

Design and Manufacture of a Chain Driven Motor Shaft for Formula Electric Applications

by

Jack J. Wanderman

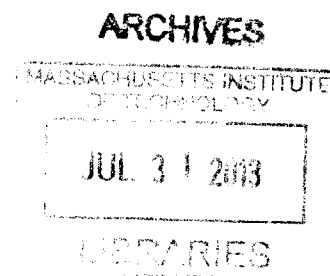
Submitted to the
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ABSTRACT

This document describes the design, manufacture, and testing of a motor shaft assembly that is a subcomponent of the drivetrain system in a Formula Electric racecar. The design covers detailed analysis of the bearing, sprocket, shaft, pins, and spacer created to meet a set of functional requirements. The process of determining these requirements is also covered, along with a set of best practices for thoughtful design in general.

An initial phase of testing is covered as well, from the design of the testing apparatus to the execution of a torsional test on the shaft. With limited data, conclusions are hard to draw with certainty, but initial results seem to indicate that the shaft performs as expected.

The intent of this document is to serve as an example of the level of detail and thoroughness that is expected when designing racecar components. To that end, this document should provide a number of unique examples of how specific problems were modeled and addressed in the design. It should also serve as a template for future documentation of such design efforts.

Thesis Supervisor: Anette Hosoi

Title: Professor of Mechanical Engineering

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The MIT Motorsports alumni, current members, faculty advisors, sponsors, and fans. This team has given me more than I can possibly give back. Through outstanding mentorship, management, and friendship, those associated with the team have provided an experience unrivaled by any course and unmatched by any job. The lessons learned in an early morning, last minute, crunch-time haze may occasionally be forgotten, but the glow of confidence, understanding, and diligence that comes with that experience will last a lifetime.

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1. Introduction

The motor shaft assembly design that is explained in the following pages was created for use by the MIT Motorsports team to compete in the Formula SAE® Electric competition in Lincoln, Nebraska. The assembly is just one part of the overall drivetrain system that will power a full-sized racecar in the event. The shaft assembly was designed to transfer torque from the electric motor to the chain via a sprocket. Although the scale of the project is small, this document will address to great depth every important decision that was made during its design, fabrication, and testing to provide a thorough example of documentation for the future design work of the team.

1.1 Formula SAE®

Formula SAE® is a collegiate level design competition in which university students design, manufacture, and race an open-wheeled racecar. An international group of teams compete in dynamic events that test a car's handling, acceleration, and reliability. In addition to racing, teams compete in a set of static events that judge design, cost, and marketing.

SAE International facilitates the completion every year by hosting two US based events. The larger completion is held in Detroit, Michigan and draws up to 140 teams every year. The smaller completion is held as of this year in Lincoln, Nebraska, and will have 80 traditional internal combustion teams, plus a new group of 20 electric-only powertrain teams. SAE International publishes a rulebook [1] that outlines regulations for safety, frame construction, powertrain limits, etc. Outside of the limits enumerated in this document, teams are free to design the car however they wish.

1.2 MIT Motorsports

The MIT Motorsports team was founded in 2001, and has produced 8 internal combustion engine cars to date. The team has achieved a high degree of success with its most recent vehicle, MY11, placing 9th in the West completion in 2011, and in 16th place in Detroit last year. This year, the team made the decision to switch to an electric drivetrain to compete in the brand new Formula SAE® Electric competition. This marks the sharpest change of design direction for the team to date, and has presented a variety of new and somewhat unforeseen challenges.

2. Design Methodology and Philosophy

This document details in depth the care and consideration that went into the design and manufacture of a select number of parts. Rather than a manual to guide future team members through a motor shaft design process, it should serve as an example of a thoughtful design exercise. The process of careful researching, planning, modeling, manufacturing, and testing described here should be present in every critical assembly that the team creates.

All extraordinary design comes as a result of maximizing desirable characteristics within the constraints of the system. The desired characteristics for most of the parts of car are clear – we want the lightest weight possible part that costs as little as possible. To achieve a high level of the design, then, the difficult task becomes accurately characterizing the constraints and developing meaningful functional requirements that will determine the boundaries for the design.

2.1 Constraints on the Designer

Before discussing the physical constraints that exist in component design, it is critical to also discuss the constraints on the designer. In fact, a good understanding and management of these constraints can make the difference between a completed component and a half-finished venture.

In spite all of the rules, cost regulations, and resource limitations that come as a part of this competition, time has consistently proven to be the biggest factor in limiting the success of designs. Many Formula SAE® teams compete on a one year design cycle, meaning that a new car is produced and brought to competition every year. Such a short cycle pushes designers to be aware of every remaining week until competition. To be an effective designer, such diligence is critical.

The tendency for many designers when faced with a pressing deadline is to rush through the details of a design in order to begin fabrication. While it can be important to know when to curtail the optimization and analysis of a component, the details of manufacturing and assembly should never be overlooked. As Durand [2] mentions, maintaining an accurate tool list for each component saves time and money for the designer and the team as a whole. Additionally, the exercise of planning out every step of the machining or assembly process for a component can help the designer highlight unforeseen problem areas.

In addition to time, the designer may also be limited in his or her own expertise or knowledge in the topic. It seems intuitive that, in order for a designer to achieve a high level of design, he or she needs to utilize highly complex models or tools, the likes of which a new team member may be unfamiliar with. In fact, this is a fallacy. In truth, almost any member of the team is capable of excellent design given a basic knowledge of physics. The critical step for a designer is to abstract the physical problem to the point where it fits within his or her body of knowledge. Doubt of one's own ability can be as great a constraint to the design as large forces or limited space.

Similarly, overconfidence is also an important constraint. It is critical for a designer to continually ask themselves or others, "How could I be wrong?" An excellent mechanism to formalize this behavior is to institute team or sub-team wide design review meetings, where members of the team are forced to defend their design choices in front of a group.

Finally, a designer is constrained by the design tools and software that is available to him or her. MIT Motorsports uses the most current version of SolidWorks, as provided by the mechanical engineering department at MIT, as its primary CAD package. SolidWorks Simulation is the attached FEA package that is used to analyze the strength of designed parts. Although this software is quite powerful, results are only as useful as the original test setup allows them to be. With that in mind, parts are designed with high safety factors. For parts in the driveline, the team adheres to a factor of safety of 2 at minimum. Furthermore, physics-based, "by hand" calculations should be executed wherever possible to check the validity of FEA calculations. As a general rule, the team suggests completing these calculations first, before doing any FEA.

2.2 Constraints on the Assembly

In a generic assembly for the racecar, designs are typically constrained by some combination of the following: external loading, geometry of pre-built parts, and rules specific to the assembly found in the rules document. Different systems in the car tend to have varying levels of

dependence on these constraints. For example, the internal combustion engine used in the previous iterations of the car was a stock, purchased motorcycle engine. As a result the entire driveline system needed to be designed to fit and operate around its geometry, but there were relatively few rules governing that assembly's construction. By contrast, many of the safety systems on the car are completely custom-made, but their design is heavily regulated in the rules.

In the following sections, this paper will use the motor shaft assembly for the MY13 Electric Vehicle as an example of how to identify and interpret these physical constraints.

2.3 Functional Requirements

Once the constraints of the system have been identified, the designer can construct a list of functional requirements for the part or assembly. These requirements should be specific and measurable. For example "Motor shaft is lightweight" would not be an adequate requirement. Instead, a statement such as "Motor shaft has a dynamic moment of inertia less than 1/100 that of the motor itself" would be a good requirement. Some may draw a distinction between functional requirements (characteristics that directly relate to the explicit performance and behavior of the part) and so-called non-functional requirements (other relevant characteristics like cost and serviceability) however this distinction will not be addressed in this paper. For the purposes of these parts, all relevant characteristics are relevant to the function of the part, and will thusly be referred to simply as functional requirements.

3. Mechanical Design

The overall powertrain package is based around a dual motor setup, with each motor independently powering each of the two rear wheels. The motors selected are Enstroj EMRAX liquid cooled motors. These motors are capable of supplying well over the 42.5 kilowatts of power that we expect to be our maximum draw¹ at 300 Volts. The rated maximum torque for the motors is 220 Newton meters for the purposes of this analysis.²

A direct load path from the large sprocket to the motor is desired in order to avoid passing the relatively large chain tension loads through the frame. Further description of the benefits of this direct load path can be seen in [2]. To achieve this, each motor is bolted to an outboard plate, which is in turn bolted to the frame. The driveshaft is connected to a universal joint housing, which is in turn connected to a large sprocket. This housing is supported by bearings in the outboard plate, as well as an inboard plate to provide moment support for the chain tension loads. The large sprocket is attached by chain to a small sprocket mounted on a shaft that connects to the motor. The shaft is also supported by a bearing on the inboard plate. In this way, the chain tension load is passed through the compression of the plates, rather than the bending of the frame, allowing for a lightweight, robust design.

¹ The rules [1] specify a maximum power draw of 85kW. This will be split between both motors.

² This was the listed maximum torque at the time that the analysis was completed. Since then, Enstroj has made more detailed analysis available [3], showing that our expected torque will be somewhat lower than this original guess. For the purposes of this design, that difference simply adds an additional safety factor.

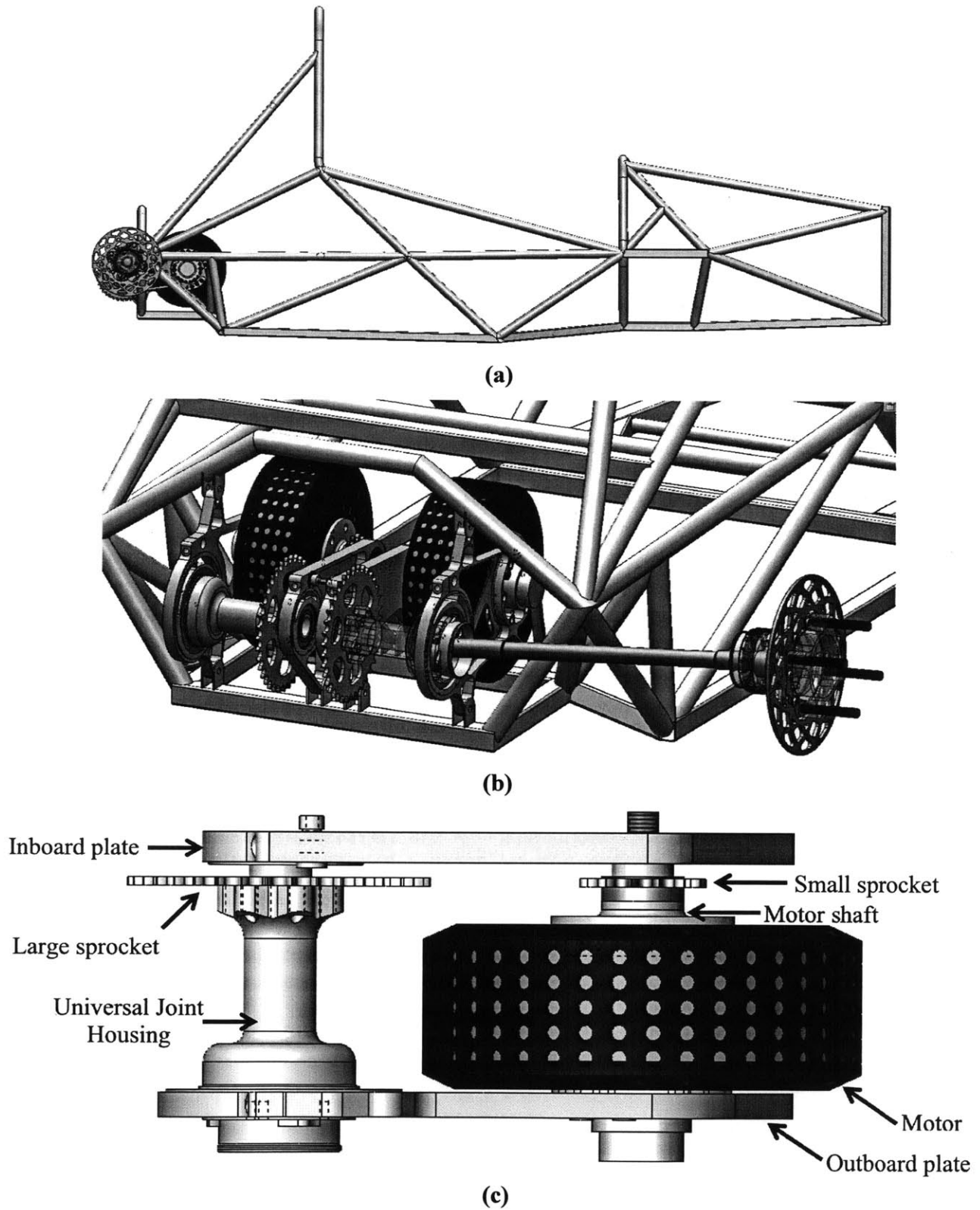


Figure 1: Showing the powertrain package in CAD. (a) Relative to the rest of the frame; (b) In perspective, and showing attachment to the driveshaft and wheel hub; (c) On of the symmetrical modules, with parts labeled.

The sub-assembly of this system that will be the focus of this paper includes the motor shaft, the small sprocket, the bearing that supports the shaft, the axial retention nut, the spacer that transmits axial load, and the pins that constrain the sprocket rotationally. From here on, this sub-assembly will be known as the motor shaft assembly.

The constraints mentioned in Section 2.2 include external loads, geometry of the surrounding parts, and any constraints imposed by the rules. For the motor shaft assembly, the contest rules do not specify any requirements, so we are left with external loading and the geometry of the rest of the system as constraints.

To determine the external loading that shaft would encounter, several strategies were considered. Previously published theses by Durand [2] and Scelfo [4], calculate chain tension by estimating vehicle acceleration, and working physically backwards from there. Our chassis team estimated a vehicle weight of 660 pounds and a maximum forward acceleration of 1.5g. Our tire diameter is 20 inches. The acceleration load is split between both motors, which effectively cuts the accelerating mass of each system in half, so torque at each large sprocket is

$$T_{large} = M \cdot a \cdot R_{tire} = \left(\frac{660 \text{ lbs}}{2}\right) \cdot \left(1.5 \cdot 9.8 \frac{m}{s^2}\right) \cdot (10 \text{ in}) = 559 \text{ Nm} \quad (1)$$

where T_{large} is the torque at the large sprocket, M is the mass of the car, a is the forward acceleration, and R_{tire} is the radius of the tire. Our large sprocket has 37 teeth with a $\frac{1}{2}$ inch pitch. This means that the chain tension force would be

$$F_{chain} = \frac{T_{large}}{(n_{teeth} \cdot Pitch) \cdot \frac{1}{2\pi}} = \frac{559 \text{ Nm}}{(37 \cdot 0.5 \text{ in}) \cdot \frac{1}{2\pi}} = 7470 \text{ N} \quad (2)$$

where F_{chain} is the chain tension force, n_{teeth} is the number of teeth in the large sprocket, and $Pitch$ is the pitch of the large sprocket. The torque at the small sprocket with 14 teeth, T_{small} , would be

$$T_{small} = T_{large} \cdot \left(\frac{14}{37}\right) = 559 \text{ Nm} \cdot \left(\frac{14}{37}\right) = 211.5 \text{ Nm} \quad (3)$$

Alternatively, we can recognize that the motor provides the only reaction torque in each powertrain system. To that end, the maximum torque that the small sprocket could see would be maximum reaction torque that the motor is capable of producing. This information can be found using data from the motor manufacturer. In this case, Enstroj listed 220 Newton meters for the EMRAX's peak torque. The fact that this value lines up well with the calculated values from Equation (3) helps to validate our assumptions.

Using Equation (2) with the new, slightly higher torque yields a final chain tension of 7770 N. This tension load is supported through the bolted connection to the motor from the shaft, and by a bearing on the inboard plate. Due to the spacing of the plates and the placement of the large sprocket, the small sprocket teeth will be biased toward the inboard side, meaning that the bearing will take up most of the tension load. By creating a ratio of the lever arms on either side of the sprocket, we can say that the bearing will support at least $\frac{2}{3}$ of the load, or 5180 N.

The lifetime and reliability of the parts must also be considered. We can get a rough estimate of the number of cycles we expect at the motor shaft estimating the total runtime of the car and an average speed. Typically, even with extensive testing, the team's cars see less than 10

hours of runtime, $t_{runtime}$. An aggressive estimate for an average speed over that time, v_{car} , is 40 mph. With a tooth ratio of 37:14 and a tire diameter, d_{wheel} , of 20 inches, we can say

$$N_{small} = \frac{t_{runtime} \cdot v_{car}}{d_{wheel} \cdot \pi} \left(\frac{37}{14} \right) = \frac{10 \text{ hours} \cdot 40 \text{ mph}}{20 \text{ in} \cdot \pi} \left(\frac{37}{14} \right) \cong 10^6 \quad (4)$$

where N_{small} is the number of expected cycles of the small sprocket and shaft. This tells us that all of the motor shaft components must survive 10^6 cycles of loading.

Regarding the geometry constraints, there are a few parts that were purchased, and the overall architecture of the powertrain system had already been designed. The motor shaft needed to bolt on to the stock motor bolt pattern. The sprocket location between the two plates was set with the architecture of the full powertrain system. Whatever bearing that was selected needed to be adequately held and retained axially by the $\frac{3}{4}$ inch thick inboard plate. An off-the-shelf sprocket was chosen (more about this in Section 3.2) which had an inner bore of $\frac{3}{4}$ inch, which set the diameter for part of the shaft.

Armed with an enumerated set of constraints, we began constructing functional requirements for each part in the assembly. In the following sections, the functional requirements are listed, followed by a description of the design choices that attempted to meet those requirements, and finally comments about the relevant manufacturing processes used in the production of the part.

3.1 Bearing Selection

3.1.1 Functional Requirements

Functionality:

- Must support 5180 Newtons for 10^6 cycles without failure
- Must be sealed to an IP65 standard to survive rainy conditions

Manufacturability:

- Must be available from NSK America, a sponsor of the team

Integration:

- Must be less than 0.7 inches in width to allow room for an axial retention flange in the mounting plate
- Must have a bore large enough to support a shaft diameter that will not deflect more than 12.5 micron under a 7770 N in three point bending

3.1.2 Design Process

The NSK Bearing catalog [5] contains a wealth of information regarding bearing life. There are detailed equations that relate a bearing life constant, C_r , to a specific loading condition. In this case however, since C_r is originally calculated using 10^6 cycles, no extra calculation is needed. We can simply read the maximum radial load value out of the catalog – again looking for a load that exceeds 5180 N. Applying our safety factor of two, we search for C_r values that are larger than 10360 N.

The two smallest bearings in the catalogue that meet this criterion are the 6302 and the 6303 bearing, the latter being a slightly larger in bore, outer diameter, and thickness. This has the

disadvantage of adding weight, but may have the advantage of supporting a thicker and therefore stiffer shaft.

A rough estimate for the sensitivity of the shaft can be done by approximating its bending deflection between the bearing and the motor as simple three point bending. This does not accurately match the real world situation, but the simple calculation is useful for comparative purposes. In three point bending of a cylinder, the deflection of the beam is given by

$$\delta = \frac{FL^3}{48E} \cdot \frac{4}{\pi \cdot r^4} \quad (5)$$

Using this relationship, we can estimate the deflection of in the shaft due to bending, and we can compare the 15mm bore bearing (6302) with the 17mm bore bearing (6303). In this case, while the larger bore does improve the deflection by close to 40 percent, both of the deflections are in the single micron range, which is well within the bounds of our functional requirements.

In this case, while the 6302 bearing seems to be the best option, we must consider the availability from our sponsor, NSK. We needed the sealed DDU-type bearing in order to safely operate in the rain, and the 6302DDU was unavailable at the time of order. The 6303DDU was substituted as the next best option.

3.1.3 Manufacturing

The bearing came fully assembled, and required no maintenance. It did need to be pressed in to the inboard mounting plate, though. It should be noted that the NSK suggested fit for bearings that will have the shaft applying torque is that the bearing be pressed onto the shaft, not into the housing. In this case however, the bearing was pressed into the plate for ease of assembly. As explained in Section 3.3.3, great care was used to make the shaft as close a sliding fit as possible to insure that there would be no slipping between the shaft and the bearing.

Guidelines for pressing bearings into the housing are given in [5]. It is particularly important that the housing bore is appropriately sized using the fit tables in [5] and adjusted for the fact that the housing material is aluminum instead of steel. For this application, the appropriate interference was determined to be approximately .001 inches. When pressing bearings in, force should never be applied through the rolling surfaces, so an annular spacer was used to insure that the force from the press was only applied through the outer race. Heating the housing prior to pressing the bearing in will also allow a shrink fit, which can achieve a tight interference without the application of much force.

3.2 Sprocket

3.2.1 Functional Requirements

Functionality:

- Must transmit 220 Newton meters of torque from shaft to chain without failure

Manufacturability:

- All machining and hardening processes must be achievable with available tools (e.g. no EDM is allowed.)

Integration:

- Must locate axially to specified location on shaft

- Must accept size 415 chain

3.2.2 Design Process

There are a number of methods of connecting a sprocket to a shaft in order to transmit torque. The usual strategies involve either a keyway or a splined shaft connection. A keyway involves a rectangular channel cut into the outer diameter of the shaft and the inner diameter of the sprocket. A metal “key” is inserted into this channel to couple one part to the other.

This design is simple to machine and assemble. However it introduces large stress concentrations at the bottom edges of the channels. Even ignoring these concentrations, it is relatively simple to estimate the pressure, P , that would exist on a large keyway if one were implemented. If we assume that a keyway is 0.25 inches deep, x , and 0.8 inches long, L , and that the inside radius, r , of the sprocket is 0.375 inches, we get

$$P = \frac{T}{r} \cdot \frac{1}{x \cdot L} = \frac{220 \text{ Nm}}{.375 \text{ in}} \cdot \frac{1}{0.25 \text{ in} \cdot 0.8 \text{ in}} = 179 \text{ MPa} \quad (6)$$

where T is the applied torque. This value is approximately half of the yield strength of 6061 Aluminum, and doesn't factor in any of the stress concentration or the uneven loading due to the geometry of the key. Further FEA confirmed that a keyway would likely result in yielding and failure in the shaft part.

For higher torque applications, many designers implement toothed, interlocking connections called splines to transmit torque. Although they are effective, it is very difficult to machine splines. Typically, inside splines must be broached, and outside spline hobbled or extruded. Since we had none of these capabilities in-house, we looked to other options for attaching the sprocket and shaft.

The final sprocket design transmits torque to the shaft primarily via friction. By using a nut on a threaded section of the shaft, force can be transmitted through the inner race of the bearing, a spacer, the sprocket, and back into the flange section of the shaft. The amount of force, F , needed to prevent slipping between the sprocket and shaft can be calculated by

$$F = T \cdot \frac{(r_{outer}^2 - r_{inner}^2)}{(r_{outer}^3 - r_{inner}^2) \cdot \mu} \cdot \frac{3}{2} \quad (7)$$

where μ is the coefficient of friction between the sprocket and the shaft, and r_{inner} and r_{outer} correspond to the dimensions of the sprocket annulus in contact with the shaft.

In the case that this friction hold slips, it is important that a backup form of torque transmission exists. To that end, two shear pins are pressed into the shaft and interface with the sprocket. These pins are rated for their shear strength. To withstand the torque,

$$SS \geq \frac{T}{d} \quad (8)$$

where SS is the shear strength, T is the torque, and d is the distance between the two pins.

The teeth of small sprockets must support a large load over a relatively small area, meaning that they tend to be a highly stressed section of the part. It is for that reason that typical small sprockets are made of steel, and the teeth are hardened. While making such a sprocket from scratch would have been possible, it would have diverted engineering and manufacturing effort

away from other projects, so in this case the decision was made to purchase an off-the-shelf sprocket and modify it to suit the design application.

In this case, that modification involved facing the sprocket to length such that it would line up with the large sprocket in the assembly and drilling holes for the shear pins that will carry the torque load should the friction hold slip.

Since size 415 chain is relatively uncommon, there were very few choices for sprockets available. A Martin sprocket was chosen because the teeth were hardened, but the body was still machinable. This gave us increased manufacturing flexibility. The nominal bore of the purchased sprocket was $\frac{3}{4}$ inch, and the outer diameter was 1.68 inches. The sprocket came with a keyway, but it was not used.

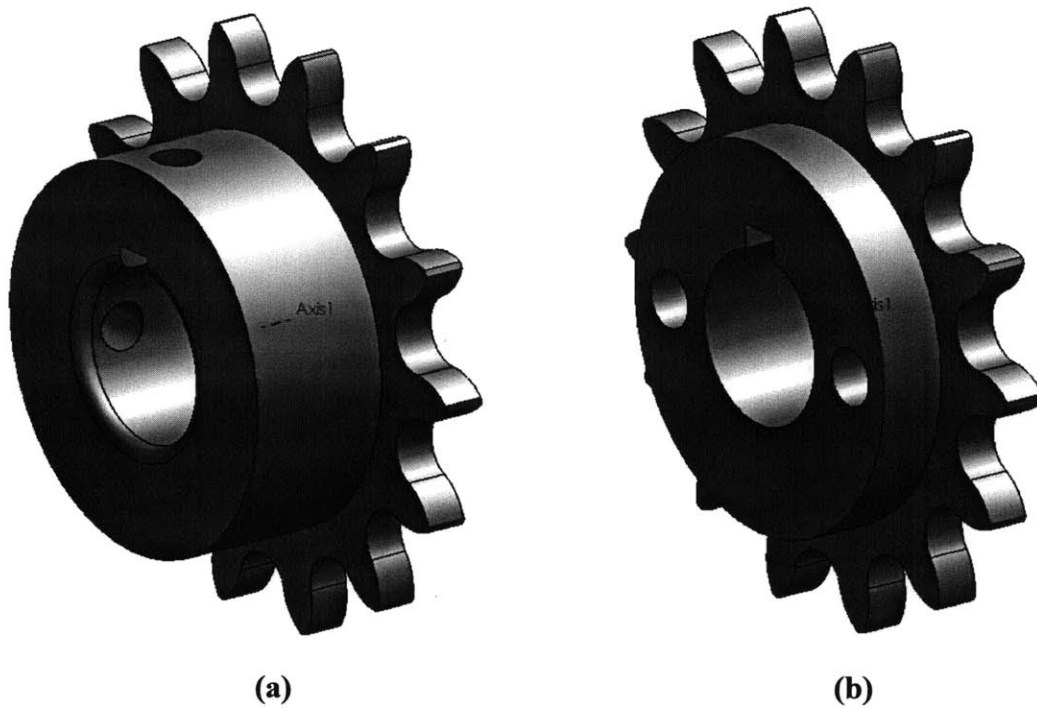


Figure 2: Showing design of sprocket (a) as an off-the-shelf part; (b) with modified thickness and shear pin holes.

3.2.3 Manufacturing

Since this part was intended to slide on and the motor shaft, properly tolerancing the shear pin holes was crucial. Additionally, since the part was meant to be held by friction on the motor shaft, the flatness of the faced surface was also important.

To achieve these machining steps, a single setup on a mill was utilized. A jig was created by machining an aluminum plate with a bolt circle along the pitch diameter of the sprocket. This allowed machine screws to be fastened between every other tooth with a washer underneath. By tightening the screws, the sprocket was held firmly to the plate.

The plate was then placed in the mill, and the exact center of the sprocket was located with a dial indicator. The flatness of the sprocket was also checked, and adjusted with a rubber mallet on the jig plate until the part was within a 0.001 inch flatness variance. The sprocket was faced to its final depth. The pin holes were then drilled, being careful to use a spotting drill first, then a

roughly 0.030 inch undersized drill, and finally a 0.001 inch oversized reamer, which was the closest available to the Machinery's Handbook [6] suggested sliding fit clearance.

3.3 Shaft

3.3.1 Functional Requirements

Functionality:

- Must transmit 220 Newton meters of torque from motor bolt circle to small sprocket without failure
- Must support a 34 kilo-Newton clamping force for the sprocket without failure

Manufacturability:

- All machining processes must be achievable with available tools (e.g. no EDM is allowed.)

Integration:

- Must support shear pins for the sprocket interface
- Must span the 1.844 inch distance from motor to the inboard edge of the bearing
- Must achieve a close sliding fit (RC1 in [6]) with the bearing bore
- Must be concentric to within .001 inches with the sprocket.

3.3.2 Design Process

To begin the design of the motor shaft, an initial concept was modeled in CAD. It incorporated all of the features that were known to be necessary including a bearing seat, a sprocket seat, holes for shear pins, and a flange to bolt on to the motor. None of the thicknesses or diameters were optimized to start.

Due to the complexity of the geometry, FEA was used extensively to guide the optimization of the part. Naturally, the proper setup of the FEA test is vital to the tests usefulness. It is important to consider worst-case loading scenarios. When using FEA, this means making assumptions that, while still physically plausible, air on the side of more stress. To that end, the shaft part was constrained by setting the bearing seat and a single pin hole as fixed hinges, and applying torque to the top three bolt holes on the part. This assumes that, because the sprocket is asymmetrically loaded, the majority of the load is transferred into the top half of the part.

The material chosen was 7075 aluminum, a strong alloy frequently used by the team for parts where a high yield strength is desired. Although this material is much more expensive than more common alloys like 6061, all aluminum alloys have the same price in the cost event in the competition. The only downside, therefore, is that it costs the team more to purchase the stock.

As shown in Figure 4, the original part had an unacceptably low factor of safety. As a result, the flange and the friction surface were thickened and the radius between them was increased. These changes increased the weight of the part by 0.2 pounds, which represented a roughly 20% increase in the weight of the assembly, and almost doubled the weight of the part.

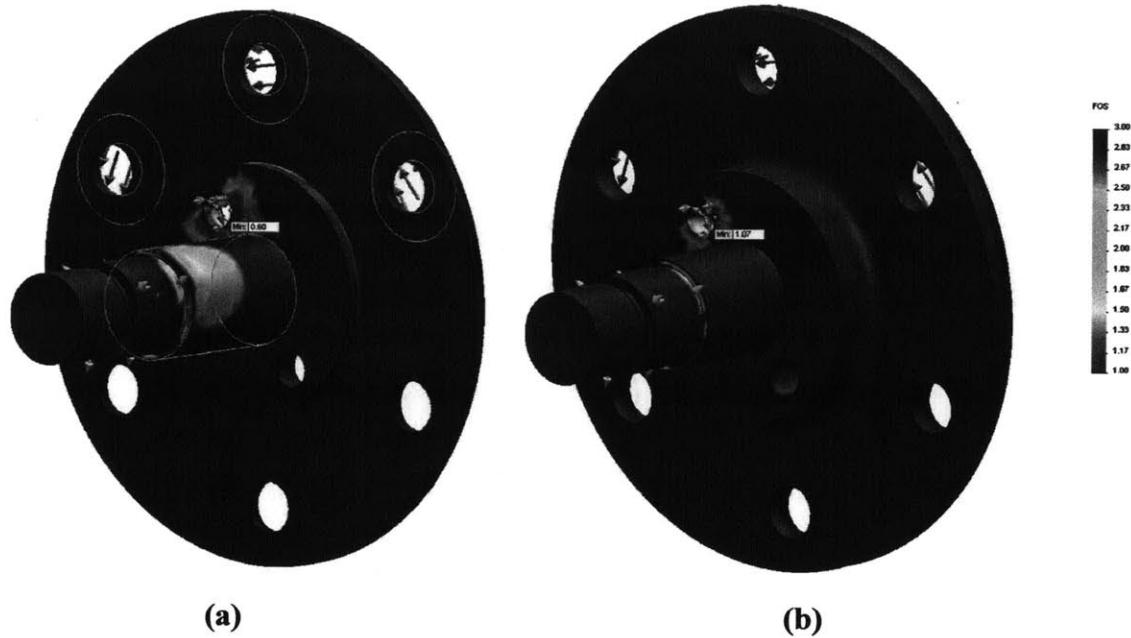


Figure 3: Showing finite element analysis of the shaft under torsional load. One of the pin holes and the bearing seat is held as a fixed hinge while 220 Nm of torque is applied to the top three bolt holes. In (a) the minimum factor of safety is 0.6, indicating yielding; by increasing the flange thickness, (b) increased the factor of safety to 1.07.

Another important takeaway at this stage is the role of “gut checks” and intuition when designing with FEA. Due to the simple nature of the loading conditions, it can be possible for FEA to indicate that a part will not yield, even when it is unrealistically small in a dimension. Perhaps the FEA setup failed to capture all of the forces that will be at work. It is the duty of the designer to insure that all of the dimensions of the part are reasonable.

For example, the flange on the shaft where it is bolted to the motor was originally 3/32 inch thick. While, the FEA indicates that the safety factor is high, intuition raises issue with the wall thickness. There will be some bending load in the shaft due to chain tension, and less than 0.1 inch thick aluminum may seem unreliable to the designer. Furthermore, such a thin flange would prove very difficult to machine to a high tolerance. A small increase in weight in favor of a slightly thicker flange was accepted to alleviate these concerns.

Special attention was paid to insure that the outside radii on the shaft were small enough not to interfere with the radii on the bore of the bearing or the sprocket. Furthermore, a threaded section on the end of the shaft was added to facilitate clamping through the bearing. A 5/8-18 thread was used as it was the largest standard size that would accommodate the bearing sliding over it. Fine threads were used to allow for the most flexibility with setting the clamping force. A thread relief was modeled, but was unnecessary due to the thread cutting capability of the lathe.

3.3.3 Manufacturing

This part was machined from a piece of 4 inch diameter 7075 aluminum round stock. The part was held in a CNC lathe by roughly ½ inch of material behind the flange that would be faced off later in the machining process. The CNC lathe operations turned all of the outside diameters and cut the threads in the end of the shaft. To insure a close sliding fit for the sprocket

and bearing, these diameters were finished a few ten-thousandths of an inch at a time, checking the fit with the actual parts in between.

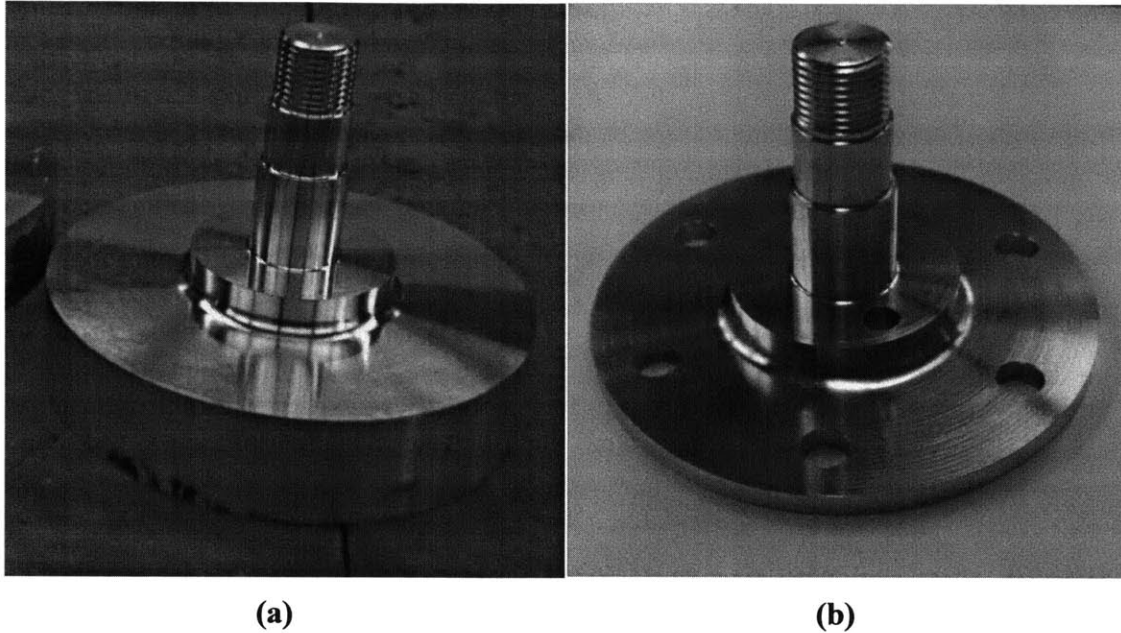


Figure 4: Showing the machining process (a) after turning; (b) the finished part.

After the profile was cut, the part was placed in a 4-jaw chuck that was bolted down to the table of a mill. In this way we could hold the part securely in the mill to drill the bolt circle and the holes for the shear pins. The part was checked for flatness and zeroed about the shaft with a dial indicator. The shear pin holes were .001 inch undersized as per press fit guidelines in [6].

Finally, the part was placed in a lathe once again to face the flange to its final thickness. The dowel pins were pressed in using an arbor press.

3.4 Pin, spacer, and clamping design

The shear pin that was selected for was a $\frac{1}{4}$ in hardened alloy pin. The size was the largest available that still left a reasonable wall thickness on the sprocket. They were located by centering them between the inside and outside diameter of the sprocket to keep wall thicknesses even.

The pins were rated from McMaster-Carr to break at 10000 pounds in double shear. From equation (8), we can calculate that, given their center-to-center distance of 1.215 inches, the shear strength of the pins must be 1603 pounds in single shear, giving us a factor of safety of 3.1.

The design intent is for the sprocket and the motor shaft to be joined by the pins with no perceptible play. This will insure that, should the pins be loaded, they will be loaded in pure shear. To achieve this fit the pins were pressed into the shaft, and a sliding contact fit was reamed into the holes in the sprocket. This arraignment was chosen because, aluminum is softer than steel, allowing a wider tolerance for the press fit. In the actual assembly, the sprocket is a light interference fit, likely due to small machining error regarding hole placement. This fit is acceptable, though, since the sprocket is still easily tapped off with a mallet if there is a need for repair.

As stated in Section 3.2, the pins should only be used as backup from for a friction hold between the shaft and the sprocket. From equation (7), we calculate that a clamping force of 34kN is necessary to hold the sprocket with friction alone. While this seems somewhat high compared to the other forces on the system, it still represents only a third of the yield stress of 7075 aluminum when spread over a 5/8 inch diameter surface.

A spacer was required to fill the space between the bearing and the sprocket. To evenly transmit clamping force, flatness needed to be tightly controlled. For this reason, the part was machined on a mill instead of a lathe. The surface finish was not ideal, but the part was flat to within .001 inch. The stress that the spacer would need to undergo is well within the elastic range of 6061 aluminum, which was chosen for its light weight and availability.

It was also important to insure that, as the shaft elongates under load, the bearing would not bottom out on the sprocket seat, which has a larger diameter. The thickness of the spacer determines a gap between the bearing and the sprocket seat. To determine how large the gap should be, the axial stiffness of the shaft was considered. Axial elongation can be determined by

$$\delta = \frac{FL}{EA} \quad (9)$$

The diameter of the shaft varies some, but it can be reasonably estimated as a single 0.7 inch diameter, 1.375 inch long cylinder. In that case, under 34kN of load the extension of the shaft would be 0.0027 inches. To give this value a factor of safety, the spacer was size such that there was a .007 inch gap between the sprocket seat and the bearing. Furthermore, with the extension calculated, it can now be measured with a micrometer as the nut is tightened to insure the proper compression load is achieved.

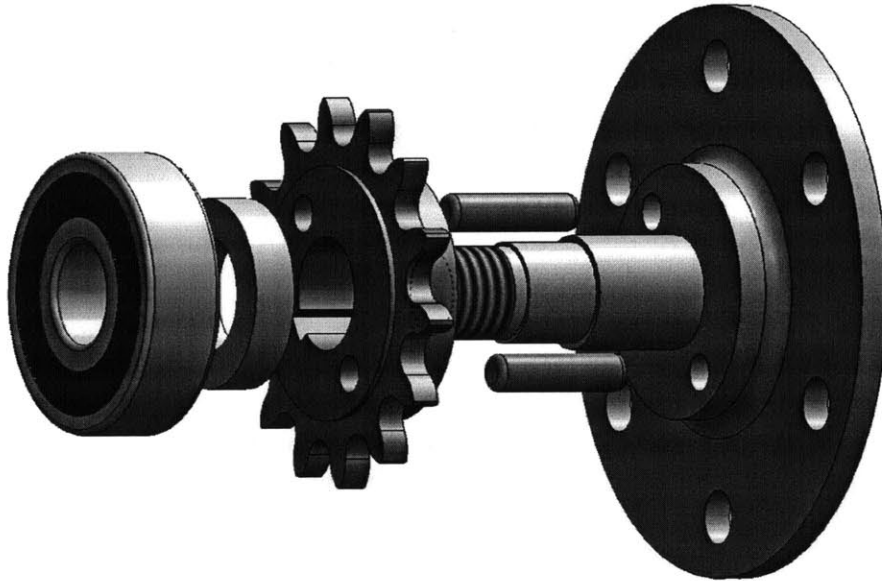


Figure 5: Showing the full shaft assembly, exploded view. Components from left to right: bearing, spacer, sprocket, dowel pins, shaft (includes threads, bearing seat, sprocket seat, friction surface, bolt flange).

4. Testing and Validation

Modeling and analysis are most meaningful when coupled with real-world validation. Since it would be extremely time intensive to test every aspect of an assembly, it is best to focus on the areas that are the highest risk. For the shaft assembly, the biggest question surrounds the analysis of the shaft itself. This analysis relied on FEA which made assumptions about the interface between the sprocket and the friction surface of the shaft which may be incorrect. More specifically, although the design and analysis claim that friction will be enough to hold the sprocket and shaft together, the load is probably shared to a degree between friction and the shear pins.

To test the part, an Instron machine capable of applying torque and measuring displacement was used. The sprocket, spacer, and bearing were removed from the assembly and replaced with a thick-walled steel tube. The tube was clamped down with a nut just like the sprocket, and there were holes that accepted the shear pins as well. The top end of the tube was made to fit into a collet in the machine. Another steel part was made to bolt on to the flange and fit into the other collet.

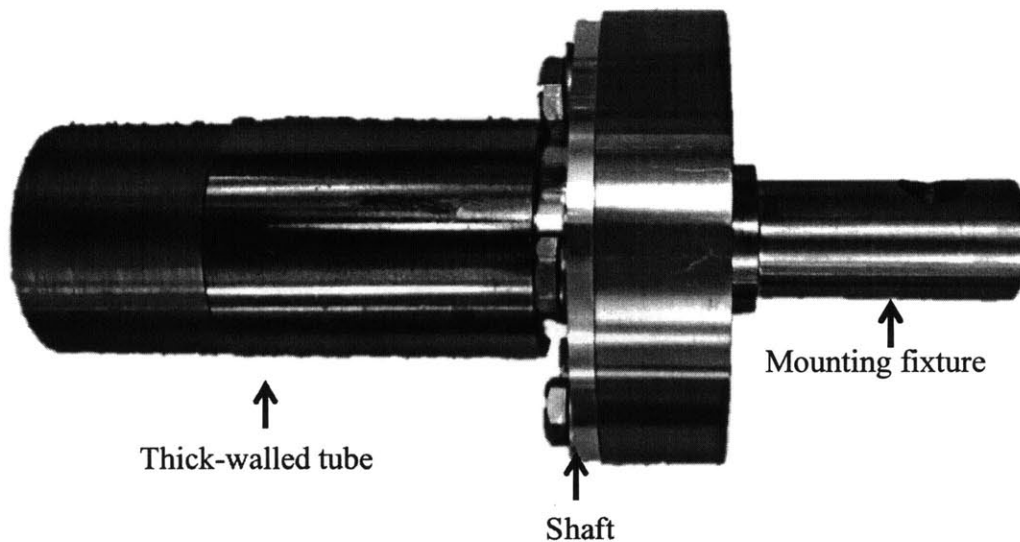


Figure 6: Showing the shaft testing assembly. A thick-walled tube is internally bolted to the friction surface of the shaft. The tube, like the sprocket, also has holes for the shear pins. The flange of the shaft is bolted on to a thick steel mounting fixture. The fixture is held while the tube is turned, and displacement is recorded.

In this way, torque is transferred to the shaft in the same manner as with the sprocket. By twisting the assembly and recording torque and rotational displacement, we can see evidence of any plastic deformation in the part. If the curve remains linear, then the part is operating within its limits.

To execute this test, the assembly was held as described above in an Instron machine capable of measuring torsion. Due to limitations of the machine's controller, the test was based

on position control rather than torque control. The assembly was turned to 0.9 degrees then turned backwards to 0. That process was repeated once more. That displacement was chosen by testing a variety of displacements to find one that would achieve a 220 Newton meter response. In this test, the actual peak load was 230 Newton meters.

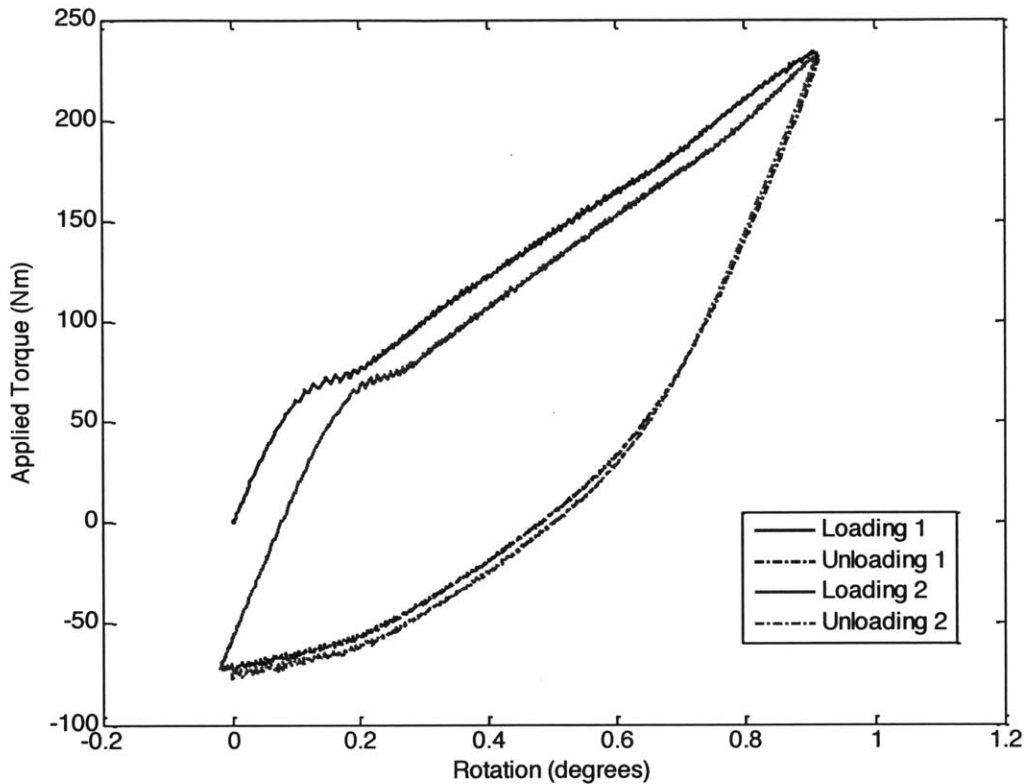


Figure 7: Showing the relationship of torque and rotational displacement over two consecutive loading and unloading cycles. The loading path for each cycle is mostly linear except for the first section, which may be an artifact of the machine. No evidence of yielding is seen near the peak torque, 230 Nm.

The data contain features that are somewhat unexpected. The unloading pattern does not match the loading pattern and in fact quickly begins applying negative torque. Also unexpected is the small transition plateau around 0.15 degrees. This feature seems consistent with slipping of some kind, but it is somewhat unclear what part of the assembly or the machine itself was slipping.

What can be determined with some certainty is that, once the curve exists the plateau, it appears to behave linearly. If the part were plastically deforming, one would expect to see a characteristic tail with decreasing slope at the high end of the plot. Instead, the torque goes linearly with the rotation all the way to the maximum load, indicating that the part is responding in its elastic regime.

5. Conclusion

There is still work to be done and improvements that can be made to this assembly in the coming years. Further testing, ideally involving strain readings directly from the part, would be very useful in gaining a more detailed understanding of the sprocket-shaft interface. This interface could also likely be made much lighter by implementing a splined shaft as opposed to a friction and pin hold. Splines are more difficult to machine, but allow the sprocket to get even smaller, saving weight.

A complete assembly-wide finite element model would also be useful to characterize forces. These kinds of models typically take much longer to solve, and may require the use of a more powerful FEA package. Nevertheless, this is an assembly that is limited enough in scope to make such analysis possible, and the completion of the exercise would provide a wonderful example of how to properly execute such analysis for team members in the future.

This motor shaft assembly is just one small part of a much larger powertrain system, which itself is only part of the entire racecar. This design exercise is meant to serve as an example and guide for parts and assemblies in general. There was a great deal of modeling, analysis, and thought that went into the creation of each part. This detailed focus is critical to achieving successful design. Cutting corners when it comes to analysis will only lead to delays in the long run when parts must be redesigned and remade. Ultimately, the goal of every team member should be the same: achieve good design. If that mentality is diligently followed, a high performance product will follow.

7. References

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