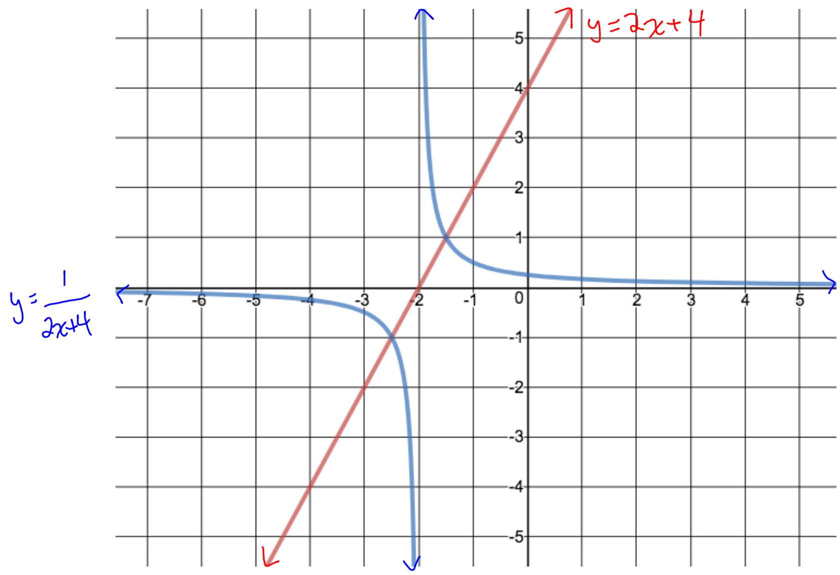
L1 – 3.1/3.2 Reciprocal of Linear and Quadratic Functions
MHF4U

Part 1: Analyze the Reciprocal of a Linear Function

Example 1:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

b) What graphical characteristic in the reciprocal function does the zero (x-int) of the original function correspond to?

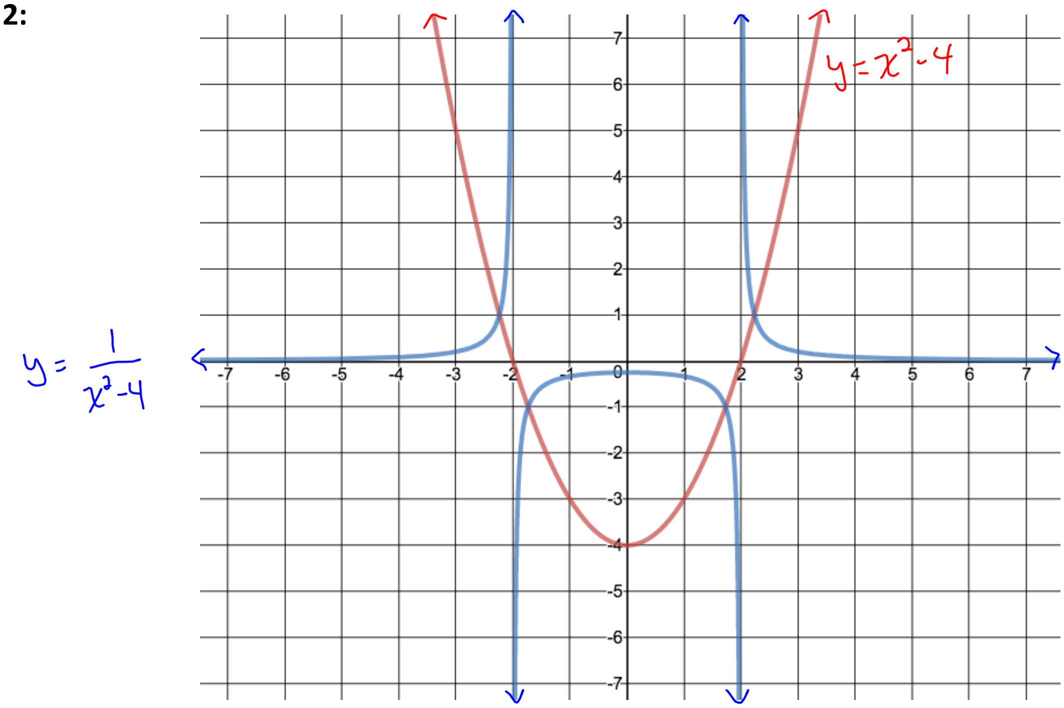
c) When the original function is increasing, what is happening to the reciprocal function?

d) What are the y-coordinates of the points of intersection?

e) Label a point on the graph of both functions at $x = 2$. What do you notice about the y values of each point?

Part 2: Analyze the Reciprocal of a Quadratic Function

Example 2:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

b) What graphical characteristic in the reciprocal function do the zeros (x-int) of the original function correspond to?

c) When the original function is decreasing, what is happening to the reciprocal function?

d) What are the y-coordinates of the points of intersection?

f) Label the local min or max point on each function. What do you notice about them?

Properties of Reciprocal Functions

- All the y-coordinates of the reciprocal function are the reciprocals of the y-coordinates of the original function
- The graph of the reciprocal function has a vertical asymptote at the x-intercepts (zeros) of the original
    - This is because it makes the denominator of the reciprocal $= 0$
- $y = 0$ will always be a horizontal asymptote
- The reciprocal function has the same positive/negative intervals as the original function
- Intervals of increase on the original function are intervals of decrease on the reciprocal
- Intervals of decrease on the original function are intervals of increase on the reciprocal
- If 1 is in the range of the original function, this is where the functions will intersect
- If the original function has a local min point, the reciprocal will have a local max at the same x-value (and vice versa)

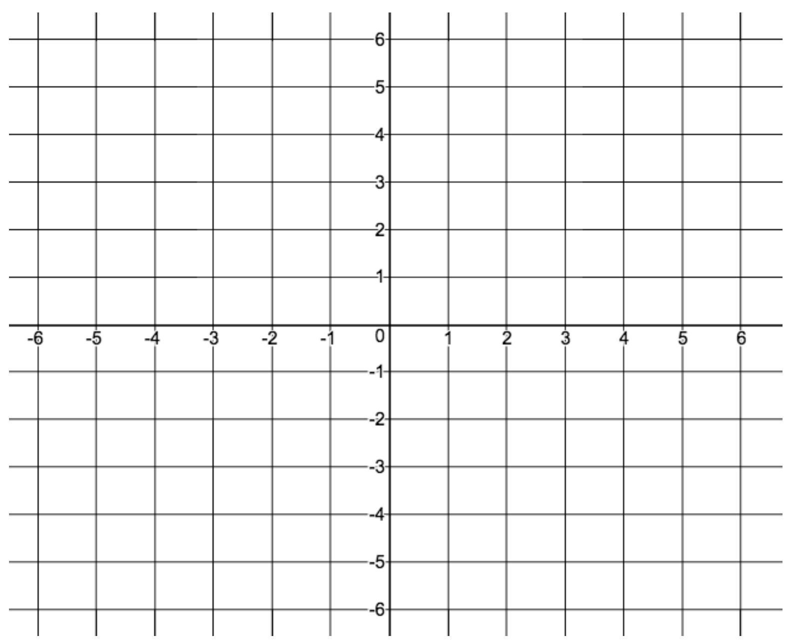
Part 3: Graphing Reciprocal Functions

Process:

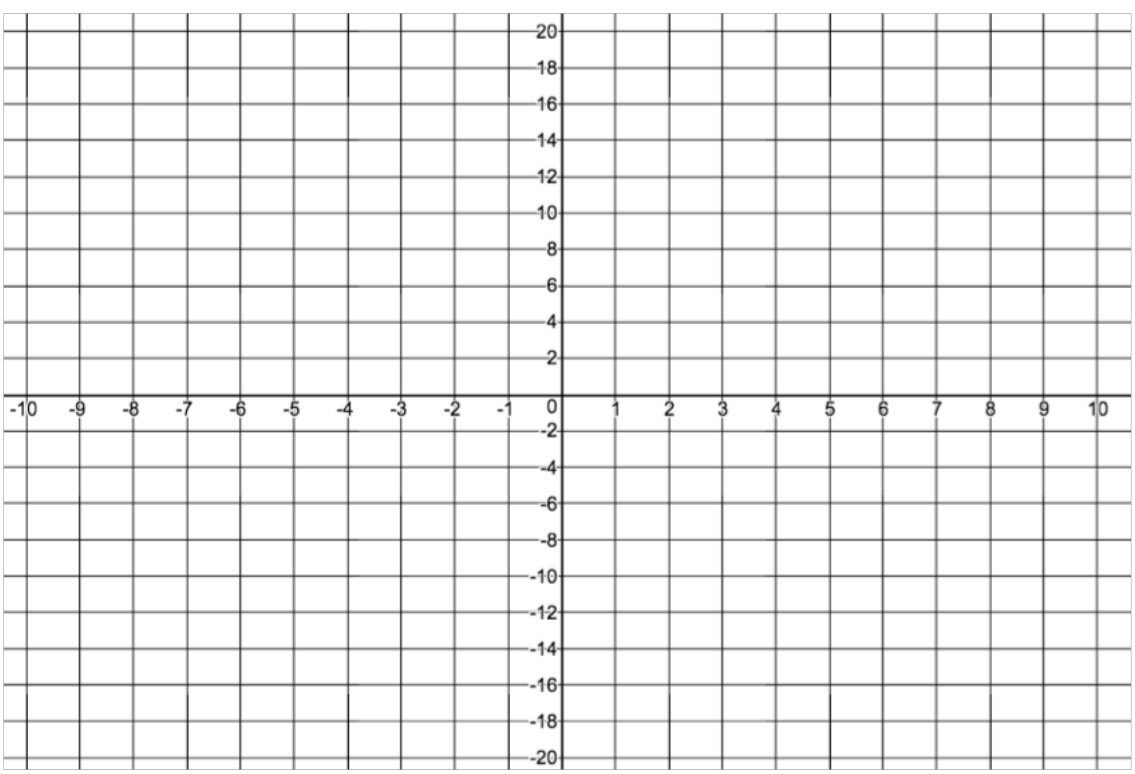
- Find key features of the function in the denominator and graph it using a table of values
- Create a table of values for the reciprocal function by keeping the same x values but using the reciprocal of all y values
- Draw vertical asymptotes at any point that is a zero of the original linear/quadratic function
    - Reciprocal of 0 is undefined
- If the numerator is something other than 1, multiply the y-values by this stretch factor

Example 3: Graph each of the following reciprocal functions. Start by graphing the function in the denominator.

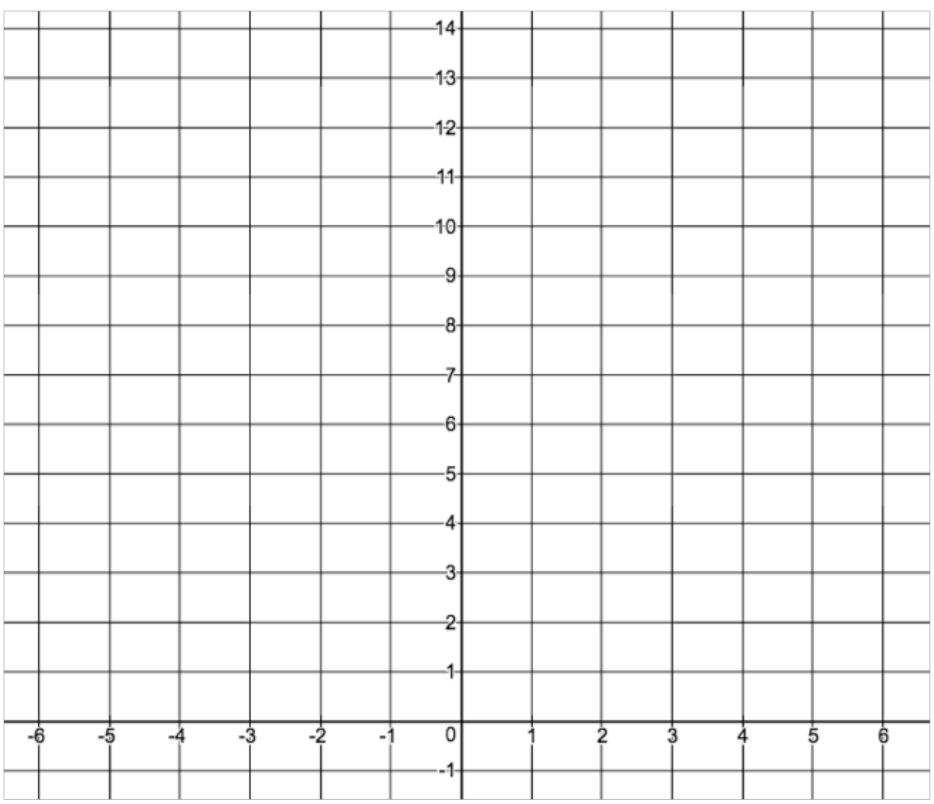
a) $y = \frac{1}{2x-1}$



b) $y = \frac{1}{x^2-2x-15}$



c) $y = \frac{1}{x^2+4}$



d) $y = \frac{2}{x^2-6x+9} = 2\left[\frac{1}{(x-3)^2}\right]$