W5 – 7.4 – Solving Logarithmic Equations
MHF4U

1) Find the roots of each equation

a) $2 = \log(x + 25)$

$10^2 = x+25$

$100 = x+25$

$x = 75$

b) $1 - \log(w - 7) = 0$

$1 = \log(w-7)$

$10^1 = w-7$

$10 = w-7$

$w = 17$

c) $6 - 3\log(2n) = 0$

$6 = 3\log(2n)$

$2 = \log(2n)$

$10^2 = 2n$

$100 = 2n$

$n = 50$

2) Solve

a) $5 = \log_2(2x - 10)$

$2^5 = 2x-10$

$32 = 2x-10$

$42 = 2x$

$x = 21$

b) $9 = \log_5(x + 100) + 6$

$3 = \log_5(x+100)$

$125 = x+100$

$x = 25$

c) $\log_3(n^2 - 3n + 5) = 2$

$3^2 = n^2-3n+5$

$9 = n^2-3n+5$

$0 = n^2-3n-4$

$0 = (n-4)(n+1)$

$n = 4$ $\quad$ $n = -1$

3) Solve. Make sure to reject any extraneous roots.

a) $\log x + \log(x - 4) = 1$

$\log[x(x-4)] = 1$

$10^1 = x(x-4)$

$10 = x^2-4x$

$0 = x^2-4x-10$

$x = \frac{4 \pm \sqrt{(-4)^2-4(1)(-10)}}{2(1)}$

$x = \frac{4 \pm \sqrt{56}}{2}$

$x = \frac{4 \pm 2\sqrt{14}}{2}$

$x = \frac{2(2 \pm \sqrt{14})}{2}$

$x = 2 \pm \sqrt{14}$

Reject $2-\sqrt{14}$

$x = 2+\sqrt{14}$

b) $\log x^3 - \log 2 = \log(2x^2)$

$\log x^3 - \log(2x^2) = \log 2$

$\log\left(\frac{x^3}{2x^2}\right) = \log 2$

$\log\left(\frac{x}{2}\right) = \log 2$

$\frac{x}{2} = 2$

$x = 4$

c) $\log(v-1) = 2 + \log(v-16)$

$\log(v-1) - \log(v-16) = 2$

$\log\left(\frac{v-1}{v-16}\right) = 2$

$10^2 = \frac{v-1}{v-16}$

$100(v-16) = v-1$

$100v - 1600 = v - 1$

$99v = 1599$

$v = \frac{533}{33}$

d) $\log(k+2) + \log(k-1) = 1$

$\log[(k+2)(k-1)] = 1$

$10^1 = (k+2)(k-1)$

$10 = k^2 - 1k + 2k - 2$

$0 = k^2 + k - 12$

$0 = (k+4)(k-3)$

$k = -4$ Reject

$k = 3$

4) Solve. Check for extraneous roots.

a) $\log\sqrt{x^2-3x} = \frac{1}{2}$

$\log(x^2-3x)^{1/2} = \frac{1}{2}$

$\frac{1}{2}\log(x^2-3x) = \frac{1}{2}$

$\log(x^2-3x) = 1$

$10^1 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$

b) $\log\sqrt{x^2+48x} = 1$

$\log(x^2+48x)^{1/2} = 1$

$\frac{1}{2}\log(x^2+48x) = 1$

$\log(x^2+48x) = 2$

$10^2 = x^2 + 48x$

$0 = x^2 + 48x - 100$

$0 = (x+50)(x-2)$

$x = -50$

$x = 2$

5) Solve. Check for extraneous roots.

a) $\log_2(x+5) - \log_2(2x) = 8$

$\log_2\left(\frac{x+5}{2x}\right) = 8$

$2^8 = \frac{x+5}{2x}$

$256 = \frac{x+5}{2x}$

$512x = x + 5$

$511x = 5$

$x = \frac{5}{511}$

b) $\log(2k+4) = 1 + \log k$

$\log(2k+4) - \log(k) = 1$

$\log\left(\frac{2k+4}{k}\right) = 1$

$10^1 = \frac{2k+4}{k}$

$10k = 2k + 4$

$8k = 4$

$k = \frac{1}{2}$

ANSWER KEY

1)a) 75 **b)** 17 **c)** 50 **2)a)** 21 **b)** 25 **c)** 4, -1 **3)a)** $2 + \sqrt{14}$ b) 4 **c)** $\frac{533}{33}$ **d)** 3

4)a) 5, -2 **b)** -50, 2 **5)a)** $\frac{5}{511}$ **b)** $\frac{1}{2}$