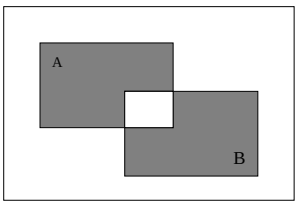
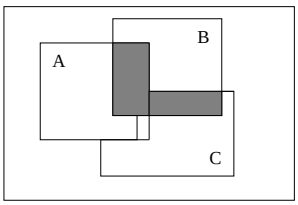
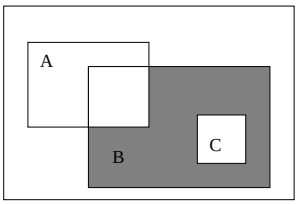
Midterm 1 Practice Problems

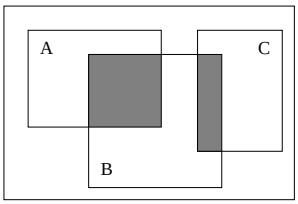
Below, you will find a number of practice problems on the same topics that will be covered on Midterm 1. If you don't know how to start a problem or you get stuck, ask for help on piazza, during office hours or during the review session. You can find answers but not complete solutions to these problems posted on Canvas.

1. For each of the following four figures, find the expression in set notation that describes the shaded set.









2. A library has 50 textbooks. Of these, 30 discuss probability, 25 discuss statistics and 10 discuss neither probability nor statistics.
   (a) Find the probability that a book selected at random discusses only statistics.
   (b) Find the probability that a book selected at random discusses exactly one of the subjects.

3. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35% . Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

4. Consider two events A and B. Suppose $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Find $P(A)$ (a Venn diagram may help).

5. Suppose $P(E) = 0.25, P(F) = 0.60$, and $P(E \cap F) = 0.13$. Compute the following probabilities (a Venn diagram may help):
   (a) $P(E|F)$
   (b) $P(F|E)$
   (c) $P(F|E')$

(d) $P(E \cap F|$ at least one event occurs)

6. You and a friend go out for dinner to a Pizza restaurant. Each person can choose from 20 different toppings for their pizza. You choose to have a pizza with three different toppings and your friend will have four. In how many different ways is that possible?

7. Consider a random reordering of the letters in "TENNESSEE". What is the probability of each of the following?

   (a) All the E's are together?

   (b) Both of the S's come before the T.

   (c) The N's are in first and last place.

8. Each state of the 50 in the U.S. has 2 senators. What is the probability that in a random committee of 50 senators

   (a) California is represented?

   (b) All 50 states are represented?

9. A machine selects five coins at random from a purse containing ten quarters and four nickels.

   (a) Find the probability that there is at least one nickel selected.

   (b) Find the probability that there are at least two nickels selected.

   (c) Given that at least one nickel is selected, find the probability that there are at least two nickels selected.

10. Three married couples have purchased theater tickets and are seated in a row consisting of just six seats. If they take their seats randomly,

    (a) What is the probability that Andy and Betty (husband and wife) sit in the two seats on the far left?

    (b) What is the probability that Andy and Betty end up sitting next to one another?

11. Crate #1 contains 3 apples, 4 oranges, and 2 pears. Crate #2 contains 5 apples and 3 pears. Julia wants a piece of fruit and to select it she will roll a fair die, selecting a random fruit from Crate #1 if the die shows a 3 or 4 and otherwise selecting it from Crate #2.

    (a) Find the probability that the selected fruit is a pear.

    (b) Given that the selected fruit is a pear, find the probability that it came from Crate #1.

12. Two balanced dice are rolled.

    (a) What is the probability that at least one is a six?

    (b) If the two faces are different, what is the probability that at least one is a six?

13. There are 3 drawers in a dresser‘, and you are equally likely to pick any of the three. In drawer 1, there are 2 black socks and 3 red socks. In drawer 2, there are 3 black socks and 2 red socks. In drawer 3, there are 3 black socks and 3 red socks. Once you have randomly selected a drawer, you randomly pull out a sock of that drawer.
    (a) What is probability you pull out a black sock?
    (b) Given that you pull out a red sock, what is the probability that you got it from drawer 2?
    (c) If you were to randomly choose a drawer and the draw **two** socks from that drawer, what is the probability that you get a pair (red or black)?
14. If four fair dice are tossed, what is the probability that they will show
    (a) four different faces?
    (b) all four the same face?
    (c) all four the number "2"?
    (d) three times the number "2" and one different number?
15. A fair coin is flipped three times. What is the probability that there will be at least two heads? What is the probability of flipping exactly two heads, given that there are at least two heads?
16. English and American spellings are *rigour* and *rigor* respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 40% of the English-speaking men at the hotel are English and 60% are American, what is the probability that the writer is an American?
17. An urn contains 3 white and 10 black balls. A fair die is rolled and that number of balls are chosen from the urn. What is the probability that all of the balls selected are white? What is the conditional probability that the die landed on 3 if all the balls selected are white?
18. In a probability class there are 4 junior boys, 6 junior girls, and 6 senior boys. How many senior girls must be present, so that sex and class are to be independent when a student is selected at random from this class?
19. A single card is drawn from a standard 52 card deck. Consider the events A = card is an ace, S = card is a spade, Q = card is a queen, F = card is a face-card (that is, either Jack, Queen, or King).
    (a) Are the events A and Q mutually exclusive? Explain why or why not.
    (b) Are the events A and Q independent? Explain why or why not.
    (c) Are the events Q and F mutually exclusive? Explain why or why not.
    (d) Are the events Q and F independent? Explain why or why not.
    (e) Are the eventsA and S mutually exclusive? Explain why or why not.
    (f) Are the events A and S independent? Explain why or why not.