CHAPTER

2 Sets

INTRODUCTION

The concept of set serves as a fundamental part of the present day mathematics. It permeates virtually every branch of mathematics. Yet, most mathematics students receive only a cursory overview of the theory of sets.

DEFINITION

A set is a well defined collection of objects.

By well defined we mean there should be no ambiguity regarding the inclusion and exclusion of the objects.

		
Set of Fruits in a Basket	Set of Coin in a Bank	Set of Books in a Library

Collection	Set
1. Consonant in English Alphabet	Set
2. Difficult topics in Mathematics	Not a Set
3. Collection of past Presidents of India	Set
4. Group of Intelligent Students in JEE Batch	Not a Set

Note:

1. Generally, If you can see an adjective like good, difficult intelligent, brave in a sentence then it does not describe a set.
2. Sets are usually denoted by capital letters A, B, …, X, Y, Z. Collection of objects or things in a set called as elements

   The elements of the set are denoted by small letters.
3. In a set, the order in which elements are written makes no difference.
4. In a set, the repetition of elements not allowed.

REPRESENTATION OF SET

There are two methods for representing a set.

(*i*) **Tabulation method or Roster form**

All the elements belonging to the set are written in curly brackets and separated by commas

If A is the set of days of a week, then

A = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

(*ii*) **Set Builder Method or Set rule method**

In this method, we use the definition, which is satisfied by all the elements of set.

In above example set A may be written as

$A = \{x : x \text{ is a day of week}\}$

NOTATIONS OF SET OF DIFFERENT NUMBERS

(*i*) Set of all natural numbers $N = \{1, 2, 3, \ldots\}$

(*ii*) Set of all integers Z or $I = \{0, \pm1, \pm2, \ldots\}$

(*iii*) Set of non zero integers Z_0 or $I_0 = \{\pm1, \pm2, \pm3, \ldots\}$

(*iv*) Set of all rational numbers

$Q = \{x : x = p/q$, where p and q relatively prime integers and $q \neq 0\}$

(*v*) Set of real numbers is denoted by R

(*vi*) Set of complex numbers is denoted by C

CARDINAL NUMBER

The number of distinct elements in a set A is denoted by $n(A)$ and it is known as cardinal number of the set A.

- ❖ Two finite sets A and B are equivalent if their cardinal number are same.

Example:

$A = \{x \in Z \text{ and } x^2 - 5x + 6 = 0\} \Rightarrow A = \{2, 3\}$

$\therefore\ n(A) = 2$

TYPES OF SETS

(*i*) **Null set or Void set or Empty set:**

A set having no element is called as null set or empty set or void set. It is denoted by ϕ or { }. The null set is the subset of every set.

Example:

Set of even prime numbers less than 2.

Set of natural numbers strictly lying between 5 and 6.

$A = \{x : x \in N, 5 < x < 6\} = \phi$

(*ii*) **Finite Set**

A set which is empty or consists of a definite number of elements

Example:

Set of all natural numbers less than 6 = {1, 2, 3, 4, 5}

(*iii*) **Infinite sets**

A set whose elements can not be counted.

Example

Set of real numbers and natural numbers.

(*iv*) **Singleton Set**

A set having one and only one element is called singleton set or unit set.

Example:

Set of all positive integral roots of the equation

$x^2 - 2x - 15 = 0.$

$\Rightarrow (x + 3)(x - 5) = 0$

$\Rightarrow x = -3$ or $x = 5$ i.e., only one positive integral root is 5.

$C = \{x \in R : x - 5 = 0\}$

$\Rightarrow x = 5 \Rightarrow C = \{5\}$ i.e., only one element.

Set of all positive integral roots = {5}.

Train Your Brain

Example 1: Solve $3x^2 - 12x = 0$.

When

(*i*) $x \in N$

(*ii*) $x \in Z$

Sol. $3x^2 - 12x = 0 \Leftrightarrow 3x(x - 4) = 0 \Leftrightarrow x = 0$ or $x = 4$

(*i*) when $x \in N \Rightarrow x = 4$

(*ii*) when $x \in Z \Rightarrow x = 0$ or $x = 4$

Example 2: If $Q = \left\{x : x = \frac{1}{y}, \text{where } y \in N\right\}$, then

(*a*) $0 \in Q$ (*b*) $1 \in Q$

(*c*) $2 \in Q$ (*d*) $\frac{2}{3} \in Q$

Sol. Here $\frac{1}{y} \neq 0, \frac{1}{y} \neq 2, \frac{1}{y} \neq \frac{2}{3}$, $[\because y \in N]$

$\therefore \frac{1}{y}$ can be 1, $[\because y$ can be 1].

Concept Application

1. A = $\{x : x \neq x\}$ represents

(*a*) {0} (*b*) { }

(*c*) {1} (*d*) $\{x\}$

2. The set $\{x : x$ is a positive integer less than 6} in roster form is

(*a*) {1, 2, 3, 4, 5}

(*b*) {1, 2, 3, 4, 5, 6}

(*c*) {2, 4, 6}

(*d*) {1, 3, 5}

3. Which of the following sets is a finite set?

(*a*) $A = \{x ; x \in Z \text{ and } x^2 - 5x + 6 = 0\}$

(*b*) $B = \{x ; x \in Z \text{ and } x^2 \text{ is even}\}$

(*c*) $D = \{x ; x \in Z \text{ and } x > -10\}$

(*d*) All of these

EQUAL AND EQUIVALENT SETS

- Two sets A and B are said to be Equivalent sets if their cardinalities are same i.e., $n(A) = n(B)$.
- Two finite sets A and B are said to be Equal sets if their cardinalities are same, and the members in both the sets are the same, i.e., A = B.
- If A and B are equal sets, we denote it by $A = B$.
- If A and B are unequal sets, we denote it by $A \neq B$.

SUBSET AND SUPERSET

If A and B are two sets such that every element of A is also an element of B, then A is a subset of B and B is superset of A. We write $A \subseteq B$.

Note:

- Every set is a subset of itself i.e. $A \subseteq A$ for all A.
- Empty set ϕ is subset of every set
- If $A \subseteq B$ and $B \subseteq A$ then A and B are said to be equal sets, i.e. $A = B$.
- If $A \subseteq B$ and $A \neq B$, there A is called as Proper subset of B and denoted by $A \subset B$.
- If a set A has n elements, then the number of subsets of $A = 2^n$.

POWER SET

Power set of a set A is the collection of all subsets of A and is denoted by $P(A)$.

Let A be a finite set containing m elements i.e., $n(A) = m$, then

- The number of elements in the power set of A, $n(P(A)) = 2^m$.

- The number of non-void/non-empty subsets of $A = (2^m) - 1$.
- The number of proper subsets of $A = 2^m - 1$.
- The number of non-void proper subsets of $A = 2^m - 2$.

  Example:

  The number of elements in the power set of set $A = \{1, 2\}$ is 2^2.

UNIVERSAL SET

The universal set is the superset for all the sets under the consideration.

The set of complex numbers is the universal set for all possible sets related to numbers.

Train Your Brain

Example 3: The number of non-empty subsets of the set $\{1, 2, 3, 4\}$ is

(*a*) 15 (*b*) 14

(*c*) 16 (*d*) 17

Sol. The number of non- empty subsets

$= 2^n - 1 = 2^4 - 1 = 16 - 1 = 15.$

Example 4: Consider the following sets.

$A = \{0\}$

$B = \{x : x > 15 \text{ and } x < 5\}$,

$C = \{x : x - 5 = 0\}$,

$D = \{x : x^2 = 25\}$,

$E = \{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$

Choose the pair of equal sets

(*a*) *A* and *B* (*b*) *C* and *D*

(*c*) *C* and *E* (*d*) *B* and *C*

Sol. Since, $0 \in A$ and 0 does not belong to any of the sets *B*, *C*, *D* and *E*, it follows that $A \neq B$, $A \neq C$, $A \neq D$, $A \neq E$. Since, $B = \phi$, but none of the other sets are empty. Therefore $B \neq C$, $B \neq D$ and $B \neq E$. Also, $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.

Since, $E = \{5\}$, $C = E$. Further, $D = \{-5, 5\}$ so *C* & *E* are equal sets.

Example 5: If $A = \{x, y\}$ then the power set of A is

(*a*) $\{x^y, y^x\}$

(*b*) $\{\phi, x, y\}$

(*c*) $\{\phi, \{x\}, \{2y\}\}$

(*d*) $\{\phi, \{x\} \{y\}, \{x, y\}\}$

Sol. The collection of all the subsets of the set *A* is called the power set of *A*. It is denoted by $P(A)$

Given $A = \{x, y\}$; $P(A) = \{\phi, \{x\}, \{y\}, \{x, y\}\}$

Concept Application

4. Consider the following sets.

I. $A = \{1, 2, 3\}$

II. $B = \{x \in R : x^2 - 2x + 1 = 0\}$

III. $C = \{1, 2, 2, 3\}$

IV. $D = \{x \in R : x^3 - 6x^2 + 11x - 6 = 0\}$

Which of the following sets are equal?

(*a*) $A = B = C$ (*b*) $A = C = D$

(*c*) $A = B = D$ (*d*) $B = C = D$

5. The number of the proper subsets of $\{a, b, c\}$ is:

(*a*) 3 (*b*) 8 (*c*) 6 (*d*) 7

6. Given the sets

$A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Which of the following sets may be considered as universal set for all the three sets *A*, *B* and *C*?

(*a*) $\{0, 1, 2, 3, 4, 5, 6\}$

(*b*) ϕ

(*c*) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(*d*) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

7. The cardinality of the set $P\{P[P(\phi)]\}$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

INTERVALS AS SUBSETS OF R

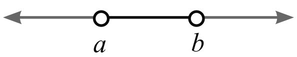
Four type of intervals can be defined as *a* subsets of *R*.

Let $a, b \in R$, such that $a < b$

1. Open Interval

(a, b) or$]a, b[= \{x : a < x < b\}$

= Set of all real numbers between *a* and *b*, not including *a* and *b* both.

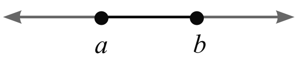


$]a, b[$ or (a, b)

2. Closed Interval

$[a, b] = \{x : a \leq x \leq b\}$

= Set of all real numbers between *a* and *b* as well as including *a* and *b* both.

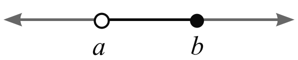


$[a, b]$

3. Open-closed Interval (semi closed or semi open interval)

$(a, b]$ or $]a, b] = \{x : a < x \leq b\}$

= Set of all real numbers between *a* and *b*, *a* is not included but *b* is included.



$(a, b]$ or $]a, b]$

4. **Closed-open interval (semi closed or semi open interval)**

$[a, b)$ or $[a, b[\ = \{x : a \le x < b\}$

= Set of all real numbers between a and b including a but excluding b.

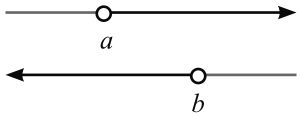
$[a, b)$ or $[a, b[$

Some More Representations on Number Line

Infinite open interval

$x > a$

$x < b$



Infinite close interval

$x \le b$

$x \ge a$

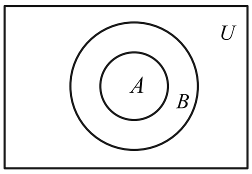
$(0, \infty) = R^+$

$(-\infty, 0) = R^-$

$(-\infty, \infty) = R$

VENN-DIAGRAMS

Venn – diagram is a systematic representation of sets in pictorial form. A set is represented by circle inside the universal set which is represented by rectangular region.



$A \subset B$

OPERATIONS ON SETS

(*a*) **Union of sets**

If A and B are two sets, then the union of two sets is denoted by $A \cup B$ (read as "A union B") and defined as

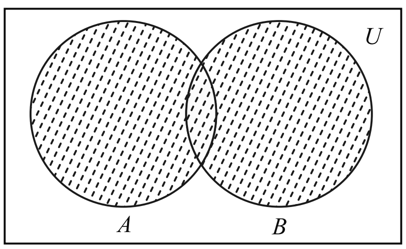
$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Union is also known as join or "logical sum" of A and B.

Example:

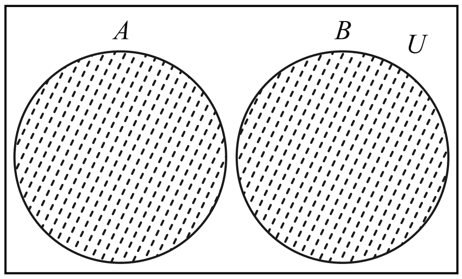
$A = \{1, 2, 3\}$, $B = \{1, 3, 5, 7\}$ then $A \cup B = \{1, 2, 3, 5, 7\}$

Case-I: If A and B are not equal sets but they have some elements in common.



A∪B

Case-II: If A and B have no elements in common.



$A \cup B$

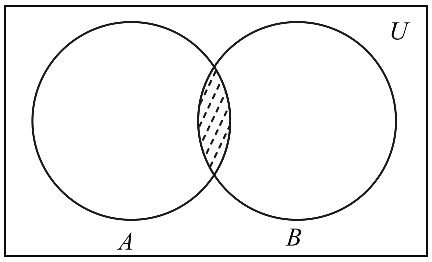
(*b*) **Intersection of sets**

The intersection of sets A and B is denoted by $A \cap B$ (read as "A intersection B") and defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Example:

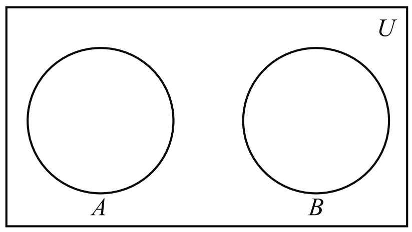
$A = \{1, 2, 3, 5, 6, 9\}$, $B = \{2, 3, 4, 5, 7, 8, 10\}$ then $A \cap B = \{2, 3, 5\}$

Case-I: If A and B are not equal sets but they have some elements in common.



$A \cap B$

Case-II: If A and B have no elements in common.



Note:

Disjoint sets

Two sets are said to be disjoint sets if they have no elements in common, that is $A \cap B = \phi$

Example: $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{7, 8, 9, 10, 11\}$ then A and B are disjoint sets

(*c*) **Difference of sets**

The difference of A and B, i.e.

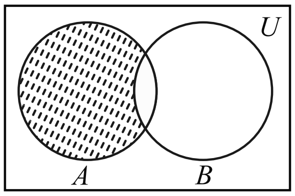
$A - B$ = {all those elements of A which do not belong to B}

$A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, $B - A$ = {all those elements of B that do not belong to A}

$B - A = \{x : x \in B \text{ and } x \notin A\}$

Example: $A = \{2, 3, 4, 5, 6, 7\}$, $B = \{3, 5, 7, 9, 11, 13\}$ then $A - B = \{2, 4, 6\}$



$A - B$

Properties

If A and B are any two sets, then

(*i*) $A - B = A \cap B'$

(*ii*) $B - A = B \cap A'$

(*iii*) $A - B = A \Leftrightarrow A \cap B = \phi$

(*iv*) $(A - B) \cup B = A \cup B$

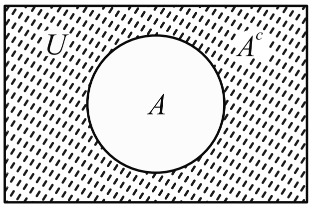
(*v*) $(A - B) \cap B = \phi$

(*vi*) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

(*d*) **Complement of Set**

Let U be the universal set and A be a subset of U, then complement of set A represented as A' or A^c is the set of all elements of U which do not belong to set A.

Thus $A' = \{x : x \in U \text{ and } x \notin A\}$ OR $A' = \{x : x \notin A\}$



A'

Example: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 3, 5, 7, 9\}$ then $A^c = \{2, 4, 6, 8\}$

Properties:

- $A \cap A^c = \phi$ (A and A^c are disjoint set)
- $A \cup A^c = U$
- $(A^c)^c = A$
- $U^c = \phi$
- $\phi^c = U$

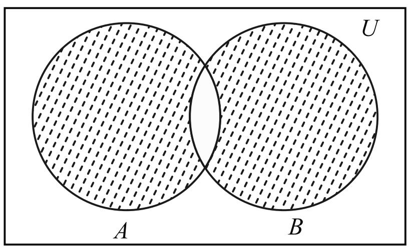
ADVANCED LEARNING

(*e*) **Symmetric difference of two sets**

The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$

Thus, $A \Delta B = \{x : x \in (A - B) \cup (B - A)\} = \{x : x \notin A \cap B\}$

Example: $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{1, 3, 5, 6, 7, 8, 9\}$ then $A \Delta B = \{2, 4, 9\}$



$A \Delta B$

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Example 6: If $N = \{ax : x \in N\}$ describe the set $3N \cap 7N$.

Sol. $3N = \{3x : x \in N\} = \{3, 6, 9, \ldots\}$

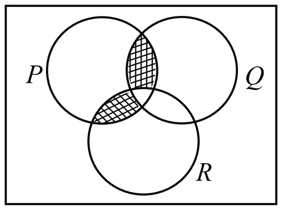
$7N = \{7x : x \in N\} = \{7, 14, 21, 28, \ldots\}$

$\therefore$ $3N \cap 7N = \{y : y$ is a multiple of 3 and y is a multiple of 7$\}$

$= \{y : y$ is a multiple of 21$\}$

$= \{21, 42, 63, \ldots\} = 21\,N$

Example 7: What does the shaded portion of the Venn diagram given below represent?



(*a*) $(P \cap Q) \cap (P \cap R)$

(*b*) $((P \cap Q) - R) \cup ((P \cap R) - Q)$

(*c*) $((P \cup Q) - R) \cap ((P \cap R) - Q)$

(*d*) $((P \cap Q) \cup R) \cap ((P \cup R) - R)$

Sol. In the given Venn diagram, shaded area between sets P and Q is $(P \cap Q) - R$ and shaded area between P and R is $(P \cap R) - Q$. So, both the shaded area is union of these two area and is represented by $((P \cap Q) - R) \cup ((P \cap R) - Q)$.

Example 8: If $A = \{x \in R : 0 < x < 3\}$ and $B = \{x \in R : 1 \le x \le 5\}$ then $A \Delta B$ is

(*a*) $\{x \in R : 0 < x < 1\}$

(*b*) $\{x \in R : 3 \le x \le 5\}$

(*c*) $\{x \in R : 0 < x < 1 \text{ or } 3 \le x \le 5\}$

(*d*) ϕ

Sol. From the given we have in interval notation $A = (0, 3)$ and $B = [1, 5]$

Clearly $A - B = (0, 1) = \{x \in R : 0 < x < 1\}$ and $B - A = [3, 5] = \{x \in R : 3 \le x \le 5\}$

$\therefore$ $A \Delta B = (A - B) \cup (B - A) = (0, 1) \cup [3, 5]$

$= \{x \in R : 0 < x < 1 \text{ or } 3 \le x \le 5\}$

Concept Application

8. Let $A = \{(n, 2n) : n \in N\}$ and $B = \{(2n, 3n) : n \in N\}$. What is $A \cap B$ equal to?

(*a*) $\{(n, 6n) : n \in N\}$ (*b*) $\{(2n, 6n) : n \in N\}$

(*c*) $\{(n, 3n) : n \in N\}$ (*d*) ϕ

9. Let X = {Ram, Geeta, Akbar} be the set of students of Class XI, who are in school hockey team and Y = {Geeta, David, Ashok} be the set of students from class XI, who are in the school football team, Then $X \cap Y$ is

(a) {Ram, Geeta} (b) {Ram}

(c) {Geeta} (d) {None of these}

10. Let $A = \{3, 6, 9, 12, 15, 18, 21\}$

$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

and $D = \{5, 10, 15, 20\}$

(I) $(A \cup B) - C = \{3, 9, 18, 21\}$

(II) $(A - B) \cap (C - D) = \{6\}$

(III) $(A \cap B \cap C) \cup D = \{12, 5, 15, 10, 20\}$

Which of the following is correct?

(a) Only I and II

(b) Only II and III

(c) Only III and I

(d) None of these

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A$ and B are disjoint sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) No. of elements in exactly two of the sets A, B, C
$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(vii) No. of elements in exactly one of the sets A, B, C
$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(viii) $n(A' \cup B') = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n(U) - n(A \cup B)$

LAWS OF ALGEBRA OF SETS

(a) **Idempotent laws**

(i) $A \cup A = A$

(ii) $A \cap A = A$

(b) **Identity laws**

(i) $A \cup \phi = A$

(ii) $A \cap U = A$

i.e. ϕ and U are identity elements for union and intersection respectively.

(c) **Commutative laws**

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative

(d) **Associative laws**

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

i.e. union and intersection are associative

(e) **Distributive laws**

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

(f) **De–Morgan's laws**

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(iii) $(A')' = A$

CARTESIAN PRODUCT

ADVANCED LEARNING

The Cartesian product of two sets A and B is the set $\{(a, b): a \in A$ and $b \in B\}$ and is denoted by $A \times B$. If A has m elements and B has n elements, then $A \times B$ has mn elements

Example: $A = \{1, 2, 3\}$ and $B = \{a, b\}$ then $A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$n(A \times B) = 3 \times 2 = 6$

Properties

For three sets A, B, and C

(i) $n(A \times B) = n(A) \times n(B)$

(ii) $A \times B = \phi$, if either A or B is an empty set

(iii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(iv) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(v) $A \times (B - C) = (A \times B) - (A \times C)$

(vi) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(vii) $A \times B = B \times A \Leftrightarrow A = B$

Train Your Brain

Example 9: If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.

Sol. We have $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

Case-I

When $n(A \cap B)$ is minimum, i.e., $n(A \cap B) = 0$

This is possible only when $A \cap B = \phi$. In this case,

$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9$

So maximum number of elements in $A \cup B$ is 9.

Case-II

When $n(A \cap B)$ is maximum.

This is possible only when $A \subseteq B$.

In this case $n(A \cap B) = 3$.

$\therefore\ n(A \cup B) = n(A) + n(B) - n(A \cap B) = (3 + 6 - 3) = 6$

So, minimum number of elements in $A \cup B$ is 6.

Example 10: If A, B and C are three sets and U is the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Find $n(A' \cap B')$.

Sol. We have $A' \cap B' = (A \cup B)'$

$\therefore\ n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

$= n(U) - [n(A) + n(B) - n(A \cap B)]$

$= 700 - (200 + 300 - 100) = 300$

Example 11: Prove that for non empty sets A and B.

$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Sol. Consider

$(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)'$

$= (A \cup B) \cap (A' \cup B')$

$= [(A \cup B) \cap A'] \cup [(A \cup B) \cap B']$

$= [(A \cap A') \cup (B \cap A')] \cup [(A \cap B') \cup (B \cap B')]$

$= [\phi \cup (B \cap A')] \cup [(A \cap B') \cup \phi]$

$= (B \cap A') \cup (A \cap B')$

$= (B - A) \cup (A - B)$ ...(1)

Now R.H.S. $= (A - B) \cup (B - A) \cup (A \cap B)$

$= [(A \cup B) - (A \cap B)] \cup (A \cap B)$ using (1)

$= [(A \cup B) \cap (A \cap B)' \cup (A \cap B)$

$= [(A \cup B) \cup (A \cap B)] \cap [(A \cap B)' \cup (A \cap B)]$

$= (A \cup B) \cap U$, where U is universal set.

$= A \cup B =$ L.H.S.

Example 12: In a certain city, only two newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements, while 50% of those who read both A and B, look into advertisements. What % of the population read an advertisement?

Sol. Let L = Set of people who read paper A

M = Set of people who read paper B

Then $n(L) = 25$, $n(M) = 20$, $n(L \cap M) = 8$

$n(L - M) = n(L) - n(L \cap M) = 25 - 8 = 17$

$n(M - L) = n(M) - n(L \cap M) = 20 - 8 = 12$

$\therefore$ % of people reading an advertisement

$= (30\% \text{ of } 17) + (40\% \text{ of } 12) + (50\% \text{ of } 8)$

$$= \frac{51}{10} + \frac{24}{5} + 4 = 13.9\%$$

Concept Application

11. A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products?

(*a*) 170 (*b*) 280
(*c*) 220 (*d*) None of these

12. A set A has 3 elements and another set B has 6 elements. Then

(*a*) $3 \le n(A \cup B) \le 6$ (*b*) $3 \le n(A \cup B) \le 9$
(*c*) $6 \le n(A \cup B) \le 9$ (*d*) $0 \le n(A \cup B) \le 9$

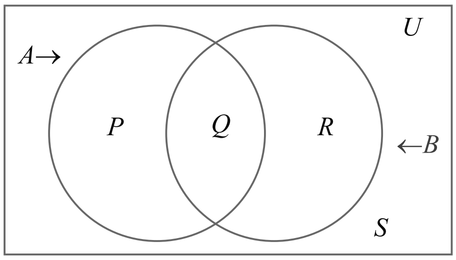
13. At a certain conference of 100 people, there are 29 Indian women and 23 Indian men of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. How many foreigners and women doctors are attending the conference?

(*a*) 48, 1 (*b*) 34, 3 (*c*) 46, 4 (*d*) 42, 2

14. The number of elements in the set $\{(a, b) : 2a^2 + 3b^2 = 35,\ a, b \in Z\}$ where Z is the set of all integers, is

(*a*) 2 (*b*) 4 (*c*) 8 (*d*) 12

15. Find P, Q, R and S in given Venn diagram in terms of A, B where A and B are non empty sets



16. The universal set U and the sets O, P and S are given by

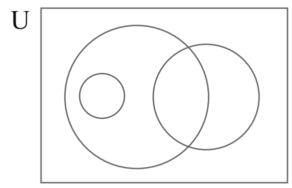
$U = \{x : x \text{ is an integer such that } 3 \le x \le 100\}$,

$O = \{x : x \text{ is an odd number}\}$,

$P = \{x : x \text{ is a prime number}\}$,

$S = \{x : x \text{ is a perfect square}\}$.

In the Venn diagram below, each of the sets O, P and S is represented by a circle.

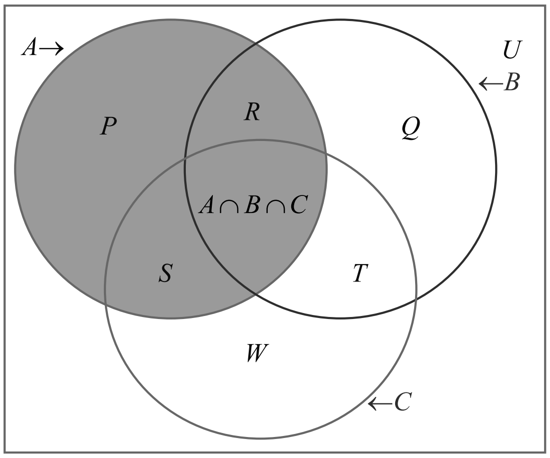


(*i*) Copy the Venn diagram and label each circle with the appropriate letter.

(*ii*) Place each of the numbers 34, 35, 36 and 37 in the appropriate part of your diagram.

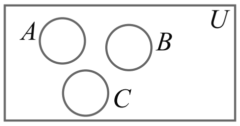
(*iii*) State the value of $n(O \cap S)$ and of $n(O \cup S)$.

17. Find P, Q, R, S, T, U and V in given Venn diagram in terms of A, B, C where A, B and C:

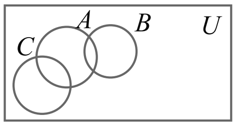


18. Shade $(A \cup B) \cap (A \cup C)$ in the following diagrams.

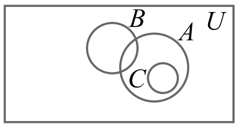
(*a*)



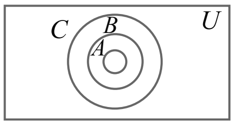
(*b*)



(*c*)



(*d*)



1. Among employee of a company taking vacations last years, 90% took vacations in the summer, 65% in the winter, 10% in the spring, 7% in the autumn, 55% in winter and summer, 8% in the spring and summer, 6% in the autumn and summer, 4% in winter and spring, 4% in winter and autumn, 3% in the spring and autumn, 3% in the summer, winter and spring 3% in the summer, winter and autumn, 2% in the summer, autumn and spring, and 2% in the winter, spring and autumn. Percentage of employee that took vacations during every season

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) 8

Sol. Suppose that number of employee taking vacations is 100.

S_u – set of employee taking leave in summer

S_w – set of employee taking leave in Winter

S_p – set of employee taking leave in Spring

S_a – set of employee taking leave in Autumn

$n(S_u) = 90$, $n(S_w) = 65$, $n(S_p) = 10$, $n(S_a) = 7$

$n(S_w \cap S_u) = 55$, $n(S_p \cap S_u) = 8$, $n(S_a \cap S_u) = 6$

$n(S_w \cap S_p) = 4$, $n(S_w \cap S_a) = 4$, $n(S_p \cap S_a) = 3$

$n(S_u \cap S_a) = 3$, $n(S_u \cap S_w \cap S_a) = 3$

$n(S_u \cap S_w \cap S_p) = 3$, $n(S_u \cap S_a \cap S_p) = 2$

$n(S_w \cap S_p \cap S_a) = 2$

$\Rightarrow n(S_u \cap S_p \cap S_w \cap S_a)$

$= n(S_u) + n(S_p) + n(S_w) + n(S_a) - n(S_u \cap S_p) - n(S_p \cap S_w) - n(S_w \cap S_a) - n(S_u \cap S_a) - n(S_u \cap S_w) - n(S_p \cap S_a) + n(S_u \cap S_p \cap S_w) + n(S_u \cap S_w \cap S_a) + n(S_w \cap S_a \cap S_u) + n(S_u \cap S_p \cap S_a) - n(S_p \cup S_u \cup S_a \cup S_w)$

$= 90 + 65 + 10 + 7 - 55 - 8 - 6 - 4 - 4 - 3 + 3 + 3 + 2 + 2 - 100 = 2$

Therefore, option (*c*) is the correct answer.

2. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$ then the number of elements in $(A \cup B) \times (A \cap B) \times (A \,\Delta\, B)$ is

(*a*) 5 (*b*) 30

(*c*) 10 (*d*) 4

Sol. $A \cup B = \{1, 2, 3, 4, 5\}$, $n(A \cup B) = 5$

$A \cap B = \{3, 4\}$, $n(A \cap B) = 2$

$\therefore A \,\Delta\, B = (A - B) \cup (B - A) = \{1, 2\} \cup \{5\} = \{1, 2, 5\}$

$n(A \,\Delta\, B) = 3$. Hence $n((A \cup B) \times (A \cap B) \times (A \,\Delta\, B))$

$= 5 \times 2 \times 3 = 30$.

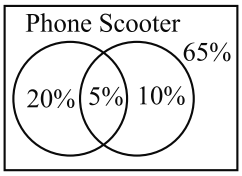
Therefore, option (*b*) is the correct answer.

3. In a certain town 25% families own a cell phone, 15% families own a scooter and 65% families own neither a cell phone nor a scooter. If 1500 families own both a cell phone and a scooter. then total number of families in the town is

(*a*) 10000 (*b*) 20000

(*c*) 30000 (*d*) 40000

Sol. Let the total population of town be x.



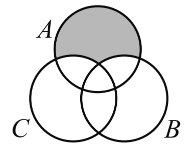
$$\therefore \frac{25x}{100} + \frac{15x}{100} - 1500 + \frac{65x}{100} = x$$

$$\Rightarrow \frac{105x}{100} - x = 1500 \qquad \Rightarrow \frac{5x}{100} = 1500$$

$\Rightarrow x = 30000$

Therefore, option (*c*) is the correct answer.

4. The shaded region in the given figure represents



(*a*) $A \cap (B \cup C)$ (*b*) $A \cup (B \cap C)$

(*c*) $A \cap (B - C)$ (*d*) $A - (B \cup C)$

Sol. Shaded region contain elements of A not in B and not in C hence it is $A - (B \cup C)$

Therefore, option (*d*) is the correct answer.

5. If $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$, then $A \times (B \cap C)$ is

(*a*) $\{(2, 4), (3, 4)\}$

(*b*) $\{(4, 2), (4, 3)\}$

(*c*) $\{(2, 4), (3, 4), (4, 4)\}$

(*d*) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

Sol. Clearly, $A = \{2, 3\}$, $B = \{2, 4\}$, $C = \{4, 5\}$

$\Rightarrow B \cap C = \{4\}$

$\therefore A \times (B \cap C) = \{(2, 4), (3, 4)\}$

Therefore, option (*a*) is the correct answer.

6. The set $(B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

(*a*) $B \cap C'$ (*b*) $A \cap C$

(*c*) $B' \cap C'$ (*d*) None of these

Sol. $(B \cup C) \cap (A \cap B' \cap C')' \cap C'$

$= (B \cup C) \cap (A' \cup B \cup C) \cap C'$

$= (B \cup C) \cap C' = (B \cup C) \cap C'$

$= (B \cap C') \cup \phi = B \cap C'$

Therefore, option (*a*) is the correct answer.

7. Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$. Then the number of sets C such that $A \cap B \subseteq C \subseteq A \cup B$ is

(*a*) 6 (*b*) 9

(*c*) 8 (*d*) 10

Sol. Here, $A \cap B = \{2, 4\}$

and $A \cup B = \{1, 2, 3, 4, 6\}$

$\because A \cap B \subseteq C \subseteq A \cup B$

$\therefore$ C can be $\{2, 4\}$, $\{1, 2, 4\}$, $\{3, 2, 4\}$, $\{6, 2, 4\}$, $\{1, 6, 2, 4\}$, $\{6, 3, 2, 4\}$, $\{1, 3, 2, 4\}$, $\{1, 2, 3, 4, 6\}$

Thus, number of set C which satisfy the given condition is 8.

Therefore, option (c) is the correct answer.

8. If $A = \phi$, $P(A)$ denotes power set of A, then number of elements in $P(P(P(P(P(A)))))$ is

(a) 1 (b) 2^4

(c) 2^5 (d) 2^{16}

Sol. $n(P(A)) = 2^{\circ} = 1$

$\Rightarrow n(P(P(A))) = 2^1 = 2$

$\Rightarrow n(P(P(P(A)))) = 2^2 = 4$

$\Rightarrow n(P(P(P(P(A))))) = 2^4 = 16$

$\Rightarrow n(P(P(P(P(P(A)))))) = 2^{16}$

Therefore, option (d) is the correct answer.

9. If A and B be two sets such that $n(A) = 15$, $n(B) = 25$, then number of possible values of $n(A \Delta B)$ (symmetric difference of A and B) is

(a) 30 (b) 16 (c) 26 (d) 40

Sol. $n(A \Delta B) = n(A \cup B) - n(A \cap B)$ for maximum $n(A \Delta B)$, $n(A \cup B)$ should be maximum and $n(A \cap B)$ is minimum

For $n(A \cap B)$ to be minimum, $A \cap B = \phi$

$\Rightarrow n(A \cup B) = 25 + 15 = 40$

$n(A \Delta B) = 40$

For $n(A \cap B)$ to be maximum, $A \subset B$

$n(A \cap B) = 15$

$\Rightarrow n(A \cup B) = 25$

$n(A \Delta B) = 25 - 15 = 10$

$\Rightarrow$ Range of $(A \Delta B) = \{10, 12, 14, 16, \ldots\ldots, 40\}$

Therefore, option (b) is the correct answer.

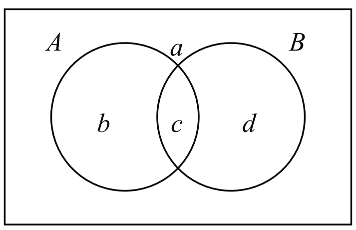
10. $|X|$ represent number of elements in region X. Now the following conditions are given

$|U| = 14$, $|(A - B)^c| = 12$, $|A \cup B| = 9$ and $|A \Delta B| = 7$, where A and B are two subsets of the universal set U and A^c represents complement of set A, then

(a) $|A| = 2$ (b) $|B| = 5$

(c) $|A| = 4$ (d) $|B| = 7$

Sol.



$a + b + c + d = 14$ ...(i)

$a + c + d = 12$ ...(ii)

$b + c + d = 9$ ...(iii)

$b + d = 7$

$b = 2, a = 5, d = 5, c = 2$

$|A| = b + c = 4$

$|B| = d + c = 7$

Therefore, option (c,d) is the correct answers.

11. For any three sets. P, Q and R, S is an element of $(P \Delta Q) \Delta R$, then S may belong to

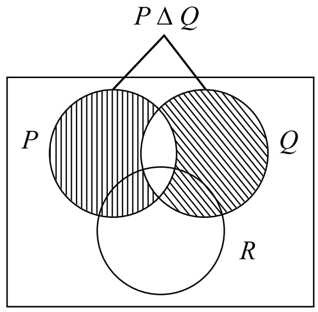
(a) Exactly one of P, Q and R, if P, Q and R disjoint sets

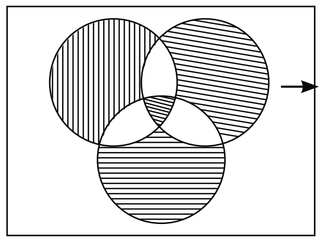
(b) At least one of P, Q and R, but not in all three of them at the same time

(c) Exactly two of P, Q and R

(d) Exactly one of P, Q and R or in all the three of them

Sol.





Therefore, option (a,d) is the correct answers.

12. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots, B_n$ are n sets each having 3 elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each elements of S belongs to exactly 10 of A_i's and exactly 9 of B_j's. The value of n is equal to

Sol. Number of elements in

$A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_{30}$ is 30×5 but each element is used 10 times, so

$$n(S) = \frac{30 \times 5}{10} = 15 \quad \ldots(i)$$

Similarly, number of elements in $B_1 \cup B_2 \ldots \cup B_n$ is 3 n but each element is repeated 9 times, so

$$n(S) = \frac{3n}{9}$$

$\Rightarrow 15 = \frac{3n}{9}$ [from Eq. (*i*)]

$\Rightarrow n = 45$

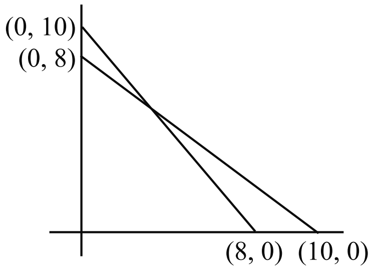
Therefore, 45 is the correct answer.

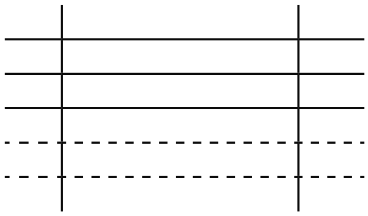
13. $A = \{(x, y) : x, y \in I, x \geq 0, y \geq 0 \text{ and } 4x + 5y \leq 40\}$

$B = \{(x, y) : x, y \in I, x \geq 0, y \geq 0 \text{ and } 5x + 4y \leq 40\}$

where I denotes the set of integers, then $n(A \cap B) =$

Sol.





Total $= \frac{9 \times 10}{2} = 45$

Therefore, 45 is the correct answer.

14. In a survey it was found that 21 persons liked product P_1, 26 liked product P_2 and 29 liked product P_3. If 14 persons liked products P_1 and P_2; 12 persons liked product P_3 and P_1; 14 persons liked products P_2 and P_3, and 8 liked all the three products. Find how many liked product P_3 only.

Sol. Let $n(P_1)$ be a number of people liking product P_1.

Let $n(P_2)$ be a number of people liking product P_2.

Let $n(P_3)$ be a number of people liking product P_3.

Then, according to the question:

$n(P_1) = 21$, $n(P_2) = 26$,

$n(P_3) = 29$, $n(P_1 \cap P_2) = 14$

$n(P_1 \cap P_3) = 12$,

$n(P_2 \cap P_3) = 14$,

$n(P_1 \cap P_2 \cap P_3) = 8$

$\therefore$ Number of people liking product P_3 only:

$= 29 - (4 + 8 + 6)$

$= 29 - 18 = 11$

Therefore, 11 is the correct answer.

SCHOOL LEVEL PROBLEMS

SINGLE CORRECT TYPE QUESTIONS

1. Write solution set of equation $x^2 - 3x + 2 = 0$ in roster form

(*a*) {1, 3} (*b*) {2, 4}
(*c*) {1, 4} (*d*) {1, 2}

2. Which one of the following is the correct representation of set $A = \{2, 4, 8, 16, ...$ in set builder form?

(*a*) $\{x : x = 2n \text{ where } n \in N\}$
(*b*) $\{x : x = 2^n \text{ where } n \in N\}$
(*c*) $\{x : x = 4n \text{ where } n \in N\}$
(*d*) $\{x : x = 2n + 4 \text{ where } n \in N$

3. Two finite sets have M and N elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of M and N are respectively.

(*a*) 6,3 (*b*) 8,5
(*c*) 4,1 (*d*) none of these

4. Every set is of itself

(*a*) Proper subset
(*b*) Improper subset
(*c*) Compliment
(*d*) None of the above

5. If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then

(*a*) $B = C$
(*b*) $A = B$
(*c*) $A = C$
(*d*) None of these

6. A survey shows that 63% of the americans like cheese whereas 76% like apples. If X% of the americans like both cheese and apples, then we have

(*a*) $39 \le x \le 63$ (*b*) $x \le 63$
(*c*) $x \le 39$ (*d*) none of these.

7. What is the interval of $f(x) = (x - 1)(x - 2)(x - 3).(x^3 + 6x^2 + 11x + 6)$ where $f(x)$ is positive?

(*a*) $(-\infty, -3) \cup (3, \infty)$
(*b*) $(3, -2) \cup (1, 1) \cup (2, 3)$
(*c*) $(-\infty, -3) \cup (-2, -1) \cup (1, 2) \cup (3, \infty)$
(*d*) $(-\infty, \infty)$

8. In a family of 10 members, 7 of them like tea or coffee, 4 of them like tea and 5 of them like coffee. How many of them like only tea?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

9. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 5\}$, $B = \{6, 7\}$ then $A \cap B'$ is

(*a*) B' (*b*) A (*c*) A' (*d*) B

VERY SHORT ANSWER TYPE QUESTIONS

10. Write the following sets in roster form:

(i) $A = \{x : x \text{ is an integer and } -3 \le x < 7\}$

(ii) $B = \{x : x \text{ is a natural number less than } 6\}$

11. Given that $N = \{1, 2, 3, ..., 100\}$, then

(i) Write the subset A of N, whose elements are odd numbers.

(ii) Write the subset B of N, whose elements are represented by $x + 2$, where $x \in N$.

12. Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, $B = \{3, 5\}$ and $C = \{1, 2, 4, 7\}$, find $A' \cup (B \cap C')$

13. In a class of 60 students, 23 play hockey, 15 play basketball, 20 play cricket and 7 play hockey and basketball, 5 play cricket and basketball, 4 play hockey and cricket, 15 do not play any of the three games. Find

How many play hockey but not cricket

14. Let $U = \{x : x \in N, x \le 9\}$; $A = \{x : \text{is an even number}, 0 < x < 10\}$; $B = \{2, 3, 5, 7\}$. Write the set $(A \cup B)'$.

15. Given $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$

Verify that $L - (M \cup N) = (L - M) \cap (L - N)$

16. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Find the number of students who play neither.

LONG ANSWER TYPE QUESTIONS

17. Let A, B and C be sets. Then show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

18. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science; 4 in English and Science; 4 in all the three. Find how many passed

(i) in English and Mathematics but not in Science

(ii) in Mathematics and Science but not in English

(iii) in Mathematics only

(iv) in more than one subject only

19. If A and B are subsets of the universal set U, then show that

(*i*) $A \subset A \cup B$

(*ii*) $A \subset B \Leftrightarrow A \cup B = B$

(*iii*) $(A \cap B) \subset A$

20. In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3%

buy B and C and 4% buy A and C. If 2% families buy all the three papers. Find the no. of families which buy

(i) A only

(ii) B only

(iii) none of A, B, and C.

21. If $P(A) = P(B)$, Show that $A = B$

CASE STUDY BASED QUESTIONS

Case Study-I

The school organised a farewell party for 100 students and school management decided three types of drinks will be distributed in farewell party i.e. Milk (M), Coffee (C) and Tea (T).

Organiser reported that 10 students had all the three drinks M, C, T. 20 students had M and C; 30 students had C and T; 25 students had M and T. 12 students had M only; 5 students had C only; 8 students had T only.

Based on the above information, answer the following questions.

22. The number of students who did not take any drink, is

(a) 20 (b) 30 (c) 10 (d) 25

23. The number of students who prefer Milk and Coffee but not tea is

(a) 12 (b) 10 (c) 15 (d) 20

Case Study-II

In a library, 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.

Based on the above information, answer the following questions.

24. The number of students who reading only chemistry is

(a) 5 (b) 4 (c) 2 (d) 1

25. The number of students who reading only , one of the subjects is

(a) 5 (b) 8 (c) 11 (d) 6

PRARAMBH (TOPICWISE)

DEFINITION AND TYPE OF SETS

1. The set of intelligent students in a class is

(a) A null set

(b) A singleton set

(c) A finite set

(d) Not a well defined collection

2. Which of the following is the empty set

(a) $\{x : x$ is a real number and $x^2 - 1 = 0\}$

(b) $\{x : x$ is a real number and $x^2 + 1 = 0\}$

(c) $\{x : x$ is a real number and $x^2 - 9 = 0\}$

(d) $\{x : x$ is a real number and $x^2 = x + 2\}$

3. The set $A = \{x : x \in \mathrm{R}, x^2 = 16$ and $2x = 6\}$ equals

(a) ϕ (b) $\{14,3,4\}$

(c) $\{3\}$ (d) $\{4\}$

4. If a set A has n elements, then the total number of subsets of A is

(a) n (b) n^2 (c) 2^n (d) $2n$

5. The number of proper subsets of the set $\{1, 2, 3\}$ is

(a) 8 (b) 7 (c) 6 (d) 5

6. If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n - 1) : n \in N\}$, then

(a) $X \subseteq Y$ (b) $Y \subseteq X$

(c) $X = Y$ (d) None of these

OPERATIONS ON SETS

7. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is

(a) $\{3\}$ (b) $\{1, 2, 3, 4\}$

(c) $\{1, 2, 4, 5\}$ (d) $\{1, 2, 3, 4, 5, 6\}$

8. If $A \subseteq B$, then $A \cup B$ is equal to

(a) A (b) $B \cap A$

(c) B (d) None of these

9. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to

(a) A (b) B

(c) A^c (d) B^c

10. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to

(a) A (b) B

(c) ϕ (d) $A \cap B^c$

11. If $N_a = \{an : n \in N\}$ then $N_3 \cap N_4 =$

(a) N_7 (b) N_{12}

(c) N_3 (d) N_4

12. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then

(a) $d = bc$ (b) $c = bd$

(c) $b = cd$ (d) None of these

13. If the sets A and B are defined as

$A = \{(x, y) : y = \dfrac{1}{x},\ 0 \neq x \in R\}$

$B = \{(x, y) : y = -x, x \in R\}$, then

(a) $A \cap B = A$ (b) $A \cap B = B$

(c) $A \cap B = \phi$ (d) None of these

14. Let $A = [x : x \in R, |x| < 1]$; $B = [x : x \in R, |x - 1| \geq 1]$ and $A \cup B = R - D$, then the set D is

(a) $[x : 1 < x \leq 2]$ (b) $[x : 1 \leq x < 2]$

(c) $[x : 1 \leq x \leq 2]$ (d) None of these

15. If the sets A and B are defined as

$A = \{(x , y) : y = e^x, x \in R\}$

$B = \{x, y\} : y = x, x \in R$, then

(a) $B \subseteq A$ (b) $A \subseteq B$

(c) $A \cap B = \phi$ (d) $A \cup B = A$

16. If $X = \{4^n - 3n - 1: n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$ then $X \cup Y$ is equal to

(a) X (b) Y

(c) N (d) None of these

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS

17. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$

(a) 3 (b) 6

(c) 9 (d) 18

18. If A and B are two sets such that $n(A) = 70$, $n(B) = 60$, and $n(A \cup B) = 110$, then $n(A \cap B)$ is equal to

(a) 240 (b) 50

(c) 40 (d) 20

19. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$ then $n(A^c \cap B^c)$

(a) 400 (b) 600 (c) 300 (d) 200

20. If $A = [(x, y) : x^2 + y^2 = 25]$ and $B = [(x, y) : x^2 + 9y^2 = 144]$, then $A \cap B$ contains

(a) One point

(b) Three points

(c) Two points

(d) Four points

21. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is

(a) 80 percent (b) 40 percent

(c) 60 percent (d) 70 percent

22. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is

(a) 6 (b) 9 (c) 7 (d) 22

LAWS OF ALGEBRA OF SETS

23. If A, B and C are any three sets, then $A \times (B \cap C)$ is equal to

(a) $(A \times B) \cup (A \times C)$ (b) $(A \times B) \cap (A \times C)$

(c) $(A \cup B) \times (A \cup C)$ (d) $(A \cap B) \times (A \cap C)$

24. If A, B and C are any three sets, then $A \times (B \cup C)$ is equal to

(a) $(A \times B) \cup (A \times C)$ (b) $(A \cup B) \times (A \cup C)$

(c) $(A \times B) \cap (A \times C)$ (d) None of these

25. If A, B and C are any three sets, then $A - (B \cup C)$ is equal to

(a) $(A - B) \cup (A - C)$ (b) $(A - B) \cap (A - C)$

(c) $(A - B) \cup C$ (d) $(A - B) \cap C$

26. If $A = [x : x$ is a multiple of 3] and $B = [x : x$ is a multiple of 5], then $A - B$ is ($\overline{A}$ means complement of A)

(a) $\overline{A} \cap B$ (b) $A \cap \overline{B}$

(c) $\overline{A} \cap \overline{B}$ (d) $\overline{A \cap B}$

27. If A, B and C are non-empty sets, then $(A - B) \cup (B - A)$ equals

(a) $(A \cup B) - B$ (b) $A - (A \cap B)$

(c) $(A \cup B) - (A \cap B)$ (d) $(A \cap B) \cup (A \cup B)$

CARTESIAN PRODUCT OF SETS

28. If $A = \{0, 1\}$, and $B = \{1, 0\}$, then $A \times B$ is equal to

(a) $\{0, 1, 1, 0\}$ (b) $\{(0, 1), (1, 0)\}$

(c) $\{0, 0\}$ (d) $\{(0,1),(0,0),(1,1),(1,0)\}$

29. If $A = \{2, 4, 5\}$, $B = \{7, 8, 9\}$ then $n(A \times B)$ is equal to

(a) 6 (b) 9 (c) 3 (d) 0

30. If the set A has p elements, B has q elements, then the number of elements in $A \times B$ is

(a) $p + q$ (b) $p + q + 1$
(c) pq (d) p^2

31. If $A = \{a, b\}$, $B = \{c, d\}$, $C = \{d, e\}$ then $\{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$ is equal to

(a) $A \cap (B \cup C)$ (b) $A \cup (B \cap C)$
(c) $A \times (B \cup C)$ (d) $A \times (B \cap C)$

32. If $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$ then $A \times (B \cap C)$ is

(a) $\{(2, 4), (3, 4)\}$
(b) $\{(4, 2), (4, 3)\}$
(c) $\{(2, 4), (3, 4), (4, 4)\}$
(d) $\{(2,2), (3,3), (4,4), (5,5)\}$

33. If P, Q and R are subsets of a set A, then $R \times (P^c \cup Q^c)^c =$

(a) $(R \times P) \cap (R \times Q)^c$ (b) $(R \times Q) \cap (R \times P)$
(c) $(R \times P) \cup (R \times Q)$ (d) None of these

34. Let $S = \{x \in R : (x-3)^2 + (x-2)^2 + 5(x-4)^2 = 0\}$. Then S is not

(a) Singleton set (b) Empty set
(c) Void set (d) null set

35. Let $S = \{x \in R : 4^x + 2^{x+1} - 8 = 0\}$. Then $x =$

(a) 0 (b) 1 (c) 2 (d) 3

36. The set $(A \cap B')' \cup (B \cap C)$ is equal to

(a) $A' \cup B \cup C$ (b) $A' \cup B$
(c) $A' \cap C'$ (d) $A' \cap B$

37. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to

(a) A (b) B (c) ϕ (d) $A \cap B^c$

38. Let $A = \{(x, y): x \in R, y \in R, x^3 + y^3 = 1]\}$, $B = \{(x, y): x \in R, y \in R, x - y = 1\}$ and $C = \{(x, y): x \in R, y \in R, x + y = 1\}$. If $A \cap B$ contains 'p' elements and $A \cap C$ contains 'q' elements then find $(q - p)$.

(a) 0 (b) 1 (c) 2 (d) 3

39. In a examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects?

(a) 9%
(b) 7%
(c) 15%
(d) Cannot be determined due to insufficient data

40. **Statement-I:** $N \cup (B \cap Z) = (N \cup B) \cap Z$ for any subset B of R, where N is the set of positive integers, Z is the set of integers, R is the set of real numbers.

Statement-II: Let $A = \{n \in N : 1 \le n \le 24$, n is a multiple of $3\}$. There exists no subset B of N such that the number of elemets in A is equal to the number of elements in B.

Which of the above statements is/are correct?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

41. In year 10 of a Kuala Lumpur International School, there are 198 students.

C = {students who like chilli}

D = {students who like durian}

90 like durian and 130 like chilli. The number who like both is x and the number who like neither is $212 - 2x$. Find the value of x.

(a) 76 (b) 78
(c) 80 (d) None of these

PRABAL (JEE MAIN LEVEL)

SINGLE CORRECT TYPE QUESTIONS

1. Let $A = \{x : x \in R, x \ge 2\}$ and $B = \{x : x \in R, x < 4\}$. Then $A \cap B =$

(a) $\{x : x \in R, 2 < x < 4\}$, (b) $\{x : x \in R, 2 \le x < 4\}$
(c) B (d) A

2. If $A = \{\phi, \{(\phi)\}\}$, then the power set $P(A)$ of A is

(a) A (b) $\{\phi, \{\phi\}, A\}$
(c) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$ (d) $\{\phi, \{\phi\}, \{\{\phi\}\}\}$

3. If $A = \{2, 3\}$ and $B = \{x \mid x \in N$ and $x < 3\}$, then $A \times B$ is

(a) $\{(2, 1), (2, 2), (3, 1), (3, 2)\}$
(b) $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$
(c) $\{(1, 2), (2, 2), (3, 3), (3, 2)\}$
(d) $\{(1, 1), (2, 2), (3, 3), (3, 2)\}$

4. Let U be the universal set containing 700 elements. If A, B are subsets of U such that $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B') =$

(a) 400 (b) 600 (c) 300 (d) 200

5. Let $R: x \to y$ be a relation on N defined by $x + 2y = 8$. The domain of R is

(a) $\{2, 4, 8\}$ (b) $\{2, 4, 6, 8\}$
(c) $\{2, 4, 6\}$ (d) $\{1, 2, 3, 4\}$

6. If $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{x \in N : 30 < x^2 < 70\}$, $B = \{x : x$ is a prime number less than $10\}$, then which of the following is incorrect?

(a) $A \cup B = \{2, 3, 5, 6, 7, 8\}$

(b) $A \cap B = \{7, 8\}$

(c) $A - B = \{6, 8\}$

(d) $A \Delta B = \{2, 3, 5, 6, 8\}$

7. Let X be the universal set for sets A and B, if $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A' \cap B')$ is equal to 300 provided $n(X)$ is equal to

(a) 600 (b) 700

(c) 800 (d) 900

8. If two sets A and B are having 80 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are

(a) 2^{80} (b) 80^2

(c) 81 (d) 79

9. If A and B be two universal sets and $A \cup B \cup C = U$. Then, $((A - B) \cup (B - C) \cup (C - A))'$ is equal to

(a) $A \cup B \cup C$ (b) $A \cup (B \cap C)$

(c) $A \cap B \cap C$ (d) $A \cap (B \cup C)$

10. The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

(a) $B \cup C'$ (b) $A \cap C$

(c) $B' \cap C'$ (d) None of these

11. If there are three athelitic teams in a school 21 are in the basketball team,26 in hockey team and 29 in the football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the games. The total number of members is

(a) 42 (b) 43

(c) 45 (d) None of these

12. If $P(A)$ denotes the power set of A and A is the void set, then what is number of elements in $P\{P\{P\{P(A)\}\}\}$?

(a) 0 (b) 1

(c) 4 (d) 16

13. There are 4 prime numbers between n and $2n$. Then, possible values of n is

(a) $n = 4$ (b) $n = 13$

(c) $n = 10$ (d) $n = 50$

14. Let $S = \{x \in R : 2^{333x-2} + 2^{111x+1} = 2^{222x+2} + 1\}$. Then Sum of all elements of S is

(a) $\frac{111}{2}$ (b) $\frac{2}{111}$

(c) 2 (d) 111

15. Let F_1 be the set of all parallelograms, F_2 the set of all rectangles, F_3 the set of all rhombuses, F_4 the set of all squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to

(a) $F_2 \cap F_4$

(b) $F_3 \cap F_4$

(c) $F_2 \cap F_3$

(d) $F_2 \cup F_3 \cup F_4 \cup F_1$

16. Let $A = \{(x, y) : a^x = a^y;\ a > 0 \text{ and } a \neq 1;\ a, x, y \in R\}$

$B = \{(x, y);\ xy = 1;\ x, y \in R_0\}$

Choose the correct statements amongst the following.

(a) $A \cap B = B$

(b) $A \cap B = A$

(c) $n(B) > n(A)$

(d) A and B are non-comparable

17. Let A_1, A_2 and A_3 be subsets of a set X. Which one of the following is correct?

(a) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing elements of each of A_1, A_2 and A_3.

(b) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing either A_1 or $(A_2 \cup A_3)$ but not both.

(c) The smallest subset of X containing $A_1 \cup A_2$ and A_3 equals the smallest subset of X containing both A_1, and $A_2 \cup A_3$ only if $A_2 = A_3$.

(d) $A_1 \cup A_2 \cup A_3$ is the largest subset of X containing elements of each of A_1, A_2 and A_3.

INTEGER TYPE QUESTIONS

18. Universal set, $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

and $A = \{x : x^2 - 5x + 6 = 0\}$

$B = \{x : x^2 - 3x + 2 = 0\}$

Then $n(A \cap B)'$ is equal to

19. If $n(A \cap B) = 5$, $n(A \cap C) = 7$ and $n(A \cap B \cap C) = 3$, then the minimum possible value of $n(B \cap C)$ is

20. If $A = \{1,2,3,4\}$, then the number of subsets of set A containing element 3, is

21. If $A = \left\{n : \frac{n^3 + 5n^2 + 2}{n} \text{is an integer and } n \text{ itself is an integer}\right\}$, then the number of elements in the set A, is

22. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examinations is

PARIKSHIT (JEE ADVANCED LEVEL)

SINGLE CORRECT TYPE QUESTIONS

1. Let $A = \{n \in N$: H.C.F. $(n, 45) = 1\}$ and let $B = \{2k : k \in \{1, 2, ..., 100\}\}$, Then the sum of all the elements of $A \cap B$ is given by

(*a*) 5265 (*b*) 4372

(*c*) 5264 (*d*) 5371

2. Let $A = \{n \in N \mid n^2 \leq n + 10{,}000\}$, $B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ________.

(*a*) 831 (*b*) 832

(*c*) 732 (*d*) 730

3. Let Z be the set of all integers,

$A = \{(x, y) \in Z \times Z : (x - 2)^2 + y^2 \leq 4\}$

$B = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$ and

$C = \{(x, y) \in Z \times Z : (x - 2)^2 + (y - 2)^2 \leq 4\}$

If the total number of relations from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is ________.

(*a*) 16 (*b*) 49

(*c*) 25 (*d*) 9

MULTIPLE CORRECT TYPE QUESTIONS

4. The number of elements in the set $\{n \in N: 10 \leq n \leq 100$ and $3^n - 3$ is a multiple of $7\}$ is k, then k is divisible by

(*a*) 3 (*b*) 5

(*c*) 15 (*d*) 21

5. Let $A = \{1,2,3,4,5,6,7\}$. Define $B = \{T \subseteq A$: either $1 \notin T$ or $2 \in T\}$ and $C = \{T \subseteq A$: T the sum of all the elements of T is a prime number.$\}$ If the number of elements in the set $B \cup C$ is α, then sum of digits of α is divisible by

(*a*) 2 (*b*) 4

(*c*) 8 (*d*) 9

COMPREHENSION BASED QUESTIONS

Passage-I

In a survey of 100 students, the number of students studying the various languages is found as: English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find

6. How many students are studying Hindi,

(*a*) 18 (*b*) 9

(*c*) 4 (*d*) 15

7. How many students are studying English and Hindi both.

(*a*) 3 (*b*) 6

(*c*) 9 (*d*) 2

Passage-II

In a survey of 25 students, it was found that 12 have taken physics, 11 have taken chemistry and 15 have taken mathematics; 4 have taken physics and chemistry; 9 have taken physics and mathematics; 5 have taken chemistry and mathematics while 3 have taken all the three subjects. Find the number of students who have taken.

8. Mathematics only;

(*a*) 8 (*b*) 4

(*c*) 16 (*d*) 2

9. Physics and Mathematics but not Chemistry;

(*a*) 6 (*b*) 12

(*c*) 14 (*d*) 3

MATCH THE COLUMN TYPE QUESTIONS

10. Match the following sets for all sets A, B and C

	Column-I		Column-II
A.	$((A' \cup B')' - A)'$	p.	$A - B$
B.	$[B' \cup (B' - A)]'$	q.	U
C.	$(A - B) - (B - C)$	r.	B
D.	$(A - B) \cap (C - B)$	s.	$(A \cap C) - B$

(*a*) A → p, B → q, C → r, D → s

(*b*) A → q, B → r, C → p, D → s

(*c*) A → q, B → r, C → s, D → p

(*d*) A → p, B → r, C → q, D → s

11. Match the set P in Column-I with its super set Q in Column-II

	Column-I		Column-II
A.	$[3^{2n} - 8n - 1 : n \in N]$	p.	$\{49\,(n-1) : n \in N\}$
B.	$\{2^{3n} - 1 : n \in N\}$	q.	$\{64\,(n-1) : n \in N\}$
C.	$\{3^{2n} - 1 : n \in N\}$	r.	$\{7n : n \in N\}$
D.	$\{2^{3n} - 7n - 1 : n \in N\}$	s.	$\{8n : n \in N\}$

(*a*) A → p, B → q, C → r, D → s

(*b*) A → q, B → r, C → p, D → s

(*c*) A → q, B → r, C → s, D → p

(*d*) A → p, B → r, C → q, D → s

NUMERICAL TYPE QUESTIONS

12. An investigator interviewed 100 students to determine their preferences for the three drinks: milk (M), coffee (C) and tea (T). He reported the following : 10 students had all the three drinks M, C and T; 20 had M and C; 30 had C and T; 25 had M and T; 12 had M only, 5 had C only; and 8 had T only. If number of students who did not take any the three drinks is n, then $\frac{n}{5}$ is

13. In a class of 80 students numbered 1 to 80, all odd numbered students opt of Cricket, students whose numbers are divisible by 5 opt for Football and those whose numbers are divisible by 7 opt for Hockey. If the number of students who do not opt any of the three games is n, then $\frac{n}{4}$ is equal to

14. A survey shows that 61%, 46% and 29% of the people watched "3 idiots", "Rajneeti" and "Avatar" respectively. 25% people watched exactly two of the three movies and 3% watched none. What percentage of people watched all the three movies?

15. If $n(A) = 4$ and $n(B) = 7$, then the difference between maximum and minimum value of $n(A \cup B)$ is

16. Let $S = \{x \in R : |x-2|^{10x^2-1} = |x-2|^{3x}\}$. Then $n(S)$ is

17. Let $S = \{x \in R : 5^x \sqrt[x]{8^{x-1}} = 500\}$. Then $n(S)$ is

18. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, and 3 read all the three newspapers. Find the number of people who read at least one of the newspapers.

19. Times, Mirror and Sun are three newspapers

(*i*) All readers of the Times read the Sun

(*ii*) Every person either reads the Sun or does not read the Mirror.

(*iii*) 11 people read the Sun but do not read the Mirror

(*iv*) 8 people read either the Times or the Mirror but not both.

(*v*) 10 people read the Sun and either read the Mirror or do not read the Times

(*vi*) 14 people either read the Sun and not the Mirror or read both the Sun and Times

(*vii*) 9 people neither read the Times nor the Mirror

Find the number of people who read Times and Mirror both

20. Let $S = \{x \in R : 5^x + \frac{125}{5^x} = 30\}$. Then sum of all possible value of x equal to.

21. Let $T = \left\{x \,|\, \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$. Then number of elements in T equal to.

22. Given that $n(U) = 40$, $n(A) = 28$, $n(B) = 25$, $n(A \cap B) = x$ and $n(A' \cap B') = y$. Find the sum of greatest and least values of x and y both.

PYQ'S (PAST YEAR QUESTIONS)

1. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events? **[11 April, 2023 (Shift-I)]**

(*a*) 10 (*b*) 9 (*c*) 21 (*d*) 15

2. The number of elements in the set $\{n \in \mathrm{N} : 10 \le n \le 100$ and $3^n - 3$ is a multiple of $7\}$ is _______.

[15 April, 2023 (Shift-I)]

3. The number of elements in the set $\{n \in \mathbb{Z} : |n^2 - 10n + 19| < 6\}$ is _________. **[10 April, 2023 (Shift-I)]**

4. Let $S = \left\{x \in [-6, 3] - \{-2, 2\} : \frac{|x+3|-1}{|x|-2} \ge 0\right\}$ and

$T = \{x \in Z : x^2 - 7|x| + 9 \le 0\}$. Then the number of elements of $S \cap T$ is **[28 July, 2022 (Shift-II)]**

(*a*) 7 (*b*) 5 (*c*) 4 (*d*) 3

5. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A$: either $1 \notin T$ or $2 \in T\}$ and $C = \{T \subseteq A$: T is the sum of all the elements of T is a prime number$\}$. Then the number of elements in the set $B \cup C$ is _______ **[25 July, 2022 (Shift-II)]**

6. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \ne \phi\}$ is __________ **[26 July, 2022 (Shift-II)]**

7. The sum of all the elements of the set $\{\alpha \in \{1, 2, ..., 100\}$: HCF$(\alpha, 24) = 1\}$ is **[24 June, 2022 (Shift-II)]**

8. Let $A = \{n \in N$: H.C.F. $(n, 45) = 1\}$ and Let $B = \{2k : k \in \{1, 2, ..., 100\}\}$. Then the sum of all the elements of $A \cap B$ is_______. **[26 June, 2022 (Shift-I)]**

9. Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, ..., 1000\}$. If $A = \{a_1 + a_2 + ... + a_k : k \in N, a_1, a_2, a_3, ..., a_k \in S\}$, then the sum of all the elements in the set $T - A$ is equal to

[29 July, 2022 (Shift-I)]

10. Let $S = \{(x, y) \in N \times N : 9(x-3)^2 + 16(y-4)^2 \le 144\}$ and

$T = \{(x, y) \in R \times R : (x-7)^2 + (y-4)^2 \le 36\}$.

Then $n(S \cap T)$ is equal to **[29 July, 2022 (Shift-II)]**

11. If $A = \{x \in R; |x-2| > 1\}$, $B = \{x \in R : \sqrt{x^2 - 3} > 1\}$, $C = \{x \in R; |x-4| \ge 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap Z$ is _____.

[27 Aug, 2021 (Shift-I)]

12. The number of elements in the set $\{x \in R : (|x| - 3)\,|x + 4| = 6\}$ is equal to **[16 March, 2021 (Shift-I)]**

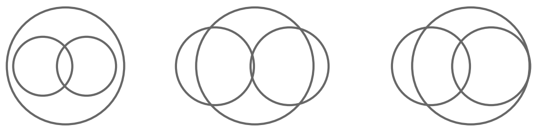
(*a*) 3 (*b*) 2 (*c*) 4 (*d*) 1

13. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set:

[26 Aug, 2021 (Shift-I)]

(*a*) {80, 83, 86, 89} (*b*) {84, 87, 90, 93}
(*c*) {84, 86, 88, 90} (*d*) {79, 81, 83, 85}

14. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement? **[17 March, 2021 (Shift-I)]**



(*a*) *Q* and *R* (*b*) None of these
(*c*) *P* and *R* (*d*) *P* and *Q*

15. Let $S = \{1, 2, 3, 4, 5, 6, 9\}$. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of $3\}$ is **[27 Aug, 2021 (Shift-II)]**

16. The number of elements in the set $\left\{n \in \{1,2,3,\ldots.100\} \mid (11)^n > (10)^n + (9)^n\right\}$ is ______

[22 July, 2021 (Shift-II)]

17. Let $A = \{n \in N : n$ is a 3-digit number$\}$ $B = \{9k + 2 : k \in N\}$ and $C = \{9k + \ell : k \in N\}$ for some $\ell (0 < \ell < 9)$. If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then ℓ is equal to___________. **[24 Feb, 2021 (Shift-I)]**

18. Let $A = \{n \in N | n^2 \leq n + 10{,}000\}$, $B = \{3k + 1 | k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ____. **[27 July, 2021 (Shift-II)]**

19. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If *x* denotes the percentage of them, who like both coffee and tea, then *x* cannot be:

[5 Sep, 2020 (Shift-I)]

(*a*) 63 (*b*) 36 (*c*) 38 (*d*) 54

20. If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x - 2| \geq 3\}$: then:

[9 Jan, 2020 (Shift-II)]

(*a*) $B - A = R - (-2, 5)$ (*b*) $A \cap B = (-2, -1)$
(*c*) $A - B = [-1, 2)$ (*d*) $A \cup B = R - (2, 5)$

21. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$ where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets $X_i's$ and exactly 6 of sets $Y_i's$, then n is equal to **[4 Sep, 2020 (Shift-II)]**

(*a*) 50 (*b*) 15 (*c*) 30 (*d*) 45

22. A survey shows that 63% of the people in a city read newspaper *A* whereas 76% read newspaper *B*. If *x*% of the people read both the newspapers, then a possible value of *x* can be: **[4 Sep, 2020 (Shift-I)]**

(*a*) 37 (*b*) 55 (*c*) 29 (*d*) 65

23. Set *A* has *m* elements and Set *B* has *n* elements. If the total number of subsets of *A* is 112 more than total number of subsets of *B*, then the value of $m \cdot n$ is ________.

[6 Sep, 2020 (Shift-I)]

24. Let $X = \{n \in N : 1 \leq n \leq 50\}$. If $A = \{n \in X : n$ is a multiple of $2\}$; $B = \{n \in X : n$ is a multiple of $7\}$, then the number of elements in the smallest subset of *X* containing both *A* and *B* is _____________. **[7 Jan, 2020 (Shift-II)]**

25. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is: **[10 Jan, 2019 (Shift-I)]**

(*a*) 102 (*b*) 42 (*c*) 1 (*d*) 38

26. Let *A*, *B* and *C* be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

[12 April, 2019 (Shift-II)]

(*a*) If $(A - C) \subseteq B$ then $A \subseteq B$

(*b*) $(C \cup A) \cap (C \cup B) = C$

(*c*) If $(A - B) \subseteq C$, then $A \subseteq C$

(*d*) $B \cap C \neq \phi$

27. Let *Z* be the set of integers. If $A = \{X \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is: **[12 Jan, 2019 (Shift-II)]**

(*a*) 2^{15} (*b*) 2^{18}
(*c*) 2^{12} (*d*) 2^{10}

28. Let $S = \{1, 2, 3, \ldots\ldots, 100\}$. The number of non-empty subsets *A* of *S* such that the product of elements in *A* is even is: **[12 Jan, 2019 (Shift-I)]**

(*a*) $2^{100} - 1$ (*b*) $2^{50}(2^{50} - 1)$
(*c*) $2^{50} - 1$ (*d*) $2^{50} + 1$

PW CHALLENGERS

1. Let X be a set of 56 elements. Find the least positive integer n such that for any 15 subsets of X, if the union of every 7 sets of these subsets contains at least n elements, then there exist 3 of the 15 subsets whose intersection is nonempty.
2. Let n be a positive integer such that $1 \le n \le 1000$. Let M_n be the number of integers in the set X_n = $\left\{\sqrt{4n+1}, \sqrt{4n+2}, ..., \sqrt{4n+1000}\right\}$. Let $a = \max\{M_n : 1 \le n \le 1000\}$, and $b = \min\{M_n : 1 \le n \le 1000\}$. Find $a - b$.
3. Consider a set $S = \left\{a_1^{2025}, a_2^{2025}, a_3^{2025}, ..., a_n^{2025}\right\}$ and Say S_i denote the subsets of S $\forall i$, then, the total number of unordered pairs of dispoint subset S is equal to, (where a_i's are distinct $\forall i = 1, \ldots, n$)

   (*a*) $\dfrac{3^n+1}{2}$ (*b*) $\dfrac{3^{n+1}}{2}$

   (*c*) $\dfrac{3^n-1}{2}+2025$ (*d*) $\dfrac{3^{n-1}}{2}$
4. Consider the set X_1, X_2, X_3 such that $X_2, X_3, \subseteq X_1$, and X_1 contain 7! elements, $n(X_2 \cup X_3) = 793$.
   If $n\left(X_2' \cap X_3'\right) = \alpha_1^3 + \alpha_2^3 + \alpha_3^3 + \alpha_4^3 = \beta_1^3 + \beta_2^3 + \beta_3^3 + \beta_4^3$ for some positive integers $\alpha_1 < \alpha_3 < \beta_1 \le \beta_3 < \beta_2 < \alpha_2 < \beta_4 < \alpha_4$

   Then $\dfrac{\left(\alpha_3^2+\alpha_4\right)-\left(\alpha_1^2+\alpha_2\right)}{(\beta_2-\beta_1)(\beta_4-\beta_3)} =$

   (*a*) $\dfrac{6}{7}$ (*b*) $\dfrac{8}{7}$ (*c*) $\dfrac{7}{6}$ (*d*) $\dfrac{4}{7}$
5. If $A_1 \times A_2 \times A_3 \times A_4 =$
   $\{(1, 1, 1, 1), (2, 4, 8, 16), (3, 9, 27, 81), \ldots\}$
   and $\forall a_i \in A\,|, \forall b_i \in A_2, \forall c_i \in A_3, \forall d_i \in A_4$

   Define $\alpha = 7\sum_{i=1}^{10} a_i$; $\beta = 2\sum_{i=1}^{10} b_i$

   $\eta = 8\sum_{i=1}^{10} c_i$; $\delta = \sum_{i=1}^{10} d_i$

   Then, $\alpha + \beta + \eta - \delta =$
6. A, B, C be three sets such that $n(A) = 2$, $n(B) = 3$, $n(C) = 4$. If $P(X)$ denotes power set of X,

   $K = \dfrac{n(P(P(C)))}{n(P(P(A))) \times n(P(P(B)))}$. Sum of digits of K is____.
7. Let U be set with number of elements in it is 2009. A is a subset of U with $n(A) = 1681$ and out of these 1681 elements, exactly 1075 elements belong to a subset B of U. If $n(A - B) = m^2 + p_1 p_2 p_3$ for some positive integer m and distinct primes $p_1 < p_2 < p_3$ then for least m find $7\dfrac{p_1 p_3}{p_2}$.
8. Consider $A_1, A_2, ..., A_{2025}$ be 2025 sets, such that $A_1 \subset A_2 \subset A_3, ... \subset A_{2025}$, then choose the correct statement from following:

   (*a*) If $n(A_i) = i + 1$, then $\bigcup_{i=1}^{2025} A_i$ contains 2026 elements

   (*b*) If $n(A_i) = i + 1$, then $\bigcup_{i=1}^{2025} A_i$ contains 2025 elements

   (*c*) If $n(A_i) = i + 2$, and $\bigcup_{i=3}^{2025} A_i = A$ then $n(A) = 5!$

   (*d*) If $n(A_i) = i + 2$, and $\bigcup_{i=3}^{2025} A_i = A$ then $n(A) = 6!$
9. $A = \{x : x \in N$, G.C.D. $(x, 36) = 1, x < 36\}$, $B = \{y : y \in N$, G.C.D. $(y, 40) = 1, y < 40\}$; (G.C.D. stands for greatest common divisors)

	Column-I		Column-II
A.	$n(A \cap B)$	p.	10
B.	$C = \{x : x \in A \cup B, x$ is prime $\}, n(C)=$	q.	9
C.	$n(A \Delta B)$	r.	21
D.	$n((A - B) \times (B - A))$	s.	11

10. Consider the set X, such that

$$X = \left\{x : x = \frac{(n^2)!\,((n!)^2 + 2026n!) + (n!)^{n+1}(14n^5 + 30n^4 + 10n^3 + 123n^2 + 36n + 24)(n! + 2026)}{\left((n!)^2 + 2026n!\right)(n!)^{n+1}}, x \in Z\right\}$$

Then the number of elements in set A =

Answer Key

CONCEPT APPLICATION

1. (*b*) **2.** (*a*) **3.** (*a*) **4.** (*b*) **5.** (*d*) **6.** (*c*) **7.** (*d*) **8.** (*d*) **9.** (*c*)

10. (*b*) **11.** (*a*) **12.** (*c*) **13.** (*a*) **14.** (*c*) **15.** P $\rightarrow A\cap\overline{B}$, Q $\rightarrow A\cap B$, R $\rightarrow B\cap\overline{A}$ and S $\rightarrow \overline{A}\cap\overline{B}$

16. (*iii*) 4,54

SCHOOL LEVEL PROBLEMS

1. (*d*) **2.** (*b*) **3.** (*a*) **4.** (*b*) **5.** (*a*) **6.** (*a*) **7.** (*c*) **8.** (*a*) **9.** (*b*) **22.** (*a*)

23. (*b*) **24.** (*a*) **25.** (*c*)

PRARAMBH (TOPICWISE)

1. (*d*) **2.** (*b*) **3.** (*a*) **4.** (*c*) **5.** (*b*) **6.** (*a*) **7.** (*b*) **8.** (*c*) **9.** (*a*) **10.** (*d*)

11. (*b*) **12.** (*a*) **13.** (*c*) **14.** (*b*) **15.** (*c*) **16.** (*b*) **17.** (*b*) **18.** (*d*) **19.** (*c*) **20.** (*d*)

21. (*c*) **22.** (*d*) **23.** (*b*) **24.** (*a*) **25.** (*b*) **26.** (*b*) **27.** (*c*) **28.** (*d*) **29.** (*b*) **30.** (*c*)

31. (*c*) **32.** (*a*) **33.** (*b*) **34.** (*a*) **35.** (*b*) **36.** (*b*) **37.** (*d*) **38.** (*b*) **39.** (*b*) **40.** (*a*)

41. (*b*)

PRABAL (JEE MAIN LEVEL)

1. (*b*) **2.** (*c*) **3.** (*a*) **4.** (*c*) **5.** (*c*) **6.** (*b*) **7.** (*b*) **8.** (*b*) **9.** (*c*) **10.** (*d*)

11. (*b*) **12.** (*d*) **13.** (*c*) **14.** (*b*) **15.** (*d*) **16.** (*d*) **17.** (*d*) **18.** [3] **19.** [3] **20.** [8]

21. [4] **22.** [14]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*c*) **2.** (*b*) **3.** (*c*) **4.** (*a*, *b*, *c*) **5.** (*a*, *b*, *c*) **6.** (*a*) **7.** (*a*) **8.** (*b*) **9.** (*a*) **10.** (*b*)

11. (*c*) **12.** [4] **13.** [7] **14.** [7] **15.** [4] **16.** [5] **17.** [1] **18.** [52] **19.** [3] **20.** [3]

21. [1] **22.** [50]

PYQ's (PAST YEAR QUESTIONS)

1. (*c*) **2.** [15] **3.** [6] **4.** (*d*) **5.** [107] **6.** [112] **7.** [1633] **8.** [5264] **9.** [11] **10.** [27]

11. [256] **12.** (*b*) **13.** (*d*) **14.** (*b*) **15.** [80] **16.** [96] **17.** [5] **18.** [832] **19.** (*b*) **20.** (*a*)

21. (*c*) **22.** (*b*) **23.** [28] **24.** [29] **25.** (*d*) **26.** (*a*) **27.** (*a*) **28.** (*b*)

PW CHALLENGERS

1. [41] **2.** [22] **3.** (*a*) **4.** (*c*) **5.** [22] **6.** [7] **7.** [86] **8.** (*a*)

9. A $\rightarrow$ q, B $\rightarrow$ s, C $\rightarrow$ p, D $\rightarrow$ r **10.** [3]