BEE 4750 Homework 3: Uncertain Sea-Level Rise and Levee Reliability

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> **Due Date**
>
> Friday, 10/06/23, 9:00pm

Overview

Instructions

This assignment asks you to conduct a Monte Carlo analysis of levee reliability in the face of uncertain changes to local sea levels. You will propagate uncertainty in equilibrium climate sensitivity through the energy balance model to obtain a distribution of temperatures, which will then drive a model of sea-level rise. You will finally use this distribution to assess the probability that a planned levee will achieve its desired reliability standard.

Load Environment

The following code loads the environment and makes sure all needed packages are installed. This should be at the start of most Julia scripts.

In [ ]:
```
import Pkg
Pkg.activate(@__DIR__)
Pkg.instantiate()
```

```
  Activating project at `~/Downloads/hw03`
```

In [ ]:
```
using Random
using Plots
using LaTeXStrings
using Distributions
using CSV
using DataFrames
using Statistics
```

Problems (Total: 40 Points)

Problem 1 (12 points)

Recall from class that the simple energy balance model (EBM) of planetary energy balance links changes in radiative forcing (F) to global mean temperature (T) changes through the discretized equation

$$T_{i+1} = T_i + \frac{F_i - \lambda T_i}{cd} \times \Delta t,$$

where i is the current time step, $c = 4.184 \times 10^6$ J/K/m^2 is the heat capacity of water per unit area, d is the (uncertain) depth of the mixing layer, Δt is the annual time step in seconds and $\lambda = F_{2\text{xCO}_2}/S$ is the climate feedback parameter in W/m^2/$^\circ$ C, where S is the equilibrium climate sensitivity (the uncertain equilibrium temperature change resulting from a doubling of atmospheric CO_2). Finally, while total radiative forcing can be the result of non-aerosol and aerosol effects, we do not know the relative intensity of aerosol forcing, so we represent this with an uncertain aerosol scaling factor α.

We can implement this model with the following Julia function. We will assume an ocean mixing depth $d = 100$ m and an aerosol scaling factor $\alpha = 1.3$ so we can focus on the uncertainty in S .

The last technical concern is that “global mean temperature” does not make sense in absolute terms as a marker of climate change. Instead, we typically refer to temperature changes relative to some historical pre-industrial baseline. In this case, we will use the period from 1880-1900, though this choice can vary.

In [ ]:

```julia
# we need to split up the aerosol and non-aerosol forcings when we call the func
function energy_balance_model(S, forcing_aerosol, forcing_non_aerosol)
    d = 100 # ocean mixing depth [m]
    α = 1.3 # aerosol scaling factor
    F2xCO₂ = 4.0 # radiative forcing [W/m²] for a doubling of CO₂
    λ = F2xCO₂/S

    c = 4.184e6 # heat capacity/area [J/K/m²]
    C = c*d # heat capacity of mixed layer (per area)

    F = forcing_non_aerosol + α*forcing_aerosol # radiative forcing

    Δt = 31558152.0 # annual timestep [s]

    T = zero(F)
    for i in 1:length(F)-1
        T[i+1] = T[i] + (F[i] - λ*T[i])/C * Δt
    end
    # return temperature anomaly relative to 1880-1900 baseline
    return T .- mean(T[1:21])
end
```

```
energy_balance_model (generic function with 1 method)
```

Finally, we need to load some radiative forcing data. We will use the radiative forcing scenario RCP 8.5. We can load this data, which is in a `.csv` (comma-delimited) file, into a `DataFrame` , which is a tabular data structure. Rows and columns in a `DataFrame` can be accessed using their numerical index (like a matrix), but columns also have names; you can access a particular column in a dataframe `df` by name using `df.colname` or `df[:, "colname"]` .

Of note: this data set goes from 1750–2500, so you will need to take care to make sure you are using the right years at each step. For example, here we will constrain the data to 1880–2100, which is the period we are interested in.

In [ ]:
```
# The CSV is read into a DataFrame object, and we specify that it is comma delim
forcings_all_85 = CSV.read("data/ERF_ssp585_1750-2500.csv", DataFrame, delim=","

# get the years corresponding to the forcings
t = Int64.(forcings_all_85[!,"year"]) # Ensure that years are interpreted as int
# find the indices of the years 1880 and 2100
# we can do this with the indexin function
time_bounds = indexin([1880, 2100], t)
years = time_bounds[1]:time_bounds[2] # create range of years

# Separate out the individual components
forcing_co2_85 = forcings_all_85[years,"co2"]
# Get total aerosol and non-aerosol forcings
forcing_aerosol_rad_85 = forcings_all_85[years,"aerosol-radiation_interactions"]
forcing_aerosol_cloud_85 = forcings_all_85[years,"aerosol-cloud_interactions"]
forcing_aerosol_85 = forcing_aerosol_rad_85 + forcing_aerosol_cloud_85 # aerosol
forcing_total_85 = forcings_all_85[years,"total"]
forcing_non_aerosol_85 = forcing_total_85 - forcing_aerosol_85 # non-aerosol for
```

```
221-element Vector{Float64}:
  0.42741479112315905
  0.4487940147601447
  0.4900144276528058
 -0.019811270078689047
 -1.480725700367619
 -0.3154905163510021
  0.233186559774844
  0.2923308952663089
  0.45833872365810924
  0.5168189581089915
  ⋮
  9.879065698371564
  9.946850640754889
 10.002169546141578
 10.061009359425011
 10.115684195707905
 10.182946497594184
 10.249699157847772
 10.331407709334023
 10.424544495740134
```

For this assignment, you can use the `forcing_aerosol_85` and `forcing_non_aerosol_85` vectors as is to correspond to the relevant forcings. You will need to use the vector `t` to find the appropriate years for analysis.

Problem 1.1 (3 points)

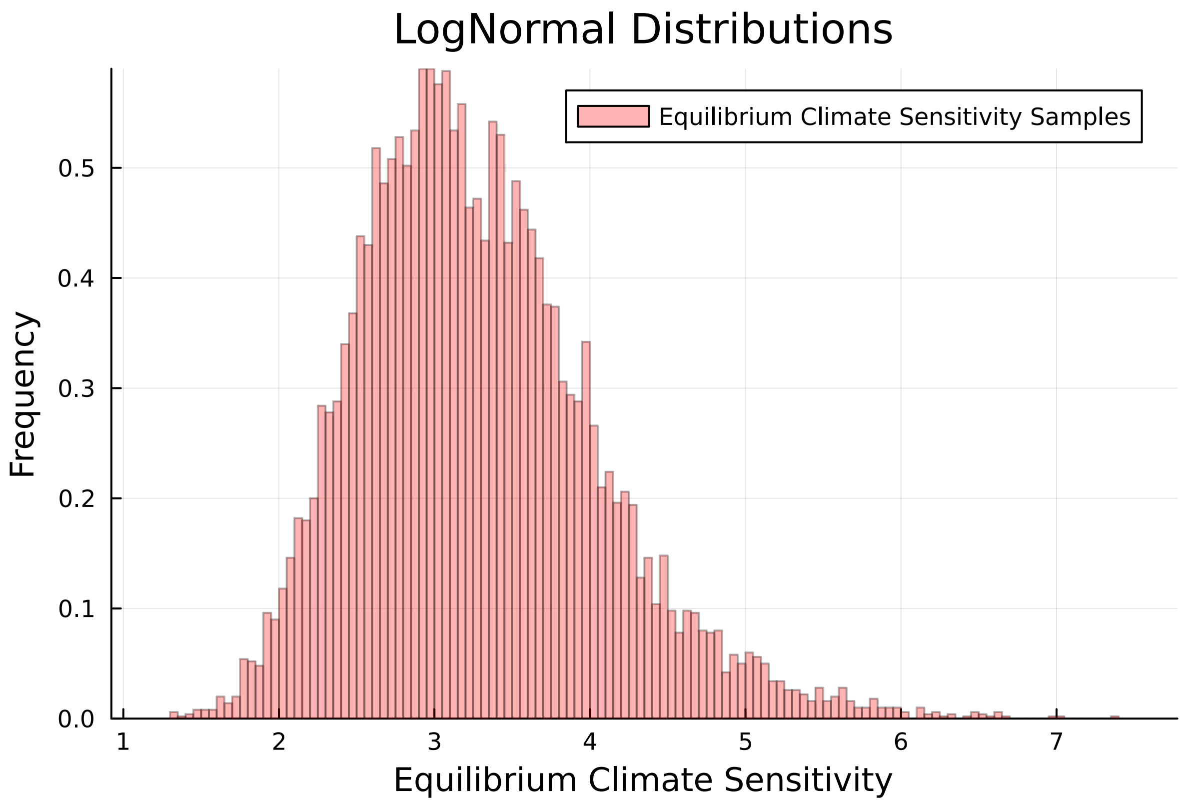
Assume that S is distributed according to $\mathrm{LogNormal}(\log(3.2), \log 2/3)$ (as in class). Draw 10,000 samples from this distribution and make a histogram.

In [ ]:
```
using Random
using Plots

#parameters for the LogNormal distribution
μ = log(3.2)
σ = log(2) / 3
num_samples = 10000
```

```
#generate random samples from LogNormal distribution
samples = rand(LogNormal(μ, σ), num_samples)

#plot histogram
histogram(samples, bins=150, normalize=true,
xlabel="Equilibrium Climate Sensitivity", ylabel="Frequency",
label="Equilibrium Climate Sensitivity Samples", alpha=0.3,
title = "LogNormal Distributions",
color=:red)
```



Problem 1.2 (5 points)

Use the EBM to propagate your samples of S to a distribution of global mean temperature. Plot the median and 90% predictive distribution (between the .05 and .95 quantiles) from 1880-2100.

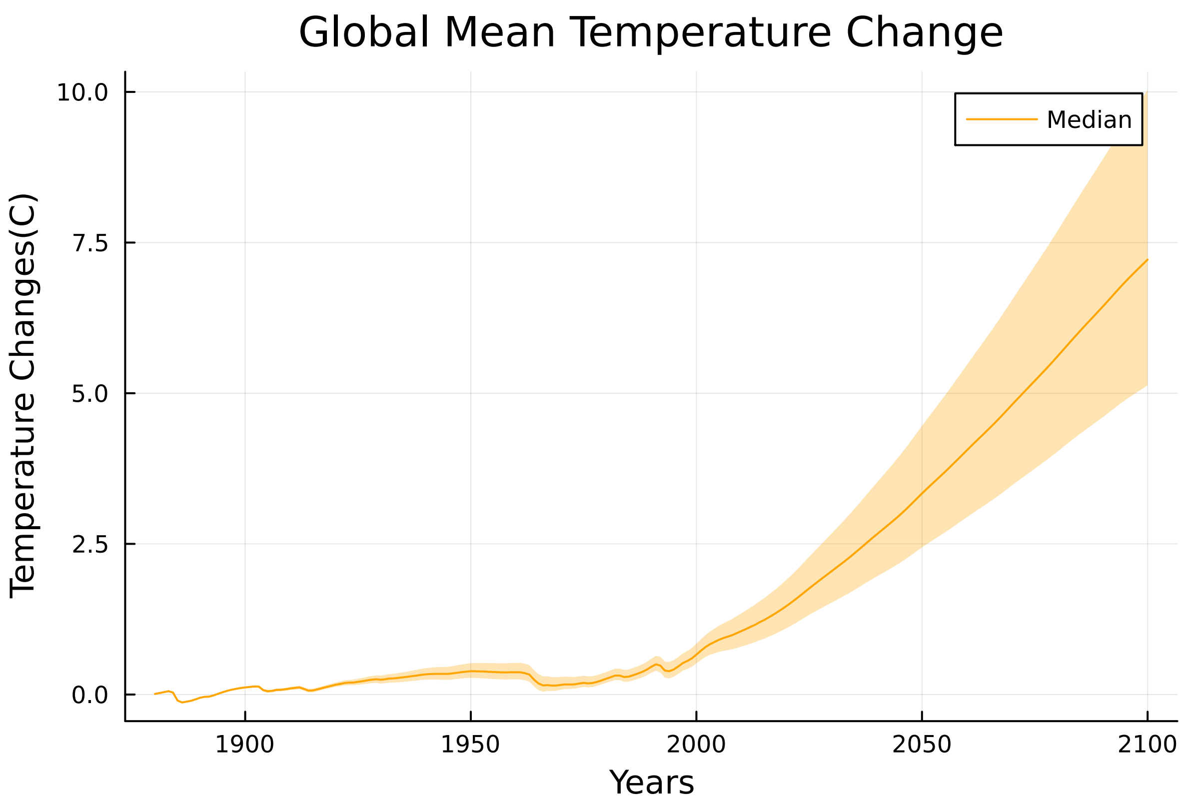
In [ ]:
```
years = 1880:2100

#ensure lengths are the same so mathemcatical functions can work
temperature_simulations = zeros(length(samples), length(years))

#generates temperature data from function above
for i in 1:length(samples)
    temperature_simulations[i, :] = energy_balance_model(samples[i],
    forcing_aerosol_85, forcing_non_aerosol_85)
end

#calculate medians and distributions
median_temp = median(temperature_simulations, dims = 1)[:]
lower_quantile = [quantile(temperature_simulations[:, i], 0.05) for
i in 1:size(temperature_simulations, 2)]
upper_quantile = [quantile(temperature_simulations[:, i], 0.95) for
i in 1:size(temperature_simulations, 2)]
```

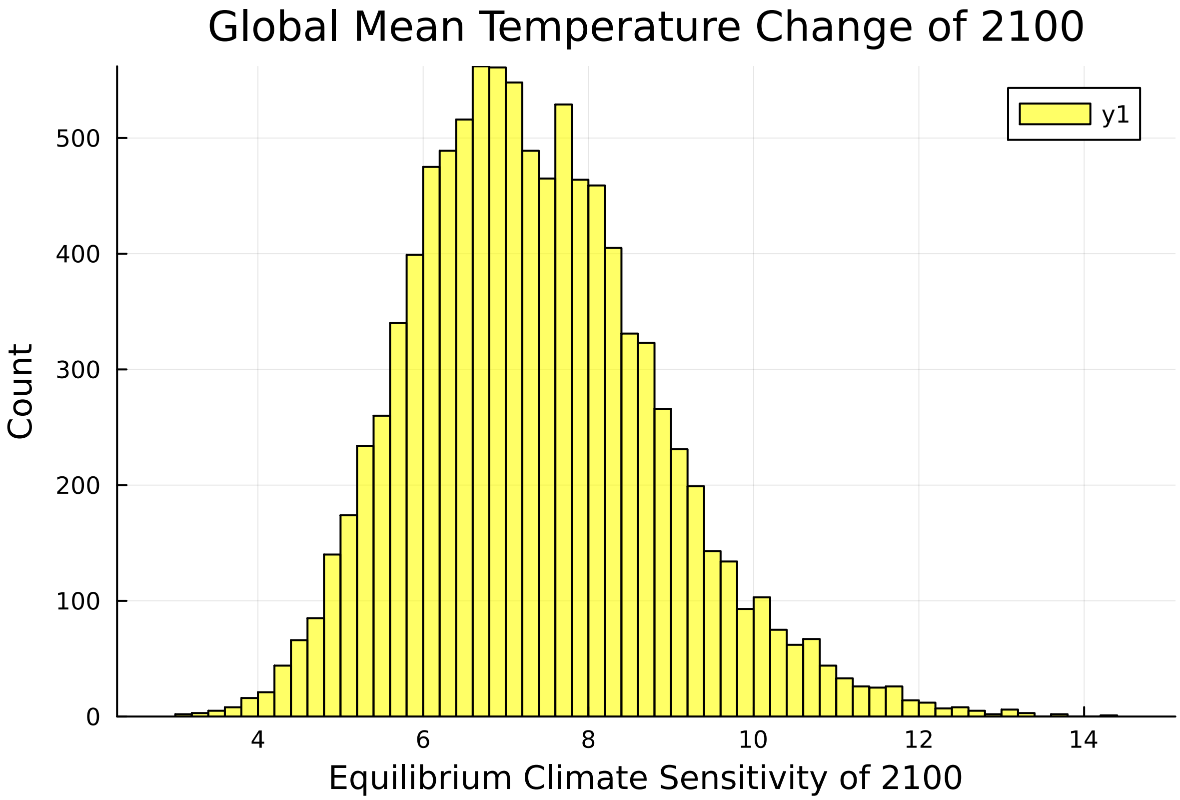
```
#plot mean temperature by pushing difference of the
#medians and quantiles
plot(years, median_temp,
    ribbon = (median_temp .- lower_quantile,
    upper_quantile .- median_temp),
    fillalpha = 0.3,
    label = "Median",
    legend=:topright,
    xlabel = "Years",
    ylabel= "Temperature Changes(C)",
    title = "Global Mean Temperature Change",
    color=:orange)
```



Problem 1.3 (4 points)

Make a histogram of global mean temperature projections in 2100. If you compare this distribution to the distribution of S from Problem 1.1, what do you observe?

In [ ]:
```
#use temp data from 1.2 & apply all parameters
histogram(temperature_simulations[:, 221],
fillalpha = 0.6,
xlabel = "Equilibrium Climate Sensitivity of 2100",
ylabel = "Count",
title = "Global Mean Temperature Change of 2100",
color=:yellow)
```



The histogram above has twice the x distrubution compared to the distribution of S. Both distributions are more towards the left. Additionally, I used 10,000 samples to have some variability.

Problem 2 (15 points)

Changes to global temperatures cause changes in global sea levels through several mechanisms, including thermal expansion (the change in ocean volume due to increased heat content) and melting land-based ice. One simple way to represent this link is through the following model, proposed by Rahmstorf (2007).

$$\frac{dH}{dt} = a(T - T_0),$$

where H is the global mean sea level in mm, T is global mean temperature, T_0 is an equilibrium temperature (where there is no change in sea levels), and a is a proportionality constant. This model can be discretized to give

$$H_{i+1} - H_i = a(T_i - T_0).$$

Note that, like with global mean temperature, the notion of "global mean sea level" does not make sense in absolute terms (were sea levels ever at "zero"?). Instead, we want to normalize this relative to some historical baseline. In this case (with a view towards Problem 3), we will compute our sea levels relative to the 2010 sea level. Note that in addition to the model parameters, we also need an initial sea-level parameter H_0 which will give us the right anomaly level.

The best estimates for these parameters are:

- $a = 1.86$;
- $H_0 = -223$;
- $T_0 = -0.62$

Problem 2.1 (5 points)

Write a function `sea_level_model()` to implement the mathematical sea-level rise model described above. It should take in needed parameters and a vector of temperatures and return a vector of sea levels. To test your function, use the provided temperature series `historical_temps` (read in below) to compute the global mean sea level anomaly in 2022 (the last year of the dataset) with the parameter values above; you should get a value of approximately 40mm.

In [ ]:
```
historical_temp_data = CSV.read("data/HadCRUT.5.0.1.0.analysis.summary_series.gl
# column 2 is the temperature anomaly, column 1 is the year
temp_bds = indexin([1880, 1900], historical_temp_data[!, 1]) # find the index of
historical_temp_data[:, 2] .-= mean(historical_temp_data[temp_bds[1]:temp_bds[2]
historical_temps = historical_temp_data[temp_bds[1]:end, 2]
```

```
143-element Vector{Float64}:
  0.07297116761904765
  0.15655772761904765
  0.09327316761904769
  0.04232884761904765
 -0.10351681238095234
 -0.08232033238095232
 -0.03210037238095231
 -0.10998251238095236
  0.00942435761904764
  0.13890768761904768
  ⋮
  1.0616748976190478
  1.2139176176190476
  1.3217303776190477
  1.2339774976190476
  1.1514572476190477
  1.2798758476190477
  1.3115970476190477
  1.1506591476190478
  1.1896871476190476
```

In [ ]:
```
using CSV

#define model and ensuring length for var/func are same
function sea_level_model(a, H0, T0, historical_temps)
    sea_levls = zeros(length(historical_temps))
    sea_levls[1] = H0 #H = sea levls

    #implement sea level equation to run through temp numbers
    for i in 2:length(historical_temps)
        sea_levls[i] = sea_levls[i-1] + a *
        (historical_temps[i] - T0)
    end

    #normalize sea level data with 2010 as ref point and
    #use element-wise subtraction to show sea level changes
```

```
    sea_levls .-= sea_levls[findfirst(s -> s === 2010, years)]

end

a = 1.86
#global mean constants
H0 = 223.0
T0 = -.62

#calling function and returning last value of array
H = sea_level_model(a, H0, T0, historical_temps)
H_2022 = H[end]

println("Global Sea level anomaly in 2022: $H_2022 mm")
```

```
Global Sea level anomaly in 2022: 39.54335225065722 mm
```

Problem 2.2 (5 points)

Evaluate `sea_level_model()` using the projected temperature ensemble from Problem 1. Plot the 90% projection interval of the sea levels.

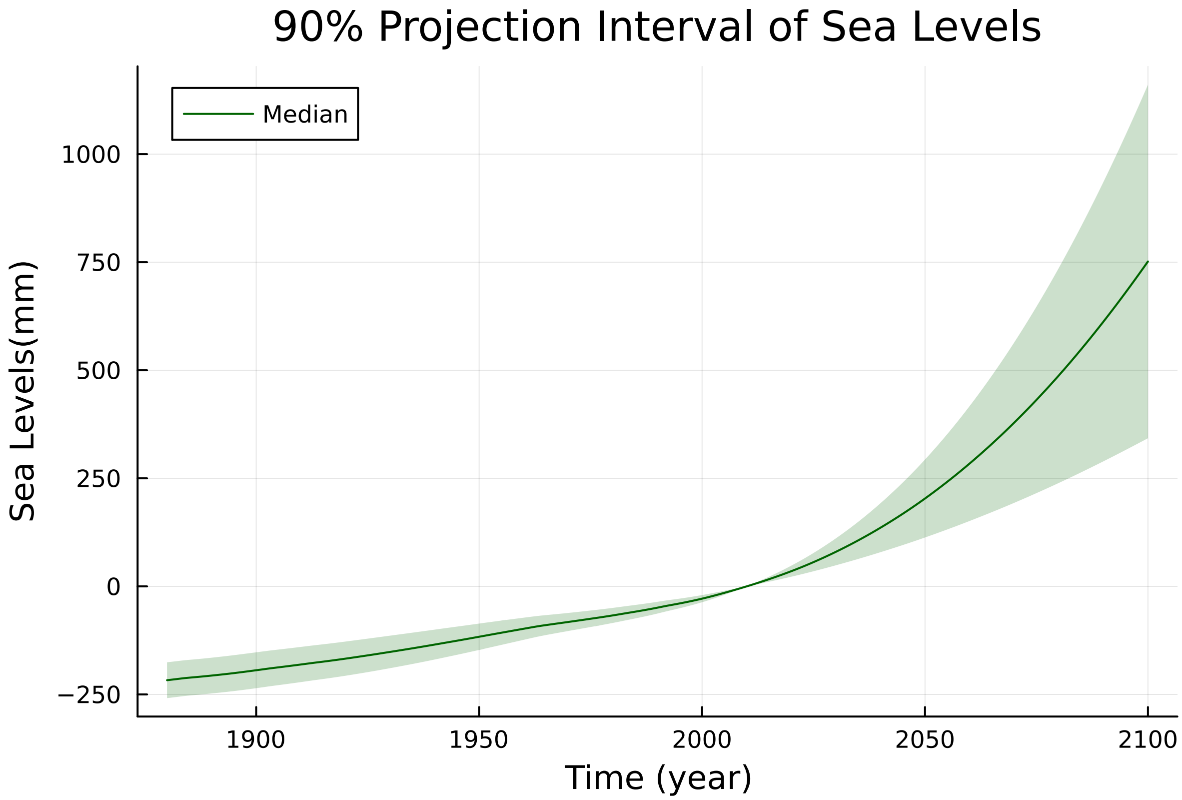
In [ ]:

```
yrs = 1880:2100
#ensure data inputted and collected is same dimensions
sea_level_proj = zeros(length(samples), length(yrs))

#calculate projected sea level for ea sample and store data
for i in 1:length(samples)
    sea_level_proj[i, :] = sea_level_model(a, H0, T0,
    temperature_simulations[i, :])
end

#calculate the quantiles and median
lower_quantile2 = quantile.(eachcol(sea_level_proj), 0.05)
upper_quantile2 = quantile.(eachcol(sea_level_proj), 0.95)
median_temp2 = quantile.(eachcol(sea_level_proj), 0.50)

#plotting results and data received
plot(yrs, median_temp2,
    ribbon = (upper_quantile2 .- lower_quantile2),
    fillalpha = .2,
    label = "Median",
    legend = :topleft,
    xlabel = "Time (year)",
    ylabel = "Sea Levels(mm)",
    title="90% Projection Interval of Sea Levels",
    color=:darkgreen)
```



Problem 2.3 (5 points)

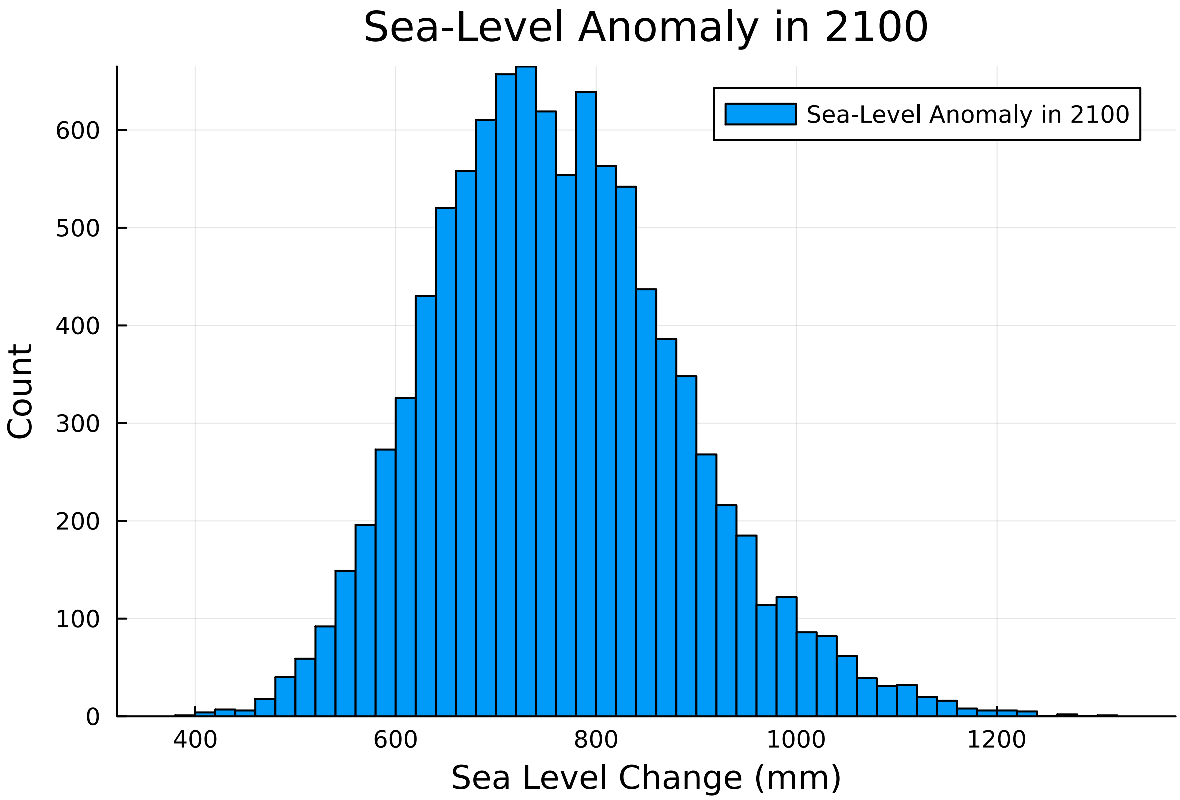
Make a histogram of the sea-level anomaly in 2100. What can you observe about how the ECS uncertainty has impacted sea-level uncertainty under this radiative forcing scenario? What might the implications be of only using the best-estimate ECS value?

In [ ]:

```
years_20 = 1880:2100

#grab sea level data needed and store it a var
sea_levels_2100 = sea_level_proj[:, findfirst(years_20.== 2100)]

#plot histogram with information
histogram(sea_levels_2100,
    label = "Sea-Level Anomaly in 2100",
    xlabel = "Sea Level Change (mm)",
    ylabel = "Count",
    title="Sea-Level Anomaly in 2100")
```



The ECS uncertainity has been impacted by centralizing it around the 700 mm sea level change. The implications are that having the best estimate can give you the most accurate data for the sea level anomaly and its change.

Problem 3 (13 points)

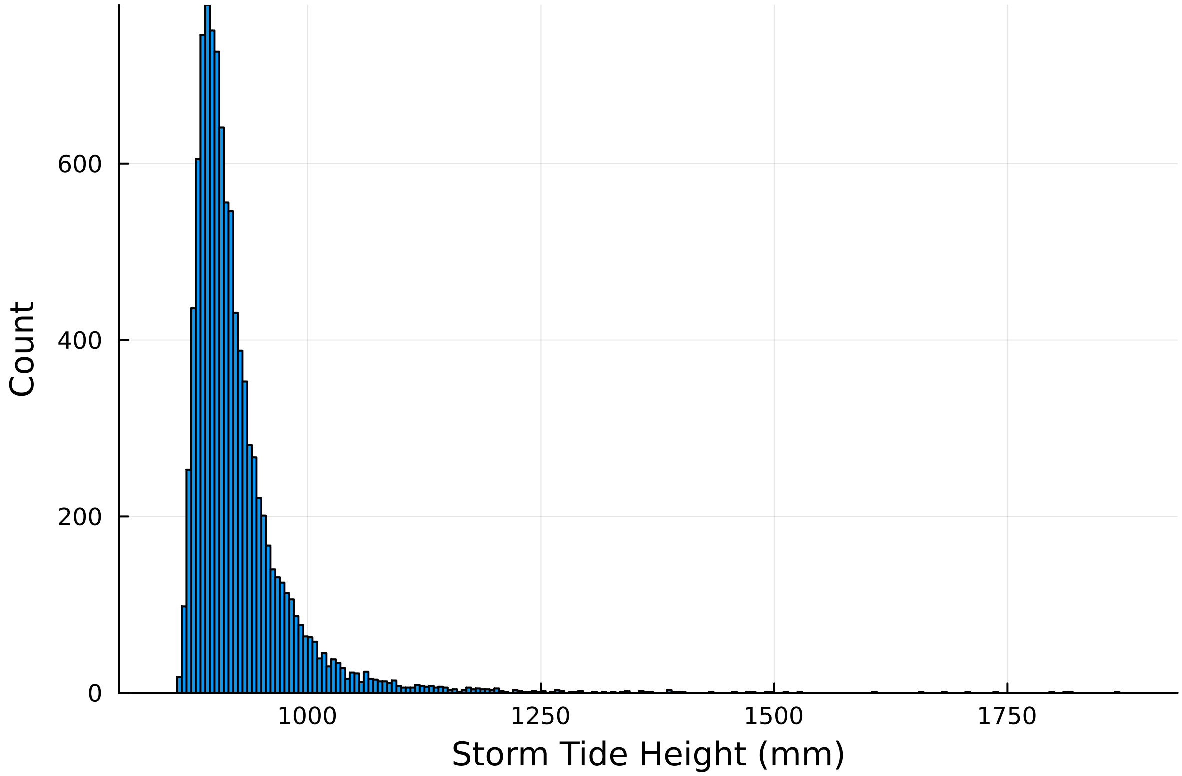
You've been asked to consult on a levee reliability analysis. For context, levees in the United States are supposed to only fail once in 100 years, or, in other words, to have at most a 1% chance of failure in a given year. We will assume for this problem that the only way in which a levee fails is by being overtopped (note: this is unrealistic).

We can assess the probability of levee overtopping by comparing its height to a distribution of extreme sea levels. A common approach is to look at the distribution of the highest sea level each year. These extreme sea levels can be obtained by combining the absolute sea level (we will use our distribution of global sea levels for this), the rate of subsidence (how much the ground sinks), and the distribution of storm tides (the highest tide level, which is often the result of storm surges combining with high tide).

Assume for this problem that:

1. the annual rate of subsidence ν is 1.2mm/yr;
2. the distribution of annual storm tide maxima, above the mean sea level, is (and is expected to continue to be) given by a $\text{GeneralizedExtremeValue}(900, 25, 0.3)$ distribution, which looks like this:

In [ ]:
```
tide_distribution = GeneralizedExtremeValue(900, 25, 0.3)
    histogram(rand(tide_distribution, 10000), xlabel="Storm Tide Height (mm)", y
```



Feel free to just sample from `tide_distribution` in your solution below.

Problem 3.1 (2 points)

How would you use your sea-level simulations and the above information to compute a distribution of extreme sea levels in 2100 relative to 2010 mean sea level? Write down the approach in clear steps, with equations as needed.

In order to get the distribution of sea levels, there are 4 pivotal steps:

1) Find the max tide using the rand(lognormal) method from problem 1:

$$maxtides = rand(tidedistribution, 10000)$$

2) Calculate the total subsidence with the following equation:

$$totalsubsidence = maxtides + 1.2 * yearrange$$

3) Find sea level max by using data from problem 2.3 (sea_levels_2100)
and adding totalsubsidence:

$$sealevelmax = sealevels2100 + totalsubsidence$$

4) Incorporate the substidence and the max tide for 3.2 histogram by getting
the distribution of 2100 sea levels relative to 2010 mean sea level (maxsealevels).

Problem 3.2 (3 points)

Follow the steps above and produce a histogram of the extreme sea levels in 2100 relative to 2010.

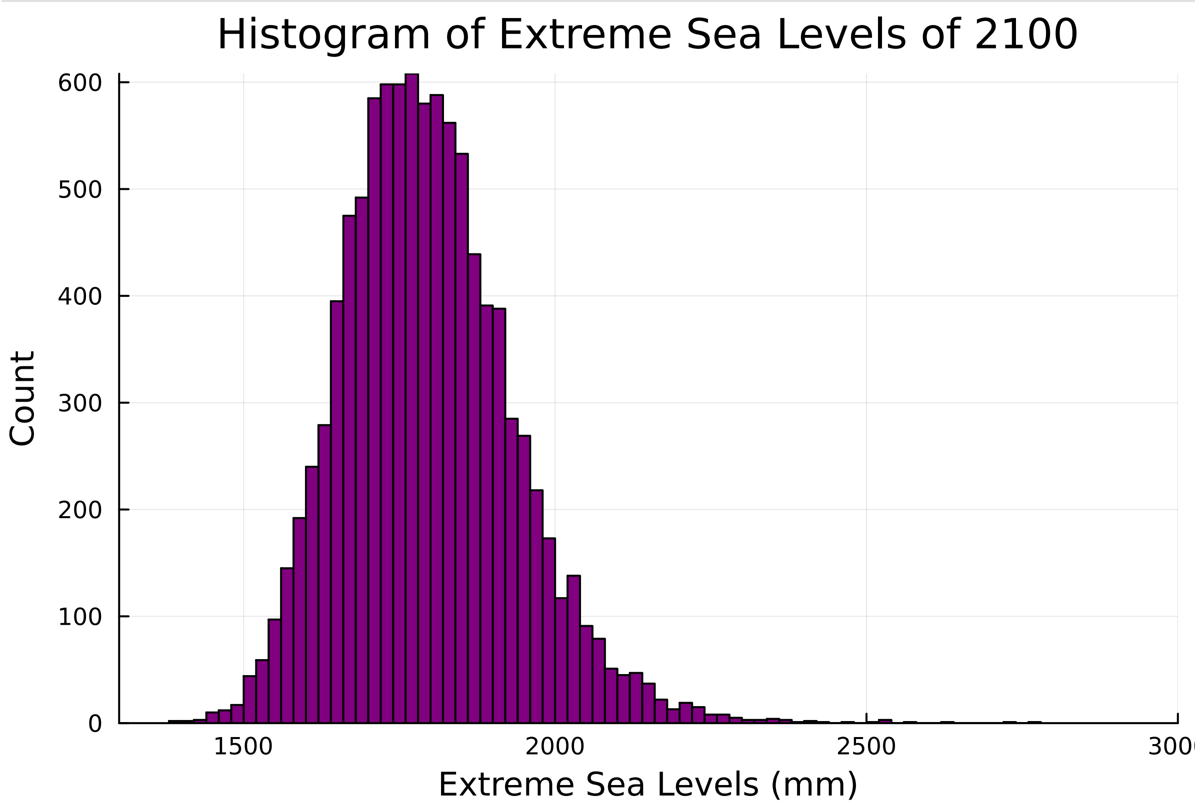
In [ ]:
```
subs = 1.2

#generate random values to get max tides
max_tides = rand(tide_distribution, 10000)

#calculate total subsidence in model
total_subs =  max_tides .+ subs * (2100 - 2010 + 1)

#find max sea level
max_sea_levels = sea_levels_2100 .+ total_subs

#plot distribution of 2100 sealvl relative to 2010 data
histogram(max_sea_levels,
    title = "Histogram of Extreme Sea Levels of 2100",
    xlabel = "Extreme Sea Levels (mm)",
    ylabel = "Count",
    legend=:false,
    xlims = (1300, 3000),
    color=:purple)
```



Problem 3.3 (5 points)

The current levee was heightened in 2010 to 2m above the 2010 mean sea level. Based on your analysis above, what is the probability that the levee will be overtopped in 2100 (remember that the reliability standard is 1%)?

In [ ]:
```
height = 2000 #mm

#initialize frequency var
nums = 0

#keeps track of amount of times max lvls
```

```
#in 2100 greater than the levee height
for i in max_sea_levels
    if i > height
        nums += 1
    end
end

#calculates % levee being overtopped
prob = nums / 10000 * 100
println("Probability of levee overtopping in 2100:
$prob%")
```

```
Probability of levee overtopping in 2100:
7.21%
```

Problem 3.4 (3 points)

Based on your analysis, would you recommend that the levee be heightened again in the future, and if so, how high? What other information might you need, if any, to make your recommendation?

In [ ]:

```
height = 2000

#est condition where height is increased
#when condition is met
while prob > 1
    nums = 0
    height += 1


    #est condition where frequency increased
    #when condition is met
    for i in max_sea_levels
        if i > height
            nums += 1
        end
    end
    prob = nums / 10000 * 100
end

#shows ultimate height when its less than
#reliability standard

println("Recommend levee to be heightened to:
$height mm")
```

```
Recommended to levee to be heightened to:
2173 mm
```

Seeing that the realibility standard is 1%, I would aim the levee to be heightened so that the probability comes out to be less than 1%. According to the code I wrote above, the levee can be heightened to 2173 mm. However knowing the weather, surrounding area, the flora and fauna by the levee could allow me to make a more accurate suggestion.

References

List any external references consulted, including classmates.

Worked with Ella Bear and Ruby Pascual.