BEE 4750/5750 Homework 2

Jason Shao (jls647)

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Problem 1

Problem 1.1

```
julia> #A function which takes distance x in km, and inital DO,BOD, and NOD
concetrations and outputs the DO concentration at that distance
       function C(x,Co,CBODo,NBODo)
       #given values
       ka=.55
       kc=.35
       kn=.25
       Cs=10
       U=6

       #alpha value calculations
       a1=exp(-ka*x/U)
       a2=(kc/(ka-kc))*(exp(-kc*x/U)-exp(-ka*x/U))
       a3=(kn/(ka-kn))*(exp(-kn*x/U)-exp(-ka*x/U))
       DO=Cs*(1-a1)+Co*a1-CBODo*a2-NBODo*a3
       return DO
       end
C (generic function with 1 method)

julia> #Decay of CBOD function
       function CBOD(x,CBODo)
       kc=.35
       U=6
       CBOD=CBODo*exp(-kc*x/U)
       return CBOD
       end
CBOD (generic function with 1 method)

julia> #Decay of NBOD function
       function NBOD(x,NBODo)
       kn=.35
       U=6
       NBOD=NBODo*exp(-kn*x/U)
       return NBOD
       end
NBOD (generic function with 1 method)
```

To find initial concetration of DO:

$$C_{o1} = \frac{C_{River} * Q_{River} + C_{Waste1} * Q_{Waste1}}{Q_{River} + Q_{Waste1}}$$

$$C_{o1} = \frac{7.5\frac{mg}{L} * 10^8\frac{L}{d} + 5\frac{mg}{L} * 10^7\frac{L}{d}}{10^8\frac{L}{d} + 10^7\frac{L}{d}} = 7.27\frac{mg}{L}$$

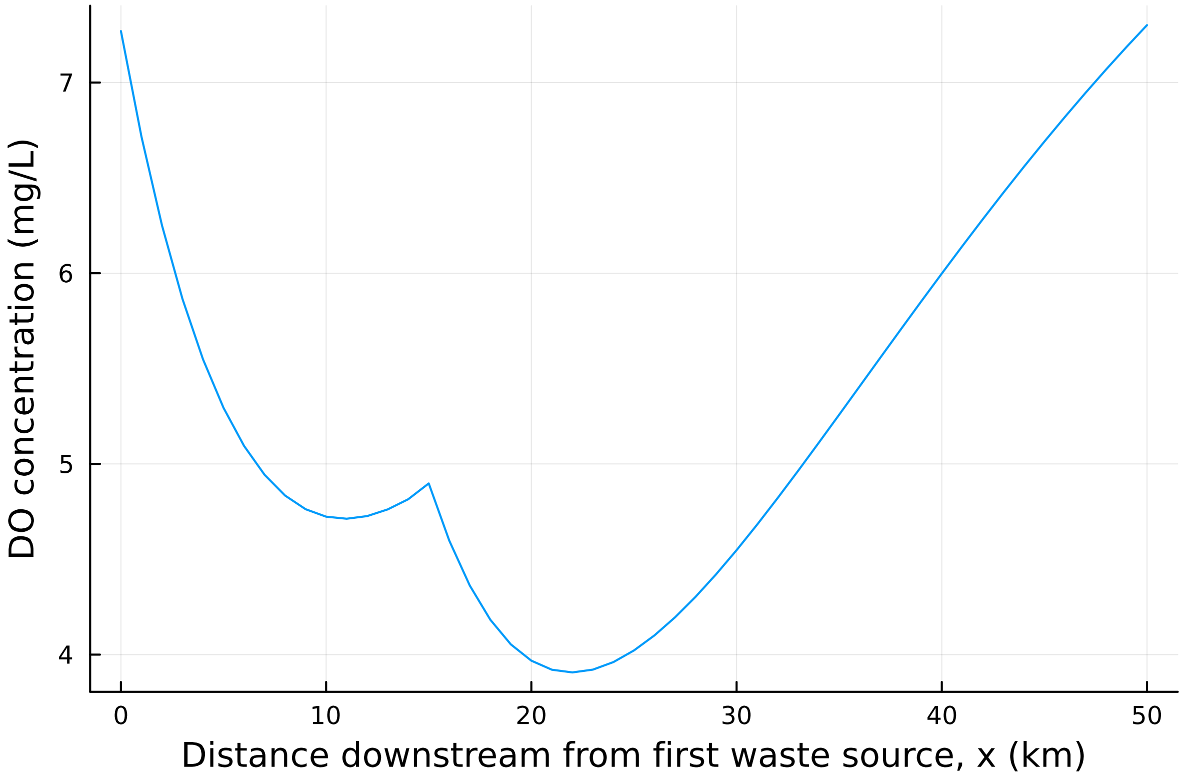
The same process is done for CBOD and NBOD at the start of the first inflow and again at the second waste flow

$$CBOD_{o1} = \frac{5\frac{mg}{L} * 10^8\frac{L}{d} + 50\frac{mg}{L} * 10^7\frac{L}{d}}{10^8\frac{L}{d} + 10^7\frac{L}{d}} = 9.09\frac{mg}{L}$$

$$NBOD_{o1} = \frac{5\frac{mg}{L} * 10^8\frac{L}{d} + 35\frac{mg}{L} * 10^7\frac{L}{d}}{10^8\frac{L}{d} + 10^7\frac{L}{d}} = 7.72\frac{mg}{L}$$

```
julia> using Plots, Distributions

julia> DO=zeros(51); #initialize DO vector

julia> DO[1]=7.27 ; #set initial DO concentration with both river inflow and
waste source 1

julia> for i=1:14
       DO[i+1]=C(i,7.27,9.09,7.72); #calculate DO concentration for 1 to 14 km
downstream
       end

julia> #calculate inital DO, BOD, and NOD concentrations of inflow diverging
with waste source 2
       CBODo2=(1.1*10^8*CBOD(15,9.09)+45*1.5*10^7)/(1.25*10^8);

julia> NBODo2=(1.1*10^8*NBOD(15,7.72)+35*1.5*10^7)/(1.25*10^8);

julia> DO[16]=(C(15,7.27,9.09,7.72)*1.1*10^8+1.5*10^7*5)/(1.25*10^8);

julia> #using these initial values, calculate the DO concentration after waste
source 2
       for i=16:50
       DO[i+1]=C(i-15,DO[16],CBODo2,NBODo2);
```

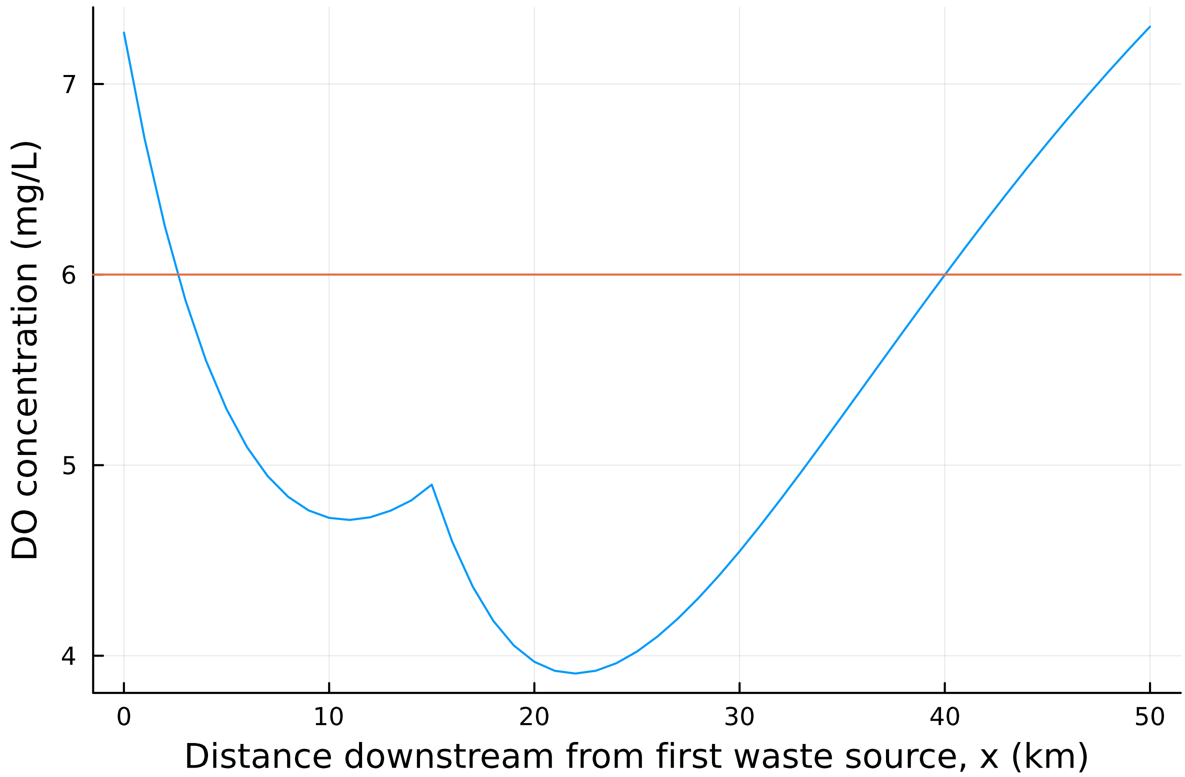
```
    end

julia> x=0:1:50; #vector of distance values

julia> plot(x,DO, xguide="Distance downstream from first waste source, x
(km)", yguide= "DO concentration (mg/L)", legend=false)
```



Problem 1.2

Using the conditions at the input of the second waste stream as the initial conditions to find DO concentration, you can solve for the distance which achieves 6 mg/L. Since the equation seems difficult to solve algebraically, you can find the distance finding the root of f(x)=C(x)-6. This can be one using the Roots package in Julia. A plot with a horizontal line at DO=6mg/L is shown to provide an inital guess of x.

```
julia> using Roots

julia> plot(x,DO, xguide="Distance downstream from first waste source, x
(km)", yguide= "DO concentration (mg/L)", legend=false);

julia> hline!([6])
```




```
julia> ka=.55;

julia> kc=.35;

julia> kn=.25;

julia> Cs=10;

julia> U=6;

julia> Co=DO[16];

julia> CBODo=(1.1*10^8*CBOD(15,9.09)+45*1.5*10^7)/(1.25*10^8);

julia> NBODo=(1.1*10^8*NBOD(15,7.72)+35*1.5*10^7)/(1.25*10^8);

julia>
f(x)=begin(Cs*(1-(exp(-ka*x/U)))+Co*(exp(-ka*x/U))-CBODo*((kc/(ka-kc))*(exp(-kc*x/U)-
       exp(-ka*x/U)))-NBODo*((kn/(ka-kn))*(exp(-kn*x/U)-exp(-ka*x/U))))-6 end
f (generic function with 1 method)

julia> #redefine function with only one input, x
       find_zero(f, 30)
25.01130063650589
```

Finding the root of the equation shows that at 25.0 km downstream from waste stream 2, the dissolved oxygen concentration rebounds back to 6 mg/L.

Problem 1.3

In order to find the treatment needed to prevent the dissolved oxygen concentration from dropping below 4 mg/L, you can test a range of treatment percentages. Then you can select the treatment which has a minimum DO level just at or above 4 mg/L.

```
julia> Co=DO[16]; #set initial DO concentration

julia> minConc=zeros(20,2); #initialize matrix of minimum DO concentrations
and their corresponding treatment percentage values

julia> for x=1:20 #test 20 values
       DOtreat=zeros(45);
       CBODo=(1.1*10^8*CBOD(15,9.09)+45*(1-x*.01)*1.5*10^7)/(1.25*10^8);
       NBODo=(1.1*10^8*NBOD(15,7.72)+35*(1-x*.01)*1.5*10^7)/(1.25*10^8);
       for i=1:45
       DOtreat[i]=C(i,Co,CBODo,NBODo)
       end
       minConc[x,1]=minimum(DOtreat);
       minConc[x,2]=x*.01;


       end

julia> minConc
20×2 Matrix{Float64}:
 3.92745  0.01
 3.94825  0.02
 3.96904  0.03
 3.98983  0.04
 4.01063  0.05
 4.03142  0.06
 4.05222  0.07
 4.07301  0.08
 4.09301  0.09
 4.11215  0.1
 4.13129  0.11
 4.15042  0.12
 4.16956  0.13
 4.18869  0.14
 4.20783  0.15
 4.22697  0.16
 4.2461   0.17
 4.26524  0.18
 4.28437  0.19
 4.30351  0.2
```

A treatment of 5% removal will achieve a minimum DO concentration of 4.01 mg/L, just over 4 mg/L.

Problem 1.4

Now the same can be done with treatment done at both waste streams. Treatment done at the first waste stream will affect the inital DO, CBOD, and NBOD concentrations at the second waste stream.

```
julia> minConc=zeros(11,2); #initialize matrix of minimum DO concentrations
and their corresponding treatment percentage values

julia> for x=0:10
       DOtreat=zeros(45);
       CBODo1=(5*10^8+50*(1-x*.01)*10^7)/(1.1*10^8); #calculate new initial
value of CBOD from mixing of river inflow and waste stream 1
       NBODo1=(5*10^8+35*(1-x*.01)*10^7)/(1.1*10^8); #calculate new initial
value of NBOD from mixing of river inflow and waste stream 1
       Co2=(C(15,7.27,CBODo1,NBODo1)*1.1*10^8+1.5*10^7*5)/(1.25*10^8);
#calculate DO conc after 15 km from waste stream 1 and mixing with waste
stream 2
       CBODo2=(1.1*10^8*CBOD(15,CBODo1)+45*(1-x*.01)*1.5*10^7)/(1.25*10^8);
#calculate initial CBOD at waste stream 2 after decay
       NBODo2=(1.1*10^8*NBOD(15,NBODo1)+35*(1-x*.01)*1.5*10^7)/(1.25*10^8);
#calculate initial CBOD at waste stream 2 after decay
       for i=1:45
       DOtreat[i]=C(i,Co2,CBODo2,NBODo2);
       end
       minConc[x+1,1]=minimum(DOtreat);
       minConc[x+1,2]=x*.01;
       end

julia> minConc
11×2 Matrix{Float64}:
 3.90517  0.0
 3.94166  0.01
 3.97815  0.02
 4.01464  0.03
 4.05113  0.04
 4.08762  0.05
 4.12411  0.06
 4.1606   0.07
 4.19709  0.08
 4.23358  0.09
 4.27007  0.1
```

A 3% treatment of both streams achieves a minimum DO concentration of 4.01 mg/L.

Problem 1.5

I would treat each waste stream equally. This is because both waste streams contribute to the depletion of dissolved oxygen. While waste stream 2 brings the DO concentration to its minimum point, this minimum is still affected by the DO depletion from waste stream 1. Additional information may change this strategy however. For example, if the cost for treatment is in USD/mg/d, treating only the second waste stream would be less costly.

Problem 1.6

To estimate the probability that treating both waste streams equally, you can run 100000 random trials with the CBOD and NBOD values of the river inflow varying uniformly.

```
julia> using Random, Distributions

julia> function isPos(x) #a function that takes a vector x as an input and outputs vector y of the same length, with values of 1 if element in x is greater than or equal to 0 and 0 otherwise
       n=length(x);
       y=zeros(n);
       for i=1:n
       if x[i]>= 0
       y[i]=1;
       else
       y[i]=0;
       end
       end
       return y
       end
isPos (generic function with 1 method)

julia> n=100000; #set number of random trials

julia> minTrials=zeros(n);

julia> NBODin=zeros(n);

julia> CBODin=zeros(n);

julia> NBODo1=zeros(n);

julia> CBODo1=zeros(n);

julia> NBODo2=zeros(n);

julia> CBODo2=zeros(n);

julia> Co2=zeros(n);

julia> for i=1:n
       CBODin[i]=rand(4:.01:7);
       NBODin[i]=rand(3:.01:8);
       CBODo1[i]=(10^8*CBODin[i]+50*.97*10^7)/(1.1*10^8);
       NBODo1[i]=(10^8*NBODin[i]+35*.97*10^7)/(1.1*10^8);
       Co2[i]=(C(15,7.27,CBODo1[i],NBODo1[i])*1.1*10^8+5*1.5*10^7)/(1.25*10^8);
       CBODo2[i]=(CBOD(15,CBODo1[i])*1.1*10^8+45*.97*1.5*10^7)/(1.25*10^8);
       NBODo2[i]=(NBOD(15,NBODo1[i])*1.1*10^8+35*.97*1.5*10^7)/(1.25*10^8);
       DO=zeros(35);
       for x=1:35
       DO[x]=C(x,Co2[i],CBODo2[i],NBODo2[i])
       end
       minTrials[i]=minimum(DO);
       end

julia> diff=minTrials-4*ones(n); #how much above or below 4 mg/L is the minimum for each trial

julia> p=isPos(diff); #if positive then the treatment was able to keep DO concentration above 4 mg/L
```

```
julia> successRate=(sum(p))/n #sum up all the successes and divide by total
number of trials to get success rate
0.29636
```

Running 100000 random trials with uniformly distributed values for CBOD and NBOD of river inflow, gives a success rate of about .3, meaning about 30% percent of the time the treatment keeps the DO concentration from going below 4 mg/L. This means there is about a 70% chance of failure with the treatment plan of 3% at each waste source.

Problem 1.7

Assuming the values of CBOD and NBOD for the river inflow are correlated produces different results than if they are independent as they were in Problem 1.6.

```
julia> n=100000;

julia> minTrials=zeros(n);

julia> NBODin=zeros(n);

julia> CBODin=zeros(n);

julia> NBODo1=zeros(n);

julia> CBODo1=zeros(n);

julia> NBODo2=zeros(n);

julia> CBODo2=zeros(n);

julia> Co2=zeros(n);

julia> BODin=sample_correlated_uniform(n,[4,7],[3,8],.7);

julia> for i=1:n
       CBODin[i]=BODin[i,1];
       NBODin[i]=BODin[i,2];
       CBODo1[i]=(10^8*CBODin[i]+50*.97*10^7)/(1.1*10^8);
       NBODo1[i]=(10^8*NBODin[i]+35*.97*10^7)/(1.1*10^8);
       Co2[i]=(C(15,7.27,CBODo1[i],NBODo1[i])*1.1*10^8+5*1.5*10^7)/(1.25*10^8);
       CBODo2[i]=(CBOD(15,CBODo1[i])*1.1*10^8+45*.97*1.5*10^7)/(1.25*10^8);
       NBODo2[i]=(NBOD(15,NBODo1[i])*1.1*10^8+35*.97*1.5*10^7)/(1.25*10^8);
       DO=zeros(35);
       for x=1:35
       DO[x]=C(x,Co2[i],CBODo2[i],NBODo2[i])
       end
       minTrials[i]=minimum(DO);
       end

julia> diff=minTrials-4*ones(n);

julia> p=isPos(diff);

julia> successRate=(sum(p))/n
0.36105
```

When the values of CBOD and NBOD of the river inflow are correlated, the success rate increases to about 36%. This means the failure probability decreases to about 64% as opposed to 70% when the BOD values of river inflow were independent.

Problem 1.8

The treatment plan of 3% for both waste streams from problem 1.5 is slightly more reliable if the values of CBOD and NBOD of the river inflow are positively correlated as opposed to independent. You could calculate the failure probability for only treating waste stream 2 under both conditions of dependence for CBOD and NBOD values of river inflow. Then, if you know which scenario of dependence of BOD values best fits the actual conditions, you could choose the treatment plan with the lowest probability of failure.

References