L6 – 6.5 – Applications of Logarithms in Physical Sciences
MHF4U

Part 1: Review of Solving Logarithmic Equations

Example 1: Solve for x in the following equation

$\log_2(x-6) = 4 - \log_2 x$

$\log_2(x-6) + \log_2 x = 4$

$\log_2[(x-6)(x)] = 4$

$2^4 = (x-6)(x)$

$16 = x^2 - 6x$

$0 = x^2 - 6x - 16$

$0 = (x-8)(x+2)$

$x = 8$

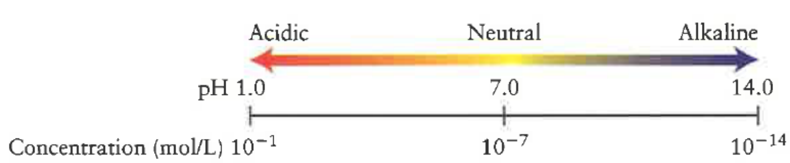
Reject $x = -2$ as bot original logarithmic expressions are undefined for this value

Part 2: pH Scale

The pH scale is used to measure the acidity or alkalinity of a chemical solution. It is defined as:

$$pH = -\log[H^+]$$

where $[H^+]$ is the concentration of hydronium ions, measured in moles per liter.



pH = 0	battery acid, strong hydrofluoric acid
pH = 1	hydrochloric acid secreted by stomach lining
pH = 2	lemon juice, gastric acid, vinegar
pH = 3	grapefruit, orange juice, soda
pH = 4	tomato juice, acid rain
pH = 5	soft drinking water, black coffee
pH = 6	urine, saliva
pH = 7	"pure" water
pH = 8	seawater
pH = 9	baking soda
pH = 10	Great Salt Lake, milk of magnesia
pH = 11	ammonia solution
pH = 12	soapy water
pH = 13	bleaches, oven cleaner
pH = 14	liquid drain cleaner

Example 2: Answer the following pH scale questions

a) Tomato juice has a hydronium ion concentration of approximately 0.0001 mol/L. What is its pH?

$pH = -\log 0.0001$

$pH = -(-4)$

$pH = 4$

b) Blood has a hydronium ion concentration of approximately 4×10^{-7} mol/L. Is blood acidic or alkaline?

$pH = -\log(4 \times 10^{-7})$

$pH \cong 6.4$

Since this is below the neutral value of 7, blood is acidic.

c) Orange juice has a pH of approximately 3. What is the concentration of hydronium ions in orange juice?

$3 = -\log[H^+]$

$-3 = \log[H^+]$

$10^{-3} = [H^+]$

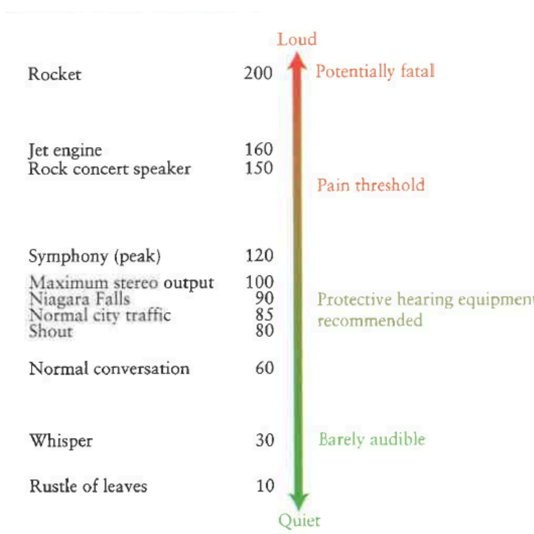
$[H^+] = 0.001$ mol/L

Part 3: Decibel Scale

Some common sound levels are indicated on the decibel scale shown. The difference in sound levels, in decibels, can be found using the equation:

$$\boldsymbol{\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)}$$

where, $\beta_2 - \beta_1$ is the difference in sound levels, in decibels, and $\frac{I_2}{I_1}$ is the ratio of their sound intensities, where I is measured in watts per square meter (W/m^2)



Example 3: Answer the following questions about decibels

a) How many times as intense as a whisper is the sound of a normal conversation

$$60 - 30 = 10\log\left(\frac{I_2}{I_1}\right)$$

$$30 = 10\log\left(\frac{I_2}{I_1}\right)$$

$$3 = \log\left(\frac{I_2}{I_1}\right)$$

$$10^3 = \frac{I_2}{I_1}$$

$$\frac{I_2}{I_1} = 1000$$

A conversation sounds 1000 times as intense as a whisper.

b) The sound level in normal city traffic is approximately 85 dB. The sound level while riding a snowmobile is about 32 times as intense. What is the sound level while riding a snowmobile, in decibels?

$$\beta_2 - 85 = 10\log(32)$$

$$\beta_2 = 10\log(32) + 85$$

$$\beta_2 \cong 100 \text{ dB}$$

<u>Part 4: Richter Scale</u>

The magnitude, M, of an earthquake is measured using the Richter scale, which is defined as:

$$\boldsymbol{M = \log\left(\frac{I}{I_0}\right)}$$

where I is the intensity of the earthquake being measured and I_0 is the intensity of a standard, low-level earthquake.

Example 4: Answer the following questions about the Richter Scale

a) How many times as intense as a standard earthquake is an earthquake measuring 2.4 on the Richter scale?

$$2.4 = \log\left(\frac{I}{I_0}\right)$$

$$10^{2.4} = \frac{I}{I_0}$$

$$\frac{I}{I_0} = \cong 251.19$$

It is about 251 times as intense as a standard earthquake.

b) What is the magnitude of an earthquake 1000 times as intense as a standard earthquake?

$$M = \log(1000)$$

$$M = 3$$