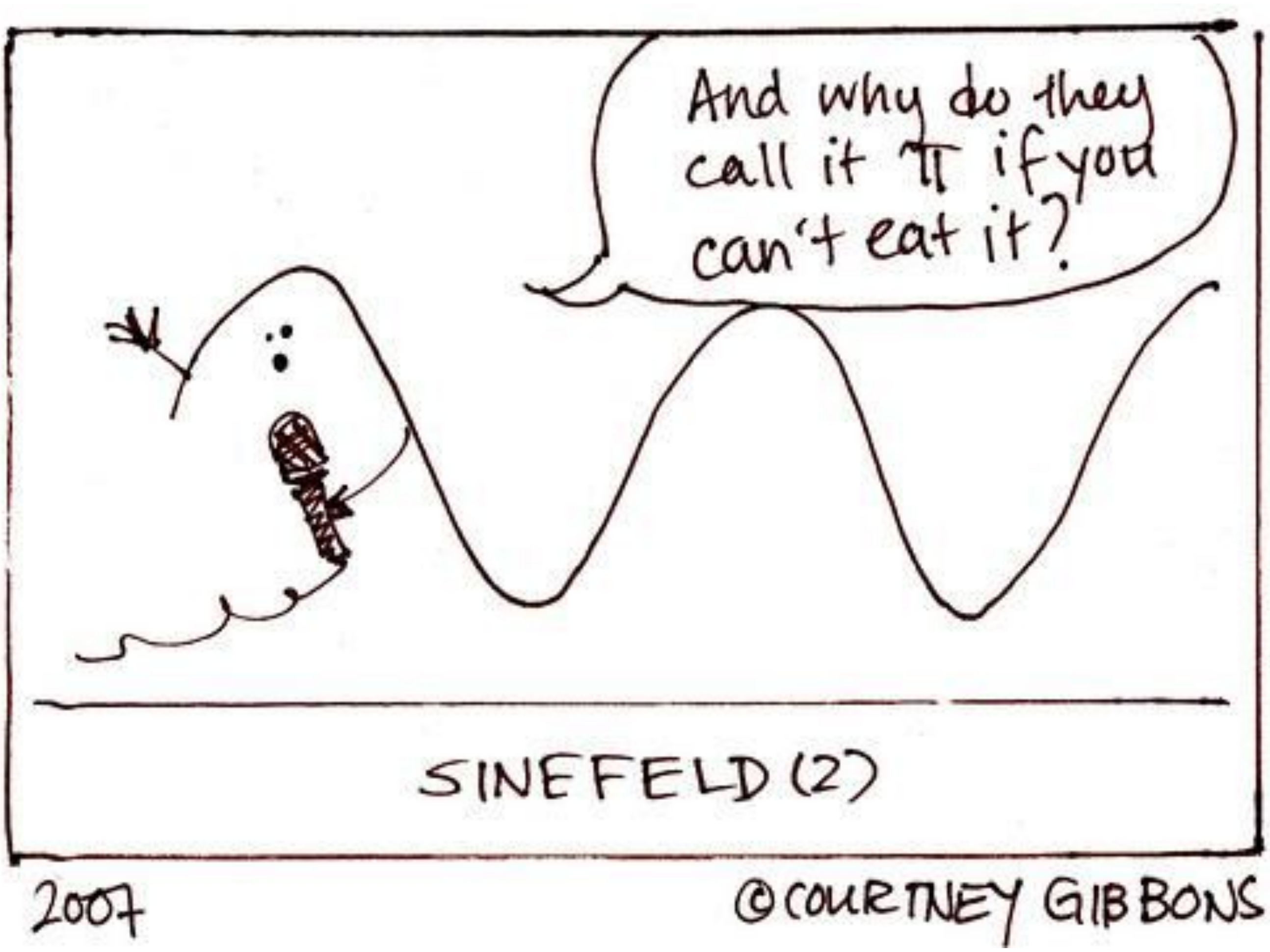
UNIT 4

Chapter 4/5 Part 1- Trigonometry in Radians

Lesson Package

MHF4U



Chapter 4/5 Part 1 Outline

Unit Goal: By the end of this unit, you will be able to demonstrate an understanding of meaning and application of radian measure. You will be able to make connections between trig ratios and the graphical and algebraic representations of the corresponding trig functions.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Radian Measure	- recognize the radian as an alternative unit to the degree for angle measurement - convert between degree and radian measures	B1.1, 1.2
L2	Trig Ratios and Special Angles	- Determine, without technology, the exact values of trig ratios for special angles	B1.3, 1.4
L3	Graphing Trig Functions	- sketch the graphs of all 6 trig ratios and be able to describe key properties	B2.1, 2.2, 2.3
L4	Transformations of Trig Functions	- Given equation state properties of sinusoidal function, and graph it using transformations - Given graph, write the equation of the transformed function	B2.4, 2.5, 2.6
L5	Applications of Trig Functions	- Solve problems arising from real world applications involving trig functions	B2.7

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Finding Trig Ratios	F		P	
PreTest Review	F/A		P	
Test – Trig in Radians	O	B1.1, 1.2, 1.3, 1.4 B2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7	P	K(21%), T(34%), A(10%), C(34%)

L1 – 4.1 Radian Measure
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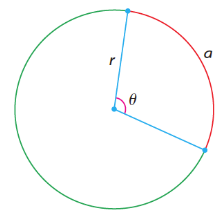
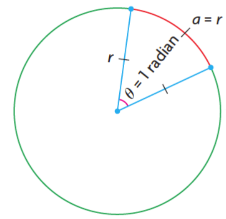
Part 1: What is a Radian?

Angles are commonly measured in **degrees**. However, in mathematics and physics, there are many applications in which expressing an angle as a pure number, without units, is more convenient than using degrees.

When measuring in **radians**, the size of an angle is expressed in terms of the length of an arc, a, that subtends the angle, θ, where $\theta = \frac{a}{r}$.

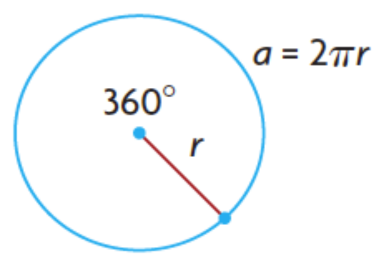


1 Radian is defined as the size of an angle that is subtended by an arc with a length **equal to the radius** of the circle.



Therefore, when the arc length and radius are equal, $\theta = \frac{a}{r} = \frac{r}{r} = 1$ radian

How many radians are in a full circle? Or in other words, how many times can an arc length equal to the radius fit around the circumference of a circle?



Remember: $C = 2\pi r$

So if we use the full circumference of the circle for the arc length,
$\theta = \frac{a}{r} = \frac{2\pi r}{r} = 2\pi$ radians

Part 2: Switching Between Degrees and Radians

The key relationship you need to know in order to switch between degrees and radians is:

$$360° = 2\pi \text{ radians}$$

1 degree = $\frac{\pi}{180}$ radians To switch from degrees to radians, multiply by $\frac{\pi}{180}$	1 radian = $\frac{180}{\pi}$ degrees To switch from radians to degrees, multiply by $\frac{180}{\pi}$

Example 1: Start by converting the following common degree measures to radians

a) $180°$

$= \cancel{180°} \times \frac{\pi}{\cancel{180}}$

$= \pi$ radians

b) $90°$

$= 90° \times \frac{\pi}{180}$

$= \frac{\pi}{2}$ radians

c) $60°$

$= 60° \times \frac{\pi}{180}$

$= \frac{\pi}{3}$ radians

d) $45°$

$= 45° \times \frac{\pi}{180}$

$= \frac{\pi}{4}$ radians

e) $30°$

$= 30° \times \frac{\pi}{180}$

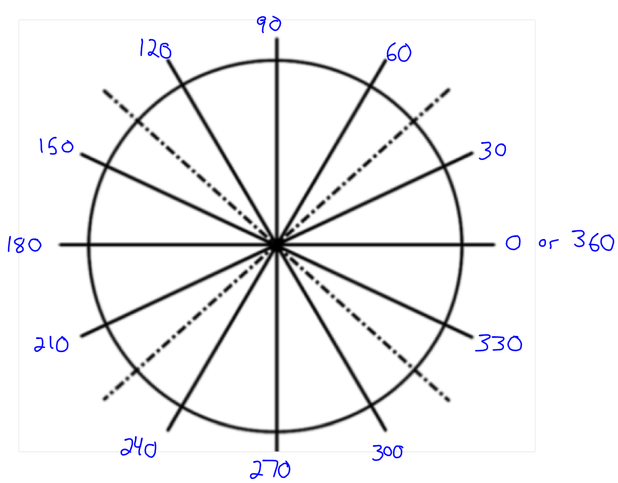
$= \frac{\pi}{6}$ radians

f) $1°$

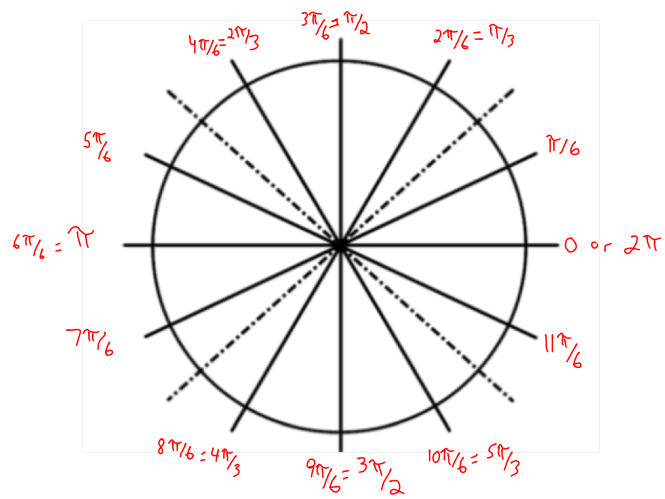
$= 1° \times \frac{\pi}{180}$

$= \frac{\pi}{180}$ radians

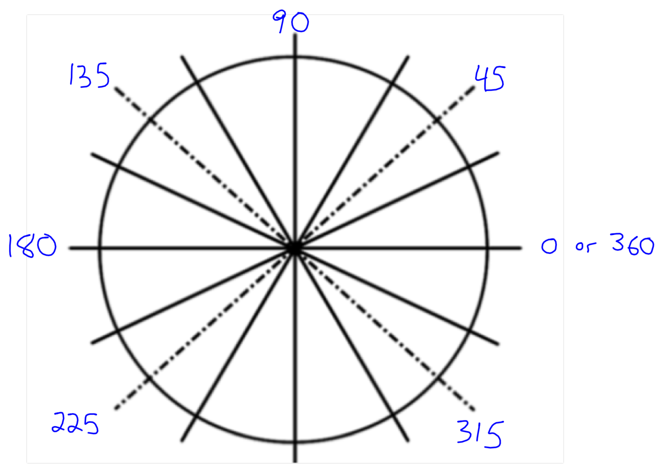
Degrees



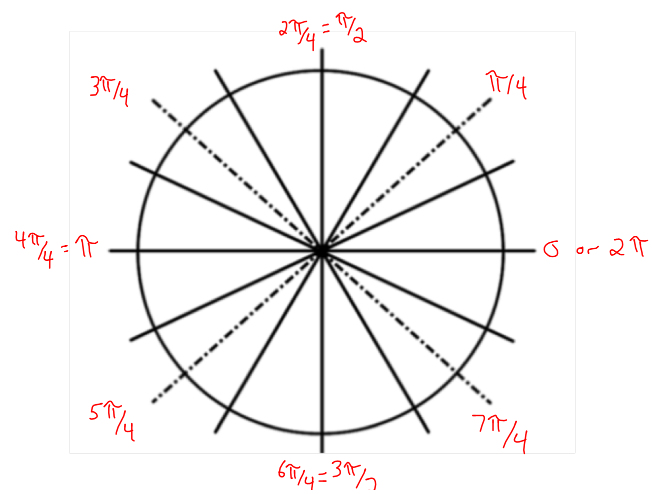
Radians



Degrees



Radians



Example 2: Convert each of the following degree measures to radian measures

a) $225°$

$= 225° \times \frac{\pi}{180}$

$= \frac{5\pi}{4}$ radians

b) $80°$

$= 80° \times \frac{\pi}{180}$

$= \frac{4\pi}{9}$ radians

c) $450°$

$= 450° \times \frac{\pi}{180}$

$= \frac{5\pi}{2}$ radians

Example 3: Convert each of the following radian measures to degree measures

a) $\frac{2\pi}{3}$ radians

$= \frac{2\pi}{3} \times \frac{180}{\pi}$

$= 120°$

b) $\frac{9\pi}{4}$ radians

$= \frac{9\pi}{4} \times \frac{180}{\pi}$

$= 405°$

c) 1 radian

$= 1 \times \frac{180}{\pi}$

$= 57.296°$

Part 3: Application

Example 4: Suzette chooses a camel to ride on a carousel. The camel is located 9 m from the center of the carousel. It the carousel turns through an angle of $\frac{5\pi}{6}$, determine the length of the arc travelled by the camel, to the nearest tenth of a meter.

$\theta = \frac{a}{r}$

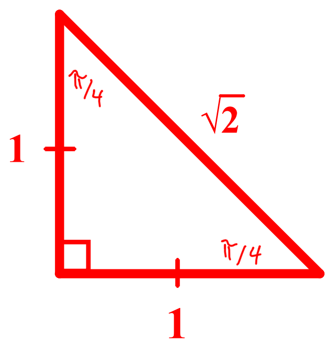
$a = \theta r$

$a = \frac{5\pi}{6}(9)$

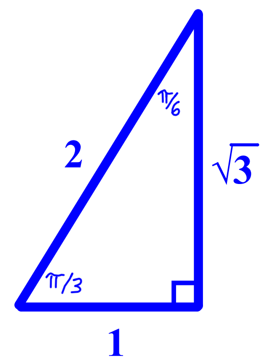
$a \cong 23.6$ m

<u>Part 4: Special Triangles Using Radian Measures</u>

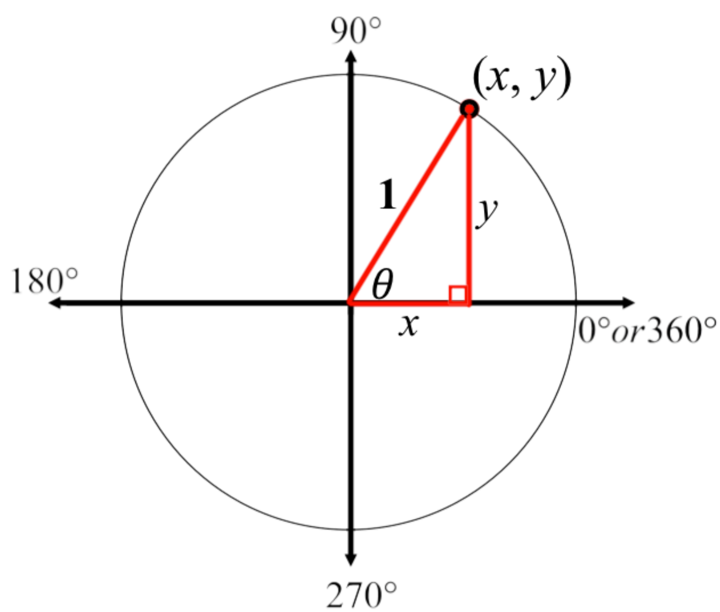
Draw the 45-45-90 triangle



Draw the 30-60-90 triangle



Also, you will need to remember the UNIT CIRLCE which is a circle that has a radius of 1. Use the unit circle to write expressions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ in terms of x, y, and r.



$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$

On the unit circle, the sine and cosine functions take a simple form:

$\sin\theta = y$

$\cos\theta = x$

The value of $\sin\theta$ is the <u>**y-coordinate**</u> of each point on the unit circle

The value of $\cos\theta$ is the <u>**x-coordinate**</u> of each point on the unit circle

Therefore, we can use the points where the terminal arm intersects the unit circle to get the sine and cosine ratios just by looking at the y and x co-ordinates of the points.

Example 5: Fill out chart of ratios using special triangles and the unit circle

	$0, 2\pi$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	0	undefined

L2 – 4.2 Trig Ratios and Special Angles
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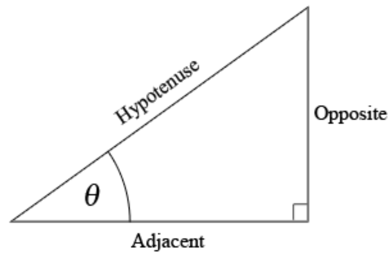
Part 1: Review of Last Year Trig

What is SOHCAHTOA?

If we know a right angle triangle has an angle of θ, all other right angle triangles with an angle of θ are **similar** and therefore have equivalent ratios of corresponding sides. The three primary ratios are shown in the diagram to the right.



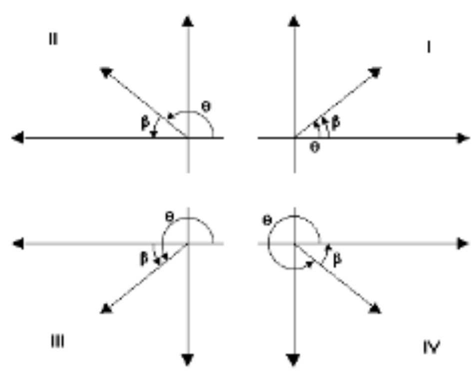
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

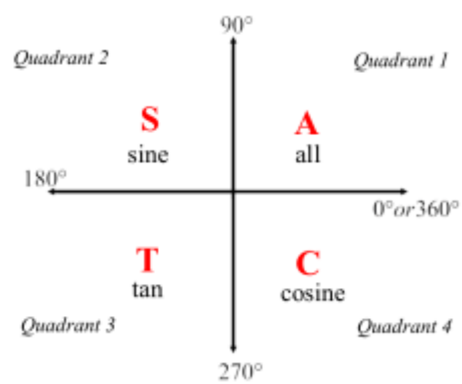
What is a reference angle?

Any angle over 90 has a **reference angle**. The reference angle is between 0° and 90° and helps us determine the exact trig ratios when we are given an obtuse angle (angle over 90 degrees). The reference angle is the angle between the terminal arm and the **closest x-axis** (0/360 or 180).



What is the CAST rule?

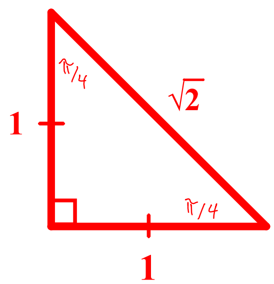
When finding the trig ratios of positive angles, we are rotating counter clockwise from 0 degrees toward 360. The **CAST rule** helps us determine which trig ratios **are positive** in each **quadrant**

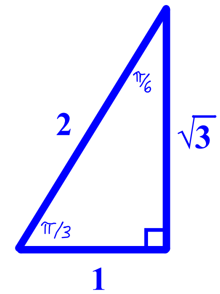


Note: *There are multiple angles that have the same trig ratio. You can use reference angles and the cast rule to find them.*

Part 2: Finding Exact Trig Ratios for Special Angles

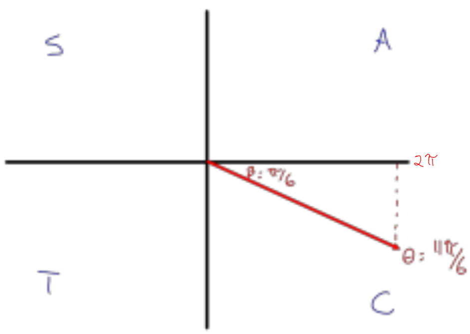
Start by drawing both special triangles using radian measures





Example 1: Find the exact value for each given trig ratio.

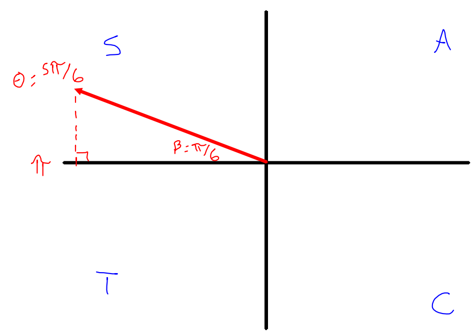
a) $\tan\frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$



$\beta = 2\pi - 11\pi/6$
$= 12\pi/6 - 11\pi/6$
$= \pi/6$

$\tan 11\pi/6 = -\tan \pi/6$
$= -1/\sqrt{3}$

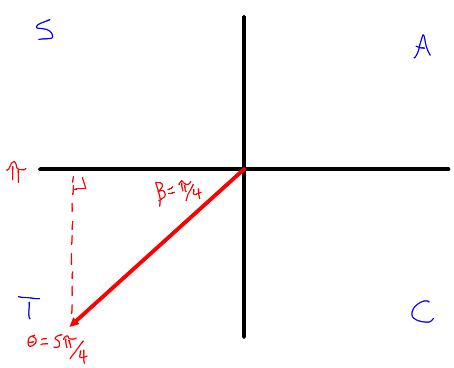
b) $\sin\frac{5\pi}{6} = \frac{1}{2}$



$\beta = \pi - 5\pi/6$
$= 6\pi/6 - 5\pi/6$
$= \pi/6$

$\sin 5\pi/6 = \sin \pi/6$
$= 1/2$

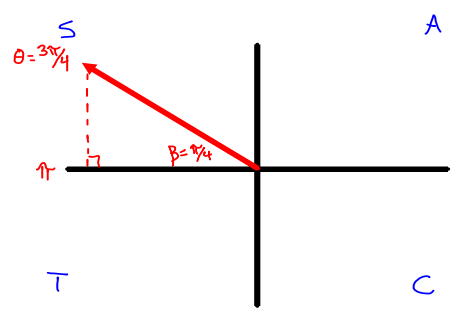
c) $\cos\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$



$\beta = 5\pi/4 - \pi$
$= 5\pi/4 - 4\pi/4$

$\cos 5\pi/4 = -\cos \pi/4$
$= -\frac{1}{\sqrt{2}}$

d) $\sec\frac{3\pi}{4} = -\sqrt{2}$



$\beta = \pi - 3\pi/4$
$= 4\pi/4 - 3\pi/4$
$= \pi/4$

$\sec 3\pi/4 = \frac{1}{\cos 3\pi/4}$
$= -\frac{1}{\cos \pi/4}$
$= -\frac{1}{\left(\frac{1}{\sqrt{2}}\right)}$
$= -\sqrt{2}$

Example 2: Find the value of all 6 trig ratios for $\frac{5\pi}{3}$

$$\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

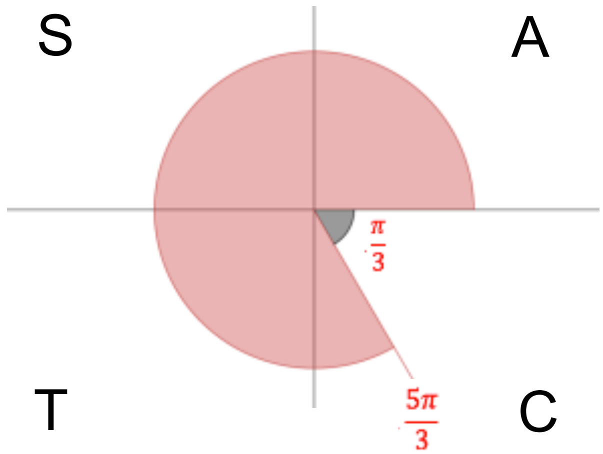
$$\csc\frac{5\pi}{3} = -\frac{2}{\sqrt{3}}$$

$$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\sec\frac{5\pi}{3} = 2$$

$$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$$

$$\cot\frac{5\pi}{3} = -\frac{1}{\sqrt{3}}$$



Part 3: Application Question

Justin is flying a kite at the end of a 50-m string. The sun is directly overhead, and the string makes an angle of $\frac{\pi}{6}$ with the ground. The wind speed increases, and the kite flies higher until the string makes an angle of $\frac{\pi}{3}$ with the ground. Determine an exact expression for the horizontal distance between the two positions of the kite along the ground.

$$\cos\frac{\pi}{6} = \frac{x_1}{50}$$

$$\frac{\sqrt{3}}{2} = \frac{x_1}{50}$$

$$x_1 = \frac{50\sqrt{3}}{2}$$

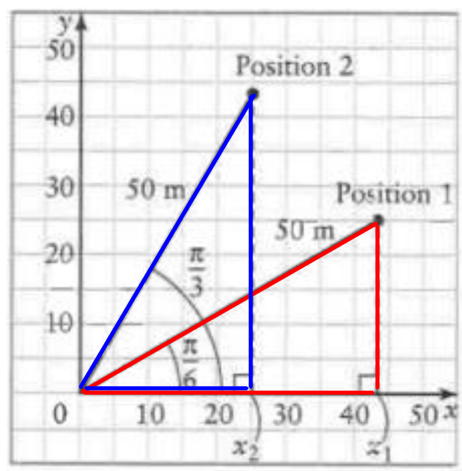
$$x_1 = 25\sqrt{3}$$

$$\cos\frac{\pi}{3} = \frac{x_2}{50}$$

$$\frac{1}{2} = \frac{x_2}{50}$$

$$x_2 = \frac{50}{2}$$

$$x_2 = 25$$



The horizontal distance between the two kites $= x_1 - x_2 = 25\sqrt{3} - 25 = 25(\sqrt{3} - 1)$ meters

L3 – 5.1/5.2 Graphing Trig Functions
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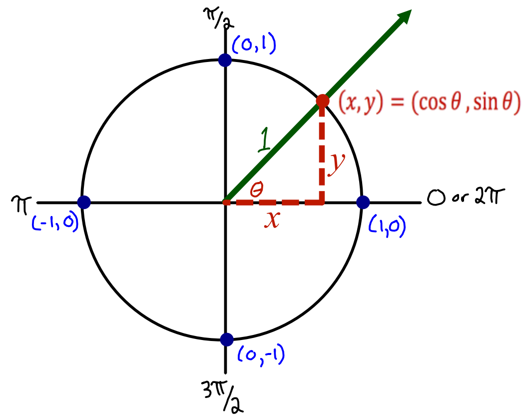
Part 1: Remember the Unit Circle https://www.geogebra.org/m/tKkYHMXC

The unit circle is a circle is a circle that is centered at the origin and has a radius of **1 unit**. On the unit circle, the sine and cosine functions take a simple form:

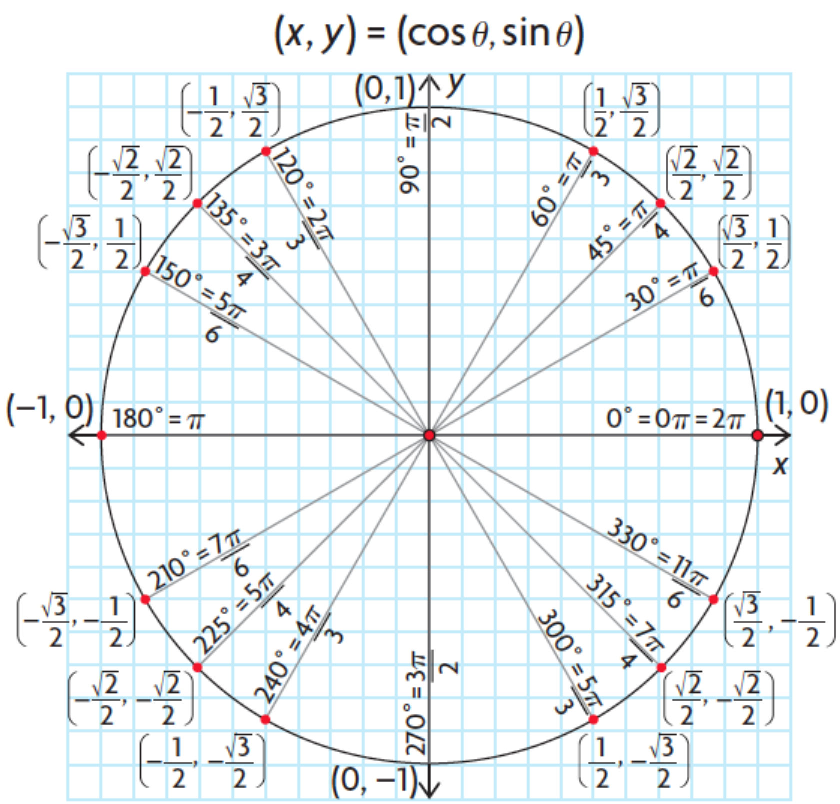
$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$



The value of sin θ is the ***y*-coordinate** of each point on the unit circle

The value of cos θ is the ***x*-coordinate** of each point on the unit circle

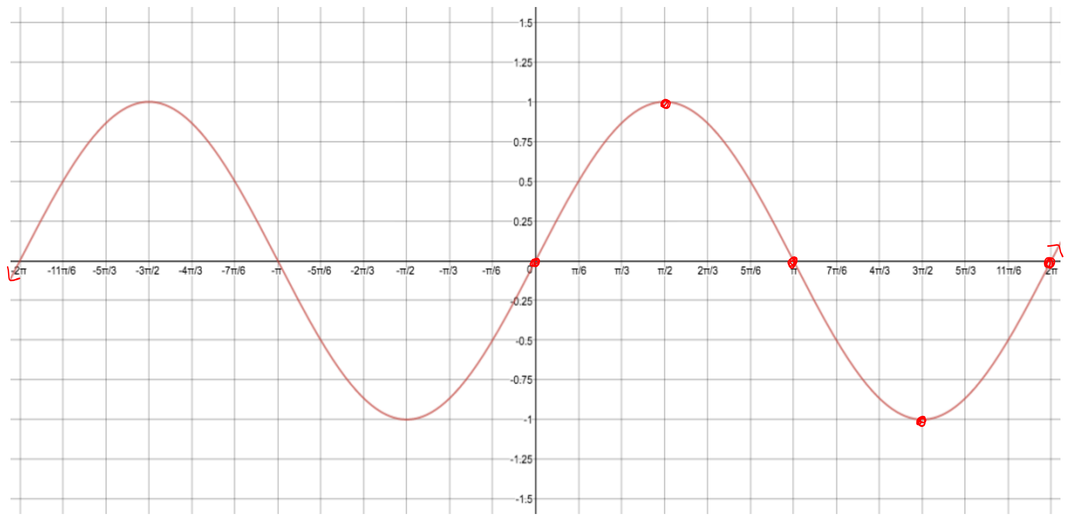


Part 2: Graphing Sine and Cosine

To graph sine and cosine, we will be using a Cartesian plane that has angles for x values.

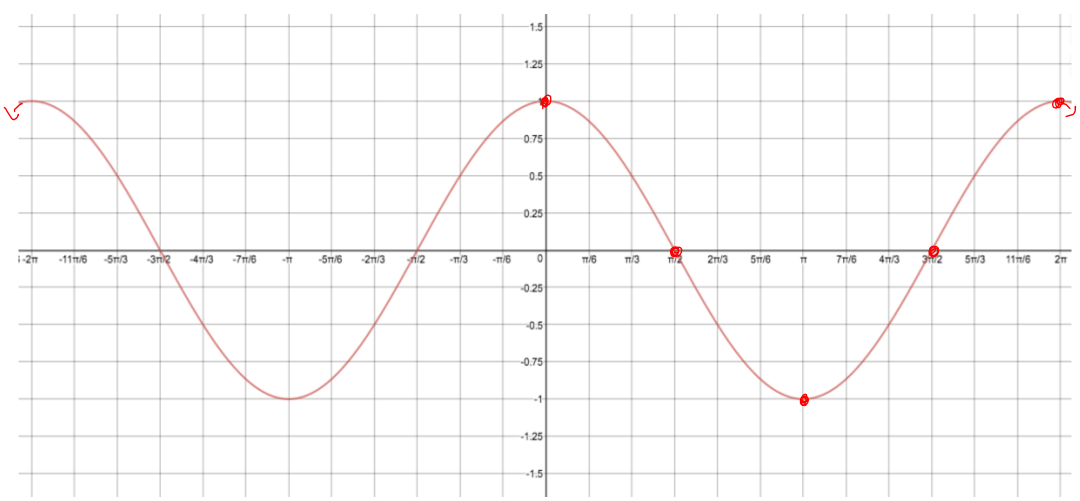
Example 1: Complete the following table of values for the function $f(x) = \sin(x)$. Use special triangles, the unit circle, or a calculator to find values for the function at $30° = \frac{\pi}{6}$ radian intervals.

x	$\sin x$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{5\pi}{6}$	$\frac{1}{2}$
$\frac{6\pi}{6} = \pi$	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{11\pi}{6}$	$-\frac{1}{2}$
$\frac{12\pi}{6} = 2\pi$	0



Example 2: Complete the following table of values for the function $f(x) = \cos(x)$. Use special triangles, the unit circle, or a calculator to find values for the function at $30° = \frac{\pi}{6}$ radian intervals.

x	$\cos x$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{2}$
$\frac{3\pi}{6} = \frac{\pi}{2}$	0
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{6\pi}{6} = \pi$	-1
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{1}{2}$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$\frac{1}{2}$
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{12\pi}{6} = 2\pi$	1



Properties of both Sine and Cosine Functions

Domain: $\{X \in \mathbb{R}\}$

Range: $\{Y \in \mathbb{R} | -1 \leq y \leq 1\}$

Period: 2π radians

Amplitude: $\frac{max-min}{2} = \frac{1-(-1)}{2} = 1$

PERIOD: *the horizontal length of one cycle on a graph.*

AMPLITUDE: *half the distance between the maximum and minimum values of a periodic function.*

Part 3: Graphing the Tangent Function

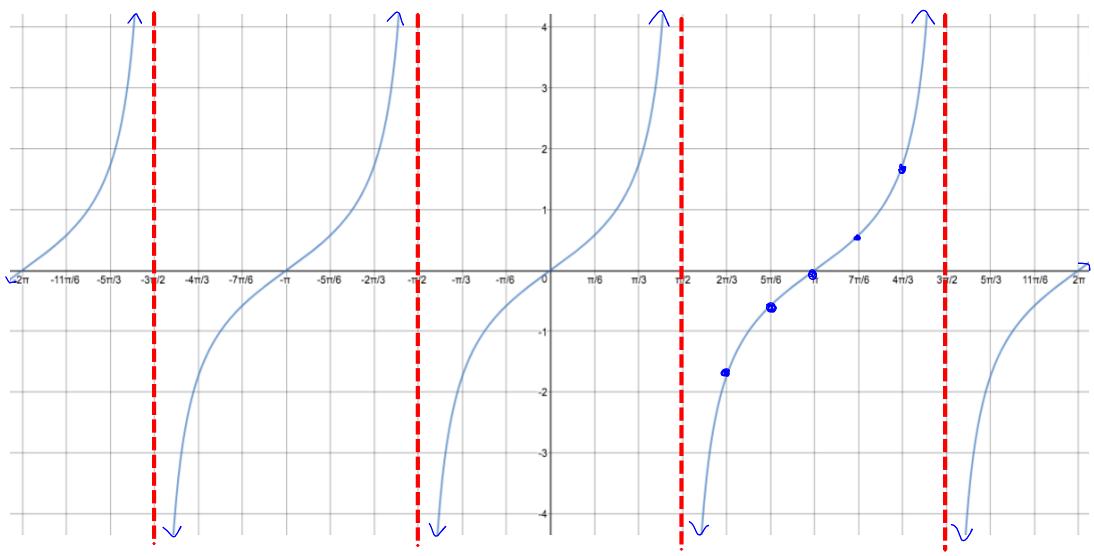
Recall: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Note: Since $\cos\theta$ is in the denominator, any time $\cos\theta = 0$, $\tan\theta$ will be undefined which will lead to a vertical asymptote.

Since $\sin\theta$ is in the numerator, any time $\sin\theta = 0$, $\tan\theta$ will equal 0 which will be an x-intercept.

Example 3: Complete the following table of values for the function $f(x) = \tan(x)$. Use the quotient identity to find y-values.

x	$\tan x$
0	$\frac{0}{1} = 0$
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\sqrt{3} \sim 1.73$
$\frac{3\pi}{6} = \frac{\pi}{2}$	$\frac{1}{0} =$ und.
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\sqrt{3} \sim -1.73$
$\frac{5\pi}{6}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{6\pi}{6} = \pi$	$\frac{0}{-1} = 0$
$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\sqrt{3} \sim 1.73$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	$\frac{-1}{0} =$ und.
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\sqrt{3} \sim -1.73$
$\frac{11\pi}{6}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{12\pi}{6} = 2\pi$	$\frac{0}{1} = 0$



Properties of the Tangent Function

Domain: $\{X \in \mathbb{R} \mid x \neq \frac{k\pi}{2}$ *when k is odd*$\}$

Range: $\{Y \in \mathbb{R}\}$

Period: π radians

Amplitude: none (no max or min)

Part 4: Graphing Reciprocal Trig Functions

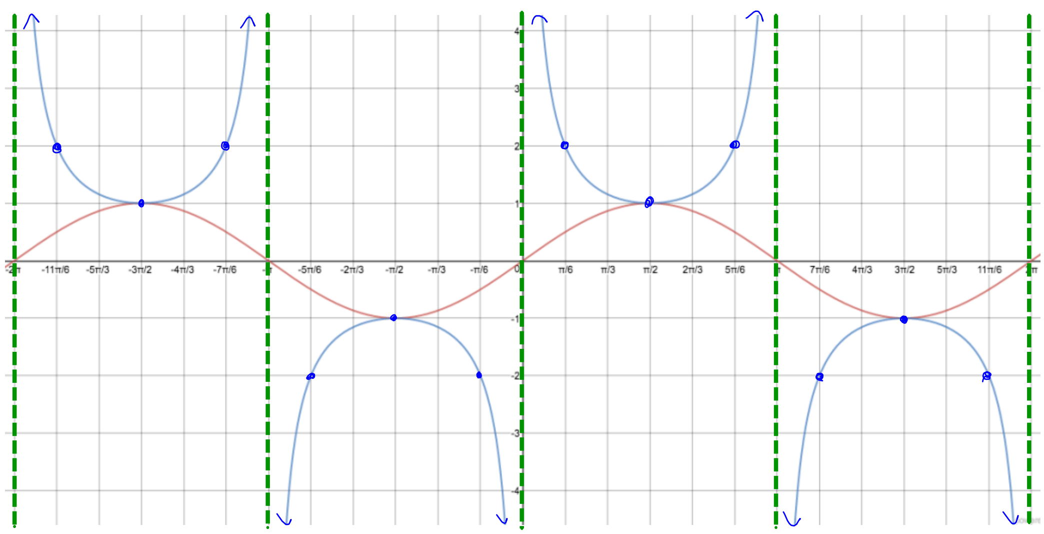
Reciprocal Identities		
$csc\ \theta = \dfrac{1}{sin\ \theta}$	$sec\ \theta = \dfrac{1}{cos\ \theta}$	$cot\ \theta = \dfrac{1}{tan\ \theta}$

The graph of a reciprocal trig function is related to the graph of its corresponding primary trig function in the following ways:

- Reciprocal has a vertical asymptote at each zero of its primary trig function
- Reciprocal has a zero at each vertical asymptote of its primary trig function
- Has the same positive/negative intervals but intervals of increasing/decreasing are reversed
- y-values of 1 and -1 do not change and therefore this is where the reciprocal and primary intersect
- Local min points of the primary become local max of the reciprocal and vice versa.

Example 4: Complete the following table of values for the function $f(x) = \csc(x)$. Use the reciprocal identity to find y-values.

x	$\csc x$
0	und.
$\frac{\pi}{6}$	2
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{5\pi}{6}$	2
$\frac{6\pi}{6} = \pi$	UNDEFINED
$\frac{7\pi}{6}$	-2
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{11\pi}{6}$	-2
$\frac{12\pi}{6} = 2\pi$	und.



Properties of the Cosecant Function

Domain: $\{X \in \mathbb{R} \mid x \neq k\pi\}$ where $k \in Z$

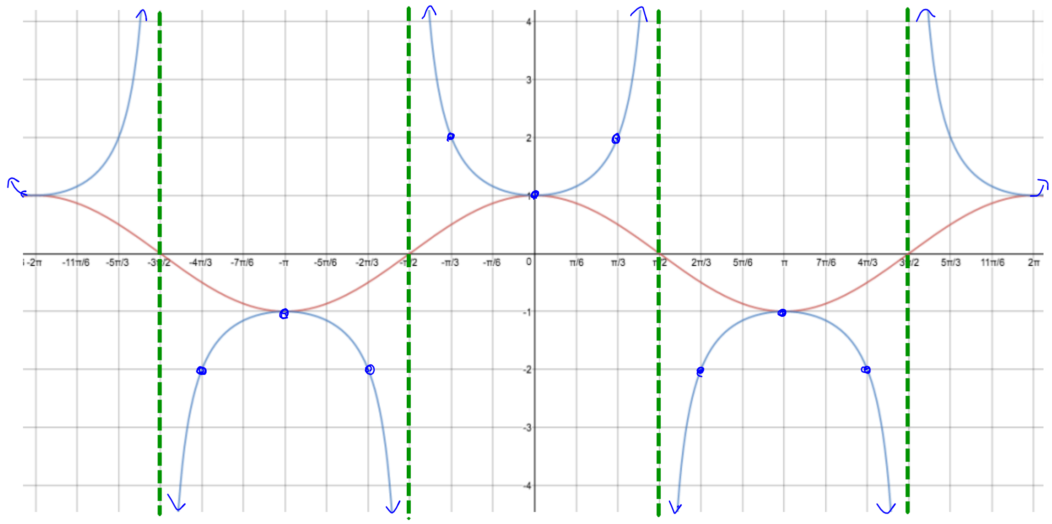
Range: $\{Y \in \mathbb{R} | y \leq -1\ or\ y \geq 1\}$

Period: 2π radians

Amplitude: none (no max or min)

Example 5: Complete the following table of values for the function $f(x) = sec\,(x)$. Use the reciprocal identity to find y-values.

x	$\sec x$
0	1
$\frac{\pi}{6}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{2\pi}{6} = \frac{\pi}{3}$	2
$\frac{3\pi}{6} = \frac{\pi}{2}$	und.
$\frac{4\pi}{6} = \frac{2\pi}{3}$	-2
$\frac{5\pi}{6}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{6\pi}{6} = \pi$	-1
$\frac{7\pi}{6}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	-2
$\frac{9\pi}{6} = \frac{3\pi}{2}$	und.
$\frac{10\pi}{6} = \frac{5\pi}{3}$	2
$\frac{11\pi}{6}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{12\pi}{6} = 2\pi$	1



Properties of the Secant Function

Domain: $\{X \in \mathbb{R} \mid x \neq \frac{\pi+2k\pi}{2}\}$ where $k \in Z$

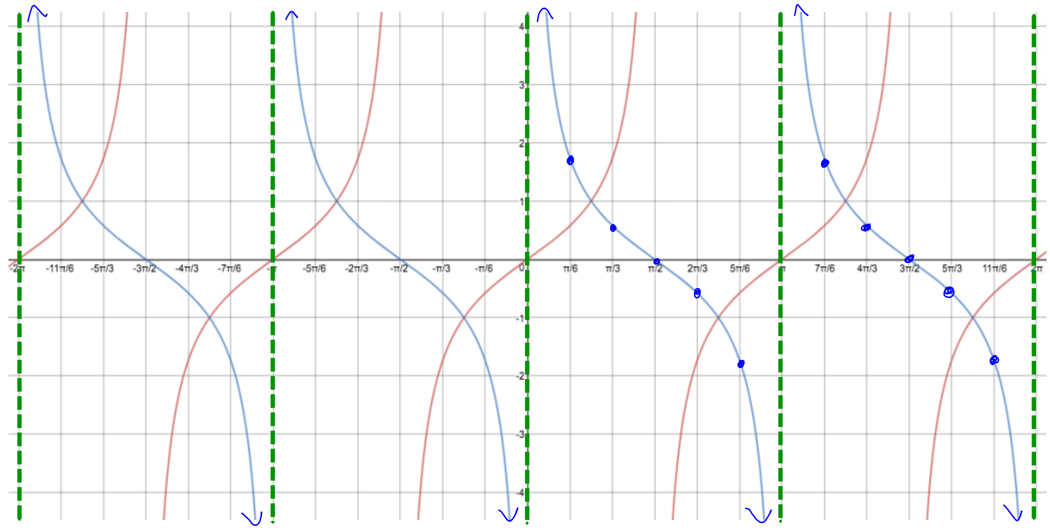
Range: $\{Y \in \mathbb{R} | y \leq -1 \; or \; y \geq 1\}$

Period: 2π radians

Amplitude: none (no max or min)

Example 6: Complete the following table of values for the function $f(x) = \cot(x)$. Use the reciprocal identity to find y-values.

x	$\cot x$
0	und.
$\frac{\pi}{6}$	$\sqrt{3}\sim 1.73$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{\sqrt{3}}\sim 0.58$
$\frac{3\pi}{6} = \frac{\pi}{2}$	0
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{\sqrt{3}}\sim -0.58$
$\frac{5\pi}{6}$	$-\sqrt{3}\sim -1.73$
$\frac{6\pi}{6} = \pi$	und.
$\frac{7\pi}{6}$	$\sqrt{3}\sim 1.73$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\frac{1}{\sqrt{3}}\sim 0.58$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{1}{\sqrt{3}}\sim -0.58$
$\frac{11\pi}{6}$	$-\sqrt{3}\sim -1.73$
$\frac{12\pi}{6} = 2\pi$	und.



Properties of the Cotangent Function

Domain: $\{X \in \mathbb{R} \mid x \neq k\pi\}$ where $k \in Z$

Range: $\{Y \in \mathbb{R}\}$

Period: π radians

Amplitude: none (no max or min)

L4 – 5.3 Transformations of Trig Functions
MHF4U

Part 1: Transformation Properties

$y = a\sin[k(x-d)] + c$

Desmos Demonstration

a	k	d	c
Vertical stretch or compression by a factor of $\lvert a\rvert$. Vertical reflection if $a < 0$ $\lvert a\rvert = amplitude$	Horizontal stretch or compression by a factor of $\frac{1}{\lvert k\rvert}$. Horizontal reflection if $k < 0$. $\frac{2\pi}{\lvert k\rvert} = period$	Phase shift $d > 0; shift\ right$ $d < 0; shift\ left$	Vertical shift $c > 0; shift\ up$ $c < 0; shift\ down$

Example 1: For the function $y = 3\sin\left[\frac{1}{2}\left(\theta + \frac{\pi}{3}\right)\right] - 1$, state the…

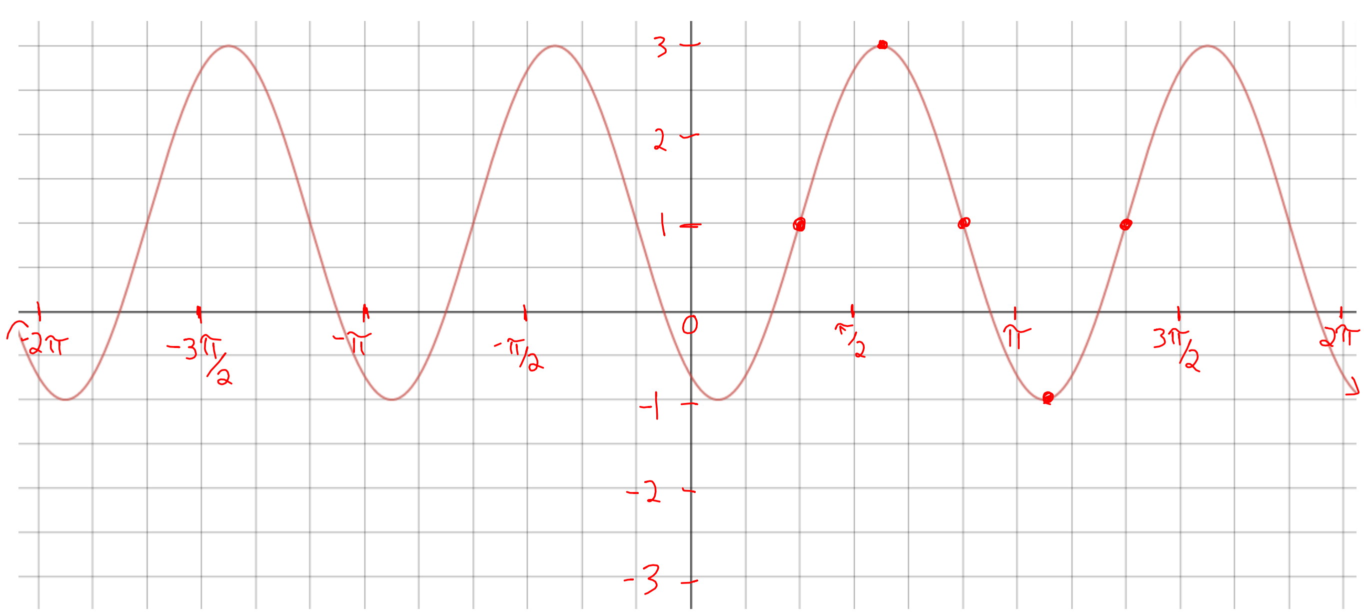
Amplitude: $amp = \lvert a\rvert = 3$	Period: $period = \frac{2\pi}{\lvert k\rvert} = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$
Phase shift: $d = -\frac{\pi}{3}$; shift left $\frac{\pi}{3}$	Vertical shift: $c = -1$; shift down 1
Max: $max = c + amp = -1 + 3 = 2$	Min: $min = c - amp = -1 - 3 = -4$

Part 2: Given Equation → Graph Function

Example 2: Graph $y = 2\sin\left[2\left(x - \frac{\pi}{3}\right)\right] + 1$ using transformations. Then state the amplitude and period of the function.

$y = \sin x$	
$\boldsymbol{x}$	$\boldsymbol{y}$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$y = 2\sin\left[2\left(x - \frac{\pi}{3}\right)\right] + 1$	
$\boldsymbol{\frac{x}{2} + \frac{\pi}{3}}$	$\boldsymbol{2y + 1}$
$\frac{\pi}{3} = \frac{2\pi}{6}$	1
$\frac{7\pi}{12} = \frac{3.5\pi}{6}$	3
$\frac{5\pi}{6}$	1
$\frac{13\pi}{12} = \frac{6.5\pi}{6}$	-1
$\frac{4\pi}{3} = \frac{8\pi}{6}$	1



Amplitude: $\frac{max - min}{2} = \frac{3-(-1)}{2} = 2$

Period: π radians

Part 3: Given the Graph → Write the Equation

$y = a\sin[k(x-d)] + c$

a	k	d	c
Find the amplitude of the function: $a = \dfrac{max - min}{2}$	Find the period (in radians) of the function using a starting point and ending point of a full cycle. $k = \dfrac{2\pi}{period}$	**for sin x:** x-coordinate of a rising mid-line. **for cos x:** x-coordinate of a maximum point. $d_{sin} = d_{cos} - \dfrac{\pi}{2k}$ $d_{cos} = d_{sin} + \dfrac{\pi}{2k}$	Find the vertical shift $c = max - amplitude$ OR $c = \dfrac{max + min}{2}$ (this finds the 'middle' of the function)

Example 3: Determine the equation of a sine and cosine function that describes the following graph

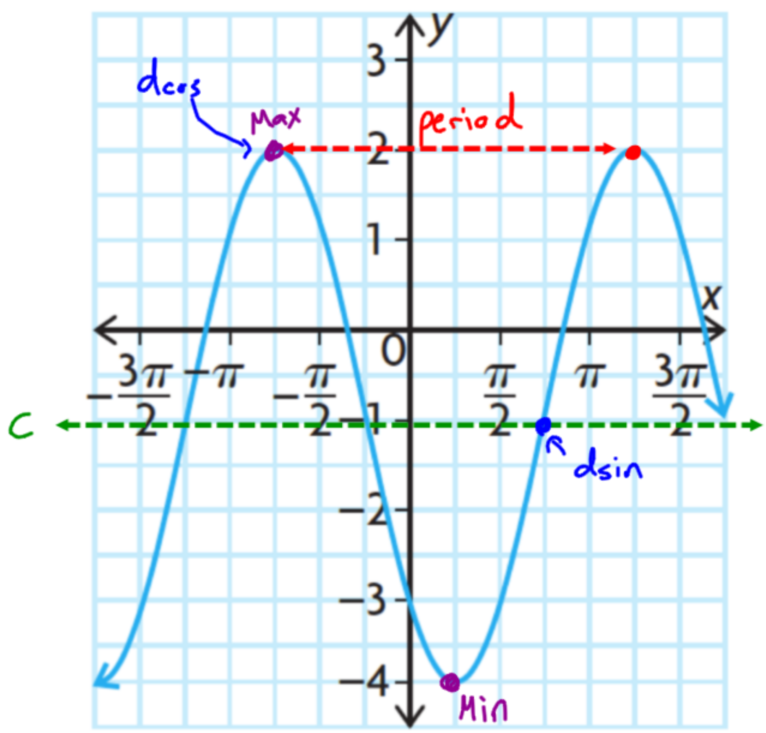
$a = \dfrac{max - min}{2} = \dfrac{2-(-4)}{2} = 3$

$k = \dfrac{2\pi}{period} = \dfrac{2\pi}{\left(\frac{5\pi}{4} - \left(-\frac{3\pi}{4}\right)\right)} = \dfrac{2\pi}{2\pi} = 1$

$c = max - |a| = 2 - 3 = -1$

$d_{cos} = \dfrac{-3\pi}{4}$

$d_{sin} = \dfrac{3\pi}{4}$



$$\boxed{\begin{aligned} y &= 3\sin\left(x - \tfrac{3\pi}{4}\right) - 1 \\ y &= 3\cos\left(x + \tfrac{3\pi}{4}\right) - 1 \end{aligned}}$$

Example 4: Determine the equation of a sine and cosine function that describes the following graph

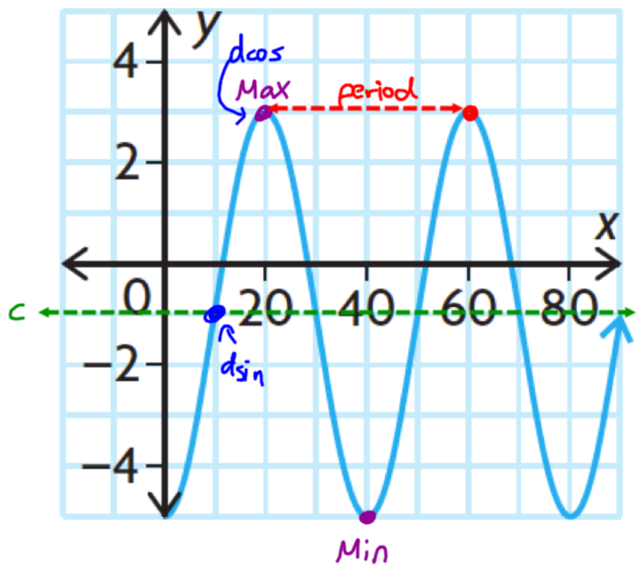
$a = \frac{max - min}{2} = \frac{3-(-5)}{2} = 4$

$k = \frac{2\pi}{period} = \frac{2\pi}{40} = \frac{\pi}{20}$

$c = max - |a| = 3 - 4 = -1$

$d_{cos} = 20$

$d_{sin} = 10$



$$\boxed{\begin{aligned} y &= 4\cos\left[\frac{\pi}{20}(x-20)\right] - 1 \\ y &= 4\sin\left[\frac{\pi}{20}(x-10)\right] - 1 \end{aligned}}$$

Example 5:

a) Create a sine function with an amplitude of 7, a period of π, a phase shift of $\frac{\pi}{4}$ right, and a vertical displacement of -3.

$a = 7$

$k = \frac{2\pi}{period} = \frac{2\pi}{\pi} = 2$

$c = -3$

$d = \frac{\pi}{4}$

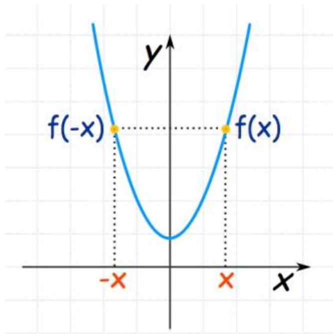
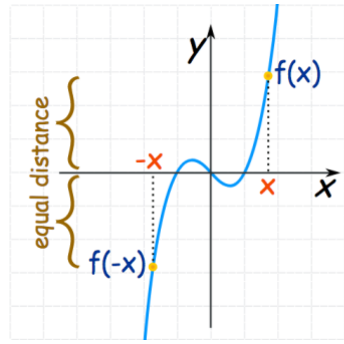
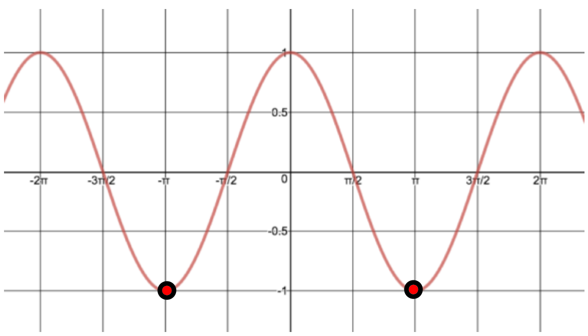
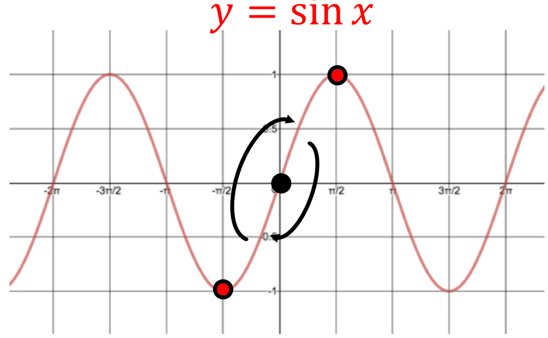
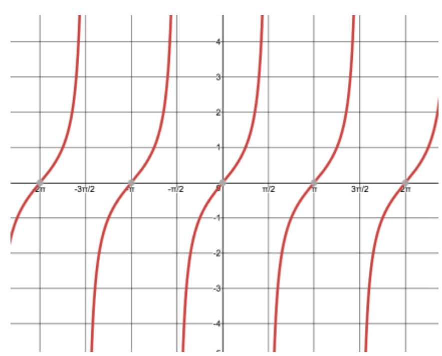
$$\boxed{y = 7\sin\left[2\left(x - \frac{\pi}{4}\right)\right] - 3}$$

b) What would be the equation of a cosine function that represents the same graph as the sine function above?

$d_{cos} = d_{sin} + \frac{\pi}{2k} = \frac{\pi}{4} + \frac{\pi}{2(2)} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\boxed{y = 7\cos\left[2\left(x - \frac{\pi}{2}\right)\right] - 3}$$

Part 4: Even and Odd Functions

Even Functions	Odd Functions
EVEN FUNCTION if: Line symmetry over the y-axis	ODD FUNCTION if: Point symmetry about the origin (0, 0)
Rule: $f(-x) = f(x)$  	Rule: $-f(x) = f(-x)$ 
Example: $y = \cos x$  $f(\pi) = -1$ $f(-\pi) = -1$ Therefore, $f(\pi) = f(-\pi)$	Example: $y = \sin x$  $f\left(\frac{\pi}{2}\right) = 1$ $f\left(-\frac{\pi}{2}\right) = -1$ Therefore, $-f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$ $y = \tan x$ is also an odd function 

L5 – 5.3 Trig Applications
MHF4U

Example 1: A Ferris wheel has a diameter of 15m and is 6m above ground level at its lowest point. It takes the rider 30s from their minimum height above the ground to reach the maximum height of the ferris wheel. Assume the rider starts the ride at the min point.

a) Model the vertical displacement of the rider vs. time using a sine function.

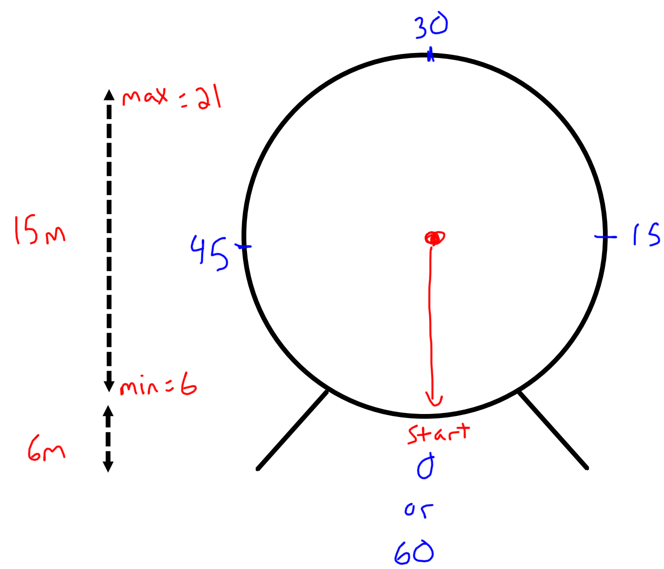
$$a = \frac{max - min}{2} = \frac{21 - 6}{2} = 7.5$$

$$k = \frac{2\pi}{period} = \frac{2\pi}{60} = \frac{\pi}{30}$$

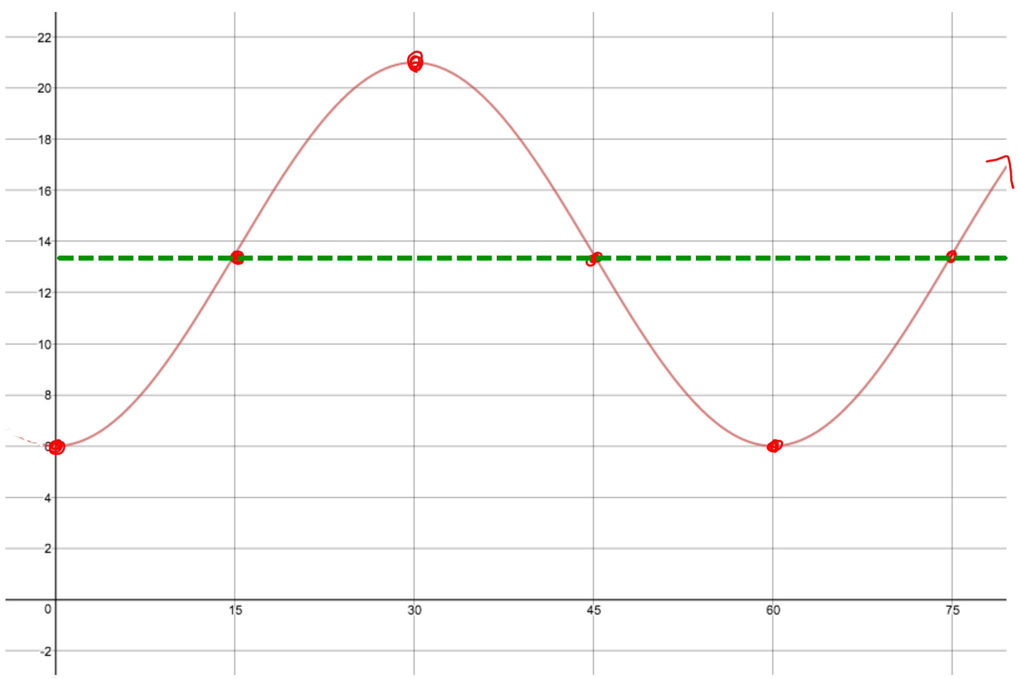
$$c = max - a = 21 - 7.5 = 13.5$$

$$d_{sin} = 15$$

$$h(t) = 7.5 \sin\left[\frac{\pi}{30}(t - 15)\right] + 13.5$$



b) Sketch a graph of this function



Example 2: Mr. Ponsen has the heating system, in this room, turn on when the room reaches a min of 66°F, it heats the room to a maximum temperature of 76°F and then turns off until it returns to the minimum temperature of 66°F. This cycle repeats every 4 hours. Mr. Ponsen ensures that this room is at its max temperature at 9 am.

a) Write a cosine function that gives the temperature at King's, *T*, in °F, as a function of *h* hours after midnight.

$$a = \frac{max - min}{2} = \frac{76 - 66}{2} = 5$$

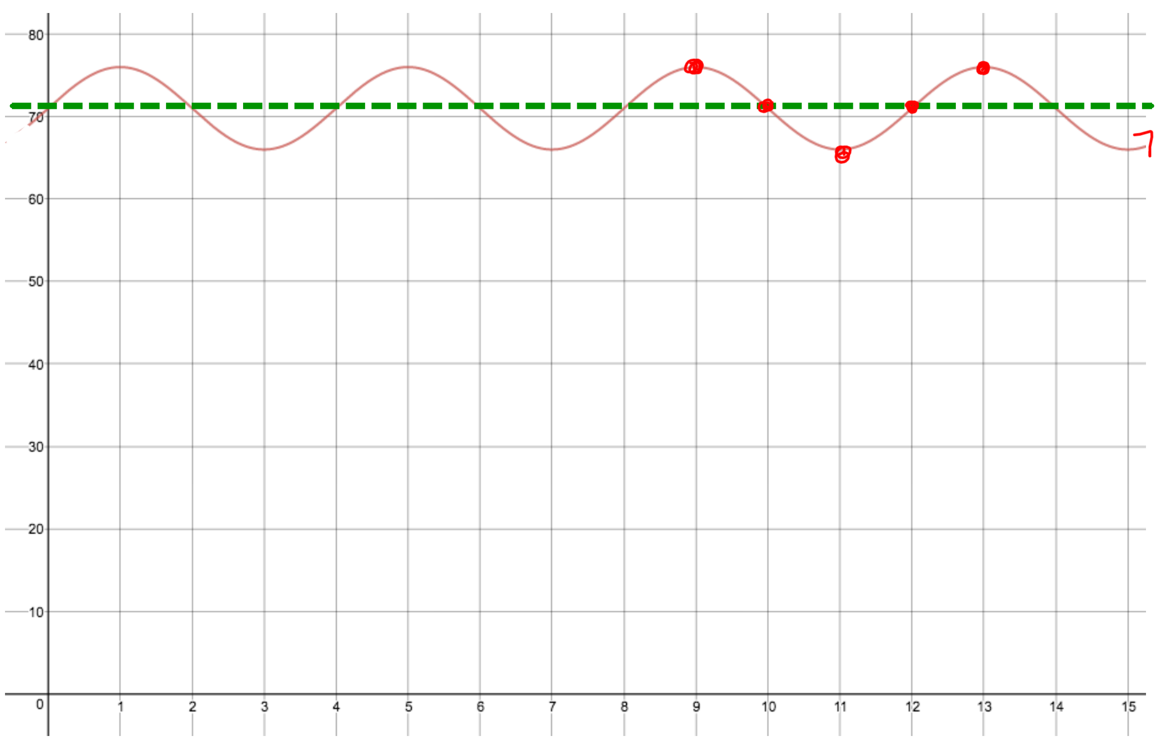
$$k = \frac{2\pi}{period} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$c = max - a = 76 - 5 = 71$$

$$d_{cos} = 9$$

$$T(h) = 5\cos\left[\frac{\pi}{2}(h - 9)\right] + 71$$

b) Sketch a graph of the function.



Example 3: The tides at Cape Capstan, New Brunswick, change the depth of the water in the harbor. On one day in October, the tides have a high point of approximately 10 meters at 2 pm and a low point of 1.2 meters at 8:15 pm. A particular sailboat has a draft of 2 meters. This means it can only move in water that is atleast 2 meters deep. The captain of the sailboat plans to exit the harbor at 6:30 pm. Assuming $t = 0$ is noon, find the height of the tide at 6:30 pm. Is it safe for the captain to exit the harbor at this time?

$$a = \frac{max - min}{2} = \frac{10 - 1.2}{2} = 4.4$$

$$k = \frac{2\pi}{period} = \frac{2\pi}{12.5} = \frac{4\pi}{25}$$

> ***Note:*** *Horizontal distance between max and min points represent half a cycle. Since max and min tide are 6.25 hours apart, one cycle must be 12.5 hours.*

$$c = max - a = 10 - 4.4 = 5.6$$

$$d_{cos} = 2$$

$$h(t) = 4.4\cos\left[\frac{4\pi}{25}(t - 2)\right] + 5.6$$

$$h(6.5) = 4.4\cos\left[\frac{4\pi}{25}(6.5 - 2)\right] + 5.6$$

$$h(6.5) = 4.4\cos\left[\frac{4\pi}{25}(4.5)\right] + 5.6$$

$$h(6.5) = 4.4\cos\left[\frac{18\pi}{25}\right] + 5.6$$

$h(6.5) \cong 2.8$ meters

Since the depth of the water is greater than 2 meters, the sailboat can safely exit the harbor.