# Unit 5: Exponential and Logarithmic Functions
# 5.4 Techniques for Solving Exponential & Logarithmic Equations

**Part A: Solving Exponential Equations:**

Exponential equations have variables in the exponents or bases.

**Ex1.** $8^x = 4^{2x+1}$

**Ex2.** $5^{3x} = 63$

**Ex3.** $64^{3x-5} = \left(5^{6x+8}\right)$

**Ex4.** $4(2^x) = 3^{x+1}$

**Ex5.** Solve a) $5^{2x} - 5^x - 20 = 0$

b) $3^{2x} - 6(3)^x - 7 = 0$

**Ex.6:** All radioactive substances decrease in mass over time. Jamie works in a laboratory that uses radioactive substances. The laboratory received a shipment of 200 g of radioactive radon, and 16 days later, 12.5 g of the radon remained. What is the half-life of radon?

**Ex.7**: Solve $2(5^{6x})-9(5^{4x})+10(5^{2x})-3=0$.

**Ex.8:** Solve $2^{x+1}=3^{x-1}$ to three decimal places.

**Ex.9:** Express $\dfrac{2^6 \times \left(\frac{1}{4}\right)^5}{\left(\sqrt[4]{16}\right)^3}$ as a power with a base of 4.

## Exit Card!

Solve for x.

a) $4\left(\sqrt{2^x}\right) - \dfrac{5}{\sqrt{2^x}} = -1$

b) $\log_{(m-1)}(m^2 - 1) = 3$

c) $5^{4x} = 7(4^{x-2})$ (Round to 2dp)

d) $\left(3^{2x}\right) + 2\left(3^{x+1}\right) - 27 = 0$

**Practice**

Solve each of the following equations

a) $2^{2x} - 8(2^{x}) + 16 = 0$

b) $5^{2x} - 26(5^{x}) + 25 = 0$

c) $(27 \times 3^{x}) = 27^{x} \times 3^{0.25}$

d) $2^{2x} - 8(2^{x}) + 16 = 0$

e) $2^{2x+3} - 3(2^{x+1}) + 1 = 0$

f) $x^{\frac{4}{3}} - 13x^{\frac{2}{3}} = -36$

g) $2^{2+x} - 2^{2-x} = 15$

h) $2^{x} + 2^{2-x} = 5$

i) $(x-5)^{\frac{2}{3}} = (27)^{-\frac{1}{9}}$

Answer

a) 2

b) 0 or 2

c) $\frac{11}{8}$

d) -1 or 3

e) -1 or -2

f) ±8 or ±27

g) 2

h) 0 or 2

i) $\frac{15 \pm \sqrt{3}}{3}$

**Part B: Solving Logarithmic Equations**

The properties of logarithms we learned in the last sections can help us solve equations involving logarithmic expressions. We must remember that $y = \log_a x$ is defined only for x > 0. Some of the logarithmic equations we solve will appear to have a root that is less that zero. Such a root is inadmissible. This means that every time we solve a logarithmic equation, we must check that the roots obtained are admissible.

**Ex.1 Solve for x.**

a) $\log_x 0.01 = -2$

b) $\log_5(2x - 4) = \log_5 36$

c) $\log_6 x + \log_6(x+1) = 1$

d) $\log_8\left(x^3\right) + 6\log_4\left(x\right) = -1$

e) $\log_3\left(x\right) - 4\log_x\left(9\right) = 2$

f) $\log_2\left(\log_4\left(x\right)\right) = 2$

g) $\left[\log_4(x)\right]^2 + 18\log_{x^3}(4) = 7$

h) $\log\sqrt[3]{x^2 + 48x} = \dfrac{2}{3}$

i) $\log_3\left(x^2 + 5x - 36\right) - \log_3\left(x^2 - 2x - 8\right) + \log_3\left(x + 2\right) = 3$

j) $\log_{\sqrt{x}} 5^{\log_x 5} + 3(\log_x 5) - 2 = 0$

**Ex2**. Solve the system:

$$\begin{cases} y = 2\log_3(x) \\ y + 1 = \log_3(9x) \end{cases}$$

**Ex.3** If the population of a colony of bacteria doubles every 30 minutes, how long will it take for the population to triple?

**Ex.4*** Determine the values of $x$ and $y$ given the following information.

- $\log_x\left(-\frac{1}{4}\log_y\left(\log_{x^y} x\right)^2\right)=1$
- $(\log_2 x)(\log_2 y)-3\log_2(4y)-\log_2(8x)=-16$

Warm Up

Solve for x.

a) $2\log_x(x)\log_2(x-6) = \log_{(x-6)}16 \bullet \log_{(x-6)}\left(x^2 - 12x + 36\right)$

b) $\log_3(x) + \log_2(x) = 5$ (Round to 3dp)

## Mid-Review: Logarithmic Functions

1. Evaluate each of the following exactly.

a) $\log_a \frac{1}{\sqrt[5]{a}}$

b) $7^{-4\log_7 x^3}$

c) $\log_3 81 - 3\log_3 27$

d) $\log_9 81^{2x}$

e) $\log_2\left(\sin\left(\frac{\pi}{4}\right)\right)$

f) $\left(\frac{1}{5}\right)^{\log_{\sqrt{5}} 100}$

g) $\log_{64}(4096) - \frac{1}{2}\log_6(46656)$

h) $\log_{36} 2 - \frac{1}{2}\log_{\frac{1}{6}} 3$

i) $125^{\log_5 125}$

2. State

a) the domain and range of $f(x) = \log\sqrt{x^2 - 9}$ .

b) the $\frac{1}{2}\log_4 x^2 + \frac{3}{2}\log_4 y^4 - \frac{\log xy}{\log 4}$ as a single logarithm.

c) the $3^3 \times \left(\sqrt{729}\right)^5$ as a single power of 9.

3. Express $\log_3(x-2) + \log_3 y - \log_3(x^2 - 4)$ as a single logarithm. State your final answer in the simplest form possible.

4. Evaluate using properties of logarithms.

a) $\left(\sqrt{10}\right)^{4\log\sqrt[8]{6} - \log 4}$

b) $(0.2)^{-2 + \log_{\sqrt{5}} 10}$

c) $\log_6 3 + \left(\frac{1}{2}\right)\log_6 5 - \log_6 2$

d) $\log_8\left(\frac{\sqrt{2}}{4}\right)$

5. Using $\log_a\left(\frac{x+y}{5}\right) = \frac{1}{2}(\log_a x + \log_a y)$ , show that $x^2 + y^2 = 23xy$.

6. Solve for x $\log_b x = 2\log_b(1-a) + \frac{2}{\log_{(1+a)} b} - \log_b\left(\frac{1}{a} - a\right)^2$.

7. Explain why $\log_{-2}\left(-\frac{1}{8}\right) = -3$ is not a valid logarithmic equation, but it does make algebraic sense exponentially.

8. Explain the steps used to solve the equation $\sqrt[3]{256^2} \times 16^x = 64^{x-3}$.

9. If $\log_a 2 = x$ and $\log_a 3 = y$ find the value of $\log_{\sqrt{6}} 12$ in terms of x and y.

10. If $\log_{12} 3 = m$, find the value of $\log_{\sqrt{3}} 16$ in terms of $m$ .

11. How many digits are there in the number $3^{2015}$?