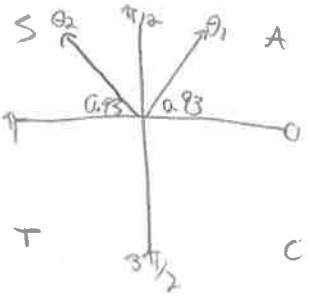
W6 – 5.4 Solve Double Angle Trigonometric Equations
MHF4U

SOLUTIONS

Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible.

a) $\sin(2x) - 0.8 = 0$ Let $\theta = 2x$

$\sin\theta = 0.8$



$\theta_1 = \sin^{-1}(0.8)$

$\theta_1 = 0.93$

$\theta_2 = \pi - 0.93$

$\theta_2 = 2.21$

$2x = \theta$

$2x = 0.93$ $\quad$ $2x = 2.21$

$x_1 = 0.47$ $\quad$ $x_2 = 1.11$

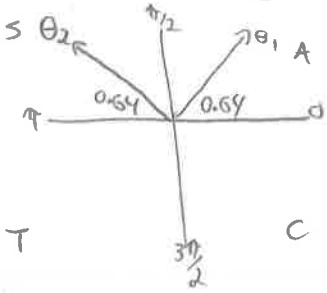
*Add the period of π to find other solutions

$x_3 = x_1 + \pi$ $\quad$ $x_4 = x_2 + \pi$

$x_3 = 3.61$ $\quad$ $x_4 = 4.25$

b) $5\sin(2x) - 3 = 0$ Let $\theta = 2x$

$\sin\theta = \frac{3}{5}$



$\theta_1 = \sin^{-1}\left(\frac{3}{5}\right)$

$\theta_1 = 0.64$

$\theta_2 = \pi - 0.64$

$\theta_2 = 2.5$

$2x = \theta$

$2x = 0.64$ $\quad$ $2x = 2.5$

$x_1 = 0.32$ $\quad$ $x_2 = 1.25$

* Add period of π to find other solutions *

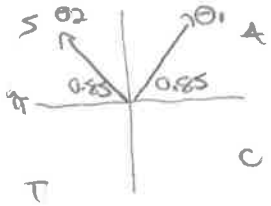
$x_3 = x_1 + \pi$ $\quad$ $x_4 = x_2 + \pi$

$x_3 = 3.46$ $\quad$ $x_4 = 4.39$

c) $-4\sin(2x) + 3 = 0$

Let $\theta = 2x$

$\sin\theta = \frac{3}{4}$



$\theta_1 = \sin^{-1}\left(\frac{3}{4}\right)$

$\theta_1 = 0.85$

$\theta_2 = \pi - 0.85$

$\theta_2 = 2.29$

$2x = \theta$

$2x = 0.85$ $\quad$ $2x = 2.29$

$x_1 = 0.43$ $\quad$ $x_2 = 1.15$

* add period of π to find other solutions *

$x_3 = x_1 + \pi$ $\quad$ $x_4 = x_2 + \pi$

$x_3 = 3.57$ $\quad$ $x_4 = 4.29$

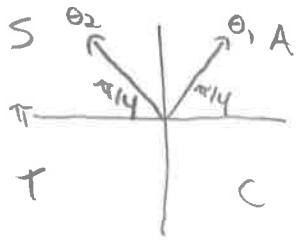
d) $\sin(2x) = \frac{1}{\sqrt{2}}$

Let $\theta = 2x$

$\sin\theta = \frac{1}{\sqrt{2}}$

From Δ; $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Place in Q1+Q2



$\theta_1 = \frac{\pi}{4}$

$\theta_2 = \pi - \frac{\pi}{4}$

$\theta_2 = \frac{3\pi}{4}$

$2x = \theta$

$2x = \frac{\pi}{4}$ $\quad$ $2x = \frac{3\pi}{4}$

$x_1 = \frac{\pi}{8}$ $\quad$ $x_2 = \frac{3\pi}{8}$

$x_3 = x_1 + \pi$ $\quad$ $x_4 = x_2 + \pi$

$x_3 = \frac{9\pi}{8}$ $\quad$ $x_4 = \frac{11\pi}{8}$

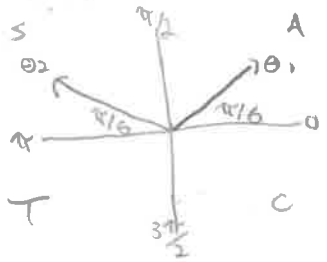
Let $\theta = 4x$

e) $\sin(4x) = \frac{1}{2}$

From Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

Place in Q1+Q2



$\theta_1 = \frac{\pi}{6}$

$\theta_2 = \pi - \frac{\pi}{6}$

$\theta_2 = \frac{5\pi}{6}$

$4x = \theta$

$4x = \frac{\pi}{6}$ ✓ $\quad 4x = \frac{5\pi}{6}$

$x_1 = \frac{\pi}{24}$ $\quad x_2 = \frac{5\pi}{24}$

* add period of $\frac{\pi}{2} = \frac{12\pi}{24}$ to find other solutions *

$x_3 = \frac{13\pi}{24}$ $\quad x_6 = \frac{17\pi}{24}$

$x_4 = \frac{25\pi}{24}$ $\quad x_7 = \frac{29\pi}{24}$

$x_5 = \frac{37\pi}{24}$ $\quad x_8 = \frac{41\pi}{24}$

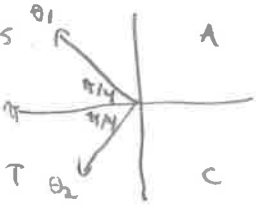
g) $\cos(4x) = -\frac{1}{\sqrt{2}}$

Let $\theta = 4x$

$\cos\theta = -\frac{1}{\sqrt{2}}$

From Δ; $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

place in Q2+Q3



$\theta_1 = \pi - \frac{\pi}{4}$

$\theta_1 = \frac{3\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

$4x = \theta$

$4x = \frac{3\pi}{4}$ $\quad 4x = \frac{5\pi}{4}$

$x_1 = \frac{3\pi}{16}$ $\quad x_2 = \frac{5\pi}{16}$

* add period of $\frac{\pi}{2} = \frac{8\pi}{16}$ to find other solutions *

$x_3 = \frac{11\pi}{16}$ $\quad x_6 = \frac{13\pi}{16}$

$x_4 = \frac{19\pi}{16}$ $\quad x_7 = \frac{21\pi}{16}$

$x_5 = \frac{27\pi}{16}$ $\quad x_8 = \frac{29\pi}{16}$

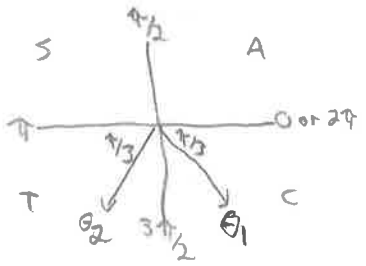
Let $3x = \theta$

f) $\sin(3x) = -\frac{\sqrt{3}}{2}$

From Δ; $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

Place in Q3+Q4



$\theta_1 = 2\pi - \frac{\pi}{3}$

$\theta_1 = \frac{5\pi}{3}$

$\theta_2 = \pi + \frac{\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$

$3x = \theta$

$3x = \frac{5\pi}{3}$ $\quad 3x = \frac{4\pi}{3}$

$x_1 = \frac{5\pi}{9}$ $\quad x_2 = \frac{4\pi}{9}$

* add period of $\frac{2\pi}{3} = \frac{6\pi}{9}$ to find other solutions *

$x_3 = \frac{11\pi}{9}$ $\quad x_5 = \frac{10\pi}{9}$

$x_4 = \frac{17\pi}{9}$ $\quad x_6 = \frac{16\pi}{9}$

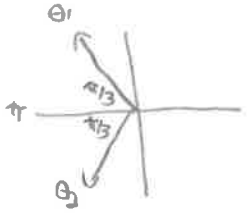
h) $\cos(2x) = -\frac{1}{2}$

Let $\theta = 2x$

$\cos\theta = -\frac{1}{2}$

From Δ; $\cos\frac{\pi}{3} = \frac{1}{2}$

place in Q2+Q3



$\theta_1 = \pi - \frac{\pi}{3}$

$\theta_1 = \frac{2\pi}{3}$

$\theta_2 = \pi + \frac{\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$

$2x = \theta$

$2x = \frac{2\pi}{3}$ $\quad 2x = \frac{4\pi}{3}$

$x_1 = \frac{\pi}{3}$ $\quad x_2 = \frac{2\pi}{3}$

* add period of $\pi = \frac{3\pi}{3}$ to find other solutions *

$x_3 = \frac{4\pi}{3}$ $\quad x_4 = \frac{5\pi}{3}$