2.1 A Little More Elastic, Please?

LEARNING GOALS

Learning Objectives

1. Explain the Poisson's ratio in terms of axial and radial strains
2. Given any two of the axial strain, radial strain, and Poisson's ratio, determine the other
3. Demonstrate when the shear stress, shear strain, and shear moduli are appropriately used
4. Compare and contrast the shear and tensile stress, strain, and moduli
5. Perform calculations involving elastic shear loading

The Poisson's Ratio - (Because Nobody Wants a Square Salmon)

Alright, the Poisson's ratio has nothing to do with fish, it's named after a French fellow. When we stretch a sample: that is, pull it in tension, we find that the diameter decreases, as shown in Figure 1.

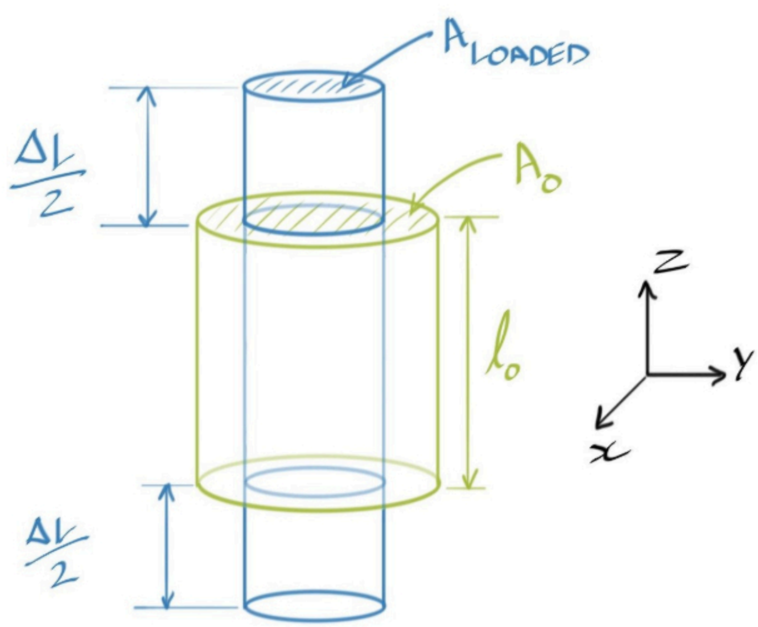

Figure 1: A hypothetical cylinder of material loaded *elastically* in tension will experience a decrease in radius for an increase in length along its long axis.

The extent to which any given material "shrinks" in cross-section for a given elongation, or vice-versa, "fattens" for a given compression, is given by the Poisson's ratio. We define the Poisson's ratio as the negative ratio of the radial strain to the axial strain: $\nu = -\frac{\epsilon_R}{\epsilon_Z} = -\frac{\epsilon_x}{\epsilon_Z} = -\frac{\epsilon_y}{\epsilon_Z} \qquad (1)$

where ν is the lower case Greek letter nu, and the radial strain is equal to the strain in the x or y directions and ϵ_Z could be called the *axial strain*.

You may well wonder about the apparently irritating negative sign in this equation. This is included in the definition so that the value for Poisson's ratio is almost always positive. This saves us having to include negative signs on all of our Poisson's ratio values when we tabulate them. Us engineers love it when we can tabulate useful values. We also pride ourselves in being efficient, which is why we define the Poisson's ratio already having the negative sign built in. Consider a typical material that is elongated by a small elastic strain. The diameter would decrease. Assuming we established our sign convention for elongation being positive we would then have a negative value in the numerator and a positive value in the denominator. Without the negative sign in our definition we would have a negative ratio. How irritating would that be?

Shear Stress and Shear Strain

Until now we have considered only *uniaxial tensile* loads, causing normal stresses (here *normal* means perpendicular, not *usual*, although, arguable normal stresses are the usual ones we consider. Stated another way, we have considered only loading a sample along a single axis, perpendicular to an area, to cause the sample to get longer. Another way that loads may be applied is in *shear* loading. In shear loading the force is applied parallel to an area and causes the sample to become skewed, as shown in Figure 2.

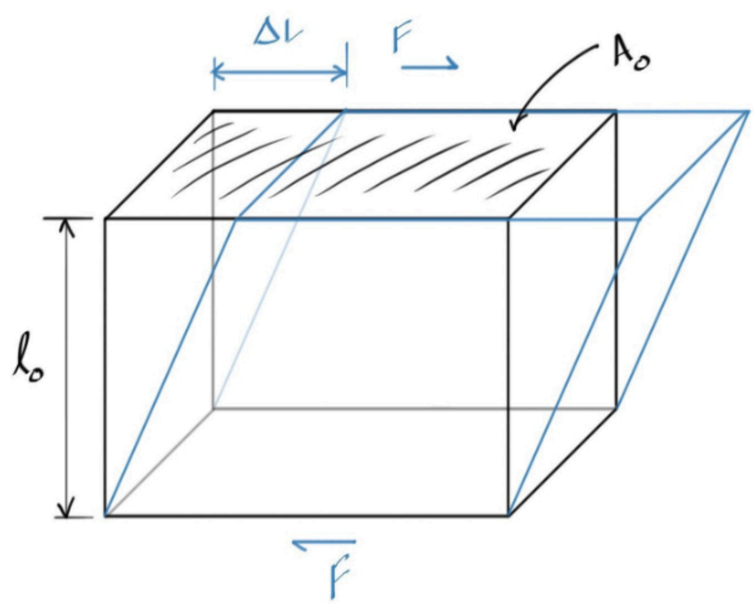

Figure 2: A hypothetical rectangular solid sample of material loaded in shear

I find that a good way to imagine this type of loading is to consider a stack of papers on a desk that you want to spread out across the desk. You place the palm of your hand on top of the stack and then move your hand parallel to the desk surface. The top paper is dragged by your hand sideways. The paper beneath that one is dragged slightly less by the top paper, and so on until the bottom paper, that remains stationary on the desk. Your pile of papers that originally had straight sides, is now shaped like remains stationary on the desk. Your pile of papers that originally had straight sides, is now shaped like a parallelogram. You just loaded the stack of papers in shear. Fasteners securing components to walls may be loaded largely in shear. Shear stress and strain are experienced differently by materials than are uniaxial tensile loads. Shear stress is defined as

$$\tau = \frac{F}{A_0} \qquad \sigma = \frac{F}{A_0} \qquad (2)$$

where τ is the lower case Greek letter *tau*. Shear strain is defined as

$$\gamma = \frac{\Delta l}{l_0} \qquad \varepsilon = \frac{\Delta l}{l_0} \qquad (3)$$

where γ is the lower case Greek letter *gamma*.

Now that we have established shear stress and strain, you may well imagine that they might be related elastically, as the normal stress and strain are. You would be correct in imagining this and in fact we can write Hooke's Law for shear stress and strain as follows

$$\tau = G\gamma \qquad (4)$$

where G is the *shear modulus*. The shear modulus is a different material property from the Young's modulus. However, the shear modulus and the Young's modulus are related through the Poisson's ratio in this expression

$$E = 2G(1 + \nu) \qquad (5)$$

shear modulus

Young's modulus