

## W6 – 6.5 – Applications of Logarithms AND Exponentials in Physical Sciences

MHF4U

### *Exponential Formulas*

$$A(t) = A_0(1+i)^t$$

general, where  $i$  is  
percent growth(+) or  
decay(-)

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$$

half-life,  $H$  is  
the half-life period

$$A(t) = A_0(2)^{\frac{t}{D}}$$

doubling,  $D$  is  
the doubling period

### *Logarithmic Formulas*

$$pH = -\log[H^+]$$

Where pH is acidity and  
[H<sup>+</sup>] is concentration of  
hydronium ions mol/L

$$\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$$

Where  $\beta$  is loudness in dB  
and  $I$  is intensity of sound  
in W/m<sup>2</sup>

$$M = \log\left(\frac{I}{I_0}\right)$$

Where  $M$  is magnitude  
measure by richters,  
 $I$  is intensity

1) The half-life of a radioactive form of tritium is about 2 years. How much of a 5-kg sample of this material would remain after ...

a) 8 years

b) 12 months

2) The population of Littleton is currently (2014) 23000, and is increasing exponentially with a growth rate of 2% per year. Estimate when Littleton will have a population of 30000.

**3)** The population of purple martins in Algonquin park was estimated to be 35000 in 1992. Ten years later, in 2002, the population had risen to 44400.

**a)** What is the annual growth rate for the purple martin population?

**b)** Estimate the population for 2010 to the nearest hundred.

**4)** After an accident at a nuclear plant, the radiation level in the plant was 950 R (roentgens). Five hours later the level was 800 R. How long will it take before safe levels of radiation are reached, which is less than 0.01 R?

**5)** The value of a new minivan drops 40% after the first year, and then decreases exponentially at a rate of 12% per year after that. When will a minivan that cost \$35000 new be worth less than \$10000?

**6)** A crab fossil contains 38.6% of its original Carbon14 isotope, which has a half life of 5370 years. Approximately how old is the crab fossil?

**7)** A Trimark mutual fund has track record of 4.2% growth per year. What is the doubling period for this investment?

**8)** A treatment to help a patient stop smoking involves chewing nicotine gum. Each gum introduces 1.5 mg of nicotine into the patient's system. Nicotine has a half life of 3 hours. The patient will feel the urge to smoke when the level of nicotine drops below 0.45 mg in her system. If she chewed a gum at 8:00 a.m., and another at 10:00 a.m., at what time will she next feel the urge to smoke?

**9)** Determine the pH of a solution with hydronium ion concentration:

**a)** 0.01

**b)**  $1.5 \times 10^{-10}$

**10)** Determine the hydronium concentration, in moles per litre, of a solution with pH:

**a)** 8.5

**b)** 3

**11)** How many times as intense is the sound of a shout as the sound of a whisper?

**12)** A loud car stereo has a decibel level of 110 dB. How many times as intense as the sound of a loud car stereo is the sound of a rock concert speaker?

**13)** The sound intensity of a pin drop is about  $1/30\,000$  of the sound intensity of a normal conversation. What is the decibel level of a pin drop?

**14)** On September 26, 2001, an earthquake in North Bay, Ontario, occurred that was 10 000 times as intense as  $I_0$ . What was the measure of this earthquake on the Richter scale?

**15)** On February 10, 2000, an earthquake happen in Welland, Ontario, that measured 2.3 on the Richter scale.  
**a)** How many times as intense was this as a standard low-level earthquake?

**b)** On July 22, 2001, an earthquake in St. Catharines measured 1.1 on the Richter scale. How many times as intense as the St.Catharines earthquake was the Welland earthquake?

**16)** The stellar magnitude scale compares the brightness of stars using the equation  $m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$ , where  $m_2$  and  $m_1$  are the apparent magnitude of the two stars being compared (how bright they appear in the sky) and  $b_1$  and  $b_2$  are their brightness (how much light they actually emit). This relationship does not factor in how far from Earth the stars are.

**a)** Sirius is the brightest-appearing star in our sky, with an apparent magnitude of -1.5. How much brighter does Sirius appear than Betelgeuse, whose apparent magnitude is 0.12?

**b)** The Sun appears about  $1.3 \times 10^{10}$  times as brightly in our sky as does Sirius. What is the apparent magnitude of the Sun?