2.6 Making Connections with Rational Functions and Equations

A Quick Review... Steps to Graph a Rational Function:

1. Factor the numerator and the denominator and simplify if possible.
2. Find the x-intercept(s) by setting the numerator equal to zero and solving.
3. Find the y-intercept [Let $x = 0$ and evaluate]
4. Find the vertical asymptotes by setting the denominator equal to zero and solving.

NOTE THAT THERE ARE CASES THAT THERE ARE NO VERTICAL ASYMPTOTE FOR RATIONAL FUNCTION, eg. $f(x)=\dfrac{2}{x^2+2}$.

Determine and illustrate the behaviour of the graph near the vertical asymptote(s). Also, consider any **"holes"** in the graph.

The rational function will have a **"HOLE"** in the graph instead of a vertical asymptote for the zero(s) in the denominator equal to the zero(s) in the numerator.

Examples:

a) $$\begin{aligned} f(x) &= \frac{2x-2}{x-1} \\ &= \frac{2(x-1)}{x-1} \\ &= 2,\ x \neq 1 \end{aligned}$$

b) $$f(x)=\frac{x^2-x-2}{x^2+4x+3}$$

Let's look at what will happen in each of these cases.

- There are more (x-a) factors in the denominator. After dividing out all duplicate factors, the (x-a) is still in the denominator. Factors in the denominator result in vertical asymptotes. Therefore, there will be a vertical asymptote at x=a.

- There are more (x-a) factors in the numerator. After dividing out all the duplicate factors, the (x-a) is still in the numerator. Factors in the numerator result in x-intercepts. But, because you can't use x=a, there will be a hole in the graph on the x-axis.

5. Find the horizontal asymptote, if it exists, using the rules. Determine and illustrate the end behaviour of the graph near the horizontal asymptote.

6. Check for any points that cross the horizontal asymptote. [If $y = L$ then solve $f(x) = \dfrac{P(x)}{Q(x)} = L$ or if $y = mx + b$ then solve $f(x) = \dfrac{P(x)}{Q(x)} = mx + b$

7. The vertical asymptote will divide the x number line into regions. If necessary, graph at least one point in each region. This point will tell us whether the graph will be above or below the horizontal asymptote and if you should need several points to determine the general shape of the graph.

GRAPHING RATIONAL FUNCTIONS WITH SPECIAL CASES OF "HOLE"

Graph $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3}$

As $x \rightarrow$	$f(x) \rightarrow$
$+\infty$	
$-\infty$