Finance Quantitative

Modèle de Black-Litterman
Solution

Patrick Hénaff

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- Effectuer une lecture attentive de l'article de He et Litterman.
- A partir de la note de cours, reproduire les autres exemples de l'article, comparer les résultats avec ceux obtenus avec le package BLCOP.
- Comparer avec une allocation MV classique.

Données

```
 data =
'1,0.4880,0.4780,0.5150,0.4390,0.5120,0.4910
 0.4880,1,0.6640,0.6550,0.3100,0.6080,0.7790
 0.4780,0.6640,1,0.8610,0.3550,0.7830,0.6680
 0.5150,0.6550,0.8610,1,0.3540,0.7770,0.6530
 0.4390,0.3100,0.3550,0.3540,1,0.4050,0.3060
 0.5120,0.6080,0.7830,0.7770,0.4050,1,0.6520
 0.4910,0.7790,0.6680,0.6530,0.3060,0.6520,1'

  Corrmat = matrix( as.double(spl( gsub('\n', ',', data), ',')),
                    nrow = length(spl(data, '\n')), byrow=TRUE)

  stdevs = c(16.0, 20.3, 24.8, 27.1, 21.0,  20.0, 18.7)/100
  w.eq = c(1.6, 2.2, 5.2, 5.5, 11.6, 12.4, 61.5)/100
  # Prior covariance of returns
  Sigma = Corrmat * (stdevs %*% t(stdevs))
```

Calcul des rendements d'équilibre:

```
# risk aversion parameter
delta = 2.5
Pi = delta * Sigma %*% w.eq
```

Fonction de calcul des poids optimaux

Table 1: Résumé des données de marché

Assets	σ	w_{eq}	Π
Australia	16	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9
Japan	21	11.6	4.3
UK	20	12.4	6.8
USA	18.7	61.5	7.6

```
bl.weights <- function(P, Q, tau.s, tau.o) {
# one tau per view
x = tau.o * diag(P %*% Sigma %*% t(P))
Omega = diag(x, nrow=length(x))
tau.Sigma.inv = solve(tau.s*Sigma)
M.inverse = solve(tau.Sigma.inv + (t(P) %*% solve(Omega) %*% P))
mu.bar = M.inverse %*% (tau.Sigma.inv %*% Pi + t(P) %*% solve(Omega) %*% Q)
Sigma.bar = M.inverse + Sigma

w.star = (1/delta) * solve(Sigma.bar) %*% mu.bar

A = (1/tau.s)*Omega + 1/(1+tau.s) * P %*% Sigma %*% t(P)
APS <- 1/(1+tau.s) * solve(A) %*% P %*% Sigma
OIQ <- (tau.s/delta) * solve(Omega) %*% Q
Lambda =  OIQ - APS %*% w.eq - APS %*% t(P) %*% OIQ

Hmisc::llist(mu.bar, w.star, Lambda)
}
```

Point de vue 1: Le marché action Allemand surperforme le reste du marché action Européen de 5% par an.

Portefeuille exprimant le point de vue:

```
P.1 = matrix(c(0, 0, -29.5, 100, 0, -70.5, 0)/100, nrow=1)
Q.1 = 5/100
tau.s = 0.05
tau.o = 0.05

res <- bl.weights(P.1, Q.1, tau.s, tau.o)
df = data.frame(100*cbind(t(P.1), res$mu.bar, res$w.star, res$w.star-w.eq/(1+tau.s)))
df <- rbind(df, c(100*Q.1, rep(NA, 4)))
df <- rbind(df, c(100*res$Lambda[1], rep(NA, 4)))

row.names(df) = c(AssetNames, 'q', '$\\lambda \\times 100$')
names(df) =  c('P', "$\\bar{\\mu}$", '$w^*$','$w^* - \\frac{W_{eq}}{1+\\tau}$')
```

```r
tmp <- kable(df, digits = 1, format="latex", booktabs=T, escape=F,
      caption="Solution avec PdV 1. P: matrice du PdV, $\\bar{\\mu}$: rendement ex-post,
      $w^*$: poids optimaux, $\\frac{W_{eq}}{1+\\tau}$: poids ex-ante") %>%
  kable_styling(latex_options="HOLD_position")
kableExtra::row_spec(tmp, 7, hline_after = TRUE)
```

Table 2: Solution avec PdV 1. P: matrice du PdV, $\bar{\mu}$: rendement ex-post, w^*: poids optimaux, $\frac{W_{eq}}{1+\tau}$: poids ex-ante

	P	$\bar{\mu}$	w^*	$w^* - \frac{W_{eq}}{1+\tau}$
Australia	0.0	4.3	1.5	0.0
Canada	0.0	7.6	2.1	0.0
France	-29.5	9.3	-3.9	-8.9
Germany	100.0	11.0	35.4	30.2
Japan	0.0	4.5	11.0	0.0
UK	-70.5	7.0	-9.5	-21.3
USA	0.0	8.1	58.6	0.0
q	5.0			
$\lambda \times 100$	31.7			

Point de vue 2: le marché action Canadien surperforme le marché US de 3% par an.

Solution Litterman & He

Portefeuille exprimant le point de vue:

```r
P.2 = matrix(c(0, 100, 0, 0, 0, 0, -100)/100, nrow=1)
Q.2 = 3/100

P <- rbind(P.1, P.2)
Q <- matrix(c(Q.1, Q.2), nrow=2)
tau.o <- rep(0.05,2)
res <- bl.weights(P, Q, tau.s, tau.o)
df = data.frame(100*cbind(t(P), res$mu.bar, res$w.star, res$w.star-w.eq/(1+tau.s)))
df <- rbind(df, c(100*t(Q), rep(NA, 4)))
df <- rbind(df, c(t(100*res$Lambda), rep(NA, 4)))

row.names(df) = c(AssetNames, 'q', '$\\lambda \\times 100$')
names(df) =  c('$P_1$', '$P_2$', "$\\bar{\\mu}$", '$w^*$','$w^* - \\frac{W_{eq}}{1+\\tau}$')
tmp <- kable(df, digits = 1, format="latex", booktabs=T, escape=F,
      caption="Solution avec PdV 1 and 2.") %>%
  kable_styling(latex_options="HOLD_position")
kableExtra::row_spec(tmp, 7, hline_after = TRUE)
```

Table 3: Solution avec PdV 1 and 2.

	P_1	P_2	$\bar{\mu}$	w^*	$w^* - \frac{W_{eq}}{1+\tau}$
Australia	0.0	0.0	4.4	1.5	0.0
Canada	0.0	100.0	8.7	41.9	39.8
France	-29.5	0.0	9.5	-3.4	-8.4
Germany	100.0	0.0	11.2	33.6	28.3
Japan	0.0	0.0	4.6	11.0	0.0
UK	-70.5	0.0	7.0	-8.2	-20.0
USA	0.0	-100.0	7.5	18.8	-39.8
q	5.0	3.0			
$\lambda \times 100$	29.8	41.8			

Solution BLCOP

La solution obtenue en resolvant directement le portefeuille tangent avec les rendements et la matrice de covariance ex-post est globalement en accord avec le résultat de Litterman & He.

```r
# rendement ex-ante
delta = 2.5
Pi = delta * Sigma %*% w.eq

# Point de vue
tau.pdv = 0.05

PDV.1 = matrix(c(0, 0, -29.5, 100, 0, -70.5, 0)/100, nrow=1)
colnames(PDV.1) <- AssetNames
# niveau de confiance
sd <- as.numeric(tau.pdv * PDV.1  %*% Sigma %*% t(PDV.1))
views <- BLViews(P = PDV.1, q = 0.05,
                 confidences =  1/sd,
                 assetNames = AssetNames)

PDV.2 = matrix(c(0, 100, 0, 0, 0, 0, -100)/100, nrow=1)
colnames(PDV.2) <- AssetNames
# niveau de confiance
sd <- as.numeric(tau.pdv * PDV.2  %*% Sigma %*% t(PDV.2))
views <- addBLViews(PDV.2, q = 0.03,
                 confidences =  1/sd,
                 views)

dist.expost <- posteriorEst(views=views, sigma=Sigma, mu=as.vector(Pi), tau=0.05)

mu <- dist.expost@posteriorMean
S <- dist.expost@posteriorCovar
res <- solve.QP(Dmat=S, dvec=rep(0, length(mu)), Amat=as.matrix(mu, ncol=1), bvec=1, meq=1)
w.QP <- round(100*res$solution/sum(res$solution),1)
df <- data.frame(w=w.QP)
row.names(df) <- AssetNames
names(df) <- "$w^*$"
kable(df, caption = "Portefeuille tangent avec BLCOP et solve.QP, incorporant les PDV 1 et 2",
```

```
      format="latex", booktabs=T, escape=F) %>%
    kable_styling(latex_options="HOLD_position")
```

Table 4: Portefeuille tangent avec BLCOP et solve.QP, incorporant les PDV 1 et 2

	w^*
Australia	1.6
Canada	44.0
France	-3.6
Germany	35.3
Japan	11.6
UK	-8.6
USA	19.7

Point de vue 3: Optimiste sur le marché action Canadien

Le seul changement est le paramètre q_2:

```
Q.2 = 4/100

Q <- matrix(c(Q.1, Q.2), nrow=2)

res <- bl.weights(P, Q, tau.s, tau.o)
df = data.frame(100*cbind(t(P), res$mu.bar, res$w.star, res$w.star-w.eq/(1+tau.s)))
df <- rbind(df, c(100*t(Q), rep(NA, 4)))
df <- rbind(df, c(t(100*res$Lambda), rep(NA, 4)))

row.names(df) = c(AssetNames, 'q', '$\\lambda \\times 100$')
names(df) =  c('$P_1$', '$P_2$', "$\\bar{\\mu}$", '$w^*$','$w^* - \\frac{W_{eq}}{1+\\tau}$')
tmp <- kable(df, digits = 1, format="latex", booktabs=T, escape=F,
      caption="Actions Allemandes surperforment de 4\\%") %>%
  kable_styling(latex_options="HOLD_position")
kableExtra::row_spec(tmp, 7, hline_after = TRUE)
```

Table 5: Actions Allemandes surperforment de 4%

	P_1	P_2	$\bar{\mu}$	w^*	$w^* - \frac{W_{eq}}{1+\tau}$
Australia	0.0	0.0	4.4	1.5	0.0
Canada	0.0	100.0	9.1	53.3	51.3
France	-29.5	0.0	9.5	-3.3	-8.2
Germany	100.0	0.0	11.3	33.1	27.8
Japan	0.0	0.0	4.6	11.0	0.0
UK	-70.5	0.0	7.0	-7.8	-19.6
USA	0.0	-100.0	7.3	7.3	-51.3
q	5.0	4.0			
$\lambda \times 100$	29.2	53.8			

Point de vue 4: Moindre confiance dans le PdV "Allemagne vs reste de l'Europe"

L'écart type du rendement du portefeuille 1 double ($\tau = 0.1$):

```
tau.o <- c(0.1, .05)
res <- bl.weights(P, Q, tau.s, tau.o)
df = data.frame(100*cbind(t(P), res$mu.bar, res$w.star, res$w.star-w.eq/(1+tau.s)))
df <- rbind(df, c(100*t(Q), rep(NA, 4)))
df <- rbind(df, c(t(100*res$Lambda), rep(NA, 4)))

row.names(df) = c(AssetNames, 'q', '$\\lambda \\times 100$')
names(df) =  c('$P_1$', '$P_2$', "$\\bar{\\mu}$", '$w^*$','$w^* - \\frac{W_{eq}}{1+\\tau}$')
tmp <- kable(df, digits = 1, format="latex", booktabs=T, escape=F,
      caption="Moindre confiance dans le PdV 1.") %>%
  kable_styling(latex_options="HOLD_position")
kableExtra::row_spec(tmp, 7, hline_after = TRUE)
```

Table 6: Moindre confiance dans le PdV 1.

	P_1	P_2	$\bar{\mu}$	w^*	$w^* - \frac{W_{eq}}{1+\tau}$
Australia	0.0	0.0	4.3	1.5	0.0
Canada	0.0	100.0	8.9	53.9	51.8
France	-29.5	0.0	9.3	-0.5	-5.4
Germany	100.0	0.0	10.6	23.6	18.4
Japan	0.0	0.0	4.6	11.0	0.0
UK	-70.5	0.0	6.9	-1.1	-13.0
USA	0.0	-100.0	7.1	6.8	-51.8
q	5.0	4.0			
$\lambda \times 100$	19.3	54.4			

Ajout d'un point de vue redondant.

Le point de vue "Le marché action Canadien surperforme le marché Nippon de 4.12%" est implicite aux points de vue précédents. L'ajout du PdV ne change pas l'allocation.

```
P.3 = matrix(c(0, 100, 0, 0, -100, 0, 0 )/100, nrow=1)
Q.3 = 4.12/100

P <- rbind(P.1, P.2, P.3)
Q <- matrix(c(Q.1, Q.2, Q.3), nrow=3)
tau.o <- c(0.1, .05, 0.05)
res <- bl.weights(P, Q, tau.s, tau.o)
df = data.frame(100*cbind(t(P), res$mu.bar, res$w.star, res$w.star-w.eq/(1+tau.s)))
df <- rbind(df, c(100*t(Q), rep(NA, 4)))
df <- rbind(df, c(t(100*res$Lambda), rep(NA, 4)))

row.names(df) = c(AssetNames, 'q', '$\\lambda \\times 100$')
```

```r
names(df) =  c('$P_1$', '$P_2$', '$P_3$', "$\\bar{\\mu}$", '$w^*$','$w^* - \\frac{W_{eq}}{1+\\tau}$')
tmp <- kable(df, digits = 1, format="latex", booktabs=T, escape=F,
      caption="PdV redondant Canada/Japon.") %>%
  kable_styling(latex_options="HOLD_position")
kableExtra::row_spec(tmp, 7, hline_after = TRUE)
```

Table 7: PdV redondant Canada/Japon.

	P_1	P_2	P_3	$\bar{\mu}$	w^*	$w^* - \frac{W_{eq}}{1+\tau}$
Australia	0.0	0.0	0.0	4.3	1.5	0.0
Canada	0.0	100.0	100.0	8.8	53.9	51.8
France	-29.5	0.0	0.0	9.2	-0.5	-5.4
Germany	100.0	0.0	0.0	10.6	23.6	18.4
Japan	0.0	0.0	-100.0	4.6	11.0	0.0
UK	-70.5	0.0	0.0	6.9	-1.1	-13.0
USA	0.0	-100.0	0.0	7.1	6.8	-51.8
q	5.0	4.0	4.1			
$\lambda \times 100$	19.3	54.4	0.0			