# Chapter.  The Linear Search Theorem.

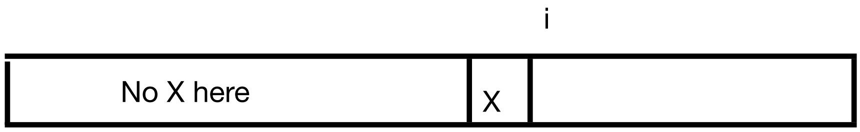
**An example.**

Given an array f[0..N) of int where $1 \le N$, and a target int value X. We are asked to find the smallest index i in $0 \le i < N$ where f.i = X. We are guaranteed that X is present in f.

*Precondition*

$$\text{Pre} : \langle \exists\ j : 0 \le j < N : f.j = X \rangle$$

*Postcondition.*




$$\text{Post} : \langle\ \forall j\ : 0 \le j < i : f.j \ne X \rangle \wedge f.i = X$$

*Domain model.*

* (0) C.i $\equiv$ $\langle\ \forall j\ : 0 \le j < i : f.j \ne X \rangle$ , $0 \le i \le N$

Consider

C.0
= {(0)}
$\langle\ \forall j\ : 0 \le j < 0 : f.j \ne X \rangle$
= { Empty range }
Id$\wedge$

which gives us

\- (1) C.0 $\equiv$ True

Consider

C.(i+1)
= {(0)}
$\langle\ \forall j\ : 0 \le j < i+1 : f.j \ne X \rangle$
= { Split off j=i term }
$\langle\ \forall j\ : 0 \le j < i : f.j \ne X \rangle \wedge f.i \ne X$
= {(0)}
C.i $\wedge$ f.i $\ne$ X

which gives us

- (2) $C.(i+1) \equiv C.i \wedge f.i \neq X$ , $0 \leq i < N$

*Rewrite postcondition.*

Post: $C.i \wedge f.i = X$

*Choose invariants.*

P0: $C.i$
P1: $0 \leq i \leq N$

*Guard.*

$f.i \neq X$

*Establish invariants.*

$i := 0$

*Variant.*[1]

$K - i$

*Loop body.*

$(i := i+1).P0$
= { Text Substitution }
$C.(i+1)$
= {(2)}
$C.i \wedge f.i \neq X$
= { P0 & guard }
True

*Finished algorithm.*

```
  i := 0
; do f.i ≠ X →
       i := i+1
  od
```

[1] K is the , as yet unknown, smallest index where f.K=X

This problem is an example of a more general type of problem called a Linear Search. We will now look at the more abstract case and construct the generic algorithm.

**Generic linear search.**

Given an *ordered, non-empty domain* $f[\alpha..\beta)$ of some type, and a predicate Q defined on the individual elements of the domain, find the smallest index i where $\alpha \le i < \beta$ where $Q.(f.i)$ holds true, if such a place exists.

Postcondition $\langle \forall\, j : \alpha \le j < i : \neg\, Q.(f.j) \rangle \wedge Q.(f.i)$

*Domain model.*

* (0) $C.i \equiv \langle \forall\, j : \alpha \le j < i : \neg\, Q.(f.j) \rangle$ , $\alpha \le i \le \beta$

Consider

$$
\begin{array}{ll}
 & C.\alpha \\
= & \quad \{(0)\} \\
 & \langle \forall\, j : \alpha \le j < \alpha : \neg\, Q.(f.j) \rangle \\
= & \quad \{ \text{ Empty range } \} \\
 & Id\wedge
\end{array}
$$

which gives us

\- (1) $C.\,\alpha \equiv True$

Consider

$$
\begin{array}{ll}
 & C.(i+1) \\
= & \quad \{(0)\} \\
 & \langle \forall\, j : \alpha \le j < i+1 : \neg Q.(f.j) \rangle \\
= & \quad \{ \text{ Split off } j=i \text{ term } \} \\
 & \langle \forall\, j : \alpha \le j < i : \neg Q.(f.j) \rangle \wedge \neg Q.(f.i) \\
= & \quad \{(0)\} \\
 & C.i \wedge \neg\, Q.(f.i)
\end{array}
$$

which gives us us

\- (2) $C.(i+1) \equiv C.i \wedge \neg Q.(f.i)$ , $\alpha \le i < \beta$

*Rewrite postcondition.*

Post: $C.i \wedge Q.(f.i)$

*Choose invariants.*

P0: C.i
P1: $\alpha \leq i \leq \beta$

*Guard.*

¬Q.(f.i)

*Establish invariants.*

i := $\alpha$

*Variant.*[2]

K - i

*Loop body.*

(i := i+1).P0
= { Text Substitution }
C.(i+1)
= {(2)}
C.i ∧ ¬Q.(f.i)
= { P0 & guard }
True

*Finished algorithm.*

```
 i := 0
;do ¬Q.(f.i) →
        i := i+1
 od
```

[2] K is the smallest index where f.K = X