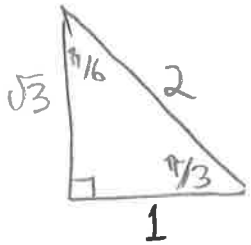
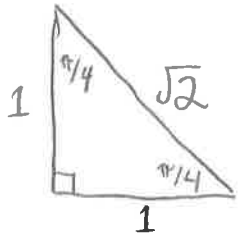
W2 – 4.2 Trig Ratios and Special Angles
MHF4U

ANSWERS

1) Draw both special triangles using radian measures.





2) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

a) cos 3.43 $\simeq -0.9587$

b) sin 2.92 $\simeq 0.2198$

c) tan 5.61 $\simeq -0.7975$

d) csc 1.27 $= \frac{1}{\sin 1.27} \simeq 1.0470$

e) cot 4.53 $= \frac{1}{\tan 4.53} \simeq 0.1844$

f) sec 0.98 $= \frac{1}{\cos 0.98} \simeq 1.7953$

3) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

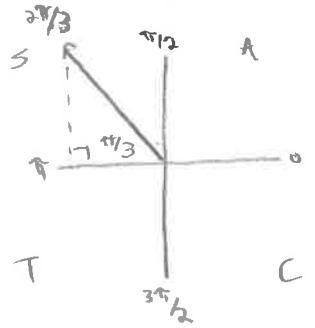
a) cot $\frac{3\pi}{7}$ $= \frac{1}{\tan\left(\frac{3\pi}{7}\right)} \simeq 0.2282$

b) sec $\frac{16\pi}{3}$ $= \frac{1}{\cos\left(\frac{16\pi}{3}\right)} = -2$

c) csc $\frac{5\pi}{11}$ $= \frac{1}{\sin\left(\frac{5\pi}{11}\right)} \simeq 1.0103$

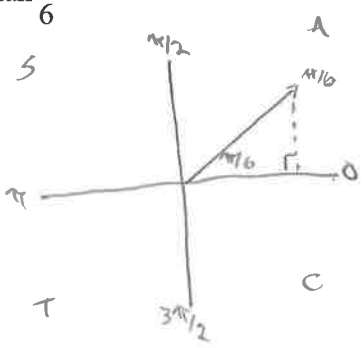
4) Use the unit circle and the cast rule to find exact expressions for each ratio

a) $\sin\frac{2\pi}{3}$



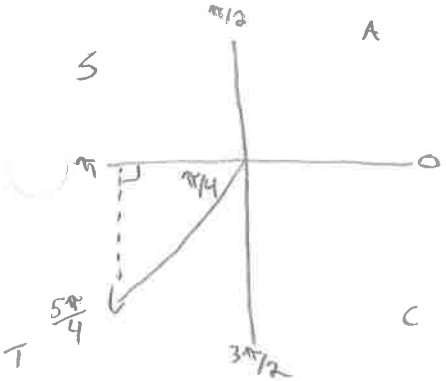
$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

b) $\tan\frac{\pi}{6}$



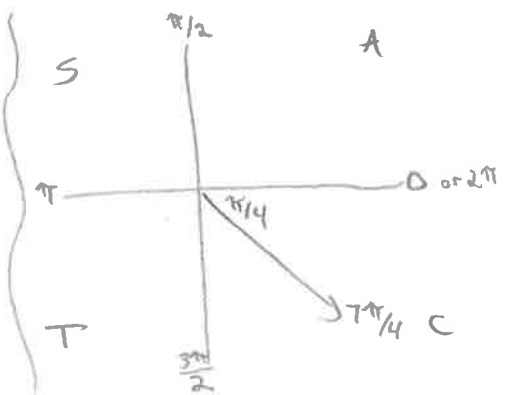
$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

c) $\cos\frac{5\pi}{4}$



$\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

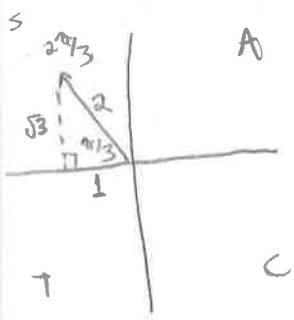
d) $\tan\frac{7\pi}{4}$



$\tan\left(\frac{7\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$

5) Use the unit circle and cast rule to determine exact values of the primary trig ratios for each angle.

a) $\frac{2\pi}{3}$

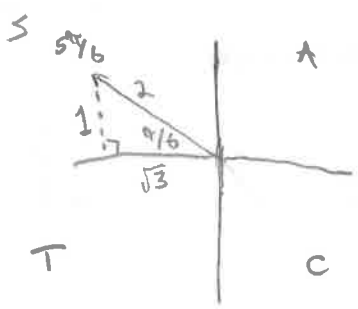


$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

b) $\frac{5\pi}{6}$

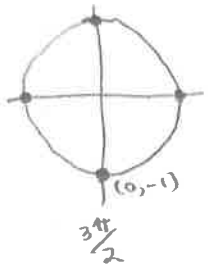


$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

c) $\frac{3\pi}{2}$

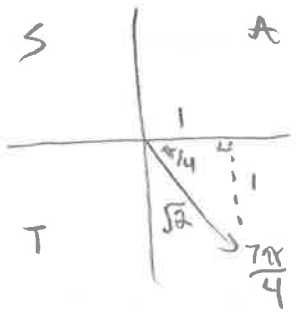


$\sin\theta = \frac{y}{r} = \frac{-1}{1} = -1$

$\cos\theta = \frac{x}{r} = \frac{0}{1} = 0$

$\tan\theta = \frac{y}{x} = \frac{-1}{0} = \text{undefined}$

d) $\frac{7\pi}{4}$

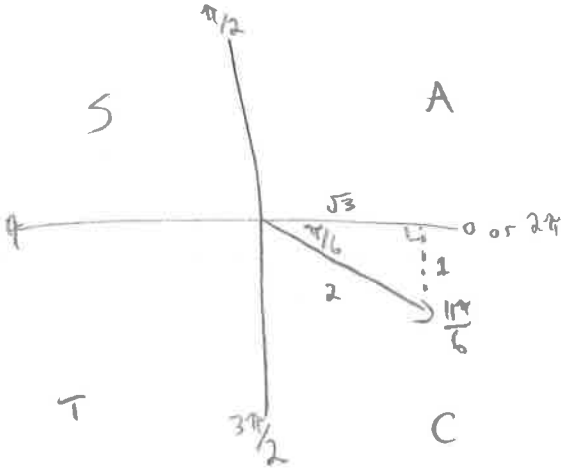


$\sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$\cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\tan\left(\frac{7\pi}{4}\right) = -1$

6) Use the special triangles determine exact values for the six trigonometric ratios for $\frac{11\pi}{6}$.



$\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

$\csc\left(\frac{11\pi}{6}\right) = -2$

$\cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

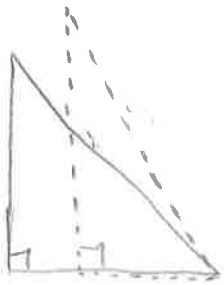
$\sec\left(\frac{11\pi}{6}\right) = \frac{2}{\sqrt{3}}$

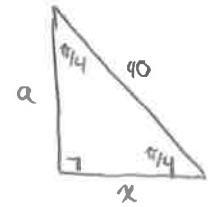
$\tan\left(\frac{11\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = \frac{-1}{\sqrt{3}}$

$\cot\left(\frac{11\pi}{6}\right) = -\sqrt{3}$

7) Lynda is flying her kite at the end of a 40-m string. The string makes an angle of $\frac{\pi}{4}$ with the ground. The wind speed increases, and the kite flies higher until the string makes and angle of $\frac{\pi}{3}$ with the ground.

Determine an exact expression for the horizontal distance that the kite moves between the two positions.

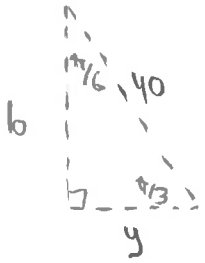




$\cos\frac{\pi}{4} = \frac{x}{40}$

$\frac{1}{\sqrt{2}} = \frac{x}{40}$

$x = \frac{40}{\sqrt{2}} = 20\sqrt{2}$



$\cos\frac{\pi}{3} = \frac{y}{40}$

$\frac{1}{2} = \frac{y}{40}$

$y = 20$

horizontal distance $= x - y$

$= 20\sqrt{2} - 20$

$= 20(\sqrt{2}-1)$ m

b) Determine and exact expression for the vertical distance that the kite moves between the two positions.

$\sin\frac{\pi}{4} = \frac{a}{40}$

$\frac{1}{\sqrt{2}} = \frac{a}{40}$

$a = \frac{40}{\sqrt{2}}$

$a = 20\sqrt{2}$

$\sin\frac{\pi}{3} = \frac{b}{40}$

$\frac{\sqrt{3}}{2} = \frac{b}{40}$

$b = 20\sqrt{3}$

vertical distance $= b - a$

$= 20\sqrt{3} - 20\sqrt{2}$

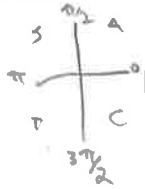
$= 20(\sqrt{3} - \sqrt{2})$ m.

8) Determine an exact value for each expression

a) $\frac{\sin\frac{\pi}{3}\tan\frac{\pi}{6}}{\cos\frac{\pi}{4}} = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

$= \frac{\sqrt{2}}{2}$



b) $\cot\frac{5\pi}{4} + \tan\frac{11\pi}{6}\tan\frac{5\pi}{3}$

$= \cot\frac{\pi}{4} - \tan\frac{\pi}{6}\left(-\tan\frac{\pi}{3}\right)$

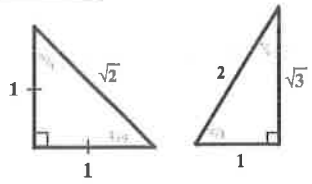
$= 1 - \left(\frac{1}{\sqrt{3}}\right)\left(-\frac{\sqrt{3}}{1}\right)$

$= 1 - (-1)$

$= 2$

Answer Key

1)



2) a) –0.9587 **b)** 0.2198 **c)** –0.7975 **d)** 1.0470 **e)** 0.1844 **f)** 1.7953

3) a) 0.2282 **b)** –2.0000 **c)** 1.0103

4)a) $\frac{\sqrt{3}}{2}$ **b)** $\frac{1}{\sqrt{3}}$ **c)** $-\frac{1}{\sqrt{2}}$ **d)** -1

5)a) $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$; $\cos\frac{2\pi}{3}=-\frac{1}{2}$; $\tan\frac{2\pi}{3}=-\sqrt{3}$

b) $\sin\frac{5\pi}{6}=\frac{1}{2}$; $\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}$; $\tan\frac{5\pi}{6}=-\frac{1}{\sqrt{3}}$

c) $\sin\frac{3\pi}{2}=-1$; $\cos\frac{3\pi}{2}=0$; $\tan\frac{3\pi}{2}=$ undefined

d) $\sin\frac{7\pi}{4}=-\frac{1}{\sqrt{2}}$; $\cos\frac{7\pi}{4}=\frac{1}{\sqrt{2}}$; $\tan\frac{7\pi}{4}=-1$

6) $\sin\frac{11\pi}{6}=-\frac{1}{2}$; $\cos\frac{11\pi}{6}=\frac{\sqrt{3}}{2}$; $\tan\frac{11\pi}{6}=\frac{-1}{\sqrt{3}}$; $\csc\frac{11\pi}{6}=-2$; $\sec\frac{11\pi}{6}=\frac{2}{\sqrt{3}}$; $\cot\frac{11\pi}{6}=-\sqrt{3}$

7)a) $20(\sqrt{2}-1)$ meters **b)** $20(\sqrt{3}-\sqrt{2})$ meters

8)a) $\frac{\sqrt{2}}{2}$ **b)** 2