L5 – 1.3 – Symmetry in Polynomial Functions
MHF4U

In this section, you will learn about the properties of even and odd polynomial functions.

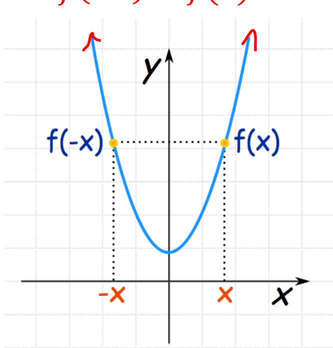
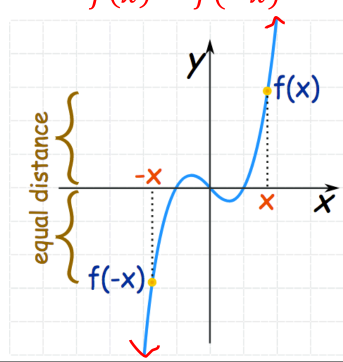
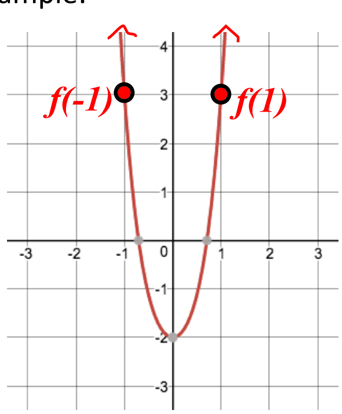
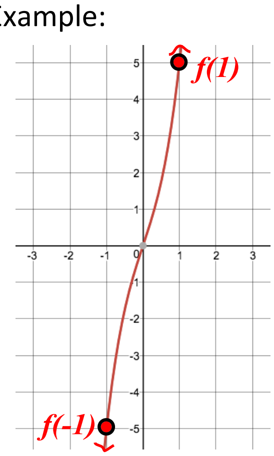
Symmetry in Polynomial Functions

Line Symmetry – there is a vertical line over which the polynomial remains unchanged when reflected.

Point symmetry / Rotational Symmetry – there is a point about which the polynomial remains unchanged when rotated 180°

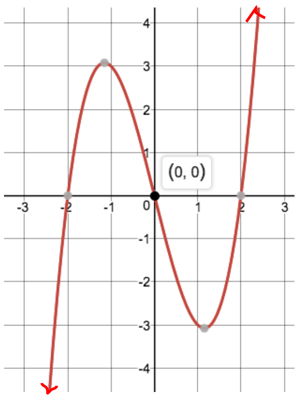
Section 1: Properties of Even and Odd Functions

A polynomial function of even or odd degree is NOT necessarily and even or odd function. The following are properties of all even and odd functions:

Even Functions	Odd Functions
An even degree polynomial function is an EVEN FUNCTION if: • Line symmetry over the y-axis • The exponent of each term is even • May have a constant term	An odd degree polynomial function is an ODD FUNCTION if: • Point symmetry about the origin (0, 0) • The exponent of each term is odd • No constant term
Rule: $f(-x) = f(x)$ 	Rule: $-f(x) = f(-x)$ 
Example:  $f(x) = 2x^4 + 3x^2 - 2$ Notice: $f(1) = 3$ $f(-1) = 3$ $\therefore f(1) = f(-1)$	Example:  $f(x) = 2x^3 + 3x$ Notice: $f(1) = 5$ $f(-1) = -5$ $\therefore -f(1) = f(-1)$

Example 1: Identify each function as an even function, odd function, or neither. Explain how you can tell.

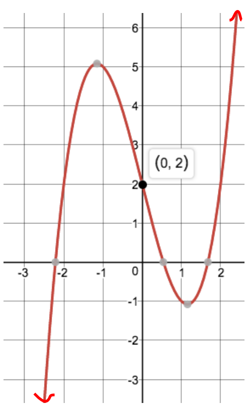
a) $y = x^3 - 4x$



This is an odd function because:

- It has point symmetry about the origin
- All terms in the equation have an odd exponent and there is no constant term

b) $y = x^3 - 4x + 2$

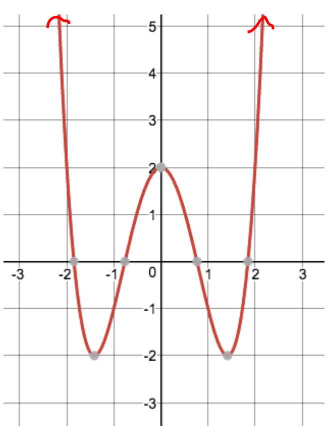


Neither

This function has point symmetry. However, the origin is not the point about which the function is symmetrical. Therefore, it is not an odd or even function.

From the equation we can tell it is NOT an odd function because there is a constant term.

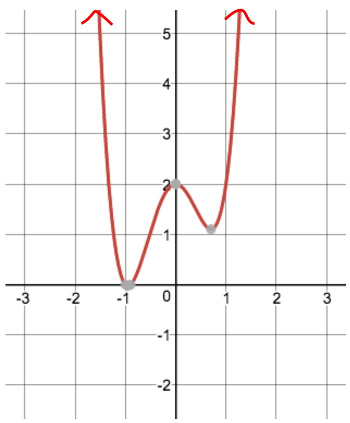
c) $y = x^4 - 4x^2 + 2$



This is an even function because:

- It has line symmetry about the y-axis
- All terms in the equation have an even exponent. Even functions are allowed to have a constant term.

d) $y = 3x^4 + x^3 - 4x^2 + 2$

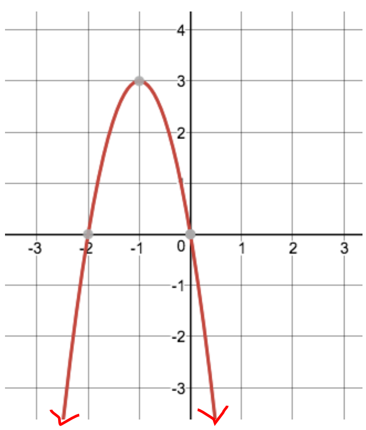


Neither

This function does not have line or point symmetry.

From the equation we can tell it is NOT an even or odd function because there is a mix of even and odd exponents.

e) $y = -3x^2 - 6x$



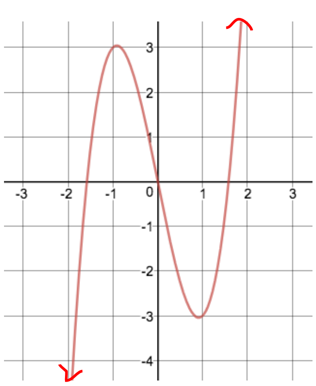
Neither

This function has line symmetry. However, the y-axis is not the line about which the function is symmetrical. Therefore, it is not an odd or even function.

From the equation we can tell it is NOT an even or odd function because there is a mixture of even and odd exponents.

<u>**Example 2:**</u> Choose all that apply for each function

a)



i) no symmetry

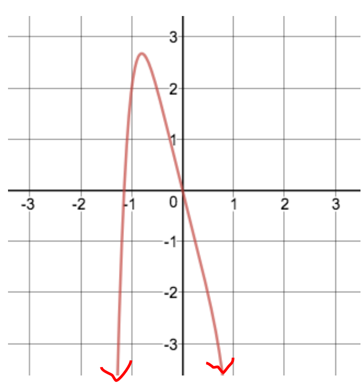
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

b)



i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

c) $P(x) = 5x^3 + 3x^2 + 2$

Note: *all cubic functions have point symmetry*

i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

d) $P(x) = x^6 + x^2 - 11$

i) no symmetry

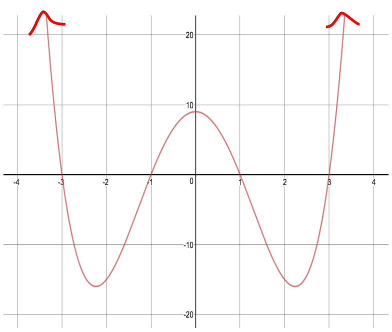
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

e)



i) no symmetry

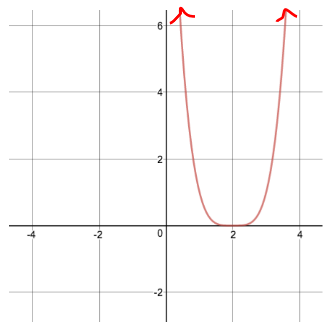
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

f)



i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

g) $P(x) = 5x^5 - 4x^3 + 8x$

i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

<u>Example 3:</u> Without graphing, determine if each polynomial function has line symmetry about the y-axis, point symmetry about the origin, or neither. Verify your response algebraically.

a) $f(x) = 2x^4 - 5x^2 + 4$

The function is even since the exponent of each term is even. The function has line symmetry about the y-axis.

Verify $f(x) = f(-x)$

$f(-x) = 2(-x)^4 - 5(-x)^2 + 4$
$f(-x) = 2x^4 - 5x^2 + 4$
$f(-x) = f(x)$

b) $f(x) = -3x^5 + 9x^3 + 2x$

The function is odd since the exponent of each term is odd. The function has point symmetry about the origin.

Verify $-f(x) = f(-x)$

$-f(x) = -(-3x^5 + 9x^3 + 2x)$
$-f(x) = 3x^5 - 9x^3 - 2x$

$f(-x) = -3(-x)^5 + 9(-x)^3 + 2(-x)$
$f(-x) = 3x^5 - 9x^3 - 2x$

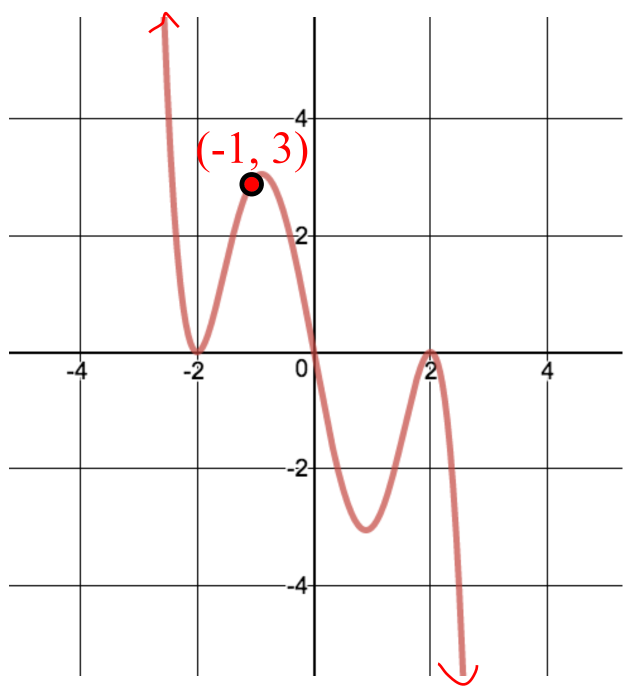
$\therefore\ -f(x) = f(-x)$

c) $x^6 - 4x^3 + 6x^2 - 4$

Some exponents are even and some are odd, so the function is neither even nor odd. It does not have line symmetry about the y-axis or point symmetry about the origin.

Section 2: Connecting from throughout the unit

Example 4: Use the given graph to state:



a) x-intercepts

-2 (order 2), 0 (order 1), and 2(order 2)

b) number of turning points

2 local min and 2 local max
4 turning points

c) least possible degree

Least possible degree is 5

b) any symmetry present

Point symmetry about the origin. Therefore, this is an odd function.

c) the intervals where $f(x) < 0$

$(0, 2) \cup (2, \infty)$

d) Find the equation in factored form

$$P(x) = k(x)(x+2)^2(x-2)^2$$

$$3 = k(-1)(-1+2)^2(-1-2)^2$$

$$3 = k(-1)(1)^2(-3)^2$$

$$3 = -9k$$

$$k = -\frac{1}{3}$$

$$P(x) = -\frac{1}{3}x(x+2)^2(x-2)^2$$