

W5 - 5.4 Solve Linear Trigonometric Equations MHF4U

SOLUTIONS

1) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\sin x - \frac{1}{4} = 0$

$$\sin x = \frac{1}{4}$$

$$x_1 = \sin^{-1}\left(\frac{1}{4}\right)$$

$$\approx 0.25$$

$$x_2 = \pi - 0.25$$

$$\approx 2.89$$

$$\sin 0.25 = \sin 2.89 = \frac{1}{4}$$

c) $\tan x - 5 = 0$

$$\tan x = 5$$

$$x_1 = \tan^{-1}(5)$$

$$\approx 1.37$$

$$x_2 = \pi + 1.37$$

$$\approx 4.51$$

$$\tan 1.37 = \tan 4.51 = 5$$

b) $\cos x + 0.75 = 0$

$$\cos x = -0.75$$

$$x_1 = \cos^{-1}(-0.75)$$

$$\approx 2.42$$

$$x_2 = \pi - 2.42$$

$$\approx 0.72$$

$$x_2 = \pi + 0.72$$

$$\approx 3.86$$

$$\cos 0.72 = \cos 3.86 = -0.75$$

d) $\sec x - 4 = 0$

$$\sec x = 4$$

$$\cos x = \frac{1}{4}$$

$$x_1 = \cos^{-1}\left(\frac{1}{4}\right)$$

$$\approx 1.32$$

$$x_2 = 2\pi - 1.32$$

$$x_2 \approx 4.96$$

$$\sec 1.32 = \sec 4.96 = 4$$

e) $3 \cot x + 2 = 0$

$$3 \cot x = -2$$

$$\cot x = -\frac{2}{3}$$

$$\tan x = -\frac{3}{2}$$

$$x_1 = \tan^{-1}\left(-\frac{3}{2}\right)$$

$$\approx -0.98 + 2\pi$$

$$\approx 5.3$$

$$x_2 = \pi - 0.98$$

$$\approx 2.16$$

$$\cot 5.3 = \cot 2.16 = -\frac{2}{3}$$

f) $2 \csc x + 5 = 0$

$$2 \csc x = -5$$

$$\csc x = -\frac{5}{2}$$

$$\sin x = -\frac{2}{5}$$

$$x_1 = \sin^{-1}\left(-\frac{2}{5}\right)$$

$$\approx -0.41 + 2\pi$$

$$\approx 5.87$$

$$x_2 = \pi + 0.41$$

$$\approx 3.55$$

$$\csc 5.87 = \csc 3.55 = -\frac{5}{2}$$

2) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin x + \frac{\sqrt{3}}{2} = 0$ $\sin x = -\frac{\sqrt{3}}{2}$

A from special Δ : $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q3+Q4

$$x_1 = 2\pi - \frac{\pi}{3}$$

$$x_1 = \frac{5\pi}{3}$$

$$x_2 = \pi + \frac{\pi}{3}$$

$$x_2 = \frac{4\pi}{3}$$

c) $\tan x - 1 = 0$

$$\tan x = 1$$

from special Δ : $\tan \frac{\pi}{4} = 1$

place in Q1+Q3

$$x_1 = \frac{\pi}{4}$$

$$x_2 = \pi + \frac{\pi}{4}$$

$$x_2 = \frac{5\pi}{4}$$

b) $\cos x - 0.5 = 0$ $\cos x = \frac{1}{2}$

from special Δ : $\cos \frac{\pi}{3} = \frac{1}{2}$

place $\frac{\pi}{3}$ in Q1+Q4

$$x_1 = \frac{\pi}{3}$$

$$x_2 = 2\pi - \frac{\pi}{3}$$

$$x_2 = \frac{5\pi}{3}$$

d) $\cot x + 1 = 0$

$$\cot x = -1$$

$$\tan x = -1$$

from special Δ : $\tan \frac{\pi}{4} = 1$

place $\frac{\pi}{4}$ in Q2+Q4

$$x_1 = \pi - \frac{\pi}{4}$$

$$x_1 = \frac{3\pi}{4}$$

$$x_2 = 2\pi - \frac{\pi}{4}$$

$$x_2 = \frac{7\pi}{4}$$

3) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

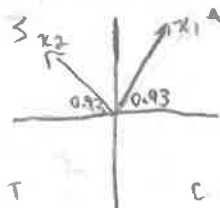
a) $\sin^2 x - 0.64 = 0$

$$\sin^2 x = 0.64$$

$$\sin x = \pm \sqrt{0.64}$$

$$\sin x = \pm 0.8$$

$$\sin x = 0.8$$



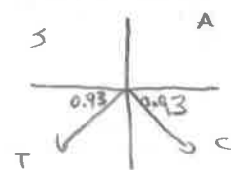
$$x_1 = \sin^{-1}(0.8)$$

$$x_1 = 0.93$$

$$x_2 = \pi - 0.93$$

$$x_2 = 2.21$$

$$\sin x = -0.8$$



$$x_3 = \sin^{-1}(-0.8)$$

$$x_3 = -0.93 + 2\pi$$

$$x_3 = 5.36$$

$$x_4 = \pi + 0.93$$

$$x_4 = 4.07$$

b) $\cos^2 x - \frac{4}{9} = 0$

$$\cos^2 x = \frac{4}{9}$$

$$\cos x = \pm \sqrt{\frac{4}{9}}$$

$$\cos x = \pm \frac{2}{3}$$

$$\cos x = \frac{2}{3}$$



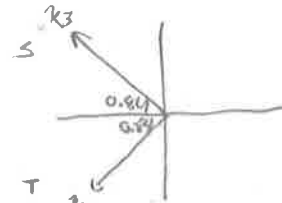
$$x_1 = \cos^{-1}\left(\frac{2}{3}\right)$$

$$x_1 = 0.84$$

$$x_2 = 2\pi - 0.84$$

$$x_2 = 5.44$$

$$\cos x = -\frac{2}{3}$$



$$x_3 = \cos^{-1}\left(-\frac{2}{3}\right)$$

$$x_3 = 2.30$$

$$x_4 = \pi + 0.84$$

$$x_4 = 3.98$$

c) $\tan^2 x - 1.44 = 0$

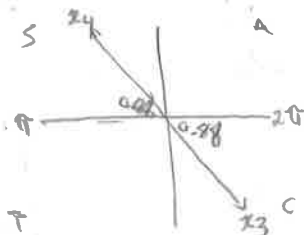
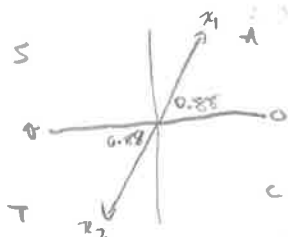
$\tan^2 x = 1.44$

$\tan x = \pm \sqrt{1.44}$

$\tan x = \pm 1.2$

$\tan x = 1.2$

$\tan x = -1.2$



$x_1 = \tan^{-1}(1.2)$

$x_3 = \tan^{-1}(-1.2)$

$x_1 = 0.88$

$x_3 = -0.88 + 2\pi$

$x_2 = \pi + 0.88$

$x_3 = 5.4$

$x_2 = 4.02$

$x_4 = \pi - 0.88$

$x_4 = 2.26$

d) $\sec^2 x - 2.5 = 0$

$\sec^2 x = 2.5$

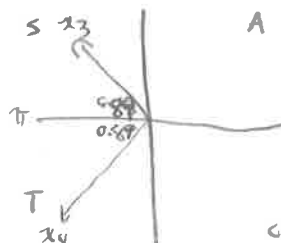
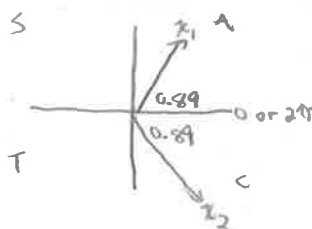
$\cos^2 x = \frac{1}{2.5}$

$\cos x = \pm \sqrt{\frac{1}{2.5}}$

$\cos x = \pm 0.63$

$\cos x = 0.63$

$\cos x = -0.63$



$x_1 = \cos^{-1}(0.63)$

$x_3 = \cos^{-1}(-0.63)$

$x_1 = 0.89$

$x_3 = 2.25$

$x_2 = 2\pi - 0.89$

$x_4 = \pi + 0.89$

$x_2 = 5.39$

$x_4 = 4.03$

Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin^2 x - \frac{1}{4} = 0$

$\sin^2 x = \frac{1}{4}$

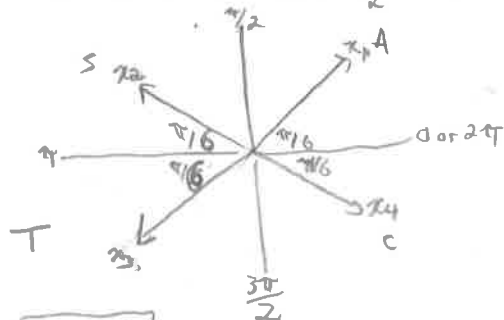
$\sin x = \pm \sqrt{\frac{1}{4}}$

$\sin x = \pm \frac{1}{2}$

from special Δ ; $\sin \frac{\pi}{6} = \frac{1}{2}$

Place in Q1+Q2 for $\sin x = \frac{1}{2}$

Place in Q3+Q4 for $\sin x = -\frac{1}{2}$



$x_1 = \frac{\pi}{6}$

$x_3 = \pi + \frac{\pi}{6}$

$x_2 = \pi - \frac{\pi}{6}$

$x_3 = \frac{7\pi}{6}$

$x_2 = \frac{5\pi}{6}$

$x_4 = 2\pi - \frac{\pi}{6}$

$x_4 = \frac{11\pi}{6}$

b) $\cos^2 x - \frac{3}{4} = 0$

$\cos^2 x = \frac{3}{4}$

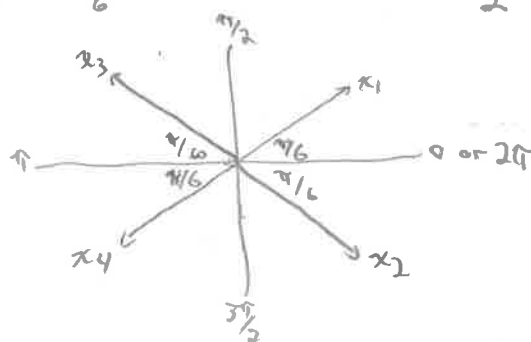
$\cos x = \pm \sqrt{\frac{3}{4}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

from special Δ ; $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{6}$ in Q1+Q4 for $\cos x = \frac{\sqrt{3}}{2}$

Place $\frac{5\pi}{6}$ in Q2+Q3 for $\cos x = -\frac{\sqrt{3}}{2}$



$x_1 = \frac{\pi}{6}$

$x_3 = \pi - \frac{\pi}{6}$

$x_2 = 2\pi - \frac{\pi}{6}$

$x_3 = \frac{5\pi}{6}$

$x_2 = \frac{11\pi}{6}$

$x_4 = \pi + \frac{\pi}{6}$

$x_4 = \frac{7\pi}{6}$

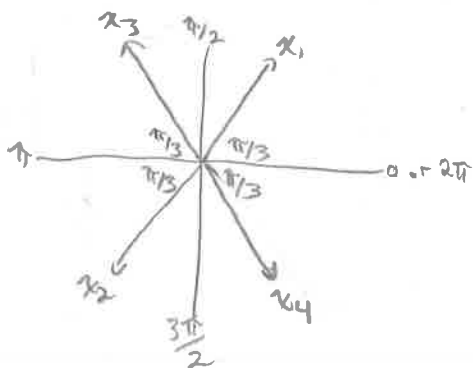
$$c) \tan^2 x - 3 = 0 \quad \tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$

From special Δ ; $\tan \frac{\pi}{3} = \sqrt{3}$

place $\frac{\pi}{3}$ in Q1+Q3 for $\tan x = \sqrt{3}$

place $\frac{\pi}{3}$ in Q2+Q4 for $\tan x = -\sqrt{3}$



$$x_1 = \frac{\pi}{3}$$

$$x_3 = \pi - \frac{\pi}{3}$$

$$x_2 = \pi + \frac{\pi}{3}$$

$$x_3 = \frac{2\pi}{3}$$

$$x_2 = \frac{4\pi}{3}$$

$$x_4 = 2\pi - \frac{\pi}{3}$$

$$x_4 = \frac{5\pi}{3}$$

5) Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$.

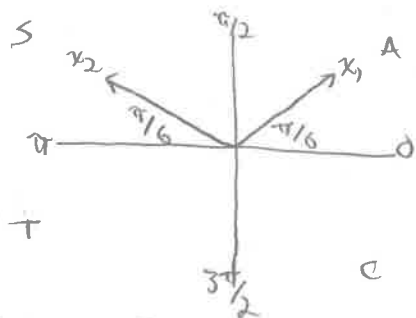
$$a) 3 \sin x = \sin x + 1$$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

From special Δ ; $\sin \frac{\pi}{6} = \frac{1}{2}$

Place in Q1+Q2



$$x_1 = \frac{\pi}{6}$$

$$x_2 = \pi - \frac{\pi}{6}$$

$$x_2 = \frac{5\pi}{6}$$

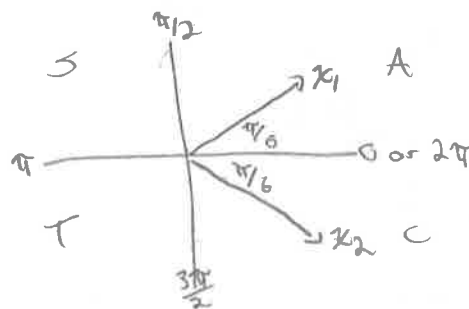
$$b) 5 \cos x - \sqrt{3} = 3 \cos x$$

$$2 \cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

From special Δ ; $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Place in Q1+Q4



$$x_1 = \frac{\pi}{6}$$

$$x_2 = 2\pi - \frac{\pi}{6}$$

$$x_2 = \frac{11\pi}{6}$$

$$d) 3 \csc^2 x - 4 = 0$$

$$\csc^2 x = \frac{4}{3}$$

$$\sin^2 x = \frac{3}{4}$$

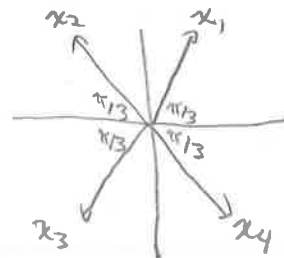
$$\sin x = \pm \sqrt{\frac{3}{4}}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

From special Δ ; $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q1+Q2 for $\sin x = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q3+Q4 for $\sin x = -\frac{\sqrt{3}}{2}$



$$x_1 = \frac{\pi}{3}$$

$$x_3 = \frac{4\pi}{3}$$

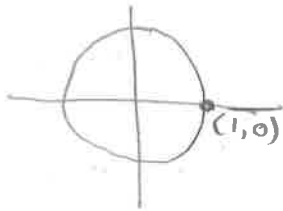
$$x_2 = \frac{2\pi}{3}$$

$$x_4 = \frac{5\pi}{3}$$

c) $7 \sec x = 7 \quad \sec x = 1$

$\cos x = 1$

use unit circle
where each point is $(\cos x, \sin x)$



$x_1 = 0$

$x_2 = 2\pi$

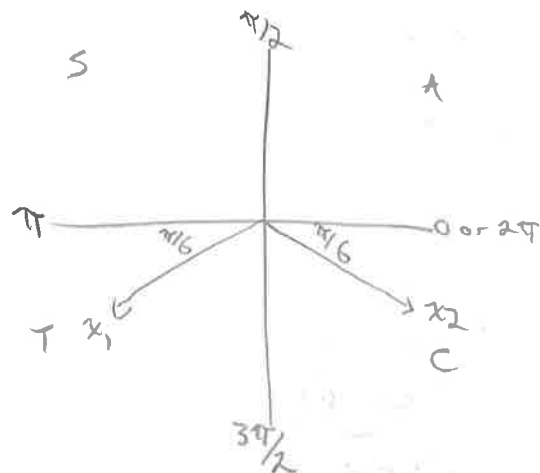
d) $2 \csc x + 17 = 15 + \csc x$

$\csc x = -2$

$\sin x = -\frac{1}{2}$

From special 4: $\sin \frac{\pi}{6} = \frac{1}{2}$

Place $\frac{\pi}{6}$ in Q3 + Q4



$x_1 = \pi + \frac{\pi}{6}$

$x_1 = \frac{7\pi}{6}$

$x_2 = 2\pi - \frac{\pi}{6}$

$x_2 = \frac{11\pi}{6}$