W3 – Newton Quotient
MHF4U

SOLUTIONS

, Find the equation of the derivative for each of the following functions. Also, find the instantaneous rate of change for the function when $x = 4$ and $x = -1$.

a) $f(x) = 3x - 8$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{3(x+h)-8-(3x-8)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{3x+3h-8-3x+8}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{3h}{h}$$

$f'(x) = 3$

$f'(4) = 3$

$f'(-1) = 3$

b) $y = 20x + x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{20(x+h)+(x+h)^2-(20x+x^2)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{20x+20h+x^2+2xh+h^2-20x-x^2}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{20h+2xh+h^2}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h(20+2x+h)}{h}$$

$f'(x) = 20+2x+0$

$f'(x) = 2x+20$

$f'(4) = 2(4)+20$

$f'(4) = 28$

$f'(-1) = 2(-1)+20$

$f'(-1) = 18$

c) $y = 2x^3 + 4$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^3+4-(2x^3+4)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)(x^2+2xh+h^2)+4-2x^3-4}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2(x^3+2x^2h+xh^2+x^2h+2xh^2+h^3)-2x^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2(x^3+3x^2h+3xh^2+h^3)-2x^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2x^3+6x^2h+6xh^2+2h^3-2x^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h(6x^2+6xh+2h^2)}{h}$$

$f'(x) = 6x^2+6x(0)+2(0)^2$

$f'(x) = 6x^2$

$f'(4) = 6(4)^2 = 96$

$f'(-1) = 6(-1)^2 = 6$

d) $f(x) = x^2 - 9x + 17$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2-9(x+h)+17-(x^2-9x+17)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{x^2+2xh+h^2-9x-9h+17-x^2+9x-17}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2xh+h^2-9h}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h(2x+h-9)}{h}$$

$f'(x) = 2x+0-9$

$f'(x) = 2x-9$

$f'(4) = 2(4)-9 = -1$

$f'(-1) = 2(-1)-9 = -11$

e) $f(x) = \frac{x(x+1)}{2}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{(x+h)(x+h+1)}{2} - \frac{x(x+1)}{2}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{x^2+xh+x+xh+h^2+h-x^2-x}{2h}$$

$$f'(x) = \lim_{h\to 0} \frac{2xh+h^2+h}{2h}$$

$$F'(x) = \lim_{h\to 0} \frac{h(2x+h+1)}{2h}$$

$$f'(x) = \frac{2x+0+1}{2}$$

$$F'(x) = \frac{2x+1}{2}$$

$F'(x) = x + \frac{1}{2}$

$f'(4) = 4 + \frac{1}{2} = \frac{9}{2}$

$f'(-1) = -1 + \frac{1}{2} = -\frac{1}{2}$

f) $f(x) = \frac{1}{x}$

$$F'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$F'(x) = \lim_{h\to 0} \frac{\frac{x-1(x+h)}{x(x+h)}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{x-x-h}{h(x)(x+h)}$$

$$F'(x) = \lim_{h\to 0} \frac{-1h}{h(x)(x+h)}$$

$$f'(x) = \frac{-1}{x(x+0)}$$

$$F'(x) = \frac{-1}{x^2}$$

2) State whether the functions are increasing, decreasing, or neither when $x = 4$ for each function in #1. How do you know?

a, b, c, e are increasing since $F'(4)$ is positive

d, f are decreasing since $F'(4)$ is negative

3)a) State the derivative of $f(x) = x^3$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$F'(x) = \lim_{h\to 0} \frac{(x+h)(x^2+2xh+h^2) - x^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{x^3+2x^2h+xh^2+x^2h+2xh^2+h^3-x^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h(3x^2+3xh+h^2)}{h}$$

$$f'(x) = 3x^2 + 3x(0) + (0)^2$$

$f'(x) = 3x^2$

b) Evaluate $f'(-6)$ $= 3(-6)^2$

$= 108$

c) Determine the equation of the tangent line at $x = 6$

$f(6) = (6)^3$
$= 216$ ∴ point $(6, 216)$ is on the tangent line

$f'(6) = 3(6)^2$
$= 108$ ∴ slope of the tangent line is 108

$y = mx + b$

$216 = 108(6) + b$

$b = -432$

$y = 108x - 432$