**Exercises.**

Determine whether the following Hoare triples are valid.

$\{y > 16\}\ x := x + 2\ \{y > 15\}$ Yes

$\{x = 4\}\ x := x + 1\ \{x > 4\}$ Yes

$\{y = z\}\ z := z + 1\ \{y - 1 > z\}$ No

$\{17 = 18\}\ x := x+1\ \{ x > z\}$ Yes

That one is a bit unusual. Anything that has a false state as the starting point is going to be valid simply because you could never start in such a state. So you can say that if you could then you could achieve anything you liked.

$\{x = 4\}\ x := 4\ \{x = 4\}$ Yes

$\{x = 4\}\ x := x + 1\ \{x = 4\}$ No

Determine the Weakest Preconditions of the following.

$\{x = 11\}x := x + 1\ \{x = 12\}$

$\{ y > 15\}\ x := y + 2\ \{x > 17\}$

$\{ x = 6 \}\ y := 4\ \{x = y + 2\}$

$\{ y \leq 0 \}\ x := y\ \{x \leq 0\}$

$\{74 = 73 \}\ x := 74\ \{x = 73\}$

What assignments would make the following into valid Hoare triples?

| | | |
|---|---|---|
| $\{x = y + z\}$ | $z := y + z$ | $\{x = z\}$ |
| $\{y * 2 = 12\}$ | $y := y * 4$ | $\{y = 24\}$ |
| $\{true\}$ | $y := 12$ | $\{y = 12\}$ |
| $\{x > y\}$ | $x, y := 2, 7$ | $\{x < y\}$ |

There are many solutions to that one.

| | | |
|---|---|---|
| $\{x = X \wedge y = Y\}$ | $x, y := y, x$ | $\{x = Y \wedge y = X\}$ |

This shows how nice our language is, we can swap values in 1 step.

Given an array f[0..100) of int. Express the following in Quantified form

r is the sum of the values in f

$$r = \langle +j : 0 \le j < 100 : f.j \rangle$$

p is the product of the values in the 2nd half of f

$$p = \langle *\, j : 50 \le j < 100 : f.j \rangle$$

r is the largest value in f

$$r = \langle \uparrow j : 0 \le j < 100 : f.j \rangle$$

s is the smallest value in f

$$s = \langle \downarrow j : 0 \le j < 100 : f.j \rangle$$

k is the sum of the last 20 elements in f

$$k = \langle +j : 80 \le j < 100 : f.j \rangle$$

v is the product of the middle 20 elements in f

$$v = \langle *\, j : 40 \le j < 60 : f.j \rangle$$

r is the sum of the even elements in f

$$r = \langle +j : 0 \le j < 100 : g.(f.j) \rangle$$

where
$$\begin{aligned} g.x &= x \Leftarrow \text{even}.x \\ g.x &= 0 \Leftarrow \text{odd}.x \end{aligned}$$

p is the product of the negative elements in f

$$p = \langle *\, j : 0 \le j < 100 : g.(f.j) \rangle$$

where
$$\begin{aligned} g.x &= x \Leftarrow x < 0 \\ g.x &= 1 \Leftarrow 0 \le x \end{aligned}$$

v is the smallest positive element in the first half of f

$$v = \langle \downarrow j : 0 \le j < 50 : g.(f.j) \rangle$$

where
$$\begin{aligned} g.x &= x \Leftarrow 0 \le x \\ g.x &= id\!\downarrow\, \Leftarrow x < 0 \end{aligned}$$

p is the largest even value in the 2nd half of f

$$p = \langle \uparrow j : 50 \leq j < 100 : g.(f.j) \rangle$$

$$\text{where} \qquad \begin{aligned} g.x &= x &\Leftarrow\ & even.x \\ g.x &= Id\uparrow &\Leftarrow\ & odd.x \end{aligned}$$

i is the smallest index in f where f.i = X

$$\langle \wedge j : 0 \leq j < i : f.j \neq X \rangle \wedge f.i = X$$

or

$$\langle \forall j : 0 \leq j < i : f.j \neq X \rangle \wedge f.i = X$$

k is the largest index in f where f.k = 7

$$\langle \wedge j : k < j < 100 : f.j \neq X \rangle \wedge f.k = X$$

or

$$\langle \forall j : k < j < 100 : f.j \neq X \rangle \wedge f.k = X$$

All of the elements in f are greater than 10

$$\langle \wedge j : 0 \leq j < 100 : 10 < f.j \rangle$$

or

$$\langle \forall j : 0 \leq j < 100 : 10 < f.j \rangle$$

All of the elements in f are even numbers

$$\langle \wedge j : 0 \leq j < 100 : even.f.j \rangle$$

or

$$\langle \forall j : 0 \leq j < 100 : even.f.j \rangle$$

None of the elements in f is larger than 123

$$\langle \wedge j : 0 \leq j < 100 : f.j \leq 123 \rangle$$

or

$$\langle \forall j : 0 \leq j < 100 : f.j \leq 123 \rangle$$