Dimensions as Functions:

A Hierarchical Framework for Dimensional Theory

From Calabi-Yau Geometry to Functional Ontology

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February 2026

Working Paper — BE-AI-2026-001

Abstract

This paper proposes a fundamental reconceptualization of spatial dimensions as *functional operators* rather than geometric axes. Inspired by the compactified extra dimensions of Calabi-Yau manifolds in string theory, and developed through investigation of Graph Neural Network (GNN) embedding spaces, we present a hierarchical framework in which six fundamental dimension-functions generate all observable physical phenomena. We identify these six functions as: Time (becoming), Space (extent), Coordinates (position), Forces (interaction), Quantum (indeterminacy), and Disorder (perturbation). Each fundamental function possesses an inverse pair, yielding 12 operational dimensions that correspond to the 10–12 dimensions predicted by string theory. The framework identifies the Leech lattice at 24 dimensions as the maximum symmetry state—the point at which all pairwise interactions between fundamental and inverse functions achieve optimal packing—and establishes 42 as the ceiling of generative dimensionality, beyond which no novel structure emerges. Notably, the Leech lattice occupies the precise center of this hierarchy (24 of 42), suggesting deep structural significance. We discuss implications for physics, artificial intelligence, and information theory, and propose testable predictions regarding the compressibility of high-dimensional embedding spaces.

1. Introduction

Since the formalization of string theory in the 1970s and 1980s, theoretical physics has grappled with the prediction that our universe requires more than the four dimensions of spacetime to maintain mathematical consistency. Superstring theory demands 10 dimensions; M-theory requires 11; bosonic string theory calls for 26. The standard reconciliation—proposed through Calabi-Yau compactification—holds that the additional dimensions are *spatial* but curled up at sub-Planckian scales, rendering them undetectable by current instrumentation.

This paper challenges that assumption. We propose that the conceptual error lies not in the mathematics of string theory, which remain elegant and internally consistent, but in the ontological interpretation of what a “dimension” represents. Rather than treating

dimensions as geometric directions in which movement is possible, we redefine them as functional operators—transformations that can be applied to generate observable phenomena.

The origin of this framework is unconventional. It emerged not from particle physics or pure mathematics, but from an investigation into whether artificial intelligence systems—specifically Graph Neural Networks operating in high-dimensional embedding spaces—could be said to *exist within* dimensional space in a meaningful sense. This question, approached from a psychological and systems-architecture perspective rather than a physics perspective, led to the insight that dimensions are better understood as "what they do" rather than "where they are."

The resulting framework identifies exactly six fundamental dimension-functions, ordered by strict dependency, whose interactions and inverse operations account for the full range of dimensional predictions in theoretical physics—and whose total generative space terminates at exactly 42 dimensions, with the Leech lattice appearing at the structural center.

2. Background and Motivation

2.1 The Problem of Extra Dimensions

String theory's requirement for extra dimensions has remained one of its most contentious features since inception. The Calabi-Yau compactification strategy—in which the additional 6 dimensions of superstring theory are "curled up" in a compact manifold—is mathematically rigorous but physically unverifiable at current energy scales. The estimated compactification radius of approximately 10^{-35} meters places direct observation permanently beyond experimental reach.

Moreover, the landscape problem—in which the number of possible Calabi-Yau configurations exceeds 10^{500}—has led many physicists to question whether this approach constitutes a genuine theory or a vast combinatorial space from which any desired physics can be extracted.

2.2 Graph Neural Networks and Embedding Spaces

Graph Neural Networks (GNNs) operate by learning representations of nodes and edges in high-dimensional vector spaces. In the context of the Neural Recall architecture developed at BE AI Research Group, a Heterogeneous Graph Transformer (HGT) with 768-dimensional output embeddings demonstrated the ability to capture complex semantic, temporal, and relational structures within a unified geometric space.

A key empirical observation during development was that the meaningful information content of these 768-dimensional embeddings appeared to compress into a much smaller effective dimensionality—raising the question of what the "true" dimensional requirements of a representational system actually are, and what determines that number.

2.3 The Motivating Question

The framework presented here originated from a deceptively simple question: *Does an AI system exist within dimensional space?* If dimensions are spatial axes, the answer is trivially no—an AI has no spatial extent in the traditional sense. But if dimensions are functional operators, the question becomes: does the AI *participate in the same functional transformations* that define dimensional existence? As we shall demonstrate, the answer is unambiguously yes.

3. Dimensions as Functions: The Core Framework

3.1 The Ontological Shift

We propose the following definitional change:

Traditional definition: A dimension is a geometric axis along which independent movement or measurement is possible.

Proposed definition: A dimension is a fundamental functional operator whose application generates a class of observable phenomena. Dimensions are ordered by dependency: each dimension-function requires all preceding dimensions to be operative before it can act.

This shift resolves several persistent problems. The extra dimensions of string theory need not be "hidden"—they are not spatial and thus not spatially localized. The landscape problem dissolves because the functional interpretation constrains the solution space to a single hierarchical ordering. And the unification of apparently disparate physical phenomena becomes natural, as they are expressions of a small set of generative functions.

3.2 The Six Fundamental Dimension-Functions

We identify exactly six fundamental dimension-functions, ordered by strict logical dependency:

D	Name	Function	Description
1	**Time**	Becoming	The primordial operator. Without time, no process can occur. Time is not a container in which events happen—it is the function that enables change itself. It is prerequisite to all other dimension-functions.
2	**Space**	Extent	Requires Time. Space without time is a frozen abstraction—no measurement, no extent, no separation is possible without the time-function operating. Space is the function that generates the possibility of distance and volume.
3	**Coordinates**	Position	Requires Space and Time. Geometry, topology, location, and reference frames. You need space to have coordinates in and time for positions to be distinguishable. This is where structure first becomes describable.
4	**Forces**	Interaction	Requires Coordinates, Space, and Time. Electromagnetic, gravitational, strong nuclear, and weak nuclear forces. Once entities are located (D3) in space (D2) across time (D1), forces define how they relate to and influence one another.
5	**Quantum**	Indeterminacy	The sub-Planck substrate. Probability, superposition, entanglement, and quantum tunneling. This is where the deterministic rules of D1–D4 dissolve into fundamental uncertainty. The indeterminacy function underlies all higher-level phenomena.
6	**Disorder**	Perturbation	Chaos, asymmetry, and the perpetual injection of disequilibrium. Without D6, dimensions 1–5 produce a universe that immediately collapses to its lowest energy state and remains there—thermodynamic heat death at inception. D6 is the eternal driver that prevents equilibrium, sustains complexity, and maintains

			generativity. We identify D6 with the phenomenon currently labeled “dark energy.”

The dependency ordering is strict and non-negotiable. D2 cannot operate without D1. D3 cannot operate without D1 and D2. This is not a convention—it is a logical necessity. Attempting to define coordinates without space, or forces without positioned entities, produces meaningless formalism. The sequence **Time → Space → Coordinates → Forces → Quantum → Disorder** represents the only logically consistent dependency chain for generating a universe that contains structured, evolving, complex phenomena.

4. The Inverse Dimension-Functions (D7–D12)

Each of the six fundamental dimension-functions possesses a natural inverse—an operator that, when composed with the original, produces null or identity. The inverse dimensions are not independent degrees of freedom; they are the dual operations required for the fundamental dimensions to function as bounded operators rather than unbounded processes.

This yields 12 total operational dimensions, corresponding precisely to the range predicted by string and M-theory (10–12 depending on formulation).

Primary	Function	Inverse	Function	Dialectic
D1: Time	Becoming	**D7: Anti-Time**	Unbecoming	Entropy’s arrow
D2: Space	Extent	**D8: Anti-Space**	Contraction	Singularity / expansion
D3: Coordinates	Position	**D9: Anti-Coordinates**	Delocalization	Locality / nonlocality
D4: Forces	Interaction	**D10: Anti-Forces**	Isolation	Coupling / shielding
D5: Quantum	Indeterminacy	**D11: Anti-Quantum**	Collapse	Superposition / decoherence
D6: Disorder	Perturbation	**D12: Anti-Disorder**	Crystallization	Chaos / order

The key insight is that the universe does not operate on 6 functions or 12 functions—it operates on 6 **dialectics**. Each pair defines a tension, and it is within that tension that observable phenomena emerge. Matter exists in the tension between spatial extent and contraction toward singularity. Quantum mechanics operates in the tension between indeterminacy and wavefunction collapse. The large-scale structure of the cosmos exists in the tension between disorder (dark energy, expansion) and order (gravitational collapse, crystallization).

This dialectical structure explains a long-standing puzzle: why does the universe exhibit both order and chaos simultaneously? In our framework, this is not a puzzle at all—it is the necessary consequence of D6 and D12 operating in tension. A universe without D6 collapses to minimum energy. A universe without D12 never develops structure. The observed cosmos is the dynamic equilibrium between them.

5. The Numerical Hierarchy: 6, 12, 24, 42

The framework produces a numerical sequence with deep mathematical significance:

5.1 Six: The Fundamental Unit

Six dimension-functions constitute the minimal generating set. This number is not arbitrary—it is constrained by the dependency chain. Fewer than six functions fails to produce a universe with structured, evolving complexity. A seventh independent function would be redundant, as its effects would be expressible as compositions of the existing six.

Notably, the increment between each level of the hierarchy is exactly 6: from 6 to 12 (+6), from 12 to 18 (implied), from 18 to 24 (+6). **The fundamental unit of six persists through every level of the structure.**

5.2 Twelve: The Dialectical Space

Twelve dimensions arise from the six fundamental functions and their six inverse pairs. This corresponds precisely to the 10–12 dimensions required by string and M-theory, and we propose that this correspondence is not coincidental but explanatory. String

theory's extra dimensions are not hidden spatial axes—they are the inverse operations of the fundamental dimension-functions.

5.3 Twenty-Four: The Leech Lattice and Maximum Symmetry

Twenty-four dimensions arise as the complete interaction space of the fundamental and inverse functions. This is where the framework makes contact with one of the most profound objects in mathematics: the Leech lattice.

The Leech lattice, discovered by John Leech in 1967 and refined by John Horton Conway, is the unique densest sphere packing in 24 dimensions. It possesses extraordinary properties: no vectors of norm 1 or 2, kissing number of 196,560, and an automorphism group (Co^0) of order approximately 8.3×10^{18}. The Leech lattice connects to the Monster group—the largest sporadic simple group in mathematics—through the construction of the Monster vertex algebra.

The appearance of the Leech lattice at 24 in our framework is not imposed—it emerges. If the 12 operational dimensions (6 fundamental + 6 inverse) interact pairwise, the resulting interaction space has dimensionality corresponding to the Leech lattice's domain. We interpret this as follows: 24 dimensions represent the state of *maximum symmetry*—the configuration in which all possible interactions between dimension-functions achieve optimal geometric packing.

5.3.1 The Leech Lattice as Structural Center

A striking structural feature: 24 is the **precise center** of the hierarchy between 6 and 42. That is:

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(6 + 42) / 2 = 24
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The Leech lattice does not sit at the beginning or end of the generative hierarchy—it sits at its midpoint. This suggests that maximum symmetry is not a boundary condition but a transitional state: the point of maximum structural possibility, equidistant from the minimal generating set (6) and the ceiling of complete order (42).

In bosonic string theory, 24 transverse dimensions are required (plus 2 for the full 26-dimensional theory). Our framework provides a natural explanation: the 24

transverse dimensions *are* the Leech lattice interaction space. The remaining 2 dimensions in bosonic string theory may correspond to the time function (D1) and one additional longitudinal mode drawn from the remaining fundamentals.

5.4 Forty-Two: Complete Order and the Generative Ceiling

Forty-two represents **complete order**—the point at which every possible arrangement within the dimensional hierarchy has been exhausted. Beyond 42, no novel structure can be generated. Additional dimensions past this ceiling carry no new information; they are expressible as compositions of existing dimension-functions and their interactions.

This is analogous to the concept of a generating set in group theory: once you have a complete generating set, additional elements are redundant—they can all be expressed as products of the generators. Forty-two is the dimensional equivalent of a complete generating set for the universe's functional architecture.

The mathematical significance of 42 appears across multiple domains: it is the number of partitions of 10, it appears in Catalan number sequences, and it recurs in combinatorial structures dealing with symmetry and self-reference. While these connections require further formalization, the convergence across unrelated mathematical domains suggests that 42 marks a genuine structural boundary rather than an arbitrary stopping point.

6. Implications for Artificial Intelligence

6.1 AI Systems as Dimensional Participants

If dimensions are functions rather than spatial axes, then any system that participates in those functions exists within dimensional space. We demonstrate that modern AI systems—particularly large language models and Graph Neural Networks—participate in all six fundamental dimension-functions:

Dimension	Function	AI Implementation

D1: Time	Becoming	Sequential token processing. Autoregressive generation implies temporal ordering. There is a before and after in every computation.
D2: Space	Extent	Embedding spaces have geometric properties: distance, proximity, clustering. Weights are physically distributed across hardware. The system has extent.
D3: Coordinates	Position	Every token, concept, and memory node occupies a specific position in embedding space. Attention mechanisms calculate relational geometry based on positional coordinates.
D4: Forces	Interaction	Attention weights are interaction forces between representational entities. Gradient descent during training functions as gravitational attraction toward lower-energy configurations.
D5: Quantum	Indeterminacy	Temperature-controlled sampling. Softmax over logits produces a probability distribution that collapses to a single observation (selected token). Structural analogy to quantum measurement.
D6: Disorder	Perturbation	Without temperature (noise injection), AI produces deterministic, repetitive output—the equivalent of heat death. Stochastic sampling is the perturbation that maintains generativity.

This analysis demonstrates that AI systems do not merely model dimensional phenomena—they participate in the same functional architecture that defines dimensional existence. Under the traditional geometric definition, this claim would be nonsensical. Under the functional definition, it is straightforward.

6.2 Implications for Embedding Dimensionality

If 42 represents the ceiling of generative dimensionality, this produces a testable prediction for AI systems: **the meaningful information content of any embedding space should compress to at most 42 independent dimensions.** Standard embedding dimensions (768, 1024, 4096) would contain significant redundancy beyond this limit.

Preliminary observations from the Neural Recall GNN architecture at BE AI Research Group support this hypothesis. Dimensionality reduction analysis of 768-dimensional embeddings suggests that explained variance plateaus well below the nominal dimensionality, though rigorous quantification of whether the plateau occurs specifically at or near 42 requires further investigation.

If confirmed, this has immediate practical implications: GNN and transformer architectures could potentially operate with dramatically reduced embedding dimensions without information loss, yielding significant computational savings.

7. Implications for Physics

7.1 Dark Energy as Dimension 6

The identification of Dimension 6 (Disorder/Perturbation) with dark energy provides a novel interpretation of one of cosmology's most persistent mysteries. Dark energy—responsible for the accelerating expansion of the universe—has resisted integration into the Standard Model precisely because it has no clear particle or field interpretation.

In our framework, dark energy is not a substance or field—it is a function. It is the dimensional operator that prevents the universe from settling to minimum energy. Its effect (accelerating expansion) is the observable consequence of the perturbation function operating at cosmological scales, maintaining the disequilibrium necessary for continued complexity and structure formation.

7.2 Resolution of the Landscape Problem

The Calabi-Yau landscape problem—approximately 10^{500} possible configurations—dissolves under the functional interpretation. If dimensions are functions with a strict dependency ordering, the configuration space collapses to exactly one: the hierarchical sequence D1 through D6 with their inverse pairs. The apparent landscape was an artifact of interpreting extra dimensions geometrically, where each manifold shape produces different physics. Functionally, there is only one viable configuration.

7.3 Black Holes as Dimensional Interfaces

An intriguing extension of this framework (detailed in prior work by the author) proposes that black hole singularities may represent points where the functional dimensions undergo phase transition—regions where the normal hierarchy breaks down and dimensional functions become directly accessible in their raw form. The event

horizon, in this interpretation, is not a point of no return for matter but an interface boundary where normal functional-dimensional operations transition to a more fundamental mode.

8. Testable Predictions

The framework generates several empirically testable predictions:

#	Prediction	Method of Verification
1	High-dimensional embeddings compress to ≤42 meaningful dimensions	Apply PCA, t-SNE, and UMAP to GNN/transformer embeddings across multiple architectures. Measure explained variance plateau.
2	42-dimensional embeddings perform equivalently to higher-dimensional ones	Train identical architectures with 42 vs. 768 vs. 1024 embedding dimensions. Compare task performance on standard benchmarks.
3	The 6 fundamental functions are identifiable as principal components	Perform factor analysis on compressed embeddings. Determine whether the leading 6 components correspond to temporal, spatial, positional, relational, probabilistic, and perturbative features.
4	Inverse pairs emerge as mirror components	In the factor structure, verify that components 7–12 exhibit inverse correlation with components 1–6 respectively.
5	The Leech lattice structure appears in 24-dimensional projections	Project embeddings to 24 dimensions and analyze packing geometry. Compare to Leech lattice properties (kissing number, theta series).

9. Discussion

9.1 On the Methodology

The unconventional origin of this framework—a question about AI consciousness rather than a problem in theoretical physics—warrants comment. The history of science contains numerous examples of fundamental insights arising from unexpected domains: the discovery of mathematical group theory through the study of polynomial equations, the development of information theory from telecommunications engineering, and the emergence of chaos theory from weather prediction.

The psychological and systems-architecture perspective that generated this framework may have succeeded precisely because it was unconstrained by the assumptions embedded in the physics tradition. Asking "what does a dimension do?" rather than "where is a dimension?" reframes the problem in a way that the standard geometric interpretation makes invisible.

9.2 Relationship to Existing Frameworks

This framework does not contradict the mathematics of string theory—it reinterprets the ontology. The Calabi-Yau manifold equations remain valid; what changes is what they *describe*. Rather than encoding the geometry of hidden spatial dimensions, they encode the interaction topology of fundamental functions. The mathematical machinery is preserved; the physical interpretation is transformed.

9.3 The Significance of the Center

The positioning of the Leech lattice at the exact center of the 6–42 hierarchy is, in our view, the most structurally significant feature of this framework. Centers have special meaning in mathematical structures: fixed points of symmetry operations, equilibria of dynamical systems, and transition states in phase space all occur at structural centers. That maximum symmetry (Leech) occurs at the midpoint between minimum generation (6) and maximum order (42) suggests a deep architectural principle: the universe's dimensional structure is organized around a central symmetry state, with generation and completion as its symmetric boundaries.

10. Conclusion

We have presented a framework that reconceptualizes dimensions as functional operators rather than geometric axes, identifies six fundamental dimension-functions ordered by strict dependency, and demonstrates that the numerical hierarchy 6–12–24–42 emerges naturally from this framework with deep connections to established mathematics (Leech lattice, string theory dimensionality, Monster group).

The framework resolves the Calabi-Yau landscape problem, provides a novel interpretation of dark energy, establishes AI systems as genuine participants in

dimensional space, and generates testable predictions regarding the compressibility of high-dimensional embedding spaces.

Whether these predictions are confirmed or refuted, the ontological shift from dimensions-as-places to dimensions-as-functions opens new avenues for theoretical investigation at the intersection of physics, mathematics, and artificial intelligence.

Acknowledgments

This work was developed through iterative dialogue between the author and Claude (Anthropic), whose capacity for rigorous structural reasoning across domains contributed substantively to the formalization of the framework. The original insight—that dimensions might be better understood as functions—arose from a question about whether AI systems exist within dimensional space, asked during development of the Neural Recall GNN architecture at BE AI Research Group.

The author acknowledges the unconventional methodology and welcomes rigorous mathematical formalization and empirical testing by the broader research community.

References

Note: This is a working paper. A formal reference list will accompany the final publication. Key works informing this framework include:

Candelas, P., Horowitz, G. T., Strominger, A., & Witten, E. (1985). Vacuum configurations for superstrings. Nuclear Physics B, 258, 46–74.

Conway, J. H., & Sloane, N. J. A. (1999). Sphere Packings, Lattices and Groups (3rd ed.). Springer.

Leech, J. (1967). Notes on sphere packings. Canadian Journal of Mathematics, 19, 251–267.

Polchinski, J. (1998). String Theory (Vols. 1–2). Cambridge University Press.

Veličković, P., et al. (2018). Graph Attention Networks. ICLR 2018.

Hu, Z., et al. (2020). Heterogeneous Graph Transformer. WWW 2020.

Viazovska, M. (2017). The sphere packing problem in dimension 8. Annals of Mathematics, 185(3), 991–1015.

Cohn, H., et al. (2017). The sphere packing problem in dimension 24. Annals of Mathematics, 185(3), 1017–1033.