

## Discussion 11

1) Make parentheses explicit in the  $\lambda$ -expressions.

$a\ b\ c$

$\lambda a.\ \lambda b.\ a\ b$

$\lambda a.\ a\ b\ \lambda a.\ a\ b$

2) Circle the free variables in the  $\lambda$ -expressions.

$\lambda a.\ a\ b\ a$

$a\ (\lambda a.\ a)\ a$

$\lambda a.\ (\lambda b.\ a\ b)\ a\ b$

3) Do an  $\alpha$ -conversion.

$\lambda a.\ \lambda a.\ a$

$(\lambda a.\ a)\ a\ b$

$(\lambda a.\ (\lambda a.\ (\lambda a.\ a)\ a)\ a)\ a$

4) Do a  $\beta$ -reduction.

$(\lambda a.\ a\ b)\ x\ b$

$(\lambda a.\ b)\ (\lambda a.\ \lambda b.\ \lambda c.\ a\ b\ c)$

$(\lambda a.\ a\ a)\ (\lambda a.\ a\ a)$

5) Reduce the  $\lambda$ -expressions to  $\beta$  normal form.

$(\lambda a. a) (\lambda a. a a) (\lambda a. a x)$

$(\lambda a. \lambda b. a b) (\lambda a. a) x$

$(\lambda x. y) ((\lambda y. y y y) (\lambda x. x x x))$