Time Series Analysis

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```
library(xts)
library(tseries)
library(xts)
library(zoo)
library(forecast)
library(readxl)
```

1 Exercise 1

```
Fund=read.csv("8 Funds.csv",header=TRUE)
head(Fund,5)
```

```
      Date LP60083956 LP60071398 LP60088034 LP60081330 LP60094177 LP60058385
1 1/4/2005      46.60     102.58      10.96      25.03      10.85       5.47
2 1/5/2005      46.60     102.01      10.93      24.98      10.80       5.52
3 1/6/2005      46.60     101.55      10.97      25.01      10.82       5.51
4 1/7/2005      46.60     101.63      11.07      25.08      10.88       5.54
```

```
5 1/10/2005      46.72     101.72      11.09      25.23      10.91       5.55
  LP60071418 LP60070661
1      93.48       8.43
2      90.09       8.38
3      90.62       8.40
4      90.19       8.51
5      91.45       8.52
```

```r
time=as.Date(Fund$Date,format="%m/%d/%Y")
head(time,5)
```

```
[1] "2005-01-04" "2005-01-05" "2005-01-06" "2005-01-07" "2005-01-10"
```

```r
Fundnew=Fund[,-1]
head(Fundnew,5)
```

```
  LP60083956 LP60071398 LP60088034 LP60081330 LP60094177 LP60058385 LP60071418
1      46.60     102.58      10.96      25.03      10.85       5.47      93.48
2      46.60     102.01      10.93      24.98      10.80       5.52      90.09
3      46.60     101.55      10.97      25.01      10.82       5.51      90.62
4      46.60     101.63      11.07      25.08      10.88       5.54      90.19
5      46.72     101.72      11.09      25.23      10.91       5.55      91.45
  LP60070661
1       8.43
2       8.38
3       8.40
4       8.51
5       8.52
```

```r
Fund.xts=xts(Fundnew,order.by=time)
head(Fund.xts,5)
```

```
           LP60083956 LP60071398 LP60088034 LP60081330 LP60094177 LP60058385
2005-01-04      46.60     102.58      10.96      25.03      10.85       5.47
2005-01-05      46.60     102.01      10.93      24.98      10.80       5.52
2005-01-06      46.60     101.55      10.97      25.01      10.82       5.51
2005-01-07      46.60     101.63      11.07      25.08      10.88       5.54
2005-01-10      46.72     101.72      11.09      25.23      10.91       5.55
           LP60071418 LP60070661
2005-01-04      93.48       8.43
```

```
2005-01-05      90.09      8.38
2005-01-06      90.62      8.40
2005-01-07      90.19      8.51
2005-01-10      91.45      8.52
```

```
colnames(Fund.xts)
```

```
[1] "LP60083956" "LP60071398" "LP60088034" "LP60081330" "LP60094177"
[6] "LP60058385" "LP60071418" "LP60070661"
```

```
Fund2=Fund.xts[,2]

summary(Fund2)
```

```
     Index              LP60071398
 Min.   :2005-01-04   Min.   : 93.52
 1st Qu.:2008-04-03   1st Qu.:152.69
 Median :2011-07-04   Median :172.48
 Mean   :2011-07-03   Mean   :178.84
 3rd Qu.:2014-10-01   3rd Qu.:201.23
 Max.   :2017-12-29   Max.   :299.00
```

```
class(Fund2)
```

```
[1] "xts" "zoo"
```

```
dim(Fund2)
```

```
[1] 3389    1
```

```
colnames(Fund2)
```

```
[1] "LP60071398"
```

```
head(Fund2)
```

```
           LP60071398
2005-01-04     102.58
2005-01-05     102.01
2005-01-06     101.55
2005-01-07     101.63
2005-01-10     101.72
2005-01-11     101.46
```

```r
tail(Fund2)
```

```
           LP60071398
2017-12-22     289.85
2017-12-25     289.85
2017-12-26     289.85
2017-12-27     287.69
2017-12-28     288.87
2017-12-29     288.36
```

```r
start(Fund2)
```

```
[1] "2005-01-04"
```

```r
end(Fund2)
```

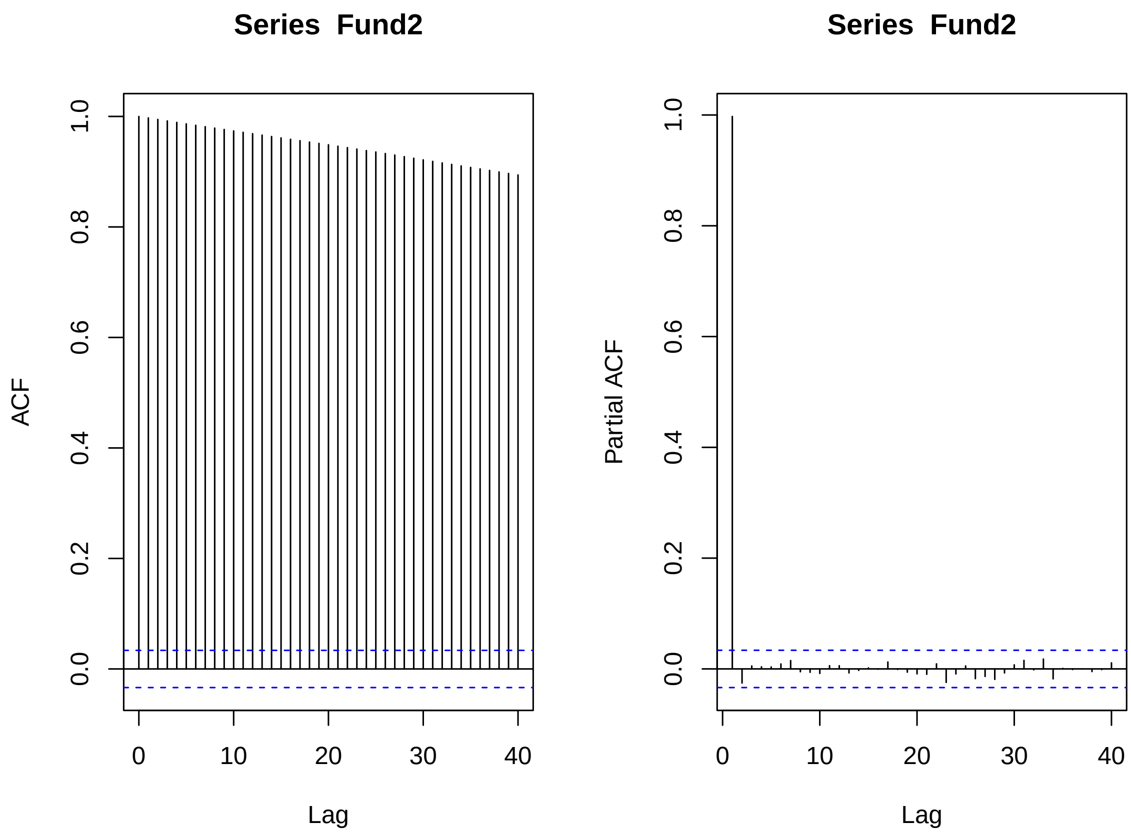
```
[1] "2017-12-29"
```

```r
length(Fund2)
```

```
[1] 3389
```

```r
plot(Fund2,main="Fund2",ylab="Price",xlab="Date")
```


```
par(mfrow=c(1,2))
acf(Fund2,lag.max=40,type="correlation")
pacf(Fund2,lag.max=40)
```




```r
adf.test(Fund2)
```

```
	Augmented Dickey-Fuller Test

data:  Fund2
Dickey-Fuller = -1.8825, Lag order = 15, p-value = 0.6281
alternative hypothesis: stationary
```

```r
return=diff(log(Fund2))
head(return,5)
```

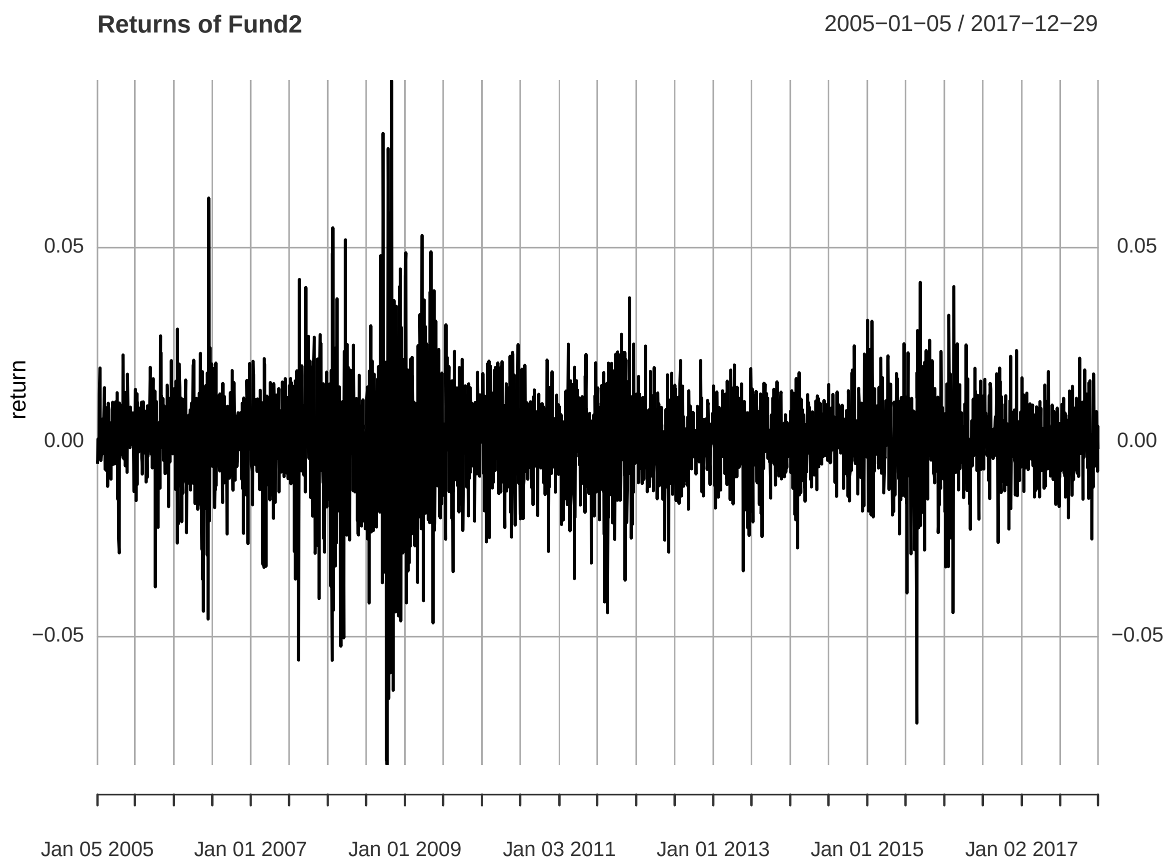
```
              LP60071398
2005-01-04            NA
2005-01-05 -0.0055721343
2005-01-06 -0.0045195597
```

```
2005-01-07  0.0007874791
2005-01-10  0.0008851734
```

```r
return=na.omit(return)
head(return,5)
```

```
              LP60071398
2005-01-05 -0.0055721343
2005-01-06 -0.0045195597
2005-01-07  0.0007874791
2005-01-10  0.0008851734
2005-01-11 -0.0025593084
```

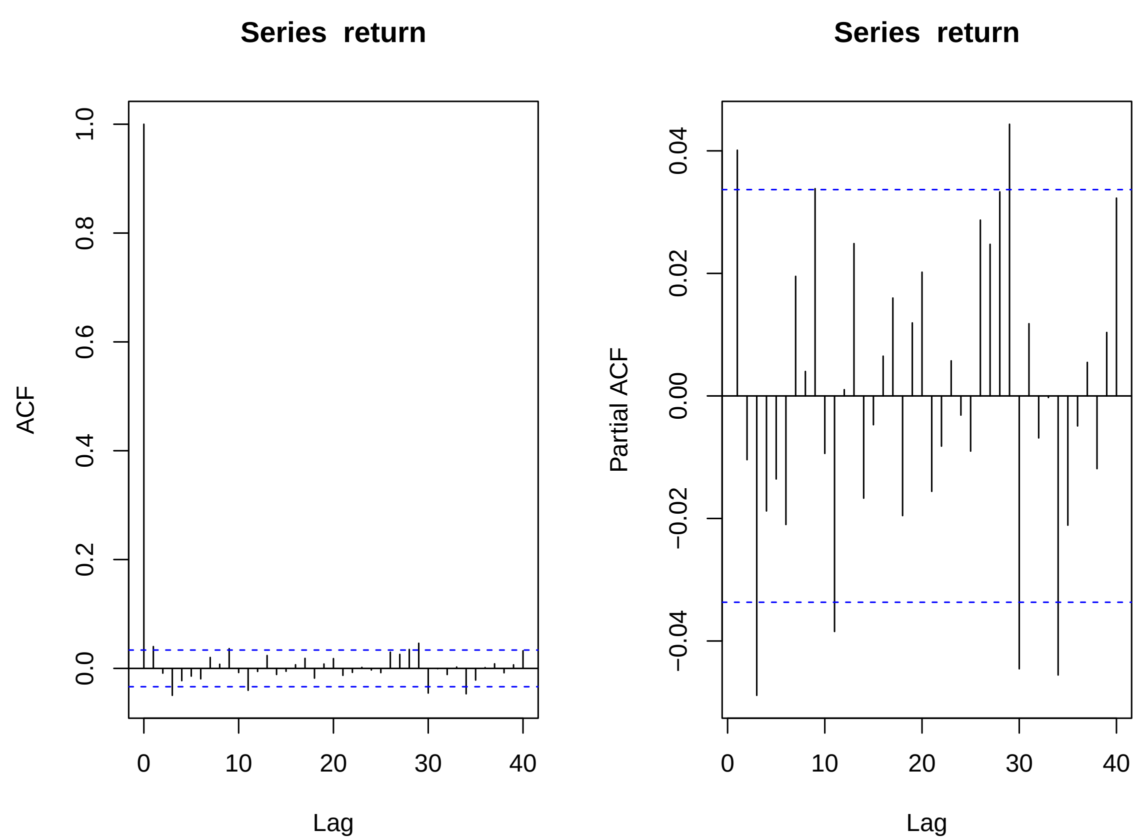
```r
par(mfrow = c(1, 1))
plot(return,main="Returns of Fund2",ylab="return",xlab="Date")
```



```r
adf.test(return)
```

```
    Augmented Dickey-Fuller Test

data:  return
Dickey-Fuller = -14.741, Lag order = 15, p-value = 0.01
alternative hypothesis: stationary
```

```r
par(mfrow=c(1,2))
acf(return,lag.max=40,type="correlation")
pacf(return,lag.max=40)
```




```r
model1=arima(return,order=c(0,0,1))
model1
```

```
Call:
arima(x = return, order = c(0, 0, 1))

Coefficients:
         ma1  intercept
      0.0406      3e-04
s.e.  0.0172      2e-04

sigma^2 estimated as 0.0001504:  log likelihood = 10103.5,  aic = -20200.99
```

```
model2=arima(return,order=c(1,0,0))
model2
```

```
Call:
arima(x = return, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.0401      3e-04
s.e.  0.0172      2e-04

sigma^2 estimated as 0.0001504:  log likelihood = 10103.45,  aic = -20200.91
```

```
model3=arima(return,order=c(0,0,2))
model3
```

```
Call:
arima(x = return, order = c(0, 0, 2))

Coefficients:
         ma1      ma2  intercept
      0.0400  -0.0052      3e-04
s.e.  0.0173   0.0177      2e-04

sigma^2 estimated as 0.0001504:  log likelihood = 10103.54,  aic = -20199.08
```

```r
model4=arima(return,order=c(1,0,2))
model4
```

```
Call:
arima(x = return, order = c(1, 0, 2))

Coefficients:
         ar1      ma1      ma2  intercept
      0.6809  -0.6442  -0.0539      3e-04
s.e.  0.1467   0.1469   0.0166      2e-04

sigma^2 estimated as 0.0001502:  log likelihood = 10106,  aic = -20202
```

```r
AIC(model1,model2,model3,model4)
```

```
       df       AIC
model1  3 -20200.99
model2  3 -20200.91
model3  4 -20199.08
model4  5 -20202.00
```

1.1 Answers for exercise 1

The original price series is not stationary, but the returns are:

- Price Series (Fund2):
  The (ADF) test yielded a p-value of 0.6281 (> 0.05). We cannot reject the null hypothesis of a unit root, it is non-stationary.
- Return Series (return):
  After taking the log-difference, the ADF test yielded a p-value of 0.01 (< 0.05). We reject the null hypothesis, it is stationary.

Since differencing the data once produces a stationary series (the return), the original process is integrated of order 1 (random walk).

The ARMA(1,2) model is the best because it has the lowest AIC (-20202.00).
The plots of ACF and PACF confirm stationarity and mostly stay within the significance bounds

2 Exercise 2

```r
MSFT=read.csv("MSFT.csv",header=TRUE)
head(MSFT,5)
```

```
      Date Adj.Close
1 1/1/2010  22.12004
2 2/1/2010  22.50467
3 3/1/2010  23.09885
4 4/1/2010  24.08464
5 5/1/2010  20.34655
```

```r
time=as.Date(MSFT$Date,format="%m/%d/%Y")
head(time,5)
```

```
[1] "2010-01-01" "2010-02-01" "2010-03-01" "2010-04-01" "2010-05-01"
```

```r
MSFTnew=MSFT[,-1]
head(MSFTnew,5)
```

```
[1] 22.12004 22.50467 23.09885 24.08464 20.34655
```

```r
MSFT.xts=xts(MSFTnew,time)
head(MSFT.xts,5)
```

```
               [,1]
2010-01-01 22.12004
2010-02-01 22.50467
2010-03-01 23.09885
2010-04-01 24.08464
2010-05-01 20.34655
```

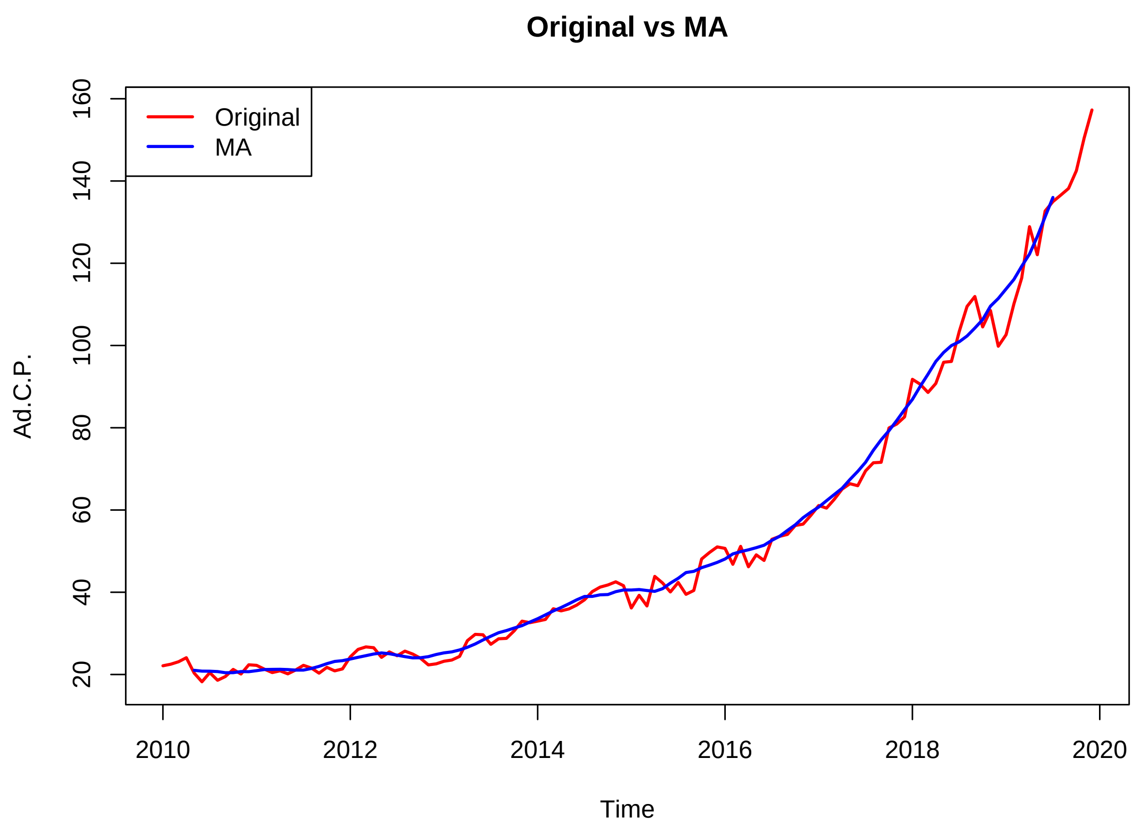
```r
MSFT.ts=ts(MSFT[,2],start=c(2010,1),end=c(2019,12),frequency=12)
head(MSFT.ts,5)
```

```
          Jan      Feb      Mar      Apr      May
2010 22.12004 22.50467 23.09885 24.08464 20.34655
```

```
MSFT_MA=rollmean(MSFT.ts,10)
head(MSFT_MA,5)
```

```
          May      Jun      Jul      Aug      Sep
2010 21.01661 20.81634 20.80231 20.71444 20.43584
```

```
plot(MSFT.ts,main="Original vs MA",ylab="Ad.C.P.",xlab="Time",col ="red",lwd=2)
lines(MSFT_MA,col="blue",lwd=2)
legend("topleft",legend=c("Original","MA"),col=c("red","blue"),lty=1,lwd=2)
```




```
dec=decompose(MSFT.ts,type="additive")
dec
```

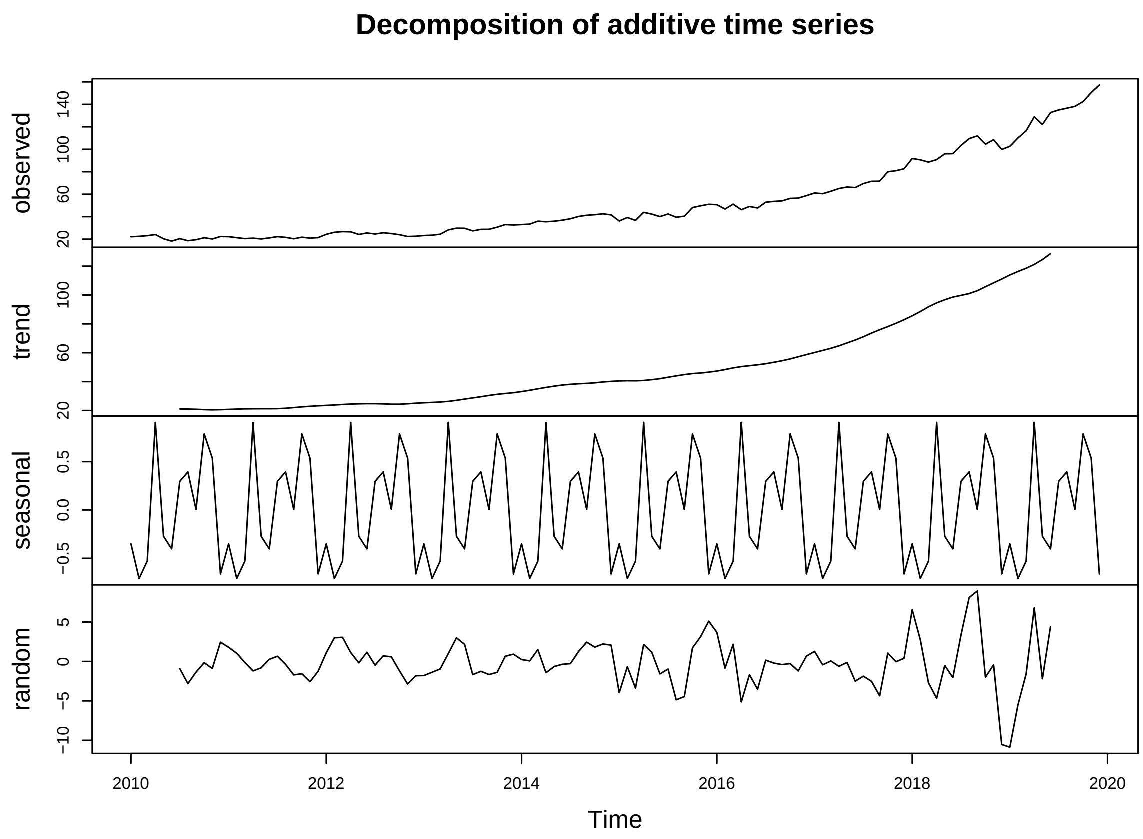
```
$x
           Jan       Feb       Mar       Apr       May       Jun       Jul
```

```
2010  22.12004  22.50467  23.09885  24.08464  20.34655  18.22817  20.44628
2011  22.22015  21.29865  20.46536  20.89256  20.15906  21.09441  22.23026
2012  24.29114  26.10907  26.71152  26.51280  24.16954  25.49495  24.56149
2013  23.21874  23.51479  24.40029  28.22963  29.76477  29.66431  27.34545
2014  32.97568  33.38527  35.98862  35.47060  35.94472  36.87027  38.16118
2015  36.17065  39.25947  36.66249  43.85794  42.25294  40.06810  42.38234
2016  50.66554  46.79367  51.15899  46.19408  49.09338  47.72978  52.86933
2017  61.08928  60.45619  62.60993  65.08163  66.39352  65.90403  69.50853
2018  91.78175  90.58389  88.58300  90.76675  95.93014  96.12175 103.40325
2019 102.62807 110.09694 116.39996 128.89465 122.06501 132.70018 134.98845
           Aug       Sep       Oct       Nov       Dec
2010  18.59257  19.50409  21.24027  20.11733  22.36439
2011  21.58120  20.32129  21.74191  20.88464  21.35449
2012  25.68664  24.96750  23.94397  22.33317  22.59281
2013  28.68523  28.78357  30.62580  32.97832  32.60095
2014  40.16826  41.24656  41.77148  42.53664  41.58728
2015  39.49634  40.43281  48.08818  49.65031  51.02423
2016  53.59690  54.06235  56.23986  56.55898  58.71752
2017  71.48766  71.59940  79.95219  80.90376  82.63354
2018 109.49554 111.91309 104.51549 108.50786  99.81742
2019 136.56349 138.18109 142.49458 150.45569 157.27043

$seasonal
              Jan          Feb          Mar          Apr          May
2010 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2011 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2012 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2013 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2014 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2015 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2016 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2017 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2018 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
2019 -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
              Jun          Jul          Aug          Sep          Oct
2010 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2011 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2012 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2013 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2014 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2015 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2016 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2017 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
```

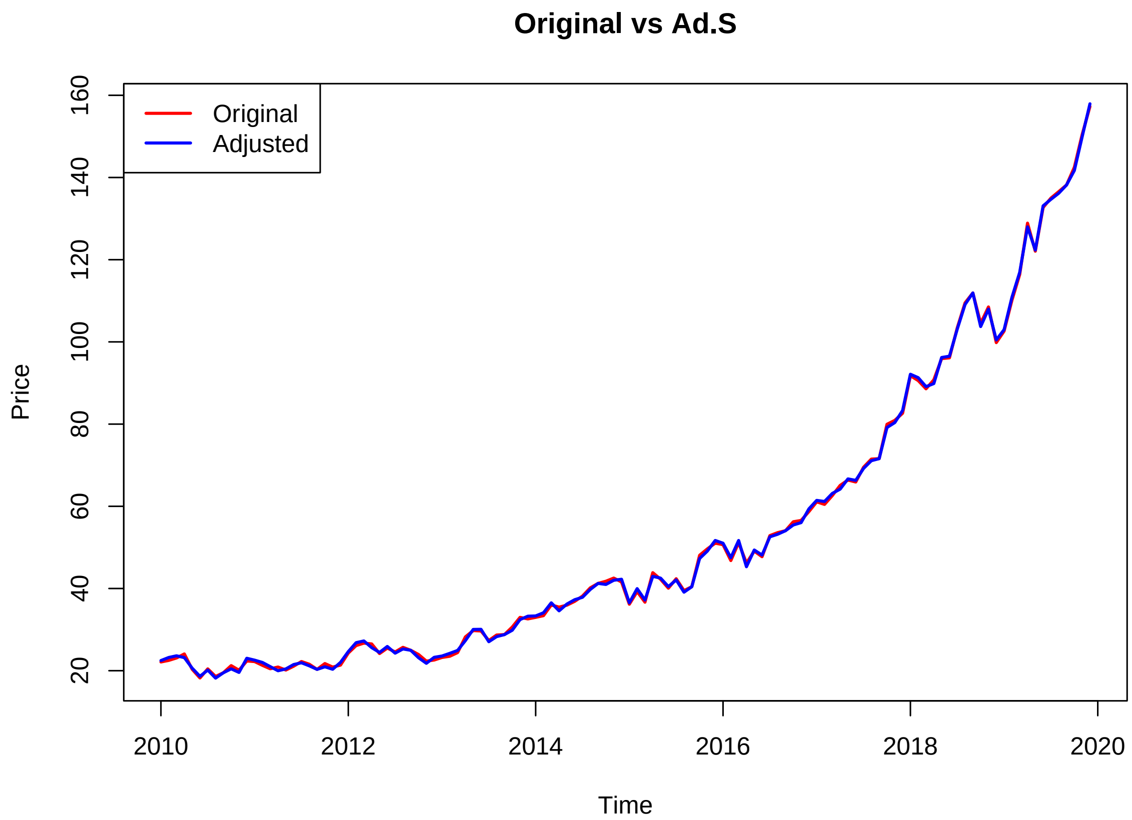
```
2018 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
2019 -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
             Nov          Dec
2010  0.536440778 -0.662038384
2011  0.536440778 -0.662038384
2012  0.536440778 -0.662038384
2013  0.536440778 -0.662038384
2014  0.536440778 -0.662038384
2015  0.536440778 -0.662038384
2016  0.536440778 -0.662038384
2017  0.536440778 -0.662038384
2018  0.536440778 -0.662038384
2019  0.536440778 -0.662038384

$trend
           Jan       Feb       Mar       Apr       May       Jun       Jul
2010        NA        NA        NA        NA        NA        NA  21.05816
2011  20.77393  20.97278  21.13136  21.18631  21.23919  21.22908  21.27329
2012  23.54737  23.81556  24.18022  24.46556  24.61767  24.72962  24.73653
2013  25.35584  25.59678  25.88073  26.31814  27.04010  27.90065  28.72419
2014  33.08853  34.01765  35.01540  35.99909  36.86176  37.63445  38.14201
2015  40.48780  40.63568  40.57378  40.80307  41.36267  42.05228  43.04944
2016  47.32943  48.35391  49.50933  50.41688  51.04440  51.65281  52.40769
2017  60.15826  61.59701  63.07317  64.79189  66.79427  68.80514  71.08049
2018  85.56664  88.56259  91.82599  94.52919  96.70283  98.56900  99.73692
2019 113.85251 116.29639 118.51872 121.19568 124.52597 128.66767        NA
           Aug       Sep       Oct       Nov       Dec
2010  21.01208  20.85210  20.60937  20.46855  20.58017
2011  21.56001  22.02071  22.51514  22.91642  23.26688
2012  24.58375  24.37936  24.35459  24.65926  25.06612
2013  29.54200  30.43612  31.22068  31.77988  32.33763
2014  38.51989  38.79272  39.17027  39.78259  40.17867
2015  43.96732  44.88526  45.58662  45.96898  46.57323
2016  53.41128  54.45768  55.72178  57.22960  58.70770
2017  73.61467  75.95220  78.10462  80.40553  82.89529
2018 101.00190 102.97398 105.72168 108.39930 111.01235
2019        NA        NA        NA        NA        NA

$random
             Jan          Feb          Mar          Apr          May
2010          NA           NA           NA           NA           NA
2011  1.79647418   1.03512788  -0.13695327  -1.20063244  -0.80804880
2012  1.09401893   3.00276992   3.06035431   1.14036135  -0.17605101
```

```
2013  -1.78685145  -1.37273050  -0.95138377   1.00461518   2.99675562
2014   0.23740026   0.07688680   1.50227756  -1.43536365  -0.64496188
2015  -3.96690278  -0.66695229  -3.38223706   2.14799443   1.16235133
2016   3.68635984  -0.85098429   2.17870969  -5.12968003  -1.67893888
2017   1.28126472  -0.43156070   0.06581006  -0.61714028  -0.12867417
2018   6.56535518   2.73056559  -2.71393698  -4.66931607  -0.50061788
2019 -10.87418495  -5.49018850  -1.58970664   6.79209543  -2.18888042
              Jun          Jul          Aug          Sep          Oct
2010           NA  -0.90815056  -2.81334275  -1.35329719  -0.15592891
2011   0.26820446   0.66069606  -0.37264596  -1.70470389  -1.56005920
2012   1.16819817  -0.47131360   0.70904812   0.58285344  -1.19745291
2013   2.16653225  -1.67502148  -1.25060442  -1.65783664  -1.38170491
2014  -0.36130533  -0.27709681   1.25453471   2.44854852   1.81438338
2015  -1.58130554  -0.96337181  -4.86481113  -4.45774464   1.71472863
2016  -3.52015854   0.16537273  -0.20821496  -0.40061294  -0.26874620
2017  -2.49823479  -1.86823431  -2.52084213  -4.35808548   1.06073772
2018  -2.04437800   3.37005369   8.09981242   8.93381273  -1.99302370
2019   4.43538121           NA           NA           NA           NA
              Nov          Dec
2010  -0.88766328   2.44626118
2011  -2.56822011  -1.25035195
2012  -2.86252819  -1.81127524
2013   0.66199626   0.92536418
2014   2.21760647   2.07064822
2015   3.14489435   5.11303147
2016  -1.20706224   0.67186055
2017  -0.03820632   0.40029063
2018  -0.42788303 -10.53289512
2019           NA           NA

$figure
 [1] -0.350245676 -0.709259171 -0.529048398  0.906877028 -0.272078662
 [6] -0.402872004  0.296273436  0.393833084  0.005291061  0.786826908
[11]  0.536440778 -0.662038384

$type
[1] "additive"

attr(,"class")
[1] "decomposed.ts"
```

```
plot(dec)
```




```
ad_dec=MSFT.ts-dec$seasonal
plot(MSFT.ts,main="Original vs Ad.S",ylab="Price",xlab="Time",col="red",lwd=2)
lines(ad_dec,col="blue",lwd=2)
legend("topleft",legend=c("Original","Adjusted"),col=c("red","blue"),lty=1,lwd=2)
```

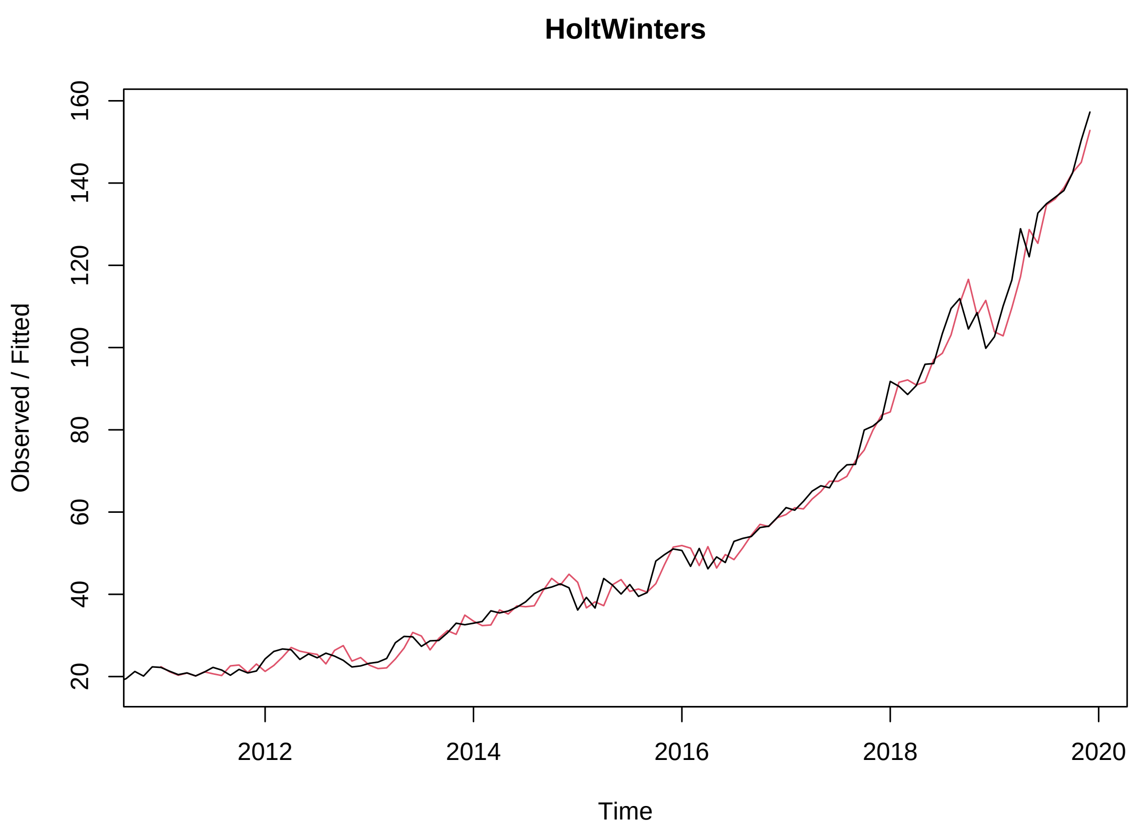



```r
HW=HoltWinters(MSFT.ts,seasonal="additive")
HW
```

```
Holt-Winters exponential smoothing with trend and additive seasonal component.

Call:
HoltWinters(x = MSFT.ts, seasonal = "additive")

Smoothing parameters:
 alpha: 0.8081951
 beta : 0.06711894
 gamma: 0.3070682

Coefficients:
            [,1]
a   155.72063694
b     3.14592341
```

```
s1     1.83644081
s2     1.13833328
s3     0.47781402
s4     0.95540145
s5    -0.65718658
s6    -0.04486145
s7     0.35617170
s8    -1.05558191
s9    -1.38982893
s10    0.27759949
s11    0.52428442
s12    0.95696130
```

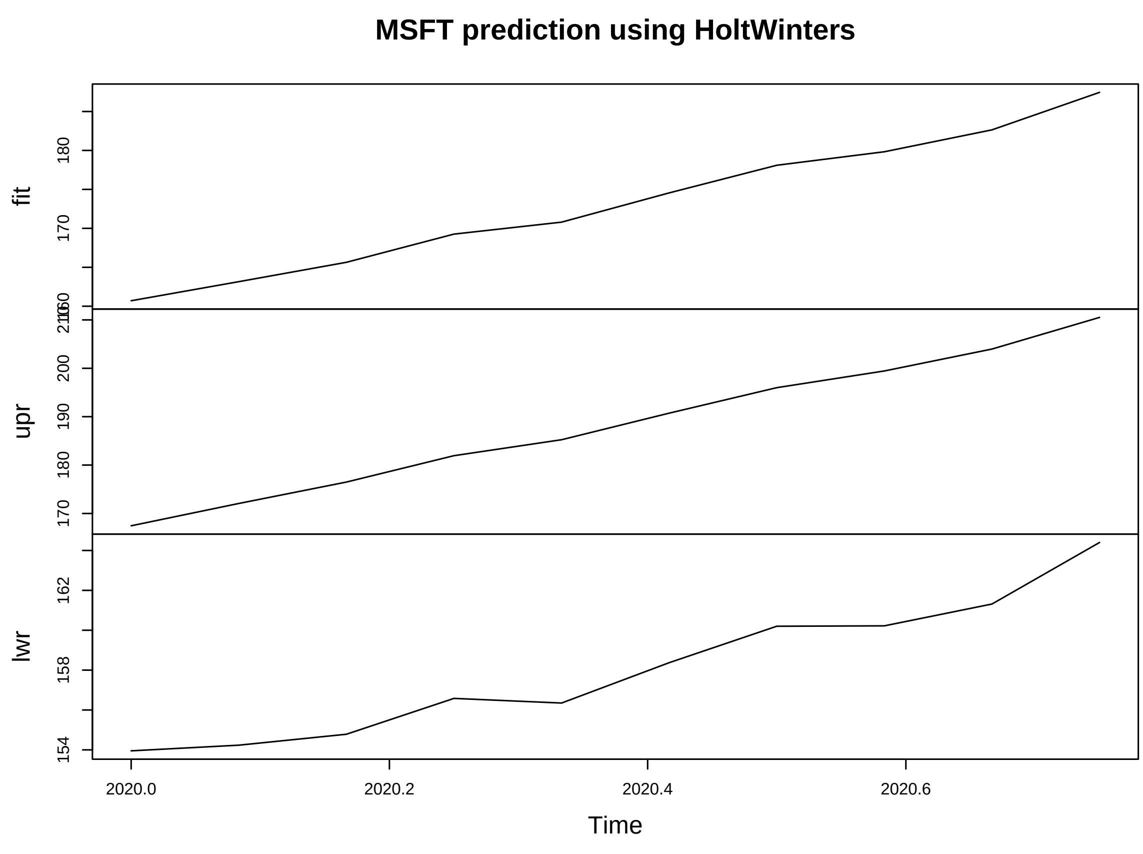
```r
plot(HW,main="HoltWinters")
```



```r
prediction=predict(HW,n.ahead=10,prediction.interval=T,level=0.95)
prediction
```

```
              fit      upr      lwr
Jan 2020 160.7030 167.4541 153.9519
Feb 2020 163.1508 172.0659 154.2357
Mar 2020 165.6362 176.4888 154.7837
Apr 2020 169.2597 181.9382 156.5812
May 2020 170.7931 185.2377 156.3485
Jun 2020 174.5513 190.7300 158.3726
Jul 2020 178.0983 195.9960 160.2006
Aug 2020 179.8324 199.4448 160.2201
Sep 2020 182.6441 203.9741 161.3141
Oct 2020 187.4575 210.5132 164.4017
```

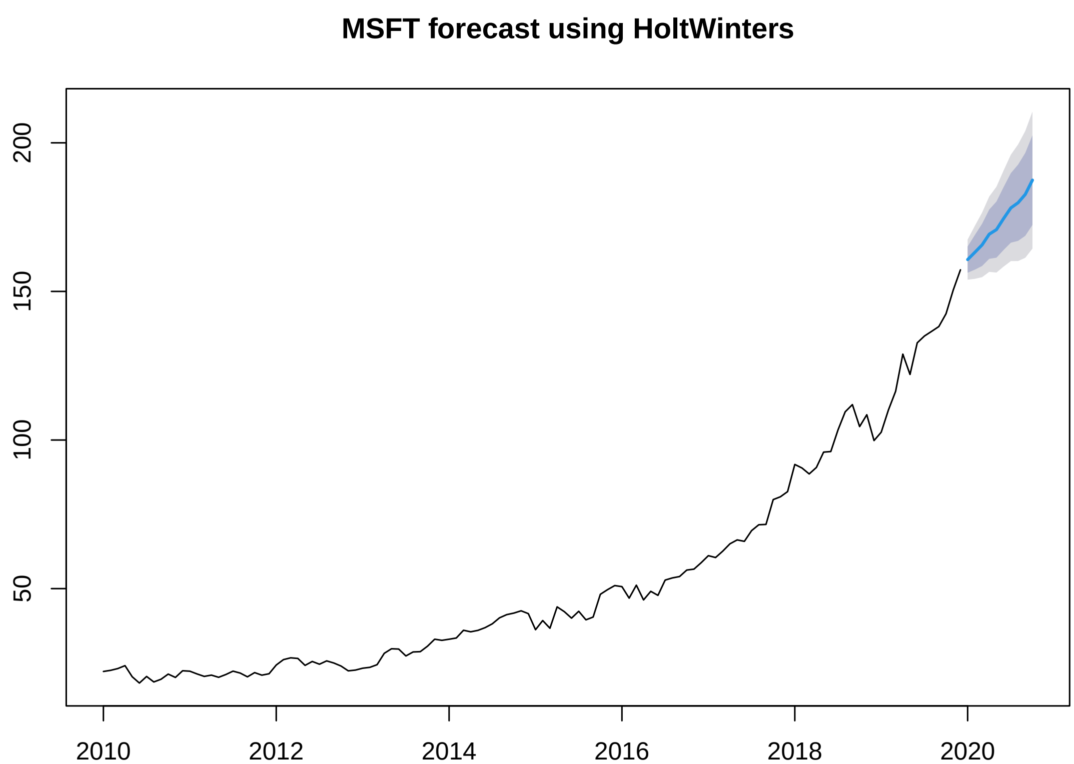
```r
plot(prediction,main="MSFT prediction using HoltWinters")
```



```r
forecast=forecast(HW,h=10,level=c(0.80,0.95))
forecast
```

```
         Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
Jan 2020       160.7030 156.2887 165.1173 153.9519 167.4541
Feb 2020       163.1508 157.3216 168.9801 154.2357 172.0659
Mar 2020       165.6362 158.5401 172.7323 154.7837 176.4888
Apr 2020       169.2597 160.9697 177.5498 156.5812 181.9382
May 2020       170.7931 161.3483 180.2379 156.3485 185.2377
Jun 2020       174.5513 163.9726 185.1300 158.3726 190.7300
Jul 2020       178.0983 166.3956 189.8009 160.2006 195.9960
Aug 2020       179.8324 167.0086 192.6563 160.2201 199.4448
Sep 2020       182.6441 168.6972 196.5910 161.3141 203.9741
Oct 2020       187.4575 172.3821 202.5328 164.4017 210.5132
```

```r
plot(forecast,main="MSFT forecast using HoltWinters")
```



```
residuals=forecast$residuals
residuals
```

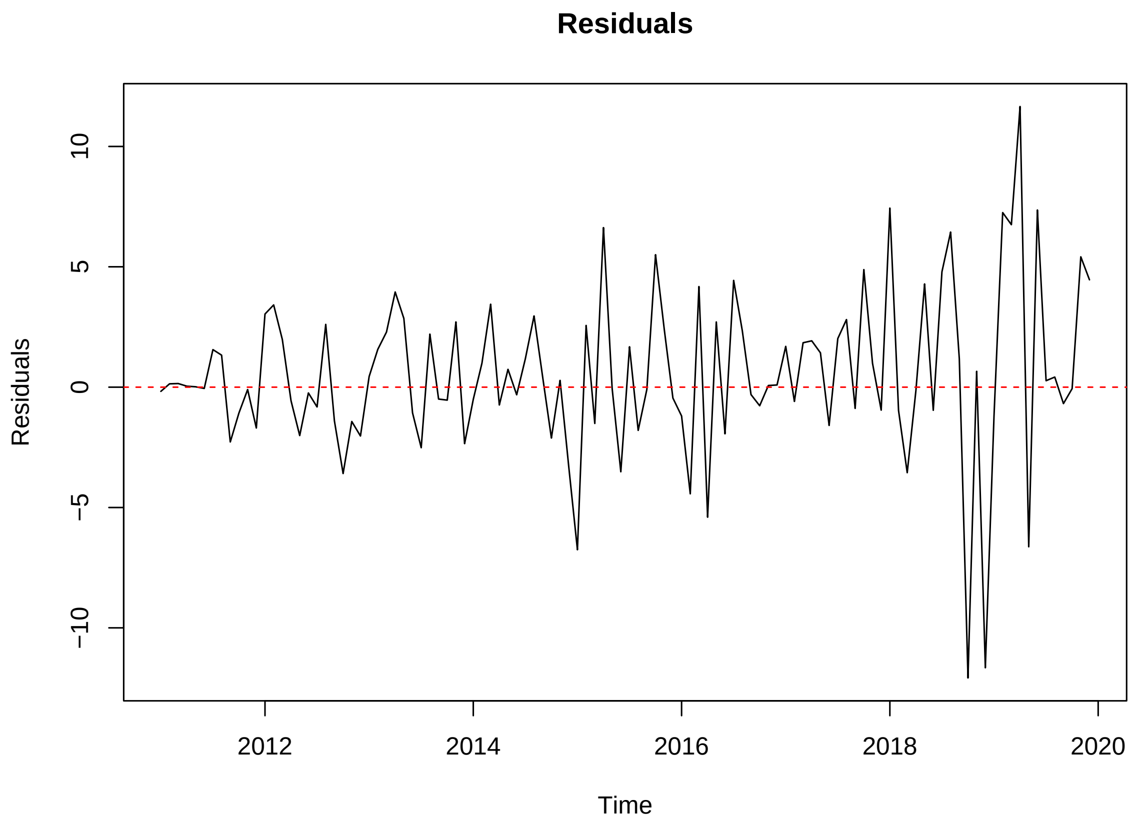
```
             Jan          Feb          Mar          Apr          May
2010          NA           NA           NA           NA           NA
2011 -0.17519775   0.13776566   0.15003803   0.04062846   0.01536056
2012  3.03842884   3.41689596   1.97787305  -0.56354860  -2.01131031
2013  0.44164249   1.57206492   2.28932898   3.95453191   2.85446572
2014 -0.50756377   0.99679407   3.44562931  -0.74080660   0.74064937
2015 -6.75471685   2.56395157  -1.50859102   6.62747834  -0.05364583
2016 -1.19743366  -4.43146176   4.17950023  -5.40193867   2.70852045
2017  1.69378571  -0.59202716   1.84452872   1.92514867   1.42535370
2018  7.43904225  -0.96808999  -3.55323072  -0.13754823   4.28785331
2019 -1.19828689   7.25311882   6.74986015  11.65955203  -6.63185675
             Jun          Jul          Aug          Sep          Oct
2010          NA           NA           NA           NA           NA
2011 -0.05330014   1.55958818   1.32986484  -2.27632408  -1.07140669
2012 -0.24050241  -0.82303670   2.60741976  -1.40058968  -3.58853543
2013 -1.06581934  -2.51609739   2.20346113  -0.49444173  -0.53917042
2014 -0.31667078   1.17981868   2.95822594   0.40479909  -2.11452837
2015 -3.51526032   1.67689865  -1.79581997  -0.09218546   5.50224893
2016 -1.93850731   4.43514045   2.33800567  -0.30821192  -0.77238619
2017 -1.59065625   2.01870871   2.80796901  -0.88304470   4.88403473
2018 -0.96095374   4.79167857   6.44386795   1.17764272 -12.08242555
2019  7.36085131   0.26793761   0.41916818  -0.68335614  -0.06200813
             Nov          Dec
2010          NA           NA
2011 -0.09903535  -1.69760837
2012 -1.42606738  -2.02965854
2013  2.71028962  -2.34658456
2014  0.28166169  -3.30253177
2015  2.41855149  -0.45411946
2016  0.06817592   0.09091387
2017  0.99328282  -0.95470247
2018  0.65766656 -11.65767240
2019  5.41486643   4.46049187
```

```
residuals=na.omit(residuals)
residuals
```

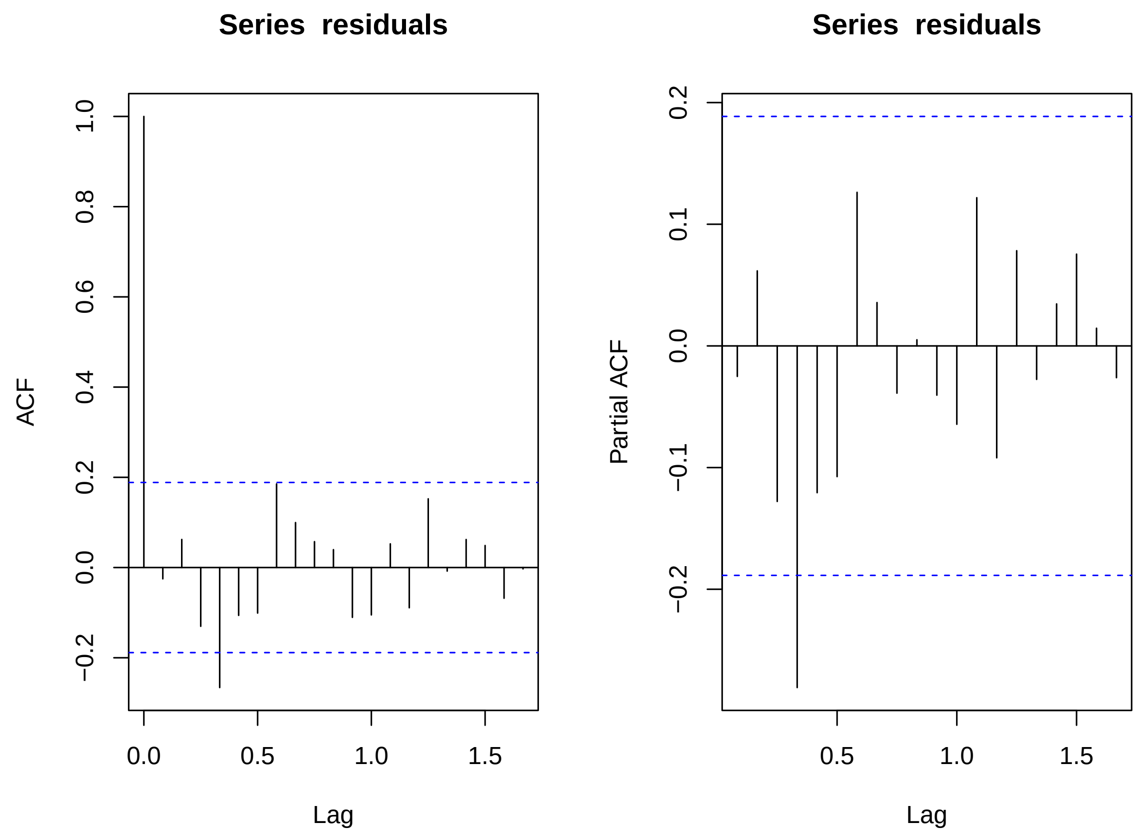
```
             Jan          Feb          Mar          Apr          May
```

```
2011  -0.17519775   0.13776566   0.15003803   0.04062846   0.01536056
2012   3.03842884   3.41689596   1.97787305  -0.56354860  -2.01131031
2013   0.44164249   1.57206492   2.28932898   3.95453191   2.85446572
2014  -0.50756377   0.99679407   3.44562931  -0.74080660   0.74064937
2015  -6.75471685   2.56395157  -1.50859102   6.62747834  -0.05364583
2016  -1.19743366  -4.43146176   4.17950023  -5.40193867   2.70852045
2017   1.69378571  -0.59202716   1.84452872   1.92514867   1.42535370
2018   7.43904225  -0.96808999  -3.55323072  -0.13754823   4.28785331
2019  -1.19828689   7.25311882   6.74986015  11.65955203  -6.63185675
              Jun          Jul          Aug          Sep          Oct
2011  -0.05330014   1.55958818   1.32986484  -2.27632408  -1.07140669
2012  -0.24050241  -0.82303670   2.60741976  -1.40058968  -3.58853543
2013  -1.06581934  -2.51609739   2.20346113  -0.49444173  -0.53917042
2014  -0.31667078   1.17981868   2.95822594   0.40479909  -2.11452837
2015  -3.51526032   1.67689865  -1.79581997  -0.09218546   5.50224893
2016  -1.93850731   4.43514045   2.33800567  -0.30821192  -0.77238619
2017  -1.59065625   2.01870871   2.80796901  -0.88304470   4.88403473
2018  -0.96095374   4.79167857   6.44386795   1.17764272 -12.08242555
2019   7.36085131   0.26793761   0.41916818  -0.68335614  -0.06200813
              Nov          Dec
2011  -0.09903535  -1.69760837
2012  -1.42606738  -2.02965854
2013   2.71028962  -2.34658456
2014   0.28166169  -3.30253177
2015   2.41855149  -0.45411946
2016   0.06817592   0.09091387
2017   0.99328282  -0.95470247
2018   0.65766656 -11.65767240
2019   5.41486643   4.46049187
```

```r
plot(residuals,main="Residuals",ylab="Residuals",xlab="Time")
abline(h = 0, col = "red", lty = 2)
```




```
par(mfrow=c(1,2))
acf(residuals)
pacf(residuals)
```




```
Box.test(residuals,lag=20,type="Ljung-Box")
```

```
	Box-Ljung test

data:  residuals
X-squared = 27.429, df = 20, p-value = 0.1236
```

```
shapiro.test(residuals)
```

```
	Shapiro-Wilk normality test

data:  residuals
W = 0.94243, p-value = 0.0001494
```

```r
jarque.bera.test(residuals)
```

```

	Jarque Bera Test

data:  residuals
X-squared = 35.857, df = 2, p-value = 1.636e-08
```

```r
ks.test(residuals,"pnorm",mean=mean(residuals),sd=sd(residuals))
```

```

	Asymptotic one-sample Kolmogorov-Smirnov test

data:  residuals
D = 0.10089, p-value = 0.2216
alternative hypothesis: two-sided
```

2.1 Answers for exercise 2

- ACF & PACF Plot:

The residuals show no significant autocorrelation, as almost all lags fall within the significance bounds

- Ljung-Box Test:

With a p-value of 0.1236 (> 0.05), we fail to reject the null hypothesis of no autocorrelation (white noise).
The residuals are not perfectly normal (Shapiro test # jurqe bera < 0.05)

3 Exercise 3

```r
data=read_excel("c7ex5.xls")
head(data,5)
```

```
# A tibble: 5 x 4
  OBS      X1    X2    X3
  <chr> <dbl> <dbl> <dbl>
1 1      22.1 101.   53.8
2 2      24.4 101.   55
3 3      19.7 100.0  54.8
4 4      13.2  96.0  55.0
5 5      22.8  99.0  56.6
```

```r
ser1=data$X1
ser1.ts=ts(ser1,start=1,frequency=1)
head(ser1,5)
```

```
[1] 22.10000 24.40000 19.66114 13.17239 22.76479
```

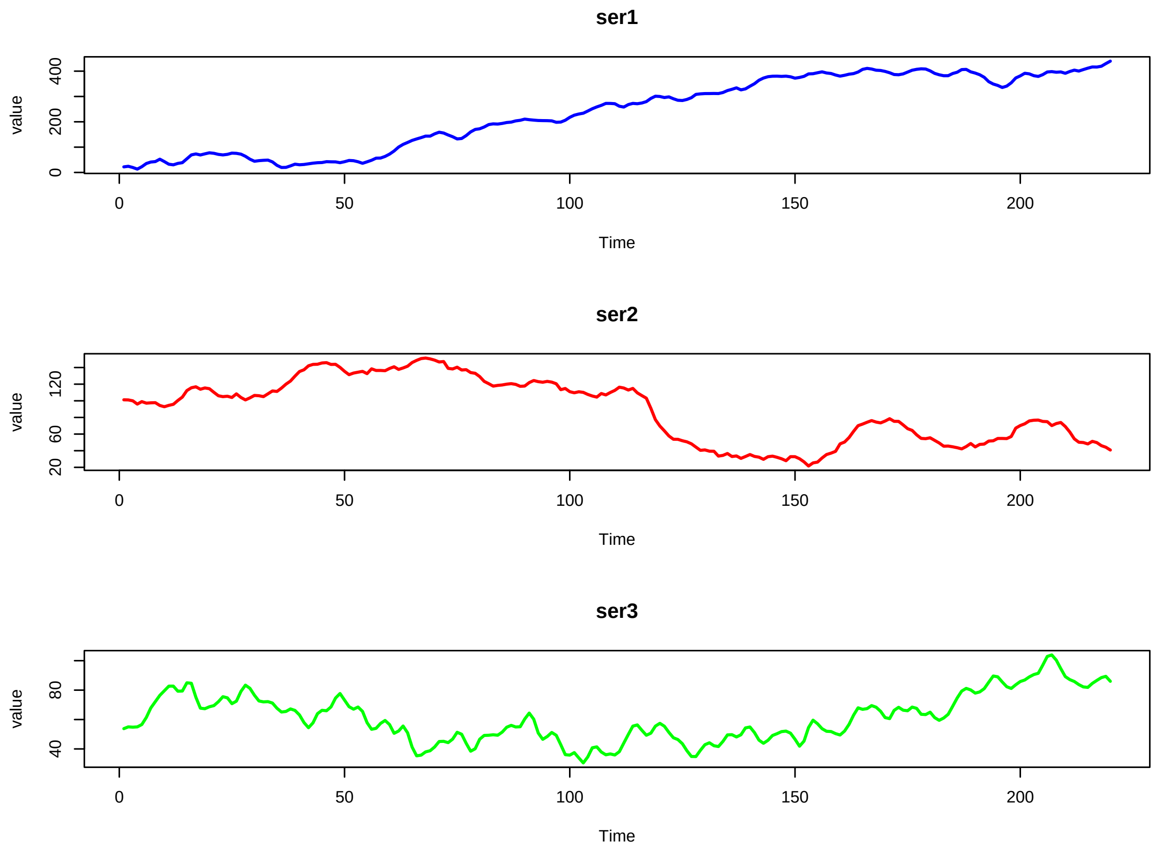
```r
ser2=data$X2
ser2.ts=ts(ser2,start=1,frequency=1)
head(ser2,5)
```

```
[1] 101.20000 101.07199  99.97172  96.03309  99.04480
```

```r
ser3=data$X3
ser3.ts=ts(ser3,start=1,frequency=1)
head(ser3,5)
```

```
[1] 53.80000 55.00000 54.79635 55.00262 56.59452
```

```r
par(mfrow = c(3,1))
plot(ser1.ts,main="ser1",ylab="value",col="blue",lwd=2)
plot(ser2.ts,main="ser2",ylab="value",col="red",lwd=2)
plot(ser3.ts,main="ser3",ylab="value",col="green",lwd=2)
```




```
summary(ser1.ts)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  13.17   73.43  269.67  237.41  382.05  439.50
```

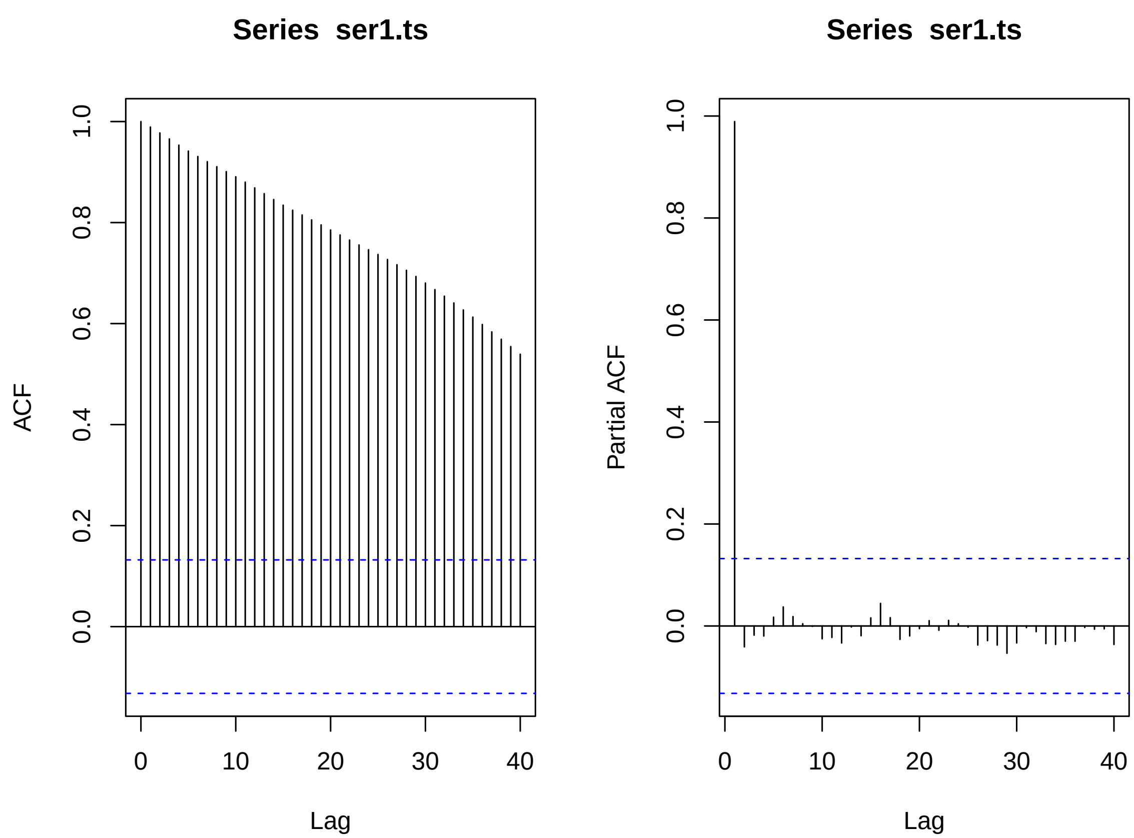
```
summary(ser2.ts)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  21.59   51.51   97.66   88.53  119.80  151.40
```

```
summary(ser3.ts)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  30.49   49.31   57.92   61.17   72.26  103.91
```

```
par(mfrow=c(1,2))
acf(ser1.ts,lag.max=40,type="correlation")
pacf(ser1.ts,lag.max=40)
```




```
adf.test(ser1.ts)
```

```

	Augmented Dickey-Fuller Test

data:  ser1.ts
Dickey-Fuller = -1.5931, Lag order = 6, p-value = 0.7467
alternative hypothesis: stationary
```

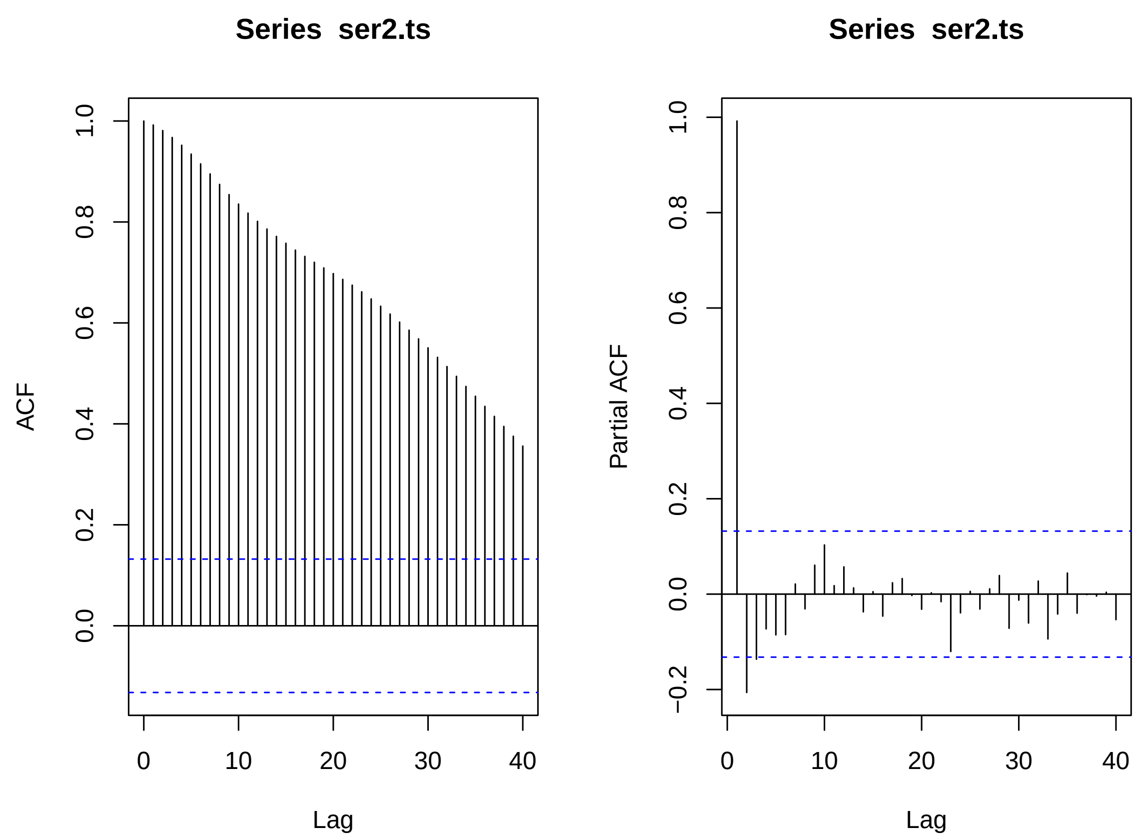
```
pp.test(ser1.ts)
```

```

	Phillips-Perron Unit Root Test

data:  ser1.ts
Dickey-Fuller Z(alpha) = -5.6692, Truncation lag parameter = 4, p-value
= 0.7914
alternative hypothesis: stationary
```

```r
kpss.test(ser1.ts)
```

```

	KPSS Test for Level Stationarity

data:  ser1.ts
KPSS Level = 4.3617, Truncation lag parameter = 4, p-value = 0.01
```

```r
par(mfrow=c(1,2))
acf(ser2.ts,lag.max=40,type="correlation")
pacf(ser2.ts,lag.max=40)
```




```
adf.test(ser2.ts)
```

```
	Augmented Dickey-Fuller Test

data:  ser2.ts
Dickey-Fuller = -2.4621, Lag order = 6, p-value = 0.3821
alternative hypothesis: stationary
```

```
pp.test(ser2.ts)
```

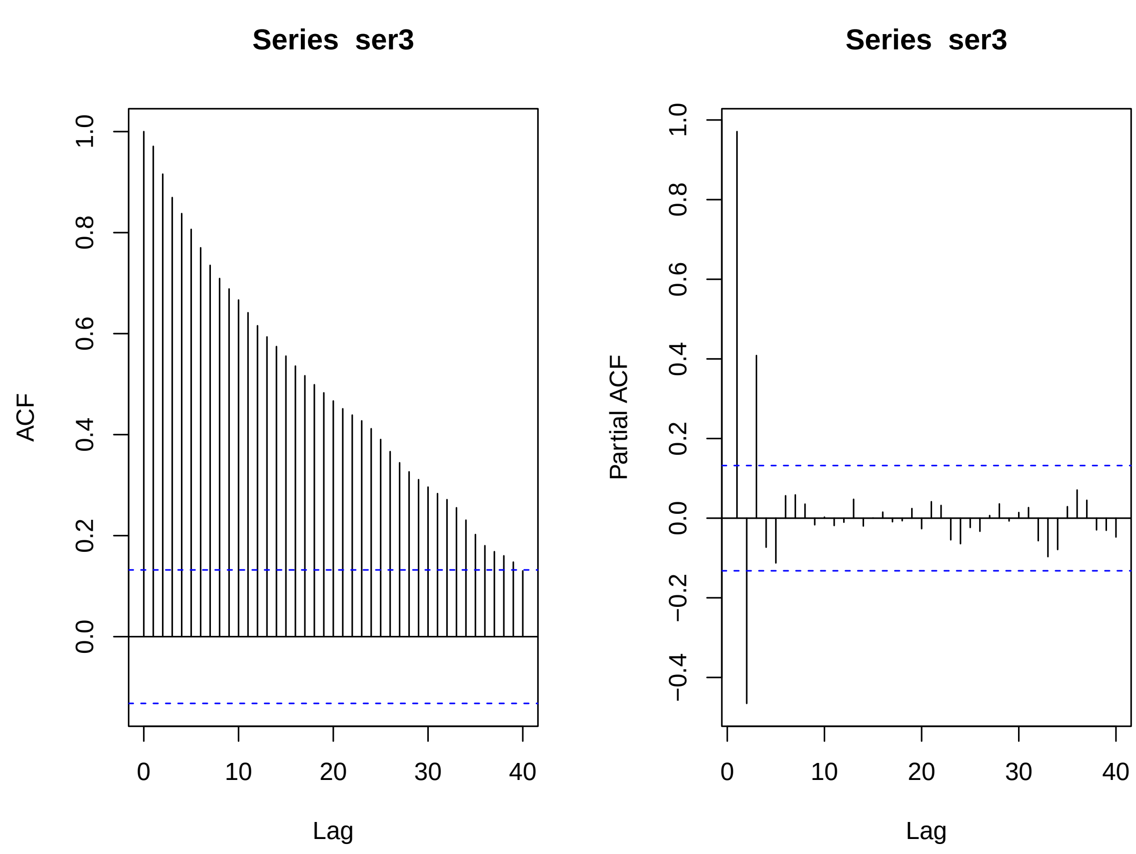
```
	Phillips-Perron Unit Root Test

data:  ser2.ts
```

```
Dickey-Fuller Z(alpha) = -5.4487, Truncation lag parameter = 4, p-value
= 0.8038
alternative hypothesis: stationary
```

```
kpss.test(ser2.ts)
```

```
    KPSS Test for Level Stationarity

data:  ser2.ts
KPSS Level = 2.9049, Truncation lag parameter = 4, p-value = 0.01
```

```
par(mfrow=c(1,2))
acf(ser3,lag.max=40,type="correlation")
pacf(ser3,lag.max=40)
```



```r
adf.test(ser3.ts)
```

```
	Augmented Dickey-Fuller Test

data:  ser3.ts
Dickey-Fuller = -1.5941, Lag order = 6, p-value = 0.7463
alternative hypothesis: stationary
```

```r
pp.test(ser3.ts)
```

```
	Phillips-Perron Unit Root Test

data:  ser3.ts
Dickey-Fuller Z(alpha) = -7.0839, Truncation lag parameter = 4, p-value
= 0.7114
alternative hypothesis: stationary
```

```r
kpss.test(ser3.ts)
```

```
	KPSS Test for Level Stationarity

data:  ser3.ts
KPSS Level = 1.095, Truncation lag parameter = 4, p-value = 0.01
```

All 3 series are non-stationary and correlation:
ACF stays positive & pacf has a few significant spekes adf & pp p-value > 0.05 - kpss p-value < 0.05 we should keep going with return and difference

```r
return1=diff(ser1.ts)
head(return1,5)
```

```
Time Series:
Start = 2
End = 6
Frequency = 1
[1]  2.300000 -4.738855 -6.488752  9.592396 12.620457
```

```r
return2=diff(ser2.ts)
head(return2,5)
```

```
Time Series:
Start = 2
End = 6
Frequency = 1
[1] -0.1280097 -1.1002707 -3.9386341  3.0117132 -1.8804590
```

```r
return3=diff(ser3.ts)
head(return3,5)
```

```
Time Series:
Start = 2
End = 6
Frequency = 1
[1]  1.2000000 -0.2036476  0.2062637  1.5919036  4.8195657
```

```r
adf.test(return1)
```

```

	Augmented Dickey-Fuller Test

data:  return1
Dickey-Fuller = -5.3548, Lag order = 6, p-value = 0.01
alternative hypothesis: stationary
```

```r
pp.test(return1)
```

```

	Phillips-Perron Unit Root Test

data:  return1
Dickey-Fuller Z(alpha) = -87.323, Truncation lag parameter = 4, p-value
= 0.01
alternative hypothesis: stationary
```

```r
kpss.test(return1)
```

```
    KPSS Test for Level Stationarity

data:  return1
KPSS Level = 0.10123, Truncation lag parameter = 4, p-value = 0.1
```

```r
adf.test(return2)
```

```
    Augmented Dickey-Fuller Test

data:  return2
Dickey-Fuller = -4.063, Lag order = 6, p-value = 0.01
alternative hypothesis: stationary
```

```r
pp.test(return2)
```

```
    Phillips-Perron Unit Root Test

data:  return2
Dickey-Fuller Z(alpha) = -131.79, Truncation lag parameter = 4, p-value
= 0.01
alternative hypothesis: stationary
```

```r
kpss.test(return2)
```

```
    KPSS Test for Level Stationarity

data:  return2
KPSS Level = 0.1717, Truncation lag parameter = 4, p-value = 0.1
```

```r
adf.test(return3)
```

```
    Augmented Dickey-Fuller Test

data:  return3
Dickey-Fuller = -5.9952, Lag order = 6, p-value = 0.01
alternative hypothesis: stationary
```

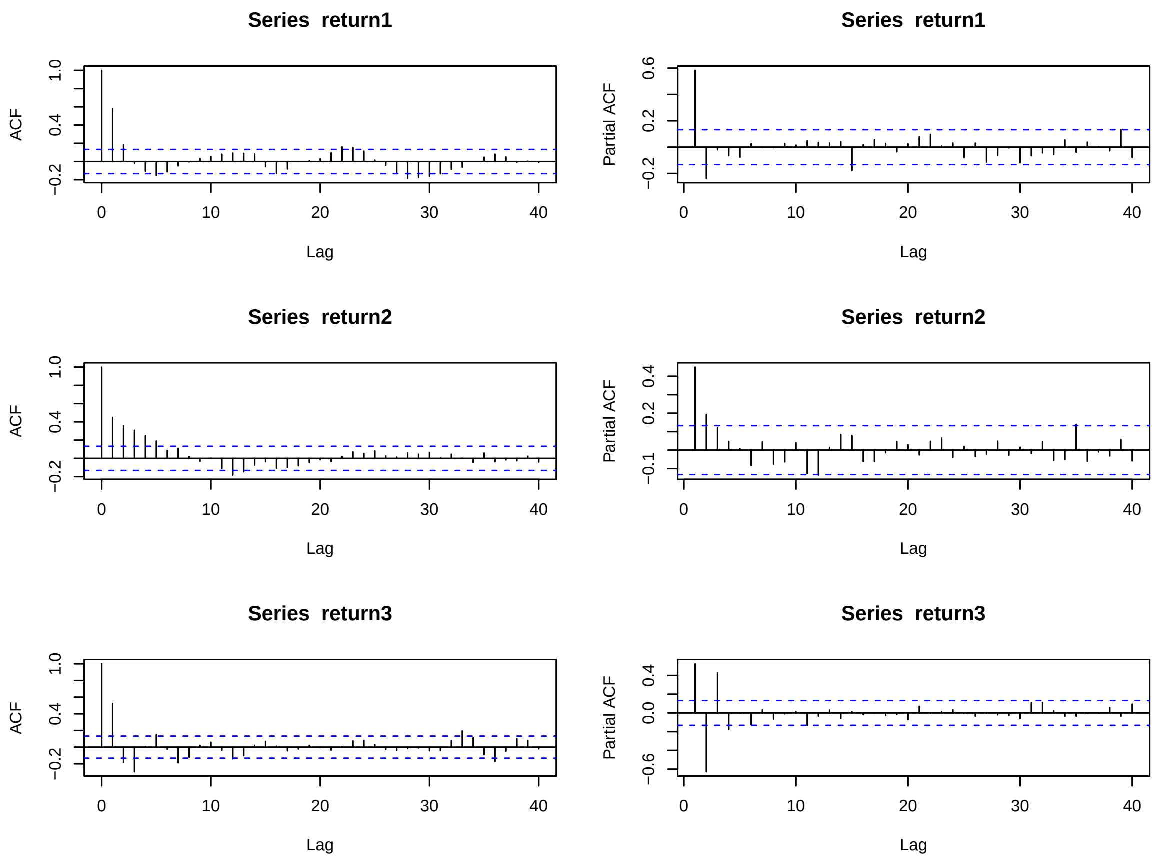
```r
pp.test(return3)
```

```
    Phillips-Perron Unit Root Test

data:  return3
Dickey-Fuller Z(alpha) = -77.821, Truncation lag parameter = 4, p-value
= 0.01
alternative hypothesis: stationary
```

```r
kpss.test(return3)
```

```
    KPSS Test for Level Stationarity

data:  return3
KPSS Level = 0.094388, Truncation lag parameter = 4, p-value = 0.1
```

```r
par(mfrow=c(3,2))
acf(return1,lag.max=40,type="correlation")
pacf(return1,lag.max=40)
acf(return2,lag.max=40,type="correlation")
pacf(return2,lag.max=40)
acf(return3,lag.max=40,type="correlation")
pacf(return3,lag.max=40)
```



Or we can do it:

```
returns_total=cbind(return1,return2,return3)
apply(returns_total,2,Box.test,lag=20,type="Ljung-Box")
```

```
$return1

    Box-Ljung test

data:  newX[, i]
X-squared = 108.59, df = 20, p-value = 3.553e-14


$return2

    Box-Ljung test
```

```
data:  newX[, i]
X-squared = 145.85, df = 20, p-value < 2.2e-16


$return3

	Box-Ljung test

data:  newX[, i]
X-squared = 114.88, df = 20, p-value = 2.554e-15
```

```r
model1=auto.arima(return1)
model1
```

```
Series: return1
ARIMA(0,0,2) with non-zero mean

Coefficients:
         ma1     ma2    mean
      0.7137  0.2551  1.9348
s.e.  0.0649  0.0629  0.6628

sigma^2 = 25.31:  log likelihood = -663.3
AIC=1334.6   AICc=1334.79   BIC=1348.16
```

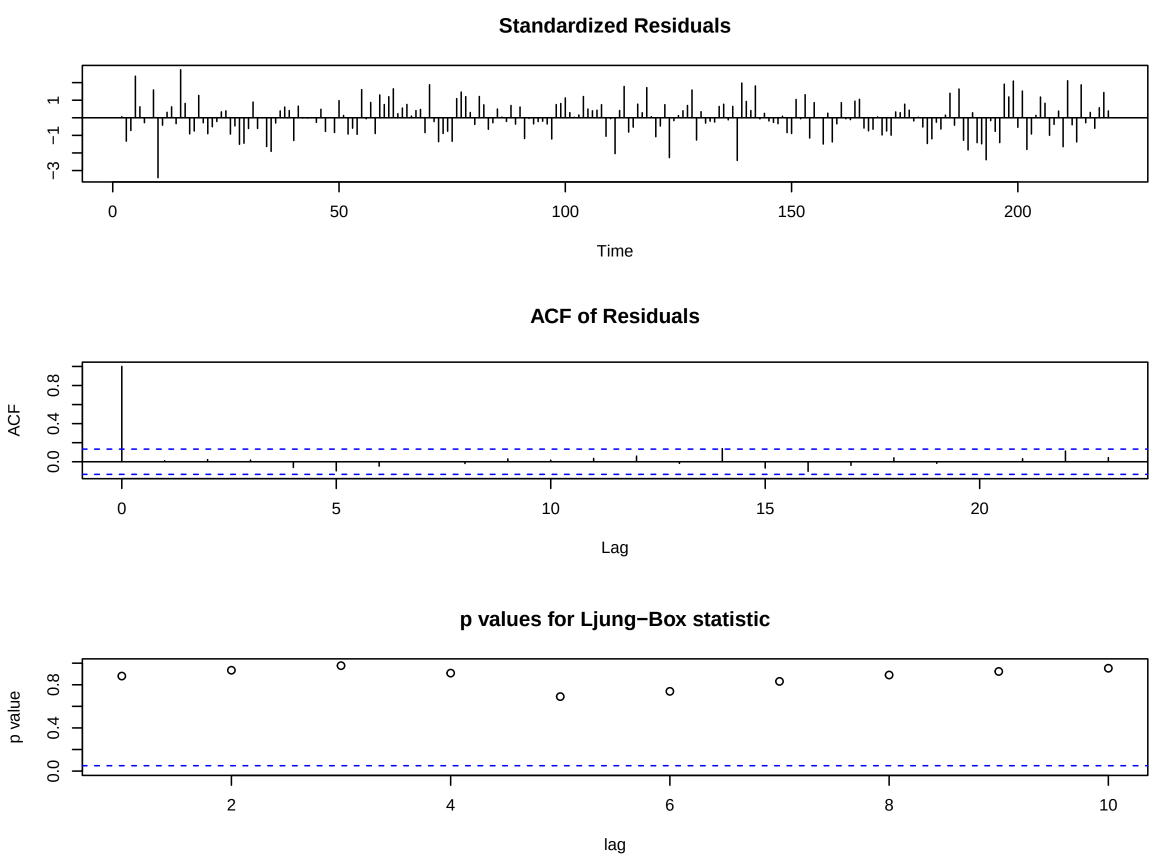
```r
model2=auto.arima(return2)
model2
```

```
Series: return2
ARIMA(1,0,1) with zero mean

Coefficients:
         ar1      ma1
      0.8021  -0.4615
s.e.  0.0692   0.1001

sigma^2 = 9.025:  log likelihood = -550.82
AIC=1107.63   AICc=1107.75   BIC=1117.8
```

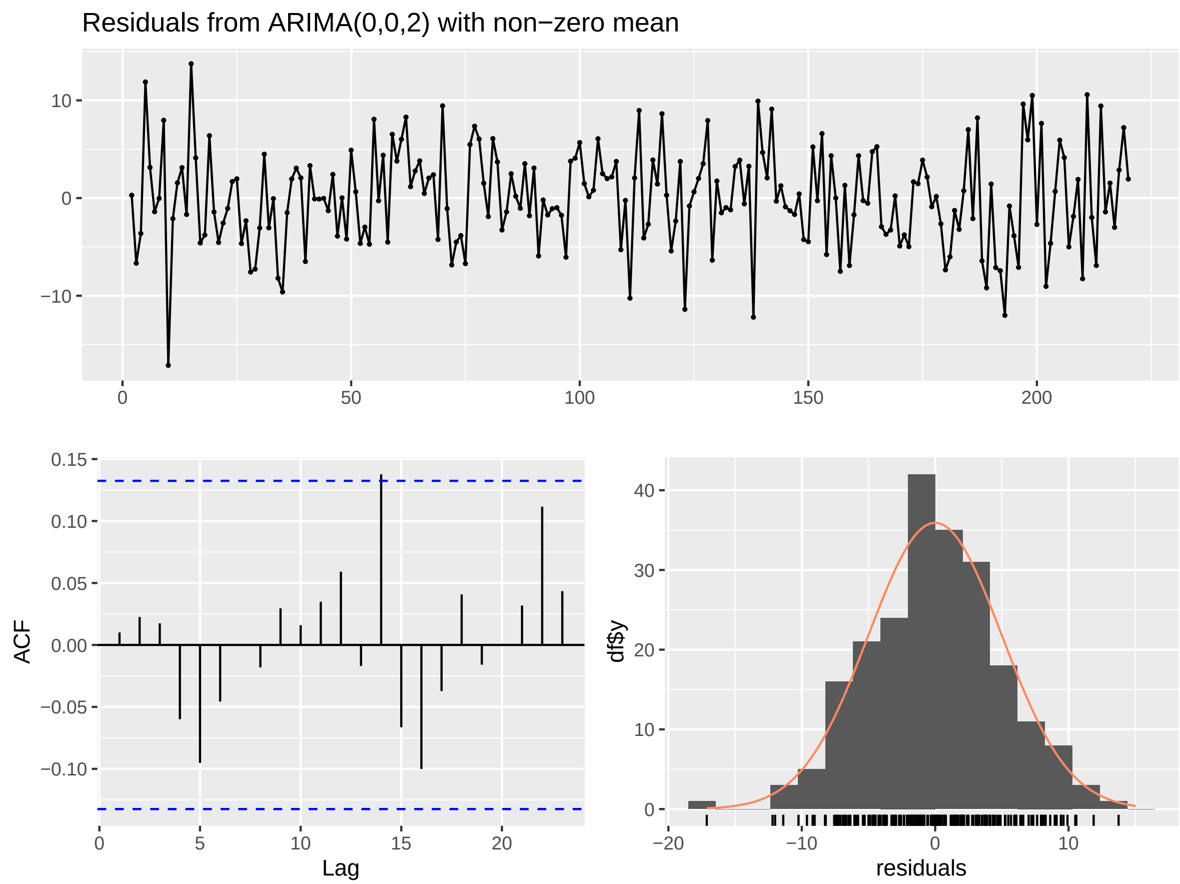
```r
model3=auto.arima(return3)
model3
```

```
Series: return3
ARIMA(3,0,1) with zero mean

Coefficients:
         ar1      ar2     ar3     ma1
      0.7791  -0.7022  0.2142  0.4871
s.e.  0.1176   0.1143  0.0993  0.1100

sigma^2 = 4.133:  log likelihood = -465.25
AIC=940.5   AICc=940.79   BIC=957.45
```

```r
tsdiag(model1)
```

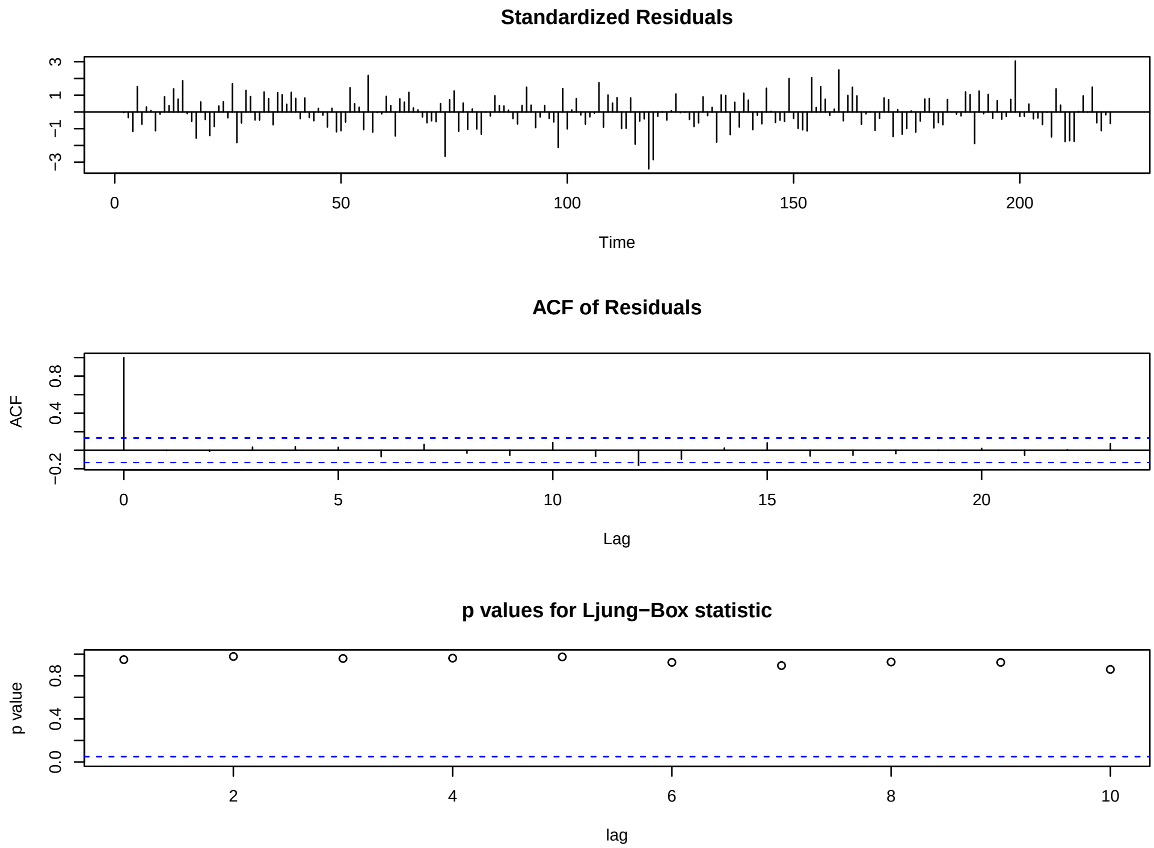


```r
checkresiduals(model1)
```

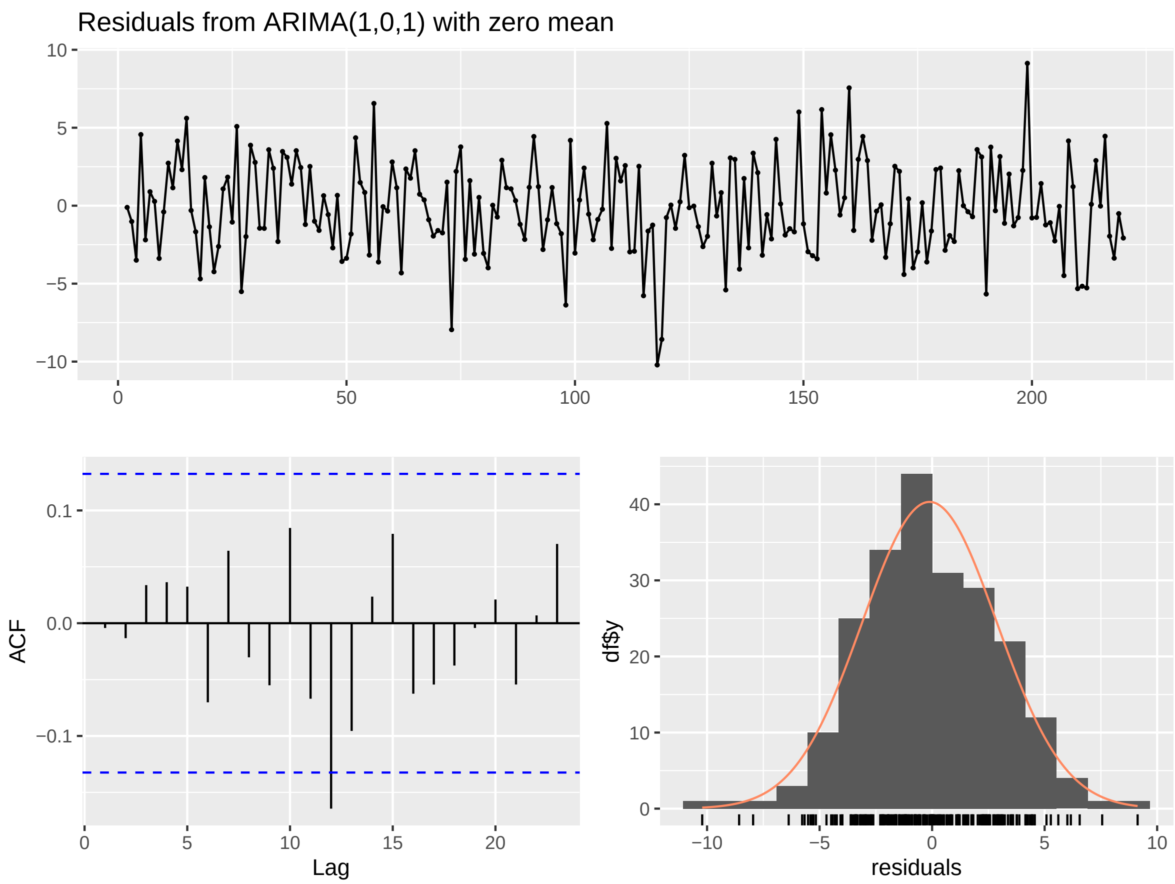



```
    Ljung-Box test

data:  Residuals from ARIMA(0,0,2) with non-zero mean
Q* = 3.8734, df = 8, p-value = 0.8684

Model df: 2.   Total lags used: 10
```

```r
tsdiag(model2)
```

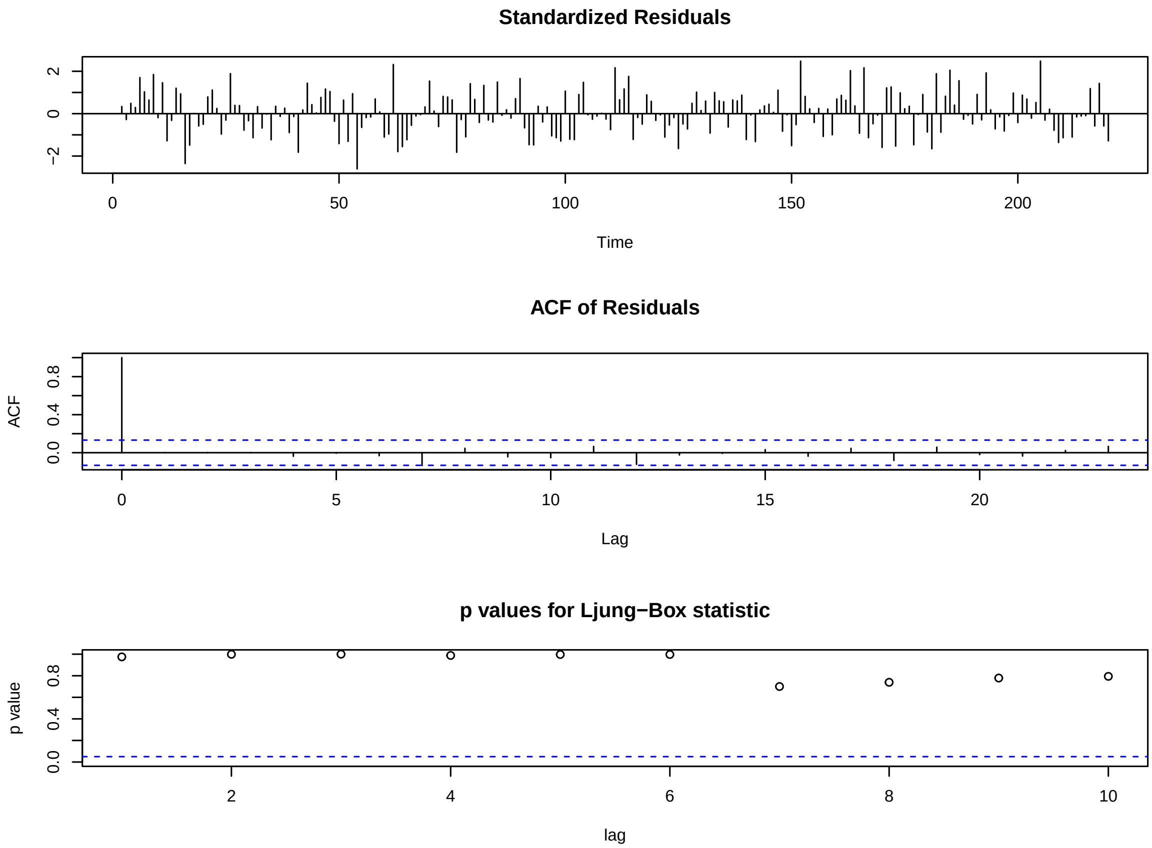



```r
checkresiduals(model2)
```

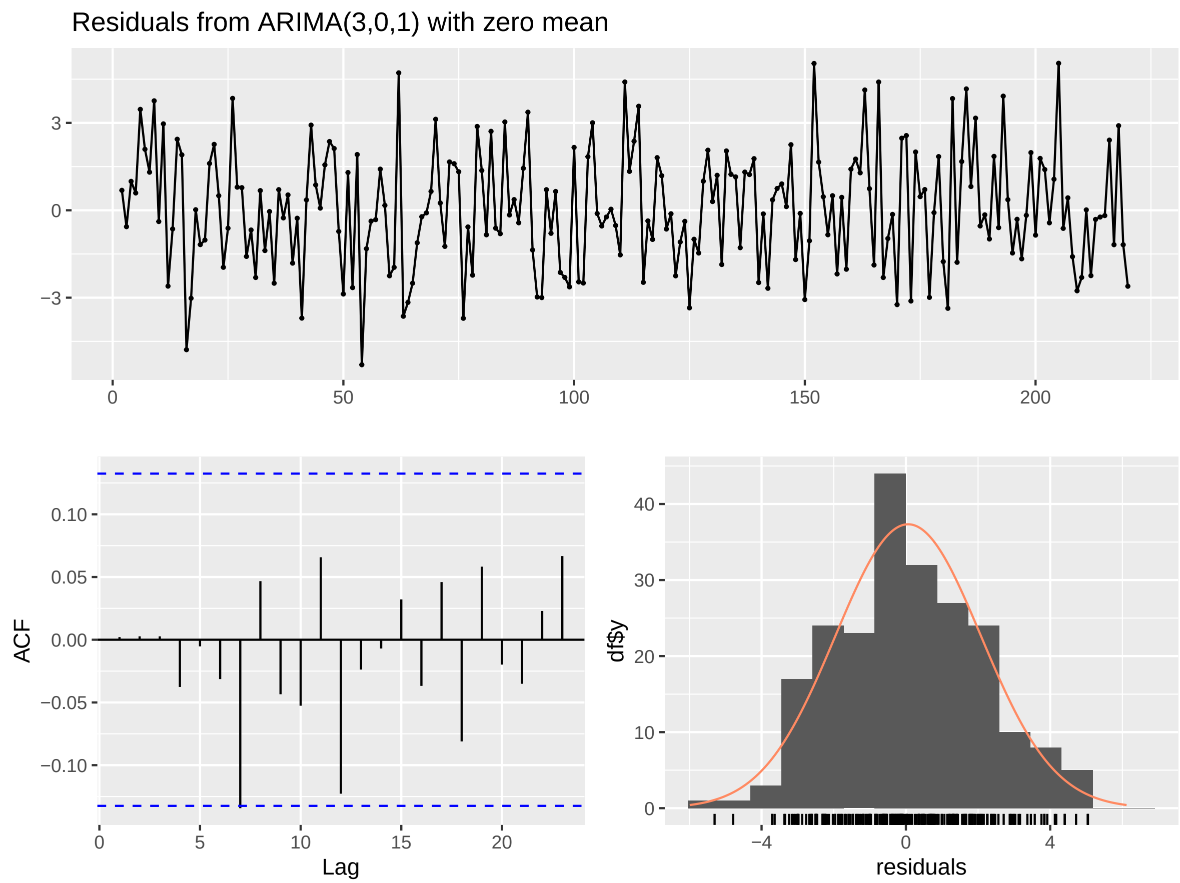



```
    Ljung-Box test

data:  Residuals from ARIMA(1,0,1) with zero mean
Q* = 5.4566, df = 8, p-value = 0.7078

Model df: 2.   Total lags used: 10
```

```
tsdiag(model3)
```




```
checkresiduals(model3)
```

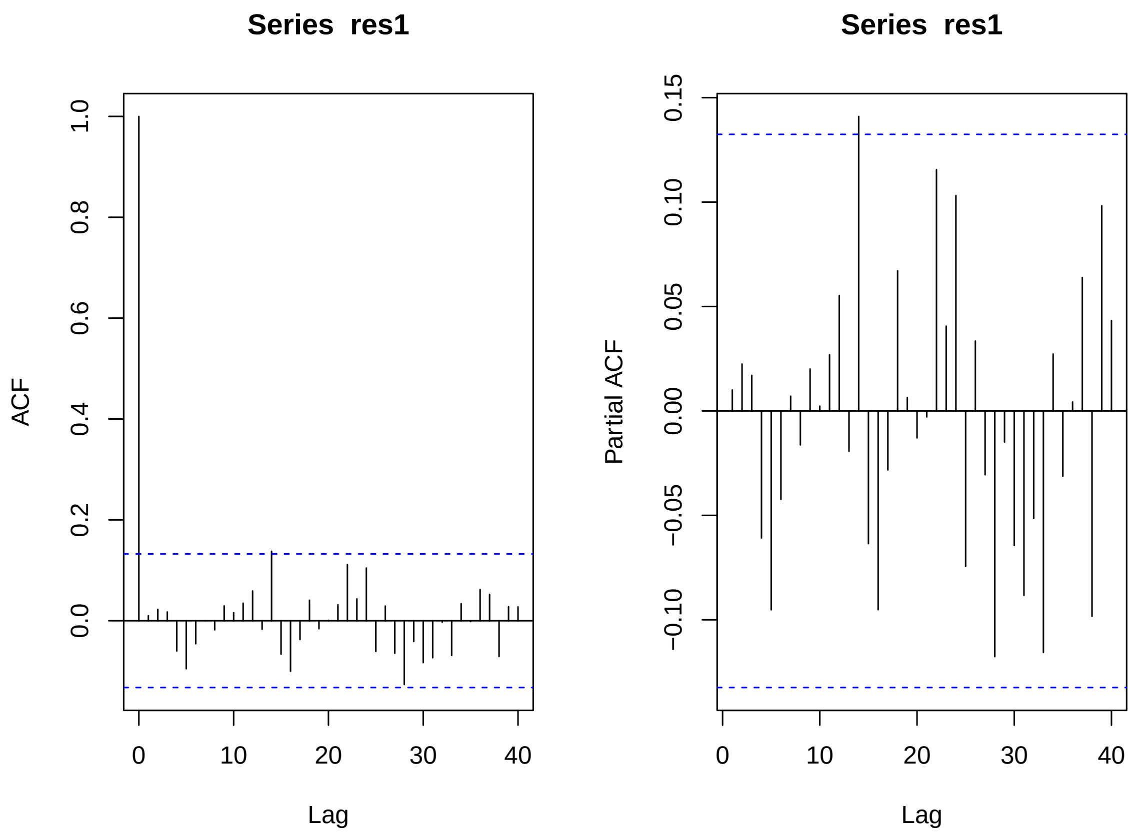



```
    Ljung-Box test

data:  Residuals from ARIMA(3,0,1) with zero mean
Q* = 6.2461, df = 6, p-value = 0.3962

Model df: 4.   Total lags used: 10
```

```r
res1=residuals(model1)
head(res1,5)
```

```
Time Series:
Start = 2
End = 6
Frequency = 1
[1]  0.2910655 -6.6686829 -3.6241980 11.8796853  3.1371996
```

```r
par(mfrow=c(1,2))
acf(res1,lag.max=40,type="correlation")
pacf(res1,lag.max=40)
```



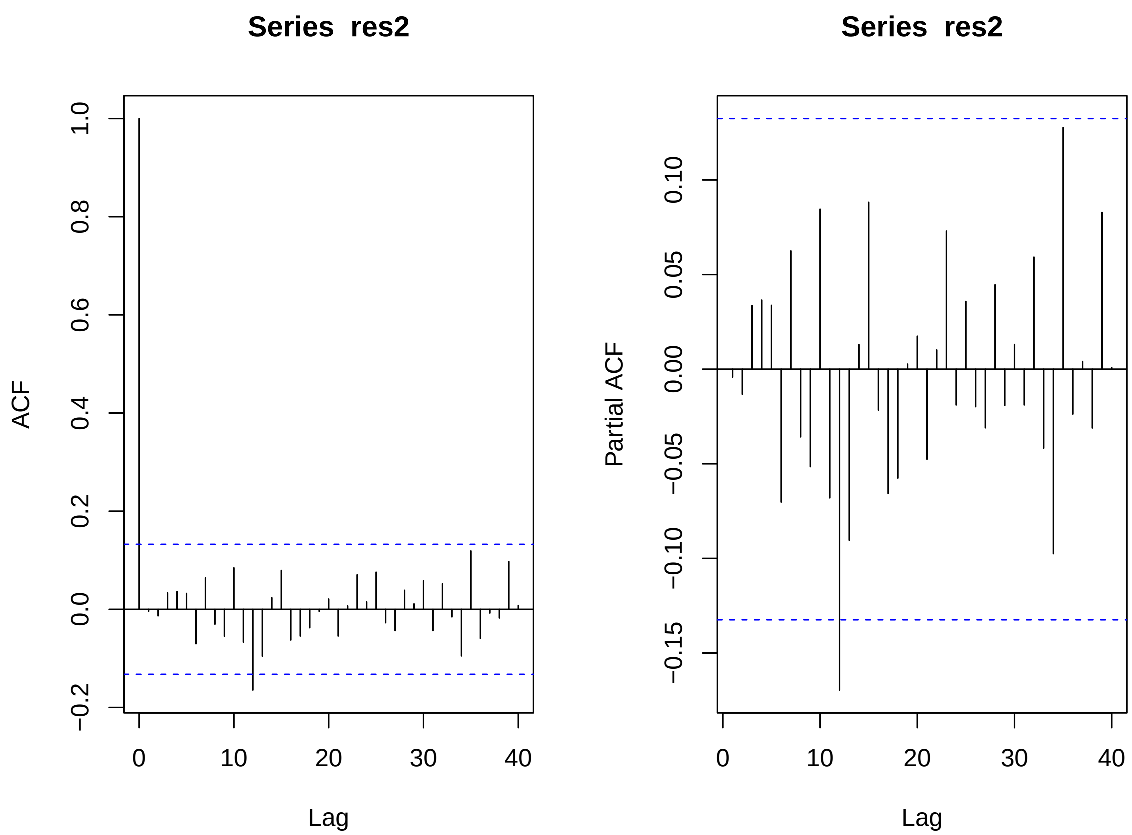

```r
Box.test(res1,lag=20,type="Ljung-Box")
```

```
	Box-Ljung test

data:  res1
X-squared = 13.748, df = 20, p-value = 0.843
```

```r
res2=residuals(model2)
head(res2,5)
```

```
Time Series:
Start = 2
End = 6
Frequency = 1
[1] -0.1111951 -1.0159559 -3.4947238  4.5615291 -2.1929673
```

```
par(mfrow=c(1,2))
acf(res2,lag.max=40,type="correlation")
pacf(res2,lag.max=40)
```



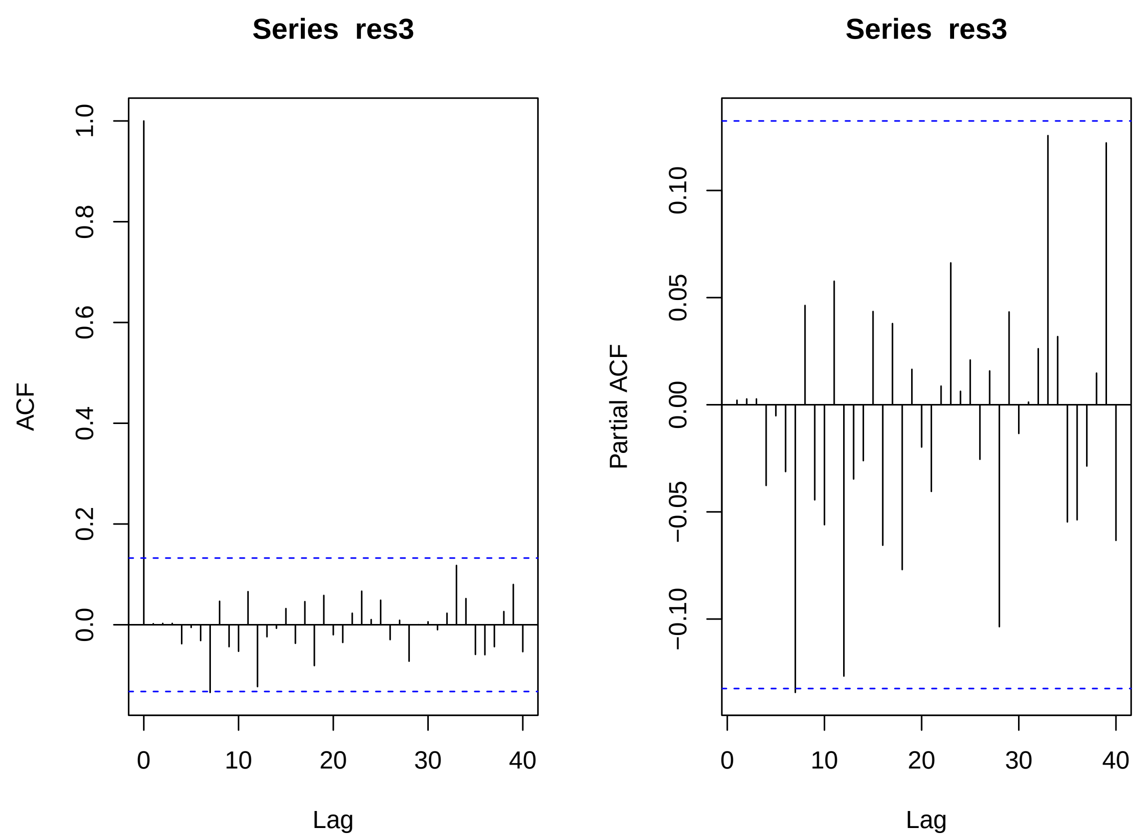

```
Box.test(res2,lag=20,type="Ljung-Box")
```

```
    Box-Ljung test

data:  res2
X-squared = 18.68, df = 20, p-value = 0.5427
```

```
res3=residuals(model3)
head(res3,5)
```

```
Time Series:
Start = 2
End = 6
Frequency = 1
[1]  0.6851210 -0.5637058  0.9933426  0.5935008  3.4643470
```

```
par(mfrow=c(1,2))
acf(res3,lag.max=40,type="correlation")
pacf(res3,lag.max=40)
```




```
Box.test(res3,lag=20,type="Ljung-Box")
```

```
	Box-Ljung test

data:  res3
X-squared = 14.492, df = 20, p-value = 0.8047
```

```r
shapiro.test(res1)
```

```
	Shapiro-Wilk normality test

data:  res1
W = 0.99692, p-value = 0.9468
```

```r
ks.test(res1, "pnorm", mean = mean(res1), sd = sd(res1))
```

```
	Asymptotic one-sample Kolmogorov-Smirnov test

data:  res1
D = 0.028853, p-value = 0.9932
alternative hypothesis: two-sided
```

```r
shapiro.test(res2)
```

```
	Shapiro-Wilk normality test

data:  res2
W = 0.99427, p-value = 0.5711
```

```r
ks.test(res2, "pnorm", mean = mean(res2), sd = sd(res2))
```

```
	Asymptotic one-sample Kolmogorov-Smirnov test

data:  res2
D = 0.045046, p-value = 0.7658
alternative hypothesis: two-sided
```

```r
shapiro.test(res3)
```

```

    Shapiro-Wilk normality test

data:  res3
W = 0.9934, p-value = 0.4414
```

```r
ks.test(res3, "pnorm", mean = mean(res3), sd = sd(res3))
```

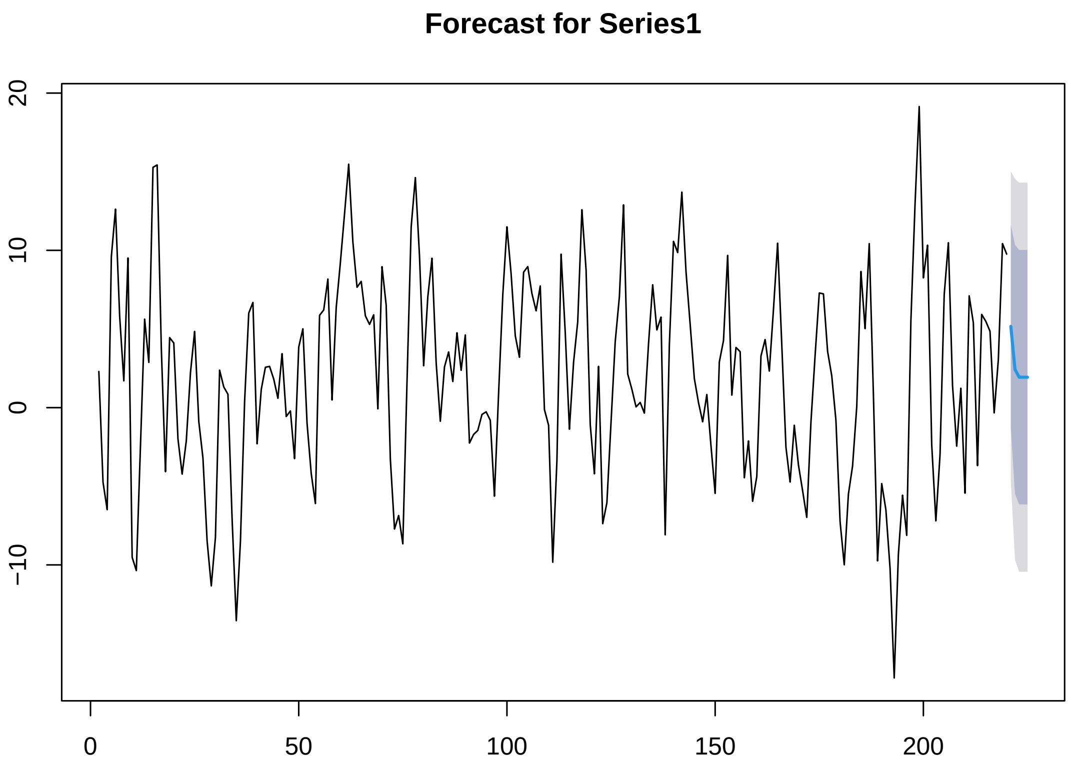
```

    Asymptotic one-sample Kolmogorov-Smirnov test

data:  res3
D = 0.039753, p-value = 0.8794
alternative hypothesis: two-sided
```

All 3 series are stationary. Using `auto.arima` on series:
We obtain low-order ARIMA(p,1,q) models:
ARIMA(0,1,2), ARIMA(1,1,1) and ARIMA(3,1,1) for the series
The ACF/PACF of the residuals show no significant autocorrelation.
The `Ljung-Box` tests on residuals (lag 20) have large p-values (> 0.5) (white noise).

```r
forecast1=forecast(model1,h=5)
forecast1
```

```
    Point Forecast     Lo 80    Hi 80      Lo 95    Hi 95
221       5.165362 -1.281541 11.61227  -4.694326 15.02505
222       2.431788 -5.488819 10.35240  -9.681735 14.54531
223       1.934769 -6.154838 10.02438 -10.437218 14.30676
224       1.934769 -6.154838 10.02438 -10.437218 14.30676
225       1.934769 -6.154838 10.02438 -10.437218 14.30676
```

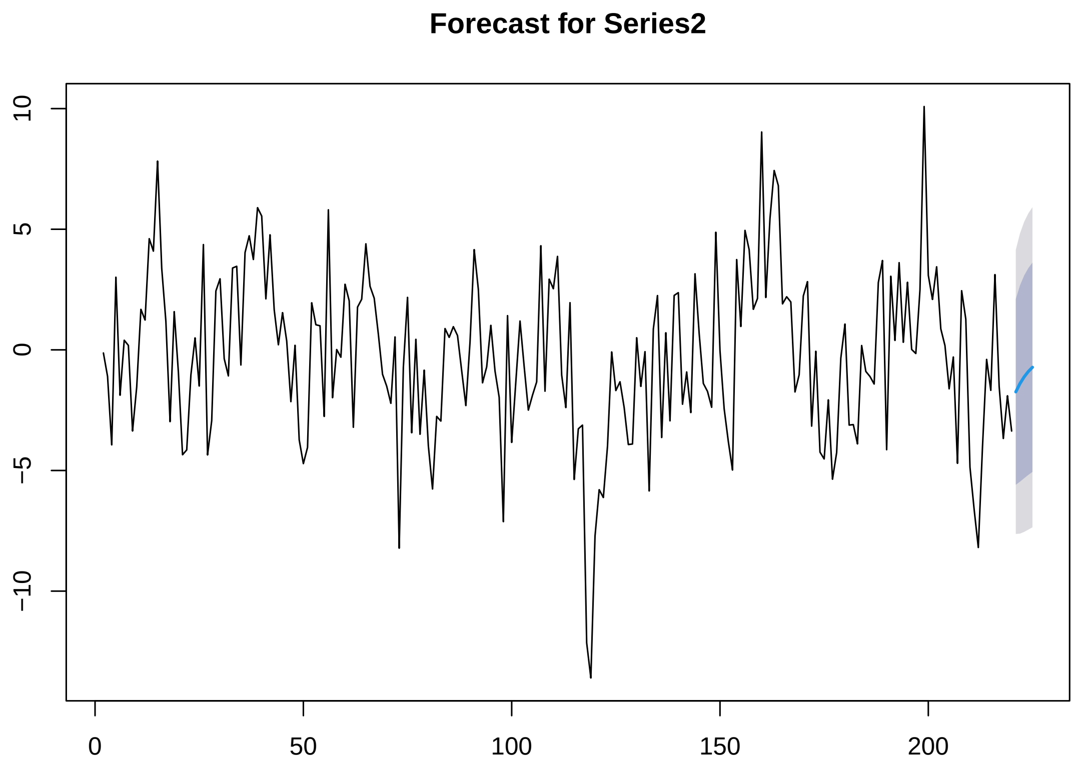
```r
plot(forecast1,main="Forecast for Series1")
```




```
forecast2=forecast(model2,h=5)
forecast2
```

```
    Point Forecast     Lo 80    Hi 80     Lo 95    Hi 95
221     -1.7425637 -5.592582 2.107454 -7.630658 4.145531
222     -1.3976751 -5.464913 2.669563 -7.617978 4.822628
223     -1.1210469 -5.322096 3.080002 -7.545997 5.303903
224     -0.8991690 -5.184095 3.385757 -7.452398 5.654059
225     -0.7212053 -5.059235 3.616824 -7.355648 5.913238
```

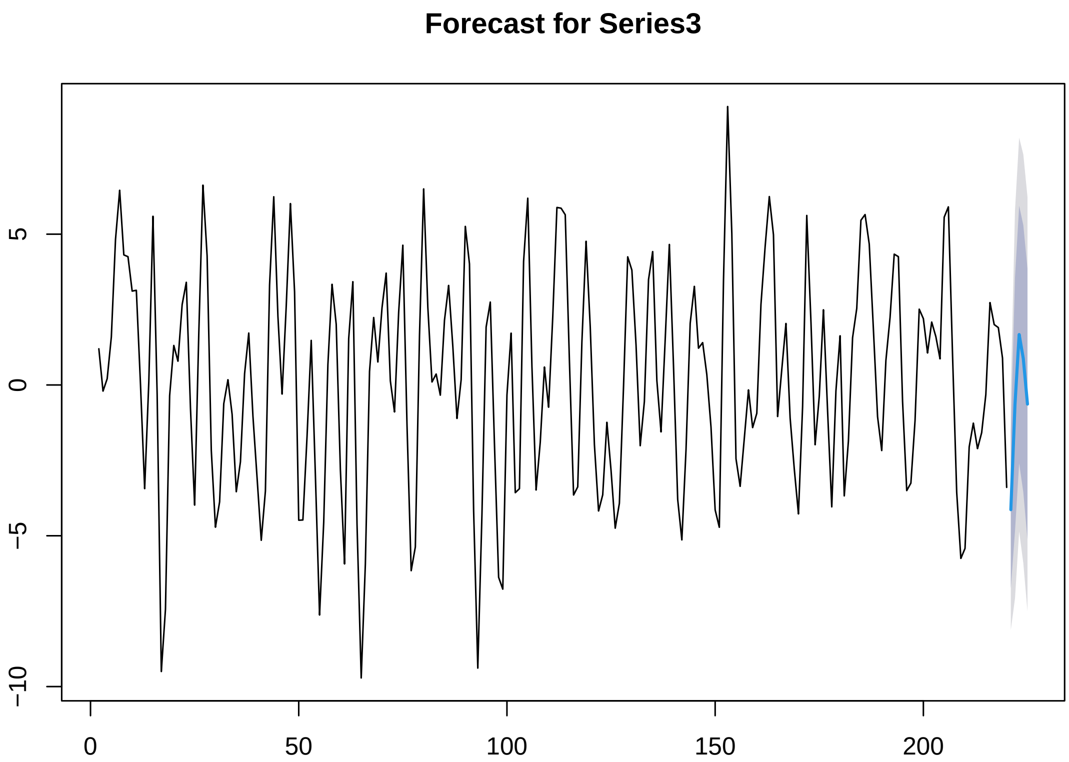
```
plot(forecast2,main="Forecast for Series2")
```




```
forecast3=forecast(model3,h=5)
forecast3
```

```
    Point Forecast     Lo 80     Hi 80     Lo 95      Hi 95
221     -4.1349545 -6.740425 -1.529484 -8.119678 -0.1502309
222     -0.6454578 -4.849203  3.558288 -7.074532  5.7836161
223      1.6729206 -2.595563  5.941404 -4.855162  8.2010033
224      0.8706380 -3.558273  5.299549 -5.902797  7.6440731
225     -0.6346691 -5.123908  3.854570 -7.500367  6.2310290
```

```
plot(forecast3,main="Forecast for Series3")
```



We then computed 5-step-ahead forecasts for each return series using a forecasting function obtaining point forecasts and 95% prediction intervals for the next 5 period.