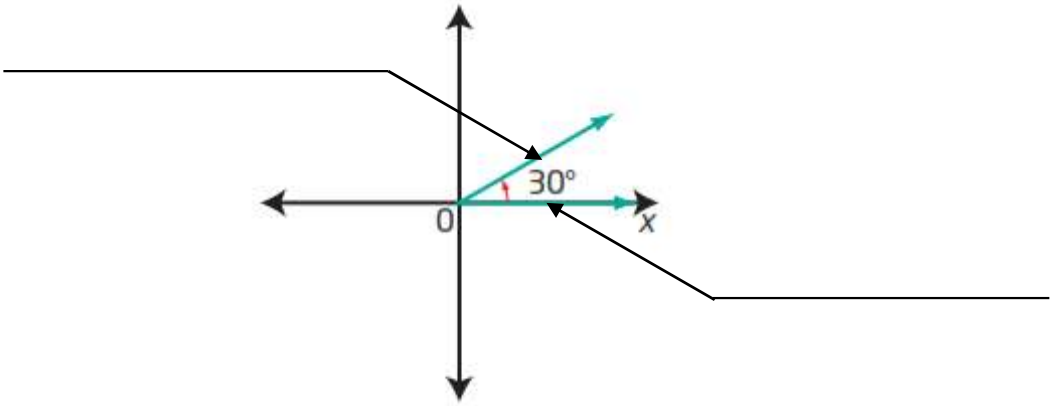
Grade 11 Trigonometry -Review

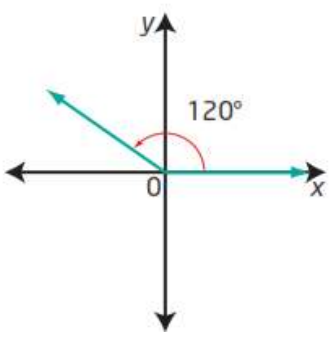
Part A: Standard Position and Co-terminal Angles

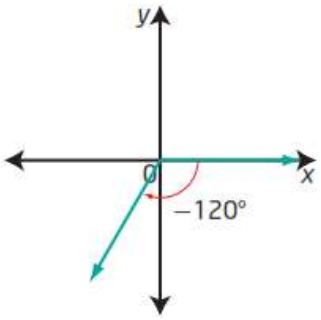
STANDARD POSITION An angle in standard position has its center at the origin and its initial arm along the positive *x*-axis.

Label the initial arm and terminal arm for the angle 30° below.



An angle can be either positive or negative. If the terminal arm of an angle opens in a counter-clockwise direction, the angle is positive. If the terminal arm opens in a clock-wise direction, the angle is negative.

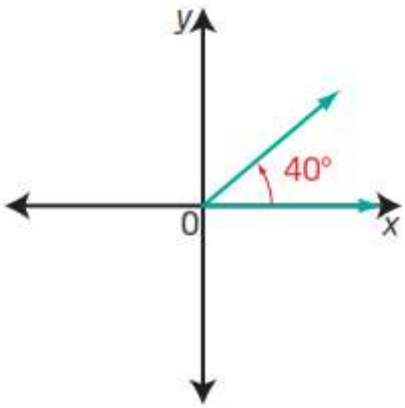




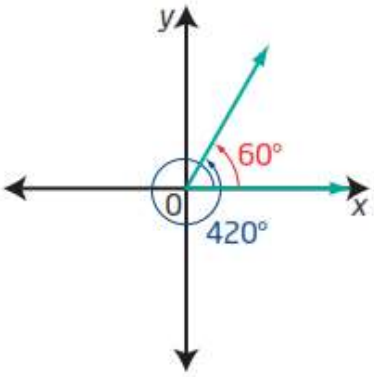
Exercises:

1. Draw the following angles in standard position. The first one has been done for you.

a) 40 ° b) 120° c) -45° d) 430°



COTERMINAL ANGLES are angles in standard position with the same terminal arms. coterminal angles, as are 40° and -320°.



Any given angle has an infinite number of angles coterminal with it, since each time you make one full rotation from the terminal arm, you arrive back at the same terminal arm. Angles coterminal with any
angle θ can be described using the expression $\theta \pm (360°)n$, where n is a natural number. This way of expressing an answer is called the **general form**

Exercises:
2. For each angle below, find one positive coterminal angle and one negative coterminal angle.
a) 30° b) 310° c) -90°

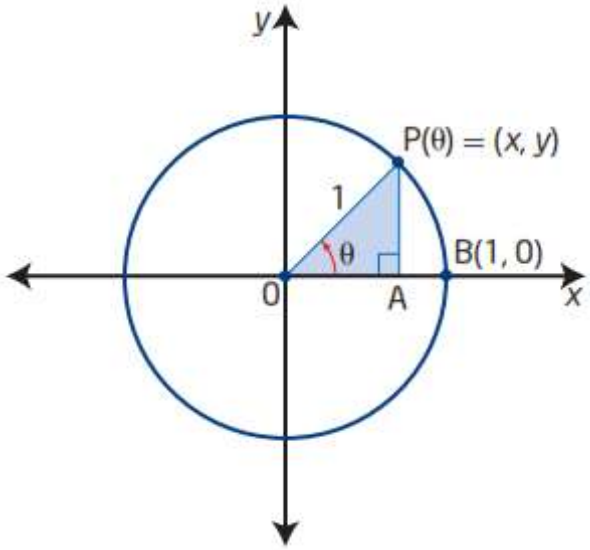
Part B: Definitions of the Primary & Reciprocal Trigonometric Ratios

The **Standard Position** of angles allows us to **<u>define</u>** trigonometric ratios for ANY angle, even angles bigger than 90°. To find the trig ratios for θ , pick a point P(x, y) on the terminal arm. Drop a <u>vertical</u> line to the x-axis to construct a right triangle.

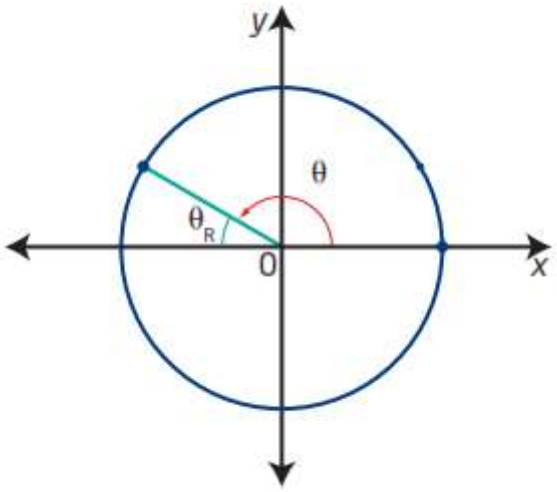
$\text{Sin}(\theta) = \frac{y}{r}$

$\cos(\theta) =$

$\tan(\theta) =$



When the terminal arm of θ lies in the 2nd, 3rd, or 4th quadrant then θ will not be contained in the right triangle. So we *define* the sine, cosine and tangent for such angles to be **the related acute angle, θ_R ,** the acute angle formed between the terminal arm and the x-axis taking into account whether the trig ratio is positive or negative as indicated by the CAST rule (see Part D).



The sine, cosine and tangent ratios for an angle are called the PRIMARY TRIGONOMETRIC RATIOS. The RECIPROCAL TRIGONOMETRIC RATIOS are defined as follows:

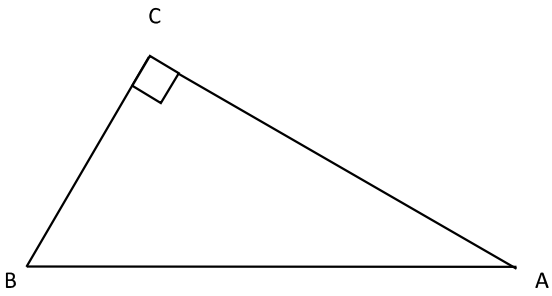
cosecant θ or csc(θ) = $\frac{1}{\sin(\theta)} = \frac{r}{y}$

secant θ or sec(θ) =

cotangent θ or cot(θ) =

Exercises:

3. In the triangle drawn at the right, ∠C = 90°, c = 13, and a = 5. State all six trig ratios for ∠B.



Part C: Special Angles

Certain angles are considered 'special' because finding their trig ratios is relatively easy as such angles are readily formed in 'special' triangles. Sketch the special triangles that allow you find the primary and reciprocal trig ratios for 45°, 30° & 60°, and 0° & 90°. For example, the special triangle for 45° is a right isosceles triangle with side lengths 1, 1, $\sqrt{2}$. Use your triangles to complete the chart below.

Special Angle	0°	30°	45°	60°	90°	180°
Sin(θ)						
cos(θ)						
tan(θ)						
sec(θ)						
csc(θ)						

Part D: The CAST Rule

For each diagram below, P is a point on the terminal arm of an angle **θ.**

i) Construct a right triangle by dropping a perpendicular from P to the x-axis.

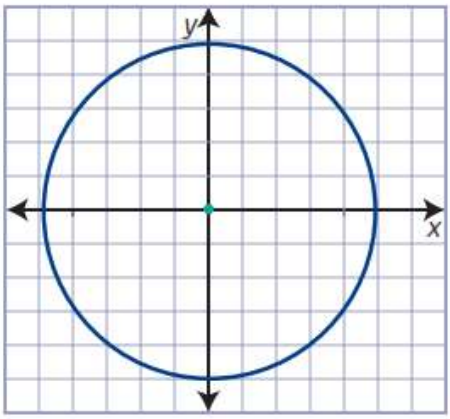
ii) Determine the 'lengths' of all three sides of the right triangle constructed, including whether the 'length' is positive or negative.

iii) Determine the three primary trig ratios, sine, cosine and tangent. (NOTE: some of these trig ratios WILL be negative)

iv) Determine **θ** in degrees.

QUADRANT 1

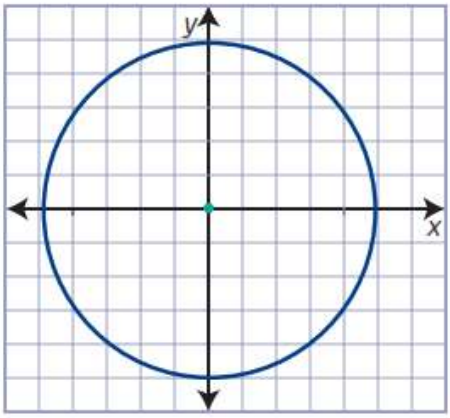
P(4, 3)



sin**(θ)** =
cos**(θ)** =
tan**(θ)** =

QUADRANT 2

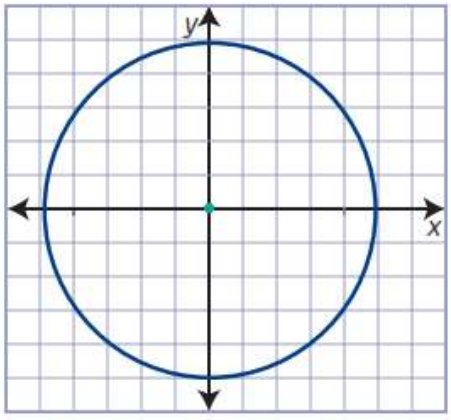
P(-4, 3)



sin**(θ)** =
cos**(θ)** =
tan**(θ)** =

QUADRANT 3

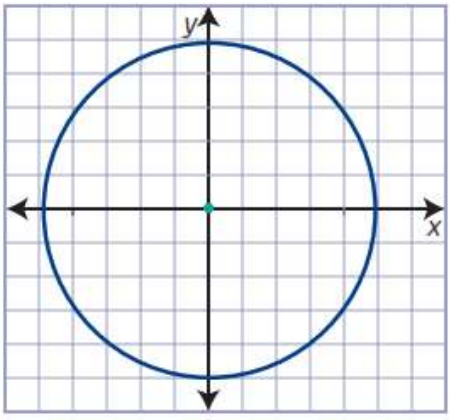
P(-4, -3)



sin**(θ)** =
cos**(θ)** =
tan**(θ)** =

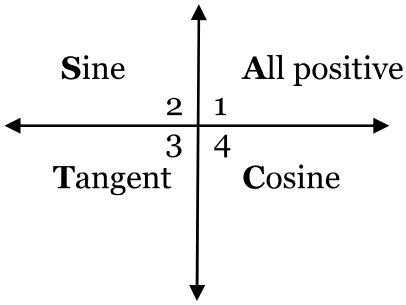
QUADRANT 4

P(4, -3)



sin**(θ)** =
cos**(θ)** =
tan**(θ)** =

CAST rule
Notice that exactly one trig ratio is positive in every quadrant except of the first quadrant in which all the ratios are positive. The CAST rule helps us remember in which quadrants a particular ratio is positive.

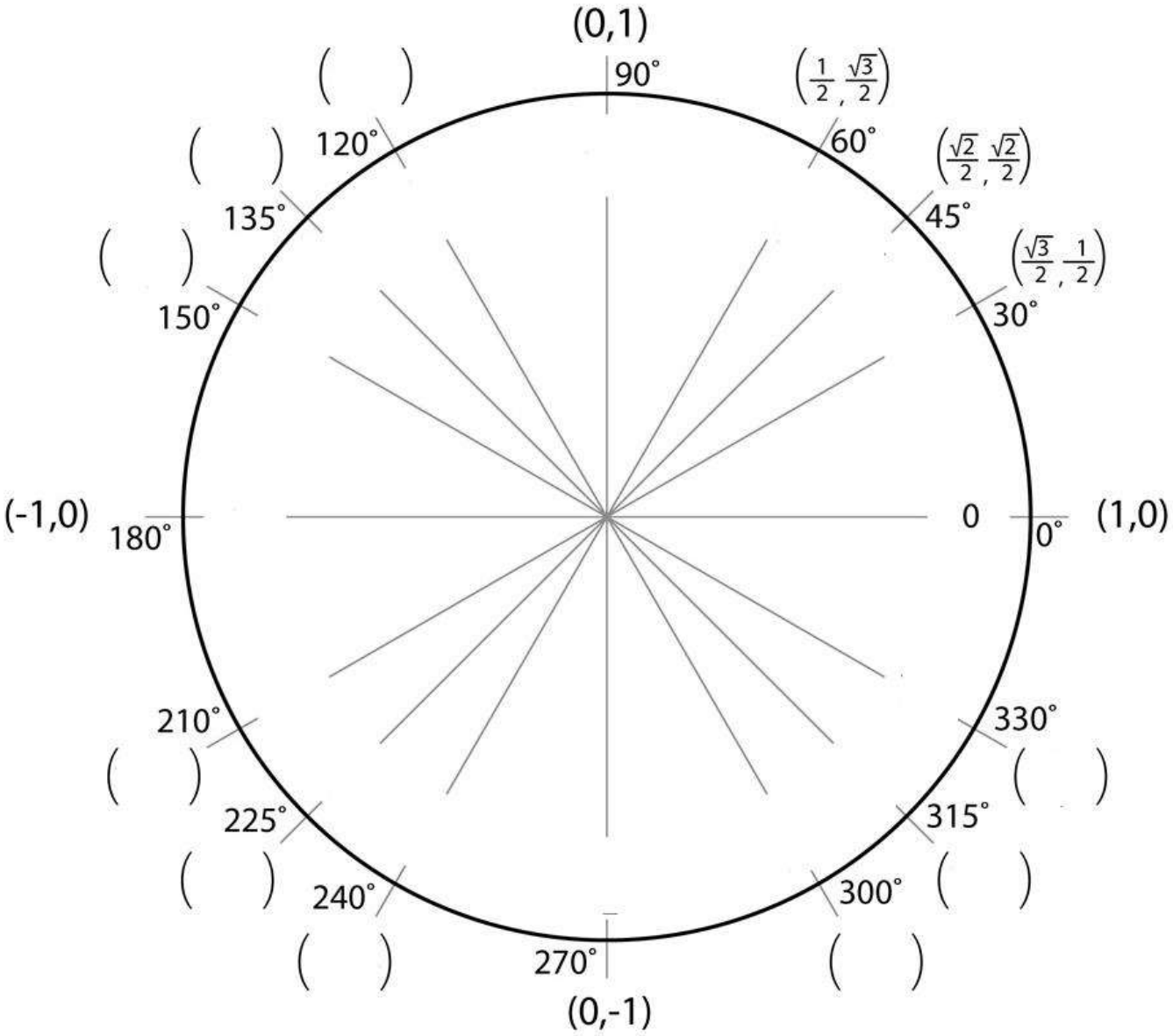


Exercise:

Complete the following chart for the trig ratios listed in the top row. You will have to use your knowledge of SPECIAL ANGLES. Do NOT use your calculator.

Trig Ratio	**a) cos120°**	**b) sin150°**	**c) tan330°**
Standard Position of Angle ⇒Construct right triangle ⇒Determine the reference angle θ_R			
Positive or Negative?			
Exact Value of Trig Ratio			

Unit Circle


(0,1)
90°
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
60°
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
45°
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
30°
120°
135°
150°
(-1,0)
180°
0
0°
(1,0)
210°
330°
225°
315°
240°
300°
270°
(0,-1)

Grade 11 Trig Review

1. Predict whether each value will be positive or negative. Sketch each angle on a coordinate grid.
   a) tan 167° b) sin 99° c) cos 132°

2. Given that ($0 \le \theta \le 180°$), determine the value(s) of θ *to* 1 decimal place.
   a) cos θ = 0.4772 b) tan θ = -0.2272 c) sin θ = 0.5476
   d) tan θ = 1.6191 e) sin θ = 0.3486 f) cos θ = 0.5577

3. Angle θ is obtuse.
   a) $\tan\theta$ = -0.4452; calculate $\sin\theta$ to 4 decimal places.
   b) $\sin\theta$ = 0.9707; calculate $\cos\theta$ to 4 decimal places.

4. Sketch each angle θ in standard position, then write a coterminal angle.
   a) θ =170° b) θ =293° c) θ =-30° d) θ =-320°
   e) θ =450° f) θ =600° g) θ =-370° h) θ =200°

5. Determine two angles between 0° and 360° that have each trigonometric function value. Write the angle to the nearest degree.
   a) $\sin\theta$ = 0.42 b) $\cos\theta$ = -0.31 c) $\tan\theta$ = 3.46

6. The point P(4, -15) lies on the terminal arm of an angle θ in standard position. Determine each trigonometric function value to 3 decimal places.
   a) $\sin\theta$ b) $\cos\theta$ c) $\tan\theta$

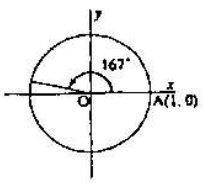
7. The terminal arm of an angle θ lies in Quadrant II on the line with equation 4x + 3y = 0. Determine each trigonometric function value.
   a) $\sin\theta$ b) $\cos\theta$ c) tan θ

8. State each exact value. Do not use a calculator.
   a) cos 135° b) tan 225° c) sin 210° d) $\dfrac{1}{\tan 60°}$

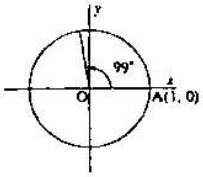
9. Simplify each expression. Do not use a calculator.
   a) sin 30° + cos 60° b) tan 45° + tan 225° c) sin 240° + cos 300°

Answers

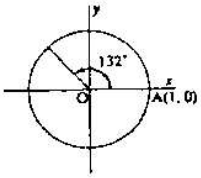
1. a) Negative



b) Positive



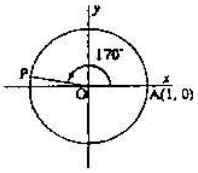
c) Negative



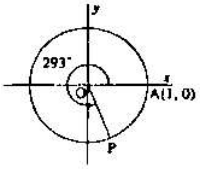
2. a) 61.5° b) 167.2° c) 33.2°, 146.8°
d) 58.3° e) 20.4°, 159.6° f) 123.9°

4. Coterminal angles may vary.

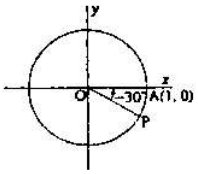
a) 530°



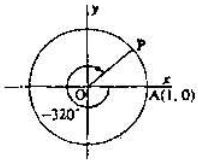
b) 653°



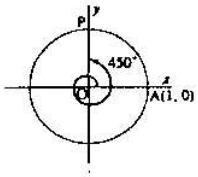
c) 330°



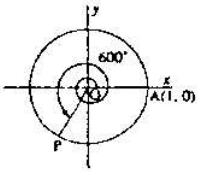
d) 40°



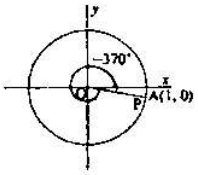
e) 90°



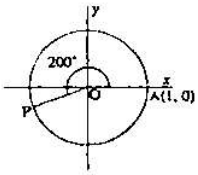
f) 240°



g) 350°



h) 560°



5. a) 25°, 155° b) 108°, 252° c) 74°, 254°

6. a) −0.966 b) 0.258 c) −3.750

7. a) $\frac{4}{5}$ b) $-\frac{3}{5}$ c) $-\frac{4}{3}$

8. a) $-\frac{1}{\sqrt{2}}$ b) 1 c) $-\frac{1}{2}$ d) $\frac{1}{\sqrt{3}}$

9. a) 1 b) 2 c) $\frac{1-\sqrt{3}}{2}$