Semester One of Academic Year (2014---2015) of BJUT

《Discrete Mathematics》 Exam Paper A/B

Exam Instructions： Answer ALL Questions

Honesty Pledge：

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger： ________ **Class No：** ________

BJUT Student ID： ________ **UCD Student ID** ________

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Notes：

The exam paper has 7 parts on 1 pages, with a full score of 100 points. You are required to use the given Examination Book only. Choose **six** items of seven to answer.

Total Score of the Exam Paper（For teachers' use only）

Item	1	2	3	4	5	6	7			**Total Score**
Full Score	16	17	16	16	17	17	17			
Obtained Score										

Obtained score

Part 1: Let R be a binary relation on a finite set A. Prove that

$$|A/R| \cdot |R| \geq |A|^2$$

Obtained score

Part 2: Let R, S be two relations on a nonempty set A such that R, S are both symmetric. Prove that $R \circ S$ is a symmetric relation if and only if

$$R \circ S = S \circ R$$

Obtained score

Part 3: Compute the Principle Conjunctive Normal Form of
$(P \to \neg Q) \to (Q \leftrightarrow \neg R)$**.**

Obtained score

Part 4: Compute the Prenex Normal Form of.
$\forall y(\exists z A(x,y,z)) \vee \forall u B(x,u)) \to \exists x C(y,x)$

Obtained score

Part 5: Let t be a prime number and let m be a positive integer. Prove that any group of order t^m has a subgroup of order t.

Obtained score

Part 6: Let G be a finite group. Suppose that G is non-commutative. Prove that $|G| \geq 6$

Obtained score

Part 7: Let G be a tournament, and let V be the vertex-set of G. Prove the following identity:

$$\sum_{x \in \boldsymbol{V}} (\deg^-(x))^2 = -|\boldsymbol{V}|^2 + \sum_{x \in \boldsymbol{V}} (\deg^+(x) - |\boldsymbol{V}|)^2$$