Assignment - 1
Module - I

1) Define Signal :

⇒ signal is anything that is visible, audible, observable or measurable with the help of some machine.

Example : audio, light, radio, TV etc

2) Types of signals :

A) 1> One dimentional signal ; Example ⇒ speech, audio

2> Multidimentional signal ; Example ⇒ Image

2-D signal

3-D signal

B) One channel & Multichannel signals : Example : ECG

C) Continuous and Discrete Signals : ~~Example~~

D) Analog & Digital Signals

E) Real & Complex signals :

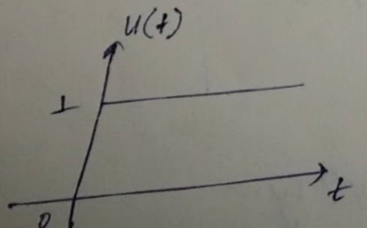
F) Even & Odd Signals

G) Deterministic & Random signals

3) What does following signal signifies .

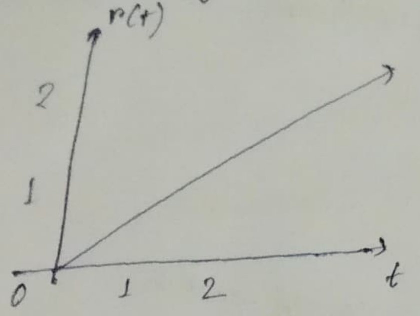
(i) Step signal :

Unit step signal is denoted by $u(t)$. It is defined as $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$



(ii) Ramp signal:

Ramp signal is denoted by $r(t)$. and it is defined as $r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$

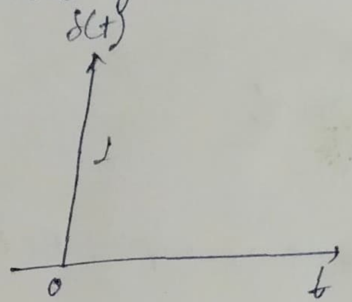


$\int u(t) = \int 1 = t = r(t)$

$u(t) = \frac{dr(t)}{dt}$

(iii) Impulse Signal:

Impulse signal is denoted by $\delta(t)$. and It is defined as $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$

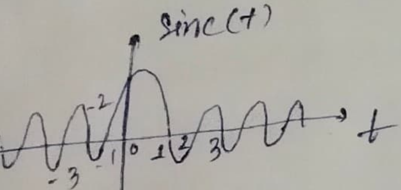


$\int_{-\infty}^{\infty} \delta(t)\, dt = u(t)$

$\delta(t) = \frac{du(t)}{dt}$

(iv) sinc signal:

It is denoted as sinc(t) and it is defind as sinc



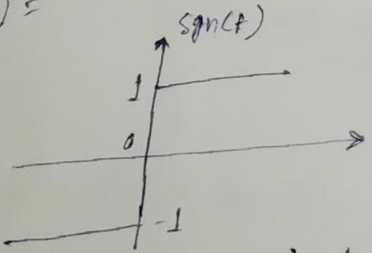
$(t) = \frac{\sin \pi t}{\pi t}$

$= 0$ for $t = \pm 1, \pm 2, \pm 3$

(v) Signum signal:

Signum function is denoted as sgn(t).

It is defined as $sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$



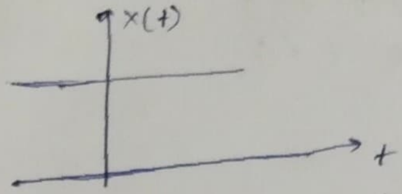
$sgn(t) = 2u(t) - 1$

(vi) Exponential signal:

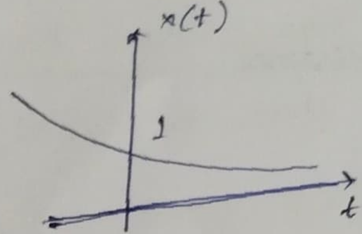
It is in the form of $x(t) = e^{\alpha t}$

The shape of exponential can be defind by $\alpha$

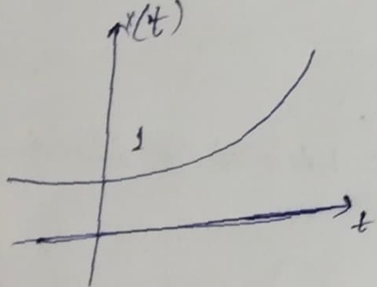
Case i : if $\alpha = 0 \rightarrow x(t) = e^{0} = 1$




Case ii : if $\alpha < 0$ i.e -ve then $x(t) = e^{-\alpha t}$. The shape is called decaying exponential.



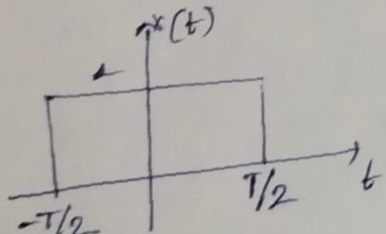

Case iii : if $\alpha > 0$ i.e +ve then $x(t) = e^{\alpha t}$. The shape is called raising exponential



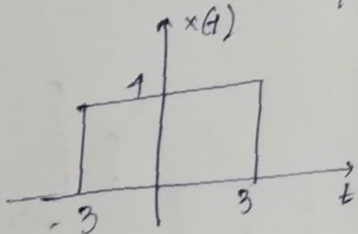

(vii) Rectangular Signal:

let it be denoted as x(t) and it is defined as

$x(t) = A\ \text{rect}\left[\frac{r}{t}\right]$



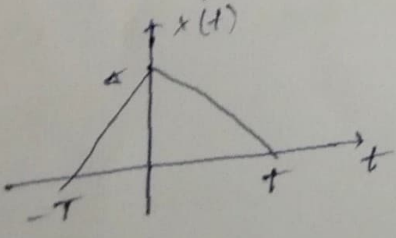

$4\ \text{rect}\left[\frac{r}{6}\right]$



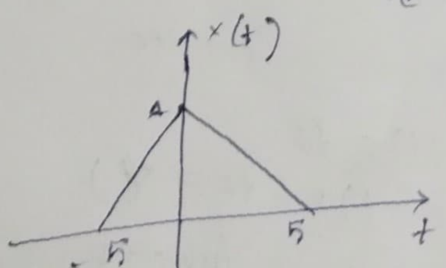

(viii) Triangular Signal:

let it be denoted as x(t)

$x(t) = A\left[1 - \frac{|t|}{T}\right]$

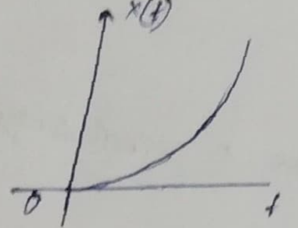



$x(t) = 4\left[1 - \frac{|t|}{5}\right]$



## (ix) Parabolic Signal :

Parabolic signal can be defined as $x(t) = \begin{cases} t^2/2 & t \geq 0 \\ 0 & t < 0 \end{cases}$




$$\int\int u(t)dt = \int r(t)\,dt = \int t\,dt = \frac{t^2}{2} = \text{parabolic signal}$$

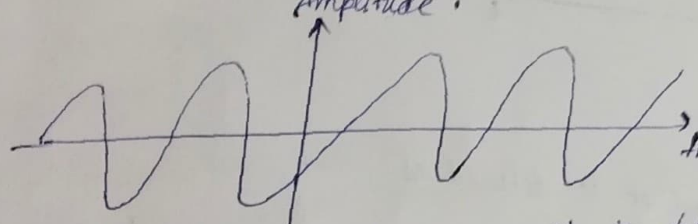
$$\Rightarrow u(t) = \frac{d^2x(t)}{dt^2}$$

$$\Rightarrow r(t) = \frac{dx(t)}{dt}$$

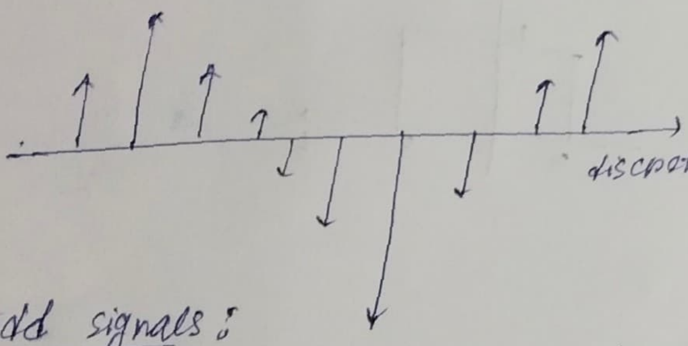
## 5) Following signals defination

### (i) Continuous & Discrete Time Signals :

A signal is said to be continuous when it is defined for all instants of time.




A signal is said to be discrete when it is defined at only discrete instants of time.




### Even & odd signals :

A signal is said to be even when it satisfies the condition $x(t) = x(-t)$

Example: $t^2, t^4 \ldots \cos t$ etc

let $x(t) = t^2$

$x(-t) = (-t)^2 = t^2 = x(t)$

$\therefore t^2$ is even function

A signal is said to be odd when it satisfies the condition $x(t) = -x(-t)$

Example: $t, t^3$ ... And $\sin t$

let $x(t) = \sin t$

$x(-t) = \sin(-t) = -\sin t = -x(t)$

$\therefore \sin t$ is odd function

(iii) Energy and Power Signals

A signal is said to be energy signal when it has finite energy

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

A signal is said to be power signal when it has finite power

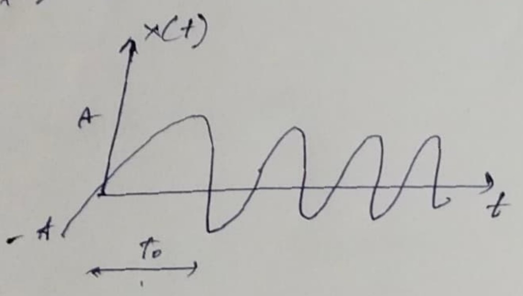
$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$

(iv) Periodic and Aperiodic signals:

A signal is said to be periodic if it satisfies the condition $x(t) = x(t+T)$ or $x(n) = x(n+N)$

$\therefore T =$ fundamental time period,

$1/T = f =$ fundamental frequency



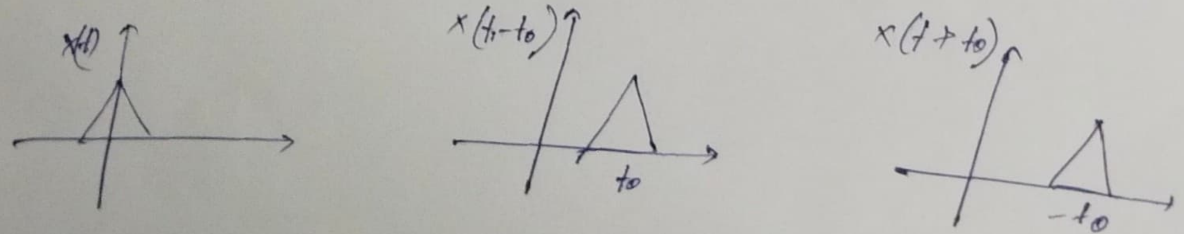

This signal will repeat for every time interval To hence it is periodic with Period To

Time shifting:

(i) $x(t \pm t_0)$ is time shifted version of signal $x(t)$

$x(t + t_0) \rightarrow$ negative shift

$x(t - t_0) \rightarrow$ positive shift

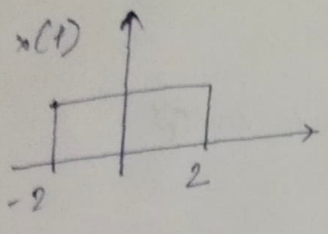


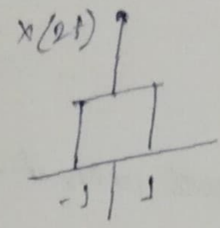
## Time Scaling:

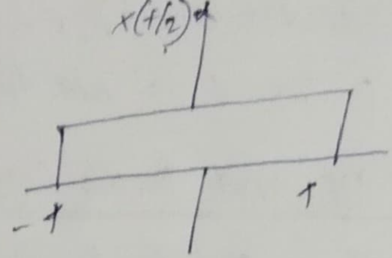
$x(At)$ is time scaled version of the signal $x(t)$. where $A$ is always positive

$|A| > 1 \rightarrow$ compression of the signal

$|A| < 1 \rightarrow$ Expansion of the signal

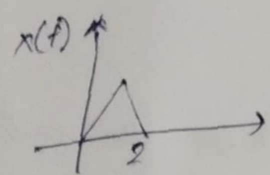


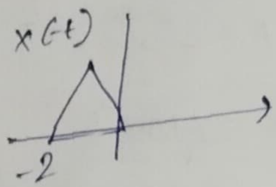




## Time Reversal:

$x(-t)$ is the time reversal of the signal $x(t)$



9) Power & Energy determination: Energy $= E_\infty$ , Power $= P_\infty$

(i) $x(t) = e^{-2t} u(t)$

$E_\infty = \int_0^\infty e^{-4t} dt = \frac{1}{4}$. $P_\infty = 0$, because $E_\infty < \infty$.

(ii) $x(t) = e^{j(2t + \pi/4)}$

$|x(t)| = 1$

Therefore, $E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} dt = \infty$

$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} dt = \lim_{T \to \infty} 1 = 1$

(iii) $x(n) = \left(\frac{1}{4}\right)^n u[n]$, $[x[n]]^2 = \left(\frac{1}{16}\right)^n u[n]$

therefore,

$E_\infty = \sum_{n=-\infty}^{\infty} [x[n]]^2 = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \frac{16}{15}$

$P_\infty = 0$, because $E_\infty < \infty$

(v) $x[n] = \cos\left(\frac{\pi}{3} n\right)$ therefore

$E_\infty = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \sum_{n=-\infty}^{\infty} \cos^2\left(\frac{\pi}{4} n\right) = \infty$

$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} [x_3[n]]^2$

$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4} n\right) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{\left(1 + \cos\left(\frac{\pi}{2} n\right)\right)}{2}$

$= \frac{1}{2}$

# Module-II

1. System: A group of components or subsystems that integrate & function together in order to acheive specific goal.

Ex: A disk subsystem is a component/part of a computer system

2) and 3)

## Basic types of systems:

(i) Linear or non linear system:

A system is linear if it satisfies the following property, where signals $x_1(t)$ and $x_2(t)$ output $y_1(t)$ and $y_2(t)$, respectively

$$T[a_1x_1(t)+a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)] = a_1y_1(t)+a_2y_2(t)$$

Linear systems are typically much simpler than their non-linear counterparts. They are used in automatic control theory, signal processing and telecommunications specifically, wireless communication can be modeled by linear systems.

(ii) Time varient and time-invariant systems

A system is time-varient if its input and output relationship varies with time. The equations that define these classes are as follows.

When $y(n,t) = T[x(n-t)]$ = input change and $y(n-t)$ = output chang

$y(n,t) = y(n-t)$ for time-invariant systems

$y(n,t) \neq y(n-t)$ time-varient system

(iii) Static & Dynamic system:

Static systems are memory-less systems.

An example eqn $\quad y[t] = 2^{x[t]}$

Dynamic system might have follows eqn

$$y[t] = 2 \cdot x[t-1]$$

(iv) Casual and Non-casual :

Similar to the distinction between static & dynamic systems, a casual system is one that depends on only present & past inputs. So, $y[t] = 2 \cdot x[t-1]$ still described a casual system. A non-casual system depends on future inputs. $y[t] = x[t+1]$ is a non-casual system.

(v) Stable & unstable system :

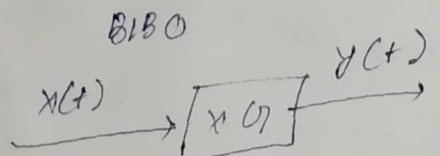
- A stable system is one that has bounded outputs for bounded inputs. In other words, for a bounded signal the output amplitude is finite.

$$y[n] = 2 \cdot x[n]$$

- An unstable system has a unbounded output for a bounded input. these systems, when implemented correctly, will cause a 'stack overflow' in computer programs.

(v) BIBO:

If a ~~sig~~ system is BIBO stable, then the output will be bounded for every input to the system that is bounded.




$\rightarrow |x(t)| < a$ for all $t$ $\leftarrow y(t) = (t)\, x(t)$

$\rightarrow |y(t)| < b$ for all $t$

$|y(t)| = |a\, x(t)|$

$= |a|\, |x(t)|$

$< \underbrace{|a|\, a}_{b}$

$\xrightarrow{u(t)}$ $a = 2$

1) Impulse response :

The impulse response of a system is defined as its output when an impulse of zero duration, infinite magnitude, and unit area is applied at the input. The response demonstrates the behaviour of the system to a temporary signal and provides insight into the characteristics and properties of the system.

Properties :

(i) Linear-time invarient systems
(ii) Convolution & impulse response
(iii) Frequency & domain representation

* Zero width
* Infinite height
* Integral of one (area)

2) 'Linear shift Invariant' systems operate "independently" on each sine wave, and merely scale & shift them. A simplified model of neurons in the visual system, the linear receptive field, results in a neural image that is linear and shift - invarient

Commulative Property :

- $x[n] * h[n] = h[n] * x[n]$

Distributive Property :

- $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$

Associative Property :

- $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$

Memoryless

- If $h[n] = 0$ for $n$ not equal 0.

(6) Linear or non linear

(i) $y(t) = t x(t)$

$y_1(t)$ $t x_1(t)$ and $y_2(t) = t x_2(t)$

$y_1(t) + y_2(t) = t x_1(t) + t x_2(t)$

$y_1(t) + y_2(t) = t(x_1(t) + x_2(t))$

Thus the system is linear

(ii) $y(t) = x(t^2)$

$y(t,k) = x(t^2 - k)$

shifting output

$y(t-k) = x((t-k)^2)$

$y(t,k) \neq y(t-k)$

no non-linear operator, so its linear

(iii) $y(t) = e^{x(t)}$

We make input 0 but output will 1. In addition exponential non-linear operator is applied in input. So it's Non-Linear

7) Casuality check:

(i) $y(t) = x(t) - x(t-1)$ : it is casual as the output is dependent on only present and past inputs

(ii) $y(t) = t x(t)$ : it is casual as the output is dependent on only present inputs

(iii) $y(t) = 2t + \frac{d}{dt} x(t)$ : Casual, output depends on past & present input

(iv) $y(t) = x(t^2)$ ; system depends on future inputs so non-casual

8) Static or not :

(i) $y(t) = x(t)$

It has no dependency on future or past values so it's State system

(ii) $y(t) = \pi \cos[x(t-2)]$ it is not static because it depends on past values

(iii) $y(t) = x(t) + x(t-2)$ it is non-static system

9) Convolution Theorem :

Discrete :

Let $x_1[n]$ and $x_2[n]$ be two discrete signals. If

$$y[n] = x_1[n] * x_2[n]$$

$$Y[w] = X_1(w) \cdot X_2(w)$$

Proof .

$$Y(w) = \sum_{n=-\infty}^{\infty} y[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \left( \sum_{m=-\infty}^{\infty} x_1[m] \cdot x_2[n-m] \right) e^{-jwn}$$

$$= \sum_{m=-\infty}^{\infty} x_1[m] \left( \sum_{n=-\infty}^{\infty} x_2[n-m] e^{-jwn} \right)$$

$$= \sum_{m=-\infty}^{\infty} x_1[m] \left( e^{-jwn} X_2(w) \right)$$

$$= X_2(w) \sum_{m=-\infty}^{\infty} x_1[m] e^{-jwn}$$

$$= X_2(w) X_1(w)$$

Convolution is a mathematical operation used to express the relation between input & output of an LTI system. It relates input, output and impulse response of an LTI system as

$$y(t) = x(t) * h(t)$$

10) Various properties of Convolution :

(i) Commutative $x(n) * h(n) = h(n) * x(n)$

(ii) Associative $\{x(n) * h_1(n)\} * h_2(n) = x(n) * \{h_1(n) * h_2(n)\}$

(iii) distributive $x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

Condition for BIBO stability :

A system is called BIBO stable if and only if every bounded input to the system produces a bounded output.

Condition

(i) bounded input that leads to an unbounded output.

(ii) In practical terms, a BIBO stable system is well behaved in the sense that, as long as the system input remains finite for all time, the output will also remain finite for all time.

10) i) $x_1(t) * x_2(t) = x_2(t) * x_1(t)$

PROOF

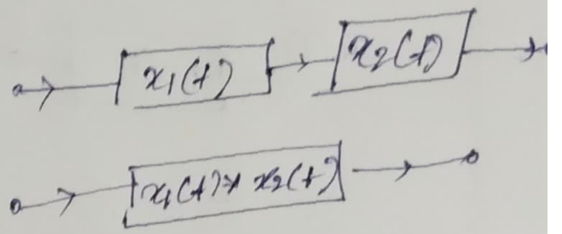
$y(k) = x_1(k) * x_2(k)$

$= \sum_{m=-\infty}^{\infty} x_1(m)\, x_2(k-m)$

let, $n = k-m \Rightarrow m = k-n$

when $m = -\infty \Rightarrow n = k-(-\infty) = \infty$

$m = +\infty \Rightarrow n = k-\infty = -\infty$




$y(k) = \sum_{n=\infty}^{-\infty} x_1(k-n)\, x_2(n)$

$= \sum_{n=-\infty}^{\infty} x_1(k-n)\, x_2(n) = \sum_{n=-\infty}^{\infty} x_2(n)\, x_1(k-n)$

$= x_2(n) * x_1(n)$

$\Rightarrow x_1(n) * x_2(n) = x_2(n) * x_1(n)$.

$\Rightarrow$ substitute $t$ with $n$

$x_1(t) * x_2(t) = x_2(t) * x_1(t)$

(ii) $[x_1(t) * x_2(t)] * x_3(t)$

$= x_1(t) * [x_2(t) * x_3(t)]$

**Proof**

$\cancel{y[k] = x_3(k)}$

$x[n] * (h_1(n) + h_2(n))$

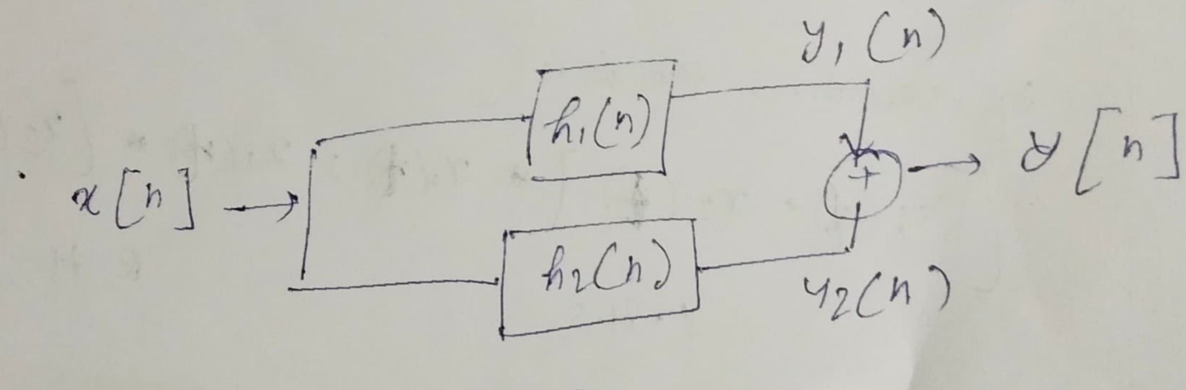
let $h[n] = h_1[n] + h_2[n]$

$x[n] * h[n]$

$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$= \sum_{k=-\infty}^{\infty} x[k] \left(h_1[n-k] + h_2[n-k]\right)$

$= \sum_{k=-\infty}^{\infty} x[k]\, h_1[n-k] + \sum_{k=-\infty}^{\infty} x[k]\, h_2[n-k]$




$y_1[n] = x[n] * h_1(n)$
$y_2[n] = x[n] * h_2[n]$
$y[n] = y_1[n] + y_2[n]$

$x[n] \rightarrow \boxed{h_1[n] + h_2[n]} \longrightarrow y[n]$

(iii)

$x_1(n)$
$x_2(n)$ } Discrete time signal
$x_3(n)$

$y_1(n) = x_1(n) * x_2(n)$ —(i)

Replace n by t

$y_1(t) = x_1(t) * x_2(t)$

$= \sum_{m=-\infty}^{\infty} x_1(m) * x_2(\cancel{t}-m)$ — (ii).

$y_2(n) = x_2(n) * x_3(n)$

$y_2(n) = \sum_{q=-\infty}^{\infty} x_1(q) x_2(n-q)$

$y_2(n-m) = \sum_{q=-\infty}^{\infty} x_1(q) x_2(n-m-q)$ — (iii)

p, q, m $\longrightarrow$ dummy variables

by associative properties

$\underbrace{[x_1(t) * x_2(t)] * x_3(t)}_{L.H.S} = \underbrace{x_1(t) * [x_2(t) * x_3(t)]}_{R.H.S}$

L.H.S

$[x_1(n) * x_2(n)] * x_3(n)$

$\underbrace{x_1(n) * x_2(n)}_{y_1(t)}$

$= y_1(n) * x_3(n)$

$= \sum_{p=-\infty}^{\infty} y_1(p) * x_3(n-p)$

$= \sum_{p=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} x_1(m) x_2(p-m) x_3(n-p)$

$= \sum_{m=-\infty}^{+\infty} x_1(m) \sum_{p=-\infty}^{+\infty} x_2(p-m) x_3(n-p)$ —— (iv)

$= \sum_{m=-\infty}^{+\infty} x_1(m) \sum_{p=-\infty}^{+\infty} x_2(p-m) x_3$

$= p-m = q$

when $p = -\infty$ $q = p-m = -\infty - m = -\infty$

$p = +\infty$ $q = p-m = +\infty - m = +\infty$

Now replace $(p-m) \to q$ $p \to (q+m)$ in eq 4

$$\text{L.H.S} = \sum_{m=-\infty}^{\infty} x_1(m) \underbrace{\sum_{q=-\infty}^{+\infty} x_2(q)\, x_3(n-q-m)}_{y_2(n-m)}$$

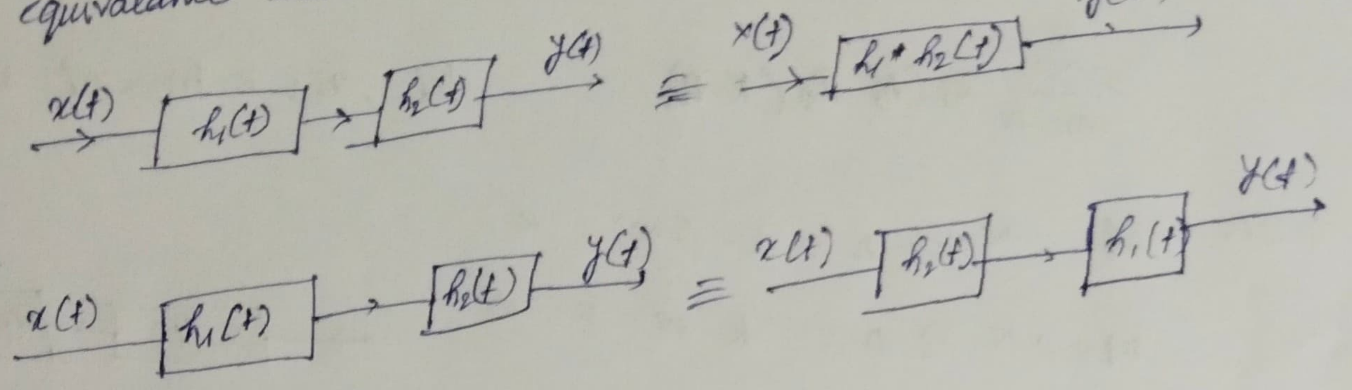
$$= \underbrace{\sum_{m=-\infty}^{+\infty} x_1(m)\, y_2(n-m)}_{x_1(n) * y_2(n)}$$

$= x_1(n) * [x_2(n) * x_3(n)]$ R.H.S

Proved

12) **LTI system in series:**

The series interconnection of the LTI systems with impulse responses $h_1$ a... is the LTI system with impulse response $h = h_1 * h_2$. That is, we ha... $h_2$ is the LTI system with impulse response ... the equivalance shown below.




**LTI system in Parallel:**

The parallel interconnection of the LTI systems with impulse responses $h_1$ and h... is a LTI systems with impulse response $h = h_1 + h_2$. That is we have ... the equivalance shown below.