CHAPTER 1
Basic Mathematics

NUMBER SYSTEM

(*i*) **Natural numbers:** The counting numbers 1, 2, 3, 4, ... are called Natural Numbers. The set of natural numbers is denoted by N.

Thus $N = \{1, 2, 3, 4, ...\}$.

(*ii*) **Whole numbers:** Natural numbers including zero are called whole numbers. The set of whole numbers is denoted by W.

Thus W = $\{0, 1, 2, ...\}$

(*iii*) **Integers:** The numbers ... – 3, – 2, – 1, 0, 1, 2, 3 .... are called integers and the set is denoted by I or Z. Thus I (or Z) = $\{.. – 3, – 2, – 1, 0, 1, 2, 3...\}$

Note: (*a*) Positive integers $I^+ = \{1, 2, 3 ....\} = N$

(*b*) Negative integers $I^- = \{....., –3, –2, –1\}$.

(*c*) Non-negative integers (whole numbers) = $\{0, 1, 2, ...\}$.

(*d*) Non-positive integers = $\{......, –3, –2, –1, 0\}$.

(*iv*) **Even integers:** Integers which are divisible by 2 are called even integers.

e.g. $0, \pm 2, \pm 4$,.......

(*v*) **Odd integers:** Integers which are not divisible by 2 are called odd integers.

e.g. $\pm 1, \pm 3, \pm 5, \pm 7$......

(*vi*) **Prime numbers:** Natural numbers which are divisible by 1 and itself only are called prime numbers.

e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

(*vii*) **Composite number:** Let 'a' be a natural number, '*a*' is said to be composite if, it has atleast three distinct factors.

e.g. 4, 6, 8, 9, 10, 12, 14, 15 ....

Note: (*a*) 1 is neither a prime number nor a composite number.

(*b*) Numbers which are not prime are composite numbers (except 1).

(*c*) '4' is the smallest composite number.

(*d*) '2' is the only even prime number.

(*viii*) **Co-prime numbers:** Two natural numbers (not necessarily prime) are called coprime, if their H.C.F (Highest common factor) is one.

e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) (15, 16) etc.

These numbers are also called as **relatively prime** numbers.

Note:

(*a*) Two prime numbers are always co-prime but converse need not be true.

(*b*) Consecutive natural numbers are always co-prime numbers.

(*ix*) **Twin prime numbers:** If the difference between two prime numbers is two, then the numbers are called twin prime numbers.

e.g. $\{3, 5\}, \{5, 7\}, \{11, 13\}, \{17, 19\}, \{29, 31\}$

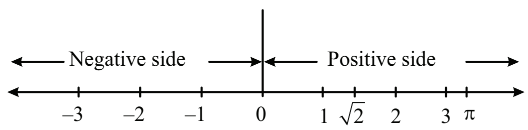
(*x*) **Rational numbers:** All the numbers that can be represented in the form p/q, where p and q are integers and $q \neq 0$, are called rational numbers and their set is denoted by Q. Thus $Q = \{p/q : p, q \in \text{I and } q \neq 0\}$. It may be noted that every integer is a rational number since it can be written as p/q. It may be noted that all recurring decimals are rational numbers.

Note: Maximum number of different decimal digits in $\frac{p}{q}$ is equal to q, i.e. $\frac{11}{9}$ will have maximum of 9 different decimal digits.

(*xi*) **Irrational numbers:** The numbers which can not be expressed in p/q form where $p, q \in$ I and $q \neq 0$ i.e. the numbers which are not rational are called irrational numbers and their set is denoted by Q^c. (i.e. complementary set of Q) e.g. $\sqrt{2}$, $1 + \sqrt{3}$ etc. Irrational numbers can not be expressed as recurring decimals.

Note: $e \approx 2.71$ (is called Napier's constant) and $\pi \approx 3.14$ are irrational numbers.

(*xii*) **Real numbers:** Numbers which can be expressed on number line are called real numbers. The complete set of rational and irrational numbers is the set of real numbers and is denoted by R. Thus $R = Q \cup Q^C$.



Real line

All real numbers follow the order property i.e. if there are two distinct real numbers a and b then either $a < b$ or $a > b$.

Note:

(a) Integers are rational numbers, but converse need not be true.

(b) Negative of an irrational number is an irrational number.

(c) Sum of a rational number and an irrational number is always an irrational number

e.g. $2 + \sqrt{3}$

(d) The product of a non zero rational number and an irrational number will always be an irrational number.

(e) If $a \in Q$ and $b \notin Q$, then ab = rational number, only if $a = 0$.

(f) Sum, difference, product and quotient of two irrational numbers need not be a irrational number or we can say, result may be a rational number also.

ADVANCED LEARNING

($xiii$) **Complex number:** A number of the form $a + ib$ is called a complex number, where $a, b \in R$ and $i = \sqrt{-1}$. Complex number is usually denoted by Z and the set of complex number is represented by C. Thus $C = \{a + ib : a, b \in R$ and $i = \sqrt{-1}\}$

Note: It may be noted that $N \subset W \subset I \subset Q \subset R \subset C$.

Train Your Brain

Example 1: The value of $1.\overline{285714} \div 1.\overline{714285} =$ ______.

(a) $\frac{3}{4}$ (b) $\frac{7}{8}$

(c) $\frac{7}{12}$ (d) $\frac{3}{7}$

Sol.

$$1.\overline{285714}$$
$$= 1 + 0.\overline{285714}$$
$$= 1 + \frac{2}{7} = \frac{9}{7}$$
$$1.\overline{714285}$$
$$= 1 + \frac{5}{7} = \frac{12}{7}$$
$$\therefore 1.\overline{285714} \div 1.\overline{714285}$$
$$= \frac{9}{7} \div \frac{12}{7}$$
$$= \frac{9}{7} \times \frac{7}{12}$$
$$= \frac{3}{4}$$

Example 2: Prove that the difference $10^{25} - 7$ is divisible by 3.

Sol. Write the given difference in the form $10^{25} - 7 = (10^{25} - 1) - 6$. The number $10^{25} - 1 = \underbrace{99..9}_{25\ digits}$ is divisible by 3 (and 9). Since the numbers $(10^{25} - 1)$ and 6 are divisible by 3, the number $10^{25} - 7$, being their difference, is also divisible by 3 without a remainder.

Concept Application

1. The product of $1.\overline{142857}$ and $0.\overline{63}$ = _____.

(a) $\frac{8}{11}$ (b) $\frac{7}{11}$

(c) $\frac{11}{7}$ (d) $\frac{8}{7}$

2. If $x = \sqrt{12} - \sqrt{9}$, $y = \sqrt{13} - \sqrt{10}$, and $z = \sqrt{11} - \sqrt{8}$, then which of the following is true?

(a) $z > x > y$

(b) $z > y > x$

(c) $y > x > z$

(d) $y > z > x$

SOME IMPORTANT IDENTITIES

1. $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$
2. $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
5. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
6. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$
7. $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$
9. $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$
10. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$
If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
11. $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$
12. $a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)$

Train Your Brain

Example 3: Show that the expression, $(x^2-yz)^3+(y^2-zx)^3+(z^2-xy)^3-3(x^2-yz).(y^2-zx).(z^2-xy)$ is a perfect square and find its square root.

Sol. $(x^2-yz)^3+(y^2-zx)^3+(z^2-xy)^3-3(x^2-yz)(y^2-zx)(z^2-xy)=a^3+b^3+c^3-3abc$

where $a=x^2-yz,\ b=y^2-zx,\ c=z^2-xy$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=\dfrac{1}{2}(a+b+c)((a-b)^2+(b-c)^2+(c-a)^2)$

$=\dfrac{1}{2}(x^2+y^2+z^2-xy-yz-zx)[(x^2-yz-y^2+zx)^2+(y^2-zx-z^2+xy)^2+(z^2-xy-x^2+yz)^2]$

$=\dfrac{1}{2}(x^2+y^2+z^2-xy-yz-zx)[\{x^2-y^2+z(x-y)\}^2+\{y^2-z^2+x(y-z)\}^2+\{z^2-x^2+y(z-x)\}^2]$

$=\dfrac{1}{2}(x^2+y^2+z^2-xy-yz-zx)(x+y+z)^2[(x-y)^2+(y-z)^2+(z-x)^2]$

$=(x+y+z)^2(x^2+y^2+z^2-xy-yz-zx)^2$

$=(x^3+y^3+z^3-3xyz)^2$

(which is a perfect square) its square roots are

$\pm(x^3+y^3+z^3-3xyz)$

Example 4: If $x^2-4x+1=0$, then what is the value of $x^3+\dfrac{1}{x^3}$?

Sol. $x^2-4x+1=0\Rightarrow x+\dfrac{1}{x}=4$

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=4^3-3\times 4=52$$

Example 5: If $x+\dfrac{1}{x}=a$, then what is the value of $x^3+x^2+\dfrac{1}{x^3}+\dfrac{1}{x^2}$?

(a) a^3+a^2 (b) a^3+a^2-5a

(c) a^3+a^2-3a-2 (d) a^3+a^2-4a-2

Sol.

Given, $x+\dfrac{1}{x}=a$

Now, $x^3+x^2+\dfrac{1}{x^3}+\dfrac{1}{x^2}=\left(x^3+\dfrac{1}{x^3}\right)+\left(x^2+\dfrac{1}{x^2}\right)$

$=\left(x+\dfrac{1}{x}\right)^3-3\left(x+\dfrac{1}{x}\right)+\left(x+\dfrac{1}{x}\right)^2-2$

$=a^3-3a+a^2-2=a^3+a^2-3a-2.$

Concept Application

3. If $x^{1/3}+y^{1/3}+z^{1/3}=0$, then what is $(x+y+z)^3$ equal to?
(a) 1 (b) 3
(c) $3xy$ (d) $27xyz$

4. If $a+b+c=0$, then what is the value of
$$\frac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$$
(a) 1 (b) 3 (c) $\dfrac{1}{3}$ (d) 0

5. If $x+\dfrac{1}{x}=p$ then $x^6+\dfrac{1}{x^6}$ equals to :
(a) p^6+6p (b) p^6-6p
(c) $p^6+6p^4+9p^2+2$ (d) $p^6-6p^4+9p^2-2$

6. If $x+\dfrac{1}{x}=4,$ then find values of
(i) $x^2+\dfrac{1}{x^2}$ (ii) $x^3+\dfrac{1}{x^3}$
(iii) $x^4+\dfrac{1}{x^4}$

7. Prove that $(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})=\dfrac{(1-x^{32})}{(1-x)}$

8. If x, y, z are all different real numbers, then prove that
$$\frac{1}{(x-y)^2}+\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}=\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2$$

9. If $\dfrac{a}{b}+\dfrac{b}{a}=-1,$ then find value of a^3-b^3.

10. If $a-b=-8$, $ab=-12$ then a^3-b^3 will be

11. The product $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$ simplifies to

12. Find the real values of p, q, r satisfying $(2p-3)^8+(1-q)^6+(4-3r)^4=0$.

ADVANCED LEARNING

INDICES

If 'a' is any non zero real or imaginary number and 'm' is the positive integer, then $a^m=a\cdot a\cdot a. \ldots a$ (m times). Here a is called the base and m is called the index, power or exponent.

Law of indices:

1. $a^0=1,\ (a\neq 0)$
2. $a^{-m}=\dfrac{1}{a^m},\ (a\neq 0)$
3. $a^{m+n}=a^m\cdot a^n$, where m and n are rational numbers

4. $a^{m-n} = \dfrac{a^m}{a^n}$, where m and n are rational numbers, $a \neq 0$
5. $(a^m)^n = a^{mn}$
6. $a^{p/q} = \sqrt[q]{a^p}$
7. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$, where $m, n \in N$ and ($m, n \geq 2$) and a is positive rational number
8. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, $a, b \in R$ and atleast one of a and b should be positive

SURDS

If a is a positive rational number, which is not the nth power (n is any natural number) of any rational number, then the irrational number $\pm\sqrt[n]{a}$ are called simple surds or monomial surds.

Every surd is an irrational number (but every irrational number is not a surd). So, the representation of monomial surd on a number line is same as that of irrational numbers.

Examples:

1. $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.

2. $\sqrt[3]{5}$ is a surd and $\sqrt[3]{5}$ is an irrational number.

3. π is an irrational number, but it is not a surd.

4. $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

Train Your Brain

Example 6: Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$

Sol. $a^{9(1/6)(1/3)4} \cdot a^{9(1/3)(1/6)4} = a^2 \cdot a^2 = a^4$.

Example 7: Arrange the following in ascending or descending order of magnitude:

$\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

Sol. $\sqrt[4]{6} = 6^{1/4}, \sqrt[3]{7} = 7^{1/3}, \sqrt{5} = 5^{1/2}$

LCM of the denominators of the exponents of these three terms, 4, 3 and 2 is12.

Now express the exponent of each term, as a fraction in which then denominator is 12.

$$6^{\frac{1}{4}} = 6^{\frac{3}{12}} = \left(6^3\right)^{\frac{1}{12}} = \sqrt[12]{216}$$

$$7^{\frac{1}{3}} = 7^{\frac{4}{12}} = \left(7^4\right)^{\frac{1}{12}} = \sqrt[12]{2401}$$

$$5^{\frac{1}{2}} = 5^{\frac{6}{12}} = \left(5^6\right)^{\frac{1}{12}} = \sqrt[12]{15625}$$

Now, $\sqrt[4]{6} = \sqrt[12]{216}, \sqrt[3]{7} = \sqrt[12]{2401}, \sqrt{5} = \sqrt[12]{15625}$

Hence, their ascending order is

$\sqrt[12]{216}, \sqrt[12]{2401}, \sqrt[12]{15625}$, *i.e.*, $\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

$\therefore$ The descending order of magnitude of the given radical is $\sqrt{5}, \sqrt[3]{7}, \sqrt[4]{6}$.

Example 8: Find the square root of $10+\sqrt{24}+\sqrt{60}+\sqrt{40}$

Sol. $= \sqrt{10+\sqrt{24}+\sqrt{60}+\sqrt{40}}$

$= \sqrt{10+2\sqrt{6}+2\sqrt{15}+2\sqrt{10}}$

$= \sqrt{(2+3+5)+2\sqrt{2(3)}+2\sqrt{3(5)}+2\sqrt{2(5)}}$

$= \sqrt{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)^2}$

$= \sqrt{2}+\sqrt{3}+\sqrt{5}$.

Concept Application

13. If the surds $\sqrt[4]{4}, \sqrt[6]{5}, \sqrt[8]{6}$ *and* $\sqrt[12]{8}$ are arranged in ascending order from left to right, then the third surd from the left is

(*a*) $\sqrt[12]{8}$ (*b*) $\sqrt[4]{4}$ (*c*) $\sqrt[8]{6}$ (*d*) $\sqrt[6]{5}$

14. $\sqrt[6]{15-2\sqrt{56}}\,\sqrt[3]{\sqrt{7}+2\sqrt{2}} =$ ______.

(*a*) 0 (*b*) $\sqrt{2}$ (*c*) 1 (*d*) $\sqrt[6]{2}$

15. If $x = 2^{1/3} - 2$, then $x^3 + 6x^2 + 12x =$ _________.

(*a*)6 (*b*) –6 (*c*) 8 (*d*) –8

16. If $a = 4^{\frac{-3}{2}}$, $b = (125)^{\frac{-2}{3}}$, $c = 81^{\frac{-3}{4}}$ then $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c} =$

17. If $p = 8^{\frac{2}{3}}$, $q = 9^{\frac{3}{2}}$, $r = 16^{\frac{5}{4}}$, $s = \dfrac{1}{6^{-2}}$ then $p+q+r+s =$

18. If $u = \left(\dfrac{1}{27}\right)^{\frac{-4}{3}}$, $v = \left(\dfrac{1}{216}\right)^{\frac{-2}{3}}$, $w = (8)^{5/3} \cdot (4)^{-5/2}$, $t = (9)^{3/2} \cdot (81)^{-3/4}$ then $\dfrac{vwt}{\sqrt{u}} =$

19. Which of the following number is greater than 1

(*a*) $\left(\dfrac{1}{216}\right)^{-2/3} \cdot \left(\dfrac{1}{6^2}\right)$ (*b*) $(125)^{-2/3} \cdot (625)^{1/2}$

(*c*) $\log_{(1/2)} 16$ (*d*) $\log_{16} 32$

20. Simplify $a\left(\dfrac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b\left(\dfrac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}$

ADVANCED LEARNING

RATIO

(*i*) If A and B be two quantities of the same kind, then their ratio is $A : B$; which may be denoted by the fraction $\frac{A}{B}$ (This may be an integer or fraction)

(*ii*) A ratio may represented in a number of ways e.g. $\frac{a}{b} = \frac{ma}{mb} = \frac{na}{nb} =$...... where $m, n,$...... are non-zero numbers.

(*iii*) To compare two or more ratio, reduce them to common denominator.

(*iv*) Ratio between two ratios may be represented as the ratio of two integers

e.g. $\frac{a}{b} : \frac{c}{d} : \frac{a/b}{c/d} = \frac{ad}{bc}$ or $ad : bc$

(*v*) Ratios are compounded by multiplying them together i.e.

$\frac{a}{b} \cdot \frac{c}{d} \cdot \frac{e}{f}\,..... = \frac{ace}{bdf}\,.....$

(*vi*) If $a : b$ is any ratio then its duplicate ratio is $a^2 : b^2$; triplicate ratio is $a^3 : b^3$..... etc.

(*vii*) If $a : b$ is any ratio, then its sub-duplicate ratio is $a^{1/2} : b^{1/2}$; sub-triplicate ratio is $a^{1/3} : b^{1/3}$ etc.

PROPORTION

When two ratios are equal, then the four quantities compositing them are said to be proportional. If $\frac{a}{b} = \frac{c}{d}$, then it is written as $a : b = c : d$ or $a : b :: c : d$

(*i*) 'a' and 'd' are known as extremes and 'b and c' are known as means.

(*ii*) An important property of proportion Product of extremes = product of means.

(*iii*) If $a : b = c : d$, then $b : a = d : c$ (Invertando)

(*iv*) If $a : b = c : d$, then $a : c = b : d$ (Alternando)

(*v*) If $a : b = c : d$, then $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

(*vi*) If $a : b = c : d$, then $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

(*vii*) If $a : b = c : d$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo and dividendo)

(*viii*) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} =$..... , then each $\frac{a+c+e+.....}{b+d+f+......}$

$= \frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$

(*ix*) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} =$..... , then each $= \frac{xa+yc+ze+......}{xb+yd+zf+......}$

(*x*) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} =$..... , then each $= \left(\frac{xa^n+yc^n+ze^n}{xb^n+yd^n+zf^n}\right)^{1/n}$

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Example 9: If $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4}$, then find $x : y : z$.

Sol. Each $= \frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$

$$= \frac{2(x+y+z)}{9} = \frac{x+y+z}{9/2} \text{ and therefore each}$$

$$= \frac{(x+y+z)-(y+z)}{\frac{9}{2}-3} = \frac{(x+y+z)-(x+z)}{\frac{9}{2}-4}$$

$$= \frac{(x+y+z)-(x+y)}{\frac{9}{2}-2}$$

$$= \frac{x}{3/2} = \frac{y}{1/2} = \frac{z}{5/2} \Rightarrow x : y : z = 3 : 1 : 5$$

Example 10: If $\frac{a+3b+2c+6d}{-a-3b+2c+6d} = \frac{3a+b+6c+2d}{-3a-b+6c+2d}$, then the correct statement is

(*a*) $ad = bc$ (*b*) $ac = bd$

(*c*) $c = \frac{ab}{d}$ (*d*) $a+d = b+c$

Sol. Apply C and D the the given equation

$$\Rightarrow \frac{4c+12d}{2a+6d} = \frac{4d+12c}{2b+6a} \quad \Rightarrow \frac{c+3d}{a+3b} = \frac{d+3c}{b+3a}$$

$\Rightarrow bc + 3ac + 3bd + 9ad = ad + 3ac + 3bd + 9bc$

$\Rightarrow 8ad = 8bc$

$\Rightarrow ad = bc$

Concept Application

21. Solve the equations $\dfrac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}} = \dfrac{2x^2-1}{2}$

22. Solve: $\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}} = 3$

23. If $(4x - 3y) : (2x + 5y) = 12 : 19$ then $x : y$ is ......

(*a*) 2 : 3 (*b*) 1 : 2
(*c*) 9 : 4 (*d*) 2 : 1

INTERVALS

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows :

Open Interval (a, b)
$\{x : a < x < b\}$ i.e. extreme points are not included
Closed Interval $[a, b]$
$\{x : a \le x \le b\}$ i.e. extreme points are included
It can possible when a and b are finite
Semi-Open Interval $(a, b]$
$\{x : a < x \le b\}$ i.e. a is not included and b is included
Semi-Closed Interval $[a, b)$
$\{x : a \le x < b\}$ i.e. a is included and b is not included

Note:

1. The infinite intervals are defined as follows :
   (*i*) $(a, \infty) = \{x : x > a\}$
   (*ii*) $[a, \infty) = \{x : x \ge a\}$
   (*iii*) $(-\infty, b) = \{x : x < b\}$
   (*iv*) $(\infty, b] = \{x : x \le b\}$
   (*v*) $(-\infty, \infty) = \{x : x \in R\}$
2. $x \in \{1, 2\}$ denotes some particular values of x, i.e. $x = 1, 2$
3. If there is no value of x, then we say $x \in \phi$ (null set)

GENERAL METHOD TO SOLVE INEQUALITIES

Method of Intervals (Wavy Curve Method)

Let $g(x) = \left(\dfrac{(x-b_1)^{k_1}(x-b_2)^{k_2} --- (x-b_n)^{k_n}}{(x-a_1)^{r_1}(x-a_2)^{r_2} --- (x-a_n)^{r_n}}\right)$ ...(*i*)

Where $k_1, k_2 ..... k_n$ and $r_1, r_2 .... r_n \in N$ and $b_1, b_2 ..... b_n$ and $a_1, a_2 ... a_n$ are real numbers.
Then to solve the inequality following steps are taken.

Steps: Points where numerator becomes zero are called zeros or roots of the function and where denominator becomes zero are called poles of the function.

(*i*) First we find the zeros and poles of the function.
(*ii*) Then we mark all the zeros and poles on the real line and form a curve to divide the real line in many intervals.
(*iii*) Determine sign of the function in any of the interval and then alternates the sign in the neghbouring interval if the poles or zeros dividing the two interval has appeared odd number of times otherwise retain the sign.
(*iv*) Thus we consider all the intervals. The solution of the $g(x) > 0$ is the union of the intervals in which we have put the plus sign and the solution of $g(x) < 0$ is the union of all intervals in which we have put the minus sign.

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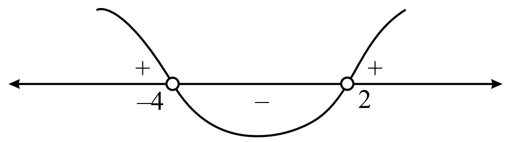
Example 11: Solution $\dfrac{(x-1)^2(x+4)}{(2-x)} < 0$ is

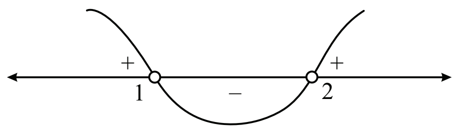
Sol. $\dfrac{(x-1)^2(x+4)}{(2-x)} < 0 \Rightarrow \dfrac{(x+4)}{(x-2)} > 0$



$\Rightarrow (-\infty, -4) \cup (2, \infty)$.

Example 12: The solution of $\dfrac{\sqrt[3]{(x+4)^4}(x-1)^3}{(x-2)} > 0$ is

Sol. $\dfrac{(x+4)^{4/3}(x-1)^3}{(x-2)} > 0 \Rightarrow \dfrac{x-1}{x-2} > 0$

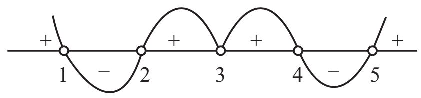


$\Rightarrow (-\infty, 1) \cup (2, \infty)$ Excluding – 4.

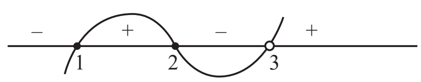
Example 13: Find the range of x, so that following expressions are defined.

(*a*) $(x - 1)(x - 2)(x - 3)^2(x - 4)^5(x - 5) > 0$
(*b*) $\dfrac{(x-1)(x-2)}{(x-3)} \ge 0$

Sol. (*a*) $\Rightarrow x \in (-\infty, 1) \cup (2, 3) \cup (3, 4) \cup (5, \infty)$



(*b*) $\Rightarrow x \in [1, 2] \cup (3, \infty)$



Concept Application

24. Solve the inequality $\dfrac{x^3-4x^2-12x}{x+3} \geq 0$

25. Solve the inequality $\dfrac{(x+1)^4(x-2)(x-3)^3(x-4)^2}{x^3-36x} \geq 0$

26. Find the number of integer values of variable x satisfying the following pair of inequalities.

$\dfrac{(x-1)(x+4)}{x-3} < 0 \; \& \; x^2+6x-27 \leq 0$

27. The solution of the inequality $2x - 1 \leq x^2 + 3 \leq x - 1$ is

(*a*) $x \in R$ (*b*) $(-2, 2]$

(*c*) $(-2, 2)$ (*d*) $x \in \phi$

28. Solve for x: $-1 \leq \dfrac{1+x^2}{2x} \leq 1$

29. Solve for x: $\dfrac{(x-\sin 1)(x-\sin 2)}{(x-\sin 3)(x-\sin 4)} \leq 0$

30. Solve for x: $\sqrt{(x-1)}(x-2)x \leq 0$

31. Solve for x: $\sqrt{x-5}-\sqrt{9-x} > 1 : x \in Z$

32. Solve for x: $\sqrt{x-1} > \sqrt{3-x}$

LOGARITHM FUNCTION

Definition

The logarithm of the number N to the base 'a' is the exponent indicating the power to which the base 'a' must be raised to obtain the number N.

This number is designated as $\log_a N$.

Hence $\log_a N = x \Leftrightarrow a^x = N$, $a > 0$, $a \neq 1$ and $N > 0$

If $a = 10$, then we write $\log b$ rather than $\log_{10} b$

$a = e$, we write $\ln b$ rather than $\log_e b$

The existence and uniqueness of the number $\log_a N$ follows from the properties of an exponential functions.

Domain

The existence and uniqueness of the number $\log_a N$ can be determined with the help of set of conditions, $a > 0$ & $a \neq 1$ & $N > 0$.

FUNDAMENTAL IDENTITY

(*i*) $\log_a 1 = 0$ $(a > 0,\ a \neq 1)$

(*ii*) $\log_a a = 1$ $(a > 0,\ a \neq 1)$

(*iii*) $\log_{1/a} a = -1$ $(a > 0,\ a \neq 1)$

Remember

$\log_{10} 2 \approx 0.3010$; $\log_{10} 3 \approx 0.4771$

$\ln 2 \approx 0.693$; $\ln 10 \approx 2.303$

FUNDAMENTAL LOGARITHMIC IDENTITY

$a^{\log_a N} = N$, $a > 0$, $a \neq 1$ & $N > 0$

Proof :

$\log_a N = x$ ... (i)

$N = (a)^x$ ... (ii)

by equation (i) & (ii)

$N = (a)^{\log_a N}$

PRINCIPAL PROPERTIES

Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, and x, y are any real numbers, then:

(*i*) $\log_a (M \cdot N) = \log_a M + \log_a N$;

Proof:

Let $\log_a M = x$ and $\log_a N = y$

$\Rightarrow M = a^x$ and $N = a^y$

Now, $MN = a^x a^y = a^{x+y}$

$\Rightarrow \log_a MN = x + y$

In general

$\log_a(x_1 x_2 \ldots x_n) = \log_a x_1 + \log_a x_2 + \ldots + \log_a x_n$

(*ii*) $\log_a(M/N) = \log_a M - \log_a N$

Proof:

Let $\log_a M = x$ and $\log_a N = y$

$\Rightarrow M = a^x$ and $N = a^y$

Now, $M/N = a^x/a^y = a^{x-y}$

$\Rightarrow \log_a(M/N) = x - y$

(*iii*) $\log_a M^\alpha = \alpha \cdot \log_a M$

BASE CHANGING THEOREM

It states that ratio of logarithm of two numbers is independent of their common base

Symbolically

$$\frac{\log_a M}{\log_a b} = \log_b M \ (a > 0, M > 0, b > 0)$$

Proof:

Let $\log_b M = x$

$\Rightarrow M = b^x$

$\Rightarrow \log_a M = \log_a b^x$

$\Rightarrow \log_a M = x \cdot \log_a b$

$\Rightarrow \dfrac{\log_a M}{\log_a b} = x = \log_b M$

Important Results

(*i*) **Base power formula:** $\log_{a^k} M = \dfrac{1}{k}\log_a M$

Proof:

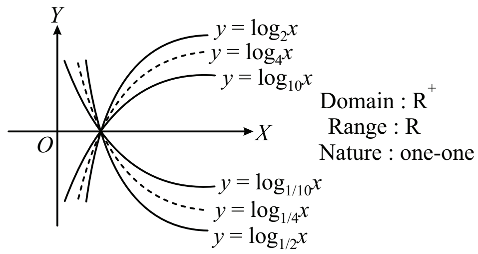
$$\log_{a^k}(M) = \frac{\log_a M}{\log_a a^k} = \frac{\log_a M}{k\log_a a} = \frac{1}{k}\log_a M$$

(*ii*) $a^{\log_b c} = c^{\log_b a}$

Proof: $a^{\log_b c} = a^{\log_a c.\log_b a} = (a^{\log_a c})^{\log_b a} = (c)^{\log_b a}$

GRAPH OF LOGARITHMIC FUNCTIONS

If $a > 0$, $a \neq 1$, then the function $y = \log_a x$, $x \in R^+$ (set of positive real numbers) is called the logarithmic Function with base a.



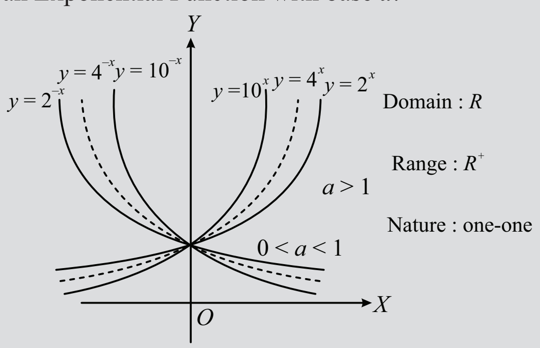
Note: (*i*) If the number and the base are on the same side of the unity, then the logarithm is positive.

(*ii*) If the number and the base are on the opposite sides of unity, then the logarithm is negative.

EXPONENTIAL FUNCTION

ADVANCED LEARNING

If $a > 0$, $a \neq 1$ then the function defined by $f(x) = a^x$, $x \in R$ is called an Exponential Function with base a.



LOGARITHMIC EQUATION

The equality $\log_a x = \log_a y$ is possible if and only if $x = y$

i.e. $\boldsymbol{\log_a x = \log_a y \quad \Leftrightarrow \quad x = y}$

Always check validity of given equation, ($x > 0$, $y > 0$, $a > 0$, $a \neq 1$)

LOGARITHMIC INEQUALITY

Let 'a' is a real number such that

(*i*) If $a > 1$, then $\log_a x > \log_a y \Rightarrow x > y$

(*ii*) If $a > 1$, then $\log_a x < \alpha \Rightarrow 0 < x < a^\alpha$

(*iii*) If $a > 1$, then $\log_a x > \alpha \Rightarrow x > a^\alpha$

(*iv*) If $0 < a < 1$, then $\log_a x > \log_a y \Rightarrow 0 < x < y$

(*v*) If $0 < a < 1$, then $\log_a x < \alpha \Rightarrow x > a^\alpha$

Form - I: $f(x) > 0$, $g(x) > 0$, $g(x) \neq 1$

Form	Collection of system
(*a*) $\log_{g(x)} f(x) \geq 0 \Leftrightarrow$	$\begin{cases} f(x) \geq 1, & g(x) > 1 \\ 0 < f(x) \leq 1, & 0 < g(x) < 1 \end{cases}$
(*b*) $\log_{g(x)} f(x) \leq 0 \Leftrightarrow$	$\begin{cases} f(x) \geq 1, 0 < g(x) < 1 \\ 0 < f(x) \leq 1, g(x) > 1 \end{cases}$
(*c*) $\log_{g(x)} f(x) \geq a \Leftrightarrow$	$\begin{cases} f(x) \geq g(x)^a, g(x) > 1 \\ 0 < f(x) \leq g(x)^a, 0 < g(x) < 1 \end{cases}$
(*d*) $\log_{g(x)} f(x) \leq a \Leftrightarrow$	$\begin{cases} 0 < f(x) \leq g(x)^a, g(x) > 1 \\ f(x) \geq g(x)^a, 0 < g(x) < 1 \end{cases}$

From - II: $f(x) > 0$, $g(x) > 0$, $\phi(x) > 0$, $\phi(x) \neq 1$

Form	Collection of system
(*a*) $\log_{\phi(x)} f(x) \geq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} f(x) \geq g(x), \phi(x) > 1, \\ 0 < f(x) \leq g(x); 0 < \phi(x) < 1 \end{cases}$
(*b*) $\log_{\phi(x)} f(x) \leq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} 0 < f(x) \leq g(x), \phi(x) > 1, \\ f(x) \geq g(x) > 0, 0 < \phi(x) < 1 \end{cases}$

COMMON AND NATURAL LOGARITHM

$\log_{10} N$ is referred as a common logarithm and $\log_e N$ is called as natural logarithm of N to the base Napierian and is popularly written as $\ell n\, N$. Note that e is an irrational quantity lying between 2.7 to 2.8 **Note that** $e^{\ell n\, x} = x$.

Train Your Brain

Example 14: How many solutions are there for equation

$\log_4 (x - 1) = \log_2 (x - 3)$?

Sol. $\log_4 (x - 1) = \log_2 (x - 3)$

$\Rightarrow \log_{2^2} (x - 1) = \log_2 (x - 3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1)^{1/2} = \log_2(x-3)$

$\Rightarrow (x-1)^{1/2} = (x-3)$

$\Rightarrow x - 1 = x^2 - 6x + 9$

$\Rightarrow (x-2)(x-5) = 0$

$\Rightarrow x = 2, 5$

But $x - 1 > 0$ and $x - 3 > 0$

$x > 1$ and $x > 3$

So only one solution $x = 5$

Example 15: Solve the logarithmic inequality

$\log_{1/5}\frac{4x+6}{x} \geq 0$.

Sol. Since $\log_{1/5} 1 = 0$, the given inequality can be written as.

$$\log_{1/5}\frac{4x+6}{x} \geq \log_{1/5} 1$$

When the domain of the function is taken into account the inequality is equivalent to the system of inequalities.

$$\begin{cases} \dfrac{4x+6}{x} > 0, \\ \dfrac{4x+6}{x} \leq 1 \end{cases}$$

Solving the inequalities by using method of intervals

$$x \in \left[-2, \frac{-3}{2}\right)$$

Example 16: For $x \geq 0$, what is the smallest possible value of the expression $\log(x^3 - 4x^2 + x + 26) - \log(x + 2)$?

Sol. $\log\dfrac{(x^3-4x^2+x+26)}{(x+2)}$

$= \log\dfrac{(x^2-6x+13)(x+2)}{(x+2)}$

$= \log(x^2 - 6x + 13)$ $\quad [\because x \neq -2]$

$= \log\{(x-3)^2 + 4\}$

$\therefore$ Minimum value is $\log 4$ when $x = 3$

Example 17: Given $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8) = \dfrac{2}{s^3+1}$.

Write $\log_2\dfrac{a^2b^5}{c^4}$ as a function of 's' ($a, b, c > 0, c \neq 1$).

Sol. Given $\log_2 a = s$ ...(i)

$\log_2 b = 2s^2$ ...(ii)

$\log_8 c^2 = \dfrac{s^3+1}{2}$ ...(iii)

$\Rightarrow \dfrac{2\log c}{3\log 2} = \dfrac{s^3+1}{2} \Rightarrow 4\log_2 c = 3(s^3+1)$ ...(iv)

to find $2\log_2 a + 5\log_2 b - 4\log_2 c$

$\Rightarrow \quad 2s + 10s^2 - 3(s^3 + 1)$

Example 18: If $\frac{1}{4}\log_4 \mathrm{M} + 4\log_4 \mathrm{N} = 1 + \log_{.008} 5$ then the value of $\mathrm{MN}^{16} = \mathrm{k}.2^{1/3}$, where k is equal to

(*a*) 8 (*b*) 32

(*c*) 36 (*d*) 40

Sol. $\frac{1}{8}\log_2 M + 2\log_2 N = 1 + \dfrac{\log_2 5}{\log_2(.008)}$

$\Rightarrow \log_2 M^{1/8} + \log_2 N^2 = 1 + \dfrac{\log_2 10 - 1}{3 - 3\log_2 10}$

$\Rightarrow \log_2\left(MN^{16}\right)^{1/8} = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow \left(MN^{16}\right)^{1/8} = 2^{2/3}$

$\Rightarrow MN^{16} = 2^{16/3} = 32(2^{1/3})$

Concept Application

33. Solve the inequality $\log_{1/3}(5x - 1) > 0$.

34. Suppose that a and b are positive real numbers such that $\log_{27}a + \log_9 b = \dfrac{7}{2}$ and $\log_{27}b + \log_9 a = \dfrac{2}{3}$. Find the value of the ab.

35. If $m_1 = \log_8 16$, $m_2 = \log_{81} 27$, $m_3 = \log_{1/3} 1/9$, $m_4 = \log_{1/3} 9\sqrt{3}$ then $m_1 \cdot m_2 \cdot m_3 \cdot m_4$

36. If $p = \log_{2\sqrt{3}} 1728$, $q = \log_2(\cos 45°)$, $r = \log_2(\log_2 4)$, $s = \log_3(\tan 30°)$, $t = \log_{625} 125$ then $\dfrac{prt}{qs} =$

37. If $\log_7(\log_3(\log_2 x)) = 0$, then find $\log_{0.125} x$.

38. Solve for x:

(*i*) $\log_3 x > 0$ (*ii*) $\log_5 x \geq 0$

(*iii*) $\log_6 x < 0$ (*iv*) $\log_2 x \leq 0$

(*v*) $\log_{1/7} x > 0$ (*vi*) $\log_{1/8} x \geq 0$

(*vii*) $\log_{1/9} x < 0$ (*viii*) $\log_{1/e} x \leq 0$

(*ix*) $\log_2(x - 1) > 1$ (*x*) $\log_{1/2}(x - 2) \leq 1$

39. Solve for x:

(*i*) $\log_4(2x - 3) < 2$

(*ii*) $\log_{1/2}(3x - 2) \geq 3$

(*iii*) $\log_{16}(\log_4(x)) > 1$

(*iv*) $\log_{1/2}(\log_{1/4}(x)) < 1$

40. $\log_2 (a^2 - 5) = 2$

41. $\log_{1/3} (a^2 - 1) = -1$

42. Prove that : $2^{\sqrt{\log_2 3}} = 3^{\sqrt{\log_3 2}}$

43. If $\log_{12}27 = a$ find the value of $\log_6 16$ in term of a.

44. Solve for x:

(*i*) $\log_x (2) \,.\, \log_{2x} 2 = \log_{4x} 2$

(*ii*) $5^{\log_a x} + 5x^{\log_a 5} = 3 (a > 0)$

(*iii*) $x^{(\log_2 x) + 4} = 32$

(*iv*) $\log_{x+1} (x^2 + x - 6)^2 = 4$

(*v*) $x + \log_{10} (1 + 2^x) = x \cdot \log_{10} 5 + \log_{10} 6$

45. Prove that $\log_a N \cdot \log_b N + \log_b N \cdot \log_c N + \log_c N \cdot \log_a N = \dfrac{\log_a N \cdot \log_b N \cdot \log_c N}{\log_{abc} N}$

46. Prove that: $2^{\left(\sqrt{\log_a \sqrt[4]{ab} + \log_b \sqrt[4]{ab}} - \sqrt{\log_a \sqrt[4]{\frac{b}{a}} + \log_b \sqrt[4]{\frac{a}{b}}}\right) \cdot \sqrt{\log_a b}}$

$$= \left[\begin{array}{lll} 2 & \text{if} & b \geq a > 1 \\ 2^{\log_a b} & \text{if} & 1 < b < a \end{array} \right.$$

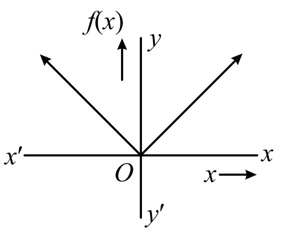
47. Simplify: $\log_{1/3} \sqrt[4]{729 \cdot \sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$

ABSOLUTE VALUE FUNCTION / MODULUS FUNCTION

This is also known as absolute value function and denoted by

$$f(x) = |x| \text{ i.e. } f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Domain of this function is set of all real numbers because $f(x)$ exists for all $x \in R$ but $|x| \geq 0$ so range is all non-negative real numbers.



Domain = R; Range = $[0, \infty)$ or $R^+ \cup \{0\}$

Properties of modulus : For any $a, b \in R$

(*i*) $|a| \geq 0$

(*ii*) $|a| = |-a|$

(*a*) $|a|^n = |a^n|$

(*b*) $|a^n| = a^n$, where n is even and $n \in z$

(*iii*) $|a| \geq a, |a| \geq -a$

(*iv*) $|ab| = |a||b|$

(*v*) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$

Note: $|f(x)| + |g(x)| = |f(x) + g(x)|$

$\Rightarrow f(x) \cdot g(x) \geq 0$

Train Your Brain

Example 19: The absolute value of sum of real solutions of $\log_2 |x^2 + 5x + 4| = \log_2 3 + \log_2 |x + 1|$ is

(*a*) 8 (*b*) 6 (*c*) 7 (*d*) 5

Sol. $\log_2 \dfrac{|(x+1)(x+4)|}{|x+1|} = \log_2 3$

$|x + 4| = 3$

$x + 4 = -3, +3$

$x = -7, -1$ (rejected); $\Rightarrow x = -7$

Example 20: Solve the following linear equation

(*i*) $x\,|x| = 4$

(*ii*) $|x - 3| + 2\,|x + 1| = 4$

Sol. (*i*) $x\,|x| = 4$

If $x > 0$

$\therefore x^2 = 4 \quad \Rightarrow \quad x = \pm 2 \quad \therefore x = 2 \; (\because x \geq 0)$

If $x < 0$

$\Rightarrow \quad -x^2 = 4 \Rightarrow x^2 = -4$ which is not possible

(*ii*) $|x - 3| + 2\,|x + 1| = 4$

Case-I: If $x \leq -1$

$\therefore -(x - 3) - 2(x + 1) = 4$

$\Rightarrow -x + 3 - 2x - 2 = 4 \Rightarrow \quad -3x + 1 = 4$

$\Rightarrow -3x = 3 \quad \Rightarrow \quad x = -1$

Case-II: If $-1 < x \leq 3$

$\therefore -(x - 3) + 2(x + 1) = 4$

$\Rightarrow -x + 3 + 2x + 2 = 4$

$\Rightarrow x = -1$ which is not possible

Case-III: If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4 \quad \Rightarrow \quad x = 5/3$ which is not possible

$\therefore x = -1$

Example 21: Number of real solutions of $|x - 1| = |x - 2| + |x - 3|$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) more than 2

Sol.

Case-I: $x \le 1$, $1 - x = 2 - x + 3 - x$

$x = 4$ (rejected)

Case-II: $1 < x \le 2$, $x - 1 = 2 - x + 3 - x$, $x = 2$

Case-III: $2 < x < 3$, $x - 1 = x - 2 + 3 - x$, $x = 2$

Case-IV: $x \ge 3$, $x - 1 = x - 2 + x - 3$

$x = 4 \Rightarrow x = 2, 4$

Concept Application

48. Let $x_1, x_2, x_3, x_4, x_5 \in R$ and $|x_1 - x_2| = 2$, $|x_2 - x_3| = 4$, $|x_3 - x_4| = 3$, $|x_4 - x_5| = 5$.

Then the sum of all distinct possible values of $|x_5 - x_1|$ is

49. The number of integers which does NOT satisfy $\log_{|2x|}(|x+2| + |x-2|) = 1$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

50. The real solutions of the equation where $|x|^2 - 3|x| + 2 = 0$ where $x_1 < x_2 < x_3 < x_4$ then

(*a*) $|x_1| = |x_3|$ (*b*) $|x_2| = |x_3|$

(*c*) $x_1 + x_4 = x_2 + x_3$ (*d*) $-x_2 + x_4 = x_1 - x_3$

51. Solve for x:

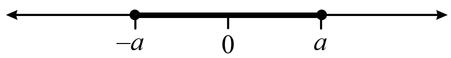
(*i*) $||x-2| -1| = 2$ (*ii*) $||x-3| -5| = 1$

(*iii*) $|||x-5| -4| -3| = 2$

INEQUALITIES INVOLVING ABSOLUTE VALUE

(*i*) $|x| \le a$ (where $a > 0$)

It implies those values of x on real number line which are at distance a or less than a from zero.



$\Rightarrow -a \le x \le a$

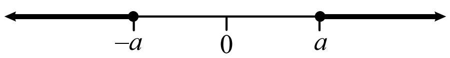
e.g. $|x| \le 2 \Rightarrow -2 \le x \le 2$

$|x| < 3 \Rightarrow -3 < x < 3$

In general, $|f(x)| \le a$ (where $a > 0$) $\Rightarrow -a \le f(x) \le a$.

(*ii*) $|x| \ge a$ (where $a > 0$)

It implies those values of x on real number line which are at distance a or more than a from zero



$\Rightarrow x \le -a$ or $x \ge a$

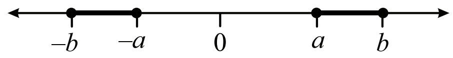
e.g. $|x| \ge 3 \Rightarrow x \le -3$ or $x \ge 3$

$|x| > 2 \Rightarrow x < -2$ or $x > 2$

In general, $|f(x)| \ge a \Rightarrow f(x) \le -a$ or $f(x) \ge a$

(*iii*) $a \le |x| \le b$ (where $a, b > 0$)

It implies those value of x on real number line whose distance from zero is equal to a or b or lies between a and b



$\Rightarrow [-b, -a] \cup [a, b]$

e.g. $2 \le |x| \le 4 \Rightarrow x \in [-4, -2] \cup [2, 4]$

(*iv*) If $|x + y| = |x| + |y|$, $xy \ge 0$

If $|x - y| = |x| + |y|$, $xy \le 0$

If $|x + y| = ||x| - |y||$, $xy \le 0$

If $|x - y| = ||x| - |y||$, $xy \ge 0$

Train Your Brain

Example 22: Solve $x^2 - 4|x| + 3 < 0$.

Sol. $x^2 - 4|x| + 3 < 0$

$\Rightarrow (|x| - 1)(|x| - 3) < 0$

$\Rightarrow 1 < |x| < 3$

$\Rightarrow -3 < x < -1$ or $1 < x < 3$

$\Rightarrow x \in (-3, -1) \cup (1, 3)$

Example 23: Solve $1 \le |x - 2| \le 3$

Sol. $1 \le |x - 2| \le 3$

$\Rightarrow -3 \le x - 2 \le -1$ or $1 \le x - 2 \le 3$

$\Rightarrow -1 \le x \le 1$ or $3 \le x \le 5$

$\Rightarrow x \in [-1, 1] \cup [3, 5]$

Example 24: Solve $|x - 1| + |x - 2| + |x - 3| \ge 6$,

Sol. For $x \le 1$, the given inequation becomes

$1 - x + 2 - x + 3 - x \ge 6 \Rightarrow -3x \ge 0$

$\Rightarrow x \le 0$ and for $x \ge 3$, the given equation becomes

$x - 1 + x - 2 + x - 3 \ge 6 \Rightarrow 3x \ge 12 \Rightarrow x \ge 4$

For $1 < x \le 2$

we get $x - 1 + 2 - x + 3 - x \ge 6$

$\Rightarrow -x + 4 \ge 6$

i.e. $-x \ge 2 \Rightarrow x \le -2$ Not possible

For $2 < x < 3$,

We get $x - 1 + x - 2 + 3 - x \ge 6$

$\Rightarrow x \ge 6$ not possible

Hence solution set is $(-\infty, 0] \cup [4, \infty)$

i.e. $x \le 0$ or $x \ge 4$

Concept Application

52. Solve $||x-1| - 2| < 5$

53. Number of non-positive integral values of 'x' satisfying the given inequality, $|x^2 - 1| \le |2x - 1|$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

54. Solve $|x^2 - 2x| + |x - 4| > |x^2 - 3x + 4|$.

55. Solve $\left|\frac{x-3}{x+1}\right| \le 1$.

56. Solve for x:

(i) $|x| > 1$ (ii) $|x| \ge 5$ (iii) $|x| < 7$

(iv) $|x| \le 10$ (v) $|x| \ge 0$ (vi) $|x| < -8$

(vii) $|x| > -4$ (viii) $|x| \ge -5$ (ix) $|x| \le -10$

57. Solve for x:

(i) $|x-1| > 1$ (ii) $|x-2| < 1$

(iii) $1 < |2x+1| < 3$ (iv) $1 \le |1-2x| \le 3$

(v) $-1 \le |3x-1| \le 5$ (vi) $-6 \le |1-3x| \le -1$

58. Solve for x:

(i) $||x-2|-1| \le 2$ (ii) $||x-3|-5| \ge 1$

(iii) $|||x-5|-4|-3| \le 2$

59. Solve for x:

(i) $\frac{1}{2} \le |2x-1| \le \frac{3}{5}$ (ii) $-\frac{1}{3} \le |3x-4| \le 2$

(iii) $2 \le |4-5x| \le \frac{10}{3}$ (iv) $3 < |x^2-1| < 8$

60. Solve for x:

(i) $|||x+5|-3|-1| = 2$

(ii) $||||x-5|-7|-3|-2| = 1$

61. Solve for x:

(i) $2 \le |||x-7|-3|+2| \le 5$

(ii) $||||x-5|-7|-3|-2| \le 1$

62. Solve for x:

(i) $\frac{(|x|-1)}{(|x|-2)} \le 0$ (ii) $\frac{(|x|-1)(|x|-3)}{|x|^2-2|x|} \ge 0$

(iii) $\frac{(|x|^2-5|x|+6)}{(4-|x|^2)} \le 0$

(iv) $(||x-1|-2|-3)(|x-2|-3) \ge 0$

(v) $(|||x-1|-2|-1|-2)(|x-2|) \ge 0$

63. Solve for x:

(i) $\left|\frac{x-3}{x+1}\right| \le 1$ (ii) $\left|\frac{x+1}{x}\right| + |x+1| = \frac{(x+1)^2}{|x|}$

(iii) $\left|1+\frac{3}{x}\right| > 2$ (iv) $|2^x-1| + |4-2^x| < 3$

(v) $\left(\frac{1}{3}\right)^{\frac{|x+2|}{2-|x|}} > 9$

64. Solve: $||x^2-2x+6|-|x+6|| = |x^2-3x|$

AARAMBH (SOLVED EXAMPLES)

1. The value of $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$ is equal to

(*a*) 49 (*b*) 625 (*c*) 216 (*d*) 890

(JEE Arjuna Mathematics M-1)

Sol. $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$

$= 3^{4\log_3 5} + 3^{3\cdot\frac{1}{2}\log_3 36} + 3^{4\log_9 7}$

$= 3^{\log_3 5^4} + 3^{\log_3 36^{3/2}} + 3^{\log_3 7^{4/2}}$

$= 5^4 + 36^{3/2} + 7^2 = 890$

Therefore, option (*d*) is the correct answer.

2. The largest integral value of x satisfying

$\sqrt{18^x - 5} \le \sqrt{2(18^x + 12)} - \sqrt{18^x + 5}$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) no integral value of x possible

(JEE Arjuna Mathematics M-1)

Sol. Let $18^x = p$

$\sqrt{p-5} + \sqrt{p+5} \le \sqrt{2(p+12)}$

$\Rightarrow p - 5 + p + 5 + 2\sqrt{p^2 - 25} \le 2p + 24$

$\Rightarrow \sqrt{p^2 - 25} \le 12 \Rightarrow p^2 \le 169 \Rightarrow p \le 13$

Also $p \ge 5$

Thus $5 \le p \le 13 \Rightarrow \log_{18}5 \le x \le \log_{18}13$

Therefore, option (*d*) is the correct answer.

3. Solve if $|x - 5| + |x + 4| = 9$

(*a*) $[-4, 5]$ (*b*) $(-4, 5)$ (*c*) $(-4, 5]$ (*d*) $[-4, 5)$

(JEE Arjuna Mathematics M-1)

Sol. Given equation is of form $|a| + |b| = |a - b|$

It is true for $ab \le 0$

$(x - 5)(x + 4) \le 0$

So $x \in [-4, 5]$

Therefore, option (*a*) is the correct answer.

4. Solve $\dfrac{(e - \sin x)(x-2)}{(x+4)} \ge 0.$

(*a*) $(-\infty, -4) \cup [2, \infty)$ (*b*) $(-\infty, -4] \cup (2, \infty)$

(*c*) $(-\infty, -4) \cup (2, \infty)$ (*d*) None of these

(JEE Arjuna Mathematics M-1)

Sol. Zeros $x = 2$, Pole $x \ne -4$

$e - \sin x > 0$ always positive

$\dfrac{(e - \sin x)(x-2)}{(x+4)} \ge 0$

Final solution $x \in (-\infty, -4) \cup [2, \infty)$

Therefore, option (*a*) is the correct answer.

5. Values of x satisfying the equation

$\log_5^2 x + \log_{5x}\left(\dfrac{5}{x}\right) = 1$ are

(*a*) 1 (*b*) 5

(*c*) $\dfrac{1}{25}$ (*d*) 3

(JEE Arjuna Mathematics M-1)

Sol. $(\log_5 x)^2 + \log_{5x}\dfrac{5}{x} = 1$

$\Rightarrow (\log_5 x)^2 + \log_{5x}5 - \log_{5x}x = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{\log_5 5}{\log_5 5 + \log_5 x} - \dfrac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{1}{1 + \log_5 x} - \dfrac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore\ t^2 + \dfrac{1}{1+t} - \dfrac{t}{1+t} = 1$

$\Rightarrow \dfrac{t^2(1+t) + 1 - t}{1+t} = 1$

$\Rightarrow t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0$

$t(t^2 + t - 2) = 0$

$t(t-1)(t+2) = 0$

$t = 0, 1, -2$

$\therefore\ \log_5 x = 0, 1, -2$

$\therefore\ x = 1, 5, \dfrac{1}{25}$

Therefore, option (*a,b,c*) is the correct answers.

6. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has

(*a*) One irrational solution

(*b*) No prime solution

(*c*) Two real solutions

(*d*) One integral solution

(JEE Arjuna Mathematics M-1)

Sol. $\dfrac{4}{2}\log_x 2 + \dfrac{\log_x 64}{\log_x 2x}$

$\Rightarrow\ 2\log_x 2 + \dfrac{6\log_x 2}{1 + \log_x 2} = 3$

Let $\alpha = \log_x 2$

$\therefore\ 2\alpha + \dfrac{6\alpha}{1+\alpha} = 3$

$\Rightarrow 2\alpha + 2\alpha^2 + 6\alpha - 3 - 3\alpha = 0$

$\Rightarrow 2\alpha^2 + 5\alpha - 3 = 0$

$\Rightarrow (\alpha + 3)(2\alpha - 1) = 0 \Rightarrow a = -3, 1/2$

$\therefore \log_x 2 = -3 \Rightarrow x = 2^{-1/3}$ (Irrational)

or $\log_x 2 = \dfrac{1}{2} \Rightarrow x = 4$ (Integer)

Therefore, option (*a,b,c,d*) is the correct answers.

7. If $\dfrac{1}{2} \le \log_{0.1} x \le 2$, then

(*a*) Maximum value of x is $\dfrac{1}{\sqrt{10}}$

(*b*) x lies between $\dfrac{1}{100}$ and $\dfrac{1}{\sqrt{10}}$

(*c*) Minimum value of x is $\dfrac{1}{10}$

(*d*) Minimum value of x is $\dfrac{1}{100}$

(JEE Arjuna Mathematics M-1)

Sol. $\dfrac{1}{2} \le \log_{0.1} x \le 2 \Rightarrow \left(\dfrac{1}{10}\right)^{1/2} \ge x \ge \left(\dfrac{1}{10}\right)^{2}$

Therefore, option (*a,b,d*) is the correct answers.

8. Let $N = \dfrac{\log_3 135}{\log_{15} 3} - \dfrac{\log_3 5}{\log_{405} 3}$. Then N is

(*a*) a natural number (*b*) a prime number

(*c*) a rational number (*d*) an integer

(JEE Arjuna Mathematics M-1)

Sol. $= \log_3 135 \log_3 15 - \log_3 5 \log_3 405$

$= \log_3(5 \times 3^3) \cdot \log_3(5 \times 3) - \log_3 5 . \log_3 (5 \times 3^4)$

$= (\log_3 5 + \log_3 3^3)(\log_3 5 + \log_3 3) - \log_3 5 (\log_3 5 + \log_3 3^4)$

$= (x + 3)(x + 1) - x(x + 4)$

{Let $\log_3 5 = x$}

$= x^2 + 4x + 3 - x^2 - 4x = 3$

which is Prime, rational Integer and natural number

Therefore, option (*a,b,c,d*) is the correct answers.

9. If $|x - 5| + |x + 5| = 10$, then

(*a*) The number of integral solutions is 10

(*b*) The number of integral solutions is 11

(*c*) The sum of all the integral solutions is 0

(*d*) All the solutions of the equation are rational numbers

(JEE Arjuna Mathematics M-1)

Sol. $|x - 5| + |x + 5| = 10$

Case-I: $x \ge 5$, the equation becomes

$(x - 5) + (x + 5) = 10$

$\Rightarrow 2x = 10$

$\Rightarrow x = 5$ which satisfies the case, therefore accepted.

Case-II: $-5 < x < 5$ The above equation becomes

$-(x - 5) + (x + 5) = 10$

$\Rightarrow -x + 5 + x + 5 = 10$

$\Rightarrow 10 = 10$ which is true.

So, the solution is $x \in (-5, 5)$

Case-III: $x \le -5$, The above equation becomes

$-(x - 5) - (x + 5) = 10$

$\Rightarrow -x + 5 - x - 5 = 10$

$\Rightarrow -2x = 10$

$\Rightarrow x = -5$ which satisfies the above case so, accepted.

$\therefore$ final answer is $x \in [-5, 5]$

Therefore, option (*b,c*) is the correct answers.

10. If $\log_a x = b$ for permissible values of a and x then identify the statements(*s*) which can be correct?

(*a*) If a and b are two irrational numbers then x can be rational.

(*b*) If a rational and b irrational then x can be rational.

(*c*) If a irrational and b rational then x can be rational.

(*d*) If a rational and b rational then x can be rational.

(JEE Arjuna Mathematics M-1)

Sol. (*a*) $a = (\sqrt{2})^{\sqrt{2}}$ is irrational

$b = \sqrt{2}$ is also irrational

but $a^b = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = 2$ which is rational $\Rightarrow$ (*a*) is correct.

(*b*) $a = 2 \in Q$; $b = \log_2 3 \notin Q$

$a^b = 2^{\log_2 3} = 3 \in Q \Rightarrow$ (*b*) is correct

Therefore, option (*a,b,c,d*) is the correct answers.

11. Match the column:

	Column-I		Column-II
A.	The roots of $\log_2(x + e) = \log_2 x + \log_2 e$ is a	p.	Positive Number
B.	The solution of $\log_{1/5}(2x^2 + 5x + 1) < 0$ contains	q.	Rational Number
C.	$\log_{\sin\frac{\pi}{6}} \pi$ is	r.	Irrational Number
D.	$\log_{10}5.\log_{10}20 + \log_{10}^2 2$ simplifies to	s.	Negative Number

(*a*) A→(p, r) ; B→(p, q, r, s) ; C→(r, s) ; D→(p, q)

(*b*) A→(p, q, r, s) ; B→(p, r) ; C→(r, s) ; D→(p, q)

(*c*) A→(r, s) ; B→(p, q, r, s) ; C→(p, r) ; D→(p, q)

(*d*) A→(p, q) ; B→(p, q, r, s) ; C→(r, s) ; D→(p, r)

(JEE Arjuna Mathematics M-1)

Sol. (*A*) $x + e = xe$

$x(e - 1) = e \Rightarrow x = \dfrac{e}{e-1}$

(*B*) $2x^2 + 5x + 1 > 1$ and $2x^2 + 5x + 1 > 0$

$\Rightarrow 2x^2 + 5x + 1 > 1$

$\Rightarrow (x)(2x + 5) > 0$

$\Rightarrow x \in \left(-\infty, \frac{-5}{2}\right) \cup (0, \infty)$

(*D*) $(1-\log_{10} 2)(1+\log_{10} 2) + \log_{10}^2 2$

$\Rightarrow 1 - \log_{10}^2 2 + \log_{10}^2 2 = 1$

Therefore, option (*a*) is the correct answer.

12. Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10.$

Find $y \in \mathbb{N}$.

(JEE Arjuna Mathematics M-1)

Sol. $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$

$= \sqrt{\log_2 3 \cdot (2+\log_2 3)(4+\log_2 3)(6+\log_2 3) + 16} - (2+\log_2 3)(4+\log_2 3) + 10$

Let us put $\log_2 3 = x$

$= \sqrt{x(2+x)(4+x)(6+x)+16} - (2+x)(4+x) + 10$

$= \sqrt{(x^2+6x)(x^2+6x+8)+16} - (x^2+6x+8) + 10$

Put again $x^2 + 6x = \alpha$

$= \sqrt{\alpha(\alpha+8)+16} - (\alpha+8) + 10$

$= \sqrt{\alpha^2 + 8\alpha + 16} - (\alpha+8) + 10$

$= \sqrt{(\alpha+4)^2} - (\alpha+8) + 10$

$= (\alpha+4) - (\alpha+8) + 10 = y = 6.$

Therefore, 6 is the correct answer.

13. If 'x' and 'y' are real numbers such that,

$2\log(2y - 3x) = \log x + \log y$, find $\frac{y}{x}$.

(truncated upto two decimal)

(JEE Arjuna Mathematics M-1)

Sol. $\log(2y - 3x)^2 = \log xy$

$\Rightarrow (2y - 3x)^2 = xy$

$\Rightarrow 4y^2 - 12xy + 9x^2 = xy$

Dividing the equation by y^2

$9\left(\frac{x}{y}\right)^2 - 13\frac{x}{y} + 4 = 0$

$\left(\frac{x}{y} - 1\right)\left(\frac{9x}{y} - 4\right) = 0$

$\frac{x}{y} = 1, \frac{x}{y} = \frac{4}{9}.$

$x = y$ disregarded as for $x = y$, $2y - 3x$ is negative.

Hence $\frac{y}{x} = \frac{9}{4}.$

Therefore, 2.25 is the correct answer.

14. Sum of all the solutions of the equation

$\log_6(x^2-1) - \log_6\sqrt{(x-6)^2} = \log_6(x+1)^2$ is $a + \sqrt{b}$, $(a, b \in N)$.

Then $a + b$ is equal to

(JEE Arjuna Mathematics M-1)

Sol. $\log_6(x^2 - 1) - \log_6\sqrt{(x-6)^2} = \log_6(x+1)^2$

$\Rightarrow \log_6 \frac{(x-1)(x+1)}{(x+1)^2} = \log_6 |x-6|$

$\Rightarrow \log_6 \left[\frac{(x-1)}{(x+1)}\right] = \log_6 |x-6|$

$\Rightarrow \frac{x-1}{x+1} = |x-6|$

Case-I: $x \geq 6$

$\Rightarrow x - 1 = x^2 - 5x - 6$

$\Rightarrow x^2 - 6x - 5 = 0$

$\Rightarrow (x-3)^2 = 14$

$\Rightarrow x = 3 \pm \sqrt{14}$

$x = 3 - \sqrt{14} < 1$ rejected

$x = 3 + \sqrt{14}$ accepted

Case-II: $x < 6$

$x - 1 = -(x^2 - 5x - 6)$

$\Rightarrow x^2 - 4x - 7 = 0$

$(x-2)^2 = 11$

$x = 2 \pm \sqrt{11}$

$x = 2 + \sqrt{11}$ (accepted)

$x = 2 - \sqrt{11}$ (accepted)

Sum of roots $= 7 + \sqrt{14}$

$\Rightarrow a = 7, b = 14$

$a + b = 21$

Therefore, 21 is the correct answer.

SCHOOL LEVEL PROBLEMS

SINGLE CORRECT TYPE QUESTIONS

1. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

(*a*) an even number (*b*) an odd number

(*c*) An odd prime number (*d*) a prime number

(JEE Arjuna Mathematics M-1)

2. The smallest number by which $\sqrt{27}$ should be multiplied to get a rational number is

(*a*) $\sqrt{27}$ (*b*) $3\sqrt{3}$ (*c*) $\sqrt{3}$ (*d*) 3

(JEE Arjuna Mathematics M-1)

3. If $a + ib = c + id$, then

(*a*) $a^2 + c^2 = 0$ (*b*) $b^2 + c^2 = 0$

(*c*) $b^2 + d^2 = 0$ (*d*) $a^2 + b^2 = c^2 + d^2$

(JEE Arjuna Mathematics M-1)

4. If $(\sqrt{5})^5 \times 25^2 = 5^x \times 5\sqrt{5}$, then x is equal to

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

(JEE Arjuna Mathematics M-1)

5. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) $\frac{7}{2}$

(JEE Arjuna Mathematics M-1)

6. If $\frac{2}{3}$ of A = 75% of B = 0.6 of C, then $A : B : C$ is

(*a*) 2 : 3 : 3 (*b*) 3 : 4 : 5

(*c*) 4 : 5 : 6 (*d*) 9 : 8 : 10

(JEE Arjuna Mathematics M-1)

7. If $\log_{10}5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$, then x is equal to:

(*a*) 1 (*b*) 3 (*c*) 5 (*d*) 10

(JEE Arjuna Mathematics M-1)

8. If $\log_8 p = 25$ and $\log_2 q = 5$, then

(*a*) $p = q^{15}$ (*b*) $p^2 = q^3$

(*c*) $p = q^5$ (*d*) $p^3 = q$

(JEE Arjuna Mathematics M-1)

9. If $4x + 3 < 6x + 7$, then x belongs to the interval

(*a*) $(2, \infty)$ (*b*) $(-2, \infty)$

(*c*) $(-\infty, 2)$ (*d*) $(-4, \infty)$

(JEE Arjuna Mathematics M-1)

VERY SHORT ANSWER TYPE QUESTIONS

10. If $\log_7 2 = m$, then $\log_{49}28$ is

(JEE Arjuna Mathematics M-1)

11. $\frac{4}{x+1} \le 3 \le \frac{6}{x+1}$

(JEE Arjuna Mathematics M-1)

12. Simplify $\frac{11^{n+3} + 3\times 11^{n+1}}{5\times 11^{n+1} - 11^n \times 3}$

(JEE Arjuna Mathematics M-1)

13. Solve modulus and find the interval of x for $|x^2 - 5x + 6|$

(JEE Arjuna Mathematics M-1)

14. If $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4}$, then find $x : y : z$.

(JEE Arjuna Mathematics M-1)

15. $\sqrt{\left(\frac{5}{7}\right)^{6x-4}} = \frac{2401}{875}$, find value of x

(JEE Arjuna Mathematics M-1)

16. If the sum of the first 20 terms of the series

$\log_{7^{1/2}} x + \log_{7^{1/3}} x + \log_{7^{1/4}} x + \ldots$ is 460, then x is equal to:

LONG ANSWER TYPE QUESTIONS

(JEE Arjuna Mathematics M-1)

17. Solve $\frac{2x-1}{3} \ge \frac{3x-2}{4} - \frac{2-x}{5}$

(JEE Arjuna Mathematics M-1)

18. Evaluate the following

$\frac{1}{2}\log 9 + \frac{1}{4}\log 81 + 2\log 6 - \log 12$

(JEE Arjuna Mathematics M-1)

19. Simplify $\left(6a^{-2}bc^{-3} / 4ab^{-3}c^2\right) \div \left(5a^{-3}b^2c^{-1} / 3ab^{-2}c^3\right)$

(JEE Arjuna Mathematics M-1)

20. Solve for x, $|x - 1| - |x - 2| = 10$

(JEE Arjuna Mathematics M-1)

21. Solve for x : $\log_2 x - 3\log_{1/2} x = 6$

(JEE Arjuna Mathematics M-1)

PRARAMBH (TOPICWISE)

BASIC CONCEPTS AND NUMBER SYSTEM

1. Let $x \in Q$, $y \in Q^c$, Which of the following statement is always WRONG?

(*a*) $xy \in Q^c$

(*b*) $y/x \in Q$, whenever defined

(*c*) $\sqrt{2}\,x + y \in Q$

(*d*) $x/y \in Q^c$, whenever defined

(JEE Arjuna Mathematics M-1)

2. If x and y are two rational numbers such that $(x + y) + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$, then:

(*a*) $x = 1, y = 1$

(*b*) $x = 2, y = 1$

(*c*) $x = 5, y = 1$

(*d*) x and y can take infinitely many values

(JEE Arjuna Mathematics M-1)

3. Which of the following statement is incorrect:

(*a*) rational number + rational number = rational number

(*b*) irrational number + rational number = irrational number

(*c*) integer + rational number = rational number

(*d*) irrational number + irrational number = Irrational number

(JEE Arjuna Mathematics M-1)

4. The number of real roots of the equation $(x - 1)^2 + (x - 2)^2 + (x - 3)^2 = 0$ is:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

(JEE Arjuna Mathematics M-1)

5. If $x - a$ is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to

(*a*) 0 (*b*) 2 (*c*) –2 (*d*) 1

(JEE Arjuna Mathematics M-1)

6. Every irrational number can be expressed on the number line. This statement is

(*a*) Always true

(*b*) Never true

(*c*) True subject to some condition

(*d*) None of these

(JEE Arjuna Mathematics M-1)

7. The multiplication of a rational number 'x' and an irrational number 'y' is

(*a*) Always rational

(*b*) Rational except when $y = \pi$

(*c*) Always irrational

(*d*) Irrational except when $x = 0$

(JEE Arjuna Mathematics M-1)

8. If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then value of $|x + y|$ is

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 2 or 4 or 6

(JEE Arjuna Mathematics M-1)

9. If a, b, c are real, then $a(a - b) + b(b - c) + c(c - a) = 0$, only if

(*a*) $a + b + c = 0$

(*b*) $a = b = c$

(*c*) $a = b$ or $b = c$ or $c = a$

(*d*) $a - b - c = 0$

(JEE Arjuna Mathematics M-1)

10. If $2x^3 - 5x^2 + x + 2 = (x - 2)(ax^2 - bx - 1)$, then a & b are respectively

(*a*) 2, 1 (*b*) 2, – 1

(*c*) 1, 2 (*d*) –1, 1/2

(JEE Arjuna Mathematics M-1)

11. The value of $[e] - [-\pi]$ is, where [.] denotes greatest integer function.

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 8

(JEE Arjuna Mathematics M-1)

12. If $L = \frac{1}{\sqrt{7}-\sqrt{8}} + \frac{1}{\sqrt{7}-\sqrt{6}} + \frac{1}{3-\sqrt{8}} + \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{5}-\sqrt{6}}$ $= 1 + 2\sqrt{a} + 2\sqrt{b}$, then $a \times b$ is equal to

(*a*) 30 (*b*) 45

(*c*) 8 (*d*) 0

(JEE Arjuna Mathematics M-1)

13. If a, b, c are real and distinct numbers, then the value of $\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$ is

(*a*) 1 (*b*) $a\,b\,c$

(*c*) 2 (*d*) 3

(JEE Arjuna Mathematics M-1)

14. The remainder obtained when the polynomial $1 + x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by $x - 1$ is

(*a*) 3 (*b*) 5

(*c*) 7 (*d*) 11

(JEE Arjuna Mathematics M-1)

LOGARITHM AND ITS PRINCIPLE PROPERTIES

15. $\dfrac{1}{1+\log_b a+\log_b c}+\dfrac{1}{1+\log_c a+\log_c b}+\dfrac{1}{1+\log_a b+\log_a c}$ has the value equal to

(*a*) abc (*b*) $\dfrac{1}{abc}$

(*c*) 0 (*d*) 1

(JEE Arjuna Mathematics M-1)

16. $\log_7 \log_7 \sqrt{7(\sqrt{7\sqrt{7}})} =$

(*a*) $3 \log_2 7$ (*b*) $1 - 3 \log_3 7$

(*c*) $1 - 3\log_7 2$ (*d*) $1 - 10 \log_2 7$

(JEE Arjuna Mathematics M-1)

17. $\dfrac{1}{\log_{\sqrt{bc}} abc}+\dfrac{1}{\log_{\sqrt{ca}} abc}+\dfrac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(*a*) 1/2 (*b*) 1

(*c*) 2 (*d*) 4

(JEE Arjuna Mathematics M-1)

18. If $\log_x \log_{18}(\sqrt{2}+\sqrt{8})=\dfrac{1}{3}$. Then the value of 1000 x is equal to

(*a*) 8 (*b*) 1/8

(*c*) 1/125 (*d*) 125

(JEE Arjuna Mathematics M-1)

19. Number of real solutions of the equation

$\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is :

(*a*) none (*b*) exactly 1

(*c*) exactly 2 (*d*) 4

(JEE Arjuna Mathematics M-1)

20. Greatest integer less than or equal to the number $\log_2 15.\log_{1/6} 2.\log_3 1/6$ is

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

(JEE Arjuna Mathematics M-1)

21. The ratio $\dfrac{2^{\log_{2^{\frac{1}{4}}} a}-3^{\log_{27}(a^2+1)^3}-2a}{7^{4\log_{49} a}-a-1}$ simplifies to

(*a*) $a^2 - a - 1$ (*b*) $a^2 + a - 1$

(*c*) $a^2 - a + 1$ (*d*) $a^2 + a + 1$

(JEE Arjuna Mathematics M-1)

22. If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

(*a*) zero (*b*) 1

(*c*) 2 (*d*) More than 2

(JEE Arjuna Mathematics M-1)

23. The number $\log_2 7$ is

(*a*) an integer (*b*) a rational number

(*c*) an irrational number (*d*) a prime number

(JEE Arjuna Mathematics M-1)

24. Anti logarithm of 0.75 to the base 16 has the value equal to

(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 12

(JEE Arjuna Mathematics M-1)

25. The sum of all the solutions to the equation $2\log x - \log(2x - 75) = 2$:

(*a*) 30 (*b*) 350 (*c*) 75 (*d*) 200

INEQUALITIES

(JEE Arjuna Mathematics M-1)

26. If the solution set of the inequality $\log_{\sqrt{0.9}} \log_5 (\sqrt{x^2+5+x}) > 0$ contains 'n' integral values, then n equals to

(*a*) 7 (*b*) 8 (*c*) 6 (*d*) 10

(JEE Arjuna Mathematics M-1)

27. If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then '$x$' lies in the interval

(*a*) $\left(-3,-\sqrt{5}\right)\cup\left(\sqrt{5},3\right)$ (*b*) $\left(-3,-\sqrt{5}\right)\cup\left(\sqrt{5},3\sqrt{5}\right)$

(*c*) $\left(\sqrt{5},3\sqrt{5}\right)$ (*d*) ϕ

(JEE Arjuna Mathematics M-1)

28. Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is:

(*a*) $[-4, 1]$

(*b*) $[-4, -3) \cup (0, 1]$

(*c*) $(-\infty, -3) \cup (1, \infty)$

(*d*) $(-\infty, -4) \cup [1, \infty)$

MODULUS FUNCTION

(JEE Arjuna Mathematics M-1)

29. Solutions of $|4x + 3| + |3x - 4| = 12$ are

(*a*) $x = -\dfrac{7}{3}, \dfrac{3}{7}$ (*b*) $x = -\dfrac{5}{2}, \dfrac{2}{5}$

(*c*) $x = -\dfrac{11}{7}, \dfrac{13}{7}$ (*d*) $x = -\dfrac{3}{7}, \dfrac{7}{5}$

(JEE Arjuna Mathematics M-1)

30. If $|x^2 - 2x - 8| + |x^2 + x - 2| = 3\,|x + 2|$, then the set of all real values of x is

(*a*) $[1, 4] \cup \{-2\}$ (*b*) $[1, 4]$

(*c*) $[-2, 1] \cup [4,\infty)$ (*d*) $(-\infty, -2] \cup [1, 4]$

(JEE Arjuna Mathematics M-1)

31. The complete set of real 'x' satisfying $||x-1|-1| \le 1$ is:

(*a*) [0, 2] (*b*) [– 1, 3]

(*c*) [– 1, 1] (*d*) [1, 3]

(JEE Arjuna Mathematics M-1)

32. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

(JEE Arjuna Mathematics M-1)

33. Number of real solution (x) of the equation $|x-3|^{3x^2-10x+3} = 1$ is

(*a*) exactly four (*b*) exactly three

(*c*) exactly two (*d*) exactly one

(JEE Arjuna Mathematics M-1)

MISCELLANEOUS

34. Simplify: $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

(*a*) 0 (*b*) 1

(*c*) 3 (*d*) 4

(JEE Arjuna Mathematics M-1)

35. The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has a factor

(*a*) $x + y + z + 1$ (*b*) $-x + y + z$

(*c*) $x + y - z + 1$ (*d*) $x - y + z + 1$

(JEE Arjuna Mathematics M-1)

36. Solve the equation $\dfrac{3x^4 + x^2 - 2x - 3}{3x^4 - x^2 + 2x + 3} = \dfrac{5x^4 + 2x^2 - 7x + 3}{5x^4 - 2x^2 + 7x - 3}$

(*a*) $x = 5, 2$ (*b*) $x = 4, 1$

(*c*) $x = 3, 8$ (*d*) $x = 1, 5$

(JEE Arjuna Mathematics M-1)

37. If x, y, z are positive real number and a, b, c are rational numbers, then the value of $\dfrac{1}{1 + x^{b-a} + x^{c-a}} + \dfrac{1}{1 + x^{a-b} + x^{c-b}} + \dfrac{1}{1 + x^{b-c} + x^{a-c}}$ is

(*a*) –1 (*b*) 1 (*c*) 0 (*d*) 2

(JEE Arjuna Mathematics M-1)

38. If $a^x = \sqrt{b}, b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$ then the value of xyz is

(*a*) $\dfrac{1}{2}$ (*b*) $\dfrac{1}{3}$ (*c*) $\dfrac{1}{6}$ (*d*) $\dfrac{1}{12}$

(JEE Arjuna Mathematics M-1)

39. If $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$, then $a^a \cdot b^b \cdot c^c =$

(*a*) 3 (*b*) 1

(*c*) 4 (*d*) 2

(JEE Arjuna Mathematics M-1)

40. The number of prime numbers satisfying the inequality $\dfrac{x^2 - 1}{2x + 5} < 3$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

(JEE Arjuna Mathematics M-1)

PRABAL (JEE MAIN LEVEL)

1. If A & B are two rational numbers and AB, A + B and A – B are rational numbers, then A/B is

(*a*) Always rational (*b*) Never rational

(*c*) Rational when B ≠ 0 (*d*) Rational when A ≠ 0

(JEE Arjuna Mathematics M-1)

2. If $x^{x\sqrt[3]{x}} = \left(x.\sqrt[3]{x}\right)^x$, then $x =$

(*a*) 1 (*b*) –1

(*c*) 0 (*d*) 2

(JEE Arjuna Mathematics M-1)

3. The equation $4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$ has the solution

(*a*) $x = \pm 1$ (*b*) $x = 10$

(*c*) $x = \pm\sqrt{2}$ (*d*) $x = \sqrt{3}$

(JEE Arjuna Mathematics M-1)

4. Logarithm of $32\sqrt[5]{4}$ to the base $2\sqrt{2}$ is

(*a*) 3.6 (*b*) 5

(*c*) 5.6 (*d*) 10

(JEE Arjuna Mathematics M-1)

5. If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$, then which of the following is equal to 1

(*a*) $x + y + z$

(*b*) $(1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1}$

(*c*) xyz

(*d*) $x + y - z$

(JEE Arjuna Mathematics M-1)

6. The solution of the equation $\log_7 \log_5 \left(\sqrt{x^2 + 5 + x}\right) = 0$.

(*a*) $x = 2$ (*b*) $x = 3$

(*c*) $x = 4$ (*d*) $x = -2$

(JEE Arjuna Mathematics M-1)

7. The value of $\left(0.05\right)^{\log_{\sqrt{20}}(0.1+0.01+0.001+........)}$ is

(*a*) 81 (*b*) $\frac{1}{81}$

(*c*) 20 (*d*) $\frac{1}{20}$

(JEE Arjuna Mathematics M-1)

8. The value of $\log_2 .\log_3 ....\log_{100} 100^{99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}}}$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 100!

(JEE Arjuna Mathematics M-1)

9. The number of solution of $\log_2(x + 5) = 6 - x$ is

(*a*) 2 (*b*) 0 (*c*) 3 (*d*) 1

(JEE Arjuna Mathematics M-1)

10. If $\log_{10}2 = 0.30103$, $\log_{10}3 = 0.47712$, the number of digits in $3^{12} \times 2^8$ is

(*a*) 7 (*b*) 8 (*c*) 9 (*d*) 10

(JEE Arjuna Mathematics M-1)

11. Exhaustive set of values of x satisfying $\log_{|x|}(x^2 + x + 1) \geq 0$ is

(*a*) $(-1, 0)$ (*b*) $(-\infty, -1) \cup (1, \infty)$

(*c*) $(-\infty, \infty) - \{-1, 0, 1\}$ (*d*) $(-\infty,-1)\cup(-1,0)\cup(1,\infty)$

(JEE Arjuna Mathematics M-1)

12. The set of real values of x satisfying $\log_{1/2}\left(x^2 - 6x + 12\right) \geq -2$ is

(*a*) $\left(-\infty, 2\right]$ (*b*) $[2, 4]$

(*c*) $\left[4, +\infty\right)$ (*d*) $[3, 8]$

(JEE Arjuna Mathematics M-1)

13. If $\log_{0.04}(x - 1) \geq \log_{0.2}(x - 1)$ then x belongs to the interval

(*a*) $(1, 2]$ (*b*) $(-\infty, 2]$

(*c*) $[2, \infty)$ (*d*) $(-\infty, 2)$

(JEE Arjuna Mathematics M-1)

14. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the interval

(*a*) $(2, \infty)$ (*b*) $(-2, -1)$

(*c*) $(1, 2)$ (*d*) $(-2, 2)$

(JEE Arjuna Mathematics M-1)

15. The minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$ is equal to

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

(JEE Arjuna Mathematics M-1)

16. The set of real value(s) of p for which the equation $|2x + 3| + |2x - 3| = px + 6$ has more than two solutions is :

(*a*) $[0, 4)$ (*b*) $(-4, 4)$

(*c*) $R - \{4, -4, 0\}$ (*d*) $\{0\}$

(JEE Arjuna Mathematics M-1)

17. Let $3^a = 4, 4^b = 5, 5^c = 6, 6^d = 7, 7^e = 8$ and $8^f = 9$. The value of the product $(abcdef)$ is :

(*a*) 1 (*b*) 2 (*c*) $\sqrt{6}$ (*d*) 3

(JEE Arjuna Mathematics M-1)

18. There are two positive solutions to the equation $\log_{2x}2 + \log_4 2x = -\frac{3}{2}$. The product of these two solution is:

(*a*) $\frac{1}{32}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{21}$

(JEE Arjuna Mathematics M-1)

19. Number of real value of x satisfying the equation $\log_2(2x^2 + \sqrt{2}) = \frac{\sqrt{x^2+1}}{x^2+2}$ is :

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

(JEE Arjuna Mathematics M-1)

20. Number of values of x satisfying the equation $x^4 = |x|^{\log_2(x^2+12)}$ is:

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

(JEE Arjuna Mathematics M-1)

21. The number of zeros after decimal before the start of any significant digit in the number $N = (0.15)^{20}$ are :

(*a*) 15 (*b*) 16 (*c*) 17 (*d*) 18

(JEE Arjuna Mathematics M-1)

22. If $n, k \in N$, then the smallest value of k such that $k + 2k + 3k + ... + 24k = n^3$ is

(*a*) 100 (*b*) 90 (*c*) 120 (*d*) 60

(JEE Arjuna Mathematics M-1)

23. If $a, b, c \in R$ and $a, b, c \neq 0$ such that $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=6$ and $\frac{b}{a}+\frac{c}{b}+\frac{a}{c}=8$ then $\frac{a^3}{b^3}+\frac{b^3}{c^3}+\frac{c^3}{a^3}-3$ is equal to

(*a*) 81 (*b*) 48 (*c*) 72 (*d*) 84

(JEE Arjuna Mathematics M-1)

24. If $3^x = 5^y$, then how many ordered pairs $(x, y), x, y \in R$ satisfy the given equation?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) Infinite

(JEE Arjuna Mathematics M-1)

25. The value of $x + y + z$ satisfying the system of equations

$\log_2 x + \log_4 y + \log_4 z = 2$ is

$\log_3 y + \log_9 z + \log_9 x = 2$

$\log_4 z + \log_{16} x + \log_{16} y = 2$

(*a*) $\frac{175}{12}$ (*b*) $\frac{349}{24}$ (*c*) $\frac{353}{24}$ (*d*) $\frac{112}{3}$

(JEE Arjuna Mathematics M-1)

26. Which is the correct order for a given number α, $\alpha > 1$

(*a*) $\log_2 \alpha < \log_3 \alpha < \log_e \alpha > \log_{10} \alpha$

(*b*) $\log_{10} \alpha < \log_3 \alpha < \log_e \alpha > \log_2 \alpha$

(*c*) $\log_{10} \alpha < \log_e \alpha < \log_2 \alpha > \log_3 \alpha$

(*d*) $\log_e \alpha < \log_3 \alpha < \log_2 \alpha > \log_{10} \alpha$

(JEE Arjuna Mathematics M-1)

27. The smallest integral value of x such that $\sqrt{x+2} - \sqrt{x-2} < \frac{1}{10}$ is

(*a*) 400 (*b*) 20 (*c*) 401 (*d*) 399

(JEE Arjuna Mathematics M-1)

28. $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q(\log_r(\log_p x)) = 0$ then 'p' equals

(*a*) $r^{q/r}$ (*b*) rq (*c*) 1 (*d*) $r^{r/q}$

(JEE Arjuna Mathematics M-1)

29. Which one of the following is the smallest?

(*a*) $\log_{10}\pi$ (*b*) $\sqrt{\log_{10} \pi^2}$

(*c*) $\left(\frac{1}{\log_{10} \pi}\right)^3$ (*d*) $\left(\frac{1}{\log_{10} \sqrt{\pi}}\right)$

(JEE Arjuna Mathematics M-1)

30. The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set

(*a*) $(-\infty, 0)$ (*b*) $(-\infty, 1)$ (*c*) $(1, \infty)$ (*d*) $(3, \infty)$

(JEE Arjuna Mathematics M-1)

31. The value of b satisfying the equation,

$\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is

(*a*) 5 (*b*) 7 (*c*) 9 (*d*) 10

(JEE Arjuna Mathematics M-1)

32. The solution set of the system of equation

$\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is:

(*a*) $\{6, 3\}$ (*b*) $\{9, 6\}$

(*c*) $\{6, 12\}$ (*d*) $\{12, 6\}$

(JEE Arjuna Mathematics M-1)

33. Which of the following statements are true?

(*a*) $\log_2 3 < \log_{12} 10$ (*b*) $\log_6 5 < \log_7 8$

(*c*) $\log_3 26 > \log_2 9$ (*d*) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

(JEE Arjuna Mathematics M-1)

34. If $x + y = a$ and $x^2 + y^2 = b$, then the value of $(x^3 + y^3)$ is

(*a*) ab (*b*) $a^2 + b$

(*c*) $a + b^2$ (*d*) $\frac{3ab - a^3}{2}$

(JEE Arjuna Mathematics M-1)

35. If $x + y + z = 0$, then a factor of the expression $(x + y)^3 + (y + z)^3 + (z + x)^3$ is

(*a*) $3(x + y)(y + z)(z + x)$ (*b*) $3xyz$

(*c*) $(x + y - z)$ (*d*) $(x - y + z)$

(JEE Arjuna Mathematics M-1)

36. The number of real solution/s of the equation $9^{\log_3(\log_e x)} = \log_e x - (\log_e x)^2 + 1$ is:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

(JEE Arjuna Mathematics M-1)

37. The set of all the solutions of the inequality $\log_{1-x}(x-2) \geq -1$ is

(*a*) $(-\infty, 0)$ (*b*) $(2, \infty)$

(*c*) $(-\infty, 1)$ (*d*) ϕ

(JEE Arjuna Mathematics M-1)

38. The complete solution of $\frac{x^2-1}{x+3} \geq 0$ & $x^2 - 5x + 2 \leq 0$ is:

(*a*) $x \in \left[\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right]$ (*b*) $x \in \left[1, \frac{5+\sqrt{17}}{2}\right]$

(*c*) $x \in (-3, -1]$ (*d*) $(-3, -1) \cup [1, \infty)$

(JEE Arjuna Mathematics M-1)

39. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is:

(*a*) 1 (*b*) 3 (*c*) 4 (*d*) 5

(JEE Arjuna Mathematics M-1)

40. Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

(JEE Arjuna Mathematics M-1)

41. The solution set of inequality $\log_{(3x^2+1)} 2 < \frac{1}{2}$

(*a*) $|x| > 1$

(*b*) $|x| < 1$

(*c*) ϕ

(*d*) None of these

(JEE Arjuna Mathematics M-1)

42. The solution set of $\log 4+\left(1+\frac{1}{2x}\right)\log 3=\log\left((3)^{1/x}+27\right)$

(a) (1/4, 1/2) (b) {1/4, 1/2}

(c) (1/4, 1/2] (d) None of these

(JEE Arjuna Mathematics M-1)

43. The solution set of $\log_7\left(\frac{2x-6}{2x-1}\right)>0$

(a) $\left(-\infty, \frac{1}{2}\right)$ (b) $\left(-\infty, \frac{1}{2}\right]$

(c) $\left(-\frac{1}{2}, \frac{1}{2}\right]$ (d) None of these

(JEE Arjuna Mathematics M-1)

44. If $\log_{x-3}(2x-3)$ is a meaningful quantity then find the interval in which x must lie.

(a) $x \in (3, 4] \cup (4, \infty)$ (b) $x \in [3, 4) \cup (4, \infty)$

(c) $x \in (3, 4) \cup (4, \infty)$ (d) None of these

(JEE Arjuna Mathematics M-1)

45. $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}=$

(a) $(a+b)(b+c)(c+a)$ (b) 1

(c) 3 (d) None of these

INTEGER TYPE QUESTIONS

(JEE Arjuna Mathematics M-1)

46. Suppose $x, y, z > 0$ and different than one and $\ln x+\ln y+\ln z=0$. If $\frac{1}{e^k}=x^{\frac{1}{\ln y}+\frac{1}{\ln z}}\cdot y^{\frac{1}{\ln z}+\frac{1}{\ln x}}\cdot z^{\frac{1}{\ln x}+\frac{1}{\ln y}}$. The $k=$

(JEE Arjuna Mathematics M-1)

47. If $\log_2(\log_3(\log_4(x)))=0$ and $\log_3(\log_4(\log_2(y)))=0$ and $\log_4(\log_2(\log_3(z)))=0$ then the sum of x, y and z is

(JEE Arjuna Mathematics M-1)

48. Let $\log_2 x+\log_4 x+\log_8 x=\log_k x$ for all $x \in R^+$. If $k=\sqrt[b]{a}$ where $a, b \in N$ then find the smallest positive value of $(a+b)$. **(JEE Arjuna Mathematics M-1)**

49. Find the value of the expression

$$6\left(\frac{2}{\log_4(2000)^6}+\frac{3}{\log_5(2000)^6}\right).$$

(JEE Arjuna Mathematics M-1)

50. If $N=7^{\log_{49}900}, A=2^{\log_2 4}+3^{\log_2 4}+4^{\log_2 2}-4\log_2 3,$

$D=(\log_5 49)(\log_7 125)$

Find $P=\log_{\left(A-\frac{N}{10}\right)}|N+A+D+6|-\log_5 2,$

(JEE Arjuna Mathematics M-1)

51. If $a+b+c=1$, $a^2+b^2+c^2=9$, $a^3+b^3+c^3=1$, then find value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

(JEE Arjuna Mathematics M-1)

52. $a+b+c=10$ and $ab+bc+ac=20$ then find the value of $a^3+b^3+c^3-3abc$

(JEE Arjuna Mathematics M-1)

53. $(a-b)^3+(b-c)^3+(c-a)^3=p(a-b)(b-c)(c-a)$, then $p=$

(JEE Arjuna Mathematics M-1)

54. The value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5........}}}}}$ is

(JEE Arjuna Mathematics M-1)

55. If $x=\sqrt[3]{7+5\sqrt{2}}-\frac{1}{\sqrt[3]{7+5\sqrt{2}}}$, then the value of $x^3+3x-14$ is equal to **(JEE Arjuna Mathematics M-1)**

56. $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4}=7^x$ then $-3x=$

(JEE Arjuna Mathematics M-1)

57. Number of cyphers after decimal before a significant figure starts in $\left(\frac{5}{4}\right)^{-100}$ is equal to [**Use:** $\log_{10}2=0.3010$]

(JEE Arjuna Mathematics M-1)

58. Number of real solution of $\log_5[2+\log_3(x+3)]=0$ is

(JEE Arjuna Mathematics M-1)

59. If $4^A+9^B=10^C$, where $A=\log_{16}4$, $B=\log_3 9$ & $C=\log_x 83$, then find x.

SINGLE CORRECT TYPE QUESTIONS

1. The expression $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\ldots\sqrt[p]{p}}}}}_{n \text{ radical sign}}$, where $p \geq 2$, $p \in N$; $n \in N$ when simplifies is

(*a*) Independent of p

(*b*) Independent of p and of n

(*c*) Dependent on both p and n

(*d*) Positive

(JEE Arjuna Mathematics M-1)

2. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$ is

(*a*) $\left(\frac{1}{2}, 2\right)$ (*b*) $\left(1, \frac{5}{2}\right)$

(*c*) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (*d*) $(1, 2)$

(JEE Arjuna Mathematics M-1)

3. If $\sqrt{\log_4\{\log_3\{\log_2(x^2 - 2x + a)\}\}}$ is defined $\forall\, x \in R$, then the set of values of 'a' is

(*a*) $[9, \infty)$ (*b*) $[10, \infty)$

(*c*) $[15, \infty)$ (*d*) $[2, \infty)$

(JEE Arjuna Mathematics M-1)

4. Number of integers, which satisfy the inequality, $\frac{(16)^{1/x}}{(2^{x+3})} > 1$ is equal to:

(*a*) Infinite (*b*) Zero (*c*) 1 (*d*) 4

(JEE Arjuna Mathematics M-1)

5. The solution set of inequality $\frac{(3^x - 4^x) \cdot \ln(x+2)}{x^2 - 3x - 4} \leq 0$ is

(*a*) $(-\infty, 0] \cup (4, \infty)$ (*b*) $(-2, 0] \cup (4, \infty)$

(*c*) $(-1, 0] \cup (4, \infty)$ (*d*) $(-2, -1) \cup (-1, 0) \cup (4, \infty)$

(JEE Arjuna Mathematics M-1)

6. Number of values of x in the interval $(0, 5)$ satisfying the equation $\frac{\ln(\sqrt{\sqrt{x^2+1} + x}) + \ln(\sqrt{\sqrt{x^2+1} - x})}{\ln x} = x$, is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

(JEE Arjuna Mathematics M-1)

7. If $A = \log_{\sqrt{5}}\left(\left(5^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}\cdot^{\cdot^{\cdot 10 \text{ times}}}}$, then value of $\log_{\sqrt{3}}(1024A + 1)$, is equal to

(*a*) 1 (*b*) 3 (*c*) 5 (*d*) 2

(JEE Arjuna Mathematics M-1)

MULTIPLE CORRECT TYPE QUESTIONS

(JEE Arjuna Mathematics M-1)

8. The roots of the equations $|x| = 49^{\left(\frac{1}{2} + \log_{\frac{1}{7}} 27 + \log_{343} 81\right)}$ include

(*a*) One positive number greater than 1 only

(*b*) Two real number

(*c*) Two irrational number

(*d*) One negative rational number

(JEE Arjuna Mathematics M-1)

9. Which of the following is true?

(*a*) $(\log_{10}2)^2 + 1 > \log_{10}4$

(*b*) $\log_{10}90 > \log_5 50$

(*c*) $\log_4 \log_3 \log_2 16 > \log_{16} 4$

(*d*) $2(\log_{10} 3)^2 - 3(\log_{10} 2)^2 > (\log_{10} 2) \times (\log_{10} 3)$

(JEE Arjuna Mathematics M-1)

10. Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has:

(*a*) At least one real solution

(*b*) Exactly three real solutions

(*c*) Exactly one irrational solution

(*d*) Imaginary roots

(JEE Arjuna Mathematics M-1)

11. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(*a*) Exactly three real solution

(*b*) At least one real solution

(*c*) Exactly one irrational solution

(*d*) Complex roots

(JEE Arjuna Mathematics M-1)

12. Solution set of the inequality

$$(\log_2 x)^4 - \left(\log_{1/2}\frac{x^3}{8}\right)^2 + 9\log_2\left(\frac{32}{x^2}\right) < 4(\log_{1/2} x)^2 \text{ is}$$

$(a, b) \cup (c, d)$ then the correct statement is

(*a*) $a = 2b$ and $d = 2c$

(*b*) $b = 2a$ and $d = 2c$

(*c*) $\log_c d = \log_b a$

(*d*) there are 4 integers in (c, d)

(JEE Arjuna Mathematics M-1)

13. Choose the correct from the following

(*a*) $\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right) = 1$

(*b*) $5^{\log_{1/5}\frac{1}{2}} + \log_{\sqrt{2}}\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right) + \log_{1/2}\left(\frac{1}{(10+2\sqrt{21})}\right) = 6$

(*c*) $\sqrt{10^{2+\frac{1}{2}\log(16)}} = 20$

(*d*) None of these

(JEE Arjuna Mathematics M-1)

14. Choose the correct from the following

(*a*) $\log_2(\log_{1/2}(x)) < 2$, for all $x \in \left(\frac{1}{16}, 1\right)$

(*b*) $\log_{1/2}(\log_3(x)) > 3$, for all $x \in (1, 3^{1/8})$

(*c*) $(\log_2(x) - 1)(\log_3(x) - 2) \le 0$, for all $x \in [2, 9]$

(*d*) $(\log_2(x) - 1)(\log_{1/2}(x) - 2) \le 0$, for all $x \in \left(0, \frac{1}{4}\right] \cup [2, \infty)$

COMPREHENSION BASED QUESTIONS

Comprehension (Q. No. 15 to 17): Let α and β are the solutions of the equation $\left(\sqrt{x}\right)^{\log_5 x - 1} = 5$ where $\alpha \in$ I and $\beta \in$ Q Then

[**Use:** $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

(JEE Arjuna Mathematics M-1)

15. The number of significant digits before decimal in $(\alpha)^{10}$ is

(*a*) 13 (*b*) 14

(*c*) 15 (*d*) None of these

(JEE Arjuna Mathematics M-1)

16. Number of zeros after decimal before a significant digit in $(\beta)^{10}$ is

(*a*) 5 (*b*) 7 (*c*) 8 (*d*) 6

(JEE Arjuna Mathematics M-1)

17. The value of $(\beta)^{\log_{25} 9}$ is

(*a*) $\frac{1}{3}$ (*b*) 5 (*c*) $\frac{1}{5}$ (*d*) 9

(JEE Arjuna Mathematics M-1)

MATCH THE COLUMN TYPE QUESTIONS

(JEE Arjuna Mathematics M-1)

18. Match the Column:

	Column-I		Column-II
A.	The value(s) of x, which does not satisfy the equation $\log_2^2(x^2 - x) - 4\log_2(x-1)\log_2 x = 1$, is (are)	p.	2
B.	The value of x satisfying the equation $2^{\log_2 e^{\ln 5^{\log_5 7^{\log_7 10^{\log_{10}(8x-3)}}}}} = 13$, is	q.	3
C.	The number $N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right)$ is less than	r.	4
D.	Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1+9^{\log_3 8}\right)^{\log_{65} 5}$ then $(l + m)$ is divisible by	s.	5
		t.	6

(*a*) A → r, t, s; B → q; C → r, s, q; D → q, r

(*b*) A → q, r, s, t; B → p; C → q, r; D → r, s

(*c*) A → q, r, s, t; B → p; C → q, r, s, t; D → p, r, s

(*d*) A → t; B → s; C → q, t; D → r, s

(JEE Arjuna Mathematics M-1)

19. Match the column:

	Column-I		Column-II
A.	$\log_{\sin 30°}(\cos 60°) + 1$	p.	3
B.	$\log_{4/3}(1.\overline{3}) + 3$	q.	5
C.	$\log_{2-\sqrt{3}}(2+\sqrt{3}) + 6$	r.	4
D.	$\log_{\tan 20°} \tan 70° + 4$	s.	2
E.	$\log_{\cot 40°} \tan 50°$	t.	0
F.	$\log_{0.125}(8) + 8$	u.	–1
G.	$\log_{1.5}(0.\overline{6}) + 9$	v.	8
H.	$\log_{2.25}(0.\overline{4})$	w.	7
I.	$\log_{10}(0.\overline{9})$	x.	1

(*a*) A → q, B → p, C → s, D → v, E → u, F → u, G → q, H → w, I → x

(*b*) A → s, B → r, C → q, D → p, E → x, F → w, G → v, H → u, I → t

(*c*) A → s, B → v, C → t, D → p, E → t, F → u, G → w, H → x, I → w

(*d*) A → q, B → s, C → r, D → v, E → u, F → v, G → v, H → w, I → x

(JEE Arjuna Mathematics M-1)

20. Match the following columns:

	Column-I		Column-II
A.	If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	p.	–1
B.	If $a = \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	q.	1

C.	If $a=\sqrt{3+2\sqrt{2}}, b=\sqrt{3-2\sqrt{2}}$ then the value of $\log_a b$ is equal to	r.	2
D.	If $a=\sqrt{7+\sqrt{7^2-1}}, b=\sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to	s.	$2+2\log_2 3$

(*a*) $A\to s, B\to p, C\to q, D\to p$

(*b*) $A\to r, B\to p, C\to r, D\to p$

(*c*) $A\to r, B\to s, C\to p, D\to p$

(*d*) $A\to p, B\to q, C\to p, D\to r$

(JEE Arjuna Mathematics M-1)

NUMERICAL TYPE QUESTIONS

21. Find the number of integral solution of the equation $\log_{\sqrt{x}}(x+|x-2|) = \log_x(5x-6+5|x-2|)$.

(JEE Arjuna Mathematics M-1)

22. If *a*, *b* are co-prime numbers and satisfying $(2+\sqrt{3})^{\frac{1}{\log_a(2-\sqrt{3})}+\frac{1}{\log_b\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}} = \frac{1}{12}$, then $(a+b)$ can be is equal to

(JEE Arjuna Mathematics M-1)

23. The sum of all integral values of *x* satisfying the equation $2\log_8(2x)+\log_8(x^2-2x+1)=\frac{4}{3}$ is.

(JEE Arjuna Mathematics M-1)

24. If the complete solution set of the inequality $(\log_{10}x)^2 \ge \log_{10}x+2$ is $(0, a] \cup \left[\frac{1}{a^2}, \infty\right)$ then find the value of $10a$.

(JEE Arjuna Mathematics M-1)

25. If complete solution set of inequality $\log_{1/2}(x+5)^2 > \log_{1/2}(3x-1)^2$ is $(-\infty, p)\cup(q, r)\cup(s, \infty)$ then find $3\left(\frac{p^2+q^2+r^2}{s^2}\right)$

(JEE Arjuna Mathematics M-1)

26. Solve the equation $x^{0.5\log_{\sqrt{x}}(x^2-x)} = 3^{\log_9 4}$.

(JEE Arjuna Mathematics M-1)

27. If the solution set of $(0.3)^{\log_{\frac{1}{3}}\log_2\frac{3x+6}{x^2+2}} > 1$ is $\left(\frac{-1}{\alpha}, \alpha\right)$ then $\alpha =$

(JEE Arjuna Mathematics M-1)

28. If the solution set of $\log_{0.5}\left(\log_6\frac{x^2+x}{x+4}\right) < 0$ is $(\alpha, \beta)\cup(-2\alpha, \infty)$ then $-\alpha+\beta =$

(JEE Arjuna Mathematics M-1)

29. If the solution set of $\log_3\frac{|x^2-4x|+3}{x^2+|x-5|} \ge 0$ is $\left(-\infty, -\frac{\alpha}{\beta}\right]\cup\left[\frac{1}{\alpha}, \alpha\right]$, then $\alpha\beta =$

(JEE Arjuna Mathematics M-1)

30. For the equation

$(0.4)^{\log^2 x+1} = (6.25)^{2-p\log x}$

(base 10)

If $p = 2$, number of real roots *m*,

If $p = 3$, number of real roots *n*,

Then $m + n =$

(JEE Arjuna Mathematics M-1)

PYQ'S (PAST YEAR QUESTIONS)

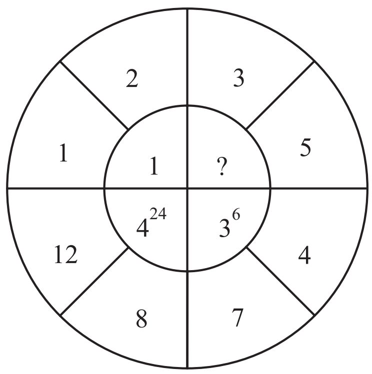
INEQUALITIES

1. Let the point $(p, p+1)$ lie inside the region $E=\left\{(x, y): 3-x\le y\le\sqrt{9-x^2}, 0\le x\le 3\right\}$. If the set of all values of *p* is the interval (a, b). then b^2+b-a^2 is equal to _________ **[6 April, 2023 (Shift-I)]**

(JEE Arjuna Mathematics M-1)

2. The missing value in the following figure is

[18 Mar, 2021 (Shift-I)]



Use the logic which gives answer in single digit.

(JEE Arjuna Mathematics M-1)

3. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is **[10 April, 2019 (Shift-II)]**

(*a*) 3 (*b*) 2 (*c*) 4 (*d*) 1

LOGARITHM

4. The number of integral solutions x of

$$\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0 \text{ is}$$ **[11 April, 2023 (Shift-I)]**

(*a*) 6 (*b*) 8 (*c*) 5 (*d*) 7

(JEE Arjuna Mathematics M-1)

5. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to _________ .

[27 June, 2022 (Shift-I)]

(JEE Arjuna Mathematics M-1)

6. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is **[26 Feb, 2021 (Shift-I)]**

(JEE Arjuna Mathematics M-1)

7. If for $x \in \left(0, \frac{\pi}{2}\right)$ $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1), n > 0$, then the value of n is equal to: **[16 March, 2021 (Shift-I)]**

(*a*) 16 (*b*) 12 (*c*) 9 (*d*) 20

(JEE Arjuna Mathematics M-1)

8. The inverse of $y = 5^{\log x}$ is: **[17 March, 2021 (Shift-I)]**

(*a*) $x = y^{\log 5}$ (*b*) $x = y^{\frac{1}{\log 5}}$ (*c*) $x = e^{\log_5 y}$ (*d*) $x = 5^{\frac{1}{\log y}}$

(JEE Arjuna Mathematics M-1)

9. The sum of the roots of the equation,

[31 Aug, 2021 [Shift-II]

$x + 1 - 2\log_2\left(3 + 2^x\right) + 2\log_4\left(10 - 2^{-x}\right) = 0$, is:

(*a*) $\log_2 12$ (*b*) $\log_2 13$

(*c*) $\log_2 11$ (*d*) $\log_2 14$

(JEE Arjuna Mathematics M-1)

10. The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0$, is.

[20 July, 2021 (Shift-II)]

(JEE Arjuna Mathematics M-1)

11. The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is __________ **[9 Jan, 2020 (Shift-I)]**

(JEE Arjuna Mathematics M-1)

12. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____. **[JEE Adv, 2020]**

(JEE Arjuna Mathematics M-1)

PW CHALLENGERS

1. If $\log_4(x + 2y) + \log_4(x - 2y) = 1$, then the minimum value of $|x| - |y|$ is ____.

(JEE Arjuna Mathematics M-1)

2. Let a, b, c, d be positive integers and $\log_a b = \frac{3}{2}, \log_c d = \frac{5}{4}$. If $a - c = 9$, then $b - d =$

(JEE Arjuna Mathematics M-1)

3. Let $x \in N$ such that $2^{1+[\log_2(x-2)]} - x = 20$. ([·] is G.I.F.) The smallest value of x, is

(JEE Arjuna Mathematics M-1)

4. If $\sqrt{4 + \sqrt{8 - \sqrt{32 + \sqrt{768}}}} = a\sqrt{2}\cos\left(\frac{11\pi}{b}\right)$, where a and b are natural numbers then find $a + b$.

(JEE Arjuna Mathematics M-1)

5. Let $r_1, r_2, r_3 \ldots r_n$ be n positive integers, not necessarily distinct, such that $(x + r_1)(x + r_2)(x + r_3)\ldots(x + r_n) = x^n + 56x^{n-1} + \ldots + 2009$ then the value of n is equal to

(JEE Arjuna Mathematics M-1)

6. If $(a + 1)(b + 1)(c + 1)(d + 1) = 1$

$(a + 2)(b + 2)(c + 2)(d + 2) = 2$

$(a + 3)(b + 3)(c + 3)(d + 3) = 3$

$(a + 4)(b + 4)(c + 4)(d + 4) = 4$

Then the value of $(a + 5)(b + 5)(c + 5)(d + 5)$ is equal to.

(JEE Arjuna Mathematics M-1)

7. Find sum of all possible natural numbers 'n' for which $\frac{5n^2 - 7n + 84}{n}$ is divisible by 5

(JEE Arjuna Mathematics M-1)

8. The value of

$$\left[2008 + \log_{\left(\frac{6561}{256}\right)}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\ldots}}}\right)\right]$$ is

(where [·] is G.I.F.)

(JEE Arjuna Mathematics M-1)

9. Let a, b and c be distinct non zero real numbers such that $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c}$. The value of $10(a^3 + b^3 + c^3)$, is

(JEE Arjuna Mathematics M-1)

10. Match the Column:

	Column-I		Column-II
A.	Number of integral pair of the form (x, y) satisfying $\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}=\frac{1}{\sqrt{20}}$ is/are equal to	p.	16
B.	Number of positive integral solutions of the equation $3x + 5y = 1008$ is/are equal to	q.	2
C.	Number of integers n such that $\sqrt{\frac{3n-5}{n+1}}$ is also an integer, is/are equal to	r.	0
D.	Number of integers n (positive, negative or 0) such that $n^2 + 73$ is divisible by $(n + 73)$, is/are equal to	s.	67
		t.	3

(*a*) A $\rightarrow$ t; B $\rightarrow$ s; C $\rightarrow$ q; D $\rightarrow$ p
(*b*) A $\rightarrow$ r; B $\rightarrow$ p; C $\rightarrow$ q; D $\rightarrow$ s
(*c*) A $\rightarrow$ q; B $\rightarrow$ p; C $\rightarrow$ r; D $\rightarrow$ s
(*d*) A $\rightarrow$ s; B $\rightarrow$ p; C $\rightarrow$ q; D $\rightarrow$ p

(JEE Arjuna Mathematics M-1)

11. $\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}=a$ then find the absolute value of $a - 2023$.

(JEE Arjuna Mathematics M-1)

Answer Key

CONCEPT APPLICATION

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*d*) **6.** (*i*) 14 (*ii*) 52 (*iii*) 194 **9.** [0] **10.** [–224]

11. $x^6 - y^6$ **12.** $p = 3/2, q = 1, r = 4/3$ **13.** (*d*) **14.** (*c*) **15.** (*b*) **16.** [60] **17.** [99] **18.** [4]

19. (*d*) **20.** [2*ab*] **21.** ϕ **22.** $x = \pm\sqrt{\frac{3}{5}}$ **23.** (*c*) **24.** $x \in (-\infty, -3) \cup [-2, 0] \cup [6, \infty)$

25. $x \in (-6, 0] \cup [2, 3] \cup (6, \infty) \cup \{4\}$ **26.** [6] **27.** (*d*) **28.** [1, –1] **29.** $[x \in (\sin 4, \sin 3) \cup [\sin 1, \sin 2]]$

30. $x \in [1, 2]$ **31.** {9} **32.** $x \in (2, 3)$

33. $\left(\frac{1}{5}, \frac{2}{5}\right)$ **34.** (243) **35.** [–5] **36.** [18]

37. [–1] **38.** (*i*) $(1, \infty)$ (*ii*) $[1, \infty)$ (*iii*) $(0, 1)$ (*iv*) $(0, 1]$ (*v*) $(0, 1)$ (*vi*) $(0, 1]$ (*vii*) $(1, \infty)$ (*viii*) $[1, \infty)$ (*ix*) $(3, \infty)$ (*x*) $[5/2, \infty)$

39. (*i*) $x \in \left(\frac{3}{2}, \frac{19}{2}\right)$ (*ii*) $x \in \left(\frac{2}{3}, \frac{17}{24}\right]$ (*iii*) $x \in (4^{16}, \infty)$ (*iv*) $x \in \left(0, \frac{1}{2}\right)$ **40.** {3, –3} **41.** {–2, 2} **43.** $\frac{12 - 4a}{3 + a}$

44. (*i*) $\left\{2^{\pm\sqrt{2}}\right\}$ (*ii*) $x = a^{-\log_5 2}$ (*iii*) {1/32, 2} (*iv*) {1} (*v*) {1}

47. [–1] **48.** [44] **49.** (*d*) **50.** (*b*, *c*)

51. (*i*) {–1, 5} (*ii*) {–3, –1, 7, 9} (*iii*) (14, –4, 0, 10, 2, 8) **52.** (–6, 8) **53.** (*d*) **54.** $(0, 2) \cup (4, \infty)$ **55.** $x \geq 1$

56. (*i*) $x \in (-\infty, 1) \cup (1, \infty)$ (*ii*) $x \in (-\infty, -5] \cup [5, \infty)$ (*iii*) $x \in (-7, 7)$ (*iv*) $x \in [-10, 10]$ (*v*) $x \in R$ (*vi*) $x \in \phi$ (*vii*) $x \in R$ (*viii*) $x \in R$ (*ix*) $x \in \phi$

57. (*i*) $x \in (-\infty, 0) \cup (2, \infty)$ (*ii*) $1 < x < 3$ (*iii*) $x \in (-2, -1) \cup (0, 1)$ (*iv*) $x \in [-1, 0] \cup [1, 2]$ (*v*) $-4/3 \leq x \leq 2$ (*vi*) $x \in \phi$

58. (*i*) $-1 \leq x \leq 5$ (*ii*) $x \in (-\infty, -3] \cup [9, \infty) \cup [-1, 7]$ (*iii*) $x \in [-4, 0] \cup [2, 8] \cup [10, 14]$

59. (*i*) $x \in \left[\frac{1}{5}, \frac{1}{4}\right] \cup \left[\frac{3}{4}, \frac{4}{5}\right]$ (*ii*) $x \in \left[\frac{2}{3}, 2\right]$ (*iii*) $x \in \left[\frac{2}{15}, \frac{2}{5}\right] \cup \left[\frac{6}{5}, \frac{22}{15}\right]$ (*iv*) $x \in (-3, -2) \cup (2, 3)$

60. (*i*) $x \in \{-11, -5, -1\}$ (*ii*) $x \in \{-8, -6, -2, -4, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$

61. (*i*) $x \in [1, 13]$ (*ii*) $x \in [-8, -6] \cup [-4, 0] \cup [2, 4] \cup [6, 8] \cup [10, 14] \cup [16, 18]$

62. (*i*) $x \in (-2, -1] \cup [1, 2)$ (*ii*) $x \in (-\infty, -3] \cup (-2, -1] \cup [3, \infty) \cup [1, 2)$ (*iii*) $x \in (-\infty, -3] \cup [3, \infty)$
(*iv*) $x \in (-\infty, -4] \cup [-1, 5] \cup [6, \infty)$ (*v*) $x \in (-\infty, -4] \cup [6, \infty] \cup \{2\}$

63. (*i*) $x \in [1, \infty]$ (*ii*) $x \in (0, \infty) \cup \{-1\}$ (*iii*) $x \in (-1, 0) \cup (0, 3)$ (*iv*) $x \in \phi$ (*v*) $x \in (2, 6)$

64. $[-6, \infty)$

SCHOOL LEVEL PROBLEMS

1. (*a*) **2.** (*c*) **3.** (*d*) **4.** (*d*) **5.** (*c*) **6.** (*d*) **7.** (*b*) **8.** (*a*) **9.** (*b*) **22.** (*b*)
23. (*a*) **24.** (*c*) **25.** (*c*)

PRARAMBH (TOPICWISE)

1. (*b*)	**2.** (*b*)	**3.** (*d*)	**4.** (*a*)	**5.** (*c*)	**6.** (*a*)	**7.** (*d*)	**8.** (*b*)	**9.** (*b*)	**10.** (*a*)
11. (*b*)	**12.** (*d*)	**13.** (*d*)	**14.** (*c*)	**15.** (*d*)	**16.** (*c*)	**17.** (*b*)	**18.** (*d*)	**19.** (*c*)	**20.** (*c*)
21. (*d*)	**22.** (*b*)	**23.** (*c*)	**24.** (*c*)	**25.** (*d*)	**26.** (*b*)	**27.** (*a*)	**28.** (*b*)	**29.** (*c*)	**30.** (*a*)
31. (*b*)	**32.** (*d*)	**33.** (*b*)	**34.** (*a*)	**35.** (*d*)	**36.** (*c*)	**37.** (*b*)	**38.** (*d*)	**39.** (*b*)	**40.** (*d*)

PRABAL (JEE MAIN LEVEL)

1. (*c*)	**2.** (*a*)	**3.** (*a*)	**4.** (*a*)	**5.** (*b*)	**6.** (*c*)	**7.** (*a*)	**8.** (*b*)	**9.** (*d*)	**10.** (*c*)
11. (*d*)	**12.** (*b*)	**13.** (*c*)	**14.** (*a*)	**15.** (*b*)	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*b*)	**20.** (*d*)
21. (*b*)	**22.** (*b*)	**23.** (*c*)	**24.** (*d*)	**25.** (*c*)	**26.** (*c*)	**27.** (*c*)	**28.** (*a*)	**29.** (*a*)	**30.** (*d*)
31. (*a*)	**32.** (*a*)	**33.** (*b*)	**34.** (*d*)	**35.** (*a*)	**36.** (*b*)	**37.** (*d*)	**38.** (*b*)	**39.** (*d*)	**40.** (*d*)
41. (*a*)	**42.** (*b*)	**43.** (*a*)	**44.** (*c*)	**45.** (*a*)	**46.** [3]	**47.** [89]	**48.** [75]	**49.** [1]	**50.** [2]
51. [1]	**52.** [400]	**53.** [3]	**54.** [5]	**55.** [0]	**56.** [1]	**57.** [9]	**58.** [1]	**59.** [10]	

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*a*)	**2.** (*c*)	**3.** (*a*)	**4.** (*a*)	**5.** (*d*)	**6.** (*d*)	**7.** (*d*)	**8.** (*b, c*)	**9.** (*a, d*)	**10.** (*a, b, c*)
11. (*a, b, c, d*)		**12.** (*b, c*)	**13.** (*a*,*b*,*c*)	**14.** (*a*,*b*,*c*,*d*)		**15.** (*b*)	**16.** (*d*)	**17.** (*a*)	**18.** (*c*)
19. (*b*)	**20.** (*c*)	**21.** [1]	**22.** [7]	**23.** [2]	**24.** [10]	**25.** [17]	**26.** [2]	**27.** [2]	**28.** [1]
29. [6]	**30.** [2]								

PYQ's (PAST YEAR QUESTIONS)

1. [3]	**2.** [4]	**3.** (*d*)	**4.** (*a*)	**5.** [45]	**6.** [1]	**7.** (*b*)	**8.** (*c*)	**9.** (*c*)
10. [1]	**11.** [8]	**12.** [8]						

PW CHALLENGERS

1. [$\sqrt{3}$]	**2.** [93]	**3.** [44]	**4.** [50]	**5.** [4]	**6.** [29]	**7.** [63]	**8.** [2007]	**9.** [30]	**10.** (*a*)
11. [2019]									