

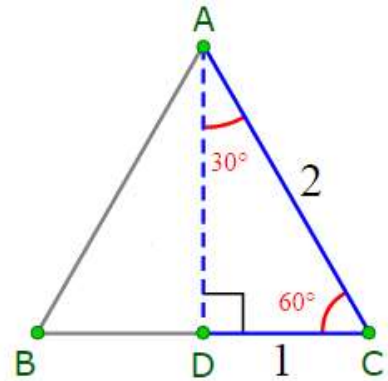
Unit 3: Trigonometry

3.2 Trigonometric Ratios and Special Angles

Angles that measure 30° , 45° , and 60° occur often in trigonometry. They are sometimes called **special angles**.

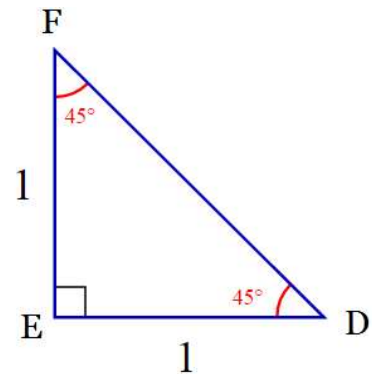
- 30° - 60° - 90° Triangle**

$\triangle ABC$ is an equilateral triangle with side lengths of 2 units. AD bisects $\angle BAC$ to form two congruent triangles. $\triangle ABD$ and $\triangle ACD$ are called 30° - 60° - 90° triangles. What is the length of the side lengths of $\triangle ACD$? Express the length of AD in radical form.



- 45° - 45° - 90° Triangle**

$\triangle DEF$ is an isosceles right triangle whose equal sides, DE and EF , have a length of 1 unit. $\triangle DEF$ is called 45° - 45° - 90° triangle. Determine the length of DF . Express the answer in radical form.



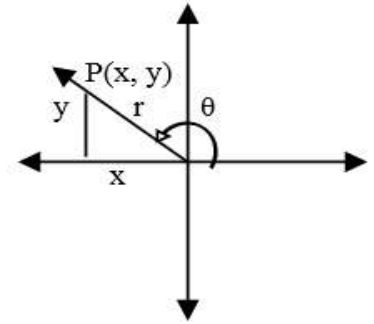
Use the 30° - 60° - 90° and 45° - 45° - 90° Triangle triangle to determine the exact values of the sine, cosine and tangent of 30° , 45° and 60° .

θ in degrees	θ in radians	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
30°				
45°				
60°				

Angle Measures and the Coordinate Plane

Let $P(x, y)$ be any point on the terminal arm of angle θ in standard position.

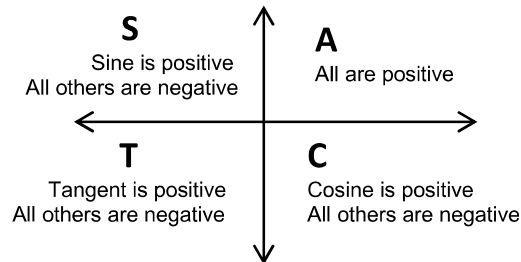
By the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$. State the three Primary Trigonometric Ratios in terms of x , y and r for angle θ .



Recall that supplementary angles have the same ratio.

Recall the CAST rule. We can use this to help us remember the signs of the trigonometric ratios in each of the quadrants.

CAST Rule:



Sine, Cosine and Tangent of Any Angle

Ex 1: The point $P(-3, -6)$ lies on the terminal arm of an angle θ in standard position. Determine the exact values of $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$.

Special Cases

Ex 2: Find the sine, cosine and tangent of any angle that measures 0 , $\frac{\pi}{2}$, $\frac{3\pi}{2}$ and 2π .

Case 1: Angle of 0	Case 2: Angle of $\frac{\pi}{2}$	Case 3: Angle of π	Case 4: Angle of $\frac{3\pi}{2}$

Exact Trigonometric Ratios From Radian Measures

Ex 3: Determine the exact values of

a) $\sin\left(\frac{7}{4}\pi\right)$

b) $\cos\left(\frac{5}{6}\pi\right)$

c) $\tan\left(-\frac{11\pi}{6}\right)$

Recall: Reciprocal Trigonometric Functions

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Ex 4: Determine the exact values of $\csc\left(\frac{2\pi}{3}\right)$, $\sec\left(\frac{2\pi}{3}\right)$ and $\cot\left(\frac{2\pi}{3}\right)$.

Ex 5: Determine the angle θ , where $0 \leq \theta \leq 2\pi$.

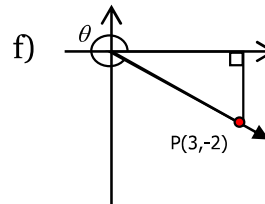
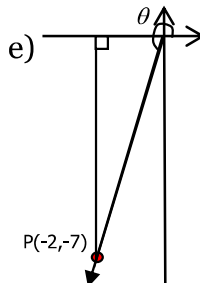
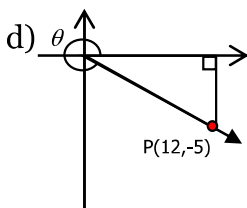
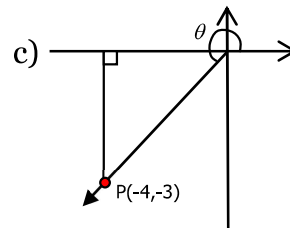
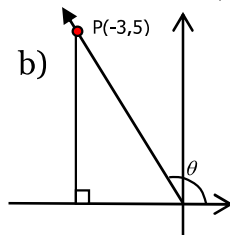
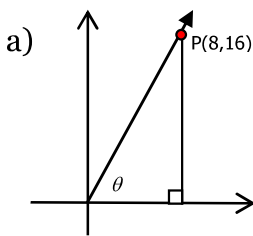
a) $\cos(\theta) = -\frac{\sqrt{3}}{2}$

b) $\sin(\theta) = -\frac{1}{\sqrt{2}}$

3.2 Practice

1. The co-ordinates of point P on the terminal arm of each $\angle\theta$ in standard position are shown,

where $0 \leq \theta \leq 2\pi$. Determine the exact values of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.



2. The co-ordinates of point P on a terminal arm of each $\angle\theta$ in standard position are shown, where $0 \leq \theta \leq 2\pi$. Determine the exact values of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

a) $P(6, 5)$

b) $P(-1, 8)$

c) $P(-2, -5)$

d) $P(6, -1)$

e) $P(2, -4)$

f) $P(-3, -9)$

g) $P(3, 3)$

h) $P(-2, 6)$

3. Find the exact value of each trigonometric ratio.

a) $\sin\left(\frac{5\pi}{4}\right)$

b) $\tan\left(\frac{11\pi}{6}\right)$

c) $\cos\left(\frac{\pi}{6}\right)$

d) $\cos\left(\frac{7\pi}{4}\right)$

e) $\tan\left(\frac{4\pi}{3}\right)$

f) $\cos\left(\frac{7\pi}{6}\right)$

g) $\sin\left(\frac{5\pi}{6}\right)$

h) $\cos\left(\frac{3\pi}{4}\right)$

4. $\angle\theta$ in standard position with its terminal arm in the stated quadrant, and $0 \leq \theta \leq 2\pi$.

A trigonometric ratio is given. Find the exact values of the other two trigonometric ratios.

a) $\sin(\theta) = \frac{4}{5}$, quadrant II

b) $\cos(\theta) = -\frac{2}{3}$, quadrant III

c) $\tan(\theta) = -\frac{5}{2}$, quadrant IV

d) $\sin(\theta) = -\frac{3}{7}$, quadrant III

5. Determine $\angle\theta$, given $0 \leq \theta \leq 2\pi$.

a) $\sin(\theta) = \frac{1}{2}$

b) $\cos(\theta) = \frac{\sqrt{3}}{2}$

c) $\tan(\theta) = -1$ d) $\sec(\theta) = -2$

e) $\cot(\theta) = -\sqrt{3}$

f) $\sin(\theta) = -\frac{5}{6}$

Warm up

1. For each function find the exact value of the given function.

(a) $\tan\left(\frac{-11\pi}{6}\right)$

(b) $\sec\left(-\frac{4\pi}{3}\right)$

(c) $\sin\left(\frac{13\pi}{6}\right)$

2. Given:

$$13\sin(\alpha) + 5 = 0 \quad \left(0 < \alpha < \frac{3\pi}{2}\right)$$

$$13\cos(\beta) - 12 = 0 \quad \left(\frac{3\pi}{2} < \beta < 2\pi\right)$$

Draw a sketch and determine the following, without the use of a calculator:

(a) $\tan(\alpha)$

(b) $\tan(\beta)$

(c) $\sin(\beta)$

(d) $\cos(\alpha)$

