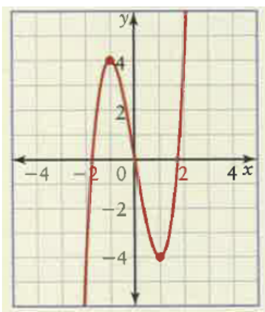
L2 – 1.2 – Characteristics of Polynomial Functions Lesson
MHF4U

In section 1.1 we looked at power functions, which are single-term polynomial functions. Many polynomial functions are made up of two or more terms. In this section we will look at the characteristics of the graphs and equations of polynomial functions.

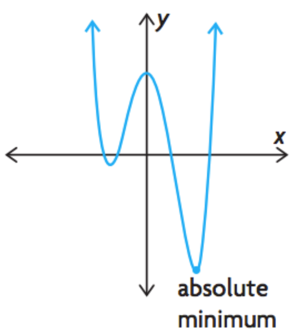
New Terminology – Local Min/Max vs. Absolute Min/Max

Local Min or Max Point – Points that are minimum or maximum points on some interval around that point.

Absolute Max or Min – The greatest/least value attained by a function for ALL values in its domain.



In this graph, (-1, 4) is a ________________ and (1, -4) is a ________________. These are not absolute min and max points because there are other points on the graph of the function that are smaller and greater. Sometimes local min and max points are called _________________________.



On the graph of this function…

There are ___ local min/max points. ___ are local min and ___ is a local max.

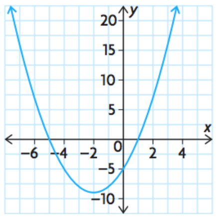
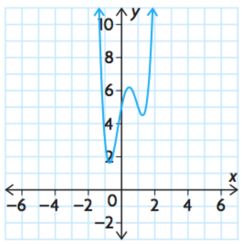
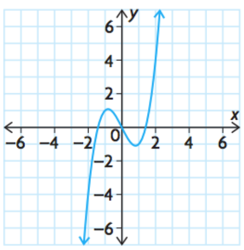
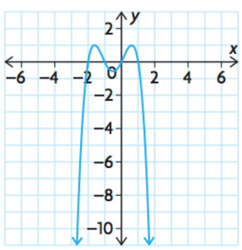
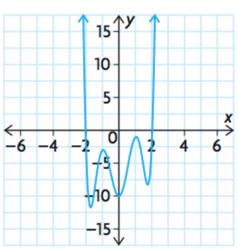
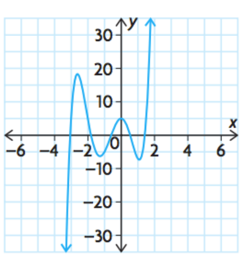
One of the local min points is also an absolute min (it is labeled).

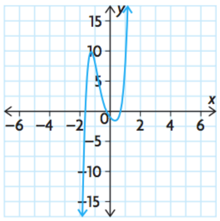
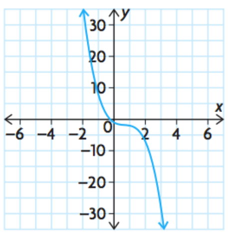
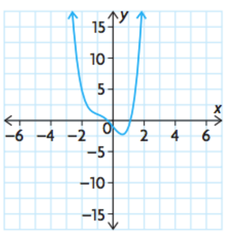
Investigation: Graphs of Polynomial Functions

The degree and the leading coefficient in the equation of a polynomial function indicate the end behaviours of the graph.

The degree of a polynomial function provides information about the shape, turning points (local min/max), and zeros (x-intercepts) of the graph.

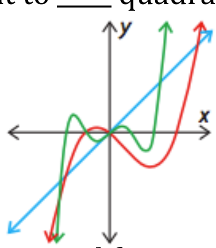
Complete the following table using the equation and graphs given:

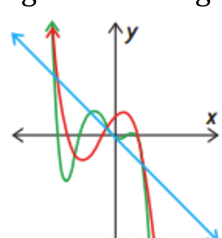
Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviour	Number of turning points	Number of x-intercepts
$f(x) = x^2 + 4x - 5$ 						
$f(x) = 3x^4 - 4x^3 - 4x^2 + 5x + 5$ 						
$f(x) = x^3 - 2x$ 						
$f(x) = -x^4 - 2x^3 + x^2 + 2x$ 						
$f(x) = 2x^6 - 12x^4 + 18x^2 + x - 10$ 						
$f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$ 						

Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviour	Number of turning points	Number of x-intercepts
$f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$ 						
$f(x) = -2x^3 + 4x^2 - 3x - 1$ 						
$f(x) = x^4 + 2x^3 - 3x - 1$ 						

Summary of Findings:

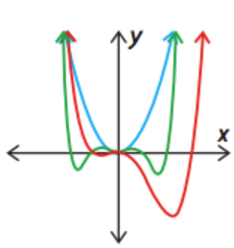
- A polynomial function of degree n has at most ____________ local max/min points (turning points)
- A polynomial function of degree n may have up to ____ distinct zeros (x-intercepts)
- If a polynomial function is ________ degree, it must have at least one x-intercept, and an even number of turning points
- If a polynomial function is _______ degree, it may have no x-intercepts, and an odd number of turning points
- An odd degree polynomial function extends from…
    - ____ quadrant to ____ quadrant if it has a positive leading coefficient
    - ____ quadrant to ____ quadrant if it has a negative leading coefficient

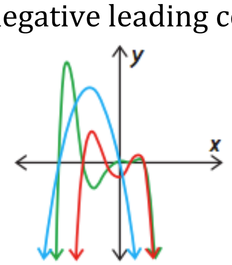




Note: *Odd degree polynomials have OPPOSITE end behaviours*

- An even degree polynomial function extends from…
    - ____ quadrant to ____ quadrant if it has a positive leading coefficient
    - ____ quadrant to ____ quadrant if it has a negative leading coefficient





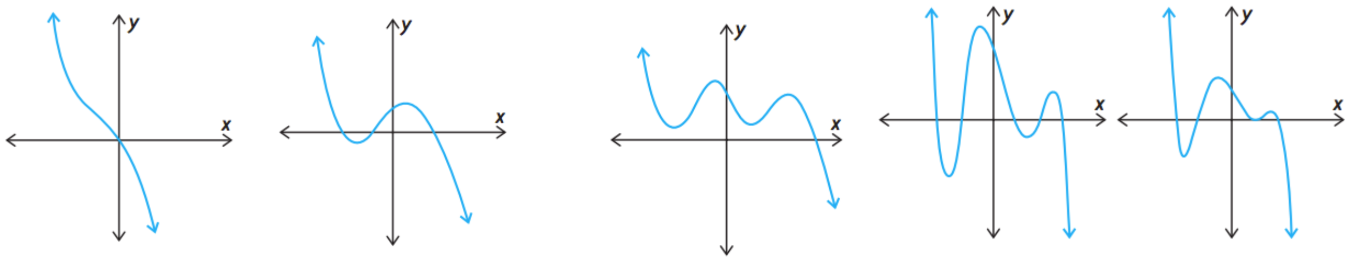
Note: *Even degree polynomials have THE SAME end behaviour.*

Example 1: Describe the end behaviours of each function, the possible number of turning points, and the possible number of zeros. Use these characteristics to sketch possible graphs of the function

a) $f(x) = -3x^5 + 4x^3 - 8x^2 + 7x - 5$

Note: *Odd degree functions must have an even number of turning points.*

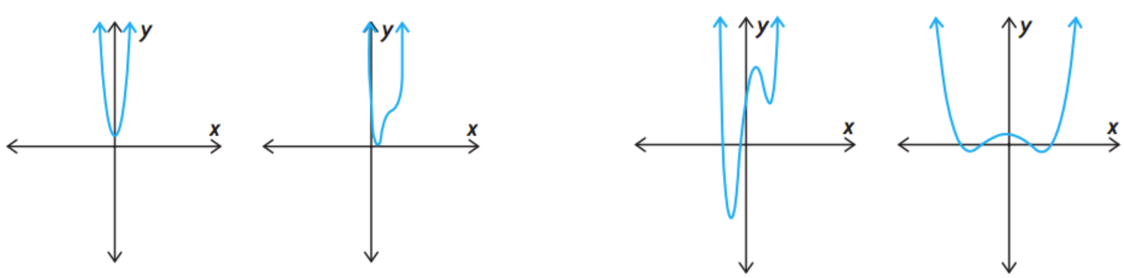
Possible graphs of 5th degree polynomial functions with a negative leading coefficient:



b) $g(x) = 2x^4 + x^2 + 2$

Note: *Even degree functions must have an odd number of turning points.*

Possible graphs of 4th degree polynomial functions with a positive leading coefficient:

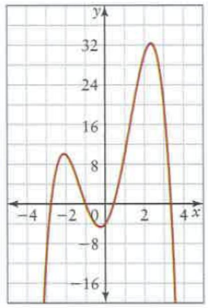


Example 2: Fill out the following chart

Degree	Possible # of x-intercepts	Possible # of turning points
1		
2		
3		
4		
5		

Example 3: Determine the key features of the graph of each polynomial function. Use these features to match each function with its graph. State the number of x-intercepts, the number of local max/min points, and the number of absolute max/min points for the graph of each function. How are these features related to the degree of each function?

a) $f(x) = 2x^3 - 4x^2 + x + 1$
b) $g(x) = -x^4 + 10x^2 + 5x - 4$
c) $h(x) = -2x^5 + 5x^3 - x$
d) $p(x) = x^6 - 16x^2 + 3$

i)


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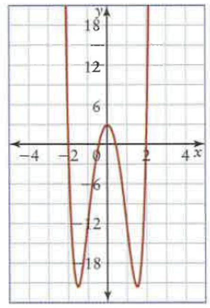

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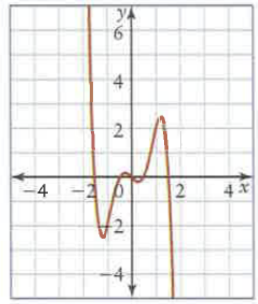

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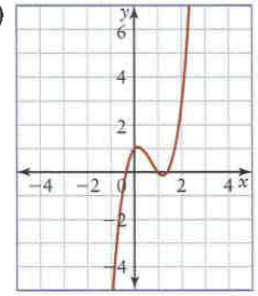

a)

b)

c)

d)

Finite Differences

For a polynomial function of degree n, where n is a positive integer, the n^{th} differences…

- are equal
- have the same sign as the leading coefficient
- are equal to $a \cdot n!$, where a is the leading coefficient

Note:

$n!$ is read as n factorial.

$n! = n\times(n-1)\times(n-2)\times \ldots \times 2\times 1$

$5! = 5\times4\times3\times2\times1 = 120$

Example 4: The table of values represents a polynomial function. Use finite differences to determine

- **a)** the degree of the polynomial function
- **b)** the sign of the leading coefficient
- **c)** the value of the leading coefficient

x	y	**First differences**	**Second differences**	**Third differences**
-3	-36			
-2	-12			
-1	-2			
0	0			
1	0			
2	4			
3	18			
4	48			

a)

b)

c)

Example 5: For the function $2x^4 - 4x^2 + x + 1$ what is the value of the constant finite differences?

$Finite\ differences =$