Rational Functions

Workbook
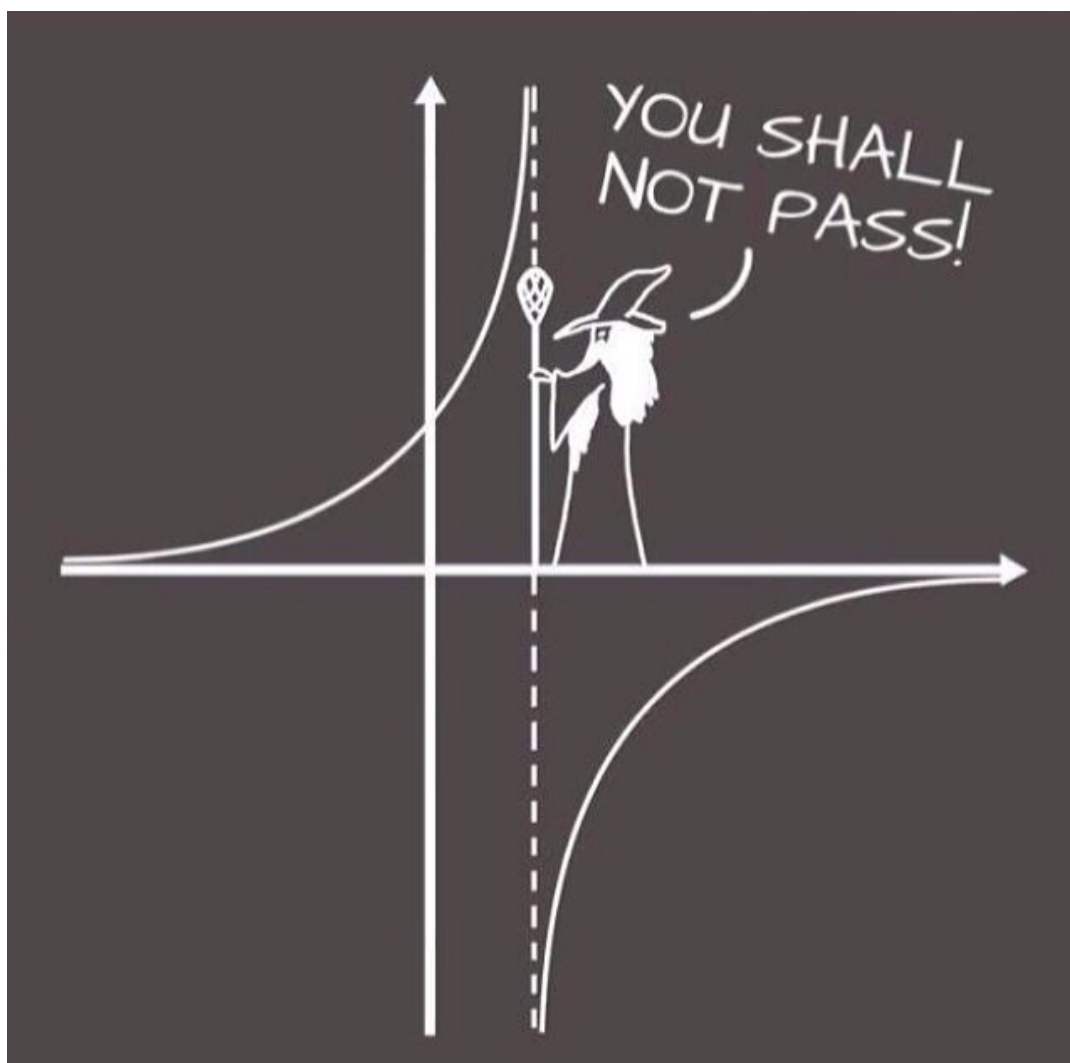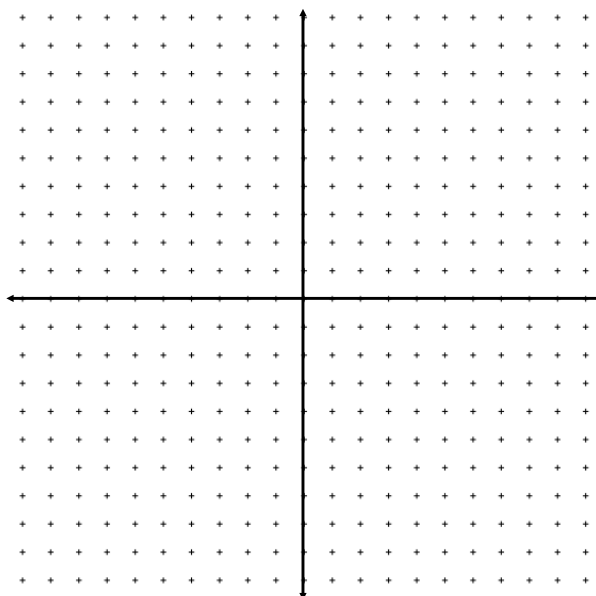
MHF4U

1) Graph each of the following reciprocal functions. Start by graphing the function in the denominator. Show as much characteristic information about the graph as you can (e.g. intercepts, asymptotes with equations, other defining points, etc).
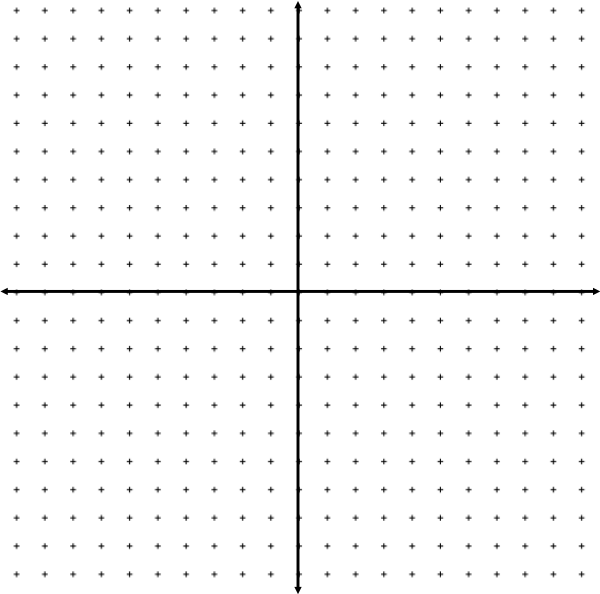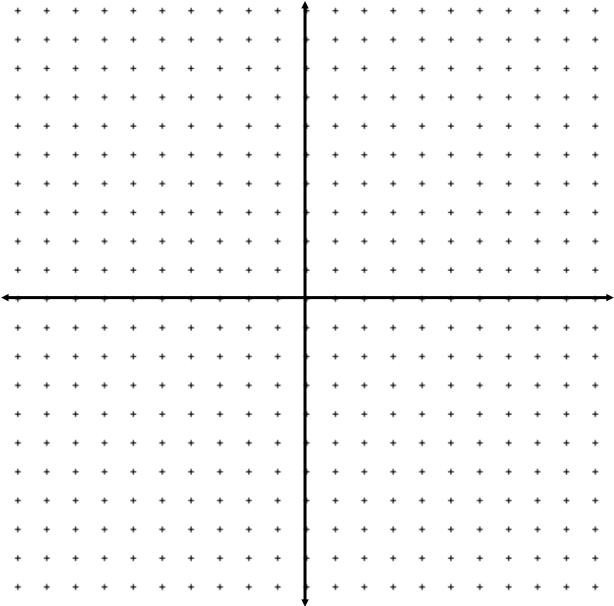
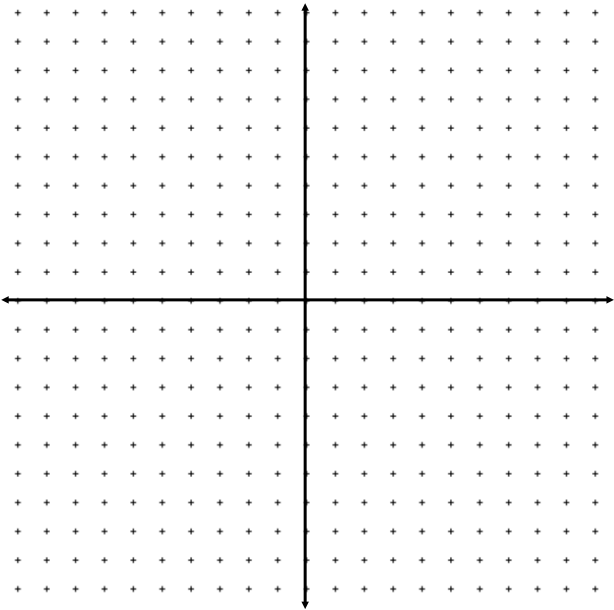**a)** $f(x) = \frac{1}{x-1}$
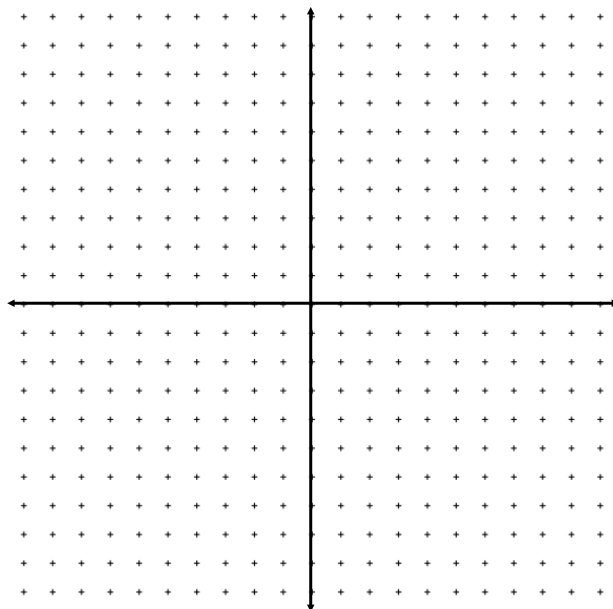
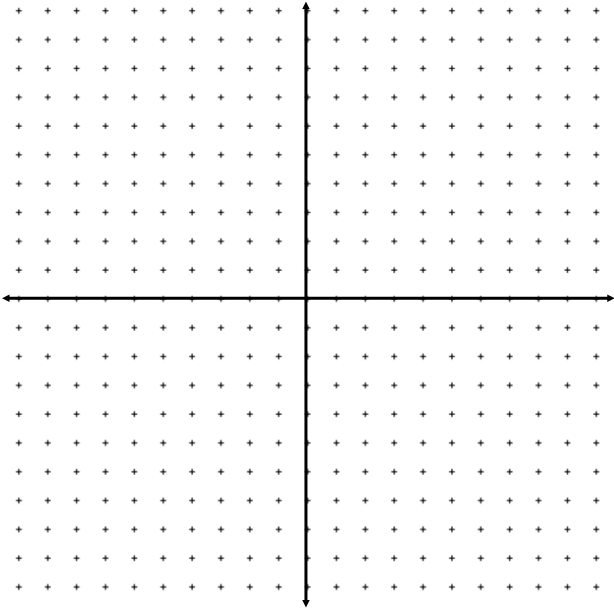**b)** $g(x) = -\frac{2}{x+4}$

c) $h(x) = \frac{1}{x^2 - 9}$
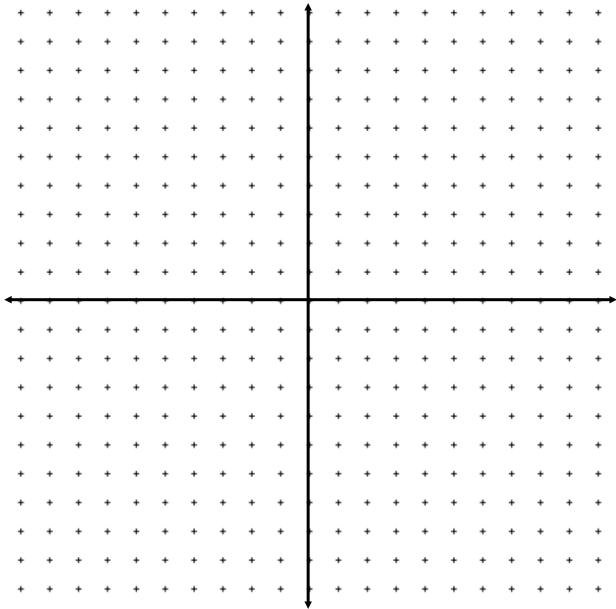
d) $j(x) = \frac{1}{x^2 - 2x - 15}$

e) $k(x) = \frac{1}{x^2+2}$

f) $m(x) = \frac{4}{x^2+x-6}$

g) $n(x) = -\dfrac{1}{4x^2-4x-3}$

h) $p(x) = \dfrac{4}{2x^2-8x+9}$
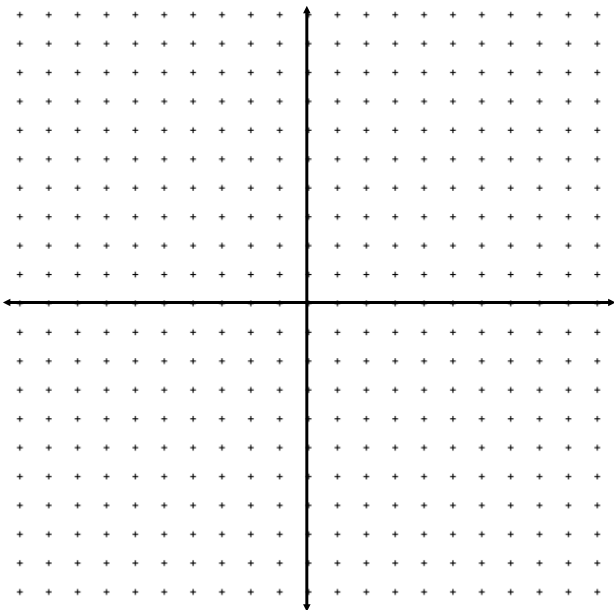
1) State the equation of the vertical and horizontal asymptotes for each function.

a) $p(x) = \frac{x}{x-6}$
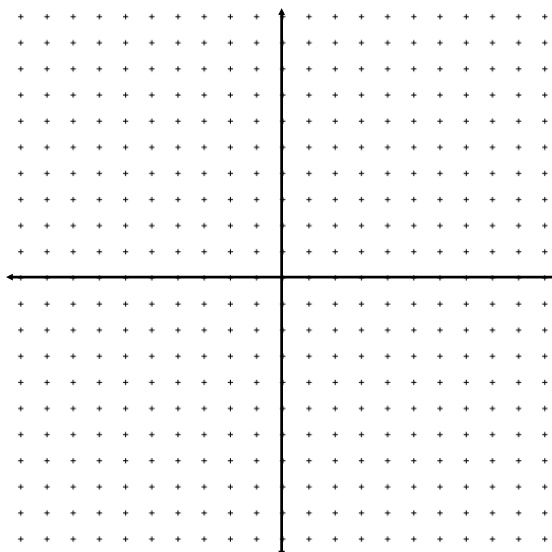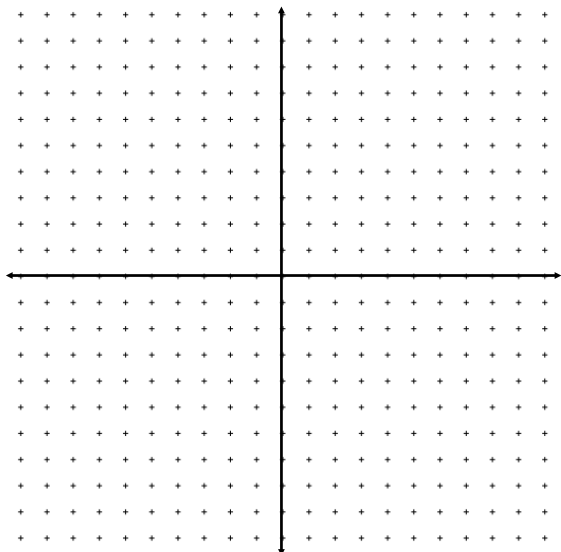
b) $q(x) = \frac{3x}{x+4}$

c) $r(x) = \frac{x-1}{x+1}$
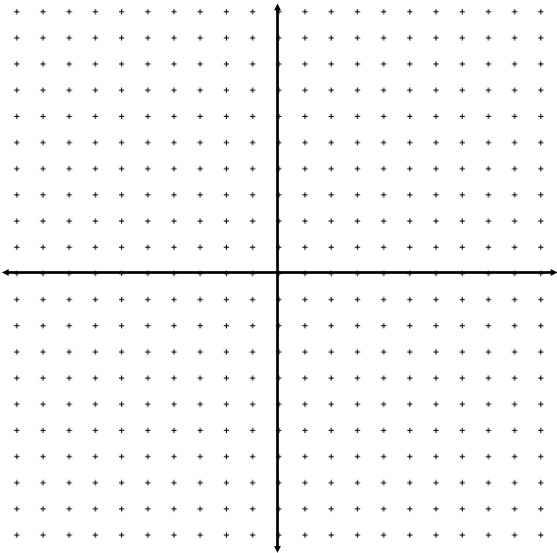
d) $\frac{5x-2}{2x+3}$

2) Graph each of the following functions. Make sure to identify key characteristics of the functions including intercepts and asymptotes.
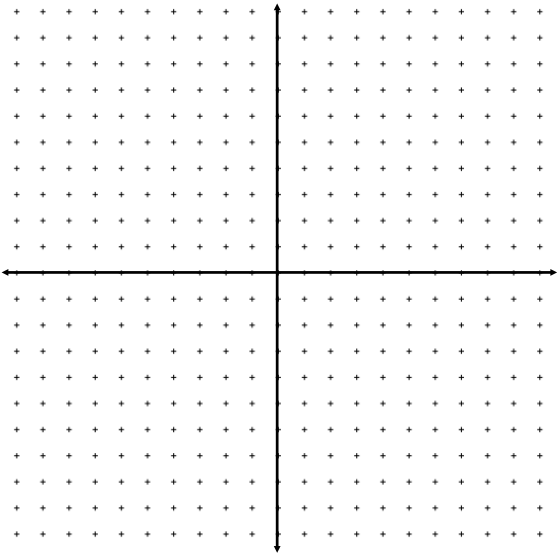
a) $f(x) = \frac{x}{x-5}$
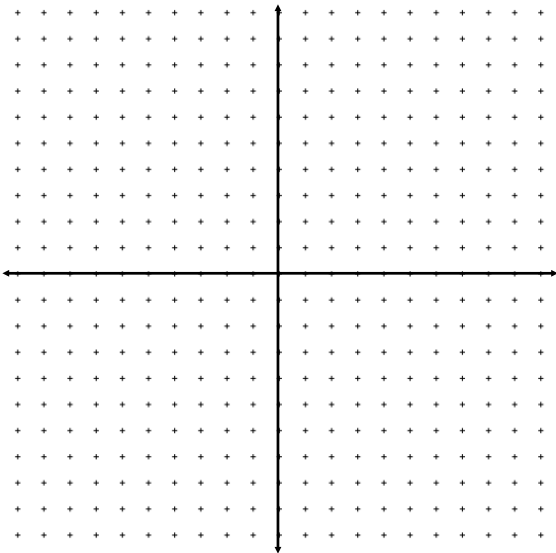
b) $c(x) = \frac{4x}{x+8}$
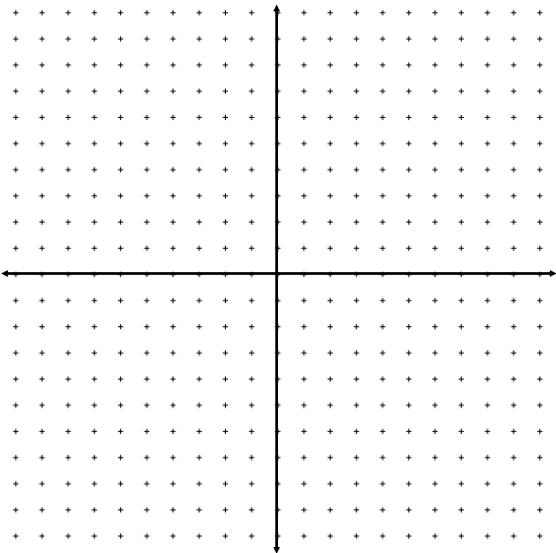
c) $k(x) = \frac{x+1}{4-x}$

d) $w(x) = \frac{x+2}{4x-5}$

e) $d(x) = \frac{-2x-3}{x+5}$

f) $m(x) = \frac{3x+1}{2x+1}$

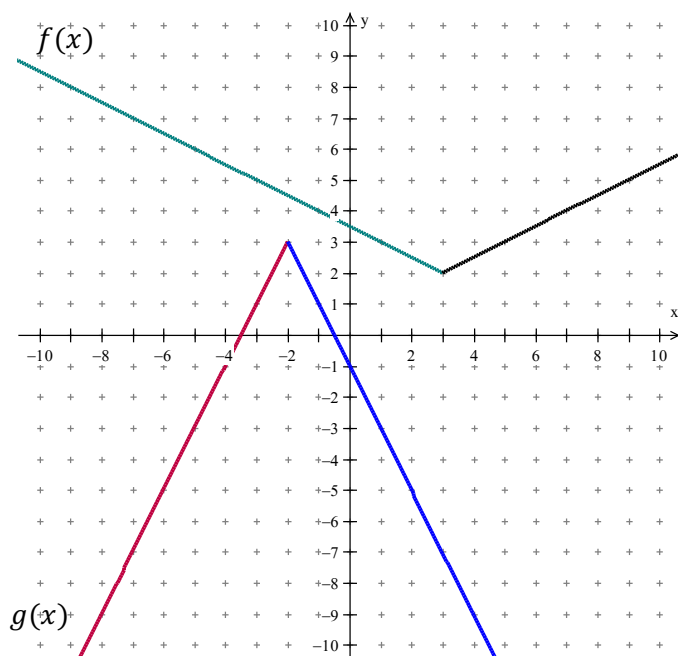**g)** $g(x) = \dfrac{x-2}{x^2+3x+2}$
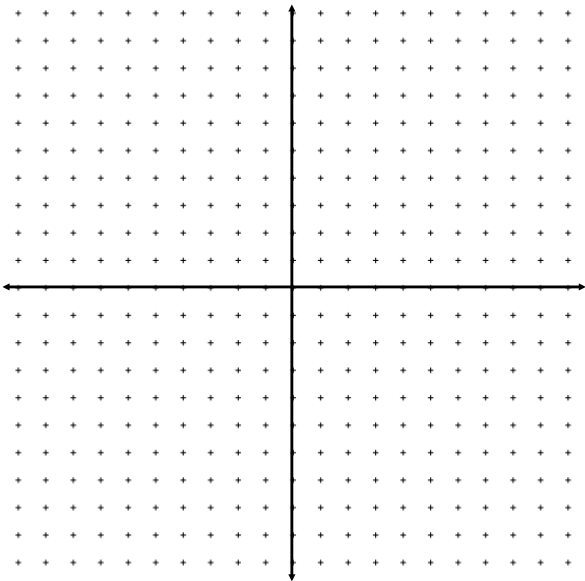
1) Let $f(x) = 3x - 5$ and $g(x) = 2x + 3$.

a) Write the equation for $h(x) = f(x) + g(x)$ and determine the value of $h(2)$.
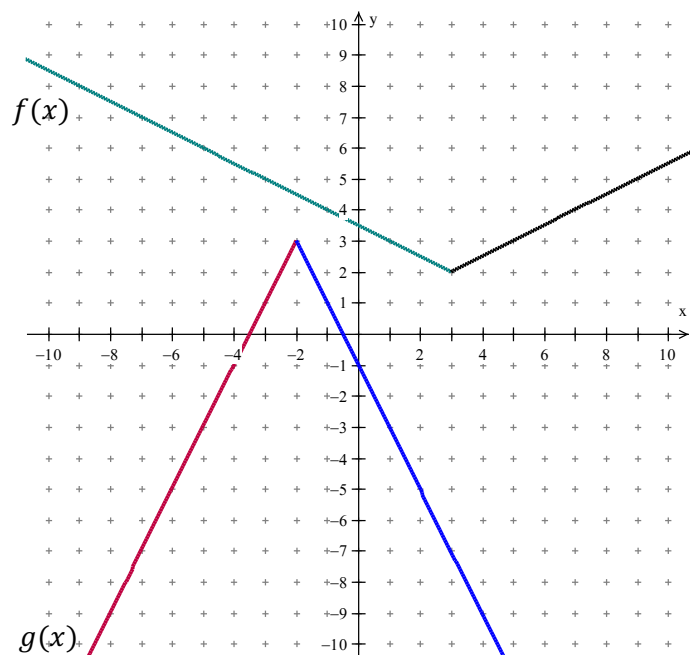
b) Write the equation for $k(x) = f(x) - g(x)$ and determine the value of $k(2)$.

2) Use the functions $f(x)$ and $g(x)$ as shown. Apply the superposition principle to graph
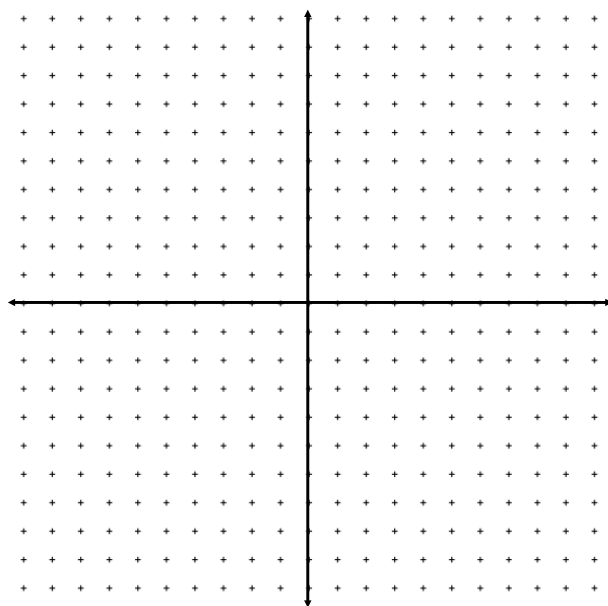
a) $y = f(x) + g(x)$
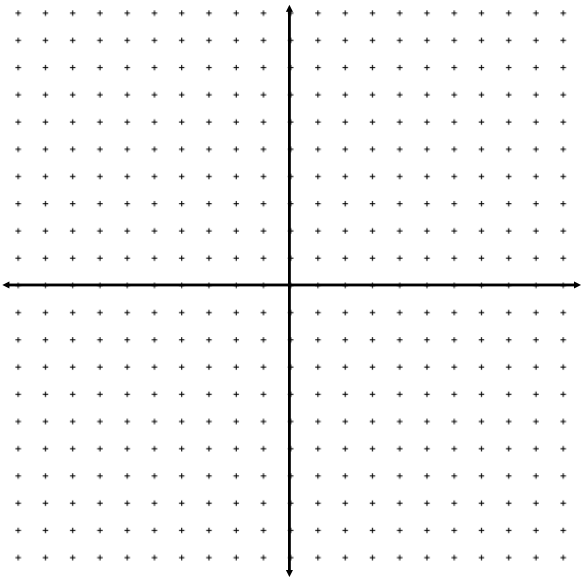
b) $y = f(x) - g(x)$.

3) Let $f(x) = x - 2$ and $g(x) = x^2 + 3x - 3$. Determine an algebraic and graphical model for $h(x) = f(x) + g(x)$.
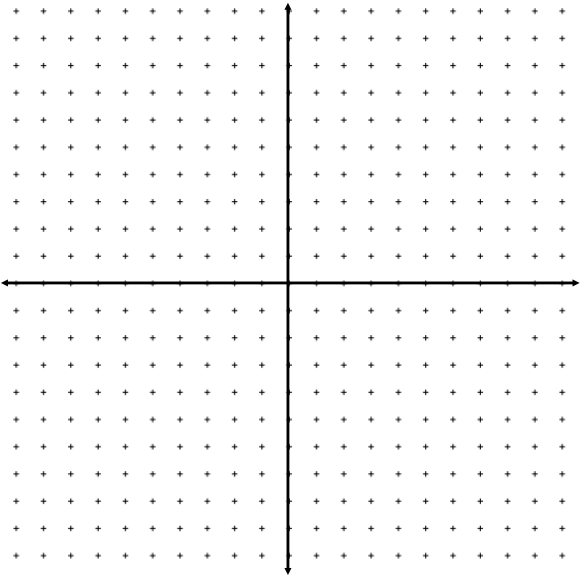
4) Let $f(x) = x - 2$ and $g(x) = x^2 - 4$. Develop an algebraic and graphical model for each of the following:

a) $y = f(x)g(x)$

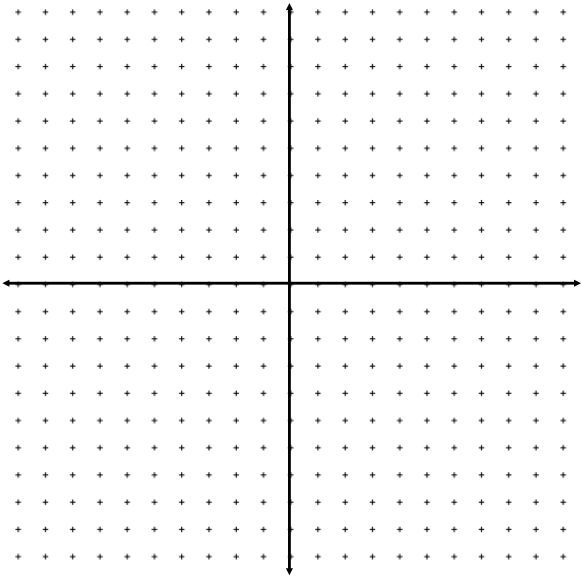**b)** $y = \dfrac{f(x)}{g(x)}$

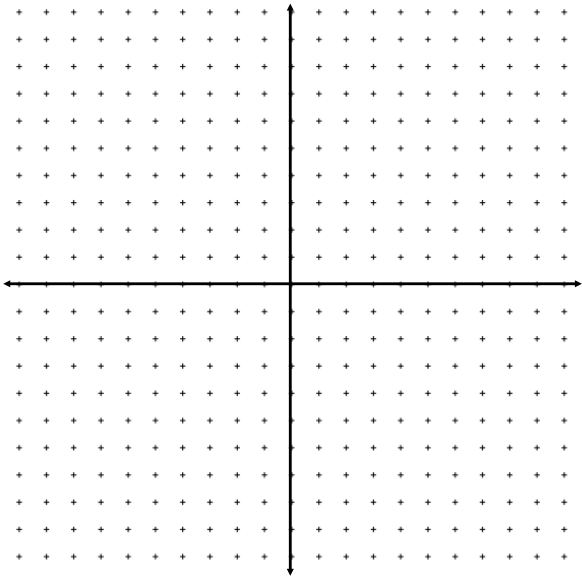**c)** $y = \dfrac{g(x)}{f(x)}$

5) Let $f(x) = x^2 + 2x - 4$ and $g(x) = \frac{1}{x+1}$.

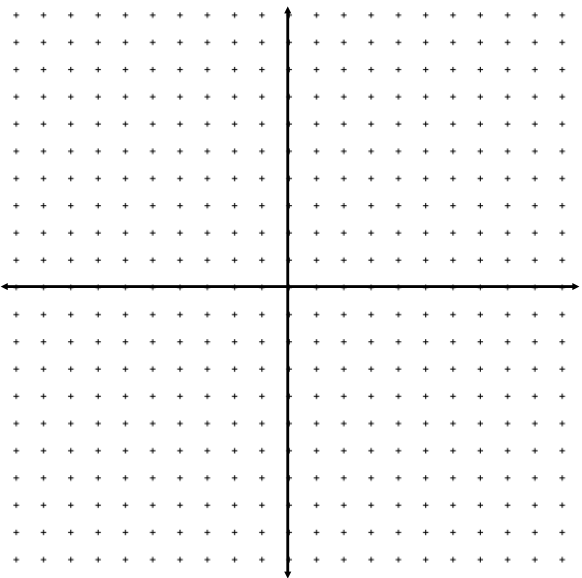**a)** Evaluate $g(f(0))$

b) Evaluate $f(g(-2))$

6) Let $f(x) = x^2 + 3x$ and $g(x) = 2x - 5$. Determine an equation for each composite function and graph it.

a) $y = f(g(x))$

b) $y = g(f(x))$

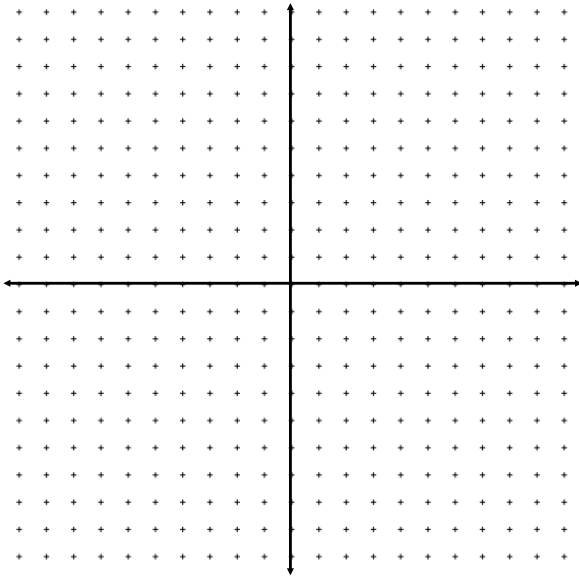**c)** $y = g\big(g(x)\big)$
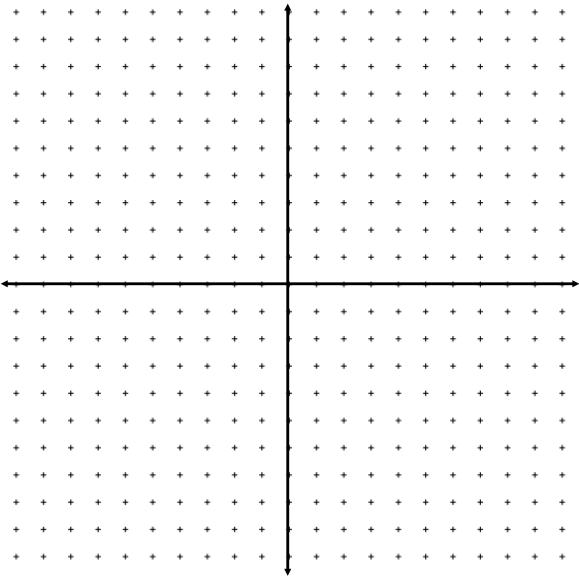
d) $y = g^{-1}\big(g(x)\big)$

1) Solve each equation algebraically.

a) $\dfrac{6}{2x-1} = 5$

b) $\dfrac{6}{x} = x - 5$

c) $1 = \dfrac{5}{3x^2 - 8x + 2}$

d) $\dfrac{x+5}{x-1} = \dfrac{x+1}{x-3}$

e) $\dfrac{3}{x+5} + \dfrac{4}{x} = 0$

f) $2x = 5 - \dfrac{3}{x}$

g) $\dfrac{2}{1-x} + \dfrac{3}{x+1} = \dfrac{1}{x}$

h) $\dfrac{3}{x-1} + 5 + \dfrac{2}{x} = 0$

2) Solve each inequality without using technology.

a) $\dfrac{4}{2x-3} < \dfrac{1}{x+4}$

b) $\dfrac{2x+3}{x-3} \geq \dfrac{6x-5}{3x+1}$

c) $\dfrac{(x-3)(2x-1)}{(x+4)(x-5)} > 0$

d) $\dfrac{2x^2+5x-3}{x^2+5x+4} \leq 0$

e) $\frac{4}{x-3} < 1$

f) $\frac{2x^2+5x-3}{x^2+8x+16} < 0$

g) $\frac{x}{x+3} > \frac{x}{x-1}$

h) $\frac{2x+3}{x} > \frac{x+1}{x}$