**Unit 2 Pre-Test Review – Factor Theorem and Inequalities**
**MHF4U**

SOLUTIONS

## Section 1: 2.1 - Long and Synthetic Division / Remainder Theorem

**1)** What is the remainder when $x^4 - 4x^2 - 2x + 3$ is divided by $x + 1$? Do not divide. Support your answer with an explanation.

$$P(-1) = (-1)^4 - 4(-1)^2 - 2(-1) + 3$$
$$= 1 - 4 + 2 + 3$$
$$= 2$$

Based on the remainder theorem, the remainder will be 2.

**2)** Is $x - 3$ a factor of the polynomial $3x^2 - 8x - 3$? Do not divide. Support your answer with an explanation.

$$P(3) = 3(3)^2 - 8(3) - 3$$
$$= 27 - 24 - 3$$
$$= 0$$

Yes because the remainder is 0.

**3)** Divide $\frac{f(x)}{g(x)}$ and state the answer in quotient form. Use synthetic division where possible.

**a)** $f(x) = x^4 - 4x^2 - 2x + 3, g(x) = x - 2$

$$\begin{array}{r|rrrrr}
2 & 1 & 0 & -4 & -2 & 3 \\
 & \downarrow & 2 & 4 & 0 & -4 \\
\hline
x & 1 & 2 & 0 & -2 & -1 \\
 & x^3 & x^2 & x & \# & R
\end{array}$$

$$\boxed{\frac{x^4-4x^2-2x+3}{x-2} = x^3+2x^2-2 - \frac{1}{x-2}}$$

**b)** $f(x) = x^5 - x^4 + 2x^3 + 3x - 2, \quad g(x) = x^2 + 2$

$$\begin{array}{r|l}
 & x^3 - 1x^2 + 0x + 2 \\
\hline
x^2+0x+2 & x^5 - x^4 + 2x^3 + 0x^2 + 3x - 2 \\
 & x^5 + 0x^4 + 2x^3 \\
\hline
 & -1x^4 + 0x^3 + 0x^2 \\
 & -1x^4 + 0x^3 - 2x^2 \\
\hline
 & 0x^3 + 2x^2 + 3x \\
 & 0x^3 + 0x^2 + 0x \\
\hline
 & 2x^2 + 3x - 2 \\
 & 2x^2 + 0x + 4 \\
\hline
 & R = 3x - 6
\end{array}$$

$$\boxed{\frac{x^5-x^4+2x^3+3x-2}{x^2+2} = x^3 - x^2 + 2 + \frac{3x-6}{x^2+2}}$$

**4)** Perform each division. Express the answer in quotient form and write the statement that could be used to check the division.

**a)** $x^3 + 9x^2 - 5x + 3$ divided by $x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 9 & -5 & 3 \\ & \downarrow & 2 & 22 & 34 \\ \hline \times & 1 & 11 & 17 & 37 \\ & x^2 & x & \# & R \end{array} \quad +$$

Q.F.: $\dfrac{x^3+9x^2-5x+3}{x-2} = x^2+11x+17 + \dfrac{37}{x-2}$

As product:

$x^3+9x^2-5x+3 = (x-2)(x^2+11x+7)+37$

**b)** $12x^3 - 2x^2 + x - 11$ divided by $3x + 1$

$$\begin{array}{r|l} & 4x^2 - 2x + 1 \\ 3x+1 & 12x^3 - 2x^2 + x - 11 \\ & \underline{12x^3 + 4x^2} \\ & \quad -6x^2 + x \\ & \quad \underline{-6x^2 - 2x} \\ & \qquad 3x - 11 \\ & \qquad \underline{3x + 1} \\ & \qquad R = -12 \end{array}$$

Q.F.: $\dfrac{12x^3-2x^2+x-11}{3x+1} = 4x^2-2x+1 - \dfrac{12}{3x+1}$

Product:

$12x^3-2x^2+x-11 = (3x+1)(4x^2-2x+1) - 12$

**c)** $-8x^4 - 4x + 10x^3 - x^2 + 15$ divided by $2x - 1$

$$\begin{array}{r|l} & -4x^3 + 3x^2 + 1x - \frac{3}{2} \\ 2x-1 & -8x^4 + 10x^3 - x^2 - 4x + 15 \\ & \underline{-8x^4 + 4x^3} \\ & \quad 6x^3 - 1x^2 \\ & \quad \underline{6x^3 - 3x^2} \\ & \qquad 2x^2 - 4x \\ & \qquad \underline{2x^2 - 1x} \\ & \qquad\quad -3x + 15 \\ & \qquad\quad \underline{-3x + \frac{3}{2}} \\ & \qquad\quad R = \frac{27}{2} \end{array}$$

Q.F.: $\dfrac{-8x^4+10x^3-x^2-4x+15}{2x-1} = -4x^3+3x^2+x-\frac{3}{2} + \dfrac{27}{2(2x-1)}$

Product:

$-8x^4+10x^3-x^2-4x+15 = (2x-1)(-4x^3+3x^2+x-\frac{3}{2}) + \frac{27}{2}$

**d)** $x^3 + 4x^2 - 3$ divided by $x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & 0 & -3 \\ & \downarrow & 2 & 12 & 24 \\ \hline \times & 1 & 6 & 12 & 21 \\ & x^2 & x & \# & R \end{array} \quad +$$

Q.F.: $\dfrac{x^3+4x^2-3}{x-2} = x^2+6x+12 + \dfrac{21}{x-2}$

Product:

$x^3+4x^2-3 = (x-2)(x^2+6x+12)+21$

**5)** Determine the value of $k$ such that when $f(x) = x^4 + kx^3 - 3x - 5$ is divided by $x - 3$, the remainder is $-10$.

$$f(3) = (3)^4 + k(3)^3 - 3(3) - 5$$
$$-10 = 81 + 27k - 9 - 5$$
$$-10 = 27k + 67$$
$$-77 = 27k$$
$$\boxed{k = \frac{-77}{27}}$$

**Section 2: 2.2 – Factor Theorem**

**6)** Suppose the cubic polynomial $8x^3 + mx^2 + nx - 6$ has both $2x + 3$ and $x - 1$ as factors. Find $m$ and $n$. Do not divide.

$$0 = 8\left(-\tfrac{3}{2}\right)^3 + m\left(-\tfrac{3}{2}\right)^2 + n\left(-\tfrac{3}{2}\right) - 6$$
$$0 = -27 + \tfrac{9}{4}m - \tfrac{3}{2}n - 6$$
$$33 = \tfrac{9}{4}m - \tfrac{3}{2}n$$
$$(1)\quad 132 = 9m - 6n$$

$$0 = 8(1)^3 + m(1)^2 + n(1) - 6$$
$$0 = 8 + m + n - 6$$
$$(2)\quad -2 = m + n$$

$$(1)\quad 132 = 9m - 6n$$
$$6\times(2)\quad -12 = 6m + 6n \quad +$$
$$120 = 15m$$
$$\boxed{m = 8}$$

sub $m=8$ into (2)

$$-2 = 8 + n$$
$$\boxed{-10 = n}$$

**7)** Factor each of the following

**a)** $x^3 - 4x^2 + x + 6$

Possible Factors: $\pm1, \pm2, \pm3, \pm6$

$f(2) = 0 \therefore x-2$ is a factor

| | | | | |
|---|---|---|---|---|
| 2 | 1 | -4 | 1 | 6 |
| | ↓ | 2 | -4 | -6 + |
| x | 1 | -2 | -3 | 0 |
| | $x^2$ | $x$ | # | R |

$$x^3 - 4x^2 + x + 6 = (x-2)(x^2 - 2x - 3)$$
$$\boxed{= (x-2)(x-3)(x+1)}$$

**b)** $3x^3 - 5x^2 - 26x - 8$

Possible factors: $\pm1, \pm\frac{1}{3}, \pm2, \pm\frac{2}{3}, \pm4, \pm8, \pm\frac{4}{3}, \pm\frac{8}{3}$

$f(-2) = 0 \therefore x+2$ is a factor

| | | | | |
|---|---|---|---|---|
| -2 | 3 | -5 | -26 | -8 |
| | ↓ | -6 | 22 | 8 + |
| x | 3 | -11 | -4 | 0 |
| | $x^2$ | $x$ | # | R |

P: $-12$, S: $-11$

$\left(\frac{-4}{1}\right) \quad \frac{-12}{3} \quad \left(\frac{1}{3}\right)$

$$3x^3 - 5x^2 - 26x - 8 = (x+2)(3x^2 - 11x - 4)$$
$$\boxed{= (x+2)(x-4)(3x+1)}$$

c) $-4x^3 - 4x^2 + 16x + 16$

$= -4(x^3 + x^2 - 4x - 4)$

$= -4\left[x^2(x+1) - 4(x+1)\right]$

$= -4(x+1)(x^2-4)$

$\boxed{= -4(x+1)(x-2)(x+2)}$

DOC: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

d) $x^3 - 64 = x^3 - 4^3$

$= (x-4)(x^2+4x+4^2)$

$\boxed{= (x-4)(x^2+4x+16)}$

**Section 3: 2.3&2.6 – Factoring to Solve Equations and Inequalities**

8) Determine the real roots of each equation.

a) $(5x^2 + 20)(3x^2 - 48) = 0$

$5x^2 + 20 = 0$

$5x^2 = -20$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$\varnothing$ No solutions

$3x^2 - 48 = 0$

$3x^2 = 48$

$x^2 = 16$

$x = \pm\sqrt{16}$

$\boxed{x_1 = 4}$ $\boxed{x_2 = -4}$

b) $(2x^2 - x - 13)(x^2 + 1) = 0$

NOT Factorable

So use QF

$x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(2)(-13)}}{2(2)}$

$x = \dfrac{1 \pm \sqrt{105}}{4}$

$\boxed{x_1 = \dfrac{1 + \sqrt{105}}{4}}$ $\boxed{x_2 = \dfrac{1 - \sqrt{105}}{4}}$

$x^2 + 1 = 0$

$x = \pm\sqrt{-1}$

$\varnothing$ No solutions

**9)** Solve the following polynomial equations.

**a)** $2x^3 + 1 = x^2 + 2x$

$2x^3 - x^2 - 2x + 1 = 0$

$x^2(2x-1) - 1(2x-1) = 0$

$(2x-1)(x^2-1) = 0$

$(2x-1)(x-1)(x+1) = 0$

$\boxed{x_1 = \frac{1}{2}}$ $\boxed{x_2 = 1}$ $\boxed{x_3 = -1}$

**b)** $x^3 + 6x^2 + 11x + 6 = 0$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 6$

$f(-1) = 0$, $\therefore$ $x+1$ is a factor

$$\begin{array}{r|rrrrl} -1 & 1 & 6 & 11 & 6 & \\ & \downarrow & -1 & -5 & -6 & + \\ \hline x & 1 & 5 & 6 & 0 & \\ & x^2 & x & \# & R & \end{array}$$

$(x+1)(x^2+5x+6) = 0$

$(x+1)(x+2)(x+3) = 0$

$\boxed{x_1 = -1}$ $\boxed{x_2 = -2}$ $\boxed{x_3 = -3}$

$x^5 - 4x^3 - x^2 + 4 = 0$

$x^3(x^2-4) - 1(x^2-4) = 0$

$(x^2-4)(x^3-1) = 0$

$(x-2)(x+2)(x-1)(x^2+1x+1) = 0$

R

No solutions

$b^2 - 4ac = -3$

$\boxed{x_1 = 2 \quad x_2 = -2 \quad x_3 = 1}$

**d)** $3x^3 + 2x^2 - 11x - 10 = 0$

Possible Factors: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{3}, \pm 10, \pm \frac{10}{3}$

$f(-1) = 0$, $\therefore$ $x+1$ is a factor

$$\begin{array}{r|rrrrl} -1 & 3 & 2 & -11 & -10 & \\ & \downarrow & -3 & 1 & 10 & + \\ \hline x & 3 & -1 & -10 & 0 & \\ & x^2 & x & \# & R & \end{array}$$

P: $-30$

S: $-1$

$\frac{-2}{1}$ $\frac{-6}{3}$ $\frac{5}{3}$

$(x+1)(3x^2 - x - 10) = 0$

$(x+1)(x-2)(3x+5) = 0$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 2}$ $\boxed{x_3 = \frac{-5}{3}}$

**10)** Solve the following polynomial inequalities. (Refer to #9 where you factored the polynomials)

**a)** $2x^3 + 1 < x^2 + 2x$

$2x^3 - x^2 - 2x + 1 < 0$

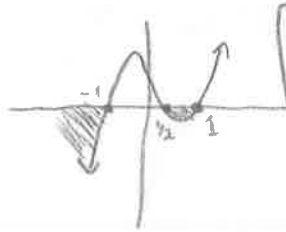
$(2x-1)(x-1)(x+1) < 0$

x-int at $x = -1, \frac{1}{2}, 1$

+ L.C., odd degree

Q3 → Q1

| | $-\infty$ to $-1$ | $-1$ to $0.5$ | $0.5$ to $1$ | $1$ to $\infty$ |
|---|---|---|---|---|
| | $-2$ | $0$ | $0.75$ | $2$ |
| $2x-1$ | $-$ | $-$ | $+$ | $+$ |
| $x-1$ | $-$ | $-$ | $-$ | $+$ |
| $x+1$ | $-$ | $+$ | $+$ | $+$ |
| overall | $-$ | $+$ | $-$ | $+$ |




Solution:

when $x < -1$ or $0.5 < x < 1$

OR

when $x \in (-\infty, -1) \cup (0.5, 1)$

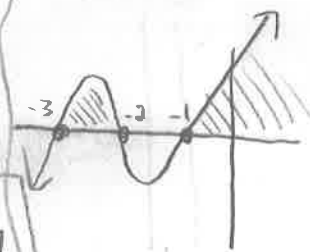
**b)** $x^3 + 6x^2 + 11x + 6 > 0$

$(x+1)(x+2)(x+3) > 0$

x-int at $x = -3, -2, -1$

+ L.C., odd degree

Q3 → Q1




| | $-\infty$ to $-3$ | $-3$ to $-2$ | $-2$ to $-1$ | $-1$ to $\infty$ |
|---|---|---|---|---|
| | $-4$ | $-2.5$ | $-1.5$ | $0$ |
| $x+1$ | $-$ | $-$ | $-$ | $+$ |
| $x+2$ | $-$ | $-$ | $+$ | $+$ |
| $x+3$ | $-$ | $+$ | $+$ | $+$ |
| overall | $-$ | $+$ | $-$ | $+$ |

SOLUTION:

when $-3 < x < -2$ OR $x > -1$

when $x \in (-3, -2) \cup (-1, \infty)$

**11)** Where is the polynomial $y = 8x^3 + 1$ positive? Justify your solution.

SOC

$8x^3 + 1 > 0$

$(2x+1)\left[(2x)^2 - (2x)(1) + (1)^2\right] > 0$

$(2x+1)(4x^2 - 2x + 1) > 0$

$\boxed{x = -\frac{1}{2}}$

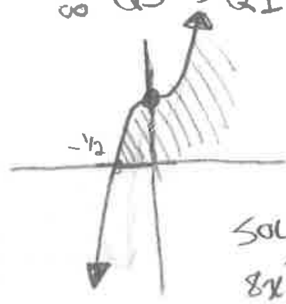
check $b^2 - 4ac = (-2)^2 - 4(4)(1)$

$= -12$

∴ NO ROOTS

Positive L.C. and odd degree.

∴ Q3 → Q1.




SOLUTION:

$8x^3 + 1 > 0$ when $x > -\frac{1}{2}$

when $x \in \left(-\frac{1}{2}, \infty\right)$

**12)** Solve $6x^3 + 13x^2 - 41x + 12 \leq 0$ using a sign chart.

Possible zeros: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 2, \pm\frac{2}{3}, \pm 3, \pm\frac{3}{2}, \pm 4, \pm\frac{4}{3}, \pm 6, \pm 12$

$f(-4) = 0$; ∴ $x+4$ is a factor

$$\begin{array}{r|rrrr} -4 & 6 & 13 & -41 & 12 \\ & \downarrow & -24 & 44 & -12 \\ \hline \times & 6 & -11 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

P: 18, S: -11; $-\frac{9}{6} = -\frac{3}{2}$, $-\frac{2}{6} = -\frac{1}{3}$

$(x+4)(6x^2 - 11x + 3) \leq 0$

$(x+4)(2x-3)(3x-1) \leq 0$

x-int at $x = -4, \frac{1}{3}, \frac{3}{2}$

| | $-\infty$ to $-4$ | $-4$ to $\frac{1}{3}$ (0.33) | $\frac{1}{3}$ to $\frac{3}{2}$ (1.5) | $\frac{3}{2}$ to $\infty$ |
|---|---|---|---|---|
| | $-5$ | $0$ | $1$ | $2$ |
| $x+4$ | $-$ | $+$ | $+$ | $+$ |
| $2x-3$ | $-$ | $-$ | $-$ | $+$ |
| $3x-1$ | $-$ | $-$ | $+$ | $+$ |
| overall | $-$ | $+$ | $-$ | $+$ |

SOLUTION:

when $x \leq -4$ or $\frac{1}{3} \leq x \leq \frac{3}{2}$

when $x \in (-\infty, -4] \cup \left[\frac{1}{3}, \frac{3}{2}\right]$

## Section 4: 2.4 – Families of Polynomials

**13)** Find the equation for the family of quartic polynomials that have real roots of 3 (order 2) and $2 \pm \sqrt{2}$.

factors:

$x = 3$

$x - 3 = 0$

$(x-3)$ order 2

$x = 2 \pm \sqrt{2}$

$x - 2 = \pm\sqrt{2}$

$(x-2)^2 = 2$

$x^2 - 4x + 4 = 2$

$x^2 - 4x + 2 = 0$

$(x^2 - 4x + 2)$

Equation for family:

$$\boxed{y = k(x-3)^2(x^2-4x+2)}$$

**14)** A family of cubic polynomials has roots of -2, -3 and -5. Find the member of this family that passes through the point (2,-35). What is this polynomials *y*-intercept?

$f(x) = k(x+2)(x+3)(x+5)$

$-35 = k(2+2)(2+3)(2+5)$

$-35 = k(4)(5)(7)$

$-35 = 140k$

$k = -\frac{1}{4}$

$f(0) = -\frac{1}{4}(0+2)(0+3)(0+5)$

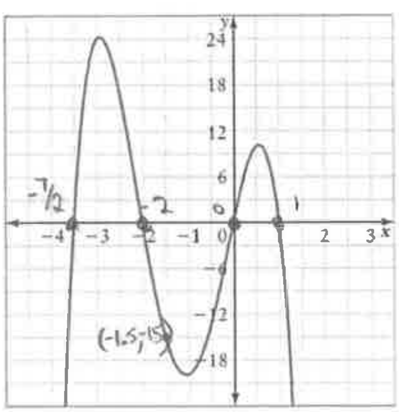
$f(0) = -\frac{1}{4}(30)$

$f(0) = -\frac{15}{2}$

The equation is $f(x) = -\frac{1}{4}(x+2)(x+3)(x+5)$.
It has a y-intercept of $(0, -\frac{15}{2})$

**15)** Find an equation for each of the following functions

**a)**




$f(x) = k(2x+7)(x+2)(x)(x-1)$

$-15 = k[2(-1.5)+7](-1.5+2)(-1.5)(-1.5-1)$

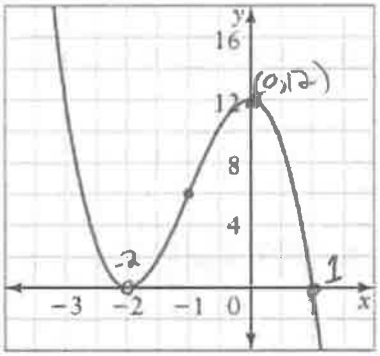
$-15 = k(4)(0.5)(-1.5)(-2.5)$

$-15 = 7.5k$

$k = -2$

$$\boxed{f(x) = -2(x)(2x+7)(x+2)(x-1)}$$

**b)**




$f(x) = k(x+2)^2(x-1)$

$12 = k(0+2)^2(0-1)$

$12 = k(4)(-1)$

$12 = -4k$

$k = -3$

$$\boxed{f(x) = -3(x+2)^2(x-1)}$$

## ANSWER KEY

**1)** $P(-1) = 2$ =remainder . This was found using remainder theorem.

**2)** $P(3) = 0$, so $x - 3$ is a factor because remainder is 0 (Factor Theorem)

**3)a)** $\frac{x^4-4x^2-2x+3}{x-2} = x^3 + 2x^2 - 2 - \frac{1}{x-2}$ **b)** $\frac{x^5-x^4+2x^3+3x-2}{x^2+2} = x^3 - x^2 + 2 + \frac{3x-6}{x^2+2}$

**4)a)** $\frac{x^3+9x^2-5x+3}{x-2} = x^2 + 11x + 17 + \frac{37}{x-2}$; $x^3 + 9x^2 - 5x + 3 = (x-2)(x^2 + 11x + 17) + 37$

**b)** $\frac{12x^3-2x^2+x-11}{3x+1} = 4x^2 - 2x + 1 - \frac{12}{3x+1}$; $12x^3 - 2x^2 + x - 11 = (3x+1)(4x^2 - 2x + 1) - 12$

**c)** $\frac{-8x^4-4x+10x^3-x^2+15}{2x-1} = -4x^3 + 3x^2 + x - \frac{3}{2} + \frac{27}{2(2x-1)}$; $-8x^4 - 4x + 10x^3 - x^2 + 15 = (2x-1)\left(-4x^3 + 3x^2 + x - \frac{3}{2}\right) + \frac{27}{2}$

**d)** $\frac{x^3+4x^2-3}{x-2} = x^2 + 6x + 12 + \frac{21}{x-2}$; $x^3 + 4x^2 - 3 = (x-2)(x^2 + 6x + 12) + 21$

**5)** $k = -\frac{77}{27}$

**6)** $m = 8$ , $n = -10$

**7)a)** $(x+1)(x-3)(x-2)$ **b)** $(x+2)(3x+1)(x-4)$ **c)** $-4(x+1)(x+2)(x-2)$ **d)** $(x-4)(x^2 + 4x + 16)$

**8)a)** (-4, 0) and (4, 0) **b)** $\left(\frac{1-\sqrt{105}}{4}, 0\right)$ and $\left(\frac{1+\sqrt{105}}{4}, 0\right)$

**9) a)** $x = -1, 1, \frac{1}{2}$ **b)** $x = -1, -2, -3$ **c)** $x = 1, -2, 2$ **d)** $x = -1, -\frac{5}{3}, 2$

**10)a)** $x \in (-\infty, -1) \cup (0.5, 1)$ **b)** $x \in (-3, -2) \cup (-1, \infty)$

**11)** $x \in \left(-\frac{1}{2}, \infty\right)$

**12)** $x \in (-\infty, -4] \cup \left[\frac{1}{3}, \frac{3}{2}\right]$

**13)** $P(x) = k(x-3)^2(x^2 - 4x + 2)$

**14)** $f(x) = -\frac{1}{4}(x+2)(x+3)(x+5)$, $y$-int is $\left(0, -\frac{15}{2}\right)$

**15)a)** $P(x) = -2x(x-1)(x+2)(2x+7)$ **b)** $P(x) = -3(x+2)^2(x-1)$