

### W3 - 2.2 - Factor Theorem

MHF4U

Determine if  $x + 3$  is a factor of each polynomial:

a)  $x^3 + x^2 - x + 6$

$$\begin{aligned} f(-3) &= (-3)^3 + (-3)^2 - (-3) + 6 \\ &= -27 + 9 + 3 + 6 \\ &= -9 \end{aligned}$$

∴ Not a factor

b)  $2x^3 + 9x^2 + 10x + 3$

$$\begin{aligned} f(-3) &= 2(-3)^3 + 9(-3)^2 + 10(-3) + 3 \\ &= -54 + 81 - 30 + 3 \\ &= 0 \end{aligned}$$

∴ a factor

c)  $x^3 + 27$

$$\begin{aligned} f(-3) &= (-3)^3 + 27 \\ &= 0 \end{aligned}$$

∴ a factor

2) Find possible factors of the following polynomials using integral zero theorem. Then, factor the polynomial.

a)  $x^3 + 3x^2 - 6x - 8$

Possible factors:  $\pm 1, \pm 2, \pm 4, \pm 8$

$f(-1) = 0$ ; ∴  $x+1$  is a factor

-1		1	3	-6	-8	
		↓	-1	-2	8	
⊗		1	2	-8	0	
		$x^2$	$x$	#	R	

$$x^3 + 3x^2 - 6x - 8 = (x+1)(x^2 + 2x - 8)$$

$$= (x+1)(x+4)(x-2)$$

b)  $x^3 + 4x^2 - 15x - 18$

Possible factors:  $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$f(-1) = 0$ ; ∴  $x+1$  is a factor

-1		1	4	-15	-18	
		↓	-1	-3	18	
x		1	3	-18	0	
		$x^2$	$x$	#	R	

$$x^3 + 4x^2 - 15x - 18 = (x+1)(x^2 + 3x - 18)$$

$$= (x+1)(x+6)(x-3)$$

c)  $x^3 - 3x^2 - 10x + 24$

Possible factors:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$f(2) = 0$ ; ∴  $x-2$  is a factor

2		1	-3	-10	24	
		↓	2	-2	-24	
x		1	-1	-12	0	
		$x^2$	$x$	#	R	

$$x^3 - 3x^2 - 10x + 24 = (x-2)(x^2 - x - 12)$$

$$= (x-2)(x-4)(x+3)$$

3) Factor by grouping:

a)  $x^3 + x^2 - 9x - 9$

$$= (x^3 + x^2) + (-9x - 9)$$

$$= x^2(x+1) - 9(x+1)$$

$$= (x+1)(x^2 - 9)$$

$$= (x+1)(x-3)(x+3)$$

b)  $2x^3 - x^2 - 72x + 36$

$$= (2x^3 - x^2) + (-72x + 36)$$

$$= x^2(2x-1) - 36(2x-1)$$

$$= (2x-1)(x^2 - 36)$$

$$= (2x-1)(x-6)(x+6)$$

4) Determine a value of  $k$  so that  $x+2$  is a factor of  $x^3 - 2kx^2 + 6x - 4$ .

$$f(-2) = (-2)^3 - 2k(-2)^2 + 6(-2) - 4$$

$$0 = -8 - 8k - 12 - 4$$

$$0 = -24 - 8k$$

$$24 = -8k$$

$$k = -3$$

5) Find possible factors of the following polynomials using integral zero theorem. Then, factor the polynomial.

a)  $3x^3 + x^2 - 22x - 24$

Possible factors:  $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm 4, \pm \frac{4}{3}, \pm 6, \pm 8, \pm \frac{8}{3}, \pm 12, \pm 24$

$f(-2) = 0$ ; so  $x+2$  is a factor

$$\begin{array}{r|rrrrr} -2 & 3 & 1 & -22 & -24 & \\ & \downarrow & -6 & 10 & 24 & + \\ x & 3 & -5 & -12 & 0 & \\ & x^2 & x & \# & R \end{array}$$

$$3x^3 + x^2 - 22x - 24 = (x+2)(3x^2 - 5x - 12)$$

$$= (x+2)(x-3)(3x+4)$$

$$\begin{array}{c} P \\ -36 \\ \hline \begin{array}{c} \left(\frac{-3}{1}\right) = \frac{-9}{3} \times \left(\frac{4}{3}\right) \\ -5 \\ \hline S \end{array} \end{array}$$

b)  $2x^3 - 9x^2 + 10x - 3$

Possible factors:  $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}$

$f(1) = 0$ ; so  $x-1$  is a factor

$$\begin{array}{r|rrrrr} 1 & 2 & -9 & 10 & -3 & \\ & \downarrow & 2 & -7 & 3 & + \\ x & 2 & -7 & 3 & 0 & \\ & x^2 & x & \# & R \end{array}$$

$$2x^3 - 9x^2 + 10x - 3 = (x-1)(2x^2 - 7x + 3)$$

$$= (x-1)(x-3)(2x-1)$$

$$\begin{array}{c} P \\ 6 \\ \hline \begin{array}{c} \left(\frac{-3}{1}\right) = \frac{-6}{2} \times \left(\frac{-1}{2}\right) \\ -7 \\ \hline S \end{array} \end{array}$$

c)  $6x^3 - 11x^2 - 26x + 15$

Possible factors:  $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{3}, \pm \frac{5}{6}, \dots$

$f(3) = 0$ ; so  $x-3$  is a factor

$$\begin{array}{r|rrrrr} 3 & 6 & -11 & -26 & 15 & \\ & \downarrow & 18 & 21 & -15 & + \\ x & 6 & 7 & -5 & 0 & \\ & x^2 & x & \# & R \end{array}$$

$$6x^3 - 11x^2 - 26x + 15 = (x-3)(6x^2 + 7x - 5)$$

$$= (x-3)(3x+5)(2x-1)$$

$$\begin{array}{c} P \\ -36 \\ \hline \begin{array}{c} \left(\frac{5}{3}\right) = \frac{10}{6} \times \left(\frac{-3}{2}\right) \\ 7 \\ \hline S \end{array} \end{array}$$

d)  $4x^3 + 3x^2 - 4x - 3$

Possible factors:  $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$f(1) = 0$ ; so  $x-1$  is a factor

$$\begin{array}{r|rrrrr} 1 & 4 & 3 & -4 & -3 & \\ & \downarrow & 4 & 7 & 3 & + \\ x & 4 & 7 & 3 & 0 & \\ & x^2 & x & \# & R \end{array}$$

$$4x^3 + 3x^2 - 4x - 3 = (x-1)(4x^2 + 7x + 3)$$

$$= (x-1)(x+1)(4x+3)$$

$$\begin{array}{c} P \\ 12 \\ \hline \begin{array}{c} \left(\frac{1}{1}\right) = \frac{4}{4} \times \left(\frac{3}{4}\right) \\ 7 \\ \hline S \end{array} \end{array}$$

# 6) Factor each polynomial

a)  $2x^3 + 5x^2 - x - 6$

Possible Factors:  $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

$f(1) = 0$  ;  $\therefore x-1$  is a factor

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -1 & -6 \\ & \downarrow & & & \\ x & 2 & 7 & 6 & 0 \\ & x^2 & 7x & 6 & R \end{array}$$

$$\begin{array}{c} P \\ \begin{array}{c} 12 \\ \begin{array}{c} \frac{3}{1} = \frac{4}{2} \\ \frac{3}{2} \end{array} \end{array} \end{array}$$

$$2x^3 + 5x^2 - x - 6 = (x-1)(2x^2 + 7x + 6)$$

$$= (x-1)(x+2)(2x+3)$$

b)  $4x^3 - 7x - 3$

Possible Factors:  $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$f(-1) = 0$  ;  $\therefore x+1$  is a factor

$$\begin{array}{r|rrrr} -1 & 4 & 0 & -7 & -3 \\ & \downarrow & & & \\ x & 4 & -4 & -3 & 0 \\ & 4x^2 & -4x & -3 & R \end{array}$$

$$\begin{array}{c} P \\ \begin{array}{c} -12 \\ \begin{array}{c} \frac{-3}{2} = \frac{-6}{4} \\ \frac{2}{4} = \frac{1}{2} \end{array} \end{array} \end{array}$$

$$4x^3 - 7x - 3 = (x+1)(4x^2 - 4x - 3)$$

$$= (x+1)(2x-3)(2x+1)$$

c)  $x^4 - 15x^2 - 10x + 24$

Possible factors:  $\pm 1, \pm 2, \pm 4$

$f(1) = 0$  ;  $\therefore x-1$  is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -15 & -10 & 24 \\ & \downarrow & & & & \\ x & 1 & 1 & -14 & -24 & 0 \\ & x^3 & x^2 & x & 6 & R \end{array}$$

$$x^4 - 15x^2 - 10x + 24 = (x-1)(x^3 + x^2 - 14x - 24)$$

$$= (x-1)(x+2)(x^2 - x - 12)$$

$$= (x-1)(x+2)(x-4)(x+3)$$

Possible factors:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$f(-2) = 0$  ;  $\therefore x+2$  is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & \downarrow & & & \\ x & 1 & -1 & -12 & 0 \\ & x^2 & x & 6 & R \end{array}$$

## ANSWER KEY

1) **a)** No **b)** Yes **c)** Yes

2) **a)**  $(x - 2)(x + 1)(x + 4)$  **b)**  $(x - 3)(x + 1)(x + 6)$  **c)**  $(x - 4)(x - 2)(x + 3)$

3) **a)**  $(x - 3)(x + 1)(x + 3)$  **b)**  $(x - 6)(x + 6)(2x - 1)$

4)  $k = -3$

5) **a)**  $(x - 3)(x + 2)(3x + 4)$  **b)**  $(x - 3)(x - 1)(2x - 1)$  **c)**  $(x - 3)(2x - 1)(3x + 5)$  **d)**  $(x - 1)(x + 1)(4x + 3)$

6) **a)**  $(x - 1)(x + 2)(2x + 3)$  **b)**  $(x + 1)(2x - 3)(2x + 1)$  **c)**  $(x - 4)(x - 1)(x + 2)(x + 3)$