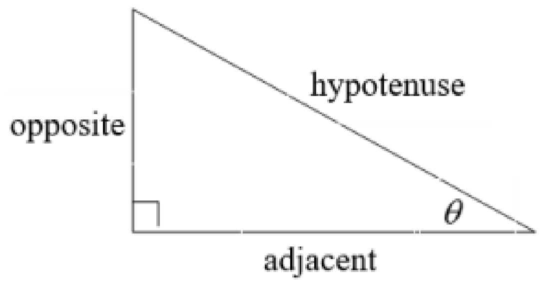
Trig. Toolbox

Right triangle definition

For this definition we assume that $0 < \theta < \frac{\pi}{2}$ or $0^\circ < \theta < 90^\circ$.



$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad$ $\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}}$

$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\quad$ $\sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ $\quad$ $\cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$

Law of Sines

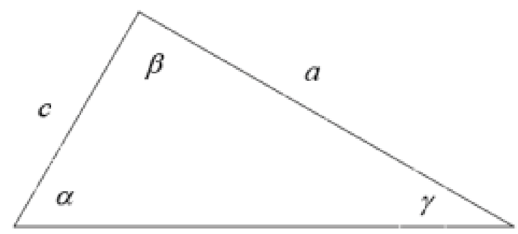
$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Law of Cosines

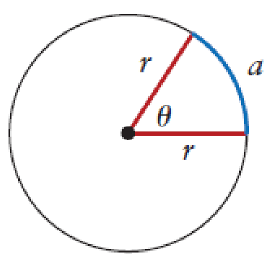
$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$



- The radian measure of angle θ is defined as the length, a, of the arc that subtends the angle divided by the radius, r, of the circle: $\theta = \frac{a}{r}$.



- 2π rad $= 360^\circ$ or π rad $= 180^\circ$.
- To convert degree measure to radian measure, multiply the degree measure by $\frac{\pi}{180}$ radians.
- To convert radian measure to degree measure, multiply the radian measure by $\left(\frac{180}{\pi}\right)^\circ$.

Range

The range is all possible values to get out of the function.

$$-1 \leq \text{sin}(\theta) \leq 1 \qquad -1 \leq \text{cos}(\theta) \leq 1$$

$$-\infty < \text{tan}(\theta) < \infty \qquad -\infty < \text{cot}(\theta) < \infty$$

$$\text{sec}(\theta) \geq 1 \text{ and sec}(\theta) \leq -1 \qquad \text{csc}(\theta) \geq 1 \text{ and csc}(\theta) \leq -1$$

Domain

The domain is all the values of θ that can be plugged into the function.

sin(θ), θ can be any angle

cos(θ), θ can be any angle

tan(θ), $\theta \neq \left(n + \frac{1}{2}\right)\pi, \; n = 0, \pm 1, \pm 2, \ldots$

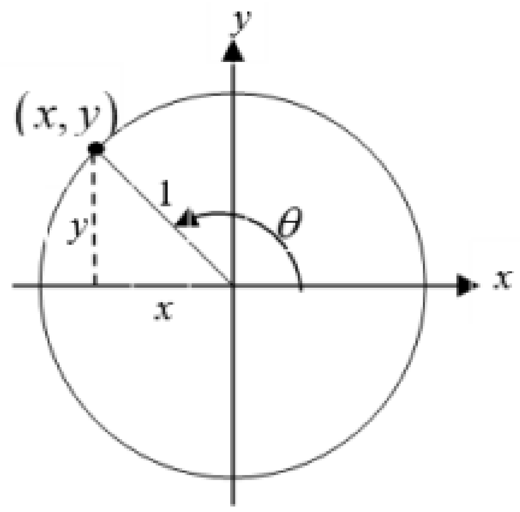
csc(θ), $\theta \neq n\pi, \; n = 0, \; \pm 1, \; \pm 2, \ldots$

sec(θ), $\theta \neq \left(n + \frac{1}{2}\right)\pi, \; n = 0, \pm 1, \pm 2, \ldots$

cot(θ), $\theta \neq n\pi, \; n = 0, \pm 1, \pm 2, \ldots$

Unit Circle Definition

For this definition θ is any angle.



$$\sin(\theta) = \frac{y}{1} = y \qquad \csc(\theta) = \frac{1}{y}$$

$$\cos(\theta) = \frac{x}{1} = x \qquad \sec(\theta) = \frac{1}{x}$$

$$\tan(\theta) = \frac{y}{x} \qquad \cot(\theta) = \frac{x}{y}$$

Let θ be the acute angle, then by CAST Rule

In QII

$$\sin(\pi-\theta) = \sin\theta$$
$$\cos(\pi-\theta) = -\cos\theta$$
$$\tan(\pi-\theta) = -\tan\theta$$

In Q III

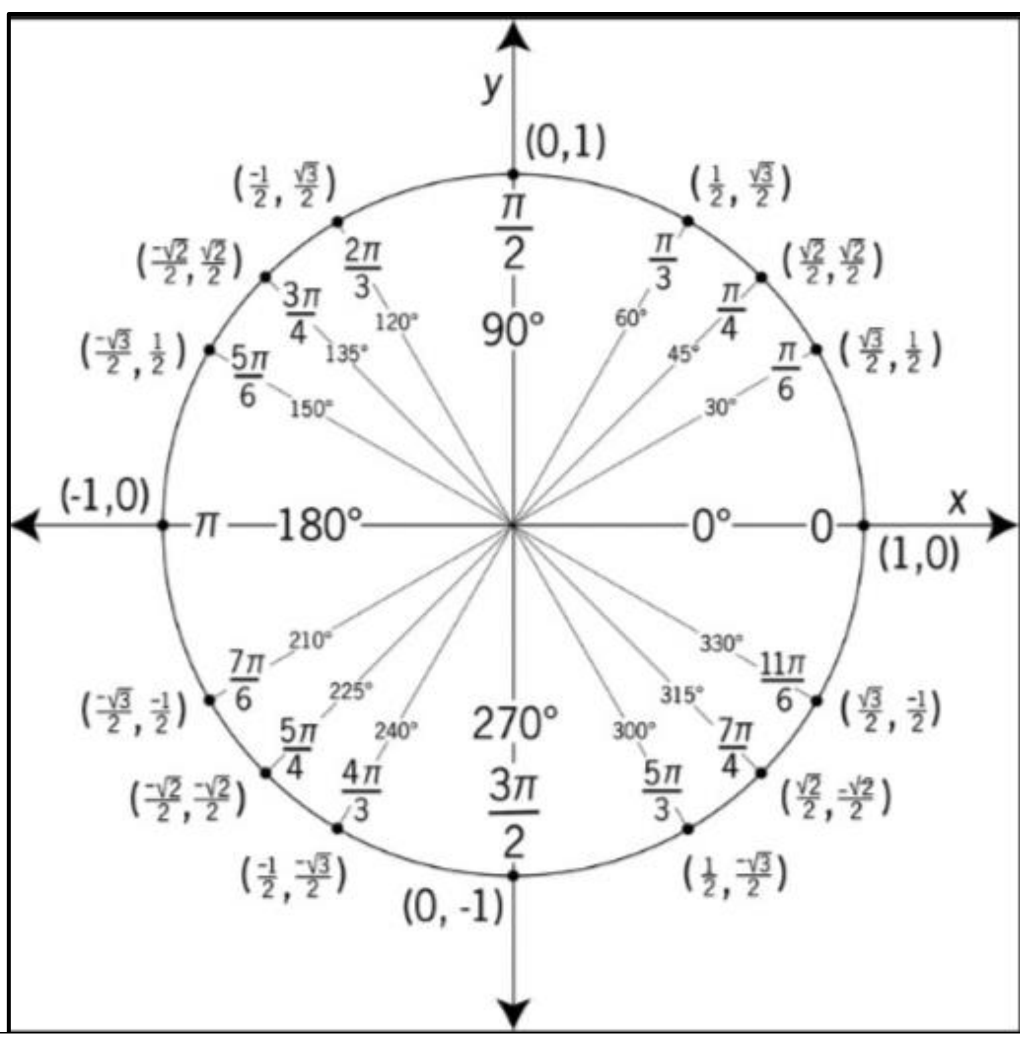
$$\sin(\pi+\theta) = -\sin\theta$$
$$\cos(\pi+\theta) = -\cos\theta$$
$$\tan(\pi+\theta) = \tan\theta$$

In Qiv

$$\sin(2\pi-\theta) = -\sin\theta$$
$$\cos(2\pi-\theta) = \cos\theta$$
$$\tan(2\pi-\theta) = -\tan\theta$$

In QI

$$\sin(2\pi+\theta) = \sin\theta$$
$$\cos(2\pi+\theta) = \cos\theta$$
$$\tan(2\pi+\theta) = \tan\theta$$



Reciprocal Identities	**Quotient Identities**	**Pythagorean Identity**	**Reflection Identities**
$\csc A = \frac{1}{\sin A}$	$\tan(A) = \frac{\sin(A)}{\cos(A)}$	$\sin^2(A) + \cos^2(A) = 1$	$\sin(-A) = -\sin(A)$
$\sec(A) = \frac{1}{\cos(A)}$	$\cot(A) = \frac{\cos(A)}{\sin(A)}$	$\tan^2(A) + 1 = \sec^2(A)$	$\cos(-A) = \cos(A)$
$\cot(A) = \frac{1}{\tan(A)}$		$\cot^2(A) + 1 = \csc^2(A)$	$\tan(-A) = -\tan(A)$
$\tan(A) = \frac{1}{\cot(A)}$			
$\cot(A) = \frac{1}{\tan(A)}$			

Co-Function Identities	
$\sin\left(\frac{\pi}{2} - x\right) =$	$\csc\left(\frac{\pi}{2} - x\right) =$
$\cos\left(\frac{\pi}{2} - x\right) =$	$\sec\left(\frac{\pi}{2} - x\right) =$
$\tan\left(\frac{\pi}{2} - x\right) =$	$\cot\left(\frac{\pi}{2} - x\right) =$

Other Co-Function Identities	
$\sin\left(\frac{\pi}{2} + x\right) =$	$\csc\left(\frac{\pi}{2} + x\right) =$
$\cos\left(\frac{\pi}{2} + x\right) =$	$\sec\left(\frac{\pi}{2} + x\right) =$
$\tan\left(\frac{\pi}{2} + x\right) =$	$\cot\left(\frac{\pi}{2} + x\right) =$

More Co-Function Identities	
$\sin\left(\frac{3\pi}{2} - x\right) =$	$\csc\left(\frac{3\pi}{2} - x\right) =$
$\cos\left(\frac{3\pi}{2} - x\right) =$	$\sec\left(\frac{3\pi}{2} - x\right) =$
$\tan\left(\frac{3\pi}{2} - x\right) =$	$\cot\left(\frac{3\pi}{2} - x\right) =$

More Co-Function Identities	
$\sin\left(\frac{3\pi}{2} + x\right) =$	$\csc\left(\frac{3\pi}{2} + x\right) =$
$\cos\left(\frac{3\pi}{2} + x\right) =$	$\sec\left(\frac{3\pi}{2} + x\right) =$
$\tan\left(\frac{3\pi}{2} + x\right) =$	$\cot\left(\frac{3\pi}{2} + x\right) =$

Reflection Identities	
$\sin(-x) = -\sin(x)$	$\csc(-x) = -\csc(x)$
$\cos(-x) = \cos(x)$	$\sec(-x) = \sec(x)$
$\tan(-x) = -\tan(x)$	$\cot(-x) = -\cot(x)$

Compound Angel Identities

$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$$

$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$$

$$\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$$

$$\tan(\alpha+\beta)=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)} \qquad (*)$$

$$\tan(\alpha-\beta)=\frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)} \qquad (*)$$

Double Angle Formula for Cosine

$\cos(2A)=\cos^2(A)-\sin^2(A)$ $\qquad$ $\cos(2A)=2\cos^2(A)-1$ $\qquad$ $\cos(2A)=1-2\sin^2(A)$

Double Angle Formula for Tangen

$$\tan(2A)=\frac{2\tan(A)}{1-\tan^2(A)}$$

Double Angle Formula for Sine

$\sin(2A)=2\sin(A)\cos(A)$