Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION - 2018/2019

BDIC2002J Discrete Mathematics

Time Allowed: 95 minutes

Instructions for Candidates

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.
No rough-work paper is to be provided for candidates.

The Full Score of All Items of the Exam Paper

Item	1	2	3	4	5	6	7	Full
Full score	**14**	**14**	**14**	**14**	**14**	**14**	**16**	100

Obtained score

Question 1: Suppose that R is a binary relation on the set A such that R is reflexive and transitive. Denote $\circ$ to be the composition of two relations. Prove that $R \circ R = R$

Obtained score

Question 2: Let $a_1, a_2, \ldots, a_n$ be arbitrary n positive integers. Prove that two integers $i \geq 0, k \geq 1$ exist such that $a_{i+1} + a_{i+2} + \ldots + a_{i+k}$ is divisible by n (i.e. $a_{i+1} + a_{i+2} + \ldots + a_{i+k} = mn$ for some positive integer m).

Obtained score

Question 3: Prove the following tautological implication using Conclusion Premise (CP) rule

$$A \to (B \to C),\ (C \wedge D) \to E,\ \neg F \to (D \wedge \neg E) \Rightarrow A \to (B \to F)$$

Obtained score

Question 4: Compute the Prenex Normal Form of

$$(\forall y C(y) \to \forall z D(y,z)) \to \exists x (A(x) \to B(x,y))$$

Obtained score

Question 5: Let $(G, *)$ be a group such that

$$(a * b)^5 = a^5 * b^5$$
$$(a * b)^3 = a^3 * b^3$$

hold for any element a, b of G. Prove that G is a commutative group.

Obtained score

Question 6: Let $(G, *)$ be a group of order $n \geq 5$, such that $a * a = e$ holds for any element a of G. Prove that G has at least a subgroup of order four.

Obtained score

Question 7: Let G be a simple undirected graph which has six vertices and thirteen edges.

(1) **Prove that G is a Hamiltonian graph.**

(2) **Prove that G is a non-planar graph.**