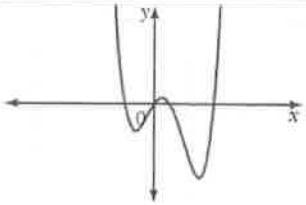
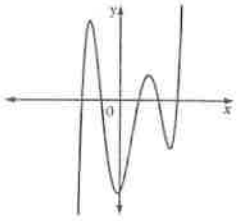
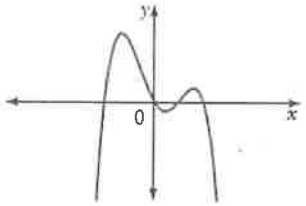
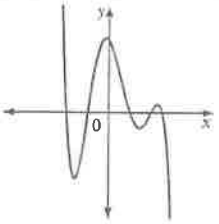
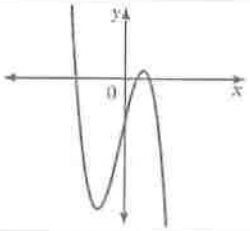
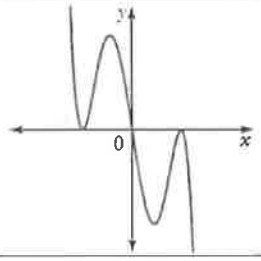
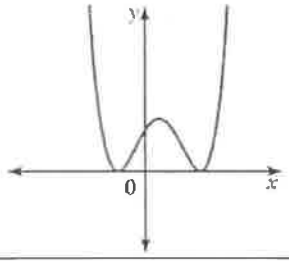
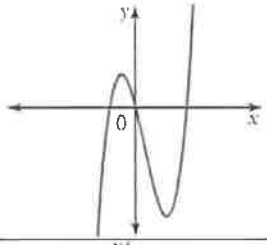
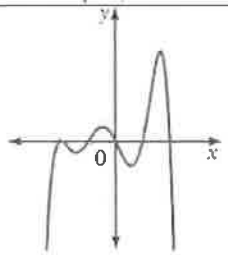
W2 – 1.2 – Characteristics of Polynomial Functions
MHF4U

ANSWERS

1) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q2 to Q1	None	3	4	4
	+	Odd	Q3 to Q1	None	4	5	5
	−	Even	Q3 to Q4	None	3	4	4
	−	Odd	Q2 to Q4	None	4	5	5
	−	Odd	Q2 to Q4	Point	2	3	3

2) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	−	Odd	Q2 to Q4	Point	4	3	5
	+	Even	Q2 to Q1	Line	3	2	4
	+	Odd	Q3 to Q1	Point	2	3	3
	−	Even	Q3 to Q4	None	5	5	6

3) Complete the following table

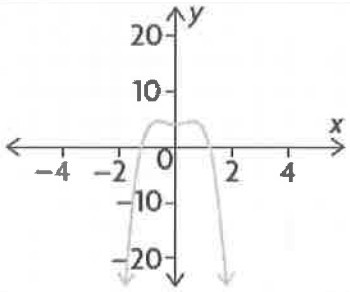
Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	−	Even	Q3 to Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	+	Odd	Q3 to Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	−	Odd	Q2 to Q4	2, 0	3, 2, 1
$h(x) = 2x(x-5)(3x+2)(4x-3)$	4	+	Even	Q2 to Q1	3, 1	4, 3, 2, 1, 0

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

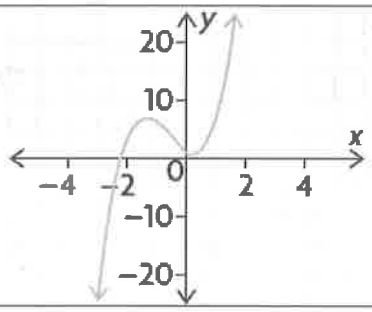
A) $y = 2x^3 - 4x^2 + 3x + 2$ ↘ **B)** $y = -4x^4 + 3x^2 + 4$ ↘ **C)** $y = x^2 + 3x - 5$ ↘

D) $y = x^4 - x^3 - 4x^2 + 5x$ ↘ **E)** $y = -2x^5 + 3x^4 + 6x^3 - 10x^2 + 2x + 5$

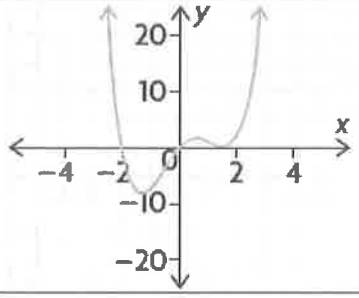
F) $y = 3x^3 + 5x^2 - 3x + 1$ ↘



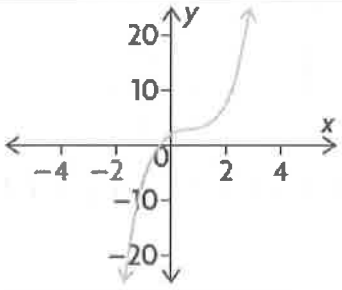
B



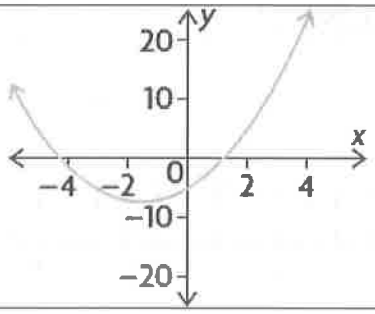
A



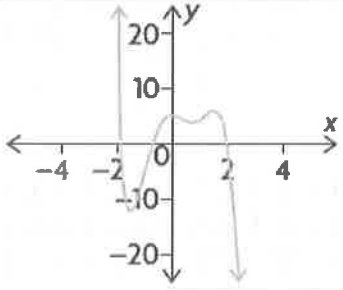
D



F



C



E

5) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) second differences = -8 ↓ degree 2

$-8 = a(2!)$

$-8 = 2a$

$-4 = a$

b) fourth differences = 24 ↓ degree 4

$24 = a(4!)$

$24 = 24a$

$1 = a$

6) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
−3	−45
−2	−16
−1	−3
0	0
1	−1
2	0
3	9
4	32

1st 2nd 3rd

29 → −16 → 6

13 → −10 → 6

3 → −4 → 6

−1 → 2 → 6

1 → 8 → 6

9 → 14 → 6

23

Degree = 3

$6 = a(3!)$

$6 = 6a$

$\boxed{1 = a}$

b)

x	y
−2	−40
−1	12
0	20
1	26
2	48
3	80
4	92
5	30

1st 2nd 3rd 4th

52 → −44 → 42 → −24

8 → −2 → 18 → −24

6 → 16 → −6 → −24

22 → 10 → −30 → −24

32 → −20 → −54 → −24

12 → −74

−62

Degree = 4

$-24 = a(4!)$

$-24 = 24a$

$\boxed{-1 = a}$

7) By analyzing the impact of growing economic conditions, a demographer establishes that the predicted population, P, of a town t years from now can be modelled by the function
$P(t) = 6t^4 - 5t^3 + 200t + 12000$

a) What is the value of the constant finite differences

Finite Differences $= a(n!)$

$= 6(4!)$

$= 144$

b) What is the current population of the town $P(0) = 6(0)^4 - 5(0)^3 + 200(0) + 12000$

$= 12000$

c) What will the population of the town be 10 years from now

$P(10) = 6(10)^4 - 5(10)^3 + 200(10) + 12000$

$= 69000$

ANSWER KEY

1)

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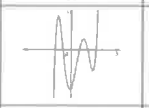
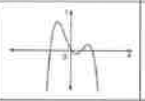
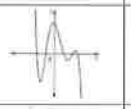
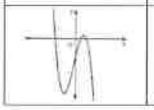
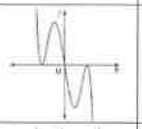
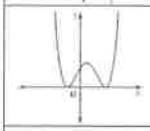
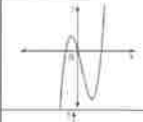
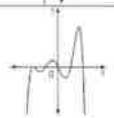
	POS	EVEN	Q2 to Q1	NONE	3	4	4
	POS	ODD	Q3 to Q1	NONE	4	5	5
	NEG	EVEN	Q3 to Q4	NONE	3	4	4
	NEG	ODD	Q2 to Q4	NONE	4	5	5
	NEG	ODD	Q2 to Q4	POINT	2	3	3

2)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	NEG	ODD	Q2 to Q4	Point	4	3	5
	POS	EVEN	Q2 to Q1	Line	3	2	4
	POS	ODD	Q3 to Q1	Point	2	3	3
	NEG	EVEN	Q3 to Q4	None	5	5	6

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4) B F D
A C E

5) a) degree 2, $a = -4$ **b)** degree 4, $a = 1$

6) a) degree 3, $a = 1$ **b)** degree 4, $a = -1$

7) a) 144 **b)** 12 000 **c)** 69 000