



Beijing-Dublin International College



SEMESTER I FINAL EXAMINATION - 2016/2017

MODULE CODE: BDIC2002J

MODULE TITLE: Discrete Mathematics

Time Allowed: 95 minutes

Instructions for Candidates

The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: _____

UCD Student ID: _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Answer ALL seven questions.

The Full Score of All Items of the Exam Paper

Item	1	2	3	4	5	6	7	Full
Full score	14	14	14	16	14	14	14	100

Obtained score

Question 1: Let A,B,C be three sets. Prove the following identity

$$(A - B) \cup (B - C) \cup (C - A) = (A \cup B \cup C) - (A \cap B \cap C)$$

Obtained score

Question 2: Prove the following tautological implication

$$\forall x(A(x) \leftrightarrow B(x)) \Rightarrow \forall xA(x) \leftrightarrow \forall xB(x)$$

Obtained score

Question 3: Compute the Principle Disjunctive Normal Form of $(P \wedge Q) \leftrightarrow R$.

Obtained score

Question 4: Let (A, \preceq) be a poset, with $A = \{2, 3, 4, 6, 8, 9, 12, 16, 18\}$.

Let \preceq be the division relation (i.e. $a \preceq b$ iff $b = ma$ for some integer m).

- (1) Draw the Hasse diagram of the poset (A, \preceq)**
- (2) List the greatest element and the least element of the poset (A, \preceq) (if it exists)**
- (3) List all maximal elements and all minimal elements of the poset (A, \preceq)**

Obtained score

Question 5: Denote $G := \{(a, b) | a \in \mathbb{R}, b \in \mathbb{R}, a \neq 0\}$. The operator $*$ is defined by: $(a, b) * (c, d) = (ac, ad + b)$. Prove that $(G, *)$ is a group.

Obtained score

Question 6: Let $G = \{1, -1, \frac{1+\sqrt{3}i}{2}, \frac{-1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\}$ be a collection of six complex numbers, and let \times be the complex product operator. Prove that (G, \times) is a cyclic group.

Obtained score

Question 7:

- (1) Write down the Kuratowski theorem.
- (2) Using this theorem, show that the following graph is a non-planar graph.

