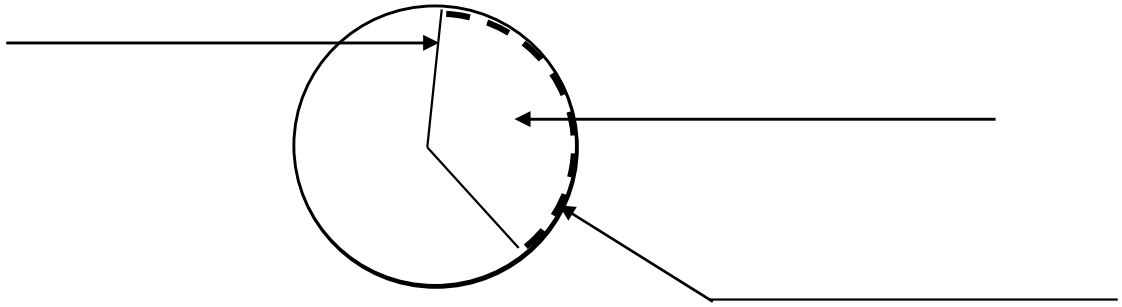
3.1 Investigation- RADIANS

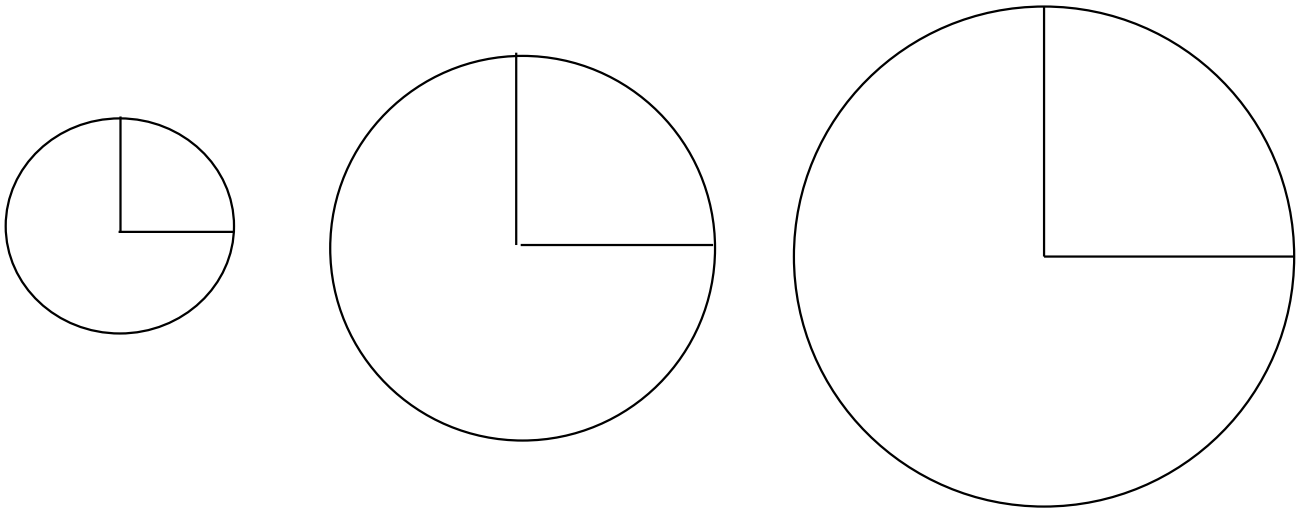
PART A: TERMINOLOGY
Fill in each blank with one of the following terms: arc length, radius, sector



PART B: Investigate Angle Measure
Materials

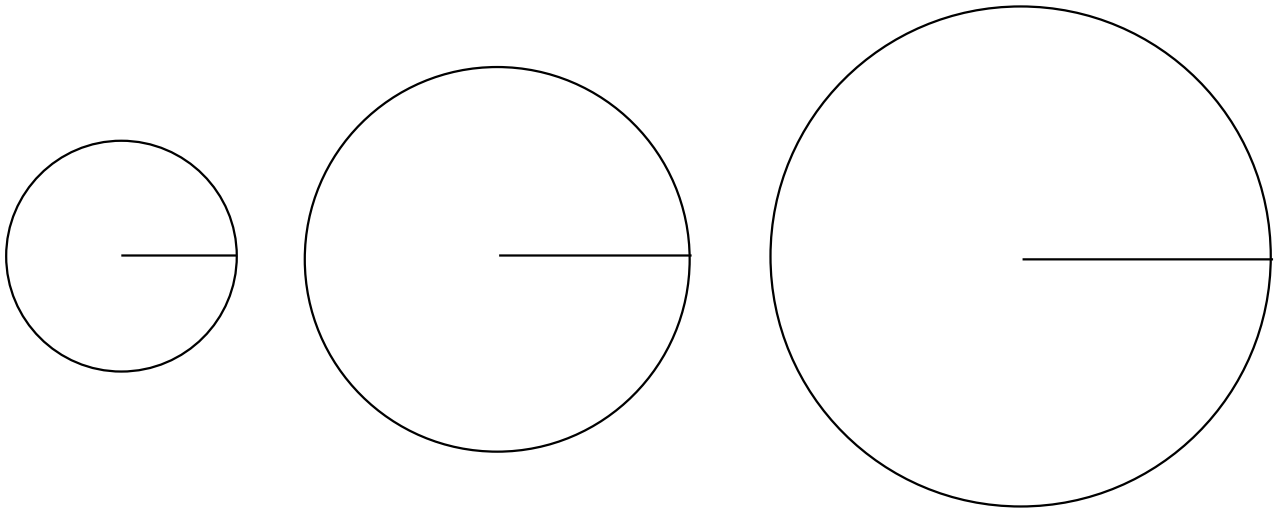
- protractor
- ruler
- string
- calculator

1. Use a protractor to confirm that in each of the 3 circles below the sector drawn has an angle measuring 90°.
2. Use a ruler to measure the radius of each circle and record in the chart on the next page.
3. Use a string to measure the arc length of each sector and record in the chart.
4. Complete the 5th column using a calculator. Provide an answer to **1 decimal place**.



Size of Angle	Size of Circle	arc length (**a**)	radius (**r**)	$\frac{a}{r}$
90°	small			
	medium			
	large			
120°	small			
	medium			
	large			
57°	small			
	medium			
	large			

5. In the following circles, use a protractor to construct sectors that have a sector angle of 120°. Measure the arc length and radius for each angle you constructed and record in the chart.
6. Repeat #5 but this time construct angles of 57° . Use the same circles.



PART C: SUMMARY
Answer the following questions on a separate sheet of paper.

1. What do you notice about the chart?

2. How does the size of the circle affect the ratio, $\frac{a}{r}$?

3. How does the size of the angle affect the ratio, $\frac{a}{r}$?

4. Use your answers to questions 2 and 3 to describe another way to give an angle's size other than by using degrees.

Part E: Definition of a Radian

Radians are an alternative way to measure angles, other than using degrees. The number of radians contained in an angle is equal to the ratio $\frac{a}{r}$, where a is the arc length of the sector containing the angle and r is the radius of that circle.

One radian is defined to be the measure of the angle where $\frac{a}{r} = 1$. In other words, one radian is the size an angle that has an arc length equal to the radius, which makes the angle about 57°. *(See the last column in the chart of the investigation.)*

Converting Between Degrees and Radians

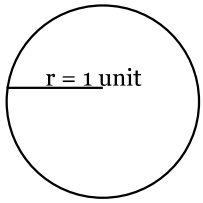
Question:

How many radians are in a circle? (i.e. How many radians equal 360°?)

Solution:

Step 1: To find the number of radians, we need to know the arc length and radius. Since the size of a circle does not matter, consider a circle with a radius of 1 unit.

Radius equals _____.



Step 2: Now we need to find the arc length. Hmmmm…what is another name for the arc length of a sector whose angle measures 360°? _____.

Find the arc length. Do not use decimals. Leave your answer in terms of π.

Arc length equals _____.

Step 3: Remember that: # of radians $= \frac{a}{r}$

$= \frac{\text{\_\_\_\_\_}}{\text{\_\_\_\_\_}}$

$=$

Step 4: Therefore, 360° = _____ # of radians.

OR:

180° = _____ # of radians

Rules for conversion between
DEGREES ⟺ RADIANS

Since $180^0 = \pi$ radians…

- Dividing both sides by 180 gives that $1^0 =$_____radians.
- Dividing both sides by π gives that 1 radian =_____^{0}.

These deductions give us the following nice conversion shortcuts:	
DEGREES=_____ RADIANS	**RADIANS =_____DEGREES**

Example 1: Convert from degrees to radians.

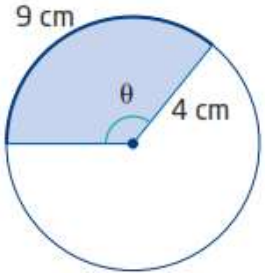
a) 90° b) 270° c) 138°

Example 2: Convert from radians to degrees.

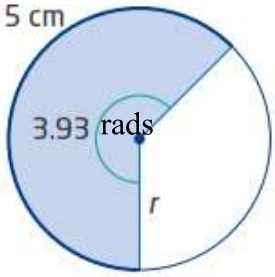
a) 3π b) $\frac{3}{4}\pi$ c) 1.45 rads.

Example 3 : Use the information in each diagram to determine the value of the variable. Give your answers to the nearest hundredth of a unit.

a)



b)



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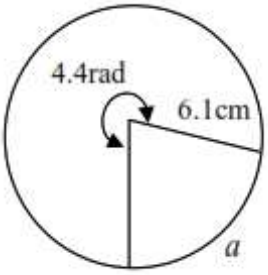


Exit Card!

1. Change each degree to radian measure in terms of π and vice versa.

Degree	Radian	Radian	Degree
(a) -315°		(c) $\frac{\pi}{12}$	
(b) 135°		(d) 6 rad	

2. Sara is taking a course in industrial engineering. For an assignment, she is designing the interface of a DVD player. In her plan, she includes a decorative arc below the on/off button. The arc has central angle 130° in a circle with radius 6.7 mm. Determine the length of the arc, to the nearest tenth of a millimeter.

3.1 Practice

1. Find the exact number of degrees in the angles with the following radian measures.

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) 2π d) $\frac{\pi}{2}$ e) $\frac{3}{4}\pi$ f) $\frac{3}{2}\pi$

g) 4π h) $\frac{5}{6}\pi$ i) $\frac{\pi}{18}$ j) $\frac{11}{3}\pi$ k) $\frac{7}{6}\pi$ l) 5π

2. Find the exact radian measure in terms of π for each of the following angles.

a) 40^o b) 75^o c) 10^o d) 120^o e) 225^o

f) 315^o g) 330^o h) 240^o i) 540^o j) 1080^o

3. Find the approximate number of degrees, to the nearest tenth, in the angles with the following radian measures.

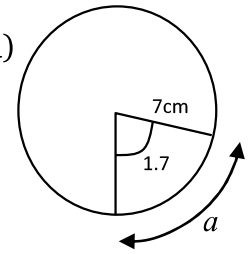
a) 2.5 b) 1.75 c) 0.35 d) $\frac{17}{13}\pi$ e) $\frac{5}{7}\pi$

4. Find the approximate number of radians (nearest hundredth), in the angles with the following degree measures.

a) 60^o b) 150^o c) 310.5^o d) 145^o e) 230^o

5. Determine the length of each arc, a, to the nearest tenth.

a)



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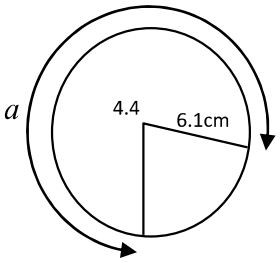
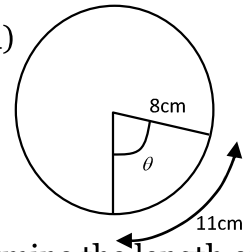


6. Determine the approximate measure of $\angle\theta$, (nearest hundredth) of a radian and to the tenth of a degree.

a)



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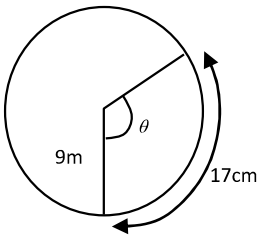
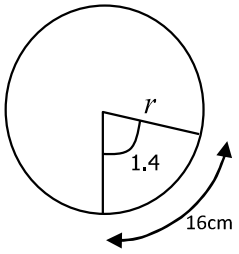


7. Determine the length of each radius, r, to the nearest tenth.

a)



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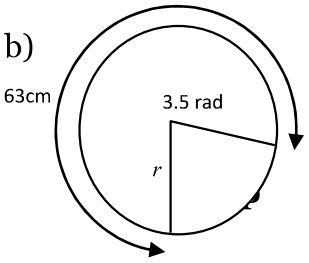

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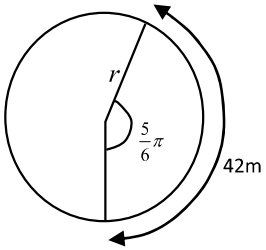
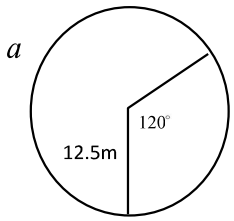


Warm up

1. Determine the missing value in each diagram

a)



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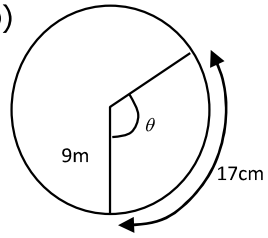
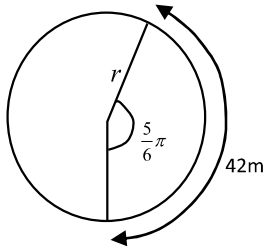


c)



2. State two co-terminal angles for each of the following. Your answers must include π.

a) $\frac{2\pi}{3}$

b) $-\frac{3\pi}{4}$

3. Change each degree to radian measure in terms of π and vise versa.

Degree	Radian	Radian	Degree
(c) - 315º		(d) $\frac{20\pi}{3}$	
(d) 135º		(e) 6 rad	
(e) 540º		(f) $\frac{3\pi}{4}$	