L3 – 1.3 – Factored Form Polynomial Functions Lesson
MHF4U

In this section, you will investigate the relationship between the factored form of a polynomial function and the x-intercepts of the corresponding graph, and you will examine the effect of repeated factor on the graph of a polynomial function.

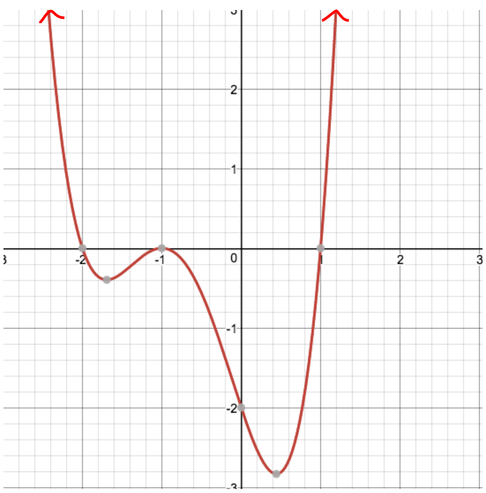
Factored Form Investigation

If we want to graph the polynomial function $f(x) = x^4 + 3x^3 + x^2 - 3x - 2$ accurately, it would be most useful to look at the factored form version of the function:

$f(x) = (x+1)^2(x+2)(x-1)$

Let's start by looking at the graph of the function and making connections to the factored form equation.

Graph of $f(x)$:



From the graph, answer the following questions…

a) What is the degree of the function?

b) What is the sign of the leading coefficient?

c) What are the x-intercepts?

d) What is the y-intercept?

e) The x-intercepts divide the graph in to into four intervals. Write the intervals in the first row of the table. In the second row, choose a test point within the interval. In the third row, indicate whether the function is positive (above the x-axis) or negative (below the y-axis).

Interval				
Test Point				
Sign of $f(x)$				

f) What happens to the sign of the of $f(x)$ near each x-intercept?

Conclusions from investigation:

The x-intercepts of the graph of the function correspond to the roots (zeros) of the corresponding equation. For example, the function $f(x) = (x-2)(x+1)$ has x-intercepts at ___ and ___. These are the roots of the equation $(x-2)(x+1) = 0$.

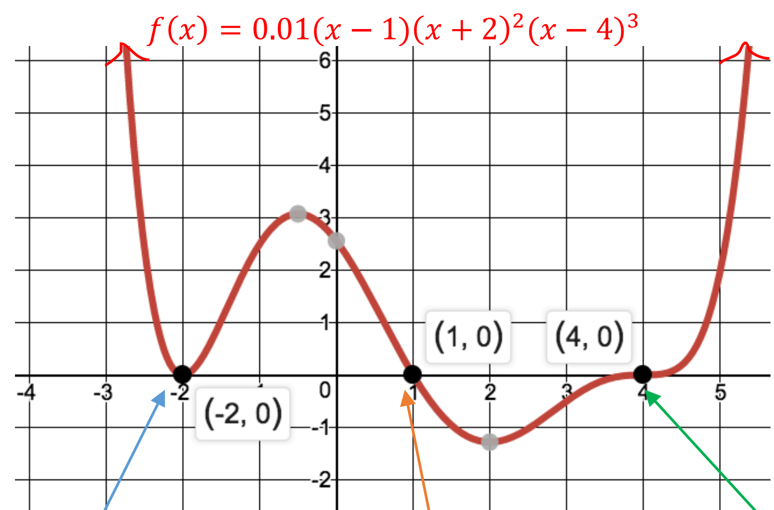
If a polynomial function has a factor $(x-a)$ that is repeated n times, then $x = a$ is a zero of __________ n.

Order – the exponent to which each factor in an algebraic expression is raised.

For example, the function $f(x) = (x-3)^2(x-1)$ has a zero of order _______ at $x = 3$ and a zero of order _______ at $x = 1$.

The graph of a polynomial function changes sign at zeros of __________ order but does not change sign at zeros of __________ order.

Shapes based on order of zero:



ORDER 2

(-2, 0) is an x-intercept of order 2. Therefore, it doesn't change sign.

"Bounces off" x-axis.

Parabolic shape.

ORDER 1

(1, 0) is an x-intercept of order 1. Therefore, it changes sign.

"Goes straight through" x-axis.

Linear Shape

ORDER 3

(4, 0) is an x-intercept of order 3. Therefore, it changes sign.

"S-shape" through x-axis.

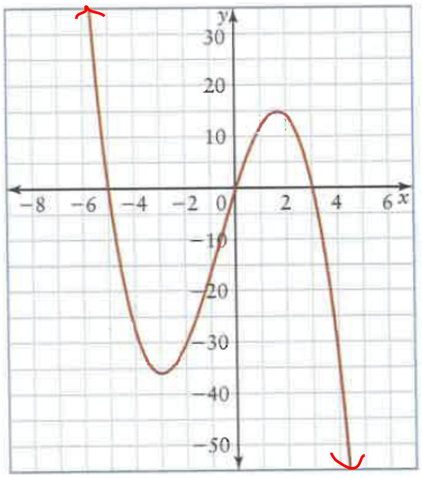
Cubic shape.

Example 1: Analyzing Graphs of Polynomial Functions

For each graph,

i) the least possible degree and the sign of the leading coefficient
ii) the x-intercepts and the factors of the function
iii) the intervals where the function is positive/negative

a)



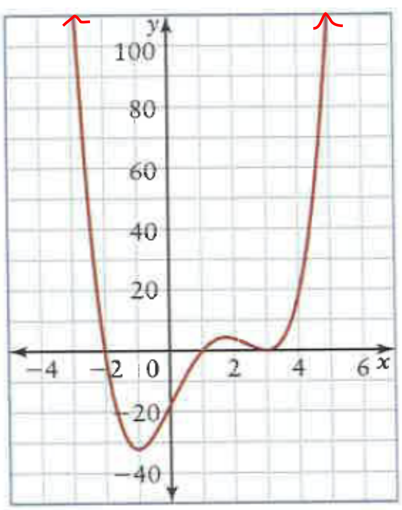
i)

ii)

iii)

Interval				
Sign of $f(x)$				

b)



i)

ii)

iii)

Interval				
Sign of $f(x)$				

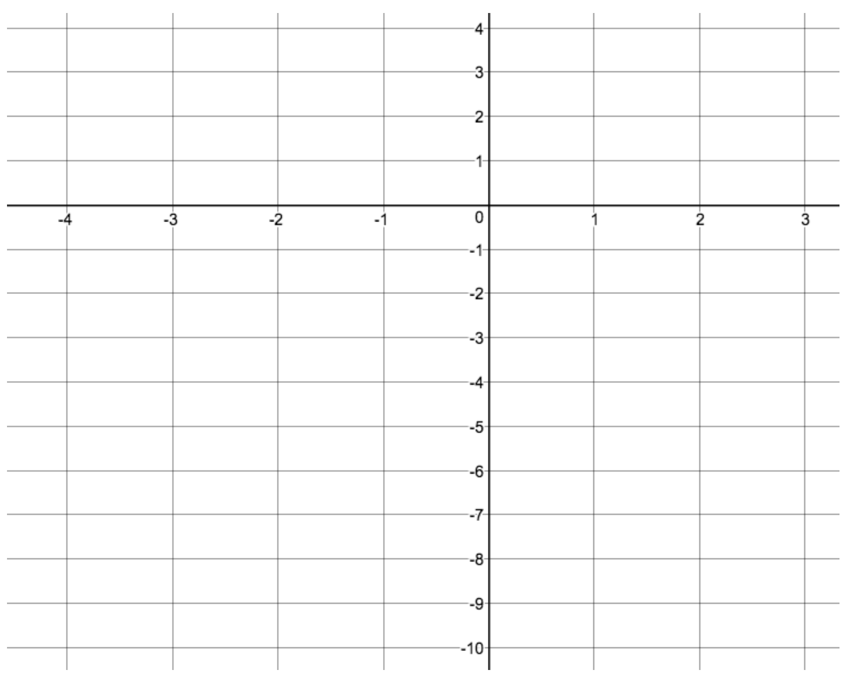
Example 2: Analyze Factored Form Equations to Sketch Graphs

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
The exponent on x when all factors of x are multiplied together OR Add the exponents on the factors that include an x.	The product of all the x coefficients	Use degree and sign of leading coefficient to determine this	Set each factor equal to zero and solve for x	Set $x = 0$ and solve for y

Sketch a graph of each polynomial function:

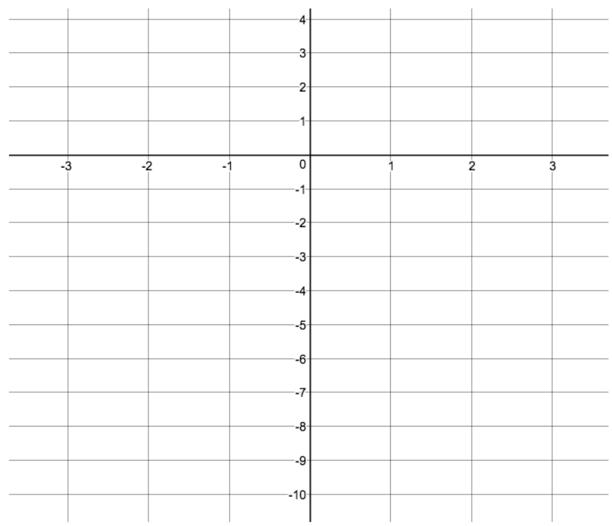
a) $f(x) = (x-1)(x+2)(x+3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept



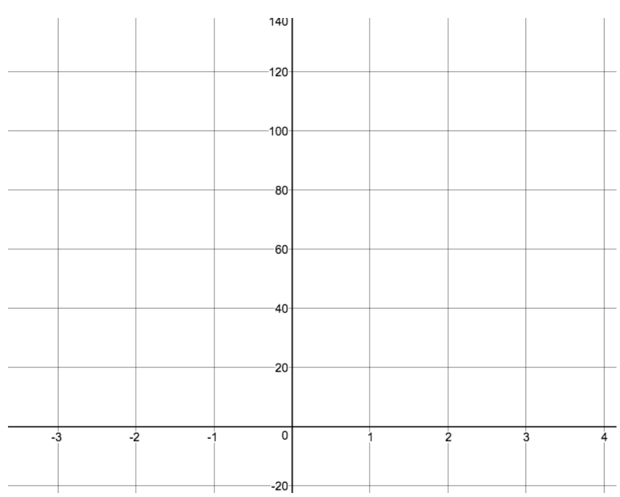
b) $g(x) = -2(x-1)^2(x+2)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept



c) $h(x) = -(2x+1)^3(x-3)$

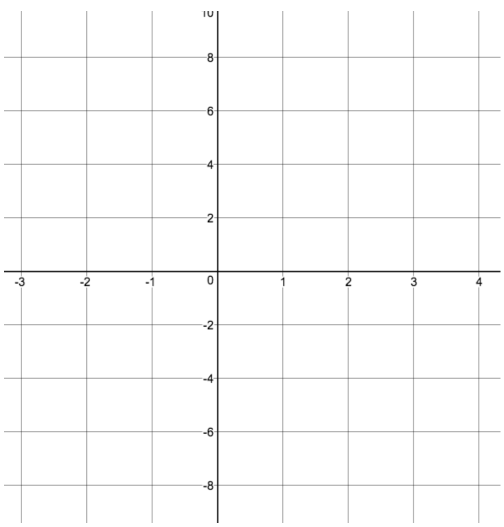
Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept



d) $j(x) = x^4 - 4x^3 + 3x^2$

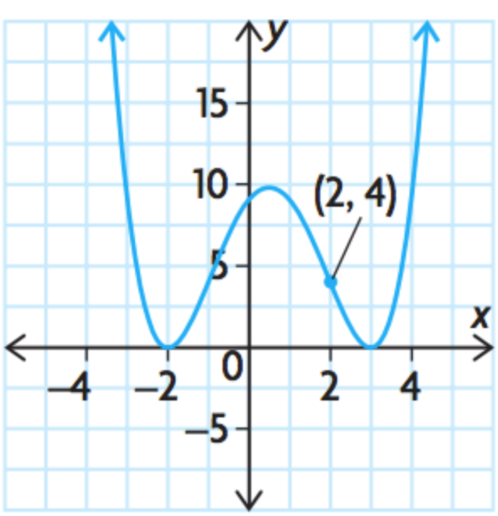
> ***Note:*** *must put in to factored form to find x-intercepts*

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept



Example 3: Representing the Graph of a Polynomial Function with its Equation

a) Write the equation of the function shown below:



> **Steps:**
>
> **1)** Write the equation of the family of polynomials using factors created from x-intercepts
>
> **2)** Substitute the coordinates of another point (x, y) into the equation.
>
> **3)** Solve for a
>
> **4)** Write the equation in factored form

b) Find the equation of a polynomial function that is degree 4 with zeros -1 (order 3) and 1, and with a y-intercept of -2.