HYDRA
BOOKS

KARMA

ARJUNA
for
JEE
MAIN & ADVANCED
PHYSICS
FULL COURSE STUDY MATERIAL

Class XI

- Thermal Properties of Matter
- Kinetic Theory of Gases and Thermodynamics
- Simple Harmonic Motion
- Waves

Module-4

CONTENTS

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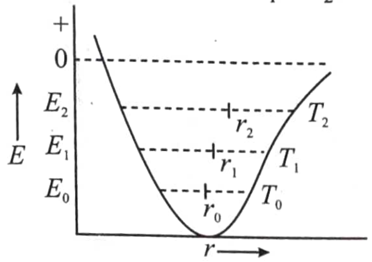
CHAPTER

15 Thermal Properties of Matter

THERMAL EXPANSION

When matter is heated without change in state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. With rise in temperature say from T_1 to T_2 the amplitude of vibration and energy of atoms increases from them increases E_1 to E_2 and hence the average distance between the from r_1 to r_2.



Due to this increase in distance between atoms, the matter as a whole expands.

Thermal Expansion in Solid

To varying extents most materials expand when heated and contract when cooled. The increase in any one dimension of a solid is called linear expansion, linear in the sense that the expansion occurs along a line. The expansion in length is called linear expansion. The expansion in area is called areal expansion. The expansion in volume is called volume expansion.

Linear Expansion

A rod whose length is L_0 when the temperature is T_0, when the temperature increases to $T_0 + \Delta T$, the length becomes $L_0 + \Delta L$, where ΔT and ΔL are the magnitude of the changes in temperature and length, respectively.

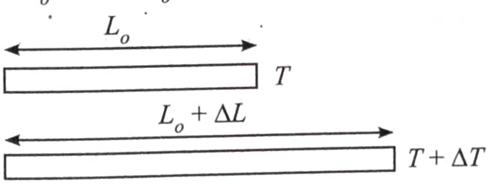
Conversely, when the temperature decreases to $T_0 - \Delta T$, the length decreases to $L_0 - \Delta L$.

For small temperature changes, experiments show that

$\Delta L = L_o$ and $\Delta L \propto \Delta T$

$\Rightarrow \Delta L \propto L_o \Delta T \Rightarrow \Delta L = \alpha L_o \Delta T$

$\Rightarrow L = L_o + \Delta L = L_o(1 + \alpha \Delta L)$



Proportionality constant α is called the coefficient of linear expansion.

Unit for the coefficient of linear expansion $= \dfrac{1}{°C} = (°C)^{-1}$

Measurement of length by metallic scale:

Case (i)

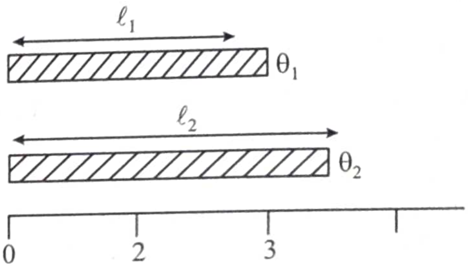
When only object expands

$\ell_2 = \ell_1 \{1 + \alpha_0(\theta_2 - \theta_1)$

ℓ_1 = actual length of object at θ_1°C = measure length of object at θ_1°C.

ℓ_2 = actual length of object at θ_2°C = measure length of object at θ_2°C.

α_0 = coefficient of linear expansion of object.



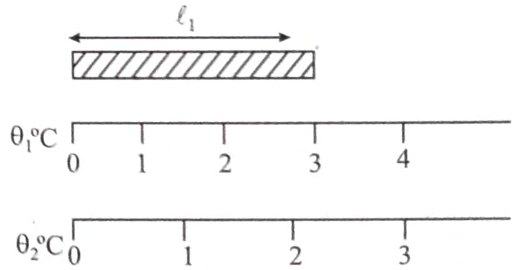
Case (ii)

When only measuring instrument expands actual length of object will not change but measured value (MV) decreases.

MV $= \ell_1 \{1 - \alpha_S(\theta_2 - \theta_1)\}$

α_S = coefficient of linear expansion measuring instrument.

at θ_1 °C, MV = 3



at θ_2 °C MV = 2.2

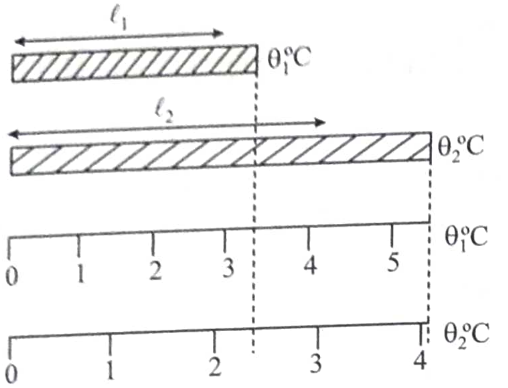
Case (iii)

If both expand simultaneously

MV $= \{1 + (\alpha_0 - \alpha_s)(\theta_2 - \theta_1)\}$

(i) If $\alpha_0 > \alpha_s$, then measured value is more then the actual value at θ_1°C

(ii) If $\alpha_0 < \alpha_s$, then measured value is less then the actual value at θ_1°C



at θ_1°C MV = 3.4

θ_2°C MV = 4.1

Measured value = calibrated value × {1 + α Δ θ}

where $\alpha = \alpha_0 - \alpha_s$

α_o = coefficient of linear expansion of object material,

α_s = coefficient of linear expansion of scale material

$\Delta\theta = \theta - \theta_C$

θ = temperature at the time of measurement

θ_C = temperature at the time of calibration.

For scale,

true measurement = scale reading $[1 + \alpha(\theta - \theta_0)]$

Variation of Time Period of Pendulum Clocks

The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum Every time it reaches to its extreme position the second hand of the clock advances by one second, that means second hand moves by two seconds when one oscillation in complete

Let $T = 2\pi\sqrt{\dfrac{L_0}{g}}$ at temperature θ_0 and $T' = 2\pi\sqrt{\dfrac{L}{g}}$ at temperature θ.

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L[1+\alpha\Delta\theta]}{L}} = 1 + \frac{1}{2}\alpha\Delta\theta$$

Therefore change (loss or gain) in time per unit time lapsed is

$$\frac{T'-T}{T} = \frac{1}{2}\alpha\Delta\theta$$

gain or loss in time in duration of 't' in

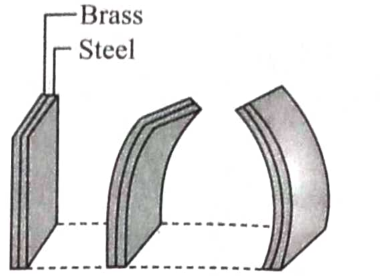
$$\Delta t = \frac{1}{2}\alpha\Delta\theta\, t, \text{ if } T \text{ is the correct time then}$$

(a) $\theta < \theta_0$, $T' < T$ clock becomes faster and gain time

(b) $\theta > \theta_0$, $T' > T$ clock becomes slower and loose time

THE BIMETALLIC STRIP

A bimetallic strip is made from two thin strips of metal that have different coefficients of linear expansion as figure shows.

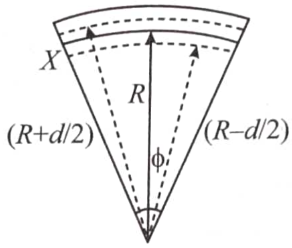


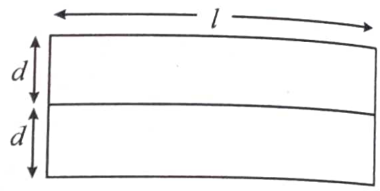
(a) (b) Heated (c) Cooled

(a) A bimetallic strip (b) on heated and (c) on cooled.

Often brass [$\alpha = 19 \times 10^{-6}$ °C^{-1}] and steel [$\alpha = 12 \times 10^{-6}$ °C^{-1}] are selected. The two pieces are welded or riveted together. When the bimetallic strip is heated, the brass having the larger value of α expands more than the steel. Since, the two metals are bonded together, the bimetallic strips bends into an arc as in part b, with the longer brass piece having a larger radius than the steel piece. When the strip is cooled, the bimetallic strips bends in the opposite direction, as in part c.

Mathematical Solution





$$\left(R+\frac{d}{2}\right)\phi = L_0(1+\alpha_1\Delta\theta)$$

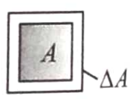
$$\left(R-\frac{d}{2}\right)\phi = L_0(1+\alpha_2\Delta\theta)$$

$$\frac{R+\frac{d}{2}}{R-\frac{d}{2}} = \frac{1+\alpha_1\Delta\theta}{1+\alpha_2\Delta\theta}$$

From the above equation we can find the value of R.

Areal Expansion

Expansion in area on heating is known as areal expansion.



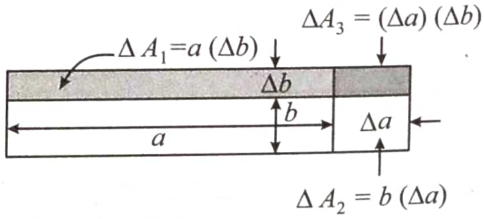
$\Delta A \propto A_o$

$\Delta A \propto \Delta T$

$\Rightarrow \Delta A \propto A_o \Delta T$; $\Delta A = \beta A_o \Delta T$

$A = A_o + \Delta A$; $A = A_o(1 + \beta\Delta T)$

Relation Between Coefficient of Linear expansion (α) and Coefficient of areal expansion (β)



Consider a rectangular sheet of the solid material of length a and breadth b in figure. When the temperature increases by ΔT, a increases by $\Delta a = a\alpha\Delta T$ and b increases $\Delta b = b\alpha\Delta T$. From figure the increase in area.

$$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$$

$$\Delta A = a\,\Delta b + b\,\Delta a + (\Delta a)(\Delta b)$$

$$= a\,\alpha b\,\Delta T + b\,\alpha a\Delta T + (\alpha_1)^2\,ab\,(\Delta T)^2$$

$$= 2\alpha\,ab\,\Delta T$$

$$= 2\alpha\,A\,\Delta T$$

$$\because \quad \Delta A = \beta A\Delta T$$

$$\Rightarrow \quad \beta = 2\alpha$$

Therefore the coefficient of areal expansion β of a rectangular sheet of the solid is twice its linear expansion, α.

VOLUME EXPANSION

The volume of a normal material increases as the temperature increases. Most solids and liquids behave in this fashion. By analogy with linear thermal expansion, the change in volume ΔV is proportional to the change in temperature ΔT and to the initial volume V_0, provided the change in temperature is not too large.

$$\Delta V \propto V_o\,;\ \Delta V \propto \Delta T$$

$$\Rightarrow \quad \Delta V \propto V_o\Delta T\,;\ \Delta V = \gamma V_o\Delta T$$

$$V = V_o + \Delta V\,;\ V = V_o(1 + \gamma\Delta T)$$

γ = coefficient of volume expansion.

If a cavity exists within a solid object, the volume of the cavity increases when the object expands, just as if the cavity were filled with the surrounding material. The expansion of the cavity is analogous to the expansion of a hole in a sheet of material. Accordingly, the change in volume of a cavity can be found using the relation $\Delta V = \gamma V_0\Delta T$, where γ is the coefficient of volume expansion of the material that surrounds the cavity.

Relation Between γ and α

For an isotropic solid (which has the same value of α in all directions) $\gamma = 3\alpha$. To see that $\gamma = 3\alpha$ for a solid, consider a cube of length l and volume $V = l^3$.

When the temperature of the cube is increased by dT, the side length increases by dl and the volume increase by an amount dV given by

$$dV = \left(\frac{dV}{dl}\right)\cdot dl = 3l^2 \cdot dl$$

Now, $dl = l\alpha dT$

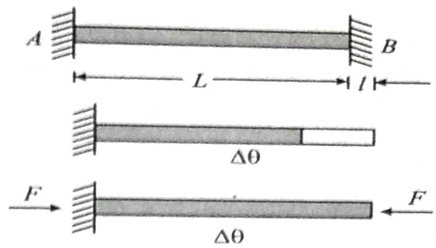
$$\therefore \quad dV = 3l^3\alpha dT = (3\alpha)\,VdT$$

$$\because \quad dV = \gamma VdT, \text{ only if}$$

$$\gamma = 3\alpha \qquad \text{...(iii)}$$

Thermal Stress

When a metal rod is heated or cooled it tends to expand or contract. If it is left free to expand or contract, no temperature induced stresses will develop. However, if the rod is not allowed to change its length then temperature stresses are generated within it. Stress induced due to temperature change can be understood as follows:



Consider a uniform rod AB fixed rigidly between two supports (figure). If L be its length, α the coefficient of linear expansion, then a change in temperature of $\Delta\theta$, would tend to bring a change in its length by $\Delta L = L\alpha\Delta\theta$. Had the rod been free (say one of its ends) its length would have changed by ΔL. Now, let a force be gradually applied so as to restore the natural length. Since the rod, tends to remain in the new state, due to a change in temperature, so when a force F is applied, thermal stress is induced. In equilibrium.

$$\frac{F}{A} = \frac{\Delta l\ Y}{(L \pm l)} \qquad [\because \text{stress} = \text{strain} \times Y]$$

Neglecting l in comparison to L,

$$F = \frac{\Delta l A\ Y}{L} = AY\alpha\,\Delta\theta$$

Now, if the two ends remain fixed, then this external force is provided from the support.

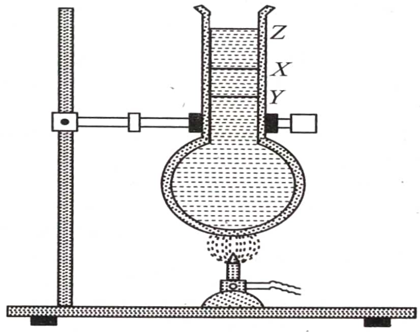
$$\text{Clearly, strain} = \frac{\Delta l}{L} = \alpha\,\Delta\theta$$

FLUID EXPANSION

Expansion of Liquids

Like solids, liquids also, expand on heating; however, their expansion is much larger compared to solids for the same temperature rise. A note worthy point to be taken into account during the expansion of liquid is that they are always contained in a vessel or a container and hence the expansion of the vessel also comes into picture. Further, linear or superficial (Area) expansion in case of a liquid does not carry any sense.

Consider a liquid contained in a round bottomed flask fitted with a long narrow stem as shown in figure. Let the initial level of the liquid be X. When it is heated, the level falls initially to Y.



However, after sometime, the liquid level eventually rises to Z. The entire phenomenon can be understood as follows: upon being heated, the container gets heated first and hence expands. As a result, the capacity of the flask increases and hence the liquid level falls.

After sometime, the heat gets conducted from the vessel to the liquid and hence liquid also expands thereby rising its level eventually to Z. Since, the volume expansivity of liquids, in general, are far more than that of solids, so the level Z will be above the level X.

Effect of Temperature on Density

When a solid or liquid is heated, it expands, with mass remaining constant. Density being the ratio of mass to volume, it decreases. Thus, if V_0 and V_1 be the respective volumes of a substance at 0°C and t°C and if the corresponding values of densities be ρ_0 and ρ_t, then the mass m of the substance is given by

$$m = V_0\rho_0 = V_t\rho_t$$

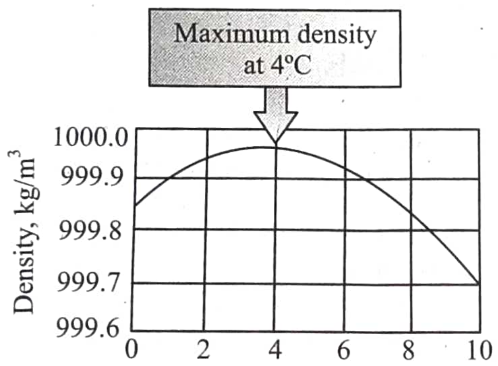
But $V_t = V_0(1 + \gamma t)$,

So $\rho_t = \rho_0(1 + \gamma t)^{-1} = \rho_0(1 - \gamma t)$ $\quad [\because \gamma << 1]$

ANOMALOUS BEHAVIOUR OF WATER

While most substances expand when heated, a few do not. For instant, if water at 0°C is heated, its volume decreases until the temperature reaches 4°C. Above 4°C water behaves normally, and its volume increases as the temperature increases.

Because a given mass of water has a minimum volume at 4°C, the density (mass per unit volume) of water is greatest at 4°C, as figure shows.



The density of water in the temperature range from 0 to 10°C.

Water has a maximum density of 999.973 kg/m³ at 4°C. (This value for the density is equivalent to the often quoted density of 1.000 grams per milliliter.)

When the air temperature drops, the surface layer of water is chilled. As the temperature of the surface layer drops toward 4°C, this layer becomes more dense than the warmer water below. The denser water sinks and pushes up the deeper and warmer water, which in turn is chilled at the surface. This process continues until the temperature of the entire lake reaches 4°C. Further cooling of the surface water below 4°C makes it less dense than the deeper layers; consequently, the surface layer does not sink but stays on top. Continued cooling of the top layer to 0°C leads to the formation of ice that floats on the water, because ice has a smaller density than water at any temperature. Below the ice, however, the water temperature remains above 0°C. The sheet of ice acts as an insulator that reduces the loss of heat from the lake, especially if the ice is covered with a blanket of snow, which is also an insulator. As a result, lakes usually do not freeze solid, even during prolonged cold spells, so fish and other aquatic life can survive.

Train Your Brain

Example 1: A brass scale correctly calibrated at 15°C is employed to measure a length at a temperature of 35°C. If the scale gives a reading of 75 cm, find the true length. [Linear expansively of brass = 2.0×10^{-5} °C^{-1}]

Sol. Let the distance between two fixed divisions on the scale at 15°C be L_1 and that at 35°C be L_2.

Clearly, $(L_2 - L_1) = \alpha L_1(35 - 15)$

or $\quad L_2 = L_1(1 + 20 \times 2.0 \times 10^{-5})$

$= L_1(1.0004)$

i.e. at 35°C, an actual length of L_2 will be read as L_1 (note) due to the increased separation of the divisions of the scale. In other words, the observed length will be less than the actual length.

$L_1 = 75$ cm

$L_2 = 75\ (1.0004)$ cm $= 75.03$ cm

Example 2: Estimate the time lost or gained by a pendulum clock at the end of a week when the atmospheric temperature rises to 40°C. The clock is known to give correct time at 15°C and the pendulum is of steel. (Linear expansively of steel is 12×10^{-6} / °C).

Sol. Rise in temperature = 40 – 15 = 25°C

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{\Delta T}{T} = -\frac{1}{2}\frac{\Delta \ell}{\ell}$$

Time lost per second $= \frac{1}{2}\alpha\Delta\theta$

$= \frac{1}{2} \times (12 \times 10^{-6}/°C) \times (25°C) = 150 \times 10^{-6}$ s/s

Therefore, time lost per week (i.e., 7×86400 s)

$= 150 \times 10^{-6} \times 7 \times 86400$ s $= 90.72$.

Example 3: A glass rod when measured with a zinc scale, both being at 30°C, appears to be of length 100 cm. If the scale shows correct reading at 0°C, determine the true length of the glass rod at (*a*) 30°C and (*b*) 0°C.

['α' for glass = 8×10^{-6}/°C and for zinc 26×10^{-6}/°C]

Sol. At 30°C, although the reading shown by the zinc scale corresponding to the length of the glass rod is 100cm, but the actual length would be more than 100cm, the reason being the increased separation between the markings, owing to a rise in temperature (from 0°C to 30°C).

Now, an actual (at 0°C) length of 100cm on the zinc scale (or more precisely, two markings or divisions on the scale, separated by a distance of 100cm) would, at a temperature of 30°C, correspond to a length given by:
$l = 100\,(1 + 26 \times 10^{-6} \times 30)$ cm = 100.078 cm

$\therefore$ The true length of the glass rod at 30°C is 100.078 cm,

Now, at 0°C, the length of glass rod would be lesser than that at 30°C.

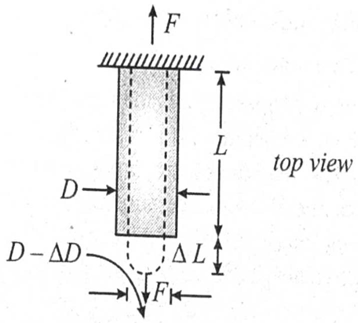
$\therefore$ Using $l_t = l_0\,(1 + \alpha t)$, $l_0 = \dfrac{l_t}{1+\alpha t}$

$\therefore$ The length of the rod at 0°C, will be

$$l_0 = \frac{100.078 \text{ cm}}{(1+8\times10^{-6}\times30)} = 100.054 \text{ cm}$$

Example 4: A brass rod of length 1m is fixed to a vertical wall, at one end, with the other end free to expand. When the temperature of the rod is increased by 120°C, the length increases by 2 cm. What is the strain?

Sol. After the rod expands, to the new length there are no elastic forces developed internally in it.
So, strain = 0.



Example 5: A rod of length 2m is at a temperature of 20°C. Find the free expansion of the rod, if the temperature is increased to 50°C. Find the thermal stresses produced when the rod is

(*i*) fully prevented to expand

(*ii*) permitted to expand by 0.4 mm. ($Y = 2 \times 10^{11}$ Nm^{-2}; $\alpha = 15 \times 10^{-6}$ per°C)

Sol. Free expansion of the rod = $\alpha\Delta\theta L$

$= (15 \times 10^{-6}\ /°\text{C}) \times (2\text{m}) \times (50° - 20°)$ C

$= 9 \times 10^{-4}$ m = 0.9 mm

(*i*) If the expansion is fully prevented, then strain

$= \dfrac{\Delta l}{l}$

$\dfrac{9\times10^{-4}}{2} \Rightarrow 4.5 \times 10^{-4}$

$\therefore$ stress = strain × $Y = 4.5 \times 10^{-4} \times 2 \times 10^{11}$
$= 9 \times 10^{7}$ Nm^{-2}

(*ii*) If 0.4 mm. expansion is allowed, then length restricted to expand = 0.9 – 0.4 = 0.5 mm

$\therefore$ Strain = $\dfrac{5\times10^{-4}}{2} = 2.5 \times 10^{-4}$

$\therefore$ Temperature stress = strain × Y = $2.5 \times 10^{-4} \times 2 \times 10^{11}$
$= 5 \times 10^{7}$ Nm^{-2}.

Example 6: Two rods made of different materials are placed between massive walls. The cross section of the rods are A_1 and A_2, their lengths L_1 and L_2 coefficients of linear expansion α_1 and α_2 and the modulus of elasticity of their materials Y_1 and Y_2 respectively. If the rods are heated by t°C. Find the force F with which the rods act on each other.

Sol. Let the first rod expand slightly (say by length δl) and the second rod get compressed by the same amount (since net elongation / compression of the rods is zero.

$\therefore$ Natural increase in length of the first rod (after being heated) when free to expand would have been $\alpha_1 L_1 t$. The expansion allowed is just δl (where $\delta l < \alpha_1 L_1 t$).

$\therefore$ Amount of elongation restricted = $\alpha_1 L_1 t - \delta l$

$$\therefore \text{ Strain} = \frac{\text{elongation restricted}}{\text{original length}}$$

$$= \frac{\alpha_1 L_1 t - \delta l}{L_1\,(1+\alpha_1 t)}$$

Since $\alpha_1 t << 1$

$\therefore$ $1 + \alpha_1 t \simeq 1$

$$\therefore \text{ Strain} = \frac{\alpha_1 L_1 t - \delta l}{L_1}$$

$$\therefore \text{ Stress} = \text{strain} \times Y = \left(\frac{\alpha_1 L_1 t - \delta l}{L_1}\right) Y_1.$$

$$\text{or } F = \text{stress} \times A = \left(\frac{\alpha_1 L_1 t - \delta l}{L_1}\right) Y_1 A_1 \quad \ldots(1)$$

$$\text{Similarly, } F = \left(\frac{\alpha_2 L_2 t - \delta l}{L_2}\right) Y_2 t_2$$

$$\text{or } \delta l = \alpha_1 L_1 t - \frac{FL_1}{Y_1 A_1} = \frac{FL_2}{Y_1 L_2} - \alpha_2 L_2 t$$

$$\text{or } F = \frac{(\alpha_1 L_1 + \alpha_2 L_2)\,t}{\left(\dfrac{L_1}{Y_1 A_1} + \dfrac{L_2}{Y_1 A_2}\right)}$$

Concept Application

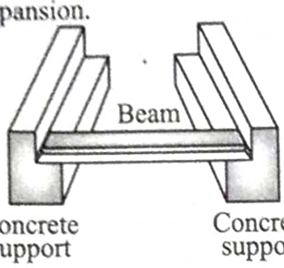
1. A bar measured with a Vernier caliper is found to be 180mm long. The temperature during the measurement is 10°C. The measurement error will be if the scale of the Vernier caliper has been graduated at a temperature of 20°C : ($\alpha = 1.1 \times 10^{-5}$ °C^{-1}. Assume that the length of the bar does not change.)
   (*a*) 1.98×10^{-1} mm
   (*b*) 1.98×10^{-2} mm
   (*c*) 1.98×10^{-3} mm
   (*d*) 1.98×10^{-4} mm

2. A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct

time at 20°C, how fast or slow will it go in 24 hours at 40°C? Coefficient of linear expansion of iron = 1.2 × 10^{-6}/°C.

(*a*) Slow by 1.04 sec (*b*) Slow by 1.08 sec
(*c*) Fast by 1.06 sec (*d*) Fast by 1.08 sec

3. A steel beam is used in the road bed of a bridge. The beam is mounted between two concrete supports when the temperature is 22°C, with no room provided for thermal expansion.



What compressional stress must the concrete supports apply to each end of the beam, if they are to keep the beam from expanding when the temperature rises to 42°C?

$Y = 2 \times 10^{11}$ N/m^2; $\alpha = 12 \times 10–5$ °C^{-1}

(*a*) 5.2×10^7 N/m^2 (*b*) 4.8×10^7 N/m^2
(*c*) 5.6×10^8 N/m^2 (*d*) 7.2×10^6 N/m^2

4. The volume of a glass vessel filled with mercury is 500 cc at 25°C. What volume of mercury will overflow at 45°C? The coefficients of volume expansion of mercury and glass are 1.8×10^{-4} /°C and 9.0×10^{-6} /°C respectively.

(*a*) 2.1 cc (*b*) 1.7 cc
(*c*) 1.4 cc (*d*) 1.2 cc

THERMOMETRY

Thermometry is the branch of Physics that deals with the measurement of temperature. The device which is used to measure temperature is called thermometer. The concept of temperature is derived from the zeroth law of thermodynamics which states that, if two bodies *A* and *B* are separately in thermal equilibrium with a third body *C*, then *A* and *B* are in thermal equilibrium with each other. When two bodies are left for a long time so that they reach thermal equilibrium, the property that becomes common to the two bodies is temperature.

Temperature Scales

Defining a temperature scale involves

(*i*) Choosing the thermometric substance
(*ii*) The choice of the thermometric property of the substance.
(*iii*) Choosing the upper fixed point and lower fixed point.
(*iv*) Choosing the number of divisions between the two fixed points.

Let N be the number of divisions between the fixed upper and fixed lower point. If measure of the thermodynamic property at the upper fixed point be x_N and that at lower fixed point is x_0 then if temperature at the lower fixed point is t_0 and at an unknown temperature t the measure of the thermometric property is x then,

$$t = \left(\frac{x - x_0}{x_N - x_0}\right)N + t_0$$

This equation defines temperature t.

Some of the most common thermometric properties are the length of the liquid column (like mercury) in a glass capillary tube.

Based on the choice of upper and lower fixed points we define various scales e.g., Celsius scale, Fahrenheit scale, etc.

For any types of scale

$$\frac{\text{Reading point} - \text{ice point}}{\text{Steam point} - \text{ice point}} = \text{constant}$$

The relation between these scales is given by the following equation.

$$\frac{C - 0°}{100° - 0°} = \frac{k - 273}{373 - 273} = \frac{F - 32}{212 - 32} \Rightarrow \frac{C}{5} = \frac{K - 273}{5} = \frac{F - 32}{9}$$

C = Reading in Celcuis
K = Reading in Kelvin
F = Reading in Forenheit

❖ **Different Thermometers**

Thermometric property : It is the property that can be used to measure the temperature. It is represented by any physical quantity such as length, volume, pressure and resistance etc., which varies linearly with a certain range of temperature. Let X denotes the thermometric physical quantity and X_0, X_{100} and Xt be tis values at 0°C, 100°C and t°C respectively. Then,

$$t = \left(\frac{X_t - X_0}{X_{100} - X_0}\right) \times 100°\text{C}$$

(*i*) **Constant volume gas thermometer :** The pressure of a gas at constant volume is the thermometric property. Therefore,

$$t = \left(\frac{p_t - p_0}{p_{100} - p_0}\right) \times 100°\text{C}$$

(*ii*) **Platinum resistance thermometer :** The resistance of a platinum wire is the thermometric property. Hence,

$$t = \left(\frac{R_t - R_0}{R_{100} - R_0}\right) \times 100°\text{C}$$

(*iii*) **Mercury thermometer :** In this thermometer, the length of a mercury column from some fixed point is taken as thermometric property. Thus,

$$t = \left(\frac{l_t - l_0}{l_{100} - l_0}\right) \times 100°\text{C}$$

❖ Two other thermometers, commonly used are thermocouple thermometer and total radiation pyrometer.

❖ Total radiation pyrometer is used to measure very high temperatures. When a body is at a high temperature, it glows brightly and the radiation emitted per second from unit area of the surface of the body is proportional to the fourth power of the absolute temperature of the body. If this radiation is measured by some device, the temperature of the body is calculated. This is the principle of a total radiation pyrometer. The main advantage of this thermometer is that the experimental body is not kept in contact with it. Hence, there is no definite higher limit of its temperature range. It can measure temperature from 800°C to 3000°C–4000°C. However, it cannot be used to measure temperatures below 800°C because at low temperatures the emission of radiation is so poor that it cannot be measured directly.

❖ **Range of different thermometers**

Thermometer	Lower limit	Upper limit
Mercury thermometer	–30°C	300°C
Gas thermometer	–268°C	–1500°C
Platinum resistance thermometer	–200°C	1200°C
Thermocouple thermometer	–200°C	1600°C
Radiation thermometer	800°C	No limit

❖ A substance is found to exist in three states solid, liquid and gas. For each substance, there is a set of temperature and pressure at which all the three states may coexist. This is called triple point of that substance. For water, the values of pressure and temperature corresponding to triple point are 4.58 mm of Hg and 273.16°K.

Train Your Brain

Example 7: Express a temperature of 60°F in degrees Celsius and in Kelvin.

Sol. Substituting $TF = 60°F$ in Eq. (ii),

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(60° - 32°) = 15.55°\text{ C}$$

From Eq. (i), $T = TC + 273.15 = 15.55°C + 273.15 = 288.7\text{ K}$

Example 8: What is the temperature which has the same numerical value in centigrade scale and Fahrenheit Scale?

Sol. Let x be the required temperature

$$\text{Now } \frac{x-0}{100-0} = \frac{x-32}{212-32}$$

$$\text{or } \frac{x}{100} = \frac{x-32}{180} \quad (or) \quad \frac{x}{5} = \frac{x-32}{9}$$

$9x = 5x - 160$; $4x = -160$; $x = -40$ $\therefore$ $-40°C$

Example 9: A constant volume gas thermometer shows pressure reading of 50 cm and 90 cm of mercury at 0°C and 100°C respectively. When the pressure reading is 60 cm of mercury, the temperature is

(*a*) 25°C (*b*) 40°C
(*c*) 15°C (*d*) 12.5°C

Sol (*a*) $t = \left(\dfrac{P_t - P_0}{P_{100} - P_0}\right) \times 100°C$

$$= \left(\frac{60-50}{90-50}\right) \times 100 = 25°C$$

Concept Application

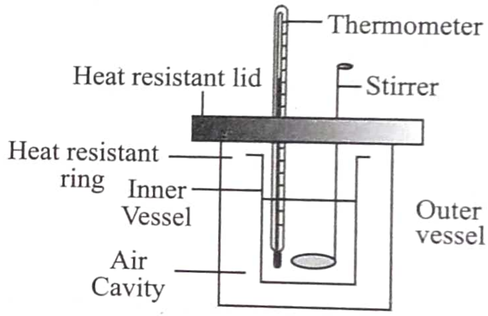
5. A thermometer has its lower and upper fixed points marked as 10° and 80°. When it reads 38° what is the corresponding temperature on Centigrade scale?
(*a*) 40°C (*b*) 30°C
(*c*) 45°C (*d*) 35°C

6. On the Celsius scale the absolute zero or temperature is at
(*a*) 0°C (*b*) –32°C
(*c*) 100°C (*d*) –273.15°C

7. On centigrade scale the temperature of a body increases by 30 degrees. The increase in temperature on Fahrenheit scale is
(*a*) 50° (*b*) 40°
(*c*) 30° (*d*) 54°

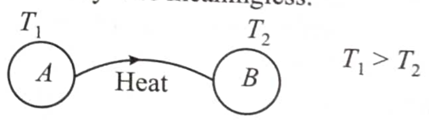
CALORIMETRY

The act or science of measuring the changes in the state variables of a body in order to calculate the heat transfer associated with changes in its states, such as physical changes or phase transitions under specific conditions, is known as calorimetry. Calorimetry is performed with the help of a calorimeter. Calorimeter is a setup which does not allow heat exchange with surrounding.



Heat

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of receiving body. It should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of body" are meaningless.



When we say that a body is heated it means that its molcules begin to move with greater kinetic energy.

S.I. unit of heat energy is joule (*J*). Another practical unit of heat energy is calorie (cal).

MECHANICAL EQUIVALENT OF HEAT

In early days heat was not recognised as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiment were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write,

$$W = JH$$

where J is called mechanical equivalent of heat. J is expressed in joule/calories. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

1 calorie: The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 °C at one atmospheric pressure is 1 calorie.

1 calorie = 4.186 Joule

SPECIFIC HEAT

Specific heat of substances is equal to heat gain by that substance to raise its temperature by 1°C for a unit mass of substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to:

(*a*) the mass of the body $\Delta Q \propto m$

(*b*) rise or fall of temperature of the body $\Delta Q \propto \Delta T$

$\Delta Q \propto m\,\Delta T$

or $\Delta Q = m\,s\,\Delta T$ or $dQ = m\,s\,d\,T$ or $Q = m\int s\,dT$

where s is a constant and is known as the specific heat of the body $s = \dfrac{Q}{m\Delta T}$.

S.I. unit of s is joule/kg-kelvin and C.G.S unit is cal/gm °C

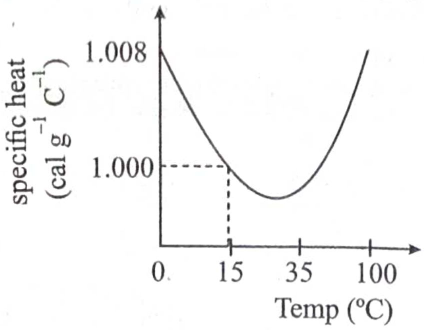
Specific heat of water: s = 1 cal/gm°C

Specific heat of ice: s = 0.5 cal/gm°C

Specific heat of steam: s = 0.46 cal/gm°C ≈ 0.5 cal/g,°C

Important Points

- We know, $s = \dfrac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature (ΔT = 0), and constant pressure (ΔP = 0) then $s = Q/0 = \infty$. Thus the specific heat of a substance when it melts or boils at constant temperature is infinite.
- If the temperature of the substance changes without the transfer of heat (Q = 0) then $s = \dfrac{Q}{m\Delta T} = 0$. Thus when liquid in the Thermos flask is shaken, its temperature increases without the transfer of heat and hence and the specific heat of liquid in the Thermos flask is zero.
- To raise the temperature of saturated water vapour, heat (Q) is withdrawn. Hence, specific heat of saturated water vapour is negative. (This is for your information only and not in the course)
- The slight variation of specific heat of water with temperature is shown in the graph at one atmosphere pressure. Its variation is less than 1% over the interval form 0 to 100°C.



Heat Capacity or Thermal Capacity

Heat capacity of a body is defined as the amount of heat required to rasie the temperature of that body by 1°C. If 'm' is the mass and 's' the specific heat of the body, then

Heat capaicty = m s.

Units of heat capacity in: CGS system is, cal °C^{-1}; SI unit is JK^{-1}

Relation between Specific Heat and Water Equivalent

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$ms\,\Delta T = m_w\,sw\,\Delta T \Rightarrow m_w = \dfrac{ms}{s_w}$

In calorie $sw = 1$

$\therefore\ m_w = ms$

m_w is also represented by W

so $W = ms$

Law of Mixture (When phase of substance does not change)

When two substances at different temperatures are mixed together the exchange of heat continues to take place till their temperature become equal. This temperature is then called final temperature of mixture. Here,

Heat taken by one substance = Heat given by another substance

$\Rightarrow m_1s_1(T_1 - T_m) = m_2s_2(T_m - T_2)$

Train Your Brain

Example 10: Heat required to increases the temperature of 1 kg water by 20°C

Sol. heat required = $\Delta Q = ms\Delta\theta$

$= 1 \times 20 = 20$ Kcal.

$\therefore$ S = 1 cal/gm°C = 1 Kcal/kg°C

Example 11: An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it loses 25% energy to surroundings. Then find the temperature rise of the block (Take sp. heat of iron 470 J/kg°C)

Sol. $mS\Delta\theta = \frac{1}{4}mgh \Rightarrow \Delta\theta = \frac{10\times 10}{4\times 470} = 0.053$

Example 12: The temperature of equal masses of three different liquids A, B, and C are 10°C 15°C and 20°C respectively. The temperature when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?

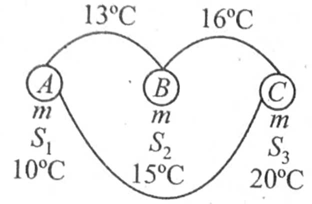
Sol. When A and B are mixed

$mS_1 \times (13-10) = m \times S_2 \times (15-13)$

$3S_1 = 2S_2$ ...(1)

When B and C are mixed

$S_2 \times 1 = S_3 \times 4$ ...(2)



When C and A are mixed

$S_1(\theta - 10) = S_3 \times (20-\theta)$ ...(3)

by using equation (1), (2) and (3)

We get $\theta = \frac{140}{11}$°C

Example 13: If three different liquid of different masses, specific heats and temperature are mixed with each other, then what is the temperature mixture at thermal equilibrium.

$m_1, s_1, T_1 \rightarrow$ specification for 1st liquid

$m_2, s_2, T_2 \rightarrow$ specification for 2nd liquid

$m_3, s_3, T_3 \rightarrow$ specification for 3rd liquid

Sol. Total heat lost or gain by all substance is equal to zero

$\Delta Q = 0$

$m_1s_1(T-T_1) + m_2s_2(T-T_2) + m_3s_3(T-T_3) = 0$

So $T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$

Concept Application

8. When a hot liquid is mixed with a cold liquid, the temperature of the mixture –
(a) First decrease then becomes constant.
(b) First increases then become constant.
(c) Continuously increases.
(d) Is undefined for some time and then becomes nearly constant.

9. Utensils used for efficient cooking should have-
(a) Large heat capacity
(b) Small heat capacity
(c) Medium heat capacity
(d) Any heat capacity

10. An electric heater of power 1000W raises the temperature of 5 kg of a liquid from 25°C to 31°C in 2 minutes. Heat capacity of the liquid is –
(a) 2×10^4 J/°C (b) 1×10^4 J/°C
(c) 3×10^4 J/°C (d) 4×10^4 J/°C

11. The water equivalent of a copper calorimeter of mass 400 g (specific heat = 0.1 cal/g°C) is
(a) 40 g (b) 4000 g
(c) 200 g (d) 4 g

PHASE CHANGE

Heat required for the change of phase or state, at constant temperature and pressure is

$$Q = mL, L = \text{latent heat.}$$

(a) **Latent heat (L):** The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

(b) **Latent heat of Fusion (L_f):** The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion.

(c) **Latent heat of vaporisation (L_v):** The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization.

If in question latent heat of water are not mentioned and to solve the problem it require to assume that we should consider following values.

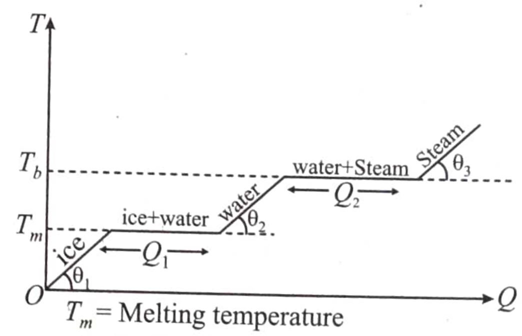
Latent heat of ice:

$L = 80$ cal/gm = 80 Kcal/kg = 4200 × 80 J/kg

Latent heat of steam:

$L = 540$ cal/gm = 540 Kcal/kg = 4200 × 540 J/kg

The given figure, represents the change of state by different lines



T_m = Melting temperature

T_b = Boiling temperature

If heat is supplied at constant rate

$dQ = msdT$

$$\text{Slope} = \frac{dT}{dQ} = \frac{1}{ms}$$

Since $s_{ice} = s_{steam} = 0.5$ cal/g°C and $s_{water} = 1$ cal/g°C

$\Rightarrow \theta_1 = \theta_3 > \theta_2$

and $Q_2 > Q_1$

Train Your Brain

Example 14: Find amount of heat released if 100 g ice at –10°C is converted into 120°C, 100 g steam.

Sol. – 10° C, 100 gm ice ⟶ 120° 100 gm steam

$Q = ms\Delta T$

$= \frac{1}{2} \times 100 \times 10$

$= 500$ cal.

0°, 100 gm ice

$Q = mL_f$

$= 100 \times 80$

$= 8000$ cal

0°, 100 gm water

$Q = ms\Delta T$

$= 1 \times 100 \times 100$

$= 10000$ cal.

100°, 100 gm water

$Q = mL_v$

$= 100 \times 540$

$= 54000$ cal

100°, 100 gm Steam

$Q = ms\Delta T$

$= 0.46 \times 100 \times 20$

$= 920$ cal.

$Q_{net} = 73.5$ Kcal.

$Q_{net} = 73.5$ Kcal.

Example 15: 500 gm of water at 80°C is mixed with 100 gm steam at 120°C. Find out the final mixture.

Sol. 120°C steam ⟶ 100°C steam

Req. heat $= 100 \times 0.46 \times 20 = 920$ cal

80°C water ⟶ 100°C water

Req. heat $= 500 \times 1 \times 20 = 10$ kcal

100gm steam ⟶ 100 gm water at 100°C

Req. heat $= 100 \times 540 = 54$ kcal

Total heat = 55 kcal.

Remaining heat = 55 – 10 = 45 kcal

Now we have 600 gm water at 100°C

$$\Rightarrow 4500 = m \times 540 \Rightarrow m = \frac{250}{3}\text{gm}$$

So at last we have $\frac{250}{3}$ gm steam and $\left(600 - \frac{250}{3}\right)$gm of water

Example 16: How should one kg of water at 10°C be so divided that one part of it when converted into ice at 0°C, would by this change of state provide a quantity of heat that would be sufficient to vaporise the other part?

Sol. Initially 1000 g of water is at 10°C.

Let m gram of it be cooled to ice at 0°C.

Heat released due to this $= (m \times 1 \times 10) + (m \times 80)$

$= 10$ m $+ 80$ m $= 90$ m cal.

The heat required by $(1000 - m)$ g of water at 10°C to become steam at 100°C

$= (1000 - m)(100 - 10) + (1000 - m)$ 540 cal

$= (1000 - m)(90 + 540)$ cal

$= (1000 - m)(630)$ cal

Now, 90 m = (1000 – m) 630 or, 720 m = 630 × 1000

$$m = \frac{630 \times 1000}{720} = 875 \text{ g}$$

Hence 875 g of water by turning into at 0°C will supply heat to evaporate 125 g of water.

Example 17: The specific heat of a substance is given by $C = a + bT$, where $a = 1.12$ kJ kg^{-1}c^{-1} and $b = 0.016$ kJ – kg $c^{-1}k^{-1}$. The amount of heat required to raise the temperature of 1.2 kg of the material from 280 K to 312 K is:

(*a*) 205 kJ (*b*) 215 kJ

(*c*) 225 kJ (*d*) 235 kJ

Sol. (*c*) Heat required

$$= Q = \int_{280}^{312} mCdT = \int_{280}^{312} (1.2)(a + bT)dT$$

$$= 1.2\left[aT + \frac{bT^2}{2}\right]_{280}^{312}$$

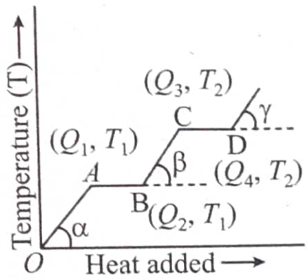
$$= 1.2\,[1.12\,(312 - 280) + \frac{0.016}{2}\{(312)^2 - (280)^2\}]$$

$$= 1.2\,[1.12 \times 32 + \frac{0.016}{2} \times (97344 - 78400)]$$

$$= 1.2\,[35.84 + 0.008 \times 18944]$$

$$= 224.8704 \text{ KJ} = 225\text{KJ}$$

Example 18: The accompanying graph shows the variation of temperature (T) of one kilogram material with Heat (Q) supplied to it. At O, the substance is in solid state. Which of the following interpretation from the graph is correct –



(*a*) T_2 is the melting point of the solid

(*b*) BC represents the change of state from solid to liquid.

(*c*) $(Q_2 - Q_1)$ represent the latent heat of fusion of the substance.

(*d*) $(Q_3 - Q_1)$ represents the latent heat of vaporisation of the liquid.

Sol. (*c*) The graph suggests the following processes:

Substance in solid state
(below melting point)
↓
Substance at its melting point
↓
Substance in liquid state at melting point
↓
Substance at its boiling point
↓
Substance at its boiling point in vapour state
↓
Substance at some higher temperature in vapour state

Clearly T_2 is boiling point of substance. BC represents the increase in temperature remaining in same liquid state. $Q_3 - Q_1$ represent heat required to change temperature of substance from its melting point to its boiling point. But $Q_1 - Q_2$ represents heat released when substance changes from liquid state to solid state at its melting point, i.e. latent heat of fusion

Concept Application

12. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on large ice block. The maximum amount of ice that can melt is (Specific heat of copper = 0.39 $Jg^{-1}\,°C^{-1}$, latent heat of fusion of water = 335 Jg^{-1})

(*a*) 1.2 kg (*b*) 1.455 kg
(*c*) 1 kg (*d*) 2.5 kg

13. When vapour condenses into liquid:

(*a*) It absorbs heat
(*b*) It liberates heat
(*c*) Its temperature increases
(*d*) Its temperature decreases

14. Heat required to convert one gram of ice at 0°C into steam at 100°C is (given L_{steam} = 536 cal/gm, L_{ice} = 80 cal/g):

(*a*) 100 calorie (*b*) 0.01 kilocalorie
(*c*) 716 calorie (*d*) 1 kilocalorie

15. A lead ball at 30°C is dropped from a height of 6.2 km. The ball is heated due to the air resistance and it completely melts just before reaching the ground. The molten substance falls slowly on the ground. Calculate the latent heat of fusion of lead. Specific heat capacity of lead = 126 J/kg°C and melting point of lead = 330°C. Assume that any mechanical energy lost is used to heat the ball. Use $g = 10\ m/s^2$.

(*a*) 24 kJ/kg
(*b*) 20 kJ/kg
(*c*) 28 kJ/kg
(*d*) 26 kJ/kg

HEAT TRANSFER

Heat is energy in transit which flows due to temperature difference; from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three types.

(*i*) Conduction
(*ii*) Convection
(*iii*) Radiation

(*a*) **CONDUCTION**

Transfer of heat through a substance in which heat is transferred without direct mass transfer is called conduction. For example : Heat transfer in solid

(*i*) Requires Medium to propagate the heat
(*ii*) Energy is transmitted from one particle to another particle because of thermal collision.
(*iii*) No transfer of particle from their mean position.

(*b*) **CONVECTION**

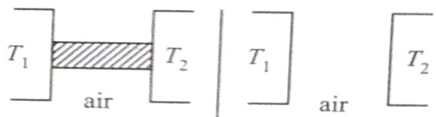
When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection. For example : Heat transfer in fluid (liquid and gas)

(*i*) Requires Medium to propagate the heat
(*ii*) Energy is transferred through movement of the particles of the medium.

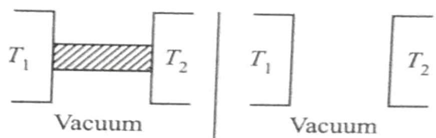
(*c*) **RADIATION**

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. For example : Transfer of heat from Sun to Earth.

(*i*) Does not require any medium
(*ii*) Energy is transferred through Electromagnetic waves.



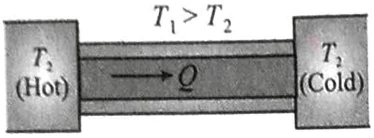
Conduction	✓	×
Correction	✓	✓
Radiation	✓	✓



Conduction	✓	×
Correction	×	×
Radiation	✓	✓

CONDUCTION

Figure shows a rod whose ends are in thermal contact with a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2. The sides of the rod are covered with insulating medium, so the transport of heat is along the rod, not through the sides. The molecules at the hot reservoir have greater vibrational energy. This energy is transferred by collisions to the atoms at the end face of the rod. These atoms transfer energy to their neighbours further along the rod. Such transfer of heat through a substance in which heat is transferred without direct mass transfer is called conduction.



The free electrons in metals, which move throughout the metal can rapidly carry energy from the hotter to cooler regions, so metals are generally good conductors of heat. The presence of 'free' electrons also causes most metals to be good electrical conductors. A metal rod at 5°C feels colder than a piece of wood at 5°C because heat can flow more easily from your hand into the metal.

Heat transfer occurs only between regions that are at different temperatures, and the rate of heat flow is $\frac{dQ}{dt}$. This rate is also called the heat current, denoted by H. Experiments show that the heat current is proportional to the cross-section area A of the rod and to the temperature gradient $\frac{dT}{dx}$, which is the rate of change of temperature with distance along the bar. In general

$$H = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

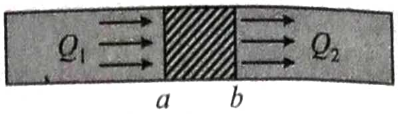
The negative sign is used to make $\frac{dQ}{dt}$ a positive quantity since $\frac{dT}{dt}$ is negative. The constant k, called the thermal conductivity is a measure of the ability of a material to conduct heat.

A substance with a large thermal conductivity k is a good heat conductor. The value of k depends on the temperature, increasing slightly with increasing temperature, but k can be taken to be practically constant throughout a substance if the temperature difference between its ends is not too great.

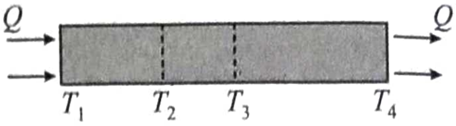
Important Points in Conduction

1. Consider a section ab of a rod as shown in figure. Suppose Q_1 heat enters into the section at 'a' and Q_2 leaves at 'b', then $Q_2 < Q_1$. Part of the energy $Q_1 - Q_2$ is utilized in raising the tempeature of section ab and the remaining is lost to atmosphere through ab. This is called **transient state.**

   If heat is continuously supplied from the left end of the rod, a stage comes when temperature of the section becomes constant. In that case, $Q_1 = Q_2$ if rod is insulated from the surroundings (or loss thorugh ab is zero). This is called the **steady state** condition. Thus, in steady state temperature of different sections of the rod becomes constant (but not uniform).



transient state



Insulated rod in steady state

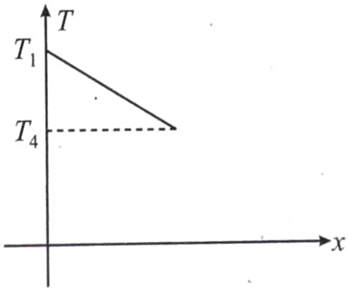
T_1 = constant, T_2 = constant etc.

and $T_1 > T_2 > T_3 > T_4$

Now, a natural question arises, why the temperature of whole rod not becomes equal when heat is being continuously supplied?

Here in the rod, there must be a temperature difference in the rod for the heat flow, same as we require a potential difference across a resistance for the current to flow through it.

In steady state, the temperature varies linearly with distance along the rod of uniform cross-section if it is insulated.



2. A rod of length L and constant cross sectional area A in which a steady state has been reached. In a steady state the temperature at each point is constant in time. Hence.

$$-\frac{dT}{dx} = \frac{T_1 - T_2}{L}$$

Therefore, the heat ΔQ transferred in time Δt is

$$\Delta Q = kA\left(\frac{T_1 - T_2}{L}\right)\Delta t$$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta T}{R}$$

Here, ΔT = temperature difference (TD) and $R = \frac{L}{kA}$ = thermal resistance of the rod.

We find the following similarities in heat flow through a rod and current flow through a resistance.

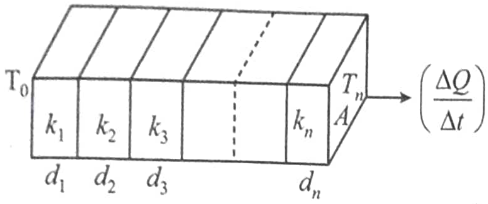
Heat flow through a conducting rod	Current flow through a resistance
Heat current $H = \frac{dQ}{dt}$ = rate of heat flow	Electric current $i = \frac{dq}{dt}$ = rate of charge flow
$H = \frac{\Delta T}{R}$	$i = \frac{\Delta V}{R}$
$R = \frac{L}{kA}$	$R = \frac{L}{\sigma A}$
k = thermal conductivity	s = electrical conductivity

From the above table it is evident that flow of heat through rods in series and parallel is analogous to the flow of current through resistances in series and parallel. This analogy is of great importance in solving complicated problems of heat conduction.

COMBINATION OF LAYERS

Layers in Series

Let us consider a multilayer medium consisted of layers having area of cross-section A each and length d_1, d_2, ... d_n as shown. The thermal conductivities of various layers are K_1, K_2 ..., K_n. If a temperature difference of magnitude $T_0 - T_n$ is maintained between the near and far faces of the multiplayer (where T_0 and T_n are the temperatures of the near and far faces respectively), The rate of heat flow is given by



$$\frac{\Delta Q}{\Delta t}=\frac{A(T_0-T_n)}{\frac{d_1}{k_1}+\frac{d_2}{k_2}+...+\frac{d_n}{k_n}}$$

which can be obtained directly using the concept of thermal resistance

Effective thermal Conductivity in Series Combination (Area will be uniform)

$$\frac{\Delta\alpha}{\Delta t}=\frac{A(T_0-T_n)}{\frac{d_1}{k_1}+\frac{d_2}{k_2}+...+\frac{d_n}{k_n}}=k_{eq}\frac{A(T_0-T_n)}{d}=\frac{A(T_0-T_n)}{\frac{d}{k_{eq}}}$$

$$\frac{d_1}{k_1}+\frac{d_2}{k_2}+...+\frac{d_n}{k_n}=\frac{d}{k_{eq}}$$

$$k_{eq}=\frac{d}{\frac{d_1}{k_1}+\frac{d_2}{k_2}+\frac{d_3}{k_3}+...\frac{d_n}{k_n}}$$

Alternate Method :

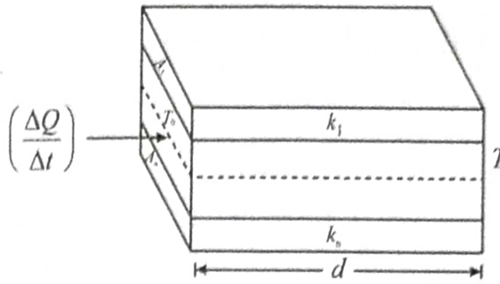
$$Req = R_1 + R_2 + R_3 + ... Rn$$

$$\frac{d}{k_{eq}A}=\frac{d_1}{k_1A}+\frac{d_2}{k_2A}+...\frac{d_n}{k_nA}$$

$$k_{eq}=\frac{d}{\frac{d_1}{k_1}+\frac{d_2}{k_2}+\frac{d_3}{k_3}+...\frac{d_n}{k_n}}$$

Layers in Parallel

Now let us consider a multilayer, this time having same width d but different areas of cross-section A_1, A_2, ... An as shown. The thermal conductivities of the various layers are k_1, k_2, ... k_n. If a temperature difference of magnitude $(T_0 - T_n)$ is maintained between the near and far faces of the multiplayer (where T_0 and Tn are the temperatures of near and far faces respectively, the rate of heat flow is given by



$$\frac{\Delta Q}{\Delta t}=\frac{(T_0-T_n)(k_1A_1+k_2A_2+....+k_nA_n)}{d}$$

which also can be obtained directly using the concept of thermal resistance.

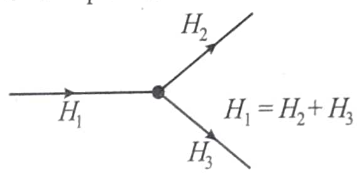
Effective thermal Conductivity in Parallel Combination : [length will be uniform]

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+...\frac{1}{R_n}\ ;\ \frac{l}{k_{eq}A}=\frac{l}{k_1A}+\frac{l}{k_2A_2}+\frac{l}{k_3A_3}+...\frac{l}{k_4A_4}$$

$$k_{eq}=\frac{\frac{1}{A}}{\frac{1}{k_1A_1}+\frac{1}{k_2A_2}+...\frac{1}{k_nA_n}}$$

Junction Law

According to the Junction law the sum of all the heat current directed towards a point is equal to the sum of all the heat currents directed away from the points.



Cooling by Conduction

A body P of mass m and specific heat c is connected to a large body Q (of infinite heat capacity) through a rod of length l, thermal conductivity K and area of cross-section A. Temperature of Q is T_0 $(< T_i)$. This temperature will remain constant as its heat capacity is infinite. Heat will flow from P to Q through the rod. If we neglect the loss of heat due to radiation then due to this heat transfer, temperature of P will decrease but temperature of Q will remain almost constant. At time t, suppose temperature of P becomes T then due to temperature difference heat transfer through the rod.

$$\frac{dQ}{dt}=H=\frac{\Delta T}{R}=\frac{T-T_0}{R} \qquad ...(i)$$

Here, $R=\frac{l}{KA}$

Now, it we apply equation of calorimetry in P, then

$$Q=mc(-\Delta T) \text{ or } \frac{dQ}{dt}=mc\left(\frac{-dT}{dt}\right) \qquad ...(ii)$$

Equating Equation (i) and (ii), we have

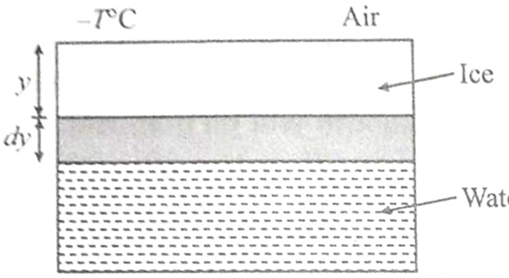
$$-\frac{dT}{dt}=\frac{\Delta T}{mcR}=\frac{T-T_0}{mcR}=\text{Rate of cooling} \qquad ...(iii)$$

$$\text{or } -\frac{dT}{dt}\propto\Delta T \qquad ...(iv)$$

GROWTH OF ICE ON LAKE

When the temperature of the air is less than 0°C, the cold air near the surface of the pond takes heat (latent) from the water which freezes it and a layer of ice is formed.

Consequently, the thickness of the ice layer keeps increasing with time. Let y be the thickness of ice layer in the lake at any instant t and atmospheric temperature be $-T$°C. If the thickness is increased by dy in time dt, then the amount of heat flowing through the slab in time dt is given by $dQ_2 = mL = \rho(dt\ A)L$ (see figure).



The temperature of water in contact with lower surface of ice will be zero. If A is the area of lake, heat escaping through ice in time dt,

$$dQ_1 = \frac{KA[0-(-T)]dt}{y}$$

As $dQ_1 = dQ_2$, hence, rate of growth of ice will be $\frac{dy}{dt} = \frac{kT}{\rho L y}$

So, the time taken by ice to grow to a thickness y is

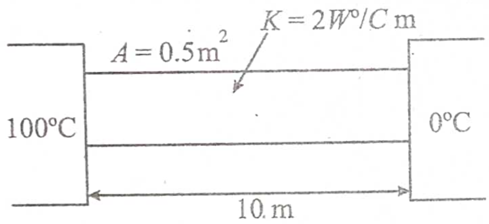
$$t = \frac{\rho L}{kT}\int_0^y y\,dy = \frac{\rho L}{2KT}y^2$$

If the thickness is increased from y_1 to y_2 then time taken,

$$t = \frac{\rho L}{kT}\int_{y_1}^{y_2} y\,dy = \frac{\rho L}{2KT}(y_2^2 - y_1^2)$$

Train Your Brain

Example 19:

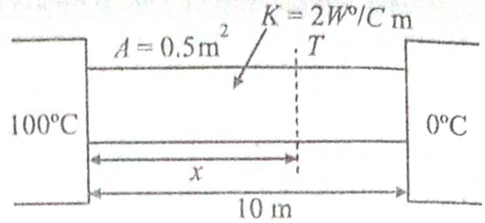


Find out the heat current and temperature at any distance x.

Sol. $R = \frac{L}{KA} = \frac{10}{2\times 0.5} = 10$

$$i = \frac{100}{10}$$

temperature at any distance x.



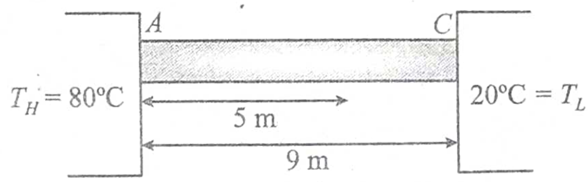
$$\Delta Q = \frac{KA(100-T)}{x} = \frac{KA\Delta T}{\ell}$$

$$\Rightarrow \frac{(100-T)}{x} = \frac{(100-0)}{\ell}$$

$$100\,\ell - T\,\ell = 100\,x$$

$$T = \frac{100(\ell - x)}{\ell}$$

Example 20:

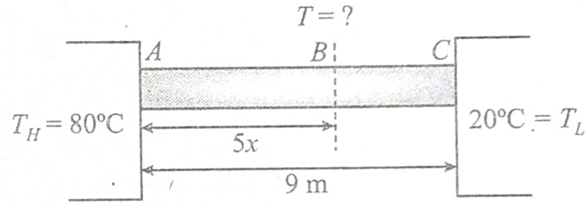


Find out the temperature at distance 5 m.

Sol. Heat current is same. so,

$$\frac{T_H - T_L}{\ell} = \frac{T_H - T}{x} \Rightarrow \frac{80-20}{9} = \frac{80-T}{5}$$

$$T = ?$$



$$T = \frac{140}{3}\,°C$$

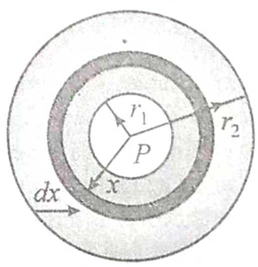
Example 21: Two thin concentric shells made from copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures TH and TC respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P.

Sol. Heat flowing per second through each cross-section of the sphere = i = heat current

Thermal resistance of the

Spherical shell of radius

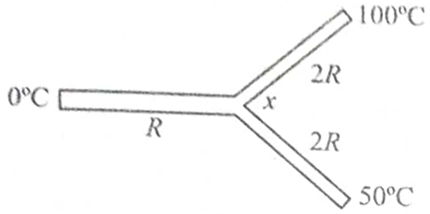
x and thickness dx, $dR = \frac{dx}{K.4\pi x^2}$



$$\Rightarrow R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2 . K} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \text{ thermal current}$$

$$i = \frac{T_H - T_C}{R} = \frac{4\pi K (T_H - T_C) r_1 r_2}{(r_2 - r_1)}$$

Example 22:

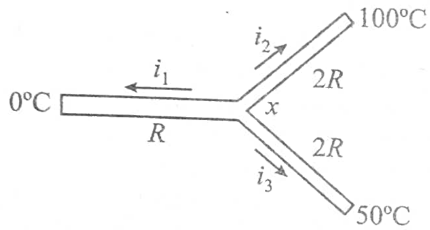


Find out the temperature at point x.

Sol. As per junction Law

$$i_1 + i_2 + i_3 = 0$$

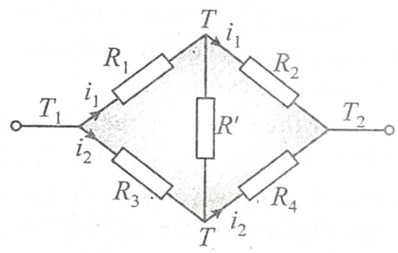
$$i_1 = \frac{(x-0)}{R}, i_2 = \frac{(x-100)}{2R}, i_3 = \frac{(x-50)}{2R}$$



$4x = 150 \quad \Rightarrow x = 37.5°C$

Wheatstone Bridge

Example 23: Find out the relation between R_1, R_2, R_3 and R_4, so there is no that current in R' and also find the expression of heat current.



Sol. $\Delta T = T_1 - T_2$

$T_1 - T = i_1 R_1$ ...(i)

$T_1 - T = i_2 R_3$ ...(ii)

Eq. (i)/(ii)

$\Rightarrow \frac{i_1 R_1}{i_2 R_3} = 1$ ...(iii)

$T - T_2 = i_1 R_2$ ...(iv)

$T - T_2 = i_2 R_4$ ...(v)

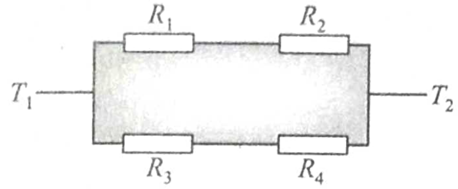
Eq. (iv)/eq. (v)

$i_1 R_2 = i_2 R_4$ ...(vi)

Eq. (iii)/(vi)

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \Rightarrow R_1 R_4 = R_2 R_3$$

Now Equivalent circuit of wheatstone bridge can be represented as follow

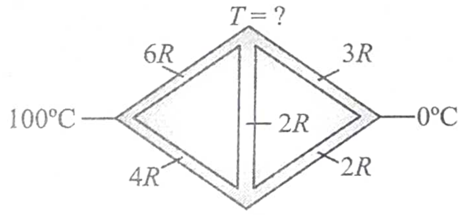


$$R_{eq} = \frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2) + (R_3 + R_4)}$$

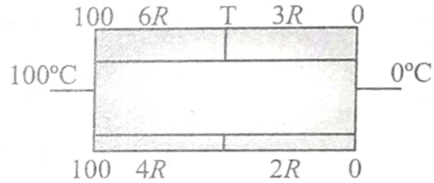
Now

$$\frac{dQ}{dt} = i = \frac{T_1 - T_2}{R_{eq}}$$

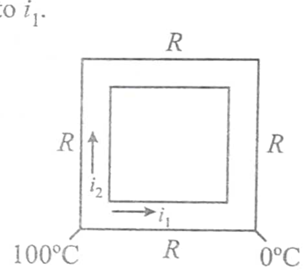
Example 24:



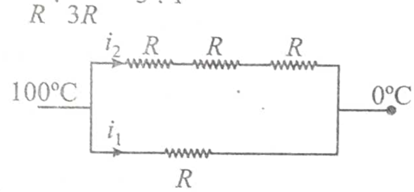
Sol. $\frac{100 - T}{6R} = \frac{T - 0}{3R} \quad \Rightarrow T = 100/3°C$



Example 25: If some rod circuit as shown in figure then find ratio of i_2 to i_1.



Sol. $i_1 : i_2 = \frac{1}{R} : \frac{1}{3R} = 3 : 1$



$$i_1 = \frac{3}{4} \times 100 = 75$$

$$\Rightarrow i_2 = \frac{1}{4} \times 100 = 25$$

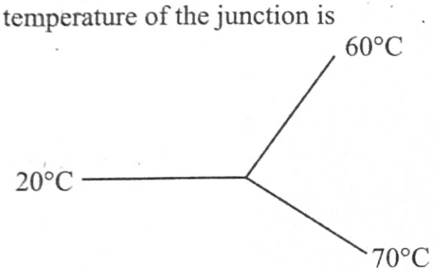
Concept Application

16. A body of length 1 m having cross-sectional area $0.75 \, m^2$ has heat flow through it at the rate of 6000 J/s. The temperature difference between two ends of conductor if $K = 200 \, J \, m^{-1} K^{-1}$ is
(a) 20°C (b) 40°C
(c) 80°C (d) 100°C

17. A cylindrical rod has temperature T_1 and T_2 at its ends. The rate of flow of heat is Q_1 cal/s. If all the linear dimensions are doubled keeping temperature constant then rate of flow of heat Q_2 will be
(a) $4Q_1$ (b) $2Q_1$
(c) $\frac{Q_1}{4}$ (d) $\frac{Q_1}{2}$

18. The temperature gradient in a rod of 0.5 m long is 80°C/m. If the temperature of hotter end of the rod is 30°C, then the temperature of the colder end is
(a) 40°C (b) –10°C
(c) 10°C (d) 0°C

19. Three identical thermal conductors are connected as shown in figure. Consider no heat lost due to radiation, the temperature of the junction is

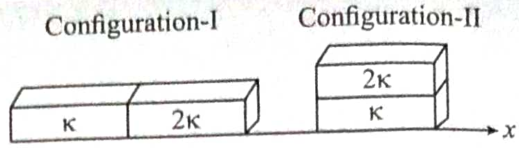


(a) 60°C (b) 20°C
(c) 50° (d) 10°C

20. A slab consists of two parallel layers of two different materials of same thickness having thermal conductivities K_1 and K_2. The equivalent conductivity of the combination is
(a) $K_1 + K_2$ (b) $\frac{K_1 + K_2}{2}$
(c) $\frac{2K_1K_2}{K_1 + K_2}$ (d) $\frac{K_1 + K_2}{2K_1K_2}$

21. The ratio of thermal conductivity of two rods of different material is 5 : 4. The two rods of same area of cross-section and same thermal resistance will have the length in the ratio
(a) 4 : 5 (b) 9 : 1
(c) 1 : 9 (d) 5 : 4

22. Two rectangular blocks, having identical dimensions, can be arranged either in configuration-I or in configuration-II as shown in the figure.



(a) 2.0 s (b) 3.0 s
(c) 4.5 s (d) 6.0 s

23. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is
(a) $K_1 + K_2$ (b) $K_1K_2/(K_1 + K_2)$
(c) $(K_1 + 3K_2)/4$ (d) $(3K_1 + K_2)/4$

CONVECTION

When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection. Convection occurs through the aid of earth's gravity. Normally the portion of fluid at greater temperature is less dense, while that at lower temperature is denser. Hence hot fluid rises up while colder fluid sink down, accounting for convection. In the absence of gravity convection would not be possible.

Also, the anomalous behaviour of water (its density increases with temperature in the range 0-4°C) give rise to interesting consequences the process of convection. One of these interesting consequences is the presence of aquatic life in temperate and polar waters. The other is the rain cycle.

Can you now see how the following facts can be explained by thermal convection?

(a) Oceans freeze top to down and not bottom to up. (this fact is singularly responsible for presence of aquatic life is temperate and polar waters.)
(b) The temperature in the bottom of deep oceans is invariably 4°C, whether it is winter or summer.
(c) You cannot illuminate the interior of a lift in free fall or an artificial satellite of earth with a candle.
(d) You can Illuminate your room with a candle.

RADIATION

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Properties of Radiation

(a) All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of the radiation that falls on them from other objects.

(b) Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.

(c) More radiations are emitted at higher temperature of a body and lesser at lower temperature.

(d) The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing. Radiations from a body at NTP has predominantly infrared waves.

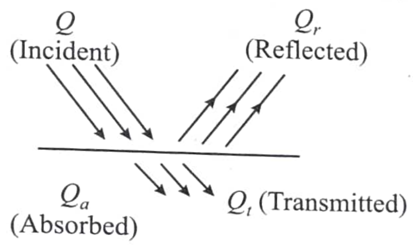
(e) Thermal radiations travels with the speed of light and move in a straight line.

(f) Radiations are electromagnetic waves and can also travel through vacuum.

(g) Similar to light, thermal radiations can be reflected, refracted, diffracted and polarized.

(h) Radiation from a point source obeys inverse square law (intensity $\alpha \frac{1}{r^2}$).

Absorption, Reflection and Emission of Radiations



$Q = Qr + Qt + Qa$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$1 = r + t + a$

where r = reflecting power, a = absorptive power and t = transmission power.

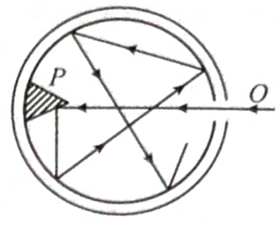
(i) $r = 0, t = 0, a = 1$, perfect black body

(ii) $r = 1, t = 0, a = 0$, perfect reflector

(iii) $r = 0, t = 1, a = 0$, perfect transmitter

Perfectly Black Body and Black Body Radiation (Fery's black body)

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.



In actual practice, no natural object possesses strictly the properties of a perfectly black body except sun. Sun is only one natural object which is perfect black body. But the lamp-black and platinum black are good approximation of black body. They absorb about 99% of the incident radiation. The most simple and commonly used black body was designed by Ferry. It consists of an enclosure with a small opening which is painted black from inside. The opening acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little chance of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.

Absorptive Power

In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As all the radiations incident on a black body are absorbed, $a = 1$ for a black body.

Emissive power (E)

Energy radiated per unit time per unit area per unit solid angle is known as emissive power.

$$E = \frac{Q}{(\Delta A)(\Delta t)(\Delta \omega)}$$

(Notice that unlike absorptive power, emissive power is not a dimensionless quantity).

Spectral Emissive Power (E_λ)

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, E_λ. If E is the total emissive power and E_λ is spectral emissive power, they are related as follows,

$$E = \int_0^\infty E_\lambda \, d\lambda \text{ and Emissive Power } (E_\lambda) = \frac{dE}{d\lambda}$$

Emissivity

$$e = \frac{\text{Emissive power of a body at temperature } T}{\text{Emissive power of a black body at same temperature } T}$$

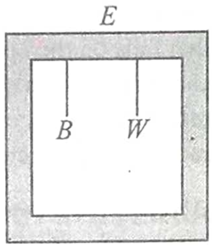
$$= \frac{E}{E_0}.$$

Mathematically absorptive power (a) + emissivity (e) are same but conceptually they are different.

Prevost Theory of Heat Exchange

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings it radiates at the same rate as it absorbs.

KIRCHHOFF'S LAW



One of the most important conclusions of the Prevost's theory of heat exchanges is the Kirchhoff's law. Let us try to arrive this law first qualitatively and then quantitatively. Let us suspend two balls, one black (B) and the other white (W) inside an enclosure E, with the help of insulating the threads as shown in the figure. The chamber is maintained at a constant temperature T. The walls of the chamber are continuously emitting and absorbing radiation. Due to the exchange of radiation with the walls of the chamber (or with themselves), the balls will also attain the temperature will remain constant as long as the temperature of the enclosure does not change.

We know that the black ball is a good absorber and as such it absorbs more radiation than the white ball. How are then their temperatures equal? It is possible only if the black ball also radiates more heat than the white ball. Thus, a good absorber (black ball) is a good emitter and vice-versa. It was Kirchhoff, who derived the exact quantities form of this statement which after him is known as Kirchhoff's law.

According to this law, the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature. Hence,

$$\frac{e_1}{a_1} = \frac{e_2}{a_2} = \ldots = \left(\frac{E}{A}\right)_{\text{Perfectly black body}}$$

But for perfectly black body, $A = 1$, i.e., $\frac{e}{a} = E$

If emissive and absorptive powers are considered for a particular wavelength λ,

$$\left(\frac{e_\lambda}{a_\lambda}\right) = (E_\lambda)_{\text{black}}$$

Now since $(E_l)_{\text{black}}$ is constant at a given temperature, according to this law, if a surface is a good absorber of a particular wavelength, then it is also a good emitter of that wavelength. This in turn implies that a good absorber is a good emitter (or radiator).

The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of a perfectly black body for the same wavelength and temperature.

$$\frac{\text{E(body)}}{\text{a(body)}} = \text{E(black body)}$$

Hence we can conclude that good emitters are also good absorbers.

STEFAN-BOLTZMANN'S LAW

Based on several experimental observations on the rate of emission of radiant heat by surfaces of black bodies, Stefan gave this law which states that 'the emissive power (the rate of emission of radiant energy per unit area) of a perfectly black body varies directly as the fourth power of its absolute temperature.' This is know as Stefan's law.

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$$\boxed{(dQ/dt)_{\text{Radiation}} = \sigma A T^4}$$

A body which in not a black body absorbs and hence emits less radiation then

For such a body,

$$\boxed{(dQ/dt)_{\text{Radiation}} = e\sigma AT^4}$$

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

Similarly

$$\boxed{(dQ/dt)_{\text{absorption}} = \sigma eAT_0^4}$$

T_o = Surrounding Temperature.

Here one thing to keep in mind that radiation depends on body's temperature and absorption on surrounding temperature only.

With the surroundings of temperature T_0, net energy radiated by an area A per unit time.

$$\left(\frac{dQ}{dt}\right)_{\text{net}} = \left(\frac{dQ}{dt}\right)_{\text{Radiation}} - \left(\frac{dQ}{dt}\right)_{\text{Absorption}}$$

$$= \sigma eAT^4 - \sigma eAT_0^4$$

$$\boxed{\left(\frac{dQ}{dt}\right)_{\text{net}} = \sigma eA(T^4 - T_0^4)}$$

where σ is a constant called **Stefan's constant** having **dimension** $[MT^{-3}\theta^{-4}]$ and value 5.67×10^{-8} W/m$^2K^4$.

Important Points

Total radiant energy and power radiated by the ordinary body.

- **Radiant energy :** Let A be the surface area of a body of emissivity e, at an absolute temperature T and Q be the total energy radiated by the ordinary body. Then

$$\frac{dQ}{dt} = \frac{Q}{A \times t} = e\sigma T^4 \Rightarrow Q = A\,e\sigma T^4 t$$

- **Radiant power (P) :** It is defined as energy radiated per unit area, i.e., $P = \frac{Q}{T} = Ae\sigma T^4$.

Important Points

- If two bodies are made of same material, have same surface area and are at the same initial temperature then

$$\frac{dQ}{dt} \propto A \Rightarrow \frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{A_1}{A_2}$$

- Net power radiated by black body, $P_{net} = \sigma A(T^4 - T_0^4)$

For small temperature difference, $\Delta T = T - T_0$

$$P_{net} = \sigma A[(T_0 + \Delta T)^4 - T_0^4]$$

$$= \sigma A T_0^4\left[\left(1+\frac{\Delta T}{T_0}\right)^4 - 1\right] = \sigma A T_0^4\left[\frac{4\Delta T}{T_0}\right]$$

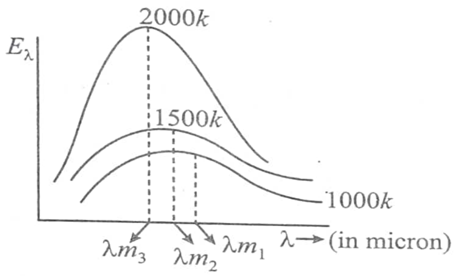
$$P_{net} = 4\sigma A T_0^3(T - T_0) = 4\sigma A T_0^3 \Delta T = 4A\sigma T_0^4\ \frac{\Delta T}{T_0}$$

Nature of Thermal Radiations (Wien's Displacement Law)

From the energy distribution curve of black body radiation, the following conclusions can be drawn:

(a) The higher the temperature of a body, the higher is the area under the curve i.e. more amount of energy is emitted by the body at higher temperature.

(b) The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.



(c) For a given temperature, there is a particular wavelength (λ_m) for which the energy emitted ($E\lambda$) is maximum.

(d) With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.

From the study of energy distribution of black body radiation discussed as above, it was established experimentally that the wavelength (λm) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body. i.e.

$$\lambda m \propto \frac{1}{T} \quad \text{or } \lambda m\ T = b$$

This is called Wien's displacement law.

Here $b = 0.282$ cm-K, is the Wien's constant.

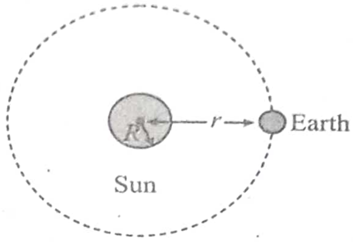
Temperature of the Sun and Solar Constant

If R is the radius of the sun and T is its temperature, then the energy emitted by the sun per second through radiation in accordance with Stefan's law will be given by

$$P = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

By the time it reaches the earth, this energy will spread over a sphere of radius r (= average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant S) will be given by

$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$



i.e., $T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{1/4}$

$$= \left[\left(\frac{1.5\times10^8}{7\times10^5}\right)^2 \times \frac{1.4\times10^3}{5.67\times10^{-8}}\right]^{1/4} = 5800\text{ K}$$

As $r = 1.5 \times 10^8$ km, $R = 7 \times 10^5$ km,

$$S = 2\frac{\text{cal}}{\text{cm}^2\text{-min}} = 1.4\frac{\text{kW}}{\text{m}^2} \text{ and } s = 5.67\times10^{-8}\frac{W}{\text{m}^2\text{-}K^4}$$

This result is in good agreement with the experimental value of temperature of sun, i.e. 6000 K.

Train Your Brain

Example 26: A body of emissivity ($e = 0.75$), surface area of 300 cm^2 and temperature 227°C is kept in a room at temperature 27°C. Calculate the initial value of net power emitted by the body.

Sol. $P = e\sigma A\ (T^4 - T_0^4)$

$= (0.75)\ (5.67 \times 10^{-8})\ (300 \times 10^{-4}) \times \{(500 - 300)^4\}$

$= 69.4$ Watt.

Example 27: A hot black body emits the energy at the rate of 16 J m^{-2} s^{-1} and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in Jm^{-2} s^{-1}.

Sol. Wein's displacement law is:

$\lambda_m . T = b$

i.e. $T \propto \dfrac{1}{\lambda_m}$

Here, λ_m becomes half.

$\therefore$ Temperature doubles.

Also $e = \sigma T^4$

$$\Rightarrow \frac{e_1}{e_2} = \left(\frac{T_1}{T_2}\right)^4 \Rightarrow e_2 = \left(\frac{T_2}{T_1}\right)^4 .e_1 = (2)^4 . 16$$

$= 16.16 = 256$ J m^{-2} s^{-1}

Example 28: A thin square steel plate with each side equal to 10 cm is heated by a blacksmith. The rate of radiated energy by the heated plate is 1134 watts. The temperature of the hot steel plate is:

(Stefan's constant $\sigma = 5.67 \times 10^{-8}$ watts $m^{-2}K^{-4}$ emissivity of the plate = 1)

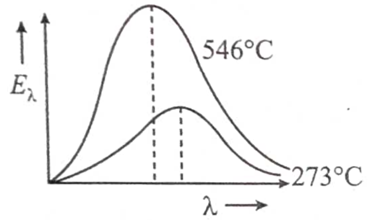
(*a*) 1000K
(*b*) 1189K
(*c*) 2000K
(*d*) 2378K

Sol. (*b*) $\sigma eAT^4 = 1134$

$$\Rightarrow T = \left(\frac{1134}{\sigma eA}\right)^{1/4} = \left[\frac{1134}{5.67\times10^{-8}\times1\times(.1)^2}\right]^{1/4}$$

$T = 1189$ K

Example 29: The spectra of a black body at temperatures 273°C and 546°C are shown in the figure. If A_1 and A_2 be the areas under the two curves respectively, the value of $\frac{A_2}{A_1}$ is



(*a*) $\frac{81}{16}$ (*b*) $\frac{16}{1}$
(*c*) $\frac{27}{8}$ (*d*) $\frac{16}{81}$

Sol. (*a*) $\lambda mT =$ constant

Area under E_λ v/s l represents total emissive power (E) and $E \propto T^4$

$$\text{So, } \frac{A_1}{A_2} = \frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{273+273}{546+273}\right)^4 = \left(\frac{2}{3}\right) = \frac{16}{81}$$

$$\Rightarrow \frac{A_2}{A_1} = \frac{81}{16}$$

Concept Application

24. Solar radiation is found to have an intensity maximum near the wavelength range of 470 nm. Assuming the surface of sun to be perfectly absorbing ($a = 1$), calculate the temperature of solar surface.

(*a*) 5000 K (*b*) 6000 K
(*c*) 4200 K (*d*) 3200 K

25. A block of ice at –10°C is placed in a room which has temperature of 27°C. Then the ice:

(*a*) Does not emit any radiation
(*b*) Emits some radiation but absorbs an equal amount of radiation
(*c*) Absorbs more radiation than it emits
(*d*) Emits more radiation then it absorbs

26. Two sphere made of same substance have radii 1m and 4 m, and temperatures 4000°K and 2000°K respectively. The ratio of power radiated by two spheres is:

(*a*) 1/2 (*b*) 1/4
(*c*) 4 (*d*) 1

27. The rate of emission of radiation of a body at temperature 27°C is 20 watt. If the temperature of a body is increased to 327°C, the rate of emission of radiation will be:

(*a*) 20 watt (*b*) 160 watt
(*c*) 320 watt (*d*) 327 watt

Cooling by Radiation

Consider a hot body at temperature T placed in an environment at a lower temperature T_0. The body emits more radiation than it absorbs and cools down while the surroundings absorb radiation from the body and warm up. The body is losing energy by emitting radiations at a rate

$P_1 = eA\sigma T^4$

and is receiving energy by absorbing radiations at a rate

$P_2 = aA\sigma T_0^4$

Here, 'a' is a pure number between 0 and 1 indicating the relative ability of the surface to absorb radiation from its surroundings. Note that this 'a' is different from the absorptive power 'a'. In thermal equilibrium, both the body and the surroundings have the same temperature (sat Tc) and,

$P_1 = P_2$ or $eA\sigma Tc^4 = aA\sigma Tc^4$ or $e = a$

Thus, when $T > T_0$, the net rate of heat transfer from the body to the surroundings is

$$\frac{dQ}{dt} = eA\sigma(T^4 - T_0^4) \text{ or } mc\left(-\frac{dT}{dt}\right) = eA\sigma(T^4 - T_0^4)$$

Rate of cooling

$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}(T^4 - T_0^4) \text{ or } -\frac{dT}{dt} \propto (T^4 - T_0^4)$$

For two bodies of the same material under identical environments the ratio of their rates of cooling is

$$\frac{(R_e)_1}{(R_2)_2} = \frac{A_1}{A_2}\cdot\frac{V_2}{V_1}$$

Dependence of Rate of Cooling

When a body cools by radiation, the rate of cooling depends on

(*i*) nature of radiating surface, i.e., greater the emissivity, faster will be the cooling.

(*ii*) area of radiating surface, i.e., greater the area of radiating surface, faster will be the cooling.

(*iii*) mass of radiating body, i.e., greater the mass of radiating body slower will be the cooling.

(*iv*) specific heat of radiating body, i.e., greater the specific heat of radiating body slower will be cooling.

(*v*) temperature of radiating body, i.e., greater the temperature of body faster will be cooling.

(*vi*) temperature of surrounding, i.e., greater the temperature of surrounding slower will be cooling.

NEWTON'S LAW OF COOLING

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

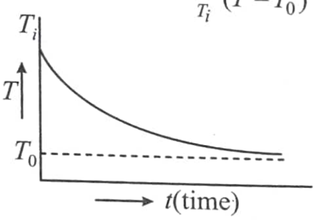
$\frac{dT}{dt} \propto (T - T_0)$, where T and T_0 are temperature corresponding to object and surroundings.

From above expression, $\frac{dT}{dt} = -k(T - T_0)$ ...(*i*)

This expression represents Newton's law of cooling. It can be derived directly from stefan's law, when cooling is accur only from radiations.

$$k = \frac{4e\sigma T_0^3}{mc}A$$

$$\text{Now } \frac{dT}{dt} = -k\,[T - T_0] \Rightarrow \int_{T_i}^{T_f} \frac{dT}{(T - T_0)} = \int_0^t -k\,dt$$



where T_i = initial temperature of object and

T_f = final temperature of object.

$$\Rightarrow \ell n\frac{(T_f - T_0)}{(T_i - T_0)} = -kt \Rightarrow (T_f - T_0) = (T_i - T_0)\,e^{-kt}$$

$$\Rightarrow T_f = T_0 + (T_i - T_0)\,e^{-kt} \quad \text{...(ii)}$$

Limitations of Newton's Law of Cooling

(*a*) The difference in temperature between the body and surroundings must be small

(*b*) The loss of heat from the body should be by radiation only.

(*c*) The temperature of surroundings must remain constant during the cooling of the body.

Approximate Method for Applying Newton's Law of Cooling

Sometime when we need only approximate values from Newton's law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

$$\left(\frac{dT}{dt}\right) = -k(T_{avg} - T_0)$$

$$\frac{T_f - T_i}{t} = -k\left(\frac{T_i + T_f}{2} - T_0\right) \quad \text{...(iii)}$$

If T_i & T_f be initial and final temperature of the body then,

$$T_{avg} = \frac{T_i + T_f}{2}$$

Remember equation (*iii*) is only an approximation and equation (*ii*) must be used for exact values.

Train Your Brain

Example 30: A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30°C.

Sol. $\Delta\theta f = \Delta\theta i\, e^{-kt}$ for the interval in which temperature falls from 40 to 35° C.

$(35 - 20) = (40 - 20)\, e^{-k \cdot 10}$

$$\Rightarrow e^{-10}k = \frac{3}{4} \Rightarrow k = \frac{\ell n\frac{4}{3}}{10} = 0.0285$$

for the next interval

$(30 - 20) = (35 - 20)e^{-kt}$

$$\Rightarrow e^{-kt} = \frac{2}{3} \Rightarrow kt = \ell n\frac{3}{2}$$

$$\Rightarrow \frac{\left(\ell n\frac{4}{3}\right)t}{10} = \ell n\frac{3}{2} \Rightarrow t = 10\frac{\left(\ell n\frac{3}{2}\right)}{\left(\ell n\frac{4}{3}\right)}$$

$\Rightarrow t = 10$ minute

$= 14.096$ min

Example 31: A liquid initially at 70°C cools to 55°C in 5 min and 45°C in 10 min. What is the temperature of the surroundings?

(*a*) 22°C (*b*) 21°C

(*c*) 30°C (*d*) 25°C

Sol. (*d*) As we know that, $\frac{dT}{dt} = -K(T - T_o)$

$$\frac{dT}{dt} = -K\left(\frac{70+55}{2} - T_o\right) \Rightarrow \frac{15}{5} = -K(62.5 - T_o)$$

$$\frac{dT}{dt} = -K\left(\frac{45+55}{2} - T_o\right) \Rightarrow \frac{10}{5} = -K(50 - T_o)$$

Dividing (i) & (ii) we get

$$\frac{3}{2} = \frac{-K(62.5 - T_o)}{-K(50 - T_o)} \Rightarrow 3(50 - T_o) = 2(62.5 - T_o)$$

$$\therefore 150 - 3T_o = 125 - 2T_o$$

$$T_o = 25^\circ C$$

Example 32: A body takes 4 minutes to cool from 100°C to 70°C. If temperature of the surrounding is 20°C. The time taken by it to cool from 70°C to 50°C is

(*a*) 13/3 min
(*b*) 13 min
(*c*) 3 min
(*d*) 6 min

Sol. (*a*) $\frac{dT}{t} = -K\left(\frac{T_1+T_2}{2} - T_0\right) \Rightarrow \frac{30}{4} = -K\left(\frac{170}{2} - 20\right)$

$$\frac{20}{t} = -K\left(\frac{120}{2} - 20\right)$$

Divide (i) & (ii) we get

$$\frac{t}{20} \times \frac{30}{4} = \frac{-K(85-20)}{-K(60-20)} \Rightarrow t = \frac{13}{3} \text{ min}$$

Example 33: The initial temperature of a body is 80°C. If its temperature falls to 64°C in 5 minute and in 8 minutes to 54°C, then the temperature of surrounding will be

(*a*) 60°C (*b*) 70°C
(*c*) 50.66°C (*d*) 81°C

Sol. (*b*) $\frac{80-64}{5} = -K\left(\frac{80+64}{2} - T_0\right) \Rightarrow \frac{16}{5} = -K(72 - T_0)$

$$\Rightarrow \frac{64-54}{8} = -K\left(\frac{64+54}{2} - T_0\right) \Rightarrow \frac{10}{8} = -K(59 - T_0)$$

From eq (1) & (2) we get

$$\frac{4}{5} \times \frac{16}{5} = \frac{-K(72-T_o)}{-K(59-T_0)} \Rightarrow 64(59-T_o) = 25(72-T_0)$$

$$\Rightarrow 3776 - 64T_0 = 1800 - 25T_0 \Rightarrow T_o = 50.66^\circ C$$

Example 34: A beaker of boiled water cools from 80°C to 40°C in 6 minutes. What is the time taken to cool from 40°C to 30°C. The temperature of the surrounding is 20°C

(*a*) 8 min (*b*) 5 min
(*c*) 3.7 min (*d*) 4 min

Sol. (*b*) $t_0 = \frac{2.3026}{K} \log_{10} \frac{T_1 - T_o}{T_2 - T_o}$

$$t_0 = \frac{2.3026}{K} \log_{10} \frac{80-20}{40-20} \Rightarrow t_0 = \frac{2.3026}{K} \log_{10} \frac{60}{20}$$

$$6 = \frac{2.303}{K} \log_{10}(3)$$

$$t = \frac{2.303}{K} \log_{10} \frac{40-20}{30-20} \Rightarrow t = \frac{2.303}{K} \log_{10} \frac{20}{10}$$

$$t = \frac{2.303}{K} \log_{10}(2)$$

From eq (i) & (ii) we get

$$\frac{6}{t} = \frac{\log_{10}(3)}{\log_{10}(2)}$$

$$t = 6 \times 0.631 = 3.7 \text{ minutes}$$

Concept Application

28. A tub of hot water cools from 80°C to 75°C in time t_1 from 75°C to 70°C in time t_2, and from 70°C to 65°C in time t_3 then

(*a*) $t_1 > t_2 > t_3$ (*b*) $t_3 > t_2 > t_1$
(*c*) $t_2 > t_1 > t_3$ (*d*) $t_1 > t_2 > t_4$

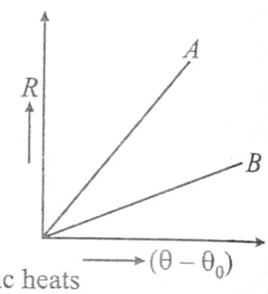
29. A body cools from 62°C to 52°C in 10 min and to 40°C in the next 10 minutes. The temperature of the surrounding is:

(*a*) 40°C (*b*) 42°C
(*c*) 107°C (*d*) 30°C

30. A body takes 5 minutes to cool from 100°C to 60°C and takes time *t* to cool from 80 to 40°C. If surrounding temperature is 20°C, Find *t* :

(*a*) 9 min (*b*) 4 min
(*c*) 6 min (*d*) 8 min

31. Two circular discs *A* and *B* with equal radii are blackened. They are heated to same temperature and are cooled under identical conditions. What inference do you draw from their cooling curves?



(*a*) *A* and *B* have same specific heats
(*b*) Specific heat of *A* is less
(*c*) Specific heat of *B* is less
(*d*) Nothing can be said

Short Notes

Thermal Expansion

(i) Linear Expansion

$L = L_o(1 + \alpha\Delta T)$

(ii) Areal Expansion

$A = A_o(1 + \beta\Delta T)$

(iii) Volume Expansion

$V = V_o(1 + \gamma\Delta T)$

For an isotropic material

$\alpha : \beta : \gamma = 1 : 2 : 3$

Thermometry

(i) For any types of scale

$$\frac{\text{Reading point} - \text{Ice Point}}{\text{Steam point} - \text{Ice Point}} = \text{Constant}$$

(ii) Relation among's different standard scale

$$\frac{C}{5} = \frac{K - 273}{5} = \frac{F - 32}{9}$$

Heat Transfer

Energy in Transite due to temperature difference.

Unit → Calorie or Joule

Mechanical Equivalent

$\boxed{W = JH}$ W = Mechanical work

H = Heat

J = Mechanical Equivalent

J = 1 calorie = 4.18 joule

Specific Heat

$$\boxed{S = \frac{\Delta Q}{m\Delta t}}$$

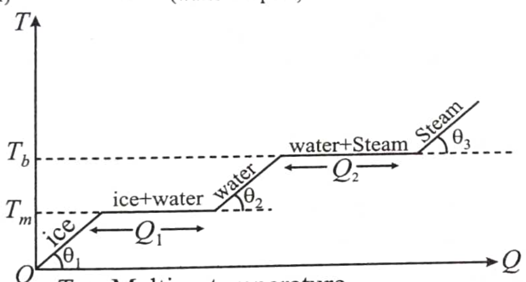
$s_{water} = 1$ cal/g°C ; $s_{ice} = s_{steam} = 1/2$ cal/g°C

Latent heat capacity

$$L = \frac{Q}{m} \Rightarrow Q = mL$$

Heat required during phase change

$L_{(ice - water)} = 80$ cal/g ; $L_{(water - vapour)} = 540$ cal/g



T_m = Melting temperature

T_b = Boiling temperature

- $\theta_3 = \theta_2 > \theta_2$
- $Q_2 > Q_1$

Law of Mixture

Heat taken by one substance = Heat given by another substance.

Heat Transfer

Conduction

$$\text{Heat current} = i = \left(\frac{dQ}{dt}\right) = -kA\frac{dT}{dx}$$

$$i = kA\frac{\Delta T}{L}$$

$$i = \frac{\Delta T}{\left(\frac{L}{kA}\right)} = \frac{\Delta T}{R}$$

$$R = \frac{L}{kA} = \text{Thermal Resistance}$$

$$k_{eq} = \frac{d}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3} + \ldots \frac{d_n}{k_n}}$$

$$k_{eq} = \frac{\frac{1}{A}}{\frac{1}{k_1 A_1} + \frac{1}{k_2 A_2} + \ldots \frac{1}{k_n A_n}}$$

Radiation

- Absorptive Power $(a) = \dfrac{\text{Energy Absorbed}}{\text{Energy incident}}$
- Emissive Power (E)

  Energy radiated per unit area (ΔA), per unit time (Δt) & per unit solid angle $(\Delta\omega)$

  $$E = \frac{\Delta U}{\Delta A \cdot \Delta\omega \cdot \Delta t}$$

- Spectral Emissive Power (E_λ) -

  Emissive Power per unit wavelength

  $$E_\lambda = \frac{dE}{d\lambda}$$

- Emissivity $(e) = \dfrac{\text{Emissive Power of a body}}{\text{Emissive Power of a black body}}$

  $$e = \frac{E}{E_0}$$

- Stefan–Boltzman's Law

  $$\left(\frac{dQ}{dt}\right)_{\text{Radiation}} = \sigma e A T^4$$

  T = body's Temperature

$$\left(\frac{dQ}{dt}\right)_{\text{Absorption}} = \sigma e A T_0^4$$

T_o = Surrounding Temperature

- Law of cooling

$$\left(\frac{dQ}{dt}\right)_{\text{net}} = \left(\frac{dQ}{dt}\right)_{\text{Radiation}} - \left(\frac{dQ}{dt}\right)_{\text{Absorption}}$$

$$= \sigma e A(T^4 - T_0^4)$$

- Newton's Law of Cooling

$$\frac{dT}{dt} = \sigma e A T_0^3 - \Delta T$$

$$\Rightarrow \quad \frac{dT}{dt} \propto \Delta T$$

- Approximation in Newton's Law of Cooling

$$\frac{T_1 - T_2}{t} = \alpha\left[\frac{T_1 + T_2}{2} - T_o\right]$$

α = Constant

T_1 = initial body's Temperature

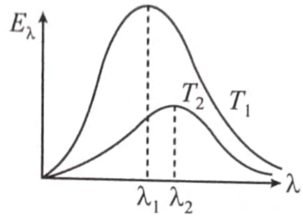
T_2 = final body's Temperature

T_o = Surrounding Temperature

- Wien's displacement Law

$$\lambda_m \cdot T = b$$

$b = 0.282$ cm-K



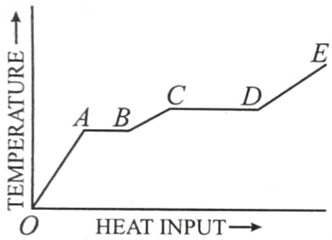
$T_1 > T_2$

$\lambda_1 < \lambda_2$

Area under curve gives emissive power (E) and Emissive power (E) is proportional to T^4.

Solved Examples

1. A Solid material is supplied with heat at a constant rate. The temperature of the material is changing with the heat input as shown in the graph in figure. Study the graph carefully and answer the following questions:



(*a*) What do the horizontal regions AB and CD represent?

(*b*) If CD is equal to $2AB$, what do you infer?

(*c*) What does the slope of DE represent?

(*d*) The slope of $OA >$ the slope of BC. What does this indicate?

Sol. (*i*) **Region AB:** Heat is absorbed by the material at a constant temperature called the melting point.

The phase changes from solid to liquid.

Region CD: Heat is absorbed by the material at a constant temperature called the boiling point. The phase changes from liquid to gas.

(*ii*) Latent heat of vaporisation = 2 (latent heat of fusion)

(*iii*) $Q = mc_g \Delta T$.

The slope $DE = \dfrac{\Delta T}{Q} = \dfrac{1}{mc_g}$

Note: The slope DE indicates that the temperature of the solid begins to rise.

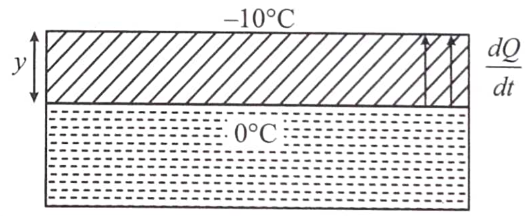
(*iv*) The reciprocal of heat capacity in solid state is greater than the reciprocal of heat capacity in liquid state

$$\left(\frac{1}{mc}\right)_{\text{solid}} > \left(\frac{1}{mc}\right)_{\text{liquid}} \Rightarrow (mc)_{\text{liquid}} > (mc)_{\text{solid}}$$

$$\Rightarrow \quad c_{\text{liouid}} > c_{\text{solid}}$$

2. A pond of water at 0°C is covered with a layer of ice 4.00 cm thick. If the air temperature stays constant at –10° C how long does it take the ice's thickness to increase to 8.00 cm. Thermal conductivity of ice is 2 W/m°C and latent heat of fusion is 80 caVg. Density of ice is 900 kg/m³ .

Sol. Let A be the area of the pond. Transfer of heat will take place from bottom to top.



K = thermal conductivity of ice = 2 W/m°C

ρ = density of ice = 900 kg/m³

L = latent heat = 80 cal/g

= $(80 \times 10^3 \times 4.18)$ J/kg

Now, rate of heat flow $\dfrac{dQ}{dt} = L \cdot \dfrac{dm}{dt}$

or $\dfrac{\text{Temperature difference}}{\text{Thermal resistance}}$

or $= L\rho A \dfrac{dy}{dt}$

$$\text{or}\ \frac{10}{y/KA} = L\rho A\frac{dy}{dt}$$

$$y \cdot dy = \frac{10K}{L\rho} \cdot dt$$

$$dt = \frac{L\rho}{10K} ydy$$

$$\text{or}\ \int_0^t dt = \frac{L\rho}{10K}\int_{4\text{ cm}}^{8\text{ cm}} ydy = \frac{L\rho}{10.K}\left(\frac{y^2}{2}\right)_{4\times10^{-2}\ m}^{8\times10^{-2}\ m}$$

$$\text{or}\ t = \frac{L\rho}{10K}(64-16)\frac{(10^{-4})}{2}$$

Substituting the values, we have

$$t = \frac{80\times4.18\times(10^3)(48)(10^{-4})(900)}{10\times2\times2}$$

or $t = 36115.2$ s $= 10.03$ hr

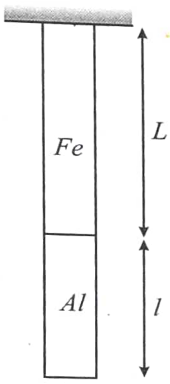
3. An aluminium rod of length $l = 0.5$ m is attached to the bottom of an iron rod of length $L = 1$ m as shown in figure. Find the decrease in height per unit change in temperature of the centre of mass of both the rods.

Assume that cross-sectional area of both the rods are equal.

Given: density of iron $= 7.86$ g /cc,

density of aluminium $= 4.4$ g/cc

$\alpha_{\text{iron}} = 11\alpha \times 10^{-6}$ per°C and $\alpha = 25 \times 10^{-6}$ per°C.



Sol. Let m_1 and m_2 be the masses of iron rod and aluminium rod respectively. Then

$m_1 = LA\rho_{\text{iron}}$ and $m_2 = lA\rho_{Al}]$ (ρ = density)

The centre of mass of iron rod is at its centre, i.e. at a distance $y_1 = \frac{L}{2}$ from the top and the centre of mass of aluminium rod is at a distance $y_2 = \left(L + \frac{l}{2}\right)$ from the top. From definition of centre of mass, the centre of mass of iron and aluminium rods from the top is given by :

$$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{(m_1)\left(\frac{L}{2}\right) + m_2\left(L + \frac{l}{2}\right)}{m_1 + m_2}$$

$$\therefore \frac{dy}{d\theta} = \frac{\left(\frac{m_1}{2}\right)\frac{dL}{d\theta} + m_2\left(\frac{dL}{d\theta} + \frac{1}{2}\frac{dl}{d\theta}\right)}{m_1 + m_2} \quad ...(1)$$

Here, $\frac{dL}{d\theta} = L\alpha_{\text{iron}}$ and $\frac{dL}{d\theta} = l\alpha_{Al}$

Equation (1) can be written as

$$\frac{dy}{d\theta} = \frac{\frac{LA\rho_{\text{iron}}}{2}(L\alpha_{\text{iron}}) + (lA\rho_{\text{Al}})(L\alpha_{\text{iron}}) + \frac{lA\rho_{Al}}{2}(l\alpha_{Al})}{LA\rho_{iron} + lA\rho_{Al}}$$

$$\frac{dy}{d\theta} = \frac{\frac{L^2}{2}\rho_{\text{iron}}\alpha_{\text{iron}} + Ll\rho_{Al}\alpha_{\text{iron}} + \frac{l^2\rho_{Al}\alpha_{Al}}{2}}{L\rho_{\text{iron}} + l\rho_{Al}}$$

Substituting the values, we get

$$\frac{dy}{d\theta} = 8.06 \times 10\ 6\ \text{m /°C}$$

4. Two identical adiabatic vessels A and B, each containing n moles of a monoatomic and diatomic ideal gas respectively, are connected by a rod of length l and cross-sectional area A. Thermal conductivity of rod is K and lateral surface of the rod is insulated. At time $t = 0$, temperature of the gases in the vessel are T_1 and T_2 respectively with $T_1 > T_2$. Find the temperature difference between the vessels at any time t.

Sol. Let θ_1 and θ_2 be the temperatures of the vessels at any time t. Then the temperature difference will be $\theta = \theta_1 = \theta_2$ or

$$\theta = \theta_1 + \theta_2$$

$$\text{or}\ -\frac{d\theta}{dt} = -\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt} \quad ...(1)$$

with time, θ and θ_1 will decrease while θ_2 will increase

$$\frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{KA\theta}{l} \quad ...(2)$$

The vessels are adiabatic and volume is constant Hence heat will transfer from A to B through the rod. Work done by both the gases will be zero. Temperature of A will decrease while that of B will increase.

$$\text{Hence}\ -\frac{d\theta_1}{dt} = \frac{dQ/dt}{nC_{V_1}} = \frac{2}{3nR}\cdot\frac{dQ}{dt}$$

$$\frac{d\theta_2}{dt} = \frac{dQ/dt}{nC_{V_2}} = \frac{2}{5nR}\cdot\frac{dQ}{dt} \quad ...(3)$$

From (1), (2) and (3)

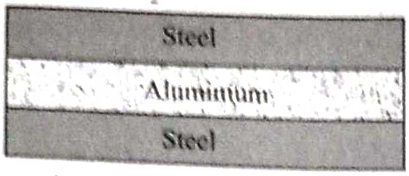
$$-\frac{d\theta}{dt} = \frac{16}{15nR}\cdot\frac{dQ}{dt} = \frac{16}{15nR}\cdot\frac{KA}{l}\theta$$

$$\frac{d\theta}{\theta} = -\frac{16KA}{15nRl}\cdot dt$$

$$\int_{T_1}^{\theta}\frac{d\theta}{\theta} = \frac{-16KA}{15nRl}\int_0^t dt$$

$$\theta = (T_1 - T_2)e^{-\frac{16KA}{15nRl}t}$$

5. Two steel rods and an aluminium rod of equal length l_0 and equal cross-section are joined rigidly at their ends as shown in the figure. All the rods are in a state of zero tension at 0°C Find the length of the system when the temperature is raised to θ. Coefficient of linear expansion of aluminium and steel are α_1 and θ_2 respectively. Young's modulus of aluminium is Y_1 and of steel is Y_2.



Sol. At increased temperature, let Δl_1 and Δl_2 the increase in length of aluminium and steel respectively (if they are free). Then

$\Delta l_1 = l_0 \alpha_1 \theta$

and $\Delta l_2 = l_0 \alpha_2 \theta$

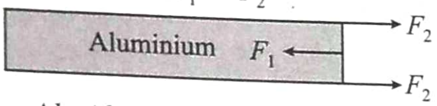
Suppose $\Delta l_1 < \Delta l_2$

Therefore, the composite rod will increase in between of Δl_1 and Δl_2. Say it is Δl, where

$\Delta l_1 < \Delta l_2 < \Delta l_3$

Due to this, aluminium rod has a length $(\Delta l_1 - \Delta l_2)$ more than its natural length at temperature 9 and steel rod (s) will have a length $(\Delta l_2 - \Delta l)$ less than its natural length at temperature θ. Due to this steel rods will exert a force F_2 on aluminium rod from two sides, which in equilibrium be balanced by internal restoring force F_1 Thus,

$$F_1 = 2F_2$$



$$\therefore \Delta l = Y_1 A \frac{\Delta l - \Delta l_1}{l_0} = 2Y_2 A \frac{\Delta l_2 - \Delta l}{l_0}$$

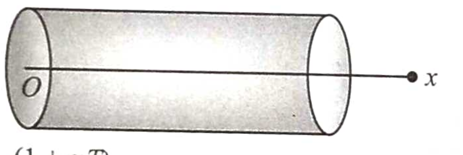
$$= \frac{Y_1 l_0 \alpha_1 \theta + 2Y_2 l_0 \alpha_2 \theta}{Y_1 + 2Y_2}$$

$$\therefore = I_0 + \Delta l$$

$$= I_0 \left[1 + \frac{Y_1\alpha_1 + 2Y_2\alpha_2}{Y_1 + 2Y_2} . \theta\right]$$

Note: On steel rod, force exerted by aluminium rod is F_2 (in opposite direction), which is being balanced by its own restoring force.

6. A solid cylindrical rod of length L and cross-sectional area A lies with its axis along the x-axis and one of its ends at the origin O. The conductivity of the material of the cylinder varies with temperature T as.



$K = K_0 (1 + \alpha T)$

If the end O is maintained at a temperature $2T_0$ and the other end is at T_0. Find the rate of heat flow across the rod assuming no loss of heat from the sides of the rods.

Sol. Let H be the rate of heat flow. Then let us take a strip of length dx at a distance x where temperature is T. Then thermal resistance of strip.

$$dR = \frac{dx}{KA} = \frac{dx}{K_0(1+\alpha T)A}$$

$$(-dT) = H \cdot dR = \frac{H \cdot dx}{K_0 A(1+\alpha T)}$$

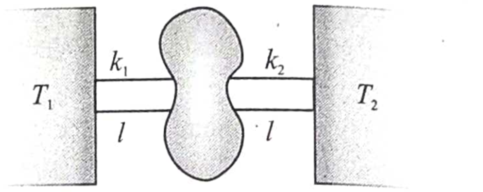
$$\therefore \int_{2T_0}^{T_0} -\frac{K_0 A}{H}(1+\alpha T)dT = \int_0^{L_0} dx$$

Solving this equation we get,

$$H = \frac{K_0 A T_0}{L_0}\left[1 + \frac{3\alpha T_0}{2}\right]$$

7. A body of heat capacity C is connected by 2 rods of same cross-sectional area A and length I but different thermal conductivities k_1 and k_2 as shown in the figure. The free ends of both the rods are maintained at constant temperatures T_1 and T_2. The temperature of the body is T_n at time $t = 0$, such that $T_2 < T_0 < T_1$. Find the temperature of the body after time

$$t_0 = \frac{1C}{A(k_1 + k_2)}$$



Sol. $$\frac{k_1 A(T_1 - T)}{l} - \frac{k_2 A(T - T_2)}{l} = C\frac{dT}{dt}$$

$$\Rightarrow \left[(k_1 T_1 + k_2 T_2) - (k_1 + k_2)T\right] = \frac{IC}{A}\frac{dT}{dt}$$

$$\Rightarrow 1 - \frac{(k_1 + k_2)T}{(k_1 T_1 + k_2 T_2)} = \frac{IC}{A(k_1 T_1 + k_2 T_2)}\frac{dT}{dt}$$

$$\Rightarrow (1 - \lambda T) = \lambda' \frac{dT}{dt} \text{ where } \lambda = \frac{k_1 + k_2}{k_1 T_1 + k_2 T_2}$$

and $$\lambda' = \frac{IC}{A(k_1 T_1 + k_2 T_2)}$$

$$\Rightarrow \int_{T_0}^{T} \frac{dT}{1 - \lambda T} = \int_0^t \frac{dt}{\lambda'} \Rightarrow \ln\frac{1 - \lambda T}{1 - \lambda T_0} = \frac{-\lambda}{\lambda'} t$$

$$\Rightarrow T = \frac{\left[1 - (1 - \lambda T_0)e^{\frac{-\lambda}{\lambda'}t}\right]}{\lambda}$$

By putting values of λ, λ' and t_0 we can see that

$$\frac{\lambda t_0}{\lambda'} = 1$$

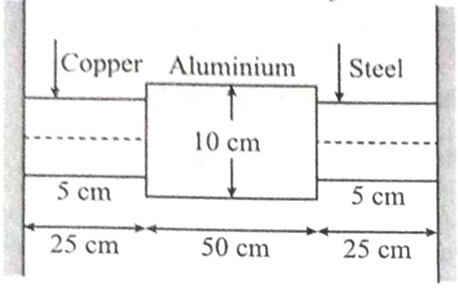
$$\Rightarrow T_{t=t_0} = \frac{1}{\lambda} - \frac{(1 - \lambda T_0)e^{-1}}{\lambda}$$

$$\Rightarrow T_{t=t_0} = \frac{1}{\lambda} 1 - \frac{1}{e} + \frac{T_0}{e}$$

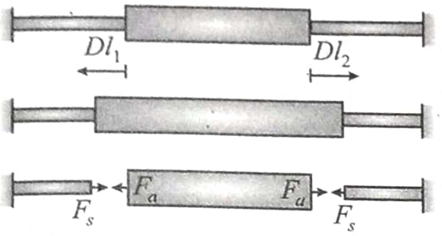
$$\Rightarrow T_{t=t_0} = \frac{T_0}{e} + 1 - \frac{1}{e}\frac{(k_1 T_1 + k_2 T_2)}{k_1 + k_2}$$

8. A composite bar is rigidly attached to the end supports. The temperature of the composite system is raised by 60°C. Find out the stress in three portions of the bar.

[$Ys = 200$ GPa, $\alpha s = 12 \times 10^{-6}$/°C, $Y_a = 90$ GPa, $\alpha_a = 20 \times 10^{-6}$/°C, $Y_c = 100$ GPa, $\alpha_c = 16 \times 10$/°C]



Sol.



In equilibrium $F_a = F_c$

$$\frac{\Delta l_a - \Delta l_1 - \Delta l_2}{l_a}(Y_a A_a) = \frac{\Delta l_c + \Delta l_1}{l_c}(Y_c A_c)$$

$$\therefore \frac{(50)(20\times10^{-6})(60) - \Delta l_1 - \Delta l_2}{50}(90)(10)^2$$

$$= \frac{(25)(16\times10^{-6})(60) + \Delta l_1}{25}(100)(5)^2$$

$$\therefore 1.8(6\times10^{-2} - \Delta l_1 - \Delta l_2) = (2.4\times10^{-2} + \Delta l_1)$$

$$\therefore 2.8\Delta l_1 + 1.8\Delta l_2 = 8.4\times10^{-2}$$

Similarly, $F_a = F_s$

$$\therefore \frac{(50)(20\times10^{-6})(60) - \Delta l_1 - \Delta l_2}{50}(90)(10)^2$$

$$= \frac{(25)(12\times10^{-6})(60) + \Delta l_2}{25} = (200)(5)^2$$

$$\therefore 0.9(6\times10^{-2} - \Delta l_1 - \Delta l_2)$$

$$= (1.8\times10^{-2} + \Delta l_2)$$

$$0.9\Delta l_1 + 1.9\Delta l_2 = 3.6\times10^{-2}$$

Solving Eqs. (1) and (2), we get

$$\Delta l_1 = 2.56\times10^{-2}\ cm$$

and $\Delta l_2 = 0.68\times10^{-2}\ cm$

Stress in three portions,

$$\sigma_c = \frac{(25)(16\times10^{-6})(60) + 2.56\times10^{-2}}{25}\times(100\times10^9)\mathrm{N/m^2}$$

$$= 19.84\times10^7\ \mathrm{N/m^2}$$

$$\sigma_a = \frac{50\times(20\times10^{-6})(60-2.56)\times10^{-2} - 0.68\times10^{-2}(90\times10^9)}{50}$$

$$= 4.97\times10^7\ \mathrm{N/m^2}\ \text{(compressive)}$$

$$\text{and } \sigma_s = \frac{(25)(12\times10^{-6})(60) + 0.68\times10^{-2}}{25}\times(200\times10^9)$$

$$= 19.84\times10^8\ \mathrm{N/m^2}\ \text{(compressive)}$$

9. A vertical hollow cylinder of height 1.52 m is fitted with a movable piston of negligible mass and thickness. The lower half portion of the cylinder contains an ideal gas and the upper half is filled with mercury. The cylinder is initially at 300 K. When the temperature is raised, half of the mercury comes out of the cylinder. Find this temperature, assuming the thermal expansion of mercury to be negligible.

(P_0 = atmospheric pressure = 76 cm of Hg)

Sol. Initial pressure of gas $P_1 = P_0 + \frac{H}{2}dg$,

where P_0 is atmospheric pressure

Initial volume of gas $V_1 = \frac{V}{2}$ Initial temperature $T_1 = 300$ K

When half of mercury comes out of the cylinder, final pressure of gas

$$P_2 = P_0 + \frac{H}{4}dg$$

and final volume of gas

$$V_2 = \frac{V}{2} + \frac{V}{4} = \frac{3V}{4}$$

we have $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\therefore T_2 = \frac{P_2V_2}{P_1V_1}T_1 = \frac{P_0 + \frac{H}{4}dg\cdot\frac{3V}{4}\times300}{P_0 + \frac{H}{2}dg\cdot\frac{V}{2}}$$

Putting $P_0 = 0.76dg$ (atmospheric pressure) we get

$T_2 = 337.5$ K

10. A vessel is completely filled with 500 g of water and 1000g of mercury. When 2.12×10^4 cal of heat is given to it, water of mass 3.52g overflows. Calculate the coefficient of volume expansion of mercury. The expansion of vessel may be neglected. Coefficient of volume expansion of water is 1.5×10^{-4}per °C. Density of mercury is 13.6 g/cc and density of water is 1 g/cc before heating and specific heat of mercury and water are 0.03 and 1 cal/g–°C respectively.

Sol. Let θ be the rise in temperature, γ_{Hg} and γ_w the coefficients of volume expansion of mercury and water respectively.

Then,

$\therefore$ volume of water outflow = final volume of water + final volume of mercury – volume of vessel.

Initial volumes of water and mercury are $\frac{500}{1}$ cc and $\frac{1000}{13.6}$ cc respectively.

Then volume of water outflow

$$= \frac{500}{1}(1+\gamma_w\theta)+\frac{1000}{13.6}(1+\gamma_{Hg}\theta)-\frac{500}{1}+\frac{1000}{13.6}$$

or $\dfrac{3.52}{\rho_{final}} = 500\gamma_w\cdot\theta+\dfrac{1000}{13.6}\gamma_{Hg}\cdot\theta$

or $\dfrac{3.52}{1}(1+\gamma_w\theta) = 500\gamma_w\theta+73.53\gamma_{Hg}\theta$

$496.48\gamma_w\theta+73.53\gamma_{Hg}\theta = 3.52$

Another equation can be formed from calorimetry

$21200 = 500\times 1\times\theta+1000\times 0.03\times\theta$

or $\theta = 40°C$

Substituting the values of θ and γ_w in equation (1)

we get $\gamma_{Hg} = 1.84\times10^{-4}$ per°C.

11. A cube of coefficient of linear expansion as is floating in a bath containing a liquid of coefficient of volume expansion γ_1 When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_1, showing all the steps.**(2004)**

Sol. When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same, hence the upthrust will not change.

$F = F'$

$\therefore V_i\rho_L g = V_i'\rho_L' g$ $\qquad$ (V_i = volume immersed)

$$\therefore (Ah_i)(\rho_L)(g) = A(1+2\alpha_s\Delta T)(h_i)\times\frac{\rho L}{1+\gamma\Delta T}g$$

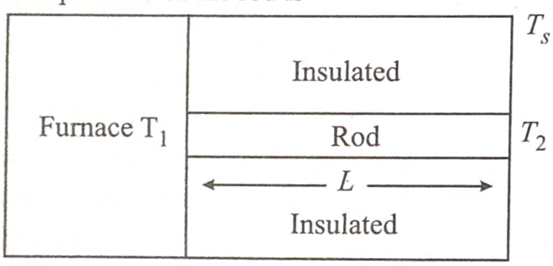
$$= \frac{5.28\times10^5\times2\times10^{-3}}{\frac{1}{R}\times R}$$

Solving this equation, we get

$\gamma_l = 2\alpha_s$

12. One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e. It is given that $T_2 = T_s + \Delta T$, where $\Delta T \le T_s$, T_s being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_s)$, find the proportionality constant.

Consider that heat is lost only by radiation at the end where the temperature of the rod is



Sol. Rate of heat conduction through rod = rate of heat lost from the right end of the rod.

$$\therefore \frac{KA(T_1-T_2)}{L} = eA\sigma\left(T_2^4-T_s^4\right) \qquad ...(i)$$

Given that $T_2 = T_s+\Delta T$

$$\therefore T_2^4 = (T_s+\Delta T)^4 = T_s^4\left(1+\frac{\Delta T}{T_s}\right)^4$$

Using binomial expansion we have,

$$T_2^4 \cong T_s^4\left(1+4\frac{\Delta T}{T_s}\right) \text{ as } \Delta T << T_s \;\therefore\; T_2^4 - T_s^4 = 4(\Delta T)(T_s^3)$$

Substituting in equation (i) we get

$$\frac{K(T_1-T_s-\Delta T)}{L} = 4e\sigma T_s^3\cdot\Delta T$$

or $$\frac{K(T_1-T_s)}{L} = \left(4e\sigma T_s^3+\frac{K}{L}\right)\cdot\Delta T$$

$$\therefore \Delta T = \frac{K(T_1-T_s)}{(4e\sigma LT_s^3+K)}$$

Comparing with the given relation, proportionality constant

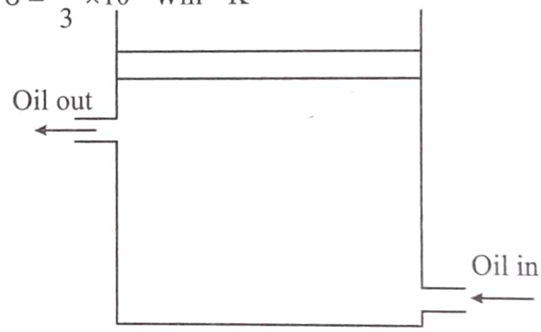
$$= \frac{K}{4e\sigma LT_s^3+K}$$

13. The top of an insulated cylindrical container is covered by a disc having emissivity 0.6 and conductivity 0.167 W/Km and thickness 1cm. The temperature is maintained by circulating oil as shown :

(a) Find the radiation loss to the surroundings in J/m²s if temperature of the upper surface of disc is 127°C and temperature of surroundings is 27°C.

(b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection.

$\sigma = \dfrac{17}{3}\times10^{-8}\,\text{Wm}^{-2}\,\text{K}^{-4}$



Sol. (a) Rate of heat loss per unit area due to radiation

$$I = e\sigma\left(T^4-T_0^4\right)$$

Here $T = 127+273 = 400$ K

and $T_0 = 27+273 = 300$ K

$$\therefore I = 0.6\times\frac{17}{3}\times10^{-8}\left[(400)^2-(300)^4\right] = 595 \text{ watt/m}^2$$

(b) Let θ be the temperature of the oil. Then rate of heat flow through conduction = rate of heat loss due to radiation

$$\therefore \frac{\text{Temperature difference}}{\text{Thermal resistance}} = (595)\text{A}$$

$$\therefore \frac{(\theta-127)}{\frac{l}{kA}} = (595)A$$

Here A = area of disc; k = thermal conductivity and l = thickness (or length) of disc

$$\therefore (\theta - 127)\frac{k}{l} = 595$$

$$\therefore \theta = 595\frac{l}{k} + 127$$

$$= \frac{595 \times 10^{-2}}{0.167} + 127 = 162.6°C$$

14. A solid body X of heat capacity C is kept in an atmosphere whose temperature is T_0 = 300 K. At time t = 0, the temperature of X is T_0 = 400 K. It cools according to Newton's law of cooling. At time its temperature is found to be 350 K.

At this time (t_1)the body X is connected to a large body Y at atmospheric temperature T_A through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$.

Sol. In the first part of the question ($t \le 0$);

At t = 0; $T_X = T_0$=T0 = 400 K and at $t = t_1$;

$T_X = T_1 = 350$ K

Temperature of atmosphere, $T_A = 3000$ K

$T_A = 300$ K (constant).

This cools down according to Newton's law of cooling. Therefore, rate of cooling $\propto$ temperature difference.

$$\therefore \left(-\frac{dT}{dt}\right) = k(T - T_A)$$

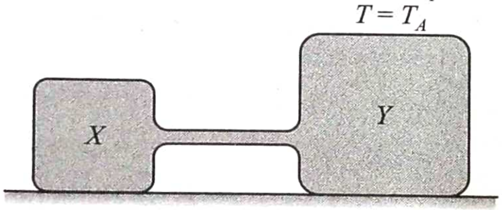
$$\Rightarrow \frac{dT}{T - T_A} = -k \cdot dt \quad \Rightarrow \int_{T_0}^{T_1} \frac{dT}{T - T_A} = -k\int_0^{t_1} dt$$

$$\Rightarrow \ln\left(\frac{T_1 - T_A}{T_0 - T_A}\right) = -kt_1 \Rightarrow kt_1 = -\ln\left(\frac{350 - 300}{400 - 300}\right)$$

kt$_1$ $n - 1$ = ln (2) ...(1)

In the second part, body X cools by radiation (according to Newton's law) as well as by conduction ($t > t_1$).

$T = T_A$



Therefore, Rate of cooling

= (cooling by radiation) + (cooling by conduction)

$$\therefore \left(-\frac{dT}{dt}\right) = k(T - T_A) + \frac{KA}{CL}(T - T_A) \quad ...(2)$$

In conduction,

$$\frac{dQ}{dt} = \frac{KA(T - T_A)}{L} = C\left(-\frac{dT}{dt}\right)$$

$$\therefore \left(-\frac{dT}{dt}\right) = \frac{KA}{LC}(T - T_A)$$

where C = heat capacity of body X

$$\therefore \left(-\frac{dT}{dt}\right) = \left(k + \frac{KA}{CL}\right)(T - T_A) \quad ...(3)$$

Let at $t = 3t_1$, temperature of X becomes T_2

Therefore, from equation (3) we have

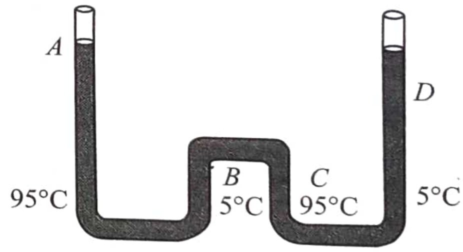
$$\int_{T_1}^{T_2} \frac{dT}{T - T_A} = -\left(k + \frac{KA}{LC}\right)\int_{t_1}^{3t_1} dt$$

$$\ln\left(\frac{T_2 - T_A}{T_1 - T_A}\right) = -\left(k + \frac{KA}{LC}\right)(2t_1) = -\left(2kt_1 + \frac{2KA}{LC}t_1\right)$$

$$\ln\left(\frac{T_2 - 300}{350 - 300}\right) = -2\ln(2) - \frac{2KAt_1}{LC}$$

$$\therefore T_2 = \left(300 + 12.5e^{\frac{-2KAt_1}{CL}}\right)\text{kelvin}$$

15. The apparatus shown in figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95° C while the columns B and D are maintained at 5°C The height of the liquid in A and D measured from the base line are 52.8 cm and 51 cm respectively. Determine the linear coefficient of thermal expansion of the liquid.



Sol. Density of a liquid varies with temperature as

$$\rho_{t°C} = \frac{\rho_{0°C}}{1 + \gamma t}$$

Here γ is the coefficient of volume expansion

In the figure

$h_1 = 52.8$ cm, $h_2 = 51$cm and $h = 49$cm

Now pressure at B = pressure at C.

Therefore, $P_0 + h_1\rho_{95°}g - h_1\rho_{5°}g$

$= P_0 + h_2\rho_{5°}g - h_2\rho_{95°}$

$$\rho_{95^\circ}(h_1+h)=\rho_{5^\circ}(h_2+h)$$

$$\Rightarrow \frac{\rho_{95^\circ}}{\rho_{5^\circ}}=\frac{h_2+h}{h_1+h}$$

$$\Rightarrow \frac{\frac{\rho_{0^\circ}}{(1+95\gamma)}}{\frac{\rho_{0^\circ}}{(1+5\gamma)}}=\frac{h_2+h}{h_1+h}$$

$$\Rightarrow \frac{1+5\gamma}{1+95\gamma}=\frac{51+49}{52.8+49}=\frac{100}{101.8}$$

Solving this equation, we get

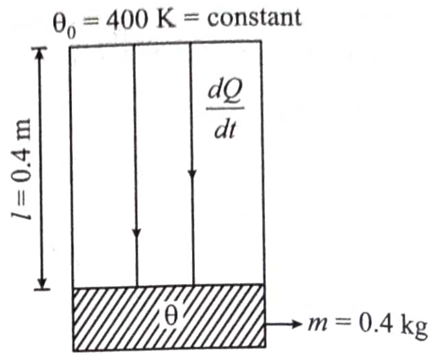
$\gamma = 2\times 10^{-4}$ per °C

Coefficient of linear expansion of temperature,

$$\alpha=\frac{Y}{3}=6.7\times10^{-5}\text{ per}^\circ\text{C}$$

16. A cylindrical block of length 0.4 m and area of cross-section 0.04 m² is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 watt/m K and the specific heat capacity of the material of the disc is 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K ? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

Sol. Let at any time temperature of the disc be θ. At this moment rate of heat flow



$$\frac{dQ}{dt}=\frac{KA(\ddot{A}\theta)}{l}=\frac{KA}{l}(\theta_0-\theta)$$

This heat is utilised in increasing temperature of the disc. Hence

$$\frac{dQ}{dt}=ms\frac{d\theta}{dt}$$

Equating (1) and (2), we have

$$ms.\frac{d\theta}{dt}=\frac{KA}{l}(\theta_0-\theta)$$

Therefore, $\dfrac{d\theta}{\theta_0-\theta}=\dfrac{KA}{msl}dt$

or $\displaystyle\int_{300\,K}^{350\,K}\frac{d\theta}{\theta_0-\theta}=\frac{KA}{msl}\int_0^t dt$

or $\left[-\ln(\theta_0-\theta)\right]_{300\,K}^{350\,K}=\dfrac{KA}{msl}t$

$$\therefore t=\frac{msl}{KA}\ln\frac{\theta_0-300}{\theta_0-350}$$

Substituting the values, we have

$$t=\frac{(0.4)(600)(0.4)}{(10)(0.04)}\ln\frac{400-300}{400-350}$$

$t = 166.32$ s

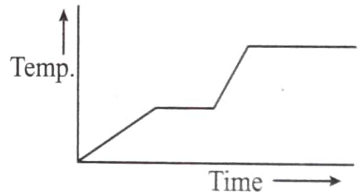
Exercise-1 (Topicwise)

THERMAL EXPANSION

1. From a black body, radiation is not:
   (a) Emitted (b) Absorbed
   (c) Reflected (d) None of these
2. The energy radiated by a body depends on:
   (a) Temperature of body (b) Nature of surface
   (c) Area of body (d) All of above
3. The rate of cooling of a body by radiation depends on:
   (a) Area and mass of body
   (b) Temperature of body and surrounding
   (c) Specific heat of body
   (d) All of above
4. A rod is fixed between two points at 20°C. The coefficient of linear expansion of material of rod is 1.1×10^{-5} /°C and Young's modulus is 1.2×10^{11} N/m. Find the stress developed in the rod if temperature of rod becomes 10°C
   (a) 1.32×10^{7} N/m^2 (b) 1.10×10^{15} N/m^2
   (c) 1.32×10^{8} N/m^2 (d) 1.10×10^{6} N/m^2

CALORIMETRY

5. 5 kg of water at 10°C is added to 10 kg of water at 40°C. Neglecting heat capacity of vessel and other losses, the equilibrium temperature will be close to:
   (a) 30°C (b) 25°C
   (c) 35°C (d) 33°C
6. The water equivalent of a 400 g copper calorimeter (specific heat = 0.1 cal/g°C)
   (a) 40 g (b) 4000 g
   (c) 200 g (d) 4 g
7. There are four objects A, B, C and D. It is observed that A and B are in thermal equilibrium and C and D are also in thermal equilibrium. However, A and C are not in thermal equilibrium. We can conclude that:
   (a) B and D are in thermal equilibrium
   (b) B and D could be in thermal equilibrium but might not be
   (c) B and D cannot be in thermal equilibrium
   (d) The zeroth law of thermodynamics does not apply here because there are more than three objects
8. Heat required to convert 1 g of ice at 0°C into steam at 100°C is (Given L_{steam} = 536 cal/gm)
   (a) 100 cal (b) 0.01 cal/°C
   (c) 716 cal (d) 1 kilocal
9. A substance of mass m kg requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the substance completely solidifies in t seconds. The latent heat of fusion of the substance is:
   (a) $\frac{pt}{\pi M}$ (b) $\frac{2pt}{\pi M}$
   (c) $\frac{pt}{M}$ (d) $\frac{pt}{2M}$
10. One kg of ice at 0°C is mixed with 1 kg of water at 10°C. The resulting temperature will be
    (a) Between 0°C and 10°C (b) 0°C
    (c) Less than 0°C (d) Greater than 0°C
11. The ratio of densities of two bodies is 3 : 4 and ratio of their specific heats is 4 : 3. Then ratio of their heat capacity per unit volume is
    (a) 1 : 1 (b) 3 : 4
    (c) 9 : 16 (d) 16 : 9
12. 100 kg solid iron at 730°C is cooled by immersing it into vessel filled by water at 15°C. In the process, final temperature of water is 80°C. How much was mass of water? (Neglect vaporisation)
    S_{water} = 4200 J/kg°C, S_{iron} = 420 J/kg°C.
    (a) 10 kg (b) 100 kg
    (c) 500 kg (d) 1000 kg
13. Heat is supplied to a certain homogeneous sample of matter at a uniform rate. Its temperature is plotted against time as shown in the figure. Which of the following conclusions can be drawn?



    (a) Its specific heat capacity is greater in the solid state than in the liquid state
    (b) Its specific heat capacity is greater in the liquid state than in the solid state
    (c) Its latent heat of vaporization is smaller than its latent heat of fusion
    (d) None of these

HEAT TRANSFER

Conduction

14. If a water in a bucket is heated by immersion rod, then heat is transferred just below the immersion rod is
    (a) Convection (b) Conduction
    (c) Radiation (d) None of these

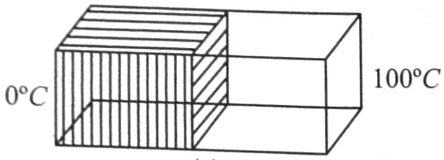
15. The lengths and radii of two rods made of same material are in the ratios 1 : 2 and 2 : 3 respectively; If the temperature difference between the ends for the two rods be the same, then in the steady state, the amount of heat flowing per second through them will be in the ratio:

(a) 1 : 3 (b) 4 : 3
(c) 8 : 9 (d) 3 : 2

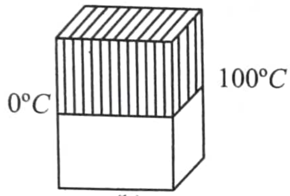
16. Which of following qualities suit for a cooking utensil?

(a) High specific heat and low thermal conductivity
(b) High specific heat and high thermal conductivity
(c) Low specific heat and low thermal conductivity
(d) Low specific heat and high thermal conductivity

17. Two identical square rods of metal are welded end to end as shown in figure (a). Assume that 10 cal of heat flows through the rods in 2 min. Now the rods are welded as shown in figure. (b) The time it would take for 10 cal to flow through the rods now, is



(a)



(b)

(a) 0.75 min (b) 0.5 min
(c) 1.5 min (d) 1 min

18. Consider a compound slab consisting of two different materials having equal length, thicknesses and thermal conductivities K and 2K respectively. The equivalent thermal conductivity of the slab is–

(a) $\sqrt{2}K$ (b) $\frac{1}{3}K$
(c) $\frac{3}{2}K$ (d) $\frac{2}{3}K$

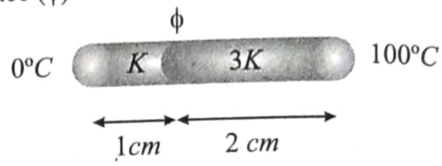
19. When two ends of a rod wrapped with cotton are maintained at different temperatures and after some time every point of the rod attains a constant temperature, then

(a) Conduction of heat at different points of the rod stops because the temperature is not increasing
(b) Rod is bad conductor of heat
(c) Heat is being radiated from each point of the rod
(d) Each point of the rod is giving heat to its neighbour at the same rate at which it is receiving heat

20. Wires A and B have identical lengths and have circular cross-sections. The radius of A is twice the radius of B i.e. $r_A = 2r_B$. For a given temperature difference between the two ends, both wires conduct heat at the same rate. The relation between the thermal conductivities is given by

(a) $K_A = 4K_B$ (b) $K_A = 2K_B$
(c) $K_A = K_B/2$ (d) $K_A = K_B/4$

21. Two bars of thermal conductivities K and 3K and lengths 1 cm and 2 cm respectively have equal cross-sectional area, they are joined lengths wise as shown in the figure. If the temperature at the ends of this composite bar is 0°C and 100°C respectively (see figure), then the temperature of the interface (ϕ) is



(a) 50°C (b) $\frac{100}{3}$°C
(c) 60°C (d) $\frac{200}{3}$°C

Radiation Stefan's Law and Wein's Law

22. The velocity of heat radiation in vacuum is

(a) Equal to that of light
(b) Less than that of light
(c) Greater than that of light
(d) Equal to that of sound

23. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by

(a) Wien's law
(b) Rayleigh jeans law
(c) Planck's law of radiation
(d) Stefan's law of radiation

24. Certain substance emits only the wavelengths $\lambda_1, \lambda_2, \lambda_3$ and λ_4 when it is at a high temperature. When this substance is at a colder temperature, it will absorb only the following wavelengths

(a) λ_1 (b) λ_2
(c) λ_1 and λ_2 (d) $\lambda_1, \lambda_2, \lambda_3$ and λ_4

25. A black body has maximum wavelength λ_m at temperature 2000 K. Its corresponding wavelength at temperature 3000 K will be

(a) $\frac{3}{2}\lambda_m$ (b) $\frac{2}{3}\lambda_m$ (c) $\frac{4}{9}\lambda_m$ (d) $\frac{9}{4}\lambda_m$

26. The amount of radiation emitted by a perfectly black body is proportional to

(a) Temperature on ideal gas scale
(b) Fourth root of temperature on ideal gas scale
(c) Fourth power of temperature on ideal gas scale
(d) Source of temperature on ideal gas scale

27. The rate of radiation of a black body at 0°C is EJ/sec. The rate of radiation of this black body at 273°C will be

(a) 16 E (b) 8 E
(c) 4 E (d) 12 E

28. A black body radiates energy at the rate of E W/m^2 at a high temperature TK. When the temperature is reduced to $\frac{T}{2}K$, the radiant energy will be

(a) $\frac{E}{16}$ (b) $\frac{E}{4}$

(c) $4E$ (d) $16E$

29. Two spheres P and Q, of same colour having radii 8 cm and 2 cm are maintained at temperatures 127°C and 527°C respectively. The ratio of energy radiated by P and Q is

(a) 0.054 (b) 0.0034

(c) 1 (d) 2

30. The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. The power radiated by the black body will now increase by a factor of

(a) 256/81 (b) 64/27

(c) 16/9 (d) 4/3

31. If the sun's surface radiates heat at 6.3×10^7 Wm^{-2}. Calculate the temperature of the sun assuming it to be a black body ($\sigma = 5.7 \times 10^{-8}$ Wm^{-2} K^{-4})

(a) 5.8×10^3 K (b) 8.5×10^3 K

(c) 3.5×10^8 K (d) 5.3×10^8 K

32. Two identical objects A and B are at temperatures T_A and T_B respectively. Both objects are placed in a room with perfectly absorbing walls maintained at temperatures T ($T_A > T > T_B$). The objects A and B attain temperature T eventually which one of the following is correct statement

(a) 'A' only emits radiations while B only absorbs them until both attain temperature T

(b) A loses more radiations than it absorbs while B absorbs more radiations that it emits until temperature T is attained

(c) Both A and B only absorb radiations, but do not emit it until they attain temperature T

(d) Both A and B only emit radiations until they attain temperature T

33. Newton's law of cooling is a special case of

(a) Stefan's law (b) Kirchhoff's law

(c) Wien's law (d) Planck's law

34. Equal masses of two liquids are filled in two similar calorimeters. The rate of cooling will

(a) Depend on the nature of the liquids

(b) Depend on the specific heats of liquids

(c) Be same for both the liquids

(d) Depend on the mass of the liquids

35. A body cools from 60° C to 50°C in 10 minutes. If the room temperature is 25°C and assuming Newton's law of cooling to hold good, the temperature of the body at the end of the next 10 minutes will be

(a) 38.5°C (b) 40°C

(c) 42.85°C (d) 45°C

Convection and Radiation

36. A spherical black body with a radius of 12 cm radiates 450 W power of 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(a) 225 (b) 450

(c) 900 (d) 1800

37. Three discs, A, B and C having radii 2m, 4 m and 6m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C respectively.

(a) Q_A is maximum (b) Q_B is maximum

(c) Q_C is maximum (d) $Q_A = Q_B = Q_C$

38. Two spheres, one solid and other hollow are kept in atmosphere at same temperature. They are made of same material and their radii are also same. Which sphere will cool at a faster rate initially?

(a) Hollow sphere (b) Solid sphere

(c) Both will same rate (d) None of these

39. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{1}$ (d) $\left(\frac{1}{3}\right)^{1/3}$

Newton's Law of Cooling

40. According to 'Newton's law of cooling', the rate of cooling of a body is proportional to the

(a) Temperature of the body

(b) Temperature of the surrounding

(c) Fourth power of the temperature of the body

(d) Difference of the temperature of the body and the surroundings.

41. If a metallic sphere gets cooled from 60°C to 50°C in 10 minutes and in the next 10 minutes gets cooled to 42°C, then the temperature of the surroundings is

(a) 30°C (b) 36°C

(c) 26°C (d) 20°C

42. The excess temperature of a body falls from 12°C to 6°C in 5 minutes, then the time to fall the excess temperature from 6°C to 3°C is (assume the Newton's cooling is valid)

(a) 10 minute (b) 7.5 minute

(c) 5 minute (d) 2.5 minute

43. Newton's law of cooling is used in laboratory for the determination of the
(a) Specific heat of the gases
(b) The latent heat of gases
(c) Specific heat of liquids
(d) Latent heat of liquids

44. The readings of a thermometer at 0°C and 100°C are 50 cm and 75 cm of mercury column respectively. Find the temperature at which its reading is 80 cm of mercury column?
(a) 120°C
(b) 140°C
(c) 110°C
(d) 105°C

Exercise-2 (Learning Plus)

1. A steel scale is to be prepared such that the millimeter intervals are to be accurate within 6×10^{-5} mm. The maximum temperature variation from the temperature of calibration during the reading of the millimeter marks is ($\alpha = 12 \times 10^{-6}$ /°C)
(a) 4.0°C
(b) 4.5°C
(c) 5.0°C
(d) 5.5°C

2. A steel rod 25 cm long has a cross-sectional area of 0.8 cm^2. Force that would be required to stretch this rod by the same amount as the expansion produced by heating it through 10°C is:
(Coefficient of linear expansion of steel is 10^{-5}/°C and Young's modulus of steel is 2×10^{10} N/m^2.)
(a) 160 N
(b) 360 N
(c) 106 N
(d) 260 N

3. If I is the moment of inertia of a solid body having α-coefficient of linear expansion then the change in I corresponding to a small change in temperature ΔT is
(a) $\alpha I \Delta T$
(b) $\alpha I \Delta T$
(c) $2\alpha I \Delta T$
(d) $3\alpha I \Delta T$

4. A metallic wire of length L is fixed between two rigid supports. If the wire is cooled through a temperature difference ΔT (Y = young's modulus, ρ = density, α = coefficient of linear expansion) then the frequency of transverse vibration is proportional to:
(a) $\dfrac{\alpha}{\sqrt{\rho Y}}$
(b) $\sqrt{\dfrac{Y\alpha}{\rho}}$
(c) $\dfrac{\sigma}{\sqrt{Y\alpha}}$
(d) $\sqrt{\dfrac{\sigma\alpha}{Y}}$

5. A rod of length 20 cm is made of metal. It expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature, a third rod of the same length is composed of two parts one of metal A and the other of metal B. Thus rod expand by 0.06 cm for the same change in temperature. The portion made of metal A has the length.
(a) 20 cm
(b) 10 cm
(c) 15 cm
(d) 18 cm

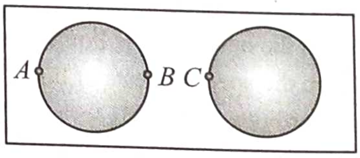
6. A sphere of diameter 7 cm and mass 266.5 gm floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature 35°C. If the density of a liquid at 0°C is 1.527 gm/cc, then neglecting the expansion of the sphere, the coefficient of cubical expansion of the liquid (f) is :
(a) 8.32×10^{-4} per °C
(b) 8.486×10^{-5} per °C
(c) 8.486×10^{-6} per °C
(d) 8.486×10^{-3} per °C

7. A metal ball immersed in Alcohol weights W_1 at 0°C and W_2 at 50°C. The coefficient of cubical expansion of the metal $(\gamma)_m$ is less than that of alcohol $(\gamma)_{Al}$. Assuming that density of metal is large compared to that of alcohol, it can be shown that
(a) $W_1 > W_2$
(b) $W_1 = W_2$
(c) $W_1 < W_2$
(d) Any of (a), (b) or (c)

8. The volume thermal expansion coefficient of an ideal gas at constant pressure is (Here T = absolute temperature of gas)
(a) T
(b) T^2
(c) $\dfrac{1}{T}$
(d) $\dfrac{1}{T^2}$

9. If two rods of length L and $2L$ having coefficients of linear expansion α and 2α respectively are connected so that total length becomes $3L$, the average coefficient of linear expansion of the composition rod equals:
(a) $\dfrac{3}{2}\alpha$
(b) $\dfrac{5}{2}\alpha$
(c) $\dfrac{5}{3}\alpha$
(d) None of these

10. Two large holes are cut in a metal sheet. If this is heated distances AB and BC, (as shown)

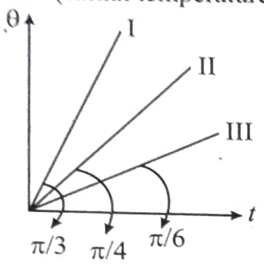


(a) Both will increase
(b) Both will decrease
(c) AB increases, BC decreases
(d) AB decreases, BC increases

11. Two thermometers x and y have fundamental intervals of 80° and 120°. When immersed in ice, they show the reading of 20° and 30°. If y measures the temperature of a body as 120°, the reading of x is:

(a) 59° (b) 65°
(c) 75° (d) 80°

12. Three bodies A, B and C of masses m, m and $\sqrt{3}$m respectively are supplied heat at same rate. The change in temperature θ versus time t graph for A, B and C are shown by I, II and III respectively. If their specific heat capacities are S_A, S_B and S_C respectively then which of the following relation is correct? (Initial temperature of body is 0°C)



(a) $S_A > S_B > S_C$ (b) $S_B = S_C < S_A$
(c) $S_A = S_B = S_C$ (d) $S_B = S_C > S_A$

13. 10 gm of ice at 0°C is kept in a calorimeter of water equivalent 10 gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)

(a) 6200 cal (b) 7200 cal
(c) 13600 cal (d) 8200 cal

14. A bullet of mass 70 gm traveling at 250 m/s strikes and gets embedded in ice at 0°C. Assuming that temperature of bullet does not change, find the amount of ice that melts.

(a) $\frac{2}{127}$ kg (b) $\frac{3}{424}$ kg
(c) $\frac{5}{768}$ kg (d) $\frac{7000}{256}$ kg

15. A block of ice with mass m falls into a lake. After impact, a mass of ice m/s melts. Both the block of ice and the lake have a temperature of 0°C. If L represents the heat of fusion, the minimum distance the ice fell before striking the surface is

(a) $\frac{L}{5g}$ (b) $\frac{5L}{g}$
(c) $\frac{gL}{5m}$ (d) $\frac{mL}{5g}$

16. Water at 0°C, contained in a closed vessel, is abruptly opened in an evacuated chamber. If the latent heats of fusion and vaporization at 0°C are in the ratio $\lambda : 1$, the fraction of water evaporated will be:

(a) $\lambda / 1$ (b) $\lambda / (\lambda + 1)$
(c) $(\lambda - 1) / \lambda$ (d) $(\lambda - 1) / (\lambda + 1)$

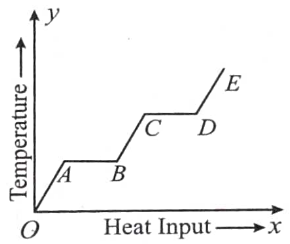
17. The density of a material A is 1500 kg/m³ and that of another mateial B is 2000 kg/m³. It is found that the heat capacity of 8 volumes of A is equal to heat capacity of 12 volumes of B. The ratio of specific heats of A and B will be

(a) 1 : 2 (b) 3 : 1
(c) 3 : 2 (d) 2 : 1

18. The specific heat of a solid at low temperature varies according to the relation $c = kT^3$ where k is a constant and T is temperature in kelvin. The heat required to raise the temperature of a mass m of such a solid from $T = 0$ K to $T = 20$ K is :

(a) 2×10^4 mk (b) 4×10^4 mk
(c) 8×10^4 mk (d) 16×10^4 mk

19. A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represent.



(a) Latent heat of liquid
(b) Latent heat of vapour
(c) Heat capacity of vapour
(d) Inverse of heat capacity of vapour

20. 20g of water at 30°C is mixed with 5 gm of ice at –10°C. Find the final temperature of mixture (in °C) if latent heat of fusion of ice is 80 cal/gm, specific heat of water = 1 cal/gm°C, specific heat of ice = 0.5 cal/gm°C

(a) 0°C (b) –5°C
(c) 7°C (d) 10°C

21. Which of the law can be understood in terms of Stefan's law

(a) Wien's displacement law
(b) Kirchoff's law
(c) Newton's law of cooling
(d) Planck's law

22. A boiler is made of a copper plate 2.4 mm thick with an inside coating of a 0.2 mm thick layer of tin. The surface area exposed to gases at 700°C is 400 cm². The amount of steam that could be generated per hour at atmospheric pressure is (K_{cu} = 0.9 and K_{tin} = 0.15 cal/cm/s/°C and L_{steam} = 540 cal/g)

(a) 5000 Kg (b) 1000 kg
(c) 4000 kg (d) 200 k

23. A pot with a steel bottom 1.2 cm thick rests on a hot stove. The area of the bottom of the pot is 0.150 m². The water inside the pot is at 100°C and 0.440 kg vapourise in every 5 minutes. The temperature of the lower surface of the pot, which is in contact with the stove is (Given : L_v = 2.256 × 10^6 J/kg and K_{steel} = 50.2 W/m-K)

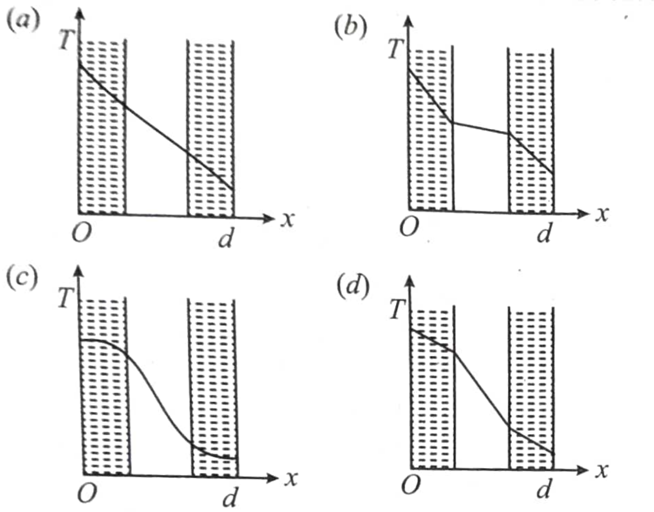
(a) 105.3 °C (b) 205.3 °C
(c) 185.3 °C (d) 115.3 °C

24. Four rods of same material with different radii r and length l are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat?

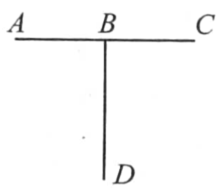
(a) $r = 2$ cm, $l = 0.5$ m (b) $r = 2$ cm, $l = 2$ m

(c) $r = 0.5$ cm, $l = 0.5$ m (d) $r = 1$ cm, $l = 1$ m

25. The wall with a cavity consists of two layers of brick separated by a layer of air. All three layers have the same thickness and the thermal conductivity of the brick is much greater than that of air. The left layer is at a higher temperature than the right layer and steady state condition exists. Which of the following graphs predicts correctly the variation of temperature T with distance d inside the cavity?

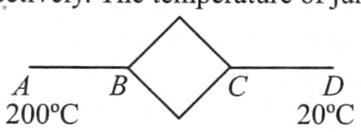


26. Three conducting rods of same material and cross-section are shown in figure. Temperatures of A, D and C are maintained at 20°C, 90°C and 0°C. The ratio of lengths of BD and BC if there is no heat flow in AB is



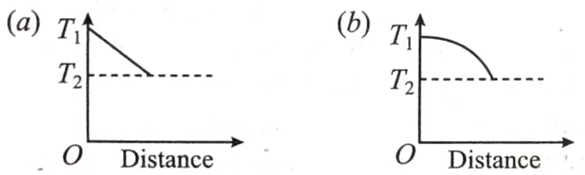
(a) 2/7 (b) 7/2 (c) 9/2 (d) 2/9

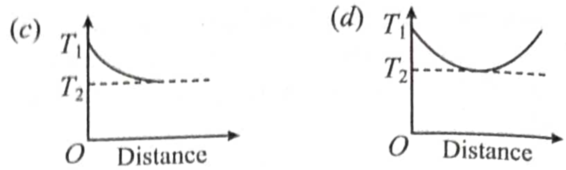
27. Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperature of 200°C and 20°C respectively. The temperature of junction B will be:



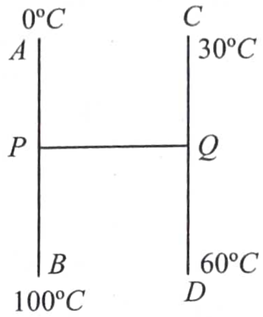
(a) 120°C (b) 100°C (c) 140°C (d) 80°C

28. The ends of a metal bar of constant cross-sectional area are maintained at temperatures T_1 and T_2 which are both higher than the temperature of the surroundings. If the bar is untagged, which one of the following sketches best represents the variation of temperature with distance along the bar?





29. Three identical rods AB, CD and PQ are joined as shown. P an Q are mid points of AB and CD respectively. Ends A, B, C and D are maintained at 0°C, 100°C, 30°C and 60°C respectively. The direction of heat flow in PQ is



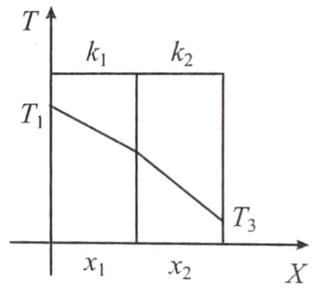
(a) From P to Q

(b) From Q to P

(c) Heat does not flow in PQ

(d) Data not sufficient

30. The temperature drop through each layer of two layer furnace wall is shown in figure. Assume that the external temperature T_1 and T_3 are maintained constant and $T_1 > T_3$. If the thickness of the layers x_1 and x_2 are the same, which of the following statements are correct.



(a) $k_1 > k_2$

(b) $k_1 < k_2$

(c) $k_1 = k_2$ but heat flow through material (a) is larger then through (b)

(d) $k_1 = k_2$ but heat flow through material (a) is less than that through (b)

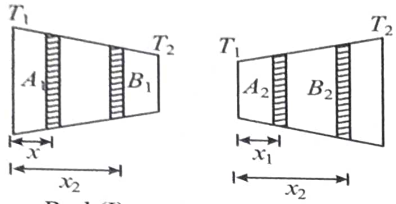
31. Two sheets of thickness d and $3d$, are touching each other. The temperature just outside the thinner sheet side is A, and on the side of the thicker sheet is C. The interface temperature is B. A, B and C are in arithmetic progression, then the ratio of thermal conductivity of thinner sheet and thicker sheet is

(a) 1 : 3 (b) 3 : 1 (c) 2 : 3 (d) 1 : 9

32. A cylindrical rod with one end in a steam chamber and the outer end in ice results in melting of 0.1 gm of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is 1/4 that of first, the rate at which ice melts is gm/sec will be

(a) 3.2 (b) 1.6 (c) 0.2 (d) 0.1

33. The two ends of two similar non-uniform rods of length ℓ each and thermal conductivity 'K' are maintained at different but constant temperature. The temperature gradient at any point on the rod is $\frac{\Delta T}{\Delta \ell}$. The heat flow per unit time through the rod is I: **Given** $T_1 > T_2$. Then which of the following is true:



Rod (I) Rod (II)

(a) I of Rod (I) = I of Rod (II)
(b) I of Rod (I) > I of Rod (II)
(c) I of Rod (I) < I of Rod (II)
(d) Data is insufficient

34. A metallic sphere having radius 0.08 m and mass $m = 10$ kg is heated to a temperature of 227°C and suspended inside a box whose walls are at a temperature of 27°C. The maximum rate at which its temperature will fall is: (Take $e = 1$, Stefan's constant $\sigma = 5.8 \times 10^{-8}$ $Wm^{-2}K^{-4}$ and specific heat of the metal $s = 90$ cal/kg/deg J = 4.2 Joules/Calorie)

(a) .055 °C/sec (b) .066 °C/sec
(c) .044 °C/sec (d) 0.03 °C/sec

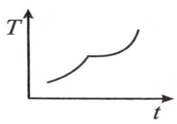
35. A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity for A is twice that of B. Under steady state, the temperature difference across the whole wall is 36°C. Then the temperature difference across the layer A is

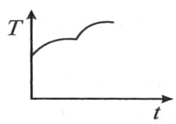
(a) 6°C (b) 12°C
(c) 18°C (d) 24°C

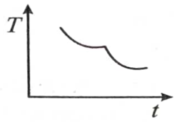
36. Star S_1 emits maximum radiation of wavelength 420 nm and the star S_2 emits maximum radiation of wavelength 560 nm, what is the ratio of the temperature of S_1 and S_2:

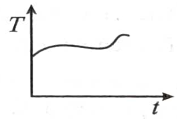
(a) 4/3 (b) $(4/3)^{1/4}$
(c) 3/4 (d) $(3/4)^{1/2}$

37. An ice cube at temperature –20°C is kept in a room at temperature 20°C. The variation of temperature of the body with time is given by

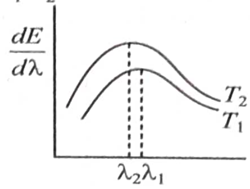
(a)


(b)


(c)


(d)


38. The spectral emissive power E_λ for a body at temperature T_1 is plotted against the wavelength and area under the curve is found to be A. At a different temperature T_2 the area is found to be 9A Then λ_1/λ_2 =



(a) 3 (b) 1/3
(c) $1/\sqrt{3}$ (d) $\sqrt{3}$

39. Two bodies P and Q have thermal emissivities of ε_p and ε_Q respectively. Surface areas of these bodies are same and the total radiant power is also emitted at the same rate. If temperature of P is θ_P kelvin then temperature of Q i.e. θ_Q is

(a) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4}\theta_P$ (b) $\left(\frac{\varepsilon_P}{\varepsilon_Q}\right)^{1/4}\theta_P$

(c) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4}\times\frac{1}{\theta_P}$ (d) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{4}\theta_P$

40. The rate of emission of radiation of a black body at 273°C is E, then the rate of emission of radiation of this body at 0°C will be

(a) $\frac{E}{16}$ (b) $\frac{E}{4}$
(c) $\frac{E}{8}$ (d) 0

41. A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S becomes constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1 °C to 34.9 °C in 1 minute. The heat capacity of S is

(a) 100 J/°C (b) 300 J/°C
(c) 750 J/°C (d) 1500 J/°C

42. A sinker of weight w_0 has an apparent weight w_1 when weighed in a liquid at a temperature t_1 and w_2 when weight in the same liquid at temperature t_2. The coefficient of cubical expansion of the material of sinker is β. What is the coefficient of volume expansion of the liquid.

(a) $\gamma_\ell = \frac{w_2 - w_1}{(w_0 - w_2)(t_2 - t_1)} + \frac{\beta(w_0 - w_1)}{(w_0 - w_2)}$

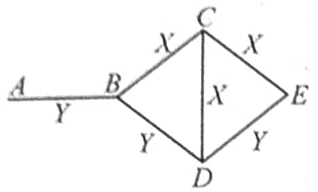
(b) $\gamma_\ell = \frac{w_1 + w_2}{(w_0 - w_2)(t_2 - t_1)} - \frac{\beta(w_0 + w_1)}{(w_0 - w_2)}$

(c) $\gamma_\ell = \frac{w_2 - w_1}{(w_2 - w_0)(t_2 - t_1)} + \frac{\beta(w_0 + w_1)}{(w_0 - w_2)}$

(d) $\gamma_\ell = \frac{w_2 - w_1}{(w_2 - w_0)(t_2 - t_1)} + \frac{\beta(w_0 - w_1)}{(w_0 - w_2)}$

43. Three rods of material X and three rods of material Y are connected as shown in the figure. All the rods are of identical length and cross-sectional area. If the end A is maintained at

60°C and the junction E at 10°C. What are the temperature of the junctions B and C? The thermal conductivity of X is 0.92 cal/sec-cm-°C and that of Y is 0.46 cal/sec-cm-°C.



(a) 30°C & 20°C
(b) 30°C & 15°C
(c) 50°C & 20°C
(d) 40°C & 20°C

44. A cylindrical block of length 0.4 m and area of Cross-section 0.004 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 watt/mK and the specific heat capacity of the material of the disc is 600 J/kg–K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purpose of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

(a) 1610 seconds
(b) 1580 seconds
(c) 1663 seconds
(d) 1593 seconds

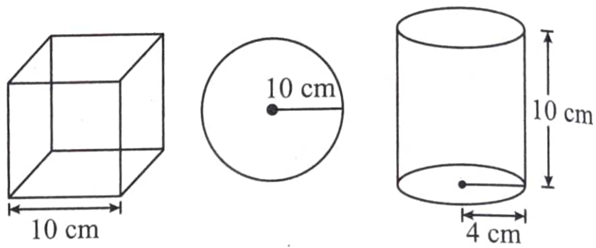
45. A metallic cylindrical vessel whose inner and outer radii are r_1 and r_2 is filled with ice at 0°C. The mass of the ice in the cylinder is m. Circular portions of the cylinder is sealed with completely adiabatic walls The vessel is kept in air. Temperature of the air is 50°C. How long will it take for the ice to melt completely? Thermal conductivity of the cylinder is k and its length is l. Latent heat of fusion is L.

(a) $\dfrac{mL}{\pi kl}\ln\left(\dfrac{r_2}{r_1}\right)$
(b) $\dfrac{mL}{50\,\pi kl}\ln\left(\dfrac{r_2}{r_1}\right)$
(c) $\dfrac{100\,mL}{\pi kl}\ln\left(\dfrac{r_2}{r_1}\right)$
(d) None of these

46. Calculate the rate of flow of the heat through a tapering rod of length l, tapering from radius r_1 to r_2 when temperatures of the end are maintained at T_1 and T_2. Thermal conductivity of the material of the rod is K. $(T_1 > T_2)$

(a) $\dfrac{T_1+T_2}{l}\cdot\pi k(r_1r_2)$
(b) $\dfrac{T_2+T_1}{2l}\cdot\pi k(r_1+r_2)$
(c) $\dfrac{T_1-T_2}{l}\cdot\pi k(r_1\cdot r_2)$
(d) $\dfrac{T_2+T_1}{l}\cdot\pi k\sqrt{r_1r_2}$

47. A cube, sphere and a cylinder made of same material shown in figure are allowed to cool under identical conditions. Determine which of these will cool at a faster rate?



(a) Cylinder
(b) Cube
(c) Sphere
(d) All will cool at the same rate

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. An experiment is performed to measure the specific heat of copper. A lump of copper is heated in an oven, then dropped into a beaker of water. To calculate the specific heat of copper, the experimenter must know or measure the value of all of the quantities below EXCEPT the

(a) Heat capacity of water and beaker
(b) Original temperature of the copper and the water
(c) Final (equilibrium) temperature of the copper and the water
(d) Time taken to achieve equilibrium after the copper is dropped into the water

2. When m, gm of water at 10°C is mixed with m, gm of ice at 0°C, which of the following statements are false?

(a) The temperature of the system will be given by the equation $m \times 80 + m \times 1 \times (T-0) = m \times 1 \times (10-T)$
(b) Whole of ice will melt and temperature will be more than 0°C but lesser than 10°C
(c) Whole of ice will melt and temperature will be 0°C
(d) Whole of ice will not melt and temperature will be 0°C

3. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

(a) The temperature of B is 1934 K
(b) $\lambda_B = 1.5$ μm
(c) The temperature of B is 11604 K
(d) The temperature of B is 2901 K

4. The solar constant is the amount of heat energy received per second per unit area of a perfectly black surface placed at a mean distance of the Earth from the Sun, in the absence of Earth's atmosphere, the surface being held perpendicular to the direction of Sun's rays. Its value is 1388 W/m^2. If the solar constant for the earth is 's'. The surface temperature of the sun is TK. The sun subtends a small angle 'θ' at the earth. Then correct options is/are:–

(*a*) $s \propto T^2$ (*b*) $s \propto T^4$
(*c*) $s \propto \theta^2$ (*d*) $s \propto \theta$

5. A force of 200 N is applied at one end of a wire of length 2 m and having area of cross-section 10^{-2} cm^2. The other end of the wire is rigidly fixed. If coefficient of linear expansion of the wire $\alpha = 8 \times 10^{-6}$/°C and Young's modulus $Y = 2.2 \times 10^{11}$ N/m^2 and its temperature is increased by 5°C, then the increase in the tension of the wire will be

(*a*) 4.2 N (*b*) 4.4 N
(*c*) 2.4 N (*d*) 8.8 N1

6. In accordance with Kirchhoff's law:

(*a*) Bad absorber is bad emitter
(*b*) Bad absorber is good reflector
(*c*) Bad reflector is good emitter
(*d*) Bad emitter is good absorber

7. A wire of cross-sectional area 3mm^2 is first stretched between two fixed points at a temperature of 20°C. Determine the tension when the temperature falls to 10°C. Coefficient of linear expansion $\alpha = 10^{-5o}$ C^{-1} and $Y = 2 \times 10^{11}$ N/m^2

(*a*) 20 N (*b*) 30 N
(*c*) 60 N (*d*) 120 N

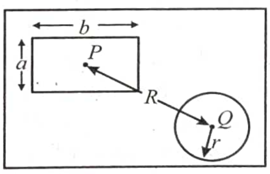
8. The coefficient of linear expansion of brass and steel are α_1 and α_2. If we take a brass rod of length l_1 and steel rod of length l_2 at 0°C, their difference in length $(l_2 - l_1)$ will remain the same at a temperature if

(*a*) $\alpha_1 l_2 = \alpha_2 l_1$ (*b*) $\alpha_1 l_2^2 = \alpha_2 l_1^2$
(*c*) $\alpha_1^2 l_1 = \alpha_2^2 l_2$ (*d*) $\alpha_1 l_1 = a_2 l_2$

9. A polished metallic piece and a black painted wooden piece are kept in open in bright sun for a long time:

(*a*) The wooden piece will absorbs less heat than the metallic piece
(*b*) The wooden piece will have a lower temperature than the metallic piece
(*c*) If touched, the metallic piece will feel hotter than the wooden piece
(*d*) When the two pieces are removed from the open to a cold room, the wooden piece will lose heat at a faster rate than the metallic piece

10. There is a rectangular metal plate in which two cavities in the shape of rectangle and circle are made, as shown with dimensions. P and Q are the centres of these cavities. On heating the plate, which of the following quantities increase?



(*a*) πr^2 (*b*) ab
(*c*) R (*d*) b

11. Four rods A, B, C, D of same length and material but of different radii r, $r\sqrt{2}$, $r\sqrt{3}$ and $2r$ respectively are held between two rigid walls. The temperature of all rods is increased by same amount. If the rods do not bend, then

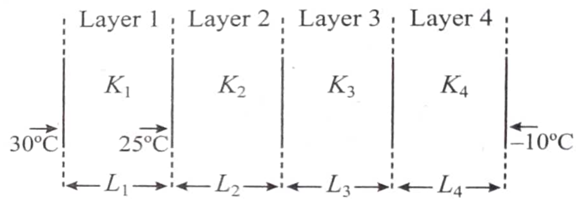
(*a*) The stress in the rods are in the ratio 1 : 2 : 3 : 4
(*b*) The force on the rod exerted by the wall are in the ratio 1 : 2 : 3 : 4
(*c*) The energy stored in the rods due to elasticity are in the ratio 1 : 2 : 3 : 4
(*d*) The strains produced in the rods are in the ratio 1 : 2 : 3 : 4

12. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%.

(*a*) Percentage rise in the area of a face is 0.4%
(*b*) Percentage rise in the thickness is 0.4%
(*c*) Percentage rise in the volume is 0.6%
(*d*) Coefficient of linear expansion of copper is 0.25×10^{-4}C^{o-1}.

COMPREHENSION BASED QUESTIONS

Comprehension–1 (No. 13 to 15): Figure shows in cross section a wall consisting of four layers with thermal conductivities K_1=0.06 W/m°C; K_3=0.04 W/m°C and K_4=0.10 W/m°C. The layer thicknesses are L_1 = 1.5 cm; L_3 = 2.8 cm and L_4 = 3.5 cm. The temperature of interfaces is as shown in figure. Energy transfer through the wall is steady.



13. The temperature of the interface between layers 3 and 4 is:

(*a*) – 1°C (*b*) – 3°C
(*c*) 2°C (*d*) 0°C

14. The temperature of the interface between layers 2 and 3 is:

(*a*) 11°C (*b*) 8°C
(*c*) 7.2°C (*d*) 5.4°C

15. If layer thickness L_2 is 1.4 cm, then its thermal conductivity K_2 will have value (in W/mK):

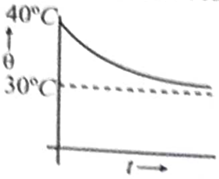
(*a*) 2×10^{-2} (*b*) 2×10^{-3}
(*c*) 4×10^{-2} (*d*) 4×10^{-3}

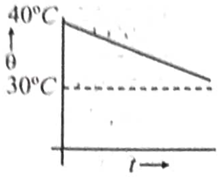
Comprehension–2 (No. 16 to 18): A body cools in a surrounding of constant temperature 30°C. Its heat capacity is 2J/°C. Initial temperature of the body is 40°C. Assume Newton's law of cooling is valid. The body cools to 38°C in 10 minutes.

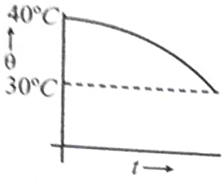
16. In further 10 minutes it will cool from 38°C to:

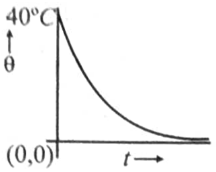
(a) 36°C
(b) 36.4°C
(c) 37°C
(d) 37.5°C

17. The temperature of the body in °C denoted by θ the variation of θ versus time t is best denoted as

(a)


(b)


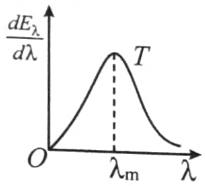
(c)


(d)


18. When the body temperature has reached 38°C, it is heated again so that it reaches to 40°C in 10 minutes. The total heat required from a heater by the body is:

(a) 3.6J
(b) 0.364J
(c) 8 J
(d) 4 J

Comprehension–3 (No. 19 to 21): The figure shows a radiant energy spectrum graph for a black body at a temperature T.



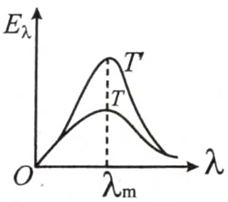
19. Choose the correct statement(s)

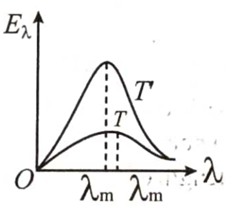
(a) The radiant energy is not equally distributed among all the possible wavelengths
(b) For a particular wavelength the spectral intensity is maximum
(c) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature
(d) None of these

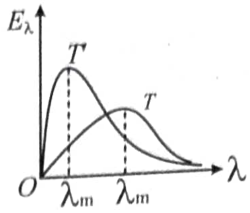
20. If the temperature of the body is raised to a higher temperature T,' then choose the correct statement(s)

(a) The intensity of radiation for every wavelength increases
(b) The maximum intensity occurs at a shorter wavelength
(c) The area under the graph increases
(d) The area under the graph is proportional to the fourth power of temperature

21. Identify the graph which correctly represents the spectral intensity versus wavelength graph at two temperatures T' and $T(T < T')$

(a)


(b)


(c)


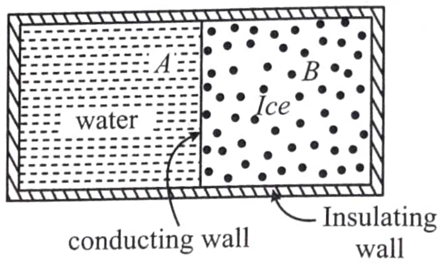
(d) None of these

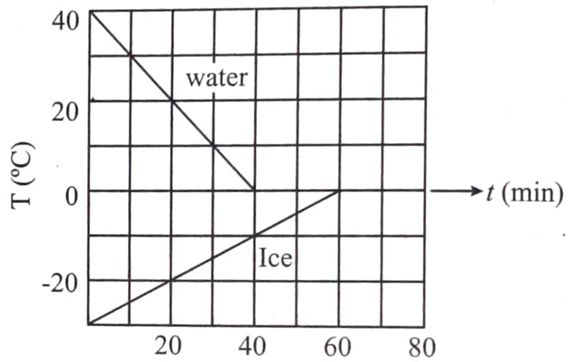
Comprehension–4 (No. 22 to 24): A 0.60 kg sample of wate and a sample of ice are placed in two compartments A an B that are separated by a conducting wall, in a thermall insulated container. The rate of heat transfer from the wate to the ice though the conducting wall is constant P, unt thermal equilibrium is reached. The temperature T of the liqui water and the ice are given in graph as functions of time Temperature of the compartments remain homogeneous durin whole heat transfer process.

Given specific heat of ice = 2100 J/kg-K

Given specific heat of water = 4200 J/kg-K

Latent heat of fusion of ice = 3.3×10^5 J/kg





22. The value of rate P is

(a) 42.0 W
(b) 36.0 W
(c) 21.0 W
(d) 18.0 W

23. The initial mass of the ice in the container is equal to

(a) 0.36 kg
(b) 1.2 kg
(c) 2.4 kg
(d) 3.6 kg

24. The mass of the ice formed due to conversion from the wate till thermal equilibrium is reached, is equal to

(a) 0.12 kg
(b) 0.15 kg
(c) 0.25 kg
(d) 0.40 kg

Comprehension–6 (No. 25 to 27): In a container of negligibl heat capacity, 200 gm ice at 0°C and 100 gm steam at 100° are added to 200 gm of water that has temperature 55°C Assume no heat is lost to the surroundings and the pressur in the container is constant 1.0 atm. (Latent heat of fusio of ice = 80 cal/gm, Latent heat of vaporization of wate

= 540 cal/gm, Specific heat capacity of ice = 0.5 cal/gm-K, Specific heat capacity of water = 1 cal/gm-K)

25. What is the final temperature of the system ?

(*a*) 48°C (*b*) 72°C

(*c*) 94°C (*d*) 100°C

26. At the final temperature, mass of the total water present in the system, is

(*a*) 472.6 gm (*b*) 483.3 gm

(*c*) 493.6 gm (*d*) 500 gm

27. Amount of the steam left in the system, is equal to

(*a*) 16.7 gm

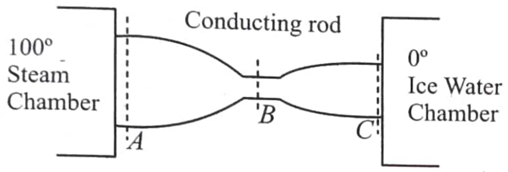
(*b*) 12.0 gm

(*c*) 8.4 gm

(*d*) 0 gm, as there is no steam left.

MATCH THE COLUMN TYPE QUESTIONS

28. A copper rod (initially at room temperature 20°C) of non-uniform cross section is placed between a steam chamber at 100°C and ice-water chamber at 0°C.



A.	Initially rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	p.	maximum at section *A*
B.	At steady state rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	q.	maximum at section *B*
C.	At steady state temperature gradient $\left\|\left(\frac{dT}{dx}\right)\right\|$ will be	r.	minimum at section *A*
D.	At steady state rate of change of temperature $\left(\frac{dT}{dt}\right)$ at a certain point will be	s.	minimum at section *B*
		t.	same for all section

(*a*) A → (p, s); B → (t); C → (q, r); D → (t)

(*b*) A → (t, r); B → (p); C → (s, q); D → (r)

(*c*) A → (r, s); B → (q); C → (s, r); D → (p)

(*d*) A → (t, q); B → (s); C → (q, p); D → (r)

29. In the following question column-I represents some physical quantities & column-II represents their units, match them

	Column-I		Column-II
A.	Coefficient of linear expansion	p.	Cal/°C
B.	Water equivalent	q.	gm
C.	Heat capacity	r.	$(°C)^{-1}$
D.	Specific heat	s.	Cal/g°C

(*a*) A → (q); B → (r); C → (s); D → (p)

(*b*) A → (r); B → (q); C → (p); D → (s)

(*c*) A → (p); B → (q); C → (r); D → (s)

(*d*) A → (s); B → (r); C → (q); D → (p)

30. Match the following two columns.

	Column-I		Column-II
A.	Stefan's constant	p.	$[L, \theta]$
B.	Wien's constant	q.	$(M^{-1}L^{-2}T^{+3}\theta^2)$
C.	Emissive power	r.	$[MT^{-3}]$
D.	Thermal resistance	s.	None of these

(*a*) A → (p); B → (q); C → (r); D → (s)

(*b*) A → (r); B → (q); C → (p); D → (s)

(*c*) A → (s); B → (p); C → (r); D → (s)

(*d*) A → (s); B → (r); C → (q); D → (p)

31. Match the following two columns.

	Column-I		Column-II
A.	Some liquid is filled in a container. On heating container height of liquid in container.	p.	Must increase
B.	When heat is supplied to ice at 0°*C*. Internal energy of the system.	q.	Must decrease
C.	The rate of heat loss from a body on increasing temperature of surrounding	r.	May increase
D.	On increasing temperature density of water	s.	May decrease
		t.	May be constant

(*a*) A → (r,s,t); B → (p); C → (q); D → (r,s)

(*b*) A → (q,r,s); B → (p); C → (t); D → (s)

(*c*) A → (p,q,r); B → (s); C → (r); D → (t)

(*d*) A → (s,t,p); B → (r); C → (q); D → (p)

NUMERICAL TYPE QUESTIONS

32. A 30 cm long metal rod expands by 0.0650 cm when its temperature is raised from 0°C to 100°C. A second rod of different metal and of the same length expands by 0.0350 cm for the same rise in temperature. A third composite rod, also 30 cm long, is made-up of pieces of each of the above metals placed end to end and expands by 0.0580 cm when temperature is increased from 0°C to 100°C. Find the length of the longer portion of the composite bar in cm at 0°C.

33. A steel rod at 25°C is welded at both ends and then cooled. By how many °C should the rod be cooled so that it will rupture? Assume that till rupture, Hooke's law is obeyed. ($\alpha_{steel} = 10 \times 10^{-6}/°C$, $Y = 2 \times 10^{11} N/m^2$ and σ_b (breaking stress of steel rod) $= 4 \times 10^8 N/m^2$)

34. In the past, temperature was measured in laboratories using the so called "weight" thermometer, which consists of a hollow platinum sphere filled with mercury and provided with a small hole. An increase in temperature was estimated from the amount of mercury flowing out of the hole. How much temperature (in °C) is increased if 0.0153% of mercury flows out of the hole of such a thermometer, if the completely filled sphere of thermometer contained mercury at 0°C. The coefficient of volume expansion of platinum is $\gamma_1 = 2.7 \times 10^{-5}/°C$ and of mercury is $\gamma_2 = 1.8 \times 10^{-4}/°C$.

35. A spherical black body of diameter 0.10 m is held at constant temperature. If the power radiated by the body is $5.67\pi \times 10^2 Js^{-1}$, the temperature of the body (in K) is

36. The filament of an incandescent lamp of power 64 W is made of Tungsten. The operation temperature of the lamp is 2000K. Consider the filament a black body and find its radius (in mm).

[Given : $\sigma = 6 \times 10^{-8} W/m^2$ & Length of filament is $\frac{10}{3\pi}$ cm]

37. A thermos contains 140 gm coffee at 80°C. To cool it, we drop two 10 gm ice cubes at 0°C into thermos. Ice cubes are at 0°C initially. What is the final temperature of the coffee (in °C), if its specific heat capacity is same as that of water.

38. A liquid A is kept flowing through a tube while it is cooled by water surrounding the tube. The temperature of the incoming liquid A is 60°C and that of the outgoing liquid A is 30°C. The capacity of the cooler tank is 60 liter and the water in it is completely changed in every hour. The initial temperature of the cooling water is 10°C and it is 20°C when it is changed. What is the amount of the liquid (in kg) that goes through the tube in one hour? (Take: Specific heat capacity of water $S_{water} = 4200$ J/kg-K, $S_{liquid\ A} = 140$ J/kg-K, density of water $= 1000 kg/m^3$)

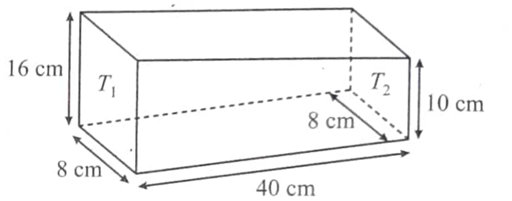
39. A metal rod AB of length $10x$ has its one end A in ice at 0°C, and the other end B in water at 100 °C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value λ. [Neglect any heat loss to the surrounding]

40. A piece of ice (heat capacity $= 2100 Jkg^{-1}°C$ and latent heat $= 3.36 \times 10^5 Jkg^{-1}$) of mass m grams is at $-5°C$ at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1gram of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

41. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

42. A long rod has one end at 0°C and other end at a high temperature. The coefficient of thermal conductivity varies with distance from the low temperature end as $k = k_0(1 + ax)$, where $kQ = 102$ S.I. unit and $\alpha = 1 m^{-1}$. At what distance from the first end the temperature will be 100°C? [The area of cross-section is $1 cm^2$ and rate of heat conduction is 1W.]

43. A slab of conductivity k is in the shape of trapezoid as shown in figure. If the two end faces be maintained at temperature $T_1 = 50°C$ and $T_2 = 25°C$, then determine the thermal current through the slab by ignoring any heat loss through the lateral surfaces is $\frac{xk}{\ln(8/5)}$ J/s ten x is



44. A rod of length L with thermal insulated lateral surface consists of material whose heat conductivity coefficient varies with temperature as $k = \frac{\alpha}{T}$ where α is a constant. The ends of the rod are kept at temperatures T_1 and T_2. If the function $T(x)$ where x is the distance from the end whose temperature is T_1, is $T = T_1\left(\frac{T_2}{T_1}\right)^{\frac{nx}{L}}$ then x is

Exercise-4 (Past Year Questions)

JEE MAIN

1. A pendulum clock loses 12 sec a day if the temperature is 40°C and gains 4 sec a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively: **(2016)**

(a) 25^0C; $\alpha = 1.85 \times 10^{-5}/°C$
(b) 60^0C; $\alpha = 1.85 \times 10^{-4}/°C$
(c) 30^0C; $\alpha = 1.85 \times 10^{-3}/°C$
(d) 55^0C; $\alpha = 1.85 \times 10^{-2}/°C$

2. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with

170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by: (Given : room temperature = 30°C, specific heat of copper = 0.1 cal/gm°C. **(2017)**

(*a*) 1250°C (*b*) 825°C
(*c*) 800°C (*d*) 885° C

3. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in ^{0}C, if this thermometer in the contact with the object reads $x_0/2$? **(2019)**

(*a*) 25 (*b*) 60
(*c*) 40 (*d*) 45

4. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water? [Specific heat Capacitites of water and metal are, respectively 4200 $Jkg^{-1}K^{-1}$ and 400 $Jkg^{-1}K^{-1}$] **(2019)**

(*a*) 15% (*b*) 30%
(*c*) 25% (*d*) 20%

5. When 100g of a liquid *A* at 100°C is added to 50g of a liquid *B* at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100g of liquid *A* at 100°C is added to 50g of liquid *B* at 50°C, will be: **(2019)**

(*a*) 85°C (*b*) 60°C
(*c*) 80°C (*d*) 70°C

6. An unknown metal of mass 192 g heated to a temperature of 100ºC was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J kg^{-1} K^{-1}): **(2019)**

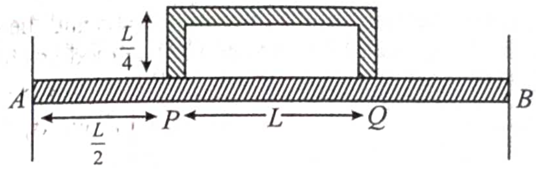
(*a*) 458 J kg^{-1} K^{-1}
(*b*) 1232 J kg^{-1} K^{-1}
(*c*) 916 J kg^{-1} K^{-1}
(*d*) 654 J kg^{-1} K^{-1}

7. Ice at – 20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to

(Specific heat of water = 4.2 J/g/°C). Heat of fusion of water at 0°C = 334 J/g) specific heat of ice = 2.1 J/g/°C **(2019)**

(*a*) 50 g (*b*) 100 g
(*c*) 60 g (*d*) 40 g

8. Temperature difference of 120ºC is maintained between two ends of a uniform rod *AB* of length 2L. Another bent rod *PQ*, of same cross-section as *AB* and length $\frac{3L}{2}$, is connected across *AB* (See figure). In steady state, temperature difference between *P* and *Q* will be close to: **(2019)**



(*a*) 45º C (*b*) 75º C
(*c*) 60º C (*d*) 35º C

9. A cylinder of radius *R* is surrounded by a cylindrical shell of inner radius *R* and outer radius 2*R*. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is: **(2019)**

(*a*) $\frac{K_1 + K_2}{2}$ (*b*) $K_1 + K_2$
(*c*) $\frac{2K_1 + 3K_2}{5}$ (*d*) $\frac{K_1 + 3K_2}{4}$

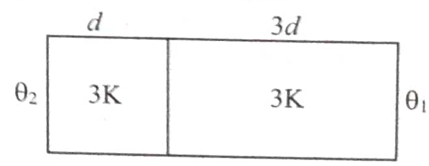
10. A heat source at $T = 10^3$ *K* is connected to another heat reservoir at $T = 10^2$ *K* by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 W $K^{-1}m^{-1}$, the energy flux through it in the state is: **(2019)**

(*a*) 90 Wm^{-2} (*b*) 120 Wm^{-2}
(*c*) 65 Wm^{-2} (*d*) 200 Wm^{-2}

11. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K) **(2019)**

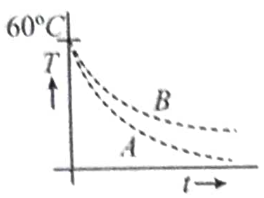
(*a*) 10^{-3} K (*b*) 10^{-4} K
(*c*) 10^{-1} K (*d*) 10^{-5} K

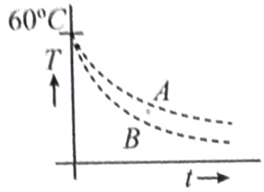
12. Two materials having coefficients of thermal conductivity '3*K*' and '*K*' and thickness '*d*' and '3*d*', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is: **(2019)**

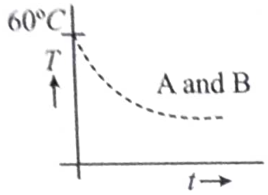


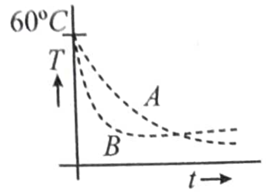
(*a*) $\frac{\theta_2 + \theta_1}{2}$ (*b*) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$
(*c*) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$ (*d*) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$

13. Two identical breakers *A* and *B* contain equal volumes of two different liquids at 60ºC each and left to cool down. Liquid in *A* has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same) **(2019)**

(a) 

(b) 

(c) 

(d) 

14. A thermally insulated vessel contains 150g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to:
(Latent heat of vaporization of water $= 2.10\times10^6$ J kg^{-1} and Latent heat of Fusion of water $= 3.36\times10^5$ J kg^{-1}) **(2019)**

(a) 130 g (b) 35 g
(c) 20 g (d) 150 g

15. When M_1 gram of ice at –10° C (specific heat = 0.5 cal g^{-1} °C^{-1}) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is: **(2019)**

(a) $\dfrac{5M_1}{M_2}-50$ (b) $\dfrac{50M_2}{M_1}$
(c) $\dfrac{50M_2}{M_1}-5$ (d) $\dfrac{5M_2}{M_1}-5$

16. M grams of steam at 100°C is mixed with 200g of ice at its melting point in a thermally insulated container. If the produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is ________. **(2020)**

17. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process) **(2020)**

C_1	C_2	C_3	T
1ℓ	2ℓ	–	$60°C$
–	1ℓ	2ℓ	$30°C$
2ℓ	–	1ℓ	$60°C$
1ℓ	1ℓ	1ℓ	θ

The value of θ (in °C to the nearest integer) is ______

18. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of 'm' is close to
(Latent heat of water = 540 cal g^{-1}, Specific heat of water = 1 cal g^{-1} °C^{-1}] **(2020)**

(a) 2 (b) 3.2
(c) 2.6 (d) 4

19. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? **(2020)**

(a) 6 (b) 7
(c) 3 (d) 5

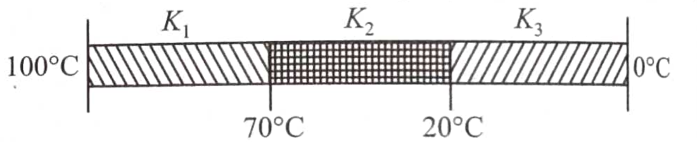
20. The specific heat of water = 4200 J kg^{-1} K^{-1} and the latent heat of ice $= 3.4\times10^5$ J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams) **(2020)**

(a) 69.3 (b) 63.8
(c) 64.6 (d) 61.7

21. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to **(2020)**

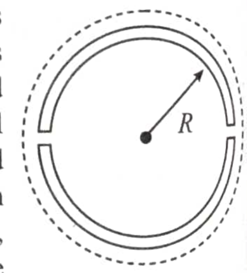
(a) 28°C (b) 35°C (c) 33°C (d) 31°C

22. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is **(2020)**



(a) $K_1 : K_3 = 2:3$; $K_2 : K_3 = 2:5$
(b) $K_1 < K_2 < K_3$
(c) $K_1 : K_2 = 5:2$; $K_1 : K_3 = 3:5$
(d) $K_1 > K_2 > K_3$

23. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is: **(2012)**



(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$
(c) $\pi SY\alpha\Delta T$ (d) $2SY\alpha\Delta T$

24. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by: **(2017)**

(a) $\frac{3\alpha}{PK}$ (b) $3PK\alpha$
(c) $\frac{P}{3\alpha K}$ (d) $\frac{P}{\alpha K}$

25. Two rods A and B of identical dimensions are at temperature 30°C. If A heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of lienar expansion of A and B is 4 : 3, then the value of T is: **(2019)**

(a) 230°C (b) 270°C
(c) 200°C (d) 250°C

26. A rod of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when it temperature rises by ΔT°K. Young's modulus, Y, for this metal is **(2019)**

(a) $\frac{F}{A\alpha\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T-273)}$
(c) $\frac{F}{2A\alpha\Delta T}$ (d) $\frac{2F}{A\alpha\Delta T}$

27. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultanously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equals to: **(2019)**

(a) $F/(3\pi r^2 YT)$ (b) $3F/(\pi r^2 YT)$
(c) $6F/(\pi r^2 YT)$ (d) $9F/(\pi r^2 YT)$

28. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to:

(Coefficient of linear expansion and Young' s modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively; $g = 10$ ms^{-2}) **(2019)**

(a) 1.5 kg (b) 9 kg
(c) 0.9 kg (d) 0.5 kg

29. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(\text{beaker})} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(\text{mercury})} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to ________. **(2020)**

30. A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to

(Given bulk modulus of metal, $B = 8 \times 10^{10}$ Pa) **(2020)**

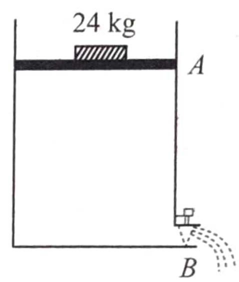
(a) 0.6 (b) 20
(c) 1.67 (d) 5

31. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is **(2020)**

(a) $2\sqrt{\alpha_1\alpha_2}$ (b) $\frac{4\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2L_1}{(L_2+L_1)^2}$
(c) $\frac{\alpha_1+\alpha_2}{2}$ (d) $\frac{\alpha_1L_1+\alpha_2L_2}{L_1+L_2}$

32. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross-section area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}.

The value of v, to the nearest integer, is ________ [Take value of g to be 10 ms^{-2}] **(2021)**



33. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm.

Then the thickness of oleic acid Layer will be $x \times 10^{-14}$ m. Where x is **(2021)**

34. At what temperature a gold ring of diameter 6.230 cm be heated so that it can be fitted on a wooden bangle of diameter 6.241 cm? Both the diameters have been measured at room temperature (27°C). (Given: coefficient of linear thermal expansion of gold $\alpha_L = 1.4 x 10^{-5}$ K^{-1}) **(2022)**

(a) 125.7°C (b) 91.7°C
(c) 425.7° (d) 152.7°C

35. A solid metallic cube having total surface area 24 m^2 is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube (Given : $\alpha = 5.0 x 10^{-4}$ °C^{-1}) **(2022)**

(a) 2.4×10^6 cm^3 (b) 1.2×10^5 cm^3
(c) 6.0×10^4 cm^3 (d) 4.8×10^5 cm^3

36. A copper block of mass 5.0 kg is heated to a temperature of 500°C and is placed on a large ice block. What is the maximum amount of ice that can melt? [Specific heat of copper: 0.39 J g^{-1} °C^{-1} and latent heat of fusion of water : 335 J g^{-1} °C^{-1} **(2022)**

(*a*) 1.5 kg (*b*) 5.8 kg
(*c*) 2.9 kg (*d*) 3.8 kg

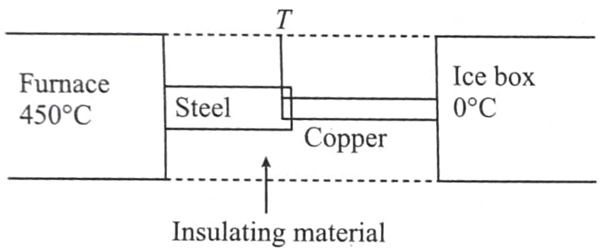
37. A block of ice of mass 120 g at temperature 0°C is put in 300 gm of water at 25°C. The xg of ice melts as the temperature of the water reaches 0°C. The value of x is

[Use: Specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$. Latent heat of ice = 3.5×10^5 Jkg^{-1}) **(2022)**

38. A steam engine intakes 50g of steam at 100°C per minute and cools it down to 20°C. If latent heat of vaporization of steam is 540 cal g^{-1}. then the heat rejected by the steam engine per minute is________ $\times 10^3$ cal. **(2022)**

39. If K_1 and K_2 are the thermal conductivities L_1 and L_2 are the lengths and A_1 and A_2 are the cross sectional areas of steel and copper rods respectively such that $\frac{K_2}{K_1} = 9, \frac{A_1}{A_2} = 2, \frac{L_1}{L_2} = 2$

Then, for the arrangement as shown in the figure. The value of temperature T of the steel – copper junction in the steady state will be : **(2022)**



(*a*) 18°C (*b*) 14 °C
(*c*) 45°C (*d*) 150 °C

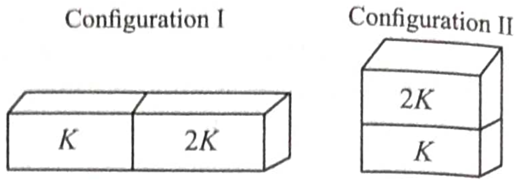
JEE ADVANCED

40. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is **(2012)**

(*a*) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (*b*) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$

(*c*) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (*d*) $(97)^{\frac{1}{4}} T$

41. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or II as shown in the figure, On of the blocks has thermal conductivity K and the other $2K$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is: **(2013)**

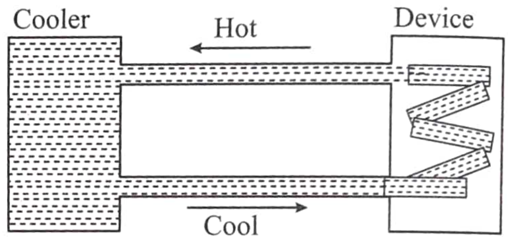


(*a*) 2.0 s (*b*) 3.0 s
(*c*) 4.5 s (*d*) 6.0 s

42. Parallel rays of light of intensity $I = 912$ Wm^{-2} are incident a spherical black body kept in surroundings of temperat 300 K. Take Stefan-Biltzmann constant $\sigma = 5.7 \times 1($ Wm^{-2} K^{-4} and assume that the energy exchange with surroundings is only through radiation. The final steady st temperature of the black body is close to: **(201**

(*a*) 330 K (*b*) 660 K
(*c*) 990 K (*d*) 1550 K

43. A water cooler of storage capacity 120 litres can cool water a constant rate of P watts. In a closed circulation system (shown schematically in the figure), the water from the cool is used to cool an external device that generates constant 3 kW of heat (thermal load). The temperature of water f into the device cannot exceed 30°C and the entire stored 1 litres of water is initially cooled to 10°C. The entire syste is thermally insulated. The minimum value of P (in watt for which the device can be operated for 3 hours is **(201**



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density c water is 1000 kgm^{-3})

(*a*) 1600 (*b*) 2067
(*c*) 2533 (*d*) 3933

44. An incandescent bulb has a thin filament of tungsten that i heated to high temperature by passing an electric current The hot filament emits black-body radiation. The filament i observed to break up at random locations after a sufficientl long time of operation due to non-uniform evaporation o tungsten from the filament. If the bulb is powered at constan voltage, which of the following statement(s) is (are) true **(2016**

(*a*) The temperature distribution over the filament is uniforn

(*b*) The resistance over small sections of the filamen decreases with time

(*c*) The filament emits more light at higher band of frequencies before it breaks up

(*d*) The filament consumes less electrical power towards the end of the life of the bulb.

45. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2$ (P/P_0), where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C? **(2016)**

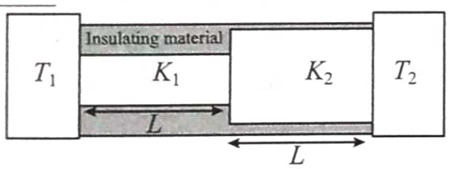
46. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of the linear thermal expansion of PQ is $1.2 \times 10^{-5}\ K^{-1}$, the change in length of the wire PQ is **(2016)**

(*a*) 0.78 mm (*b*) 0.90 mm
(*c*) 1.56 mm (*d*) 2.34 mm

47. A human body has surface area of approximately 1m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be $T_0 = 300$ K. For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460 Wm^{-2}$ (where σ is the Stefan-Boltzmann constant.).
Which of the following options is /are correct? **(2017)**

(*a*) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.
(*b*) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
(*c*) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths
(*d*) The amount of energy radiated by the body in 1 second is close to 60 joules

48. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperature T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady is 200 K, then K_1/K_2 = ______ **(2018)**



49. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as: **(2019)**

$T(t) = T_0(1 + \beta t^{1/4})$

Where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature.
The heat capacity of the metal is :

(*a*) $\dfrac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$ (*b*) $\dfrac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

(*c*) $\dfrac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (*d*) $\dfrac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

50. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be _______ °C.
[Neglect the heat exchange with surrounding] **(2019)**

51. The filament of a light bulb has surface area 64 mm^2. The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m. Assume the pupil of the eyes of the observer to be circular with radius 3 mm. Then
(Take Stefan-Boltzmann constant = 5.67×10^{-8} $\text{Wm}^{-2}\ \text{K}^{-4}$, Wien's displacement constant = 2.90×10^{-3} m-K, Planck's constant = 6.63×10^{-34} Js, speed of light in vacuum = $3.00 \times 10^8\ \text{ms}^{-1}$)- **(2020)**

(*a*) Power radiated by the filament is in the range 642 W to 645 W
(*b*) Radiated power entering into one eye of the observer is in the range 3.15×10^{-8} W to 3.25×10^{-8} W
(*c*) The wavelength corresponding to the maximum intensity of light is 1160 nm
(*d*) Taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75×10^{11} to 2.85×10^{11}

52. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm^{-2} and it is absorbed by the water over an effective area of 0.05 m^2. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be ____________.
(Ignore effect of the container, and take constant for Newton's law of cooling = $0.001\ \text{s}^{-1}$, Heat capacity of water = $4200\ \text{J kg}^{-1}\ \text{K}^{-1}$) **(2020)**

ANSWER KEY

CONCEPT APPLICATION

1. (*b*)	**2.** (*a*)	**3.** (*b*)	**4.** (*b*)	**5.** (*a*)	**6.** (*d*)	**7.** (*d*)	**8.** (*d*)	**9.** (*b*)	**10.** (*a*)
11. (*a*)	**12.** (*b*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)	**16.** (*b*)	**17.** (*b*)	**18.** (*b*)	**19.** (*c*)	**20.** (*b*)
21. (*d*)	**22.** (*a*)	**23.** (*c*)	**24.** (*b*)	**25.** (*c*)	**26.** (*d*)	**27.** (*c*)	**28.** (*b*)	**29.** (*c*)	**30.** (*d*)
31. (*b*)									

EXERCISE-1 (TOPICWISE)

1. (*c*)	**2.** (*d*)	**3.** (*d*)	**4.** (*a*)	**5.** (*a*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*b*)
11. (*d*)	**12.** (*b*)	**13.** (*a*)	**14.** (*b*)	**15.** (*c*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)	**19.** (*d*)	**20.** (*d*)
21. (*c*)	**22.** (*a*)	**23.** (*c*)	**24.** (*d*)	**25.** (*b*)	**26.** (*c*)	**27.** (*a*)	**28.** (*a*)	**29.** (*c*)	**30.** (*a*)
31. (*a*)	**32.** (*b*)	**33.** (*a*)	**34.** (*b*)	**35.** (*c*)	**36.** (*d*)	**37.** (*b*)	**38.** (*a*)	**39.** (*d*)	**40.** (*d*)
41. (*c*)	**42.** (*c*)	**43.** (*c*)	**44.** (*a*)						

EXERCISE-2 (LEARNING PLUS)

1. (*c*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*b*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*a*)
11. (*d*)	**12.** (*d*)	**13.** (*d*)	**14.** (*c*)	**15.** (*a*)	**16.** (*b*)	**17.** (*d*)	**18.** (*b*)	**19.** (*d*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*a*)	**24.** (*a*)	**25.** (*d*)	**26.** (*b*)	**27.** (*c*)	**28.** (*c*)	**29.** (*a*)	**30.** (*a*)
31. (*a*)	**32.** (*c*)	**33.** (*a*)	**34.** (*b*)	**35.** (*b*)	**36.** (*a*)	**37.** (*a*)	**38.** (*d*)	**39.** (*b*)	**40.** (*a*)
41. (*d*)	**42.** (*a*)	**43.** (*a*)	**44.** (*c*)	**45.** (*d*)	**46.** (*c*)	**47.** (*a*)			

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*d*)	**2.** (*a,b,c*)	**3.** (*a,b*)	**4.** (*b,c*)	**5.** (*d*)	**6.** (*a,b,c*)	**7.** (*c*)	**8.** (*d*)	**9.** (*c,d*)	
10. (*a,b,c,d*)	**11.** (*b,c*)	**12.** (*a,c,d*)	**13.** (*b*)	**14.** (*a*)	**15.** (*a*)	**16.** (*b*)	**17.** (*a*)	**18.** (*c*)	**19.** (*a,b*)
20. (*a,b,c,d*)	**21.** (*b*)	**22.** (*a*)	**23.** (*c*)	**24.** (*b*)	**25.** (*d*)	**26.** (*b*)	**27.** (*a*)	**28.** (*a*)	**29.** (*b*)
30. (*c*)	**31.** (*a*)	**32.** [0023]	**33.** [0200]	**34.** [0001]	**35.** [1000]	**36.** [0001]	**37.** [0060]	**38.** [600]	**39.** [9]
40. [8]	**41.** [9]	**42.** [1.71]	**43.** [30]	**44.** [1]					

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*a*)	**2.** (*d*)	**3.** (*a*)	**4.** (*d*)	**5.** (*c*)	**6.** (*c*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*a*)
11. (*d*)	**12.** (*b*)	**13.** (*a*)	**14.** (*c*)	**15.** (*c*)	**16.** [40]	**17.** [50]	**18.** (*a*)	**19.** (*a*)	**20.** (*d*)
21. (*c*)	**22.** (*a*)	**23.** (*d*)	**24.** (*c*)	**25.** (*a*)	**26.** (*a*)	**27.** (*b*)	**28.** (*b*)	**29.** [20.00]	**30.** (*c*)
31. (*d*)	**32.** [3]	**33.** [25]	**34.** (*d*)	**35.** (*b*)	**36.** (*c*)	**37.** [90]	**38.** [31]	**39.** (*c*)	

JEE Advanced

40. (*c*)	**41.** (*a*)	**42.** (*a*)	**43.** (*b*)	**44.** (*c,d*)	**45.** [9]	**46.** (*a*)	**47.** (*b*)	**48.** [4.00]	**49.** (*a*)
50. [270.00]	**51.** (*b,c,d*)	**52.** [8.33]							

CHAPTER

16 Kinetic Theory of Gases and Thermodynamics

IDEAL GAS

Ideal gases are molecules having definite mass but undefined volume. It has no shape and size and can be contained in a vessel of any shape or size.

IDEAL GAS EQUATION

For n mole $\quad PV = nRT$

For 1 gm. mole $\quad PV = RT$

$PV = N_A kT \quad (N_A \cdot k = R)$

[k is Boltzman's constant]

For one gram of gas $PV = \dfrac{RT}{M_w}$

Values of R, its unit and dimensions:

- $R = 2$ Calorie/mol/°C,
- $R = 8.31$ Joule/mol/°C
- $R = 8.31 \times 10^7$ erg/mol/°C

Unit S I: Joule/mol/K.

Dimensions: $ML^2T^{-2}\theta^{-1}$ mol^{-1}

$M = mN$

m : Mass of the gas molecule

N : Number of molecule in gas

M : Mass of the gas

Some Definition

Avogadro Number (N_A) :

The number of molecules present in 1 gm mole of a gas is defined as Avogadro's Number.

$N_A = 6.02 \times 10^{23}$ per gm/mole

Moleculer weight (M_w) :

Mass of one mole gas is known as molecular weight of gas.

Number of moles

$$n = \frac{\text{mass of gas}}{\text{mol wt. of gas}} \quad \text{or} \quad n = \frac{M}{M_w}$$

If $n = 1$ mol then $M = M_w$

$$\Rightarrow M_w = N_A \cdot m \quad \Rightarrow m = \frac{M_w}{N_A}$$

KINETIC THEORY OF GASES

The kinetic theory of gases is based on the following assumptions:

- All the molecules of a gas are identical as regarding their shape and mass. The molecules of different gases are different.
- The molecules are rigid and perfectly elastic spheres of very small diameters.
- Gas molecules occupy very small space. The actual volume occupied by the molecule is very small compared to the total volume of the gas. Therefore volume of the gas is equal to volume of the vessel.
- The molecules are in a state of random motion, i.e., they are constantly moving with all possible velocities in all possible directions.
- Normally no force acts between the molecules. Hence they move in straight lines with constant speeds.
- The molecules can have all possible velocities lying between zero and infinity.
- The molecules collide with one another and also with the walls of the container and change their direction and speed due to collision. These collisions are perfectly elastic i.e., there is no loss of kinetic energy in these collisions.
- The molecules do not exert any force of attraction or repulsion on each other except during collision. So, the molecules do not posses any potential energy. Their energy is total kinetic.
- The collisions are instantaneous i.e., the time spent by a molecule in a collision is very small as compared to the time elapsed between two consecutive collisions.
- Though the molecules are constantly moving from one place to another, the average number of molecules per unit volume of the gas remains constant.
- The molecules inside the vessel keep on moving continuously in all possible directions, the distribution of molecules in the whole vessel remains uniform.
- The mass of a molecule is negligibly small and the speed is very large, there is no effect of gravity on the motion of the molecules. If this effect were there, the density of the gas would have been greater at the bottom of the vessel.

Train Your Brain

Example 1: The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be:

(*a*) 2.5×10^{25} (*b*) -2.5×10^{25}
(*c*) -1.61×10^{23} (*d*) 1.38×10^{25}

Sol. (*b*) Using ideal gas equation

$PV = nRT$ (N is number of moles)

$P_0V_0 = n_iR \times 290$ ..... (*i*)

$[T_i = 273 + 17 = 290\ K]$

After heating

$P_0V_0 = n_fR \times 300$ .... (*ii*)

$[T_f = 273 + 27 = 300\ K]$

from equation (*a*) and (*b*), $n_f - n_i = \dfrac{P_0V_0}{R\times 300} - \dfrac{P_0V_0}{R\times 290}$

difference in number of moles $-\dfrac{P_0V_0}{R}\left[\dfrac{10}{290\times 300}\right]$

Hence $n_f - n_i$ is $= -\dfrac{P_0V_0}{R}\times\left[\dfrac{10}{290\times 300}\right]\times 6.023\times 10^{23}$

putting $P_0 = 10^5$ *PA* and $V_0 = 30$ m^3

Number of molecules $n_f - n_i = -2.5 \times 10^{25}$

Example 2: The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 *K* and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$ then value of C in (*K*/atm.) is ______.

Sol. [150] For temperature to be constant

$PV = NRT$

$\Rightarrow\ PdV + VdP = 0 \Rightarrow |\Delta V| = \left|\dfrac{VdP}{P}\right|$

And at constant pressure

$PdV = NRdT \Rightarrow |dV| = \left|\dfrac{NRdT}{P}\right|$

$\dfrac{VdP}{P} = \dfrac{NRdT}{P}$

$\Rightarrow VdP = NRC\,dP$

$\therefore\ V = NRC$

$\Rightarrow C = \dfrac{V}{NR} = \dfrac{T}{P}$

$\Rightarrow\ C = \dfrac{300K}{2\text{ atm}} = 150\left(\dfrac{K}{\text{atm}}\right)$

Example 3: A balloon is filled at 27°C and 1 atm pressure by 500 m^3 He. At –3°C and 0.5 atm pressure, the volume of He gas contained in balloon will be

(*a*) 500 m^3 (*b*) 850 m^3
(*c*) 900 m^3 (*d*) 895 m^3

Sol. (*c*) $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

$V_2 = \dfrac{P_1V_1T_2}{P_2T_1}$

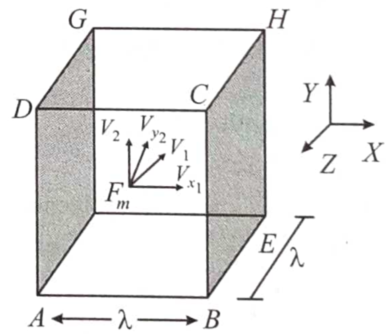
$V_2 = \dfrac{1\times 500\times(273-3)}{0.5\times(273+27)} = 900\ \text{m}^3$

Concept Application

1. Consider the quantity MkT/pV of an ideal gas where M is the mass of the gas. It depends on the
   (*a*) Temperature of the gas
   (*b*) Volume of the gas
   (*c*) Pressure of the gas
   (*d*) Nature of the gas
2. A gas behaves more closely like an ideal gas at
   (*a*) Low pressure and low temperature
   (*b*) Low pressure and high temperature
   (*c*) At all pressure and temperature
   (*d*) None of these
3. Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3×10^5 pascal
   (*a*) 2.4×10^{23} (*b*) 3×10^{23}
   (*c*) 6×10^{23} (*d*) 4.8×10^{23}

PRESSURE EXERTED BY AN IDEAL GAS ON THE WALL OF CONTAINER

Let us suppose that a gas is enclosed in a cubical box having length ℓ. Let there are '*N*' identical molecules, each having mass '*m*'. Only collisions with the walls of the container contribute to the pressure by the gas molecules. Let us focus on a molecule having velocity v_1 and components of velocity $v_{x_1}, v_{y_1}, v_{z_1}$ along *x*, *y* and *z*-axis as shown in figure.



$$v_1^2 = v_{x_1}^2 + v_{y_1}^2 + v_{z_1}^2$$

The change in momentum of the molecule after one collision with wall *BCHE* $= mv_{x_1} - (-mv_{x_1}) = 2mv_{x_1}$.

The time taken between the successive impacts on the face $BCHE$

$$BCHE = \frac{\text{distance}}{\text{velocity}} = \frac{2\ell}{v_{x_1}}$$

Time rate of change of momentum due to collision

$$= \frac{\text{change in momentum}}{\text{time taken}} = \frac{2mv_{x_1}}{2\ell / v_{x_1}} = \frac{mv^2 x_1}{\ell}$$

Hence the net force on the wall $BCHE$ due to the impact of nN' molecules of the gas is:

$$F_x = \frac{mv_{x_1}^2}{\ell} + \frac{mv_{x_2}^2}{\ell} + \frac{mv_{x_3}^2}{\ell} + \ldots\ldots + \frac{mv_{x_n}^2}{\ell}$$

$$= \frac{m}{\ell}\left(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + \ldots.. + v_{x_n}^2\right) = \frac{mN}{\ell} < v_x^2 > = \text{mean}$$

Where $\langle v_x^2 \rangle$ = square velocity in x - direction. Since molecules do not favour any particular direction therefore $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$

But $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$

$$\Rightarrow \langle v_x^2 \rangle = \frac{\langle v^2 \rangle}{3}$$

Pressure is equal to force divided by area. Therefore $P = \frac{F_x}{\ell^2} = \frac{M}{3\ell^3}\langle v^2 \rangle = \frac{M}{3V}\langle v^2 \rangle$. Pressure is independent of x, y, z directions

where ℓ^3= of the container = V

M = total mass of the gas, $< v^2 >$ = mean square velocity of molecules

$P = \frac{1}{3}\langle v^2 \rangle$ from $PV = nRT$

$$\because n = \frac{\text{Mass}}{\text{Molecular Weight}} = \frac{M}{M_0} \text{(in kg /mole)}$$

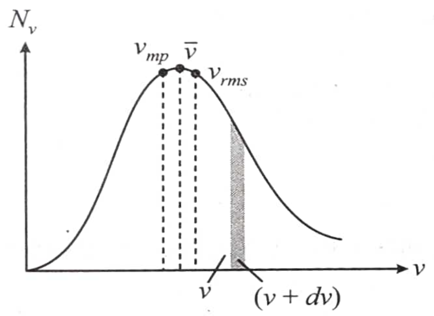
$$P = \frac{M}{M_0 V} RT = \frac{\rho RT}{M_0} \Rightarrow \frac{\rho RT}{M_0}; \quad V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$P = \frac{1}{3}\rho V_{rms}^2$$

$$v_{rms} = \sqrt{\frac{3RT}{M_0}} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3kT}{m}}$$

$k = \frac{R}{N_A}$ is called Boltzman's constant.

Maxwell Law for Distribution of Molecular Speed



From the graph, following important conclusions emerge:

(*i*) The area under the graph represents the total number of molecules.

Area under the graph $= \int N_v dv = \int \frac{dN}{dv} dv = N$

(*ii*) The peak of the curve suggests that there exists a speed corresponding to which the number of molecules is maximum. This speed is called the most probable speed, v_{mp}.

Most Probable Speed :

The most probable speed v_p or v_{mp} is the speed possessed by the maximum number of molecules, and corresponds to the maximum (peak) of the distribution curve. Mathematically, it is obtained by the condition.

$\frac{dN(v)}{dv} = 0$ [by substitution of formula of $dN(v)$ (which is not in the course)]

Hence the most probable speed is $v_p = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M_0}}$

From the above expression, we can see that $v_{rms} > \overline{v} < v_p$.

$R = 8.314$ J/mole

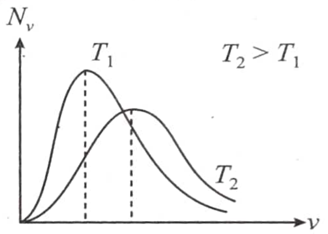
k = Boltzmann counstant ($k = 1.38 \times 10^{-23}\, JK^{-1}$)

(*iii*) The shape of the curve is such that area (shown shaded) enclosed by its portion on right side of v_{mp} is more than the area on the left side of v_{mp}. Thus, the number of molecules having speeds less than v_{mp} is less than the number of molecules having speeds more than v_{mp}.

(*iv*) The number of molecules with too low and too high speeds is very small as the function N_v is zero when $v = 0$ and approaches zero as v approaches infinity.

(*v*) The number of molecules in the range v and $(v + dv)$ is equal to the area of the shaded rectangle.

(*vi*) As the temperature increases,



(*a*) the distribution curve shifts to the right and average speed increases.

(*b*) the distribution curve broadens.

Different Types of Speeds of Gas Molecules

Most probable speed v_{mp}: It is the speed which maximum number of molecules in a gas have at constant temperature and is:

$$v_{mp} = \sqrt{\frac{2RT}{M_w}} = \left(\sqrt{\frac{2}{3}}\right) v_{\text{rms}} = 0.816\, v_{\text{rms}}$$

Average speed v_{av}: It is the arithmetic mean of the speeds of molecules in a gas at a given temperature, i.e,

$$v_{av} = \frac{v_1 + v_2 + v_3 + \ldots. + v_N}{N}$$

and according to kinetic theory of gases

$$v_{av} = \sqrt{\frac{8RT}{\pi M_w}} = \left(\sqrt{\frac{8}{3\pi}}\right) v_{\text{rms}} = 0.92\, v_{\text{rms}}$$

Root Mean Square Velocity (R.M.S. Velocity)

The square root of the average of the squares of the velocities of gas molecules is called the R.M.S. velocity.

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + ... + v_N^2}{N}} = \sqrt{\overline{v^2}}$$

Note : $v_{rms} = \sqrt{\frac{3RT}{m}} = 1.73\sqrt{\frac{RT}{m}}$

$$\bar{v} = \sqrt{\frac{8RT}{\pi m}} = 1.60\sqrt{\frac{RT}{m}} = 0.92v_{rms}$$

$$v_{mp} = \sqrt{\frac{2RT}{m}} = 1.41\sqrt{\frac{RT}{m}} = 0.82v_{rms}$$

Thus, $v_{rms} > \bar{v} > v_{mp}$.

Relation between Pressure and Kinetic Energy

$\because\ P = \frac{1}{3}\frac{Nm}{V}v_{rms}^2$ or $PV = \frac{2}{3}N\left(\frac{1}{2}mv_{rms}^2\right)$

$\therefore\ N\left(\frac{1}{2}mv_{rms}^2\right) = \frac{3}{2}PV$

But $\frac{1}{2}mv_{rms}^2$ = average K.E. of a gas molecule.

$\therefore$ Total K.E. of a gas $E = N\left(\frac{1}{2}mv_{rms}^2\right)$

$E = \frac{3}{2}PV$

$\therefore$ K.E. per unit volume of the gas $E = \frac{3}{2}P$

The pressure exerted by a gas is numerically equal to $\frac{2}{3}$rd of the kinetic energy of the molecules present per unit volume of the gas.

KINETIC INTERPRETATION OF TEMPERATURE

Mean Kinetic Energy

According to kinetic theory of gases

$$PV = \frac{1}{3}mN\overline{v^2} \text{ or } PV = nRT$$

So $\frac{1}{3}mN\overline{v^2} = nRT$ or $\frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{nRT}{N}$

As $n = \frac{\text{mass}}{\text{mol. wt.}} = \frac{Nm}{M}$ and $R = N_A k$

$\therefore\ \frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{Nm}{M}\frac{N_A kT}{N} = \frac{3}{2}kT \quad (\because N_A m = M)$

i.e., Translational KE of a molecule

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

Mean kinetic energy per molecule, $E = \frac{3}{2}kT$.

Further, mean kinetic energy per mole is given by :

$$E_{\text{mole}} = \left[\frac{1}{2}m\overline{v^2}\right]N_A = \frac{3}{2}kTN_A$$

$$E_{\text{mole}} = \frac{3}{2}RT$$

- Average translational KE of a gas molecule depends only on its temperature and is independent of its nature, i.e., molecules of different gases say He, H_2 and O_2, etc, at same temperature will have same translational kinetic energy though their rms speeds are different.
- Mean kinetic energy of molecules is zero at absolute zero if gas continues to remain a gas at that temperature.

KINETIC INTERPRETATION OF PRESSURE

According to kinetic theory of gases,

$$P = \frac{1}{3}\frac{mN}{V}(\overline{v^2}) \text{ with } \overline{v^2} \propto T$$

So the pressure of a gas is controlled by following three independent factors:

- **If volume and temperature of a gas are constant,** $P \propto mN$, i.e., $P \propto$ mass of gas (as mN = mass of gas), i.e., if mass of gas is increased, number of molecules and hence number of collisions per sec increases, i.e., pressure will increase.
- **If mass and temperature of a gas are constant,** $P \propto \frac{1}{V}$, i.e., if volume decreases, number of collisions per sec will increase due to lesser effective distance between the walls resulting in greater pressure.
- **If mass and volume of gas are constant,** $P \propto \overline{v^2} \propto T$, i.e., if temperature increases, the mean square speed of gas molecules will increase and as gas molecules are moving faster, they will collide with the walls more often with greater momentum resulting in greater pressure.

KINETIC INTERPRETATION OF EVAPORATION

Evaporation is a slow process which takes place from the surface of a liquid at all temperatures.

Evaporation depends on:

(*a*) nature of liquid (boiling point)

(*b*) temperature of liquid

(*c*) temperature of surroundings

(*d*) air currents

(*e*) relative humidity

(*f*) pressure

Cooling Results Due to Process of Evaporation

According to kinetic theory of gases at a given temperature in a liquid, molecules are moving in all possible directions with all possible speeds; so at all temperatures high speed molecules can escape from the surface, i.e., evaporation takes place at all temperatures.

As high speed molecules have escaped, the energy and hence the temperature of remaining liquid will decrease, i.e., evaporation produces cooling. Further, greater the area of liquid surface, greater will be the probability of escaping of high speed molecules and so greater will be the rate of evaporation.

MEAN FREE PATH AND KINETIC INTERPRETATION OF BROWNIAN MOTION

Mean Free Path

According to kinetic theory of gases, the molecules of a gas are in a state of random motion and collide frequently with each other. The path of an individual molecule is random and resembles that shown in Fig. (*a*). The path transversed by a molecule between two successive collisions with other molecules is called the free path. Since all the free paths are not of the same length, we define a more useful quantity known as mean free path. The average distance travelled by a molecule between two successive collisions is called the mean free path.

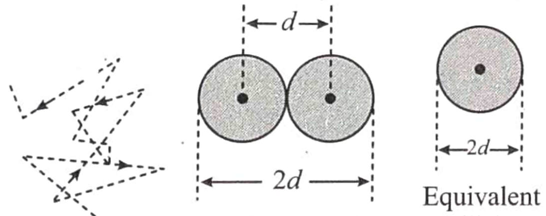
Obviously, mean free path,

$$\lambda = \frac{\left(\begin{array}{c}\text{Total distance travelled by a gas molecule} \\ \text{between successive collisions}\end{array}\right)}{\text{Total number of collisions}}$$

To calculate the mean free path, we make the following two assumptions:

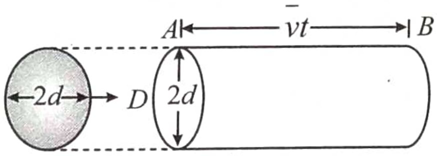
(*i*) Only the particular molecule that we are considering is in motion, all others being at rest.

(*ii*) The molecules are spheres, each of diameter *d*. Therefore, a collision will take place when the centres of two molecules are at a distance d apart as shown in Fig. (*b*). An equivalent description of collisions made by the moving molecule is to regard that this molecule has a diameter 2*d* and all the other molecules at rest are point masses as shown in Fig. (*c*).



Actual collision | Equivalent collision

(*a*) (*b*) (*c*)



(*d*)

Let this molecule of diameter 2*d* travel from *A* to *B* in time *t*. Let $\bar{v}$ be the average speed of this molecule. Clearly, it will sweep out a cylinder of diameter 2*d* (i.e., cross-sectional area πd^2) and length $\bar{v}t$ as shown in Fig. (*d*).

Volume of the cylinder, $AB = \pi d^2\left(\bar{v}t\right) = \pi d^2 \bar{v}t$

If *n* is the number of molecules per unit volume, number of molecules in the cylinder $= \left(\pi d^2 \bar{v}t\right)n$

Since the moving molecule meets $\left(\pi d^2 \bar{v}t\right)n$ molecules on its way while travelling a distance $\bar{v}t$, it will collide with all these molecules thereby making $\left(\pi d^2 \bar{v}t\right)n$ collision in time *t*.

From Eq. (*i*), $\lambda = \dfrac{\bar{v}t}{\left(\pi d^2 \bar{v}t\right)n} = \dfrac{1}{\pi d^2 n}$

Equation (*iv*) has been derived on the assumption that all the molecules in the cylinder are at rest. Actually all these molecules are moving with speeds governed by the Maxwell law of distribution of speeds. Applying this law, it can be shown that

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Collision frequency: Since the number of collisions in time *t* is $\left(\pi d^2 \bar{v}t\right)n$.

Number of collisions per unit time i.e., collision frequency,

$$f = \frac{\left(\pi d^2 \bar{v}t\right)n}{t} = \pi d^2 \bar{v}n$$

Applying Maxwell's law of distribution of speeds,

$$f = \sqrt{2}\pi d^2 \bar{v}n = \frac{\bar{v}}{\lambda}$$

Mean free time: The inverse of collision frequency is the average time between collisions and is called the mean free time.

As $PV = Nk_BT, n = \dfrac{N}{V} = \dfrac{P}{k_BT}$

$$\therefore \lambda = \frac{1}{\sqrt{2}\pi d^2}\frac{k_BT}{P} = \frac{k_BT}{\sqrt{2}\pi d^2 P}$$

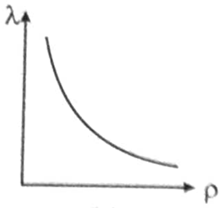
From Eq. (viii) it follows that

- $\lambda \propto \dfrac{1}{d^2}$, i.e., mean free path is inversely proportional to the square of the molecular diameter. Thus, smaller the diameter of the molecule, longer is the mean free path.
- $\lambda \propto T$, i.e., the mean free path varies directly with the absolute temperature of the gas.
- $\lambda \propto \dfrac{1}{P}$, i.e., mean free path varies inversely with the pressure of the gas.

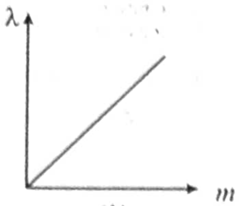
During two successive collisions, a molecule of a gas moves in a straight line with constant velocity.

Important Points

- As $\lambda \propto 1/\rho$ and $\lambda \propto m$, i.e., the mean free path is inversely proportional to the density of a gas and directly proportional to the mass of each molecule.

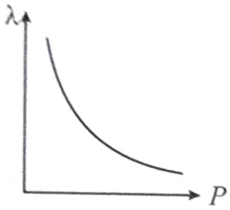


(a)

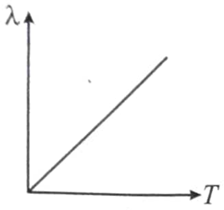


(b)

❖ As $\lambda = \frac{1}{\sqrt{2}}\frac{kT}{\pi d^2 P}$. For constant volume and hence constant number density n of gas molecules, $\frac{P}{T}$ is constant so that λ will not depend on P and T. But if volume of given mass of a gas is allowed to change with P or T then $\lambda \propto T$ at constant pressure and $\lambda \propto 1/P$ at constant temperature.



(a)



(b)

Train Your Brain

Example 4: The ratio of the mean speed of hydrogen molecules to the mean speed of nitrogen molecules in a sample containing a mixture of the two gases.

(a) $\sqrt{14}$ (b) $\sqrt{7}$
(c) $\sqrt{28}$ (d) None of these

Sol. (a) $V_{avr} = \overline{V} = \sqrt{\frac{8\,RT}{\pi M}}$

$$= \frac{\overline{V}_{H_2}}{\overline{V}_{N_2}} = \sqrt{\frac{1/2}{1/28}} = \sqrt{\frac{14}{1}}$$

Example 5: r.m.s. velocity of nitrogen molecules at NTP is

(a) 492 m/s (b) 517 m/s
(c) 546 m/s (d) 33 m/s

Sol. (b) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.3\times 10^7 \times 300}{28}} = 517$ m/sec

Example 6: An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to

(a) 2×10^{-7} s (b) 4×10^{-8} s
(c) 0.5×10^{-8} s (d) 3×10^{-6} s

Sol. (b) The mean time between two collision $\propto \frac{P}{\sqrt{T}}$

$$\frac{\Delta t_1}{\Delta t_2} = \frac{P_1}{P_2}\times\frac{\sqrt{T_2}}{\sqrt{T_1}} \Rightarrow \frac{6\times 10^{-8}}{\Delta t_2} = \left(\frac{\sqrt{3}}{5}\right)\times 2$$

$$\Rightarrow \Delta t_2 = 6 \times 10^{-8} \times \frac{1}{2}\sqrt{\frac{5}{3}} \approx 4 \times 10^{-8} \text{ sec.}$$

Example 7: The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to: [Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K Avogadro Number $N_A = 6.02 \times 10^{26}$/kg Radius of Earth: 6.4×10^6 m Gravitational acceleration of Earth = 10 ms^{-2}]

(a) 650 K
(b) 3×10^5 K
(c) 10^4 K
(d) 800 K

Sol. (c) $v_{\text{rms}} = \sqrt{\frac{3RT}{m}}$ $\quad v_{\text{escape}} = \sqrt{2gR_e}$

$v_{\text{rms}} = v_{\text{escape}}$

$$\frac{3RT}{m} = 2gR_e\,; \quad \frac{3\times 1.38\times 10^{-23}\times 6.02\times 10^{26}}{2}\times T$$

$= 2 \times 10 \times 6.4 \times 10^6$

$$T = \frac{4\times 10\times 6.4\times 10^6}{3\times 1.38\times 6.02\times 10^3} = 10 \times 10^3 = 10^4 k$$

Note: Question gives avogadro Number $N_A = 6.02 \times 10^{26}$/kg but we take $N_A = 6.02 \times 10^{26}$/kmol.

Example 8: The ratio of number of collisions per second at the walls of containers by H_2 and Ne gas molecules kept at same volume and temperature is

(a) 10 : 1 (b) 1: 10
(c) $1:\sqrt{10}$ (d) $\sqrt{10}:1$

Sol. (d) Let's consider the wall perpendiculars to x-axis number of collisions per second are given by

$\frac{V_x}{2L}$ Now, $v_X = v_{rms}/\sqrt{3}$

'L' and T is same of both. Then. Ratio of speed given by $\frac{V_{rms}(H_2)}{V_{rms}(H_e)} = \sqrt{\frac{M_{r_{He}}}{M_{H_2}}} = \sqrt{\frac{20}{2}} = \sqrt{10}:1$

Example 9: The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to

(a) $n_0\alpha^{1/4}$ (b) $n_0\alpha^{-3}$
(c) $n_0\alpha^{-3/4}$ (d) $\sqrt{n_0}\,\alpha^{1/2}$

Sol. (c) Given number density of molecules of gas as a function of r is

$$n(r) = n_0 e^{-\alpha r^4}$$

∴ Total number of molecule $\int_0^\infty n(r)\,dV$

$$= \int_0^\infty n_0 e^{-\alpha r^4} 4\pi r^2 dr$$

∴ Number of molecules is proportional to $n_0\alpha^{-3/4}$

Concept Application

4. The rms speed of oxygen molecules in a gas is v. If the temperature is doubled and the O_2 molecule dissociate into oxygen atoms, the rms speed will become
   (a) v (b) $v\sqrt{2}$
   (c) 2 v (d) 4v
5. The root mean square velocity of a gas molecule of mass m at a given temperature is proportional to
   (a) m^0 (b) m
   (c) $\sqrt{m}$ (d) $\frac{1}{\sqrt{m}}$
6. The temperature at which the r.m.s velocity of oxygen molecules equal that of nitrogen molecules at 100°C is nearly:
   (a) 426.3 K (b) 456.3 K
   (c) 436.3 K (d) 446.3 K
7. A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{arg on})}\right]$, is close to:

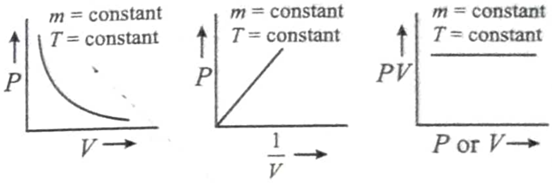
   $T = \frac{273 \times 32}{28} = 426.3\ K.$

   (a) 3.16 (b) 0.32
   (c) 0.45 (d) 2.24
8. For a given gas at 1 atm pressure, rms speed of the molecule is 200 m/s at 127° C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be
   (a) 80 m/s (b) $100\sqrt{5}$ m/s
   (c) $80\sqrt{5}$ m/s (d) 100 m/s
9. Three closed vessels A, B, and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is V_1, that of the N_2 molecules in vessel B is V_2 the average speed of the O_2 molecules in vessel C will be:
   (a) $(V_1 + V_2)/2$
   (b) V_1
   (c) $(V_1V_2)^{1/2}$
   (d) $\sqrt{3kT/M}$

DERIVATION OF GAS-LAWS FROM KTG

Boyle's Law

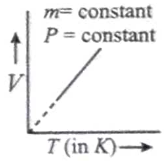
According to Boyle's law for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e., $V \propto 1/P$ if mass of gas and T = constant



Charles' Law

According to it for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature,

i.e., $V \propto T$. If m and P = constant

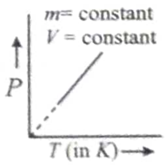


Gay-Lussac's Law

According to it for a given mass of an ideal gas at constant volume,

pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$

if m and V = constant



Avogadro's Law

According to it, at same temperature and pressure, equal volumes of all the gases contain equal number of molecules, i.e., $N_1 = N_2$ if P, V and T are same

Graham's Law

According to it, at constant pressure and temperature, the rate of diffusion of a gas is inversely proportional to the square root of its density.

Rate of diffusion $\propto \frac{1}{\sqrt{\rho}}$ and if P and T = constant

Dalton's Law

According to it, the pressure exerted by a gaseous mixture is equal to the sum of partial pressure of each component present in the mixture, i.e., $P = P_1 + P_2 + ...$

READING OF P–V DIAGRAM

Isobaric Process

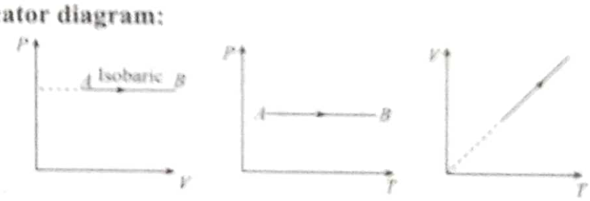
When a thermodynamic system undergoes a physical change in such a way that its pressure remains constant, then the change is known as isobaric process.

Equation of state: In this process V and T changes but P remains constant. Hence Charle's law is obeyed in this process.

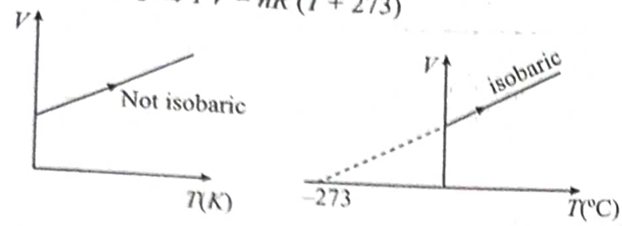
Hence if pressure remains constant

$V \propto T \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_1}$

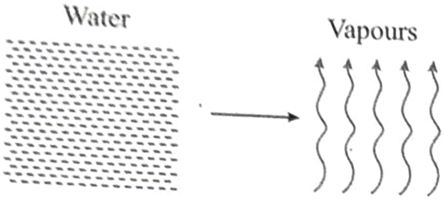
Indicator diagram:



When T in °$C \Rightarrow PV = nR\,(T + 273)$



Examples of isobaric process: All state changes occurs at constant temperature and pressure.



Boiling of water

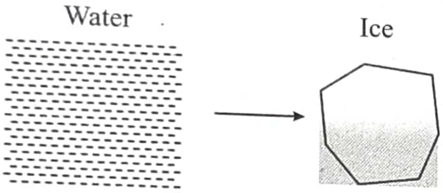
(i) Water ⟶ vapours

(ii) Temperature ⟶ constant

(iii) Volume ⟶ increases

(iv) A part of heat supplied is used to change volume (expansion) against external pressure and remaining part is used to increase it's potential energy (kinetic energy remains constant)

Freezing of water



(i) Water ⟶ ice

(ii) Temperature ⟶ constant

(iii) Volume ⟶ increases

(iv) Heat is given by water it self. It is used to do work against external atmospheric pressure and to decreases the internal potential energy.

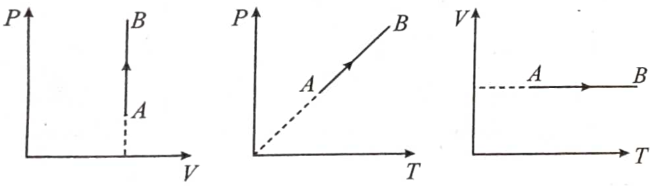
Isochoric or Isometric Process

When a thermodynamic process undergoes a physical change in such a way that its volume remains constant, then the change is known as isochoric process.

Equation of state: In this process P and T changes but V = constant. Hence Gay-Lussac's law is obeyed in this process *i.e.* $P \propto T$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{constant}$$

Indicator diagram: Graph 1 and 2 represent isometric increase in pressure at volume V_1 and isometric decrease in pressure at volume V_2 respectively and slope of indicator diagram $\frac{dP}{dV} = \infty$



Isothermal Process

When a thermodynamic system undergoes a physical change i such a way that its temperature remains constant, then the chang is known as isothermal process.

Essential condition for isothermal process

The walls of the container must be perfectly conducting to allo free exchange of heat between the gas and its surrounding.

The process of compression or expansion should be so slow so a to provide time for the exchange of heat.

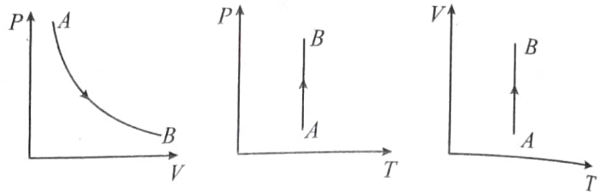
Since these two conditions are not fully realised in practic therefore, no process is perfectly isothermal.

Equation of state: In this process, P and V change bu T = constant *i.e.* change in temperature $\Delta T = 0$.

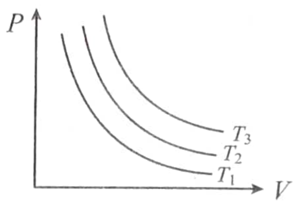
Boyle's law is obeyed *i.e.* PV = constant

$\Rightarrow P_1V_1 = P_2V_2$

Indicator diagram:



According to PV = constant, graph between P and V is a par of rectangular hyperbola. The graphs at different temperature ar parallel to each other are called isotherms.

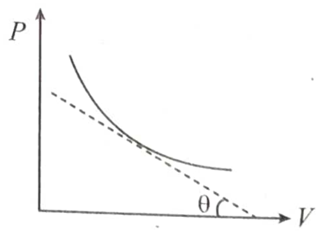


$T_1 < T_2 < T_3$

Two isotherms never intersect

(i) Slope of isothermal curve: By differentiating PV = constant. We get

$PdV + VdP = 0$



$\Rightarrow PdV = -VdP$

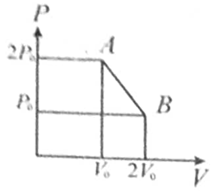
$\Rightarrow \text{Slope} = \tan\theta = \frac{dP}{dV} = -\frac{P}{V}$

(ii) If volume increases $\Delta W = +$ (Area under curve) and if volume decreases $\Delta W = -$ (Area under curve)

Example of isothermal process: Melting of ice (at 0°C) and boiling of water (at 100°C) are common example of this process.

Train Your Brain

Example 10: 'n' moles of an ideal gas undergoes a process $A \to B$ as shown in the figure. The maximum temperature of the gas during the process will be:



(a) $\dfrac{9P_0V_0}{4nR}$ (b) $\dfrac{3P_0V_0}{2nR}$

(c) $\dfrac{9P_0V_0}{2nR}$ (d) $\dfrac{9P_0V_0}{nR}$

Sol. (a) $P - P_0 = \dfrac{-P_0}{V_0}(V - 2V_0)_b$

$$T = \frac{P_0}{V_0} \times \frac{V^2}{nR} + \frac{3P_0V}{nR}$$

$$\frac{dT}{dV} = \frac{-P_0}{nRV_0} \times 2V + \frac{3P_0}{nR} = 0$$

$$\Rightarrow V = \frac{3V_0}{2} \ \& \ P = \frac{3P_0}{2}$$

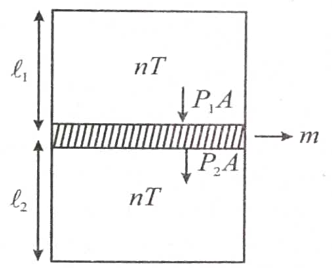
$$T_{\max} = \frac{PV}{nR} = \frac{9P_0V_0}{4nR}$$

Example 11: A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by: (R is universal gas constant and g is the acceleration due to gravity)

(a) $\dfrac{RT}{ng}\left[\dfrac{l_1 - 3l_2}{l_1l_2}\right]$ (b) $\dfrac{RT}{g}\left[\dfrac{2l_1 - l_2}{l_1l_2}\right]$

(c) $\dfrac{nRT}{g}\left[\dfrac{1}{l_2} + \dfrac{1}{l_1}\right]$ (d) $\dfrac{nRT}{g}\left[\dfrac{l_2 - l_1}{l_1l_2}\right]$

Sol. (d)



$$P_2A - P_1A = mg$$

$$m = \frac{1}{g}\left(\frac{P_1A\ell_1}{\ell_1} - \frac{P_2A\ell_2}{\ell_2}\right); \ m = \frac{1}{g}\left(\frac{nRT}{\ell_1} - \frac{nRT}{\ell_2}\right)$$

$$m = \frac{nRT}{g}\left(\frac{1}{\ell_1} - \frac{1}{\ell_2}\right)$$

Example 12: One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1 - \dfrac{1}{2}\left(\dfrac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes from V_0 to $2V_0$.

(a) $\dfrac{1}{2}\dfrac{P_0V_0}{R}$ (b) $\dfrac{3}{4}\dfrac{P_0V_0}{R}$

(c) $\dfrac{5}{4}\dfrac{P_0V_0}{R}$ (d) $\dfrac{1}{4}\dfrac{P_0V_0}{R}$

Sol. (c) $P = P_0\left[1 - \dfrac{1}{2}\left(\dfrac{V_0}{V}\right)^2\right]$

Pressure at $V_0 = P_0\left(1 - \dfrac{1}{2}\right) = \dfrac{P_0}{2}$

Pressure at $2V_0 = P_0\left(1 - \dfrac{1}{2} \times \dfrac{1}{4}\right) = \dfrac{7}{8}P_0$

Temperature at $V_0 = \dfrac{\left(\frac{7}{8}P_0\right)(2V_0)}{nR} = \dfrac{\frac{7}{4}P_0V_0}{nR}$

Change in temperature $= \left(\dfrac{7}{4} - \dfrac{1}{2}\right)\dfrac{P_0V_0}{nR}$

$$= \frac{5}{4}\frac{P_0V_0}{nR} = \frac{5P_0V_0}{4R}$$

Example 13: A closed container of volume $0.02\,m^3$ Contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of $1 \times 10^5\ Nm^{-2}$. The total mass of the mixture is 28 g. If the molar masses of neon and argon are 20 and 40 g mol^{-1} respectively, find the masses of the individual gases in the container assuming them to be ideal (universal gas constant, $R = 8.314$ J/mol-K

Sol. Given temperature of the mixture,

$T = 27°C = 300K$

Let m be the mass of the neon gas in the mixture. Then mass of argon would be (28-m)

Number of gram moles of neon,

Number of gram moles of Argon,

$$n_2 = \frac{(28 - m)}{40}$$

From Dalton's law of partial pressures, Total pressure of the mixture (P) = Pressure due to Neon (P_1) + Pressure due to Argon (P_2) or Substituting the values

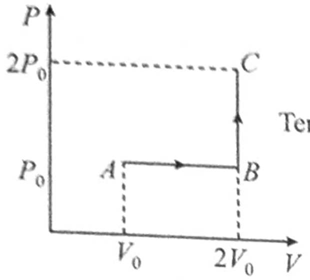
$$1.0 \times 10^5 = \left(\frac{m}{20} + \frac{28 - m}{40}\right)\frac{(8.314)(300)}{0.02}$$

Solving this equation, we get

$m = 4.074$ g and $28 - m = 23.926$ g

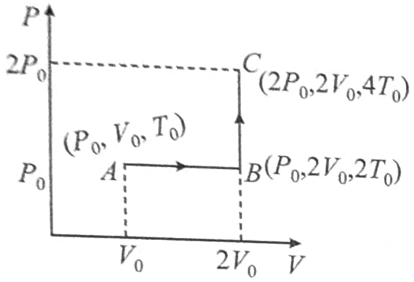
Therefore, in the mixture, 4.074 g Neon is present and the rest i. e. 23.926 g argon is present.

Example 14: Find out the values of co-ordinates at point A, B, C in terms of pressure, volume and temperature and draw curve.



Temp. At point $A = T_0$

Sol.



$A \rightarrow B$ (Isobaric)

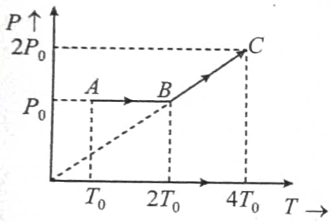
$$\frac{V_0}{T_0} = \frac{2V_0}{T_B}$$

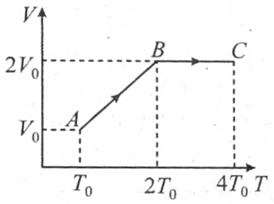
$$T_b = 2T_0$$

$B \rightarrow C$ (Isochoric)

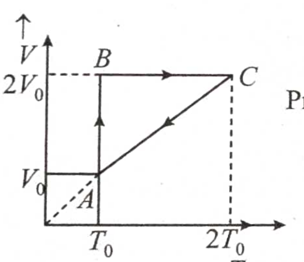
$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \frac{P_0}{2T_0} = \frac{2P_0}{T_2} \quad T_2 = 4T_0$$





Example 15: Find out the values of co-ordinates at point A, B, C in terms of pressure, volume and temperature and draw curve.



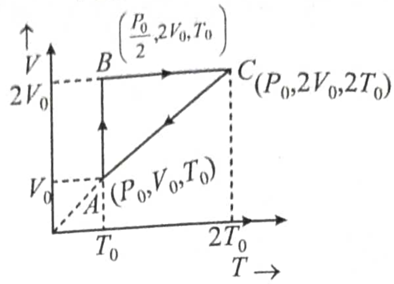
Pressure at point $A = T_0$

Sol. $A \rightarrow B$ Temp. Constant (isothermal)

$$P_1V_2 = P_2V_2$$

$$2P_0V_0 = 2V_0P_2 \Rightarrow P_2 = \frac{P_0}{2}$$

$B \rightarrow C$ volume constant (Isochoric)



$$\frac{P_B}{T_B} = \frac{P_C}{T_C}$$

$$\Rightarrow \frac{P_0}{2T_0} = \frac{P_C}{2T_0} \Rightarrow P_C = P_0$$

Concept Application

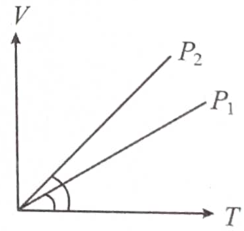
10. A flask is filled with 13 gm of an ideal gas at 27°C and its temperature is raised to 52°C. The mass of the gas that has to be released to maintain the temperature of the gas in the flask at 52°C and the pressure remaining the same is

(*a*) 2.5 g (*b*) 2.0 g
(*c*) 1.5 g (*d*) 1.0 g

11. Keeping the number of moles, volume and pressure the same, which of the following are the same for all ideal gas?

(*a*) Rms speed of a molecule
(*b*) Density
(*c*) Temperature
(*d*) Average of magnitude of momentum.

12. In the following V-T diagram what is the relation between P_1 and P_2:



(*a*) $P_2 = P_1$ (*b*) $P_2 > P_1$
(*c*) $P_2 < P_1$ (*d*) cannot be predicted

13. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is ________.

(*a*) 2 (*b*) 4
(*c*) 5 (*d*) 6

14. When an ideal gas is compressed isothermally then its pressure increases because :
(a) Its potential energy decreases
(b) Its kinetic energy increases and molecules move apart
(c) Its number of collisions per unit area with walls of container increases
(d) Molecular energy increases

DEGREES OF FREEDOM (f)

The term degrees of freedom of a system refers to the possible independent motions a system can have or number of possible independent ways in which a system can have energy.

The independent motions of a system can be translational, rotational or vibrational or any combination of these.

A particle in motion confined to a straight line has only one translational degree of freedom while

If same particle is confined to move in a plane, it will have two translational degrees of freedom.

If the particle is free to move in space, it will have three translational degrees of freedom.

Possible translational degrees of freedom are three i.e.

$$\left(\frac{1}{2}mv_x^2+\frac{1}{2}mV_y^2+\frac{1}{2}mV_z^2\right)$$

Maximum possible rotational degrees of freedom are three $\left(\frac{1}{2}I_x\omega_x^2+\frac{1}{2}I_y\omega_y^2+\frac{1}{2}I_z\omega_z^2\right)$

Vibrational degrees of freedom are two i.e. (Kinetic energy, of vibration and Potential energy of vibration)

Monoatonic

Eg : (all inert gases, He, Ar, etc.)

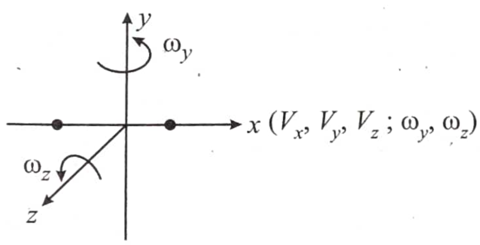
$f=3$ (translational)

(V_x, V_y, V_z)

Diatomic

Eg : (gases like H_2, N_2, O_2 etc)

$f=5$ (3 translational + 2 rotational)



Note : For diatomic gas :

(a) At low temperature

$f=3$(transalational)

(b) At room temperature

$f=5$(3 translational + 2 rotational)

(c) At high temperature

$f=7$ [3 translational + 2 rotational + 2 vibrational]

Triatomic (Non-linear)

$f=6$ [3 translational + 3 rotational]

For Example : H_2O

Triatomic (linear)

$f=5$ [3 translational + 2 rotational]

For Example : CO_2

Equivalent degrees of freedom for a gaseous mixture

- We know if two substances at same temperature are connected or mixed, they do not exchange any thermal energy and the temperature of mixture remains same.
- If A' gases with degrees of freedom $f_1, f_2, f_3 \ldots f_N$ are mixed with $n_1, n_2, n_3, \ldots n_N$ moles at same temperature T then their total internal energy before mixing can be given as

$$U_i=\frac{f_1}{2}n_1RT+\frac{f_2}{2}n_2RT+\cdots+\frac{f_N}{2}n_NRT \quad \ldots(i)$$

- If after homogeneous mixing, for analytical purpose we assume f_{eq} are the number of degrees of freedom for the mixture then after mixing the internal energy of this mixture can be given as

$$U_f=\frac{f_{eq}}{2}(n_1+n_2+\cdots n_N)RT \quad \ldots(ii)$$

As no energy loss is taking place during mixing of gases, we have from Eqs. (i) and (ii)

$$U_i=U_f$$

$$\frac{f_1}{2}n_1RT+\frac{f_2}{2}n_2RT+\cdots+\frac{f_N}{2}n_NRT=\frac{f_{eq}}{2}(n_1+n_2+\cdots n_N)RT$$

or

$$\boxed{f_{eq}=\frac{f_1n_1+f_2n_2+\cdots f_Nn_N}{n_1+n_2+\cdots n_N}} \quad \ldots(iii)$$

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY

For a dynamical system in thermal equilibrium, the energy of the system is equally distributed amongst the various degrees of freedom and the energy associated with each degree of freedom per molecule is $=\frac{1}{2}kT$,

Translational kinetic energy of a molecule

$$\frac{1}{2}mv_{rms}^2=\frac{3}{2}kT$$

we have $\frac{1}{2}m(v_{rmx}^2)_x+\frac{1}{2}m(v_{rmx}^2)_y+\frac{1}{2}m(v_{rmx}^2)_z=\frac{3}{2}kT$

The molecular motion is a random one and as no direction of motion is a preferred one, the average kinetic energy corresponding to each degree of freedom is the same, i.e.

$$\frac{1}{2}m(v_{rmx}^2)_x=\frac{1}{2}m(v_{rmx}^2)_y=\frac{1}{2}m(v_{rmx}^2)_z$$

$$\Rightarrow\quad \frac{1}{2}m(v_{rmx}^2)_x=\frac{1}{2}m(v_{rmx}^2)_y=\frac{1}{2}m(v_{rmx}^2)_z=\frac{1}{2}kT$$

Mean kinetic energy per molecule per degree of freedom $=\frac{1}{2}kT$

Energy associated with each degree of freedom $=\frac{1}{2}KT$.

If degree of freedom of a molecule is f then

Mean energy of that molecule $=\frac{f}{2}KT$

Monoatomic

Energy of one particle $=\frac{3}{2}KT$; of one mole $=\frac{3}{2}RT$; of n mole $=\frac{3}{2}nRT$

Diatomic

Energy of one Particle $=\frac{5}{2}KT$; of one mole $=\frac{5}{2}RT$, of n mole $=\frac{5}{2}nRT$

General degree of freedom

Energy of one particle $=\frac{f}{2}KT$; of one mole $=\frac{f}{2}RT$, of n mole $=\frac{f}{2}nRT$

Internal energy of a gas only depends on the temperature of the gas. It doesn't depend on process taken by the gas to reach the temperature. It is a state function.

INTERNAL ENERGY

The internal energy of a system is the sum of kinetic and potential energies of the molecules of the system. It is denoted by U. Internal energy (U) of the system is the function of its absolute temperature (T) and its volume (V). i.e. $U=f(T, V)$

In case of an ideal gas, intermolecular force is zero. Hence its potential energy is also zero. In this case, the internal energy is only due to kinetic energy, which depends on the absolute temperature of the gas. i.e. $U=f(T)$.

For an ideal gas internal energy $(U)=\frac{f}{2}nRT$

Train Your Brain

Example 16: A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is:

(*a*) 1 : 1
(*b*) 1 : 2
(*c*) 2 : 1
(*d*) depends on the moments of inertia of the two molecule

Sol. (*a*) Average rotational $K.E.=\frac{1}{2}KT\times2=KT$

So it will be same for both the gases.

Example 17: The quantity $\frac{2U}{fkT}$ represents (where U = internal energy of gas f = degree of freedom, k = Boltzmann constant & T = absolute temperature of gas)

(*a*) Mass of the gas
(*b*) Kinetic energy of the gas
(*c*) Number of moles of the gas
(*d*) Number of molecules in the gas

Sol. (*d*) $U=\frac{nfRT}{2}=\frac{nfN_AkT}{2}$; $\frac{2U}{fkT}=nN_A=N$

Example 18: Two kg of a monoatomic gas is at a pressure 4×10^4 N/m². The density of the gas is 8 kg/m³. What is the order of energy of the gas due to its thermal motion?

(*a*) 10^3 J (*b*) 10^5 J (*c*) 10^4 J (*d*) 10^6 J

Sol. (*c*) $E=\frac{1}{2}Mv_m^2$

$$=\frac{1}{2}\times2\times\left(\frac{3P}{\rho}\right)=\frac{3\times4\times10^4}{8}=1.5\times10^4 J$$

Example 19: At what temperature will the root mean square velocity of oxygen molecules be sufficient so as the escape from the earth?

(*a*) 1.6×10^5 K (*b*) 16×10^5 K
(*c*) 16×10^5 K (*d*) 160×10^5 K

Sol. (*a*) $\therefore\frac{3}{2}kT=\frac{1}{2}mv_e^2$

Where v_c = escape velocity of earth = 11.1 km/sec

m = mass of 1 molecule of oxygen = 5.34×10^{-26}

$$T=\left(\frac{mv_c^2}{3k}\right)$$

$$\therefore T=\left(\frac{\left(5.3210^{-3}\times\left(111\times10^{b}\right)\right)}{3\times\left(.38\times10^{-2}\right)}\right)$$

$\Rightarrow T=1.6\times10^5$ K

Example 20: The pressure of an ideal gas is written as $P=\frac{2}{3}E$. Where E refers to -

(*a*) Translational kinetic energy
(*b*) Rotational kinetic energy
(*c*) Vibrational kinetic energy
(*d*) Total kinetic energy

Sol. (*a*) Pressure of the gas $P=\frac{1}{3}\frac{mN}{v}v_m^2$

Energy $E=E=\frac{1}{2}mv_m^2$

$$=\frac{1}{2}mN\left(\frac{v_1^2+v_2^2+\ldots+v_1^2}{N}\right)$$

$$= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \ldots + \frac{1}{2}mv_n^2$$

So energy is basically the sum of energies of all molecules which represents translational kinetic energy.

Concept Application

15. A mono-atomic gas molecule has
(*a*) Three degrees of freedom
(*b*) Four degrees of freedom
(*c*) Five degrees of freedom
(*d*) Six degrees of freedom

16. The ratio of mean kinetic energy of hydrogen and oxygen at a given temperature is
(*a*) 1 : 16 (*b*) 1 : 8
(*c*) 1 : 4 (*d*) 1 : 1

17. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3 J/K mole]
(*a*) 0.9 kJ (*b*) 6 kJ
(*c*) 10 kJ (*d*) 14 kJ

18. An HC1 molecule has rotational, translational and vibrational motions. If the rms velocity of HC1 molecules in its gaseous phase is $\bar{v}$. m is its mass and k_B is Boltzmann constant, then its temperature will be:
(*a*) $\frac{m\bar{v}^2}{6k_B}$ (*b*) $\frac{m\bar{v}^2}{5k_B}$
(*c*) $\frac{m\bar{v}^2}{3k_B}$ (*d*) $\frac{m\bar{v}^2}{7k_B}$

19. The first excited state of hydrogen atom is 10.2 eV above its ground state. What temperature is needed to excite hydrogen atoms to first excited level -
(*a*) 7.88×10^4K
(*b*) $.788 \times 10^4$ K
(*c*) 78.8×10^4 K
(*d*) 788×10^4 K

20. At what temperature does the average translational kinetic energy of a molecule in a gas equal to the kinetic energy of an electron accelerated from rest through a potential difference of 5 volt.
(*a*) 386.5×10^3K
(*b*) 3.865×10^3K
(*c*) $.38 \times 10^3$K
(*d*) 38.65×10^3 K

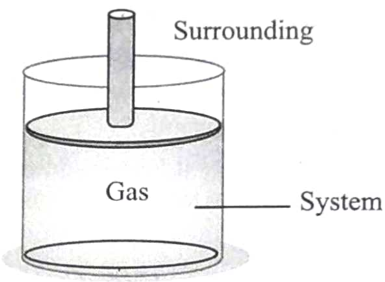
THERMODYNAMICS

Introduction

Thermodynamics is concerned with the work done by a system and the heat it exchanges with its surroundings. We are concerned only with work done by a system on its surroundings or on the system by the surroundings. We are not concerned with internal work done by one part of a system on another.

Some Definitions

1. **Thermodynamic system:**
   (*i*) It is a collection of an extremely large number of atoms or molecules
   (*ii*) It is confined within certain boundaries.
   (*iii*) Anything outside the thermodynamic system with which energy or matter is exchanged is called its surroundings.



   (*iv*) Thermodynamic system may be of three types
   (*a*) Open system: It exchanges both energy and matter with the surroundings.
   (*b*) Closed system: It exchanges only energy (not matter) with the surroundings.
   (*c*) Isolated system: It exchanges neither energy nor matter with the surroundings.
2. **Thermodynamic variables and equation of state:** A thermodynamic system can be described by specifying its pressure, volume, temperature, internal energy and the number of moles. These parameters are called thermodynamic variables. The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.
   For n moles of an ideal gas, equation of state is $PV = nRT$ and for 1 mole of an ideal gas it is $PV = RT$.
3. **Thermodynamic equilibrium:** In steady state thermodynamic variables are independent of time and the system is said to be in the state of thermodynamic equilibrium. For a system to be in thermodynamic equilibrium, the following conditions must be fulfilled.
   (*i*) Mechanical equilibrium: There is no unbalanced force between the system and its surroundings.
   (*ii*) Thermal equilibrium: There is a uniform temperature in all parts of the system and is same as that of surrounding.
   (*iii*) Chemical equilibrium: There is a uniform chemical composition through out the system and the surrounding.
4. **Thermodynamic process:** The process of change of state of a system involves change of thermodynamic variables such as pressure P, volume V and temperature T of the system.

The process is known as thermodynamic process. Some important processes are

(i) Isobaric process: Pressure remains constant

(ii) Isochoric process: Volume remains constant

(iii) Isothermal process: Temperature remain constant

(iv) Adiabatic process: No transfer of heat

(v) Cyclic process: In a cyclic process Initial and final states are same.

5. **Indicator diagram:** Whenever the state of a gas (P, V, T) is changed, we say the gaseous system is undergone a thermodynamic process.

HEAT, INTERNAL ENERGY AND WORK IN THERMODYNAMICS

Heat (ΔQ)

It is the energy that is transferred between a system and its environment because of the temperature difference between them.

Heat transferred may be different even if initial and final state are same. It is a path dependent quantity.

By Convention, ΔQ is positive if heat is added to the system and ΔQ is negative if heat is extracted from the system.

Internal energy (ΔU)

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

The energy due to molecular motion is called internal kinetic energy U_K and that due to molecular configuration is called internal potential energy U_P *i.e.* Total internal energy $U = U_K + U_P$

(i) For an ideal gas, as there is no molecular attraction $U_P = 0$ *i.e.* internal energy of an ideal gas is only kinetic and is given by $U = U_K = \frac{f}{2}nRT$ and change in internal energy $\Delta U = \frac{f}{2}nR\Delta T = nC_v\Delta T$ where C_v is molar specific heat at constant volume.

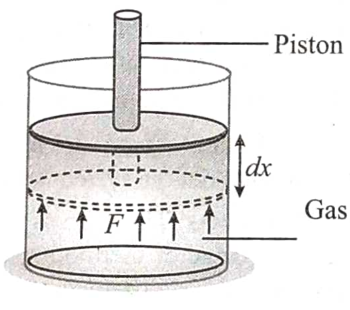
(ii) Change in internal energy does not depend on the path of the process. So it is called a point or state function *i.e.* it depends only on the initial and final states of the system, *i.e.*

$$\Delta U = U_f - U_i$$

Work (W)

Suppose a gas is confined in a cylinder that has a movable piston at one end. If P be the pressure of the gas in the cylinder, then force exerted by the gas on the piston of the cylinder

$F = PA$ (A = Area of cross-section of piston)



When the piston is pushed outward an infinitesimal distance dx the work done by the gas

$dW = F.dx = P(Adx) = PdV$

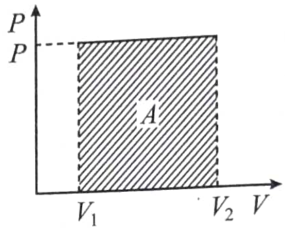
For a finite change in volume from V_i to V_f

Total amount of work done

$W = \int_{V_i}^{V_f} PdV = P(V_f - V_i)$

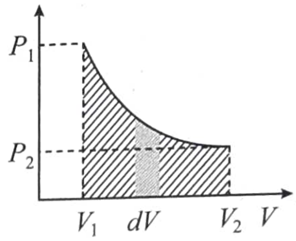
(i) If we draw indicator diagram, the area bounded by PV-graph and volume axis represents the work done

(a)



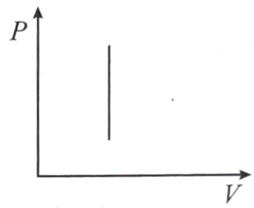
Work = Area = $P(V_2 - V_1)$

(b)



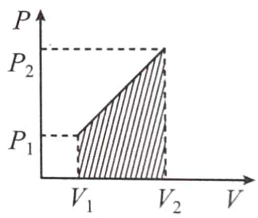
Work = $\int_{V_1}^{V_2} PdV$ = Area under the curve.

(c)



Work = 0

(d)



Work = Area of the shown trapezium

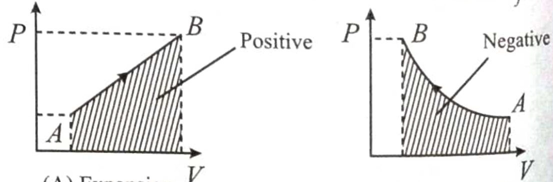
$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

(ii) From $\Delta W = \int_{V_i}^{V_f} PdV$

If system expands against some external force then $V_f > V_i$

$\Rightarrow \Delta W$ = positive

If system contracts because of external force then $V_f < V_i$



(A) Expansion (B) Compression

(iii) Like heat, work done also depends upon initial and final state of the system and path adopted for the process

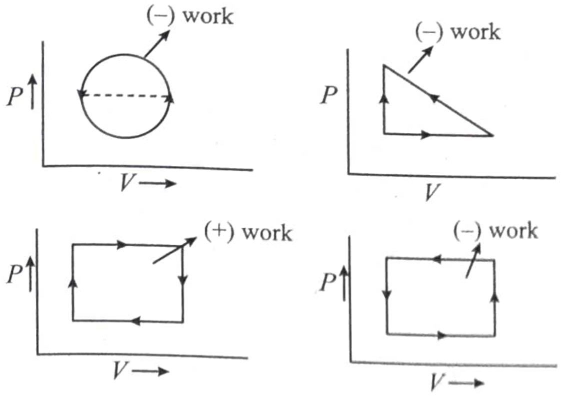
Work Done for Cyclic Process

In the cyclic process initial and final states are same therefore initial state = final state

Work done = Area enclosed under P-V diagram.

If the process on P-V curve is clockwise, then net work done is (+ve) and vice-versa.

The graphs shown below explains when work is positive and when it is negative



FIRST LAW OF THERMODYNAMICS

1. It is a statement of conservation of energy in thermodynamical process.
2. According to it, heat given to a system (ΔQ) is equal to the sum of increase in its internal energy (ΔU) and the work done (ΔW) by the system against the surroundings.
$$\Delta Q = \Delta U + \Delta W$$
3. It makes no distinction between work and heat as according to it the internal energy (and hence temperature) of a system may be increased either by adding heat to it or doing work on it or both.
4. ΔQ and ΔW are the path functions but ΔU is the point function.
5. In the above equation all three quantities ΔQ, ΔU and ΔW must be expressed either in Joule or in calorie.
6. Limitation: First law of thermodynamics does not indicate the direction of heat transfer. It does not tell anything about the conditions, under which heat can be transformed into work and also it does not indicate as to why the whole of heat energy cannot be converted into mechanical work continuously.

Heat Capacity

Specific heat Capacity: Specific heat capacity of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance by unit degree. This definition holds good in case of gases as well.

Specific heat capacity, $s = \dfrac{\Delta Q}{m\Delta T}$

Molar specific heat: Molar specific heat of a substance may be defined as the amount of heat required to raise the temperature of one gram mole of the substance by a unit degree,

Molar specific heat, $C = \dfrac{\Delta Q}{n\Delta T}$

Molar heat capacity at Constant Volume (C_v)

Consider, n moles of an ideal gas, confined in a cylinder, fitted with a fixed piston. If ΔQ be the heat supplied to the system and increase in temperature be ΔT, then

$$C_v = \left.\frac{\Delta Q}{n\Delta T}\right|_v$$

From first law of thermodynamics

$\Delta Q = \Delta U + W$

But work done by the gas is zero as volume remains constant

$$C_v = \left.\frac{\Delta Q}{n\Delta T}\right|_v = \frac{\Delta U}{n\Delta T} = \frac{f}{2}\frac{R\Delta T}{\Delta T} = \frac{f}{2}R \qquad \left[\because \Delta U = n\frac{fR}{2}\Delta T\right]$$

$$\boxed{C_v = \frac{f}{2}R}$$

For a monatomic gas: According to kinetic energy of gases, an ideal (or monatomic) gas has three degrees of freedom, hence, $C_v = \dfrac{3}{2}R$.

For a diatomic gas: The diatomic gas has five degrees of freedom, hence $C_v = \dfrac{5}{2}R$.

For a polyatomic (non-linear) gas: The degrees of freedom possessed by a polyatomic gas, at ordinary temperature, is six hence $C_v = 3R$.

Molar heat capacity at Constant Pressure (C_p)

Consider n moles of a gas contained in a cylinder fitted with a frictionless movable piston bearing some load. Let heat be supplied to the gas slowly so that the pressure of the gas never exceeds the external pressure more than by an infinitesimal amount. Assuming, no loss of heat to the surrounding, if ΔQ be total heat supplied to the gas and ΔT the total rise in temperature then,

$$C_p = \left.\frac{\Delta Q}{n\Delta T}\right|_p$$

Work done by the gas at constant pressure is

$$W = \int_{V_1}^{V_2} PdV = P(V_2 - V_1) = nR(T_2 - T_1)$$

$= nR\,\Delta T$

From First law of thermodynamics

$\Delta Q = \Delta U + W$

$= nC_v\Delta T + nR\Delta T = n(C_v + R)\Delta T$

$$C_p = \left.\frac{\Delta Q}{n\Delta T}\right|_p = C_v + R$$

This reaction $C_p - C_v = R$ is called Mayer's law.

(i) **Specific Heat of a Gas at Constant Pressure in Terms of Degree of Freedom**

We know, $C_v = \dfrac{fR}{2}$

But $C_p = C_v + R \Rightarrow C_p = \frac{fR}{2} + R$

$$\boxed{C_p = \left(\frac{f}{2}+1\right)R}$$

For a monoatomic gas: According to kinetic energy of freedom. Hence, $C_p = R\left(\frac{f}{2}+1\right) = R\left(\frac{3}{2}+1\right) = \frac{5R}{2}$

For a diatomic gas: The diatomic gas has five degrees of freedom. Hence, $C_p = R\left(\frac{f}{2}+1\right) = R\left(\frac{5}{2}+1\right) = \frac{7R}{2}$

For a polyatomic (non-linear) gas: The degrees of freedom possessed by a polyatomic six hence $C_p = R\left(\frac{f}{2}+1\right) = R\left(\frac{6}{2}+1\right) = 4R$ at ordinary temperatures.

(ii) **Adiabatic Constant**

It is the ratio of molar specific heat at constant pressure to molar specific hat at constant volume.

$$\gamma = \frac{C_p}{C_v} = \frac{R\left(\frac{f}{2}+1\right)}{\frac{fR}{2}} = \left(1+\frac{2}{f}\right)$$

For a monatomic gas:

$$\gamma = \left(1+\frac{2}{f}\right) = \left(1+\frac{2}{3}\right) = \frac{5}{3}$$

For a diatomic gas:

$$\gamma = \left(1+\frac{2}{f}\right) = \left(1+\frac{2}{5}\right) = \frac{7}{5}$$

For a nonlinear polyatomic gas:

$$\gamma = \left(1+\frac{2}{f}\right) = \left(1+\frac{2}{6}\right) = \frac{4}{3}$$

(iii) **Internal energy of an ideal gas in terms of adiabatic constant:**

We have the relations: $C_p - C_v = R$ ...(i)

and $\gamma = \frac{C_P}{C_v} \Rightarrow C_P = \gamma C_v$ ...(ii)

Solving Eqs. (i) and (ii)

$\gamma C_v - C_v = R$

$\Rightarrow C_v(\gamma - 1) = R \Rightarrow C_v = \frac{R}{(\gamma-1)}$ and $C_p = \frac{\gamma R}{(\gamma-1)}$

We can express internal energy of an ideal gas as:

$$U = nC_vT = n\left(\frac{R}{(\gamma-1)}\right)T$$

Hence we can write, $\boxed{U = \frac{nRT}{(\gamma-1)} = \frac{PV}{(\gamma-1)}}$

Also we can express change in internal energy of an ideal gas as:

$\boxed{\Delta U = \frac{nR\Delta T}{(\gamma-1)} = \frac{(P_2V_2 - P_1V_1)}{(\gamma-1)}}$ C_p is greater than C_v

(iv) When a gas is to be heated at constant volume, the pressure on the gas has to be increased. As the gas is not allowed to expand (volume being kept constant), therefore, in case of C_v heat is required only for raising the temperature of one mole of gas through 1 K.

$$\boxed{Q = nC_v\Delta T}$$

(v) When a gas is heated at constant pressure, it expands. In expanding, the gas has to do some work against the external pressure and it gets cooled. More heat has therefore to be supplied for raising its temperature through $1K$. Thus, in case of C_p, heat is required for two purposes:

1. For increasing the temperature of one mole of gas through $1K$.
2. For performing work whole increasing the volume of the gas against external pressure.

Clearly, heat required at constant pressure is more than that at constant volume, i.e., $C_p > C_v$. The difference of the two is the amount of heat spent in doing work for increasing the volume of the gas when heated at constant pressure.

$$\boxed{Q = nC_p\Delta T}$$

(vi) **C_p, C_v & γ for non-reactive gaseous mixture**

1. If two non-reactive gases are enclosed in a vessel of volume V. In the mixture n_1 moles of one gas are mixed with n_2 moles of another gas. If N_A is Avogadro's number then

Number of molecules of first gas $N_1 = n_1N_A$ and number of molecules of second gas $N_2 = n_2N_A$

Total mole fraction $n = (n_1 + n_2)$.

If M_1 is the molecular weight of first gas and M_2 that of second gas. Then molecular weight of mixture $M = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$

As there is no loss of internal energy, the internal energy should be same before mixing and after mixing. Let specific heat of the mixture at constant volume be $C_{v_{mix}}$ and adiabatic constant of mixture be γ.

$$(n_1+n_2)C_{v_{mix}}T = n_1C_{v_1}T + n_2C_{v_2}T$$

$$\boxed{C_{V_{mix}} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1+n_2}}$$

In Eq. (i), we can substitute $C_{v_{mix}} = \frac{R}{\gamma-1}$,

$C_{v_1} = \frac{R}{\gamma_1 - 1}$ and $C_{v_2} = \frac{R}{\gamma_2 - 1}$

$$(n_1+n_2)\left(\frac{R}{\gamma-1}\right)T = n_1\left(\frac{R}{\gamma_1-1}\right)T + n_2\left(\frac{R}{\gamma_2-1}\right)T$$

$$\frac{(n_1+n_2)}{\gamma-1} = \left(\frac{n_1}{\gamma_1-1}\right) + \left(\frac{n_2}{\gamma_2-1}\right)$$

From Eq. (ii), we can calculate γ of mixture.

Similarly, we can calculate specific heat of the mixture at constant pressure.

$\boxed{C_{P_{mix}} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1+n_2}}$ and $\boxed{\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}}}$

Train Your Brain

Example 21: The quantity of heat required to raise one mole through one degree Kelvin for a monoatomic gas at constant volume is

(*a*) $\frac{3}{2}R$ (*b*) $\frac{5}{2}R$ (*c*) $\frac{7}{2}R$ (*d*) $4R$

Sol. (*a*) $(\Delta Q)_v = \mu C_v \Delta T \Rightarrow (\Delta Q)_v = 1 \times C_v \times 1 = C_v$

For monoatomic gas $C_v = \frac{3}{2}R \Rightarrow (\Delta Q)_v = \frac{3}{2}R$

Example 22: C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is:

(*a*) $a = 14\,b$ (*b*) $a = 28\,b$

(*c*) $a = \frac{1}{14}b$ (*d*) $a = b$

Sol. (*a*) $C_p - C_v = R$

where C_p and C_v are molar specific heat capacities

As per the question

$a = \frac{R}{2}$ $b = \frac{R}{28}$

$a = 14\,b$

Example 23: The specific heats, C_p and C_v of a gas of diatomic molecules, A are given (in units of $J\,mol^{-1}\,K^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules, B has the corresponding values 30 and 21. If they are treated as ideal gases, then:

(*a*) A has one vibrational mode and B has two
(*b*) Both A and B have a vibrational mode each
(*c*) A is rigid but B has a vibrational mode
(*d*) A has a vibrational mode but B has none

Sol. (*d*) For A

$R = C_p - C_v = 7$

$C_v = \frac{fR}{2} = 22 \Rightarrow f = \frac{44}{7} = 6.3$

$f \cong 6$ → 5 (Rotation + Translational), 1 (Vibration)

For B

$R = C_p - C_v = 9$

$C_v = \frac{fR}{2} = 21 \Rightarrow f = \frac{42}{9}$

$f \cong 5$ → 5 (Rotation + Translational), 0 (Vibration)

Example 24: When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is:

(*a*) $\frac{7}{5}Q$ (*b*) $\frac{3}{2}Q$

(*c*) $\frac{5}{3}Q$ (*d*) $\frac{2}{3}Q$

Sol. (*a*) $Q = nC_v\Delta T$

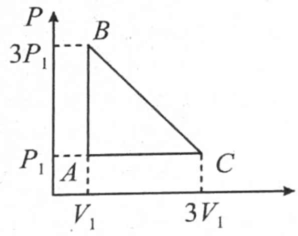
$Q' = nC_p\Delta T$

$\therefore \frac{Q'}{Q} = \frac{C_p}{C_v}$

For diatomic gas: $\frac{C_p}{C_v} = \gamma = \frac{7}{5}$

$Q' = \frac{7}{5}Q$

Example 25: An ideal gas is taken around the cycle $ABCA$ as shown in the P-V diagram. The net work done by the gas during the cycle is equal to

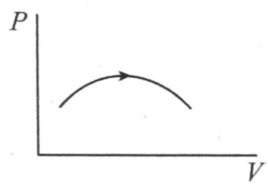


(*a*) $12P_1V_1$ (*b*) $6P_1V_1$

(*c*) $3P_1V_1$ (*d*) $2P_1V_2$

Sol. (*d*) Work done $= \frac{1}{2} \times 2P_1 \times 2V_1 = 2P_1V_1$

Example 26: In figure, P-V curve of an ideal gas is given. During the process, the cumulative work done by the gas



(*a*) Continuously increases
(*b*) Continuously decreases
(*c*) First increases then decreases
(*d*) First decreases then increases

Sol. (*a*) As volume increases

$\therefore$ *WD* continuously increases

Pressure P, volume V and temperature T for a certain material are related by $P = \frac{\alpha T - \beta T^2}{V}$

where α and β are constants. Find the work done by the material if the temperature changes from T_1 to T_2 while the pressure remains the constant.

$P = \frac{\alpha T - \beta T^2}{V}$ (P = constant)

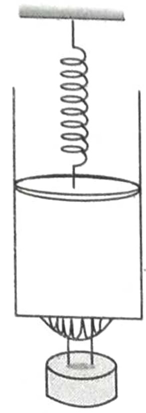
$$V = \frac{\alpha T - \beta T^2}{P} \text{ or } dV = \frac{\alpha - 2\beta T}{P} \cdot dT$$

$$W = \int P dV = \int_{T_1}^{T_2} P \frac{\alpha - 2\beta T}{P} dT$$

$$W = [\alpha T - \beta T^2]_{T_1}^{T_2}$$

$$W = \alpha(T_2 - T_1) - \beta(T_2^2 - T_1^2)$$

Example 27: A gas is inside a cylinder closed by a piston. The piston is held from above by a spring whose elastic properties obey Hooke's law. Plot a rough $P - V$ diagram. If the gas is heated then determine the work done by the gas in the process if volume of the gas varies from V_1 to V_2 and the pressure varies from P_1 to P_2.



Sol. When the piston moves upward by Δx: pressure inside the gas is

$$P = P_0 + \frac{mg}{A} + \frac{k\Delta x}{A} \; ; \; P = P_0 + \frac{mg}{A} + \frac{k\Delta V}{A^2}$$

$$P = P_0 + \frac{mg}{A} + \frac{k}{A^2}(V - V_i)$$

Here k = force constant of spring

P_0 = atmospheric pressure,

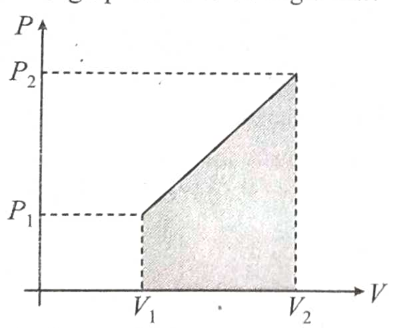
m = mass of piston

A = Area of cross-section of piston

and V_i = initial volume of gas

Equation (1) is a equation of straight line.

Hence $P - V$ graph will be a straight line.



The work done will be the area under graph or the hatched area.

$$\text{Hence } W = \frac{(P_1 + P_2)(V_2 - V_1)}{2}$$

Concept Application

21. Supposing the distance between the atoms of a diatomic gas to be constant, its specific heat at constant volume per mole (gram mole) is

(a) $\frac{5}{2}R$
(b) $\frac{3}{2}R$
(c) R
(d) $\frac{7}{2}R$

22. For an ideal gas, the heat capacity at constant pressure is larger than at constant volume because

(a) Positive work is done during expansion of the gas by the external pressure
(b) Positive work is done during expansion by the gas against external pressure
(c) Positive work is done during expansion by the gas against intermolecular forces of attraction
(d) More collisions occur per unit time when volume is kept constant.

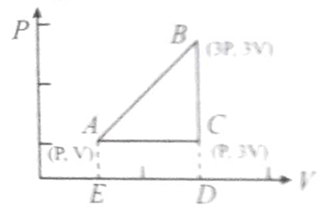
23. A gas has:

(a) One specific heat only
(b) Two specific heats only
(c) Infinite number of specific heats
(d) No specific heat

24. The quantities which remains same for all ideal gases at the same temperature is/are?

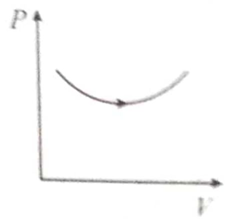
(a) The kinetic energy of equal moles of gas
(b) The kinetic energy of equal mass of gas
(c) The number of molecules of equal moles of gas
(d) The number of molecules of equal mass of gas

25. An ideal gas is taken around $ABCA$ as shown in the above P-V diagram. The work done during a cycle is



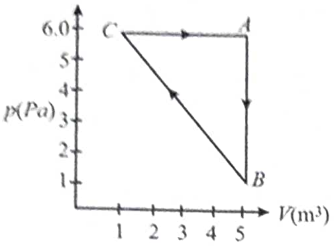
(a) $2PV$
(b) PV
(c) $1/2PV$
(d) Zero

26. Consider the process on a system shown in fig. During the process, the cumulative work done by the system



(a) Continuously increase
(b) Continuously decreases
(c) First increases then decreases
(d) First decreases then increases

27. For the given cyclic process CAB as shown for a gas, the work done is:



(a) 30 J
(b) 10 J
(c) 1 J
(d) 5 J

WORK DONE AND HEAT TRANSFERRED FOR VARIOUS THERMODYNAMICAL PROCESSES

1. Isobaric Process:

Pressure remains constant in isobaric process

$\therefore\ P = \text{constant} \Rightarrow \dfrac{V}{T} = \text{constant}$

Work done in isobaric process:

$\Delta W = \int_{V_i}^{V_f} PdV = P\int_{V_i}^{V_f} dV = P[V_f - V_i]$

[As P = constant]

$\Rightarrow\ \Delta W = P(V_f - V_i) = nR[T_f - T_i] = nR\Delta T$

Heat transferred : $\Delta Q = \Delta U + \Delta W$

$= n(C_v + R)\Delta T = nC_p\Delta T$

Specific heat: Specific heat of gas during isobaric process

$C_p = \left(\dfrac{f}{2}+1\right)R$

Bulk modulus of elasticity: $B = \dfrac{\Delta P}{\dfrac{-\Delta V}{V}} = 0$

2. Iso-Choric Process :

V = constant

$\Rightarrow$ change in volume is zero

$\dfrac{P}{T}$ = const. (Gay-Lussac's law)

Work done in isochoric process:

Since change in volume is zero therefore

$\int dW = \int PdV = 0$

Heat absorbed in isochoric process

$\Delta Q = \Delta U = nC_v\Delta T$

Specific heat: Specific heat of gas during isochoric process

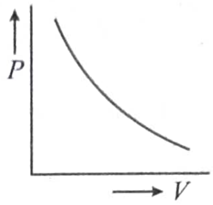
$C_v = \dfrac{f}{2}R$

Bulk modulus of elasticity: $B = \dfrac{\Delta P}{\dfrac{-\Delta V}{V}} = 0$

3. Isothermal Process:

T = constant (Boyle's law)

PV = constant

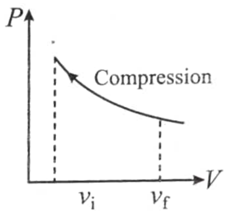


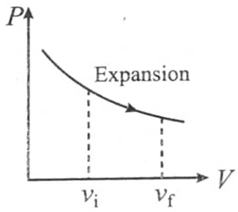
Work done in isothermal process:

$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} \dfrac{nRT}{V} dV$ [As $PV = nRT$]

$W = nRT\log_e\left(\dfrac{V_f}{V_i}\right) = 2.303\,nRT\log_{10}\left(\dfrac{V_f}{V_i}\right)$

or $W = nRT\log_e\left(\dfrac{P_i}{P_f}\right) = 2.303\,nRT\log_{10}\left(\dfrac{P_i}{P_f}\right)$





Heat absorbed in isothermal process

$\Delta Q = \Delta U + W = W$

as $\Delta U = 0$

Specific heat: Specific heat of gas during isothermal change is infinite.

As $C = \dfrac{\Delta Q}{m\Delta T} = \dfrac{\Delta Q}{m \times 0} = \infty$ [As $\Delta T = 0$]

Bulk Modulus of Elasticity (B): For this process PV = constant.

$\Rightarrow\ P\Delta V = -V\Delta P \Rightarrow\ P = \dfrac{\Delta P}{-\Delta V / V} = P$

$\Rightarrow\ B = P$ *i.e.* isothermal elasticity is equal to pressure

At N.T.P., B = Atmospheric pressure

$= 1.01 \times 10^5$ N/m^2

4 Adiabatic Process:

When a thermodynamic system undergoes a change in such a way that no exchange of heat takes place between System and surroundings, the process is known as adiabatic process.

In this process P, V and T changes but $\Delta Q = 0$.

Essential conditions for adiabatic process

(i) There should not be any exchange of heat between the system and its surroundings. All walls of the container and the piston must be perfectly insulating.

(ii) The system should be compressed or allowed to expand suddenly so that there is no time for the exchange of heat between the system and its surroundings.

In adiabalic process $dQ = O$

From first law of thermodynamics

$0 = nC_v\, dT + PdV$

But $P = \dfrac{nRT}{V}$

$\Rightarrow 0 = nC_v dT + \dfrac{nRT}{V} dV \Rightarrow \dfrac{dT}{T} + \dfrac{R}{C_v}\dfrac{dV}{V} = 0$

$\Rightarrow \int \dfrac{dT}{T} + \dfrac{R}{C_v}\int \dfrac{dV}{V} = \text{const.} \Rightarrow \ln T + \dfrac{R}{C_v}\ln V = \text{const.}$

$\ln (TV^{R/C_v}) = \text{const.}$

$TV^{R/C_v} = \text{const.}$

$TV^{\gamma-1} = \text{const.}$

Using $PV = nRT$, we can also express it as

or $PV^{\gamma} = \text{const.}$

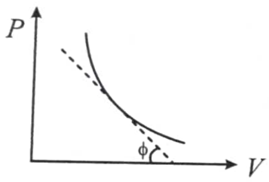
or $P^{1-\gamma}T^{\gamma} = \text{const.}$

Table: Special cases of adiabatic process

Types of gas	$P \propto \dfrac{1}{V^{\gamma}}$	$P \propto T^{\frac{\gamma}{\gamma-1}}$	$T \propto \dfrac{1}{V^{\gamma-1}}$
Monoatomic $\gamma = 5/3$	$P \propto \dfrac{1}{V^{5/3}}$	$P \propto T^{5/2}$	$T \propto \dfrac{1}{V^{2/3}}$
Diatomic $\gamma = 7/5$	$P \propto \dfrac{1}{V^{7/5}}$	$P \propto T^{7/2}$	$T \propto \dfrac{1}{V^{2/5}}$
Polyatomic $\gamma = 4/3$	$P \propto \dfrac{1}{V^{4/3}}$	$P \propto T^{4}$	$T \propto \dfrac{1}{V^{1/3}}$

Indicator diagram

(*i*) Curve obtained on PV graph are called adiabatic curve.



(*ii*) Slope of adiabatic curve : From $PV^{\gamma} = \text{constant}$

By differentiating, we get

$dPV^{\gamma} + P\gamma V^{\gamma-1}dV = 0$

$\dfrac{dP}{dV} = -\gamma\dfrac{PV^{\gamma-1}}{V^{\gamma}} = -\gamma\left(\dfrac{P}{V}\right)$

$\therefore$ Slope of adiabatic curve $\tan\phi = -\gamma\left(\dfrac{P}{V}\right)$

(*iii*) But we also know that slope of isothermal curve $\tan\theta = \dfrac{-P}{V}$

Hence $(\text{Slope})_{Adi} = \gamma \times (\text{Slope})_{Iso}$ or $\dfrac{(\text{Slope})_{Adi}}{(\text{Slope})_{Iso}} > 1$

Work done in adiabatic process:

$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \dfrac{K}{V^{\gamma}}dV$

$\Rightarrow W = \dfrac{[P_iV_i - P_fV_f]}{(\gamma-1)} = \dfrac{nR(T_i - T_f)}{(\gamma-1)}$

(As $PV^{\gamma} = K$, $P_fV_f = nRT_f$ and $P_iV_i = nRT_i$)

(*i*) $W \propto$ quantity of gas (either M or n)

(*ii*) $W \propto$ temperature difference $(T_i - T_f)$

(*iii*) $W \propto \dfrac{1}{\gamma-1}$ $\because \gamma_{mono} > \gamma_{di} > \gamma_{tri} \Rightarrow W_{\text{mono}} < W_{\text{dia}} < W_{\text{tri}}$

Specific heat: Specific heat of a gas during adiabatic change is zero As $C = \dfrac{\Delta Q}{m\Delta T} = \dfrac{0}{m\Delta T} = 0$ [As $Q = 0$]

Bulk Modulus of Elasticity = Adiabatic elasticity (B):

$PV^{\gamma} = \text{constant}$

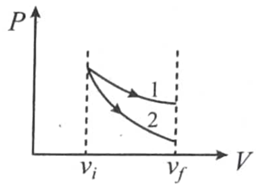
Differentiating both sides $d\,PV^{\gamma} + P\gamma V^{\gamma-1}\,dV = 0$

$\gamma P = \dfrac{\Delta P}{-\Delta V/V} \Rightarrow B = \gamma P$

i.e. adiabatic elasticity is γ times that of pressure

Comparison between isothermal and adiabatic indicator diagrams: Always remember that adiabatic curves are more steeper than isothermal curves

(*i*) **Equal expansion:** Graph 1 represent isothermal process and 2 represent adiabatic process



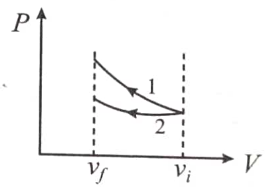
$W_{\text{isothermal}} > W_{\text{adiabatic}}$

$P_{\text{isothermal}} > P_{\text{adiabatic}}$

$T_{\text{isothermal}} > T_{\text{adiabatic}}$

$(\text{Slope})_{\text{Isothermal}} < (\text{Slope})_{\text{adi}}$

(*ii*) **Compression:** Graph 1 represent adiabatic process and 2 represent isothermal process



$W_{\text{Adiabatic}} > W_{\text{Isothermal}}$

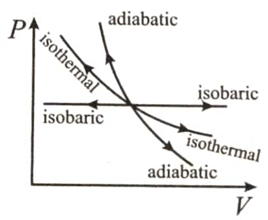
$P_{\text{Adiabatic}} > P_{\text{Isothermal}}$

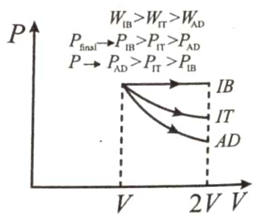
$T_{\text{Adiabatic}} > T_{\text{Isothermal}}$

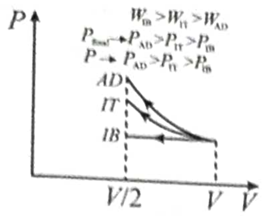
$(\text{Slope})_{\text{Isothermal}} < (\text{Slope})_{\text{Adiab}}$

Important point

When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.







5. Polytropic Process

A polytropic process is one in which the equation of the pressure versus volume curve is of the form PV^N constant; where N is a constant.

The thermodynamic processes discussed earlier, may, in some instances, be regarded as particular cases of the polytropic process, where each process has a corresponding value for the exponent N = constant.

It is easy to see that, for the isochoric process, $N \rightarrow \pm\infty$ for the isobaric process, $N = 0$ for the isothermal process, $N = 1$; and for the adiabatic process, $N = \gamma = C_p / C_v$ (the adiabatic index).

The values of N for the last three above mentioned processes are apparent; for the isochoric process, if we extract the N the root on both sides of the equation PV^N = constant, we get $P^{1/N} \times V$ = constant.

For V = constant, $P^{1/N} = 1$ or $\dfrac{1}{N} = 0 \Rightarrow N \rightarrow \pm\infty$

Thus, for a polytropic process, in the general case, PV^N = constant

Work Done in Polytropic Process

Let $PV^N = C$

$$W = \int PdV = \int_{V_i}^{V_f} \frac{C}{V^N} dV$$

$$= \frac{1}{(-N+1)} \frac{C}{V^{N-1}}\Bigg|_{V_i}^{V_f} = PV$$

$$= \frac{1}{1-N} PV\Big|_{V_i}^{V_f} = \frac{1}{1-N}\left(P_f V_f - P_i V_i\right)$$

$$= \frac{nR}{1-N}\left(T_f - T_i\right) = \frac{nR\Delta T}{1-N}$$

Heat absorbed :

$$\Delta Q = \Delta U + W = n\left(\frac{R}{\gamma-1} + \frac{R}{1-N}\right)\Delta T$$

Moler specific heat : $C = \dfrac{\Delta Q}{n\Delta T} = \dfrac{R}{\gamma-1} + \dfrac{R}{1-N}$

STANDARD PROCESSES AS A SPECIAL CASE OF POLYTROPIC PROCESS

All the standard processes are the special cases of general polytropic processes. Let us discuss these processes again in terms of polytropic process

Isochoric Process: In an isochoric process, the process equation for the process is given by V = constant

The general equation for polytropic process PV^N = constant, approaches this result when $N \rightarrow \infty$ and in this case The molar specific heat of the process is given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-N} \text{ as } N \rightarrow \infty \; C = \frac{R}{\gamma-1} = C_V$$

which is true for the isochoric process

Isobaric process: In isobaric process the process equation is given as P = constant

The general equation of polytropic process PV^N = constant, above equation is obtained by substituting polytropic constant $N = 0$

Hence the molar specific heat of this process can be given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-N} \text{ or } C = \frac{R}{\gamma-1} + R = C_V + R = C_P$$

which is true for the isobaric process

Isothermal process: In isothermal process equation is given as: PV = constant

If we compare above equation with general equation of polytropic process, we can see that for an isothermal process the polytropic constant $N = 1$. Hence, we can find molar specific heat for isothermal process as

$$C = \frac{R}{\gamma-1} + \frac{R}{1-N} \text{ or } C \rightarrow \infty$$

The temperature of gas in isothermal process never changes and molar heat capacity is the amount of heat required to change the temperature for one mole of gas by one degree. Hence if we continuously supply heat to a gas infinitely then also its temperature will not change then it is undergoing an isothermal process.

Adiabatic process: The process equation of an adiabatic process is given as PV^γ = Constant

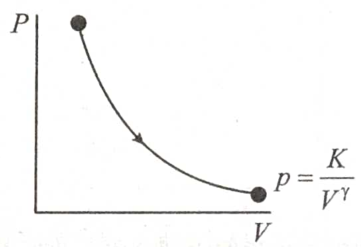
Hence, for an adiabatic process, the polytropic constant is equal to the adiabatic exponent of the gas, $N = \gamma$. Now the molar heat capacity for a gas in adiabatic process is

$$C = \frac{R}{\gamma-1} + \frac{R}{1-\gamma} = 0$$

Now we can say that in an adiabatic expansion or compression, temperature of gas changes without any supply of heat. Thus, in this process no heat is required to raise the temperature of gas.

Train Your Brain

Example 28: The molar heat capacity for the process shown in fig. is



(a) $C = C_p$ (b) $C = C_v$

(c) $C > C_v$ (d) $C = 0$

Sol. (d) For polytropic process

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-x} \Rightarrow \text{As } PV^{\gamma} = K$$

$\Rightarrow$ Put $x = \gamma$ $\therefore$ $C = 0$

Example 29: In a process, temperature and volume of one mole of an ideal monoatomic gas *are* varied according to the relation $VT = K$, where K is a constant. In this process the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant)

(a) $\frac{1}{2}R\Delta T$ (b) $\frac{1}{2}KR\Delta T$

(c) $\frac{3}{2}R\Delta T$ (d) $\frac{2K}{3}\Delta T$

Sol. (a) $VT = K$

$$\Rightarrow V\left(\frac{PV}{nR}\right) = K \Rightarrow PV^2 = K$$

$$\therefore \quad C = \frac{R}{1-x} + C_V \qquad \text{(For polytropic process)}$$

$$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$$

$$\therefore \quad \Delta Q = nC\,\Delta T = \frac{1R}{2}\Delta T$$

Example 30: Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by has is close to: (Gas constant R = 8.31 J/mol-K)

(a) 581 J

(b) 291 J

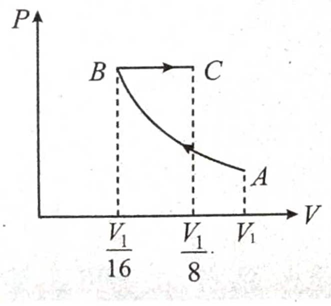
(c) 146 J

(d) 73 J

Sol. (b) $W = nR\Delta T = \frac{1}{2}\times 8.31\times 70 = 290.85\ J$

Example 31: Starting at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2 = V_1/16$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in K) is (to the nearest integer) ________.

Sol. [1819]



PV^{γ} = constant ; $TV^{\gamma-1}$ = constant

$$300(V_1)^{1.4-1} = T_B\left(\frac{V_1}{16}\right)^{2/5}$$

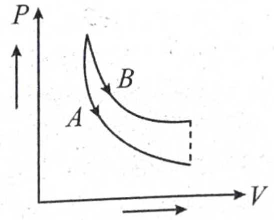
$T_B = 300 \times 2^{8/5}$

Now for process BC

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}\ ;\ T_C = \frac{V_C T_B}{V_B} = 2\times 300\times 2^{8/5}$$

$T_C = 1818.859$; $T_C = 1819\ K$

Example 32: A and B are two adiabatic curves for two different gases. Then A and B corresponds to:



(a) Ar and He respectively

(b) He and H_2 respectively

(c) O_2 and H_2 respectively

(d) H_2 and He respectively

Sol. (b) Slope $= -\gamma\frac{dP}{dV}$

As slope of $A >$ slope of B

$\therefore$ γ of $A > \gamma$ of B

or $A \rightarrow$ Helium

$B \rightarrow$ Hydrogen

Example 33: Which is the correct statement

(a) For an isothermal change PV = constant

(b) In an isothermal process the change in internal energy must be equal to the work done

(c) For an adiabatic change $\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{\gamma}$, where γ is the ratio of specific heats

(d) In an adiabatic process work done must be equal to the heat entering the system

Sol. (a) In thermodynamic processes.

Work done = Area covered by PV diagram with V-axis

Example 34: Two samples A and B of same gas have equal volumes and pressures. The gas in sample A is expanded isothermally to double its volume and the gas in sample B is expanded to double its volume adiabatically. If work done by the gas is same in the two processes. Show that γ $(= C_p/C_v)$ satisfies the equation

$$1 - 2^{1-\gamma} = (\gamma - 1)\ \text{In}2$$

Sol. Suppose initially pressure and volume of each gas is P_0 and V_0 and n_1 and n_2 are the number of moles in A and B respectively. Similarly T_1 and T_2 are the initial temperature of sample A and B then

$$P_0V_0 = n_1RT_1 = n_2RT_2 \qquad ...(1)$$

Work done in isothermal process is

$$W_1 = n_1 RT_1 \ln\frac{2V_0}{V_0} \quad \text{(Volume is doubled)}$$

or $W_1 = n_1 RT_1 \ln(2)$ ...(2)

Let T_2' be the temperature of sample B after expansion. Then

$$W_2 = \frac{n_2 RT_2 - n_2 RT_2'}{\gamma - 1} \quad ...(3)$$

For adiabatic process

$TV^{\gamma-1}$ = constant

$T_2 V_0^{\gamma-1} = T_2'(2V_0)^{\gamma-1}$

$T_2' = T_2(2)^{1-\gamma}$

Substituting this in equation (3) we get

$$W_2 = \frac{n_2 RT_2\left(1-2^{1-\gamma}\right)}{\gamma-1} \quad ...(4)$$

Given that $W_1 = W_2$

Hence equating (2) and (4) we get

$$n_1 RT_1 \ln(2) = \frac{n_2 RT_2\left(1-2^{1-\gamma}\right)}{\gamma-1}$$

But $n_1 RT_1 = n_2 RT_2$ (from equation 1)

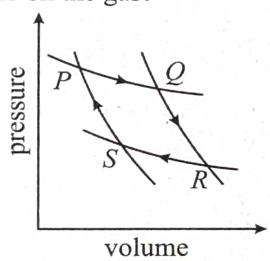
Concept Application

28. Certain perfect gas is found obey $PV^{3/2}$ = const, during adiabatic process. If such a gas at initial temperature T is adiabatically compressed to half the initial volume, in final temperature will be

(a) $\sqrt{2}T$ (b) $2T$

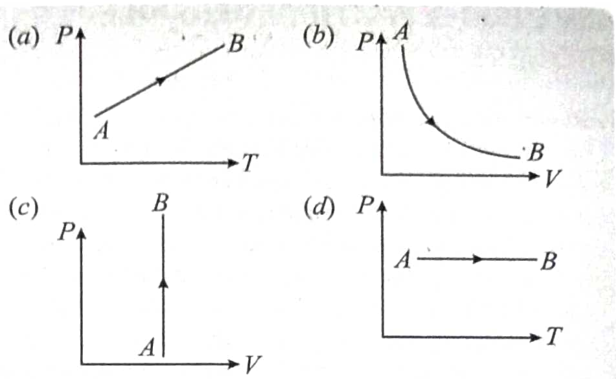
(c) $2\sqrt{2}T$ (d) $4T$

29. A fixed mass of gas undergoes the cycle of changes represented by $PQRSP$ as shown in Figure. In some of the changes, work is done on the gas and in others, work is done by the gas. In which pair of the changes work is done on the gas?



(a) PQ and RS
(b) PQ and QR
(c) OR and RS
(d) RS and SP.

30. The process $\Delta U = 0$, for an ideal gas can be best represented in the form of a graph:



31. The adiabatic bulk modulus of hydrogen gas ($\gamma = 1.4$) at NTP is:

(a) $1 \times 10^5\ \text{N/m}^2$ (b) $1 \times 10^{-5}\ \text{N/m}^2$

(c) $1.4\ \text{N/m}^2$ (d) $1.4 \times 10^5\ \text{N/m}^2$

32. An ideal gas at 27°C is compressed adiabatically to $\frac{8}{27}$ of its original volume. If $\gamma = \frac{5}{3}$, then the rise in temperature is

(a) 450 K (b) 375 K

(c) 225 K (d) 405 K

33. The amount of work done in an adiabatic expansion from temperature T to T_1 is

(a) $R(T-T_1)$ (b) $\frac{R}{\gamma-1}(T-T_1)$

(c) RT (d) $R(T-T_1)(\gamma-1)$

34. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively):

(a) $n = \frac{C_P}{C_V}$ (b) $n = \frac{C - C_P}{C - C_V}$

(c) $n = \frac{C_P - C}{C - C_V}$ (d) $n = \frac{C - C_V}{C - C_P}$

QUASI STATIC PROCESS

When we perform a process on a given system, its state is changed. Suppose the initial state of the system is described by the values P_1, V_1, T_1 and the final state by P_2, V_2, T_2. If the process is performed in such a way that at any instant during the process, the system is very nearly in thermodynamic equilibrium, the process is called quasistatic. This means, we can specify the parameters P, V, T uniquely at any instant during such a process.

Actual processes are not quasi-static. To change the pressure of a gas, we can move a piston inside the enclosure. The gas near the piston is acted upon by piston. The pressure of the gas may not be uniform everywhere while the piston is moving. However, we can move the piston very slowly to make the process as close to quasistatic as we wish. Thus, a quasistatic process is an idealised process in which all changes take place infinitely slowly.

REVERSIBLE AND IRREVERSIBLE PROCESS

1. **Reversible process:** A reversible process is one which can be reversed in such a way that all changes occurring in the direct process are exactly repeated in the opposite order and inverse sense and no change is left in any of the bodies taking part in the process or in the surroundings. For example if heat is absorbed in the direct process, the same amount of heat should be given out in the reverse process, if work is done on the working substance in the direct process then the same amount of work should be done by the working substance in the reverse process. The conditions for reversibility are
   (*i*) There must be complete absence of instantaneow forces such as friction, viscosity, electric resistance etc.
   (*ii*) The direct and reverse processes must take place infinitely slowly.
   (*iii*) The temperature of the system must not differ appreciably from its surroundings.

   Some examples of reversible process are
   (*a*) All isothermal and adiabatic changes are reversible if they are performed very slowly.
   (*b*) When a certain amount of heat is absorbed by ice, it melts. If the same amount of heat is removed from it, the water formed in the direct process will be converted into ice.
   (*c*) An extremely slow extension or contraction of a spring without setting up oscillations.
   (*d*) When a perfectly elastic ball falls from some height on a perfectly elastic horizontal plane, the ball rises to the initial height.
   (*e*) If the resistance of a thermocouple is negligible there will be no heat produced due to Joule's heating effect. In such a case heating or cooling is reversible. At a junction where a cooling effect is produced due to Peltier effect when current flows in one direction and equal heating effect is produced when the current is reversed.
   (*f*) Very slow evaporation or condensation. It should be remembered that the conditions mentioned for a reversible process can never be realised in practice. Hence, a reversible process is only an ideal concept. In actual process, there is always loss of heat due to friction, conduction, radiation *etc.*

2. **Irreversible process:** Any process which is not reversible exactly is an irreversible process. All natural processes such as conduction, radiation, radioactive decay *etc.* are irreversible. All practical processes such as free expansion, Joule-Thomson expansion, electrical heating of a wire are also irreversible. Some examples of irreversible processes are given below
   (*i*) When a steel ball is allowed to fall on an inelastic lead sheet, its kinetic energy changes into heat energy by friction. The heat energy raises the temperature of lead sheet. No reverse transformation of heat energy occurs.
   (*ii*) The sudden and fast stretching of a spring may produce vibrations in it. Now a part of the energy is dissipated. This is the case of irreversible process.
   (*iii*) Sudden expansion or contraction and rapid evaporation or condensation are examples of irreversible processes.
   (*iv*) Heat produced by the passage of an electric current through a resistance is irreversible.
   (*v*) Heat transfer between bodies at different temperatures is also irreversible.
   (*vi*) Joule-Thomson effect is irreversible because on reversing the flow of gas a similar cooling or heating effect is not observed.

Application of First Law Thermodynamics for Different Process

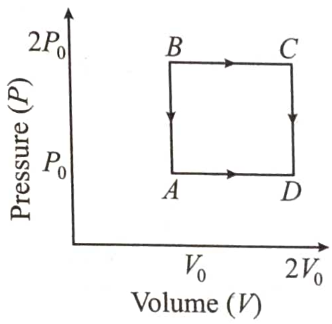
Quantity	Sign	Condition
ΔQ	+	When heat is supplied to a system
	–	When heat is drawn from the system
ΔW	+	When work done by the gas (expansion)
	–	When work done on the gas (compression)
ΔU	+	With temperature rise, internal energy increases
	–	With temperature fall, internal energy decreases

Table: First Law of Thermodynamics Applied to Different Processes

Process	$\Delta U = nC_V\Delta T$	W	$\Delta Q = \Delta U + W$
Cyclic	$nC_v\Delta T = 0$	Area of the closed curve	ΔW
Isochoric	$nC_V\Delta T$ (n mole of gas)	0	ΔU
Isothermal	$nC_v\Delta T = 0$	$nRT\log_e\left[\frac{V_f}{V_i}\right]$	ΔW
Adiabatic	$nC_v\Delta T$	$\frac{nR(T_f - T_i)}{1-\gamma}$	0
Isobaric	$nC_v\Delta T$	$P(V_f - V_i) = nR(T_f - T_i)$	$nC_p\Delta T$

EFFICIENCY OF CYCLIC PROCESS

When a system is subjected to a cyclic process, heat is supplied during some part of the process, while heat is abstracted during other part.



Evidently, the net heat supplied will be the work done by the system ($\because Q = \Delta W$). However, the gross heat supplied will be more than that of the net heat.

Efficiency (η) of a cycle is defined as the ratio of the work performed (net heat given) to the gross heat supplied to the system per cycle,

Thus, $\eta = \dfrac{\text{work done per cycle}}{\text{gross heat supplied per cycle}}$

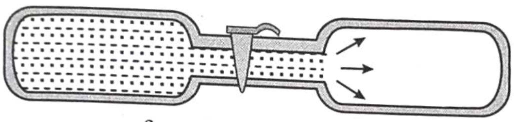
FREE EXPANSION

Take a thermally insulated bottle with ideal gas at some temperature T_1 and, by means of a pipe with a stopcock, connect this to another insulated bottle which is evacuated.

If we suddenly open the stopcock, the gas will rush from the first bottle into the second until the pressures are equalized.

Experimentally, we find that this process of free expansion does not change the temperature of the gas-when the gas attains equilibrium and stops flowing, the final temperature of both bottles are equal to the initial temperature T_1.

This process is called a free expansion.



free expansion of a gas

The change in the internal energy of the gas can be calculated by applying the first law of thermodynamics to the free-expansion process.

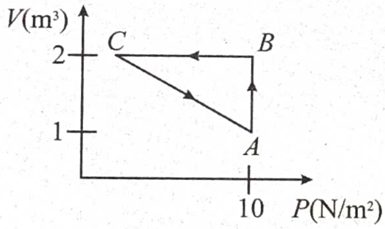
The process is adiabatic because of the insulation,

so $Q = 0$.

No part of the surroundings moves so the system does no work on its surroundings.

Train Your Brain

Example 35: An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 *J*, the work done by the gas in the process $C \to A$ is



(*a*) $-5\ J$ (*b*) $-10\ J$
(*c*) $-15\ J$ (*d*) $-20\ J$

Sol. (*a*) For cyclic process. Total work done

$= W_{AB} + W_{BC} + W_{CA}$

$\Delta W_{AB} = P\Delta V = 10\,(2-1) = 10$ J and $\Delta W_{BC} = 0$

(as V = constant)

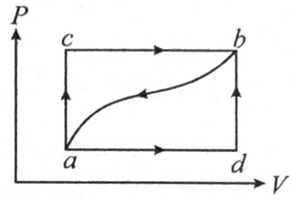
$\Delta Q = \Delta U + \Delta W$

$\Delta U = 0$ (Processs *ABCA* is cyclic)

$\Rightarrow \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$\Rightarrow 5 = 10 + 0 + \Delta W_{CA} \Rightarrow \Delta W_{CA} = -5\ J$

Example 36: When a system is taken from state '*a*' to state '*b*' along the path '*acb*', it is found that a quantity of heat $Q = 200\ J$ is absorbed by the system and a work $W = 80\ J$ is done by it. Along the path '*adb*', $Q = 144\ J$. The work done along the path '*adb*' is



(*a*) $6\ J$ (*b*) $12\ J$
(*c*) $18\ J$ (*d*) $24\ J$

Sol. (*d*) ΔU = same is both process

$Q_{acb} - W_{acb} = Q_{adb} - W_{adb}$.

$200 - 80 = 144 - W_{adb}$.

$W_{adb} = 24$ J.

Example 37: In the above question, if the work done on the system along the curved path '*ba*' is 52J, heat absorbed is

(*a*) $-140\ J$ (*b*) $-172\ J$
(*c*) $140\ J$ (*d*) $172\ J$

Sol. (*b*) $\Delta U = Q_{acb} - W_{acb} = 200 - 80 = 120$ J

$\Delta U = Q_{ba} - W_{ba}, -120 = Q_{ba} + 52,$

$Q_{ba} = -172$ J.

Example 38: In above question, if $U_a = 40\ J$, value of U_b will be

(*a*) $-50\ J$ (*b*) $100\ J$
(*c*) $-120\ J$ (*d*) $160\ J$

Sol. (*d*) $U_b - U_a = 120$

$U_b = 120 + 40 = 160$J

Example 39: In above question, if $U_d = 88\ J$, heat absorbed for the path '*db*' is

(*a*) $-72\ J$ (*b*) $72\ J$
(*c*) $144\ J$ (*d*) $-144\ J$

Sol. (*b*) In *db*.

$W_{db} = 0\ ;\ U_b - U_d = Q_{db}$.

$160 - 88 = Q_{db}$

$Q_{db} = 72$J.

Example 40: Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume $2V$. Calculate (*i*) the final temperature of the gas and (*ii*) change in its internal energy.

(*a*) (*i*) 195 *K* (*ii*) –2.7 kJ (*b*) (*i*) 189 *K* (*ii*) –2.7 kJ
(*c*) (*i*) 195 *K* (*ii*) 2.7 kJ (*d*) (*i*) 189 *K* (*ii*) 2.7 kJ

Sol. (*b*) In an adiabatic process

PV^{γ} = constant and, $PV = nRT$, gives

$\Rightarrow \quad V^{\gamma-1} \propto \frac{1}{T}\ ;\ \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

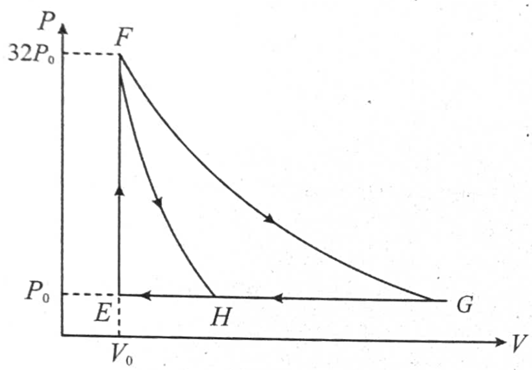
monoatomic gas : $\gamma = \frac{5}{3}$

$\Rightarrow \quad T_2 = (300\ K)\left(\frac{V}{2V}\right)^{\frac{5}{3}-1} = 189\ K$ (final temperature)

change in internal energy

$\Delta U = n\frac{f}{2}R\ \ \Delta T = 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7$ kJ

Example 41: One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



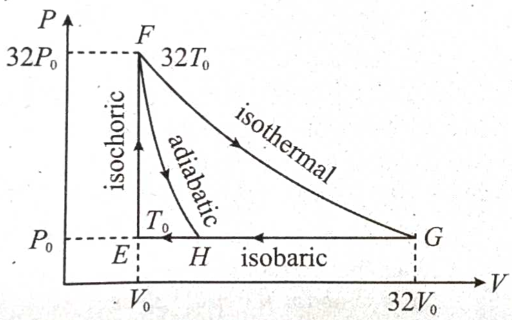
Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

	List I		List II
P.	$G \to E$	1.	$60\ P_0V_0$
Q.	$G \to H$	2.	$36\ P_0V_0$
R.	$F \to H$	3.	$24\ P_0V_0$
S.	$F \to G$	4.	$31\ P_0V_0$

Codes:

	P	Q	R	S
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	1	2	4
(d)	1	3	2	4

Sol. (a)



In $F \to G$ work done in isothermal process is $nRT \ln \left(\frac{V_f}{V_i}\right) = 32\ P_0V_0 \ln \left(\frac{32V_0}{V_0}\right)$

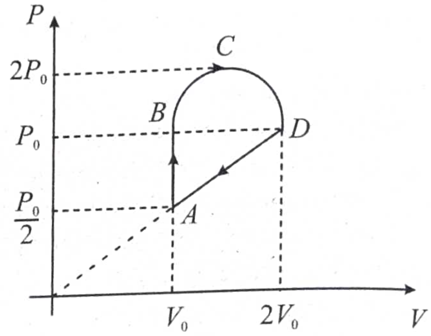
$= 32\ P_0V_0\ ln\ 2^5 = 160\ P_0V_0\ ln\ 2$

In $G \to E$, $\Delta W = P_0\ \Delta V = P_0\ (31\ V_0) = 31\ P_0V_0$

In $G \to H$ work done is less than $31\ P_0V_0$ i.e. $24\ P_0V_0$

In $F \to H$ work done is $36\ P_0V_0$

Example 42: Two moles of a monoatomic ideal gas undergo a cyclic process $ABCDA$ as shown in figure. BCD is a semicircle. Find the efficiency of the cycle.



Sol. Process AB is isochoric (V = constant). Hence

$\Delta W_{AB} = 0$

$\Delta W_{BCD} = P_0V_0 + \frac{\pi}{2}(P_0)\frac{V_0}{2} = \frac{\pi}{4}+1P_0V_0$

$\Delta W_{DA} = -\frac{1}{2}\frac{P_0}{2} + P_0(2V_0 - V_0) = -\frac{3}{4}P_0V_0$

$\Delta U_{AB} = nC_V\Delta T = (2)\frac{3}{2}R(T_B - T_A) \quad \left(n = 2, C_V = \frac{3}{2}R\right)$

$= 3R\frac{P_0V_0}{2R} - \frac{P_0V_0}{4R} = \frac{3}{4}P_0V_0 = \Delta Q_{AB} \quad \left(T = \frac{PV}{nR}\right)$

$\Delta U_{BCD} = nC_V\Delta T = (2)\frac{3}{2}R(T_D - T_B)$

$= (3R)\frac{2P_0V_0}{2R} - \frac{P_0V_0}{2R} = \frac{3}{2}P_0V_0$

Hence, $\Delta Q_{BCD} = \Delta U_{BCD} + \Delta W_{BCD}$

$= \frac{\pi}{4} + \frac{5}{2}P_0V_0 = (2)\frac{3}{2}R(T_A - T_D)$

$= (3R)\frac{P_0V_0}{4R} - \frac{2P_0V_0}{2R} = -\frac{9}{4}P_0V_0$

$\therefore\ \Delta Q_{DA} - \Delta U_{DA} + \Delta W_{DA}$

$= -\frac{9}{4}P_0V_0 - \frac{3}{4}P_0V_0 = -3\ P_0V_0$

Net work done is $W_{net} = \frac{\pi}{4} + 1 - \frac{3}{4}P_0V_0 = 1.04\ P_0V_0$

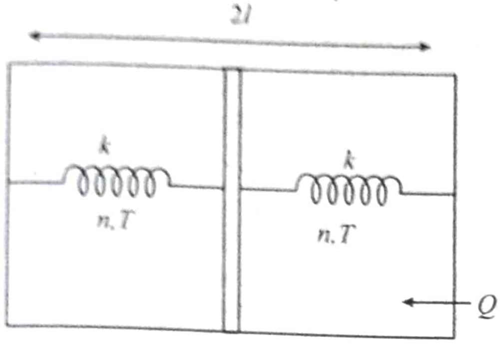
and heat absorbed is $Q_{ab} = \Delta Q_{+ve}$

$= \frac{3}{4} + \frac{\pi}{4} + \frac{5}{2}P_0V_0 = 0.03\ P_0\ V_0$

Hence efficiency of the cycle is $\eta = \frac{W_{net}}{Q_{ab}} \times 100$

$= \frac{1.04P_0V_0}{4.03P_0V_0} \times 100 = 25.8\%$

Example 43: A horizontal insulated cylindrical vessel of length $2l$ is separated by a thin insulating piston into two equal parts each of which contains n moles of an ideal monoatomic gas at temperature T. The piston is connected to the end faces of the vessel by undeformed springs of force constant k each. The left part is in contact with a thermostat (a device which maintains a constant temperature). When an amount of heat Q is supplied to the gas in the right part, the piston is displaced to the left by a distance $x = l/2$. Determine the heat Q' given away at the temperature T to the thermostat by the left part of the piston.



Sol. Total heat given to the system is $Q - Q'$. So from first law of thermodynamics, $Q - Q'$ = total work done by the gas in both the chambers (W) + change in internal energies of both gases (ΔU) ...(1)

Here W = sum of potential energies stored in the springs

$$W = 2\left[\frac{1}{2}k\left(\frac{l}{2}\right)^2\right] = \frac{kl^2}{4} \quad ...(2)$$

Since the temperature of left part remains constant (piston does not conduct heat), internal energy of left part does not change. ΔU of right part can be given as

$$\Delta U = nC_V\Delta T = \frac{3}{2}nR\Delta T \quad ...(3)$$

ΔT can be found from the condition of equilibrium at the end of the process.

pressure on right side pressure on left side

$$\text{or} \quad \frac{nR(T+\Delta T)}{A(l+l/2)} = \frac{nRT}{A(l-l/2)} + \frac{2k\frac{l}{2}}{A}$$

simplifying this we get

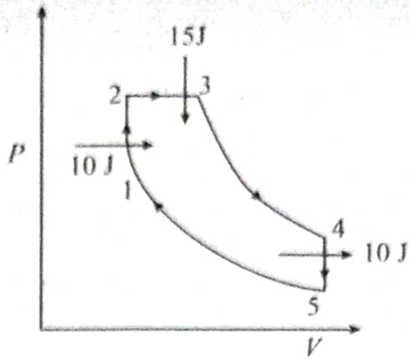
$$\text{or } \Delta T = \frac{3kl^2}{2nR} + 2T \quad ..(4)$$

From equations (1), (2), (3) and (4), we get

$$Q' = Q - \frac{kl^2}{4} - \frac{3}{2}nR\frac{3kl^2}{2nR} + 2T$$

$$\text{or } Q' = Q - \frac{5}{2}kl^2 - 3nRT$$

Example 44: A gas undergoes cyclic process as shown in the figure, 5 – 1 and 3 – 4 are adiabatic process, 1 – 2 and 4 – 5 are isochoric process, 2 – 3 is isobaric process. Find efficiency of the cycle.



Sol. $W_{net} = (\Sigma Q)_{cycle} = 10 + 15 - 10 = 15\ J$

$\Delta Q_{in} = 10 + 15 = 25\ J$

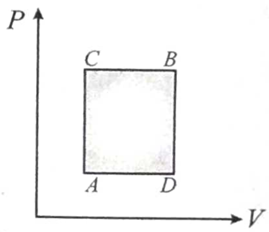
$$\eta = \frac{15}{25} \times 100 = 60\%$$

Concept Application

35. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as
(*a*) Irreversible adiabatic
(*b*) Reversible adiabatic
(*c*) Irreversible isothermal
(*d*) Reversible isothermal

36. A thermodynamic cycle takes in heat energy at a high temperature and rejects energy at a lower temperature. If the amount of energy rejected at the low temperature is 3 times the amount of work done by the cycle, the efficiency of the cycle is
(*a*) 0.25 (*b*) 0.33
(*c*) 0.67 (*d*) 0.9

37. A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work down by the system is 10J. The heat flow into the system in path ADB is



(*a*) 40 J (*b*) 80 J
(*c*) 100 J (*d*) 20 J

38. For free expansion of a gas in an adiabatic container which of the following is true?
(*a*) $Q = W = 0$ and $\Delta U = 0$
(*b*) $Q = 0$, $W > 0$ and $\Delta U = Q$
(*c*) $W = 0$, $Q > 0$ and $\Delta U = Q$
(*d*) $W = 0$, $Q < 0$ and $\Delta U = 0$

39. An ideal gas is allowed to expand freely against a vacuum in a rigid insulated container. The gas undergoes:
(a) an increase in its internal energy
(b) a decrease in its internal energy
(c) neither an increase nor decrease in temperature or internal energy
(d) an increase in temperature

40. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabatically to $1/10^{th}$ of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is______.
(a) 40 (b) 48
(c) 46 (d) 52

41. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for the process is TV^x = constant, then x is:
(a) 3/5 (b) 2/5
(c) 2/3 (d) 5/3

SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy to include heat energy. It tells us that heat and mechanical work are mutually interconvertible.

Second law of thermodynamics tells us in what conditions heat can be converted into useful work.

The following three conditions must be fulfilled to utilize heat for useful work :

1. A device called engine with a working substance is essential.
2. The engine must work in a reversible cyclic process.
3. The engine must operate between two temperatures. It will absorb heat from a hot body (called source), convert a part of into useful work and reject the rest to a cold body (called sink).

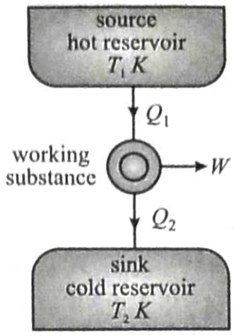
There are two conventional statements of second law

(i) **Kelvin-Planck Statement:** It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work. In other words, 100% conversion of heat into work is impossible.

(ii) **Clausius Statement:** It is impossible for a self-acting machine, unaided by any external agency to transfer heat from a cold to hot reservoir. In other words heat can not in itself flow from a colder to a hotter body.

HEAT ENGINE

Heat engine is a device which converts heat into work.



Three parts of a heat engine:

(i) Source of high temperature reservoir at temperature T_1
(ii) Sink or low temperature reservoir at temperature T_2
(iii) Working substance.

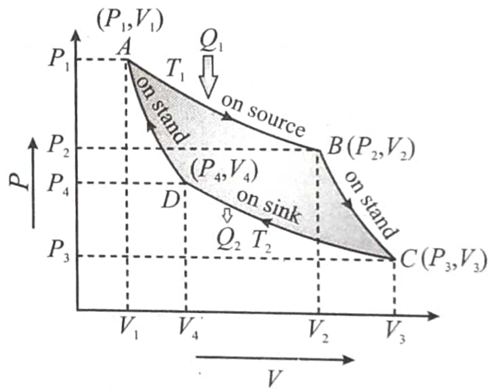
In a cycle of heat engine the working substance extracts hea Q_1 from source, does some work W and rejects remainin heat Q_2 to the sink.

Efficiency of heat engine

$$\eta = \frac{\text{work done}\,(W)}{\text{heat taken from soruce}\,(Q_1)}$$

CARNOT CYCLE (CARNOT ENGINE)

Carnot devised an ideal engine which is based on a reversibl cycle of four operations in succession:



(i)	isothermal expansion	$A \rightarrow B$
(ii)	adiabatic expansion	$B \rightarrow C$
(iii)	isothermal compression	$C \rightarrow D$
(iv)	adiabatic compression	$D \rightarrow A$

Working

A set of reversible processes through them working substance taken back to initial condition to get maximum work from th type of ideal engine.

Processes of Carnot's cycle can be denoted by an indicat diagram.

Isothermal expansion $A \rightarrow B$

Gas expand and receive heat Q_1 from source and gets stat $B(P_2, V_2, T_1)$

This heat input Q_1 to the gas from path A to B is utilized fo doing work W_1.

By path A to B the heat input to the gas = the work done against the external pressure.

$$W_1 = Q_1 = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT_1 \ell n \frac{V_2}{V_1}$$

$$\Rightarrow W_1 = nRT_1 \ell n \frac{V_2}{V_1}$$

$$\Rightarrow W_1 = 2.303 nRT_1 \log_{10} \frac{V_2}{V_1}$$

Adiabatic expansion $B \to C$

The tempeature falls to T_2 and gas describes the adiabatic from B to **point** C (P_3, V_3, T_2) during this expansion more work is done (W_2) at the expense of the internal energy.

Work done in adiabatic path BC is

$$W_2 = \frac{nR}{\gamma - 1}(T_1 - T_2)$$

Isothermal compression $C \to D$

The gas is isothermally compressed from C to **point D**. (P_4, V_4, T_2)

The heat rejected Q_2 to the cold reservoir (sink) at T_2 occurs over this path.

Amount of work done on gas W_3 = amount of heat rejected to the sink

$$W_3 = Q_2 = nRT_2 \ell n \frac{V_4}{V_3} \Rightarrow W_3 = Q_2 = -nRT_2 \ell n \frac{V_3}{V_4}$$

Adiabatic compression $D \to A$

Now work is done on the gas during adiabatic compression from state D to initial point A (P_1, V_1, T_1).

No heat exchanges occur over the adiabatic path.

Work done on the system W_4

$$= \frac{nR}{\gamma - 1}(T_2 - T_1) = -\frac{nR}{\gamma - 1}(T_1 - T_2)$$

This cycle of operations is called a Carnot cycle.

In first two steps work is done by engine W_1 and W_2 are positive

In last two steps work is done on gas

W_3 and W_4 are negative

The work done in complete cylce W = the area of the closed part of the P–V cycle.

$$W = W_1 + W_2 + W_3 + W_4$$

$$\therefore W = nRT_1 \ell n \frac{V_2}{V_1} + \frac{nR}{\gamma - 1}(T_1 - T_2) - nRT_2 \ell n \frac{V_3}{V_4} - \frac{nR}{\gamma - 1}(T_1 - T_2)$$

$$\therefore W = nRT_1 \ell n \frac{V_2}{V_1} - nRT_2 \ell n \frac{V_3}{V_4}$$

B to C and D to A are adiabatic paths

so $T_1 V_2^{(\gamma - 1)} = T_2 V_3^{(\gamma - 1)}$ and $T_1 V_1^{(\gamma - 1)} = T_2 V_4^{(\gamma - 1)}$

$$\Rightarrow \frac{V_3}{V_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma - 1}} = \rho \text{ and } \frac{V_4}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma - 1}} = \rho$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Where ρ = adiabatic expansion ratio

$\therefore$ Work done in Carnot cycle $W = nR(T_1 - T_2)\, \ell n \left(\frac{V_2}{V_1}\right)$

Efficiency of Carnot Engine

$$\eta = \frac{W}{Q_1} \Rightarrow \eta = \frac{nR(T_1 - T_2)\ell n \frac{V_2}{V_1}}{nRT_1 \ell n \frac{V_2}{V_1}}$$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\Rightarrow \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\boxed{\eta = 1 - \frac{T_2}{T_1}} ; \ \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \text{ (for reversible process)}$$

$$\boxed{\eta = 1 - \frac{T_{Lower}}{T_{Higher}}}$$

- The efficiency for the Carnot engine is the best that can be obtained for any heat engine.
- The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink T_2 = 0K or $T_1 = \infty$ which is impossible.

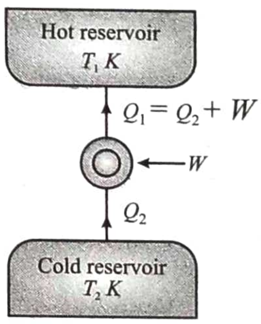
REFRIGERATOR

It is inverse of heat engine. It extracts heat (Q_2) from a cold reservoir, same external work W is done on it and rejects heat (Q_1) to hot reservoir.

The coefficient of performance (COP) of a refrigerator.

$$\beta = \frac{\text{heat extracted from cold reservoir}}{\text{work done on refrigerator}}$$

$$= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$



For Carnot reversible refrigerator $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\therefore \beta = \frac{Q_2}{W} = \frac{1}{\left[\frac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\frac{T_1}{T_2} - 1\right]}$$

$$\Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

ENTROPY

It is a measure of molecular disorder of a system. Greater is disorder, greater is entropy.

The change in entropy,

$$\Delta S = \frac{\text{heat absorbed}}{\text{absolute temperature}}$$

$$\Delta S = \frac{\Delta Q}{T} \Rightarrow \Delta Q = T\Delta S$$

Mathematical form of second law of thermodynamics $\Delta Q = T\Delta S$

For solids and liquids

The change in entropy during a change of state of mass m of substance $\Delta S = \pm \frac{mL}{T}$

where (+) sign refers to heat absorption and (–) sign for heat evolution.

Train Your Brain

Example 45: A carnot engine has the same efficiency between 800 K to 500 K and x K to 600 K. The value of x is

(*a*) 1000 K (*b*) 960 K (*c*) 846 K (*d*) 754 K

Sol. (*b*) In first case, $(\eta_1) = 1 - \frac{500}{800} = \frac{3}{8}$

and in Second case, $(\eta_2) = 1 - \frac{600}{x}$

Since $n_1 = n_2$ therefore $\frac{3}{8} = 1 - \frac{600}{x}$

or $\frac{600}{x} = 1 - \frac{3}{8} = \frac{5}{8}$.

or $x = \frac{600 \times 8}{5} = 960\ K$

Example 46: A Cannot engine works between 200°C and 0°C. Another Carnot engine works between 0°C and –200°C. In both cases the working substance absorbs 4 kilocalories of heat from the source. The efficiency of first engine will be:

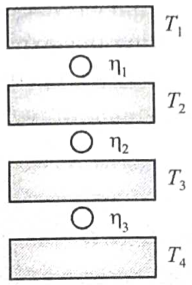
(*a*) $\frac{100}{473}$ (*b*) $\frac{200}{473}$

(*c*) $\frac{200}{273}$ (*d*) $\frac{273}{373}$

Sol. (*b*) $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100 = \left(1 - \frac{273}{473}\right) \times 100$

$= \frac{200}{473} \times 100$ in % or $\eta\ \frac{200}{473}$ in fraction

Example 47: Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if:



(*a*) $T_2 = (T_1T_4)^{1/2}$; $T_3 = (T_1^2T_4)^{1/3}$

(*b*) $T_2 = (T_1^2T_4)^{1/3}$; $T_3 = (T_1T_4^2)^{1/3}$

(*c*) $T_2 = (T_1T_4^2)^{1/3}$; $T_3 = (T_1^2T_4)^{1/3}$

(*d*) $T_2 = (T_1^3T_4)^{1/4}$; $T_3 = (T_1T_4^3)^{1/4}$

Sol. (*b*) $\eta_1 = \eta_2 = \eta_3$

$$\Rightarrow 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow T_2\,T_3 = T_1\,T_4 \text{ and } \frac{T_3^2}{T_2} = T_4$$

Solve for T_2 and T_3.

Example 48: A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10J, the amount of heat absorbed from the reservoir at lower temperature is:

(*a*) 90 J (*b*) 1 J

(*c*) 99 J (*d*) 100 J

Sol. (*a*) For Carnot engine using as refrigerator

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

It is given $\eta = \frac{1}{10} \Rightarrow \eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{9}{10}$

So, $Q_2 = 90\ J$ (as $W = 10\ J$)

Example 49: If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer ?

Sol. [8791]

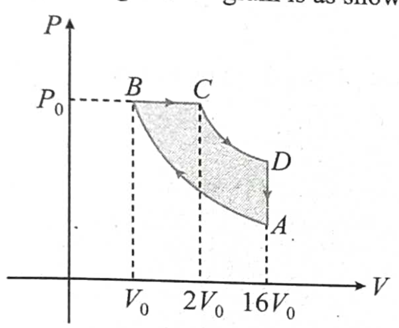
$$\frac{Q_1}{T_1}=\frac{Q_2}{T_2}\ ;\ Q_1=(100\times 80)=8000 \text{ cal}$$

$$\therefore\quad Q_2=\frac{8000\times 300}{273}\approx 8791$$

Example 50: One mole of a diatomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A. The process $A \to B$ is an adiabatic compression. $B \to C$ is isobaric expansion, $C \to D$ an adiabatic expansion and $D \to A$ is isochoric.

The volume ratios are $V_A/V_B = 16$ and $V_c/V_B = 2$ and the temperature at A is $T_A = 300$ K. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle.

Sol. The corresponding P-V diagram is as shown :



Given : $T_A = 300$ K,

$n = \gamma = 1.4$, $V_A/V_B, = 16$

and $V_c/V_B = 2$

Let $V_B = V_0$

and $P_B = P_0$

Then $V_C = 2V_0$

and $V_A = 16V_0$

Temperature at B

Process A - B is adiabatic. Hence

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$$

$$\text{or}\quad T_B = T_A\left(\frac{V_A}{V_B}\right)^{\gamma-1} = (300)(16)^{1.4-1}$$

$T_B = 909$ K

Temperature at D

$B \to C$ is an isobaric process (P = constant)

$T \propto V$

$V_C = 2V_B$

$T_C = 2T_B = (2)\ (909)$ K

$T_C = 1818$ K

Now the process C - D is adiabatic.

$$\text{Therefore,}\ T_D = T_C\left(\frac{V_C}{V_D}\right)^{\gamma-1} = (1818)\left(\frac{2}{16}\right)^{1.4-1}$$

$T_D = 791.4$ K

Efficiency of cycle

Efficiency of cycle (in percentage) is defined as

$$\eta = \frac{\text{net work done in the cycle}}{\text{heat absorbed in the cycle}}\times 100$$

$$\text{or}\ \eta = \frac{W_{\text{Total}}}{Q_{+ve}}\times 100 = \frac{Q_{+ve}-Q_{-ve}}{Q_{+ve}}\times 100$$

$$=\left(1-\frac{Q_1}{Q_2}\right)\times 100 \qquad ...(1)$$

where Q_1 = Negative heat in the cycle (heat released) and Q_2 = Positive heat in the cycle (heat absorbed) in the cycle

$Q_{AB} = Q_{CD} = 0$ (Adiabatic process)

$$Q_{BA} = nC_V\Delta T = (1)\left(\frac{5}{2}R\right)(T_A - T_D)$$

$$\left(C_V = \frac{5}{2}R \text{ for a diatomic gas}\right)$$

$$=\frac{5}{2}\times 8.31(300-791.4)\text{J} = -10208.8 \text{ J}$$

and $Q_{BC} = nC_p\Delta T$

$$=(1)\frac{7}{2}R(T_C - T_B) \qquad \left(C_P = \frac{7}{2}R \text{ for a diatomic gas}\right)$$

or $Q_{BC}(831)(1818\text{-}909)$ J

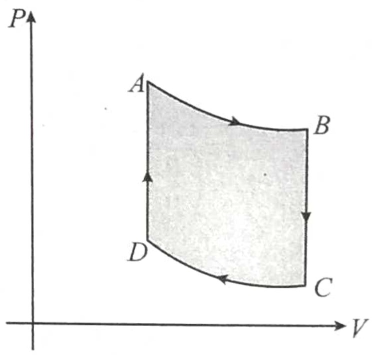
$Q_{BC} = 264383$ J

Therefore, substituting $Q_1 = 10208.8$ J and $Q_2 = 26438.3$ J in equation (1), we get

$$\therefore \eta = 1-\frac{10208.8}{26438.3}\times 100$$

or $\eta = 61.4\%$

Example 51: One mole of a monoatomic ideal gas is taken through the cycle shown in figure :



$A \to B$: adiabatic expansion

$B \to C$: cooling at constant volume

$C \to D$: adiabatic compression

$D \to A$: heating at constant volume.

The pressure and temperature at A, B etc. are denoted by P_A, T_A, P_B, T_B etc., respectively. Given that T_A = 1000 K, P_B = (2/3) P_A and P_C = (1/3) P_A, calculate the following quantities :

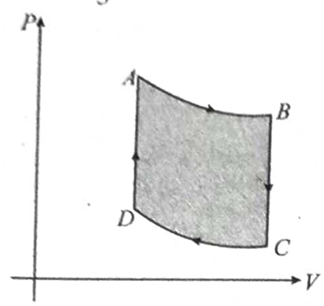
(a) The work done by the gas in the process $A \rightarrow B$.

(b) The heat lost by the gas in the process $B \rightarrow C$.

(c) The temperature T_D. [Given : $(2/3)^{2/5} = 0.85$]

Sol. Given T_A = 1000 K,

$$P_B = (2/3)P_A, P_C = \frac{1}{3}P_A$$



Number of moles, $n = 1$

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3}R \qquad \text{(monoatomic)}$$

(i) $A \rightarrow B$ is an adiabatic process, therefore,

$$P_A^{1-\gamma}T_A^{\gamma} = P_B^{1-\gamma}T_B^{\gamma}$$

$$\therefore T_B = T_A\left(\frac{P_A}{P_B}\right)^{\frac{1-\gamma}{\gamma}} = (1000)\left(\frac{3}{2}\right)^{\frac{1-5/3}{5/3}}$$

$$= (1000)\left(\frac{3}{2}\right)^{-2/5} = (1000)\left(\frac{2}{3}\right)^{2/5}$$

T_B = (1000) (0.85)

$\therefore$ T_B = 850 K $\qquad \left[\because \text{Given}\left(\frac{2}{3}\right)^{2/3} = 0.85\right]$

Now work done in the process A–B will be

$$W_{AB} = \frac{R}{1-\gamma}(T_B - T_A) = \frac{8.31}{1-5/3}(850-1000)$$

or W_{AB} = 1869.75 J

(ii) B–C is an isochoric process

(V = constant)

$$\therefore \frac{T_B}{T_C} = \frac{P_B}{P_C} \quad \therefore T_C = \left(\frac{P_C}{P_B}\right)T_B = \left(\frac{1/3)P_A}{(2/3)P_A}\right)850\,K$$

T_C = 425 K

Therefore

$$Q_{BC} = nC_V\Delta T = (1)\left(\frac{3}{2}R\right)(T_C - T_B)$$

$$= \left(\frac{3}{2}\right)(8.31)(425-850)$$

Q_{BC} = – 5297.6 J

Therefore, heat lost in the process BC is 5297.6 J.

(iii) C–D and A–B are adiabatic processes. Therefore,

$$P_C^{1-\gamma}T_C^{\ \gamma} = P_D^{1-\gamma}T_D^{\ \gamma}$$

$$\frac{P_C}{P_D} = \left(\frac{T_D}{T_C}\right)^{\frac{\gamma}{1-\gamma}} \qquad \text{...(1)}$$

$$P_A^{1-\gamma}T_A^{\ \gamma} = P_B^{1-\gamma}T_B^{\ \gamma}$$

$$\frac{P_A}{P_B} = \left(\frac{T_B}{T_A}\right)^{\frac{\gamma}{1-\gamma}} \qquad \text{...(2)}$$

Multiplying (1) and (2), we get

$$\frac{P_CP_A}{P_DP_B} = \left(\frac{T_DT_B}{T_CT_A}\right)^{\frac{\gamma}{1-\gamma}} \qquad \text{...(3)}$$

Processes B–C and D–A are isochoric

(V = constant)

Therefore,

$$\frac{P_C}{P_B} = \frac{T_C}{T_B} \text{ and } \frac{P_A}{P_D} = \frac{T_A}{T_D}$$

Multiplying these two equations, we get

$$\frac{P_CP_A}{P_DP_B} = \frac{T_CT_A}{T_BT_D} \qquad \text{...(4)}$$

From (3) and (4), we have

$$\left(\frac{T_DT_B}{T_CT_A}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_CT_A}{T_BT_D}\right)$$

$$\Rightarrow \frac{T_CT_A}{T_DT_B} = 1$$

$$T_D = \frac{T_CT_A}{T_B} = \frac{(425)(1000)}{850}K$$

or T_D = 500 K

Concept Application

42. A Carnot engine operates between 227° C and 27°C. Efficiency of the engine will be

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$

(c) $\frac{3}{4}$ (d) $\frac{3}{5}$

43. Two Carnot engines A and B are operated in series. The first one, A, receive heat at T_1 (= 600 K) and rejects to a reservoir at temperature T_2. The seconds engine B receives heat rejected by the first engine and, in turns, rejects to a heat reservoir at T_3 (= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal:

(a) 600 K (b) 400 K
(c) 300 K (d) 500 K

44. In the above problem, the ratio of efficiencies of two engines will be:
(a) 0.18 (b) 0.38
(c) 0.58 (d) 0.78

45. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C. Its efficiency is doubled. The temperatures of the source and the sink are, respectively
(a) 124°C, 62° C (b) 37°C, 99° C
(c) 62°C, 124° C (d) 99°C, 37° C

46. A carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is
(a) 500 J (b) 560 J
(c) 600 J (d) 650

47. A heat engine is involved with exchange of heat of 1915 J, – 40 J, + 125 J and $-Q$ J, during one cycle achieving an efficiency of 50.0%. The value of Q is
(a) 980 J (b) 40 J
(c) 400 J (d) 640 J

Short Notes

❖ **Boyle's law:** If m and T are constant

$$V \propto \frac{1}{P}$$

$$P_1V_1 = P_2V_2$$

❖ **Charles's law:** If m and P are constant

$$V \propto T$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

❖ **Gay-Lussac's law:** If m and V are constant

$$P \propto T$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

❖ **Avogadro's law:** If P, V and T are same

$$N_1 = N_2$$

where, N_1 and N_2 are the number of molecules

$V \propto n$ (no. of molecules of gas)

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

❖ **Graham's law:** If P and T are constant

rate of diffusion $r \propto \frac{1}{\sqrt{\rho}}$

ρ is density

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

❖ **Dalton's law:** $P = P_1 + P_2 + P_3 \dots\dots$

P = Total pressure

$P_1, P_2, P_3 \dots$ = Pressure exerted by each component present in the mixture.

❖ **Ideal gas equation:** $PV = nRT = k_BNT$

n = number of moles

N = number of molecules

R = universal gas constant

k_B = Boltzmann's constant

❖ Pressure exerted by ideal gas

$$P = \frac{1}{3}\frac{mN}{v}\overline{v^2}$$

$\overline{v^2}$ = mean square velocity $\left\{\overline{v^2} = \frac{v_1^2 + v_2^2 + v_3^2 + \dots\dots}{N}\right\}$

m = mass of each molecule

or $P = \frac{1}{3}nm\overline{v^2}$

n = number density *i.e.,* $n = \frac{N}{V}$

❖ $V_{ms} = \left(\overline{v^2}\right)^{1/2}$

$$\text{K.E.} = \frac{3}{2}k_BT = \frac{1}{2}m\overline{v^2}$$

$$\overline{v^2} = \frac{3k_BT}{m}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{3k_BT}{m}}$$

❖ $V_{av} = \sqrt{\frac{8K_BT}{\pi m}}$

❖ $V_{mp} = \sqrt{\frac{2K_BT}{m}}$

❖ Mean free path $(\bar{l}) = \frac{1}{\sqrt{2}n\pi d^2}$

n = number density

d = diameter of molecule

❖ $\gamma = \frac{C_p}{C_v}$

γ = ratio of specific heats

C_p = specific heat at constant pressure

C_v = specific heat at constant volume

$C_p - C_v = R$

R = universal gas constant

S.No.	Atomicity	No. of degree of freedom	C_P	C_v	$\gamma = C_p/C_v$
1	Monoatomic	3	$\frac{5}{2}R$	$\frac{3}{2}R$	$\frac{5}{3}$
2	Diatomic	5	$\frac{7}{2}R$	$\frac{5}{2}R$	$\frac{7}{5}$

- $C_{v(\text{mix})} = \dfrac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$

where n_1 and n_2 are number of moles of gases mixed together C_{v_1} and C_{v_2} are molar specific heat at constant volume of the two gas and $C_{v(\text{mix})}$: Molar specific heat at constant volume for mixture.

THERMODYNAMICS

- First law of themodynamics

  $\Delta Q = \Delta U + \Delta W$

- Work done.

  $\Delta W = P\Delta V$

  $\therefore \Delta Q = \Delta U + P\Delta V$

- Relation between specific heats for a gas

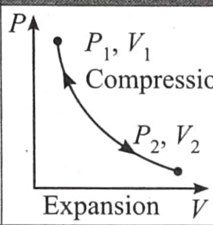
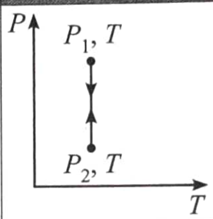
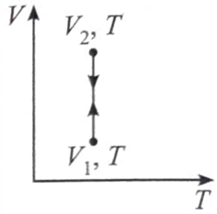
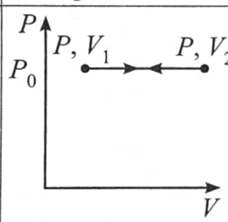
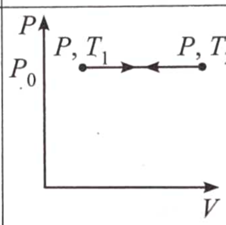
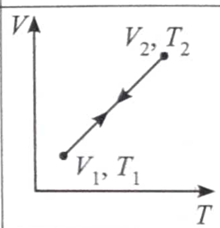
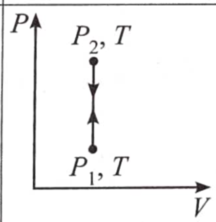
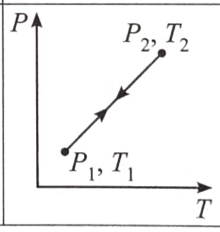
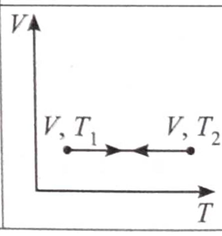
  $C_p - C_v = R$

Polytropic Process

It is a thermodynamic process that can be expressed as follows:

PV^x = Constant

x (Polytropic exponent)	Type of standard process	Expression
0	Isobaric ($dP = 0$)	P = Constant
1	Isothermal ($dT = 0$)	PV = Constant
γ	Adiabatic ($dQ = 0$)	PV^g = Constant
∞	Isochoric ($dV = 0$)	V = Constant

Process	Definition	P-V graph	P-T graph	V-T graph
Isothermal	Temperature constant			
Isobaric	Pressure constant			
Isochoric	Volume constant			

For any General Process

$\Delta Q = \Delta U + W$

$\Rightarrow \; nC\Delta T = \dfrac{f}{2}nR\Delta T + \dfrac{nR\Delta T}{1-x}$

[$\because$ Work done in a general polytropic process = [$nR\Delta T/(1-x)$]

$\Rightarrow \; C = \dfrac{f}{2}R + \dfrac{R}{1-x}$

For infinitesimal changes in Q, U, and W, we ca write,

$dQ = dU + dW$

$\Rightarrow \; nCdT = \dfrac{f}{2}nRdT + PdV$

$\Rightarrow \; C = \dfrac{f}{2}R + \dfrac{P}{n}\dfrac{dV}{dT}$

Process	Equation of State	W	ΔU
Isobaric ($dP = 0$)	$\frac{V}{T} = c$	$P(V_f - V_i) = nR(T_f - T_i)$	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}P(V_f - V_i)$
Isochoric ($dV = 0$)	$\frac{P}{T} = c$	0	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}V(P_f - P_i)$
Isothermal ($dT = 0$)	$PV = c$	$nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{P_i}{P_f}\right)$	0
Adiabatic ($dQ = 0$)	$PV^{\gamma} = c$	$\frac{f}{2}nR(T_i - T_f) = \frac{f}{2}(P_iV_i - P_fV_f)$	$\frac{f}{2}nR(T_f - T_i)$

Process	ΔQ
Isobaric ($dP = 0$)	$\left(\frac{f}{2}+1\right)nR\Delta T = \left(\frac{f}{2}+1\right)P(V_f - V_i)$
Isochoric ($dV = 0$)	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}V(P_f - P_i)$
Isothermal ($dT = 0$)	$nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{P_i}{P_f}\right)$
Adiabatic ($dQ = 0$)	0

- Slope of adiabatic = γ (slope of isotherm)
- Carnót engine

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

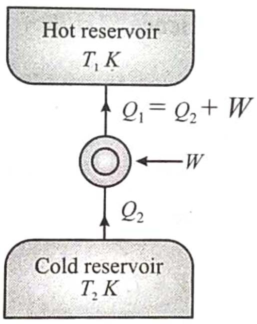
$$W = Q_1 - Q_2$$

(efficiency) $\eta = \frac{W}{Q_1}$

- Refrigerator

Coefficient of performance is β

$$\beta = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W}; \ \beta = \frac{1-\eta}{\eta}$$



- Heat pump

$$\alpha = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{\eta}$$

SUMMARY

- Zeroth law of thermodynamics states that 'two systems in thermal equilibrium with a third system are in thermal equilibrium with each other.
- Zeroth law leads to the idea of temperature.
- Heat and work are two modes of energy transfer to the system.
- Heat gets transferred due to temperature difference between the system and its environment (surroundings).
- Work is energy transfer which arises by other means, such as moving the piston of a cylinder containing the gas, by raising or lowering some weight connected to it.
- Internal energy of any thermodynamic system depends only on its state. The internal energy change in a process depends only on the initial and final states, not on the path, i.e. it is state dependent.
- The internal energy of an isolated system is constant.
- (*a*) For isothermal process $\Delta T = 0$

  (*b*) For adiabatic process $\Delta Q = 0$
- First law of thermodynamics states that when heat Q is added to a system while the system does work W, the internal energy U changes by an amount equal to $Q - W$. This law can also be expressed for an infinitesimal process.
- First law of thermodynamics is general law of conservation of energy.
- Second law of thermodynamics does not allow some processes which are consistent with the first law of thermodynamics.

  It states

  Clausius statement: No process is possible whose only result is the transfer of heat from a colder object to a hotter object.

  Kelvin-Plank statement: No process is possible whose only result is the absorption of heat from a reservoir and complete conversion of the heat into work.
- No engine can have efficiency equal to 1 or no refrigerator can have co-efficient of performance equal to infinity.
- Carnot engine is an ideal engine.
- The Carnot cycle consists of two reversible isothermal process and two reversible adiabatic process.
- If $Q > 0$, heat is added to the system.

  If $Q < 0$, heat is removed from the system.

  If $W > 0$, work is done by the system (Expansion).

  If $W < 0$, work is done on the system (Compression).

Solved Examples

1. Three moles of an ideal gas initially at temperature $T = 273$ K is expanded isothermally so that its volume increases 5 times. It is then heated at constant volume till the final pressure becomes equal to its initial value. If the total amount of heat transferred is $Q = 80$ kJ. Find the ratio $\gamma = \frac{C_p}{C_v}$ of the gas.

Sol. For isothermal process $\Delta U_1 = 0$

$$\Delta W_1 = nRT_1 \ln\frac{V_2}{V_1} = \Delta Q_1$$

In the second step, gas is heated isochorically.

So, $\Delta W_2 = 0$

and $\Delta Q_2 = \Delta U_2 = nC_v(T_3 - T_2)$

$$= \frac{nR}{\gamma - 1}(T_3 - T_2) \qquad \left(C_V = \frac{R}{\gamma - 1}\right)$$

or $\Delta Q_2 = \dfrac{P_3V_3 - P_2V_2}{\gamma - 1}$

$\Delta Q = \Delta Q_1 + \Delta Q_2$

Given that $\Delta Q = 80$ kJ, $n = 3$,

$T_1 = T_2 = 273$ K,

$V_2 = 5V_1 = V_3$

$P_3 = P_1$

and $nRT_3 = P_3V_3 = 5P_1V_1 = 5nRT_1$

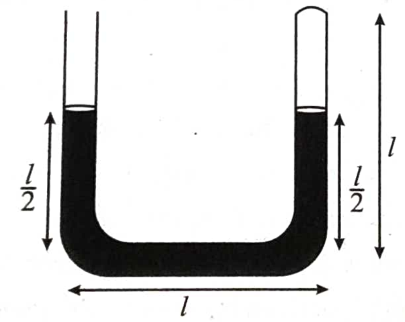
We get $\Delta Q = nRT_1 \ln(5) + \dfrac{4nRT_1}{\gamma - 1}$

or $\gamma = \dfrac{4nRT_1}{\Delta Q - nRT_1 \ln(5)} + 1$

Substituting the values we get

$$\gamma = \frac{(4)(3)(8.31)(273)}{80,000 - (3)(8.31)(273)(\ln 5)} + 1 \quad \text{or } \gamma = 1.4$$

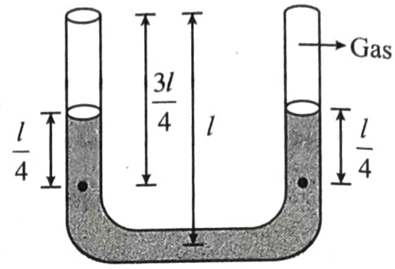
2. A thin U-tube sealed at one end consists of three bends of length $l = 250$ mm each forming right angles. The vertical parts of the tube are filled with mercury to half the height. All of mercury is displaced from the tube by heating slowly the gas in the sealed end of the tube. Determine the work done by the gas if atmospheric pressure is $P_0 = 10^5$ N/m^2, the density of mercury in $\rho_{Hg} = 13.6 \times 10^3$ kg/m^3 and the cross-sectional area of tube is $S = 1$cm^2 ($g = 9.8$ m/s^2)



Sol. The work done by the gas is

$W = W_1 + W_2$

Here W_1 is work done against the force of atmospheric pressure and



W_2 = work done against the force of gravity.

The mercury gas interface is shifted by

$(l + l + l) - l/2$ or $\frac{5l}{2}$ upon the complete displacement of mercury. Hence

$$W_1 = \frac{5}{2}P_0 Sl$$

The work done against the force of gravity is equal to the change in the potential energy of mercury as a result of its displacement.

COM of the vertical portion of the mercury of mass $\frac{m}{2}$ rises $3l/4$ while that of horizontal portion of mass $\frac{m}{2}$ rises by l. Hence

$$W_2 = \frac{m}{2}(g)\frac{3l}{4} + \frac{m}{2}g(l) = \frac{7}{8}mgl$$

Here, $m = 2l\rho_{Hg}S$

Hence $W = W_1 + W_2 = \frac{5}{2}P_0Sl + \frac{7}{8}mgl$

$$W = \left[\frac{5}{2}P_0S + \frac{7}{8}\rho_{Hg}(2l)gS\right]l$$

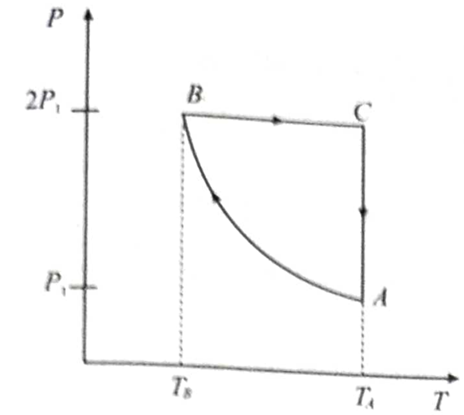
Substituting the values

$$W = \left(\frac{5}{2}\times10^5\times10^{-4} + \frac{7}{8} + 13.6\times10^3\times2\times0.25\times9.8\times10^{-4}\right)0.25$$

$W \approx 7.7$J

3. Three moles of an ideal diatomic gas is taken through a cyclic process $ABCA$ as shown in the $P - T$ diagram. During the process AB pressure and temperature of the gas varies such that PT^2 is constant. If $T_B = 300$ K, calculate :
   (a) The work done on the gas in the process AB.
   (b) The heat absorbed or released by the gas in each of the processes.

   Give answer in terms of the gas constant R.



Sol. (*a*) $PT^2 = \text{constant}$

$$(2P_1)T_B^2 = (P_1)T_A^2$$

$$T_A = T_B\sqrt{2}$$

$$T_A = T_C = 600 \text{ K}$$

During the process $A \to B$

$$PT^2 = C\,;\; P^3V^2 = k$$

$$P = \frac{k^{1/3}}{V^{2/3}}$$

$$W_{AB} = \int_A^B PdV \;= k^{1/3}\left[\frac{V^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}\right]_{V_A}^{V_B}$$

$$= 3\left[k^{1/3}\cdot\left(V_B^{1/3} - V_A^{1/3}\right)\right]$$

$$W_{AB} = 3(P_BV_B - P_AV_A) = 3nR(T_B - T_A)$$

$$= 3\times3\times R(300-600) = -2700R$$

(*b*) Heat evolved in different processes are

$$Q_{AB} = W_{AB} + \Delta U$$

$$= W_{AB} + nC_V\Delta T$$

$$= -2700R + 3\times\frac{5}{2}R\times(-300)$$

$$= -2700R - 15\times150R = -4950\text{R}$$

$$Q_{BC} = nC_P\Delta T = 3\times\frac{7}{2}R(300R)$$

$$= 21\times150R = 3150R$$

$$Q_{CA} = \Delta U + W = 0 + W_{CA}$$

[As *CA* is an isothermal process]

$$Q_{CA} = nRT_A\ln\frac{V_A}{V_C} = nRT_A\ln\frac{P_C}{P_A}$$

$$Q_{CA} = 3\times R\times600\ln(2) = 1247.4R$$

4. A gas is undergoing an adiabatic process. At a certain stage A the values of volume and temperature = (V_0, T_0) and the magnitude of the slope of *VT* curve is m Find the value of C_p and C_V.

Sol. We have, for an adiabatic process

$TV^{\gamma-1} = \text{constant}$

Differentiating with respect to T

$$V^{\gamma-1}\cdot 1 + T(\gamma-1)V^{\gamma-2}\frac{dV}{dT} = 0$$

$$\Rightarrow \frac{dV}{dT} = -\frac{V}{T(\gamma-1)}$$

Also, $\left.\dfrac{dV}{dT}\right|_{T_0,V_0} = m \;\;\therefore\; \dfrac{V_0}{T_0(\gamma-1)} = m$

or $\gamma - 1 = \dfrac{V_0}{T_0m}$ and $C_V = \dfrac{R}{\gamma-1} = \dfrac{RT_0m}{V_0}$

and $C_P = C_V + R = 1 + \dfrac{T_0m}{V_0}R$

5. 2.8 grams of an ideal gas is taken in a container of volume 100 litres at temperature $T_1 = 288$ K and pressure $P = 1.5\times10^3$ Pa. The gas in the tube is heated isobarically to a temperature T_2. Standing waves of frequency 5 kHz is produced in tube. The separation between the nodes at temperatures T_1 and T_2 are 3 cm and 4 cm respectively find

(*a*) The final temperature T_2

(*b*) Adiabatic constant γ

(*c*) Amount of heat supplied

Sol. Under isobaric process,

$$W = P\Delta V = nR\Delta T$$

$$\Delta U = nC_V\Delta T = \frac{nR\Delta T}{\gamma-1}$$

$$Q = W + \Delta U$$

$$Q = \frac{n\gamma R\Delta T}{\gamma-1} \qquad ...(1)$$

At T_1, distance between nodes = 3 cm

$$\frac{\lambda_1}{2} = 3 \Rightarrow \lambda_1 = 6\text{ cm} \Rightarrow v_1 = n\lambda_1 = 300\text{ m/s}$$

at $T_2 = \dfrac{\lambda_2}{2} = 4 \Rightarrow \lambda_2 = 8\text{ cm} \Rightarrow v_2 = n\lambda_2 = 400\text{ m/s}$

$$V = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma P}{\rho}}$$

$$v \propto \sqrt{T}$$

i.e. $\dfrac{v_1}{v_2} = \sqrt{\dfrac{T_1}{T_2}}$

hence $\dfrac{T_1}{T_2} = \dfrac{v_1}{v_2}^2 = \dfrac{9}{16}$

Hence, $T_2 = \dfrac{16}{9}T_1 = 512$ K

Density $= \dfrac{\text{mass}}{\text{volume}}$ and $v = 300 = \sqrt{\dfrac{\gamma\times1.5\times10^3}{2.8\times10^{-2}}}$

which gives $\gamma = \dfrac{(300)^2\times2.8\times10^{-2}}{1.5\times10^3} = 1.68$

$$n = \frac{PV}{RT} = \frac{1.5\times10^3\times10^2\times10^{-3}}{8.31\times288} = 6.27\times10^{-2}$$

Substituting the values of n, γ and ΔT in equation (1).

Amount of the heat supplied

$$Q=\frac{6.27\times1.68\times8.31\times224\times10^{-2}}{(1.68-1)}=288.34\,\text{J}$$

6. The minimum velocity of projection of a body to send it to infinity from the surface of a planet is $1/\sqrt{6}$ times that is required from the surface of the earth. The radius of the planet is 1/36 times the radius of the earth. The planet is surrounded by an atmosphere which contains monatomic inert gas ($\gamma = 5/3$) of constant density up to a height h(h << radius of the planet). Find the velocity of sound on the surface of the planet in terms of g_e (the acceleration due to gravity on earth surface) and h.

Sol. Escape velocity from the surface of the planet

$$v_p=\sqrt{2g_pR_p}$$

Given $v_p=\frac{v_e}{\sqrt{6}}=\sqrt{\frac{2g_eR_e}{6}}$

$$\sqrt{\frac{g_eR_e}{3}}=\sqrt{\frac{2g_pR_e}{36}}\Rightarrow g_p=6g_e$$

Pressure exerted by the atmospheric column of height h on the surface of the planet

$$P=\rho g_p h$$

Therefore, $\frac{P}{\rho}=g_p h$

Hence, speed of the sound $v=\sqrt{\frac{\gamma P}{\rho}}=\sqrt{\gamma g_p h}$

$$=\sqrt{6\gamma g_e h}=\sqrt{10g_e h}$$

7. One mole of an ideal gas is taken from an initial state P_0, V_0 and temperature T_0 through the following cycle : (C_0/C_v for the gas is 1.4)

(*a*) Heating at constant volume to a temperature $3T_0$.

(*b*) Adiabatic expansion to a volume $2V_0$.

(*c*) Cooling at constant volume.

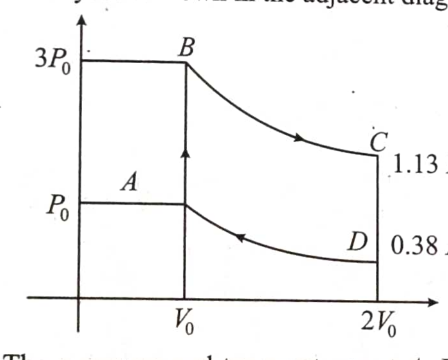
(*d*) Adiabatic compression so that it is returned to its initial state.

(*i*) Show this cycle on a *PV* diagram,

(*ii*) Calculate efficiency of the cycle.

Given : $(2)^{-1.4}=0.38$

Sol. (*a*) The cycle is shown in the adjacent diagram:



The pressures and temperatures at A, B, C and D are found by using the equation of the process and the equation of state.

$$\frac{P_0}{T_0}=\frac{P_B}{3T_0}\Rightarrow P_B=3P_0$$

$$P_BV_B^{\gamma}=P_CV_C^{\gamma}$$

$$\Rightarrow P_C=P_B\left(\frac{V_B}{V_C}\right)^{\gamma}=1.13P_0$$

Similarly using $P_AV_A^{\gamma}=P_DV_D^{\gamma}$ we can find

$$P_D=0.38P_0$$

The temperatures are: $(\text{as}\,T\propto PV)$

$$T_A=T_0,\ T_B=3T_0,$$

$$T_C=2.26T_0,\ T_D=0.76T_0$$

(*b*) $Q_{AB}=\frac{5}{2}R(3T_0-T_0)=5RT_0$

$$Q_{BC}=Q_{DA}=0$$

$$Q_{CD}=-\frac{5}{2}R(1.5T_0)=-3.75RT_0$$

$$Q_{\text{Total}}=1.25RT_0$$

Efficiency, $\eta=\frac{1.25RT_0}{5RT_0}\times100=25\%$

8. An ideal gas has a specific heat at constant pressure $C_P=\frac{5R}{2}$. The gas is kept in a closed vessel of volume $V_0\,\text{m}^3$ at a temperature T_0 K and pressure P_0 N/m². An amount of $10\,P_0V_0$ J of heat is supplied to the gas.

(*a*) Calculate the final pressure and temperature of the gas.

(*b*) Show the process on $P-V$ diagram.

Sol. (*a*) First law of thermodynamics for the given process from state 1 to state 2

Here, $Q_{12}-W_{12}=U_2-U_1$

$$Q_{12}=+10P_0V_0\ \text{J}$$

$W_{12}=0$ (Volume remains constant)

$U_2-U_1=nC_V(T_2-T_1)$; $nC_V(T_2-T_1)=10P_0V_0$

For an ideal gas $P_0V_0=nRT_0$ and $C_P-C_V=R$

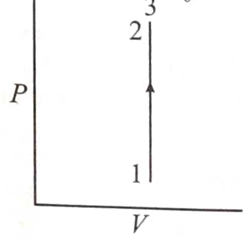
$$\therefore\ C_V=C_P-R=\frac{5R}{2}-R=\frac{3R}{2}$$

$$\therefore\ n\left(\frac{3R}{2}\right)(T_2-T_0)=10nRT_0$$

$$T_2=\frac{23}{3}T_0$$

As $P\propto T$ for constant volume

(*b*) $P_2=\frac{23}{3}P_0$

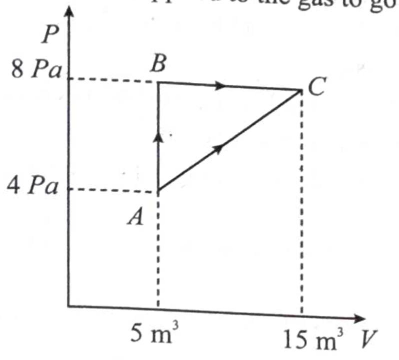


9. In the given figure, an ideal gas changes its state from state A to state C by two paths ABC and AC.

(*a*) Find the path along which work done is less.

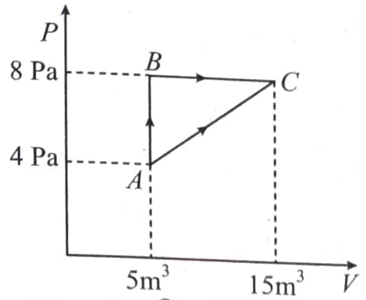
(*b*) The internal energy of gas at A is 10 J and the amount of heat supplied to change its state to C through path AC is 200 J. Calculate the internal energy of gas at C.

(*c*) The internal energy of gas at state B is 20 J. Find the amount of heat supplied to the gas to go from A to B.



Sol. (*a*) W_{AC} is less than W_{ABC} as area under graph is less.

(*b*) For process A to C



$Q_{AC} = 200$ J

Work done W_{AC} = area under AC

$$= (10 \times 4) + \frac{10 \times 4}{2} = 60 \text{ J}$$

From first law of thermodynamics.

$$\Delta U = Q - W_{AC}$$

$$U_C - U_A = 200 - 60$$

$$U_C = U_A + 140 = 10 + 140 = 150 \text{ J}$$

(*c*) From A to B

$$U_B = 20 \text{ J}$$

$$\therefore \ \Delta U = Q - W_{AB}$$

$$U_B - U_A = Q - 0$$

$$20 - 10 = Q$$

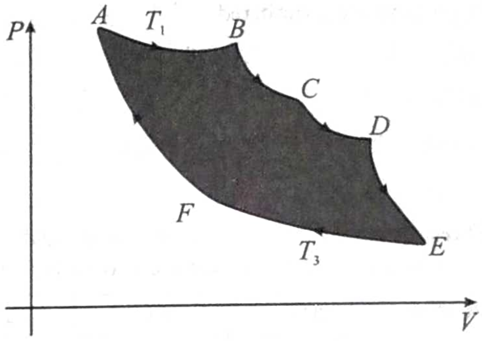
$$\therefore \ Q = 10 \text{ J}$$

10. One mole of an ideal gas goes through a cycle consisting of alternate isothermal and adiabatic curves. AB, CD, EF are isothermal and BC, DE, FA are adiabatic curves. Temperature of each isothermal curve is given in the graph. The volume changes two fold in every isothermal expansion. Find

(*a*) Work done by the gas in the cycle

(*b*) Heat absorbed by the gas

(*c*) Efficiency of the cyde



Sol. (*a*) Given $\dfrac{V_B}{V_A} = 2$ and $\dfrac{V_D}{V_C} = 2$

In the process AB (isothermal)

$$\Delta U_{AB} = 0$$

Hence, $\Delta Q_{AB} = \Delta W_{AB}$

$$= nRT_1 \ln \frac{V_B}{V_A} = RT_1 \ln(2) \ (n = 1)$$

In process BC (adiabatic)

$$\Delta Q_{BC} = 0$$

hence $\Delta W_{BC} = -\Delta U_{BC}$

$$= -nC_V (T_C - T_B) = C_V (T_1 - T_2)$$

In process CD (isothermal)

$$\Delta U_{CD} = 0$$

Hence $\Delta Q_{CD} = \Delta W_{CD} = nRT_2 \ln \dfrac{V_D}{V_C} = RT_2 \ln(2)$

In process DE (adiabatic)

$\Delta Q_{DE} = 0$ hence $\Delta W_{DE} = -\Delta U_{DE}$

$$= -nC_V (T_E - T_D) = C_V (T_2 - T_3)$$

In adiabatic process, we know that

$TV^{\gamma - 1}$ = constant or $T \propto V^{1-\gamma}$

Hence in process BC, DE and FA:

$$\frac{T_1}{T_2} = \left(\frac{V_B}{V_C}\right)^{1-\gamma} \quad ...(1)$$

$$\frac{T_2}{T_3} = \left(\frac{V_D}{V_E}\right)^{1-\gamma} \quad ...(2)$$

and $$\frac{T_3}{T_1} = \left(\frac{V_F}{V_A}\right)^{1-\gamma} \quad ...(3)$$

Multiplying equations (1), (2) and (3), we get

$$1 = \left(\frac{V_B}{V_A} \cdot \frac{V_D}{V_C} \cdot \frac{V_F}{V_E}\right)^{1-\gamma}$$

or $1 = \left(2 \times 2 \times \dfrac{V_F}{V_E}\right)^{1-\gamma} = \left(\dfrac{4V_F}{V_E}\right)^{1-\gamma}$ or $\dfrac{4V_F}{V_E} = 1 \Rightarrow \dfrac{V_F}{V_E} = \dfrac{1}{4}$

Now in process EF (isothermal)

$\Delta U_{EF} = 0$ and $= -2RT_3 \ln(2)$

In process FA (adiabatic)

$\Delta Q_{FA} = 0$

$\Delta W_{FA} = -\Delta U_{FA}$

$= -nC_V\left(T_A - T_F\right) = C_V\left(T_3 - T_1\right)$

$\therefore\ \Delta W_{FA} = -\Delta U_{FA}$

Three quantities viz. ΔQ, ΔU and ΔW in tabular form in different processes are shown below:

Process	ΔQ	ΔU	ΔW
AB	$RT_1 \ln(2)$	O	$RT_1 \ln(2)$
BC	O	$C_V(T_2 - T_1)$	$C_V(T_1 - T_2)$
CD	$RT_2 \ln(2)$	O	$RT_2 \ln(2)$
DE	O	$C_V(T_3 - T_2)$	$C_T(T_2 - T_3)$
EF	$-2RT_3 \ln(2)$	O	$-RT_3 \ln(2)$
FA	O	$C_V(T_1 - T_3)$	$C_V(T_3 - T_1)$

(*a*) From the table

$$W_{\text{net}} = R\left(T_1 + T_2 - 2T_3\right)\ln(2)$$

(*b*) Heat absorbed by the gas

$$Q_{ab} = Q_{+\text{ve}} = R\left(T_1 + T_2\right)\ln(2)$$

(*c*) Heat absorbed by the gas

$$\eta = \frac{\text{net work done}}{\text{heat absorbed}}$$

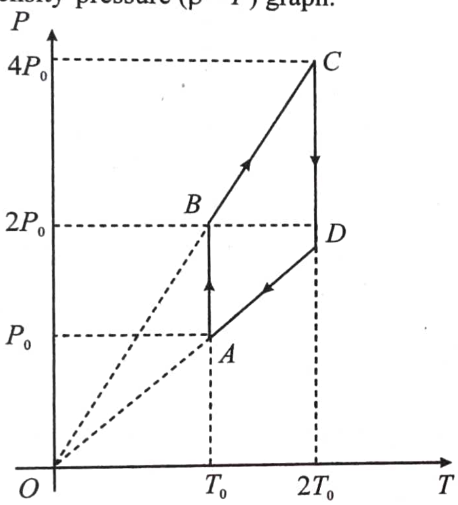
$$= \frac{R\left(T_1 + T_2 - 2T_3\right)\ln(2)}{R\left(T_1 + T_2\right)\ln(2)} = 1 - \frac{2T_3}{T_1 + T_2}$$

11. Pressure-temperature ($P - T$) graph of n moles of an ideal gas is shown in figure. Plot the corresponding

(*a*) density-volume ($\rho - V$) graph

(*b*) pressure-volume ($P - V$) graph and

(*c*) density-pressure ($\rho - P$) graph.



Sol. Process $A - B$ is an isothermal process i.e. T = constant

Hence $P \propto \dfrac{1}{V}$ or $P - V$ graph will be a rectangular hyperbola with increasing V and decreasing ρ.

$\rho \propto \dfrac{1}{V}$. Hence $\rho - V$ graph is also a rectangular hyperbola with decreasing V and hence increasing ρ.

$$\rho \propto P\left[\rho = \frac{PM}{RT}\right]$$

Hence $\rho - V$ graph will be a straight line passing through origin, with increasing V and ρ.

Process $B - C$ is an isochoric process, because $P - T$ graph is a straight line passing through origin

i.e. V = constant

Hence $P - V$ graph will be a straight line parallel to P-axis with increasing P.

Since V = constant hence ρ will also be constant

Hence $\rho - P$ graph will be a dot.

$\rho - P$ graph will be a straight line parallel to P-axis with increasing P, because

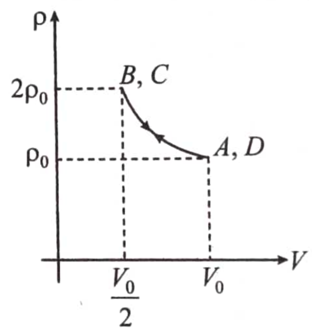
Process $C - D$ is inverse of $A - B$ and $D - A$ is inverse of $B - C$.

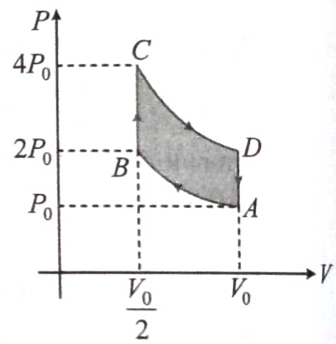
Different values of and in tabular form are shown below

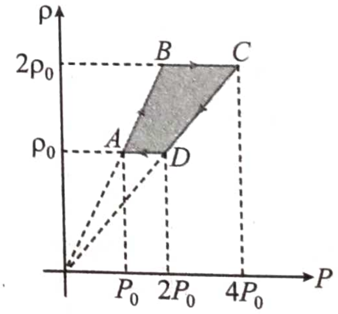
	P	V	T	ρ
A	P_0	V_0	T_0	ρ_0
B	$2P_0$	$\dfrac{V_0}{2}$	T_0	$2\rho_0$
C	$4P_0$	$\dfrac{V_0}{2}$	$2T_0$	$2\rho_0$
D	$2P_0$	V_0	$2T_0$	ρ_0

Here $V_0 = nR\dfrac{T_0}{P_0}$ and $\rho_0 = \dfrac{P_0 M}{RT_0}$

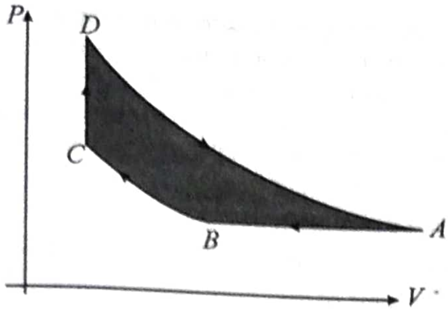
The corresponding graphs are as follows:







12. Helium is used as working substance in an engine working on the cycle as shown in figure. Process AB is isobaric, BC is adiabatic, CD is isochoric and DA is isothermal. The ratio of maximum to minimum volume of helium during the cycle is $8\sqrt{2}$ and that of maximum to minimum absolute temperature is 4. Calculate efficiency of the cycle in percentage.



Sol. Let $V_C = V_D = V_0$ (minimum)

Then $V_A = 8\sqrt{2}V_0$ (maximum)

$T \propto PV$

Hence temperature at A and D will be maximum and at B will be minimum. (In adiabatic compression temperature is increased. Hence $T_C - T_B$)

So, let $T_B = T_0$

Then $T_A = T_D = 4T_0$

Process BC is adiabatic, so

$$T_C = T_B\left(\frac{V_B}{V_C}\right)^{\gamma-1} = T_0\left(\frac{2\sqrt{2}V_0}{V_0}\right)^{5/3-1} = 2T_0$$

(γ for He is 5/3)

So, finally $T_A = 4T_0$, $T_B = T_0$, $T_C = 2T_0$ and $T_D = 4T_0$

Now in process AB (isobaric)

$$\Delta W_{AB} = \Delta Q_{AB} - \Delta U_{AB}$$

$$= nC_p\Delta T - nC_V\Delta T = nR\Delta T$$

$$= nR(T_B - T_A) = -3nRT_0$$

and ΔQ_{AB} is negative (released)

In process BC (adiabatic)

$$\Delta Q_{BC} = 0$$

$$\Delta W_{BC} = -\Delta U_{BC} = -nC_V\Delta T$$

$$= n\frac{3}{2}R(T_B - T_C) = -\frac{3}{2}nRT_0$$

In process CD (isochoric)

$\Delta W_{CD} = 0$ and $\Delta Q_{CD} = \Delta U_{CD} = nC_V(T_D - T_C)$

$$= n\frac{3}{2}R(2T_0) = 3nRT_0$$

In process DA (isothermal) $\Delta U_{DA} = 0$

Hence $\Delta Q_{DA} = \Delta W_{DA} = nRT_D \ln\dfrac{V_A}{V_D}$

$$= nR(4T_0)\ln(8\sqrt{2}) = 14nRT_0\ln(2)$$

$\therefore$ Total work done is

$$W_{\text{net}} = -3nRT_0 - \frac{3}{2}nRT_0 + 14nRT_0\ln(2)$$

$$W_{\text{net}} = nRT_0\left[14\ln(2) - \frac{9}{2}\right] = 5.202nRT_0$$

and heat absorbed is:

$$Q_{ab} = 3nRT_0 + 14nRT_0\ln(2)$$

$$= 12.702nRT_0$$

Hence efficiency of the cycle is

$$\eta = \frac{W_{\text{net}}}{Q_{\text{ab}}} \times 100$$

$$= \frac{5.202\,nRT_0}{12.702\,nRT_0} \times 100 = 41\%$$

13. A vertical thermally insulated cylinder of volume V contains n moles of an ideal monoatomic gas under a weightless piston. A load of weight. W is placed on the piston as a result of which the piston is displaced by a distance h. Determine the final temperature of the gas. The area of the piston is A and atmospheric pressure is P_0.

Sol. Let T_1 and T_2 be the initial and final temperatures then

$$P_0V = nRT_1 \quad \text{...(1)}$$

$$\text{and } \left(P_0 + \frac{W}{A}\right)(V - Ah) = nRT_2 \quad \text{...(2)}$$

Since the gas is thermally insulated, the entire work done on the gas is spent to change its internal energy. Work done on the gas is Wh. Hence

$$Wh = \Delta U = nC_V\Delta T = \frac{3}{2}nR(T_2 - T_1) \quad \text{...(3)}$$

$C_V = \dfrac{3}{2}R$ for a monoatomic gas

Equation (2) – (1) gives:

$$nR(T_2 - T_1) = \frac{W}{A}.V - Wh - P_0Ah$$

Substituting $(T_2 - T_1)$ from equation (3), we get

$$\frac{2}{3}WAh = WV - WAh - P_0A^2h$$

$$\text{or } Ah = \frac{WV}{P_0A + \frac{5}{3}W}$$

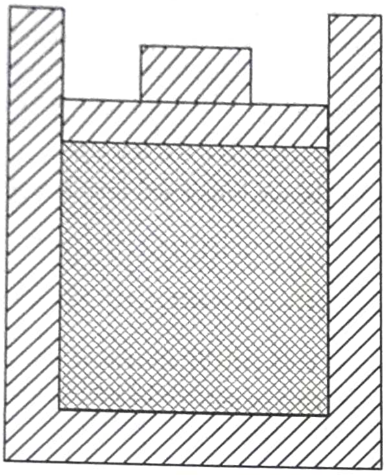
Substituting value of Ah in equation (2), we get

$$T_2 = \frac{\left(P_0 + \frac{W}{A}\right)\left(V - \frac{WV}{P_0A + \frac{5}{3}W}\right)}{nR}$$

$$\text{or } T_2 = \frac{(P_0A + W)\left(P_0AV + \frac{2}{3}WV\right)}{nAR\left(P_0A + \frac{5}{3}W\right)}$$

$$\text{or } T_2 = \frac{(P_0A + W)(3P_0AV + 2WV)}{nAR(3P_0A + 5W)}$$

14. An ideal gas has specific heat at constant pressure $C_p = \frac{5R}{2}$. The gas is kept in a cylindrical vessel fitted with a movable piston as shown in the figure. Mass of the frictionless piston is 9 kg. Initial volume of the gas is 0.027 m^3 and cross-section area of the piston is 0.09 m^2. The initial temperature of the gas is 300 K. An amount of 2.5×10^4 J of heat energy is supplied to the gas. Calculate the initial pressure, final pressure, final temperature and the work done by the gas. The walls of the cylinder and piston are thermally insulated. ($P_{atm} = 1.05 \times 10^5$ N/m^2)



Sol. Initial pressure, $P = P_{atm} + \frac{(mg)}{A}$

$= 1.05\times10^5 + \frac{9\times10}{0.09}$

$= 1.05\times10^5 + \frac{90\times10^2}{9}$

$= 1.05\times10^5 + 1\times10^3$

$= 1.06\times10^5 \text{ N/m}^2$

If n is the number of moles of the gas

$n = \frac{PV}{RT}$

$= \frac{1.06\times10^5\times0.027}{8.31\times300} = 1.15$

When heat is given to the system piston will move under constant pressure.

$\Delta Q = nC_P\Delta T$

$2.5\times10^4 = 1.15\times\frac{5}{2}\times8.31\times(\Delta T)$

$\Delta T = 1046$ K

Final temperature = 1046 + 300 = 1346 K

Final pressure will be same as initial = 1.06×10^5 N/m^2

$W = Q - \Delta U = nC_P\Delta T - nC_V\Delta T = nR\Delta T$

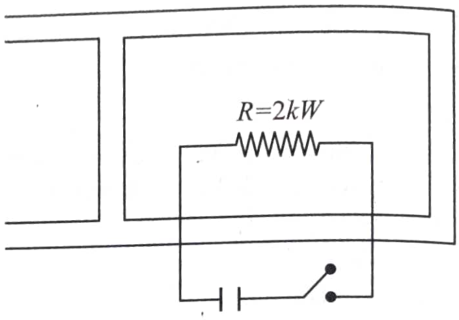
$= (1.15)(8.31)(1046)\text{J} = 10^4$ J

15. One mole of a gas is taken in a cylinder with a movable piston. A resistor R connected to a capacitor through a key is immersed in the gas. Initial potential difference across the plates of the capacitor is equal to 640/3 V. When the key is closed for (2.5 In 4) minutes, the gas expands isobarically and its temperature changes by 22 K

(a) Find the work done by the gas

(b) Increment in the internal energy of the gas

(c) The value of γ.



Sol. Initial charge on the capacitor

$q_0 = CV_0 = 75\times10^{-3}\times\frac{640}{3} = 16C$

The charge on the capacitor decays as

$q = q_0e^{-t/RC}$

At $t = 2.5\ln(4)$ minutes $= 150\ln(4)$sec

$q = 16\times e^{-\ln(4)} = 4C$

$\because RC = 150$ s

Total heat dissipated in the resistor in the given time

$= \frac{q_0^2 - q^2}{2C} = 1.6$ kJ

= heat imparted to the gas

(a) Work done by the gas at constant pressure

$= P\Delta V = nR\Delta T \approx 0.182\,kJ$

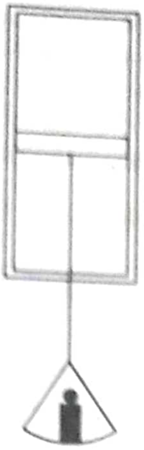
(b) Increment in the internal energy

$\Delta U = Q - W = 1.6 - 0.182 = 1.418$ kJ

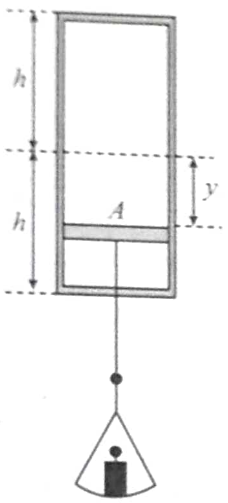
(c) $\gamma = \frac{C_P}{C_V} = \frac{nC_P\Delta T}{nC_V\Delta T} = \frac{Q}{\Delta U} = 1.13$

16. A horizontal frictionless piston, of negligible mass and heat capacity, divides a vertical insulated cylinder into two halves. Each half of the cylinder contains 1 mole of air at standard temperature and pressure p_0.

A load of weight W is now suspended from the piston, as shown in the figure. It pulls the piston down and comes to rest after a few oscillations. How large a volume does the compressed air in the lower part of the cylinder ultimately occupy if W is very large?



Sol. Let A denote the cross-sectional area of the piston and y the vertical displacement between its initial and final equilibrium positions. (See fig.)



The decrease in potential energy of the weight W increases the internal energy of the air inside the cylinder. Conservation of energy between the initial and the final states gives

$$Wy = \frac{5}{2}\left[p_1A(h-y)+p_2A(h+y)\right]-\left[2p_0Ah\right] \qquad ...(i)$$

where p_1 is the final pressure in the lower part of the cylinder and that p_2 in the upper part. The internal energy of a gas made up of diatomic molecules has been written in the form $\frac{5}{2}pV$. If W is very large, the decrease in its potential energy (and the corresponding increase in the internal energies of the gases) is very large, and the initial internal energy of the air can be neglected. Thus

$$Wy = \frac{5}{2}\left[p_1A(h-y)+p_2A(h+y)\right] \qquad ...(ii)$$

When the load is finally at rest, $(p_1 - p_2)A = W$ ...(iii)

The temperatures and the masses of the gases in the two halves are identical, and so their internal energies must be equal:

$$\frac{5}{2}p_1A(h-y) = \frac{5}{2}p_2A(h+y) \qquad ...(iv)$$

Equations (*ii*), (*iii*) and (*iv*) yield $y = \sqrt{\frac{5}{7}}h$ for the displacement of the piston, i.e., the gas in the lower part is compressed to $1-\sqrt{\frac{5}{7}} \approx 15$ per cent of its original volume.

Note: The surprising result is that the volume of air in the lower part does not tend to zero, however large the weight is, even though gases are supposed to be compressible! The large load increases the internal energy, and hence the temperature, of the enclosed gas. This causes considerable increases in not only the absolute pressures, but also in the difference between the upper and lower pressures.

17. $1/R$ mole (where R is the magnitude of the gas constant in the SI system) of an ideal diatomic gas is enclosed in a container of volume 4 litres at a pressure of 2×10^5 Pa. The container is attached to a piston by which its volume can be changed. Initially the gas undergoes adiabatic compression to a volume of 2 litres. Then the gas is given 200 J of heat at constant pressure.

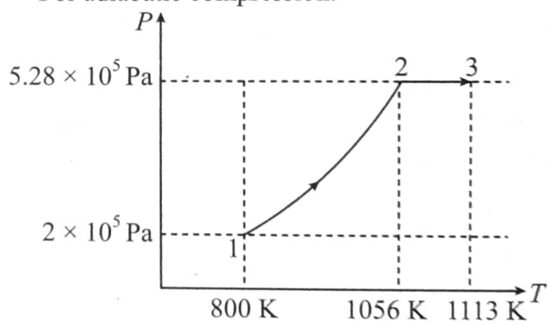
(*a*) Plot the complete process on a *PT* diagram.

(Take $2^{1.4} = 2.63$)

(*b*) Find the total work done by the gas.

Sol. (*a*) $V_1 = 4\times10^{-3}\ \text{m}^3$; $P_1 = 2\times10^5$ Pa ; $V_2 = 2\times10^{-3}\ \text{m}^3$

For adiabatic compression.



$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 2\times10^5\left(\frac{4\times10^{-3}}{2\times10^{-3}}\right)^{1.4} = 5.28\times10^5\,\text{Pa}$$

$$T_1 = \frac{P_1V_1}{nR} = \frac{2\times10^5\times4\times10^{-3}}{\frac{1}{R}\times R} = 800\,\text{K}$$

$$T_2 = \frac{P_2V_2}{nR} = \frac{5.25\times10^5\times2\times10^{-3}}{\frac{1}{R}\times R} = 1056\,\text{K}$$

In the second process heat given in 200 J.

$\Delta Q = nC_p\Delta T$

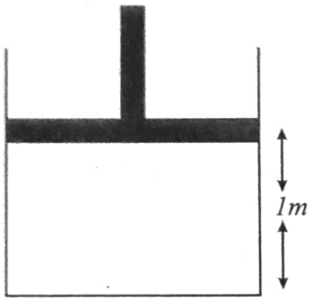
$$200 = \frac{1}{R} \times \frac{7}{2} R\Delta T$$

$$\Rightarrow \Delta T = 57\,\text{K}, T = 1113\,\text{K}$$

(*b*) $W_{\text{total}} = W_{\text{adiabatic}} + W_{\text{isobaric}}$

$$= \frac{1}{R}\frac{R(1056-800)}{1-1.4} + \frac{1}{R}.R(57) = -583\,\text{J}$$

18. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is 1m^2. Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brough back to its original height without any heat loss, find the new equilibrium temperature of the gas.



Sol. At constant pressure $V \propto T$

or $\frac{V_2}{V_1} = \frac{T_2}{T_1}$

or $\frac{Ah_2}{Ah_1} = \frac{T_2}{T_1}$

$$\therefore\ h_2 = h_1\left(\frac{T_2}{T_1}\right) = (1.0)\left(\frac{400}{300}\right)\text{m} = \frac{4}{3}\text{m}$$

As there is no heat loss, process is adiabatic. For adiabatic process,

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$\therefore\ T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = (400)\frac{h_i}{h_f}$$

$$= 400\left(\frac{4}{3}\right)^{1.4-1} = 448.8\ \textbf{K}$$

19. An insulated box containing a monoatomic gas of molar mass M moving with a speed v_0 is suddenly stopped. Find the increment in gas temperature as a result of stopping the box.

Sol. Decrease in kinetic energy = increase in internal energy of the gas

$$\therefore\ \frac{1}{2}mv_0^2 = nC_v\Delta T = \frac{m}{M}\frac{3}{2}R\Delta T$$

$$\therefore\ \Delta T = \frac{Mv_0^2}{3R}$$

20. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom traveling with the root-mean-square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, 25 qq without any collision with other atoms. Take $R = \frac{25}{3}$ J/mol – K and $k = 1.38 \times 10^{-23}$ J/K.

(*a*) Evaluate the temperature of the gas.

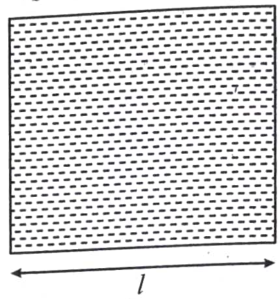
(*b*) Evaluate the average kinetic energy per atom.

(*c*) Evaluate the total mass of helium gas in the box.

Sol. Volume of the box = 1 m^3

Pressure of the gas = 100 N/m^2

Let T be the temperature of the gas. Then



(*a*) Time between two consecutive collisions with one wall $= \frac{1}{500}$ s. This time should be equal $\frac{2l}{v_{\text{rms}}}$, where l is the side of the cube.

$$\frac{2l}{v_{\text{rms}}} = \frac{1}{500}$$

or $v_{\text{rms}} = 1000$ m/s (as $l = 1$ m)

or $\sqrt{\frac{3RT}{M}} = 1000$

$$\therefore\ T = \frac{(1000)^2 M}{3R} = \frac{(10)^6(4\times10^{-3})}{3(25/3)} = 160\,\text{K}$$

(*b*) Average kinetic energy per atom $= \frac{3}{2}kT$

$$= \frac{3}{2}(1.38\times10^{-23})(160)\,\text{J} = 3.312\times10^{-21}\,\text{J}$$

(*c*) From $PV = nRT = \frac{m}{M}RT$

we get mass of helium gas in the box,

$$m = \frac{PVM}{RT}$$

substituting the value we get,

$$m = \frac{(100)(1)(4)}{(25/3)(160)} = 0.3\,\text{g}$$

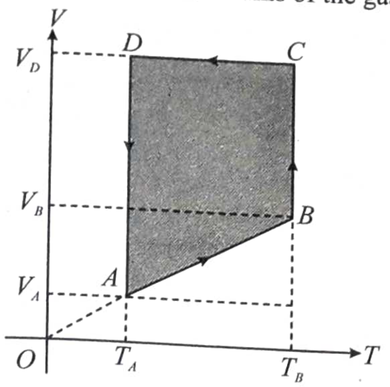
21. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The volume ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature

T_A at A is 27°C, calculate

(*a*) The temperature of the gas at point B.

(*b*) Heat absorbed or released by the gas in each process.

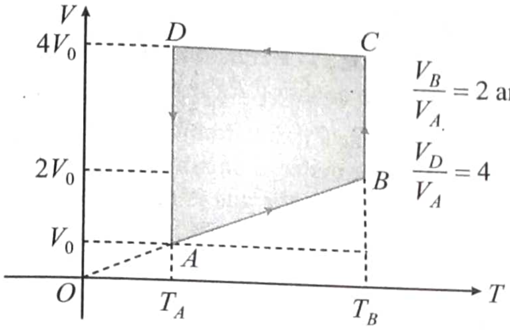
(*c*) The total work done by the gas during the complete cycle. Express your answer in terms of the gas constant R.



Sol. Given: number of moles, $n = 2$

$C_v = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$ (Monoatomic)

$T_A = 27°C = 300$ K



$\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$

Let $V_A = V_0$ then $V_B = 2V_0$ and $V_D = V_C = 4\,V_0$

(*a*) Process $A \to B$

$$V \propto T \Rightarrow \frac{T_B}{T_A} = \frac{V_B}{V_A}$$

$$\therefore\ T_B = T_A\left(\frac{V_B}{V_A}\right) = (300)(2) = 600\text{ K}$$

$$\therefore\ T_B = 600\text{ K}$$

(*b*) Process $A \to B$

$V \propto T \Rightarrow P = $ constant

$$\therefore\ Q_{AB} = nC_P\,dT = nC_P(T_B - T_A)$$

$$= (2)\frac{5}{2}R(600 - 300)$$

$Q_{AB} = 1500R$ (absorbed)

Process $B \to C$

$T = $ constant $\therefore\ dU = 0$

$$\therefore\ dU = 0\ \therefore\ Q_{BC} = W_{BC} = nRT_B\ln\frac{V_C}{V_B}$$

$$= (2)(R)(600)\ln\frac{4V_0}{2V_0}$$

$$= (1200R)\ln(2) = (1200R)(0.693)$$

or $Q_{BC} \approx 831.6R$ (absorbed)

Process $C \to D$

$V = $ constant

$$\therefore\ Q_{CD} = nC_V dT = nC_V(T_D - T_C) = n\frac{3}{2}R(T_A - T_B)$$

$(T_D = T_A$ and $T_C = T_B)$

$Q_{CD} = -900R$

Process $D \to A$

$T = $ constant $\Rightarrow dU = 0$

$$\therefore\ Q_{DA} = W_{DA} = nRT_D\ln\frac{V_A}{V_D}$$

$$= 2(R)(300)\ln\frac{V_0}{4V_0} = 600R\ln\frac{1}{4}$$

$Q_{DA} \approx -831.6R$ (released)

(*c*) In the complete cycle $dU = 0$

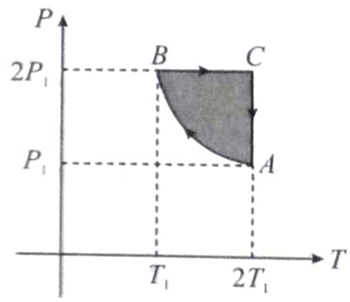
therefore, from conservation of energy

$W_{\text{net}} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$

$W_{\text{net}} = 1500\text{ R} + 831.6\text{ R} - 900\text{ R} - 831.6\text{ R}$

or $W_{\text{net}} = W_{\text{total}} = 600$ R

22. Two moles of an ideal monoatomic gas is taken through a cycle $ABCA$ as shown in the $P-T$ diagram. During the process AB, pressure and temperature of the gas vary such that PT = constant. If T_1 = 300 K, calculate

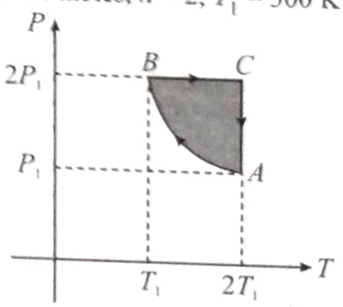


(*a*) The work done on the gas in the process AB and

(*b*) The heat absorbed or released by the gas in each of the processes.

Give answers in terms of the gas constant R.

Sol. (*a*) Number of moles, $n = 2$, $T_1 = 300$ K



During the process $A \to B$

$PT = $ constant

or $P^2V = $ constant $= K$ (say)

$$\therefore\ P = \frac{\sqrt{K}}{\sqrt{V}}$$

$$\therefore W_{A\to B} = \int_{V_A}^{V_B} P\cdot dV = \int_{V_A}^{V_B}\frac{\sqrt{K}}{\sqrt{V}}dV$$

$$= 2\left[\sqrt{(P_B^2 V_B)V_B} - \sqrt{(P_A^{\,2} V_A)V_A}\right](K = P^2 V)$$

$$= 2[P_B V_B - P_A V_A] = 2[nRT_B - nRT_A]$$

$$= 2nR[T_1 - 2T_1] = (2)(2)(R)[300-600] = -1200R$$

Work done on the gas in the process AB is 1200 R.

$W_{A\to B} = -\,1200$ K

Alternate Solution

$PV = nRT$

$PdV + VdP = nRdT$

$$\text{or } PdV + \frac{(nRT)}{P}\cdot dP = nRdT \qquad ...(1)$$

From the given condition

$PT = $ constant

$$PdT + TdP = 0 \qquad ...(2)$$

From equations (1) and (2), we get

$PdV = 2nRdT$

$$\therefore W_{A\to B} = \int PdV = 2nR\int_{T_A}^{T_B} dT$$

$= 2nR(T_B - T_A)$

$= 2nR(T_1 - 2T_1)$

$= (2)\,(2)\,(R)\,(300 - 600)$

or $W_{A\to B} = -\,1200\ R$

(*b*) Heat absorbed/released in different processes : since the gas is monoatomic,

therefore $C_v = \dfrac{3}{2}R$

and $C_P = \dfrac{5}{2}R$ and $\gamma = 5/3$

Process $A \to B$

$$\Delta U = nC_V\,\Delta T == = (2)\frac{3}{2}R(T_B - T_A)$$

$$= (2)\frac{3}{2}R(300-600) = -\,900\text{ R}$$

$Q_{A\to B} = W_{A\to B} + \Delta AU$

$= (-1200\ R) - (900\ R)$

$Q_{A\to B} = -\,2100\ R$ (released)

Alternate Solution :

In the process $PV^x = $ constant

Molar heat capacity,

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-x}$$

Here the process is $\quad P^2V = $ constant

or $PV^{1/2} = $ constant

i.e. $x = \dfrac{1}{2}$

$$\therefore C = \frac{R}{(5/3)-1} + \frac{R}{1-1/2}$$

$C = 3.5\ R$

$Q_{A\to B} = mC\Delta T = (2)(3.5\text{ R})\,(300 - 600)$

or $Q_{A\to B} = -\,2100$ R

Process $B \to C$: Process is isobaric

$$Q_{B\to C} = nC_p\Delta T = (2)\frac{5}{2}R(T_C - T_B)$$

$$= 2\frac{5}{2}R(2T_1 - T_1) = (5\text{R})\,(600 - 300)$$

$Q_{B\to C} = 1500\ R$ (absorbed)

Process $C \to A$: Process is isothermal

$\therefore\ \Delta U = 0$

$$\text{and } Q_{C\to A} = W_{C\to A} = nRT_C \ln\frac{P_C}{P_A} = nR(2T_1)\ln\frac{2P_1}{P_1}$$

$= (2)\ (R)\ (600)$ In (2)

$Q_{C\to A} = 831.6$ R (absorbed)

In first law of thermodynamics, $(dQ = dU + dW)$ we come cross three terms dQ, dU and dW.

$dU = nC_VdT$ for all the processes whether it is isobaric, isochoric or else,

and $dQ = nCdT$ where $C = \dfrac{R}{\gamma-1} + \dfrac{R}{1-x}$ the process $PV^x =$ constant.

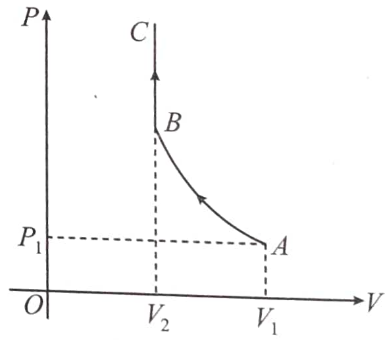
In both the terms we require $dT\ (= T_f - T_i)$ only. The third terms dW is obviously $dQ - dU$. Therefore, if in any process change in temperature (dT) and $P - V$ relation is known, then the above method is the simplest one. Note that even if we have $V - T$ or $T - P$ relation, it can be converted into PV relation by the equation $PV = nRT$.

23. Two moles of an ideal monoatomic gas, initially at pressure P_1 and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2.

(*a*) Sketch the complete process on a $P - V$ diagram.

(*b*) Find the total work done by the gas, the total change in internal energy and the final temperature of the gas. [Give your answer in terms of $P_1 : V_1 : V_2 : Q$ and R]

Sol. (*a*) The P–V diagram for the complete process will be as shown :



Process $A \to B$ is adiabatic compression, and Process $B \to C$ is isochoric.

(*b*) (*i*) Total work done by the gas

Process *A-B*

$$W_{AB} = \frac{P_AV_A - P_BV_B}{\gamma - 1} \quad \left[W_{\text{adiabatic}} = \frac{P_iV_i - P_fV_f}{\gamma - 1}\right]$$

$$= \frac{P_1V_1 - P_2V_2}{(5/3)-1}$$

$\gamma = 5/3$ for monoatomic gas

$$= \frac{P_1V_1 - P_1\left(\frac{V_1}{V_2}\right)^{\gamma} V_2}{2/3} \quad \left[\begin{array}{l} P_1V_1^{\gamma} = P_2V_2^{\gamma} \\ \therefore P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} \end{array}\right]$$

$$= \frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right]$$

$$= -\frac{3}{2}P_1V_1\left[\left(\frac{V_1}{V_2}\right)^{\frac{5}{3}-1} - 1\right]$$

$$= -\frac{3}{2}P_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Process B-C

$W_{BC} = 0$ (V = constant)

$W_{\text{Total}} = W_{AB} + W_{BC}$

$$= -\frac{3}{2}P_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

(*ii*) Total change in internal energy Process A-B (Process is adiabatic)

$Q_{AB} = 0$

$$\therefore \Delta U_{AB} = -W_{AB} = \frac{3}{2}P_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Process B-C

$W_{BC} = 0$

$\Delta U_{BC} = Q_{BC} = Q$ (Given)

$\Delta U_{\text{Total}} = \Delta U_{AB} + \Delta U_{BC}$

$$= \frac{3}{2}P_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q$$

(*iii*) Final temperature of the gas

$$\Delta U_{Total} = nC_V\Delta T = 2\frac{R}{\gamma - 1}(T_C - T_A)$$

$$\therefore \frac{3}{2}P_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q = \frac{2R}{(5/3)-1}\left[T_C - \frac{P_AV_A}{2R}\right]$$

$$\text{or } \frac{3}{2}P_1V_1\frac{V_1}{V_2} - 1 + Q = 3RT_C - \frac{P_1V_1}{2R}$$

$$\therefore T_C = \frac{Q}{3R} + \frac{P_1V_1}{2R}\frac{V_1}{V_2} = T_{\text{final}}$$

24. A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with γ ($= C_p/C_v = 5/3$) and another gas B with $\gamma = 7/5$ at a certain temperature T. The gram molecular weights of the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic process.

(*a*) Find the number of gram moles of the gas B in the gaseous mixture.

(*b*) Compute the speed of sound in the gaseous mixture at 300 K.

(*c*) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

(*d*) The mixture is compressed adiabatically to 1/5 of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.

Sol. (*a*) Number of moles of gas A are $n_A = 1$ (Given)

Let the number of moles of gas B be $n_B = n$ The internal energy of the mixture = internal energy of gas A + internal energy of gas B. and since $U = \frac{nRT}{\gamma - 1}$, therefore,

$$(n_A + n_B)\frac{R}{\gamma_{\text{mixture}} - 1}T$$

$$= n_A\frac{R}{\gamma_A - 1}T + n_B\frac{R}{\gamma_B - 1}T \quad ..(1)$$

Since the mixture obeys the law

$PV^{19/13}$ = constant (in adiabatic process)

Therefore,

$\gamma_{\text{mixture}} = 19/13$ (PV^{γ} = constant)

Substituting the values in equation (1), we have

$$\frac{(1+n)}{(19/13)-1} = \frac{1}{(5/3)-1} + \frac{n}{(7/5)-1}$$

Solving this, we get $n = 2$

Note: For γ_{mixture} we can directly use the formula:

$$\frac{n}{\gamma_{\text{mixture}} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

(*b*) Molecular weight of the mixture will be given by

$$M = \frac{n_AM_A + n_BM_B}{n_A + n_B} = \frac{(1)(4)+2(32)}{1+2} = 22.67$$

Speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v = \sqrt{\frac{(19/13)(8.31)(300)}{(22.67\times10^{-3})}}\text{ m/s or } v = 401 \text{ m/s}$$

Note: It is a common mistake that the students generally put $M = 22.67$ in $v = \sqrt{\frac{\gamma RT}{M}}$ **Note** that if you are writing the $R = 8.31$ J/mole-K, substitute value of $M = 22\ 67 \times 10^{-3}$kg.

$$v \propto \sqrt{T} \quad ....(2)$$

$$\Rightarrow dv = K\left(\frac{dT}{2\sqrt{T}}\right)$$

$$\Rightarrow \frac{dv}{v} = \frac{K}{v}\cdot\left(\frac{dT}{2\sqrt{T}}\right)$$

$$\frac{K}{v} = \frac{1}{\sqrt{T}} \qquad \text{(from 2)}$$

$$\Rightarrow \frac{dv}{v} = \frac{1}{\sqrt{T}}\left(\frac{dT}{2\sqrt{T}}\right) = \frac{1}{2}\left(\frac{dT}{T}\right)$$

$$\Rightarrow \frac{dv}{v}\times 100 = \frac{1}{2}\left(\frac{dT}{T}\right)\times 100$$

$$= \frac{1}{2}\left(\frac{1}{300}\right)\times 100 = 0.167$$

Therefore, percentage change in speed is 0.167%.

(*d*) Compressibility $= \dfrac{1}{\text{Bulk modulus}} = \beta$ (say)

Adiabatic bulk modulus is given by

$$\beta = \gamma P \left(B = -\frac{dP}{dV/V}\right)$$

Adiabatic compressibility will be given by

$$\beta = \frac{1}{\gamma P}$$

$$\beta' = \frac{1}{\gamma P'} = \frac{1}{\gamma P(5)^{\gamma}} \qquad [PV^{\gamma} = \text{constant}]$$

$$\left[\because PV^{\gamma} = P'(V/5)^{\gamma} \Rightarrow P' = P(5)^{\gamma}\right]$$

$$= \frac{V}{\gamma(n_A + n_B)RT}\left[1-\left(\frac{1}{5}\right)^{\gamma}\right]\left(P = \frac{nRT}{V}\right)$$

$$= \frac{V}{\left(\frac{19}{13}\right)(1+2)(8.31)(300)}\left[1-\left(\frac{1}{5}\right)^{19/13}\right]$$

$$\left(\gamma = \gamma_{mixture} = \frac{19}{13}\right)$$

$$\Delta\beta = 8.27\times 10^{-5}\, V$$

Exercise-1 (Topicwise)

RMS AND KTG

1. The volume of a gas at 20°C is 200 ml. If the temperature is reduced to – 20°C at constant pressure, its volume will be
(a) 172.6 ml (b) 17.26 ml
(c) 192.7 ml (d) 19.27 ml

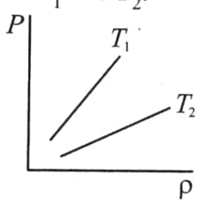
2. Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium ?
(a) kinetic energy (b) momentum
(c) density (d) speed

3. The mean free path of gas molecules depends on (d = molecular diameter)
(a) d (b) d^2
(c) d^{-2} (d) d^{-1}

4. A gas behaves more closely as an ideal gas at
(a) Low pressure and low temperature
(b) Low pressure and high temperature
(c) High pressure and low temperature
(d) High pressure and high temperature

5. Which of the following statements about kinetic theory of gases is wrong
(a) The molecules of a gas are in continuous random motion
(b) The molecules continuously undergo inelastic collisions
(c) The molecules do not interact with each other except during collisions
(d) The collisions amongst the molecules are of short duration

6. Figure shows graphs of pressure vs density for an ideal gas at two temperatures T_1 and T_2.



(a) $T_1 > T_2$ (b) $T_1 = T_2$
(c) $T_1 < T_2$
(d) Any of the three is possible

7. Suppose a container is evacuated to leave just one molecule of a gas in it. Let v_a and v_{rms} represent the average speed and the *rms* speed of the gas.
(a) $v_a > v_{rms}$ (b) $v_a < v_{rms}$
(c) $v_a = v_{rms}$ (d) v_{rms} is undefined

8. Consider a mixture of oxygen and hydrogen kept at room temperature. As compared to a hydrogen molecule an oxygen molecule hits the wall
(a) With greater average speed
(b) With smaller average speed
(c) With greater average kinetic energy
(d) With smaller average kinetic energy

9. Three containers of the same volume contain three different gases. The masses of the molecules are m_1, m_2 and m_3 and the number of molecules in their respective containers are N_1, N_2 and N_3. The gas pressure in the containers are P_1, P_2 and P_3 respectively. All the gases are now mixed and put in one of the containers. The pressure P of mixture will be
(a) $P < (P_1 + P_2 + P_3)$ (b) $P = \frac{P_1 + P_2 + P_3}{3}$
(c) $P = P_1 + P_2 + P_3$ (d) $P > (P_1 + P_2 + P_3)$

10. Air is pumped into an automobile tube upto a pressure of 200 *kPa* in the morning when the air temperature is 22°C. During the day, temperature rises to 42°C and the tube expands by 2%. The pressure of the air in the tube at this temperature, will be approximately
(a) 212 kPa (b) 209 kPa
(c) 206 kPa (d) 200 kPa

11. To what temperature should the hydrogen at 327°C be cooled at constant pressure, so that the root mean square velocity of its molecules become half of its previous value
(a) – 123°C (b) 123°C
(c) – 100°C (d) 0°C

12. At room temperature, the r.m.s. speed of the molecules of certain diatomic gas is found to be 1930 m/s. The gas is
(a) H_2 (b) F_2
(c) O_2 (d) Cl_2

13. The root mean square speed of the molecules of a gas is
(a) Independent of its pressure but directly proportional to its Kelvin temperature
(b) Directly proportional to the square roots of both its pressure and its Kelvin temperature
(c) Independent of its pressure but directly proportional to the square root of its Kelvin temperature
(d) Directly proportional to both its pressure and its Kelvin temperature

14. Let A and B the two gases and given: $\frac{T_A}{M_A} = 4\frac{T_B}{M_B}$; where T is the temperature and M is molecular mass. If C_A and C_B are the r.m.s. speed, then the ratio $\frac{C_A}{C_B}$ will be equal to
(a) 2 (b) 4
(c) 1 (d) 0.5

GAS LAW'S

15. To double the volume of a given mass of an ideal gas at 27°C keeping the pressure constant, one must raise the temperature in °C to

(*a*) 54° (*b*) 270°

(*c*) 327° (*d*) 600°

16. A mass of an ideal gas undergoes a reversible isothermal compression. Its molecules will then have compared with initial state, the same

(*i*) Root mean square velocity

(*ii*) Mean momentum

(*iii*) Mean kinetic energy

(*a*) (*i*), (*ii*), (*iii*) correct (*b*) (*i*), (*ii*) correct

(*c*) (*ii*), (*iii*) correct (*d*) (*i*) correct

INTERNAL ENERGY SPECIFIC HEAT OF GAS AND DEGREES OF FREEDOM

17. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is

(*a*) $\frac{2}{5}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{5}{7}$

18. Boiling water is changing into steam. Under this condition, the specific heat of water is

(*a*) zero (*b*) one

(*c*) Infinite (*d*) less than one

19. If the degree of freedom of a gas are *f*, then the ratio of two specific heats C_P/C_V is given by

(*a*) $\frac{2}{f}+1$ (*b*) $1-\frac{2}{f}$

(*c*) $1+\frac{1}{f}$ (*d*) $1-\frac{1}{f}$

20. The relation between two specific heats of a gas is

(*a*) $C_P - C_V = \frac{R}{J}$ (*b*) $C_V - C_P = \frac{R}{J}$

(*c*) $C_P - C_V = J$ (*d*) $C_V - C_P = J$

21. For a solid with a small expansion coefficient,

(*a*) $C_p - C_v = R$

(*b*) $C_p - C_v = R$

(*c*) C_p is slightly greater than C_v

(*d*) C_p is slightly less than C_v

22. The degrees of freedom of a stationary rigid body about its axis will be

(*a*) One (*b*) Two

(*c*) Three (*d*) Four

23. A gaseous mixture consists of 16g of helium and 16g of oxygen. The ratio $\frac{C_P}{C_V}$ of the mixture is

(*a*) 1.4 (*b*) 1.54

(*c*) 1.59 (*d*) 1.62

24. Relationship between *P*, *V*, and *E* for a gas is

(*a*) $P = \frac{3}{2}EV$ (*b*) $V = \frac{2}{3}EP$

(*c*) $PV = \frac{3}{2}E$ (*d*) $PV = \frac{2}{3}E$

25. The mean kinetic energy of a gas at 300 *K* is 100 *J*. The mean kinetic energy of the gas at 450 *K* is equal to

(*a*) 100 *J* (*b*) 3000 *J*

(*c*) 450 *J* (*d*) 150 *J*

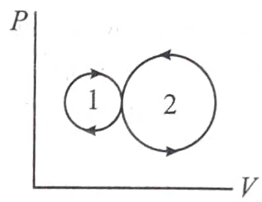
26. Energy of all molecules of a monoatomic gas having a volume *V* and pressure *P* is $\frac{3}{2}PV$. The total translational kinetic energy of all molecules of a diatomic gas as the same volume and pressure is

(*a*) $\frac{1}{2}PV$ (*b*) $\frac{3}{2}PV$

(*c*) $\frac{5}{2}PV$ (*d*) 3 *PV*

WORK

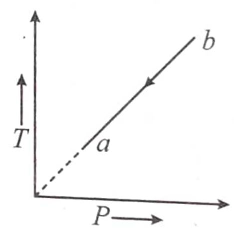
27. In the following indicator diagram, the net amount of work done will be



(*a*) Positive (*b*) Negative

(*c*) Zero (*d*) Infinity

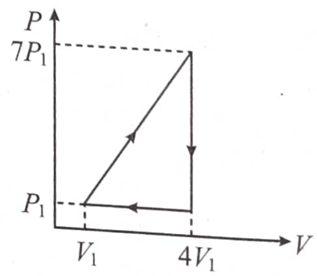
28. An ideal gas changes from state a to state b as shown in Fig. What is the work done by the gas in the process?



(*a*) zero (*b*) positive

(*c*) negative (*d*) infinite

29. In the cyclic process shown in the figure, the work done by the gas in one cycle is

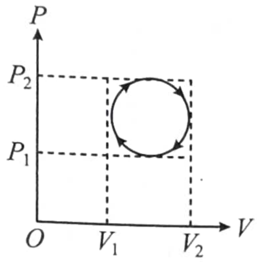


(*a*) 28 P_1V_1 (*b*) 14 P_1V_1

(*c*) 18 P_1V_1 (*d*) 9 P_1V_1

30. A gas is contained in a metallic cylinder fitted with a piston. The piston is suddenly moved in to compress the gas and is maintained at this position. As time passes, after this pressure of the gas in the cylinder
(a) Increases
(b) Decreases
(c) Remains constant
(d) Increases or decreases depending on the nature of the gas.

31. In the cyclic process shown on the $P-V$ diagram the magnitude of the work done is:

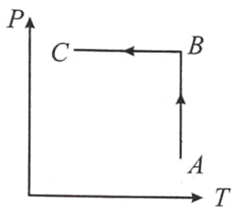


(a) $\pi\left(\frac{P_2-P_1}{2}\right)^2$
(b) $\pi\left(\frac{V_2-V_1}{2}\right)^2$
(c) $\frac{\pi}{4}(P_2-P_1)(V_2-V_1)$
(d) $\pi(P_2V_2-P_1V_1)$

FIRST LAW OF THERMODYNAMICS

32. First law of thermodynamics is given by
(a) $dQ = dU + PdV$
(b) $dQ = dU \times PdV$
(c) $dQ = (dU + dV)P$
(d) $dQ = PdU + dV$

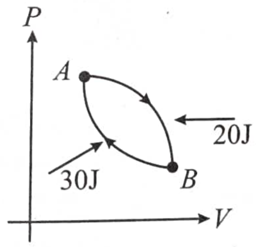
33. Ideal gas is taken through process shown in figure:



(a) In process AB, work done by system is positive
(b) In process AB, heat is rejected out of the system.
(c) In process AB, internal energy increases
(d) In process AB internal energy decreases and in process BC internal energy increases.

34. If heat is supplied to an ideal gas in an isothermal process,
(a) The internal energy of the gas will increase
(b) The gas will do positive work
(c) The gas will do negative work
(d) The said process is not possible

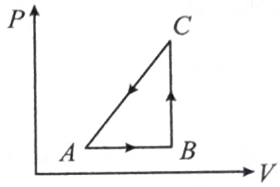
35. In a cyclic process shown in the figure an ideal gas is adiabatically taken from B and A, the work done on the gas during the process $B \rightarrow A$ is $30\ J$, when the gas is taken from $A \rightarrow B$ the heat absorbed by the gas is $20\ J$. The change in internal energy of the gas in the process $A \rightarrow B$ is:



(a) $20\ J$
(b) $-30\ J$
(c) $50\ J$
(d) $-10\ J$

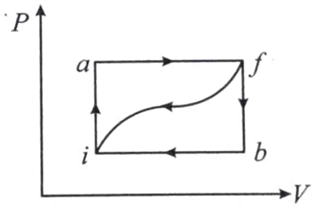
36. A system can be taken from the initial state p_1, V_1 to the final state p_2, V_2 by two different methods. Let ΔQ and ΔW represent the heat given to the system and the work done by the system. Which of the following must be the same in both the methods?
(a) ΔQ
(b) ΔW
(c) $\Delta Q + \Delta W$
(d) $\Delta Q - \Delta W$

37. The P-V diagram of a system undergoing thermodynamic transformation is shown in figure. The work done on the system in going from $A \rightarrow B \rightarrow C$ is $50\ J$ and 20 cal heat is given to the system. The change in internal energy between A and C is



(a) $34\ J$
(b) $70\ J$
(c) $84\ J$
(d) $134\ J$

38. When a system is taken from state i to a state f along path iaf, $Q = 50\ J$ and $W = 20\ J$. Along path ibf, $Q = 35\ J$. If $W = -13\ J$ for the curved return path f_i, Q for this path is

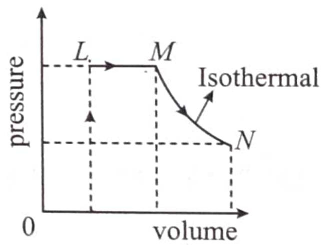


(a) $33\ J$
(b) $17\ J$
(c) $-7\ J$
(d) $-43\ J$

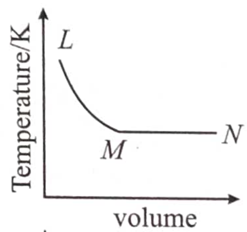
DIFFERENT TYPES OF THERMODYNAMICS PROCESS

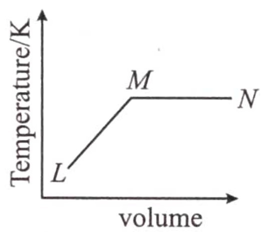
39. For an ideal gas, in an isothermal process
(a) Heat content remains constant
(b) Heat content and temperature remain constant
(c) Temperature remains constant
(d) None of these

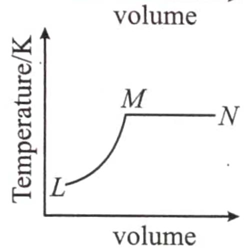
40. A fixed mass of ideal gas undergoes changes of pressure and volume starting at L, as shown in Figure.

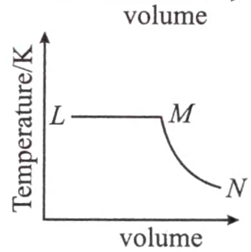


Which of the following is correct:

(a)


(b)


(c)


(d)


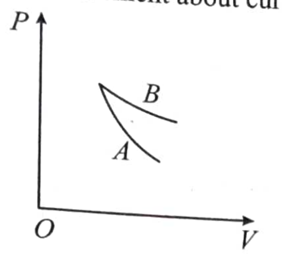
41. The gas law $\frac{PV}{T}$ = constant is true for

(a) Isothermal changes only
(b) Adiabatic changes only
(c) Both isothermal and adiabatic changes
(d) Neither isothermal nor adiabatic changes

42. The isothermal Bulk modulus of an ideal gas at pressure P is

(a) P
(b) γP
(c) $P/2$
(d) P/γ

43. Two curves A and B, are drawn in the Fig. for a given amount of gas. The correct statement about curve is



(a) Both are isothermal
(b) Both are adiabatic
(c) A is adiabatic B is isothermal
(d) B is adiabatic and A is isothermal

44. If a cylinder containing a gas at high pressure explodes, the gas undergoes

(a) Reversible adiabatic change and fall of temperature
(b) Reversible adiabatic change and rise of temperature
(c) Irreversible adiabatic change and fall of temperature
(d) Irreversible adiabatic change and rise of temperature

45. A given quantity of a gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is:

(a) $\frac{2}{3}P$
(b) P
(c) $\frac{3}{2}P$
(d) $2P$

SECOND LAW OF THERMODYNAMICS

46. If the door of a refrigerator is kept open then which of the following is true

(a) Room is cooled
(b) Room is heated
(c) Room is either cooled or heated
(d) Room is neither cooled nor heated

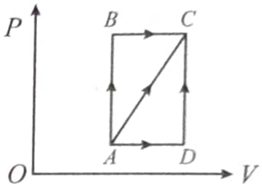
47. An ideal gas heat engine exhausting heat at 77° C is not have a 30% efficiency. It must take heat at

(a) 127°C
(b) 227°
(c) 327°C
(d) 673°C

48. A Carnot engine works between ice point and steam point. Its efficiency will be -

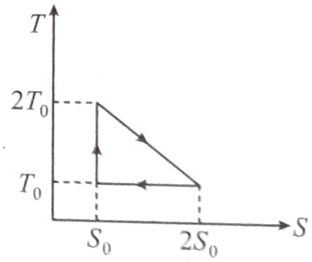
(a) 26.81 %
(b) 53.36 %
(c) 71.23 %
(d) 85.42 %

49. A thermodynamic process is shown in the figure. The pressures and volumes corresponding to some points in the figure are: $P_A = 3 \times 10^4$ Pa, $P_B = 8 \times 10^4$ Pa and $V_A = 2 \times 10^{-3}$ m^3, $V_D = 5 \times 10^{-3}$ m^3. In process AB, 600 J of heat is added to the system and in process BC, 200 J of heat is added to the system. The change in internal energy of the system in process AC would be



(a) 560 J
(b) 800 J
(c) 600 J
(d) 640 J

50. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



(a) 1/3
(b) 2/3
(c) 1/2
(d) 1/4

Exercise-2 (Learning Plus)

1. A diatomic ideal gas is used in a carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is

(a) 25%
(b) 5%
(c) 75%
(d) 99%

2. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

(a) 35 J
(b) 40 J
(c) 25 J
(d) 30 J

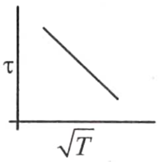
3. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is: [Given that $R = 8.31$ J $mol^{-1} K^{-1}$]
   (a) 748 J
   (b) 374 J
   (c) 350 J
   (d) 700 J

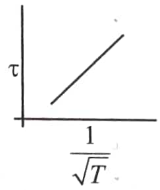
4. A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm, and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule?
   (a) $\sim 10^{11}$
   (b) $\sim 10^{13}$
   (c) $\sim 10^{10}$
   (d) $\sim 10^{12}$

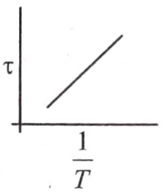
5. In moles of an ideal gas with constant volume heat capacity C_v undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is:
   (a) $\dfrac{4nR}{C_v - nR}$
   (b) $\dfrac{nR}{C_v - nR}$
   (c) $\dfrac{nR}{C_v + nR}$
   (d) $\dfrac{4nR}{C_v + nR}$

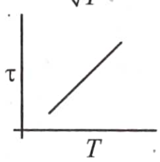
6. Two moles of an ideal gas with $\dfrac{C_P}{C_V} = \dfrac{5}{3}$ are mixed with 3 moles of another ideal gas with $\dfrac{C_P}{C_V} = \dfrac{4}{3}$. The value of $\dfrac{C_P}{C_V}$ for the mixture is:
   (a) 1.42
   (b) 1.47
   (c) 1.50
   (d) 1.45

7. The plot that depicts the behaviour of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale)

(a) 

(b) 

(c) 

(d) 

8. Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_p/C_v value will be:
   (a) $\dfrac{23}{15}$
   (b) $\dfrac{40}{27}$
   (c) $\dfrac{67}{45}$
   (d) $\dfrac{19}{13}$

9. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\dfrac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B, respectively is:
   (a) 3 : 5
   (b) 5 : 9
   (c) 7 : 9
   (d) 5 : 7

10. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to:
   (a) 1.07
   (b) 2.3
   (c) 3.67
   (d) 4.67

11. Which of the following is correct for the molecules of a gas in thermal equilibrium?
   (a) All have the same speed
   (b) All have different speeds which remain constant
   (c) They have a certain constant average speed
   (d) They do not collide with one another.

12. The average speed of nitrogen molecules in a gas is v. If the temperature is doubled and the N_2 molecule dissociate into nitrogen atoms, then the average speed will be
   (a) v
   (b) $v\sqrt{2}$
   (c) $2v$
   (d) $4v$

13. In a mixture of nitrogen and helium kept at room temperature. As compared to a helium molecule nitrogen molecule hits the wall
   (a) With greater average speed
   (b) With smaller average speed
   (c) With greater average kinetic energy
   (d) With smaller average kinetic energy.

14. The pressure of an ideal gas is written as $E = \dfrac{3PV}{2}$. Here E stands for
   (a) Total translational kinetic energy
   (b) Rotational kinetic energy
   (c) Average translational kinetic energy
   (d) Total kinetic energy.

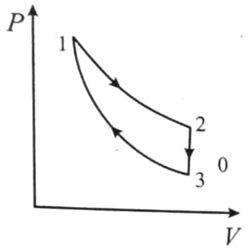
15. A gas has:
   (a) One specific heat only
   (b) Two specific heats only
   (c) Infinite number of specific heats
   (d) No specific heat

16. An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43 m. The length of air column above mercury level in the tube will be: Take $P_{atm} = 76$ cm of mercury
   (a) 0.215 m
   (b) 0.2 m
   (c) 0.1 m
   (d) 0.4 m

17. A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube?

(*a*) 33.3 *kPa* (*b*) 66.7 *kPa*
(*c*) 3.33 *MPa* (*d*) 6.67 *MPa*

18. Three processes compose a thermodynamics cycle shown in the *P-V* diagram. Process $1 \rightarrow 2$ takes place at constant temperature. Process $2 \rightarrow 3$ takes place at constant volume, and process $3 \rightarrow 1$ is adiabatic. During the complete cycle, the total amount of work done is 10 *J*. During process $2 \rightarrow 3$, the internal energy decrease by 20 *J* and during process $3 \rightarrow 1$, 20 *J* of work is done on the system. How much heat is added to the system during process $1 \rightarrow 2$?

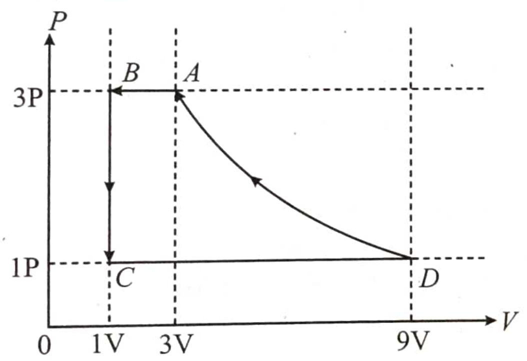


(*a*) 0 (*b*) 10 *J*
(*c*) 20 *J* (*d*) 30 *J*

19. 5.6 liter of helium gas at *STP* is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is:

(*a*) $\frac{9}{8}RT_1$ (*b*) $\frac{3}{2}RT_1$
(*c*) $\frac{15}{8}RT_1$ (*d*) $\frac{9}{2}RT_1$

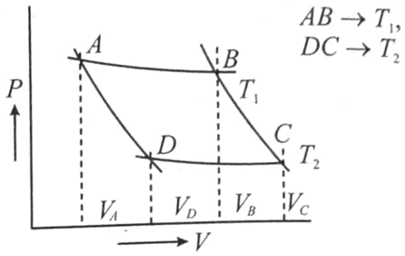
20. One mole of a monatomic ideal gas is taken through a cycle *ABCDA* as shown in the *P-V* diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



	Column-I		Column-II
A.	Process $A \rightarrow B$	p.	Internal energy decreases
B.	Process $B \rightarrow C$	q.	Internal energy increases
C.	Process $C \rightarrow D$	r.	Heat is lost
D.	Process $D \rightarrow A$	s.	Heat is gained
		t.	Work is done on the gas.

(*a*) A→ (p, r, t) B → (p, r) C → (q, s) D → (r, t)
(*b*) A → (s); B → (r); C → (p); D → (s)
(*c*) A → (p); B → (r); C → (q); D → (t)
(*d*) A → (p); B → (q); C → (t); D → (s)

21. In the following *P–V* diagram of an ideal gas, *AB* and *CD* are isothermal where as *BC* and *DA* are adiabatic process. The value of V_B/V_C is



(*a*) $= V_A / V_D$ (*b*) $< V_A / V_D$
(*c*) $> V_A / V_D$ (*d*) cannot say

22. Two samples 1 and 2 are initially kept in the same state. The sample 1 is expanded through an isothermal process where as sample 2 through an adiabatic process upto the same final volume. The final temperature in process 1 and 2 are T_1 and T_2 respectively, then

(*a*) $T_1 > T_2$
(*b*) $T_1 = T_2$
(*c*) $T_1 < T_2$
(*d*) The relation between T_1 and T_2 cannot be deduced.

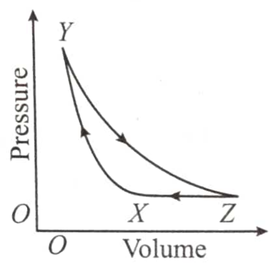
23. Let P_1 and P_2 be the final pressure of the samples 1 and 2 respectively in the previous question then:

(*a*) $P_1 < P_2$
(*b*) $P_1 = P_2$
(*c*) $P_1 > P_2$
(*d*) The relation between P_1 and P_2 cannot be deduced.

24. In an adiabatic process on a gas with $\gamma = 1.4$, the pressure is increased by 0.5%. The volume decreases by about

(*a*) 0.36 % (*b*) 0.5 %
(*c*) 0.7 (*d*) 1 %

25. In given figure, a fixed mass of an ideal gas undergoes the change represented by *XYZX* below. Which one of the following sets could describe these of changes?



	XY	*YZ*	*ZX*
(*a*)	Isothermal expansion	adiabatic compression	compression at constant pressure
(*b*)	Adiabatic expansion	isothermal compression	pressure reduction constant volume

(*c*) Isothermal compression | adiabatic expansion | compression at constant pressure

(*d*) Adiabatic compression | isothermal expansion | compression at constant pressure

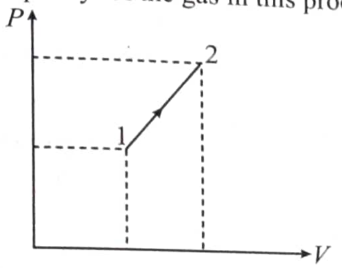
26. The internal energy of an ideal gas is equal to negative of the work done by the system, then

(*a*) The process must be adiabatic
(*b*) The process must be isothermal
(*c*) The process must be isobaric
(*d*) The temperature must decrease

27. A spherical soap bubble (surface tension = S) encloses n moles a monoatomic ideal gas. The gas is heated slowly so that the surface area of the bubble increases by $\frac{3nR}{2S}$ per unit increment in temperature. The specific heat for this process is :

(*a*) $\frac{3R}{2}$ (*b*) $\frac{5R}{2}$
(*c*) $\frac{7R}{2}$ (*d*) $\frac{9R}{2}$

28. A process $1 \rightarrow 2$ using diatomic gas is shown on the P-V diagram below. $P_2 = 2P_1 = 10^6$ N/m², $V_2 = 4V_1 = 0.4$ m³. The molar heat capacity' of the gas in this process will be:

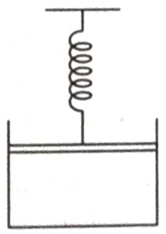


(*a*) $35R/12$ (*b*) $25R/13$
(*c*) $35R/11$ (*d*) $22R/7$

29. The C_P/C_V ratio for a gas mixture consisting of 4gms helium and 32gms of oxygen is:

(*a*) 1.45 (*b*) 1.6
(*c*) 1.5 (*d*) 1.66

30. One mole of an ideal gas is kept enclosed under a light piston (area $=10^{-2}$ m²) connected by a compressed spring (spring constant 100 N / m). The volume of gas is 0.83 m³ and its temperature is 100 K. The gas is heated so that it compresses the spring further by 0.1 m. The work done by the gas in the process is (Take $R = 8.3$ J / mole and suppose there is no atmosphere) :



(*a*) 3 J (*b*) 6 J
(*c*) 9 J (*d*) 1.5 J

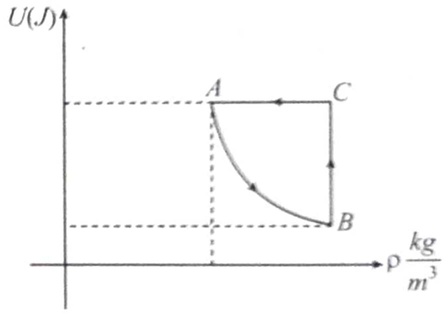
31. A long glass tube of length L, sealed at both ends, contains a small column of mercury (density = ρ) of length 'a' ($a << L$) at its middle and air at pressure P on both sides. The tube is fixed horizontally. If the mercury column gets a small displacement, the time period of its oscillations would be (assuming that the air on the sides undergoes isothermal expansion or compression):

(*a*) $\pi\,[\rho La/P]^{1/2}$ (*b*) $2\pi\,[\rho La/P]^{1/2}$
(*c*) $\pi\,[2\rho La/P]^{1/2}$ (*d*) $\pi\,[\rho La/2P]^{1/2}$

32. Find the pressure at which temperature attains its maximum value if the relation between pressure and volume for an ideal is $P = P_0 + (1-\alpha)V^2; x > 1$

(*a*) $\frac{2P_0}{3}$ (*b*) $\frac{P_0}{3}$
(*c*) P_0 (*d*) $\frac{4P_0}{3}$

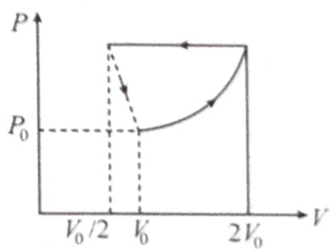
33. Figure shows the variation of the internal energy U with density ρ of one mole of an ideal monatomic gas for thermodynamic cycle $ABCA$. Here process AB is a part of rectangular hyperbola:

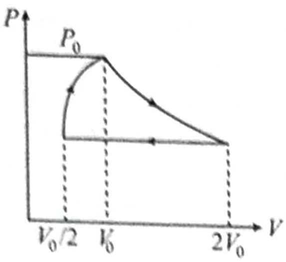


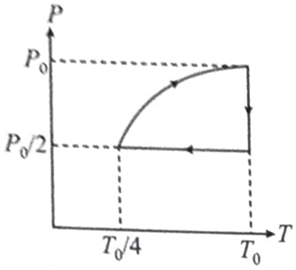
(*a*) Process AB is isothermal and net work in cycle is done by gas.
(*b*) Process AB is isobaric and net work in cycle is done by gas
(*c*) Process AB is isobaric and net work in cycle is done on the gas
(*d*) Process AB is adiabatic and net work in cycle is done by gas

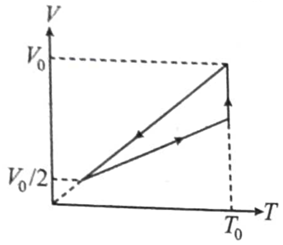
34. One mole of an ideal gas at pressure P_0 and temperature T_0 volume V_0 is expanded isothermally to twice its volume and then compressed at constant pressure to $(V_0/2)$ and the gas is brought to original state by a process in which $P \propto V$ (Pressure is directly proportional to volume). The correct representation of process is :

(*a*)

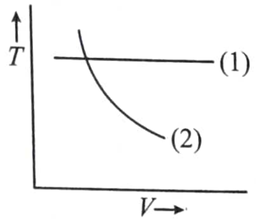


(b) 

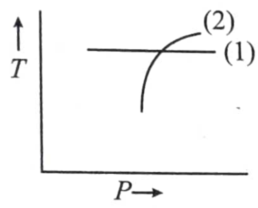
(c) 

(d) 

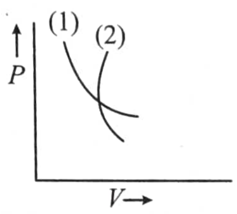
35. If (1) represents isothermal and (2) represents adiabatic, which of the graphs given above in respect of an ideal gas are correct ?



(I)



(II)



(III)

(a) I and II (b) II and III
(c) I and III (d) I, II and III

36. For an ideal gas the equation of a process for which the heat capacity of the gas varies with temperature as $C = \alpha / T$ (α is a constant) is given by :

(a) $V \ln T$ = constant

(b) $VT^{1/(\gamma-1)} e^{\alpha/RT}$ = constant

(c) $V^{\frac{1}{\gamma-1}} T e^{\alpha/RT}$ = constant

(d) $V^{\gamma-1}$ = constant

37. Three samples of the same gas A, B and C (γ =3/2) have initially equal volume. Now the volume of each sample is doubled. The process is adiabatic for A, isobaric for B and isothermal for C. If the final pressure are equal for all three samples, the ratio of their initial pressures are :

(a) $2\sqrt{2}:2:1$ (b) $2\sqrt{2}:1:2$
(c) $\sqrt{2}:1:2$ (d) $2:1:\sqrt{2}$

38. The efficiency of a Carnot's engine is 0.6. It rejects total 20 J of heat. The work done by the engine is :

(a) 40 J (b) 50 J
(c) 20 J (d) 30 J

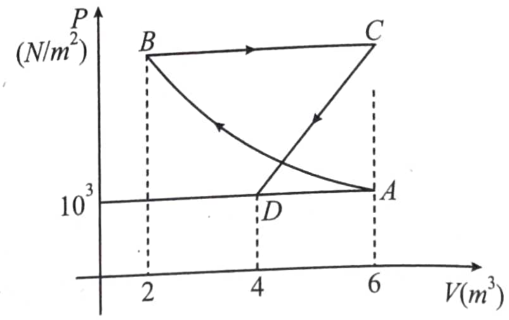
39. Two moles of an ideal monoatomic gas is taken through the process as shown in figure. The process is

$A - B$ Adiabatic

$B - C$ isobaric

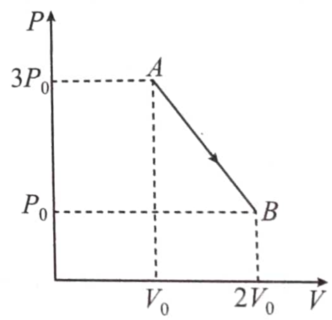
$C - D$ Linear relation between P and V.

The net change in internal energy of the gas is:



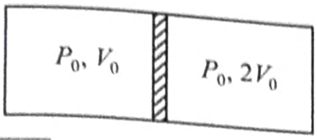
(a) 100 KJ (b) –200 KJ
(c) –300 KJ (d) 500 KJ

40. One moles of an ideal monoatomic gas undergoes a process $A \to B$ as shown in the P–V diagram. The volume at which the process changes from endothermic to exothermic is



(a) $\frac{25}{16}V_0$ (b) $\frac{20}{19}V_0$
(c) $\frac{4}{25}V_0$ (d) $\frac{21}{26}V_0$

41. A horizontal cylinder is divided into two chambers by a thin smooth heat insulating piston of mass 'm' and cross section area 'A' initially. When the piston is in equilibrium, pressure of gas in each chamber is P_0 and volumes of the chambers are V_0 and $2V_0$ as shown in the figure. The ideal gases in the two chambers are kept at constant temperatures T_1 and T_2. The piston is slightly displaced from equilibrium position and then released. Then the time period of small oscillations of the piston is



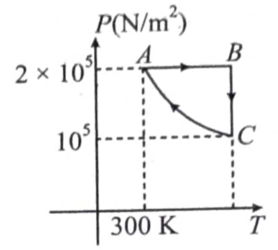
(*a*) $2\pi\sqrt{\dfrac{mV_0}{3P_0A^2}}$

(*b*) $2\pi\sqrt{\dfrac{2mV_0}{3P_0A^2}}$

(*c*) $2\pi\sqrt{\dfrac{3mV_0}{2P_0A^2}}$

(*d*) $2\pi\sqrt{\dfrac{mV_0}{2P_0A^2}}$

42. Two moles of a monoatomic ideal gas is taken through a cyclic process shown on pressure (P) temperature (T) diagram in figure. Process CA is represented as PT = Constant. If efficiency of given cyclic process is $1-\dfrac{3x}{12\ln 2+15}$, then find the value of 'x'.



(*a*) 2 (*b*) 7 (*c*) 4 (*d*) 5

43. In a thermally isolated system, two boxes filled with an ideal gas are connected by a valve. When the valve is in closed position, states of the box 1 and 2, respectively, are (1 atm, V, T) and (0.5 atm, $4V$, T). When the valve is opened, what is approximately the final pressure (in atm) of the system?

(*a*) 0.6 (*b*) 0.3 (*c*) 0.06 (*d*) 0.9

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. Consider a collision between an argon molecule and a nitrogen molecule in a mixture of argon and nitrogen kept at room temperature. Which of the following are possible?

(*a*) The kinetic energies of both the molecules decrease.
(*b*) The kinetic energies of both the molecules increase
(*c*) The kinetic energy of the argon molecule increases and that of the nitrogen molecules decrease.
(*d*) The kinetic energy of the nitrogen molecules increases and that of the argon molecule decrease.

2. In case of hydrogen and oxygen at N.T.P., which of he following quantities is/are the same?

(*a*) Average momentum per molecule
(*b*) Average kinetic energy per molecule
(*c*) Kinetic energy per unit volume
(*d*) Kinetic energy per unit mass

3. Two vessels of the same volume contain the same gas at same temperature. If the pressure in the vessel be in the ratio of 1 : 2, then

(*a*) The ratio of the average kinetic energy is 1 : 2
(*b*) The ratio of the root mean square velocity is 1 : 1
(*c*) The ratio of the average velocity is 1 : 2
(*d*) The ratio of number of molecules is 1 : 2

4. A mixture of ideal gases 7 kg of nitrogen and 11 kg of CO_2. Then

(*a*) Equivalent molecular weight of the mixture is 36.
(*b*) Equivalent molecular weight of the mixture is 18.
(*c*) γ for the mixture is 5/2
(*d*) γ for the mixture is 47/35

(Take γ for nitrogen and CO_2 as 1.4 and 1.3 respectively)

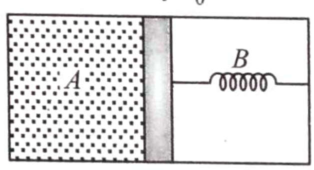
5. Specific heat of a substance can be:

(*a*) Finite (*b*) Infinite
(*c*) Zero (*d*) Negative

6. The following sets of values for C_v and C_p f a gs have been reported by different students. The units are cal mole^{-1} K^{-1}. Which of these sets is most reliable?

(*a*) $C_v = 3, C_p = 5$ (*b*) $C_v = 4, C_p = 6$
(*c*) $C_v = 3, C_p = 2$ (*d*) $C_v = 3, C_p = 4.2$

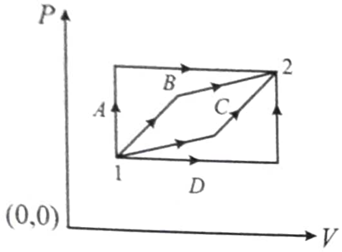
7. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless piston of area S & mass m into two equal parts A and B. Part A has an ideal gas at pressure P_0 and temperature T_0 and in part B is vacuum. A massless spring of force constant K is connected with the piston and the wall of the container as shown. Initially the spring is undeformed. The gas in chamber A is allowed to expand. Let in equilibrium the spring is compressed by x_0. Then:



(*a*) Pressure of the gas at equilibrium is $\dfrac{Kx_0}{S}$
(*b*) Work done by the gas is $\dfrac{1}{2}Kx_0^2$
(*c*) Increase in internal energy of the gas is $\dfrac{1}{2}Kx_0^2$
(*d*) Temperature of the gas is decreased

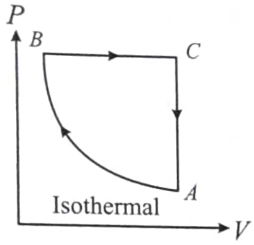
8. A gas kept in a container of finite conductivity is suddenly compressed. The process
   (a) Must be very nearly adiabatic
   (b) Must be very nearly isothermal
   (c) May be very nearly adiabatic
   (d) May be very nearly isothermal

9. An ideal gas is taken from state 1 to state 2 through optional path A, B, C & D as shown in P-V diagram. Let Q, W and U represent the heat supplied, work done & internal energy of the gas respectively. Then



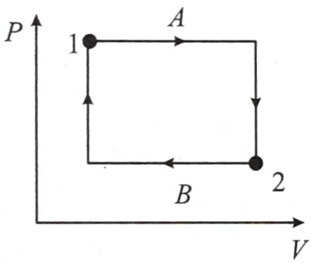
(a) $Q_B - W_B > Q_C - W_C$ (b) $Q_A - Q_D = W_A - W_D$
(c) $W_A < W_B < W_C < W_D$ (d) $Q_A > Q_B > Q_C > Q_D$

10. A cyclic process of an ideal monoatomic gas is shown in figure. The correct statement is (are):



(a) Work done by gas in process AB is more than that of the process BC.
(b) Net heat energy has been supplied to the system.
(c) Temperature of the gas is maximum in state B.
(d) In process CA, heat energy is rejected out by system.

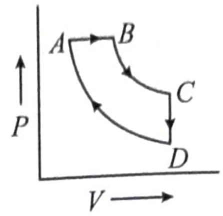
11. In given figure, let ΔU_1 and ΔU_2 be change in internal energy in process A and B respectively. ΔQ and W be the net heat given and net work done by the system in the process $A + B$, then



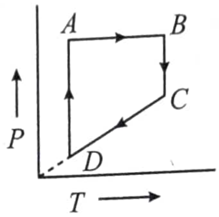
(a) $\Delta U_1 + \Delta U_2 = 0$ (b) $\Delta U_1 - \Delta U_2 = 0$
(c) $\Delta Q - W = 0$ (d) $\Delta Q + W = 0$

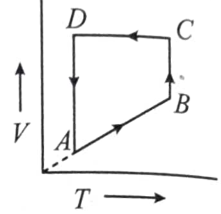
12. An ideal gas of one mole is kept in a rigid container of negligible heat capacity. If 25 J of heat is supplied the gas temperature raises by 2^0 C. Then the gas may be
(a) Helium (b) Argon
(c) Oxygen (d) Carbon dioxide

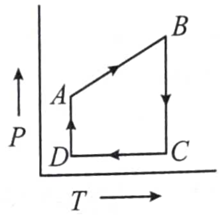
13. A cyclic process $ABCD$ is shown in the P–V diagram. (BC and DA are isothermal)

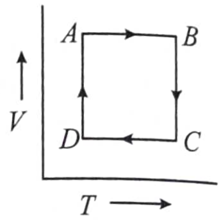


Which of the following curves represents the same process?

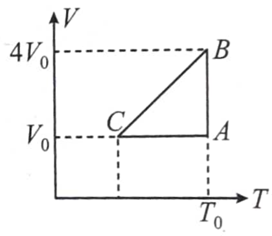
(a)


(b)


(c)


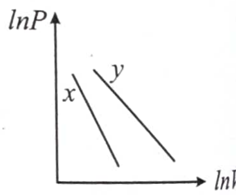
(d)


14. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following:



(a) Internal energies at A and B are the same
(b) Work done by the gas in process AB is $P_0V_0 \ell n\, 4$
(c) Pressure at C is $P_0/4$
(d) Temperature at C is $T_0/4$

15. For two different gases X and Y, having degrees of freedom f_1 and f_2 and molar heat capacities at constant volume and respectively, the $ln\ P$ versus $ln\ V$ graph is plotted of adiabatic process, as shown



(a) $f_1 > f_2$ (b) $f_2 > f_1$
(c) $C_{v_2} > C_{v_1}$ (d) $C_{v_1} > C_{v_2}$

16. A system undergoes a cyclic process in which it absorbs Q_1 heat and gives out Q_2 heat. The efficiency of the process is η and work done is W. Select correct statement:

(a) $W = Q_1 - Q_2$ (b) $\eta = \dfrac{W}{Q_1}$
(c) $\eta = \dfrac{Q_2}{Q_1}$ (d) $\eta = 1 - \dfrac{Q_2}{Q_1}$

17. Coefficient of volume expansion of an ideal gas is $\gamma = \frac{2}{T}$ (T is the absolute temperature) in a process. C_P and C_V are the molar specific heat capacities at constant pressure and constant volume respectively. If the molar heat capacity of the process is C. Then choose the correct option(s).

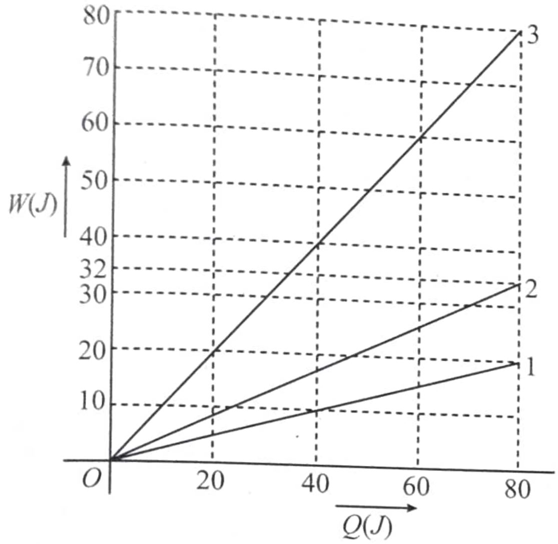
(*a*) $C = C_V + 2R$

(*b*) $C = C_P + R$

(*c*) $C = 3C_V + 2R$

(*d*) $C = C_V - 2R$

18. In the figure shown, the amount of heat supplied to one mole of an ideal gas is plotted on the horizontal axis and the amount work performed by the gas is drawn on the vertical axis. One of the straight lines in the Fig. is an isotherm and the other two are isobars of two gases. The initial states of both gases are same.



Select the correct statements.

(*a*) Curve 3 corresponds to isothermal process

(*b*) Curve 1 corresponds to a polyatomic gas

(*c*) Curve 2 corresponds to a monoatomic gas

(*d*) Process 1 and 2 are isobaric process.

19. On the $P - T$ graph of an ideal gas:

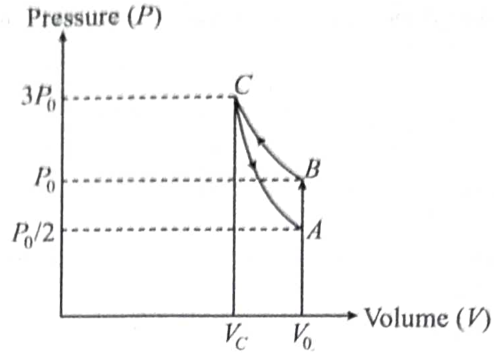
(*a*) Adiabatic process will be a straight line

(*b*) Isochoric process will be a straight line passing through the origin

(*c*) Adiabatic curve will have a positive slope

(*d*) The slope of adiabatic curve will decrease with increase in T

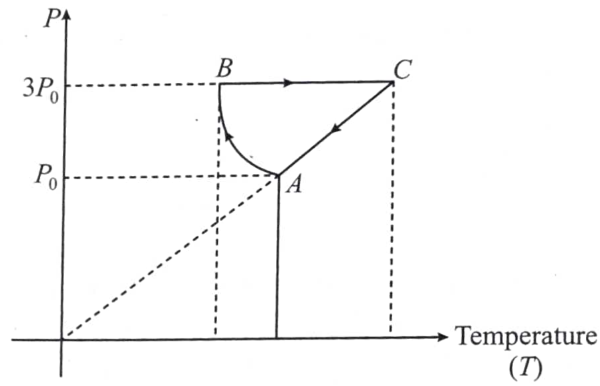
20. One mole of an ideal gas is carried through : thermodynamic cycle as shown in the figure. The cycle consists of an isochoric, an isothermal and a adiabatic processes. The adiabatic exponent of the gas is γ. Choose the correct option(s).



(*a*) $\gamma = \frac{ln\,6}{ln3}$ (*b*) $\gamma = \frac{ln\,5}{ln3}$

(*c*) BC is adiabatic (*d*) AC is adiabatic

21. One mole of monoatomic gas is taken through cyclic process shown below. $T_A = 300$ K. Process AB is defined as $PT =$ constant



(*a*) Work done in process AB is $-400R$

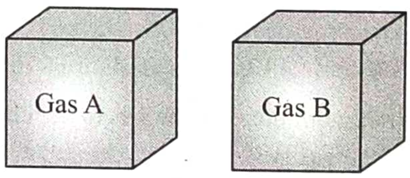
(*b*) Change in internal energy in process CA is $900R$

(*c*) Heat transferred in the process BC is $2000R$

(*d*) Change in internal energy in process CA is $-900R$

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 22 to 25): Two closed identical conducting containers are found in the laboratory of an old scientist. For the verification of the gas some experiments are performed on the two boxes and the results are noted.



Experiment 1.

When the two containers are weighed $W_A = 225$ g, $W_B = 160$ g and mass of evacuated container $W_C = 100$ g.

Experiment 2.

When the two containers are given same amount of heat same temperature rise is recorded. The pressure changes found are

$\Delta P_A = 2.5$ atm. $\Delta P_B = 1.5$ atm.

Required data for unknown gas:

Mono	*He*	*Ne*	*Ar*	*Kr*	*Xe*	*Rd*
(molar mass)	4 g	20 g	40 g	84 g	131 g	222 g
Dia	H_2	F_2	N_2	O_2	Cl_2	
(molar mass)	2 g	19 g	28 g	32 g	71 g	

22. Identify the type of gas filled in container A and B respectively.

(*a*) Mono, Mono
(*b*) Dia, Dia
(*c*) Mono, Dia
(*d*) Dia, Mono

23. Identify the gas filled in the container A and B.

(*a*) N_2, Ne (*b*) He, H_2
(*c*) O_2, Ar (*d*) Ar, O_2

24. Total number of molecules in 'A' (here N_A = Avagadro number)

(*a*) $\frac{125}{64}N_A$ (*b*) $3.125\ N_A$
(*c*) $\frac{125}{28}N_A$ (*d*) $31.25\ N_A$

25. The initial internal energy of the gas in container 'A', If the container were at room temperature 300 K initially.

(*a*) 1406.25 cal (*b*) 1000 cal
(*c*) 2812.5 cal (*d*) none of these

Comprehension (Q. 26 to 28): A monoatomic ideal gas is filled in a non conducting container. The gas can be compressed by a movable non conducting piston. The gas is compressed slowly to 12.5% of its initial volume.

26. The percentage increase in the temperature of the gas is

(*a*) 400% (*b*) 300%
(*c*) – 87.5% (*d*) 0%

27. The ratio of initial adiabatic bulk modulus of the gas to the final value of adiabatic bulk modulus of the gas is

(*a*) 32 (*b*) 1
(*c*) 1/32 (*d*) 4

28. The ratio of work done by the gas to the change in internal energy of the gas is

(*a*) 1 (*b*) –1
(*c*) ∞ (*d*) 0

MATCH THE COLUMN TYPE QUESTIONS

29. An ideal monoatomic gas undergoes different types of processes which are described in column-I. Match the corresponding effects in column-II. The letters have usual meaning.

	Column-I		Column-II
A.	$P = 2V^2$	p.	If volume increases then temperature will also increases.
B.	PV^2 = constant	q.	If volume increases then temperature will decreases.
C.	$C = C_V + 2R$	r.	For expansion, heat will have to be supplied to the gas.
D.	$C = C_V - 2R$	s.	If temperature increases then work done by gas is positive.

(*a*) A→ (p, r, s) B → (q) C → (p, r, s) D → (q, r)
(*b*) A→ (p, r, q) B → (s) C → (p, s, r) D → (q, r)
(*c*) A→ (q, r, s) B → (p) C → (p, q, r) D → (p, s)
(*d*) A→ (s, q, r) B → (r) C → (q, r, p) D → (s, r)

30. The figures given below show different processes (relating pressure P and volume V) for a given amount for an ideal gas. W is work done by the gas and ΔQ is heat absorbed by the gas.

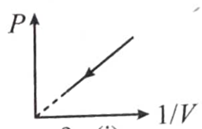


fig (i)

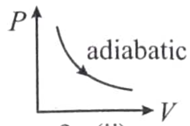


fig (ii)

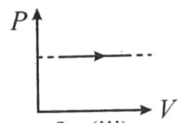


fig (iii)

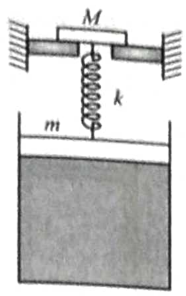
fig (iv)

	Column-I		Column-II
A.	In Figure (i)	p.	$\Delta Q > 0$.
B.	In Figure (ii)	q.	$W < 0$.
C.	In Figure (iii)	r.	$\Delta Q < 0$.
D.	In Figure (iv) (for complete cycle)	s.	$W > 0$.

(*a*) A → (q, r); B → (p); C → (p, s); D → (p, r)
(*b*) A → (p, r); B → (q); C → (r, s); D → (q, r)
(*c*) A → (r, s); B → (r); C → (p, q); D → (s, r)
(*d*) A → (p, s); B → (s); C → (p, s); D → (q, r)

NUMERICAL TYPE QUESTIONS

31. 0.01 moles of an ideal diatomic gas is enclosed in a adiabatic cylinder of cross-sectional area $A = 10^{-4} m^2$. In the arrangement shown, a block of mass $M = 0.8$ k is placed on a horizontal support, and piston of mas $m = 1$ kg is suspended from a spring of stiffness constar $k = 16$ N/m. Initially, the spring is relaxed and the volum of the gas is $V = 1.4 \times 10^{-4}\ m^3$.

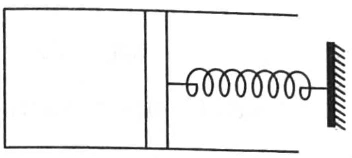


When the gas in the cylinder is heated up the piston starts moving up and the spring gets compressed so that the block M is just lifted up. Determine the heat supplied (in Joule).

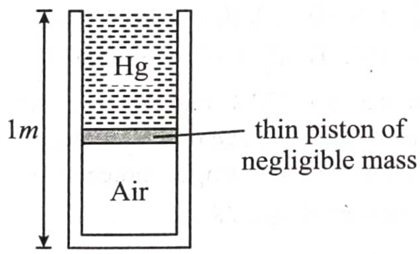
Take atmospheric pressure $P_0 = 10^5$ Nm^{-2}, $g = 10$m/s^2.

32. A 5000 cm^3 tank contains an ideal gas ($M = 40$ kg/kmol) at a pressure of 500 kPa and a temperature of 27°C. What mass of gas (in gm) is in the tank? (Take $R =$ J/mol K).

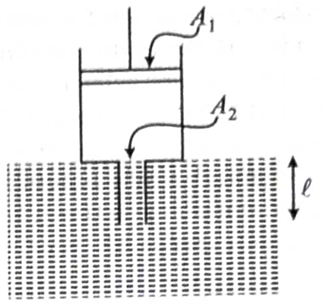
33. An ideal gas is enclosed in a container as shown in figure. The spring constant is 100 N/m, the area of the piston is 1 cm^2 and the length of the gas column is 100 cm. The compression of the spring is 10 cm. There is no friction between the walls and the piston. The whole system is heated to make the temperature 2.4 times the original temperature. Find the distance moved by the piston (in cm) Take atmospheric pressure = 10^5 *Pa*.



34. The 1 m tall conducting cylinder in figure contains air at a pressure of 76 cm of Hg. A very thin piston of negligible mass is placed at the top of the cylinder to prevent any air from air escaping, then mercury is very slowly poured into the cylinder until no more can be added without the cylinder over flowing. What is the height (in cm) of the compressed air?

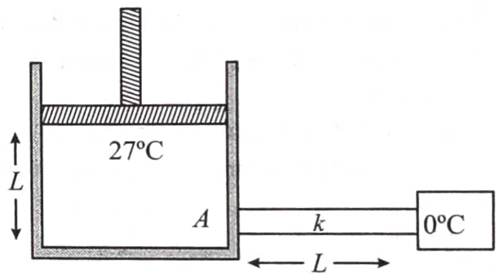


35. How far can I move (in cm) the piston of cross-section A_1, in the device shown in figure, so that 40% of the air enclosed remains. (The atmospheric pressure $p_0 = 10$ N/cm^2, density of liquid = 5 g/cm^3, $\ell = 20$cm, $A_1 = 4$cm^2, $A_2 = 1$ cm^2. The temperature is constant. The piston is massless and the initial height of gas in device is 110 cm.

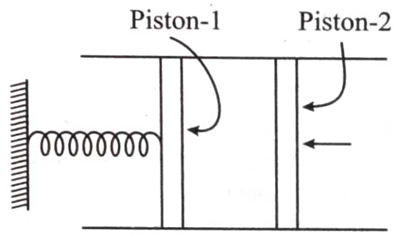


36. In a chemical processing plant, a reaction chamber of fixed volume V_0 is connected to a reservoir chamber of fixed volume $4V_0$ by a passage containing a thermally insulating porous plug. The plug permits the chambers to be at different temperatures. The plug allows gas to pass from either chamber to the other, ensuring that the pressure is the same in both. At one point in the processing, both chambers contain gas at a pressure of 100 kPa and a temperature of 27°C. The reservoir is heated to 127°C while the reaction chamber is maintained at 27°C. What is the pressure in both chambers (in kPa) after this is done?

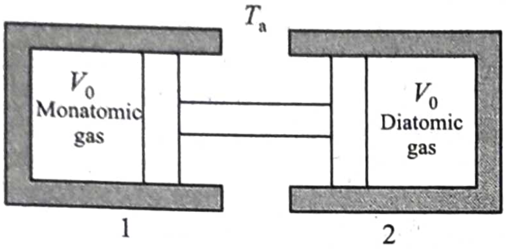
37. 0.5 mole of an ideal gas is kept inside an adiabatic cylinder of length 'L' and cross-sectional area 'A' closed by massless adiabatic piston. The cylinder is attached with a conducting rod of length 'L', cross-sectional area $\frac{1}{900}$ m^2 and thermal conductivity 415.5 W/m-K, whose other end is maintained at 0°C. The piston is moved such that the temperature of the gas remains constant at 27°C. Find the velocity (in mm/sec) of piston when it is at height $L/2$ from the bottom of cylinder. Rod is well lagged and has negligible heat capacity. $R = 8.31$ J/mol-K.



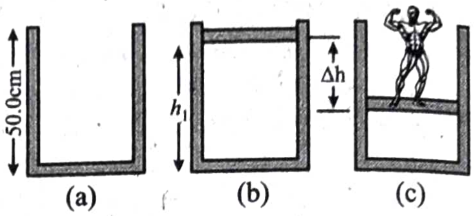
38. A long container has air enclosed inside at room temperature and atmospheric pressure (10^5 *pa*). It has a volume of 20,000 cc. The area of cross section is 100 cm^2 and force constant of spring is $k_{\text{spring}} = 1000$N/m. We push the right piston isothermally and slowly till it reaches the original position of the left piston which is movable. What is the final length of air column in cm. Assume that spring is initially relaxed.



39. The two conducting cylinder-piston systems shown below are linked. Cylinder 1 is filled with a certain molar quantity of a monoatomic ideal gas, and cylinder 2 is filled with an equal molar quantity of a diatomic ideal gas. The entire apparatus is situated inside an oven whose temperature is $T_a = 27°C$. The cylinder volumes have the same initial value $V_0 = 100$cc. When the oven temperature is slowly raised to $T_b = 127°C$. What is the volume change ΔV (in *cc*) of cylinder 1?



40. A cylinder that has a cross-section area of 100 cm^2 and is 50 cm deep is filled with air at 21.0°C and 1.00 atm (figure-*a*) A 20.0 kg piston is now lowered into the cylinder, compressing the air trapped inside (figure-*b*). Finally, a 80.0 kg man stands on the piston, further compressing the air, which remains at 21°C (figure-*c*). To what temperature (in°C) should the gas be heated to raise the piston and the man back to h_i? ($g = 10$ m/s^2)



41. Three insulated vessels of equal volumes V are connected by thin tubes that can transfer gas but do not transfer heat. Initially all vessels are filled with the same type of gas at a temperature T_0 and pressure P_0. Then the temperature in the first vessel is doubled and the temperature in the second vessel is tripled. The temperature in the third vessel remains unchanged. If initial pressure is 77 cm of Hg. Find the final pressure P' in the system in cm of Hg.

Exercise-4 (Past Year Questions)

JEE MAIN

1. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true? **(2020)**

A. The mean free path of the molecules decreases.

B. The mean collision time between the molecules decreases.

C. The mean free path remains unchanged.

D. The mean collision time remains unchanged

(*a*) C and D (*b*) A and D

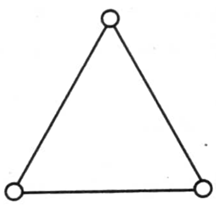
(*c*) B and C (*d*) A and B

2. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is **(2020)**

(*a*) 13 (*b*) 15

(*c*) 20 (*d*) 11

3.



Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is **(2020)**

(*a*) $3RT$ (*b*) $\frac{9}{2}RT$

(*c*) $\frac{3}{2}RT$ (*d*) $\frac{5}{2}RT$

4. Match the C_p/C_v ratio for ideal gases with different type of molecules **(2020)**

Molecule Type	C_p/C_v
A. Monatomic	(I) 7/5
B. Diatomic rigid molecules	(II) 9/7
C. Diatomic non-rigid molecules	(III) 4/3
D. Triatomic rigid molecules	(IV) 5/3

(*a*) (A)–(II), (B)–(III), (C)–(I), (D)–(IV)

(*b*) (A)–(IV), (B)–(II), (C)–(I), (D)–(III)

(*c*) (A)–(IV), (B)–(I), (C)–(II), (D)–(III)

(*d*) (A)–(III), (B)–(IV), (C)–(II), (D)–(I)

5. Nitrogen gas is at 300°C temperature. The temperature (in *K*) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is ——. **(2020)** (Molar mass of N_2 gas 28 g).

6. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, $g = 980$ cm/s^2, density of mercury = 13.6 g/cm^3) **(2020)**

(*a*) 5.8×10^{18} (*b*) 4.0×10^{16}

(*c*) 5.8×10^{16} (*d*) 4.0×10^{18}

7. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T as **(2020)**

(a) $\sqrt{T}$ (b) T

(c) $\frac{1}{\sqrt{T}}$ (d) $\frac{1}{T}$

8. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(\frac{C_p}{C_v}\right)$ are given, respectively, by **(2020)**

(a) $U = 5RT$ and $\gamma = \frac{7}{5}$ (b) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

(c) $U = 5RT$ and $\gamma = \frac{6}{5}$ (d) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

9. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.40$, the work done by air is: $(3^{1.4} = 4.6555)$ [Take air to be an ideal gas] **(2020)**

(a) 90.5 J (b) 48 J (c) 60.7 J (d) 100.8 J

10. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecules changes from τ_1 to τ_2. If for $\frac{C_P}{C_V} = \gamma$ this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by: **(2020)**

(a) $\left(\frac{1}{2}\right)^{\gamma}$ (b) $2\frac{1+\nu}{2}$

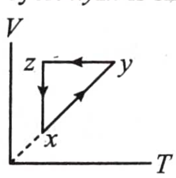
(c) $\frac{1}{2}$ (d) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

11. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work? **(2020)**

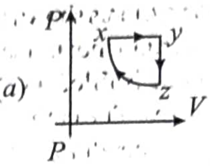
(a) $T = \sqrt{T_1T_2}$ (b) $T = \frac{T_1 + T_2}{2}$

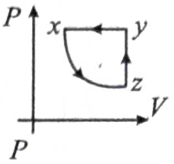
(c) $T = \frac{2T_1T_2}{T_1 + T_2}$ (d) $T = 0$

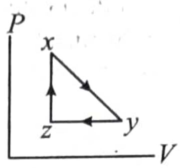
12. A thermodynamic cycle $xyzx$ is shown on a V-T diagram.

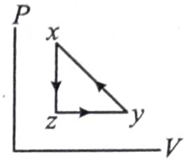


The P-V diagram that best describes this cycle is: (Diagrams as schematic and not to scale) **(2020)**

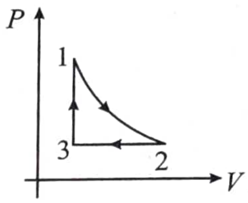
(a)


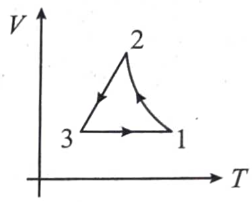
(b)


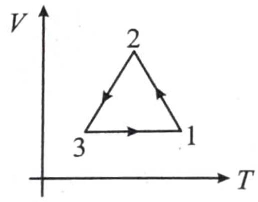
(c)


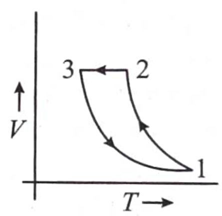
(d)


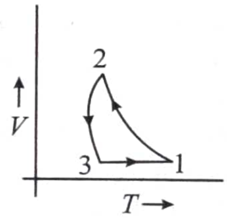
13. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure? Where, $1 \to 2$ is adiabatic. (Graphs are schematic and are not to scale) **(2020)**



(a)


(b)


(c)


(d)


14. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to ________. **(2020)**

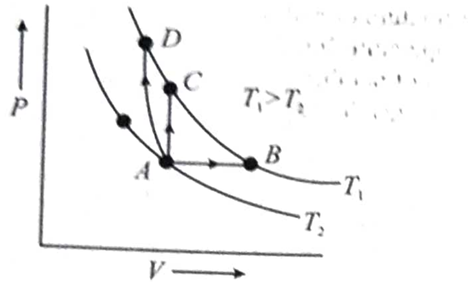
15. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is **(2020)**

(a) 128 (b) 32

(c) $\frac{1}{32}$ (d) 326

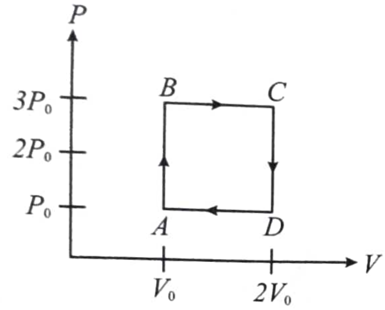
16. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as $A \to B$, $A \to C$ and $A \to D$. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the work done as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are

(2020)



(a) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(b) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
(c) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$
(d) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$

17. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to ________. **(2020)**



18. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of $2N$. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of $a : b$. Then a/b can be expressed as $1/x$ where x is **(2021)**

19. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be °C. (Round off to the Nearest Integer) **(2021)**

20. Let η_1 is the efficiency of an engine at T_1 = 447°C and T_2 = 147°C while η_2 is the efficiency at T_1 = 447°C and $T_2 = 47°C$. The ratio $\frac{\eta_1}{\eta_2}$ will be: **(2022)**

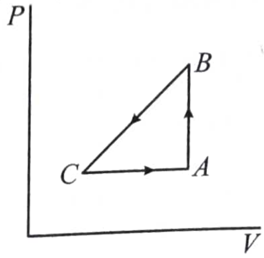
(a) 0.41 (b) 0.56
(c) 0.73 (d) 0.70

21. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats 1.4. Vessel is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surrounding and vessel temperature of the gas increases by : (R = universal gas constant) **(2022)**

(a) $\frac{Mv^2}{7R}$ (b) $\frac{Mv^2}{5R}$
(c) $2\frac{M^2}{7R}$ (d) $7\frac{Mv^2}{5R}$

22. The efficiency of a Carnot's engine, working between steam point and ice point, will be : **(2022)**
(a) 26.81% (b) 37.81% (c) 47.81% (d) 57.81%

23. A sample of an ideal gas is taken through the cyclic process $ABCA$ as shown in figure. It absorbs, 40J of heat during the part AB, no heat during BC and rejects 60J of heat during CA. A work 50J is done on the gas during the part BC. The internal energy of the gas at A is 1560J. The work done by the gas during the part CA is: **(2022)**



(a) 20J (b) 30J (c) –30J (d) –60J

24. A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be: **(2022)**
(a) P (b) $8P$ (c) $32P$ (d) $64P$

25. Read the following statements : **(2022)**
(A) When small temperature difference between a liquid and its surrounding is doubled the rate of loss of heat of the liquid becomes twice.
(B) Two bodies P and Q having equal surface areas are maintained at temperature 10°C and 20°C. The thermal radiation emitted in a given time by P and Q are in the ratio 1:1.15
(C) A carnot Engine working between 100K and 400K has an efficiency of 75%
(D) When small temperature difference between a liquid and its surrounding is quadrupled, the rate of loss of heat of the liquid becomes twice.

Choose the correct answer from the options given below :
(a) A, B, C only (b) A, B only
(c) A, C only (d) B, C, D only

26. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal. W_2. if the process is purely adiabatic and W_3 if the process is purely isobaric. Then, choose the coned option **(2022)**
(a) $W_1 < W_2 < W_3$ (b) $W_2 < W_3 < W_1$
(c) $W_3 < W_1 < W_2$ (d) $W_2 < W_1 < W_3$

27. The ratio of specific heats $\left(\frac{C_P}{C_V}\right)$ in terms of degree of freedom (f) is given by: **(2022)**

(a) $\left(1+\frac{f}{3}\right)$ (b) $\left(1+\frac{2}{f}\right)$
(c) $\left(1+\frac{f}{2}\right)$ (d) $\left(1+\frac{1}{f}\right)$

28. The pressure P_1 and density d_1 of diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes suddenly to P_2 ($> P_1$) and d_2 respectively during an adiabatic process. The temperature of the gas increases and becomes times of its initial temperature.

(given $\frac{d_2}{d_1} = 32$) **(2022)**

29. One mole of a monoatomic gas is mixed with three moles of a diatomic gas. The molecular specific heat of mixture at constant volume is $\frac{\alpha^2}{4} RJ/molK$; then the value of α will be (Assume that the given diatomic gas has no vibrational mode.) **(2022)**

30. In a carnot engine, the temperature of reservoir is 527°C and that of sink is 200 K. If the work done by the engine when it transfers heat from reservoir to sink is 12000 kJ, the quantity of heat absorbed by the engine from reservoir is $\times 10^6$ J **(2022)**

31. A diatomic gas ($\gamma = 1.4$) does 400 J of work when it is expanded isobarically. The heat given to the gas in the process is *J*. **(2022)**

32. What will be the effect on the root mean square velocity of oxygen molecules if the temperature is doubled and oxygen molecule dissociates into atomic oxygen? **(2022)**
(*a*) The velocity of atomic oxygen remains same
(*b*) The velocity of atomic oxygen doubles
(*c*) The velocity of atomic oxygen becomes half
(*d*) The velocity of atomic oxygen becomes four times

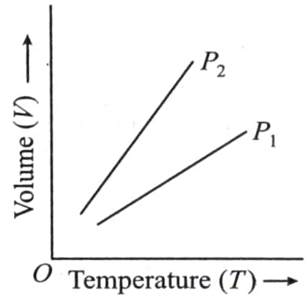
33. 7 mole of certain monoatomic ideal gas undergoes a temperature increase of 40 K at constant pressure. The increase in the internal energy of the gas in this process is (Given $R = 8.3 JK^{-1} mol^{-1}$) **(2022)**
(*a*) 5810 J (*b*) 3486 J
(*c*) 11620 J (*d*) 6972 J

34. Same gas is filled in two vessels of the same volume at the same temperature. If the ratio of the number of molecules is 1:4, then **(2022)**
(*a*) The r.m.s. velocity of gas molecules in two vessels will be the same.
(*b*) The ratio of pressure in these vessels will be 1:4
(*c*) The ratio of pressure will be 1:1
(*d*) The r.m.s. velocity of gas molecules in two vessels will be in the ratio of 1:4
(*a*) *a* and *c* only (*b*) *b* and *d* only
(*c*) *a* and *b* only (*d*) *c* and *d* only

35. A vessel contains 16 g of hydrogen and 128 g of oxygen at standard temperature and pressure. The volume of the vessel in cm^3 is : **(2022)**
(*a*) 72×10^5 (*b*) 32×10^5
(*c*) 27×10^4 (*d*) 54×10^4

36. A mixture of hydrogen and oxygen has volume 2000 cm^3, temperature 300 K, pressure 100 KPa and mass 0.76 g The ratio of number of moles of hydrogen to number of moles of oxygen in the mixture will be : **(2022)**
(*a*) $\frac{1}{3}$ (*b*) $\frac{3}{1}$ (*c*) $\frac{1}{16}$ (*d*) $\frac{16}{1}$

37. For a perfect gas, two pressures P_1 and P_2 are shown in figure. The graph shows: **(2022)**



(*a*) $P_1 > P_2$
(*b*) $P_1 < P_2$
(*c*) $P_1 = P_2$
(*d*) Insufficient data to draw any conclusion

38. According to kinetic theory of gases, **(2022)**
(*a*) The motion of the gas molecules freezes at 0°C
(*b*) The mean free path of gas molecules decreases if the density of molecules is increased.
(*c*) The mean free path of gas molecules increases if temperature is increased keeping pressure constant.
(*d*) Average kinetic energy per molecule per degree of freedom is $\frac{3}{2}k_B T$ (for monoatomic gases)

Choose the most appropriate answer from the options given below:
(*a*) *a* and *c* only (*b*) *b* and *c* only
(*c*) *a* and *b* only (*d*) *c* and *d* only

39. The relation between root mean square speed (V_{rms}) and most probable speed (V_p) for the molar mass M of oxygen gas molecule at the temperature of 300*K* will be :- **(2022)**
(*a*) $v_{rms} = \sqrt{\frac{2}{3}} v_p$ (*b*) $v_{rms} = \sqrt{\frac{3}{2}} v_p$
(*c*) $V_{rs} = V_p$ (*d*) $v_{rms} = \sqrt{\frac{1}{3}} v_p$

40. When a gas filled in a closed vessel is heated by raising the temperature by 1°C, its pressure increase by 0.4%. The initial temperature of the gas is K. **(2022)**

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41. A mixture of 2 moles of helium gas (atomic mass = 4 amu), and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 *K* in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is: **(2012)**
(*a*) 0.32 (*b*) 0.45 (*c*) 2.24 (*d*) 3.16

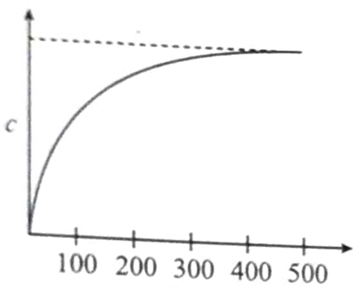
42. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take $R = 8.31$ J/mol.K) **(2012)**

(a) 62 J (b) 104 J
(c) 124 J (d) 208 J

43. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is: **(2013)**

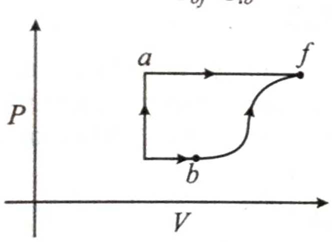
(a) 1 : 4 (b) 1 : 2
(c) 6 : 9 (d) 8 : 9

44. The figure below shows the variation of specific heat capacity (c) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. **(2013)**



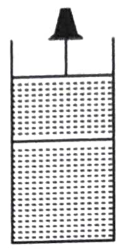
(a) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.
(b) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.
(c) there is no change in the rate of heat absorption in the range 400–500 K.
(d) the rate of heat absorption increases in the range 200–300 K.

45. A thermodynamic system is taken form an initial state i with internal energy $U_i = 100\ J$ to the final state f along two different paths i_{af} and i_{bf}, as schematically shown in the fire. The work done by the system along the paths af, ib and bf are $W_{af} = 200\ J$, $W_{ib} = 50\ J$ and $W_{bf} = 100\ J$ respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{bf} and Q_{ib} respectively. If the internal energy of the sytem in the state b is $U_b = 200\ J$ and $Q_{iaf} = 500\ J$, the ratio Q_{bf}/Q_{ib} is: **(2014)**



Paragraph For Questions 46 to 47

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monoatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monoatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$. **(2014)**



46. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(a) 550 K (b) 525 K
(c) 513 K (d) 490 K

47. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be:

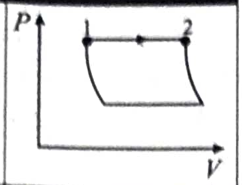
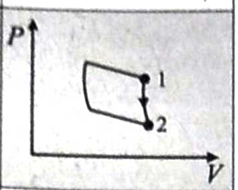
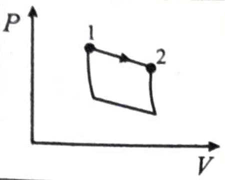
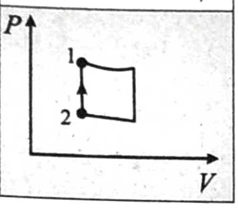
(a) 250 R (b) 200 R
(c) 100 R (d) –100 R

48. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5\ Pa$ and volume $V_i = 10^{-3}\ m^3$ changes to a final state at $P_f = (1/32) \times 10^5\ Pa$ and $V_f = 8 \times 10^{-3}\ m^3$ in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately. **(2016)**

(a) 112 J (b) 294 J
(c) 588 J (d) 813 J

49, 50 and 51 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding $P - V$ diagrams in column 3 of the table. Consider only path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic process. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column-1		Column-2		Column-3	
(I)	$W_{1\to 2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$	(i)	Isothermal	(P)	
(II)	$W_{1\to 2} = -PV_2 + PV_1$	(ii)	Isochoric	(Q)	
(III)	$W_{1\to 2} = 0$	(iii)	Isobaric	(R)	
(IV)	$W_{1\to 2} = -nRT \ln\left(\frac{V_2}{V_1}\right)$	(iv)	Adiabatic	(S)	

49. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$? **(2017)**

(*a*) (II) (iii) (P) (*b*) (II) (iii) (S)

(*c*) (III) (iii) (P) (*d*) (II) (iv) (R)

50. Which one of the following options is the correct combination? **(2017)**

(*a*) (II) (iv) (P) (*b*) (IV) (ii) (S)

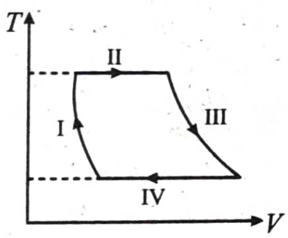
(*c*) (II) (iv) (R) (*d*) (III) (ii) (S)

51. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas? **(2017)**

(*a*) (III) (iv) (R) (*b*) (I) (ii) (Q)

(*c*) (IV) (ii) (R) (*d*) (I) (iv) (Q)

52. One mole of a monoatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statement below is (are) true? **(2018)**



(*a*) Process I is an isochoric process.

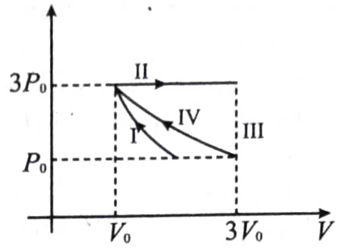
(*b*) In process II, gas absorbs heat.

(*c*) In process IV, gas releases heat.

(*d*) Process I and II are not isobaric

53. One mole of a monoatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0\ J\ mol^{-1}\ K^{-1}$, the decrease in its internal energy, in Joule, is ...... **(2018)**

54. One mole of a monoatomic ideal agas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II **(2018)**



List-I	List-II
P. In process I	1. Work done by the gas is zero
Q. In process II	2. Temperature of the gas remains unchanged
R. In process III	3. No heat is exchanged between the gas and its surroundings
S. In process IV	4. Work done by the gas is $6P_0V_0$

(*a*) $P \to 4; Q \to 3; R \to 1; S \to 2$

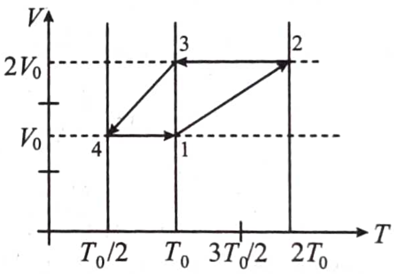
(*b*) $P \to 1; Q \to 3; R \to 2; S \to 4$

(*c*) $P \to 3; Q \to 4; R \to 1; S \to 2$

(*d*) $P \to 3; Q \to 4; R \to 2; S \to 1$

55. One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are: **(2019)**

[R is the gas constant]



(*a*) Work done in this thermodynamic cycle ($1 \to 2 \to 3 \to 4 \to 1$) is $|W| = \frac{1}{2}RT_0$

(*b*) The ratio of heat transfer during processes $1 \to 2$ and $2 \to 3$ is $\left|\frac{Q_{1\to 2}}{Q_{2\to 3}}\right| = \frac{5}{3}$

(*c*) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(*d*) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to 2}}{Q_{3\to 4}}\right| = \frac{1}{2}$

56. A mixture of ideal gas containing 5 moles of monoatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are: **(2019)**

(Give $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

(*a*) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

(*b*) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(*c*) The work $|W|$ done during the process is $13RT_0$

(*d*) Adiabatic constant of the gas mixture is 1.6

(Direction: 57 to 58) Answer the following by appropriately matching the lists based on the information given in the paragraph.

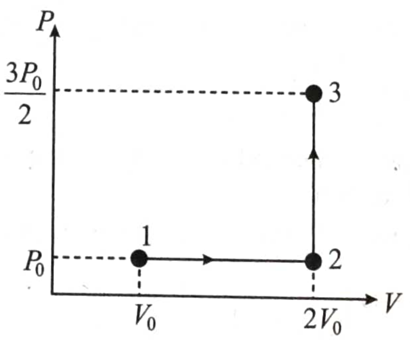
In a thermodynamics process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monoatomic ideal gas $X = \frac{3}{2}R \text{ in}\left(\frac{T}{T_A}\right) + R \text{ In}\left(\frac{V}{V_A}\right)$.

Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved and List-II gives some possible values of these quantities.

List-I	List-II
(I) Work done by the system in process $1 \rightarrow 2 \rightarrow 3$	(P) $\frac{1}{3}RT_0 \text{ In } 2$
(II) Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \rightarrow 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3 + In2)$
	(U) $\frac{5}{6}RT_0$

57. If the process carried out on one mole of monoatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is, **(2019)**



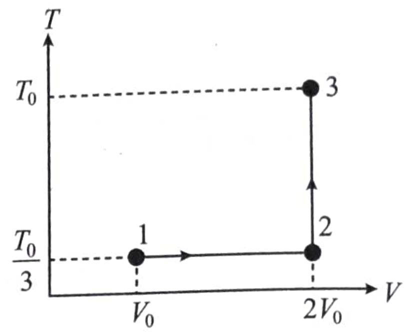
(*a*) I → Q, II → R, III → P, IV → U

(*b*) I → S, II → R, III → Q, IV → T

(*c*) I → Q, II → R, III → S, IV → U

(*d*) I → Q, II → S, III → R, IV → U

58. If the process on one mole of monoatomic ideal gas is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is **(2019)**



(*a*) I → S, II → T, III → Q, IV → U

(*b*) I → P, II → R, III → T, IV → S

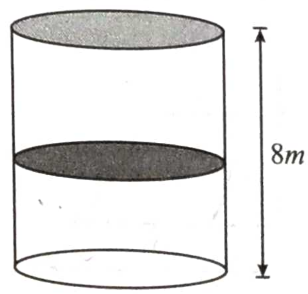
(*c*) I → P, II → T, III → Q, IV → T

(*d*) I → P, II → R, III → T, IV → P

59. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ ln\ 2$, then f is ...... **(2020)**

60. A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus the partition is held initially at a distance of 4 m from the top, as shown in the schematic figure below. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now released and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be _______

(Take the acceleration due to gravity $=10\ ms^{-2}$ and the universal gas constant $= 8.3\ J\ mol^{-1}\ K^{-1}$). **(2020)**

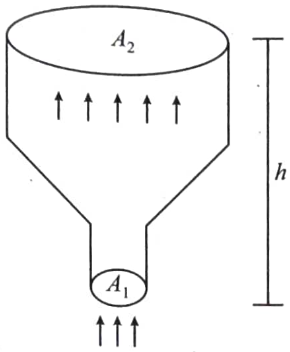


61. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes $(R-a)$. For

$a << R$ the magnitude of the work done in the process is given by $(4\pi p_0 R a^2)X$, where X is a constant and $\gamma = C_p/C_V = 41/30$. The value of X is **(2020)**

62. A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time $T = 0$, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and at 50 K. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2/t_1) is **(2021)**

63. An ideal gas of density $\rho = 0.2$ kg m^{-3} enters a chimney of height h at the rate of $\alpha = 0.8$ kg s^{-3} from its lower end, and escapes through the upper end as shown in the figure. The cross-sectional area of the lower end is $A_1 = 0.1$ m^{-2} and $A_2 = 0.4$ m^{-2} the upper end is . The pressure and the temperature of the gas at the lower end are 600 Pa and 300K respectively, while its temperature at the upper end is 150K. The chimney is heat insulated so that the gas undergoes adiabatic expansion. Take g = 10 ms^{-2} and the ratio of specific heats of the gas $\gamma = 2$. Ignore atmospheric pressure.



Which of the following statement(s) is (are) correct?

(*a*) The pressure of the gas at the upper end of the chimney is 300Pa.

(*b*) The velocity of the gas at the lower end of the chimney is 40 ms^{-1} and at the upper end is 20 ms^{-1}.

(*c*) The height of the chimney is 590m.

(*d*) The density of the gas at the upper end is 0.05 kg m^{-3}.

64. List I describes thermodynamic processes in four different systems. List II gives the magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process.

	List-I		List-II
A.	10^{-3} kg of water at 100°C is converted to steam at the same temperature, at a pressure of 10^5 Pa. The volume of the system changes from 10^{-6} m^3 to 10^{-3} m^3 in the process. Latent heat of water = 2250 kJ/kg.	p.	2 kJ
B.	0.2 moles of a rigid diatomic ideal gas with volume V at temperature 500 K undergoes an isobaric expansion to volume 3 V. Assume $R = 8.0$Jmol^{-1} K^{-1}.	q.	7 kJ
C.	On mole of a monoatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3}$m^3 and pressure 2kPa to volume $\frac{V}{8}$.	r.	4 kJ
D.	Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 kJ of heat and undergoes isobaric expansion.	s.	5 kJ
		t.	

(*a*) A-(t); B-(r); C-(s); D-(q) (*b*) A-(p); B-(r); C-(t); D-(q)

(*c*) A-(s); B-(p); C-(t); D-(p) (*d*) A-(q); B-(r); C-(s); D-(t)

65. A bubble has surface tension S. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is P_{a1}, the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1. When the atmospheric pressure is P_{a2}, the radius of the bubble and the temperature of the enclosed gas are r_2 and T_2, respectively.

Which of the following statement(s) is(are) correct?

(*a*) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{r_1}{r_2}\right)^5 = \dfrac{P_{a2} + \frac{2S}{r_2}}{P_{a1} + \frac{2S}{r_1}}$

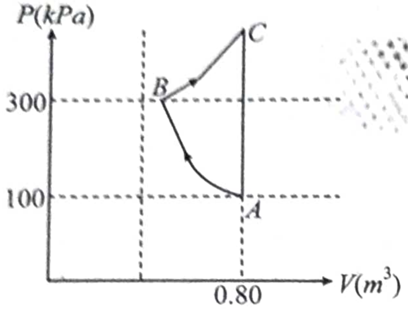
(*b*) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

(*c*) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\frac{r_1}{r_2}\right)^3 = \dfrac{P_{a2} + \frac{4S}{r_2}}{P_{a1} + \frac{4S}{r_1}}$.

(*d*) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \dfrac{P_{a_2} + \frac{4S}{r_2}}{P_{a_1} + \frac{4S}{r_1}}$.

66. In the given $P - V$ diagram, a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is first compressed adiabatically from state A to state B. Then it expands isothermally from state B to state C.

[Given: $\left(\frac{1}{3}\right)^{0.6}$ 0.5, in 2 = 0.7].



Which of the following statement(s) is(are) correct?

(a) The magnitude of the total work done in the process $A \to B \to C$ is 144 kJ.

(b) The magnitude of the work done in the process $B \to C$ is 84 kJ.

(c) The magnitude of the work done in the process $A \to B$ is 60 kJ.

(d) The magnitude of the work done in the process $C \to A$ is zero.

ANSWER KEY

CONCEPT APPLICATION

1. (d)	2. (b)	3. (a)	4. (c)	5. (d)	6. (a)	7. (a)	8. (b)	9. (b)	10. (d)
11. (c)	12. (c)	13. (c)	14. (c)	15. (a)	16. (d)	17. (c)	18. (c)	19. (a)	20. (d)
21. (a)	22. (b)	23. (c)	24. (c)	25. (a)	26. (a)	27. (b)	28. (a)	29. (d)	30. (b)
31. (d)	32. (b)	33. (b)	34. (b)	35. (a)	36. (a)	37. (a)	38. (a)	39. (c)	40. (c)
41. (b)	42. (b)	43. (d)	44. (c)	45. (b)	46. (c)	47. (a)			

EXERCISE-1 (TOPICWISE)

1. (a)	2. (b)	3. (c)	4. (b)	5. (b)	6. (a)	7. (c)	8. (b)	9. (c)	10. (b)
11. (a)	12. (a)	13. (c)	14. (a)	15. (c)	16. (a)	17. (d)	18. (c)	19. (a)	20. (a)
21. (c)	22. (c)	23. (d)	24. (d)	25. (d)	26. (b)	27. (b)	28. (a)	29. (d)	30. (a)
31. (c)	32. (a)	33. (b)	34. (b)	35. (b)	36. (d)	37. (d)	38. (b)	39. (c)	40. (b)
41. (c)	42. (a)	43. (c)	44. (c)	45. (b)	46. (b)	47. (b)	48. (a)	49. (a)	50. (a)

EXERCISE-2 (LEARNING PLUS)

1. (c)	2. (a)	3. (a)	4. (c)	5. (c)	6. (a)	7. (b)	8. (d)	9. (d)	10. (a)
11. (c)	12. (c)	13. (b)	14. (a)	15. (c)	16. (c)	17. (a)	18. (d)	19. (a)	20. (a)
21. (a)	22. (a)	23. (c)	24. (a)	25. (d)	26. (a)	27. (c)	28. (d)	29. (c)	30. (d)
31. (a)	32. (a)	33. (b)	34. (c)	35. (d)	36. (b)	37. (b)	38. (d)	39. (c)	40. (a)
41. (b)	42. (b)	43. (a)							

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (c,d)	2. (a,b,c)	3. (b,d)	4. (a,d)	5. (a,b,c,d)	6. (a,b)	7. (a,d)	8. (c,d)	9. (b,d)	10. (b,d)
11. (a,c)	12. (a,b)	13. (a,b)	14. (a,b,c,d)	15. (b,c)	16. (a,b,d)	17. (a,b)	18. (a,b,c,d)	19. (b,c)	20. (a,d)
21. (a,c,d)	22. (c)	23. (d)	24. (b)	25. (c)	26. (b)	27. (c)	28. (b)	29. (a)	30. (d)
31. [0075]	32. [40]	33. [0020]	34. [76]	35. [0075]	36. [0125]	37. [0005]	38. [0100]	39. [0000]	40. [217]
41. [0126]									

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (c)	2. (b)	3. (a)	4. (c)	5. [40.93]	6. (d)	7. (c)	8. (d)	9. (a)	
10. (b)	11. (b)	2. (a)	13. (a)	14. [266.67]	15. (a)	16. (None of the option)		17. [19]	18. [32]
19. [113]	20. (a)	21. (b)	22. (a)	23. (b)	24. (c)	25. (a)	26. (d)	27. (b)	28. [4]
29. [3]	30. [16]	31. [1400]	32. (b)	33. (b)	34. (c)	35. (c)	36. (b)	37. (a)	38. (b)
39. (b)	40. [250]								

JEE Advanced

41. (d)	42. (d)	43. (d)	44. (a,b,c,d)	45. [2]	46. (d)	47. (d)	48. (d)	49. (a)	50. (d)
51. (d)	52. (b,c,d)	53. [900]	54. (c)	55. (a,b)	56. (a,c,d)	57. (c)	58. (d)	59. [1.77 to 1.78]	
60. [6]	61. [2.05]	62. [9]	63. (b)	64. (c)	65. (c,d)	66. (b,c,d)			

CHAPTER

17 Simple Harmonic Motion

PERIODIC MOTION

Any motion that repeats itself in equal intervals of time is called **periodic motion**. For example, rotation of the earth around the sun, oscillation of pendulum in vacuum, etc. are periodic motions.

If a particle in periodic motion moves back and forth (or to and fro) over the same path, then its motion is called **oscillatory**. The examples of oscillatory motion are:

- The motion of a pendulum
- The motion of a spring fixed at one end, which is stretched or compressed and then released
- The motion of a violin string
- The motion of atoms in molecules or in a solid lattice
- The motion of air molecules as a sound wave passes by

SIMPLE HARMONIC MOTION

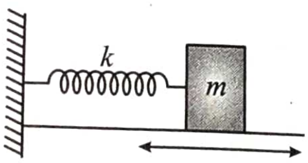
If the restoring force/ torque acting on the body in oscillatory motion is directly proportional to the displacement of body / particle and is always directed towards equilibrium position then the motion is called Simple Harmonic Motion (SHM). It is the simplest (easy to analyze) form of oscillatory motion.

Restoring force:

When a system in SHM is displaced from its equilibrium position, then a force acts on it in such a way so as to restore the system back to its equilibrium position. This force is called restoring force. It is directed opposite to the displacement from the mean position.

Types of SHM

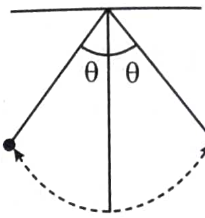
(i) **Linear SHM:**



When a particle moves to and fro (or up and down) about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.

Example: Motion of a block connected to a spring.

(ii) **Angular SHM:**



When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion

Example: Motion of a bob of a simple pendulum.

Necessary Condition to execute SHM

- In linear SHM, the restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle from mean position (x) and directed towards the equilibrium condition

  $\therefore\ F \propto -x \Rightarrow a \propto -x$

  Negative sign shows that direction of force & acceleration is towards equilibrium position.
- In angular SHM, the restoring torque (or acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium condition.

  $\therefore\ \tau \propto -\theta \Rightarrow \alpha \propto -\theta.$

Comparison of Linear and Angular SHM

S. No.	Linear SHM	Angular SHM
(i)	$F \propto -x = -kx$ where k is restoring force constant	$\tau \propto -\theta = -C\theta$ where C is restoring torque constant.
(ii)	$a = -\dfrac{k}{m}x$	$\alpha = -\dfrac{C}{I}\theta$
(iii)	$\dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$	$\dfrac{d^2\theta}{dt^2} + \dfrac{C}{I}\theta = 0$
(iv)	$a = -\omega^2 x$	$\alpha = -\omega^2\theta$
(v)	$\omega^2 = \dfrac{k}{m},\ \omega = \sqrt{\dfrac{k}{m}} = \dfrac{2\pi}{T}$	$\omega = \sqrt{\dfrac{C}{I}} = 2\pi f = \dfrac{2\pi}{T}$

EQUATION OF SHM

The necessary and sufficient condition for SHM is

$F = -kx$

where k = positive constant for a SHM = force constant, and x = displacement from mean position.

or $m\dfrac{d^2x}{dt^2} = -kx$

$\Rightarrow \dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$

$\Rightarrow \dfrac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \left(\omega = \sqrt{\dfrac{k}{m}}\right)$

The solution of above equation is $x = A\sin(\omega t + \phi)$. It can be proved using a little mathematics as below.

We have, $a = -\omega^2 x = v\dfrac{dv}{dx}$

Therefore, $\int_0^v v\,dv = -\int_A^x \omega^2 x\,dx \Rightarrow \dfrac{v^2}{2} = -\dfrac{\omega^2}{2}[x^2 - A^2]$

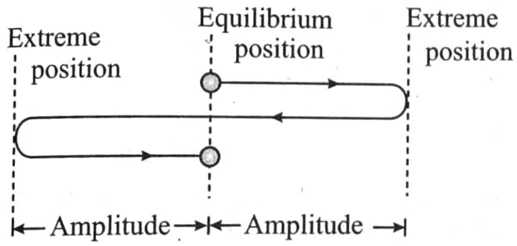
Thus $v = \pm\,\omega\sqrt{A^2 - x^2}$

$\Rightarrow \dfrac{dx}{dt} = \pm\,\omega\sqrt{A^2 - x^2}$

Further integrating leads to $x = A\sin(\omega t + \phi)$ where A and ϕ are constants determined from initial conditions.

PHYSICAL QUANTITIES IN SHM

In the figure shown, path of the particle is on a straight line.



1. **Displacement:** It is defined as the distance of the particle from the mean position at that instant. Displacement in SHM at time t is given by $x = A\sin(\omega t + \phi)$
2. **Amplitude (A):** It is the maximum value of displacement of the particle from its equilibrium position.
   Amplitude = 1/2 (distance between extreme points/position)
   It depends on energy of the system.
3. **Time period (T):** Smallest time interval after which the oscillatory motion gets repeated is called time period.
4. **Angular Frequency (ω):** $\omega = \dfrac{2\pi}{T}$ and its units is rad/sec.
5. **Frequency (f):** Number of oscillations completed in unit time interval is called frequency of oscillations; $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$. Its units is sec^{-1} or Hz.
6. **Phase:** The physical quantity which represents the state of motion of particle (e.g. its position and direction of motion at any instant). Argument of sine function is called phase.
7. **Phase constant (ϕ):** Constant ϕ in equation of SHM is called phase constant or initial phase. It depends on initial position and direction of velocity.
8. **Velocity (v):**
   (i) It is defined as the rate of change of the displacement of the particle with respect to time at the given instant.
   (ii) Velocity in SHM is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t) = A\omega\cos\omega t$$

$$v = \pm A\omega\sqrt{1 - \sin^2\omega t}$$

$$\Rightarrow v = \pm A\omega\sqrt{\left(1 - \frac{x^2}{A^2}\right)} \qquad [\because x = A\sin\omega t]$$

$$v = \pm\omega\sqrt{(A^2 - x^2)}$$

Squaring both the sides

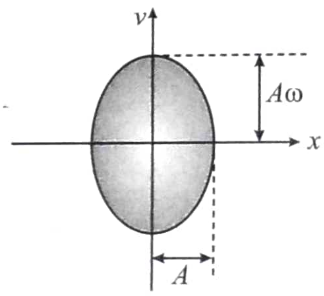
$$v^2 = \omega^2(A^2 - x^2)$$

$$\Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2$$

$$\Rightarrow \frac{v^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2} \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

This is equation of ellipse. So, curve between displacement and velocity of particle executing SHM is ellipse.

   (iii) The graph between velocity and displacement is shown in figure.



If particle oscillates with unit angular frequency ($\omega = 1$) then curve between v and x would be circular.

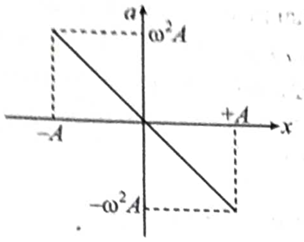
Note:

- The direction of velocity of a particle in SHM is either towards or away from the mean position
- At mean position ($x = 0$), velocity is maximum ($= A\omega$) and at extreme position ($x = \pm A$), the velocity of particle executing SHM is zero.

9. **Acceleration:**
   (i) It is defined as the rate of change of the velocity of the particle with respect to time at given instant.
   (ii) Acceleration in SHM is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t) = -\omega^2 A\sin\omega t = -\omega^2 x$$

(iii) The graph between acceleration and displacement.



Note:

- The acceleration of a particle executing SHM is always directed towards the mean position.
- The acceleration of the particle executing SHM is maximum at extreme position ($=\omega^2A$) and minimum at mean position (= zero)

Train Your Brain

Example 1: For a particle performing SHM, equation of motion is given as $\frac{d^2x}{dt^2} + 4x = 0$. Find the time period.

Sol. $\frac{d^2x}{dt^2} = -4x \Rightarrow \omega^2 = 4 \Rightarrow \omega = 2$

Time period: $T = \frac{2\pi}{\omega} = \pi$

Example 2: If amplitude of harmonic oscillator is a, when velocity of particle is half of the maximum velocity, then find the position of particle.

Sol. $v^2 = \omega^2(a^2 - x^2)$

But $v = \frac{v_{max}}{2} = \frac{\omega a}{2}$

$\therefore \frac{\omega^2 a^2}{4} = \omega^2(a^2 - x^2) \Rightarrow a^2 = 4(a^2 - x^2)$

$x^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4} \Rightarrow x = \pm\frac{\sqrt{3}}{2}a$

Example 3: If equation of displacement of a particle is $y = A\sin\omega t + B\cos\omega t$, then motion of particle is

(a) Simple harmonic (b) Linear
(c) Uniform circular (d) Uniform elliptical

Sol. (a) $y = A\sin\omega t + B\cos\omega t$

Differentiate w.r.t. to t,

$\frac{dy}{dt} = \omega A\cos\omega t - \omega B\sin\omega t$

Again differentiate w.r.t. to t,

$\frac{d^2y}{dt^2} = -\omega^2A\sin\omega t - \omega^2B\cos\omega t$

$\frac{d^2y}{dt^2} = -\omega^2[A\sin\omega t + B\cos\omega t]$

$\Rightarrow \frac{d^2y}{dt^2} + \omega^2 y = 0$, this is equation for S.H.M

Example 4: The maximum velocity of a harmonic oscillator is α and its maximum acceleration is β. Find its time period.

Sol. $v_{max} = \omega A = \alpha$ ...(i)

$a_{max} = \omega^2 A = \beta$ ...(ii)

Dividing equation (ii) by equation (i),

$\frac{\beta}{\alpha} = \omega$

$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi\alpha}{\beta}$

Concept Application

1. The time period of a particle in simple harmonic motion is equal to the time between consecutive appearances of the particle at a particular point in its motion. This point is
   (a) The mean position
   (b) An extreme position
   (c) Between the mean position and the positive extreme
   (d) Between the mean position and the negative extreme.
2. A particle executing simple harmonic motion along y-axis has its motion described by the equation $y = A\sin(\omega t) + B$. The amplitude of the simple harmonic motion is
   (a) A (b) B
   (c) $A + B$ (d) $\sqrt{A+B}$
3. The displacement of a particle in simple harmonic motion in one time period is
   (a) A (b) $2A$
   (c) $4A$ (d) zero
4. The distance moved by a particle in simple harmonic motion in one time period is
   (a) A (b) $2A$
   (c) $4A$ (d) Zero

GRAPHICAL REPRESENTATION OF DISPLACEMENT, VELOCITY AND ACCELERATION IN SHM

Displacement, $x = A\sin\omega t$

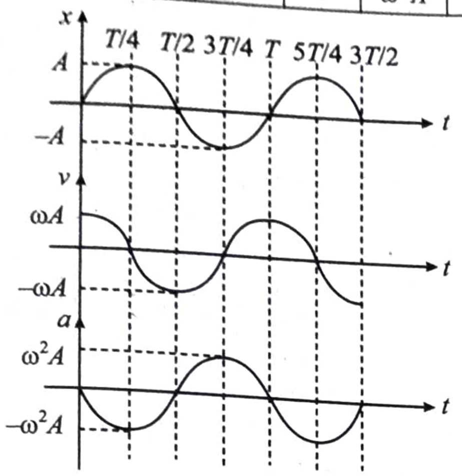
Velocity, $v = A\omega\cos\omega t = A\omega\sin(\omega t + \frac{\pi}{2})$

Acceleration, $a = -\omega^2A\sin\omega t = \omega^2A\sin(\omega t + \pi)$

Note: $v = \omega\sqrt{A^2 - x^2} \Rightarrow a = -\omega^2 x$

These relations are true for any equation of x.

Time, t	0	$T/4$	$T/2$	$3T/4$	T
Displacement, x	0	A	0	$-A$	0
Velocity, v	$A\omega$	0	$-A\omega$	0	$A\omega$
Acceleration, a	0	$-\omega^2 A$	0	$\omega^2 A$	0



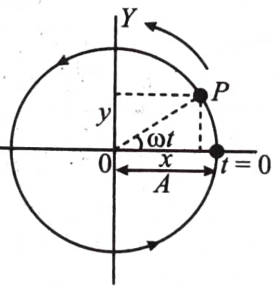
Note:

- All the three quantities displacement, velocity and acceleration vary harmonically with time, having same period.
- The velocity amplitude is ω times the displacement amplitude ($v_{max} = \omega A$).
- The acceleration amplitude is ω^2 times the displacement amplitude ($a_{max} = \omega^2 A$).
- In SHM, the velocity is ahead of displacement by a phase angle of $\frac{\pi}{2}$.
- In SHM, the acceleration is ahead of velocity by a phase angle of $\frac{\pi}{2}$.

GEOMETRICAL INTERPRETATION OF SHM

If a particle is moving with uniform speed along the circumference of a circle, then the straight-line motion of the foot of the perpendicular drawn from the particle on the diameter of the circle is called 'SHM'

Description of SHM Based on Circular Motion



(i) Draw a circle, having radius equal to amplitude (A) of SHM

(ii) Suppose particle is executing circular motion with angular velocity ω on the circumference of the circle.

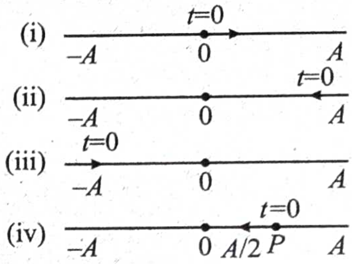
(iii) Shadow or projection of particle performs SHM on vertical or horizontal diameter of circle.

(iv) By joining centre of circle to particle position, angle θ is determined from horizontal or vertical diameter. After time t, radius vector will turn by an angle ωt. Based on these unknown values can be calculated.

(v) As per the position of particle at $t = 0$ in above diagram, equation of SHM along x-axis can be given as $x = A\cos(\omega t + \phi)$ and along y-axis as $y = A\sin(\omega t + \phi)$

Train Your Brain

Example 5: A particle starts from mean position and moves towards positive extreme as shown. Find the equation of the SHM. Amplitude of SHM is A. Also write the equation of *SHM* for the situations (ii) to (iv)



Sol. (i) Let $x = A\sin(\omega t + \phi)$, then $v = \omega A\sin(\omega t + \phi)$

To find ϕ, we use initial conditions.

$x(t = 0) = A\sin(\phi) = 0 \Rightarrow \phi = 0$ or π

$v(t = 0) = \omega A\cos(\phi) > 0 \Rightarrow \cos(\phi) > 0$

Therefore $\phi = 0 \Rightarrow x = A\sin(\omega t)$

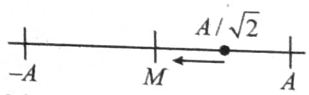
(ii) $x = A\cos\omega t$ (since at $t = 0$, $x = +A$ and $v = -$ve)

(iii) $x = -A\cos\omega t$ (since at $t = 0$, $x = -A$ and $v = +$ve)

(iv) $x = A\sin\left(\omega t + \frac{5\pi}{6}\right)$ ($\because t = 0, x = A/2$ and $v = -$ve)

Example 6: Write equation of SHM of ω angular frequency and A amplitude if the particle is situated at $\frac{A}{\sqrt{2}}$ at $t = 0$ and is going towards mean position.

Sol. At $t = 0$ particle was at $\frac{A}{\sqrt{2}}$ and was going towards mean position as shown in the figure below.



Putting $t = 0$ in equation $x = A\sin(\omega t + \phi)$

We get $\frac{A}{\sqrt{2}} = A\sin\phi$

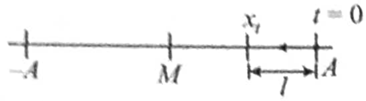
Thus, $\phi = \frac{\pi}{4}, \frac{3\pi}{4}$ but at $t = 0$; $v < 0$ so $\frac{\pi}{4}$ is rejected as

$v = \omega A\cos(\omega t + \phi)$

$\therefore x = A\sin(\omega t + 3\pi/4)$

Example 7: Find distance traveled by a particle of time period T and amplitude A in time $T/12$ starting from rest.

Sol. The situation is as shown



Equation of SHM: At $t = 0, x = A$

$\Rightarrow A = A\sin(\delta) \Rightarrow \delta = \pi/2$

$\therefore x = A\sin(\omega t + \pi/2) = A\cos(\omega t)$

Now, $x_t = A\cos\left(\frac{2\pi}{T}\times\frac{T}{12}\right) = \frac{A\sqrt{3}}{2}$

So, $l = A - \frac{A\sqrt{3}}{2} = \frac{A(2-\sqrt{3})}{2}$

Example 8: Two particles execute SHM in same straight line, Their amplitudes and frequencies are equal. When their displacement is half of amplitude, they pass each other in opposite direction. Find the phase difference.

Sol. Equation of SHM $y = A\sin(\omega t + \phi)$

When displacement is half of amplitude,

$\Rightarrow y = \frac{A}{2}$

$\Rightarrow \frac{A}{2} = A\sin(\omega t + \phi)$

$\Rightarrow \sin(\omega t + \phi) = 1/2$

$\Rightarrow \omega t + \phi = 30^\circ$ or 150°

$\Rightarrow \omega t + \phi_1 = 30^\circ$ and $\omega t + \phi_2 = 150^\circ$

$\Rightarrow \phi_2 - \phi_1 = 120^\circ$

Example 9: The position of a particle moving along x-axis is given by $x = 0.08\sin(12t + 0.3)$ m, where t is in seconds.

(i) What is the amplitude and the period of the motion?

(ii) Determine the position, velocity, and acceleration at $t = 0.6$ s.

Sol. (i) On comparing the given equation with the standard equation of SHM, i.e. $y = A\sin(\omega t + \phi)$, we see that the amplitude is $A = 0.08$ m and the angular frequency $\omega = 12$ rad/s. Thus, the period is $T = 2\pi/\omega = 0.524$ s.

(ii) The velocity and acceleration at any time are given by

$$v = \frac{dx}{dt} = 0.96\cos(12t + 0.3) \text{ m/s}$$

$$a = \frac{dv}{dt} = -11.5\sin(12t + 0.3) \text{ m/s}^2$$

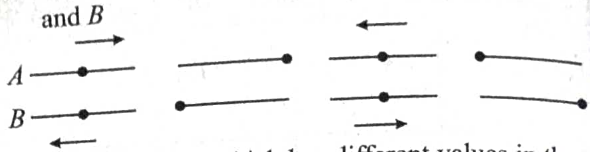
At $t = 0.6$ s, the phase of motion is

$(12 \times 0.6 + 0.3) = 7.5$ rad.

When this is used in the above expressions, we find $x = 0.075$ m, $v = 0.333$ m/s, and $a = -10.8$ m/s^2.

Concept Application

5. Figure represents two simple harmonic motions A and B



The parameter which has different values in the two motions is

(a) Amplitude

(b) Frequency

(c) Phase

(d) Maximum velocity

6. The equation of SHM is $y = a\sin(2\pi ft + \alpha)$, then its phase at time t is

(a) $2\pi ft$ (b) α

(c) $2\pi ft + \alpha$ (d) $2\pi t$

7. Two SHM's are represented by $y = a\sin(\omega t - kx)$ and $y = b\cos(\omega t - kx)$. The phase difference between the two is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

ENERGY IN SHM

- In a spring-mass system, the instantaneous potential energy and kinetic energy are expressed as

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$$

and $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$

- Since $\omega^2 = \frac{k}{m}$, therefore,

$$K = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

- The total mechanical energy is given by

$E = K + U$

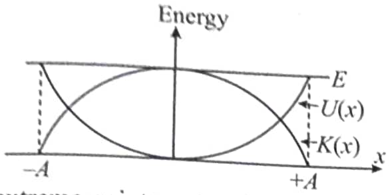
$$E = \frac{1}{2}kA^2\left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right]$$

or $E = \frac{1}{2}kA^2 = \text{constant}$

Thus, the total energy of SHM is constant and proportional to the square of the amplitude.

- The variation of K and U as function of x is shown in figure When $x = \pm A$, the kinetic energy is zero and the total energy is equal to the maximum potential energy.

$$E = U_{max} = \frac{1}{2}kA^2$$

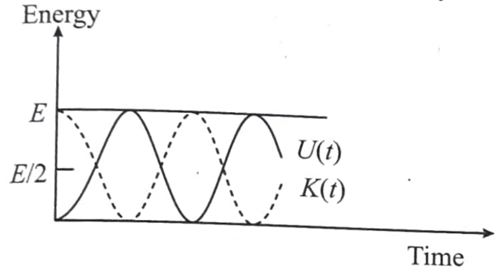


There are extreme points or turning points of the SHM.

At $x = 0$, $U = 0$ and the energy is purely kinetic,

i.e. $E = K_{max} = \frac{1}{2}m(\omega A)^2$

- The variation of K and U as function of t is shown in figure. Note that the frequency of variation of the kinetic and potential energy is twice that of the frequency of oscillation.



- The instantaneous total mechanical energy of the spring-mass system may be written as

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Differentiating it w.r.t. time, we get

$$\frac{dE}{dt} = \frac{1}{2}m\frac{d}{dt}\left(v^2\right) + \frac{1}{2}k\frac{d}{dx}\left(x^2\right) \text{ or } 0 = mv\frac{dv}{dt} + kx\frac{dx}{dt}$$

Since $v = \frac{dx}{dt}$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$, therefore,

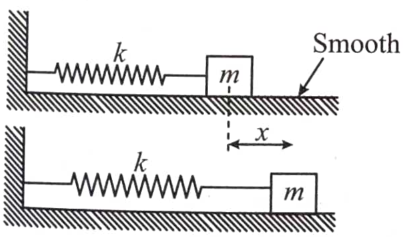
$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

This is the differential equation of SHM.

SPRING MASS SYSTEM

Let us find out the time period of a spring-mass system oscillating on a smooth horizontal surface as shown in the figure.

At the equilibrium position the spring is relaxed. When the block is displaced through x towards right, it experiences a net restoring force $F = -kx$ towards left.



The negative sign shows that the restoring force is always opposite to the displacement. That is, when x is positive, F is negative, the force is directed to the left. When x is negative, F is positive, the force is directed to the right. Thus, the force always tends to restore the block to its equilibrium position $x = 0$.

So $F = -kx$

Applying Newton's Second Law,

$$F = m\frac{d^2x}{dt^2} = -kx$$

$$\text{or } \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Comparing the above equation with standard equation, we get

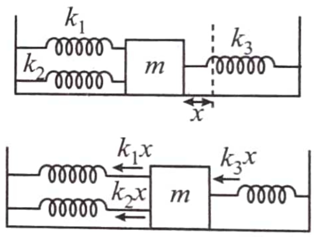
$$\omega^2 = \frac{k}{m} \text{ or } T = 2\pi\sqrt{\frac{m}{k}}$$

Note that the time period is independent of the amplitude. For a given spring constant, the period increases with the mass of the block a more massive block oscillates more slowly. For a given block, the period decreases as k increases. A stiffer spring produces quicker oscillations.

COMBINATIONS OF SPRINGS

Springs in Parallel

Springs are in parallel when they undergo equal deformation when block moves (see figure) If block is displaced by x

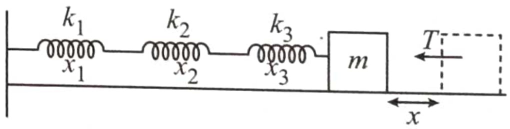


$$F = -(k_1 + k_2 + k_3)x = -k_{eq}x$$

$$\Rightarrow k_{eq} = k_1 + k_2 + k_3$$

Springs in Series

Spring are said to be in series if they have same tension (see figure).



Clearly, $F = -T = k_{eq}x$

$$x_1 + x_2 + x_3 = x$$

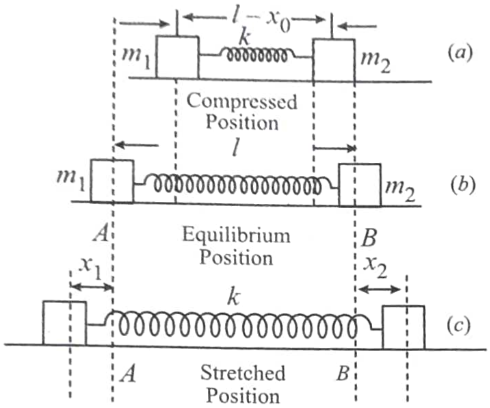
$$k_1x_1 + k_2x_2 + k_3x_3 = T$$

$$\Rightarrow \frac{T}{k_1} + \frac{T}{k_2} + \frac{T}{k_3} = \frac{T}{k_{eq}}$$

$$\Rightarrow \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

OSCILLATION OF TWO BLOCK SYSTEM

Two blocks of masses m_1 and m_2 are connected with a spring of natural length l and spring constant k. The system is lying on a frictionless horizontal surface. Initially spring is compressed by a distance x_0 as shown in figure.



The two blocks will perform SHM about their equilibrium position. We will discuss in this section about the oscillation of the system. If we release the blocks.

(a) **Time period of the blocks:** Here both the blocks will be in equilibrium at the same time when spring is in its natural length. Let A and B be equilibrium positions of blocks m_1 and m_2 as shown in figure.

Let at any time during oscillations, blocks are at a distance of x_1 and x_2 from their equilibrium positions (A and B) in figure (c)

As no external force is acting on the spring block system, the displacement of centre of mass of the system $\Delta x_{CM} = 0$

$(m_1 + m_2)\Delta x_{CM} = m_1x_1 - m_2x_2 \Rightarrow m_1x_1 = m_2x_2$

For first particle, force equation can be written as

$$-k(x_1 + x_2) = m_1\frac{d^2x_1}{dt^2} \text{ or, } -k(x_1\frac{m_1}{m_2} + x_1) = m_1a_1$$

$$\text{or, } a_1 = \frac{-k(m_1 + m_2)}{m_1m_2}x_1$$

$$\therefore\ \omega^2 = \frac{k(m_1 + m_2)}{m_1m_2}$$

Using vector we can write $\vec{a}_1 = -\dfrac{k(m_1 + m_2)}{m_1m_2}\vec{x}_1$

Negative sign is include as $\vec{a}_1$ and $\vec{x}_1$ are opposite to each other.

$$\text{Hence, } T = 2\pi\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}} = 2\pi\sqrt{\frac{\mu}{K}}$$

where $\mu = m_1m_2/(m_1 + m_2)$ which is known as reduced mass. Similarly time period of 2nd particle can be found. Both will have the same time period.

(b) **Amplitude of the particles:** Let the amplitude of blocks be A_1 and A_2, $m_1A_1 = m_2A_2$

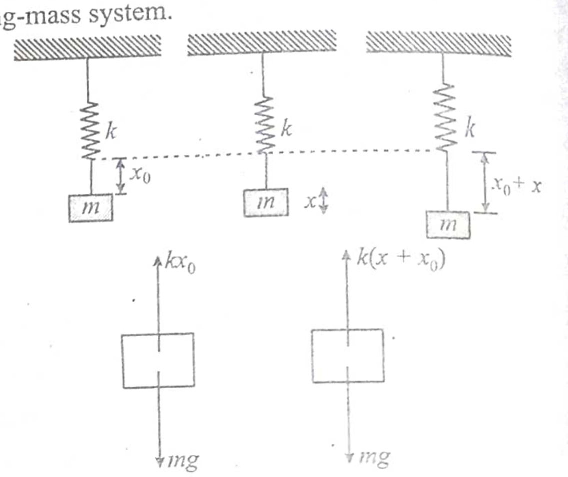
By energy conservation, $\frac{1}{2}k(A_1 + A_2)^2 = \frac{1}{2}kx_0^2$

or $A_1 + A_2 = x_0$ or $A_1 + \dfrac{m_1}{m_2}A_1 = x_0$ or, $A_1 = \dfrac{m_2x_0}{m_1 + m_2}$

Similarly, $A_2 = \dfrac{m_1x_0}{m_1 + m_2}$

Train Your Brain

Example 10: Find the period of oscillation of a vertical spring-mass system.



Sol. Let x_0 be the deformation in the spring in equilibrium.

Then $kx_0 = mg$

When the block is further displaced by x, the ne[t] restoring force is given by

$F = -[k(x + x_0) - mg]$ or $F = -kx$ (because $kx_0 = mg$)

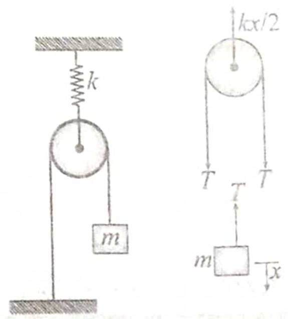
Using second law of motion,

$$m\frac{d^2x}{dt^2} = -kx \text{ or } \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$\text{Thus, } \omega^2 = \frac{k}{m} \text{ or } T = 2\pi\sqrt{\frac{m}{k}}$$

Note that gravity does not influence the time peri[od] of the spring-mass system, it merely changes t[he] equilibrium position.

Example 11: For the arrangement shown in the figure, fi[nd] the period of oscillation.



Sol. Obviously, when the block is displaced down by x, the spring will stretch by $\frac{x}{2}$.

From the free body diagram of the pulley,

$$\frac{kx}{2} = 2T \text{ or } T = \frac{kx}{4}$$

The net restoring force on the block is T.

Using the Second Law of motion, we get

$$m\frac{d^2x}{dt^2} = -T = -\frac{kx}{4}$$

$$\text{or } \frac{d^2x}{dt^2} + \left(\frac{k}{4m}\right)x = 0$$

Thus, the period of SHM is given by

$$T = 2\pi\sqrt{\frac{4m}{k}}$$

Note that we have not taken gravity into account as it does not affect the time period.

Example 12: A 2 kg block is attached to a spring for which $k = 200$ N/m. It is held at an extension of 5 cm and then released at $t = 0$. Find

(i) The displacement as a function of time

(ii) The velocity when $x = +A/2$

(iii) The acceleration when $x = +A/2$

Sol. (i) We need to find A, ω, and ϕ in SHM equation. The amplitude is the maximum extension, that is,

$A = 0.05$ m

We know the angular frequency of the spring-mass system is given by

$$\omega = \sqrt{\frac{k}{m}} = 10 \text{ rad/s}$$

To find ϕ we note that at $t = 0$ we are given $x = +A$ and $v = 0$.

Thus, from the equation of displacement and velocity, we get

$x = A \sin(\omega t + \phi)$

$\Rightarrow A = A \sin(0 + \phi)$

$v = \omega A(\omega t + \phi)$

$\Rightarrow 0 = 10A \cos(0 + \phi)$

Since $\sin\phi = 1$ and $\cos\phi = 0$,

it follows that $\phi = \pi/2$ rad. Thus,

$$x = 0.05 \sin\left(10t + \frac{\pi}{2}\right) \text{ m} \quad \ldots\text{(i)}$$

(ii) In order to find the velocity we have to find the time t, when $x = A/2$. Equation (i) yields

$$\frac{1}{2} = \sin\left(10t + \frac{\pi}{2}\right),$$

From which we infer that $\left(10t + \frac{\pi}{2}\right) = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

The velocity is given by

$$v = \frac{dx}{dt} = 0.5 \cos\left(10t + \frac{\pi}{2}\right)$$

$$= 0.5 \cos\frac{\pi}{6} \text{ or } 0.5 \cos\frac{5\pi}{6}$$

$$= +0.43 \text{ m/s} \quad \text{or} -0.43 \text{ m/s}$$

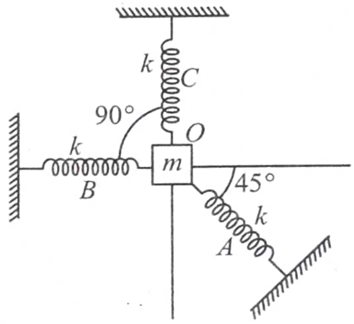
At a given position, there are two velocities of equal magnitude but opposite directions.

(iii) The acceleration at $x = A/2$ may be found from the equation,

$$a = -\frac{k}{m}x = -\omega^2 x$$

$$= -(10 \text{ rad/s})^2(0.05/2 \text{ m}) = -2.5 \text{ m/s}^2$$

Example 13: A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass k is pushed slightly against the spring A and released, find the time period of oscillations.



Sol. When the particle of mass m at O is pushed by y in the direction of A, spring A will be compressed by y while B and C will be stretched by $y' = y \cos 45°$, so the total restoring force on the mass m along OA is given by

$F = -[F_A + F_B \cos 45° + F_C \cos 45°]$

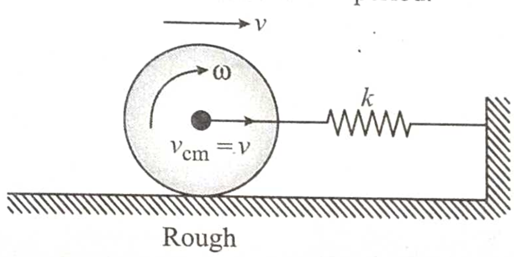
i.e. $F = -[ky + 2(ky') \cos 45°]$

or $F = -[ky + 2k(y \cos 45°) \cos 45°]$

i.e., $F = -k'y$ where $k' = 2k$

$$\text{so } T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

Example 14: A solid cylinder of mass M and radius R is attached to a spring of stiffness k as shown in the figure. The cylinder can roll without slipping on a rough horizontal surface. Show that the centre of mass of the cylinder executes SHM and determine its time period.



Sol. Consider the situation when the spring is extended by x.

The total energy of the cylinder and spring system is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2 = \text{constant}$$

Here, $I = \frac{1}{2}MR^2$ and $\omega = \frac{v}{R}$

$$\therefore \frac{3}{4}Mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Differentiating w.r.t. time, we get

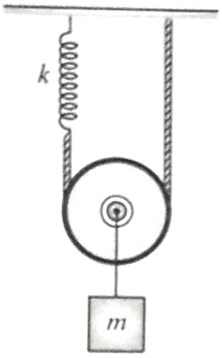
$$\left(\frac{3}{4}M\right)2v\frac{dv}{dt} + \left(\frac{1}{2}k\right)2x\frac{dx}{dt} = 0$$

Noting that $v = \frac{dx}{dt}$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$

We get $\frac{d^2x}{dt^2} + \frac{2k}{3M}x = 0$

$$\therefore T = 2\pi\sqrt{\frac{3M}{2k}}$$

Example 15: Find the time period of oscillation of m if pulley P is light and small.

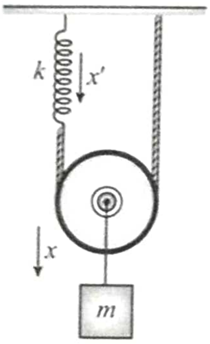


Sol. $T = kx_0$

Initially, $2T = 2kx_0$.

$\therefore 2kx_0 = mg$

Let the displacement of block at any time is x and stretch of spring is x', then



$$\frac{x'+0}{2} = x \Rightarrow x' = 2x$$

Energy of the system is

$$\frac{1}{2}mv^2 + \frac{1}{2}k(x_0 + x')^2 - mgx = \text{constant}$$

Differentiating w.r.t. time

$$mv\frac{dv}{dt} + k(x_0 + x')\frac{dx'}{dt} - mg\frac{dx}{dt} = 0$$

$$mva + k(x_0 + x')\frac{dx'}{dt} - mgv = 0$$

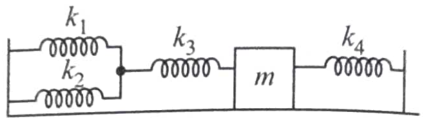
$$mva + k(x_0 + 2x)2v - mgv = 0$$

$$ma + 2k(x_0 + 2x) - mg = 0 \quad [\text{As } 2kx_0 = mg, \text{ initial}$$

$$ma + 4kx = 0$$

$$a = -\left(\frac{4k}{m}\right)x \Rightarrow \omega = \sqrt{\frac{4k}{m}}$$

Example 16: Find the time period of SHM of the spri block system. ($k_1 = k_2 = k_3 = k_4 = k$)



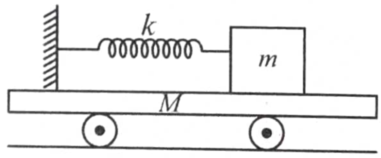
Sol. k_1 and k_2 are parallel and their combination is in seri with k_3. Therefore

$$k' = \frac{(k_1 + k_2)k_3}{k_1 + k_2 + k_3} = \frac{2k}{3} \quad \text{and } k' \text{ is parallel with } k_4. \text{ So}$$

$$k_{eq} = k' + k_4 = \frac{2k}{3} + k = \frac{5k}{3}$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{3m}{5k}}$$

Example 17: Following figure represents spring bloc system. If mass 'm' is slightly displaced, find the tim period of oscillation.

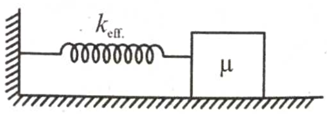


Sol. Using the concept of reduced mass:

For two blocks of mass m_1 and m_2

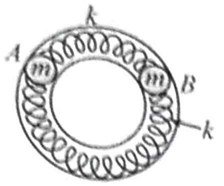
Reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$

In this case $\mu = \frac{mM}{(M + m)}$

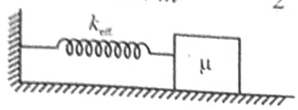


Now time period of oscillation is

Example 18: In the following figure two spherical balls each of masses 'm' attached with spring of stiffness 'k' find time period of oscillation.



Sol. Reduced mass $(\mu) = \dfrac{m \times m}{m + m}$ or $\mu = \dfrac{m}{2}$



Now, $k_{eff.} = 2k$

$$T = 2\pi\sqrt{\frac{\mu}{k_{eff.}}} = 2\pi\sqrt{\frac{m}{4k}} \text{ or } T = \pi\sqrt{\frac{m}{k}}$$

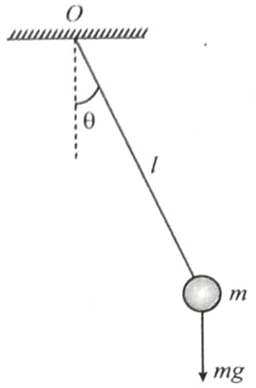
Concept Application

8. The total mechanical energy of a spring-mass system in simple harmonic motion is $E = \frac{1}{2}m\omega^2A^2$. Suppose the oscillating particle is replaced by another particle of double the mass while the amplitude A remains the same. The new mechanical energy will
   (*a*) Become $2E$ (*b*) Become $E/2$
   (*c*) Become $\sqrt{2}E$ (*d*) Remain E.
9. A particle executes simple harmonic motion along a straight line with an amplitude A. The potential energy is maximum when the displacement is
   (*a*) $\pm A$ (*b*) Zero
   (*c*) $\pm\frac{A}{2}$ (*d*) $\pm\frac{A}{\sqrt{2}}$
10. In SHM particle oscillates with frequency f then the frequency of oscillation of its kinetic energy is
    (*a*) f (*b*) $f/2$ (*c*) $2f$ (*d*) Zero
11. A spring-mass system oscillates with a frequency f. If it is taken in an elevator slowly accelerating upward, the frequency will
    (*a*) Increase (*b*) Decrease
    (*c*) Remain same (*d*) Become zero.
12. One-forth length of a spring of force constant k is cut away. The force constant of the remaining spring will be
    (*a*) $\frac{3}{4}k$ (*b*) $\frac{4}{3}k$ (*c*) k (*d*) $4k$

PENDULUMS

Simple Pendulum

Ideally, a simple pendulum consists of a point mass suspended at the end of a massless string as shown in the figure.



At the equilibrium position, net torque about the point 'O' is zero, at $\theta = 0$.

When the pendulum is displaced through an angle θ, the restoring torque about the point O is given by, $\tau_0 = -mgl\sin\theta$

For small angles $\sin\theta \approx \theta$, therefore, $\tau_0 = -mgl\theta$

Using Newton's Second Law, $\tau_0 = I_0\alpha = ml^2\dfrac{d^2\theta}{dt^2}$

Thus, $\dfrac{d^2\theta}{dt^2} + \left(\dfrac{g}{l}\right)\theta = 0$

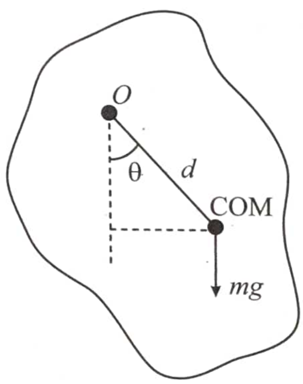
Comparing the above equation with the standard differential equation, we get $\omega^2 = \dfrac{g}{l}$

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

The pendulum having a time period equal to two seconds is called a second's pendulum.

Physical Pendulum

In figure, an extended body is pivoted freely about an axis that does not pass through its center of mass. Such an arrangement forms a physical pendulum that executes simple harmonic motion for small angular displacements. If d is the distance from the pivot to the center of mass, the restoring torque is $-mgd\sin\theta$.



Using Newton's Second law, $\tau = I\alpha$, we get $-mgd\sin\theta = I\dfrac{d^2\theta}{dt^2}$

where I is the moment of inertia about the given axis. If we use the small-angle approximation, $\sin\theta \approx \theta$, then

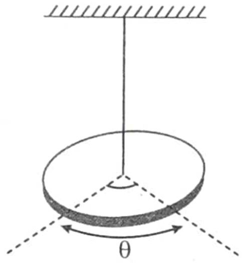
$$\frac{d^2\theta}{dt^2} + \frac{mgd}{I}\theta = 0$$

Thus, $\omega = \sqrt{\dfrac{mgd}{I}}$ and $T = 2\pi\sqrt{\dfrac{I}{mgd}}$

If the location of the center of mass and d are known, then a measurement of the period allows us to determine the moment of inertia of the body.

Torsional Pendulum

In torsional pendulum, an extended object is suspended at the centre by a light torsion wire. A torsion wire is essentially inextensible, but is free to twist about its axis. When the lower end of the wire is rotated by a slight amount the wire applies a restoring torque causing the body to oscillate rotationally



Let the twist given = θ, when released, the restoring torque produced is given by

$\tau = -C\theta$ where, C = Torsional constant or, $I\alpha = -C\theta$

Where, I = Moment of inertia about the vertical axis. or, $\alpha = -\dfrac{C}{I}\theta$

$\therefore$ Time period, $T = 2\pi\sqrt{\dfrac{I}{C}}$

Train Your Brain

Example 19: The angular displacement of simple pendulum is given by $\theta = 0.1\,\pi\sin\left(2\pi t + \dfrac{\pi}{6}\right)$ rad. The mass of the bob is 0.4 kg. Calculate

(i) The length of the simple pendulum; and

(ii) The velocity of the bob at $t = 0.25$ s

Sol. (i) We are given $\theta_0 = 0.1\,\pi$ rad, $\phi = \pi/6$ rad, and $\omega = 2\,\pi$ rad/s. Since $\omega^2 = (g/L)$, we have

$$L = \frac{g}{\omega^2} = \frac{9.8\,\text{m/s}^2}{(2\times 3.14\,\text{rad/s})^2} = 0.25\text{ m}$$

(ii) Since $s = L\theta$, the velocity of the bob, $v = \dfrac{ds}{dt}$, is

$$v = L\frac{d\theta}{dt} = (0.25\text{ m})(0.1\,\pi)(2\pi)\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$$

$= -0.247$ m/s

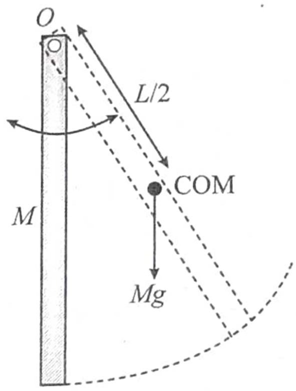
Example 20: A rod of mass M and length L is pivoted about its end O as shown in the figure. Find the period of SHM.

Sol. Restoring torque is $\tau_O = -Mg\dfrac{L}{2}\sin\theta$

Using Newton's Second Law and the small angle approximation, we get

$$I_O\frac{d^2\theta}{dt^2} + Mg\frac{L}{2}\theta = 0$$

Using parallel-axes theorem .

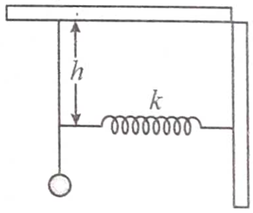


$$I_O = I_{COM} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$$

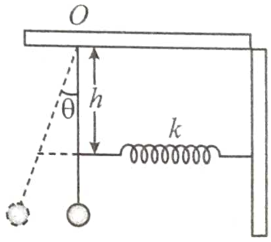
$$\therefore \frac{d^2\theta}{dt^2} + \left(\frac{3}{2}\frac{g}{L}\right)\theta = 0$$

Thus, $T = 2\pi\sqrt{\dfrac{2L}{3g}}$

Example 21: A simple pendulum of length L and mass m has a spring of force constant k connected to it at a distance h below its point of suspension. Find the frequency of vibrations of the system for small values of amplitude.



Sol. As shown in figure, if the pendulum is given a small angular displacement θ, the spring will also stretch by $y\,(= h\tan\theta)$.



So the restoring torque about O will be due to both force of gravity and elastic force of the spring.

i.e. $\tau = -[mg(L\sin\theta) + k(h\tan\theta)h]$

[Here $RF_{\text{spring}} = -k(h\tan\theta)$]

Now for small θ, $\tan\theta \approx \sin\theta \approx \theta$

so, $\tau = -(mgL + kh^2)\theta$

i.e. restoring torque is linear, so motion is angular SHM.

Now as $\tau = I\alpha = mL^2 \dfrac{d^2\theta}{dt^2}$ (as $I = mL^2$)

So $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ with $\omega^2 = \left[\dfrac{mgL + kh^2}{mL^2}\right]$

This is the standard equation of angular SHM with frequency $f = (\omega/2\pi)$; so here

$$f = \frac{1}{2\pi}\sqrt{\frac{mgL + kh^2}{mL^2}}$$

Example 22: A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.

Sol. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200\text{ kg})(5.0 \times 10^{-2}\text{ m})^2}{2}$$

$$= 2.5 \times 10^{-4}\text{ kg-m}^2$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{or, } C = \frac{4\pi^2 I}{T^2}$$

$$= \frac{4\pi^2(2.5 \times 10^{-4})}{(0.20)^2} = 0.25\ \frac{\text{kg m}^2}{\text{s}^2}$$

Example 23: One solid sphere, having mass 1 kg and diameter 0.3 m is suspended from a wire. If the twisting couple per unit twist for the wire is 6×10^{-3} N-m/radian, then the time period of small oscillations will be

Sol. $T = 2\pi\sqrt{\dfrac{I}{C}}$ ...(i)

$I = \dfrac{2}{5}MR^2$ ...(ii)

By equation (i) and (i)

$$T = 2\pi\sqrt{\frac{2MR^2}{5C}}$$

Substitute the provided values

$$\Rightarrow T = 6.28\sqrt{\frac{0.4 \times 1 \times (0.15)^2}{6 \times 10^{-3}}}$$

$$= 6.28\sqrt{\frac{9 \times 10^{-3}}{6 \times 10^{-3}}}$$

Concept Application

13. A wall clock uses simple pendulum. It is accurate at earth surface. If it is taken to high altitude

(*a*) It will run fast

(*b*) Its length should be increased to keep it accurate

(*c*) Its length should be decreased to keep it accurate

(*d*) Even if length is changed it cannot be accurate

14. A pendulum clock that keeps correct time on the earth is taken to the moon. It will run

(*a*) At correct rate (*b*) 6 times faster

(*c*) $\sqrt{6}$ times faster (*d*) $\sqrt{6}$ times slower

15. The free end of a simple pendulum is attached to the ceiling of a box. The box is taken to a height and the pendulum is oscillated. When the bob is at its lowest point, the box is released to fall freely. As seen from the box during this period, the bob will

(*a*) Continue its oscillation as before

(*b*) Stop

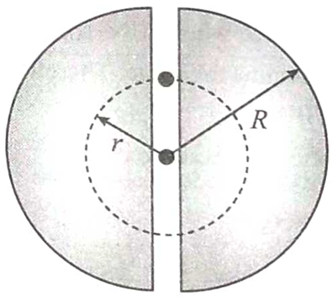
(*c*) Will go in a circular path

(*d*) Move on a straight line.

OTHER SHM EXAMPLES

Motion of Particle in a Tunnel Along the Diameter of Earth

Suppose a tunnel could be dug through the earth from one side to the other along a diameter, as shown in figure and a particle is dropped along it.



The gravitational force on the particle at a distance r from the centre of the earth arises entirely from that portion of matter of the earth in shells internal to the position of the particle. The external shells exert no force on the particle.

The value of gravity at a distance r is given by

$$g = g_0\frac{r}{R}$$

Where $g_0 = \dfrac{GM}{R^2}$ is the gravity on the surface of earth.

The net force on the particle is $F = -mg = -mg_0\dfrac{r}{R}$

Using Newton's Law,

$$F = m\frac{d^2r}{dt^2} = \frac{-mg_0}{R}r \quad \text{or} \quad \frac{d^2r}{dt^2} + \left(\frac{g_0}{R}\right)r = 0$$

This is the differential equation of SHM.

Thus, the motion of the particle is simple harmonic.

The period of oscillation is

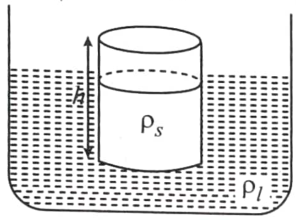
$$T = 2\pi\sqrt{\frac{R}{g_0}}$$

Taking $R = 6400$ km $= 6.4 \times 10^6$ m and $g_0 = 9.81$ m/s^2, we get $T = 84.2$ min.

Oscillations of a Floating Cylinder in a Liquid

A floating body is in a stable equilibrium. When it is displaced up and released, it accelerates down and when it is pushed down and released, it accelerates up. It means, a floating body experiences a net restoring force towards its stable equilibrium position. Hence, a floating body oscillates when displaced up or down from its mean position.

Consider a solid cylinder of density ρ_s, cross-sectional area A and height h floating in a liquid of density ρ_l as shown in ($\rho_l > \rho_s$). It is displaced slightly by x and allowed to oscillate vertically.



At equilibrium the net force on the cylinder is zero in the vertical direction.

$F_{net} = B - W = 0$; B = the buoyancy and W = the weight of the cylinder.

When the cylinder is depressed slightly by x, the buoyancy increase from B to $B + \delta B$, where $\delta B = |x|\rho_l Ag$

The weight W remains the same. Therefore the net force,

$F_{net} = B + \delta B - W = \delta B = |x|\rho_l Ag$

The equation of motion is, therefore,

$$\rho_s Ah\frac{d^2x}{dt^2} = -x\rho_l Ag$$

The negative sign takes into account the fact that x and restoring force are in opposite directions. Therefore

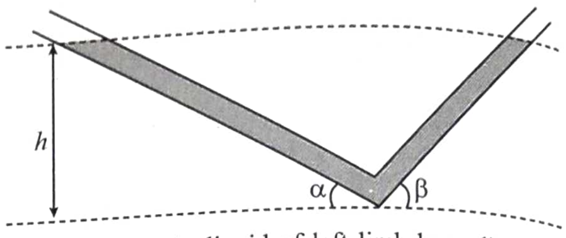
$$\frac{d^2x}{dt^2} = -x\frac{\rho_l g}{\rho_s h}$$

and the angular frequency, ω, is

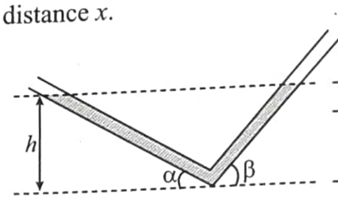
$$\omega = \sqrt{\frac{g\rho_l}{h\rho_s}}$$

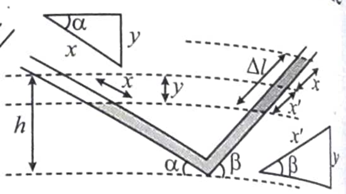
Oscillations of Liquid Column In a U-Tube

A V-shaped glass tube of uniform cross section is kept in a vertical plane as shown. A liquid is poured in the tube. In equilibrium the level of liquid in both limbs of tube are equal.



Let us displace the liquid of left limb by a distance 'x' the liquid of right limb will rise relative to equilibrium line by same distance x.





Excess length of the liquid on the right limb

$\Delta l = x + x'$ ...(i)

From the figure

$y = x\sin\alpha = x'\sin\beta$ ...(ii)

From equation (i)

$$x' = \frac{x\sin\alpha}{\sin\beta} \quad \text{...(iii)}$$

From equation (i) and (iii),

$$\Delta l = \left(\frac{\sin\alpha + \sin\beta}{\sin\beta}\right)x$$

This excess length Δl in right limb will provide restoring force to the liquid in tube. Weight of the excess liquid column

$$W = \rho A \cdot \Delta l g = \left(\frac{\sin\alpha + \sin\beta}{\sin\beta}\right)\rho Axg$$

ρ is the density of liquid and A is the area of cross section of tube. The component of W along the tube will be equal to restoring force

$F = W\sin\beta = \rho(\sin\alpha + \sin\beta)Axg$

This force help the entire liquid to restore its original position by moving it with an acceleration $a = F/m$; m = mass of total liquids

$$a = \frac{\rho(\sin\alpha + \sin\beta)Axg}{m}$$

Here m = mass of liquid in the tube $= A\rho l$

l = length of total liquid column

$$l = \frac{h}{\sin\alpha} + \frac{h}{\sin\beta} = h\left[\frac{\sin\alpha + \sin\beta}{\sin\alpha \cdot \sin\beta}\right]$$

Hence, $a = \dfrac{\rho(\sin\alpha + \sin\beta)Axg}{A\rho\left[h\dfrac{(\sin\alpha + \sin\beta)}{\sin\alpha \cdot \sin\beta}\right]}$ or $\vec{a} = -\left(\dfrac{g\sin\alpha \cdot \sin\beta}{h}\right) \cdot \vec{x}$

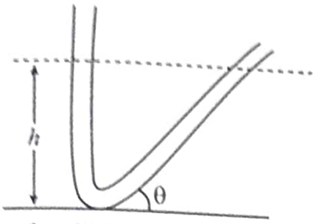
Compare with $\vec{a} = -\omega^2\vec{x}$ which give

$$\omega = \sqrt{\frac{g\sin\alpha \cdot \sin\beta}{h}} \Rightarrow T = 2\pi\sqrt{\frac{h}{g\sin\alpha \cdot \sin\beta}}$$

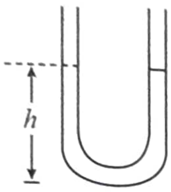
Note:

- If we modify the tube as shown in the figure below, the modified time period will be

$T = 2\pi\sqrt{\dfrac{h}{g\sin\theta}}$ $\quad (\because \sin\alpha = 1, \sin\beta = \sin\theta)$

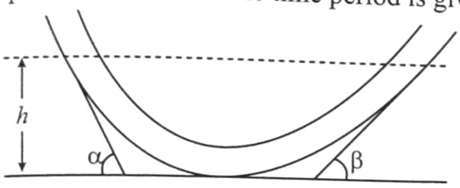


- For U-tube, $\alpha = \beta = 90°$



Hence $T = 2\pi\sqrt{\dfrac{h}{g}}$

- For an arbitrary tube if α and β are the angles made by the targets (with horizontal) drawn at the tube at the free surface of the liquid in both limbs. The time period is given by



$$T = 2\pi\sqrt{\frac{h}{g\sin\alpha\cdot\sin\beta}}$$

SMALL OSCILLATIONS IN STABLE EQUILIBRIUM

When a particle under the influence of some arbitrary potential $U(x)$ is sitting at a point of minima/stable equilibrium (say $x = x_0$), and is then slighty disturbed, it performs small oscillations which can be approximated to an SHM with angular frequency given by

$$\omega = \sqrt{\frac{U''(x_0)}{m}}$$

SUPERPOSITION OF TWO SHMs

1. **In the same direction and of same frequency:**

$x_1 = A_1 \sin\omega t$, $x_2 = A_2 \sin(\omega t + \theta)$, then resultant displacement

$x = x_1 + x_2 = A_1 \sin\omega t + A_2 \sin(\omega t + \theta) = A \sin(\omega t + \phi)$

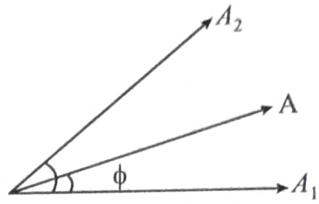
where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$ and

$$\phi = \tan^{-1}\left[\frac{A_2\sin\theta}{A_1 + A_2\cos\theta}\right]$$

If $\theta = 0$, both SHMs are in phase and $A = A_1 + A_2$

If $\theta = \pi$, both SHMs are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHMs of this case can also be found by vector diagram also called **Phasor diagram.**



(i) Amplitude of SHM is taken as length (magnitude) of vector

(ii) Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant of vector's give resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.

2. **In same direction but are of different frequencies: (special case, if $A_1 = A_2$)**

$x_1 = A_1 \sin\omega_1 t$ and $x_2 = A_2 \sin\omega_2 t$

Then resultant displacement $x = x_1 + x_2 = A_1 \sin\omega_1 t + A_2 \sin\omega_2 t$

3. **In two perpendicular directions:**

$x = A \sin\omega t$ and $y = B \sin(\omega t + \phi)$

(i) If $\phi = 0$ or π then $y = \pm (B/A)\, x$. So path will be straight line and resultant displacement will be

For $\phi = 0$ or π, $R = x + y = (A + B) \sin\omega t$

(ii) If $\theta = \dfrac{\pi}{2}$ then, $x = A \sin\omega t$

$y = B \sin(\omega t + \pi/2) = B \cos\omega t$

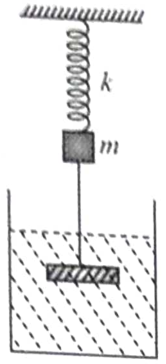
So, resultant will be $\dfrac{x^2}{A^2} + \dfrac{y^2}{B^2} = 1$. i.e. equation of an ellipse and if $A = B$, then superposition will be an equation of circle.

DAMPED SIMPLE HARMONIC MOTION

In each of the several examples of oscillatory systems we have considered case the amplitude of the oscillation does not depend on time and the system, once set to oscillate, will continue to do so forever. Physical systems are never as ideal as that. Even in the most controlled case there will be dissipative elements. In such cases the amplitudes of the oscillation will gradually decrease and the motion will be 'damped'.

Most of the time we are interested in the motion of an object through air or other viscous fluids. The motion of the object is then subject to a resistive force called aerodynamic or viscous drag. Experimentally the resistive force is found to be proportional to the velocity of the body for low relative speed with respect to the medium.

A laboratory model for a damped spring-mass system is shown in figure where a vertical spring has at its end a mass m which is connected to massless piston which moves through a liquid.



For low speeds it is reasonable to write the frictional force as

$F_f = -bv$ (b in kg/s)

where b is a positive constant known as damping constant.

The equation of motion of a harmonic oscillator becomes

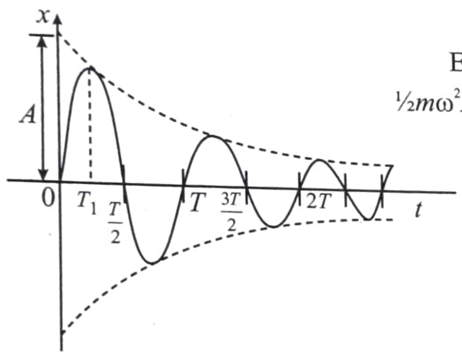
$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

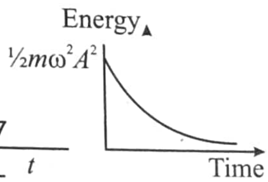
On solving

$x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)$ where $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$ and

amplitude of oscillation $= Ae^{-bt/2m}$.

Total energy $E = \dfrac{1}{2}kA^2e^{-bt/m}$





when $b << \sqrt{km}$, then $\omega' \approx \omega$

FORCED OSCILLATIONS AND RESONANCE

The oscillations in a system can be indefinitely maintained by supplying energy continuously. In a mechanical system this can be done by subjecting it to an external force which itself has harmonic time dependence.

An interesting phenomenon occurs if the frequency of the external source is equal or nearly equal to the natural frequency of the system. The amplitude of oscillations is found to increase many folds in such cases. This is called **resonance.**

In the presence of resistive forces proportional to velocity and the applied periodic force $F = F_0 \sin \omega_d t$ differential equation becomes

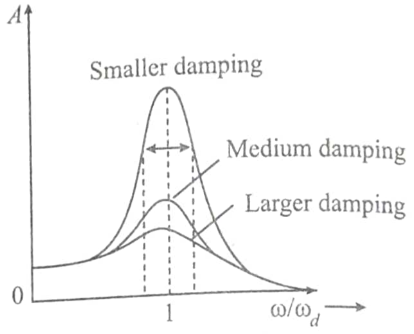
$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = F_0 \sin \omega_d t$$

The solution is, $x = A\sin(\omega_d t + \phi)$

where $A = \dfrac{F_0}{m\sqrt{(\omega^2 - \omega_d^2)^2 + \left(\dfrac{b\omega_d}{m}\right)^2}}$

is a function of natural frequency ω and driving frequency ω_d.

The phenomena of increase in amplitude when the driving force is close to the natural frequency of the oscillator is called **resonance.**



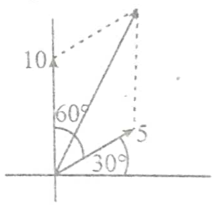
Train Your Brain

Example 24: Consider $x_1 = 5\sin(\omega t + 30°)$ and $x_2 = 10\cos(\omega t)$ suppose. Find amplitude of resultant SHM.

Sol. $x_1 = 5\sin(\omega t + 30°)$, $x_2 = 10\sin\left(\omega t + \dfrac{\pi}{2}\right)$

$\Rightarrow A_1 = 5, A_2 = 10, \phi = 90 - 30 = 60°$.

Draw the phasor diagram as shown below



$$A = \sqrt{5^2 + 10^2 + 2\times5\times10\cos 60°}$$

$$= \sqrt{25+100+50} = \sqrt{175} = 5\sqrt{7}$$

Example 25: $x_1 = 3\sin\omega t$, $x_2 = 4\cos\omega t$. Find

(i) Amplitude of resultant SHM and

(ii) Equation of the resultant SHM.

Sol. First write all SHMs in terms of sine functions with positive amplitude.

$\therefore x_1 = 3\sin\omega t$, $x_2 = 4\sin(\omega t + \pi/2)$

Then, $A = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\dfrac{\pi}{2}}$

$= \sqrt{9+16} = \sqrt{25} = 5$

and $\tan\phi = \dfrac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}}$ $\phi = 53°$

Therefore, equation $x = 5\sin(\omega t + 53°)$

Example 26: Equation of two SHM

$x_1 = 5\sin(2\pi t + \pi/4)$

$x_2 = 5\sqrt{2}\,(\sin 2\pi t + \cos 2\pi t)$

Ratio of amplitudes and phase difference will be

(*a*) 2 : 1, 0

(*b*) 1 : 2, 0

(*c*) 1 : 2, $\pi/2$

(*d*) 2 : 1, $\pi/2$

Sol. $x_1 = 5\sin(2\pi t + \pi/4)$

$x_2 = 5\sqrt{2}\,(\sin 2\pi t + \cos 2\pi t)$

$x_2 = 10(\sin 2\pi t \cos \pi/4 + \cos 2\pi t \sin \pi/4)$

$x_2 = 10\sin(2\pi t + \pi/4)$

Ratio of amplitude = 5 : 10

= 1 : 2

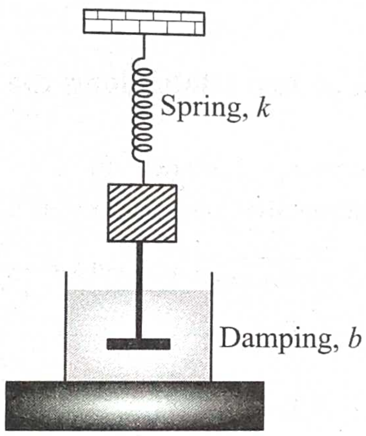
Phase difference $= (2\pi t + \pi/4) - (2\pi t + \pi/4) = 0$

Example 27. For the damped oscillator shown in figure, the mass (m) of the block is 200 g, $k = 80\ \text{Nm}^{-1}$ and the damping constant b is 40 gs^{-1}. Calculate:

(i) The period of oscillation,

(ii) Time taken for its amplitude of vibrations to drop to half of its initial value

(iii) The time for the mechanical energy to drop to half initial value.



Sol. (i) As the damping constant,

$b\,(= 0.04\ \text{kg s}^{-1}) << \sqrt{km}$

The time period T from the equation

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \approx \sqrt{\frac{k}{m}}$$

is given by,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2\text{kg}}{80\ \text{Nm}^{-1}}} = 0.314\ \text{s}$$

(ii) From the equation $x(t) = Ae^{-\frac{bt}{2m}}$ the time $t_{1/2}$ for the amplitude to drop to half of its initial value is

$$\frac{1}{2} = e^{-\frac{bt_{1/2}}{2m}}$$

or $\log_e 2 = \dfrac{bt_{1/2}}{2m}$

or $0.693 = \dfrac{bt_{1/2}}{2m}$

or $t_{1/2} = \dfrac{0.693 \times 2 \times 0.2}{40 \times 10^{-3}} = 6.93\text{s}$

(iii) $E(t) = E(0)e^{-\frac{bt}{m}}$;

$\dfrac{1}{2} = e^{-\frac{bt_{1/2}}{m}}$; $\log_e 2 = \dfrac{bt_{1/2}}{m}$

$t_{1/2} = \dfrac{0.693 \times 0.2}{40 \times 10^{-3}} = 3.46\,\text{sec}$

Concept Application

16. The resultant of two rectangular simple harmonic motions of the same frequency and unequal amplitudes but differing in phase by $\pi/2$ is

(*a*) Simple harmonic

(*b*) Circular

(*c*) Elliptical

(*d*) Parabolic

17. In case of a forced vibration, the resonance wave becomes very sharp when the

(*a*) Restoring force is small

(*b*) Applied periodic force is small

(*c*) Quality factor is small

(*d*) Damping force is small

18. A vertical U-tube of uniform cross-section contains water upto a height of 2.45 cm. If the water on one side is depressed and then released, its up and down motion in tube is simple harmonic. Calculate its time period in seconds. (Given $g = 980\ \text{cm s}^{-2}$).

19. A weighted glass tube is floating in a liquid with 20 cm of its length immersed. It is pushed down through a certain distance and then released. Show that up and down motion executed by the glass tube is SHM and find the time period of vibration in seconds. Given, $g = 980\ \text{cm s}^{-2}$.

Short Notes

Simple Harmonic Motion

$F = -kx$

General equation of SHM is $x = A\sin(\omega t + \phi)$; $(\omega t + \phi)$ is phase of the motion and ϕ is initial phase of the motion.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Speed : $v = \omega\sqrt{A^2 - x^2}$

Acceleration : $a = -\omega^2 x$

Energy in SHM

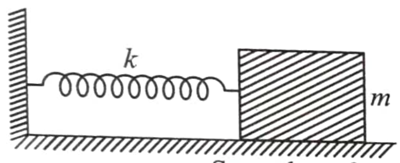
Kinetic Energy: $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}k\left(A^2 - x^2\right)$

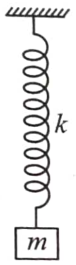
Potential Energy: $\text{PE} = \frac{1}{2}kx^2$

Total Mechanical Energy:

$$E = \text{KE} + \text{PE} = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}Kx^2 = \frac{1}{2}KA^2 = \text{constant}$$

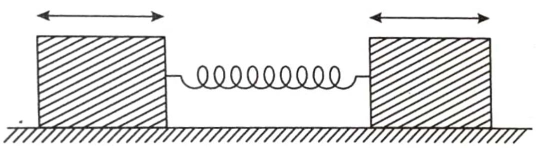
Spring-Mass System

❖ 



Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

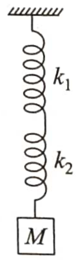
❖ $T = 2\pi\sqrt{\frac{\mu}{k}}$, where $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$ is known as reduced mass.



❖ Series combination

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

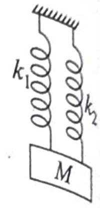
$$T = 2\pi\sqrt{\frac{M}{k_s}}$$



❖ Parallel combination,

$$k_p = k_1 + k_2$$

$$T = 2\pi\sqrt{\frac{M}{k_p}}$$



Simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$= 2\pi\sqrt{\frac{l}{g_{eff.}}} \quad \text{(in accelerating Reference Frame)}$$

where g_{eff} is net acceleration due to psuedo force and gravitational force.

Compound Pendulum/Physical Pendulum

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

where, $I = I_{cm} + ml^2$; l is distance between point of suspension and centre of mass.

Torsional Pendulum

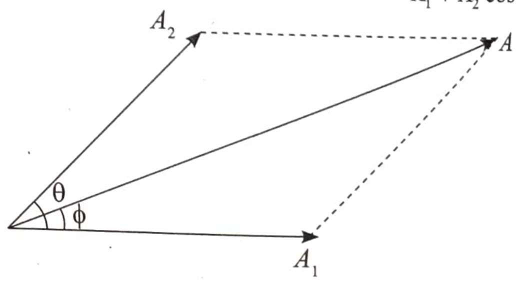
$T = 2\pi\sqrt{\frac{I}{C}}$ where, C = Torsional constant

Superposition of Two SHMs Along the Same Direction

$x_1 = A_1 \sin\omega t$ and $x_2 = A_2\sin(\omega t + \theta)$

If equation of resultant SHM is taken as $x = A\sin(\omega t + \phi)$, then

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta} \quad \text{and} \quad \tan\phi = \frac{A_2\sin\theta}{A_1 + A_2\cos\theta}$$



Small Oscillations

$$\omega = \sqrt{\frac{U''(x_0)}{m}}$$

where $U''(x_0)$ is second derivative of potential at the point x_0 i.e. the point of stable equilibrium

Damped Harmonic Oscillations

If the damping force is given by $\vec{F}_d = -b\vec{v}$, where $\vec{v}$ is the velocity of the oscillator and b is a damping constant, then the displacement of the oscillator is given by

$$x(t) = A_0 e^{-bt/2m}\cos(\omega' t + \phi),$$

where ω' is the angular frequency of the damped oscillator, is given as $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

If the damping constant is small $\left(b \ll \sqrt{km}\right)$, then $\omega' \approx \omega$, where ω is the angular frequency of the undamped oscillator.

For small b, the mechanical energy E of the oscillator is given by

$$E(t) = \frac{1}{2}kA_0^2 e^{-bt/m}.$$

Forced Oscillations and Resonance

If an external driving force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω_0, the system oscillates with angular frequency ω_d. The velocity amplitude v_m of the system is greatest when

$$\omega_d = \omega,$$

a condition called **resonance**. The amplitude A_0 of the system is (approximately) greatest under this condition.

Solved Examples

1. Periodic time of a simple pendulum is 2 second and it can travel to and fro from equilibrium position upto maximum 5 cm. At the starting of motion pendulum is at maximum displacement in right side from equilibrium position. Then, the equation of SHM can be

(a) $y = 5\sin \pi t$ (b) $y = 5\cos \pi t$

(c) $y = 5\cos 2\pi t$ (d) $y = 5\sin 2\pi t$

Sol. Displacement expression for SHM be $y = a\sin(\omega t + \phi)$

Time period of simple pendulum is

$T = 2\pi/\omega = 2\text{sec}$

$\therefore \omega = \pi$ and $a = 5$ cm

$\therefore y = 5\sin(\pi t + \phi)$ cm

Now at $t = 0$, displacement $y = 5$ cm

$\therefore 5 = 5\sin(\pi \times 0 + \phi)$

$\Rightarrow \sin\phi = 1 \quad \Rightarrow \phi = \pi/2$

$\Rightarrow y = 5\sin(\pi t + \pi/2)$

$\Rightarrow y = 5\cos \pi t$

2. A simple pendulum, which is meant to beat seconds (i.e., each half oscillation takes 1 second), gains 1 minute per day. By what percentage of its length should be increased to make it accurate?

Sol. The pendulum makes $(24 \times 60 \times 60 + 60)$ half oscillations in 24 hours.

The time for half an oscillation is therefore,

$$\frac{T'}{2} = \frac{24\times 60\times 60}{60(24\times 60+1)}$$

$$\text{or } \frac{T'}{2} = \frac{1440}{1441} = \pi\sqrt{\frac{l}{g}} \qquad \text{...(i)}$$

Let kl be the extra length required for an accurate 1 second beat. Then

$$1 = \pi\sqrt{\frac{(l+kl)}{g}} = \pi\sqrt{\frac{l}{g}}\left(\sqrt{k+1}\right) \qquad \text{...(ii)}$$

Dividing equation (ii) by (i), we get

$\sqrt{k+1} = \left(\frac{1441}{1440}\right)$ or $k = 0.0014$

$\therefore$ Percentage increase in length will be $100k$ or 0.14%.

3. A particle is moving in a straight line with SHM Its velocity has values 3 ms^{-1} and 2 ms^{-1} when its distance from the mean positions are 1 m and 2 m respectively. Find the period of its motion and length of its path.

Sol. When $x = 1\text{m} \Rightarrow v = 3\text{ ms}^{-1}$

When $x = 2m \Rightarrow v = 2ms^{-1}$

$v = 3 = \omega\sqrt{A^2 - 1}$

$3 = \omega\sqrt{A^2 - 1}$

$2 = \omega\sqrt{A^2 - 4}$

$\Rightarrow \frac{9}{4} = \frac{A^2 - 1}{A^2 - 4} \Rightarrow 9A^2 - 36 = 4A^2 - 4$

$\Rightarrow 5A^2 = 32 \Rightarrow A = \sqrt{\frac{32}{5}} = \sqrt{6.4} \Rightarrow A = 2.53$ m

$3 = \omega\sqrt{6.4 - 1}$

$\omega = \frac{3}{\sqrt{5.4}} = 1.29\text{ rad/s}$

Period of motion: $T = \frac{2\pi}{\omega} = 4.86$ s

Length of path $= 2A = 5.06$ m

4. Consider a spring that exerts the following restoring force

$$F = \begin{cases} -kx & \text{for } x > 0 \\ -2kx & \text{for } x < 0 \end{cases}$$

A mass m on a frictionless surface is attached to this spring, displaced to $x = A$ by stretching the spring and released.

(a) Find the period of motion.

(b) What is the most negative value of x that the mass m reaches?

Sol. (a) $T = \frac{T_1}{2} + \frac{T_2}{2} = \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{2k}}$

$$= \pi\sqrt{\frac{m}{k}}\left(1 + \frac{1}{\sqrt{2}}\right) = 5.36\sqrt{\frac{m}{k}}$$

$$T = 5.36\sqrt{\frac{m}{k}}$$

(b) From conservation of energy

$$\frac{1}{2}kA^2 = \frac{1}{2}(2k)\left(x_{max}^2\right)$$

$$\Rightarrow x_{max} = \frac{A}{\sqrt{2}}$$

i.e. the most negative value of x is $-\frac{A}{\sqrt{2}}$

5. A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1:3 and again oscillated from the two parts separately, then the periods are T_1 and T_2 then find T_1/T_2.

Sol. $T \propto \frac{1}{k^{1/2}}; T = 2\pi\sqrt{\frac{m}{k}}$

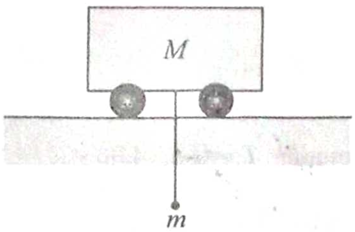
$$k_1 = 4k_i; k_2 = \frac{4k}{3}$$

By $k_1\lambda_1 = k_2\lambda_2 = kl$

$$T_1 = \frac{T}{2}; T_2 = \frac{T\sqrt{3}}{2}$$

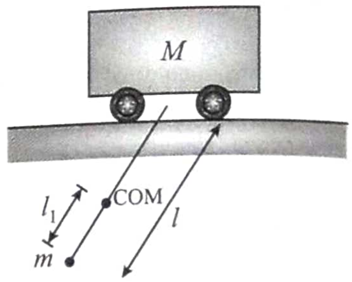
$$\frac{T_1}{T_2} = \frac{1}{\sqrt{3}}$$

6. A load of mass M is on horizontal rails. A pendulum made of ball of mass m tied to a weightless inextensible thread is suspended to the load. The load can move only along the rails. Determine the ratio of the periods T_2/T_1 of small oscillations of the pendulum in vertical planes parallel and perpendicular to rails. Neglect friction everywhere.



Sol. The period of oscillations of the pendulum in the direction perpendicular to the rails is

$$T_1 = 2\pi\sqrt{\frac{l}{g}}\ (l = \text{length of thread})$$



Time period (T_2) in the direction parallel to the rails:

The period of oscillations in the plane parallel to the rails for small oscillations can be found from the condition that the centre of mass (COM) of the system remains stationary. The distance of COM from m can be found by

$$ml_1 = M(l - l_1)$$

$$l_1 = \frac{Ml}{M+m}$$

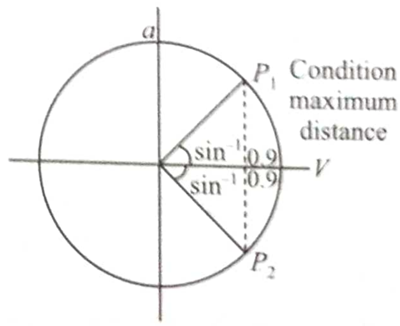
Thus the ball performs oscillations about COM with time period

$$T_2 = 2\pi\sqrt{\frac{l_1}{g}} = 2\pi\sqrt{\frac{Ml}{(M+m)g}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{M}{M+m}}$$

7. Two particles A and B execute SHM along the same line with the same amplitude a, same frequency and same equilibrium position O. If the phase difference between them is $\phi = 2\sin^{-1}(0.9)$, then find the maximum distance between the two.

Sol. $\phi = 2\sin^{-1}(0.9)$



$P_1P_2 \,||\, y$-axis

Max. Distance = $1.8a$

8. A block of mass 2 kg is resting on a smooth horizontal floor of a truck attached to its front by a spring of force constant $k = 40$ N/m. At time $t = 0$, the truck begins to move with constant acceleration 4 m/s². Find the amplitude and period of oscillations of the block relative to floor of the truck.

Sol. Relative to floor of the truck, block is acted upon by a pseudo force.

$F = ma$ (backwards)

In equilibrium position $kx_0 = F$ or $kx_0 = ma$

or $x_0 = \dfrac{ma}{k}$

This gives the new position of equilibrium. However, block acquires a velocity, till it comes to its new equilibrium position, which can be obtained by $\int_0^v v \cdot dv = \int_0^{x_0} \left(a - \dfrac{kx}{m}\right) dx$

or $\dfrac{v^2}{2} = ax_0 - \dfrac{kx_0^2}{2m} = \dfrac{1}{2}\dfrac{ma^2}{k} \quad \left(x_0 = \dfrac{ma}{k}\right)$

The kinetic energy of block at equilibrium position is therefore, $\text{KE} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{m^2a^2}{k}$

Let A be the amplitude then $\dfrac{1}{2}kA^2 = K.E$ at mean position

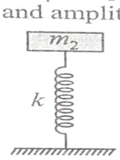
or $\dfrac{1}{2}kA^2 = \dfrac{1}{2}\dfrac{m^2a^2}{k}$ or $A = \dfrac{ma}{k}$

Substituting the value, we have $A = \dfrac{(2)(4)}{40} = 0.2\,\text{m}$

Period of oscillation will remain unchanged. Only the mean position is changed. Therefore,

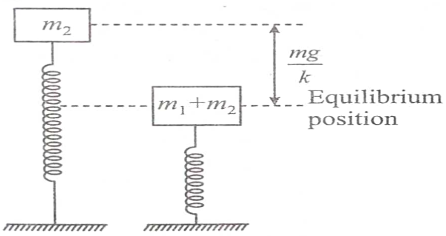
$T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{2}{40}}$ or $T = 1.4$ s

9. Block of mass m_2 is in equilibrium as shown in figure. Another block of mass m_1 is kept gently on m_2. Find the time period of oscillation and amplitude.



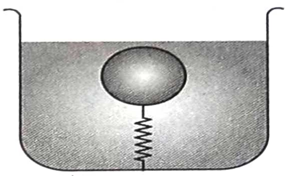
Sol. Time period $T = 2\pi\sqrt{\dfrac{m_1 + m_2}{k}}$.

At initial position since velocity is zero it is the extreme position.



$\therefore$ Amplitude $A = \dfrac{m_1 g}{k}$

10. As a submerged body moves through a fluid, the particles of the fluid flow around the body and thus acquire kinetic energy. In the case of a sphere moving in an ideal fluid, the total kinetic energy acquired by the fluid is $\dfrac{1}{4}\rho V v^2$ where ρ is the mass density of fluid, V the volume of sphere and v is the velocity of the sphere. Consider a 0.5 kg hollow spherical shell of radius 8 cm which is held submerged in a tank of water by a spring of force constant 500 N/m.

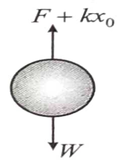


(*a*) Neglecting fluid friction, determine the period of vibration of the shell when it is displaced vertically and then released.

(*b*) Solve part (a) assuming that the tank is accelerated upward at the constant rate of 10^3 kg/m³. Density of water is 20 kg.

Sol. (*a*) Let F be the thrust and W the weight of the sphere. In equilibrium let x_0 be the compression of the spring,

Then, $F + kx_0 = W$ or $kx_0 = W - F$ ...(i)



If the sphere is further compressed by x, then total energy of the system will be

$$E = -(W - F)\cdot x + \frac{1}{2}k(x + x_0)^2 + \frac{1}{2}mv^2 + \frac{1}{4}\rho V v^2$$

Since friction is absent, total energy remains constant, hence

$$\frac{dE}{dt} = 0$$

$$\text{or } 0 = -(W - F)\cdot\frac{dx}{dt} + k(x + x_0)\frac{dx}{dt} + mv\left(\frac{dv}{dt}\right) + \frac{1}{2}\rho V v\left(\frac{dv}{dt}\right) \quad \text{...(ii)}$$

From (i) and (ii), with substitutions $\dfrac{dx}{dt} = v$ and $\dfrac{dv}{dt} = a$, we get

$$a = -\frac{k}{\frac{1}{2}\rho V + m}\cdot x$$

$a \propto -x$

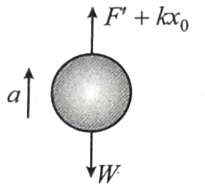
Oscillations are simple harmonic, time period of which will be $T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m+\frac{1}{2}\rho V}{k}}$

$$= 2\pi\sqrt{\frac{0.5+\frac{1}{2}\times 10^3\times\frac{4}{3}\times\pi\times(0.08)^3}{500}}$$

$T = 0.352$ s

(*b*) When it is accelerated upwards with an acceleration 'a'.

$$F' = \frac{F(g+a)}{g}$$



Now $F' + kx_0 - W = \left(\frac{W}{g}\right)a$

or $kx_0 = \frac{W}{g}\cdot a + W - F\left(1+\frac{a}{g}\right)$

or $kx_0 = (W-F) + \frac{a}{g}(W-F)$

$$kx_0 = (W-F)\left(1+\frac{a}{g}\right) \quad \text{...(iii)}$$

When displaced downwards, total energy would be

$$E = -(W-F)\frac{(g+a)}{g}\cdot x + \frac{1}{2}K(x+x_0)^2 + \frac{1}{2}mv^2 + \frac{1}{4}\rho V v^2$$

Substituting $\frac{dE}{dt} = 0$

or $0 = -(W-F)\left(1+\frac{a}{g}\right)\frac{dx}{dt} + k(x+x_0)\frac{dx}{dt}$

$$+mv\cdot\frac{dv}{dt} + \frac{1}{2}\rho vV\cdot\frac{dv}{dt} \quad \text{...(iv)}$$

From (iv) and (iii) we get the same result as was obtained in part (*a*) i.e., $T = 0.352$ s

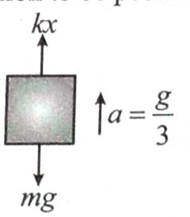
11. A block with mass of 2 kg hangs without vibrating at the end of a spring of spring constant 500 N/m, which is attached to the ceiling of an elevator.

The elevator is moving upwards with acceleration $g/3$. At time $t = 0$, the acceleration suddenly ceases.

(*a*) What is the angular frequency of oscillation of the block after the acceleration ceases?

(*b*) By what amount is the spring stretched during the time when the elevator is accelerating?

(*c*) What is the amplitude of oscillation and initial phase angle observed by a rider in the elevator? Take the upward direction to be positive. Take $g = 10.0\text{m/s}^2$.



Sol. The angular frequency of the spring block system in vertical oscillations does not depend on the acceleration due to gravity or the acceleration of the elevator. The equilibrium position depends on the acceleration due to gravity and the elevator. When the acceleration of the elevator ceases the block moves to the new equilibrium position.

(*a*) Angular frequency

$$\omega = \sqrt{\frac{k}{m}} \text{ or } \omega = \sqrt{\frac{500}{2}}$$

or $\omega = 15.81$ rad/s

(*b*) Equation of motion of the block (while elevator is accelerating) is,

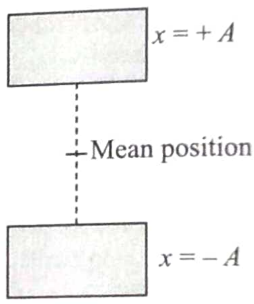
$$kx - mg = ma = m\frac{g}{3}$$

$$\therefore x = \frac{4mg}{3k} = \frac{(4)(2)(10)}{(3)(500)} = .053m$$

or $x = 5.3$ cm

(*c*) (i) In equilibrium when the elevator has zero acceleration, the equation of motion is

$$kx_0 = mg \text{ or } x_0 = \frac{mg}{k} = \frac{(2)(10)}{500} = 0.04\,\text{m} = 4\,\text{cm}$$



$\therefore$ Amplitude $A = x - x_0$

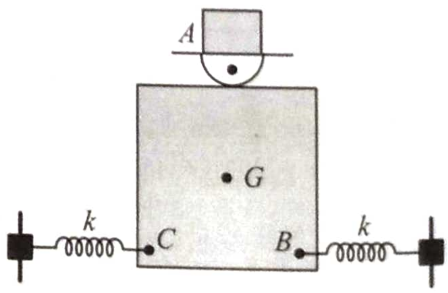
$= 5.3 - 4.0 = 1.3$ cm

(ii) At time $t = 0$, block is at $x = -A$. Therefore, substituting $x = -A$ and $t = 0$ in equation,

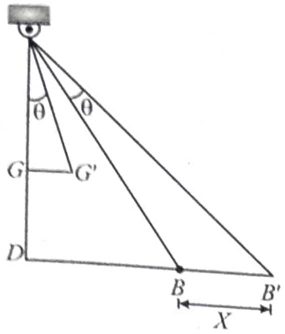
$x = A\sin(\omega t + \phi)$, we get initial phase $\phi = \frac{3\pi}{2}$

12. A 20 kg uniform square plate is suspended from a pin located at the midpoint A of one of its edges of length 0.4 and is attached to two springs, each of constant $k = 1.4\times 10^3$ N/m. If corner B is given a small displacement and released. Determine the frequency of the resulting vibration.

($g = 9.8$ m/s²)



Sol. Let '$2a$' be the sides of the square. If it is displaced by θ, then $2a = 0.4$ m, $a = 0.2$ m



Compression/elongation in each spring: $x = (AB)\theta = \sqrt{5}a\theta$

Net restoring torque about A will be:

$$\tau = -\{mg(GG') + 2kx(AD)\}$$

$$\tau = -\{mg(AG.\theta) + 2kx(2a)\}$$

$$\tau = -\{mg(a\theta) + 2\sqrt{5}ka\theta(2a)\}$$

Since $\tau \propto -\theta$

$\therefore$ Oscillations are simple harmonic in nature

Hence, $I_A \cdot \alpha = -\{mga + 4\sqrt{5}ka^2\}\theta$

Here, $I_A = I_G + m(GA)^2$

$$= m\left(\frac{4a^2 + 4a^2}{12}\right) + ma^2 = \frac{5}{3}ma^2$$

$$\left(\frac{5}{3}ma^2\right)\alpha = -\{mga + 4\sqrt{5}ka^2\}\theta$$

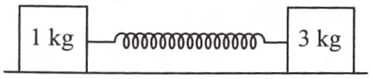
$$\therefore\ f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{mga + 4\sqrt{5}ka^2}{\frac{5}{3}ma^2}}$$

Substituting the values, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{(20)(9.8)(0.2) + \left[4\sqrt{5}\times(1.4\times10^3)(0.2)^2\right]}{\frac{5}{3}(20)(0.2)^2}}$$

or $f = 3.2$ Hz

13. Two point masses of 3.0 kg and 1.0 kg are attached to opposite ends of a horizontal spring whose spring constant in 300 Nm^{-1} as shown in figure. Find the natural frequency of vibration of the system.



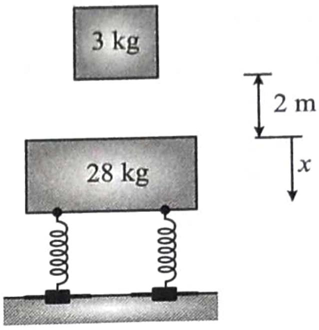
Sol. For two mass system. We take effective mass instead of mass to calculate frequency.

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{1\times3}{1+3} = \frac{3}{4}\text{ kg}$$

$$\omega^2 = \frac{k}{\mu} = \frac{300}{3/4} = 400,\ \omega = 20\text{rad/sec.}$$

$$f = \frac{10}{\pi}\text{Hz} \cong 3.2\text{ Hz}$$

14. A 3 kg block is dropped from a height of 2 m onto the initially stationary 28 kg block which is supported by four springs, each of which has a constant $k = 800$ N/m. The two blocks stick after collision. Determine the displacement x as a function of time during the resulting vibration, where x is measured from the initial position of the block as shown. Two springs which are not shown in the figure are behind the two springs shown in figure. ($g = 9.8$ m/s^2)



Sol. $k_{eq} = 4k = 4 \times 800$ N/m $= 3200$ N/m

$$\omega = \sqrt{\frac{k_{eq}}{M+m}} = \sqrt{\frac{3200}{28+3}}$$

or $\omega = 10.16$ rad/s

The equilibrium position of the system will be at a distance below the initial position where

$$k_{eq}\cdot x = mg \text{ or } x = \frac{mg}{k_{eq}} = \frac{(3)(9.8)}{3200}$$

or $x = 0.0092$ m

Velocity of 3 kg block just before collision will be:

$$v_0 = \sqrt{2gh}$$

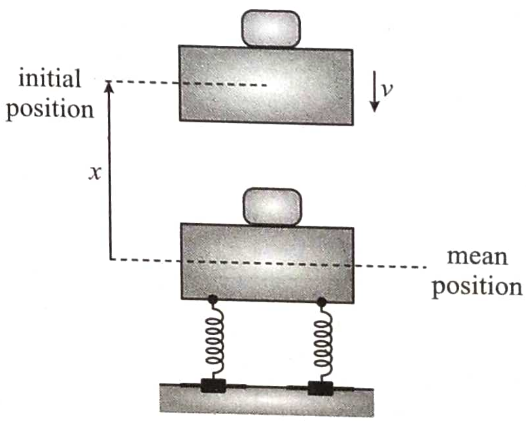
$\therefore$ From conservation of linear momentum velocity of both the blocks just after collision would be:

$$v = \frac{mv_0}{(M+m)} = \frac{(3)\sqrt{2gh}}{(28+3)}$$

$$= \frac{3\sqrt{2\times9.8\times2}}{31} = 0.606\text{ m/s}$$

Now at time $t = 0$, displacement of blocks from mean position is $x = 0.0092$ and velocity is $v = -0.606$ m/s.

Hence let the equation of displacement from mean position is



$x = a\sin(\omega t+\phi)$ then $v = a\omega\cos(\omega t+\phi)$

$\Rightarrow 0.0092 = a\sin\phi$ ...(i)

and $-0.606 = 10.16\, a\cos\phi$ ($\omega = 10.16$ rad/s)

or $-0.06 = a\cos\phi$ ...(ii)

Squaring and adding (i) and (ii), we get

$a^2 = 0.00368464$

$a = 0.060701235$ m

And from equation (i) and (ii), we get

$\phi = 171.28°$ or 2.99 rad

Therefore, displacement from its mean position at any time would be $x = 0.06\sin(10.16t + 2.99)$

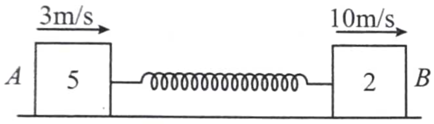
$\therefore$ Displacement at any time from the initial position would be

$s = 0.0092 - x = \{0.0092 - 0.06\sin(10.16t + 2.99)\}$m

or $s = \{9.2 - 60\sin(10.16t + 2.99)\}$mm

Note: For displacement s from initial position downward direction is positive.

15. Two blocks A (5kg) and B (2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



(a) Find the maximum extension of the spring.

(b) When does the first maximum compression occurs after start.

Sol. (a) $v_{cm} = \dfrac{5\times3+10\times2}{7} = 5\text{ ms}^{-1}$

From energy conservation $E_i = E_f$

$$\Rightarrow \frac{1}{2}5\times3^2 + \frac{1}{2}\times2\times10^2 = \frac{1}{2}7\times5^2 + \frac{1}{2}kx^2$$

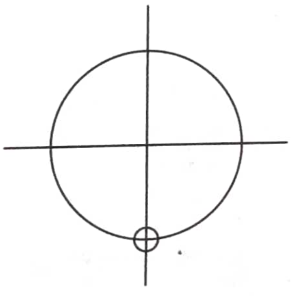
$45 + 200 = 175 + kx^2 \Rightarrow kx^2 = 70$

$$x^2 = \frac{70}{1120} = \frac{1}{16}\text{ m}$$

Maximum extension = 0.25 m

(b) $t = \dfrac{3}{4}T$

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$



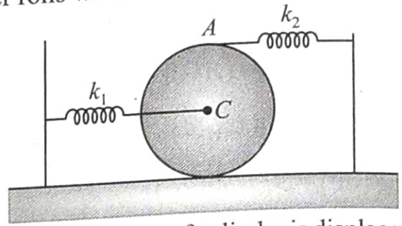
$$\mu = \frac{5\times2}{7} = \frac{10}{7}$$

$$T = 2\pi\sqrt{\frac{10}{7\times1120}} = \frac{2\pi}{28} = \frac{\pi}{14}$$

Time for first maximum compression

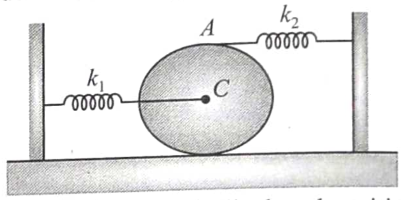
$$= \frac{3}{4}\times\frac{\pi}{14} = \frac{3\pi}{56}\text{sec}$$

16. A solid cylinder of mass $m = 1$ kg is kept in equilibrium on a horizontal surface. Two unstretched springs of force constant $k_1 = 20$ N/m and $k_2 = 20$ N/m are attached to the cylinder shown in figure. Find the period of small oscillations. Cylinder rolls without sliding.



Sol. Let at any instant centre of cylinder is displaced by x (towards left).

Then spring attached at C is compressed by x and spring attached at A elongates by $2x$. Let v be the velocity of centre of cylinder and ω its angular velocity.



Total mechanical energy in displaced position is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I_C\omega^2 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2(2x)^2$$

But $\omega = \dfrac{v}{R}$ and $I_C = \dfrac{1}{2}mR^2$

Hence, $E = \dfrac{3}{4}mv^2 + \dfrac{1}{2}k_1x^2 + 2k_2x^2$

In case of pure rolling energy is conserved

$$\therefore \frac{dE}{dt} = 0 \text{ or } \frac{3}{2}mv\left(\frac{dv}{dt}\right) + k_1x\left(\frac{dx}{dt}\right) + 4k_2x\left(\frac{dx}{dt}\right) = 0$$

$\dfrac{dx}{dt} = v$ and $\dfrac{dv}{dt} = a$ (acceleration), with these

Substitutions we get, $\dfrac{3}{2}ma = -(k_1 + 4k_2)x$

Since $a \propto -x$ oscillations are simple harmonic in nature.

Time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\left|\frac{3m}{2(k_1+4k_2)}\right|}$$

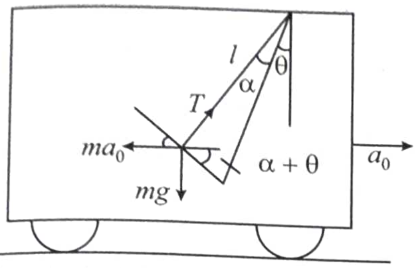
Substituting the values we get

$$T = 2\pi\sqrt{\frac{3\times1}{2(10+4\times20)}} \text{ or } T = 0.81\text{ s}$$

17. A simple pendulum is suspended from the ceiling of a car accelerating uniformly on a horizontal road. If the acceleration is a_0 and the length of the pendulum is l, find the time period of small oscillations about the mean position.

Sol. The car accelerates with acceleration a. In the reference frame of car the effective value of acceleration due to gravity is

$$g_{eff} = |\vec{g} - \vec{a}| = \sqrt{g^2 + a^2}$$



We shall work in the car frame. As it is accelerated with respect to the road, we shall have to apply a pseudo force $m a_0$ on the bob of mass m.

For mean position, the acceleration of the bob with respect to the car should be zero. If θ be the angle made by the string with the vertical, then tension, weight and the pseudo force will add to zero in this position.

Suppose at some instant during oscillation, the string is further deflected by an angle α so that the displacement of the bob is x. Taking the components perpendicular to the string, component of $T = 0$, component of $mg = mg \sin(\alpha + \theta)$ and component of $ma_0 = -ma_0 \cos(\alpha + \theta)$. Thus, the resultant component $F = m[g \sin(\alpha + \theta) - a_0 \cos(\alpha + \theta)]$.

Expanding the sine and cosine and putting $\cos\alpha \approx 1$, $\sin\alpha \approx x/l$, we get,

$$F = m\left[g\sin\theta - a_0\cos\theta + (g\cos\theta + a_0\sin\theta)\frac{x}{l}\right]$$

At $x = 0$, the force F on the bob should be zero, as this is the mean position. Thus by (i),

$$0 = m[\sin\theta - a_0 \cos\theta]$$

$$\text{Giving } \tan\theta = \frac{a_0}{g}$$

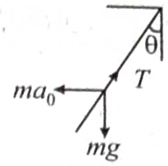
$$\text{Thus, } \sin\theta = \frac{a_0}{\sqrt{a_0^2 + g^2}}$$

$$\cos\theta = \frac{g}{\sqrt{a_0^2 + g^2}}$$

$$F = m\sqrt{g^2 + a_0^2}\,\frac{x}{l} \text{ or, } F = m\omega^2 x, \text{ where } \omega^2 = \frac{\sqrt{g^2 + a_0^2}}{l}.$$

This is an equation of simple harmonic motion with time period $t = \frac{2\pi}{\omega} = 2\pi \times \frac{\sqrt{l}}{\left(g^2 + a_0^2\right)^{1/4}}$.

As easy working rule may be found out as follows. In the mean position, the tension, the weight and the pseudo force balance. From figure, the tension is



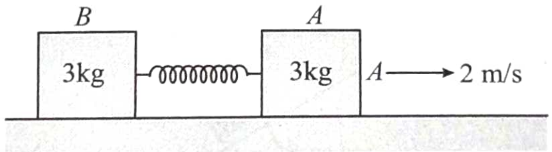
$$T = \sqrt{(ma_0)^2 + (mg)^2}$$

$$\text{or, } \frac{T}{m} = \sqrt{a_0^2 + g^2}.$$

This plays the role of effective 'g' Thus the time period is

$$t = 2\pi\sqrt{\frac{l}{T/m}} = 2\pi\frac{\sqrt{l}}{\left[g^2 + a_0^2\right]^{1/4}}.$$

18. Two blocks A (2 kg) and B (3 kg) rest up on a smooth horizontal surface are connected by a spring of stiffness 120 N/m. Initially the spring is undeformed. A is imparted a velocity of 2 m/s along the line of the spring away from B. Find the displacement of A, t second later.



Sol. $V_{CM} = \dfrac{2\times 2 + 3\times 0}{2+3} = 0.8 \text{ m/s}$

Energy of oscillation, $E = \frac{1}{2}\times 2\times (2)^2 - \frac{1}{2}\times(2+3)\times V_{CM}^2$

or $E = 4 - \frac{1}{2}\times 5\times(0.8)^2 = 2.4$ J

Let A be the maximum extension in the spring, then

or $E = \frac{1}{2}kA^2$

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\times 2.4}{120}} = 0.2 \text{ m}$$

Angular frequency of oscillation, $\omega = \sqrt{\dfrac{k}{\propto}}$

where $\propto$ = reduced mass $= \dfrac{3\times 2}{3+2} = 1.2$ kg

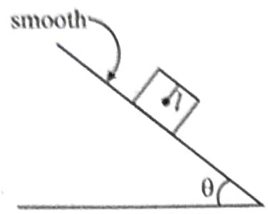
$$\therefore\ \omega = \sqrt{\frac{120}{1.2}} = 10 \text{ rad/s}$$

Amplitude of 2 kg $= \left(\dfrac{3}{3+2}\right)A = 0.12$ m

After time t, centre of mass will move a distance $d = 0.8t$. From centre of mass frame, at $t = 0$, block 2 kg is in its mean position and travelling towards positive x-axis with amplitude 0.12 m and angular frequency 10 rad/s. Hence, displacement of 2 kg after t seconds

= displacement of centre of mass × displacement of 2 kg w.r.t to centre of mass $= 0.8t + 1.2\sin 10t$

19. A box is placed on a smooth inclined plane and it is free to move. A simple pendulum is attached in the block. Find its time period.



Sol. Let F_p = Pseudo force

For equilibrium position, $T \sin\alpha + F_p = mg\sin\theta$

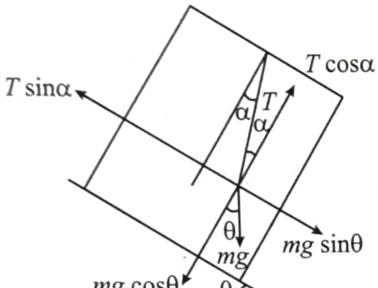
$T\sin\alpha + ma = mg\sin\theta$

Since $a = g\sin\theta$

$T\sin\theta + mg\sin\theta = mg\sin\theta$

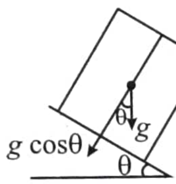
$T\sin\alpha = 0$

$\therefore a = 0$



Hence in equilibrium position the string is perpendicular to incline plane.

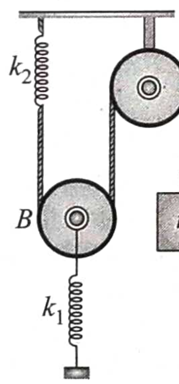
Therefore effective 'g' is $g_{eff} = g\cos\theta$



Time period $T = 2\pi\sqrt{\dfrac{\ell}{g_{eff}}}$

$$T = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}$$

20. A block of mass m is tied to one end of a string which passes over a smooth fixed pulley A and under a light smooth movable pulley B as shown in figure. The other end of the string is attached to the lower end of a spring of spring constant k_2. Find the period of small oscillations of mass m about its equilibrium position.



Sol. Let F be the extra tension (net restoring force) in the string when mass m is displaced by 'x' from its mean position. Then, $F = k_2x_2$ and $2F = k_1x_1$

where x_1 and x_2 are further (in displaced position) extension in the springs.

$$\therefore\ x_1 = \frac{2F}{k_1} \text{ and } x_2 = \frac{F}{k_2}$$

Further, $x = 2x_1 + x_2$

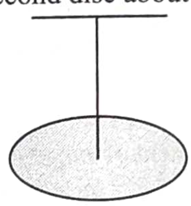
$$= \frac{4F}{k_1} + \frac{F}{k_2} = F\left(\frac{4k_2 + k_1}{k_1k_2}\right) \text{ or } F = \left(\frac{k_1k_2}{4k_2 + k_1}\right)x$$

As $F \propto x$, motion is simple harmonic in nature.

$$k_{\text{eff}} = \frac{k_1k_2}{4k_2 + k_1}$$

$$T = 2\pi\sqrt{\frac{m(4k_2 + k_1)}{k_1k_2}}$$

21. The moment of inertia of the disc used in torsional pendulu about the suspension wire is 0.2 kg - m². It oscillates with period of 2 s. Another disc is played over the first one and t time period of the system becomes 2.5 s. Fine the mome of inertia of the second disc about the wire.



Sol. As another disc is placed on the first disc moment of iner about the axis passing through the wire increases and th time period increases.

Let the torsional constant of the wire be k. The moment inertia of the first disc about the wire is $0.2\ \text{kg} - m^2$. hen the time period is

$$2s = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{0.2\,\text{kg} - \text{m}^2}{k}} \quad ...$$

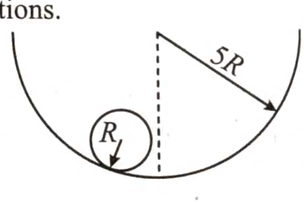
When the second disc having moment of inertia I_1 abo The wire is added, the time period is

$$2.5\,\text{s} = 2\pi\sqrt{\frac{0.2\,\text{kg} - \text{m}^2 + I_1}{k}} \quad ...($$

From (i) and (ii), $\dfrac{6.25}{4} = \dfrac{0.2\text{kgm} - \text{m}^2 + \text{I}_1}{0.2\ \text{kg} - \text{m}^2}$.

This gives $I_1 = 0.11\ \text{kg} - m^2$.

22. A solid sphere (radius = R) rolls without slipping i cylindrical through (radius = $5R$). Find the time period small oscillations.



Sol. The sphere executes pure rolling in the cylinder. The mean position is at the lowest point in the cylinder. Find the acceleration for small displacement from the mean position and compare with standard equation of SHM to find angular frequency.

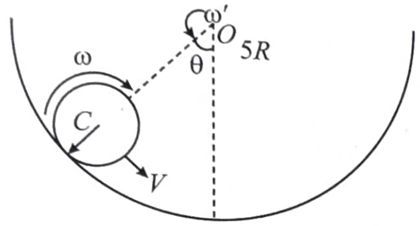
For pure rolling to take place, $v = R\omega$

ω = Angular velocity of COM of sphere C about O

$= \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$

$\therefore \frac{d\omega'}{dt} = \frac{1}{4}\frac{d\omega}{dt}$ or $\alpha' = \frac{\alpha}{4}$

$\alpha = \frac{a}{R}$ for pure rolling;



Where, $a = \frac{g\sin\theta}{I + mR^2} = \frac{5\,g\sin\theta}{7}$

As, $I = \frac{2}{5}mR^2 \therefore \alpha' = \frac{5\,g\sin\theta}{28R}$

For small θ, $\sin\theta \approx \theta$, being restoring in nature,

$\alpha' = -\frac{5g}{28R}\theta \therefore T = 2\pi\sqrt{\left|\frac{\theta}{\alpha'}\right|} = 2\pi\sqrt{\frac{28R}{5g}}$

23. Potential Energy (U) of a body of unit mass moving in one-dimension conservative force field is given by, $U = (x^2 - 4x + 3)$. All units are in S.I.

(i) Find the equilibrium position of the body.

(ii) Show that oscillations of the body about this equilibrium position are simple harmonic motion & find its time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}\,\text{m/s}$

Sol. $U = (x^2 - 4x + 3)$

(i) $F = -\frac{dU}{dx}$

$F = -2x + 4$

At equilibrium $F = 0$

$-2x + 4 = 0 \Rightarrow x = 2m$

(ii) $dF = -2dx$ similar to $dF = -\frac{\omega^2}{m}dx$ (for unit mass) as in SHM

$2 = \frac{\omega^2}{m} = \omega^2$

$\omega = \sqrt{2}$

$T = \frac{2\pi}{\omega} = \sqrt{2}\pi\text{sec}$

(iii) $A\omega = 2\sqrt{6}$

$A = \frac{2\sqrt{6}}{\sqrt{2}} \Rightarrow A = 2\sqrt{3}$ m

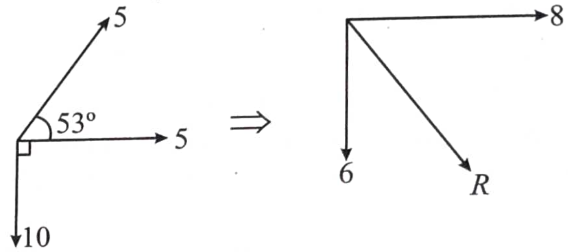
24. $x_1 = 5\sin\omega t$; $x_2 = 5\sin(\omega t + 53°)$; $x_3 = -10\cos\omega t$. Find amplitude of resultant SHM

Sol. $x_1 = 5\sin\omega t$

$x_2 = 5\sin(\omega t + 53°)$

$x_3 = -10\cos\omega t$

we can write $x_3 = 10\sin(\omega t + 270°)$



Finding the resultant amplitude by vector notation.

Resultant Amplitude

$|R| = \sqrt{8^2 + 6^2} = 10$

25. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals.

(a) 0.81 (b) 0.729

(c) 0.6 (d) 0.7

Sol. (b) $A = A_0 e^{-kt}$

$\Rightarrow 0.9\,A_0 = A_0\,e^{-5k}$ and $\alpha\,A_0 = A_0\,e^{-15k}$

Solving

$\Rightarrow \alpha = 0.729$

At mean position, K.E. is maximum where as P.E. is minimum.

Exercise-1 (Topicwise)

DISPLACEMENT, PHASE VELOCITY AND ACCELERATION IN SHM

1. A particle moving along the x-axis executes simple harmonic motion, then the force acting on it is given by Where A and k are positive constants
   (a) $-Akx$ (b) $A\cos(kx)$
   (c) $A\exp(-kx)$ (d) $A\,kx$

2. Which of the following equation does not represent a simple harmonic motion
   (a) $y = a\sin\omega t$ (b) $y = a\cos\omega t$
   (c) $y = a\sin\omega t + b\cos\omega t$ (d) $y = a\tan\omega t$

3. A simple harmonic oscillator has a period of 0.01 sec and an amplitude of 0.2 m. The magnitude of the velocity in m sec^{-1} at the centre of oscillation is
   (a) 20π (b) 100
   (c) 40π (d) 100π

4. A particle executes SHM with a period of 6 second and amplitude of 3 cm. Its maximum speed in cm/sec is
   (a) $\pi/2$ (b) π
   (c) 2π (d) 3π

5. A particle is performing simple harmonic motion with amplitude A and angular velocity ω. The ratio of maximum velocity to maximum acceleration is
   (a) ω (b) $1/\omega$
   (c) ω^2 (d) $A\omega$

6. The maximum displacement of a particle executing SHM is 1 cm and the maximum acceleration is $(1.57)^2$ cm per sec^2. Then the time period is
   (a) 0.25 sec (b) 4.00 sec
   (c) 1.57 sec (d) $(1.57)^2$ sec

7. A body executing simple harmonic motion has a maximum acceleration equal to 24 m/s^2 and maximum velocity equal to 16 m/s. The amplitude of the simple harmonic motion is
   (a) $\frac{32}{3}$ metres (b) $\frac{3}{32}$ metres
   (c) $\frac{1024}{9}$ metres (d) $\frac{64}{9}$ metres

8. A particle in SHM is described by the displacement equation $x(t) = A\cos(\omega t + \theta)$. If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is π cm/s, what is its amplitude? The angular frequency of the particle is πs^{-1}
   (a) 1 cm (b) $\sqrt{2}$ cm (c) 2 cm (d) 2.5 cm

9. The displacement x (in metre) of a particle in, simple harmonic motion is related to time t (in seconds) as
   $x = 0.01\cos\left(\pi t + \frac{\pi}{4}\right)$
   The frequency of the motion will be
   (a) 0.5 Hz (b) 1.0 Hz
   (c) $\frac{\pi}{2}Hz$ (d) π Hz

10. A simple harmonic motion having an amplitude A and time period T is represented by the equation
    $y = 5\sin\pi(t+4)\,m$ Then the values of A (in m) and T (in sec) are
    (a) $A = 5; T = 2$ (b) $A = 10; T = 1$
    (c) $A = 5; T = 1$ (d) $A = 10; T = 2$

11. A particle has simple harmonic motion. The equation of its motion is $x = 5\sin\left(4t - \frac{\pi}{6}\right)$, where x is its displacement. If the displacement of the particle is 3 units, then it velocity is
    (a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) 20 (d) 16

12. A simple pendulum performs simple harmonic motion about $X = 0$ with an amplitude A and time period T. The speed of the pendulum at $X = \frac{A}{2}$ will be
    (a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$
    (c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

13. The amplitude of a particle executing SHM is 4 cm. At the mean position the speed of the particle is 16 cm/sec. The distance of the particle from the mean position at which the speed of the particle becomes half will be
    (a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm (c) 1 cm (d) 2 cm

14. The displacement of a body executing SHM is given by $x = A\sin(2\pi t + \pi/3)$. The first time from $t = 0$ when the velocity is maximum is
    (a) 0.33 sec (b) 0.16 sec (c) 0.25 sec (d) 0.5 sec

15. The instantaneous displacement of a simple pendulum oscillator is given by $x = A\cos\left(\omega t + \frac{\pi}{4}\right)$. Its speed will be maximum at time
    (a) $\frac{\pi}{4\omega}$ (b) $\frac{\pi}{2\omega}$ (c) $\frac{\pi}{\omega}$ (d) $\frac{2\pi}{\omega}$

16. The amplitude and the periodic time of a SHM are 5cm and 6 sec respectively. At a distance of 2.5cm away from the mean position, the phase will be

(a) $5\pi/12$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/6$

17. How long after the beginning of motion is the displacement of a harmonically oscillating particle equal to one half its amplitude if the period is 24s and particle starts from rest

(a) 12s (b) 2s (c) 4s (d) 6s

18. A particle is executing SHM of amplitude 'A' and time period = 4 second. Then the time taken by it to move from the extreme position to half the amplitude is

(a) 1 sec (b) $\frac{1}{3}$ sec

(c) $\frac{2}{3}$ sec (d) $\frac{4}{3}$ sec

ENERGY IN SHM

19. The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement x from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is

(a) $x^2\omega^2/(a^2 - x^2\omega^2)$ (b) $x^2/(a^2 - x^2)$

(c) $(a^2 - x^2\omega^2)/x^2\omega^2$ (d) $(a^2 - x^2)/x^2$

20. Consider the following statements. The total energy of a particle executing simple harmonic motion depends on its

A. Amplitude

B. Period

C. Displacement from mean position

(a) A and B are correct (b) B and C are correct

(c) A and C are correct (d) A, B and C are correct

21. A particle of mass m is hanging vertically by an ideal spring of force constant K. If the mass is made to oscillate vertically, total energy of system is

(a) Maximum at extreme position

(b) Maximum at mean position

(c) Minimum at mean position

(d) Same at all position

22. For a particle executing simple harmonic motion, the kinetic energy K is given by $K = K_0 \cos^2 \omega t$. The maximum value of potential energy is

(a) K_0 (b) Zero

(c) $\frac{K_0}{2}$ (d) Not obtainable

23. The kinetic energy of a particle executing SHM is 16 J when it is in its mean position. If the amplitude of oscillations is 25 cm and the mass of the particle is 5.12 kg, the time period of its oscillation is

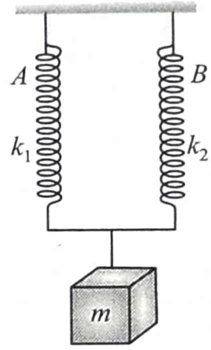
(a) $\frac{\pi}{5}$ sec (b) 2π sec

(c) 20π sec (d) 5π sec

SPRING MASS SYSTEMS

24. A mass m is suspended by means of two coiled spring which have the same length in unstretched condition as in figure. Their force constant are k_1 and k_2 respectively. When set into vertical vibrations, the period will be

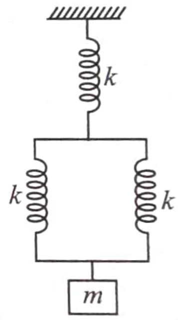
(The rod from which the mass is hanging remains horizontal)



(a) $2\pi\sqrt{\left(\frac{m}{k_1 k_2}\right)}$ (b) $2\pi\sqrt{m\left(\frac{k_1}{k_2}\right)}$

(c) $2\pi\sqrt{\left(\frac{m}{k_1 - k_2}\right)}$ (d) $2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

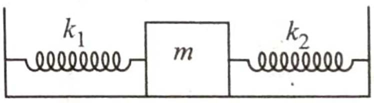
25. A body of mass 'm' hangs from three springs, each of spring constant 'k' as shown in the figure. If the mass is slightly displaced and let go, the system will oscillate with time period



(a) $2\pi\sqrt{\frac{m}{3k}}$ (b) $2\pi\sqrt{\frac{3m}{2k}}$

(c) $2\pi\sqrt{\frac{2m}{3k}}$ (d) $2\pi\sqrt{\frac{3k}{m}}$

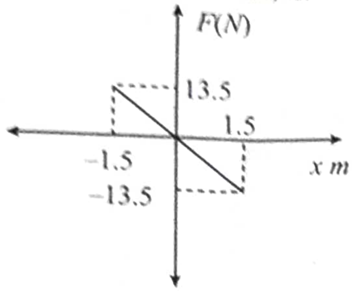
26. A block of mass m is connected between two springs (constants k_1 and k_2) as shown in the figure and is made to oscillate, the frequency of oscillation of the system shall be



(a) $\frac{1}{2\pi}\left(\frac{m}{k_1 + k_2}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{k_1 k_2}{(k_1 + k_2)m}\right)^{1/2}$

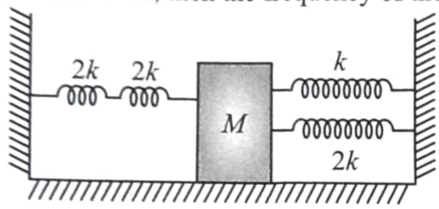
(c) $\frac{1}{2\pi}\left(\frac{k_1 + k_2}{m}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{(k_1 + k_2)m}{k_1 k_2}\right)^{1/2}$

27. A particle of mass 1 kg is undergoing SHM, for which graph between force and displacement (from mean position) as shown. Its time period, in seconds, is.



(a) $\pi/3$ (b) $2\pi/3$
(c) $\pi/6$ (d) $3/\pi$

28. Four massless springs whose force constants are $2k$, $2k$, k and $2k$ respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of the system.



(a) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$
(c) $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

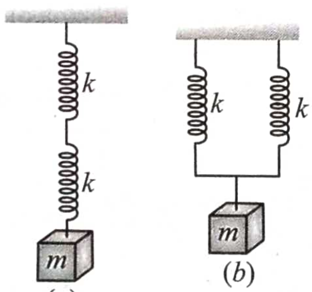
29. A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half:

(a) 1 kg (b) 2 kg (c) 3 kg (d) 4 kg

30. Two bodies M and N of equal masses are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude M to that of N is

(a) $\frac{k_1}{k_2}$ (b) $\sqrt{\frac{k_1}{k_2}}$ (c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

31. Two identical spring of constant k are connected in series and parallel as shown in figure. a mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be



(b)
(a)

(a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 4 : 1

32. A weightless spring which has a force constant oscillates with frequency n when a mass m is suspended from it. The spring is cut into two equal halves and a mass 2m is suspended from one half. The frequency of oscillation will now become

(a) n (b) $2n$
(c) $n/\sqrt{2}$ (d) $n(2)^{1/2}$

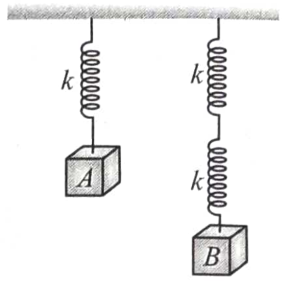
33. Infinite springs with force constant k, $2k$, $4k$ and $8k$.... respectively are connected in series. The effective force constant of the spring will be

(a) $2k$ (b) k (c) $k/2$ (d) $4k$

34. The force constants of two springs are k_1 and k_2. Both are stretched till their elastic energies are equal. If the stretching forces are F_1 and F_2, then $F_1 : F_2$ is

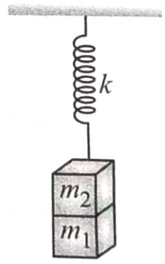
(a) $k_1 : k_2$ (b) $k_2 : k_1$
(c) $\sqrt{k_1} : \sqrt{k_2}$ (d) $k_1^2 : k_2^2$

35. The springs shown are identical. When A = 4 kg, the elongation of spring is 1 cm. If B = 6 kg, the elongation produced by it is



(a) 4 cm (b) 3 cm (c) 2 cm (d) 1 cm

36. Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. The amplitude of oscillations is



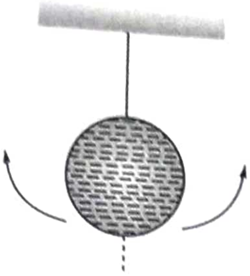
(a) $\frac{m_1 g}{k}$ (b) $\frac{m_2 g}{k}$
(c) $\frac{(m_1+m_2)g}{k}$ (d) $\frac{(m_1-m_2)g}{k}$

PENDULUM

37. To show that a simple pendulum executes simple harmonic motion, it is necessary to assume that

(a) Length of the pendulum is small
(b) Mass of the pendulum is small
(c) Amplitude of oscillation is small
(d) Acceleration due to gravity is small

38. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will



(a) Remains unchanged (b) Increase

(c) Decrease (d) Become erratic

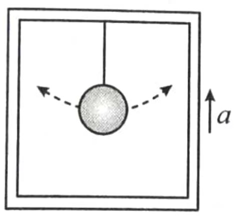
39. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitude is decreased by 5%?

(a) 6 % (b) 3 % (c) 1.5 % (d) 0 %

40. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (If it is a second's pendulum on earth)

(a) $\frac{1}{\sqrt{2}}$ sec (b) $2\sqrt{2}$ sec (c) 2 sec (d) $\frac{1}{2}$ sec

41. A scientist measures the time period of a simple pendulum as T in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is



(a) $\frac{T}{4}$ (b) $4T$ (c) $\frac{2}{\sqrt{5}}T$ (d) $\frac{\sqrt{5}}{2}T$

42. The length of the second pendulum on the surface of earth is 1 m. The length of seconds pendulum on the surface of moon, where g is 1/6th value of g on the surface of earth, is

(a) 1/6 m (b) 6 m (c) 1/36 m (d) 36 m

43. The time period of a simple pendulum in a lift descending with constant acceleration g is

(a) $T = 2\pi\sqrt{\frac{l}{g}}$ (b) $T = 2\pi\sqrt{\frac{l}{2g}}$

(c) Zero (d) Not defined

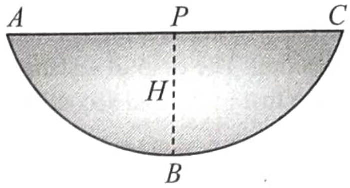
44. The ratio of frequencies of two pendulums are 2 : 3, then their length are in ratio

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

45. Two pendulums begin to swing simultaneously. The first pendulum makes 9 full oscillations when the other makes 7. Find the ratio of length of the two pendulums.

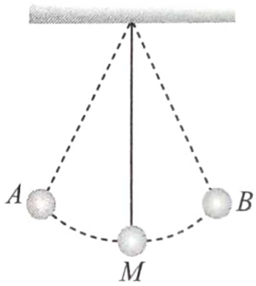
(a) $\frac{49}{81}$ (b) $\frac{7}{9}$ (c) $\frac{50}{81}$ (d) $\frac{1}{2}$

46. A simple pendulum with a bob of mass 'm' oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is 'g', then the velocity of the bob as it passes through B is



(a) mgH (b) $\sqrt{2gH}$ (c) $2gH$ (d) Zero

47. What is the velocity of the bob of a simple pendulum at its mean position, if it is able to rise to vertical height of 10 cm ($g = 9.8$ m/s^2)



(a) 2.2 m/s (b) 1.8 m/s (c) 1.4 m/s (d) 0.6 m/s

Exercise-2 (Learning Plus)

1. According to a scientist, he applied a force $F = -cx^{1/3}$ on a particle and the particle is performing SHM. No other force acted on the particle. He refuses to tell whether c is a constant or not. Assume that he had worked only with positive x then

(a) As x increases c also increases

(b) As x increases c decreases

(c) As x increases c remains constant

(d) The motion cannot be SHM

2. A particle is made to under go simple harmonic motion. Find its average acceleration in one time period.

(a) $\omega^2 A$ (b) $\frac{\omega^2 A}{2}$

(c) $\frac{\omega^2 A}{\sqrt{2}}$ (d) Zero

3. Two particles are in SHM on same straight line with amplitude A and $2A$ and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles.

(a) 45° (b) 90°
(c) 135° (d) 180°

4. Equation of SHM is $x = 10 \sin 10\pi t$. Find the distance between the two points where speed is 50π cm/sec. x is in cm and t is in seconds.

(a) 10 cm (b) 20 cm
(c) 17.32 cm (d) 8.66 cm

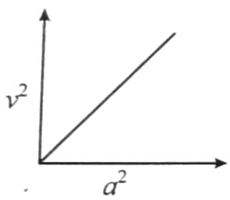
5. Two particles execute SHM of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement is half of their amplitude. The phase-difference between them is

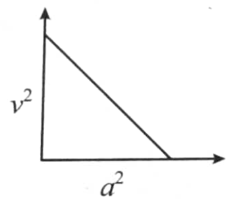
(a) 0° (b) 120°
(c) 180° (d) 135°

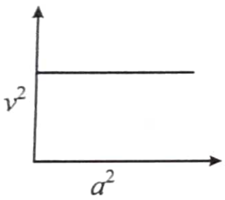
6. A particle executes SHM with time period 8 s. Initially, it is at its mean position. The ratio of distance travelled by it in the 1st second to that in the 2nd second is

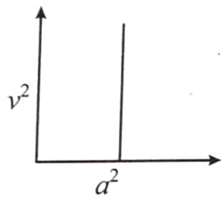
(a) $\sqrt{2}:1$ (b) $1:(\sqrt{2}-1)$
(c) $(\sqrt{2}+1):\sqrt{2}$ (d) $(\sqrt{2}-1):1$

7. A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity v is

(a) 

(b) 

(c) 

(d) 

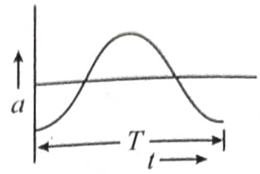
8. The $K.E.$ and $P.E$ of a particle executing SHM with amplitude A will be equal when its displacement is

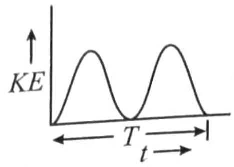
(a) $\sqrt{2}A$ (b) $\frac{A}{2}$
(c) $\frac{A}{\sqrt{2}}$ (d) $\sqrt{\frac{2}{3}}A$

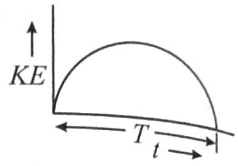
9. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds)

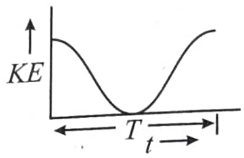
(a) 2 (b) 8
(c) 6 (d) 4

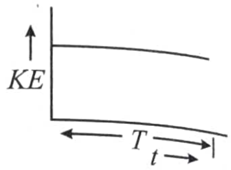
10. Acceleration versus time t graph of a body in SHM is given by a curve shown below. T is the time period. Then corresponding graph between kinetic energy KE and time t is correctly represented by



(a) 

(b) 

(c) 

(d) 

11. A particle of mass m oscillating with amplitude A and angular frequency ω. Its average energy in one time period is?

(a) $\frac{1}{2}m\omega^2A^2$ (b) $\frac{1}{4}m\omega^2A^2$
(c) $m\omega^2A^2$ (d) Zero

12. A body is executing simple harmonic motion. At a displacement x, its potential energy is E_1 and at a displacement y, its potential energy is E_2. The potential energy E at a displacement $(x + y)$ is

(a) $E_1 + E_2$ (b) $\sqrt{E_1^2 + E_2^2}$
(c) $E_1 + E_2 + 2\sqrt{E_1 E_2}$ (d) $\sqrt{E_1 E_2}$

13. A particle performing SHM is found at its equilibrium at $t = 1$ sec. and it is found to have a speed of 0.25 m/s at $t = 2$ sec. If the period of oscillation is 6 sec. Calculate amplitude of oscillation

(a) $\frac{3}{2\pi}$ m (b) $\frac{3}{4\pi}$ m (c) $\frac{6}{\pi}$ m (d) $\frac{3}{8\pi}$ m

14. A cork floating on the pond water executes a simple harmonic motion, moving up and down over a range of 4 cm. The time period of the motion is 1 s. At $t = 0$, the cork is at its lowest position of oscillation, the position and velocity of the cork at $t = 10.5$ s, would be

(a) 2 cm above the mean position, 0 m/s
(b) 2 cm below the mean position, 0 m/s
(c) 1 cm above the mean position, $2\sqrt{3}\pi$ m/s up
(d) 1 cm below the mean position, $2\sqrt{3}\pi$ m/s up

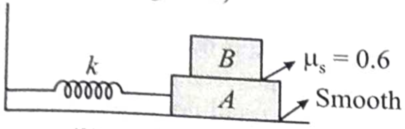
15. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

16. A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance of $a/2$ from the extreme position is

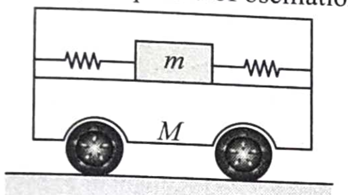
(a) a/T (b) $2a/T$ (c) $3a/T$ (d) $a/2T$

17. A block A is connected to spring and performs simple harmonic motion with a time period of 2 s. Another block B rests on A. The coefficient of static friction between A and B is $\mu_s = 0.6$. the maximum amplitude of oscillation which the system can have so that there is no relative motion between A and B is (take $\pi^2 = g = 10$)



(a) 0.3 m (b) 0.6 m (c) 0.4 m (d) 0.52 m

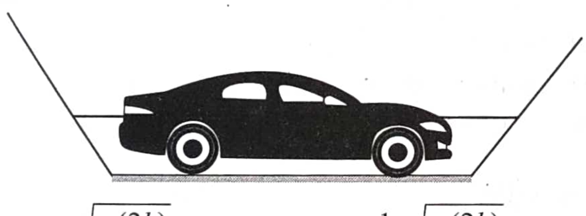
18. Two springs, each of spring constant k, are attached to a block of mass m as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass M. If the block is displaced by x cm and released, the period of oscillation is:



(a) $T = 2\pi\sqrt{\dfrac{Mm}{2k}}$ (b) $T = 2\pi\sqrt{\dfrac{(M+m)}{kmM}}$

(c) $T = 2\pi\sqrt{\dfrac{mM}{2k(M+m)}}$ (d) $T = 2\pi\sqrt{\dfrac{(M+m)^2}{k}}$

19. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is k and surface is frictionless. The car is displaced from mean position by x cm and released. At the mean position the ribbons are undeformed. Vibration period is



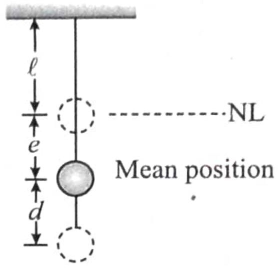
(a) $2\pi\sqrt{\dfrac{m(2k)}{k^2}}$ (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{m(2k)}{k^2}}$

(c) $2\pi\sqrt{\dfrac{m}{k}}$ (d) $2\pi\sqrt{\dfrac{m}{k+k}}$

20. A vertical spring carries a 5 kg body and is hanging in equilibrium, an additional force is applied so that the spring is further stretched. When released from this position, it performs 50 complete oscillations in 25 s, with an amplitude of 5 cm. The additional force applied is

(a) 80 N (b) $80\pi^2$ N (c) $4\pi^2$ N (d) 4 N

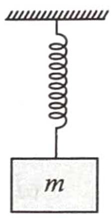
21. An elastic string of length ℓ supports a heavy particle of mass m and the system is in equilibrium with elongation produced being e as shown in figure. The particle is now pulled down below the equilibrium position through a distance d ($<e$) and released. The angular frequency and amplitude for SHM is



(a) $\sqrt{\dfrac{g}{e}}, d$ (b) $\sqrt{\dfrac{g}{\ell}}, 2e$

(c) $\sqrt{\dfrac{g}{d+e}}, d$ (d) $\sqrt{\dfrac{g}{e}}, 2d$

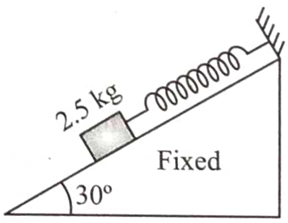
22. A block of mass 'm' is suspended from a spring and executes vertical SHM of time period T as shown in Figure. The amplitude of the SHM is A and spring is never in compressed state during the oscillation. The magnitude of minimum force exerted by spring on the block is



(a) $mg - \dfrac{4\pi^2}{T^2}mA$ (b) $mg + \dfrac{4\pi^2}{T^2}mA$

(c) $mg - \dfrac{\pi^2}{T^2}mA$ (d) $mg + \dfrac{\pi^2}{T^2}mA$

23. A smooth inclined plane having angle of inclination 30° with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end as shown in figure. If the mass is taken 2.5 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is

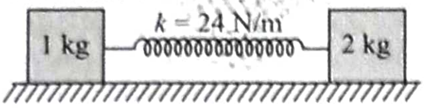


(a) 0.707 (b) 7.07 (c) 1.414 (d) 14.14

24. The potential energy of a simple harmonic oscillator of mass 2 kg in its mean position is 5 J. If its total energy is 9 J and its amplitude is 0.01 m, its time period would be

(a) $\dfrac{\pi}{10}$ sec (b) $\dfrac{\pi}{20}$ sec (c) $\dfrac{\pi}{50}$ sec (d) $\dfrac{\pi}{100}$ sec

25. The two blocks shown here rest on a frictionless surface. If they are pulled apart by a small distance and released at $t = 0$, the time when

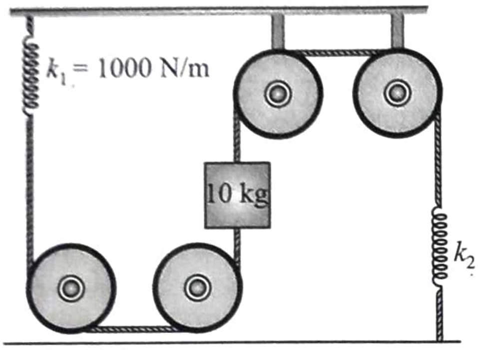


1 kg block comes to rest can be

(a) $\frac{2\pi}{3}$ sec (b) π sec.

(c) $\frac{\pi}{2}$ sec (d) $\frac{\pi}{3}$ sec

26. A block of mass 10 kg is in equilibrium as shown in figure. Initially the springs have same stretch. If the block is displaced in vertical direction by a small amount, then the angular frequency of resulting motion is (assume strings are never slack)



(a) $10\sqrt{2}$ (b) $10\sqrt{5}$

(c) $5\sqrt{2}$ (d) $5\sqrt{5}$

27. A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then maintain the orientation. If the sphere is then released, its time period of the oscillation will be:

(Assume restoring Torque is proportional to angle turned)

(a) π second (b) $\sqrt{2}\,\pi$ second

(c) 2π second (d) 4π second

28. Two pendulums have time periods T and $5T/4$. They start SHM at the same time from the mean position. After how many oscillations of the smaller pendulum they will be again in the same phase

(a) 5 (b) 4 (c) 11 (d) 9

29. A ball is hung vertically by a thread of length ℓ from a point 'P' of an inclined wall that makes an angle 'α' with the vertical. The thread with the ball is then deviated through a small angle 'β' '($\beta > \alpha$) and set free. Assuming the wall to be perfectly elastic, the period of such pendulum is/are

(a) $2\sqrt{\frac{\ell}{g}}\left[\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$ (b) $2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2}+\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(c) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(\frac{\alpha}{\beta}\right)\right]$ (d) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(-\frac{\alpha}{\beta}\right)\right]$

30. A ring is suspended at a point on its rim and it behaves as a second's pendulum when it oscillates such that its centre move in its own plane. The radius of the ring would be $(g = \pi^2)$

(a) 0.5 m (b) 1.0 m (c) 0.67 m (d) 1.5 m

31. A circular disc has a tiny hole in it, at a distance z from its center. Its mass is M and radius R $(R > z)$. A horizontal shaft is passed through the hole and held fixed so that the disc can freely swing in the vertical plane. For small disturbance, the disc performs SHM whose time period is the minimum for $z =$

(a) $R/2$ (b) $R/3$ (c) $R/\sqrt{2}$ (d) $R/\sqrt{3}$

32. A simple pendulum of length l and a mass m of the bob is suspended in a car that is travelling with a constant speed v around a circle of radius R. If the pendulum undergoes small oscillations about its equilibrium position, the frequency of its oscillation will be

(a) $\frac{1}{2\pi}\sqrt{\frac{g}{l}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{\left(g^2+\frac{v^4}{R^2}\right)^{\frac{1}{2}}}{l}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{v^2}{Rl}}$

33. When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed

(a) The resulting motion is uniform circular motion

(b) The resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones

(c) The resulting motion is an elliptical motion, symmetrical about the lines of motion of the components

(d) The two SHM will cancel each other

34. The position of a particle in motion is given by $y = C\sin\omega t + D\cos\omega t$ w.r.t. origin. Then motion of the particle is

(a) SHM with amplitude $C + D$

(b) SHM with amplitude $\sqrt{C^2 + D^2}$

(c) SHM with amplitude $\frac{(C+D)}{2}$

(d) Not SHM

35. The position vector of a particle moving in x-y plane is given by $\vec{r} = (A\sin\omega t)\,\hat{i} + (A\cos\omega t)\,\hat{j}$ then motion of the particle is

(a) SHM (b) On a circle

(c) On a straight line (d) With constant acceleration

36. The displacement of a particle executing periodic motion is given by $y = 4\cos^2(0.5t)\sin(1000t)$. The given expression is composed by minimum

(a) Four SHMs (b) Three SHMs

(c) One SHM (d) None of these

37. The amplitude of the vibrating particle due to superposition of two SHMs, $y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin\omega t$ is

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. A particle moves on the x-axis according to the equation $x = x_0 \sin^2 \omega t$. The motion is simple harmonic
   (*a*) With amplitude $x_0/2$
   (*b*) With amplitude $2x_0$
   (*c*) With time period $\frac{2\pi}{\omega}$
   (*d*) With time period $\frac{\pi}{\omega}$

2. The potential energy of a particle of mass 0.1 kg, moving along x-axis, is given by $U = 5x(x-4)$ J where x is in metres. It can be concluded that
   (*a*) The particle is acted upon by a constant force
   (*b*) The speed of the particle is maximum at $x = 2$ m
   (*c*) The particle executes simple harmonic motion
   (*d*) The period of oscillation of the particle is $\pi/5$ s

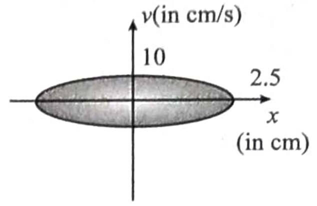
3. If y, v and a represent displacement, velocity and acceleration at any instant for a particle executing SHM, which of the following statements are true?
   (*a*) v and y may have same direction
   (*b*) v and a may have same direction
   (*c*) a and y may have same direction
   (*d*) a and v may have opposite direction

4. The equation of motion for an oscillating particle is given by $x = 3\sin(4\pi t) + 4\cos(4\pi t)$, where x is in mm and t is in second
   (*a*) The motion is simple harmonic
   (*b*) The period of oscillation is 0.5 s
   (*c*) The amplitude of oscillation is 5 mm
   (*d*) The particle starts its motion from the equilibrium

5. Which of the following functions represent SHM?
   (*a*) $\sin 2\omega t$
   (*b*) $\sin^2 \omega t$
   (*c*) $\sin \omega t + 2\cos \omega t$
   (*d*) $\sin \omega t + \cos 2\omega t$

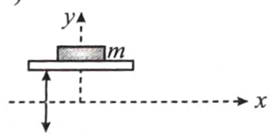
6. The displacement of a particle varies according to the relation $x = 3\sin 100t + 8\cos^2 50t$. Which of the following is/are correct about this motion?
   (*a*) The motion of the particle is not SHM
   (*b*) The amplitude of the SHM of the particle is 5 units
   (*c*) The amplitude of the resultant SHM is $\sqrt{73}$ units
   (*d*) The maximum displacement of the particle from the origin is 9 units

7. The figure shows a graph between velocity and displacement (from mean position) of a particle performing SHM



   (*a*) The time period of the particle is 1.57 s
   (*b*) The maximum acceleration will be 40cm/s^2
   (*c*) The velocity of particle is $2\sqrt{21}$ cm/s when it is at a distance 1 cm from the mean position
   (*d*) Angular frequency is 2 rad/s.

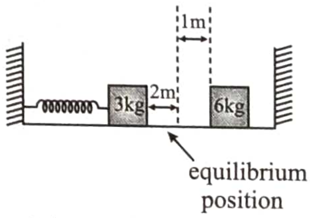
8. As shown in figure a horizontal platform with a mass m placed on it is executing SHM along y-axis. If the amplitude of oscillation is 2.5 cm, the minimum period of the motion for the mass not to be detached from the platform is close to ($g = 10$ m/sec^2)



   (*a*) $\frac{10}{\pi}$ s (*b*) $\frac{\pi}{10}$ s (*c*) $\frac{\pi}{\sqrt{10}}$ s (*d*) $\frac{1}{\sqrt{10}}$ s

9. If a SHM is given by $y = (\sin \omega t + \cos \omega t)$ m, which of the following statements are true?
   (*a*) The amplitude is 1 m
   (*b*) The amplitude is $\sqrt{2}$ m
   (*c*) Time is considered from $y = 1$ m
   (*d*) Time is considered from $y = 0$ m

10. Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg block is attached to a spring with a force constant $k = 900$ Nm^{-1} which is compressed 2 m from beyond the equilibrium position. The 6 kg mass is at rest at 1m from mean position. 3 kg mass strikes the 6 kg mass and the two stick together.



   (*a*) Velocity of the combined masses immediately after the collision is 10 m s^{-1}
   (*b*) Velocity of the combined masses immediately after the collision is 5 m s^{-1}
   (*c*) Amplitude of the resulting oscillation is $\sqrt{2}$ m
   (*d*) Amplitude of the resulting oscillation is $\sqrt{5}/2$ m

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 11 to 13): A 2 kg block hangs without vibrating at the bottom end of a spring with a force constant of 400 N/m. The top end of the spring is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of 5 m/s^2 when the acceleration suddenly ceases at time $t = 0$ and the car moves upward with constant speed. ($g = 10$ m/s^2)

11. What is the angular frequency of oscillation of the block after the acceleration ceases?

(*a*) $10\sqrt{2}$ rad/s

(*b*) 20 rad/s

(*c*) $20\sqrt{2}$ rad/s

(*d*) 32 rad/s

12. The amplitude of the oscillations is

(*a*) 7.5 cm (*b*) 5 cm

(*c*) 2.5 cm (*d*) 1 cm

13. The initial phase angle observed by a rider in the elevator, taking upward direction to be positive and positive extreme position to have $\pi/2$ phase constant, is equal to

(*a*) Zero

(*b*) $\pi/2$ rad

(*c*) π rad

(*d*) $3\pi/2$ rad

Comprehension (Q. 14 to 16): Equations of displacements during three different SHMs of a particle can be written as $x_1 = A_1\cos\omega_1 t$, $x_2 = A_2\cos(\omega_2 t + \delta_2)$ and $y = A_3\cos(\omega_3 t + \delta_3)$. Here x_1 and x_2 are the displacement of the particle along x-axis during two different SHMs. y is the displacement of the particle during SHM along y-axis. x and y axis are orthogonal to each other.

Consider the superposition of two SHMs along x-axis. The resultant displacement is $x = x_1 + x_2$ and resultant amplitude is $A = [A_1^2 + A_2^2 + 2A_1A_2\cos(\omega_1 - \omega_2)t]^{1/2}$ if $\omega_1 \neq \omega_2$ and $\delta_2 = 0$.

Superposition of two SHMs in perpendicular direction may result in SHM along a straight line or motion along a circle or an ellipse in clockwise or counter clockwise direction.

14. In superposition of two SHMs along x-axis, identify the correct options.

(*a*) If $A_1 = A_2$ and $\delta_2 = \pi$ the particle is always at rest

(*b*) If $\delta_2 = 0$, $A_1 = A_2$ and $\omega_1 \neq \omega_2$ then the resultant motion is a harmonic motion with angular frequency $\omega = \dfrac{\omega_1 + \omega_2}{2}$.

(*c*) If $\omega_1 \neq \omega_2$, $\delta_2 = 0$ and $A_1 = A_2$ then the particle is at origin at time 't' $= \dfrac{3\pi}{\omega_1 - \omega_2}$

(*d*) If $A_1 = A_2$, $\omega_1 = \omega_2$ and $\delta_2 = 0$ then the amplitude of resultant SHM is $2A_1$

15. In superposition of two SHMs along mutually perpendicular directions. Identify the correct options

(*a*) If $\omega_1 = \omega_3$ and $\delta_3 = 0$ then the resulting motion is SHM with amplitude $\sqrt{A_1^2 + A_3^2}$ along line $y = \dfrac{A_3}{A_1}x$

(*b*) If $\omega_1 = \omega_3$, $A_1 = A_3$ and $\delta_3 = \dfrac{3\pi}{2}$ then the resulting motion is a circular motion in counter clockwise direction

(*c*) If $\omega_1 = \omega_2$, $A_1 \neq A_3$ and $\delta_3 = \dfrac{\pi}{2}$ then the resulting motion is on elliptical path in clockwise direction

(*d*) If $\omega_1 = \omega_3$ and $\delta_3 = \pi$ then the resulting motion is on elliptical path in counter clockwise direction

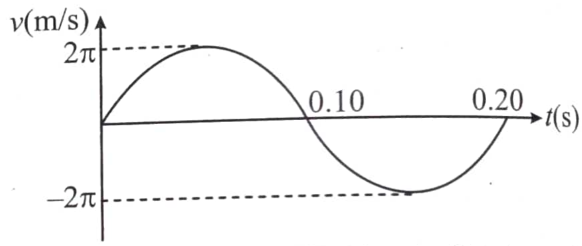
16. In superposition of two SHMs of same frequency along x-axis the initial phase angle of resultant SHM is given as

(*a*) $\tan^{-1}\dfrac{A_1\sin\delta_2}{A_2 + A_1\cos\delta_2}$ (*b*) $\tan^{-1}\dfrac{A_2\sin\delta_2}{A_1 + A_2\cos\delta_2}$

(*c*) $\tan^{-1}\dfrac{A_2\cos\delta_2}{A_2 + A_1\sin\delta_2}$ (*d*) $\tan^{-1}\dfrac{A_2\cos\delta_2}{A_1 + A_2\sin\delta_2}$

MATCH THE COLUMN TYPE QUESTIONS

17. A simple harmonic oscillator consists of a block attached to a spring with $k = 200$ N/m. The block slides on a frictionless horizontal surface with equilibrium point $x = 0$. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in the left column with the values given in the right column. (use $\pi^2 = 10$)



	Column-I		Column-II
A.	The block's mass in kg	p.	– 0.20
B.	The block's displacement at $t = 0$ in metres	q.	–200
C.	The block's acceleration at $t = 0.10$ s in m/s^2	r.	0.20
D.	The block's maximum kinetic energy in joule	s.	4.0

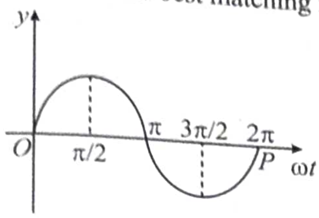
(*a*) A-(r); B-(p); C-(q); D-(s)

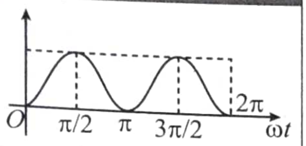
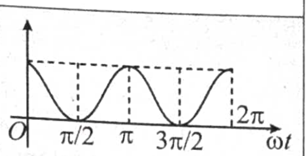
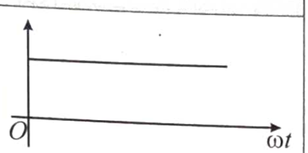
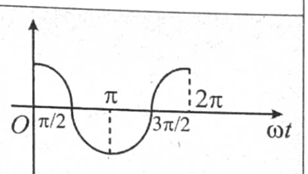
(*b*) A-(q); B-(p); C-(r); D-(s)

(*c*) A-(p); B-(q); C-(q); D-(r)

(*d*) A-(r); B-(q); C-(r); D-(q)

18. The graph plotted between phase angle (ϕ) and displacement of a particle from equilibrium position (y) is a sinusoidal curve as shown below. Then the best matching is



	Column-I		Column-II
A.	KE versus phase angle curve	p.	
B.	PE versus phase angle curve	q.	
C.	TE versus phase angle curve	r.	
D.	Velocity versus phase angle curve	s.	

(a) A-(p); B-(q); C-(r); D-(s)
(b) A-(q); B-(p); C-(r); D-(s)
(c) A-(q); B-(p); C-(s); D-(r)
(d) A-(q); B-(r); C-(s); D-(p)

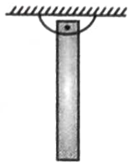
NUMERICAL TYPE QUESTIONS

19. A 50 g block is attached to a vertical spring whose stiffness constant is 4 N/m. The block is released at the position where the spring is unextended. What is the maximum extension of the spring in meter?

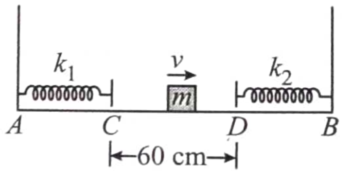
20. A block is resting on a piston which is moving vertically with a SHM of period 1.0 s. At what amplitude (in m) of motion will the block and piston separate?

21. In an oscillating block-spring system, the spring constant is 2.45 N/m, the amplitude is 16 cm, and the maximum speed is 56 cm/s. What is the mass of the block in kg?

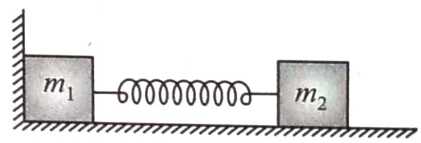
22. If the time period of simple harmonic motion of a uniform rod shown here is π sec, what is its length (in cm)?



23. Two light springs of force constants k_1 and k_2 and a very small block of mass m are in one line AB on smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the springs is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation (in sec) of the block to nearest integer. ($k_1 = 1.8$ N/m, $k_2 = 3.2$ N/m, $m = 200$ g) (Take $\pi = \frac{22}{7}$)



24. On a corner on a smooth horizontal surface are kept 2 masses m_1 and m_2 attached to a spring as shown. The spring has a spring constant of 150 N/m. Now m_2 is suddenly given a velocity to the left. It is seen that spring first comes to natural length at $\frac{\pi}{5\sqrt{2}}$ sec. It next comes to natural length at $\frac{\pi}{10\sqrt{2}}$ sec. What is mass m_1 (in kg).



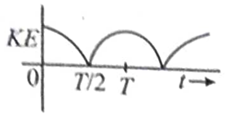
Exercise-4 (Past Year Questions)

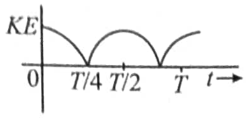
JEE MAIN

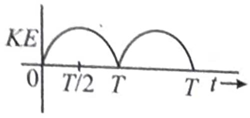
1. A particle performs simple harmonic motion with amplitude A. Its speed is tripled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is **(2016)**

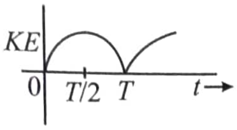
(a) $\frac{A}{3}\sqrt{41}$
(b) $3A$
(c) $A\sqrt{3}$
(d) $\frac{7A}{3}$

2. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like **(2017)**

(a) 

(b) 

(c) 

(d) 

3. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **(2018)**

(a) 7.1 N/m (b) 2.2 N/m
(c) 5.5 N/m (d) 6.4 N/m

4. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by **(2019)**

(a) 10^{-3} rad/s (b) 1 rad/s
(c) 10^{-1} rad/s (d) 10^{-5} rad/s

5. A simple haromic motion is represented by:

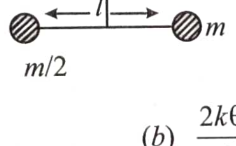
$y = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$ cm

The amplitude and time period of the motion are **(2019)**

(a) 10 cm, $\frac{2}{3}s$ (b) 10 cm, $\frac{3}{2}s$

(c) 5 cm, $\frac{3}{2}s$ (d) 5 cm, $\frac{2}{3}s$

6. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be **(2019)**



(a) $\frac{3k\theta_0^2}{l}$ (b) $\frac{2k\theta_0^2}{l}$

(c) $\frac{k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

7. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2 then: **(2019)**

(a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$

(c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

8. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of the particle at $t = 210$ s will be **(2019)**

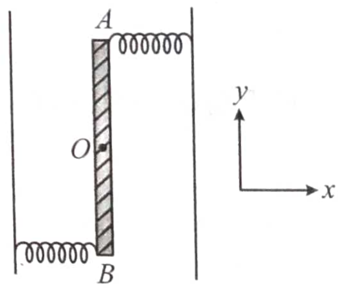
(a) 1/9 (b) 1
(c) 2 (d) 1/3

9. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The oscillation of the same pendulum on the planet would be **(2019)**

(a) $\frac{\sqrt{3}}{2}s$ (b) $\frac{2}{\sqrt{3}}s$

(c) $\frac{3}{2}s$ (d) $2\sqrt{3}\,s$

10. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is: **(2019)**



(a) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

11. A rod of mass 'M' and length '$2L$' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance '$L/2$' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to: **(2019)**

(a) 0.77 (b) 0.57
(c) 0.37 (d) 0.17

12. A particle executes simple harmonic motion with an amplitude of 5 cm. when the particle is at 4 cm from the mean position, the magnitude of its velocity is SI units is equal to that of its acceleration. Then, its periodic time in seconds is: **(2019)**

(a) $\frac{4\pi}{3}$ (b) $\frac{3}{8}\pi$

(c) $\frac{8\pi}{3}$ (d) $\frac{7}{3}\pi$

13. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to: **(2019)**

(a) 100 s (b) 20 s

(c) 10 s (d) 50 s

14. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is: **(2019)**

(a) $4T\sqrt{\frac{1}{15}}$ (b) $2T\sqrt{\frac{1}{10}}$

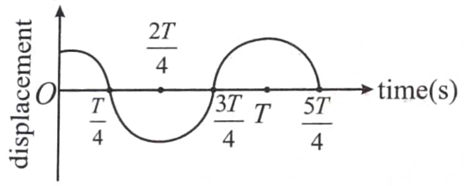
(c) $4T\sqrt{\frac{1}{14}}$ (d) $2T\sqrt{\frac{1}{14}}$

15. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to **(2019)**

(a) 13s (b) 7 s

(c) 27 s (d) 4 s

16. The displacement time graph of a particle executing S.H.M is given in figure (sketch is schematic and not to scale) **(2020)**



Which of the following statements is/are true for this motion?

A. The force is zero at $t = \frac{3T}{4}$

B. The acceleration is maximum at $t = T$

C. The speed is maximum at $t = \frac{T}{4}$

D. The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(a) B, C and D (b) A and D

(c) A, B and C (d) A, B and D

17. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is **(2020)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 1 (d) $\frac{1}{2}$

18. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. The ratio $\frac{T_1}{T_2}$ will be **(2020)**

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{3}$ (d) $\frac{3}{\sqrt{2}}$

19. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0\sin^2\omega t$, where 'y' is measured from the lower end of unstretched spring. Then ω is **(2020)**

(a) $\sqrt{\frac{5g}{y_0}}$ (b) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$

(c) $\sqrt{\frac{g}{2y_0}}$ (d) $\sqrt{\frac{g}{y_0}}$

20. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is **(2021)**

(a) $\cos(\omega t)+\cos(2\omega t)+\cos(3\omega t)$

(b) $\sin^2(\omega t)$

(c) $\sin(\omega t)+\cos(\omega t)$

(d) $3\cos\left(\frac{\pi}{4}-2\omega t\right)$

21. The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$.

The measured value of the length of pendulum is 10 cm known to a 1 mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1 s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to be nearest integer is **(2021)**

(a) 2% (b) 3%

(c) 5% (d) 4%

22. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion.
(Take ln 2 = 0.693) **(2021)**

(a) 0.69×10^2 kg s^{-1} (b) 3.3×10^2 kg s^{-1}
(c) 1.16×10^{-2} kg s^{-1} (d) 5.7×10^{-3} kg s^{-1}

23. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is **(2021)**

(a) $\frac{K_1}{K_2}$ (b) $\sqrt{\frac{K_1}{K_2}}$
(c) $\frac{K_2}{K_1}$ (d) $\sqrt{\frac{K_2}{K_1}}$

24. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to drop to half of its initial value? (ln 2 = 0.693) **(2021)**

(a) 17.32s (b) 34.65s
(c) 0.034s (d) 15.01s

25. Time period of a simple pendulum is inside a lift when the lift is stationary. If the lift moves upwards with an acceleration $g/2$, the time period of pendulum will be: **(2021)**

(a) $\sqrt{3}T$ (b) $\sqrt{\frac{2}{3}}T$
(c) $\frac{T}{\sqrt{3}}$ (d) $\sqrt{\frac{3}{2}}T$

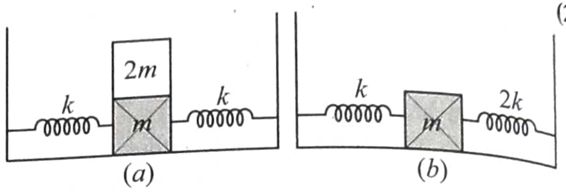
26. A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as **(2021)**

(a) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$ (b) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$
(c) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$ (d) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

27. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position. **(2021)**

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

28. In figure (a), mass '$2m$' is fixed on mass 'm' which is attached to two springs of spring constant k. In figure (b), mass 'm' is attached to two spring of spring constant 'k' and '$2k$'. If mass 'm' in (a) and (b) are displaced by distance 'x' horizontally and then released, then time period T_1 and T_2 corresponding to (a) and (b) respectively follow the relation. **(2022)**



(a) $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$ (b) $\frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$
(c) $\frac{T_1}{T_2} = \sqrt{\frac{2}{3}}$ (d) $\frac{T_1}{T_2} = \frac{\sqrt{3}}{2}$

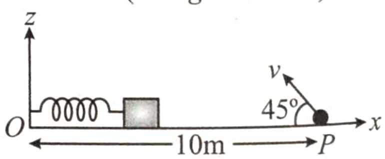
29. A mass 0.9 kg, attached to a horizontal spring, executes SHM with an amplitude A_1. When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move together with amplitude A_2. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha - 1}$, then the value of α will be ______. **(2022)**

30. The time period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination α, is given by. **(2022)**

(a) $2\pi\sqrt{L/(g\cos\alpha)}$ (b) $2\pi\sqrt{L/(g\sin\alpha)}$
(c) $2\pi\sqrt{L/g}$ (d) $2\pi\sqrt{L/(g\tan\alpha)}$

JEE ADVANCED

31. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3} rad/s$. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 cm from O. If the pebble hits the block at $t =$ 1s, the value of v is (take $g = 10$ m/s^2) **(2012)**



(a) $\sqrt{50}$ m/s (b) $\sqrt{51}$ m/s
(c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

32. A particle of mass m is attached to one end of a mass–less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5\ u_0$, it collies elastically with a rigid wall. After this collision **(2013)**

(*a*) The speed of the particle when it returns to its equilibrium position is u_0.

(*b*) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(*c*) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(*d*) The time at which the particle passes throughout the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

33. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (*i*) when the block is at x_0; and (*ii*) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m ($< M$) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M? **(2016)**

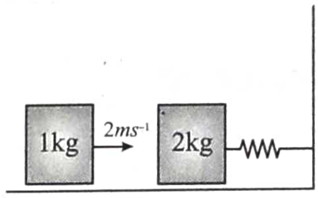
(*a*) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged.

(*b*) The final time period of oscillation in both the cases is same

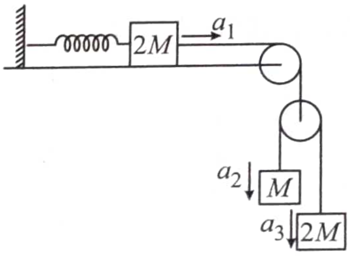
(*c*) The total energy decreases in both the cases

(*d*) The instantaneous speed at x_0 of the combined masses decreases in both the cases

34. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is ______. **(2018)**



35. A block of mass $2M$ is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and $2M$ using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2, and a_3 as shown in figure. The system is released from rest with the spring in its unstretched stale. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct? [g is the acceleration due to gravity. Neglect friction] **(2019)**



(*a*) $x_0 = \frac{4Mg}{k}$

(*b*) When spring achieves an extension of $x_0/2$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(*c*) $a_2 - a_1 = a_1 - a_3$

(*d*) At an extension of $x_0/4$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

36. On a frictionless horizontal plane, a bob of mass $m = 0.1$ kg is attached to a spring with natural length $l_0 = 0.1$ m. The spring constant is $k_1 = 0.009$ Nm^{-1} when the length of the spring $l > l_0$ and is $k_2 = 0.016$ Nm^{-1} when $l < l_0$. Initially the bob is released from $l = 0.15$ m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T = (n\pi)$ s, then the integer closest to n is__________. **(2022)**

ANSWER KEY

CONCEPT APPLICATION

1. (*b*)	**2.** (*a*)	**3.** (*d*)	**4.** (*c*)	**5.** (*c*)	**6.** (*c*)	**7.** (*a*)	**8.** (*d*)	**9.** (*a*)	**10.** (*c*)
11. (*c*)	**12.** (*b*)	**13.** (*c*)	**14.** (*d*)	**15.** (*c*)	**16.** (*c*)	**17.** (*d*)	**18.** [0.314]	**19.** [0.898]	

EXERCISE-1 (TOPICWISE)

1. (*a*)	**2.** (*d*)	**3.** (*c*)	**4.** (*b*)	**5.** (*b*)	**6.** (*b*)	**7.** (*a*)	**8.** (*b*)	**9.** (*a*)	**10.** (*a*)
11. (*d*)	**12.** (*a*)	**13.** (*a*)	**14.** (*a*)	**15.** (*a*)	**16.** (*d*)	**17.** (*c*)	**18.** (*c*)	**19.** (*d*)	**20.** (*a*)
21. (*d*)	**22.** (*a*)	**23.** (*a*)	**24.** (*d*)	**25.** (*b*)	**26.** (*c*)	**27.** (*c*)	**28.** (*b*)	**29.** (*c*)	**30.** (*d*)
31. (*c*)	**32.** (*a*)	**33.** (*c*)	**34.** (*c*)	**35.** (*b*)	**36.** (*a*)	**37.** (*c*)	**38.** (*b*)	**39.** (*d*)	**40.** (*b*)
41. (*c*)	**42.** (*a*)	**43.** (*d*)	**44.** (*d*)	**45.** (*a*)	**46.** (*b*)	**47.** (*c*)			

EXERCISE-2 (LEARNING PLUS)

1. (*a*)	**2.** (*d*)	**3.** (*c*)	**4.** (*c*)	**5.** (*b*)	**6.** (*b*)	**7.** (*b*)	**8.** (*c*)	**9.** (*b*)	**10.** (*a*)
11. (*a*)	**12.** (*c*)	**13.** (*a*)	**14.** (*a*)	**15.** (*c*)	**16.** (*c*)	**17.** (*b*)	**18.** (*c*)	**19.** (*c*)	**20.** (*c*)
21. (*a*)	**22.** (*a*)	**23.** (*d*)	**24.** (*d*)	**25.** (*d*)	**26.** (*a*)	**27.** (*d*)	**28.** (*a*)	**29.** (*d*)	**30.** (*a*)
31. (*c*)	**32.** (*c*)	**33.** (*b*)	**34.** (*b*)	**35.** (*b*)	**36.** (*b*)	**37.** (*c*)			

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*a,d*)	**2.** (*b,c,d*)	**3.** (*a,b,d*)	**4.** (*a,b,c*)	**5.** (*a,c*)	**6.** (*b,d*)	**7.** (*a,b,c*)	**8.** (*b,d*)	**9.** (*b,c*)	**10.** (*a,c*)
11. (*a*)	**12.** (*c*)	**13.** (*d*)	**14.** (*b, c, d*)	**15.** (*a, b, c*)	**16.** (*b*)	**17.** (*a*)	**18.** (*b*)	**19.** [0.25]	**20.** [0.25]
21. [0.2]	**22.** [375]	**23.** [3]	**24.** [1]						

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*d*)	**2.** (*b*)	**3.** (*a*)	**4.** (*a*)	**5.** (*a*)	**6.** (*c*)	**7.** (*a*)	**8.** (*d*)	**9.** (*d*)	**10.** (*c*)
11. (*c*)	**12.** (*c*)	**13.** (*b*)	**14.** (*a*)	**15.** (*b*)	**16.** (*c*)	**17.** (*a*)	**18.** (*b*)	**19.** (*c*)	**20.** (*d*)
21. (*b*)	**22.** (*c*)	**23.** (*d*)	**24.** (*b*)	**25.** (*b*)	**26.** (*d*)	**27.** (*b*)	**28.** (*a*)	**29.** [16]	**30.** (*a*)

JEE Advanced

31. (*a*)	**32.** (*a, d*)	**33.** (*a, b, d*)	**34.** [2.09]	**35.** (*c*)	**36.** [6]

CHAPTER

18 Waves

INTRODUCTION

A wave is a disturbance which travels or propagates and transports energy and momentum from one point to another without the transport of matter. The ripples on a pond, the sound we hear, visible light, radio and TV signals are a few examples of waves.

Types of Waves

(i) **Mechanical Waves:** Require material medium (elasticity and inertia) for their propagation. These waves are also called elastic waves, water waves and sound waves are example of mechanical waves.

(ii) **Electromagnetic or non-mechanical Waves:** Do not require any material medium for their propagation, such as light and TV signals. Elasticity or inertia of medium do not affect the propagation of such waves.

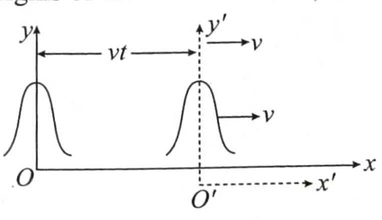
Classification of waves based an direction of particle vibration and direction of propagation:

- **Transverse wave:** Velocity of wave and velocity of particles are perpendicular to each other. Ex. EM waves.
- **Longitudinal wave:** Velocity of wave and velocity of particles are in same direction. Ex. Sound waves.

WAVE FUNCTION

The disturbance created by a wave is represented by **wave function**. For a string, the wave function is a displacement; whereas for sound waves, it is displacement, pressure or density fluctuation. In the case of light or radio waves, the wave function is either an electric or magnetic field vector.

(*a*) **Mathematical Representation of Wave Function:** Consider a disturbance or a pulse travelling along x-direction with a velocity v. Let us look at this pulse from two different frames of reference. The xy-frame is stationary, whereas the other $x'y'$ frame is moving with velocity v along x-axis, as shown in the figure. We assume that the origins of the two frames coincide at $t = 0$.



In the moving frame, the pulse appears to be at rest, since both the pulse and the $x'y'$-frame are moving with the same velocity v. Therefore, at any time the vertical displacement y' at position x' is given by some function $f(x')$ that describes the shape of the pulse; $y' = f(x')$ ...(i)

In the stationary frame, the pulse has the same shape but it is moving with a velocity v. It means that the displacement y is a function of both x and t.

The coordinates of any point on the pulse as measured in the two frame are related as, $y' = y$, $x' = x - vt$

Thus, Eq. (i) may be modified as, $y = f(x - vt)$ ...(ii)

This equation represents a wave motion along +ve x-direction.

Any given feature (**phase**) of the pulse, for example, its peak, has a fixed value of x'. It means that

$x' = x - vt = \text{constant}$ ...(iii)

The quantity $(x - vt)$ is called the **phase** of the wave function.

Differentiating Eq. (iii) w.r.t. time, we get $\frac{dx}{dt} = v$

where v is the **wave velocity** or **phase velocity**. It is the velocity at which a particular phase of the disturbance travels through space. If the wave is travelling along the negative x-axis, the wave function is given by Eq. (ii) modified as $y = f(x + vt)$

In general, the wave motion in one dimension is given by $y = f(x \mp vt)$

(*b*) **The Wave Equation:** A travelling wave satisfies a differential equation, called the linear wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{dt^2}$$

Any function of space and time which satisfies above differential equation is a travelling wave.

Functions such as $y = A \sin \omega t$ or $y = A \sin kx$ do not satisfy above equation, hence do not represent waves. On the other hand; functions such as

$A \sin (kx - \omega t)$, $A \sin kx \sin \omega t$, $[A \sin (kx - \omega t) + B \cos (kx + \omega t)]$, $\sqrt{(ax + bt)}$, $(ax - bt)^2$, $Ae^{-B(x-vt)^2}$ or $A \cos^2 (kx - \omega t)$ satisfy the wave equation, and hence these are wave functions.

Note:

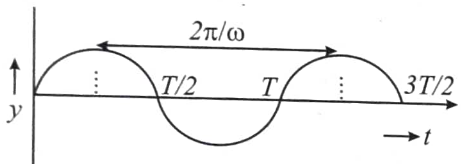
- For a function to be wave function, the three quantities x, t and v must appear in the linear combinations $(x - vt)$ or $(x + vt)$. Thus, $(x - vt)^2$ is acceptable but $(x^2 - v^2t^2)$ is not (because it is not linear).
- Negative sign between t and x implies that the wave is travelling along positive x-axis and vice-versa.

VARIOUS TERMS RELATED TO WAVE MOTION

The general equation of a travelling sinusoidal wave is

$$y = A\sin(kx - \omega t + \delta) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \delta\right]$$

1. **Amplitude (A):** The maximum displacement of a vibrating particle of the medium from the mean position. 'A' represents amplitude in $y = A\sin(\omega t - kx)$
2. **Time period (T):** Time taken to complete one oscillation and denoted by T.



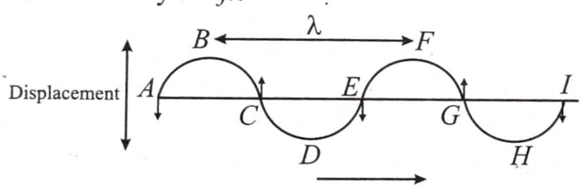
x = constant

3. **Wave Frequency (f):** Number of vibrations made per second by the particles and is denoted by f.

   $f = 1/T$; Unit: Hz

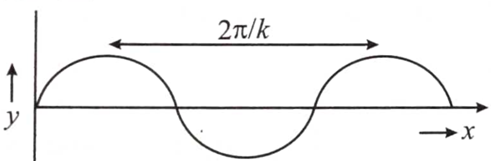
   Angular Frequency: $\omega = 2\pi f$; Unit: rad/sec
4. **Wave Length (λ):** The distance between two consecutive particle in the same phase or the distance travelled by the wave in one periodic time. It is denoted by λ

   $\because$ Wave Velocity $v = f\lambda$



Direction of Propagation of Wave

Other Relations:



t = constant

Position of different particles at some instant in a string

$$v = f\lambda = \frac{\lambda}{T} = \frac{\omega}{2\pi}\lambda = \frac{\omega}{k}$$

where k is defined as wave number.

The term δ is called phase constant is determined from initial conditions.

Note:

- As $v = f\lambda$

  f = frequency of wave, which is a constant.

  λ = wavelength

  $\Rightarrow \lambda \propto v$

  As a wave changes medium its speed and wavelength changes but frequency remains same.

5. **Relation between slope and velocity of particle:**

$$y = f(x - vt),\ v_p = -\frac{\partial y}{\partial t} = -vf'(x - vt),\ \frac{\partial y}{\partial x} = \text{slope} = f'(x - vt)$$

$$\Rightarrow v_p = -v\frac{\partial y}{\partial x} = -v \times \text{slope}$$

6. **Relation between phase difference and path difference**

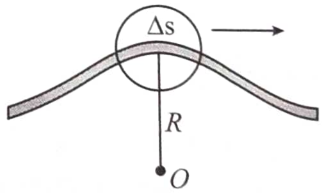
$$\frac{\Delta\phi}{\Delta x} = \frac{2\pi}{\lambda}$$

where $\Delta\phi$ = Phase difference and Δx = Path difference

VELOCITY OF A WAVE

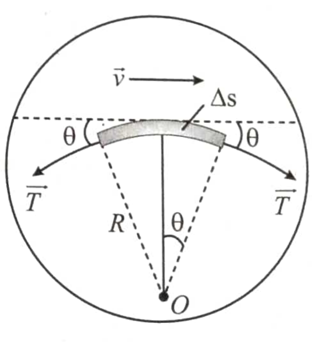
Speed of Transverse Wave on a Stretched String

Consider a pulse moving on a taut string to the right with a uniform speed v measured relative to a stationary frame of reference. Instead of staying in this reference frame, it is more convenient to choose a different inertial reference frame that moves along with the pulse with the same speed as the pulse so that the pulse is at rest within the frame. This change of reference frame is permitted because of validity of Newton's laws in both a stationary frame, and one that moves with constant velocity. In our new reference frame, all elements of the string move to the left: a given element of the string initially to the right of the pulse moves to the left, rises up and follows the shape of the pulse, and then continues to move to the left. Figure (a) shows such an element at the instant it is located at the top of the pulse.



(a)

The small element of the string of length as shown in figure (a) and magnified in figure (b), forms an approximate arc of a circle of radius R.



(b)

In the moving frame of reference (which moves to the right at a speed v along with the pulse), the shaded elements moves to the left with a speed v. This element has a centripetal acceleration equal to v^2/R, which is supplied by components of the force $\vec{T}$ whose magnitude is the tension in the string. The force $\vec{T}$ acts on both sides of the element and is tangent to the arc as shown in figure (b). The horizontal components of $\vec{T}$ cancel, and each vertical component $T\sin\theta$ acts on radially towards the arc's centre. Hence, the total radial force on the element is $2T\sin\theta$. Since the element is small, θ is small, and we can therefore use the small-angle approximation $\sin\theta \approx \theta$. So the total radial force is $F_r = 2T\sin\theta \approx 2T\theta$.

Further, the element has a mass $m = \mu\Delta s$. Because the element forms part of a circle and subtends an angle 2θ at the centre, $\Delta s = R(2\theta)$, and $m = \mu\Delta s = 2\mu R\theta$.

Applying Newton's second law to this element in the radial direction gives

$$F_r = \frac{mv^2}{R}$$

$$2T\theta = \frac{2\mu R\theta v^2}{R} \rightarrow v = \sqrt{\frac{T}{\mu}}$$

Note that this derivation is based on the assumption that the pulse height is small relative to the length of the string. Using this assumption, we were able to use the approximation $\sin\theta \approx \theta$. Furthermore, the model assumes the tension T is not affected by the presence of the pulse; therefore, T is the same at all points on the string. Finally, this proof does not assume any particular shape for the pulse. Therefore, a pulse of any shape travels along the string with speed $v = \sqrt{T/\mu}$ without any change in pulse shape.

Further,

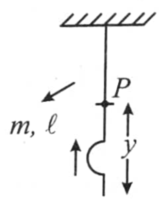
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\text{stress}}{\text{density}}} = \frac{2}{D}\sqrt{\frac{T}{\pi\rho}}$$

$$\boxed{\mu = A\rho = \frac{\pi D^2}{4}\rho}$$

D = diameter of string, ρ = density

Wave Speed on a Hanging Rope

A uniform rope of mass m and length ℓ hangs from a ceiling. The mass per unit length of the rope is $\mu = \frac{m}{\ell}$.



The speed of the wave at point P at a distance y from the lower end is $v = \sqrt{\frac{T}{\mu}}$ where Tension T at point $P = \mu \times yg$.

Thus,

$$\Rightarrow v = \sqrt{\frac{\mu yg}{\mu}} = \sqrt{yg} \quad \text{...(i)}$$

Note that it is a function of y.

Further, rewrite equation (i) as $\frac{dy}{dt} = \sqrt{yg}$

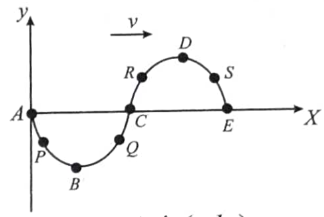
Integrating the above equation within applicable limits, we get

$$\int_0^\ell \frac{dy}{\sqrt{y}} = \sqrt{g}\int_0^t dt \Rightarrow t = 2\sqrt{\ell/g}$$

Thus, the time a transverse wave takes to travel the length of the rope is given by $t = 2\sqrt{\frac{\ell}{g}}$.

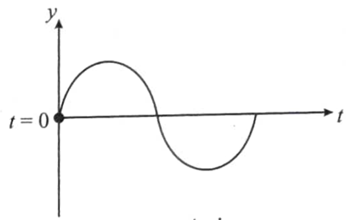
ANALYSIS OF HARMONIC WAVES

1. For a transverse wave
   $y = A\sin(\omega t - kx)$
   At $t = 0$, snapshot of the wave is



$y = A\sin(-kx)$

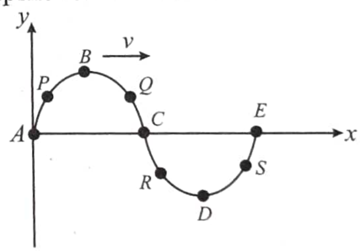
For the particle at $x = 0$



$y = A\sin\omega t$

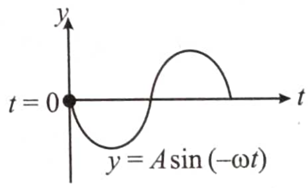
- Particles at A, P, S, E are moving upwards at $t = 0$ (slope negative)
- Particles at Q, C, R are moving downwards (slope positive)
- Particles at B and D are at rest at $t = 0$ (slope = 0)

2. For wave $y = A\sin(kx - \omega t)$
   At $t = 0$, snapshot of the wave is



$y = A\sin(kx)$

For the particle at $x = 0$



$y = A\sin(-\omega t)$

- Particles at A, P, S, E are moving downwards at $t = 0$
- Particles at Q, C, R are moving upwards at $t = 0$
- Particles at B and D are at rest at $t = 0$

Important Points:

- Particle velocity, $v_P = -v\left(\frac{\partial y}{\partial x}\right)$ where v = wave velocity and $\frac{\partial y}{\partial x}$ = slope of the wave
- In general, points with positive slope $\left(\frac{\partial y}{\partial x}\right)$ move downward.
- The points with negative slope move upward.
- The points with maximum slope (A, C, E) have maximum velocity.
- The points with zero slope are at rest.

Train Your Brain

Example 1: The wave function of a pulse is given by $y = \frac{3}{2+(x-4t)^2}$, where y is in metres and t is seconds. Determine the wave velocity of the pulse and indicate the direction of propagation of the wave.

Sol. On comparing the given expression with, $y = f(x - vt)$ we get the velocity of the wave as, $v = 4$ m/s

Since, there occurs negative sign between x and t in the given expression, the wave propagates along the +ve x-axis.

Example 2: The displacement of a particle of a string carrying a traveling wave is given by: $y = (3.0 \text{ cm}) \sin 6.28 (0.50x - 50t)$, where x is in cm and t in s. Find (a) amplitude, (b) wavelength, (c) frequency, (d) speed of the wave.

Sol. On comparing with the standard wave equation

$$y = A\sin(kx - \omega t) = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

we see that, Amplitude = A = 3.0 cm,

$$\text{Wavelength} = \lambda = \frac{1}{0.50} \text{ cm} = 2.0 \text{ cm},$$

$$\text{and the frequency } f = \frac{1}{T} = 50 \text{ Hz}$$

The speed of the wave is $v = f\lambda$

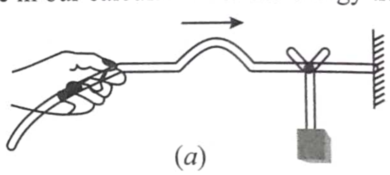
$$= \left(50 \text{ s}^{-1}\right)(2.0 \text{ cm}) = 100 \text{ cm s}^{-1}$$

Concept Application

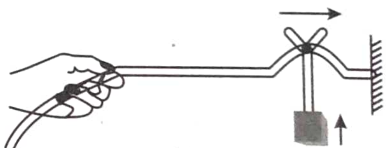
1. A transverse wave travels along the Z-axis. The particles of the medium must move
   (a) Along the Z-axis (b) Along the X-axis
   (c) Along the Y-axis (d) In the X-Y plane.
2. A wave going in a solid
   (a) Must be longitudinal (b) May be longitudinal
   (c) Must be transverse (d) May be transverse.
3. A wave moving in a gas
   (a) Must be longitudinal (b) May be longitudinal
   (c) Must be transverse (d) May be transverse.
4. Two particles A and B have a phase difference of π when a sine wave passes through the region.
   (a) A oscillates at half the frequency of B.
   (b) A and B move in opposite directions.
   (c) A and B must be separated by half of the wave-length.
   (d) The displacements at A and B have equal magnitudes.

POWER TRANSMITTED BY A WAVE

Waves transport energy through medium as they propagate. For example, consider an object is hanging on a stretched string and a pulse is sent down the string as in figure (a). When the pulse meets the suspended object, the object is momentarily displaced upward as in figure (b). In the process, energy is transferred to the object and appears as an increase in the gravitational potential energy of the object-earth system. We will examine the rate at which energy is transported along a string, assuming a one dimensional sinusoidal wave in our calculation of the energy transferred.

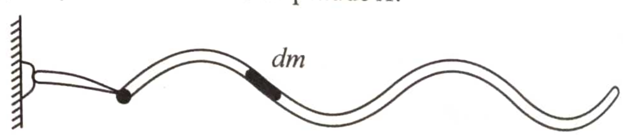


(a)



(b)

Consider a sinusoidal wave travelling on a string. The source of the energy is some external agent at the left end of the string, which docs work in producing the oscillations. We can consider the string to be a non-isolated system. As the external agent performs work on the end of the string, moving it up and down, energy enters the system of the string and propagates along its length. Let us focus our attention on an infinitesimal element of the string of length dx and mass dm. Each such element moves vertically with simple harmonic motion. Therefore, we can model each element of the string as a simple harmonic oscillator, with the oscillation in the y direction. All elements have the same angular frequency ω and the same amplitude A.



The average power transmitted by the wave equals the energy contained in one wavelength divided by the period of the wave.

The kinetic energy K associated with a moving particle is $K = 1/2\ mv^2$. If we apply this equation to the infinitesimal element, the kinetic energy dK of this element is

$$dK = \frac{1}{2}(dm)v_y^{\,2}$$

where v_y is the transverse speed of the element. If μ is the mass per unit length of the string, the mass dm of the element of length dx is equal to μdx.

Substituting for the general transverse speed of a simple harmonic oscillator.,

$$dK = \frac{1}{2}\mu[-\omega A\cos(kx-\omega t)]^2\,dx$$

$$= \frac{1}{2}\mu\omega^2 A^2\cos^2(kx-\omega t)dx$$

If we take a snapshot of the wave at time $t = 0$, the kinetic energy of a given element is

$$dK = \frac{1}{2}\mu\omega^2 A^2\cos^2(kx)dx$$

Integrating the above expression over all the siring elements in a wavelength of the wave gives the total kinetic energy K, in one wavelength.

$$K_A = \int dK = \int_0^{\lambda}\frac{1}{2}\mu\omega^2 A^2\cos^2(kx)dx = \frac{1}{2}\mu\omega^2 A^2\int_0^{\lambda}\cos^2(kx)dx$$

$$= \frac{1}{2}\mu\omega^2 A^2\left[\frac{1}{2}x+\frac{1}{4k}\sin 2kx\right]_0^{\lambda} = \frac{1}{2}\mu\omega^2 A^2\left[\frac{1}{2}\lambda\right] = \frac{1}{4}\mu\omega^2 A^2\lambda$$

In addition to kinetic energy, there is potential energy associated with each clement of the string due to its stretching when it moves from its mean position to say a height h where its length changes to $dl = \sqrt{dx^2+dy^2}$. The work by tension in stretching the element from dx to dl is stored as potential energy in the string. A similar analysis to that above for the total potential energy U_A in one wavelength gives

$$U_A = \frac{1}{4}\mu\omega^2 A^2\lambda$$

The total energy in one wavelength of the wave is the sum of the potential and kinetic energies:

$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda$$

As the wave moves along the string, this amount of energy passes through a point on the siring during a time interval of one period of the oscillation. Therefore, the average power P (or rate of energy transfer) associated with the mechanical wave, is

$$P = \frac{E_\lambda}{T} = \frac{1/2\mu\omega^2 A^2\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2\left(\frac{\lambda}{T}\right)$$

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

INTENSITY OF THE WAVE

The flow of energy per unit area of cross section of the string in the unit time is known as the intensity of the wave. Thus,

$$I = \frac{\text{power}}{\text{area of cross section}} = \frac{(1/2)\mu\omega^2 A^2 v}{s} = \frac{(1/2)\rho s\omega^2 A^2 v}{s}$$

$$I = \frac{1}{2}\rho\omega^2 A^2 v$$

This is the average intensity transmitted through the siring.

ENERGY DENSITY

Energy per unit volume is called energy density.

$$E = \frac{\text{Energy Intensity}}{\text{Energy Velocity}} = \frac{\frac{1}{2}\rho\omega^2 A^2 v}{v} = \frac{1}{2}\rho\omega^2 A^2$$

Train Your Brain

Example 3. A string with linear mass density $\mu = 5 \times 10^{-2}$ kg m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60 Hz and an amplitude of 6 cm?

Sol. The speed of the wave on the string is

$$v = \sqrt{\frac{T}{\mu}} = \left(\frac{80.0\text{ N}}{5.00\times10^{-2}\text{kg/m}}\right)^{1/2} = 40.0\text{ m/s}$$

The angular frequency ω of the sinusoidal waves on the string is

$\omega = 2\pi f = 2\pi(60.0\text{ Hz}) = 377\text{ s}^{-1}$

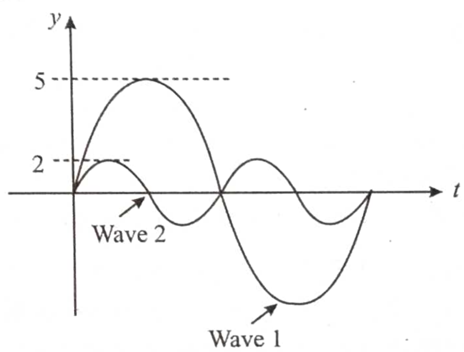
Thus, the required power is

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

$$= \frac{1}{2}(5.00\times10^{-2}\text{ kg/m})(377\text{s}^{-1})^2$$

$$\times (6.00 \times 10^{-2}\text{ m})^2 (40.0\text{ m/s}) = 512\text{ W}$$

Example 4: y-t curves in the figure shown represent two waves in the same medium. Find the ratio of their average intensities.



Sol. $\dfrac{I_1}{I_2} = \dfrac{\omega_1^2 A_1^2}{\omega_2^2 A_2^2} = \dfrac{f_1^2\times A_1^2}{f_2^2\times A_2^2} = \dfrac{1\times25}{4\times4} = \dfrac{25}{16}$

Concept Application

5. Sinusoidal waves of 5 cm amplitude are to be transmitted along a string that has a linear mass density of 4×10^{-2} kg/m. The source can deliver a maximum power of 300 W and the string is under a tension of 100 N. Find the highest frequency (in Hz, rounded off to nearest integer) at which the source can operate.

6. A sine wave on a string is described by the wave function, $y = 0.15\sin(0.80x - 50t)$ where x and y are in meters and t is in seconds. The mass per unit length of this string is 12.0 g/m. Determine (i) the speed of the wave, (ii) the wavelength, (iii) the frequency and (iv) the power transmitted to the wave. Quote the results in SI units.

PRINCIPLE OF SUPERPOSITION OF WAVES & INTERFERENCE

Principle of Superposition of Waves

Superposition principle states that the net disturbance at a given place and time caused by a number of waves in the same space is the vector sum of the disturbances which would have been produced by each wave independent of the other, that is, resultant displacement is

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \_\_\_\_\_\_$$

The value of resultant displacement depends on

(i) Amplitude of waves

(ii) Phase difference and path difference of waves

(iii) Frequency of waves

(iv) Direction of wave motion

Phenomenon observed due to superposition depends on above points (i), (ii), (iii), (iv).

Interference

When waves of equal frequency and nearly equal amplitude are superimposed, interference occurs.

At a time t, at point x two waves of equal frequency $y_1 = a_1\sin(\omega t - k_1x + \delta_1)$ and $y_2 = a_2\sin(\omega t - k_2x + \delta_2)$ are super-imposed, then for the resultant wave

Amplitude A: $A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\Delta\phi$

Intensity I: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$

where $\Delta\phi = -(k_2 - k_1)x + \delta_2 - \delta_1$ is the phase difference between the waves at the point of interference.

Intensity of resultant wave changes periodically from minimum to maximum and maximum to minimum from one point to another point

(i) **Constructive Interference:** It occurs at points where phase difference is

$\Delta\phi = 2n\pi \qquad (n = 0,1,2,3 .....)$

or path difference $\Delta x = (n\lambda)$

$\Rightarrow I_{max} \propto (a_1 + a_2)^2$

(ii) **Destructive Interference:** It occurs at point where phase difference is

$\Delta\phi = (2n - 1)\pi \qquad (n = 0,1,2,3 .....)$

or path difference $\Delta x = \left(n - \frac{1}{2}\right)\lambda$

$\Rightarrow I_{min} \propto (a_1 - a_2)^2$

Train Your Brain

Example 5: Two waves of the same frequency but of amplitude in the ratio 1 : 3 are superimposed. What is the ratio of maximum to minimum intensity.

Sol. Let the two amplitudes be a and $3a$.

Maximum amplitude $= a + 3a = 4a$

$I_{max} = (4a)^2 = 16a^2$

Minimum amplitude $= 3a - a = 2a$

$I_{min} = (2a)^2 = 4a^2$

$\therefore \frac{I_{max}}{I_{min}} = \frac{16a^2}{4a^2} = \frac{4}{1}$

Example 6: Consider interference between waves from two sources of intensities I and $4I$. Find the intensities at points where the phase difference is (i) $\pi/2$ (ii) π

Sol. Resultant intensity

$I_R = I + 4I + 4I\cos\Delta\phi$

(i) $\Delta\phi = \pi/2$

$\therefore I_R = 5I + 4I\cos(\pi/2) = 5I$

(ii) $\Delta\phi = \pi$

$I_R = 5I + 4I\cos\pi = I$

Example 7: Two sources of intensities I and $4I$ are used in interference experiment. Find the intensity at points where the waves from the sources super - impose with a phase difference of (i) 0, (ii) $\pi/2$, (iii) π.

Sol. Resultant intensity are

(i) $I_R = I_1 + I_2 + 2\sqrt{(I_1 I_2)}\cos\Delta\phi$

$= I + 4I + 4I\cos 0° = 9I$

(ii) $I_R = I + 4I + 4I\cos(\pi/2) = 5I$

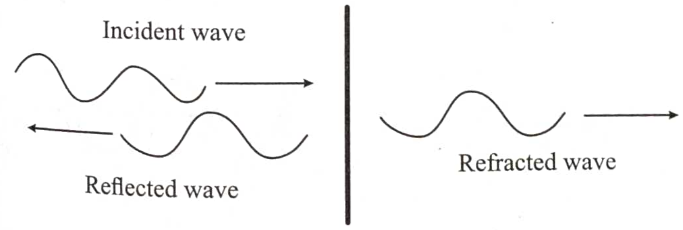
(iii) $I_R = I + 4I + 4I\cos\pi = I$

Concept Application

7. Consider two waves passing through the same string. Principle of superposition for displacement says that the net displacement of a particle on the string is sum of the displacements produced by the two waves individually. Suppose we state similar principles for the net velocity of the particle and the net kinetic energy of the particle. Such a principle will be valid for
   (a) Both the velocity and the kinetic energy
   (b) The velocity but not for the kinetic energy
   (c) The kinetic energy but not for the velocity
   (d) Neither the velocity nor the kinetic energy.
8. Two wave pulses travel in opposite directions on a string and approach each other. The shape of one pulse is inverted with respect to the other.
   (a) The pulses will collide with each other and vanish after collision.
   (b) The pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
   (c) The pulses will pass through each other but their shapes will be modified.
   (d) The pulses will pass through each other without any change in their shapes.
9. Two sine waves travel in the same direction in a medium. The amplitude of each wave is A and the phase difference between the two waves is 120°. The resultant amplitude will be
   (a) A (b) $2A$ (c) $4A$ (d) $32A$
10. When two waves with same frequency and constant phase difference interfere,
    (a) There is a gain of energy
    (b) There is a loss of energy
    (c) The energy is redistributed and the distribution changes with time
    (d) The energy is redistributed and the distribution remains constant in time.

REFLECTION AND REFRACTION OF WAVE

Consider a plane progressive wave travelling in a certain medium (say 1) having the equation $y_1 = A_i \sin(\omega t - k_1 x)$ where A_i is the amplitude.



If A_r and A_t be the amplitudes of the reflected and transmitted waves, then their respective equations can be written as

$$y_2 = A_r \sin(\omega t + k_1 x) \qquad \text{...(ii)}$$

$$y_3 = A_t \sin(\omega t - k_2 x) \qquad \text{...(iii)}$$

For simplicity let us assume that the wave gets reflected at $x = 0$.

Since the wave is continuous, the resultant displacement at the two sides of the interface should be equal, i.e.,

$y_1 + y_2 = y_3$ at $x = 0$

From Eqs. (i), (ii) and (iii), we get on substituting $x = 0$

$$A_i + A_r = A_t \qquad \text{...(iv)}$$

From the condition of continuity of slope at the interface (i.e., $x = 0$), we have,

$$A_i + A_r = A_t \qquad \text{...(v)}$$

$$\frac{d(y_1)}{dx} + \frac{d(y_2)}{dx} = \frac{d(y_3)}{dx}$$

Obtaining $\frac{d(y_1)}{dx}, \frac{d(y_2)}{dx}, \frac{d(y_3)}{dx}$ from equation (i), (ii) and (iii), and substituting these in equation (v) and remembering that $x = 0$, we get, $-A_i k_1 + A_r k_1 = -A_t k_2$

$$A_i\left(\frac{2\pi}{\lambda_1}\right) - A_r\left(\frac{2\pi}{\lambda_1}\right) = A_t\left(\frac{2\pi}{\lambda_2}\right) \qquad \text{...(vi)}$$

Since $v = f\lambda$ and f is constant, so,

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \qquad \text{...(vii)}$$

From equation (vi) and (vii),

$$A_i - A_r = \frac{v_1}{v_2} A_t \qquad \text{...(viii)}$$

Solving equation (iv) and (viii) for A_r and A_t, we have

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_i \text{ and } A_t = \left(\frac{2v_2}{v_2 + v_1}\right) A_i \qquad \text{...(ix)}$$

Cases:

- When $v_1 > v_2$, i.e., medium 1 is rare and 2 is denser, $|A_i|$ = negative and $|A_r|$ and $|A_t|$ both are individually less than $|A_i|$. Negative value of A_r indicates that the reflected wave suffers a phase change of π.
- When $v_1 < v_2$, both A_r and A_t are positive. Also $A_t > A_i$.
- **Fixed end of a string:** Since the fixed end is equivalent to a string of infinite linear mass density, $v_2 = \sqrt{(T/\mu)} = 0$, and we obtain,

  $A_t = 0$ and $A_r = -A_i$
- **Free end of a string:** In this case $\mu_2 \to 0$, so $v_2 \to \infty$ and we can show that,

  $A_r = 2A_i$ and $A_r = A_i$

STANDING WAVES

In previous section we've discussed that when two coherent waves superpose on a medium particle, phenomenon of interference takes place. Similarly when two coherent waves travelling in opposite direction superpose then simultaneous interference of all the medium particles takes place. These waves interfere to produce a pattern of all the medium particles what we call, a **stationary wave.** If the two interfering waves which travel in opposite direction carry equal energies then no net flow of energy takes place in the region of superposition. Within this region redistribution of energy takes place between medium particles. There are some medium particles where constructive interference takes place and hence energy increases and on the other hand there are some medium particles where destructive interference takes place and energy decreases.

Let us discuss the stationary waves analytically. Let two waves of equal amplitude are travelling in opposite direction along x-axis. The wave equation of the two waves can be given as

$y_1 = A \sin(\omega t - kx)$ [wave travelling in $+x$ direction] ...(i)

$y_2 = A \sin(\omega t + kx)$ [wave travelling in $-x$ direction] ...(ii)

When the two waves superpose on medium particles, the resultant displacement of the medium particles can be given as

$y = y_1 + y_2$ or $y = A \sin(\omega t - kx) + A \sin(\omega t + kx)$

or $y = A$ [$\sin\omega t \cos kx - \cos \omega t \sin kx + \sin \omega t \cos kx + \cos \omega t \sin kx$]

or $y = 2A \cos kx \sin \omega t$ ...(iii)

Equation (iii) can be rewritten as

$y = R \sin \omega t$ ...(iv)

where $R = 2A \cos kx$ ...(v)

- Equation (iv) is an equation of SHM. It implies that after superposition of the two waves the medium particles executes SHM with same frequency ω and amplitude R which is given by equation (v).
- We can see that the oscillation amplitude of medium particles depends on x i.e. the position of medium particles. Thus in stationary waves, the oscillation amplitude of the medium particle at different positions is different.
- At some point of medium the resultant amplitude R is maximum due to constructive interference.

R is maximum when

$\cos kx = \pm 1$

or $\frac{2\pi}{\lambda}x = N\pi$ [$N \in I$]

or $x = \frac{N\lambda}{2}$

or $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ....

and the maximum value of R is given as

$R_{max} = \pm 2A$ ...(vi)

- Similarly, at some points of the medium, the waves interfere destructively, the oscillation amplitude become minimum i.e. zero in this case. These are the points where R is minimum, when

$\cos kx = 0$ or $\frac{2\pi x}{\lambda} = (2N+1)\frac{\pi}{2}$

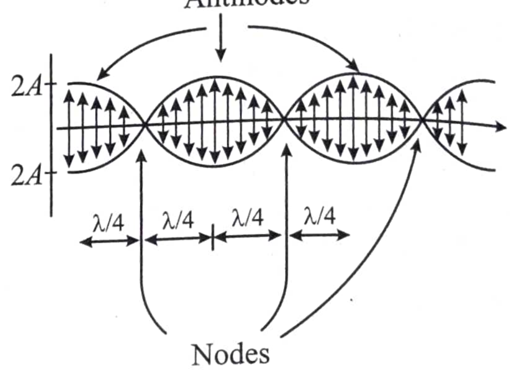
or $x = (2N+1)\frac{\lambda}{4}$ [$N \in I$] or $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$..........

and the minimum value of R is given as

$R_{min} = 0$ ...(vii)

These points always remain at rest.

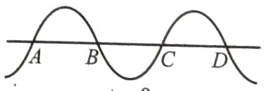
- Figure (a) shows the oscillation amplitude of different medium particles in a stationary wave.



(a)

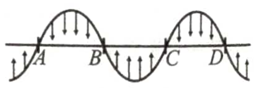
In figure (a) we can see that the points where displacement amplitude is maximum are called anti-nodes and the points of destructive interference are called nodes of stationary waves which always remain at rest.

- Figure (b) explains the movement of medium particles with time in the region where stationary waves are formed. Let us assume that at an instant $t = 0$ all the medium particles are at their extreme positions as shown in figure ($b1$). Here points $ABCD$ are the nodes of stationary waves where medium particles remains at rest. All other particles starts moving towards their mean positions and at $t = T/4$ all particles cross their mean position as shown in figure ($b3$). Now the medium particles crosses their mean position and start moving on other side of mean position toward the other extreme position. At time $t = T/2$, all the particles reach their other extreme position as shown in figure ($b5$) and at time $t = 3T/4$ again all these particles cross their mean position in opposite direction as shown in figure ($b7$).



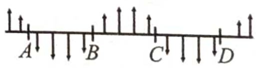
$t = 0$

(1)



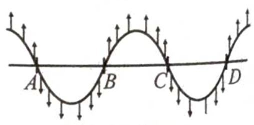
$T/4 > t > 0$

(2)



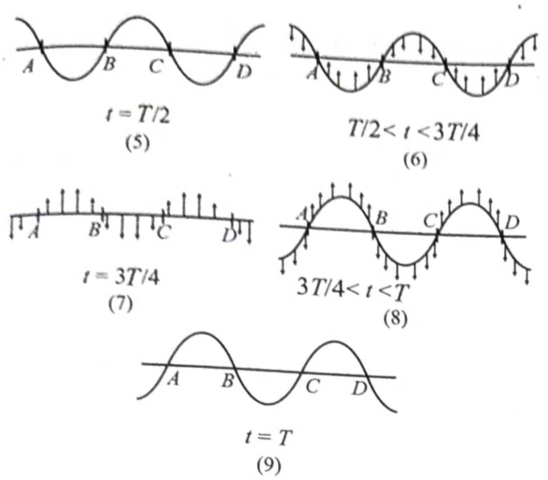
$t = T/4$

(3)



$T/4 < t < T/2$

(4)



(b)

Comparison Between Travelling and Stationary Waves

S.No	Travelling waves	Stationary waves
1.	These waves advance in a medium with a definite velocity	These waves remain stationary between two boundaries in the medium.
2.	In these waves, all particles of the medium oscillate with same frequency and amplitude.	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at anti-nodes.
3.	At any instant phase of vibration varies continuously from one particle to the other i.e., phase difference between two particles can have any value between 0 and 2π	At any instant the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node, i.e, phase difference between any two particles can be either 0 or π
4.	In these wave, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves all particles of the medium pass through their mean position simultaneously twice in each time period.
5.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.

Different Equations for a Stationary Wave

Consider two equal amplitude waves travelling in opposite direction as

$$y_1 = A \sin(\omega t - kx) \quad \text{...(i)}$$

and $$y_2 = A \sin(\omega t + kx) \quad \text{...(ii)}$$

The result of superposition of these two waves is

$$y = 2A \cos kx \sin \omega t \quad \text{...(iii)}$$

Which is the equation of stationary wave where $2A \cos kx$ represents the amplitude of medium particle situated at position x and $\sin \omega t$ is the time sinusoidal factor. This equation (iii) can be written in several ways depending on initial phase differences in the component waves given by equation (i)) can (ii). If the superposing waves are having an initial phase difference π, then the component waves can be expressed as

$$y_1 = A \sin(\omega t - kx) \quad \text{...(iv)}$$

$$y_2 = -A \sin(\omega t + kx) \quad \text{...(v)}$$

Superposition of the above two waves will result

$$y = 2A \sin kx \cos \omega t \quad \text{...(vi)}$$

Equation (vi) is also an equation of stationary wave but here amplitude of different medium particles in the region of interference is given by, $R = 2A \sin kx$ ...(vii)

Similarly the possible equations of a stationary wave can be written as

$$y = A_0 \sin kx \cos(\omega t + \phi) \quad \text{....(viii)}$$

$$y = A_0 \cos kx \sin(\omega t + \phi) \quad \text{...(ix)}$$

$$y = A_0 \sin kx \sin(\omega t + \phi) \quad \text{...(x)}$$

$$y = A_0 \cos kx \cos(\omega t + \phi) \quad \text{...(xi)}$$

Here A_0 is the amplitude of anti-nodes. In a pure stationary wave it is given as $A_0 = 2A$

where A is the amplitude of component waves. If we care fully look at equation (viii) to (xi), we can see that in equation (viii) and (x), the particle amplitude is given by

$$R = A_0 \sin kx \quad \text{...(xii)}$$

Here at $x = 0$, there is nodes as $R = 0$ and in equation (ix) and (xi) the particle amplitude is given as

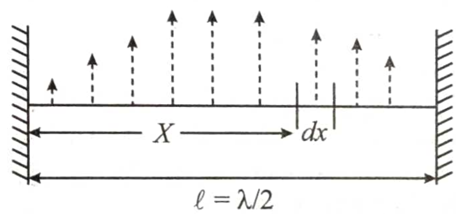
$$R = A_0 \cos kx \quad \text{...(xiii)}$$

Here at $x = 0$, there is an anti-node as $R = A_0$. Thus we can state that in a given system of co-ordinates when origin of system is at a node we use either equation (viii) or (x) for analytical representation of a stationary wave and we use equation (ix) or (xi) for the same when an anti-node is located at the origin of system.

Energy of Standing Wave in One Loop

(i) When all the particles of one loop are at extreme position then total energy in the loop is in the form of potential energy only when the particles reaches its mean position then total potential energy converts into kinetic energy of the particles so we can say total energy of the loop remains constant

Total kinetic energy at mean position is equal to total energy of the loop because potential energy at mean position is zero. Consider small element dx of the string which has mass dm.



Kinetic energy of element is $dK = \frac{1}{2} dmv^2$

where $dm = \mu\, dx$

Velocity of particle at mean position $= 2A \sin kx\, \omega$

then $dK = \frac{1}{2}\mu dx.\ 4A^2\omega^2 \sin^2 kx$

$\Rightarrow dK = 2A^2\omega^2\mu.\ \sin^2 kx\, dx$

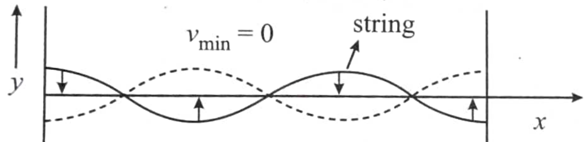
$$\int dK = 2A^2\omega^2\mu \int_0^{\lambda/2} \sin^2 kx dx$$

$$\text{Total kinetic energy} = A^2\omega^2\mu \int_0^{\lambda/2} (1 - \cos 2kx)dx$$

$$= A^2\omega^2\mu \left[x - \frac{\sin 2kx}{2k} \right]_0^{\lambda/2}$$

$$= \frac{1}{2}\lambda A^2\omega^2\mu$$

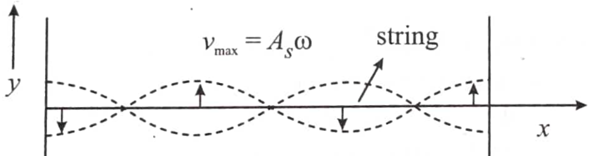
(ii) In stationary waves nodes are permanently at rest, so no energy can be transmitted across them, i.e., energy of one region (segment) is confined in that region. However, this energy oscillates between elastic potential energy and kinetic energy of the particles of the medium. When all the particles are at their extreme positions KE is minimum while elastic PE is maximum (as shown in figure (*a*)), and when all the particles (simultaneously) pass through their mean position KE will be maximum while elastic PE minimum (Figure (*b*)). The total energy confined in a segment (elastic PE + KE), always remains the same.



Max elastic PE = E

Min kinetic energy = 0

(*a*)



Min elastic PE = 0

Max kinetic energy = E

(*b*)

STATIONARY WAVES IN STRINGS

When Both Ends of String are Fixed

A string of length L is stretched between two points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the other fixed end. The incident and the reflected waves interfere to produce a stationary transverse wave in which **the ends are always nodes, as both ends of string are fixed**.

Fundamental Mode (0^{th} overtone)

In the simplest form, the string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and frequency of vibration is known as the fundamental frequency or first harmonic.

Since the distance between consecutive nodes is $\frac{\lambda}{2}$

$\therefore L = \frac{\lambda}{2}$

$\therefore \lambda = 2L$

If f_1 is the fundamental frequency of vibration, then the velocity of transverse waves is given as,

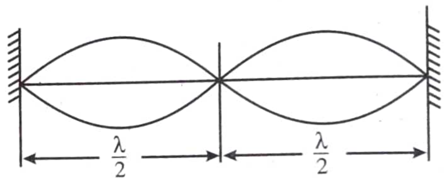
$$v = \lambda f_1 \text{ or } f_1 = \frac{v}{2L} \quad ...(i)$$

First Overtone

The same string under the same conditions may also vibrate in two loops, such that the centre is also the node.

$\therefore L = \frac{2\lambda}{2}$

$\therefore \lambda = L$



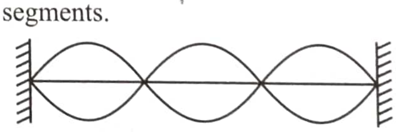
If f_2 is frequency of vibrations

$$\therefore f_2 = \frac{v}{\lambda} = \frac{v}{L} = 2f_1 \quad ...(ii)$$

The frequency f_2 is known as second harmonic or first overtone.

Second Overtone

The same string under the same conditions may also vibrate in three segments.



$\therefore L = \frac{3\lambda}{2}$

$\therefore \lambda_3 = \frac{2}{3}L$

If f_3 is the frequency in this mode of vibration, then,

$$f_3 = \frac{3v}{2L} = 3f_1 \quad ...(iii)$$

The frequency f_3 is known as third harmonic or second overtone.

Thus a stretched string vibrates with frequencies, which are integral multiples of the fundamental frequencies. These frequencies are known as **harmonics**.

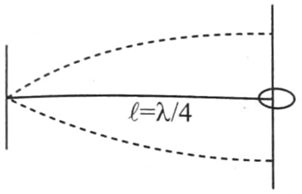
$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad \left(\because v = \sqrt{\frac{T}{\mu}}\right)$$

In general $f_n = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$ (n^{th} harmonic or $(n-1)^{th}$ overtone)

In general, any integral multiple of the fundamental frequency is an allowed frequency. These higher frequencies are called overtones. Thus, $f_1 = 2f_0$ is the first overtone, $f_2 = 3f_0$ is the second overtone etc. An integral multiple of a frequency is called its harmonic. Thus, for a string fixed at both the ends, all the overtones are harmonics of the fundamental frequency and all the harmonics of the fundamental frequency are overtones.

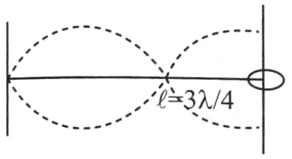
When one End of the String is Fixed and Other is Free

In this case, the free end acts as antinode.

1. 

$$f_0 = \frac{1}{4\ell}\sqrt{\frac{T}{\mu}}$$

Fundamental or Ist harmonic

2. 

$$f_1 = \frac{3}{4\ell}\sqrt{\frac{T}{\mu}}$$

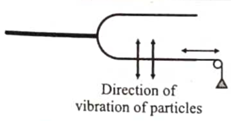
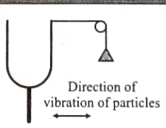
IIIrd harmonic or Ist overtone

In general : $f = \frac{(2n+1)}{4\ell}\sqrt{\frac{T}{\mu}}\Bigg\}$ ($(2n+1)^{th}$ harmonic, n^{th} overtone). In this case, even harmonics are not there.

EXPERIMENT OF MELDE'S

1. This is experimental illustration of transverse stationary wave.
2. Unknown frequency of tuning fork can be determined by this experiment.
3. Laws of transverse vibration in stretched string can be proved by this experiment.
4. One stretched string is tied to arm of tuning fork. Now it is set into vibration.

There are two arrangements:

S.No.	Transverse arrangement	Longitudinal arrangement
1.		
2.	Oscillation of tuning fork arms is perpendicular to string length	Oscillation of tuning fork arms is parallel to string length
3.	To complete one oscillation, tuning fork and string both take same time. It means frequency is same for both.	String complete one oscillation in same time in which tuning fork complete two oscillations. Frequency tuning fork = 2 × frequency of string
4.	If P are number of loops in string then $L = \frac{P\lambda}{2}; f = \frac{P}{2L}\sqrt{\frac{T}{\mu}}$	If P are number of loops in string, then $L = \frac{P\lambda}{2}; f = \frac{P}{2L}\sqrt{\frac{T}{\mu}}$ Frequency of tuning fork: $f_t = \frac{2P}{2L}\sqrt{\frac{T}{\mu}}$
5.	If L, μ and frequency remains constant, $P\sqrt{T} =$ constant	If L, μ and frequency remain constant, $P\sqrt{T} =$ constant

Train Your Brain

Example 8: Standing waves are produced by superposition of two waves $y_1 = 0.05 \sin(3\pi t - 2x)$, $y_2 = 0.05 \sin(3\pi t + 2x)$ where x and y are measured in m and t in s. Find the amplitude of particle at $x = 0.5$ m [cos 57.3 = 0.54]

Sol. $A = 2a \cos kx = 2 \times 0.5 \cos 2 \times 0.5 = 0.1 \cos(1)$

$$= 0.1\cos\left(\frac{180^\circ}{\pi}\right) = 0.1\cos\left(\frac{180^\circ}{3.14}\right)$$

$$= 0.1 \cos(57.3^\circ) = 0.1 \times 0.54 = 0.054 \text{ m}$$

Example 9: A progressive wave travels in a medium M_1 and enters into another medium M_2 in which its speed decreases to 25%. What is the ratio of the amplitude of the

(*a*) Reflected and the incident waves, and

(*b*) Transmitted and the incident waves?

Sol. Given that, velocity in the medium refracted is 25% of that in the initial medium $v_2 = \frac{1}{4}v_1$

From equation (viii) and (ix)

$$(a)\quad \frac{A_r}{A_i} = \frac{v_2 - v_1}{v_2 + v_1} = \frac{\frac{v_2}{v_1} - 1}{\frac{v_2}{v_1} + 1} = \frac{\frac{1}{4} - 1}{\frac{1}{4} + 1} = -\frac{3}{5}$$

i.e., the required ratio is $\left|\frac{A_r}{A_i}\right| = 3:5$

$$(b)\quad \frac{A_t}{A_i} = \frac{2v_2}{v_2 + v_1} = \frac{2v_2/v_1}{\frac{v_2}{v_1} + 1} = \frac{2(1/4)}{\left(\frac{1}{4}\right) + 1} = \frac{2}{5}$$

i.e., the required ratio is $\left|\frac{A_t}{A_i}\right| = 2:5$

Example 10: A stretched string vibrates with a frequency of 30 Hz in its fundamental mode when the support are 60 cm apart the velocity of transverse waves in the string is

(a) 9 m/s (b) 18 m/s

(c) 36 m/s (d) 72 m/s

Sol. (c) Given $f = 30$ Hz, $\ell = 60$ cm

velocity of wave $v = f\lambda$

$\lambda = 2l = 2 \times 60 = 120$ cm

$f = 30$ Hz × 120 cm

$$= 30\text{ Hz} \times \frac{120}{100}\text{m} = 36\text{ m}$$

Example 11: The mathematical form of Meldes law is

(a) $P\sqrt{T}$ = constant (b) PT = constant

(c) $\frac{P}{\sqrt{T}}$ = constant (d) $\frac{P}{T}$ = constant

Sol. From Melde's law if I and n are constant then $P\sqrt{T}$ = constant.

Example 12: In Melde's experiment it was found that the string vibrate in 3 loops when 8 gm were placed in the pan. What mass must be placed in the pan to make the string vibrate in 5 loops. (Neglect the mass of string)

Sol. $$f_0 = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{P}{2\ell}\sqrt{\frac{mg}{\mu}}$$

($\because$ All are constant except P and m)

So, $P\sqrt{m}$ = constant

$P_1\sqrt{m_1} = P_2\sqrt{m_2}$

$3\sqrt{8} = 5\sqrt{m}$

$m = 0.36 \times 8 = 2.88$ gm

Example 13: A sonometer wire under tension T is in unison with a tuning fork of frequency 250 Hz. If now the sonometer wire is replaced by another hollow wire of the same diameter but low density, the number of loops shall

(a) Decrease

(b) Increase

(c) Remain the same

(d) Incomplete information

Sol. (a) $$v = \frac{p}{2\ell}\sqrt{\frac{F}{\mu}}$$

or $$v = \frac{p}{2\ell}\sqrt{\frac{F}{\pi r^2 \rho}}$$

$$p = v(2\ell)\frac{\sqrt{\pi r^2 \rho}}{\sqrt{F}} \text{ or } p \propto \sqrt{\rho}$$

As density of hollow wire is less than that of sonometer wire, so number of loops shall decrease.

Example 14: When a stretched string of length L is vibrating in a particular mode, the distance between two nodes on the string is ℓ. The sound produced in this mode of vibration constitutes the nth overtone of the fundamental frequency of the string then

(a) $L = (n+1)\ell$ (b) $L = (n-1)\ell$

(c) $L = n\ell$ (d) $L = (n + \frac{1}{2})\ell$

Sol. (a) Here, ℓ = distance between the nodes

$$\ell = \frac{\lambda}{2}$$

Here, no. of loops = No. of overtones +1 = n+1

So, length of string = $L = (n+1)\ell$

Example 15: Two similar wires on a sonometer are tuned to unison. One wire is 25 cm long and is stretched by 100 gram-weight. The length of the other wire which is stretched by 400 gram-weight is

Sol. Here, $\ell = 0.25$ m, $m = 100$ g $wt = 0.1$ kg

So, $$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$f_A = \frac{1}{2\ell_A}\sqrt{\frac{T_A}{\mu}} = \frac{1}{2(0.25)}\sqrt{\frac{0.1g}{\mu}}$$

$$f_B = \frac{1}{2\ell_B}\sqrt{\frac{T_B}{\mu}} = \frac{1}{2\ell_B}\sqrt{\frac{0.4g}{\mu}}$$

$$\therefore \frac{f_A}{f_B} = \frac{2\ell_B}{0.5}\sqrt{\frac{1}{4}}$$

$f_A = f_B = f$ = frequency of tuning fork

So, $$1 = \frac{2\ell_B}{0.5(2)}$$

$\Rightarrow \ell_B = 0.5$ m = 50 cm

Concept Application

11. A standing wave is produced on a string clamped at one end and free at the other. The length of the string
(a) Must be an integral multiple of $\lambda/4$
(b) Must be an integral multiple of $\lambda/2$
(c) Must be an integral multiple of λ
(d) May be an integral multiple of $\lambda/2$.

12. In a stationary wave,
(a) All the particles of the medium vibrate in phase
(b) All the anti-nodes vibrate in phase
(c) All the alternate anti-nodes vibrate in phase
(d) All the particles between consecutive nodes vibrate in phase.

SOUND WAVES

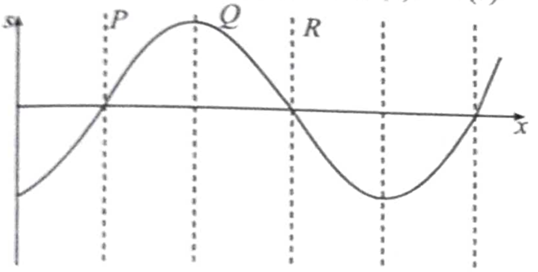
Sound is produced in material medium by a vibrating source. When a vibrating source (a piston or loud speaker cone) executes simple harmonic motion, it produces a harmonic wave. As the piston moves, it causes the particles of medium next to it to oscillate with simple harmonic motion about their equilibrium position. These particles collide with neighbouring particles, forcing them to oscillate. Suppose that at a time t, the particle at the undisturbed position x suffers a displacement $s(x, t)$ in x-direction. The harmonic wave then can be described by the equation.

$$s(x, t) = s_0 \sin(kx - \omega t)$$

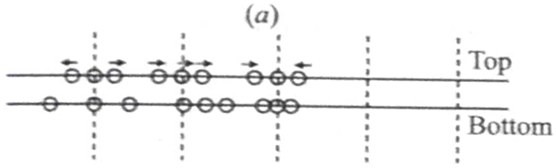
where s_0 is the maximum displacement of the particle from its equilibrium position. Since the displacement is along the direction of propagation, **sound is a longitudinal wave.**

The displacement of the particles of the medium causes the density of the medium to increase or decrease, depending on whether the gas is displaced towards a point or away from it. If the density increases (or decreases) at the point, then the pressure also increases (or decreases) at that point, giving a pressure wave that is related to the displacement wave. Thus, the **propagation of sound waves in air can be visualized as the propagation of pressure fluctuation**. Typically, pressure fluctuation are of the order of 1 Pa, whereas atmosphere pressure is 10^5Pa.

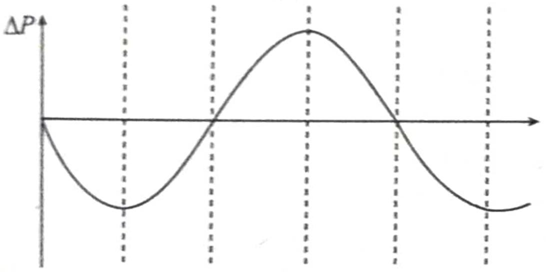
The figure below shows three graphs (a), (b) and (c)



(a)



(b)



(c)

(a) It shows the displacement S of the particles of the medium at some instant t. At positions P and R, the particles are at their equilibrium positions and at Q, they have maximum displacement.

(b) It shows the equilibrium position (top) and displacement (bottom) of the three particles near each of point P, Q and R. Note that density (pressure) is maximum at R and minimum at point P.

(c) It shows the pressure changes ΔP above and below the ambient pressure. Note that the maximum pressure deviation occur at point P and Q which are positions of zero displacement and no pressure change occurs at point Q, the position maximum displacement. Thus the pressure fluctuations are $\frac{\pi}{2}$ out of phase with displacements.

Relationship Between Pressure Waves and Displacement Waves

For a harmonic wave, the longitudinal displacement (s) is given by

$$s = s_0 \sin(kx - \omega t)$$

We know $\Delta P = -B\dfrac{\Delta V}{V}$

Since change in volume is produced by the displacement of the particles, therefore,

$$\frac{\Delta V}{V} = \frac{\partial s}{\partial x}$$

Thus, $\Delta P = -B\dfrac{\partial s}{\partial x}\cos(kx - \omega t)$

or $\Delta P = -P_0 \cos(kx - \omega t)$

where $\Delta P_0 = Bs_0 k$ is the pressure amplitude

Note: that displacement and pressure amplitudes are $\pi/2$ out of phase.

Characteristics of Sound

Loudness

It is sensation of sound produced in human ear due to amplitude. It depends upon intensity, density of medium, presence of surrounding bodies,

(i) **Intensity of sound wave:**

$$I = \frac{P_0^2}{2\rho v}$$

P_0 = Pressure amplitude

ρ = density of medium

v = speed of sound in medium

(ii) **Intensity Level or Sound Level (β):**

- $\beta = 10\log_{10}\left(\dfrac{I}{I_0}\right)$ dB

  [I_0 = minimum intensity of audible sound = 10^{-12} W/m^2
  I = measured intensity]
- Unit of sound level β is decibel (dB)
- Sound level range for audible sound: 0 dB to 120 dB
- $\beta_2 - \beta_1 = 10\log_{10}\left(\dfrac{I_2}{I_1}\right)$

Quality

Sensation produced in human ear due to shape of wave. Quality is that characteristic of sound by which we can differentiate between the sound of same pitch and loudness coming from different sources.

Pitch

Sensation produced in human ear due to frequency.

- Pitch is the characteristics of sound that depends on frequency.
- Smaller the frequency smaller the pitch, higher the frequency higher the pitch.
- Humming of mosquito has high pitch (high frequency) but low intensity (low loudness) while the roar of a lion has high intensity (loudness) but low pitch.

Velocity of Sound in a Medium

Medium is necessary for sound propagation. In a medium, velocity of sound wave and mechanical wave depends on property of medium. It does not depends on amplitude and nature of wave. Wave velocity is affected by following properties of medium:

(i) Elasticity

(ii) Inertia

In a medium, velocity of sound is given by

$$v = \sqrt{\frac{E}{\rho}} \text{ where}$$

E = Elasticity coefficient of medium

ρ = Density of medium.

1. **Velocity of Sound in Solid:** for solid $E = Y$ (Young's modulus)

$$\therefore v = \sqrt{\frac{Y}{\rho}}$$

2. **Velocity in Liquid:** for liquid $E = B$ (Bulk modulus),

$$v = \sqrt{\frac{B}{\rho}}$$

3. **Velocity of sound in gas**

(i) **Newton's Formula:** According to it, in a gaseous medium, compression and rarefaction are formed slowly during wave propagation. So medium has sufficient time for energy transfer. Thus, temperature of medium remains constant. Sound wave motion is isothermal phenomenon.

$$B = \frac{\Delta P}{-\Delta V/V},$$

For isothermal $E = P$,

$$\therefore v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{RT}{M}}$$

(ii) **Laplace's Correction:** According to it, compression and rarefactions are formed so fast that sufficient time is not available to energy transfer. Thus, sound wave motion in gaseous medium is adiabatic process.

For adiabatic process,

$$B = -\frac{dP}{dV/V} = \gamma P$$

$$\therefore v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

Factors Affecting Velocity of Sound in Gaseous Medium

(i) **Effect of pressure at constant temperature:** No effect, because density increases on pressure increasing.

(ii) **Effect of temperature:**

- On increasing temperature, velocity of sound increases.

$$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \alpha \sqrt{T}$$

where T = Temperature in kelvin.

$\Rightarrow v \alpha \sqrt{273+t}$,

where t = temperature in °C

- For a given gas:

$$\frac{v_1}{v_2} = \sqrt{\frac{273+t_1}{273+t_2}},$$

where t_1 and t_2 are in °C

- If velocity of sound at 0 °C in gas is v_0, then at t °C

$v_t = v_0 + 0.61\, t$ m/sec,

where $v_0 = 331$ m/sec.

Note:

1. On increasing temperature by 1°C velocity of sound increases by 0.61 m/sec
2. At same temperature velocity in different gases

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 M_2}{\gamma_2 M_1}}$$

(iii) **Effect of Humidity:** On increasing humidity, velocity of sound increases because density decreases with humidity increase. Dry air has more density than humid air.

$\therefore$ Velocity of sound in dry air is less than in humid air.

Interference of Sound Waves

When two coherent sound waves superpose in a certain region interference takes place. For waves to be coherent, the phase difference between the two waves should not change with time. This requires that the wavelength (frequency) of the two waves must be the same. Furthermore to have a better contrast between the maximum and minimum intensities, the two waves producing interference and should propagate in the same direction.

Consider two waves:

$$y_1 = a\sin(\omega t + kx)$$

$$y_2 = a\sin(\omega t + kx + \phi)$$

where ϕ is the phase difference between the two waves. This phase difference may arise because of the path difference [$\phi = 2\pi$ (path diff)/λ] or may be some initial phase difference or may be both. Then the resultant wave obtained from their superposition is written as,

$$y = y_1 + y_2$$

$$= A \sin(\omega t + kx + \theta)$$

where the resultant amplitude A is given by,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$$

The phase ϕ of the resultant wave is not of any importance in our discussions. The intensity of the resultant wave is,

$I = KA^2$, or $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

The intensity will be maximum or minimum depending on the value of ϕ.

Constructive interference: The constructive interference takes place when the phase difference between the two waves is,

$\phi = 0, 2\pi, 4\pi, 6\pi..... = 2n\pi$,

Then $\cos\phi = +1$

and the resultant wave amplitude and intensity are maximum

$A_{max} = a_1 + a_2$ and

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

Destructive interference: The destructive interference takes place when the phase difference between the two waves is,

$\phi = \pi, 3\pi, 5\pi,..... = (2n-1)\pi$, where $n = 1, 2, 3,.....$

Then the resultant wave amplitude and intensity are minimum,

$A_{min} = a_1 - a_2$

$$I_{min} = I_1 + I_2 - 2\left(\sqrt{I_1 I_2}\right) = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

Note:

1. If the two interfering waves have the same amplitudes, then $I_{max} = 4I_0, A_{max} = 2A_0, I_{min} = 0, A_{min} = 0$ where A_0 is the amplitude and I_0 is the intensity of one wave.
2. If the initial phase difference is zero then the condition on path difference for observing constructive or destructive interference is,

   Path difference $= f\lambda$ for constructive

   Path difference $= \left(n - \dfrac{1}{2}\right)\lambda$ for destructive

Reflection of Sound Waves

(i) Because sound propagates in the form of waves, it shows both the phenomenon of reflection and refraction. When sound wave travelling in a medium strikes the surface separating the two media, a part of incident wave is reflected back into initial medium obeying ordinary laws of reflection while the rest is partly absorbed and partly refracted or transmitted into second medium.

(ii) When a sound wave gets reflected from a rigid boundary, the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the oncoming wave to produce zero displacement at the rigid boundary. At these points of zero displacement, the pressure variation is maximum. This implies that the phase of wave is reversed but the nature of sound wave does not change i.e. on reflection the compression is reflected back as compression and rarefaction as rarefaction.

If the incident wave is represented by the equation: $y = a\sin(\omega t - kx)$, then the equation of reflected wave takes the form $y = A'\sin(\omega t + kx + \pi) = -A'\sin(\omega t + kx)$ where A' is the amplitude of reflected wave.

(iii) A sound wave is also reflected if it encounters a rarer medium or free boundary or low pressure region. A practical example is when a sound wave travels in a narrow open tube. When the wave reaches an open end, it gets reflected. The force on the particles there due to the outside air is quite small and hence, the particles vibrate with the increasing amplitude. As a result, the pressure there remains at the average value. This implies that there is no change in the phase of wave but the nature of sound wave is changed i.e. on reflection the compression is reflected back as rarefaction and vica versa.

If the incident wave is $y = A\sin(\omega t - kx)$, then the equation of reflected wave take the form $y = A'\sin(\omega t + kx)$ where A' is the amplitude of reflected wave.

Stationary Waves in Sound

- Two progressive sound wave of equal amplitude and frequency, travelling in opposite direction are super-imposed. Due to superposition one new waves form which do not travel in either direction, this wave is called stationary wave.
- Energy does not propagate in medium.
- Medium expands and contracts relative to its mean position.

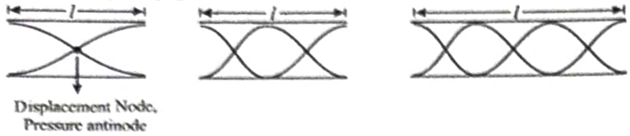
Characteristics of Stationary Waves

1. Certain point in the bounded medium, situated at equal distances are always in the position of rest, i.e., their displacement remain zero. These points are called nodes. At these nodes the change in pressure and density is maximum as compared to other points.
2. The displacement of the midpoints between the nodes is always maximum as compared to other points. These points are called anti-nodes. There is no change in pressure and density at these points.
3. All points between two successive nodes vibrate in the same phase. They reach simultaneously their positions of maximum displacement and pass simultaneously through their mean position.
4. Except nodes, all points of the medium vibrate but the amplitude of vibration is different from one point to the other. It is zero at the nodes and maximum at the anti-nodes.
5. The distance between two consecutive nodes, or between two consecutive anti-nodes is $\lambda/2$. The distance between a node and its neighbouring antinode is $\lambda/4$.
6. At any instant, the phase of vibration of the points on one side of a node is opposite from the phase of vibration of the points on the other side.
7. All points of the medium pass through their mean positions simultaneously twice in each period.
8. The nodes are found alternately in the state of maximum compression and maximum rarefaction twice in each period.

VIBRATIONS OF AIR COLUMNS IN PIPES

Organ pipes are the musical instrument which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves

1. **Open organ pipe:**



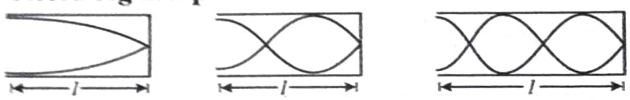
$l=\frac{\lambda}{2}$ or $\lambda=2l$ | $l=\lambda$ | $l=\frac{3\lambda}{2}$ or $\lambda=\frac{2l}{3}$

$f_1=\frac{v}{\lambda}=\frac{v}{2l}$ | $f=\frac{v}{l}=2f_1$ | $f_0=\frac{3v}{2l}=3f_1$

1st harmonic, Fundamental mode | 2nd harmonic, Ist overtone | 3rd harmonic, 2nd overtone

Note: All harmonic (odd/even) are present in open organ pipe.

2. **Closed organ Pipe:**



$l=\frac{\lambda}{4}\Rightarrow\lambda=4l$ | $l=\frac{3\lambda}{4}\Rightarrow\lambda=\frac{4l}{3}$ | $l=\frac{5\lambda}{4}$ $\lambda=\frac{4l}{5}$

$f=\frac{v}{4l}$ | $f=\frac{3v}{4l}$ | $f=\frac{5v}{4l}$

Ist harmonic, Fundamental mode | 3rd harmonic, Ist overtone | 5th harmonic, 2nd overtone

Pipe	Fundamental mode	1st overtone	$(n-1)^{th}$ overtone	Ratio of successive frequency
Open	$f=\frac{v}{2l}$ 1st Harmonic	$f=\frac{v}{l}$ 2nd Harmonic	$f=n\frac{v}{2l}$ nth Harmonic	1:2:3:4
Closed	$f=\frac{v}{4l}$ 1st Harmonic	$f=\frac{3v}{4l}$ 3rd Harmonic	$f=(2n-1)\frac{v}{l}$ $(2n-1)^{th}$ Harmonic	1:3:5:7

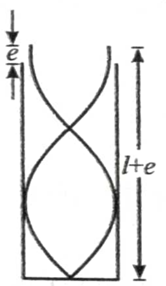
Note: Even numbered (i.e., 2nd, 4th .....) harmonics do not exist in close organ pipe.

End Correction

Because of the finite momentum of air molecules in organ pipe, reflection occurs not exactly at open end but slightly above it. Hence, an antinode is not formed exactly at the open end of the pipe but instead at a little distance away from open end, outside it. End correction (e) is the distance of the antinode from the open end. It is given as $e=0.6\,r$, where r is the radius of pipe.

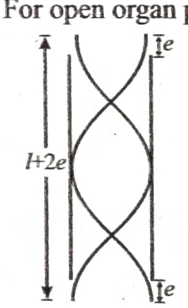
Thus, we have,

For closed organ pipe



$$f=\frac{(2n-1)v}{4(l+e)}$$

For open organ pipe



$$f=\frac{nv}{2(l+2e)}$$

Train Your Brain

Example 16: Two sources S_1 and S_2 separated by 2.0 m, vibrate according to equation $y_1=0.03\sin(\pi t)$ and $y_2=0.02\sin(\pi t)$

Where y_1, y_2 and t are in M.K.S. units. They send out waves of velocity 1.5 m/s.

Calculate the amplitude of the resultant motion of the particle collinear with S_1 and S_2 and located at a point.

(i) To the right of S_2

(ii) To the left of S_1

(iii) In the middle of S_1 and S_2

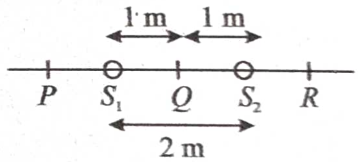
Sol. The phase difference between the two waves is given by $\phi=\frac{2\pi x}{\lambda}$ where $x=2.0$ m is the path difference between the two waves at points near to S_1 or S_2. The resultant amplitude of the superimposed wave is $a=\sqrt{a_1^2+a_2^2+2a_1a_2\cos\phi}$.

Let P and R be respective points to the left of S_1 and right of S_2, respectively.

The oscillations y_1 and y_2 have amplitude $a_{1=0.03}$ and $a_{2=0.02}$, respectively. These have equal period $T=2$ s and same frequency $f=\frac{1}{T}=\frac{1}{2}=0.5\ s^{-1}$.

The wavelength of each vibration,

$$\lambda=\frac{v}{f}=\frac{1.5}{0.5}=3.0\text{ m}$$



(i) The path difference for point R to the right of $S_2=\Delta x=(S_1R-S_2R)=S_1S_2=2$ m

$\therefore$ Phase difference $\phi=\frac{2\pi}{\lambda}x=\frac{2\pi}{3}\times2.0=\frac{4\pi}{3}$

The resultant amplitude for point R is given by

$$\sqrt{a_1^2+a_2^2+2a_1a_2\cos\phi}$$
$$=\sqrt{\{(0.03)^2+(0.02)^2+2\times0.03\times0.02\times\cos(4\pi/3)\}}$$

Solving, we obtain $a=0.02565$ m.

(ii) The path difference for all point p to the left of S_1 is $\Delta x=S_2P-S_1P=2.0$ m.

Hence, the resultant amplitude for all points to the left of S_1 is 0.0265 m.

(iii) for a point Q, midway between S_1 and S_2, the path difference is zero i.e., $\phi = 0$

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}$$

$$= \sqrt{\{(0.03)^2 + (0.02)^2 + 2(0.03)(0.02)\}}$$

$$= 0.03 + 0.02 = 0.05 \text{ m}$$

Example 17: Find the fundamental frequency and the first four overtone of a 15 cm pipe

(i) If the pipe is closed at one end,

(ii) If the pipe is open at both ends,

(iii) How many overtones may be heard by a person of normal hearing in each of the above cases? Velocity of sound in air = 330 ms^{-1}

Sol. For the organ pipe closed at one end, the fundamental frequency of the wave of wavelength λ is given by,

$f_0 = \frac{v}{4L}$. The frequency of i^{th} over tone is given by

$f_1 = (i + 1) \times f_0$ where $i = 1, 2, 3$ ... etc.

(i) $f_0 = \frac{v}{4L}$ where f_0 = frequency of the fundamental

$$\Rightarrow f_0 = \frac{330}{4 \times 0.15} = 550 \text{ Hz}$$

$f_1 = 3f_0, f_2 = 5f_0, f_3 = 7f_0, f_4 = 9f_0$

(ii) The first four overtones are $2f_0$, $3f_e$, $4f_0$ and $5f_a$. So, the required frequencies are 1100, 2200, 3300, 4400, and 5500 Hz.

(iii) Open pipe $nf_0 = 20{,}000$

$n(100) = 20{,}000$

$n = 18.18 \simeq 18$

Closed pipe

the frequency of the nth overtone is $(2n + 1)r$

$\therefore (2n+1) f_0 = 20000;$ or $(2n+1)550 = 20000$

or $n = 17.68$

Acceptable value is 17.

Example 18: A tube closed at one end has a vibrating diaphragm at the other end, which may be assumed to be displacement node. It is found that when the frequency of the diaphragm is 2000 Hz, a stationary wave pattern is set up in which the distance between adjacent nodes is 8 cm. When the frequency is gradually reduced, then the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate

(i) The speed of sound in air.

(ii) The distance between adjacent nodes at a frequency of 1600 Hz.

(iii) The distance between the diaphragm and the closed end.

(iv) The next lower frequencies at which stationary wave pattern will be obtained.

Sol. The standing waves generated inside the tube closed at one end, have the wavelength $\lambda = 2L$ where L is length of the tube. The velocity of the wave in air is given by $v = f\lambda$, where f is the frequency of the sound wave.

Since the node-to-node distance is

or $\frac{\lambda}{2} = 0.08$ or $\lambda = 0.16$ m

(i) $v = f\lambda \quad \therefore v = 2000 \times 0.16 = 320 \text{ m/s}$

(ii) $320 = 1600 \times \lambda$ or $\lambda = 0.2$ m

$\therefore$ Distance between nodes $= \frac{0.2}{2} = 0.1 \text{ m} = 10 \text{ cm}$

(iii) Since there are nodes at the ends, the distance between the closed end and the diaphragm must be an integral multiple of $\lambda/2$

$\therefore \; L = n\lambda/2 = n \times 0.2/2 = n' \times 0.16/2$

$\Rightarrow \frac{n}{n'} = \frac{4}{5}$ (when $n' = 5, n = 4$)

$L = \frac{n' \times 0.16}{2} = 0.4 \text{ m} = 40 \text{ cm}$

(iv) For the next lower frequency $n = 3$

$\therefore \; 0.4 = 3\lambda/2$ or $\lambda = 0.8/3$

since $v = f\lambda, f = \frac{320}{0.8/3} = 1200$ Hz

Concept Application

13. When sound wave is refracted from air to water, which of the following will remain unchanged?
    (a) Wave number
    (b) Wavelength
    (c) Wave velocity
    (d) Frequency

14. The speed of sound in a medium depends on
    (a) The elastic property but not on the inertia property
    (b) The inertia property but not on the elastic property
    (c) The elastic property as well as the inertia property
    (d) Neither the elastic property nor the inertia property.

15. Consider the following statements about sound passing through a gas.
    A. The pressure of the gas at a point oscillates in time.
    B. The position of a small layer of the gas oscillates in time.

    (a) Both A and B are correct.
    (b) A is correct but B is wrong.
    (c) B is correct but A is wrong.
    (d) Both A and B are wrong.

16. The bulk modulus and the density of water are greater than those of air. With this much of information, we can say that velocity of sound in air
   (a) Is larger than its value in water
   (b) Is smaller than its value in water
   (c) Is equal to its value in water
   (d) Cannot be compared with its value in water.
17. An organ pipe, open at both ends, contains
   (a) Longitudinal stationary waves
   (b) Longitudinal travelling waves
   (c) Transverse stationary waves
   (d) Transverse travelling waves.

BEATS

When two sound waves of slightly different frequencies travelling along the same path in the same direction in a medium superpose upon each other, the intensity of the resultant sound at any point in the medium rises and falls alternately with time. These periodic variations in the intensity of sound caused by the superposition of two sound waves of slightly different frequencies are called **beats**. One rise and one fall of intensity constitute one beat. The number of beats produced per second is called **beat frequency**, i.e.

Beat frequency = Difference in frequencies of the two superposing waves

Proof: Consider two harmonic waves of frequencies f_1 and f_2 (f_1 being slightly greater than f_2) and each of amplitude A travelling in a medium in the same direction. The displacements due to the two waves at a given observation point may be represented as

$$y_1 = A \sin \omega_1 t = A \sin 2\pi f_1 t$$

$$y_2 = A \sin \omega_2 t = A \sin 2\pi f_2 t$$

From the principle of superposition, the resultant displacement at the given point will be

$$y = y_1 + y_2 = A \sin 2\pi f_1 t + A \sin 2\pi f_2 t$$

$$= 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t . \sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

Let us write

$$f_{mod} = \frac{f_1 - f_2}{2} \text{ and } f_{av} = \frac{f_1 + v_2}{2}$$

then $y = 2A\cos(2\pi f_{mod} t)\sin(2\pi f_{av} t)$

or $y = R\sin(2\pi f_{av} t)$

where $R = 2A\cos(2\pi f_{mod} t)$ is the amplitude of the resultant wave. As f_1 is slightly greater than f_2, so $f_{mod} << f_{av}$, i.e. R varies very slowly with time. Hence, the above equation represents a wave of periodic rapid oscillation of average frequency f_{av} 'modulated' by a slowly varying oscillation of frequency f_{mod}.

The amplitude R of the resultant wave will be maximum, when

$\cos 2\pi t f_{mod} t = \pm 1$

or $2\pi f_{mod} t = n\pi$

or $\pi(f_1 - f_2)\, t = n\pi$

or $t = \frac{n}{f_1 - f_2} = 0, \frac{1}{f_1 - f_2}, \frac{2}{f_1 - f_2}, \ldots$

$\therefore$ Time interval between two successive maxima

$$= \frac{1}{f_1 - f_2}$$

Similarly, the amplitude R will be minimum, when

$\cos 2\pi f_{mod} t = 0$

or $2\pi f_{mod} t = (2n + 1)\pi/2$

or $\pi(f_1 - f_2)t = (2\pi + 1)\pi/2$

or $t = \frac{(2n+1)}{2(f_1 - f_2)} = \frac{1}{f_1 - f_2}, \frac{3}{2(f_1 - f_2)}, \frac{5}{2(f_1 - f_2)}, \ldots$

$\therefore$ The time interval between successive minima $= \frac{1}{f_1 - f_2}$

Clearly, both maxima and minima of intensity occur alternately. Technically, one maximum of intensity followed by a minimum is called a beat. Hence the time interval between two successive beats or the beat period is

$$t_b = \frac{1}{f_1 - f_2}$$

The number of beats produced per second is called beat frequency.

$$f_b = \frac{1}{t_{beat}} \text{ or } f_{beat} = f_1 - f_2$$

$\therefore$ Beat frequency = Difference between the frequencies of two superposing waves.

Application of Beats: Determination of an Unknown Frequency

Suppose f_1 is the known frequency of tuning fork A and f_2 is the unknown frequency of tuning fork B. When the two tuning forks are sounded together, suppose they produce b beats per second. Then

$$f_2 = f_1 + b \quad \text{or} \quad f_1 - b$$

The exact frequency may be determined by any of the following two methods:

(i) **Loading method:** Attach a little wax to the prong of the tuning fork B. Again, find the number of beats produced per second. If the frequency of B is greater than that of A i.e., $(f_1 + b)$, then the attaching of a little wax lowers its frequency and reduces the difference in frequencies of A and B. This would decrease the beat frequency. That is,

$f_2 = f_1 + b$

On the other hand, if the frequency of B is less than that of A i.e. $(f_1 - b)$ then the attaching of a little wax further lowers its frequency and increases the difference in frequencies of A and B. This would increase the beat frequency. That is,

$f_2 = f_1 - b$

(ii) **Filing method:** If a prong of the tuning fork B is filled, its frequency increases. Again, note the number of beats produced per second.

If on filing the prong of B, the beat frequency decreases, then $f_2 = f_1 - b$

If on filing the prong of B the beat frequency increases, then $f_2 = f_1 + b$

Train Your Brain

Example 19: The points of the prong of a tuning fork B originally in unison with tuning fork A of frequency 384 Hz are filed and the fork produces 3 beats per second, when sounded together with A. What is the pitch of B after filing?

Sol. Frequency of tuning fork $A = 384$ Hz.

As tuning fork B is in unison with A, so its original frequency = 384 Hz

Beat frequency =3 s^{-1}

Possible frequencies of B after filing

$= 384 \pm 3 = 387$ or 381 Hz

As the frequency increases on filing, so frequency of B after filing =387 Hz.

Example 20: A tuning fork arrangement (pair) produces 4 beats s^{-1} with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats s^{-1}. What is the frequency of the unknown fork?

Sol. Unknown frequency = Known frequency ± Beat frequency

$= 288 \pm 4 = 292$ Hz or 284 Hz

On loading with wax, the frequency decreases, the beat frequency also decreases to 2.

Unknown frequency = 292 cps (higher one).

Example 21: A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.

Sol. Unknown frequency

= Known frequency ± Beat frequency

$= 310 \pm 4 = 314$ or 306 Hz

Out of these two possible frequencies, one must be the initial value and the other final value. As frequency increases on filing, therefore initial unknown frequency = 306 Hz.

Example 22: A set of 24 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats per second with the preceding one and the last sounds the octave of the first, find the frequencies of the first and the last forks.

Sol. Let the frequency of the first fork $= f$

Then the frequency of the second fork $= f + 4$

Frequency of the third fork $= f + 2 \times 4$

Frequency of the fourth fork $= f + 3 \times 4$

Frequency of the 24th fork $= f + 23 \times 4$

But the frequency of the last octave is the octave of the first.

Therefore,

$2f = f + 23 \times 4$ or $f = 92$ Hz

Thus, frequency of the first fork $= f = 92$ Hz and frequency of the last fork $= 2f = 184$ Hz.

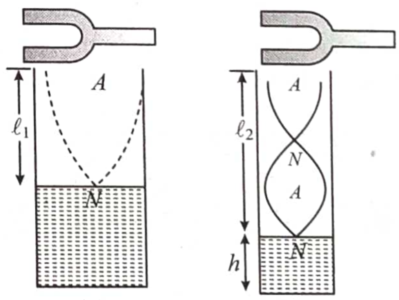
Concept Application

18. A timing fork produces 4 beats/s when sounded with a tuning fork of frequency 512 Hz. The same tuning fork when sounded with another tuning fork of frequency 514 Hz produces 6 beats/s. Find the frequency of the tuning fork (in Hz).

19. A tuning fork of known frequency of 256 Hz makes 5 beat s^{-1} with the vibrating string of a piano. The beat frequency decreases to 2 beats s^{-1}, when the tension in the piano string is slightly increased. What was the frequency (in Hz) of the piano string before increasing the tension?

RESONANCE TUBE

It is used to determined velocity of sound in air by the help of tunning fork. A resonance column or a resonance tube is merely a pipe closed at one end with a convenient provision for varying the length of the column. A simple form is shown in figure, where the portion of the pipe above water level acts as the resonance tube. The surface of water inside the pipe serves as a closed end of the air column.



(a) (b)

A tuning fork of known frequency is struck with a rubber or cork hammer and is held near the mouth of the tube. The water level is gradually adjusted till for a particular position the air column resonates.

For resonance, the natural frequency of the air column must be equal to that of the tuning fork (f_t). If ℓ_1 is the length of the resonating column, then if end corrections are ignored,

$$\ell_1 = \frac{\lambda}{4} \text{ and, } f = \frac{v}{4\ell_1} = f_t$$

Thus a knowledge of f_t and ℓ_1 determines the velocity of sound as

$$v = f\lambda = f_t(4\ell_1)$$

If end correction is not neglected, then the true length that appears in the formula will be

$$L = \ell_1 + e = \frac{\lambda}{4}$$

So that unless e is known, v can not be determined. To eliminate e, the experiment is repeated for a greater length. This time resonance occurs for the mode shown in fig (b) i.e., when f_t = frequency of the first overtone. Let now the length is ℓ_2 then,

$$L' = \ell_2 + e = \frac{3\lambda}{4}$$

From the relations $f_t = \frac{v}{4L}$ and $f_t = 3\frac{v}{4L}$, we expect

$$L = \frac{L'}{3} \text{ or } \ell_1 + e = \frac{\ell_2 + e}{3}$$

This gives, $e = \frac{(\ell_2 - 3\ell_1)}{2}$

and **velocity of sound is determined from the formula**

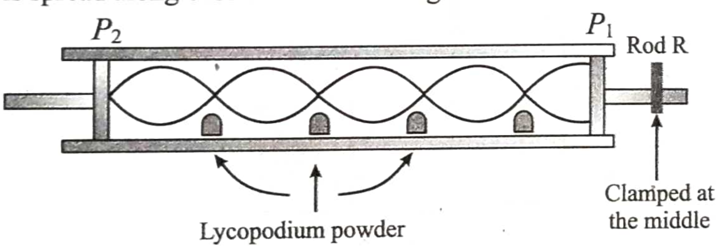
$$f = \frac{v}{2(\ell_2 - \ell_1)}$$

The wavelength of the sound in the air column is,

$\lambda = 2(\ell_2 - \ell_1)$

KUNDT'S TUBE METHOD

The apparatus consists of a long glass tube about 5cm in diameter, fixed horizontally. A metal rod R clamped firmly at the centre is mounted so that its one end carrying a light disc P_1 (of cork or card board) projects some distance into the glass tube. The other end of the glass tube is closed with a movable piston P_2. Any desired length of the air or gas can be enclosed in between the two discs P_1 and P_2. A small amount of dry lycopodium powder or cork dust is spread along base of the entire length of the tube.



The free end of the metal rod R is rubbed (stroked) along the length with resined cloth. The rod begins to vibrate longitudinally and emits a very high pitched shrill note. These vibrations are impressed upon the air column in the tube through disc P_1. Let disc P_2 is so adjusted that the stationary waves are formed in the air (gas) column in the tube. At anti-nodes powder is set into oscillations vigorously while it remains uneffected at nodes. Heaps of powder are formed at nodes.

Theory of Kundt's tube: Let f is the frequency of vibration of the rod, then this is also the frequency of sound wave in the air column in the tube.

For rod: $\frac{\lambda_{rod}}{2} = \ell_{rod}$, For air: $\frac{\lambda_{air}}{2} = \ell_{air}$

Where ℓ_{air} is the distance between two heaps of powder in the tube (i.e. distance between two nodes). If v_{air} and v_{rod} are velocity of sound waves in the air and rod respectively, then

$$f = \frac{v_{air}}{\lambda_{air}} = \frac{v_{rod}}{\lambda_{rod}}$$

Therefore, $\frac{v_{air}}{v_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}} = \frac{\ell_{air}}{\ell_{rod}}$

Thus knowledge of v_{rod}, determines v_{air}.

Kundt's tube may be used for

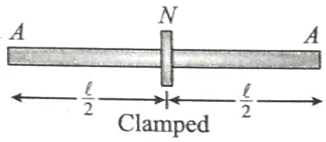
(i) Comparison of velocities of sound in different gases.
(ii) Comparison of velocities of sound in different solids.
(iii) Comparison of velocities of sound in a solid and in a gas.
(iv) Comparison of density of two gases.
(v) Determination of γ of a gas.
(vi) Determination of velocity of sound in a liquid.

LONGITUDINAL VIBRATIONS OF ROD

Rod Free at Both Ends

A rod clamped at the middle and free at both ends, vibrate longitudinally with the node in middle and anti-nodes at the free end. The first two possible modes are shown.

(i) The fundamental mode (lowest frequency note) is shown in figure.(a)

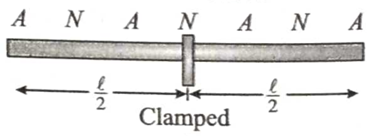


(a)

$$\frac{2\lambda_1}{4} = \ell \Rightarrow \lambda_1 = 2\ell$$

Its frequency is, $f_1 = \frac{v}{\lambda_1} = \frac{v}{2\ell}$

(ii) The next mode is shown below.



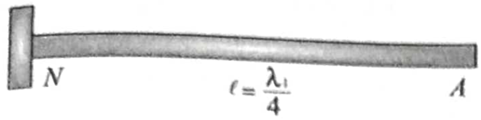
(b)

$$\frac{6\lambda_2}{4} = \ell \Rightarrow \lambda_2 = 2\ell/3$$

It frequency is $f_2 = \frac{v}{\lambda_2} = \left(\frac{v}{2\ell/3}\right) = 3f_1$

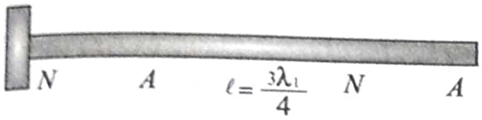
Rod Clamped at One End

Fundamental mode:



Frequency: $f_1 = \frac{v}{\lambda_1} = \frac{v}{4\ell}$

First overtone:



Frequency: $f_2 = \frac{v}{\lambda_2} = 3\frac{v}{4\ell} = 3f_1$

Thus again the overtone frequencies are 3,5,7,......times that of the fundamental.

DOPPLER'S EFFECT

If a wave source and a receiver are moving relative to each other, the frequency observed by the receiver (f) is different from the actual source frequency (f_0) given by,

$$f = f_0\left(\frac{v \pm v_0}{v \mp v_s}\right)$$

where v = speed of sound, v_0 = speed of observer, v_s = speed of source

Various Cases

1. **Source at rest, observer moves:**
   (i) Observer moves away from source,
   $$f = f_0\left(\frac{v - v_0}{v}\right)$$
   (ii) Observer moves towards source,
   $$f = f_0\left(\frac{v + v_0}{v}\right)$$
2. **Observer at rest, source moves:**
   (i) Source moves towards observer,
   $$f = f_0\left(\frac{v}{v - v_s}\right)$$
   (ii) Source moves away from observer,
   $$f = f_0\left(\frac{v}{v + v_s}\right)$$
3. **Both move:**
   (i) Both approaching each other

$$f = f_0\left(\frac{v + v_0}{v - v_S}\right)$$

(ii) Source following the observer

$$f = f_0\left(\frac{v - v_0}{v - v_S}\right)$$

(iii) Observer following the source

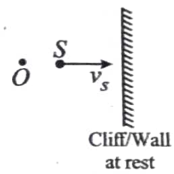
$$f = \left(\frac{v + v_0}{v + v_S}\right)f_0$$

(iv) Both moving away from each other

$$f = \left(\frac{v - v_0}{v + v_S}\right)f_0$$

APPLICATIONS OF BEATS AND DOPPLER EFFECT

Case-I: In figure, O represents observer and S represent source which is moving toward a cliff/wall at rest f = frequency of source. Let



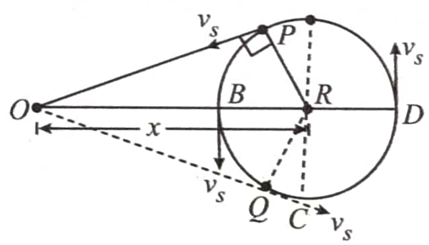
v_s = velocity of source
f' = frequency of direct sound
v = velocity of sound
f'' = frequency of reflected sound

Now, $f' = \left(\frac{v}{v + v_s}\right)f$ (source moving away)

$f'' = \left(\frac{v}{v - v_s}\right)f$ (source moving towards)

Beat frequency $= f'' - f' = \left(\frac{v}{v - v_s} - \frac{v}{v + v_s}\right)f = \left(\frac{2v_s v}{v^2 - v_s^2}\right)f$

Case-II: A source of frequency 'f' is revolving in a circle of radius R with speed v_s. An observer is standing at a distance x from the centre.



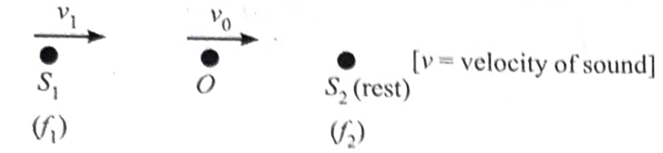
At B and D, observed frequency is 'f'. At 'P', frequency is maximum as $OP \perp PR$, i.e.,

$$f' = \left(\frac{v}{v - v_s}\right)f \qquad \left(OP = \sqrt{OR^2 - PR^2} = \sqrt{x^2 - R^2}\right)$$

At Q frequency is minimum as $OQ \perp QR$, i.e.,

$$f'' = \left(\frac{v}{v + v_s}\right)f$$

Case-III: An observer is moving between two sources



Frequency of sound heard from S_1

$$f_1' = \left(\frac{v - v_0}{v - v_1}\right) f_1$$

Frequency of sound heard from S_2

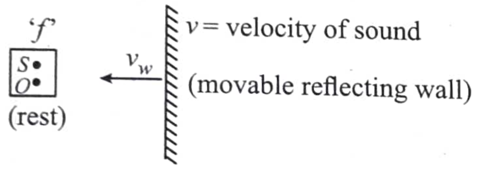
$$f_2' = \left(\frac{v + v_0}{v}\right) f_2,$$

Beat frequency $= |f_1' - f_2'|$

Case-IV: An observer at rest observers the sound after reflection from a mirror moving towards source and directly from the source.

Direct frequency $= f$

Reflected frequency $= \left(\frac{v + v_w}{v - v_w}\right) f$



Beat frequency $= -f\left(\frac{v + v_w}{v - v_w} - 1\right) = f\left(\frac{2v_w}{v - v_w}\right)$

DOPPLER'S EFFECT IN LIGHT

The velocity of light in free space is independent of the motion of source or observer and it is a universal constant given as $c = 3 \times 10^8$m/s. Thus the doppler's effect in light depends only upon the relative motion of the light source and the observer and it does not matter which one is moving.

When a light source and an observer are approaching each other with a velocity v, then the apparent frequency of light will be.

$f' = f$ and $f' = f(1 - v/c)$ if $v << c$

or $\Delta f = -fv/c$

Case (*a*): When the light source is going away from the earth then

$\Delta f = f$

$f' < f$ and $\Delta\lambda = \lambda$ or $\lambda' > \lambda$ i.e. λ is increased or spectral line will shift towards the red end of the spectrum. This is known as red shift.

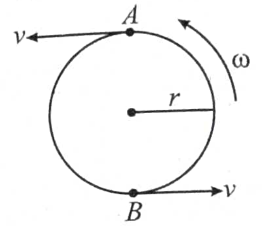
Case (*b*): When the light source is coming nearer to earth.

$\Delta f = f$ or $f' > v$ and $\Delta\lambda = -\lambda$ or $\lambda' < \lambda$

⇒ wavelength appears to be decreasing i.e. the spectral line in electromagnetic spectrum gets displaced towards violet end, hence it is known as violet shift.

Train Your Brain

Example 23: A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotates with an angular velocity of 20 rad/s in the horizontal plane. Calculate the range of frequency heard by an observer stationed at a large distance from the whistle, will be (v = 330 m/s)



Sol. As the whistle is moved in the circle in horizontal plane, it sometimes moves away and sometimes towards the stationary observer. Thus the observer will hear the minimum frequency of $f_{min} = f\left(\frac{v}{v + v_s}\right)$ when whistle is moving away from him. The observer will hear maximum frequency of $f_{max} = f\left(\frac{v}{v - v_s}\right)$ when the whistle is moving towards him.

Velocity of source $= v_s = r\omega = 1.5 \times 20 = 30$ m/s

Frequency $f = 440$ Hz.

And speed of sound, $v = 330$ m/s, the maximum frequency f_{max} will correspond to a position when source is approaching the observer

$$f_{max} = n\left(\frac{v}{v - v_s}\right) = 440\left(\frac{330}{330 - 30}\right)$$

$$= \frac{440 \times 330}{300} = 484$$

The minimum frequency f_{min} will correspond to a position when source is receding the observer.

$$f_{min} = f\left(\frac{v}{v + v_s}\right) = 440\left(\frac{330}{330 + 30}\right)$$

$$= \frac{440 \times 330}{360} = 403 \text{ Hz}$$

The range of frequency is from 403 Hz to 484 Hz.

Example 24: A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 600 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find,

(i) The frequency of the whistle as heard by an observer on the hill.

(ii) The distance from the hill at which the echo from the hill is heard by the driver and its frequency. (Velocity of sound in air c = 1200 km/hr.)

Sol. A train is moving towards a hill with speed v_z with respect to the ground. The speed of sound in air, i.e. the speed of sound with respect to medium (air) is c, while air itself is blowing towards hill with velocity $v_m = 40$ km/hr (as observed from ground). For an observer standing on the ground, which is the inertial frame, the speed of sound towards hill is given by

$v = c + v_m = 1240$ km/hr

(i) The observer on the hill is stationary while source is approaching him. Hence, frequency of whistle heard by him is

$$f' = f\frac{v}{v - v_s} \quad (v_s \text{ is speed of the train})$$

$\therefore f = 600$ Hz

$v_s = 40$ km/hr,

and $v = 1240$ km/hr,

we get

$$f' = 600 - \frac{1240}{1240 - 40} = 620 \text{ Hz.}$$

(ii) The train sounds the whistle when it is at distance x from the hill. Sound moving with velocity v with respect to ground, takes time t to reach the hill, such that,

$$t = \frac{x}{v} = \frac{x}{c + v_m} \quad \text{...(i)}$$

After reflection from hill, sound waves move backwards, towards the train. The sound is now moving opposite to the wind direction. Hence, its velocity with respect to the ground is

$v' = c - v_m$

Suppose when this reflected sound (or echo) reaches the train, it is at distance x' from hill. The time taken by echo to travel distance x' is given by

$$t' = \frac{x'}{v'} = \frac{x'}{c - v_m} \quad \text{...(ii)}$$

Thus, total time $(t + t')$ elapses between sounding the whistle and echo reaching back. In the same time $(t + t')$ the train moves a distance $(x - x')$ with constant speed v_s, as observed from ground. That is,

$$t + t' = \frac{x - x'}{v_s}$$

or $x - x = (t + t')v_s$. ...(iii)

Substituting from (i) and (ii) in equation (iii) for t and t', we find

$$x - x' = \frac{v_s}{c + v_m}x + \frac{v_s}{c - v_m}x'$$

$$\text{or, } \frac{c + v_m - v_s}{c + v_m}x = \frac{v_s + c - v_m}{c - v_m}x'$$

For $x = 1$ km, $c = 1200$ km/hr, $v_s = 40$ km/hr, and $v_m = 40$ km/hr, we get

$$\frac{1200 + 40 - 40}{1200 + 40} \times 1 = \frac{40 + 1200 - 40}{1200 - 40}x'$$

$$\text{or, } x' = \frac{1160}{1240} = 0.935 \text{ km.}$$

Thus, the echo is heard when train is 935 m from the hill.

Now, for the observer moving along with train, echo is a sound produced by a stationary source, i.e., the hill. Hence as observed from ground, source is stationary and observer is moving towards source with speed 40 km/hr . Hence $v_0 = -40$ km/hr. On the other hand, reflected sound travels opposite to wind velocity. That is, velocity of echo with respect to ground is v'. Further, the source (hill) is emitting sound of frequency f' which is the frequency observed by the hill.

Thus, frequency of echo as heard by observer on train, is given by

$$f'' = f'\frac{v' + v_0}{v'}$$

$$\Rightarrow f'' = \frac{(1160 - (-40))}{1160} \times 620$$

$$= 641 \text{ Hz}$$

Concept Application

20. The phenomenon of beats can take place

(*a*) For longitudinal waves only

(*b*) For transverse waves only

(*c*) For both longitudinal and transverse waves

(*d*) For sound waves only

21. The change in frequency due to Doppler effect does not depend on

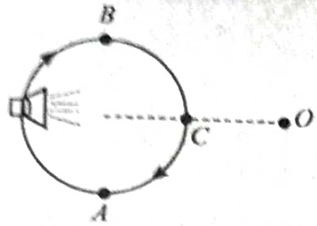
(*a*) The speed of the source

(*b*) The speed of the observer

(*c*) The frequency of the source

(*d*) Separation between the source and the observer.

22. A small source of sound moves on a circle as figure and an observer is sitting at O. Let v_1, v_2, v_3 be the frequencies heard when the source is at A, B and C respectively.



(a) $v_1 > v_2 > v_3$
(b) $v_1 = v_2 > v_3$
(c) $v_2 > v_3 > v_1$
(d) $v_1 > v_3 > v_2$.

23. A source of sound moves towards an observer.
(a) The frequency of the source is increased.
(b) The velocity of sound in the medium is increased.
(c) The wavelength of sound in the medium towards the observer is decreased.
(d) The amplitude of vibration of the particles is increased.

24. A listener is at rest with respect to the source of sound. A wind starts blowing along the line joining the source and the observer. Which of the following quantities do not change?
(a) Frequency
(b) Velocity of sound
(c) Wavelength
(d) Time period

Short Notes

General Equation of Wave Motion

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

$$y(x,t) = f\left(t \pm \frac{x}{v}\right)$$

where, $y(x, t)$ should be finite everywhere.

$f\left(t+\frac{x}{v}\right)$ represents wave travelling in –ve x-axis.

$f\left(t-\frac{x}{v}\right)$ represents wave travelling in + ve x-axis.

$$y = A\sin(\omega t \pm kx + \phi)$$

Terms Related to Wave Motion (For 1-D Progressive Sine Wave)

Wave Number (or Propagation Constant) (k)

$$k = 2\pi/\lambda = \frac{\omega}{v}\left(\text{rad m}^{-1}\right)$$

Phase of Wave

The argument of harmonic function $(\omega t \pm kx + \phi)$ is called phase of the wave.

Phase difference $(\Delta\phi)$: difference in phases of two particles at any time t.

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \text{ where } \Delta x \text{ is path difference}$$

Also $$\Delta\phi = \frac{2\pi}{T}.\Delta t$$

Speed of Transverse Wave Along the String

$v = \sqrt{\frac{T}{\mu}}$ where

T = Tension

μ = mass per unit length

Power Transmitted Along the String

Average Power $\langle P\rangle = 2\pi^2 f^2 A^2 \mu v$

Intensity $I = \frac{\langle P\rangle}{s} = 2\pi^2 f^2 A^2 \rho v$

Reflection of Waves

If we have a wave

$y_i(x, t) = a\sin(kx - \omega t)$ then,

(i) Equation of wave reflected at a rigid boundary

$y_r(x, t) = a\sin(kx + \omega t + \pi)$

or $y_r(x, t) = -a\sin(kx + \omega t)$

i.e. the reflected wave is 180° out of phase.

(ii) Equation of wave reflected at an open boundary

$y_r(x, t) = a\sin(kx + \omega t)$

i.e. the reflected wave is in phase with the incident wave.

Standing/stationary Waves

$y_1 = A\sin(\omega t - kx + \theta_1)$

$y_2 = A\sin(\omega t + kx + \theta_2)$

$$y_1 + y_2 = 2A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)\sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$$

The quantity $2A\cos\left(kx+\frac{\theta_2-\theta_1}{2}\right)$ represents resultant amplitude at x. At some position resultant amplitude is zero these are called **nodes**. At some positions resultant amplitude is $2A$, these are called **anti-nodes**.

Distance between successive nodes or anti-nodes $=\frac{\lambda}{2}$

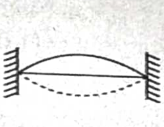
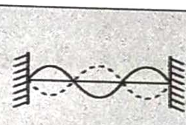
Distance between adjacent node and anti-node $= \lambda/4$.

All the particles in same segment (portion between two successive nodes) vibrate in same phase.

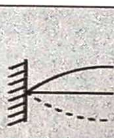
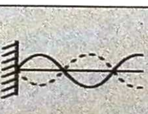
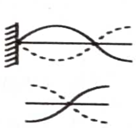
Since nodes are permanently at rest so energy can not be transmitted across these.

Vibrations of Strings (standing wave)

Fixed at Both Ends

First harmonics or Fundamental frequency	$L=\frac{\lambda}{2},\ f_1=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$	
Second harmonics or First overtone	$L=\frac{2\lambda}{2},\ f_2=\frac{2}{2L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or Second overtone	$L=\frac{3\lambda}{2},\ f_3=\frac{3}{2L}\sqrt{\frac{T}{\mu}}$	
n^{th} harmonics or $(n-1)^{th}$ overtone	$L=\frac{n\lambda}{2},\ f_n=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$	

String Free at One End

First harmonics or Fundamental frequency	$L=\frac{\lambda}{4},\ f_1=\frac{1}{4L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or First overtone	$L=\frac{3\lambda}{4},\ f_3=\frac{3}{4L}\sqrt{\frac{T}{\mu}}$	
Fifth harmonics or Second overtone	$L=\frac{5\lambda}{4},\ f_5=\frac{5}{4L}\sqrt{\frac{T}{\mu}}$	
$(2n+1)^{th}$ harmonic or n^{th} overtone	$L=\frac{(2n+1)\lambda}{4}$, $f_{2n+1}=\frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}}$	

Sound Waves

Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed v of a sound wave in a medium having bulk modulus B and density ρ is

$$v=\sqrt{\frac{B}{\rho}} \text{ (speed of sound).}$$

In air at 20°C, the speed of sound is 343 m/s. A sound wave causes a longitudinal displacement s of a mass element in a medium as given by

$$s=s_m\cos(kx-\omega t),$$

where s_m is the displacement amplitude (maximum displacement) from equilibrium, $k = 2\pi/\lambda$, and $\omega = 2\pi f$, λ and f being the wavelength and frequency of the sound wave. The wave also causes a pressure change Δp from the equilibrium pressure:

$$\Delta p=\Delta p_m\sin(kx-\omega t),$$

where the pressure amplitude is $\Delta p_m=(v\rho\omega)s_m$.

Sound Intensity

The intensity I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface:

$$I=\frac{P}{A},$$

where P is the time rate of energy transfer (power) of the sound wave and A is the area of the surface intercepting the sound. The intensity I is related to the displacement amplitude s_m of the sound wave by

$$I=\frac{1}{2}\rho v\omega^2 s_m^2.$$

The intensity at a distance r from a point source that emits sound waves of power P_x is

$$I=\frac{P_x}{4\pi r^2}.$$

Sound Level in Decibels: The sound level β in decibels (dB) is defined as

$$\beta=(10\text{ dB})\log_{10}\frac{I}{I_0},$$

where $I_0(=10^{-12}\text{ W/m}^2)$ is a reference intensity level to which all intensities are compared. For every factor-of-10 increase in intensity, 10 dB is added to the sound level.

Standing Wave in Open and Closed Pipes

Patterns in Pipes Standing sound wave patterns can be set up in pipes. A pipe open at both ends will resonate at frequencies

$$f=\frac{v}{\lambda}=\frac{nv}{2L},\ n=1,2,3,\ldots.$$

where v is the speed of sound in the air in the pipe. For a pipe closed at one end and open at the other, the resonant frequencies are

$$f=\frac{v}{\lambda}=\frac{nv}{4L},\ n=1,3,5,\ldots.$$

Beats

Beats arise when two waves having slightly different frequencies, f_1 and f_2, are detected together. The beat frequency is

$$f_{beat} = f_1 - f_2.$$

Doppler Effect

Doppler effect is a change in the observed frequency of a wave when the source or the detector moves relative to the transmitting medium (such as air). For sound the observed frequency f' is given in terms of the source frequency f by

$$f' = f\frac{v \pm v_o}{v \pm v_s} \quad \text{(general Doppler effect)},$$

where v_o is the speed of the detector relative to the medium, v_s is that of the source, and v is the speed of sound in the medium. The signs are chosen such that f' tends to be greater for motion toward and less for motion away.

Solved Examples

1. One end of a long string is attached to an oscillator moving in transverse direction at a frequency of 20 Hz. The string has a cross-section area of 0.80 mm^2 and a density of 12.5 g cm^{-3}. It is subjected to a tension of 64 N along the X axis. At $t = 0$, the source is at a maximum displacement $y = 1.0$ cm.
   (i) Find the speed of the wave traveling on the string.
   (ii) Write the equation for the wave.
   (iii) What is the displacement of the particle of the string at $x = 50$ cm at time $t = 0.05$ s?
   (iv) What is the velocity of this particle at this instant?

Sol. As the wave is under tension T, the maximum wave velocity of wave is $v = \sqrt{\frac{T}{\mu}}$ where μ is the mass per unit length of the string. The wave equation is $y = A\cos(\omega t)$ where ω is angular frequency and A is amplitude of wave.

(i) The mass of 1 m long part of the string is

$$m = (0.80\text{ mm}^2) \times (1\text{ m}) \times (12.5\text{ gcm}^{-3})$$
$$= (0.80 \times 10^{-6}\text{ m}^3) \times (12.5 \times 10^3\text{ kgm}^{-3})$$
$$= 0.01\text{ kg}$$

So, the linear mass density is $\mu = 0.01$ kgm^{-1}.

The wave speed is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{64\text{ N}}{0.01\text{ kgm}^{-1}}} = 80\text{ ms}^{-1}$

(ii) The amplitude of the source is $A = 1.0$ cm and the frequency is $= 20$ Hz. The angular frequency is $\omega = 2\pi f = 40\,\pi$ rad.s^{-1}. Also at $t = 0$, the displacement is equal to its amplitude, i.e., at $t = 0$, $x = A$. The equation of motion of the source is, therefore,

$$y = (1.0\text{ cm})\cos\left[\left(40\pi\text{ rad.s}^{-1}\right)t\right] \quad \text{...(i)}$$

The equation of the wave traveling on the string along the position X-axis is obtained by replacing t with $t - x/v$ in equation (i). It is, therefore,

$$y = (1.0\text{ cm})\cos\left[\left(40\pi\text{ rad.s}^{-1}\right)\left(t - \frac{x}{v}\right)\right]$$

$$= (1.0\text{cm})\cos\left[\left(40\pi\text{ rad.s}^{-1}\right)t - \left(\frac{x}{2}\text{m}^{-1}\right)\right]$$

$(\because v = 80\text{ ms}^{-1})$

(iii) The displacement of the particle at $x = 50$ cm at time $t = 0.05$ s is by

$$y = (1.0\text{ cm})\cos\left[(40\pi\text{ rad.s}^{-1})(0.05\text{ s}) - \left(\frac{\pi}{2}\text{ m}^{-1}\right)(0.5\text{ m})\right]$$

$$= (1.0\text{ cm})\cos\left[2\pi - \frac{\pi}{4}\right]$$

$$= \frac{1.0}{\sqrt{2}} = 0.71\text{ cm}$$

(iv) The velocity of the particle at position x at time t is, by

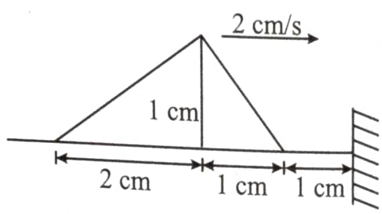
$$v = \frac{\partial y}{\partial t} = -(1.0\text{ cm})\left(40\pi\text{s}^{-1}\right)\sin\left[\left(40\pi\text{s}^{-1}\right)t - \left(\frac{\pi}{2}\text{ m}^{-1}\right)x\right]$$

Putting the values of $x = 50$ cm and $t = 0.05$ sec

$$v = -\left(40\pi\text{cms}^{-1}\right)\sin\left(2\pi - \frac{\pi}{4}\right)$$

$$= \frac{40\pi}{\sqrt{2}}\text{cms}^{-1} \approx 89\text{cms}^{-1}$$

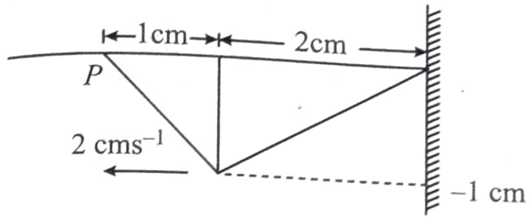
2. The figure shown a triangle pulse on a rope at $t = 0$. It is approaching a fixed end at 2 cm/s



   (i) Draw the pulse at $t = 2$ sec.
   (ii) The particle speed on the leading edge at the instant depicted is________.

Sol. (i) String wave will change its phase by π on striking with rigid support

The wave will move 4 cm in $t = 2$ sec, so the profile of wave after $t = 2$ sec will look like:



(ii) The particle speed on the leading edge at $t = 2$ sec

$v_P = (\text{slope}) \times (v_{\text{wave}})$

$= (-1) \times (2\ \text{cms}^{-1})$

$v_P = -2\text{cms}^{-1}$

where '–ve' sign indicates particle moving leftwards

3. The vibrations of a string fixed at both ends are described by the equation

$y = (5.00\ \text{mm}) \sin [(1.57\ \text{cm}^{-1})x] \sin [(314\ \text{rad.s}^{-1})t]$

(i) What is the maximum displacement of the particle at $x = 5.66$ cm?

(ii) What are the wavelengths and the wave speeds of the two transverse waves that combine to give the above vibration?

(iii) What is the velocity of the particle at $x = 5.66$ cm at time $t = 2.00$ s?

(iv) If the length of the string is 10.0 cm, locate the nodes and the anti-nodes. How many loops are formed in the vibration?

Sol. The transverse velocity of particle of string is $u = \dfrac{\partial y}{\partial t}$. The wave velocity is $v = f\lambda$. Comparing wave equation with $y = A \sin kx \sin \omega t$, we get the amplitude A and angular frequency of the wave.

(i) The amplitude of the vibration of the particle at position x is

$A = (5.00\ \text{mm}) \sin [(1.57\ \text{cm}^{-1})x]$

putting $x = 5.66$ cm, and re-writing the expression

$$A = (5.00\ \text{mm})\sin\left[\frac{\pi}{2} \times 5.66\right]$$

$$\text{or} \quad = \left|(5.00\ \text{mm})\sin\left(\frac{5\pi}{2} + \frac{\pi}{3}\right)\right|$$

$$= \left|(5.00\ \text{mm})\cos\frac{\pi}{3}\right|$$

$= 2.50$ mm

(ii) From the given equation, the wave number $k = 1.57\ \text{cm}^{-1}$ and the angular frequency $\omega = 314\ \text{rad.s}^{-1}$. Thus, the wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{1.57\ \text{cm}^{-1}} = 4.00\ \text{cm}$$

and Frequency is $f = \dfrac{\omega}{2\pi} = \dfrac{314\ \text{rad.s}^{-1}}{2 \times 3.14} = 50\ \text{s}^{-1}$

The wave speed is

$v = f\lambda = (50\ \text{s}^{-1})(4.00\ \text{cm}) = 2.00\ \text{ms}^{-1}$.

(iii) The velocity of the particle at position x at time t is given by

$$u = \frac{\partial y}{\partial t} = (5.00\ \text{mm})\sin\left[\left(1.57\ \text{cm}^{-1}\right)x\right]\left[314\ \text{s}^{-1} \times \cos\left(314\ \text{s}^{-1}\right)t\right]$$

$$\frac{\partial y}{\partial t} = (5.00\ \text{mm})\sin\left(\frac{\pi}{2}\text{cm}^{-1}x\right)\{200\pi\cos(200\pi t)\}$$

putting $x = 5.66$ cm & $t = 2$ sec, we have

$$\frac{\partial y}{\partial t} = (5\ \text{mm} \times 200\pi)\sin\left(\frac{\pi}{2}\text{cm}^{-1} \times 5.66\ \text{cm}\right)\cos(400\pi)$$

$$= (5 \times 10^{-3}\ \text{m} \times 200 \times 3.14\ \text{sec}^{-1})\sin\left(\frac{5\pi}{2} + \frac{\pi}{3}\right).1$$

$$= (3.14\ \text{ms}^{-1})\cos\frac{\pi}{3}$$

or $v = 1.57$ m/s $= 157\ \text{cms}^{-1}$

(iv) The nodes occur where the amplitude is zero, i.e.,

$\sin (1.57\ \text{cm}^{-1})x = 0$ or $\left(\dfrac{\pi}{2}\ \text{cm}^{-1}\right)x = n\pi$

Where n is an integer. Thus, $x = 2n$ cm. $(n = 0, 1, 2 \ldots)$

The nodes, therefore, occur at $x = 0$, 2 cm, 4 cm, 6 cm, 8 cm and 10 cm. Anti-nodes occur in between them, i.e., at $x = 1$ cm, 3 cm, 5 cm, 7 cm and 9 cm. The string vibrates in 5 loops.

4. The loudness level at a distance R from a long linear source is found to be 40dB. At this point, the amplitude of oscillation of air molecules is 0.01cm. Then find the loudness level and amplitude at a point at distance '10R' from the source.

Sol. For a linear source we know.

Intensity $(I) \propto \dfrac{1}{R}$ (R : distance from source)

and amplitude $(A) \propto \dfrac{1}{\sqrt{R}}$

Loudness $(L) = 10\log_{10}\left(\dfrac{I}{I_0}\right)$

where I is intensity at 'R' i.e., $I = I_0/R$

and I_0 original intensity

Let the intensities at $r_1 = 10R$ and $r_2 = R$ be I_1 & I_2 respectively

Hence, loudness at $r = 10R$, $L_1 = 10\log_{10}\left(\frac{I_1}{I_0}\right)$

and loudness at $r = R$, $L_2 = 10\log_{10}\left(\frac{I_2}{I_0}\right)$

$\Rightarrow L_1 - L_2 = 10\log_{10}\left(\frac{I_1}{I_2}\right)$

or $L_1 - 40\text{ dB} = 10\log_{10}\left(\frac{1/10R}{1/R}\right)$

$L_1 - 40\text{ dB} = -10\text{ dB}$

$\Rightarrow$ Loudness at $r = 10R$,

$L_1 = 30\text{ dB}$

$\because$ Amplitude at $r = R$ i.e.,

$$A = \frac{A_0}{\sqrt{R}} = 0.01\text{ cm}$$

or original amplitude $(A_0) = 0.1\sqrt{R}$ cm

$\Rightarrow$ Amplitude at $r = 10R$

i.e., $A' = \frac{A_0}{\sqrt{10R}} = \frac{0.01\sqrt{R}}{\sqrt{10R}}$ cm

or amplitude $A' = \frac{0.01}{\sqrt{10}}$ cm $= 10\sqrt{10}\mu$m

5. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the length of the pipes. (take speed of sound $v_s = 330$ m/s)

Sol. The beat are produced when the wave of same amplitude but different frequencies, resonate with each other.

Let the length of open and closed pipes be l_1 and l_2 respectively.

The frequency of first over tone of open organ pipe is

$$f_1 = \frac{2v}{2l_1} = \frac{v}{l_1}$$

The frequency of first over tone of closed organ pipe is

$$f_2 = \frac{3v}{4l_2}$$

Fundamental frequency of closed organ pipe

$$f = \frac{v}{4l_2};\quad \therefore = \frac{v}{4l_2} = \frac{330}{4l_2} = 110\text{ Hz (given)}$$

$$l_2 = \frac{330}{4\times 110} = 0.75\text{ m}$$

As beat frequency = 2.2 Hz

$$= \frac{v}{l_1} - \frac{3v}{4l_2} \Rightarrow \frac{330}{l_1} - \frac{3\times 330}{4\times 0.75} = 2.2\text{ Hz}$$

$\therefore l_1 = \frac{330}{332.2} = 0.993$ m; (which is the length of closed organ pipe)

Also, beat frequency $= \frac{3v}{4l_2} - \frac{v}{l_1} = 2.2$ Hz

or $\frac{3\times 330}{4\times 0.75} - \frac{330}{l_1} = 2.2; \frac{330}{l_1} = 327.8$

$l_1 = 1.006$ m (which is the length of open organ pipe)

6. A tuning fork of frequency 256 Hz and an open orange pipe of slightly lower frequency are at 17°C. When sounded together, they produce 4 beats per second. On altering the temperature of the air in the pipes, it is observed that the number of beats per second first diminishes to zero and then increases again to 4. By how much and in what direction has the temperature of the air in the pipe been altered?

Sol. In a open organ pipe the frequency of the wave is $f = \frac{v_t}{\lambda}$

where v_t is the velocity of wave at temperature t and $\lambda = 2L$ is the wavelength of the vibrating wave. If temperature of air inside the organ pipe changes, the velocity of wave also changes, since $v \propto \sqrt{T}$.

$f = \frac{v_{17}}{2L}$ where L = length of the pipe

$$\therefore 256 - \frac{v_{17}}{2L} = 4 \text{ or } \frac{v_{17}}{2L} = 252\text{ Hz} \quad \text{...(i)}$$

Since beats decrease first and then increase to 4, the frequency of the pipe increases. This can happen only if the temperature increases.

Let t be the final temperature, in Celsius,

$$f = \frac{v_t}{2L} - 256 = 4 \text{ or } \frac{v_t}{2L} = 260\text{ Hz} \quad \text{...(ii)}$$

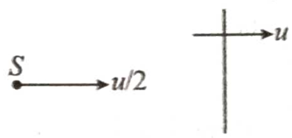
dividing equation (ii) and (i)

$$\frac{v_t}{v_{17}} = \frac{260}{252} \text{ or } \sqrt{\frac{273+t}{273+17}} = \frac{260}{252} \quad (\because v \propto \sqrt{T})$$

or $t = 308.7 - 273 = 35.7°\text{C} - 17°\text{C} = 18.7°\text{C}$.

$\therefore$ Rise in temperature = 18.7°C

7. A wall is moving with velocity u and a source of sound moves with velocity $u/2$ in the same direction as shown in figure. Assuming that the sound travels with velocity $10u$. The ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall, is equal to (assume observer for reflected sound is at rest):



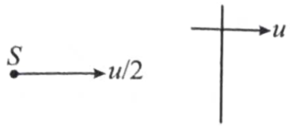
(a) 9 : 11
(b) 11 : 19
(c) 4 : 5
(d) 5 : 4

Sol. (*a*) Frequency of incident wave is

$$f' \text{ wall (recieved)} = \frac{10u - u}{10u - u/2} f = \frac{9u}{9.5u} f$$

Wave length of incident wave is

$$\Rightarrow \lambda_1 = \frac{10u - u/2}{f} = \frac{9.5u}{f} \quad ...(i)$$



Frequency of reflected wave is

$$f'' \text{ wall (recieved)} = \frac{10.u}{10u + u} f' = \frac{10u}{11u} \times \frac{9u}{9.5u} f$$

Wave length of reflected wave is

$$\Rightarrow \lambda_2 = \frac{10u + u}{f'} = \frac{11u \times 9.5}{9f} \quad ...(ii)$$

From equation (i) and (ii)

$$\frac{\lambda_1}{\lambda_2} = \frac{9.5}{11 \times 9.5} \times 9 = \frac{9}{11}$$

8. The equation for a wave traveling in the direction x on a string is

$$y = (3.0 \text{ cm}) \sin\left[\left(\pi \text{ cm}^{-1}\right)x - \left(10\pi \text{ s}^{-1}\right)t\right].$$

(i) Find the maximum velocity of a particle of the string.

(ii) Find the acceleration of a particle at $x = 6.0$ cm at time $t = 0.1$ s

Sol. The maximum velocity is $v = \frac{\partial y}{\partial t}$ While the acceleration $a = \frac{\partial v}{\partial t}$

(i) The velocity of the particle at x at time t is

$$v = \frac{\partial y}{\partial t}$$

So, $v = \frac{\partial y}{\partial t} = 3\cos(\pi x - 10\pi t)(-10\pi) \times 10^{-2}$

For maximum velocity,

So, $v_{max} = \frac{3\pi}{10}$ m/s

(ii) The acceleration of the particle at x at time t is

$$a = \frac{\partial v}{\partial t}$$

So, $a = -3(-10\pi)\sin(\pi x - 10\pi t)(-10\pi)$

at $x = 6$ cm

and $t = 0.1$ s

then, $a = 0$ m/s^2

9. Two waves of the same frequency and amplitudes $2a$ and $3a$ are super-imposed on each other.

(*i*) For what values of the phase difference ϕ the amplitude A of the resultant wave will be maximum and for what values minimum?

(*ii*) Calculate the ratio of maximum and minimum intensities of the resultant wave.

Sol. (i) The intensity is maximum for $\Delta\phi = 0, 2\pi, 4\pi$,...........

The intensity is minimum for $\Delta\phi = \pi, 3\pi, 5\pi$, ...

(ii) The maximum intensity is

$I_{max} \propto (a_1 + a_2)^2 = 25a^2$

and minimum intensity is

$I_{min} \propto (a_1 - a_2)^2 = a^2$

$\because \frac{I_{max}}{I_{min}} = 25$

10. The equation of a standing wave produced on a string fixed at both the ends is

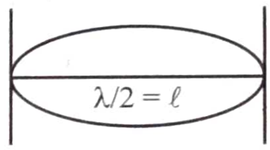
$y = (0.4)\sin[(0.314 \text{ cm}^{-1})x] \cos[(600\,\pi \text{ s}^{-1})t]$

What would be the smallest length of the string?

(*a*) 20 cm (*b*) 30 cm

(*c*) 10 cm (*d*) 5 cm

Sol. (*c*) Both ends of strings are fixed, so both ends are node. So it looks as,



So, possible smallest length

$$\ell = \frac{\lambda}{2}$$

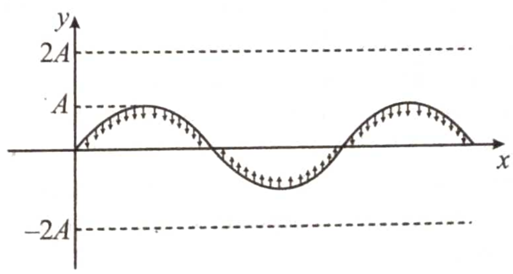
$$k = \frac{2\pi}{\lambda}; \Rightarrow \lambda = \frac{2\pi}{k}$$

From equation,

$$\lambda = \frac{2\pi}{0.314} = \frac{2 \times 3.14}{0.314} = 20 \text{ cm}$$

$$\ell = \frac{\lambda}{2} = \frac{20}{2} = 10 \text{ cm}$$

11. Figure shows the standing waves pattern in a string at $t = 0$. Find out the equation of the standing wave where the amplitude of anti-node is $2A$.



Sol. Let we assume the equation of standing waves is:

$y = 2A\sin(kx + \phi_1)\sin(\omega t + \phi_2)$

$\because$ At $x = 0$, $y = 0$ (node)

$y = A\sin(\phi_1)\sin(\omega t + \phi_2) = 0$

$\phi_1 = 0$

Now,

At $x = \dfrac{\lambda}{4}$

$y = 2A\sin(\omega t + \phi_2)$

at $t = 0$, $y = A$

$A = 2A\sin(\omega t + \phi_2)$

$\sin(\phi_2) = \dfrac{1}{2} \qquad (\because t = 0)$

$\phi_2 = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$

As seen from figure velocity is negative

so, $\phi_2 = \dfrac{5\pi}{6}$

$y = 2A\sin(kx)\sin\left(\omega t + \dfrac{5\pi}{6}\right)$

12. A wire under tension vibrates with a frequency of 450 per second. What would be the fundamental frequency if the wire were half as long, twice as thick and under one-fourth tension

(*a*) 225 cps (*b*) 190 cps

(*c*) 247 cps (*d*) 174 cps

Sol. (*a*) We have,

$$v = \frac{1}{2\ell}\sqrt{\frac{F}{\pi r^2 \rho}}$$

$$\text{So, } 450 = \frac{1}{2\ell_1}\sqrt{\frac{T_1}{\pi r_1^2 \rho}}$$

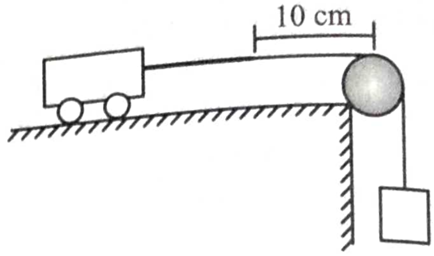
$$v_2 = \frac{1}{2\ell_2}\sqrt{\frac{T_2}{\pi r_2^2 \rho}}$$

$$\frac{450}{v_2} = \frac{\ell_2}{\ell_1}\sqrt{\frac{T_1}{T_2} \times \frac{\pi r_2^2 \rho}{\pi r_1^2 \rho}}$$

$$\frac{450}{v_2} = \frac{1}{2}\sqrt{4 \times 4}$$

$$v_2 = \frac{450 \times 2}{4} = 225 \text{ cps}$$

13. A heavy string is tied at one end to a movable support and to a light thread at the other end as shown in figure. The thread goes over a fixed pulley and supports a weight to produce a tension. The lowest frequency with which the heavy string resonates is 120 Hz. If the movable support is pushed to the right by 10 cm so that the joint is placed on the pulley, then the minimum frequency at which the heavy string can resonate is



(*a*) 120 Hz (*b*) 60 Hz

(*c*) 240 Hz (*d*) 480 Hz

Sol. (*c*) Initially the condition at the two ends of rod are different one is node (fixed) other is anti-node (free). So behaviour is just like a closed pipe closed at one end.

Hence fundamental or lowest frequency is $\dfrac{v}{4L}$ where L is length of heavy string, so

$$\frac{v}{4L} = 120 \qquad \text{...(i)}$$

Now, in second case, conditions at the two ends of rod become similar (both ends are nodes (fixed), So behaviour is just like a open pipe open at both ends.

So minimum frequency $= \dfrac{v}{4L}$

According to question

$$\therefore \quad f_1\lambda_1 = f_2\lambda_2$$

$$120 \times 4L = f_2 \times 2L$$

$$f_2 = 240 \text{ Hz}$$

14. One end of a string of length L is tied to the ceiling of a lift accelerating upwards with an acceleration $2g$. The other end of the string is free. The linear mass density of the string varies linearly from 0 to λ from bottom to top

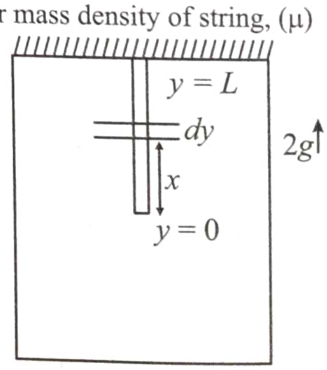
(*a*) The velocity of the wave in the string will be 0

(*b*) The acceleration of the wave on the string will be $3g/4$ every where

(*c*) The time taken by a pulse to reach from bottom to top will be $\sqrt{8L/3g}$

(*d*) The time taken by a pulse to reach from bottom to top will be $\sqrt{4L/3g}$

Sol. (*b,c*) Linear mass density of string, (μ)



$$\mu = \frac{\lambda}{L}y$$

$$\mu = \frac{dm}{dy} \Rightarrow dm = \mu dy$$

Mass of string of length y units is

$$\int_0^{m_y} dm = \int_0^{y} \mu dy = \int_0^{y} \frac{\lambda}{L} y dy = \frac{\lambda}{L}\frac{y^2}{2}$$

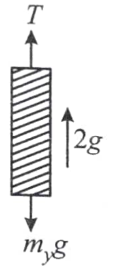
$$m_y = \frac{\lambda}{L}\frac{y^2}{2}$$

$T \rightarrow$ Tension in spring.

Applying Newton's II law on this segment of string

$$T - m_y g = m_y(2g)$$

$$T = m_y(3g) = \frac{3}{2}\frac{\lambda y^2}{L} g$$



Speed of wave in string

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{3\lambda y^2 g}{2L} \times \frac{L}{\lambda y}} = \sqrt{\frac{3yg}{2}}$$

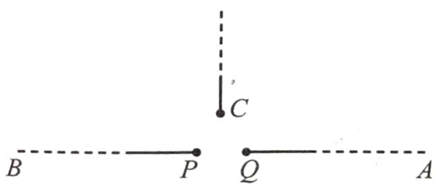
$$a = \frac{vdv}{dx} = \frac{3g}{4}$$

Also, $v = \dfrac{dy}{dt} = \sqrt{\dfrac{3yg}{2}}$

$$\int_0^{l} \frac{1}{\sqrt{y}} dy = \int_0^{t} \sqrt{\frac{3g}{2}} dt$$

$$t = \sqrt{\frac{8L}{3g}}$$

15. Two monochromatic sources of electromagnetic wave, P and Q emit waves of wavelength $\lambda = 20$ m and separated by 5 m as shown. A, B and C are three points where interference of these waves is observed. If phase of a wave generated by P is ahead of wave generated by Q by $\pi/2$ then (given intensity of both waves is I):



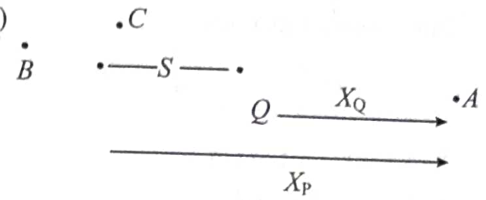
(*a*) Phase difference of these waves at B is 180^0

(*b*) Intensities at A, B and C are in the ratio 2 : 0 : 1 respectively

(*c*) Intensities at A, B and C are in the ratio 1 : 2 : 0 respectively

(*d*) Phase difference at A is 0^0

Sol. (*a,b,d*)



Wave emitted from Q $y = A \sin(\omega t - kx_Q)$

Wave emitted from P $y = A \sin(\omega t - kx_p + \frac{\pi}{2})$

$$\Delta\phi = \phi_p - \phi_Q = \omega t - kx_p + \frac{\pi}{2} - (\omega t - kx_Q)$$

$$= k(x_Q - x_p) + \frac{\pi}{2}$$

$$= \frac{2\pi}{\lambda}(x_Q - x_p) + \frac{\pi}{2}$$

$$= \frac{\pi}{10}(x_Q - x_p) + \frac{\pi}{2}$$

For A $x_Q - x_P = -5$

$$\Rightarrow (\Delta\phi)_A = \frac{\pi}{10}(-5) + \frac{\pi}{2} = 0$$

For B $x_Q - x_P = 5$

$$\Rightarrow (\Delta\phi)_B = \pi$$

For C $x_Q - x_P = 0 \Rightarrow (\Delta\phi)_C = \dfrac{\pi}{2}$

$I_A : I_B : I_C = (I + I + 2I\cos 0) : (I + I + 2I\cos\pi) : (I + I + 2I\cos\frac{\pi}{2}) = 2 : 0 : 1$

16. In a resonance tube experiment, a closed organ pipe of length 120 cm is used. Initially it is completely filled with water. It is vibrated with tuning fork of frequency 340 Hz. To achieve resonance the water level is lowered then (given $v_{air} = 340$ m/sec., neglect end correction):

(*a*) Minimum length of water column to have the resonance is 45 cm.

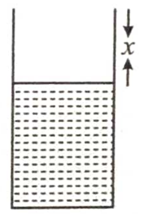
(*b*) The distance between two successive nodes is 50 cm.

(*c*) The maximum length of water column to create the resonance is 95 cm.

(*d*) The distance between two successive nodes is 25 cm.

Sol. (*a,b,c*)

$$\lambda = \frac{v}{f} = \frac{340}{340} = 1 \text{ m}$$



For resonance position

$x = (2n-1)\dfrac{\lambda}{4}$ $\qquad n = 1, 2, 3 \ldots\ldots$

So $x = 25, 75, 125$ cm ....

Tube length is only 120 cm so get resonance with minimum length of water $= 120 - 75 = 45$

Maximum length $= 120 - 25 = 95$

Distance between two successes nodes $= \dfrac{\lambda}{2} = \dfrac{100}{2} = 50$

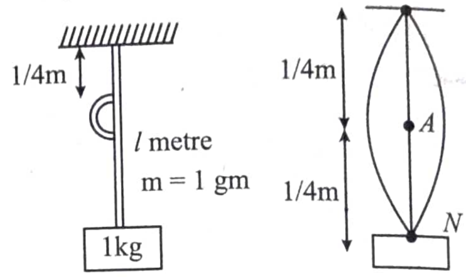
Ans A, B, C

17. Imagine a very thin uniform wire of length ℓ meter and total mass 1 gram. Fix one end of the wire in the ceiling, hang a large mass of 1 kg from the other end, and then pluck the wire $\dfrac{\ell}{4}$ m from the top. Assume that as the wire vibrates the large mass is not moving. The lowest frequency heard by us is 100 Hz. What is the length of the wire (in meter). (You may ignore the effect of gravity on the mass of the string)

Sol. [1] $\mu = \dfrac{10^{-3}\,\text{kg}}{\ell\text{ (metre)}} = \dfrac{10^{-3}}{\ell}\text{kg/m}$

$T \approx mg = (a)(10) = 10\text{N}$

$\therefore\ v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{10}{10^{-3}/\ell}} = \sqrt{10^4\ell} = 10^2\sqrt{\ell}$



$\dfrac{1}{4} = \dfrac{\lambda}{4} \qquad \therefore \lambda = 1\text{ m}$

$v = f\lambda \ \Rightarrow 10^2\sqrt{1} = 100\lambda$

$\therefore\ \ell = 1\text{ m}$

Exercise-1 (Topicwise)

WAVE EQUATION

1. A sine wave has amplitude A and wavelength λ. If V is the wave velocity and v be the maximum particle velocity, then

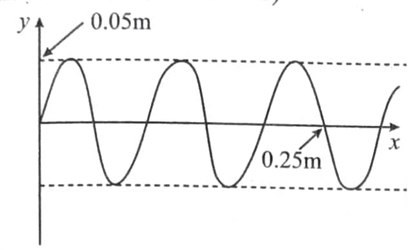
   (*a*) $V = v$ if $A = \dfrac{\lambda}{2\pi}$

   (*b*) $V = v$ if $A = 2\pi\lambda$

   (*c*) $V = v$ if $A = \pi\lambda$

   (*d*) V can never be equal to v

2. If the speed of the wave shown in the figure is 330 m/s in the given medium, then the equation of the wave propagating in the positive x-direction will be (all quantities are in MKS units)



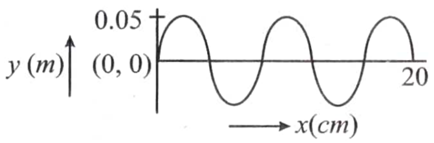
   (*a*) $y = 0.05 \sin 2\pi\,(4000\,t - 12.5\,x)$

   (*b*) $y = 0.05 \sin 2\pi\,(4000\,t - 122.5\,x)$

   (*c*) $y = 0.05 \sin 2\pi\,(3300\,t - 10\,x)$

   (*d*) $y = 0.05 \sin 2\pi\,(3300\,x - 10\,t)$

3. For the wave shown in figure, the equation for the wave, travelling along +x axis with velocity 350 ms^{-1} when its position at t = 0 is as shown



   (*a*) $0.05\sin\left(\dfrac{314}{4}x - 27475t\right)$

   (*b*) $0.05\sin\left(\dfrac{379}{5}x - 27000t\right)$

   (*c*) $1\sin\left(\dfrac{314}{4}x - 27500t\right)$

   (*d*) $0.05\sin\left(\dfrac{289}{5}x - 25700t\right)$

4. The distance between two consecutive crests in a wave train produced in a string is 5 cm. If 2 complete waves pass through any point per second, the velocity of the wave is

   (*a*) 10 cm/sec (*b*) 2.5 cm/sec

   (*c*) 5 cm/sec (*d*) 15 cm/sec

5. The phase difference between two points separated by 1m in a wave of frequency 120 Hz is 90°. The wave velocity is

   (*a*) 180 m/s (*b*) 240 m/s

   (*c*) 480 m/s (*d*) 720 m/s

6. The plane wave represented by an equation of the form $y = f(x - vt)$ implies the propagation along the positive x-axis without change of shape with constant velocity v:

   (*a*) $\dfrac{\partial y}{\partial t} = -v\left(\dfrac{\partial y}{\partial x}\right)$

   (*b*) $\dfrac{\partial y}{\partial t} = -v\left(\dfrac{\partial^2 y}{\partial x^2}\right)$

   (*c*) $\dfrac{\partial^2 y}{\partial t^2} = -v^2\left(\dfrac{\partial^2 y}{\partial x^2}\right)$

   (*d*) $\dfrac{\partial^2 y}{\partial t^2} = v^2 \Big/ \left(\dfrac{\partial^2 y}{\partial x^2}\right)$

7. A plane wave is represented by:

   $x = 1.2\sin(314t + 12.56y)$

   Where x and y are distances measured along in x and y direction in meters and t is time in seconds. This wave has

   (*a*) A wavelength of 0.25 m and travels in + ve x direction

   (*b*) A wavelength of 0.25 m and travels in + ve y direction

   (*c*) A wavelength of 0.5 m and travels in – ve y direction

   (*d*) A wavelength of 0.5 m and travels in – ve x direction

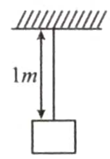
8. If the wave equation $y = 0.08\sin\dfrac{2\pi}{\lambda}(200t - x)$ then the velocity of the wave will be

   (*a*) $400\sqrt{2}$ (*b*) $200\sqrt{2}$

   (*c*) 400 (*d*) 200

VELOCITY OF WAVE, ENERGY AND POWER

9. A block of mass 1 kg is hanging vertically from a string of length 1m and Mass/length =0.001 kg/m. A small pulse is generated at its lower end. The Pulse reaches the top end in approximately.



   (*a*) 0.2 sec

   (*b*) 0.1 sec

   (*c*) 0.02 sec

   (*d*) 0.01 sec

10. A progressive wave is represented by

$y = 0.1 \sin \frac{8\pi}{7}\left(0.1t - \frac{x}{20}\right)$.

Where all the observations are in MKS system. The wave velocity will be

(a) 2 m/sec
(b) 15 m/sec
(c) 20 m/sec
(d) 40 m/sec

11. The equation of plane progressive wave propagating in positive X-direction is $y = a \sin 2\pi\left[t - \frac{x}{\lambda}\right]$. If the maximum particle velocity is three times the wave velocity then the wavelength of wave is

(a) $\frac{\pi a}{3}$ (b) πa
(c) $\frac{\pi a}{2}$ (d) $\frac{2\pi a}{3}$

12. When a sound wave of frequency 300Hz passes through a medium, the maximum displacement of a particle of the medium is 0.1 cm. The maximum velocity of the particle is equal to :

(a) 60 π cm/sec (b) 30 π cm/sec
(c) 30 cm/sec (d) 60 cm /sec

13. The rate of transfer of energy in a wave depends

(a) Directly on the square of the wave amplitude and square of the wave frequency.
(b) Directly on the square of the wave amplitude and square root of the wave frequency.
(c) Directly on the wave frequency and square of the wave amplitude.
(d) Directly on the wave amplitude and square of the wave frequency.

14. For a wave displacement amplitude is 10^{-8} m, density of air 1.3 kg m^{-3}, velocity in air 340 ms^{-1} and frequency is 2000 Hz. The intensity of wave is -

(a) 5.3×10^{-4} Wm^{-2} (b) 5.3×10^{-6} Wm^{-2}
(c) 3.5×10^{-8} Wm^{-2} (d) 3.5×10^{-6} Wm^{-2}

INTERFERENCE OF WAVE

15. The amplitude of a wave represented by displacement equation : $y = \frac{1}{\sqrt{a}} \sin \omega t \pm \frac{1}{\sqrt{b}} \cos \omega t$ will be -

(a) $\frac{a+b}{ab}$ (b) $\frac{\sqrt{a}+\sqrt{b}}{ab}$
(c) $\frac{\sqrt{a}\pm\sqrt{b}}{ab}$ (d) $\sqrt{\frac{a+b}{ab}}$

16. The resultant amplitude, when two waves of same frequency but with amplitudes a_1 and a_2 superimpose at phase difference of $\frac{\pi}{2}$, will be

(a) $a_1 + a_2$ (b) $a_1 - a_2$
(c) $\sqrt{a_1^2 + a_2^2}$ (d) $a_1^2 + a_2^2$

17. The ratio of sound intensities of two waves of the same frequency is 1 : 16. Then the ratio of the amplitudes will be

(a) 1 : 2 (b) 1 : 4
(c) 1 : 8 (d) 1 : 16

18. If two progressive waves of amplitude A_1 and A_2 superpose, while moving in the same direction, then the amplitude of the resultant wave will be

(a) $A_1 + A_2$
(b) $\sqrt{A_1^2 + A_2^2}$
(c) Between $A_1 + A_2$ and $A_1 - A_2$
(d) $A_1 - A_2$

19. When two waves of the same amplitude and frequency but having a phase difference of ϕ, travelling with the same speed in the same direction (positive x), interfere, then

(a) Their resultant amplitude will be twice that of a single wave but the frequency will be same.
(b) Their resultant amplitude and frequency will both be twice that of a single wave.
(c) Their resultant amplitude will depend on the phase angle while the frequency will be the same.
(d) The frequency and amplitude of the resultant wave will depend upon the phase angle.

20. Two wave pulses travel in opposite directions on a string and approach each other. The shape of the one pulse in inverted with respect to the other

(a) The pulses will collide with each other and vanish after collision.
(b) The pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
(c) The pulses will pass through each other but their shapes will be modified.
(d) The pulses will pass through each other without any change in their shape.

REFLECTION OF WAVES AND STANDING WAVES

21. A stationary wave is represented by

(a) $y = 2A \cos kx \sin \omega t$.
(b) $y = 2A \sin k(x - vt) \sin \omega t$
(c) $y = 2A \cos kx \cos(\omega t - kx)$
(d) $y = 2A \cos k(x - vt) \cos \omega t$

22. The following equations represent progressive transverse waves $z_1 = A\cos(kx - \omega t)$, $z_2 = A\cos(ky + \omega t)$ and $z_3 = A\cos(kz - \omega t)$. Which two waves can produce stationary waves?

(a) 1 and 2

(b) 2 and 3

(c) 1 and 3

(d) None of these

23. In a stationary wave the phase difference between the two particles situated on both the sides of a node is

(a) 0° (b) 90°

(c) 180° (d) 360°

24. Four wires of same length and same material, whose diameters are in the ratio 4 : 3 : 2 :1, are clamped in such a way that each wire produces note of fundamental frequency double that of the preceding wire. If the tension in the first wire is 2 Kg-*wt*, then tension in the second wire will be

(a) 4.5 (b) 8

(c) 9 (d) 16

25. In Melde's experiment, 8 loops are formed with a tension of a 0.75 N. If the tension is increased to four times then the number of loops produced will be

(a) 4 (b) 8

(c) 2 (d) 12

26. A 1 cm long string vibrates with fundamental frequency of 256 Hz. If the length is reduced to $\frac{1}{4}$cm keeping the tension unaltered, the new fundamental frequency will be

(a) 64 (b) 256

(c) 512 (d) 1024

27. A steel wire of mass 4.0 g and length 80 cm is fixed at the two ends. The tension in the wire is 50 N. The wavelength of the fourth harmonic of the fundamental will be :

(a) 80 cm (b) 60 cm

(c) 40 cm (d) 20 cm

28. A string is producing transverse vibration whose equation is $y = 0.021\sin(x + 30t)$, Where x and y are in meters and t is in seconds. If the linear density of the string is 1.3×10^{-4} kg/m, then the tension in the string in N will be

(a) 10 (b) 0.5

(c) 1 (d) 0.117

29. If vibrations of a string are to be increased by a factor of two, then tension in the string must be made

(a) Half

(b) Twice

(c) Four times

(d) Eight times

30. In order to double the frequency of the fundamental note emitted by a stretched string, the length is reduced to $\frac{3}{4}$th of the original length and the tension is changed. The factor by which the tension is to be changed, is

(a) $\frac{3}{8}$ (b) $\frac{2}{3}$

(c) $\frac{8}{9}$ (d) $\frac{9}{4}$

SOUND PROPAGATION AND PRESSURE WAVE

31. A stone is dropped into a lake from a tower 500 metre high. The sound of the splash will be heard by the man approximately after

(a) 11.5 seconds

(b) 21 seconds

(c) 10 seconds

(d) 14 seconds

32. When sound waves travel from air to water, which of the following remains constant

(a) Velocity

(b) Frequency

(c) Wavelength

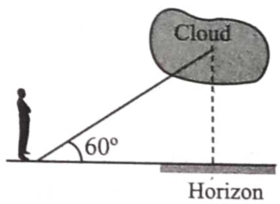
(d) All of these

33. The frequency of a man's voice is 300 Hz and its wavelength is 1 meter. If the wavelength of a child's voice is 1.5 m, then the frequency of the child's voice is:

(a) 200 Hz (b) 150 Hz

(c) 400 Hz (d) 350 Hz.

34. The elevation of a cloud is 60°above the horizontal. A thunder is heard 8 s after the observation of lighting. The speed of sound is 330 ms^{-1}. The vertical height of cloud from ground is:



(a) 2826 m (b) 2682 m

(c) 2286 m (d) 2068 m

35. The frequency of a tunning fork is 384 per second and velocity of sound in air is 352 m/s. How far the sound has traversed while fork completes 36 vibration

(a) 3 m (b) 13 m

(c) 23 m (d) 33 m

36. If amplitude of waves at distance r from a point source is A, the amplitude at a distance $2r$ will be

(a) $2A$ (b) A

(c) $A/2$ (d) $A/4$

VELOCITY OF SOUND WAVE, ENERGY AND INTENSITY

37. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is

(a) $\sqrt{2/7}$ (b) $\sqrt{1/7}$

(c) $\sqrt{3}/5$ (d) $\sqrt{6}/5$

38. At what temperature velocity of sound is double than that at 0°C

(a) 819 K

(b) 819°C

(c) 600°C

(d) 600 K

39. When the temperature of an ideal gas is increased by 600 K, the velocity of sound in the gas becomes $\sqrt{3}$ times the initial velocity in it. The initial temperature of the gas is

(a) –73°C (b) 27°C

(c) 127°C (d) 327°C

40. Quality of a musical note depends on

(a) Harmonics present

(b) Amplitude of the wave

(c) Fundamental frequency

(d) Velocity of sound in the medium

41. The loudness and pitch of a sound depends on

(a) Intensity and velocity

(b) Frequency and velocity

(c) Intensity and frequency

(d) Frequency and number of harmonics

42. If separation between screen and point source is increased by 2% what would be the effect on the intensity

(a) Increases by 4%

(b) Increases by 2%

(c) Decreases by 2%

(d) Decreases by 4%

43. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distances of 2m and 3m respectively from the source. The ratio of the intensities of the waves at P and Q is

(a) 9 : 4 (b) 2 : 3

(c) 3 : 2 (d) 4 : 9

44. The sound intensity is 0.008 W/m^2 at a distance of 10 m from an isotropic point source of sound. The power of the source is approximately :

(a) 2.5 watt

(b) 0.8 watt

(c) 8 watt

(d) 10 watt

INTERFERENCE AND REFLECTION OF SOUND WAVE

45. When two sound waves with a phase difference of $\pi/2$, and each having amplitude A and frequency ω, are superimposed on each other, then the maximum amplitude and frequency of resultant wave is

(a) $\frac{A}{\sqrt{2}}:\frac{\omega}{2}$ (b) $\frac{A}{\sqrt{2}}:\omega$

(c) $\sqrt{2}A:\frac{\omega}{2}$ (d) $\sqrt{2}A:\omega$

46. If the phase difference between the two wave is 2π during superposition, then the resultant amplitude is

(a) Maximum

(b) Minimum

(c) Maximum or minimum

(d) None of these

47. Two waves of same frequency and intensity superimpose with each other in opposite phases, then after superposition the

(a) Intensity increases by 4 times

(b) Intensity increases by two times

(c) Frequency increases by 4 times

(d) None of these

VIBRATION IN AIR COLUMN AND BEATS

48. The frequencies of two sound sources are 256 Hz and 260 Hz. At $t = 0$, the intensity of sound is maximum. Then the phase difference at the time $t = 1/16$ sec will be

(a) Zero (b) π

(c) $\pi/2$ (d) $\pi/4$

49. A tuning fork whose frequency as given by manufacturer is 512 Hz is being tested with an accurate oscillator. It is found that the fork produces a beat of 2 Hz when oscillator reads 514 Hz but produces a beat of 6 Hz when oscillator reads 510 Hz. The actual frequency of fork is

(a) 508 Hz (b) 512 Hz

(c) 516 Hz (d) 518 Hz

50. For the stationary wave $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\,\pi t)$, the distance between a node and the next anti-node is

(a) 7.5 (b) 15

(c) 22.5 (d) 30

51. At a certain instant a stationary transverse wave is found to have maximum kinetic energy. The appearance of string at that instant is

(a) Sinusoidal shape with amplitude $A/3$

(b) Sinusoidal shape with amplitude $A/2$

(c) Sinusoidal shape with amplitude A

(d) Straight line

52. A standing wave having 3 nodes and 2 anti-nodes is formed between two atoms having a distance 1.21 Å between them. The wavelength of the standing wave is
(*a*) 1.21 Å
(*b*) 2.42 Å
(*c*) 6.05 Å
(*d*) 3.63 Å

53. If the velocity of sound in air is 350 m/s. Then the fundamental frequency of an open organ pipe of length 50 cm, will be
(*a*) 350 Hz (*b*) 175 Hz
(*c*) 900 Hz (*d*) 750 Hz

54. An organ pipe P_1 closed at one end vibrating in its first overtone and another pipe P_2 open at both ends vibrating in its third overtone are in resonance with a given tuning fork. The ratio of lengths of P_1 and P_2 is
(*a*) 1 : 2 (*b*) 1 : 3
(*c*) 3 : 8 (*d*) 3 : 4

55. Two closed pipe produce 10 beats per second when emitting their fundamental nodes. If their length are in ratio of 25 : 26. Then their fundamental frequency in Hz, are
(*a*) 270, 280
(*b*) 260, 270
(*c*) 260, 250
(*d*) 260, 280

56. Fundamental frequency of pipe is 100 Hz and other two frequencies are 300 Hz and 500 Hz then
(*a*) Pipe is open at both the ends
(*b*) Pipe is closed at both the ends
(*c*) One end open and another end is closed
(*d*) None of these

DOPPLER'S EFFECT

57. Doppler shift in frequency does not depend upon
(*a*) The frequency of the wave produced
(*b*) The velocity of the source
(*c*) The velocity of the observer
(*d*) Distance from the source to the listener

58. A source of sound of frequency 450 cycles/sec is moving towards a stationary observer with 34 m/sec speed. If the speed of sound is 340 m/sec, then the apparent frequency will be
(*a*) 410 cycles/sec
(*b*) 500 cycles/sec
(*c*) 550 cycles/sec
(*d*) 450 cycles/sec

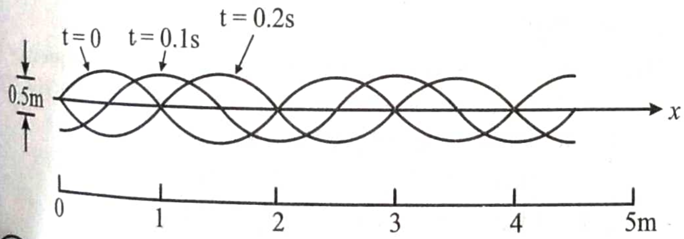
59. The speed of sound in air at a given temperature is 350 m/s. An engine blows whistle at a frequency of 1200 cps. It is approaching the observer with velocity 50 m/s. The apparent frequency in cps heard by the observer will be
(*a*) 600 (*b*) 1050
(*c*) 1400 (*d*) 2400

60. A man sitting in a moving train hears the whistle of the engine. The frequency of the whistle is 600 Hz
(*a*) The apparent frequency as heard by him is smaller than 600 Hz
(*b*) The apparent frequency is larger than 600 Hz
(*c*) The frequency as heard by him is 600 Hz
(*d*) None of these

61. What should be the velocity of a sound source moving towards a stationary observer so that apparent frequency is double the actual frequency (Velocity of sound is v)
(*a*) v (*b*) $2v$ (*c*) $\frac{v}{2}$ (*d*) $\frac{v}{4}$

Exercise-2 (Learning Plus)

1. Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct. (Mass per unit length of the string = 3 g/cm.)

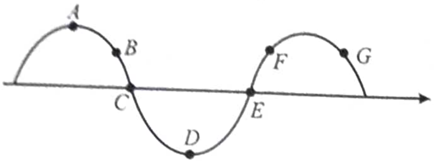


(*a*) Displacement amplitude of the wave is 0.25 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
(*b*) Displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.7/s.
(*c*) Displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 /s.
(*d*) Displacement amplitude of the wave is 0.5 m, wavelength is 2m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

2. The velocity of a wave propagating along a stretched string is 10m/s and its frequency is 100 Hz. The phase difference between the particles situated at a distance of 2.5 cm on the string will be

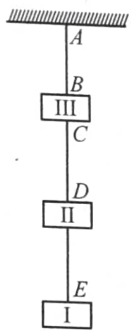
(*a*) $\pi/8$ (*b*) $\pi/4$ (*c*) $3\pi/8$ (*d*) $\pi/2$

3. The following figure depicts a wave travelling in a medium. Which pair of particles are in phase.



(*a*) *A* and *D* (*b*) *B* and *F* (*c*) *C* and *E* (*d*) *B* and *G*

4. Three blocks I, II, and III having mass of 1.6 kg, 1.6 kg and 3.2 kg respectively are connected as shown in the figure. The linear mass density of the wire *AB*, *CD* and *DE* are 10 g/m, 8 g/m and 10 g/m respectively. The speed of a transverse wave pulse produced in *AB*, *CD* and *DE* are: ($g = 10$ m/sec^2)



(*a*) 80 m/s, $20\sqrt{10}$ m/s, 40 m/s

(*b*) $20\sqrt{10}$ m/s, 80 m/s, 40 m/s

(*c*) $20\sqrt{10}$ m/s in all

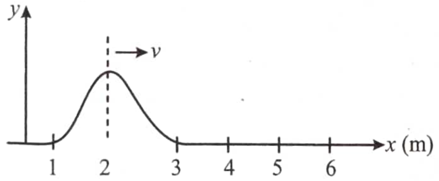
(*d*) 80 m/s in all

5. A copper wire is held at the two ends by rigid supports. At 30°C the wire is just taut, with negligible tension. The speed of transverse waves in this wire at 10°C is: ($\alpha = 1.7 \times 10^{-5}$ / °C, $Y = 1.3 \times 10^{11}$ N/m^2, $d = 9 \times 10^3$ kg/m^3).

(*a*) 80 m/sec (*b*) 90 m/sec

(*c*) 100 m/sec (*d*) 70 m/sec

6. Wave pulse on a string shown in figure is moving to the right without changing shape. Consider two particles at positions $x_1 = 1.5$ m and $x_2 = 2.5$ m. Their transverse velocities at the moment shown in figure are along directions :



(*a*) Positive *y*–axis and positive *y*–axis respectively

(*b*) Negative *y*–axis and positive *y*–axis respectively

(*c*) Positive *y*–axis and negative *y*–axis respectively

(*d*) Negative *y*–axis and negative *y*–axis respectively

7. The displacement produced by a simple harmonic wave is : $y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{17}\right)$ cm . The time period and maximum velocity of the particle will be respectively

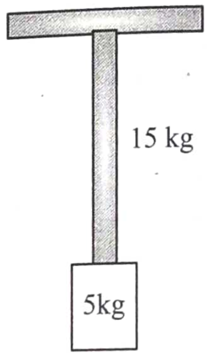
(*a*) 10^{-3} second and 200 m/s

(*b*) 10^{-2} second and 2000 m/s

(*c*) 10^{-3} second and 330 m/s

(*d*) 10^{-4} second and 20 m/s

8. A uniform rope of length 10 m and mass 15 kg hangs vertically from a rigid support. A block of mass 5 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.08 m is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope will be



(*a*) 0.08 m (*b*) 0.04 m

(*c*) 0.16 m (*d*) 0 m

9. A wire of 10^{-2}kgm^{-1} passes over a frictionless light pulley fixed on the top of a frictionless inclined plane, which makes an angle of 30° with the horizontal. Masses m and *M* are tied at two ends of wire such that m rests on the plane and *M* hangs freely vertically downwards. The entire system is in equilibrium and a transverse wave propagates along the wire with a velocity of 100 ms^{-1}.

(*a*) $M = 5$ kg (*b*) $\frac{m}{M} = \frac{1}{4}$ kg

(*c*) m=20 kg (*d*) $\frac{m}{M} = 4$

10. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a average power of 90 W and the string is under a tension of 100 N, then the highest frequency at which the source can operate is (take $\pi^2 = 10$):

(*a*) 45 Hz

(*b*) 50 Hz

(*c*) 30 Hz

(*d*) 62 Hz

11. A stretched sonometer wire resonates at a frequency of 350 Hz and at the next higher frequency of 420 Hz. The fundamental frequency of this wire is

(*a*) 350 Hz (*b*) 5 Hz

(*c*) 70 Hz (*d*) 170 Hz

12. Two stretched wires A and B of the same lengths vibrate independently. If the radius, density and tension of wire A are respectively twice those of wire B, then the fundamental frequency of vibration of A relative to that of B is

(a) 1 : 1 (b) 1 : 2
(c) 1 : 4 (d) 1 : 8

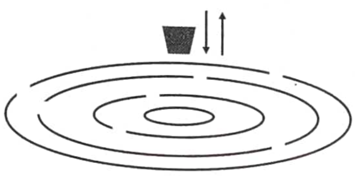
13. A string 1m long is drawn by a 300Hz vibrator attached to its end. The string vibrates in 3 segments. The speed of transverse waves in the string is equal to

(a) 100 m/s
(b) 200 m/s
(c) 300 m/s
(d) 400 m/s

14. A sonometer wire resonates with a given tuning fork forming standing waves with five anti-nodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M, the wire resonates with the same tuning fork forming three nodes and anti-nodes for the same position of the bridges. The value of M is

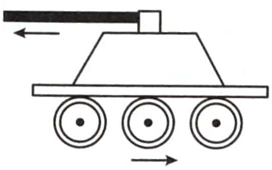
(a) 25kg (b) 5kg
(c) 12.5kg (d) 1/25kg

15. A piece of cork is floating on water in a small tank. The cork oscillates up and down vertically when small ripples pass over the surface of water. The velocity of the ripples being 0.21 ms^{-1}, wave length 15 mm and amplitude 5 mm, the maximum velocity of the piece of cork is $\left(\pi = \frac{22}{7}\right)$



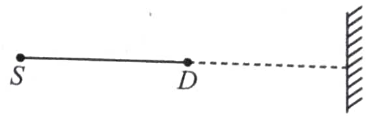
(a) 0.44 ms^{-1} (b) 0.24 ms^{-1}
(c) 2.4 ms^{-1} (d) 4.4 ms^{-1}

16. A machine gun is mounted on an armored car moving with a speed of 20 ms^{-1} as shown in figure. The gun can point against the direction of motion of car. The muzzle speed of bullet is equal to speed of sound in air i.e., 340 ms^{-1}. The time difference between bullet actually reaching and sound of firing reaching at a target 544 m away from car at the instant of firing is



(a) 1.2 s (b) 0.1 s
(c) 1 s (d) 10 s

17. A source of sound S and a detector D are placed at some distance from one another. A big cardboard is placed near the detector and perpendicular to the line SD as shown in figure. It is gradually moved away and it is shown that the intensity change from a maximum to a minimum as the board is moved through a distance of 20cm. What will be the frequency of the sound emitted. Velocity of sound in air is 336 ms^{-1}.



(a) 420 Hz (b) 422 Hz
(c) 450 Hz (d) 410 Hz

18. The ratio of speed of sound in Neon to that in water vapours at any temperature (when molecular weight of neon is 2.02×10^{-2} kg mol^{-1} and for water vapours is 1.8×10^{-2} kg mol^{-1})

(a) 1.06 (b) 1.60
(c) 6.10 (d) 15.2

19. A Firecracker exploding on the surface of a lake is heard as two sounds a time interval t apart by a man on a boat close to water surface. Sound travels with a speed u in water and a speed v in air. The distance from the exploding firecracker to the boat is

(a) $\frac{uvt}{u+v}$ (b) $\frac{t(u+v)}{uv}$
(c) $\frac{t(u-v)}{uv}$ (d) $\frac{uvt}{u-v}$

20. Under similar conditions of temperature and pressure, In which of the following gases the velocity of sound will be largest

(a) H_2 (b) N_2
(c) He (d) CO_2

21. How many times more intense is 90 dB sound than 40 dB sound?

(a) 5 (b) 50
(c) 500 (d) 10^5

22. Sound waves from a tuning fork F reach a point P by two separate routes FAP and FBP. When FBP is greater than FAP by 12 cm there is silence at P. If the difference is 24 cm the sound becomes maximum at P but at 36 cm there is silence again and so on. If velocity of sound in air is 330 ms^{-1}, the least frequency of tuning fork is :

(a) 1537 Hz (b) 1735 Hz
(c) 1400 Hz (d) 1375 Hz

23. The ratio of intensities between two coherent sound sources is 4:1 The difference of loudness in dB between maximum and minimum intensities when they interfere in space is

(a) 10 log 2 (b) 20 log 3
(c) 10 log 3 (d) 20 log 2

24. In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5 cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (when displacement is complete). The wavelength of sound is:

(a) 10/9 cm (b) 1 cm (c) 1/2 cm (d) 5/9 cm

25. A cylindrical tube, open at one end and closed at the other, is in acoustic unison with an external source of frequency held at the open end of the tube, in its fundamental note. Then

(a) The displacement wave from the source gets reflected with a phase change of π at the closed end

(b) The pressure wave from the source get reflected without a phase change at the closed end

(c) The wave reflected from the closed end again gets reflected at the open end

(d) All the above

26. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particle has maximum amplitude of vibration is (velocity of sound in air is 330 m\s)

(a) 0.125 m (b) 0.5 m
(c) 0.25 m (d) 2 m

27. If λ_1, λ_2, λ_3 are the wavelengths of the waves giving resonance in the fundamental, first and second overtone modes respectively in a open organ pipe, then the ratio of the wavelengths $\lambda_1 : \lambda_2 : \lambda_3$, is :

(a) 1 : 2 : 3 (b) 1 : 3 : 5
(c) 1 : 1/2 : 1/3 (d) 1 : 1/3 : 1/5

28. The maximum variation of pressure in an open organ pipe of length ℓ vibrating in fundamental mode is at.

(a) ends (b) middle of pipe
(c) $\frac{L}{4}$ from centre (d) $\frac{3L}{8}$ from centre

29. An open organ pipe of length L vibrates in second harmonic mode. The pressure vibration is maximum

(a) At the two ends

(b) At a distance $L/4$ from either end inside the tube

(c) At the mid-point of the tube

(d) None of these

30. The second overtone of an open pipe A and a closed pipe B have the same frequencies at a given temperature. Both pipes contain air. The ratio of fundamental frequency of A to the fundamental frequency of B is:

(a) 3 : 5 (b) 5 : 3
(c) 5: 6 (d) 6 : 5

31. A resonance tube is resonated with tuning fork of frequency 256 Hz. If the length of first and second resonating air columns are 32 cm and 100 cm, then end correction will be

(a) 1 cm (b) 2 cm
(c) 4 cm (d) 6 cm

32. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of air column that resonates in the fundamental mode, with a tuning fork is 0.1m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.

(a) 0.012m (b) 0.025m
(c) 0.05 m (d) 0.024

33. A closed organ pipe and an open pipe of same length produce 4 beats when they are set into vibrations simultaneously. If the length of each of them were twice their initial lengths, the number of beats produced will be [Assume same mode of vibration in both cases]

(a) 2 (b) 4
(c) 1 (d) 8

34. A pipe's lower end is immersed in water such that the length of air column from the top open end has a certain length 25 cm. The speed of sound in air is 350 m/s. The air column is found to resonate with a tuning fork of frequency 1750 Hz. By what minimum distance should the pipe be raised in order to make the air column resonate again with the same tuning fork

(a) 7 cm (b) 5 cm
(c) 35 cm (d) 10 cm

35. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then

(a) $18 > x$

(b) $x > 54$

(c) $54 > x > 36$

(d) $36 > x > 18$

36. A closed organ pipe has length 'l'. The air in it is vibrating in 3[rd] overtone with maximum displacement amplitude 'a'. The displacement amplitude at distance $l/7$ from closed end of the pipe is:

(a) 0 (b) a
(c) $a/2$ (d) None of these

37. A tuning fork of frequency 280 Hz produces 10 beats per sec when sounded with a vibrating sonometer string. When the tension in the string increases slightly, it produces 11 beats per sec. The original frequency of the vibrating sonometer string is:

(a) 269 Hz

(b) 291 Hz

(c) 270 Hz

(d) 290 Hz

38. A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats/s when excited along with a tuning fork of frequency f. Now when the tension of the string is slightly increased, the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency f of the tuning fork in Hz is

(a) 344 (b) 336

(c) 117.3 (d) 109.3

39. The speed of sound in a gas, in which two waves of wavelength 1.0 m and 1.02 m produce 6 beats per second, is approximately :

(a) 350 m/s (b) 300 m/s

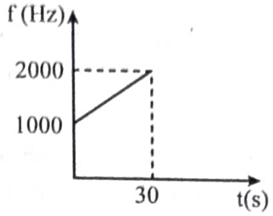
(c) 380 m/s (d) 410 m/s

40. A source S of frequency f_0 and an observer O, moving with speeds v_1 and v_2 respectively, are moving away from each other. When they are separated by distance $a(t=0)$, a pulse is emitted by the source. This pulse is received by O at time t_1 then t_1, is equal to

(a) $\dfrac{a}{v_s+v_2}$ (b) $\dfrac{a}{v_1+v_s}$

(c) $\dfrac{a}{v_s-v_2}$ (d) $\dfrac{a}{v_1+v_2+v_s}$

41. A detector is released from rest over a source of sound of frequency $f_0 = 10^3$ Hz. The frequency observed by the detector at time t is plotted in the graph. The speed of sound in air is ($g = 10$ m/s^2)



(a) 330 m/s (b) 350 m/s

(c) 300 m/s (d) 310 m/s

42. A source of sound S having frequency f. Wind is blowing from source to observer O with velocity u. If speed of sound with respect to air is C, the wavelength of sound detected by O is:

(a) $\dfrac{C+u}{f}$

(b) $\dfrac{C-u}{f}$

(c) $\dfrac{C(C+u)}{(C-u)f}$

(d) $\dfrac{C}{f}$

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. The particle displacement in a wave is given by
$y = 0.2 \times 10^{-5} \cos(500\,t - 0.025\,x)$
Where the distances are measured in meters and time in seconds. Now

(a) Wave velocity is 2×10^4 ms^{-1}

(b) Particle velocity is 2×10^4 ms^{-1}

(c) Initial phase difference is $\pi/2$

(d) Wavelength of the wave is (80π) m

2. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. if $a = 10^{-3}$ m and $v = 10$ m/s, then λ and f are given by

(a) $\lambda = 2\pi \times 10^{-2}$ m (b) $\lambda = 10^{-3}$ m

(c) $f = 10^3/(2\pi)$ Hz (d) $f = 10^4$ Hz

3. A wave equation which gives the displacement along the Y direction is given by $Y = 10^{-4} \sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave

(a) Travelling with a velocity of 30 m/s in the negative x direction

(b) Of wavelength π metre

(c) Of frequency $30/\pi$ hertz

(d) Of amplitude 10^{-4} metre travelling along the negative x direction

4. A sinusoidal progressive wave is generated in a string. It's equation is given by $y = (2\text{ mm}) \sin(2\pi x - 100\,\pi t + \pi/3)$. The time when particle at $x = 4$ m first passes through mean position, will be

(a) $\dfrac{1}{150}$ sec (b) $\dfrac{1}{12}$ sec

(c) $\dfrac{1}{300}$ sec (d) $\dfrac{1}{100}$ sec

5. A wave equation is given as $y = \cos(500t - 70x)$, where y is in mm, x in m and t is in sec

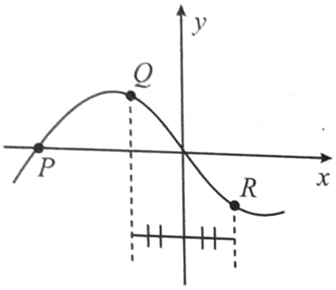
(a) The wave must be a transverse propagating wave

(b) The speed of the wave is 50/7 m/s

(c) The frequency of oscillations 1000π Hz

(d) Two closest points which are in same phase have separation $20\pi/7$ cm

6. At a certain moment, the photograph of a string on which a harmonic wave is travelling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.



(a) v_P is upwards

(b) $v_Q = -v_R$

(c) $|v_P| > |v_Q| = |v_R|$

(d) $v_Q = v_R$

7. Two waves of equal frequency f and velocity v travel in opposite directions along the same path. The waves have amplitudes A and $3A$. Then :

(a) The amplitude of the resulting wave varies with position between maxima of amplitude 4A and minima of zero amplitude

(b) The distance between a maxima and adjacent minima of amplitude is $V/2f$

(c) At point on the path the average displacement is zero

(d) The position of a maxima or minima of amplitude does not change with time

8. The vibrations of a string of length 600 cm fixed at both ends are represented by the equation

$$y = 4 \sin\left(\pi \frac{x}{15}\right) \cos(96\pi t)$$

Where x and y are in cm and t in seconds.

(a) The maximum displacement of a point $x = 5$ cm is $2\sqrt{3}$ cm

(b) The nodes located along the string are at $15n$ cm where integer n varies from 0 to 40

(c) The velocity of the particle at $x = 7.5$ cm at $t = 0.25$ sec is 0

(d) The equations of the component waves whose superposition gives the above wave are $2 \sin 2\pi\left(\frac{x}{30} + 48t\right)$, $2 \sin 2\pi\left(\frac{x}{30} - 48t\right)$

9. The length, tension, diameter and density of a wire B are double than the corresponding quantities for another stretched wire A. Then.

(a) Fundamental frequency of B is $\frac{1}{2\sqrt{2}}$ times that of A

(b) The velocity of wave in B is $\frac{1}{\sqrt{2}}$ times that of velocity in A

(c) The fundamental frequency of A is equal to the third overtone of B

(d) The velocity of wave in B is half that of velocity in A

10. A clamped string is oscillating in nth harmonic, then

(a) Total energy of oscillations will be n^2 times that of fundamental frequency

(b) Total energy of oscillations will be $(n-1)^2$ times that of fundamental frequency

(c) Average kinetic energy of the string over a complete oscillations is half of that of the total energy of the string

(d) None of these

11. Which one of the following statements is incorrect for stable interference to occur between two waves?

(a) The waves must have the same wave length.

(b) The waves must have a constant phase difference.

(c) The waves must be transverse only.

(d) The waves must have equal amplitudes.

12. Two sound waves move in the same direction in the same medium. The pressure amplitude of the waves are equal but the wavelength of the first wave is double that of the second. Let the average power transmitted across a cross section by the two wave be P_1 and P_2 and their displacement amplitudes are s_1 and s_2 then

(a) $P_1/P_2 = 1$ (b) $P_1/P_2 = 2$

(c) $s_1/s_2 = 1/2$ (d) $s_1/s_2 = 2/1$

13. The effect of making a hole exactly at ($1/3^{rd}$) of the length of the pipe from its closed end is such that :

(a) Its fundamental frequency is an octave higher than the open pipe of same length.

(b) Its fundamental frequency is thrice of that before making a hole.

(c) The fundamental frequency is 3/2 time of that before making a hole.

(d) The fundamental alone is changed while the harmonics expressed as ratio of fundamentals remain the same.

14. Two identical straight wires are stretched so as to produce 6 beats/sec. when vibrating simultaneously. When changing the tension slightly in one of them the beat frequency remains unchanged. Denoting by T_1, T_2, the higher & the lower initial tensions in the strings, then it could be said that while making the above changes in tension:

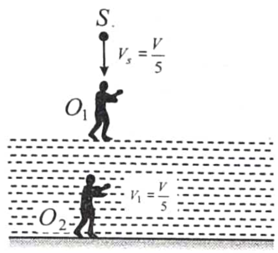
(a) T_2 was decreased (b) T_2 was increased

(c) T_1 was increased (d) T_1 was decreased

15. At the closed end of an organ pipe :
   (a) The displacement is zero
   (b) The displacement amplitude is maximum
   (c) The pressure amplitude is zero
   (d) The pressure amplitude is maximum

16. A girl stops singing a pure note. She is surprised to hear an echo of higher frequency, i.e., a higher musical pitch. Then:
   (a) There could be some warm air between the girl and the reflecting surface.
   (b) There could be two identical fixed reflecting surfaces, one half a wavelength of the sound wave away from the other.
   (c) The girl could be moving towards a fixed reflector.
   (d) The reflector could be moving towards the girl.

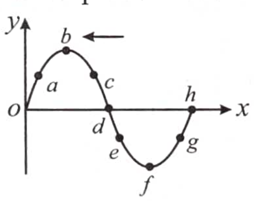
17. In the figure shown an observer O_1 floats (static) on water surface with ears in air while another observer O_2 is moving upwards with constant velocity $V_1 = V/5$ in water. The source moves down with constant velocity $V_s = V/5$ and emits sound of frequency 'f'. The velocity of sound in air is V and that in water is 4V. For the situation shown in figure:



   (a) The wavelength of the sound received by O_1 is $\frac{4V}{5f}$
   (b) The wavelength of the sound received by O_1 is V/f
   (c) The frequency of the sound received by O_2 is $\frac{21f}{16}$
   (d) The wavelength of the sound received by O_2 is $\frac{16V}{5f}$

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 18 to 21): The figure represents the instantaneous picture of a transverse harmonic wave traveling along the negative x-axis. Choose the correct alternative(s) related to the movement of the nine points shown in the figure.



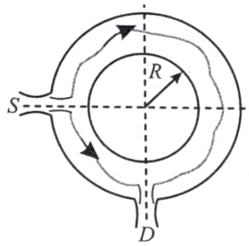
18. The points moving upward is/are
   (a) a (b) c
   (c) f (d) g

19. The points moving downwards is/are
   (a) o (b) b
   (c) d (d) h

20. The stationary points is/are
   (a) o (b) b
   (c) f (d) h

21. The points moving with maximum speed is/are
   (a) b (b) c
   (c) d (d) h

Comprehension (Q. 22 to 26): A narrow tube is bent in the form of a circle of radius R, as shown in the figure. Two small holes S and D are made in the tube at the positions right angle to each other. A source placed at S generated a wave of intensity I_0 which is equally divided into two parts : One part travels along the longer path, while the other travels along the shorter path. Both the part waves meet at the point D where a detector is placed



22. If a maxima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by
   (a) πR (b) $\frac{\pi R}{2}$
   (c) $\frac{\pi R}{4}$ (d) $\frac{2\pi R}{3}$

23. If the minima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by
   (a) $2\pi R$ (b) $\frac{3\pi R}{2}$
   (c) $\frac{2\pi R}{3}$ (d) $\frac{2\pi R}{5}$

24. The maximum intensity produced at D is given by
   (a) $4I_0$ (b) $2I_0$
   (c) I_0 (d) $3I_0$

25. The maximum value of λ to produce a maxima at D is given by
   (a) πR (b) $2\pi R$
   (c) $\frac{\pi R}{2}$ (d) $\frac{3\pi R}{2}$

26. The maximum value of λ to produce a minima at D is given by
   (a) πR (b) $2\pi R$
   (c) $\frac{\pi R}{2}$ (d) $\frac{3\pi R}{2}$

Comprehension (Q. 27 to 30): In an organ pipe (may be closed or open) of 99 cm length standing wave is setup, whose equation is given by longitudinal displacement

$$\xi = (0.1 \text{ mm}) \cos \frac{2\pi}{80}(y + 1 \text{ cm}) \cos 2\pi(400)t$$

Where y is measured from the top of the tube in centimeters and t in second. Here 1 cm is the end correction.

27. The upper end and the lower end of the tube are respectively:
(*a*) open – closed (*b*) closed – open
(*c*) open – open (*d*) closed – closed

28. The air column is vibrating in
(*a*) First overtone (*b*) Second overtone
(*c*) Third harmonic (*d*) Fundamental mode

29. Equation of the standing wave in terms of excess pressure is – (Bulk modulus of air $B = 5 \times 10^5$ N/m²)

(*a*) $P_{ex} = (125\,\pi \text{N/m}^2) \sin \frac{2\pi}{80}(y + 1 \text{ cm}) \cos 2\pi(400t)$

(*b*) $P_{ex} = (125\,\pi \text{ N/m}^2) \cos \frac{2\pi}{80}(y + 1 \text{ cm}) \sin 2\pi(400t)$

(*c*) $P_{ex} = (225\,\pi \text{N/m}^2) \sin \frac{2\pi}{80}(y + 1 \text{ cm}) \cos 2\pi(200t)$

(*d*) $P_{ex} = (225\,\pi \text{N/m}^2) \cos \frac{2\pi}{80}(y + 1 \text{ cm}) \sin 2\pi(200t)$

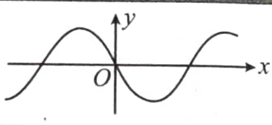
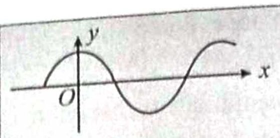
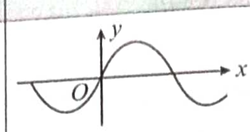
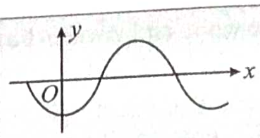
30. Assume end correction approximately equals to (0.3) × (diameter of tube), estimate the approximate number of moles of air present inside the tube (Assume tube is at NTP, and at NTP, 22.4 litre contains 1 mole)

(*a*) $\frac{10\pi}{36 \times 22.4}$ (*b*) $\frac{10\pi}{18 \times 22.4}$

(*c*) $\frac{10\pi}{72 \times 22.4}$ (*d*) $\frac{10\pi}{60 \times 22.4}$

MATCH THE COLUMN TYPE QUESTIONS

31. For four sine waves, moving on a string along positive x direction, displacement-distance curves (y-x curves) are shown at time $t = 0$. In the right column, expressions for y as function of distance x and time t for sinusoidal waves are given. All terms in the equations have general meaning. Correctly match y-x curves with corresponding equations:

	Column-I		Column-II
A.		p.	$y = A \cos(\omega t - kx)$
B.		q.	$y = -A \cos(kx - \omega t)$
C.		r.	$y = A \sin(\omega t - kx)$
D.		s.	$y = A \sin(kx - \omega t)$

(*a*) A-(p); B-(q); C-(s); D-(r)
(*b*) A-(r); B-(p); C-(s); D-(q)
(*c*) A-(q); B-(s); C-(p); D-(r)
(*d*) A-(r); B-(p); C-(q); D-(s)

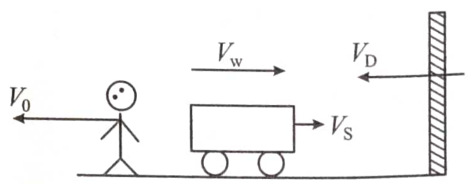
32. Match the statements in column-I with the statements in column-II.

	Column-I		Column-II
A.	$y = 4\sin(5x - 4t) + 3\cos(4t - 5x + \pi/6)$	p.	Particles at every position are performing SHM
B.	$y = 10\cos\left(t - \frac{x}{330}\right)$. $\sin(100)\left(t - \frac{x}{330}\right)$	q.	Equation of travelling wave
C.	$y = 10\sin(2\pi x - 120t) + 10\cos(120t + 2\pi x)$	r.	Equation of standing wave
D.	$y = 10\sin(2\pi x - 120t) + 8\cos(118t - 59/30\pi x)$	s.	Equation of Beats

(*a*) A-(p,r); B-(r,s); C-(p,q); D-(q)
(*b*) A-(p,r); B-(r,s); C-(q); D-(s,q)
(*c*) A-(p,q); B-(s); C-(p,r); D-(s)
(*d*) A-(p,r); B-(r); C-(p,q); D-(q,r,s)

33. S, O and W represent source of sound (of frequency f), observer & wall respectively. V_O, V_S, V_D, V are velocity of observer, source, wall & sound (in still air) respectively. V_W is the velocity of wind. They are moving as shown. Then match the following :

where $f_r = \frac{V + V_w + V_D}{V + V_w - V_S} f$



	Column-I		Column-II
A.	The wavelength of the waves coming towards the observer from source	p.	$(V - V_W - V_D)/f_r$
B.	The wavelength of the waves incident on the wall	q.	$(V - V_W - V_O)f_r/(V - V_W - V_D)$
C.	The wavelength of the waves coming towards observer from the wall	r.	$(V - V_W + V_S)/f$
D.	Frequency of the waves (as detected by O) coming from wall after reflection.	s.	$(V + V_W - V_S)/f$

(*a*) A-(p); B-(r); C-(s); D-(q)
(*b*) A-(r); B-(s); C-(p); D-(q)
(*c*) A-(q); B-(r); C-(s); D-(p)
(*d*) A-(r); B-(s); C-(q); D-(p)

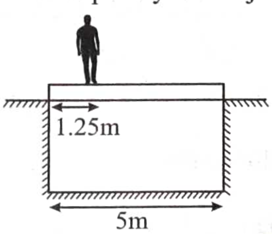
NUMERICAL TYPE QUESTIONS

34. A telephone cord is 4.00m long and has a mass of 0.20kg A transverse wave pulse is produced by plucking one end of the that cord. The pulse makes four complete trips forwards and backwards along the cord in 0.800s. What is the tension (in N) in the cord.

35. The n^{th} harmonic on a taut string has a frequency of 720 Hz. The next higher harmonic has a frequency of 840 Hz. What is the value of n?

36. A motorcycle's horn is putting out a 3015 Hz sound as it moves eastward towards a stationary observer. A train sounds its horn at 855 Hz as it heads westward away from that same observer. If the observer hears the train as 850 Hz and the motorcycle's horn as 3060 Hz, what is the speed of the motorcycle relative to the train (in m/s) ? Use 340 m/s as the speed of sound in air.

37. A wooden flexible plank is placed over a pit which is 5 m wide. Student begins to jump up and down at the center. Such that she jumps at a frequency of 2 Hz. The plank forms a standing wave pattern with minimum possible loops. Now student moves at 1.25 m from edge of the pit, and jumps at this position such that this point has maximum amplitude. What is minimum frequency of her jumping?



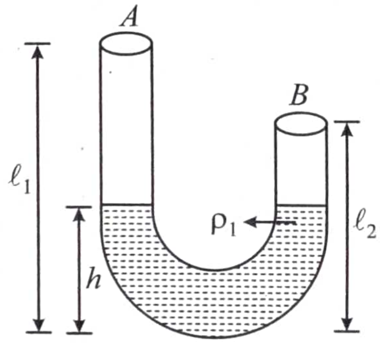
38. A point source of sound having a frequency of 165 Hz is located at the origin. Find the phase difference between points (3,4,0) and (8,6,0) at any given time. The velocity of sound in the medium is 330 m/s. If answer is nπ, fill n in OMR sheet.

39. A point source of sound is located somewhere along the x-axis. Experiments show that the same wave front simultaneously reaches listeners at $x = -8$ m and $x = +2.0$ m.

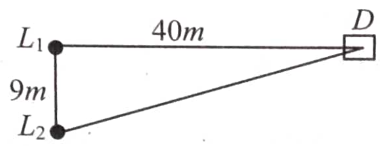
A third listener is positioned along the positive y-axis. What is her y-coordinate (in m) if the same wave front reaches her at the same instant as it does the first two listeners?

40. A U-tube having uniform cross-section but unequal arm length $l_1 = 100$ cm and $l_2 = 50$ cm has same liquid of density ρ_1 filled in it upto a height h = 30 cm as shown in figure. Another liquid of density $\rho_2 = \rho_1/2$ is poured in arm A. Both liquids are immiscible. What length of the second liquid (in cm) should be poured in A so that second overtone of A is in unison with fundamental tone of B.

(Neglect end correction)



41. Two loudspeakers are, driven by a common oscillator and amplifier, are arranged as shown. The frequency of the oscillator is gradually increased from zero and the detector at D records a series of maxima and minima. If speed of sound is 330 m/s, then what is the frequency in Hz at which the first time a maxima is heard.



42. Straight line AB connects two point sources that are 5.00 m apart, emit 165 Hz sound waves of the same amplitude, and emit exactly in same phase. What is the shortest distance (in cm) between the midpoint of AB and a point on AB where the interfering waves cause maximum oscillation of the air molecules?

(Take: velocity of sound = 330 m/s)

Exercise-4 (Past Year Questions)

JEE MAIN

1. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: **(2016)**

   (a) $2\pi\sqrt{2}\,s$ (b) $2s$

   (c) $2\sqrt{2}\,s$ (d) $\sqrt{2}\,s$

2. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$ where distance and time are measured in *SI* units. The tension in the string is: **(2019)**

   (a) 10 N (b) 7.5 N

   (c) 12.5 N (d) 5 N

3. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100Hz. The separation between successive nodes on the string is close to: **(2019)**

   (a) 10.0 cm (b) 33.3 cm

   (c) 16.6 cm (d) 20.0 cm

4. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is: **(2019)**

   (a) 18/17 (b) 19/18

   (c) 20/19 (d) 20/21

5. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin(50t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave? **(2019)**

   (a) The wave is propagating along the negative x-axis with speed 25 ms^{-1}.

   (b) The wave is propagating along the positive x-axis with speed 100 ms^{-1}.

   (c) The wave is propagating along the positive x-axis with speed 25 ms^{-1}.

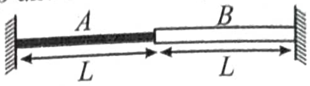
   (d) The wave is propagating along the negative x-axis with speed 100 ms^{-1}.

6. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is: **(2019)**

   (a) 320 m/s, 120 Hz (b) 180 m/s, 80 Hz

   (c) 180 m/s, 120 Hz (d) 320 m/s, 80 Hz

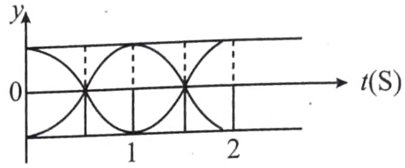
7. A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of anti-nodes in wire A is p and that in B is q then the ratio $p : q$ is: **(2019)**



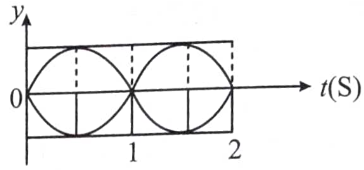
   (a) 4 : 9 (b) 3 : 5 (c) 1 : 4 (d) 1 : 2

8. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is: **(2019)**

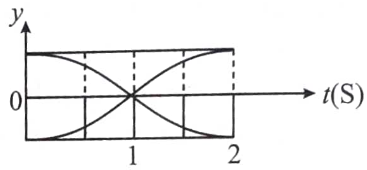
(a)



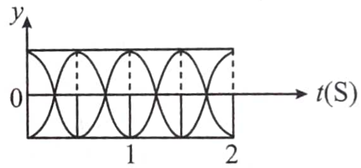
(b)



(c)



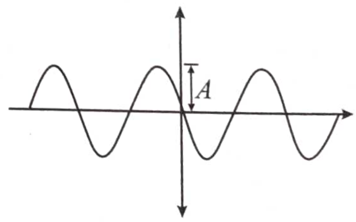
(d)



9. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is: (All quantities are in *SI* units.) **(2019)**

   (a) 20 m (b) 80 m (c) 60 m (d) 40 m

10. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure: **(2019)**



For this wave, the phase ϕ is:

   (a) 0 (b) $-\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{2}$

11. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to $v/2$. The value of T is close to: **(2020)**

(a) 5.15×10^3 N (b) 10.2×10^2 N
(c) 2.50×10^4 N (d) 30.5×10^4 Ns

12. A wire of length L and mass per unit length 6.0×10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are 420 Hz and 490 Hz. Then L in meters is: **(2020)**

(a) 8.1 m (b) 2.1 m
(c) 1.1 m (d) 5.1 m

13. Two identical strings X and Z made of same material have tension T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X/T_Z is **(2020)**

(a) 1.5 (b) 2.25 (c) 0.44 (d) 1.25

14. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope? **(2020)**

(a) 6 (b) 3 (c) 12 (d) 9

15. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are **(2020)**

(a) 1, 2, 3, ... (b) 1, 3, 5, ...
(c) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$, ... (d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$, ...

16. A pipe open at both ends, has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now: **(2016)**

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$
(c) $2f$ (d) f

17. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer?
(Speed of light = 3×10^8 ms^{-1}) **(2017)**

(a) 17.3 GHz (b) 15.3 GHz
(c) 10.1 GHz (d) 12.1 GHz

18. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **(2018)**

(a) 2.5 kHz (b) 10 kHz (c) 7.5 kHz (d) 5 kHz

19. A musician using an open flute of length 50 cm produces second harmonic sound waves.
A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to: **(2019)**

(a) 660 Hz (b) 753 Hz
(c) 500 Hz (d) 333 Hz

20. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be: (Assume that the highest frequency a person can hear is 20,000 Hz) **(2019)**

(a) 6 (b) 4
(c) 7 (d) 5

21. A person standing on an open ground hears the sound of a Jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If υ is the speed of sound, speed of the plane is: **(2019)**

(a) $\frac{\sqrt{3}}{2}\upsilon$ (b) $\frac{2\upsilon}{\sqrt{3}}$
(c) υ (d) $\frac{\upsilon}{2}$

22. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to: **(2019)**

(a) 322 ms^{-1} (b) 341 ms^{-1}
(c) 335 ms^{-1} (d) 328 ms^{-1}

23. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B? (speed of sound in air = 340 ms^{-1}): **(2019)**

(a) 2250 Hz (b) 2060 Hz
(c) 2150 Hz (d) 2300 Hz

24. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take velocity of sound in air is 350 m/s) **(2019)**

(a) 857 Hz (b) 807 Hz
(c) 750 Hz (d) 1143 Hz

25. The pressure wave, $P = 0.01 \sin [1000t - 3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is $0°C$. On some other day, when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be $336 ms^{-1}$. Approximate value of T is: **(2019)**

(*a*) $15° C$ (*b*) $12° C$
(*c*) $4° C$ (*d*) $11° C$

26. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30$ cm and $l_2 = 70$ cm. Then v is equal to: **(2019)**

(*a*) $332 ms^{-1}$ (*b*) $379 ms^{-1}$
(*c*) $384 ms^{-1}$ (*d*) $338 ms^{-1}$

27. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330m/s. Then, u equals: **(2019)**

(*a*) 2.5 m/s (*b*) 15.0 m/s
(*c*) 5.5 m/s (*d*) 10.0 m/s

28. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound ? (Given reference intensity of sound as 10^{-12} W/m^2] **(2019)**

(*a*) 10 cm (*b*) 30 cm
(*c*) 40 cm (*d*) 20 cm

29. A submarine (*a*) travelling at 18 km/hr be being chased along the line of its velocity by another submarine (*b*) travelling at 27 km/hr. B sends a sonar signal of 500Hz to detect A and receives a reflected sound of frequency v. The value of v is close to: (Speed of sound in water = 1500 ms^{-1}) **(2019)**

(*a*) 499 Hz (*b*) 502 Hz
(*c*) 507 Hz (*d*) 504 Hz

30. A stationary source emits sound waves of frequency 500Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz. and 530 Hz. Their respective speeds are, in ms^{-1}, (Given speed of sound = 300 m/s) **(2019)**

(*a*) 16,14 (*b*) 12, 18
(*c*) 12, 16 (*d*) 8, 18

31. A stationary observer receives sound from two identical turning forks, one of which approaches and the other one recedes with the same speed (much less that the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is $v_0 = 1400$ Hz and the velocity of sound in air is 350m/s. The speed of each tuning fork is close to: **(2020)**

(*a*) $\frac{1}{4}$ m/s (*b*) $\frac{1}{2}$ m/s (*c*) 1 m/s (*d*) $\frac{1}{8}$ m/s

32. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ________Hz. **(2020)**

33. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1} **(2020)**

(*a*) 61 kmh^{-1} (*b*) 91 kmh^{-1}
(*c*) 81 kmh^{-1} (*d*) 71 kmh^{-1}

34. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is **(2020)**

(*a*) 36 km/hr (*b*) 54 km/hr
(*c*) 24 km/hr (*d*) 18 km/hr

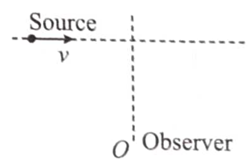
35. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is **(2020)**

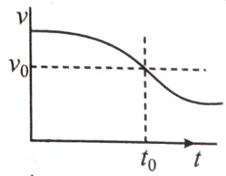
(*a*) 2200 Hz (*b*) 3300 Hz (*c*) 1100 Hz (*d*) 550 Hz

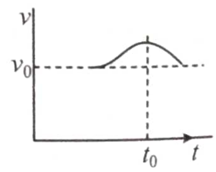
36. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$ Pa, $v \sim 300$ m/s, $\rho \sim 1$ kg/m^3 and $f \sim 1000$ Hz, then s will be of the order of (take the multiplicative constant to be 1) **(2020)**

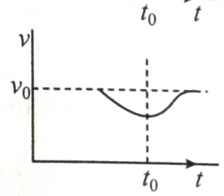
(*a*) $\frac{3}{100}$ mm (*b*) 10 mm (*c*) 1 mm (*d*) $\frac{1}{10}$ mm

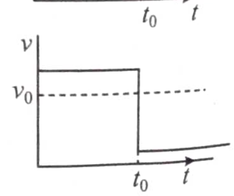
37. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency v_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by (t_0 represents the instant when the distance between the source and observer is minimum) **(2020)**



(*a*)


(*b*)


(*c*)


(*d*)


38. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively.

The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3} L \sqrt{\frac{\rho_1}{\rho_2}}$ where x is (Round off to the Nearest Integer) **(2021)**

39. Two waves propagating along x-axis are given by
$y_1 = 5 \sin (\omega t - kx)$; $y_2 = 3 \sin (\omega t - kx + 1.57)$
Find resultant amplitude due to superposition of two waves **(2022)**

(*a*) 8 (*b*) 2
(*c*) 4 (*d*) $\sqrt{34}$

40. Intensities of two waves are I and $9I$ meets at points P and Q. If phase difference between two waves at point P is $\frac{\pi}{2}$ and at point Q is π. Then ratio of intensity at P and Q is. **(2022)**

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{2}$
(*c*) $\frac{5}{2}$ (*d*) $\frac{4}{2}$

41. In resonance tube, first resonance is obtain at 20 cm, then third resonance length will be: (frequency of source = 400 Hz, speed of sound in air = 336 m/s) **(2022)**

(*a*) 60 cm (*b*) 104 cm
(*c*) 60 cm (*d*) 100 cm

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42. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01 \text{ m}) \sin [(62.8 \text{ m}^{-1}) x] \cos [(628 \text{ s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are): **(2013)**

(*a*) The number of nodes is 5
(*b*) The length of the string is 0.25 m
(*c*) The maximum displacement of the midpoint of the string its equilibrium position is 0.01 m
(*d*) The fundamental frequency is 100 Hz

43. One end of a taut string of length 3m along the x-axis is fixed at $x = 0$. The speed of the waves in the string is 100 m/s. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are) **(2014)**

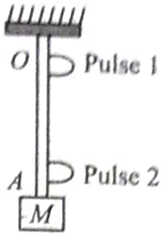
(*a*) $y(t) = A \sin \frac{\pi x}{6} \cos \frac{50\pi t}{3}$

(*b*) $y(t) = A \sin \frac{\pi x}{3} \cos \frac{100\pi t}{3}$

(*c*) $y(t) = A \sin \frac{5\pi x}{6} \cos \frac{250\pi t}{3}$

(*d*) $y(t) = A \sin \frac{5\pi x}{2} \cos 250\pi t$

44. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct: **(2017)**



(*a*) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint at the midpoint of rope
(*b*) The velocity of any pulse along the rope is independent of its frequency and wavelength
(*c*) The wavelength of Pulse 1 becomes longer when it reaches point A
(*d*) The time $T_{AO} = T_{OA}$

Comprehension (Q. 45 to 46): A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

Column-I gives the above four strings while Column-II lists the magnitude of some quantity. **(2019)**

Column-I		Column-II	
A.	String-1(μ)	p.	1
B.	String-2(2μ)	q.	1/2
C.	String-3(3μ)	r.	$1/\sqrt{2}$
D.	String-4(4μ)	s.	$1/\sqrt{3}$
		t.	3/16
		u.	1/16

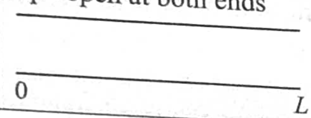
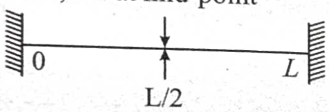
45. If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be, **(2019)**

(*a*) A-(p); B-(r); C-(s); D-(q)
(*b*) A-(q); B-(p); C-(r); D-(s)
(*c*) A-(s); B-(p); C-(q); D-(r)
(*d*) A-(q); B-(p); C-(r); D-(s)

46. The length of the string 1,2,3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1,2,3 and 4 are vibrated at their 1st, 3rd, 5th and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be. **(2019)**

(*a*) A-(p); B-(q); C-(t); D-(u)
(*b*) A-(q); B-(u); C-(r); D-(t)
(*c*) A-(s); B-(p); C-(q); D-(r)
(*d*) A-(q); B-(p); C-(s); D-(r)

47. Column-I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column-II describing the nature and wavelength of the standing waves.

	Column-I		Column-II
A.	Pipe closed at one end 	p.	Longitudinal waves
B.	Pipe open at both ends 	q.	Transverse waves
C.	Stretched wire clamped at both ends 	r.	$\lambda_f = L$
D.	Stretched wire clamped at both ends,and at mid-point 	s.	$\lambda_f = 2L$
		t.	$\lambda_f = 4L$

(*a*) A-(p,t); B-(p,s); C-(q,s); D-(q,r)
(*b*) A-(p,t); B-(q,s); C-(q,t); D-(q,r)
(*c*) A-(p,r); B-(s); C-(p,r,q); D-(r,s)
(*d*) A-(p,t); B-(r); C-(p,q); D-(q,r,s)

48. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe. **(2012)**

(*a*) A high-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.
(*b*) A low-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.
(*c*) A low-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.
(*d*) A high-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.

49. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4 cm. The distance frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top and of the Resonance column. When first resonance occurs, the reading of the water level in the column is **(2012)**

(*a*) 14.0 (*b*) 15.2
(*c*) 16.4 (*d*) 17.6

50. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are): **(2013)**

(*a*) If the wind blows from the observer to the source, $f_2 > f_1$
(*b*) If the wind blows from the source to the observer, $f_2 > f_1$
(*c*) If the wind blows from the observer to the source, $f_2 < f_1$
(*d*) If the wind blows from the source to the observer, $f_2 < f_1$

51. A student is performing an experiment using a resonance column and a tuning fork of frequency $244 s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(Useful information):

$\sqrt{167RT} = 640 j^{1/2}$ mole$^{-1/2}$;

$\sqrt{140RT} = 590 j^{1/2}$ mole$^{-1/2}$. The molar masses M in grams are given in the options. Take the value of $\sqrt{\frac{10}{M}}$ for each gas as given there.) **(2014)**

(*a*) Neon ($M = 20$, $\sqrt{\frac{10}{20}} = \frac{7}{10}$)

(*b*) Nitrogen ($M = 28$, $\sqrt{\frac{10}{28}} = \frac{3}{5}$)

(*c*) Oxygen ($M = 32$, $\sqrt{\frac{10}{32}} = \frac{9}{16}$)

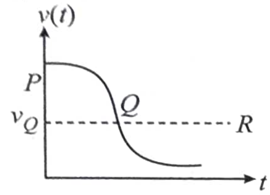
(*d*) Argon ($M = 36$, $\sqrt{\frac{10}{36}} = \frac{17}{32}$)

52. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Let $v(t)$

represent the beat frequency measured by a person sitting in the car at time t. Let ν_p, ν_Q and ν_R be the beat frequencies measured at locations P, Q, and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is (are) true regarding the sound heard by the person? **(2016)**

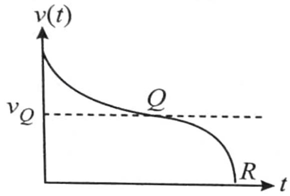
(*a*) The rate of change in beat frequency is maximum when the car passes through Q

(*b*) The plot below represents schematically the variation of beat frequency with time



(*c*) $\nu_P + \nu_R = 2\nu_Q$

(*d*) The plot below represents schematically the variation of beat frequency with time



53. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received). **(2017)**

54. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is ______. **(2018)**

55. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air column of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true? **(2018)**

(*a*) The speed of sound determined from this experiment is 332 ms^{-1}

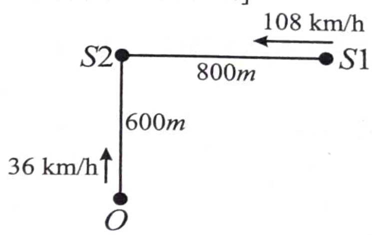
(*b*) The end correction in this experiment is 0.9 cm

(*c*) The wavelength of the sound wave is 66.4 cm

(*d*) The resonance at 50.7 cm corresponds to the fundamental harmonic

56. A train $S1$, moving with a uniform velocity of 108 km/h, approaches another train $S2$ standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards $S2$, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from $S2$ and distance between $S1$ and $S2$ is 800 m, the number of beats heard by O is ______.

[Speed of the sound = 330 m/s] **(2019)**



57. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1}, the smallest value of the percentage change required in the length of the pipe is... **(2020)**

58. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_s, will the sound reaching the pipe lead to a resonance? **(2021)**

(*a*) $u = 0.8v$ and $f_s = f_0$ (*b*) $u = 0.8v$ and $f_s = 2f_0$

(*c*) $u = 0.8v$ and $f_s = 0.5f_0$ (*d*) $u = 0.5v$ and $f_s = 1.5f_0$

ANSWER KEY

CONCEPT APPLICATION

1. (d) 2. (b,d) 3. (a) 4. (b,d) 5. [55] 6. (i) 62.5, (ii) 7.85, (iii) 7.96, (iv) 21.1 7. (b) 8. (d)
9. (a) 10. (d) 11. (a) 12. (c,d) 13. (d) 14. (c) 15. (a) 16. (d) 17. (a) 18. [508]
19. [251] 20. (c) 21. (d) 22. (c) 23. (c) 24. (c)

EXERCISE-1 (TOPICWISE)

1. (a) 2. (c) 3. (a) 4. (a) 5. (c) 6. (a) 7. (c) 8. (d) 9. (d) 10. (a)
11. (d) 12. (a) 13. (a) 14. (d) 15. (d) 16. (c) 17. (b) 18. (c) 19. (c) 20. (d)
21. (a) 22. (d) 23. (c) 24. (a) 25. (a) 26. (d) 27. (c) 28. (d) 29. (c) 30. (d)
31. (a) 32. (b) 33. (a) 34. (c) 35. (d) 36. (c) 37. (c) 38. (b) 39. (b) 40. (a)
41. (c) 42. (d) 43. (a) 44. (d) 45. (d) 46. (a) 47. (d) 48. (c) 49. (c) 50. (a)
51. (d) 52. (a) 53. (a) 54. (c) 55. (c) 56. (c) 57. (d) 58. (b) 59. (c) 60. (c)
61. (c)

EXERCISE-2 (LEARNING PLUS)

1. (c) 2. (d) 3. (d) 4. (a) 5. (d) 6. (b) 7. (a) 8. (c) 9. (c) 10. (c)
11. (c) 12. (b) 13. (b) 14. (a) 15. (a) 16. (b) 17. (a) 18. (a) 19. (d) 20. (a)
21. (d) 22. (d) 23. (b) 24. (b) 25. (d) 26. (a) 27. (c) 28. (b) 29. (b) 30. (b)
31. (b) 32. (b) 33. (a) 34. (d) 35. (b) 36. (b) 37. (d) 38. (a) 39. (b) 40. (c)
41. (c) 42. (a)

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (a,d) 2. (a,c) 3. (a,b,c,d) 4. (c) 5. (a,b,d) 6. (c,d) 7. (c,d) 8. (a,b,c,d) 9. (c,d) 10. (a,c)
11. (c,d) 12. (a,d) 13. (b,d) 14. (b,d) 15. (a,d) 16. (c,d) 17. (a,c,d) 18. (a,d) 19. (c) 20. (b,c)
21. (c,d) 22. (a,b,c) 23. (a,c,d) 24. (b) 25. (a) 26. (b) 27. (a) 28. (b) 29. (a) 30. (a)
31. (b) 32. (c) 33. (b) 34. [80] 35. [6] 36. [7] 37. [4] 38. [5] 39. [4] 40. [60]
41. [0330] 42. [0100]

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (c) 2. (c) 3. (d) 4. (b) 5. (a) 6. (d) 7. (d) 8. (d) 9. (b) 10. (c)
11. (a) 12. (b) 13. (b) 14. (c) 15. (c) 16. (d) 17. (a) 18. (d) 19. (a) 20. (a)
21. (d) 22. (d) 23. (a) 24. (c) 25. (c) 26. (c) 27. (a) 28. (c) 29. (b) 30. (b)
31. (a) 32. [106] 33. (b) 34. (b) 35. (a) 36. (a) 37. (a) 38. [4] 39. (d) 40. (c)
41. (b)

JEE Advanced

42. (b,c) 43. (a,c,d) 44. (b,d) 45. (a) 46. (a) 47. (a) 48. (b,d) 49. (b) 50. (a,b) 51. (d)
52. (a,b,c) 53. [6] 54. [5.00Hz] 55. (a,b,c) 56. [8] 57. [0.62 to 0.63] 58. (a,d)