W1 – 1.5 Average Rates of Change
MHF4U

SOLUTIONS

1) Calculate the average rate of change for the function $g(x) = 4x^2 - 5x + 1$ over each interval.

a) $2 \le x \le 4$

$m = \frac{g(4) - g(2)}{4-2}$

$= \frac{45-7}{2}$

$= 19$

b) $2 \le x \le 3$

$m = \frac{g(3) - g(2)}{3-2}$

$= \frac{22-7}{1}$

$= 15$

c) $2 \le x \le 2.5$

$m = \frac{g(2.5) - g(2)}{2.5-2}$

$= \frac{13.5-7}{0.5}$

$= 13$

d) $2 \le x \le 2.25$

$m = \frac{g(2.25) - g(2)}{2.25-2}$

$= \frac{10-7}{0.25}$

$= 12$

e) $2 \le x \le 2.1$

$m = \frac{g(2.1) - g(2)}{2.1-2}$

$= \frac{8.14-7}{0.1}$

$= 11.4$

f) $2 \le x \le 2.01$

$m = \frac{g(2.01) - g(2)}{2.01-2}$

$= \frac{7.1104-7}{0.01}$

$= 11.04$

2) An emergency flare is shot into the air. Its height, in meters, above the ground at various times in its flight is given in the following table:

Time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (m)	2.00	15.75	27.00	35.75	42.00	45.75	47.00	45.75	42.00

Determine the average rate of change in the height of the flare during each interval

a) $1.0 \le t \le 2.0$

$m = \frac{h(2) - h(1)}{2-1}$

$m = \frac{42-27}{1}$

$m = 15$ m/s

b) $3.0 \le t \le 4.0$

$m = \frac{h(4) - h(3)}{4-3}$

$= \frac{42-47}{1}$

$= -5$ m/s

3) What is the average rate of change in the values of the function $f(x) = 4x$ from $x = 2$ to $x = 6$? What about from $x = 2$ to $x = 26$? What do your results indicate about $f(x)$?

$2 \leq x \leq 6$

$m = \frac{f(6) - f(2)}{6 - 2}$

$= \frac{24 - 8}{4}$

$= 4$

$2 \leq x \leq 26$

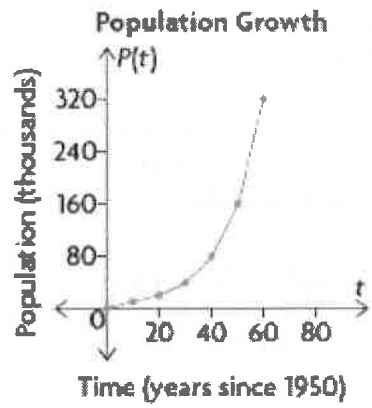
$m = \frac{f(26) - f(2)}{26 - 2}$

$= \frac{104 - 8}{24}$

$= 4$

Average rate of change will always be 4 because it is a linear function with a slope of 4.

4) The population of a city has continued to grow since 1950. The population, P, in thousands, and the time t, in years, since 1950 are given in the table below and in the graph.



Time (years)	0	10	20	30	40	50	60
Population (thousands)	5	10	20	40	80	160	320

a) Calculate the average rate of change in the population for the following intervals of time.

i) $0 \leq t \leq 20$

$m = \frac{P(20) - P(0)}{20 - 0}$

$= \frac{20000 - 5000}{20}$

$= 750$ ppl/year

ii) $20 \leq t \leq 40$

$m = \frac{P(40) - P(20)}{40 - 20}$

$= \frac{80000 - 20000}{20}$

$= 3000$ ppl/year

iii) $40 \leq t \leq 60$

$m = \frac{P(60) - P(40)}{60 - 40}$

$= \frac{320000 - 80000}{20}$

$= 12000$ ppl/year

iv) $0 \leq t \leq 60$

$m = \frac{P(60) - P(0)}{60 - 0}$

$= \frac{320000 - 5000}{60}$

$= 5250$ ppl/year

b) Is the population growth constant?

NO.

5) A company that sells sweatshirts finds that the profit can be modelled by $P(s) = -0.30s^2 + 3.5s + 11.15$, where $P(s)$ is the profit, in thousands of dollars, and s is the number of sweatshirts sold (expressed in thousands).

Calculate the average rate of change in the profit for the following intervals.

i) $1 \leq s \leq 2$

$$m = \frac{P(2) - P(1)}{2 - 1}$$

$$= \frac{16.95 - 14.35}{1}$$

$= \$2.6$ / sweatshirt

ii) $2 \leq s \leq 3$

$$m = \frac{P(3) - P(2)}{3 - 2}$$

$$= \frac{18.95 - 16.95}{1}$$

$= \$2$ / sweatshirt

iii) $3 \leq s \leq 4$

$$m = \frac{P(4) - P(3)}{4 - 3}$$

$$= \frac{20.35 - 18.95}{1}$$

$= \$1.40$ / sweatshirt

iv) $4 \leq s \leq 5$

$$m = \frac{P(5) - P(4)}{5 - 4}$$

$$= \frac{21.15 - 20.35}{1}$$

$= \$0.80$ / sweatshirt

b) As the number of sweatshirts sold increases, what do you notice about the average rate of change in profit on each sweatshirt? What does this mean?

Rate of change is positive but decreasing. Profits are going up but at a decreasing rate.

c) Predict if the rate of change in profit will stay positive. Explain what this means.

$$x\text{-vertex} = \frac{-b}{2a} = \frac{-3.5}{2(-0.3)} \doteq 5.83$$

~~so~~ at around 6000 sweatshirts sold, profits will start to decrease.