CHAPTER 1
Functions: Characteristics and Properties

Getting Started, p. 2

1. $f(x) = x^2 + 3x - 4$

a) $f(2) = (2)^2 + 3(2) - 4$
$= 4 + 6 - 4$
$= 6$

b) $f(-1) = (-1)^2 + 3(-1) - 4$
$= 1 - 3 - 4$
$= -6$

c) $f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) - 4$
$= \frac{1}{16} + \frac{3}{4} - 4$
$= -\frac{51}{16}$

d) $f(a + 1) = (a + 1)^2 + 3(a + 1) - 4$
$= (a + 1)(a + 1) + 3a + 3 - 4$
$= a^2 + 2a + 1 + 3a - 1$
$= a^2 + 5a$

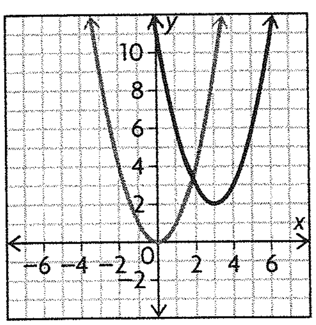
2. a) $x^2 + 2xy + y^2 = (x + y)(x + y)$

b) $5x^2 - 16x + 3 = 5x^2 - 15x - 1x + 3$
$= 5x(x - 3) + (-1)(x - 3)$
$= (5x - 1)(x - 3)$

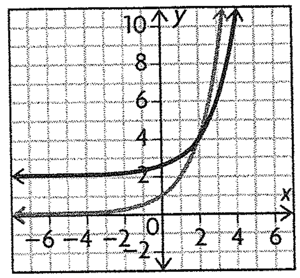
c) $(x + y)^2 - 64 = (x + y)^2 - (8)^2$
$= (x + y + 8)(x + y - 8)$

d) $ax + bx - ay - by = x(a + b) + (-y)(a + b)$
$= (a + b)(x - y)$

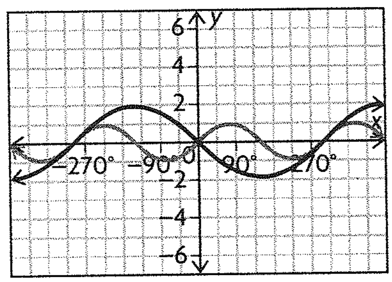
3. a) horizontal translation 3 units to the right, vertical translation 2 units up;



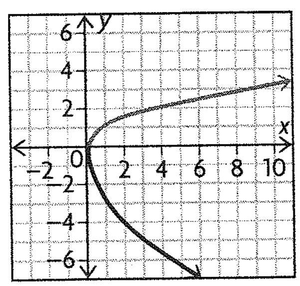
b) horizontal translation 1 unit to the right, vertical translation 2 units up;



c) horizontal stretch by a factor of 2, vertical stretch by a factor of 2, reflection across the x-axis;



d) horizontal compression by a factor of $\frac{1}{2}$, vertical stretch by a factor of 2, reflection across the x-axis;



4. a) $D = \{x \in \mathbf{R} \mid -2 \le x \le 2\}$, $R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$

b) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \ge -19\}$

c) $D = \{x \in \mathbf{R} \mid x \ne 0\}$, $R = \{y \in \mathbf{R} \mid y \ne 0\}$

d) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid -3 \le y \le 3\}$

e) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y > 0\}$

5. a) This is not a function; it does not pass the vertical line test.
b) This is a function; for each x-value, there is exactly one corresponding y-value.
c) This is not a function; for each x-value greater than 0, there are two corresponding y-values.
d) This is a function; for each x-value, there is exactly one corresponding y-value.
e) This is a function; for each x-value, there is exactly one corresponding y-value.
6. a) $y = x^3$
$y = 2^3$
$y = 8$
b) $y = x^3$
$20 = x^3$
$\sqrt[3]{20} = x$
$2.71 \doteq x$
7. If a relation is represented by a set of ordered pairs, a table, or an arrow diagram, one can determine if the relation is a function by checking that each value of the independent variable is paired with no more than one value of the dependent variable. If a relation is represented using a graph or scatter plot, the vertical line test can be used to determine if the relation is a function. A relation may also be represented by a description/rule or by using function notation or an equation. In these cases, one can use reasoning to determine if there is more than one value of the dependent variable paired with any value of the independent variable.

1.1 Functions, pp. 11–13

1. a) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | -4 \leq y \leq -2\}$; This is a function because it passes the vertical line test.
b) $D = \{x \in \mathbf{R} | -1 \leq x \leq 7\}$;
$R = \{y \in \mathbf{R} | -3 \leq y \leq 1\}$; This is a function because it passes the vertical line test.
c) $D = \{1, 2, 3, 4\}$; $R = \{-5, 4, 7, 9, 11\}$; This is not a function because 1 is sent to more than one element in the range.
d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; This is a function because every element in the domain produces exactly one element in the range.
e) $D = \{-4, -3, 1, 2\}$; $R = \{0, 1, 2, 3\}$; This is a function because every element of the domain is sent to exactly one element in the range.
f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y \leq 0\}$; This is a function because every element in the domain produces exactly one element in the range.
2. a) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y \leq -3\}$; This is a function because every element in the domain produces exactly one element in the range.
b) $D = \{x \in \mathbf{R} | x \neq -3\}$; $R = \{y \in \mathbf{R} | y \neq 0\}$; This is a function because every element in the domain produces exactly one element in the range.
c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y > 0\}$; This is a function because every element in the domain produces exactly one element in the range.
d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | 0 \leq y \leq 2\}$; This is a function because every element in the domain produces exactly one element in the range.
e) $D = \{x \in \mathbf{R} | -3 \leq x \leq 3\}$;
$R = \{y \in \mathbf{R} | -3 \leq y \leq 3\}$; This is not a function because $(0, 3)$ and $(0, -3)$ are both in the relation.
f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | -2 \leq y \leq 2\}$; This is a function because every element in the domain produces exactly one element in the range.
3. a) $D = \{1, 3, 5, 7\}$; $R = \{2, 4, 6\}$; This is a function because each element of the domain has exactly one corresponding element in the range.
b) $D = \{0, 1, 2, 5\}$; $R = \{-1, 3, 6\}$; This is a function because each element of the domain has exactly one corresponding element in the range.
c) $D = \{0, 1, 2, 3\}$; $R = \{2, 4\}$; This is a function because each element of the domain has exactly one corresponding element in the range.
d) $D = \{2, 6, 8\}$; $R = \{1, 3, 5, 7\}$; This is not a function because 2 is sent to both 5 and 7 in the range.
e) $D = \{1, 10, 100\}$; $R = \{0, 1, 2, 3\}$; This is not a function because 1 is sent to both 0 and 1 in the range.
f) $D = \{1, 2, 3, 4\}$; $R = \{1, 2, 3, 4\}$; This is a function because each element of the domain has exactly one corresponding element in the range.
4. a) This is a function because it passes the vertical line test; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y \geq 2\}$
b) This is not a function because it fails the vertical line test; $D = \{x \in \mathbf{R} | x \geq 2\}$; $R = \{y \in \mathbf{R}\}$
c) This is a function because every element of the domain produces exactly one element in the range; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y \geq -0.5\}$
d) This is not a function because $(1, 1)$ and $(1, -1)$ are both in the relation; $D = \{x \in \mathbf{R} | x \geq 0\}$;
$R = \{y \in \mathbf{R}\}$

e) This is a function because every element of the domain produces exactly one element in the range; $D = \{x \in \mathbf{R} | x \neq 0\}$; $R = \{y \in \mathbf{R} | y \neq 0\}$

f) This is a function because every element of the domain produces exactly one element in the range; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

5. a) $y = x + 3$

b) $y = 2x - 5$

c) $y = 3(x - 2)$

d) $y = -x + 5$

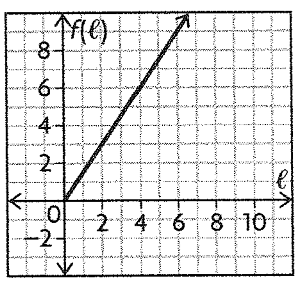
6. a) The length is twice the width.

b) Since $l = 2w$, $w = \frac{1}{2}l$

$$f(l) = l + w = l + \frac{1}{2}l$$

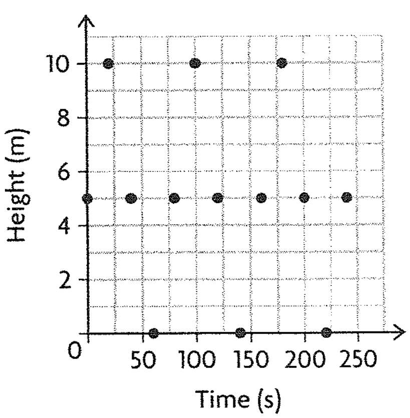
$$f(l) = \frac{3}{2}l$$

c)



d) Since $l = 2w$, the length must be 8 m and the width 4 m in order to use all 12 m of material.

7. a)

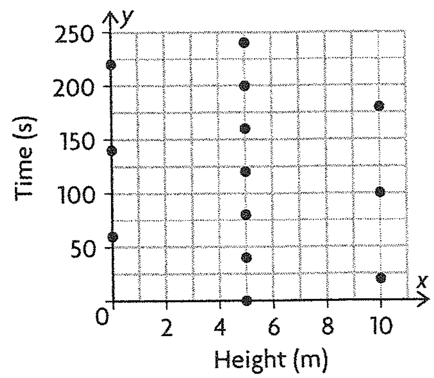


b) D = {0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240}

c) R = {0, 5, 10}

d) It is a function because it passes the vertical line test.

e)



f) It is not a function because $(5, 0)$ and $(5, 40)$ are both in the relation.

8. a) $\{(1, 2), (3, 4), (5, 6)\}$

b) $\{(1, 2), (3, 2), (5, 6)\}$

c) $\{(2, 1), (2, 3), (5, 6)\}$

9. If a vertical line passes through a function and hits two points, those two points have identical x-coordinates and different y-coordinates. This means that one x-coordinate is sent to two different elements in the range, violating the definition of *function*.

10. a)
$$\begin{aligned} d &= \sqrt{(4 - 0)^2 + (3 - 0)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Yes, because the distance from $(4, 3)$ to $(0, 0)$ is 5.

b)
$$\begin{aligned} d &= \sqrt{(1 - 0)^2 + (5 - 0)^2} \\ &= \sqrt{1^2 + 5^2} \\ &= \sqrt{26} \\ 5 &\neq \sqrt{26} \end{aligned}$$

No, because the distance from $(1, 5)$ to $(0, 0)$ is not 5.

c) No, because $(4, 3)$ and $(4, -3)$ are both in the relation.

11. a) $g(x) = x^2 + 3$

b)
$$\begin{aligned} g(3) - g(2) &= 12 - 7 \\ &= 5 \\ g(3 - 2) &= g(1) \\ &= 4 \end{aligned}$$

So, $g(3) - g(2) \neq g(3 - 2)$

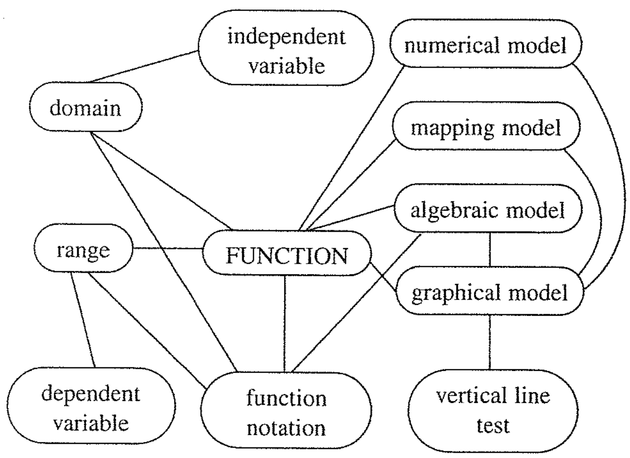
12. a) $f(6) = 1 + 2 + 3 + 6$
$= 12$
$f(7) = 1 + 7$
$= 8$
$f(8) = 1 + 2 + 4 + 8$
$= 15$

b) $f(15) = 1 + 3 + 5 + 15$
$= 24$
$f(3) \times f(5) = (1 + 3) \times (1 + 5)$
$= 4 \times 6$
$= 24$
$f(15) = f(3) \times f(5)$

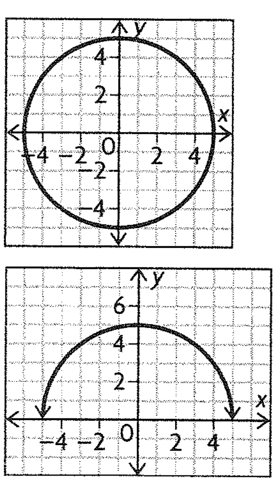
c) $f(12) = 1 + 2 + 3 + 4 + 6 + 12$
$= 28$
$f(3) \times f(4) = (1 + 3) \times (1 + 2 + 4)$
$= 4 \times 7$
$= 28$
$f(12) = f(3) \times f(4)$

d) Yes, there are others that will work.
$f(a) \times f(b) = f(a \times b)$ whenever a and b have no common factors other than 1.

13. Answers may vary. For example:



14.



The first is not a function because it fails the vertical line test: $D = \{x \in \mathbf{R} \mid -5 \le x \le 5\}$; $R = \{y \in \mathbf{R} \mid -5 \le y \le 5\}$. The second is a function because it passes the vertical line test: $D = \{x \in \mathbf{R} \mid -5 \le x \le 5\}$; $R = \{y \in \mathbf{R} \mid 0 \le y \le 5\}$.

15. x is a function of y if the graph passes the horizontal line test. This occurs when any horizontal line hits the graph at most once.

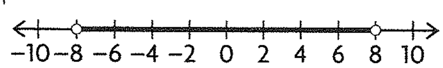
1.2 Exploring Absolute Value, p. 16

1. $|-5| = 5$, $|20| = 20$, $|-15| = 15$, $|12| = 12$, $|-25| = 25$
From least to greatest, 5, 12, 15, 20, 25, or $|-5|$, $|12|$, $|-15|$, $|20|$, $|-25|$

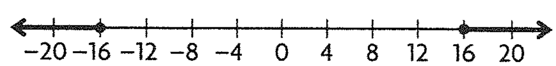
2. a) $|-22| = 22$
b) $-|-35| = -35$
c) $|-5 - 13| = |-18|$
$= 18$
d) $|4 - 7| + |-10 + 2| = |-3| + |-8|$
$= 3 + 8$
$= 11$
e) $\dfrac{|-8|}{-4} = \dfrac{8}{-4}$
$= -2$
f) $\dfrac{|-22|}{|-11|} + \dfrac{-16}{|-4|} = \dfrac{22}{11} + \dfrac{-16}{4}$
$= 2 - 4$
$= -2$

3. a) $|x| > 3$
b) $|x| \le 8$
c) $|x| \ge 1$
d) $|x| \ne 5$

4. a)

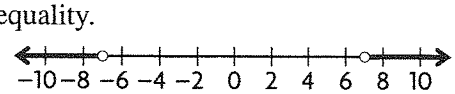


b)

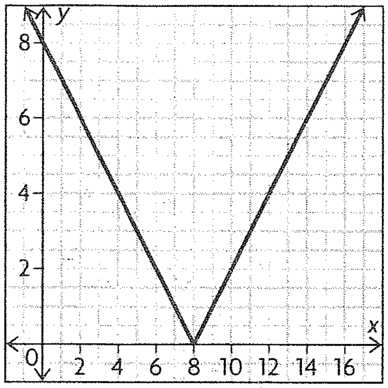


c) The absolute value of a number is always greater than or equal to 0. There are no solutions to this inequality.

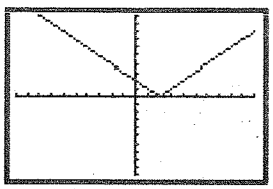
d)

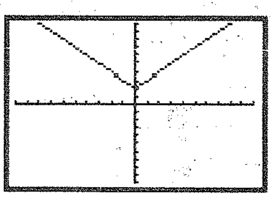


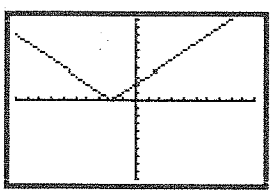
5. a) $|x| \le 3$
b) $|x| > 2$
c) $|x| \ge 2$
d) $|x| < 4$

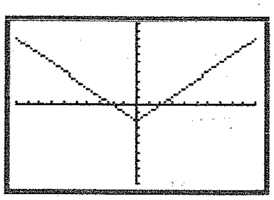
6. 

a) The graphs are the same.
b) Answers may vary. For example, $x - 8 = -(-x + 8)$, so they are negatives of each other and have the same absolute value.

7. a) 

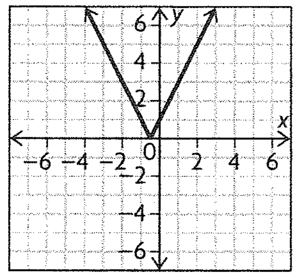
b) 

c) 

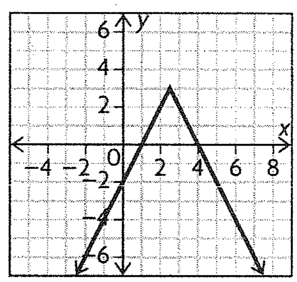
d) 

8. When the number you are adding or subtracting is inside the absolute value signs, it moves the function to the left (when adding) or to the right (when subtracting) of the origin. When the number you are adding or subtracting is outside the absolute value signs, it moves the function down (when subtracting) or up (when adding) from the origin. The graph of the function will be the absolute value function moved to the left 3 units and down 4 units from the origin.

9. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$ and translated $\frac{1}{2}$ unit to the left.



10. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$, reflected over the x-axis, translated $2\frac{1}{2}$ units to the right, and translated 3 units up.

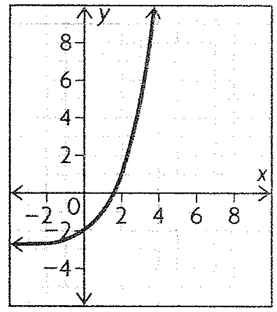


1.3 Properties of Graphs of Functions, pp. 23–25

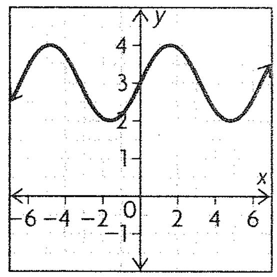
1. Answers may vary. For example, domain because most of the parent functions have all real numbers as a domain.
2. Answers may vary. For example, the end behaviour because the only two that match are x^2 and $|x|$.
3. Given the horizontal asymptote, the function must be derived from 2^x. But the asymptote is at $y = 2$, so it must have been translated up two. Therefore, the function is $f(x) = 2^x + 2$.
4. a) Both functions are odd, but their domains are different.
b) Both functions have a domain of all real numbers, but $\sin(x)$ has more zeros.
c) Both functions have a domain of all real numbers, but different end behaviour.
d) Both functions have a domain of all real numbers, but different end behaviour.

5. a) $f(x) = x^2 - 4$
$f(-x) = (-x)^2 - 4 = x^2 - 4$
$-f(-x) = -x^2 + 4$
Since $f(x) = f(-x)$, the function is even.
b) $f(x) = \sin(x) + x$
$f(-x) = \sin(-x) + (-x) = -\sin x - x$
$= -(\sin x + x) = -f(x)$
$-f(-x) = \sin x + x$
Since $f(-x) = -f(x)$, the function is odd.
c) $f(x) = \frac{1}{x} - x$
$f(-x) = \frac{1}{-x} - (-x) = -\frac{1}{x} + x = -f(x)$
$-f(-x) = \frac{1}{x} - x$
Since $f(-x) = -f(x)$, the function is odd.
d) $f(x) = 2x^3 + x$
$f(-x) = 2(-x)^3 + (-x) = -2x^3 - x$
$= -(2x^3 + x) = -f(x)$
$-f(-x) = 2x^3 + x$
Since $f(-x) = -f(x)$, the function is odd.
e) $f(x) = 2x^2 - x$
$f(-x) = 2(-x)^2 - (-x) = 2x^2 + x$
$-f(-x) = -2x^2 - x$
Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is neither even nor odd.
f) $f(x) = |2x + 3|$
$f(-x) = |2(-x) + 3| = |-2x + 3|$
$-f(-x) = -|-2x + 3|$
Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is neither even nor odd.
6. a) $|x|$, because it is a measure of distance from a number
b) $\sin(x)$, because the heights are periodic
c) 2^x, because population tends to increase exponentially
d) x, because there is $1 on the first day, $2 on the second, $3 on the third, etc.
7. a) $f(x) = \sqrt{x}$, because the domain of x must be greater than 0 for the function to be defined and $f(0) = \sqrt{0} = 0$
b) $f(x) = \sin x$, because the function is periodic and is at 0 at 0°, 180°, 360°, 540°, 720°, etc.
c) $f(x) = x^2$; It is even because $f(-x) = (-x)^2 = x^2 = f(x)$. The graph of the function is a smooth curve without any sharp corners.
d) $f(x) = x$, because $y = x$ in this function and, therefore, y and x have the same behaviour.

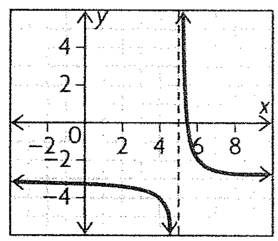
8. a) $f(x) = 2^x - 3$



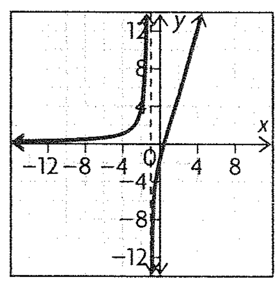
b) $g(x) = \sin x + 3$



c) $h(x) = \frac{1}{x - 5} - 3 = \frac{16 - 3x}{x - 5}$

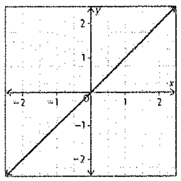
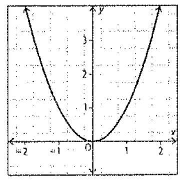
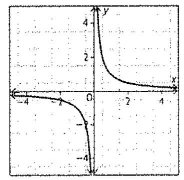
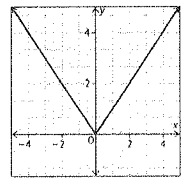
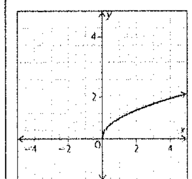
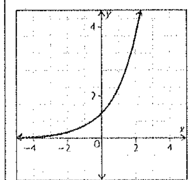
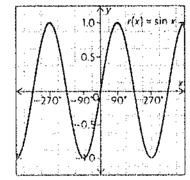


9.



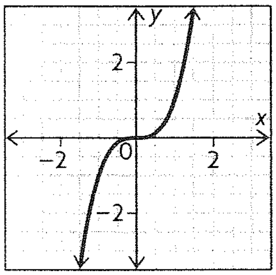
10. a) The quadratic is a parabola opening upward with its vertex at (2, 0). Using the vertex form, the function would be $f(x) = (x - 2)^2$.
b) There is not only one function.
$f(x) = \frac{3}{4}(x - 2)^2 + 1$ works as well.
c) There is more than one function that satisfies the property. $f(x) = |x - 2| + 2$ and $f(x) = 2|x - 2|$ both work.
11. x^2 is a smooth curve, while $|x|$ has a sharp, pointed corner at (0, 0).

12.

Parent Function	$f(x) = x$	$g(x) = x^2$	$h(x) = \frac{1}{x}$	$k(x) = \lvert x \rvert$	$m(x) = \sqrt{x}$	$p(x) = 2^x$	$r(x) = \sin x$
Sketch							
Domain	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \neq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \geq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$
Range	$\{f(x) \in \mathbf{R}\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \neq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) > 0\}$	$\{f(x) \in \mathbf{R} \mid -1 \leq f(x) \leq 1\}$
Intervals of Increase	$(-\infty, \infty)$	$(0, \infty)$	None	$(0, \infty)$	$(0, \infty)$	$(-\infty, \infty)$	$[90(4k + 1), 90(4k + 3)]$ $K \in \mathbf{Z}$
Intervals of Decrease	None	$(-\infty, 0)$	$(-\infty, 0)(0, \infty)$	$(-\infty, 0)$	None	None	$[90(4k + 3), 90(4k + 1)]$ $K \in \mathbf{Z}$
Location of Discontinuities and Asymptotes	None	None	$y = 0$ $x = 0$	None	None	$y = 0$	None
Zeros	$(0, 0)$	$(0, 0)$	None	$(0, 0)$	$(0, 0)$	None	$180k$ $K \in \mathbf{Z}$
Y-Intercepts	$(0, 0)$	$(0, 0)$	None	$(0, 0)$	$(0, 0)$	$(0, 1)$	$(0, 0)$
Symmetry	Odd	Even	Odd	Even	Neither	Neither	Odd
End Behaviours	$x \to \infty, y \to \infty$ $x \to -\infty, y \to -\infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to 0$ $x \to -\infty, y \to 0$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to \infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to 0$	Oscillating

13. It is important to name parent functions in order to classify a wide range of functions according to similar behaviour and characteristics.

14.

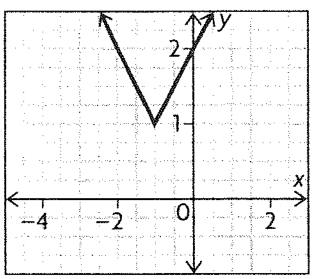


$D = \{x \in \mathbf{R}\}$, $R = \{f(x) \in \mathbf{R}\}$; interval of increase $= (-\infty, \infty)$, no interval of decrease, no discontinuities, x- and y-intercept at $(0, 0)$, odd, $x \to \infty$, $y \to \infty$, and $x \to -\infty$, $y \to -\infty$. It is very similar to $f(x) = x$. It does not, however, have a constant slope.

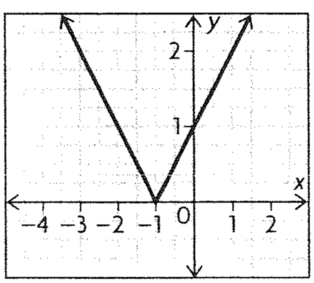
15. No, $\cos x$ is a horizontal translation of $\sin x$.

16. The graph can have 0, 1, or 2 zeros.

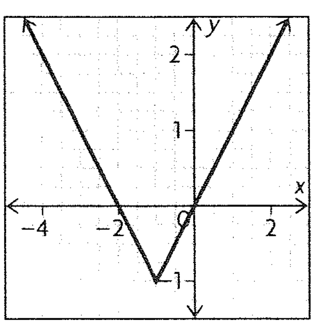
0 zeros:



1 zero:



2 zeros:



Mid-Chapter Review, p. 28

1. a) This is a function because every value in the domain goes to only one value in the range; $D = \{0, 3, 15, 27\}$, $R = \{2, 3, 4\}$

b) This is a function because every value in the domain goes to only one value in the range; $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$

c) This is not a function. It fails the vertical line test; $D = \{x \in \mathbf{R} | -5 \le x \le 5\}$, $R = \{y \in \mathbf{R} | -5 \le y \le 5\}$

d) This is not a function because 2, in the domain, goes to both 6 and 7 in the range; $D = \{1, 2, 10\}$, $R = \{-1, 3, 6, 7\}$

2. a) Yes. Every element in the domain gets sent to exactly one element in the range.

b) $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c) $R = \{10, 20, 25, 30, 35, 40, 45, 50\}$

3. a) $D = \{x \in \mathbf{R}\}$, $R = \{f(x) \in \mathbf{R}\}$; function

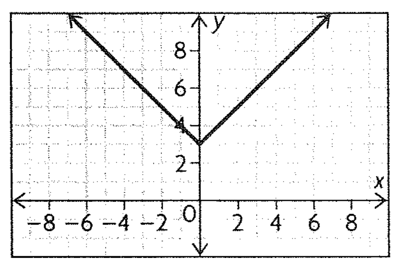
b) $D = \{x \in \mathbf{R} | -3 \le x \le 3\}$, $R = \{y \in \mathbf{R} | -3 \le y \le 3\}$; not a function

c) $D = \{x \in \mathbf{R} | x \le 5\}$, $R = \{y \in \mathbf{R} | y \ge 0\}$; function

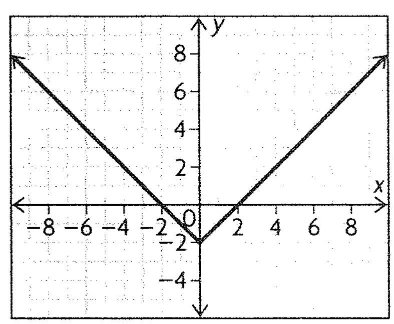
d) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} | y \ge -2\}$; function

4. $|-3| = 3$, $-|3| = -3$, $|5| = 5$, $|-4| = 4$, $|0| = 0$
$-|3| < |0| < |-3| < |-4| < |5|$

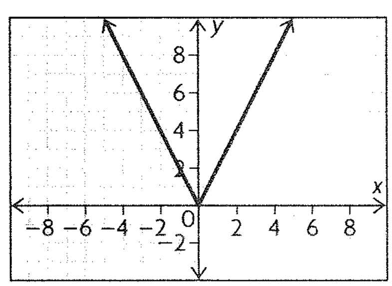
5. a)



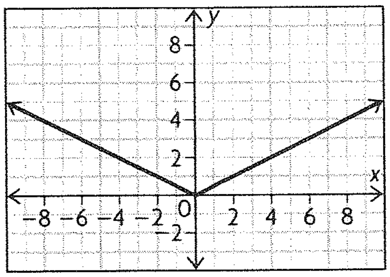
b)



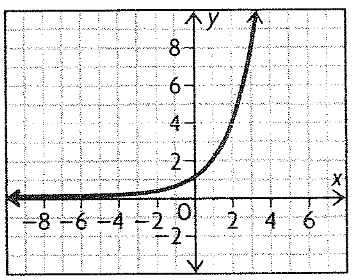
c)



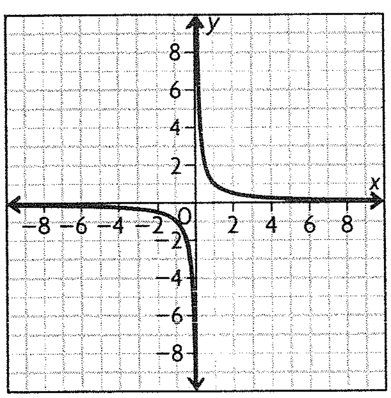
d)



6. a) The graph of $f(x) = 2^x$ is not symmetric about the y-axis nor the origin, and, therefore, is neither even nor odd. Looking at the graph we notice that $x \to \infty$ and $y \to \infty$.



b) $(-\infty, 0)$ and $(0, \infty)$ are both intervals of decrease for the function $f(x) = \frac{1}{x}$.



c) The function $f(x) = \sqrt{x}$ must have a domain greater than or equal to 0 because the square root of a negative number is undefined.

7. a) $f(x) = |2x|$

$f(-x) = |2(-x)| = |2x| = f(x)$

Since $f(x) = f(-x)$, the function is even.

b) $f(x) = (-x)^2$

$f(-x) = (-(-x))^2 = x^2 = (-x)^2 = f(x)$

Since $f(x) = f(-x)$, the function is even.

c) $f(x) = x + 4$

$f(-x) = (-x) + 4 = -x + 4$

Since $f(x) \neq f(-x)$ and $f(x) \neq -f(x)$, the function is neither odd nor even.

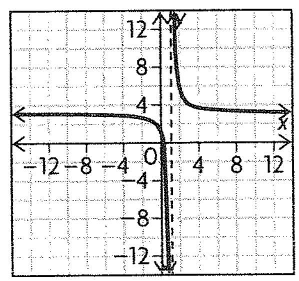
d) $f(x) = 4x^5 + 3x^3 - 1$

$f(-x) = 4(-x)^5 + 3(-x)^3 - 1$

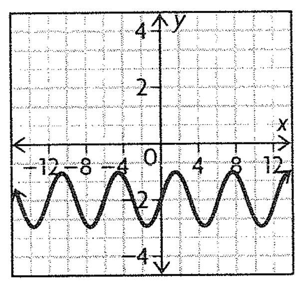
$= -4x^5 - 3x^3 - 1$

Since $f(x) \neq f(-x)$ and $f(x) \neq -f(x)$, the function is neither odd nor even.

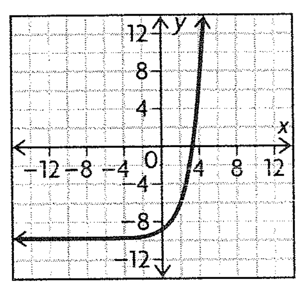
8. a) This is $f(x) = \frac{1}{x}$ translated right 1 and up 3; discontinuous



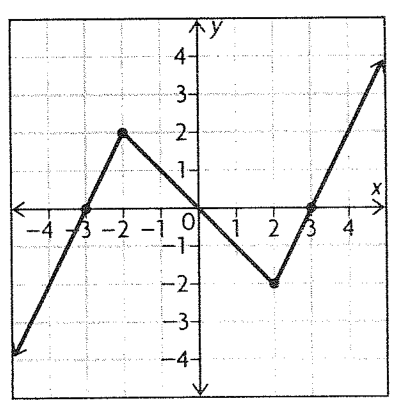
b) This is $f(x) = \sin x$ translated down 2; continuous



c) This is $f(x) = 2^x$ translated down 10; continuous



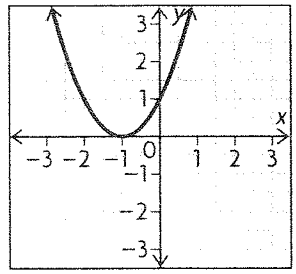
9.



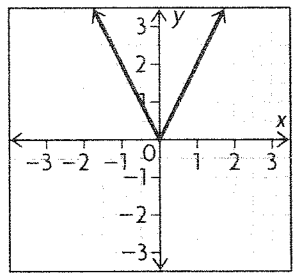
1.4 Sketching Graphs of Functions, pp. 35–37

1. a) translation 1 unit down
b) horizontal compression by a factor of $\frac{1}{2}$, translation 1 unit right
c) reflection over the x-axis, translation 2 units up, translation 3 units right
d) reflection over the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of $\frac{1}{4}$
e) reflection over the x-axis, translation 3 units down, reflection over the y-axis, translation 2 units left
f) vertical compression by a factor of $\frac{1}{2}$, translation 6 units up, horizontal stretch by a factor of 4, translation 5 units right
2. a) Representing the reflection in the x-axis: $a = -1$, representing the horizontal stretch by a factor of 2: $k = \frac{1}{2}$, representing the horizontal translation: $d = 0$, representing the vertical translation 3 units up: $c = 3$. The function is $y = -\sin\left(\frac{1}{2}x\right) + 3$.
b) Representing the amplitude: $a = 3$, representing the horizontal stretch by a factor of 2: $k = \frac{1}{2}$, representing the horizontal translation: $d = 0$, representing the vertical translation 3 units down: $c = -2$. The function is $y = 3\sin\left(\frac{1}{2}x\right) - 2$.
3. Consider the transformations of $f(x)$: horizontal compression by a factor of $\frac{1}{2}$, vertical stretch by a factor of 2, reflection across the x-axis, horizontal translation 5 units left, and vertical translation 4 units down. These transformations take $(2, 3)$ to $(1, 3)$, $(1, 6)$, $(1, -6)$, $(-4, -6)$, and finally to $(-4, -10)$.
4. a) Each y-coordinate gets multiplied by 2. $(2, 6)$, $(4, 14)$, $(-2, 10)$, $(-4, 12)$
b) Each x-coordinate gets increased by 3. $(5, 3)$, $(7, 7)$, $(1, 5)$, $(-1, 6)$
c) Each y-coordinate gets increased by 2. $(2, 5)$, $(4, 9)$, $(-2, 7)$, $(-4, 8)$
d) Each x-coordinate gets decreased by 1, and each y-coordinate gets decreased by 3. $(1, 0)$, $(3, 4)$, $(-3, 2)$, $(-5, 3)$
e) The points are reflected across the y-axis, so for x-coordinates that differ in sign switch the y-coordinates. $(2, 5)$, $(4, 6)$, $(-2, 3)$, $(-4, 7)$
f) The x-coordinates are reduced by a factor of $\frac{1}{2}$, and the y-coordinates are decreased by 1. $(1, 2)$, $(2, 6)$, $(-1, 4)$, $(-2, 5)$

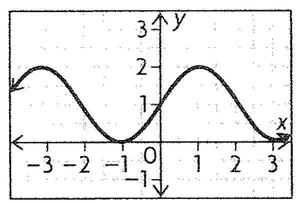
5. a) $f(x) = x^2$, translated left 1



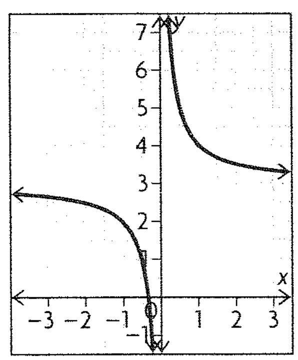
b) $f(x) = |x|$, vertical stretch by 2



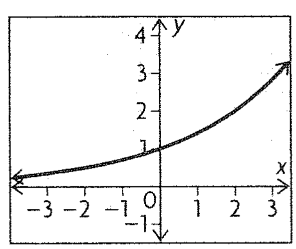
c) $f(x) = \sin(x)$, horizontal compression of $\frac{1}{3}$ translation up 1



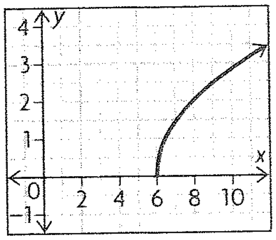
d) $f(x) = \frac{1}{x}$, translation up 3



e) $f(x) = 2^x$, horizontal stretch by 2

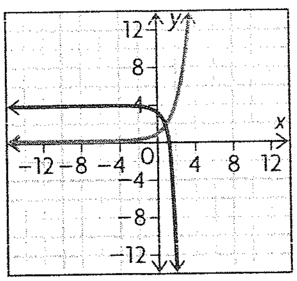


f) $f(x) = \sqrt{x}$, horizontal compression by $\frac{1}{2}$, translation right 6



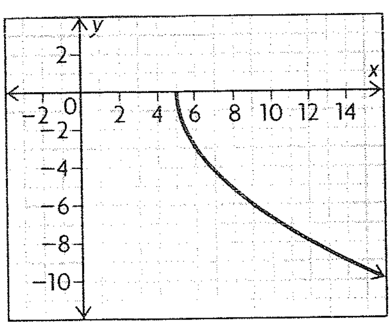
6. a) $D = \{x \in \mathbf{R}\}$, $R = \{f(x) \in \mathbf{R} | f(x) \geq 0\}$
b) $D = \{x \in \mathbf{R}\}$, $R = \{f(x) \in \mathbf{R} | f(x) \geq 0\}$
c) $D = \{x \in \mathbf{R}\}$, $R = \{f(x) \in \mathbf{R} | 0 \leq f(x) \leq 2\}$
d) $D = \{x \in \mathbf{R} | x \neq 0\}$, $R = \{f(x) \in \mathbf{R} | f(x) \neq 3\}$
e) $D = \{x \in \mathbf{R}\}$, $R = \{f(x) \in \mathbf{R} | f(x) > 0\}$
f) $D = \{x \in \mathbf{R} | x \geq 6\}$, $R = \{f(x) \in \mathbf{R} | f(x) \geq 0\}$

7. a)



b) The domain remains unchanged at $D = \{x \in \mathbf{R}\}$. The range must now be less than 4: $R = \{f(x) \in \mathbf{R} | f(x) < 4\}$. It changes from increasing on $(-\infty, \infty)$ to decreasing on $(-\infty, \infty)$. The end behaviour becomes as $x \to -\infty$, $y \to 4$, and as $x \to \infty$, $y \to -\infty$.
c) $g(x) = -2(2^{3(x-1)} + 4)$

8. $y = -3\sqrt{x-5}$;

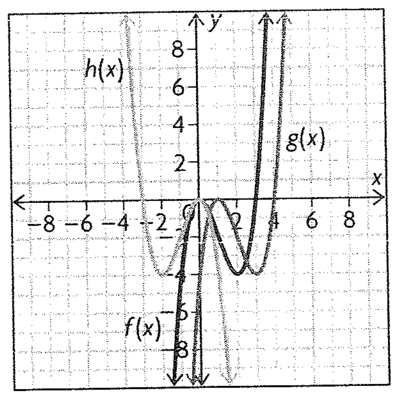


9. a) $(1, 8) \to (1 + 2, 8 \times 3) = (3, 24)$
b) $(1, 8) \to \left(\frac{1}{2}(1) - 1, 8 - 4\right) = (-0.5, 4)$
c) $(1, 8) \to \left(\frac{1}{-1}, 8(2) - 7\right) = (-1, 9)$
d) $(1, 8) \to \left(\frac{1}{4}(1) - 1, 8 \times -1\right) = (-0.75, -8)$
e) $(1, 8) \to \left(\frac{1}{-1}, 8 \times -1\right) = (-1, -8)$
f) $(1, 8) \to \left(\frac{1}{0.5}(1) - 3, 0.5(8) + 3\right) = (-1, 7)$

10. a) $g(x) = \sqrt{x-2}$
$D = \{x \in \mathbf{R} | x \geq 2\}$, $R = \{g(x) \in \mathbf{R} | g(x) \geq 0\}$
b) $h(x) = 2\sqrt{x-1} + 4$
$D = \{x \in \mathbf{R} | x \geq 1\}$, $R = \{h(x) \in \mathbf{R} | h(x) \geq 4\}$
c) $k(x) = \sqrt{-x} + 1$
$D = \{x \in \mathbf{R} | x \leq 0\}$, $R = \{k(x) \in \mathbf{R} | k(x) \geq 1\}$
d) $j(x) = 3\sqrt{2(x-5)} - 3$
$D = \{x \in \mathbf{R} | x \geq 5\}$, $R = \{j(x) \in \mathbf{R} | j(x) \geq -3\}$

11. $y = 5(x^2 - 3)$ is the same as $y = 5x^2 - 15$, not $y = 5x^2 - 3$.

12.



13. a) a vertical stretch by a factor of 4
b) a horizontal compression by a factor of $\frac{1}{2}$
c) $(2x)^2 = 2^2x^2 = 4x^2$

14. Answers may vary. For example:

horizontal stretch or compression, based on value of k

↓

vertical stretch or compression, based on value of a

↓

reflection in x-axis if $a < 0$; reflection in y-axis if $k < 0$

↓

horizontal translation, based on value of d

↓

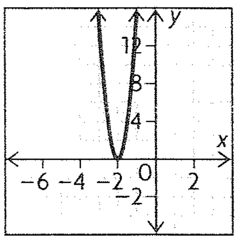
vertical translation, based on value of c

15. The new y-coordinate was produced by translating down 4 after a stretch by a factor of 2. To go backwards, we must translate up 4, which takes the 6 to 10, and then compress by a factor of $\frac{1}{2}$, which takes 10 to 5. The new x-coordinate was produced by translating left 1 unit. To go backwards, we translate right 1 unit, so 3 becomes 4. The original point is (4, 5).

16. a) horizontal compression by a factor of $\frac{1}{3}$, translation 2 units to the left

b) Because they are equivalent expressions:
$3(x + 2) = 3x + 6$

c)



1.5 Inverse Relations, pp. 43–45

1. a) (5, 2)
b) (−6, −5)
c) (−8, 4)
d) $f(1) = 2 \rightarrow (1, 2)$
So, (2, 1) is on the inverse.
e) $g(-3) = 0 \rightarrow (-3, 0)$
So, (0, −3) is on the inverse.
f) $h(0) = 7 \rightarrow (0, 7)$
So, (7, 0) is on the inverse.

2. The domain and the range of the original functions are switched for the inverses.
a) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$
b) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \geq 2\}$
c) $D = \{x \in \mathbf{R} \mid x < 2\}$, $R = \{y \in \mathbf{R} \mid y \geq -5\}$
d) $D = \{x \in \mathbf{R} \mid -5 < x < 10\}$, $R = \{y \in \mathbf{R} \mid y < -2\}$

3. Function A: $y = \frac{1}{2}x - 2$
The inverse of function A is:

$$x = \frac{1}{2}y - 2$$

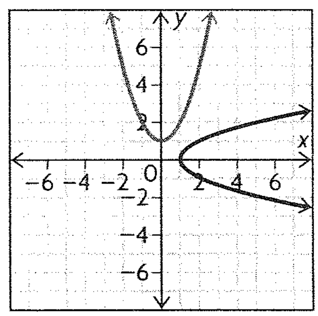
$$x + 2 = \frac{1}{2}y$$

$2x + 4 = y$
Functions A and D match.
Function B: $y = x^2 + 2$ for $x \geq 0$
The inverse of function E is:
$x = y^2 + 2$
$x - 2 = y^2$
$\sqrt{x - 2} = y$, where $x \geq 2$
Functions B and F match.
Function C: $y = (x + 3)^2$ where $x \geq -3$
The inverse of function F is:
$x = (y + 3)^2$
$\sqrt{x} = y + 3$
$\sqrt{x} - 3 = y$
Functions C and E match.

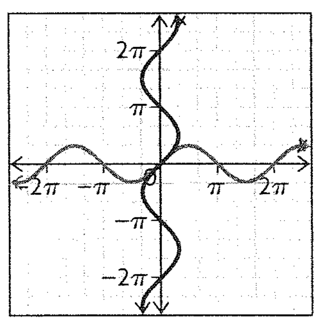
4. a) (4, 129)
b) (129, 4)
c) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$
d) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$
e) Yes; it passes the vertical line test.

5. a) (4, 248)
b) (248, 4)
c) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \geq -8\}$
d) $D = \{x \in \mathbf{R} \mid x \geq -8\}$, $R = \{y \in \mathbf{R}\}$
e) No; (248, 4) and (248, −4) are both on the inverse relation.

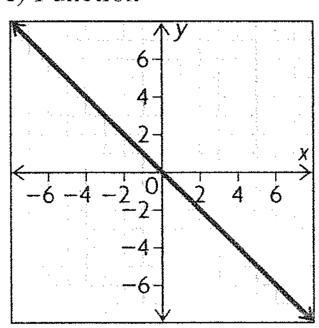
6. a) Not a function



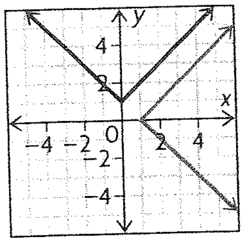
b) Not a function



c) Function



d) Not a function



7. a) $F = \frac{9}{5}C + 32$

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9}(F - 32) = C$$

$C = \frac{5}{9}(F - 32)$; this allows you to convert from Fahrenheit to Celsius.

b) $F = \frac{9}{5}C + 32$

$$F = \frac{9}{5}(20) + 32 = 36 + 32 = 68$$

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}(68 - 32) = \frac{5}{9}(36) = 20$$

20 °C = 68 °F

8. a) $A = \pi r^2$

$$\frac{A}{\pi} = r^2$$

$$\sqrt{\frac{A}{\pi}} = r$$

$r = \sqrt{\frac{A}{\pi}}$; this can be used to determine the radius of a circle when its area is known.

b) $A = \pi r^2 = \pi(5)^2 = 25\pi$

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{25\pi}{\pi}} = \sqrt{25} = 5$$

$A = 25\pi$ cm², $r = 5$ cm

9. $y = kx^3 - 1$

$$x = ky^3 - 1$$

$$x + 1 = ky^3$$

$$\sqrt[3]{\frac{x+1}{k}} = y$$

$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{k}}$$

$$f^{-1}(15) = \sqrt[3]{\frac{x+1}{k}} = 2$$

$$\sqrt[3]{\frac{15+1}{k}} = 2$$

$$\frac{16}{k} = 2^3$$

$$16 = 8k$$

$$k = 2$$

10. $h(x) = 2x + 7$

$h^{-1}(x)$:

$$x = 2y + 7$$

$$\frac{x - 7}{2} = y$$

$$h^{-1}(x) = \frac{x - 7}{2}$$

a) $h(3) = 2(3) + 7 = 13$

b) $h(9) = 2(9) + 7 = 25$

c) $\frac{h(9) - h(3)}{9 - 3} = \frac{25 - 13}{6} = 2$

d) $h^{-1}(3) = \frac{3 - 7}{2} = \frac{-4}{2} = -2$

e) $h^{-1}(9) = \frac{9 - 7}{2} = \frac{2}{2} = 1$

f) $\frac{h^{-1}(9) - h^{-1}(3)}{9 - 3} = \frac{1 - (-2)}{6} = \frac{3}{6} = \frac{1}{2}$

11. No; several students could have the same grade point average.

12. a) $f(x) = 3x + 4$

$$x = 3y + 4$$

$$x - 4 = 3y$$

$$\frac{x - 4}{3} = y$$

$$f^{-1}(x) = \frac{1}{3}(x - 4)$$

b) $h(x) = -x$

$$x = -y$$

$$-x = y$$

$$h^{-1}(x) = -x$$

c) $g(x) = x^3 - 1$

$$x = y^3 - 1$$

$$x + 1 = y^3$$

$$\sqrt[3]{x + 1} = y$$

$$g^{-1}(x) = \sqrt[3]{x + 1}$$

d) $m(x) = -2(x + 5)$

$$x = -2(y + 5)$$

$$\frac{x}{-2} = y + 5$$

$$-\frac{x}{2} - 5 = y$$

$$m^{-1}(x) = -\frac{x}{2} - 5$$

13. a) $g(x) = 4(x-3)^2 + 1$

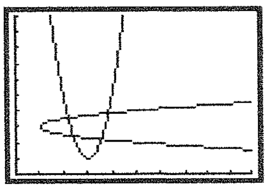
$$x = 4(y-3)^2 + 1$$

b) $$\frac{x-1}{4} = (y-3)^2$$

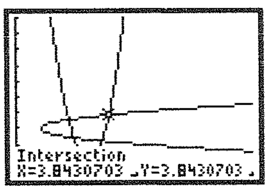
$$\pm\sqrt{\frac{x-1}{4}} + 3 = y$$

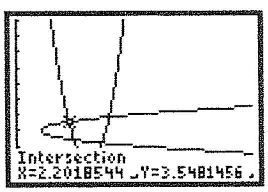
$$y = \pm\sqrt{\frac{x-1}{4}} + 3$$

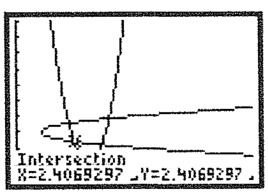
c)

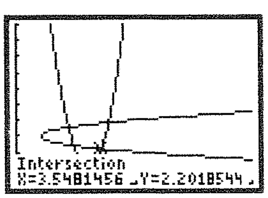


d) The points of intersection are approximately (2.20, 3.55), (2.40, 2.40), (3.55, 2.20), and (3.84, 3.84).









e) $x \geq 3$ because a negative square root is undefined.

f) $g(2) = 5$, but $g^{-1}(5) = 2$ or 4; the inverse is not a function if this is the domain of g.

14. For $y = -\sqrt{x+2}$, $D = \{x \in \mathbf{R} | x \geq -2\}$ and $R = \{y \in \mathbf{R} | y \leq 0\}$. For $y = x^2 - 2$, $D = \{x \in \mathbf{R}\}$ and $R = \{y \in \mathbf{R} | y \geq -2\}$. The student would be correct if the domain of $y = x^2 - 2$ is restricted to $D = \{x \in \mathbf{R} | x \leq 0\}$.

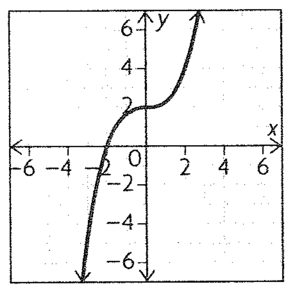
15. Yes; the inverse of $y = \sqrt{x+2}$ is $y = x^2 - 2$ so long as the domain of this second function is restricted to $D = \{x \in \mathbf{R} | x \geq 0\}$.

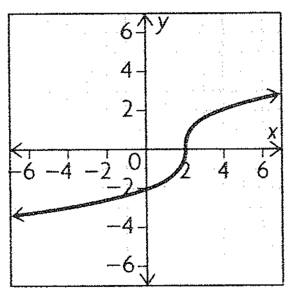
16. John is correct.

Algebraic: $y = \dfrac{x^3}{4} + 2$; $y - 2 = \dfrac{x^3}{4}$; $4(y-2) = x^3$; $x = \sqrt[3]{4(y-2)}$.

Numeric: Let $x = 4$. $y = \dfrac{4^3}{4} + 2 = \dfrac{64}{4} + 2$ $= 16 + 2 = 18$; $x = \sqrt[3]{4(y-2)} = \sqrt[3]{4(18-2)}$ $= \sqrt[3]{4(16)} = \sqrt[3]{64} = 4$.

Graphical:





The graphs are reflections over the line $y = x$.

17. $f(x) = k - x$ works for all $k \in \mathbf{R}$.

$$y = k - x$$

Switch variables and solve for y: $x = k - y$

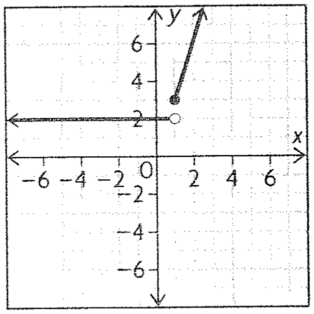
$y = k - x$

So the function is its own inverse.

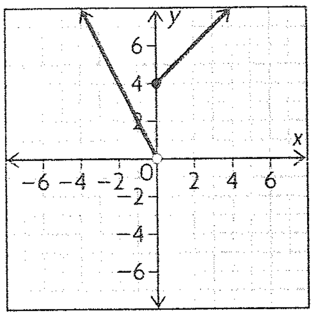
18. If a horizontal line hits the function in two locations, that means there are two points with equal y-values and different x-values. When the function is reflected over the line $y = x$ to find the inverse relation, those two points become points with equal x-values and different y-values, thus violating the definition of a function.

1.6 Piecewise Functions, pp. 51–53

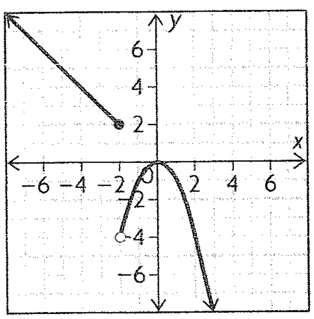
1. a)



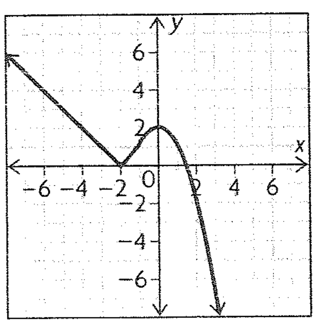
b)



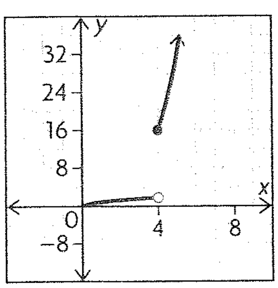
c)



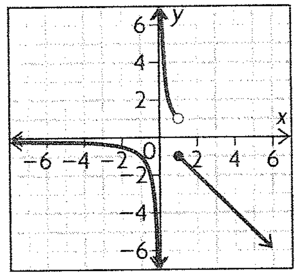
d)



e)



f)



2. a) Discontinuous at $x = 1$
b) Discontinuous at $x = 0$
c) Discontinuous at $x = -2$
d) Continuous
e) Discontinuous at $x = 4$
f) Discontinuous at $x = 1$ and $x = 0$

3. a) The function changes at $x = 1$. When $x \le 1$, the function is a parabola represented by the equation $y = x^2 - 2$. When $x > 1$, it is a line represented by the equation $y = x + 1$.

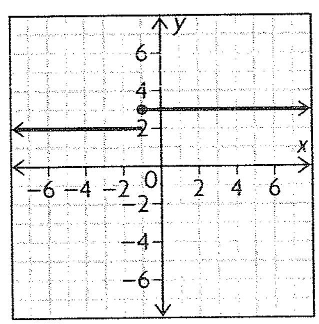
$$f(x) = \begin{cases} x^2 - 2, \text{ if } x \le 1 \\ x + 1, \text{ if } x > 1 \end{cases}$$

b) The function changes at $x = 1$. When $x < 1$, the function is an absolute value function represented by the equation $y = |x|$. When $x \ge 1$, it is a radical function represented by the equation $y = \sqrt{x}$.

$$f(x) = \begin{cases} |x|, \text{ if } x < 1 \\ \sqrt{x}, \text{ if } x \ge 1 \end{cases}$$

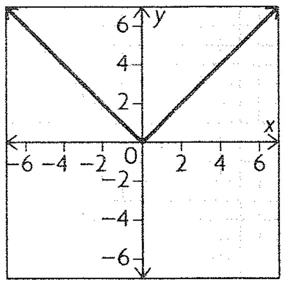
4. a) $D = \{x \in \mathbf{R}\}$; the function is discontinuous at $x = 1$.
b) $D = \{x \in \mathbf{R}\}$; the function is continuous.

5. a)



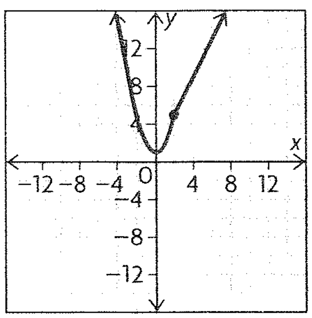
The function is discontinuous at $x = -1$.
$D = \{x \in \mathbf{R}\}$
$R = \{2, 3\}$

b)



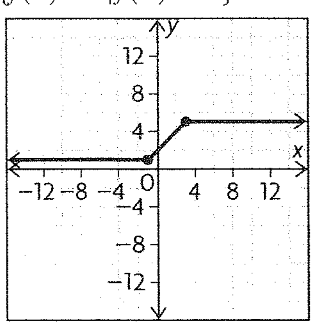
The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} | f(x) \geq 0\}$

c)



The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} | f(x) \geq 1\}$

d)



The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} | 1 \leq f(x) \leq 5\}$

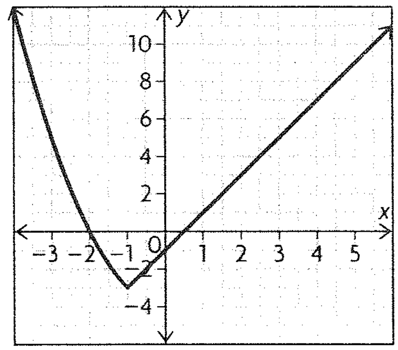
6. There is a flat fee of \$15 for the first 500 minutes which is represented by the top equation. Over 500 minutes results in a rate represented by the bottom equation.

$$f(x) = \begin{cases} 15, \text{ if } 0 \leq x \leq 500 \\ 15 + 0.02x, \text{ if } x \geq 500 \end{cases}$$

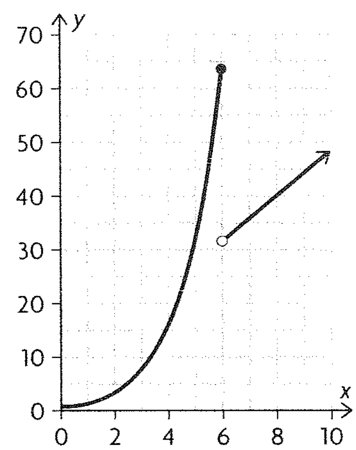
7.

$$f(x) = \begin{cases} 0.35x, \text{ if } 0 \leq x \leq 100\,000 \\ 0.45x - 10\,000, \text{ if } 100\,000 < x \leq 500\,000 \\ 0.55x - 60\,000, \text{ if } x > 500\,000 \end{cases}$$

8. In order for the function to be continuous the two pieces must have the same value for $x = -1$.
$1 - k = -2 - 1$, or $k = 4$.



9. a)



b) The function is discontinuous at $x = 6$.
c) $2^x - (4x + 8)$ at $x = 6$
$2^6 - (4(6) + 8) = 64 - 32 = 32$ fish
d) Using the function that represents the time after the spill, $4x + 8 = 64$; $4x = 56$; $x = 14$
e) Answers may vary. For example: three possible events are environmental changes, introduction of a new predator, and increased fishing.
10. Answers may vary. For example:

Plot the function for the left interval.

↓

Plot the function for the right interval.

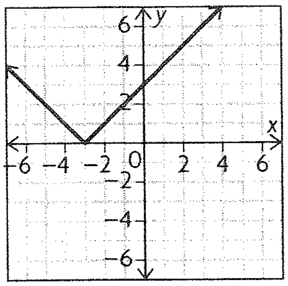
↓

Determine if the plots for the left and right intervals meet at the x-value that serves as the common endpoint for the intervals; if so, the function is continuous at this point.

↓

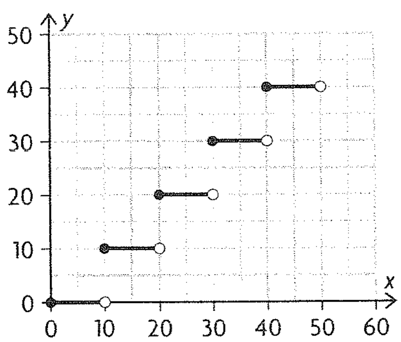
Determine continuity for the two intervals using standard methods.

11. $f(x) = |x+3| = \begin{cases} x+3, \text{ if } x \geq -3 \\ -x-3, \text{ if } x < -3 \end{cases}$



12. The function is discontinuous at $p = 0$ and $p = 15$; continuous at $0 < p < 15$ and $p > 15$.

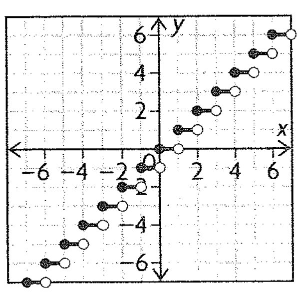
13. $f(x) = \begin{cases} 0, \text{ if } \ 0 \leq x < 10 \\ 10, \text{ if } 10 \leq x < 20 \\ 20, \text{ if } 20 \leq x < 30 \\ 30, \text{ if } 30 \leq x < 40 \\ 40, \text{ if } 40 \leq x < 50 \end{cases}$



It is often referred to as a step function because the graph looks like steps.

14. To make the first two pieces continuous, $5(-1) = -1 + k$, so $k = -4$. But if $k = -4$, the graph is discontinuous at $x = 3$.

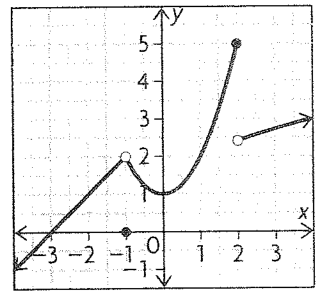
15.



16. Answers may vary. For example:

a) $f(x) = \begin{cases} x+3, \text{ if } x < -1 \\ x^2+1, \text{ if } -1 \leq x \leq 2 \\ \sqrt{x}+1, \text{ if } x > 2 \end{cases}$

b)



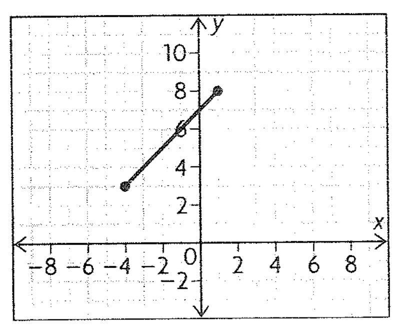
c) The function is not continuous. The last two pieces do not have the same value for $x = 2$.

d) $f(x) = \begin{cases} x+3, \text{ if } x < -1 \\ x^2+1, \text{ if } -1 \leq x \leq 1 \\ \sqrt{x}+1, \text{ if } x > 1 \end{cases}$

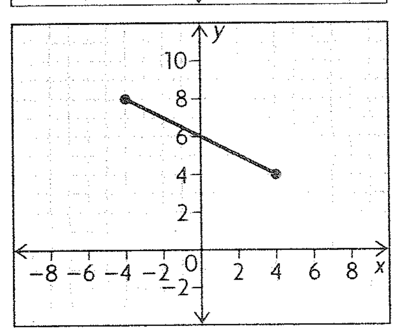
1.7 Exploring Operations with Functions, pp. 56–57

1. a) Add y-coordinates for the same x-coordinates of f and g.
$f + g = \{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
b) Subtract the y-coordinate of g from the y-coordinate of f for the same x-coordinates of f and g.
$f - g = \{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$
c) Subtract the y-coordinate of f from the y-coordinate of g for the same x-coordinates of f and g.
$g - f = \{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$
d) Multiply y-coordinates for the same x-coordinates of f and g.
$fg = \{(-4, 8), (-2, 4), (1, 6), (4, 24)\}$

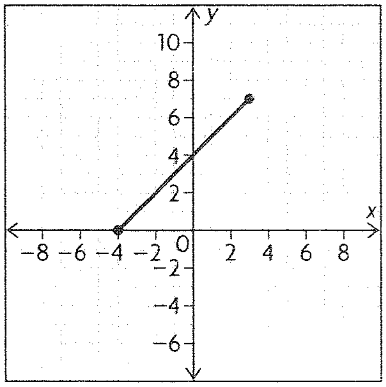
2. a)



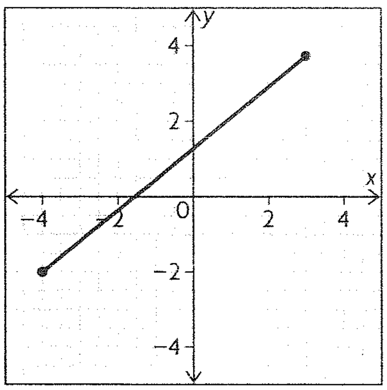
b)



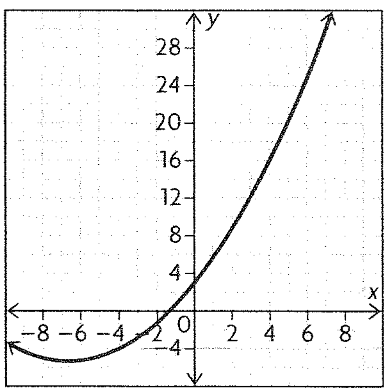
3. a)



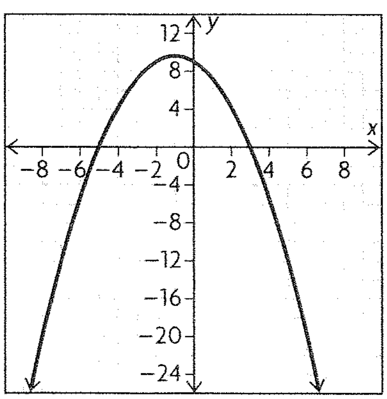
b)



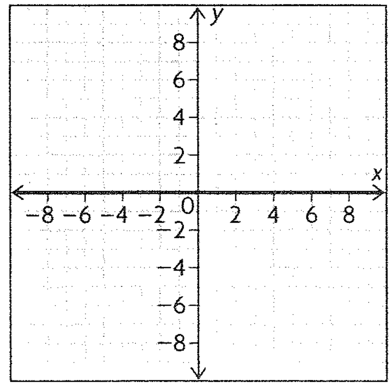
4. a)



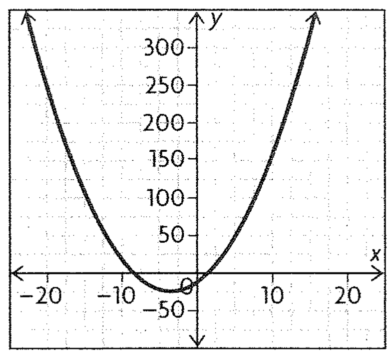
b)



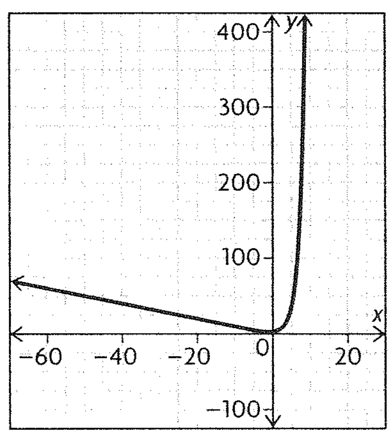
5. a) $h(x) = f(x) + g(x)$
$= x^2 + (-x^2)$
$= 0$



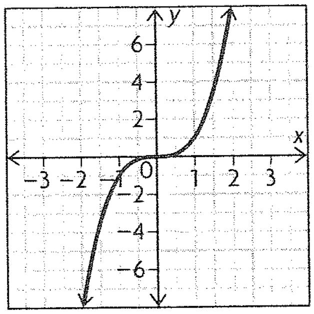
b) $p(x) = m(x) - n(x)$
$= x^2 - (-7x + 12)$
$= x^2 + 7x - 12$



c) $r(x) = s(x) + t(x)$
$= |x| + 2^x$

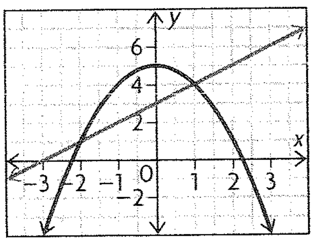


d) $a(x) = b(x) \times c(x)$
$= x \times x^2$
$= x^3$



6. a)–b) Answers may vary. For example, properties of the original graphs such as intercepts and sign at various values of the independent variable figure prominently in the shape of the new function.

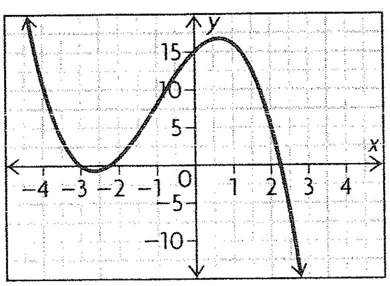
7. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
−3	0	−4	0
−2	1	1	1
−1	2	4	8
0	3	5	15
1	4	4	16
2	5	1	5
3	6	−4	−24

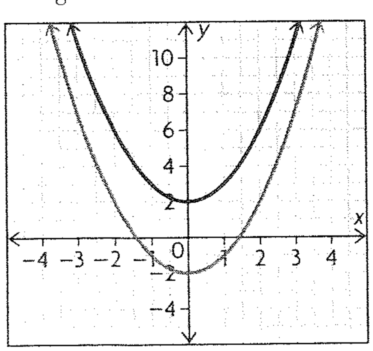
c)



d) $h(x) = (x + 3)(-x^2 + 5)$
$= -x^3 - 3x^2 + 5x + 15$; degree is 3

e) $D = \{x \in \mathbf{R}\}$; this is the same as the domain of both f and g.

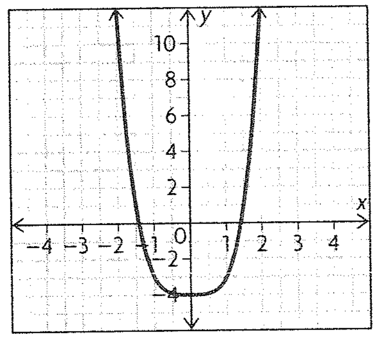
8. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
−3	11	7	77
−2	6	2	12
−1	3	−1	−3
0	2	−2	−4
1	3	−1	−3
2	6	2	12
3	11	7	77

c)



d) $h(x) = (x^2 + 2)(x^2 - 2) = x^4 - 4$; degree is 4

e) $D = \{x \in \mathbf{R}\}$

Chapter Review, pp. 60–61

1. a) This is a function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

b) This is a function; $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid y \le 3\}$

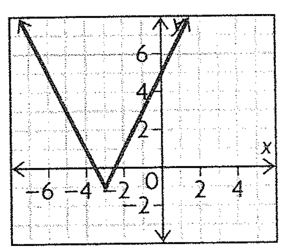
c) This is not a function; $D = \{x \in \mathbf{R} \mid -1 \le x \le 1\}$;
$R = \{y \in \mathbf{R}\}$

d) This is a function; $D = \{x \in \mathbf{R} \mid x > 0\}$
$R = \{y \in \mathbf{R}\}$

2. a) $C(t) = 30 + 0.02t$

b) $D = \{t \in \mathbf{R} \mid t \ge 0\}$; $R = \{C(t) \in \mathbf{R} \mid C(t) \ge 30\}$

3. $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} \mid f(x) \ge 1\}$



4. The number line has open circles at 2 and −2.
$|x| < 2$

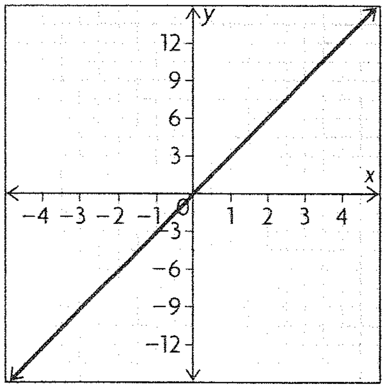
5. a) Both functions have a domain of all real numbers, but the ranges differ.

b) Both functions are odd but have different domains.

c) Both functions have the same domain and range, but x^2 is smooth and $|x|$ has a sharp corner at (0, 0).

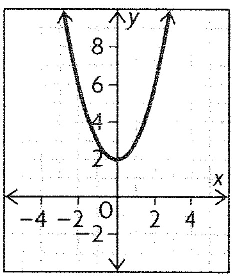
d) Both functions are increasing on the entire real line, but 2^x has a horizontal asymptote while x does not.

6. a)



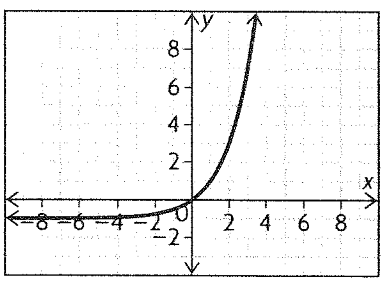
Increasing on $(-\infty, \infty)$; odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R}\}$

b)



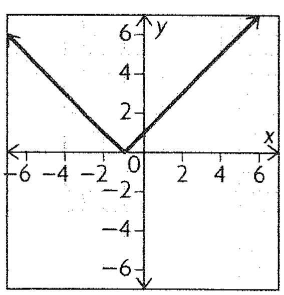
Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; even; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} | f(x) \geq 2\}$

c)

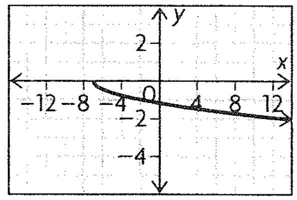


Increasing on $(-\infty, \infty)$; neither even nor odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} | f(x) > -1\}$

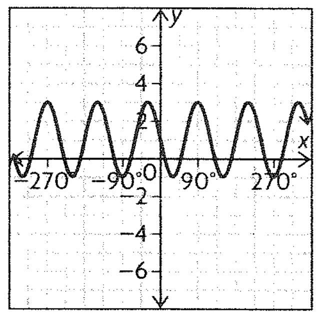
7. a) Parent: $y = |x|$; translated left 1



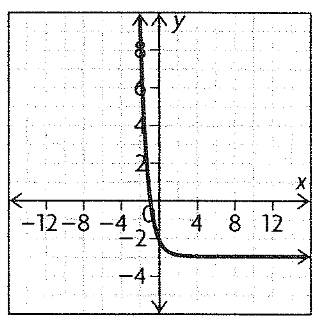
b) Parent: $y = \sqrt{x}$; compressed vertically by a factor of 0.25, reflected across the x-axis, compressed horizontally by a factor of $\frac{1}{3}$, and translated left 7



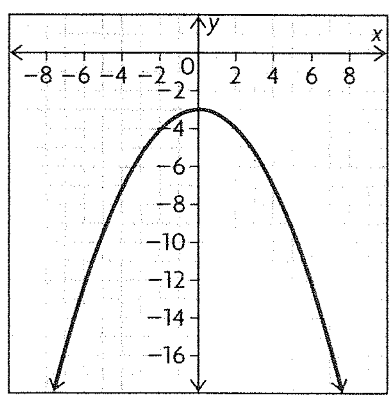
c) Parent: $y = \sin x$; reflected across the x-axis, expanded vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{3}$, translated up by 1



d) Parent: $y = 2^x$; reflected across the y-axis, compressed horizontally by a factor of $\frac{1}{2}$, and translated down by 3.



8. $y = -\left(\frac{1}{2}x\right)^2 - 3$;



9. a) $(2, 1) \rightarrow \left(\frac{2}{-1}, -1 + 2\right) = (-2, 1)$

b) $(2, 1) \rightarrow \left(-\frac{1}{2}(2) - 9, 1 - 7\right) = (-10, -6)$

c) $(2, 1) \rightarrow (2 + 2, 1 + 2) = (4, 3)$

d) $(2, 1) \rightarrow \left(\frac{1}{5}(2) + 3, 0.3(1)\right) = \left(\frac{17}{5}, 0.3\right)$

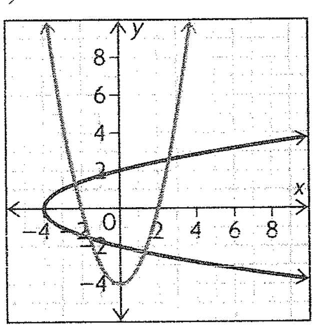
e) $(2, 1) \rightarrow (-2 + 1, -1 + 1) = (-1, 0)$

f) $(2, 1) \rightarrow \left(\frac{1}{2}(2) + 8, -1 \times 1\right) = (9, -1)$

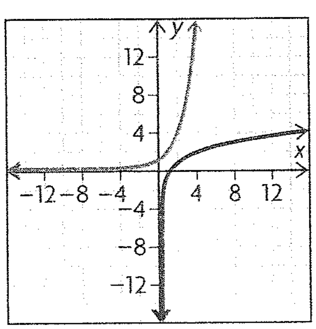
10. a) $(1, 2) \rightarrow (2, 1)$
b) $(-1, -9) \rightarrow (-9, -1)$
c) $(0, 7) \rightarrow (7, 0)$
d) $f(5) = 7 \rightarrow (5, 7)$
So, $(7, 5)$ is on the inverse.
e) $g(0) = -3 \rightarrow (0, -3)$
So, $(-3, 0)$ is on the inverse.
f) $h(1) = 10 \rightarrow (1, 10)$
So, $(10, 1)$ is on the inverse.

11. The domain and the range of the original functions are switched for the inverses.
a) $D = \{x \in \mathbf{R} \mid -2 < x < 2\}$, $R = \{y \in \mathbf{R}\}$
b) $D = \{x \in \mathbf{R} \mid x < 12\}$, $R = \{y \in \mathbf{R} \mid y \geq 7\}$

12. a) The inverse relation is not a function.



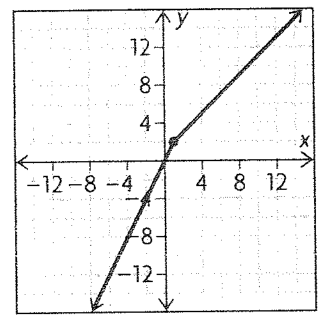
b) The inverse relation is a function.



13. a) $f(x) = 2x + 1$
$x = 2y + 1$
$x - 1 = 2y$
$\frac{x - 1}{2} = y$
$f^{-1}(x) = \frac{x - 1}{2}$

b) $g(x) = x^3$
$x = y^3$
$\sqrt[3]{x} = y$
$g^{-1}(x) = \sqrt[3]{x}$

14.



The function is continuous; $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$

15. $f(x) = \begin{cases} 3x - 1, \text{ if } x \leq 2 \\ -x, \text{ if } x > 2 \end{cases}$;
The function is discontinuous at $x = 2$.

16. In order for $f(x)$ to be continuous at $x = 1$, the two pieces must have the same value when $x = 1$. When $x = 1$, $x^2 + 1 = 2$, and $3x = 3$. The two pieces are not equal when $x = 1$, so the function is not continuous at $x = 1$.

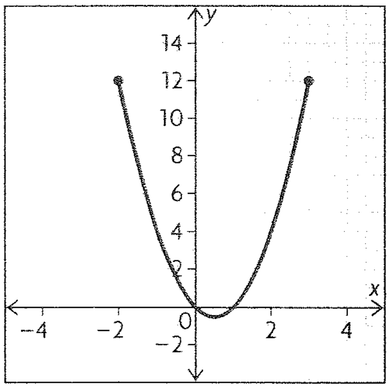
17. a) For any number of minutes up to 200, the cost is \$30. For any number above 200 minutes, the charge is \$30 plus \$0.03 per minute above 200.
$30 + 0.03(x - 200) = 30 + 0.03x - 6$
$= 24 + 0.03x$

$$f(x) = \begin{cases} 30, \text{ if } x \leq 200 \\ 24 + 0.03x, \text{ if } x > 200 \end{cases}$$

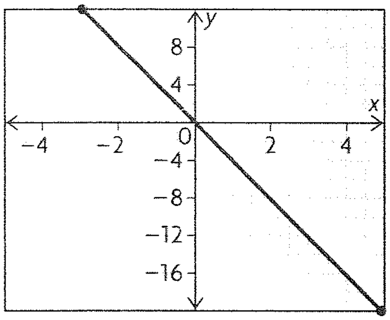
b) $24 + 0.03(350) = \$34.50$
c) $180 < 200$, so the cost is \$30.

18. a) For x-coordinates that f and g have in common, add the corresponding y-coordinates.
$f + g = \{(1, 7), (4, 15)\}$
b) For x-coordinates that f and g have in common, subtract the corresponding y-coordinates.
$f - g = \{(1, -1), (4, -1)\}$
c) For x-coordinates that f and g have in common, multiply the corresponding y-coordinates.
$fg = \{(1, 12), (4, 56)\}$

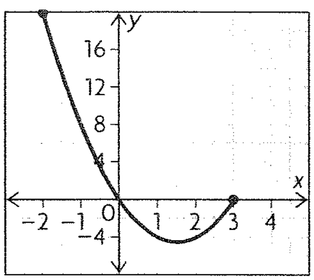
19. a)



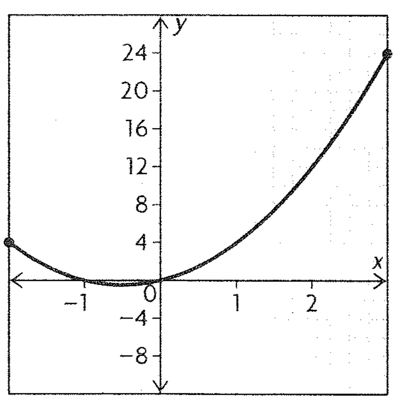
b)



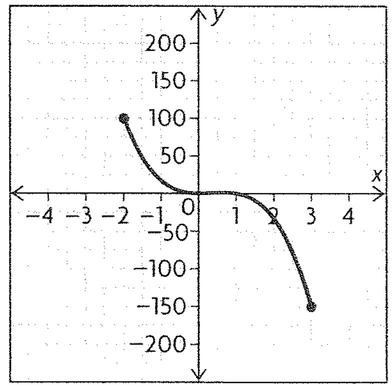
c) $f + g = 2x^2 - 2x + (-4x)$
$= 2x^2 - 6x, -2 \le x \le 3$



d) $f - g = 2x^2 - 2x - (-4x)$
$= 2x^2 + 2x, -2 \le x \le 3$



e) $fg = (2x^2 - 2x)(-4x)$
$= -8x^3 + 8x^2, -2 \le x \le 3$



20. $f(x) = x^2 + 2x$, $g(x) = x + 1$

A $f(x) + g(x) = x^2 + 2x + x + 1$
$= x^2 + 3x + 1$

B $f(x) - g(x) = x^2 + 2x - (x + 1)$
$= x^2 + x - 1$

C $g(x) - f(x) = x + 1 - (x^2 + 2x)$
$= -x^2 - x + 1$

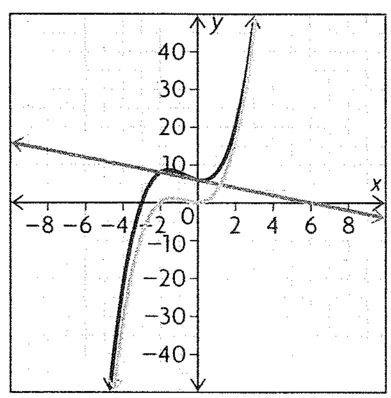
D $f(x) \times g(x) = (x^2 + 2x)(x + 1)$
$= x^3 + 3x^2 + 2x$

a) D
b) C
c) A
d) B

21. a)

x	−3	−2	−1	0	1	2
$f(x)$	−9	0	1	0	3	16
$g(x)$	9	8	7	6	5	4
$(f + g)(x)$	0	8	8	6	8	20

b)–c)

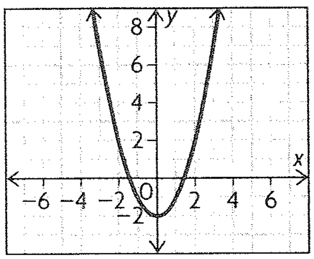


d) $(f + g)(x) = x^3 + 2x^2 + (-x + 6)$
$= x^3 + 2x^2 - x + 6$

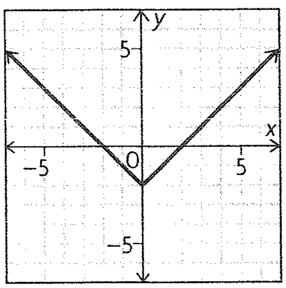
e) Answers may vary. For example, (0, 0) belongs to f, (0, 6) belongs to g, and (0, 6) belongs to $f + g$. Also, (1, 3) belongs to f, (1, 5) belongs to g, and (1, 8) belongs to $f + g$.

Chapter Self-Test, p. 62

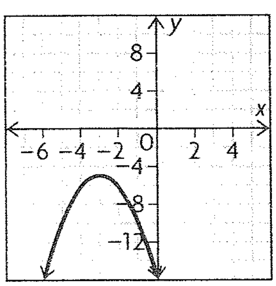
1. a) Yes. It passes the vertical line test.
b) D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R} \mid y \geq 0\}$
2. a) $f(x) = x^2$ or $f(x) = |x|$
b)



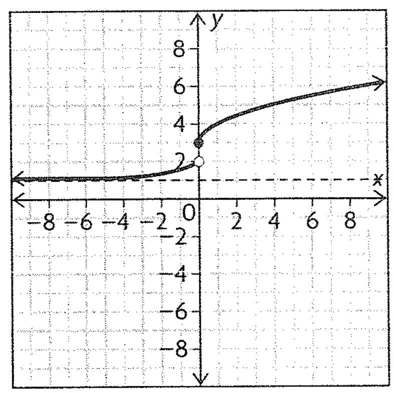
or



c) The graph was translated 2 units down.
3. $f(-x) = |3(-x)| + (-x)^2 = |3x| + x^2 = f(x)$
4. 2^x has a horizontal asymptote while x^2 does not. The range of 2^x is $\{y \in \mathbf{R} \mid y > 0\}$ while the range of x^2 is $\{y \in \mathbf{R} \mid y \geq 0\}$. 2^x is increasing on the whole real line and x^2 has an interval of decrease and an interval of increase.
5. reflection over the x-axis, translation down 5 units, translation left 3 units



6. horizontal stretch by a factor of 2, translation 1 unit up; $f(x) = |\frac{1}{2}x| + 1$
7. a) $(3, 5) \rightarrow (-3, -1, 5(3) + 2) = (-4, 17)$
b) $(3, 5) \rightarrow (5, 3)$
8.
$$f(x) = -2(x + 1)$$
$$x = -2(y + 1)$$
$$-\frac{x}{2} = y + 1$$
$$-\frac{x}{2} - 1 = y$$
$$f^{-1}(x) = -\frac{x}{2} - 1$$
9. a) $0.12(125000) - 6000 = \$9000$
b) $f(x) = \begin{cases} 0.05x, \text{ if } x \leq 50\,000 \\ 0.12x - 6\,000, \text{ if } x > 50\,000 \end{cases}$
10. a)



b) $f(x)$ is discontinuous at $x = 0$ because the two pieces do not have the same value when $x = 0$. When $x = 0$, $2^x + 1 = 2$ and $\sqrt{x} + 3 = 3$.
c) intervals of increase: $(-\infty, 0)$, $(0, \infty)$; no intervals of decrease
d) D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} \mid 0 < y < 2 \text{ or } y \geq 3\}$