L1 – 6.1/6.2 – Intro to Logarithms and Review of Exponentials
MHF4U

In this section you will learn about how a logarithmic function is the inverse of an exponential function. You will also learn how to express exponential equations in logarithmic form.

Part 1: Review of Exponential Functions

Equation: $\boldsymbol{y = a(b)^x}$

$a =$ initial amount

$b =$ growth $(b > 1)$ or decay $(0 < b < 1)$ factor

$y =$ future amount

$x =$ number of times a has increased or decreased

To calculate x, use the equation: $x = \dfrac{total\ time}{time\ it\ takes\ for\ one\ growth\ or\ decay\ period}$

Example 1: An insect colony has a current population of 50 insects. Its population doubles every 3 days.

a) What is the population after 12 days?

$y = 50(2)^{\frac{12}{3}}$

$y = 50(2)^4$

$y = 800$

b) How long until the population reaches 25 600?

$25\ 600 = 50(2)^{\frac{t}{3}}$

$512 = 2^{\frac{t}{3}}$

$\log 512 = \log 2^{\frac{t}{3}}$

$\log 512 = \left(\frac{t}{3}\right)\log 2$

$\dfrac{\log 512}{\log 2} = \dfrac{t}{3}$

$9 = \dfrac{t}{3}$

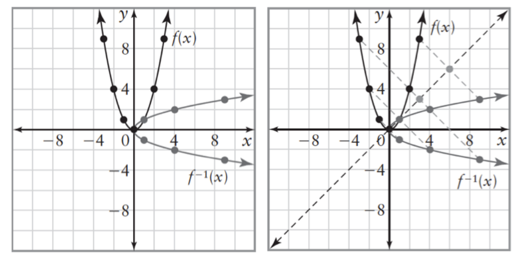
$t = 27$ days

Part 2: Review of Inverse Functions

Inverse of a function:

· The inverse of a function f is denoted as f^{-1}
· The function and its inverse have the property that if f(a) = b, then $f^{-1}(b) = a$
· So if $f(5) = 13$, then $f^{-1}(13) = 5$
· More simply put: The inverse of a function has all the same points as the original function, except that the x's and y's have been reversed.

The **graph** of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$. This is true for all functions and their inverses.



Example 2: Determine the equation of the inverse of the function $f(x) = 3(x-5)^2 + 1$

$$y = 3(x-5)^2 + 1$$

$$x = 3(y-5)^2 + 1$$

$$\frac{x-1}{3} = (y-5)^2$$

$$\pm\sqrt{\frac{x-1}{3}} = y - 5$$

$$5 \pm \sqrt{\frac{x-1}{3}} = y$$

Algebraic Method for finding the inverse:

1. Replace $f(x)$ with "y"
2. Switch the x and y variables
3. Isolate for y
4. replace y with $f^{-1}(x)$

Equation of inverse:

$$f^{-1}(x) = 5 \pm \sqrt{\frac{x-1}{3}}$$

Part 3: Review of Exponent Laws

Name	Rule
Product Rule	$x^a \cdot x^b = x^{a+b}$
Quotient Rule	$\frac{x^a}{x^b} = x^{a-b}$
Power of a Power Rule	$(x^a)^b = x^{a \times b}$
Negative Exponent Rule	$x^{-a} = \frac{1}{x^a}$
Exponent of Zero	$x^0 = 1$

Part 4: Inverse of an Exponential Function

Example 3:

a) Find the equation of the inverse of $f(x) = 2^x$.

$y = 2^x$

$x = 2^y$

$\log x = \log 2^y$

$\log x = y \log 2$

$y = \dfrac{\log x}{\log 2}$

This step uses the 'change of base' formula that we will cover later in the unit.

$\log_b m = \dfrac{\log m}{\log b}$

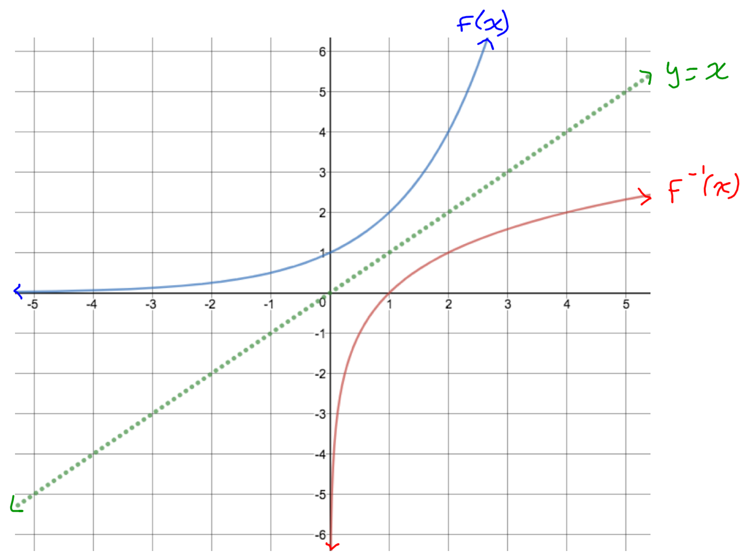
$y = \log_2 x$

$f^{-1}(x) = \log_2 x$

b) Graph the both $f(x)$ and $f^{-1}(x)$.

$f(x) = 2^x$	
x	y
-2	0.25
-1	0.5
0	1
1	2
2	4

$f^{-1}(x) = \log_2 x$	
x	y
0.25	-2
0.5	-1
1	0
2	1
4	2



Note: just swap x and y coordinates to get key points for the inverse of a function. The graph should appear to be a reflection across the line $y = x$.

c) Complete the chart of key properties for both functions

$y = 2^x$	$y = \log_2 x$
x-int: none	x-int: (1, 0)
y-int: (0, 1)	y-int: none
Domain: $\{X \in \mathbb{R}\}$	Domain: $\{X \in \mathbb{R} \mid x > 0\}$
Range: $\{Y \in \mathbb{R} \mid y > 0\}$	Range: $\{Y \in \mathbb{R}\}$
Asymptote: horizontal asymptote at $y = 0$	Asymptote: vertical asymptote at $x = 0$

Part 5: What is a Logarithmic Function?

The logarithmic function is the **inverse** of the exponential function with the same base.

The **logarithmic function** is defined as $y = \log_b x$, or y equals the logarithm of x to the base b.

The function is defined only for $\boldsymbol{b > 0, b \neq 1}$

In this notation, $\boldsymbol{y}$ is the exponent to which the base, $\boldsymbol{b}$, must be raised to give the value of $\boldsymbol{x}$.

In other words, the solution to a logarithm is always an **EXPONENT**.

The logarithmic function is most useful for solving for unknown **exponents**

Common logarithms are logarithms with a base of 10. It is not necessary to write the base for common logarithms: $\log x$ means the same as $\log_{10} x$

Part 6: Writing Equivalent Exponential and Logarithmic Expressions

Exponential equations can be written in logarithmic form, and vice versa

$y = b^x \rightarrow \mathrm{x} = \log_b y$

$y = \log_b x \rightarrow x = b^y$

Example 4: Rewrite each equation in logarithmic form

a) $16 = 2^4$

$\log_2 16 = 4$

b) $m = n^3$

$\log_n m = 3$

c) $3^{-2} = \frac{1}{9}$

$\log_3 \left(\frac{1}{9}\right) = -2$

Example 5: Write each logarithmic equation in exponential form

a) $\log_4 64 = 3$

$4^3 = 64$

b) $y = \log x$

$10^y = x$

Note: because there is no base written, this is understood to be the common logarithm of x.

Part 7: Evaluate a Logarithm

Example 6: Evaluate each logarithm without a calculator

Rule: if $x^a = x^b$, then $a = b$

a) $y = \log_3 81$

$3^y = 81$

$3^y = 3^4$

$y = 4$

Rule: $\log_a(a^b) = b$

a) $y = \log_4 64$

$y = \log_4(4^3)$

$y = 3$

Note: *either of the rules presented above are appropriate to use for evaluating logarithmic expressions*

b) $y = \log\left(\frac{1}{100}\right)$

$10^y = \frac{1}{100}$

$10^y = \left(\frac{1}{10}\right)^2$

$10^y = 10^{-2}$

$y = -2$

c) $y = \log_2\left(\frac{1}{8}\right)$

$y = \log_2\left(\frac{1}{2}\right)^3$

$y = \log_2 2^{-3}$

$y = -3$