

## Robust ICP Registration using Biunique Correspondence

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**Abstract**—In this paper, a novel variant of the ICP algorithm is proposed for registration of partially overlapping range images. Biunique correspondence is introduced to enhance the performance of ICP by searching multiple closest points. A new kind of outlier is defined, called No-Correspondence (NC) Outlier, which is the point that is not assigned to a biunique correspondence. In order to maintain efficiency, a coarse-to-fine approach is adopted. Experiments show that the proposed algorithm can find the correct rigid transformation with the existence of large non-overlapping area and poor initial alignment. The proposed algorithm is also applied to SLAM with the use of odometry information.

**Keywords**—ICP registration; multiple closest points; biunique correspondence; 6DOF SLAM

### I. INTRODUCTION

The registration of range images is a fundamental problem in the fields of object modeling, robotics, medical imaging and recognition. Due to the limitations the field of view of range sensors, most of these applications require that multiple range images be aligned into a single model automatically and accurately. The iterative closest point (ICP) algorithm is one of the most popular methods to accomplish the registration task when the correspondences are unknown. After the ICP was proposed by Besl and McKay [1], and Chen and Medioni [2], many variants have been introduced based on the original ICP concept. Rusinkiewicz and Levoy classified these variants according to six stages [3]:

1. Select subset of points in one or both range images
2. Find correspondence points.
3. Weight the corresponding pairs.
4. Reject certain corresponding pairs based on a given criterion.
5. Assign an error metric.
6. Minimize the error metric.

Step 2 and 4 are the most important ones. Many researches have been conducted to improve the ICP's performance by increasing the robustness of these two steps. The Picky ICP [4] prevents that a model point is present in more than one corresponding pairs. All pairs containing the same model point are rejected except the one with smallest distance. The method increases the robustness to noise, but has the disadvantage of slow convergence. In addition, the proof of convergence in the original ICP does not hold for Picky ICP. Almhdie et al. proposed the Comprehensive ICP

[5] which utilizes a comprehensive lookup matrix to ensure that each point in the data set will have a different correspondence in the model set. For each point in the data set, this method computes the Euclidean distance between each point in the model set, and then sort the distances in ascending order. Clearly, this method cannot be used in real-time applications due to high computational cost.

Phillips et al. proposed the Fractional ICP [6] which minimizes the fractional root mean squared distance. After the closest points are searched for every point in the data set, only a fraction of the corresponding pairs are used to compute the rigid transformation. This method is claimed by the authors to be robust to outliers. However, when high level sensor noise exists, it's difficult to distinguish outliers from inliers accurately. Thus, the Fractional ICP can fall into a local minimum easily in this case. Chetverikov et al. proposed a Trimmed ICP [7] algorithm which uses the Least Trimmed Squares approach and estimates a degree of overlap between two range images to increase the robustness against outliers. Pomerleau et al. proposed an adaptive rejection technique called Relative Motion Threshold [8]. A maximum authorized error is defined based on a simulated annealing ratio for each iteration. Since the simulated annealing ratio only depends on the translation vector, this technique may fail when large rotation error exists.

Recently, ICP has been successfully adapted to the applications of Simultaneous Localization and Mapping (SLAM) which is an important research area in mobile robotics. Nüchter et al. evaluated the approximated data association to speed up the ICP-based 6D SLAM [9]. The performance of approximated k-D trees and box decomposition trees was investigated. Tomono proposed a variant of ICP which employs the edge-points to improve the performance of the feature-based SLAM in non-texture environments [10] [11]. Valencia et al. adopted a strategy of hierarchical correspondence search, using point-to-plane metric at the coarsest level and point-to-point metric at finer level [12].

Since ICP is a non-linear optimization method, it can fall into a local minimum due to outliers, occlusions and poor initial alignment. The ICP variants explained above only solve part of these problems under certain constraints. In this paper, we propose a new variant of ICP, called Biunique Correspondence (BC) ICP, which guarantees the uniqueness of corresponding pairs by searching multiple closest points. Unlike Picky ICP or Comprehensive ICP, the proof of

convergence still holds for the algorithm proposed in this paper, while the computational efficiency is kept the same level as the original ICP. A new kind of outlier is defined as No-Correspondence (NC) Outlier, which is the point that cannot be assigned to a biunique correspondence. Discarding NC Outliers greatly improve the robustness in the case that large rotation error and non-overlapping areas exist. We also propose a coarse-to-fine approach to increase the flexibility of the BC-ICP algorithm.

We compare BC-ICP with other variants of ICP by conducting experiments using range images captured from real-world scenes. BC-ICP yields better registration results. BC-ICP is also applied to 6D SLAM with the use of odometry information, and experiments show that the proposed algorithm is suitable for fast SLAM applications.

The paper is organized as follows. Section 2 presents an overview of biunique correspondence. Section 3 gives an outline of BC-ICP algorithm. Section 4 presents experiment results that evaluate the performance of the proposed algorithm.

## II. BIUNIQUE CORRESPONDENCE

Assume that two partially overlapping range images are given: the *data set* with  $n_P$  points,  $P = \{p_i, i = 1, \dots, n_P\}$ , and the *model set* with  $n_M$  points,  $M = \{m_j, j = 1, \dots, n_M\}$ . The registration of  $P$  and  $M$  is to compute a rigid transformation  $(R, t)$  which under initial estimation. The following object function is to be minimized:

$$T = (R, t) = \arg \min_{(R, t)} \left\{ \sum_{i=1}^{n_P} \|m_j - Rp_i - t\|^2 \right\}, \quad (1)$$

where  $p_i \in P$ ,  $m_j \in M$  and  $m_j$  is the closest point of  $p_i$ .

The original ICP framework utilizes the closest point in  $M$  as the correspondence for each point in  $P$ . Intuitively, one point in the *data set* can correspond to at most one point in the *model set*. However, the criterion of closest point may cause the many-to-one correspondence problem: the same point in the *model set* is assigned to different points in the *data set*, as illustrated in Figure 1(a). The correspondences are concentrated on only a few of points of  $M$ . Even false matches are rejected using distance threshold, the registration can still be trapped into a local minimum easily due to poor initial alignment. In order to spread the correspondences more uniformly, the uniqueness of correspondences must be considered.

### A. Multiple Closest Points Search

Instead of single closest point, multiple closest points are used to establish biunique correspondence. For a given point  $p_i \in P$ , we search for multiple closest points in  $M$ :  $q = \{q_k, k = 1, \dots, N_{mc}\}$  which has  $N_{mc}$  points sorted in ascending order with respect to their distance to  $p_i$ . If  $q_1$ , which is the closest point, is not used as correspondence by another point in  $P$ , then  $q_1$  is assigned to  $p_i$  as a corresponding pair. Otherwise, the second closest point  $q_2$  is assigned to  $p_i$  if  $q_2$  is not already used. Every point in  $q$  is checked in the sorted order until a biunique correspondence is established for  $p_i$  so that one model point can be assigned to only one data point. If all

points in  $q$  are not available,  $p_i$  has to find a model point outside the neighborhood  $q$  for a biunique correspondence, which increases the possibility of  $p_i$  being an outlier.

**Definition 1:** For a given point  $p_i \in P$ ,  $N_{mc}$  closest points are searched in  $M$ :  $q = \{q_k, k = 1, \dots, N_{mc}\}$ . The point  $p_i$  is defined as a No-Correspondence (NC) Outlier if every point in  $q$  is assigned to another point in  $P$  with a biunique correspondence.

NC Outlier is a direct outcome of the use of biunique correspondence. When  $P$  and  $M$  are almost converged, most of the biunique correspondences are established between the overlapping area of  $P$  and  $M$ , while most of the NC Outliers arise in the non-overlapping area, as shown in Figure 1(b). If  $P$  and  $M$  are aligned under a poor initial alignment, it's difficult to distinguish the overlapping and the non-overlapping area only using distance between corresponding point pairs. Therefore the definition of NC Outlier is extremely useful. When large non-overlapping areas exist, this definition provides a better criterion to estimate outliers, which greatly improves the robustness of ICP algorithm. We can discard this kind of outlier before rejection of unreliable correspondence with large Euclidean distance explained in the next section.

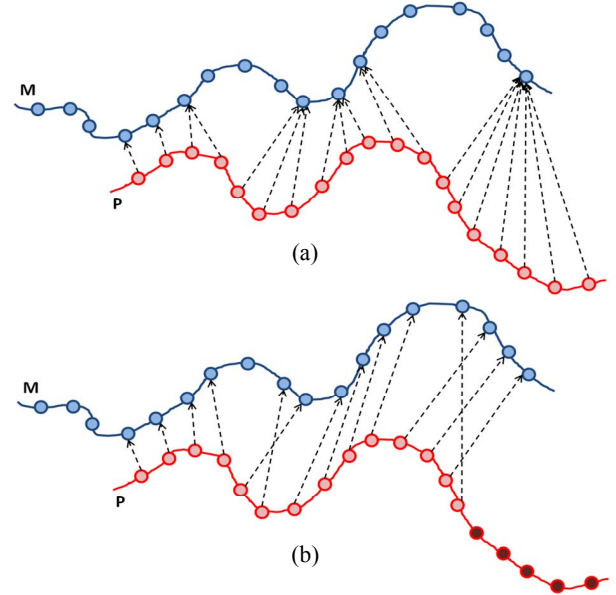


Figure 1. (a) Correspondences searched by Original ICP. The dash line arrows represent the closest-point relation from  $P$  to  $M$ . (b) Biunique correspondences as explained in section II.A. The dark red points are NC Outliers.

### B. Rejection of False Matches

After discarding NC Outliers, subsets of  $P$  and  $M$ , i.e.  $p_{bc}$  and  $m_{bc}$  are obtained, which only contain biunique correspondences. The correspondences with squared Euclidean distance larger than a threshold  $t$  are rejected. The threshold  $t$  is calculated using the following equations:

$$t = \begin{cases} (N_{mc})^\lambda \cdot \text{mean}_{SD} + s \cdot c^2, & \text{if } \lambda > \lambda_C \\ \text{mean}_{SD}, & \text{otherwise} \end{cases} \quad (2)$$

Here  $N_{mc}$  is the point number during multiple closest points search explained in the previous section;  $mean_{SD}$  is the mean of squared distance of biunique correspondences;  $s$  is the uniform subsampling factor;  $c$  is the distance between centroids of  $p_{bc}$  and  $m_{bc}$ .  $\lambda$  is the ratio between number of NC Outliers and number of the *data set* points after uniform sampling, which is always less than 1.  $\lambda_c$  is a constant with the interval (0, 1).

As explained in the previous section, the number of NC Outliers decreases during the process of registration. Therefore  $\lambda$  can be considered as an approximated numerical expression under current  $N_{mc}$  for how well  $P$  and  $M$  are aligned. If  $\lambda$  falls into the interval (0,  $\lambda_c$ ) where  $\lambda_c$  is a relative small value, i.e. 0.1,  $P$  and  $M$  are assumed to be “close enough”. In this case,  $mean_{SD}$  is a strong constraint and a much simpler way to determine the threshold.

The term  $(N_{mc})^k$  is used as a balance factor. When  $N_{mc}$  increases, the number of NC Outliers will decrease since there will be more options for a point in  $P$  to establish a biunique correspondence. Therefore  $(N_{mc})^k$  will be relatively stable for a range of  $N_{mc}$ , which makes the threshold robust and flexible.

### C. Coarse-to-Fine Approach

As explained in the previous section, the number of NC Outliers increases as  $N_{mc}$  decreases. This property can be utilized by a coarse-to-fine approach. When  $P$  and  $M$  are aligned under a poor initial transformation with large rotation error, it is expected to bring  $P$  to  $M$  as close as possible first. In this case,  $N_{mc}$  is assigned with a large number to start the registration in a coarse manner. As  $P$  and  $M$  are converged gradually, more outliers should be rejected in order to keep  $P$  from being trapped into a local minimum. If  $N_{mc}$  is decreased step-by-step between iterations, fine registration is expected since more NC Outliers will be rejected than the case of  $N_{mc}$  being not decreased. Clearly, the lower bound of  $N_{mc}$  is set to 1, since there exists at most one correspondence for every point in  $P$ . Also, by decreasing the number of multiple closest points, the computation efficiency is maintained at the same level of that only one closest point is searched.

Assume that the point number of  $P$  is less than the one of  $M$ . If we set  $N_{mc}$  to be the point number of  $M$ , we can establish a biunique correspondence for every point in  $P$ . However, the distances of some corresponding point pairs will be very large, which increase the difficulty of the calculation of the threshold for outlier rejection, not to mention the highly increased computational cost. An upper bound for  $N_{mc}$  must be defined to avoid these problems. Alternatively, setting  $N_{mc}$  a very small value for initialization will produce less biunique correspondences for the first few iterations, which makes the algorithm fall into a local minimum if there is not enough overlapping area. Generally, since the initial positions and the accurate size of overlapping area between two range images are unknown,  $N_{mc}$  can be set a number between 7 and 10 initially to adapt most cases.

As explained in the previous sections,  $\lambda$  depends on both the current state of registration and  $N_{mc}$ , using  $\lambda$  as a criterion to decrease  $N_{mc}$  is not a good idea. It is more reasonable to

use the ratio between the number of inliers which are the biunique correspondences that pass the distance threshold test, and the total point number of the *data set*. It is expected that the number of inliers is approximated equal to the number of points in the overlapping area of  $P$  and  $M$  when they are converged to the global minimum. Since the process of pair-wise range images registration can be considered as the process of maximize the overlapping area of two range images, the number of inliers is approximately maximized when two range images are registered correctly, and so is the inlier ratio.

## III. OUTLINE OF BC-ICP ALGORITHM

The pseudo code of BC-ICP algorithm is proposed in *Algorithm 1* and some details of implementation are also discussed.

### Algorithm 1: BC-ICP

**Input:** two range images: the *data set*  $P = \{p_i, i = 1, \dots, n_P\}$ , and the *model set*  $M = \{m_j, j = 1, \dots, n_M\}$ .

**Output:** the rigid transformation  $T$ , which is shown in (1).

```

1  Subsample the data set  $P$  uniformly ;
2  IterNum = 0.
3  while( IterNum < MaxIterNum && not converged ) do
4    IterNum = IterNum + 1;
5    for ( every point  $p_i$  in the sampled  $P$  ) do
6      Find  $N_{mc}$  closest points  $q = \{q_k, k = 1, \dots, N_{mc}\}$ 
        for  $p_i$ , where  $q \subset M$ ;
7      Sort the distance between  $p_i$  and every point in  $q$ 
        in ascending order;
8       $k = 1$ ;
9      while (  $k \leq N_{mc}$  ) do
10       if (  $q_k$  is assigned to another point in  $P$  ) then
11          $k = k + 1$ ;
12       else
13         assign  $q_k$  to  $p_i$ ;
14         break ;
15       end
16     end
17     if ( no biunique correspondence for  $p_i$  ) then
18       define  $p_i$  as a NC Outlier ;
19     end
20   end
21   Discard all of the NC Outliers ;
22   Compute the threshold  $t$  using (2) ;
23   Reject biunique correspondences with squared
     distance larger than  $t$  ;
24   Compute the rigid-body transformation matrix
      $T$  using Least-Squares minimization ;
25   Apply  $T$  to the data set  $P$  ;
26   if ( Inlier ratio increases over a constant value ) then
27      $N_{mc} = N_{mc} - 1$ ;
28   end
29 end

```

### A. Implementation

The BC-ICP algorithm is implemented using C++. To keep the algorithm simple and easy to implement, uniform sampling is preferred in this paper. We use the Approximate Nearest Neighbor (ANN) Library [13], which is originally programmed by Mount and Arya, to implement the k-D tree in order to accelerate the closest point search. After biunique correspondences are established, the rigid transformation is calculated using the SVD-based Least-Squares minimization [14]. The algorithm terminates when the difference of Mean Square Error between two consecutive iterations is close to zero, or the number of iterations exceeds a predefined maximum.

## IV. EXPERIMENTAL RESULTS

The range images used in experiments were grabbed from a Bumblebee2 stereovision camera manufactured by Point Grey Research Inc. In our case, the sensor noise level is much higher than the use of 3D laser scanner, which makes the evaluations more convincing under practical purposes. All the range images used as the *data set* were uniformly sampled and we only used a few percentages of total points to increase the efficiency. All experiments were done off-line on a PC with Intel Core i5 2.66GHz CPU, 4GB main memory and Window 7 OS.

### A. Pair-wise Registration

We tested the performance of BC-ICP comparing with other three algorithms: the original ICP (OICP), Picky ICP (PICP), and Fractional ICP (FICP). Since the matching step and the outlier rejection step are the most critical steps in ICP framework, we only tested these two steps and set other steps the same for all four algorithms. The Root Mean Square Error (RMSE) was evaluated with respect to large rotation error and different overlapping area between pair-wise range images.

Two real-world scenes were tested for pair-wise registration. First, range images from a statue of Beethoven were registered under poor initial alignment with large rotation error, as shown in Figure 2(b). The *data set* (red) has been rotated 50 degrees with respect to both X-axis and Y-axis. There was approximately 60% overlapping area between the *data set* and the *model set*. Although massive outliers existed, BC-ICP still found the correct solution while all other three algorithms trapped into a local minimum. Initially,  $N_{mc}$  was set to 7. Applying the coarse-to-fine approach,  $N_{mc}$  was decreased by 1 for each time until it reached the lower bound as mentioned in Section II.C. After sampling uniformly, 1700 points were used to find the rigid transformation between two range images. The algorithm took 1017.7ms and 108 iterations to converge.

Second, range images from a corner of an indoor working environment were tested. Since this kind of environment contains many planes, point-to-plane metric usually produces much better results than point-to-point metric. However, the normal vector information is not available only by using the stereovision camera. Moreover, calculations for the normal vectors increase the

computational cost drastically; it can hardly be applied to real-time applications. In our experiment, the *data set* (red) has been rotated 40 degrees with respect to all three axes, as shown in Figure 3(b). BC-ICP greatly increased the robustness of registration without using any normal vector information. Each range image used here contains approximately 50000 points and we subsampled the *data set* to 2000 points for registration. The algorithm took 1339.89ms and 114 iterations to find the correct solution. Again, all other three algorithms converged to a bad local minimum.

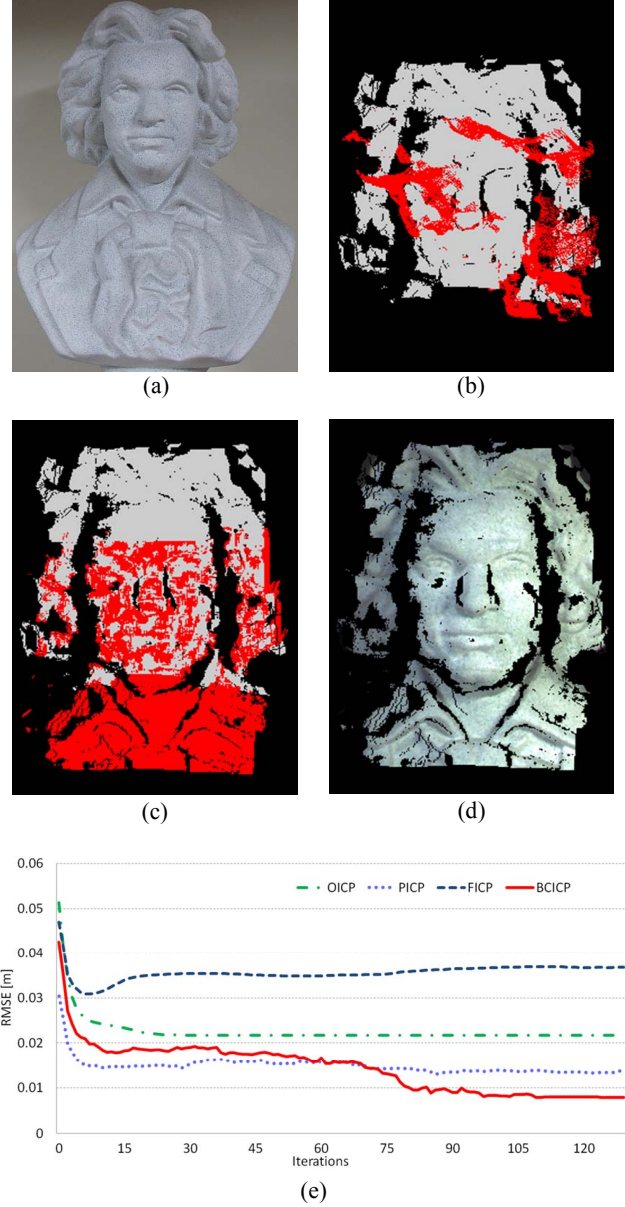


Figure 2. (a) Picture of the Beethoven statue. (b) Initial position of two overlapping range images. The red one is the *data set* and the grey one is the *model set*. (c) Registration result using BC-ICP with pseudo color. (d) Same result of (c) with real textures. (e) The evaluation for RMSE.



## B. 6DOF SLAM

For 6DOF SLAM experiments, we compared the performance of BC-ICP with OICP. The stereovision camera was installed on a mobile robot which was controlled by human. We conducted experiments in a long corridor environment, as shown in Figure 4(a). The robot traveled approximately 25 meters while 430 frames were captured. Each frame contained average 11000 points and about 1300 points were used for registration.

We first considered the case without using odometry information. In order to find the robot motion between time  $t$  and  $t+1$ , we applied the registration result between time  $t-1$  and  $t$  as the initial estimation, and then run the OICP/BC-ICP algorithm to get the final transformation result. Due to lack of textures in the corridor, range images captured from this kind of environment contained numerous noises, which make the task of localization extremely difficult. Without odometry information, small error accumulated between consecutive frames in a long sequence. The quality of localization and mapping was degraded, which led to inconsistent scenes and trajectories, as shown in Figure 4(c) for the case of BC-ICP.

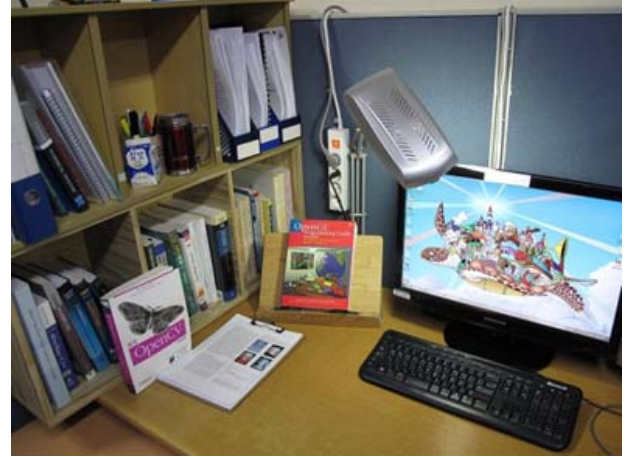
Applying odometry information as the initial estimation, we had much better SLAM results using BC-ICP, as shown in Figure 4(d). The standard deviation of RMSE for BC-ICP is smaller for the case without using odometry as shown in Table I, indicating that the accumulated error was spread over the whole set. While for the case of OICP, the task of localization and mapping clearly failed since motions between consecutive range images were not able to be retrieved, as shown in Figure 4(e). For each motion with the use of odometry, BC-ICP used average 63ms to find the correct solution. Considering the time spending on range image acquisition, BC-ICP is quite suitable for fast 6DOF SLAM applications.

TABLE I. COMPARISON OF BCICP AND OICP WITH AND WITHOUT USING ODOMETRY.

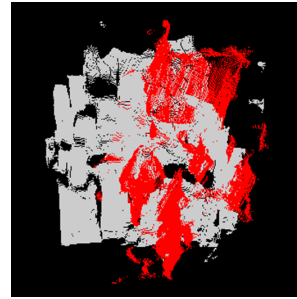
Odometry	Algorithm	Aver. Time [ms]	Aver. # Iter.	Aver. RMSE [m]	Std. Dev. RMSE
No	OICP	107.78	35.13	0.038	0.048
	BCICP	99.25	24.58	0.026	0.016
Yes	OICP	226.02	69.3	0.138	0.063
	BCICP	63.72	15.34	0.024	0.012

## V. CONCLUSIONS

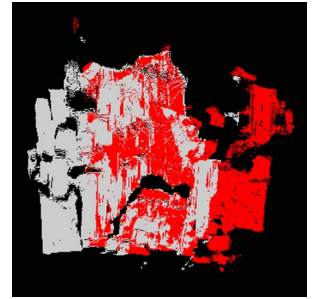
In this paper, we have proposed a novel variant of ICP algorithm, called BC-ICP, which establishes biunique correspondences to increase the performance of the original ICP. A new kind of outlier, called No-Correspondence Outlier, is defined, which is the point that cannot be assigned to a biunique correspondence. By discarding NC Outliers, the robustness of ICP has been greatly increased when large rotation error and non-overlapping areas exist. In addition, the threshold for rejection of correspondences with large distance is calculated based on NC Outliers. A coarse-to-fine approach is adopted to make the proposed algorithm more flexible and maintain the computation efficiency.



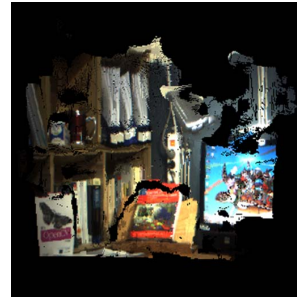
(a)



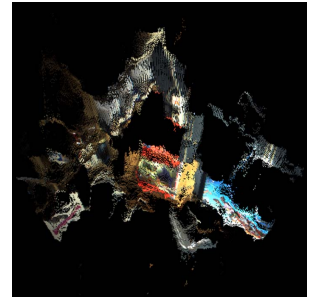
(b)



(c)



(d)



(e)

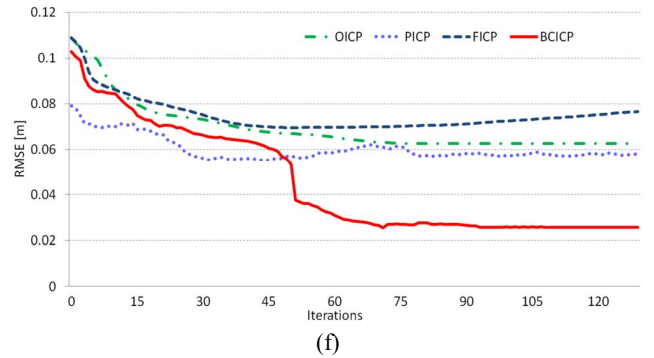


Figure 3. (a) The indoor environment for experiment. (b) Initial position of two overlapping range images. The red one is the *data set* and the grey one is the *model set*. (c) Registration result using BC-ICP with pseudo color. (d), (e) Registration result with real texture, front view and top view. (f) The evaluation for RMSE.

Comparing with other variants of ICP algorithm, better convergence and stability are provided by BC-ICP. Experiments show that BC-ICP can converge to the global minimum under approximately 40-50 degrees rotation error. We also apply BC-ICP to fast applications of 6DOF SLAM for indoor environments. Future work includes increment of computation efficiency and loop closing for SLAM.

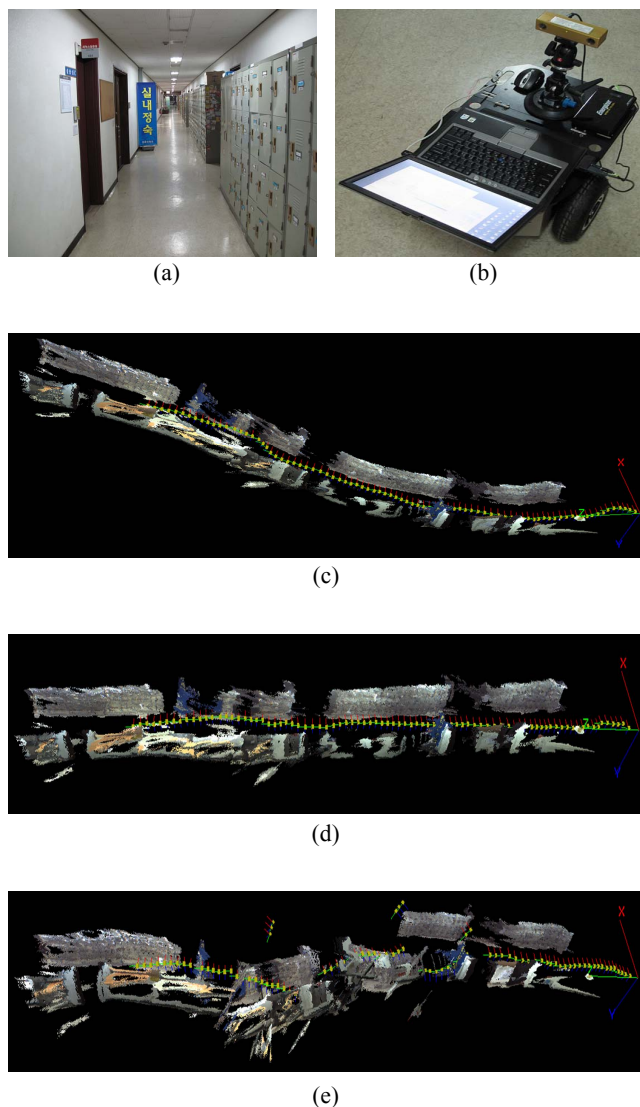


Figure 4. (a) The environment of experimented corridor. (b) The stereovision camera was installed on the robot. (c) SLAM experiment result using BCICP without odometry. (d) SLAM experiment result using BCICP with odometry. (e) SLAM experiment result using OICP with odometry.

## ACKNOWLEDGMENT

This work was supported by the FM Electronics and the Agency for Defense Development, Daejeon, Korea.

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