Unit 2 Pre-Test Review – Factor Theorem and Inequalities
MHF4U

Section 1: 2.1 - Long and Synthetic Division / Remainder Theorem

1) What is the remainder when $x^4 - 4x^2 - 2x + 3$ is divided by $x + 1$? Do not divide. Support your answer with an explanation.

2) Is $x - 3$ a factor of the polynomial $3x^2 - 8x - 3$? Do not divide. Support your answer with an explanation.

3) Divide $\frac{f(x)}{g(x)}$ and state the answer in quotient form. Use synthetic division where possible.

a) $f(x) = x^4 - 4x^2 - 2x + 3, g(x) = x - 2$ **b)** $f(x) = x^5 - x^4 + 2x^3 + 3x - 2, \ g(x) = x^2 + 2$

4) Perform each division. Express the answer in quotient form and write the statement that could be used to check the division.

a) $x^3 + 9x^2 - 5x + 3$ divided by $x - 2$

b) $12x^3 - 2x^2 + x - 11$ divided by $3x + 1$

c) $-8x^4 - 4x + 10x^3 - x^2 + 15$ divided by $2x - 1$

d) $x^3 + 4x^2 - 3$ divided by $x - 2$

5) Determine the value of k such that when $f(x) = x^4 + kx^3 - 3x - 5$ is divided by $x - 3$, the remainder is -10.

Section 2: 2.2 – Factor Theorem

6) Suppose the cubic polynomial $8x^3 + mx^2 + nx - 6$ has both $2x + 3$ and $x - 1$ as factors. Find m and n. Do not divide.

7) Factor each of the following

a) $x^3 - 4x^2 + x + 6$

b) $3x^3 - 5x^2 - 26x - 8$

c) $-4x^3 - 4x^2 + 16x + 16$

d) $x^3 - 64$

Section 3: 2.3&2.6 – Factoring to Solve Equations and Inequalities

8) Determine the real roots of each equation.

a) $(5x^2 + 20)(3x^2 - 48) = 0$

b) $(2x^2 - x - 13)(x^2 + 1) = 0$

9) Solve the following polynomial equations.

a) $2x^3 + 1 = x^2 + 2x$

b) $x^3 + 6x^2 + 11x + 6 = 0$

c) $x^5 - 4x^3 - x^2 + 4 = 0$

d) $3x^3 + 2x^2 - 11x - 10 = 0$

10) Solve the following polynomial inequalities. (Refer to #9 where you factored the polynomials)

a) $2x^3 + 1 < x^2 + 2x$

b) $x^3 + 6x^2 + 11x + 6 > 0$

11) Where is the polynomial $y = 8x^3 + 1$ positive? Justify your solution.

12) Solve $6x^3 + 13x^2 - 41x + 12 \leq 0$ using a sign chart.

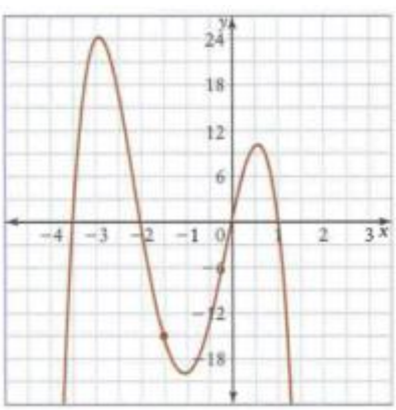
Section 4: 2.4 – Families of Polynomials

13) Find the equation for the family of quartic polynomials that have real roots of 3 (order 2) and $2 \pm \sqrt{2}$.

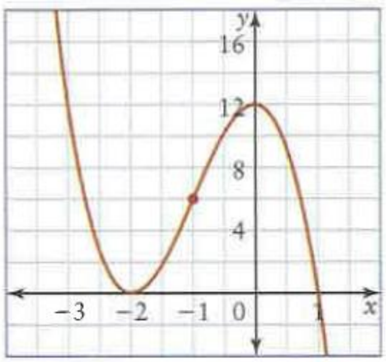
14) A family of cubic polynomials has roots of -2, -3 and -5. Find the member of this family that passes through the point (2,-35). What is this polynomials *y*-intercept?

15) Find an equation for each of the following functions

a)



b)



ANSWER KEY

1) $P(-1) = 2$ =remainder . This was found using remainder theorem.

2) $P(3) = 0$, so $x-3$ is a factor because remainder is 0 (Factor Theorem)

3)a) $\frac{x^4-4x^2-2x+3}{x-2} = x^3 + 2x^2 - 2 - \frac{1}{x-2}$ **b)** $\frac{x^5-x^4+2x^3+3x-2}{x^2+2} = x^3 - x^2 + 2 + \frac{3x-6}{x^2+2}$

4)a) $\frac{x^3+9x^2-5x+3}{x-2} = x^2 + 11x + 17 + \frac{37}{x-2}$; $x^3 + 9x^2 - 5x + 3 = (x-2)(x^2 + 11x + 17) + 37$

b) $\frac{12x^3-2x^2+x-11}{3x+1} = 4x^2 - 2x + 1 - \frac{12}{3x+1}$; $12x^3 - 2x^2 + x - 11 = (3x+1)(4x^2 - 2x + 1) - 12$

c) $\frac{-8x^4-4x+10x^3-x^2+15}{2x-1} = -4x^3 + 3x^2 + x - \frac{3}{2} + \frac{27}{2(2x-1)}$; $-8x^4 - 4x + 10x^3 - x^2 + 15 = (2x-1)\left(-4x^3 + 3x^2 + x - \frac{3}{2}\right) + \frac{27}{2}$

d) $\frac{x^3+4x^2-3}{x-2} = x^2 + 6x + 12 + \frac{21}{x-2}$; $x^3 + 4x^2 - 3 = (x-2)(x^2 + 6x + 12) + 21$

5) $k = -\frac{77}{27}$

6) $m = 8$, $n = -10$

7)a) $(x+1)(x-3)(x-2)$ **b)** $(x+2)(3x+1)(x-4)$ **c)** $-4(x+1)(x+2)(x-2)$ **d)** $(x-4)(x^2+4x+16)$

8)a) (-4, 0) and (4, 0) **b)** $\left(\frac{1-\sqrt{105}}{4}, 0\right)$ and $\left(\frac{1+\sqrt{105}}{4}, 0\right)$

9) a) $x = -1, 1, \frac{1}{2}$ **b)** $x = -1, -2, -3$ **c)** $x = 1, -2, 2$ **d)** $x = -1, -\frac{5}{3}, 2$

10)a) $x \in (-\infty, -1) \cup (0.5, 1)$ **b)** $x \in (-3, -2) \cup (-1, \infty)$

11) $x \in \left(-\frac{1}{2}, \infty\right)$

12) $x \in (-\infty, -4] \cup \left[\frac{1}{3}, \frac{3}{2}\right]$

13) $P(x) = k(x-3)^2(x^2 - 4x + 2)$

14) $f(x) = -\frac{1}{4}(x+2)(x+3)(x+5)$, y-int is $\left(0, -\frac{15}{2}\right)$

15)a) $P(x) = -2x(x-1)(x+2)(2x+7)$ **b)** $P(x) = -3(x+2)^2(x-1)$