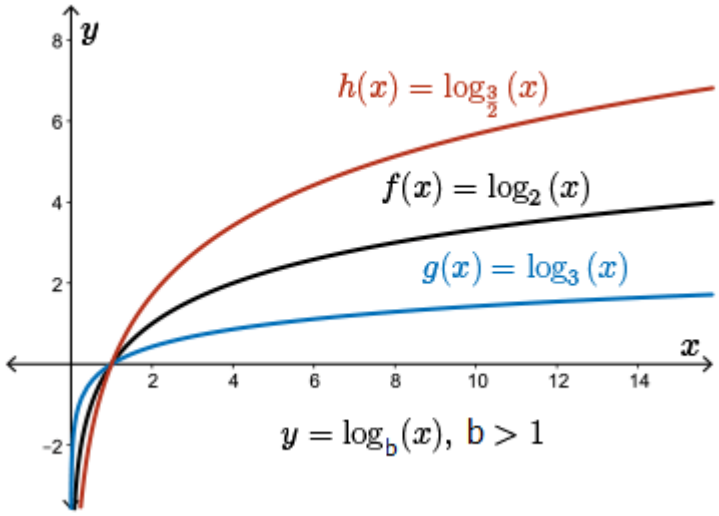
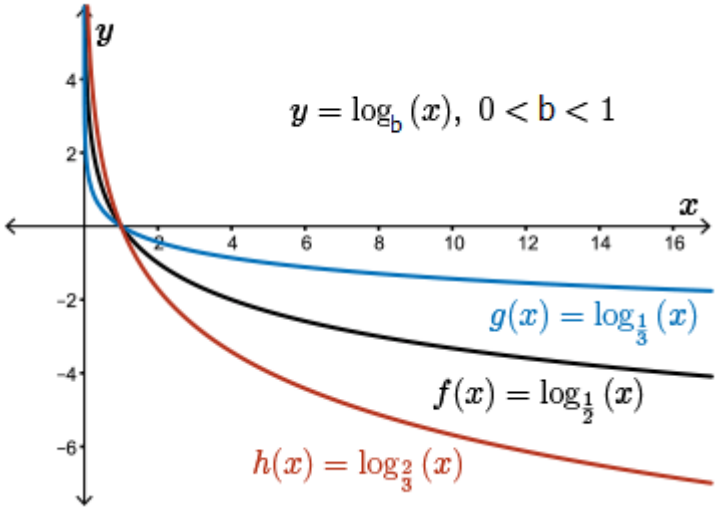
Unit 5: Exponential and Logarithmic Functions
5.3 Transformations of Logarithmic Functions

General Observations

The graph of a logarithmic function where $b > 1$ is always increasing. The greater the value of the base, b, the slower the curve increases as x increases.



The graph of a logarithmic function where $0 < b < 1$ is always decreasing. The smaller the value of the base, b, the slower the curve decreases as x increases.



Transformations of Logarithmic Functions

The parameters a, b, d, and c in the equation $y = a\log_b(k(x-d))+c$ correspond to the following transformations:

- If $a<0, y=log_b(x)$ is reflected in the x-axis.
- $y=log_b(x)$ is stretched vertically about the x-axis by a factor of $|a|$
- If $k<0, y=log_b(x)$ is reflected in the y-axis.
- $y=log_b(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{|k|}$
- $y=log_b(x)$ is translated horizontally d units.
  - If $d>0$, *then* $y=log_b(x)$ is translated right.
  - If $d<0$, *then* $y=log_b(x)$ is translated left.
- $y=log_b(x)$ is translated vertically c units.
  - If $c>0$, *then* $y=log_b(x)$ is translated up.
  - If $c<0$, *then* $y=log_b(x)$ is translated down.

The transformation of each point is defined by the mapping $(x,y)\rightarrow\left(\frac{1}{k}x+d, ay+c\right)$

Example 1

Graph the function $f(x)=2\log_5(3-x)+1$. Identify the domain, range, and any asymptote of the function. Is the function increasing or decreasing?

Solution:

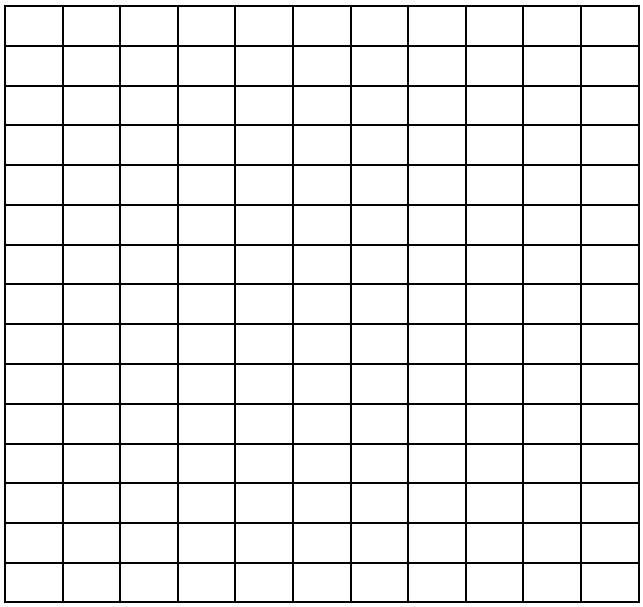
Start with the graph of $y=\log_5(x)$.

Mapping rule: $(x,y)\rightarrow(\_\_\_\_\_,\_\_\_\_\_)$

x	$y=\log_5(x)$.
$\frac{1}{5}$	
1	
5	
25	

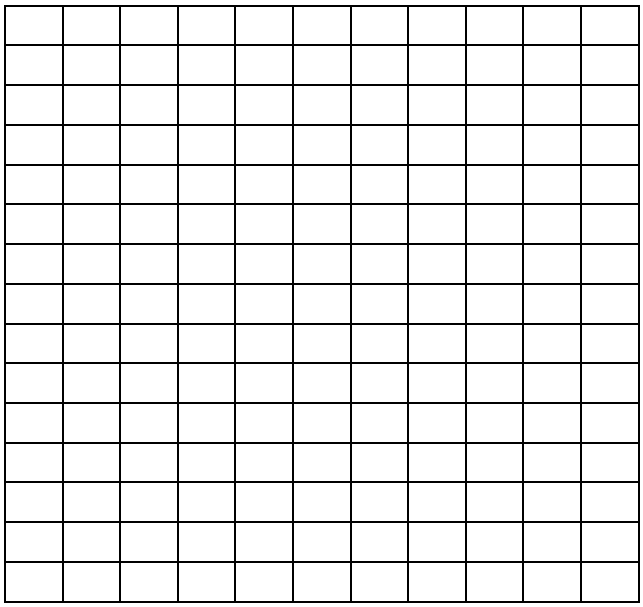
⇨

x	$f(x)$

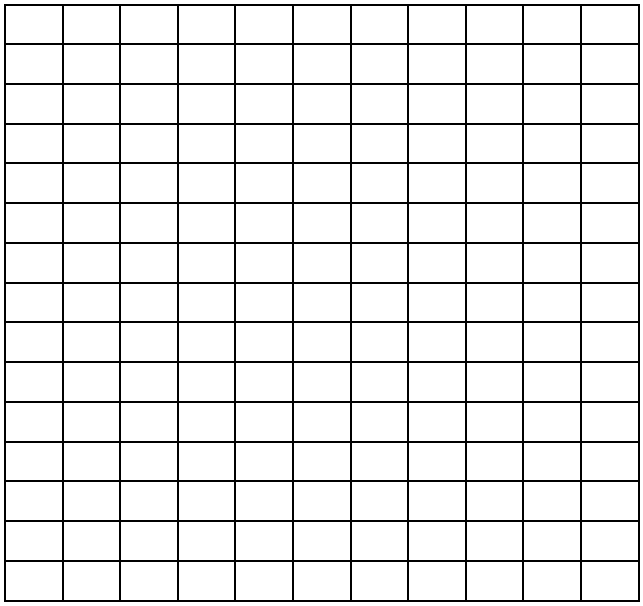


Practice- Using a mapping rule, sketch the following functions on the grids provided.

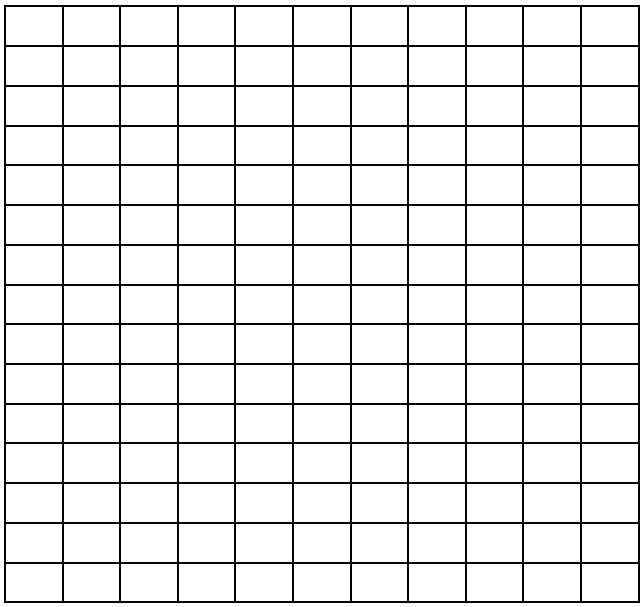
1. a) $f(x) = 2\log_2(x+3)$



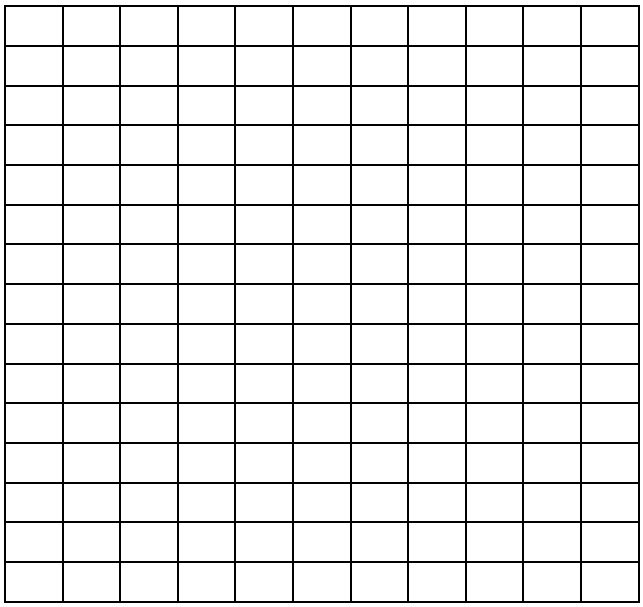
b) $f(x) = -\log_5\left(\sqrt{x}\right) - 3$



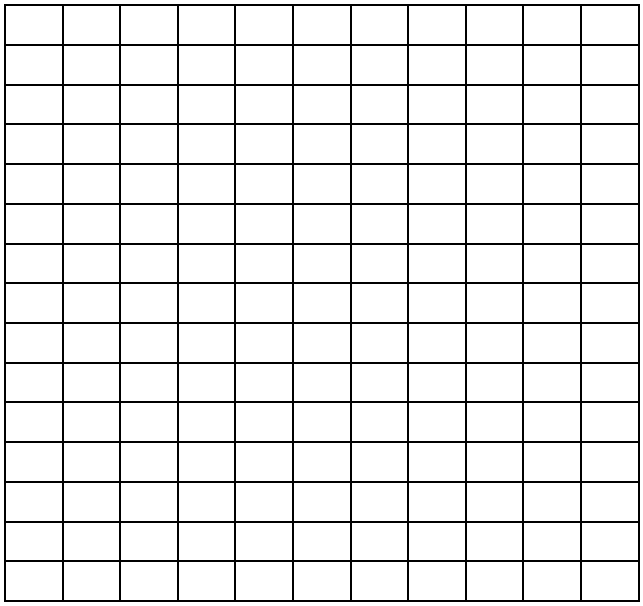
c) $f(x) = 3\log_2(-x) + 1$



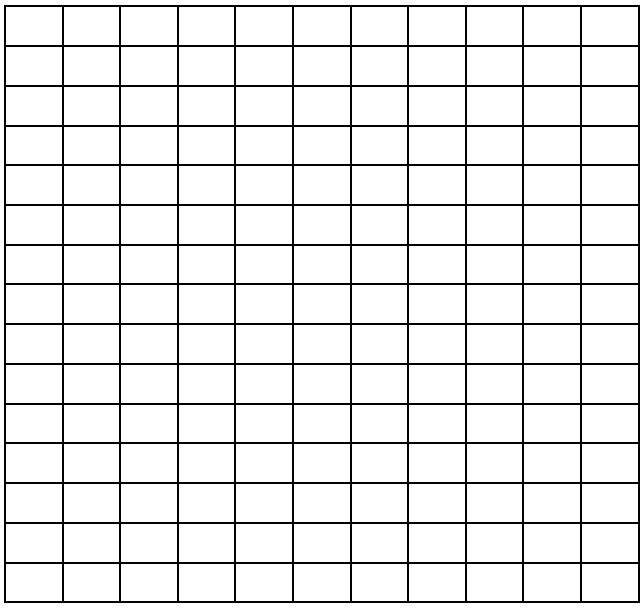
d) $f(x) = -\log\left(\frac{1}{3}x\right) + 4$



e)* $f(x) = 1 - \log_2\left(x^2\right)$



f) $y = 2\log_4(3x+12) - 4$



Exit Card!

Graph the function $y = \frac{6}{\log_{(5x-15)}(4)}$ on the grid below.

Mapping statement: $(x, y) \rightarrow$