Probability and Statistics for Engineers

Dr. Sanae KOUISMI

AUI, School of Science and Engineering
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Lecture 4 :
2.6 Conditional probability, Independance, and the product rule

Outline

1. Conditional probability, Independance, and the product rule
   - Conditional probability
   - Independent Events

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Conditional probability

Definition (Conditional probability)

The ***conditional probability of B, given A****, denoted by $P(B|A)$, is defined by*

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ provided } P(A) > 0.$$

Remark

According to the previous relation, we can also calculate $P(A \cap B)$:

$$P(A \cap B) = P(B|A) \times P(A)$$
$$P(A \cap B) = P(A|B) \times P(B)$$

Conditional probability

Example

The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$.
Find the probability that a plane
(a) arrives on time, given that it departed on time,
(b) departed on time, given that it has arrived on time.

Conditional probability

Solution

1. The probability that a plane arrives on time, given that it departed on time, is

$$P(A|D) = \frac{P(D \cap A)}{P(D)} = \frac{0.78}{0.83} = 0.94.$$

2. The probability that a plane departed on time, given that it has arrived on time, is

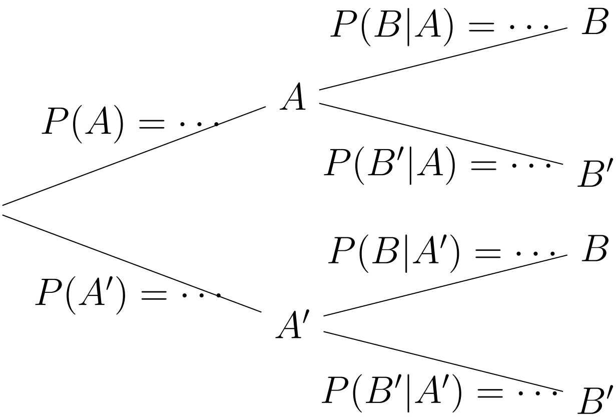
$$P(D|A) = \frac{P(D \cap A)}{P(A)} = \frac{0.78}{0.82} = 0.95.$$

Exercise 1

The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient files a lawsuit ?

Conditional probability

Tree diagram (of probability)

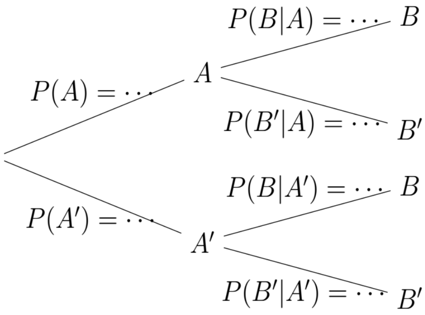


Conditional probability

Tree diagram (of probability)

Rule 1

The sum of the probabilities assigned to branches with the same origin is equal to 1

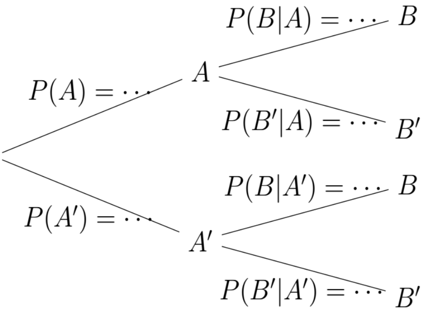


Conditional probability

Tree diagram (of probability)

Rule 2

The conditional probabilities are assigned to the branches of the second level

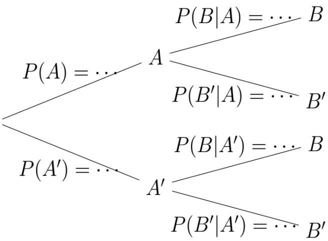


Conditional probability

Tree diagram (of probability)

Rule 3

The product of the probabilities assigned to branches belonging to the same path is equal to the probability of the intersection of events located on the nodes of the same path



Conditional probability

Tree diagram

Steps to follow :

1. Translate the statement into the 'language of probabilities'
2. Construct the probability tree
3. Calculate the requested probability

Exercise 2

A machine produces different types of products, including a product of type A. Due to unexpected breakdowns in the production line, defective products may occur. We randomly select a product.
One-third of type A products are defective, and one out of four products is both of type A and defective.
What is the probability of having a product that is not of type A ?

Exercise 2

Consider the events :

- A : "The chosen product is of type A"
- D : "The chosen product is defective"

One-third of type A products are defective :
$P(D|A) = \frac{1}{3}$
and one out of four products is both of type A and defective : $P(A \cap D) = \frac{1}{4}$
What is the probability of having a product that is not of type A : $P(A')$

Exercise 2

$$P(A \cap D) = P(A) \times P(D/A) = P(D) \times P(A/D)$$
$$P(A) = \frac{P(A \cap D)}{P(D/A)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$
$$P(A') = 1 - P(A) = \frac{1}{4}$$

Independent Events

A and B are independent means that having information about the occurrence of A does not provide any information about the occurrence of B. Furthermore, the occurrence of one event does not influence the occurrence of the other.

Definition (Indenpendant events)

Two events A and B are ***independent*** *if and only if*

$$P(B|A) = P(B) \quad or \quad P(A|B) = P(A),$$

assuming the existence of conditional probabilities. Otherwise, A and B are dependent.

Independent Events

Theorem

Two events A and B are independent if and only if

$$P(A \cap B) = P(A) \cdot P(B)$$

Independent Events

Example

A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Let A and B represent the respective events that the fire engine and the ambulance are available. Then

$$P(A \cap B) = P(A) \cdot P(B) = (0.98)(0.92) = 0.9016.$$

Independent Events

Definition

A collection of events $A = \{A_1, \ldots, A_n\}$ *are mutually independent if for any subset of* A, $A_{i_1}, \ldots, A_{i_k}$, *for* $k \leq n$, *we have*

$$P(A_{i_1} \cap \ldots \cap A_{i_k}) = P(A_{i_1}) \cdot \cdot P(A_{i_k}).$$

Exercise 1

A town has two fire engines operating independently. The probability that a specific engine is available when needed is 0.96.
(a) What is the probability that a fire engine is available when needed ?
(b) What is the probability that neither is available when needed ?