Unit 3: Trigonometry
3.4 Double Angle Formulas

We will extend our knowledge of compound angle formulas to include the double angle formulas. These formulas are special cases of the angle sum formulas studied in the previous lesson.

Double Angle Formula for Sine

$\sin(2A) = 2\sin(A)\cos(A)$

Proof:

Double Angle Formula for Cosine

$\cos(2A) = \cos^2(A) - \sin^2(A)$ $\quad$ $\cos(2A) = 2\cos^2(A) - 1$ $\quad$ $\cos(2A) = 1 - 2\sin^2(A)$

Proof:

Double Angle Formula for Tangent

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

.

Proof:

Example 1: Express the following as a single trigonometric ratio.

a) $2\sin(5y)\cos(5y)$

b) $1-2\sin^2(3x)$

c) $4\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

d) $\sin(6x)\cos(6x)$

e) $2\cos^2(3\theta-2)-1$

f) $\cos^2\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)$

Example 2: Express the following as a single trigonometric ratio and then evaluate.

a) $\cos^2\left(\frac{\pi}{12}\right)-\sin^2\left(\frac{\pi}{12}\right)$

b) $\dfrac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)}$

c) $\sin^2(75^\circ)-\cos^2(75^\circ)$

d) $\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$

Example 3: Determine the exact values of $\sin(2\theta)$ and $\cos(2\theta)$ if $\sin(\theta)=\frac{4}{5}$ for $\frac{\pi}{2}\leq\theta\leq\pi$.

Example 4: If $\tan(\theta) = -\frac{\sqrt{5}}{2}$ for $\frac{\pi}{2} \leq \theta \leq \pi$, determine the exact value of $\sin(2\theta)$.

Example 5: Use a double angle formula to rewrite each of the following:

a) $\sin(6x) =$

b) $\sin(x)=$

c) $\cos(x) =$

Example 6*: Use the fact that $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ to determine the exact value of $\cos\left(\frac{\pi}{8}\right)$.

3.4 Practice

1. **Multiple Choice:** Select the best answer for each of the following

i. Given $\cos(\theta)=\frac{2}{3}$ in the first quadrant, the value of $\sin\left(\frac{\theta}{2}\right)$ is: ____

a. $\frac{\sqrt{3}}{2}$ b. $\frac{\sqrt{6}}{2}$ c. $\frac{\sqrt{6}}{6}$ d. $\frac{\sqrt{3}}{6}$

ii. $2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$ is equivalent to : ____

a. $\sin\left(\frac{\pi}{8}\right)$ b. $\cos\left(\frac{\pi}{8}\right)$ c. $\sin\left(\frac{\pi}{4}\right)$ d. $\cos\left(\frac{\pi}{4}\right)$

iii. The exact value of $1-2\sin^2\left(\frac{\pi}{8}\right)$ is: ____

a. $-\sqrt{2}$ b. $\frac{\sqrt{2}}{2}$ c. $-\frac{\sqrt{2}}{2}$ d. $\sqrt{2}$

2. Express as a single sine or cosine function.

a) $10\sin(x)\cos(x)$

b) $1-2\sin^2\left(\frac{2\theta}{3}\right)$

c) $5\sin(2x)\cos(2x)$

d) $2\cos^2(5\theta)-1$

3. Simplify each expression. State any restrictions on the variable in the domain $[0,2\pi]$

a) $\frac{\sin(2a)}{\cos(a)}=$

b) $2\tan(a)\cos^2(a)=$

c) $2\sin^2(a)+\cos(2a)=$

4. Expand using a double angle formula.

a) $3\sin(4x)=$

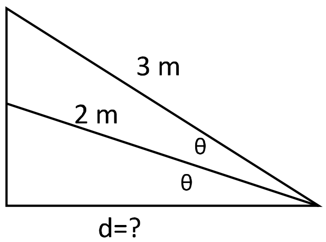
b) $6\cos(6x)=$

c) $1-\cos(8x)=$

d) $\tan(4x)=$

e) $\cos(2x)-\frac{\sin(2x)}{\sin(x)}=$

5. Two ropes (2m and 3m long) used to stabilize a pole for a volleyball net are anchored to the ground. The angle between the two ropes is equal to the angle between the ground and the lower rope. Determine the distance from the base of the pole to the point at which the ropes are anchored to the ground.



6. a) Express $\sec(2\theta)$ in terms of $\sec(\theta)$ and $\tan(\theta)$.

b) Express $\csc(2\theta)$ in terms of $\csc(\theta)$ and $\sec(\theta)$.

7. Determine formulas for

a) $\sin(3\theta)$ in terms of $\sin(\theta)$

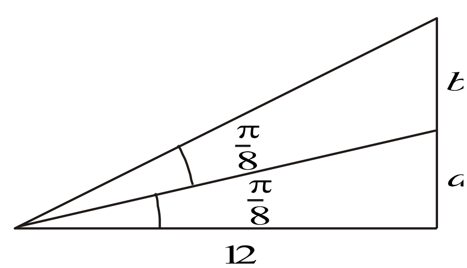
b) $\cos(3\theta)$ in terms of $\cos(\theta)$

d) $\tan(3\theta)$ in terms of $\tan(\theta)$

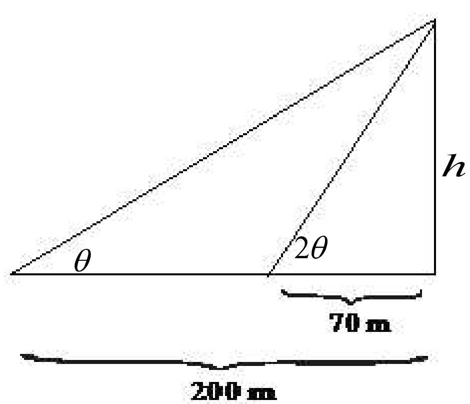
8. Express $\sin(2\theta)$ and $\cos(2\theta)$ in terms of $\tan(\theta)$.

9. If x and y are all first quadrant angles and $\sec(x) = \frac{5}{3}$ and $\sin(y) = \frac{1}{3}$, then determine the **exact** value of $\sin(2x + y)$.

10. Find the exact value of *b*.



11. Find the value of h in the below diagram.



Warm up

1. Express each of the following as a single trigonometric ratio, <u>and then</u> evaluate the exact value of the ratio.

b) $\sin^2\left(\dfrac{7\pi}{3}\right)-\cos^2\left(\dfrac{7\pi}{3}\right)$

c) $1-2\sin^2\left(\dfrac{5\pi}{12}\right)$

d) $\sin\left(\dfrac{5\pi}{6}\right)\cos\left(\dfrac{3\pi}{8}\right)-\cos\left(\dfrac{5\pi}{6}\right)\sin\left(\dfrac{3\pi}{8}\right)$

e) $2\cos^2(\dfrac{5\pi}{8})-1$

f) $\cos\dfrac{4\pi}{3}-\csc\dfrac{5\pi}{6}$

2. If $\cos\alpha = \frac{4}{5}$, and $\sin\beta = -\frac{12}{13}$, $0 < \alpha < \frac{\pi}{2}$, $\pi < \beta < \frac{3\pi}{2}$, evaluate $\cos(2\alpha + \beta)$.

3. If $\frac{\pi}{2} < x < \pi$ and $\cos^2 x = \frac{8}{9}$, determine the **exact** value of $\sin(4x)$.