# Appendix A

# Liquid Targets for Run91

## A.1    Density determination

### A.1.1    Chemical composition

Different orientations of the two nuclear spins in the diatomic molecules of $D_2$ and $H_2$ give rise to the molecular modifications designated by the prefixes ortho and para. The equilibrium composition is temperature dependent. Close to the boiling point of liquid hydrogen (20.4 deg K ) the concentrations are : ortho $\sim$ 0.21 % and para $\sim$ 99.79 % (equilibrium $H_2$ ). At 20.4 deg K the deuterium composition is $\sim$ 98 % ortho (equilibrium $D_2$ ). Most of the physical properties like vapor pressure, density of the liquid, etc. are mildly dependent upon the composition. The boiling point temperature in deg K for equilibrium $H_2$ is 20.27, for equilibrium $D_2$ is 23.52 and for hydrogen deuteride (HD) is 22.13 (at atmospheric pressure) from [121].

### A.1.2    Measuring the E665 Liquid Target Densities

The target densities are derived indirectly from the measurement of the saturated vapor pressure in the cryogenic liquid reservoir ( a description of the target setup and

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geometry can be found in [60] ). The vapor pressure was measured by a pressure to voltage transducer. The raw pressure has to be adjusted according to the pressure transducer calibration data [123]. The raw pressure values were recorded on the E665 raw data tapes from the EPICURE data logger. This datum is part of the 91 Spill Database record [69]. The information from each tape and for each spill is merged to create an n-tuple and it is checked for readout failures [94]. The data integrated over all Run blocks and after the transducer calibration correction are shown in figures A.1 and  A.2, for $D_2$ and $H_2$ respectively.
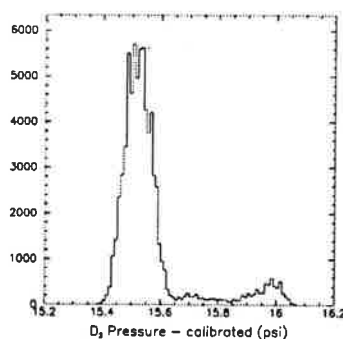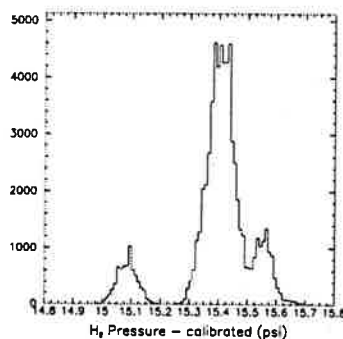


Figure A.1: $D_2$ Pressure



Figure A.2: $H_2$ Pressure

The Run dependence of the pressure values is shown in figure A.3 and figure A.4. The big fluctuations shown for some run blocks correspond to periods that either one

of the targets had problems. This was verified from the information found in the logbook of the experiment (Table A.1).
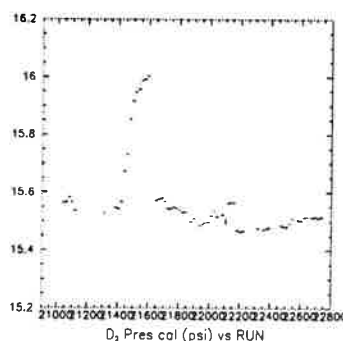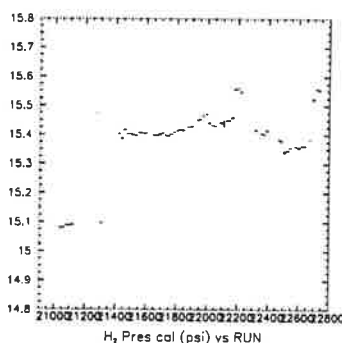


Figure A.3: $D_2$ Pressure vs Run



Figure A.4: $H_2$ Pressure vs Run

The target pressures are converted to temperatures using the parametrization for $D_2$ in equilibrium ( from reference [121] ) and the temperatures to density using the parametrizations from [124] as described in references [94] and [123]. The parametrizations for $D_2$ are:

$$\log_{10}P = a_0 + a_{-1}/T + a_1 \times T \qquad \text{(A.1)}$$

where P and T are the pressure and temperature (mm Hg, deg Kelvin), $a_0 = 4.7367$,

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$a_{-1} = -58.4440$, and $a_1 = 0.02670$.

$$\rho = (b_0 - T)/b_1 \qquad (A.2)$$

where $\rho$ and T are the density and temperature (gr/$cm^3$, Kelvin) and $b_0 = 86.981$ and $b_1 = 389.97$. For $H_2$ the parametrizations used are:

$$P = c_a^{c_0 + c_i/(T+c_b) + c_1 \times T} \qquad (A.3)$$

where P and T are the pressure and temperature (atm, Kelvin) and $c_a = 10., c_0 = 2.00062$, $c_i = -50.09708$, $c_b = 1.0044$ and $c_1 = 0.01748495$.

$$\rho = \rho_c + a_1 \times \delta^{0.38} + a_2 \times \delta + a_3 \times \delta^{1.33333} + a_4 \times \delta^{1.6666667} + a_5 \times \delta^2 \qquad (A.4)$$

where $\rho$ is in mol/$cm^3$, $\delta = T_c - T$, $T_c$ is 32.976 degrees Kelvin and T is in Kelvin, $\rho_c = 0.01559$ mol/$cm^3$, $a_1 = 7.3234603 \times 10^{-3}, a_2 = -4.407426 \times 10^{-4}, a_3 = 6.6207946 \times 10^{-4}$, $a_4 = -2.9226363 \times 10^{-4}$ and $a_5 = 4.00844907 \times 10^{-5}$.

The conversion factors for pressure units are: 1 mm Hg $\rightarrow$ 1 atm $1.3157895 \times 10^{-3}$ and atm $\rightarrow$ psi 14.6960. It is worth emphasizing the fact that the densities are insensitive to small pressure changes. The slope for $H_2$ is 0.00025 gm/$cm^3$/psi and for $D_2$ is 0.00055 gm/$cm^3$/psi. The measured densities are shown in figure A.5 and figure A.6. The Run dependence is shown in figures A.7 and A.8. The arrows correspond to the bad Run Blocks listed in Table A.1 for either $H_2$ or $D_2$.

## A.1.3   Results

After the Run blocks listed in Table A.1 are removed, the density distributions for the two targets are fitted to a gaussian shape. The result of the fit for the $H_2$ density is:

$\rho = 70.598 \pm 0.79164 \times 10^{-3}$ with $\sigma_\rho = 0.0160 \pm 0.40909 \times 10^{-4}$ (in g/$cm^3$).

The result of the fit for the $D_2$ density is:
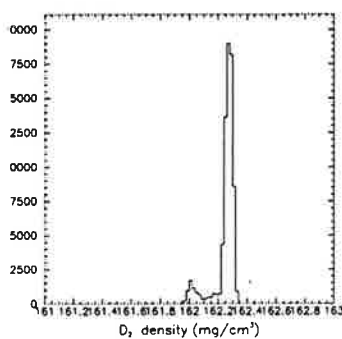
Figure A.5: $D_2$ Density
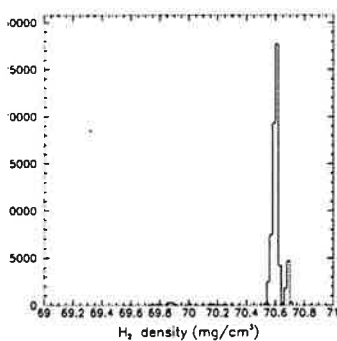


Figure A.6: $H_2$ Density
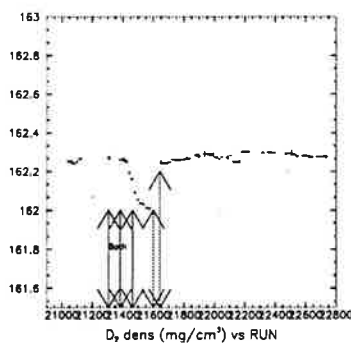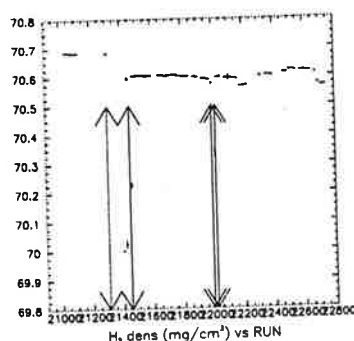


Figure A.7: $D_2$ Density vs Run

Figure A.8: $H_2$ Density vs Run

$\rho = 162.27 \pm 0.17169 \times 10^{-3}$ with $\sigma_\rho = 0.023666 \pm 0.52230 \times 10^{-4}$ . The values are in g/$cm^3$.

The measured mean values of the pressures are $P = 15.409 \pm 0.00024$ for $H_2$ (in psi) and $P = 15.522 \pm 0.00021$ for $D_2$ (psi). The number of points entering these fits is $\sim 670000$, since the Spill Database contains one pressure measurement per spill.

## A.1.4  Systematic Errors

- Pressure to Density conversion. The resulting uncertainty of the conversion due to the different parametrizations is of the order of 0.3 % for $D_2$ and 0.04 % for $H_2$ ([124]).

- Pressure Transducer. The accuracy is quoted to be 0.11 % of the full reading (50 psi) ([123]). This gives a relative error in the density measurement $\sim 0.01$ % for $H_2$ and $D_2$.

## A.2 Measuring the Target Composition - Effects on the $D_2$ rate

### A.2.1 Temperature determination

The chemical contents of the targets were analyzed using a boil-off test. Hydrogen deuteride (HD) in the $D_2$ target is the only impurity causing concern. The results for the $D_2$ composition from the Argonne Chemistry Lab are tabulated in Table A.2. The big difference between the last two measurements and the first one ( and also the value for the 1990 Run Period) can be blamed on the $D_2$ target failure of the 6th of September 1991. For the 1991 Run Period the procedure used for the $D_2$ target was to collect the liquid in a tank and then reuse it to fill the target. The explanation for the large HD contamination after the 6th of September is that this tank was contaminated when the target liquid was collected after the target failed.

Since the $D_2$ is not pure the $D_2$ density computed from the measured vapor pressure is not correct since the pressure really corresponds to the $D_2$ and HD equilibrium mixture. The following procedure determines the correct $D_2$ density:

- A temperature $T_i$ is obtained from the measured vapor pressure $P_{meas}$, using the $D_2$ P to T relation (equation A.1).

- This temperature is used with the parametrization (for HD)

$$\log_{10}P_{HD} = 5.04964 - \frac{55.2495}{T} + 0.01479 \times T \qquad (A.5)$$

  from [121] to find the vapor pressure $P_{HD}$ (in mm Hg). Note that NMC [111] is using the same parametrization from [125] .

- The actual $D_2$ vapor pressure is computed from $nP_{meas} = n_{HD}P_{HD} + n_{D_2}P_{D_2}$, ($n = n_{HD} + n_{D_2}$ with n, $n_{HD}$ and $n_{D_2}$ being the number of moles). The HD molar volume is given by

$$U_{HD} = 24.886 - 0.30911T + 0.01717T^2 \qquad (A.6)$$

in $cm^3/mole$ from [125] and $n_{D_2}$ is from equation A.2 converted to $moles/cm^3$. Note that the number of moles in the determination of the number $n$ are converted to molar densities (dividing with the constant target volume $V$).

- From $P_{D_2}$ a new T is found from equation A.1 and the procedure is repeated, until HD and $D_2$ are found to have the same T.

The first T measurement $(T_i)$ was 23.718 deg K with $P_{meas} = 15.522$ psi. After a few iterations, for the first period (HD contamination of 4.2 %) the temperature is evaluated to be 23.602 deg K with $P_{D_2} = 15.061$ psi. The molar density from that value is 0.040346 moles/$cm^3$ and the mass density is 0.16225 gm/$cm^3$. For HD the molar density at 23.602 deg K is 0.039644 moles/$cm^3$.

The values for the second period (averaging the HD from the second and third measurements of Table A.2 ) starting with values $T = 23.718$ deg K and $P_{meas} = 15.522$ psi are: $T = 23.38163$ deg K, $P_{D_2} = 14.049$ psi and molar density 0.040487 moles/$cm^3$ and the mass density 0.16309 gm/$cm^3$ for the target $D_2$. For HD the molar density is 0.039788 moles/$cm^3$. The error from averaging measurements 2 and 3 of Table A.2 is $\sim 0.53$ % . The results for the two periods (after the iterative procedure ) are presented in Table A.3.

## A.2.2   neutron scattering rate from the $D_2$ target

Two terms ($\alpha$ and $\beta$ in the following formula ), which are related to the deuterium target composition are required in order to obtain the relation between the scattering rate ($R_{D_2}$) from the target, as well as $\sigma_p$ and $\sigma_n$, the proton and neutron cross sections (see [126]). The molar densities of electrons, protons and neutrons in the target are:

$$n_e = n_p = 2(1 - f)n_{D_2} + 2fn_{HD} \tag{A.7}$$

$$n_n = 2(1 - f)n_{D_2} + 1fn_{HD} \tag{A.8}$$

where $n_{D_2}$ and $n_{HD}$ are the molar densities of $D_2$ and HD respectively and f is the HD fraction per volume in the target. These are evaluated from equation A.2 converting to $moles/cm^3$ and equation A.6, using $n_{HD}=1/U_{HD}$.

Assuming that $\sigma$ from the actual $D_2$ target is the sum of $\sigma_p$ and $\sigma_n$ (weighted with the molar densities for p and n in the target), the scattering rate is:

$$R = LN_A N_{flux}(n_p \sigma_p + n_n \sigma_n) = LN_A N_{flux} n_0 (\alpha \sigma_n + \beta \sigma_p) \qquad (A.9)$$

where $n_0 = 2n_{D_2}$.

The results for the two different periods ( the first period and the average of the second and the third periods as defined in Table A.2 ) are given in Table A.3.

## A.2.3 Systematic error from the target composition determination

The total systematic error due to the target quantities in the cross-section measurement has a contribution from the pressure to density conversion (calibration, conversion parametrizations - section 1 of this appendix) and a contribution from the HD contamination correction (measurement of f - section 2). This last error affects both the determination of $n_{D_2}$ and $n_p$, $n_n$ from equation A.9. The fractional errors on each quantity are given below.

- $\delta_{n_{H_2}}/n_{H_2} = 0.05\%$. This is the error of the $H_2$ molar densisty.

- $\delta_{n_{D_2}}/n_{D_2} = \sqrt{0.31\% + 0.53\%} = 0.58\%$. This is the error of the $D_2$ molar densisty. The second number is the effect of the HD contamination correction (see section 2 of this appendix). The 2 errors are independent, so they add in quadrature.

- $\delta_f/f = 6.0\%$. This is the error assigned to the evaluation of the HD per volume fraction. The assumption is that there are 2 periods in the $D_2$ target composition, before and after the target problems. There are 2 measurements of the

target composition for period 2 (the 2 last 1991 values from Table A.2). The mean value is used as the f value for period 2 and the error on the mean as the error in f due to the chemical analysis method ($\delta_f = 0.006$).

From equations A.7 and A.8, the error in the number of neutrons and number of protons in the $D_2$ target depends on the error in f and the error in $n_{D_2}$. The errors are correlated since the value of $n_{D_2}$ depends on f.

$$\delta_{n_i} = \sqrt{(\frac{\partial n_i}{\partial f}\delta_f)^2 + (\frac{\partial n_i}{\partial n_{D_2}}\delta_{n_{D_2}})^2 + 2\frac{\partial n_i}{\partial f}\frac{\partial n_i}{\partial n_{D_2}}\delta_f\delta_{n_{D_2}}} \qquad (A.10)$$

with i=p or n, $\frac{\partial n_p}{\partial n_{D_2}} = \frac{\partial n_n}{\partial n_{D_2}} = 2(1-f)$, $\frac{\partial n_p}{\partial f} = -2n_{D_2} + 2n_{HD}$ and $\frac{\partial n_n}{\partial f} = -2n_{D_2} + n_{HD}$. Substituting these expressions in equation A.10, results in:

$$\delta_{n_p} = \sqrt{((-2n_{D_2} + 2n_{HD})\delta_f)^2 + (2(1-f)\delta_{n_D})^2 + 8(1-f)(-n_{D_2} + n_{HD})\delta_f\delta_{n_D}}$$

$$\qquad (A.11)$$

$$\delta_{n_n} = \sqrt{((-2n_{D_2} + n_{HD})\delta_f)^2 + (2(1-f)\delta_{n_D})^2 + 4(1-f)(-2n_{D_2} + n_{HD})\delta_f\delta_{n_D}}$$

$$\qquad (A.12)$$

The result is $\delta_{n_p} = 0.0004275 moles/cm^3$ and $\delta_{n_n} = 0.00019 moles/cm^3$. These errors correspond to $\delta_\alpha = 0.24$ % and $\delta_\beta$ 0.53 % . In order to find the normalization error from equation A.9 we need to know $\sigma_p$ and $\sigma_n$. For the $\sigma_n/\sigma_p$ ratio, with $N_H$ and $N_D$ the number of events from each target, $N_f^H$ and $N_f^D$ the number of beams, and given the equal lengths of the 2 targets we have ($n_H = 2n_{H_2}$):

$$\sigma_n/\sigma_p = \frac{1}{\alpha}(\frac{N_D n_H N_f^H}{N_H n_0 N_f^D}) - \frac{\beta}{\alpha} \qquad (A.13)$$

Calling $A \equiv \frac{1}{\alpha}(\frac{N_D n_H N_f^H}{N_H n_0 N_f^D})$ and $r \equiv \sigma_n/\sigma_p$, the error due to the determination of the target composition is given by:

$$\delta_r = \sqrt{(\frac{\partial r}{\partial \alpha}\delta_\alpha)^2 + (\frac{\partial r}{\partial \beta}\delta_\beta)^2 + (\frac{\partial r}{\partial n_H}\delta_{n_H})^2 + +2(\frac{\partial r}{\partial \alpha}\frac{\partial r}{\partial \beta}\delta_\alpha\delta_\beta)} \qquad (A.14)$$

Note that the $n_0$ that appears in equation A.13 is a scale factor. The effect of the error in the $n_0$ measurement has been taken into account in the $\alpha$ and $\beta$ error calculation.

The derivatives are : $\frac{\partial r}{\partial \alpha} = -\frac{(A-\frac{\beta}{\alpha})}{\alpha}$, $\frac{\partial r}{\partial \beta} = -1/\alpha$ and $\frac{\partial r}{\partial n_H} = \frac{(A+\frac{\beta}{\alpha})}{n_H}$. Substituting these expressions to equation A.14 we get:

$$\delta_r = \sqrt{((A-\frac{\beta}{\alpha})\frac{\delta_\alpha}{\alpha})^2 + (\frac{\beta}{\alpha}\frac{\delta_\beta}{\beta})^2 + ((A+\frac{\beta}{\alpha})\frac{\delta_{n_H}}{n_H})^2 + +2(A-\frac{\beta}{\alpha})\frac{\beta}{\alpha}\frac{\delta_\alpha}{\alpha}\frac{\delta_\beta}{\beta}} \quad (A.15)$$

which for $A \sim 2$ and $\frac{\beta}{\alpha} \sim 1$, gives $\delta_{\sigma_n/\sigma_p} \sim .85\%$.

Table A.1: Run 91 Liquid Target Performance vs Run Block

| Run Blocks | Time | $H_2$ status | $D_2$ status |
|---|---|---|---|
| 20935-21304 | 07jul91-12aug91 | OK | OK |
| 21305-21382 | 12aug91-20aug91 | no | no |
| 21383-21428 | 20aug91-24aug91 | no | OK |
| 21429-21449 | 24aug91-25aug91 | OK | OK |
| 21450-21463 | 25aug91-27aug91 | no | OK |
| 21464-21600 | 27aug91-06sep91 | OK | OK |
| 21601-21642 | 06sep91-15sep91 | OK | no |
| 21643-21990 | 15sep91-21oct91 | OK | OK |
| 21991-22009 | 21oct91-25oct91 | no | OK |
| 22015-22726 | 25oct91-08jan92 | OK | OK |
| ——— | 15oct91 move | targets | 2cm east |
| ——— | 18nov91 swap | targets | — |

Table A.2: Run 91 $D_2$ Target Composition

| Date | $H_2$ | HD (f) | $D_2$ |
|---|---|---|---|
| 06sep91 | 0.0042 | 0.032 | 0.963 |
| 15nov91 | 0.0080 | 0.088 | 0.906 |
| 08jan92 | 0.0021 | 0.100 | 0.898 |
| run 90 | 0.0006 | 0.044 | 0.955 |

Table A.3: Density and Scattering rate

| Period | $n_{H_2}$ moles/$cm^3$ | $n_{D_2}$ moles/$cm^3$ | $n_{HD}$ moles/$cm^3$ | $\alpha$ | $\beta$ |
|---|---|---|---|---|---|
| I | 0.035022 | 0.040346 | 0.039644 | 0.98372 | 0.99944 |
| II | 0.035022 | 0.040487 | 0.039788 | 0.95219 | 0.99838 |

gh/m²    .0700    .01616

σρht    .0700    .01600

$L_R$    .114    .130