6.2 Product and Quotient of two functions

If you are given the functions $f(x)$ and $g(x)$

- the **product of two functions** is defined as $f(x)g(x)$ or $(fg)(x)$
- the **quotient of two functions** is defined as $\frac{f(x)}{g(x)}$ or $\left(\frac{f}{g}\right)(x)$

Similarly, the product of two functions can be found by multiplying the y-coordinates with the same x-coordinates. I.e.)$\big(x, f(x)g(x)\big)$

- **The Domain of fg=**

To find the quotient of two functions, we need to divide the y-coordinates with the same x-coordinates.

For the functions f and g, $\frac{f}{g}$ is defined by $\frac{f}{g} = \left\{\left(x, \frac{f(x)}{g(x)}\right) \middle| \frac{f(x)}{g(x)} \; is \; defined\right\}$

Note: The quotient of two functions may not be a function due to possible Holes and V.A.(s)

- **The Domain of $\frac{f}{g}$ =**

Note: The product of two functions will remain a function!

Ex 1) Given $f = \{(-3,7),(-2,5),(-1,-4),(2,8)\}$ and $g = \{(-4,6),(-2,3),(-1,0),(1,5),(2,-4)\}$
Find

a) $f+g$

b) $f-g$

c) fg

d) $\frac{f}{g}$

e) $\frac{g}{f}$

Ex 2) Let $f(x) = \sqrt{x+6}$ and $g(x) = x^2 + x - 6$

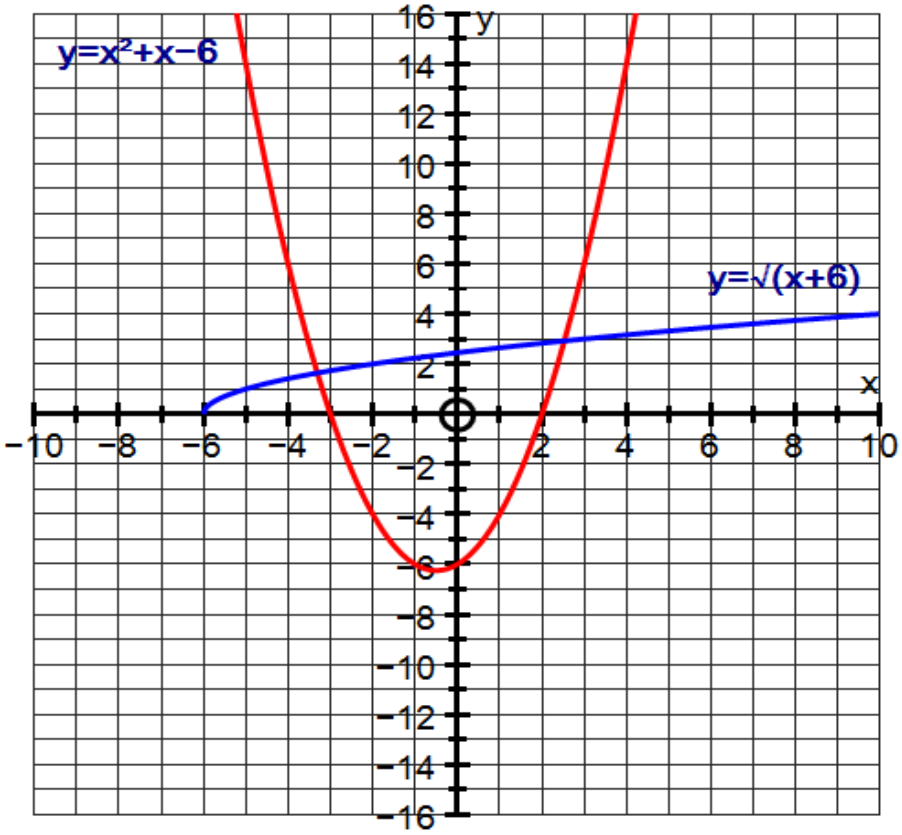
a) Determine the equation of $h(x) = f(x)g(x)$

b) Determine the equation of $k(x) = \frac{g(x)}{f(x)}$

c) Sketch the graphs of the combined functions

d) State the domain and range of $h(x)$ and $k(x)$.

x	$f(x)$	$g(x)$	$h(x)$	$k(x)$



Ex 3) Consider the following table illustrating the product of functions $f(x)$ and $g(x)$. Let the product be defined as $h(x) = f(x)g(x)$ and the quotient be defined as $k(x) = \frac{f(x)}{g(x)}$. Determine if $h(x)$ and $k(x)$ are even or odd in the following scenarios.

$f(x)$	$g(x)$	$f(x)g(x)$	$\frac{f(x)}{g(x)}$
Even	Even		
Even	Odd		
Odd	Even		
Odd	Odd		

Ex 4) For each of the following pairs of functions, write the equation of $h(x) = \left(\frac{f}{g}\right)(x)$ and state the domain of $h(x)$

1. $f(x) = 10, g(x) = x$

2. $f(x) = 4x, g(x) = 3x - 2$

3. $f(x) = 4x^2, g(x) = x^2 - 4$

4. $f(x) = x^2 - x - 12, g(x) = x^2 + 2x - 3$

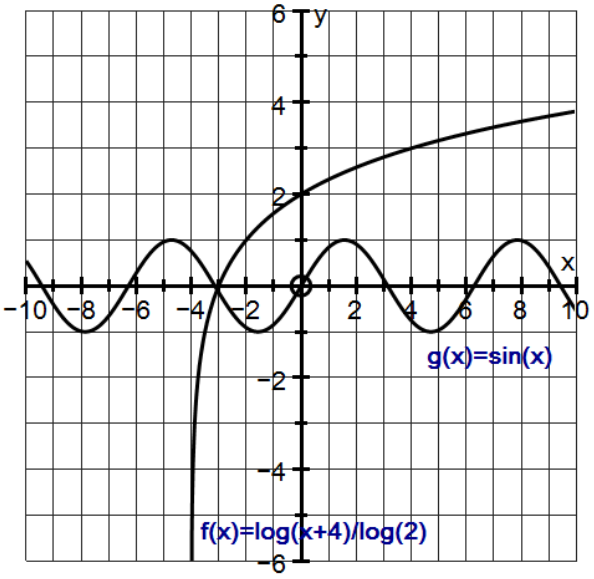
Practice: Product and Quotient of Functions of Different Families

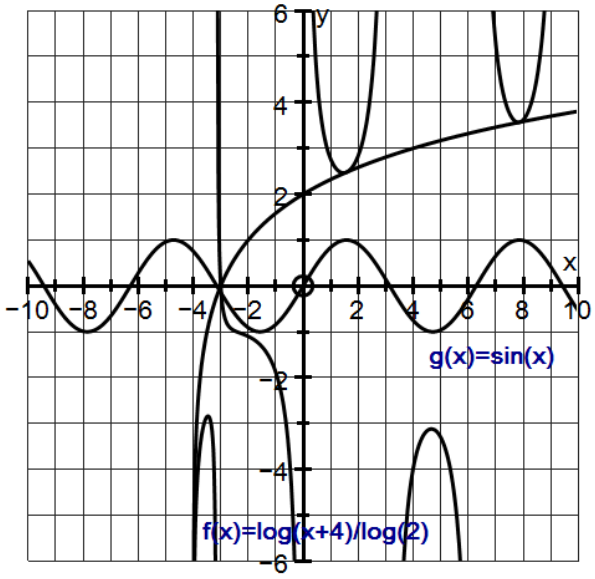
Given $f(x) = \log_2(x+4)$ and $g(x) = \sin(x)$.

a) Complete the table below

b) Sketch $h(x) = f(x)g(x)$ on the grid to the left

Note: The grid to the right illustrates $k(x) = \frac{f(x)}{g(x)}$



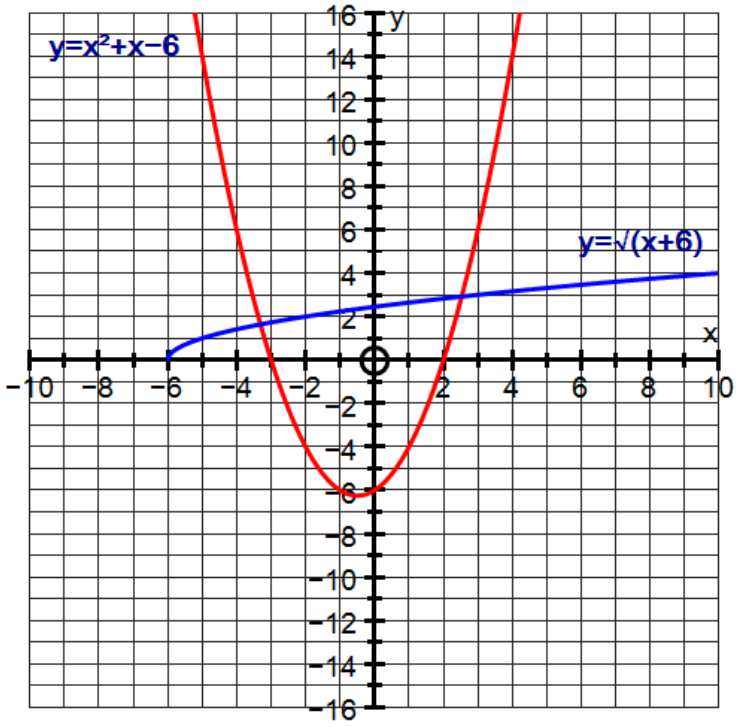


x	$f(x) = \log_2(x+4)$	$g(x) = \sin(x)$	$h(x) = f(x)g(x)$	$k(x) = \frac{f(x)}{g(x)}$

Warm Up

3. Determine the graph of $h(x) = f(x)g(x)$, given the graphs of $f(x) = x^2 + x - 6$ and $g(x) = \sqrt{x+6}$ by creating a table of values

x	$f(x)$	$g(x)$	$h(x)$
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			



$h(x)$ in un-simplified form:_____ Domain of $h(x)$:_____

4. Let $f(x) = mx^2 + 2x + 5$ and $g(x) = 2x^2 - nx - 2$. The functions are combined to form the new function $h(x) = (fg)(x)$. Points $(1, -40)$ and $(-1, 24)$ satisfy the new function. Determine the values of m and n. [m=3, n=4]