L5 – 5.4 Solve Linear Trigonometric Equations
MHF4U

In the previous lesson we have been working with identities. Identities are equations that are true for ANY value of x. In this lesson, we will be working with equations that are not identities. We will have to solve for the value(s) that make the equation true.

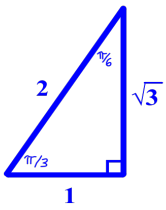
Remember that 2 solutions are possible for an angle between 0 and 2π with a given ratio. Use the reference angle and CAST rule to determine the angles.

When solving a trigonometric equation, consider all 3 tools that can be useful:

1. Special Triangles
2. Graphs of Trig Functions
3. Calculator

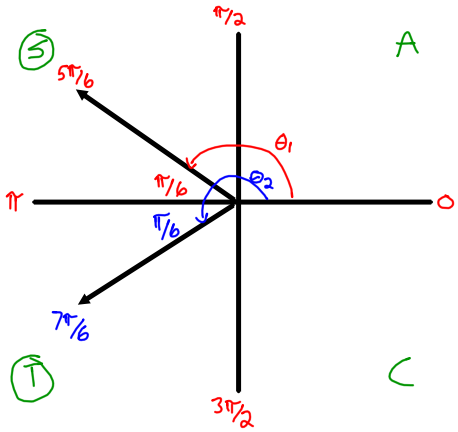
Example 1: Find all solutions for $\cos\theta = -\frac{\sqrt{3}}{2}$ in the interval $0 \le x \le 2\pi$

$-\frac{\sqrt{3}}{2}$ → Graph → NO → Yes



$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Put reference angle in to quadrants 2 and 3 to make the ratio negative

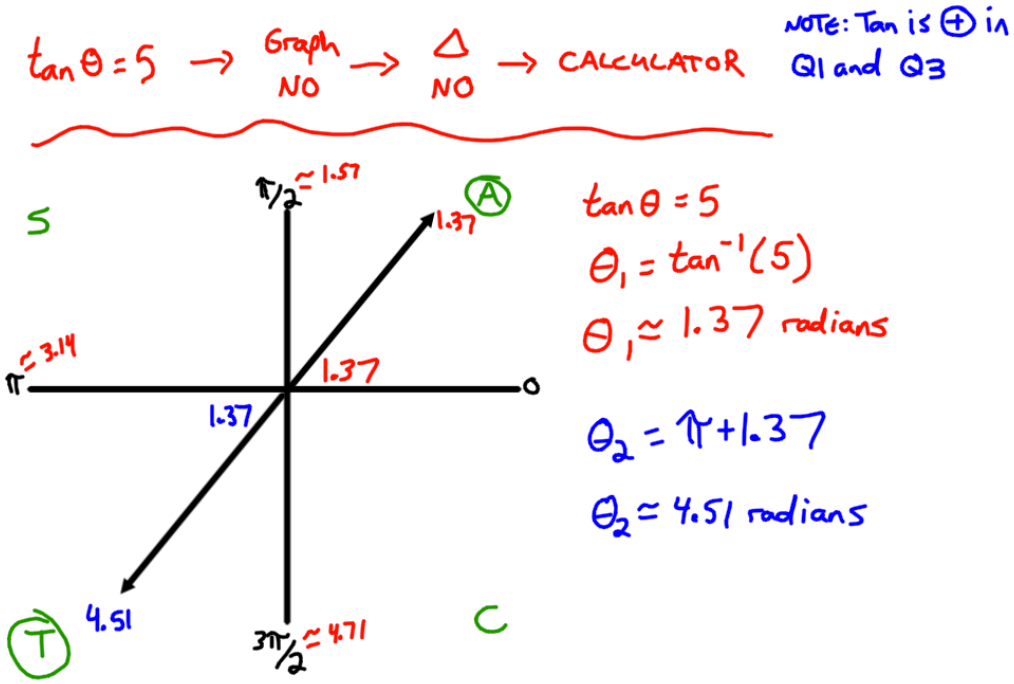


$\theta_1 = \pi - \pi/6$
$= 5\pi/6$

$\theta_2 = \pi + \pi/6$
$= 7\pi/6$

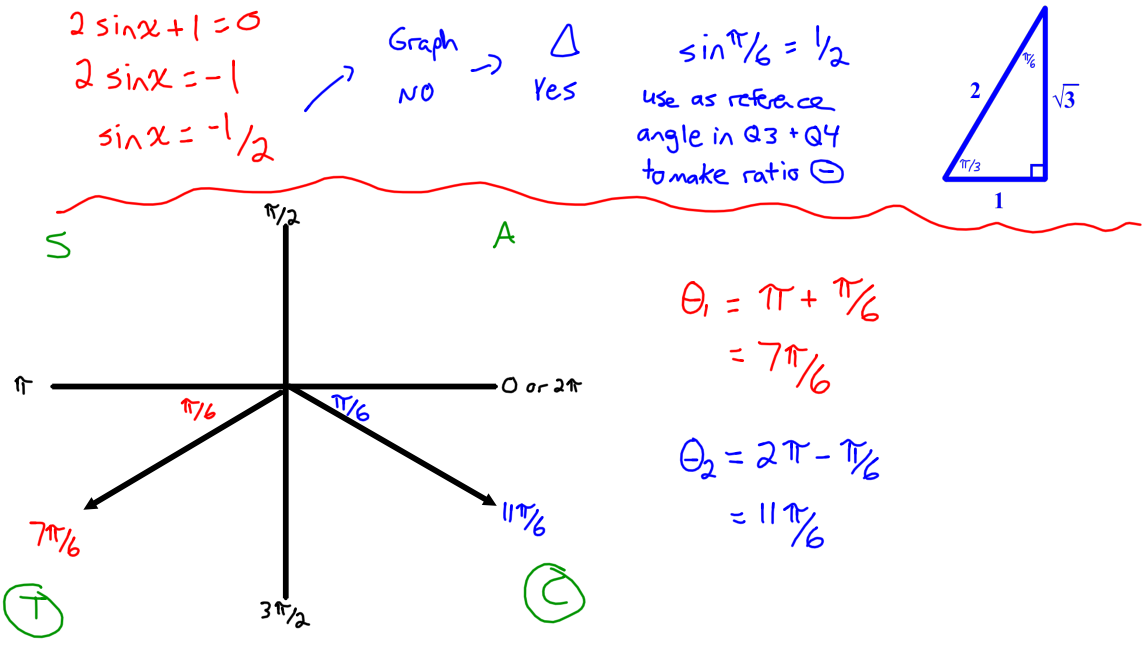
$\cos 5\pi/6 = \cos 7\pi/6 = -\frac{\sqrt{3}}{2}$

Example 2: Find all solutions for $\tan\theta = 5$ in the interval $0 \le x \le 2\pi$



$$\tan 1.37 = \tan 4.51 = 5$$

Example 3: Find all solutions for $2\sin x + 1 = 0$ in the interval $0 \le x \le 2\pi$



$$\sin\frac{7\pi}{6} = \sin\frac{11\pi}{6} = -\frac{1}{2}$$

Example 4: Solve $3(\tan x + 1) = 2$, where $0 \leq x \leq 2\pi$

$3(\tan x + 1) = 2$

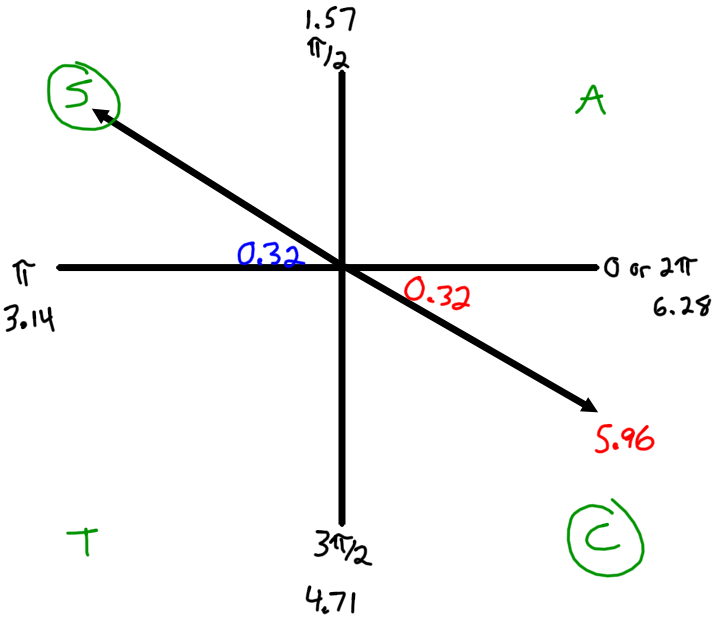
$\tan x + 1 = \frac{2}{3}$

$\tan x = \frac{2}{3} - 1$

$\tan x = -\frac{1}{3}$

Graph → NO → Δ → NO → CALCULATOR

NOTE: tan is ⊖ in Q2 + Q4



$\tan x = -\frac{1}{3}$

$x_1 = \tan^{-1}\left(-\frac{1}{3}\right)$

$x_1 = -0.32175$

$x_1 = 2\pi - 0.32175$

$x_1 \simeq 5.96$ radians

$x_2 = \pi - 0.32$

$x_2 \simeq 2.82$ radians

$\tan 5.96 = \tan 2.82 = -\frac{1}{3}$