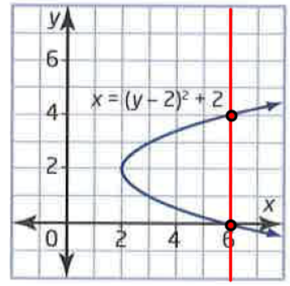
L1 – 1.1 – Power Functions Lesson
MHF4U

Things to Remember About Functions

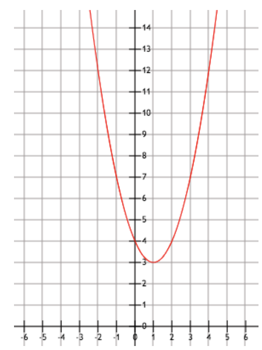


- A relation is a function if for every x-value there is only 1 corresponding y-value. The graph of a relation represents a function if it passes the **vertical line test**, that is, if a vertical line drawn anywhere along the graph intersects that graph at no more than one point.

- The **DOMAIN** of a function is the complete set of all possible values of the independent variable (x)
  - Set of all possible x-vales that will output real y-values

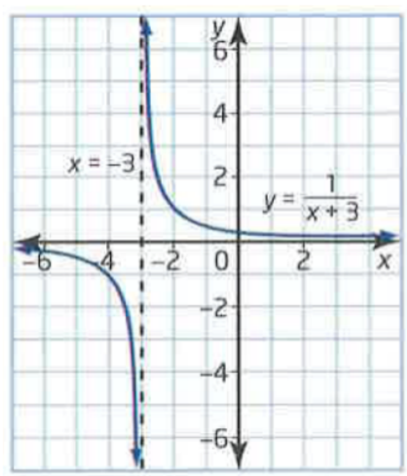
- The **RANGE** of a function is the complete set of all possible resulting values of the dependent variable (y)
  - Set of all possible y-values we get after substituting all possible x-values

- For the function $f(x) = (x-1)^2 + 3$
  - D: $\{X \in \mathbb{R}\}$
  - R: $\{Y \in \mathbb{R} | y \geq 3\}$



- The degree of a function is the highest exponent in the expression
  - $f(x) = 6x^3 - 3x^2 + 4x - 9$ has a degree of **3**

- An **ASYMPTOTE** is a line that a curve approaches more and more closely but never touches.



The function $\boldsymbol{y = \frac{1}{x+3}}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq -3$. This is why the vertical line $x = -3$ is an asymptote for this function.

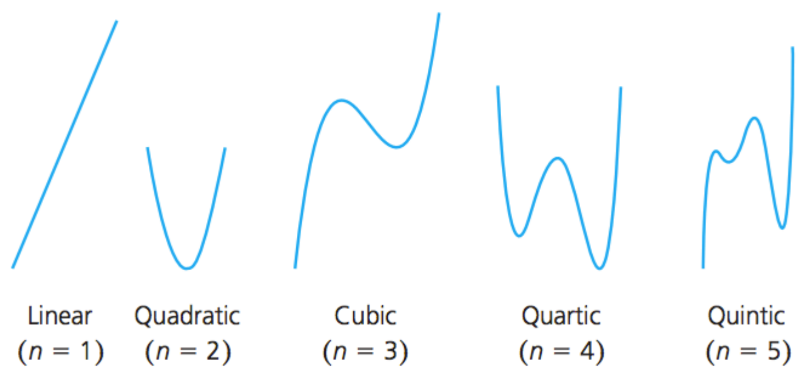
Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

Polynomial Functions

A ***polynomial function*** has the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x^1 + a_0$$

- n Is a whole number
- x Is a variable
- the **coefficients** $a_0, a_1, \ldots, a_n$ are real numbers
- the **degree** of the function is n, the exponent of the greatest power of x
- a_n, the coefficient of the greatest power of x, is the **leading coefficient**
- a_0, the term without a variable, is the **constant term**
- The domain of a polynomial function is the set of real numbers D: $\{X \in \mathbb{R}\}$
- The range of a polynomial function may be all real numbers, or it may have a lower bound or an upper bound (but not both)
- The graph of polynomial functions do not have horizontal or vertical asymptotes
- The graphs of polynomial functions of degree 0 are **horizontal lines**. The shapes of other graphs depends on the degree of the function. Five typical shapes are shown for various degrees:



A **power function** is the simplest type of polynomial function and has the form:

$$f(x) = ax^n$$

- a is a real number
- x is a variable
- n is a whole number

Example 1: Determine which functions are polynomials. State the degree and the leading coefficient of each polynomial function.

a) $g(x) = \sin x$

This is a trigonometric function, not a polynomial function.

b) $f(x) = 2x^4$

This is a polynomial function of degree 4.
The leading coefficient is 2

c) $y = x^3 - 5x^2 + 6x - 8$

This is a polynomial function of degree 3.
The leading coefficient is 1.

d) $g(x) = 3^x$

This is not a polynomial function but an exponential function, since the base is a number and the exponent is a variable.

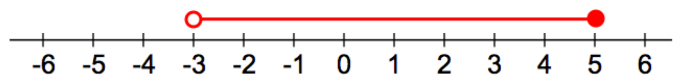
Interval Notation

In this course, you will often describe the features of the graphs of a variety of types of functions in relation to real-number values. Sets of real numbers may be described in a variety of ways:

1) as an inequality $-3 < x \le 5$

2) interval (or bracket) notation $(-3, 5]$

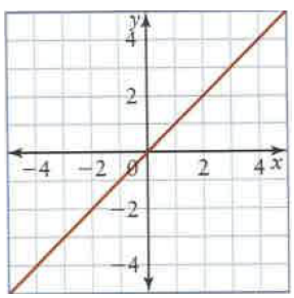
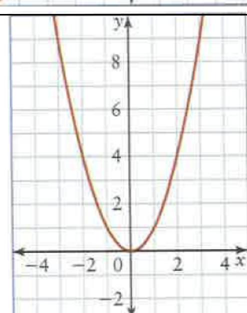
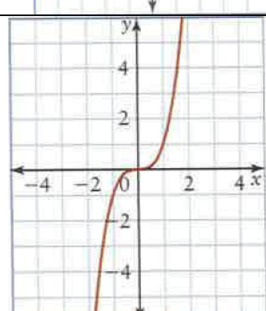
3) graphically on a number line



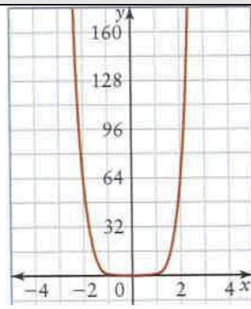
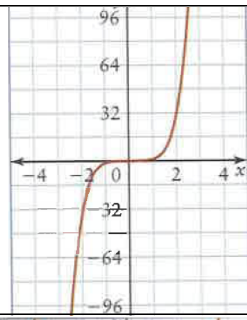
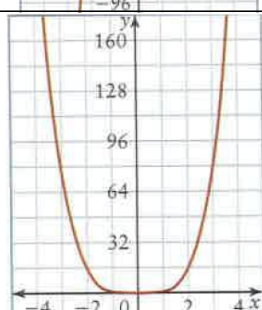
Note:

- Intervals that are infinite are expressed using ∞ (infinity) or $-\infty$ (negative infinity)
- **Square brackets** indicate that the end value is included in the interval
- **Round brackets** indicate that the end value is NOT included in the interval
- A **round** bracket is always used at infinity and negative infinity

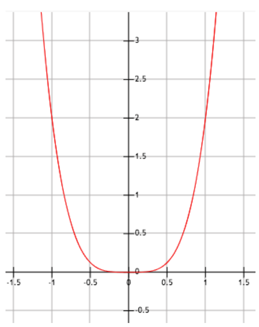
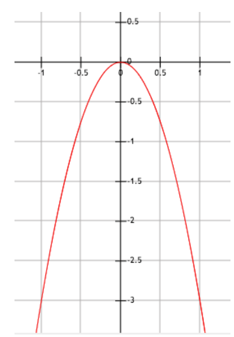
Example 2: Below are the graphs of common power functions. Use the graph to complete the table.

Power Function	Special Name	Graph	Domain	Range	End Behaviour as $x \to -\infty$	End Behaviour as $x \to \infty$
$y = x$	Linear		$(-\infty, \infty)$	$(-\infty, \infty)$	$y \to -\infty$ Starts in quadrant 3	$y \to \infty$ Ends in quadrant 1
$y = x^2$	Quadratic		$(-\infty, \infty)$	$[0, \infty)$	$y \to \infty$ Starts in quadrant 2	$y \to \infty$ Ends in quadrant 1
$y = x^3$	Cubic		$(-\infty, \infty)$	$(-\infty, \infty)$	$y \to -\infty$ Starts in quadrant 3	$y \to \infty$ Ends in quadrant 1

[redacted]

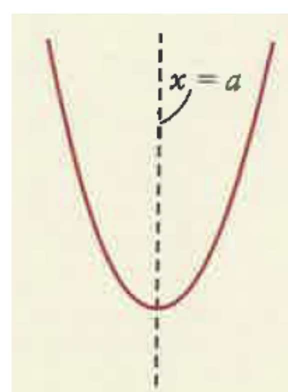
Power Function	**Special Name**	**Graph**	**Domain**	**Range**	**End Behaviour as $x \to -\infty$**	**End Behaviour as $x \to \infty$**
$y = x^4$	Quartic		$(-\infty, \infty)$	$[0, \infty)$	$y \to \infty$ Starts in quadrant 2	$y \to \infty$ Ends in quadrant 1
$y = x^5$	Quintic		$(-\infty, \infty)$	$[-\infty, \infty)$	$y \to -\infty$ Starts in quadrant 3	$y \to \infty$ Ends in quadrant 1
$y = x^6$	Sextic		$(-\infty, \infty)$	$[0, \infty)$	$y \to \infty$ Starts in quadrant 2	$y \to \infty$ Ends in quadrant 1
[redacted]						

Key Features of EVEN Degree Power Functions

When the leading coefficient (a) is positive		When the leading coefficient (a) is negative	
End behaviour	as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to \infty$ Q2 to Q1	**End behaviour**	as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to -\infty$ Q3 to Q4
Domain	$(-\infty, \infty)$	**Domain**	$(-\infty, \infty)$
Range	$[0, \infty)$	**Range**	$[0, -\infty)$
Example: $f(x) = 2x^4$		**Example:** $f(x) = -3x^2$	

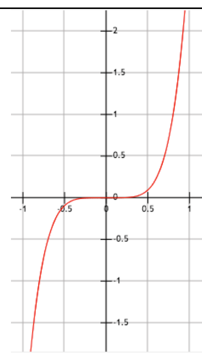
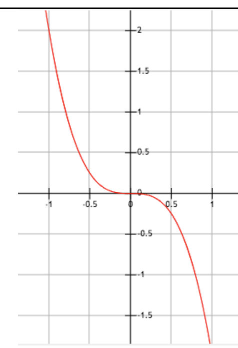
Line Symmetry

A graph has line symmetry if there is a vertical line $x = a$ that divides the graph into two parts such that each part is a reflection of the other.



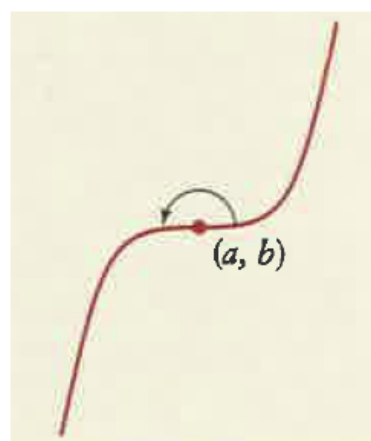
Note: The graphs of even degree power functions have line symmetry about the vertical line $x = 0$ (the y-axis).

Key Features of ODD Degree Power Functions

When the leading coefficient (a) is positive		When the leading coefficient (a) is negative	
End behaviour	as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$ Q3 to Q1	**End behaviour**	as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$ Q2 to Q4
Domain	$(-\infty, \infty)$	**Domain**	$(-\infty, \infty)$
Range	$(-\infty, \infty)$	**Range**	$(-\infty, \infty)$
Example: $f(x) = 3x^5$		**Example:** $f(x) = -2x^3$	

Point Symmetry

A graph has point point symmetry about a point (a, b) if each part of the graph on one side of (a, b) can be rotated 180° to coincide with part of the graph on the other side of (a, b).



Note: The graph of odd degree power functions have point symmetry about the origin (0, 0).

Example 3: Write each function in the appropriate row of the second column of the table. Give reasons for your choices.

$y = 2x$ $\qquad$ $y = 5x^6$ $\qquad$ $y = -3x^2$ $\qquad$ $y = x^7$

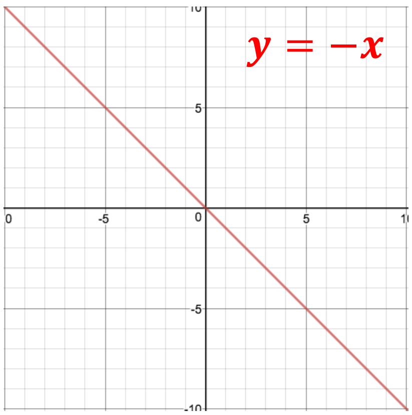
$y = -\frac{2}{5}x^9$ $\qquad$ $y = -4x^5$ $\qquad$ $y = x^{10}$ $\qquad$ $y = -0.5x^8$

End Behaviour	Functions	Reasons
Q3 to Q1	$y = 2x$ $y = x^7$	Odd exponent Positive leading coefficient
Q2 to Q4	$y = -\frac{2}{5}x^9$ $y = -4x^5$	Odd exponent Negative leading coefficient
Q2 to Q1	$y = 5x^6$ $y = x^{10}$	Even exponent Positive leading coefficient
Q3 to Q4	$y = -3x^2$ $y = -0.5x^8$	Even exponent Negative leading coefficient

Example 4: For each of the following functions

i) State the domain and range
ii) Describe the end behavior
iii) Identify any symmetry

a)

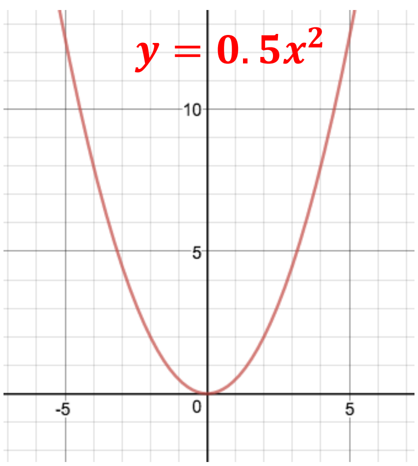


i) Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

ii) As $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$
The graph extends from quadrant 2 to 4

iii) Point symmetry about the origin (0, 0)

b)

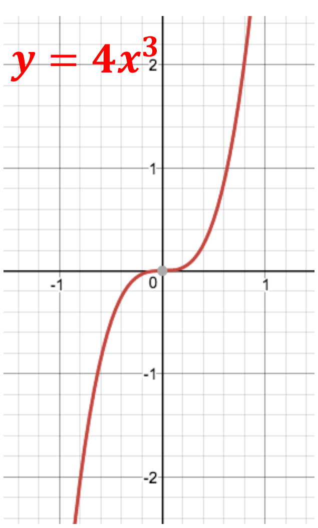


i) Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

ii) As $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to \infty$
The graph extends from quadrant 2 to 1

iii) Line symmetry about the line $x = 0$ (the y-axis)

c)



i) Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

ii) As $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$
The graph extends from quadrant 3 to 1

iii) Point symmetry about the origin (0, 0)