

# Fusion: Rank-Based

## **COMP3009J: Information Retrieval**

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# Rank-Based Fusion: Interleaving

- **Interleaving** is perhaps the simplest fusion algorithm of all\*
- Here, we take one document from the top of each input set in a "**round robin**" fashion and add it to the fused result set.
  - The document chosen is the highest-ranked document that is not yet included in the fused result set.
- The effectiveness of this technique is, however, poor.
- There is an assumption that every result set is of equal quality, which can have the result that the better result sets are diluted by being merged with non-relevant documents from poorer systems.

\* Voorhees, E. M., Gupta, N. K., & Johnson-Laird, B. (1994). The Collection Fusion Problem. In *Proceedings of the Third Text REtrieval Conference (TREC-3)* (pp. 95–104)

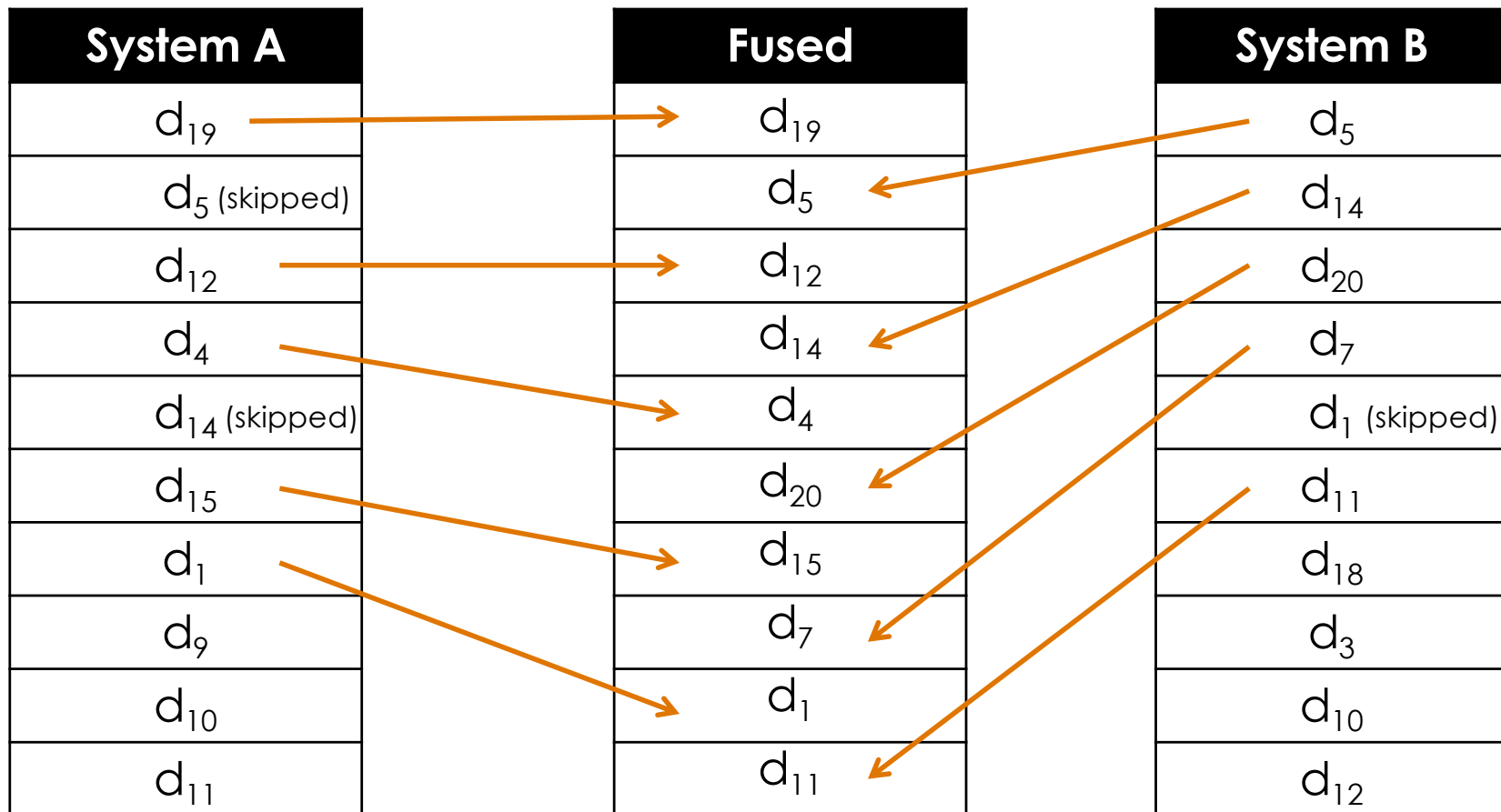
# Rank-Based: Interleaving (Example)

System A
$d_{19}$
$d_5$
$d_{12}$
$d_4$
$d_{14}$
$d_{15}$
$d_1$
$d_9$
$d_{10}$
$d_{11}$

Fused

System B
$d_5$
$d_{14}$
$d_{20}$
$d_7$
$d_1$
$d_{11}$
$d_{18}$
$d_3$
$d_{10}$
$d_{12}$

# Rank-Based: Interleaving (Example)



# Rank-Based: Borda-Fuse

- **Borda-Fuse** is based on an election system for when a few voters(input systems) vote for many candidates (documents).
  - Each voter ranks a set of  $c$  candidates in order of preference.
  - For each voter, the top-ranked candidate is given  $c$  points, the second ranked is given  $c-1$  points, etc.
  - If a candidate is not ranked by a voter, the voter's remaining points are divided evenly among unranked candidates.

\* Aslam, J. A., & Montague, M. (2001). Models for metasearch. In *SIGIR '01: Proceedings of the 24th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval* (pp. 276–284). New York, NY, USA.

# Rank-Based: Borda-Fuse

- **Borda-Fuse** is based on an election system for when a few voters(input systems) vote for many candidates

**Inventor: Jean-Charles de Borda**

*Born 4<sup>th</sup> May, 1773*

*French Mathematician, Physicist, Naval Officer*

*There is a moon crater named after him*

*Developed "Borda Count" in 1770 to elect members to the French Academy of Science.*

*Fought in the American Revolution*

*Helped to define the standard metre*

*Developed a technique for computing longitude in 1778*



\* Aslam, J. A., & Montague, M. (2001). Models for metasearch. In *SIGIR '01: Proceedings of the 24th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval* (pp. 276–284). New York, NY, USA.

# Rank-Based: Borda-Fuse

System A	Points
$d_{19}$	14
$d_5$	13
$d_{12}$	12
$d_4$	11
$d_{14}$	10
$d_{15}$	9
$d_1$	8
$d_9$	7
$d_{10}$	6
$d_{11}$	5

System B	Points
$d_5$	14
$d_{14}$	13
$d_{20}$	12
$d_7$	11
$d_1$	10
$d_{11}$	9
$d_{18}$	8
$d_3$	7

- 14 unique documents:  $c = 14$
- System A gives 2.5 points to any documents it didn't choose:  
 $(4+3+2+1)/4$ .
- System B gives 3.5 points to documents it didn't choose:  
 $(6+5+4+3+2+1)/6$

# Rank-Based: Borda-Fuse

System A	Points
d <sub>19</sub>	14
d <sub>5</sub>	13
d <sub>12</sub>	12
d <sub>4</sub>	11
d <sub>14</sub>	10
d <sub>15</sub>	9
d <sub>1</sub>	8
d <sub>9</sub>	7
d <sub>10</sub>	6
d <sub>11</sub>	5

System B	Points
d <sub>5</sub>	14
d <sub>14</sub>	13
d <sub>20</sub>	12
d <sub>7</sub>	11
d <sub>1</sub>	10
d <sub>11</sub>	9
d <sub>18</sub>	8
d <sub>3</sub>	7

Fused	Points (A)	Points (B)	Points (Borda)
d <sub>5</sub>	13	14	27
d <sub>14</sub>	10	13	23
d <sub>1</sub>	8	10	18
d <sub>19</sub>	14	3.5	17.5
d <sub>12</sub>	12	3.5	15.5
d <sub>4</sub>	11	3.5	14.5
d <sub>20</sub>	2.5	12	14.5
d <sub>11</sub>	5	9	14
d <sub>7</sub>	2.5	11	13.5
d <sub>15</sub>	9	3.5	12.5
d <sub>18</sub>	2.5	8	10.5
d <sub>9</sub>	7	3.5	10.5
d <sub>10</sub>	6	3.5	9.5
d <sub>3</sub>	2.5	7	9.5



# Comparison

System A	System B	Interleaving	Borda Fuse
$d_{19}$	$d_5$	$d_{19}$	$d_5$
$d_5$	$d_{14}$	$d_5$	$d_{14}$
$d_{12}$	$d_{20}$	$d_{12}$	$d_1$
$d_4$	$d_7$	$d_{14}$	$d_{19}$
$d_{14}$	$d_1$	$d_4$	$d_{12}$
$d_{15}$	$d_{11}$	$d_{20}$	$d_4$
$d_1$	$d_{18}$	$d_{15}$	$d_{20}$
$d_9$	$d_3$	$d_7$	$d_{11}$
$d_{10}$	$d_{10}$	$d_1$	$d_7$
$d_{11}$	$d_{12}$	$d_{11}$	$d_{15}$

# Reciprocal Rank Fusion

- Reciprocal Rank Fusion is a simple rank-based method that has been shown to be effective in practice\*.
- Given a set of documents  $D$  to be ranked, and a set of results  $R$ , the score for each document is calculated as follows:
  - $RRFscore(d \in D) = \sum_{r \in R} \frac{1}{k+r(d)}$
  - where  $r(d)$  is the rank of document  $d$  in result set  $r$ , and  $k=60$  (set by experiment)

\* Cormack, G. V., Clarke, C. L. A., Büttcher, S. (2009). Reciprocal Rank Fusion outperforms Condorcet and Individual Rank Learning Models. In *Proceedings of the 32nd International ACM SIGIR Conference on Research and Development in Information Retrieval* (pp. 758-759)

# Useful Maths

- ▣ RRF uses  $k=60$ .
- ▣ If a document,  $d$ , is not ranked, it does not contribute to the RRF Score
  - ▣ i. e. the score for the term related to document  $d$  in the summation is 0.

Rank	Calculation	RRF Score
1	$1/61$	0.01639
2	$1/62$	0.01612
3	$1/63$	0.01587
4	$1/64$	0.01562
5	$1/65$	0.01538
6	$1/66$	0.01515
7	$1/67$	0.01492
8	$1/68$	0.01470
9	$1/69$	0.01449
10	$1/70$	0.01428
Not Ranked	0	0.0

# Rank-Based: RRF

System A	System B
d <sub>19</sub>	d <sub>5</sub>
d <sub>5</sub>	d <sub>14</sub>
d <sub>12</sub>	d <sub>20</sub>
d <sub>4</sub>	d <sub>7</sub>
d <sub>14</sub>	<b>d<sub>1</sub></b>
d <sub>15</sub>	d <sub>11</sub>
<b>d<sub>1</sub></b>	d <sub>18</sub>
d <sub>9</sub>	d <sub>3</sub>
d <sub>10</sub>	d <sub>10</sub>
d <sub>11</sub>	d <sub>12</sub>

$$RRFscore(d_1) = \sum_{r \in R} \frac{1}{k+r(d)} = \frac{1}{67} + \frac{1}{65} = 0.0303$$

# Rank-Based: RRF

System A	System B
d <sub>19</sub>	d <sub>5</sub>
d <sub>5</sub>	d <sub>14</sub>
d <sub>12</sub>	d <sub>20</sub>
d <sub>4</sub>	d <sub>7</sub>
d <sub>14</sub>	d <sub>1</sub>
d <sub>15</sub>	d <sub>11</sub>
d <sub>1</sub>	d <sub>18</sub>
d <sub>9</sub>	<b>d<sub>3</sub></b>
d <sub>10</sub>	d <sub>10</sub>
d <sub>11</sub>	d <sub>12</sub>

$$RRFscore(d_1) = \sum_{r \in R} \frac{1}{k+r(d)} = \frac{1}{67} + \frac{1}{65} = 0.0303$$

$$RRFscore(d_3) = \sum_{r \in R} \frac{1}{k+r(d)} = 0 + \frac{1}{68} = 0.0147$$

# Rank-Based: RRF

System A	System B
$d_{19}$	$d_5$
$d_5$	$d_{14}$
$d_{12}$	$d_{20}$
$d_4$	$d_7$
$d_{14}$	$d_1$
$d_{15}$	$d_{11}$
$d_1$	$d_{18}$
$d_9$	$d_3$
$d_{10}$	$d_{10}$
$d_{11}$	$d_{12}$

Fused	RRF(A)	RRF(B)	RRF
$d_1$	0.01492	0.01538	0.0303
$d_3$		0.01470	0.0147
$d_4$	0.01562		0.0156
$d_5$	0.01612	0.01639	0.0325
$d_7$		0.01562	0.0156
$d_9$	0.01470		0.0147
$d_{10}$	0.01449	0.01449	0.0290
$d_{11}$	0.01428	0.01515	0.0294
$d_{12}$	0.01587	0.01428	0.0302
$d_{14}$	0.01538	0.01612	0.0315
$d_{15}$	0.01515		0.0152
$d_{18}$		0.01492	0.0149
$d_{19}$	0.01639		0.0164
$d_{20}$		0.01587	0.0159

# Rank-Based: RRF

System A	System B
d <sub>19</sub>	d <sub>5</sub>
d <sub>5</sub>	d <sub>14</sub>
d <sub>12</sub>	d <sub>20</sub>
d <sub>4</sub>	d <sub>7</sub>
d <sub>14</sub>	d <sub>1</sub>
d <sub>15</sub>	d <sub>11</sub>
d <sub>1</sub>	d <sub>18</sub>
d <sub>9</sub>	d <sub>3</sub>
d <sub>10</sub>	d <sub>10</sub>
d <sub>11</sub>	d <sub>12</sub>

Fused	RRF(A)	RRF(B)	RRF
d <sub>5</sub>	0.01612	0.01639	0.0325
d <sub>14</sub>	0.01538	0.01612	0.0315
d <sub>1</sub>	0.01492	0.01538	0.0303
d <sub>12</sub>	0.01587	0.01428	0.0302
d <sub>11</sub>	0.01428	0.01515	0.0294
d <sub>10</sub>	0.01449	0.01449	0.0290
d <sub>19</sub>	0.01639		0.0164
d <sub>20</sub>		0.01587	0.0159
d <sub>4</sub>	0.01562		0.0156
d <sub>7</sub>		0.01562	0.0156
d <sub>15</sub>	0.01515		0.0152
d <sub>18</sub>		0.01492	0.0149
d <sub>9</sub>	0.01470		0.0147
d <sub>3</sub>		0.01470	0.0147

# Comparison

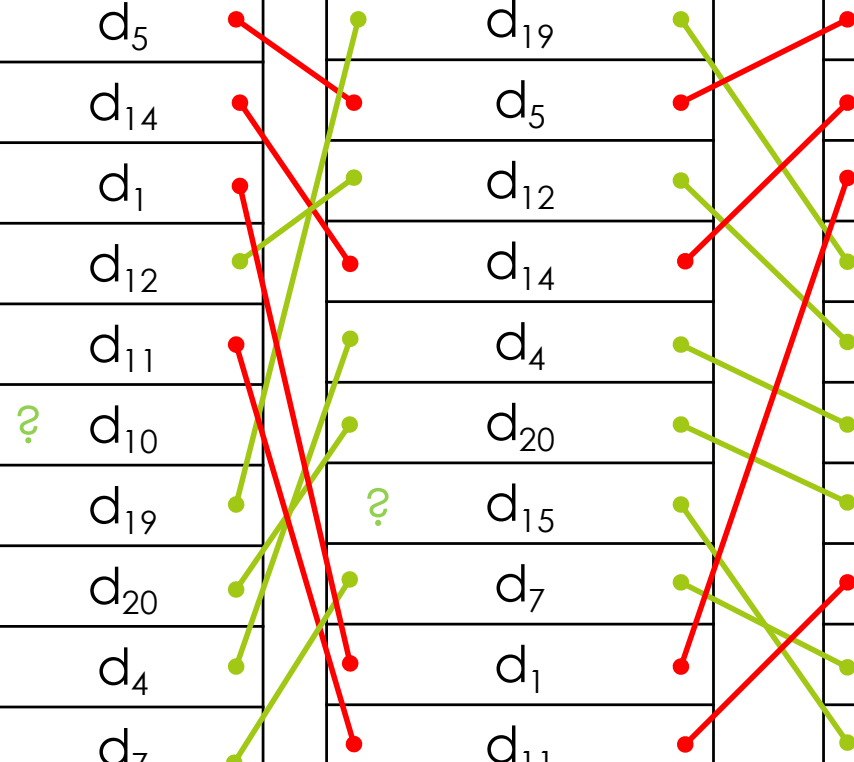
System A
$d_{19}$
$d_5$
$d_{12}$
$d_4$
$d_{14}$
$d_{15}$
$d_1$
$d_9$
$d_{10}$
$d_{11}$

System B
$d_5$
$d_{14}$
$d_{20}$
$d_7$
$d_1$
$d_{11}$
$d_{18}$
$d_3$
$d_{10}$
$d_{12}$

RRF
$d_5$
$d_{14}$
$d_1$
$d_{12}$
$d_{11}$
$d_{10}$
$d_{19}$
$d_{20}$
$d_4$
$d_7$

Interleaving
$d_{19}$
$d_5$
$d_{12}$
$d_{14}$
$d_4$
$d_{20}$
$d_{15}$
$d_7$
$d_1$
$d_{11}$

BF
$d_5$
$d_{14}$
$d_1$
$d_{19}$
$d_{12}$
$d_4$
$d_{20}$
$d_{11}$
$d_7$
$d_{15}$





# Other Rank-Based Techniques

- A variation on interleaving is to use **historical data** to estimate which input system(s) tends to perform better.
- A weighted version of interleaving is then used so that **more documents are taken from the better systems.** \*
- Another election-based approach from the same authors as Borda-Fuse is the *Condorcet-Fuse*\*\* algorithm. A weighted version of Borda-Fuse is also proposed, where the points from each input system are multiplied by some weight.

\* Voorhees et al. (1994)

\*\* Montague, M., & Aslam, J. A. (2002). Condorcet fusion for improved retrieval. In *CIKM '02: Proceedings of the eleventh international conference on Information and knowledge management* (pp. 538–548). New York, NY, USA.

# Class Question 1

System A
$d_{10}$
$d_{18}$
$d_4$
$d_6$
$d_5$
$d_{17}$
$d_{11}$
$d_{14}$

System B
$d_{18}$
$d_6$
$d_1$
$d_2$
$d_{17}$

System C
$d_6$
$d_4$
$d_3$
$d_{18}$
$d_5$
$d_{10}$
$d_{15}$
$d_{19}$

■ What is the fused ranking using:

1. Interleaving?
2. Borda Fuse?
3. Reciprocal Rank Fusion?