# Unit 5: Exponential and Logarithmic Functions
## 5.2 Laws of Logarithms

There are some basic properties of logarithms:

**Basic Property** **Proof**

#1: $\boxed{\log_b 1 = 0}$

#2: $\boxed{\log_b b = 1}$

#3: $\boxed{\log_b b^x = x}$

#4: $\boxed{b^{\log_b x} = x}$

Logarithms are exponents and the Exponent Laws give rise to corresponding Laws of Logarithms.
**Recall**: the power law of exponents

$$\left(b^x\right)^y = b^{xy}$$

We can use the power law of exponents to develop a power law of logarithms.

**The Power Law**: the logarithm of a power of a number is equal to the exponent multiplied by the logarithm of the number. $\boxed{\log_a\left(b^n\right) = n\log_a b}$

Proof:

**Ex. 1**: Rewrite the following:

a. $\log_6 36^4$ b. $3\log_5 25$ c. $\log_3 \sqrt{27}$

**Ex. 2**: Determine $\log_3 81^{\frac{51003}{2}}$.

**Ex. 3**: **Change of base Formula**: Show that $\log_b x = \dfrac{\log_a x}{\log_a b}$, where a > 0.

**Ex. 4:** Find the value of $\log_3 23$, correct to two decimal places.

**Ex. 5:** Prove that $\log_t b = \dfrac{1}{\log_b t}$.

**The Product Law**

The logarithm of a product is equal to the sum of the logarithms of the factors.

$$\boxed{\log_a (mn) = \log_a m + \log_a n}$$

Proof:

**Ex. 6:** Evaluate using properties of logarithms.

a. $\log_6 12 + \log_6 3$

b. $\log 25 + \log 4$

**c.** $\log_{\frac{1}{9}} \left(\sqrt[3]{81}\right)^2$

**Ex. 7:** Write $\log_a x^3 y^4$ in terms of $\log_a x$ and $\log_a y$.

**The Quotient Law**

The logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator.

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Proof:

**Ex. 8:** Evaluate using properties of logarithms.

a. $\log_2\left(\frac{25.6}{6.4}\right)$ b. $\log_4 48 - \log_4 3$ c. $\log_5\left(\frac{1}{25}\right)$

**Ex. 9:** Write the following expressions as a single logarithm.

a) $3\log(x+3)-2\log(x-1)$

b) $2\log(x+5)-\log(x^2-25)+3\log x$

c) $\log\left(\frac{\sqrt[6]{x^5}}{x^2}\right)-\log\sqrt[3]{x}$

**Ex. 10:** Fully expand and simplify the following expressions.

$log_k\left(\sqrt{\frac{x^3y^2}{w}}\right)$

**Practice**

**1.** Evaluate using properties of logarithms.

a) $4\log_4(\log 100)$

b) $\frac{3}{\log_8 2}-\frac{5}{\log_2 2}-\frac{2}{\log_4 2}$

c) $\sqrt{25^{\frac{1}{\log_6(5)}}+49^{\frac{1}{\log_8(7)}}}$

d) $\frac{\log_3\sqrt{243\sqrt{81\sqrt[3]{3}}}}{\log_2\sqrt[4]{64}+\log_5 5^{-10}}$

e) $\log_9(243)^{-4}-\log_6\left(\frac{1}{1296}\right)+\log_5\sqrt[3]{78125}$

**2.** If $(x+y)^2=4xy$, prove that $\log_a\left(\frac{x+y}{2}\right)=\frac{1}{2}(\log_a x+\log_a y)$.

**3.** If $\log_8 5=m$ and $\log_4 3=n$ find an expression for $\log_2 15$ in terms of *m* and *n*.

4. If $\log_a b=\frac{2}{x^3}$ and $\log_b\sqrt{a}=4x^5$, determine x.

## Exit Card!

1. Write $\log(x) - 4\log(x-5) + \frac{2}{3}\log\sqrt{x+1}$ as a single logarithm.

2. Evaluate $\left(\frac{1}{5}\right)^{-2+\log_{\sqrt{5}}10}$.

# Warm Up- Exponential and Logarithmic Functions

Multiple Choice: Write the capital letter corresponding to the most correct answer on the line provided

_____1. The inverse of $f(x)=1+3^x$ is :

A) $f^{-1}(x)=\log_{\frac{1}{3}}(x-1)$ B) $f^{-1}(x)=\left(\frac{1}{3}\right)^x-1$

C) $f^{-1}(x)=\log_3(x-1)$ D) $f^{-1}(x)=3\log(x-1)$

_____2. If $\log(2)=m$, and $\log(3)=n$, $\log_6(5)$ in terms of m and n is equivalent to .....

A) $\frac{m}{n}$ B) $1-\frac{m}{n+m}$ C) $\frac{1-m}{n+m}$ D) $\frac{m-1}{n+m}$

_____ 3. $\log_8\left(\frac{\sqrt{2}}{4}\right)$ is equal to:

A) $-\frac{1}{2}$ B) $-\frac{8}{6}$ C) $\frac{1}{2}$ D) $-\frac{3}{5}$

4. Evaluate. Show steps. Leave answers in exact form.

a) $\log_3\left(\log_3\left(\sqrt[3]{\sqrt[3]{3}}\right)\right)$ b) $2^{\log_2 5+2\log_2 3}$ c) $\frac{\log_{0.2}(125)^2}{\log_8\sqrt{512}}$

5. Let $f(x)=\log_5(x^2-x-2)-2\log_5(x+1)$ .Determine the domain of the function and express f(x) as a single logarithm.

6. If $\log_{12}(3)=m$, find the value of $\log_{\sqrt{3}}(16)$ in terms of $m$.