Probability and Statistics for Engineers

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Lecture 1 :
2.1 Sample Space and 2.2 Events

Outline

Introduction

The probability theory is a branch of mathematics that

- allows modeling experiments whose outcome cannot be predicted with absolute certainty.
- enables making predictions, where the outcome of experiments is **uncertain**, in various fields such as :
  - biology,
  - medicine,
  - engineering,
  - psychology,
  - sociology,

Terminology

- In the study of statistics, we are concerned basically with the presentation and interpretation of **chance outcomes** that occur in a planned study or scientific investigation.

Example

We may record the number of traffic accidents that occur monthly.

Terminology

- We shall refer to any recording of information, whether it be numerical or categorical, as an **observation**.
- **An experiment** is any activity or process whose outcome is subject to uncertainty.

Example

Tossing of a coin is an example of a statistical experiment. In this experiment, there are only two possible outcomes, heads or tails.

Sample Space

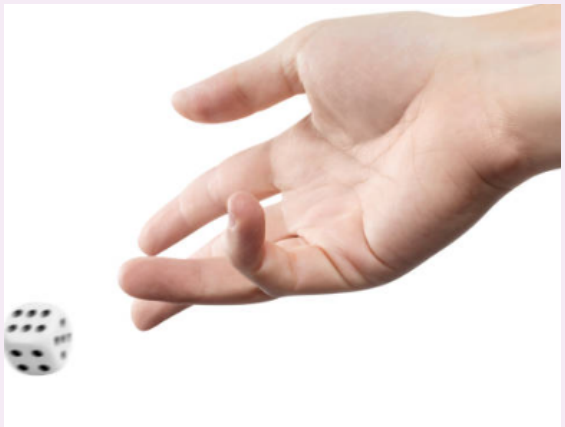
Definition (**Sample space**)

- *The set of all possible outcomes of a statistical experiment is called the* ***sample space*** *and is represented by the symbol* S.
- *Each outcome in a sample space is called an* ***element*** *or a* ***member*** *of the sample space, or simply a* ***sample point***.

Sample Space

Example 1

Consider the experiment of tossing a die. we are interested in the number that shows on the top face.



- the sample space is $S = \{1, 2, 3, 4, 5, 6\}$,
- 3 is an element of the sample space.

Sample Space

Example 2

An experiment consists of flipping a coin

- $S = \{Head, Tail\}$,
- $Tail$ is an element of the sample space.

Sample Space

Example 3

Simultaneously draw two balls from an urn containing one white ball and two black balls

- $S = \{\{B1, B2\}; \{B1, W\}; \{B2, W\}\}$
- $\{B2, W\}$ is an element of the sample space.

Sample Space

Remark

In some experiments, it is helpful to list the elements of the sample space systematically by means of a ***tree diagram****.*

Example

An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once. To list the elements of the sample space providing the most information, we construct the tree diagram of Figure 2.1.

Sample Space

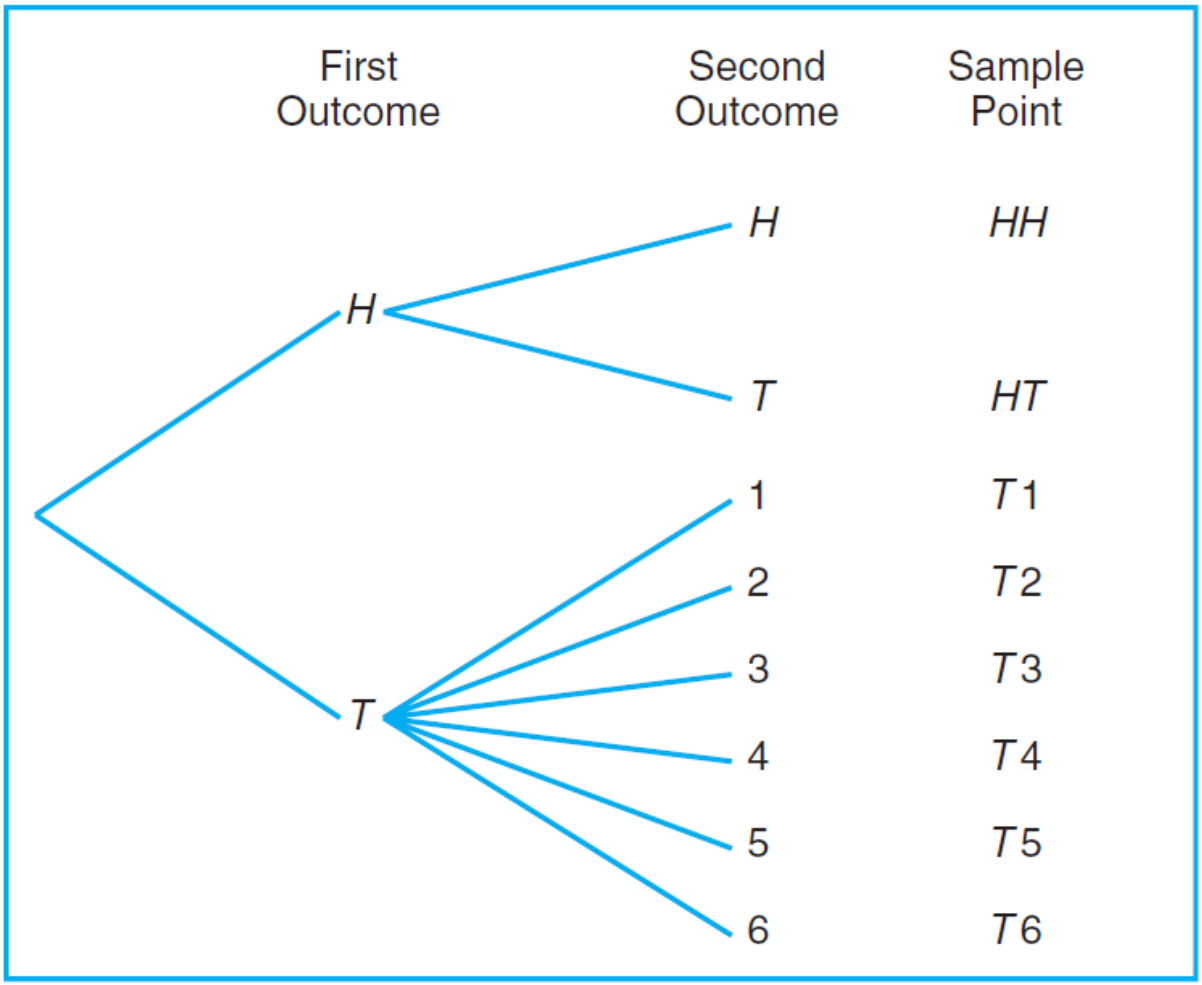


Figure 2.1: Tree diagram for Example 2.2.

Event

Definition (**Event**)

*An **event** is a subset of a sample space.*

Example

We consider the set $E = \{1, 2, 3, 4\}$.
The statistical experiment consists of choosing two numbers simultaneously from E.
The event "*Choose two numbers whose sum is greater than 5*" is an event that we can name A, with the outcomes $\{4, 2\}$ and $\{4, 3\}$. It is written as :
$A = \{\{4, 2\}; \{4, 3\}\}$.

Event

Definition (The complement of an event)

The complement of an event A *with respect to S is the subset of all elements of S that are not in A. We denote the complement of A by the symbol A'.*

Example

Let R be the event that a red card is selected from an ordinary deck of 52 playing cards, and let S be the entire deck. Then R' is the event that the card selected from the deck is not a red card but a black card.

Event

Example

Consider the sample space $S = \{\text{book, cell phone, mp3, paper, stationery, laptop}\}$. Let $A = \{\text{book, stationery, laptop, paper}\}$. Then the complement of A is $A' = \{\text{cell phone, mp3}\}$.

Event

Definition (**Intersection**)

*The **intersection** of two events A and B, denoted by the symbol $A \cap B$, is the event containing all elements that are common to A and B.*

Example

Let E be the event that *a person selected at random in a classroom is majoring in engineering*, and let F be the event that *the person is female.*
Then $E \cap F$ is the event of *all female engineering students in the classroom.*

Event

Example

Let $V = \{a, e, i, o, u\}$ and $C = \{l, r, s, t\}$, then it follows that $V \cap C = \emptyset$.
That is, V and C have no elements in common and, therefore, cannot both simultaneously occur. We can say V and C are mutually exclusive.

Event

Definition

Two events A and B are ***mutually exclusive****, or* ***disjoint****, if $A \cap B = \emptyset$, that is, if A and B have no elements in common.*

Event

Definition (**Union**)

*The **union** of the two events A and B, denoted by the symbol $A \cup B$, is the event containing all the elements that belong to A or B or both.*

Example

Let $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$, then $A \cup B = \{a, b, c, d, e\}$.

Event

Definition (**Union**)

*The **union** of the two events A and B, denoted by the symbol $A \cup B$, is the event containing all the elements that belong to A or B or both.*

Example

If $M = \{x|3 < x < 9\}$ and $N = \{y|5 < y < 12\}$, then $M \cup N = \{z|3 < z < 12\}$

Set Operators

- $A \cup B$ is read as "**event A or B**" and means that at least one of the events A or B occurs.
- $A \cap B$ is read as "**event A and B**" and means that **A and B occur simultaneously**.
- $A \setminus B$ is read as "**event A minus B**" and means that **A occurs alone (without B)**.
- $A \subseteq B$ is read as "**A is included in B**" and means that **if A occurs, then B also occurs.**

Set Operators

Example

A die is rolled and we consider the following two events : $A = \{1, 2, 3, 5\}$ and $B = \{2, 3, 4\}$.
Determine the following sets :

- $A \cup B$
- $A \cap B$
- $A \setminus B$

Event

- The relationship between events and the corresponding sample space can be illustrated graphically by means of **Venn diagrams**.
- In a Venn diagram we let the sample space be a rectangle and represent events by circles drawn inside the rectangle.

Event

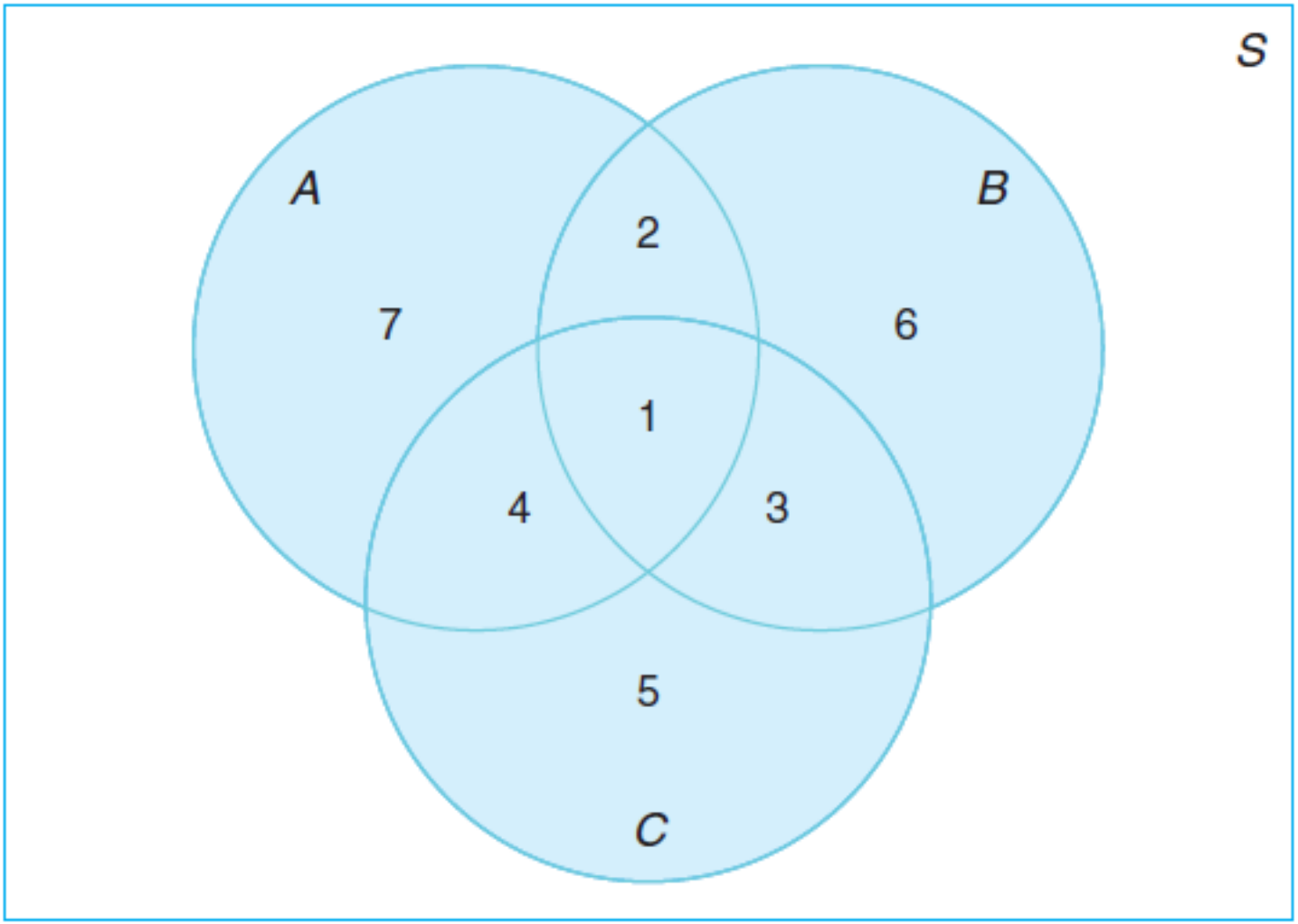


Figure 2.3: Events represented by various regions.

Event

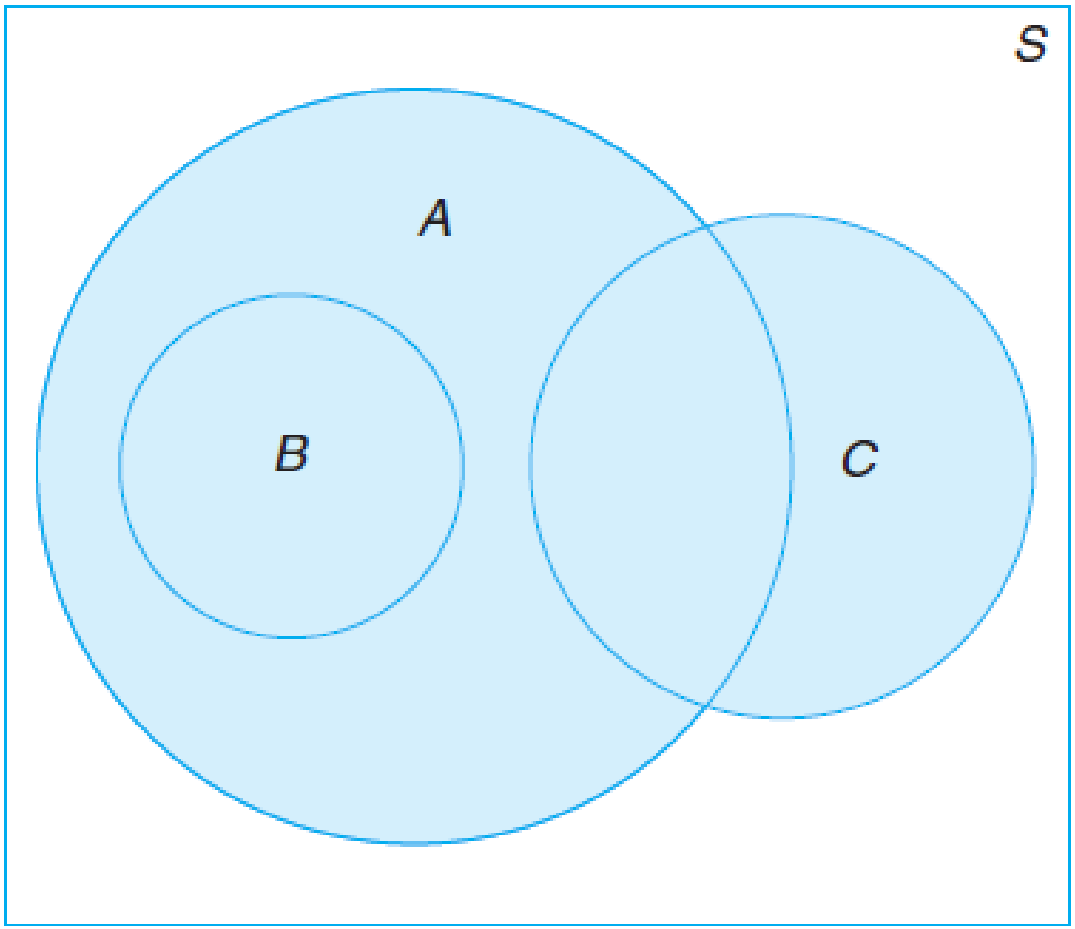


Figure 2.4: Events of the sample space S.

Event

Rules

1. $A \cap \phi = \phi$.
2. $A \cup \phi = A$.
3. $A \cap A' = \phi$.
4. $A \cup A' = S$.
5. $S' = \phi$.
6. $\phi' = S$.
7. $(A')' = A$.
8. $(A \cap B)' = A' \cup B'$.
9. $(A \cup B)' = A' \cap B'$.

Exercise 1

List the elements of each of the following sample spaces :

(a) The set of integers between 1 and 50 divisible by 8.

(b) The set $S = \{x \mid x^2 + 4x - 5 = 0\}$.

(c) The set of outcomes when a coin is tossed until a tail or three heads appear.

(d) The set $S = \{x \mid x \text{ is a continent}\}$.

(e) The set $S = \{x \mid 2x - 4 \geq 0 \text{ and } x < 1\}$.

Exercise 2

Let S be the sample space of a random experiment, and let A, B, and C be events. Express the following events using set notation :

1. All three events A, B, and C occur.
2. None of the three events occurs.
3. At least one of the events occurs.
4. At most two of the events occur.

Exercise 3

Which of the following events are equal ?

(a) $A = \{1, 3\}$;

(b) $B = \{x \mid x \text{ is a number on a die}\}$;

(c) $C = \{x \mid x^2 - 4x + 3 = 0\}$;

(d) $D = \{x \mid x$ is the number of heads when six coins are tossed$\}$.

Exercise 3

If $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 3, 4, 5\}$, and $D = \{1, 6, 7\}$, list the elements of the sets corresponding to the following events :

(a) $A \cup C$;

(b) $A \cap B$;

(c) C' ;

(d) $(C' \cap D) \cup B$;

(e) $(S \cap C)'$;

(f) $A \cap C \cap D'$.