

4.5 Rates of Change in Trigonometric Functions

(1) Average Rate of Change

$$\text{Average Rate of Change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

(2) Instantaneous Rate of Change

$$\text{Instantaneous Rate of Change} = \frac{f(a+h) - f(a)}{h}, h \rightarrow 0$$

Example #1: Consider the trigonometric function $y = -3\sin\left[\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right]$.

(a) Sketch one cycle of the function.

(b) Determine an interval where the avg. rate of change is:

(i) positive

(ii) negative

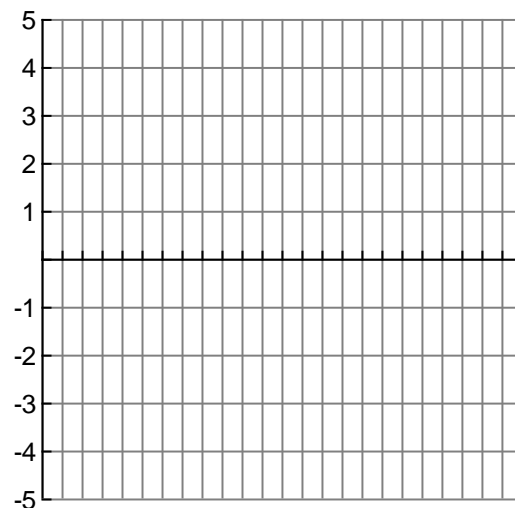
(iii) zero

(c) Determine a point where the inst. rate of change is:

(i) positive

(ii) negative

(iii) zero



(d) Calculate the average rate of change for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.

(e) Describe how the instantaneous rate of change varies over the interval $[0, 4\pi]$.

Example #2: The position of a particle as it moves horizontally is described by the equation $s(t) = 12\sin\left(\frac{\pi t}{90}\right) + 15$, where s is the displacement, in metres, and t is the time, in seconds.

(a) Calculate the average rate of change of $s(t)$ for the following intervals:

(i) 5 s to 10 s

(ii) 9 s to 10 s

(b) Estimate the instantaneous rate of change of $s(t)$ at $t = 10$ s.

(c) What physical quantity does this instantaneous rate of change represent?