L3 - 2.2 - Factor Theorem Lesson MHF4U

In this section, you will learn how to determine the factors of a polynomial function of degree 3 or greater.

Part 1: Remainder Theorem Refresher

a) Use the remainder theorem to determine the remainder when $f(x) = x^3 + 4x^2 + x - 6$ is divided by x + 2

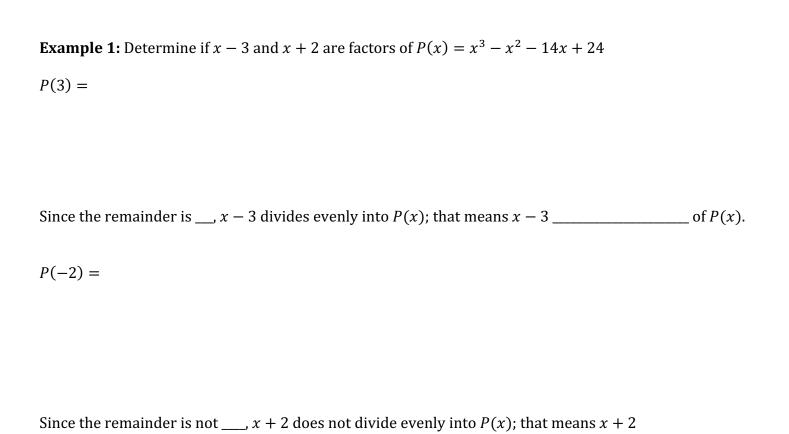
Remainder Theorem: When a polynomial function P(x) is divided by x-b, the remainder is P(b); and when it is divided by ax-b, the remainder is $P\left(\frac{b}{a}\right)$, where a and b are integers, and $a \neq 0$.

b) Verify your answer to part a) by completing the division using long division or synthetic division.

Note: I chose synthetic since it is a linear divisor of the form x - b.

Factor Theorem:

x - b is a factor of a polynomial P(x) if and only if P(b) = 0. Similarly, ax - b is a factor of P(x) if and only if $P\left(\frac{b}{a}\right) = 0$.



Part 2: How to determine a factor of a Polynomial With Leading Coefficient 1

You could guess and check values of b that make P(b) = 0 until you find one that works...

Or you can use the Integral Zero Theorem to help.

 $\underline{\hspace{1cm}}$ of P(x).

Integral Zero Theorem

If x - b is a factor of a polynomial function P(x) with leading coefficient 1 and remaining coefficients that are integers, then **b** is a factor of the constant term of P(x).

Note: Once one of the factors of a polynomial is found, division is used to determine the other factors.

Example 2: Fact	for $x^3 + 2x^2 - 5x - 6$ fully.	
$Let P(x) = x^3 +$	$2x^2 - 5x - 6$	
	such that $P(b) = 0$. Based of then divide $P(x)$ by that fac	on the factor theorem, if $P(b) = 0$, then we know that $x - b$ is tor.
The integral zero	theorem tells us to test fac	tors of
Test divide $P(x)$.	Once one f	actor is found, you can stop testing and use that factor to
P(1) =		
Since	, we know that	a factor of $P(x)$.
P(2) =		
Since	, we know that	a factor of $P(x)$.
You can now use	either long division or synt	hetic division to find the other factors

Method 2: Synthetic Division

Method 1: Long division

Example 3: Factor $x^4 + 3x^3 - 7x^2 - 27x - 18$ completely. Let $P(x) = x^4 + 3x^3 - 7x^2 - 27x - 18$ Find a value of b such that P(b) = 0. Based on the factor theorem, if P(b) = 0, then we know that x - b is a factor. We can then divide P(x) by that factor.

The integral zero theorem tells us to test factors of _______.

Test _________. Once one factor is found, you can stop testing and use that factor to divide P(x).

Since ________, this tell us that ________ is a factor. Use division to determine the other factor.

We can now further divide $x^3 + 2x^2 - 9x - 18$ using division again or by factoring by grouping.

Method 1: Division

Method 2: Factoring by Grouping

Group the first 2 terms and the last 2 terms and separate with an addition sign.

Common factor within each group

Factor out the common binomial

Therefore,

$$x^4 + 3x^3 - 7x^2 - 27x - 18 =$$

Example 4: Try Factoring by Grouping Again

$$x^4 - 6x^3 + 2x^2 - 12x$$

Note: Factoring by grouping does not always work...but when it does, it saves you time!

Part 3: How to determine a factor of a Polynomial With Leading Coefficient NOT 1

The integral zero theorem can be extended to include polynomials with leading coefficients that are not 1. This extension is known as the rational zero theorem.

Rational Zero Theorem:

Suppose P(x) is a polynomial function with integer coefficients and $x = \frac{b}{a}$ is a zero of P(x), where a and b are integers and $a \ne 0$. Then,

- b is a factor of the constant term of P(x)
- a is a factor of the leading coefficient of P(x)
- (ax b) is a factor of P(x)

Example 5: Factor
$$P(x) = 3x^3 + 2x^2 - 7x + 2$$

We must start by finding a value of $\frac{b}{a}$ where $P\left(\frac{b}{a}\right) = 0$.

b must be a factor of the constant term. Possible values for *b* are: _____

a must be a factor of the leading coefficient. Possible values of *a* are: _____

Therefore, possible values for $\frac{b}{a}$ are:

Test values of $\frac{b}{a}$ for x in P(x) to find a zero.

Since ______ of P(x). Use division to find the other factors.

