Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION - 2023/2024

BDIC2002J/2025J Discrete Mathematics

Exam Test A

Time Allowed: 95 minutes

Instructions for Candidates

BJUT Student ID: ________________ **UCD Student ID: ________________**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: ____ (Signature)

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

The Full Score of All Items of the Exam Paper

Question	Q1	Q2	Q3	Q4	Full
Full score	**2*15**	**2*10**	**1.5*10**	**35**	100
Obtained score					

Obtained score

Question 1: Single choice question (choose only one item, fill the answer in bracket, **2*15 score**)

1.1 What is the function type of $f: \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+, f(x,y) = xy$ **[ ]**

A、 not a surjection but an injection B、 not an injection but a surjection
C、 a bijection D、 neither an injection nor a surjection

1.2 What is the function type of $f: \mathbb{Z}^+ \to \{0,1,2\}, f(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$ **[ ]**

A、 not a surjection but an injection B、 not an injection but a surjection
C、 a bijection D、 neither an injection nor a surjection

1.3 What is the function type of $f: \mathbb{Z}^+ \to \mathbb{Z}, f(x) = |x|$ **[ ]**

A、 not a surjection but an injection B、 not an injection but a surjection
C、 a bijection D、 neither an injection nor a surjection

1.4 What is the function type of $f: \mathbb{Z} \to \mathbb{Z}, f(x) = x^2 - 2x - 15$ **[ ]**

A、 not a surjection but an injection B、 not an injection but a surjection
C、 a bijection D、 neither an injection nor a surjection

1.5 Let $A = \{0,1\}$, let $P(A)$ be its power set, and let $\oplus$ be symmetric difference. What is the function type of $f: P(A) \times P(A) \to P(A), f(x,y) = x \oplus y$. **[ ]**

A、 not a surjection but an injection B、 not an injection but a surjection
C、 a bijection D、 neither an injection nor a surjection

1.6 Which identity is wrong **[ ]**

A、 $\overline{A} \oplus B = A \oplus \overline{B}$ B、 $(A \oplus B) \cup (A \oplus C) = A \cup B \cup C$
C、 $\overline{A} \oplus B = \overline{\overline{A}} \oplus \overline{B}$ D、 $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$

1.7 Let R, S be two equivalence relation on a finite set A. Which conclusion is wrong? **[ ]**

A、$R \cup S$ is a reflective relation B、$R \cup S$ is a symmetric relation

C、$R \cup S$ is a transitive relation D、$R \cap S$ is an equivalence relation

1.8 Which tautological implication is wrong? **[ ]**

A、$(A \to C) \land (B \to C) \Leftrightarrow (A \land B) \to C$ B、 $\neg(A \leftrightarrow B) \Leftrightarrow (\neg A \land B) \lor (A \land \neg B)$
C、 $(A \leftrightarrow B) \to (A \land B) \Leftrightarrow A \lor B$ D、 $\neg(A \leftrightarrow B) \Leftrightarrow (A \lor B) \land (\neg A \lor \neg B)$

1.9 Which logic equivalence is wrong? [ ]

A、$\exists x A(x) \to B \Leftrightarrow \exists x(A(x) \to B)$ B、$\exists x A(x) \to B \Leftrightarrow \forall x(A(x) \to B)$

C、$A \to \exists x B(x) \Leftrightarrow \exists x(A \to B(x))$ D、$A \to \forall x B(x) \Leftrightarrow \forall x(A \to B(x))$

1.10 Which logic equivalence is wrong? [ ]

A、$\neg\forall y \exists x A(x,y) \Rightarrow \neg\exists x \forall y A(x,y)$ B、$\neg\exists x \forall y A(x,y) \Rightarrow \neg\forall y \exists x A(x,y)$

C、$\forall x \forall y A(x,y) \Rightarrow \exists y \forall x A(x,y)$ D、$\forall x \exists y A(x,y) \Rightarrow \exists y \exists x A(x,y)$

1.11 Which tautological implication is wrong? [ ]

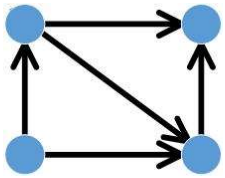
A、$\forall x\big(A(x) \to B(x)\big) \Rightarrow \forall x A(x) \to \exists x B(x)$ B、$\exists x A(x) \to \forall x B(x) \Rightarrow \forall x\big(A(x) \to B(x)\big)$

C、$\forall x\big(A(x) \leftrightarrow B(x)\big) \Rightarrow \exists x A(x) \leftrightarrow \forall x B(x)$ D、$\forall x\big(A(x) \leftrightarrow \neg B(x)\big) \Rightarrow \forall x A(x) \leftrightarrow \neg\exists x B(x)$

1.12 Which predicate formula is not a prenex normal form? [ ]

A、$\forall x \forall y\big(A(x) \to B(x,y,z)\big)$ B、$\exists x \exists y\big(A(x,y) \leftrightarrow B(x,y,z)\big)$

C、$\forall x \exists y \neg\big(P(x) \to \neg Q(x,y,z)\big)$ D、$\forall x \exists y\big(P(x) \to \exists z \neg Q(x,y,z)\big)$

1.13 Let G be the graph of . Choose the graph type of G. [ ]

A、not strongly connected graph but unilaterally connected graph B、strongly connected graph

C、not unilaterally connected graph but weakly connected graph D、none of the above

1.14 Let e be an edge of K_5, and let $G' = K_5 - \{e\}$. Choose the graph type of G'. [ ]

A、Eulerian graph B、semi-Eulerian graph C、bipartite graph D、Non-planar graph

1.15 Choose the graph type of K_8. [ ]

A、Eulerian graph B、semi-Eulerian graph C、bipartite graph D、Non-planar graph

Obtained score

Question 2: Multiple choice question (choose at least two items, fill the answer in bracket, **2*10 score**)

2.1 The principle disjunctive normal form of $(P \to Q) \leftrightarrow R$ consists of [ ]

A、$P \wedge Q \wedge R$ B、$\neg P \wedge Q \wedge R$ C、$P \wedge \neg Q \wedge R$ D、$P \wedge Q \wedge \neg R$

E、$\neg P \wedge \neg Q \wedge R$ F、$\neg P \wedge Q \wedge \neg R$ G、$P \wedge \neg Q \wedge \neg R$ H、$\neg P \wedge \neg Q \wedge \neg R$

2.2 The principle disjunctive normal form of $(P \to \neg Q)\,\overline{\vee}\,R$ consists of [ ]

A、$P \wedge Q \wedge R$ B、$\neg P \wedge Q \wedge R$ C、$P \wedge \neg Q \wedge R$ D、$P \wedge Q \wedge \neg R$
E、$\neg P \wedge \neg Q \wedge R$ F、$\neg P \wedge Q \wedge \neg R$ G、$P \wedge \neg Q \wedge \neg R$ H、$\neg P \wedge \neg Q \wedge \neg R$

2.3 The principle conjunctive normal form of $\neg Q \leftrightarrow (\neg P \leftrightarrow R)$ consists of [ ]

A、$P \vee Q \vee R$ B、$\neg P \vee Q \vee R$ C、$P \vee \neg Q \vee R$ D、$P \vee Q \vee \neg R$
E、$\neg P \vee \neg Q \vee R$ F、$\neg P \vee Q \vee \neg R$ G、$P \vee \neg Q \vee \neg R$ H、$\neg P \vee \neg Q \vee \neg R$

2.4 The principle conjunctive normal form of $(\neg P \to Q) \wedge (P \leftrightarrow \neg R)$ consists of [ ]

A、$P \vee Q \vee R$ B、$\neg P \vee Q \vee R$ C、$P \vee \neg Q \vee R$ D、$P \vee Q \vee \neg R$
E、$\neg P \vee \neg Q \vee R$ F、$\neg P \vee Q \vee \neg R$ G、$P \vee \neg Q \vee \neg R$ H、$\neg P \vee \neg Q \vee \neg R$

2.5 Let R be a binary relation on $\mathbb{Z}^+$ defined by xRy iff $4|(x+y)$ or $4|(x-y)$. Choose the relation types of R. **Note:** $\boldsymbol{x|y}$ iff $\boldsymbol{y = kx}$ for $\boldsymbol{k} \in \mathbb{Z}$. [ ]

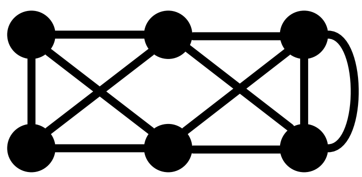
A、reflexive B、antireflexive C、symmetric D、antisymmetric E、transitive

2.6 Let R be a binary relation on $\mathbb{Z}^+$ defined by xRy iff $3|(x \div y)$. Choose the relation types of R. [ ]

A、reflexive B、antireflexive C、symmetric D、antisymmetric E、transitive

2.7 Let G be an undirected graph, with vertex number v and edge number e. Which are sufficient and necessary conditions such that G is a tree? [ ]

A、G is connected and $e = v - 1$ B、G is connected and has no circle
C、G has no circle and $e = v - 1$ D、G is connected and has no bridge

2.8 Choose the graph type of  [ ]

A、Hamiltonian graph B、Eulerian graph C、semi-Eulerian graph
D、bipartite graph E、planar graph F、non-planar graph

2.9 Choose the graph type of $K_{4,5}$ [ ]

A、Hamiltonian graph B、semi-Hamiltonian graph C、Eulerian graph
D、semi-Eulerian graph E、bipartite graph F、non-planar graph

2.10 Which of the following graphs are semi-Hamiltonian graphs but not Hamiltonian graphs [ ]

A、$K_{2,3}$ B、$K_{4,4}$ C、$K_{5,5}$ D、$K_{5,6}$ E、K_6 F、$K_{6,7}$

Obtained score

Question 3: Judgement question (fill **T** (true) or **F** (false) in bracket, **1.5*10 score**)

3.1 For arbitrary two sets A, B, $P(A - B) \neq P(A) - P(B)$. [ ]

3.2 For three sets A, B, C, if $A \subseteq B, B \in C$, then $A \subseteq C$ [ ]

3.3 Let R be a binary relation on a finite set A with $|A| = 5$. R's relation matrix is $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$.

Then R is a transitive relation on A. [ ]

3.4 Let R be a binary relation on $\mathbb{Z}^+$ defined by xRy iff $x + 5y = 22$. Then R is transitive. [ ]

3.5 For proposition formulae A, B, $(A \leftrightarrow B) \rightarrow (A \wedge B) \Leftrightarrow A \vee B$ [ ]

3.6 For propositional formulae, P, A, B, if $A \Rightarrow B$, then $\neg P \vee A \Rightarrow \neg P \vee B$ [ ]

3.7 For proposition functions $A(x), B(x)$, $\exists x\big(A(x) \rightarrow B(x)\big) \Rightarrow \exists x A(x) \rightarrow \forall x B(x)$ [ ]

3.8 Let G be a simple bipartite graph and denote v, e to be the number of vertices, edges of G. Suppose v is odd, then $e \leq \frac{1}{4}(v^2 - 1)$ [ ]

3.9 Hamiltonian graph must be a connected graph. [ ]

3.10 Let G be an n-order undirected simple graph. Suppose $\deg(x) + \deg(y) \geq n - 1$ holds for any two vertices $x \neq y$ of G. Then G is a semi-Hamiltonian graph. [ ]

Obtained score

Question 4: fill-in-the-blank question (35 score in total)

4.1 (2.5') The prenex normal form of $\neg\forall x P(x) \rightarrow \neg\exists y Q(x, y)$ is ____________________.

4.2 (2.5') The prenex normal form of $\exists x P(x) \leftrightarrow \exists x Q(x)$ is _________________________.

4.3 (3') Let R be the divide relation on $A = \{2,4,6,8,10,12\}$, i.e., xRy iff $x|y$ (i.e., $y = kx$ for $k \in \mathbb{Z}$). Then the greatest element of the poset (A, R) is _____, the least element of the poset (A, R) is _______. The maximal elements of the poset (A, R) are ______, the minimal elements of the poset (A, R) are ___.

4.4 (3') Let R be a binary relation on $A = \{1, \ldots, 17, 18\}$ (that is, A is the set of all positive integers ≤ 18) defined by xRy iff $x \times y$ is a *square number* (i.e., $1,4,9,16,25,36,49, \ldots$). So R is an equivalence relation on A. Then $[1]_R =$ ________ . The cardinality of the quotient set A/R is _______.

4.5 (3') Let A be a set with $|A| = 3$. Let R be the set of all symmetric relations on A, and let S be the set of all reflexive relations on A. Then $|R - S| =$ _________.

4.6 (3') Let A be a set with $|A| = 4$. Let R be the set of all symmetric relations on A, and let S be the set of all antisymmetric relations on A. Then $|R \cap S| =$ ________.

4.7 (3') Let $A = \{1,2,3,4,5,6,7,8\}$ be a set, and let R be a binary relation on $A \times A$ defined by $(a,b)R(c,d)$ iff $a + d = b + c, \forall(a,b),(c,d) \in A \times A$. Then R is an equivalence relation, and $[(2,6)]_R =$ ______________________.

4.8 (3') Let G be an undirected semi-Eulerian graph with 10 edges. Suppose that G has three 2-degree vertices and two 4-degree vertices, and the other vertices have odd degrees. Write down all possible combination of number of odd-degree vertices of G: ____.

(For example, Case1: one 1-degree and two 3-degree; Case2: two 3-degree and three 5-degree, etc.)

4.9 (3') After we delete at least _______ vertices from $K_{5,7}$, it becomes a graph which is both a Eulerian graph and a Hamiltonian graph.

4.10 (3') Let G be a tournament whose vertex-set is $\{v_1, v_2, \dots, v_n\}$. Then

$$\sum_{i=1}^{n} (\deg^+(v_i) - n)^2 - \sum_{i=1}^{n} (\deg^-(v_i) + n)^2 = \underline{\qquad\qquad\qquad}.$$

4.11 (3') Let e be an edge of $K_{2,2}$, and let $G' = K_{2,2} - \{e\}$. Then G' contains _______ bridges.

4.12 (3') Let T be an undirected tree with two 2-degree vertices, three 3-degree vertices, and four 4-degree vertices. Suppose the degree of each vertex of T is no greater than 4. Then T has _____ leaves.