Ball Balancing Beam

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MECA 482: Control System Design

Project Report

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Introduction

It is possible to balance a round ball on a platform powered by a single servo motor. This can be done using classical control theory. The idea of this system is that the system can recognize the ball's location and transfer that information into motor torque to control where it rolls to. By creating a mathematical model, the transfer function for this system can be found. The inputs are collected by a distance sensor that can detect where the ball is at. The mathematical model can turn this input into the correct output, if the mathematical model is correct.

Joselin Retna Kumar et el. Modeled a similar system, but instead of using a camera sensor to gather inputs, they used a touchscreen platform. They published their findings in *Design and Control of Ball on Plate System*. (https://www.researchgate.net/publication/316628039_Design_and_control_of_ball_on_plate_system).

This design report covers how the mathematical model can be found for this system, and how the system can be carried out into a physical prototype. That includes proper sensor calculations, platform and ball measurements, controller design, and simulation. All documentation and program files can be found at (https://github.com/BEN0021022/ball_balance).

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Modeling

The system consists of a ball that rests on a balancing beam. As the ball rolls along the beam, an ultrasonic sensor detects its distance from the end, (r). To correct itself so that the ball stays on the beam, a servo motor rotates causing the beam to adjust its angle relative to the horizontal, α.

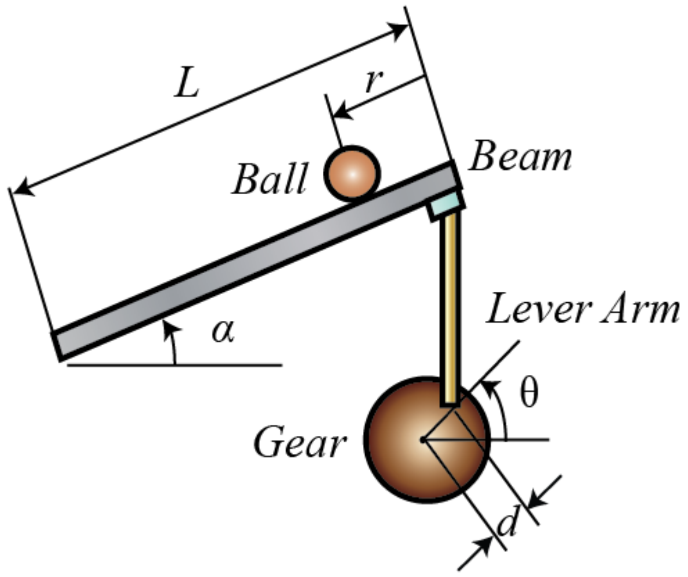


Figure 1.0 System diagram.

Physical Model Parameters

Mass of the ball	0.0027 kg
Radius of the ball	0.0201 m
Gravitational constant	- 9.81 m/s^2
Length of beam to pivot point	0.2 m
Lever arm offset	0.05 m
Moment of inertia	6.2×10^{-7} kg m^2

Force balance of the ball

Equation of motion for the ball. There is an affect on the second derivative of (r) from the second derivative of the input angle (α), however it is small, and we will ignore it.

$$0 = \left(\frac{J}{R^2} + m\right)\ddot{r} + m\,g\sin\alpha - mr\dot{\alpha}^2 \tag{1}$$

Linearizing this equation about the horizontal beam angle, $\alpha = 0$, gives us an approximation of the system.

$$\left(\frac{J}{R^2} + m\right)\ddot{r} = -mg\alpha \tag{2}$$

Relating the beam angle to the angle of the gear can be approximated as linear by the following equation.

$$\alpha = \frac{d}{L}\theta \tag{3}$$

Substituting equation (*3*) into (*2*).

$$\left(\frac{J}{R^2} + m\right)\ddot{r} = -mg\frac{d}{L}\theta \tag{4}$$

Transfer Function

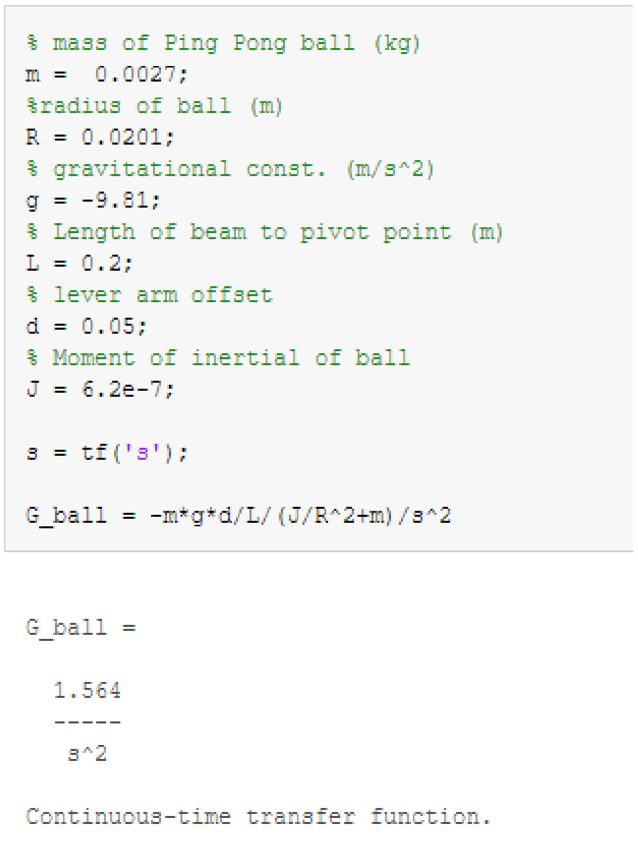
Taking the Laplace transform we get the following equation

$$\left(\frac{J}{R^2} + m\right)R(s)s^2 = -mg\frac{d}{L}\theta(s) \tag{5}$$

Rearranging gives us the transfer function from the servo angle $(\theta(s))$ to the ball position $(R(s)$.

$$G(s) = \frac{R(s)}{\theta(s)} = -\frac{mgd}{L\left(\frac{J}{R^2} + m\right)}\frac{1}{s^2} \quad \left[\frac{m}{rad}\right] \tag{6}$$

Using MATLAB to get the transfer function yields;



```
% mass of Ping Pong ball (kg)
m =  0.0027;
%radius of ball (m)
R = 0.0201;
% gravitational const. (m/s^2)
g = -9.81;
% Length of beam to pivot point (m)
L = 0.2;
% lever arm offset
d = 0.05;
% Moment of inertial of ball
J = 6.2e-7;

s = tf('s');

G_ball = -m*g*d/L/(J/R^2+m)/s^2
```

```
G_ball =

  1.564
  -----
   s^2

Continuous-time transfer function.
```

Figure 1.2 Transfer function.

State Space

With (r) for the ball's position, $(\dot{r})$ for velocity as the state variable and the servo angle (θ) as the input, we can represent the linearized system equations in the following state-space representation.

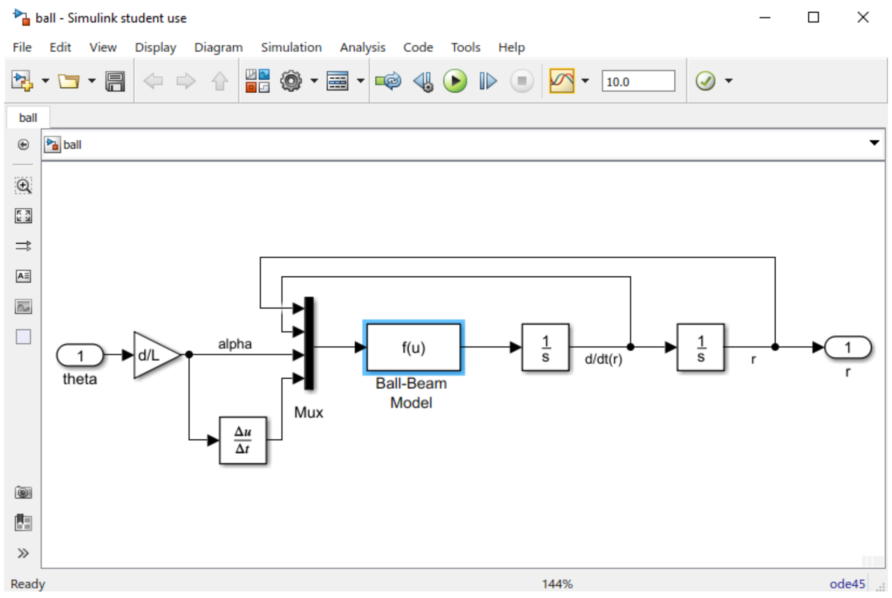
$$\begin{bmatrix} \dot{r} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{mgd}{L(\frac{J}{R^2} + m)} \end{bmatrix} \theta \tag{7}$$

However, to simplify the system even more a slight change to the model. The same equation for the ball still applies, but we control the rotation of the beam through the torque applied directly to the beam and disregard the servo angle (θ).

$$\begin{bmatrix}\dot{r}\\ \ddot{r}\\ \dot{\alpha}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & \dfrac{mg}{J/R^2+m} & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}r\\ \dot{r}\\ \alpha\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}0\\0\\0\\1\end{bmatrix}u \tag{8}$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}r\\ \dot{r}\\ \alpha\\ \dot{\alpha}\end{bmatrix} \tag{9}$$

Creating the model in Simulink



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Mux
f(u)
Ball-Beam Model
1/s
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r
1
r
Ready
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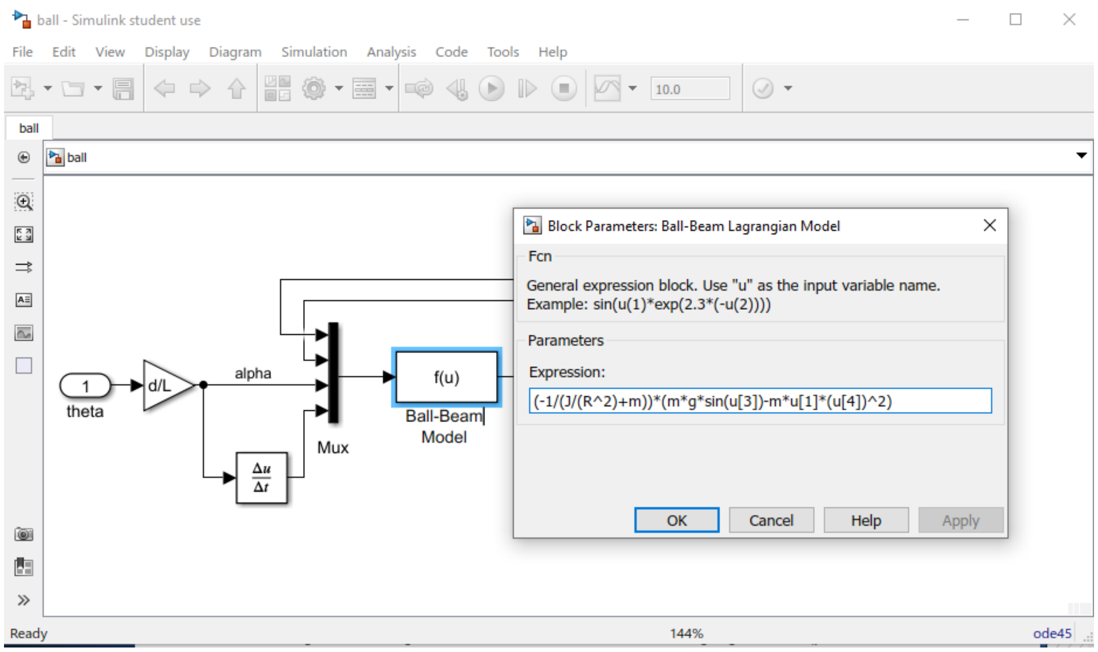
Figure 1.3 Ball-Beam Model in Simulink.

Our function block (*f(u)*) take the vector $[r \;\; \dot{r} \;\; \alpha \;\; \dot{\alpha}]$ and returns $\ddot{r}$.

$$\ddot{r} = \left[\frac{-1}{\left(\frac{J}{R^2}+m\right)}\right](mg\sin\alpha - mr\dot{\alpha}^2) \tag{10}$$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File Edit View Display Diagram Simulation Analysis Code Tools Help
10.0
ball
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theta
d/L
alpha
Mux
f(u)
Ball-Beam Model
Ready
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ode45

Block Parameters: Ball-Beam Lagrangian Model

Fcn

General expression block. Use "u" as the input variable name.
Example: sin(u(1)*exp(2.3*(-u(2))))

Parameters

Expression:

```
(-1/(J/(R^2)+m))*(m*g*sin(u[3])-m*u[1]*(u[4])^2)
```

Figure 1.4 Governing function of Ball-Beam model.

Model contained in a subsystem block (figure 1.5) with step, scope, gain block, lead compensator and feedback loop added (*See Lead Controller section for gain control and lead compensator values*).

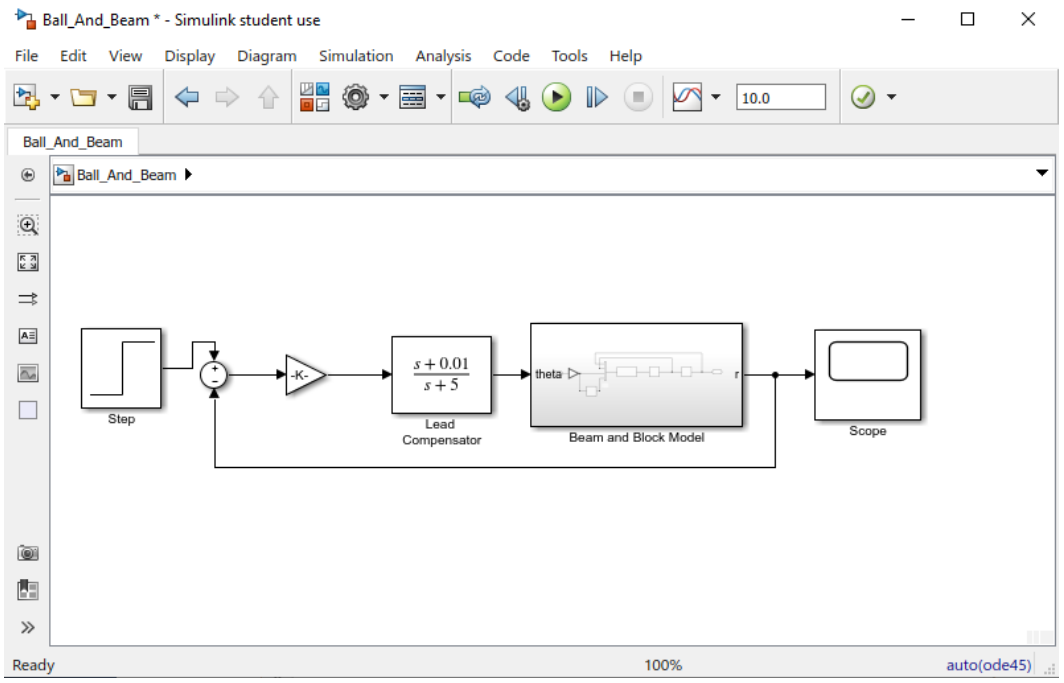


Figure 1.5 Subsystem block.

Closed loop response

With feedback control, the simulation is run and figure 1.6 gives the models response.

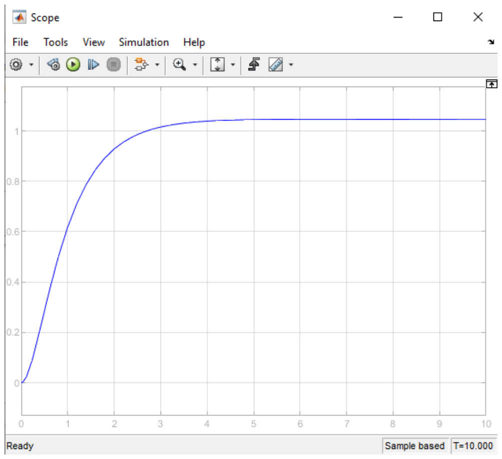


Figure 1.6 Simulation response.

Controller Design

Lead Controller

The purpose of a lead controller is to shift the root locus into the left-hand plane, contributing in overall stability in the system.

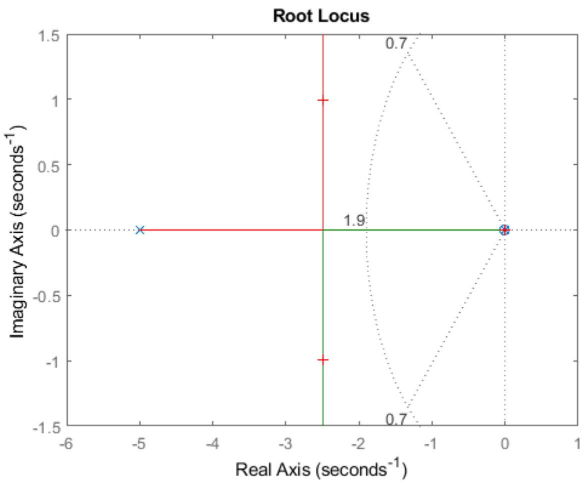
$$C(s) = K_c \frac{(s + z_0)}{(s + p_0)} \tag{11}$$

Note that the magnitude of (z_0) is less than the magnitude of (p_0).

Ball and beam simulink

```
clc
clear all
% mass of Ping Pong ball (kg)
m =  0.0027;
%radius of ball (m)
R = 0.0201;
% gravitational const. (m/s^2)
g = -9.81;
% Length of beam to pivot point (m)
L = 0.2;
% lever arm offset
d = 0.05;
% Moment of inertial of ball
J = 6.2e-7;

s = tf('s');

P_ball = -m*g*d/L/(J/R^2+m)/s^2;

rlocus(P_ball)

sgrid(0.70, 1.9)

axis([-5 5 -2 2])

zo = 0.01;
po = 5;
C=tf([1 zo],[1 po]);

rlocus(C*P_ball)
sgrid(0.70, 1.9)

[k,poles]=rlocfind(C*P_ball)

sys_cl=feedback(k*C*P_ball,1);

t=0:0.01:5;
figure
step(0.25*sys_cl,t)
```

```
Select a point in the graphics window

selected_point =

  -2.4917 - 0.9938i


k =

    4.6445


poles =

  -2.4950 + 0.9938i
  -2.4950 - 0.9938i
  -0.0101 + 0.0000i
```



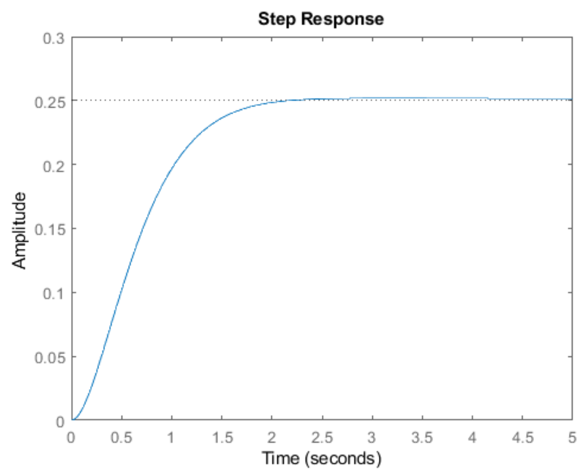


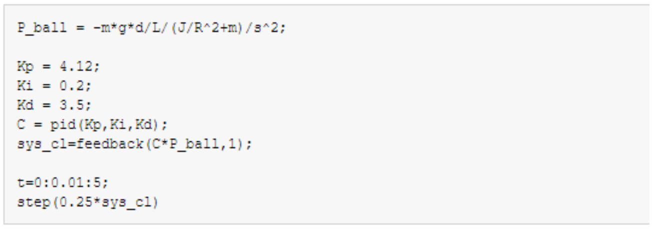
Figure 1.7 Lead controller design

Different values for (z_0) and (p_0) can be experimented with, however the stability requirements for our system were met using ($z_0 = 0.01$) and ($p_0 = 5$).

PID Control

Using MATLAB with all the physical system model parameters and equation (10), we can run multiple iterations, looking at the output step response to see which PID values generate an acceptable result (*Figure 1.8*).

Final values: $Kp = 4.21, \quad Ki = 0.2, \quad Kd = 3.5$



```
P_ball = -m*g*d/L/(J/R^2+m)/s^2;

Kp = 4.12;
Ki = 0.2;
Kd = 3.5;
C = pid(Kp,Ki,Kd);
sys_cl=feedback(C*P_ball,1);

t=0:0.01:5;
step(0.25*sys_cl)
```

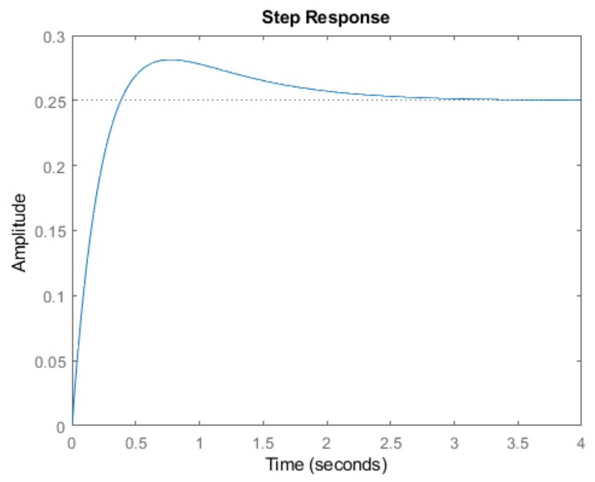


Figure 1.8 PID tuning.

Appendix

https://www.researchgate.net/publication/316628039_Design_and_control_of_ball_on_plate_system

Moment of inertia of a ping pong ball

https://physlab.lums.edu.pk/images/6/63/Moment1.pdf