Section 1: Transformation and Co-function Identities

1) Given that $\sin\left(\frac{2\pi}{7}\right) \cong 0.7818$, use equivalent trigonometric expressions to evaluate the following,

a) $\cos\frac{3\pi}{14}$

b) $\cos\frac{11\pi}{14}$

Section 2: Compound Angles

2) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each.

a) $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

b) $\cos\frac{\pi}{3}\cos\frac{5\pi}{12} - \sin\frac{\pi}{3}\sin\frac{5\pi}{12}$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

b) $\sin\left(\frac{5\pi}{4} - \frac{2\pi}{3}\right)$

4) Angle x is in the first quadrant and angle y is in the second quadrant such that $\cos x = \frac{12}{13}$ and $\sin y = \frac{7}{25}$.

Determine an exact value for...

a) $\sin x$

b) $\cos y$

c) $\sin(x + y)$

d) $\sin(x - y)$

e) $\cos(x + y)$

f) $\cos(x - y)$

5) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin \frac{11\pi}{12}$

b) $\cos \dfrac{25\pi}{12}$

c) $\tan \dfrac{7\pi}{12}$

6) A 15-m ladder leaning against a wall is in an unsafe position if it makes an angle of less than $\dfrac{\pi}{12}$ radians with the wall. Use a compound angle formula to determine an exact expression for the minimum distance that the foot of the ladder can be placed from the wall so that the ladder is standing safely.

Section 3: Double Angle Formulas

7) Express each of the following as a single trig ratio and then evaluate

a) $2 \sin \dfrac{\pi}{12} \cos \dfrac{\pi}{12}$

b) $\cos^2 \left(\dfrac{\pi}{12}\right) - \sin^2 \left(\dfrac{\pi}{12}\right)$

c) $1 - 2\sin^2\left(\frac{3\pi}{8}\right)$

d) $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\left(\frac{\pi}{6}\right)}$

7) Angle x lies in the third quadrant, and $\tan x = \dfrac{7}{24}$.

a) Determine an exact value for $\cos(2x)$

b) Determine an exact value for $\sin(2x)$

8) Given $\sin x = \dfrac{5}{13}$ and $0 \le x \le \dfrac{\pi}{2}$, find $\sin(2x)$

9) Given $\cos x = -\frac{4}{5}$ and $\pi \leq x \leq \frac{3\pi}{2}$, find $\tan(2x)$

Section 4: Trig Identities

10) Prove each of the following identities on a separate piece of paper

a) $\sin^2 x \, (1 + \cot^2 x) = 1$

b) $1 - \cos^2 x = \tan x \cos x \sin x$

c) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

d) $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = 2 \csc^2 x$

e) $\sin(2\pi - x) = -\sin x$

f) $[\sin(2x)](\tan x + \cot x) = 2$

g) $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$

h) $(\cos x - \sin x)^2 = 1 - \sin(2x)$

i) $\frac{1 - \sin^2 x}{\cot^2 x} = 1 - \cos^2 x$

j) $(\csc x - \cot x)^2 = \frac{1 - \cos x}{1 + \cos x}$

k) $\frac{1 - 2\sin^2 x}{\cos x + \sin x} + 2 \sin\frac{x}{2} \cos\frac{x}{2} = \cos x$

Section 5: Solving Linear Trigonometric Equations

11) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\cos x + 0.6 = 0$

b) $\sin x - 2.5 = 0$

c) $\csc x + 3 = 0$

d) $\cot x - 2.75 = 0$

12) Determine exact solutions for each equation in the interval $0 \le x \le 2\pi$.

a) $\tan x + 1 = 0$

b) $\sin x - 0.5 = 0$

c) $\cot x + \sqrt{3} = 0$

d) $\sec x - \sqrt{2} = 0$

Section 6: Solving Double Angle Trigonometric Equations

13) Determine solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible.

a) $\cos(2x) = -\frac{1}{4}$

b) $5\tan(3x) - 5 = 0$

Section 7: Solving Quadratic Trigonometric Equations

14) Determine approximate solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian.

a) $\sin^2 x - 0.25 = 0$

b) $\tan^2 x - 1.21 = 0$

15) For each of the following questions, determine exact solution if possible. If exact solutions are not possible, determine approximate solutions rounded to the nearest hundredth.

a) $2\sin^2 x - \sin x - 1 = 0$

b) $\tan^2 x + 3\tan x - 28 = 0$

c) $\sec^2 x - 5\sec x + 6 = 0$

d) $2\cos^2 x - 4\cos x = 0$

e) $6\sin^2 x - \sin x - 1 = 0$

16) The height, h, in meters, above the ground of a rider on a Ferris wheel can be modelled by the equation $h(t) = 10 \sin\left[\frac{\pi}{15}(t - 7.5)\right] + 12$, where t is the time in seconds. At $t = 0$, the rider is at the lowest point. Determine the first two times that the rider is 20 meters above the ground, to the nearest hundredth of a second.

17) The height, in meters, of a nail in a water wheel above the surface of the water, as a function of time, can be modelled by the function $h(t) = -4 \sin\left[\frac{\pi}{4}(t - 1)\right] + 2.5$, where t is the time in seconds. During what periods of time is the nail below the water in the first 24 seconds that the wheel is rotating?