Semester One of Academic Year (2015---2016) of BJUT

《 Discrete Mathematics》

Module Code: BDIC2002J

Exam Paper B

Exam Instructions: Answer ALL Questions

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: ___________ **Class No:** ___________

BJUT Student ID: ___________ **UCD Student ID**___________

Notes:

The exam paper has 7 parts on 1 page, with a full score of 100 points. You are required to use the given Examination Book only.

Instructions for Candidates

Full marks will be awarded for complete answer to **ALL** the questions.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Total Score of the Exam Paper（For teachers' use only）

Item	1	2	3	4	5	6	7			Total Score
Full Score	14	14	14	14	14	16	14			
Obtained Score										

Obtained score

Part 1: Let R be a binary relation on a set A. Prove that $R \circ R^{-1}$ is a symmetric relation on A

Obtained score

Part 2: Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let R be a binary relation on the set A defined by: aRb iff there exists an integer number m such that $b = ma$. Prove that R is not an equivalence relation

Obtained score

Part 3: Prove the following tautological implication

$$\Big((A \vee B) \to (C \vee D)\Big) \wedge \Big((D \vee F) \to E\Big) \Rightarrow A \to E$$

Obtained score

Part 4: Compute the Principle Disjunctive Normal Form of

$$(\neg P \to Q) \leftrightarrow (Q \to \neg R)$$

Obtained score

Part 5: Compute the Prenex Normal Form of

$$\forall y \Big(\exists z A(x, y, z) \wedge \exists z B(x, z)\Big) \to \neg \forall x C(x, y)$$

Obtained score

Part 6: Let $\boldsymbol{N}_6 = \{0, 1, 2, 3, 4, 5\}$ be a set of natural numbers smaller than six. Let $\oplus_6$ be the plus operator modulo six and let $\otimes_6$ be the product operator modulo six. That is,

$$a \oplus_6 b = \begin{cases} a + b & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases} \qquad a \otimes_6 b = \begin{cases} ab & \text{if } ab < 6 \\ ab - 6\left[\frac{ab}{6}\right] & \text{if } ab \geq 6 \end{cases}$$

where $[c]$ denotes the largest integer which is no greater than c.

(1) Write down the identity element, zero elements and the inverse element of each element of the algebra $(\boldsymbol{N}_6, \oplus_6)$

(2) Write down the identity element, zero elements and the inverse element of each element of the algebra $(\boldsymbol{N}_6, \otimes_6)$

Obtained score

Part 7: Let G be a simple connected planar graph. Denote e to be the number of all edges of G and denote v to be the number of all vertices of G . Suppose that each face of G has at least k edges. Prove that $e \leq \frac{k(v-2)}{k-2}$