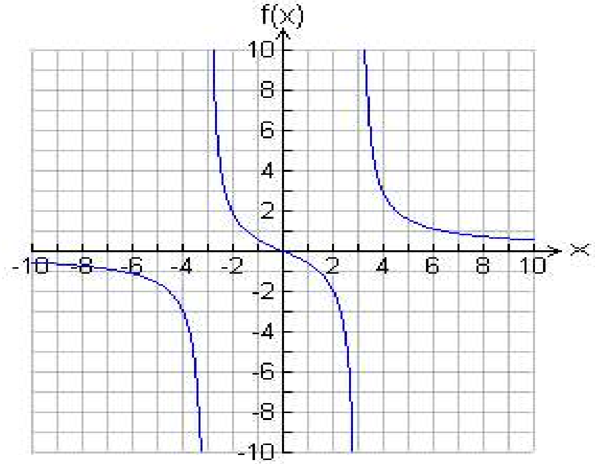
2.5 Solving Rational Function Inequalities

Solving rational inequalities is very similar to solving polynomial inequalities.
We need to consider rational functions are not defined at the zeros of the denominator.

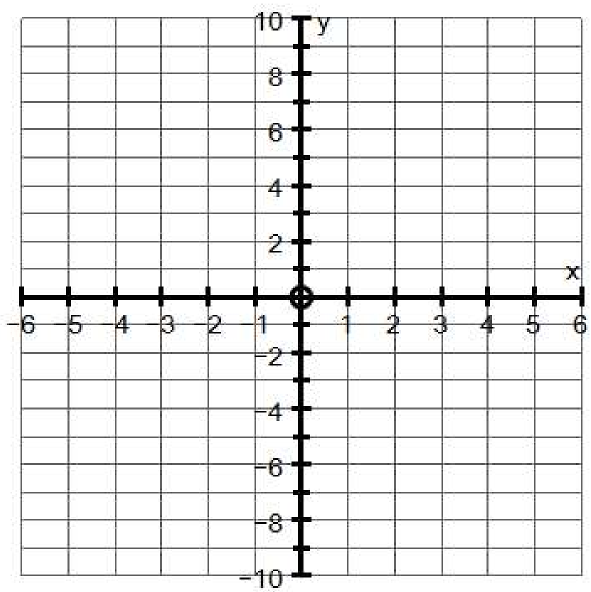
Example 1: For the rational function, f(x), solve when

i) $f(x) > 0$

ii) $f(x) \leq 0$



Example 2: Solve $\frac{2}{x} \geq x - 1$ graphically.



Solving Rational Inequalities Algebraically
Rational inequalities can be solved algebraically, graphically, or comparing values in a table.

- Move everything to the left side and set equal to zero.
- Rewrite the left side as a single fraction.
- Factor the numerator and denominator.
- Use the numerator to find the zero points.

  < 0 or > 0 open

  ≤ 0 or ≥ 0 closed
- Use the denominator to find discontinuity points.
- Locate the points on a number line.
- Label intervals formed as positive (+) or negative (–) by testing points.
- Shade appropriate intervals.

  < 0 or ≤ 0 negative

  > 0 or ≥ 0 positive
- Give a numeric solution.

Example 3 : Solve each of the following, $x \in R$

a) $\dfrac{2}{x-1} > 0$

b) $\dfrac{x-2}{x^2+x} \geq 0$

c) $\dfrac{x^2+4x-5}{1-x} < 0$

d) $\frac{x}{x+1} \le \frac{4x-3}{x-7}$

e) $\frac{2x}{x-1} \le \frac{x}{x+2}$

f) $\frac{x^2+3x}{\left(x^2+10x+21\right)\left(x^2+4\right)} \ge \frac{-x}{x^3-2x^2+4x-8}$

Exit Card!

Solve $\dfrac{2x}{x+1} \geq \dfrac{x+6}{x+3}$.

Collect all terms to the left side of the inequality and 0to the right side.

Simplify the expression on the left side by finding a common denominator and adding the terms.

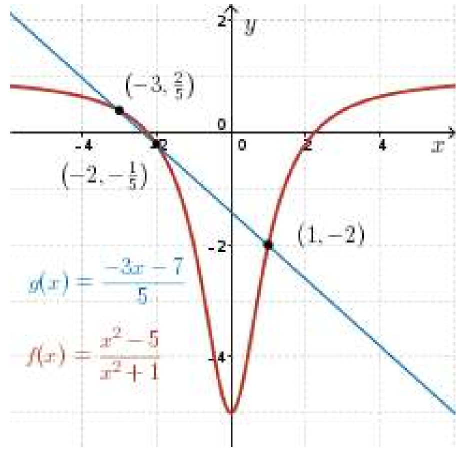
Once simplified, factor the numerator, if possible, always leaving the denominator in factored form

Create an interval table and identify the sign of each factor in the rational expression within each interval.

The zero value of each factor in the numerator and denominator indicates when a sign change may occur for that factor.

2.5 Practice

1. a. Given the graphs of $f(x)=\dfrac{x^2-5}{x^2+1}$ and $g(x)=\dfrac{-3x-7}{5}$, determine the solution of $f(x)<g(x)$.

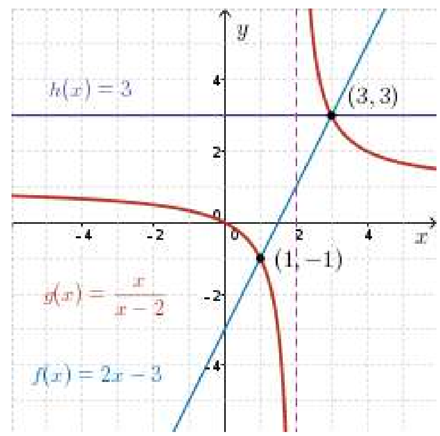


b. Given the graphs of $f(x)=2x-3$, $g(x)=\dfrac{x}{x-2}$ and $h(x)=3$, determine the solution of

i. $\dfrac{x}{x-2}\geq 2x-3$

ii. $\dfrac{x}{x-2}\leq 3$

iii. $2x-3\leq\dfrac{x}{x-2}\leq 3$



2. Solve each inequality graphically, where $x \in \mathbb{R}$

a. $\dfrac{3}{x+2}\geq x$

b. $\dfrac{x}{x+7}<\dfrac{-x}{x-2}$

3. Solve each inequality algebraically. State the solution using interval notation, where $x \in \mathbb{R}$.

a. $\dfrac{3x+4}{2x-1}>0$ b. $\dfrac{12x^2+11x+2}{2x^2-7x+3}\leqslant 0$ c. $\dfrac{3-x}{2x+2}>\dfrac{x}{2}$

d. $\dfrac{1}{-x^2-1}<-\dfrac{1}{4}$ e. $\dfrac{2x}{x-1}\leqslant\dfrac{x}{x+2}$ f. $\dfrac{3}{x-2}-\dfrac{x-3}{x+1}>\dfrac{x}{x-2}$

g. $\dfrac{x}{x^2-4}\geqslant\dfrac{-1}{x+1}$

4. Given $f(x)=\dfrac{2x}{1-x}$ determine the values of x for which $f(f(x))<-\dfrac{3}{2}, x\in\mathbb{R}$.

Solving Rational Inequalities

Answers

1. a. $x \in (-\infty, -3) \cup (-2, 1), x \in \mathbb{R}$

   b. i. $x \in (-\infty, 1] \cup (2, 3], x \in \mathbb{R}$

      ii. $x \in (-\infty, 2) \cup [3, \infty), x \in \mathbb{R}$

      iii. $x \in (-\infty, 1], x \in \mathbb{R}$

2. a. $x \leq -3$ or $-2 < x \leq 1, x \in \mathbb{R}$

   b. $-7 < x < -2.5$ or $0 < x < 2, x \in \mathbb{R}$

3. a. $x \in \left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{1}{2}, \infty\right), x \in \mathbb{R}$

   b. $x \in \left[-\frac{2}{3}, -\frac{1}{4}\right] \cup \left(\frac{1}{2}, 3\right), x \in \mathbb{R}$

   c. $x \in (-\infty, -3) \cup (-1, 1), x \in \mathbb{R}$

   d. $x \in \left(-\sqrt{3}, \sqrt{3}\right), x \in \mathbb{R}$

   e. $x \in \left[-5, -2\right) \cup \left[0, 1\right), x \in \mathbb{R}$

   f. $x \in \left(-1, \frac{1}{2}\right) \cup (2, 3), x \in \mathbb{R}$

   g. $x \in \left(-2, \frac{-1-\sqrt{33}}{4}\right] \cup \left(-1, \frac{-1+\sqrt{33}}{4}\right] \cup (2, \infty), x \in \mathbb{R}$

4. $\frac{1}{3} < x < 1$ or $1 < x \leq 3, x \in \mathbb{R}$

Warm up

Solve $\dfrac{3}{x-2} - \dfrac{x-3}{x+1} > \dfrac{x}{x-2}$.