W5 – 1.3 – Symmetry in Polynomial Functions
MHF4U

ANSWERS

1) Determine whether each function is even, odd, or neither. Does it have line symmetry about the y-axis, point symmetry about the origin, or neither?

a) $y = x^4 - x^2$

Even

Line symmetry about y-axis

b) $y = -2x^3 + 5x$

Odd

Point symmetry about origin

c) $y = -4x^5 + 2x^2$

Neither

d) $y = x(2x+1)^2(x-4)$

$= x(4x^2+4x+1)(x-4)$
$= (4x^3+4x^2+x)(x-4)$
$= 4x^4 - 16x^3 + 4x^3 - 16x^2 + x^2 - 4x$
$= 4x^4 - 12x^3 - 15x^2 - 4x$

Neither

e) $y = -2x^6 + x^4 + 8$

Even

Line symmetry about the y-axis

2) State whether each function is even or odd. Verify algebraically.

a) $f(x) = x^4 - 13x^2 + 36$ Even

$f(-x) = (-x)^4 - 13(-x)^2 + 36$
$= (-1)^4(x)^4 - 13(-1)^2(x)^2 + 36$
$= x^4 - 13x^2 + 36$

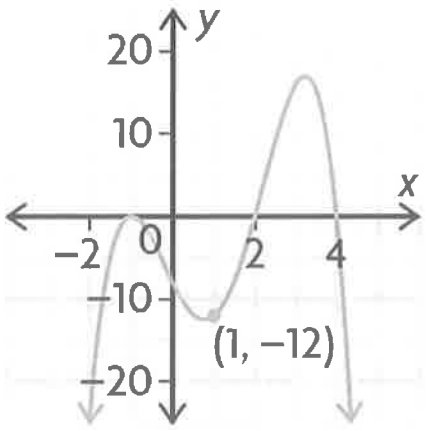
so $f(x) = f(-x)$

b) $g(x) = 6x^5 - 7x^3 - 3x$ ODD

$-g(x) = -1[6x^5 - 7x^3 - 3x]$
$= -6x^5 + 7x^3 + 3x$ & $-g(x) = g(-x)$

$g(-x) = 6(-x)^5 - 7(-x)^3 - 3(-x)$
$= 6(-1)^5(x)^5 - 7(-1)^3(x)^3 - 3(-1)(x)$
$= -6x^5 + 7x^3 + 3x$

3) Use the given graph to state:



a) x-intercepts -1 (order 2), 2, and 4

b) number of turning points 3

c) least possible degree 4

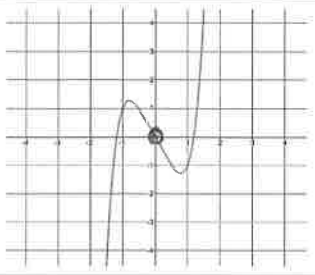
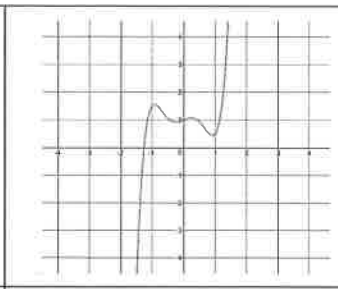
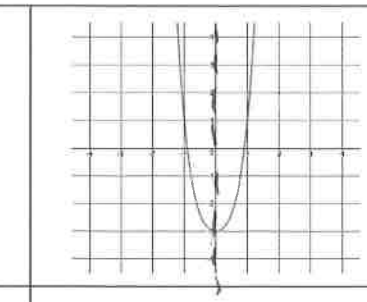
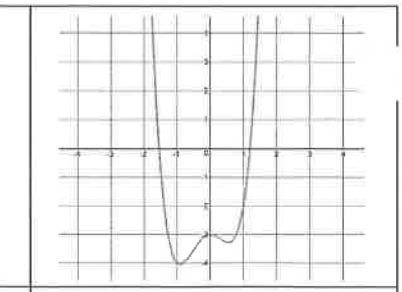
d) any symmetry present; even or odd function?

None, Neither

the intervals where $f(x) < 0$

$F(x) < 0$ when $x \in (-\infty, -1) \cup (-1, 2) \cup (4, \infty)$

4) Label each function as even, odd, or neither

			
ODD	NEITHER	EVEN	NEITHER

ANSWER KEY

1) a) even, line symmetry about y-axis **b)** odd, point symmetry about origin **c)** neither

d) neither **e)** even, line symmetry about y-axis

2) a) even, $f(-x) = f(x)$ **b)** odd, $f(-x) = -f(x)$

3) a) -1 (order 2), 2, and 4 **b)** 3 **c)** 4 **d)** no symmetry, neither **e)** $X \in (-\infty, -1) \cup (-1,2) \cup (4, \infty)$

4) odd, neither, even, neither