Introduction to particle physics
Lecture 7

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Outline

Elastic scattering

Scattering on extended objects: Form factors

- Extended objects have a matter density $\rho(\vec{r})$.

$$\text{Normalisation:} \int \mathrm{d}^3 r\, \rho(\vec{r}) = 1$$

- Its Fourier transform is called a form factor:

$$F(\vec{q}) = \int \mathrm{d}^3 r\, \exp[-i\vec{q}\vec{r}]\rho(\vec{r}) \Longrightarrow F(0) = 1$$

- Naive modification of cross sections for scattering on such objects:

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}^2\Omega}\right|_{\mathrm{ptlike}} \Longrightarrow \left.\frac{\mathrm{d}\sigma}{\mathrm{d}^2\Omega}\right|_{\mathrm{extended}} \approx \left.\frac{\mathrm{d}\sigma}{\mathrm{d}^2\Omega}\right|_{\mathrm{ptlike}} |F(q)|^2$$

Elastic *ep* scattering and the Rosenbluth formula

(Details of the formula not examinable)

- Simple test of proton's charge distribution: elastic *ep* scattering (exchange of a photon). Elastic: **Nucleon remains intact**.
- Rosenbluth-formula (E and E' are energies of electron before and after scattering, M is the proton mass, q^2 is the space-like momentum transfer, and θ is the scattering angle):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\Omega} = \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left[\left(F_1^2(q^2)-\frac{\kappa^2q^2}{4M^2}F_2^2(q^2)\right)\right.$$
$$\left.-\frac{q^2}{2M^2}\left(F_1(q^2)+\kappa F_2(q^2)\right)^2\tan^2\frac{\theta}{2}\right]$$

Compare with Rutherford scattering (on very massive objects):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}$$

Elastic *ep* scattering and charge radius of the proton

(Details of the formula not examinable)

- Differences due to relativistic kinematics plus recoil of the protons (in Rutherford scattering, the nuclei stay at rest).
- Also inner structure: there are two form factors $F_{1,2}$. They are related to the electric and magnetic form factors, and are parametrised as

$$F_{1,2} \approx \left[\frac{1}{1-q^2/0.71\mathrm{GeV}^2}\right]^2$$

- Connection to charge radius: Assume $F_1 = F_2$ and

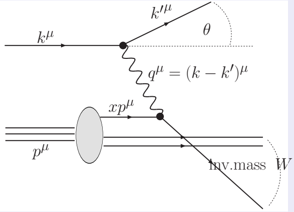
$$F(q^2) = \int \mathrm{d}^3r\,\rho(\vec{r})\exp[-i\vec{q}\vec{r}] \approx 1 - \frac{\vec{q}^2}{6}\langle r^2\rangle + \ldots$$

- Therefore: $r_{\rm proton} \equiv \langle r^2\rangle^{1/2} \approx 0.75 \pm 0.25$ fm.

Revealing the inner structure: Deep inelastic scattering

The process

- Terminology arises because in contrast to elastic scattering the **nucleon nearly always disintegrates**.
- Typically in DIS proton is probed with γ's.
  From $p \approx 1/\lambda$: If momentum transfer larger than 1 GeV, ($\approx 1/0.2\mathrm{fm}$) then inner structure revealed.
- Kinematics:



$$\nu = \frac{2pq}{m_p} \longrightarrow E - E'$$

(energy transfer)

$$x = \frac{Q^2}{2pq} \longrightarrow \frac{Q^2}{E - E'}$$

(momentum fraction of parton)

$$Q^2 = -q^2 = -2EE'(1 - \cos\theta)$$

(momentum transfer squared)

Measurement and two basic ideas

- Typically, the behaviour of the cross section with varying x (or, alternatively ν) and Q^2 is being measured.
  In addition, νp-scattering with W exchange is considered.
- Two basic ideas:
  - The **parton model** (by R.Feynman):
    The nucleon is made of smaller bits (partons). Later knowledge: Can be identified with quarks and gluons. But: In addition to the three **valence** quarks, carrying the quantum numbers (e.g. $|p\rangle = |uud\rangle$), there are virtual quarks and gluons, the **sea**.
  - The **scaling hypothesis** (by J.D.Bjorken):
    At large energies and momentum transfers, the cross section depends on one variable only. Reason: The photon ceases to scatter coherently off the nucleon, but solely sees the individual, point-like partons.

Bjorken-scaling

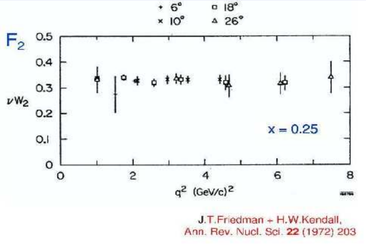
- Equation for cross section (cf. elastic scattering, replacing form factors $F_{1,2}(q^2)$ with **structure functions** $W_{1,2}(\nu, Q^2)$):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\Omega} = \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\right]$$

- Bjorken scaling implies that with no special scale present in the dynamics of the scattering the $W_{1,2}(\nu, Q^2)$ can be replaced:

$$\begin{aligned} m_p W_1(\nu, Q^2) &\longrightarrow F_1(x) \\ \frac{Q^2}{2m_p x} W_2(\nu, Q^2) &\longrightarrow F_2(x)\,, \end{aligned}$$

Independence of W_2 on q^2:



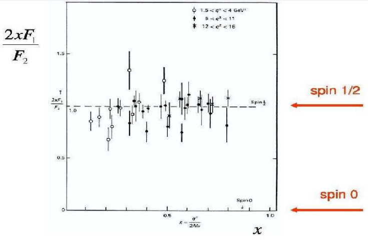
The Callan-Gross relation

- Bjorken scaling established that DIS in fact must be described in terms of parton-photon processes.
  But what are the properties of these point-like constituents?
- In 1969 Callan and Gross suggested that Bjorken's scaling functions are related:

$$2xF_1(x) = F_2(x)\,.$$

- This reflects the assumption that the partons inside the proton are indeed quarks, i.e. spin-1/2 particles (spin-0 for example would lead to $2xF_1(x)/F_2(x) = 0$.)

Measuring the quark spin



Deriving the Callan-Gross relation

- Basic idea: Compare *eq*-scattering cross section (free quark) with the DIS *ep* cross section and assume identity:

$$\frac{\mathrm{d}^2\sigma_{eq}}{\mathrm{d}^2\Omega\mathrm{d}E'} = \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\left[1+\frac{Q^2}{2m_p^2}\tan^2\frac{\theta}{2}\right]\delta\left(\nu-\frac{Q^2}{2m_p x}\right)$$

$$\frac{\mathrm{d}^2\sigma_{ep}}{\mathrm{d}^2\Omega\mathrm{d}E'} = \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\left[\frac{1}{\nu}F_2(x)+\frac{2}{m_p}\tan^2\frac{\theta}{2}F_1(x)\right]$$

Parton distributions and sum rules

- Define **probabilities** $f_a(x)$ to find a parton of type a with energy fraction between x and $x + dx$:

$$F_1(x) = \sum_a q_a^2 f_a(x)\,, \quad q_a = \text{parton's charge.}$$

- The parton momenta must add to the proton momentum:

$$\int_0^1 \mathrm{d}x\, x\, [f_u(x) + f_{\bar{u}}(x) + f_d(x) + f_{\bar{d}}(x) + f_s(x) + f_{\bar{s}}(x) + \dots] = 1\,.$$

- The parton types must yield a "net proton", $p\rangle = |uud\rangle$:

$$\begin{aligned}
\int\limits_0^1 \mathrm{d}x\, [f_u(x) - f_{\bar{u}}(x)] &= 2 & \int\limits_0^1 \mathrm{d}x\, [f_d(x) - f_{\bar{d}}(x)] &= 1 \\
\int\limits_0^1 \mathrm{d}x\, [f_s(x) - f_{\bar{s}}(x)] &= 0 & \int\limits_0^1 \mathrm{d}x\, [f_c(x) - f_{\bar{c}}(x)] &= 0\,.
\end{aligned}$$

From partons to QCD

Experimental evidences in the late 60's and beginning 70's:

- Scaling behaviour of structure functions supports the assumption of point-like particles with relatively weak interactions between them at short distance/large momentum transfers - the partons in DIS behave nearly like free particles.
- Callan-Gross relation supports spin-1/2 fermions.
- Comparing ep and νp scattering supports assignment of fractional charges for the partons, like the quarks.
- Checking the momentum sum rule suggests that only about 50% of the momentum of the proton is carried by the quarks - the other half must be carried by charge neutral objects. These are identified with the gluons, the force carriers of the strong force, binding the quarks together.

Conclusion: Quarks are real objects, need to find an interacting theory.

The invention of QCD

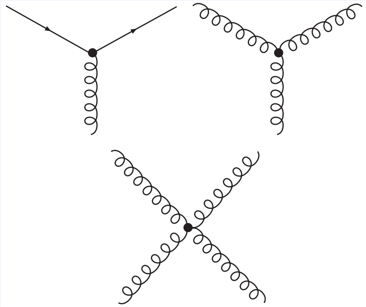
- Quantum chromodynamics (QCD) was proposed in 1973 by Fritzsch, Leutwyler and Gell-Mann. Main problem: What is the charge the gluons couple to (like electrical charge for the photons and weak isospin for the W bosons)?
- It is colour. Quarks come in three colours. This becomes apparent, when considering, e.g, the Δ^{++} particle. Since it is a fermion its wave function must be antisymmetric. But it consists of three u quarks (symmetric only), its spin-3/2 state has all spins aligned (also symmetric). Therefore: A new quantum number, in which the quarks are completely antisymmetric, i.e. $\propto \epsilon_{ijk}$.
- Construction principle as before: Gauge theory.

$$\mathbf{G}^{SU(3)_{\rm colour}}\mathcal{L}(\psi_q, G) \longrightarrow \mathcal{L}(\psi_q^*, G^*)\ ,$$

where $\psi_q = (\psi_q^r, \psi_q^g, \psi_q^b)$. and G is the colourful gluon field.

Feynman rules and colour factors

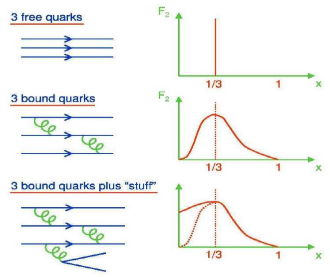
- Gluons coupling to, say, a "red" quarks transform it into a blue one. Because of charge conservation, therefore, gluons carry a colour and an anti-colour.
- Because gluons are charged under $SU(3)$, they experience self-interactions (*ggg* and *gggg* vertex).



- In calculations colours are encoded in Gell-Mann matrices, representing the algebraic structure of the symmetry group (like Pauli matrices in $SU(2)$).
- There are 8 such matrices, therefore 8 gluons (similar to $SU(2)$, where the 3 Pauli matrices correspond to the 3 weak gauge bosons).

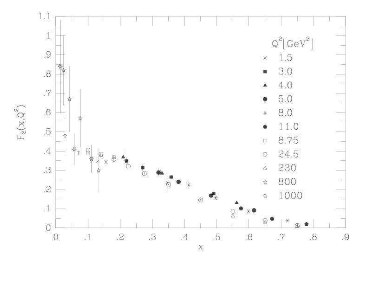
Effect on structure functions: Scaling violations

- Now it is possible to quantify the picture of "proton = quarks + stuff"



- Leads to evolution equations: "Russian dolls"

- This implies dependence of $F_{1,2}$ on the momentum transfer.
- Therefore: $F_{1,2}$ depend on both x and Q^2 - not constant in Q^2 any more.



Quantifying scaling violations: Evolution equations

(Details not examinable)

- Explanation: As the proton is hit harder and harder (i.e. at larger Q^2), the virtual photon starts resolving gluons and quark-antiquark fluctuations (partons in partons!).
- The scale Q^2 plays the role of a "resolution parameter".
- Described by the DGLAP equations. Basic structure:

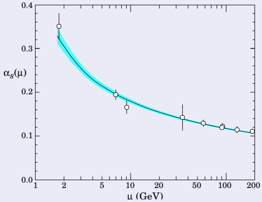
$$\begin{aligned}\frac{\mathrm{d}q(x,Q^2)}{\mathrm{d}\ln Q^2} &= \alpha_s(Q^2)\int\limits_x^1 \mathrm{d}y\left[q(y,Q^2)\,P_{q\to qg}\left(\frac{x}{y}\right)\right.\\ &\qquad\left.+g(y,Q^2)\,P_{g\to q\bar{q}}\left(\frac{x}{y}\right)\right]\end{aligned}$$

Here the quark at x can come from a quark (gluon) at y, the functions P encode the details of the decays $q\to qg$ ($g\to q\bar{q}$) responsible for it.

The "running" coupling in QCD: Asymptotic freedom

- Reassuring: Can understand the proton structure at large Q^2 in terms of perturbative objects (quarks and gluons). This implies that the coupling g_s is sufficiently small there:
  Asymptotic freedom.
- But measurements (left) and calculation show that the coupling becomes stronger the lower the scale ($\simeq Q^2$), i.e. the larger the distance.
- In fact, the perturbative α_s diverges for $\mu = \Lambda_{\rm QCD} \approx 300$ MeV, signalling the breakdown of the expansion.

$\alpha_s = g_s^2/(4\pi)$

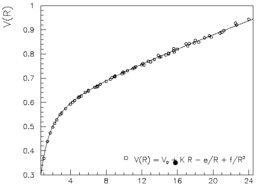


- Non-perturbative regime, where **only colour-less states** can exist:
  Confinement.
- Therefore, only hadrons (no quarks or gluons) as observable initial and final states in experiments.

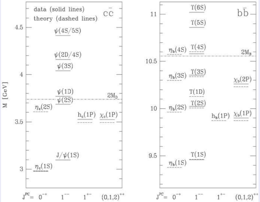
Confinement

- Strong interactions are different to electromagnetic ones:
  They have Coulomb potential at short distances (asymptotic freedom) plus linear term at large distances (confinement).
- Also, have to add factors due to colour.
- Test potential with bound states of heavy quarks (c, b): $m_c = 1.5$ GeV, $m_b = 5$ GeV. For light quarks (u, d, s), $m_{u,d,s} < \Lambda_{\rm QCD}$, the bound state energy is larger than the quark mass. There, the potential is so strong that the quarks move very fast, hard to calculate the bound states.

Potential:

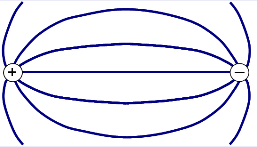


Quarkonia spectra ($Q\bar{Q}$):

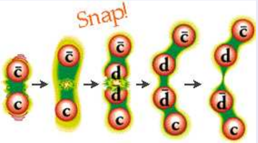


The QCD potential visualised

- Compare the potential due to a dipole in electrodynamics and in chromodynamics (right).
- The colour of the gluons "pulls" the field lines (the gluons) together: A "string" or flux tube emerges.
- It is characterised by constant energy density per unit length.



- There is a similar effect in superconductivity: Nielsen-Olsen string.
- A breakup of this string releases energy. This translates into "popping" $q\bar{q}$ pairs, which form the endpoints of smaller strings.



Summary

- Discussed probes of the nucleon (proton) structure: elastic vs. deeply inelastic scattering.
- Emergent: The parton picture of the proton.
- Partons can be identified with quarks and gluons, their interactions described by QCD.
- Consequences of QCD: Scaling violations (partons in partons).
- Also: Turned back to running coupling, asymptotic freedom and confinement.
- To read: Coughlan, Dodd & Gripaios, "The ideas of particle physics", Sec 25-26, Sec 30-33.
- Further reading on DIS (recommended): Coughlan, Dodd & Gripaios, "The ideas of particle physics", Sec 27-29.