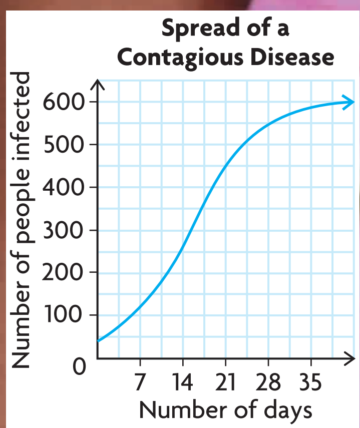
Chapter 9

Combinations of Functions

GOALS

You will be able to

- Consolidate your understanding of the characteristics of functions
- Create new functions by adding, subtracting, multiplying, and dividing functions
- Investigate the creation of composite functions numerically, graphically, and algebraically
- Determine key characteristics of these new functions
- Solve problems using a variety of function models



? *Epidemiology* is the scientific study of contagious diseases. A combination of functions is often used to model the way that a contagious disease spreads through a population. What types of functions could be combined to create an algebraic model that represents the graph shown?

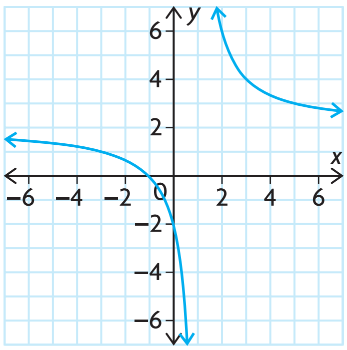
9 Getting Started

SKILLS AND CONCEPTS *You Need*

Study Aid

- For help, see the Review of Essential Skills found at the Nelson Advanced Functions website.

Question	Appendix
3	R-8



1. Evaluate each of the following functions for $f(-1)$ and $f(4)$. Round your answers to two decimal places, if necessary.

a) $f(x) = x^3 - 3x^2 - 10x + 24$

b) $f(x) = \dfrac{4x}{1 - x}$

c) $f(x) = 3\log_{10}(x)$

d) $f(x) = -5(0.5^{(x-1)})$

2. Identify the following characteristics of functions for the graph displayed.

- domain and range
- maximum or minimum values
- interval(s) where the function is increasing
- interval(s) where the function is decreasing
- end behaviour
- equations of asymptotes

3. For each parent function, apply the given transformation(s) and write the equation of the new function.

a) $y = |x|$; vertical stretch by a factor of 2, shift 3 units to the right

b) $y = \cos(x)$; reflection in the x-axis, horizontal compression by a factor of $\frac{1}{2}$

c) $y = \log_3 x$; reflection in the y-axis, shift 4 units left and 1 unit down

d) $y = \frac{1}{x}$; vertical stretch by a factor of 4, reflection in the x-axis, shift 5 units down

4. Solve each equation for x, $x \in \mathbf{R}$. State any restrictions on x, as required.

a) $2x^3 - 7x^2 - 5x + 4 = 0$

b) $\dfrac{2x + 3}{x + 3} + \dfrac{1}{2} = \dfrac{x + 1}{x - 1}$

c) $\log x + \log(x - 3) = 1$

d) $10^{-4x} - 22 = 978$

e) $5^{x+3} - 5^x = 0.992$

f) $2\cos^2 x = \sin x + 1$, $0 \le x \le 2\pi$

5. Solve each inequality for x, $x \in \mathbf{R}$.

a) $x^3 - x^2 - 14x + 24 < 0$

b) $\dfrac{(2x - 3)(x - 4)}{(x + 2)} \ge 0$

6. Identify each function as even, odd, or neither.

a) $f(x) = 2\sin(x - \pi)$

b) $f(x) = \dfrac{3}{4 - x}$

c) $f(x) = 4x^4 - 3x^2$

d) $f(x) = 2^{3x-1}$

7. Classify the types of functions you have studied (polynomial, rational, exponential, logarithmic, and trigonometric) as continuous or not.

APPLYING What You Know

Building a Sandbox

Duncan is planning to build a rectangular sandbox in his backyard for his son to play in during the summer. He has designed the sandbox so that it will have an open top and a volume of 2 m^3. The length of the base will measure four times the height of the sandbox. The wood for the base will cost $5/$m^2$, and the wood for the sides will cost $4/$m^2$.

? What dimensions should Duncan use to minimize the cost of the sandbox he has designed?

A. Let h represent the height (in metres) and let w represent the width of the sandbox. Determine an expression for the width of the sandbox in terms of its height.

B. Write an expression for the cost of the wood for the base of the sandbox in terms of its height.

C. Express the cost of the wood for the two longer sides in terms of the height. Is the cost for the two shorter sides the same?

D. Let $C(h)$ represent the total cost of the wood for the sandbox as a function of its height. Determine the equation for $C(h)$.

E. What types of functions are added in your equation for $C(h)$?

F. What would be a reasonable domain and range for this cost function? Explain.

G. Using graphing technology, graph the cost function using window settings that correspond to its domain and range.

H. Determine the height of the sandbox that will minimize the total cost.

I. What dimensions would you recommend that Duncan use to build the sandbox? Justify your answer.

9.1 Exploring Combinations of Functions

YOU WILL NEED

- graphing calculator or graphing software

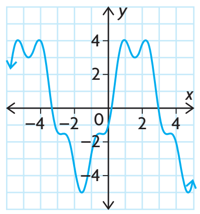
GOAL

Explore the characteristics of new functions created by combining functions.

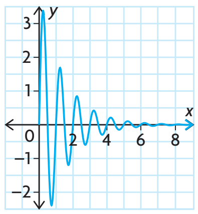
Explore the Math

Ahmad was given the graphs pictured below. They were created by combining two familiar functions.

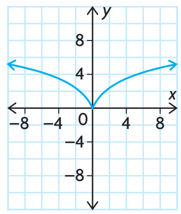
Graph 1



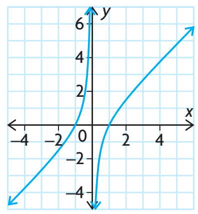
Graph 2



Graph 3



Graph 4



Ahmad does not recognize these new functions and wonders which type of functions have been combined to create them. He also wonders whether any of these graphs could model a real-life situation.

? How can two functions be combined to create a new function?

A. Compare each of the graphs above with the function equations in the table below.

$y = x\sqrt{x - 1}$	$y = 4 \sin x - \cos 4x$	$y = x - \frac{1}{x}$	$y = 5 \log (\lvert x \rvert + 1)$
$y = (x^2)(\sin (x))$	$y = \begin{cases} -0.5(x - 2)^2 + 2, x < 0 \\ 0.5(x - 2)^2 - 2, x \geq 0 \end{cases}$	$y = (0.5^x)(4 \sin (2\pi x))$	$y = x^3 \div (x + 1)$

Predict which equations will match each graph. Copy the table on the next page, and record your predictions and your rationale for each.

Graph	Equation of Function	Rationale
1		
2		
3		
4		

B. Compare your predictions with a partner's predictions. Explain to each other why you made each prediction.

C. Using graphing technology in radian mode, graph the equation that you predicted would match graph 1. Use a domain and range in the window settings that match the scale given on each of the given graphs.

D. Does the graph of your equation match graph 1? If it does not, choose another equation from the table and try again.

E. Once you have correctly matched the equation with graph 1, repeat parts C and D until all the graphs have been correctly matched.

F. Examine the equation that matches each graph.
- List the parent functions in each equation.
- State the transformations that were applied to each parent function.
- Explain how the parent functions were combined.

Reflecting

G. Which of the four given graphs is periodic? How does it differ from other periodic functions you have seen before? What type of combination produced this effect?

H. Do any of the graphs represent an even function? Do any represent an odd function? Explain how you know.

I. Which graph contains an asymptote? Describe the functions that were combined to produce this graph. Explain how you can tell from the equation where the vertical asymptote occurs.

J. Which graph could be used to model the motion of a swaying building moments after an earthquake? Explain why.

In Summary

Key Idea

- Many interesting functions can be created by combining two or more simpler functions. This can be done by adding, subtracting, multiplying, or dividing functions to create more complex functions.

Need to Know

- The characteristics of the functions that are combined affect the properties and characteristics of the resulting function.

FURTHER Your Understanding

1. Using graphing technology (in radian mode) and the functions given in the chart below, experiment to create new functions by combining different types of functions. Each time, use different operations and different types of functions. You may need to experiment with the window settings to get a clear picture of what the graph looks like. Include a sketch of your new graphs and the equations that were used for the models.

$y = 2 - 0.5x$	$y = 2^x$	$y = \sin 2\pi x$	$y = \cos 2\pi x$
$y = \log x$	$y = \left(\frac{1}{2}\right)^x$	$y = x^4 - x^2$	$y = 2x$

2. Using the functions in the chart above, create a new function that has each of the characteristics given below. Include a sketch of your new graphs and the equations that were used for the models.

a) a function that has a vertical asymptote and a horizontal asymptote

b) a function that is even

c) a function that is odd

d) a function that is periodic

e) a function that resembles a periodic function with decreasing maximum values and increasing minimum values

f) a function that resembles a periodic function with increasing maximum values and decreasing minimum values

3. Select any two functions that you have studied in this course. Experiment by combining these functions in various ways and graphing them on a graphing calculator. Include a sketch of your new graphs and the equations of the functions you selected. Challenge your classmates to see who can produce the most interesting graph.

9.2 Combining Two Functions: Sums and Differences

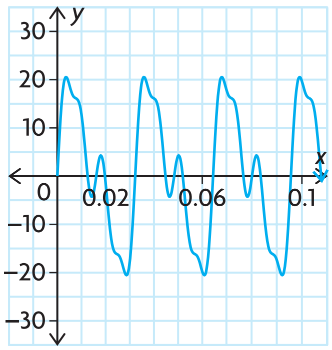
GOAL

Represent the sums and differences of two functions graphically and algebraically, and determine their properties.

YOU WILL NEED

- graphing calculator or graphing software

INVESTIGATE the Math



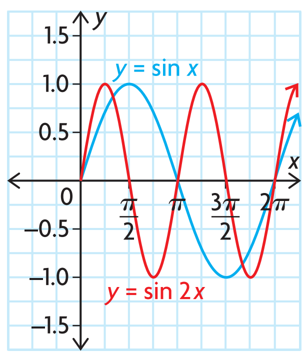
The sound produced when a person strums a guitar chord represents the combination of sounds made by several different strings. The sound made by each string can be represented by a sine function. The period of each function is based on the frequency of the sound, whereas the loudness of the individual sounds varies and is related to the amplitude of each function. These sine functions are literally added together to produce the desired sound.
The sound of a G chord played on a six-string acoustic guitar can be approximated by the following combination of sine functions:

$y = 16 \sin 196x + 9 \sin 392x + 4 \sin 784x$

? When functions are added or subtracted, how do the resulting characteristics of the new function compare with those of the original functions?

A. Explore a similar but simpler combination of sine functions by examining the properties of the sum defined by $y = \sin x + \sin 2x$. Copy and complete the table of values, and use your results and the graphs shown to sketch the graph of $y = \sin x + \sin 2x$, where $0 \le x \le 2\pi$.

x	$\sin x$	$\sin 2x$	$\sin x + \sin 2x$
0	0	0	
$\frac{\pi}{4}$	0.7071	1	
$\frac{\pi}{2}$	1	0	
$\frac{3\pi}{4}$	0.7071	−1	
π	0	0	
$\frac{5\pi}{4}$	−0.7071	1	
$\frac{3\pi}{2}$	−1	0	
$\frac{7\pi}{4}$	−0.7071	−1	
2π	0	0	



B. Set the calculator to radian mode. Adjust the window settings so that $0 \leq x \leq 4\pi$ using an Xscl $= \frac{\pi}{4}$, and $-2 \leq y \leq 2$ using a Yscl $= 1$. Verify your graph in part A by graphing $y = \sin x + \sin 2x$.

C. What is the period of $y = \sin x + \sin 2x$? How does it compare with the periods of $y = \sin x$ and $y = \sin 2x$?

D. What is the amplitude of $y = \sin x + \sin 2x$? How does it compare with the amplitudes of $y = \sin x$ and $y = \sin 2x$?

E. Create a new table of values, and use your results and the graphs of $y = \sin x$ and $y = \sin 2x$ to sketch the graph of $y = \sin x - \sin 2x$, where $0 \leq x \leq 2\pi$. Repeat parts B to D using this difference function.

F. Do you think that the graph of $y = \sin 2x - \sin x$ will be the same as the graph you created in part E? Explain. Check your conjecture by using graphing technology to graph this function.

G. Investigate the sum of other types of functions. Use graphing technology to graph each set of functions, and describe how the characteristics of the functions are related.

i) $y_1 = -x, y_2 = x^2, y_3 = -x + x^2$
ii) $y_1 = \sqrt{x}, y_2 = \sqrt{x+2}, y_3 = \sqrt{x} + \sqrt{x+2}$
iii) $y_1 = 2^x,\ y_2 = 2^{-x}, y_3 = 2^x + 2^{-x}$
iv) $y_1 = \cos x, y_2 = \cos 2x, y_3 = \cos x + \cos 2x$

H. Investigate the difference of each set of functions in part G by graphing y_1 and y_2, and changing y_3 to $y_3 = y_1 - y_2$. Describe how the characteristics of the functions are related.

Reflecting

I. How does the degree of the sum or difference of two polynomial functions compare with the degree of the individual functions?

J. How does the period of the sum or difference of two trigonometric functions compare with the periods of the individual functions?

K. When looking at the sum of two functions, does the phrase "for each x, add the corresponding y-values together" describe the result you observed for every pair of functions? What phrase would you use to describe finding the difference of two functions?

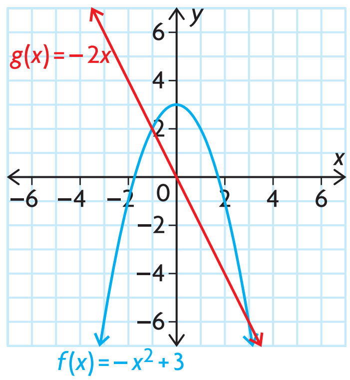
L. Looking at the graphs of the two square root functions, explain why the domain of the graph of their sum is $x \geq 0$.

M. Determine the y-intercept of y_3, where y_3 represents the difference of the two exponential functions. What does this point represent with respect to y_1 and y_2?

APPLY the Math

EXAMPLE 1 Selecting a strategy to combine functions by addition and subtraction

Given $f(x) = -x^2 + 3$ and $g(x) = -2x$, determine the graphs of $f(x) + g(x)$ and $f(x) - g(x)$. Discuss the key characteristics of the resulting graphs.

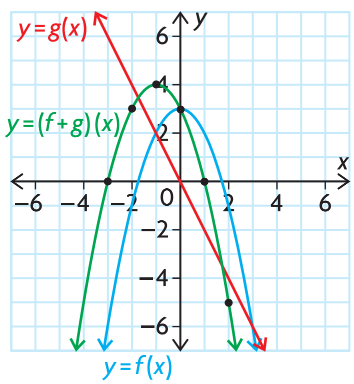


Solution A: Using a graphical strategy

x	$f(x)$	$g(x)$	$f(x) + g(x)$	$f(x) - g(x)$
-3	-6	6	$-6 + 6 = 0$	$-6 - 6 = -12$
-2	-1	4	3	-5
-1	2	2	4	0
-0.5	2.75	1	3.75	1.75
0	3	0	3	3
1	2	-2	0	4
2	-1	-4	-5	3
3	-6	-6	-12	0

Make a table of values for $f(x)$ and $g(x)$, for selected values of x. Create $f + g$ by adding the y-coordinates of f and g together. Create $f - g$ by subtracting the y-coordinates of g from f.

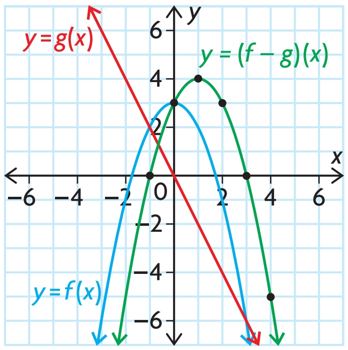
These functions can be added or subtracted over their entire domains since they both have the same domain $\{x \in \mathbf{R}\}$.



Plot the ordered pairs $(x, f(x) + g(x))$. Join the plotted points with a smooth curve.

Observe that the zeros of the new function occur when the y-values of f and g are the same distance from the x-axis, but on opposite sides. When a zero occurs for either f or g, the value of $f + g$ is the value of the other function.

At any point where f and g intersect, the value of $f + g$ is double the value of f (or g) for the corresponding x.



Plot the ordered pairs $(x, f(x) - g(x))$ from the table, and join them with a smooth curve to produce the graph of $f - g$.

Observe that the zeros of this $f - g$ graph occur when the graphs of f and g intersect.

Where g has a zero, the value of $f - g$ is the same as the value of f. Where f has a zero, the value of $f - g$ is the opposite of the value of g.

Solution B: Using an algebraic strategy

$f(x) = -x^2 + 3$ and $g(x) = -2x$

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (-x^2 + 3) + (-2x)\\ &= -x^2 - 2x + 3\end{aligned}$$

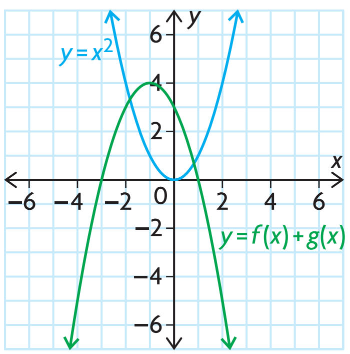
Remember that adding two functions means adding their y-values for a given value of x.

Since the expressions for $f(x)$ and $g(x)$ represent the y-values for each function, we determine an expression for $f + g$ by adding the two expressions.

$$\begin{aligned}(f+g)(x) &= -[x^2 + 2x] + 3\\ &= -[x^2 + 2x + 1 - 1] + 3\\ &= -[(x+1)^2 - 1] + 3\\ &= -(x+1)^2 + 4\end{aligned}$$

Recognizing that $f + g$ is a quadratic function, we can complete the square to change the expression into vertex form.

The graph of $f + g$ can be sketched by starting with the graph of $y = x^2$ and applying the following transformations: reflection in the x-axis, followed by a shift of 1 unit to the left and 4 units up.



The graph of $y = (f + g)(x)$ has the following characteristics: it is neither odd nor even; it is increasing on the interval $(-\infty, -1)$ and decreasing on the interval $(-1, \infty)$; it has zeros at $(-3, 0)$ and $(1, 0)$; it has a maximum value of $y = 4$ when $x = -1$; its domain is $\{x \in \mathbf{R}\}$; its range is $\{y \in \mathbf{R} \mid y \leq 4\}$.

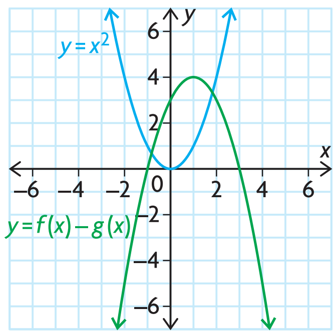
$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= (-x^2 + 3) - (-2x)\\ &= -x^2 + 2x + 3\end{aligned}$$

Similarly, we obtain the expression for $f - g$ by subtracting $g(x)$ from $f(x)$.

In vertex form,

$$\begin{aligned}(f-g)(x) &= -[x^2 - 2x] + 3\\ &= -[x^2 - 2x + 1 - 1] + 3\\ &= -(x-1)^2 + 4\end{aligned}$$

Again, we can rewrite the quadratic expression in vertex form to graph it.



The graph of $f - g$ resembles the graph of $f + g$, except it has been shifted 1 unit to the right instead of 1 unit left.

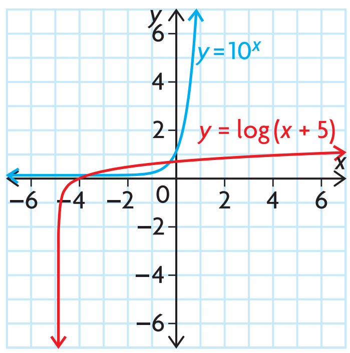
The graph of $y = (f - g)(x)$ has the following characteristics: it is neither odd nor even; it is increasing on the interval $(-\infty, 1)$ and decreasing on the interval $(1, \infty)$; it has zeros at $(-1, 0)$ and $(3, 0)$; it has a maximum value of $y = 4$ when $x = 1$; its domain is $\{x \in \mathbf{R}\}$; its range is $\{y \in \mathbf{R} | y \leq 4\}$.

EXAMPLE 2 Connecting the domains of the sum and difference of two functions

Determine the domain and range of $(f - g)(x)$ and $(f + g)(x)$ if $f(x) = 10^x$ and $g(x) = \log(x + 5)$.

Solution

Sketch the graphs of f and g.



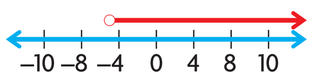
$f(x) = 10^x$ is an exponential function that has the x-axis as its horizontal asymptote. Exponential functions are defined for all real numbers, so its domain is $\{x \in \mathbf{R}\}$.

$g(x) = \log(x + 5)$ is a logarithmic function in base 10. Logarithmic functions are only defined for positive values: $x + 5 > 0$, so $x > -5$. This function has a vertical asymptote defined by $x = -5$. Its domain is $\{x \in \mathbf{R} | x > -5\}$.

$$(f - g)(x) = f(x) - g(x)$$
$$= 10^x - \log(x + 5)$$
$$(f + g)(x) = f(x) + g(x)$$
$$= 10^x + \log(x + 5)$$

The domain of the functions $(f - g)(x)$ and $(f + g)(x)$ is $\{x \in \mathbf{R} | x > -5\}$.

Values for the functions $f - g$ and $f + g$ can only be determined when functions f and g are both defined. This occurs for all values of x that are common to the domains of both f and g.



This is the intersection of the domains of f and g.
$\{x \in \mathbf{R}\} \cap \{x \in \mathbf{R} | x > -5\}$
$= \{x \in \mathbf{R} | x > -5\}$

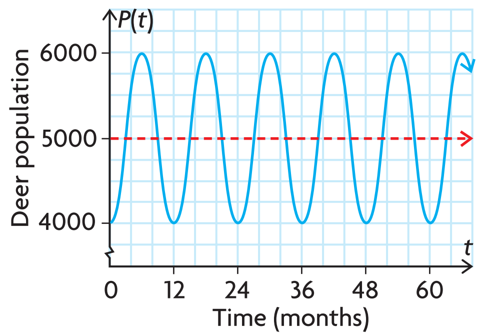
intersection
a set that contains the elements that are common to both sets; the symbol for intersection is $\cap$

EXAMPLE 3 Modelling a situation using a sum of two functions

In the past, biologists have found that the function $P(t) = 5000 - 1000 \cos\left(\frac{\pi}{6}t\right)$ models the deer population in a provincial park, which undergoes a seasonal fluctuation. In this case, $P(t)$ is the size of the deer population t months after January. A disease in the wolf population has caused its population to decline, and the biologists have discovered that the deer population is increasing by 50 deer each month. Assuming that this pattern continues, determine the new function that will model the deer population over time and discuss its characteristics.

Solution

Graph $P(t)$.



In the past, the deer population varied around its average yearly size of 5000. This is represented by the horizontal axis, or midline, of the graph shown.

This suggests that the function $P(t)$ can be thought of in the following way:

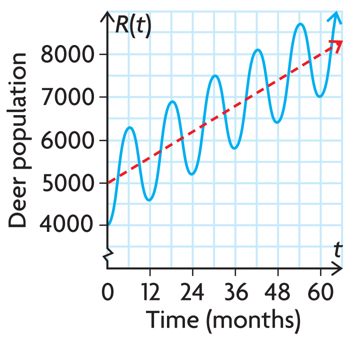
$$P(t) = \underbrace{5000}_{\text{average yearly population}} - \underbrace{1000 \cos\left(\frac{\pi}{6}t\right)}_{\text{seasonal variation}}$$

$$R(t) = \underbrace{5000 + 50t}_{\text{average yearly population increasing by 50 every month}} - \underbrace{1000 \cos\left(\frac{\pi}{6}t\right)}_{\text{seasonal variation}}$$

If the population of deer is increasing by 50 per month, then the average population could be represented by the expression $5000 + 50t$.

The new function $R(t)$ has been created by adding the function $f(t) = 50t$ to $P(t)$.

Therefore, $R(t) = P(t) + f(t)$.



The new population model has the following characteristics: it is neither odd nor even; it is increasing during the first six months of each year and decreasing during the last six months of each year; it has no zeros; it has no maximum or minimum value and its domain is $\{t \in \mathbf{R} \mid t \geq 0\}$; its range is $\{R(t) \in \mathbf{R} \mid R(t) \geq 4000\}$.

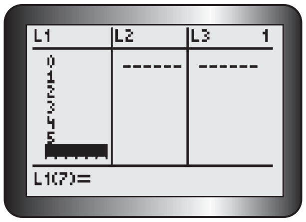
EXAMPLE 4 Reasoning about families of functions

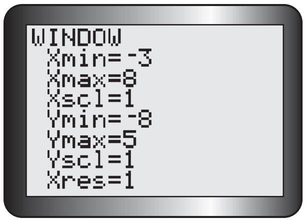
Use graphing technology to explore the graph of $f - g$, where $f(x) = x^2$ and $g(x) = nx$, and $n \in \mathbf{W}$. Discuss your results with respect to the type of function, its shape and symmetry, zeros, maximum and minimum values, intervals of increase/decrease, and domain and range.

Solution

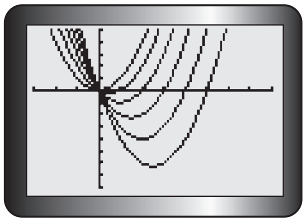
$(f - g)(x) = f(x) - g(x)$,
then for $f(x) = x^2$ and $g(x) = nx$,
$(f - g)(x) = x^2 - nx$, where $n \in \mathbf{W}$

$f - g$ will always be a quadratic function, regardless of the value of n.





Enter several values of $n \in \{0, 1, 2, 3, 4, 5\}$ into list 1 (L1) on a graphing calculator. Enter the equation $X^2 - L_1X$ into the equation editor. Then graph using the window settings shown.



The series of parabolas that $f - g$ produces will have identical shapes, since $a = 1$. It appears that, as n increases, the parabola is shifted to the right and down.

Most of these functions are neither odd nor even since their graphs are not symmetrical about the origin or y-axis.

The exception is when $n = 0$, which produces the even function $(f - g)(x) = x^2$.

The zeros of each parabola occur at $x = 0$ and $x = n$.

In factored form, $(f - g)(x) = x(x - n)$
$x = 0$ or $x - n = 0$
$x = n$

Since the parabola opens upward, the minimum value is $-\frac{n^2}{4}$, and the axis of symmetry is $x = \frac{n}{2}$.

The vertex of each parabola will occur at $\left(\frac{n}{2}, -\frac{n^2}{4}\right)$.

These functions are decreasing when $x \in \left(-\infty, \frac{n}{2}\right)$ and increasing when $x \in \left(\frac{n}{2}, \infty\right)$. The domain of $f - g$ is $\{x \in \mathbf{R}\}$, and the range is $y \in \left[-\frac{n^2}{4}, \infty\right)$.

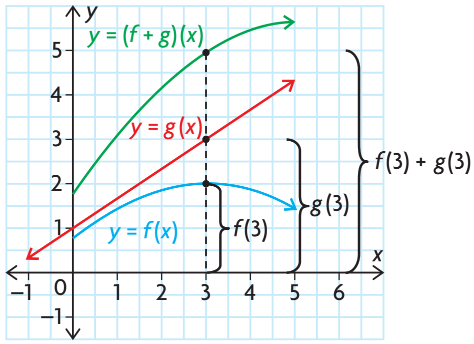
Complete the square to put the function into vertex form.

$$(f - g)(x) = x^2 - nx + \left(\frac{n}{2}\right)^2 - \left(\frac{n}{2}\right)^2$$
$$= \left(x - \frac{n}{2}\right)^2 - \frac{n^2}{4}$$

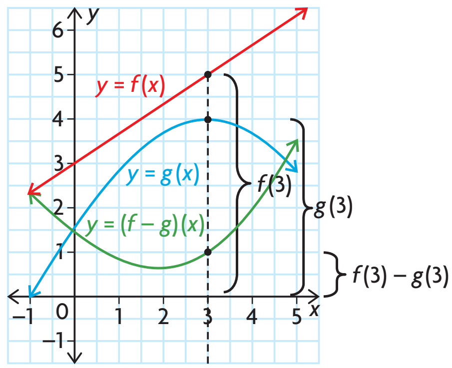
In Summary

Key Ideas

- When two functions $f(x)$ and $g(x)$ are combined to form the function $(f + g)(x)$, the new function is called the sum of f and g. For any given value of x, the value of the function is represented by $f(x) + g(x)$. The graph of $f + g$ can be obtained from the graphs of functions f and g by adding corresponding y-coordinates.



- Similarly, the difference of two functions, $f - g$, is $(f - g)(x) = f(x) - g(x)$. The graph of $f - g$ can be obtained by subtracting the y-coordinate of g from the y-coordinate of f for every pair of corresponding x-values.



Need to Know

- Algebraically, $(f + g)(x) = f(x) + g(x)$ and $(f - g)(x) = f(x) - g(x)$.
- The domain of $f + g$ or $f - g$ is the intersection of the domains of f and g. This means that the functions $f + g$ and $f - g$ are only defined where the domains of both f and g overlap.

CHECK Your Understanding

1. Let $f = \{(-4, 4), (-2, 4), (1, 3), (3, 5), (4, 6)\}$ and $g = \{(-4, 2), (-2, 1), (0, 2), (1, 2), (2, 2), (4, 4)\}$. Determine:

a) $f + g$ **c)** $f - g$ **e)** $f + f$
b) $g + f$ **d)** $g - f$ **f)** $g - g$

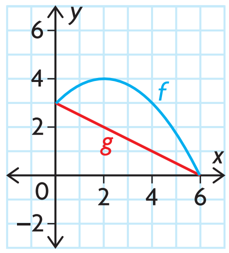
2. a) Determine $(f + g)(4)$ when $f(x) = x^2 - 3$ and $g(x) = -\frac{6}{x - 2}$.

b) For which value of x is $(f + g)(x)$ undefined? Explain why.

c) What is the domain of $(f + g)(x)$ and $(f - g)(x)$?

3. What is the domain of $f - g$, where $f(x) = \sqrt{x + 1}$ and $g(x) = 2\log[-(x + 1)]$?

4. Make a reasonable sketch of the graph of $f + g$ and $f - g$, where $0 \leq x \leq 6$, for the functions shown.



5. **a)** Given the function $f(x) = |x|$ (which is even) and $g(x) = x$ (which is odd), determine $f + g$.

b) Is $f + g$ even, odd, or neither?

PRACTISING

6. K $f = \{(-9, -2), (-8, 5), (-6, 1), (-3, 7), (-1, -2), (0, -10)\}$ and $g = \{(-7, 7), (-6, 6), (-5, 5), (-4, 4), (-3, 3)\}$. Calculate:

a) $f + g$ **c)** $f - g$ **e)** $f - f$
b) $g + f$ **d)** $g - f$ **f)** $g + g$

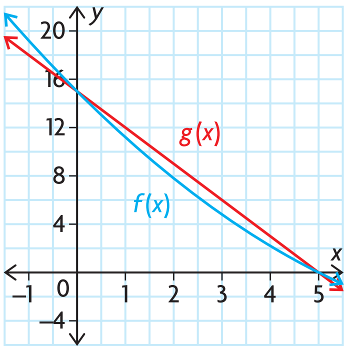
7. **a)** If $f(x) = \frac{1}{3x + 4}$ and $g(x) = \frac{1}{x - 2}$, what is $f + g$?

b) What is the domain of $f + g$?

c) What is $(f + g)(8)$?

d) What is $(f - g)(8)$?

8. The graphs of $f(x)$ and $g(x)$, where $0 \leq x \leq 5$, are shown. Sketch the graphs of $(f + g)(x)$ and $(f - g)(x)$.



9. For each pair of functions, determine the equations of $f(x) + g(x)$ and $f(x) - g(x)$. Using graphing technology, graph these new functions and discuss each of the following characteristics of the resulting graphs: symmetry, intervals of increase/decrease, zeros, maximum and minimum values, period (where applicable), and domain and range.

a) $f(x) = 2^x, g(x) = x^3$

b) $f(x) = \cos(2\pi x), g(x) = x^4$

c) $f(x) = \log(x), g(x) = 2x$

d) $f(x) = \sin(2\pi x), g(x) = 2\sin(\pi x)$

e) $f(x) = \sin(2\pi x) + 2, g(x) = \frac{1}{x}$

f) $f(x) = \sqrt{x - 2}, g(x) = \frac{1}{x - 2}$

10. a) Is the sum of two even functions even, odd, or neither? Explain.
    b) Is the sum of two odd functions even, odd, or neither? Explain.
    c) Is the sum of an even function and an odd function even, odd, or neither? Explain.

11. **A** Recall, from Example 3, the function $P(t) = 5000 - 1000 \cos\left(\frac{\pi}{6}t\right)$, which models the deer population in a provincial park. A disease in the deer population has caused it to decline. Biologists have discovered that the deer population is decreasing by 25 deer each month.
    a) Assuming that this pattern continues, determine the new function that will model the deer population over time and discuss its characteristics.
    b) Estimate when the deer population in this park will be extinct.

12. When the driver of a vehicle observes an obstacle in the vehicle's path, the driver reacts to apply the brakes and bring the vehicle to a complete stop. The distance that the vehicle travels while coming to a stop is a combination of the reaction distance, r, in metres, given by $r(x) = 0.21x$, and the braking distance, b, also in metres, given by $b(x) = 0.006x^2$. The speed of the vehicle is x km/h. Determine the stopping distance of the vehicle as a function of its speed, and calculate the stopping distance if the vehicle is travelling at 90 km/h.

13. **T** Determine a sine function, f, and a cosine function, g, such that $y = \sqrt{2}\sin(\pi(x - 2.25))$ can be written in the form of $f - g$.

14. Use graphing technology to explore the graph of $f + g$, where $f(x) = x^3$, $g(x) = nx^2$, and $n \in \mathbf{W}$. Discuss your results with respect to the type of function, its shape and symmetry, zeros, maximum and minimum values, intervals of increase/decrease, and domain and range.

15. **C** Describe or give an example of
    a) two odd functions whose sum is an even function
    b) two functions whose sum represents a vertical stretch applied to one of the functions
    c) two rational functions whose difference is a constant function

Extending

16. Let $f(x) = x^2 - nx + 5$ and $g(x) = mx^2 + x - 3$. The functions are combined to form the new function $h(x) = f(x) + g(x)$. Points $(1, 3)$ and $(-2, 18)$ satisfy the new function. Determine the values of m and n.

9.3 Combining Two Functions: Products

GOAL

Represent the product of two functions graphically and algebraically, and determine the characteristics of the product.

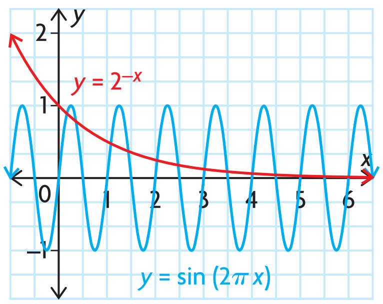
YOU WILL NEED

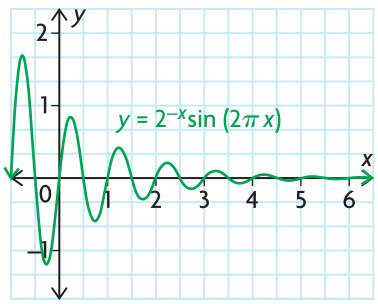
- graphing calculator or graphing software

LEARN ABOUT the Math

In the previous section, you learned that music is made up of combinations of sine waves. Have you ever wondered how sound engineers cause the music to fade out, gradually, at the end of a song? The music fades out because the sine waves that represent the music are being squashed or **damped**. Mathematically, this can be done by multiplying a sine function by another function.

The functions defined by $g(x) = \sin(2\pi x)$ and $f(x) = 2^{-x}$, where $\{x \in \mathbf{R} | x \geq 0\}$, are shown below. Observe what happens when these functions are multiplied to produce the graph of $(f \times g)(x) = 2^{-x} \sin(2\pi x)$.





? Can the product of two functions be constructed using the same strategies that are used to create the sum or difference of two functions?

EXAMPLE 1 Connecting the values of a product function to the values of each function

Investigate the product of the functions $f(x) = 2^{-x}$ and $g(x) = \sin(2\pi x)$.

Solution

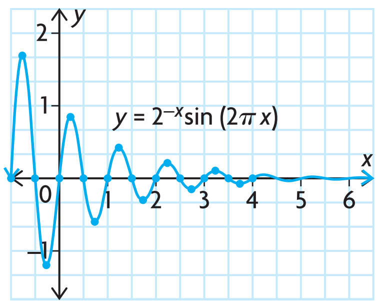
	A	B	C	D
1	x	f(x)=2^–x	g(x)=sin(2πx)	(fxg)(x)=(2^–x)sin(2πx)
2	0.00	1.00	0.00	0.00
3	0.25	0.84	1.00	0.84
4	0.50	0.71	0.00	0.00
5	0.75	0.59	–1.00	–0.59
6	1.00	0.50	0.00	0.00
7	1.25	0.42	1.00	0.42
8	1.50	0.35	0.00	0.00
9	1.75	0.30	–1.00	–0.30
10	2.00	0.25	0.00	0.00
11	2.25	0.21	1.00	0.21
12	2.50	0.18	0.00	0.00
13	2.75	0.15	–1.00	–0.15
14	3.00	0.13	0.00	0.00
15	3.25	0.11	1.00	0.11
16	3.50	0.09	0.00	0.00
17	3.75	0.07	–1.00	–0.07
18	4.00	0.06	0.00	0.00

In a spreadsheet, enter some values of x in column A, and enter the formulas for f, g, and $f \times g$ in columns B, C, and D, respectively.

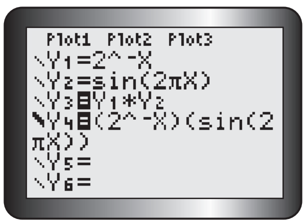
The values in the table have been rounded to two decimal places.

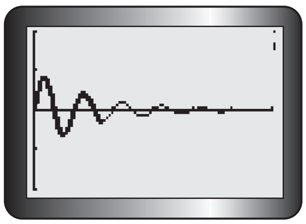
Looking at each row of the table, for any given value of x, the function value of $(f \times g)(x)$ is represented by $f(x) \times g(x)$.

This makes sense since the new function is created by multiplying the original functions together.

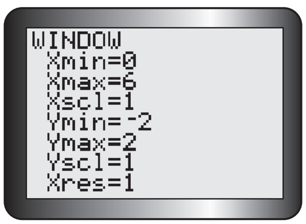


Plotting the ordered pairs $(x, (f \times g)(x))$ results in the graph of the dampened sine wave. This means that the graph of $f \times g$ can be obtained from the graphs of functions f and g by multiplying corresponding y-coordinates.





Use a graphing calculator to verify the results. Enter the functions into the equation editor as shown. Turn off the first two functions, and choose a bold line to graph the third function.



Use window settings that match the given graph of $(f \times g)(x)$.

The graph of Y4 traces over the graph of the product function Y3. This confirms that the product function is identical to, and obtained by, multiplying the expressions of the two functions together.

The graph of Y3 shows the graph produced by multiplying the corresponding *y*-values of the functions stored in Y1 and Y2.

Reflecting

A. If $(0.4, 0.76) \in f(x)$ and $(0.4, 0.59) \in g(x)$, what ordered pair belongs to $(f \times g)(x)$?

B. If $f(1) = 0.5$ and $(f \times g)(1) = 0$, what do you know about the value of $g(1)$? Explain.

C. Look at the original graphs of $f(x)$ and $g(x)$. How can you predict the locations of the zeros of $(f \times g)(x)$ before you construct a table of values or a graph? Explain.

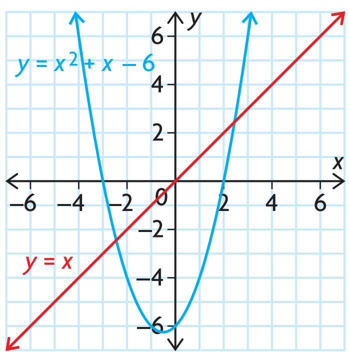
D. What is the domain of $f \times g$? How does it compare with the domains of f and g?

E. If function $f(x)$ was replaced by $f(x) = \sqrt{x}$, explain how this would change the domain of $(f \times g)(x)$.

APPLY the Math

EXAMPLE 2 Constructing the product of two functions graphically

Determine the graph of $y = (f \times g)(x)$, given the graphs of $f(x) = x^2 + x - 6$ and $g(x) = x$.



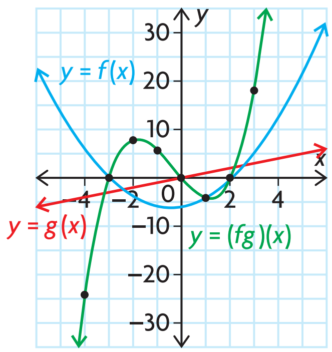
Solution

x	$f(x)$	$g(x)$	$(f \times g)(x)$
-4	6	-4	-24
-3	0	-3	0
-2	-4	-2	8
-1	-6	-1	6
0	-6	0	0
1	-4	1	-4
2	0	2	0
3	6	3	18
4	14	4	56

Use the graph to determine some of the points on the graphs of f and g, and create a table of values.

The graphs indicate that both functions have the same domain, $\{x \in \mathbf{R}\}$.

Determine the values of $(f \times g)(x)$ by multiplying the y-coordinates of f and g together for the same value of x.



The domain of the product function is the intersection of the domains of f and g, $\{x \in \mathbf{R}\}$.

Plot some of the ordered pairs $(x, (f \times g)(x))$, and use these to sketch the graph of the product function.

Notice that the zeros of the two functions, f and g, result in points that are also zeros of $f \times g$. This makes sense since the product of zero and any number is still zero.

Also notice that $(f \times g)(1) = f(1)$ because $g(1) = 1$. As a result, $(f \times g)(1) = f(1) \times 1 = -4 \times 1 = -4$. Similarly, $(f \times g)(-1) = -f(-1)$ because $g(-1) = -1$, so $(f \times g)(-1) = f(-1) \times (-1)$ $= -6 \times -1 = 6$.

Functions f and g are second and first degree polynomial functions, so the product function fg is a third degree polynomial function (also called a cubic function).

EXAMPLE 3 Constructing the product of two functions algebraically

Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{2}x - 2$.

a) Find the equation of the function $(f \times g)(x)$.
b) Determine $(f \times g)(4)$.
c) Find the domain of $y = (f \times g)(x)$.
d) Use graphing technology to graph $y = (f \times g)(x)$, and discuss the key characteristics of the graph.

Solution

a) $(f \times g)(x) = f(x) \times g(x)$

$$= \sqrt{x}\left(\frac{1}{2}x - 2\right)$$

To find the formula for the product of the functions, take the expression for $f(x)$ and multiply it by the expression for $g(x)$.

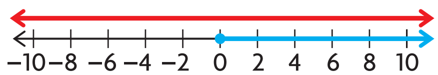
b) $(f \times g)(4) = \sqrt{4}\left(\frac{1}{2}(4) - 2\right)$

$$= 2(0)$$
$$= 0$$

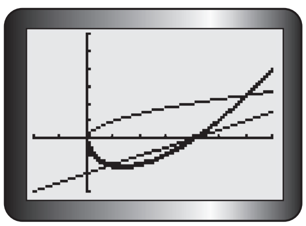
Calculate the value of $(f \times g)(4)$ by substituting $x = 4$ into the expression $(f \times g)(x)$.

c) The domain of g is $\{x \in \mathbf{R}\}$, but the domain of f is $\{x \in \mathbf{R} | x \geq 0\}$. So, the domain of $f \times g$ is $\{x \in \mathbf{R} | x \geq 0\}$.

The domain of $f \times g$ can only consist of x-values that exist in the domains of both f and g.



d)



The graph of $f \times g$ is the bold line.

The graph of $f \times g$
- lies below the x-axis when $x \in (0, 4)$, since $f(x) > 0$ and $g(x) < 0$ in that interval
- has zeros occurring at $x = 0$ when $f(x) = 0$ and at $x = 4$ when $g(x) = 0$; no other zeros will occur, since both functions are positive
- is neither odd nor even since it has no symmetry about the origin or the y-axis

EXAMPLE 4 Modelling a situation using a product function

The rate at which a contaminant leaves a storm sewer and enters a lake depends on two factors: the concentration of the contaminant in the water from the sewer and the rate at which the water leaves the sewer. Both of these factors vary with time. The concentration of the contaminant, in kilograms per cubic metre of water, is given by $c(t) = t^2$, where t is in seconds. The rate at which water leaves the sewer, in cubic metres per second, is given by $w(t) = \frac{1}{t^4 + 20}$. Determine the time at which the contaminant leaves the sewer and enters the lake at the maximum rate.

Solution

$c(t)$ is in $\frac{\text{kg}}{\text{m}^3}$ and $w(t)$ is in $\frac{\text{m}^3}{\text{s}}$

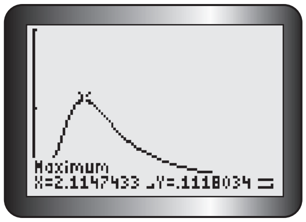
$$c(t) \times w(t) \rightarrow \left(\frac{\text{kg}}{\cancel{\text{m}^3}}\right)\left(\frac{\cancel{\text{m}^3}}{\text{s}}\right) = \frac{\text{kg}}{\text{s}}$$

The product of the concentration function and the water rate function results in a function that describes the rate of contaminant flow into the lake.

Analyze the units of both functions to help you determine the relationship between the functions that can be used to determine a function for the rate at which the contaminant flows into the lake.

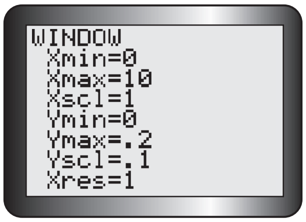
$$c(t) \times w(t) = (t^2)\left(\frac{1}{t^4 + 20}\right)$$
$$= \frac{t^2}{t^4 + 20}$$

In this context, the domain of both functions is $\{t \in \mathbf{R} \mid t \geq 0\}$ since both functions have time as the independent variable. Thus, $\{t \in \mathbf{R} \mid t \geq 0\}$ is also the domain of $c(t) \times w(t)$.



The contaminant is flowing into the lake at a maximum rate of about 0.11 kg/s. This occurs at about 2 s after the water begins to flow into the lake.

Use the maximum operation on a graphing calculator to graph $c(t) \times w(t)$ on its domain and estimate when its maximum value occurs.



Tech Support

For help determining the maximum value of a function using a graphing calculator, see Technical Appendix, T-9.

In Summary

Key Idea

- When two functions, $f(x)$ and $g(x)$, are combined to form the function $(f \times g)(x)$, the new function is called the product of f and g. For any given value of x, the function value is represented by $f(x) \times g(x)$. The graph of $f \times g$ can be obtained from the graphs of functions f and g by multiplying each y-coordinate of f by the corresponding y-coordinate of g.

Need to Know

- Algebraically, $f \times g$ is defined as $(f \times g)(x) = f(x) \cdot g(x)$.
- The domain of $f \times g$ is the intersection of the domains of f and g.
- If $f(x) = 0$ or $g(x) = 0$, then $(f \times g)(x) = 0$.
- If $f(x) = \pm 1$, then $(f \times g)(x) = \pm g(x)$. Similarly, if $g(x) = \pm 1$, then $(f \times g)(x) = \pm f(x)$.

CHECK *Your Understanding*

1. For each of the following pairs of functions, determine $(f \times g)(x)$.

a) $f(x) = \{(0, 2), (1, 5), (2, 7), (3, 12)\}$,
$g(x) = \{(0, -1), (1, -2), (2, 3), (3, 5)\}$

b) $f(x) = \{(0, 3), (1, 6), (2, 10), (3, -5)\}$,
$g(x) = \{(0, 4), (2, -2), (4, 1), (6, 3)\}$

c) $f(x) = x, g(x) = 4$

d) $f(x) = x, g(x) = 2x$

e) $f(x) = x + 2, g(x) = x^2 - 2x + 1$

f) $f(x) = 2^x, g(x) = \sqrt{x - 2}$

2. a) Graph each pair of functions in question 1, parts c) to f), on the same grid.

b) State the domains of f and g.

c) Use your graph to make an accurate sketch of $y = (f \times g)(x)$.

d) State the domain of $f \times g$.

3. If $f(x) = \sqrt{1 + x}$ and $g(x) = \sqrt{1 - x}$, determine the domain of $y = (f \times g)(x)$.

PRACTISING

4. Determine $(f \times g)(x)$ for each of the following pairs of functions.

K **a)** $f(x) = x - 7, g(x) = x + 7$

b) $f(x) = \sqrt{x + 10}, g(x) = \sqrt{x + 10}$

c) $f(x) = 7x^2, g(x) = x - 9$

d) $f(x) = -4x - 7, g(x) = 4x + 7$

e) $f(x) = 2 \sin x, g(x) = \dfrac{1}{x - 1}$

f) $f(x) = \log (x + 4), g(x) = 2^x$

5. For each of the problems in question 4, state the domain and range of $(f \times g)(x)$.

6. For each of the problems in question 4, use graphing technology to graph $(f \times g)(x)$ and then discuss each of the following characteristics of the graphs: symmetry, intervals of increase/decrease, zeros, maximum and minimum values, and period (where applicable).

7. The graph of the function $f(x)$ is a line passing through the origin with a slope of -4, whereas the graph of the function $g(x)$ is a line with a y-intercept of 1 and a slope of 6. Sketch the graph of $(f \times g)(x)$.

8. For each of the following pairs of functions, state the domain of $(f \times g)(x)$.

a) $f(x) = \dfrac{1}{x^2 - 5x - 14}, g(x) = \sec x$

b) $f(x) = 99^x, g(x) = \log(x - 8)$

c) $f(x) = \sqrt{x + 81}, g(x) = \csc x$

d) $f(x) = \log(x^2 + 6x + 9), g(x) = \sqrt{x^2 - 1}$

9. If the function $f(t)$ describes the per capita energy consumption in a particular country at time t, and the function $p(t)$ describes the population of the country at time t, then explain what the product function $(f \times p)(t)$ represents.

10. A An average of 20 000 people visit the Lakeside Amusement Park each day in the summer. The admission fee is \$25.00. Consultants predict that, for each \$1.00 increase in the admission fee, the park will lose an average of 750 customers each day.

a) Determine the function that represents the projected daily revenue if the admission fee is increased.

b) Is the revenue function a product function? Explain.

c) Estimate the ticket price that will maximize revenue.

11. A water purification company has patented a unique process to remove contaminants from a container of water at the same time that more contaminated water is added for purification. The percent of contaminated material in the container of water being purified can be modelled by the function $c(t) = (0.9)^t$, where t is the time in seconds. The number of litres of water in the container can be modelled by the function $l(t) = 650 + 300t$. Write a function that represents the number of litres of contaminated material in the container at any time t, and estimate when the amount of contaminated material is at its greatest.

12. Is the following statement true or false? "If $f(x) \times g(x)$ is an odd function, then both $f(x)$ and $g(x)$ are odd functions." Justify your answer.

T

13. Let $f(x) = mx^2 + 2x + 5$ and $g(x) = 2x^2 - nx - 2$. The functions are combined to form the new function $h(x) = f(x) \times g(x)$. Points $(1, -40)$ and $(-1, 24)$ satisfy the new function. Determine $f(x)$ and $g(x)$.

14. Let $f(x) = \sqrt{-x}$ and $g(x) = \log(x + 10)$.

C

a) Determine the equation of the function $y = (f \times g)(x)$, and state its domain.

b) Provide two different strategies for sketching $y = (f \times g)(x)$. Discuss the merits of each strategy.

c) Choose one of the strategies you discussed in part b), and make an accurate sketch.

15. **a)** If $f(x) = x^2 - 25$, determine the equation of the product function $f(x) \times \frac{1}{f(x)}$.

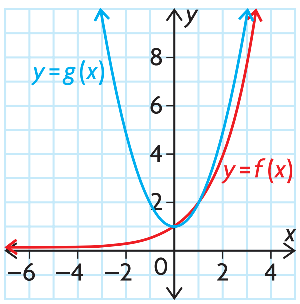
b) Determine the domain, and sketch the graph of the product function you found in part a).

c) If $f(x)$ is a polynomial function, explain how the domain and range of $f(x) \times \frac{1}{f(x)}$ changes as the degree of $f(x)$ changes.

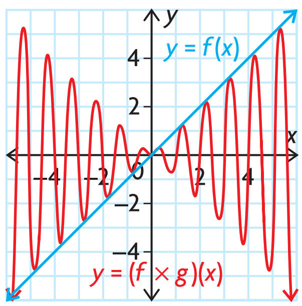
Extending

16. Given the following graphs, determine the equations of $y = f(x)$, $y = g(x)$, and $y = (f \times g)(x)$.

a)



b)



17. Determine two functions, f and g, whose product would result in each of the following functions.

a) $(f \times g)(x) = 4x^2 - 81$

b) $(f \times g)(x) = 8 \sin^3 x + 27$

c) $(f \times g)(x) = 4x^{\frac{5}{2}} - 3x^{\frac{3}{2}} + x^{\frac{1}{2}}$

d) $(f \times g)(x) = \dfrac{6x - 5}{2x + 1}$

9.4 Exploring Quotients of Functions

YOU WILL NEED

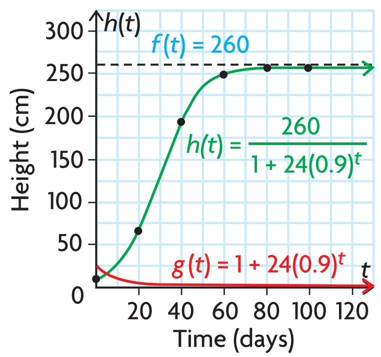
- graph paper
- graphing calculator or graphing software

GOAL

Represent the quotient of two functions graphically and algebraically, and determine the characteristics of the quotient.

EXPLORE the Math

The logistic function is often used to model growth. This function has the general equation $P(t) = \frac{c}{1 + ab^t}$, where $a > 0$, $0 < b < 1$, and $c > 0$. In this function, t is time. For example, the height of a sunflower plant can be modelled using the function $h(t) = \frac{260}{1 + 24(0.9)^t}$, where $h(t)$ is the height in centimetres and t is the time in days. The function $h(t) = \frac{f(t)}{g(t)}$ is the quotient of two functions, where $f(t) = 260$ (a constant function) and $g(t) = 1 + 24(0.9)^t$ (an exponential function). The table and graphs show that the values of a quotient function can be determined by dividing the values of the two functions.



t (days)	$f(t) = 260$	$g(t) = 1 + 24(0.9)^t$	$h(t) = \frac{260}{1 + 24(0.9)^t}$
0	260	25	$\frac{260}{25} = 10.4$
20	260	3.92	66.3
40	260	1.35	192.6
60	260	1.04	250.0
80	260	1.01	257.4
100	260	1.00	260.0

This function shows slow growth for small values of t, then rapid growth, and then slow growth again when the height of the sunflower approaches its maximum height of 260 cm.

The logistic function is an example of a quotient function. In function notation, we can express this as $(f \div g)(x) = f(x) \div g(x)$.

? What are the characteristics of functions that are produced by quotients of other types of functions?

A. Consider the function defined by $y = \frac{4}{x + 2}$ in the form $y = \frac{f(x)}{g(x)}$. Write the expressions for functions f and g.

B. On graph paper, draw and label the graphs of $y = f(x)$ and $y = g(x)$, and state their domains.

C. Locate any points on your graph of g where $g(x) = 0$. What will happen when you calculate the value of $f \div g$ for these x-coordinates? How would this appear on a graph?

D. Locate any points on your graph where $g(x) = \pm 1$. What values of x produced these results? Explain how you could determine these x-values algebraically.

E. Determine the value of $f \div g$ for each of the x's in part D. How do your answers compare with the corresponding values of f? Explain.

F. Over what interval(s) is $g(x) > 0$? Over what interval(s) is $f(x) > 0$?

G. Determine all the intervals where both f and g are positive or where both are negative. Will the function $f \div g$ be positive in the same intervals? Justify your answer.

H. Determine any intervals where either f or g is positive and the other is negative. Discuss the behaviour of $f \div g$ over these intervals. If no such intervals exist, what implication would this have for $f \div g$? Explain.

I. For what values of x is $(f \div g)(x) = f(x)$? For what values of x is $(f \div g)(x) = -f(x)$?

J. Using all the information about $f \div g$ that you have determined, make an accurate sketch of $y = (f \div g)(x)$ and state its domain.

K. Verify your results by graphing f, g, and $f \div g$ using graphing technology.

L. Repeat parts A to K using the following functions.

i) $y = \dfrac{x + 1}{(x + 3)(x - 1)}$

ii) $y = \dfrac{4}{x^2 + 1}$

iii) $y = \dfrac{\sin x}{x}$

iv) $y = \dfrac{2^x}{\sqrt{x}}$

Reflecting

M. The graphs of $y = \dfrac{4}{x + 2}$, $y = \dfrac{x + 1}{(x + 3)(x - 1)}$, and $y = \dfrac{2x}{\sqrt{x}}$ have vertical asymptotes, but the graphs of $h(t) = \dfrac{260}{1 + 24(0.9)^t}$, $y = \dfrac{4}{x^2 + 1}$, and $y = \dfrac{\sin x}{x}$ do not. Explain.

N. The graph of $y = \dfrac{x + 1}{(x + 3)(x - 1)}$ lies above the x-axis in the interval $x \in (-3, -1)$. By examining the behaviour of functions f and g, explain how you can reach this conclusion.

In Summary

Key Idea

- When two functions, $f(x)$ and $g(x)$, are combined to form the function $(f \div g)(x)$, the new function is called the quotient of f and g. For any given value of x, the value of the function is represented by $f(x) \div g(x)$. The graph of $f \div g$ can be obtained from the graphs of functions f and g by dividing each y-coordinate of f by the corresponding y-coordinate of g.

Need to Know

- Algebraically, $(f \div g)(x) = f(x) \div g(x)$.
- $f \div g$ will be defined for all x-values that are in the intersection of the domains of f and g, except in the case where $g(x) = 0$. If the domain of f is A, and the domain of g is B, then the domain of $f \div g$ is $\{x \in \mathbf{R} \mid x \in A \cap B, g(x) \neq 0\}$.
- If $f(x) = 0$ when $g(x) \neq 0$, then $(f \div g)(x) = 0$.
- If $f(x) = \pm 1$, then $(f \div g)(x) = \pm \frac{1}{g(x)}$. Similarly, if $g(x) = \pm 1$, then $(f \div g)(x) = \pm f(x)$. Also, if $f(x) = \pm g(x)$, then $(f \div g)(x) = \pm 1$

Further Your Understanding

1. For each of the following pairs of functions, write the equation of $y = (f \div g)(x)$.

a) $f(x) = 5, g(x) = x$

b) $f(x) = 4x, g(x) = 2x - 1$

c) $f(x) = 4x, g(x) = x^2 + 4$

d) $f(x) = x + 2, g(x) = \sqrt{x - 2}$

e) $f(x) = 8, g(x) = 1 + \left(\frac{1}{2}\right)^x$

f) $f(x) = x^2, g(x) = \log(x)$

2. **a)** Graph each pair of functions in question 1 on the same grid.

b) State the domains of f and g.

c) Use your graphs to make an accurate sketch of $y = (f \div g)(x)$.

d) State the domain of $f \div g$.

3. Recall that the function $h(t) = \frac{260}{1 + 24(0.9)^t}$ models the growth of a sunflower, where $h(t)$ is the height in centimetres and t is the time in days.

a) Calculate the average rate of growth of the sunflower over the first 20 days.

b) Determine when the sunflower has grown to half of its maximum height.

c) Estimate the instantaneous rate of change in height at the time you found in part b).

d) What happens to the instantaneous rate of change in height as the sunflower approaches its maximum height? How does this relate to the shape of the graph?

9 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: If you are given the graphs of two functions, *f* and *g*, how can you determine the location of a point that would appear on the graphs of $f + g$, $f - g$, $f \times g$, and $f \div g$?

A: For any particular x-value, determine the y-value on each graph, separately. For $f + g$, add these two y-values together. For $f - g$, subtract the y-value of g from the y-value of f. For $f \times g$, multiply these two y-values together. For $f \div g$, divide the y-value of f by the y-value of g. Each of these points has, as its coordinates, the same x-value and the new y-value.

Study Aid

- See Lessons 9.1 to 9.4.
- Try Mid-Chapter Review Question 2.

Q: If you are given the equations of two functions, *f* and *g*, how can you determine the equations of the functions $f + g$, $f - g$, $f \times g$, and $f \div g$?

A: Every time you combine two functions in one of these ways, you are simply performing a different arithmetic operation on every pair of y-values, one from each of the functions being combined, provided that the x-values are the same. Since the equation of each function defines the y-values of each function, the new equation can be determined by adding, subtracting, multiplying, or dividing the y-value expressions as required.

For example, if $f(x) = x^2 + 8$ and $g(x) = 5^x$, then

$$\begin{aligned}(f + g)(x) &= f(x) + g(x)\\ &= x^2 + 8 + 5^x\end{aligned} \qquad \begin{aligned}(f \times g)(x) &= f(x) \times g(x)\\ &= (x^2 + 8)(5^x)\end{aligned}$$

$$\begin{aligned}(f - g)(x) &= f(x) - g(x)\\ &= x^2 + 8 - 5^x\end{aligned} \qquad \begin{aligned}(f \div g)(x) &= f(x) \div g(x)\\ &= \frac{x^2 + 8}{5^x}\end{aligned}$$

Study Aid

- See Lessons 9.1 to 9.4.
- Try Mid-Chapter Review Questions 5 and 7.

Q: How can you determine the domain of the combined functions $f + g$, $f - g$, $f \times g$, and $f \div g$?

A: Since you can only combine points from two functions when they share the same x-value, the domain of the combined function must consist of the set of x-values where the domains of the two given functions intersect. The only exception occurs when you are dividing two functions. The function $f \div g$ is not defined when its denominator is equal to zero, since division by zero is undefined. As a result, x-values that cause $g(x)$ to equal zero must be excluded from the domain.

Study Aid

- See Lessons 9.1 to 9.4.
- Try Mid-Chapter Review Questions 5 and 7.

PRACTICE Questions

Lesson 9.1

1. Given the functions $f(x) = \cos x$ and $g(x) = \sin x$, which operations can be used to combine the two functions to create a new function with an amplitude that is less than 1?

Lesson 9.2

2. Let $f(x) = \{(-9, -2), (-6, -3), (-3, 0), (0, 2), (3, 7)\}$ and $g(x) = \{(-12, 9), (-9, 4), (-8, 1), (-7, 10), (-6, -6), (0, 12)\}$. Determine

a) $(f + g)(x)$
b) $(g + f)(x)$
c) $(f - g)(x)$
d) $(g - f)(x)$

3. The cost, in thousands of dollars, for a company to produce x thousand of its product is given by the function $C(x) = 10x + 30$. The revenue from the sales of the product is given by the function $R(x) = -5x^2 + 150x$.

a) Write the function that represents the company's profit on sales of x thousand of its product.
b) Graph the cost, revenue, and profit functions on the same coordinate grid, where $0 \leq x \leq 40$.
c) What is the company's profit on the sale of 7500 of its product?

4. Steve earns \$24.39/h operating an industrial plasma torch at a rail-car manufacturing plant. He receives \$0.58/h more for working the night shift, as well as \$0.39/h more for working weekends.

a) Write a function that describes Steve's daily earnings under regular pay.
b) What function shows his daily earnings under the night-shift premium?
c) What function shows his daily earnings under the weekend premium?
d) What function represents his earnings for the night shift on Saturday?
e) How much does Steve earn for working 11 h on Saturday night, if he earns time and a half on that day's rate for more than 8 h of work?

Lesson 9.3

5. Determine $(f \times g)(x)$ for each of the following pairs of functions, and state its domain.

a) $f(x) = x + \frac{1}{2}, g(x) = x + \frac{1}{2}$
b) $f(x) = \sqrt{x - 10}, g(x) = \sin(3x)$
c) $f(x) = 11x^3, g(x) = \frac{2}{x + 5}$
d) $f(x) = 90x - 1, g(x) = 90x + 1$

6. A diner is open from 6 a.m. to 6 p.m., and the average number of customers in the diner at any time can be modelled by the function $C(h) = -30 \cos\left(\frac{\pi}{6}h\right) + 34$, where h is the number of hours after the 6 a.m. opening time. The average amount of money, in dollars, that each customer in the diner will spend can be modelled by the function $D(h) = -3 \sin\left(\frac{\pi}{6}h\right) + 7$.

a) Write the function that represents the diner's average revenue from the customers.
b) Graph the function you wrote in part a).
c) What is the average revenue from the customers in the diner at 2 p.m.?

Lesson 9.4

7. Calculate $(f \div g)(x)$ for each of the following pairs of functions, and state its domain.

a) $f(x) = 240, g(x) = 3x$
b) $f(x) = 10x^2, g(x) = x^3 - 3x$
c) $f(x) = x + 8, g(x) = \sqrt{x - 8}$
d) $f(x) = 14x^2, g(x) = 2 \log x$

8. Recall that $y = \tan x$ can be written as the quotient of two functions: $f(x) = \sin x$ and $g(x) = \cos x$. List as many other trigonometric functions as possible that could be written as the quotient of two functions.

9.5 Composition of Functions

GOAL

Determine the composition of two functions numerically, graphically, and algebraically.

LEARN ABOUT the Math

Sometimes you will find a situation in which two related functions are present. Often both functions are needed to analyze the situation or solve a problem.

Forest fires often spread in a roughly circular pattern. The area burned depends on the radius of the fire. The radius, in turn, may increase at a constant rate each day.

Suppose that $A(r) = \pi r^2$ represents the area, A, of a fire as a function of its radius, r. If the radius of the fire increases by 0.5 km/day, then $r(t) = 0.5t$ represents the radius of the fire as a function of time, t. The area is measured in square kilometres, the radius is measured in kilometres, and the time is measured in days.

? How can the area burned be determined on the sixth day of the fire?

EXAMPLE 1 Reasoning numerically, graphically, and algebraically about a composition of functions

Determine the area burned by the fire on the sixth day.

Solution A: Using graphical and numerical analysis

Use the given functions to make tables of values.

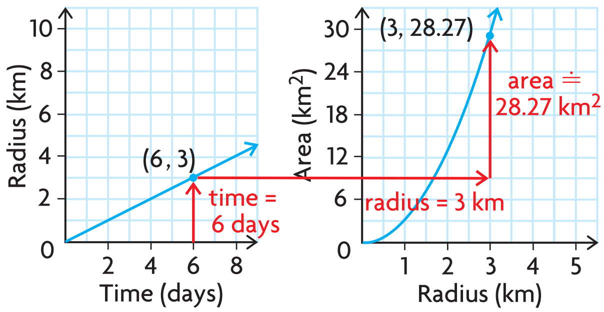
t	$r(t) = 0.5t$
0	0
2	1
4	2
6	3
8	4

r	$A(r) = \pi r^2$
0	0
1	3.14
2	12.57
3	28.27
4	50.27

Both time and radius must be positive, so $t \geq 0$ and $r \geq 0$.

$r(t)$ is a linear function, and $A(r)$ is a quadratic function.

Use the tables of values to sketch the graphs.



To find the radius of the area burned by the forest fire, the length of time that the fire has been burning must be known. Once the radius is known, the total area burned can be determined.

Reading from the first graph, the radius is 3 km when $t = 6$ days. Then reading from the second graph, a radius of 3 km indicates an area of about 28.3 km^2.

In the tables of values, time corresponds with radius, and radius corresponds with area.

$\boldsymbol{r}$: time $\rightarrow$ radius
$\boldsymbol{A}$: radius $\rightarrow$ area

The output in the first table becomes the input in the second table.

$$6 \xrightarrow[r(6)]{} 3 \xrightarrow[]{} 28.3$$

$$\begin{aligned} &A(r(6)) \\ &= A(3) \\ &\doteq 28.3 \end{aligned}$$

Determine the radius after six days, $r(6)$, and use it as the input for the area function, $A(r(6))$, to determine the area burned after six days.

$r(6) = 0.5(6) = 3$ and $A(3) = \pi(3)^2 \doteq 28.3$

The fire has burned about 28.3 km^2 on the sixth day.

Solution B: Using algebraic analysis

$$r = g(t) = 0.5t$$
$$A = f(r) = \pi r^2$$

The radius of the fire, r, grows at 0.5 km per day, so it is a function of time.

The area, A, of the fire increases in a circular pattern as its radius, r, increases, so it is a function of the circle's radius.

Since $r = g(t)$
$A = f(r) = f(g(t))$

To solve the problem, combine the area function with the radius function by using the output for the radius function as the input for the area function.

$$
\begin{aligned}
A &= f(0.5t)\\
&= \pi(0.5t)^2\\
&= 0.25\pi t^2
\end{aligned}
$$

This process yields the **composite function** for the area of the fire as a function of time.

$$
\begin{aligned}
A(6) &= 0.25\pi(6)^2\\
&= 0.25\pi(36)\\
&= 9\pi\\
&\doteq 28.3
\end{aligned}
$$

To determine the size of the burn area after six days, substitute $t = 6$ into this new function.

The fire has burned an area of about 28.3 km^2 after six days.

composite function

a function that is the composite of two other functions; the function $f(g(t))$ is called the composition of f with g; the function $f(g(t))$ is denoted by $(f \circ g)(t)$ and is defined by using the output of the function g as the input for the function f

Reflecting

A. A point on the second graph was used to solve the problem. Explain how the x-coordinate of this point was determined.

B. What connection was observed between the tables of values for the two functions? Why does it make sense that there is a function that combines the two functions to solve the forest fire problem?

C. Explain how the two functions were combined algebraically to determine a single function that predicts the area burned for a given time. How is the range of r related to the domain of A in this combination?

Communication *Tip*

$f \circ g$ is read as "f operates on g" while $f(g(x))$ is read as "f of g of x."

APPLY the Math

EXAMPLE 2 Reasoning about the order in which two functions are composed

Given the functions $f(x) = 2x + 3$ and $g(x) = \sqrt{x}$, determine whether $(f \circ g)(x) = (g \circ f)(x)$.

Solution

$$(f \circ g)(x) = \underbrace{f}_{\text{outer function}}(\underbrace{g(x)}_{\text{inner function}})$$

When f is composed with g, take the output for the inner function g and use it as the input for the outer function f.

$$x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$$

$$g(x) = \sqrt{x}$$

$$\begin{aligned} f(g(x)) &= f(\sqrt{x}) \\ &= 2(\sqrt{x}) + 3 \\ &= 2\sqrt{x} + 3 \end{aligned}$$

The output for g is the expression $\sqrt{x}$. Use this as the input for f, replacing x everywhere it occurs with $\sqrt{x}$.

$f(g(x)) = 2\sqrt{x} + 3$

Algebraically, the composition of f with g is the function $y = 2\sqrt{x} + 3$.

In terms of transformations, $f \circ g$ represents the function $y = g(x)$ stretched vertically by a factor of 2 and translated 3 units up. Its domain is $\{x \in \mathbf{R} \mid x \geq 0\}$.

$$(g \circ f)(x) = \underbrace{g(f(x))}$$

outer function — inner function

When g is composed with f, take the output from the inner function f and use it as the input for the outer function g.

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

$$f(x) = 2x + 3$$

$$\begin{aligned} g(f(x)) &= g(2x + 3) \\ &= \sqrt{(2x + 3)} \end{aligned}$$

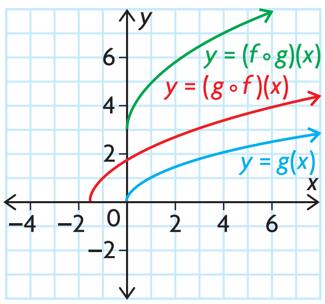
The output from f is the expression $2x + 3$. Use this as the input for g, and replace x everywhere it occurs with $2x + 3$.

$g(f(x)) = \sqrt{2x + 3}$

Algebraically, the composition of g with f is the function

$g(f(x)) = \sqrt{2x + 3} = \sqrt{2(x + 1.5)}$

In terms of transformations, $y = g(x)$ is compressed horizontally by a factor of $\frac{1}{2}$ and translated 1.5 units to the left. Its domain is $\left\{x \in \mathbf{R} \mid x \geq -\frac{3}{2}\right\}$.



Clearly, the expressions for $y = (f \circ g)(x)$ and $y = (g \circ f)(x)$ are different. Comparing their graphs illustrates the result of applying different sequences of transformations to $y = g(x)$.

$(f \circ g)(x) \neq (g \circ f)(x)$. The compositions of these two functions generate different answers depending on the order of the composition.

EXAMPLE 3 Reasoning about the domain of a composite function

Let $f(x) = \log_2 x$ and $g(x) = x + 4$.

a) Determine $f \circ g$, and find its domain.

b) What is the relationship between the domain of $f \circ g$ and the domain and range of f and g?

Solution

a) $(f \circ g)(x) = f(g(x))$
$= f(x + 4)$
$= \log_2(x + 4)$

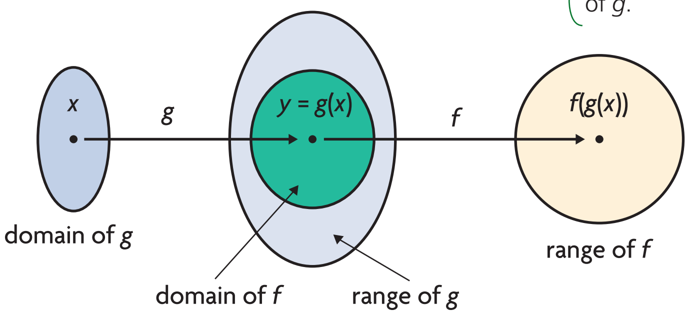
Use the output for g as the input for f.

Since $x + 4 > 0 \Rightarrow x > -4$.
The domain of $f \circ g$ is $x \in (-4, \infty)$.

The domain of a logarithmic function with base a contains only positive real numbers, so the expression $x + 4$ must be greater than zero.

b) Domain of f: $x \in (0, \infty)$ Range of f: $y \in \mathbf{R}$
Domain of g: $x \in \mathbf{R}$ Range of g: $y \in \mathbf{R}$

Looking at the domain of $f \circ g$, we can see that it is not equal to either the domain of f or the domain of g.



Recall that the output values (range of g) for $y = g(x)$, are used as the input values (domain) for f.

In this example, the domain of f is $x > 0$ and the domain of g is $x \in \mathbf{R}$, so the only y-values of g that can be used occur when $g(x) > 0$.
Since $g(x) = x + 4$, $x + 4 > 0$
$x > -4$

The domain of $f \circ g$ is the set of values, x, in the domain of g for which $g(x)$ is in the domain of f.

EXAMPLE 4 Reasoning about a function composed with its inverse

Show that, if $f(x) = \frac{1}{x - 2}$ then $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x)$.

Solution

$$\begin{aligned} x &= \frac{1}{y - 2} \\ x(y - 2) &= 1 \\ y - 2 &= \frac{1}{x} \\ y &= \frac{1}{x} + 2 \text{ or } f^{-1}(x) = \frac{1}{x} + 2 \end{aligned}$$

To find the inverse of f, switch x and y and then solve for y.

$$\begin{aligned} (f \circ f^{-1})(x) &= f(f^{-1}(x)) \\ &= f\left(\frac{1}{x} + 2\right) \\ &= \frac{1}{\left(\frac{1}{x} + 2\right) - 2} \\ &= \frac{1}{\left(\frac{1}{x}\right)} \\ &= x \end{aligned}$$

So, $(f \circ f^{-1})(x) = x$

The composition of f with its inverse maps a number in the domain of f onto itself. In other words, the result of this composition is the line $y = x$.

$$\begin{aligned} (f^{-1} \circ f)(x) &= f^{-1}(f(x)) \\ &= f^{-1}\left(\frac{1}{x - 2}\right) \\ &= \frac{1}{\left(\frac{1}{x - 2}\right)} + 2 \\ &= x - 2 + 2 \\ &= x \end{aligned}$$

The composition of f^{-1} with f maps a number in the domain of f^{-1} onto itself. In other words, the result of this composition is also the line $y = x$.

So, $(f^{-1} \circ f)(x) = x$

Therefore, $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x)$

Even though the order of the functions in the composition is reversed, the results are the same.

EXAMPLE 5 Working backward to decompose a composite function

Given $h(x) = |x^3 - 1|$, find two functions, f and g, such that $h = f \circ g$.

Solution

To evaluate h for any value of x, take that value, cube it, and subtract 1. This defines a sequence of operations for the inner function. Then, take the absolute value. This defines the outer function.

Let $g(x) = x^3 - 1$ and $f(x) = |x|$.

When evaluating the composition of f with g, you start by evaluating g for some value of x. So, it makes sense to define the inner function g that h performs on any input value. Then define the outer function f to represent the remaining operation(s) required by h.

$$\begin{aligned}
\text{Then } (f \circ g)(x) &= f(g(x)) \\
&= f(x^3 - 1) \\
&= |x^3 - 1| \\
&= h(x) \\
h(x) &= (f \circ g)(x)
\end{aligned}$$

Another solution would be to let $g(x) = x^3$ and $f(x) = |x - 1|$.

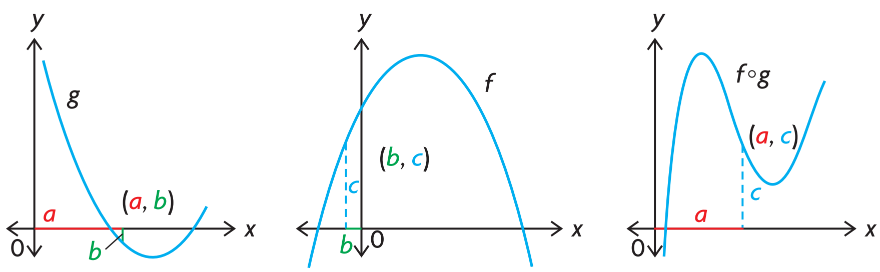
In Summary

Key Idea

- Two functions, f and g, can be combined using a process called composition, which can be represented by $f(g(x))$. The output for the inner function g is used as the input for the outer function f. The function $f(g(x))$ can be denoted by $(f \circ g)(x)$.

Need to Know

- Algebraically, the composition of f with g is denoted by $(f \circ g)(x)$, whereas the composition of g with f is denoted by $(g \circ f)(x)$. In most cases, $(f \circ g)(x) \neq (g \circ f)(x)$ because the order in which the functions are composed matters.
- Let $(a, b) \in g$ and $(b, c) \in f$. Then $(a, c) \in f \circ g$. A point in $f \circ g$ exists where an element in the range of g is also in the domain of f. The function $f \circ g$ exists only when the range of g overlaps the domain of f.



- The domain of $(f \circ g)(x)$ is a subset of the domain of g. It is the set of values, x, in the domain of g for which $g(x)$ is in the domain of f.
- If both f and f^{-1} are functions, then $(f^{-1} \circ f)(x) = x$ for all x in the domain of f, and $(f \circ f^{-1})(x) = x$ for all x in the domain of f^{-1}.

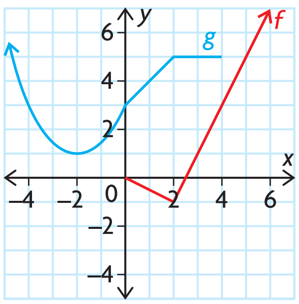
CHECK Your Understanding

1. Use $f(x) = 2x - 3$ and $g(x) = 1 - x^2$ to evaluate the following expressions.

a) $f(g(0))$
b) $g(f(4))$
c) $(f \circ g)(-8)$
d) $(g \circ g)\left(\frac{1}{2}\right)$
e) $(f \circ f^{-1})(1)$
f) $(g \circ g)(2)$

2. Given $f = \{(0, 1), (1, 2), (2, 5), (3, 10)\}$ and $g = \{(2, 0), (3, 1), (4, 2), (5, 3), (6, 4)\}$, determine the following values.

a) $(g \circ f)(2)$
b) $(f \circ f)(1)$
c) $(f \circ g)(5)$
d) $(f \circ g)(0)$
e) $(f \circ f^{-1})(2)$
f) $(g^{-1} \circ f)(1)$



3. Use the graphs of f and g to evaluate each expression.

a) $f(g(2))$
b) $g(f(4))$
c) $(g \circ g)(-2)$
d) $(f \circ f)(2)$

4. For a car travelling at a constant speed of 80 km/h, the distance driven, d kilometres, is represented by $d(t) = 80t$, where t is the time in hours. The cost of gasoline, in dollars, for the drive is represented by $C(d) = 0.09d$.

a) Determine $C(d(5))$ numerically, and interpret your result.
b) Describe the relationship represented by $C(d(t))$.

PRACTISING

5. **K** In each case, functions f and g are defined for $x \in \mathbf{R}$. For each pair of functions, determine the expression and the domain of $f(g(x))$ and $g(f(x))$. Graph each result.

a) $f(x) = 3x^2, g(x) = x - 1$
b) $f(x) = 2x^2 + x, g(x) = x^2 + 1$
c) $f(x) = 2x^3 - 3x^2 + x - 1, g(x) = 2x - 1$
d) $f(x) = x^4 - x^2, g(x) = x + 1$
e) $f(x) = \sin x, g(x) = 4x$
f) $f(x) = |x| - 2, g(x) = x + 5$

6. For each of the following,

- determine the defining equation for $f \circ g$ and $g \circ f$
- determine the domain and range of $f \circ g$ and $g \circ f$

a) $f(x) = 3x, g(x) = \sqrt{x - 4}$
b) $f(x) = \sqrt{x}, g(x) = 3x + 1$
c) $f(x) = \sqrt{4 - x^2}, g(x) = x^2$
d) $f(x) = 2^x, g(x) = \sqrt{x - 1}$
e) $f(x) = 10^x, g(x) = \log x$
f) $f(x) = \sin x, g(x) = 5^{2x} + 1$

7. For each function h, find two functions, f and g, such that $h(x) = f(g(x))$.

a) $h(x) = \sqrt{x^2 + 6}$

b) $h(x) = (5x - 8)^6$

c) $h(x) = 2^{(6x+7)}$

d) $h(x) = \dfrac{1}{x^3 - 7x + 2}$

e) $h(x) = \sin^2(10x + 5)$

f) $h(x) = \sqrt[3]{(x + 4)^2}$

8. **a)** Let $f(x) = 2x - 1$ and $g(x) = x^2$. Determine $(f \circ g)(x)$.

b) Graph f, g, and $f \circ g$ on the same set of axes.

c) Describe the graph of $f \circ g$ as a transformation of the graph of $y = g(x)$.

9. Let $f(x) = 2x - 1$ and $g(x) = 3x + 2$.

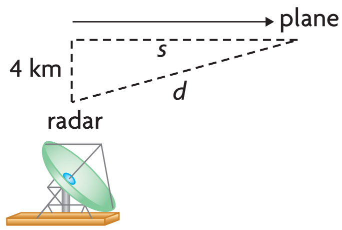
a) Determine $f(g(x))$, and describe its graph as a transformation of $g(x)$.

b) Determine $g(f(x))$, and describe its graph as a transformation of $f(x)$.

10. **A** A banquet hall charges \$975 to rent a reception room, plus \$39.95 per person. Next month, however, the banquet hall will be offering a 20% discount off the total bill. Express this discounted cost as a function of the number of people attending.

11. The function $f(x) = 0.08x$ represents the sales tax owed on a purchase with a selling price of x dollars, and the function $g(x) = 0.75x$ represents the sale price of an item with a price tag of x dollars during a 25% off sale. Write a function that represents the sales tax owed on an item with a price tag of x dollars during a 25% off sale.

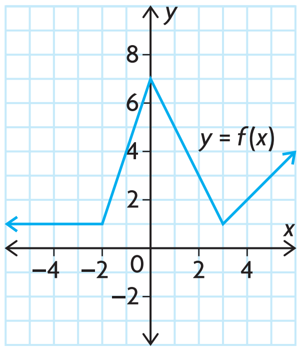
12. An airplane passes directly over a radar station at time $t = 0$. The plane maintains an altitude of 4 km and is flying at a speed of 560 km/h. Let d represent the distance from the radar station to the plane, and let s represent the horizontal distance travelled by the plane since it passed over the radar station.



a) Express d as a function of s, and s as a function of t.

b) Use composition to express the distance between the plane and the radar station as a function of time.

13. In a vehicle test lab, the speed of a car, v kilometres per hour, at a time of t hours is represented by $v(t) = 40 + 3t + t^2$. The rate of gasoline consumption of the car, c litres per kilometre, at a speed of v kilometres per hour is represented by $c(v) = \left(\frac{v}{500} - 0.1\right)^2 + 0.15$. Determine algebraically $c(v(t))$, the rate of gasoline consumption as a function of time. Determine, using technology, the time when the car is running most economically during a 4 h simulation.



14. Given the graph of $y = f(x)$ shown and the functions below, match the correct composition with each graph. Justify your choices.

T

i) $g(x) = x + 3$ **iii)** $h(x) = x - 3$ **v)** $k(x) = -x$

ii) $m(x) = 2x$ **iv)** $n(x) = -0.5x$ **vi)** $p(x) = x - 4$

a) $y = (f \circ g)(x)$ **g)** $y = (g \circ f)(x)$

b) $y = (f \circ h)(x)$ **h)** $y = (h \circ f)(x)$

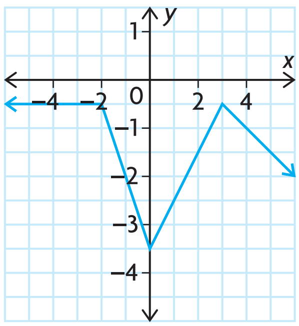
c) $y = (f \circ k)(x)$ **i)** $y = (k \circ f)(x)$

d) $y = (f \circ m)(x)$ **j)** $y = (m \circ f)(x)$

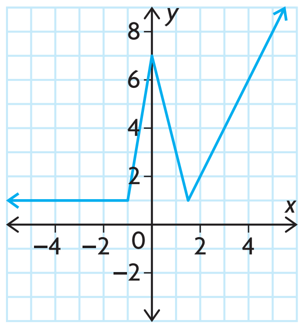
e) $y = (f \circ n)(x)$ **k)** $y = (n \circ f)(x)$

f) $y = (f \circ p)(x)$ **l)** $y = (p \circ f)(x)$

A



C



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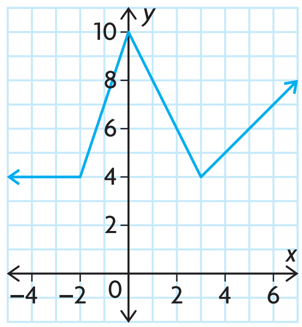
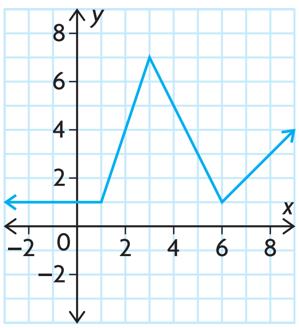
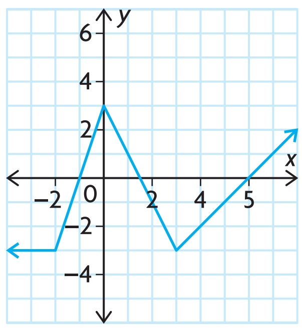


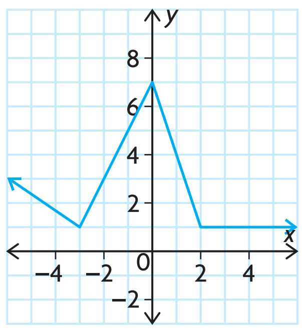
B



D



F



Sum	Product
$y = \frac{4}{x - 3} + 1$	
Quotient	Composition

15. Find two functions, f and g, to express the given function in the centre box of the chart in each way shown.

C

Extending

16. a) If $y = 3x - 2$, $x = 3t + 2$, and $t = 3k - 2$, find an expression for $y = f(k)$.

b) Express y as a function of k if $y = 2x + 5$, $x = \sqrt{3t - 1}$, and $t = 3k - 5$.

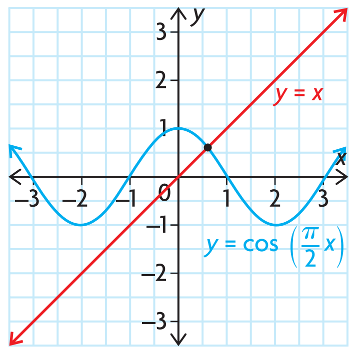
9.6 Techniques for Solving Equations and Inequalities

GOAL

Solve equations and inequalities that involve combinations of functions using a variety of techniques.

YOU WILL NEED

- graphing calculator



LEARN ABOUT the Math

On the graph are the functions $y = \cos\left(\frac{\pi}{2}x\right)$ and $y = x$. The point of intersection of the two functions is the point where $\cos\left(\frac{\pi}{2}x\right) = x$.

? How can the equation $\cos\left(\frac{\pi}{2}x\right) = x$ be solved to determine the point of intersection of these two functions?

EXAMPLE 1 Selecting tools and strategies to solve an equation

Solve the equation $\cos\left(\frac{\pi}{2}x\right) = x$ to the nearest hundredth.

Solution A: Selecting a guess and improvement strategy that involves a numerical approach

$$\cos\left(\frac{\pi}{2}x\right) = x$$

Using the given graph, the point of intersection looks like it occurs when x is about 0.5.

$$\cos\left(\frac{\pi}{2}x\right) - x = 0$$

Subtract x from both sides of the equation so that one side is equal to zero.

$$\cos\left(\frac{\pi}{2}(0.5)\right) - 0.5$$

Check the estimate by substituting the value $x = 0.5$ into the equation.

$$= \cos\left(\frac{\pi}{4}\right) - 0.5$$

$$= \frac{1}{\sqrt{2}} - 0.5$$

$$\doteq 0.207$$

0.207 is close to zero, but there may be some other values close to 0.5 that give a better answer.

When $x = 0.4$,

$$\cos\left(\frac{\pi}{2}(0.4)\right) - 0.4 \doteq 0.409$$

Repeat the process for $x = 0.4$.

The result is farther away from zero than the previous estimate, so try a number larger than 0.5.

When $x = 0.6$,

$$\cos\left(\frac{\pi}{2}(0.6)\right) - 0.6 \doteq -0.0122$$

Repeat the process for $x = 0.6$.

The result is closer to zero than the previous two estimates, but is a little below zero. Try a number a bit smaller than 0.6.

When $x = 0.59$,

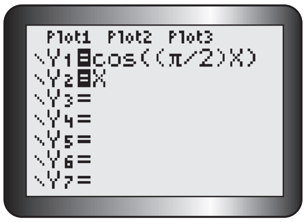
$$\cos\left(\frac{\pi}{2}(0.59)\right) - 0.59 \doteq 0.0104$$

$$\cos\left(\frac{\pi}{2}x\right) = x \text{ when } x \doteq 0.59$$

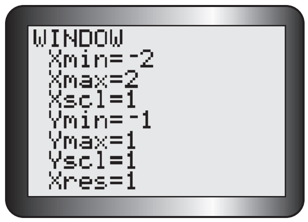
Repeat the process for $x = 0.59$.

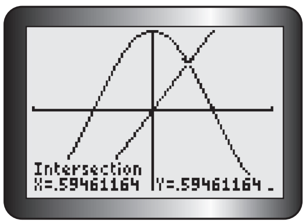
$x = 0.59$ is a much better answer because it gives a y-value that is almost equal to zero.

Solution B: Selecting a graphical strategy that involves the points of intersection



Enter the function equations $y = \cos\left(\frac{\pi}{2}x\right)$ and $y = x$ into the equation editor on a graphing calculator, as Y1 and Y2. Graph using a suitable window in radian mode.





Use the intersect operation to determine the point of intersection.

Tech Support

For help using a graphing calculator to determine points of intersection, see Technical Appendix, T-12.

$$\cos\left(\frac{\pi}{2}x\right) = x \text{ when } x \doteq 0.59$$

This is the only point of intersection since $y = \cos\left(\frac{\pi}{2}x\right)$ alternates between 1 and -1, while $y = x$ has the following end behaviours:

As $x \rightarrow \infty$, $y \rightarrow \infty$, and as $x \rightarrow -\infty$, $y \rightarrow -\infty$.

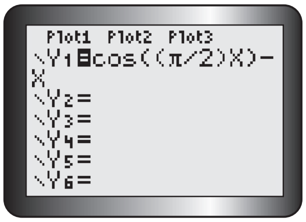
Solution C: Selecting a graphical strategy that involves the zeros

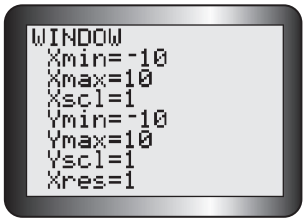
Recall that solving for the roots of an equation is related to finding the zeros of a corresponding function.

$\cos\left(\frac{\pi}{2}x\right) = x$ is equivalent to

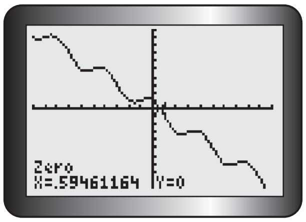
$$\cos\left(\frac{\pi}{2}x\right) - x = 0$$

Sketch the graph of $y = \cos\left(\frac{\pi}{2}x\right) - x$ on a graphing calculator, using a suitable window.





Use the zero operation to determine the function's zeros.



$$\cos\left(\frac{\pi}{2}x\right) = x \text{ when } x \doteq 0.59$$

Tech Support

For help using a graphing calculator to determine the zeros of a function, see Technical Appendix, T-8.

Reflecting

A. What are the advantages of using a guess and improvement strategy versus a graphing strategy? What are the disadvantages?

B. When using a guess and improvement strategy, how will you know when a given value of x gives you an accurate answer?

C. Which graphical strategy do you prefer? Explain.

APPLY the Math

EXAMPLE 2 | Using an equation to solve a problem

According to data collected from 1996 to 2001, the average price of a new condominium in Toronto was \$144 144 in 2001 and increased by 6.6% each year. A new condominium in Regina cost \$72 500 on average, but prices were growing by 10% per year there. If these trends continue, when will a new condominium in Regina be the same price as one in Toronto?

Solution

Let x be the number of years since 2001.
Let y be the price of a new condominium.

Toronto: $y = 144\,144(1.066)^x$

Regina: $y = 72\,500(1.10)^x$

These are the exponential functions that model the average price of a new condominium in Toronto and Regina since 2001.

Solve $144\,144(1.066)^x = 72\,500(1.10)^x$.

To determine when the prices are the same, set the two functions equal to each other.

$$\frac{144\,144(1.066)^x}{72\,500} = \frac{72\,500(1.10)^x}{72\,500}$$

$$1.9882(1.066)^x \doteq (1.10)^x$$

$$\frac{1.9882\,\cancel{(1.066)^x}}{\cancel{(1.066)^x}} = \frac{(1.10)^x}{(1.066)^x}$$

$$1.9882 = \left(\frac{1.10}{1.066}\right)^x$$

This exponential equation can be solved algebraically.

Divide both sides by 72 500.

Divide both sides by 1.066^x.

$$\log(1.9882) = \log\left(\frac{1.10}{1.066}\right)^x$$

$$\log(1.9882) = x\log\left(\frac{1.10}{1.066}\right)$$

$$\log(1.9882) = x(\log(1.10) - \log(1.066))$$

$$\frac{\log(1.9882)}{\log(1.10) - \log(1.066)} = \frac{x\,\cancel{\log(1.10) - \log(1.066))}}{\cancel{(\log(1.10) - \log(1.066))}}$$

Take the log of both sides.

Rewrite the right side using the logarithm laws.

Divide both sides by $\log(1.10) - \log(1.066)$.

$$21.89 \doteq x$$

Evaluate the left side.

If these trends continue, the price of a new condominium in Regina will be the same as the price of a new condominium in Toronto by the end of the year 2023.

$2001 + 21.89 \doteq 2023$

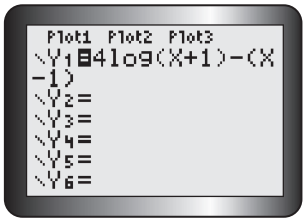
EXAMPLE 3 | Selecting a graphing strategy to solve an inequality

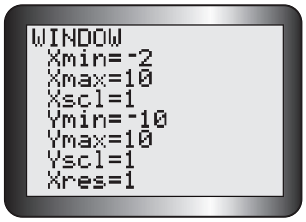
Given $f(x) = 4\log(x + 1)$ and $g(x) = x - 1$, determine all values of x such that $f(x) > g(x)$.

Solution A: Using a single function and comparing its position to the *x*-axis

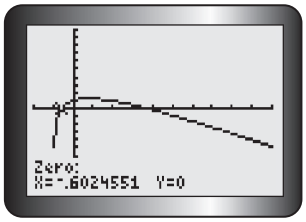
If $f(x) > g(x)$, then $f(x) - g(x) > 0$.

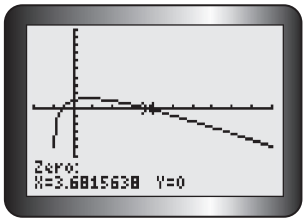
Let $y_1 = (f - g)(x) = 4\log(x + 1) - (x - 1)$.





Enter the function into the equation editor, and graph the function using an appropriate window. In this example, $x > -1$ since the log function is undefined for negative values.



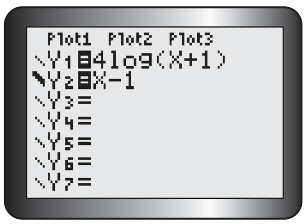


Use the zero operation to determine its zeros.

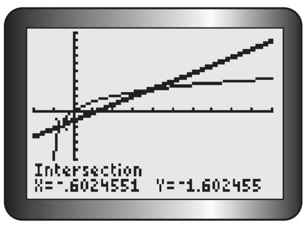
$f(x) > g(x)$ when $x \in (-0.602, 3.681)$.

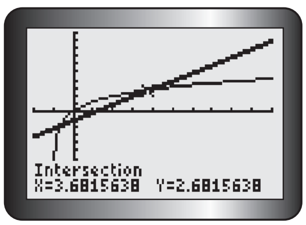
$f(x) - g(x) > 0$ when its graph is above the x-axis. This occurs in the interval between the two zeros.

Solution B: Using both functions and comparing the position of one to the other



Enter the two functions, f and g, into Y1 and Y2, respectively, in the equation editor on a graphing calculator. Use a bold line for Y2.





Determine the points of intersection using the intersect operation.

$f(x) > g(x)$ when $x \in (-0.602, 3.681)$.

This means that f lies above g in the interval between the two intersection points.

In Summary

Key Ideas

- The equation $f(x) = g(x)$ can be solved using a guess and improvement strategy. Estimate where the intersection of $f(x)$ and $g(x)$ will occur, and substitute this value into both sides of the equation. Based on the outcome, adjust your estimate. Repeat this process until the desired degree of accuracy is found.
- If graphing technology is available, the equation $f(x) = g(x)$ can be solved by graphing the two functions and using the intersect operation to determine the point of intersection.
- The equation $f(x) = g(x)$ can also be solved by rewriting the equation in the form $f(x) - g(x) = 0$ to obtain the corresponding function, $h(x) = f(x) - g(x)$. The zeros of this function are also the roots of the equation. These can be determined using a guess and improvement strategy when graphing technology is not available. Graphing technology can also be used to graph the function $h(x) = f(x) - g(x)$ and determine its zeros using the zero operation.
- Inequalities can be solved by using these strategies to solve the corresponding equation, and then selecting the intervals that satisfy the inequality.

Need to Know

- The method used to solve equations and inequalities depends on the degree of accuracy required and the access to graphing technology. A solution using graphing technology will usually result in a closer approximation to the root (zero) of the equation than a solution generated by a numerical strategy with the aid of a scientific calculator.
- The difference between the solution to a strict inequality, $f(x) > g(x)$, and an inclusive inequality, $f(x) \geq g(x)$, is that the value of each root (zero) is included in the solution to the inclusive inequality.

CHECK Your Understanding

1. For each graph shown below, state the solution to each of the following:

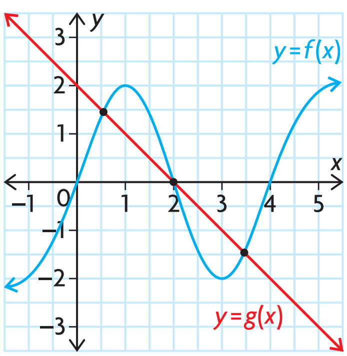
a) $f(x) = g(x)$

b) $f(x) > g(x)$

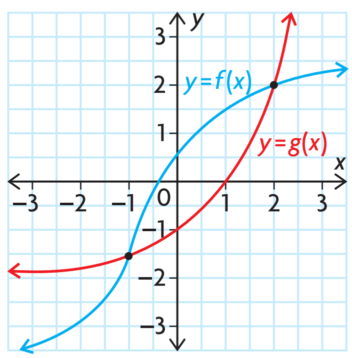
c) $f(x) \leq g(x)$

d) $f(x) \geq g(x)$

i)



ii)

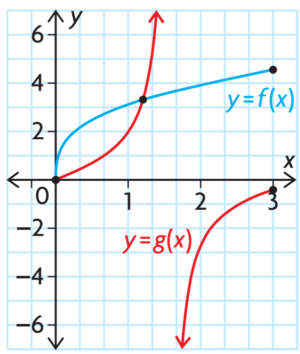


2. Use a guess and improvement strategy to determine the best one-decimal-place approximation to the solution of each equation in the interval provided.
   a) $3 = 2^{2x}$, when $x \in [0, 2]$
   b) $0 = \sin(0.25x^2)$, when $x \in [0, 5]$
   c) $3x = 0.5x^3$, when $x \in [-8, -1]$
   d) $\cos x = x$, when $x \in \left[0, \frac{\pi}{2}\right]$

3. Use graphing technology to determine the solution to $f(x) = g(x)$, where $f(x) = 2\sqrt{x + 3}$ and $g(x) = x^2 + 1$, in two different ways.

PRACTISING

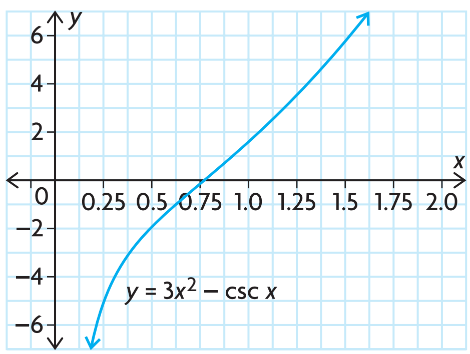
4. **K** In the graph shown, $f(x) = 3\sqrt[3]{x}$ and $g(x) = \tan x$. State the values of x in the interval $[0, 3]$ for which $f(x) < g(x)$, $f(x) = g(x)$, and $f(x) > g(x)$. Express the values to the nearest tenth.



5. Solve each of the following equations for x in the given interval, using a guess and improvement strategy. Express your answers to the nearest tenth.
   a) $5 \sec x = -x^2$, $0 \le x \le \pi$
   b) $\sin^3 x = \sqrt{x} - 1$, $0 \le x \le \pi$
   c) $5^x = x^5$, $-2 \le x \le 2$
   d) $\cos x = \frac{1}{x}$, $-4 \le x \le 0$
   e) $\log(x) = (x - 10)^2 + 1$, $0 \le x \le 10$
   f) $\sin(2\pi x) = -4x^2 + 16x - 12$, $0 \le x \le 5$

6. Use graphing technology to solve each of the following equations. Round to two decimal places, if necessary.
   a) $2^x - 1 = \log(x + 2)$
   b) $\sqrt{x + 5} = x^2$
   c) $\sqrt{x + 3} - 5 = -x^4$
   d) $\sqrt[3]{\sin x} = 2x^3$ for x in the interval $-3 \le x \le 3$
   e) $\cos(2\pi x) = -x + 0.5$ in the interval $0 \le x \le 1$
   f) $\tan(2\pi x) = 2\sin(3\pi x)$ in the interval $0 \le x \le 1$

7. To solve the equation $-\csc x = -3x^2$ for x in the interval $0 \le x \le 2$, the graph shown can be used. Determine the coordinates of the point where the graphs of the functions $f(x) = -\csc x$ and $g(x) = -3x^2$ intersect in the interval $0 \le x \le 2$.



8. Two jurisdictions in Canada and the United States are attempting
A to decrease the numbers of mountain pine beetles that have been damaging their national forests. A section of forest under study in British Columbia at the beginning of 1997 had an estimated 2.3 million of the pests, while there were about 1.95 million of the pests in a similar-sized section of forest in the state of Washington. British Columbia has been decreasing the number of mountain pine beetles by 4% per year, while Washington has been decreasing the number by 3% per year. When will there be about the same number of pests in the sections of forest under study in each jurisdiction?

9. Solve each of the following inequalities using graphing technology. State your solutions using interval notation, rounding to the nearest hundredth as required.
   a) $2x^2 < 2^x$
   b) $\log (x + 1) \geq x^3$
   c) $\left(\frac{1}{2}\right)^x > \frac{1}{x}$
   d) $\sin (\pi x) > \cos (2\pi x)$, where $x \in [0, 1]$
   e) $\cos (\pi x) \leq \left(\frac{1}{10}\right)^x$, where $x \in [0, 2]$
   f) $\tan (\pi x) > \sqrt{x}$, where $x \in [0, 1]$

10. Give an example of two functions, f and g, such that $f(x) > g(x)$
T when $x \in [-4, -2]$ or $x \in [1, \infty)$.

11. Give an example of two functions, f and g, such that $f(x) > 0$ when $x \in [-5, 5]$ and $f(x) > g(x)$ when $x \in [-4, 5]$.

12. Two of the solutions to the equation $a \cos x = bx^3 + 6$, where a and b are integers, are $x = -1.2$ and $x = -0.7$. These solutions are rounded to the nearest tenth. What are the values of a and b?

13. Construct a flow chart to describe the process of finding the solutions
C to an equation using your preferred strategy.

Extending

14. Determine the general solution to the equation $\tan (0.5\pi x) = 2 \sin (\pi x)$.

15. Determine the general solution to the inequality $\sin (\pi x) > 0$.

9.7 Modelling with Functions

GOAL

Use a variety of functions to model real-life situations.

YOU WILL NEED

- graphing calculator or graphing software/dynamic statistical software

LEARN ABOUT the Math

About 5000 people live in Sanjay's town. One person in his school came back from their March Break trip to Florida with a virus. A week later, 70 additional people have the virus, and doctors in the town estimate that about 8% of the town's residents will eventually get this virus.

? What types of functions could be used to model the spread of the virus in this town?

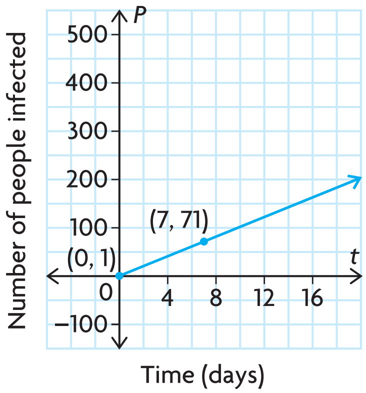
EXAMPLE 1 | Selecting a function to model the situation

Select an appropriate function to model the spread of the virus in Sanjay's town.

Solution A: Selecting a linear model

Use the given data to sketch a graph.

Time, *t* (days)	People Infected, *P*
0	1
7	71



The general equation of the linear model is $y = mx + b$, where m is the slope of the line and b is the y-intercept.

Time, t, is the independent variable. The number of people infected, P, is the dependent variable.

In this case, the vertical or y-intercept is 1 and the slope is

$$\frac{\Delta P}{\Delta t} = \frac{71 - 1}{7 - 0}$$
$$= 10$$

Two points are sufficient to determine the equation of a line.

The linear model is $P(t) = 10t + 1$ and predicts that the number of people infected by the virus will grow at a constant rate of 10 people per day.

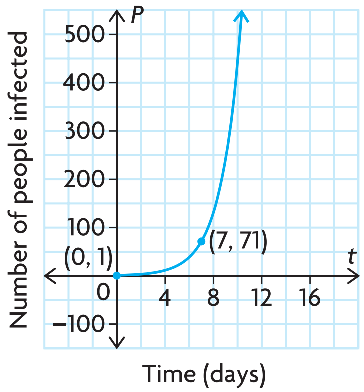
$$P(t) = 400$$
$$400 = 10t + 1$$
$$399 = 10t$$
$$39.9 = t$$

8% of 5000 is 400.
At a rate of 10 people per day, it will take about 40 days for the virus to spread to the expected number of 400 people.

Solution B: Selecting an exponential model

Use the given data to sketch a graph.

Time, *t* (days)	People Infected, *P*
0	1
7	71



The general equation of the exponential model is $y = ab^t$, where *a* is the initial value, or *y*-intercept, and *b* is $(1 + \text{growth rate})$.

Time, *t*, is the independent variable. The number of people infected, *P*, is the dependent variable.

$$P(t) = P_0(b)^t$$

Substituting gives

$$71 = 1(b)^7$$
$$71 = b^7$$
$$\sqrt[7]{71} = b$$
$$1.8385 \doteq b$$

Two points are enough to determine an exponential model. The initial value of the exponential function is $P_0 = 1$, and we know that $P(7) = 71$.

The exponential model is $P(t) = 1(1.8385)^t$.

The exponential model predicts slow initial growth followed by much faster growth.

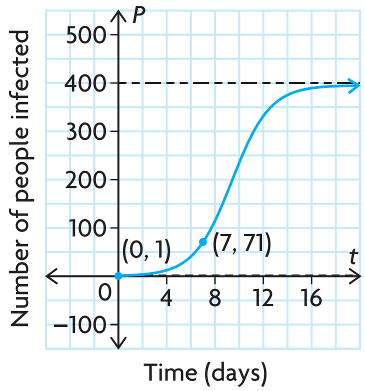
$$P(t) = 400$$
$$400 = 1(1.8385)^t.$$
$$\log(400) = \log(1.8385)^t$$
$$\frac{\log(400)}{\log(1.8385)} = \frac{t\,\cancel{\log(1.8385)}^{1}}{\cancel{\log(1.8385)}_{1}}$$
$$9.84 \doteq t$$

This model predicts that it will take about 10 days for the virus to infect the expected number of 400 people.

Solution C: Selecting a logistic model

Use the given data to sketch a graph.

Time, *t* (days)	People Infected, *P*
0	1
7	71



The general equation of the logistic model is $P(t) = \frac{c}{1 + ab^t}$ where c is the carrying capacity, or maximum value, that the function attains.

Time, t, is the independent variable. The number of people infected, P, is the dependent variable.

The carrying capacity, c, or maximum number of people infected, is 8% of $5000 = 400$.

Substituting $P(0) = 1$ gives

$$1 = \frac{400}{1 + ab^0}$$

$$1 = \frac{400}{1 + a}$$

$$a = 399$$

Substituting $P(7) = 71$ gives

$$71 = \frac{400}{1 + 399b^7}$$

$$1 + 399b^7 = \frac{400}{71}$$

$$399b^7 \doteq 5.6338 - 1$$

$$b^7 \doteq 0.011\,614$$

$$b \doteq 0.5291$$

The parameters a and b can be determined if two points on the function are known.

The logistic model is $P(t) = \frac{400}{1 + 399(0.5291)^t}$.

The logistic model predicts slow growth followed by rapid growth, and then a slowing of the growth rate again as the maximum number of infected people nears 400.

The graph approaches a horizontal asymptote at $P = 400$ when t is close to 12.

This model predicts that it will take about 12 days for the virus to infect the expected number of 400 people.

Reflecting

A. Compare the growth curves for the three mathematical models. How do the graphs differ? How are they similar?

B. How do the growth rates for the three mathematical models compare?

C. No mathematical model is perfect; what we hope for is a useful description of the situation. Which of these models do you think is the least realistic, and which one the most realistic? Why?

D. What could you do in a situation like this to improve the accuracy of your mathematical model?

E. Are there any other types of functions that you think could be used to model this situation? Explain.

APPLY the Math

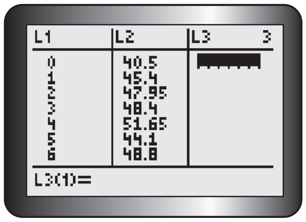
EXAMPLE 2 Selecting a function model to fit to a data set

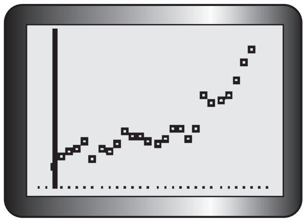
The table shows the median annual price for unleaded gasoline in Toronto for a 26-year period. Determine a mathematical model for the data, compare the values with the given values, and use the values to predict the median price of unleaded gasoline in 2010.

Year	Years since 1981	Price (cents/L)
1981	0	40.5
1982	1	45.4
1983	2	47.95
1984	3	48.4
1985	4	51.65
1986	5	44.1
1987	6	48.8
1988	7	47.6
1989	8	51.5
1990	9	56.55
1991	10	54.4
1992	11	54.35
1993	12	52.3

Year	Years since 1981	Price (cents/L)
1994	13	50.65
1995	14	53.5
1996	15	58.0
1997	16	58.05
1998	17	53.45
1999	18	58.1
2000	19	72.75
2001	20	69.85
2002	21	70.85
2003	22	72.45
2004	23	79.55
2005	24	88.25
2006	25	93.65

Solution A: Selecting a cubic model using regression on a graphing calculator





Enter the data into lists, and create a scatter plot.

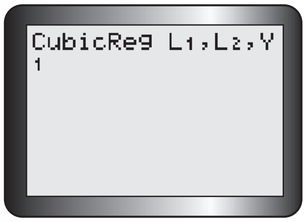
Tech Support

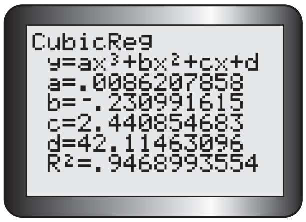
For help creating a scatter plot using a graphing calculator, see Technical Appendix, T-11.

The scatter plot clearly shows a non-linear trend. The graph increases, so possible functions include an exponential model, a quadratic model, and a cubic model.

Other functions are possible too, but a relatively simple model is preferred for ease of computation and use.

Since the data indicate that gas prices rose, then dropped a little, and then rose again, try a cubic model.





Perform a cubic regression on L1 and L2.

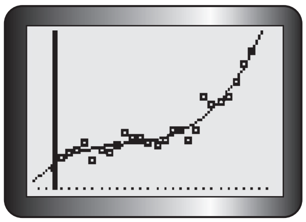
Note that the value of R^2 in the calculator output is 0.947. This means that 94.7% of the variation in gasoline prices is explained by our mathematical model.

The output is displayed, and the coefficients in the cubic polynomial are rounded.

Tech Support

For help with regression to determine the equation of a curve of best fit using a graphing calculator, see Technical Appendix, T-11.

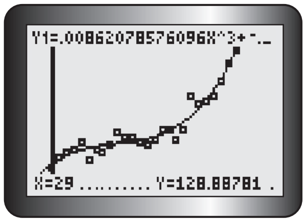
$$f(x) = 0.0086x^3 - 0.2310x^2 + 2.4409x + 42.1146$$



The regression curve fits the scatter plot well.

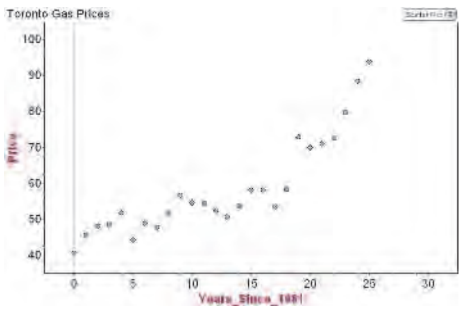
$$\begin{aligned} f(29) &= 0.0086(29)^3 - 0.2310(29)^2 \\ &\quad + 2.4409(29) + 42.1146 \\ &\doteq 128.38 \end{aligned}$$

The year 2010 is 29 years after 1981, so substitute $t = 29$ to obtain a prediction of the price of gasoline.



The cubic model predicts that the price for 2010 will be about $1.28/L.

Solution B: Selecting an exponential model using Fathom



Create a case table, and enter the years since 1981 as the independent variable and the price as the dependent variable.

Create a scatter plot of the data.

Tech | *Support*

For help creating a scatter plot using Fathom, see Technical Appendix, T-21.

The exponential function model is of the form $P(t) = k + a(b)^t$, where t is years since 1981 and P is the price in cents.

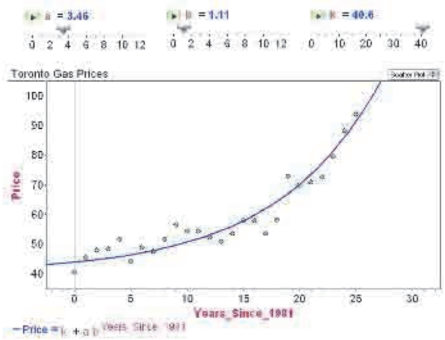
Estimate the values of the parameters based on the scatter plot created and the data given.

k is approximately 40.

The shape of the scatter plot suggests that the horizontal asymptote for the exponential model is at about 40 cents per litre, so the parameter k is approximately 40.

$b > 1$

b must be greater than 1, since the exponential function increases.



Create sliders for the parameters of a, b, and k, and enter the function as Price $= k + a(b)^t$. Adjust the sliders using a trial-and-error process until the curve fits the scatter plot.

Tech | *Support*

For help graphing functions and creating sliders using Fathom, see Technical Appendix, T-21.

An exponential model is

$$P(t) = 40.59 + 3.46(1.1134)^t.$$
$$P(29) = 40.59 + 3.46(1.1134)^{29}$$
$$= 118.57 \text{ cents per litre}$$
$$= \$1.19/\text{L}$$

The year 2010 is 29 years after 1981, so substitute $t = 29$ to obtain a prediction of the price of gasoline in 2010.

In Summary

Key Ideas

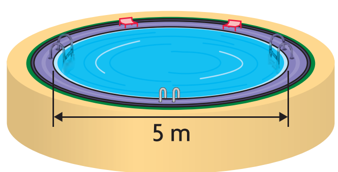
- A mathematical model is just that—a model. It will not be a perfect description of a real-life situation; but if it is a good model, then you will be able to use it to describe the real-life situation and make predictions.
- Increasing the amount of data you have for creating a mathematical model improves the accuracy of the model.
- A scatter plot gives you a visual representation of the data. Examining the scatter plot may give you an idea of what kind of function could be used to model the data. Graphing your mathematical model on the scatter plot is a visual way to confirm that it is a good fit.

Need to Know

- If you have to choose between a simple function and a complicated function, and if both fit the data equally well, the simple function is generally preferred.
- The function you choose should make sense in the context of the problem; for the growth of a population, you may want to consider an exponential model or a logistic model.
- One way to compare mathematical models created using regression analysis is to examine the value of R^2. This is the fraction of the variation in the response variable (y), which is explained by the mathematical model based on the predictor variable (x).
- Mathematical models are useful for **interpolating**. They are not necessarily useful for **extrapolating** because they assume that the trend in the data will continue. Many factors can affect the relationship between the independent variable and the dependent variable and change the trend.
- It is often necessary to restrict the domain of a mathematical model to represent a realistic situation.

CHECK Your Understanding

1. An above-ground swimming pool in the shape of a cylinder, with diameter 5 m, is filled at a constant rate to a depth of 1 m. It takes 4 h to fill the pool with a hose.



a) Make a graph showing volume of water in the pool as a function of time.

b) Determine the equation of a mathematical model for volume as a function of time.

c) When will the volume of the water be 8 m^3?

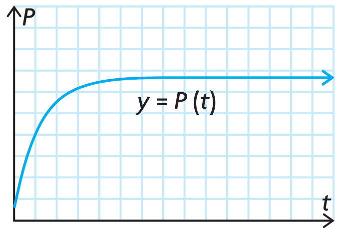
2. After being filled, the swimming pool in question 1 is accidentally punctured at the bottom and water leaks out. The volume of the pool reaches zero in 8 h. The volume of water remaining at time t follows a quadratic model, with the minimum point (vertex) at the time when the last of the water drains out.
   a) Make a graph showing the volume of water in the pool versus time.
   b) Find the equation for the quadratic model.
   c) Use the model to predict the volume of water at the 2 h mark.
   d) What is the average rate of change in the volume of the water during the first 2 h?
   e) How does the rate of change in volume vary as time elapses?

3. An abandoned space station in orbit contains 200 m^3 of oxygen. It is punctured by a piece of space debris, and oxygen begins to leak out. After 4 h, there is 80 m^3 of oxygen remaining in the space station.
   a) Make a graph showing the two data points provided. Sketch two or three possible graphs that might show how volume decreases with time.
   b) The simplest model would be linear. Determine the equation of the linear model, and use this model to find the amount of time it will take for the last of the oxygen to escape.
   c) A more realistic model would be an exponential model, since the rate of change in volume is likely to be proportional to the volume of oxygen remaining. Determine the equation of an exponential model of the form $V(t) = a(b)^t$. Use this model to estimate the time it will take for 90% of the original volume of oxygen to escape.

PRACTISING

4. **K** A lake in Northern Ontario has recovered from an acid spill that killed all of its trout. A restocking program puts 800 trout in the lake. Ten years later, the population is estimated to be 6000. The carrying capacity of the lake is believed to be 8000.
   a) Make a graph to show the given information. Extend the time scale to 20 years.
   b) Determine the parameters for a logistic model of the form $P(t) = \frac{c}{1 + a(b)^t}$ to model the growth of the trout population, and graph the function for $t \in [0, 20]$.
   c) Use the model to estimate the number of trout that were in the lake four years after restocking.
   d) Use the model to estimate the average rate of change in the number of trout over the first four years of the restocking program.

5. Consider again the population of trout in question 4. Another possible model for the trout situation is a transformed exponential function of the form $P(t) = c - a(b)^t$. A graph of this type of model, $y = P(t)$, is shown below.



a) What feature of the graph does the parameter c represent? What is the value of c for the trout population?

b) Determine the values of a and b by substituting the two known ordered pairs.

c) Graph this exponential model of the trout population for $t \in [0, 20]$.

d) Use the model to estimate the number of trout that were in the lake four years after restocking.

e) Use the model to estimate the average rate of change in the number of trout over the first four years of the restocking program.

f) Explain how this model differs from the logistic model in question 4.

6. Recall the cubic and exponential model equations for gasoline prices in Example 2. Which model more accurately calculates the current price of gasoline?

7. The following table shows the velocity of air, in litres per second, of a typical person's breathing while at rest.

Time (s)	0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
Velocity (L/s)	0	0.22	0.45	0.61	0.75	0.82	0.85	0.83	0.74	0.61	0.43	0.23	0

a) Graph the data, and determine an equation that models the situation.

b) Use a graphing calculator to draw a scatter plot of the data. Enter your equation into the equation editor, and graph. Comment on the closeness of fit between the scatter plot and the graph.

c) At $t = 6$, what is the velocity of a typical person's breathing?

d) Estimate when the rate of change in the velocity of a person's breathing is the smallest during the first 3 s.

e) What is the significance of the value you found in part d)?

f) Estimate when the rate of change in the velocity of a person's breathing is the greatest during the first 3 s.

8. The following table shows the average number of monthly hours of sunshine for Toronto.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Average Monthly Sunshine (h)	95.5	112.6	150.5	187.7	229.7	254.9	278.0	244.0	184.7	145.7	82.3	72.6

Source: Environment Canada

a) Create a scatter plot of the number of hours of sunshine versus time, where $t = 1$ represents January, $t = 2$ represents February, and so on.

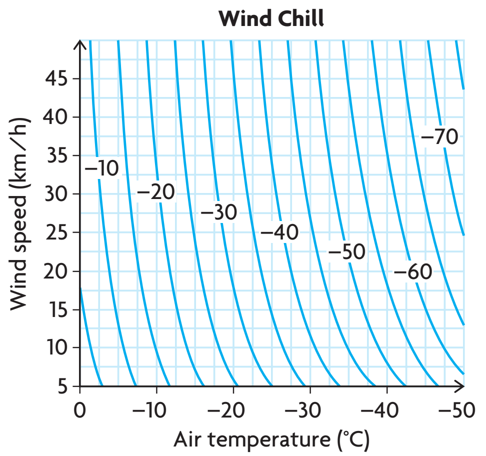
b) Draw the curve of best fit.

c) Determine a function that models this situation.

d) When will the number of monthly hours of sunshine be at a maximum according to the function? When will it be a minimum according to the function?

e) Discuss how well the model fits the data.

9. T The wind chill index measures the sensation of cold on the human skin. In October 2001, Environment Canada introduced the wind chill index shown. Each curve represents the combination of air temperature and wind speed that would produce the given wind chill value.



The following table gives the wind chill values when the temperature is $-20\,°C$.

Wind Speed (km/h)	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Wind Chill (°C)	−24	−27	−29	−31	−32	−33	−33	−34	−35	−35	−36	−37	−37	−37	−38	−38

Source: Environment Canada

a) Create a graphical model for the data.

b) Determine an algebraic model for the data.

c) Use your model from part b) to predict the wind chill for a wind speed of 0 km/h, 100 km/h, and 200 km/h (hurricane force winds). Comment on the reasonableness of each answer.

10. **A** The population of Canada is measured on a regular basis by taking a census. The table shows the population of Canada at the end of each period. From 1851 to 1951, each period is a 10-year interval. From 1951 to 2006, each period is a five-year interval.

Period	Census Population at the End of a Period (in thousands)	Period	Census Population at the End of a Period (in thousands)
1851–1861	3 230	1951–1956	16 081
1861–1871	3 689	1956–1961	18 238
1871–1881	4 325	1961–1966	20 015
1881–1891	4 833	1966–1971	21 568
1891–1901	5 371	1971–1976	23 450
1901–1911	7 207	1976–1981	24 820
1911–1921	8 788	1981–1986	26 101
1921–1931	10 377	1986–1991	28 031
1931–1941	11 507	1991–1996	29 672
1941–1951	13 648	1996–2001	30 755
		2001–2006	31 613

Source: Statistics Canada, Demography Division

a) Use technology to investigate polynomial and exponential models for the relationship of the population and years since 1861. Describe how well each model fits the data.

b) Use each model to estimate Canada's population in 2016.

c) Which model gives the most realistic answer? Explain.

d) Use the model you chose in part c) to estimate the rate at which Canada's population was increasing in 2000.

11. The data shown model the growth of a rabbit population in an environment where the rabbits have no natural predators.

a) Determine an algebraic model for the data.

b) The original population of rabbits was 75; when does the model predict this was?

c) Discuss the growth rate of the rabbit population between 1955 and 1990.

d) Predict the rabbit population in 2020.

Year	Rabbit Population
1955	650
1958	2 180
1960	5 300
1961	8 200
1962	12 400
1965	35 500
1968	66 300
1975	91 600
1980	92 900
1986	92 800
1990	93 100

12. Household electrical power in North America is provided in the form of alternating current. Typically, the voltage cycles smoothly between $+155.6$ volts and -155.6 volts 60 times per second. Assume that at time zero the voltage is $+155.6$ volts.

a) Determine a sine function to model the alternating voltage.

b) Determine a cosine function to model the alternating voltage.

c) Which sinusoidal function was easier to determine? Explain.

Time, t (min)	Pressure, P (kPa)
0	400
5	335
10	295
15	255
20	225
25	195
30	170

13. The pressure of a car tire with a slow leak is given in the table of values.

a) Use technology to investigate linear, quadratic, and exponential models for the relationship of the tire pressure and time. Describe how well each model fits the data.

b) Use each model to predict the pressure after 60 min.

c) Which model gives the most realistic answer? Explain.

14. Explain why population growth is often exponential.

15. C Consider the various functions that could be used for mathematical models.

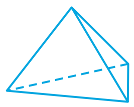
a) Which functions could be used to model a situation in which the values of the dependent variable increase toward infinity? Explain.

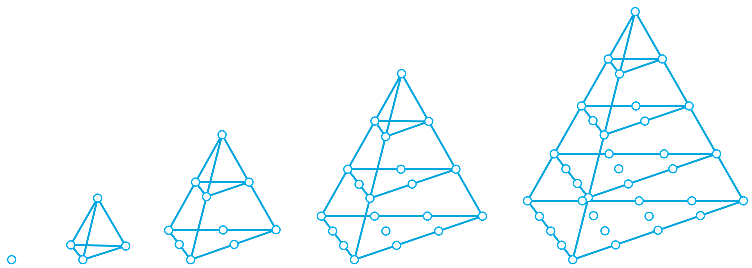
b) Which functions could be used to model a situation in which the values of the dependent variable decrease to zero? Explain.

c) Which functions could be used to model a situation in which the values of the dependent variable approach a non-zero value? Explain.

Extending

16. The numbers 1, 4, 10, 20, and 35 are called tetrahedral numbers because they are related to a four-sided shape called a tetrahedron.


tetrahedron



a) Determine a mathematical model that you can use to generate the nth tetrahedral number.

b) Is 47 850 a tetrahedral number? Justify your answer.

17. According to Statistics Canada, Canada's population reached 30.75 million on July 1, 2000—an increase of 256 700 from the previous year. The rate of growth for that year was the same as the rate of growth for the year before. Both Ontario and Alberta, however, recorded 1.3% growth rates in 2000.

a) Create algebraic and graphical models for the population growth of Canada. Assume that the percent rate of growth was the same for every year.

b) How does the growth rate for Canada's population compare with the growth rate reported by Ontario and Alberta?

9 Chapter Review

FREQUENTLY ASKED Questions

Q: How can you determine the composition of two functions, *f* and *g*?

A1: The composition of f with g can be determined numerically by evaluating g for some input value, x, and then evaluating f using $g(x)$ as the input value.

A2: The composition of f with g can be determined graphically by interpolating on the graph of g to determine its output for some input value, x, and then interpolating on the graph of f using the input value $g(x)$.

A3: The composition of f with g can be determined algebraically by taking the expression for g and then substituting this into the function f.

Study *Aid*

- See Lesson 9.5, Examples 1 and 2.
- Try Chapter Review Questions 8, 9, and 10.

Q: How do you solve an equation or inequality when an algebraic strategy is difficult or not possible?

A1: If you have access to graphing technology, there are two different strategies you can use to solve an equation:

- Represent the two sides of the equation/inequality as separate functions. Then graph the functions together using a graphing calculator or graphing software, and apply the intersection operation to determine the solution(s).
- Rewrite the equation/inequality so that one side is zero. Graph the nonzero side as a function. Use the zero operation to determine each of the zeros of the function.

A2: If you do not have access to graphing technology, you can use a guess and improvement strategy to solve an equation. Estimate where the intersection of $f(x)$ and $g(x)$ will occur, and substitute this value into both sides of the equation. Based on the outcome, adjust your estimate. Repeat this process until the desired degree of accuracy is found.

A3: Solving an inequality requires using either of the three previous strategies to find solutions to either $f(x) - g(x) = 0$ or $f(x) = g(x)$. Use these values to construct intervals. Test each interval to see whether it satisfies the inequality.

Study *Aid*

- See Lesson 9.6, Example 3.
- Try Chapter Review Question 12.

PRACTICE Questions

Lesson 9.1

1. Given the functions $f(x) = x + 5$ and $g(x) = x^2 - 6x - 55$, determine which of the following operations can be used to combine the two functions into one function that has both a vertical asymptote and a horizontal asymptote: addition, subtraction, multiplication, division.

Lesson 9.2

2. A franchise owner operates two coffee shops. The sales, S_1, in thousands of dollars, for shop 1 are represented by $S_1(t) = 700 - 1.4t^2$, where $t = 0$ corresponds to the year 2000. Similarly, the sales for shop 2 are represented by $S_2(t) = t^3 + 3t^2 + 500$.

a) Which shop is showing an increase in sales after the year 2000?

b) Determine a function that represents the total sales for the two coffee shops.

c) What are the expected total sales for the year 2006?

d) If sales continue according to the individual functions, what would you recommend that the owner do? Explain.

3. A company produces a product for \$9.45 per unit, plus a fixed operating cost of \$52 000. The company sells the product for \$15.80 per unit.

a) Determine a function, $C(x)$, to represent the cost of producing x units.

b) Determine a function, $I(x)$, to represent income from sales of x units.

c) Determine a function that represents profit.

Lesson 9.3

4. Calculate $(f \times g)(x)$ for each of the following pairs of functions.

a) $f(x) = 3 \tan (7x), g(x) = 4 \cos (7x)$

b) $f(x) = \sqrt{3x^2}, g(x) = 3\sqrt{3x^2}$

c) $f(x) = 11x - 7, g(x) = 11x + 7$

d) $f(x) = ab^x, g(x) = 2ab^{2x}$

5. A country projects that the average amount of money, in dollars, that it will collect in taxes from each taxpayer over the next 50 years can be modelled by the function $A(t) = 2850 + 200t$, where t is the number of years from now. It also projects that the number of taxpayers over the next 50 years can be modelled by the function $C(t) = 15\,000\,000(1.01)^t$.

a) Write the function that represents the amount of money, in dollars, that the country expects to collect in taxes over the next 50 years.

b) Graph the function you wrote in part a).

c) How much does the country expect to collect in taxes 26 years from now?

Lesson 9.4

6. Calculate $(f \div g)(x)$ for each of the following pairs of functions.

a) $f(x) = 105x^3, g(x) = 5x^4$

b) $f(x) = x - 4, g(x) = 2x^2 + x - 36$

c) $f(x) = \sqrt{x + 15}, g(x) = x + 15$

d) $f(x) = 11x^5, g(x) = 22x^2 \log x$

7. State the domain of $(f \div g)(x)$ for each of your answers in the previous question.

Lesson 9.5

8. Let $f(x) = \dfrac{1}{\sqrt{x + 1}}$ and $g(x) = x^2 + 3$.

a) What are the domain and range of $f(x)$ and $g(x)$?

b) Find $f(g(x))$.

c) Find $g(f(x))$.

d) Find $f(g(0))$.

e) Find $g(f(0))$.

f) State the domain of each of the functions you found in parts b) and c).

9. Let $f(x) = x - 3$. Determine each of the following functions:

a) $(f \circ f)(x)$

b) $(f \circ f \circ f)(x)$

c) $(f \circ f \circ f \circ f)(x)$

d) f composed with itself n times

10. A circle has radius r.

a) Write a function for the circle's area in terms of r.

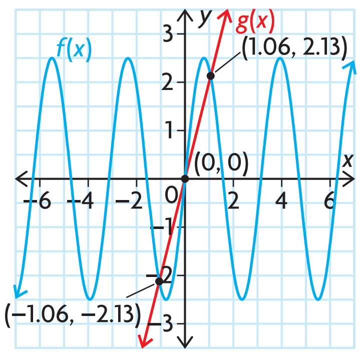
b) Write a function for the radius in terms of the circumference, C.

c) Determine $A(r(C))$.

d) A tree's circumference is 3.6 m. What is the area of the cross-section?

Lesson 9.6

11. In the graph shown below, $f(x) = 5 \sin x \cos x$ and $g(x) = 2x$. State the values of x in which $f(x) < g(x)$, $f(x) = g(x)$, and $f(x) > g(x)$. Express the values to the nearest tenth.



12. Solve each of the following equations for x in the given interval, using a guess and improvement strategy. Express your answers to the nearest tenth, and verify them using graphing technology.

a) $-3 \csc x = x,\ \pi \le x \le \dfrac{3\pi}{2}$

b) $\cos^2 x = 3 - 2\sqrt{x},\ 0 \le x \le \pi$

c) $8^x = x^8,\ -1 \le x \le 1$

d) $7 \sin x = \dfrac{3}{x},\ 0 \le x \le 2$

Lesson 9.7

13. Let P represent the size of the frog population in a marsh at time t, in years. At $t = 0$, a species of frog is released into a marsh. When $t = 5$, biologists estimate that there are 2000 frogs in the marsh. Two years later, the biologists estimate that there are 3200 frogs.

a) Find a formula for $P = f(t)$, assuming linear growth. Interpret the slope and the P-intercept of your formula in terms of the frog population.

b) Find a formula for $P = g(t)$, assuming exponential growth. Interpret the parameters of your formula in terms of the frog population.

14. The population of the world from 1950 to 2000 is shown. Create a scatter plot of the data, and determine an algebraic model for this situation. Use your model to estimate the world's population in 1963, 1983, and 2040.

Year	Population (millions)
1950	2555
1955	2780
1960	3039
1965	3346
1970	3708
1975	4088
1980	4457
1985	4855
1990	5284
1995	5691
2000	6080

Source: U.S. Census Bureau

9 Chapter Self-Test

1. A sphere has radius r.
   a) Write a function for the sphere's surface area in terms of r.
   b) Write a function for the radius in terms of the volume, V.
   c) Determine $A(r(V))$.
   d) A mother wrapped a ball in wrapping paper and gave it to her son on his birthday. The volume of the ball was 0.75 m^3. Assuming that she used the minimum amount of wrapping paper possible to cover the ball, how much wrapping paper did she use?

2. Solve $x \sin x \geq x^2 - 1$. Use any strategy.

3. Let $f(x) = (2x + 3)^7$. Find at least two different pairs of functions, $g(x)$ and $h(x)$, such that $f(x) = (g \circ h)(x)$.

n	$N(n)$
0	400
2	520
4	752
6	1144
8	1744
10	2600
15	6175

4. In the table at the left, $N(n)$ is the number, in thousands, of Canadian home computers sold, where n is the number of years since 1990.
   a) Determine the equation that best models this relationship.
   b) How many home computers were sold in June 1993?

5. The graph of the function $f(x)$ is a line passing through the point $(2, -3)$ with a slope of 6. The graph of the function $g(x)$ is the graph of the function $h(x) = x^2$ vertically stretched by a factor of 5, horizontally translated 8 units to the left, and vertically translated 1 unit down. Find $(f \times g)(x)$.

6. The height of a species of dwarf evergreen tree, in centimetres, as a function of time, in months, can be modelled by the logistic function $h(t) = \frac{275}{1 + 26(0.85)^t}$.
   a) If this function is graphed, are there any asymptotes? If so, name each asymptote and describe what it means.
   b) Determine when this tree will reach a height of 150 cm.

7. The cost, in dollars, to produce a product can be modelled by the function $C(x) = 5x + 18$, where x is the number of the product produced, in thousands. The revenue generated by producing and selling x units of this product can be modelled by the function $R(x) = 2x^2$. How much of the product must the company produce in order to break even?

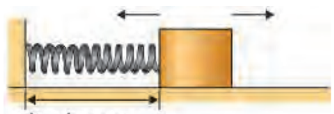
8. Solve $\frac{\cot x}{x} = x^3 + 3$. Use any strategy. Round your answer(s) to the nearest tenth, if necessary.

9. Given $f(x) = \sin x$ and $g(x) = \cos x$, which of the following operations make it possible to combine the two functions into one function that is not sinusoidal: addition, subtraction, multiplication, or division?

9 Chapter Task

Modelling a Situation Using a Combination of Functions

A mass is attached to a spring at one end and secured to a wall at the other end. When the mass is pulled away from the wall and released, it moves back and forth (oscillates) along the floor.



If there is no friction between the mass and the floor, and no drag from the air, then the displacement of the mass versus time could be modelled by a sinusoidal function. Because of friction, however, the speed of the mass is reduced, which causes the displacement to decrease exponentially with each oscillation.

The displacement function $d(t)$ is a combination of functions: $d(t) = f(t)g(t) + r$.

Consider the following situation:

- The mass is at a resting position of $r = 30$ cm.
- The spring provides a period of 2 s for the oscillations.
- The mass is pulled to $d = 50$ cm and released.
- After 10 s, the spring is at $d = 33$ cm.

? How would the displacement and speed of the mass at time $t = 7.7$ s differ if there were no friction between the mass and the floor?

A. Make a sketch of the displacement versus time graph to ensure that you understand this situation.

B. Write the general equation of the function that models this situation, with the necessary parameters.

C. Use the information provided to determine the values of the parameters, and write the equation of the model.

D. Graph the function you determined in part C using graphing technology. Check that it models the motion of the mass correctly.

E. Write the function for displacement that would be correct if there were no damping of the motion due to friction.

F. Calculate the displacement at 7.7 s for each model you determined in parts C and E, and compare your results.

G. Estimate the instantaneous speed of the mass at 7.7 s for each model, and compare your results.

Task *Checklist*

- ✔ Did you draw and label your displacement versus time graph accurately?
- ✔ Did you show all your steps when determining both models?
- ✔ Did you show all your steps when determining the displacements and speeds?
- ✔ Did you discuss the difference between the displacements and speeds?

Chapters 7–9 Cumulative Review

Multiple Choice

1. Which of these is an equivalent trigonometric ratio for $\sin \frac{2\pi}{5}$?
   a) $\cos \frac{\pi}{10}$
   b) $\sin \frac{3\pi}{5}$
   c) $-\cos \frac{9\pi}{10}$
   d) all of these

2. What is the exact value of $\cos \frac{\pi}{12}$?
   a) $\frac{\sqrt{3}}{4}$
   b) $\frac{\sqrt{2} + \sqrt{6}}{4}$
   c) $\frac{\sqrt{6}}{4}$
   d) $\frac{\sqrt{6} - \sqrt{2}}{4}$

3. If α and β are acute angles with $\sin \alpha = \frac{12}{13}$ and $\sin \beta = \frac{8}{17}$, what is the value of $\tan(\alpha + \beta)$?
   a) $-\frac{220}{21}$
   b) $\frac{220}{21}$
   c) $\frac{220}{221}$
   d) $\frac{220}{123}$

4. Given that $\sin \theta = \frac{3}{8}$ and θ is obtuse, what is the value of $\tan 2\theta$?
   a) $-\frac{3\sqrt{55}}{23}$
   b) $\frac{3\sqrt{55}}{46}$
   c) $-\frac{3\sqrt{55}}{55}$
   d) $\frac{3\sqrt{55}}{55}$

5. What is the exact value of $\cos \frac{\pi}{8}$?
   a) $\frac{2 + \sqrt{2}}{2}$
   b) $\frac{\sqrt{2 - \sqrt{2}}}{2}$
   c) $\frac{\sqrt{2 - \sqrt{2}}}{4}$
   d) $\frac{\sqrt{2 + \sqrt{2}}}{2}$

6. Which expression is equivalent to $\cos x$?
   a) $\frac{2\cos^2\left(\frac{1}{2}x\right) - 1}{\cos^2\left(\frac{1}{2}x\right)}$
   b) $2\cos^2(2x) - 1$
   c) $\frac{2 - \sec^2\left(\frac{1}{2}x\right)}{\sec^2\left(\frac{1}{2}x\right)}$
   d) $1 - 2\sin^2(2x)$

7. Which of the following identities could you use to help you prove that $\frac{2\tan x}{1 + \tan^2 x} = \sin 2x$?
   a) $1 + \tan^2 x = \sec^2 x$
   b) $\sin 2x = 2\sin x \cos x$
   c) $\tan x = \frac{\sin x}{\cos x}$
   d) all of these

8. Which set of value(s), in radians, is the solution of $5 + 7\sin\theta = 0$, where $-\pi \le \theta \le \pi$?
   a) $\theta = -0.80$
   b) $\theta = -0.80, -2.35$
   c) $\theta = 0.80, 2.35$
   d) $\theta = -0.80, 0.80$

9. The height of the tip of one blade of a wind turbine above the ground, $h(t)$, can be modelled by $h(t) = 18\cos\left(\pi t + \frac{\pi}{4}\right) + 23$, where t is the time passed in seconds. Which time interval describes a period when the blade tip is at least 30 m above the ground?
   a) $5.24 \le t \le 7.33$
   b) $0.42 \le t \le 1.08$
   c) $1.37 \le t \le 2.12$
   d) $0.08 \le t \le 1.42$

10. Which set of values is the solution of $(2\sin x + 1)(\cos x - 1) = 0$, where $0° \le x \le 360°$?
    a) $x = 180°, 210°, 330°$
    b) $x = 30°, 180°, 150°$
    c) $x = 0°, 150°, 210°, 360°$
    d) $x = 0°, 210°, 330°, 360°$

11. The equation $\cos 2\theta + d\cos\theta + e = 0$ has solutions $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ in the interval $0 \leq \theta \leq 2\pi$. What are the values of d and e?

a) $d = -3, e = 2$ **c)** $d = 1, e = 3$
b) $d = 2, e = 3$ **d)** $d = -3, e = 1$

12. What is the exponential form of $y = \log_7 x$?

a) $x = \log_7 y$ **c)** $y = 7^x$
b) $x = 7^y$ **d)** $y = x^7$

13. The function $f(x) = \log_{10} x$ is reflected in the x-axis, stretched horizontally by a factor of 3, and translated up 2 units. Which of these functions is the result?

a) $g(x) = -\log_{10}(3x) - 2$

b) $g(x) = \log_{10}\left(-\frac{1}{3}x\right) + 2$

c) $g(x) = -\log_{10}\left(\frac{1}{3}(x - 2)\right) + 2$

d) $g(x) = -\log_{10}\left(\frac{1}{3}x\right) + 2$

14. What is the value of $7^{\log_7 49}$?

a) 7 **b)** 2 **c)** 14 **d)** 49

15. The equation $\log_{10} T = 1.5 \log_{10} d - 0.45$ describes the orbit of a planet around the star Gliese 581. In this equation, T is the length of the planet's year in days, and d is its average distance in millions of kilometres from Gliese 581. The earth-like planet Gliese 581c is 11 000 000 km from Gliese 581. How long is its year?

a) 16.1 days **c)** 12.9 days
b) 1.1 days **d)** 3.9 days

16. What is the solution of the equation $\log_4 x + 3 = \log_4 1024$?

a) 16 **b)** 4 **c)** 128 **d)** $\frac{16}{3}$

17. A transformation that takes the graph of $f(x) = \log_5 x$ to that of $g(x) = \log_5 25x$ is

a) horizontal translation 2 units left
b) vertical translation 2 units up
c) vertical stretch by a factor of 25
d) horizontal stretch by a factor of 25

18. Solve $x = \log_3 27\sqrt{3}$.

a) $2\frac{1}{2}$ **b)** $3\frac{1}{2}$ **c)** $3\frac{1}{3}$ **d)** $9\frac{1}{2}$

19. An investment of \$1600 grows at a rate of 1% per month, compounded monthly. How long will it take for the investment to be worth more than \$6400? Recall that the formula for compound interest is $A = P(1 + i)^n$.

a) 11 years 7 months **c)** 11 years 8 months
b) 33 years 3 months **d)** 33 years 4 months

20. The loudness of a sound in decibels, L, is $L = 10 \log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound in watts per square metre $(\mathrm{W/m^2})$ and $I_0 = 10^{-12}\ \mathrm{W/m^2}$. If the loudness of a jet taking off is 133 dB, what is the intensity of this sound?

a) $2.00 \times 10^{13}\ \mathrm{W/m^2}$ **c)** $10^{-1}\ \mathrm{W/m^2}$
b) $10.0\ \mathrm{W/m^2}$ **d)** $20.0\ \mathrm{W/m^2}$

21. Solve the following:
$\log_a(x - 3) + \log_a(x - 2) = \log_a(5x - 15)$

a) $x = 3$ **c)** $x = -3$ or 7
b) $x = 7$ **d)** $x = 2$

22. Carbon-14 has a half-life of 5730 years. A fossil human jawbone that contains 0.017 g of carbon-14 is estimated to have contained 3.9 g when the person was alive. How old is the fossil?

a) 45 000 years **c)** 1 300 000 years
b) 13 500 years **d)** 12 000 years

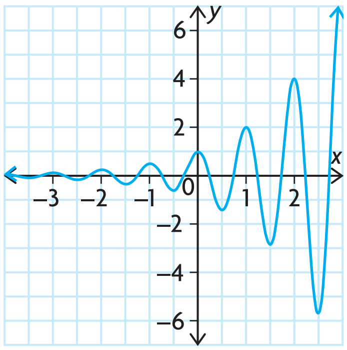
23. Assume that the annual rate of inflation will average 3.1% over the next 5 years. For a product that currently costs P dollars, which is the best model for the approximate cost, C, of goods and services during any year in the next 5 years?

a) $C = P(1 + 0.031^t)$
b) $C = (1.031)^t$
c) $C = P(1.031)^t$
d) $C = P(1 + 3.1^t)$

24. The population of a city is currently 150 000 and is increasing at a rate of 2.3%/a. Predict the instantaneous rate of growth in the population 7 years from now.

a) 175 900 people/a
b) 25 900 people/a
c) 4000 people/a
d) 3700 people/a

25. Which combination of functions could result in this graph?



a) $y = x^2 \cos(2\pi x)$
b) $y = \sin(2\pi x) + \log x$
c) $y = 2^x \cos(2\pi x)$
d) $y = \sin(2\pi|x|)0.5^x$

26. If $f(x) = \log x$ and $g(x) = \frac{1}{x-3}$, which set is the domain of $f - g$?

a) $\{x \in \mathbf{R} | x > 3\}$
b) $\{x \in \mathbf{R} | x > 0, x \neq 3\}$
c) $\{x \in \mathbf{R} | x > 0, x \neq -3\}$
d) $\{x \in \mathbf{R} | x < 3\}$

27. Which combination is always an odd function?

a) the sum of two odd functions
b) the difference of an odd function and an even function
c) the sum of an odd function and an even function
d) the difference of two even functions

28. For which pair of functions, $f(x)$ and $g(x)$, is the range of $f \times g$ equal to $\{y \in \mathbf{R} | y \geq 1\}$?

a) $f(x) = g(x) = \sec x$
b) $f(x) = \sec x, g(x) = \csc x$
c) $f(x) = 2^x, g(x) = |x| + 1$
d) $f(x) = 2^x, g(x) = x^2 + 1$

29. Given $f(x) = ax^2 + 3$ and $g(x) = bx - 1$, the graph of the product $f \times g$ passes through the points $(-1, -3)$ and $(1, 9)$. What are the values of a and b?

a) $a = -6, b = 10$
b) $a = 6, b = 2$
c) $a = -8, b = 2$
d) $a = -6, b = -2$

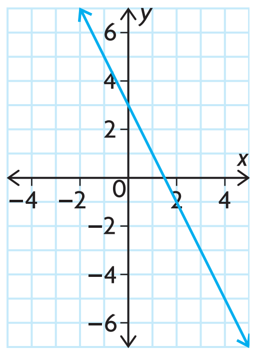
30. What is the domain of $f \div g$, where $f(x) = \log x$ and $g(x) = |x - 2|$?

a) $\{x \in \mathbf{R} | x \neq 0, 2\}$
b) $\{x \in \mathbf{R} | x > 2\}$
c) $\{x \in \mathbf{R} | 0 < x < 2\}$
d) $\{x \in \mathbf{R} | x > 0, x \neq 2\}$

31. If $f(x) = \sqrt{3 - x}$ and $g(x) = 3x^2$, what is the domain of $f \circ g$?

a) $\{x \in \mathbf{R} | -3 \leq x \leq 3\}$
b) $\{x \in \mathbf{R} | x \leq 3\}$
c) $\{x \in \mathbf{R} | -1 \leq x \leq 1\}$
d) $\{x \in \mathbf{R} | x \geq 0\}$

32. Which combination of the functions $f(x) = 2x$, $g(x) = x + 5$, and $h(x) = 3 - x$ has this graph?



a) $f \circ g$
b) $f \circ h$
c) $h \circ g$
d) $h \circ f$

33. Which values are solutions of the equation $x^3 = \sqrt[3]{\tan x}$?

a) $x = 0$
b) $x = -1.07$
c) $x = 1.07$
d) all of these

34. Given $f(x) = 4 - x^2$, for which function $g(x)$ is $f(x) < g(x)$ when $x \in (-\infty, -1)$ or $(4, \infty)$?

a) $g(x) = 4x$
b) $g(x) = -3x$
c) $g(x) = 4x - 8$
d) $g(x) = -4x$

Investigations

Touchdown Pass

35. The horizontal distance, d, in metres, that a football can be thrown from its release point to the point where it hits the ground can be modelled by the equation $d = \frac{v^2}{9.8} \sin 2\theta + 1.8$, where v is the initial speed of the football in metres per second and θ is the angle relative to the horizontal at which the football leaves the quarterback's hand. If the football is thrown at 20 m/s and travels 35 m, determine the possible angles at which the football could be thrown. Give your answer to the nearest degree.

Projecting Populations

36. The data below were collected by the Ontario Ministry of Finance and released in July 2000. It shows the projected populations (in thousands) of the Regional Municipalities of Niagara and Waterloo.

	Historical		Projections						
Regional Municipality	**1996**	**1999**	**2001**	**2006**	**2011**	**2016**	**2021**	**2026**	**2028**
Niagara	414.8	421.7	426.4	435.9	445.3	455.1	464.9	473.8	476.8
Waterloo	418.3	438.4	452.1	483.6	512.6	541.4	569.8	596.3	606.1

a) Determine suitable models that the Ministry of Finance might have used to make these projections.

b) Use your models to estimate the doubling time of the population in each region.

c) Use your models to predict which region's population will be increasing the fastest in 2025. Support your answer with the necessary calculations.

It's Rocket Science

37. The mass of a rocket just before launch is 30 000 kg. During its ascent, the rocket burns 100 kg of fuel every second, and therefore decreases in mass at a rate of 100 kg/s. The mass m, acceleration a, and thrust T are related by the equation $T - 10m = ma$. The velocity v is related to the mass by the equation $m = 30\,000(2.72)^{-v-gt}$. Determine the functions $m(t)$, $a(t)$, and $v(t)$, in terms of the variable t (time measured in seconds) and the constants T and g. Use the fact that $a(0) > 0$ for the rocket to lift off, to determine the constraint on T.

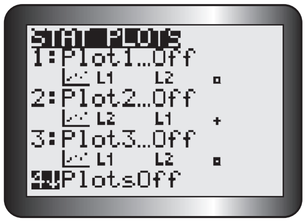
Appendix T

Review of Technical Skills

PART 1 USING THE TI-83 PLUS AND TI-84 GRAPHING CALCULATORS

T–1 Preparing the Calculator

Before you graph a function, be sure to clear any information left on the calculator from the last time it was used. You should always do the following:



1. **Clear all data in the lists.**

   Press 2nd + 4 ENTER.

2. **Turn off all stat plots.**

   Press 2nd Y= 4 ENTER.

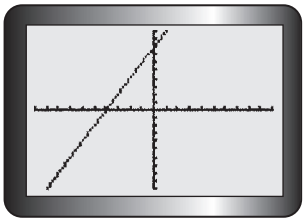
3. **Clear all equations in the equation editor.**

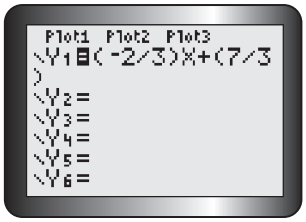
   Press Y=, and then press CLEAR for each equation.

4. **Set the window so that the axes range from -10 to 10.**

   Press ZOOM 6. Press WINDOW to verify.

T–2 Entering and Graphing a Function





1. **Enter the equation of the function in the equation editor.**

   To graph $y = 2x + 8$, press Y= 2 X, T, Θ, *n* + 8 GRAPH. The graph will be displayed as shown.

2. **Enter all linear equations in the form $y = mx + b$.**

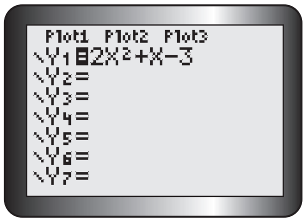
   If m or b are fractions, enter them between brackets. For example, write $2x + 3y = 7$ in the form $y = -\frac{2}{3}x + \frac{7}{3}$, and enter it as shown.

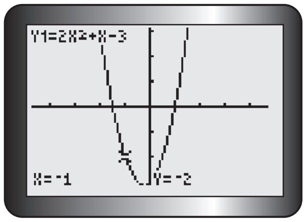
3. **Press GRAPH to view the graph.**

4. **Press TRACE to find the coordinates of any point on the graph.**

   Use the left and right arrow keys to cursor along the graph. Press ZOOM 8 ENTER TRACE to trace using integer intervals. If you are working with several graphs at the same time, use ▲ and ▼ to scroll between graphs.

T–3 Evaluating a Function





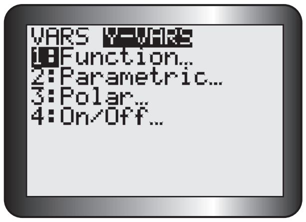
1. **Enter the function in the equation editor.**

   To enter $y = 2x^2 + x - 3$, press Y= 2 X, T, Θ, *n* x^2 + X, T, Θ, *n* − 3.

2. **Use the value operation to evaluate the function.**

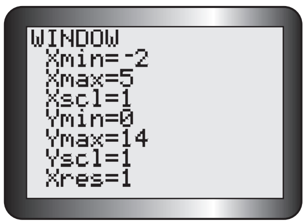
   To find the value of the function at $x = -1$, press 2nd TRACE ENTER, enter (−) 1 at the cursor, and then press ENTER.

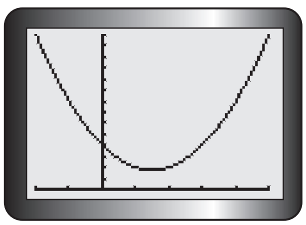
3. **Use function notation and the Y-VARS operation to evaluate the function.**
   This is another way to evaluate the function. To find the value of the function at $x = 37.5$, press CLEAR VARS. Then cursor right to **Y-VARS**, and press ENTER. Press 1 to select **Y1**. Finally, press (3 7 . 5), and then ENTER.



T–4 Changing Window Settings

The window settings can be changed to show a graph for a given domain and range.





1. **Enter the function in the equation editor.**
   For example, enter $y = x^2 - 3x + 4$ in the equation editor.

2. **Use the WINDOW function to set the domain and range.**

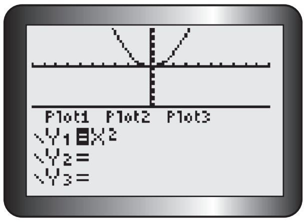
   To display the function over the domain $\{x \mid -2 \leq x \leq 5\}$ and range $\{y \mid 0 \leq y \leq 14\}$, press WINDOW (−) 2 ENTER, then 5 ENTER, then 1 ENTER, then 0 ENTER, then 1 4 ENTER, then 1 ENTER, and finally 1 ENTER.

3. **Press** GRAPH **to show the function with this domain and range.**

T–5 Using the Split Screen

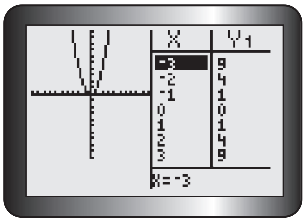
1. **The split screen can be used to see a graph and the equation editor at the same time.**

   Press MODE and cursor to **Horiz**. Press ENTER to select this, and then press 2nd MODE to return to the home screen. Enter $y = x^2$ in **Y1** of the equation editor, and then press GRAPH.

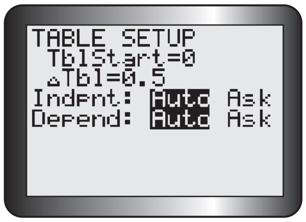


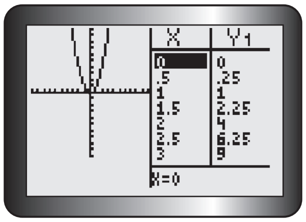
2. **The split screen can also be used to see a graph and a table at the same time.**

   Press MODE, and move the cursor to **G–T** (Graph-Table). Press ENTER to select this, and then press GRAPH.



   It is possible to view the table with different increments. For example, to see the table start at $x = 0$ and increase in increments of 0.5, press 2nd WINDOW and adjust the settings as shown. Then press GRAPH.





T–6 Using the TABLE Feature

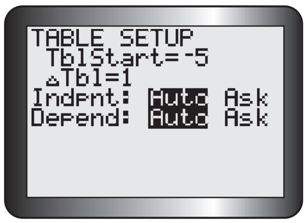
A function can be displayed in a table of values.

1. **Enter the function in the equation editor.**

   To enter $y = -0.1x^3 + 2x + 3$, press Y= (–) . 1 X, T, Θ, *n* ^ 3 + 2 X, T, Θ, *n* + 3.

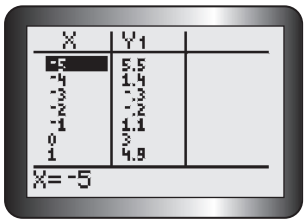
2. **Set the start point and step size for the table.**

   Press 2nd WINDOW. The cursor is beside "TblStart=." To start at $x = -5$, press (–) 5 ENTER. The cursor is now beside **ΔTbl=**. To increase the x-value in increments of 1, press 1 ENTER.



3. **To view the table, press 2nd GRAPH.**

   Use ▲ and ▼ to move up and down the table. Notice that you can look at higher or lower x-values than those in the original range.

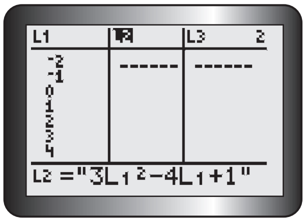


T–7 Making a Table of Differences

To make a table with the first and second differences for a function, use the STAT lists.

1. **Press** STAT 1**, and enter the *x*-values into L1.**

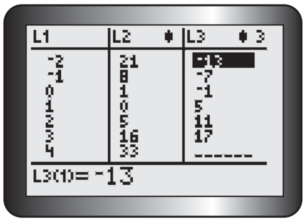
   For the function $f(x) = 3x^2 - 4x + 1$, use x-values from -2 to 4.



2. **Enter the function.**

   Scroll right and up to select **L2**. Enter the function $f(x)$, using **L1** as the variable x. Press ALPHA + 3 2nd 1 x^2 – 4 2nd 1 + 1 ALPHA +.

3. **Press** ENTER **to display the values of the function in L2.**

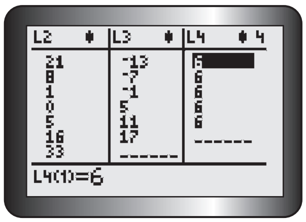


4. **Find the first differences.**

   Scroll right and up to select **L3**. Then press 2nd STAT.

   Scroll right to **OPS** and press 7 to choose **ΔList(**.

   Enter **L2** by pressing 2nd 2). Press ENTER to see the first differences displayed in **L3**.



5. **Find the second differences.**

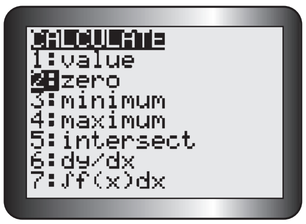
   Scroll right and up to select **L4**. Repeat step 4, using **L3** instead of **L2**. Press ENTER to see the second differences displayed in **L4**.

T–8 Finding the Zeros of a Function

To find the zeros of a function, use the **zero** operation.

1. **Start by entering the function in the equation editor.**

   For example, enter $y = -(x + 3)(x - 5)$ in the equation editor. Then press GRAPH ZOOM 6.

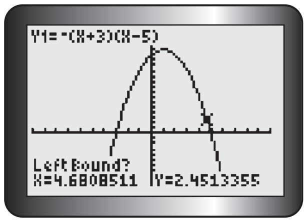


2. **Access the zero operation.**

   Press 2nd TRACE 2.

3. **Use the left and right arrow keys to cursor along the curve to any point that is left of the zero.**

   Press ENTER to set the left bound.

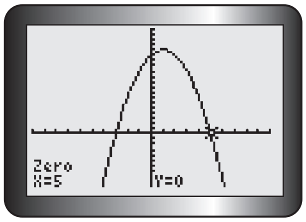


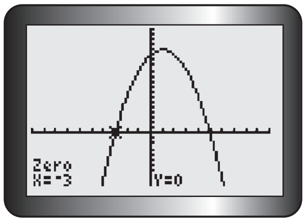
4. **Cursor along the curve to any point that is right of the zero.**

   Press ENTER to set the right bound.

5. **Press ENTER again to display the coordinates of the zero (the *x*-intercept).**

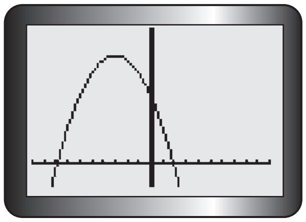
6. **Repeat to find the second zero.**





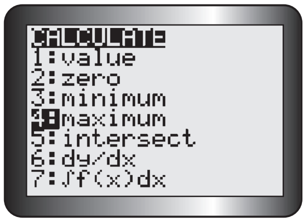
T–9 Finding the Maximum or Minimum Value of a Function

The least or greatest value can be found using the **minimum** operation or the **maximum** operation.



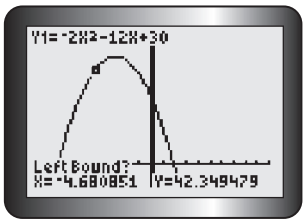
1. **Enter and graph the function.**
   For example, enter $y = -2x^2 - 12x + 30$.
   Graph the function, and adjust the window as shown. This graph opens downward, so it has a maximum.

2. **Use the maximum operation.**



   Press 2nd TRACE 4. For parabolas that open upward, press 2nd TRACE 3 to use the **minimum** operation.

3. **Use ◀ and ▶ to cursor along the curve to any point that is left of the maximum value.**

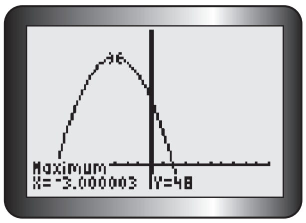


   Press ENTER to set the left bound.

4. **Cursor along the curve to any point that is right of the maximum value.**

   Press ENTER to set the right bound.

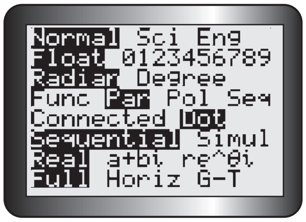
5. **Press ENTER again to display the coordinates of the optimal value.**

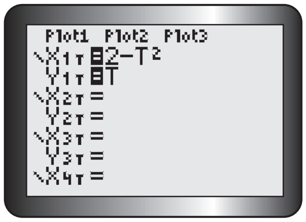


T–10 Graphing the Inverse of a Function

Parametric equations allow you to graph any function and its inverse.
For example, the function $y = 2 - x^2$, with domain $x \geq 0$, can be graphed using parametric mode. For a parametric equation, both x and y must be expressed in terms of a parameter, t. Replace x with t. Then $x = t$ and $y = 2 - t^2$. The inverse of this function can now be graphed.

1. **Clear the calculator, and press** MODE**.**
   Change the setting to the parametric mode by scrolling down to the fourth line and to the right to **Par**, as shown on the screen below. Press ENTER.



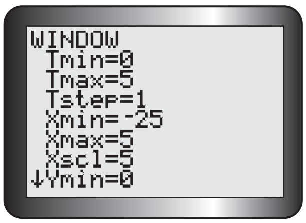


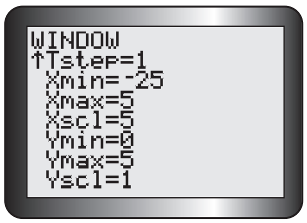
2. **Enter the inverse function by changing the parametric equations $x = t$ and $y = 2 - t^2$ to $x = 2 - t^2$ and $y = t$.**

   Press Y=. At X1T=, enter 2 – X, T, Θ, *n* x^2 ENTER. At Y1T=, enter X, T, Θ, *n*.

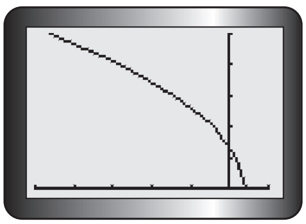
3. **Press** WINDOW**.**

   The original domain, $x \geq 0$, is also the domain of t. Use window settings, such as those shown below, to display the graph.





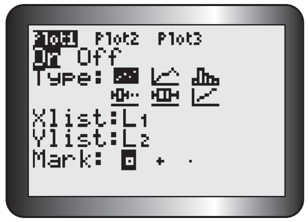
4. **Press** GRAPH **to display the inverse function.**



T–11 Creating a Scatter Plot and Determining a Line or Curve of Best Fit Using Regression

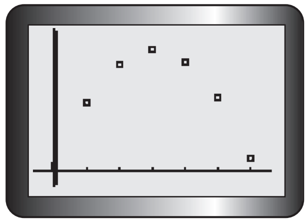
This table gives the height of a baseball above ground, from the time it was hit to the time it touched the ground.

Time (s)	0	1	2	3	4	5	6
Height (m)	2	27	42	48	43	29	5



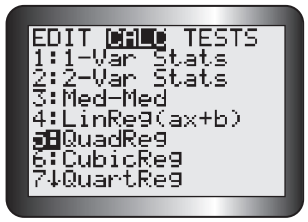
1. **Start by entering the data into lists.**

   Press STAT ENTER. Move the cursor over to the first position in **L1**, and enter the values for time. Press ENTER after each value. Repeat this for height in **L2**.



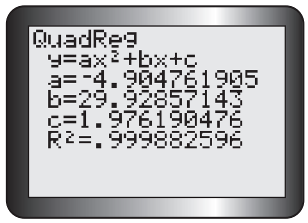
2. **Create a scatter plot.**

   Press 2nd Y= and 1 ENTER. Turn on Plot 1 by making sure that the cursor is over **On**, the **Type** is set to the graph type you prefer, and **L1** and **L2** appear after **Xlist** and **Ylist**.



3. **Display the graph.**

   Press ZOOM 9 to activate **ZoomStat**.



4. **Apply the appropriate regression analysis.**

   To determine the equation of the line or curve of best fit, press STAT and scroll over to **CALC**. Press

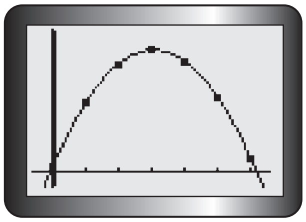
   - 4 to enable **LinReg(ax+b)**
   - 5 to enable **QuadReg**
   - 6 to enable **CubicReg**
   - 7 to enable **QuartReg**
   - 0 to enable **ExpReg**
   - ALPHA C to enable **SinReg**

   Then press 2nd 1 , 2nd 2 , VARS. Scroll over to **Y-VARS**. Press 1 twice. This action stores the equation of the line or curve of best fit into **Y1** of the equation editor.

5. **Display and analyze the results.**

Press ENTER. In this example, the letters a, b, and c are the coefficients of the general quadratic equation $y = ax^2 + bx + c$ for the curve of best fit. R^2 is the percent of data variation represented by the model. The equation is about $y = -4.90x^2 + 29.93x + 1.98$.

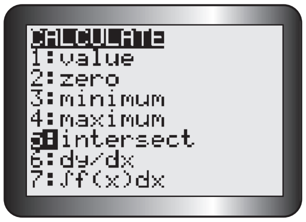
Note: For linear regression, if r is not displayed, turn on the diagnostics function. Press 2nd 0, and scroll down to **DiagnosticOn**. Press ENTER twice. Repeat steps 4 to 6.



6. **Plot the curve.**

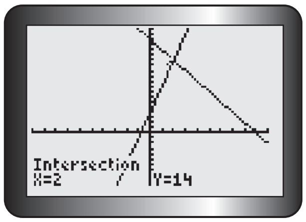
Press GRAPH

T–12 Finding the Points of Intersection of Two Functions



1. **Enter both functions in the equation editor.**
For example, enter $y = 5x + 4$ and $y = -2x + 18$.
2. **Graph both functions**.

Press GRAPH. Adjust the window settings until one or more points of intersection are displayed.



3. **Use the intersect operation.**

Press 2nd TRACE 5.

4. **Determine a point of intersection**.
You will be asked to verify the two curves and enter a guess (optional) for the point of intersection. Press ENTER after each screen appears.

The point of intersection is exactly (2, 14).

5. **Determine any additional points of intersection.**

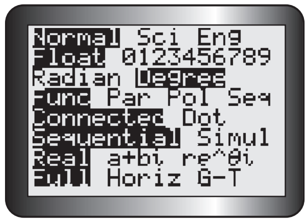
Press TRACE, and move the cursor close to the other point you wish to identify. Repeat step 4.

T–13 Evaluating Trigonometric Ratios and Finding Angles

Working with Degrees

1. **Put the calculator in degree mode.**

   Press MODE. Scroll down and across to **Degree**. Press ENTER.

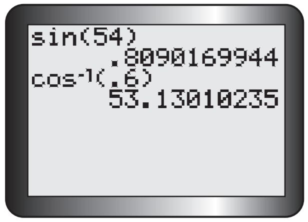


2. **Use the SIN, COS, or TAN key to calculate a trigonometric ratio.**

   To find the value of sin 54°, press SIN 5 4) ENTER.

3. **Use SIN^{-1}, COS^{-1}, or TAN^{-1} to calculate an angle.**

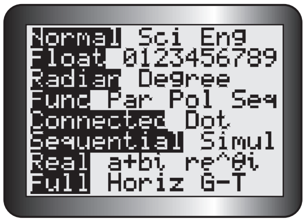
   To find the angle whose cosine is 0.6, press 2nd COS . 6) ENTER.



Working with Radians

1. **Put the calculator in radian mode.**

   Press MODE. Scroll down and across to **Radian**. Press ENTER.

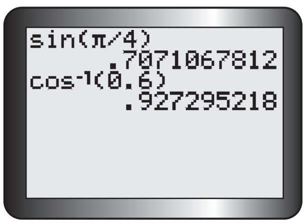


2. **Use the SIN, COS, or TAN key to calculate a trigonometric ratio.**

   To find the value of sin $\frac{\pi}{4}$, press SIN 2nd ^ ÷ 4) ENTER.

3. **Use SIN^{-1}, COS^{-1}, or TAN^{-1} to calculate an angle.**

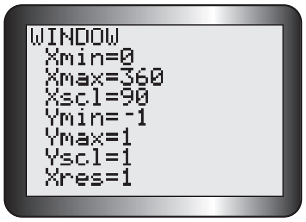
   To find the angle whose cosine is 0.6, press 2nd COS . 6) ENTER.



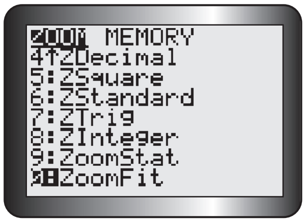
T–14 Graphing a Trigonometric Function

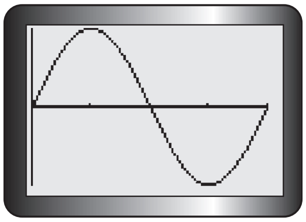
Working with Degrees

You can graph a trigonometric function in degree measure using the TI-83 Plus or TI-84 calculator.



step 3





step 4

1. **Put the calculator in degree mode.**

   Press [MODE]. Scroll down and across to **Degree**. Press [ENTER].

2. **Enter the function in the equation editor.**

   For example, to graph the function $y = \sin x$, for $0° \leq x \leq 360°$, press

   [Y=] [SIN] [X, T, Θ, *n*] [ )].

3. **Adjust the window to correspond to the given domain.**

   Press [WINDOW]. Set **Xmin** $=$ **0**, **Xmax** $=$ **360**, and **Xscl** $=$ **90**. These settings display the graph from 0° to 360°, using an interval of 90° on the *x*-axis. Then set **Ymin** $= -$**1** and **Ymax** $=$ **1**, since the sine function being graphed lies between these values. If the domain is not known, this step can be omitted.

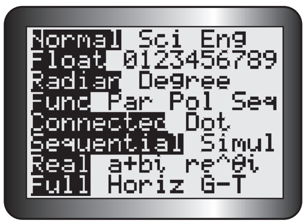
4. **Graph the function using ZoomFit.**

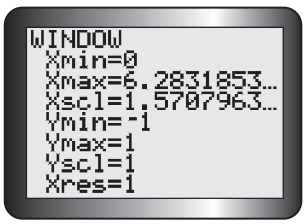
   Press [ZOOM] [0]. The graph is displayed over the domain, and the calculator determines the best values to use for **Ymax** and **Ymin** in the display window.

   Note: You can use **ZoomTrig** (press [ZOOM] [7]) to graph the function in step 4. **ZoomTrig** will always display the graph in a window where **Xmin** $= -$**360°**, **Xmax** $=$ **360°**, **Ymin** $= -$**4**, and **Ymax** $=$ **4**.

Working with Radians

You can also graph a trigonometric function in radians using the TI-83 Plus or TI-84 calculator.





1. **Put the calculator in radian mode.**

   Press [MODE]. Scroll down and across to **Radian**. Press [ENTER].

2. **Enter the function in the equation editor.**

   For example, to graph the function $y = \sin x$, for $0 \leq x \leq 2\pi$, press

   [Y=] [SIN] [X, T, Θ, *n*] [ )].

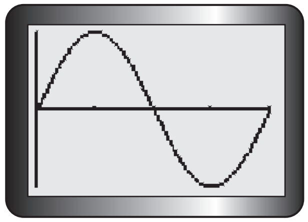
3. **Adjust the window to correspond to the given domain.**

   Press [WINDOW]. Set **Xmin** $=$ **0**, **Xmax** $= \mathbf{2\pi}$, and **Xscl** $= \mathbf{\frac{\pi}{2}}$. These settings display the graph from 0 to 2π, using an interval of $\frac{\pi}{2}$ on the *x*-axis. Then set **Ymin** $= -$**1** and **Ymax** $=$ **1**, since the sine function being graphed lies between these values. If the domain is not known, this step can be omitted.

4. **Graph the function using ZoomFit.**

   Press ZOOM 0. The graph is displayed over the domain, and the calculator determines the best values to use for **Ymax** and **Ymin** in the display window.

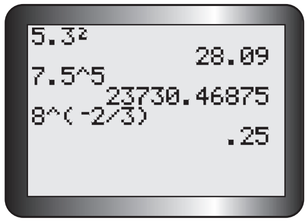
   Note: You can use **ZoomTrig** (press ZOOM 7) to graph the function in step 4. **ZoomTrig** will always display the graph in a window where **Xmin** $= -2\pi$, **Xmax** $= 2\pi$, **Ymin** $= -4$, and **Ymax** $= 4$.



T–15 Evaluating Powers and Roots

1. **Evaluate the power $(5.3)^2$.**

   Press 5 . 3 x^2 ENTER.

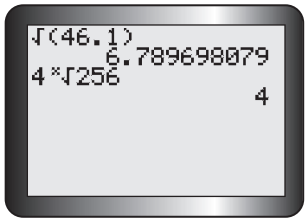


2. **Evaluate the power 7^5.**

   Press 7 ^ 5 ENTER.

3. **Evaluate the power $8^{-\frac{2}{3}}$.**

   Press 8 ^ (− 2 ÷ 3) ENTER.



4. **Evaluate the square root of 46.1.**

   Press 2nd x^2 4 6 . 1) ENTER.

5. **Evaluate $\sqrt[4]{256}$.**

   Press 4 MATH 5 2 5 6 ENTER.

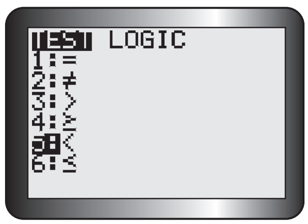
T–16 Graphing a Piecewise Function

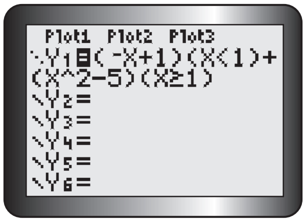
Follow these steps to graph the piecewise function defined by

$$f(x) = \begin{cases} -x + 1, \text{ if } x < 1 \\ x^2 - 5, \text{ if } x \geq 1 \end{cases}$$

1. **Enter the first equation.**

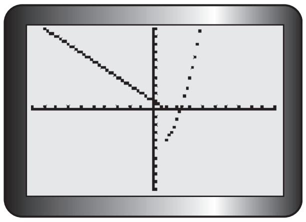
   In the equation editor for **Y1**, enter the first equation in brackets. Then enter its corresponding interval in brackets. The inequality signs can be accessed in the **Test** menu by pressing 2nd MATH.





2. **Enter the second equation.**

   Press +, and repeat step 1 for the second equation and its interval. Scroll to the left of **Y1**, and press ENTER until the dotted graphing mode appears.



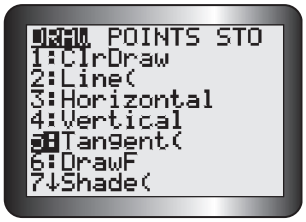
3. **Display the graph.**

   Press GRAPH to display the graph.
   Each equation produces a different graph on each interval. This function is discontinuous at $x = 1$.

T–17 Drawing Tangent Lines

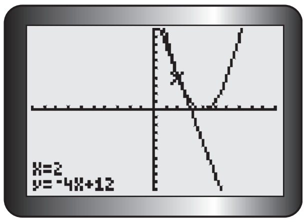
1. **Enter the function, and display the graph.**

   Enter $y = (4 - x)^2$ into **Y1** of the equation editor, and display the graph.



2. **Draw the tangent line, and estimate its slope.**
   Use the **Tangent** command in the **Draw** menu to draw a tangent line at point (2, 4) and estimate its slope.
   Press 2nd PRGM. Choose **5:Tangent(** and then press 2 and ENTER.
   The tangent line is drawn, and its equation is displayed.

   The slope of the tangent line is -4, and its y-intercept is 12.



3. **Clear the tangent line.**
   Press 2nd PRGM 1 to clear the tangent line. The function will be graphed again, without the tangent line.

PART 2 USING A SPREADSHEET

T–18 Introduction to Spreadsheets

A spreadsheet is a computer program that can be used to create a table of values and then graph the values. It is made up of cells that are identified by column letter and row number, such as A2 or B5. A cell can hold a label, a number, or a formula.

Creating a Table

Use a spreadsheet to solve a problem like this:

> How long will it take to double your money if you invest $1000 at 5%/a, compounded quarterly?

To create a spreadsheet, label cell A1 as Number of Quarters, Cell B1 as Time (years), and cell C1 as Amount ($). Enter the initial values of 0 in A2, 0 in B2, and 1000 in C2. Enter the formula = A2 + 1 in A3, the formula = A3/4 in B3, and the formula = 1000*(1.0.125)^A3 in C3 to generate the next values in the table.

	A	B	C
1	**Number of Quarters**	**Time (years)**	**Amount ($)**
2	0	0	1000
3	=A2+1	=A3/4	=1000*(1.0125^A3)
4			

Notice that an equal sign is in front of each formula, an asterisk (*) is used for multiplication, and a caret (^) is used for an exponent.

	A	B	C
1	**Number of Quarters**	**Time (years)**	**Amount ($)**
2	0	0	1000
3	1	0.25	
4			

Use the cursor to select cells A3 to C3 and several rows of cells below them. Then use the **Fill Down** command to insert the appropriate formula into the selected cells. The computer will automatically calculate and enter the values in the cells, as shown in the screen on the left.

Continue to select the cells in the last row of the table. Use the **Fill Down** command to generate more values until the solution appears, as shown below in the screen on the right.

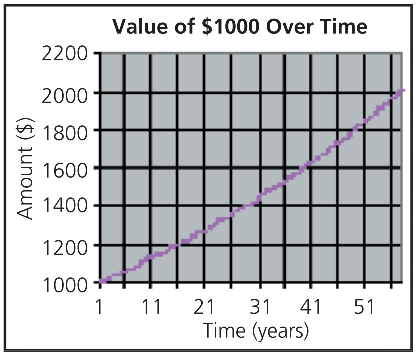
	A	B	C
1	**Number of Quarters**	**Time (years)**	**Amount ($)**
2	0	0	1000
3	1	0.25	1012.50
4	2	0.5	1025.16
5	3	0.75	1037.97
6	4	1	1050.94

	A	B	C
1	**Number of Quarters**	**Time (years)**	**Amount ($)**
2	0	0	1000
3	1	0.25	1012.50
4	2	0.5	1025.1563
⋮	⋮	⋮	⋮
56	54	13.5	1955.8328
57	55	13.75	1980.2807
58	56	14	2005.0342

Creating a Graph

Use the spreadsheet's graphing command to graph the results. Use the cursor to highlight the portion of the table you would like to graph. In this example, Time versus Amount is graphed.

	A	B	C
1	Number of Quarters	Time (years)	Amount ($)
2	0	0	1000
3	1	0.25	1012.50
4	2	0.5	1025.1563
⋮	⋮	⋮	⋮
56	54	13.5	1955.8328
57	55	13.75	1980.2807
58	56	14	2005.0342



Different spreadsheets have different graphing commands. Check the instructions for your spreadsheet to find the proper command.

Determining the Equation of the Curve of Best Fit

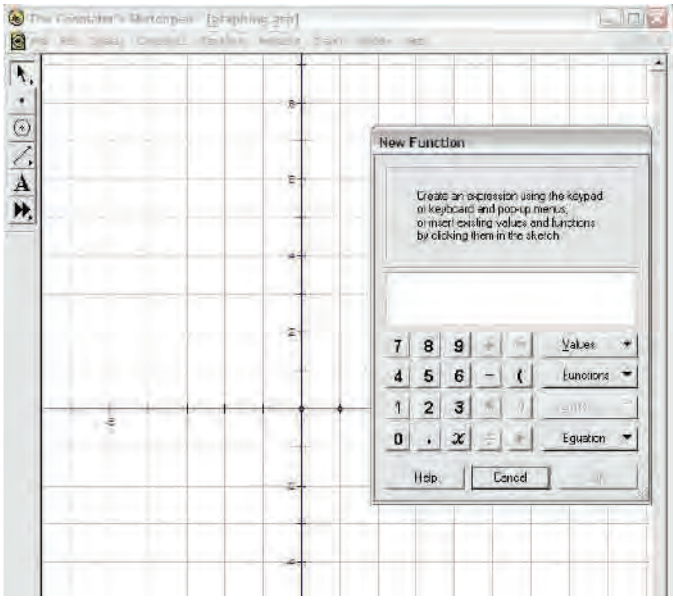
Different spreadsheets have different commands for finding the equation of the curve of best fit using regression. Check the instructions for your spreadsheet to find the proper command for the type of regression that suits the data.

PART 3 *USING* THE GEOMETER'S SKETCHPAD

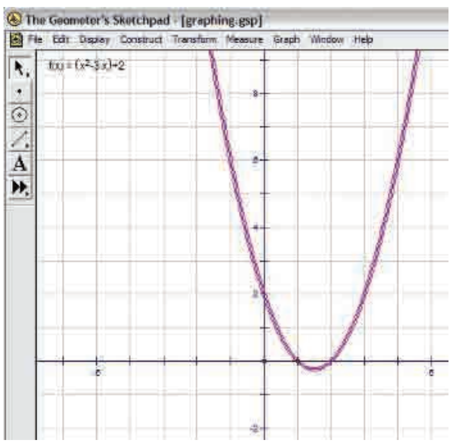
T–19 Graphing a Function

1. **Turn on the grid.**
   From the **Graph** menu, choose **Show Grid**.

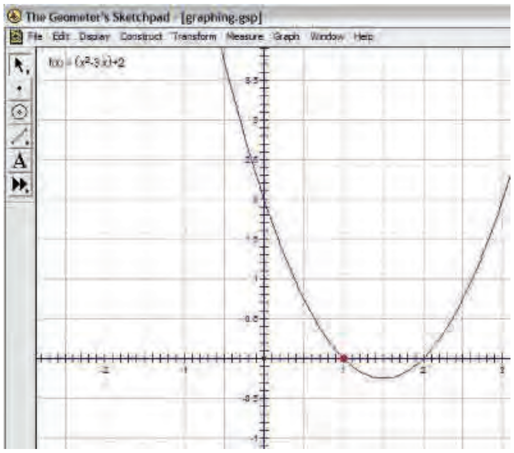
2. **Enter the function.**
   From the **Graph** menu, choose **Plot New Function**. The function calculator should appear.



3. **Graph the function.**
   To graph $y = x^2 - 3x + 2$, use either the calculator keypad or the keyboard to enter **x ^ 2 − 3 * x + 2**. Then press OK on the calculator keypad. The graph of $y = x^2 - 3x + 2$ should appear on the grid.



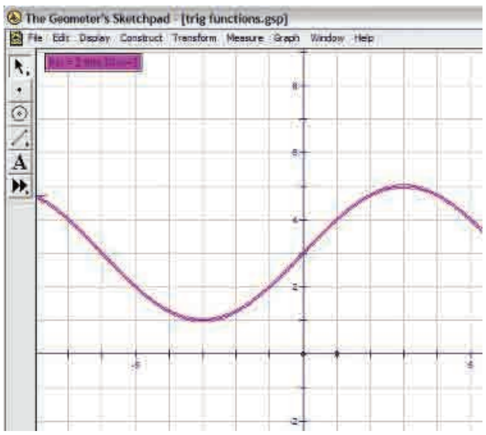
4. **Adjust the origin and/or scale.**
   To adjust the origin, left-click on the point at the origin to select it. Then left-click and drag the origin as desired.
   To adjust the scale, left-click in blank space to deselect the origin, and then left-click on the point at (1, 0) to select it. Left-click and drag this point to change the scale.



T–20 Graphing a Trigonometric Function

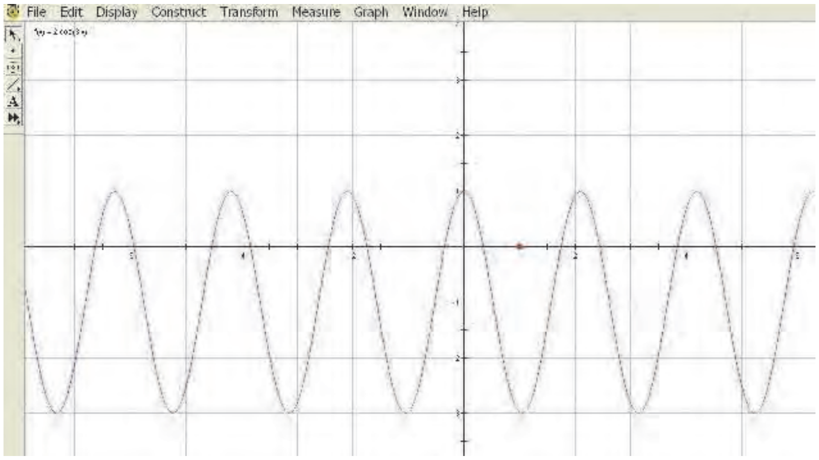
1. **Turn on the grid.**
   From the **Graph** menu, choose **Show Grid**.

2. **Graph the function $y = 2 \sin (30x) + 3$ using degrees.**
   From the **Graph** menu, choose **Plot New Function**. The function calculator should appear.
   Use either the calculator keypad or the keyboard to enter **2 * sin (30 * *x*) + 3**. To enter sin, use the pull-down **Functions** menu on the calculator keypad. Click OK on the calculator keypad.
   Click on **No** in the pop-up panel to keep degrees as the unit. The graph of $y = 2 \sin (30x) + 3$ should appear on the grid.



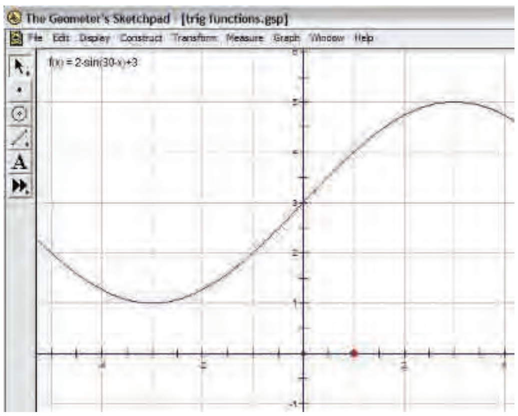
3. Graph the function $y = 2 \cos (3x) - 1$ using radians.
From the **Graph** menu, choose **Plot New Function**. The function calculator should appear.
Use either the calculator keypad or the keyboard to enter **2 * cos (3 * *x*) − 1**.
To enter cos, use the pull-down **Functions** menu on the calculator keypad.

Click OK on the calculator keypad.
Click on **Yes** in the pop-up panel to change the unit to radians. The graph of $y = 2 \cos (3x) - 1$ should appear on the grid.



Note: Selecting **Preferences** from the **Edit** menu will also allow you to change from radians to degrees or from degrees to radians.

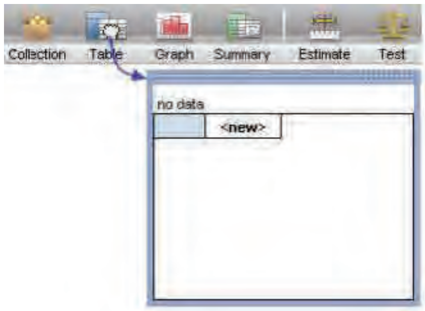
4. Adjust the origin and/or scale.
Left-click on and drag either the origin or the point $(1, 0)$.



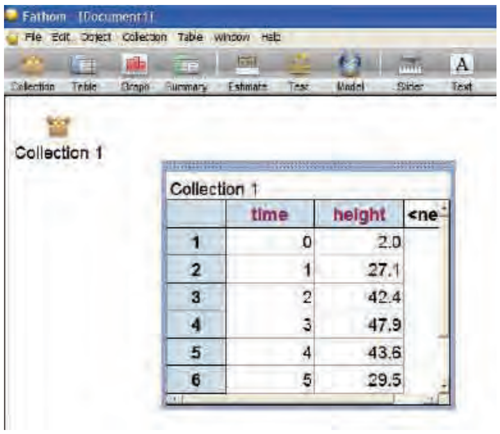
PART 4 USING FATHOM

T–21 Creating a Scatter Plot and Determining the Equation of a Line or Curve of Good Fit

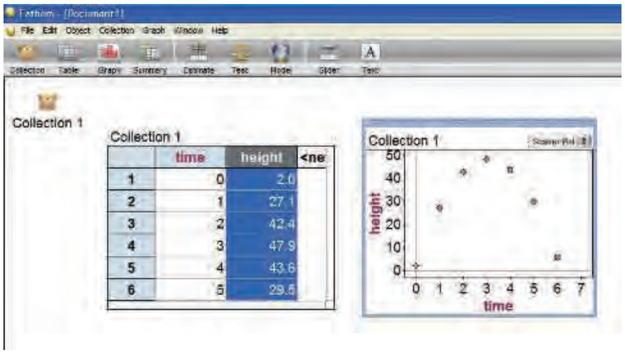
1. **Create a case table.**
   Drag a case table from the object shelf, and drop it in the document.



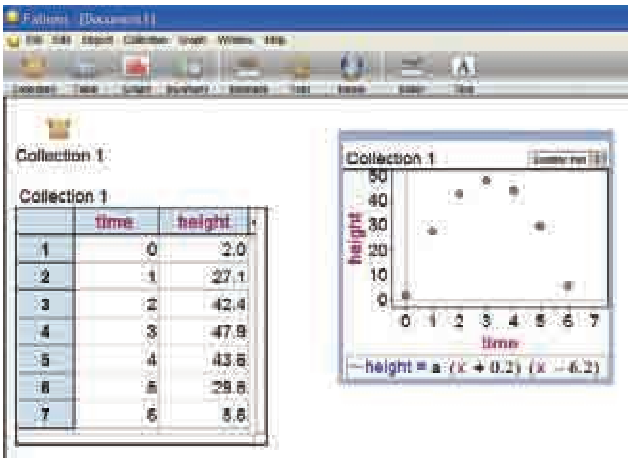
2. **Enter the Variables and Data.**
   Click on **<new>**, type a name for the new variable or attribute, and press ENTER . (If necessary, repeat this step to add more attributes. Pressing TAB instead of ENTER moves you to the next column.) When you name your first attribute, *Fathom* creates an empty collection to hold your data (a little, empty box). This is where your data are actually stored. Deleting the collection deletes your data. When you add cases by typing values, the collection icon fills with gold balls. To enter the data, click in the blank cell under the attribute name and begin typing values. (Press TAB to move from cell to cell.)



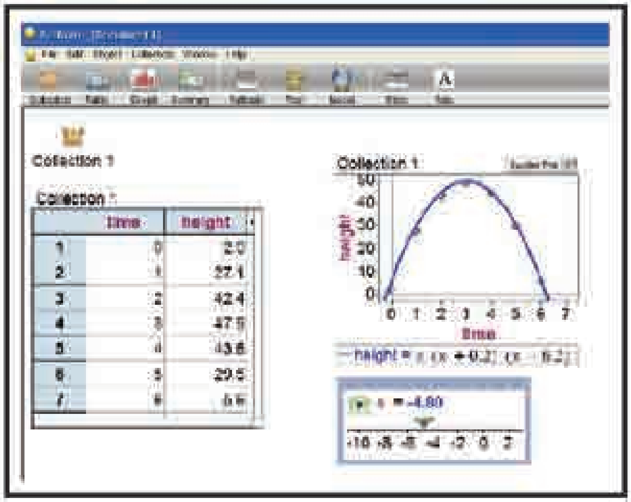
3. **Graph the data.**
Drag a new graph from the object shelf at the top of the *Fathom* window, and drop it in a blank space in your document. Drag an attribute from the case table, and drop it on the prompt below and/or to the left of the appropriate axis in the graph.



4. **Create a function.**
Right-click the graph, and select **Plot Function.** Enter your function using a parameter that can be adjusted to fit the curve to the scatter plot (**a** was used below).



5. Create a slider for the parameter.
Drag a new slider from the object shelf at the top of the *Fathom* window, and drop it in a blank space below your graph. Over **V1**, type the letter of the parameter used in step 4. Click on the number, and then adjust the value of the slider until you are satisfied with the fit.



The equation of a curve of good fit is $y = -4.8(x + 0.2)(x - 6.2)$.

Glossary

Instructional Words

C

calculate: Figure out the number that answers a question; compute

clarify: Make a statement easier to understand; provide an example

classify: Put things into groups according to a rule and label the groups; organize into categories

compare: Look at two or more objects or numbers and identify how they are the same and how they are different (e.g., Compare the numbers 6.5 and 5.6. Compare the size of the students' feet. Compare two shapes.)

conclude: Judge or decide after reflection or after considering data

construct: Make or build a model; draw an accurate geometric shape (e.g., Use a ruler and a protractor to construct an angle.)

create: Make your own example

D

describe: Tell, draw, or write about what something is or what something looks like; tell about a process in a step-by-step way

determine: Decide with certainty as a result of calculation, experiment, or exploration

draw: 1. Show something in picture form (e.g., Draw a diagram.)
2. Pull or select an object (e.g., Draw a card from the deck. Draw a tile from the bag.)

E

estimate: Use your knowledge to make a sensible decision about an amount; make a reasonable guess (e.g., Estimate how long it takes to cycle from your home to school. Estimate how many leaves are on a tree. What is your estimate of $3210 + 789$?)

evaluate: 1. Determine if something makes sense; judge
2. Calculate the value as a number

explain: Tell what you did; show your mathematical thinking at every stage; show how you know

explore: Investigate a problem by questioning, brainstorming, and trying new ideas

extend: 1. In patterning, continue the pattern
2. In problem solving, create a new problem that takes the idea of the original problem further

J

justify: Give convincing reasons for a prediction, an estimate, or a solution; tell why you think your answer is correct

M

measure: Use a tool to describe an object or determine an amount (e.g., Use a ruler to measure the height or distance around something. Use a protractor to measure an angle. Use balance scales to measure mass. Use a measuring cup to measure capacity. Use a stopwatch to measure the time in seconds or minutes.)

model: Show, represent, or demonstrate an idea or situation using a diagram, graph, table of values, equation, formula, physical model, or computer model

P

predict: Use what you know to work out what is going to happen (e.g., Predict the next number in the pattern 1, 2, 4, 7, ...)

R

reason: Develop ideas and relate them to the purpose of the task and to each other; analyze relevant information to show understanding

relate: Describe how two or more objects, drawings, ideas, or numbers are similar

represent: Show information or an idea in a different way that makes it easier to understand (e.g., Draw a graph. Make a model.)

S

show (your work): Record all calculations, drawings, numbers, words, or symbols that make up the solution

sketch: Make a rough drawing (e.g., Sketch a picture of the field with dimensions.)

solve: Develop and carry out a process for finding a solution to a problem

sort: Separate a set of objects, drawings, ideas, or numbers according to an attribute (e.g., Sort 2-D shapes by the number of sides.)

V

validate: Check an idea by showing that it works

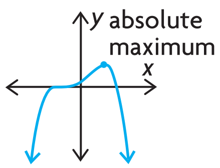
verify: Work out an answer or solution again, usually in another way; show evidence of

visualize: Form a picture in your head of what something is like; imagine

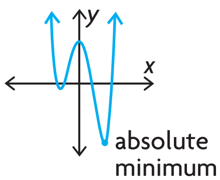
Mathematical Words

A

absolute maximum: The greatest value of a function for all values in its domain



absolute minimum: The least value of a function for all values in its domain

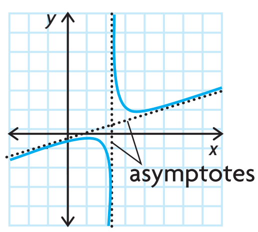


absolute value: Written as $|x|$; describes the distance of x from 0; equals x when $x \geq 0$ and equals $-x$ when $x < 0$; for example, $|3| = 3$ and $|-3| = -(-3) = 3$

amplitude: Half the difference between the maximum and minimum values of a sinusoidal function; also the vertical distance from the axis of a sinusoidal function to the maximum or minimum value

argument: The expression on which a function operates; for example, in $\sin(x + \pi)$, sin is the function and $x + \pi$ is the argument

asymptote: A line that the graph of a relation or function gets closer and closer to, but never meets, on some part of its domain



average rate of change: In a relation, the change in the quantity represented by the dependent variable (Δy) divided by the corresponding change in the quantity represented by the independent variable (Δx); for a function $y = f(x)$, the average rate of change in the interval $x_1 \leq x \leq x_2$ is $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

C

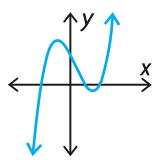
centred interval: An interval of the independent variable of the form $a - h \leq x \leq a + h$, where h is a small positive value; used to determine an average rate of change

composite function: A function that is the composite of two other functions; the function $f(g(x))$, denoted by $(f \circ g)(x)$, is called the composition of f with g and is defined using the output of the function g as the input for the function f

compound angle: An angle that is created by adding or subtracting two or more angles

conjecture: A guess or prediction based on limited evidence

continuous function: A function that does not contain any holes or breaks over its entire domain



counterexample: An example that shows a general statement to be false

cubic function: A polynomial function whose degree is three; for example, $y = 5x^3 + 6x^2 - 4x + 7$

curve of best fit: The curve that best describes the distribution of points in a scatter plot; typically found using regression analysis

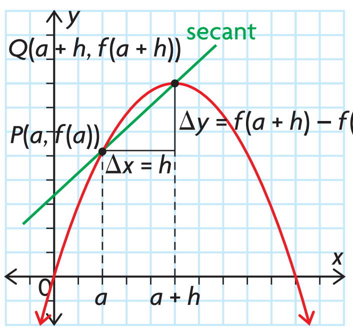
D

damped motion: Motion where a restriction is placed on an oscillating system that results in a decrease in amplitude over time

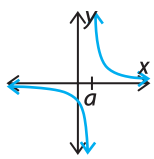
decreasing function: A function $f(x)$ whose y values get continually smaller as x gets continually larger

degree: The size of an angle that is subtended at the centre of a circle by an arc with a length equal to $\frac{1}{360}$ of the circumference of the circle

difference quotient: If $P(a, f(a))$ and $Q(a + h, f(a + h))$ are two points on the graph of $y = f(x)$, then the instantaneous rate of change of y with respect to x at P can be estimated using the average rate of change $\frac{\Delta y}{\Delta x} = \frac{f(a + h) - f(a)}{h}$, where h is a very small number; the expression $\frac{f(a + h) - f(a)}{h}$ is the difference quotient



discontinuity: A value for x, on an x-y graph, for which a value for y is not defined. In the graph below the y-value is not defined when $x = a$



displacement: A translation from one position to another, without consideration of any intervening positions; the minimal distance between two points

domain: The set of all values of the independent variable of a relation

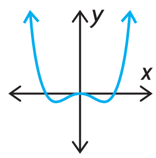
E

end behaviour: A description of the values of $f(x)$ as $x \to \infty$ or $x \to -\infty$

equation of the axis: The equation of the horizontal line that is halfway between the maximum value and minimum value of a sinusoidal function; determined using

$$y = \frac{\text{maximum value} + \text{minimum value}}{2}$$

even function: A function that is symmetric about the y-axis; algebraically, all even functions have the property $f(-x) = f(x)$



exponential function: A function of the form $y = a(b^x)$

extrapolation: The process of using a graphical or algebraic model to predict the value of a function beyond the known values

F

factor theorem: A theorem stating that $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$

family of polynomial functions: A set of polynomial functions whose equations have the same degree and whose graphs have common characteristics; for example, one quadratic family may have the same zeros and another quadratic family may have the same x-intercepts

finite difference: The difference between two consecutive values in a table that has a constant difference between the values of the independent variable; first differences are the differences between the values of the dependent variable, second differences are the differences between the first differences, and so on

following interval: An interval of the independent variable of the form $a \leq x \leq a + h$, where h is a small positive value; used to determine an average rate of change

function: A relation in which each value of the independent variable corresponds to only one value of the dependent variable

function notation: Notation, such as $f(x)$, that is used to represent the value of the dependent variable, y (the output) for a given value of the independent variable, x (the input)

H

half-life: The time that is required for a quantity to decay to half of its initial value

horizontal asymptote: An asymptote that takes the form of a horizontal line

I

identity: A mathematical statement that is true for all values of the given variables; any restrictions on the variables must be stated; for example, if an identity involves fractions, the denominator cannot be zero

increasing function: A function $f(x)$ whose y values get continually larger as x gets continually larger

independent variable: In an algebraic relation, a variable whose values may be freely chosen and upon which the values of the other variables depend; often represented by x

instantaneous rate of change: The exact rate of change of a function $y = f(x)$ at a specific value of the independent variable, $x = a$; estimated using average rates of change for small intervals of the independent variable that are very close to the value $x = a$

interpolation: The process of using a graphical or algebraic model to predict the value of a function between known values

intersection: A set that contains the elements that are common to both sets; the symbol for intersection is $\cap$

interval of decrease: The interval(s) within the domain of a function where the y values of the function get smaller, moving from left to right

interval of increase: The interval(s) within the domain of a function where the y values of the function get larger, moving from left to right

inverse of a function: The reverse of the original function; undoes what the original function has done

L

leading coefficient: The coefficient of the term with the highest degree in a polynomial

linear inequality: An inequality that contains an algebraic expression whose degree is one; for example, $5x + 3 > 6x - 2$

linear relation: A relation between two variables that appears as a straight line when graphed on a coordinate system; can be represented by an equation whose degree is one; also called a *linear function*

logarithm: The exponent required on base a to give the value x; written as $\log_a x$, where $a > 0$ and $a \neq 1$

logarithmic function: The inverse of the exponential function $y = a^x$ is the function with exponential equation $x = a^y$. We write y as a function of x using the logarithmic form of this equation, $y = \log_a x$. As with the exponential function, $a > 0$ and $a \neq 1$

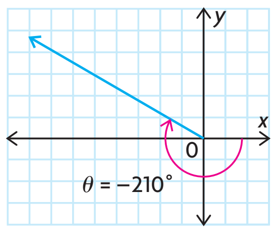
lowest common denominator: The smallest multiple that is shared by two or more denominators

M

magnitude: The absolute value of a quantity

N

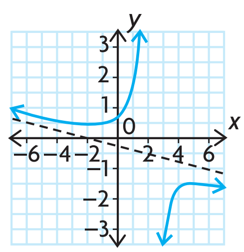
negative angle: An angle that is measured *clockwise* from the positive x-axis



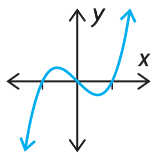
nonlinear relation: A relation whose graph is not a straight line

O

oblique asymptote: An asymptote that is neither vertical nor horizontal, but slanted



odd function: A function that has rotational symmetry about the origin; algebraically, all odd functions have the property $f(-x) = -f(x)$

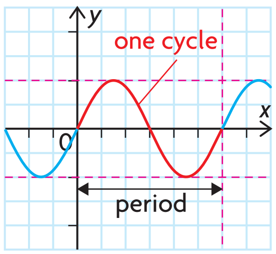


order: The exponent to which each factor in an algebraic expression is raised; for example, in $f(x) = (x - 3)^2(x - 1)$, the order of $(x - 3)$ is 2 and the order of $(x - 1)$ is 1

P

parent function: The simplest, or base, function in a family; for example, $y = x^2$ is the parent function for all quadratic functions

period: The change in the independent variable (typically x) that corresponds to one cycle of a sinusoidal function; the cycle of a periodic function is the part of the graph that repeats



piecewise function: A function that is defined using two or more rules on two or more intervals; as a result, the graph consists of two or more pieces of similar or different functions

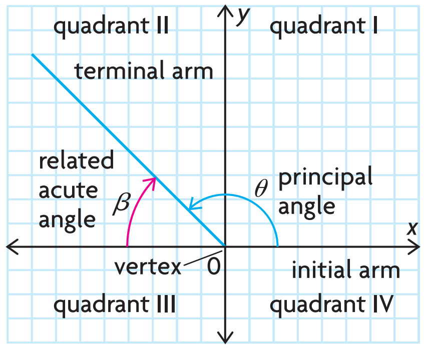
polynomial equation: An equation in which one polynomial expression is set equal to another polynomial expression; for example, $x^3 - 5x^2 = 4x - 3$ or $5x^4 - 3x^3 + x^2 - 6x = 9$

polynomial function: A function of the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0$, where $a_0, a_1, a_2, \dots, a_{n-1}$, and a_n are real numbers and n is a whole number; the equation of a polynomial function is defined by a polynomial expression, as in $f(x) = 5x^3 + 6x^2 - 3x + 7$

polynomial inequality: An inequality that contains a polynomial expression; for example, $5x^3 + 3x^2 - 6x \leq 2$

preceding interval: An interval of the independent variable of the form $a - h \leq x \leq a$, where h is a small positive value; used to determine an average rate of change

principal angle: The counterclockwise angle between the initial arm and terminal arm of an angle in standard position; its value is between 0° and 360° (0 and 2π)



Q

quadratic function: A function that can be represented by a quadratic equation whose degree is two; for example, $y = x^2 + 3x - 2$

quartic function: a polynomial function whose degree is four; for example, $y = 8x^4 - 5x^3 + 6x^2 - 4x + 7$

quintic function: a polynomial function whose degree is five; for example, $y = -2x^5 + 8x^4 - 5x^3 + 6x^2 - 4x + 7$

R

radian: The size of an angle that is subtended at the centre of a circle by an arc with a length equal to the radius of the circle; both the arc length and the radius are measured in units of length (such as centimetres) and, as a result, the angle is a real number without any units

range: The set of all values of the dependent variable of a relation

rational expression: A quotient of polynomials; for example, $\frac{2x - 1}{3x}$, $x \neq 0$

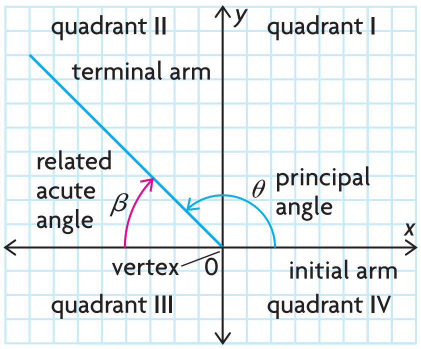
rational function: A function that can be expressed as $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions, $q(x) \neq 0$; for example, $f(x) = \frac{3x^2 - 1}{x + 1}$, $x \neq -1$, and $f(x) = \frac{1 - x}{x^2}$, $x \neq 0$, are rational functions, but $f(x) = \frac{1 + x}{\sqrt{2 - x}}$, $x \neq 2$, is not because its denominator is not a polynomial

rational inequality: A statement that one rational expression is less than or greater than (or as well as equal to in some cases) another rational expression; for example, $\frac{2x}{x+3} > \frac{x-1}{5x}$

rational number: a number that can be expressed exactly as the ratio of two integers; $\left\{\frac{a}{b} \middle| a, b \in I, b \neq 0\right\}$

real numbers: Numbers that are either rational or irrational; include positive and negative integers, zero, fractions, and irrational numbers such as $\sqrt{2}$ and π

related acute angle: The acute angle between the terminal arm of an angle in standard position and the x-axis, when the terminal arm lies in quadrant II, III, or IV.



relation: A set of ordered pairs; values of the independent variable are paired with values of the dependent variable

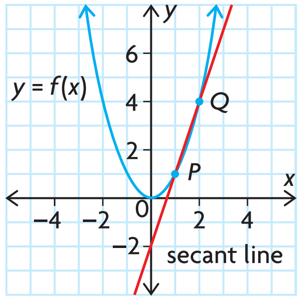
remainder theorem: A theorem stating that when a polynomial $f(x)$ is divided by $x - a$, the remainder is equal to $f(a)$; if the remainder is zero, then $x - a$ is a factor of the polynomial; the remainder theorem can be used to factor polynomials

restrictions: The values of the variable(s) in a function or expression that cause the function or expression to be undefined; the zeros of the denominator, or the numbers that are not in the domain of the function

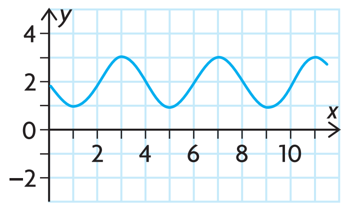
S

scatter plot: A graph that attempts to show a relationship between two variables using points plotted on a coordinate grid

secant line: A line that passes through two points on the graph of a relation

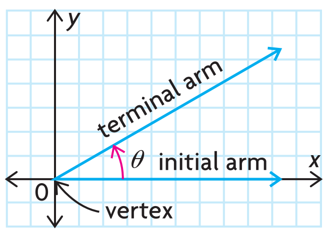


sinusoidal function: A periodic function whose graph looks like smooth symmetrical waves, if any part of the wave can be horizontally translated onto another part of the wave; a graph of a sinusoidal function can be created by transforming the graph of $y = \sin x$ or $y = \cos x$



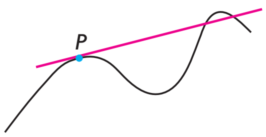
special triangle: A right triangle whose angles measure 45°, 45°, and 90° $\left(\frac{\pi}{4}, \frac{\pi}{4}, \text{ and } \frac{\pi}{2}\right)$ or 30°, 60°, and 90° $\left(\frac{\pi}{6}, \frac{\pi}{3}, \text{ and } \frac{\pi}{2}\right)$; used to determine the exact values of trigonometric ratios that include these as principal or related angles

standard position: An angle in the Cartesian plane whose vertex lies at the origin and whose initial arm (the arm that is fixed) lies on the positive x-axis; angle θ is measured from the initial arm to the terminal arm (the arm that rotates)



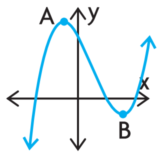
T

tangent line: A line that touches a graph at only one point, P, within a small interval of the relation; the tangent line could, but does not have to, cross the graph at another point outside this interval; it goes in the same direction as the relation at point P (called the point of tangency)



transformation: A geometric operation, such as a translation, a rotation, a dilation, or a reflection

turning point: A point on a curve where the function changes from increasing to decreasing, or vice versa; for example, A and B are turning points on the following curve



V

vertical asymptote: An asymptote that takes the form of a vertical line

vertical line test: A test that can be used to determine whether a relation is a function; if any vertical line intersects the graph of a relation more than once, then the relation is not a function

Answers

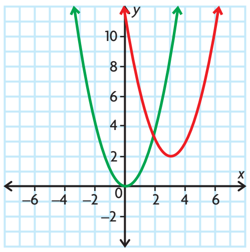
Chapter 1

Getting Started, p. 2

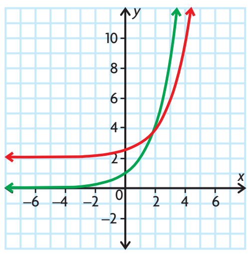
1. **a)** 6 **c)** $-\frac{51}{16}$
b) -6 **d)** $a^2 + 5a$

2. **a)** $(x + y)(x + y)$
b) $(5x - 1)(x - 3)$
c) $(x + y + 8)(x + y - 8)$
d) $(a + b)(x - y)$

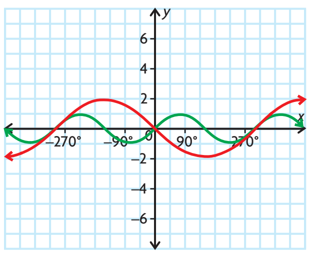
3. **a)** horizontal translation 3 units to the right, vertical translation 2 units up;



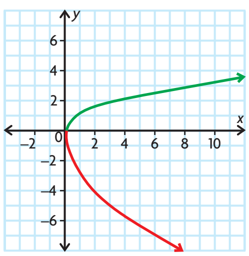
b) horizontal translation 1 unit to the right, vertical translation 2 units up;



c) horizontal stretch by a factor of 2, vertical stretch by a factor of 2, reflection across the x-axis;



d) horizontal compression by a factor of $\frac{1}{2}$, vertical stretch by a factor of 2, reflection across the x-axis;



4. **a)** $D = \{x \in \mathbf{R} \mid -2 \le x \le 2\}$, $R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$
b) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \ge -19\}$
c) $D = \{x \in \mathbf{R} \mid x \ne 0\}$, $R = \{y \in \mathbf{R} \mid y \ne 0\}$
d) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid -3 \le y \ge 3\}$
e) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y > 0\}$

5. **a)** This is not a function; it does not pass the vertical line test.
b) This is a function; for each x-value, there is exactly one corresponding y-value.
c) This is not a function; for each x-value greater than 0, there are two corresponding y-values.
d) This is a function; for each x-value, there is exactly one corresponding y-value.
e) This is a function; for each x-value, there is exactly one corresponding y-value.

6. **a)** 8
b) about 2.71

7. If a relation is represented by a set of ordered pairs, a table, or an arrow diagram, one can determine if the relation is a function by checking that each value of the independent variable is paired with no more than one value of the dependent variable. If a relation is represented using a graph or scatter plot, the vertical line test can be used to determine if the relation is a function. A relation may also be represented by a description/rule or by using function notation or an equation. In these cases, one can use reasoning to determine if there is more than one value of the dependent variable paired with any value of the independent variable.

Lesson 1.1, pp. 11–13

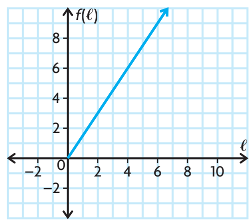
1. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid -4 \le y \le -2\}$; This is a function because it passes the vertical line test.
b) $D = \{x \in \mathbf{R} \mid -1 \le x \le 7\}$; $R = \{y \in \mathbf{R} \mid -3 \le y \le 1\}$; This is a function because it passes the vertical line test.
c) $D = \{1, 2, 3, 4\}$; $R = \{-5, 4, 7, 9, 11\}$; This is not a function because 1 is sent to more than one element in the range.
d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; This is a function because every element in the domain produces exactly one element in the range.
e) $D = \{-4, -3, 1, 2\}$; $R = \{0, 1, 2, 3\}$; This is a function because every element of the domain is sent to exactly one element in the range.
f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le 0\}$; This is a function because every element in the domain produces exactly one element in the range.

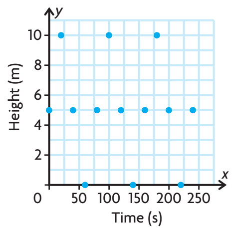
2. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le -3\}$; This is a function because every element in the domain produces exactly one element in the range.
b) $D = \{x \in \mathbf{R} \mid x \ne -3\}$; $R = \{y \in \mathbf{R} \mid y \ne 0\}$; This is a function because every element in the domain produces exactly one element in the range.
c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 0\}$; This is a function because every element in the domain produces exactly one element in the range.
d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.
e) $D = \{x \in \mathbf{R} \mid -3 \le x \le 3\}$; $R = \{y \in \mathbf{R} \mid -3 \le y \le 3\}$; This is not a function because (0, 3) and (0, 3) are both in the relation.
f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid -2 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.

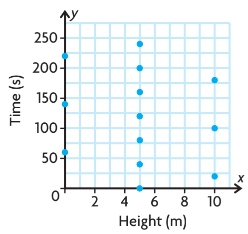
3. **a)** function; $D = \{1, 3, 5, 7\}$; $R = \{2, 4, 6\}$
b) function; $D = \{0, 1, 2, 5\}$; $R = \{-1, 3, 6\}$
c) function; $D = \{0, 1, 2, 3\}$; $R = \{2, 4\}$
d) not a function; $D = \{2, 6, 8\}$; $R = \{1, 3, 5, 7\}$
e) not a function; $D = \{1, 10, 100\}$; $R = \{0, 1, 2, 3\}$
f) function; $D = \{1, 2, 3, 4\}$; $R = \{1, 2, 3, 4\}$

4. **a)** function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \ge 2\}$.
b) not a function; $D = \{x \in \mathbf{R} \mid x \ge 2\}$; $R = \{y \in \mathbf{R}\}$
c) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \ge -0.5\}$
d) not a function; $D = \{x \in \mathbf{R} \mid x \ge 0\}$; $R = \{y \in \mathbf{R}\}$
e) function; $D = \{x \in \mathbf{R} \mid x \ne 0\}$; $R = \{y \in \mathbf{R} \mid y \ne 0\}$
f) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

5. **a)** $y = x + 3$ **c)** $y = 3(x - 2)$
b) $y = 2x - 5$ **d)** $y = -x + 5$

6. a) The length is twice the width.
b) $f(l) = \frac{3}{2}l$
c)

d) length = 8 m; width = 4 m

7. a)


b) D = {0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240}
c) R = {0, 5, 10}
d) It is a function because it passes the vertical line test.
e)


f) It is not a function because (5, 0) and (5, 40) are both in the relation.

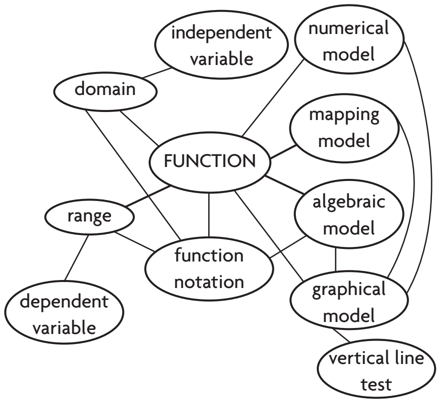
8. a) $\{(1, 2), (3, 4), (5, 6)\}$
b) $\{(1, 2), (3, 2), (5, 6)\}$
c) $\{(2, 1), (2, 3), (5, 6)\}$

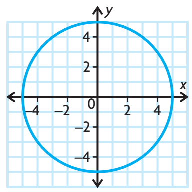
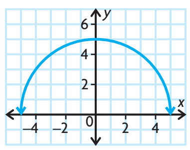
9. If a vertical line passes through a function and hits two points, those two points have identical *x*-coordinates and different *y*-coordinates. This means that one *x*-coordinate is sent to two different elements in the range, violating the definition of *function*.

10. a) Yes, because the distance from (4, 3) to (0, 0) is 5.
b) No, because the distance from (1, 5) to (0, 0) is not 5.
c) No, because (4, 3) and (4, −3) are both in the relation.

11. a) $g(x) = x^2 + 3$
b) $g(3) - g(2) = 12 - 7$
$= 5$
$g(3 - 2) = g(1)$
$= 4$
So, $g(3) - g(2) \neq g(3 - 2)$.

12. a) $f(6) = 12; f(7) = 8; f(8) = 15$
b) Yes, $f(15) = f(3) \times f(5)$
c) Yes, $f(12) = f(3) \times f(4)$
d) Yes, there are others that will work. $f(a) \times f(b) = f(a \times b)$ whenever a and b have no common factors other than 1.

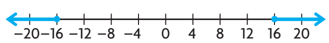
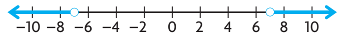
13. Answers may vary. For example,


14.


The first is not a function because it fails the vertical line test:
$D = \{x \in \mathbf{R} \mid -5 \leq x \leq 5\}$;
$R = \{y \in \mathbf{R} \mid -5 \leq y \leq 5\}$.
The second is a function because it passes the vertical line test:
$D = \{x \in \mathbf{R} \mid -5 \leq x \leq 5\}$;
$R = \{y \in \mathbf{R} \mid 0 \leq y \leq 5\}$.

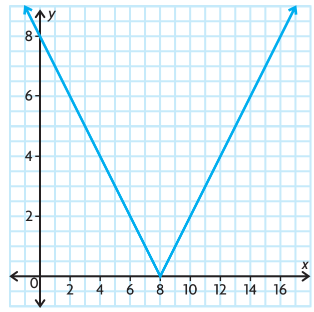
15. x is a function of y if the graph passes the horizontal line test. This occurs when any horizontal line hits the graph at most once.

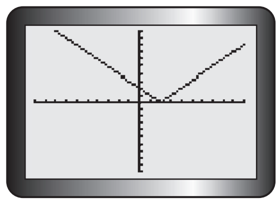
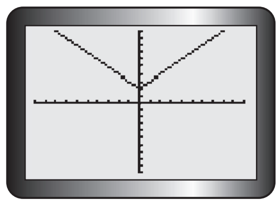
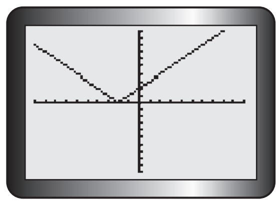
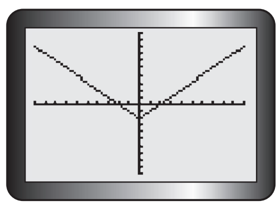
Lesson 1.2, p. 16

1. $|-5|, |12|, |-15|, |20|, |-25|$
2. a) 22 c) 18 e) −2
b) −35 d) 11 f) −2
3. a) $|x| > 3$ c) $|x| \geq 1$
b) $|x| \leq 8$ d) $|x| \neq 5$
4. a)

b)

c) The absolute value of a number is always greater than or equal to 0. There are no solutions to this inequality.
d)


5. a) $|x| \leq 3$ c) $|x| \geq 2$
b) $|x| > 2$ d) $|x| < 4$

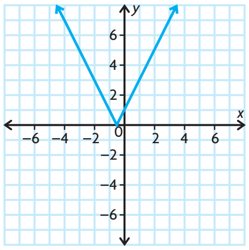
6.

a) The graphs are the same.
b) Answers may vary. For example, $x - 8 = -(-x + 8)$, so they are negatives of each other and have the same absolute value.

7. a)

b)

c)

d)


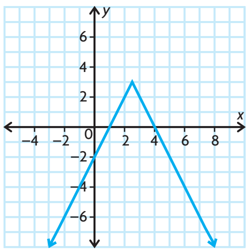
8. When the number you are adding or subtracting is inside the absolute value signs, it moves the function to the left (when adding) or to the right (when subtracting) of the origin. When the number you are

adding or subtracting is outside the absolute value signs, it moves the function down (when subtracting) or up (when adding) from the origin.
The graph of the function will be the absolute value function moved to the left 3 units and up 4 units from the origin.

9. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$ and translated $\frac{1}{2}$ unit to the left.



10. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$, reflected over the x-axis, translated $2\frac{1}{2}$ units to the right, and translated 3 units up.



Lesson 1.3, pp. 23–25

1. Answers may vary. For example, domain because most of the parent functions have all real numbers as a domain.

2. Answers may vary. For example, the end behaviour because the only two that match are x^2 and $|x|$.

3. Given the horizontal asymptote, the function must be derived from 2^x. But the asymptote is at $y = 2$, so it must have been translated up two. Therefore, the function is $f(x) = 2^x + 2$.

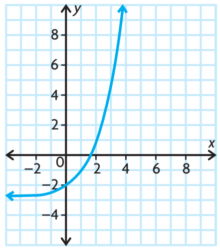
4. **a)** Both functions are odd, but their domains are different.
b) Both functions have a domain of all real numbers, but sin (x) has more zeros.
c) Both functions have a domain of all real numbers, but different end behaviour.
d) Both functions have a domain of all real numbers, but different end behaviour.

5. **a)** even **d)** odd
b) odd **e)** neither even nor odd
c) odd **f)** neither even nor odd

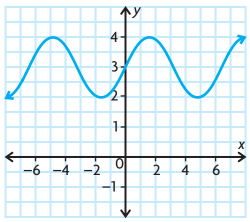
6. **a)** $|x|$, because it is a measure of distance from a number
b) sin (x), because the heights are periodic
c) 2^x, because population tends to increase exponentially
d) x, because there is \$1 on the first day, \$2 on the second, \$3 on the third, etc.

7. **a)** $f(x) = \sqrt{x}$ **c)** $f(x) = x^2$
b) $f(x) = \sin x$ **d)** $f(x) = x$

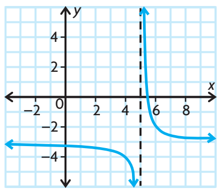
8. **a)** $f(x) = 2^x - 3$



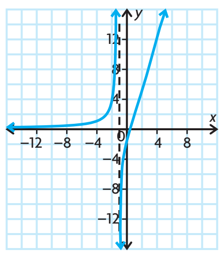
b) $g(x) = \sin x + 3$



c) $h(x) = \dfrac{1}{x-5} - 3 = \dfrac{16 - 3x}{x - 5}$



9.



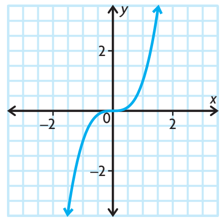
10. **a)** $f(x) = (x - 2)^2$
b) There is not only one function. $f(x) = \frac{3}{4}(x - 2)^2 + 1$ works as well.
c) There is more than one function that satisfies the property. $f(x) = |x - 2| + 2$ and $f(x) = 2|x - 2|$ both work.

11. x^2 is a smooth curve, while $|x|$ has a sharp, pointed corner at (0, 0).

12. See next page.

13. It is important to name parent functions in order to classify a wide range of functions according to similar behaviour and characteristics.

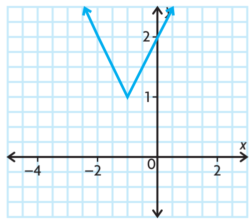
14.



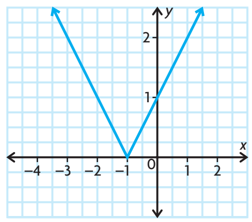
D $= \{x \in \mathbf{R}\}$, R $= \{f(x) \in \mathbf{R}\}$; interval of increase $= (-\infty, \infty)$, no interval of decrease, no discontinuities, x- and y-intercept at (0, 0), odd, $x \to \infty$, $y \to \infty$, and $x \to -\infty$, $y \to -\infty$. It is very similar to $f(x) = x$. It does not, however, have a constant slope.

15. No, cos x is a horizontal translation of sin x.

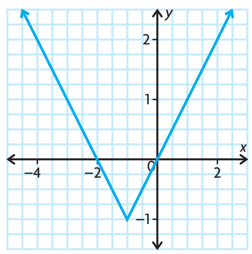
16. The graph can have 0, 1, or 2 zeros.
0 zeros:



1 zero:



2 zeros:

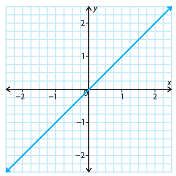
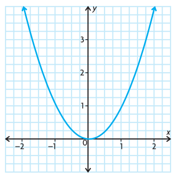
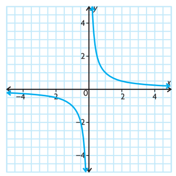
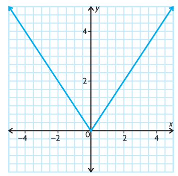
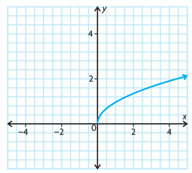
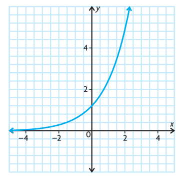
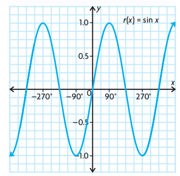


Mid-Chapter Review, p. 28

1. **a)** function; D $= \{0, 3, 15, 27\}$, R $= \{2, 3, 4\}$
b) function; D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R}\}$
c) not a function; D $= \{x \in \mathbf{R} \mid -5 \le x \le 5\}$, R $= \{y \in \mathbf{R} \mid -5 \le y \le 5\}$
d) not a function; D $= \{1, 2, 10\}$, R $= \{-1, 3, 6, 7\}$

2. **a)** Yes. Every element in the domain gets sent to exactly one element in the range.
b) D $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
c) R $= \{10, 20, 25, 30, 35, 40, 45, 50\}$

12.

Parent Function	$f(x) = x$	$g(x) = x^2$	$h(x) = \frac{1}{x}$	$k(x) = \lvert x \rvert$	$m(x) = \sqrt{x}$	$p(x) = 2^x$	$r(x) = \sin x$
Sketch							
Domain	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \neq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \geq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$
Range	$\{f(x) \in \mathbf{R}\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \neq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) > 0\}$	$\{f(x) \in \mathbf{R} \mid -1 \leq f(x) \leq 1\}$
Intervals of Increase	$(-\infty, \infty)$	$(0, \infty)$	None	$(0, \infty)$	$(0, \infty)$	$(-\infty, \infty)$	$[90(4k + 1), 90(4k + 3)]$ $K \in \mathbf{Z}$
Intervals of Decrease	None	$(-\infty, 0)$	$(-\infty, 0)$ $(0, \infty)$	$(-\infty, 0)$	None	None	$[90(4k + 3), 90(4k + 1)]$ $K \in \mathbf{Z}$
Location of Discontinuities and Asymptotes	None	None	$y = 0$ $x = 0$	None	None	$y = 0$	None
Zeros	(0, 0)	(0, 0)	None	(0, 0)	(0, 0)	None	$180k$ $K \in \mathbf{Z}$
y-Intercepts	(0, 0)	(0, 0)	None	(0, 0)	(0, 0)	(0, 1)	(0, 0)
Symmetry	Odd	Even	Odd	Even	Neither	Neither	Odd
End Behaviours	$x \to \infty, y \to \infty$ $x \to -\infty, y \to -\infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to 0$ $x \to -\infty, y \to 0$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to \infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to 0$	Oscillating

3. **a)** D $= \{x \in \mathbf{R}\}$, R $= \{f(x) \in \mathbf{R}\}$; function

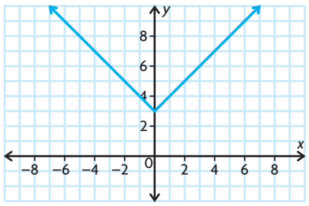
b) D $= \{x \in \mathbf{R} \mid -3 \le x \le 3\}$, R $= \{y \in \mathbf{R} \mid -3 \le y \le 3\}$; not a function

c) D $= \{x \in \mathbf{R} \mid x \le 5\}$, R $= \{y \in \mathbf{R} \mid y \ge 0\}$; function

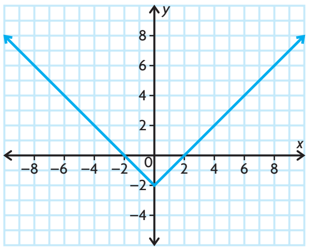
d) D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} \mid y \ge -2\}$; function

4. $-|3|, |0|, |-3|, |-4|, |5|$

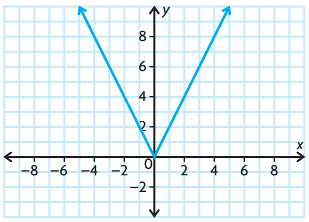
5. **a)**



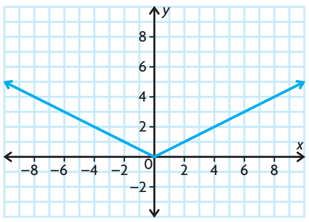
b)



c)



d)



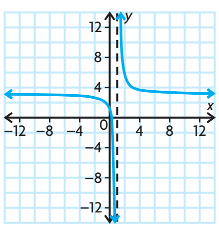
6. **a)** $f(x) = 2^x$

b) $f(x) = \frac{1}{x}$

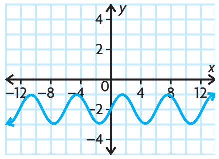
c) $f(x) = \sqrt{x}$

7. **a)** even **b)** even **c)** neither odd nor even **d)** neither odd nor even

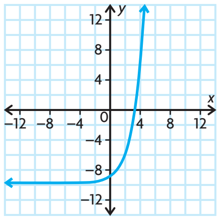
8. **a)** This is $f(x) = \frac{1}{x}$ translated right 1 and up 3; discontinuous



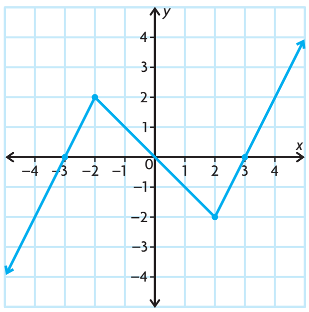
b) This if $f(x) = \sin x$ translated down 2; continuous



c) This is $f(x) = 2^x$ translated down 10; continuous



9.



Lesson 1.4, pp. 35–37

1. **a)** translation 1 unit down

b) horizontal compression by a factor of $\frac{1}{2}$, translation 1 unit right

c) reflection over the x-axis, translation 2 units up, translation 3 units right

d) reflection over the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of $\frac{1}{4}$

e) reflection over the x-axis, translation 3 units down, reflection over the y-axis, translation 2 units left

f) vertical compression by a factor of $\frac{1}{2}$, translation 6 units up, horizontal stretch by a factor of 4, translation 5 units right

2. **a)** $a = -1$, $k = \frac{1}{2}$, $d = 0$, $c = 3$

b) $a = 3$, $k = \frac{1}{2}$, $d = 0$, $c = -2$

3. $(2, 3), (1, 3), (1, 6), (1, -6), (-4, -6), (-4, -10)$

4. **a)** $(2, 6), (4, 14), (-2, 10), (-4, 12)$

b) $(5, 3), (7, 7), (1, 5), (-1, 6)$

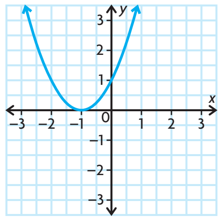
c) $(2, 5), (4, 9), (-2, 7), (-4, 8)$

d) $(1, 0), (3, 4), (-3, 2), (-5, 3)$

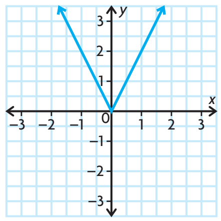
e) $(2, 5), (4, 6), (-2, 3), (-4, 7)$

f) $(1, 2), (2, 6), (-1, 4), (-2, 5)$

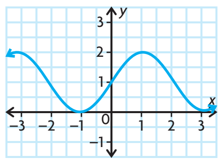
5. **a)** $f(x) = x^2$, translated left 1



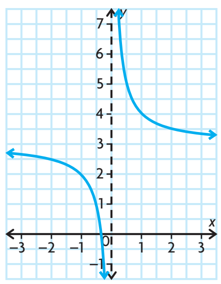
b) $f(x) = |x|$, vertical stretch by 2



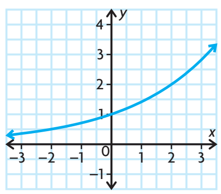
c) $f(x) = \sin x$, horizontal compression of $\frac{1}{3}$, translation up 1



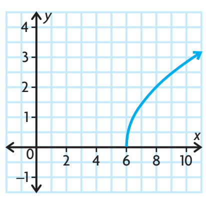
d) $f(x) = \frac{1}{x}$, translation up 3



e) $f(x) = 2^x$, horizontal stretch by 2

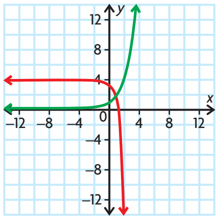


f) $f(x) = \sqrt{x}$, horizontal compression by $\frac{1}{2}$, translation right 6



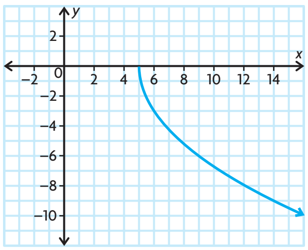
6. **a)** D $= \{x \in \mathbf{R}\}$,
R $= \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$
b) D $= \{x \in \mathbf{R}\}$,
R $= \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$
c) D $= \{x \in \mathbf{R}\}$,
R $= \{f(x) \in \mathbf{R} \mid 0 \leq f(x) \leq 2\}$
d) D $= \{x \in \mathbf{R} \mid x \neq 0\}$,
R $= \{f(x) \in \mathbf{R} \mid f(x) \neq 3\}$
e) D $= \{x \in \mathbf{R}\}$,
R $= \{f(x) \in \mathbf{R} \mid f(x) > 0\}$
f) D $= \{x \in \mathbf{R} \mid x \geq 6\}$,
R $= \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$

7. **a)**



b) The domain remains unchanged at D $= \{x \in \mathbf{R}\}$. The range must now be less than 4: R $= \{f(x) \in \mathbf{R} \mid f(x) < 4\}$. It changes from increasing on $(-\infty, \infty)$ to decreasing on $(-\infty, \infty)$. The end behaviour becomes as $x \to -\infty$, $y \to 4$, and as $x \to \infty$, $y \to -\infty$.
c) $g(x) = -2(2^{3(x-1)} + 4)$

8. $y = -3\sqrt{x-5}$

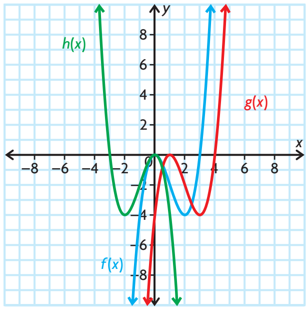


9. **a)** $(3, 24)$ **b)** $(-0.5, 4)$ **c)** $(-1, 9)$ **d)** $(-0.75, -8)$ **e)** $(-1, -8)$ **f)** $(-1, 7)$

10. **a)** D $= \{x \in \mathbf{R} \mid x \geq 2\}$,
R $= \{g(x) \in \mathbf{R} \mid g(x) \geq 0\}$
b) D $= \{x \in \mathbf{R} \mid x \geq 1\}$,
R $= \{h(x) \in \mathbf{R} \mid h(x) \geq 4\}$
c) D $= \{x \in \mathbf{R} \mid x \leq 0\}$,
R $= \{k(x) \in \mathbf{R} \mid k(x) \geq 1\}$
d) D $= \{x \in \mathbf{R} \mid x \geq 5\}$,
R $= \{j(x) \in \mathbf{R} \mid j(x) \geq -3\}$

11. $y = 5(x^2 - 3)$ is the same as $y = 5x^2 - 15$, not $y = 5x^2 - 3$.

12.



13. **a)** a vertical stretch by a factor of 4
b) a horizontal compression by a factor of $\frac{1}{2}$
c) $(2x)^2 = 2^2x^2 = 4x^2$

14. Answers may vary. For example:

horizontal stretch or compression, based on value of k

↓

vertical stretch or compression, based on value of a

↓

reflection in x-axis if $a < 0$; reflection in y-axis if $k < 0$

↓

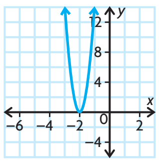
horizontal translation based on value of d

↓

vertical translation based on value of c

15. $(4, 5)$

16. **a)** horizontal compression by a factor of $\frac{1}{3}$, translation 2 units to the left
b) because they are equivalent expressions: $3(x + 2) = 3x + 6$
c)



Lesson 1.5, pp. 43–45

1. **a)** $(5, 2)$ **b)** $(-6, -5)$ **c)** $(-8, 4)$ **d)** $(2, 1)$ **e)** $(0, -3)$ **f)** $(7, 0)$

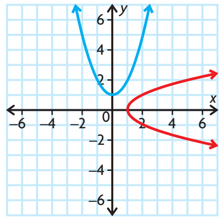
2. **a)** D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R}\}$
b) D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} \mid y \geq 2\}$
c) D $= \{x \in \mathbf{R} \mid x < 2\}$,
R $= \{y \in \mathbf{R} \mid y \geq -5\}$
d) D $= \{x \in \mathbf{R} \mid -5 < x < 10\}$,
R $= \{y \in \mathbf{R} \mid y < -2\}$

3. A and D match; B and F match; C and E match

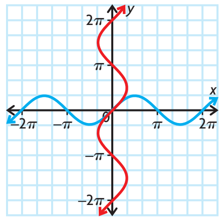
4. **a)** $(4, 129)$
b) $(129, 4)$
c) D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R}\}$
d) D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R}\}$
e) Yes; it passes the vertical line test.

5. **a)** $(4, 248)$
b) $(248, 4)$
c) D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} \mid y \geq -8\}$
d) D $= \{x \in \mathbf{R} \mid x \geq -8\}$ R $= \{y \in \mathbf{R}\}$
e) No; $(248, 4)$ and $(248, -4)$ are both on the inverse relation.

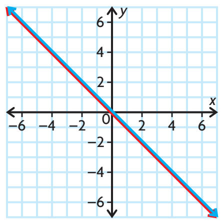
6. **a)** Not a function



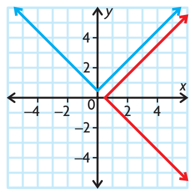
b) Not a function



c) Function



d) Not a function



7. **a)** $C = \frac{5}{9}(F - 32)$; this allows you to convert from Fahrenheit to Celsius.
b) 20 °C = 68 °F

8. **a)** $r = \sqrt{\frac{A}{\pi}}$; this can be used to determine the radius of a circle when its area is known.
b) $A = 25\pi$ cm^2, $r = 5$ cm

9. $k = 2$

10. **a)** 13 **b)** 25 **c)** 2 **d)** -2 **e)** 1 **f)** $\frac{1}{2}$

11. No; several students could have the same grade point average.

12. **a)** $f^{-1}(x) = \frac{1}{3}(x - 4)$

b) $h^{-1}(x) = -x$

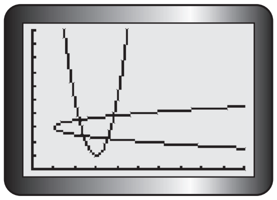
c) $g^{-1}(x) = \sqrt[3]{x + 1}$

d) $m^{-1}(x) = -\frac{x}{2} - 5$

13. **a)** $x = 4(y - 3)^2 + 1$

b) $y = \pm\sqrt{\frac{x - 1}{4}} + 3$

c)



d) (2.20, 3.55), (2.40, 2.40), (3.55, 2.20), (3.84, 3.84)

e) $x \geq 3$ because a negative square root is undefined.

f) $g(2) = 5$, but $g^{-1}(5) = 2$ or 4; the inverse is not a function if this is the domain of g.

14. For $y = -\sqrt{x + 2}$,
$D = \{x \in \mathbf{R} \mid x \geq -2\}$ and
$R = \{y \in \mathbf{R} \mid y \leq 0\}$. For $y = x^2 - 2$,
$D = \{x \in \mathbf{R}\}$ and $R = \{y \in \mathbf{R} \mid y \geq -2\}$.
The student would be correct if the domain of $y = x^2 - 2$ is restricted to
$D = \{x \in \mathbf{R} \mid x \leq 0\}$.

15. Yes; the inverse of $y = \sqrt{x + 2}$ is $y = x^2 - 2$ so long as the domain of this second function is restricted to
$D = \{x \in \mathbf{R} \mid x \geq 0\}$.

16. John is correct.

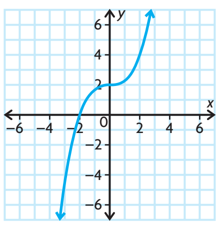
Algebraic: $y = \frac{x^3}{4} + 2$; $y - 2 = \frac{x^3}{4}$;
$4(y - 2) = x^3$; $x = \sqrt[3]{4(y - 2)}$.

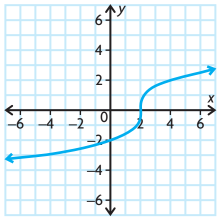
Numeric: Let $x = 4$.

$y = \frac{4^3}{4} + 2 = \frac{64}{4} + 2 = 16 + 2 = 18;$

$x = \sqrt[3]{4(y - 2)} = \sqrt[3]{4(18 - 2)}$
$= \sqrt[3]{4(16)} = \sqrt[3]{64} = 4.$

Graphical:





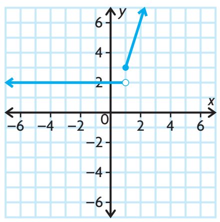
The graphs are reflections over the line $y = x$.

17. $f(x) = k - x$ works for all $k \in \mathbf{R}$.
$y = k - x$
Switch variables and solve for y: $x = k - y$
$y = k - x$
So the function is its own inverse.

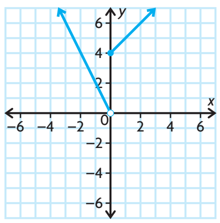
18. If a horizontal line hits the function in two locations, that means there are two points with equal y-values and different x-values. When the function is reflected over the line $y = x$ to find the inverse relation, those two points become points with equal x-values and different y-values, thus violating the definition of a function.

Lesson 1.6, pp. 51–53

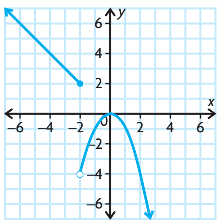
1. **a)**



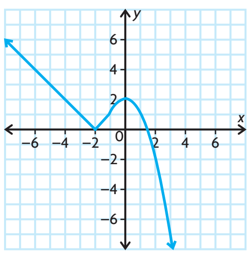
b)



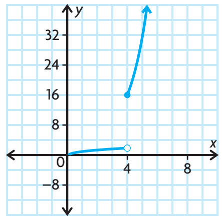
c)



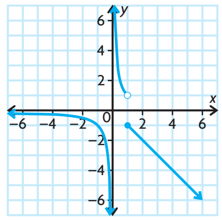
d)



e)



f)



2. **a)** Discontinuous at $x = 1$

b) Discontinuous at $x = 0$

c) Discontinuous at $x = -2$

d) Continuous

e) Discontinuous at $x = 4$

f) Discontinuous at $x = 1$ and $x = 0$

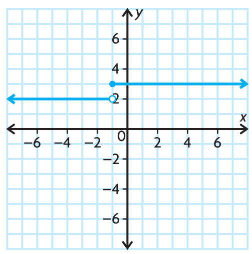
3. **a)** $f(x) = \begin{cases} x^2 - 2, \text{ if } x \leq 1 \\ x + 1, \text{ if } x > 1 \end{cases}$

b) $f(x) = \begin{cases} |x|, \text{ if } x < 1 \\ \sqrt{x}, \text{ if } x \geq 1 \end{cases}$

4. **a)** $D = \{x \in \mathbf{R}\}$; the function is discontinuous at $x = 1$.

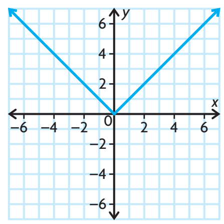
b) $D = \{x \in \mathbf{R}\}$; the function is continuous.

5. **a)**



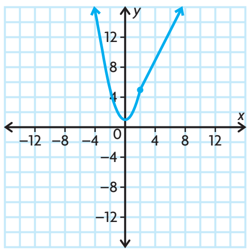
The function is discontinuous at $x = -1$.
$D = \{x \in \mathbf{R}\}$
$R = \{2, 3\}$

b)



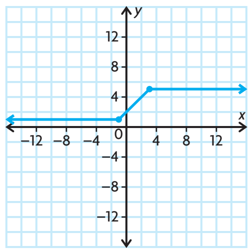
The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$

c)



The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 1\}$

d)

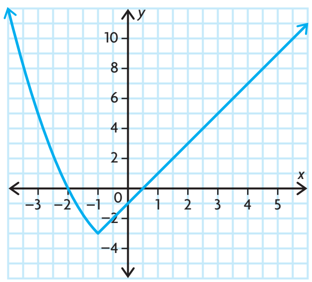


The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} \mid 1 \leq f(x) \leq 5\}$

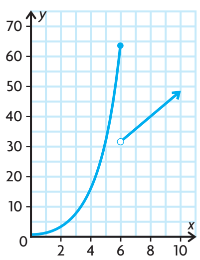
6. $f(x) = \begin{cases} 15, \text{ if } 0 \leq x \leq 500 \\ 15 + 0.02x, \text{ if } x \geq 500 \end{cases}$

7. $f(x) =$
$\begin{cases} 0.35x, \text{ if } 0 \leq x \leq 100\,000 \\ 0.45x - 10\,000, \text{ if } 100\,000 < x \leq 500\,000 \\ 0.55x - 60\,000, \text{ if } x > 500\,000 \end{cases}$

8. $k = 4$



9. a)

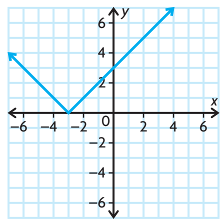


b) The function is discontinuous at $x = 6$.
c) 32 fish
d) $4x + 8 = 64$; $4x = 56$; $x = 14$
e) Answers may vary. For example, three possible events are environmental changes, introduction of a new predator, and increased fishing.

10. Answers may vary. For example:

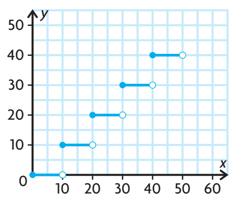
Plot the function for the left interval.
↓
Plot the function for the right interval.
↓
Determine if the plots for the left and right intervals meet at the x-value that serves as the common end point for the intervals; if so, the function is continuous at this point.
↓
Determine continuity for the two intervals using standard methods.

11. $f(x) = |x + 3| = \begin{cases} x + 3, \text{ if } x \geq -3 \\ -x - 3, \text{ if } x < -3 \end{cases}$



12. discontinuous at $p = 0$ and $p = 15$; continuous at $0 < p < 15$ and $p > 15$

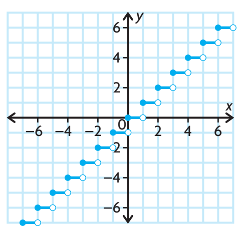
13. $f(x) = \begin{cases} 0, \text{ if } \ 0 \leq x < 10 \\ 10, \text{ if } 10 \leq x < 20 \\ 20, \text{ if } 20 \leq x < 30 \\ 30, \text{ if } 30 \leq x < 40 \\ 40, \text{ if } 40 \leq x < 50 \end{cases}$



It is often referred to as a step function because the graph looks like steps.

14. To make the first two pieces continuous, $5(-1) = -1 + k$, so $k = -4$. But if $k = -4$, the graph is discontinuous at $x = 3$.

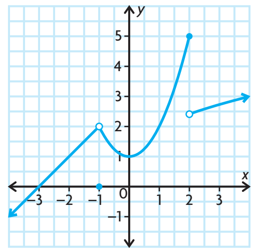
15.



16. Answers may vary. For example:

a) $f(x) = \begin{cases} x + 3, \text{ if } x < -1 \\ x^2 + 1, \text{ if } -1 \leq x \leq 2 \\ \sqrt{x} + 1, \text{ if } x > 2 \end{cases}$

b)



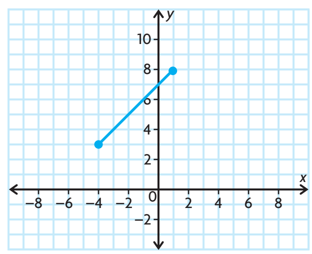
c) The function is not continuous. The last two pieces do not have the same value for $x = 2$.

d) $f(x) = \begin{cases} x + 3, \text{ if } x < -1 \\ x^2 + 1, \text{ if } -1 \leq x \leq 1 \\ \sqrt{x} + 1, \text{ if } x > 1 \end{cases}$

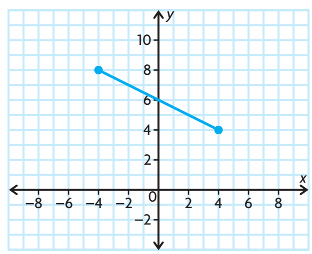
Lesson 1.7, pp. 56–57

1. **a)** $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
b) $\{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$
c) $\{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$
d) $\{(-4, 8), (-2, 4), (1, 6), (4, 24)\}$

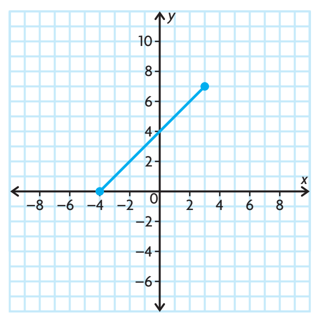
2. a)



b)

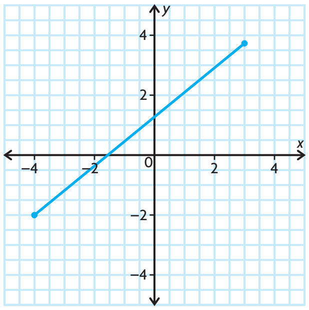


3. a)

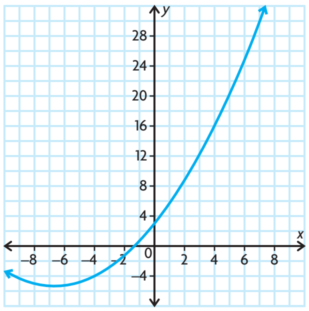


Answers

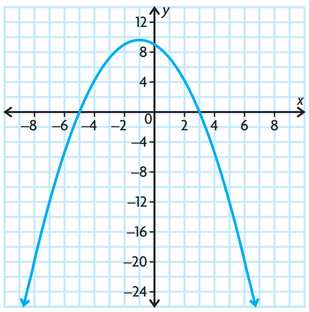
b)



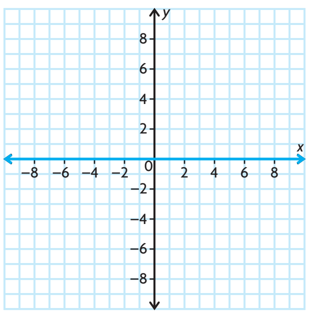
4. a)



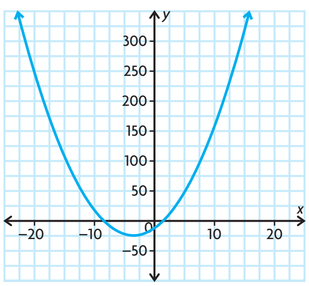
b)



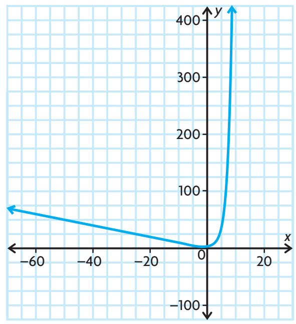
5. a) $y = 0$



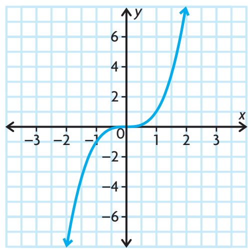
b) $y = x^2 + 7x - 12$



c) $y = |x| + 2^x$

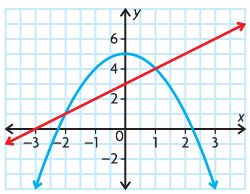


d) $y = x^3$



6. a)–b) Answers may vary. For example, properties of the original graphs such as intercepts and sign at various values of the independent variable figure prominently in the shape of the new function.

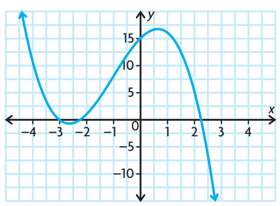
7. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
−3	0	−4	0
−2	1	1	1
−1	2	4	8
0	3	5	15
1	4	4	16
2	5	1	5
3	6	−4	−24

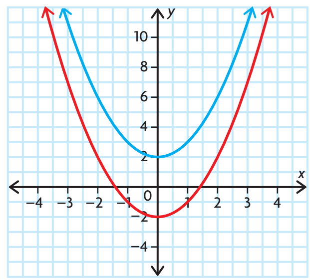
c)



d) $h(x) = (x + 3)(-x^2 + 5)$
$= -x^3 - 3x^2 + 5x + 15$; degree is 3

e) $D = \{x \in \mathbf{R}\}$; this is the same as the domain of both f and g.

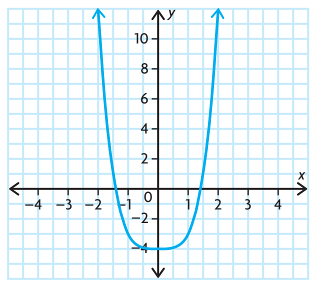
8. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
−3	11	7	77
−2	6	2	12
−1	3	−1	−3
0	2	−2	−4
1	3	−1	−3
2	6	2	12
3	11	7	77

c)



d) $h(x) = (x^2 + 2)(x^2 - 2) = x^4 - 4$; degree is 4

e) $D = \{x \in \mathbf{R}\}$

Chapter Review, pp. 60–61

1. a) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

b) function; $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid y \leq 3\}$

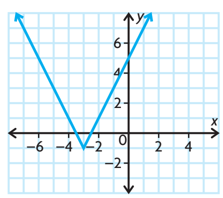
c) not a function;
$D = \{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$;
$R = \{y \in \mathbf{R}\}$

d) function; $D = \{x \in \mathbf{R} \mid x > 0\}$;
$R = \{y \in \mathbf{R}\}$

2. a) $C(t) = 30 + 0.02t$

b) $D = \{t \in \mathbf{R} \mid t \geq 0\}$,
$R = \{C(t) \in \mathbf{R} \mid C(t) \geq 30\}$

3. $D = \{x \in \mathbf{R}\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 1\}$

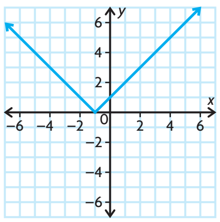


4. $|x| < 2$

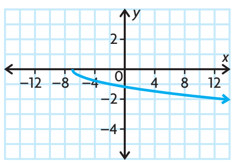
5. a) Both functions have a domain of all real numbers, but the ranges differ.
   b) Both functions are odd but have different domains.
   c) Both functions have the same domain and range, but x^2 is smooth and $|x|$ has a sharp corner at $(0, 0)$.
   d) Both functions are increasing on the entire real line, but 2^x has a horizontal asymptote while x does not.

6. a) Increasing on $(-\infty, \infty)$; odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R}\}$
   b) Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; even; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} \mid f(x) \geq 2\}$
   c) Increasing on $(-\infty, \infty)$; neither even nor odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} \mid f(x) > -1\}$

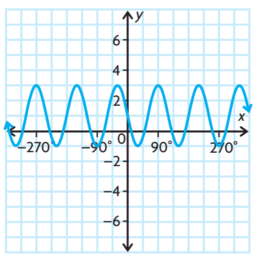
7. a) Parent: $y = |x|$; translated left 1



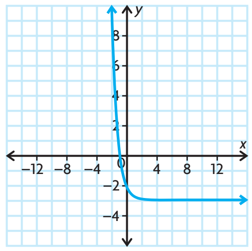
   b) Parent: $y = \sqrt{x}$; compressed vertically by a factor of 0.25, reflected across the x-axis, compressed horizontally by a factor of $\frac{1}{3}$, and translated left 7



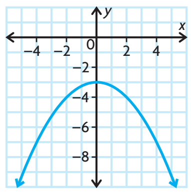
   c) Parent: $y = \sin x$; reflected across the x-axis, expanded vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{3}$, translated up by 1



   d) Parent: $y = 2^x$; reflected across the y-axis, compressed horizontally by a factor of $\frac{1}{2}$, and translated down by 3



8. $y = -\left(\frac{1}{2}x\right)^2 - 3$

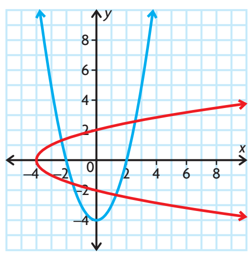


9. a) $(-2, 1)$
   b) $(-10, -6)$
   c) $(4, 3)$
   d) $\left(\frac{17}{5}, 0.3\right)$
   e) $(-1, 0)$
   f) $(9, -1)$

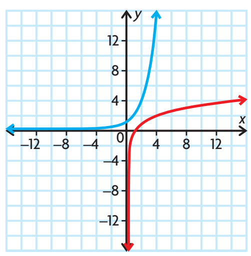
10. a) $(2, 1)$
    b) $(-9, -1)$
    c) $(7, 0)$
    d) $(7, 5)$
    e) $(-3, 0)$
    f) $(10, 1)$

11. a) $D = \{x \in \mathbf{R} \mid -2 < x < 2\}$, $R = \{y \in \mathbf{R}\}$
    b) $D = \{x \in \mathbf{R} \mid x < 12\}$, $R = \{y \in \mathbf{R} \mid y \geq 7\}$

12. a) The inverse relation is not a function.

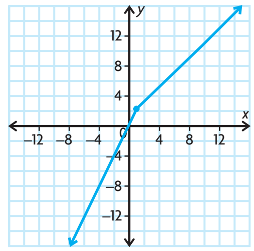


    b) The inverse relation is a function.



13. a) $f^{-1}(x) = \dfrac{x - 1}{2}$
    b) $g^{-1}(x) = \sqrt[3]{x}$

14.



The function is continuous; $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$

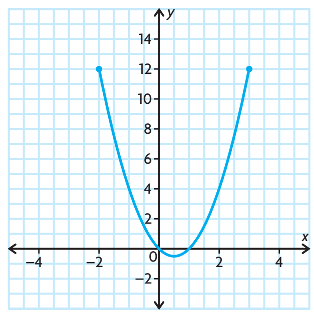
15. $f(x) = \begin{cases} 3x - 1, \text{ if } x \leq 2 \\ -x, \text{ if } x > 2 \end{cases}$; the function is discontinuous at $x = 2$.

16. In order for $f(x)$ to be continuous at $x = 1$, the two pieces must have the same value when $x = 1$.
When $x = 1$, $x^2 + 1 = 2$ and $3x = 3$.
The two pieces are not equal when $x = 1$, so the function is not continuous at $x = 1$.

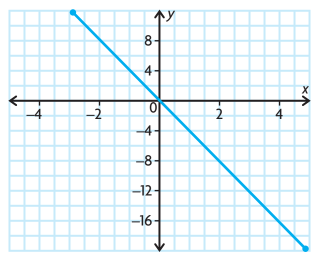
17. a) $f(x) = \begin{cases} 30, \text{ if } x \leq 200 \\ 24 + 0.03, \text{ if } x > 200 \end{cases}$
    b) \$34.50
    c) \$30

18. a) $\{(1, 7), (4, 15)\}$
    b) $\{(1, -1), (4, -1)\}$
    c) $\{(1, 12), (4, 56)\}$

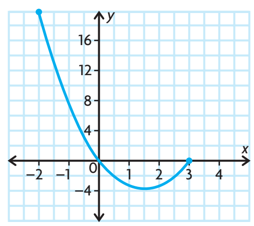
19. a)



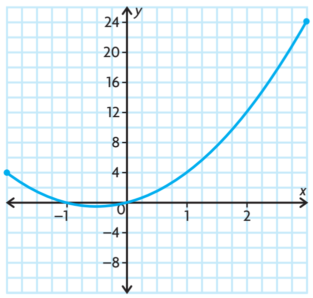
    b)



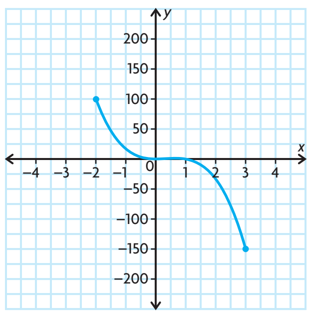
    c)



d)



e)

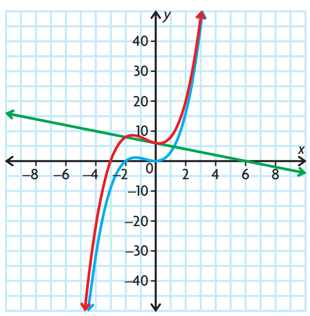


20. **a)** D
b) C
c) A
d) B

21. **a)**

x	−3	−2	−1	0	1	2
$f(x)$	−9	0	1	0	3	16
$g(x)$	9	8	7	6	5	4
$(f+g)(x)$	0	8	8	6	8	20

b)–c)

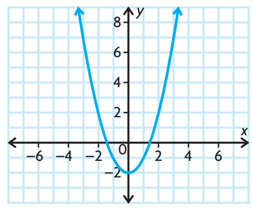


d) $x^3 + 2x^2 - x + 6$
e) Answers may vary. For example, (0, 0) belongs to f, (0, 6) belongs to g and (0, 6) belongs to $f + g$. Also, (1, 3) belongs to f, (1, 5) belongs to g and (1, 8) belongs to $f + g$.

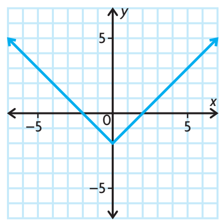
Chapter Self-Test, p. 62

1. **a)** Yes. It passes the vertical line test.
b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y \geq 0\}$

2. **a)** $f(x) = x^2$ or $f(x) = |x|$
b)



or

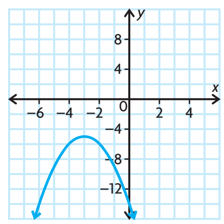


c) The graph was translated 2 units down.

3. $f(-x) = |3(-x)| + (-x)^2$
$= |3x| + x^2 = f(x)$

4. 2^x has a horizontal asymptote while x^2 does not. The range of 2^x is $\{y \in \mathbf{R} \mid y > 0\}$ while the range of x^2 is $\{y \in \mathbf{R} \mid y \geq 0\}$. 2^x is increasing on the whole real line and x^2 has an interval of decrease and an interval of increase.

5. reflection over the x-axis, translation down 5 units, translation left 3 units



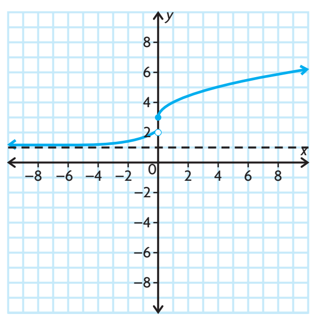
6. horizontal stretch by a factor of 2, translation 1 unit up;
$f(x) = $ if $|\frac{1}{2}x| + 1$

7. **a)** $(-4, 17)$
b) $(5, 3)$

8. $f^{1}(x) = -\frac{x}{2} - 1$

9. **a)** $9000
b) $f(x) = \begin{cases} 0.05, \text{ if } x \leq 50\,000 \\ 0.12x - 6000, \text{ if } x > 50\,000 \end{cases}$

10. **a)**



b) $f(x)$ is discontinuous at $x = 0$ because the two pieces do not have the same value when $x = 0$. When $x = 0$, $2^x + 1 = 2$ and $\sqrt{x + 3} = 3$.
c) Intervals of increase: $(-\infty, 0)$, $(0, \infty)$; no intervals of decrease
d) $D = \{x \in \mathbf{R}\}$,
$R = \{y \in \mathbf{R} \mid 0 < y < 2 \text{ or } y \geq 3\}$

Chapter 2

Getting Started, p. 66

1. **a)** $\frac{4}{3}$ **b)** $-\frac{6}{7}$

2. **a)** Each successive first difference is 2 times the previous first difference. The function is exponential.
b) The second differences are all 6. The function is quadratic.

3. **a)** $-\frac{3}{2}, 2$ **c)** 45°, 225°
b) 0 **d)** −270°, −90°

4. **a)** vertical compression by a factor of $\frac{1}{2}$
b) vertical stretch by a factor of 2, horizontal translation 4 units to the right
c) vertical stretch by a factor of 3, reflection across x-axis, vertical translation 7 units up
d) vertical stretch by a factor of 5, horizontal translation 3 units to the right, vertical translation 2 units down,

5. **a)** $A = 1000(1.08)^t$
b) $1259.71
c) No, since the interest is compounded each year, each year you earn more interest than the previous year.

6. **a)** 15 m; 1 m
b) 24 s
c) 15 m

7.

Linear relations	Nonlinear relations
constant; same as slope of line; positive for lines that slope up from left to right; negative for lines that slope down from left to right; 0 for horizontal lines.	variable; can be positive, negative, or 0 for different parts of the same relation

Rates of Change

Lesson 2.1, pp. 76–78

1. **a)** 19 **c)** 13 **e)** 11.4
b) 15 **d)** 12 **f)** 11.04

2. **a)** **i)** 15 m/s **ii)** −5 m/s

b) During the first interval, the height is increasing at 15 m/s;
during the second interval, the height is decreasing at 5 m/s.

3. $f(x)$ is always increasing at a constant rate. $g(x)$ is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$, so the rate of change is not constant.

4. **a)** 352, 138, 286, 28, 60, -34 people/h
b) the rate of growth of the crowd at the rally
c) A positive rate of growth indicates that people were arriving at the rally. A negative rate of growth indicates that people were leaving the rally.

5. **a)** 203, 193, 165, 178.5, 218.5, 146 km/day
b) No. Some days the distance travelled was greater than others.

6. 4; 4; the average rate of change is always 4 because the function is linear, with a slope of 4.

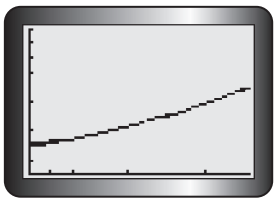
7. The rate of change is 0 for 0 to 250 min. After 250 min, the rate of change is \$0.10/min.

8. **a)** **i)** 750 people/year
ii) 3000 people/year
iii) 12 000 people/year
iv) 5250 people/year
b) No; the rate of growth increases as the time increases.
c) You must assume that the growth continues to follow this pattern, and that the population will be 5 120 000 people in 2050.

9. -2 m/s

10. **a)** **i)** \$2.60/sweatshirt
ii) \$2.00/sweatshirt
iii) \$1.40/sweatshirt
iv) \$0.80/sweatshirt
b) The rate of change is still positive, but it is decreasing. This means that the profit is still increasing, but at a decreasing rate.
c) No; after 6000 sweatshirts are sold, the rate of change becomes negative. This means that the profit begins to decrease after 6000 sweatshirts are sold.

11. **a)**



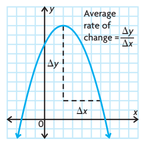
b) The rate of change will be greater farther in the future. The graph is getting steeper as the values of t increase.
c) **i)** 1500 people/year
ii) 1700 people/year
iii) 2000 people/year
iv) 2500 people/year
d) The prediction was correct.

12. Answers may vary. For example:
a) Someone might calculate the average increase in the price of gasoline over time. One might also calculate the average decrease in the price of computers over time.
b) An average rate of change might be useful for predicting the behaviour of a relationship in the future.
c) An average rate of change is calculated by dividing the change in the dependent variable by the corresponding change in the dependent variable.

13. -7.8%

14. Answers may vary. For example:

AVERAGE RATE OF CHANGE

Definition in your own words	Personal example	Visual representation
the change in one quantity divided by the change in a related quantity	I record the number of miles I run each week versus the week number. Then, I can calculate the average rate of change in the distance I run over the course of weeks.	

15. 80 km/h

Lesson 2.2, pp. 85–88

1. **a)**

Preceding Interval	$\Delta f(x)$	Δx	Average Rate of Change, $\frac{\Delta f(x)}{\Delta x}$
$1 \le x \le 2$	$13 - (-2) = 15$	$2 - 1 = 1$	15
$1.5 \le x \le 2$	8.75	0.5	17.5
$1.9 \le x \le 2$	1.95	0.1	19.5
$1.99 \le x \le 2$	0.1995	0.01	19.95

Following Interval	$\Delta f(x)$	Δx	Average Rate of Change, $\frac{\Delta f(x)}{\Delta x}$
$2 \le x \le 3$	$38 - 13 = 25$	$3 - 2 = 1$	25
$2 \le x \le 2.5$	11.25	0.5	22.5
$2 \le x \le 2.1$	2.05	0.1	20.5
$2 \le x \le 2.01$	0.2005	0.01	20.05

b) 20

2. **a)** 5.4 m/s **b)** 5.4 m/s
c) Answers may vary. For example: I prefer the centred interval method. Fewer calculations are required, and it takes into account points on each side of the given point in each calculation.

3. **a)** 200
b) 40 raccoons/month
c) 50 raccoons/month
d) The three answers represent different things: the population at a particular time, the average rate of change prior to that time, and the instantaneous rate of change at that time.

4. **a)** -24 **b)** 0 **c)** 48 **d)** 96

5. -27 m/s

6. \$11 610 per year

7. **a)** 0 people/year
b) Answers may vary. For example: Yes, it makes sense. It means that the populations in 2000 and 2024 are the same, so their average rate of change is 0.
c) The average rate of change from 2000 to 2012 is 18 000 people/year; the average rate of change from 2012 to 2024 is -18 000 people/year.
d) $t = 12$

8. About $-$\$960 per year; when the car turns five, it loses \$960 of its value.

9. **a)** 1.65 s **b)** about 14 m/s

10. 100π cm^3/cm

11. If David knows how far he has travelled and how long he has been driving, he can calculate his average speed from the beginning of the trip by dividing the distance travelled by the time he has been driving.

12. **a)** -22.5 °F/min
b) Answers may vary. For example: -25.5 °F/min
c) Answers may vary. For example, the first rate is using a larger interval to estimate the instantaneous rate.
d) Answers may vary. For example, the second estimate is better, as it uses a much smaller interval to estimate the instantaneous rate.

13. Answers may vary. For example:

Method of Estimating Instantaneous Rate of Change	Advantage	Disadvantage
series of preceding intervals and following intervals	accounts for differences in the way that change occurs on either side of the given point	must do two sets of calculations
series of centred intervals	accounts for points on either side of the given interval in same calculation	to get a precise answer, numbers involved will need to have several decimal places
difference quotient	more precise	calculations can be tedious or messy

14. **a)** 100π cm²/cm
b) 240π cm²/cm

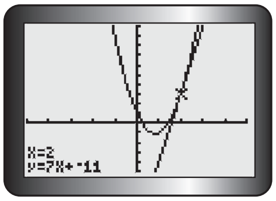
15. 36 cm²/cm

16. 160π cm²/cm

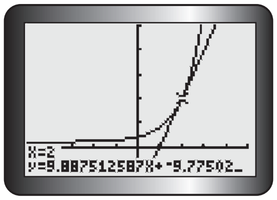
Lesson 2.3, pp. 91–92

1. **a)** about 7 **c)** about 0.25
b) about 10 **d)** 2

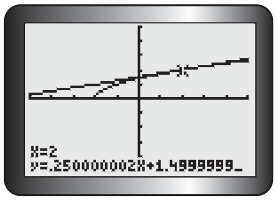
2. **a)**



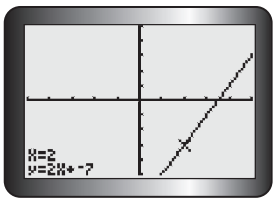
b)



c)

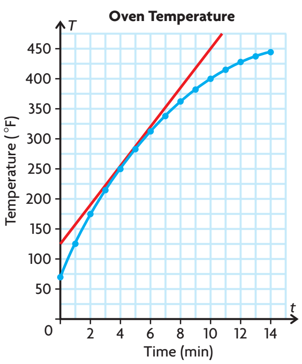


d)



3. **a)** Set A: 0, 0, 0, 0
Set B: 14, 1.4, 5, 0.009
Set C: -4, -0.69, -3, -0.009
b) Set A: All slopes are zero.
Set B: All slopes are positive.
Set C: All slopes are negative.

4. **a)** and **b)**



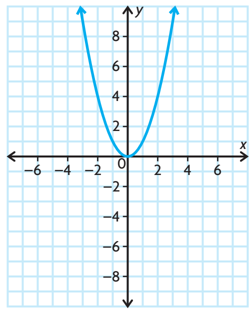
c) 31

d) Rate of change is about 30 °F/min at $x = 5$.

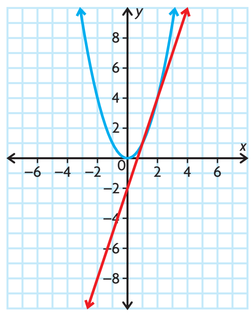
e) Answers may vary. For example: The two answers are about the same. The slope of the tangent line at the point is the same as the instantaneous rate of change at the point.

5. Answers may vary. For example: Similarity: the calculation; difference: average rate of change is over an interval; instantaneous rate of change is at a point.

6. **a)**



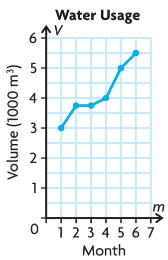
b)



c) (1.5, 2.25)

Mid-Chapter Review, p. 95

1. **a)**



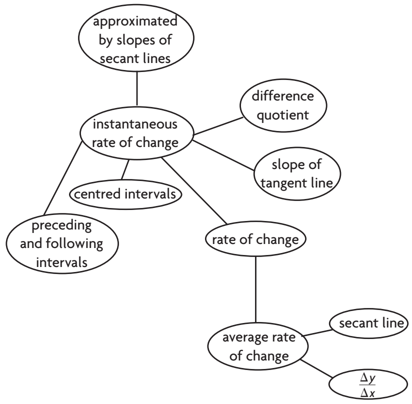
b) 750; 250; 1100; 400 m³/month
c) April and May
d) 580 m³/month

2. **a)** The equation models exponential growth. This means that the average rate of change between consecutive years will always increase.
b) The instantaneous rate of change in population in 2010 is about 950 people per year.

3. **a)** 10 m/s; -10 m/s
b) $t = 2$; Answers may vary. For example: The graph has a vertex at (2, 21). It appears that a tangent line at this point would be horizontal.
$$\frac{(f(2.01) - f(1.99))}{0.02}$$

4. 0.9 m/day

5. Answers may vary. For example:



6. Answers may vary. For example:

Points	Slope of Secant
(2, 9) and (1, 2)	7
(2, 9) and (1.5, 4.375)	9.25
(2, 9) and (1.9, 7.859)	11.41
(2, 9) and (2.1, 10.261)	12.61
(2, 9) and (2.5, 16.625)	15.25
(2, 9) and (3, 28)	19

The slope of the tangent line at (2, 9) is about 12.

7. 4

8. The instantaneous rate of change of the function whose graph is shown is 4 at $x = 2$.

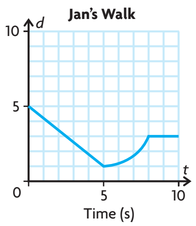
9. Answers may vary. For example:
a) 0 **b)** 4 **c)** 5 **d)** 8

Lesson 2.4, pp. 103–106

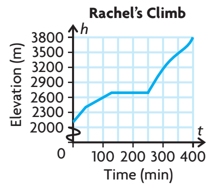
1. **a)** C **b)** A **c)** B

2. All of the graphs show that the speed is constant. In a), the speed is positive and constant. In b), the speed is negative and constant. In c), the speed is 0, which is constant.

3.

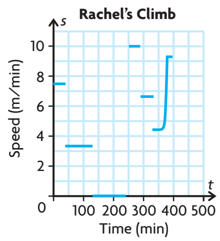


4. **a)** Answers may vary. For example:

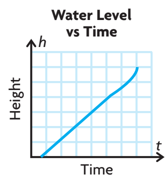


b) Average speed over first 40 min is 7.5 m/min, average speed over next 90 min is 3.3 m/min, average speed over next 120 min is 0 m/min, average speed over next 40 min is 10 m/min, average speed over next 45 min is 6.7 m/min, and average speed over last 60 min is 5.7 m/min.

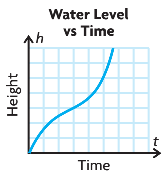
c) Answers may vary. For example:



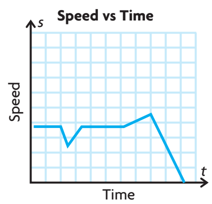
5. **a)** Answers may vary. For example:



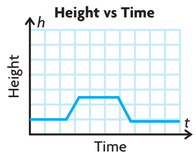
b) Answers may vary. For example:



6. **a)** Answers may vary. For example:



b) Answers may vary. For example:

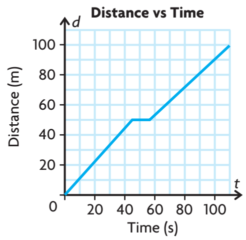


7. **a)** 1.11 m/s

b) 0.91 m/s

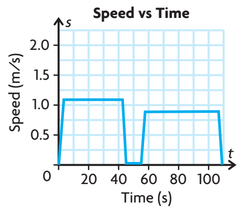
c) The graph of the first length would be steeper, indicating a quicker speed. The graph of the second length would be less steep, indicating a slower speed.

d) Answers may vary. For example:



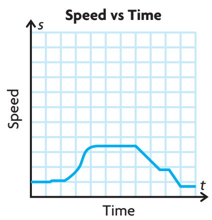
e) 0 m/s

f) Answers may vary. For example:



8. **a)** A **b)** C **c)** D **d)** B

9. Answers may vary. For example:

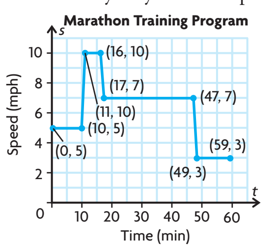


10. **a)** and **b)**

i) Start 5 m from sensor. Walk toward sensor at a constant rate of 1 m/s for 3 s. Walk away from sensor at a constant rate of 1 m/s for 3 s.

ii) Start 6 m from sensor. Walk toward sensor at a constant rate of 1 m/s for 2 s. Stand still for 1 s. Walk toward sensor at a constant rate of 1 m/s for 2 s. Walk away from sensor at a constant rate of 1.5 m/s.

11. **a)** Answers may vary. For example:



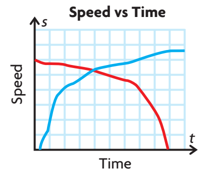
b) 5 mph/min

c) -0.1842 mph/min

d) The answer to part c) is an average rate of change over a long period, but the runner does not slow down at a constant rate during this period.

12. Answers may vary. For example: Walk from (0, 0) to (5, 5) and stop for 5 s. Then run to (15, 30). Continue walking to (20, 5) and end at (25, 0). What is the maximum speed and minimum speed on an interval? Create the speed versus time graph from these data.

13. Answers may vary. For example:



14. If the original graph showed an increase in rate, it would mean that the distance travelled during each successive unit of time would be greater—meaning a graph that curves upward. If the original graph showed a straight, horizontal line, then it would mean that the distance travelled during each successive unit of time would be greater—meaning a steady increasing straight line on the second graph. If the original graph showed a decrease in rate, it would mean that the distance travelled during each successive unit of time would be less—meaning a line that curves down.

Lesson 2.5, pp. 111–113

1. Answers may vary. For example, I used the difference quotient when $a = 1.5$ and $h = 0.001$ and got an estimate for the instantaneous rate of change in cost that was close to 0.

2. 0

3. **a)** The slopes of the tangent lines are positive, but close to 0.

b) The slopes of the tangent lines are negative, but close to 0.

4. **a)** The slopes of the tangent lines are negative, but close to 0.

b) The slopes of the tangent lines are positive, but close to 0.

5. **a)** The slope is 0.

b) The slope is 0.

c) The slope is 0.

d) The slope is 0.

6. **a)** minimum

b) maximum

c) minimum

d) maximum

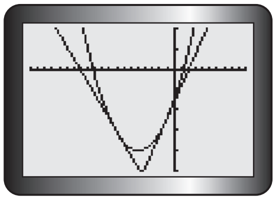
e) maximum

f) maximum

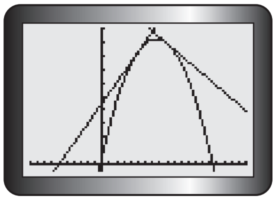
7. $t = 2.75$; Answers may vary. For example: The slopes of tangents for values of t less than about 2.75 would be positive, while slopes of tangents for values of t greater than about 2.75 would be negative.

8. a) $x = -5$; minimum
$x = 7.5$; maximum
$x = 3.25$; minimum
$x = 6$; maximum

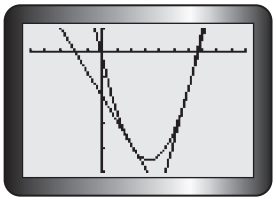
b) i)



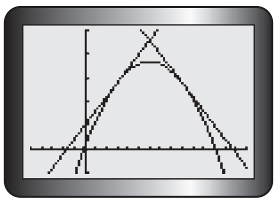
ii)



iii)



iv)



c) Answers may vary. For example, if the sign of the slope of the tangent changed from positive to negative, there was a maximum. If the sign of the slope of the tangent changed from negative to positive, there was a minimum.

9. a) i) maximum $= (0, 100)$; minimum $= (5, 44.4)$
ii) maximum $= (10, 141.6)$; minimum $= (0, 35)$

b) For an equation that represents exponential growth (where $r > 0$), the minimum value will always be at point a and the maximum value will always be at point b, because y will always increase as x increases. For an equation that represents exponential decay (where $r < 0$), the minimum value will always be at point b and the maximum value will always be at point a, because y will always decrease as x increases.

10. Answers may vary. For example, the slope of the tangent at 0.5 s is 0. The slope of the tangent at 0 s is 5, and the slope of the tangent at 1 s is -5. So, the diver reaches her maximum height at 0.5 s.

11. Answers may vary. For example, yes, this observation is correct. The slope of the tangent at 1.5 s is 0. The slopes of the tangents between 1 s and 1.5 s are negative, and the slopes of the tangent lines between 1.5 s and 2 s are positive. So, the minimum of the function occurs at 1.5 s.

12. Answers may vary. For example, estimate the slope of the tangent line to the curve when $x = 5$ by writing an equation for the slope of any secant line on the graph of $R(x)$. If the slope of the tangent is 0, this will confirm there may be a maximum at $x = 5$. If the slopes of tangent lines to the left are positive and the slopes of tangent lines to the right are negative, this will confirm that a maximum occurs at $x = 5$.

13. Answers may vary. For example, because sin 90° gives a maximum value of 1, I know that a maximum occurs when $(k(x - d)) = 90°$. Solving this equation for x will tell me what types of x-values will give a maximum. For example, when $k = 2$ and $d = 3$,
$$(2(x - 3°)) = 90°$$
$$(x - 3°) = 45°$$
$$x = 48°$$

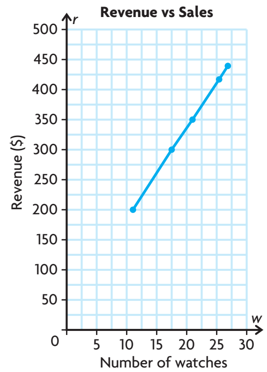
14. Myra is plotting (instantaneous) velocity versus time. The rates of change Myra calculates represent acceleration. When Myra's graph is increasing, the car is accelerating. When Myra's graph is decreasing, the car is decelerating. When Myra's graph is constant, the velocity of the car is constant; the car is neither accelerating nor decelerating.

15. -4, -2, 4, 6; The rule appears to be "multiply the x-coordinate by 2." 12, 3, 12, 27; The rule for $f(x) = x^3$ seems to be "square the x-coordinate and multiply by 3."

Chapter Review, pp. 116–117

1. a) Yes. Divide revenue by number of watches, and the slope is 17.5.

b) Answers may vary. For example:



The data represent a linear relationship.

c) $17.50 per watch

d) $17.50; this is the slope of the line on the graph.

2. a) 1.5 m/s
b) -1.5 m/s
c) The time intervals have the same length. The amount of change is the same, but with opposite signs for the two intervals. So, the rates of change are the same for the two intervals, but with opposite signs.

3. a) $E = 2500m + 10\,000$
b) $2500 per month
c) No; the equation that represents this situation is linear, and the rate of change over time for a linear equation is constant.

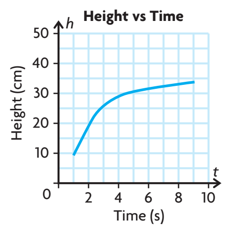
4. a) Answers may vary. For example, because the unit of the equation is years, you would not choose $3 \le t \le 4.25$ and $4 \le t \le 5$. A better choice would be $3.75 \le t \le 4.0$ and $4.0 \le t \le 4.25$.
b) Answers may vary. For example, find the average of the two interval values: $\frac{(600.56 + 621.91)}{2} = \$611.24$

5. a) Answers may vary. For example, squeezing the interval.
b) 4.19 cm/s

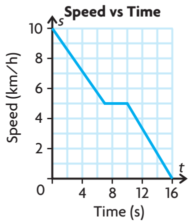
6. a) -2 b) 0 c) 4

7. a) -37 b) -17 c) 0 d) 23

8. Answers may vary. For example:



9. a) Answers may vary. For example:

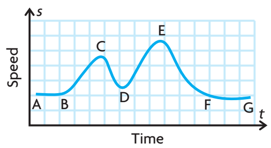


b) $-\frac{5}{7}$ km/h/s

c) From (7, 5) to (12, $\frac{10}{3}$), the rate of change of speed in $-\frac{1}{3}$ km/h/s

d) $-\frac{5}{6}$ km/h/s

10. The roller coaster moves at a slow steady speed between A and B. At B, it begins to accelerate as it moves down to C. Going uphill from C to D it decelerates. At D, it starts to move down and accelerates to E, where the speed starts to decrease until F, where it maintains a slower speed to G, the end of the track.

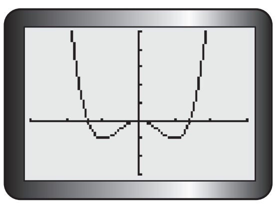


11. a) minimum
b) maximum
c) maximum
d) minimum
e) minimum
f) maximum

12. a) i) $m = h - 26$ ii) $m = -4h - 48$
b) i) $m = -26$ ii) $m = -48$

13. a) To the left of a maximum, the instantaneous rates of change are positive. To the right, the instantaneous rates of change are negative.
b) To the left of a minimum, the instantaneous rates of change are negative. To the right, the instantaneous rates of change are positive.

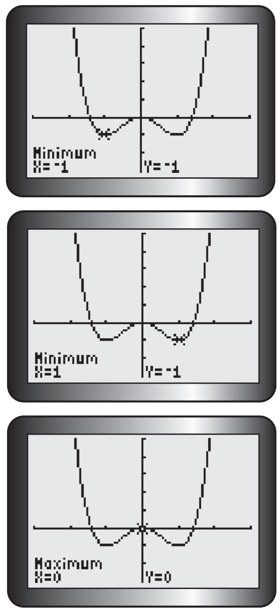
14. a)



b) minimum: $x = -1$, $x = 1$
maximum: $x = 0$

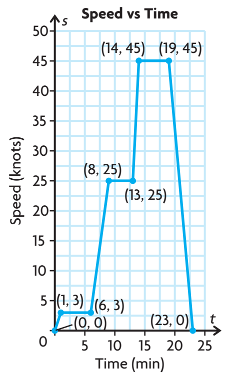
c) The slopes of tangent lines for points to the left of a minimum will be negative, while the slopes of tangent lines for points to the right of a minimum will be positive. The slopes of tangent lines for points to the left of a maximum will be positive, while the slopes of tangent lines for points to the right of a minimum will be negative.

d)



Chapter Self-Test, p. 118

1. a)



b) 11 kn/min; 0 kn/min; the two different average rates of change indicate that the boat was increasing its speed from $t = 6$ to $t = 8$ at a rate of 11knots/min and moving at a constant speed from $t = 8$ to $t = 13$.

c) 11 kn/min

2. a) -1
b) The hot cocoa is cooling by 1 °C/min on average.
c) -0.75
d) The hot cocoa is cooling by 0.75 °C/min after 30 min.
e) The rate decreases over the interval, until it is nearly 0 and constant.

3. a) \$310 per dollar spent
b) $-$\$100 per dollar spent
c) The positive sign for part a) means that the company is increasing its profit when it spends between \$8000 and \$10 000 on advertising. The negative sign means the company's profit is decreasing when it spends \$50 000 on advertising.

4. a) -1; 0 (minimum); 7
b) 4.5; -4.5; 0 (maximum)

Chapter 3

Getting Started, p. 122

1. a) $6x^3 - 22x^2$
b) $x^2 + 2x - 24$
c) $24x^3 - 44x^2 - 40x$
d) $5x^3 + 31x^2 - 68x + 32$

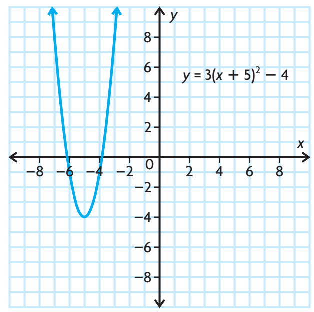
2. a) $(x + 7)(x - 4)$
b) $2(x - 2)(x - 7)$

3. a) $x = -6$
b) $x = -3, 4.5$
c) $x = -3, -8$
d) $x = \frac{1}{3}, -4$

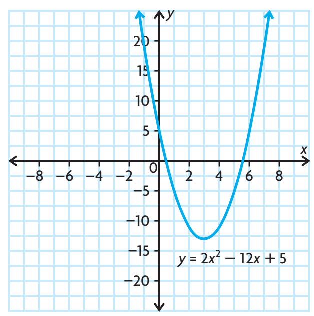
4. a) vertical compression by a factor of $\frac{1}{4}$; horizontal translation 3 units to the right; vertical translation 9 units up
b) vertical compression by a factor of $\frac{1}{4}$; vertical translation 7 units down

5. a) $y = 2(x - 5)^2 - 2$
b) $y = -2x^2 + 3$

6. a)



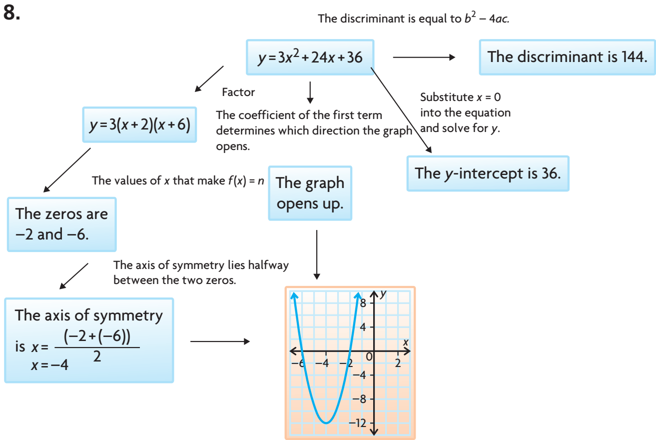
b)



7. a) quadratic
b) other
c) other
d) linear

Answers

8.



Lesson 3.1, pp. 127–128

1. **a)** This represents a polynomial function because the domain is the set of all real numbers, the range does not have a lower bound, and the graph does not have horizontal or vertical asymptotes.

b) This represents a polynomial function because the domain is the set of all real numbers, the range is the set of all real numbers, and the graph does not have horizontal or vertical asymptotes.

c) This is not a polynomial function because it has a horizontal asymptote.

d) This represents a polynomial function because the domain is the set of all real numbers, the range does not have an upper bound, and the graph does not have horizontal or vertical asymptotes.

e) This is not a polynomial function because its domain is not all real numbers.

f) This is not a polynomial function because it is a periodic function.

2. **a)** polynomial; the exponents of the variables are all natural numbers

b) polynomial; the exponents of the variables are all natural numbers

c) polynomial; the exponents of the variables are all natural numbers

d) other; the variable is under a radical sign

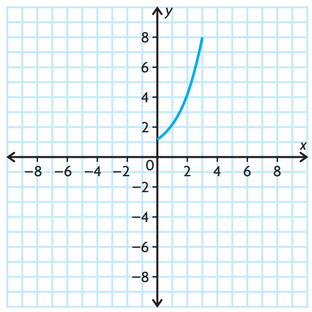
e) other; the function contains another function in the denominator

f) polynomial; the exponents of the variables are all natural numbers

3. **a)** linear **c)** linear

b) quadratic **d)** cubic

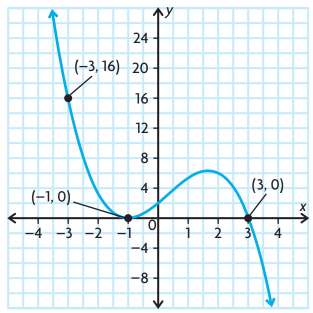
4.



a) The graph looks like one half of a parabola, which is the graph of a quadratic equation.

b) There is a variable in the exponent.

5.



6. Answers may vary. For example, any equation of the form $y = a\left(-\frac{4}{3}x^2 + \frac{8}{3}x + 4\right)$ will have the same zeros, but have a different y-intercept and a different value for $f(-3)$. Any equation of the form $y = x\left(-\frac{4}{3}x^2 + \frac{8}{3}x + 4\right)$ would have two of the same zeros, but a different value for $f(-3)$ and different positive/negative intervals.

7. $y = x + 5, y = x^2 + 5,$
$y = x^3 + 5, y = x^4 + 5$

8. Answers may vary. For example:

Definition	**Characteristics**
A polynomial is an expression of the form $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number.	The domain of the function is all real numbers, but the range can have restrictions; except for polynomial functions of degree zero (whose graphs are horizontal lines), the graphs of polynomials do not have horizontal or vertical asymptotes. The shape of the graph depends on its degree.
Polynomials	
Examples	**Non-Examples**
$x^2 + 4x + 6$	$\sqrt{x} + 1$

Lesson 3.2, pp. 136–138

1. **a)** 4; −4; as $x \to +/-\infty, y \to -\infty$

b) 5; 2; as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$

c) 3; −3; as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$

d) 4; 24; as $x \to +/-\infty, y \to \infty$

2. **a)** Turning points
- **a)** minimum 1, maximum 3
- **b)** minimum 0, maximum 4
- **c)** minimum 0, maximum 2
- **d)** minimum 1, maximum 3

b) Zeros
- **a)** minimum 0, maximum 4
- **b)** minimum 1, maximum 5
- **c)** minimum 1, maximum 3
- **d)** minimum 0, maximum 4

3. **i)** **a)** The degree is even.
b) The leading coefficient is negative.

ii) **a)** The degree is even.
b) The leading coefficient is negative.

iii) **a)** The degree is odd.
b) The leading coefficient is negative.

iv) **a)** The degree is even.
b) The leading coefficient is positive.

v) **a)** The degree is odd.
b) The leading coefficient is negative.

vi) **a)** The degree is odd.
b) The leading coefficient is positive.

4. **a)** as $x \to +/-\infty, y \to \infty$

b) as $x \to -\infty, y \to \infty$ and as $x \to \infty$, $y \to -\infty$

c) as $x \to -\infty, y \to -\infty$ and as $x \to \infty$, $y \to \infty$

d) as $x \to +/-\infty, y \to -\infty$

e) as $x \to +/- \infty$, $y \to \infty$

f) as $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$

5. **a)** D: The graph extends from quadrant III to quadrant I and the y-intercept is 2.

b) A: The graph extends from quadrant III to quadrant IV.

c) E: The graph extends from quadrant II to quadrant I and the y-intercept is -5.

d) C: The graph extends from quadrant II to quadrant I and the y-intercept is 0.

e) F: The graph extends from quadrant II to quadrant IV.

f) B: The graph extends from quadrant III to quadrant I and the y-intercept is 1.

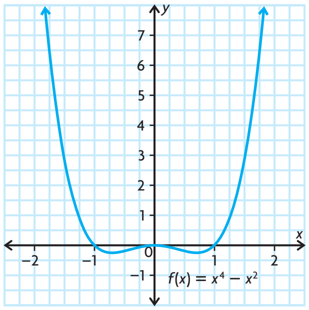
6. **a)** Answers may vary. For example, $f(x) = 2x^3 + 5$.

b) Answers may vary. For example, $f(x) = 6x^2 + x - 4$.

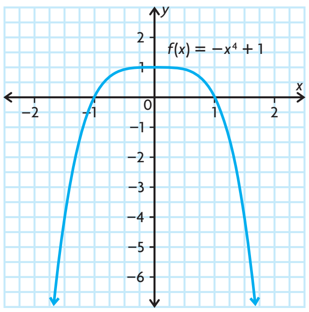
c) Answers may vary. For example, $f(x) = -x^4 - x^3 + 7$.

d) Answers may vary. For example, $f(x) = -9x^5 + x^4 - x^3 - 2$.

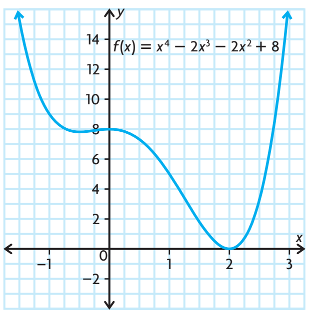
7. **a)** Answers may vary. For example:



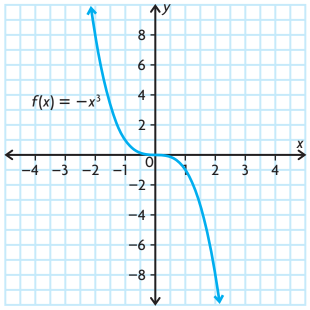
b) Answers may vary. For example:



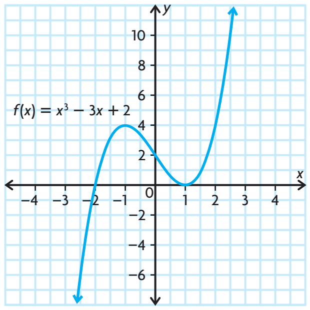
c) Answers may vary. For example:



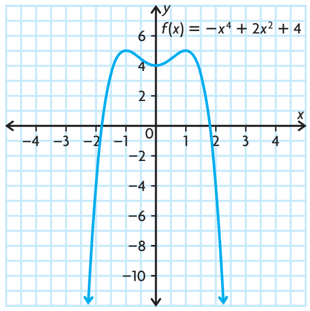
d) Answers may vary. For example:



e) Answers may vary. For example:



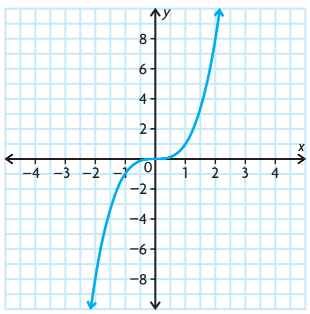
f) Answers may vary. For example:



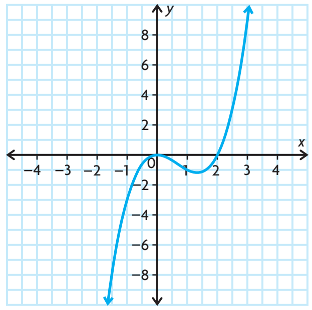
8. An odd-degree polynomial can have only local maximums and minimums because the y-value goes to $-\infty$ and ∞ at each end of the function. An even-degree polynomial can have absolute maximums and minimums because it will go to either $-\infty$ at both ends or ∞ at both ends of the function.

9. even number of turning points

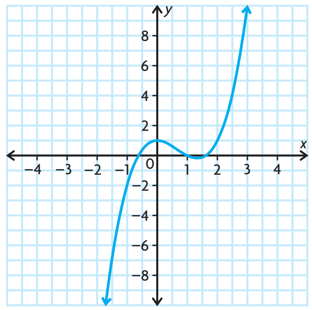
10. **a)** Answers may vary. For example: $f(x) = x^3$



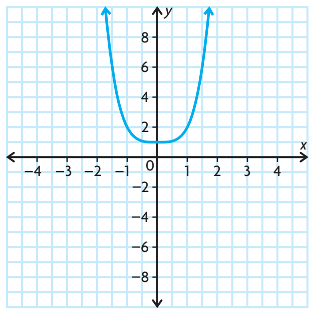
b) Answers may vary. For example: $f(x) = x^3 - 2x^2$



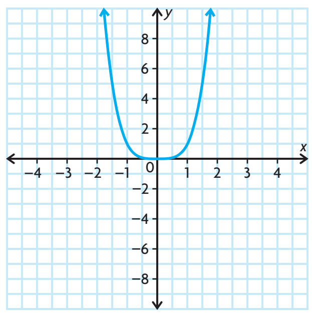
c) Answers may vary. For example: $f(x) = x^3 - 2x^2 + 1$



11. **a)** Answers may vary. For example: $f(x) = x^4 + 1$

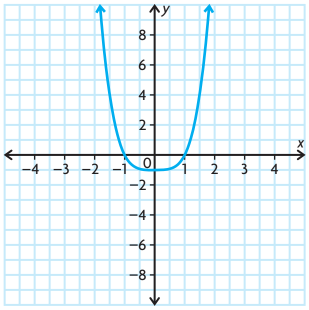


b) Answers may vary. For example: $f(x) = x^4$

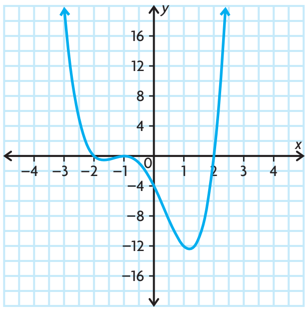


Answers

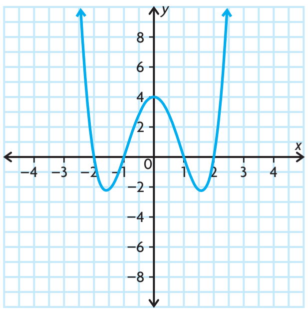
c) Answers may vary. For example:
$f(x) = x^4 - 1$



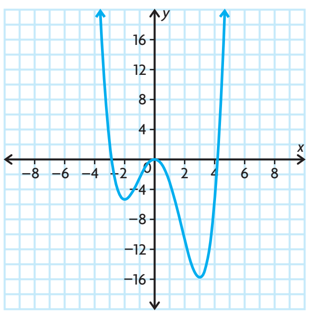
d) Answers may vary. For example:
$f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$



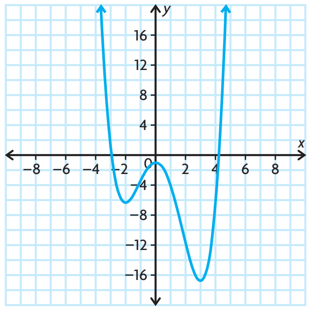
e) Answers may vary. For example:
$f(x) = x^4 - 5x^2 + 4$



12. **a)** Answers may vary. For example:
$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$



and $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 - 1$



b) zero and leading coefficient of the function

13. **a)** 700 people
b) The population will decrease because the leading coefficient is negative.

14. **a)** False; Answers may vary. For example, $f(x) = x^2 + x$ is not an even function.
b) True
c) False; Answers may vary. For example, $f(x) = x^2 + 1$ has no zeros.
d) False; Answers may vary. For example, $f(x) = -x^2$ has end behaviour opposite the behaviour stated.

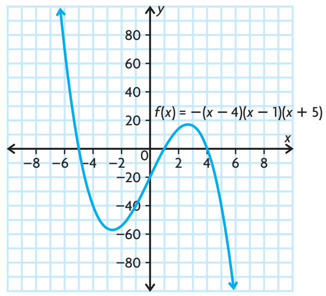
15. Answers may vary. For example, "What are the turning points of the function?", "What is the leading coefficient of the function?", and "What are the zeros of the function?" If the function has 0 turning points or an even number of turning points, then it must extend to the opposite side of the x-axis. If it has an odd number of turning points, it must extend to the same side of the x-axis. If the leading coefficient is known, it can be determined exactly which quadrants the function extends to/from and if the function has been vertically stretched. If the zeros are known, it can be determined if the function has been vertically translated up or down.

16. **a)** $b = 0$
b) $b = 0, d = 0$

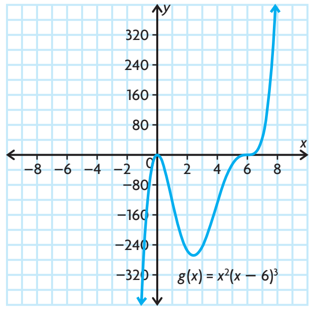
Lesson 3.3, pp. 146–148

1. **a)** C: The graph has zeros of -1 and 3, and it extends from quadrant III to quadrant I.
b) A: The graph has zeros of -1 and 3, and it extends from quadrant II to quadrant III.
c) B: The graph has zeros of -1 and 3, and it extends from quadrant II to quadrant IV.
d) D: The graph has zeros of -1, 0, 3, and 5, and it extends from quadrant II to quadrant I.

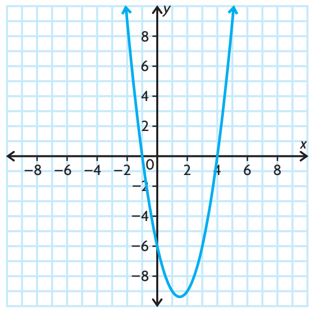
2. **a)**



b)



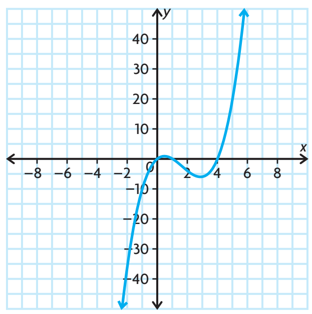
3. **a)** $f(x) = k(x + 1)(x - 4)$;
$f(x) = 4(x + 1)(x - 4)$;
$f(x) = -2(x + 1)(x - 4)$
b) $f(x) = \frac{3}{2}(x + 1)(x - 4)$



4. **a)** $y = 0.5(x + 3)(x - 2)(x - 5)$
b) $y = -(x + 1)^2(x - 2)(x - 4)$

5. Family 1: A, G, I
Family 2: B, E
Family 3: C, F, H, K
Family 4: D, J, L

6. **a)**



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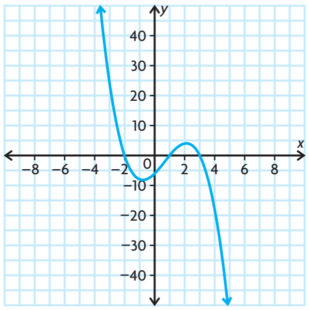
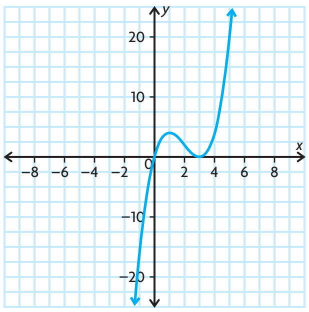


c)



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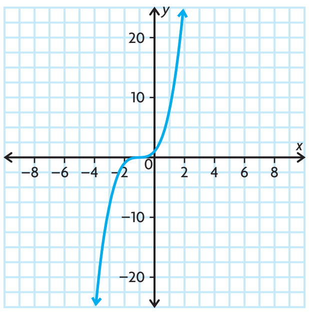
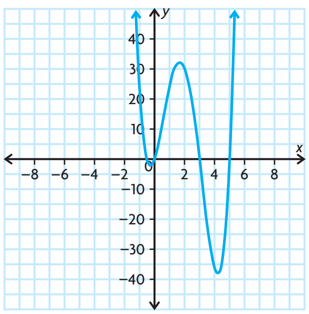
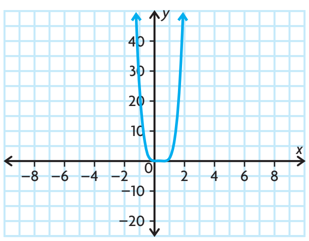


e)

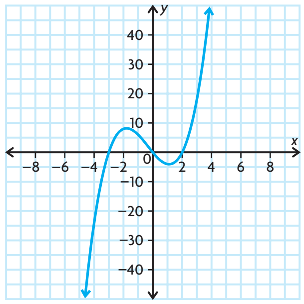


f)

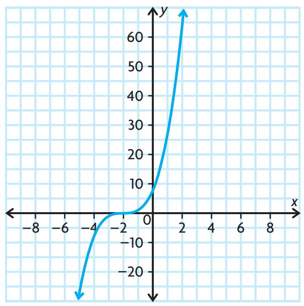


7. a) Answers may vary. For example:

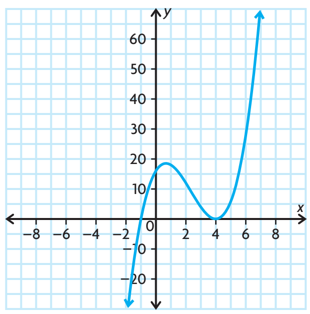
i) $y = x(x + 3)(x - 2)$



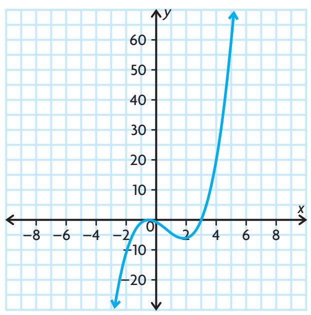
ii) $y = (x + 2)^3$



iii) $y = (x + 1)(x - 4)^2$



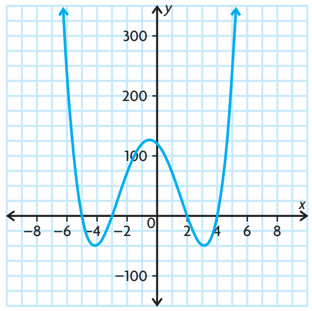
iv) $y = (x - 3)\left(x + \frac{1}{2}\right)^2$



b) No, as all the functions belong to a family of equations.

8. Answers may vary. For example:

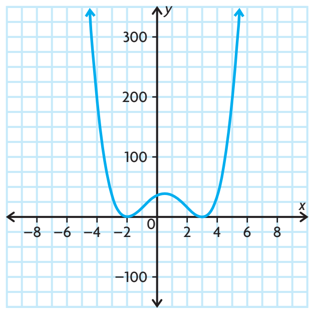
a) $y = (x + 5)(x + 3)(x - 2)(x - 4)$



$y = 2(x + 5)(x + 3)(x - 2)(x - 4)$

$y = -5(x + 5)(x + 3)(x - 2)(x - 4)$

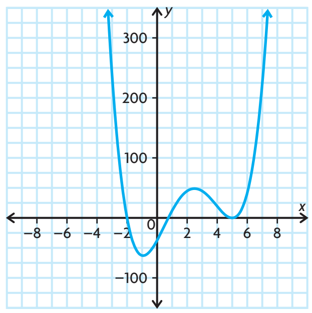
b) $y = (x + 2)^2(x - 3)^2$



$y = 10(x + 2)^2(x - 3)^2$

$y = 7(x + 2)^2(x - 3)^2$

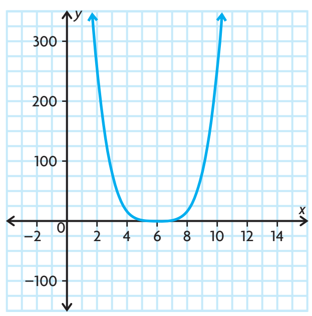
c) $y = (x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$



$y = -(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

$y = \frac{2}{5}(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

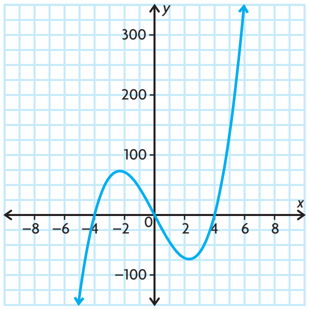
d) $y = (x - 6)^4$



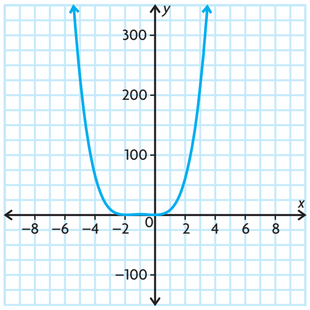
$y = 15(x - 6)^4$

$y = -3(x - 6)^4$

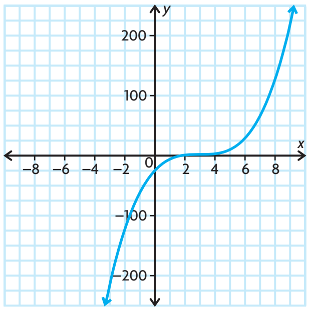
9. **a)**



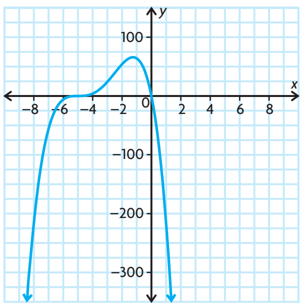
b)



c)

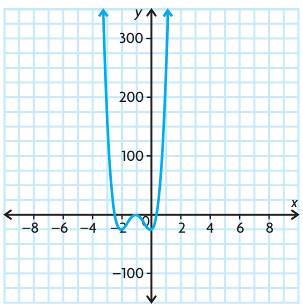


d)

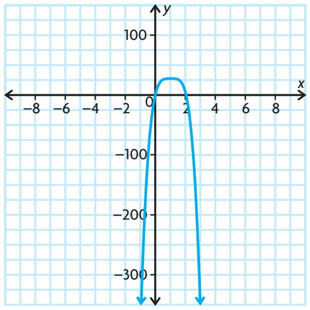


10. Answers may vary. For example:

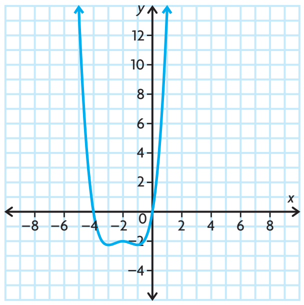
a)



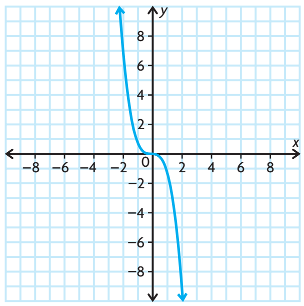
b)



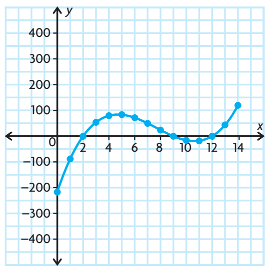
c)



d)



11. **a)**



b) $y = (x - 2)(x - 9)(x - 12)$

c) No; $\{x \in \mathbf{R} \mid 0 \le x \le 14\}$

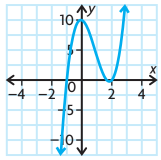
12. **a)** $y = x^3 + 2x^2 - x - 2$

b) $y = -\frac{2}{5}(x - 1)(x + 2)(x + 4)$

13. **a)** $f(x) = -6(x + 3)(x + 5)$

b) $f(x) = 2(x + 2)(x - 3)(x - 4)$

14. $k = 3$



The zeros are $\frac{5}{3}$, -1, and 2.

$f(x) = (3x - 5)(x + 1)(x - 2)$

15. **a)** It has zeros at 2 and 4, and it has turning points at 2, 3, and 4. It extends from quadrant II to quadrant I.

b) It has zeros at -4 and 3, and it has turning points at $-\frac{5}{3}$ and 3. It extends from quadrant III to quadrant I.

16. **a)** 832 cm^3

b) 2.93 cm by 24.14 cm by 14.14 cm or 5 cm by 20 cm by 10 cm

c) $0 < x < 10$; The values of x are the side lengths of squares that can be cut from the sheet of cardboard to produce a box with positive volume. Since the sheet of cardboard is 30 cm by 20 cm, the side lengths of a square cut from each corner have to be less than 10 cm, or an entire edge would be cut away, leaving nothing to fold up.

d) The square that is cut from each corner must be larger than 0 cm by 0 cm but smaller than 10 cm by 10 cm.

Lesson 3.4, pp. 155–158

1. **a)** B: $y = x^3$ has been vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 1 unit up.

b) C: $y = x^3$ has been reflected in the x-axis, vertically compressed by a factor of $\frac{1}{3}$, horizontally translated 1 unit to the left, and vertically translated 1 unit down.

c) A: $y = x^4$ has been vertically compressed by a factor of 0.2, horizontally translated 4 units to the right, and vertically translated 3 units down.

d) D: $y = x^4$ has been reflected in the x-axis, vertically stretched by a factor of 1.5, horizontally translated 3 units to the left, and vertically translated 4 units up.

2. **a)** $y = x^4$; vertical stretch by a factor of $\frac{5}{4}$ and vertical translation of 3 units up

b) $y = x$; vertical stretch by a factor of 3 and vertical translation of 4 units down

c) $y = x^3$; horizontal compression by a factor of $\frac{1}{3}$, horizontal translation of $\frac{4}{3}$ units to the left, and vertical translation of 7 units down

d) $y = x^4$; reflection in the x-axis and horizontal translation of 8 units to the left

e) $y = x^2$; reflection in the x-axis, vertical stretch by a factor of 4.8, and horizontal translation 3 units left

f) $y = x^3$; vertical stretch by a factor of 2, horizontal stretch by a factor of 5, horizontal translation of 7 units to the left, and vertical translation of 4 units down

3. a) $y = x^3$ has been translated 3 units to the left and 4 units down.
$y = (x + 3)^3 - 4$

b) $y = x^4$ has been reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 4 units to the left, and vertically translated 5 units up.
$y = -2(x + 4)^4 + 5$

c) $y = x^4$ has been vertically compressed by a factor of $\frac{1}{4}$, horizontally translated 1 unit to the right, and vertically translated 2 units down.
$y = \frac{1}{4}(x - 1)^4 - 2$

d) $y = x^3$ has been reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 4 units down.
$y = -2(x - 3)^3 - 4$

4. a) vertically stretched by a factor of 12, horizontally translated 9 units to the right, and vertically translated 7 units down

b) horizontally stretched by a factor of $\frac{8}{7}$, horizontally translated 1 unit to the left, and vertically translated 3 units up

c) vertically stretched by a factor of 2, reflected in the x-axis, horizontally translated 6 units to the right, and vertically translated 8 units down

d) horizontally translated 9 units to the left

e) reflected in the x-axis, vertically stretched by a factor of 2, reflected in the y-axis, horizontally compressed by a factor of $\frac{1}{3}$, horizontally translated 4 units to the right, and vertically translated 5 units down

f) horizontally stretched by a factor of $\frac{4}{3}$ and horizontally translated 10 units to the right

5. a) $y = 8x^2 - 11$
$y = x^2$ was vertically stretched by a factor of 8 and vertically translated 11 units down.

b) $y = -\frac{1}{4}x^2 + 1.25$
$y = x^2$ was reflected in the x-axis, vertically compressed by a factor of $\frac{1}{4}$, and vertically translated 1.25 units up.

6. a) $\left(-6\frac{1}{5}, -\frac{1}{2}\right), \left(-6, 0\right), \left(-5\frac{3}{5}, 4\right)$

b) $(2, 2), (0, 3), (-4, 11)$

c) $\left(3, 2\frac{1}{2}\right), \left(4, -\frac{1}{2}\right), \left(6, -24\frac{1}{2}\right)$

d) $\left(-7, -2\frac{1}{10}\right), \left(0, -2\right), \left(14, -1\frac{1}{5}\right)$

e) $\left(1, 1\frac{9}{10}\right), \left(0, \frac{9}{10}\right), \left(-2, -7\frac{1}{10}\right)$

f) $(-11, -8), (-4, -7), (10, 1)$

7. $y = -\frac{1}{4}(x - 1)^4 + 3$

8. $(-2, 8), (0, 0), (2, -8)$

9. a) -2 and -4

b) 4

c) -3 and 1

d) no x-intercepts

e) 6.68 and 9.32

f) -3.86

10. a) 1; $0 = 2(x - 4)^3 + 1$ has only one solution.

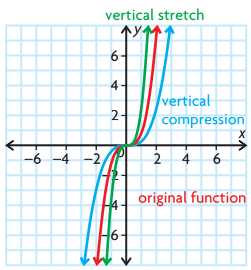
b) 0; $0 = 2(x - 4)^4 + 1$ has no solution.

c) 1 when n is odd, since an odd root results in only one value; 0 when n is even, since there is no value for an even root of a negative number.

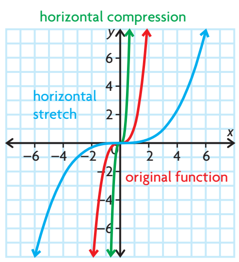
11. a) The reflection of the function $y = x^n$ in the x-axis will be the same as its reflection in the y-axis for odd values of n.

b) The reflections will be different for even values of n. The reflection in the x-axis will be $y = -x^n$, and the reflection in the y-axis will be $y = (-x)^n$. For odd values of n, $-x^n$ equals $(-x^n)$. For even values of n, $-x^n$ does not equal $(-x^n)$.

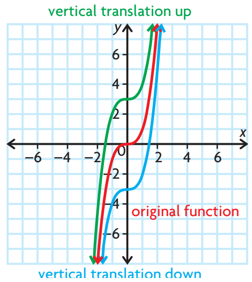
12. a) Vertical stretch and compression:
$y = ax^3$



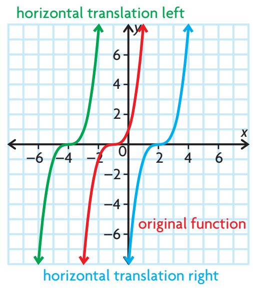
Horizontal stretch and compression:
$y = (kx)^3$



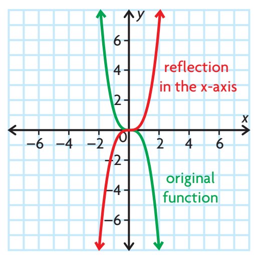
Vertical translation up or down:
$y = x^3 + c$



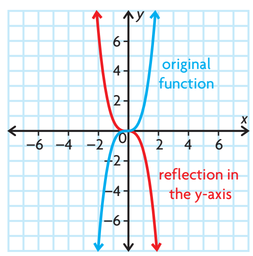
Horizontal translation left or right:
$y = (x - d)^3$



Reflection in the x-axis: $y = -x^3$



Reflection in the y-axis: $y = (-x)^3$



b) When using a table of values to sketch the graph of a function, you may not select a large enough range of values for the domain to produce an accurate representation of the function.

13. Yes, you can. The zeros of the first function have the same spacing between them as the zeros of the second function. Also, the ratio of the distances of the two curves above or below the x-axis at similar distances between the zeros is always the same. Therefore, the two curves have the same general shape, and one can be transformed into the other.

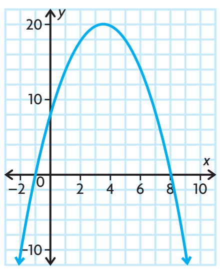
14. $y = (x - 1)^2 (x + 1)^2$ has zeroes at $x = \pm 1$ where the x-axis is tangent to these points. $y = 2(x - 1)^2(x + 1)^2 + 1$ is obtained by vertically stretching the original function by a factor of 2 and vertically translating up 1 unit. This results in a new graph that has no zeroes.

15. $f(x) = 5(2(x + 3))^2 + 1$

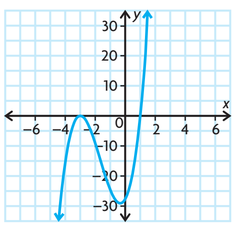
Mid-Chapter Review, p. 161

1. a) Yes
   b) No; it contains a rational exponent.
   c) Yes
   d) No; it is a rational function.
2. a) Answers may vary. For example, $f(x) = x^3 + 2x^2 - 8x + 1$.
   b) Answers may vary. For example, $f(x) = 5x^4 - x^2 - 7$.
   c) Answers may vary. For example, $f(x) = 7x^6 + 3$.
   d) Answers may vary. For example, $f(x) = -2x^5 - 4x^4 + 3x^3 - 2x^2 + 9$.
3. a) As $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$.
   b) As $x \to \pm\infty$, $y \to \infty$.
   c) As $x \to -\infty$, $y \to -\infty$ and as $x \to \infty$, $y \to \infty$.
   d) As $x \to \pm\infty$, $y \to -\infty$.
4. a) even c) odd
   b) odd d) even
5. Answers may vary. For example:

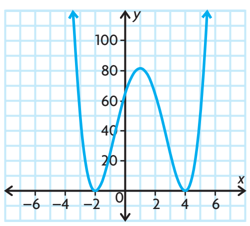
a)



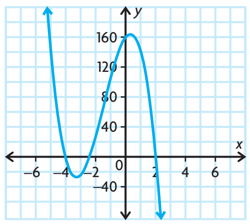
b)



c)



d)



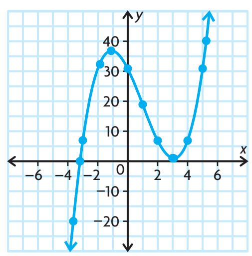
6. end behaviours
7. $y = 5(x - 2)(x + 3)^2(x - 5)$
8. a) reflection in the x-axis, vertical stretch by a factor of 25, horizontal compression by a factor of $\frac{1}{3}$, horizontal translation 4 units to the left, vertical translation 60 units down
   b) vertical stretch by a factor of 8, horizontal stretch by a factor of $\frac{4}{3}$, vertical translation 43 units up
   c) reflection in the y-axis, horizontal compression by a factor of $\frac{1}{13}$, horizontal translation 2 units to the right, vertical translation 13 units up
   d) vertical compression by a factor of $\frac{8}{11}$, reflection in the y-axis, vertical translation 1 unit down
9. vertically stretched by a factor of 5, horizontally translated 4 units to the left, and vertically translated 2 units down

Lesson 3.5, pp. 168–170

1. a) i) $x^3 - 14x^2 - 24x - 38$ remainder -87
   ii) $x^3 - 20x^2 + 84x - 326$ remainder 1293
   iii) $x^3 - 15x^2 - 11x - 1$ remainder -12
   b) No; because for each division problem there is a remainder.
2. a) 2 b) 2 c) 1 d) not possible
3. a) $x^2 - 15x + 6$ remainder $-48x + 14$
   b) $5x^2 - 19x + 60$ remainder -184
   c) $x - 6$ remainder $-6x^2 + 22x + 6$
   d) Not possible
4.

Dividend	Divisor	Quotient	Remainder
$2x^3 - 5x^2 + 8x + 4$	$x + 3$	$2x^2 - 11x + 41$	-119
$6x^4 + 12x^3 - 10x^2 - 4x + 29$	$2x + 4$	$3x^3 - 5x + 8$	-3
$6x^4 + 2x^3 + 3x^2 - 11x - 9$	$3x + 1$	$2x^3 + x - 4$	-5
$3x^3 + x^2 - 6x + 16$	$x + 2$	$3x^2 - 5x + 4$	8

5. a) $x^2 + 4x + 14$ remainder 57
   b) $x^2 - 6$ remainder 13
   c) $x^2 + 2x - 3$ remainder -2
   d) $x^2 + 3x - 9$ remainder $-16x + 62$
   e) $x + 1$ remainder $8x^2 - 8x + 11$
   f) $x + 3$ remainder $-4x^3 - 4x^2 + 8x + 14$
6. a) $x^2 + 3x + 2$ no remainder
   b) $2x^2 - 5x - 12$ remainder 7
   c) $6x^3 - 5x^2 - 19x + 10$ remainder -2
   d) $x^2 + 2x - 8$ remainder -2
   e) $6x^3 - 31x^2 + 45x - 18$ no remainder
   f) $3x^2 - 1$ no remainder
7. a) $x^3 + 4x^2 - 51x + 89$
   b) $3x^4 - 2x^3 + 3x^2 - 38x + 39$
   c) $5x^4 + 22x^3 - 17x^2 + 21x + 10$
   d) $x^6 + 8x^5 + 5x^4 - 13x^3 - 72x^2 + 49x - 3$
8. a) $r = 20$ c) $r = 0$
   b) $r = x - 22$ d) $r = 2x^2 + 2$
9. a) $x + 3$ c) $x + 4$
   b) $x + 10$ d) $x - 2$
10. a) $x + 5$ is a factor since there is no remainder.
    b) $x + 2$ is a factor since there is no remainder.
    c) $x - 2$ is not a factor since there is a remainder of 2.
    d) $x - 1$ is not a factor since there is a remainder of 1.
    e) $3x + 5$ is not a factor since there is a remainder of $-\frac{13}{3}$.
    f) $5x - 1$ is not a factor since there is a remainder of -8.
11. $(x + 1)$ cm
12. a) 7 b) 3
13. 2
14. Yes, $f(x)$ is always divisible by $x - 1$. Regardless of the value of n, $f(x) = x^n - 1$ can always be written as $f(x) = x^n + 0x^{n-1} + 0x^{n-2} + \dots 0x - 1$. Therefore, the same pattern continues when dividing $x^n - 1$ by $x - 1$, regardless of how large n is, and there is never a remainder.
15. a) $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)(x^2 + x - 6)$ remainder 7
    b) $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)(x + 3)(x - 2)$ remainder 7
    c)



16. Answers may vary. For example:

$$
\begin{array}{rl}
 & 2x^3 + 9x^2 + 2x - 1 \\
x - 3 & \overline{)\,2x^4 + 3x^3 - 25x^2 - 7x - 14} \\
2x^3(x-3) \rightarrow & \underline{2x^4 - 6x^3} \\
 & \quad 9x^3 - 25x^2 \\
9x^2(x-3) \rightarrow & \quad \underline{9x^3 - 27x^2} \\
 & \qquad 2x^2 - 7x \\
2x(x-3) \rightarrow & \qquad \underline{2x^2 - 6x} \\
 & \qquad -1x - 14 \\
-1(x-3) \rightarrow & \qquad \underline{-1x + 3} \\
 & \qquad\quad -17
\end{array}
$$

17. $r = 2x + 5$ cm

18. **a)** $x^2 + xy + y^2$
b) $x^2 - 2xy + y^2$

19. $x - y$ is a factor because there is no remainder.

20. $[q(x) + 1](x + 5)$

Lesson 3.6, pp. 176–177

1. **a)** **i)** 64
ii) 22
iii) 12
b) No, according to the factor theorem, $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

2. **a)** not divisible by $x - 1$
b) divisible by $x - 1$
c) not divisible by $x - 1$
d) divisible $x - 1$

3. $(x + 1)(x + 3)(x - 2)$

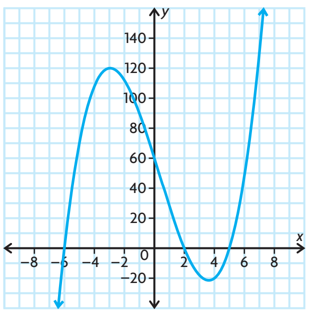
4. **a)** -1 **c)** 0 **e)** 30
b) -5 **d)** -34 **f)** 0

5. **a)** yes **c)** yes
b) no **d)** no

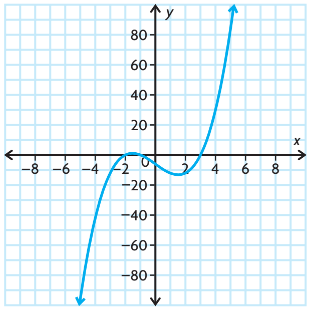
6. **a)** $(x - 2)(x - 4)(x + 3)$
b) $(x - 1)(2x + 3)(2x + 5)$
c) $x(x - 2)(x + 4)(x + 6)$
d) $(x + 2)(x + 5)(4x - 9)(x - 3)$
e) $x(x + 2)(x + 1)(x - 3)(x - 5)$
f) $(x - 3)(x - 3)(x + 4)(x + 4)$

7. **a)** $(x - 2)(x + 5)(x + 6)$
b) $(x + 1)(x - 3)(x + 2)$
c) $(x + 1)(x - 1)(x - 2)(x + 2)$
d) $(x - 2)(x + 1)(x + 8)(x - 4)$
e) $(x - 1)(x^2 + 1)$
f) $(x - 1)(x^2 + 1)(x^2 + 1)$

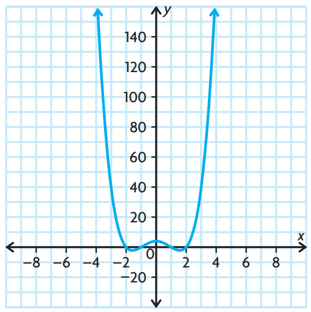
8. **a)**



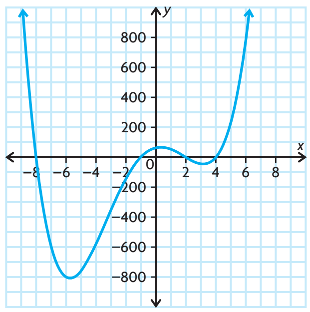
b)



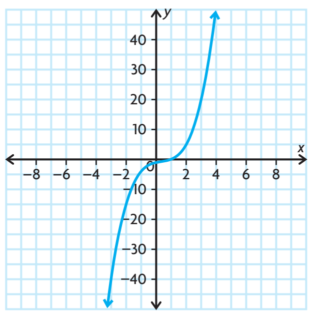
c)



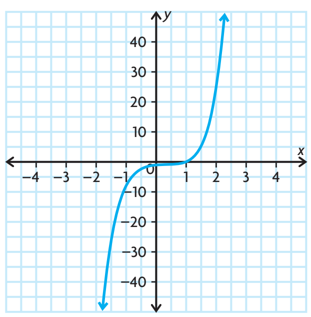
d)



e)



f)



9. 20

10. $a = 6, b = 3$

11. For $x^n - a^n$, if n is even, they're both factors. If n is odd, only $(x - a)$ is a factor. For $x^n + a^n$, if n is even, neither is a factor. If n is odd, only $(x + a)$ is a factor.

12. $a = -2, b = 22$;
The other factor is $-2x + 3$.

13. -6

14. $x^4 - a^4$
$= (x^2)^2 - (a^2)^2$
$= (x^2 + a^2)(x^2 - a^2)$
$= (x^2 + a^2)(x + a)(x - a)$

15. Answers may vary. For example: if $f(x) = k(x - a)$, then $f(a) = k(a - a)$ $= k(0) = 0$.

16. $x^2 - x - 2 = (x - 2)(x + 1)$;
If $f(x) = x^3 - 6x^2 + 3x + 10$, then $f(2) = 0$ and $f(-1) = 0$.

17. If $f(x) = (x + a)^5 + (x + c)^5 + (a - c)^5$, then $f(-a) = 0$

Lesson 3.7, p. 182

1. $(x + b)(x^2 - bx + b^2)$

2. **a)** $(x - 4)(x^2 + 4x + 16)$
b) $(x - 5)(x^2 + 5x + 25)$
c) $(x + 2)(x^2 - 2x + 4)$
d) $(2x - 3)(4x^2 + 6x + 9)$
e) $(4x - 5)(16x^2 + 20x + 25)$
f) $(x + 1)(x^2 - x + 1)$
g) $(3x + 2)(9x^2 - 6x + 4)$
h) $(10x + 9)(100x^2 - 90x + 81)$
i) $8(3x - 1)(9x^2 + 3x + 1)$

3. **a)** $(4x + 3y)(16x^2 - 12xy + 9y^2)$
b) $(-3x)(x - 2)(x^2 + 2x + 4)$
c) $(4 - x)(7x^2 + 25x + 31)$
d) $(x^2 + 4)(x^4 - 4x^2 + 16)$

4. **a)** $(x - 7)(x^2 + 7x + 49)$
b) $(6x - 1)(36x^2 + 6x + 1)$
c) $(x + 10)(x^2 - 10x + 100)$
d) $(5x - 8)(25x^2 + 40x + 64)$
e) $(4x - 11)(16x^2 + 44x + 121)$
f) $(7x + 3)(49x^2 - 21x + 9)$
g) $(8x + 1)(64x^2 - 8x + 1)$
h) $(11x + 12)(121x^2 - 132x + 144)$
i) $(8 - 11x)(64 + 88x + 121x^2)$

5. **a)** $\left(\frac{1}{3}x - \frac{2}{5}\right)\left(\frac{1}{9}x^2 + \frac{2}{15}x + \frac{4}{25}\right)$
b) $-16x^2(3x + 2)(9x^2 - 6x + 4)$
c) $7(4x - 5)(x^2 - x + 1)$
d) $\left(\frac{1}{2}x - 2\right)\left(\frac{1}{4}x^2 + x + 4\right)$
$\left(\frac{1}{64}x^6 + x^3 + 64\right)$

6. Agree; by the formulas for factoring the sum and difference of cubes, the numerator of the fraction is equivalent to $(a^3 + b^3) + (a^3 - b^3)$. Since $(a^3 + b^3) + (a^3 - b^3) = 2a^3$, the entire fraction is equal to 1.

Answers

7. a) $1^3 + 12^3 = (1 + 12)(1^2 - (1)(12) + 12^2)$
$= (13)(133) = 1729$
b) $9^3 + 10^3 = (9 + 10)(9^2 - (9)(10) + 10^2)$
$= (19)(91) = 1729$

8. $x^9 + y^9$
$= x^{18} + 2x^9y^9 + y^{18}$
$= (x^{18} + y^{18}) + 2x^9y^9$
$= (x^6 + y^6)(x^{12} - x^6y^6 + y^{12}) + 2x^9y^9$
$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x^{12} - x^6y^6 + y^{12}) + 2x^9y^9$

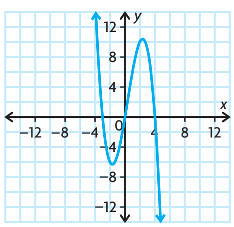
9. Answers may vary. For example, this statement is true because $a^3 - b^3$ is the same as $a^3 + (-b)^3$.

10. a) 1729 was the number of the taxicab that G. H. Hardy rode in when going to visit the mathematician Ramanujan. When Hardy told Ramanujan that the number of the taxicab he rode in was uninteresting, Ramanujan replied that the number was interesting because it was the smallest number that could be expressed as the sum of two cubes in two different ways. This is how such numbers came to be known as taxicab numbers.
b) Yes;

TN(1) = 2
TN(2) = 1729
TN(3) = 87 539 319
TN(4) = 6 963 472 309 248
TN(5) = 48 988 659 276 962 496
TN(6) = 24 153 319 581 254 312 065 344

Chapter Review, pp. 184–185

1.



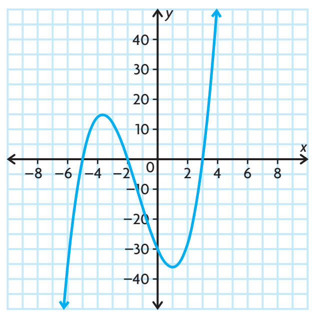
2. As $x \to -\infty$, $y \to +\infty$ and as $x \to \infty$, $y \to \infty$.

3. a) degree: 2 + 1; leading coefficient: positive; turning points: 2
b) degree: 3 + 1; leading coefficient: positive; turning points: 3

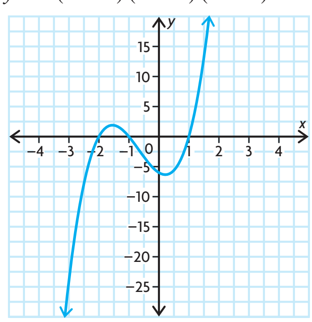
4. a) Answers may vary. For example,
$f(x) = (x + 3)(x - 6)(x - 4)$,
$f(x) = 10(x + 3)(x - 6)(x - 4)$,
$f(x) = -4(x + 3)(x - 6)(x - 4)$
b) Answers may vary. For example,
$f(x) = (x - 5)(x + 1)(x + 2)$,
$f(x) = -6(x - 5)(x + 1)(x + 2)$,
$f(x) = 9(x - 5)(x + 1)(x + 2)$
c) Answers may vary. For example,
$f(x) = (x + 7)(x - 2)(x - 3)$,
$f(x) = \frac{1}{4}(x + 7)(x - 2)(x - 3)$,
$f(x) = 3(x + 7)(x - 2)(x - 3)$
d) Answers may vary. For example,
$f(x) = (x - 9)(x + 5)(x + 4)$,
$f(x) = 7(x - 9)(x + 5)(x + 4)$,
$f(x) = -\frac{1}{3}(x - 9)(x + 5)(x + 4)$

5. a) Answers may vary. For example,
$f(x) = (x + 6)(x - 2)(x - 5)(x - 8)$,
$f(x) = 2(x + 6)(x - 2)(x - 5)(x - 8)$,
$f(x) = -8(x + 6)(x - 2)(x - 5)(x - 8)$
b) Answers may vary. For example,
$f(x) = (x - 4)(x + 8)(x - 1)(x - 2)$,
$f(x) = \frac{3}{4}(x - 4)(x + 8)(x - 1)(x - 2)$,
$f(x) = -12(x - 4)(x + 8)(x - 1)(x - 2)$
c) Answers may vary. For example,
$f(x) = x(x + 1)(x - 9)(x - 10)$,
$f(x) = 5x(x + 1)(x - 9)(x - 10)$,
$f(x) = -3x(x + 1)(x - 9)(x - 10)$
d) Answers may vary. For example,
$f(x) = (x + 3)(x - 3)(x + 6)(x - 6)$,
$f(x) = \frac{2}{5}(x + 3)(x - 3)(x + 6)(x - 6)$,
$f(x) = -10(x + 3)(x - 3)(x + 6)(x - 6)$

6.



7. $y = 3(x - 1)(x + 1)(x + 2)$



8. a) reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 1 unit to the right, and vertically translated 23 units up
b) horizontally stretched by a factor of $\frac{13}{12}$, horizontally translated 9 units to the left, and vertically translated 14 units down
c) horizontally translated 4 units to the right
d) horizontally translated $\frac{3}{7}$ units to the left
e) vertically stretched by a factor of 40, reflected in the y-axis, horizontally compressed by a factor of $\frac{1}{7}$, horizontally translated 10 units to the right, and vertically translated 9 units up

9. a) Answers will vary. For example, $(-2, -5400)$, $(3, 0)$, and $(8, 5400)$.
b) Answers will vary. For example, $(-7, -18)$, $(0, -19)$, and $(7, -20)$.
c) Answers will vary. For example, $\left(-6, \frac{182}{11}\right)$, $(-5, 16)$, and $\left(-4, \frac{170}{11}\right)$.
d) Answers will vary. For example, $(-2, -86)$, $(0, 14)$, and $(2, 114)$.
e) Answers will vary. For example, $(-1, -44)$, $(0, -45)$, and $(1, -46)$.
f) Answers will vary. For example, $(5, 1006)$, $(12, 6)$, and $(19, -994)$.

10. a) $2x^2 - 5x + 28$ remainder -144
b) $x^2 + 4x + 5$ remainder $26x + 33$
c) $2x - 6$ remainder $10x^2 + 27x - 34$
d) $x - 4$ remainder $4x^3 + 17x^2 - 8x - 18$

11. a) $(x + 2)(2x^2 + x - 3)$ remainder 1
b) $(x + 2)(3x^2 + 7x + 3)$ remainder -3
c) $(x + 2)(2x^3 + x^2 - 18x - 9)$ remainder 0
d) $(x + 2)(2x^2 - 5)$ remainder 6

12. a) $2x^3 - 7x^2 - 107x + 75$
b) $4x^4 + 3x^3 - 8x^2 + 22x + 17$
c) $3x^4 + 14x^3 - 42x^2 + 3x + 33$
d) $3x^6 - 11x^5 - 9x^4 + 47x^3 - 46x + 14$

13. 13

14. a) $(x + 1)(x - 8)(x + 2)$
b) $(x - 4)(2x + 3)(x + 3)$
c) $x(x - 2)(x - 3)(3x - 4)$
d) $(x - 1)(x + 4)(x + 4)(x + 4)$

15. a) $(x - 2)(4x + 5)(2x - 1)$
b) $(2x + 5)(x - 2)(x + 3)$
c) $(x - 3)(x - 3)(x - 3)(x + 2)$
d) $(2x + 1)(2x + 1)(x - 3)(x + 3)$

16. a) $(4x - 3)(16x^2 + 12x + 9)$
b) $(8x - 5)(64x^2 + 40x + 25)$
c) $(7x - 12)(49x^2 + 84x + 144)$
d) $(11x - 1)(121x^2 + 11x + 1)$

17. a) $(10x + 7)(100x^2 - 70x + 49)$
b) $(12x + 5)(144x^2 - 60x + 25)$
c) $(3x + 11)(9x^2 - 33x + 121)$
d) $(6x + 13)(36x^2 - 78x + 169)$

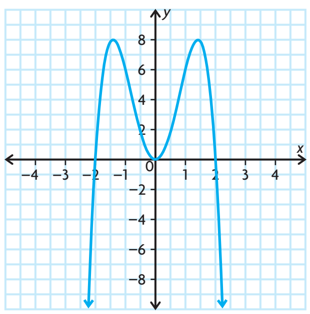
18. a) $(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$

b) $(x - y)(x + y)(x^4 + x^2y^2 + y^4)$

c) Both methods produce factors of $(x - y)$ and $(x + y)$; however, the other factors are different. Since the two factorizations must be equal to each other, this means that $(x^4 + x^2y^2 + y^4)$ must be equal to $(x^2 + xy + y^2)(x^2 - xy + y^2)$.

Chapter Self-Test, p. 186

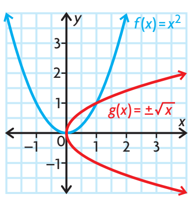
1. a) $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number. The degree of the function is n; the leading coefficient is a_n.
   b) $n - 1$
   c) n
   d) odd degree function
   e) even degree function with a negative leading coefficient
2. $y = (x + 4)(x + 2)(x - 2)$
3. a) $(x - 9)(x + 8)(2x - 1)$
   b) $(3x - 4)(3x^2 + 9x + 79)$
4. more zeros
5. $-5 < x < -3$; $x > 1$
6. yes
7. a) $y = 5(2(x - 2))^3 + 4$
   b) $(2.5, 9)$
8. $x + 5$
9. $a = -2$; zeros at $0, -2$, and 2.



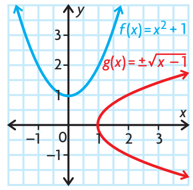
Cumulative Review Chapters 1–3, pp. 188–191

1.	(b)	**9.**	(c)	**17.**	(a)	**25.**	(c)
2.	(a)	**10.**	(d)	**18.**	(d)	**26.**	(c)
3.	(c)	**11.**	(a)	**19.**	(b)	**27.**	(d)
4.	(b)	**12.**	(a)	**20.**	(c)	**28.**	(b)
5.	(b)	**13.**	(c)	**21.**	(b)	**29.**	(c)
6.	(d)	**14.**	(d)	**22.**	(b)	**30.**	(c)
7.	(d)	**15.**	(c)	**23.**	(b)	**31.**	(c)
8.	(a)	**16.**	(c)	**24.**	(a)		

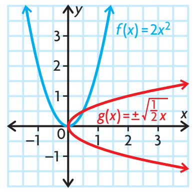
32. a)



b) Answers may vary. For example, vertical translation up produces horizontal translation of the inverse to the right.



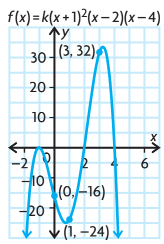
Vertical stretch produces horizontal stretch of inverse.



c) Answers may vary. For example, if the vertex of the inverse is (a, b), restrict the value of y to either $y \geq b$ or $y \leq b$.

33. Answers may vary. For example, average rates of change vary between -2 and 4, depending on the interval; instantaneous rates of change are 9 at $(0, 1)$, 0 at $(1, 5)$, -3 at $(2, 3)$, 0 at $(3, 1)$, 9 at $(4, 5)$; instantaneous rate of change is 0 at maximum $(1, 5)$ and at minimum $(3, 1)$.

34. a) $f(x) = -2(x + 1)^2(x - 2)(x - 4)$
    b) $p = 32$
    c) As $x \rightarrow \pm\infty$, $f(x) \rightarrow -\infty$; zeros: -1, 2, and 4
    d) -16
    e)

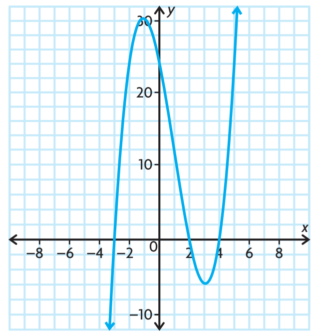


Chapter 4

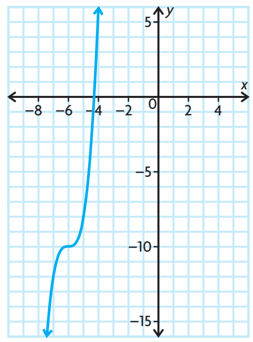
Getting Started, pp. 194–195

1. a) 3 c) 1
   b) 5 d) $\frac{64}{11}$
2. a) $x(x + 6)(x - 5)$
   b) $(x - 4)(x^2 + 4x + 16)$
   c) $3x(2x + 3)(4x^2 - 6x + 9)$
   d) $(x + 3)(x - 3)(2x + 7)$

3. a)



b)



4. 2 and 5
5. a) 3 and -3 c) $-\frac{2}{3}$ and $\frac{5}{2}$
   b) -10 and 2 d) 0.3452 and -4.345
6. a) $(3, 7)$; Answers may vary. For example, the change in distance over time from $t = 3$ to $t = 7$ is greater than at other intervals of time.
   b) $\frac{1}{3}$ m/s; $\frac{3}{4}$ m/s
   c) Answers may vary. For example, away; Erika's displacement, or distance from the sensor, is increasing.
7. a) 2 s
   b) 4.75 m/s
   c) -10.245 m/s
8. a) Disagree; You could use the quadratic formula to solve $y = x^3 + 4x^2 + 3x$ because it equals $x(x^2 + 4x + 3)$.
   b) Disagree; $y = (x + 3)^2(x - 2)$ is a cubic equation that will have two roots.
   c) Disagree; The equation $y = x^3$ will only pass through two quadrants.
   d) Agree; All polynomials are continuous and all polynomials have a y-intercept.
   e) Disagree; $f(-3) = 9$
   f) Agree; The instantaneous rates of change will tell you whether the graph is increasing, decreasing, or not changing at those points.

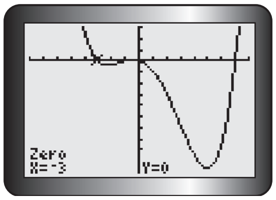
Lesson 4.1, pp. 204–206

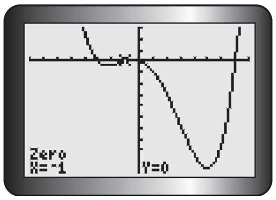
1. a) $0, 1, -2, 2$ d) $-6, \frac{5}{2}$
   b) $-\frac{3}{2}, \frac{5}{4}, -7$ e) $0, -3, 3$
   c) $3, -5, 4$ f) $-5, -2, 6$

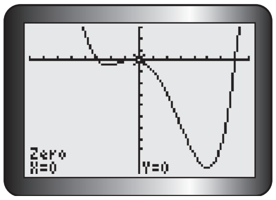
2. **a)** 0, −3, 3
b) ±3
c) $0, 2, -2, -\frac{5}{3}$
d) $0, \frac{2}{5}, 3$
e) $-3\sqrt[3]{3}$
f) $0, \pm 2\sqrt{6}$

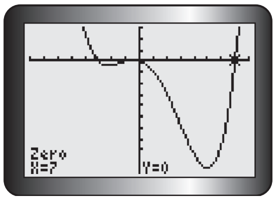
3. **a)** $6, -1, \frac{7}{2}$
b) $2x^3 - 17x^2 + 23x + 42 = 0$ or $(x - 6)(x + 1)(2x - 7)$

4. Algebraically:
$x = -1, -3, 7, 0$
Graphically:

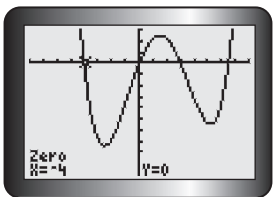


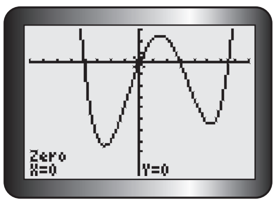


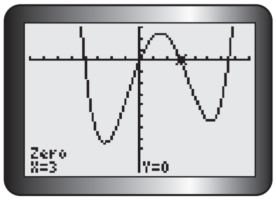


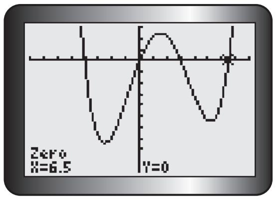


5. $0, 3, -4, \frac{13}{2}$









6. **a)** 0, 2, −5
b) −1, 17
c) 2
d) 0, −2, −5, 5
e) 0, −3, 4
f) −1

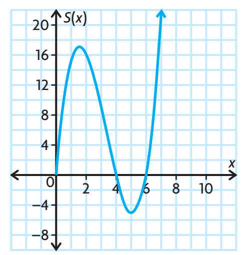
7. **a)** −3, 6, 5
b) 1, −2, −3, −5
c) $-1, \frac{1}{2}, 3$
d) $-1, \frac{3}{2}, -2$
e) $2, -4, \frac{1}{2}, \frac{5}{2}$
f) $\frac{1}{2}, \frac{5}{3}, \frac{3}{2}$

8. **a)** −3, 1, 2
b) −2, −1.24, 1, 7.24
c) −2, 1
d) −3, 0, 2
e) −0.86, 1.8, 2.33
f) −2.71, −0.16

9. **a)** 3, −2, 5
b) $0, 2, \frac{4}{3}$
c) $2, -2, -\frac{1}{3}, \frac{5}{2}$
d) 0, 3

10. 3, 4.92; either 3 cm by 3 cm or 4.92 cm by 4.92 cm.

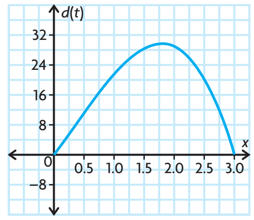
11. **a)** 4 and 6
b) 5
c) 2
d)



This is not a good model to represent Maya's score because the graph is shown for real numbers, but the number of games can only be a whole number.

12. 22.59 s

13. **a)** $d(t) = -3t(t + 2)(t - 3)$
b) 3 h after departure
c) −2, because time cannot be negative
d)



e) 1.8 h after departure

14. **a)** $0 \leq t \leq 5$
b) Answers may vary. For example, because the function involves decimals, graphing technology would be the better strategy for answering the question.
c) 0.25 L

15. All powers are even, which means every term is positive for all real numbers. Thus, the polynomial is always positive.

16. For $x = 1$, the left side is −48.
For $x = -1$, the left side is −12.

17. **a)** Answers may vary. For example, $x^3 + x^2 - x - 1 = 0$; $F(1) = 0$, so it is simple to solve using the factor theorem.
b) Answers may vary. For example, $x^2 - 2x = 0$; The common factor, x, can be factored out to solve the equation.
c) Answers may vary. For example, $x^3 - 2x^2 - 9x + 18$; An x can be factored out of the first two terms and a −2 out of the second two terms leaving you with the factors $(x - 2)(x^2 - 9)$.
d) Answers may vary. For example, $10x^2 - 7x + 1 = 0$; The roots are fractional, which makes using the quadratic formula the most sensible approach.
e) $x^3 - 8 = 0$; This is the difference of two cubes.
f) $0.856x^3 - 2.74x^2 + 0.125x - 2.89 = 0$; The presence of decimals makes using graphing technology the most sensible strategy.

18. **a)** $0 = x^4 + 10$. x^4 is non-negative for all real x, so $x^4 + 10$ is always positive.
b) A degree 5 polynomial function $y = f(x)$ has opposite end behaviour, so somewhere in the middle it must cross the x-axis. This means its corresponding equation $0 = f(x)$ will have at least one real root.

19. $y = x^5 + x + 1$; By the factor theorem, the only possible rational zeros are 1 and −1. Neither works. Because the degree is odd, the polynomial has opposite end behaviour, and hence must have at least one zero, which must be irrational.

Lesson 4.2, pp. 213–215

1. **a)** $x \le 4$; $\{x \in \mathbf{R} \mid x \le 4\}$
b) $x < 7$; $\{x \in \mathbf{R} \mid x < 7\}$
c) $x < -5$; $\{x \in \mathbf{R} \mid x < 5\}$
d) $x \ge -3$; $\{x \in \mathbf{R} \mid x \ge -3\}$
e) $x > -10$; $\{x \in \mathbf{R} \mid x > -10\}$
f) $x \ge 7$; $\{x \in \mathbf{R} \mid x \ge 7\}$

2. **a)** $x \in [-3, \infty)$
b) $x \in \left(-\infty, -\frac{2}{3}\right)$
c) $x \in [18, \infty)$
d) $x \in [1, \infty)$
e) $x \in (-\infty, 0)$
f) $x \in [-10, \infty)$

3. $-1 \le x < 6$

4. **a)** yes **b)** no **c)** no **d)** no **e)** yes **f)** no

5. **a)** $x \le 7$ **b)** $x < 0$ **c)** $x < -10$ **d)** $x \ge 5$ **e)** $x < 6$ **f)** $x \ge \frac{7}{5}$

6. **a)** yes **b)** yes **c)** no **d)** no **e)** yes **f)** no

7. **a)** $-6 < x < 2$
b) $4 < x < 8$
c) $-4 \le x \le 10$
d) $-7 \le x \le -4$
e) $7 < x < 9$
f) $-3 \le x \le -\frac{1}{2}$

8. **a)** Answers may vary. For example, $3x + 1 > 9 + x$
b) Answers may vary. For example, $3x + 1 \le 4 + x$

9. **a)** $\{x \in \mathbf{R} \mid -6 \le x \le 4\}$
b) $-13 \le 2x - 1 \le 7$

10. Attempting to solve $x - 3 < 3 - x < x - 5$ yields $3 > x > 4$, which has no solution. Solving $x - 3 > 3 - x > x - 5$ yields $3 < x < 4$.

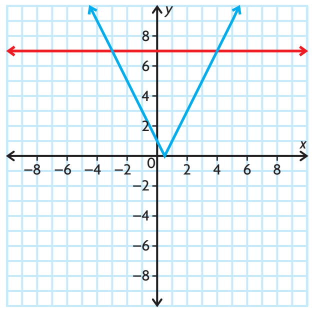
11. **a)** $\frac{1}{2}x + 1 < 3$
b) $x < 4$
c)
$$\frac{1}{2}x + 1 < 3$$
$$\frac{1}{2}x < 2$$
$$x < 4$$

12. **a)** $18 \le \frac{5}{9}(F - 32) \le 22$
b) $64.4 \le F \le 71.6$

13. 18 min

14. **a)** $\frac{9}{5}C + 32 = F$
b) $C > -40$

15. **a)**



b) $-3 < x < 4$

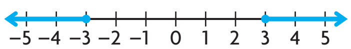
16. The solution will always have an upper and lower bound due to the manner in which the inequality is solved. The only exception to this is when there is no solution set.

17. **a)** Isolating x is very hard.
b) A graphical approach as described in the lesson yields a solution of $x > 2.75$ (rounded to two places).

18. **a)** Maintained
b) Maintained if both positive; switched if both negative; varies if one positive and one negative.
c) Maintained
d) Switched
e) Switched unless one is positive and the other is negative, in which case it is maintained. (If either side is zero, it becomes undefined.)
f) Maintained, except that $<$ and $>$ become $\le$ and $\ge$, respectively.
g) Maintained, but it is undefined for negative numbers.

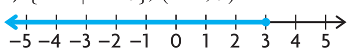
19. **a)** $\{x \in \mathbf{R} \mid -2 < x < 2\}$; $(-2, 2)$

b) $\{x \in \mathbf{R} \mid -3 \le x \ge 3\}$; $(-\infty, -3)$ or $(3, \infty)$



c) $\{x \in \mathbf{R} \mid -5 < x < 3\}$; $(-5, 3)$

d) $\{x \in \mathbf{R} \mid x \le 3\}$; $(-\infty, 3)$



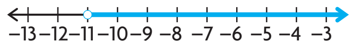
Mid-Chapter Review, p. 218

1. **a)** $0, \frac{5}{2}, 4$ **b)** -2 **c)** $1, -2, 5$ **d)** $-4, 6, 5, -5$ **e)** $0, -2, -9$ **f)** $3, -3, 2, -2$

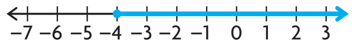
2. **a)** $h(t) = -5t^2 + 3t + 24.55$
b) 24.55 m
c) 2.5 s after jumping
d) $t > 2.5$ s; Jude is below sea level (in the water)

3. either 10 cm by 10 cm or 1.34 cm by 1.34 cm

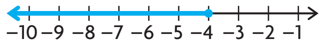
4. **a)** $x > -11$



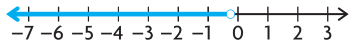
b) $x \ge -4$



c) $x \le -4$



d) $x < -\frac{1}{3}$



5. $x \in [-2, 6)$

6. **a)** Answers may vary. For example, $2x + 1 > 15$
b) Answers may vary. For example, $4x - 1 < -33$
c) Answers may vary. For example, $-3 \le 2x - 1 \le 13$
d) Answers may vary. For example, $x - 2 \le 3x - 8$

7. **a)** $f(x) = -x + 1$; $g(x) = 2x - 5$
b) $x > 2$
c)
$$f(x) < g(x)$$
$$-x + 1 < 2x - 5$$
$$-3x < -6$$
$$x > 2$$

8. **a)** $N(t) = 20 + 0.02t$; $M(t) = 15 + 0.03t$
b) $20 + 0.02t > 15 + 0.03t$
c) $0 \le t < 500$
d) Negative time has no meaning.

Lesson 4.3, pp. 225–228

1. **a)** $-2 \le x \le -1$ or $x \ge 3$
b) $-3 < x < 2$ or $x > 4$
c) $x < -\frac{2}{5}$ or $\frac{3}{4} < x < 3$
d) $-\frac{1}{4} \le x \le \frac{5}{2}$ or $x \ge 5$

2. **a)** $(-\infty, -5]$, $[-2, 0]$, and $[3, \infty)$
b) $x = 1$
c) $[-7, -3]$ and $[0, 4]$
d) $(-\infty, -4]$ and $[2, 7]$

3. $-1 < x < 2$ or $x > 3$

4. $-1.14 < x < 3$ and $x > 6.14$

5. **a)** $(-1, 2)$, $(4, \infty)$
b) $(-2, 2)$, $(2, \infty)$
c) $(-\infty, -2)$, $(0, 1)$
d) $(-\infty, 2)$, $(2, \infty)$

6. **a)** $x < -1$ or $x > 1$
b) $-3 < x < 4$
c) $x \le -\frac{1}{2}$ or $x \ge 5$
d) $-7 < x < 0$ or $x > 2$
e) $-\frac{3}{2} < x < 3$ or $x > 3$
f) $-4 \le x \le \frac{3}{2}$

Answers

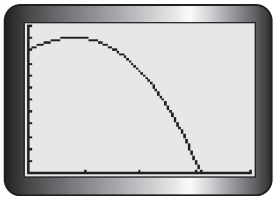
7. a) $x \leq -1$ or $x \geq 7$
b) $0 < x < 2$
c) $x \leq -3$ or $-2 \leq x \leq 1$
d) $x < -2, -1 < x < 1$ or $x > 2$
e) $x \leq -1$ or $0 \leq x \leq 3$
f) $-1 < x < -\frac{1}{2}$ or $x > 2$

8. $(-1, 1)$ and $(2, \infty)$

9. a) $x^3 + 11x^2 + 18x = 0$
b) Any values of x for which the graph of the corresponding function is above the x-axis $(y = 0)$ are solutions to the original inequality.
c) $-9 < x < -2$ or $x > 0$

10. $f(x) = -3(x + 2)(x - 1)(x - 3)^2$

11. a)



b) $0 < v < 154.77$ °C
c) 133.78 °C to 139.56 °C

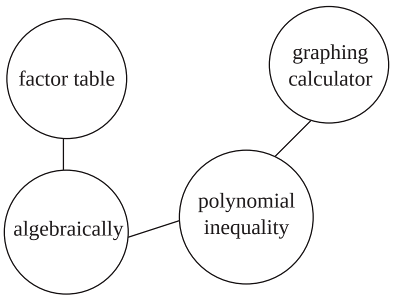
12. a) 14 m c) $0.3 < t < 2.1$
b) 3.3 s d) 1.8 s

13. $V(x) = x(50 - 2x)(30 - 2x)$;
$5 < x < 7.19$

14. a) Since all the powers are even and the coefficients are positive, the polynomial on the left is always positive.
b) Since all the powers are even and all the coefficients are negative (once all terms are brought to the left), the polynomial on the left is always negative.

15. You cannot divide by a variable expression because you do not know whether it is positive, negative, or zero.
The correct solution is $x < -1$ or $x > 4$.

16. Answers may vary. For example:



17. a) $-4 < x < -3$ or $-2 < x < 3$
b) $-1 < x < 0$ or $x > 5$

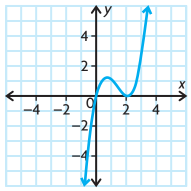
18. $x < -1$ or $x > 2$

Lesson 4.4, pp. 235–237

1. a) positive on (0, 1), (4, 7), (10, 15.5), (19, 20); negative on (1, 4), (7, 10), (15.5, 19); zero at $x = 1, 4, 7, 10$, 15.5, and 19
b) A positive slope means the cyclist's elevation is increasing, a negative slope means it is decreasing, and a zero slope means the cyclist's elevation is transitioning from increasing to decreasing or vice versa.

2. a) i) 6 ii) 12 iii) 18
b) about 12
c) The graph is increasing on (2, 6).
d) -6
e) about -6

3. a) about 0
b) It indicates that $x = 2$ is a turning point in the graph.
c)

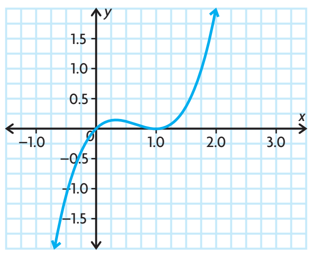


4. a) 3
b) Answers may vary. For example, $x = 4.5, 3$.

5. a) 3 c) $-\frac{1}{10}$ e) $\frac{28}{3}$
b) 17 d) -7 f) 0

6. a) 3 c) about $-\frac{1}{9}$ e) about 5.5
b) about 14 d) about -6 f) 0

7.



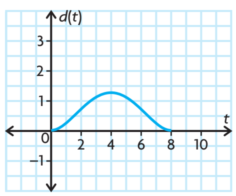
Rate of change is positive on $\left(-\infty, \frac{1}{3}\right)$ and $(1, \infty)$, negative on $\left(\frac{1}{3}, 1\right)$, and zero at $x = \frac{1}{3}$ and 1.

8. a) -55 m/s
b) about -20 m/s

9. a) about 2
b) -2
c) $y = 2x - 4$

10. a) about 10 m/s
b) about -50 m/s
c) 0 m/s

11. a)



The rate is positive for $t \in (0, 4)$, negative for $t \in (4, 8)$, and zero at $t = 0$, 4 and 8.
b) When the rate of change is zero, the boat stops.
c) When the rate of change is negative, the boat is headed back to the dock.

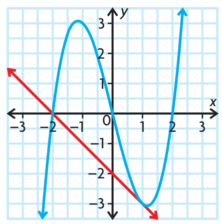
12. At $(-3, 0)$, instantaneous rate $\doteq -96$; at $(1, 0)$, instantaneous rate $\doteq 0$; at $(3, 0)$, instantaneous rate $\doteq 24$; at $(-1, 0)$, instantaneous rate $\doteq 24$

13. a) about 5 c) $2x + 3$
b) $2x + 3 + h$ d) $2(1) + 3 = 5$

14. When the instantaneous rate of change is zero, the function potentially has a local maximum or a local minimum. If the rate is positive to the left and negative to the right, it has a local maximum. If the rate is negative to the left and positive to the right, it has a local minimum.

15. a) Rate of change and $f(5)$ are both approximately 148.4.
b) Answers may vary. For example, the instantaneous rate of change at $x = 1$ is 2.7; at $x = 3$, it is 20.1; and at $x = 4$, it is 54.6.
c) The instantaneous rate of change of e^x for any value of x is e^x.

16. a) about -1
b) $y = -x - 2$
c) $(-2, 0)$



17. $x = -0.53, 2.53$

Chapter Review, pp. 240–241

1. a) ± 3 c) $0, -2, 1$
b) $\frac{1}{2}, -2$ d) $\pm 1, 2$

2. $0, 2, \frac{2}{3}, \frac{4}{5}$

3. a) $f(x) = (x - 1)(x - 2)(x + 1)(x + 2)$ or $f(x) = x^4 - 5x^2 + 4$
b) 48, 3.10

4. 2 cm by 2 cm or 7.4 cm by 7.4 cm

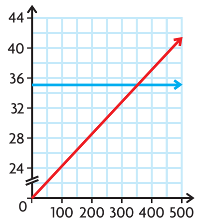
5. a) The given information states that the model is valid between 1985 and 1995, so it can be used for 1993, but not 2005.
b) Set $C(t) = 1500$ (since the units are in thousands) and solve using a graphing calculator.
c) Sales reach 1.5 million in the 8th year after 1985, so in 1993.

6. **a)** Answers may vary. For example, $2x + 1 > 17$
b) Answers may vary. For example, $3x - 4 \geq -16$
c) Answers may vary. For example, $2x + 3 \leq -21$
d) Answers may vary. For example, $-19 < 2x - 1 < -3$

7. **a)** $x \in \left(\frac{25}{2}, \infty\right)$
b) $x \in \left[-\frac{23}{8}, \infty\right)$
c) $x \in (-\infty, 2)$
d) $x \in (-\infty, 3]$

8. **a)** $\{x \in \mathbf{R} \mid -2 < x < 4\}$
b) $\{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$
c) $\{x \in \mathbf{R} \mid -3 \leq x \leq 5\}$
d) $\{x \in \mathbf{R} \mid -6 < x < -2\}$

9. **a)** The second plan is better if one calls more than 350 min per month.
b)



10. **a)** $-1 < x < 2$
b) $x \leq -\frac{3}{2}$ or $x \geq 5$
c) $x < -\frac{5}{2}$ or $1 < x < 7$
d) $x \leq -4$ or $1 \leq x \leq 5$

11. negative when $x \in (0, 5)$, positive when $x \in (-\infty, -2), (-2, 0), (5, \infty)$

12. $x \leq -3.81$

13. between January 1993 and March 1994 and between October 1995 and October 1996

14. **a)** average $= 7$, instantaneous $\doteq 8$
b) average $= 13$, instantaneous $\doteq 15$
c) average $= 129$, instantaneous $\doteq 145$
d) average $= -464$, instantaneous $\doteq -485$

15. positive when $-1 < x < 1$, negative when $x < -1$ or $x > 1$, and zero at $x = -1, 1$

16. **a)** $t \doteq 2.2$ s
b) -11 m/s
c) about -22 m/s

17. **a)** about 57.002
b) about 56.998
c) Both approximate the instantaneous rate of change at $x = 3$.

18. **a)** male: $f(x) = 0.001x^3 - 0.162x^2 + 3.394x + 72.365$; female: $g(x) = 0.0002x^3 - 0.026x^2 + 1.801x + 14.369$
b) More females than males will have lung cancer in 2006.
c) The rate was changing faster for females, on average. Looking only at 1975 and 2000, the incidence among males increased only 5.5 per 100 000, while the incidence among females increased by 31.7.
d) Between 1995 and 2000, the incidence among males decreased by 6.1 while the incidence among females increased by 5.6. Since 1998 is about halfway between 1995 and 2000, an estimate for the instantaneous rate of change in 1998 is the average rate of change from 1995 to 2000. The two rates of change are about the same in magnitude, but the rate for females is positive, while the rate for males is negative.

Chapter Self-Test, p. 242

1. $1, \frac{3}{2}, -2$

2. **a)** positive when $x < -2$ and $0 < x < 2$, negative when $-2 < x < 0$ and $x > 2$, and zero at $-2, 0, 2$
b) positive when $-1 < x < 1$, negative when $x < -1$ or $1 < x$, and zero at $x = -1, 1$
c) -1

3. **a)** Cost with card: $50 + 5n$; Cost without card: $12n$
b) at least 8 pizzas

4. **a)** $x < \frac{1}{2}$
b) $-2 \leq x \leq 1$
c) $-2 < x < -1$ or $x > 5$
d) $x \geq -3$

5. **a)** 15 m
b) 4.6 s
c) -3 m/s

6. **a)** about 5 **b)** $(1, 3)$ **c)** $y = 5x - 2$

7. Since all the exponents are even and all the coefficients are positive, all values of the function are positive and greater than or equal to 4 for all real numbers x.

8. **a)** $\{x \in \mathbf{R} \mid -2 \leq x \leq 7\}$
b) $-2 < x < 7$

9. 2 cm by 2 cm by 15 cm

Chapter 5

Getting Started, pp. 246–247

1. **a)** $(x - 5)(x + 2)$
b) $3(x + 5)(x - 1)$
c) $(4x - 7)(4x + 7)$
d) $(3x - 2)(3x - 2)$
e) $(a - 3)(3a + 10)$
f) $(2x + 3y)(3x - 7y)$

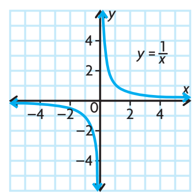
2. **a)** $3 - 2s$
b) $\frac{n^3}{3m}, m, n \neq 0$
c) $3x^2 - 4x - 1, x \neq 0$
d) $\frac{1}{5x - 2}, x \neq \frac{2}{5}$
e) $-\frac{x + 6}{3 + x}, x \neq -3, 3$
f) $\frac{a - b}{a - 3b}, a \neq -5b, \frac{3b}{2}$

3. **a)** $\frac{7}{15}$
b) $\frac{6}{x}, x \neq 0$
c) $\frac{-4x^2 + 20x - 6}{x - 3}, x \neq -2, 3$
d) $\frac{x^3 + 2x - 8x}{x^2 - 1}, x \neq -1, 0, 1, 3$

4. **a)** $1\frac{11}{21}$
b) $\frac{19x}{12}$
c) $\frac{4 + x}{x^2}, x \neq 0$
d) $\frac{3x - 6}{x^2 - 3x}, x \neq 0, 3$
e) $\frac{2x + 10 + y}{x^2 - 25}, x \neq 5, -5$
f) $\frac{-2a + 50}{(a + 3)(a - 5)(a + 3)}, x \neq -3, 4, 5$

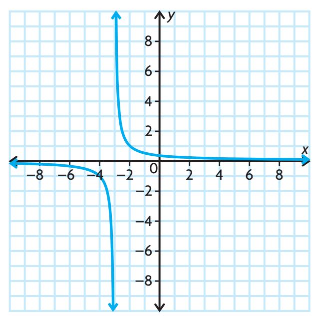
5. **a)** $x = 6$
b) $x = 2$
c) $x = 3$
d) $x = \frac{-12}{7}$

6.



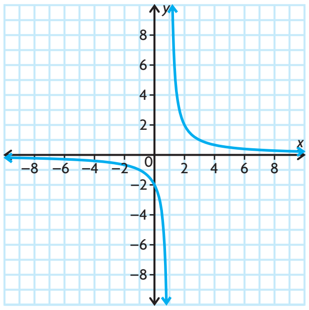
vertical: $x = 0$; horizontal: $y = 0$;
$D = \{x \in \mathbf{R} \mid x \neq 0\}$;
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

7. **a)** translated three units to the left

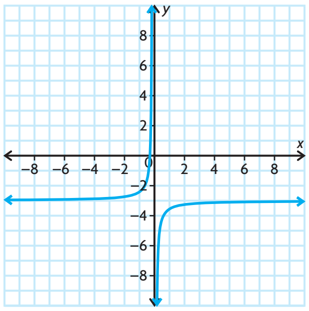


Answers

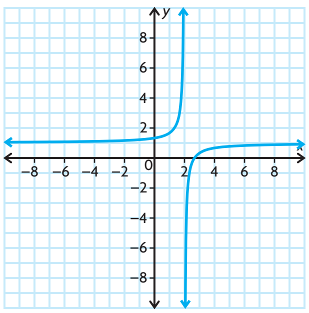
b) vertical stretch by a factor of 2 and a horizontal translation 1 unit to the right



c) reflection in the x-axis, vertical compression by a factor of $\frac{1}{2}$, and a vertical translation 3 units down



d) reflection in the x-axis, vertical compression by a factor of $\frac{2}{3}$, horizontal translation 2 units right, and a vertical translation 1 unit up



8. Factor the expressions in the numerator and the denominator. Simplify each expression as necessary. Multiply the first expression by the reciprocal of the second.

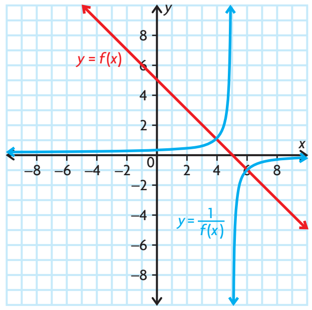
$$\frac{-3(3y-2)}{2(3y+2)}$$

Lesson 5.1, pp. 254–257

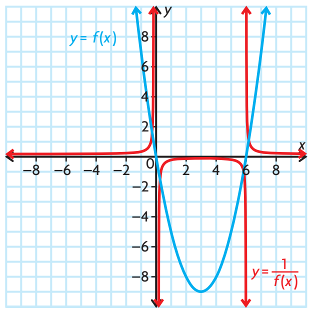
1. **a)** C; The reciprocal function is F.
b) A; The reciprocal function is E.
c) D; The reciprocal function is B.
d) F; The reciprocal function is C.
e) B; The reciprocal function is D.
f) E; The reciprocal function is A.

2. **a)** $x = 6$
b) $x = -\frac{4}{3}$
c) $x = 5$ and $x = -3$
d) $x = -\frac{5}{2}$ and $x = \frac{5}{2}$
e) no asymptotes
f) $x = -1.5$ and $x = -1$

3. **a)**



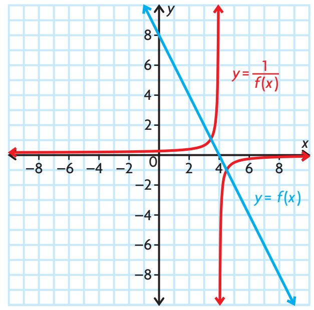
b)



4. **a)**

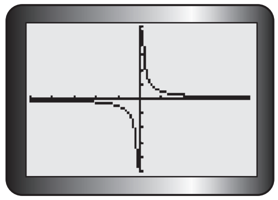
x	$f(x)$	$\frac{1}{f(x)}$
-4	16	$\frac{1}{16}$
-3	14	$\frac{1}{14}$
-2	12	$\frac{1}{12}$
-1	10	$\frac{1}{10}$
0	8	$\frac{1}{8}$
1	6	$\frac{1}{6}$
2	4	$\frac{1}{4}$
3	2	$\frac{1}{2}$
4	0	undefined
5	-2	$-\frac{1}{2}$
6	-4	$-\frac{1}{4}$
7	-6	$-\frac{1}{6}$

b)

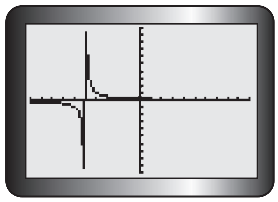


c) $f(x) = -2x + 8$, $y = \frac{1}{-2x + 8}$

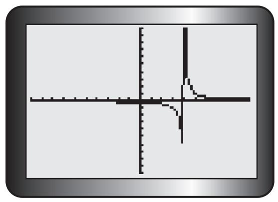
5. **a)** $y = \frac{1}{2x}$; vertical asymptote at $x = 0$



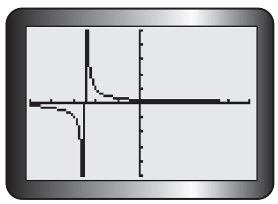
b) $y = \frac{1}{x + 5}$; vertical asymptote at $x = -5$



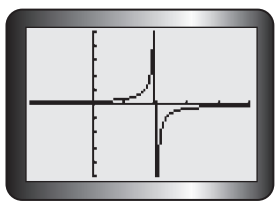
c) $y = \frac{1}{x - 4}$; vertical asymptote at $x = 4$



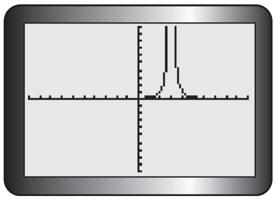
d) $y = \frac{1}{2x + 5}$; vertical asymptote at $x = -\frac{5}{2}$



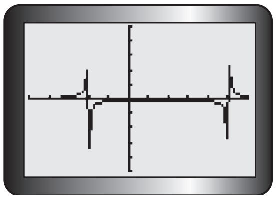
e) $y = \frac{1}{-3x + 6}$; vertical asymptote at $x = 2$



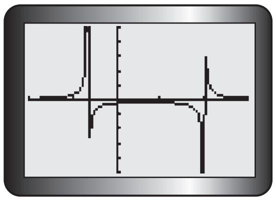
f) $y = \frac{1}{(x-3)^2}$; vertical asymptote at $x = 3$



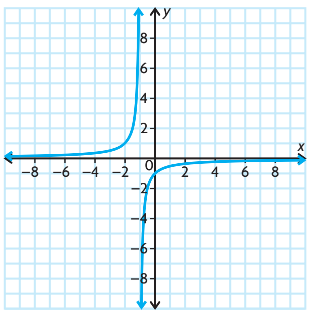
g) $y = \frac{1}{x^2 - 3x - 10}$; vertical asymptotes at $x = -2$ and $x = 5$



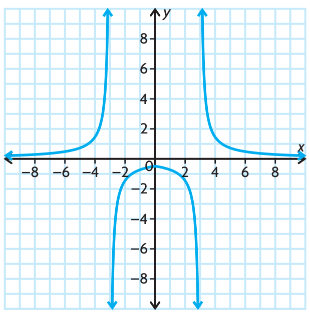
h) $y = \frac{1}{3x^2 - 4x - 4}$; vertical asymptotes at $x = -\frac{2}{3}$ and $x = 2$



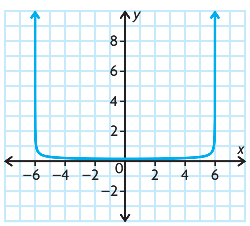
6. a)



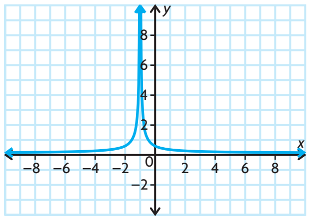
b)



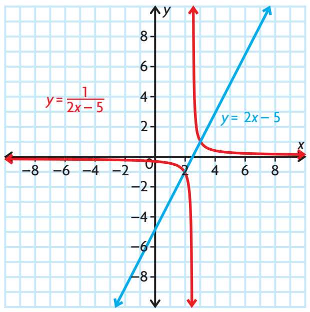
c)



d)

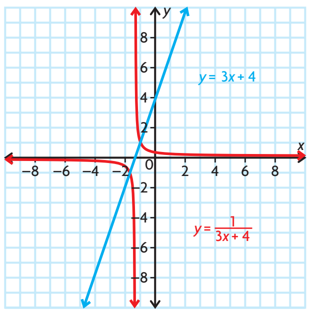


7. a)



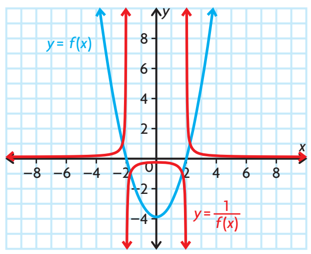
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$,
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

b)

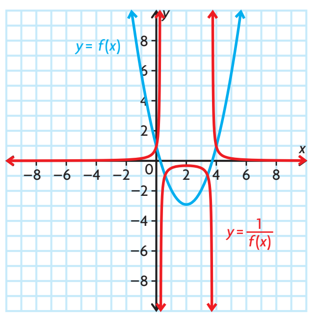


$D = \left\{x \in \mathbf{R} \mid x \neq -\frac{4}{3}\right\}$,
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

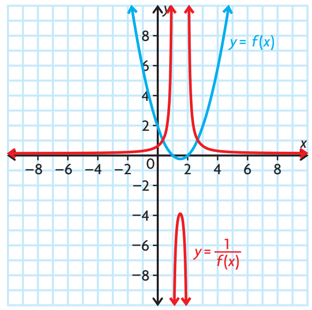
8. a)



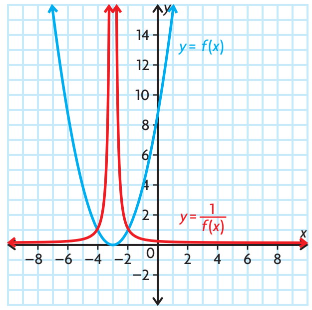
b)



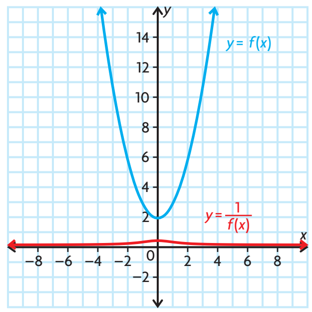
c)



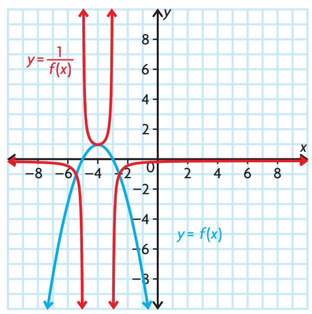
d)



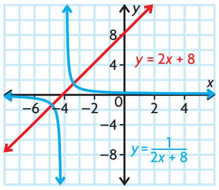
e)



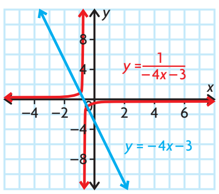
f)



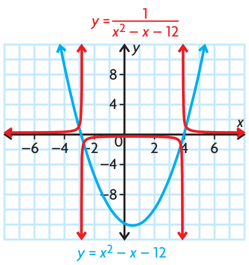
9. **a)** D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R}\}$
y-intercept $= 8$
x-intercept $= -4$
negative on $(-\infty, -4)$
positive on $(-4, -\infty)$
increasing on $(-\infty, \infty)$
equation of reciprocal $= \dfrac{1}{2x + 8}$



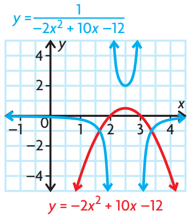
b) D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R}\}$
y-intercept $= -3$
x-intercept $= -\frac{3}{4}$
positive on $\left(-\infty, -\frac{3}{4}\right)$
negative on $\left(-\frac{3}{4}, \infty\right)$
decreasing on $(-\infty, \infty)$
equation of reciprocal $= \dfrac{1}{-4x - 3}$



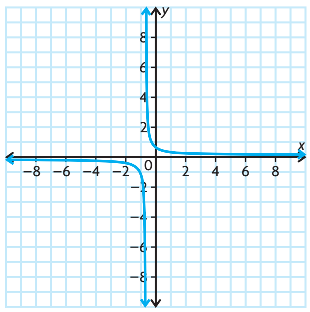
c) D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} \mid y \geq -12.25\}$
y-intercept $= 12$
x-intercepts $= 4, -3$
decreasing on $(-\infty, 0.5)$
increasing on $(0.5, \infty)$
positive on $(-\infty, -3)$ and $(4, \infty)$
negative on $(-3, 4)$
equation of reciprocal $= \dfrac{1}{x^2 - x - 12}$



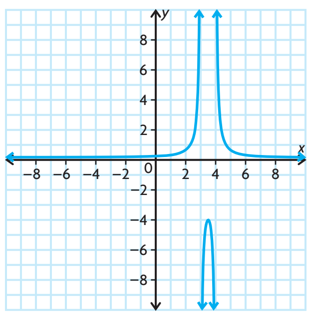
d) D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} \mid y \leq 2.5\}$
y-intercept $= -12$
x-intercepts $= 3, 2$
increasing on $(-\infty, 2.5)$
decreasing on $(2.5, \infty)$
negative on $(-\infty, 2)$ and $(3, \infty)$
positive on $(2, 3)$
equation of
reciprocal $= \dfrac{1}{-2x^2 + 10x - 12}$



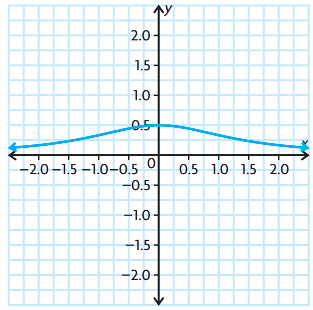
10. Answers may vary. For example, a reciprocal function creates a vertical asymptote when the denominator is equal to 0 for a specific value of x. Consider $\frac{1}{ax + b}$. For this expression, there is always some value of x that is $\frac{-b}{a}$ that will result in a vertical asymptote for the function. This is a graph of $y = \frac{1}{3x + 2}$ and the vertical asymptote is at $x = -\frac{2}{3}$.



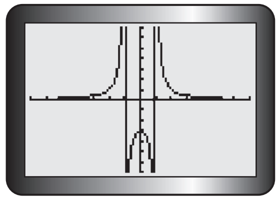
Consider the function $\frac{1}{(x - 3)(x - 4)}$. The graph of the quadratic function in the denominator crosses the x-axis at 3 and 4 and therefore will have vertical asymptotes at 3 and 4 in the graph of the reciprocal function.



However, a quadratic function, such as $x^2 + c$, which has no real zeros, will not have a vertical asymptote in the graph of its reciprocal function. For example, this is the graph of $y = \frac{1}{x^2 + 2}$.

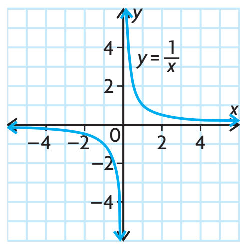


11. $y = \dfrac{3}{x^2 - 1}$



12. **a)** 500
b) $t = 2$
c) $t = 10\ 000$
d) If you were to use a value of t that was less than one, the equation would tell you that the number of bacteria was increasing as opposed to decreasing. Also, after time $t = 10\ 000$, the formula indicates that there is a smaller and smaller fraction of 1 bacteria left.
e) D $= \{x \in \mathbf{R} \mid 1 < x < 10\ 000\}$,
R $= \{y \in \mathbf{R} \mid 1 < y < 10\ 000\}$

13. **a)**

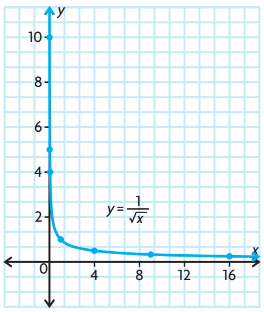


D $= \{x \in \mathbf{R} \mid x \neq -n\}$,
R $= \{y \in \mathbf{R} \mid y \neq 0\}$

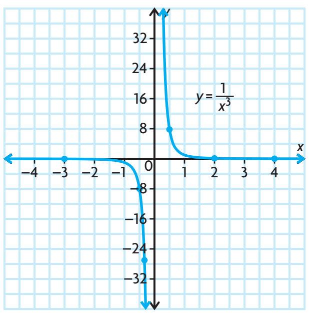
b) The vertical asymptote occurs at $x = -n$. Changes in n in the $f(x)$ family cause changes in the y-intercept—an increase in n causes the intercept to move up the y-axis and a decrease causes it to move down the y-axis. Changes in n in the $g(x)$ family cause changes in the vertical asymptote of the function—an increase in n causes the asymptote to move down the x-axis and a decrease in n causes it to move up the x-axis.
c) $x = 1 - n$ and $x = -1 - n$

14. Answers may vary. For example:
1) Determine the zero(s) of the function $f(x)$—these will be the asymptote(s) for the reciprocal function $g(x)$.
2) Determine where the function $f(x)$ is positive and where it is negative—the reciprocal function $g(x)$ will have the same characteristics.
3) Determine where the function $f(x)$ is increasing and where it is decreasing—the reciprocal function $g(x)$ will have opposite characteristics.

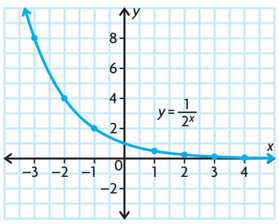
15. a)



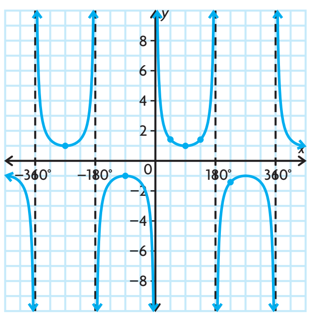
b)



c)



d)



16. $y = \dfrac{1}{x + 4} - 1$

Lesson 5.2, p. 262

1. a) A; The function has a zero at 3 and the reciprocal function has a vertical asymptote at $x = 3$. The function is positive for $x < 3$ and negative for $x > 3$.
b) C; The function in the numerator factors to $(x + 3)(x - 3)$. $(x - 3)$ factors out of both the numerator and the denominator. The equation simplifies to $y = x + 3$, but has a hole at $x = 3$.
c) F; The function in the denominator has a zero at $x = -3$, so there is a vertical asymptote at $x = -3$. The function is always positive.
d) D; The function in the denominator has zeros at $y = 1$ and $y = -3$. The rational function has vertical asymptotes at $x = 1$ and $x = -3$.
e) B; The function has no zeros and no vertical asymptotes or holes.
f) E; The function in the denominator has a zero at $x = 3$ and the rational function has a vertical asymptote at $x = 3$. The degree of the numerator is exactly 1 more than the degree of the denominator, so the graph has an oblique asymptote.

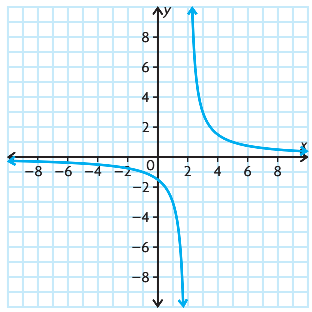
2. a) vertical asymptote at $x = -4$; horizontal asymptote at $y = 1$
b) vertical asymptote at $x = -\frac{3}{2}$; horizontal asymptote at $y = 0$
c) vertical asymptote at $x = 6$; horizontal asymptote at $y = 2$
d) hole at $x = -3$
e) vertical asymptotes at $x = -3$ and 5; horizontal asymptote at $y = 0$
f) vertical asymptote at $x = -1$; horizontal asymptote at $y = -1$
g) hole at $x = 2$
h) vertical asymptote at $x = \frac{5}{2}$; horizontal asymptote at $y = -2$
i) vertical asymptote at $x = -\frac{1}{4}$; horizontal asymptote at $y = 1$
j) vertical asymptote at $x = 4$; hole at $x = -4$; horizontal asymptote at $y = 0$
k) vertical asymptote at $x = \frac{3}{5}$; horizontal asymptote at $y = \frac{1}{5}$
l) vertical asymptote at $x = 4$; horizontal asymptote at $y = -\frac{3}{2}$

3. Answers may vary. For example:
a) $y = \dfrac{x - 1}{x^2 + x - 2}$
b) $y = \dfrac{1}{x^2 - 4}$
c) $y = \dfrac{x^2 - 4}{x^2 + 3x + 2}$
d) $y = \dfrac{2x}{x + 1}$
e) $y = \dfrac{x^3}{x^2 + 5}$

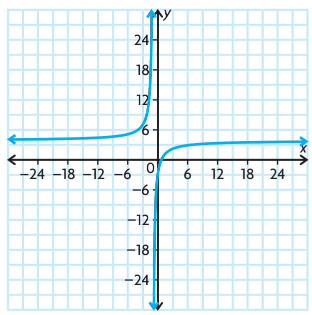
Lesson 5.3, pp. 272–274

1. a) A
b) C
c) D
d) B

2. a) $x = 2$
b) As $x \to 2$ from the right, the values of $f(x)$ get larger. As $x \to 2$ from the left, the values become larger in magnitude but are negative.
c) $y = 0$
d) As $x \to -\infty$ and as $x \to \infty$, $f(x) \to 0$.
e) $D = \{x \in \mathbf{R} \mid x \neq 3\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
f) positive: $(2, \infty)$
negative: $(-\infty, 2)$
g)

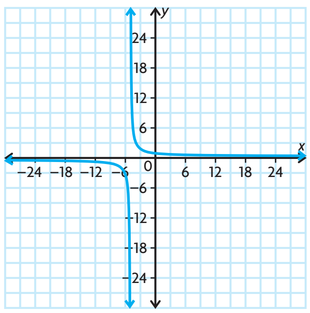


3. a) $x = -1$
b) As $x \to -1$ from the left, $y \to \infty$. As $x \to -1$ from the right, $y \to -\infty$.
c) $y = 4$
d) As $x \to \pm\infty$, $f(x)$ gets closer and closer to 4.
e) $D = \{x \in \mathbf{R} \mid x \neq -1\}$
$R = \{y \in \mathbf{R} \mid y \neq 4\}$
f) positive: $(-\infty, -1)$ and $\left(\frac{3}{4}, -\infty\right)$
negative: $\left(-1, \frac{3}{4}\right)$
g)



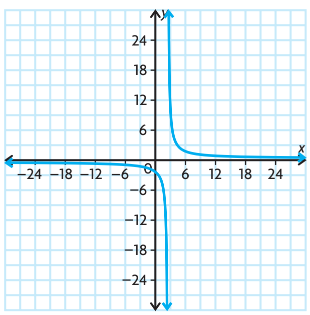
4. a) $x = -3$; As $x = -3$, $y = -\infty$ on the left.
As $x = -3$, $y = \infty$ on the right.
b) $x = 5$; As $x = 5$, $y = -\infty$ on the left.
As $x = 5$, $y = \infty$ on the right.
c) $x = \frac{1}{2}$; As $x = \frac{1}{2}$, $y = -\infty$ on the left.
As $x = \frac{1}{2}$, $y = \infty$ on the right.
d) $x = -\frac{1}{4}$; As $x = -\frac{1}{4}$, $y = -\infty$ on the left.
As $x = -\frac{1}{4}$, $y = \infty$ on the right.

5. a) vertical asymptote at $x = -5$
horizontal asymptote at $y = 0$
$D = \{x \in \mathbf{R} \mid x \neq -5\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= \frac{3}{5}$
$f(x)$ is negative on $(-\infty, -5)$ and positive on $(-5, \infty)$.



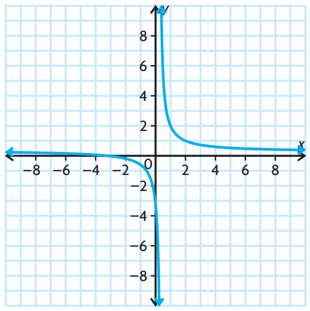
The function is decreasing on $(-\infty, -5)$ and on $(-5, \infty)$. The function is never increasing.

b) vertical asymptote at $x = \frac{5}{2}$
horizontal asymptote at $y = 0$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= -2$
$f(x)$ is negative on $\left(-\infty, \frac{5}{2}\right)$ and positive on $\left(\frac{5}{2}, \infty\right)$.



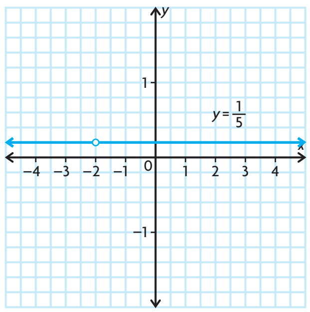
The function is decreasing on $\left(-\infty, \frac{5}{2}\right)$ and on $\left(\frac{5}{2}, \infty\right)$. The function is never increasing.

c) vertical asymptote at $x = \frac{1}{4}$
horizontal asymptote at $y = \frac{1}{4}$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{1}{4}\right\}$
$R = \left\{y \in \mathbf{R} \mid y \neq \frac{1}{4}\right\}$
x-intercept $= -5$
y-intercept $= -1$
$f(x)$ is positive on $(-\infty, -5)$ and $\left(\frac{1}{4}, \infty\right)$ and negative on $\left(-5, \frac{1}{4}\right)$.



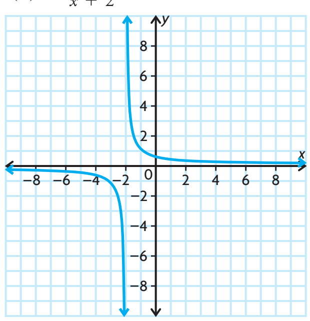
The function is decreasing on $\left(-\infty, \frac{1}{4}\right)$ and on $\left(\frac{1}{4}, \infty\right)$. The function is never increasing.

d) hole $x = -2$
$D = \{x \in \mathbf{R} \mid x \neq -2\}$
$R = \left\{y = \frac{1}{5}\right\}$
y-intercept $= \frac{1}{5}$
The function will always be positive.

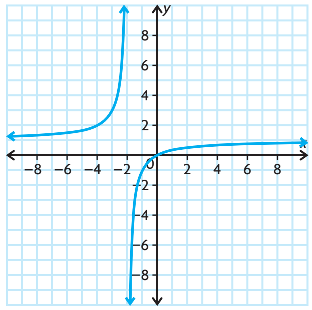


The function is neither increasing nor decreasing; it is constant.

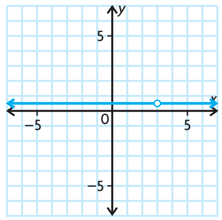
6. a) Answers may vary. For example:
$f(x) = \frac{1}{x + 2}$



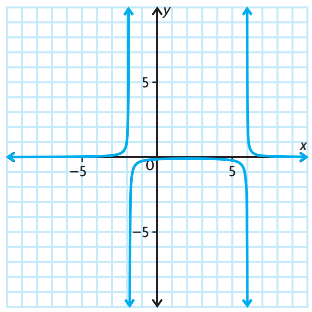
b) Answers may vary. For example:
$y = \frac{x}{x + 2}$



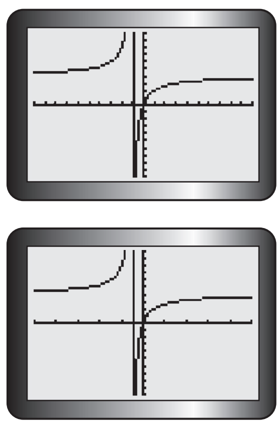
c) Answers may vary. For example:
$f(x) = \frac{x - 3}{2x - 6}$

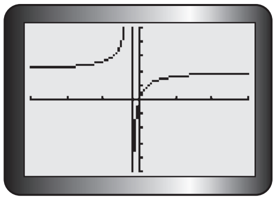


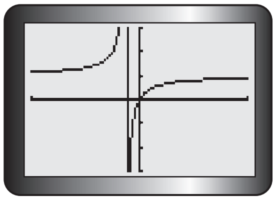
d) Answers may vary. For example:
$f(x) = \frac{1}{x^2 - 4x - 12}$



7. a)







The equation has a general vertical asymptote at $x = -\frac{1}{n}$. The function has a general horizontal asymptote at $y = \frac{8}{n}$. The vertical asymptotes are $-\frac{1}{8}$, $-\frac{1}{4}$, $-\frac{1}{2}$, and -1. The horizontal asymptotes are 8, 4, 2, and 1. The function contracts as n increases. The function is always increasing. The function is positive on $\left(-\infty, -\frac{17}{n}\right)$ and $\left(\frac{3}{10}, \infty\right)$. The function is negative on $\left(-\frac{17}{n}, \frac{3}{10}\right)$.

b) The horizontal and vertical asymptotes both approach 0 as the value of n increases; the x- and y-intercepts do not change, nor do the positive and negative characteristics or the increasing and decreasing characteristics.

c) The vertical asymptote becomes $x = \frac{17}{n}$ and the horizontal becomes $x = -\frac{10}{n}$. The function is always increasing. The function is positive on $\left(-\infty, \frac{3}{10}\right)$ and $\left(\frac{17}{n}, \infty\right)$. The function is negative on $\left(\frac{3}{10}, \frac{17}{n}\right)$. The rest of the characteristics do not change.

8. $f(x)$ will have a vertical asymptote at $x = 1$; $g(x)$ will have a vertical asymptote at $x = -\frac{3}{2}$. $f(x)$ will have a horizontal asymptote at $x = 3$; $g(x)$ will have a vertical asymptote at $x = \frac{1}{2}$.

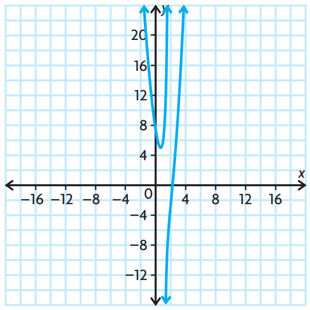
9. **a)** \$27 500
b) \$40 000
c) \$65 000
d) No, the value of the investment at $t = 0$ should be the original value invested.
e) The function is probably not accurate at very small values of t because as $t \to 0$ from the right, $x \to \infty$.
f) \$15 000

10. The concentration increases over the 24 h period and approaches approximately 1.89 mg/L.

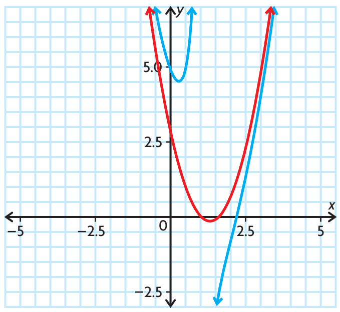
11. Answers may vary. For example, the rational functions will all have vertical asymptotes at $x = -\frac{d}{c}$. They will all have horizontal asymptotes at $y = \frac{a}{c}$. They will intersect the y-axis at $y = \frac{b}{d}$. The rational functions will have an x-intercept at $x = -\frac{b}{a}$.

12. Answers may vary. For example, $f(x) = \frac{2x^2}{2 + x}$.

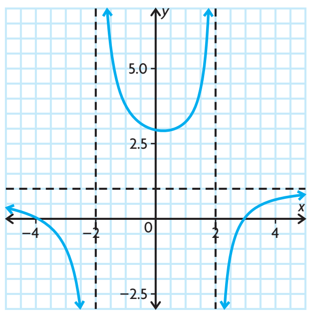
13. $f(x) = 2x^2 - 5x + 3 - \dfrac{2}{x - 1}$
As $x \to \pm\infty, f(x) \to \infty$.



vertical asymptote: $x = 1$; oblique asymptote: $y = 2x^2 - 5x + 3$



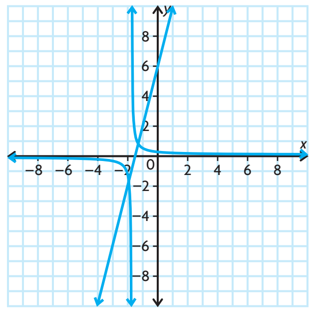
14. **a)** $f(x)$
b) $g(x)$ and $h(x)$
c) $g(x)$
d)



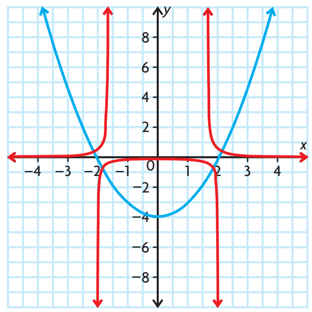
Mid-Chapter Review, p. 277

1. **a)** $\dfrac{1}{x - 3}$; $x = 3$
b) $\dfrac{1}{-4q + 6}$; $q = \dfrac{3}{2}$
c) $\dfrac{1}{z^2 + 4z - 5}$; $z = -5$ and 1
d) $\dfrac{1}{6d^2 + 7d - 3}$; $d = \dfrac{1}{3}$ and $-\dfrac{3}{2}$

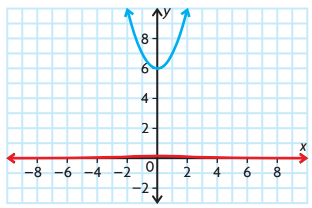
2. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{x \in \mathbf{R}\}$; y-intercept $= 6$; x-intercept $= -\frac{3}{2}$; negative on $\left(-\infty, -\frac{3}{2}\right)$; positive on $\left(-\frac{3}{2}, \infty\right)$; increasing on $(-\infty, \infty)$



b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > -4\}$; y-intercept $= -4$; x-intercepts are 2 and -2; decreasing on $(-\infty, 0)$; increasing $(0, \infty)$; positive on $(-\infty, -2)$ and $(2, \infty)$; negative on $(-2, 2)$

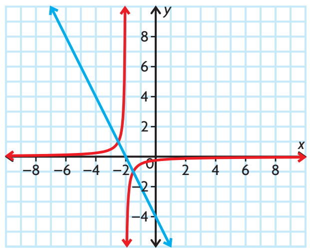


c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 6\}$; no x-intercepts; function will never be negative; decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$



Answers

d) D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R}\}$; x-intercept $= -2$; function is always decreasing; positive on $(-\infty, -2)$; negative on $(-2, \infty)$

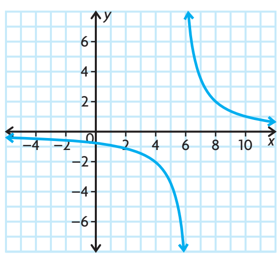


3. Answers may vary. For example: (1) Hole: Both the numerator and the denominator contain a common factor, resulting in $\frac{0}{0}$ for a specific value of x. (2) Vertical asymptote: A value of x causes the denominator of a rational function to be 0. (3) Horizontal asymptote: A horizontal asymptote is created by the ratio between the numerator and the denominator of a rational function as the function $\rightarrow \infty$ and $-\infty$. A continuous rational function is created when the denominator of the rational function has no zeros.

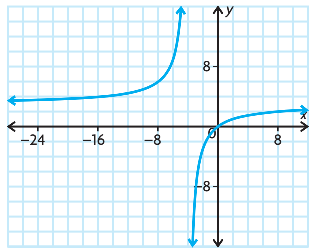
4. **a)** $x = 2$; vertical asymptote
b) hole at $x = 1$
c) $x = -\frac{1}{2}$; horizontal asymptote
d) $x = 6$; oblique asymptote
e) $x = -5$ and $x = 3$; vertical asymptotes

5. $y = \dfrac{x}{x-2}$, $y = 1$; $y = \dfrac{-7x}{4x+2}$, $y = \dfrac{-7}{4}$; $y = \dfrac{1}{x^2 + 2x - 15}$, $x = 0$

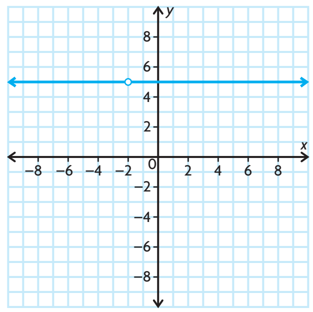
6. **a)** vertical asymptote: $x = 6$; horizontal asymptote: $y = 0$; no x-intercept; y-intercept: $-\frac{5}{6}$; negative when the denominator is negative; positive when the numerator is positive; $x - 6$ is negative on $x < 6$; $f(x)$ is negative on $(-\infty, 6)$ and positive on $(6, \infty)$; function is always decreasing



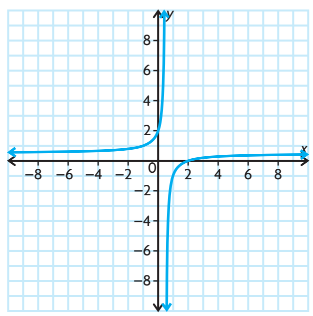
b) vertical asymptote: $x = -4$; horizontal asymptote: $y = 3$; x-intercept: $x = 0$; y-intercept: $f(0) = 0$; function is always increasing; positive on $(-\infty, -4)$ and $(0, \infty)$; negative on $(-4, 0)$



c) straight, horizontal line with a hole at $x = -2$; always positive and never increases or decreases



d) vertical asymptote: $x = \frac{1}{2}$; horizontal asymptote: $y = \frac{1}{2}$; x-intercept: $x = 2$; y-intercept: $f(0) = 5$; function is always increasing



7. Answers may vary. For example: Changing the function to $y = \frac{7x+6}{x+1}$ changes the graph. The function now has a vertical asymptote at $x = -1$ and still has a horizontal asymptote at $y = 7$. However, the function is now constantly increasing instead of decreasing. The new function still has an x-intercept at $x = -\frac{6}{7}$, but now has a y-intercept at $y = 6$.

8. $n = \dfrac{1}{3}$; $m = 35$

9. Answers may vary. For example, $f(x) = \frac{4x+8}{x+2}$.
The graph of the function will be a horizontal line at $y = 4$ with a hole at $x = -2$.

Lesson 5.4, pp. 285–287

1. 3; -2; Answers may vary. For example, substituting each value for x in the equation produces the same value on each side of the equation, so both are solutions.

2. **a)** $x = -3$ **c)** $x = -1$ and 2
b) $x = 5$ **d)** $x = -4$

3. **a)** $f(x) = \dfrac{x-3}{x+3} - 2$
b) $f(x) = \dfrac{3x-1}{x} - \dfrac{5}{2}$
c) $f(x) = \dfrac{x-1}{x} - \dfrac{x+1}{x+3}$
d) $f(x) = \dfrac{x-2}{x+3} - \dfrac{x-4}{x+5}$

4. **a)** $x = -9$ **c)** $x = 3$
b) $x = 2$ **d)** $x = -\dfrac{1}{2}$

5. **a)** $x = 3$ **d)** $x = 0$
b) $x = \dfrac{3}{4}$ **e)** $x = \dfrac{1}{4}$
c) $x = -9$ **f)** $x = -23$

6. **a)** The function will have no real solutions.
b) $x = 3$ and $x = -0.5$
c) $x = -5$
d) $x = 0$ and $x = -1$
e) The original equation has no real solutions.
f) $x = 5$ and $x = 2$

7. **a)** $x = 6$ **d)** $x = 3.25, 20.75$
b) $x = 1.30, 7.70$ **e)** $x = -1.71, 2.71$
c) $x = 10$ **f)** $x = -0.62, 1.62$

8. **a)** $\dfrac{x+1}{x-2} = \dfrac{x+3}{x-4}$

Multiply both sides of the equation by the LCD, $(x-2)(x-4)$.

$$(x-2)(x-4)\left(\frac{x+1}{x-2}\right)$$
$$= (x-2)(x-4)\left(\frac{x+3}{x-4}\right)$$
$$(x-4)(x+1) = (x-2)(x+3)$$

Simplify. $x^2 - 3x - 4 = x^2 + x - 6$

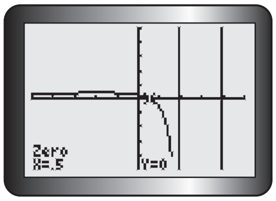
Simplify the equation so that 0 is on one side of the equation.

$$x^2 - x^2 - 3x - x - 4 + 6$$
$$= x^2 - x^2 + x - x - 6 + 6$$
$$-4x + 2 = 0$$
$$-2(2x - 1) = 0$$

Since the product is equal to 0, one of the factors must be equal to 0. It must be $2x - 1$ because 2 is a constant.

$$2x - 1 = 0$$
$$2x - 1 + 1 = 0 + 1$$
$$2x = 1$$
$$\frac{2x}{2} = \frac{1}{2}$$
$$x = \frac{1}{2}$$

b) $\frac{\frac{1}{2}+1}{\frac{1}{2}-2} = -1$ and $\frac{\frac{1}{2}+3}{\frac{1}{2}-4} = -1$

c) 

9. $w = 9.271$

10. Machine A $= 25.8$ min;
Machine B $= 35.8$ min

11. 75; \$4.00

12. a) After 6666.67 s
b) The function appears to approach $9\ \text{kg/m}^3$ as time increases.

13. a) Tom $= 4$ min; Carl $= 5$ min; Paco $= 2$ min
b) 6.4 min

14. Answers may vary. For example, you can use either algebra or graphing technology to solve a rational equation. With algebra, solving the equation takes more time, but you get an exact answer. With graphing technology, you can solve the equation quickly, but you do not always get an exact answer.

15. $x = -3.80, -1.42, 0.90, 4.33$

16. a) $x = 0.438$ and 1.712
b) $(0, 0.438)$ and $(1.712, \infty)$

Lesson 5.5, pp. 295–297

1. a) $(\infty, 1)$ and $(3, \infty)$
b) $(-0.5, 1)$ and $(2, \infty)$

2. a) Solve the inequality for x.

$$\frac{6x}{x+3} \le 4$$
$$\frac{6x}{x+3} - 4 \le 0$$
$$\frac{6x}{x+3} - 4\frac{x+3}{x+3} \le 0$$
$$\frac{6x - 4x - 12}{x+3} \le 0$$
$$\frac{2x - 12}{x+3} \le 0$$
$$\frac{2(x-6)}{x+3} \le 0$$

b) 

c) $(-3, 6]$

3. a)
$$x + 2 > \frac{15}{x}$$
$$x + 2 - \frac{15}{x} > 0$$
$$\frac{x^2}{x} + \frac{2x}{x} - \frac{15}{x} > 0$$
$$\frac{x^2 + 2x - 15}{x} > 0$$
$$\frac{(x+5)(x-3)}{x} > 0$$

b) negative: $x < -5$ and $0 < x < 3$;
positive: $-5 < x < 0$, $x > 3$

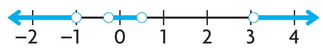
c) $\{x \in \mathbf{R} \mid -5 < x < 0 \text{ or } x > 3\}$ or $(-5, 0)$ or $(3, \infty)$

4. a) $5 < x < -4.5$
b) $-7 < x < -5$ and $x > -3$
c) $0 < x < 2$ and $x > 8$
d) $-6.8 \le x < -4$ and $x > 3$
e) $x < -1$ and $-\frac{1}{7} < x < 0$
f) $-1 < x < \frac{7}{8}$ and $x < 4$

5. a) $t < -3$ or $1 < t < 4$
b) $-3 \le t \le 2$ or $t > 4$
c) $-\frac{1}{2} < t < \frac{1}{3}$ or $t > \frac{1}{2}$
d) $t < -2$ and $-2 < t < 3$
e) $t < -5$ and $-2 < t < 0$
f) $-1 \le t < 0.25$ and $2 \le t < 9$

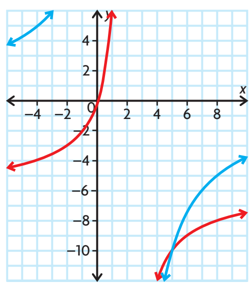
6. a) $x \in (-\infty, -6)$ or $x \in (-1, 4)$
b) $x \in (3, \infty)$
c) $x \in (-4, -2)$ or $x \in (-1, 2)$
d) $x \in (-\infty, -9)$ or $x \in [-3, -1)$ or $x \in [3, \infty)$
e) $x \in (-2, 0)$ or $x \in (4, \infty)$
f) $x \in (-\infty, -4)$ or $x \in (4, \infty)$

7. a) $x < -1$, $-0.2614 < x < 0.5$, $x > 3.065$

b) 

c) Interval notation: $(-\infty, -1)$, $(-0.2614, 0.5)$, $(3.065, \infty)$
Set notation: $\{x \in \mathbf{R} \mid x < -1, -0.2614 < x < 0.5, \text{ or } x > 3.065\}$

8. a) $t < 2$ and $t > 5$.

b) 

c) It would be difficult to find a situation that could be represented by these rational expressions because very few positive values of x yield a positive value of y.

9. The only values that make the expression greater than 0 are negative. Because the values of t have to be positive, the bacteria count in the tap water will never be greater than that of the pond water.

10. a) $\frac{(x^2 - 4x - 5)}{2x < 0}$

b)

	$x < -1$	$-1 < x < 0$	$0 < x < 5$	$x > 5$
$(x - 5)$	−	−	−	+
$(x + 1)$	−	+	+	+
$2x$	−	−	+	+
$\frac{(x-5)(x+1)}{2x}$	−	+	−	+

The inequality is true for $x < -1$ and $0 < x < 5$

11. when $x > 5$

12. a) The first inequality can be manipulated algebraically to produce the second inequality.
b) Graph the equation $y = \frac{x+1}{x-1} - \frac{x+3}{x+2}$ and determine when it is negative.
c) The values that make the factors of the second inequality zero are -5, -2, and 1. Determine the sign of each factor in the intervals corresponding to the zeros. Determine when the entire expression is negative by examining the signs of the factors.

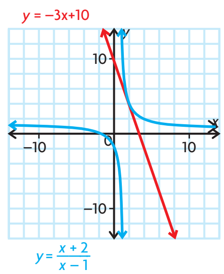
13. $[2, 4)$ and $(4, \infty)$

14. $14.48 < x < 165.52$ and $180 < x < 360$

15. $0 < x < 2$

Lesson 5.6, pp. 303–305

1. a) -0.5

b) 

slope $= -3$

2. -3

3. -3

4. -1

5. a) 0.01
b) -0.3
c) -1.3
d) 6

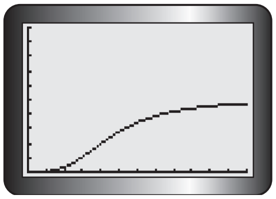
6. a) slope $= 286.1$; vertical asymptote: $x = -1.5$
b) slope $= -2.74$; vertical asymptote: $x = -5$
c) slope $= 44.65$; vertical asymptote: $x = -\frac{5}{3}$
d) slope $= -1.26$; vertical asymptote: $x = 6$

7. **a)** 0.01
b) 0.34

8. **a)** $R(x) = \dfrac{15x}{2x^2 + 11x + 5}$
b) 0.3, -0.03

9. **a)** \$5.67
b) -2

10. **a)** 68.46
b) 94.54
c)



The number of houses that were built increases slowly at first, but rises rapidly between the third and sixth months. During the last six months, the rate at which the houses were built decreases.

11. Answers may vary. For example: $14 \le x \le 15$; $x = 14.5$

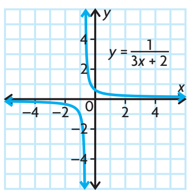
12. **a)** Find $s(0)$ and $s(6)$, and then solve $\dfrac{s(6) - s(0)}{6 - 0}$.
b) The average rate of change over this interval gives the object's speed.
c) To find the instantaneous rate of change at a specific point, you could find the slope of the line that is tangent to the function $s(t)$ at the specific point. You could also find the average rate of change on either side of the point for smaller and smaller intervals until it stabilizes to a constant. It is generally easier to find the instantaneous rate using a graph, but the second method is more accurate.
d) The instantaneous rate of change for a specific time, t, is the acceleration of the object at this time.

13. $y = -0.5x - 2.598$; $y = -0.5x + 2.598$; $y = 4x$

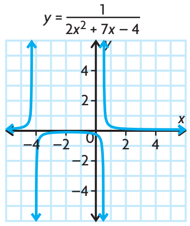
14. The instantaneous rate of change at $(0, 0) = 4$. The rate of change at this rate of change will be 0.

Chapter Review, pp. 308–309

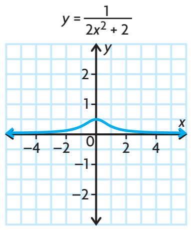
1. **a)** D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R}\}$;
x-intercept $= -\frac{2}{3}$; y-intercept $= 2$;
always increasing;
negative on $\left(-\infty, -\frac{2}{3}\right)$;
positive on $\left(-\frac{2}{3}, \infty\right)$



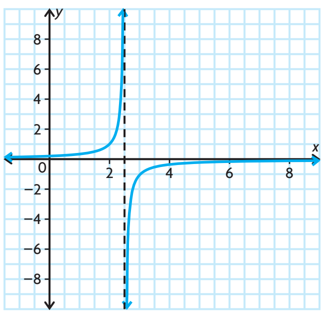
b) D $= \{x \in \mathbf{R}\}$;
R $= \{y \in \mathbf{R} \mid y > -10.125\}$;
x-intercept $= 0.5$ and -4;
positive on $(-\infty, -4)$ and $(0.5, \infty)$;
negative on $(-4, 0.5)$;
decreasing on $(-\infty, -10.125)$;
increasing on $(-10.125, \infty)$



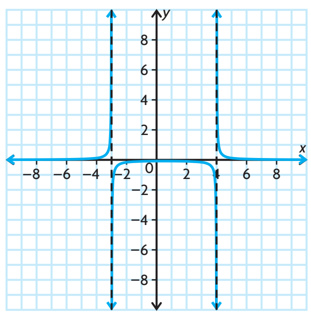
c) D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} \mid y > 2\}$; no x-intercepts; y-intercept $= 2$;
decreasing on $(-\infty, 0)$;
increasing on $(0, \infty)$; always positive, never negative



2. **a)**



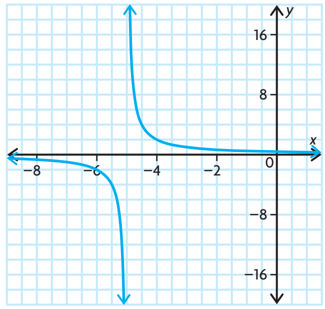
b)



3. **a)** $x = -17$
b) $x = -\frac{3}{5}$; horizontal asymptote; $y = \frac{2}{5}$
c) $x = 0.5$; hole at $x = -11$
d) $x = 1$; oblique asymptote; $y = 3x + 3$

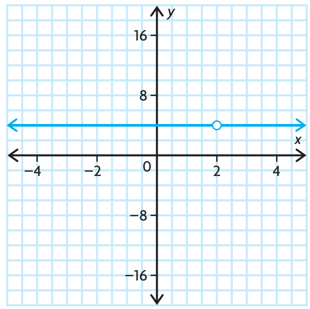
4. The locust population increased during the first 1.75 years, to reach a maximum of 1 248 000. The population gradually decreased until the end of the 50 years, when the population was 128 000.

5. **a)** x-intercept $= 2$:
horizontal asymptote: $y = 0$;
y-intercept $= \frac{2}{5}$:
vertical asymptote: $x = -5$;



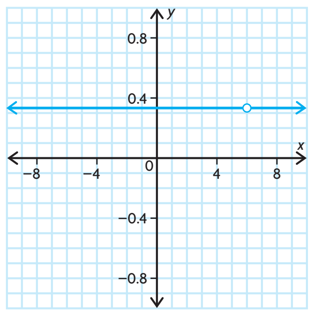
The function is never increasing and is decreasing on $(-\infty, -5)$ and $(-5, \infty)$.
D $= \{x \in \mathbf{R} \mid x \neq -5\}$;
negative for $x < -5$;
positive for $x > -5$

b) D $= \{x \in \mathbf{R} \mid x \neq 2\}$; no x-intercept; y-intercept $= 4$; positive for $x \neq 2$;



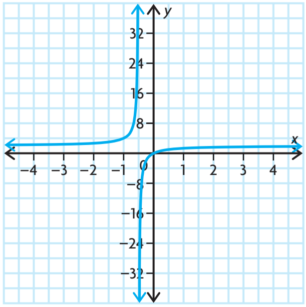
never increasing or decreasing

c) D $= \{x \in \mathbf{R} \mid x \neq 6\}$; no x-intercept; y-intercept $= \frac{1}{3}$; positive for $x \neq 6$;



never increasing or decreasing

d) $x = -0.5$; vertical asymptote: $x = -0.5$; $D = \{x \in \mathbf{R} \mid x \neq -0.5\}$; x-intercept $= 0$; y-intercept $= 0$; horizontal asymptote $= 2$; $R = \{y \in \mathbf{R} \mid x \neq 2\}$; positive on $x < -0.5$ and $x > 0$; negative on $-0.5 < x < 0$



The function is never decreasing and is increasing on $(-\infty, -0.5)$ and $(-0.5, \infty)$.

6. Answers may vary. For example, consider the function $f(x) = \frac{1}{x - 6}$. You know that the vertical asymptote would be $x = 6$. If you were to find the value of the function very close to $x = 6$ (say $f(5.99)$ or $f(6.01)$) you would be able to determine the behaviour of the function on either side of the asymptote.

$$f(5.99) = \frac{1}{(5.99) - 6} = -100$$

$$f(6.01) = \frac{1}{(6.01) - 6} = 100$$

To the left of the vertical asymptote, the function moves toward $-\infty$. To the right of the vertical asymptote, the function moves toward ∞.

7. **a)** $x = 6$
b) $x = 0.2$ and $x = -\frac{2}{3}$
c) $x = -6$ or $x = 2$
d) $x = -1$ and $x = 3$

8. about 12 min

9. $x = 1.82$ days and 3.297 days

10. **a)** $x < -3$ and $-2.873 < x < 4.873$
b) $-16 < x < -11$ and $-5 < x$
c) $-2 < x < -1.33$ and $-1 < x < 0$
d) $0 < x < 1.5$

11. $-0.7261 < t < 0$ and $t > 64.73$

12. **a)** -6; $x = 3$
b) 0.2; $x = -2$ and $x = -1$

13. **a)** 0.455 mg/L/h
b) -0.04 mg/L/h
c) The concentration of the drug in the blood stream appears to be increasing most rapidly during the first hour and a half; the graph is steep and increasing during this time.

14. $x = 5$ and $x = 8$; $x = 6.5$

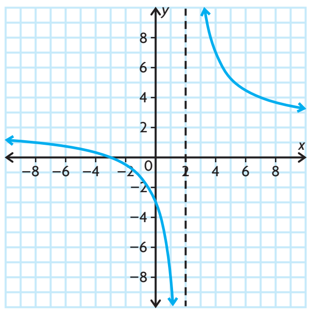
15. **a)** As the x-coordinate approaches the vertical asymptote of a rational function, the line tangent to graph will get closer and closer to being a vertical line. This means that the slope of the line tangent to the graph will get larger and larger, approaching positive or negative infinity depending on the function, as x gets closer to the vertical asymptote.
b) As the x-coordinate grows larger and larger in either direction, the line tangent to the graph will get closer and closer to being a horizontal line. This means that the slope of the line tangent to the graph will always approach zero as x gets larger and larger.

Chapter Self-Test, p. 310

1. **a)** B
b) A

2. **a)** If $f(n)$ is very large, then that would make $\frac{1}{f(n)}$ a very small fraction.
b) If $f(n)$ is very small (less than 1), then that would make $\frac{1}{f(n)}$ very large.
c) If $f(n) = 0$, then that would make $\frac{1}{f(n)}$ undefined at that point because you cannot divide by 0.
d) If $f(n)$ is positive, then that would make $\frac{1}{f(n)}$ also positive because you are dividing two positive numbers.

3.



4. 4326 kg; \$0.52/kg

5. **a)** Algebraic; $x = -1$ and $x = -3$
b) Algebraic with factor table
The inequality is true on $(-10, -5.5)$ and on $(-5, 1.2)$.

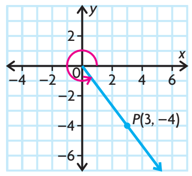
6. **a)** To find the vertical asymptotes of the function, find the zeros of the expression in the denominator. To find the equation of the horizontal asymptotes, divide the first two terms of the expressions in the numerator and denominator.
b) This type of function will have a hole when both the numerator and the denominator share the same factor $(x + a)$.

Chapter 6

Getting Started, p. 314

1. **a)** 28°
b) 332°

2. **a)**



$\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{4}{3}$,

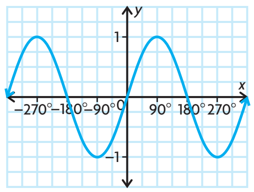
$\csc\theta = -\frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

b) 307°

3. **a)** $\frac{\sqrt{3}}{2}$ **c)** $\frac{\sqrt{3}}{2}$ **e)** $-\sqrt{2}$
b) 0 **d)** $\frac{1}{2}$ **f)** -1

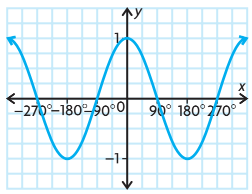
4. **a)** 60°, 300°
b) 30°, 210°
c) 45°, 225°
d) 180°
e) 135°, 315°
f) 90°

5. **a)**



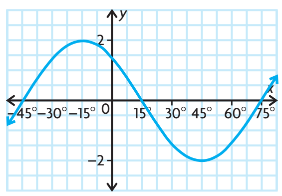
period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

b)

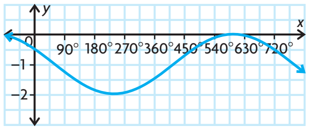


period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

6. **a)** period $= 120°$; $y = 0$; 45° to the left; amplitude $= 2$



b) period $= 720°$; $y = -1$; 60° to the right; amplitude $= 1$



7. a is the amplitude, which determines how far above and below the axis of the curve of the function rises and falls; k defines the period of the function, which is how often the function repeats itself; d is the horizontal shift, which shifts the function to the right or the left; and c is the vertical shift of the function.

Lesson 6.1, pp. 320–322

1. a) π radians; 180°

b) $\frac{\pi}{2}$ radians; 90°

c) $-\pi$ radians; $-180°$

d) $-\frac{3\pi}{2}$ radians; $-270°$

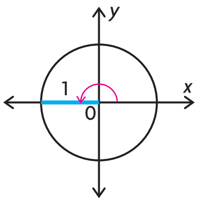
e) -2π radians; $-360°$

f) $\frac{3\pi}{2}$ radians; 270°

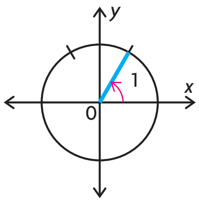
g) $-\frac{4\pi}{3}$ radians $= -240°$

h) $\frac{2\pi}{3}$ radians; 120°

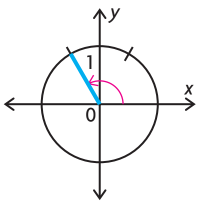
2. a)



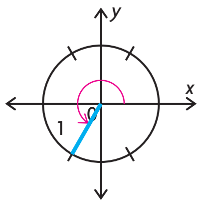
b)



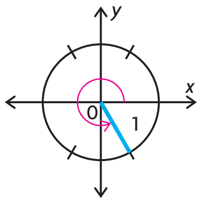
c)



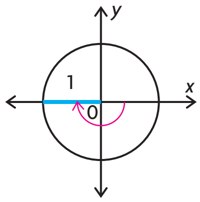
d)



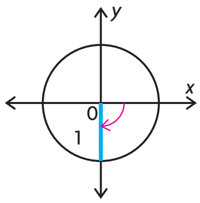
e)



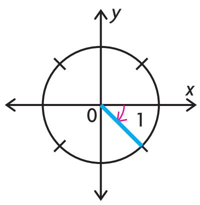
f)



g)



h)



3. a) $\frac{5\pi}{12}$ radians c) $\frac{20\pi}{9}$ radians

b) $\frac{10\pi}{9}$ radians d) $\doteq \frac{16\pi}{9}$ radians

4. a) 300° c) 171.89°

b) 54° d) 495°

5. a) 2 radians; 114.6°

b) $\frac{25\pi}{9}$ cm

6. a) 28 cm

b) $\frac{40\pi}{3}$ cm

7. a) $\frac{\pi}{2}$ radians e) $\frac{5\pi}{4}$ radians

b) $\frac{3\pi}{2}$ radians f) $\frac{\pi}{3}$ radians

c) π radians g) $\frac{4\pi}{3}$ radians

d) $\frac{\pi}{4}$ radians h) $\frac{4\pi}{3}$ radians

8. a) 120° e) 210°

b) 60° f) 90°

c) 45° g) 330°

d) 225° h) 270°

9. a) $\frac{247\pi}{4}$ m

b) 162.5 m

c) $\frac{325\pi}{6}$ cm

10. $4.50\sqrt{2}$ cm

11. a) $\doteq 0.418\ 88$ radians/s

b) $\doteq 377.0$ m

12. a) 36

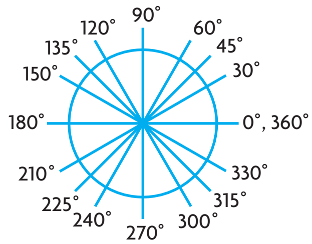
b) 0.8 m

13. a) equal to

b) greater than

c) stay the same

14.



$0° = 0$ radians; $30° = \frac{\pi}{6}$ radians;

$45° = \frac{\pi}{4}$ radians; $60° = \frac{\pi}{3}$ radians;

$90° = \frac{\pi}{2}$ radians; $120° = \frac{2\pi}{3}$ radians;

$135° = \frac{3\pi}{4}$ radians; $150° = \frac{5\pi}{6}$ radians;

$180° = \pi$ radians; $210° = \frac{7\pi}{6}$ radians;

$225° = \frac{5\pi}{4}$ radians; $240° = \frac{4\pi}{3}$ radians;

$270° = \frac{3\pi}{2}$ radians; $300° = \frac{5\pi}{3}$ radians;

$315° = \frac{7\pi}{4}$ radians; $330° = \frac{11\pi}{6}$ radians;

$360° = 2\pi$ radians

15. Circle B, Circle A, and Circle C

16. about 144.5 radians/s

Lesson 6.2, pp. 330–332

1. a) second quadrant; $\frac{\pi}{4}$; positive

b) fourth quadrant; $\frac{\pi}{3}$; positive

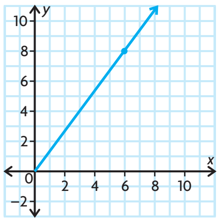
c) third quadrant; $\frac{\pi}{3}$; positive

d) second quadrant; $\frac{\pi}{6}$; negative

e) second quadrant; $\frac{\pi}{3}$; negative

f) fourth quadrant; $\frac{\pi}{4}$; negative

2. a) i)



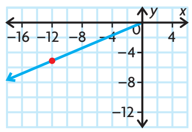
ii) $r = 10$

iii) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$,

$\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

iv) $\theta \doteq 0.93$

b) i)



ii) $r = 13$

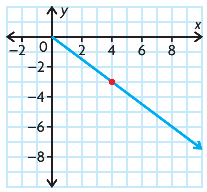
iii) $\sin\theta = -\frac{5}{13}$, $\cos\theta = -\frac{12}{13}$,

$\tan\theta = \frac{5}{12}$, $\csc\theta = -\frac{13}{5}$,

$\sec\theta = -\frac{13}{12}$, $\cot\theta = \frac{12}{5}$

iv) $\theta \doteq 3.54$

c) i)



ii) $r = 5$

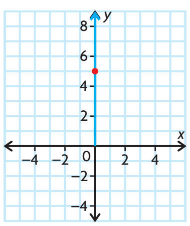
iii) $\sin\theta = -\frac{3}{5}$, $\cos\theta = \frac{4}{5}$,

$\tan\theta = -\frac{3}{4}$, $\csc\theta = -\frac{5}{3}$,

$\sec\theta = \frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

iv) $\theta \doteq 5.64$

d) i)



ii) $r = 5$

iii) $\sin\theta = \frac{5}{5} = 1$,

$\cos\theta = \frac{0}{5} = 0$,

$\tan\theta = \frac{5}{0} = \text{undefined}$,

$\csc\theta = \frac{5}{5} = 1$,

$\sec\theta = \frac{5}{0} = \text{undefined}$,

$\cot\theta = \frac{0}{5} = 0$

iv) $\theta \doteq \frac{\pi}{2}$

3. a) $\sin\left(-\frac{\pi}{2}\right) = -1$,

$\cos\left(-\frac{\pi}{2}\right) = 0$,

$\tan\left(-\frac{\pi}{2}\right) = \text{undefined}$,

$\csc\left(-\frac{\pi}{2}\right) = -1$,

$\sec\left(-\frac{\pi}{2}\right) = \text{undefined}$,

$\cot\left(-\frac{\pi}{2}\right) = 0$

b) $\sin(-\pi) = 0$,
$\cos(-\pi) = -1$,
$\tan(-\pi) = 0$,
$\csc(-\pi) = \text{undefined}$,
$\sec(-\pi) = -1$,
$\cot(-\pi) = \text{undefined}$

c) $\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$,

$\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$,

$\tan\left(\frac{7\pi}{4}\right) = -1$,

$\csc\left(\frac{7\pi}{4}\right) = -\sqrt{2}$,

$\sec\left(\frac{7\pi}{4}\right) = \sqrt{2}$,

$\cot\left(\frac{7\pi}{4}\right) = -1$

d) $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$,

$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$,

$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$,

$\csc\left(-\frac{\pi}{6}\right) = -2$,

$\sec\left(-\frac{\pi}{6}\right) = \frac{2\sqrt{3}}{3}$,

$\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$

4. a) $\sin\frac{\pi}{6}$ **c)** $\cot\frac{3\pi}{4}$

b) $\cos\frac{\pi}{3}$ **d)** $\sec\frac{5\pi}{6}$

5. a) $\frac{\sqrt{3}}{2}$ **d)** $-\frac{\sqrt{2}}{2}$

b) $-\frac{\sqrt{2}}{2}$ **e)** 2

c) $-\frac{\sqrt{3}}{3}$ **f)** 2

6. a) $\frac{4\pi}{3}$ **d)** $\frac{7\pi}{6}$

b) $\frac{11\pi}{6}$ **e)** $\frac{3\pi}{2}$

c) $\frac{5\pi}{4}$ **f)** π

7. a) $\theta \doteq 2.29$ **d)** $\theta \doteq 3.61$
b) $\theta \doteq 0.17$ **e)** $\theta \doteq 0.84$
c) $\theta \doteq 1.30$ **f)** $\theta \doteq 6.12$

8. a) $\cos\frac{5\pi}{4}$ **d)** $\cot\frac{5\pi}{3}$

b) $\tan\frac{5\pi}{6}$ **e)** $\sin\frac{7\pi}{6}$

c) $\csc\frac{4\pi}{3}$ **f)** $\sec\frac{\pi}{4}$

9. $\pi - 0.748 \doteq 2.39$

10. $x \doteq 5.55$ cm

11. $x \doteq 4.5$ cm

12. Draw the angle and determine the measure of the reference angle. Use the CAST rule to determine the sign of each of the ratios in the quadrant in which the angle terminates. Use this sign and the value of the ratios of the reference angle to determine the values of the primary trigonometric ratios for the given angle.

13. a) second or third quadrant

b) $\sin\theta = \frac{12}{13}$ or $-\frac{12}{13}$,

$\tan\theta = \frac{12}{5}$ or $-\frac{12}{5}$,

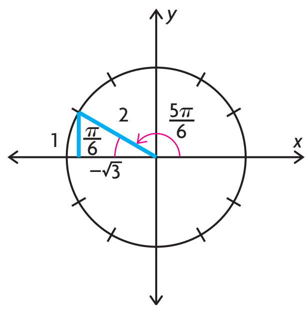
$\sec\theta = -\frac{13}{5}$,

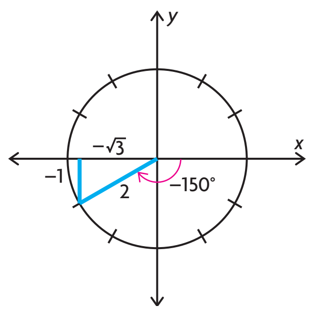
$\csc\theta = \frac{13}{12}$ or $-\frac{13}{12}$,

$\cot\theta = \frac{5}{12}$ or $-\frac{5}{12}$

c) $\theta \doteq 1.97$or 4.32

14.





By examining the special triangles, we see

$$\cos\left(\frac{5\pi}{6}\right) = \cos(-150°) = -\frac{\sqrt{3}}{2}$$

15. $$2\left(\sin^2\left(\frac{11\pi}{6}\right)\right) - 1 = 2\left(-\frac{1}{2}\right)^2 - 1$$
$$= 2\left(\frac{1}{4}\right) - 1$$
$$= -\frac{1}{2}$$

$$\left(\sin^2\frac{11\pi}{6}\right) - \left(\cos^2\frac{11\pi}{6}\right)$$
$$= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} - \frac{3}{4}$$
$$= -\frac{1}{2}$$

$$2\left(\sin^2\left(\frac{11\pi}{6}\right)\right) - 1$$
$$= \left(\sin^2\frac{11\pi}{6}\right) - \left(\cos^2\frac{11\pi}{6}\right)$$

16. $AB = 16$;

$\sin D = \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2}$;

$\cos D = \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2}$;

$\tan D = \frac{8}{8} = 1$

17. **a)** The first and second quadrants both have a positive y-value.
b) The first quadrant has a positive y-value, and the fourth quadrant has a negative y-value.
c) The first quadrant has a positive x-value, and the second quadrant has a negative x-value.
d) The first quadrant has a positive x-value and a positive y-value, and the third quadrant has a negative x-value and a negative y-value.

18. 1

19. $\cos 150° \doteq -0.26$

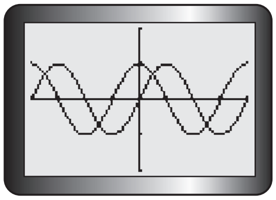
20. The ranges of the cosecant and secant functions are both $\{y \in \mathbf{R} \mid -1 \geq y$ or $y \geq 1\}$. In other words, the values of these functions can never be between -1 and 1. For the values of these functions to be between -1 and 1, the values of the sine and cosine functions would have to be greater than 1 and less than -1, which is never the case.

21. $\frac{2\sqrt{3} - 3}{4}$

Lesson 6.3, p. 336

1. **a)** $y = \sin\theta$ and $y = \cos\theta$ have the same period, axis, amplitude, maximum value, minimum value, domain, and range. They have different y- and θ-intercepts.
b) $y = \sin\theta$ and $y = \tan\theta$ have no characteristics in common except for their y-intercept and zeros.

2. **a)**



b) $\theta = -5.50, \theta = -2.36, \theta = 0.79, \theta = 3.93$
c) **i)** $t_n = n\pi, n \in \mathbf{I}$
ii) $t_n = \frac{\pi}{2} + 2n\pi, n \in \mathbf{I}$
iii) $t_n = \frac{3\pi}{2} + 2n\pi, n \in \mathbf{I}$

3. **a)** $t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$
b) $t_n = 2n\pi, n \in \mathbf{I}$
c) $t_n = -\pi + 2n\pi, n \in \mathbf{I}$

4. The two graphs appear to be identical.

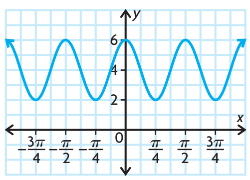
5. **a)** $t_n = n\pi, n \in \mathbf{I}$
b) $t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

Lesson 6.4, pp. 343–346

1. **a)** period: $\frac{\pi}{2}$
amplitude: 0.5
horizontal translation: 0
equation of the axis: $y = 0$
b) period: 2π
amplitude: 1
horizontal translation: $\frac{\pi}{4}$
equation of the axis: $y = 3$
c) period: $\frac{2\pi}{3}$
amplitude: 2
horizontal translation: 0
equation of the axis: $y = -1$
d) period: π
amplitude: 5
horizontal translation: $\frac{\pi}{6}$
equation of the axis: $y = -2$

2. Only the last one is cut off.

3.

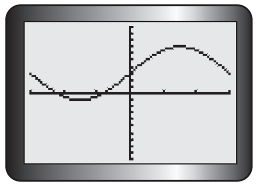


period: $\frac{\pi}{2}$
amplitude: 2
horizontal translation: $\frac{\pi}{4}$ to the left
equation of the axis: $y = 4$

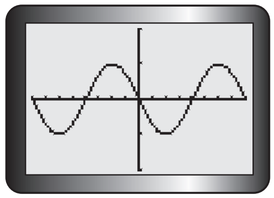
4. **a)** $f(x) = 25\sin(2x) - 4$
b) $f(x) = \frac{2}{5}\sin\left(\frac{\pi}{5}x\right) + \frac{1}{15}$
c) $f(x) = 80\sin\left(\frac{1}{3}x\right) - \frac{9}{10}$
d) $f(x) = 11\sin(4\pi x)$

5. **a)** period $= 2\pi$, amplitude $= 18$, equation of the axis is $y = 0$; $y = 18\sin x$
b) period $= 4\pi$, amplitude $= 6$, equation of the axis is $y = -2$; $y = -6\sin(0.5x) - 2$
c) period $= 6\pi$, amplitude $= 2.5$, equation of the axis is $y = 6.5$; $y = -2.5\cos\left(\frac{1}{3}x\right) + 6.5$
d) period $= 4\pi$, amplitude $= 2$, equation of the axis is $y = -1$; $y = -2\cos\left(\frac{1}{2}x\right) - 1$

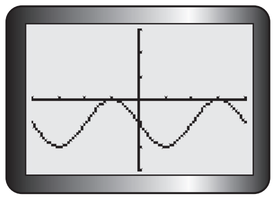
6. **a)** vertical stretch by a factor of 4, vertical translation 3 units up



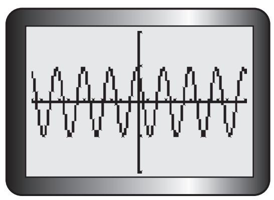
b) reflection in the x–axis, horizontal stretch by a factor of 4



c) horizontal translation π to the right, vertical translation 1 unit down



d) horizontal compression by a factor of $\frac{1}{4}$, horizontal translation $\frac{\pi}{6}$ to the left



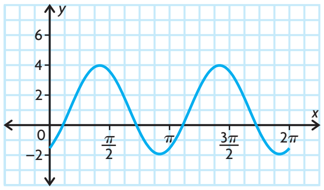
7. a) $f(x) = \frac{1}{2}\cos x + 3$

b) $f(x) = \cos\left(-\frac{1}{2}x\right)$

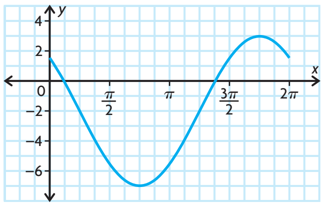
c) $f(x) = 3\cos\left(x - \frac{\pi}{2}\right)$

d) $f(x) = \cos\left(2\left(x + \frac{\pi}{2}\right)\right)$

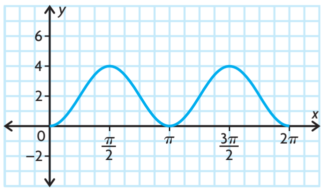
8. a)



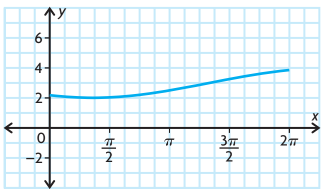
b)



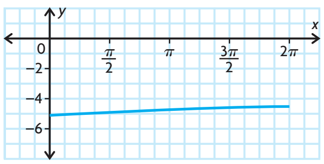
c)



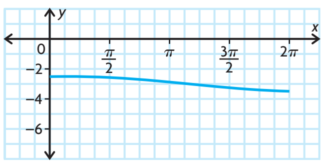
d)



e)



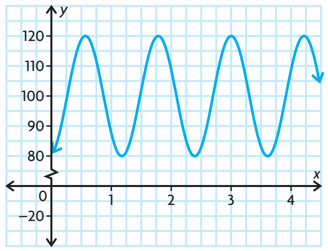
f)



9. a) The period of the function is $\frac{6}{5}$. This represents the time between one beat of a person's heart and the next beat.

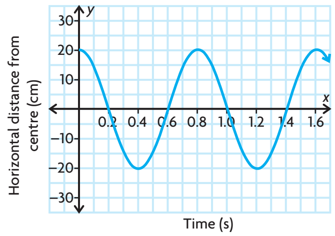
b) 80

c)



d) The range for the function is between 80 and 120. The range means the lowest blood pressure is 80 and the highest blood pressure is 120.

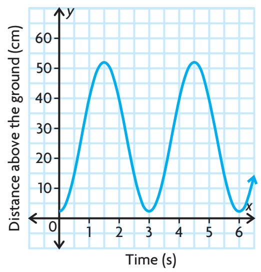
10. a)



b) There is a vertical stretch by a factor of 20, followed by a horizontal compression by a of factor of $\frac{2}{5\pi}$, and then a horizontal translation 0.2 to the left.

c) $y = 20\sin\left(\frac{5\pi}{2}(x + 0.2)\right)$

11. a)



b) vertical stretch by a factor of 25, reflection in the x-axis, vertical translation 27 units up, horizontal compression by a factor of $\frac{1}{|k|} = \frac{3}{2\pi}$

c) $y = -25\cos\left(\frac{2\pi}{3}x\right) + 27$

12. $\frac{2\pi}{7}$

13. Answers may vary. For example, $\left(\frac{14\pi}{13}, 5\right)$.

14. a) $y = \cos(4\pi x)$

b) $y = -2\sin\left(\frac{\pi}{4}x\right)$

c) $y = 4\sin\left(\frac{\pi}{20}(x - 10)\right) - 1$

15.

Start with graph of $y = \sin x$.

↓

Reflect in the x-axis and stretch vertically by a factor of 2 to produce graph of $y = -2\sin x$.

↓

Stretch horizontally by a factor of 2 to produce graph of $y = -2\sin(0.5x)$.

↓

Translate $\frac{\pi}{4}$ units to the right to produce graph of $y = -2\sin\left(0.5\left(x - \frac{\pi}{4}\right)\right)$.

↓

Translate 3 units up to produce graph of $y = -2\sin\left(0.5\left(x - \frac{\pi}{4}\right)\right) + 3$.

16. a) 100 m
b) 400 m
c) 300 m
d) 80 s
e) about 23.561 94 m/s

Mid-Chapter Review, p. 349

1. a) 22.5°
b) 720°
c) 286.5°
d) 165°

2. a) $125° \doteq 2.2$ radians
b) $450° \doteq 7.9$ radians
c) $5° \doteq 0.1$ radians
d) $330° \doteq 5.8$ radians
e) $215° \doteq 3.8$ radians
f) $-140° \doteq -2.4$ radians

3. a) 20π
b) 4π radians/s
c) 380π cm

4. a) $\frac{\sqrt{2}}{2}$
b) $-\frac{1}{2}$
c) $-\sqrt{3}$
d) $-\frac{\sqrt{3}}{3}$
e) 0
f) $-\frac{1}{2}$

5. a) about 1.78
b) about 0.86
c) about 1.46
d) about 4.44
e) about 0.98
f) about 4.91

6. a) $\sin \frac{\pi}{6}$

b) $\cot \frac{3\pi}{4}$

c) $\sec \frac{\pi}{2}$

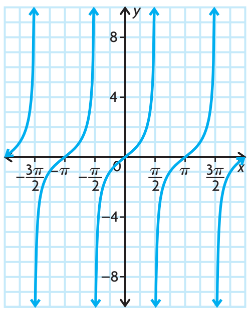
d) $\cos \frac{5\pi}{6}$

7. a) $x = 0, \pm\pi, \pm 2\pi, \ldots; y = 0$

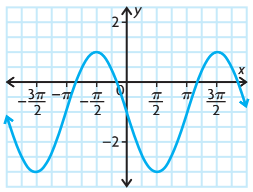
b) $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots; y = 1$

c) $x = 0, \pm\pi, \pm 2\pi, \ldots; y = 0$

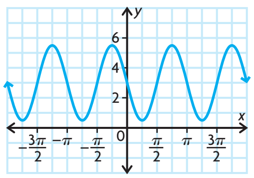
8. a)



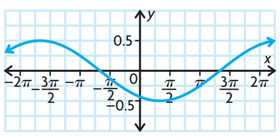
b)



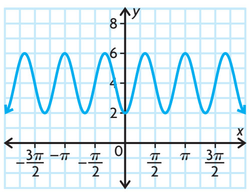
c)



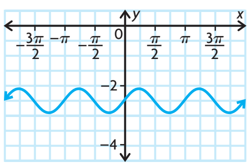
d)



e)



f)



9. $y = \frac{1}{3}\sin\left(-3\left(x + \frac{\pi}{8}\right)\right) - 23$

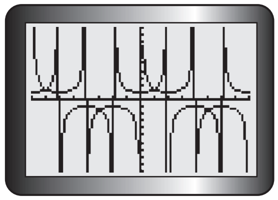
Lesson 6.5, p. 353

1. a) $t_n = n\pi, n \in \mathbf{I}$
   b) no maximum value
   c) no minimum value

2. a) $t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$
   b) no maximum value
   c) no minimum value

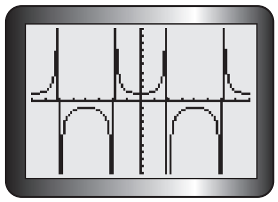
3. a) $t_n = n\pi, n \in \mathbf{I}$
   b) $t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

4.



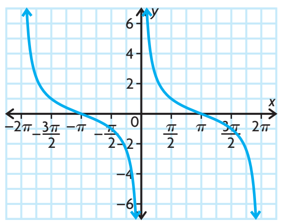
$-5.50, -2.35, 0.79, 3.93$

5. Yes, the graphs of $y = \csc\left(x + \frac{\pi}{2}\right)$ and $y = \sec x$ are identical.

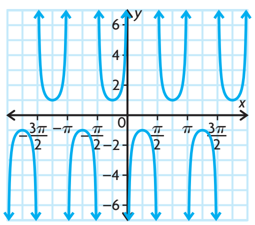


6. Answers may vary. For example, reflect the graph of $y = \tan x$ across the y-axis and then translate the graph $\frac{\pi}{2}$ units to the left.

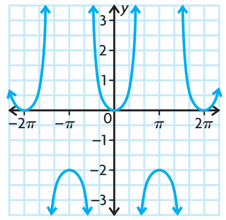
7. a) period $= 2\pi$



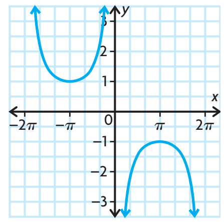
b) period $= \pi$



c) period $= 2\pi$



d) period $= 4\pi$

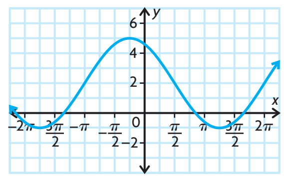


Lesson 6.6, pp. 360–362

1. $y = 3\cos\left(\frac{2}{3}\left(x + \frac{\pi}{4}\right)\right) + 2$

2. $2, 0.5, y \doteq 0.973\ 94$

3.



$x = 1.3$

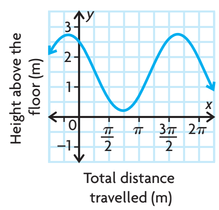
4. amplitude and equation of the axis

5. a) the radius of the circle in which the tip of the sparkler is moving
   b) the time it takes Mike to make one complete circle with the sparkler
   c) the height above the ground of the centre of the circle in which the tip of the sparkler is moving
   d) cosine function

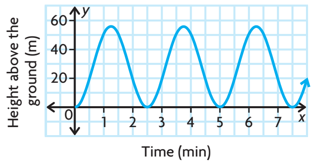
6. $y = 90\sin\left(\frac{\pi}{12}x\right) + 30$

7. $y = 250\cos\left(\frac{2\pi}{3}x\right) + 750$

8. $y = -1.25\sin\left(\frac{4}{5}x\right) + 1.5$



9. $0.98 \text{ min} < t < 1.52 \text{ min}$, $3.48 \text{ min} < t < 4.02 \text{ min}$, $5.98 \text{ min} < t < 6.52 \text{ min}$



10. **a)** $y = 3.7 \sin\left(\frac{2\pi}{365}x\right) + 12$
b) $y \doteq 13.87$ hours

11. $T(t) = 16.2 \sin\left(\frac{2\pi}{365}(t - 116)\right) + 1.4$, $0 < t < 111$ and $304 < t < 365$

12. The student should graph the height of the nail above the ground as a function of the total distance travelled by the nail, because the nail would not be travelling at a constant speed. If the student graphed the height of the nail above the ground as a function of time, the graph would not be sinusoidal.

13. minute hand:
$D(t) = 15 \cos\left(\frac{\pi}{30}t\right) + 300$;
second hand:
$D(t) = 15 \cos(2\pi t) + 300$;
hour hand:
$D(t) = 8 \cos\left(\frac{\pi}{360}t\right) + 300$

Lesson 6.7, pp. 369–373

1. **a)** $0 < x < \pi$, $\pi < x < 2\pi$
b) $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $\frac{3\pi}{2} < x < \frac{5\pi}{2}$
c) $\frac{\pi}{2} < x < \frac{3\pi}{2}$, $\frac{5\pi}{2} < x < 3\pi$

2. **a)** $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$
b) $x = \frac{\pi}{2}$, $x = \frac{5\pi}{2}$
c) $x = 0$, $x = 2\pi$

3. 0

4. **a)** about 0.465
b) 0
c) about -0.5157
d) about -1.554

5. **a)** $0 < x < \frac{\pi}{2}$, $\pi < x < \frac{3\pi}{2}$
b) $0 < x < \frac{\pi}{4}$, $\pi < x < \frac{5\pi}{4}$
c) $\frac{\pi}{4} < x < \frac{\pi}{2}$, $\frac{5\pi}{4} < x < \frac{3\pi}{2}$

6. **a)** $x = \frac{1}{4}$, $x = \frac{3}{4}$
b) $x = 0$, $x = 1$
c) $x = \frac{1}{2}$, $x = \frac{3}{2}$

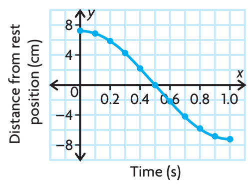
7. **a)** about -0.7459
b) about -1.310
c) 0

8. negative

9. **a)** $R(t) = 4.5 \cos\left(\frac{\pi}{12}t\right) + 20.2$
b) fastest: $t = 6$ months, $t = 18$ months, $t = 30$ months, $t = 42$ months; slowest: $t = 0$ months, $t = 12$ months, $t = 24$ months, $t = 36$ months, $t = 48$ months
c) about 1.164 mice per owl/s

10. **a)** **i)** 0.25 t/h
ii) about 0.2588 t/h
iii) 0.2612 t/h
b) The estimate calculated in part iii) is the most accurate. The smaller the interval, the more accurate the estimate.

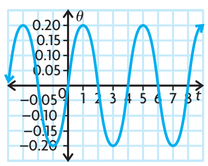
11. **a)**



b) half of one cycle
c) -14.4 cm/s
d) The bob is moving the fastest when it passes through its rest position. You can tell because the images of the balls are farthest apart at this point.
e) The pendulum's rest position is halfway between the maximum and minimum values on the graph. Therefore, at this point, the pendulum's instantaneous rate of change is at its maximum.

12. **a)** 0
b) -0.5 m/s

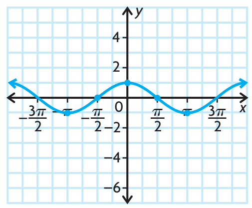
13. **a)**



b) 0.2 radians/s
c) Answers may vary. For example, about $-\frac{2}{3}$ radians/s.
d) $t = 0, 2, 4, 6$, and 8

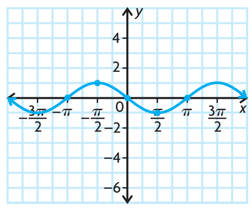
14. Answers may vary. For example, for $x = 0$, the instantaneous rate of change of $f(x) = \sin x$ is approximately 0.9003, while the instantaneous rate of change of $f(x) = 3\sin x$ is approximately 2.7009. (The interval $-\frac{\pi}{4} < x < \frac{\pi}{4}$ was used.) Therefore, the instantaneous rate of change of $f(x) = 3\sin x$ is at its maximum three times more than the instantaneous rate of change of $f(x) = \sin x$. However, there are points where the instantaneous rate of change is the same for the two functions. For example, at $x = \frac{\pi}{2}$, it is 0 for both functions.

15. **a)** $-1, 0, 1, 0$, and -1
b)



The function is $f(x) = \cos x$. Based on this information, the derivative of $f(x) = \sin x$ is $\cos x$.

16. **a)** $0, 1, 0, -1$, and 0
b)



The function is $f(x) = -\sin x$. Based on this information, the derivative of $f(x) = \cos x$ is $-\sin x$.

Chapter Review, pp. 376–377

1. $\frac{33}{16}$

2. 70π

3. **a)** $\frac{\pi}{9}$ radians
b) $\frac{-5\pi}{18}$ radians
c) $\frac{8\pi}{9}$ radians
d) $\frac{7\pi}{3}$ radians

4. **a)** 45°
b) −225°
c) 480°
d) −120°

5. **a)** $\frac{5\pi}{6}$
b) $\frac{4\pi}{3}$
c) $\frac{3\pi}{4}$
d) $\frac{7\pi}{6}$

6. **a)** $\tan\theta = \frac{12}{13}$
b) $\sec\theta = -\frac{13}{5}$
c) about 5.14

7. 2.00

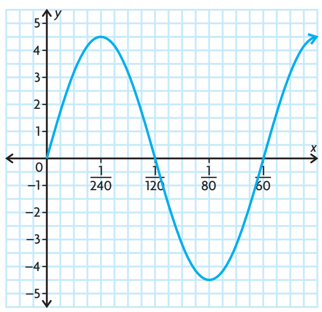
8. **a)** 2π radians
b) 2π radians
c) π radians

9. $y = 5 \sin\left(x + \frac{\pi}{3}\right) + 2$

10. $y = -3 \cos\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$

11. **a)** reflection in the x-axis, vertical stretch by a factor of 19, vertical translation 9 units down
b) horizontal compression by a factor of $\frac{1}{10}$, horizontal translation $\frac{\pi}{12}$ to the left
c) vertical compression by a factor of $\frac{10}{11}$, horizontal translation $\frac{\pi}{9}$ to the right, vertical translation 3 units up
d) reflection in the x-axis, reflection in the y-axis, horizontal translation π to the right

12. **a)**



b) $\frac{1}{60}$
c) $\frac{1}{240}$
d) $\frac{1}{80}$

13. **a)** 2π radians
b) 2π radians
c) π radians

14. **a)** the radius of the circle in which the bumblebee is flying
b) the time that the bumblebee takes to fly one complete circle
c) the height, above the ground, of the centre of the circle in which the bumblebee is flying
d) cosine function

15. $P(m) = 7250 \cos\left(\frac{\pi}{6}m\right) + 7750$

16. $h(t) = 30 \sin\left(\frac{5\pi}{3}t - \frac{\pi}{2}\right) + 150$

17. **a)** $0 < x < 5\pi$, $10\pi < x < 15\pi$
b) $2.5\pi < x < 7.5\pi$, $12.5\pi < x < 17.5\pi$
c) $0 < x < 2.5\pi$, $7.5\pi < x < 12.5\pi$

18. **a)** $x = 0$, $x = \frac{1}{2}$
b) $x = \frac{1}{8}$, $x = \frac{5}{8}$
c) $x = \frac{3}{8}$, $x = \frac{7}{8}$

19. **a)** $x = \frac{3}{4}$ s
b) the time between one beat of a person's heart and the next beat
c) 140
d) -129

Chapter Self-Test, p. 378

1. $y = \sec x$
2. $\sec 2\pi$
3. $y \doteq 108.5$
4. about 0.31 °C per day
5. $\frac{3\pi}{5}$, 110°, $\frac{5\pi}{8}$, 113°, and $\frac{2\pi}{3}$
6. $y = \sin\left(x + \frac{5\pi}{8}\right)$
7. $y \doteq -30$
8. **a)** $-3\cos\left(\frac{\pi}{12}x\right) + 22$
b) about 0.5 °C per hour
c) about 0 °C per hour

Cumulative Review Chapters 4–6, pp. 380–383

1.	(d)	**9.**	(c)	**17.**	(d)	**25.**	(b)
2.	(b)	**10.**	(c)	**18.**	(b)	**26.**	(d)
3.	(a)	**11.**	(d)	**19.**	(b)	**27.**	(a)
4.	(c)	**12.**	(a)	**20.**	(b)	**28.**	(c)
5.	(a)	**13.**	(d)	**21.**	(d)	**29.**	(b)
6.	(b)	**14.**	(c)	**22.**	(c)		
7.	(a)	**15.**	(d)	**23.**	(a)		
8.	(c)	**16.**	(a)	**24.**	(d)		

30. **a)** If x is the length in centimetres of a side of one of the corners that have been cut out, the volume of the box is $(50 - 2x)(40 - 2x)x$ cm^3.
b) 5 cm or 10 cm
c) $x \doteq 7.4$ cm
d) $3 < x < 12.8$

31. **a)** The zeros of $f(x)$ are $x = 2$ or $x = 3$. The zero of $g(x)$ is $x = 3$. The zero of $\frac{f(x)}{g(x)}$ is $x = 2$. $\frac{g(x)}{f(x)}$ does not have any zeros.
b) $\frac{f(x)}{g(x)}$ has a hole at $x = 3$; no asymptotes. $\frac{g(x)}{f(x)}$ has an asymptote at $x = 2$ and $y = 0$.
c) $x = 1$; $\frac{f(x)}{g(x)}$: $y = x - 2$, $\frac{g(x)}{f(x)}$: $y = -x$

32. **a)** Vertical compressions and stretches do not affect location of zeros; maximum and minimum values are multiplied by the scale factor, but locations are unchanged; instantaneous rates of change are multiplied by the scale factor.
Horizontal compressions and stretches move locations of zeros, maximums, and minimums toward or away from the y-axis by the reciprocal of the scale factor; instantaneous rates of change are multiplied by the reciprocal of scale factor.
Vertical translations change location of zeros or remove them; maximum and minimum values are increased or decreased by the amount of the translation, but locations are unchanged; instantaneous rates of change are unchanged.
Horizontal translations move location of zeros by the same amount as the translation; maximum and minimum values are unchanged, but locations are moved by the same amount as the translation; instantaneous rates of change are unchanged, but locations are moved by the same amount as the translation.
b) For $y = \cos x$, the answer is the same as in part **a)**, except that a horizontal reflection does not affect instantaneous rates of change. For $y = \tan x$, the answer is also the same as in part **a)**, except that nothing affects the maximum and minimum values, since there are no maximum or minimum values for $y = \tan x$.

Chapter 7

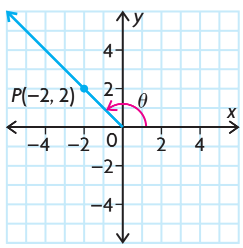
Getting Started, p. 386

1. **a)** 1
b) $-\frac{22}{7}$
c) 8 or -3
d) $\frac{2}{3}$ or $-\frac{5}{2}$
e) $-1 \pm \sqrt{2}$
f) $\frac{3 \pm \sqrt{21}}{6}$

2. To do this, you must show that the two distances are equal:
$$D_{AB} = \sqrt{(2-1)^2 + \left(\tfrac{1}{2} - 0\right)^2} = \frac{\sqrt{5}}{2};$$
$$D_{CD} = \sqrt{\left(0 - \tfrac{1}{2}\right)^2 + (6-5)^2} = \frac{\sqrt{5}}{2}.$$
Since the distances are equal, the line segments are the same length.

3. **a)** $\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\tan A = \frac{8}{15}$, $\csc A = \frac{17}{8}$, $\sec A = \frac{17}{15}$, $\cot A = \frac{15}{8}$
b) 0.5 radians
c) 61.9°

4. **a)**



b) $\frac{\pi}{4}$ radians **c)** $\frac{3\pi}{4}$ radians

5. **a)** A: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$; F: $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;
B: $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; G: $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;
C: $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; H: $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;
D: $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$; I: $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;
E: $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; J: $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

b) **i)** $-\frac{\sqrt{2}}{2}$ **ii)** $-\frac{1}{2}$ **iii)** -1 **iv)** 2

6. **a)** If the angle x is in the second quadrant:
$\sin x = \frac{3}{5}$; $\cos x = -\frac{4}{5}$;
$\csc x = \frac{5}{3}$; $\sec x = -\frac{5}{4}$; $\cot x = -\frac{4}{3}$.
If the angle x is in the fourth quadrant:
$\sin x = -\frac{3}{5}$; $\cos x = \frac{4}{5}$; $\csc x = -\frac{5}{3}$;
$\sec x = \frac{5}{4}$; $\cot x = -\frac{4}{3}$

b) If x is in the second quadrant, $x = 2.5$.
If x is in the fourth quadrant, $x = 5.6$.

7. **a)** true **d)** false
b) true **e)** true
c) false **f)** true

8.

Perform a vertical stretch/compression by a factor of $|a|$.

↓

Use $\left|\frac{1}{k}\right|$ to determine the horizontal stretch/compression.

↓

Use a and k to determine whether the function is reflected in the y-axis or the x-axis.

↓

Perform a vertical translation of c units up or down.

↓

Perform a horizontal translation of d units to the right or the left.

Lesson 7.1, pp. 392–393

1. **a)** Answers may vary. For example:
$y = \cos(\theta + 2\pi)$, $y = \cos(\theta + 4\pi)$,
$y = \cos(\theta - 2\pi)$

b) $y = \sin\left(\theta + \frac{\pi}{2}\right)$, $y = \sin\left(\theta - \frac{3\pi}{2}\right)$,
$y = \sin\left(\theta + \frac{5\pi}{2}\right)$

2. **a)** $y = \csc\theta$ is odd, $\csc(-\theta) = -\csc\theta$;
$y = \sec\theta$ is even, $\sec(-\theta) = \sec\theta$;
$y = \cot\theta$ is odd, $\cot(-\theta) = -\cot\theta$

b) $y = \cot(-\theta)$ is the graph of $y = \cot\theta$ reflected across the y-axis; $y = -\cot\theta$ is the graph of $y = \cot\theta$ reflected across the x-axis. Both of these transformations result in the same graph. $y = \csc(-\theta)$ is the graph of $y = \csc\theta$ reflected across the y-axis; $y = -\csc\theta$ is the graph of $y = \csc\theta$ reflected across the x-axis. Both of these transformations result in the same graph. $y = \sec(-\theta)$ is the graph of $y = \sec\theta$ reflected across the y-axis. This results in the same graph as $y = \sec\theta$.

3. **a)** $\cos\frac{\pi}{3}$ **c)** $\cot\frac{\pi}{8}$ **e)** $\cos\frac{3\pi}{8}$
b) $\sin\frac{\pi}{12}$ **d)** $\sin\frac{3\pi}{16}$ **f)** $\cot\frac{\pi}{3}$

4. **a)** $\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$;
$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$;
$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

b) $y = \tan\left(\frac{\pi}{2} - \theta\right) = \tan\left(-\left(\theta - \frac{\pi}{2}\right)\right)$;
This is the graph of $y = \tan\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \cot\theta$.
$y = \csc\left(\frac{\pi}{2} - \theta\right) = \csc\left(-\left(\theta - \frac{\pi}{2}\right)\right)$;
This is the graph of $y = \csc\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \sec\theta$.
$y = \sec\left(\frac{\pi}{2} - \theta\right) = \sec\left(-\left(\theta - \frac{\pi}{2}\right)\right)$;
This is the graph of $y = \sec\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \csc\theta$.

5. **a)** $\sin\frac{\pi}{8}$ **d)** $\cos\frac{\pi}{6}$
b) $-\cos\frac{\pi}{12}$ **e)** $-\sin\frac{3\pi}{8}$
c) $\tan\frac{\pi}{4}$ **f)** $-\tan\frac{\pi}{3}$

6. **a)** Assume the circle is a unit circle. Let the coordinates of Q be (x, y). Since P and Q are reflections of each other in the line $y = x$, the coordinates of P are (y, x). Draw a line from P to the positive x-axis. The hypotenuse of the new right triangle makes an angle of $\left(\frac{\pi}{2} - \theta\right)$ with the positive x-axis. Since the x-coordinate of P is y, $\cos\left(\frac{\pi}{2} - \theta\right) = y$. Also, since the y-coordinate of Q is y, $\sin\theta = y$.
Therefore, $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$.

b) Assume the circle is a unit circle. Let the coordinates of the vertex on the circle of the right triangle in the first quadrant be (x, y). Then $\sin\theta = y$, so $-\sin\theta = -y$. The point on the circle that results from rotating the vertex by $\frac{\pi}{2}$ counterclockwise about the origin has coordinates $(-y, x)$, so
$\cos\left(\frac{\pi}{2} + \theta\right) = -y$. Therefore,
$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$.

7. **a)** true

b) false; Answers may vary. For example: Let $\theta = \frac{\pi}{2}$. Then the left side is $\sin\frac{\pi}{2}$, or 1. The right side is $-\sin\frac{\pi}{2}$, or -1.

c) false; Answers may vary. For example: Let $\theta = \pi$. Then the left side is $\cos\pi$, or -1. The right side is $-\cos 5\pi$, or 1.

d) false; Answers may vary. For example: Let $\theta = \frac{\pi}{4}$. Then the left side is $\tan\frac{3\pi}{4}$, or $-\frac{\sqrt{2}}{2}$. The right side is $\tan\frac{\pi}{4}$, or $\frac{\sqrt{2}}{2}$.

e) false; Answers may vary. For example: Let $\theta = \pi$. Then the left side is $\cot\frac{3\pi}{4}$, or -1. The right side is $\tan\frac{\pi}{4}$, or 1.

f) false; Answers may vary. For example: Let $\theta = \frac{\pi}{2}$. Then the left side is $\sin\frac{5\pi}{2}$, or 1. The right side is $\sin\left(-\frac{\pi}{2}\right)$, or -1.

Lesson 7.2, pp. 400–401

1. **a)** $\sin 3a$ **b)** $\cos 7x$

2. **a)** $\tan 60°$; $\sqrt{3}$ **b)** $\cos\frac{\pi}{3}$; $\frac{1}{2}$

3. **a)** $30° + 45°$ **d)** $\frac{\pi}{4} - \frac{\pi}{6}$
b) $30° - 45°$ **e)** $60° + 45°$
c) $\frac{\pi}{6} - \frac{\pi}{3}$ **f)** $\frac{\pi}{2} + \frac{\pi}{3}$

4. **a)** $\frac{\sqrt{2} + \sqrt{6}}{4}$ **d)** $\frac{\sqrt{2} - \sqrt{6}}{4}$
b) $\frac{\sqrt{2} + \sqrt{6}}{4}$ **e)** $\frac{\sqrt{2} - \sqrt{6}}{4}$
c) $2 + \sqrt{3}$ **f)** $-2 + \sqrt{3}$

5. **a)** $-\frac{1}{2}$ **d)** $-\frac{1}{2}$
b) $-\frac{\sqrt{2}}{2}$ **e)** $\frac{\sqrt{3}}{3}$
c) 1 **f)** $-\frac{\sqrt{3}}{2}$

6. **a)** $-\sin x$ **d)** $\tan x$
b) $\sin x$ **e)** $-\sin x$
c) $-\sin x$ **f)** $-\tan x$

7. **a)** $\sin(\pi + x)$ is equivalent to $\sin x$ translated π to the left, which is equivalent to $-\sin x$.
b) $\cos\left(x + \frac{3\pi}{2}\right)$ is equivalent to $\cos x$ translated $\frac{3\pi}{2}$ to the left, which is equivalent to $\sin x$.
c) $\cos\left(x + \frac{\pi}{2}\right)$ is equivalent to $\cos x$ translated $\frac{\pi}{2}$ to the left, which is equivalent to $-\sin x$.
d) $\tan(x + \pi)$ is equivalent to $\tan x$ translated π to the left, which is equivalent to $\tan x$.
e) $\sin(x - \pi)$ is equivalent to $\sin x$ translated π to the right, which is equivalent to $-\sin x$.
f) $\tan(2\pi - x)$ is equivalent to $\tan(-x)$, which is equivalent to $\tan x$ reflected in the y-axis, which is equivalent to $-\tan x$.

8. **a)** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **d)** $\frac{\sqrt{2} - \sqrt{6}}{4}$
b) $-2 + \sqrt{3}$ **e)** $-2 - \sqrt{3}$
c) $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **f)** $-2 - \sqrt{3}$

9. **a)** $\frac{63}{65}$ **d)** $\frac{56}{65}$
b) $-\frac{16}{65}$ **e)** $-\frac{16}{63}$
c) $-\frac{33}{65}$ **f)** $-\frac{56}{33}$

10. $\frac{323}{325}; \frac{323}{36}$

11. **a)** $\cos\left(\frac{\pi}{2} - x\right)$
$= \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$
$= (0)(\cos x) + (1)(\sin x)$
$= 0 + \sin x$
$= \sin x$
b) $\sin\left(\frac{\pi}{2} - x\right)$
$= \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$
$= (1)(\cos x) - (0)(\sin x)$
$= \cos x - 0$
$= \cos x$

12. **a)** 0 **b)** $-\sqrt{3}\sin x$

13. $\tan f$, $\cos f \neq 0$, $\cos g \neq 0$

14.

Write $\sin a$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Since $a \in \left[0, \frac{\pi}{2}\right]$, choose the positive value of x and determine $\cos a$.
↓
Write $\sin b$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Since $b \in \left[0, \frac{\pi}{2}\right]$, choose the positive value of x and determine $\cos b$.
↓
Use the formula $\cos(a + b) = \cos a \cos b - \sin a \sin b$ to evaluate $\cos(a + b)$.

15. See compound angle formulas listed on p. 399.
The two sine formulas are the same, except for the operators. Remembering that the same operator is used on both the left and right sides in both equations will help you remember the formulas.
Similarly, the two cosine formulas are the same, except for the operators. Remembering that the operator on the left side is the opposite of the operator on the right side in both equations will help you remember the formulas.
The two tangent formulas are the same, except for the operators in the numerator and the denominator on the right side. Remembering that the operators in the numerator and the denominator are opposite in both equations, and that the operator in the numerator is the same as the operator on the left side, will help you remember the formulas.

16. $2\sin\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)$

$= (2)\left(\left(\sin\frac{C}{2}\right)\left(\cos\frac{D}{2}\right) + \left(\cos\frac{C}{2}\right)\left(\sin\frac{D}{2}\right)\right)\left(\left(\cos\frac{C}{2}\right)\right.$
$\times \left.\left(\cos\frac{D}{2}\right) + \left(\sin\frac{C}{2}\right)\left(\sin\frac{D}{2}\right)\right)$

$= (2)\left(\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\cos^2\frac{D}{2}\right)\right.$
$+ \left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\cos^2\frac{C}{2}\right)$
$+ \left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\sin^2\frac{C}{2}\right)$
$+ \left.\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\sin^2\frac{D}{2}\right)\right)$
$= (2)\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\cos^2\frac{D}{2} + \sin^2\frac{D}{2}\right)$
$+ 2\left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right)$
$= (2)\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)$
$+ 2\left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)$
$= \sin\left(2\left(\frac{C}{2}\right)\right) + \sin\left(2\left(\frac{D}{2}\right)\right)$
$= \sin C + \sin D$

17. $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$

18. Let $C = x + y$ and let $D = x - y$.
$\cos C + \cos D$
$= \cos(x + y) + \cos(x - y)$
$= \cos x \cos y - \sin x \sin y$
$+ \cos x \cos y + \sin x \sin y$
$= 2\cos x \cos y$
$\frac{C + D}{2} = \frac{x + y + x - y}{2} = x$
$\frac{C - D}{2} = \frac{x + y - x + y}{2} = y$
So $\cos C + \cos D$
$= 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)$

19. Let $C = x + y$ and let $D = x - y$.
$\cos C - \cos D$
$= \cos(x + y) - \cos(x - y)$
$= \cos x \cos y - \sin x \sin y$
$- (\cos x \cos y - \sin x \sin y)$
$= -2\sin x \sin y$
$\frac{C + D}{2} = \frac{x + y + x - y}{2} = x$
$\frac{C - D}{2} = \frac{x + y - x + y}{2} = y$
So $\cos C - \cos D$
$= -2\sin\left(\frac{C + D}{2}\right)\sin\left(\frac{C - D}{2}\right)$

Lesson 7.3, pp. 407–408

1. **a)** $\sin 10x$ **d)** $\tan 8x$
b) $\cos 2\theta$ **e)** $2\sin 2\theta$
c) $\cos 6x$ **f)** $\cos\theta$

2. **a)** $\sin 90°$; 1 **d)** $\cos\frac{\pi}{6}$; $\frac{\sqrt{3}}{2}$
b) $\cos 60°$; $\frac{1}{2}$ **e)** $\cos\frac{3\pi}{4}$; $-\frac{\sqrt{2}}{2}$
c) $\sin\frac{\pi}{6}$; $\frac{1}{2}$ **f)** $\sin 120°$; $\frac{\sqrt{3}}{2}$

3. a) $2\sin 2\theta \cos 2\theta$
b) $2\sin^2(1.5x) - 1$
c) $\dfrac{2\tan(0.5x)}{1-\tan^2(0.5x)}$
d) $\cos^2 3\theta - \sin^2 3\theta$
e) $2\sin(0.5x)\cos(0.5x)$
f) $\dfrac{2\tan(2.5\theta)}{1-\tan^2(2.5\theta)}$

4. $\sin 2\theta = \dfrac{24}{25}$, $\cos 2\theta = -\dfrac{7}{25}$,
$\tan 2\theta = -\dfrac{24}{7}$

5. $\sin 2\theta = -\dfrac{336}{625}$, $\cos 2\theta = \dfrac{527}{625}$,
$\tan 2\theta = -\dfrac{336}{527}$

6. $\sin 2\theta = -\dfrac{120}{169}$, $\cos 2\theta = -\dfrac{119}{169}$,
$\tan 2\theta = \dfrac{120}{119}$

7. $\sin 2\theta = -\dfrac{24}{25}$, $\cos 2\theta = \dfrac{7}{25}$,
$\tan 2\theta = -\dfrac{24}{7}$

8. $a = \dfrac{1}{2}$

9. Jim can find the sine of $\frac{\pi}{8}$ by using the formula $\cos 2x = 1 - 2\sin^2 x$ and isolating $\sin x$ on one side of the equation. When he does this, the formula becomes $\sin x = \pm\sqrt{\frac{1-\cos 2x}{2}}$. The cosine of $\frac{\pi}{4}$ is $\frac{\sqrt{2}}{2}$, so $\sin\frac{\pi}{8} = \pm\sqrt{\dfrac{1-\cos\frac{\pi}{4}}{2}}$
$= \pm\dfrac{\sqrt{2-\sqrt{2}}}{2}$.
Since $\frac{\pi}{8}$ is in the first quadrant, the sign of $\sin\frac{\pi}{8}$ is positive.

10. Marion can find the cosine of $\frac{\pi}{12}$ by using the formula $\cos 2x = 2\cos^2 x - 1$ and isolating $\cos x$ on one side of the equation. When she does this, the formula becomes $\cos x = \pm\sqrt{\frac{1+\cos 2x}{2}}$. The cosine of $\frac{\pi}{6}$ is $\frac{\sqrt{3}}{2}$, so $\cos\frac{\pi}{12} = \pm\sqrt{\dfrac{1+\cos\frac{\pi}{6}}{2}}$
$= \pm\dfrac{\sqrt{2+\sqrt{3}}}{2}$.
Since $\frac{\pi}{12}$ is in the first quadrant, the sign of $\cos\frac{\pi}{12}$ is positive.

11. a) $\sin 4x$
$= (2)(2\sin x\cos x)(\cos 2x)$
$= (2)(2\sin x\cos x)(1 - 2\sin^2 x)$
$= (4\sin x\cos x)(1 - 2\sin^2 x)$
$= 4\sin x\cos x - 8\sin^3 x\cos x$

b) $\sin\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$
$\sin 4\left(\dfrac{2\pi}{3}\right) = 4\sin\dfrac{2\pi}{3}\cos\dfrac{2\pi}{3} - 8\sin^3\dfrac{2\pi}{3}\cos\dfrac{2\pi}{3}$
$\sin\dfrac{8\pi}{3} = (4)\left(\dfrac{\sqrt{3}}{2}\right)\left(-\dfrac{1}{2}\right) - (8)\left(\dfrac{\sqrt{3}}{2}\right)^3\left(-\dfrac{1}{2}\right)$
$\sin\dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - (-4)\left(\dfrac{3\sqrt{3}}{8}\right)$
$\sin\dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - \left(-\dfrac{3\sqrt{3}}{2}\right)$
$\sin\dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - \left(-\dfrac{6\sqrt{3}}{4}\right)$
$\sin\dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} + \dfrac{6\sqrt{3}}{4}$
$\sin\dfrac{8\pi}{3} = \dfrac{2\sqrt{3}}{4}$
$\sin\dfrac{8\pi}{3} = \dfrac{\sqrt{3}}{2}$

12. a) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\sin 2\theta = 2\cos\theta\sin\theta$
$\sin 3\theta = (\sin 2\theta + \theta)$
$= (2\cos\theta\sin\theta)(\cos\theta) + (\cos^2\theta - \sin^2\theta)(\sin\theta)$
$= 2\cos^2\theta\sin\theta + \cos^2\theta\sin\theta - \sin^3\theta$
$= 3\cos^2\theta\sin\theta - \sin^3\theta$

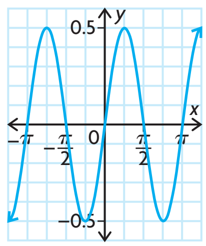
b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\sin 2\theta = 2\cos\theta\sin\theta$
$\cos 3\theta = (\cos 2\theta + \theta)$
$= (\cos^2\theta - \sin^2\theta)(\cos\theta) - (2\cos\theta\sin\theta)(\sin\theta)$
$= \cos^3\theta - \cos x\sin^2\theta - 2\cos\theta\sin^2\theta$
$= \cos^3\theta - 3\cos\theta\sin^2\theta$

c) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
$\tan 3\theta = (\tan 2\theta + \theta)$
$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\tan\theta}$$
$$= \frac{\frac{2\tan\theta + \tan\theta - \tan^3\theta}{1-\tan^2\theta}}{\frac{1-\tan^2\theta - 2\tan^2\theta}{1-\tan^2\theta}}$$
$$= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

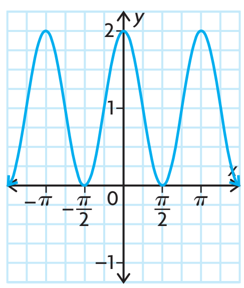
13. a) $-\dfrac{4\sqrt{2}}{9}$ b) $-\dfrac{7}{9}$ c) $\dfrac{\sqrt{3}}{3}$ d) $-\dfrac{10\sqrt{2}}{27}$

14.
Write $\sin a$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Choose the negative value of x since $a \in \left[\frac{\pi}{2}, \pi\right]$, and determine $\cos a$.
↓
Write $\cos a$ in terms of $\frac{x}{r}$.
↓
Use the formula $\sin 2a = 2\sin a\cos a$ to evaluate $\sin 2a$.

15. a) Use the formula $\sin 2x = 2\sin x\cos x$ to determine that $\sin x\cos x = \frac{\sin 2x}{2}$.
Then graph the function $f(x) = \frac{\sin 2x}{2}$ by vertically compressing $f(x) = \sin x$ by a factor of $\frac{1}{2}$ and horizontally compressing it by a factor of $\frac{1}{2}$.

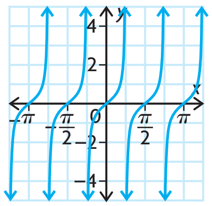


b) Use the formula $\cos 2x = 2\cos^2 x - 1$ to determine that $2\cos^2 x = \cos 2x + 1$.
Then graph the function $f(x) = \cos 2x + 1$ by horizontally compressing $f(x) = \cos x$ by a factor of $\frac{1}{2}$ and vertically translating it 1 unit up.



c) Use the formula $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ to determine that $\frac{\tan x}{1-\tan^2 x} = \frac{\tan 2x}{2}$.

Then graph the function $f(x) = \frac{\tan 2x}{2}$ by vertically compressing $f(x) = \tan x$ by a factor of $\frac{1}{2}$ and horizontally compressing it by a factor of $\frac{1}{2}$.



16. **a)** $\frac{\tan^{-1} x}{2} = \tan^{-1} y$

b) $\frac{\cos^{-1} x}{2} = \cos^{-1} y$

c) $\frac{\cos^{-1} x}{2} = \csc^{-1} y$ or

$\frac{\cos^{-1} x}{2} = \sin^{-1}\left(\frac{1}{y}\right)$

d) $\frac{\sin^{-1} x}{2} = \frac{\sec^{-1} y}{4}$ or

$\frac{\sin^{-1} x}{2} = \frac{\cos^{-1}\left(\frac{1}{y}\right)}{4}$

17. **a)** $x = \frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$

b) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, or $\frac{3\pi}{2}$

18. **a)** $\frac{2 \tan \theta}{1 + \tan^2 \theta}$

b) $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

c) $\tan \theta$

d) $\tan \theta$

Mid-Chapter Review, p. 411

1. **a)** $\cos \frac{31\pi}{16}$ **d)** $\cos \frac{7\pi}{5}$

b) $\sin \frac{2\pi}{9}$ **e)** $\sin \frac{2\pi}{7}$

c) $\tan \frac{19\pi}{10}$ **f)** $\tan \frac{7\pi}{4}$

2. $y = 6 \sin x + 4$

3. **a)** $\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$

b) $\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$

c) $\frac{1 + \tan x}{1 - \tan x}$

d) $\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x$

4. **a)** $\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$

b) $\frac{\tan x - \sqrt{3}}{1 + \sqrt{3} \tan x}$

c) $\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x$

d) $-\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x$

5. **a)** $\sqrt{3}$ **c)** $\frac{1}{2}$

b) 0 **d)** 1

6. **a)** $\tan 2x$ **d)** $\cos x$

b) $\sin x$ **e)** $\sqrt{2}(\cos x - \sin x)$

c) $\sin x$ **f)** $\frac{\tan x - 1}{1 + \tan x}$

7. $2\sqrt{3} \cos\left(x + \frac{\pi}{3}\right)$

8. **a)** $-\frac{1}{2}$ **c)** $\frac{\sqrt{2}}{2}$

b) $-\frac{1}{2}$ **d)** -1

9. **a)** $-\frac{\sqrt{11}}{11}$ **c)** $\frac{2\sqrt{10}}{11}$

b) $-\frac{\sqrt{110}}{11}$ **d)** $\frac{9}{11}$

10. $\sin 2x = \frac{24}{25}$; $\cos 2x = \frac{7}{25}$

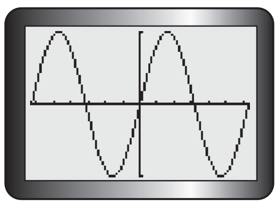
11. $\sin 2x = \frac{120}{169}$

12. $\tan 2x = \frac{24}{7}$

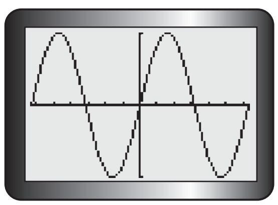
Lesson 7.4, pp. 417–418

1. Answers may vary. For example, $\sin \frac{\pi}{6} = \frac{1}{2}$; $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

2. **a)** $f(x) = \sin x$



$g(x) = \tan x \cos x$



b) $\sin x = \tan x \cos x$

c) $\tan x \cos x = \left(\frac{\sin x}{\cos x}\right) \cos x$

$= \frac{\sin x \cos x}{\cos x} = \sin x$

d) The identity is not true when $\cos x = 0$ because when $\cos x = 0$, $\tan x$, or $\frac{\sin x}{\cos x}$, is undefined.

3. **a)** C; $\sin x \cot x = \cos x$

b) D; $1 - 2 \sin^2 x = 2 \cos^2 x - 1$

c) B; $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

d) A; $\sec^2 x = \sin^2 x + \cos^2 x + \tan^2 x$

4. **a)** $\sin x \cot x = \cos x$

$$\begin{aligned} \text{LS} &= \sin x \cot x \\ &= (\sin x)\left(\frac{\cos x}{\sin x}\right) \\ &= \frac{\sin x \cos x}{\sin x} \\ &= \cos x \\ &= \text{RS} \end{aligned}$$

b) $1 - 2 \sin^2 x = 2 \cos^2 x - 1$

$$\begin{aligned} 1 - 2 \sin^2 x - 2 \cos^2 x + 1 &= 0 \\ 2 - 2 \sin^2 x - 2 \cos^2 x &= 0 \\ 2 - 2 (\sin^2 x + \cos^2 x) &= 0 \\ 2 - 2(1) &= 0 \\ 2 - 2 &= 0 \\ 0 &= 0 \end{aligned}$$

c) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$$\begin{aligned} \text{LS} &= (\sin x + \cos x)^2 \\ &= \sin^2 x + 2 \sin x \cos x \\ &\quad + \cos^2 x \\ &= (\sin^2 x + \cos^2 x) \\ &\quad + 2 \sin x \cos x \\ &= 1 + 2 \sin x \cos x \\ &= \text{RS} \end{aligned}$$

d) $\sec^2 x = \sin^2 x + \cos^2 x + \tan^2 x$

$$\begin{aligned} \text{RS} &= \sin^2 x + \cos^2 x + \tan^2 x \\ &= (\sin^2 x + \cos^2 x) + \tan^2 x \\ &= 1 + \tan^2 x \\ &= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \\ &= \text{LS} \end{aligned}$$

5. **a)** Answers may vary. For example,

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$; $\frac{1}{\cos \frac{\pi}{6}} = \frac{2\sqrt{3}}{3}$.

b) Answers may vary. For example,

$$\begin{aligned} 1 - \tan^2\left(\frac{\pi}{4}\right) &= 1 - (1)^2 \\ &= 1 - 1 = 0; \end{aligned}$$

$\sec^2\left(\frac{\pi}{4}\right) = (\sqrt{2})^2 = 2$

c) Answers may vary. For example,

$\sin\left(\frac{\pi}{2} + \pi\right) = \sin\left(\frac{3\pi}{2}\right) = -1$;

$\cos\left(\frac{\pi}{2}\right)\cos \pi + \sin\left(\frac{\pi}{2}\right)\sin \pi$

$= (0)(-1) + (1)(0)$

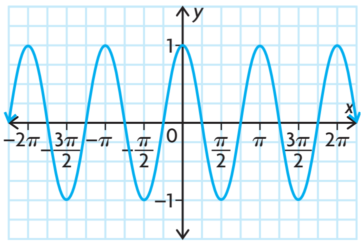
$= 0 + 0 = 0$

d) Answers may vary. For example,

$$\cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\begin{aligned} 1 + 2\sin^2\left(\frac{\pi}{3}\right) &= 1 + (2)\left(\frac{\sqrt{3}}{2}\right)^2 \\ &= 1 + (2)\left(\frac{3}{4}\right) \\ &= 1 + \frac{6}{4} = \frac{10}{4} \\ &= \frac{5}{2} \end{aligned}$$

6. Answers may vary. For example, cos 2*x*.



7. $$\begin{aligned} \frac{1 - \tan^2 x}{1 + \tan^2 x} &= \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\sec^2 x} \\ &= \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \times \cos^2 x \\ &= \cos^2 x - \sin^2 x \\ &= \cos 2x \end{aligned}$$

8. $$\begin{aligned} \text{LS} &= \frac{1 + \tan x}{1 + \cot x} & \text{RS} &= \frac{1 - \tan x}{\cot x - 1} \\ &= \frac{1 + \tan x}{1 + \frac{1}{\tan x}} & &= \frac{1 - \tan x}{\frac{1}{\tan x} - 1} \\ &= \frac{1 + \tan x}{\frac{\tan x + 1}{\tan x}} & &= \frac{1 - \tan x}{\frac{1 - \tan x}{\tan x}} \\ &= \tan x & &= \tan x \end{aligned}$$

Since the right side and the left side are equal, $\frac{1 + \tan x}{1 + \cot x} = \frac{1 - \tan x}{\cot x - 1}$

9. a) $$\begin{aligned} &\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta} \\ &= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta)(\cos\theta + \sin\theta)} \\ &= \frac{\cos\theta - \sin\theta}{\cos\theta} \\ &= \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \\ &= 1 - \tan\theta \end{aligned}$$

b) $$\begin{aligned} \text{LS} &= \tan^2 x - \sin^2 x \\ &= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x \\ &= \sin^2 x\left(\frac{1}{\cos^2 x} - 1\right) \\ &= \sin^2 x(\sec^2 x - 1) \\ &= \sin^2 x \tan^2 x \\ &= \text{RS} \end{aligned}$$

So $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$.

c) $$\begin{aligned} \tan^2 x - \cos^2 x &= \frac{1}{\cos^2 x} - 1 \\ &\quad - \cos^2 x \\ \tan^2 x - \cos^2 x + \cos^2 x &= \frac{1}{\cos^2 x} - 1 - \cos^2 x \\ &\quad + \cos^2 x \\ \tan^2 x &= \frac{1}{\cos^2 x} - 1 \\ \tan^2 x &= \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} \\ \tan^2 x &= \frac{1 - \cos^2 x}{\cos^2 x} \\ \tan^2 x &= \frac{\sin^2 x}{\cos^2 x} \\ \tan^2 x &= \tan^2 x \end{aligned}$$

d) $$\begin{aligned} &\frac{1}{1 + \cos\theta} + \frac{1}{1 - \cos\theta} \\ &= \frac{1 - \cos\theta}{(1 + \cos\theta)(1 - \cos\theta)} \\ &\quad + \frac{1 + \cos\theta}{(1 - \cos\theta)(1 + \cos\theta)} \\ &= \frac{1 - \cos\theta}{1 - \cos^2\theta} + \frac{1 + \cos\theta}{1 - \cos^2\theta} \\ &= \frac{1 - \cos\theta + 1 + \cos\theta}{1 - \cos^2\theta} \\ &= \frac{2}{1 - \cos^2\theta} \\ &= \frac{2}{\sin^2\theta} \end{aligned}$$

10. a) $$\begin{aligned} \cos x \tan^3 x &= \sin x \tan^2 x \\ \frac{\cos x \tan^3 x}{\tan^2 x} &= \frac{\sin x \tan^2 x}{\tan^2 x} \\ \cos x \tan x &= \sin x \\ \cos x\left(\frac{\sin x}{\cos x}\right) &= \sin x \\ \sin x &= \sin x \end{aligned}$$

b) $$\begin{aligned} \sin^2\theta + \cos^4\theta &= \cos^2\theta + \sin^4\theta \\ \sin^2\theta + \cos^4\theta - \sin^4\theta &= \cos^2\theta \\ &\quad + \sin^4\theta - \sin^4\theta \\ \sin^2\theta + \cos^4\theta - \sin^4\theta &= \cos^2\theta \\ \sin^2\theta + \cos^4\theta - \sin^4\theta - \sin^2\theta &\\ &= \cos^2\theta - \sin^2\theta \\ \cos^4\theta - \sin^4\theta &= \cos^2\theta - \sin^2\theta \\ (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) &\\ &= \cos^2\theta - \sin^2\theta \\ \cos^2\theta + \sin^2\theta &= 1 \\ 1 &= 1 \end{aligned}$$

c) $$\begin{aligned} &(\sin x + \cos x)\left(\frac{\tan^2 x + 1}{\tan x}\right) \\ &\quad = \frac{1}{\cos x} + \frac{1}{\sin x} \\ &(\sin x + \cos x)\left(\frac{\sec^2 x}{\tan x}\right) \\ &\quad = \frac{\sin x}{\cos x \sin x} + \frac{\cos x}{\sin x \cos x} \\ &(\sin x + \cos x)\left(\frac{1}{\cos^2 x}\right)\left(\frac{1}{\tan x}\right) \\ &\quad = \frac{\sin x + \cos x}{\cos x \sin x} \\ &(\sin x + \cos x)\left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos x}{\sin x}\right) \\ &\quad = \frac{\sin x + \cos x}{\cos x \sin x} \\ &(\sin x + \cos x)\left(\frac{1}{\cos x \sin x}\right) \\ &\quad = \frac{\sin x + \cos x}{\cos x \sin x} \\ &\frac{\sin x + \cos x}{\cos x \sin x} = \frac{\sin x + \cos x}{\cos x \sin x} \end{aligned}$$

d) $$\begin{aligned} \tan^2\beta + \cos^2\beta + \sin^2\beta &= \frac{1}{\cos^2\beta} \\ \tan^2\beta + 1 &= \frac{1}{\cos^2\beta} \\ \tan^2\beta + 1 &= \sec^2\beta \end{aligned}$$

Since $\tan^2\beta + 1 = \sec^2\beta$ is a known identity, $\tan^2\beta + \cos^2\beta + \sin^2\beta$ must equal $\frac{1}{\cos^2\beta}$.

e) $$\begin{aligned} \sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) &\\ &= \sqrt{2}\cos x; \\ \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x &\\ + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x &\\ &= \sqrt{2}\cos x; \\ 2\sin\frac{\pi}{4}\cos x &= \sqrt{2}\cos x; \\ (2)\left(\frac{\sqrt{2}}{2}\right)(\cos x) &= \sqrt{2}\cos x; \\ \sqrt{2}\cos x &= \sqrt{2}\cos x \end{aligned}$$

f) $$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right)\cot\left(\frac{\pi}{2} + x\right) &= -\sin x; \\ \sin\left(\frac{\pi}{2} - x\right)\left(\frac{\cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\right) &= -\sin x; \\ \left(\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x\right) &\\ \times\left(\frac{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x}{\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x}\right) &= -\sin x; \\ ((1)(\cos x) - (0)(\sin x)) &\\ \times\left(\frac{(0)(\cos x) - (1)(\sin x)}{(1)(\cos x) + (0)(\sin x)}\right) &= -\sin x; \\ (\cos x - 0)\left(\frac{0 - \sin x}{\cos x + 0}\right) &= -\sin x; \\ (\cos x)\left(-\frac{\sin x}{\cos x}\right) &= -\sin x; \\ -\sin x &= -\sin x \end{aligned}$$

11. a) $$\begin{aligned} \frac{\cos 2x + 1}{\sin 2x} &= \cot x \\ \frac{2\cos^2 x - 1 + 1}{2\sin x \cos x} &= \cot x \\ \frac{2\cos^2 x}{2\sin x \cos x} &= \cot x \\ \frac{\cos x}{\sin x} &= \cot x \\ \cot x &= \cot x \end{aligned}$$

Answers

b)
$$\frac{\sin 2x}{1 - \cos 2x} = \cot x$$
$$\frac{2 \sin x \cos x}{1 - (1 - 2\sin^2 x)} = \cot x$$
$$\frac{2 \sin x \cos x}{1 - 1 + 2\sin^2 x} = \cot x$$
$$\frac{2 \sin x \cos x}{2\sin^2 x} = \cot x$$
$$\frac{\cos x}{\sin x} = \cot x$$
$$\cot x = \cot x$$

c) $(\sin x + \cos x)^2 = 1 + \sin 2x;$
$$\sin^2 x + \sin x \cos x + \sin x \cos x + \cos^2 x = 1 + 2 \sin x \cos x;$$
$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = 1 + 2 \sin x \cos x;$$
$$(\cos^2 x + \sin^2 x) + 2 \sin x \cos x = 1 + 2 \sin x \cos x;$$
$$1 + 2 \sin x \cos x = 1 + 2 \sin x \cos x$$

d)
$$\cos^4 \theta - \sin^4 \theta = \cos 2\theta$$
$$(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) = \cos^2 \theta - \sin^2 \theta$$
$$(1)(\cos^2 \theta - \sin^2 \theta) = \cos^2 \theta - \sin^2 \theta$$
$$\cos^2 \theta - \sin^2 \theta = \cos^2 \theta - \sin^2 \theta$$

e) $\cot \theta - \tan \theta = 2 \cot 2\theta$
$$\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = 2\frac{\cos 2\theta}{\sin 2\theta}$$
$$\frac{\cos^2 \theta}{\sin \theta \cos \theta} - \frac{\sin^2 \theta}{\cos \theta \sin \theta} = (2)\left(\frac{\cos 2\theta}{2 \cos \theta \sin \theta}\right)$$
$$\frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} = \frac{\cos 2\theta}{\cos \theta \sin \theta}$$
$$\frac{\cos 2\theta}{\cos \theta \sin \theta} = \frac{\cos 2\theta}{\cos \theta \sin \theta}$$

f) $\cot \theta + \tan \theta = 2 \csc 2\theta$
$$\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = 2\frac{1}{\sin 2\theta}$$
$$\frac{\cos^2 \theta}{\sin \theta \cos \theta} + \frac{\sin^2 \theta}{\cos \theta \sin \theta} = (2)\left(\frac{1}{2 \cos \theta \sin \theta}\right)$$
$$\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\cos \theta \sin \theta}$$
$$\frac{1}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta}$$

g)
$$\frac{1 + \tan x}{1 - \tan x} = \tan\left(x + \frac{\pi}{4}\right)$$
$$\frac{1 + \tan x}{1 - \tan x} = \frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}}$$
$$\frac{1 + \tan x}{1 - \tan x} = \frac{\tan x + 1}{1 - (\tan x)(1)}$$
$$\frac{1 + \tan x}{1 - \tan x} = \frac{1 + \tan x}{1 - \tan x}$$

h)
$$\csc 2x + \cot 2x = \cot x;$$
$$\frac{1}{\sin 2x} + \frac{1}{\tan 2x} = \cot x;$$
$$\frac{1}{2 \sin x \cos x} + \frac{1}{\frac{2 \tan x}{1 - \tan^2 x}} = \cot x;$$
$$\frac{1}{2 \sin x \cos x} + \frac{1 - \tan^2 x}{2 \tan x} = \cot x;$$
$$\frac{1}{2 \sin x \cos x} + \frac{1 - \tan^2 x}{2\frac{\sin x}{\cos x}} = \cot x;$$
$$\frac{1}{2 \sin x \cos x} + \frac{(\cos x)(1 - \tan^2 x)}{2 \sin x} = \frac{\cos x}{\sin x};$$
$$\frac{1}{2 \sin x \cos x} + \frac{(\cos x)(1 - \tan^2 x)(\cos x)}{2 \sin x \cos x} = \frac{(\cos x)(2 \cos x)}{(\sin x)(2 \cos x)};$$
$$\frac{1}{2 \sin x \cos x} + \frac{(\cos^2 x)(1 - \tan^2 x)}{2 \sin x \cos x} = \frac{2 \cos^2 x}{2 \sin x \cos x};$$
$$\frac{1}{2 \sin x \cos x} + \frac{\cos^2 x - (\tan^2 x)(\cos^2 x)}{2 \sin x \cos x} = \frac{2 \cos^2 x}{2 \sin x \cos x};$$
$$\frac{1}{2 \sin x \cos x} + \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{2 \cos^2 x}{2 \sin x \cos x};$$
$$\frac{1 + \cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{2 \cos^2 x}{2 \sin x \cos x};$$
$$\frac{1 + \cos^2 x - \sin^2 x}{2 \sin x \cos x} - \frac{2 \cos^2 x}{2 \sin x \cos x} = \frac{2 \cos^2 x}{2 \sin x \cos x} - \frac{2 \cos^2 x}{2 \sin x \cos x};$$
$$\frac{1 + \cos^2 x - \sin^2 x - 2 \cos^2 x}{2 \sin x \cos x} = 0;$$
$$\frac{1 - \sin^2 x - \cos^2 x}{2 \sin x \cos x} = 0;$$
$$\frac{1 - (\sin^2 x + \cos^2 x)}{2 \sin x \cos x} = 0;$$
$$\frac{1 - 1}{2 \sin x \cos x} = 0;$$
$$\frac{0}{2 \sin x \cos x} = 0;$$
$$0 = 0$$

i)
$$\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$$
$$\frac{2 \tan x}{\sec^2 x} = \sin 2x$$
$$\frac{2 \tan x}{\frac{1}{\cos^2 x}} = \sin 2x$$
$$(2 \tan x)(\cos^2 x) = \sin 2x$$
$$\left(\frac{2 \sin x}{\cos x}\right)(\cos^2 x) = \sin 2x$$
$$\sin 2x = 2 \sin x \cos x$$

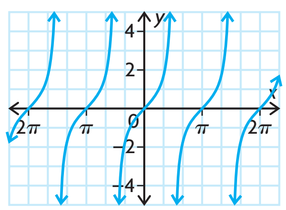
Since $\sin 2x = 2 \sin x \cos x$ is a known identity, $\frac{2 \tan x}{1 - \tan^2 x}$ must equal $\sin 2x$.

j) $\sec 2t = \frac{\csc t}{\csc t - 2 \sin t}$
$$\frac{1}{\cos 2t} = \frac{\frac{1}{\sin t}}{\frac{1}{\sin t} - 2 \sin t}$$
$$\frac{1}{\cos 2t} = \frac{\frac{1}{\sin t}}{\frac{1}{\sin t} - \frac{2 \sin^2 t}{\sin t}}$$
$$\frac{1}{\cos 2t} = \frac{\frac{1}{\sin t}}{\frac{1 - 2 \sin^2 t}{\sin t}}$$
$$\frac{1}{\cos 2t} = \frac{1}{\sin t} \times \frac{\sin t}{1 - 2 \sin^2 t}$$
$$\frac{1}{\cos 2t} = \frac{1}{1 - 2 \sin^2 t}$$
$$\frac{1}{\cos 2t} = \frac{1}{\cos 2t}$$

k) $\csc 2\theta = \frac{1}{2} \sec \theta \csc \theta$
$$\frac{1}{\sin 2\theta} = \left(\frac{1}{2}\right)\left(\frac{1}{\cos \theta}\right)\left(\frac{1}{\sin \theta}\right)$$
$$\frac{1}{\sin 2\theta} = \frac{1}{2 \cos \theta \sin \theta}$$
$$\frac{1}{2 \sin \theta \cos \theta} = \frac{1}{2 \sin \theta \cos \theta}$$

l)
$$\frac{1}{\cos t} = \frac{2 \sin t \cos t}{\sin t} - \frac{2 \cos^2 t - 1}{\cos t}$$
$$\frac{\sin t}{\cos t \sin t} = \frac{2 \sin t \cos^2 t}{\sin t \cos t} - \frac{(\sin t)(2 \cos^2 t - 1)}{\cos t \sin t}$$
$$\frac{\sin t}{\cos t \sin t} = \frac{2 \sin t \cos^2 t}{\sin t \cos t} - \frac{2 \cos^2 t \sin t - \sin t}{\cos t \sin t}$$
$$\frac{\sin t}{\cos t \sin t} = \frac{2 \sin t \cos^2 t}{\sin t \cos t} - \frac{2 \cos^2 t \sin t - \sin t}{\sin t \cos t}$$
$$\frac{\sin t}{\cos t \sin t} = \frac{2 \sin t \cos^2 t}{\sin t \cos t} + \frac{-2 \sin t \cos^2 t + \sin t}{\sin t \cos t}$$
$$\frac{\sin t}{\cos t \sin t} = \frac{\sin t}{\cos t \sin t}$$

12. Answers may vary. For example, an equivalent expression is $\tan x$.



13. $$\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$$
$$\frac{\sin x + 2 \sin x \cos x}{1 + \cos x + \cos 2x} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{1 + \cos x + \cos 2x} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{\cos x + (1 + \cos 2x)} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{\cos x + 2 \cos^2 x} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{\cos x(1 + 2 \cos x)} = \tan x$$
$$\frac{\sin x}{\cos x} = \tan x$$
$$\tan x = \tan x$$

14.

Definition	**Methods of Proof**
A statement of the equivalence of two trigonometric expressions	Both sides of the equation must be shown to be equivalent through graphing or simplifying/rewriting.
Trigonometric Identities	
Examples	**Non-Examples**
$\cos 2x + \sin^2 x = \cos^2 x$ $\cos 2x + 1 = 2 \cos^2 x$	$\cos 2x - 2 \sin^2 x = 1$ $\cot^2 x + \csc^2 x = 1$

15. She can determine whether the equation $2 \sin x \cos x = \cos 2x$ is an identity by trying to simplify and/or rewrite the left side of the equation so that it is equivalent to the right side of the equation. Alternatively, she can graph the functions $y = 2 \sin x \cos x$ and $y = \cos 2x$ and see if the graphs are the same. If they're the same, it's an identity, but if they're not the same, it's not an identity. By doing this she can determine it's not an identity, but she can make it an identity by changing the equation to $2 \sin x \cos x = \sin 2x$.

16. **a)** $a = 2, b = 1, c = 1$
b) $a = -1, b = 2, c = -2$

17. $\cos 4x + 4 \cos 2x + 3$; $a = 1$, $b = 4$, $c = 3$

Lesson 7.5, pp. 426–428

1. **a)** $\frac{\pi}{2}$
b) $\frac{3\pi}{2}$
c) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$
d) $\frac{7\pi}{6}$ or $\frac{11\pi}{6}$
e) $0, \pi$, or 2π
f) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

2. **a)** 0 or 2π
b) π
c) $\frac{\pi}{3}$ or $\frac{5\pi}{3}$
d) $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$
e) $\frac{\pi}{2}$ or $\frac{3\pi}{2}$
f) $\frac{\pi}{6}$ or $\frac{11\pi}{6}$

3. **a)** 2
b) quadrants I and II
c) $x = \frac{\pi}{3}$
d) $x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

4. **a)** 2
b) quadrants II and III
c) 30°
d) $x = 150°$ or $210°$

5. **a)** 2
b) quadrants I and III
c) 1.22
d) $\theta = 1.22$ or 4.36

6. **a)** $\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$
b) $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$
c) $\theta = \frac{\pi}{6}$ or $\frac{11\pi}{6}$
d) $\theta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$
e) $\theta = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$
f) $\theta = \frac{\pi}{3}$ or $\frac{4\pi}{3}$

7. **a)** $\theta = 210°$ or $330°$
b) $\theta = 131.8°$ or $228.2°$
c) $\theta = 56.3°$ or $236.3°$
d) $\theta = 221.8°$ or $318.2°$
e) $\theta = 78.5°$ or $281.5°$
f) $\theta = 116.6°$ or $296.6°$

8. **a)** $x = 0.52$ or 2.62
b) $x = 0.52$ or 5.76
c) $x = 1.05$ or 5.24
d) $x = 3.67$ or 5.76

9. **a)** $x = 0.79$ or 3.93
b) $x = 0.52$ or 2.62
c) $x = 0$ or 6.28
d) $x = 3.67$ or 5.76
e) $x = 1.16$ or 5.12
f) $x = 1.11$ or 4.25

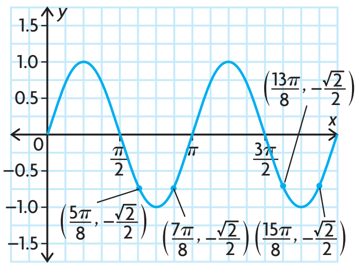
10. **a)** $x = 0.39, 1.18, 3.53$, or 4.32
b) $x = 0.13, 0.65, 1.70, 2.23, 3.27, 3.80, 4.84$, or 5.37
c) $x = 1.40, 1.75, 3.49, 3.84, 5.59$, or 5.93
d) $x = 0.59, 0.985, 2.16, 2.55, 3.73, 4.12, 5.304$, or 5.697
e) $x = 1.05, 2.09, 4.19$, or 5.24
f) $x = 1.05$

11. from about day 144 to about day 221

12. $1.86 \text{ s} < t < 4.14 \text{ s}$;
$9.86 \text{ s} < t < 12.14 \text{ s}$;
$17.86 \text{ s} < t < 20.14 \text{ s}$

13. $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

14.



15. The value of $f(x) = \sin x$ is the same at x and $\pi - x$. In other words, it is the same at x and half the period minus x. Since the period of $f(x) = 25 \sin\frac{\pi}{50}(x + 20) - 55$ is 100, if the function were not horizontally translated, its value at x would be the same as at $50 - x$. The function is horizontally translated 20 units to the left, however, so it goes through half its period from $x = -20$ to $x = 30$. At $x = 3$, the function is 23 units away from the left end of the range, so it will have the same value at $x = 30 - 23$ or $x = 7$, which is 23 units away from the right end of the range.

16. To solve a trigonometric equation **algebraically**, first isolate the trigonometric function on one side of the equation. For example, the trigonometric equation $5 \cos x - 3 = 2$ would become $5 \cos x = 5$, which would then become $\cos x = 1$. Next, apply the inverse of the trigonometric function to both sides of the equation. For example, the trigonometric equation $\cos x = 1$ would become $x = \cos^{-1} 1$. Finally, simplify the equation. For example, $x = \cos^{-1} 1$ would become $x = 0 + 2n\pi$, where $n \in \mathbf{I}$.

To solve a trigonometric equation **graphically**, first isolate the trigonometric function on one side of the equation. For example, the trigonometric equation $5 \cos x - 3 = 2$ would become $5 \cos x = 5$, which would then become $\cos x = 1$. Next, graph both sides of the equation. For example, the functions $f(x) = \cos x$ and $f(x) = 1$ would both be graphed. Finally, find the points where the two graphs intersect. For example, $f(x) = \cos x$ and $f(x) = 1$ would intersect at $x = 0 + 2n\pi$, where $n \in \mathbf{I}$.

Similarity: Both trigonometric functions are first isolated on one side of the equation.

Differences: The inverse of a trigonometric function is not applied in the graphical method, and the points of intersection are not obtained in the algebraic method.

17. $x = 0 + n\pi, \frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + n\pi$, where $n \in \mathbf{I}$

18. a) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, or $\frac{3\pi}{4}$
    b) $x = \frac{\pi}{6}, \frac{\pi}{2}$, or $\frac{5\pi}{6}$

Lesson 7.6, pp. 435–437

1. a) $(\sin\theta)(\sin\theta - 1)$
   b) $(\cos\theta - 1)(\cos\theta - 1)$
   c) $(3\sin\theta + 2)(\sin\theta - 1)$
   d) $(2\cos\theta - 1)(2\cos\theta + 1)$
   e) $(6\sin x - 2)(4\sin x + 1)$
   f) $(7\tan x + 8)(7\tan x - 8)$
2. a) $y = \pm\frac{\sqrt{3}}{3}, x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$
   b) $y = 0$ or -1, $x = 0, \pi, \frac{3\pi}{2}$, or 2π
   c) $y = 0$ or $z = \frac{1}{2}, x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$
   d) $y = 0$ or $z = 1$, $x = 0, \pi$, or 2π
3. a) $y = \frac{1}{3}$ or $\frac{1}{2}$
   b) $x = 1.05, 1.91, 4.37$, or 5.24
4. a) $\theta = 90°$ or $270°$
   b) $\theta = 0°, 180°$, or $360°$
   c) $\theta = 45°, 135°, 225°$, or $315°$
   d) $\theta = 60°, 120°, 240°$, or $300°$
   e) $\theta = 30°, 150°, 210°$, or $330°$
   f) $\theta = 45°, 135°, 225°$, or $315°$
5. a) $x = 0°, 90°, 180°, 270°$, or $360°$
   b) $x = 0°, 180°$, or $360°$
   c) $x = 90°$ or $270°$
   d) $x = 60°, 90°, 120°$, or $270°$
   e) $x = 45°, 135°, 225°$, or $315°$
   f) $x = 90°$ or $180°$
6. a) $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$
   b) $x = \frac{3\pi}{2}$
   c) $x = 0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$, or 2π
   d) $x = \frac{\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$
   e) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$
   f) $x = 0, \frac{3\pi}{2}$, or 2π
7. a) $\theta = \frac{\pi}{3}, \pi$, or $\frac{5\pi}{3}$
   b) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$
   c) $\theta = \pi$
   d) $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
   e) $\theta = \frac{\pi}{4}, 2.82, \frac{5\pi}{4}$, or 5.96
   f) $\theta = 0.73, 2.41, 3.99$, or 5.44
8. a) $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$
   b) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$
   c) $x = 0, 0.96\pi, 5.33$, or 2π
   d) $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$
   e) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}$, or $\frac{7\pi}{4}$
   f) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$, or 2π
9. a) $x = \frac{\pi}{3}, 1.98, 4.30$, or $\frac{5\pi}{3}$
   b) $x = 0.45, \frac{2\pi}{3}, \frac{4\pi}{3}$, or 5.83
   c) $x = \frac{\pi}{6}, 0.85, \frac{5\pi}{6}$, or 2.29
   d) $x = \frac{\pi}{2}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$
10. $x = 0.15, 1.02, 2.12$, or 2.99
11. $b = 1 + \sqrt{3}, c = \sqrt{3}$
12. $c = \frac{1}{2}$
13. $\frac{\pi}{3}$ km $< d < \frac{2\pi}{3}$ km, $\frac{4\pi}{3}$ km $< d < \frac{5\pi}{3}$ km
14. $x = 1.91$ or 4.37
15. a) $x = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$
    b) $x = \frac{3\pi}{4} + 2n\pi$ or $\frac{5\pi}{4} + 2n\pi$, where $n \in \mathbf{I}$
16. It is possible to have different numbers of solutions for quadratic trigonometric equations because, when factored, a quadratic trigonometric equation can be one expression multiplied by another expression or it can be a single expression squared. For example, the equation $\cos^2 x + \frac{3}{2}\cos x + \frac{1}{2}$ becomes $(\cos x + 1)\left(\cos x + \frac{1}{2}\right)$ when factored, and it has the solutions $\frac{2\pi}{3}, \pi$, and $\frac{4\pi}{3}$ in the interval $0 \le x \le 2\pi$. In comparison, the equation $\cos^2 x + 2\cos x + 1 = 0$ becomes $(\cos x + 1)^2$ when factored, and it has only one solution, π, in the interval $0 \le x \le 2\pi$. Also, different expressions produce different numbers of solutions. For example, the expression $\cos x + \frac{1}{2}$ produces two solutions in the interval $0 \le x \le 2\pi$ $\left(\frac{2\pi}{3} \text{ and } \frac{4\pi}{3}\right)$ because $\cos x = -\frac{1}{2}$ for two different values of x. The expression $\cos x + 1$, however, produces only one solution in the interval $0 \le x \le 2\pi$ (π), because $\cos x = -1$ for only one value of x.
17. $a = \frac{\pi}{4}, \frac{5\pi}{4}$
18. $x = 0.72, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, or 5.56
19. $x = 15°, 75°, 105°, 165°, 195°, 255°, 285°$, or $345°$
20. $\theta = 0.96$

Chapter Review, p. 440

1. a) Answers may vary. For example, $\sin\frac{7\pi}{10}$.
   b) Answers may vary. For example, $\cos\frac{8\pi}{7}$.
   c) Answers may vary. For example, $\sin\frac{6\pi}{7}$.
   d) Answers may vary. For example, $\cos\frac{\pi}{7}$.
2. $y = 5\cos(x) - 8$
3. a) $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$
   b) $-\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$
   c) $\frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}$
   d) $-\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$
4. a) $-\frac{\sqrt{3}}{3}$ b) $-\frac{\sqrt{3}}{2}$
5. a) $\frac{1}{2}$ c) $-\frac{\sqrt{2}}{2}$
   b) $\frac{\sqrt{3}}{2}$ d) $\sqrt{3}$
6. a) $\sin 2x = \frac{24}{25}, \cos 2x = \frac{7}{25}$, $\tan 2x = \frac{24}{7}$
   b) $\sin 2x = -\frac{336}{625}, \cos 2x = -\frac{527}{625}$, $\tan 2x = \frac{336}{527}$
   c) $\sin 2x = -\frac{120}{169}, \cos 2x = \frac{119}{169}$, $\tan 2x = -\frac{120}{119}$
7. a) trigonometric identity
   b) trigonometric equation
   c) trigonometric identity
   d) trigonometric equation
8. $\frac{\cos^2 x}{\cot^2 x} = 1 - \cos^2 x$

$$\frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}} = 1 - \cos^2 x$$

$$\frac{(\cos^2 x)(\sin^2 x)}{\cos^2 x} = 1 - \cos^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$1 - \cos^2 x = 1 - \cos^2 x$$

9. $$\frac{2(\sec^2 x - \tan^2 x)}{\csc x} = \sin 2x \sec x$$
$$\frac{2(1)}{\csc x} = \sin 2x \sec x$$
$$\frac{2}{\csc x} = \sin 2x \sec x$$
$$2 \sin x = \sin 2x \sec x$$
$$\frac{2 \sin x \cos x}{\cos x} = \sin 2x \sec x$$
$$\frac{\sin 2x}{\cos x} = \sin 2x \sec x$$
$$\sin 2x \sec x = \sin 2x \sec x$$

10. a) $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$
b) $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$
c) $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

11. a) $y = -2$ or 2
b) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

12. a) $x = \frac{\pi}{2}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$
b) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$, or 2π
c) $x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{7\pi}{4}$
d) $x = 0.95$ or 4.09

13. $x = \frac{\pi}{2}, \pi$, or $\frac{3\pi}{2}$

Chapter Self-Test, p. 441

1. $$\frac{1 - 2\sin^2 x}{\cos x + \sin x} + \sin x = \cos x$$
$$\frac{1 - 2\sin^2 x}{\cos x + \sin x} + \sin x - \sin x$$
$$= \cos x - \sin x$$
$$\frac{1 - 2\sin^2 x}{\cos x + \sin x} = \cos x - \sin x$$
$$1 - 2\sin^2 x = (\cos x - \sin x) \times (\cos x + \sin x)$$
$$\cos 2x = (\cos x - \sin x) \times (\cos x + \sin x)$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = \cos 2x$$

2. all real numbers x, where $0 \le x \le 2\pi$

3. a) $x = \frac{\pi}{6}$ or $x = \frac{11\pi}{6}$
b) $x = \frac{2\pi}{3}$ or $x = \frac{5\pi}{3}$
c) $x = \frac{5\pi}{4}$ or $x = \frac{7\pi}{4}$

4. $a = 2, b = 1$

5. $t = 7, 11, 19$, and 23

6. Nina can find the cosine of $\frac{11\pi}{4}$ by using the formula $\cos(x + y) = \cos x \cos y - \sin x \sin y$. The cosine of π is -1, and the cosine of $\frac{7\pi}{4}$ is $\frac{\sqrt{2}}{2}$. Also, the sine of π is 0, and the sine of $\frac{7\pi}{4}$ is $-\frac{\sqrt{2}}{2}$. Therefore,
$$\cos\frac{11\pi}{4} = \cos\left(\pi + \frac{7\pi}{4}\right)$$
$$= \left(-1 \times \frac{\sqrt{2}}{2}\right) - \left(0 \times -\frac{\sqrt{2}}{2}\right)$$
$$= -\frac{\sqrt{2}}{2} - 0$$
$$= -\frac{\sqrt{2}}{2}$$

7. $x = 3.31$ or 6.12

8. $-\frac{33}{65}, -\frac{16}{65}$

9. a) $-\frac{4\sqrt{5}}{9}$
c) $\sqrt{\frac{3 - \sqrt{5}}{6}}$
b) $\frac{1}{9}$
d) $\frac{22}{27}$

10. a) $x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}$, or $\frac{5\pi}{3}$
b) $x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}$, or $\frac{4\pi}{3}$
c) $x = -\pi$ and π

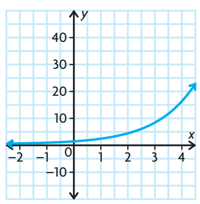
Chapter 8

Getting Started, p. 446

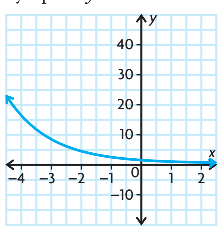
1. a) $\frac{1}{5^2} = \frac{1}{25}$
d) $\sqrt[3]{125} = 5$
b) 1
e) $-\sqrt{121} = -11$
c) $\sqrt{36} = 6$
f) $\left(\sqrt[3]{\frac{27}{8}}\right)^2 = \frac{9}{4}$

2. a) $3^7 = 2187$
d) $7^4 = 2401$
b) $(-2)^2 = 4$
e) $8^{\frac{2}{3}} = 4$
c) $10^3 = 1000$
f) $4^{\frac{1}{2}} = \sqrt{4} = 2$

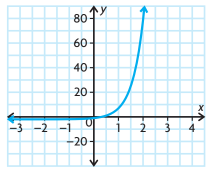
3. a) $8m^3$
d) x^3y
b) $\frac{1}{a^8b^{10}}$
e) $-d^2c^2$
c) $4|x|^3$
f) x

4. a)


D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} \mid y > 0\}$, y-intercept 1, horizontal asymptote $y = 0$

b)


D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} \mid y > 0\}$, y-intercept 1, horizontal asymptote $y = 0$

c)


D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} \mid y > -2\}$, y-intercept -1, horizontal asymptote $y = -2$

5. a) i) $y = \frac{x + 6}{3}$
ii) $y = \pm\sqrt{x + 5}$
iii) $y = \sqrt[3]{\frac{x}{6}}$
iv)
b) The inverses of (i) and (iii) are functions.

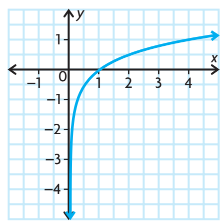
6. a) 800 bacteria
b) 6400 bacteria
c) 209 715 200
d) 4.4×10^{15}

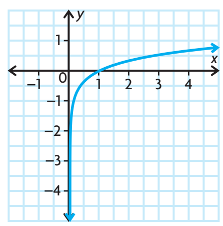
7. 12 515 people

8.

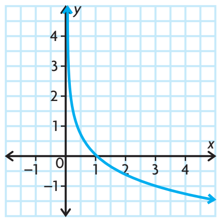
Similarities	Differences
• same y-intercept • same shape • same horizontal asymptote • both are always positive	• one is always increasing, the other is always decreasing • different end behaviour • reflections of each other across the y-axis

Lesson 8.1, p. 451

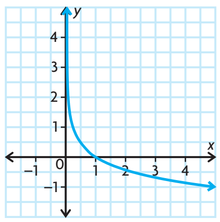
1. a) $x = 4^y$ or $f^{-1}(x) = \log_4 x$


b) $x = 8^y$ or $f^{-1}(x) = \log_8 x$


c) $x = \left(\frac{1}{3}\right)^{y}$ or $f^{-1}(x) = \log_{\left(\frac{1}{3}\right)}x$



d) $x = \left(\frac{1}{5}\right)^{y}$ or $f^{-1}(x) = \log_{\left(\frac{1}{5}\right)}x$



2. a) i) $x = 4^{y}$
ii) $\log_{4}x = y$
b) i) $x = 8^{y}$
ii) $\log_{8}x = y$
c) i) $x = \left(\frac{1}{3}\right)^{y}$
ii) $\log_{\frac{1}{3}}x = y$
d) i) $x = \left(\frac{1}{5}\right)^{y}$
ii) $\log_{\frac{1}{5}}x = y$

3. All the graphs have the same basic shape, but the last two are reflected over the x-axis, compared with the first two. All the graphs have the same x-intercept, 1. All have the same vertical asymptote, $x = 0$.

4. Locate the point on the graph that has 8 as its x-coordinate. This point is (8, 3). The y-coordinate of this point is the solution to $2^{y} = 8$, $y = 3$.

5. a) $x = 3^{y}$ c) $x = \left(\frac{1}{4}\right)^{y}$
b) $x = 10^{y}$ d) $x = m^{y}$

6. a) $\log_{3}x = y$ c) $\log_{\frac{1}{4}}x = y$
b) $\log_{10}x = y$ d) $\log_{m}x = y$

7. a) $x = 5^{y}$ c) $x = 3^{y}$
b) $x = 10^{y}$ d) $x = \frac{1}{4}^{y}$

8. a) $y = 5^{x}$ c) $y = 3^{x}$
b) $y = 10^{x}$ d) $y = \frac{1}{4}^{x}$

9. a) 2 d) 0
b) 3 e) -1
c) 4 f) $\frac{1}{2}$

10. Since 3 is positive, no exponent for 3^{x} can produce -9.

11. a) $\left(\frac{1}{4}, -2\right), \left(\frac{1}{2}, -1\right), (1, 0), (2, 1), (4, 2)$
b) $\left(\frac{1}{100}, -2\right), \left(\frac{1}{10}, -1\right), (1, 0), (10, 1), (100, 2)$

Lesson 8.2, pp. 457–458

1. a) vertical stretch by a factor of 3
b) horizontal compression by a factor of $\frac{1}{2}$
c) vertical translation 5 units down
d) horizontal translation 4 units left

2. a) (a) $\left(\frac{1}{10}, -3\right), (1, 0), (10, 3)$
(b) $\left(\frac{1}{20}, -1\right), \left(\frac{1}{2}, 0\right), (5, 1)$
(c) $\left(\frac{1}{10}, -6\right), (1, -5), (10, -4)$
(d) $\left(-3\frac{9}{10}, -1\right), (-3, 0), (6, 1)$
b) (a) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$
(b) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$
(c) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$
(d) $D = \{x \in \mathbf{R} \mid x > -4\}$, $R = \{y \in \mathbf{R}\}$

3. a) $f(x) = 5\log_{10}x + 3$
b) $f(x) = -\log_{10}(3x)$
c) $f(x) = \log_{10}(x + 4) - 3$
d) $f(x) = -\log_{10}(x - 4)$

4. i) a) reflection in the x-axis and a vertical stretch by a factor of 4; $c = 5$ resulting in a translation 5 units up
b) (1, 5), (10, 1)
c) vertical asymptote is $x = 0$
d) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$

ii) a) vertical compression by a factor of $\frac{1}{2}$; $d = 6$ resulting in a horizontal translation 6 units to the right; $c = 3$ resulting in a vertical translation 3 units up
b) $(7, 3), \left(16, 3\frac{1}{2}\right)$
c) vertical asymptote is $x = 6$
d) $D = \{x \in \mathbf{R} \mid x > 6\}$, $R = \{y \in \mathbf{R}\}$

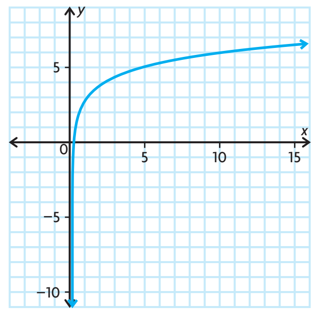
iii) a) horizontal compression by a factor of $\frac{1}{3}$; $c = -4$ resulting in a vertical shift 4 units down
b) $\left(\frac{1}{3}, -4\right), \left(3\frac{1}{3}, -3\right)$
c) vertical asymptote is $x = 0$
d) $D = \{x \in \mathbf{R} \mid x > 6\}$, $R = \{y \in \mathbf{R}\}$

iv) a) vertical stretch by a factor of 2; $k = -2$ resulting in a horizontal compression by a factor of $\frac{1}{2}$ and a reflection in the y-axis; $d = -2$ resulting in a horizontal translation 2 units to the left.
b) $\left(-2\frac{1}{2}, 0\right), (-7, 2)$
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x < -2\}$, $R = \{y \in \mathbf{R}\}$

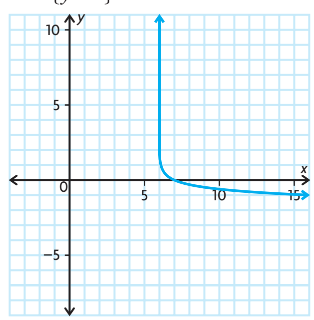
v) a) horizontal compression by a factor of $\frac{1}{2}$; $d = -2$ resulting in a horizontal translation 2 units to the left
b) $\left(-1\frac{1}{2}, 0\right), (3, 1)$
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x > -2\}$, $R = \{y \in \mathbf{R}\}$

vi) a) reflection in the x-axis; $d = -2$, resulting in a horizontal translation 2 units to the right
b) $(-3, 0), (-12, 1)$
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x < -2\}$, $R = \{y \in \mathbf{R}\}$

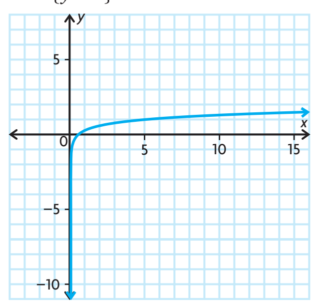
5. a) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



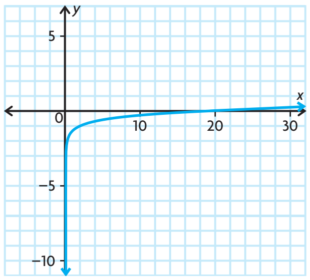
b) $D = \{x \in \mathbf{R} \mid x > -6\}$, $R = \{y \in \mathbf{R}\}$



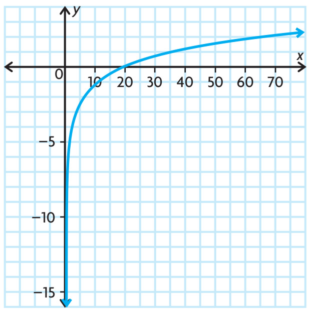
c) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



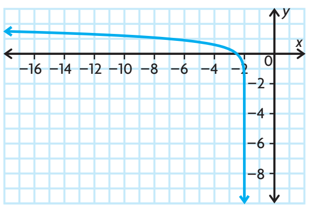
d) $D = \{x \in \mathbf{R} \mid x > 0\}$,
$R = \{y \in \mathbf{R}\}$



e) $D = \{x \in \mathbf{R} \mid x > 0\}$,
$R = \{y \in \mathbf{R}\}$



f) $D = \{x \in \mathbf{R} \mid x < -2\}$,
$R = \{y \in \mathbf{R}\}$



6. The functions are inverses of each other.

7. a) The graph of $g(x) = \log_3(x + 4)$ is the same as the graph of $f(x) = \log_3 x$, but horizontally translated 4 units to the left. The graph of $h(x) = \log_3 x + 4$ is the same as the graph of $f(x) = \log_3 x$, but vertically translated 4 units up.

b) The graph of $m(x) = 4\log_3 x$ is the same as the graph of $f(x) = \log_3 x$, but vertically stretched by a factor of 4. The graph of $n(x) = \log_3 4x$ is the same as the graph of $f(x) = \log_3 x$, but horizontally compressed by a factor of $\frac{1}{4}$.

8. a) $f(x) = -3\log_{10}\left(\frac{1}{2}x - 5\right) + 2$

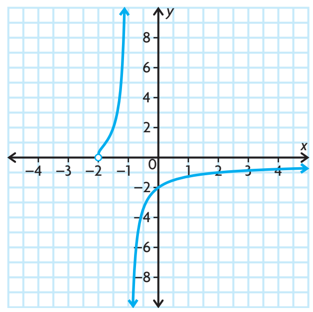
b) $(30, -1)$

c) $D = \{x \in \mathbf{R} \mid x > 5\}$,
$R = \{y \in \mathbf{R}\}$

9. vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left

10. domain, range, and vertical asymptote

11.



Lesson 8.3, pp. 466–468

1. a) $\log_4 16 = 2$ d) $\log_6 \frac{1}{36} = -2$
b) $\log_3 81 = 4$ e) $\log_{\frac{1}{3}} \frac{1}{27} = 3$
c) $\log_8 1 = 0$ f) $\log_8 2 = \frac{1}{3}$

2. a) $2^3 = 8$ d) $\left(\frac{1}{6}\right)^{-3} = 216$
b) $5^{-2} = \frac{1}{25}$ e) $6^{\frac{1}{2}} = \sqrt{6}$
c) $3^4 = 81$ f) $10^0 = 1$

3. a) 1 d) $\frac{1}{2}$
b) 0 e) 3
c) -2 f) $\frac{1}{3}$

4. a) -1 d) about 25
b) 0 e) 1.78
c) 6 f) 0.01

5. a) $\frac{1}{2}$ d) -2
b) 1 e) $\frac{1}{3}$
c) 7 f) $\frac{3}{2}$

6. a) 125 d) 16
b) 3 e) $\sqrt{5}$
c) -3 f) 8

7. a) about 2.58 c) about 4.29
b) about 3.26 d) about 4.52

8. a) about 2.50 c) about 4.88
b) about 2.65 d) about 2.83

9. a) 5 d) n
b) 25 e) b
c) $\frac{1}{16}$ f) 0

10. $\frac{4}{3}$

11. about 1.7 weeks or 12 days

12. a) 4.68 g b) 522 years

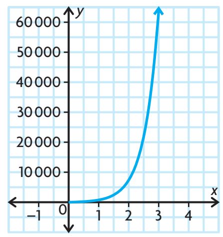
13. A: $(0.0625) = 0.017$; B: $(1) = 0.159$; B has a steeper slope.

14. a) about 233 mph b) 98 miles

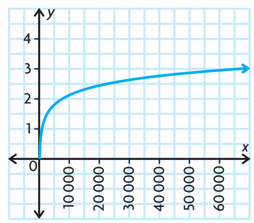
15. $\log 365 = 2.562$
$\frac{3}{2}\log 150 - 0.7 = 2.564$

16. a) about 83 years
b) about 164 years

17. a) $y = 100(2)^{\frac{x}{0.32}}$
b)



c)



d) $y = 0.32\log_2\left(\frac{x}{100}\right)$; this equation tells how many hours, y, it will take for the number of bacteria to reach x.

e) about 0.69 h; evaluate the inverse function for $x = 450$

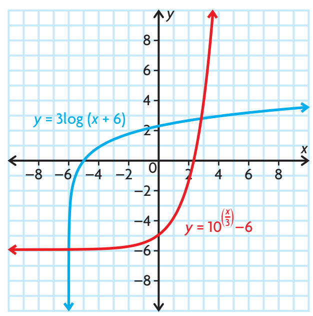
18. a) 1.0000 d) 2.1745
b) 3.3219 e) -0.5000
c) 2.3652 f) 2.9723

19. a) positive for all values $x > 1$
b) negative for all values $0 < x < 1$
c) undefined for all values $x \leq 0$

20. a) 1027
b) -27.14

21. a) $y = x^3$ c) $\sqrt[(x-2)]{0.5}$
b) $\frac{\sqrt[3]{2}}{3}$ d) $2^{\frac{x-2}{3}} + 3$

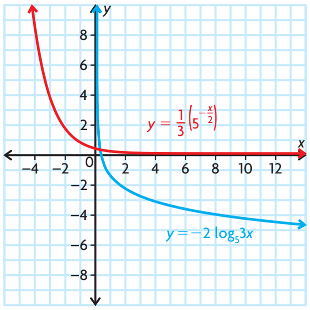
22. a)



function: $y = 3\log(x + 6)$
$D = \{x \in \mathbf{R} \mid x > -6\}$
$R = \{y \in \mathbf{R}\}$
asymptote: $x = -6$

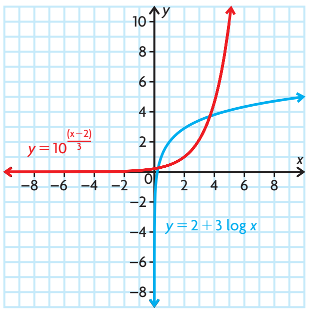
inverse: $y = 10^{\frac{x}{3}} - 6$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid y > -6\}$
asymptote: $y = -6$

b)



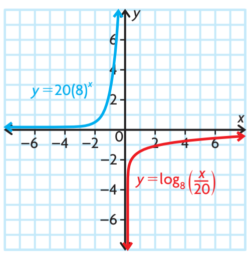
function: $y = -2\log_5 3x$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$
inverse: $y = \frac{1}{3}\left(5^{-\frac{x}{2}}\right)$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$

c)



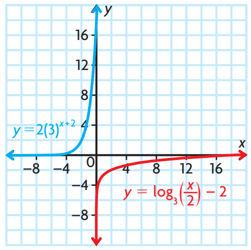
function: $y = 2 + 3\log x$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$
inverse: $y = 10^{\frac{(x-2)}{3}}$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$

d)



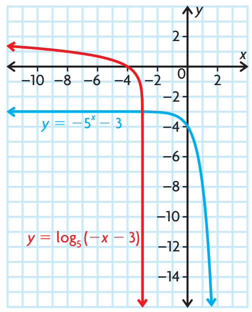
function: $y = 20(8)^x$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$
inverse: $y = \log_8\left(\frac{x}{20}\right)$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$

e)



function: $y = 2(3)^{x+2}$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$
inverse: $y = \log_3\left(\frac{x}{2}\right) - 2$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$

f)



function: $y = -5^x - 3$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y < -3\}$ asymptote: $y = -3$
inverse: $y = \log_5(-x - 3)$ $D = \{x \in \mathbf{R} \mid x < -3\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = -3$

23. Given the constraints, two integer values are possible for y, either 1 or 2. If $y = 3$, then x must be 1000, which is not permitted.

Lesson 8.4, pp. 475–476

1. **a)** $\log 45 + \log 68$
b) $\log_m p + \log_m q$
c) $\log 123 - \log 31$
d) $\log_m p - \log_m q$
e) $\log_2 14 + \log_2 9$
f) $\log_4 81 - \log_4 30$

2. **a)** $\log 35$ **d)** $\log \frac{x}{y}$
b) $\log_3 2$ **e)** $\log_6 504$
c) $\log_m ab$ **f)** $\log_4 6$

3. **a)** $2\log 5$ **d)** $\frac{1}{3}\log 45$
b) $-1\log 7$ **e)** $\frac{1}{2}\log_7 36$
c) $q\log_m p$ **f)** $\frac{1}{5}\log_5 125$

4. **a)** $\log_3 27$; 3 **d)** $7\log_4 4$; 7
b) $\log_5 25$; 2 **e)** $\log_2 32$; 5
c) $\log 100$; 2 **f)** $\frac{1}{2}\log 10$; $\frac{1}{2}$

5. $y = \log_2(4x) = \log_2 x + \log_2 4$
$= \log_2 x + 2$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ up 2 units;
$y = \log_2(8x) = \log_2 x + \log_2 8$
$= \log_2 x + 3$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ up 3 units;
$y = \log_2\left(\frac{x}{2}\right) = \log_2 x - \log_2 2$
$= \log_2 x - 1$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ down 1 unit

6. **a)** 1.5 **d)** -0.5
b) 2 **e)** 4
c) 1.5 **f)** 2

7. **a)** $\log_b x + \log_b y + \log_b z$
b) $\log_b z - (\log_b x + \log_b y)$
c) $2\log_b x + 3\log_b y$
d) $\frac{1}{2}(5\log_b x + \log_b y + 3\log_b z)$

8. $\log_5 3$ means $5^x = 3$ and $\log_5 \frac{1}{3}$ means $5^y = \frac{1}{3}$; since $\frac{1}{3} = 3^{-1}$, $5^y = 5^{x(-1)}$; therefore $\log_5 3 + \log_5 \frac{1}{3} = x + x(-1) = 0$

9. **a)** $\log_5 56$ **d)** $\log_3 4$
b) $\log_3 2$ **e)** $\log_4(3\sqrt{2})$
c) $\log_2 45$ **f)** $\log 16$

10. **a)** $\log_2 x = \log_2 245$; $x = 245$
b) $\log x = \log 432$; $x = 432$
c) $\log_4 x = \log_7 5$; $x = 5$
d) $\log_7 x = \log_7 5$; $x = 5$
e) $\log_3 x = \log_3 4$; $x = 4$
f) $\log_5 x = \log_5 384$; $x = 384$

11. **a)** $\log_2 xyz$ **d)** $\log_2 xy$
b) $\log_5 \frac{uw}{v}$ **e)** $\log_3 3x^2$
c) $\log_6 \frac{a}{bc}$ **f)** $\log_4 \frac{x^5}{v}$

12. $\log_a \frac{\sqrt{x}\sqrt{y}}{\sqrt[4]{z^3}}$

13. vertical stretch by a factor of 3, and vertical shift 3 units up

14. Answers may vary. For example,
$f(x) = 2\log x - \log 12$
$g(x) = \log \frac{x^2}{12}$
$2\log x - \log 12 = \log x^2 - \log 12$
$= \log \frac{x^2}{12}$

15. Answers may vary. For example, any number can be written as a power with a given base. The base of the logarithm is 3. Write each term in the quotient as a power of 3. The laws of logarithms make it possible to evaluate the expression by simplifying the quotient and noting the exponent.

16. $\log_x x^{m-1} + 1 = m - 1 + 1 = m$

17. $\log_b x\sqrt{x} = \log_b x + \log_b \sqrt{x}$
$$= \log_b x + \frac{1}{2}\log_b x$$
$$= 0.3 + 0.3\left(\frac{1}{2}\right)$$
$$= 0.45$$

18. The two functions have different domains. The first function has a domain of $x > 0$. The second function has a domain of all real numbers except 0, since x is squared.

19. Answers may vary; for example,
Product law
$$\log_{10}10 + \log_{10}10 = 1 + 1 = 2 = \log_{10}100 = \log_{10}(10 \times 10)$$
Quotient law
$$\log_{10}10 - \log_{10}10 = 1 - 1 = 0 = \log_{10}1 = \log_{10}\left(\frac{10}{10}\right)$$
Power law
$$\log_{10}10^2 = \log_{10}100 = 2 = 2\log_{10}10$$

Mid-Chapter Review, p. 479

1. a) $\log_5 y = x$ c) $\log x = y$
b) $\log_{\frac{1}{3}} y = x$ d) $\log_p m = q$
2. a) $3^y = x$ c) $10^k = m$
b) $10^y = x$ d) $s^t = r$
3. a) vertical stretch by a factor of 2, vertical translation 4 units down
b) reflection in the x-axis, horizontal compression by a factor of $\frac{1}{3}$
c) vertical compression by a factor of $\frac{1}{4}$, horizontal stretch by a factor of 4
d) horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 2 units to the right
e) horizontal translation 5 units to the left, vertical translation 1 unit up
f) vertical stretch by a factor of 5, reflection in the y-axis, vertical translation 3 units down
4. a) $y = -4\log_3 x$
b) $y = \log_3(x + 3) + 1$
c) $y = \frac{2}{3}\log_3\left(\frac{1}{2}x\right)$
d) $y = 3\log_3[-(x - 1)]$
5. a) $(9, -8)$
b) $(6, 3)$
c) $\left(18, \frac{4}{3}\right)$
d) $(-8, 6)$
6. It is vertically stretched by a factor of 2 and vertically shifted up 2.

7. a) 4 c) 0
b) -2 d) -3
8. a) 0.602 c) 2.130
b) 1.653 d) 2.477
9. a) $x \doteq 4.392$ c) $x \doteq 2.543$
b) $x \doteq 2.959$ d) $x \doteq 2.450$
10. a) $\log 28$ c) $\log_3 \frac{22}{3}$
b) $\log 2.5$ d) $\log_p q^2$
11. a) 1 d) -3
b) 2 e) $\frac{2}{3}$
c) 2 f) 3.5
12. Compared with the graph of $y = \log x$, the graph of $y = \log x^3$ is vertically stretched by a factor of 3.
13. a) 4.82 d) 1.69
b) 1.35 e) 3.82
c) 0.80 f) 3.49

Lesson 8.5, pp. 485–486

1. a) 4 d) $\frac{13}{9}$
b) 1 e) $-\frac{1}{3}$
c) $\frac{11}{4}$ f) -1
2. a) 4.088 d) 4.092
b) 3.037 e) -0.431
c) 1 f) 5.695
3. a) 5 d) $\frac{3}{5}$
b) 3 e) -2
c) 1.5 f) $-\frac{1}{2}$
4. a) 4.68 h c) 16 h
b) 12.68 h d) 31.26 h
5. a) 1.75 d) -4
b) $\frac{2}{3}$ e) 2
c) -4.75 f) 2
6. a) 9.12 years
b) 13.5 years
c) 16.44 quarters or 4.1 years
d) 477.9 weeks or 9.2 years
7. 13 quarter hours or 3.25 h
8. a) 2.5 d) 3
b) 6 e) 1
c) 5 f) 0
9. a) Solve using logarithms. Both sides can be divided by 225, leaving only a term with a variable in the exponent on the left. This can be solved using logarithms.
b) Solve by factoring out a power of 3 and then simplifying. Logarithms may still be necessary in a situation like this, but the factoring must be done first because logarithms cannot be used on the equation in its current form.
10. a) 1.849 c) 3.606
b) 2.931 d) 5.734

11. a) $I_f = I_o(0.95)^t$, where I_f is the final intensity, I_o is the original intensity, and t is the thickness
b) 10 mm
12. 1; 0.631
13. $a^y = x$, so $\log a^y = \log x$; $y\log a = \log x$;
$$y = \frac{\log x}{\log a}$$
A graphing calculator does not allow logarithms of base 5 to be entered directly. However, $y = \log_5 x$ can be entered for graphing, as $y = \frac{\log x}{\log 5}$.
14. a) $x = 2.5$
b) $x = 5$ or $x = 4$
c) $x = -2.45$
15. Let $\log_a 2 = x$. Then $a^x = 2$. $(a^x)^3 = 2^3$, or $a^{3x} = 8$. Since $\log_a 2 = \log_b 8$, $\log_b 8 = x$. So $b^x = 8$. Since each equation is equal to 8, $a^{3x} = b^x$ and $a^3 = b$.
16. $x = -0.737$; $y = 0.279$
17. a) $x = -1.60$
b) $x = -4.86$
c) $x = -0.42$
18. ± 1.82

Lesson 8.6, pp. 491–492

1. a) 25 d) 15
b) 81 e) 3
c) 8 f) $\sqrt{3}$
2. a) 5 d) 200.4
b) $\frac{1}{36}$ e) 5
c) 13 f) 20
3. 201.43
4. a) 9 d) 10 000
b) $\sqrt{5}$ e) -3
c) $\frac{25}{3}$ f) 4
5. a) $\frac{8}{3}$ d) 32
b) $\frac{10}{3}$ e) 3
c) $\frac{25}{6}$ f) 8.1
6. $x = 9$ or $x = -4$
Restrictions: $x > 5$ ($x - 5$ must be positive) so $x = 9$
7. a) $x = 6$ d) $x = 2.5$
b) $x = 3$ e) $x = 3$
c) $x = \frac{6}{5}$ f) $x = 16$
8. a) Use the rules of logarithms to obtain $\log_9 20 = \log_9 x$. Then, because both sides of the equation have the same base, $20 = x$.
b) Use the rules of logarithms to obtain $\log\frac{x}{2} = 3$. Then use the definition of a logarithm to obtain $10^3 = \frac{x}{2}$; $1000 = \frac{x}{2}$; $2000 = x$.

Answers

c) Use the rules of logarithms to obtain $\log x = \log 64$. Then, because both sides of the equation have the same base, $x = 64$.

9. a) 10^{-7}
b) $10^{-3.6}$

10. $x = 2.5$ or $x = 2$

11. a) $x = 0.80$ c) $x = 3.16$
b) $x = -6.91$ d) $x = 0.34$

12. $x = 4.83$

13. $\log_3(-8) = x$; $3^x = -8$; Raising positive 3 to any power produces a positive value. If $x \geq 1$, then $3^x \geq 3$. If $0 \leq x < 1$, then $1 \leq x < 3$. If $x < 0$, then $0 < x < 1$.

14. a) $x > 3$
b) If x is 3, we are trying to take the logarithm of 0. If x is less than 3, we are trying to take the logarithm of a negative number.

15. $\frac{1}{2}(\log x + \log y) = \frac{1}{2}\log xy = \log \sqrt{xy}$
so $\frac{x+y}{5} = \sqrt{xy}$ and $x + y = 5\sqrt{xy}$.
Squaring both sides gives $(x + y)^2 = 25xy$.
Expanding gives $x^2 + 2xy + y = 25xy$;
therefore, $x + y = 23xy$.

16. $x = 3$ or $x = 2$

17. 1 and 16, 2 and 8, 4 and 4, 8 and 2, and 16 and 1

18. $x = 4, y = 4.58$

19. a) $x = 3$
b) $x = 16$

20. $x = -1.75, y = -2.25$

Lesson 8.7, pp. 499–501

1. First earthquake: $5.2 = \log x$;
$10^{5.2} = 158\ 489$
Second earthquake; $6 = \log x$;
$10^6 = 1\ 000\ 000$
Second earthquake is 6.3 times stronger than the first.

2. 7.2

3. 60 dB

4. 7.9 times

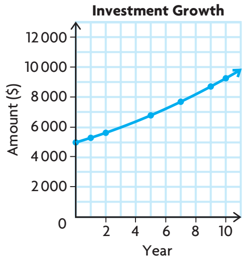
5. a) 0.000 000 001
b) 0.000 000 251
c) 0.000 000 016
d) 0.000 000 000 000 1

6. a) 3.49
b) 3.52
c) 4.35
d) 2.30

7. a) 7
b) Tap water is more acidic than distilled water as it has a lower pH than distilled water (pH 7).

8. 7.98 times

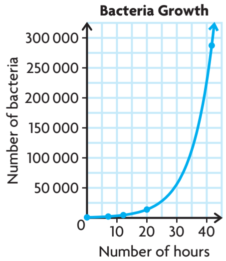
9. a) $y = 5000(1.0642)^t$



b) 6.42%
c) 11.14 years

10. 2.90 m

11. a) $y = 850(1.15)^x$



b) 4.9 h

12. a) 1.22, 1.43, 1.69, 2.00, 2.18, 2.35
b) 1.81
c) $w = 5.061\ 88(1.061\ 8)^t$
d) $w = 5.061\ 88(1.061\ 8)^t$
e) 11.5 °C

13. 33 cycles

14. 7.4 years

15. 26.2 days

16. Answers may vary. For example: (1) Tom invested $2000 in an account that accrued interest, compounded annually, at a rate of 6%. How long will it take for Tom's investment to triple? (2) Indira invested $5000 in a stock that made her $75 every month. How long will it take her investment to triple?
The first problem could be modelled using an exponential function. Solving this problem would require the use of logarithms. The second problem could be modelled using a linear equation. Solving the second problem would not require the use of logarithms.

17. 73 dB

18. a) $C = P(1.038)^t$
b) $580.80
c) $33.07

Lesson 8.8, pp. 507–508

1. a) -7.375
b) -23.25
c) -2

2. The instantaneous rate of decline was greatest in year 1. The negative change from year 1 to year 2 was 50, which is greater than the negative change in any other two-year period.

3. a) -12.378
b) -4.867
c) -1.914

4. a) $A(t) = 6000(1.075)^t$
b) 894.35
c) 461.25

5. a) i) 61.80
ii) 67.65
iii) 79.08
b) The rate of change is not constant because the value of the account each year is determined by adding a percent of the previous year's value.

6. a) 20.40 g
b) -0.111 g/h

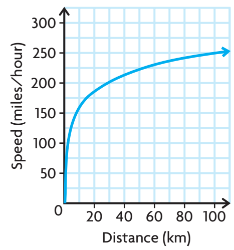
7. a) 1.59 g/day
b) $y = 0.0017(1.7698)^x$, where x is the number of days after the egg is laid
c) i) 0.0095 g/day
ii) 0.917 g/day
iii) 88.25 g/day
d) 14.3 days

8. a) 3.81 years
b) 9.5%/year

9. a) $y = 12\ 000(0.982)^t$
b) -181.7 people/year
c) -109 people/year

10. Both functions approach a horizontal asymptote. Each change in x yields a smaller and smaller change in y. Therefore, the instantaneous rate of change grows increasingly small, toward 0, as x increases.

11. a)



b) 1.03 miles/hour/hour
c) 4.03 miles/hour/hour and 0.403 miles/hour/hour
d) The rate at which the wind changes during shorter distances is much greater than the rate at which the wind changes at farther distances. As the distance increases, the rate of change approaches 0.

12. To calculate the instantaneous rate of change for a given point, use the exponential function to calculate the values of y that approach the given value of x. Do this for values on either side of the given value of x. Determine the average rate of change for these values of x and y. When the average rate of change has stabilized to a constant value, this is the instantaneous rate of change.

13. **a)** and **b)** Only a and k affect the instantaneous rate of change. Increases in the absolute value of either parameter tend to increase the instantaneous rate of change.

Chapter Review, pp. 510–511

1. **a)** $y = \log_4 x$ **c)** $y = \log_{\frac{3}{4}} x$
b) $y = \log_a x$ **d)** $m = \log_p q$

2. **a)** vertical stretch by a factor of 3, reflection in the x-axis, horizontal compression by a factor of $\frac{1}{2}$
b) horizontal translation 5 units to the right, vertical translation 2 units up
c) vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{5}$
d) horizontal stretch by a factor of 3, reflection in the y-axis, vertical shift 3 units down

3. **a)** $y = \frac{2}{5} \log x - 3$
b) $y = -\log\left[\frac{1}{2}(x - 3)\right]$
c) $y = 5 \log(-2x)$
d) $y = \log(-x - 4) - 2$

4. Compared to $y = \log x$, $y = 3 \log(x - 1) + 2$ is vertically stretched by a factor of 3, horizontally translated 1 unit to the right, and vertically translated 2 units up.

5. **a)** 3 **c)** 0
b) -2 **d)** -4

6. **a)** 3.615 **c)** 2.829
b) -1.661 **d)** 2.690

7. **a)** log 55 **c)** $\log_5 4$
b) log 5 **d)** log 128

8. **a)** 1 **c)** $\frac{2}{3}$
b) 2 **d)** 3

9. It is shifted 4 units up.

10. **a)** 5 **c)** -2
b) 3.75 **d)** -0.2

11. **a)** 2.432 **c)** 2.553
b) 3.237 **d)** 4.799

12. **a)** 0.79; 0.5
b) -0.43

13. 5.45 days

14. **a)** 63 **c)** 9
b) $\frac{10\,000}{3}$ **d)** 1.5

15. **a)** 1 **c)** 3
b) 5 **d)** $\pm\sqrt{10\,001}$

16. 10^{-2} W/m^2

17. $10^{-3.8}$ W/m^2

18. 5 times

19. 3.9 times

20. $\frac{10^{4.7}}{10^{2.3}} = 251.2$
$\frac{10^{12.5}}{10^{10.1}} = 251.2$
The relative change in each case is the same. Each change produces a solution with concentration 251.2 times the orignial solution.

21. Yes; $y = 3(2.25^x)$

22. 17.8 years

23. **a)** 8671 people per year
b) 7114; The rate of growth for the first 30 years is slower than the rate of growth for the entire period.
c) $y = 134\,322(1.03^x)$, where x is the number of years after 1950
d) **i)** 7171 people per year
ii) 12 950 people per year

24. **a)** exponential; $y = 23(1.17^x)$, where x is the number of years since 1998
b) 331 808
c) Answers may vary. For example, I assumed that the rate of growth would be the same through 2015. This is not reasonable. As more people buy the players, there will be fewer people remaining to buy them, or newer technology may replace them.
d) about 5300 DVD players per year
e) about 4950 DVD players per year
f) Answers may vary. For example, the prediction in part e) makes sense because the prediction is for a year covered by the data given. The prediction made in part b) does not make sense because the prediction is for a year that is beyond the data given, and conditions may change, making the model invalid.

Chapter Self-Test, p. 512

1. **a)** $x = 4^y$; $\log_4 x = y$
b) $y = 6^x$; $\log_6 y = x$

2. **a)** horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 4 units to the right, vertical translation 3 units up
b) vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left, vertical translation 1 unit down

3. **a)** -2 **b)** 5

4. **a)** 2 **b)** 7

5. $\log_4 xy$

6. 7.85

7. **a)** 2 **b)** $1\frac{3}{4}$

8. **a)** 50 g
b) $A(t) = 100(0.5)^{\frac{t}{5730}}$
c) 1844 years
d) -0.015 g/year

9. **a)** 6 min
b) 97°

Chapter 9

Getting Started, p. 516

1. **a)** $f(-1) = 30$, $f(4) = 0$
b) $f(-1) = -2$, $f(4) = -5\frac{1}{3}$
c) $f(-1)$ is undefined, $f(4) \doteq 1.81$
d) $f(-1) = -20$, $f(4) = -0.625$

2. $D = \{x \in \mathbf{R} \mid x \neq 1\}$
$R = \{y \in \mathbf{R} \mid y \neq 2\}$
There is no minimum or maximum value;
the function is never increasing;
the function is decreasing from $(-\infty, 1)$ and $(1, \infty)$;
the function approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right;
vertical asymptote is $x = 1$;
horizontal asymptote is $y = 2$

3. **a)** $y = 2|x - 3|$
b) $y = -\cos(2x)$
c) $y = \log_3(-x - 4) - 1$
d) $y = -\frac{4}{x} - 5$

4. **a)** $x = -1, \frac{1}{2}$, and 4
b) $x = -\frac{5}{3}$ or $x = 3$
c) $x = 5$ or $x = -2$
Cannot take the log of a negative number, so $x = 5$.
d) $x = -\frac{3}{4}$
e) $x = -3$
f) $\sin x = \frac{3}{2}$ or $\sin x = -1$. Since $\sin x$ cannot be greater than 1, the first equation does not give a solution; $x = 270°$

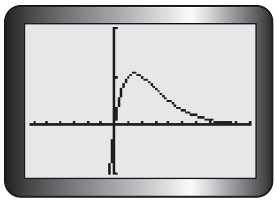
5. **a)** $(-\infty, -4) \cup (2, 3)$
b) $\left(-2, \frac{3}{2}\right) \cup [4, \infty)$

6. **a)** odd **c)** even
b) neither **d)** neither

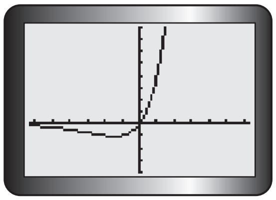
7. Polynomial, logarithmic, and exponential functions are continuous. Rational and trigonometric functions are sometimes continuous and sometimes not.

Lesson 9.1, p. 520

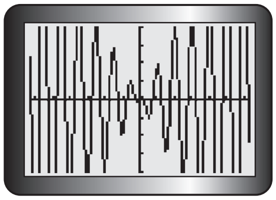
1. Answers may vary. For example, the graph of $y = \left(\left(\frac{1}{2}\right)^x\right)(2x)$ is



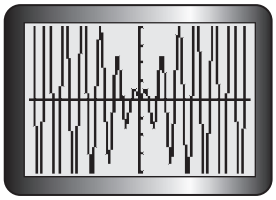
2. a) Answers may vary. For example,
$y = (2^x)(2x)$;



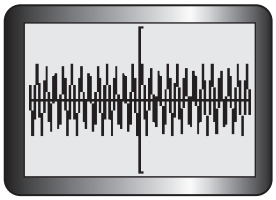
b) Answers may vary. For example,
$y = (2x)(\cos(2\pi x))$;



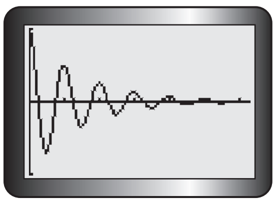
c) Answers may vary. For example,
$y = (2x)(\sin(2\pi x))$;



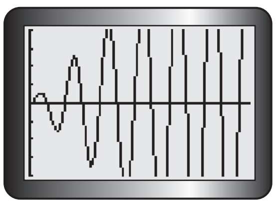
d) Answers may vary. For example,
$y = (\sin 2\pi x)(\cos 2\pi x)$;



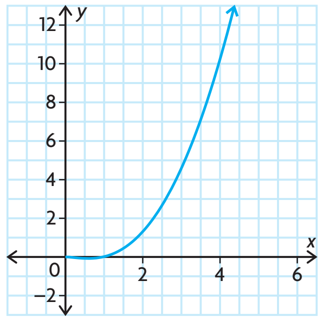
e) Answers may vary. For example,
$y = \left(\frac{1}{2}\right)^x(\cos 2\pi x)$,
where $0 \leq x \leq 2\pi$;



f) Answers may vary. For example,
$y = 2x \sin 2\pi x$, where $0 \leq x \leq 2\pi$;



3. Answers will vary. For example,
$y = x^2$
$y = \log x$
The product will be $y = x^2 \log x$.



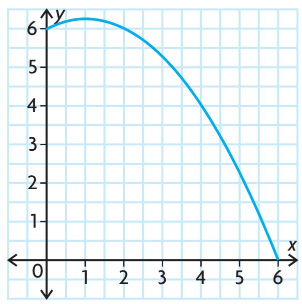
Lesson 9.2, pp. 528–530

1. a) $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
b) $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
c) $\{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$
d) $\{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$
e) $\{(-4, 8), (-2, 8), (1, 6), (3, 10), (4, 12)\}$
f) $\{(-4, 0), (-2, 0), (0, 0), (1, 0), (2, 0), (4, 0)\}$

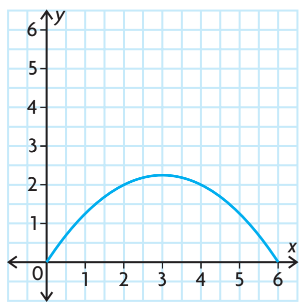
2. a) 10
b) 2; $(f + g)(x)$ is undefined at $x = 2$ because $g(x)$ is undefined at $x = 2$.
c) $\{x \in \mathbf{R} \mid x \neq 2\}$

3. $\{x \in \mathbf{R} \mid -1 \leq x < 1\}$

4. Graph of $f + g$:



Graph of $f - g$:

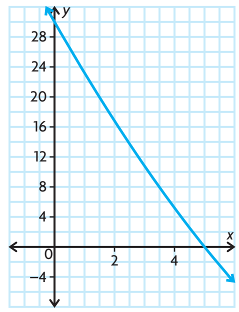


5. a) $f + g = |x| + x$
b) The function is neither even nor odd.

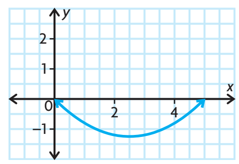
6. a) $\{(-6, 7), (-3, 10)\}$
b) $\{(-6, 7), (-3, 10)\}$
c) $\{(-6, -5), (-3, 4)\}$
d) $\{(-6, 5), (-3, -4)\}$
e) $\{(-9, 0), (-8, 0), (-6, 0), (-3, 0), (-1, 0), (0, 0)\}$
f) $\{(-7, 14), (-6, 12), (-5, 10), (-4, 8), (-3, 6)\}$

7. a) $\dfrac{2(2x + 1)}{3x^2 - 2x - 8}$
b) $\left\{x \in \mathbf{R} \middle| x \neq -\dfrac{4}{3} \text{ or } 2\right\}$
c) $\dfrac{17}{84}$
d) $-\dfrac{11}{84}$

8. The graph of $(f + g)(x)$:



The graph of $(f - g)(x)$:



9. a) $f(x) + g(x) = 2^x + x^3$
The function is not symmetric.
The function is always increasing.
zero at $x = -0.8262$
no maximum or minimum
period: N/A
The domain is all real numbers. The range is all real numbers.
$f(x) - g(x) = 2^x - x^3$
The function is not symmetric.
The function is always decreasing.
zero at $x = 1.3735$
no maximum or minimum
period: N/A
The domain is all real numbers. The range is all real numbers.
b) $f(x) + g(x) = \cos(2\pi x) + x^4$
The function is symmetric across the line $x = 0$.
The function is decreasing from $-\infty$ to -0.4882 and 0 to 0.4882 and increasing from -0.4882 to 0 and 0.4882 to ∞.
zeros at $x = -0.7092, -0.2506, 0.2506, 0.7092$

relative maximum at $x = 0$ and relative minimums at $x = -0.4882$ and $x = 0.4882$
period: N/A
The domain is all real numbers. The range is all real numbers greater than -0.1308.
$f(x) - g(x) = \cos(2\pi x) - x^4$
The function is symmetric across the line $x = 0$.
The function is increasing from $-\infty$ to -0.9180 and -0.5138 to 0 and 0.5138 to 0.9180; decreasing from -0.9180 to -0.5138 and 0 to 0.5138 and 0.9180 to ∞.
zeros at $x = -1, -0.8278, -0.2494, 0.2494, 0.8278, 1$
relative maxima at -0.9180, 0, and 0.9180; relative minima at -0.5138 and 0.5138
period: N/A
The domain is all real numbers. The range is all real numbers less than 1.

c) $f(x) + g(x) = \log(x) + 2x$
The function is not symmetric.
The function is increasing from 0 to ∞.
no zeros
no maximum or minimum
period: N/A
The domain is all real numbers greater than 0. The range is all real numbers.
$f(x) - g(x) = \log(x) - 2x$
The function is not symmetric.
The function is increasing from 0 to approximately 0.2 and decreasing from approximately 0.2 to ∞.
no zeros
maximum at $x \doteq 0.2$
period: N/A
The domain is all real numbers greater than 0. The range is all real numbers less than or equal to approximately -1.1.

d) $f(x) + g(x) = \sin(2\pi x) + 2\sin(\pi x)$
The function is symmetric about the origin.
The function is increasing from $-0.33 + 2k$ to $0.33 + 2k$ and decreasing from $0.33 + 2k$ to $1.67 + 2k$.
zero at k
minimum at $x = -0.33 + 2k$
maximum at $x = 0.33 + 2k$
period: 2
The domain is all real numbers. The range is all real numbers between -2.598 and 2.598.
$f(x) - g(x) = \sin(2\pi x) - 2\sin(\pi x)$
The function is symmetric about the origin, increasing from $0.67 + 2k$ to $1.33 + 2k$ and decreasing from $-0.67 + 2k$ to $0.67 + 2k$
zero at k
minimum at $0.67 + 2k$ and maximum at $1.33 + 2k$
period: 2
The domain is all real numbers.
The range is all real numbers between -2.598 to 2.598.

e) $f(x) + g(x) = \sin(2\pi x) + \frac{1}{x}$
The function is not symmetric.
The function is increasing and decreasing at irregular intervals.
The zeros are changing at irregular intervals.
The maximums and minimums are changing at irregular intervals.
period: N/A
The domain is all real numbers except 0.
The range is all real numbers.
$f(x) - g(x) = \sin(2\pi x) - \frac{1}{x}$
The function is not symmetric.
The function is increasing and decreasing at irregular intervals.
The zeros are changing at irregular intervals.
The maximums and minimums are changing at irregular intervals.
period: N/A
The domain is all real numbers except 0.
The range is all real numbers.

f) $f(x) + g(x) = \sqrt{x-2} + \frac{1}{x-2}$
The function is not symmetric.
The function is increasing from 3.5874 to ∞ and decreasing from 2 to 3.5874.
zeros: none
minimum at $x = 3.5874$
period: N/A
The domain is all real numbers greater than 2. The range is all real numbers greater than 1.8899.
$f(x) - g(x) = \sqrt{x-2} - \frac{1}{x-2}$
The function is not symmetric.
The function is increasing from 2 to ∞.
zero at $x = 3$
no maximum or minimum
period: N/A
The domain is all real numbers greater than 2. The range is all real numbers.

10. a) The sum of two even functions will be even because replacing x with $-x$ will still result in the original function.
b) The sum of two odd functions will be odd because replacing x with $-x$ will still result in the opposite of the original function.
c) The sum of an even and an odd function will result in neither an even nor an odd function because replacing x with $-x$ will not result in the same function or in the opposite of the function.

11. a) $R(t) = 5000 - 25t - 1000\cos\left(\frac{\pi}{6}t\right)$; it is neither odd nor even; it is increasing during the first 6 months of each year and decreasing during the last 6 months of each year; it has one zero, which is the point at which the deer population has become extinct; it has a maximum value of 3850 and a minimum value of 0, so its range is $\{R(t) \in \mathbf{R} \mid 0 \le R(t) \le 3850\}$.
b) after about 167 months, or 13 years and 11 months

12. The stopping distance can be defined by the function $s(x) = 0.006x^2 + 0.21x$. If the vehicle is travelling at 90 km/h, the stopping distance is 67.5 m.

13. $f(x) = \sin(\pi x)$; $g(x) = \cos(\pi x)$

14. The function is neither even nor odd; it is not symmetrical with respect to the y-axis or with respect to the origin; it extends from the third quadrant to the first quadrant; it has a turning point between $-n$ and 0 and another turning point at 0; it has zeros at $-n$ and 0; it has no maximum or minimum values; it is increasing when $x \in (-\infty, -n)$ and when $x \in (0, \infty)$; when $x \in (-n, 0)$, it increases, has a turning point, and then decreases; its domain is $\{x \in \mathbf{R}\}$, and its range is $\{y \in \mathbf{R}\}$.

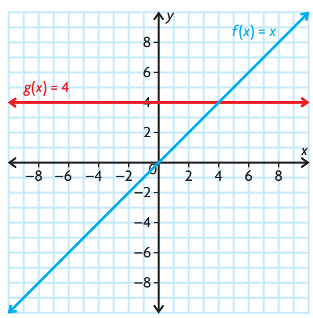
15. a) $f(x) = 0$; $g(x) = 0$
b) $f(x) = x^2$; $g(x) = x^2$
c) $f(x) = \frac{1}{x-2}$; $g(x) = \frac{1}{x-2} + 2$.

16. $m = 2, n = 3$

Lesson 9.3, pp. 537–539

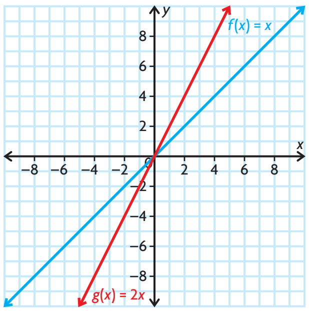
1. a) $\{(0, -2), (1, -10), (2, 21), (3, 60)\}$
b) $\{(0, 12), (2, -20)\}$
c) $4x$
d) $2x^2$
e) $x^3 - 3x + 2$
f) $2^x\sqrt{x-2}$

2. a) 1(c):

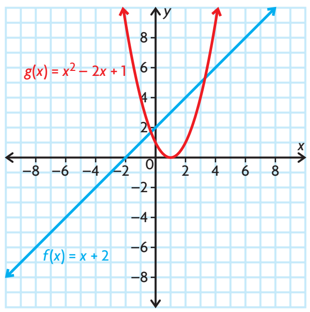


Answers

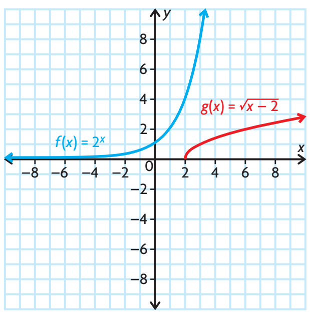
1(d):



1(e):

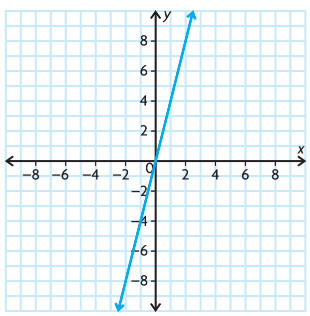


1(f):

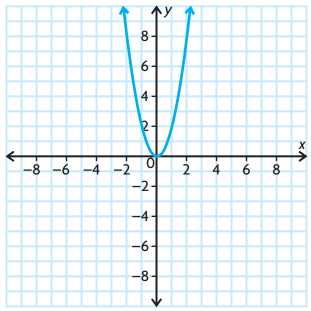


b) 1(c): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(d): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(e): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(f): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R} \mid x \geq 2\}$

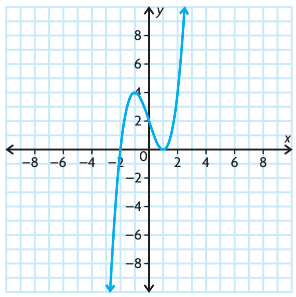
c) 1(c):



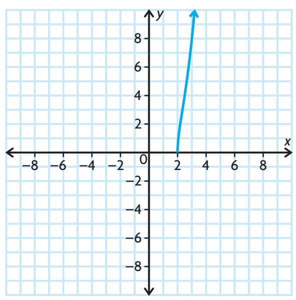
1(d):



1(e):



1(f):



d) 1(c): $\{x \in \mathbf{R}\}$
1(d): $\{x \in \mathbf{R}\}$
1(e): $\{x \in \mathbf{R}\}$
1(f): $\{x \in \mathbf{R} \mid x \geq 2\}$

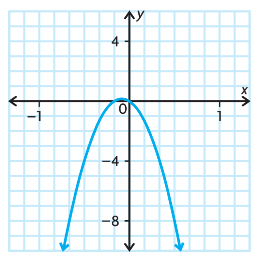
3. $\{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$

4. **a)** $x^2 - 49$
b) $x + 10$
c) $7x^3 - 63x^2$
d) $-16x^2 - 56x - 49$
e) $\dfrac{2 \sin x}{x - 1}$
f) $2^x \log(x + 4)$

5. 4(a): $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \geq -49\}$
4(b): $D = \{x \in \mathbf{R} \mid x \geq -10\}$;
$R = \{y \in \mathbf{R} \mid y \geq 0\}$
4(c): $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$
4(d): $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \leq 0\}$
4(e): $D = \{x \in \mathbf{R} \mid x \neq -1\}$; $R = \{y \in \mathbf{R}\}$
4(f): $D = \{x \in \mathbf{R} \mid x > -4\}$;
$R = \{y \in \mathbf{R} \mid y \geq 0\}$

6. 4(a): The function is symmetric about the line $x = 0$.
The function is increasing from 0 to ∞.
The function is decreasing from $-\infty$ to 0.
zeros at $x = -7, 7$
The minimum is at $x = 0$.
period: N/A
4(b): The function is not symmetric.
The function is increasing from -10 to ∞.
zero at $x = -10$
The minimum is at $x = -10$.
period: N/A
4(c): The function is not symmetric.
The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros at $x = 0, 9$
The relative minimum is at $x = -6$. The relative maximum is at $x = 0$.
period: N/A
4(d): The function is symmetric about the line $x = -1.75$.
The function is increasing from $-\infty$ to -1.75 and is decreasing from -1.75 to ∞.
zero at $x = -1.75$
The maximum is at $x = -1.75$.
period: N/A
4(e): The function is not symmetric.
The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros at $x = 0, 9$
The relative minima are at $x = -4.5336$ and 4.4286. The relative maximum is at $x = -1.1323$.
period: N/A
4(f): The function is not symmetric.
The function is increasing from -4 to ∞.
zeros: none
maximum/minimum: none
period: N/A

7.



8. **a)** $\left\{x \in \mathbf{R} \,\middle|\, x \neq -2, 7, \frac{\pi}{2}, \text{ or } \frac{3\pi}{2}\right\}$
b) $\{x \in \mathbf{R} \mid x > 8\}$
c) $\{x \in \mathbf{R} \mid x \geq -81 \text{ and } x \neq 0, \pi, \text{ or } 2\pi\}$
d) $\{x \in \mathbf{R} \mid x \leq -1 \text{ or } x \geq 1, \text{ and } x \neq -3\}$

9. $(f \times p)(t)$ represents the total energy consumption in a particular country at time t

10. **a)** $R(x) = (20\,000 - 750x)(25 + x)$ or $R(x) = 500\,000 + 1250x - 750x^2$, where x is the increase in the admission fee in dollars

b) Yes, it's the product of the function $P(x) = 20\,000 - 750x$, which represents the number of daily visitors, and $F(x) = 25 + x$, which represents the admission fee.

c) \$25.83

11. $m(t) = ((0.9)^t)(650 + 300t)$
The amount of contaminated material is at its greatest after about 7.3 s.

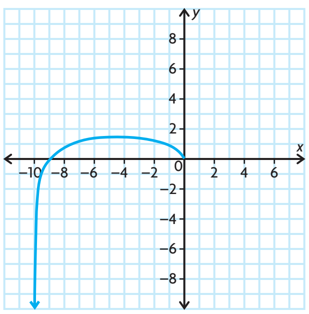
12. The statement is false. If $f(x)$ and $g(x)$ are odd functions, then their product will always be an even function. When you multiply a function that has an odd degree with another function that has an odd degree, you add the exponents, and when you add two odd numbers together, you get an even number.

13. $f(x) = 3x^2 + 2x + 5$ and
$g(x) = 2x^2 - 4x - 2$

14. a) $(f \times g)(x) = \sqrt{-x}\log(x + 10)$
The domain is $\{x \in \mathbf{R} \mid -10 < x \leq 0\}$.

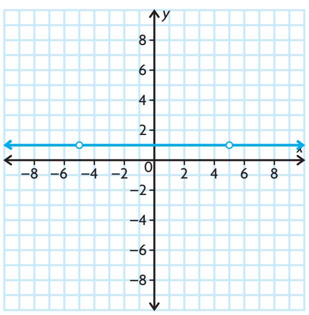
b) One strategy is to create a table of values for $f(x)$ and $g(x)$ and to multiply the corresponding y-values together. The resulting values could then be graphed. Another strategy is to graph $f(x)$ and $g(x)$ and to then create a graph for $(f \times g)(x)$ based on these two graphs. The first strategy is probably better than the second strategy, since the y-values for $f(x)$ and $g(x)$ will not be round numbers and will not be easily discernable from the graphs of $f(x)$ and $g(x)$.

c)



15. a) $f(x) \times \dfrac{1}{f(x)} = 1$

b) $\{x \in \mathbf{R} \mid x \neq -5 \text{ or } 5\}$



c) The range will always be 1. If f is of odd degree, there will always be at least one value that makes the product undefined and which is excluded from the domain. If f is of even degree, there may be no values that are excluded from the domain.

16. a) $f(x) = 2^x$
$g(x) = x^2 + 1$
$(f \times g)(x) = 2^x(x^2 + 1)$

b) $f(x) = x$
$g(x) = \sin(2\pi x)$
$(f \times g)(x) = x\sin(2\pi x)$

17. a) $f(x) = (2x + 9)$
$g(x) = (2x - 9)$

b) $f(x) = (2\sin x + 3)$
$g(x) = (4\sin^2 x - 6\sin x + 9)$

c) $f(x) = x^{\frac{1}{2}}$
$g(x) = (4x^5 - 3x^3 + 1)$

d) $f(x) = \dfrac{1}{2x + 1}$
$g(x) = 6x - 5$

Lesson 9.4, p. 542

1. a) $(f \div g)(x) = \dfrac{5}{x}, x \neq 0$

b) $(f \div g)(x) = \dfrac{4x}{2x - 1}, x \neq \dfrac{1}{2}$

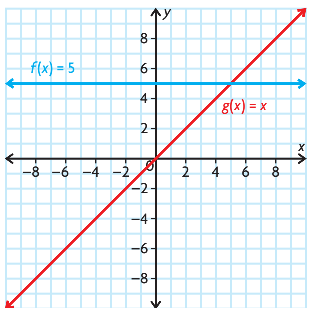
c) $(f \div g)(x) = \dfrac{4x}{x^2 + 4}$

d) $(f \div g)(x) = \dfrac{(x + 2)\left(\sqrt{x - 2}\right)}{x - 2}, x > 2$

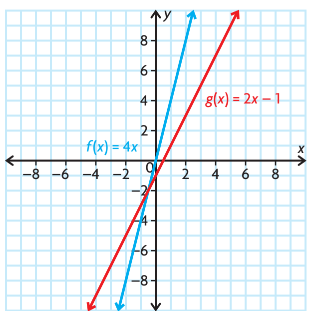
e) $(f \div g)(x) = \dfrac{8}{1 + (\frac{1}{2})^x}$

f) $(f \div g)(x) = \dfrac{x^2}{\log(x)}, x > 0$

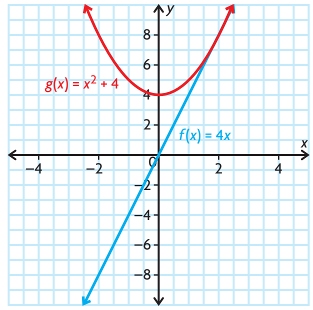
2. a) 1(a):



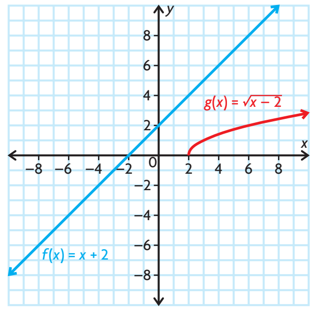
1(b):



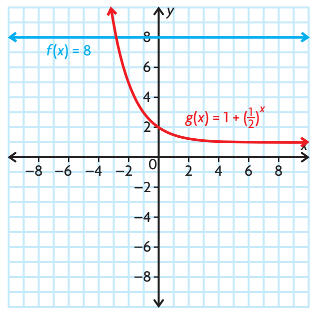
1(c):



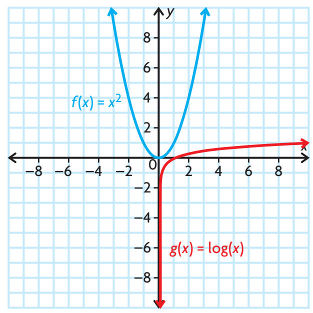
1(d):



1(e):

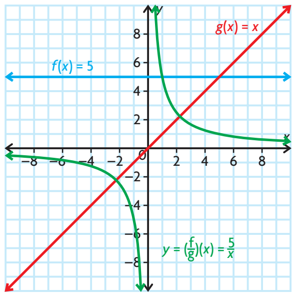


1(f):

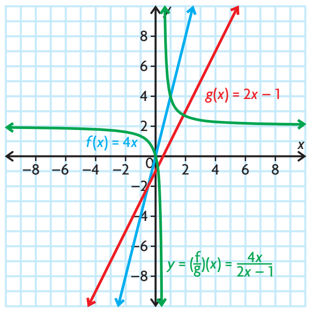


b) 1(a): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(b): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(c): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(d): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R} \mid x \geq 2\}$
1(e): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(f): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R} \mid x > 0\}$

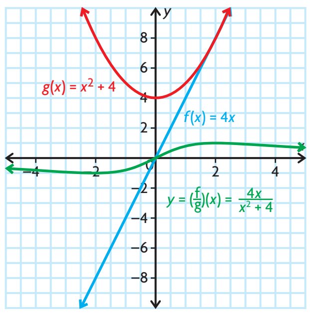
c) 1(a):



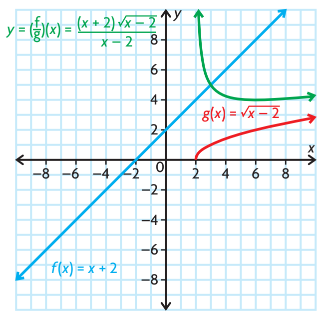
1(b):



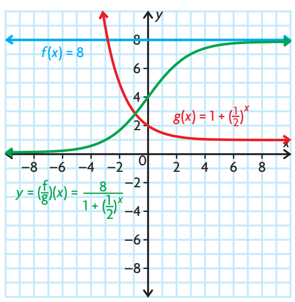
1(c):



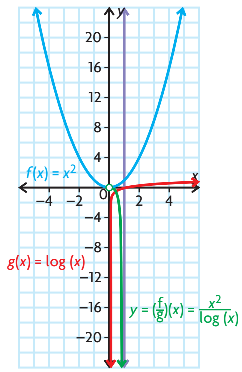
1(d):



1(e):



1(f):



d) 1(a): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x \neq 0\}$

1(b): domain of $(f \div g)$: $\left\{x \in \mathbf{R} \mid x \neq \frac{1}{2}\right\}$

1(c): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$

1(d): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 2\}$

1(e): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$

1(f): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 0\}$

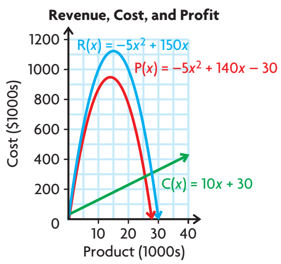
3. **a)** 2.798 cm/day
b) about 30 days
c) 6.848 cm/day
d) It slows down and eventually comes to zero. This is seen on the graph as it becomes horizontal at the top.

Mid-Chapter Review, p. 544

1. multiplication

2. **a)** $\{(-9, 2), (-6, -9), (0, 14)\}$
b) $\{(-9, 2), (-6, -9), (0, 14)\}$
c) $\{(-9, -6), (-6, 3), (0, -10)\}$
d) $\{(-9, 6), (-6, -3), (0, 10)\}$

3. **a)** $P(x) = -5x^2 + 140x - 30$
b)



c) $738 750

4. **a)** $R(h) = 24.39h$
b) $N(h) = 24.97h$
c) $W(h) = 24.78h$
d) $S(h) = 25.36h$
e) $317

5. **a)** $(f \times g)(x) = x^2 + x + \frac{1}{4}$

$D = \{x \in \mathbf{R}\}$

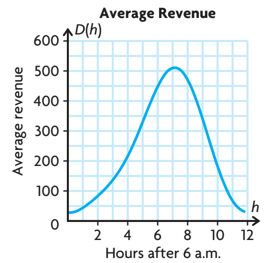
b) $(f \times g)(x) = \sin(3x)(\sqrt{x - 10})$
$D = \{x \in \mathbf{R} \mid x \geq 10\}$

c) $(f \times g)(x) = \dfrac{22x^3}{x + 5}$
$D = \{x \in \mathbf{R} \mid x \neq -5\}$

d) $(f \times g)(x) = 8100x^2 - 1$
$D = \{x \in \mathbf{R}\}$

6. **a)** $R(h) = 90\cos\left(\frac{\pi}{6}h\right)\sin\left(\frac{\pi}{6}h\right) - 102\sin\left(\frac{\pi}{6}h\right) - 210\cos\left(\frac{\pi}{6}h\right) + 238$

b)



c) about $470.30

7. **a)** $(f \div g)(x) = \dfrac{80}{x}$
$D = \{x \in \mathbf{R} \mid x \neq 0\}$

b) $(f \div g)(x) = \dfrac{10x^2}{x^2 - 3}$
$D = \{x \in \mathbf{R} \mid x \neq \pm\sqrt{3}\}$

c) $(f \div g)(x) = \dfrac{x + 8}{\sqrt{x - 8}}$
$D = \{x \in \mathbf{R} \mid x > 8\}$

d) $(f \div g)(x) = \dfrac{7x^2}{\log x}$
$D = \{x \in \mathbf{R} \mid x > 0\}$

8. $\csc x$, $\sec x$, $\cot x$

Lesson 9.5, pp. 552–554

1. **a)** -1
b) -24
c) -129
d) $\frac{7}{16}$
e) 1
f) -8

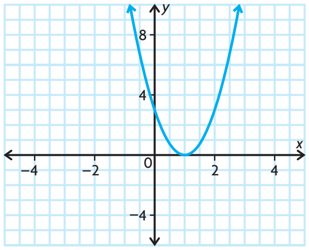
2. **a)** 3
b) 5
c) 10
d) $(f \circ g)(0)$ is undefined.
e) 2
f) 4

3. **a)** 5
b) 5
c) 4
d) $(f \circ f)(2)$ is undefined.

4. **a)** $C(d(5)) = 36$
It costs $36 to travel for 5 h.
b) $C(d(t))$ represents the relationship between the time driven and the cost of gasoline.

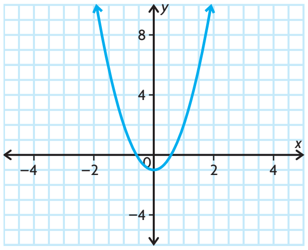
5. a) $f(g(x)) = 3x^2 - 6x + 3$

The domain is $\{x \in \mathbf{R}\}$.



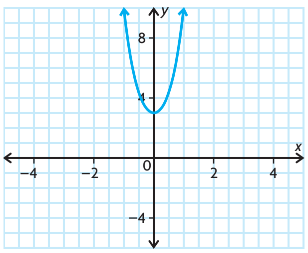
$g(f(x)) = 3x^2 - 1$

The domain is $\{x \in \mathbf{R}\}$.



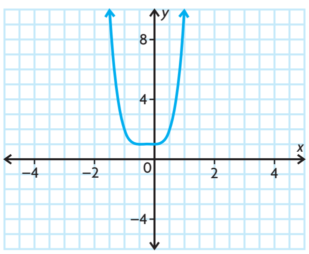
b) $f(g(x)) = 2x^4 + 5x^2 + 3$

The domain is $\{x \in \mathbf{R}\}$.



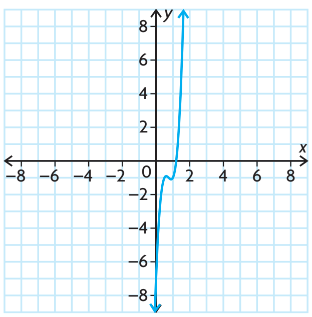
$g(f(x)) = 4x^4 + 4x^3 + x^2 + 1$

The domain is $\{x \in \mathbf{R}\}$.



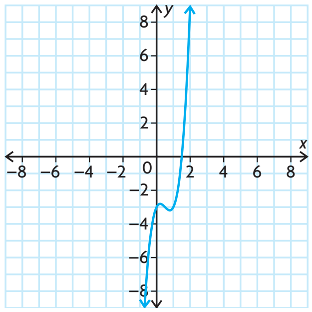
c) $f(g(x)) = 16x^3 - 36x^2 + 26x - 7$

The domain is $\{x \in \mathbf{R}\}$.



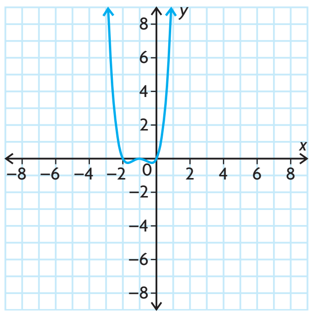
$g(f(x)) = 4x^3 - 6x^2 + 2x - 3$

The domain is $\{x \in \mathbf{R}\}$.



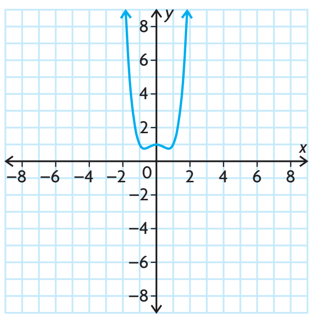
d) $f(g(x)) = x^4 + 4x^3 + 5x^2 + 2x$

The domain is $\{x \in \mathbf{R}\}$.



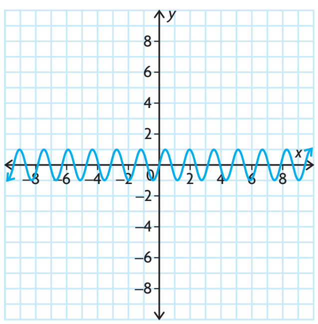
$g(f(x)) = x^4 - x^2 + 1$

The domain is $\{x \in \mathbf{R}\}$.



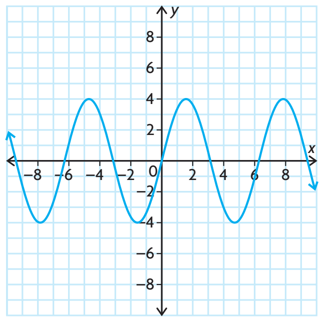
e) $f(g(x)) = \sin 4x$

The domain is $\{x \in \mathbf{R}\}$.



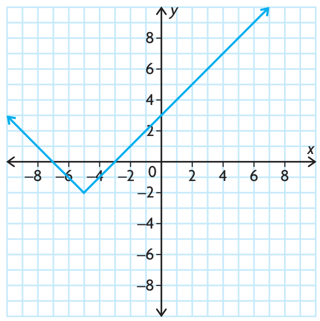
$g(f(x)) = 4 \sin x$

The domain is $\{x \in \mathbf{R}\}$.



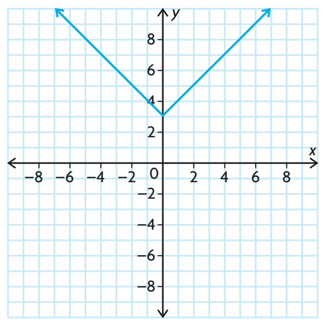
f) $f(g(x)) = |x + 5| - 2$

The domain is $\{x \in \mathbf{R}\}$.



$g(f(x)) = |x| + 3$

The domain is $\{x \in \mathbf{R}\}$.



6. a) $f \circ g = 3\sqrt{x - 4}$

$D = \{x \in \mathbf{R} \mid x \geq 4\}$

$R = \{y \in \mathbf{R} \mid y \geq 0\}$

$g \circ f = \sqrt{3x - 4}$

$D = \left\{x \in \mathbf{R} \mid x \geq \frac{4}{3}\right\}$

$R = \{y \in \mathbf{R} \mid y \geq 0\}$

b) $f \circ g = \sqrt{3x + 1}$

$D = \left\{x \in \mathbf{R} \mid x \geq -\frac{1}{3}\right\}$

$R = \{y \in \mathbf{R} \mid y \geq 0\}$

$g \circ f = 3\sqrt{x} + 1$

$D = \{x \in \mathbf{R} \mid x \geq 0\}$

$R = \{y \in \mathbf{R} \mid y \geq 1\}$

Answers

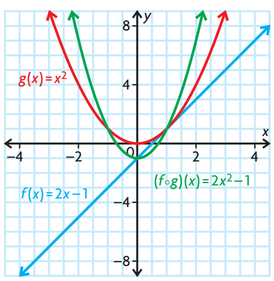
c) $f \circ g = \sqrt{4 - x^4}$
$D = \{x \in \mathbf{R} \mid -\sqrt{2} \leq x \leq \sqrt{2}\}$
$R = \{y \in \mathbf{R} \mid y \geq 0\}$
$g \circ f = 4 - x^2$
$D = \{x \in \mathbf{R} \mid -2 \leq x \leq 2\}$
$R = \{y \in \mathbf{R} \mid 0 < y < 2\}$

d) $f \circ g = 2\sqrt{x - 1}$
$D = \{x \in \mathbf{R} \mid x \geq 1\}$
$R = \{y \in \mathbf{R} \mid y \geq 1\}$
$g \circ f = \sqrt{2^x - 1}$
$D = \{x \in \mathbf{R} \mid x \geq 0\}$
$R = \{y \in \mathbf{R} \mid y \geq 0\}$

e) $f \circ g = x$
$D = \{x \in \mathbf{R} \mid x > 0\}$
$R = \{y \in \mathbf{R}\}$
$g \circ f = x$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R}\}$

f) $f \circ g = \sin(5^{2x} + 1)$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$
$g \circ f = 5^{2\sin x} + 1$
$D = \{x \in \mathbf{R}\}$
$R = \left\{y \in \mathbf{R} \mid \frac{26}{25} \leq y \leq 26\right\}$

7. a) Answers may vary. For example, $f(x) = \sqrt{x}$ and $g(x) = x^2 + 6$
b) Answers may vary. For example, $f(x) = x^6$ and $g(x) = 5x - 8$
c) Answers may vary. For example, $f(x) = 2^x$ and $g(x) = 6x + 7$
d) Answers may vary. For example, $f(x) = \frac{1}{x}$ and $g(x) = x^3 - 7x + 2$
e) Answers may vary. For example, $f(x) = \sin^2 x$ and $g(x) = 10x + 5$
f) Answers may vary. For example, $f(x) = \sqrt[3]{x}$ and $g(x) = (x + 4)^2$

8. a) $(f \circ g)(x) = 2x^2 - 1$
b)


c) It is compressed by a factor of 2 and translated down 1 unit.

9. a) $f(g(x)) = 6x + 3$
The slope of $g(x)$ has been multiplied by 2, and the y-intercept of $g(x)$ has been vertically translated 1 unit up.
b) $g(f(x)) = 6x - 1$
The slope of $f(x)$ has been multiplied by 3.

10. $D(p) = 780 + 31.96p$

11. $f(g(x)) = 0.06x$

12. a) $d(s) = \sqrt{16 + s^2}$; $s(t) = 560t$
b) $d(s(t)) = \sqrt{16 + 313\,600t^2}$, where t is the time in hours and $d(s(t))$ is the distance in kilometres

13. $c(v(t)) = \left(\frac{40 + 3t + t^2}{500} - 0.1\right)^2 + 0.15$;
The car is running most economically 2 h into the trip.

14. Graph A(k); $f(x)$ is vertically compressed by a factor of 0.5 and reflected in the x-axis. Graph B(b); $f(x)$ is translated 3 units to the left.
Graph C(d); $f(x)$ is horizontally compressed by a factor of $\frac{1}{2}$.
Graph D(1); $f(x)$ is translated 4 units down.
Graph E(g); $f(x)$ is translated 3 units up.
Graph F(c); $f(x)$ is reflected in the y-axis.

15. **Sum:** $y = f + g$
$f(x) = \frac{4}{x - 3}$; $g(x) = 1$
Product: $y = f \times g$
$f(x) = x - 3$; $g(x) = \frac{x + 1}{(x - 3)^2}$
Quotient: $y = f \div g$
$f(x) = 1 + x$; $g(x) = x - 3$
Composition: $y = f \circ g$
$f(x) = \frac{4}{x} + 1$; $g(x) = x - 3$

16. a) $f(k) = 27k - 14$
b) $f(k) = 2\sqrt{9k - 16} - 5$

Lesson 9.6, pp. 560–562

1. a) i) $x = \frac{1}{2}, 2,$ or $\frac{7}{2}$
ii) $x = -1$ or 2
b) i) $\frac{1}{2} < x < 2$ or $x > \frac{7}{2}$
ii) $-1 < x < 2$
c) i) $x \leq \frac{1}{2}$; $2 \leq x \leq \frac{7}{2}$
ii) $x \leq -1$ or $x \geq 2$
d) i) $\frac{1}{2} \leq x \leq 2$ or $x \geq \frac{7}{2}$
ii) $-1 \leq x \leq 2$

2. a) $x \doteq 0.8$
b) $x = 0$ and 3.5
c) $x \doteq -2.4$
d) $x \doteq 0.7$

3. $x = -1.3$ or 1.8

4. $f(x) < g(x)$: $1.3 < x < 1.6$
$f(x) = g(x)$: $x = 0$ or 1.3
$f(x) > g(x)$: $0 < x < 1.3$ or $1.6 < x < 3$

5. a) $x \doteq 2.5$
b) $x \doteq 2.2$
c) $x \doteq 1.8$
d) $x \doteq -2.1$
e) $x = 10$
f) $x = 1$ or 3

6. a) $x = -1.81$ or 0.48
b) $x = -1.38$ or 1.6
c) $x = -1.38$ or 1.30
d) $x = -0.8, 0,$ or 0.8
e) $x = 0.21$ or 0.74
f) $x = 0, 0.18, 0.38,$ or 1

7. $(0.7, -1.5)$

8. They will be about the same in 2012.

9. a) $x \in (-0.57, 1)$
b) $x \in [0, 0.58]$
c) $x \in (-\infty, 0)$
d) $x \in (0.17, 0.83)$
e) $x \in (0.35, 1.51)$
f) $x \in (0.1, 0.5)$

10. Answers may vary. For example, $f(x) = x^3 + 5x^2 + 2x - 8$ and $g(x) = 0$.

11. Answers may vary. For example, $f(x) = -x^2 + 25$ and $g(x) = -x + 5$.

12. $a \doteq 7$, $b \doteq 2$

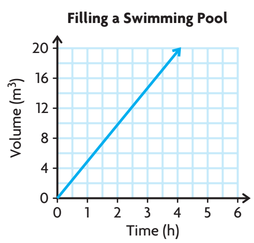
13. Answers may vary. For example:

Perform the necessary algebraic operations to move all of the terms on the right side of the equation to the left side of the equation.
↓
Construct the function $f(x)$, such that $f(x)$ equals the left side of the equation.
↓
Graph the function $f(x)$.
↓
Determine the x-intercepts of the graph that fall within the interval provided, if applicable.
↓
The x-intercepts of the graph are the solutions to the equation.

14. $x = 0 \pm 2n$, $x = -0.67 \pm 2n$ or $x = 0.62 \pm 2n$, where $n \in \mathrm{I}$

15. $x \in (2n, 2n + 1)$, where $n \in \mathrm{I}$

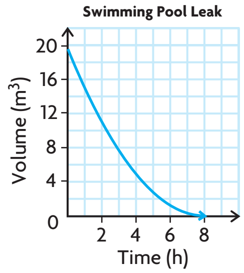
Lesson 9.7, pp. 569–574

1. a)


b) $y = 6.25\pi\left(\frac{x}{4}\right)$

c) about 1.6 h

2. a) $y = \frac{6.25\pi}{64}(x - 8)^2$



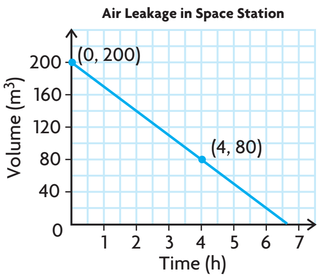
b) $V(t) = \frac{6.25\pi}{64}(t - 8)^2$

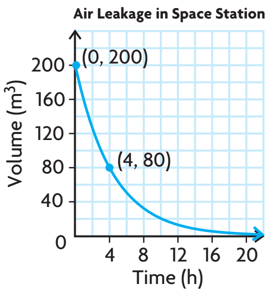
c) $V(2) \doteq 11\ \text{m}^3$

d) $-4.3\ \text{m}^3/\text{h}$

e) As time elapses, the pool is losing less water in the same amount of time.

3. a) Answers may vary. For example:

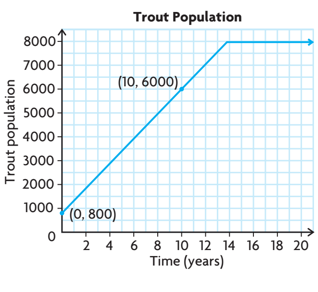




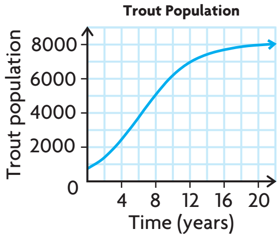
b) $V(t) = -30t + 200$;
$t \doteq 6.7$

c) $V(t) = 200(0.795)^t$;
$t \doteq 10$

4. a)



b) $P(t) = \frac{8000}{1 + 9(0.719)^t}$



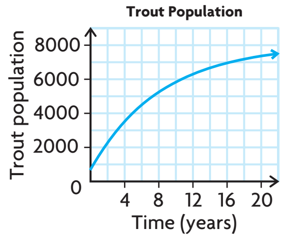
c) about 2349

d) 387.25 trout per year

5. a) the carrying capacity of the lake; 8000

b) Use (0, 800) and (10, 6000).
$a = 7200$, $b \doteq 0.88$

c)



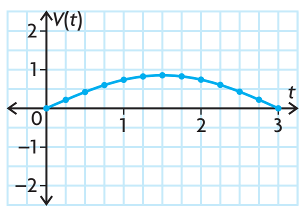
d) $P(4) \doteq 3682$

e) 720.5 trout per year

f) In the model in the previous problem, the carrying capacity of the lake is divided by a number that gets smaller and smaller, while in this model, a number that gets smaller and smaller is subtracted from the carrying capacity of the lake.

6. Answers may vary. For example, the first model more accurately calculates the current price of gasoline because prices are rising quickly.

7. a) $V(t) = 0.85\cos\left(\frac{\pi}{3}(t - 1.5)\right)$



b) The scatter plot and the graph are very close to being the same, but they are not exactly the same.

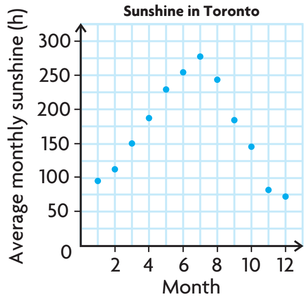
c) $V(6) = 0$ L/s

d) From the graph, the rate of change appears to be at its smallest at $t = 1.5$ s.

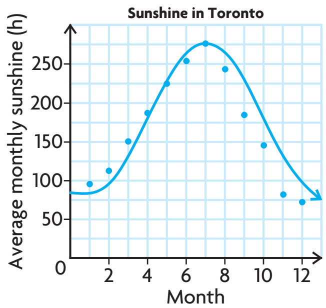
e) It is the maximum of the function.

f) From the graph, the rate of change appears to be greatest at $t = 0$ s.

8. a)



b)

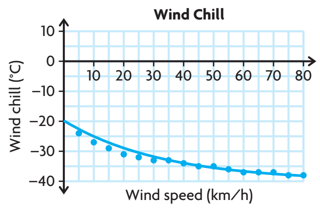


c) $S(t) = -97\cos\left(\frac{\pi}{6}(t - 1)\right) + 181$

d) From the model, the maximum will be at $t = 7$ and the minimum will be at $t = 1$.

e) It doesn't fit it perfectly, because, actually, the minimum is not at $t = 1$, but at $t = 12$.

9. a)



b) Answers may vary. For example,
$C(s) = -38 + 14(0.97)^s$

c) $C(0) = -24$ °C
$C(100) \doteq -37.3$ °C
$C(200) \doteq -38$ °C
These answers don't appear to be very reasonable, because the wind chill for a wind speed of 0 km/h should be -20 °C, while the wind chills for wind speeds of 100 km/h and 200 km/h should be less than -38 °C. The model only appears to be somewhat accurate for wind speeds of 10 to 70 km/h.

10. a) Answers will vary. For example, one polynomial model is
$P(t) = 1.4t^2 + 3230$, while an exponential model is
$P(t) = 3230(1.016)^t$. While neither model is perfect, it appears that the polynomial model fits the data better.

b) $P(155) = 1.4(155)^2 + 3230$
$\doteq 36\ 865$
$P(155) = 3230(1.016)^{155} \doteq 37\ 820$

c) A case could be made for either model. The polynomial model appears to fit the data better, but population growth is usually exponential.

d) According to the polynomial model, in 2000, the population was increasing at a rate of about 389 000 per year, while according to the exponential model, in 2000, the population was increasing at a rate of about 465 000 per year.

11. **a)** $P(t) = 3339.18(1.132\ 25)^t$
b) They were introduced around the year 1924.
c) rate of growth $\doteq$ 2641 rabbits per year
d) $P(65) \doteq 10\ 712\ 509.96$

12. **a)** $V(t) = 155.6 \sin\left(120\pi t + \frac{\pi}{2}\right)$
b) $V(t) = 155.6 \cos(120\pi t)$
c) The cosine function was easier to determine. The cosine function is at its maximum when the argument is 0, so no horizontal translation was necessary.

13. **a)** Answers will vary. For example, a linear model is $P(t) = -9t + 400$, a quadratic model is $P(t) = \frac{23}{90}(t - 30)^2 + 170$, and an exponential model is $P(t) = 400(0.972)^t$.

The exponential model fits the data far better than the other two models.

b) $P(t) = -9t + 400$
$P(60) = -140$ kPa
$P(t) = \frac{23}{90}(t - 30)^2 + 170$,
$P(60) = 400$ kPa
$P(t) = 400(0.972)^t$, $P(60) \doteq 73$ kPa

c) The exponential model gives the most realistic answer, because it fits the data the best. Also, the pressure must be less than 170 kPa, but it cannot be negative.

14. As a population procreates, the population becomes larger, and thus, more and more organisms exist that can procreate some more. In other words, the act of procreating enables even more procreating in the future.

15. **a)** linear, quadratic, or exponential
b) linear or quadratic
c) exponential

16. **a)** $T(n) = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$
b) $47\ 850 = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$

So, $n \doteq 64.975$. So, it is not a tetrahedral number because n must be an integer.

17. **a)** $P(t) = 30.75(1.008\ 418)^t$
b) In 2000, the growth rate of Canada was less than the growth rate of Ontario and Alberta.

Chapter Review, pp. 576–577

1. division

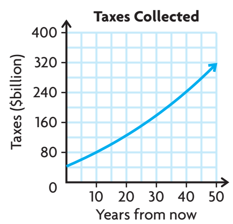
2. **a)** Shop 2
b) $S_{1+2} = t^3 + 1.6t^2 + 1200$
c) 1 473 600
d) The owner should close the first shop, because the sales are decreasing and will eventually reach zero.

3. **a)** $C(x) = 9.45x + 52\ 000$
b) $I(x) = 15.8x$
c) $P(x) = 6.35x - 52\ 000$

4. **a)** $12 \sin(7x)$
b) $9x^2$
c) $121x^2 - 49$
d) $2a^2b^{3x}$

5. **a)** $C \times A = 42\ 750\ 000\ 000(1.01)^t + 3\ 000\ 000\ 000t(1.01)^t$

b)



d) about $156 402 200 032.31

6. **a)** $\frac{21}{x}$
b) $\frac{1}{2x + 9}$
c) $\frac{\sqrt{x + 15}}{x + 15}$
d) $\frac{x^3}{2 \log x}$

7. **a)** $\{x \in \mathbf{R} \mid x \neq 0\}$
b) $\left\{x \in \mathbf{R} \mid x \neq 4, x \neq -\frac{9}{2}\right\}$
c) $\{x \in \mathbf{R} \mid x > -15\}$
d) $\{x \in \mathbf{R} \mid x > 0\}$

8. **a)** Domain of $f(x)$: $\{x \in \mathbf{R} \mid x > -1\}$
Range of $f(x)$: $\{y \in \mathbf{R} \mid y > 0\}$
Domain of $g(x)$: $\{x \in \mathbf{R}\}$
Range of $g(x)$: $\{y \in \mathbf{R} \mid y \geq 3\}$
b) $f(g(x)) = \frac{1}{\sqrt{x^2 + 4}}$
c) $g(f(x)) = \frac{3x + 4}{x + 1}$
d) $f(g(0)) = \frac{1}{2}$
e) $g(f(0)) = 4$
f) For $f(g(x))$: $\{x \in \mathbf{R}\}$
For $g(f(x))$: $\{x \in \mathbf{R} \mid x > -1\}$

9. **a)** $x - 6$
b) $x - 9$
c) $x - 12$
d) $x - 3(1 + n)$

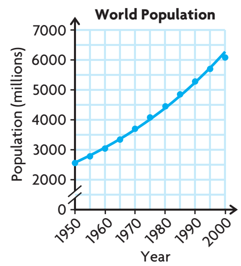
10. **a)** $A(r) = \pi r^2$
b) $r(C) = \frac{C}{2\pi}$
c) $A(r(C)) = \frac{C^2}{4\pi}$
d) $\frac{C^2}{4\pi} \doteq 1.03$ m

11. $f(x) < g(x)$: $-1.2 < x < 0$ or $x > 1.2$
$f(x) = g(x)$: $x = -1.2$, 0, or 1.2
$f(x) > g(x)$: $x < -1.2$ or $0 < x < 1.2$

12. **a)** $x \doteq 4.0$
b) $x \doteq 2.0$
c) $x \doteq -0.8$
d) $x \doteq 0.7$

13. **a)** $P(t) = 600t - 1000$. The slope is the rate that the population is changing.
b) $P(t) = 617.6(1.26)^t$, 617.6 is the initial population and 1.26 represents the growth.

14. $P(t) = 2570.99(1.018)^t$

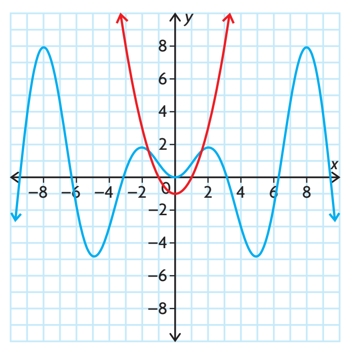


When $t = 13$, $P(t) = 3242$.
When $t = 23$, $P(t) = 3875$.
When $t = 90$, $P(t) = 12\ 806$.

Chapter Self-Test, p. 578

1. **a)** $A(r) = 4\pi r^2$
b) $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$
c) $A(r(V)) = 4\pi\left(\frac{3V}{4\pi}\right)^{\frac{2}{3}}$
d) $4\pi\left(\frac{3(0.75)}{4\pi}\right)^{\frac{2}{3}} \doteq 4\ \text{m}^2$

2.



From the graph, the solution is $-1.62 \leq x \leq 1.62$.

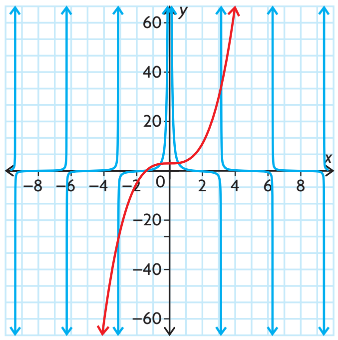
3. Answers may vary. For example, $g(x) = x^7$ and $h(x) = 2x + 3$, $g(x) = (x + 3)^7$ and $h(x) = 2x$

4. a) $N(n) = 1n^3 + 8n^2 + 40n + 400$
b) $N(3) = 619$

5. $(f \times g)(x) = 30x^3 + 405x^2 + 714x - 4785$

6. a) There is a horizontal asymptote of $y = 275$ cm. This is the maximum height this species will reach.
b) when $t \doteq 21.2$ months

7. $x = 4.5$ or 4500 items

8. 

The solutions are $x = -3.1, -1.4, -0.6, 0.5$, or 3.2.

9. Division will turn it into a tangent function that is not sinusoidal.

Cumulative Review Chapters 7–9, pp. 580–583

1.	(d)	**10.**	(d)	**19.**	(c)	**28.**	(a)
2.	(b)	**11.**	(a)	**20.**	(d)	**29.**	(d)
3.	(a)	**12.**	(b)	**21.**	(b)	**30.**	(d)
4.	(a)	**13.**	(d)	**22.**	(a)	**31.**	(c)
5.	(d)	**14.**	(d)	**23.**	(c)	**32.**	(d)
6.	(c)	**15.**	(c)	**24.**	(c)	**33.**	(d)
7.	(d)	**16.**	(a)	**25.**	(c)	**34.**	(b)
8.	(b)	**17.**	(b)	**26.**	(b)		
9.	(c)	**18.**	(b)	**27.**	(a)		

35. 27° or 63°

36. a) Answers may vary. For example, Niagara: $P(x) = (414.8)(1.0044^x)$; Waterloo: $P(x) = (418.3)(1.0117^x)$

b) Answers may vary. For example, Niagara: 159 years; Waterloo: 60 years

c) Answers may vary. For example, Waterloo is growing faster. In 2025, the instantaneous rate of change for the population in Waterloo is about 6800 people/year, compared to about 2000 people/year for Niagara.

37. $m(t) = 30\,000 - 100t$,

$$\mathrm{a}(t) = \frac{T}{30\,000 - 100t} - 10,$$

$$v(t) = -\frac{\log\left(1 - \frac{t}{300}\right)}{\log 2.72} - gt;$$

at $t = 0$, $\dfrac{T}{30\,000} - 10$ must be greater than $0\ \mathrm{m/s^2}$, so T must be greater than $300\,000\ \mathrm{kg} \times \mathrm{m/s^2}$ (or 300 000 N)

Answers

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Credits

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