L4 – 5.3 Transformations of Trig Functions
MHF4U

Part 1: Transformation Properties

$y = a\sin[k(x-d)] + c$

Desmos Demonstration

a	k	d	c
Vertical stretch or compression by a factor of $\lvert a \rvert$. Vertical reflection if $a < 0$ $\lvert a \rvert = amplitude$	Horizontal stretch or compression by a factor of $\frac{1}{\lvert k \rvert}$. Horizontal reflection if $k < 0$. $\frac{2\pi}{\lvert k \rvert} = period$	Phase shift $d > 0; shift\ right$ $d < 0; shift\ left$	Vertical shift $c > 0; shift\ up$ $c < 0; shift\ down$

Example 1: For the function $y = 3\sin\left[\frac{1}{2}\left(\theta + \frac{\pi}{3}\right)\right] - 1$, state the…

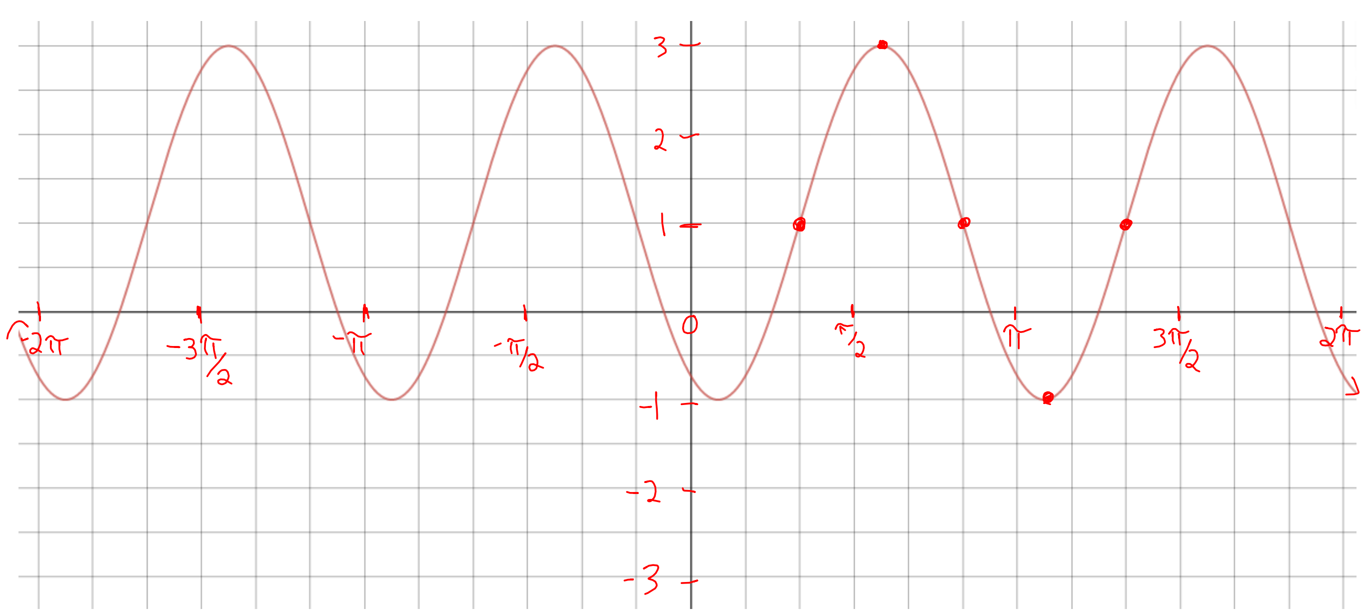
Amplitude: $amp = \lvert a \rvert = 3$	Period: $period = \frac{2\pi}{\lvert k \rvert} = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$
Phase shift: $d = -\frac{\pi}{3}$; shift left $\frac{\pi}{3}$	Vertical shift: $c = -1$; shift down 1
Max: $max = c + amp = -1 + 3 = 2$	Min: $min = c - amp = -1 - 3 = -4$

Part 2: Given Equation → Graph Function

Example 2: Graph $y = 2\sin\left[2\left(x - \frac{\pi}{3}\right)\right] + 1$ using transformations. Then state the amplitude and period of the function.

$y = \sin x$	
$\boldsymbol{x}$	$\boldsymbol{y}$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$y = 2\sin\left[2\left(x - \frac{\pi}{3}\right)\right] + 1$	
$\boldsymbol{\frac{x}{2} + \frac{\pi}{3}}$	$\boldsymbol{2y + 1}$
$\frac{\pi}{3} = \frac{2\pi}{6}$	1
$\frac{7\pi}{12} = \frac{3.5\pi}{6}$	3
$\frac{5\pi}{6}$	1
$\frac{13\pi}{12} = \frac{6.5\pi}{6}$	-1
$\frac{4\pi}{3} = \frac{8\pi}{6}$	1



Amplitude: $\frac{max - min}{2} = \frac{3-(-1)}{2} = 2$

Period: π radians

Part 3: Given the Graph → Write the Equation

$y = a\sin[k(x-d)] + c$

a	k	d	c
Find the amplitude of the function: $a = \dfrac{max - min}{2}$	Find the period (in radians) of the function using a starting point and ending point of a full cycle. $k = \dfrac{2\pi}{period}$	**for sin x:** x-coordinate of a rising mid-line. **for cos x:** x-coordinate of a maximum point. $d_{sin} = d_{cos} - \dfrac{\pi}{2k}$ $d_{cos} = d_{sin} + \dfrac{\pi}{2k}$	Find the vertical shift $c = max - amplitude$ OR $c = \dfrac{max + min}{2}$ (this finds the 'middle' of the function)

Example 3: Determine the equation of a sine and cosine function that describes the following graph

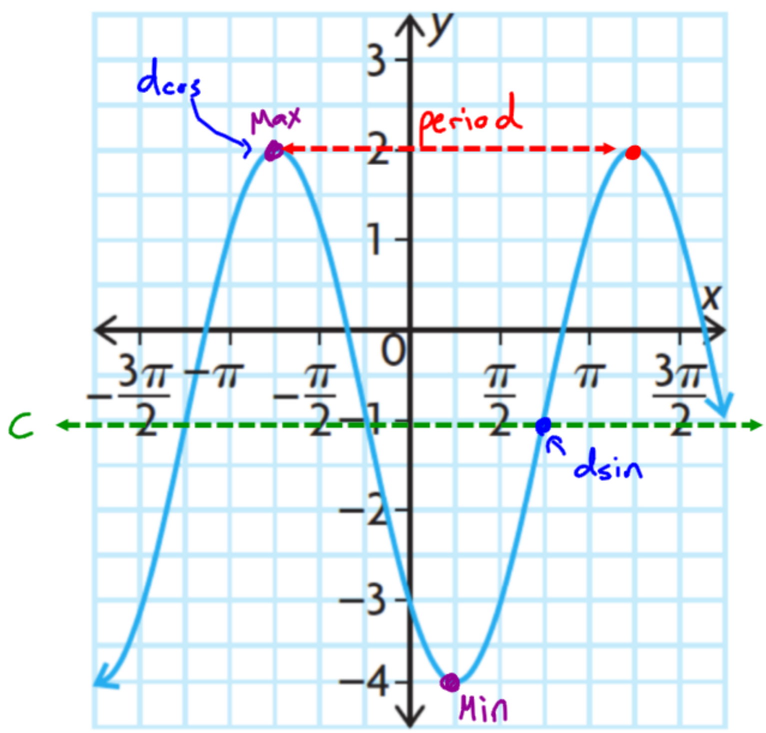
$a = \dfrac{max - min}{2} = \dfrac{2-(-4)}{2} = 3$

$k = \dfrac{2\pi}{period} = \dfrac{2\pi}{\left(\frac{5\pi}{4} - \left(-\frac{3\pi}{4}\right)\right)} = \dfrac{2\pi}{2\pi} = 1$

$c = max - |a| = 2 - 3 = -1$

$d_{cos} = \dfrac{-3\pi}{4}$

$d_{sin} = \dfrac{3\pi}{4}$



$$\boxed{\begin{aligned} y &= 3\sin\left(x - \tfrac{3\pi}{4}\right) - 1 \\ y &= 3\cos\left(x + \tfrac{3\pi}{4}\right) - 1 \end{aligned}}$$

Example 4: Determine the equation of a sine and cosine function that describes the following graph

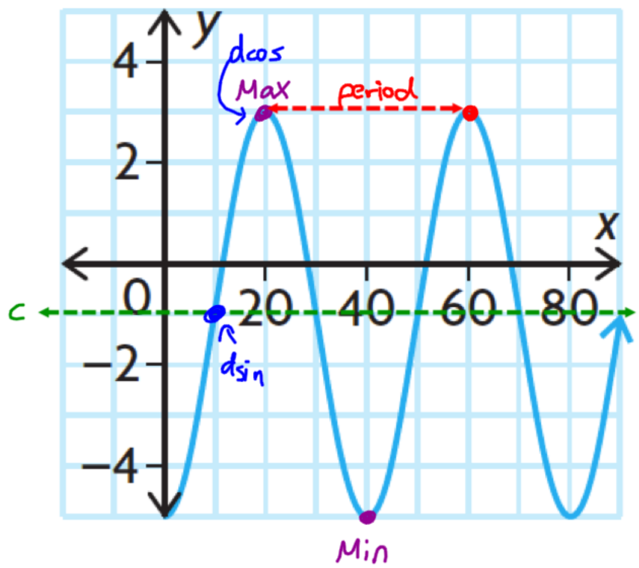
$a = \frac{max - min}{2} = \frac{3-(-5)}{2} = 4$

$k = \frac{2\pi}{period} = \frac{2\pi}{40} = \frac{\pi}{20}$

$c = max - |a| = 3 - 4 = -1$

$d_{cos} = 20$

$d_{sin} = 10$



$$\boxed{\begin{aligned} y &= 4\cos\left[\tfrac{\pi}{20}(x-20)\right]-1 \\ y &= 4\sin\left[\tfrac{\pi}{20}(x-10)\right]-1 \end{aligned}}$$

Example 5:

a) Create a sine function with an amplitude of 7, a period of π, a phase shift of $\frac{\pi}{4}$ right, and a vertical displacement of -3.

$a = 7$

$k = \frac{2\pi}{period} = \frac{2\pi}{\pi} = 2$

$c = -3$

$d = \frac{\pi}{4}$

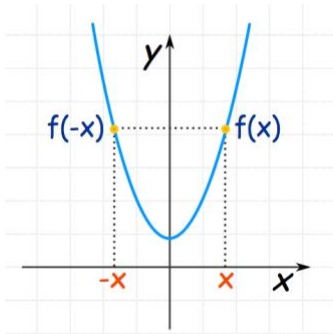
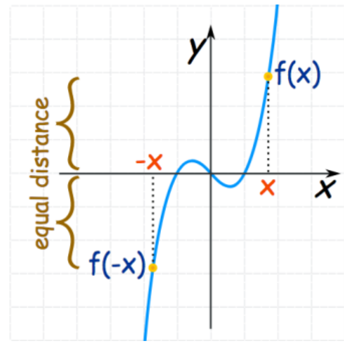
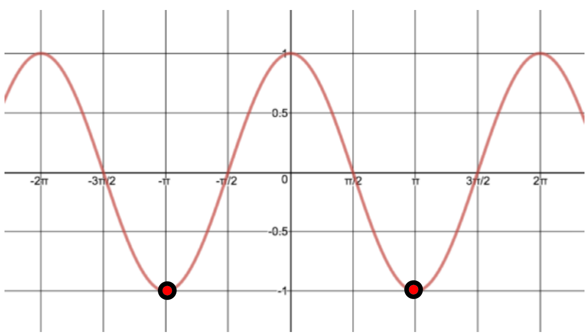
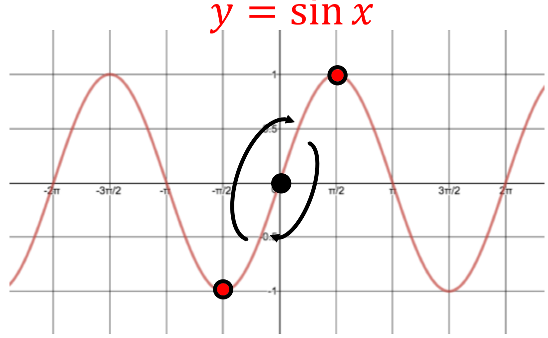
$$\boxed{y = 7\sin\left[2\left(x - \frac{\pi}{4}\right)\right] - 3}$$

b) What would be the equation of a cosine function that represents the same graph as the sine function above?

$d_{cos} = d_{sin} + \frac{\pi}{2k} = \frac{\pi}{4} + \frac{\pi}{2(2)} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\boxed{y = 7\cos\left[2\left(x - \frac{\pi}{2}\right)\right] - 3}$$

Part 4: Even and Odd Functions

Even Functions	Odd Functions
EVEN FUNCTION if: Line symmetry over the <u>y-axis</u>	ODD FUNCTION if: Point symmetry about the <u>origin (0, 0)</u>
Rule: $f(-x) = f(x)$  	Rule: $-f(x) = f(-x)$ 
Example: $y = \cos x$  $f(\pi) = -1$ $f(-\pi) = -1$ Therefore, $f(\pi) = f(-\pi)$	Example: $y = \sin x$  $f\left(\frac{\pi}{2}\right) = 1$ $f\left(-\frac{\pi}{2}\right) = -1$ Therefore, $-f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$ $y = \tan x$ is also an odd function