

# International Macroeconomics and Finance

## Assignment 3

### Instructions

Assume a small open economy inhabited by four agents: a continuum of ex-ante identical households, a continuum of identical firms, a fiscal authority, and a central bank. The economy produces one good that is traded freely in the economy. We assume the economy trades only bonds denominated in units of home goods.

### Households

The economy is inhabited by a continuum of identical infinitely-living households indexed by  $i$ . The lifetime utility of a household  $i$  is:

$$\sum_{t=0}^{\infty} \beta^t \log \left( C_t - \frac{(L_{i,t})^{1+\eta}}{1+\eta} \right) \quad (1)$$

where

$$C_t \equiv (C_{i,t}^H)^{\alpha_H} (C_{i,t}^F)^{1-\alpha_H} \quad (2)$$

and  $\beta \in (0, 1)$  is the subjective discount factor.  $C_{i,t}^H$  and  $C_{i,t}^F$  represent the consumption of household  $i$  of Home and Foreign goods, respectively.  $L_{i,t}$  is the labor of households in the country  $i$ , and  $B_{i,t}^{US}$  represents the bonds held by households in the country  $i$ , denominated in US goods. The price of the consumption basket is:

$$P_t = \left( \frac{P_t^H}{\alpha_H} \right)^{\alpha_H} \left( \frac{P_t^F}{1-\alpha_H} \right)^{1-\alpha_H} \quad (3)$$

Households are subject to the following budget constraints:

$$P_t C_t = P_t^H C_{i,t}^H + P_t^F C_{i,t}^F = (1 - \tau_t) W_t s_{i,t} L_{i,t} + P_t^H B_{i,t}^H R_t - P_t^H B_{i,t+1}^H \quad (4)$$

where  $P_{i,t}^H$  and  $P_{i,t}^F$  are the prices of Home and Foreign goods denominated in Home-country currency, and  $W_{i,t}$  is the nominal wage denominated in Home-country currency. Finally,  $\tau_t$  is the tax on labor income denominated in Home-country currency. Households' labor productivity  $\{s_t\}_{t=0}^{\infty}$  is stochastic and is characterized by an  $N$ -state Markov chain that can take on values  $s_t \in S = \{s_1, \dots, s_N\}$  with transition probability  $\gamma(s_{t+1}|s_t)$  and  $\int s = 1$ . Households are subject to the following borrowing constraint:

$$B_{i,t+1} \geq -\kappa \quad (5)$$

## Firms

There is a continuum of firms that produce under perfect competition using the following linear technology:

$$Y_t = L_t, \quad (6)$$

where  $Y_t$  is the output and  $L_t = \int_0^1 s_{i,t} L_{i,t} di$  is the aggregate labor. Firms' profits are:

$$P_t^H Y_t - W_t L_t \quad (7)$$

## Central Bank, Fiscal Policy, and Equilibrium Condition

Assume that the central bank targets the inflation of the Home goods:

$$\frac{P_t^H}{P_{t-1}^H} = 1 \quad (8)$$

The fiscal authority consumes Home goods,  $G_t^H$ , and issues bonds  $B_{g,t}^H$  in units of Home goods. Finally, the fiscal authority charges a tax on labor income,  $\tau$ . The fiscal authority budget constraint is:

$$G_t^H + B_{G,t+1}^H = \tau_t Y_t + B_{G,t}^H R_t \quad (9)$$

## Assumptions

- Assume that there is no home bias.

## Questions and Answers

1. Write down the first-order conditions for the households' and firms' problems. (20 points)

### Household Problem

$$\max \sum_{t=0}^{\infty} \beta^t \log \left( C_t - \frac{(L_{i,t})^{1+\eta}}{1+\eta} \right)$$

where

$$C_t \equiv (C_{i,t}^H)^{\alpha_H} (C_{i,t}^F)^{1-\alpha_H}$$

subject to the budget constraint:

$$P_t C_t = P_t^H C_{i,t}^H + P_t^F C_{i,t}^F = (1 - \tau_t) W_t s_{i,t} L_{i,t} + P_t^H B_{i,t}^H R_t - P_t^H B_{i,t+1}^H$$

and the borrowing constraint:

$$B_{i,t+1} \geq -\kappa$$

## Lagrangian

$$L = \sum_{t=0}^{\infty} \beta^t \left[ \log \left( (C_{i,t}^H)^{\alpha_H} (C_{i,t}^F)^{1-\alpha_H} - \frac{(L_{i,t})^{1+\eta}}{1+\eta} \right) + \lambda_t \left[ (1 - \tau_t) W_t s_{i,t} L_t + P_t^H B_{i,t}^H R_t - P_t^H B_{i,t+1}^H - P_t^H C_{i,t}^H + P_t^F C_{i,t}^F \right] + \mu_t \left[ B_{i,t+1}^H + \kappa \right] \right]$$

## Household First-Order Conditions

The first-order condition for households is obtained through maximizing the above Lagrangian equation with respect to the choice variables:

$$\begin{aligned} \frac{\partial L}{\partial C_{i,t}^H} &: \frac{\alpha_H (C_{i,t}^H)^{\alpha_H-1}}{C_t - \frac{(L_{i,t})^{1+\eta}}{1+\eta}} = \lambda_t P_t^H \\ \frac{\partial L}{\partial C_{i,t}^F} &: \frac{(1 - \alpha_H) (C_{i,t}^F)^{-\alpha_H}}{C_t - \frac{(L_{i,t})^{1+\eta}}{1+\eta}} = \lambda_t P_t^F \\ \frac{\partial L}{\partial L_{i,t}} &: \frac{L_{i,t}^\eta}{C_t - \frac{(L_{i,t})^{1+\eta}}{1+\eta}} = (1 - \tau_t) \lambda_t W_t s_{i,t} \\ \frac{\partial L}{\partial B_{i,t+1}^H} &: \beta \lambda_{t+1} P_{t+1}^H R_{t+1} = \lambda_t P_t^H - \mu_t \end{aligned}$$

## Firm problem

$$\max P_t^H Y_t - W_t L_t$$

subject to

$$Y_t = L_t$$

## Household First-Order Conditions

Through direct substitution of production function into the firm problem, we have the following FOC for firm:

$$P_t^H = W_t$$

2. Describe the conditions needed to close the model. *Hint: Foreign demand for Home goods and Foreign production of Foreign goods, for example. Add these conditions to the model.*

## More conditions to close the model

- Foreign demand for consumption of home goods:

$$C_{H,t}^* = \alpha_H^* \left( \frac{P_t^H}{P_t^F} \right)^{-\theta^*} Y_t^*$$

where  $\alpha_H^*$  is the share of home goods in foreign consumption,  $\theta^*$  represents the elasticity of substitution,  $Y_t^*$  is the foreign output, and  $P_t^H$ ,  $P_t^F$  are prices of home and foreign goods respectively.

- Foreign production of foreign goods can be assumed to be exogenous by fixing foreign labour supply  $L_t^*$ .

$$Y_t^* = L_t^*$$

## Equilibrium

An equilibrium is a sequence of interest rates  $\{R_t\}$ , a sequence of consumption and labor supply policies  $\{C_t, L_t\}$ , a sequence of taxes  $\{\tau_t\}$ , and a sequence of distributions for bond holdings and productivity levels which is consistent with the consumption and labor supply policies.

In addition to the conditions to close the model stated above, here I summarize all the conditions of the model to be fulfilled at the equilibrium.

### Household utility optimization

Household First-Order Conditions are derived above, given goods prices, tax and interest rate.

### Firm profit optimization

Firm First-Order Conditions are derived above, given factor prices/wages.

### Labour market clears

Labour demanded by firms equal labour supplied by households.

$$L_t = \int_0^1 s_{i,t} L_{i,t} di$$

### Goods market clears

- Home goods market clears:

$$Y_t = C_t^H + G_t^H + C_{H,t}^*$$

- Foreign goods market clears (where  $Y_t^*$  is exogenously assumed by fixing  $Y_t^* = L_t^*$ )

$$Y_t^* = C_{F,t}^* + G_{F,t}^* + C_{F,t}$$

**Bond market clears**

$$B_H^G + \int_0^1 B_i^H di = 0$$

**Law of one price (LOOP) holds**

$$\begin{aligned} P_t^H &= e_t P_{H,t}^* \\ P_t^F &= e_t P_{F,t}^* \end{aligned}$$

For simplicity, normalize  $P_{F,t}^*$  to unity, hence  $P_t^F = e_t$ ;  
Define terms of trade as the relative price of exports to imports:

$$x_t = \frac{P_t^H}{P_t^F} = \frac{P_t^H}{e_t}$$

**Monetary Policy**

Inflation target:

$$\frac{P_t^H}{P_{t-1}^H} = 1$$

**Fiscal Policy**

Fiscal budget constraint: the tax satisfies the government budget constraint

$$G_t^H + B_{G,t+1}^H = \tau_t Y_t + B_{G,t}^H R_t$$

3. Focus on the steady state and calibrate the model to match the US economy. Use US data to calibrate the household productivity process as well. Describe your calibration. (20 points)

**Calibration**

Baseline parameters are reported in Table 1. Time period is a quarter. Households have log preferences as given in the instruction. We set the elasticity of labor supply to 1. The discount factor  $\beta = 0.97$  to match the average real interest rate currently at 2.5 percent in the US. The curvature of utility from leisure  $\eta$  is chosen based on the average Frish elasticity from a wide range of macroeconomics literature.

The process of  $s$  assumes labor productivity follows an AR(1) process with persistence 0.967 and normal innovations with standard deviation 0.017. The income process into a seven-point Markov chain via the Rouwenhorst method. For the foreign sector, we assume no home bias and set other parameters reasonably based on our home economy and literature.

Table 1: Baseline Parameter Values and Sources

Parameter	Explanation	Value	Target/Source
$\beta$	Discount factor	0.97	Based on US interest rate on average $r = 2.5\%$
$\eta$	Curvature of utility from leisure	1.5	Average Frisch elasticity = 1
$\alpha_H$	No Home Bias	0.5	By instruction
$s$	Productivity process	0.967	Persistence of wage process in Flodén and Lindé (2001)
$\sigma_\epsilon$	Variance of productivity shock	0.017	Variance of wage process in Flodén and Lindé (2001)
$\kappa$	Borrowing limit	1	Based on recent US Debt-to-GDP ratio

4. Assume that the US does not trade bonds with the rest of the world:

$$B_{H,G} + \int_0^1 B_{H,i} di = 0 \quad (10)$$

Find the steady state and the equilibrium real interest rate. Plot the policy functions for consumption and savings and the wealth distribution. We call this case the *benchmark*. (20 points)

## Benchmark

From the value function iteration, the equilibrium interest rate is around 1.0377. Moreover, Figure 1 below suggests household policies in steady state - the optimal values of consumption, labor supply and bond as a function of the initial level of bond holdings, for two productivity levels; and the wealth distribution over different bond levels.

Consumption Policy Function in Figure 1 shows both productivity types increase their consumption as assets increase, reflecting an income effect. Moreover, High productivity agents consistently consume more than low productivity agents; this is because high productivity increases wealth more and hence results in higher consumption. Also, the gap between high and low productivity consumption narrows at higher asset levels, which might be because higher-asset-levels agents are less vulnerable to productivity shocks.

Asset Policy Function in Figure 1 Shows how agents choose next period's assets based on current assets. The nearly 45-line increasing pattern indicates that agents with higher level of bonds tend to hold more bonds, indicating positive saving behaviour. It implies that agents accumulate assets to self-insure against productivity shocks. The pattern is almost identical for both productivity types, suggesting asset accumulation is not strongly dependent on current productivity level.

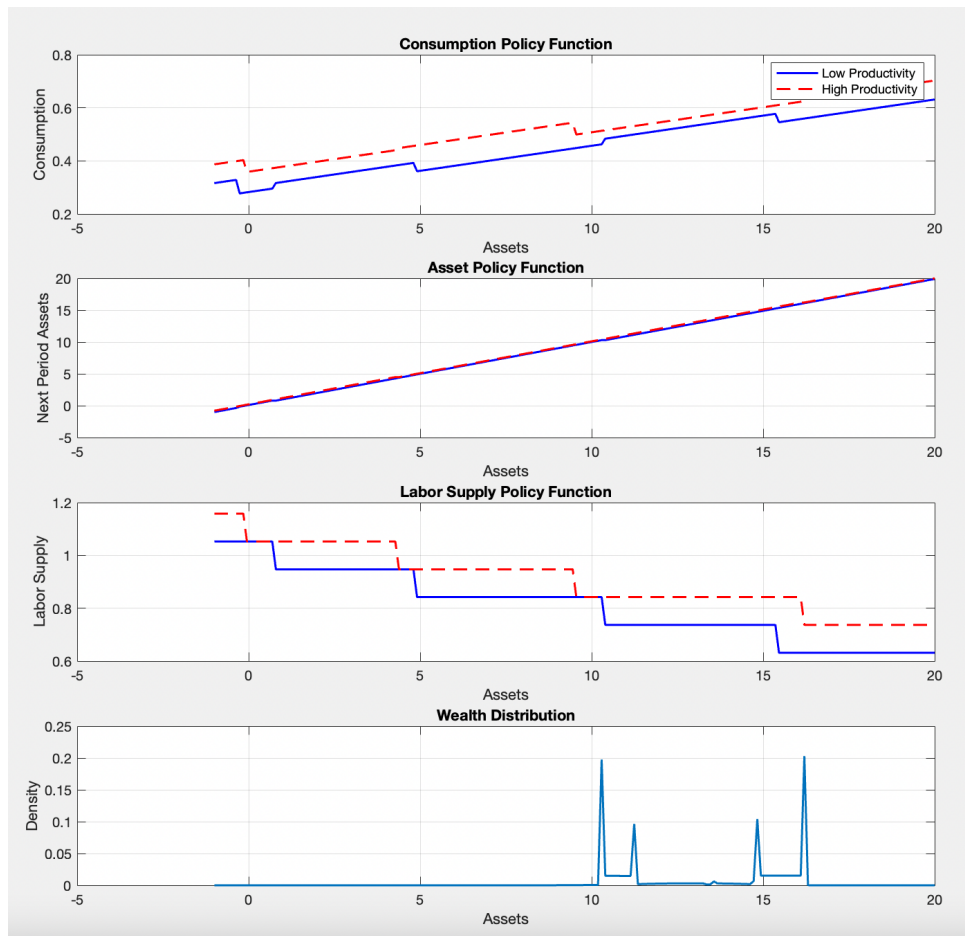


Figure 1: Benchmark Policy and Distribution Functions

Labor Supply Policy Function in Figure 1 shows that as assets increase, labor supply decreases for both types. It also supports the income effect that higher income leads to increased leisure and hence reduced labor supply. High-productivity agents supply more labor than low productivity agents at all asset levels and the negative wealth effect on labor supply is stronger for wealthier agents.

Wealth Distribution in Figure 1 shows the equilibrium distribution of assets across agents. The distribution is highly skewed to the right, with significant mass at higher asset levels and spikes in the distribution. It indicates substantial wealth inequality in the steady state. Such endogenous wealth inequality is highly likely due to productivity differences and saving behavior.

5. Assume that there is an increase in the savings of the rest of the world:

$$B_{H,G} + \int_0^1 B_{H,i} di < 0 \quad (11)$$

Assume that the fiscal authority debt is at one of the points in Question 4. Assume that the fiscal authority moves taxes to satisfy its budget constraint. What happens to the real interest rate and the terms of trade? What happens to the policy functions? Describe which households are gaining and losing from this inflow of resources. (*You need to compare this economy with the benchmark.*) (20 points)

## Tax rate adjustment

Part 5 extends the benchmark model to handle a modified bond market clearing condition with increased foreign savings added, under tax rate adjustment.

Lower interest rate: in this case, the increased foreign savings (foreign lending to domestic economy) leads to a decline in the real interest rate. This is because the increased supply of foreign savings puts downward pressure on interest rates, and lower interest rates encourage domestic borrowing and discourage domestic saving, and foreign lenders are willing to accept lower returns to invest in domestic bonds.

Terms of Trade ( $x_t = \frac{P_t^H}{P_t^F}$ ) tend to appreciate ( $x_t$  increases). With increased foreign savings, there's higher demand for domestic bonds, which tends to appreciate the terms of trade. Appreciation makes domestic goods relatively more expensive than foreign goods. This affects the trade balance and consumption patterns.

Figure 2 plots the policy function changes under higher tax rate adjustment from a tax rate of 25 percent to 35 percent.

Consumption Policy Function under tax adjustment suggests that households decrease consumption due to higher taxes. However, heterogeneous households may face offsetting effects: borrowers (usually with less assets) face positive income effect of less costly borrowing and negative income effect due to higher tax burden, while savers face negative income effect from both lower return on their savings (due to lower interest rate) and higher tax burden. Hence, savers/ wealthy agents are the 'losers', and borrowers/ poor agents are the 'winners' in this case.

All agents show reduced saving incentives due to lower interest rates. The saving/asset function becomes slightly flatter, indicating less wealth accumulation.



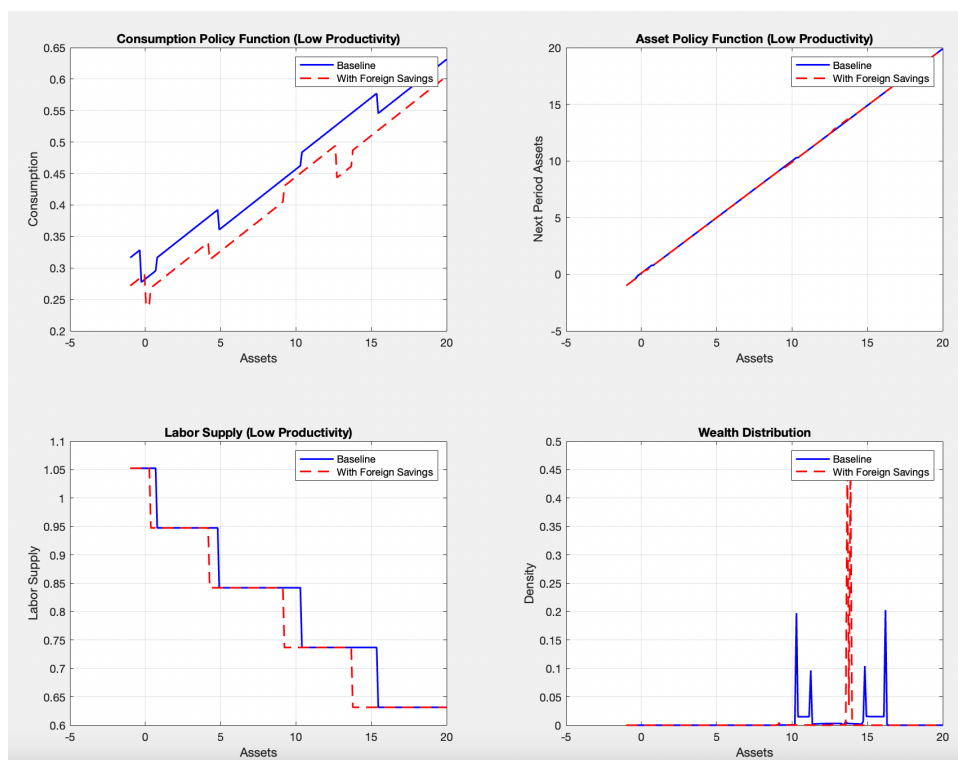


Figure 2: Policy Function Changes Under Higher Tax Rate

Labor Supply Function decreases for most households due to higher tax distortions. Wealthy agents might reduce labor more due to larger negative income effects.

6. Repeat Question 5. However, assume that the fiscal authority moves public spending instead of taxation. (20 points)

## Government spending adjustment

In Part 6, we fix the tax rate and change government spending instead. With increased foreign savings, interest rate becomes lower and terms of trade increases from the argument in Part 5.

From Figure 3, policy function changes seem small under higher government spending adjustment compare to part 5 of tax adjustment. This might be because that tax adjustment exerts more direct effect on household decisions, but government spending adjustment has more uniform and only indirect impact across households, hence leads to less distortion of household decisions. Theoretically, higher government spending results in less private consumption.

Under government spending adjustment and lower interest rate, savers are the losers and borrower are the winners. Also, public-sector workers might also be winners given higher government spending, compare to private-sector workers.

To summarize part 5 and 6, both scenarios demonstrate how financial integration of an open economy (through foreign savings) can affect domestic macroeconomic outcomes, but the transmission mechanism and distributional consequences depend crucially on the fiscal policy response chosen by the government.

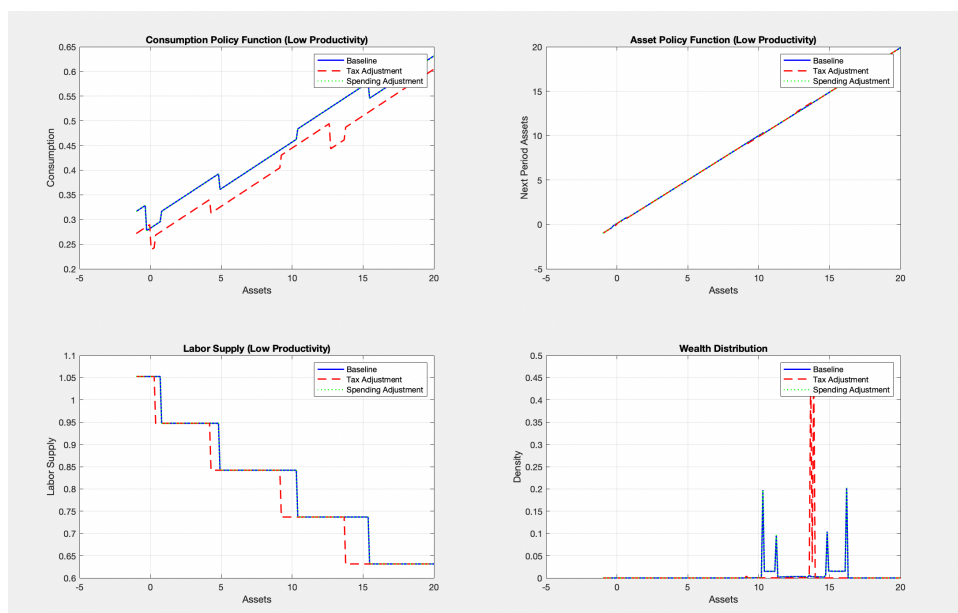


Figure 3: Policy Function Changes Under Higher Government Spending