Modèles de Black-Litterman

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Droite de Marché des Capitaux

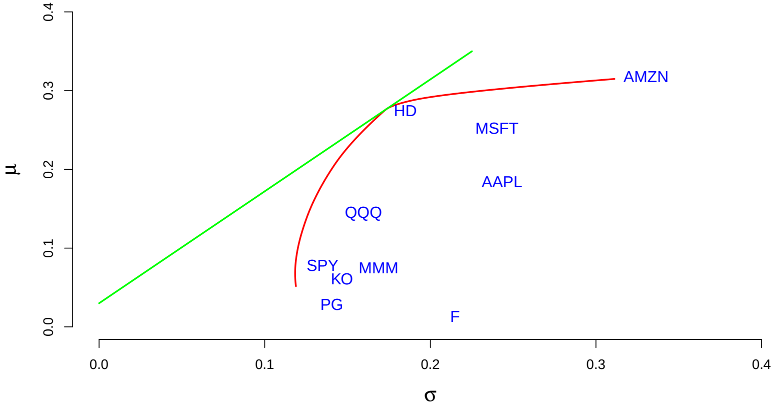

Figure 1: Droite de Marché des Capitaux

Black-Litterman (1)

- Par défaut: Accepter les espérances de rendement implicites dans le portefeuille de marché, et investir dans ce portefeuille.
- Exprimer des “vues” sur l’espérance de rendement de portefeuilles quelconques
- Utiliser ces “vues”pour modifier les espérances de rendement et la structure de covariance des actifs.

Information ex-ante

Distribution des rendements:

$$r \sim \mathcal{N}(\mu, \Sigma)$$

L'espérance de rendement μ est aussi aléatoire

$$\mu = \Pi + \epsilon^{(e)}$$

avec

$$\epsilon^{(e)} \sim \mathcal{N}(0, \tau\Sigma)$$

Optimisation inversée

On utilise le portefeuille de marché pour inférer l'espérance de rendement:

$$U(w) = w^T \Pi - \frac{\delta}{2} w^T \Sigma w$$

Solution "inversée" de Π en fonction de w:

$$\Pi = \delta \Sigma w_{eq}$$

Expression de prédictions à propos des rendements

Les prédictions sont exprimées par des portefeuilles dont on donne le rendement, avec une marge d'erreur.

$$P\mu = Q + \epsilon^{(v)}$$

avec

$$\epsilon^{(v)} \sim \mathcal{N}(0, \Omega)$$

Résumé

Deux équations pour μ

- Distribution ex-ante
$$\mu = \Pi + \epsilon^{(e)}$$
- Views
$$P\mu = Q + \epsilon^{(v)}$$

Exemple

```
##       IBM                    MS                 DELL
##  Min.   :-0.445480   Min.   :-0.53590   Min.   :-0.515656
##  1st Qu.:-0.060482   1st Qu.:-0.06699   1st Qu.:-0.086565
##  Median : 0.009032   Median : 0.02846   Median : 0.008809
##  Mean   : 0.006868   Mean   : 0.01264   Mean   : 0.002769
##  3rd Qu.: 0.070162   3rd Qu.: 0.10020   3rd Qu.: 0.079835
##  Max.   : 0.353799   Max.   : 0.50707   Max.   : 0.497706
##        C                    JPM                 BAC
##  Min.   :-0.3400743   Min.   :-0.444608   Min.   :-0.278997
##  1st Qu.:-0.0572979   1st Qu.:-0.076672   1st Qu.:-0.050389
##  Median : 0.0009806   Median : 0.013887   Median : 0.010103
##  Mean   : 0.0056924   Mean   :-0.003876   Mean   : 0.008242
##  3rd Qu.: 0.0539650   3rd Qu.: 0.082539   3rd Qu.: 0.065332
##  Max.   : 0.2533333   Max.   : 0.317181   Max.   : 0.173060
```

Correlation

	IBM	MS	DELL	C	JPM	BAC
IBM	1.0000000	0.3873395	0.4193389	0.4635322	0.4459814	0.3585381
MS	0.3873395	1.0000000	0.3981657	0.5929457	0.5226294	0.4646464
DELL	0.4193389	0.3981657	1.0000000	0.2701329	0.2671891	0.2321042
C	0.4635322	0.5929457	0.2701329	1.0000000	0.5477972	0.5070248
JPM	0.4459814	0.5226294	0.2671891	0.5477972	1.0000000	0.6832878
BAC	0.3585381	0.4646464	0.2321042	0.5070248	0.6832878	1.0000000

Exemple 1: IBM et Dell surperforme MS (sd $= 5\%$)

Rendement de (1/2 IBM - MSFT + 1/2 DELL) $= 6\% +$ terme d'erreur

```
sd <- .02
pickMatrix <- matrix(c(1/2, -1, 1/2, rep(0, 3)),
                     nrow = 1, ncol = 6 )
views <- BLViews(P = pickMatrix, q = 0.06,
                 confidences =  1/sd,
                 assetNames = colnames(monthlyReturns))
views
```

```
## 1 : 0.5*IBM+-1*MS+0.5*DELL=0.06  + eps. Confidence: 50
```

Traduction en distribution ex-post (voir note de cours)

```
## Prior means:
##  IBM   MS DELL    C  JPM  BAC
##    0    0    0    0    0    0
## Posterior means:
##           IBM            MS          DELL             C           JPM
## -0.0013812295 -0.0067000455  0.0050518582 -0.0035930342 -0.0030823049
##           BAC
## -0.0009788253
## Posterior covariance:
##              IBM          MS          DELL           C         JPM
## IBM  0.010597965 0.010272332  0.0068788065 0.005958728 0.008483623
## MS   0.010272332 0.016208544  0.0072630697 0.008228512 0.011514430
## DELL 0.006878806 0.007263070  0.0188681770 0.002517693 0.007699031
## C    0.005958728 0.008228512  0.0025176928 0.007262452 0.008141182
## JPM  0.008483623 0.011514430  0.0076990312 0.008141182 0.017521937
## BAC  0.001944320 0.001923729 -0.0002709615 0.003124811 0.005103126
##                BAC
## IBM   0.0019443201
## MS    0.0019237294
## DELL -0.0002709615
## C     0.0031248107
## JPM   0.0051031262
## BAC   0.0062264719
```

Exemple 2: Le rendement moyen du secteur financier sera de 15% (sd = .04)

Rendement de (C + JPM + BAC + MS)/4 = 15% + terme d'erreur

```
finViews <- matrix(ncol = 4, nrow = 1, dimnames = list(NULL, c("C","JPM","BAC","MS")))
finViews[,1:4] <- rep(1/4,4)
views <- addBLViews(finViews, q=0.15, confidences=1/sd, views)
views
```

```
## 1 : 0.5*IBM+-1*MS+0.5*DELL=0.06  + eps. Confidence: 50
## 2 : 0.25*MS+0.25*C+0.25*JPM+0.25*BAC=0.15  + eps. Confidence: 50
```

Traduction en distribution ex-post (voir note de cours)

```
marketPosterior <- BLPosterior(as.matrix(monthlyReturns), views,
                               tau = 1/2,
                               marketIndex = as.matrix(sp500Returns),
                               riskFree = as.matrix(US13wTB))
marketPosterior
```

```
## Prior means:
##         IBM          MS        DELL           C         JPM         BAC
## 0.020883598 0.059548398 0.017010062 0.014492325 0.027365230 0.002829908
## Posterior means:
##        IBM         MS       DELL          C        JPM        BAC
## 0.04706734 0.06682760 0.05446292 0.03021575 0.05268582 0.01692391
## Posterior covariance:
##              IBM          MS        DELL           C         JPM         BAC
## IBM  0.021741389 0.010716133 0.013042457 0.008775076 0.011014736 0.005509895
## MS   0.010716133 0.032543053 0.016985477 0.013356160 0.015376383 0.008513377
## DELL 0.013042457 0.016985477 0.048117247 0.007639836 0.009794284 0.005328471
## C    0.008775076 0.013356160 0.007639836 0.016680082 0.011539075 0.006692420
## JPM  0.011014736 0.015376383 0.009794284 0.011539075 0.028982501 0.012174496
## BAC  0.005509895 0.008513377 0.005328471 0.006692420 0.012174496 0.011460867
```

Optimisation MV classique

Portefeuille Tangent:

```
optPorts <- optimalPortfolios.fPort(marketPosterior,
            optimizer = "tangencyPortfolio")
```

Black-Litterman (7)

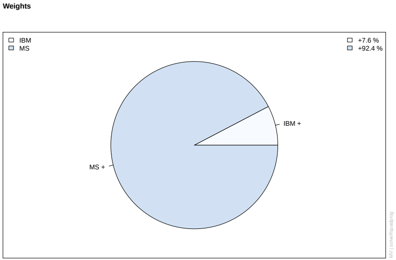

Figure 2: Prior Rdt/Risque

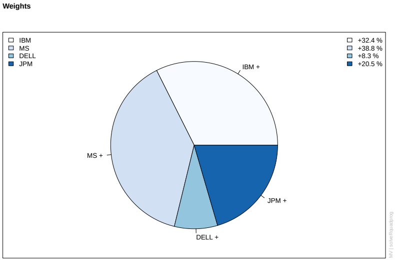

Figure 3: Posterior Rdt/Risque

Exercice

- Contraindre $w_i > 0$ en utilisant le code de la note de cours.
- BAC va surperformer Citibank (C)
- Dell aura un rendement de 0.5%