W7 – 5.4 Solve Quadratic Trigonometric Equations
MHF4U

SOLUTIONS

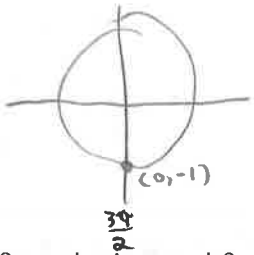
1) Solve $\sin^2 x - 2\sin x - 3 = 0$ on the interval $0 \le x \le 2\pi$

$(\sin x - 3)(\sin x + 1) = 0$

$\sin x - 3 = 0$ $\quad$ $\sin x = -1$

$\sin x = 3$ $\quad$ use unit circle where each point is $(\cos x, \sin x)$

No solutions



$x = \frac{3\pi}{2}$

2) Solve $\csc^2 x - \csc x - 2 = 0$ on the interval $0 \le x \le 2\pi$

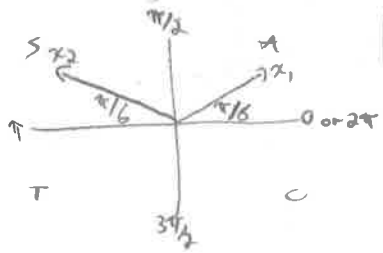
$(\csc x - 2)(\csc x + 1) = 0$

$\csc x - 2 = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$

From Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$



$x_1 = \frac{\pi}{6}$

$x_2 = \pi - \frac{\pi}{6}$

$x_2 = \frac{5\pi}{6}$

$\csc x + 1 = 0$

$\csc x = -1$

$\sin x = -1$

* refer to part a) *

$x_3 = \frac{3\pi}{2}$

3) Solve $2\sec^2 x - \sec x - 1 = 0$ on the interval $0 \le x \le 2\pi$

$2\sec^2 x - 2\sec + 1\sec x - 1 = 0$

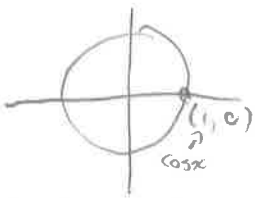
$2\sec x(\sec x - 1) + 1(\sec x - 1) = 0$

$(\sec x - 1)(2\sec x + 1) = 0$

$\sec x - 1 = 0$

$\sec x = 1$

$\cos x = 1$



$x_1 = 0$

$x_2 = 2\pi$

$2\sec x + 1 = 0$

$\sec x = -\frac{1}{2}$

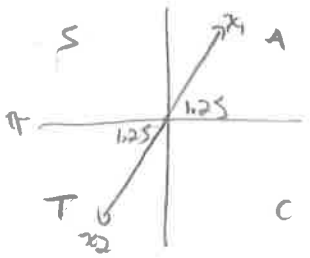
$\cos x = -2$

No solutions

4) Solve $\tan^2 x - \tan x - 6 = 0$ on the interval $0 \le x \le 2\pi$. Round answers to the nearest hundredth of a radian.

$(\tan x - 3)(\tan x + 2) = 0$

$\tan x = 3$



$x_1 = \tan^{-1}(3)$

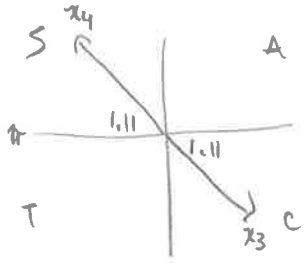
$x_1 = 1.25$

$x_2 = \pi + 1.25$

$x_2 = 4.39$

$\tan x + 2 = 0$

$\tan x = -2$



$x_3 = \tan^{-1}(-2)$

$x_3 = -1.107 + 2\pi$

$x_3 = 5.18$

$x_4 = \pi - 1.11$

$x_4 = 2.03$

5) Solve $6\cos^2 x + 5\cos x - 6 = 0$ on the interval $0 \le x \le 2\pi$

$6\cos^2 x + 9\cos x - 4\cos x - 6 = 0$

$3\cos x(2\cos x + 3) - 2(2\cos x + 3) = 0$

$(2\cos x + 3)(3\cos x - 2) = 0$

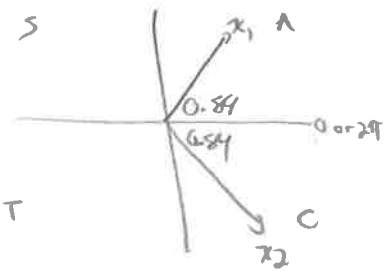
$2\cos x + 3 = 0$

$\cos x = -\frac{3}{2}$

No solutions

$3\cos x - 2 = 0$

$\cos x = \frac{2}{3}$



$x_1 = \cos^{-1}\left(\frac{2}{3}\right)$

$x_1 = 0.84$

$x_2 = 2\pi - 0.84$

$x_2 = 5.44$

6) Solve $3\csc^2 x - 5\csc x - 2 = 0$ on the interval $0 \le x \le 2\pi$

$3\csc^2 x - 6\csc x + 1\csc x - 2 = 0$

$3\csc x(\csc x - 2) + 1(\csc x - 2) = 0$

$(\csc x - 2)(3\csc x + 1) = 0$

$\csc x - 2 = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$

From Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2

$x_1 = \frac{\pi}{6}$

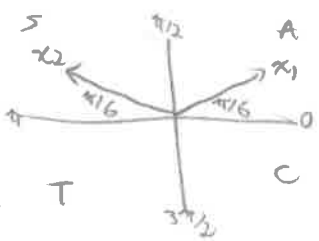
$x_2 = \frac{5\pi}{6}$

$3\csc x + 1 = 0$

$\csc x = -\frac{1}{3}$

$\sin x = -3$

No solutions



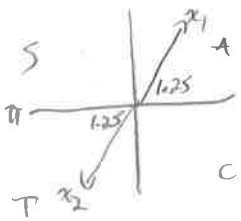
7) Solve $2\tan^2 x - 5\tan x - 3 = 0$ on the interval $0 \le x \le 2\pi$

$2\tan^2 x - 6\tan x + 1\tan x - 3 = 0$

$2\tan x(\tan x - 3) + 1(\tan x - 3) = 0$

$(\tan x - 3)(2\tan x + 1) = 0$

$\tan x = 3$



$x_1 = \tan^{-1}(3)$

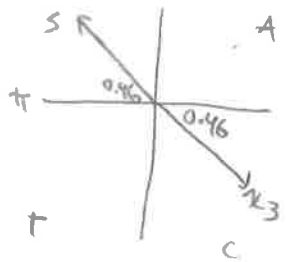
$x_1 = 1.25$

$x_2 = \pi + 1.25$

$x_2 = 4.39$

$2\tan x + 1 = 0$

$\tan x = -\frac{1}{2}$



$x_3 = \tan^{-1}\left(-\frac{1}{2}\right)$

$x_3 = -0.46364609 + 2\pi$

$x_3 = 5.82$

$x_4 = \pi - 0.46$

$x_4 = 2.68$

8) Solve $\cot x \csc^2 x = 2\cot x$ on the interval $0 \le x \le 2\pi$

$\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\sin^2 x}\right) = 2\left(\frac{\cos x}{\sin x}\right)$

$\frac{\cos x}{\sin^2 x} = 2\cos x$

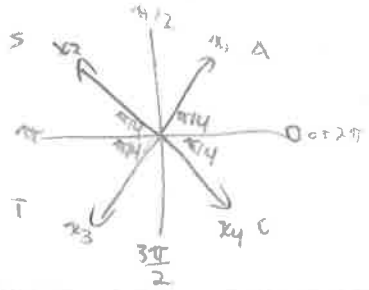
$\frac{\cos x}{\cos x} = 2\sin^2 x$

$\frac{1}{2} = \sin^2 x$

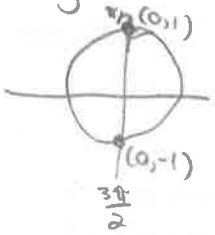
$\sin x = \pm\frac{1}{\sqrt{2}}$

from Δ; $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Place in all 4 quadrants



* Notice in 1st line of solution; if $\cos x = 0$, then LS = RS as well. OR using unit circle where a point is $(\cos x, \sin x)$



$x_1 = \frac{\pi}{4}$, $x_2 = \frac{3\pi}{4}$, $x_3 = \frac{5\pi}{4}$, $x_4 = \frac{7\pi}{4}$, $x_5 = \frac{\pi}{2}$, $x_6 = \frac{3\pi}{2}$

9) Solve for θ to the nearest hundredth, where $0 \le \theta \le 2\pi$

a) $3\tan^2\theta - 2\tan\theta = 1$

$3\tan^2\theta - 2\tan\theta - 1 = 0$

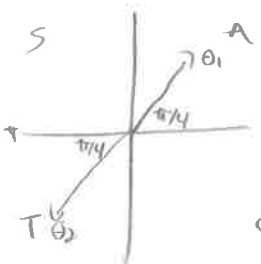
$3\tan^2\theta - 3\tan\theta + 1\tan\theta - 1 = 0$

$3\tan\theta(\tan\theta - 1) + 1(\tan\theta - 1) = 0$

$(\tan\theta - 1)(3\tan\theta + 1) = 0$

$\tan\theta - 1 = 0$

$\tan\theta = 1$



from Δ; $\tan\frac{\pi}{4} = 1$

Place in QI + QIII

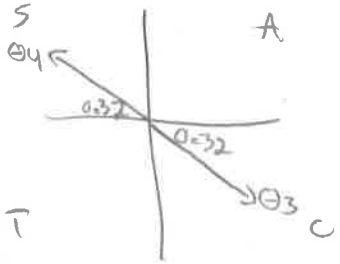
$\theta_1 = \frac{\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

$3\tan\theta + 1 = 0$

$\tan\theta = -\frac{1}{3}$



$\theta_3 = \tan^{-1}\left(-\frac{1}{3}\right)$

$\theta_3 = -0.32175 + 2\pi$

$\theta_3 = 5.96$

$\theta_4 = \pi - 0.32$

$\theta_4 = 2.82$

b) $12\sin^2\theta + \sin\theta - 6 = 0$

$12\sin^2\theta + 9\sin\theta - 8\sin\theta - 6 = 0$

$3\sin\theta(4\sin\theta + 3) - 2(4\sin\theta + 3) = 0$

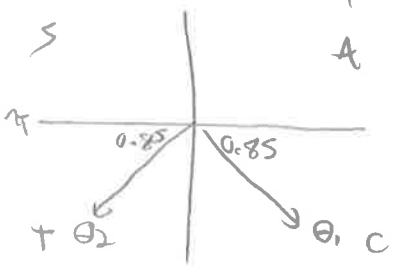
$(4\sin\theta + 3)(3\sin\theta - 2) = 0$

$4\sin\theta + 3 = 0$

$\sin\theta = -\frac{3}{4}$

$3\sin\theta - 2 = 0$

$\sin\theta = \frac{2}{3}$



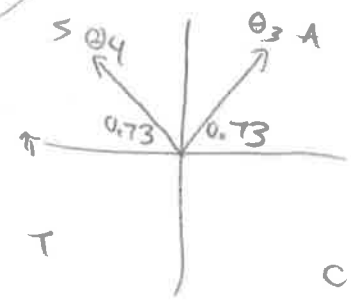
$\theta_1 = \sin^{-1}\left(-\frac{3}{4}\right)$

$\theta_1 = -0.848062 + 2\pi$

$\theta_1 = 5.44$

$\theta_2 = \pi + 0.85$

$\theta_2 = 3.99$



$\theta_3 = \sin^{-1}\left(\frac{2}{3}\right)$

$\theta_3 = 0.73$

$\theta_4 = \pi - 0.73$

$\theta_4 = 2.41$

c) $5\cos(2\theta) - \cos\theta + 3 = 0$

$5(2\cos^2\theta - 1) - \cos\theta + 3 = 0$

$10\cos^2\theta - 5 - \cos\theta + 3 = 0$

$10\cos^2\theta - \cos\theta - 2 = 0$

$10\cos^2\theta - 5\cos\theta + 4\cos\theta - 2 = 0$

$5\cos\theta(2\cos\theta - 1) + 2(2\cos\theta - 1) = 0$

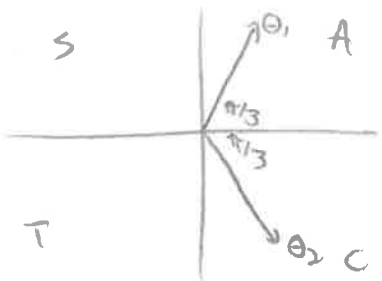
$(2\cos\theta - 1)(5\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2}$

$\cos\theta = -\frac{2}{5}$

From A; $\cos\frac{\pi}{3} = \frac{1}{2}$

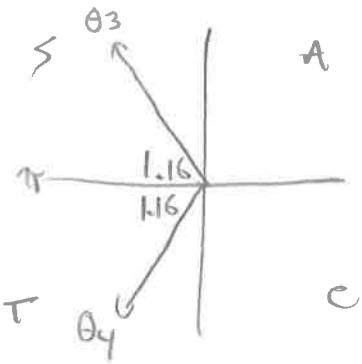
place in Q1 + Q4



$\theta_1 = \frac{\pi}{3}$

$\theta_2 = 2\pi - \frac{\pi}{3}$

$\theta_2 = \frac{5\pi}{3}$



$\theta_3 = \cos^{-1}\left(-\frac{2}{5}\right)$

$\theta_3 = 1.98$

$\theta_4 = \pi + 1.16$

$\theta_4 = 4.3$