L8 – 5.4 Applications of Trigonometric Equations
MHF4U

Part 1: Application Questions

Example 1: Today, the high tide in Matthews Cove, New Brunswick, is at midnight. The water level at high tide is 7.5 m. The depth, *d* meters, of the water in the cove at time *t* hours is modelled by the equation

$$d(t) = 3.5\cos\left(\frac{\pi}{6}t\right) + 4$$

Jenny is planning a day trip to the cove tomorrow, but the water needs to be at least 2 m deep for her to maneuver her sailboat safely. Determine the best time when it will be safe for her to sail into Matthews Cove?

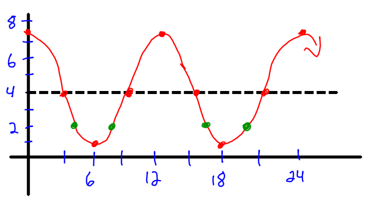
$d(t) = 3.5\cos\left(\frac{\pi}{6}t\right) + 4$

amp = 3.5

period = $\frac{2\pi}{\left(\frac{\pi}{6}\right)}$ = 12 hours

max = 4 + 3.5 = 7.5 m

min = 4 − 3.5 = 0.5 m



set d(t) = 2 and solve for t

$2 = 3.5\cos\left(\frac{\pi}{6}t\right) + 4$

$\frac{-2}{3.5} = \cos\left(\frac{\pi}{6}t\right)$

let $x = \frac{\pi}{6}t$

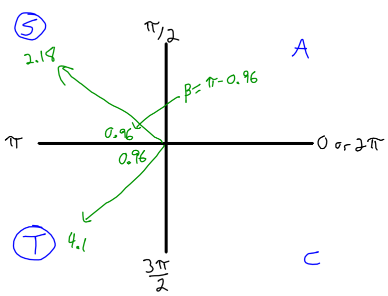
$\cos x = \frac{-2}{3.5}$ ← CALCULATOR solutions in Q2 + Q3

$x_1 = \cos^{-1}\left(\frac{-2}{3.5}\right)$

$\boxed{x_1 = 2.18 \text{ radians}}$

$x_2 = \pi + 0.96$

$\boxed{x_2 = 4.1 \text{ radians}}$



$x = \frac{\pi}{6}t$

$2.18 = \frac{\pi}{6}t$

$\frac{6(2.18)}{\pi} = t$

$t_1 = 4.16$ hours

$t_3 = t_1 + 12$

$t_3 = 16.16$ hours

$4.1 = \frac{\pi}{6}t$

$\frac{6(4.1)}{\pi} = t$

$t_2 = 7.83$ hours

$t_4 = t_2 + 12$

$t_4 = 19.83$ hours

* Add period = 12 to find other solutions

Longest interval above 2 m is between 7.83 hours and 16.16 hours.

∴ she can safely sail between 7:50 am and 4:10 pm

Example 2: A city's daily temperature, in degrees Celsius, can be modelled by the function

$$t(d) = -28\cos\left(\frac{2\pi}{365}d\right) + 10$$

where d is the day of the year and 1 = January 1. On days where the temperature is approximately 32°C or above, the air conditioners at city hall are turned on. During what days of the year are the air conditioners running at city hall?

$$t(d) = -28\cos\left(\frac{2\pi}{365}d\right) + 10$$

$$32 = -28\cos\left(\frac{2\pi}{365}d\right) + 10$$

$$-\frac{22}{28} = \cos\left(\frac{2\pi}{365}d\right) \qquad * \text{ let } x = \frac{2\pi}{365}d$$

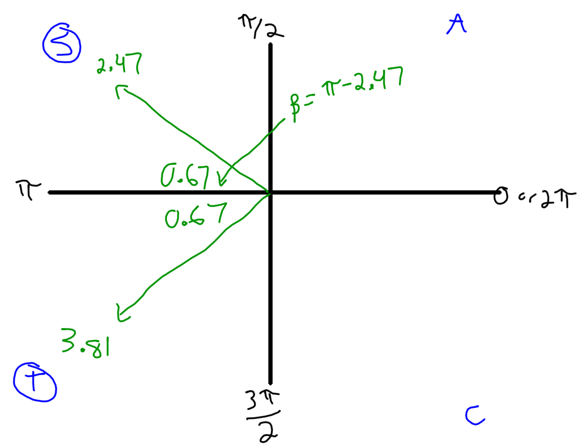
$$\cos x = -\frac{22}{28} \rightarrow$$ CALCULATOR solutions in Q2 + Q3

$$x_1 = \cos^{-1}\left(-\frac{22}{28}\right)$$

$$\boxed{x_1 = 2.47 \text{ radians}}$$

$$x_2 = \pi + 0.67$$

$$\boxed{x_2 = 3.81 \text{ radians}}$$



$$x = \frac{2\pi}{365}d$$

$$2.47 = \frac{2\pi}{365}d \qquad\qquad 3.81 = \frac{2\pi}{365}d$$

$$\frac{365(2.47)}{2\pi} = d \qquad\qquad \frac{365(3.81)}{2\pi} = d$$

$$d_1 \doteq 143 \qquad\qquad d_2 \doteq 221$$

∴ They will use the air conditioning between day 143 to day 221.

Example 3: A Ferris wheel with a 20 meter diameter turns once every minute. Riders must climb up 1 meter to get on the ride.

a) Write a cosine equation to model the height of the rider, h meters, t seconds after the ride has begun. Assume they start at the min height.

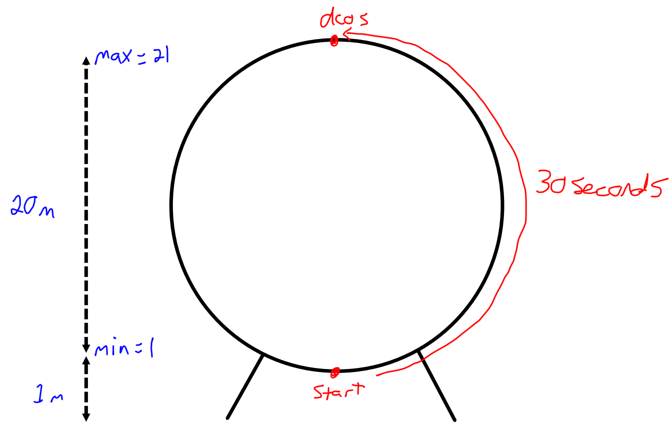
$$a = \frac{max - min}{2} = \frac{21 - 1}{2} = 10$$

$$k = \frac{2\pi}{period} = \frac{2\pi}{60} = \frac{\pi}{30}$$

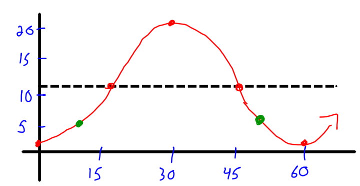
$$c = max - a = 21 - 10 = 11$$

$$d_{cos} = 30$$

$$h(t) = 10\cos\left[\frac{\pi}{30}(t - 30)\right] + 11$$



b) What will be the first 2 times that the rider is at a height of 5 meters?



Set $h(t) = 5$ and solve for t

$$5 = 10\cos\left[\frac{\pi}{30}(t-30)\right] + 11$$

$$\frac{-6}{10} = \cos\left[\frac{\pi}{30}(t-30)\right]$$

Let $x = \frac{\pi}{30}(t-30)$

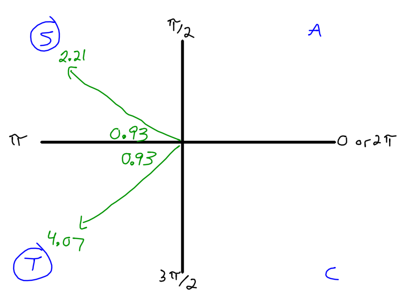
$\cos x = \frac{-6}{10}$ → CALCULATOR solutions in Q2 + Q3

$$x_1 = \cos^{-1}\left(\frac{-6}{10}\right)$$

$$\boxed{x_1 = 2.21 \text{ radians}}$$

$$x_2 = \pi + 0.93$$

$$\boxed{x_2 = 4.07 \text{ radians}}$$



$$x = \frac{\pi}{30}(t-30)$$

$$2.21 = \frac{\pi}{30}(t-30)$$

$$\frac{30(2.21)}{\pi} + 30 = t$$

$$\boxed{t_1 = 51.1 \text{ seconds}}$$

$$4.07 = \frac{\pi}{30}(t-30)$$

$$\frac{30(4.07)}{\pi} + 30 = t$$

$$t_2 = 68.87 \text{ seconds}$$

subtract period = 60 to find an earlier solution

$$t_3 = t_2 - 60$$

$$\boxed{t_3 = 8.87 \text{ seconds}}$$

∴ The first two times the rider is at a height of 5m are 8.87 seconds and 51.1 seconds.