L7 – 5.4 Solve Quadratic Trigonometric Equations
MHF4U

A quadratic trigonometric equation may have multiple solutions in the interval $0 \leq x \leq 2\pi$.

You can often **factor** a quadratic trigonometric equation and then solve the resulting two linear trigonometric equations. In cases where the equation cannot be factored, use the **quadratic formula** and then solve the resulting linear trigonometric equations.

You may need to use a Pythagorean identity, compound angle formula, or double angle formula to create a quadratic equation that contains only a single trigonometric function whose arguments all match.

Remember that when solving a linear trigonometric equation, consider all 3 tools that can be useful:

1. Special Triangles
2. Graphs of Trig Functions
3. Calculator

Part 1: Solving Quadratic Trigonometric Equations

Example 1: Solve each of the following equations for $0 \leq x \leq 2\pi$

a) $(\sin x + 1)\left(\sin x - \frac{1}{2}\right) = 0$

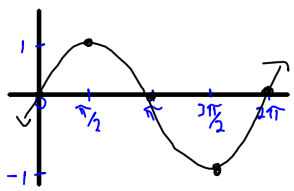
$(\sin x + 1)\left(\sin x - \frac{1}{2}\right) = 0$

* set both factors equal to zero and solve *

$\sin x + 1 = 0$

$\sin x = -1$ → Graph → Yes!

$x_1 = \frac{3\pi}{2}$

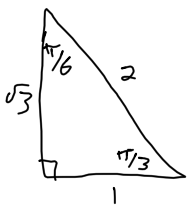


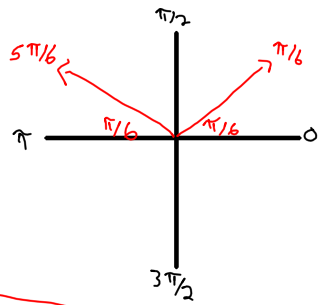
$\sin x - \frac{1}{2} = 0$

$\sin x = \frac{1}{2}$ → Graph → △ → Yes / No

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Put in Q1 + Q2





$x_2 = \pi/6$

$x_3 = \pi - \frac{\pi}{6}$

$x_3 = \frac{5\pi}{6}$

Solutions are $x = \frac{3\pi}{2}$, $\frac{\pi}{6}$, or $\frac{5\pi}{6}$ radians

b) $\sin^2 x - \sin x = 2$

$\sin^2 x - \sin x = 2$

$\sin^2 x - \sin x - 2 = 0$

Let $\sin x = x$

$x^2 - x - 2 = 0$ p: -2, s: -1 (-2 and 1)

$(x-2)(x+1) = 0$

$(\sin x - 2)(\sin x + 1) = 0$

$\sin x - 2 = 0$

$\sin x = 2$

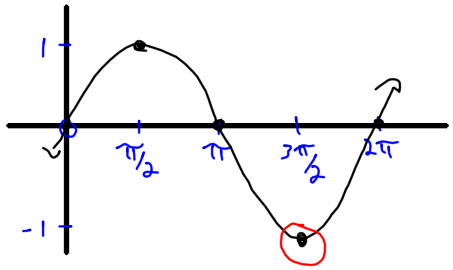
No solutions

$\sin x + 1 = 0$

$\sin x = -1$

Graph Yes

$x = \frac{3\pi}{2}$



The only solution is $x = \frac{3\pi}{2}$

c) $2\sin^2 x - 3\sin x + 1 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

Let $x = \sin x$

$2x^2 - 3x + 1 = 0$ P: 2, S: -3 (-2 and -1)

$2x^2 - 2x - 1x + 1 = 0$

$(2x^2 - 2x) + (-1x + 1) = 0$

$2x(x-1) - 1(x-1) = 0$

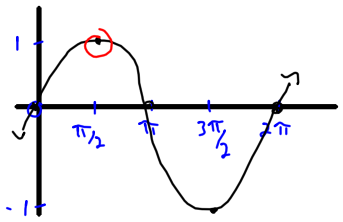
$(x-1)(2x-1) = 0$

$(\sin x - 1)(2\sin x - 1) = 0$

$\sin x - 1 = 0$

$\sin x = 1$ ← Graph

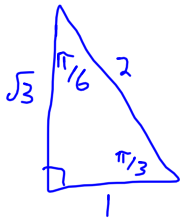
$x_1 = \frac{\pi}{2}$



$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ → △

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

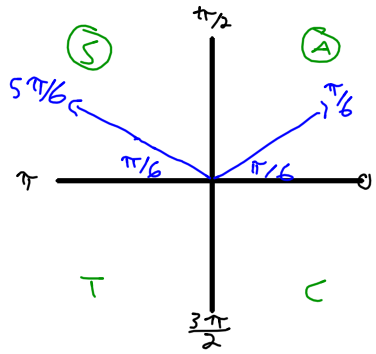


Put in Q1 + Q2

$x_2 = \frac{\pi}{6}$

$x_3 = \pi - \frac{\pi}{6}$

$x_3 = \frac{5\pi}{6}$



The solutions are $x = \frac{\pi}{2}$, $\frac{\pi}{6}$, or $\frac{5\pi}{6}$

Part 2: Use Identities to Help Solve Quadratic Trigonometric Equations

Example 2: Solve each of the following equations for $0 \leq x \leq 2\pi$

a) $2\sec^2 x - 3 + \tan x = 0$

$2\sec^2 x - 3 + \tan x = 0$

$2(\tan^2 x + 1) - 3 + \tan x = 0$

$2\tan^2 x + 2 - 3 + \tan x = 0$

$2\tan^2 x + \tan x - 1 = 0$

P.I. $\sec^2 x = \tan^2 x + 1$

Let $x = \tan x$

$2x^2 + x - 1 = 0$ P: -2 S: 1 (2 and -1)

$2x^2 + 2x - 1x - 1 = 0$

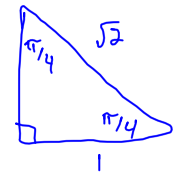
$2x(x+1) - 1(x+1) = 0$

$(x+1)(2x-1) = 0$

$(\tan x + 1)(2\tan x - 1) = 0$

$\tan x + 1 = 0$

$\tan x = -1$



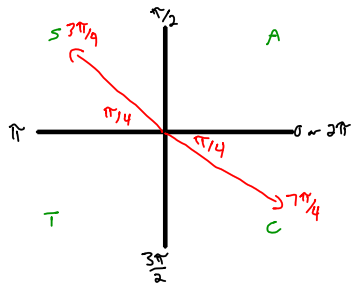
Put $\frac{\pi}{4}$ in Q2 + Q4

$x_1 = \pi - \frac{\pi}{4}$

$x_1 = \frac{3\pi}{4}$

$x_2 = 2\pi - \frac{\pi}{4}$

$x_2 = \frac{7\pi}{4}$



$2\tan x - 1 = 0$

$\tan x = \frac{1}{2}$

$x = \tan^{-1}\left(\frac{1}{2}\right)$

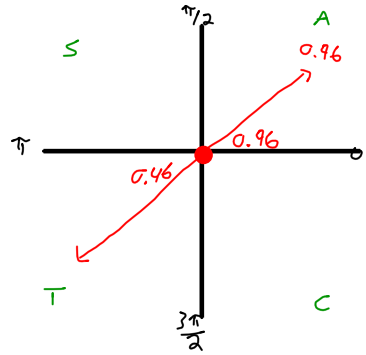
$x \simeq 0.46$ radians

Put in Q1 + Q3

$x_3 = 0.46$

$x_4 = \pi + 0.46$

$x_4 = 3.60$



The solutions are $x = \frac{3\pi}{4}, \frac{7\pi}{4}, 0.46,$ OR 3.60 RADIANS

b) $3\sin x + 3\cos(2x) = 2$

$3\sin x + 3\cos(2x) = 2$

$3\sin x + 3(1 - 2\sin^2 x) = 2$

$3\sin x + 3 - 6\sin^2 x - 2 = 0$

$-6\sin^2 x + 3\sin x + 1 = 0$

Let $x = \sin x$

$-6x^2 + 3x + 1 = 0$ P: -6 S: 3 NOT FACTORABLE USE Q.F.

$x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(-6)(1)}}{2(-6)}$

$x = \dfrac{-3 \pm \sqrt{33}}{-12}$

$x \doteq -0.23$

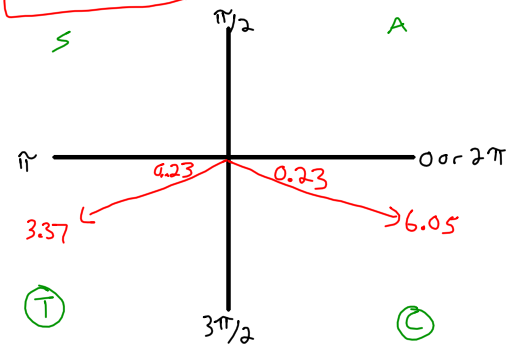
$\sin x = -0.23$ CALC

$x_1 = \sin^{-1}(-0.23)$

$x_1 = -0.23$

$x_1 = 2\pi - 0.23$

$x_1 = 6.05$



$x_2 = \pi + 0.23$

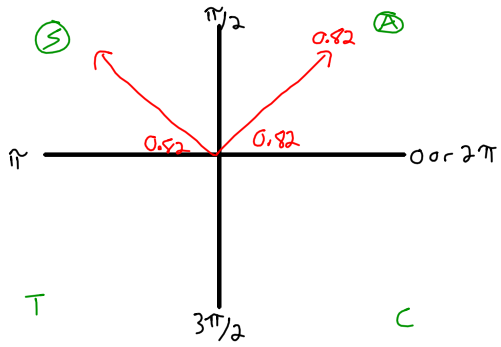
$x_2 = 3.37$

$x \doteq 0.73$

$\sin x = 0.73$ CALC

$x = \sin^{-1}(0.73)$

$x_1 = 0.82$



$x_4 = \pi - 0.82$

$x_4 = 2.32$

The solutions are $x = 0.82, 2.32, 3.37,$ OR 6.05 RADIANS