Unit 3: Trigonometry
3.5 Proving Trigonometric Identities

Fill in the blanks with the words in the box.

Counter-example	**trig-identity** **identity**	**equal**

- A statement of equality between two expressions that is true for all values of the variables for which the expressions are defined is called an ________________.
- An identity involving trigonometric expressions is called a _________________.
- Our goal is to prove that one side of an expression is ___________ to the other side of the expression.
- A ________________ can be used to show that an equation is not an identity

Strategies for Proving Trig Identities:

- Write everything in terms of sine and cosine
- Be aware of equivalent forms of the fundamental identities, ie: $\sin^2\theta+\cos^2\theta=1$ has an alternative form: $\sin^2\theta = 1-\cos^2\theta$
- Try to rewrite the more complicated side of the equation so that it is identical to the simpler side.
- Usually any factoring or indicated algebraic operations should be performed, ie: $\sin^2 x + 2\sin x + 1 = (\sin x+1)^2$ or

$$\sin^3 x+\cos^3 x = (\sin x+\cos x)(\sin^2 x - \sin x\cos x+\cos^2 x)$$
$$= (\sin x+\cos x)\left(1-\frac{1}{2}\sin 2x\right)$$

- If an expression contains $1\pm\sin x$, $\sec x\pm\tan x$ or $\csc x\pm\cot x$ multiplying both numerator and denominator by $1\mp\sin x$, $\sec x\mp\tan x$ or $\csc x\mp\cot x$ would give $\cos^2 x$,1 or -1.
- If there is more than one angle in the identity, consider using a Compound Identity

Reciprocal Identities

$$\csc A = \frac{1}{\sin A}$$
$$\sec(A) = \frac{1}{\cos(A)}$$
$$\cot(A) = \frac{1}{\tan(A)}$$
$$\tan(A) = \frac{1}{\cot(A)}$$
$$\cot(A) = \frac{1}{\tan(A)}$$

Quotient Identities

$$\tan(A) = \frac{\sin(A)}{\cos(A)}$$
$$\cot(A) = \frac{\cos(A)}{\sin(A)}$$

Pythagorean Identity

$$\sin^2(A)+\cos^2(A)=1$$
$$\tan^2(A)+1=\sec^2(A)$$
$$\cot^2(A)+1=\csc^2(A)$$

Reflection Identities

$$\sin(-A) = -\sin(A)$$
$$\cos(-A) = \cos(A)$$
$$\tan(-A) = -\tan(A)$$

COMPOUND ANGLE IDENTITIES

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

DOUBLE ANGLE IDENTITIES

$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\cos(2A) = \cos^2(A) - \sin^2(A)$$
$$\cos(2A) = 2\cos^2(A) - 1$$
$$\cos(2A) = 1 - 2\sin^2(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

1. Prove the following identity:

a) $\dfrac{1 + \sec(x)}{\tan(x) + \sin(x)} = \csc(x)$

b) $\cot^2(\theta)[\tan^2(\theta) + 1] = \csc^2(\theta)$

c) $\dfrac{\tan^2(\theta)}{\sec^2(\theta)} = \left[1 + \cos(\theta)\right]\left[1 - \cos(\theta)\right]$

d) $\dfrac{\sec(x) + \tan(x)}{\sin(x)} = \dfrac{\csc(x)}{\sec(x) - \tan(x)}$

e) $\dfrac{\sin(x)+\sin(2x)}{1+\cos(x)+\cos(2x)}=\tan(x)$

f) $\dfrac{\sin(2x)}{1+\cos(2x)}=\tan(x)$

g) $\tan(2x)-\sin(2x)=2\tan(2x)\sin^2(x)$

h) $\sin(7x)=\sin(x)\left[\cos^2(3x)-\sin^2(3x)\right]+2\cos(x)\cos(3x)\sin(3x)$

2. If $2\cos^2(x) + 4\sin(x)\cos(x)$ is expressed in the form $A\sin(2x) + B\cos(2x) + C$ where $A, B, C \epsilon R$, determine the values of A, B, and C.

3. Write $2\sin(2x) + \sqrt{12}\cos(2x)$ in the form $y = A\cos(2x - \theta)$ by finding $A > 0$ and $\theta \in [0, 2\pi]$.

3.5 PRACTICE

1. Prove each identity.

a) $\dfrac{\sec(\theta)-1}{1-\cos(\theta)}=\sec(\theta)$

b) $\tan(x)+\cot(x)=\dfrac{2}{\sin(2x)}$

$\dfrac{\sin(t)-\cos(t)}{\cos(t)}+\dfrac{\sin(t)+\cos(t)}{\sin(t)}=\sec(t)\csc(t)$

c) $\dfrac{1+\tan^2(x)}{1+\cot^2(x)}=\dfrac{1-\cos^2(x)}{\cos^2(x)}$

d) $\dfrac{1}{1+\sec(\theta)}+\dfrac{1}{1-\sec(\theta)}=-2\cot^2(\theta)$

e) $\dfrac{1+\sec(x)}{\tan(x)+\sin(x)}=\csc(x)$

f) $\cos(a+b)\cos(a-b)=\cos^2(a)-\sin^2(b)$

g) $\dfrac{\cos(x)-\sin(y)}{\cos(y)-\sin(x)}=\dfrac{\cos(y)+\sin(x)}{\cos(x)+\sin(y)}$

h) $\dfrac{1+\tan(A)}{\sin(A)}-\sec(A)=\csc(A)$

i)

j) $\tan(A)+\cot(A)=\sec^2(A)\cot(A)$

k) $\dfrac{4-\sin^2(2x)}{4\cos^4(x)}=\tan^4(x)+\tan^2(x)+1$

l) $1-\sin(x)\cos(x)=\dfrac{\sin^2(x)}{1+\cot(x)}+\dfrac{\cos^2(x)}{1+\tan(x)}$

m) $\dfrac{\sin(x-y)}{\sin(x)\sin(y)}=\cot(y)-\cot(x)$

n) $\dfrac{\sin(5x)}{\sin(x)}-\dfrac{\cos(5x)}{\cos(x)}=4-8\sin^2(x)$