

Finance Quantitative

TP-2: Modèle de Markowitz et extensions Solution

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Données

Séries de rendement quotidien pour 11 valeurs:

```
daily.ret.file <- "../GP/data/daily.ret.rda"
load(daily.ret.file)
kable(table.Stats(daily.ret), "latex", booktabs=T) %>%
  kable_styling(latex_options=c("scale_down", "HOLD_position"))
```

	AAPL	AMZN	MSFT	F	SPY	QQQ	XOM	MMM	HD	PG	KO
Observations	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000
NAs	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Minimum	-0.1792	-0.1278	-0.1171	-0.2500	-0.0984	-0.0896	-0.1395	-0.1295	-0.0822	-0.0790	-0.0867
Quartile 1	-0.0077	-0.0094	-0.0073	-0.0103	-0.0038	-0.0047	-0.0068	-0.0055	-0.0067	-0.0046	-0.0047
Median	0.0010	0.0008	0.0005	0.0000	0.0006	0.0010	0.0001	0.0008	0.0006	0.0004	0.0007
Arithmetic Mean	0.0012	0.0015	0.0008	0.0005	0.0004	0.0006	0.0001	0.0004	0.0008	0.0004	0.0005
Geometric Mean	0.0010	0.0012	0.0006	0.0001	0.0003	0.0005	0.0000	0.0003	0.0006	0.0003	0.0004
Quartile 3	0.0112	0.0123	0.0088	0.0106	0.0056	0.0070	0.0073	0.0070	0.0082	0.0055	0.0059
Maximum	0.1390	0.2695	0.1860	0.2952	0.1452	0.1216	0.1719	0.0988	0.1407	0.1021	0.1388
SE Mean	0.0003	0.0004	0.0003	0.0005	0.0002	0.0002	0.0003	0.0002	0.0003	0.0002	0.0002
LCL Mean (0.95)	0.0005	0.0006	0.0002	-0.0005	0.0000	0.0002	-0.0004	-0.0001	0.0002	0.0000	0.0001
UCL Mean (0.95)	0.0019	0.0023	0.0013	0.0014	0.0008	0.0011	0.0006	0.0009	0.0013	0.0007	0.0009
Variance	0.0004	0.0006	0.0003	0.0007	0.0001	0.0002	0.0002	0.0002	0.0003	0.0001	0.0001
Stdev	0.0196	0.0243	0.0170	0.0266	0.0121	0.0130	0.0150	0.0140	0.0162	0.0109	0.0113
Skewness	-0.2151	1.4889	0.4319	0.7627	0.1379	-0.0084	0.4199	-0.3815	0.5114	0.0555	0.5004
Kurtosis	6.2706	16.8872	10.2176	20.9458	15.2824	7.3976	15.4203	7.3856	6.4641	8.1017	14.3236

Rendement moyen:

```
kable(colMeans(daily.ret)*100, "latex", booktabs=T, col.names="Rendement",
      caption="rendement moyen journalier (\\%)") %>%
  kable_styling(latex_options=c("HOLD_position"))
```

Table 1: rendement moyen journalier (%)

	Rendement
AAPL	0.1198361
AMZN	0.1478082
MSFT	0.0755795
F	0.0452383
SPY	0.0393649
QQQ	0.0605770
XOM	0.0137287
MMM	0.0394686
HD	0.0762375
PG	0.0370556
KO	0.0494659

Matrice de covariance des rendements:

```
kable(cov(daily.ret), "latex", booktabs=T) %>%
kable_styling(latex_options=c("scale_down", "HOLD_position"))
```

	AAPL	AMZN	MSFT	F	SPY	QQQ	XOM	MMM	HD	PG	KO
AAPL	0.0003859	0.0002184	0.0001640	0.0001929	0.0001464	0.0001919	0.0001189	0.0001229	0.0001343	0.0000677	0.0000701
AMZN	0.0002184	0.0005901	0.0002075	0.0002132	0.0001661	0.0002080	0.0001406	0.0001401	0.0001711	0.0000704	0.0000836
MSFT	0.0001640	0.0002075	0.0002881	0.0001778	0.0001464	0.0001690	0.0001342	0.0001266	0.0001351	0.0000808	0.0000874
F	0.0001929	0.0002132	0.0001778	0.0007084	0.0001821	0.0001829	0.0001457	0.0001619	0.0001983	0.0000860	0.0000936
SPY	0.0001464	0.0001661	0.0001464	0.0001821	0.0001472	0.0001454	0.0001400	0.0001278	0.0001397	0.0000820	0.0000825
QQQ	0.0001919	0.0002080	0.0001690	0.0001829	0.0001454	0.0001700	0.0001251	0.0001251	0.0001389	0.0000736	0.0000764
XOM	0.0001189	0.0001406	0.0001342	0.0001457	0.0001400	0.0001251	0.0002247	0.0001258	0.0001142	0.0000858	0.0000822
MMM	0.0001229	0.0001401	0.0001266	0.0001619	0.0001278	0.0001251	0.0001258	0.0001952	0.0001248	0.0000762	0.0000747
HD	0.0001343	0.0001711	0.0001351	0.0001983	0.0001397	0.0001389	0.0001142	0.0001248	0.0002611	0.0000797	0.0000799
PG	0.0000677	0.0000704	0.0000808	0.0000860	0.0000820	0.0000736	0.0000858	0.0000762	0.0000797	0.0001195	0.0000702
KO	0.0000701	0.0000836	0.0000874	0.0000936	0.0000825	0.0000764	0.0000822	0.0000747	0.0000799	0.0000702	0.0001272

Droite de Marché des Capitaux (Capital Market Line)

- A partir des calculs présentés en cours, concevoir une méthode numérique pour déterminer le portefeuille tangent quand les poids des actifs risqués sont contraints à être positifs: $w_i \geq 0$.
- Même calcul en ajoutant des contraintes supplémentaires qui vous semblent pertinentes (ex: pas plus de 20% de l'actif risqué alloué à un seul titre, etc.)

On calcule le portefeuille optimal pour un rendement espéré quelconque, puis on normalise les poids des actifs risqués pour obtenir le portefeuille tangent.

Le taux sans risque est fixé à 1% par an.

```
mu <- colMeans(daily.ret) * 252
n <- length(mu)

mu.free <- .01
```

```

Sigma <- cov(daily.ret) * 252
# rendement espéré arbitraire
mu.star <- .07
A.sum <- matrix(mu-mu.free, ncol=1)
A.mat <- cbind(A.sum,
               diag(n))
b <- c(mu.star-mu.free, rep(0, n))
qp <- solve.QP(2*Sigma, rep(0,n), A.mat, b, meq=1)

# tangency portfolio
w.tangent <- round(matrix(qp$solution / sum(qp$solution), ncol=1),3)
names(w.tangent) <- names(mu)
sigma.tangent <- sqrt(t(w.tangent) %*% Sigma %*% w.tangent)

# utilitaire pour afficher les résultats
kable.portfolio <- function(w.opt, mu.opt, sigma.opt, title) {
w <- data.frame(w.opt, row.names = names(w.opt))
names(w) = "allocation"

x <- matrix(c(mu.opt, sigma.opt), ncol=1)
rownames(x) <- c("return", "stdev")
x = data.frame(x)
names(x) = " "

kable(list(w, x), "latex", booktabs=T, caption=title)
}

```

Table 2: Port. Tangent

allocation			
AAPL	0.349	return	0.2643473
AMZN	0.287	stdev	0.2243493
MSFT	0.000		
F	0.000		
SPY	0.000		
QQQ	0.000		
XOM	0.000		
MMM	0.000		
HD	0.099		
PG	0.000		
KO	0.265		

Pour imposer la contrainte $w_i < .2$ sur les poids normalisés, on exprime cette contrainte par

$$w_i \leq .2 \sum_{k=1}^n w_k, \quad i = 1, \dots, n$$

```

max.weight <- .2
A.weight <- matrix(max.weight, nrow=n, ncol=n)
diag(A.weight) <- max.weight-1

```

```

A.mat <- cbind(A.sum,
              A.weight,
              diag(n))
b <- c(mu.star-mu.free, rep(0,n), rep(0, n))
qp <- solve.QP(2*Sigma, rep(0,n), A.mat, b, meq=1)

# tangency portfolio
w.tangent.2 <- round(matrix(qp$solution / sum(qp$solution), ncol=1),3)
names(w.tangent.2) <- names(mu)
sigma.tangent.2 <- sqrt(t(w.tangent.2) %*% Sigma %*% w.tangent.2)

```

Table 3: Port. Tangent < 20%

allocation			
AAPL	0.200	return	0.2229422
AMZN	0.200	stdev	0.1947838
MSFT	0.062		
F	0.000		
SPY	0.000		
QQQ	0.000		
XOM	0.000		
MMM	0.000		
HD	0.200		
PG	0.138		
KO	0.200		