Finance Quantitative

Modèle Black-Scholes

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Black-Scholes Model (1)

Use the Black-Scholes model to price the option described in Table~1.

Type	Call
Strike	50
Spot	55
Maturity	3 months
Interest rate	3%
Dividend yield	0%
Volatility	.30

Table 1: Characteristics of an European option

```
K <- 50
S <- 55
r <- .03
sigma <- .3
T <- 1/4

d1 <- (log(S/K) + (r+0.5*sigma^2)*T)/(sigma*sqrt(T))
d2 <- d1 - sigma*sqrt(T)
C <- S * pnorm(d1) - K * exp(-r*T) * pnorm(d2)
```

The call price is $C = 6.52$.

Black-Scholes Model (2)

On March 21, 2012, GOOG quotes $636.91. Table~2 provides the prices of selected options expiring on 18 Jan 2013:

Type	Strike	Price
Call	635	60.70
Put	635	59.70

Table 2: Prices of options on GOOG

Assume an interest rate of 1.0%. Google does not pay any dividend.

- Compute the implied volatility for the call and the put. Comment your results.
- Use this result to price a call with strike 650.

The following function estimates the call and put prices as a function of volatility.

```
call.price <- function(sigma) {
d1 <- (log(S/K) + (r+0.5*sigma^2)*T)/(sigma*sqrt(T))
d2 <- d1 - sigma*sqrt(T)
S * pnorm(d1) - K * exp(-r*T) * pnorm(d2)
}

put.price <- function(sigma) {
d1 <- (log(S/K) + (r+0.5*sigma^2)*T)/(sigma*sqrt(T))
d2 <- d1 - sigma*sqrt(T)
-S * pnorm(-d1) + K * exp(-r*T) * pnorm(-d2)
}
```

Solve for implied volatility:

```
K <- 635
S <- 636.91
r <- .01
sigma <- .3
T <- as.numeric(dmy('18Jan2013') - dmy('21Mar2012'))/365

iv.call <- uniroot(function(s) {call.price(s)-60.70}, c(.1, .4))$root
iv.put <- uniroot(function(s) {put.price(s)-59.70}, c(.1, .4))$root
```

Use the call implied volatility to price the option:

```
K <- 650
C <- call.price(iv.call)
```

The call strike 650 is estimated at $C = 53.94$. The market price is 54.00.