L5 – 1.3 – Symmetry in Polynomial Functions
MHF4U

In this section, you will learn about the properties of even and odd polynomial functions.

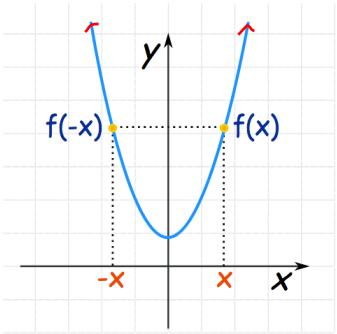
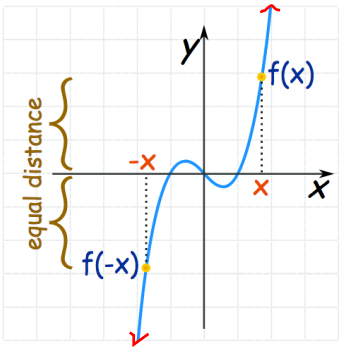
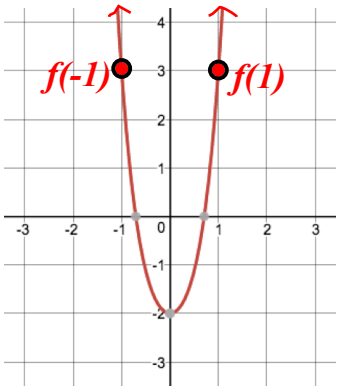
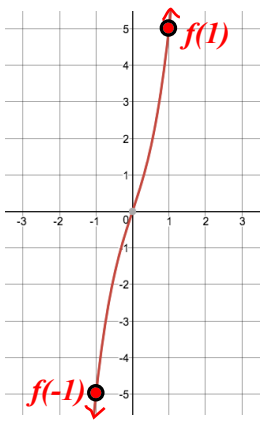
Symmetry in Polynomial Functions

_____ – there is a vertical line over which the polynomial remains unchanged when reflected.

_____ – there is a point about which the polynomial remains unchanged when rotated $180°$

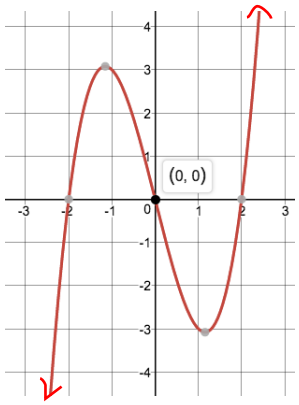
Section 1: Properties of Even and Odd Functions

A polynomial function of even or odd degree is NOT necessarily and even or odd function. The following are properties of all even and odd functions:

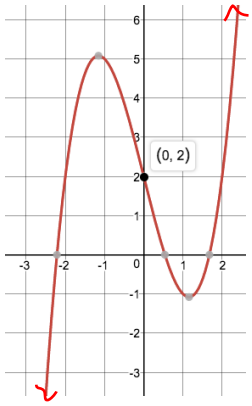
Even Functions	Odd Functions
An even degree polynomial function is an EVEN FUNCTION if: • Line symmetry over the _____ • The exponent of each term is _____ • May have a constant term	An odd degree polynomial function is an ODD FUNCTION if: • Point symmetry about the _____ • The exponent of each term is _____ • No constant term
Rule: 	Rule: 
Example:  $f(x) = 2x^4 + 3x^2 - 2$ Notice: $f(1) =$ $f(-1) =$ $\therefore$	Example:  $f(x) = 2x^3 + 3x$ Notice: $f(1) =$ $f(-1) =$ $\therefore$

Example 1: Identify each function as an even function, odd function, or neither. Explain how you can tell.

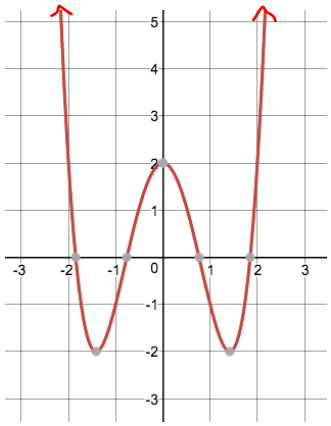
a) $y = x^3 - 4x$



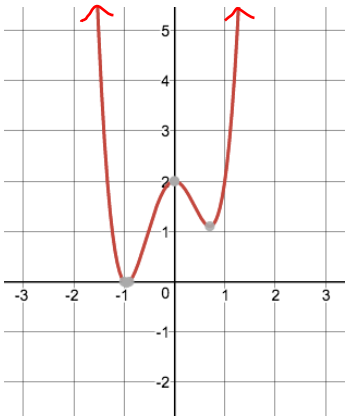
b) $y = x^3 - 4x + 2$



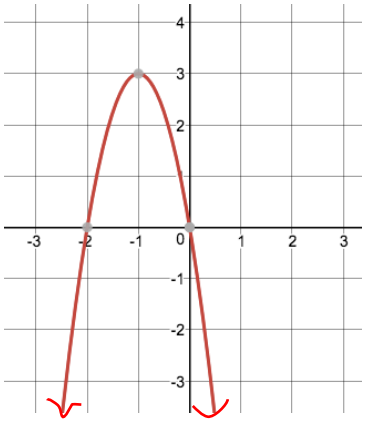
c) $y = x^4 - 4x^2 + 2$



d) $y = 3x^4 + x^3 - 4x^2 + 2$

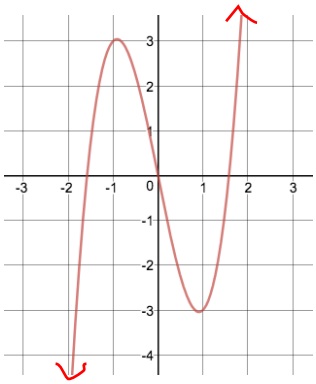


e) $y = -3x^2 - 6x$



Example 2: Choose all that apply for each function

a)



i) no symmetry

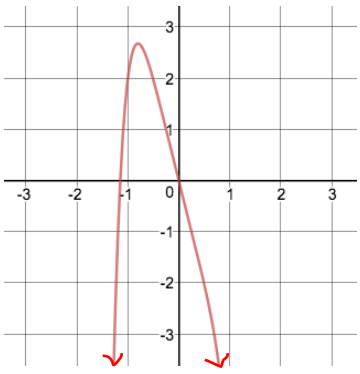
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

b)



i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

c) $P(x) = 5x^3 + 3x^2 + 2$

Note:

i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

d) $P(x) = x^6 + x^2 - 11$

i) no symmetry

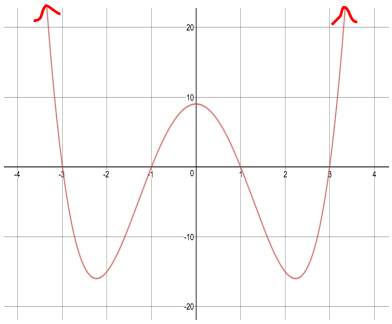
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

e)



i) no symmetry

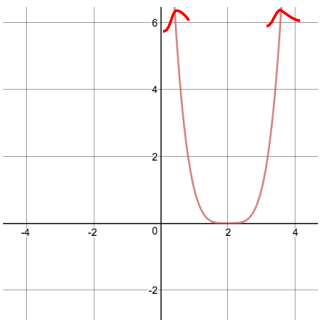
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

f)



i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

g) $P(x) = 5x^5 - 4x^3 + 8x$

i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

Example 3: Without graphing, determine if each polynomial function has line symmetry about the y-axis, point symmetry about the origin, or neither. Verify your response algebraically.

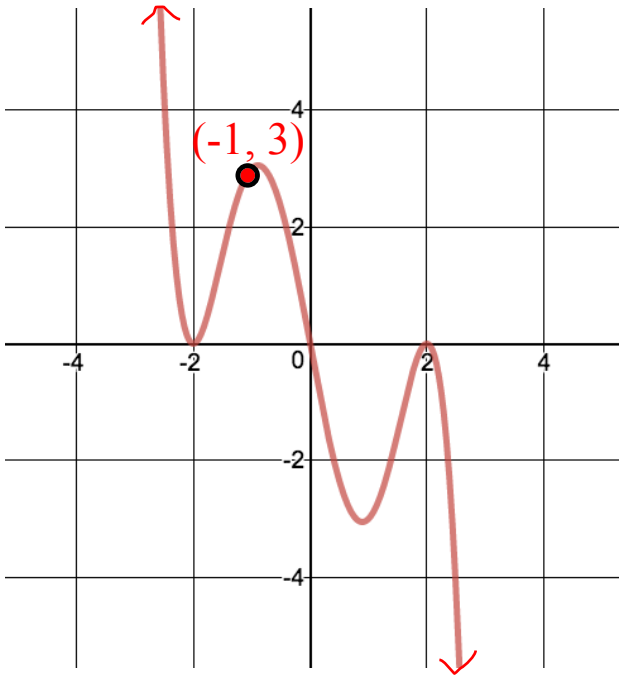
a) $f(x) = 2x^4 - 5x^2 + 4$

b) $f(x) = -3x^5 + 9x^3 + 2x$

c) $x^6 - 4x^3 + 6x^2 - 4$

Section 2: Connecting from throughout the unit

Example 4: Use the given graph to state:



a) x-intercepts

b) number of turning points

c) least possible degree

b) any symmetry present

c) the intervals where $f(x) < 0$

d) Find the equation in factored form