Probability and Statistics for Engineers

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Lecture 2 :
2.3 Counting Sample Points

Outline

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Multiplication Rule

Example

After a tour in a store, I targeted three LEDs of the same dimensions and 2 TV stands suitable for my selection criteria. How many possible choices do I have ?

Multiplication rule

If an operation can be performed in n_1 ways, and if for each of these ways a second operation can be performed in n_2 ways, then the two operations can be performed together in $n_1 n_2$ ways.

Multiplication Rule

The generalized multiplication rule

If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 n_2 .. n_k$ ways.

Multiplication Rule

Example

You are buying a suitcase with a 4-digit code. How many possibilities do you have to choose a code?

solution

Let I be the set of digits such that
$I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $|I| = 10$
$n_1 = 10$, $n_2 = 10$, $n_3 = 10$, $n_4 = 10$, so the total number of possible codes is $10 \times 10 \times 10 \times 10 = 10^4$.

Permutation

Definition (**Permutation**)

The ***permutation*** *is the selection of k distinct elements, arranged in a certain order among the n.*

Remark

A ***permutation*** *is an arrangement of all or part of a set of elements.*

Example

The number 1489 is a permutation of the number 4918.

Permutation

Definition

*For any non-negative integer n, $n!$, called "**n factorial**", is defined as*

$$n! = n(n-1)...(2)(1)$$

with special case $0! = 1$.

Permutation

Theorem

The number of permutations of n elements is $n!$.

Example

How many "words" of 4 distinct letters can we form ?"

Solution

$P_4 = 4! = 24$

Permutation

Theorem

The number of distinct permutations of n elements of which n_1 are of one kind, n_2 of a second kind, . . . , n_k of a kth kind is

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

Permutation

Example

Consider the word "CELLULE". Find the number of words that can be formed by permuting the distinct letters, i.e., the permutations of the 7 letters C ;E ;E ; L ; L ; L ; U.

Permutation

Solution

Let's calculate the number of possible words that can be written by permuting these 7 letters C, E, E, L, L, L, U :

If all the letters had been distinct, we would have had a case of "Permutation of n distinct elements," so we could have deduced from the previous result the number $P_7 = 7! = 5040$

Permutation

Solution

However, there are two groups of elements : $2 \times E$ and $3 \times L$.

Therefore, we must eliminate all cases due to the two E's (2! cases) and those due to the three L's (3! cases, which is 6 cases). Hence, we obtain the result for the possible words. $P_7 = \dfrac{7!}{2!3!} = 420$

Permutation

Theorem

The number of permutations of k distinct elements among n elements denoted by $P(k,n)$, $P_{k,n}$ or ${}_nP_k$ is

$$P_{k,n} = \frac{n!}{(n-k)!}$$

Permutation

Example

In one year, three awards (Academic Excellence, Leadership and Initiative, Innovation and Creativity) will be given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, how many possible selections are there ?

Solution :

Since the awards are distinguishable, it is a permutation problem. The total number of sample points is :

$$P_{3,25} = \frac{25!}{(25-3)!} = \frac{25!}{22!} = (25)(24)(23) = 13,800$$

Combination

Definition (Combination)

A combination is an unordered subset of k elements, chosen from a set containing n elements.

Combination

Theorem

The number of combinations k elements chosen from a set of n elements is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Combination

Example

We have 15 medicines, and we want to test their compatibility in groups of 4. How many possible groups are there ?

Solution

$$\binom{4}{15} = \frac{15!}{4!11!} = 1365$$

Exercise 1

How many different letter arrangements can be made from the letters in the word STATISTICS ?

Solution :

Using the same argument as in the discussion for the previous example :

Here we have 10 total letters, with 2 letters (S, T) appearing 3 times each, letter I appearing twice, and letters A and C appearing once each.

$$P_{10} = \frac{10!}{3!3!2!} = 50,400$$

Exercise 2

How many words of 3 distinct letters can be formed in an alphabet of 26 letters ?

Solution

The number of distinct 3-letter words that can be formed in an alphabet of 26 letters is

$P_{3,26} = \frac{26!}{(26-3)!} = \frac{26!}{23!} = 26 \times 25 \times 24 = 150600$

Exercise 3

A group of 20 people are presented for elections to form the board of an association.
This board consists of a president, a secretary, and a treasurer.
How many possible boards are there ?

Solution

The act of assigning a role to each member of the association implies **an order** in the selection of the three new members of the board.

Exercise 3

Indeed, the choice (Mohammed, Nouhaila, Soufian) is different from (Soufian, Mohammed, Nouhaila), because in the first case, Mohammed is the president, while it is Soufian in the second case.
So, a board is an arrangement of 3 members chosen from the set of 50 members. Therefore, there are $P_{3,20} = \frac{20!}{17!} = 6840$ different boards.

Exercise 4

An urn contains 3 white balls and 4 black balls.

1. What is the number of possible choices for drawing two balls of the same color at once ?
2. What is the number of possible choices for drawing two balls of different colors at once ?

Exercise 4

1) Drawing two balls of the same color means that they are drawn from either the 3 white balls or from the 4 black balls
The order is not taken into account (we want two balls of the same color without considering the order).
So the method we are going to adopt is combinations

Exercise 4

So, $\binom{3}{2}$ is the number of choices for drawing two white balls, and $\binom{4}{2}$ is the number of choices for drawing two black balls.
Therefore, the total number of choices for drawing two balls of the same color is :

$$\binom{3}{2}+\binom{4}{2}=\frac{3!}{2!(3-2)!}+\frac{4!}{2!(4-2)!}=\frac{3!}{2!.1!}+\frac{4!}{2!2!}=$$
$$\frac{6}{2}+\frac{4.3.2.1}{2.2}=3+\frac{24}{4}=3+6=9$$

So, 9 is the number of possible choices for drawing two balls from the urn that are of the same color.

Exercise 4

2)The drawing of two balls of different colors means that one is chosen from the 3 white balls, and the other is chosen from the 4 black balls.
So, the number of possible choices for drawing two balls of different colors is as follows :

$$\binom{3}{1} \times \binom{4}{1} = \frac{3!}{1!(3-1)!} \times \frac{4!}{1!(4-1)!} = \frac{3!}{2!} \times \frac{4!}{3!} = 12$$