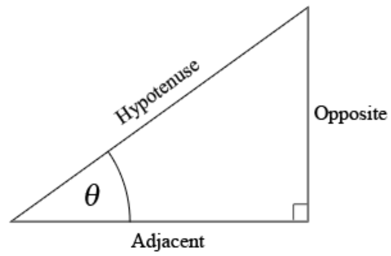
L2 – 4.2 Trig Ratios and Special Angles
MHF4U

Part 1: Review of Last Year Trig

What is SOHCAHTOA?

If we know a right angle triangle has an angle of θ, all other right angle triangles with an angle of θ are **similar** and therefore have equivalent ratios of corresponding sides. The three primary ratios are shown in the diagram to the right.



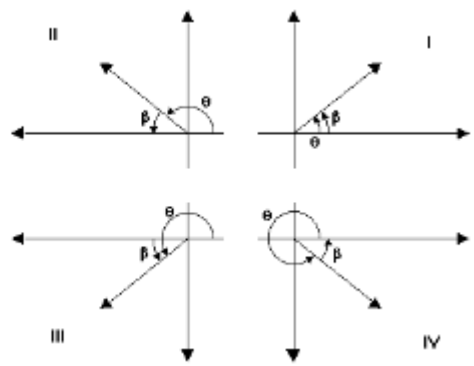
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

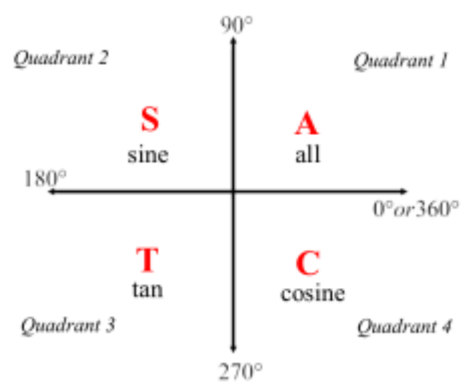
What is a reference angle?

Any angle over 90 has a **reference angle**. The reference angle is between 0° and 90° and helps us determine the exact trig ratios when we are given an obtuse angle (angle over 90 degrees). The reference angle is the angle between the terminal arm and the **closest x-axis** (0/360 or 180).



What is the CAST rule?

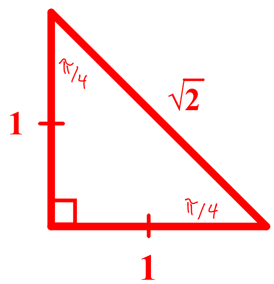
When finding the trig ratios of positive angles, we are rotating counter clockwise from 0 degrees toward 360. The **CAST rule** helps us determine which trig ratios **are positive** in each **quadrant**

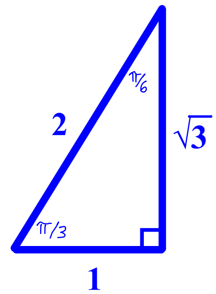


***Note:** There are multiple angles that have the same trig ratio. You can use reference angles and the cast rule to find them.*

Part 2: Finding Exact Trig Ratios for Special Angles

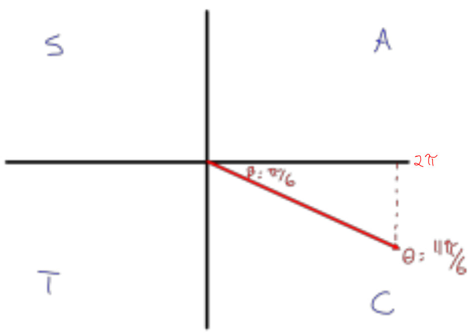
Start by drawing both special triangles using radian measures





Example 1: Find the exact value for each given trig ratio.

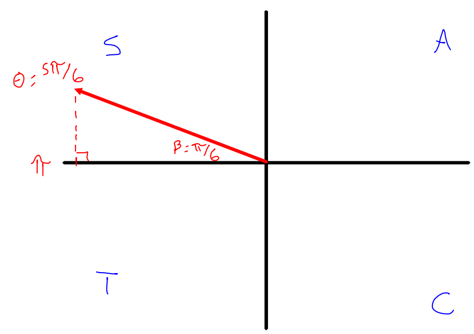
a) $\tan\frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$



$\beta = 2\pi - 11\pi/6$
$= 12\pi/6 - 11\pi/6$
$= \pi/6$

$\tan 11\pi/6 = -\tan \pi/6$
$= -1/\sqrt{3}$

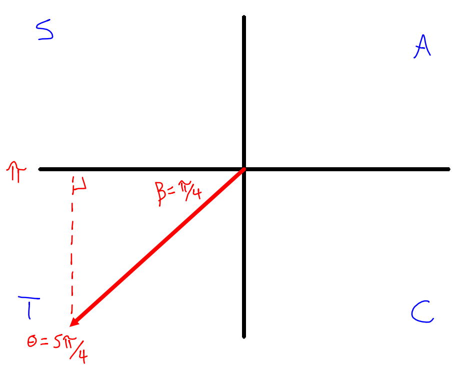
b) $\sin\frac{5\pi}{6} = \frac{1}{2}$



$\beta = \pi - 5\pi/6$
$= 6\pi/6 - 5\pi/6$
$= \pi/6$

$\sin 5\pi/6 = \sin \pi/6$
$= 1/2$

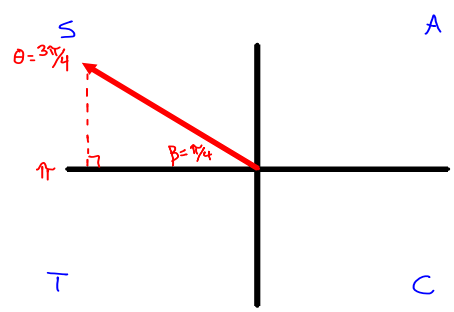
c) $\cos\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$



$\beta = 5\pi/4 - \pi$
$= 5\pi/4 - 4\pi/4$

$\cos 5\pi/4 = -\cos \pi/4$
$= -\frac{1}{\sqrt{2}}$

d) $\sec\frac{3\pi}{4} = -\sqrt{2}$



$\beta = \pi - 3\pi/4$
$= 4\pi/4 - 3\pi/4$
$= \pi/4$

$\sec 3\pi/4 = \frac{1}{\cos 3\pi/4}$
$= -\frac{1}{\cos \pi/4}$
$= -\frac{1}{\left(\frac{1}{\sqrt{2}}\right)}$
$= -\sqrt{2}$

Example 2: Find the value of all 6 trig ratios for $\frac{5\pi}{3}$

$$\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

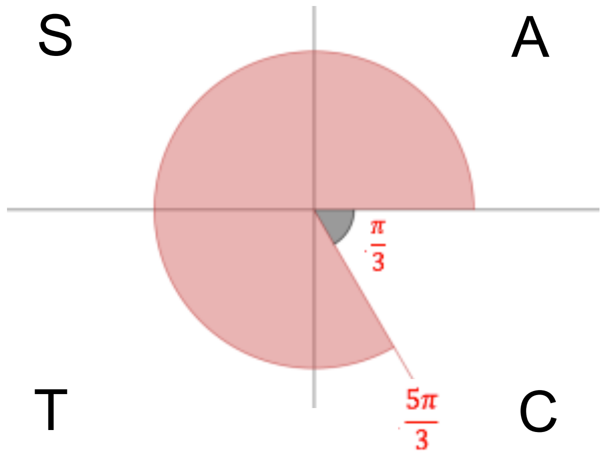
$$\csc\frac{5\pi}{3} = -\frac{2}{\sqrt{3}}$$

$$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\sec\frac{5\pi}{3} = 2$$

$$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$$

$$\cot\frac{5\pi}{3} = -\frac{1}{\sqrt{3}}$$



Part 3: Application Question

Justin is flying a kite at the end of a 50-m string. The sun is directly overhead, and the string makes an angle of $\frac{\pi}{6}$ with the ground. The wind speed increases, and the kite flies higher until the string makes an angle of $\frac{\pi}{3}$ with the ground. Determine an exact expression for the horizontal distance between the two positions of the kite along the ground.

$$\cos\frac{\pi}{6} = \frac{x_1}{50}$$

$$\frac{\sqrt{3}}{2} = \frac{x_1}{50}$$

$$x_1 = \frac{50\sqrt{3}}{2}$$

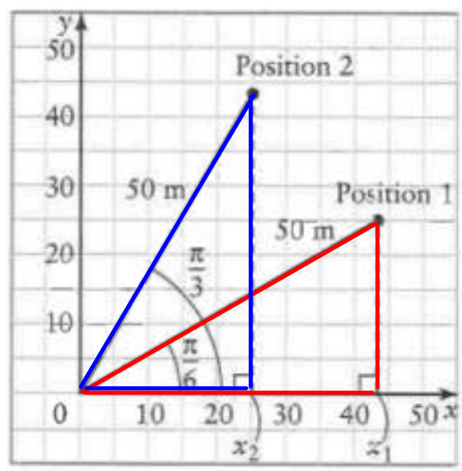
$$x_1 = 25\sqrt{3}$$

$$\cos\frac{\pi}{3} = \frac{x_2}{50}$$

$$\frac{1}{2} = \frac{x_2}{50}$$

$$x_2 = \frac{50}{2}$$

$$x_2 = 25$$



The horizontal distance between the two kites $= x_1 - x_2 = 25\sqrt{3} - 25 = 25(\sqrt{3} - 1)$ meters