BEE 4750 Homework 5: Solid Waste Disposal

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> **Due Date**
>
> Friday, 11/10/23, 9:00pm

Overview

Instructions

- In Problem 1, you will formulate, solve, and analyze a standard generating capacity expansion problem.
- In Problem 2, you will add a CO_2 constraint to the capacity expansion problem and identify changes in the resulting solution.

Load Environment

The following code loads the environment and makes sure all needed packages are installed. This should be at the start of most Julia scripts.

```
import Pkg
Pkg.activate(@__DIR__)
Pkg.instantiate()
using JuMP
using HiGHS
using DataFrames
using GraphRecipes
using Plots
using Measures
using MarkdownTables
```

In [ ]:
```
import Pkg
Pkg.activate(@__DIR__)
Pkg.instantiate()
```

```
  Activating project at `~/Desktop/BEE4750-2/hw05-jrmarcuse`
```

In [ ]:
```
using JuMP
using HiGHS
using DataFrames
```

```
using GraphRecipes
using Plots
using Measures
using MarkdownTables
```

Background

Three cities are developing a coordinated municipal solid waste (MSW) disposal plan. Three disposal alternatives are being considered: a landfill (LF), a materials recycling facility (MRF), and a waste-to-energy facility (WTE). The capacities of these facilities and the fees for operation and disposal are provided in the table below.

Disposal Facility	Capacity (Mg/d)	Fixed cost ($/d)	**Tipping Fee ** ($/Mg)	Recycling Cost ($/Mg)
Landfill	200	2000	50	
Materials Recycling Facility	350	1500	7	40 (per Mg recycled)
Waste-to-Energy Facility	210	2500	60	

Transportation costs are $1.5/Mg-km, and the relative distances between the cities and facilities are provided in the table below.

City/Facility	Landfill (km)	MRF (km)	WTE (km)
1	5	30	15
2	15	25	10
3	13	45	20
LF	-	32	18
MRF	32	-	15
WTE	18	15	-

The fixed costs associated with the disposal options are incurred only if the particular disposal option is implemented. The three cities produce 100, 90, and 120 Mg/day of solid waste, respectively, with the composition provided in the table below.

Component	% of total mass	Combustion ash (%)	MRF Recycling rate (%)
Food Wastes	15	8	0
Paper & Cardboard	40	7	55
Plastics	5	5	15
Textiles	3	10	10
Rubber, Leather	2	15	0

Component	% of total mass	Combustion ash (%)	MRF Recycling rate (%)
Wood	5	2	30
Yard Wastes	18	2	40
Glass	4	100	60
Ferrous	2	100	75
Aluminum	2	100	80
Other Metal	1	100	50
Miscellaneous	3	70	0

The information in the above table will help you determine the overall recycling and ash fractions. Note that the recycling residuals, which may be sent to either landfill or the WTE, have different ash content than the ash content of the original MSW. You will need to determine these fractions to construct your mass balance constraints.

Reminder: Use `round(x; digits=n)` to report values to the appropriate precision!

Problems (Total: 40 Points)

Problem 1 (22 points)

In this problem, you will develop an optimal disposal plan for the two cities.

In [ ]:
```
#Loading in Data from above
Disposal_Facility = [200 2000 50 0; 350 1500 7 40; 210 2500 60 0];
City_Facility = [5 30 15; 15 25 10; 13 45 20; 0 32 18; 32 0 15; 18 15 0];
Component = [15 8 0; 40 7 55; 5 5 15; 3 10 10; 2 15 0; 5 2 30; 18 2 40;
4 100 60; 2 100 75; 2 100 80; 1 100 50; 3 70 0];
```

Problem 1.1 (3 points)

Based on the information above, calculate the overall recycling and ash fractions for the waste produced by each city.

In [ ]:
```
#Overall recycling percent will be the weighted average of the
    #recycling rates
recyc_fraction = sum(0.01*Component[:,1].*Component[:,3]);
#Overall ash fraction for the city
ash_fraction_city = sum(0.01*Component[:,1].*Component[:,2]);
#Overall ash fraction for residuals
ash_fraction_residual = sum(0.01*Component[:,1].*Component[:,2]
.*[100 - i for i in Component[:,3]])/sum(0.01*Component[:,1]
.*[100 - i for i in Component[:,3]]);
```

Problem 1.2 (2 points)

What are the decision variables for your optimization problem? Provide notation and variable meaning.

My decision variables will be a 3x3 matrix of waste transported values connecting all cities to all disposals. Each city connects to 3 disposals and there are 3 cities which is why it will be a matrix W of size 3x3 where in Wij i is the city number and j is the disposal number.

Next the residual waste transported from disposal k to disposal j will be a matrix R of size 3x3 where in Rkj k is the first disposal number and j is the first disposal number. In this matrix there will be a constraint that states that when k=j R=0 as there is no transportation within the same disposal site, and then the transportation from the landfill to MRF and WTE will be 0 as the landfill is an end destination. And finally, the transportation from WTE to MRF will also be zero as you cannot recycle waste that has already been combusted in the MRF

Finally a binary decision variable for if the disposal facility is in use must be implemented to determine if fixed cost must be included. This will be variable Yj for j in 1:3 where j is the disposal number and Y is 1 if any waste is transported into the disposal facility, and 0 if no waste is transported in.

To summarize the decision variables are Wij for i in 1:3 and j in 1:3; Rkj for k in 1:3 and j in 1:3, and Yj for j in 1:3. i is the city number 1:3 and j is the disposal facility number 1:3 where 1 is LF, 2 is MRF and 3 is WTE.

Problem 1.3 (3 points)

Formulate the objective function. Make sure to include any needed derivations or justifications for your equation(s).

The objective function will be the total transportation, disposal and recycling costs summed up. The transportation cost will be the transportation cost rate (a) multiplied by the distance (l) between locations multiplied by the mass of waste transported (W) or sum(a*l*W). This will also apply to the transportation between disposal facilities so we will need a distance matrix for distances between disposal facilities (lr) and then the same equation applies - sum(a*lr*R). The Disposal cost will be the fixed cost of operating the disposal (c) facility multiplied by the decision variabel to operate the disposal facility (Y) plus the sum of the tipping fee or variable cost per mass transported (b) multiplied by the waste transported (W) as well as the waste transported between disposal facilities R or to simplify: sum(c*Y)+sum(b*W)+sum(b*W). Finally the recyclinc cost will be the total amount of waste transported into the waste recycling facility (sum(W[:,2])) multiplied by the recycing fraction multiplied by the cost of recycling or to summarize sum(W[:,2])*recyc_fraction*recyc_cost.

To put it all together:

Total Cost = sum(a*l*W)+sum(a*lr*R) +sum(c*Y)+sum(b*W) +sum(b*R)+sum(W[:,2])*recyc_fraction*recyc_cost

The final simplified objective function given by the model is as follow:

57.5 W[1,1] + 72.5 W[2,1] + 69.5 W[3,1] + 67.1 W[1,2] + 59.6 W[2,2] + 89.6 W[3,2] + 82.5 W[1,3] + 75 W[2,3] + 90 W[3,3] + 98 R[2,1] + 77 R[3,1] + 55 R[1,2] + 29.5 R[3,2] + 87 R[1,3] + 82.5 R[2,3] + 2000 Y[1] + 1500 Y[2] + 2500 Y[3] + 50 R[1,1] + 7 R[2,2] + 60 R[3,3]

Problem 1.4 (4 points)

Derive all relevant constraints. Make sure to include any needed justifications or derivations.

The constraint for the amount of ash transported from the waste to energy to landfill will be as follows: the ash fraction multiplied by the sum of all inputs into the WTE disposal facility accounting for different ash fractions for the city and the residual wastes or - ash_fraction_city*(W13+W23+W33)+ash_fraction_residual*R23

The constraint for the mass balance of waste at the recycling facility will be the total amount of waste leaving the recycling facility will be equal to one minus the recycling fraction multiplied by the mass of the total inputs to the recycling facility or - R21+R23 = (1-recyc_fraction)*(W12+W22+W32)

The general mass balance constraints will have the total waste leaving each city must be equal to the total production of solid waste from each city or - sum(W[i,:])=Si for i in 1:3 where i is the city number.

The commitments constraint (Y) will be a binary constraint that states whether or not the disposal facility is being operated. The value will be 0 only if the sum of all waste streams entering the disposal facility is 0, and otherwise it will be 1 or Yi = 0 if sum(W[i,:])+sum(R[i,:])=0, else=1 for i in 1:3. This will be implemented using the large M method.

The capacity constraints for the disposal facility will state that the sum of all waste streams into the disposal facility will be less than or equal to that facilities capacity or - sum(W[:,j])+sum(R[:,j])<=Kj for j in 1:3

The residuals that must be zero constraint will take the sum of all the R values that must be zero which is 6 out of the 9 of them and set that equal to zero or - R11+R22+R33+R12+R13+R31=0

Finally the non-negative constraint will state that W>=0 and R>=0

Problem 1.5 (3 points)

Implement your optimization problem in `JuMP`.

In [ ]:
```
s = [100 90 120]; #vector of waste produced in each city in Mg/day
k = Disposal_Facility[:,1];#Vector of capacities of each disposal facility
l = City_Facility[1:3,:];#matrix of transportation distances between cities
    #and disposals
lr = City_Facility[4:6,:]; #matrix of transportation distances between
    #disposal facilities
a = 1.5; #transport cost rate in $/Mg-km
c = Disposal_Facility[:,2]; #fixed cost of disposal operation $/day
b = [50 7 60]; #variable cost of disposal from tipping fee $/Mg
recyc_cost = Disposal_Facility[2,4];#Cost in $/Mg for recycling solid waste
M = 1000000; #M value for Big-M reformulation for Y constraint
I = 1:3; #city numbered city 1,2&3
J = 1:3; #disposal number where 1 is LF, 2 is MRF & 3 is WTE
K = 1:3; #disposal secondary number for residuals where 1 is LF,
    #2 is MRF & 3 is WTE

#initiate model
waste_mod = Model(HiGHS.Optimizer)

#initiate decision variables
@variable(waste_mod,W[i in I, j in J]>=0);
#waste transported between cities and disposal facilities
@variable(waste_mod,R[k in K, j in J]>=0);
#residual waste transported between disposal facilities
@variable(waste_mod,Y[j in J],binary=true);
#binary variable for if disposal facility is being operated

#Formulate the objective function to minimize total cost
    #Transportation costs + Disposal costs + Recycling costs
@objective(waste_mod,Min,sum(a*l.*W)+sum(a*lr.*R)+sum(c.*Y)+sum(b.*W)
+sum(b.*R)+sum(W[:,2])*0.01*recyc_fraction*recyc_cost);

#Add all constraints
@constraint(waste_mod, Ash, R[3,1]==0.01*ash_fraction_city*(sum(W[:,3]))
+0.01*ash_fraction_residual*R[2,3]); #ash fraction to landfill from WTE
@constraint(waste_mod, Rcyc, R[2,1]+R[2,3]==(1-0.01*recyc_fraction)
*sum(W[:,2])); #waste in and out mass balance at MRF
@constraint(waste_mod, Mass_Bal[i in I], sum(W[i,:])==s[i]);
#mass balance of total waste leaving each city equal to waste
    #produced by each city
@constraint(waste_mod,commit[j in J],M*Y[j]>=sum(W[:,j])+sum(R[:,j]));
#If the variable Y is 0 then the sum of waste streams into the
    #disposal facility must be 0
@constraint(waste_mod,cap[j in J],sum(W[:,j])+sum(R[:,j])<=k[j]);
#total inputs into disposal facilities must be less than the capacity
@constraint(waste_mod, residuals,R[1,1]+R[2,2]+R[3,3]+R[1,2]+R[1,3]
+R[3,2]==0);
#All of these residuals must be zero

optimize!(waste_mod)
```

```
Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
Presolving model
10 rows, 15 cols, 43 nonzeros
8 rows, 13 cols, 35 nonzeros

Solving MIP model with:
   8 rows
   13 cols (2 binary, 0 integer, 0 implied int., 11 continuous)
   35 nonzeros

        Nodes      |    B&B Tree     |            Objective Bounds
|  Dynamic Constraints |       Work
     Proc. InQueue |  Leaves   Expl. | BestBound       BestSol
Gap |   Cuts   InLp Confl. | LpIters     Time

         0       0         0   0.00%   2000            inf
inf        0      0      0         0     0.0s
 S       0       0         0   0.00%   2000            27855.482115      92.
82%        0      0      0         0     0.0s
         0       0         0   0.00%   26009.563771    27855.482115       6.
63%        0      0      0         5     0.0s
 T       0       0         0   0.00%   26009.563771    27855.482115       6.
63%       10      1      0         7     0.0s

Solving report
  Status            Optimal
  Primal bound      27855.4821151
  Dual bound        27855.4821151
  Gap               0% (tolerance: 0.01%)
  Solution status   feasible
                    27855.4821151 (objective)
                    0 (bound viol.)
                    0 (int. viol.)
                    0 (row viol.)
  Timing            0.00 (total)
                    0.00 (presolve)
                    0.00 (postsolve)
  Nodes             1
  LP iterations     7 (total)
                    0 (strong br.)
                    1 (separation)
                    0 (heuristics)
```

In [ ]:

```
display(objective_function(waste_mod))
display(value.(W))
display(value.(R))
```

$$57.5W_{1,1} + 72.5W_{2,1} + 69.5W_{3,1} + 67.1W_{1,2} + 59.6W_{2,2} + 89.6W_{3,2} + 82.5W_{1,3} + 75W_{2,3} + 90W_{3,3} + 9$$
$$+ 77R_{3,1} + 55R_{1,2} + 29.5R_{3,2} + 87R_{1,3} + 82.5R_{2,3} + 2000Y_1 + 1500Y_2 + 2500Y_3 + 50R_{1,1} + 7R_{2,2} + 6$$

```
2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, 1:3
    Dimension 2, 1:3
And data, a 3×3 Matrix{Float64}:
 100.0      0.0   0.0
  -0.0     -0.0  90.0
  78.4053   0.0  41.5947
```

```
2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, 1:3
    Dimension 2, 1:3
And data, a 3×3 Matrix{Float64}:
  0.0      0.0  0.0
  0.0      0.0  0.0
 21.5947  0.0  0.0
```

Problem 1.6 (2 points)

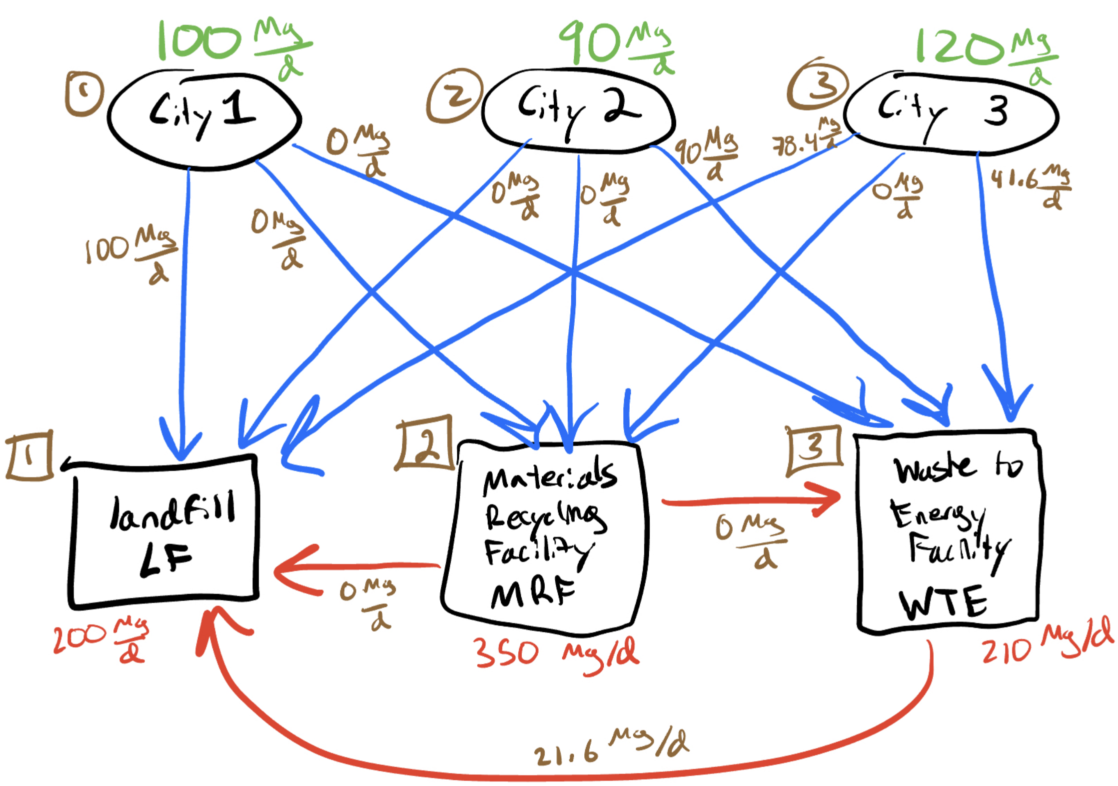
Find the optimal solution. Report the optimal objective value.

In [ ]:
```
obj = round.(objective_value(waste_mod);digits=2)
println("The objective function value is \$$obj")
```

```
The objective function value is $27855.48
```

Problem 1.7 (5 points)

Draw a diagram showing the flows of waste between the cities and the facilities. Which facilities (if any) will not be used? Does this solution make sense?



In [ ]:
```
display(value.(Y))
```

```
1-dimensional DenseAxisArray{Float64,1,...} with index sets:
    Dimension 1, 1:3
And data, a 3-element Vector{Float64}:
  1.0
 -0.0
  1.0
```

The Materials Recycling Facility will not be used which makes sense as all of the waste can go direct to the landfill or waste to energy without the model incurring additional recycling costs for waste. There should be added constraints to account for the environmental benefits of recycling. The solution therefore makes sense as it is sending the waste to the cheapest disposal stream given the loads from each city and the capacity constraints of each disposal facility.

Problem 2 (18 points)

It is projected that in the near future the state will introduce a carbon tax that will increase the cost for transportation and for disposal by incineration. It is estimated that the additional costs will be:

- tipping fee for the WTE facility will increase to$75/Mg; and
- transportation costs will increase to $2/Mg-km.

In this context, the cities are considering adding another landfill and want to know if this would be cost-effective compared to using the current facilities with the carbon tax. This landfill would have a maximum capacity of 100 Mg/day and would be located with the following distances from the existing sites (excluding LF1):

City/Facility	Distance to LF2 (km)
1	45
2	35
3	15
MRF	35
WTE	50

The fixed cost of operating this facility would be the same as the first landfill, but the tipping cost would be increased to $60/Mg-day.

Problem 2.1 (5 points)

What changes are needed to your optimization program from Problem 1 for this decision problem? Formulate any different variables, objectives, and/or constraints.

The first thing to do will be to rerun the model from before, but change the value of the tipping fee for the WTE facility and transportation costs to account for the carbon tax.

This will be used as a baseline comparison to see if it is worth adding the second landfill.

Next, the model will have to be rebuilt to account for the extra landfill. The decision variables will all include an extra dimension. Previously the j in J dimension added to each variable was 1:3 but now it will be 1:4 and the dimension k in K will also be 1:4 due to the existence of a fourth disposal facility. Additional constraints will be needed to set other residual transportations to 0, the mass balance constraint will need to account for the waste transfer to the third disposal site, and the binary commitments constraint will need an added dimensions for the second landfill. Finally, a capacity constraint will have to be included for the second landfill as well.

The input data will also have to be added to include distances between the second landfill and other sites, the increased WTE tipping fee, increased transportation cost, maximum capacity of the landfill and tipping cost at the second landfill as well.

To summarize: The changes involve adding data to the Disposal_Facility and City_Facility matrices, adding dimensions to the decision variables W, R & Y, updating the ash contrant to have the residual transport from WTE to both landfills or $R[3,1] + R[3,4] = 0.01(ashfractioncity)(sum(W[:,3])) + (ashfractionresidual)(I$, updating J and K to be 1:4 instead of 1:3 which updates the dimensions of the decision variables and constraints, adding an additional flow from the recycling facility to landfill 2 for the recycling constraint, and finally updating the R matrix to ensure all the values that must be 0 remain zero using the equation R[1,1]+R[2,2]+R[3,3]+R[4,4]+R[1,2]+R[1,3]+R[1,4]+R[3,2]+R[4,1]+R[4,2]+R[4,3] = 0

In [ ]:
```
#Baseline no extra landfill and changes transport and tipping cost:
Disposal_Facility = [200 2000 50 0; 350 1500 7 40; 210 2500 75 0];
City_Facility = [5 30 15; 15 25 10; 13 45 20; 0 32 18; 32 0 15; 18 15 0];

s = [100 90 120]; #vector of waste produced in each city in Mg/day
k = Disposal_Facility[:,1]; #Vector of capacities of each disposal facility
l = City_Facility[1:3,:]; #matrix of transportation distances between
    #cities and disposals
lr = City_Facility[4:6,:]; #matrix of transportation distances between
    #disposal facilities
a = 2; #transport cost rate in $/Mg-km
c = Disposal_Facility[:,2]; #fixed cost of disposal operation $/day
b = [50 7 75]; #variable cost of disposal from tipping fee $/Mg
recyc_cost = Disposal_Facility[2,4];#Cost in $/Mg for recycling solid waste
M = 1000000; #M value for Big-M reformulation for Y constraint
I = 1:3; #city numbered city 1,2&3
J = 1:3; #disposal number where 1 is LF, 2 is MRF & 3 is WTE
K = 1:3; #disposal secondary number for residuals where 1 is LF,
    #2 is MRF & 3 is WTE

#initiate model
waste_mod2 = Model(HiGHS.Optimizer)

#initiate decision variables
```

```
@variable(waste_mod2,W[i in I, j in J]>=0);
#waste transported between cities and disposal facilities
@variable(waste_mod2,R[k in K, j in J]>=0);
#residual waste transported between disposal facilities
@variable(waste_mod2,Y[j in J],binary=true);
#binary variable for if disposal facility is being operated

#Formulate the objective function to minimize total cost
    #Transportation costs + Disposal costs + Recycling costs
@objective(waste_mod2,Min,sum(a*l.*W)+sum(a*lr.*R)+sum(c.*Y)+sum(b.*W)
+sum(b.*R)+sum(W[:,2])*0.01*recyc_fraction*recyc_cost);

#Add all constraints
@constraint(waste_mod2, Ash, R[3,1]==0.01*ash_fraction_city*(sum(W[:,3]))
+0.01*ash_fraction_residual*R[2,3]); #ash fraction to landfill from WTE
@constraint(waste_mod2, Rcyc, R[2,1]+R[2,3]==
(1-0.01*recyc_fraction)*sum(W[:,2])); #waste in and out mass balance at MRF
@constraint(waste_mod2, Mass_Bal[i in I], sum(W[i,:])==s[i]);
#mass balance of total waste leaving each city equal to waste
    #produced by each city
@constraint(waste_mod2,commit[j in J],M*Y[j]>=sum(W[:,j])+sum(R[:,j]));
#If the variable Y is 0 then the sum of waste streams into the disposal
    #facility must be 0
@constraint(waste_mod2,cap[j in J],sum(W[:,j])+sum(R[:,j])<=k[j]);
#total inputs into disposal facilities must be less than the capacity
@constraint(waste_mod2, residuals,R[1,1]+R[2,2]+R[3,3]+R[1,2]+R[1,3]
+R[3,2]==0); #All of these residuals must be zero

optimize!(waste_mod2)
```

```
Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
Presolving model
10 rows, 15 cols, 43 nonzeros
8 rows, 13 cols, 35 nonzeros

Solving MIP model with:
   8 rows
   13 cols (2 binary, 0 integer, 0 implied int., 11 continuous)
   35 nonzeros

        Nodes      |    B&B Tree     |            Objective Bounds
|  Dynamic Constraints |       Work
     Proc. InQueue |  Leaves   Expl. | BestBound       BestSol
Gap |   Cuts   InLp Confl. | LpIters     Time

         0       0         0   0.00%   2000            inf
inf        0      0      0         0     0.0s
 S       0       0         0   0.00%   2000            31649.336045      93.
68%        0      0      0         0     0.0s
         0       0         0   0.00%   29803.417701    31649.336045       5.
83%        0      0      0         5     0.0s
 T       0       0         0   0.00%   29803.417701    31649.336045       5.
83%       10      1      0         7     0.0s

Solving report
  Status            Optimal
  Primal bound      31649.336045
  Dual bound        31649.336045
  Gap               0% (tolerance: 0.01%)
  Solution status   feasible
                    31649.336045 (objective)
                    0 (bound viol.)
                    0 (int. viol.)
                    0 (row viol.)
  Timing            0.00 (total)
                    0.00 (presolve)
                    0.00 (postsolve)
  Nodes             1
  LP iterations     7 (total)
                    0 (strong br.)
                    1 (separation)
                    0 (heuristics)
```

In [ ]:

```julia
obj2 = round.(objective_value(waste_mod2);digits=2)
println("The baseline objective function value is \$$obj2.
This is for no additional landfill, but an increase in
transportation and tipping costs.")
```

```
The baseline objective function value is $31649.34.
This is for no additional landfill, but an increase in
transportation and tipping costs.
```

Problem 2.2 (3 points)

Implement the new optimization problem in `JuMP`.

In [ ]:
```
#Baseline no extra landfill and changes transport and tipping cost:
Disposal_Facility = [200 2000 50 0; 350 1500 7 40;
210 2500 75 0; 100 2000 60 0];
City_Facility = [5 30 15 45; 15 25 10 35; 13 45 20 15;
0 32 18 0; 32 0 15 35; 18 15 0 50; 0 35 50 0];
###Start up here
s = [100 90 120]; #vector of waste produced in each city in Mg/day
k = Disposal_Facility[:,1]; #Vector of capacities of each disposal facility
l = City_Facility[1:3,:]; #matrix of transportation distances between
    #cities and disposals
lr = City_Facility[4:7,:]; #matrix of transportation distances between
    #disposal facilities
a = 2; #transport cost rate in $/Mg-km
c = Disposal_Facility[:,2]; #fixed cost of disposal operation $/day
b = [50 7 75 60]; #variable cost of disposal from tipping fee $/Mg
recyc_cost = Disposal_Facility[2,4];#Cost in $/Mg for recycling solid waste
M = 1000000; #M value for Big-M reformulation for Y constraint
I = 1:3; #city numbered city 1,2&3
J = 1:4; #disposal number where 1 is LF, 2 is MRF & 3 is WTE
K = 1:4; #disposal secondary number for residuals where 1 is LF,
#2 is MRF & 3 is WTE

#initiate model
waste_mod3 = Model(HiGHS.Optimizer)

#initiate decision variables
@variable(waste_mod3,W[i in I, j in J]>=0);
#waste transported between cities and disposal facilities
@variable(waste_mod3,R[k in K, j in J]>=0);
#residual waste transported between disposal facilities
@variable(waste_mod3,Y[j in J],binary=true);
#binary variable for if disposal facility is being operated

#Formulate the objective function to minimize total cost
    #Transportation costs + Disposal costs + Recycling costs
@objective(waste_mod3,Min,sum(a*l.*W)+sum(a*lr.*R)+sum(c.*Y)+sum(b.*W)
+sum(b.*R)+sum(W[:,2])*0.01*recyc_fraction*recyc_cost);

#Add all constraints
@constraint(waste_mod3, Ash, R[3,1]+R[3,4]==
0.01*ash_fraction_city*(sum(W[:,3]))
+0.01*ash_fraction_residual*R[2,3]); #ash fraction to landfill from WTE
@constraint(waste_mod3, Rcyc, R[2,1]+R[2,3]+R[2,4]==(1-0.01*recyc_fraction)
*sum(W[:,2])); #waste in and out mass balance at MRF
@constraint(waste_mod3, Mass_Bal[i in I], sum(W[i,:])==s[i]);
#mass balance of total waste leaving each city equal to waste produced by
    #each city
@constraint(waste_mod3,commit[j in J],M*Y[j]>=sum(W[:,j])+sum(R[:,j]));
#If the variable Y is 0 then the sum of waste streams into the disposal
    #facility must be 0
@constraint(waste_mod3,cap[j in J],sum(W[:,j])+sum(R[:,j])<=k[j]);
#total inputs into disposal facilities must be less than the capacity
@constraint(waste_mod3, residuals,R[1,1]+R[2,2]+R[3,3]+R[4,4]+R[1,2]
+R[1,3]+R[1,4]+R[3,2]+R[4,1]+R[4,2]+R[4,3]==0);
    #All of these residuals must be zero
```

```
optimize!(waste_mod3)
```

```
Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
Presolving model
12 rows, 21 cols, 59 nonzeros
12 rows, 21 cols, 59 nonzeros

Solving MIP model with:
   12 rows
   21 cols (4 binary, 0 integer, 0 implied int., 17 continuous)
   59 nonzeros

        Nodes      |    B&B Tree     |            Objective Bounds
|  Dynamic Constraints |       Work
     Proc. InQueue |  Leaves   Expl. | BestBound       BestSol
Gap |   Cuts   InLp Confl. | LpIters     Time

         0       0         0   0.00%   0               inf
inf        0      0      0         0     0.0s
 S       0       0         0   0.00%   0               30568.278502     100.
00%        0      0      0         0     0.0s
         0       0         0   0.00%   25231.281237    30568.278502      17.
46%        0      0      0         8     0.0s

25.0% inactive integer columns, restarting
Model after restart has 11 rows, 20 cols (3 bin., 0 int., 0 impl., 17 con
t.), and 53 nonzeros

         0       0         0   0.00%   29592.06329     30568.278502       3.
19%        3      0      0        16     0.0s
         0       0         0   0.00%   29592.06329     30568.278502       3.
19%        3      2      0        18     0.0s

Solving report
  Status            Optimal
  Primal bound      30568.2785022
  Dual bound        30568.2785022
  Gap               0% (tolerance: 0.01%)
  Solution status   feasible
                    30568.2785022 (objective)
                    0 (bound viol.)
                    0 (int. viol.)
                    0 (row viol.)
  Timing            0.01 (total)
                    0.00 (presolve)
                    0.00 (postsolve)
  Nodes             1
  LP iterations     25 (total)
                    0 (strong br.)
                    12 (separation)
                    3 (heuristics)
```

In [ ]:
```
display(objective_function(waste_mod3))
display(value.(W))
```

```
display(value.(R))
display(value.(Y))
```

$$60W_{1,1} + 80W_{2,1} + 76W_{3,1} + 82.1W_{1,2} + 72.1W_{2,2} + 112.1W_{3,2} + 105W_{1,3} + 95W_{2,3} + 115W_{3,3} + 150W$$
$$+ 130W_{2,4} + 90W_{3,4} + 114R_{2,1} + 86R_{3,1} + 71R_{1,2} + 37R_{3,2} + 77R_{4,2} + 111R_{1,3} + 105R_{2,3} + 175R_{4,3} +$$
$$+ 160R_{3,4} + 2000Y_1 + 1500Y_2 + 2500Y_3 + 2000Y_4 + 50R_{1,1} + 50R_{4,1} + 7R_{2,2} + 75R_{3,3} + 60R_{1,4} + 60R$$

```
2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, 1:3
    Dimension 2, 1:4
And data, a 3×4 Matrix{Float64}:
 100.0      0.0   0.0       0.0
  78.0368  -0.0  11.9632    0.0
  20.0      0.0   0.0     100.0
2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, 1:4
    Dimension 2, 1:4
And data, a 4×4 Matrix{Float64}:
 0.0      0.0  0.0  0.0
 0.0      0.0  0.0  0.0
 1.96315  0.0  0.0  0.0
 0.0      0.0  0.0  0.0
1-dimensional DenseAxisArray{Float64,1,...} with index sets:
    Dimension 1, 1:4
And data, a 4-element Vector{Float64}:
  1.0
 -0.0
  1.0
  1.0
```

In [ ]:
```
obj3 = round.(objective_value(waste_mod3);digits=2)
println("The objective value is \$$obj3 which is less
than the baseline of \$$obj2, meaning that the additional
landfill is beneficial.")
```

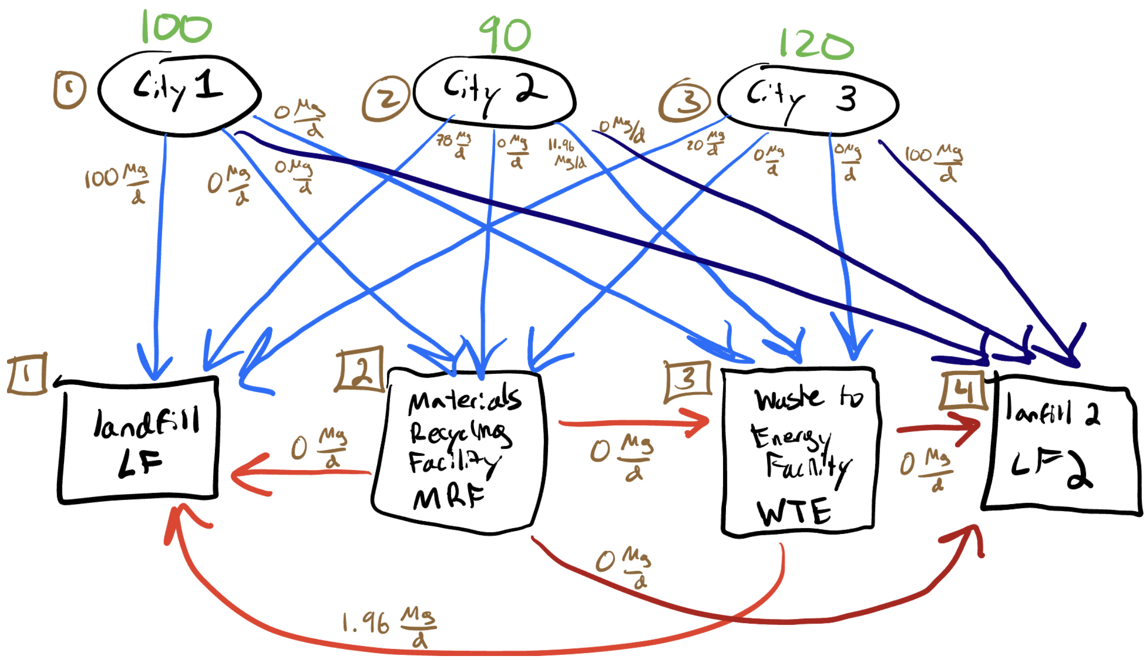
```
The objective value is $30568.28 which is less
than the baseline of $31649.34, meaning that the additional
landfill is beneficial.
```

Problem 2.3 (5 points)

Find the optimal solution and report the optimal objective value. Provide a diagram showing the new waste flows.

In [ ]:
```
println("The optimal solution has an objective value of \$$obj3
and the diagram of the new waste flows is show below:")
```

```
The optimal solution has an objective value of $30568.28
and the diagram of the new waste flows is show below:
```



Problem 2.4 (5 points)

Would you recommend that the cities build the new landfill? Why or why not? Your answer should be based on your analysis but can draw on other considerations as appropriate or desired.

In [ ]:
```
println("Yes, I would recommend that the city build another landfill as the
total cost of waste disposal decreases by \$$(round.(obj2-obj3;digits=2))
as compared to the situation without an extra landfill where the carbon tax
still exists. However, if there is no carbon tax, then the cheapest option
is to stick with the original setup. However, the situation where a
landfill is not added but the prices change does cost more but it also
incorperates more recycling which is beneficial to the environment, and
maybe this will serve to save them environmental costs in the future.")
```

```
Yes, I would recommend that the city build another landfill as the
total cost of waste disposal decreases by $1081.06
as compared to the situation without an extra landfill where the carbon tax
still exists. However, if there is no carbon tax, then the cheapest option
is to stick with the original setup. However, the situation where a
landfill is not added but the prices change does cost more but it also
incorperates more recycling which is beneficial to the environment, and
maybe this will serve to save them environmental costs in the future.
```

References

List any external references consulted, including classmates.