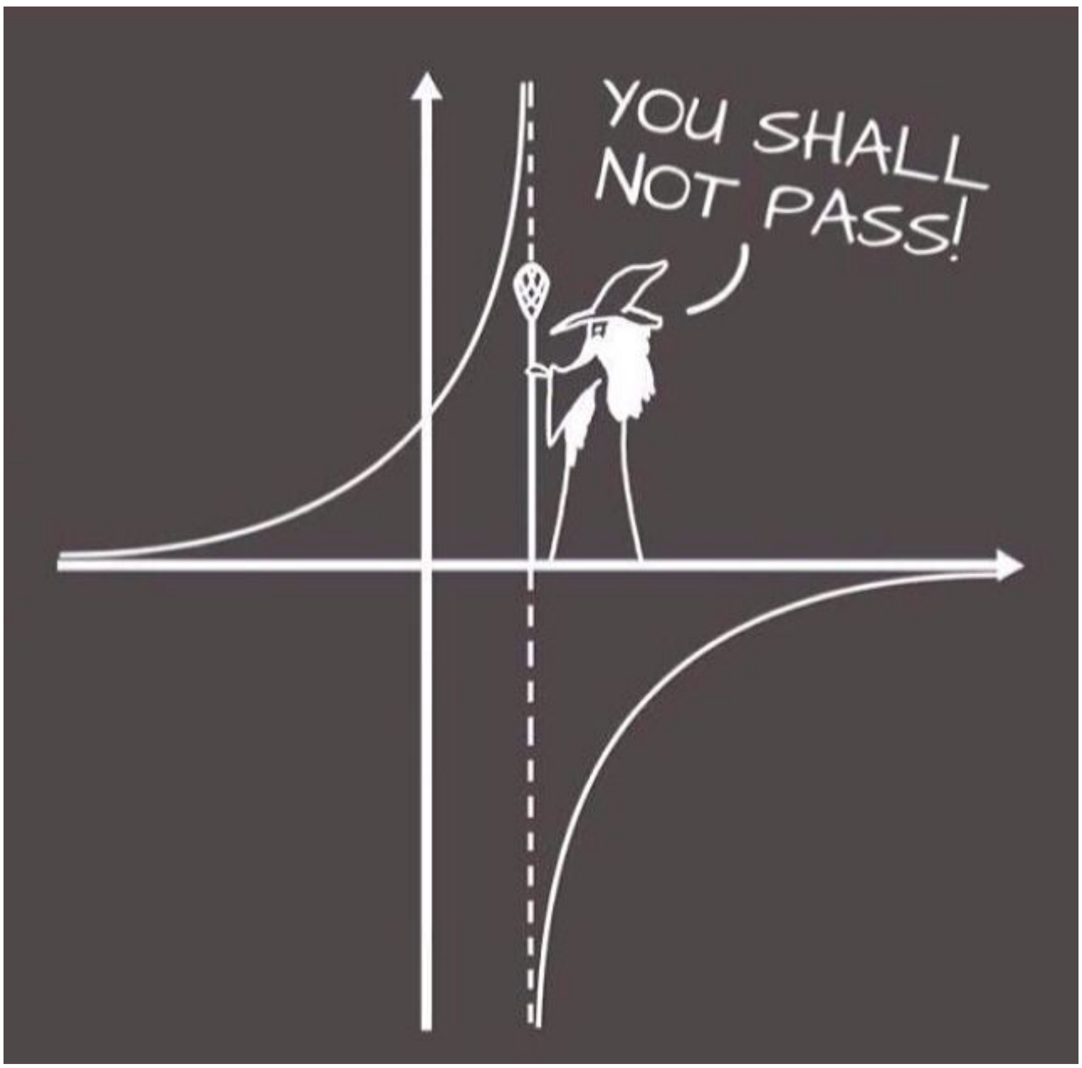
Unit 7

Rational Functions

Lesson Package

MHF4U



Unit 7 Outline

Unit Goal: By the end of this unit, you will be able to identify and describe the key features of the graphs of rational functions, and represent rational functions graphically. Also, you will be able to determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Reciprocal of Linear and Quadratic	- connect algebraic and graphical representations of the reciprocals of linear and quadratic functions	C2.1, 2.3
L2	Quotient of Linear Functions	- connect algebraic and graphical representations of the quotient of linear functions	C2.2, 2.3
L3	Combinations of Functions	- Identify key features of graphs of functions created by adding, subtracting, multiplying, and dividing two functions	D2.1, 2.2, 2.3
L4	Composite Functions	- determine algebraically the composition of two functions	D2.4, 2.5, 2.6
L5	Rational Equations and Inequalities	- Be able to solve rational equations and inequalities	C3.5, 3.6 C4.1, 4.2, 4.3

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Graphing Functions	F		P	
PreTest Review	F/A		P	
Test – Reciprocal and Combinations of Functions	O	A2.1, 2.2, 2.3, 3.4 C2.1, 2.2, 2.3 D2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 4.2	P	K(21%), T(34%), A(10%), C(34%)

L1 – 3.1/3.2 Reciprocal of Linear and Quadratic Functions
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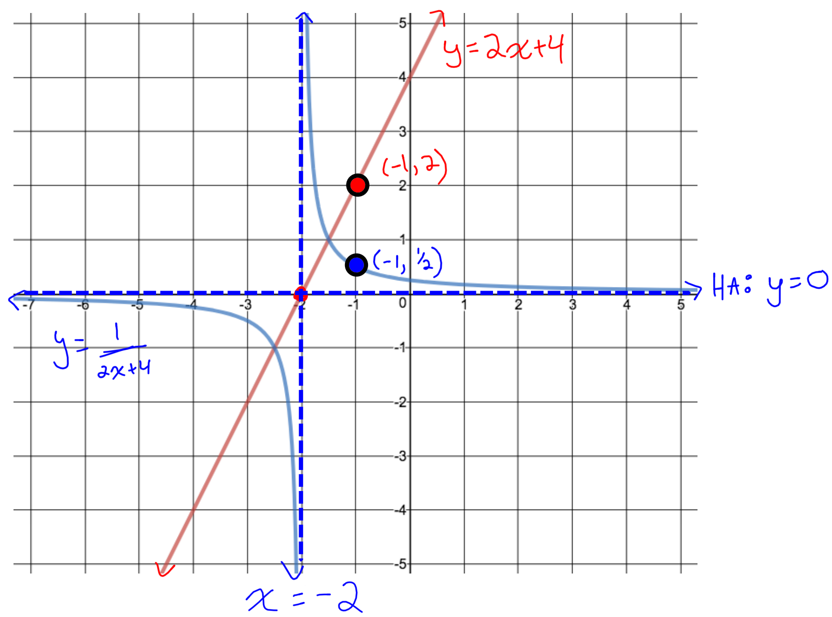
Part 1: Analyze the Reciprocal of a Linear Function

Example 1:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

HA: $y = 0$
VA: $x = -2$

b) What graphical characteristic in the reciprocal function does the zero (x-int) of the original function correspond to?

The vertical asymptote of the reciprocal function passes through the x-intercept of the linear function.

c) When the original function is increasing, what is happening to the reciprocal function?

It is DECREASING

d) What are the y-coordinates of the points of intersection?

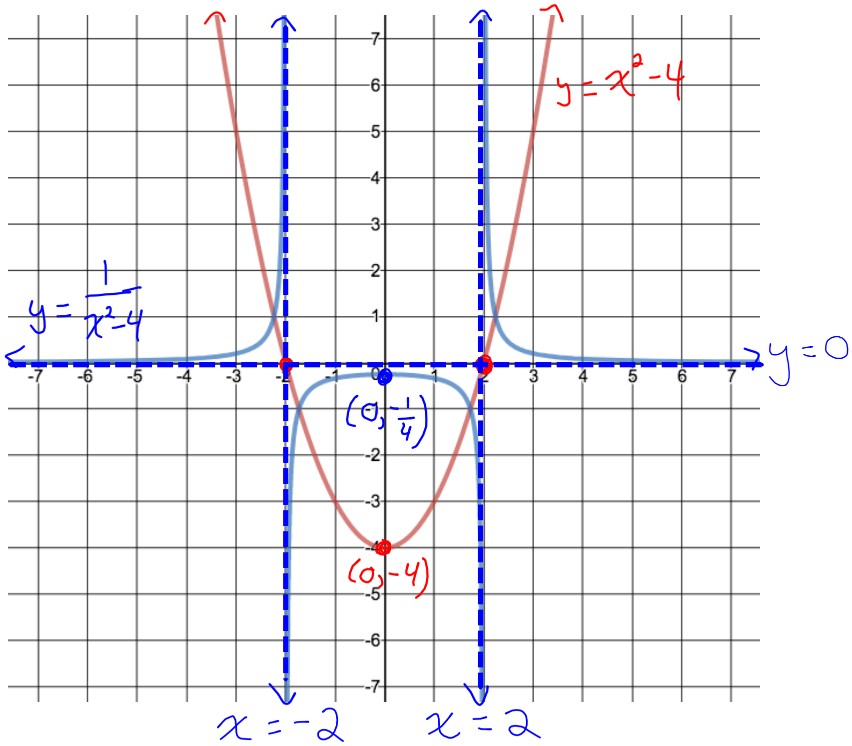
1 and -1. This is because the reciprocal of each of those does not change their value.

e) Label a point on the graph of both functions at $x = 2$. What do you notice about the y values of each point?

The y-value of the reciprocal function is the reciprocal of the y-value of the linear function

Part 2: Analyze the Reciprocal of a Quadratic Function

Example 2:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

VA: $x = 2$ and $x = -2$
HA: $y = 0$

b) What graphical characteristic in the reciprocal function do the zeros (x-int) of the original function correspond to?

The vertical asymptotes of the reciprocal function pass through the x-intercepts of the quadratic.

c) When the original function is decreasing, what is happening to the reciprocal function?

It is INCREASING

d) What are the y-coordinates of the points of intersection?

1 and -1

f) Label the local min or max point on each function. What do you notice about them?

They have the same x-coordinate which is exactly half way between the vertical asymptotes.

The quadratic has a local min but the reciprocal has a local max.

Properties of Reciprocal Functions

- All the y-coordinates of the reciprocal function are the reciprocals of the y-coordinates of the original function
- The graph of the reciprocal function has a vertical asymptote at the x-intercepts (zeros) of the original
    - This is because it makes the denominator of the reciprocal $= 0$
- $y = 0$ will always be a horizontal asymptote
- The reciprocal function has the same positive/negative intervals as the original function
- Intervals of increase on the original function are intervals of decrease on the reciprocal
- Intervals of decrease on the original function are intervals of increase on the reciprocal
- If 1 is in the range of the original function, this is where the functions will intersect
- If the original function has a local min point, the reciprocal will have a local max at the same x-value (and vice versa)

Part 3: Graphing Reciprocal Functions

Process:

- Find key features of the function in the denominator and graph it using a table of values
- Create a table of values for the reciprocal function by keeping the same x values but using the reciprocal of all y values
- Draw vertical asymptotes at any point that is a zero of the original linear/quadratic function
    - Reciprocal of 0 is undefined
- If the numerator is something other than 1, multiply the y-values by this stretch factor

Example 3: Graph each of the following reciprocal functions. Start by graphing the function in the denominator.

a) $y = \frac{1}{2x-1}$

$$\boldsymbol{y = 2x - 1}$$

$x - int: (\frac{1}{2}, 0)$

$$\boldsymbol{y = \frac{1}{2x-1}}$$

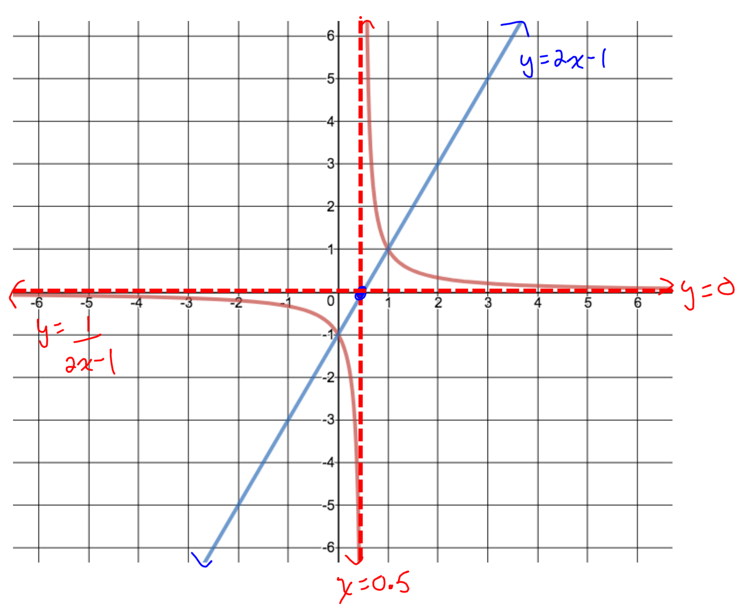
$VA: x = \frac{1}{2}$

$HA: y = 0$

x	y
-2	-5
-1	-3
0	-1
0.5	0
1	1
2	3
3	5

Center x-int

x	$\frac{1}{y}$
-2	-0.2
-1	-0.33
0	-1
0.5	$undefined$
1	1
2	0.33
3	0.2



b) $y = \frac{1}{x^2-2x-15} = \frac{1}{(x-5)(x+3)}$

$$y = x^2 - 2x - 15$$

$x^2 - 2x - 15$

$= (x-5)(x+3)$

$x - int$: $(5,0)$ and $(-3,0)$

$x - vertex = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$

$$y = \frac{1}{x^2 - 2x - 15}$$

$\frac{1}{x^2 - 2x - 15}$

$= \frac{1}{(x-5)(x+3)}$

$VA: x = 5$ and $x = -3$

$HA: y = 0$

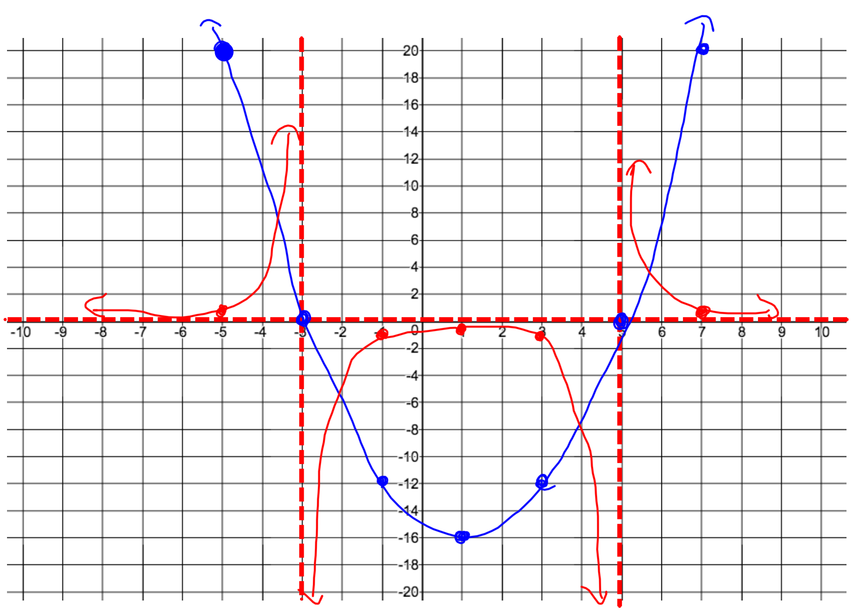
Include x-int →

Center the vertex →

Include x-int →

x	y
-5	20
-3	0
-1	-12
1	-16
3	-12
5	0
7	20

x	$\frac{1}{y}$
-5	0.05
-3	$undefined$
-1	-0.08
1	-0.0625
3	-0.08
5	$undefined$
7	0.05



c) $y = \frac{1}{x^2+4}$

$\boldsymbol{y = x^2 + 4}$

$\boldsymbol{x - int}$:

$x^2 + 4 \neq 0$

$\therefore no\ x - int$

$\boldsymbol{x - vert}$:

$$x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0$$

$\boldsymbol{y = \frac{1}{x^2 + 4}}$

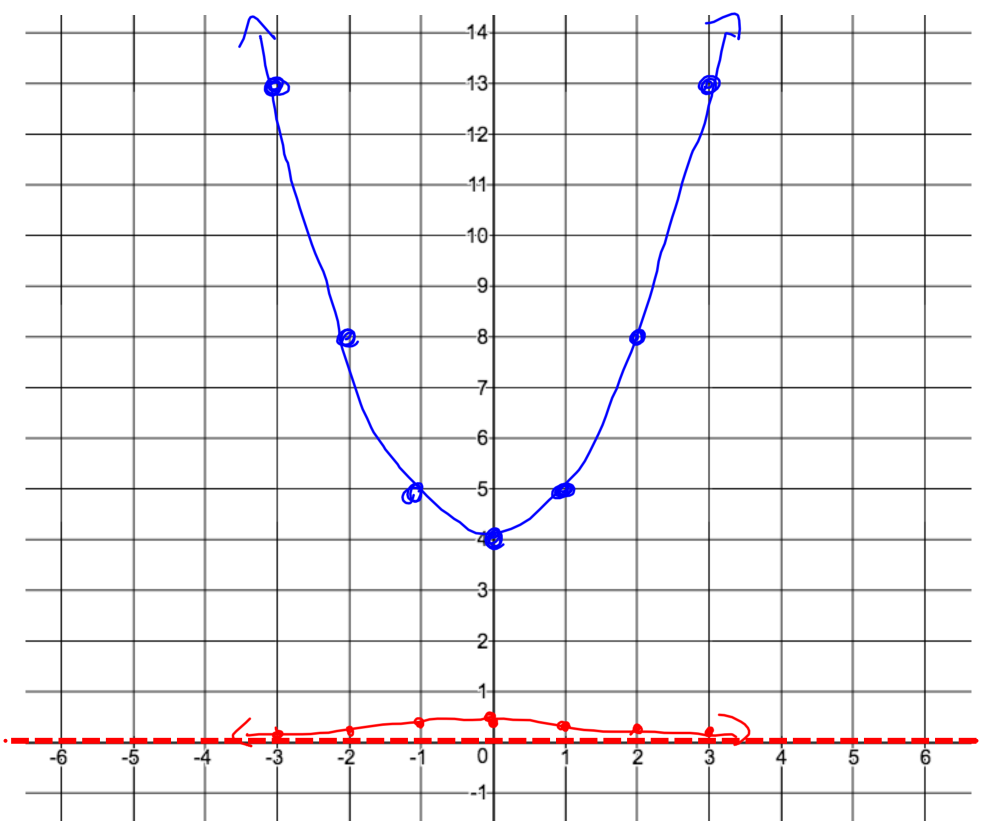
$VA: none$

$HA: y = 0$

Center the vertex

x	y
-3	13
-2	8
-1	5
0	4
1	5
2	8
3	13

x	$\frac{1}{y}$
-3	0.08
-2	0.125
-1	0.2
0	0.25
1	0.2
2	0.125
3	0.08



d) $y = \frac{2}{x^2-6x+9} = 2\left[\frac{1}{(x-3)^2}\right]$

$$\boldsymbol{y = x^2 - 6x + 9}$$

$x^2 - 6x + 9$

$= (x-3)^2$

$x - int: (3,0)$

$x - vertex = 3$

x	y
0	9
1	4
2	1
3	0
4	1
5	4
6	9

Center the vertex

$$\boldsymbol{y = \frac{2}{x^2 - 6x + 9}}$$

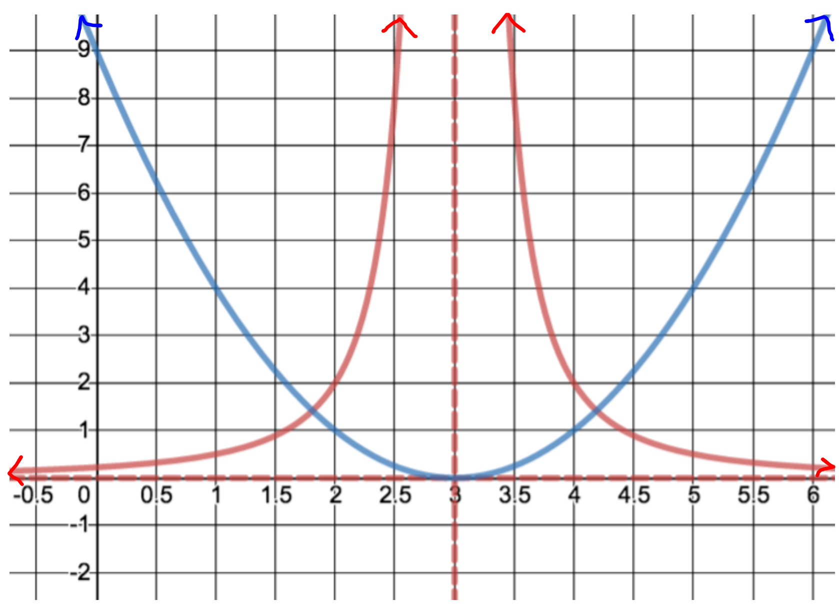
$\frac{2}{x^2 - 6x + 9}$

$= 2\left[\frac{1}{(x-3)^2}\right]$

$VA: x = 3$

$HA: y = 0$

x	$\frac{2}{y}$
0	0.22
1	0.5
2	2
3	$undefined$
4	2
5	0.5
6	0.22



L2 – 3.3 Quotient of Linear Functions
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Part 1: Key Features of the Quotient of Linear Functions

Features of $f(x) = \frac{ax+b}{cx+d}$

- If an x value is a zero of the denominator ONLY, this results in a vertical asymptote
  - Equation of vertical asymptote is $x = \frac{-d}{c}$
- If an x value is a zero of the numerator AND denominator, this results in a **hole** in the graph NOT a vertical asymptote
- There is a horizontal asymptote at the ratio of the leading coefficients
  - Equation of horizontal asymptote is $y = \frac{a}{c}$
- Forms a **Hyperbola**: the two branches of the graph of the function are equidistant from the point of intersection of the vertical and horizontal asymptotes
  - Once you know the shape of one branch, you can translate the points to graph the other branch
- You can find the x-intercept by setting $y = 0$ and solving for x
  - This results in $\left(\frac{-b}{a}, 0\right)$
- You can find the y-intercept by setting $x = 0$ and solving for y
  - This results in $\left(0, \frac{b}{d}\right)$

Part 2: Graphing a Quotient of Linear Functions

Example 1: Graph each of the following functions

a) $f(x) = \frac{x-3}{x+2}$

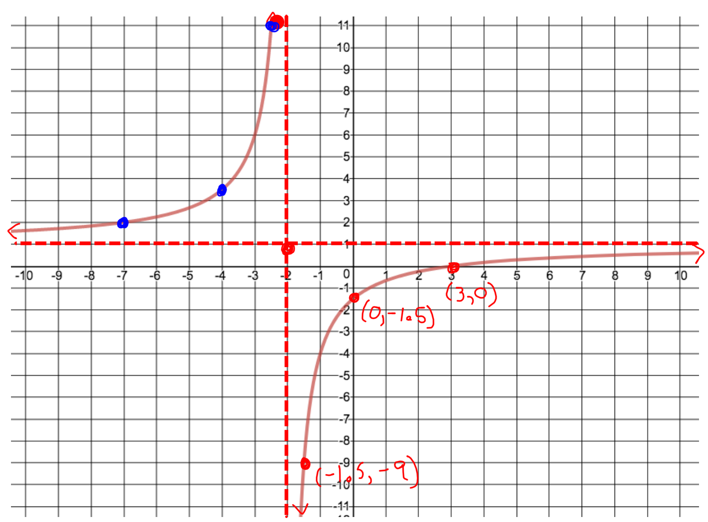
VA: $x+2=0$
$x=-2$

HA: $\frac{1}{1}=1$
$y=1$

x-int: $0=\frac{x-3}{x+2}$
$0=x-3$
$x=3$
$(3,0)$

y-int: $f(0)=\frac{0-3}{0+2}$
$=-\frac{3}{2}$
$(0,-1.5)$

Another point:
$f(-1.5)=\frac{-1.5-3}{-1.5+2}$
$=\frac{-4.5}{0.5}$
$=-9$



b) $g(x) = \frac{2x-3}{x-1}$

VA: $x-1=0$
$x=1$

HA: $\frac{2}{1}=2$
$y=2$

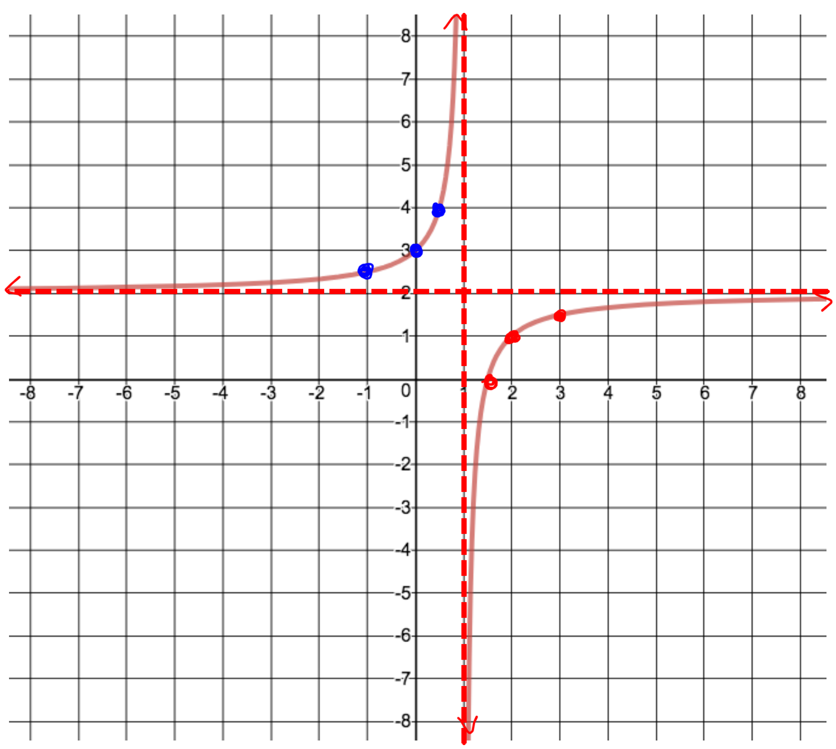
x-int: $0=\frac{2x-3}{x-1}$
$0=2x-3$
$x=\frac{3}{2}$
$(1.5, 0)$

y-int: $f(0)=\frac{2(0)-3}{0-1}$
$=3$
$(0,3)$

Other points:

$f(2)=\frac{2(2)-3}{2-1}=\frac{1}{1}=1$
$(2,1)$

$f(3)=\frac{2(3)-3}{3-1}=\frac{3}{2}$
$(3, 1.5)$



L3 – 8.1/8.2 Sum/Difference and Product/Quotient of Functions
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Part 1: Sum and Difference of Functions

When two functions $f(x)$ and $g(x)$ are combined to form the function $(f+g)(x)$ or $(f-g)(x)$, the new function is called the sum or difference of f and g.

The graph of $f+g$ or $f-g$ can be obtained by adding or subtracting corresponding y-coordinates. This is called the s***uperposition principle***.

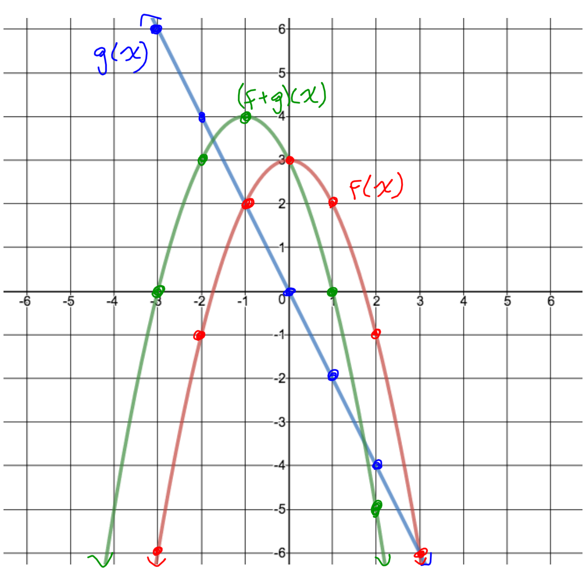
$$(f+g)(x) = f(x) + g(x)$$

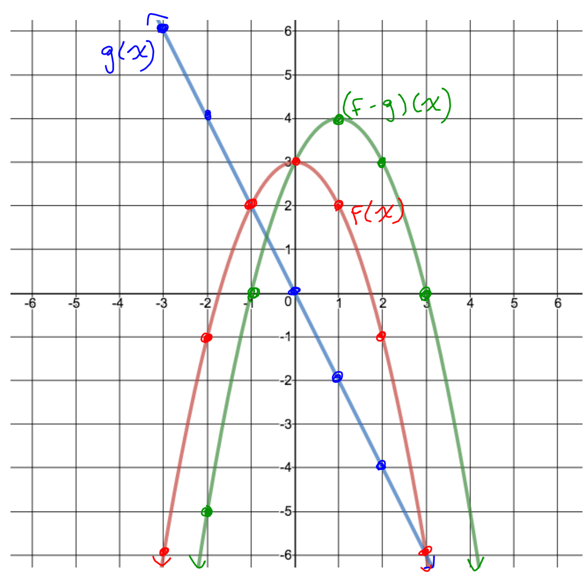
$$(f-g)(x) = f(x) - g(x)$$

Example 1: Given $f(x) = -x^2 + 3$ and $g(x) = -2x$ determine the graphs of $(f+g)(x)$ and $(f-g)(x)$.

Method 1: Graphically

x	$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)-g(x)$
-3	-6	6	0	-12
-2	-1	4	3	-5
-1	2	2	4	0
0	3	0	3	3
1	2	-2	0	4
2	-1	-4	-5	3
3	-6	-6	-12	0





Method 2: Algebraically

$(f+g)(x) = f(x) + g(x)$
$(f+g)(x) = (-x^2+3) + (-2x)$
$(f+g)(x) = -x^2 - 2x + 3$

Complete Square to find Vertex

$(f+g)(x) = -(x^2+2x) + 3$
$(f+g)(x) = -(x^2+2x+1-1) + 3$
$(f+g)(x) = -(x^2+2x+1) + 1 + 3$
$(f+g)(x) = -(x+1)^2 + 4$

vertex is (-1, 4)

x	$(f+g)(x)$
-4	-5
-3	0
-2	3
-1	4
0	3
1	0
2	-5

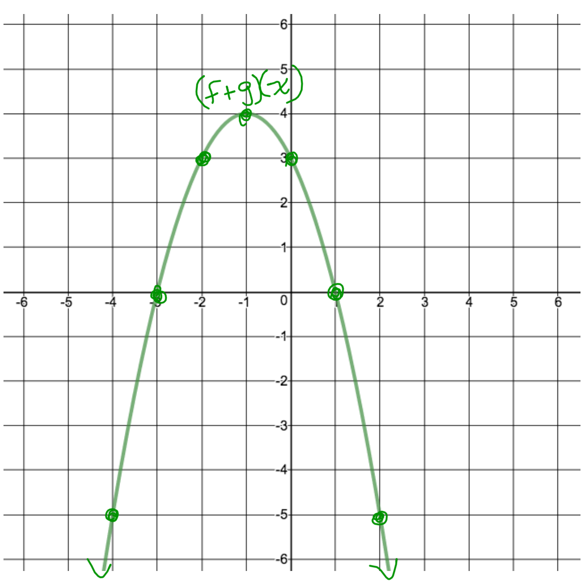
$(f-g)(x) = f(x) - g(x)$
$(f+g)(x) = (-x^2+3) - (-2x)$
$(f+g)(x) = -x^2 + 2x + 3$

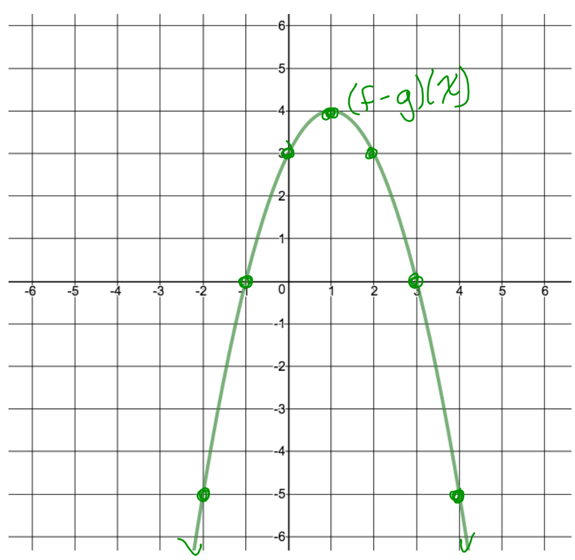
Complete Square to find Vertex

$(f+g)(x) = -(x^2-2x) + 3$
$(f+g)(x) = -(x^2-2x+1-1) + 3$
$(f+g)(x) = -(x^2-2x+1) + 1 + 3$
$(f+g)(x) = -(x-1)^2 + 4$

vertex is (1, 4)

x	$(f-g)(x)$
-2	-5
-1	0
0	3
1	4
2	3
3	0
4	-5





$(f+g)(x)$

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R} | y \leq 4\}$

$(f-g)(x)$

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R} | y \leq 4\}$

Note: *The domain of the sum or difference of functions is the intersection of the domains of f and g*

Part 2: Product and Quotient of Functions

When two functions $f(x)$ and $g(x)$ are combined to form the function $(f \cdot g)(x)$ or $(f \div g)(x)$, the new function is called the product or quotient of f and g.

The graph of $f \cdot g$ or $f \div g$ can be obtained by multiplying or dividing corresponding y-coordinates.

$$(f \times g)(x) = f(x) \times g(x)$$

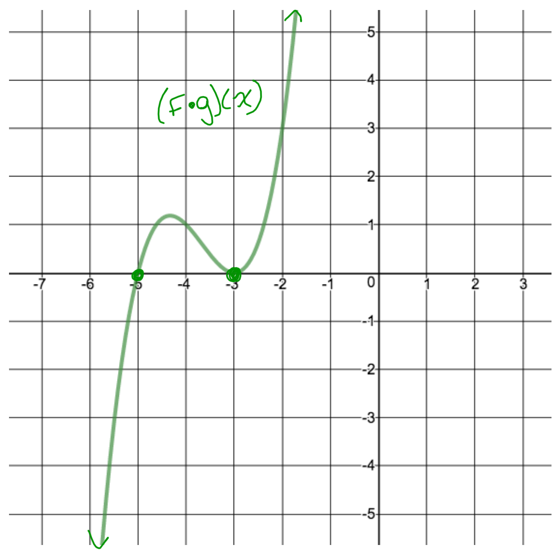
$$(f \div g)(x) = f(x) \div g(x)$$

Example 2: Let $f(x) = x + 3$ and $g(x) = x^2 + 8x + 15$. Determine an equation and graph for

a) $(f \times g)(x)$

$(f \times g)(x) = f(x)g(x)$
$(f \times g)(x) = (x+3)(x^2+8x+15)$
$(f \times g)(x) = (x+3)(x+3)(x+5)$
$(f \times g)(x) = (x+3)^2(x+5)$

x –intercepts at -3 (order 2) and -5 (order 1)
Extends from Q1 to Q3



b) $(f \div g)(x)$

$(f \div g)(x) = \frac{f(x)}{g(x)}$

$(f \div g)(x) = \frac{x+3}{(x+3)(x+5)}$

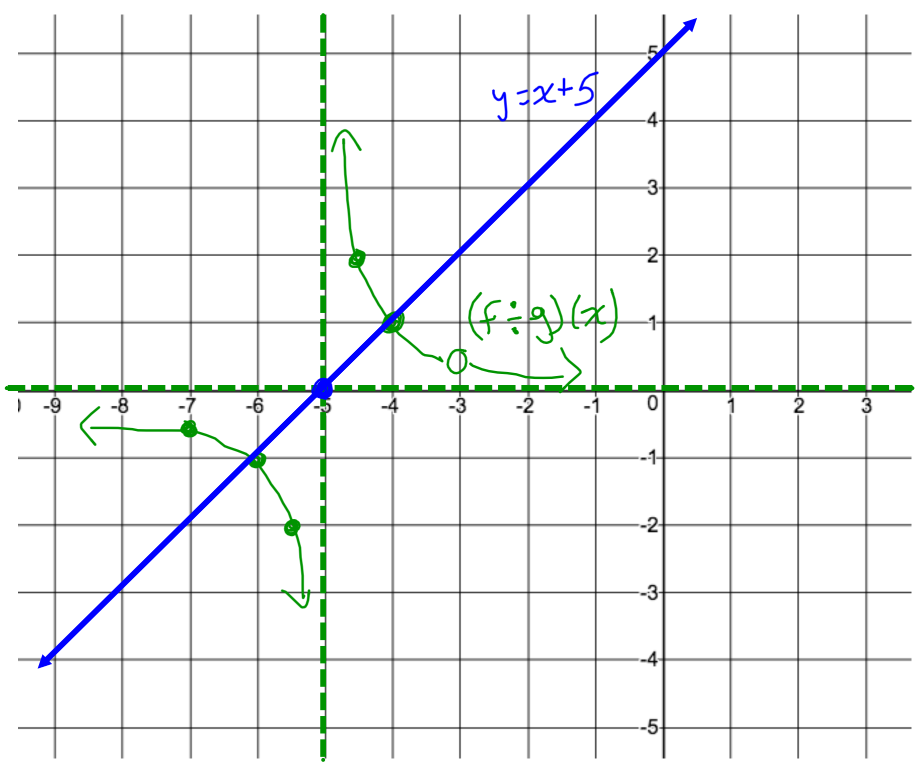
$(f \div g)(x) = \frac{1}{x+5}; x \neq -5, -3$

VA: $x = -5$
HA: $y = 0$
Hole at $x = -3$

Note: always a HA at $y = 0$ when denominator is higher degree than numerator

$y = x + 5$	
x	y
-7	-2
-6	-1
-5.5	-0.5
-5	0
-4.5	0.5
-4	1
-3	2

$y = \frac{1}{x+5}$	
x	$\frac{1}{y}$
-7	-0.5
-6	-1
-5.5	-2
-5	Und
-4.5	2
-4	1
-3	Und



c) State the domain and range of both functions

$(f \times g)(x)$

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R}\}$

$(f \div g)(x)$

D: $\{X \in \mathbb{R} | x \neq -5, -3\}$

R: $\{Y \in \mathbb{R} | y \neq 0, 0.5\}$

L4 – 8.3 Composite Functions
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Two functions, f and g can be combined using a process called composition, which can be represented by:

$$f\big(g(x)\big) \text{ OR } (f \circ g)(x)$$

This is read as "f composite g"

Part 1: Determine the Composition of Two Functions

To determine an equation for a composite function, substitute the second function into the first.

To determine $f(g(x))$, substitute $g(x)$ in for x in to $f(x)$

Example 1: If $f(x) = x^2$ and $g(x) = x + 3$, determine an equation for each composite function and then graph the function.

a) $(f \circ g)(x)$

$= f(g(x))$

$= f(x + 3)$

$= (x + 3)^2$

b) $(g \circ f)(x)$

$= g(f(x))$

$= g(x^2)$

$= x^2 + 3$

c) $g^{-1}(g(x))$

Start by finding $g^{-1}(x)$

$y = x + 3$

$x = y + 3$

$x - 3 = y$

$g^{-1}(x) = x - 3$

Now find $g^{-1}(g(x))$

$= g^{-1}(x + 3)$

$= (x + 3) - 3$

$= x$

a) $(f \circ g)(x) = (x+3)^2$

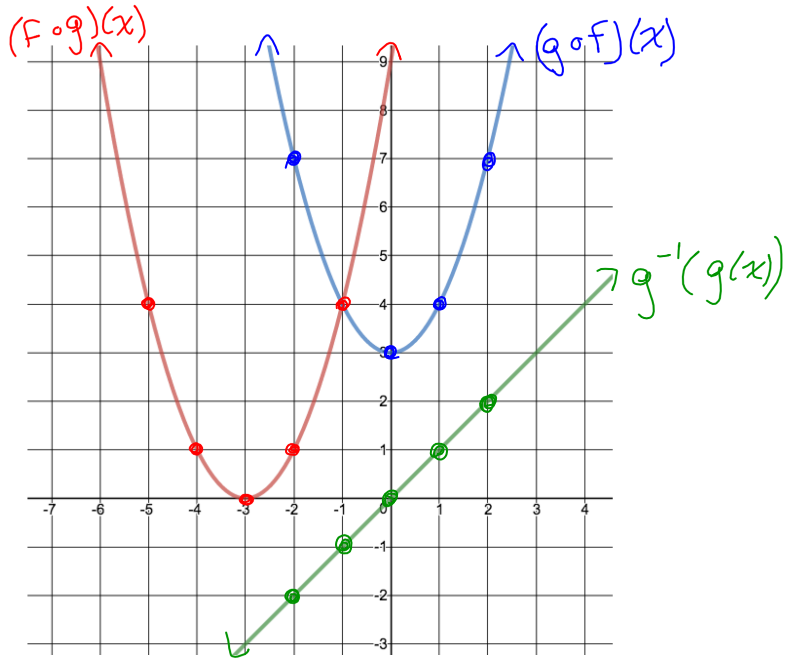
x	y
-5	4
-4	1
-3	0
-2	1
-1	4

b) $(g \circ f)(x) = x^2 + 3$

x	y
-2	7
-1	4
0	3
1	4
2	7

c) $g^{-1}(g(x)) = x$

x	y
-2	-2
-1	-1
0	0
1	1
2	2



Part 2: Evaluate a Composite Function

To evaluate a composite function $f(g(x))$ at a specific value, evaluate $g(x)$ at the specific value and then substitute the result in to $f(x)$.

Example 2: If $u(x) = x^2 + 3x + 2$ and $w(x) = \frac{1}{x-1}$

a) Evaluate $(u \circ w)(2)$

$= u(w(2))$

$= u(1)$

$= (1)^2 + 3(1) + 2$

$\boxed{= 6}$

$w(2) = \frac{1}{2-1}$

$= \frac{1}{1}$

$= 1$

b) Evaluate $w(u(-3))$

$w(u(-3)) = w(2)$

$= \frac{1}{2-1}$

$\boxed{= 1}$

$u(-3) = (-3)^2 + 3(-3) + 2$

$= 2$

Part 3: Application

Example 3: The number of rabbits, R, in a wildlife reserve as a function of time, t, in years can be modelled by the function $R(t) = 50\cos(t) + 100$. The number of wolves, W, in the same reserve can be modelled by the function $W(t) = 0.2[R(t-2)]$. Find the full equation for $W(t)$

$$R(t-2) = 50\cos(t-2) + 100$$

$$W(t) = 0.2[R(t-2)]$$

$$W(t) = 0.2[50\cos(t-2) + 100]$$

$$W(t) = 10\cos(t-2) + 20$$

L5 – 3.4 Solve Rational Equations and Inequalities
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Part 1: Rational Expressions

Rational Expression: the quotient of two polynomials, $\frac{p(x)}{q(x)}$, where $q(x) \neq 0$

Example 1: Simplify and state the restrictions of each rational expression

a) $\frac{2x^2-8}{x^2+3x+2}$

$$\frac{2x^2-8}{x^2+3x+2}$$

$$= \frac{2(x^2-4)}{(x+2)(x+1)} \leftarrow DOS$$

$$= \frac{2(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+1)}$$

$$= \frac{2(x-2)}{x+1}\ ; x \neq -2, -1$$

b) $\frac{x^3-x^2-x+1}{3x^3-3}$

$$\frac{x^3-x^2-x+1}{3x^3-3}$$

$$= \frac{(x^3-x^2)+(-x+1)}{3(x^3-1)} \leftarrow DOC$$

$$= \frac{x^2(x-1)-1(x-1)}{3(x-1)(x^2+x+1)}$$

$$= \frac{\cancel{(x-1)}(x^2-1)}{3\cancel{(x-1)}(x^2+x+1)} \leftarrow DOS$$

$$= \frac{(x-1)(x+1)}{3(x^2+x+1)}\ ; x \neq 1$$

Part 2: Solve Rational Equations

Steps for solving rational equations:

1) Fully factor both sides of the equation
2) Multiply both sides by a common denominator (cross multiply if appropriate)
3) Continue to solve as you would a normal polynomial equation
4) State restrictions throughout (values of x that would make denominator equal zero)

Example 2: Solve each equation

a) $\frac{4}{3x-5} = 4$

$$\frac{4}{3x-5} = 4$$

$$(3\cancel{x-5})\left(\frac{4}{\cancel{3x-5}}\right) = 4(3x-5) \quad ; x \neq \frac{5}{3}$$

$$4 = 4(3x-5)$$

$$4 = 12x - 20$$

$$24 = 12x$$

$$\boxed{x = 2}$$

b) $\frac{6}{x-2} = x - 1$

$$\frac{6}{x-2} = x-1$$

$$(\cancel{x-2})\left(\frac{6}{\cancel{x-2}}\right) = (x-2)(x-1) \quad ; x \neq 2$$

$$6 = x^2 - 1x - 2x + 2$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$$x - 4 = 0 \qquad x + 1 = 0$$

$$\boxed{x_1 = 4} \qquad \boxed{x_2 = -1}$$

c) $\frac{x-5}{x^2-3x-4} = \frac{3x+2}{x^2-1}$

$$\frac{x-5}{x^2-3x-4} = \frac{3x+2}{x^2-1}$$

$$\frac{x-5}{(x-4)(x+1)} = \frac{3x+2}{(x-1)(x+1)} \quad ; x \neq -1, 1, 4$$

$$\frac{(x-1)(x+1)(x-5)}{x+1} = \frac{(x-4)(x+1)(3x+2)}{x+1}$$

$$(x-1)(x-5) = (x-4)(3x+2)$$

$$x^2 - 6x + 5 = 3x^2 - 10x - 8$$

$$0 = 2x^2 - 4x - 13$$

NOT Factorable
use QF

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-13)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{120}}{4}$$

$$x = \frac{4 \pm 2\sqrt{30}}{4}$$

$$x = \frac{2(2 \pm \sqrt{30})}{4}$$

$$x = \frac{2 \pm \sqrt{30}}{2}$$

Part 3: Solve Rational Inequalities

REMEMBER: Solving inequalities is the same as solving equations. However, when both sides of an inequality are multiplied or divided by a negative number, the inequality sign must be reversed.

Steps for solving rational inequalities algebraically:

1) Use inverse operations to move all terms to one side of the inequality
2) Combine the expressions by using a common denominator
3) Factor the polynomial
4) Find the interval(s) where the function is positive or negative by making a factor table

To make a factor table:

- Use x-intercepts and vertical asymptotes to divide in to intervals
- Use a test point within each interval to find the sign of each factor
- Determine the overall sign of the product by multiplying signs of each factor within each interval.

Example 3: Solve each inequality algebraically

a) $\frac{x^2+6x+5}{2x^2-7x+3} < 0$

P: 6, S: −7; $\frac{-3}{1} = \frac{-6}{2}$, $\frac{-1}{2}$

$$\frac{x^2+6x+5}{2x^2-7x+3} < 0$$

$$\frac{(x+5)(x+1)}{(2x-1)(x-3)} < 0$$

zeros: $x = -5, -1$

restrictions: $x \neq \frac{1}{2}, 3$

	$-\infty$ to -5: -6	-5 to -1: -2	-1 to $\frac{1}{2}$: 0	$\frac{1}{2}$ to 3: 1	3 to ∞: 4
$x+5$	−	+	+	+	+
$x+1$	−	−	+	+	+
$2x-1$	−	−	−	+	+
$x-3$	−	−	−	−	+
overall	+	−	+	−	+

The inequality is true when $-5 < x < -1$ OR $0.5 < x < 3$

The inequality is true when $x \in (-5,-1) \cup (0.5, 3)$

b) $x - 2 < \frac{8}{x}$

$$x - 2 < \frac{8}{x}$$

$$x - 2 - \frac{8}{x} < 0$$

$$\frac{x^2}{x} - \frac{2x}{x} - \frac{8}{x} < 0$$

$$\frac{x^2 - 2x - 8}{x} < 0$$

$f(x)$

$$\frac{(x-4)(x+2)}{x} < 0$$

zeros: $x = -2, 4$

restriction: $x \neq 0$

	$-\infty$ to -2	-2 to 0	0 to 4	4 to ∞
	-3	-1	1	5
$x-4$	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
overall	$-$	$+$	$-$	$+$

The inequality is true when $x < -2$ OR $0 < x < 4$

The inequality is true when $x \in (-\infty, -2) \cup (0, 4)$

c) $\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$

$$\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$$

$$\frac{x+3}{x+1}\frac{(x-3)}{(x-3)} - \frac{x-2}{x-3}\frac{(x+1)}{(x+1)} \geq 0$$

$$\frac{(x+3)(x-3)-(x-2)(x+1)}{(x+1)(x-3)} \geq 0$$

$$\frac{x^2-9-(x^2-x-2)}{(x+1)(x-3)} \geq 0$$

$$\frac{x^2-9-x^2+x+2}{(x+1)(x-3)} \geq 0$$

$$\frac{x-7}{(x+1)(x-3)} \geq 0$$

zero: $x=7$

restrictions: $x \neq -1, 3$

	$-\infty$ to -1	-1 to 3	3 to 7	7 to ∞
	-2	0	4	8
$x-7$	$-$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$+$	$+$
overall	$-$	$+$	$-$	$+$

The inequality is true when $-1 < x < 3$ OR $x \geq 7$

The inequality is true when $x \in (-1,3) \cup [7,\infty)$

d) $\frac{x^3+6x^2-2x}{x^2+4} \geq 2$

$$\frac{x^3+6x^2-2x}{x^2+4} \geq 2$$

$$\frac{x^3+6x^2-2x}{x^2+4} - 2 \geq 0$$

$$\frac{x^3+6x^2-2x}{x^2+4} - \frac{2(x^2+4)}{x^2+4} \geq 0$$

$$\frac{x^3+6x^2-2x-2(x^2+4)}{x^2+4} \geq 0$$

$$\frac{x^3+6x^2-2x-2x^2-8}{x^2+4} \geq 0$$

$$\frac{x^3+4x^2-2x-8}{x^2+4} \geq 0$$

$$\frac{x^2(x+4)-2(x+4)}{x^2+4} \geq 0$$

$$\frac{(x+4)(x^2-2)}{x^2+4} \geq 0$$

zeros: $x+4=0$, $x=-4$ $\quad x^2-2=0$, $x^2=2$, $x=\pm\sqrt{2}$

no restrictions

	$-\infty$ to -4	-4 to $-\sqrt{2}$	$-\sqrt{2}$ to $\sqrt{2}$	$\sqrt{2}$ to ∞
	-5	-3	0	2
$x+4$	$-$	$+$	$+$	$+$
x^2-2	$+$	$+$	$-$	$+$
x^2+4	$+$	$+$	$+$	$+$
overall	$-$	$+$	$-$	$+$

The inequality is true when $-4 \leq x \leq -\sqrt{2}$ OR $x \geq \sqrt{2}$

The inequality is true when $x \in [-4, -\sqrt{2}] \cup [\sqrt{2}, \infty)$