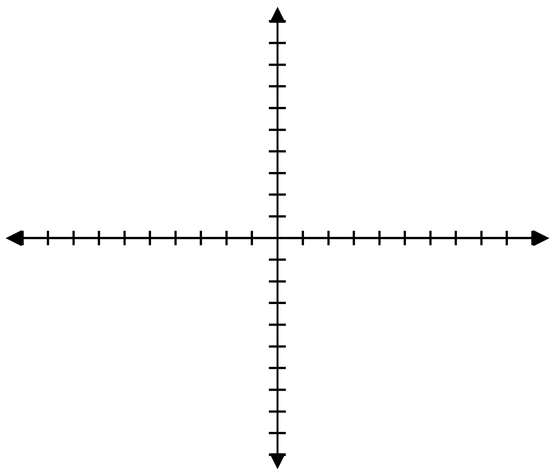
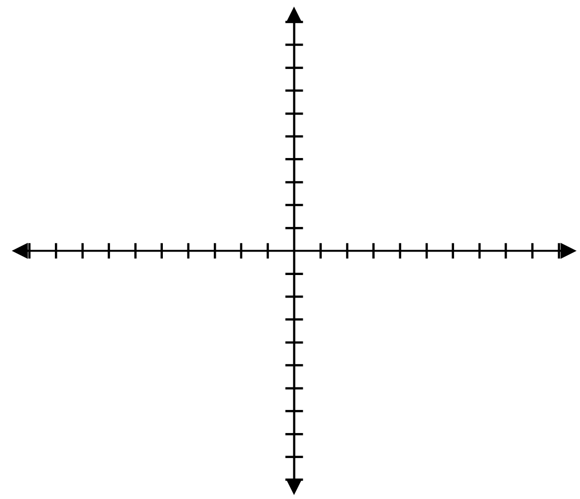
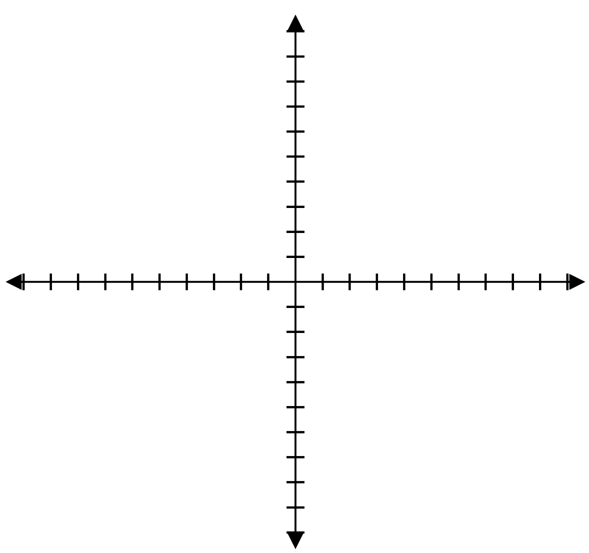
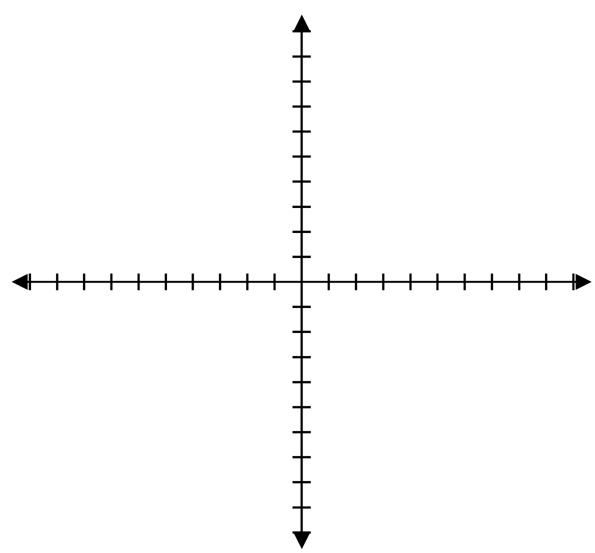
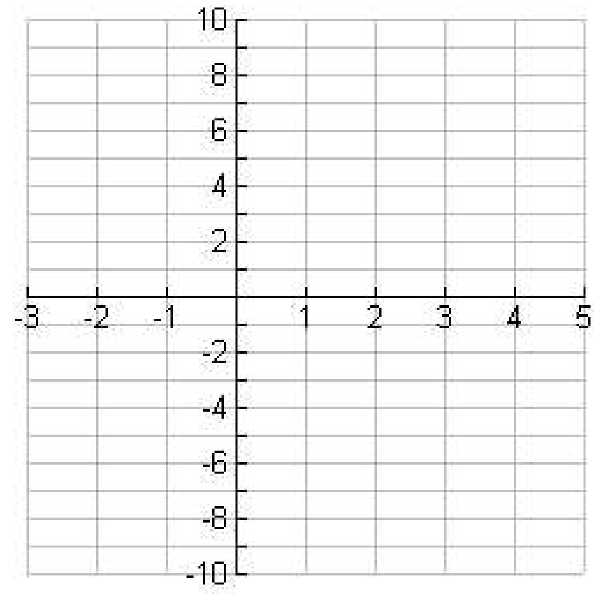
2.2 Reciprocal of Quadratic Functions

1. Use a graphing calculator to compare each of the following functions. Include a sketch of each.

$y=x^2$	$y=\dfrac{1}{x^2}$
	
comparison	

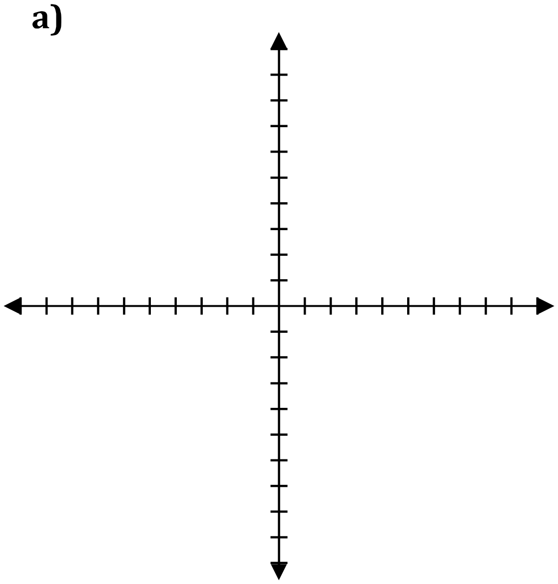
$y=(x-3)(x+1)$	$y=\dfrac{1}{(x-3)(x+1)}$
	
comparison	

2. Sketch the graph of $f(x) = -x^2 + 5x - 6$ and its reciprocal on the same axis. Clearly identify the intersection(s) between f(x) and its reciprocal.



3. Determine the following information about each of the following graphs

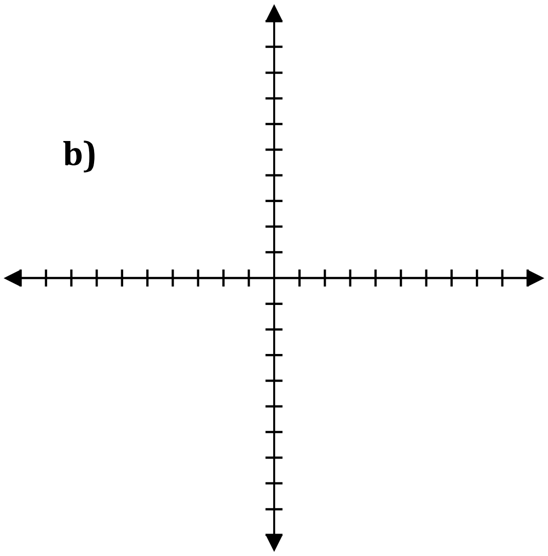
a)



Equation	$y = \dfrac{1}{(x+3)(x-3)}$		
Domain			
Range			
x-int		y-int	
Max/Min			
H. Asymptote:		V. Asymptote(s):	

As $x \rightarrow$	***f*(*x*)$\rightarrow$**
3^+	
3^-	
-3^+	
-3^-	
$+\infty$	
$-\infty$	

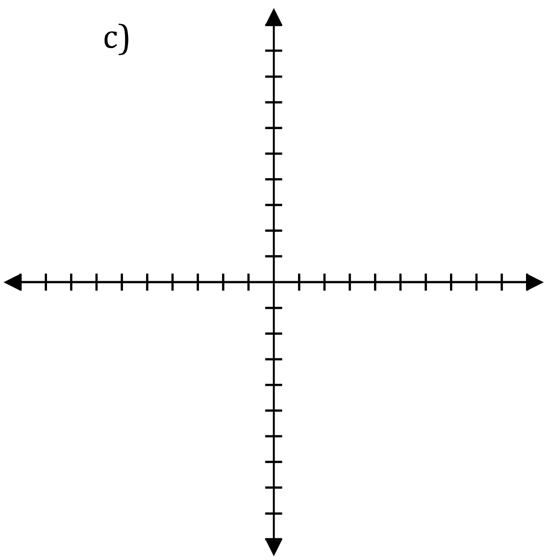
b)



Equation	$y = \dfrac{-1}{(x-2)^2}$		
Domain			
Range			
x-int		y-int	
Max/Min			
H. Asymptote:		V. Asymptote(s):	

As $x\to$	**$f(x)\to$**
2^+	
2^-	
$+\infty$	
$-\infty$	

c)

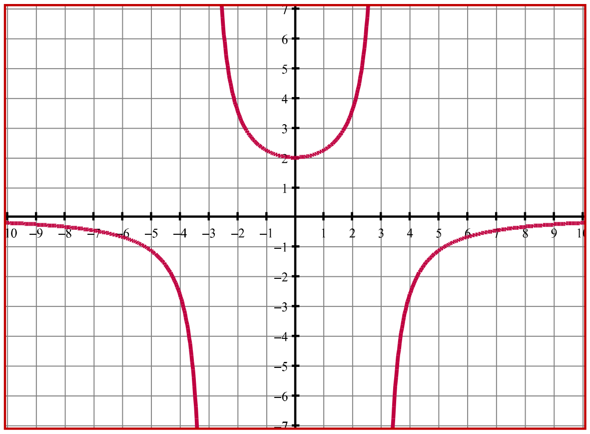


Equation	$y = \dfrac{1}{x^2+4}$		
Domain			
Range			
x-int		y-int	
Max/Min			
H. Asymptote:		V. Asymptote(s):	

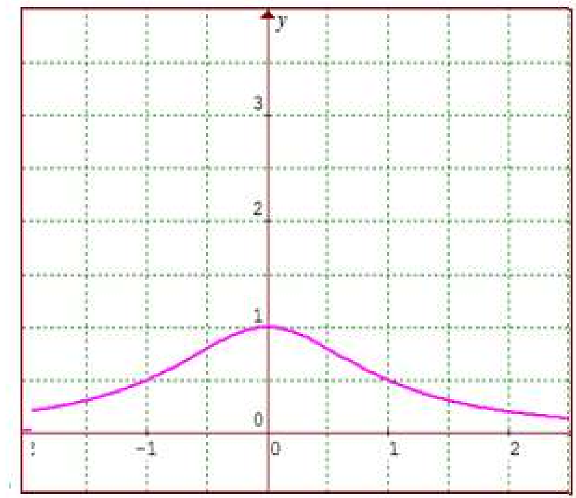
As $x\to$	**$f(x)\to$**
$+\infty$	
$-\infty$	

4. Determine the equation of each of the following graphs:

a)



b)



Summary

Reciprocals of quadratics can be classified into 3 different types:

i. $f(x) = \frac{1}{(x-a)^2}$

_______ vertical asymptotes and

_______ "branches"

ii. $f(x) = \frac{k}{(x-a)(x-b)}$

_______ vertical asymptotes and

_______ "branches"

iii. $f(x) = \frac{1}{x^2+a}$, $a > 0$

_______ vertical asymptotes (hat shaped)

- All the y-coordinates of a reciprocal function are the reciprocals of the y-coordinates of the original function.
- The graph of a reciprocal function has a vertical asymptote at each zero of the original function.
- A reciprocal function will always have as a horizontal asymptote if the original function is linear or quadratic.
- A reciprocal function has the same positive/negative intervals as the original function.
- Intervals of increase on the original function are intervals of decrease on the reciprocal function. Intervals of decrease on the original function are intervals of increase on the reciprocal function.
- If the range of the original function includes 1 and/or-1 the reciprocal function will intersect the original function at a point (or points) where the y-coordinate is 1 or -1 .
- If the original function has a local minimum point, the reciprocal function will have a local maximum point at the same x-value (and vice versa).

2.2 Practice

1) Find constants a and b that guarantee that the graph of the function defined by $h(x)=\dfrac{ax^2+7}{9-bx^2}$ will have a vertical asymptote at $x=\pm\dfrac{3}{5}$ and a horizontal asymptote at $y=-2$.

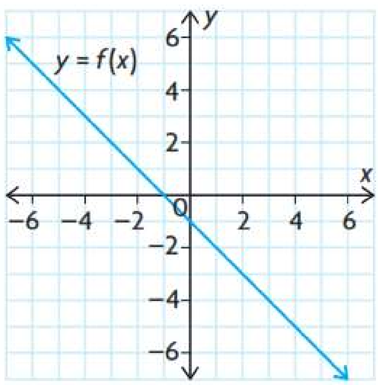
2) Sketch the graph of following functions .

   a) $y=\dfrac{-1}{x(x-5)}$ b) $y=\dfrac{-3}{(x+3)^2}$

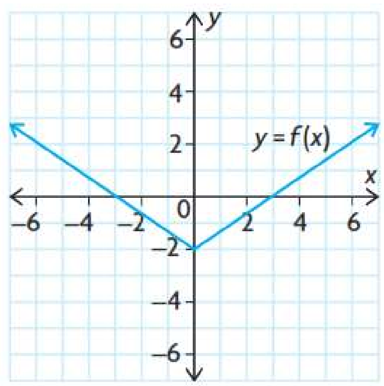
3) For each case, create a function that has a graph with the given features.
   (a) a vertical asymptote x = 1 and a horizontal asymptote y = 0
   (b) two vertical asymptotes x = -1 and x = 3, horizontal asymptote y = -1, and x-intercepts – 2 and 4.

4) Sketch the graph of the reciprocal of each function.

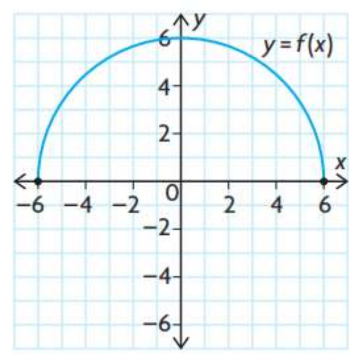
a)



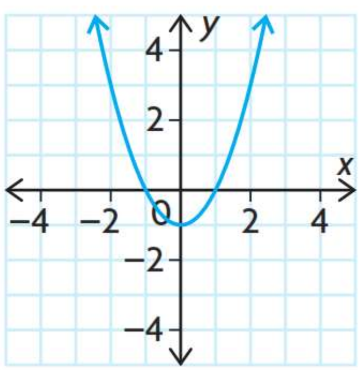
b)



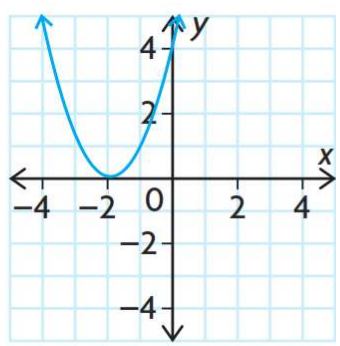
c)



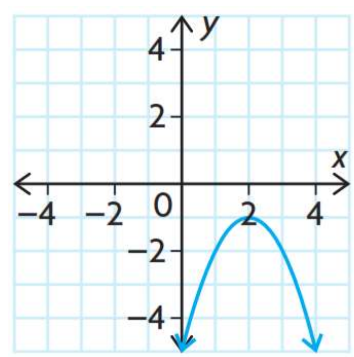
d)



e)



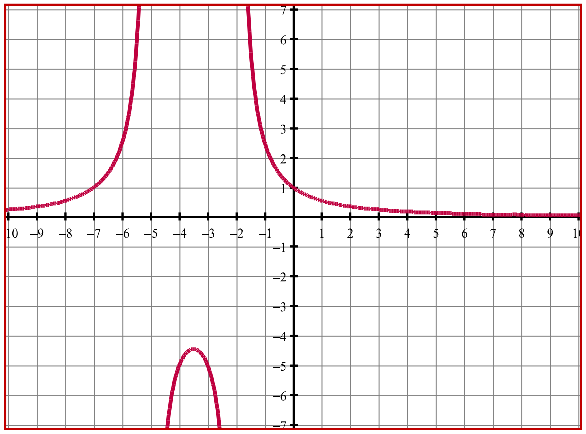
f)



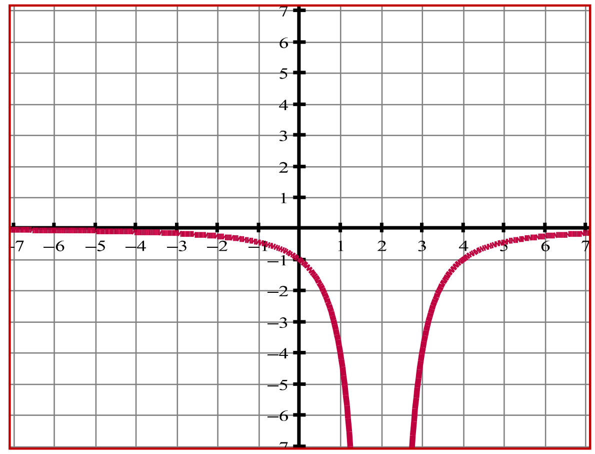
5) Sketch the graph of f(x)=-2x²+10x-12 and its reciprocal on the same axis. Clearly identify the intersection(s) between f(x) and its reciprocal.

6) Determine the equation of each of the following graphs:

a)



b)



Warm up

1. Which of the following functions does **not** have a vertical asymptote?

A) $f(x)=\dfrac{x}{x^2-x}$ B) $f(x)=\dfrac{x^2-1}{x}$

C) $f(x)=\dfrac{x-1}{x^2-x}$ D) $f(x)=\dfrac{x^2-1}{x-1}$

2. The function $f(x)=\dfrac{1}{x^2-6x-16}$ has a local maximum at

A) $f(x)=\pm 1$ B) $(3,-25)$

C) $\left(3,\dfrac{-1}{25}\right)$ D) $\left(0,\dfrac{-1}{16}\right)$

3. Which of the following has a horizontal asymptote of $y = 1$?

A) $f(x)=\dfrac{x^2+2x-24}{x^3-64}$ B) $f(x)=\dfrac{2x^2-2x-24}{4x^2-64}$

C) $f(x)=\dfrac{x^4+3x^2-40}{(x^2+1)(x+8)}$ D) $f(x)=\dfrac{(x^2-4)(2x-9)}{(x)(2x-3)(x-1)}$

4. Consider the function $f(x)=\dfrac{1}{x^2+6x+8}$.

a) Determine the point at which the slope of the tangent is 0.

b) Determine the equation of the tangent line at this point. ________________