L6 – 2.5 – Solving Inequalities Lesson
MHF4U

In this section, you will learn the meaning of a polynomial inequality and examine methods for solving polynomial inequalities.

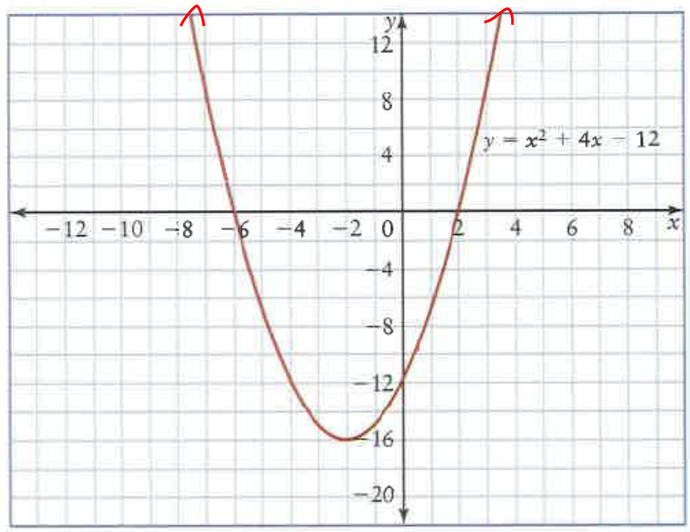
Part 1: Intro to Inequalities

Task: Read the following on your own

Examine the graph of $y = x^2 + 4x - 12$.

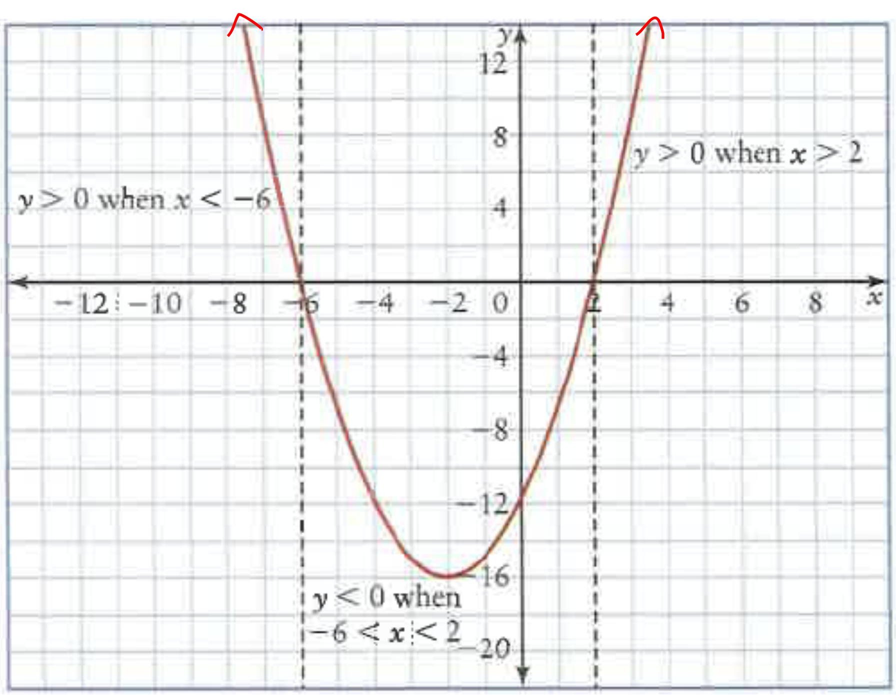
The x-intercepts are 6 and -2. These correspond to the zeros of the function $y = x^2 + 4x - 12$. Note that the factored form version of the function is $y = (x + 6)(x - 2)$. By moving from left to right along the x-axis, we can make the following observations:

- The function is positive when $x < -6$ since the y-values are positive
- The function is negative when $-6 < x < 2$ since the y-values are negative
- The function is positive when $x > 2$ since the y-values are positive.



The zeros -6 and 2 divide the x-axis into three intervals. In each interval, the function is either positive or negative. The information can be summarized in a table:

Interval	$x < -6$	$-6 < x < 2$	$x > 2$
Sign of Function	$+$	$-$	$+$



Polynomial Inequalities

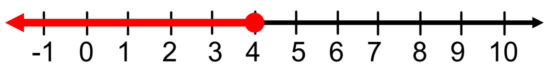
A polynomial inequality results when the equal sign in a polynomial equation is replaced with an inequality symbol.

The real zeros of a polynomial function, or x-intercepts of the corresponding graph, divide the x-axis into intervals that can be used to solve a polynomial inequality.

Part 1: Inequalities and Number Lines

Example 1: Write an inequality that corresponds to the values of x shown on each number line

a)

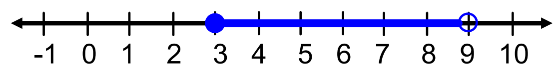


$x \leq 4$

OR

$(-\infty, 4]$

b)



$3 \leq x < 9$

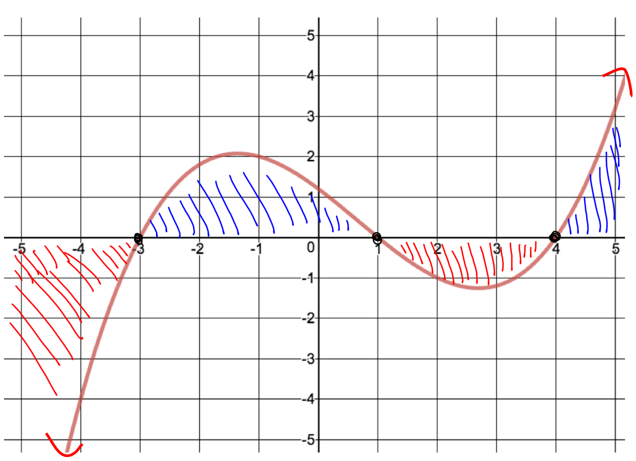
OR

$[3, 9)$

Part 2: Solve an Inequality given the Graph

Example 2: Use the graph of the function $f(x)$ to answer the following inequalities…

$f(x) = 0.1(x-1)(x+3)(x-4)$



a) $f(x) < 0$

$f(x) < 0$ when: $x < -3$ or $1 < x < 4$

$(-\infty, -3) \cup (1, 4)$

b) $f(x) \geq 0$

$f(x) \geq 0$ when: $-3 \leq x \leq 1$ or $x \geq 4$

$[-3, 1] \cup [4, \infty)$

Part 2: Solve Linear Inequalities

Note: Solving linear inequalities is the same as solving linear equations. However, when both sides of an inequality are multiplied or divided by a negative number, the inequality sign must be reversed.

Example 3: Solve each inequality

a) $x - 8 \geq 3$

$x - 8 \geq 3$

$x \geq 3 + 8$

$x \geq 11$

b) $-4 - 2x < 12$

$-4 - 2x < 12$

$\frac{-2x}{-2} < \frac{16}{-2}$

Reverse inequality when dividing by a negative

$x > -8$

Part 2: Solve Inequalities of Degree 2 and Higher

Steps for solving polynomial inequalities algebraically:

1) Use inverse operations to move all terms to one side of the inequality
2) Factor the polynomial to determine the zeros of the corresponding equation
3) Find the interval(s) where the function is positive or negative by either:
   a. Graphing the function using the zeros, leading coefficient, and degree
   b. Make a factor table and test values in each interval

Example 4: Solve each polynomial inequality algebraically

a) $2x^2 + 3x - 9 > 0$ ← when is it above the x-axis?

Method 1: Graph the inequality

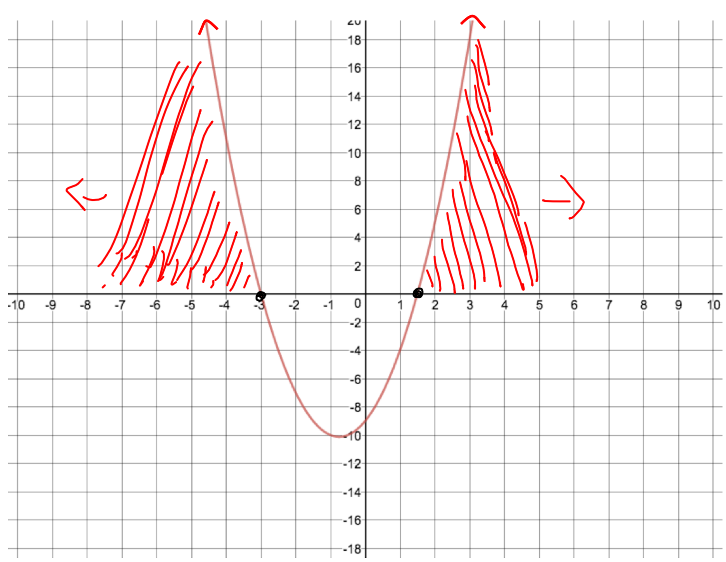
$2x^2 + 3x - 9 > 0$ P: -18 S: 3 6 and -3

$(2x^2 + 6x) + (-3x - 9) > 0$

$2x(x + 3) - 3(x + 3) > 0$

$(x + 3)(2x - 3) > 0$

x-int at -3 and 1.5



$2x^2 + 3x - 9 > 0$ when... $x < -3$ or $x > 1.5$

$(-\infty, -3) \cup (1.5, \infty)$

Method 2: Factor Table (sign chart)

To make a factor table:

- Use x-intercepts and vertical asymptotes to divide in to intervals
- Use a test point within each interval to find the sign of each factor
- Determine the overall sign of the product by multiplying signs of each factor within each interval.

$-\infty \qquad -3 \qquad 1.5 \qquad \infty$

	-4	0	2
$x+3$	$-$	$+$	$+$
$2x-3$	$-$	$-$	$+$
overall sign	$+$ *	$-$	$+$ *

$\therefore\ 2x^2+3x-9>0$ when $x<-3$ or $x>1.5$

$(-\infty,-3)\cup(1.5,\infty)$

b) $-2x^3 - 6x^2 + 12x \leq -16$

Method 1: Graph the inequality

$$-2x^3 - 6x^2 + 12x + 16 \leq 0$$

common factor

$$\frac{-2(x^3 + 3x^2 - 6x - 8)}{-2} \leq \frac{0}{-2}$$

reverse inequality

$$x^3 + 3x^2 - 6x - 8 \geq 0$$

factors of -8 are: $\pm 1, \pm 2$, and ± 3

$f(-1) = 0$, $\therefore$ $x+1$ is a factor.

-1	1	3	-6	-8
	$\downarrow$	-1	-2	8
$\otimes$	1	2	-8	0
	x^2	x	#	R

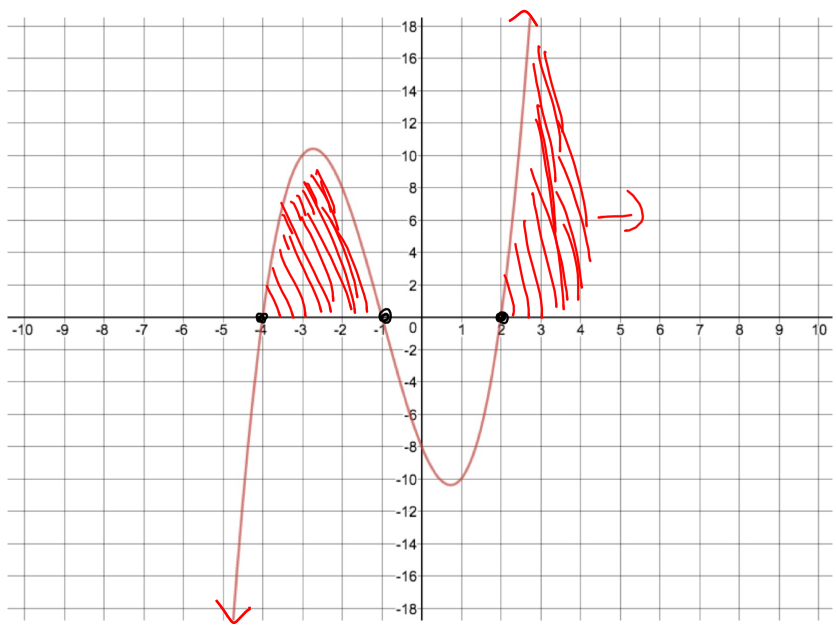
$$(x+1)(x^2 + 2x - 8) \geq 0$$

$$(x+1)(x+4)(x-2) \geq 0$$

x-int at $-1, -4$, and 2

Positive leading coefficient

Degree 3



Solution: $-4 \leq x \leq -1$ or $x \geq 2$

$[-4, -1] \cup [2, \infty)$

Method 2: Factor Table (sign chart)

$-\infty$ | -4 | -1 | 2 | ∞

	-5	-2	0	3
$x+1$	$-$	$-$	$+$	$+$
$x+4$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
overall sign	$-$	$+$ *	$-$	$+$ *

Solution: $[-4, -1] \cup [2, \infty)$

c) $x^3 + 4x^2 + 6x < -24$

$x^3+4x^2+6x+24<0$

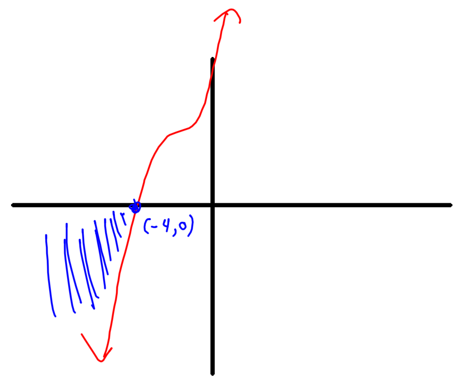
factor by grouping

$x^2(x+4)+6(x+4)<0$

$(x+4)(x^2+6)<0$

x-int at -4 | no real solutions

	$-\infty$ to -4	-4 to ∞
	-5	0
$x+4$	$-$	$+$
x^2+6	$+$	$+$
sign	$-$	$+$



Solution: $x<-4$ OR $(-\infty,-4)$

Part 2: Applications of Inequalities

3) The price, p, in dollars, of a stock t years after 1999 can be modeled by the function $p(t) = 0.5t^3 - 5.5t^2 + 14t$. When will the price of the stock be more than $90?

$0.5t^3-5.5t^2+14t>90$

$0.5t^3-5.5t^2+14t-90>0$

$0.5(t^3-11t^2+28t-180)>0$

$t^3-11t^2+28t-180>0$

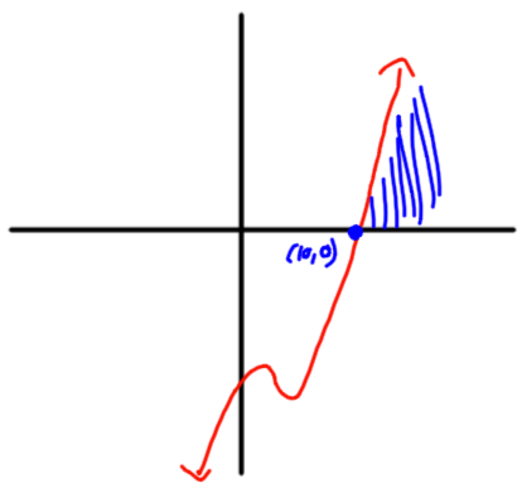
$f(10)=0$; $\therefore$ $x-10$ is a factor

$$\begin{array}{c|cccc} 10 & 1 & -11 & 28 & -180 \\ & \downarrow & 10 & -10 & 180 \\ \hline & 1 & -1 & 18 & 0 \\ & t^2 & t & \# & R \end{array}$$

$(t-10)(t^2-t+18)>0$

x-int at $(10,0)$

$b^2-4ac=(-1)^2-4(1)(18)$
$=-71$
$\therefore$ no solutions



Solution: $t>10$

$\therefore$ the price of the stock will be above $90 after year 2009