STRAIGHT LINE

01 Slope of a Straight Line

If the line makes an angle θ with positive direction of x-axis, then $\tan\theta$ is called slope of the line and is denoted by m.

02 Various forms of Line

01 **Slope intercept form:** The line with slope m and y intercept c is y=mx+c

02 **Slope point form :** The line with slope m and passing through the point (x_1, y_1) is $y-y_1=m(x-x_1)$.

03 **Two point form :** The line passing through the point (x_1, y_1) and (x_2, y_2) is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

04 **Intercept form :**

$$\frac{x}{a}+\frac{y}{b}=1$$

Here, a and b are x intercept and y intercept respectively which may be positive or negative

05 **Normal form :** The line whose normal makes an angle α with positive x axis and has length =p is

$$x\cos\alpha+\gamma\sin\alpha=p.$$

06 **6. Distance or parametric form :**

$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$$

07 **General form of line :** The equation $ax+by+c=0$ where a and b are not simultaneously zero is called general form of line.

Note

(a) x- intercept made by $ax+by+c=0$ is $-\frac{c}{a}$.

(b) y- intercept made by $ax+by+c=0$ is $-\frac{c}{b}$.

(c) Slope of the line $ax+by+c=0$ is $\left(-\frac{a}{b}\right)$.

(d) Area of triangle which the line ax+by+c=0 makes with coordinate axes $=\left|\frac{c^2}{2ab}\right|$.

03 Angle Between Two Lines

Let the slope of the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are respectively m_1 and m_2 and If the angle between these lines be θ, then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{a_1b_2-a_2b_1}{a_1a_2+b_1b_2}\right|$$

(a) Condition for the lines to be parallel is

$$m_1=m_2 \text{ or } \frac{a_1}{a_2}=\frac{b_1}{b_2}$$

(b) Condition for the lines to be coincidential is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

(c) Condition for the lines to be perpendicular is

$$m_1m_2=-1$$

$$a_1a_2+b_1b_2=0$$

04 Family of Lines

01 Family of lines which are parallel to the line $ax+by+c=0$ is $ax+by+\lambda=0$ where $\lambda\in R$

02 Family of lines which is perpendicular to the line $ax+by+c=0$ is $bx-ay+\lambda=0$ where $\lambda\in R$

03 Family of lines passing through the intersection point of $L_1=a_1x+b_1y+c_1=0$ and $L_2=a_2x+b_2y+c_2=0$ is $L_1+\lambda L_2=0$ where, $\lambda\in R$

05 Distance between a Point and a line

Let (x_1, y_1) be the given point and $ax+by+c=0$ be the given line then distance between them, is

$$p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

06 Distance between Two Parallel Lines

Let the equation of two parallel lines be $ax+by+c=0$ and $ax+by+c'=0$,

then distance between them is given by $P=\left|\frac{c-c'}{\sqrt{a^2+b^2}}\right|$

Note

01 If the foot of perpendicular drawn from point (x_1, y_1) to the line $ax+by+c=0$ be (h,k), then,

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=-\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

02 If the image of point (x_1, y_1) in the line mirror $ax+by+c=0$ be (α,β), then $\frac{\alpha-x_1}{a}=\frac{\beta-y_1}{b}=-2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$

07 Concurrent Lines

Three or more lines are said to be concurrent if they have only one point in common Let the three concurrent lines are $a_rx+b_ry+c_r=0$ where $r=1,2,3,$ then

$$\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$$

Note

If lines are concurrent Δ must be zero but $\Delta=0$ not necessarily imply the lines are concurrent.

08 Comparison of Two Points with Respect to a Line

Let the given line be $L(x,y)=ax+by+c=0$ and the points are $P(x_1,y_1)$ and $Q(x_2,y_2)$, then

1. If $L(x_1,y_1).L(x_2,y_2)>0$ points P and Q lies on the same side of line L=0

2. If $L(x_1,y_1).L(x_2,y_2)<0$ points P and Q lies on the opposite side of line L=0

09 Angle Bisectors of Angle Between Two Lines

The equations of angle bisectors of the angle between the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ is given by

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

Note

In the above equation if c_1 and c_2 are of same sign, then taking the sign same as the sign of $a_1a_2+b_1b_2$ we always get angle bisector of the given lines. Also by taking + in above formula we get the bisector of that angle region which contains origin.

1. Equation of straight line passing through given point (x_1, y_1) and making a given angle α with the given line

$y=mx+c$, are $y-y_1=\frac{m-\tan\alpha}{1+m\tan\alpha}(x-x_1)$ or $y-y_1=\frac{m+\tan\alpha}{1-m\tan\alpha}(x-x_1)$

2. The image of the line $ax+by+c=0$ in the line $x=\lambda$ is $a(2\lambda-x)+by+c=0$

3. The image of the line $ax+by+c=0$ in the line $y=\lambda$ is $ax+b(2\lambda-y)+c=0$

10 Non-homogeneous equation of degree 2

The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$... (i) is called non-homogeneous equation of degree 2.

Let, $\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

1. Equation (I) represents a pair of straight lines if $\Delta=0,\ h^2\ge ab,\ g^2\ge ac$ and $f^2\ge bc$.

2. If lines given by (I) have from $y=m_1x+c_1$ and $y=m_2x+c_2$, then $m_1+m_2=-\frac{2h}{b}, m_1m_2=\frac{a}{b}, |m_1-m_2|=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

3. If angle between the lines given by (I) be θ, then $\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

Condition for line pair (I) to represents a pair of parallel lines is $h^2=ab$

Condition for line pair (I) to represents a pair of perpendicular lines is $a+b=0$

4. If the line pair given by (I) be the pair of parallel lines, then distance between them is $=2\sqrt{\frac{g^2-ac}{a(a+b)}}$ or $2\sqrt{\frac{f^2-bc}{b(a+c)}}$

5. Condition for the line pair (I) to represent coincidental lines is $h^2=ab$ and $g^2=ac$ and $f^2=bc$

6. Point of intersection of the lines given by (I) is $\left(\frac{hf-bg}{ab-h^2}, \frac{hg-af}{ab-h^2}\right)=(\alpha,\beta)$

7. Equation of angle bisector of the angle between line pair (I) is $\frac{(x-\alpha)^2-(y-\beta)^2}{a-b}=\frac{(x-a)(y-\beta)}{h}$

Note

In homogeneous case, $ax^2+2hxy+by^2=0$ replace g, f , c by 0.

11 Points to Remember

01 Equation of lines perpendicular to the lines $ax^2+2hxy+by^2=0$ is given by $bx^2-2hxy+ay^2=0$

02 Two pair of straight lines viz. $a_1x^2+2h_1xy+b_1y^2=0$ and $a_2x^2+2h_2xy+b_2y^2=0$ have

(a) a line in common if $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}^2=4\begin{vmatrix} a_1 & h_1 \\ a_2 & h_2 \end{vmatrix}\begin{vmatrix} h_1 & b_1 \\ h_2 & b_2 \end{vmatrix}$

(b) both lines in common if $\frac{a_1}{a_2}=\frac{h_1}{h_2}=\frac{b_1}{b_2}$.

03 Equation of line pair joining the point of intersection of curve $ax^2+2hxy+by^2+2gx+2fy+c=0$ and line $lx+my=1$ with origin is given by $ax^2+2hxy+by^2+(2gx+2fy)(lx+my)+c(lx+my)^2=0$