BEE 4750/5750 Homework 3

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Problem 1

Problem 1.1

The decision variables are the installed capacities of each generator type g, the production from generator type g in period t, and the non-served energy at each time period t.

Notation:

$$x_g = \text{installed capapcity of generator type g}$$
$$x_1 - Geothermal, x_2 - Coal, x_3 - CCGT, x_4 - CT, x_5 - Wind, x_6 - Solar$$
$$\vec{x} \text{ is a vector of length 6 containing all of the x values}$$

$$y_{g,t} = \text{production of generator type g at time period t for g=1,...,6 and t=1,...,24}$$
$$Y \text{ is a 6 x 24 matrix containing the production of each generator type g at each time period t, } y_{g,t}$$

$$nse_t = \text{non-served energy at time period t for t=1,...,24}$$
$$\vec{nse} \text{ is a vector of length 24 containing the non-served energy in time period t, } nse_t$$

Problem 1.2

$$\min_{x_g, y_{g,t}, nse_t} \text{Cost} = \text{Investment Cost} + \text{Operating Cost} + \text{Non-served demand penalty}$$

$\vec{ic}$ is a vector of length 6 containing the cost per installed MW for generator type g

$\vec{oc}$ is a vector of length 6 containing the cost per MWh for each generator type g

```
ic = investment_cost = [457000, 268000, 85000, 62580, 92000, 92000];
oc = op_cost = [0, 22, 35, 45, 0, 0];
#note there are only operating costs for coal, CCGT, and CT
```

$$\min_{x_g, y_{g,t}, nse_t} \text{Cost} = \sum_{g=1}^{6} ic_g * x_g + 365 \sum_{g=1}^{6} \sum_{t=1}^{24} oc_g * y_{g,t} + 365 * 1000 \sum_{t=1}^{24} nse_t$$

Problem 1.3

Constraints:

Non-negativity, cannot have negative capacity, generation, or non-served energy

$$\begin{aligned} x_g &\geq 0 \text{ for g=1,...,6} \\ y_{g,t} &\geq 0 \text{ for g=1,...,6 and t=1,...,24} \\ nse_t &\geq 0 \text{ for t=1,...,24} \end{aligned}$$

Cannot produce more than installed capacity allows

CF is a 6x24 matrix containing the capacity factor for generator type g in time period t, $cf_{g,t}$

$$y_{g,t} \leq cf_{g,t} * x_g \text{ for g=1,...,6 and t=1,...,24}$$

Meet demands at each hour including non-served energy

$\vec{d}$ is a vector of length 24 containing demand values at each time period t

$\vec{nse}$ is a vector of length 24 containing all of the non-served energy values from each time period t

$$\sum_{g=1}^{6} y_{g,t} + nse_t = d_t \text{ for t=1,...,24}$$

Problem 1.4

```
julia> using JuMP, HiGHS

julia> gencap=Model(HiGHS.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: HiGHS

julia> generators=["geothermal", "coal", "CCGT", "CT", "wind", "solar"];

julia> periods=["hour 1","hour 2","hour 3","hour 4","hour 5","hour 6","hour
7","hour 8","hour 9","hour 10","hour 11","hour 12","hour 13","hour 14","hour
15","hour 16","hour 17","hour 18","hour 19","hour 20","hour 21","hour
22","hour 23","hour 24"];

julia> G=1:length(generators)
1:6

julia> T=1:length(periods)
1:24

julia> @variable(gencap, x[G] >=0);

julia> @variable(gencap, y[G,T]>=0);

julia> @variable(gencap, nse[T]>=0);

julia> @objective(gencap, Min,
sum(investment_cost.*x)+365*sum(y*ones(24,1).*op_cost)+sum(nse)*1000*365);

julia> @constraint(gencap, load[t in T], sum(y[:,t])+nse[t]==demand[t]);

julia> #put all capacity factors in one array
       avail=ones(6,24);

julia> for i=1:4
       avail[i,:]=avail[i,:].*thermal_cf[i];
       end

julia> avail[5,:]=wind_cf;

julia> avail[6,:]=solar_cf;

julia> @constraint(gencap, availability[g in G, t in T],
y[g,t]<=avail[g,t]*x[g]);
```

Problem 1.5

```
julia> using DataFrames

julia> optimize!(gencap)
Presolving model
156 rows, 162 cols, 420 nonzeros
156 rows, 162 cols, 420 nonzeros
```

```
Presolve : Reductions: rows 156(-12); columns 162(-12); elements 420(-24)
Solving the presolved LP
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0     0.0000000000e+00 Pr: 24(60321.5) 0s
        120     9.1214221224e+08 Pr: 0(0) 0s
Solving the original LP from the solution after postsolve
Model   status      : Optimal
Simplex   iterations: 120
Objective value     :  9.1214221224e+08
HiGHS run time      :          0.00

julia> objective_value(gencap)
9.121422122241888e8

julia> installed=value.(x).data;

julia> generated=(value.(y).data*ones(24,1))/1000;

julia> generated=vec(generated);

julia> results=DataFrame(
       "Resource" => generators,
       "Installed (MW)" => installed,
       "Generated (GWh/day)" => generated,
       )
6×3 DataFrame
 Row │ Resource    Installed (MW)  Generated (GWh/day)
     │ String      Float64         Float64
─────┼─────────────────────────────────────────────────
   1 │ geothermal          0.0                 0.0
   2 │ coal                0.0                 0.0
   3 │ CCGT             1704.26               23.2987
   4 │ CT                881.327               3.01526
   5 │ wind             1238.05                6.92072
   6 │ solar            2728.91               23.8023

julia> generatedHourly=ones(1,6)*value.(y).data;

julia> generatedHourly=vec(generatedHourly);

julia> results2=DataFrame(
       "Time Period" => periods,
       "Generated (MWh/day)" =>generatedHourly,
       "Non-served (MWh/day)" =>value.(nse).data,
       "Demand" => demand
       );

julia> show(results2, allrows=true)
24×4 DataFrame
 Row │ Time Period  Generated (MWh/day)  Non-served (MWh/day)  Demand
     │ String       Float64              Float64               Int64
─────┼───────────────────────────────────────────────────────────────
```

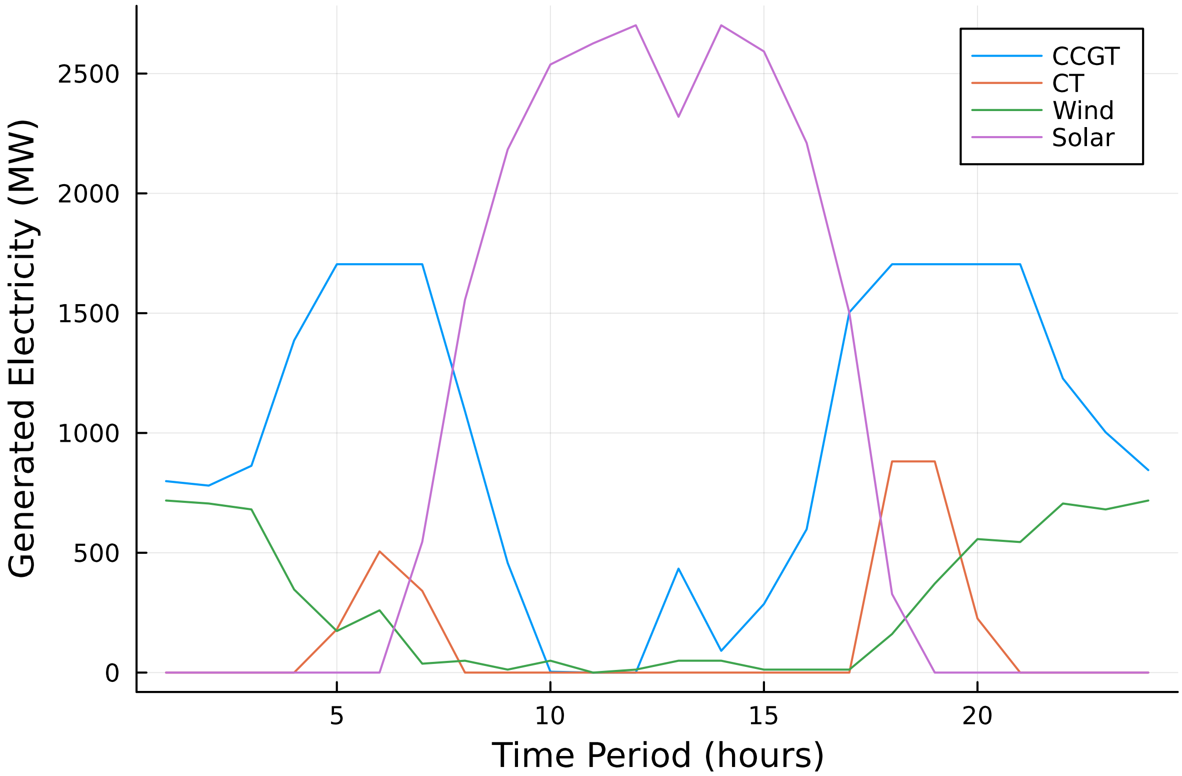
1	hour 1	1517.0	0.0	1517
2	hour 2	1486.0	0.0	1486
3	hour 3	1544.0	0.0	1544
4	hour 4	1733.0	0.0	1733
5	hour 5	2058.0	0.0	2058
6	hour 6	2470.0	0.0	2470
7	hour 7	2628.0	0.0	2628
8	hour 8	2696.0	0.0	2696
9	hour 9	2653.0	0.0	2653
10	hour 10	2591.0	0.0	2591
11	hour 11	2626.0	0.0	2626
12	hour 12	2714.0	0.0	2714
13	hour 13	2803.0	0.0	2803
14	hour 14	2842.0	0.0	2842
15	hour 15	2891.0	0.0	2891
16	hour 16	2821.0	0.0	2821
17	hour 17	3017.0	0.0	3017
18	hour 18	3074.0	0.0	3074
19	hour 19	2957.0	0.0	2957
20	hour 20	2487.0	0.0	2487
21	hour 21	2249.0	0.0	2249
22	hour 22	1933.0	0.0	1933
23	hour 23	1684.0	0.0	1684
24	hour 24	1563.0	0.0	1563

As shown in the dataframes above, in the optimal solution the utility should build 1704.26 MW of CCGT, 881.327 MW of CT, 1238.05 MW of wind, 2728.91 MW of solar, and 0 MW in both geothermal and coal. This will cost approximately $910 million for installation and operation for 1 year. In this solution, there will be no non-served energy during any time period.

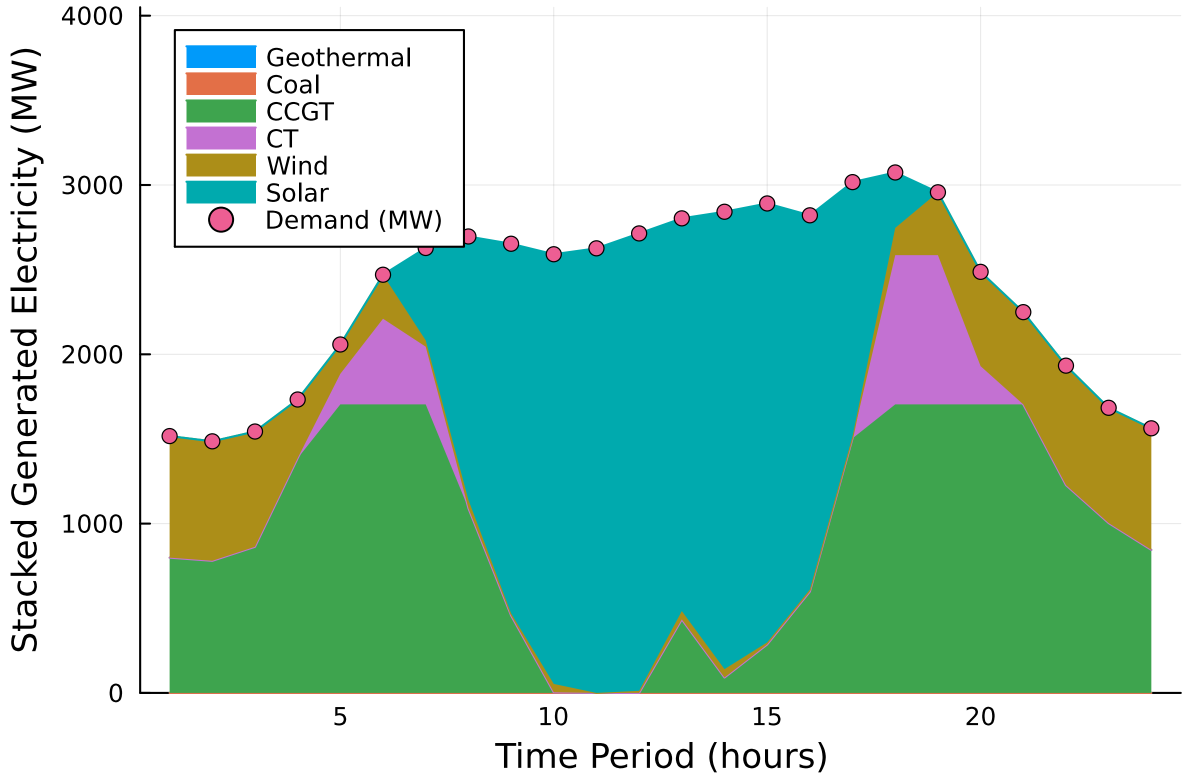
Problem 1.6

```
julia> using Plots

julia> gen=value.(y).data;

julia> geoHour=gen[1,:];

julia> coalHour=gen[2,:];

julia> CCGTHour=gen[3,:];

julia> CTHOUR=gen[4,:];

julia> windHour=gen[5,:];

julia> solarHour=gen[6,:];
```

```
julia> plot(CCGTHour, label="CCGT",legend=:topright, ylabel="Generated Electricity (MW)", xlabel="Time Period (hours)");

julia> plot!(CTHOUR, label="CT");

julia> plot!(windHour, label="Wind");

julia> plot!(solarHour, label="Solar")
```




```
julia> #Note there is no generation from Geothermal and Coal


       areaplot(gen', labels=["Geothermal"  "Coal"  "CCGT" "CT" "Wind" "Solar"]);

julia> scatter!(demand, label="Demand (MW)", legend=:topleft,ylims=(0,4050),ylabel="Stacked Generated Electricity (MW)", xlabel="Time Period (hours)")
```



Wind and solar take up a lot of the installed capacity. During daylight hours, solar generation is by far the largest. This makes sense as solar can only produce when the sun is out. This is reflected in the capacity factors for solar, with values being 0 at night and almost 1 at mid-day. Wind resource is availible at the exact opposite times as shown by the wind capacity factors, and as such, wind energy is produced when solar is not available. These two energy sources are complementary and work well for the objective because of their lower investment cost and negligible operation cost. Additionally, CCGT and CT generation occurs when solar production is lower. CCGT and CT were most likely chosen over coal and geothermal because of their low investment cost in comparison.

Problem 2

Problem 2.1

With this limit, you could still try to minimize cost, therefore the objective function would remain the same. In order to account for the limit, a new constraint could be added to the linear program which puts a max amount of carbon emissions. In order to formulate this constraint we need the values of CO2 emissions per MWh for each generator type.

$CO2_g$ is a vector of length 6 containing the CO_2 emission rate ($\frac{tCO_2}{MWh}$) associated with each generator type g

New Constraint:

$$365 * \sum_{g=1}^{6} \sum_{t=1}^{24} y_{g,t} * CO2_g \leq 1.5 * 10^6 tCO_2$$

Problem 2.2

```
julia> using JuMP, HiGHS

julia> gencapCO2=Model(HiGHS.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: HiGHS

julia> generators=["geothermal", "coal", "CCGT", "CT", "wind", "solar"];

julia> periods=["hour 1","hour 2","hour 3","hour 4","hour 5","hour 6","hour
7","hour 8","hour 9","hour 10","hour 11","hour 12","hour 13","hour 14","hour
15","hour 16","hour 17","hour 18","hour 19","hour 20","hour 21","hour
22","hour 23","hour24"];

julia> G=1:length(generators)
1:6

julia> T=1:length(periods)
1:24

julia> @variable(gencapCO2, xCO2[G] >=0);

julia> @variable(gencapCO2, yCO2[G,T]>=0);

julia> @variable(gencapCO2, nseCO2[T]>=0);

julia> @objective(gencapCO2, Min,
sum(investment_cost.*xCO2)+365*sum(yCO2*ones(24,1).*op_cost)+sum(nseCO2)*1000*365);

julia> @constraint(gencapCO2, load[t in T],
sum(yCO2[:,t])+nseCO2[t]==demand[t]);

julia> #put all capacity factors in one array
       avail=ones(6,24);

julia> for i=1:4
       avail[i,:]=avail[i,:].*thermal_cf[i];
       end

julia> avail[5,:]=wind_cf;

julia> avail[6,:]=solar_cf;

julia> @constraint(gencapCO2, availability[g in G, t in T],
yCO2[g,t]<=avail[g,t]*xCO2[g]);
```

```
julia> #new constraint
       @constraint(gencapCO2, CO2,  365*sum(yCO2*ones(24,1).*co2_emissions) <=
1.5*10^6);
```

Problem 2.3

```
julia> using DataFrames

julia> optimize!(gencapCO2)
Presolving model
157 rows, 162 cols, 492 nonzeros
157 rows, 162 cols, 492 nonzeros
Presolve : Reductions: rows 157(-12); columns 162(-12); elements 492(-24)
Solving the presolved LP
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0     0.0000000000e+00 Pr: 24(173966) 0s
        105     1.0659518571e+09 Pr: 0(0); Du: 0(1.45519e-11) 0s
Solving the original LP from the solution after postsolve
Model   status       : Optimal
Simplex   iterations: 105
Objective value      :  1.0659518571e+09
HiGHS run time       :          0.00

julia> objective_value(gencapCO2)
1.0659518570585868e9

julia> installedCO2=value.(xCO2).data;

julia> generatedCO2=(value.(yCO2).data*ones(24,1))/1000;

julia> generatedCO2=vec(generatedCO2);

julia> resultsCO2=DataFrame(
       "Resource" => generators,
       "Installed (MW)" => installedCO2,
       "Generated (GWh/day)" => generatedCO2,
       )
6×3 DataFrame
```

Row	Resource String	Installed (MW) Float64	Generated (GWh/day) Float64
1	geothermal	1029.09	21.0336
2	coal	0.0	0.0
3	CCGT	1185.43	8.6166
4	CT	444.253	0.735364
5	wind	1676.07	9.19768
6	solar	2073.25	17.4538

```
julia> generatedHourlyCO2=ones(1,6)*value.(yCO2).data;

julia> generatedHourlyCO2=vec(generatedHourlyCO2);
```

```
julia> results2CO2=DataFrame(
       "Time Period" => periods,
       "Generated (MWh/day)" =>generatedHourlyCO2,
       "Non-served (MWh/day)" =>value.(nseCO2).data,
       "Demand" => demand
       );

julia> show(results2CO2, allrows=true)
24×4 DataFrame
```

Row	Time Period String	Generated (MWh/day) Float64	Non-served (MWh/day) Float64	Demand Int64
1	hour 1	1517.0	0.0	1517
2	hour 2	1486.0	0.0	1486
3	hour 3	1544.0	0.0	1544
4	hour 4	1733.0	0.0	1733
5	hour 5	2058.0	0.0	2058
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9	hour 9	2653.0	0.0	2653
10	hour 10	2591.0	0.0	2591
11	hour 11	2626.0	0.0	2626
12	hour 12	2714.0	0.0	2714
13	hour 13	2803.0	0.0	2803
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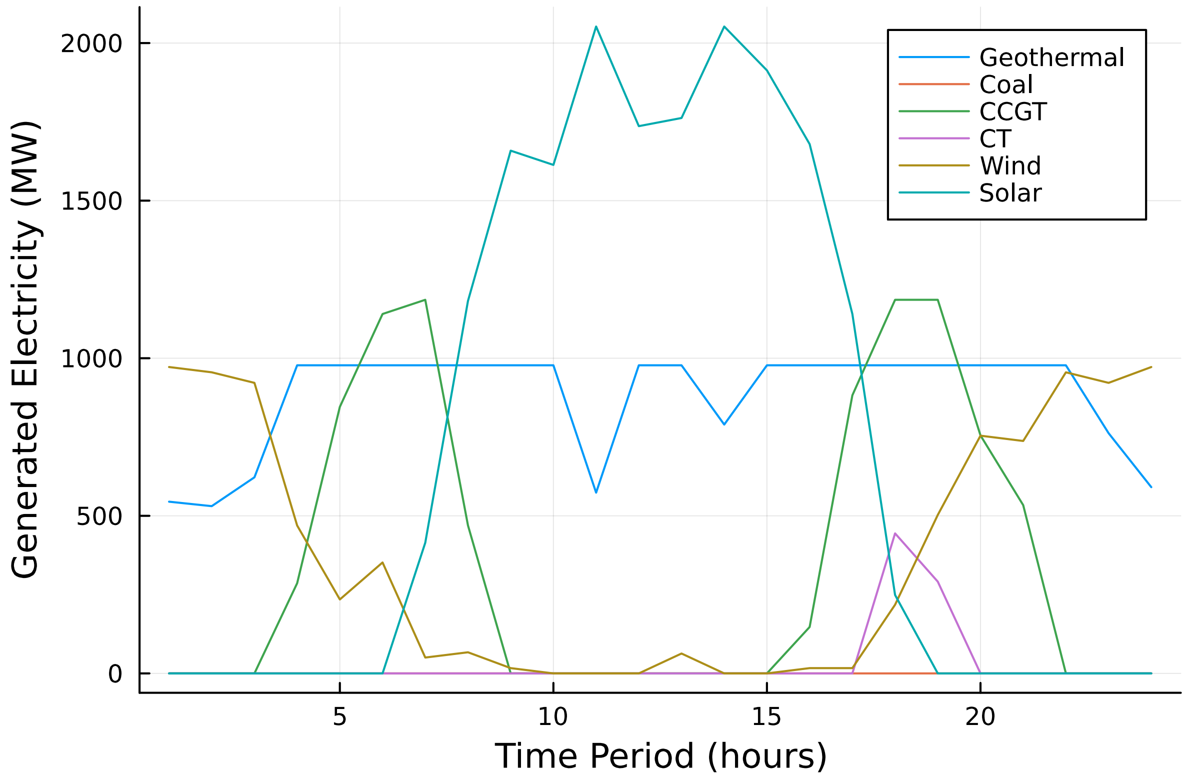
16	hour 16	2821.0	0.0	2821
17	hour 17	3017.0	0.0	3017
18	hour 18	3074.0	0.0	3074
19	hour 19	2957.0	0.0	2957
20	hour 20	2487.0	0.0	2487
21	hour 21	2249.0	0.0	2249
22	hour 22	1933.0	0.0	1933
23	hour 23	1684.0	0.0	1684
24	hour24	1563.0	0.0	1563

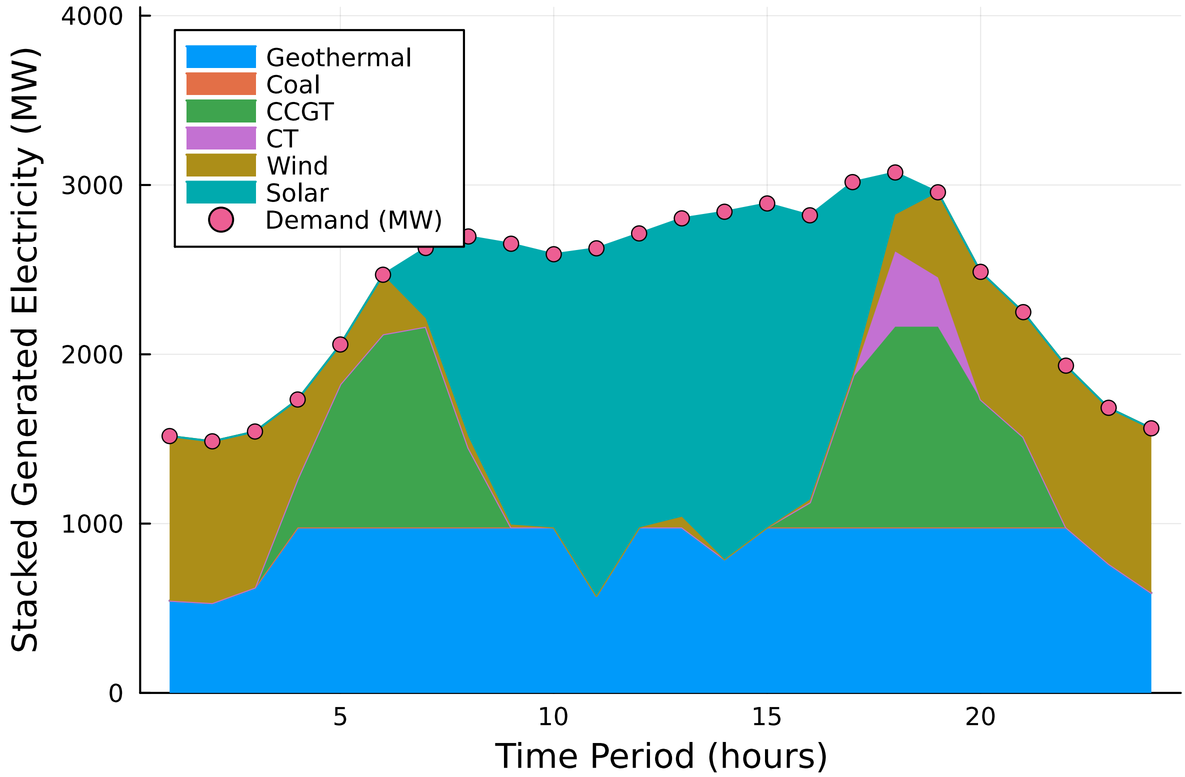
Problem 2.4

```
julia> using Plots

julia> genCO2=value.(yCO2).data;

julia> geoHourCO2=genCO2[1,:];

julia> coalHourCO2=genCO2[2,:];
```

```
julia> CCGTHourCO2=genCO2[3,:];

julia> CTHOURCO2=genCO2[4,:];

julia> windHourCO2=genCO2[5,:];

julia> solarHourCO2=genCO2[6,:];

julia> plot(geoHourCO2, label="Geothermal", legend=:topright,
ylabel="Generated Electricity (MW)", xlabel="Time Period (hours)");

julia> plot!(coalHourCO2, label="Coal");

julia> plot!(CCGTHourCO2, label="CCGT");

julia> plot!(CTHOURCO2, label="CT");

julia> plot!(windHourCO2, label="Wind");

julia> plot!(solarHourCO2, label="Solar")
```




```
julia> areaplot(genCO2', label=["Geothermal" "Coal" "CCGT" "CT" "Wind"
"Solar"]);

julia> scatter!(demand, label="Demand (MW)", legend=:topleft,ylims=(0,4050),
ylabel="Stacked Generated Electricity (MW)", xlabel="Time Period (hours)")
```



The carbon dioxide emissions constraint definitly has an effect on the linear program. The main difference compared to the previous formulation is that in this solution, there is a lot more geothermal. This geothermal generation replaces much of the CCGT and CT generation in the previous solution, as geothermal does not produce carbon dioxide while CCGT and CT do. Geothermal generation seems to provide baseline production throughout the day, while wind and solar make up a lot of the remaining demand when each resource is available. CCGT and CT make up the rest of the demand that geothermal, solar, and wind cannot achieve. Again coal is not installed, this time especially because of its high CO_2 emissions rate.

Problem 2.5

```
julia> using JuMP, HiGHS

julia> shadow_price(CO2)
-130.22112610691698
```

The shadow price of CO2 is the marginal cost of increasing CO_2 emissions limit by 1 tCO_2/yr. Therefore the value to the utility of allowing it to emit an additional 1000 tCO_2/yr is $130221.

References