CS 156:Introduction to Artificial Intelligence

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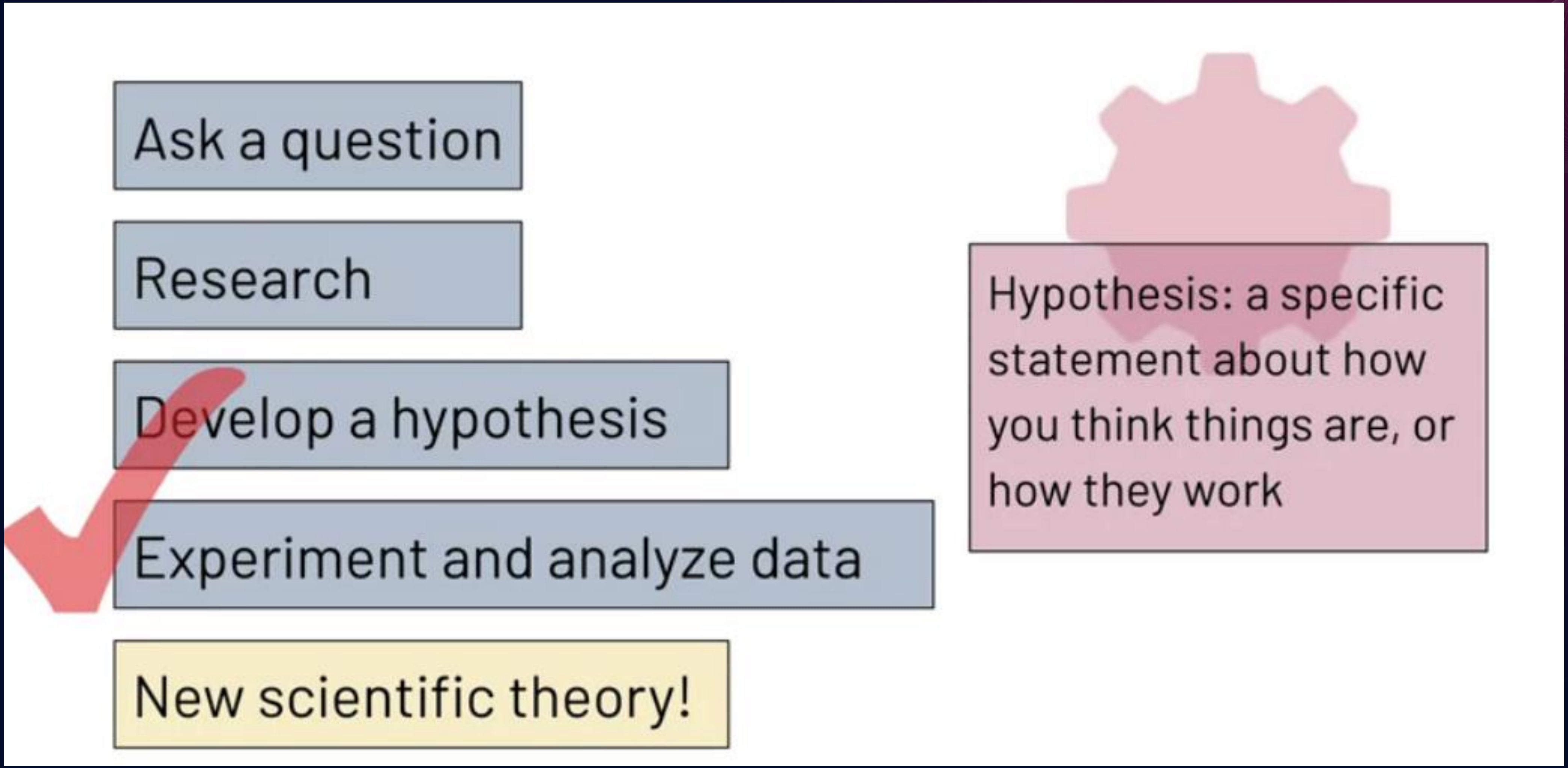
Classification Models in Machine Learning



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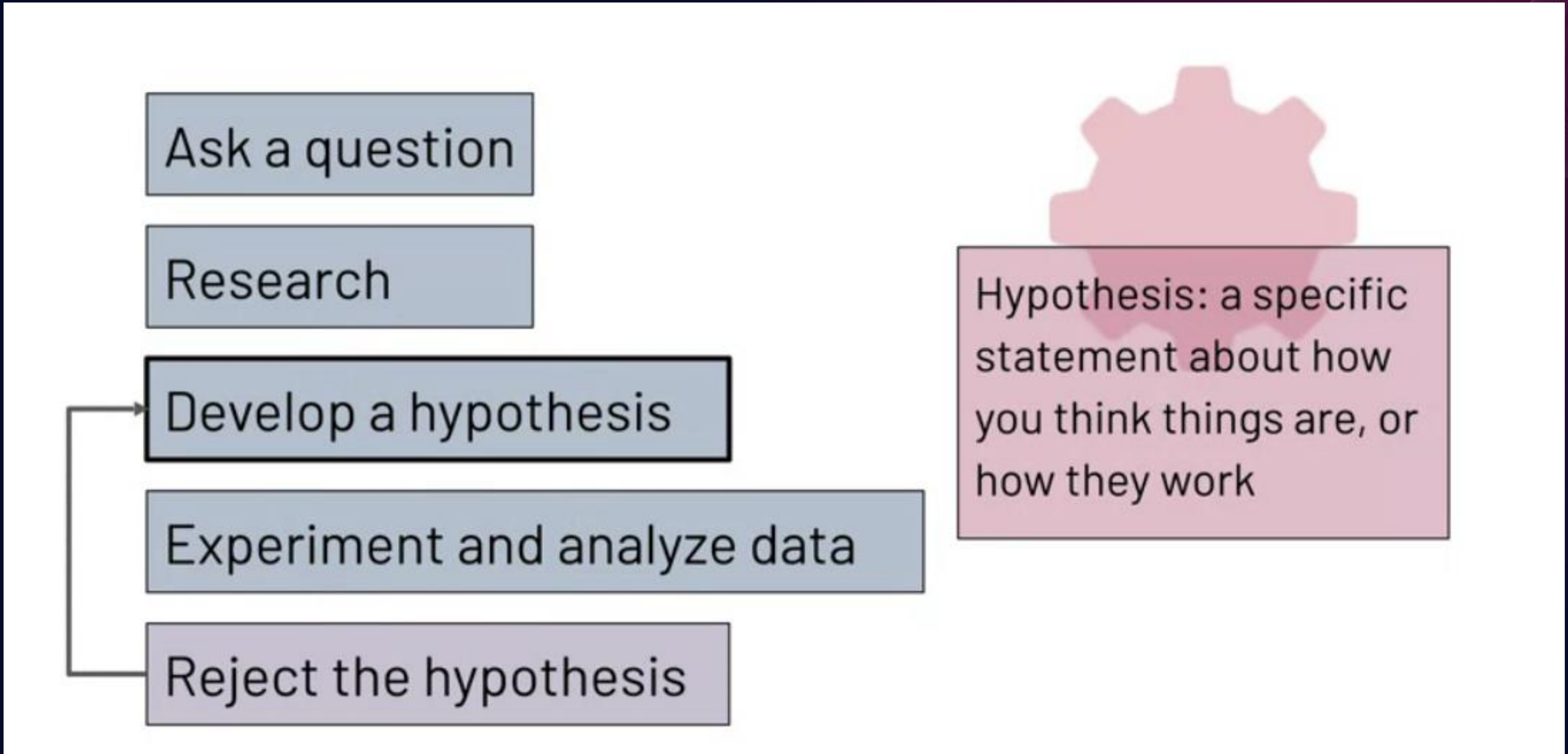
Classification Models in Machine Learning



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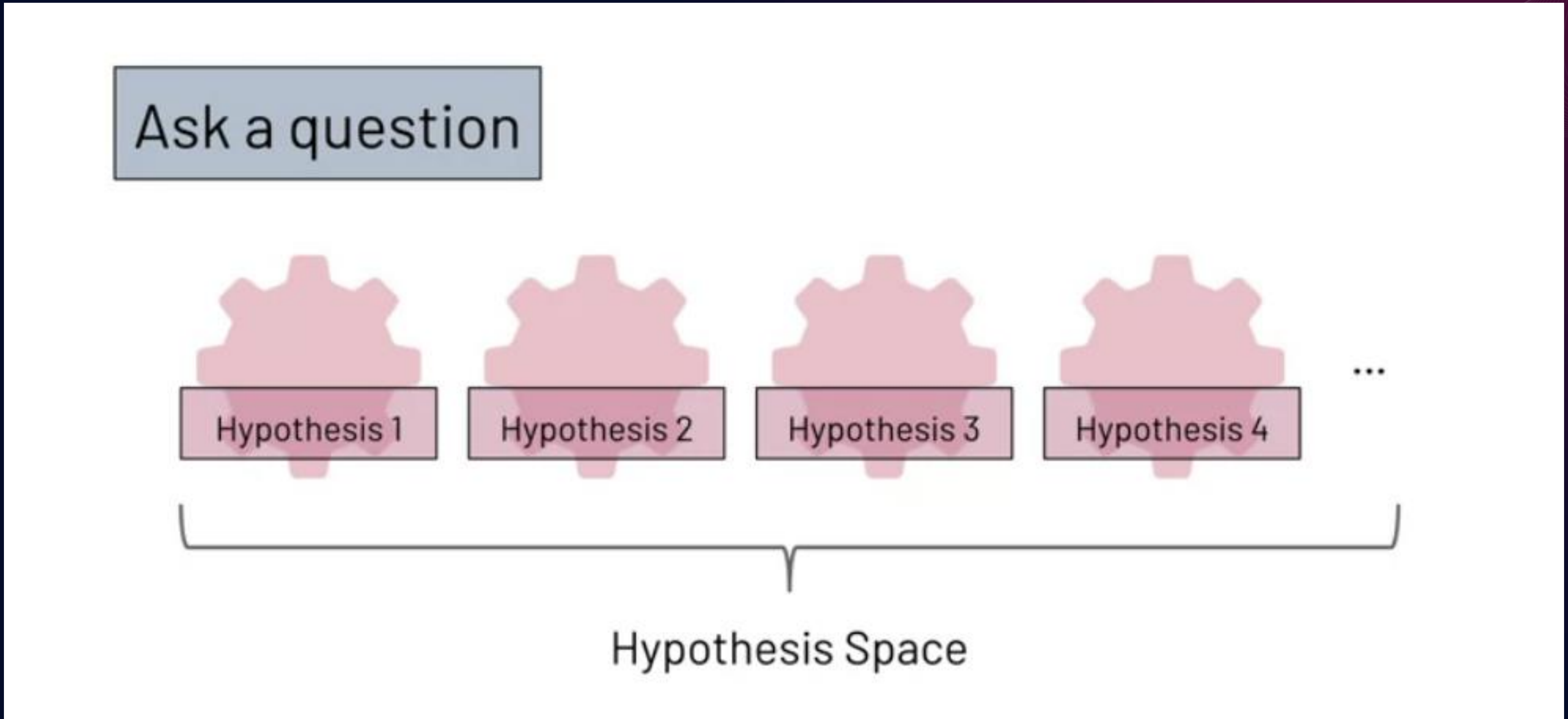

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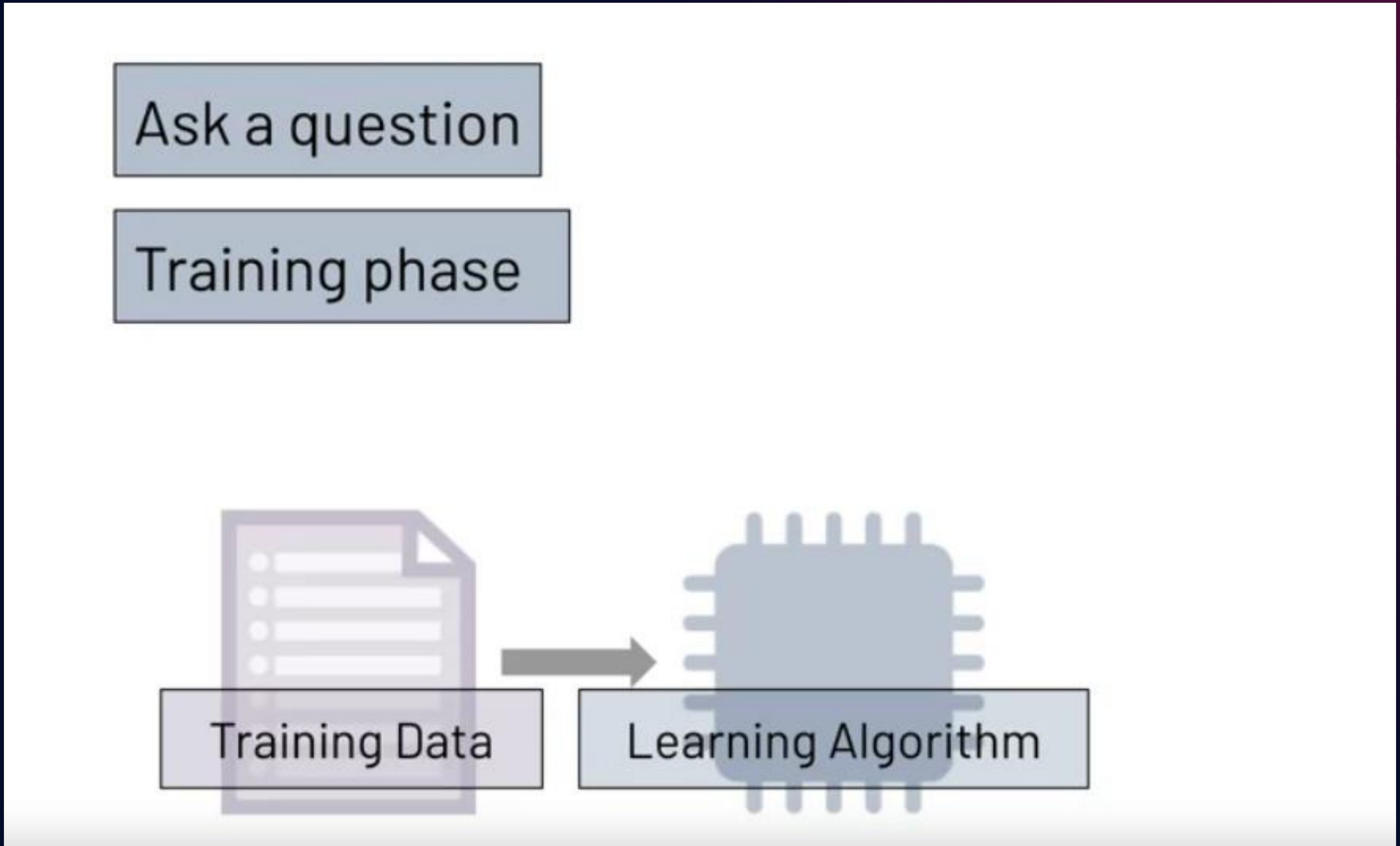

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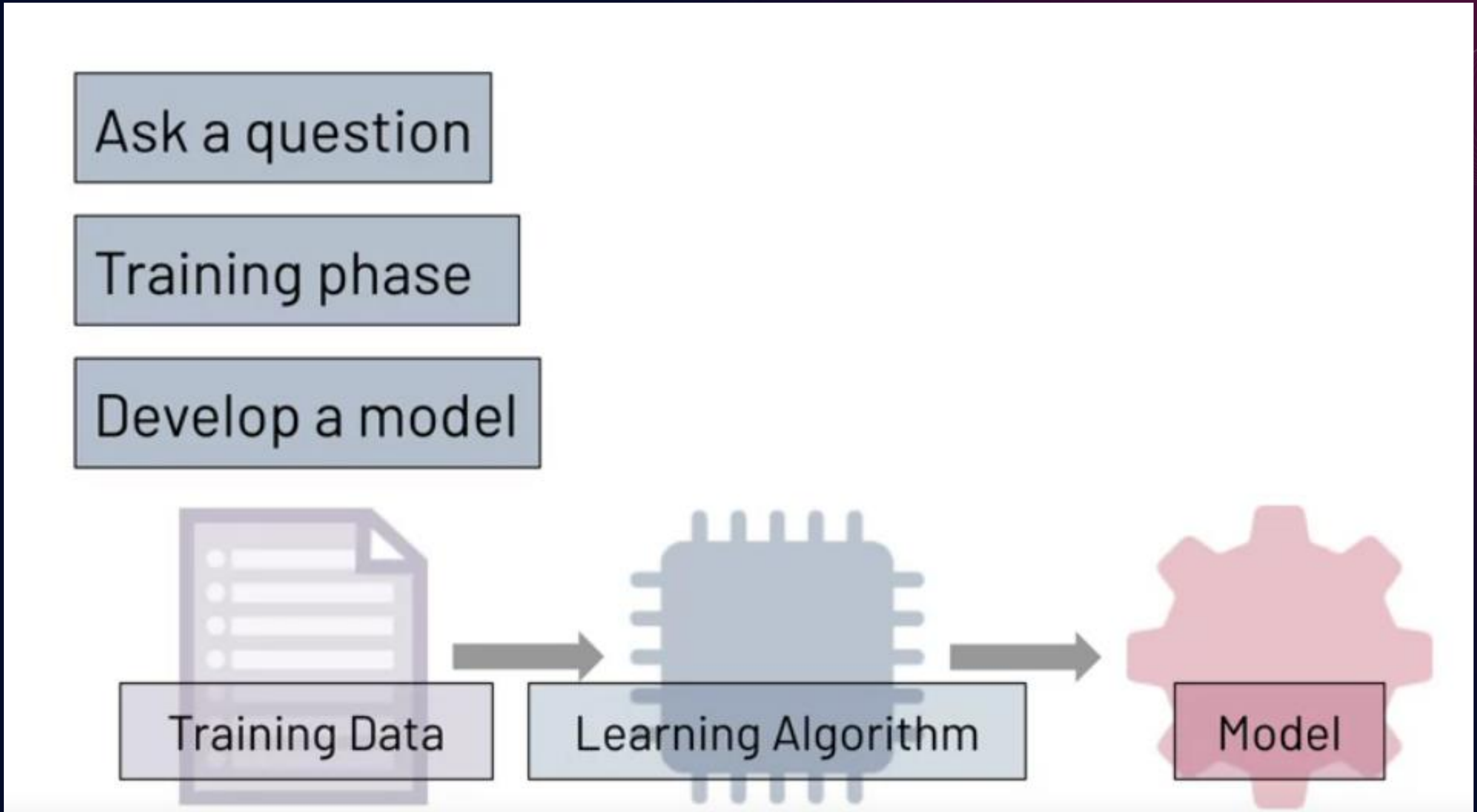

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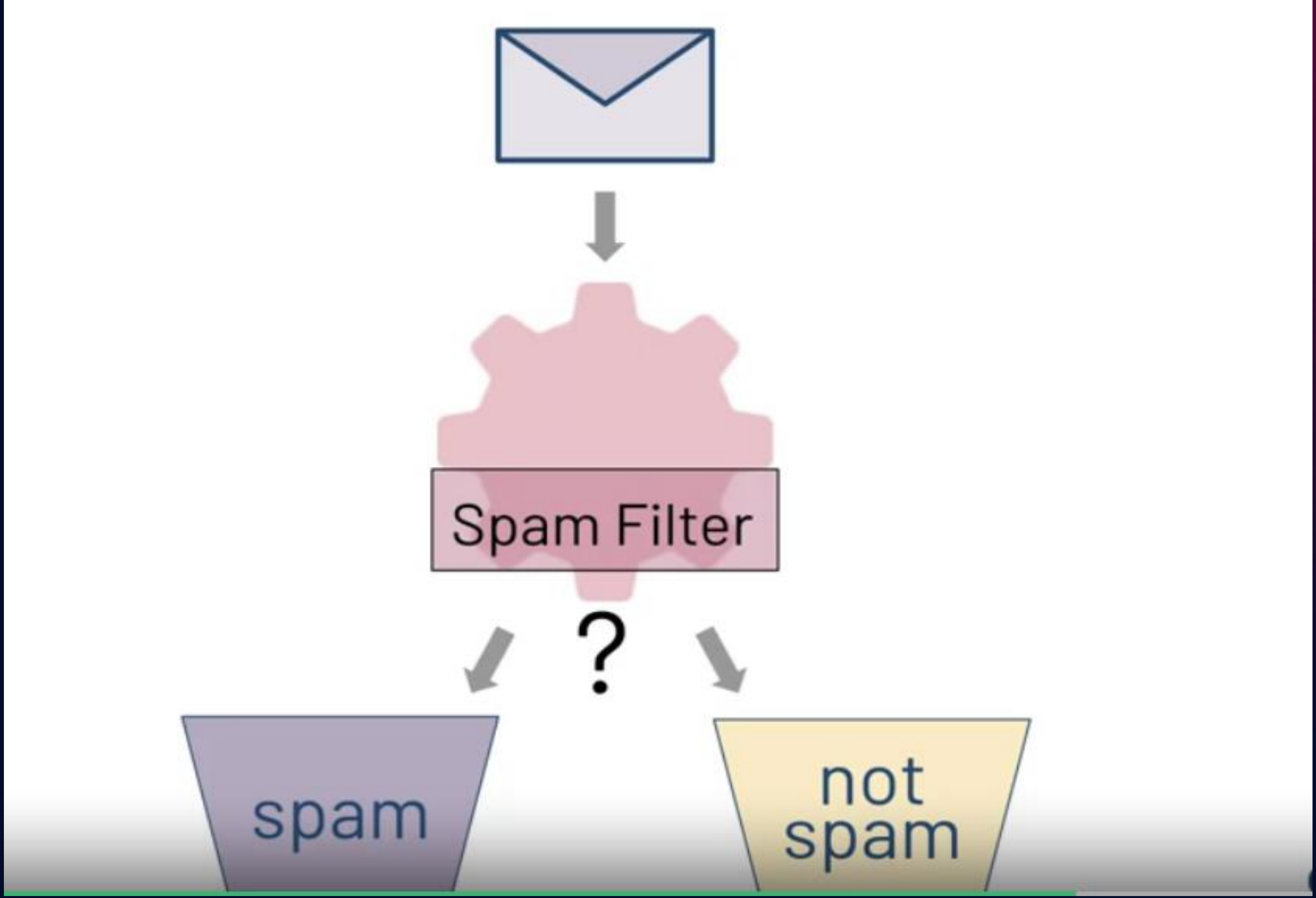
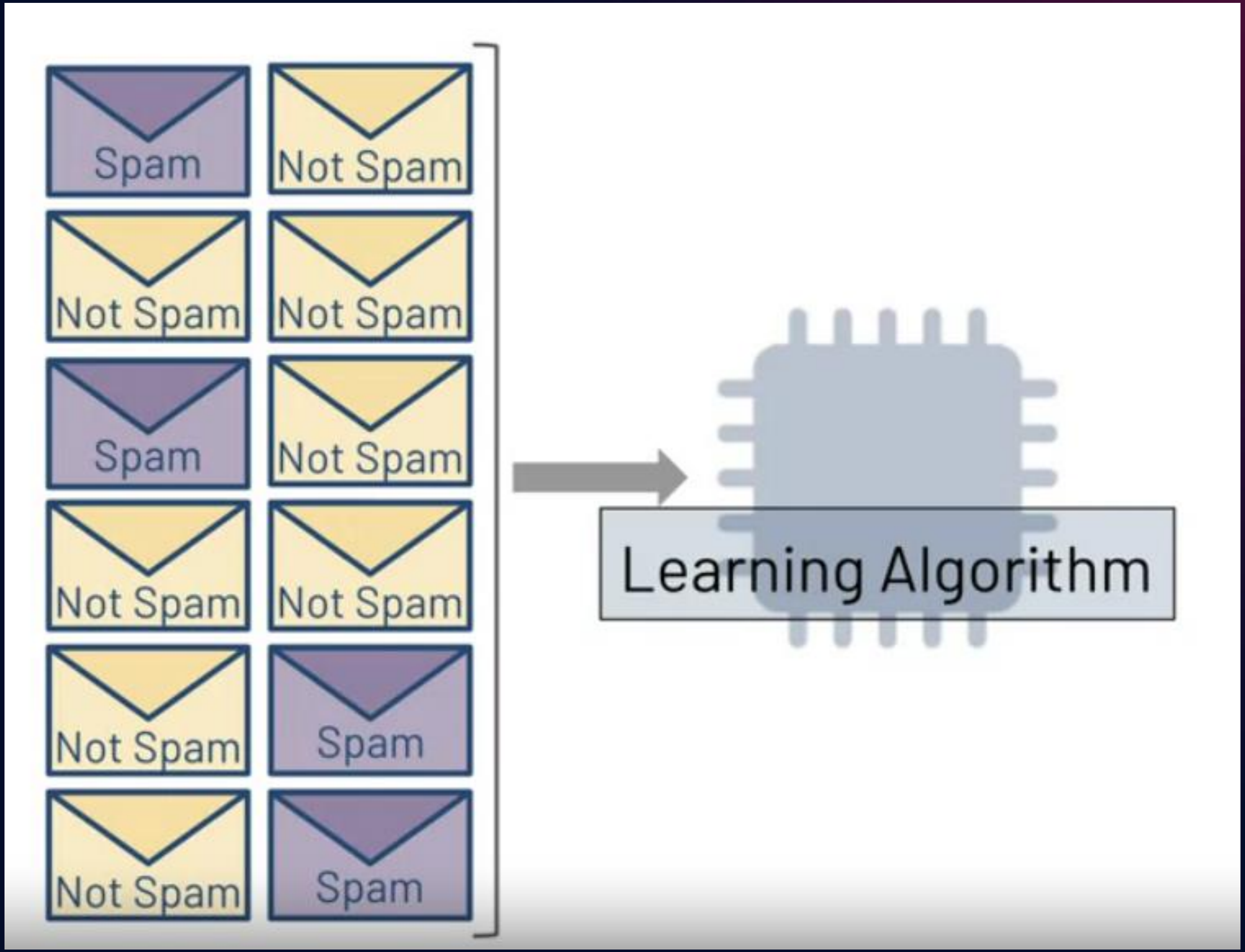
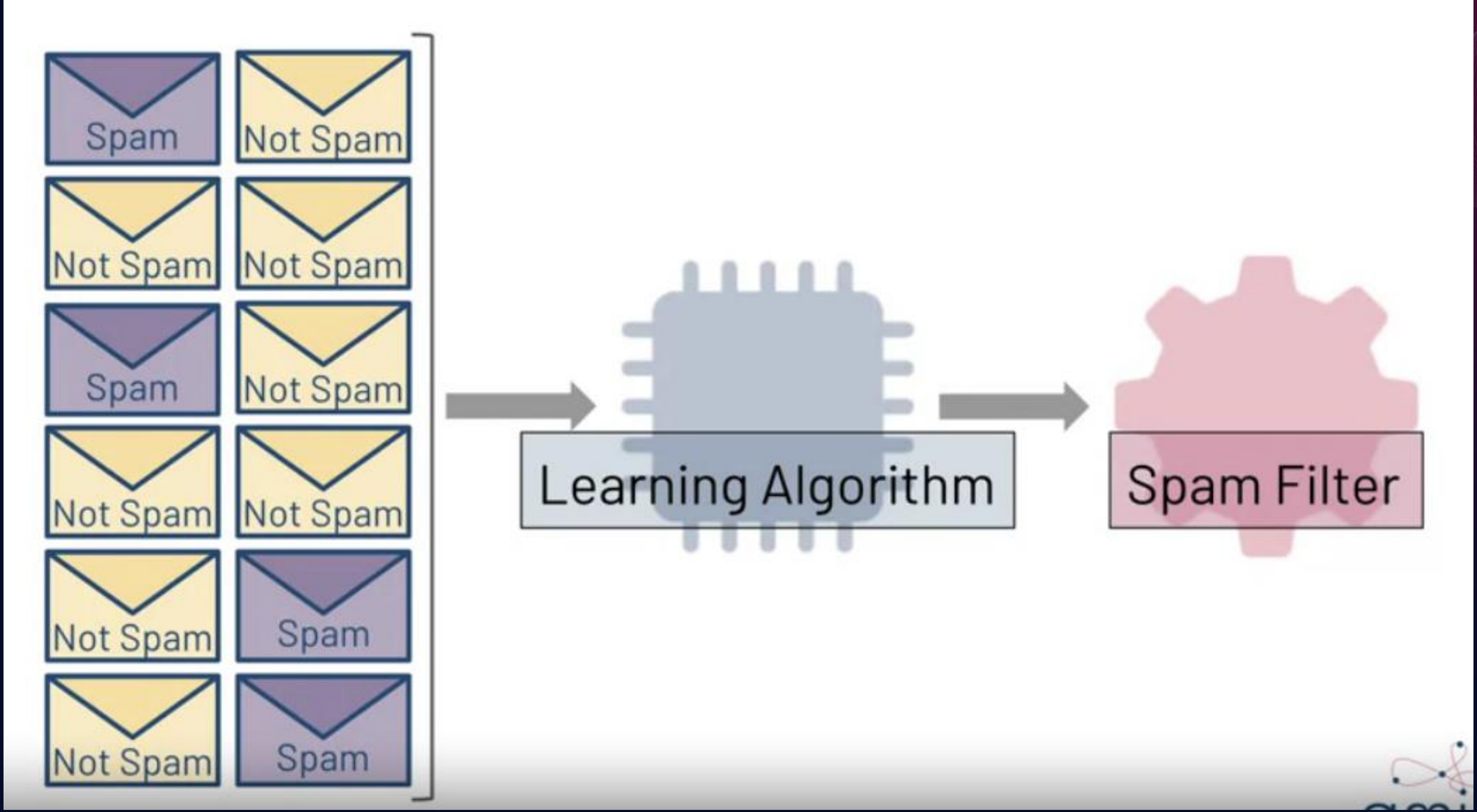


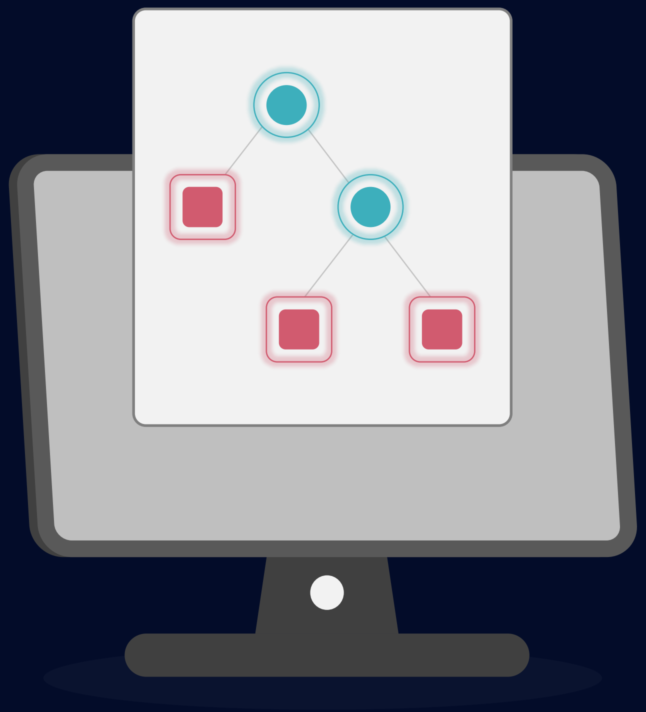
Classification Models in Machine Learning



Classification Models in Machine Learning



Decision Tree



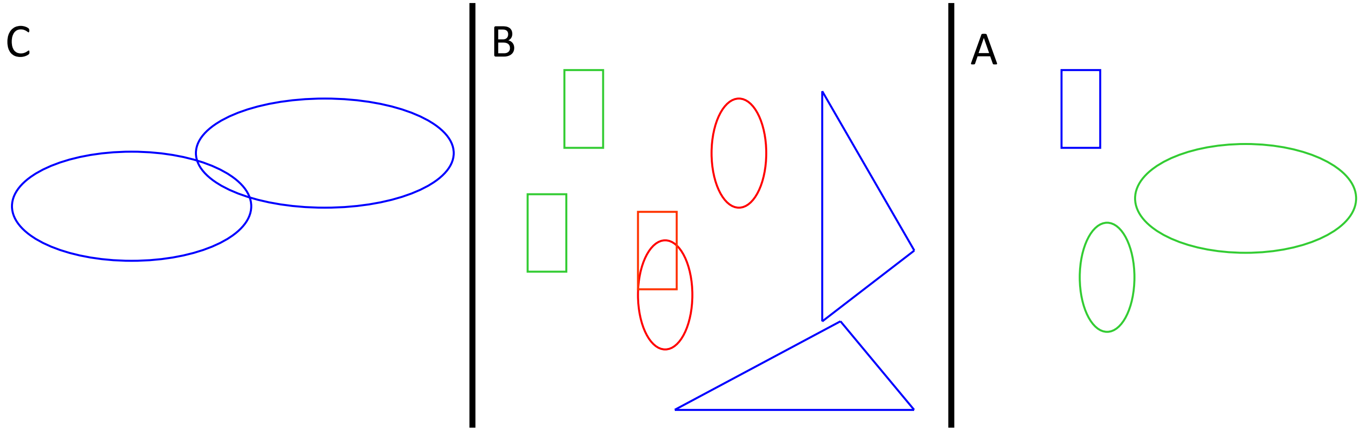
It is a handy tool with many applications. Decision trees can be used to solve classification and regression issues

The name indicates that it displays the predictions coming from a series of feature-based splits using a flowchart-like tree structure

It all starts with a root node and ends with a leaf choice

Decision Trees

- A hierarchical data structure that represents data by implementing a divide and conquer strategy
- Can be used as a non-parametric classification and regression method
- Given a collection of examples, learn a decision tree that represents it.
- Use this representation to classify new examples

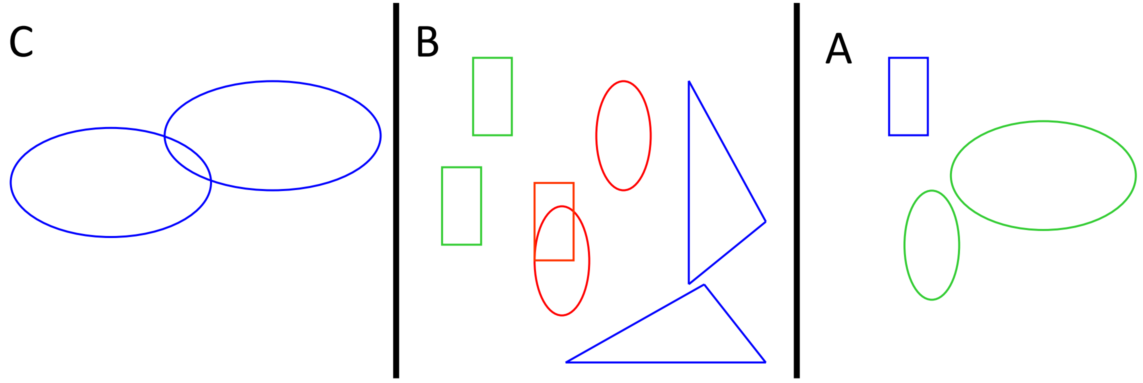


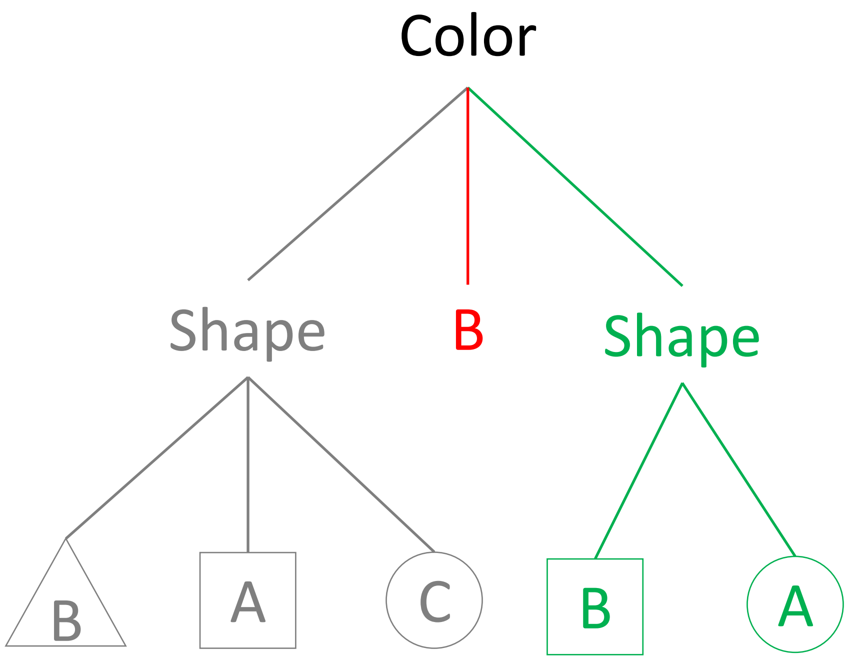
The Representation

- Decision Trees are classifiers for instances represented as feature vectors
  - color={red, blue, green} ; shape={circle, triangle, rectangle} ; label= {A, B, C}
- Nodes are tests for feature values
- There is one branch for each value of the feature
- Leaves specify the category (labels)
- Can categorize instances into multiple disjoint categories

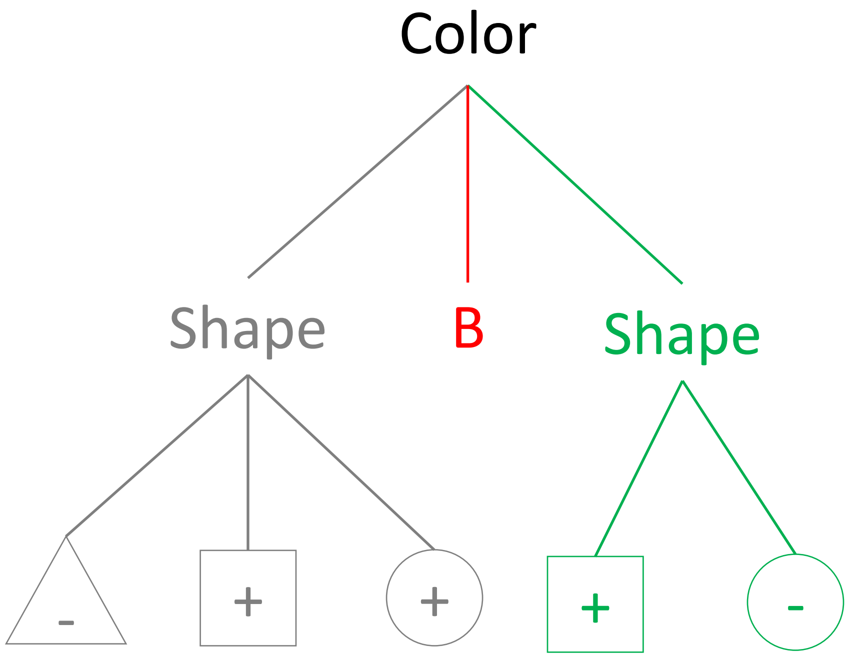
Evaluation of a Decision Tree

Learning a Decision Tree



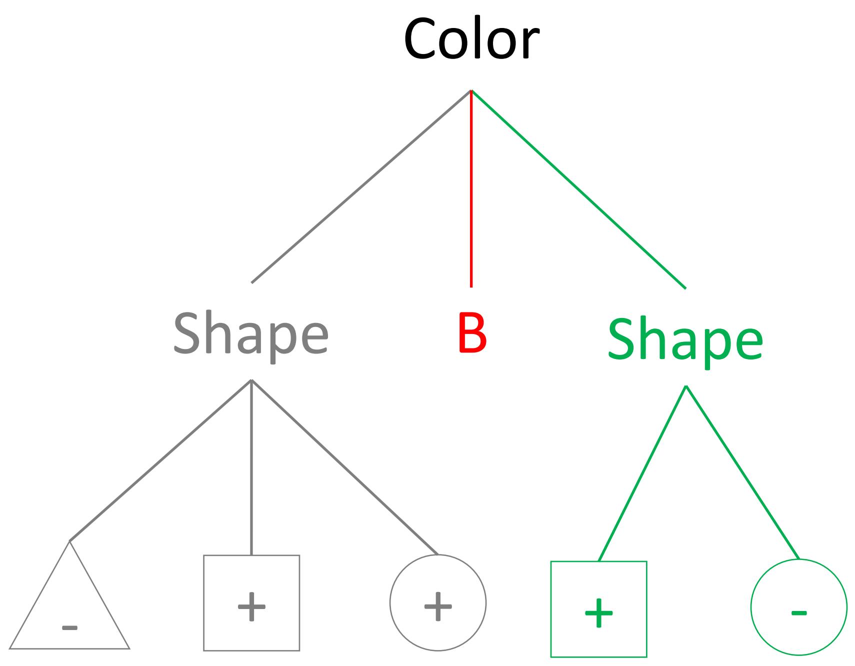


Expressivity of Decision Trees



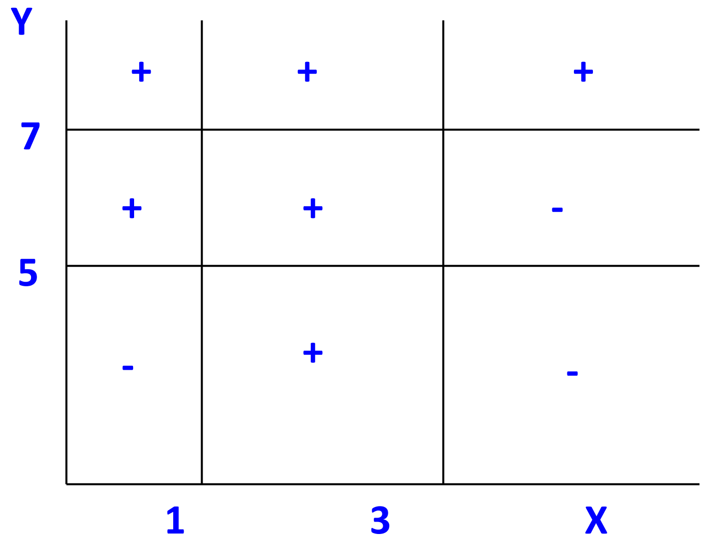
Decision Trees

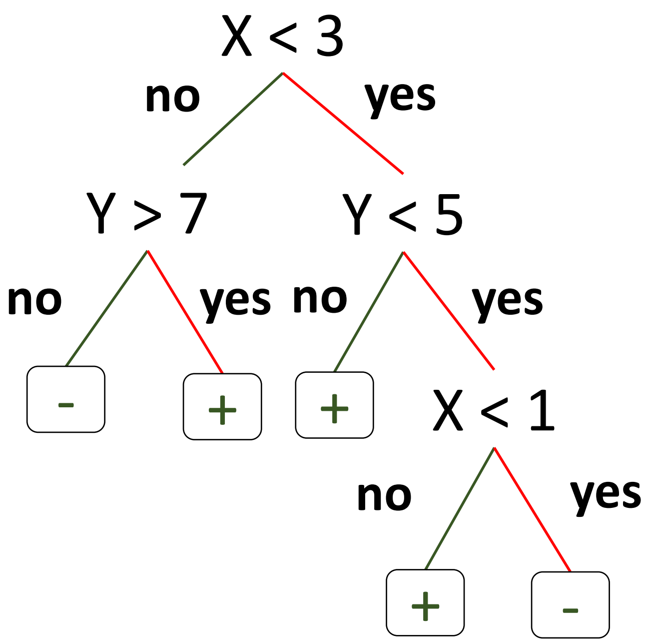
- Output is a discrete category. Real valued outputs are possible (regression trees)
- There are efficient algorithms for processing large amounts of data (but not too many features)
- There are methods for handling **noisy data** (classification noise and attribute noise) and for handling missing attribute values



Decision Boundaries

- Usually, instances are represented as attribute-value pairs (color=blue, shape = square, +)
- Numerical values can be used either by discretizing or by using thresholds for splitting nodes
- In this case, the tree divides the features space into axis-parallel rectangles, each labeled with one of the labels





Today’s key concepts

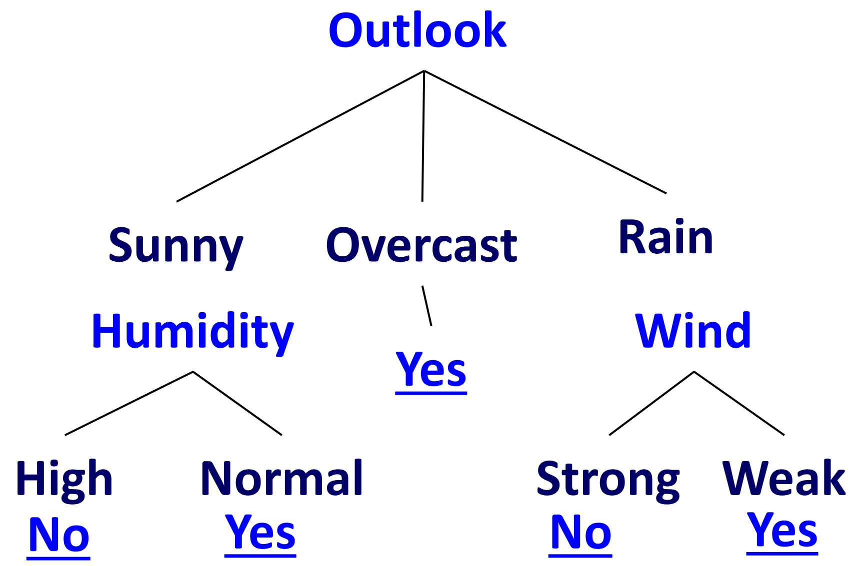
- Learning decision trees (ID3 algorithm)
  - Greedy heuristic (based on information gain)
    Originally developed for discrete features

Learning decision trees (ID3 algorithm

Decision Trees

- Can represent any Boolean Function
- Can be viewed as a way to compactly represent a lot of data.
- Natural representation: (20 questions)
- The evaluation of the Decision Tree Classifier is easy

- Clearly, given data, there are many ways to represent it as a decision tree.
- Learning a good representation from data is the challenge.



Will I play tennis today?

- **Features**
  - Outlook: {Sun, Overcast, Rain}
  - Temperature: {Hot, Mild, Cool}
  - Humidity: {High, Normal, Low}
  - Wind: {Strong, Weak}

- **Labels**
  - Binary classification task: Y = {+, -}

Will I play tennis today?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Outlook: S(unny), O(vercast), R(ainy)

Temperature: H(ot), M(edium), C(ool)

Humidity: H(igh), N(ormal), L(ow)

Wind: S(trong), W(eak)

Basic Decision Trees Learning Algorithm

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Algorithm?

LEARN-DECISION-TREE

```
function LEARN-DECISION-TREE(examples, attributes, parent_examples) returns a tree

  if examples is empty then return PLURALITY-VALUE(parent_examples)
  else if all examples have the same classification then return the classification
  else if attributes is empty then return PLURALITY-VALUE(examples)
  else
      A ← argmax_{a ∈ attributes} IMPORTANCE(a, examples)
      tree ← a new decision tree with root test A
      for each value v of A do
          exs ← {e : e ∈ examples and e.A = v}
          subtree ← LEARN-DECISION-TREE(exs, attributes − A, examples)
          add a branch to tree with label (A = v) and subtree subtree
      return tree
```

The function PLURALITY-VALUE selects the most common output value among a set of examples, breaking ties randomly.

Picking the Root Attribute

- The goal is to have the resulting decision tree as small as possible (Occam's Razor)
  - The main decision in the algorithm is the selection of the next attribute to condition on.
- We want attributes that split the examples to sets that are relatively pure in one label; this way we are closer to a leaf node.
  - The most popular heuristics is based on information gain, originated with the ID3 system of Quinlan.

Entropy

- Entropy (impurity, disorder) of a set of examples, S, relative to a binary classification is:

$$Entropy(S) = -p_+ \log(p_+) - p_- \log(p_-)$$

- p_+ is the proportion of positive examples in S and
- p_- is the proportion of negatives examples in S
  - If all the examples belong to the same category: Entropy = 0
  - If all the examples are equally mixed (0.5, 0.5): Entropy = 1
  - Entropy = Level of uncertainty.
- In general, when p_i is the fraction of examples labeled i:

$$Entropy(S[p_1, p_2, \dots, p_k]) = -\sum_1^k p_i \log(p_i)$$

- Entropy can be viewed as the number of bits required, on average, to encode the class of labels. If the probability for + is 0.5, a single bit is required for each example; if it is 0.8 – can use less then 1 bit.

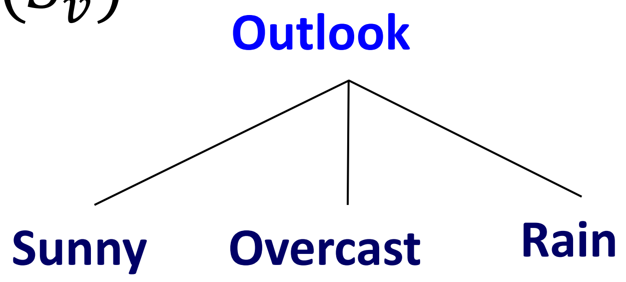
Information Gain

High Entropy – High level of Uncertainty

Low Entropy – No Uncertainty.

- The information gain of an attribute a is the expected reduction in entropy caused by partitioning on this attribute

$$Gain(S,a) = Entropy(S) - \sum_{v \in values(S)} \frac{|S_v|}{|S|} Entropy(S_v)$$



- Where:
  - S_v is the subset of S for which attribute a has value v, and
  - the entropy of partitioning the data is calculated by **weighing the entropy of each partition** by its size relative to the original set

- Partitions of low entropy (imbalanced splits) lead to high gain
- Go back to check which of the A, B splits is better

Will I play tennis today?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Outlook: S(unny), O(vercast), R(ainy)

Temperature: H(ot), M(edium), C(ool)

Humidity: H(igh), N(ormal), L(ow)

Wind: S(trong), W(eak)

Will I play tennis today?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

calculate current entropy

- $p_+ = \frac{9}{14} \quad p_- = \frac{5}{14}$
- $Entropy(Play) = -p_+ \log_2(p_+) - p_- \log_2(p_-)$

$$= -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14}$$

$$\approx 0.94$$

Information Gain: Outlook

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

$$Gain(S,a) = Entropy(S) - \sum_{v \in values(S)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Outlook = sunny:

$p_+ = 2/5 \quad p_- = 3/5$ **Entropy(O = S) = 0.971**

Outlook = overcast:

$p_+ = 4/4 \quad p_- = 0$ **Entropy(O = O) = 0**

Outlook = rainy:

$p_+ = 3/5 \quad p_- = 2/5$ **Entropy(O = R) = 0.971**

Expected entropy

= $\sum_{v \in values(S)} \frac{|S_v|}{|S|} Entropy(S_v)$

= (5/14)×0.971 + (4/14)×0 + (5/14)×0.971 = **0.694**

Information gain = 0.940 – 0.694 **= 0.246**

Information Gain: Humidity

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

$$Gain(S,a) = Entropy(S) - \sum_{v \in values(S)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Humidity = high:

$p_+ = 3/7 \quad p_- = 4/7$ **Entropy(H = H) = 0.985**

Humidity = Normal:

$p_+ = 6/7 \quad p_- = 1/7$ **Entropy(H = N) = 0.592**

Expected entropy

= $\sum_{v \in values(S)} \frac{|S_v|}{|S|} Entropy(S_v)$

= (7/14)×0.985 + (7/14)×0.592 = **0.7785**

Information gain = 0.940 – 0.7785 **= 0.1615**

Which feature to split on?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Information gain:

Outlook: 0.246
Humidity: 0.1615
Wind:?
Temperature:?

→ Split on Outlook

Which feature to split on?

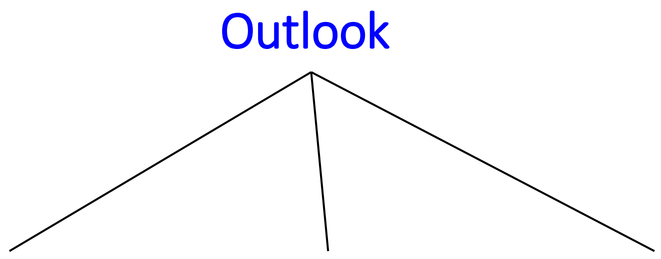
	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Information gain:

Outlook: 0.246
Humidity: 0.1615
Wind: 0.048
Temperature: 0.029

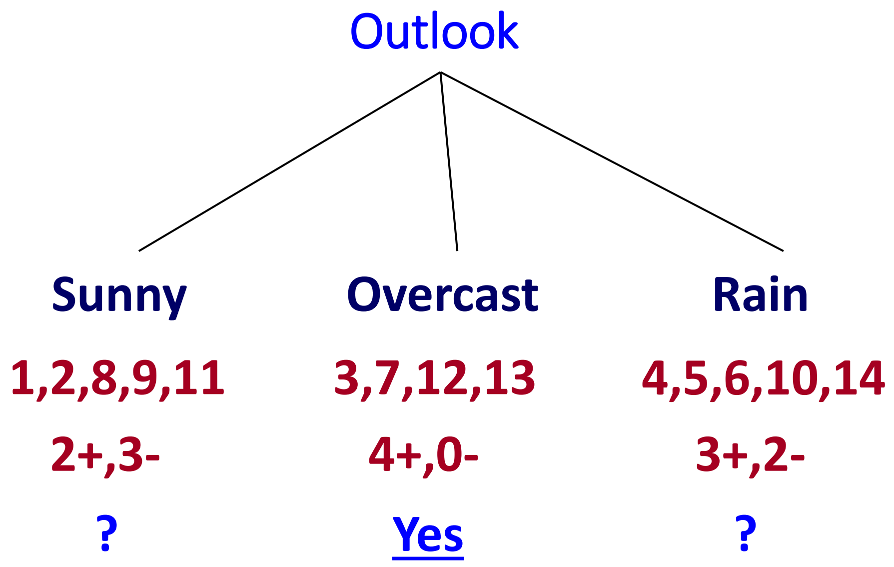
→ Split on Outlook

An Illustrative Example (III)



Gain(S,Humidity)=0.1615
Gain(S,Wind) = 0.048
Gain(S,Temperature) = 0.029
Gain(S,Outlook) = 0.246

An Illustrative Example (III)



	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
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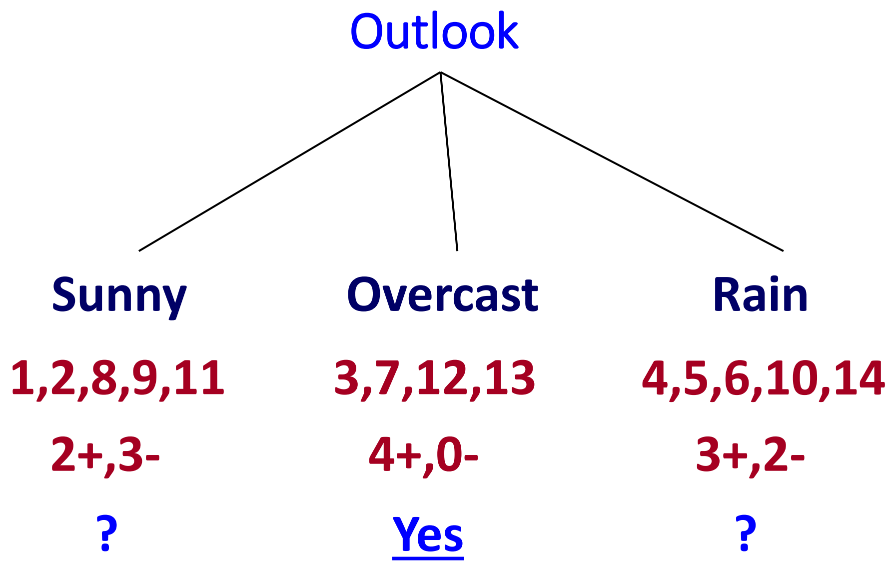
13	O	H	N	W	+
14	R	M	H	S	-

An Illustrative Example (III)



Continue until:
- Every attribute is included in path, or,
- All examples in the leaf have same label

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

An Illustrative Example (IV)

Outlook

Sunny	Overcast	Rain
1,2,8,9,11	3,7,12,13	4,5,6,10,14
2+,3-	4+,0-	3+,2-
?	Yes	?

$\text{Gain}(S_{sunny}, \text{Humidity}) =$ .97-(3/5) 0-(2/5) 0 = .97

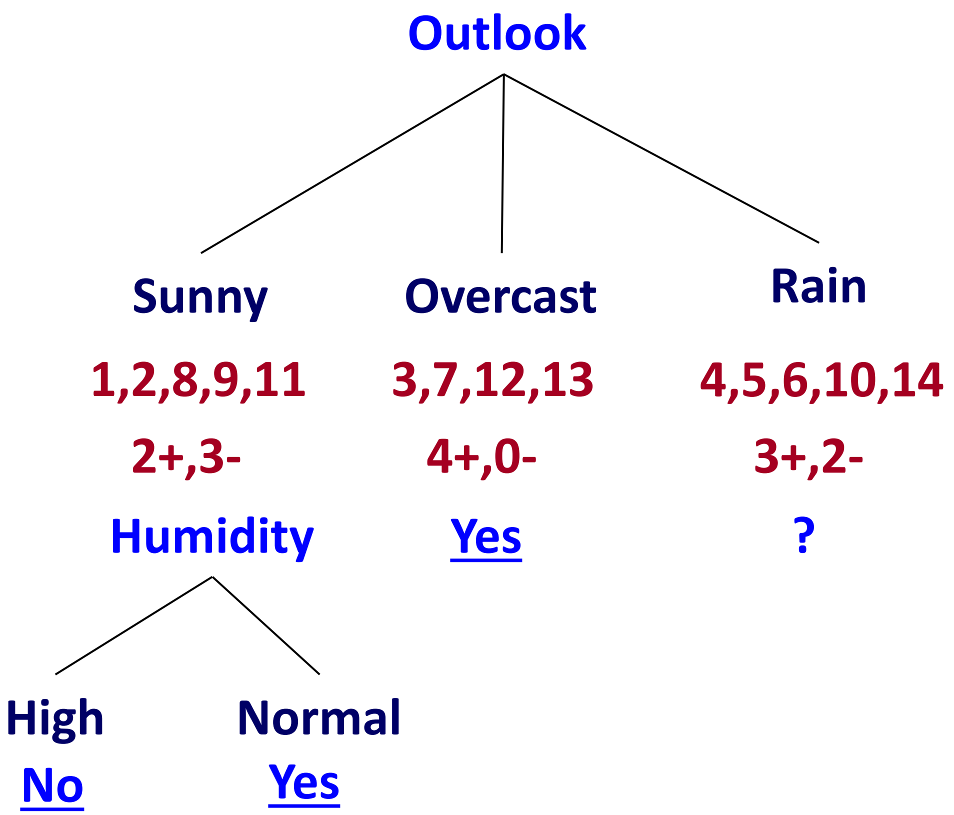
$\text{Gain}(S_{sunny}, \text{Temp}) =$ .97- 0-(2/5) 1 = .57

$\text{Gain}(S_{sunny}, \text{Wind}) =$ .97-(2/5) 1 - (3/5) .92= .02

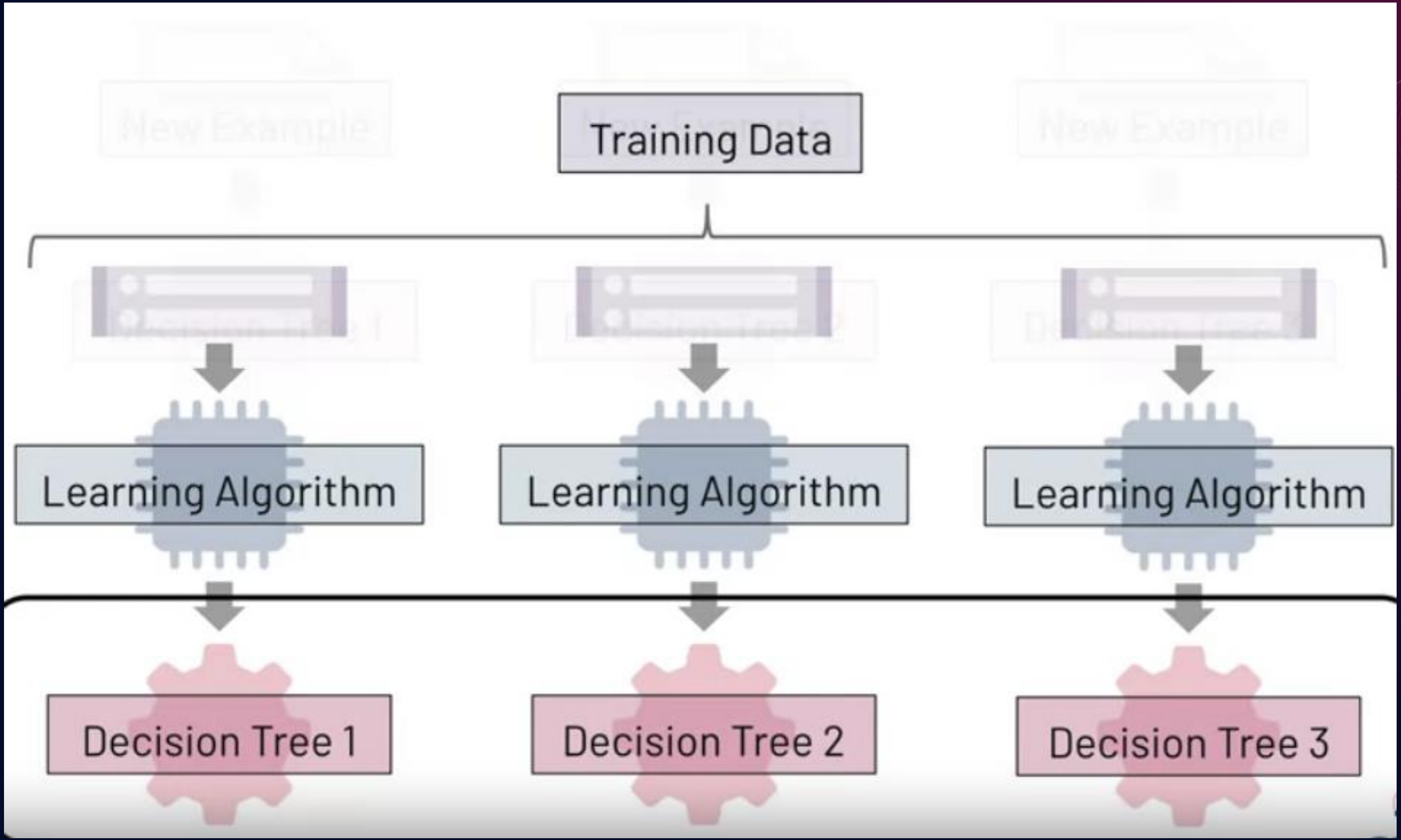
Split on Humidity

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

An Illustrative Example (V)



Classification Models: Decision Tree



Random Forest Algorithm

Ensemble Learning

Random Forest Algorithm

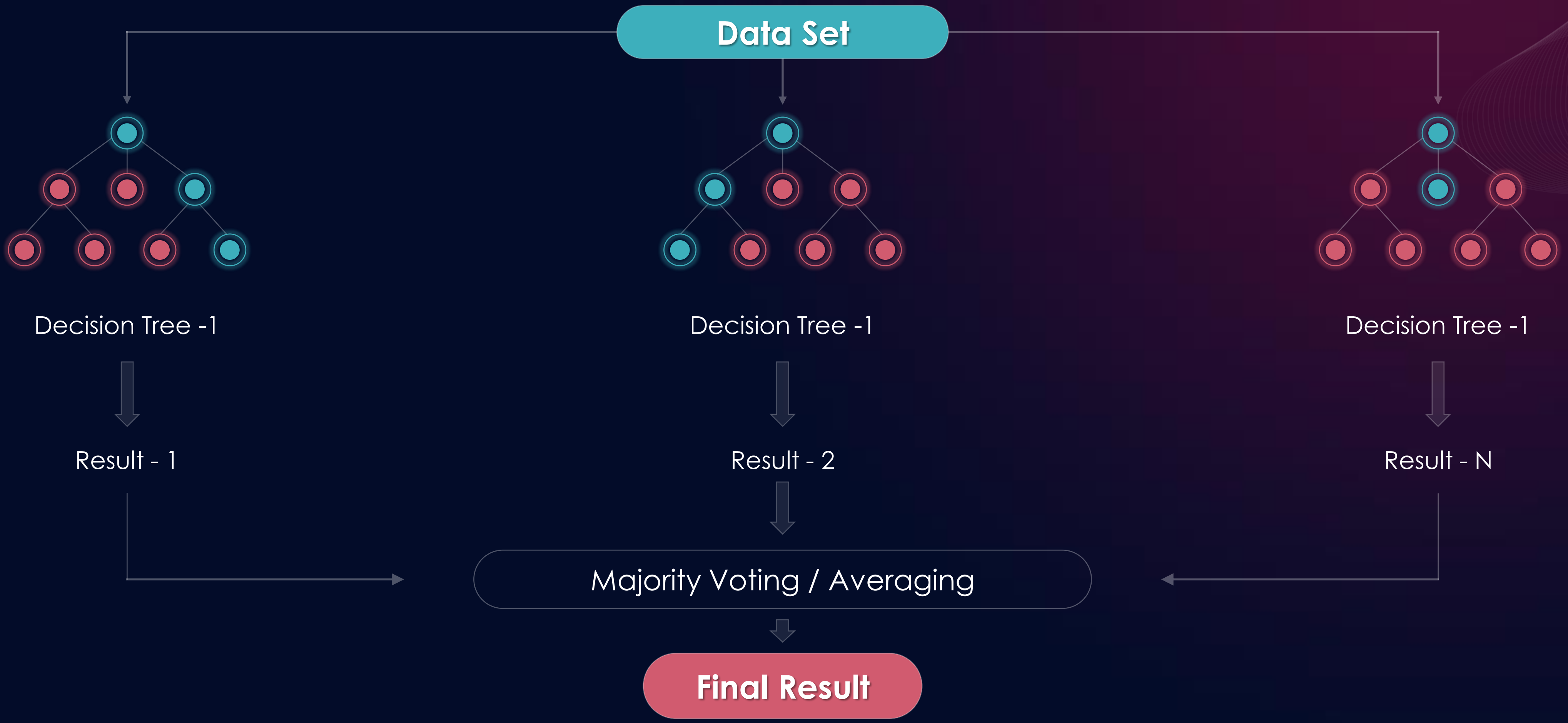
STEP 1: Pick at random K data points from the Training set.

STEP 2: Build the Decision Tree associated to these K data points.

STEP 3: Choose the number Ntree of trees you want to build and repeat STEPS 1 & 2

STEP 4: For a new data point, make each one of your Ntree trees predict the value of Y t
for the data point in question, and assign the new data point the average across all of th
predicted Y values.

Random Forest Algorithm



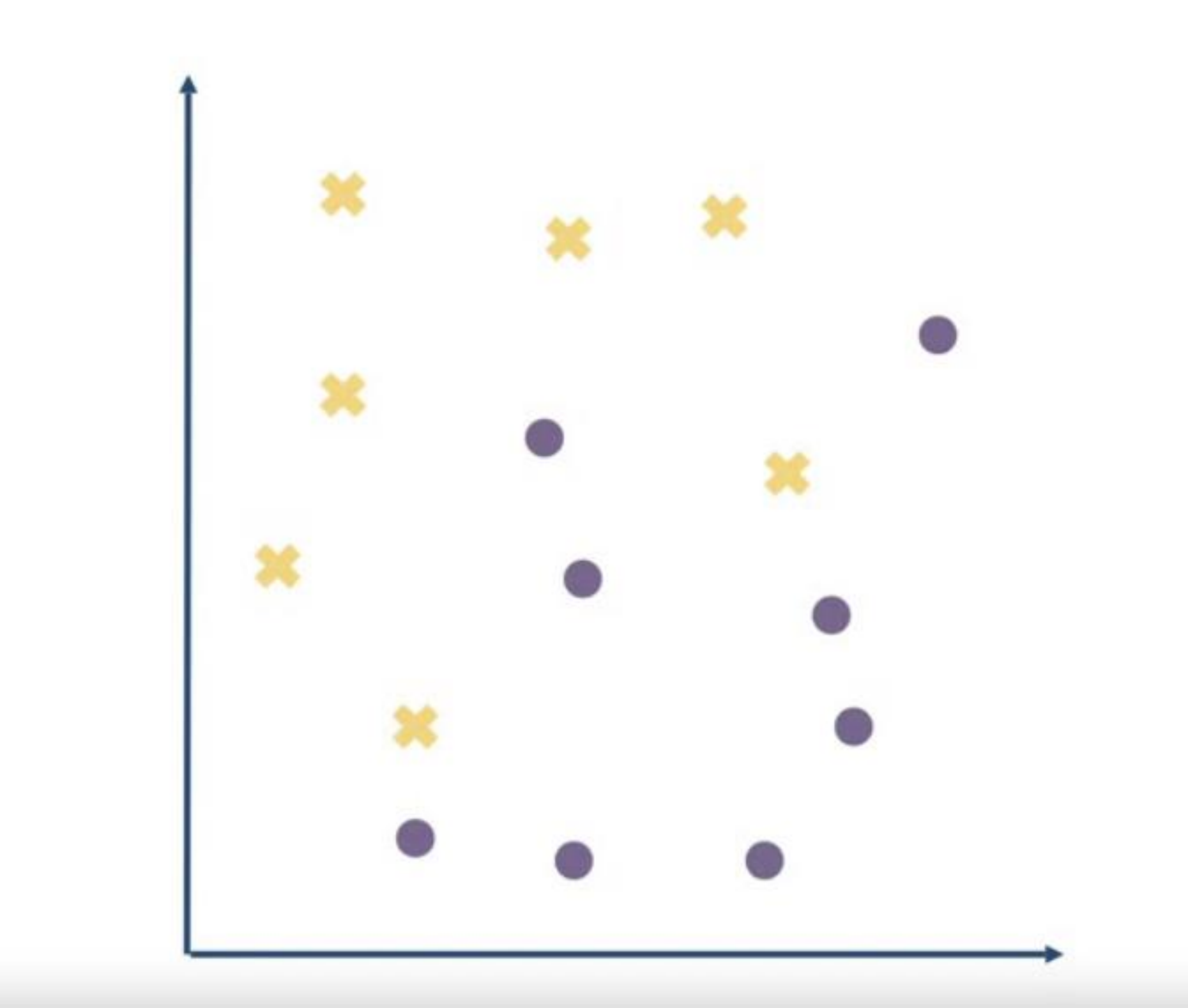
A Random Forest is a cluster of decision trees. Each tree is classed, and the tree "votes" for that class to classify a new item based on its properties. The forest chooses the categorization with the highest votes (over all the trees in the forest).

KNN (K- Nearest Neighbors) Algorithm

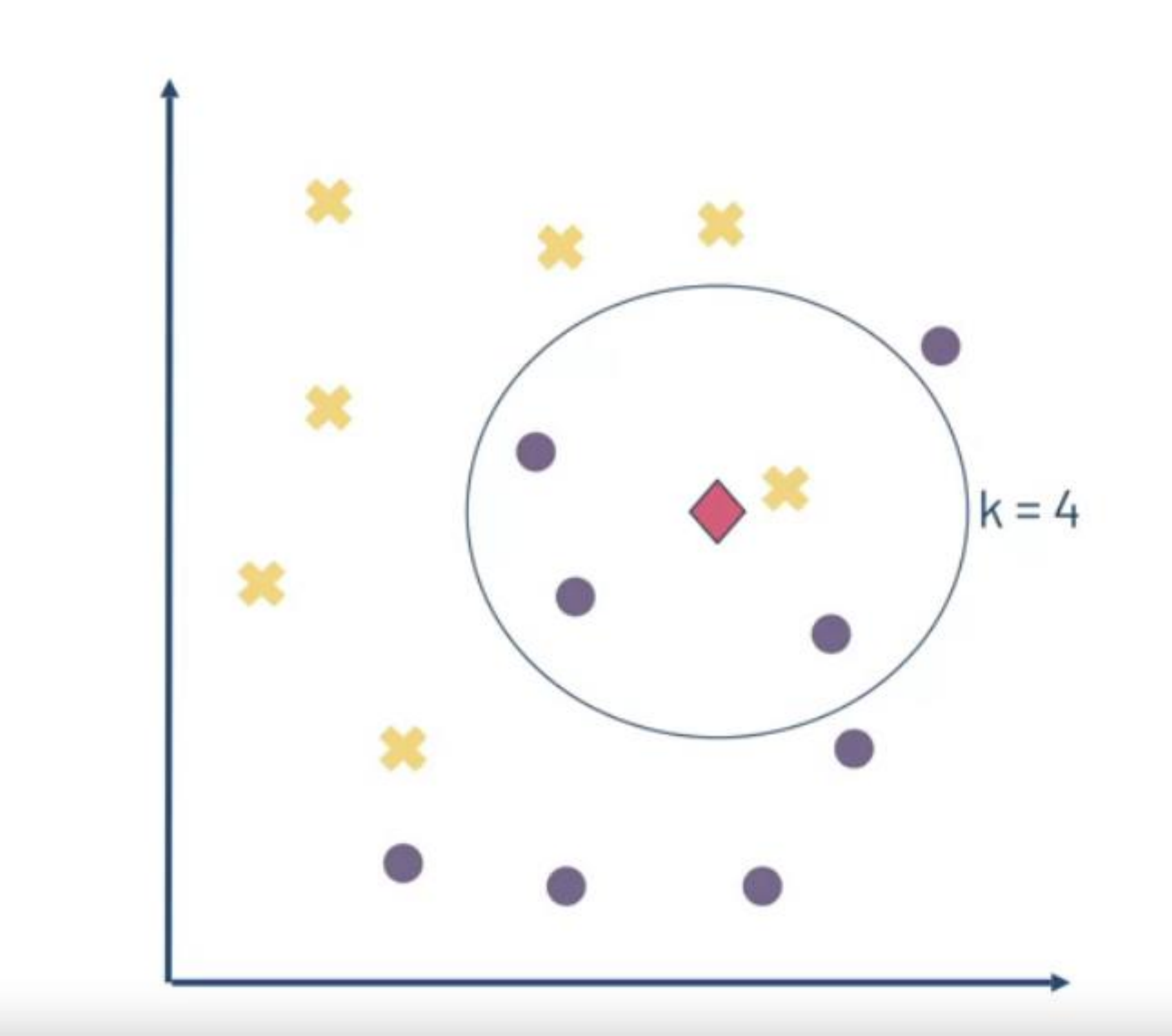
It's a simple algorithm that keeps all existing instances, and classifies new cases based on a majority vote of its k neighbors

The case is then given to the class with which it has greatest in common. A distance function is used to perform this task

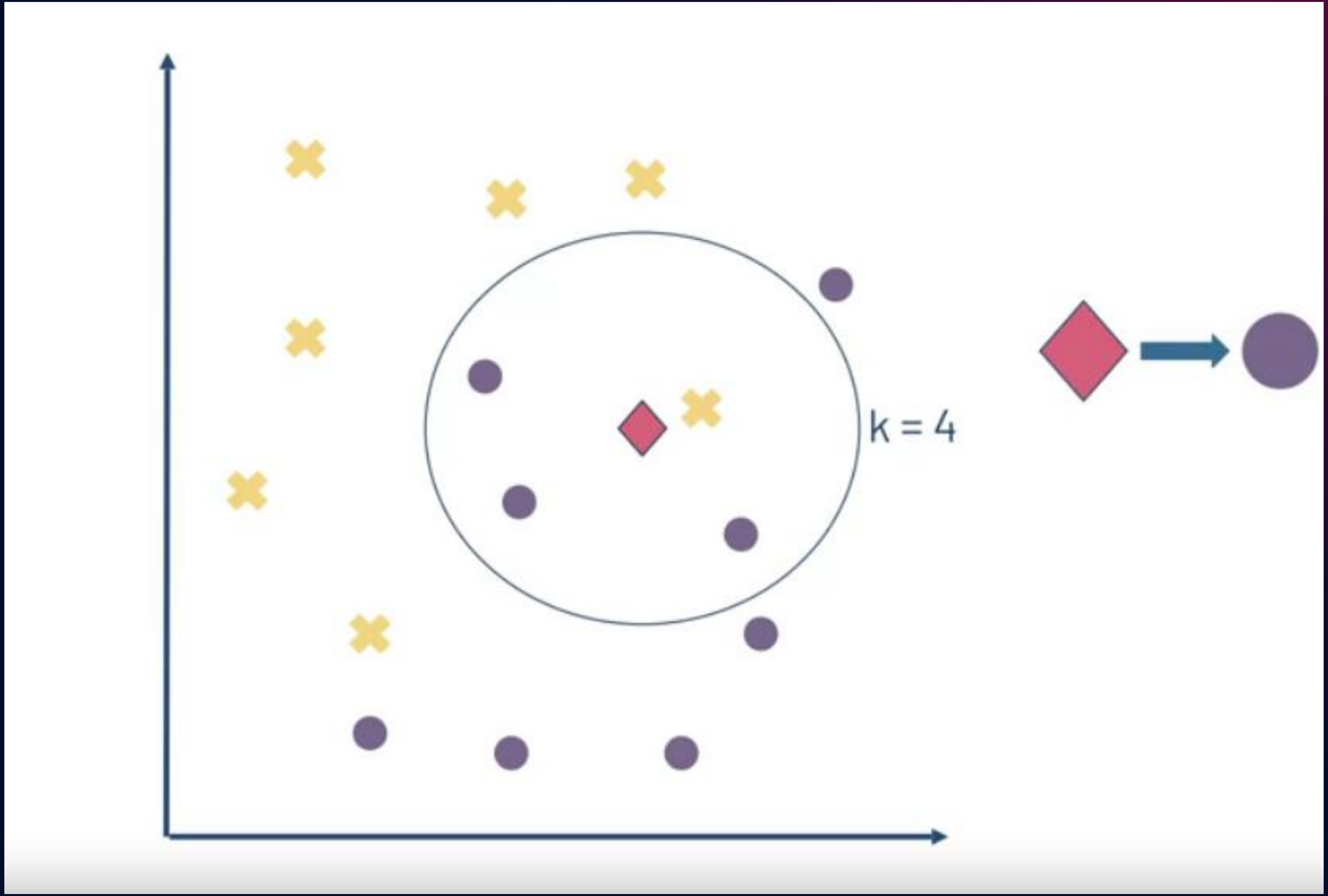
KNN (K- Nearest Neighbors) Algorithm



KNN (K- Nearest Neighbors) Algorithm



KNN (K- Nearest Neighbors) Algorithm



KNN (K- Nearest Neighbors) Algorithm

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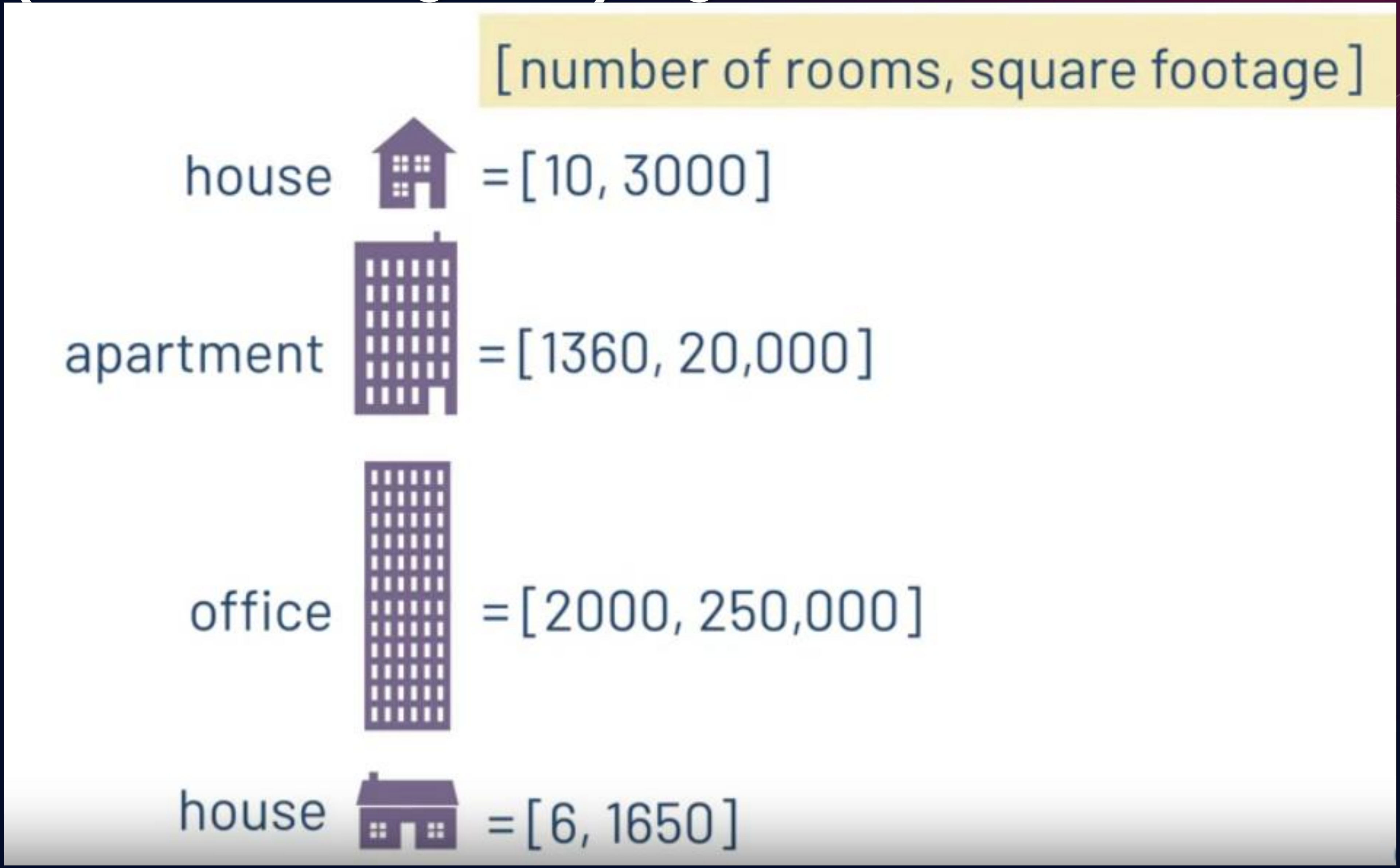
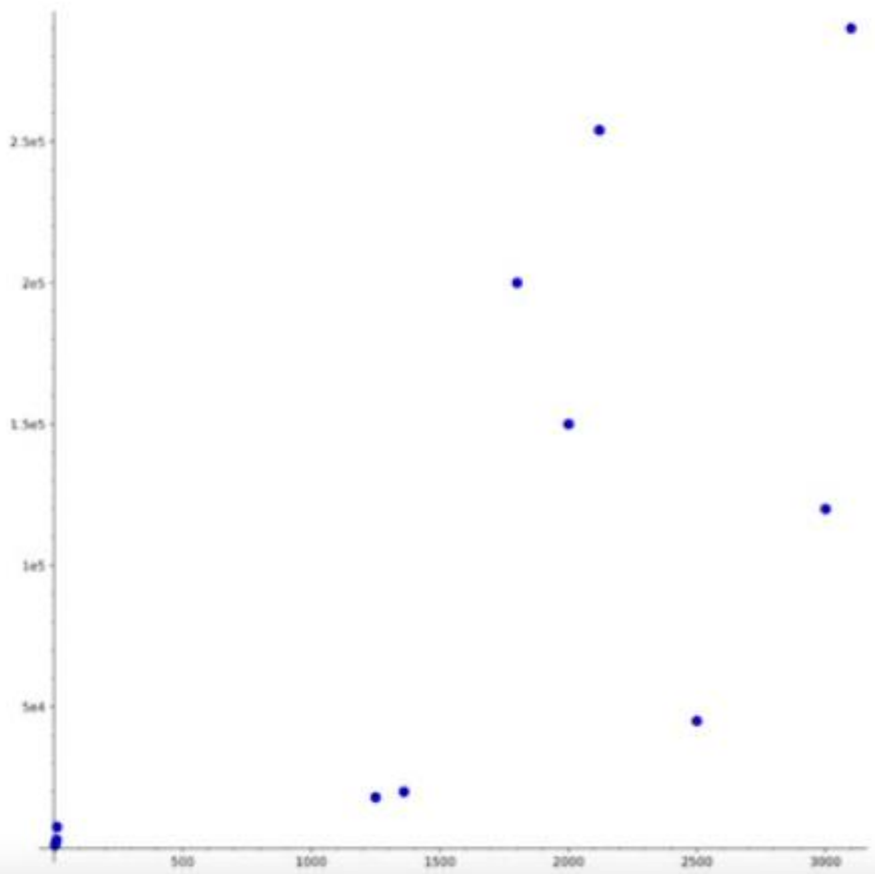


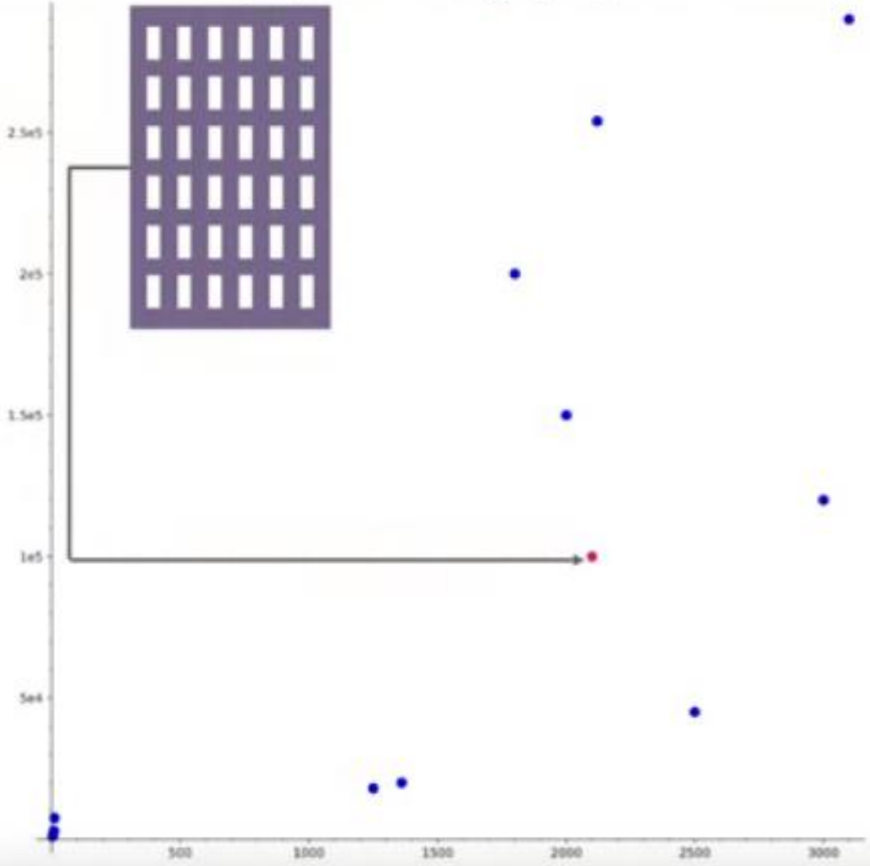
KNN (K- Nearest Neighbors) Algorithm



$$\begin{bmatrix} 10 & 3000 \\ 1360 & 20000 \\ 2000 & 250000 \\ 7 & 2050 \\ 4 & 1000 \\ 3100 & 290000 \\ 3000 & 120000 \\ 12 & 7500 \\ 2500 & 45000 \\ 1800 & 200000 \\ 2120 & 254000 \\ \vdots & \vdots \\ 1250 & 18000 \end{bmatrix} \begin{bmatrix} \text{house} \\ \text{apartment} \\ \text{office} \\ \text{house} \\ \text{house} \\ \text{office} \\ \text{apartment} \\ \text{house} \\ \text{apartment} \\ \text{office} \\ \text{office} \\ \vdots \\ \text{apartment} \end{bmatrix}$$

KNN (K- Nearest Neighbors) Algorithm

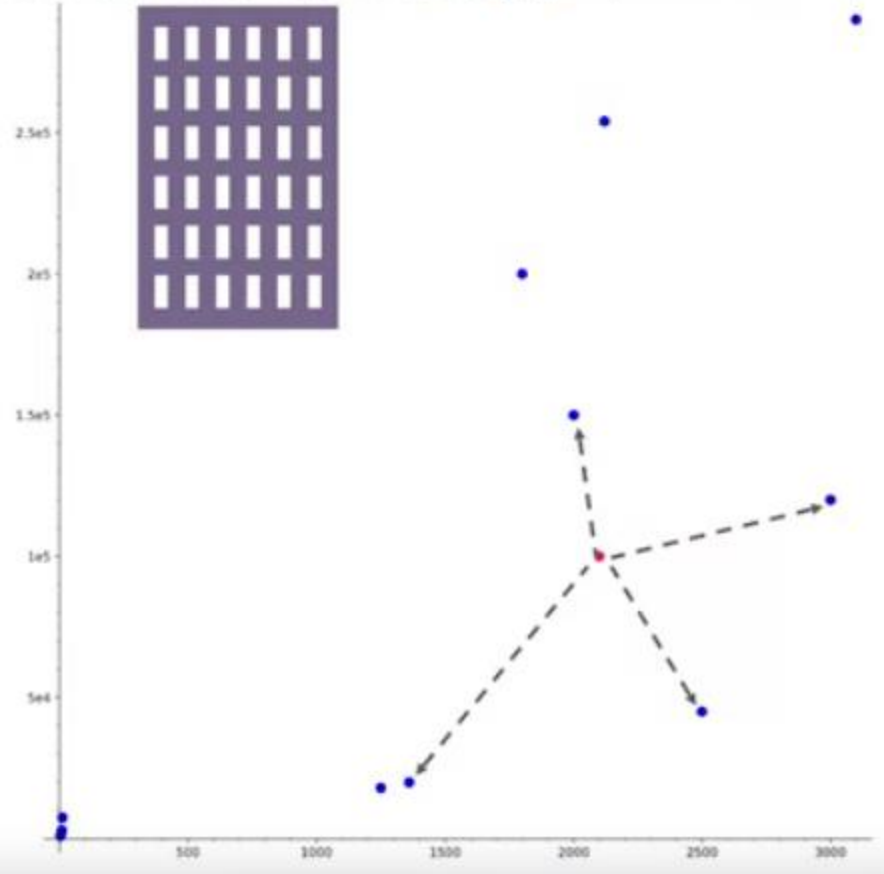
[number of rooms, square footage] = [2100 100000]



$$\begin{bmatrix} 10 & 3000 \\ 1360 & 20000 \\ 2000 & 250000 \\ 7 & 2050 \\ 4 & 1000 \\ 3100 & 290000 \\ 3000 & 120000 \\ 12 & 7500 \\ 2500 & 45000 \\ 1800 & 200000 \\ 2120 & 254000 \\ \vdots & \vdots \\ 1250 & 18000 \end{bmatrix} \begin{bmatrix} \text{house} \\ \text{apartment} \\ \text{office} \\ \text{house} \\ \text{house} \\ \text{office} \\ \text{apartment} \\ \text{house} \\ \text{apartment} \\ \text{office} \\ \text{office} \\ \vdots \\ \text{apartment} \end{bmatrix}$$

KNN (K- Nearest Neighbors) Algorithm

$$[\text{number of rooms, square footage}] = \begin{bmatrix} 2100 & 100000 \end{bmatrix}$$

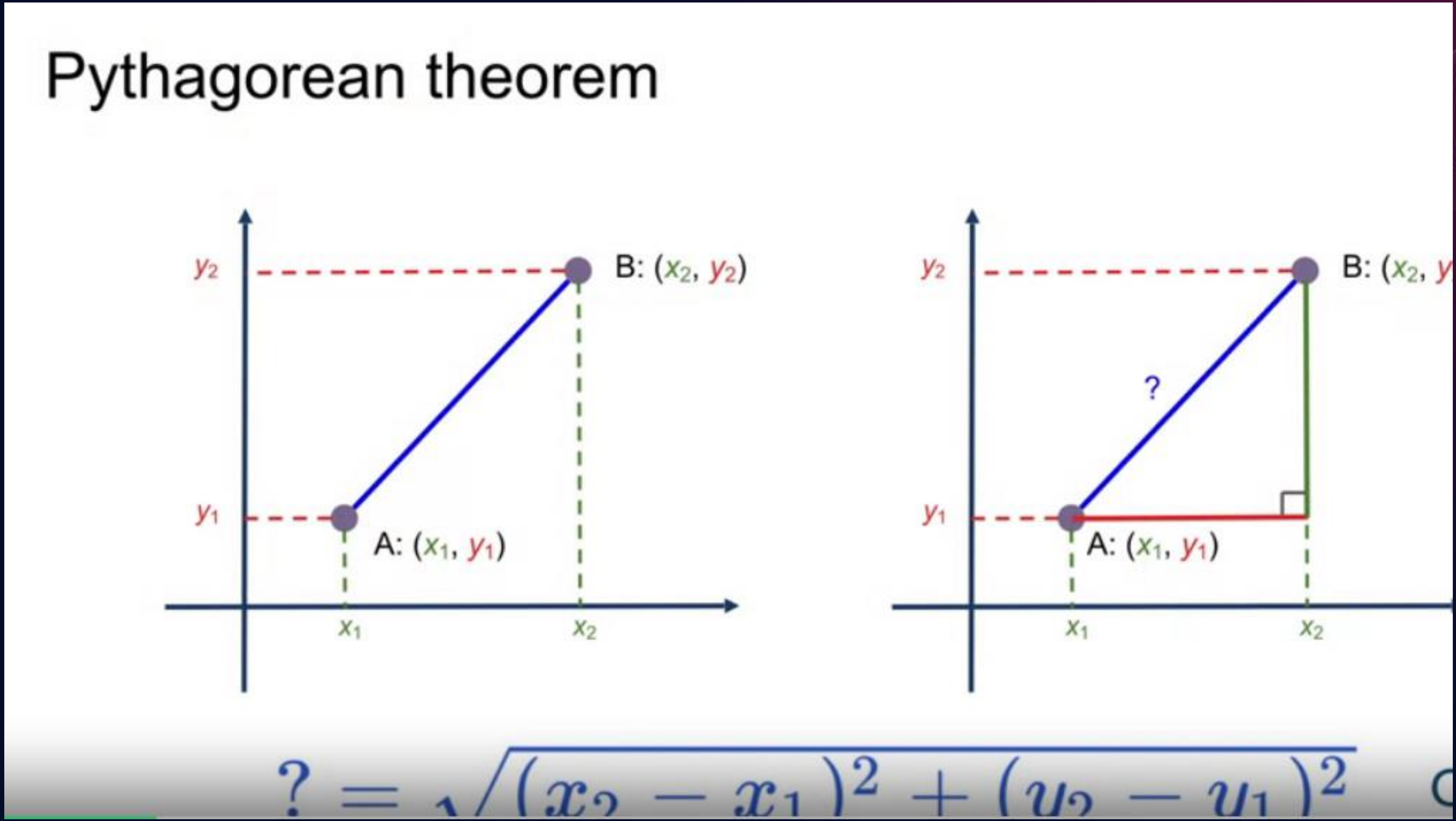


$$\begin{bmatrix} 10 & 3000 \\ 1360 & 20000 \\ 2000 & 250000 \\ 7 & 2050 \\ 4 & 1000 \\ 3100 & 290000 \\ 3000 & 120000 \\ 12 & 7500 \\ 2500 & 45000 \\ 1800 & 200000 \\ 2120 & 254000 \\ \vdots & \vdots \\ 1250 & 18000 \end{bmatrix} \quad \begin{bmatrix} \text{house} \\ \text{apartment} \\ \text{office} \\ \text{house} \\ \text{house} \\ \text{office} \\ \text{apartment} \\ \text{house} \\ \text{apartment} \\ \text{office} \\ \text{office} \\ \vdots \\ \text{apartment} \end{bmatrix}$$

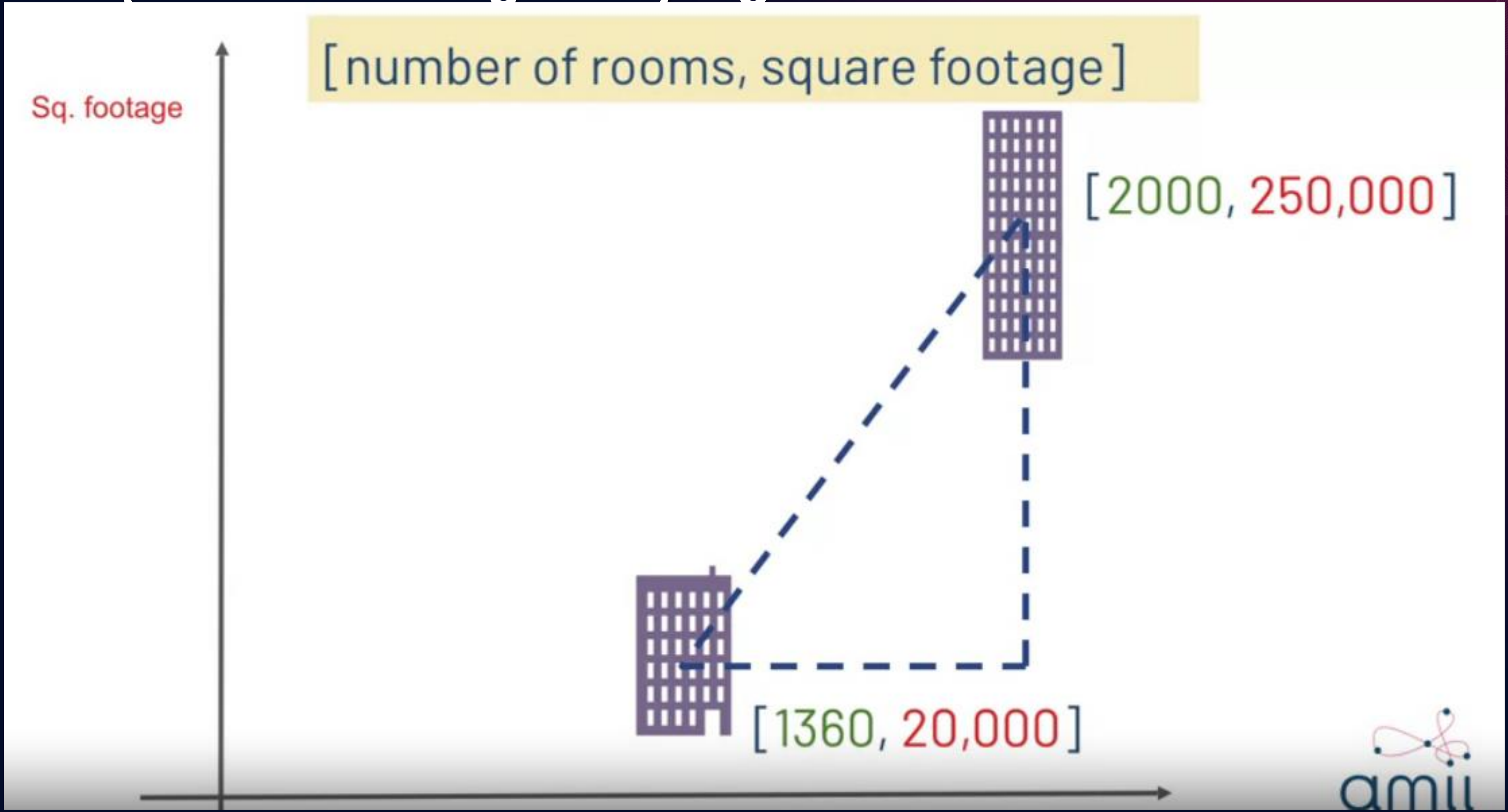
KNN (K- Nearest Neighbors) Algorithm

$$\text{m examples}\left\{\underbrace{\begin{bmatrix} 10 & 3000 & 150 & 1 & \cdots & 0 \\ 1360 & 20000 & 220 & 1 & \cdots & 1 \\ 2000 & 150000 & 300 & 0 & \cdots & 2 \\ 7 & 2050 & 45 & 0 & \cdots & 0 \\ 4 & 1000 & 13 & 1 & \cdots & 0 \\ 3100 & 290000 & 480 & 0 & \cdots & 2 \\ 3000 & 120000 & 200 & 1 & \cdots & 1 \\ 12 & 7500 & 16 & 1 & \cdots & 0 \\ 4120 & 45000 & 305 & 0 & \cdots & 1 \\ 1800 & 200000 & 122 & 1 & \cdots & 2 \\ 2120 & 254000 & 320 & 0 & \cdots & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1250 & 18000 & 100 & 0 & \cdots & 1 \end{bmatrix}}_{\text{n features}}\right.$$

KNN (K- Nearest Neighbors) Algorithm

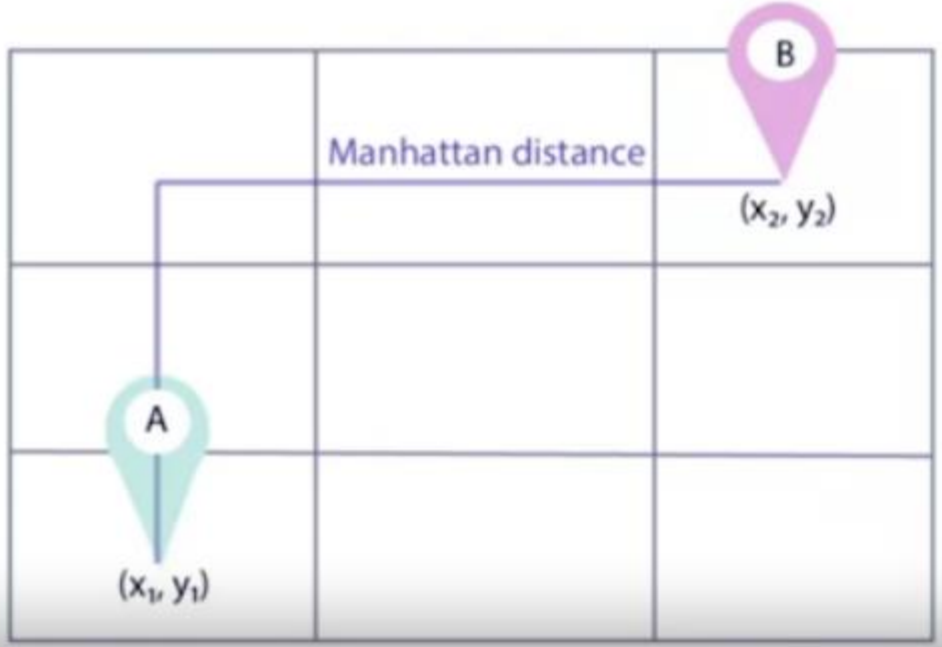


KNN (K- Nearest Neighbors) Algorithm



KNN (K- Nearest Neighbors) Algorithm

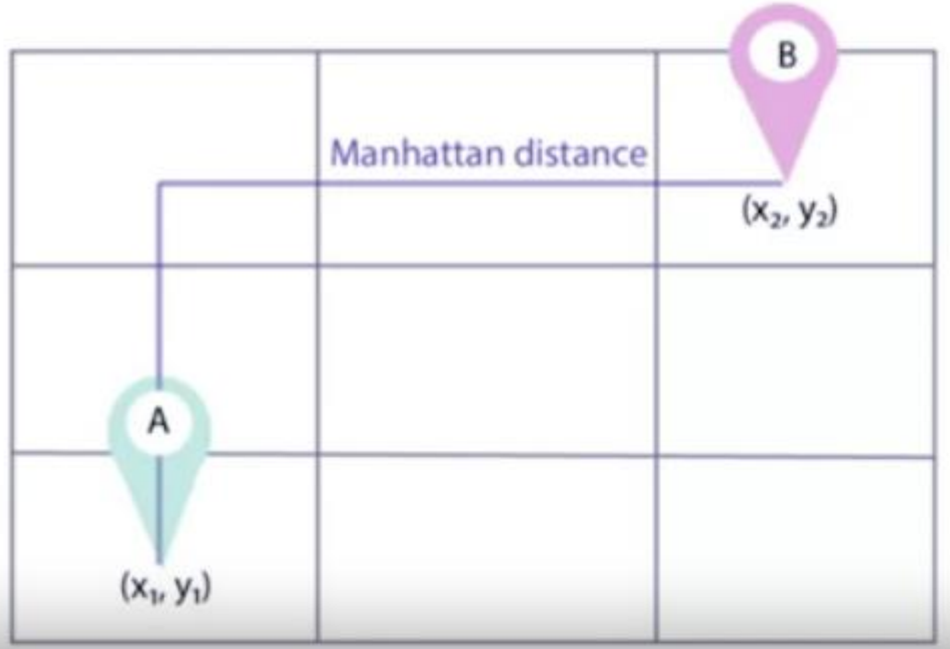
Manhattan distance



KNN (K- Nearest Neighbors) Algorithm

Manhattan distance

$$D_M(A, B) = \sum_{j=1}^{m} \left| X_{(A,j)} - X_{(B,j)} \right|$$



KNN (K- Nearest Neighbors) Algorithm

Hamming distance

$$D_H(A, B) = \sum_{j=1}^{m} \left| X_{(A,j)} - X_{(B,j)} \right|$$

$$X_{(A,j)} = X_{(B,j)} \Rightarrow D_{H_j} = 0$$

$$X_{(A,j)} \neq X_{(B,j)} \Rightarrow D_{H_j} = 1$$