L5 – 7.4 – Solving Logarithmic Equations
MHF4U

Part 1: Try and Solve a Logarithmic Equation

Solve the equation $\log(x+5) = 2\log(x-1)$

Hint: apply the power law of logarithms to the right side of the equation

$\log(x+5) = \log(x-1)^2$

$x+5 = (x-1)^2$

$x+5 = x^2 - 2x + 1$

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = 4$ or $x = -1$

Reject $x = -1$ because $\log(x-1)$ is undefined for this value of x.

Therefore, the only solution is $x = 4$

Note:

If $\log_m a = \log_m b$, then $a = b$.

Part 1: Solve Simple Logarithmic Equations

To complete this lesson, you will need to remember how to change from logarithmic to exponential:

$$y = \log_b x \rightarrow x = b^y$$

Example 2: Solve each of the following equations

a) $\log(x+4) = 1$

Method 1: re-write in exponential form

$x + 4 = 10^1$

$x + 4 = 10$

$x = 6$

Method 1: express both sides as a logarithm of the same base

$\log(x+4) = \log(10)$

$x + 4 = 10$

$x = 6$

b) $\log_5(2x-3)=2$

$5^2=2x-3$

$25=2x-3$

$28=2x$

$14=x$

Part 2: Apply Factoring Strategies to Solve Equations

Example 3: Solve each equation and reject any extraneous roots

a) $\log(x-1)-1=-\log(x+2)$

$\log(x-1)+\log(x+2)=1$

$\log[(x-1)(x+2)]=1$

$\log(x^2+x-2)=1$

$x^2+x-2=10^1$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ or $x=3$

Reject $x=-4$ because both of the original expressions are undefined for this value.

The only solution is $x=3$

b) $\log \sqrt[3]{x^2+48x} = \frac{2}{3}$

$\log(x^2+48x)^{\frac{1}{3}} = \frac{2}{3}$

$\frac{1}{3}\log(x^2+48x) = \frac{2}{3}$

$3\left[\frac{1}{3}\log(x^2+48x)\right] = 3\left(\frac{2}{3}\right)$

$\log(x^2+48x) = 2$
$x^2+48x = 10^2$

$x^2+48x-100 = 0$

$(x+50)(x-2) = 0$

$x = -50$ or $x = 2$

Both are valid solutions because they both make the argument of the logarithm positive.

c) $\log_3 x - \log_3(x-4) = 2$

$\log_3\left(\frac{x}{x-4}\right) = 2$

$\frac{x}{x-4} = 3^2$

$\frac{x}{x-4} = 9$

$x = 9(x-4)$

$x = 9x-36$

$36 = 8x$

$\frac{9}{2} = x$

Example 4: If $\log_a b = 3$, then use log rules to find the value of…

a) $\log_a ab^2$

$= \log_a a + \log_a b^2$

$= \log_a a + 2\log_a b$

$= 1 + 2(3)$

$= 7$

b) $\log_b a$

$= \frac{\log_a a}{\log_a b}$

$= \frac{1}{3}$

Hint: need to change the base

$$\log_b m = \frac{\log m}{\log b}$$