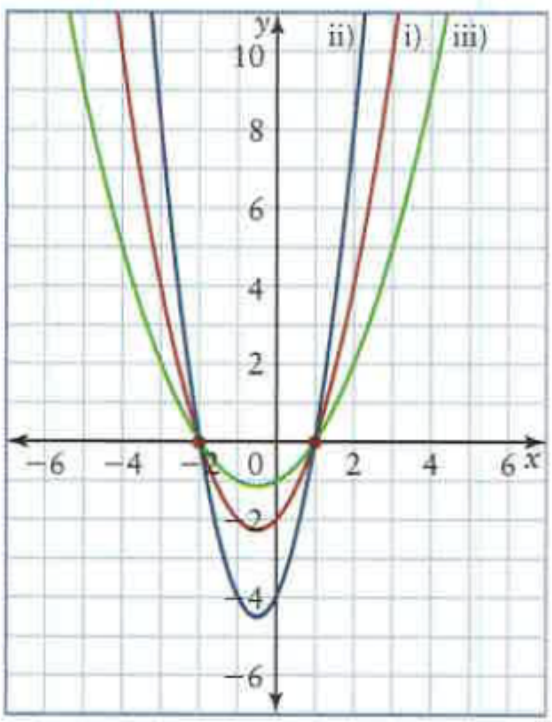
L5 – 2.4 – Families of Polynomial Functions Lesson
MHF4U

In this section, you will determine equations for a family of polynomial functions from a set of zeros. Given additional information, you will determine an equation for a particular member of the family.

Part 1: Investigation

1)

Set A

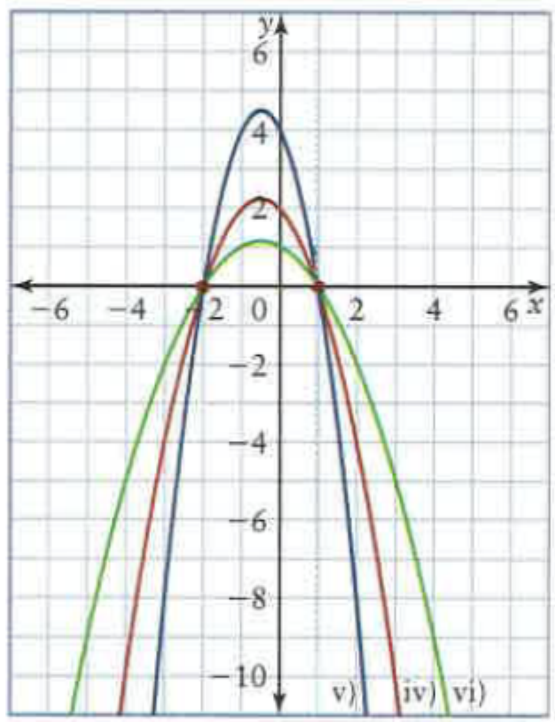


i) $y = (x - 1)(x + 2)$

ii) $y = 2(x - 1)(x + 2)$

iii) $y = \frac{1}{2}(x - 1)(x + 2)$

Set B



iv) $y = -(x - 1)(x + 2)$

v) $y = -2(x - 1)(x + 2)$

vi) $y = -\frac{1}{2}(x - 1)(x + 2)$

a) How are the graphs of the functions similar and how are they different?

Same	Different
• x-intercepts (zeros) • equations have same degree	• y-intercepts • stretch or compression factors • vertices

b) Describe the relationship between the graphs of functions of the form $y = k(x - 1)(x + 2)$, where $k \in \mathbb{R}$

They have the same x-intercepts.

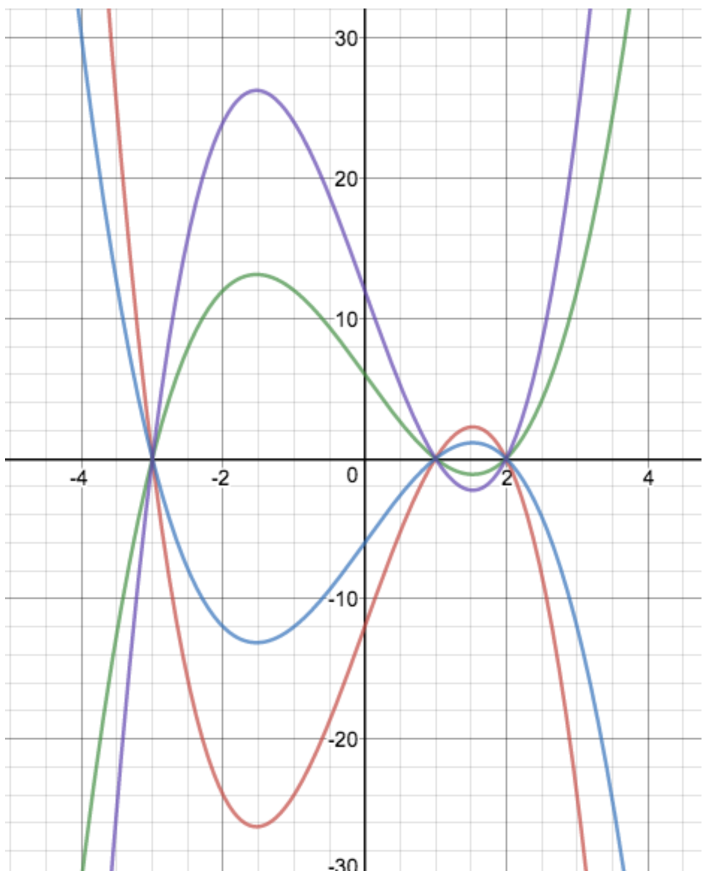
2) a) Examine the following functions. How are they similar? How are they different?

i) $y = -2(x-1)(x+3)(x-2)$
ii) $y = -(x-1)(x+3)(x-2)$
iii) $y = (x-1)(x+3)(x-2)$
iv) $y = 2(x-1)(x+3)(x-2)$

b) Predict how the graphs of the functions will be similar and how they will be different.

They will have the same x-intercepts but their shape and direction will be different due to the sign and value of the leading coefficient.

c) Use technology to help you sketch the graphs of all four functions on the same set of axes.



A **family** of functions is a set of functions that have the same characteristics. Polynomial functions with the same **zeros** are said to belong to the same family. The graphs of polynomial functions that belong to the same family have the same x-intercepts but have different y-intercepts (unless 0 is one of the x-intercepts).

An equation for the family of polynomial functions with zeros $a_1, a_2, a_3, \ldots, a_n$ is:

$$y = k(x-a_1)(x-a_2)(x-a_3)\ldots(x-a_n), \text{ where } k \in \mathbb{R}, k \neq 0$$

Part 2: Represent a Family of Functions Algebraically

1) The zeros of a family of quadratic functions are 2 and -3.

a) Determine an equation for this family of functions.

$y = k(x-2)(x+3)$

b) Write equations for two functions that belong to this family

$y = 8(x-2)(x+3)$
$y = -3(x-2)(x+3)$

c) Determine an equation for the member of the family that passes through the point (1, 4).

$y = k(x-2)(x+3)$

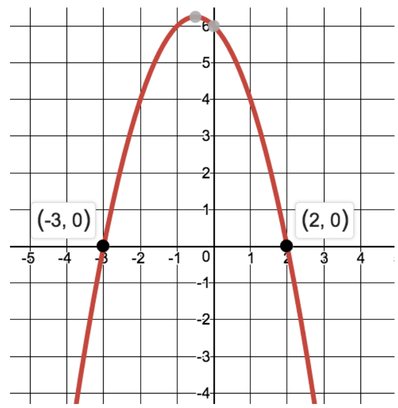
$4 = k(1-2)(1+3)$

$4 = k(-1)(4)$

$4 = -4k$

$-1 = k$

$y = -(x-2)(x+3)$



2) The zeros of a family of cubic functions are -2, 1, and 3.

a) Determine an equation for this family.

$y = k(x+2)(x-1)(x-3)$

b) Determine an equation for the member of the family whose graph has a y-intercept of -15.

$-15 = k(0+2)(0-1)(0-3)$

$-15 = k(2)(-1)(-3)$

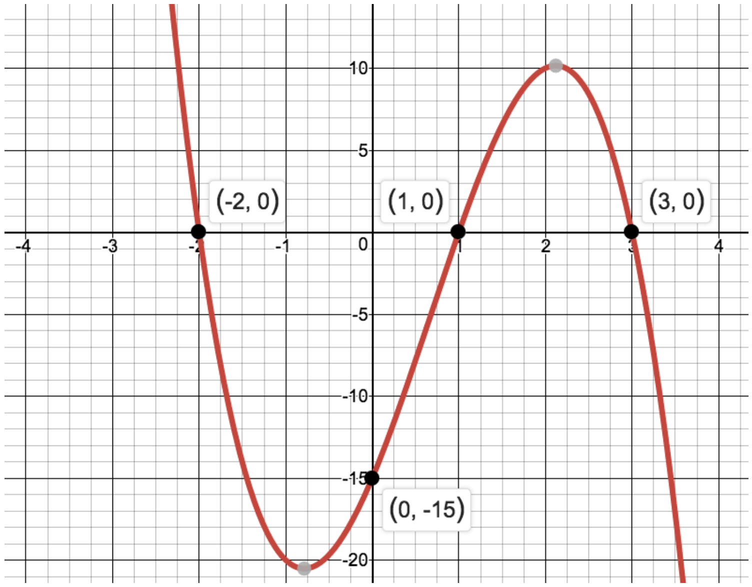
$-15 = 6k$

$k = -2.5$

$y = -2.5(x+2)(x-1)(x-3)$

d) Sketch a graph of the function

Negative leading coefficient and odd degree so it will extend from Q2 to Q4



To sketch a graph:

- Plot y-intercept
- Plot x-intercepts
- Use degree and leading coefficient to determine end behaviour

3) Determine an equation for the family of cubic functions with zeros $3 \pm \sqrt{5}$ and $-\frac{1}{2}$

Factors:

$$x = 3 \pm \sqrt{5}$$

$$x - 3 = \pm\sqrt{5}$$

$$(x-3)^2 = 5$$

$$x^2 - 6x + 9 = 5$$

$$x^2 - 6x + 4 = 0$$

$$x = -\frac{1}{2}$$

$$2x = -1$$

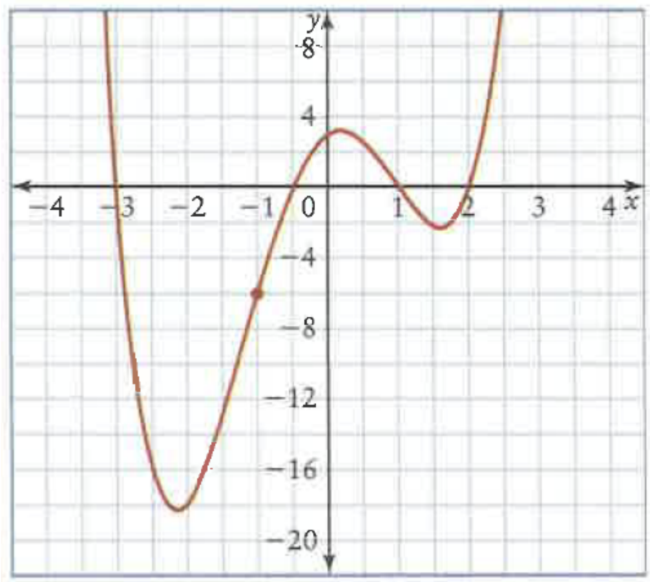
$$2x + 1 = 0$$

Equation:

$$P(x) = k(2x+1)(x^2 - 6x + 4)$$

Part 3: Determine an Equation for a Function From a Graph

3) Determine an equation for the quartic function represented by this graph.



The x-intercepts are $-3, -\frac{1}{2}, 1$, and 2

$y = k(x+3)(2x+1)(x-1)(x-2)$

The graph passes through the point (-1, -6)

$-6 = k(-1+3)(2(-1)+1)(-1-1)(-1-2)$

$-6 = k(2)(-1)(-2)(-3)$

$-6 = -12k$

$k = 0.5$

$y = 0.5(x+3)(2x+1)(x-1)(x-2)$