as a whisper is a conversation?

The pH Scale

The pH scale allows chemists to determine the concentration of hydrogen ion in a liquid. It ranges from values of 1 to 14. The higher the pH, the more basic, or less acidic the liquid. The lower the pH, the more acidic or less basic the liquid.

- A liquid with a pH of less than 7.0 is considered *acidic*
- A liquid with a pH of greater than 7.0 is considered *basic*
- A liquid with pH = 7.0 is considered to be *neutral*. Pure water has a pH of 7.0.

The relationship between pH and H^+ ion concentration is inversely proportional and can be summarized as:

Low pH = High H^+ ion concentration
High pH = Low H^+ ion concentration

The relationship between pH and hydrogen ion concentration is given by the formula

pH Scale:

$$pH = -\log[H^+]$$

where, pH = measure of acidity level of a substance
$[H^+]$ = concentration of hydrogen ion in moles/litre

Ex. 5: Find the pH of a swimming pool with a hydrogen ion concentration of 6.1×10^{-8} mol/L.

Ex. 6: The pH of a fruit juice is 3.10. What is the hydrogen ion concentration of the fruit juice?

GENERALISING THE PROCEDURE

Formula can be used to compare two pH is

$$pH_2 - pH_1 = -\log\left(\frac{[H^+]_2}{[H^+]_1}\right)$$

Ex. 7: Refer to the table at the right to answer the following questions:

a) How many times as acidic as tomato juice is lemon juice?

Solution	pH
Lemon juice	2
Tomato juice	4
Pure water	7
Baking soda	9
Oven cleaner	13

b) How many times as acidic as pure water is lemon juice?

c) How many times as acidic as pure water is baking soda?

d) How many times as acidic as baking soda is oven cleaner?

Practice

1. A sample of 500 cells in a medical research lab doubles every 20 min.
   a) Determine a formula for the number of cells at time t, where t is measured in minutes.
   b) How long will it take for the population to reach 18 000? Answer correct to 2 decimal places.

2. In 1987 there were about 130 000 cell phone users in Canada. In 1999, there were about 10 million cell phone users. What is the percent increase per year? Answer correct to 2 decimal places.

3. A sample of 700 cells in a medical research lab triples every 30 min.
   a) Determine a formula for the number of cells at time t.
   b) How long will it take for the population to reach 18 000? Answer correct to 2 decimal places

4. A sample of radioactive iodine-131 atoms has a half-life of about 8 days. Suppose that one million iodine-131 atoms are initially present.
   a) Determine a formula for the number of atoms at time t, where t represents number of days
   b) How long will it take for the sample to reach 180 000 atoms? Answer correct to 2 decimal places.

5. A new car costs $23 000. In 5 years it will be worth $9500. What is the rate of depreciation per year? Answer in percent, correct to 2 decimal places.

6. Most of Canada's earthquakes occur along the west coast. In 1949, there was an earthquake in the Queen Charlotte Islands that had a magnitude of 8.1 on the Richter Scale. In 1997 there was an earthquake in south-western B.C. with a magnitude of 4.6 on the Richter Scale. How many times as intense as the 1997 earthquake was the 1949 earthquake? Answer correct to 2 decimal places.

7. (a) The loudness level of a heavy snore is 69 dB. How many times is this more intense than conversational speech at 60 dB? Answer correct to 2 decimal places. (b) Sound is 316 times less intense if earplugs are worn. What would the decibel level of snoring be if earplugs were worn? Answer correct to the nearest dB.

8. Given the function $y = 4^x$, determine:
   (a) the average rate of change from t=5 seconds to t=6 seconds.
   (b) the instantaneous rate of change at t=5 seconds.

Answers:

1.(a) $N(t) = 500(2)^{\frac{t}{20}}$ (b) 103.40 min

2. 43.61%

3. (a) $N(t) = 700(3)^{\frac{t}{30}}$ (b) 88.67 min

4. (a) $N(t) = 1000000\left(\frac{1}{2}\right)^{\frac{t}{8}}$ (b) 19.79 days

5. 16.21%

6. 3162.28

7. (a) 7.94 (b) 44 dB

8. (a) 3072 (b) 1419.57