W5 – 3.4 Solve Rational Equations and Inequalities
MHF4U

SOLUTIONS

1) Solve each equation algebraically.

a) $\frac{6}{2x-1} = 5$

$(2x-1)\left(\frac{6}{2x-1}\right) = 5(2x-1)$

$6 = 5(2x-1)$

$6 = 10x - 5$

$11 = 10x$

$x = \frac{11}{10}$

b) $\frac{6}{x} = x - 5$

$x\left(\frac{6}{x}\right) = x(x-5)$

$6 = x(x-5)$

$6 = x^2 - 5x$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x-6=0$ $\quad$ $x+1=0$

$x_1 = 6$ $\quad$ $x_2 = -1$

c) $1 = \frac{5}{3x^2-8x+2}$

$(3x^2-8x+2)(1) = \frac{5}{3x^2-8x+2}(3x^2-8x+2)$

$3x^2 - 8x + 2 = 5$

$3x^2 - 8x - 3 = 0$

$3x^2 - 9x + 1x - 3 = 0$

$3x(x-3) + 1(x-3) = 0$

$(x-3)(3x+1) = 0$

$x-3=0$ $\quad$ $3x+1=0$

$x_1 = 3$ $\quad$ $x_2 = -\frac{1}{3}$

d) $\frac{x+5}{x-1} = \frac{x+1}{x-3}$

$(x-3)(x+5) = (x+1)(x-1)$

$x^2 + 2x - 15 = x^2 - 1$

$2x = 14$

$x = 7$

e) $\frac{3}{x+5}+\frac{4}{x}=0$

$$\frac{3}{x+5}=\frac{-4}{x}$$

$$3x=-4(x+5)$$

$$3x=-4x-20$$

$$7x=-20$$

$$\boxed{x=-\frac{20}{7}}$$

f) $2x=5-\frac{3}{x}$

$$x(2x)=x\left(5-\frac{3}{x}\right)$$

$$2x^2=5x-3$$

$$2x^2-5x+3=0$$

$$2x^2-2x-3x+3=0$$

$$2x(x-1)-3(x-1)=0$$

$$(x-1)(2x-3)=0$$

$x-1=0$ $\qquad$ $2x-3=0$

$$\boxed{x_1=1} \qquad \boxed{x_2=\frac{3}{2}}$$

$\frac{2}{1-x}+\frac{3}{x+1}=\frac{1}{x}$

$$\frac{2(x+1)+3(1-x)}{(1-x)(x+1)}=\frac{1}{x}$$

$$\frac{2x+2+3-3x}{1-x^2}=\frac{1}{x}$$

$$\frac{-1x+5}{1-x^2}=\frac{1}{x}$$

$$x(-1x+5)=1(1-x^2)$$

$$-1x^2+5x=1-1x^2$$

$$5x=1$$

$$\boxed{x=\frac{1}{5}}$$

h) $\frac{3}{x-1}+5+\frac{2}{x}=0$

$$x(x-1)\left[\frac{3}{x-1}+5+\frac{2}{x}\right]=0\,(x)(x-1)$$

$$3x+5x(x-1)+2(x-1)=0$$

$$3x+5x^2-5x+2x-2=0$$

$$5x^2=2$$

$$x^2=\frac{2}{5}$$

$$\boxed{x=\pm\sqrt{\frac{2}{5}}}$$

2) Solve each inequality without using technology.

a) $\frac{4}{2x-3} < \frac{1}{x+4}$

$\frac{4}{2x-3}(x+4) - \frac{1}{x+4}(2x-3) < 0$

$\frac{4(x+4)-1(2x-3)}{(2x-3)(x+4)} < 0$

$\frac{4x+16-2x+3}{(2x-3)(x+4)} < 0$

$\frac{2x+19}{(2x-3)(x+4)} < 0$

	$-\infty$	-9.5	-4	1.5	∞
	-10	-5	0	2	
$2x+19$	−	+	+	+	
$2x-3$	−	−	−	+	
$x+4$	−	−	+	+	
overall	−	+	−	+	

zero: $x = -9.5$

restrictions: $x \neq -4, 1.5$

Solution: $x < -9.5$ or $-4 < x < 1.5$

$x \in (-\infty, -9.5) \cup (-4, 1.5)$

b) $\frac{2x+3}{x-3} \geq \frac{6x-5}{3x+1}$

$\frac{2x+3}{x-3}(3x+1) - \frac{6x-5}{3x+1}(x-3) \geq 0$

$\frac{(3x+1)(2x+3)-(6x-5)(x-3)}{(3x+1)(x-3)} \geq 0$

$\frac{6x^2+11x+3-(6x^2-23x+15)}{(3x+1)(x-3)} \geq 0$

$\frac{34x-12}{(3x+1)(x-3)} \geq 0$

$\frac{2(17x-6)}{(3x+1)(x-3)} \geq 0$

	$-\infty$	$-\frac{1}{3}$	$\frac{6}{17}$	3	∞
	-1	0	1	4	
2	+	+	+	+	
$17x-6$	−	−	+	+	
$3x+1$	−	+	+	+	
$x-3$	−	−	−	+	
overall	−	+	−	+	

zero: $\frac{6}{17}$

restrictions: $-\frac{1}{3}, 3$

Solution: $-\frac{1}{3} < x \leq \frac{6}{17}$ or $x > 3$

$x \in (-\frac{1}{3}, \frac{6}{17}] \cup (3, \infty)$

c) $\frac{(x-3)(2x-1)}{(x+4)(x-5)} > 0$

zeros: $\frac{1}{2}, 3$

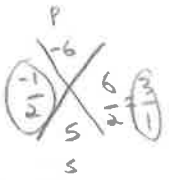
restrictions: $-4, 5$

	$-\infty$	-4	0.5	3	5	∞
	-5	0	1	4	6	
$x-3$	−	−	−	+	+	
$2x-1$	−	−	+	+	+	
$x+4$	−	+	+	+	+	
$x-5$	−	−	−	−	+	
overall	+	−	+	−	+	

Solution: $x < -4$ or $0.5 < x < 3$ or $x > 5$

$x \in (-\infty, -4) \cup (0.5, 3) \cup (5, \infty)$

d) $\frac{2x^2+5x-3}{x^2+5x+4} \leq 0$



$\frac{(2x-1)(x+3)}{(x+4)(x+1)} \leq 0$

zeros: $-3, \frac{1}{2}$

restrictions: $-4, -1$

	$-\infty$	-4	-3	-1	$\frac{1}{2}$	∞
	-5	-3.5	-2	0	1	
$2x-1$	−	−	−	−	+	
$x+3$	−	−	+	+	+	
$x+4$	−	+	+	+	+	
$x+1$	−	−	−	+	+	
overall	+	−	+	−	+	

Solution: $-4 < x \leq -3$ or $-1 < x \leq \frac{1}{2}$

$x \in (-4, -3] \cup (-1, \frac{1}{2}]$

e) $\frac{4}{x-3} < 1$

$\frac{4}{x-3} - 1 < 0$

$\frac{4}{x-3} - \frac{x-3}{x-3} < 0$

$\frac{4-(x-3)}{x-3} < 0$

$\frac{4-x+3}{x-3} < 0$

$\frac{7-x}{x-3} < 0$

	$-\infty$		3		7		∞
		0		4		8	
$7-x$		+		+		−	
$x-3$		−		+		+	
overall		−		+		−	

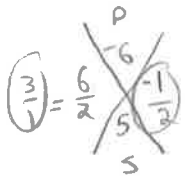
zero: $x = 7$

restriction: $x \neq 3$

solution: $x < 3$ or $x > 7$

$x \in (-\infty, 3) \cup (7, \infty)$

f) $\frac{2x^2+5x-3}{x^2+8x+16} < 0$



$\frac{(2x-1)(x+3)}{(x+4)^2} < 0$

	$-\infty$		-4		-3		$\frac{1}{2}$		∞
		-5		-3.5		0		1	
$2x-1$		−		−		−		+	
$x+3$		−		−		+		+	
$(x+4)^2$		+		+		+		+	
overall		+		+		−		+	

zeros: $x = -3, \frac{1}{2}$

restriction: $x \neq -4$

solution: $-3 < x < \frac{1}{2}$

$x \in (-3, \frac{1}{2})$

g) $\frac{x}{x+3} > \frac{x}{x-1}$

$\frac{x}{x+3} - \frac{x}{x-1} > 0$

$\frac{x(x-1) - x(x+3)}{(x+3)(x-1)} > 0$

$\frac{x^2 - x - x^2 - 3x}{(x+3)(x-1)} > 0$

$\frac{-4x}{(x+3)(x-1)} > 0$

	$-\infty$		-3		0		1		∞
		-4		-1		0.5		2	
$-4x$		+		+		−		−	
$x+3$		−		+		+		+	
$x-1$		−		−		−		+	
overall		+		−		+		−	

zero: $x = 0$

restrictions: $x \neq -3, 1$

solution: $x < -3$ or $0 < x < 1$

$x \in (-\infty, -3) \cup (0, 1)$

h) $\frac{2x+3}{x} > \frac{x+1}{x}$

$\frac{2x+3}{x} - \frac{x+1}{x} > 0$

$\frac{2x+3-(x+1)}{x} > 0$

$\frac{2x+3-x-1}{x} > 0$

$\frac{x+2}{x} > 0$

	$-\infty$		-2		0		∞
		-3		-1		1	
$x+2$		−		+		+	
x		−		−		+	
overall		+		−		+	

zero: $x = -2$

restriction: $x \neq 0$

solution: $x < -2$ or $x > 0$

$x \in (-\infty, -2) \cup (0, \infty)$