# BEE 4750 Homework 2: Systems Modeling and Simulation

2024-09-08

> **Due Date**
>
> Thursday, 09/19/24, 9:00pm

## Overview

### Instructions

- Problem 1 asks you to derive a model for water quality in a river system and use this model to check for regulatory compliance.
- Problem 2 asks you to explore the dynamics and equilibrium stability of the shallow lake model under a particular set of parameter values.
- Problem 3 (5750 only) asks you to modify the lake eutrophication example from Lecture 04 to account for atmospheric deposition.

### Load Environment

The following code loads the environment and makes sure all needed packages are installed. This should be at the start of most Julia scripts.

```julia
import Pkg
Pkg.activate(@__DIR__)
Pkg.instantiate()
```

```
using Plots
using LaTeXStrings
using CSV
using DataFrames
using Roots
```

## Problems (Total: 50/60 Points)

### Problem 1 (25 points)

A river which flows at 10 km/d is receiving discharges of wastewater contaminated with CRUD from two sources which are 15 km apart, as shown in the Figure below. CRUD decays exponentially in the river at a rate of $0.36$ $d^{-1}$.
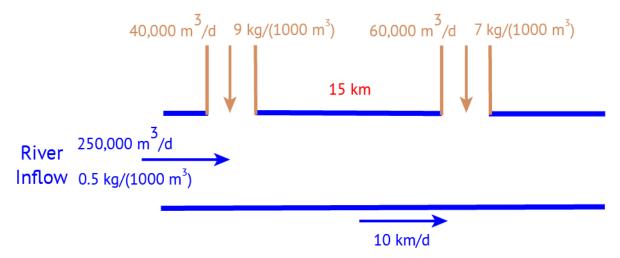


Figure 1: Schematic of the river system in Problem 1

**In this problem**:

- Assuming steady-state conditions, derive a model for the concentration of CRUD downriver by solving the appropriate differential equation(s) analytically.
- Determine if the system in compliance with a regulatory limit of $2.5$ kg/(1000 m$^3$).

> 💡 Tip
>
> Formulate your model in terms of distance downriver, rather than leaving it in terms of time from discharge.

**Problem 2 (25 points)**

Consider the shallow lake model from class:

$$X_{t+1} = X_t + a_t + y_t + \frac{X_t^q}{1 + X_t^q} - bX_t,$$

$$y_t \sim \text{LogNormal}(\mu, \sigma^2),$$

where:

- $X_t$ is the lake phosphorous (P) concentration at time $t$;
- $a_t$ is the point-source P release at time $t$;
- $y_t$ is the non-point-source P release at time $t$, which is treated as random from a Log-Normal distribution with mean $\mu$ and standard deviation $\sigma$;
- $b$ is the linear rate of P outflow;
- $q$ is a parameter influencing the rate of P recycling from the sediment.

**In this problem**:

- Make an initial conditions plot for the model dynamics for $b = 0.5$, $q = 1.5$, $y_t = 0$, and $a_t = 0$ for $t = 0, \ldots, 30$. What are the equilibria? What can you say about the resilience of the system?

  > 💡 Finding equilibria
  >
  > Use `Roots.jl` to find the equilibria by solving for values where $X_{t+1} = X_t$. For example, if you have functions `X_outflow(X,b)` and `X_recycling(X,q)`, you could create a function `X_delta(x, a) = a + X_recycling(x) - X_outflow(x)` and call `Roots.find_zero(x -> X_delta(x, a), x )`, where `x` is an initial value for the search (you might need to use your plot to find values for `x` near each of the "true" equilibria).

- Repeat the analysis with $a_t = 0.02$ for all $t$. What are the new equilibria? How have the dynamics and resilience of the system changed?

**Problem 3 (10 points)**

**This problem is only required for students in BEE 5750**.

Consider the lake eutrophication example from Lecture 04. Suppose that phosphorous is also atmospherically deposited onto the lake surface at a rate of $1.6 \times 10^{-4} \text{kg}/(\text{yr} \cdot \text{m}^2)$, which is then instantly mixed into the lake. Derive a model for the lake phosphorous concentration

and find the maximum allowable point source phosphorous loading if the goal is to keep lake concentrations below 0.02 mg/L.

## References

List any external references consulted, including classmates.