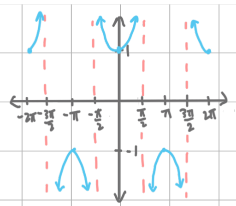
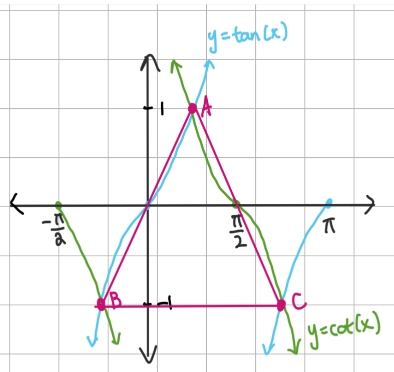
Unit 4- Review

1. State the intervals where the graph of $y = \sec(x), -2\pi \le x \le 2\pi$ is increasing.



$x \in \left(-2\pi, -\frac{3\pi}{2}\right) \cup \left(-\frac{3\pi}{2}, -\pi\right) \cup \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

2. Sketch the graphs of $y = \tan(x)$ and $y = \cot(x), -\frac{\pi}{2} \le x \le \pi$. Using the letters A,B and C, label the three intersections points of the two functions .Determine the area and perimeter of ΔABC.



P.O.I. occur when $\tan(x) = \cot(x)$:

$\tan(x) = \frac{1}{\tan(x)}$

$\tan^2(x) = 1$

$\tan(x) = \pm 1$

R.A.A $= \frac{\pi}{4}$

$A = \left(\frac{\pi}{4}, 1\right)$

$B = \left(-\frac{\pi}{4}, -1\right)$

$C = \left(\frac{3\pi}{4}, -1\right)$

Area: $d_{BC} = \sqrt{\left(\frac{3\pi}{4} - \left(-\frac{\pi}{4}\right)\right)^2 + (-1-(-1))^2}$

$= \sqrt{\pi^2}$

$= \pi$

By inspection, height is 2 units.

$\therefore A = \frac{1}{2}(\text{base})(\text{height})$

$A = \frac{1}{2}(d_{BC})(\text{height})$

$A = \frac{1}{2}(\pi)(2)$

$\boxed{A = \pi \text{ units}^2}$

Perimeter:

$d_{AB} = \sqrt{\left(-\frac{\pi}{4} - \frac{\pi}{4}\right)^2 + (-1-1)^2}$

$= \sqrt{\left(-\frac{\pi}{2}\right)^2 + (-2)^2}$

$= \sqrt{\frac{\pi^2}{4} + 4}$

$= \sqrt{\frac{\pi^2 + 16}{4}}$

$= \frac{\sqrt{\pi^2 + 16}}{2}$ units

$d_{AC} = \sqrt{\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)^2 + (-1-1)^2}$

$= \sqrt{\left(\frac{\pi}{2}\right)^2 + 4}$

$= \frac{\sqrt{\pi^2 + 16}}{2}$ units

$d_{BC} = \pi$

$\therefore P = \frac{\sqrt{\pi^2+16}}{2} + \frac{\sqrt{\pi^2+16}}{2} + \pi$

$P = \frac{2\sqrt{\pi^2+16}}{2} + \frac{2\pi}{2}$

$P = \frac{2\left(\sqrt{\pi^2+16} + \pi\right)}{2}$

$\boxed{P = \sqrt{\pi^2+16} + \pi \text{ units}}$

a p d c

3. Identify the amplitude, period, phase shift, and vertical displacement for each of the following:

a) $y = 6\cos\left[12\left(x - 30^\circ\right)\right] + 3$

$a = 6$
$p = \frac{360^\circ}{k} = \frac{360^\circ}{12} = 30^\circ$
$d = 30^\circ$
$c = 3$

b) $y = -2 + 3\sin\left(x + \frac{\pi}{4}\right)$

$a = 3$
$p = 2\pi$
$d = -\frac{\pi}{4}$
$c = -2$

c) $y = -4\cos\left(2x - \frac{\pi}{3}\right) - 2$

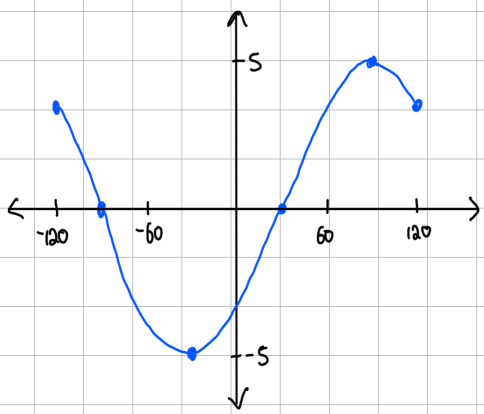
$y = -4\cos\left[2\left(x - \frac{\pi}{6}\right)\right] - 2$

$a = 4$
$p = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$
$d = \frac{\pi}{6}$
$c = -2$

4. Sketch $y = 5\sin\left[\frac{3}{2}\left(x - 30^\circ\right)\right], -120^\circ \le x \le 120^\circ$.

$(x, y) \to \left(\frac{2}{3}x + 30^\circ, 5y\right)$

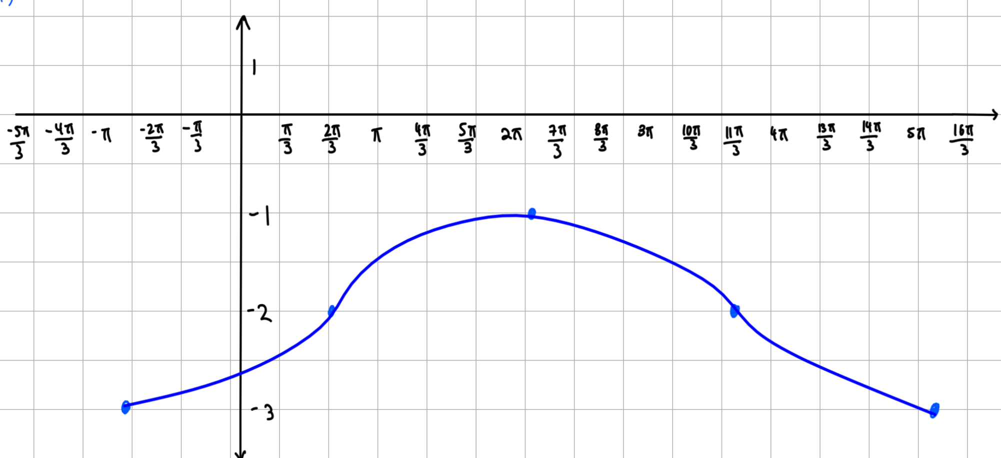
(x,y)	$\left(\frac{2}{3}x + 30^\circ, 5y\right)$
$(0,0)$	$(30,0)$
$(90,1)$	$(90,5)$
$(180,0)$	$(150,0)$
$(270,-1)$	$(210,-5)$
$(360,0)$	$(270,0)$



5. Sketch one period of the function $f(x) = -\cos\left[\frac{1}{3}\left(x + \frac{5\pi}{6}\right)\right] - 2$.

$(x, y) \to \left(3x - \frac{5\pi}{6}, -y-2\right)$

(x,y)	$\left(3x - \frac{5\pi}{6}, -y-2\right)$
$(0,1)$	$\left(-\frac{5\pi}{6}, -3\right)$
$\left(\frac{\pi}{2},0\right)$	$\left(\frac{2\pi}{3}, -2\right)$
$(\pi,-1)$	$\left(\frac{13\pi}{6}, -1\right)$
$\left(\frac{3\pi}{2},0\right)$	$\left(\frac{11\pi}{3}, -2\right)$
$(2\pi,1)$	$\left(\frac{31\pi}{6}, -3\right)$

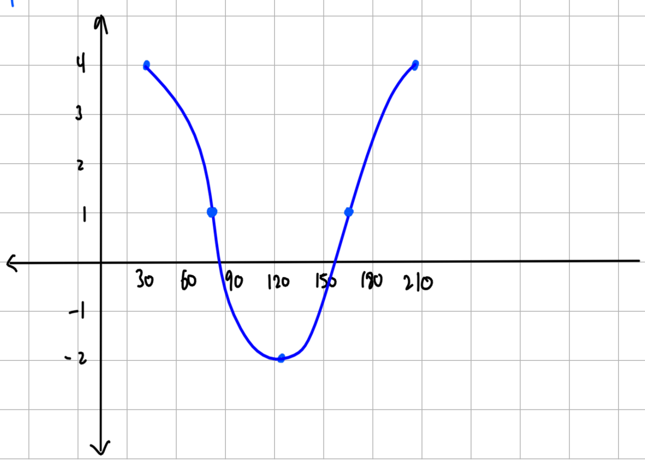


6. Sketch one period of the function $f(x) = 3\cos\left(2x - 60^\circ\right) + 1$.

$f(x) = 3\cos\left[2\left(x - 30^\circ\right)\right] + 1$

$(x, y) \longrightarrow \left(\frac{1}{2}x + 30, 3y + 1\right)$

(x,y)	$\left(\frac{1}{2}x + 30, 3y + 1\right)$
$(0,1)$	$(30,4)$
$(90,0)$	$(75,1)$
$(180,-1)$	$(120,-2)$
$(270,0)$	$(165,1)$
$(360,1)$	$(210,4)$

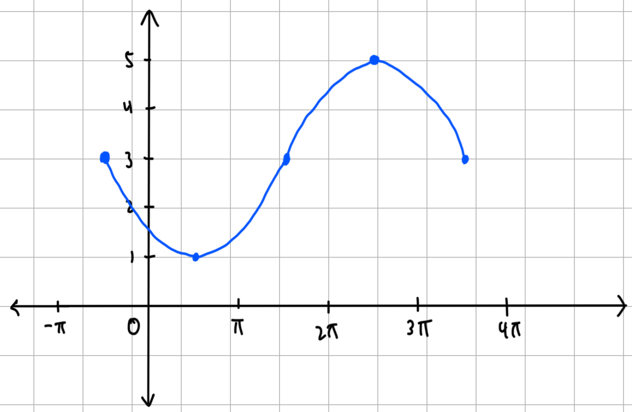


7. Sketch the graph of $y = -2\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)+3$.

$y = -2\sin\left[\frac{1}{2}\left(x+\frac{\pi}{2}\right)\right]+3$

$(x,y) \rightarrow \left(2x-\frac{\pi}{2}, -2y+3\right)$

(x,y)	$\left(2x-\frac{\pi}{2}, -2y+3\right)$
$(0,0)$	$\left(-\frac{\pi}{2}, 3\right)$
$\left(\frac{\pi}{2}, 1\right)$	$\left(\frac{\pi}{2}, 1\right)$
$(\pi, 0)$	$\left(\frac{3\pi}{2}, 3\right)$
$\left(\frac{3\pi}{2}, -1\right)$	$\left(\frac{5\pi}{2}, 5\right)$
$(2\pi, 0)$	$\left(\frac{7\pi}{2}, 3\right)$



8. a) Determine a sine function that is defined for all x≥0 and has its first minimum at (π/3,3) and its first maximum at (4π/3,9).

ANSWERS MAY VARY.

$a = \frac{max-min}{2} = \frac{9-3}{2} = 3$

$\text{Period} = \left(\frac{4\pi}{3}-\frac{\pi}{3}\right)\times 2 = 2\pi$

$k = \frac{2\pi}{p} = \frac{2\pi}{2\pi} = 1$

$d = \frac{4\pi}{3} - \frac{\pi}{2} = \frac{5\pi}{6}$ (for + sine)

OR

$d = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$ (for − sine)

$c = \frac{max+min}{2} = \frac{9+3}{2} = 6$

$\therefore y = 3\sin\left(x-\frac{5\pi}{6}\right)+6$

OR

$y = -3\sin\left(x+\frac{\pi}{6}\right)+6$

b) State an equivalent cosine function for part a).

ANSWERS MAY VARY

$\frac{p}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

$y = 3\cos\left(x-\frac{5\pi}{6}-\frac{\pi}{2}\right)+6$

$\therefore y = 3\cos\left(x-\frac{4\pi}{3}\right)+6$

OR

$y = -3\cos\left(x+\frac{\pi}{6}-\frac{\pi}{2}\right)+6$

$\therefore y = -3\cos\left(x-\frac{\pi}{3}\right)+6$

9. a) Determine a sinusoidal function f(x) that
- has a maximum of 100;
- has a minimum of 20;
- a period of 30;
- has the point (15,60) on its curve; and
- for x≥0, reaches its first maximum before its first minimum.

ANSWERS MAY VARY.

$a = \frac{max-min}{2} = \frac{100-20}{2} = 40$

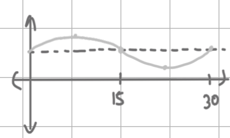
$c = \frac{max+min}{2} = \frac{100+20}{2} = 60$

$k = \frac{2\pi}{p} = \frac{2\pi}{30} = \frac{\pi}{15}$

The axis of the curve is $y=60 \Rightarrow$ the point (15,60) is on the axis of the curve

Since 15 is half the period, we can consider (0,60) as the first point on the cycle and (30,60) as the last point on the cycle.

$\Rightarrow d = 0$



$\boxed{\therefore f(x) = 40\sin\left(\frac{\pi}{15}x\right)+60}$

b) Use your function from part a) to determine the first value of x,x≥0 such that f(x)=80.

$80 = 40\sin\left(\frac{\pi}{15}x\right) + 60$

$20 = 40\sin\left(\frac{\pi}{15}x\right)$

$\frac{1}{2} = \sin\left(\frac{\pi}{15}x\right)$

Let $A = \frac{\pi}{15}x$: $\frac{1}{2} = \sin A$

R.A.A $= \frac{\pi}{6}$

$A = \frac{\pi}{6}$ | $A = \pi - \frac{\pi}{6}$

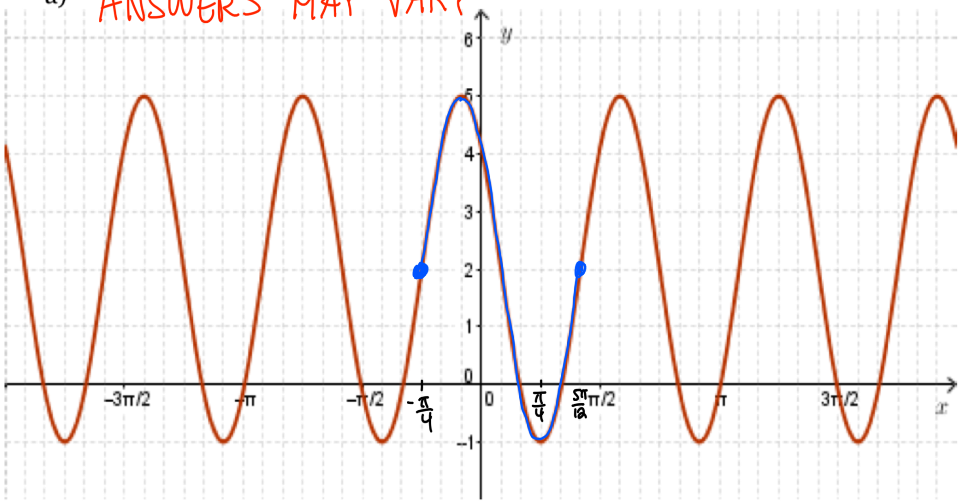
$\frac{\pi}{15}x = \frac{\pi}{6}$ | $A = \frac{5\pi}{6}$

$\boxed{x = \frac{5}{2}}$

↳ inadmissible since we're looking for the first value

10. Determine a sinusoidal function that could represent the graph drawn below.

a) ANSWERS MAY VARY



$a = \frac{\text{max} - \text{min}}{2} = \frac{5-(-1)}{2} = 3$

$\text{period} = \frac{5\pi}{12} - \left(-\frac{\pi}{4}\right) = \frac{2\pi}{3}$

$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{2\pi/3} = 3$

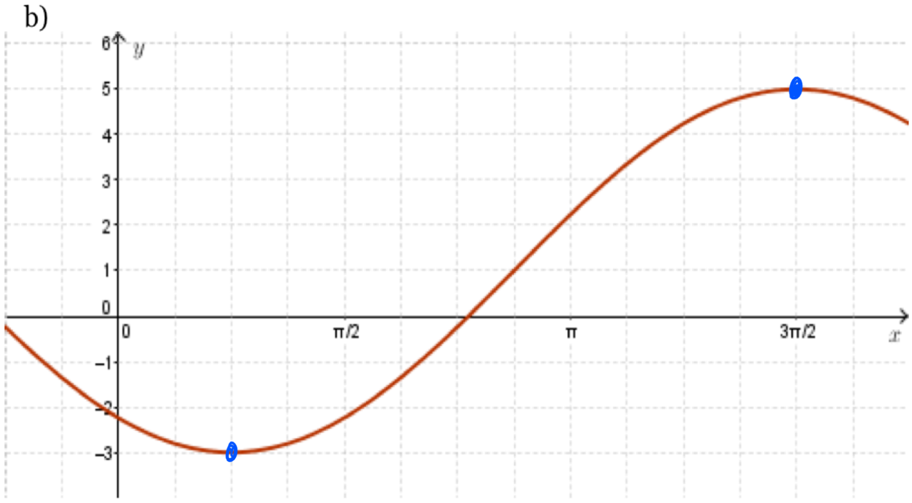
$d = -\pi/4$

$c = \frac{\text{max} + \text{min}}{2} = \frac{5+(-1)}{2} = 2$

$\boxed{\therefore y = 3\sin\left[3\left(x + \frac{\pi}{4}\right)\right] + 2}$

OR $y = 3\cos\left[3\left(x + \frac{\pi}{12}\right)\right] + 2$

b)



$a = \frac{\text{max} - \text{min}}{2} = \frac{5-(-3)}{2} = 4$

$\text{period} = \left(\frac{3\pi}{2} - \frac{\pi}{4}\right) \times 2 = \frac{5\pi}{2}$

$k = \frac{2\pi}{p} = \frac{2\pi}{5\pi/2} = \frac{4}{5}$

$d = \frac{\pi}{4}$

$c = \frac{\text{max} + \text{min}}{2} = \frac{5+(-3)}{2} = 1$

$\boxed{\therefore y = -4\cos\left[\frac{4}{5}\left(x - \frac{\pi}{4}\right)\right] + 1}$

OR $y = 4\sin\left[\frac{4}{5}\left(x - \frac{7\pi}{8}\right)\right] + 1$

$y = 4\cos\left[\frac{4}{5}\left(x - \frac{3\pi}{2}\right)\right] + 1$

11. For the function $y=-3\cos\left(2x+\frac{\pi}{2}\right)+6$:

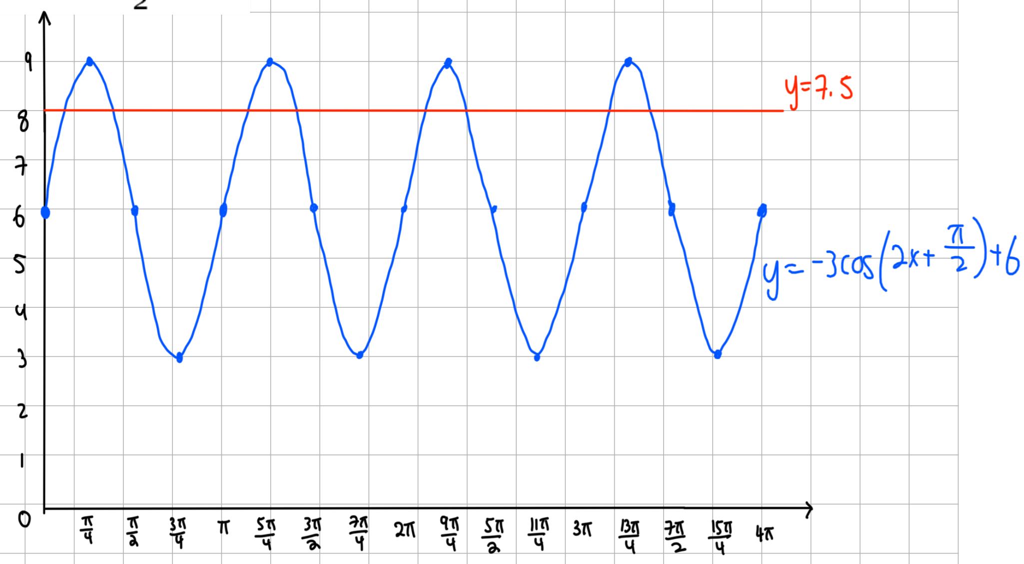
a) Graph four periods of the function for $x\geq 0$.

b) On your graph, sketch the line $y=\frac{15}{2}$.

$y=-3\cos\left[2\left(x+\frac{\pi}{4}\right)\right]+6$

$(x,y)\rightarrow\left(\frac{1}{2}x-\frac{\pi}{4}\ ,\ -3y+6\right)$

(x,y)	$\left(\frac{1}{2}x-\frac{\pi}{4}\ ,\ -3y+6\right)$
$(0,1)$	$\left(-\frac{\pi}{4},3\right)$
$\left(\frac{\pi}{2},0\right)$	$(0,6)$
$(\pi,-1)$	$\left(\frac{\pi}{4},9\right)$
$\left(\frac{3\pi}{2},0\right)$	$\left(\frac{\pi}{2},6\right)$
$(2\pi,1)$	$\left(\frac{3\pi}{4},3\right)$



c) Determine all solutions to $-3\cos\left(2x+\frac{\pi}{2}\right)+6=\frac{15}{2}$, in the interval $0\leq x\leq 4\pi$.

$-3\cos\left(2x+\frac{\pi}{2}\right)=\frac{3}{2}$

$\cos\left(2x+\frac{\pi}{2}\right)=-\frac{1}{2}$

Let $A=2x+\frac{\pi}{2}$: $\cos A=-\frac{1}{2}$

R.A.A $=\frac{\pi}{3}$

$A=\pi-\frac{\pi}{3}$ $\qquad$ $A=\pi+\frac{\pi}{3}$

$A=\frac{2\pi}{3}$ $\qquad$ $A=\frac{4\pi}{3}$

$2x+\frac{\pi}{2}=\frac{2\pi}{3}$ $\qquad$ $2x+\frac{\pi}{2}=\frac{4\pi}{3}$

$2x=\frac{\pi}{6}$ $\qquad$ $2x=\frac{5\pi}{6}$

$x_1=\frac{\pi}{12}$ $\qquad$ $x_2=\frac{5\pi}{12}$

in the first cycle

$x_3=\frac{\pi}{12}+\text{period}$
$=\frac{\pi}{12}+\pi$
$=\frac{13\pi}{12}$

$x_4=\frac{5\pi}{12}+\text{period}$
$=\frac{5\pi}{12}+\pi$
$=\frac{17\pi}{12}$

$x_5=\frac{\pi}{12}+2\pi$
$=\frac{25\pi}{12}$

$x_6=\frac{5\pi}{12}+2\pi$
$=\frac{29\pi}{12}$

$x_7=\frac{\pi}{12}+3\pi$
$=\frac{37\pi}{12}$

$x_8=\frac{5\pi}{12}+3\pi$
$=\frac{41\pi}{12}$

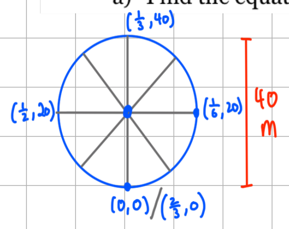
$\therefore$ The solutions are $x\in\left\{\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12},\frac{25\pi}{12},\frac{29\pi}{12},\frac{37\pi}{12},\frac{41\pi}{12}\right\}$

d) Using the graph, verify that all of the solutions in the interval $0\leq x\leq 4\pi$ have been determined in part c).

We can tell from the graph that $y=\frac{15}{2}$ and $y=-3\cos\left(2x+\frac{\pi}{2}\right)+6$ intercepts 8 times in the interval $[0,4\pi]$ and they match the values found algebraically.

12. Ashley is riding a Ferris wheel that has a diameter of 40 metres. The wheel revolves at a rate of 1.5 revolutions per minute. If Ashley's height above the ground is shown by h after t minutes

a) Find the equation of h(t).



max = 40

min = 0

period = $\frac{2}{3}$

$a = \frac{40-0}{2} = 20$

$k = \frac{2\pi}{2/3} = 3\pi$

$d = 0$

$c = \frac{40+0}{2} = 20$

1.5 revolutions / min

= 1 revolution / $\frac{2}{3}$ mins

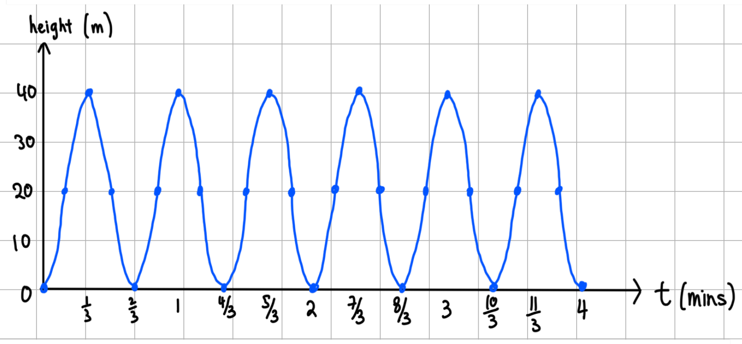
$\boxed{\therefore h(t) = -20\cos(3\pi t) + 20}$

b) What is Ashley's maximum height above the ground while she is riding the Ferris wheel? 40 m

c) If Ashley gets on the ride at its lowest point, how high above the ground will she be to start the ride? 0 m

d) How many times does Ashley go around the Ferris wheel in four minutes? $1.5 \times 4 = 6$ times

e) Draw a sketch of the rider's height above the ground at any time during the first four minutes.



f) How long after she starts riding will her height be 31 metres above the ground?

$31 = -20\cos(3\pi t) + 20$

$-\frac{11}{20} = \cos(3\pi t)$

R.A.A $= \cos^{-1}\left(\frac{11}{20}\right) \doteq 0.988$

$3\pi t \doteq \pi - 0.988$ $\qquad$ $3\pi t \doteq \pi + 0.988$

$3\pi t \doteq 2.154$ $\qquad$ $3\pi t \doteq 4.130$

$t_1 \doteq 0.23$ $\qquad$ $t_2 \doteq 0.44$

$\therefore$ She will be 31 m above the ground after 0.23 minutes

g) In the four minutes that she spends riding the Ferris wheel, what is the total amount of time that Ashley's height is above 31 metres?

$t_2 - t_1 = 0.44 - 0.23 = 0.21$

↳ Ashley is above 31 m for 0.21 minutes in the first cycle.

$0.21 \times 6 = 1.26$ mins.

$\therefore$ In the 4 minutes, Ashley's height is above 31 m for a total of 1.26 minutes

13. The minimum depth, d (in metres), of water in a harbour, t hours after midnight, can be approximated by the function $d(t)=5\cos(0.5t)+12$, where $0\le t\le 24$.

a) Determine the maximum and minimum depths of water in the harbour. max $= 12+5=17$ m ; min $= 12-5=7$ m

b) Determine the period of the depth function. period $= \frac{2\pi}{0.5} = 4\pi \doteq 12.566$ hours $=$ 12 hours and 34 mins.

c) What is the depth of water, to the nearest tenth of a metre, at 2:00 AM?

$d(2) = 5\cos(0.5(2))+12$

$= 5\cos(1)+12$

$\doteq 14.7$ m

∴ At 2 AM, the depth of water is about 14.7 m

d) A ship, which requires a minimum depth of 8.5 metres, is docked at midnight. By what time, to the nearest minute, must it leave in order to prevent being grounded?

$8.5 = 5\cos(0.5t)+12$

$-0.7 = \cos(0.5t)$

Let $A=0.5t$: $-0.7=\cos A$

R.A.A $= \cos^{-1}(0.7) \doteq 0.795$

$A = \pi - 0.795$ $\quad$ $A = \pi + 0.795$

$0.5t = 2.347$ $\quad$ $0.5t = 3.937$

$t_1 = 4.694$ $\quad$ $t_2 = 7.874$

first time the depth is 8.5 m after midnight

∴ The ship must leave at approximately 4:41 AM.

e) What is the next time, to the nearest minute, that the ship can return to the harbour?

As calculated in d), $t_2 = 7.874$

∴ The ship can return to the harbour again at approximately 7:52 AM

14. Determine the period and equation of vertical asymptotes of $y=-2\tan(50x)$.

$y = -2\left[\frac{\sin(50x)}{\cos(50x)}\right]$

$\boxed{\text{period} = \frac{\pi}{50}}$

V.A : $\cos(50x)=0$

$50x = (2k+1)\frac{\pi}{2}$

$50x = k\pi + \frac{\pi}{2}$

$\boxed{x = \frac{k\pi}{50} + \frac{\pi}{100},\ k\in\mathbb{Z}}$

15. The height of a rung on a hamster wheel can be modeled by

$h(t)=-25\cos\left[2\pi\left(\frac{t-4}{12}\right)\right]+27$, where ***h(t)*** represents the height of the rung above the bottom of the cage in centimeters and ***t*** is the time in seconds after the wheel starts moving. Show all your work for these questions.

a) Determine the height of the rung at the start of the ride.

$h(0) = -25\cos\left[2\pi\left(\frac{0-4}{12}\right)\right]+27$

$= -25\cos\left(-\frac{2\pi}{3}\right)+27$

$= 39.5$

∴ The height is 39.5 m

b) Determine the maximum height of the rung during one rotation. max = 27 + 25 = 52 cm

c) How long will it take for the wheel to complete one full rotation?

$k = \frac{2\pi}{12} = \frac{\pi}{6}$

$\text{Period} = \frac{2\pi}{\pi/6} = 12$

$\therefore$ It will take 12 seconds to complete one full rotation.

d) How long will it take for the wheel to reach its maximum height?

$52 = -25\cos\left[2\pi\left(\frac{t-4}{12}\right)\right] + 27$

$-1 = \cos\left[\pi\left(\frac{t-4}{6}\right)\right] \Rightarrow \pi = \pi\left(\frac{t-4}{6}\right)$

$1 = \frac{t-4}{6}$

$6 = t-4$

$10 = t$

$\therefore$ It will take 10 seconds.

e) Determine the average rate of change in the height of the rung between 2 and 3 seconds.

$A.R.O.C = \frac{h(3) - h(2)}{3-2}$

$\doteq 5.349 - 14.5$

$= -9.15 \text{ cm/sec}$

$h(3) = -25\cos\left(2\pi\left(-\frac{1}{12}\right)\right) + 27$

$= -25\cos\left(-\frac{\pi}{6}\right) + 27$

$\doteq 5.349$

$h(2) = -25\cos\left(2\pi\left(-\frac{2}{12}\right)\right) + 27$

$= -25\cos\left(-\frac{\pi}{3}\right) + 27$

$= 14.5$

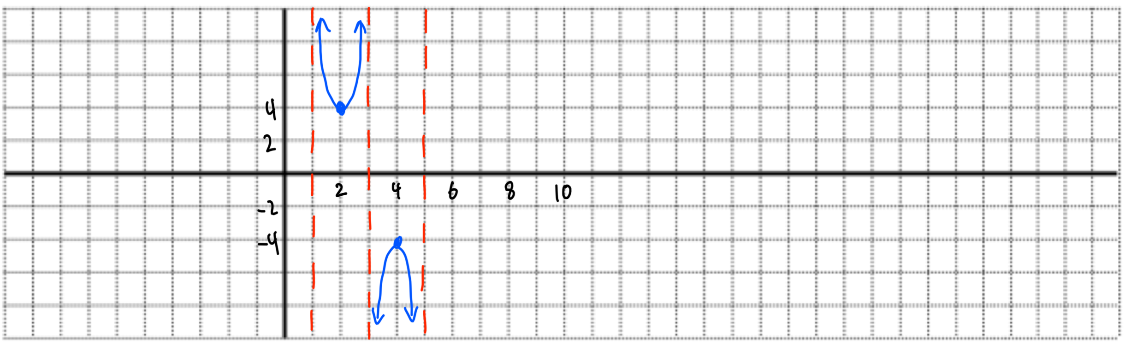
16. Graph one complete cycle of $y = 4\csc\left[\frac{\pi(t-1)}{2}\right]$.

$p = \frac{2\pi}{k} = \frac{2\pi}{\pi/2} = 4$

V.A: $\sin\left[\frac{\pi(t-1)}{2}\right] = 0 \Rightarrow \frac{\pi(t-1)}{2} = k\pi$

$\pi(t-1) = 2k\pi$

$t = 2k+1, \ k \in \mathbb{Z}$



17. Solve for x, $4\cos(x) - 3\sin(x) = 2 \quad (0 \le x \le 2\pi)$ (2 **DECIMAL PLACES**)

$[4\cos(x)]^2 = [2 + 3\sin(x)]^2$

$16\cos^2(x) = 4 + 12\sin(x) + 9\sin^2(x)$

$16[1-\sin^2(x)] = 4 + 12\sin(x) + 9\sin^2(x)$

$16 - 16\sin^2(x) = 4 + 12\sin(x) + 9\sin^2(x)$

$0 = 25\sin^2(x) + 12\sin(x) - 12$

Let $A = \sin(x)$: $0 = 25A^2 + 12A - 12$

$A = \frac{-12 \pm \sqrt{144 - 4(25)(-12)}}{2(25)}$

$= \frac{-12 \pm \sqrt{1344}}{50}$

$= \frac{-12 \pm 2\sqrt{336}}{50}$

$= \frac{-6 \pm \sqrt{336}}{25}$

$\sin(x) \doteq 0.4932$

$R.A.A = \sin^{-1}(0.4932)$

$\doteq 0.52$

$x_1 \doteq 0.52$

$x_2 \doteq \pi - 0.52$

$\doteq 2.62$

extraneous

$\sin(x) \doteq -0.973$

$R.A.A = \sin^{-1}(0.973)$

$\doteq 1.34$

$x_3 \doteq \pi + 1.34$

$\doteq 4.48$

$x_4 \doteq 2\pi - 1.34$

$\doteq 4.94$

extraneous

$\boxed{\therefore x \in \{0.52, 4.48\}}$

18. Solve for θ , $-10\cos^2(x)-3\sin(x)+9=0$, $(0\le x\le 2\pi)$ (2 **DECIMAL PLACES**)

$-10[1-\sin^2(x)]-3\sin(x)+9=0$

$-10+10\sin^2(x)-3\sin(x)+9=0$

$10\sin^2(x)-3\sin(x)-1=0$

Let $A=\sin(x)$:

$10A^2-3A-1=0$

$(5A+1)(2A-1)=0$

$[5\sin(x)+1][2\sin(x)-1]=0$

$5\sin(x)+1=0$

$\sin(x)=-\frac{1}{5}$

R.A.A $=\sin^{-1}\left(\frac{1}{5}\right)\doteq 0.201$

$x_1\doteq \pi+0.201$

$\doteq 3.34$

$x_2\doteq 2\pi-0.201$

$\doteq 6.08$

$2\sin(x)-1=0$

$\sin(x)=\frac{1}{2}$

R.A.A $=\frac{\pi}{6}$

$x_3=\frac{\pi}{6}$

$x_4=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

$\boxed{\therefore x\in\left\{\frac{\pi}{6},\frac{5\pi}{6},3.34,6.08\right\}}$

19. Find an **equation** of a function in the form $y=a\sin k(x-d)+c$ whose graph has a maximum at the point $A\left(\frac{\pi}{4},1\right)$ and a minimum at $B\left(\frac{5\pi}{4},-1\right)$.

$a=\frac{max-min}{2}=\frac{1-(-1)}{2}=1$

period $=\left(\frac{5\pi}{4}-\frac{\pi}{4}\right)\times 2=2\pi$

$k=\frac{2\pi}{2\pi}=1$

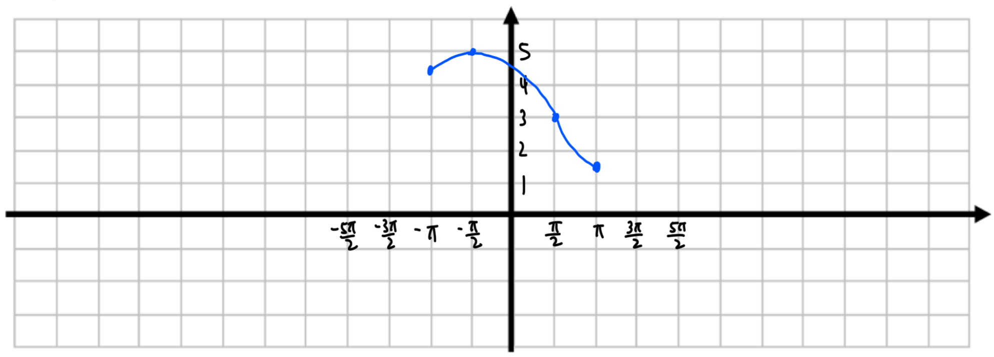
$d=\frac{\pi}{4}-\frac{2\pi}{4}=-\frac{\pi}{4}$

$c=\frac{max+min}{2}=\frac{1+(-1)}{2}=0$

$\therefore y=\sin\left(x+\frac{\pi}{4}\right)$

20. Graph $y=-2\sin\left(\frac{1}{2}\theta-\frac{\pi}{4}\right)+3$, $-\pi\le\theta\le\pi$

$y=-2\sin\left[\frac{1}{2}\left(\theta-\frac{\pi}{2}\right)\right]+3$



$(x,y)\longrightarrow\left(2\theta+\frac{\pi}{2},-2y+3\right)$

(x,y)	$\left(2\theta+\frac{\pi}{2},-2y+3\right)$
$(0,0)$	$\left(\frac{\pi}{2},3\right)$
$\left(\frac{\pi}{2},1\right)$	$\left(\frac{3\pi}{2},1\right)$
$(\pi,0)$	$\left(\frac{5\pi}{2},3\right)$
$\left(\frac{3\pi}{2},-1\right)$	$\left(\frac{7\pi}{2},5\right)$
$(2\pi,0)$	$\left(\frac{9\pi}{2},3\right)$