Beijing-Dublin International College

SEMESTER 2 FINAL EXAMINATION - 2020/2021

BDIC2025J/2002J Discrete Mathematics

Exam Test A

MODULE COORDINATOR: Shaofan Wang

Time Allowed: 95 minutes

Instructions for Candidates

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

All electronic devices, notebooks, books, work papers are strictly prohibited.

The Full Score of All Items of the Exam Paper

Item	1	2	3	4	5	6	7	8	Full
Full score	**25**	**8**	**8**	**8**	**15**	**12**	**12**	**12**	**100**

Obtained score

Question 1: Choose the single answer among A,B,C,D for each question.

Question 1-1: Let A, B be two sets with |A|=m, |B|=n. How many different binary relations from A to B?

(A) 2^{m+n} **(B)** 2^{mn} **(C)** m^2n^2 **(D)** mn

Question 1-2: Let A, B be two sets with |A|=2, |B|=m>2. How many different injection from A to B?

(A) $m(m-1)$ **(B)** $\frac{1}{2}m(m-1)$ **(C)** m^2 **(D)** $\frac{1}{2}m^2$

Question 1-3: Which function is a surjection?

(A) $f:\mathbb{Z}^+ \to \mathbb{Z}^+, f(x)=|x|$

(B) $f:\mathbb{Z} \to \mathbb{Z}\times\mathbb{Z}, f(x)=(x,x)$

(C) $f:\mathbb{Z} \to \mathbb{Z}\times\mathbb{Z}, f(x)=(0,x)$

(D) $f:\mathbb{N} \to \mathbb{N}, f(x)=2^x$

Question 1-4: Which graph is not a Eulerian graph but a semi-Eulerian graph?

(A) $K_{2,3}$ **(B)** $K_{3,3}$ **(C)** $K_{2,2}$ **(D)** K_4

Question 1-5: Let * be the minimum operator, i.e. $a*b=\min(a,b)$. All the sub-monoids of $(\{0,1,2\},*)$ are

(A) $(\{0,1\},*),(\{0,2\},*),(\{0,1,2\},*)$ **(B)** $(\{0,2\},*),(\{1,2\},*),(\{0,1,2\},*)$

(C) $(\{2\},*),(\{1,2\},*),(\{0,2\},*),(\{0,1,2\},*)$ **(D)** $(\{0\},*),(\{0,1\},*),(\{0,2\},*),(\{0,1,2\},*)$

Obtained score

Question 2: Compute the Conjunctive Normal Form of $(A\vee B)\to\big((B\wedge A)\leftrightarrow C\big)$.

Obtained score

Question 3: Compute the Prenex Normal Form of $\big(\forall xP(x)\vee\exists yQ(y)\big)\to\forall xR(x)$.

Obtained score

Question 4: Prove the following tautological implication using the Inference theory (Please give law notations in each inference step: P, T, AP, CP, US, UG, ES, EG)
$\exists x A(x) \to \forall x B(x) \Rightarrow \forall x(A(x) \to B(x))$

Obtained score

Question 5: Let A={1,2,3,4,5,6,7,8,9,10}**, and let** R **be the division relation on** A**. That is,** $(a,b) \in R$ **if** $b = ma$ **for some integer** m.

(1) Prove that R **is a partial order relation on** A **(i.e.** (A,R) **is a poset).**

(2) Draw the Hasse diagram of R**.**

(3) Compute the greatest element, the least element, maximal elements, and minimal elements of (A,R)**.**

Obtained score

Question 6: Let R_1, R_2 **be two equivalence relations on a set** A**. Prove that** $R_1 \cap R_2$ **is an equivalence relation.**

Obtained score

Question 7: Let $A = \mathbb{R} - \{2\}$ **and let * be the operator defined by**
$a * b = ab - 2(a+b-3)$**. Prove that** $(A,*)$ **is a group.**

Obtained score

Question 8: Let G be an undirected simple graph of order n**. Suppose that** $\deg(x) + \deg(y) \geq n-1$ **holds for any different two vertices** x, y **of G. Prove that G is a connected graph.**