W6 – 2.5 – Solving Inequalities
MHF4U

1) Solve each linear inequality

a) $x + 3 \leq 5$

$x \leq 2$

b) $7x < 4 + 3x$

$4x < 4$

$x < 1$

2) Solve each inequality by graphing

a) $(x + 3)(x - 2) > 0$

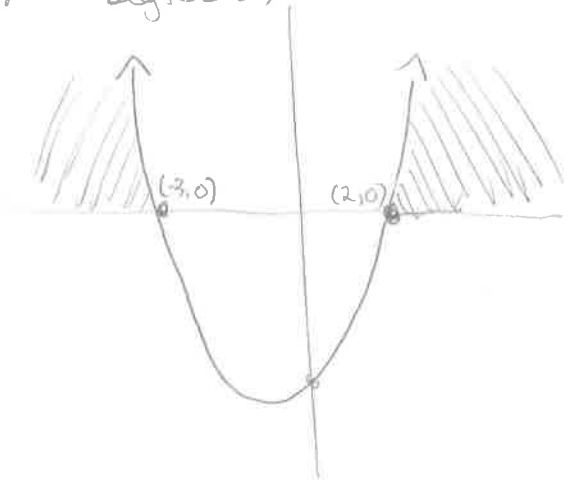
Factor Table

	$-\infty$		-3		2		∞
		-4		0		3	
$x+3$		$-$		$+$		$+$	
$x-2$		$-$		$-$		$+$	
overall		$+$		$-$		$+$	

Solution: $x < -3$ or $x > 2$

$x \in (-\infty, -3) \cup (2, \infty)$

Graph: Degree 2; Positive L.C.



b) $(x + 2)(3 - x)(x + 1) < 0$

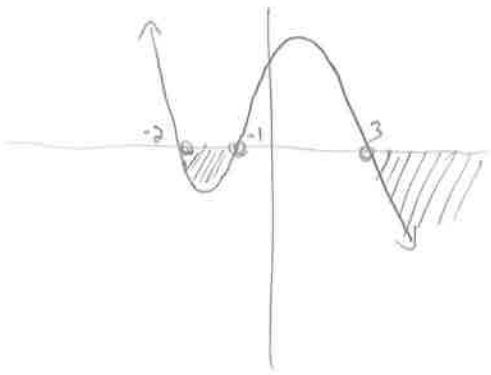
Factor Table:

	$-\infty$		-2		-1		3		∞
		-3		-1.5		0		4	
$x+2$		$-$		$+$		$+$		$+$	
$3-x$		$+$		$+$		$+$		$-$	
$x+1$		$-$		$-$		$+$		$+$	
overall		$+$		$-$		$+$		$-$	

Solution: $-2 < x < -1$ or $x > 3$

$x \in (-2, -1) \cup (3, \infty)$

Graph: Degree 3; Negative L.C.



3) Solve each of the following polynomial inequalities

a) $x^2 - 7x + 10 \geq 0$

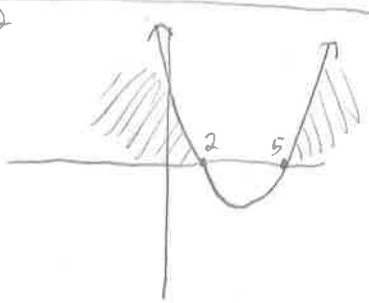
$(x-2)(x-5) \geq 0$

	$-\infty$	2	5	∞
	0	3	6	
$x-2$	−	+	+	
$x-5$	−	−	+	
overall	(+)	−	(+)	

Solution: $x \leq 2$ or $x \geq 5$

$x \in (-\infty, 2] \cup [5, \infty)$

Degree 2 + L.C.



b) $x^3 + 6x^2 - 16x > 0$

$x(x^2 + 6x - 16) > 0$

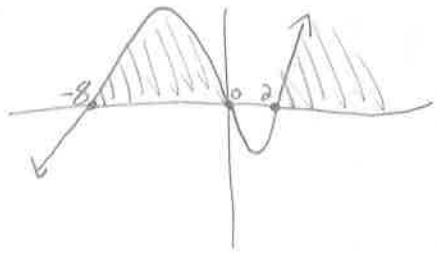
$x(x+8)(x-2) > 0$

	$-\infty$	-8	-10	-1	0	1	2	3	∞
x			−	−		+		+	
$x+8$			−	+		+		+	
$x-2$			−	−		−		+	
overall			−	(+)		−		(+)	

Solution: $-8 < x < 0$ or $x > 2$

$x \in (-8, 0) \cup (2, \infty)$

Degree 3 + L.C.



c) $-x^2 + 36 \geq 0$

$-1(x^2 - 36) \geq 0$

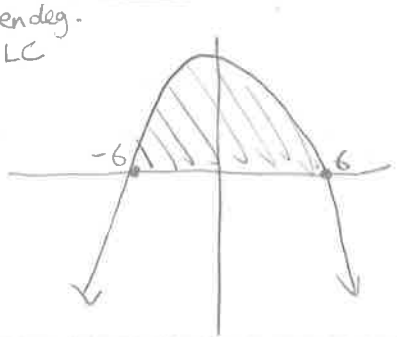
$-1(x-6)(x+6) \geq 0$

	$-\infty$	-7	-6	0	6	7	∞
-1		−		−		−	
$x-6$		−		−		+	
$x+6$		−		+		+	
overall		−		(+)		−	

Solution: $-6 \leq x \leq 6$

$x \in [-6, 6]$

Even deg. - LC



d) $x^4 - 26x^2 + 25 > 0$

$(x^2 - 25)(x^2 - 1) > 0$

$(x-5)(x+5)(x-1)(x+1) > 0$

	$-\infty$	-5	-1	1	5	∞
	-6	-2	0	2	6	
$x-5$	−	−	−	−	+	
$x+5$	−	+	+	+	+	
$x-1$	−	−	−	+	+	
$x+1$	−	−	+	+	+	
overall	(+)	−	(+)	−	(+)	

Solution: $x < -5$ or $-1 < x < 1$ or $x > 5$

$x \in (-\infty, -5) \cup (-1, 1) \cup (5, \infty)$

e) $x^3 - 3x^2 \geq 25x - 75$

$x^3 - 3x^2 - 25x + 75 \geq 0$

$[x^2(x-3) - 25(x-3)] \geq 0$

$(x-3)(x^2-25) \geq 0$

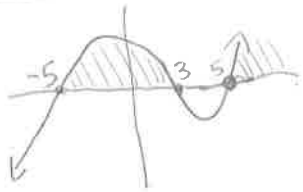
$(x-3)(x-5)(x+5) \geq 0$

	$-\infty$	-5		3		5	∞
	-6		0	4		6	
$x-3$	$-$		$-$	$+$		$+$	
$x-5$	$-$		$-$	$-$		$+$	
$x+5$	$-$		$+$	$+$		$+$	
overall	$-$		$(+)$	$-$		$(+)$	

Solution: $-5 \leq x \leq 3$ or $x \geq 5$

$x \in [-5, 3] \cup [5, \infty)$

odd deg. + LC



f) $-x^3 + 28x + 48 \geq 0$

$f(-2) = 0$; $\therefore$ $x+2$ is a factor

-2	-1	0	28	48
	$\downarrow$	2	-4	-48
	-1	2	24	0
	x^2	x	#	R

$(x+2)(-x^2+2x+24) \geq 0$

$-1(x+2)(x^2-2x-24) \geq 0$

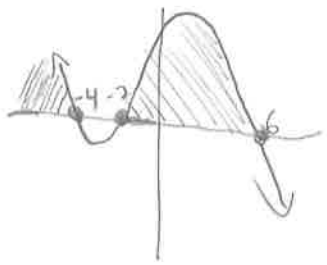
$-1(x+2)(x-6)(x+4) \geq 0$

	$-\infty$	-4		-2		6	∞
	-5		-3		0	7	
-1	$-$		$-$		$-$	$-$	
$x+2$	$-$		$-$		$+$	$+$	
$x-6$	$-$		$-$		$-$	$+$	
$x+4$	$-$		$+$		$+$	$+$	
overall	$(+)$		$-$		$(+)$	$-$	

Solution: $x \leq -4$ or $-2 \leq x \leq 6$

$x \in (-\infty, -4] \cup [-2, 6]$

odd Deg - LC



g) $x^3 - 2x^2 - 5x + 6 < 0$

$f(1) = 0$; $\therefore$ $x-1$ is a factor

$(x-1)(x^2-x-6) < 0$

$(x-1)(x-3)(x+2) < 0$

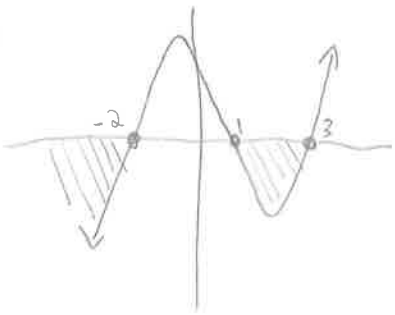
1	1	-2	-5	6
		1	-1	-6
x	1	-1	-6	0
	x^2	x	#	R

	$-\infty$	-3		-2		1		3	∞
		-3		0		2		4	
$x-1$		$-$		$-$		$+$		$+$	
$x-3$		$-$		$-$		$-$		$+$	
$x+2$		$-$		$+$		$+$		$+$	
overall		$(-)$		$+$		$(-)$		$+$	

Solution: $x < -2$ or $1 < x < 3$

$x \in (-\infty, -2) \cup (1, 3)$

odd deg + LC



h) $5x^3 - 12x^2 - 11x + 6 \leq 0$

$f(-1) = 0$; $x+1$ is a factor

-1	5	-12	-11	6
	$\downarrow$	-5	17	-6
	5	-17	6	0
	x^2	x	#	R

$(x+1)(5x^2-17x+6) \leq 0$

$(x+1)(5x-2)(x-3) \leq 0$

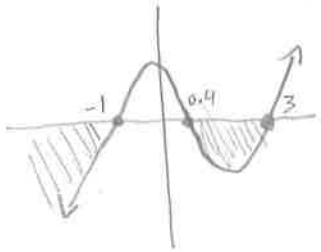
P: 30; $\frac{-2}{5}$, $\frac{-15}{5} = \frac{-3}{1}$, -17; S

	$-\infty$	-1		$2/5$		3	∞
	-2		0		1	4	
$x+1$	$-$		$+$		$+$	$+$	
$5x-2$	$-$		$-$		$+$	$+$	
$x-3$	$-$		$-$		$-$	$+$	
overall	$(-)$		$+$		$(-)$	$+$	

Solution: $x \leq -1$ or $2/5 \leq x \leq 3$

$x \in (-\infty, -1] \cup [2/5, 3]$

Odd deg. + LC

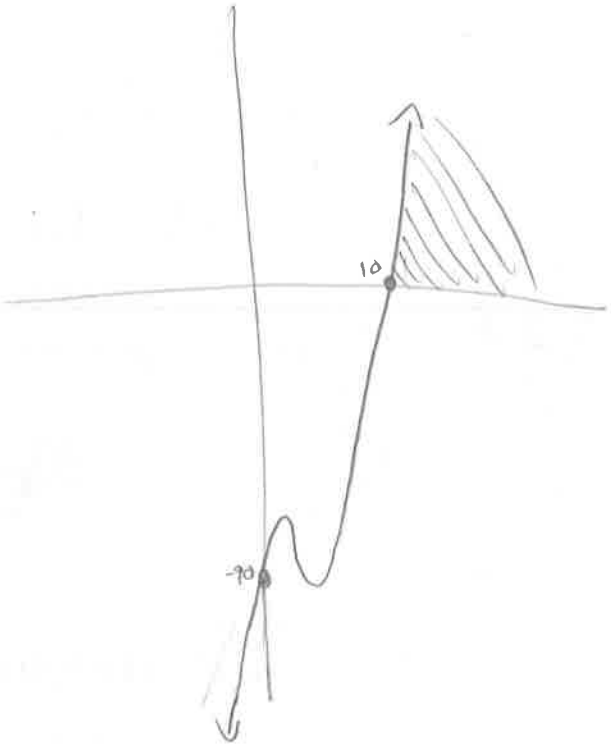


4) The price, p, in dollars, of a stock t years after 1999 can be modelled by the function $p(t) = 0.5t^3 - 5.5t^2 + 14t$. When will the stock be more than \$90? You may use technology to help you determine the solution.

$0.5t^3 - 5.5t^2 + 14t > 90$

$0.5t^3 - 5.5t^2 + 14t - 90 > 0$

Using Desmos:



Solution: $t > 10$

$t \in (10, \infty)$

∴ The stock will be more than \$90 after 10 years.

ANSWER KEY

1)a) $x \leq 2$ **b)** $x < 1$
2)a) $x < -3$ or $x > 2$ **b)** $-2 < x < -1$ or $x > 3$
3)a) $x \leq 2$ or $x \geq 5$ **b)** $-8 < x < 0$ or $x > 2$ **c)** $-6 \leq x \leq 6$ **d)** $x < -5$ or $-1 < x < 1$ or $x > 5$
e) $-5 \leq x \leq 3$ or $x \geq 5$ **f)** $x \leq -4$ or $-2 \leq x \leq 6$ **g)** $x < -2$ or $1 < x < 3$
h) $x \leq -1$ or $\frac{2}{5} < x < 3$
4) after 10 years (2009)