## International Macroeconomics and Finance

## Problem Set 4

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## 1 Based on Galí and Monacelli (2005)

Consider a small open economy integrated with the rest of the world. <sup>1</sup> Up to the first order, an aggregate supply equation describes inflation dynamics

$$\pi_{Ht} = \kappa x_t + \beta \mathbb{E}_t \pi_{Ht+1} + u_t, \tag{1}$$

where  $\pi_{Ht}$  is domestic inflation,  $x_t$  is the output gap,  $u_t$  is a cost-push shock,  $\beta \in (0,1)$  is the individual discount factor, and  $\kappa \equiv \lambda(1+\varphi)$ . In turn,  $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$ , where  $\theta \in (0,1)$  is the probability of not being able to readjust its price in any given period for a monopolistic competitive firm, and  $\varphi > 0$  is the inverse Frisch elasticity.

An aggregate demand relation describes the evolution of the output gap

$$x_t = -(i_t - \mathbb{E}_t \pi_{Ht+1} - r_t^n) + \mathbb{E}_t x_{t+1}, \tag{2}$$

where  $i_t$  is the nominal interest rate, and  $r_t^n$  is the natural rate of interest.

The terms of trade  $\tau_t$  links domestic and CPI inflation  $\pi_t$  according to

$$\pi_t = \pi_{Ht} + \alpha \Delta \tau_t, \tag{3}$$

where  $\alpha \in (0,1)$  measures the degree of openness of the economy. The relation between domestic and foreign output  $y_t^*$  (which is exogenous from the perspective of the small open economy) is  $y_t = y_t^* + \tau_t$ , which we can rewrite in terms of the domestic output gap as

$$x_t = z_t + \tau_t, \tag{4}$$

<sup>&</sup>lt;sup>1</sup>We thank Andrea Ferrero for sharing his course material.

Table 1: Calibration

Parameter		Value	Description
$\beta$	=	0.99	Individual discount factor
$\theta$	=	0.75	Probability of not adjusting prices
arphi	=	3	Inverse Frisch elasticity of labor supply
$\alpha$	=	0.4	Degree of openness
ε	=	6	Elasticity of substitution among varieties

where  $z_t \equiv y_t^* - y_t^n$  is the difference between foreign output and the level of natural output for the small open economy  $y_t^n$ . Finally, the uncovered interest rate parity condition is

$$i_t = i_t^* + \mathbb{E}_t \Delta e_{t+1}, \tag{5}$$

where  $i_t^*$  is the foreign nominal interest rate (also exogenous) and  $e_t$  is the nominal exchange rate.

All the exogenous shocks (cost-push, natural rate of interest, gap between foreign output and domestic natural level of output, and the foreign interest rate) follow stationary AR(1) processes with persistence equal to 0.9. The innovations of the exogenous shock processes are i.i.d. Normal with unit standard deviation and are mutually uncorrelated.

1. Consider a benevolent planner that minimizes the loss function

$$\mathcal{L}_0 = -\frac{\Omega}{2} \sum_{t=0}^{\infty} \beta^t (x_t^2 + \lambda_\pi \pi_{Ht}^2),$$
 (6)

with  $\Omega \equiv (1-\alpha)(1+\varphi)$ ,  $\lambda_{\pi} \equiv \varepsilon/[\lambda(1+\varphi)]$ , where  $\varepsilon > 0$  is the elasticity of substitution among varieties produced domestically. Write the optimal monetary policy problem for the small open economy and derive the targeting rule that implements the optimal plan.

- 2. Table 1 reports the calibration of the parameters. Compare the impulse response of  $\{\pi_{Ht}, x_t, \pi_t, \tau_t, i_t, \Delta e_t\}_{t=0}^{20}$  to a cost-push shock  $u_t$  under optimal policy against the case in which the central bank of the small open economy adopts
  - (a) An interest rate rule that targets domestic inflation

$$i_t = \phi_\pi \pi_{Ht}; \tag{7}$$

(b) An interest rate rule that targets CPI inflation

$$i_t = \phi_\pi \pi_t; \tag{8}$$

For the two interest rate rules (a) and (b), assume  $\phi_{\pi} = 1.5.^{2}$ 

- 3. Repeat the previous point for the other three shocks:  $r_t^n$ ,  $z_t$ , and  $i_t^*$ . Produce one chart for each shock.
- 4. Do exercise 2 and 3 under an exchange rate peg policy rule

$$\Delta e_t = 0. (9)$$

Try to understand why the model is not working on Dynare. Modify slightly the equations to make the model work. [Suggestion: the problem of inderminacy comes from the UIP condition.]

5. Compute the welfare costs of (7), (8), and (9) relative to optimal policy. Explain carefully your approach to calculate the welfare costs.

<sup>&</sup>lt;sup>2</sup>The output should take the form of a 3x2 (or 2x3) panel. Each panel should report one variable under the four different policy configurations. Make sure to appropriately label each panel and to use line styles that are easy to distinguish. Inflation rates and interest rates should be plotted in annualized percentage points. The other variables should be plotted in percentage deviations from steady state.