BEE 4750/5750 Homework 4

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Problem 1

Before tackling individual components, the given information has been entered in Julia for reference:

```
#Facility Information
facilities = ["WTE", "MRF", "LF"]; #facility types
capacities = [150, 350, 200]; #Mg/day
costs_fixed = [2500, 1500, 2000]; #dollars/day
tipping_fees = [60, 7, 50]; #dollars/Mg
costs_recycling = [0, 45, 0]; #dollars/Mg recycled

#Relative Distances
distance_city1 = [15, 5, 30]; #km
distance_city2 = [10, 15, 25]; #km
distance_WTE = [0, 15, 18]; #km
distance_MRF = [15, 0, 32]; #km
distance_LF = [18, 32, 0]; #km

#Transportation Costs
cost_transport = 1.5; #dollars/Mg/km

#Waste Production
solid_waste = [100, 170]; #Mg/day

#Waste Composition and Properties
components =
["Food","Paper","Plastic","Textile","Rubber","Wood","Yard","Glass","Fe","Al","Metal","Misc"]
mass_percent = [15, 40, 5, 3, 2, 5, 18, 4, 2, 2, 1, 3];
ash_percent = [8, 7, 5, 10, 15, 2, 2, 100, 100, 100, 100, 70];
rec_percent = [0, 55, 15, 10, 0, 30, 40, 60, 75, 80, 50, 0];
```

Problem 1.1

The following Julia code calculates the overall recycling and ash fractions for the waste produced by each city:

```
rec_frac = mass_percent'*rec_percent / 10000;
ash_frac = mass_percent'*ash_percent / 10000;
```

This gives us a recycling rate of 37.75% and an ash fraction of 16.41%.

Problem 1.2

The decision variables for this optimization problem are:

1. Waste transported from city i to disposal j in Mg/day, W_{ij}
2. Residual waste transported from disposal k to disposal j in Mg/day, R_{kj}
3. Operational status of disposal j (a binary variable), Y_j

Problem 1.3

The objective function will be to minimize the sum of disposal costs and transportation costs:

$$Cost = \sum_i \sum_j a_{ij} l_{ij} W_{ij} + \sum_j [c_j Y_j + \sum_i b_j W_{ij}]$$

where a_{ij} is the cost of transporting waste from source i to disposal j in dollars/Mg-km, l_{ij} is the distance between sources i and disposal j in km, c_j is the fixed cost of operating disposal j in dollars/day, and b_j is the variable cost of disposing waste at disposal j in dollars/Mg.

For the specifics of this problem, this translates to:

1. Waste-to-Energy: $2500Y_1 + 60(W_{11} + W_{21} + R_{21})$
2. Material Recovery: $1500Y_2 + 7(W_{21} + W_{22}) + 0.3775(45)(W_{12} + W_{22})$
3. Landfill: $2000Y_3 + 50(W_{13} + W_{23} + R_{13} + R_{23})$
4. Transportation: $1.5[15W_{11} + 5W_{12} + 30W_{13} + 10W_{21} + 15W_{22} + 25W_{23} + 18R_{13} + 15R_{21} + 32R_{23}]$

Then the total cost is: $2500Y_1 + 1500Y_2 + 2000Y_3 + 82.5W_{11} + 31.4875W_{12} + 95W_{13} + 75W_{21} + 46.4875W_{22} + 87.5W_{23} + 77R_{13} + 82.5R_{21} + 98R_{23}$

Problem 1.4

Constraints can be organized into city mass balance, capacity mass balance, recycling rate and residual ash constraints, and committment and nonnegativity constraints.

City Mass Balance Constraints: The sum of waste sent to each facility must equal the waste produced by each city.

- City 1: $W_{11} + W_{12} + W_{13} = 100$
- City 2: $W_{21} + W_{22} + W_{23} = 170$

Capacity Mass Balance Constraints: The total weights sent to each facility must not exceed their capacities.

- WTE: $W_{11} + W_{21} + R_{21} \leq 150$
- MRF: $W_{12} + W_{22} \leq 350$
- LF: $W_{13} + W_{23} + R_{23} + R_{13} \leq 200$

Recycling Rate and Residual Ash Constraints: The waste sent from the WTE must be equal to the ash weight produced, and the weight sent from the MRF must be the non-recycled waste that entered the facility.

- Residual Ash: $R_{13} = 0.1641(W_{11} + W_{21} + R_{21})$
- Recycling Rate: $R_{21} + R_{23} = (1 - 0.3775)(W_{12} + W_{22})$

Commitment and Non-Negativity: Defining the binary variables and making all waste streams nonnegative

- If $W_{11} + W_{21} + R_{21} = 0$, $Y_1 = 0$, else $Y_1 = 1$
- If $W_{21} + W_{22} = 0$, $Y_1 = 0$, else $Y_1 = 1$
- The landfill must be on: $Y_3 = 1$
- Nonnegativity: $W_{ij}, R_{ij} \geq 0$

Problem 1.5

```julia
using JuMP
using Cbc

waste = Model(Cbc.Optimizer)
I = 1:2; #number of cities
J = 1:3; #number of disposal sites

@variable(waste, W[i in I, j in J] >= 0);
@variable(waste, R[k in J, j in J] >= 0);
@variable(waste, Y[j in J], Bin);
@objective(waste, Min, sum([82.5 31.4875 95; 75 46.4875 87.5].*W)+sum([0 0 77;
82.5 0 98; 0 0 0].*R) + sum(costs_fixed .* Y));

# mass-balance
@constraint(waste, city[i in I], sum(W[i,:]) == solid_waste[i]);
@constraint(waste, wte, W[1,1] + W[2,1] + R[2,1] <= capacities[1]);
@constraint(waste, mrf, W[1,2] + W[2,2]  <= capacities[2]);
@constraint(waste, lf, W[1,3] + W[2,3] + R[2,3] + R[1,3] <= capacities[3]);

# residuals
@constraint(waste, resid1, R[1,3] == ash_frac .* (W[1,1] + W[2,1] + R[2,1]));
```