Module → III

Q ① Explain the convergence condition of Fourier series. State and explain Dirichlet's condition to Fourier series?

Ans * If f ~~st~~ satisfies a Holder codition, then its fourier series converges uniformly.

* If f is of bounded vibration, then is Fourier series converges everywhere.
* If f is of continuous, then its Fourier coefficients are absolutely summable

[* Fourier series converges uniformly]

→ Two conditions must be satisfied [along with the weak Dirichlet condition]

ⓐ In one period $f(t)$ has only a finite number of minima & maxima.

ⓑ In one period $f(t)$ has only a finit no. of discontinuities & each one is finite.

Q ② State the expression of a fourier series. Also evaluate the coefficient of fourier series?

Ans expression for a Fourier series.

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{2\pi n x}{T}\right) + b_n\sin\left(\frac{2\pi n x}{T}\right)\right).$$

where. $f(x)$ is periodic function you want to represent.

- a_0 is the average value of the function over one period.
- a_n & b_n are fourier coefficient.

$$a_n = \frac{2}{T}\int_0^T f(x)\cos\left(\frac{2\pi n x}{T}\right)dx$$

$$b_n = \frac{2}{T}\int_0^T f(x)\sin\left(\frac{2\pi n x}{T}\right)dx.$$

Q(3) Derive the expression for exponential fourier series from trigonometric fourier series

Ans. Let $f(t) = e^{jw_0 t}$, where $w_0 = \frac{2\pi}{T}$

The fourier series for $f(t)$ is:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right)\right)$$

To find the coefficients a_n & b_n, we can use.

$$a_n = \frac{2}{T} \int_0^T f(t) \cos\left(\frac{2\pi n t}{T}\right) dt$$

since $f(t) = e^{jw_0 t}$, we have

$$a_n = \frac{2}{T} \int_0^T e^{jw_0 t} \cos\left(\frac{2\pi n t}{T}\right) dt$$

Similarly

$$b_n = \frac{2}{T} \int_0^T e^{jw_0 t} \sin\left(\frac{2\pi n t}{T}\right) dt$$

Q(4) Define Fourier transform pair. what are the necessary condition for existance of Fourier Transform. state the merits & demerits of Fourier transform?

Ans A tranformation technique that tranforms signals from the continuous-time domain to the corresponding frequency domain & vice-versa.

Necessary conditions for existance of Fourier Series

- A finite no. of maxima & minima in every finite interval of time
- A finite number of discontinuities in every finite interval of time
- absolutely integrable

Advantages of fourier series

- Can improve the signal-to-noise ratio (SNR)
- Easy to do computationally.

disadvantages of fourier series

- Sensitive to noise
- Limited to analyzing signals that are continuous in time.

Q (8) Find fourier transform of the lower expression:

$$x(t) = e^{-at} u(t), \quad a > 0$$

Also draw the phase spectrum & magnitude spectrum?

Ans The Lapace transform of $e^{-at} u(t)$

$$X(s) = \int_0^\infty e^{-st} e^{-at} dt$$

Simply the integral.

$$X(s) = \int_0^\infty e^{-(s+a)t} dt$$

Now, you can evaluate the integral:

$$X(s) = \left. \frac{e^{-(s+a)t}}{-(s+a)} \right|_0^\infty .$$

Since $\lim_{t \to \infty} e^{-(s+a)t} = 0$ for $s > a$, we have

$$X(s) = \frac{1}{s+a}$$

So, Laplace transform of $e^{-at} u(t)$ is $\frac{1}{s+a}$

In case the fourier transform is $\frac{1}{s-a}$, & then phase spectrum $\phi(\omega)$ is given by.

$$\phi(\omega) = \arg\left(\frac{1}{s-a}\right)$$

The argument of $\frac{1}{s-a}$ is $a\tan 2(\mathrm{Im}(1), \mathrm{Re}(s-a))$,

where
$a\tan 2 \rightarrow$ four-quadrant inverse.
$\mathrm{Im}(1) = 0$

phase spectrum is simply the angle of $(s-a)$

$$\phi(\omega) = \arg(s-a).$$

Q ⓘ Find Fourier transform of $\delta(t)$. Also draw spectrum.

(II) Find inverse Fourier transform of $\delta(\omega)$.

Ans (I) $$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt.$$

If $f(t) = \delta(t)$

$$\delta(t) = \begin{cases} 1, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases}$$

Fourier transform.

$$F(\omega) = \int_{-\infty}^{\infty} \lambda(t)\, e^{-j\omega t}\, dt.$$

$$\boxed{F(\omega) = \int_{0}^{\infty} e^{-j\omega t}\, dt}$$

(II). $$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$F(\omega) = \int_{0}^{\infty} e^{-j\omega t}\, dt$$

$$\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\int_{0}^{\infty} e^{-j\omega t}\, dt\right) e^{j\omega t}\, d\omega}$$

Module (IV)

(1) Illustrate the basic advantages of Laplace Transform over Fourier transform. Write expression for evaluating Laplace transform & its inverse?

An

Advantages of Laplace Transform

(1) **Incorporating Transient Behavior:**

useful for analyzing systems with transient behaviour. It allows you to analyze how a system responds to initial conditions & how it evolves over time. Fourier Transform is more suited for steady-state or periodic signals & systems.

(2) **Complex functions & Systems**

Laplace transform can handle complex-valued function & systems. It provides a broader view of frequency domain by including information about system stability, damping & transient response. F.T deals primarily with real value signal

(c) **Convergence for more signals** → don't satisfies Dirichlet conditions. This makes it more applicable to broader range of real-world signals

Expression for evaluating Laplace Transform & its inverse

$$\boxed{F(s) = \int_0^{\infty} e^{-st} f(t)\,dt} \xrightarrow{\text{inverse}} \boxed{f(t) = \frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty} e^{st} F(s)\,ds}$$

are fundamental tools is engineering, control theory, & various fields of science for analyzing & solving linear-time-variant system

(2) Define Region of Convergence. State various properties of ROC?

Ans Set of points in S-plane for which the Laplace transform of a function $x(t)$ converges

Property

(I) The ROC of $X[Z]$ consists of a ring in the Z-plane centered about the origin

(II) ROC does not contain any poles

(III) If $x[n]$ is of finite duration, then the ROC is the entire z-plane, except possibly $z=0$ or $z=\infty$

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(3) Prove the Scaling property of Laplace transform?

Ans Time Scaling property of Laplace transform

$$x(t) \overset{LT}{\longleftrightarrow} X(s).$$

Then

$$x(at) \overset{LT}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{s}{a}\right)$$

Proof

$$L[x(t)] = \int_0^\infty x(t) e^{-st} dt$$

If $t \to at$, then

$$L[x(at)] = \int_0^\infty x(at) e^{-st} dt$$

Substituting $at = p$ in RHS of the above equation.

$$t = \frac{p}{a} \text{ and } dt = \frac{1}{a} dp$$

$$\therefore L[x(at)] = \int_0^\infty x(p) e^{-\left(\frac{s}{a}\right)p} \frac{dp}{a}$$

$$\boxed{= L[x(at)] = \frac{1}{a} \int_0^\infty x(p) e^{-\left(\frac{s}{a}\right)p} dp = \frac{1}{a} X\left(\frac{s}{a}\right)}$$