Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION - 2019/2020

BDIC2002J Discrete Mathematics

Time Allowed: 95 minutes

Instructions for Candidates

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

The Full Score of All Items of the Exam Paper

Item	1	2	3	4	5	6	7	Full
Full score	**12**	**14**	**14**	**14**	**14**	**14**	**18**	100

Obtained score

Question 1: Let A be a non-empty set such that $|A| = n$, and let B be a non-empty set such that $|B| = 2$, with $n \geq 3$.

(1) Write down: how many different binary relations on A can be defined.
(2) Write down: how many different functions from A to B can be defined.
(3) Write down: how many different injections from B to A can be defined.
(4) Write down: how many different surjections from A to B can be defined.

Obtained score

Question 2: Let R be a binary relation on a non-empty set A. Suppose that R is symmetric and transitive. Suppose that for any element $a \in A$, there exists $b \in A$ such that $(a, b) \in R$. Prove that R is an equivalence relation.

Obtained score

Question 3:

(1) Compute of Principle Disjunctive Normal Form of

$$(\neg P \vee \neg Q) \leftrightarrow ((P \rightarrow R) \wedge Q)$$

(2) Compute of Principle Conjunctive Normal Form of

$$(P \vee \neg Q) \leftrightarrow (((\neg P \rightarrow R) \wedge Q) \vee R)$$

Obtained score

Question 4:

(1) Compute the Prenex Normal Form of

$$\forall y\,(\exists z A(x, y, z) \vee \forall u B(x, u)) \rightarrow \exists x C(y, x)$$

(2) Select all the free variables among ①–⑦

$$\forall y\,(\exists z A(\underset{①}{x}, \underset{②}{y}, \underset{③}{z}) \vee \forall u B(\underset{④}{x}, \underset{⑤}{u})) \rightarrow \exists x C(\underset{⑥}{y}, \underset{⑦}{x})$$

Obtained score

Question 5: Let $(G, *)$ be a finite group and let $H \subseteq G$ be a non-empty subset of G. Suppose that * is close on H. Prove that $(H, *)$ is a subgroup of $(G, *)$.

Obtained score

Question 6: Let $(G, *)$ be a finite group. Prove that for any elements a, b of G, $a * b$ and $b * a$ have the same order.

Obtained score

Question 7: Note a loop counts two edges.

(1) **Draw an undirected Eulerian graph with odd number of vertices and even number of edges (both numbers are nonzero).**

(2) **Draw an undirected Eulerian graph with even number of vertices and odd number of edges (both numbers are nonzero).**

(3) **Draw a five-order undirected simple graph G such that G is isomorphic to $\overline{G}$.**

(4) **Judge whether there exists a six-order undirected simple graph G such that G is isomorphic to $\overline{G}$; and give the reason.**