

<p style="text-align: center;"><b>Unit 3: Trigonometry</b> <b>3.3 Equivalent Trigonometric Expressions</b></p>
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**Investigation**

1. Sketch  $\triangle ABC$  such that  $\angle B$  is a right angle. Mark  $\angle B$  as  $\frac{\pi}{2}$ .
2. Mark  $\angle A$  as  $x$ . Use the sum of the interior angles of a triangle to derive an expression for the measure of  $\angle C$  in terms of  $x$ .
3. Determine an expression for  $\sin(x)$  in terms of the sides of the triangle,  $a$ ,  $b$  and  $c$ .
4. Determine expressions for the sine, cosine, and tangent of  $\angle C$  in terms of the sides of the triangle. Which of these is equal to  $\sin(x)$ ?
5. You have shown that  $\sin x$  and  $\cos\left(\frac{\pi}{2} - x\right)$  are equivalent trigonometric expressions. Does the relationship between these equivalent expressions depend on the value of  $x$ ? Justify your answer.
6. What is the sum of  $x$  and  $\left(\frac{\pi}{2} - x\right)$ ? What name is given to a pair of angles such as  $x$  and  $\left(\frac{\pi}{2} - x\right)$ ?
7. Use  $\triangle ABC$  to determine equivalent trigonometric expressions for  $\cos(x)$  and  $\tan(x)$ . Use  $\triangle ABC$  to determine equivalent trigonometric expressions for  $\csc(x)$ ,  $\sec(x)$ , and  $\cot(x)$ .

8. Summarize the six relations among trigonometric functions in the table.

Co-Function Identities	
$\sin\left(\frac{\pi}{2} - x\right) =$	$\csc\left(\frac{\pi}{2} - x\right) =$
$\cos\left(\frac{\pi}{2} - x\right) =$	$\sec\left(\frac{\pi}{2} - x\right) =$
$\tan\left(\frac{\pi}{2} - x\right) =$	$\cot\left(\frac{\pi}{2} - x\right) =$

Other Co-Function Identities	
$\sin\left(\frac{\pi}{2} + x\right) =$	$\csc\left(\frac{\pi}{2} + x\right) =$
$\cos\left(\frac{\pi}{2} + x\right) =$	$\sec\left(\frac{\pi}{2} + x\right) =$
$\tan\left(\frac{\pi}{2} + x\right) =$	$\cot\left(\frac{\pi}{2} + x\right) =$

An identity involving trigonometric expressions is called a **trigonometric identity**. The trigonometric identities in the table in step 8 are known as the **co-function or co-related identities**.

Summary of the other identities are below.

More Co-Function Identities	
$\sin\left(\frac{3\pi}{2} - x\right) =$	$\csc\left(\frac{3\pi}{2} - x\right) =$
$\cos\left(\frac{3\pi}{2} - x\right) =$	$\sec\left(\frac{3\pi}{2} - x\right) =$
$\tan\left(\frac{3\pi}{2} - x\right) =$	$\cot\left(\frac{3\pi}{2} - x\right) =$

More Co-Function Identities	
$\sin\left(\frac{3\pi}{2} + x\right) =$	$\csc\left(\frac{3\pi}{2} + x\right) =$
$\cos\left(\frac{3\pi}{2} + x\right) =$	$\sec\left(\frac{3\pi}{2} + x\right) =$
$\tan\left(\frac{3\pi}{2} + x\right) =$	$\cot\left(\frac{3\pi}{2} + x\right) =$

Recall- Even and odd functions identity:

Reflection Identities	
$\sin(-x) = -\sin(x)$	$\csc(-x) = -\csc(x)$
$\cos(-x) = \cos(x)$	$\sec(-x) = \sec(x)$
$\tan(-x) = -\tan(x)$	$\cot(-x) = -\cot(x)$

### Examples

- State an equivalent trigonometric expression for  $\cos\left(\frac{\pi}{12}\right)$  using co-function identity.

2. Given that  $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$ , use an equivalent trigonometric expression to show that

$$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}.$$

3. Simplify the following expressions

a)  $\frac{\cos\left(\frac{\pi}{2} - 2x\right)}{\tan(\pi - 2x)} + \frac{\csc\left(\frac{\pi}{2} - 2x\right)}{\sec(2x + 2\pi)}$

b)  $\frac{\sin(x - \pi)}{\cos(x + \pi)} - \frac{\csc(-x - \pi)}{\sec\left(\frac{\pi}{2} - x\right)}$

4. Determine a value of  $b$  such that  $\csc\left(2b + \frac{\pi}{8}\right) = \sec\left(3b - \frac{\pi}{8}\right)$ .

### 3.3 Practice

1. Write each of the following in terms of the co-function identity:

(a)  $\sin\left(\frac{5\pi}{8}\right)$

(b)  $\csc\left(\frac{5\pi}{18}\right)$

(c)  $\cos\left(\frac{\pi}{9}\right)$

(d)  $\cos\left(\frac{7\pi}{36}\right)$

2. Find the exact value of special angles using equivalent trigonometric expressions.

a)  $\cot\left(\frac{-5\pi}{3}\right)$

b)  $\csc\left(\frac{4\pi}{3}\right)$

c)  $\sin\left(\frac{10\pi}{3}\right)$

d)  $\sec\left(\frac{2\pi}{3}\right)$

e)  $\sin\left(\frac{-7\pi}{6}\right)$

f)  $\cot\left(\frac{\pi}{2} - x\right)\cot(x)$

3. Express each of the following as a single trigonometric ratio, and then evaluate the ratio.

a)  $\sec^2\left(\frac{7\pi}{6}\right)\cot^2\left(\frac{7\pi}{6}\right)$

b)  $\csc^2\left(\frac{3\pi}{4}\right)\tan^2\left(\frac{3\pi}{4}\right)$

c)  $\cos\left(\frac{4\pi}{3}\right) - \csc\left(\frac{5\pi}{6}\right)$

4. Determine a value for  $c$  such that  $\tan\left(2c + \frac{\pi}{3}\right) - \cot\left(3c - \frac{\pi}{4}\right) = 0$ .

5. Determine a value for  $\theta$  that satisfies  $\sin\left(2\theta - \frac{\pi}{3}\right) = \cos\left(\frac{5\pi}{4} - 3\theta\right)$ .

6. Given  $\sin\left(x + \frac{3\pi}{2}\right) = \frac{2}{3}$  where  $\pi < x < \frac{3\pi}{2}$ , determine the exact value of  $\sin(x)$ .

7. Simplify.

(a)  $\cos(-\theta) + \cos(180^\circ - \theta) - \cos(180^\circ + \theta)$

(b)  $\frac{\cos(\pi + x) + \sin(\pi - x)}{\sin(2\pi - x) - \cos(\pi - x)}$

(c)  $\sin(\pi + x)\cos\left(\frac{\pi}{2} - x\right) - \cos(\pi + x)\sin\left(x + \frac{3\pi}{2}\right)$

(d)  $\frac{\cos(-x)\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{3\pi}{2} - x\right)\csc(x)}{\sin\left(\frac{\pi}{2} + x\right) - \cos\left(x - \frac{\pi}{2}\right) + \sin\left(\frac{3\pi}{2} - x\right)}$

8. Fill in the blanks with the appropriate trig function name:

(a)  $\sin\frac{2\pi}{3} = \underline{\hspace{1cm}}\left(\frac{-\pi}{6}\right)$

(b)  $\underline{\hspace{1cm}}\frac{11\pi}{60} = \sin\frac{19\pi}{60}$

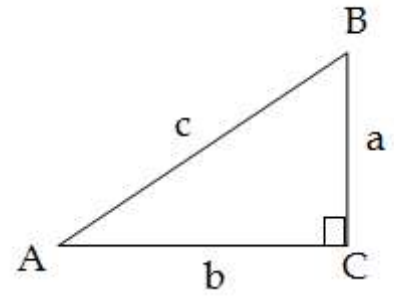
$$(c) \cos \frac{7\pi}{18} = \frac{1}{\frac{\pi}{9}}$$

9. For right triangle ABC:

(a) If  $\sin A = \frac{\sqrt{3}}{3}$ , what is the value of  $\cos B$ ?

(b) If  $\cos A = 0.109$ , what is  $\sin\left(\frac{\pi}{2} - A\right)$ ?

(c) If  $\cos \frac{11\pi}{180} = 0.9816$ , what is  $\sin \frac{79\pi}{180}$ ?



### Warm-up

1. Find without using calculators, the value of

$$\tan\left(\frac{5\pi}{4}\right) - \tan\left(\frac{\pi}{9}\right) \cot\left(\frac{\pi}{9}\right) - \sec\left(\frac{\pi}{6}\right) \csc\left(\frac{\pi}{3}\right) - \sin\left(\frac{11\pi}{6}\right) + \sin\left(\frac{\pi}{2}\right)$$

2. Given  $A = \frac{\pi}{6}$ ,  $B = \frac{\pi}{4}$ ,  $C = \frac{\pi}{3}$ , **find the value of**  $\sin^2(A) - \cos^2(C) + 2\tan(B) - \sec^2(B)$ .