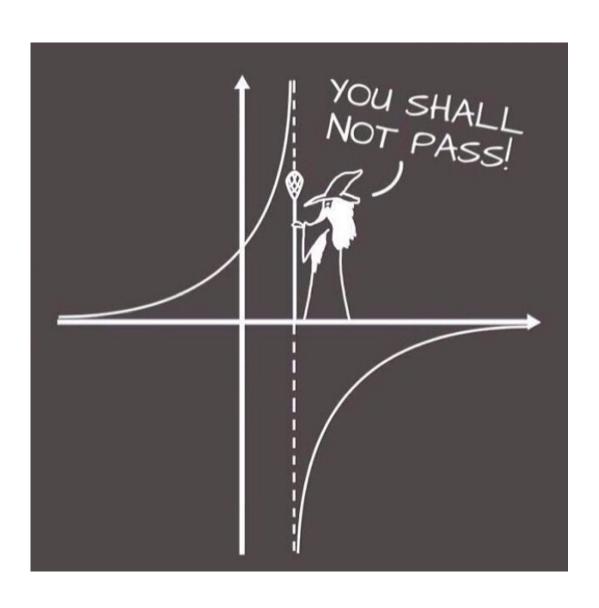
Rational Functions

Workbook

MHF4U

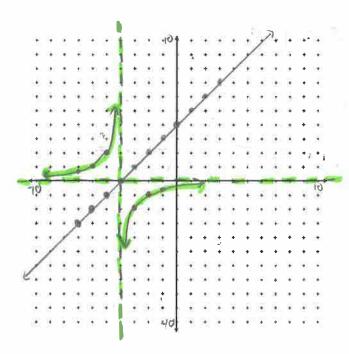


WI - 3.1/3.2 Reciprocal of Linear and Quadratic Functions MHF4U

1) Graph each of the following reciprocal functions. Start by graphing the function in the denominator. Show as much characteristic information about the graph as you can (e.g. intercepts, asymptotes with equations, other defining points, etc).

$$a) f(x) = \frac{1}{x-1}$$

b)
$$g(x) = -\frac{2}{x+4}$$



c)
$$h(x) = \frac{1}{x^2-9} = \frac{1}{(x-3)(x+3)}$$
 HA: $y=0$
VA: $x=3$ and $x=-3$

$$\chi^2 - 9 = (\chi - 0)^2 - 9$$

 $\chi - vertex at (0, -9)$

d)
$$j(x) = \frac{1}{x^2 - 2x - 15} = \frac{1}{(x-5)(x+3)}$$

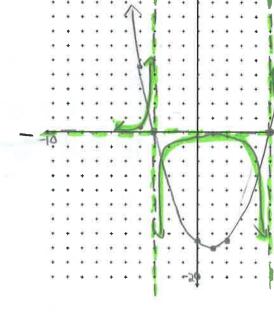
VA: x=-3 and x=5

$$x^{2} - 2x - 15$$

 $x - 15$
 $x - 15$

underned

11.0



e)
$$k(x) = \frac{1}{x^2 + 2}$$

HA: y = 0

VA: NONE

x-vertex at (0,2)

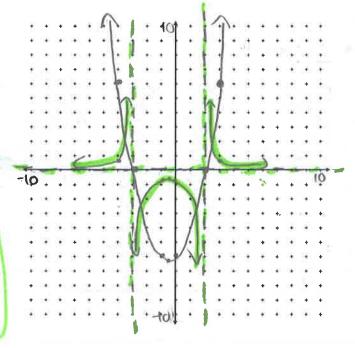
χ_{\perp}	4_
_3	II
-5	6
1	3
0	2
1	3
2	6
3	11-

k(x)	
×	7
-3	0.1
-2	0.17
-1	0.33
0	0.5
1	0.33
2	0.17
3	0.1

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f)
$$m(x) = \frac{4}{x^2 + x - 6} = \frac{4}{(x+3)(x-2)}$$
 Here $y = 0$
 $(x+3)(x-2)$ When $x = 2$

x ly
$$\frac{1}{2a} = \frac{1}{2(1)} = -0.5$$



g)
$$n(x) = -\frac{1}{4x^2 - 4x - 3}$$
 = $(2x - 3)(2x + 1)$



$$4x^2-4x-3$$

Reverted at $-\frac{b}{aa} = \frac{4}{2(9)} = 0.5$

X	4
-1	5
0	-3
0.5	-4
1 /	- 3
1.5	0
2	5

n	(K)
χ	1 - 19
-1 -0.5 0 0.65 1 10.5	-0.23 0.33 0.25 undefined -0.2

n	(K)
X	1 5
-1 -0.5 0 0.5 1 10.5	-0e2 undefined 0.33 0.25 undefined -0.2

$$p(x) = \frac{4}{2x^2 - 8x + 9}$$

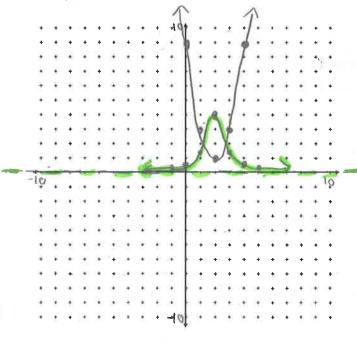
& No restrictions

V4: None

2x2	-8	2+9		
rvertex	ळ	·설 :	8	=2

χ	4
-1	19
0	9
1	3
2	3
3	3
4	3 9 19
3 4 5	19

1		p(x)
	γ	19
	-1	0,21
	2 3	1.33
l	2 3 4 5	0.44



W2 - 3.3 Quotient of Linear Functions

MHF4U

1) State the equation of the vertical and horizontal asymptotes for each function.

a)
$$p(x) = \frac{x}{x-6}$$

$$c) r(x) = \frac{x-1}{x+1}$$

b)
$$q(x) = \frac{3x}{x+4}$$

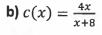
d)
$$\frac{5x-2}{2x+3}$$

2) Graph each of the following functions. Make sure to identify key characteristics of the functions including intercepts and asymptotes.

a)
$$f(x) = \frac{x}{x-5}$$

$$VA: \chi = 5 \qquad \chi - i \pi + i \qquad 0 = \frac{\chi}{\chi - 5}$$

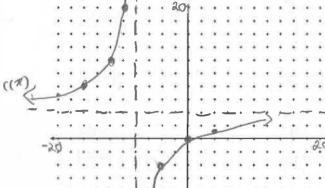
$$f(x) = \frac{2}{3} = -0.67$$
 $(2, -0.67)$



b)
$$c(x) = \frac{4x}{x+8}$$
 $\chi - i \chi$, $Q = \frac{4\chi}{\chi + \zeta}$

other points:





$$c) k(x) = \frac{x+1}{4-x} = \frac{x+1}{x+4}$$

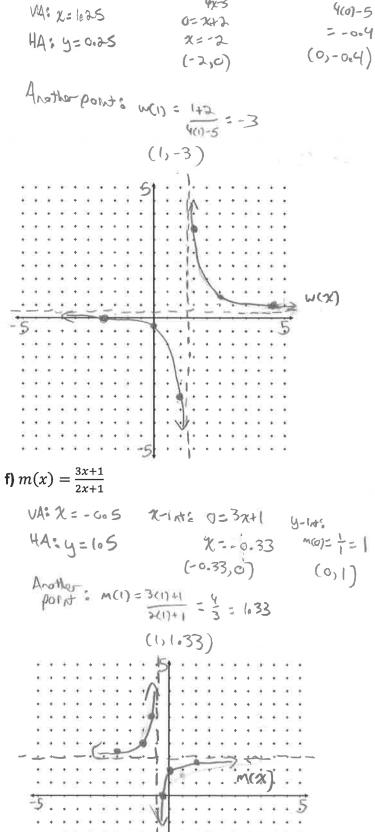
$$0 + \frac{x+4}{x+4} = \frac{x+1}{x+4}$$

$$0 + \frac{x+4}{x+4} = \frac{x+4}{x+4}$$

$$0 + \frac{x+4}{x+4} = \frac{x+4}{x+4} = \frac{x+4}{x+5}$$

$$0 + \frac{x+4}{x+4} = \frac{x+4}{x+5} = \frac{x+4}{x+5} = \frac{x+4}{x+5}$$

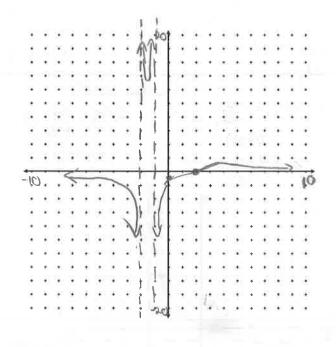
$$0 + \frac{x+4}{x+5} = \frac{x+4}{x+5}$$



9-14: Wat= 0+2

use graphing calculator / desmos

g)
$$g(x) = \frac{x-2}{x^2+3x+2} = \frac{\chi - 2}{(\chi + 2)(\chi + 1)}$$



W3&4 – Combinations of Functions and Inverse Functions

MHF4U

1) Let
$$f(x) = 3x - 5$$
 and $g(x) = 2x + 3$.

a) Write the equation for
$$h(x) = f(x) + g(x)$$
 and determine the value of $h(2)$.

$$h(x) = (3x - 5) + (2x + 3)$$

 $h(x) = 3x - 5 + 2x + 3$
 $h(x) = 5x - 2$
 $h(x) = 5x - 2$

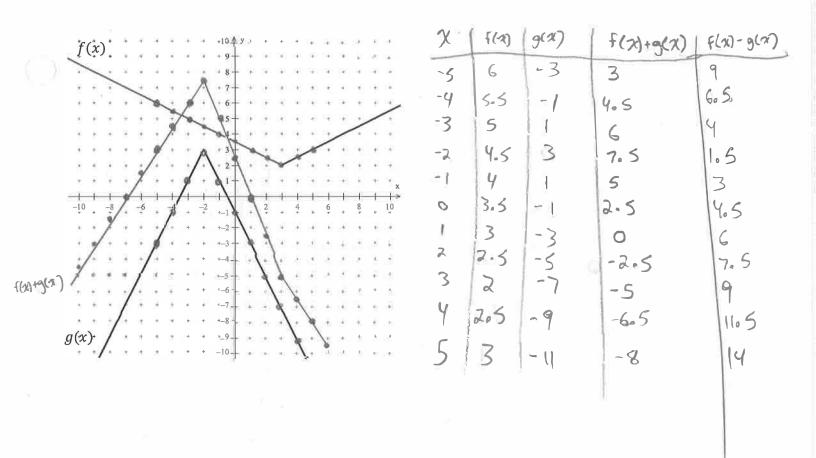
b) Write the equation for
$$k(x) = f(x) - g(x)$$
 and determine the value of $k(2)$.

$$k(x) = (3x-5) - (2x+3)$$

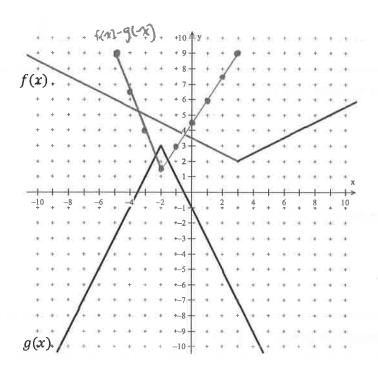
 $k(x) = 3x-5-2x-3$
 $k(2) = 2-8$
 $k(2) = -6$

2) Use the functions
$$f(x)$$
 and $g(x)$ as shown. Apply the superposition principle to graph

a)
$$y = f(x) + g(x)$$



b)
$$y = f(x) - g(x)$$
.

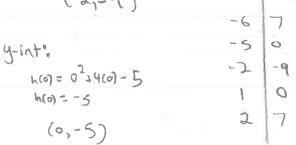


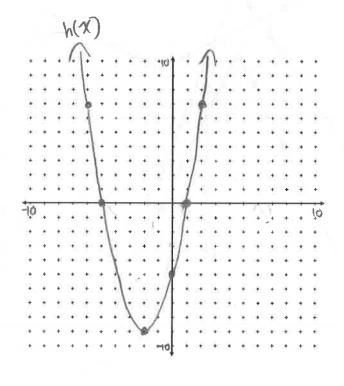
3) Let f(x) = x - 2 and $g(x) = x^2 + 3x - 3$. Determine an algebraic and graphical model for h(x) = f(x) + g(x).

$$h(x) = (x-2) + (x^2+3x-3)$$

$$h(x) = x - 2 + x^2 + 3x - 3$$

$$h(x) = x^2 + 4x - 5$$





4) Let f(x) = x - 2 and $g(x) = x^2 - 4$. Develop an algebraic and graphical model for each of the following:

a)
$$y = f(x)g(x) = (x-2)(x^2-4)$$

= $(x-2)(x-2)(x+2)$
= $(x-2)^2(x+2)$

$$\chi$$
-int: (2,0) order 2 y -int: $y=(0-2)^2(0+2)$
(-2,0) order 1 $y=8$
(0,8)

b)
$$y = \frac{f(x)}{g(x)}$$
 = $\frac{\chi - \lambda}{\chi^2 - 4}$ = $\frac{\chi - \lambda}{(\chi - \lambda)(\chi + \lambda)}$ = $\frac{1}{\chi + \lambda}$; $\chi \neq -2$, λ

Hove at 1=2	1472	242
VA: x =- 2	214	70.00
HA: 4=0	-5 -3	-5 - 0.33
	-3 -1	4 -0.5
	-5 0	-3 -1 -2 und.
	-1 1	-1 1
	0 2	0 0-5
	1/3	1 10.33

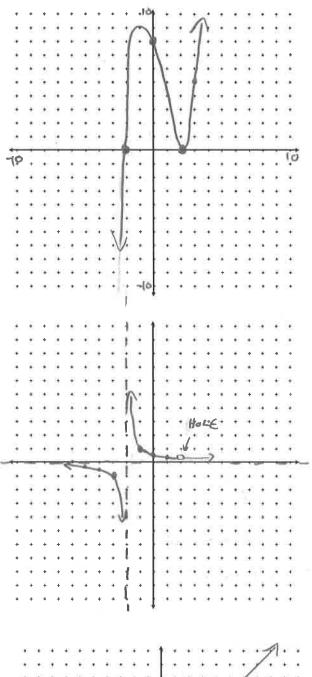
c)
$$y = \frac{g(x)}{f(x)}$$

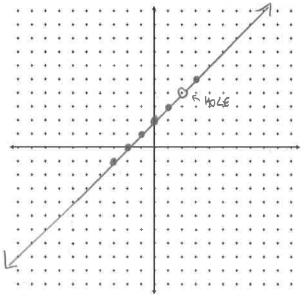
$$= \frac{\chi^{2} - 4}{\chi - 2}$$

$$= (\frac{\chi}{\chi} \times \chi + 2)$$

$$= \chi + 2 \quad | \chi \neq 2$$
Have at $\chi = 2$

X	9
-3	-1
-5	0
- }	1
0	2
1	3
2	undeflied
3	5





5) Let
$$f(x) = x^2 + 2x - 4$$
 and $g(x) = \frac{1}{x+1}$.

a) Evaluate g(f(0))

$$f(0) = (0)^{2} + 2(0) - 4$$

$$g(f(0)) = g(-4)$$

$$= \frac{1}{-4+1}$$

$$= -\frac{1}{3}$$

b) Evaluate f(g(-2))

$$g(-2) = \frac{1}{-2+1} \left(f(g(-2)) = f(-1) \right)$$

$$= (-1)^{2} + 2(-1) - 4$$

$$= -5$$

6) Let $f(x) = x^2 + 3x$ and g(x) = 2x - 5. Determine an equation for each composite function and graph it.

a)
$$y = f(g(x))$$

$$f(g(x)) = (2x-5)^{2} + 3(2x-5)$$

$$= 4x^{2} - 20x + 25 + 6x - 15$$

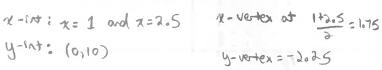
$$= 4x^{2} - 14x + 10$$

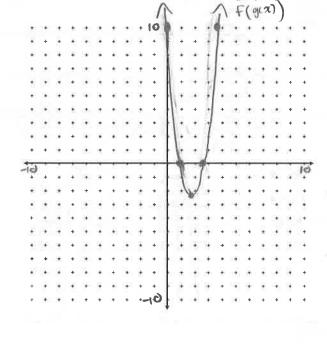
$$= 2(2x^{2} - 7x + 5)$$

$$= 2[2x^{2} - 5x - 2x + 5]$$

$$= 2[x(2x-5) - 1(2x-5)]$$

$$= 2(2x-5)(x-1)$$





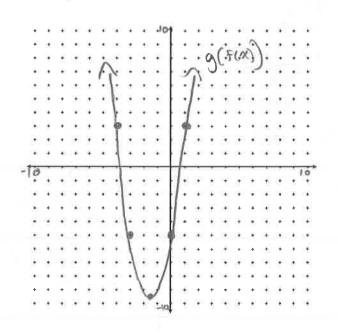
$$\mathbf{b)}\,y=g\big(f(x)\big)$$

$$g(f(x)) = 2(x^{2}+3x)-5$$

$$= 2x^{2}+6x-5$$

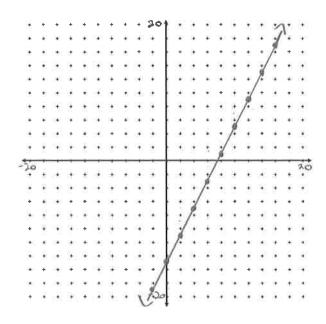
$$x-vertex at $\frac{-b}{2a} = \frac{-6}{2(2)} = -1.5$

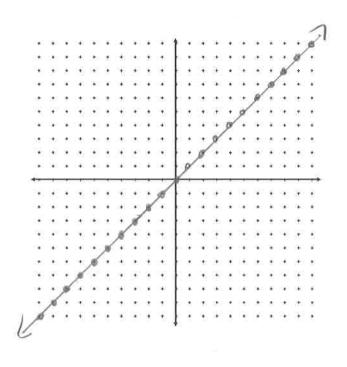
$$y-vertex = -9.5$$$$



c)
$$y = g(g(x))$$

$$y = g^{-1}\big(g(x)\big)$$





W 5 – 3.4 Solve Rational Equations and Inequalities MHF4U

1) Solve each equation algebraically.

a)
$$\frac{6}{2x-1} = 5$$

$$(2xx-1)\left(\frac{6}{2xx-1}\right) = 5(2x-1)$$

$$6 = 5(2x-1)$$

$$6 = 10x - 5$$

$$11 = 10x$$

$$2 = \frac{11}{10}$$

b)
$$\frac{6}{x} = x - 5$$
 $\chi(\frac{6}{\chi}) = \chi(\chi - 5)$
 $6 = \chi(\chi - 5)$
 $6 = \chi^2 - 5\chi$
 $0 = \chi^2 - 5\chi - 6$
 $0 = (\chi - 6)(\chi + 1)$
 $\chi - 6 = 0$
 $\chi = 6$
 $\chi = 6$

c)
$$1 = \frac{5}{3x^2 - 8x + 2}$$

 $(3x^2 - 8x + 2)(1) = \frac{5}{3x^2 - 8x + 2}$
 $3x^2 - 8x + 2 = 5$
 $3x^2 - 8x - 3 = 0$
 $3x^2 - 9x + 1x - 3 = 0$
 $3x(x-3) + 1(x-3) = 0$
 $(x-3)(3x+1) = 0$
 $(x-3) = 0$
 $(x-3)(3x+1) = 0$

d)
$$\frac{x+5}{x-1} = \frac{x+1}{x-3}$$

$$(x-3)(x+5) = (x+1)(x-1)$$

$$x^2 + 2x - (5 = x^2 - 1)$$

$$2x = 14$$

$$x = 7$$

$$e) \frac{3}{x+5} + \frac{4}{x} = 0$$

$$\frac{3}{245} = \frac{-4}{x}$$

$$3x = -4(x+5)$$

$$3x = -4x - 20$$

$$7x = -20$$

$$x = -20$$

$$f) 2x = 5 - \frac{3}{x}$$

$$\chi(2x) = \chi(5 - \frac{3}{x})$$

$$2\chi^{2} = 5x - 3$$

$$2\chi^{2} - 5x + 3 = 0$$

$$2\chi^{2} - 2x - 3x + 3 = 0$$

$$2\chi(x - 1) - 3(x - 1) = 0$$

$$(x - 1)(2x - 3) = 0$$

$$\chi - 1 = 0$$

$$\chi - 1 = 0$$

$$\chi - 1 = 0$$

$$\chi - 3 = 0$$

$$\chi - 1 = 0$$

$$\chi - 3 = 0$$

$$\frac{12}{1-x} + \frac{3}{x+1} = \frac{1}{x}$$

$$\frac{2(x+1)+3(1-x)}{(1-x)(x+1)} = \frac{1}{x}$$

$$\frac{2x+3+3-3x}{1-x^2} = \frac{1}{x}$$

$$\frac{-1x+5}{1-x^2} = \frac{1}{x}$$

$$x(-1x+5) = 1(1-x^2)$$

$$-1x^2 + 5x = 1 - 1x^2$$

$$5x = ($$

h)
$$\frac{3}{x-1} + 5 + \frac{2}{x} = 0$$

$$\chi(x-1) \left(\frac{3}{x-1} + 5 + \frac{2}{x} \right) = O(\chi)(x-1)$$

$$3\chi + 5\chi(x-1) + \lambda(\chi-1) = 0$$

$$3\chi + 5\chi^2 - 5\chi + \lambda\chi - \lambda = 0$$

$$5\chi^2 = \lambda$$

$$\chi^2 = \frac{2}{5}$$

$$\chi = \pm \sqrt{\frac{2}{5}}$$

2) Solve each inequality without using technology.

a)
$$\frac{4}{2x-3} < \frac{1}{x+4}$$

$$\frac{4(x+4)-1(2x-3)}{(2x-3)(x+4)}<0$$

$$\frac{4x+16-2x+3}{(2x-3)(x+4)} < 0$$

restalctions: XX -4,1.5

b)
$$\frac{2x+3}{x-3} \ge \frac{6x-5}{3x+1}$$

$$(3x+1)$$
 $2x+3$ $-6x-5$ $(x-3)$ $(3x+1)$ $x-3$ $-3x+1$ $(x-3)$ >0

$$(3x+1)(2x+3)-(6x-5)(x-3) \ge 0$$

$$\frac{(3x+1)(x-3)}{34x-12} \ge 0$$

$$\frac{2(17x-6)}{(3x+1)(x-3)} \ge 0$$

-00	ب	9	3	00	
	-\	B	1	-	_
2	+	+	+	+	_
17x-6	-	_	+	+	-
3n+1	-	+	+	7	-
x-3	-	-	-	+	1
ارور	-	1+	1-	1+	

Solution:
$$\chi < -9.5 \text{ or } -4 < \chi < 1.5$$

Restrictions: $\frac{1}{3}$ (3,00)

 $\chi \leq (-\alpha, -9.5) \cup (-4, 10.5)$

Restrictions: $\frac{1}{3}$, $\frac{3}{3}$
 $\chi \in (\frac{1}{3}, \frac{6}{17}] \cup (3,00)$

c)
$$\frac{(x-3)(2x-1)}{(x+4)(x-5)} > 0$$

Zeros: 1,3

restrictions : -4.5

CLION 2 :	1) -				06	Du .
-00	-5	0	1	4]	6	1.00
2-3		-	-	+	+	
2x-1	-	_	+	+	+	_
x+4	-	+	+	+	+	
v-5	-	-	-	_	+	-
overall	+	-	+	-	+	

$$d) \frac{2x^2 + 5x - 3}{x^2 + 5x + 4} \le 0$$

zeros: -3, 5

restrictions: -4,-1

90	_(1 -3	-	1 2) d	N.
Ī	-51	3.5	-2	0	1	105
2x-1	-	-	-	-	+	
743	_	-	+	+	+	L
744	-	+	+	+	+	
7×+1	-	-	-	+	+	L
avera 11	+	1-	+	\ -	1+)

e)
$$\frac{4}{x-3} < 1$$

$$\frac{4}{2x-3} < 1 < 0$$

$$\frac{4}{2x-3} < \frac{2}{2x-3} < 0$$

$$\frac{4-(2x-3)}{2x-3} < 0$$

$$\frac{4-2x+3}{2x-3} < 0$$

$$\frac{7-2x}{2x-3} < 0$$

restriction: X #3

Solution:
$$\chi < 3$$
 or $\chi > 7$

$$\chi \in (-0,3) \cup (7,0)$$

$$-\frac{x}{x+3} > \frac{x}{x-1}$$

$$\frac{\chi}{\chi_{+3}} - \frac{\chi}{\chi_{-1}} > 0$$

$$\frac{\chi(\chi-1)-\chi(\chi+3)}{(\chi+3)(\chi-1)}>0$$

$$\frac{\chi^2 - \chi - \chi^2 - 3\chi}{(\chi + 3)(\chi - 1)} > 0$$

$$\frac{-4\chi}{(\chi+3)(\chi-1)} > 0$$

-0	9 -	5 9	001		
	-4	-1	9.5	2	
-476	+	+	-	_	-
X+3	-	+	+	+	_
-x-1	-	_	-	+	
leavo	+	-	+	-	

7ero: X = G

restrictions: x = -3,1

solution:
$$\chi < -3$$
 or $0 < \chi < 1$

$$\chi \in (-\omega, -3) \cup (0, 1)$$

f)
$$\frac{2x^2 + 5x - 3}{x^2 + 8x + 16} < 0$$
 $(3) = \frac{6}{2} \times \frac{6}{5} \times \frac{7}{3}$

$$\frac{(2\chi-1)(\chi+3)}{(\chi+4)^2}<0$$

(X+4)2	-a -4 -3 1/2 00						
		-5	-3.5	O	1		
	22-1		-	-	+		
Zeros; 2= -3, 1	2+3	-	-	+	+		
5 A	(x+4)2	+	+	+	+		
restriction: X = -4	lpous	+	+	-	+		

Solutren:
$$-3$$

h)
$$\frac{2x+3}{x} > \frac{x+1}{x}$$

$$\frac{\chi+2}{\chi}>0$$

	Q 40,	gh —,	5 3) 0	0
X+2		-3	-1	1	
7 >0	2+2	-	+	+	
<i>/</i> ×	X	-		+	
₹ero; X=-2	owall	+	-	+	

KSTRICTION: X 10

 $\chi \in (-\infty, -2) \cup (0, \infty)$