**L2 – 4.4 Compound Angle Formulas**
**MHF4U**

***Compound angle:*** *an angle that is created by adding or subtracting two or more angles.*

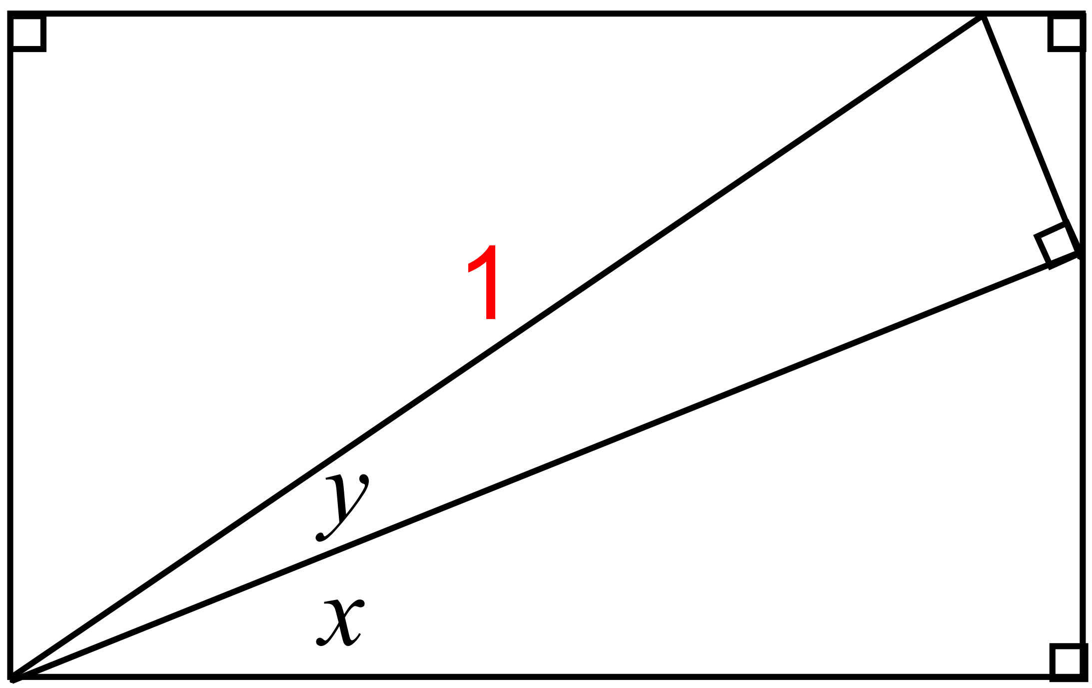
Normal algebra rules do not apply:

$$\cos(x+y) \neq \cos x + \cos y$$

**Part 1: Proof of $\cos(x+y)$ and $\sin(x+y)$**

So what does $\cos(x+y) = ?$

Using the diagram below, label all angles and sides:




$$\cos(x+y) =$$

$$\sin(x+y) =$$

## Part 2: Proofs of other compound angle formulas

**Even/Odd Properties**

$\cos(-x) = \cos x$ $\sin(-x) = -\sin x$

**Example 1:** Prove $\cos(x - y) = \cos x \cos y + \sin x \sin y$

| **LS** | **RS** |
|---|---|

**LS = RS**

**Example 2:**

**a)** Prove $\sin(x - y) = \sin x \cos y - \cos x \sin y$

| **LS** | **RS** |
|---|---|

**LS = RS**

**<u>Compound Angle Formulas</u>**

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

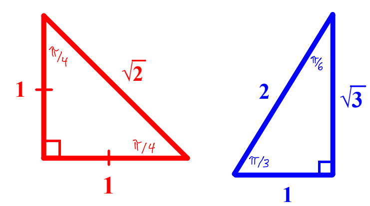
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

## <u>Part 3: Determine Exact Trig Ratios for Angles other than Special Angles</u>




By expressing an angle as a sum or difference of angles in the special triangles, exact values of other angles can be determined.

**Example 3:** Use compound angle formulas to determine exact values for

**a)** $\sin\frac{\pi}{12}$

$$\sin\frac{\pi}{12} =$$

**b)** $\tan\left(-\frac{5\pi}{12}\right)$

$$\tan\left(-\frac{5\pi}{12}\right) =$$

**Part 4: Use Compound Angle Formulas to Simplify Trig Expressions**

**Example 4:** Simplify the following expression

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12}+\sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

**Part 5: Application**

**Example 5:** Evaluate $\sin(a+b)$, where $a$ and $b$ are both angles in the second quadrant; given $\sin a=\frac{3}{5}$ and $\sin b=\frac{5}{13}$

Start by drawing both terminal arms in the second quadrant and solving for the third side.