6.1 Sum and Difference of functions

Throughout grade 11 and 12, we have expressed functions using function notation.

Ex) $f(x) = 2x + 1$ $g(x) = 2x^2 - 7x - 4$ $h(x) = 2\sin(3x)\cos(2x)$

Functions are made of terms separated by **arithmetic operators** $(+, -, \times, \div)$. Because terms can be added, subtracted, multiplied, and divided and functions are made of terms, functions may be added, subtracted, multiplied, and divided.

a) Whenever arithmetic operations are applied to functions, a new function may be created.
   - The new function is called a **combined function**.

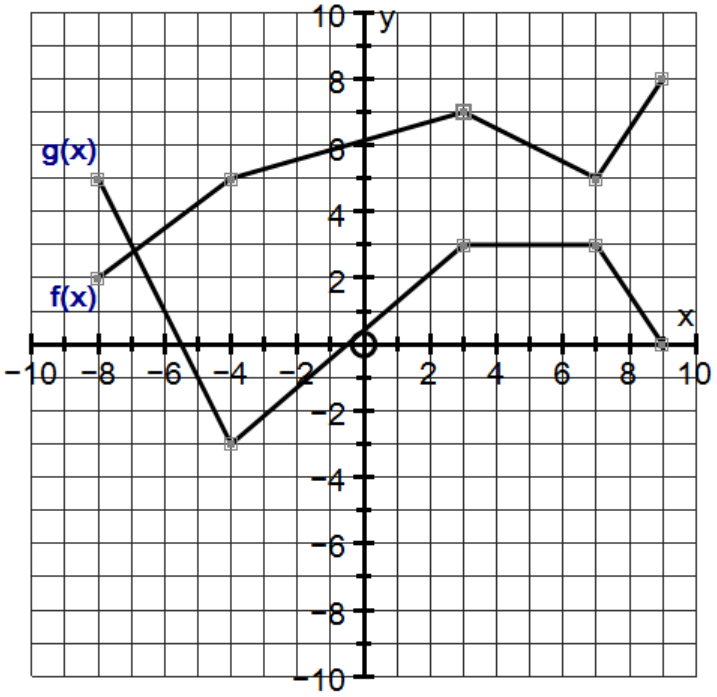
Function Notation: $f(x) + g(x)$ is often represented as $(f + g)(x)$
$f(x) - g(x)$ is often represented as $(f - g)(x)$

$$D_{f \pm g} =$$

The Superposition Principal

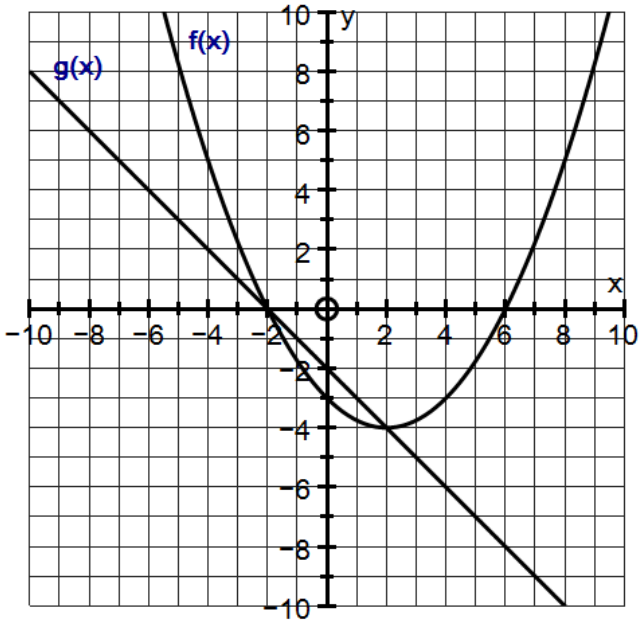
If $f(x)$ and $g(x)$ are functions then the sum of the two functions $\boldsymbol{h(x)} = f(x) + g(x)$ can be found by adding the y-coordinates at each point along the x-axis. The difference of the two functions $(f - g)(x)$ can be found by subtracting the y-coordinates at each point along the x-axis.

Ex 1) Given the following graphs, find the graph of $(f + g)(x)$ and $(f - g)(x)$.



x	$f(x)$	$g(x)$	$(f + g)(x)$	$(f - g)(x)$
-8				
-7				
-4				
0				
3				
7				
9				

Ex 2) Given the following graphs, find the graph of $h(x) = f(x) + g(x)$



x	$f(x)$	$g(x)$	$(f+g)(x)$

Ex 3) Given $f = \{(-2,6),(-1,8),(0,5),(1,0),(2,-2)\}$ *and* $g = \{(-2,2),(-1,4),(0,-6)\}$

a) State the domain of f

b) State the domain of g

c) State the domain of $f + g$

d) State the domain of $f - g$

e) List $f + g$

f) List $f - g$

Ex 4) If $f(x) = 2x^2 - 12x + 8$ and $g(x) = x^2 - 5x + 12$

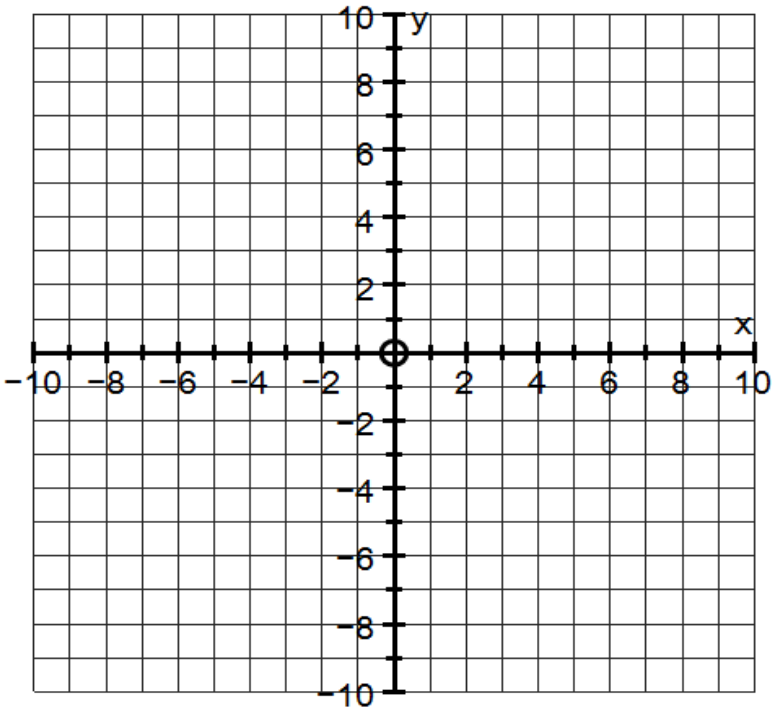
a) find the value(s) of x for which $(f + g)(x) = 0$

b) Find the optimal value of $(f - g)(x)$

Ex 5) If $f(x) = \frac{x}{x-1}$ and $g(x) = \frac{3}{x^2-1}$, find the value of x for which $(f + g)(x) = 1$

Practice: Addition and Subtraction of Functions of the Same Family

1. Given $f(x) = \frac{1}{4}x^2 - 4$ and $g(x) = -\frac{1}{2}(x-3)^2 + 8$.
   a) Graph the functions on the same set of axes with a graphing calculator and sketch the functions on the given axes with a different colour.
   b) Complete the table below
   c) Determine $h(x) = f(x) + g(x)$
   d) Determine $k(x) = f(x) - g(x)$



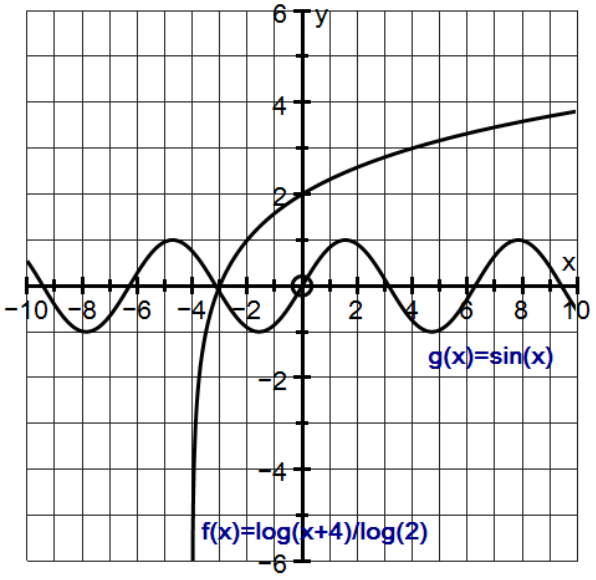
x	$f(x) = \frac{1}{4}x^2 - 4$	$g(x) = -\frac{1}{2}(x-3)^2 + 8$	$h(x) = f(x) + g(x)$	$k(x) = f(x) - g(x)$

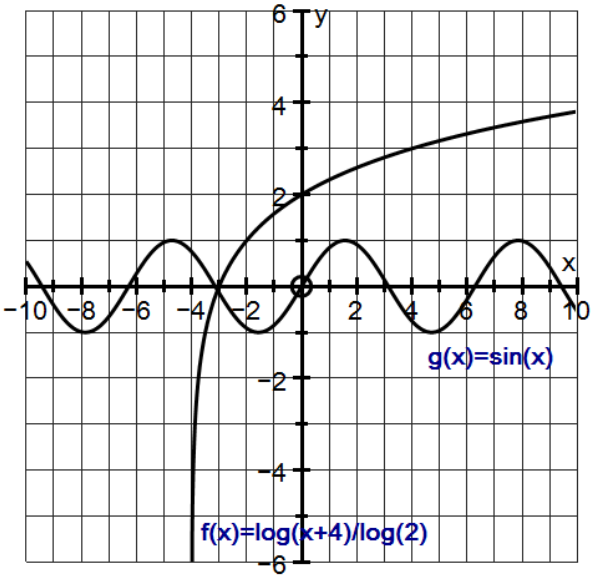
2. Given $f(x) = \log_2(x+4)$ and $g(x) = \sin(x)$.

a) Complete the table below

b) Sketch $h(x) = f(x) + g(x)$ on the grid to the left

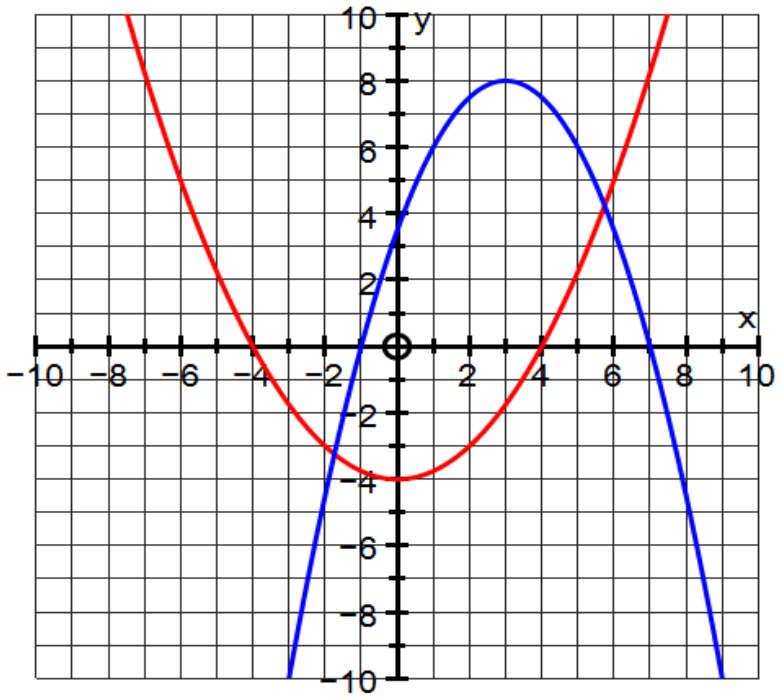
c) Sketch $k(x) = f(x) - g(x)$ on the grid to the right





x	$f(x) = \log_2(x-4)$	$g(x) = \sin(x)$	$h(x) = f(x) + g(x)$	$k(x) = f(x) - g(x)$

3. Given $f(x) = \frac{1}{4}x^2 - 4$ and $g(x) = -\frac{1}{2}(x-3)^2 + 8$.
   a) Complete the table below
   b) Determine $h(x) = f(x) + g(x)$
   c) Determine $k(x) = f(x) - g(x)$

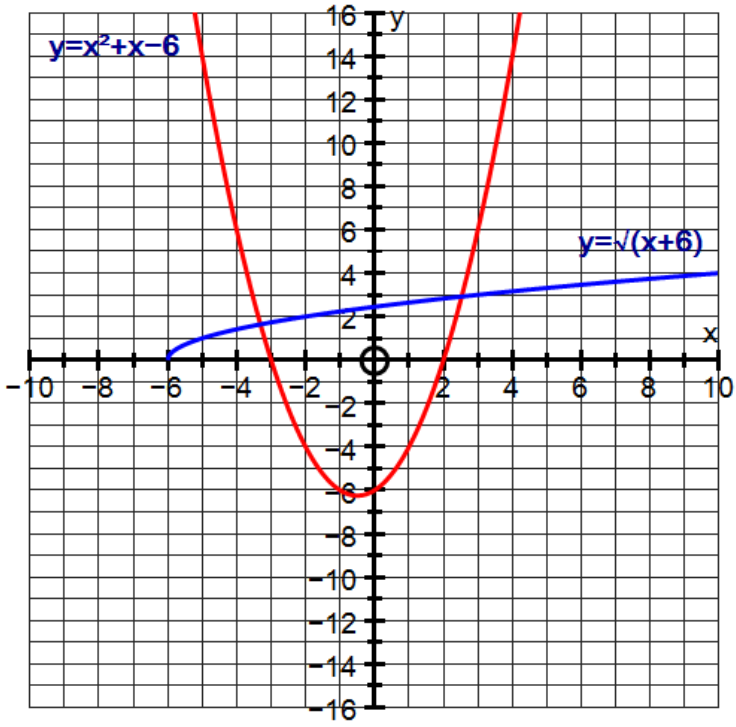


x	$f(x) = \frac{1}{4}x^2 - 4$	$g(x) = -\frac{1}{2}(x-3)^2 + 8$	$h(x) = f(x) + g(x)$	$k(x) = f(x) - g(x)$

Warm Up

1. Determine the graph of $h(x) = f(x) + g(x)$, given the graphs of $f(x) = x^2 + x - 6$ and $g(x) = \sqrt{x+6}$ by creating a table of values

x	$f(x)$	$g(x)$	$h(x)$
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			



$h(x)$ in un-simplified form:_____ Domain of $h(x)$:_____

2. Let $f(x) = mx^2 + 2x + 5$ and $g(x) = 2x^2 - nx - 2$. The functions are combined to form the new function $h(x) = (f - g)(x)$. Points $(1, 10)$ and $(-1,16)$ satisfy the new function. Determine the values of m and n. [m=8, n=-5]