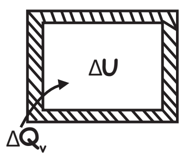
THERMODYNAMICS

Internal Energy(U)

(1) $\Delta U = \Delta Q_v = nC_v\Delta T = n\frac{f}{2}R\Delta T$

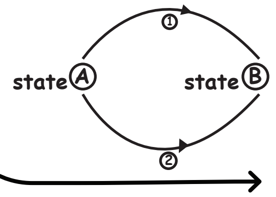
$= \frac{nR\Delta T}{\gamma-1} = \frac{\Delta(PV)}{\gamma-1}$

$= \frac{P_fV_f - P_iV_i}{\gamma-1}$



(Internal Energy is only the function of temperature of the gas)

- First law of T.D $\Longrightarrow$ $Q = \Delta U + W$
- Q, W $\Longrightarrow$ path functions
- ΔU $\Longrightarrow$ state function



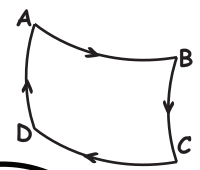
- $W_1 \neq W_2$
- $Q_1 \neq Q_2$
- $\Delta U_1 = \Delta U_2 = U_B - U_A$

Isothermal process $\Rightarrow \Delta T = 0 \longrightarrow \Delta U = 0$

Cyclic process

$A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$

$\Delta U = U_A - U_A = 0$



WORK

Work done: path function

$W = F.dx = \int P dv$

- Unit: Joule(J)

Sign Convention

W System W

-ve (compressive) work done on the gas

+ve (expansion) work done by the gas

Work

+ve Q System -ve Q

Heat

HEAT

Heat: path function

$\Rightarrow$ unit: calorie/Joule

For any process,

$Q = nC\Delta T$

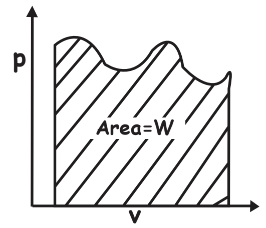
where, C= Specific heat capacity for the process

Adiabatic process $\Rightarrow \Delta Q = 0$ [No heat transfer]

At constant volume $\Rightarrow$ $Q_V = \Delta U = nC_V\Delta T$

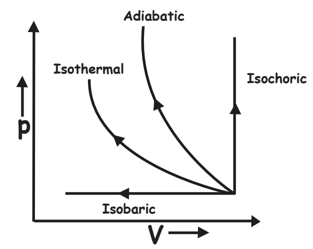
At constant pressure $\Rightarrow$ $Q_p = \Delta U + W = nC_p\Delta T$

Work done from P-V Graph



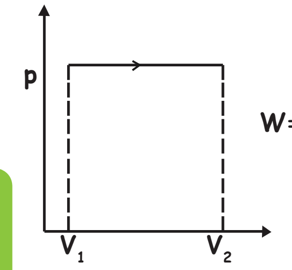
Area under P-V diagram gives work done by the gas

Compression



$|W_{adiabatic}| > |W_{isothermal}| > |W_{isobaric}| > |W_{isochoric}|$

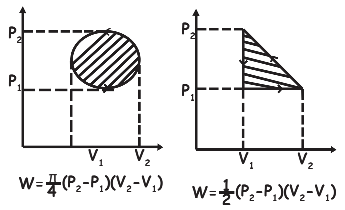
Isobaric process



$W = P(V_2 - V_1)$

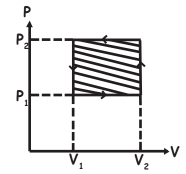
Cyclic process

- W=are inside the graph
- For clockwise process, W=-ve
- For anti-clockwise process, W=+ve



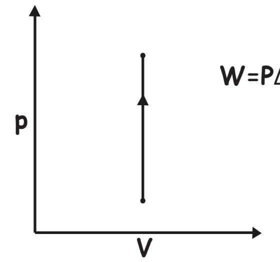
$W = \frac{\pi}{4}(P_2-P_1)(V_2-V_1)$

$W = \frac{1}{2}(P_2-P_1)(V_2-V_1)$

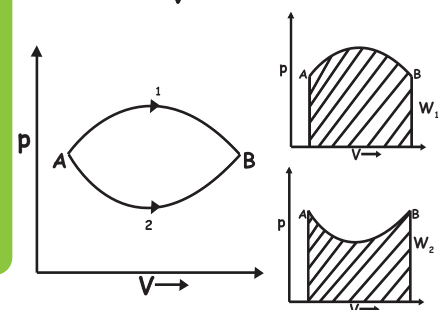


$W = (P_2-P_1)(V_2-V_1)$

Isochoric process



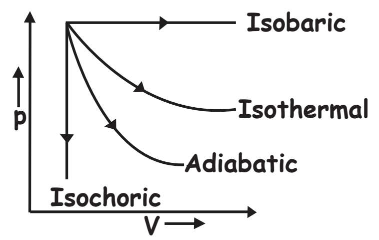
$W = P\Delta V = 0$



$W_1 \neq W_2$

$W_1 > W_2$

Expansion



$W_{isobaric} > W_{isothermal} > W_{adiabatic} > W_{Isochoric}$

Thermodynamic processes

01 Adiabatic process

- Q=0 [no exchange of heat]
- Rapid or spontaneous process/insulated vessel

$Q = \Delta U + W$

$Q = 0 \quad \Delta U = -W$

Compression

$W = -ve \quad \Delta U = +ve$

$\Delta U\uparrow \Longrightarrow$ Temperature$\uparrow$

$\Rightarrow$ Pressure$\uparrow$

Expansion

$W = -ve \quad \Delta U = +ve$

$\Delta U\downarrow \Longrightarrow$ Temperature$\downarrow$

$\Rightarrow$ Pressure$\downarrow$

Equation of state

PV^{γ} = constant

$TV^{\gamma-1}$ = constant

$PT^{\left[\frac{\gamma}{1-\gamma}\right]}$ = constant

Work done by the gas

$W = -\Delta U = nC_v(T_1 - T_2)$

$= n\frac{f}{2}R(T_i - T_f)$

$W = \frac{nR}{\gamma-1}(T_i - T_f)$

$W = \frac{P_iV_i - P_fV_f}{\gamma-1}$

Slope of adiabatic process

$= \gamma \times$ slope of isothermal process

specific heat of gas $\Rightarrow C = 0$

$C = \frac{Q}{\Delta T} \longrightarrow Q = 0$

$C = 0$

02 Isothermal process

$\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$

eg:- perfectly conducting slow process

$Q = \Delta U + W$

$Q = W$

equation of states $\Rightarrow$ PV=Constant

$P_1V_1 = P_2V_2$

Workdone by the gas

$W = 2.303\ nRT \log\left(\frac{V_2}{V_1}\right)$

$W = 2.303\ nRT \log\left(\frac{P_1}{P_2}\right)$

Slope:

Slope of adiabatic process

$= \gamma \times$ slope of isothermal process

specific heat $\quad C = \infty$

03 Isobaric process

$\Delta P = 0$

$Q = \Delta U + W$

equation of state $\Rightarrow V \propto T$

$\frac{V_1}{V_2} = \frac{T_1}{T_2}$

Work done by the gas

$W = P\Delta V = P(V_2 - V_1) = nR(T_2 - T_1)$

specific heat

$\Rightarrow C_p = \left(1 + \frac{f}{2}\right)R$

$= \frac{\gamma R}{\gamma - 1}$

04 Isochoric process

$\Delta V = 0$ or V=constant

equation of state $\Rightarrow P \propto T \Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}$

Work done by the gas

$\Delta V = 0 \Rightarrow W = 0$

specific heat $\Rightarrow C_V = \frac{f}{2}R$

$= \frac{R}{\gamma - 1}$

Heat Engine

'Device that converts heat into work'

High temp. T_1 Source

$Q_1 \rightarrow$ heat absorbed by engine

$\rightarrow W = Q_1 - Q_2$

$Q_2 \rightarrow$ heat released by engine

T_2 Sink (surrounding)

Low temp.

efficiency(η)

$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

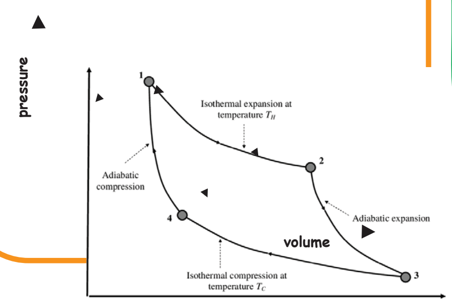
$\eta_{carnot} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$

$\eta_{max} \Longrightarrow$ When $Q_2 = 0$ or $T_2 = 0K$ (not possible)

Carnot Engine

$\rightarrow$ Ideal engine

$\rightarrow \boxed{\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}}$



Refrigerator and heat pump

Source T_1 (High Temp.)

Q_1

W (work done)

Q_2

Sink T_2 (Low Temp.)

Coefficient of performance of refgeration(β)

$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \qquad \beta = \frac{T_2}{T_1 - T_2}$

Coefficient of performance of heat pump

$= \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$

Relationship between

$\beta = \frac{1-\eta}{\eta}$

$(COP)_{heat\ pump} = 1 + (COP)_{refrigerator}$

Cascaded engine

T_1 → E_1 → W_1 → T → E_2 → W_2 → T_3

Same work output

$W_1 = W_2$

$T_1 - T = T - T_3$

$2T = T_1 + T_3$

$T = \frac{T_1 + T_3}{2}$

Same efficiency

$1 - \frac{T}{T_1} = 1 - \frac{T_3}{T}$

$T^2 = T_1T_3$

$T = \sqrt{T_1T_3}$