2.3 Rational Functions of the Form $f(x)=\frac{ax+b}{cx+d}$

1. Consider the following:

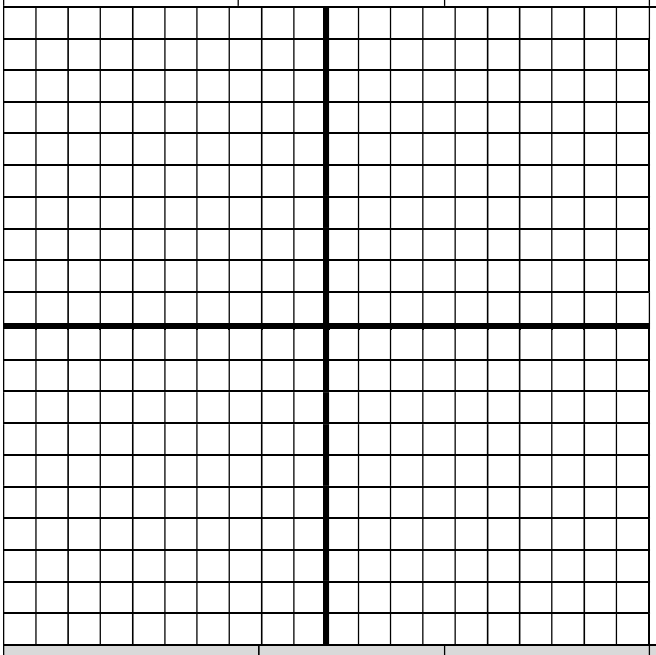
Function	H.A $y=$	V.A $x=$
$f(x)=\frac{1}{x}$	as $x \to -\infty$, $f(x) \to$ --------- as $x \to \infty$, $f(x) \to$ ---------	as $x \to$ ---------- , $f(x) \to$ ---------- as $x \to$ --------- , $f(x) \to$ ---------
Function	**H.A $y=$**	**V.A $x=$**
$f(x)=\frac{1}{3x+2}$	as $x \to -\infty$, $f(x) \to$ --------- as $x \to \infty$, $f(x) \to$ ---------	as $x \to$ --------- $f(x) \to$ --------- as $x \to$ ---------, $f(x) \to$ ---------
Function	**H.A $y=$**	**V.A $x=$**
$f(x)=\frac{1}{x-2}$	as $x \to -\infty$, $f(x) \to$ --------- as $x \to \infty$, $f(x) \to$ ---------	as $x \to$ --------- $f(x) \to$ --------- as $x \to$ ---------, $f(x) \to$ ---------

2. What happens if we change the numerator so that the number is not a constant?

Function	H.A $y=$	V.A $x=$
$f(x)=\frac{x}{x+5}$	as $x \to -\infty$, $f(x) \to$ --------- as $x \to \infty$, $f(x) \to$ ---------	as $x \to$ ---------- , $f(x) \to$ -------- as $x \to$ --------- , $f(x) \to$ -------
Function	**H.A $y=$**	**V.A $x=$**
$f(x)=\frac{x+2}{4x-5}$	as $x \to -\infty$, $f(x) \to$ --------- as $x \to \infty$, $f(x) \to$ ---------	as $x \to$ ---------- , $f(x) \to$ -------- as $x \to$ --------- , $f(x) \to$ -------
Function	**H.A $y=$**	**V.A $x=$**
$f(x)=\frac{3-2x}{4x-1}$	as $x \to -\infty$, $f(x) \to$ --------- as $x \to \infty$, $f(x) \to$ ---------	as $x \to$ ---------, $f(x) \to$ --------- as $x \to$ ---------, $f(x) \to$ --------

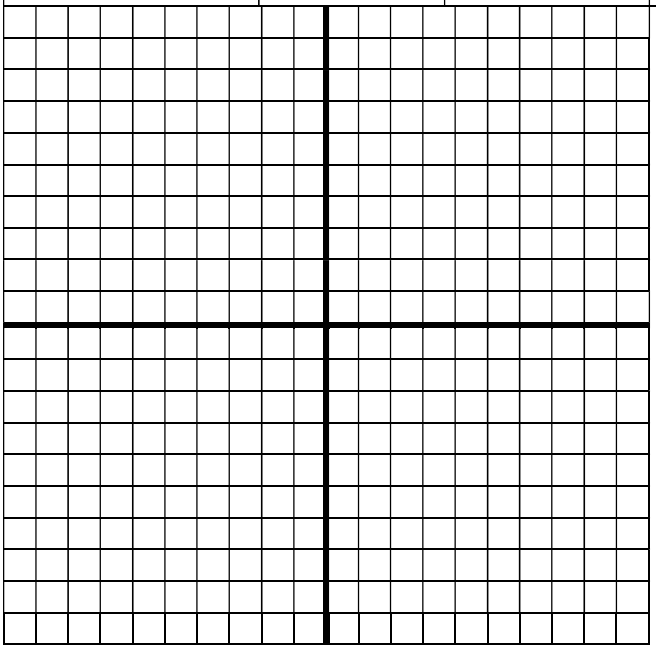
Example#1: Fill in the chart for each of the following, and then sketch the graph.

Function	x-intercept	y-intercept	Equation of the V.A.	Equation of the H.A.
$f(x)=\dfrac{x}{x-3}$				



As $x\to$	$f(x)\to$
3^+	
3^-	
$+\infty$	
$-\infty$	

Function	x-intercept	y-intercept	Equation of the V.A.	Equation of the H.A.
$f(x)=\dfrac{-4x-3}{2x+1}$				



As $x\to$	$f(x)\to$
$-\frac{1}{2}^+$	
$-\frac{1}{2}^-$	
$+\infty$	
$-\infty$	

Example#2. Determine the value of constants *a* and *b* that guarantee the graph of the function defined by $f(x) = \frac{7 - ax}{bx + 2}$ will have a vertical asymptote of $x = -3$ and a horizontal asymptote of $y = -2$.

Example#3. Find the linear over linear function , $f(x) = \frac{ax + b}{cx + d}$, satisfying the given requirements: f(10) = 20, f(30) = 25, and the graph of f(x) has y = 30 as its horizontal asymptote.

Exit Card!

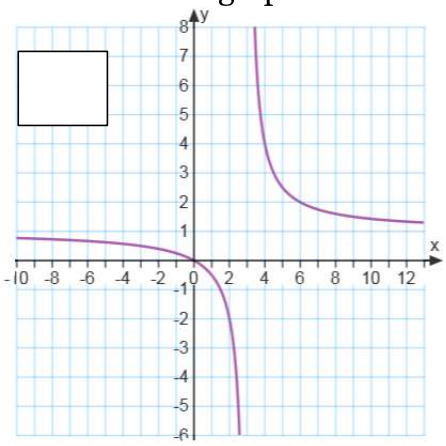
1. State the equations for all asymptotes of $y = \frac{1-2x}{3x-1}$.

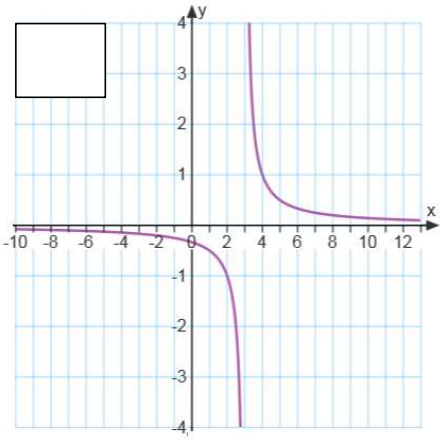
   H.A. _____ , V.A. _____

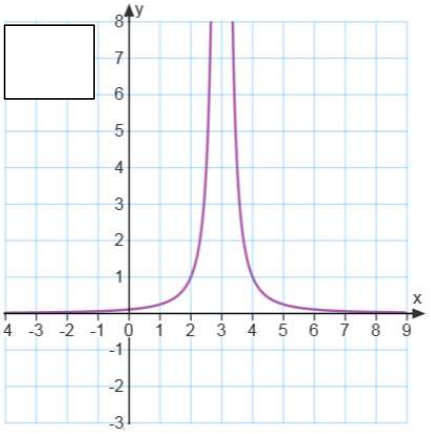
2. The following graphs represent the functions:

   a) $y = \frac{x}{x-3}$ b) $y = \frac{1}{x-3}$ c) $y = \frac{1}{(x-3)^2}$

   Match each graph with the appropriate equation.







3. Find constants **a** and **b** that guarantee that the graph of the function defined by $f(x) = \frac{ax+3}{4-bx}$ will have a vertical asymptote at x = 2 and a horizontal asymptote at y = -2.

2.3 Practice

1. The more you study for a certain exam, the better your performance on it. If you study for 10 hours, your score will be 55%. If you study for 26 hours, your score will be 95%. You can get as close as you want to a perfect score just by studying long enough. Assume your percentage score is a linear-to-linear function of the number of hours that you study. If you want a score of 80%, how long do you need to study?

2. Clyde makes extra money selling tickets in front of the Safeco Field. The amount he charges for a ticket depends on how many he has. If he only has one ticket, he charges $100 for it. If he has 10 tickets, he charges $80 a piece. But if he has a large number of tickets, he will sell them for $50 each. How much will he charge for a ticket if he holds 20 tickets?

3. Sketch he graph of the following functions.
   a. $y = \frac{2x}{x-1}$
   b. $y = \frac{4x+3}{x+2}$
   c. $y = \frac{-1}{x+3} + 2$
   d. $y = \frac{-3x+5}{2x-4}$
   e. $y = \frac{4}{2x+3} - 1$

4. Rosetta is growing a bamboo plant in her apartment. The height of the plant is a linear-over-linear function of time. Thirty days ago, the plant was 14 cm high. Today, the plant is 18 cm high. The plant always increases in height, and will approach (but never exceed) a height of 32 cm. Find a function representing the height of the plant as a function of time.

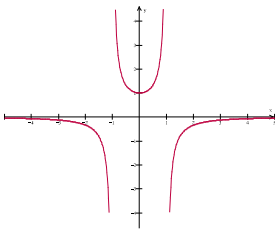
5. Sketch a possible function with the given features:
   i. Vertical asymptotes at $x = -1$ and $x = 3$
   ii. x-intercepts at $x = -2$ and $x = 4$
   iii. y-intercept at $y = -4$
   iv. the function is not defined at $x = 1$
   v. As $x \to 3^-, f(x) \to -\infty$
   vi. As $x \to -\infty, f(x) \to -2$ from above
   vii. As $x \to +\infty, f(x) \to -2$ from above

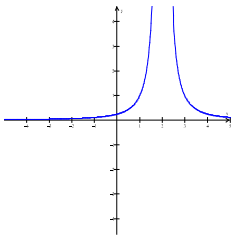
Self Assessment

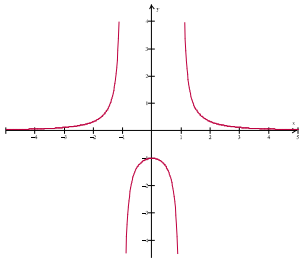
Part A:

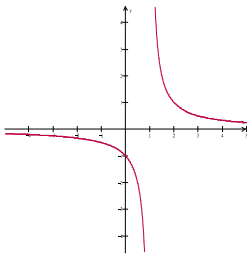
Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided. One mark per question.

1. Which graph represents $f(x)=\frac{1}{(x-2)^2}$?

A) 

B) 

C) 

D) 

2. State the interval(s) for which the function $h(x)=\frac{1}{(x-1)(x-3)}$ is above the x-axis

A) $x<3$ B) $1<x<3$

C) $x>1$ D) $x<1,\ x>3$

3. Which of the following functions does not have a horizontal asymptote?

A) $y=\frac{x^3-3x^2+3x-1}{x^2-5}$ B) $y=\frac{x+1}{x-1}$

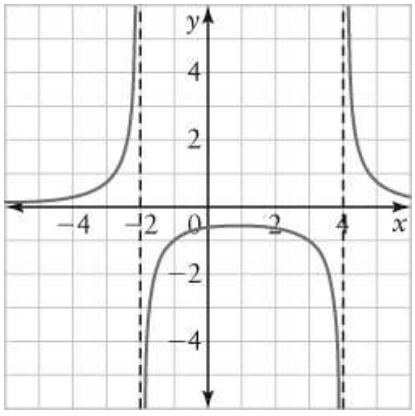
C) $y=\frac{x^2-1}{x^3+8}$ D) $y=\frac{3x^2-5x+2}{2x^2-5}$

4. What is true about the function $f(x)=\frac{2-x}{3x+5}$ as $x\to\pm\infty$?

A) $f(x)\to\frac{2}{3}$ B) $f(x)\to-\frac{1}{3}$

C) $f(x)\to 0$ D) $f(x)\to\frac{1}{3}$

5. Over what interval(s) is the graph of the rational function decreasing?



A) $x\in(-2,4)$
B) $x\in(1,4)\cup(4,\infty)$
C) $x\in(-\infty,-2)\cup(-2,1)$
D) $x\in(-\infty,-2)\cup(4,\infty)$

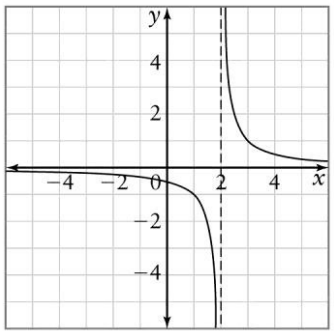
Part B: Full Solutions

1. Copy and complete the table to describe the behaviour of the function $f(x) = -\frac{1}{x+4}$.

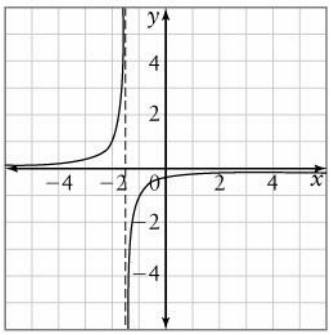
As $x \to$	$f(x) \to$
-4^+	
-4^-	
$+\infty$	
$-\infty$	

2. Determine a possible equation to represent each function shown.

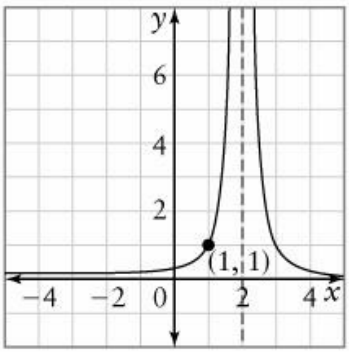
a)



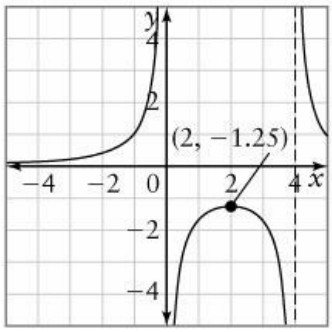
b)



c)



d)



3. Sketch a graph of each function. Label the *y*-intercept. State the domain, the range and the equations of asymptotes.

a) $f(x) = -\frac{x+2}{x-5}$ **b)** $h(x) = \frac{2x}{5-x}$

4. Copy and complete the table to describe the behaviour of the function $f(x) = \frac{1}{(x+2)(x+5)}$.

As $x \to$	$f(x) \to$
-2^+	
-2^-	
-5^+	
-5^-	
$+\infty$	
$-\infty$	

5. Determine the equations for the vertical asymptotes, if they exist, for each function. Then, state the domain.

a) $f(x) = -\dfrac{1}{x^2 - 7x + 6}$ **b)** $f(x) = \dfrac{1}{x^2 + 4x + 6}$

6. For each function,
i) determine the equations for the asymptotes, if they exist
ii) give the domain
iii) determine *y*-intercept
iv) sketch a graph of the function

a) $y = \dfrac{1}{(x-2)(x+4)}$ **b)** $y = \dfrac{-1}{(x+4)^2}$ **c)** $y = \dfrac{1}{x^2 + x + 4}$

7. Determine the **point(s)** of intersection between $f(x) = \dfrac{1}{2x^2 - 16x + 33}$ and its reciprocal, if any.