Probability and Statistics for Engineers

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Lecture 3 :
2.4 Probability of an Event and 2.5 Additive Rules

Outline

Probability of an Event

Given an experiment and a sample space S , the objective of probability is to assign to each event A a number $P(A)$, called *the probability of the event* A, which will give a precise measure of the chance that A will occur.

Probability of an Event

Definition (**Probability**)

Let S be a sample space. A probability (on S) is a set function, denoted by $P(\cdot)$, which associates a real number $P(A)$ to every event A in S, as shown below :

$$P \ : \ A \ \to \ P(A)$$

satisfying :

- *For every* $A \in S : 0 \leq P(A) \leq 1$
- $P(S) = 1.$
- *If* $A_1, A_2, \ldots$ *are pairwise mutually exclusive events* $(A_i \cap A_j = \varnothing$ *if* $i \neq j)$, *then :* $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$

Probability of an Event

Example

A coin is tossed twice. What is the probability that at least 1 head occurs ?

The sample space for this experiment is

$$S = \{HH, HT, TH, TT\}$$

If the coin is fair, each of these outcomes is **equally likely to occur**. Therefore, we assign a probability of ω to each sample point. Then $4\omega = 1$, or $\omega = \frac{1}{4}$.
If A represents the event of at least 1 head occurring, then $A = \{HH, HT, TH\}$ and $P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

Probability of an Event

Equally Likely outcomes

In the case where all sample points have the same probability, the probability of any event $A \subset S$ is equal to the ratio of n, the cardinality of A, to N, the cardinality of S.

$$P(A) = \frac{n}{N}$$

Probability of an Event

Example

We consider the random experiment of rolling a regular die. Let A be the event of "getting 3 or 6," thus $A = \{3, 6\}$.

We have : $P(A) = \dfrac{n}{N} = \dfrac{2}{6} = \dfrac{1}{3}$

1. Probability of an Event
2. Additive Rules

Additive Rules

Theorem

If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary

If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B)$$

Additive Rules

Corollary

If $A_1, A_2, .., A_n$ *are mutually exclusive, then*

$$P(A_1 \cup A_2..A_n) = P(A_1) + P(A_2) + .. + P(A_n)$$

Additive Rules

Definition (Partition)

The events $A_1, A_2, .., A_n$ *is called a partition of* S *if :*

- *For all* $i \neq j$ *:* $A_i \cap A_j = \emptyset$
- $A_1 \cup A_2 \cup .. \cup A_n = S$

Corollary

If $A_1, A_2, \ldots, A_n$ *is a partition of the sample space* S*, then*

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) =$$
$$P(A_1) + P(A_2) + \ldots + P(A_n) = P(S) = 1$$

Additive Rules

Theorem

For three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Additive Rules

Example

An industrial engineering student is going to graduate from a university by the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company A is 0.8, and his probability of getting an offer from company B is 0.6. If he believes that the probability that he will get offers from both companies is 0.5, what is the probability that he will get at least one offer from these two companies?

we have $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.6 - 0.5 = 0.9$

Additive Rules

Example

What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?

Solution

Let A be the event that 7 occurs and B the event that 11 comes up. A total of 7 occurs for 6 of the 36 sample points, and a total of 11 occurs for only 2 of the sample points. Since all sample points are equally likely, we have $P(A) = \frac{1}{6}$ and $P(B) = \frac{1}{18}$.

Additive Rules

The events A and B are mutually exclusive, since a total of 7 and 11 cannot both occur on the same toss. Therefore,

$$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{18} = \frac{2}{9}.$$

This result could also have been obtained by counting the total number of points for the event $A \cup B$, namely 8, and writing

$$P(A \cup B) = \frac{n}{N} = \frac{8}{36} = \frac{2}{9}.$$

Additive Rules

Example

If the probabilities are, respectively, 0.09, 0.15, 0.21, and 0.23 that a person purchasing a new automobile will choose the color green, white, red, or blue, what is the probability that a given buyer will purchase a new automobile that comes in one of those colors?

Solution

Let G, W, R, and B be the events that a buyer selects, respectively, a green, white, red, or blue automobile.

Additive Rules

Since these four events are mutually exclusive, the probability is
$P(G \cup W \cup R \cup B) = P(G) + P(W) + P(R) + P(B) = 0.09 + 0.15 + 0.21 + 0.23 = 0.68$

Additive Rules

Theorem

If A and A' are complementary events, then

$$P(A) + P(A') = 1$$

Additive Rules

Example

If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.12, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at work ?

Let E be the event that at least 5 cars are serviced. Now, $P(E) = 1 - P(E')$, where E' is the event that fewer than 5 cars are serviced. Since
$P(E') = 0.12 + 0.19 = 0.31$, then
$P(E) = 1 - 0.31 = 0.69$.

Exercise 1

If a letter is chosen at random from the English alphabet, find the probability that the letter
(a) is a vowel exclusive of y ;
(b) is listed somewhere ahead of the letter j ;
(c) is listed somewhere after the letter g.

Exercise 2

In a high school graduating class of 100 students, 54 studied mathematics, 69 studied history, and 35 studied both mathematics and history. If one of these students is selected at random, find the probability that
(a) the student took mathematics or history ;
(b) the student did not take either of these subjects ;
(c) the student took history but not mathematics.

Exercise 3

It is common in many industrial areas to use a filling machine to fill boxes full of product. This occurs in the food industry as well as other areas in which the product is used in the home, for example, detergent. These machines are not perfect, and indeed they may A, fill to specification, B, underfill, and C, overfill. Generally, the practice of underfilling is that which one hopes to avoid. Let $P(B) = 0.001$ while $P(A) = 0.990$.

(a) Give $P(C)$.

(b) What is the probability that the machine does not underfill ?

(c) What is the probability that the machine either overfills or underfills ?