

4.2 Properties of Sinusoidal Functions

$$y = a \sin(k(x-d)) + c \text{ and } y = a \cos(k(x-d)) + c$$

Amplitude – Half the distance between the maximum and minimum values of a periodic function.

$$\text{amplitude} = \frac{\text{max} - \text{min}}{2}$$

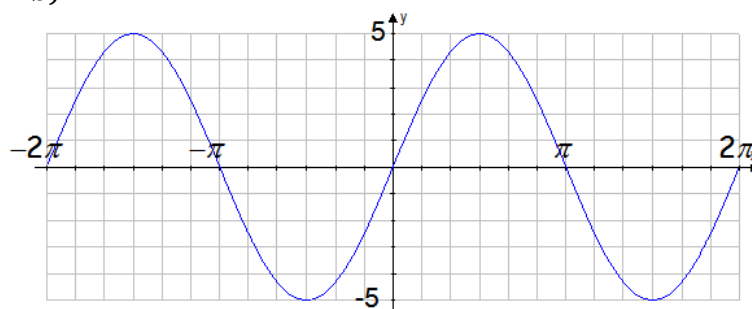
For $y = a \sin(\theta)$ or $y = a \cos(\theta)$

$$\text{Amplitude} = |a|$$

Q1. Find the amplitude for the following:

a) $y = -2 \cos(\theta)$

b)

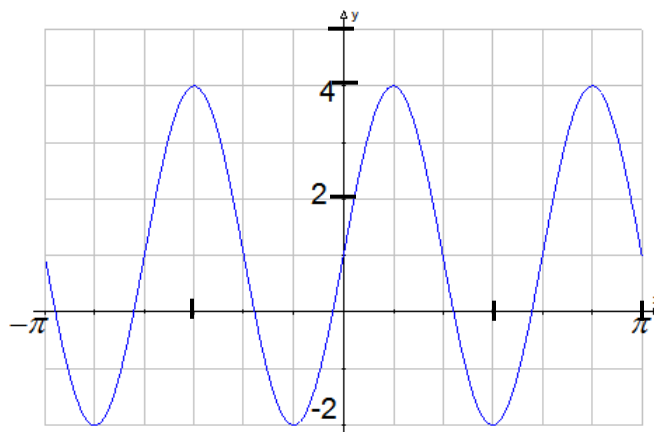


Axis of the curve – The horizontal line that is half way between maximum and minimum of values of a periodic curve is called the axis of the curve

The equation of the axis of the curve is:

$$y = \frac{\text{maximum value} + \text{minimum value}}{2}$$

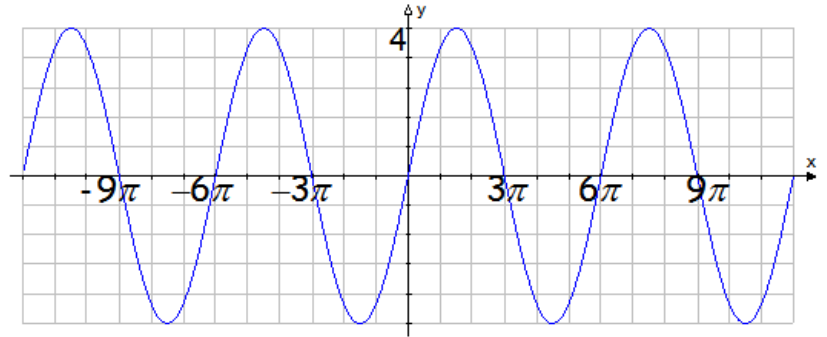
Q2. Find the equation of axis of the curve for the following graph :



Period – The horizontal length of one cycle. (The horizontal distance between two consecutive maximum or two consecutive minimum)

Recall: Pick a point on the curve. Follow the graph until it starts repeating. That is one **cycle**.

Q3. Indicate one cycle on the graph, then, state the period of the function shown:



For $y = \sin(k\theta)$ or $y = \cos(k\theta)$

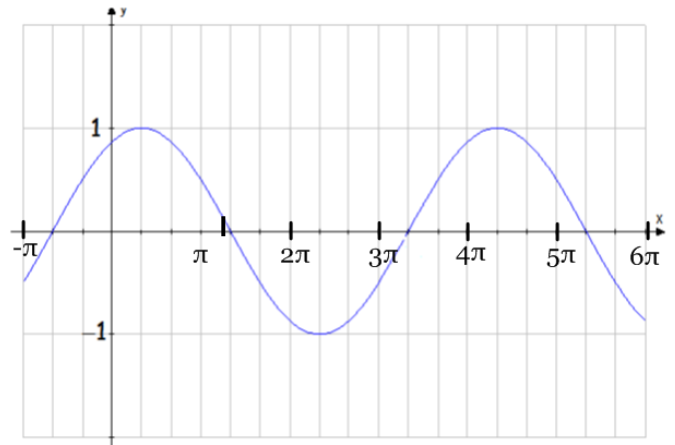
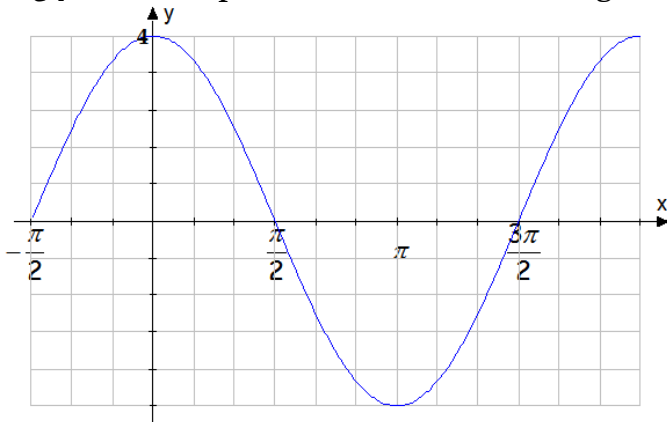
$$\text{Period} = \frac{2\pi}{|k|}$$

Phase Shift – The horizontal translation of a trigonometric function.

For $y = \sin[k(x - d)] + c$ or $y = \cos[k(x - d)] + c$

$$\text{Phase shift} = x_{\max} - \frac{P}{4}$$

Q4. State the phase shift for the following functions.

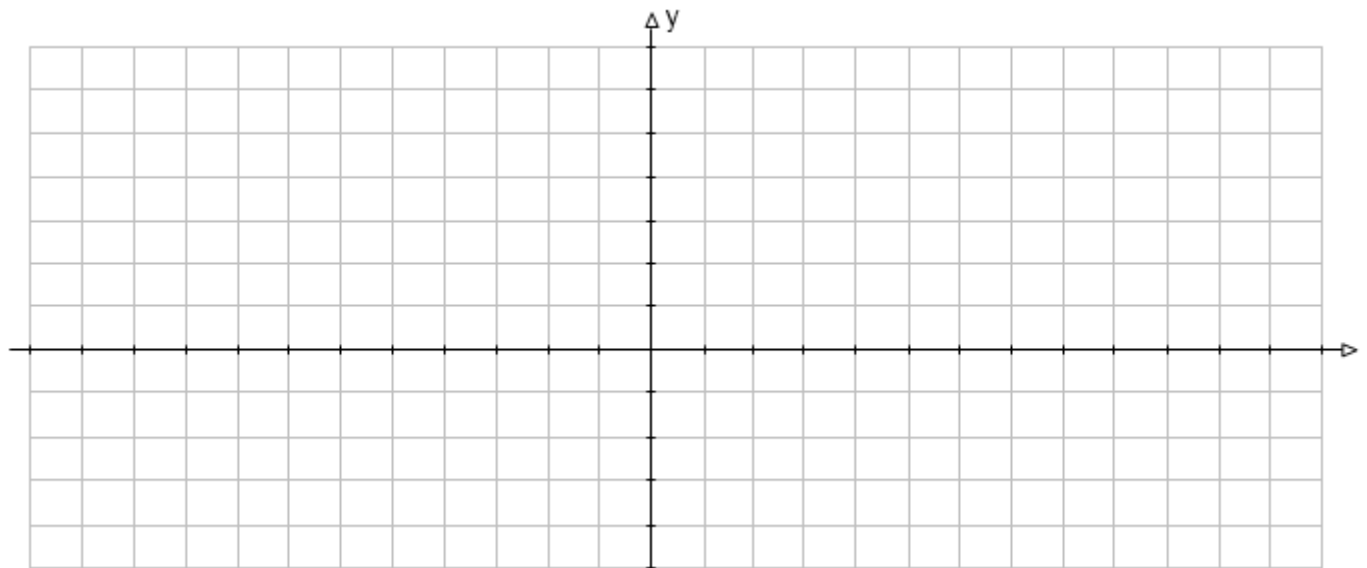


EXAMPLE 1: Complete the following table and then sketch the graph of the function using mapping rule.

a) $f(\theta) = 2 \sin(2\theta - \frac{\pi}{3})$.

$(\theta, y) \rightarrow$

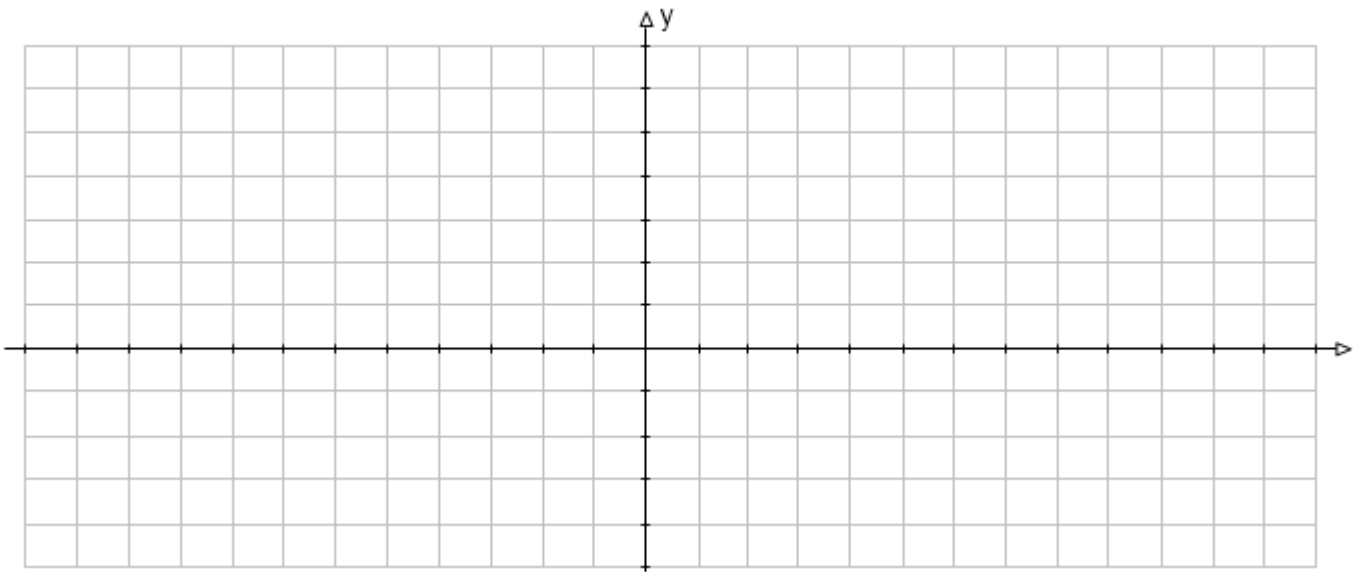
Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



b) $f(\theta) = -3\cos\frac{1}{2}(-\theta + \frac{\pi}{4})$.

$(\theta, y) \rightarrow$

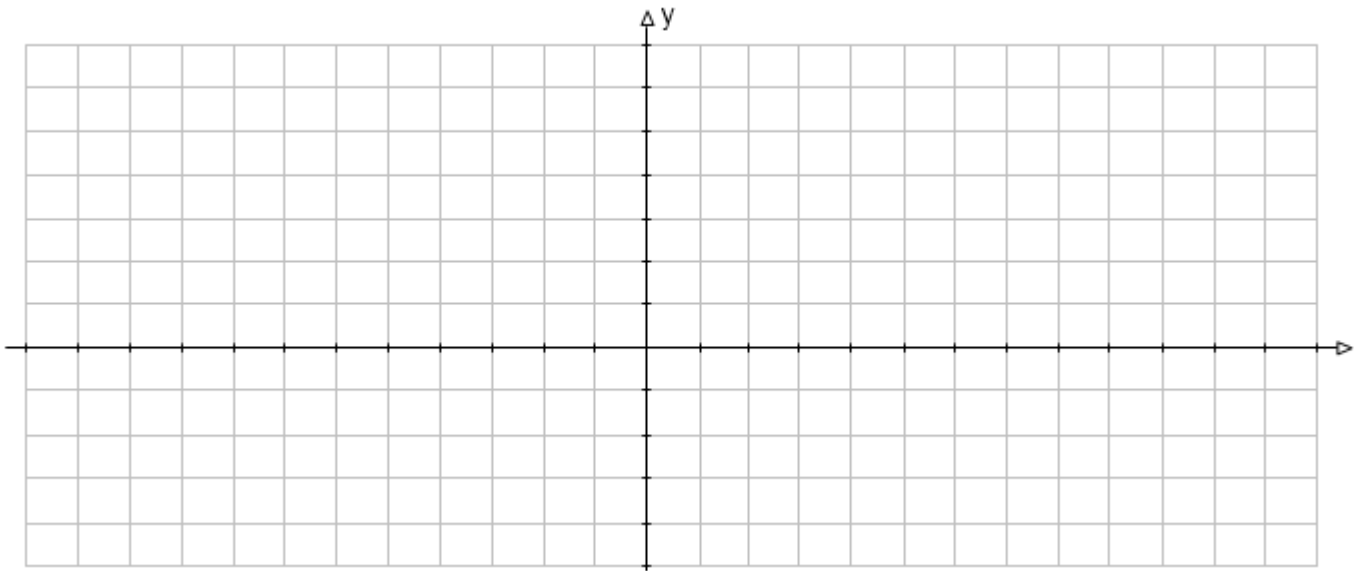
Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



c) $f(\theta) = \frac{1}{2} \cos\left(3\theta - \frac{\pi}{2}\right) + 3$.

$(\theta, y) \rightarrow$

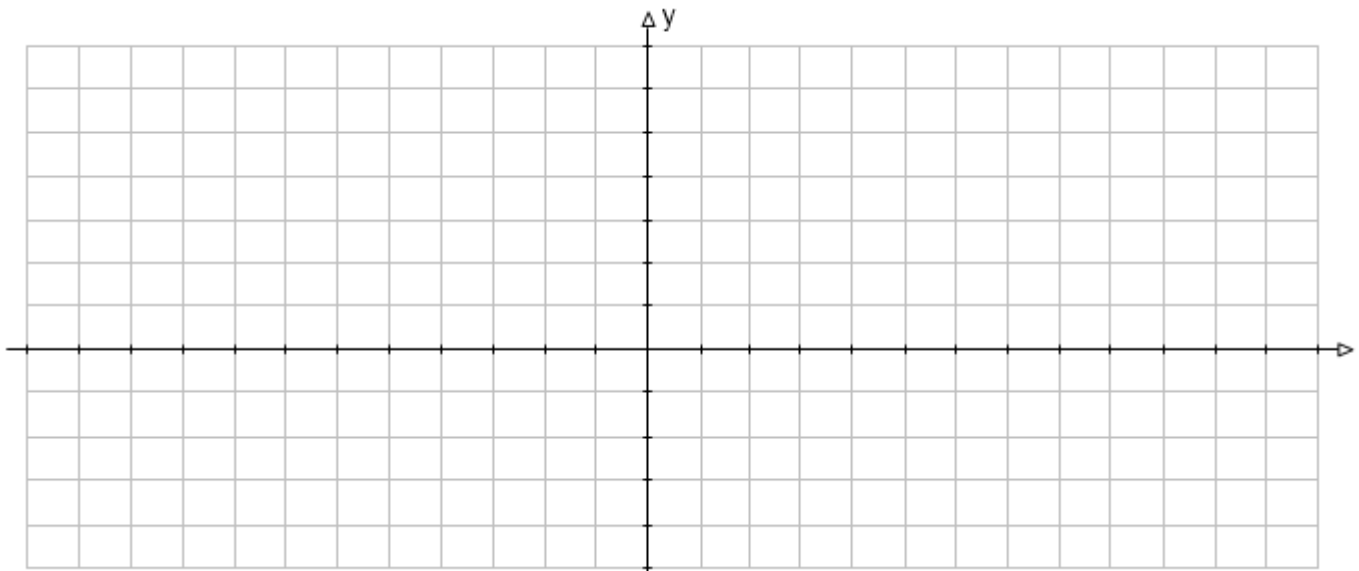
Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



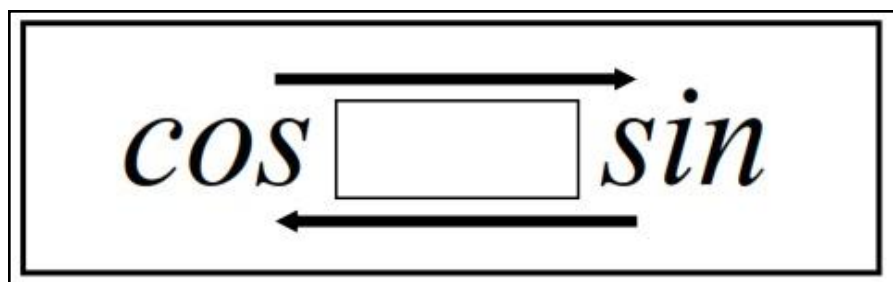
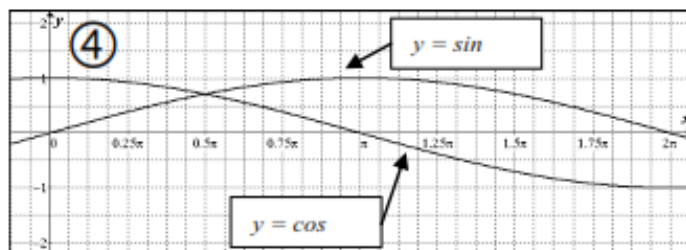
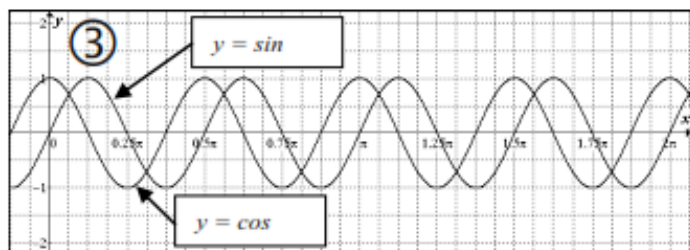
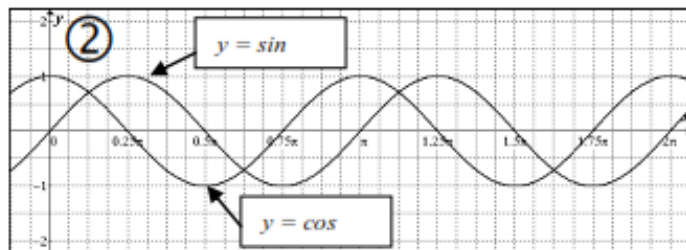
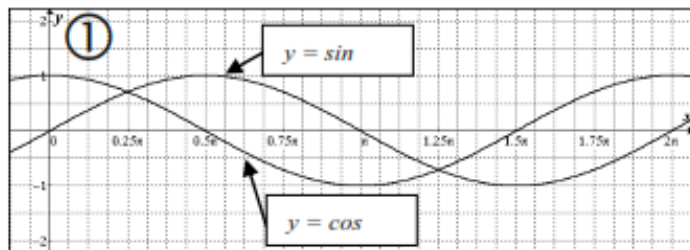
d) $f(\theta) = -2\sin\left(\frac{\theta}{2} - \frac{\pi}{3}\right) + 1.$

$(\theta, y) \rightarrow$

Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



4.2 Converting between Sine and Cosine



Q1. Convert sin to cos or cos to sin in each of the following:

#	a	P	d	c	Equation with <i>sin</i>	Same curve using <i>cos</i>
a)	1	2π	$\frac{\pi}{4}$ left	0		
b)	2	4π	$\frac{3\pi}{2}$ left	2 up		
c)	7	π	$\frac{\pi}{4}$ left	1 down		
d)	$\frac{1}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{8}$ right	$\frac{3}{5}$ up		
e)	3	8π	$\frac{5\pi}{6}$ right	$\frac{1}{8}$ down		

f) $y = \sin\left(\frac{1}{8}x\right)$ _____

g) $y = 2\cos\left(\frac{3}{4}x\right)$ _____

h) $y = \sin 4\left(x - \frac{\pi}{8}\right)$ _____

i) $y = 2\sin \frac{1}{3}\left(x - \frac{7\pi}{3}\right)$ _____

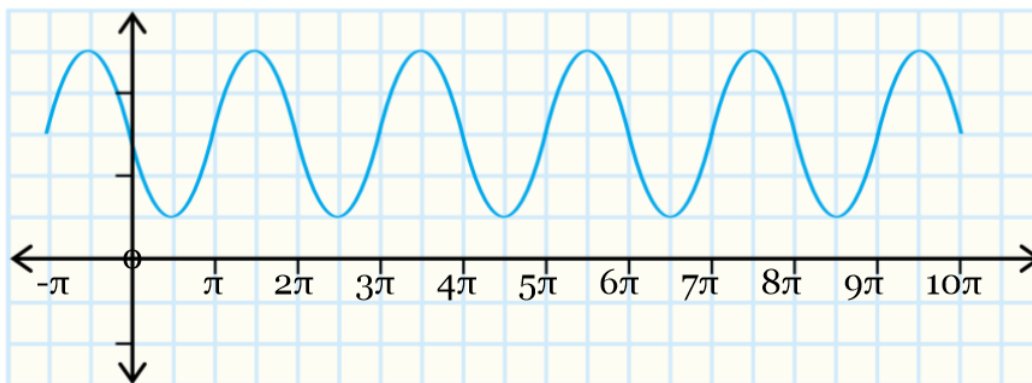
j) $y = 3\cos 3\left(x + \frac{\pi}{4}\right)$ _____

Q2. Write an equation to represent a i) Sine function, ii) Cosine function with the following properties. Maximum: 23 Minimum: 11 Period 5 Phase shift: 9

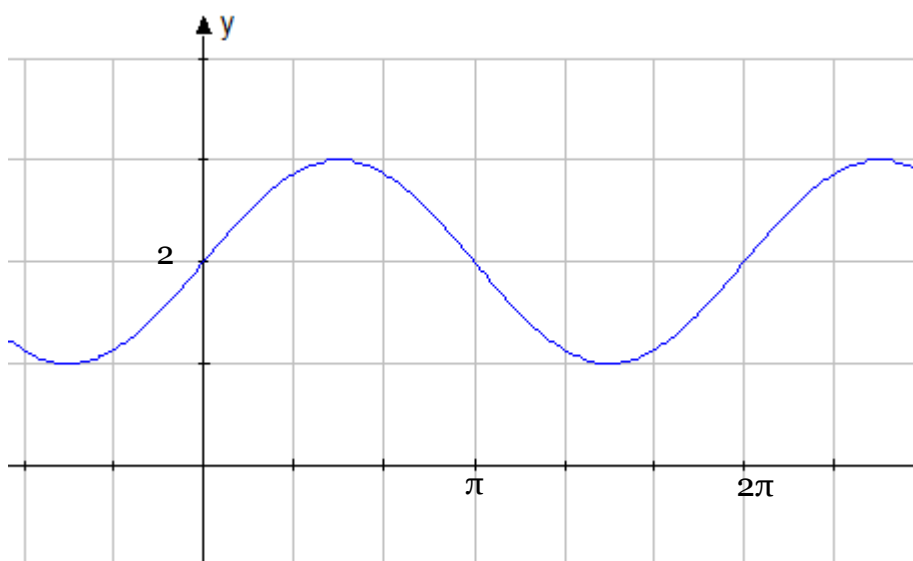
Q3. Find an equation of a function of the form $y = \cos(\theta) + q$ whose graph passes through the point $A\left(\frac{\pi}{3}, \frac{-1}{2}\right)$.

Q.4: State the amplitude and period, phase shift, and vertical shift of each function. Then, write the equation of the function as :

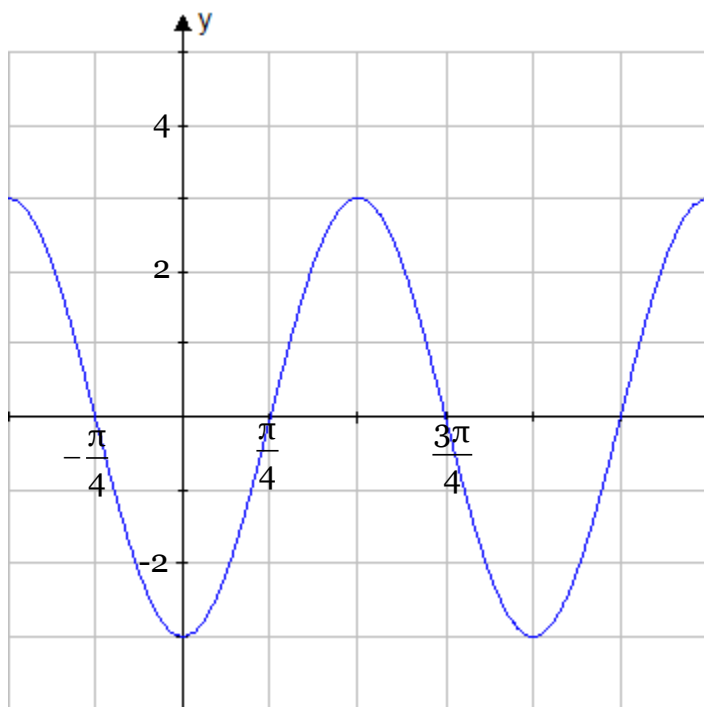
a) a sine function



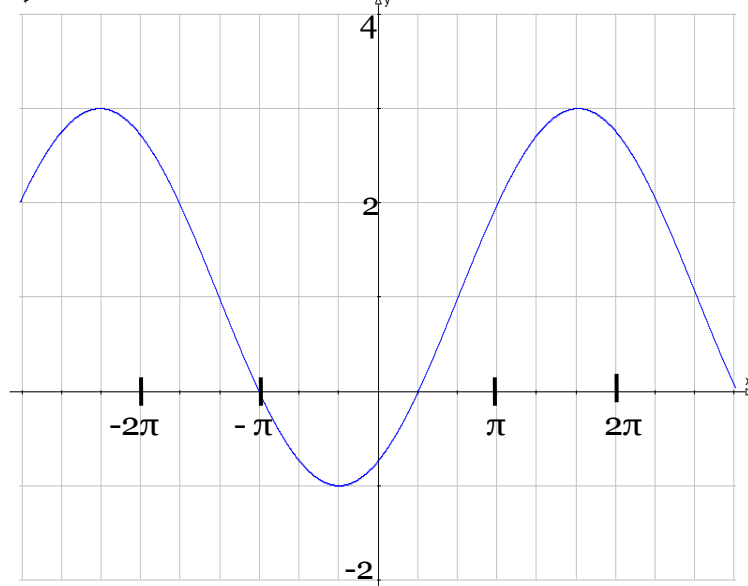
b) a cosine function:



c) A cosine function.

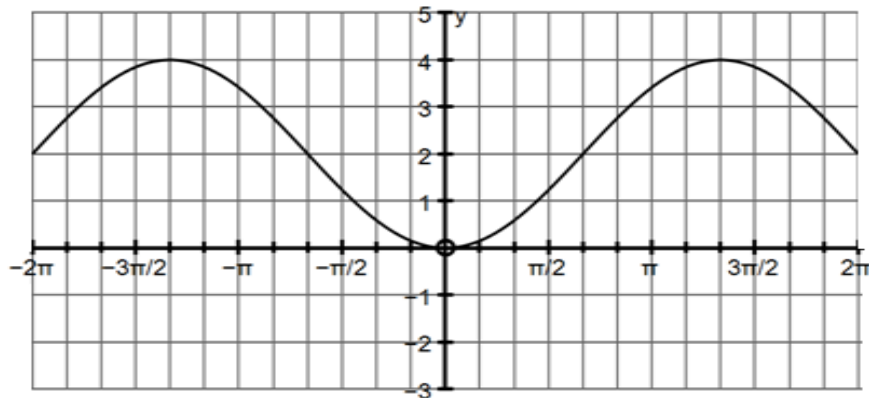


d) A convenient sinusoidal function.



Warm up

1) Write the equation of the following function as both sine and cosine function.



Sine Equation: _____

Cosine Equation: _____

2) An equation that relates the height of the tide with respect to time is given by the equation $h(t) = 1.8\sin\left[\frac{2\pi(t-4)}{12.4}\right] + 3.1$. Time is given in hours using a 24 hour clock and the height is given in meters above sea level.

(a) What is the minimum height of the tide? _____

(b) Calculate the depth of the water at 5 A.M. _____

(c) At **what time** is the first occurrence of the maximum height? _____