

# BEE 4750 Homework 2: Systems Modeling and Simulation

2024-09-08

Due Date

Thursday, 09/19/24, 9:00pm

## Overview

### Instructions

- Problem 1 asks you to derive a model for water quality in a river system and use this model to check for regulatory compliance.
- Problem 2 asks you to explore the dynamics and equilibrium stability of the shallow lake model under a particular set of parameter values.
- Problem 3 (5750 only) asks you to modify the lake eutrophication example from Lecture 04 to account for atmospheric deposition.

### Load Environment

The following code loads the environment and makes sure all needed packages are installed. This should be at the start of most Julia scripts.

```
import Pkg
Pkg.activate(@__DIR__)
Pkg.instantiate()
```

```

using Plots
using LaTeXStrings
using CSV
using DataFrames
using Roots

```

## Problems (Total: 50/60 Points)

### Problem 1 (25 points)

A river which flows at 10 km/d is receiving discharges of wastewater contaminated with CRUD from two sources which are 15 km apart, as shown in the Figure below. CRUD decays exponentially in the river at a rate of  $0.36 \text{ d}^{-1}$ .



Figure 1: Schematic of the river system in Problem 1

#### In this problem:

- Assuming steady-state conditions, derive a model for the concentration of CRUD downriver by solving the appropriate differential equation(s) analytically.
- Determine if the system is in compliance with a regulatory limit of  $2.5 \text{ kg}/(1000 \text{ m}^3)$ .

#### 💡 Tip

Formulate your model in terms of distance downriver, rather than leaving it in terms of time from discharge.

## Problem 2 (25 points)

Consider the shallow lake model from class:

$$X_{t+1} = X_t + a_t + y_t + \frac{X_t^q}{1 + X_t^q} - bX_t,$$
$$y_t \sim \text{LogNormal}(\mu, \sigma^2),$$

where:

- $X_t$  is the lake phosphorous (P) concentration at time  $t$ ;
- $a_t$  is the point-source P release at time  $t$ ;
- $y_t$  is the non-point-source P release at time  $t$ , which is treated as random from a Log-Normal distribution with mean  $\mu$  and standard deviation  $\sigma$ ;
- $b$  is the linear rate of P outflow;
- $q$  is a parameter influencing the rate of P recycling from the sediment.

**In this problem:**

- Make an initial conditions plot for the model dynamics for  $b = 0.5$ ,  $q = 1.5$ ,  $y_t = 0$ , and  $a_t = 0$  for  $t = 0, \dots, 30$ . What are the equilibria? What can you say about the resilience of the system?

### Finding equilibria

Use [Roots.jl](#) to find the equilibria by solving for values where  $X_{t+1} = X_t$ . For example, if you have functions `X_outflow(X,b)` and `X_recycling(X,q)`, you could create a function `X_delta(x, a) = a + X_recycling(x) - X_outflow(x)` and call `Roots.find_zero(x -> X_delta(x, a), x)`, where `x` is an initial value for the search (you might need to use your plot to find values for `x` near each of the “true” equilibria).

- Repeat the analysis with  $a_t = 0.02$  for all  $t$ . What are the new equilibria? How have the dynamics and resilience of the system changed?

## Problem 3 (10 points)

**This problem is only required for students in BEE 5750.**

Consider the lake eutrophication example from [Lecture 04](#). Suppose that phosphorous is also atmospherically deposited onto the lake surface at a rate of  $1.6 \times 10^{-4} \text{kg}/(\text{yr} \cdot \text{m}^2)$ , which is then instantly mixed into the lake. Derive a model for the lake phosphorous concentration

and find the maximum allowable point source phosphorous loading if the goal is to keep lake concentrations below 0.02 mg/L.

## **References**

List any external references consulted, including classmates.