2.4 Oblique (Slant) Asymptotes

If $f(x)=\frac{P(x)}{Q(x)}$ is a rational function in which the degree of the numerator is one more than the degree of the denominator, we can use the Division Algorithm to express the function in the form

$$f(x)=(mx+b)+\frac{R(x)}{Q(x)}$$

where the degree of R is less than the degree of Q and m ≠ 0. This means that as $x \to \pm\infty$, $\frac{R(x)}{Q(x)} \to 0$, so for large values of |x|, the graph of y = f(x) approaches the graph of the line y = mx + b. In this situation we say that y = mx + b is a **slant asymptote**, or an **oblique asymptote**.

To test the behaviour, we must find $\left[f(x)-(mx+b)\right]$ which is $\frac{remainder}{Q(x)}$.

- Let $x \to \infty$, sub x = 100 into $\frac{remainder}{Q(x)}$. If $\frac{remainder}{Q(x)}$ >0, the approach is from above otherwise the approach is from below.

- Let $x \to -\infty$, sub x = – 100 into $\frac{remainder}{Q(x)}$ .If $\frac{remainder}{Q(x)}$ >0, the approach is from above otherwise the approach is from below.

To conclude, we must write:
as $x \to \infty, \left[f(x)-(mx+b)\right] \to 0$ from above/below or
as $x \to -\infty, \left[f(x)-(mx+b)\right] \to 0$ from above/below

Ex.1 Find the oblique asymptote of the following rational functions.

a. $f(x)=\frac{x^2-4x-5}{x-3}$

$$\begin{array}{r} x-1 \\ x-3\overline{)x^2-4x-5} \\ \underline{-x^2+3x} \\ -x-5 \\ \underline{x-3} \\ -8 \end{array}$$

O.A. $y=x-1$

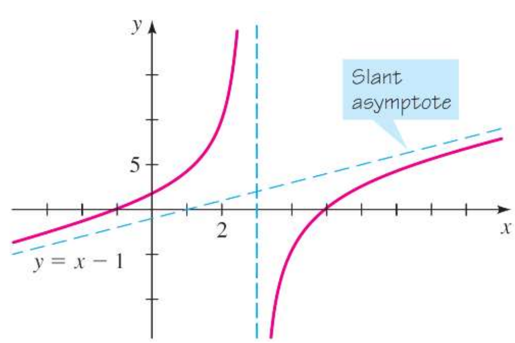
Since $f(x)=x-1+\left(\frac{-8}{x-3}\right)$

As $x \to \infty$, sub x = 100 into $\frac{-8}{x-3}$. Since $\frac{-8}{100-3}<0$ the approach is from below.

As $x \to -\infty$, sub x =- 100 into $\frac{-8}{x-3}$. Since $\frac{-8}{-100-3}>0$ the approach is from above.

Conclusion: As $x \to \infty$, $\left[f(x)-(x-1)\right] \to 0$ (below)

As $x \to -\infty$,$\left[f(x)-(x-1)\right] \to 0$ (above)



b. $g(x)=\dfrac{2x^3-3x^2+2x-7}{x^2-4x+2}$

O.A. ___________

As $x \to \infty$, (above/below)

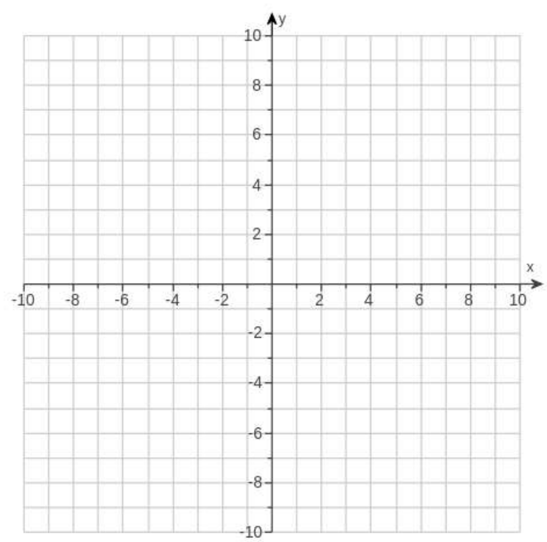
Recall: sub x = 100 into $\dfrac{remainder}{Q(x)}$. If $\dfrac{remainder}{Q(x)}$ >0, the approach is from above otherwise the approach is from below

As $x \to -\infty$, (above/below)

Recall: sub x = – 100 into $\dfrac{remainder}{Q(x)}$ .If $\dfrac{remainder}{Q(x)}$ >0, the approach is from above otherwise the approach is from below.

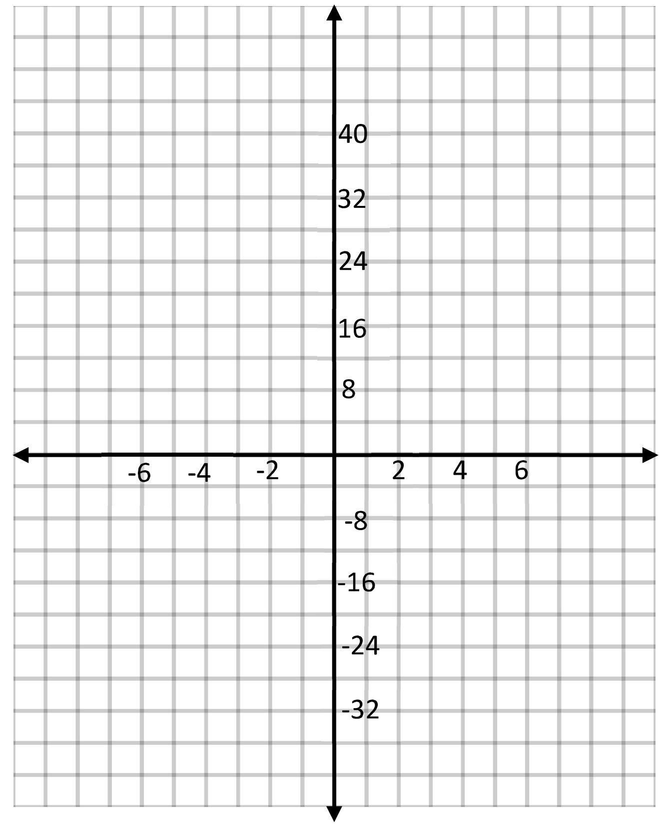
Conclusion:

c. Determine the vertical asymptote, oblique asymptote, x-intercept(s) and y-intercept of $f(x)=\dfrac{x^2-3x-4}{x-1}$. Then graph $f(x)$.



Does f(x) cross the oblique asymptote? How do you know?

d. Graph $f(x) = \frac{2x^3-4x^2-9}{4-x^2}$. Be sure to check for any cross-overs.



2.4 Practice

1) Which of the following has an oblique asymptote?

A) $f(x)=\dfrac{x^2-49}{x+7}$

B) $f(x)=\dfrac{x^2+49}{x+7}$

C) $f(x)=\dfrac{x^2-49}{x-7}$

D) both A) and C)

2) Which of the following functions crosses its horizontal asymptote?

A) $f(x)=\dfrac{x^2-2x-8}{(x+1)(x^2-16)}$

B) $f(x)=\dfrac{-5}{x^2-7x+17}$

C) $f(x)=\dfrac{x^2-49}{x+7}$

D) Horizontal asymptotes can never be crossed

3) Find all asymptotes, then analyze behavior of function near asymptotes:

a) $f(x)=\dfrac{x^2+3x+2}{x-2}$ b) $f(x)=\dfrac{2x^3-4x^2-9}{4-x^2}$ c) $f(x)=\dfrac{-x^2}{x-3}$

4) a. Given $f(x)=\dfrac{3x^3+x^2+9}{x^2+2x+1}$. Determine the equation of the oblique asymptote.

b. Determine the **exact** point(s) where $f(x)$ crosses the oblique asymptote.

5) Find constants a,b,c and n that guarantee that the graph of the function defined by $f(x)=\dfrac{ax^2+bx-3}{x^n+c}$ will have a vertical asymptote of x=−3 and an oblique asymptote of y=2x+1.