ARJUNA

for

JEE

MAIN & ADVANCED

PHYSICS

FULL COURSE STUDY MATERIAL

Class XI

- Units and Measurement
- Vector and Calculus
- Motion in a Straight Line
- Projectile Motion
- Relative Motion

Module-1

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PHYSICS

Full Course Study Material **XI**

MODULE–1

CONTENTS

Physics Module-1 1-164

CHAPTER

1 Units and Measurements

PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationships.

MEASUREMENT

Measurement is the comparison of a quantity with a standard of the same physical quantity. Different countries followed different standards.

UNITS

All physical quantities are measured with respect to standard magnitude of the same physical quantity and these standards are called UNITS. e.g. second, meter, kilogram, etc.

Four basic properties of units are:

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

Set of Fundamental Quantities

A set of physical quantities which are completely independent of each other but all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.

The Fundamental Quantities that are currently being accepted by the scientific community are mass, time, length, current, temperature, luminous intensity and amount of substance.

Derived Physical Quantities

The physical quantities that can be expressed in terms of fundamental physical quantities are called derived physical quantities. E.g. speed = distance/time.

System of Units

1. **FPS or British Engineering system:** In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.
2. **CGS or Gaussian system:** In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).
3. **MKS system:** In this system also the fundamental quantities are length, mass and time but their fundamental units are meter (m), kilogram (kg) and second (s) respectively.

Table: Units of some physical quantities in different systems

Type of physical Quantity	Physical Quantity	System		
		CGS	MKS	FPS
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s

4. **International system (SI) of units:** This system is modification over the MKS system. Besides the three base units of MKS system four other fundamental and two supplementary units are also included in this system.

Table: SI base quantities and their units

S. No.	Physical quantity	unit	Symbol
1	Length	metre	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous Intensity	candela	cd
7	Amount of substance	mole	mol

Physical Quantity (SI Unit)	Definition
Length (m)	The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit m s^{-1}, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$.

Physical Quantity (SI Unit)	Definition
Mass (kg)	The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.62607015×10^{-34} when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the metre and the second are defined in terms of c and $\Delta\nu_{Cs}$.
Time (s)	The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground state hyperfine transition frequency of the caesium-133 atom, to be 9192631770 when expressed in the unit Hz, which is equal to s^{-1}.
Electric Current (A)	The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602176634\times10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{Cs}$.
Thermodynamic Temperature (K)	The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380649×10^{-23} when expressed in the unit J K^{-1}, which is equal to kg m^2 s^{-2} k^{-1}, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.
Amount of substance (mole)	The mole, symbol mol, is the SI unit of amount of substance. One mole substance contains exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
Luminous Intensity (cd)	The candela, symbol cd, is the SI unit of luminous intensity in given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, to be 683 when expressed in the unit lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr kg^{-1} $m^{-2}s^3$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

DIMENSIONS AND DIMENSIONAL FORMULA

All the physical quantities of interest can be derived from the base quantities. The power (exponent) of base quantity that enters into the expression of a physical quantity, is called the dimension of the quantity in that base. To make it clear, consider the physical quantity force.

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$= \text{Mass}\times\frac{\text{Length / Time}}{\text{Time}} = \text{Mass} \times \text{Length} \times (\text{Time})^{-2}$$

So the dimensions of force are 1 in mass, 1 in length and –2 in time. Thus

$$[\text{Force}] = MLT^{-2}$$

Similarly, energy has dimensional formula given by

$$[\text{Energy}] = ML^2T^{-2}$$

i.e. energy has dimensions 1 in mass, 2 in length and –2 in time.

Such an expression for a physical quantity in terms of base quantities is called dimensional formula

Physical quantity can be further of four types:

1. Dimensionless constant i.e. 1, 2, 3, π etc.
2. Dimensionless variable i.e. angle θ etc.
3. Dimensional constant i.e. G, h etc.
4. Dimensional variable i.e. F, v, etc.

Table: Units and dimensions of some physical quantities

Quantity	SI Unit	Dimension
Density	kg/m^3	M/L^3
Force	newton (N)	ML/T^2
Work	joule (J)(= N m)	ML^2/T^2
Energy	joule(J)	ML^2/T^2
Power	Watt (W) (= J/s)	ML^2/T^3
Momentum	kg m/s	ML/T
Gravitational constant	N m^2/kg^2	L^3/MT^2
Angular velocity	radian/s	T^{-1}
Angular acceleration	radian/s^2	T^{-2}
Angular momentum	kg m^2/s	ML^2/T
Moment of inertia	kg m^2	ML^2
Torque	N m	ML^2/T^2
Angular frequency	radian/s	T^{-1}
Frequency	hertz (Hz)	T^{-1}
Period	s	T
Surface Tension	N/m	M/T^2
Coefficient of viscosity	N s/m^2	M/LT
Wavelength	m	L
Intensity of wave	W/m^2	M/T^3
Temperature	Kelvin (K)	K

Quantity	SI Unit	Dimension
Specific heat capacity	J/kg K	L^2/T^2K
Stefan's constant	$W/m^2\ K^4$	M/T^3K^4
Heat	J	ML^2/T^2
Thermal conductivity	W/m-K	ML/T^3K
Current density	A/m^2	I/L^2
Electrical conductivity	$1/\Omega$ m(= mho/m)	I^2T^3/ML^3
Electric dipole moment	C m	LIT
Electric field	V/m (=N/C)	ML/IT^3
Potential (voltage)	Volt (V) (=J/C)	ML^2/IT^3
Electric flux	V m	ML^3/IT^3
Capacitance	Farad (F)	I^2T^4/ML^2
Electromotive force	Volt (V)	ML^2/IT^3
Resistance	ohm (Ω)	ML^2/I^2T^3
Permittivity of space	$C^2/N\ m^2$ (=F/m)	I^2T^4/ML^3
Permeability of space	N/A^2	ML/I^2T^2
Magnetic field	Tesla (T) (= Wb/m^2)	M/IT^2
Magnetic flux	Weber (Wb)	ML^2/IT^2
Magnetic dipole moment	$A\ m^2$	IL^2
Inductance	Henry (H)	ML^2/I^2T^2

DIMENSIONAL EQUATION

Whenever the dimension of a physical quantity is equated with its dimensional formula, we get a dimensional equation.

PRINCIPLE OF HOMOGENEITY

The magnitude of a physical quantity may be added or subtracted from each other only if they have the same dimension. Also the dimension on both sides of an equation must be same. This is called as principle of homogeneity.

Train Your Brain

Example 1: The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d.

Sol. The equation contains five terms. All of them should have the same dimensions. Since $[x]$ = length, each of the remaining four must have the dimension of length.

Thus, $[a]$ = length = L

$[bt]$ = L, or $[b] = LT^{-1}$

$[ct^2]$ = L, or $[c] = LT^{-2}$

and $[dt^3]$ = L or $[d] = LT^{-3}$

Example 2: Calculate the dimensional formula of energy from the equation $E = \frac{1}{2}mv^2$.

Sol. Dimensionally, E = mass × (velocity)^2, since $\frac{1}{2}$ is a number and has no dimension.

or, $[E] = M \times \left(\frac{L}{T}\right)^2 = ML^2T^{-2}$.

Example 3: Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ where s is the distance travelled by the particle. Determine dimensions of α.

Sol. $K = \alpha s^2 \Rightarrow \alpha = \frac{k}{52}$

$$[\alpha] = \frac{(M L^2T^{-2})}{(L^2)}$$

$[\alpha] = M^1\ L^0\ T^{-2}$

$[\alpha] = M\ T^{-2}$

Example 4: The position of a particle at time t, is given by the equation, $x(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively.

(*a*) $M^0\ L^1\ T^0$ and T^{-1} (*b*) $M^0\ L^1\ T^{-1}$ and T

(*c*) $M^0\ L^1\ T^{-1}$ and T^{-1} (*d*) $M^1\ L^1\ T^{-1}$ and LT^{-2}

Sol. (*c*) $[\alpha]\ [t] = M^0L^0T^0$ and $[v_0] = [x]\ [\alpha]$

$[\alpha] = M^0L^0T^{-1}$ $= M^0L^1T^{-1}$

Example 5: If $\frac{v + A\sqrt{s}}{Bt^2} = F$; find the dimension of A and B where F = force, v = velocity, s = displacement and t = time

Sol. Using principle of homogeneity; $A\sqrt{S} = [v]$

$$AL^{\frac{1}{2}} = LT^{-1}$$

$$\therefore A = L^{\frac{1}{2}}T^{-1}$$

Also, we can write, $B = \frac{v + A\sqrt{S}}{Ft^2}$

$$[B] = \frac{LT^{-1}}{(MLT^{-2})(T^2)} \Rightarrow [B] = M^{-1}L^0T^{-1}$$

Concept Application

1. If $\frac{v}{A} - Bu = at$, find the dimension of A and B where u, v = velocity, a = acceleration, t = time.

2. If $\frac{v}{t^2} = A\sqrt{s} - B$ find the dimension of A and B.

3. If $A \sin\left(\frac{B}{t^2}+C\right)$ is equation of displacement of a body. Find dimensions of A, B, C.

4. If displacement, $y = \frac{A}{t^2} - Bt^3$ find dimension of $(A \times B)$.

USES OF DIMENSIONAL ANALYSIS

To Check the Dimensional Correctness of a Given Physical Relation

It is based on principle of homogeneity, which states that a given physical relation is dimensionally correct if the dimensions of the various terms on either side of the relation are the same.

Remark:

- Powers are dimensionless
- $\sin\theta$, e^{θ}, $\cos\theta$, $\log\theta$ give dimensionless value and in above expression θ is dimensionless
- We can add or subtract quantity having same dimensions only.

Train Your Brain

Example 6: Check the accuracy of the relation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

for a simple pendulum using dimensional analysis.

Sol. From principle of homogeneity of dimension, the dimensions of LHS = The dimension of RHS

Now, $T = [M^0L^0T^1]$

The dimensions of

$$\text{RHS} = \left(\frac{\text{dimension of length}}{\text{dimension of acceleration}}\right)^{1/2}$$

($\because$ 2π is a dimensionless constant)

$$[\text{RHS}] = \left[\frac{L}{LT^{-2}}\right]^{1/2} = \left[T^2\right]^{1/2} = [T] = [M^0L^0T^1] = [\text{LHS}].$$

So, equation is correct.

Example 7: Check whether the given relation $\frac{Fv^2}{t} = \text{KE}$ is dimensionally correct? Where F = force, v = velocity and t = time?

Sol. $$[\text{LHS}] = \left|\frac{Fv^2}{t}\right| = \frac{MLT^{-2} \times L^2T^{-2}}{T} = \left[ML^3T^{-5}\right]$$

$$[\text{RHS}] = [\text{KE}] = \left[\frac{1}{2}mv^2\right] = \left[M \times L^2T^{-2}\right] = \left[ML^2T^{-2}\right]$$

$\therefore$ [LHS] $\neq$ [RHS], so the given relation in incorrect dimensionally.

Concept Application

5. Consider the following equation: $a = qvbt^2 + c\left(\frac{t}{x}\right)^2$, where a, b, c are constants (not necessarily dimensionless) and q, v, x and t represent charge, velocity, distance and time respectively. For the equation to be dimensionally correct,

   (*a*) $\left[\frac{a}{c}\right]^{\frac{1}{2}} = \left[LT^{-1}\right]$ (*b*) $\left[\frac{a}{c}\right]^{\frac{1}{2}} = \left[L^{-1}T^{-1}\right]$

   (*c*) $\left[\frac{b}{c}\right] = \left[L^{-3}A^{-1}\right]$ (*d*) $\left[\frac{a}{b}\right] = [LTA]$

6. Consider the statements below.

   (i) A dimensionally consistent equation is a physically correct equation.

   (ii) A dimensionally consistent equation may or may not be correct.

   (iii) A dimensionally inconsistent equation is an incorrect equation.

   (iv) A dimensionally inconsistent equation may or may not be incorrect.

   The correct statement(s) is/are

   (*a*) (i), (iii) and (iv) (*b*) (ii) and (iv)

   (*c*) only (iii) (*d*) (ii) and (iv)

To Establish a Relation Between Different Physical Quantities

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors by using principle of homogeneity.

Train Your Brain

Example 8: Find an expression for the time period T of a simple pendulum. The time period T may depend upon (i) mass m of the bob of the pendulum, (ii) length ℓ of pendulum, (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Sol. Let (i) $T \propto m^a$ (ii) $T \propto \ell^b$ (iii) $T \propto g^c$

Combining all the three factors, we get

$$T \propto m^a \ell^b g^c \quad \text{or} \quad T = Km^a \ell^b g^c \theta^d \qquad \text{...(i)}$$

where K is a dimensionless constant of proportionality.

Writing down the dimensions on either side of equation (i), we get

$[T] = [M^a][L^b][LT^{-2}]^c = [M^aL^{b+c}T^{-2c}]$

Comparing dimensions, $a = 0$, $b + c = 0$, $-2c = 1$

$\therefore\ a = 0, c = -1/2, b = 1/2$

From equation (i) $T = Km^0 \ell^{1/2} g^{-1/2}$

or $\quad T = K\left(\dfrac{\ell}{g}\right)^{1/2} = K\sqrt{\dfrac{\ell}{g}}$

The value of K, as found by experiment or mathematical investigation, comes out to be 2π.

$\therefore\ T = 2\pi\sqrt{\dfrac{\ell}{g}}$

Example 9: When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Sol. Suppose the formula is $F = k\,\eta^a r^b v^c$

Then, $MLT^{-2} = [ML^{-1}T^{-1}]^a L^b \left(\dfrac{L}{T}\right)^c$

$= M^a L^{-a+b+c} T^{-a-c}$

Equating the exponents of M, L and T from both sides,

$a = 1$

$-a + b + c = 1$

$-a - c = -2$

Solving these, $a = 1$, $b = 1$ and $c = 1$

Thus, the formula for F is $F = k\eta r v$.

Example 10: If P is the pressure of a gas and ρ is its density, then find the dimension of velocity in terms of P and ρ.

(*a*) $P^{1/2}\rho^{-1/2}$ (*b*) $P^{1/2}\rho^{1/2}$

(*c*) $P^{-1/2}\rho^{1/2}$ (*d*) $P^{-1/2}\rho^{-1/2}$

Sol. (*a*) **Method - I**

$[P] = [ML^{-1}T^{-2}]$ ...(i)

$[\rho] = [ML^{-3}]$ ...(ii)

Dividing eq. (i) by (ii)

$[P\rho^{-1}] = [L^2T^{-2}]$

$\Rightarrow [LT^{-1}] = [P^{1/2}\rho^{-1/2}]$

$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$

Method - II

$v \propto P^a \rho^b$

$v = kP^a \rho^b$

$[LT^{-1}] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b$

$\Rightarrow a = \dfrac{1}{2}, b = -\dfrac{1}{2}$ (Equating dimensions)

$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$

Example 11: Find relationship between speed of sound in a medium (v), the elastic constant (E) and the density of the medium (ρ).

Sol. Let the speed depends upon elastic constant and density according to the relation

$v \propto E^a \rho^b$

or $\quad v = KE^a\rho^b$ ...(i)

Where K is a dimensionless constant of proportionality.

Considering dimensions of the quantities

$[v] = M^0 L T^{-1}$

$[E] = \dfrac{[\text{stress}]}{[\text{strain}]} = \dfrac{[\text{force}]/[\text{area}]}{[\Delta\ell]/[\ell]} = \dfrac{[M^1L^1T^{-2}]/[L^2]}{[L^1]/[L^1]} = [M^1L^{-1}T^{-2}]$

$\therefore\ [E^a] = [M^a L^{-a} T^{-2a}]$

$[\rho] = [\text{mass}]/[\text{volume}] = [M]/[L^3] = [M^1L^{-3}T^0]$

$\therefore\ [\rho^b] = [M^b L^{-3b} T^0]$

Equating the dimensions of the LHS and RHS quantities of equation (i), we get

$[M^0 L^1 T^{-1}] = [M^a L^{-a} T^{-2a}] \neq [M^b L^{-3b} T^0]$

or $[M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}]$

Comparing the individual dimensions of M, L and T

$a + b = 0,$ ...(ii)

$-a - 3b = 1$, and ...(iii)

$-2a = -1$ ...(iv)

Solving we get

$a = \dfrac{1}{2}, b = -\dfrac{1}{2}$

Therefore the required relation is

$v = K\sqrt{\dfrac{E}{\rho}}$.

Example 12: Pressure (P) acting due to a fluid kept in a container depends on, weight of liquid (w), Area of cross-section of container (A) and density of fluid (ρ). Establish a formula of pressure (P).

Sol. According to the question

$P \propto w^x A^y \rho^z$

$P = K[w^x A^y \rho^z]$

Now writing the dimension of each quantity on either side.

$ML^{-1}T^{-2} = K\,[MLT^{-2}]^x\,[L^2]^y\,[ML^{-3}]^z$

$ML^{-1}T^{-2} = K\,[M]^{x+z}\,[L]^{x+2y-3z}\,[T]^{-2x}$

Now comparing the powers

For M, $1 = x + z$ ...(i)

L, $-1 = x + 2y - 3z$ ...(ii)

T, $-2 = -2x$ ...(iii)

$\Rightarrow$ Solving we get

$\therefore\ P = KwA^{-1}\rho^0$

Concept Application

7. If c is the velocity of light, h is Planck's constant and G is Gravitational constant are taken as fundamental quantities, then the dimensional formula of mass is

8. Taking frequency f, velocity (v) and Density (ρ) to be the fundamental quantities then the dimensional formula for momentum will be
   (a) $(\rho v^4 f^{-3})$ (b) $(\rho v^3 f^{-1})$
   (c) $(\rho v f^2)$ (d) $(\rho^2 v^2 f^2)$

9. If momentum (P), mass (M) and time (T) are chosen as fundamental quantities the dimensional formula for length is ________.
   (a) $(P^1T^1M^1)$ (b) $(P^1T^1M^2)$
   (c) $(P^1T^1M^{-1})$ (d) $(P^2T^2M^1)$

10. For the equation $F = A^a v^b d^c$ where F is force, A is area, v is velocity and d is density, with the dimensional analysis gives the following values for exponents.
   (a) $a = 1, b = 2, c = 1$
   (b) $a = 2, b = 1, c = 1$
   (c) $a = 1, b = 1, c = 2$
   (d) $a = 0, b = 1, c = 1$

To Convert Units of a Physical Quantity from One System of Units to Another

It is based on the fact that,

Numerical value × unit = constant

So on changing unit, numerical value will also gets changed. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

LIMITATIONS OF DIMENSIONAL ANALYSIS

(i) It supplies no information about dimensionless constants and the nature (vector and scalar) of physical quantities.

(ii) This method fails to derive the exact form of a physical relation, if a physical quantity depends upon more than three other mechanical physical quantities.

(iii) This method is applicable only if relation is of product type. It fails in the case of exponential and trigonometric relations.

(iv) It does not predict numerical correctness of formula.

Train Your Brain

Example 13: Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is $[F] = [M^1 L^1 T^{-2}]$

Therefore if n_1, u_1 and n_2, u_2 corresponds to SI and CGS unit respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{\text{kg}}{\text{g}}\right]\left[\frac{\text{m}}{\text{cm}}\right]\left[\frac{\text{s}}{\text{s}}\right]^{-2} = 1\times(1000)(100)(1) = 10^5$$

Example 14: A calorie is a unit of heat or energy and it equals about 4.2 J, where 1 J = 1 kg m^2/s^2. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β metre, the unit of time is γ second. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^2$ in terms of the new units.

Sol. 1 cal = 4.2 kg m^{2}s^{-2}

SI	**New system**
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$ m	$L_2 = \beta$ metre
$T_1 = 1$ s	$T_2 = \gamma$ second

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^aL^bT^c]$,

We find that $a = 1$, $b = 2$, $c = -2$

$$\text{Now, } n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ s}}{\gamma\text{ s}}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

Example 15: Young's modulus of steel is 19×10^{10} N/m^2. Express it in dyne/cm^2. Here dyne is the CGS unit of force.

Sol. The unit of Young's modulus is N/m^2.

This suggest that it has dimensions of $\dfrac{\text{Force}}{(\text{Distance})^2}$.

Thus, $[Y] = \dfrac{[F]}{L^2} = \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

N/m^2 is in SI units, so, 1 N/m^2 = (1 kg)(1m)$^{-1}$ (1s)$^{-2}$

and 1 dyne/cm^2 = (1g)(1cm)$^{-1}$ (1s)$^{-2}$ so, $\dfrac{1\text{ N/m}^2}{1\text{dyne/cm}^2}$

$$= \left(\frac{1\text{kg}}{1\text{g}}\right)\left(\frac{1\text{m}}{1\text{cm}}\right)^{-1}\left(\frac{1\text{s}}{1\text{s}}\right)^{-2} = 1000\times\frac{1}{100}\times1 = 10$$

or, 1 N/m^2 = 10 dyne/cm^2

or, 19×10^{10} N/m^2 = 19×10^{11} dyne/cm^2.

Example 16: The dimensional formula for viscosity of fluids is $\eta = M^1L^{-1}T^{-1}$. Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $\eta = M^1 L^{-1} T^{-1}$

1 CGS units = g cm^{-1} s^{-1}

1 SI units = kg m^{-1} s^{-1}

= $(1000 \text{ g})(100 \text{ cm})^{-1}$ s^{-1}

= 10 g cm^{-1} s^{-1}

Thus, 1 Poiseuilli = 10 poise

Concept Application

11. If minute is the unit of time, 10 ms^{-2} is the unit of acceleration and 100 kg is the unit of mass, the new unit of work in joule is

(*a*) 10^5 (*b*) 10^6

(*c*) 6×10^6 (*d*) 36×10^6

12. The magnitude of force is 100 N. What will be its value if the units of mass and time are doubled and that of length is halved?

(*a*) 25 (*b*) 100

(*c*) 200 (*d*) 400

13. The value of universal gravitational constant *G* in CGS system is 6.67×10^{-8} dyne cm^2 g^{-2}. Its value in SI system is

(*a*) 6.67×10^{-11} Nm^2 kg^{-2}

(*b*) 6.67×10^{-5} Nm^2 kg^{-2}

(*c*) 6.67×10^{-10} Nm^2 kg^{-2}

(*d*) 6.67×10^{-9} Nm^2 kg^{-2}

14. Which equation cannot be derived using dimensional analysis from the following? (*k* is a dimensionless constant and *x* does not necessarily represent distance. Remaining variables follow standard meaning)

(*a*) $x = 2at^2$ (*b*) $x = k\sin(\omega t)$

(*c*) $x = \dfrac{r^2\rho g}{\eta}$ (*d*) All of these

MEASUREMENT OF LENGTH

You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for lengths from 10^{-3} m to 10^2 m. A vernier callipers is used for lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths as less as 10^{-5} m. To measure lengths beyond these ranges, we make use of some special indirect methods.

Range of Lengths

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-15} m of the proton to the size of the order of 10^{26} m of the extent of the observable universe.

We also use certain special length units for short and large lengths. These are

1 fermi = 1 f = 10^{-15} m

1 angstrom = 1 Å = 10^{-10} m (It is used mainly in measuring wavelength of light)

1 astronomical unit = 1 AU (average distance of the Sun from the Earth) = 1.496×10^{11} m

1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of 3×10^8 m s^{-1} in 1 year)

1 parsec = 3.08×10^{16} m (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)

MEASUREMENT OF LARGE DISTANCES

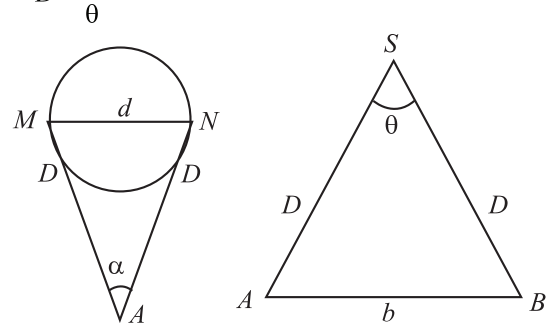
Parallax Method

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method. When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye *A* (closing the right eye) and then look at the pencil through your right eye *B* (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called parallax.

The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes. To measure the distance *D* of a far away planet *S* by the parallax method, we observe it from two different positions (observatories) *A* and *B* on the Earth, separated by distance *AB* = *b* at the same time as shown in figure. We measure the angle between the two directions along which the planet is viewed at these two points. The ΔASB in figure represented by symbol θ is called the parallax angle or parallactic angle.

As the planet is very far away, $\dfrac{b}{D} << 1$ and therefore, θ is very small. Then we approximately take *AB* as an arc of length *b* of a circle with centre at *S* and the distance *D* as the radius *AS* = *BS* so that *AB* = *b* = *D* θ where θ is in radians.

$$D = \frac{b}{\theta} \qquad \text{...(i)}$$



Having determined D, we can employ a similar method to determine the size or angular diameter of the planet. If d is the diameter of the planet and α the angular size of the planet (the angle subtended by d at the earth), we have

$\alpha = d/D$ ....(ii)

The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through the telescope. Since D is known, the diameter d of the planet can be determined using equation (ii).

ESTIMATION OF VERY SMALL DISTANCES

Size of a Molecule

To measure a very small size, like that of a molecule (10^{-8} m to 10^{-10} m), we have to adopt special methods. We cannot use a screw gauge or similar instruments. Even a microscope has certain limitations. An optical microscope uses visible light to 'look' at the system under investigation. As light has wave like features, the resolution to which an optical microscope can be used is the wavelength of light.

For visible light the range of wavelengths is from about 4000 Å to 7000 Å (1 angstrom = 1 Å = 10^{-10} m). Hence an optical microscope cannot resolve particles with sizes smaller than this. Instead of visible light, we can use an electron beam. Electron beams can be focused by properly designed electric and magnetic fields. The resolution of such an electron microscope is limited finally by the fact that electrons can also behave as waves.

The wavelength of an electron can be as small as a fraction of an angstrom. Such electron microscopes with a resolution of 0.6 Å have been built. They can almost resolve atoms and molecules in a material. In recent times, tunneling microscopy has been developed in which again the limit of resolution is better than an angstrom. It is possible to estimate the sizes of molecules.

ORDER OF MAGNITUDE

If a number P can be expressed as

$P = A \times 10^x$

where $0.5 \le A < 5$, then x is called order of magnitude of the number.

SI Prefixes: The magnitudes of physical quantities vary over a wide range. The mass of an electron is 9.1×10^{-31} kg and that of our earth is about 6×10^{24} kg. Standard prefixes for certain power of 10. Table shows these prefixes

Power of 10	Prefix	Symbol
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deka	da
–1	deci	d
–2	centi	c
–3	milli	m
–6	micro	μ
–9	nano	n
–12	pico	p
–15	femto	f

Train Your Brain

Example 17: The Sun's angular diameter is measured to be 1920''. The distance D of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun?

Sol. Sun's angular diameter α.

$= 1920''$

$= 1920 \times 4.85 \times 10^{-6}$ rad

$= 9.31 \times 10^{-3}$ rad

Sun's diameter $d = \alpha D$

$= (9.31 \times 10^{-3}) \times (1.496 \times 10^{11})$ m

$= 1.39 \times 10^{9}$ m

Example 18: The moon is observed from two diametrically opposite points on the earth with angle subtended = 1°54' given diameter of earth = 1.276×10^{7} m. Find the distance of moon from the earth.

Sol. From parallax method,

$$\theta = \frac{b}{D}$$

We have, θ = 1°54' = 60' + 54' = 114'

Also 1' = 2.91×10^{-4} rad

$\therefore \theta = 114 \times 2.91 \times 10^{-4} = 0.033$ rad

$$\therefore D = \frac{1.2760 \times 10^{7}}{0.033} = 3.8 \times 10^{8}\,\text{m}$$

Example 19: If the size of a nucleus (in the range of 10^{-15} to 10^{-11}m) is scaled up to the tip of a sharp pin, what roughly is the size of an atom? Assume tip of the pin to be in the range 10^{-5} m to 10^{-4} m.)

Sol. The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10^{-5} m and 10^{-4} m.

Thus we are scaling up by a factor of 1m. An atom roughly of size 10^{-10} m will be scaled up to a size of 1 m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.

Concept Application

15. Considering the distance between sun and moon to be 15×10^{10} m, the angular diameter of the sun as observed from the moon is close to (Take the radius of sun to be 7 × 108 m.)

(*a*) 214 rad (*b*) 107 rad
(*c*) 0.005 rad (*d*) 0.009 rad

16. The mass of an object is 75.2 × 104 kg. The order of magnitude of the mass is

(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

ERROR ANALYSIS IN EXPERIMENTS

Significant Figures or Digits

The significant figures (SF) in a measurement are the figures or digits that are known with certainty plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

1. **Rules to find out the number of significant figures:**

   I Rule: All the non-zero digits are significant E.g. 1984 has 4 SF.

   II Rule: All the zeros between two non-zero digits are significant. E.g. 10806 has 5 SF.

   III Rule: All the zeros to the left of first non-zero digit are not significant. E.g.00108 has 3 SF.

   IV Rule: If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. E.g. 0.002308 has 4 SF.

   V Rule: The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. E.g. 01.080 has 4 SF.

   VI Rule: The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant. E.g. m = 100 kg has 3 SF.

   VII Rule: When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$, each term has 3 SF only. (**Note:** It has 3 significant figure in each expression.)

2. **Rules for arithmetical operations with significant figures:**

   I Rule: In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. E.g. 12.587 – 12.5 = 0.087 = 0.1 (∵ second term contain lesser i.e. one decimal place)

   II Rule: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. E.g. 5.0 × 0.125 = 0.625 = 0.62

   To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in scientific notation (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number a is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).

   Note: The change in the unit of measurement of a quantity does not affect the number of SF. For example in 2.308 cm = 23.08 mm = 0.02308 m = 23080 μm each term has 4 SF.

ROUNDING OFF

To represent the result of any computation containing more than one uncertain digit, it is rounded off to appropriate number of significant figures.

Rules for rounding off the numbers:

I Rule : If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. 6.87≈ 6.9

II Rule : If the digit to be rounded off is less than 5, than the preceding digit is unaffected and is left unchanged. e.g. 3.94 ≈ 3.9

III Rule : If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. 14.35 ≈ 14.4 and 14.45 ≈ 14.4

Train Your Brain

Example 20: Write down the number of significant figures in the following.

(i) 165 (ii) 2.05
(iii) 34.000 m (iv) 0.005
(v) 0.02340 N m^{-1} (vi) 26900
(vii) 26900 kg

Sol. (i) 3 SF (following rule I)
(ii) 3 SF (following rules I and II)
(iii) 5 SF (following rules I and V)
(iv) 1 SF (following rules I and IV)
(v) 4 SF (following rules I, IV and V)
(vi) 3 SF (see rule VI)
(vii) 5 SF (see rule VI)

Example 21: The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Sol. Length (ℓ) = 4.234 m, Breadth (b) = 1.005 m

Thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore, area of the sheet = $2(\ell \times b + b \times t + t \times \ell)$

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) \text{m}^2$

$= 2(4.3604739) \text{ m}^2 = 8.720978 \text{ m}^2$

Since area can contain a maximum of 3 SF (Rule II of arithmetic operations) therefore, rounding off, we get

Area = 8.72 m^2

Like wise volume = $\ell \times b \times t$

$= 4.234 \times 1.005 \times 0.0201 \text{ m}^3 = 0.0855289 \text{ m}^3$

Since volume can contain 3 SF, therefore, rounding off,

Volume = 0.0855 m^3

Example 22: Find the number of significant figures in each

(i) 36.72 (ii) 0.003303

Sol. (i) It has four significant figures. All non-zero digits are significant.

(ii) Here first 3 zeros are insignificant but zeros between 3 are significant. So it has four significant figures.

Example 23: Round off following values to four significant figures.

(i) 36.879 (ii) 1.0084

(iii) 11.115 (iv) 11.1250

(v) 11.1251

Sol. The following values can be rounded off to four significant figures as follows:

(i) $36.879 \approx 36.88$ ($\because$ $9 > 5$ $\therefore 7$ is increased by one i.e. I Rule)

(ii) $1.0084 \approx 1.008$ ($\because$ $4 < 5$ $\therefore 8$ is left unchanged i.e. II Rule)

(iii) $11.115 \approx 11.12$ ($\because$ last 1 is odd it is increased by one i.e. III Rule)

(iv) $11.1250 \approx 11.12$ ($\because$ 2 is even it is left unchanged i.e. III Rule)

(v) $11.1251 \approx 11.13$ ($\because$ $51 > 50$ $\therefore$ 2 is increased by one i.e. I Rule)

Concept Application

17. The number of significant figures in 0.0006032 is

(*a*) 7 (*b*) 4 (*c*) 5 (*d*) 2

18. The radius of disc is 1.2 cm, its area according to idea of significant figures is ________

(*a*) 4.5216 cm^2 (*b*) 4.521 cm^2

(*c*) 4.52 cm^2 (*d*) 4.5 cm^2

19. The number of significant figures in 5.69×10^{15} kg is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

20. When 57.986 is rounded off to 4 significant figures, then it becomes ______.

(*a*) 58 (*b*) 57

(*c*) 57.90 (*d*) 57.99

ERRORS IN MEASUREMENT

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Classification of Errors

Errors may arise from different sources and are usually classified as follows:

Systematic or controllable errors: Systematic errors are the errors whose causes are known. They can be either positive or negative. Systematic errors can further be classified into three categories:

(i) Instrumental errors: These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.

(ii) Environmental errors: These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic and electrostatic fields.

(iii) Observational errors: These errors arise due to improper setting of the apparatus or carelessness in taking observations. This can be reduced by proper setting of the instrument before we start using it.

Random errors: These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely they can not be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random errors can be reduced by repeating the observation a large number of times and taking the arithmetic mean of all the observations. This mean value would be very close to the most accurate reading.

Note: If the number of observations is made n times then the random error reduces to $\left(\frac{1}{n}\right)$ times. E.g. If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$

For example:

(i) Reading instrument without proper initial settings.

(ii) Taking the observations wrongly without taking necessary precautions.

(iii) Exhibiting mistakes in recording the observations.

(iv) Putting improper values of the observations in calculations.

These errors can be minimised by increasing the sincerity and alertness of the observer.

REPRESENTATION OF ERRORS

Errors can be expressed in the following ways:

1. **Mean Absolute Error:** The mean of measurements of a physical quantity (a_1, a_2, ... a_n) is expressed as

$$a_m = \frac{a_1 + a_2 + ...+ a_n}{n} = \sum_{i=1}^{n} \frac{a_i}{n}$$

a_m is taken as the true value of a quantity if the same is not known.

The absolute error in the measurements are expressed as

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
....
$$\Delta a_n = a_m - a_n$$

Mean absolute error is defined as

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \sum_{i=1}^{n} \frac{\Delta a_i}{n}$$

Final result of measurement may be written as:

$$a = a_m \pm \overline{\Delta a}$$

2. **Relative Error or Fractional Error:** It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

Train Your Brain

Example 24: The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. Find (i) mean time period (ii) absolute error in each observation and percentage error.

Sol. (i) Mean time period is given by

$$\overline{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.62s$$

(ii) The absolute error in each observation is

2.62 – 2.63 = –0.01, 2.62 – 2.56

= 0.06, 2.62 – 2.42

= 0.20, 2.62 – 2.71

= –0.09, 2.62 – 2.80

= –0.18

Mean absolute error, $\overline{\Delta T} = \dfrac{\Sigma |\Delta T|}{5}$

$$= \frac{0.01 + 0.06 + 0.2 + 0.09 + 0.18}{5} = 0.11\text{sec}$$

∴ Percentage error is

$$= \frac{\overline{\Delta T}}{\overline{T}} \times 100 = \frac{0.11}{2.62} \times 100 = 4.2\%$$

Example 25: In an experiment the values of refractive indices of glass material recorded as 1.56, 1.53, 1.54, 1.45, 1.44 and 1.43 in repeated measurements. Find

(i) Mean value of μ (ii) Mean absolute error

(iii) Relative error (iv) Percentage error

Sol. (i) Mean value of μ

$$\overline{\mu} = \frac{\mu_1 + \mu_2 + ... + \mu_n}{n}$$

$$\overline{\mu} = \frac{1.56 + 1.54 + 1.53 + 1.45 + 1.44 + 1.43}{6} = 1.491 \approx 1.50$$

(ii) Absolute error $|\Delta \text{i}|$

$$|\Delta\mu_1| = |\overline{\mu} - \mu_1| = 0.06$$

$$|\Delta\mu_2| = |\overline{\mu} - \mu_2| = 0.04$$

$$|\Delta\mu_3| = |\overline{\mu} - \mu_3| = 0.03$$

$$|\Delta\mu_4| = |\overline{\mu} - \mu_4| = 0.05$$

$$|\Delta\mu_5| = |\overline{\mu} - \mu_5| = 0.06$$

$$|\Delta\mu_6| = |\overline{\mu} - \mu_6| = 0.07$$

∴ mean absolute error

$$|\Delta\overline{\mu}| = \frac{|\Delta\mu_1| + |\Delta\mu_2| + ...|\Delta\mu_6|}{6}$$

$$= \frac{0.06 + 0.04 + 0.03 + 0.05 + 0.06 + 0.07}{6}$$

= 0.051

∴ Reading = $\overline{\mu} + |\Delta\,\overline{\mu}|$ = 1.50 ± 0.05

(iii) Relative error = $\pm\dfrac{0.05}{1.50} = \pm\, 0.033$

(iv) Relative percentage error = Relative error × 100%

= ± 0.033 × 100%

= ± 3.3% (approx)

Concept Application

21. The diameter of a wire as measured by a screw gauge was found to be, 1.004 cm, 1.000 cm. Find the absolute error in first reading.

22. The diameter of a thick wire measured by a screw-gauge.

$D_1 = 1.006$ cm, $D_2 = 1.004$ cm, $D_3 = 1.002$ cm

Find mean absolute error and write the reading.

COMBINATION OF ERRORS

(i) **In Sum:** If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$.

Maximum fractional error in this case is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A+B} + \frac{\Delta B}{A+B}$$

i.e. when two physical quantities are added then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

(ii) **In Difference:** If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A-B} + \frac{\Delta B}{A-B}$$

(iii) **In Product:** If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where $\Delta Z/Z$ is known as fractional error.

(iv) **In Division:** If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(v) **In Power:** If $Z = A^n$ then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

In more general form if $Z = \frac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

Applications:

1. For a simple pendulum, $T \propto l^{1/2}$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

2. For a sphere, area and volume are given as

$$A = 4\pi r^2, V = \frac{4}{3}\pi r^3$$

So, relative errors in them are given as

$$\frac{\Delta A}{A} = 2\frac{\Delta r}{r} \text{ and } \frac{\Delta V}{V} = 3\frac{\Delta r}{r}$$

3. When two resistors R_1 and R_2 are connected

(i) In series $R_s = R_1 + R_2 \Rightarrow \Delta R_s = \Delta R_1 + \Delta R_2$

$$\Rightarrow \frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

(ii) In parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

Train Your Brain

Example 26: In an experiment of simple pendulum, the errors in the measurement of length of the pendulum (L) and time period (T) are 3% and 2% respectively. The maximum percentage error in the value of L/T^2 is

(*a*) 5% (*b*) 7% (*c*) 8% (*d*) 1%

Sol. (*c*) Maximum percentage in the value of L/T^2

$$= \frac{\Delta L}{L} \times 100\% + 2\frac{\Delta T}{T} \times 100\%$$

$= 3 + 2 \times 2 = 7\%$

Example 27: If $X = \frac{A^2\sqrt{B}}{C}$, then

(*a*) $\Delta X = \Delta A + \Delta B + \Delta C$

(*b*) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

(*c*) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$

(*d*) $\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

Sol. (*c*) $\because X = A^2 B^{1/2} C$

$$\therefore \frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$$

Example 28: A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity?

Sol. Given distance, $s = (13.8 \pm 0.2)$ m

and time $t = (4.0 \pm 0.3)$ s

Velocity $v = \frac{s}{t} = \frac{13.8}{4.0} = 3.45\,\text{m s}^{-1} = 3.5\,\text{m s}^{-1}$

$$\frac{\Delta v}{v} = \pm\left(\frac{\Delta s}{s} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right) = \pm\left(\frac{0.8 + 4.14}{13.8 \times 4.0}\right)$$

$\therefore \Delta v = \pm\, 0.0895 \times v = \pm\, 0.0895 \times 3.45 = \pm\, 0.3087 = \pm\, 0.31$

Hence $v = (3.5 \pm 0.31)\ \text{ms}^{-1}$

Percentage error in velocity

$$= \frac{\Delta v}{v} \times 100 = \pm 0.0895 \times 100 = \pm\, 8.95\% = \pm 9\%$$

Example 29: The heat generated in a circuit is given by $Q = I^2 Rt$, where I is current, R is resistance and t is time. If the percentage errors in measuring I, R and t are 2%, 1% and 1% respectively, then the maximum error in measuring heat will be

(*a*) 2 % (*b*) 3 % (*c*) 4 % (*d*) 6 %

Sol. (*d*) The percentage error in heat is given as

$$\frac{\Delta Q}{Q}\% = 2\frac{\Delta I}{I}\% + \frac{\Delta R}{R}\% + \frac{\Delta t}{t}\%$$

Substituting the values, maximum possible percentage error is

$= 2 \times 2\% + 1 \times 1\% + 1 \times 1\% = 6\%$

Example 30: Given: Resistance, $R_1 = (8 \pm 0.4)\ \Omega$ and Resistance, $R_2 = (8 \pm 0.6)\ \Omega$. What is the net resistance when R_1 and R_2 are connected in series?

(*a*) $(16 \pm 0.4)\ \Omega$ (*b*) $(16 \pm 0.6)\ \Omega$

(*c*) $(16 \pm 1.0)\ \Omega$ (*d*) $(16 \pm 0.2)\ \Omega$

Sol. (*c*) $R_1 = (8 \pm 0.4)\Omega$, $R_2 = (8 \pm 0.6)\ \Omega$

$R_s = R_1 + R_2 = (16 \pm 1.0)\Omega$

Example 31: The following observations were taken for determining surface tension of water by capillary tube method: Diameter of capillary, $D = 1.25 \times 10^{-2}$ m and rise of water in capillary, $h = 1.45 \times 10^{-2}$ m.

Taking $g = 9.80\ \text{ms}^{-2}$ and using the relation

$T = (rgh/2) \times 10^3\ \text{Nm}^{-1}$, what is the possible error in surface tension T?

(*a*) 2.4 % (*b*) 15 %

(*c*) 1.6 % (*d*) 0.15 %

Sol.(*c*) Given $T = (rgh/2) \times 10^3\ \text{Nm}^{-1}$,

$D = 1.25 \times 10^{-2}$ m, $h = 1.45 \times 10^{-2}$ m,

$g = 9.80\ \text{ms}^{-2}$

$$\frac{\delta T}{T} = \frac{\delta r}{r} + \frac{\delta h}{h} + \frac{\delta g}{g}$$

after applying the above values in this relation we get $\delta T\% = 1.6\%$

Concept Application

23. If the length of cylinder is measured to be 4.28 cm with an error of 0.01 cm. The percentage error in the measured length is approximately.

(*a*) 0.4% (*b*) 0.5% (*c*) 0.2 % (*d*) 0.1 %

24. The pressure on square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in measurement of force and length are respectively 4% and 2% then the maximum error in measurement of pressure is

(*a*) 1% (*b*) 2% (*c*) 6 % (*d*) 8%

25. The length and breadth of rectangular object are 25.2 cm and 16.8 cm respectively and have been measured to an accuracy of 0.1 cm. Relative error and percentage error in the area of the object are

(*a*) 0.01 and 1% (*b*) 0.02 and 2%

(*c*) 0.03 and 3% (*d*) 0.04 and 4%

26. The error in the measurement of length of a simple pendulum is 0.1% and error in the time period is 2%. The possible maximum error in the quantity having dimensional formula LT^{-2} is

(*a*) 1.1% (*b*) 2.1%

(*c*) 4.1% (*d*) 6.1%

MEASURING INSTRUMENTS

Measurement is an important aspect of physics. Whenever we want to know about a physical quantity, we take its measurement first of all. Instruments used in measurement are called measuring instruments.

Least Count: The least value of a quantity, which the instrument can measure accurately, is called the least count of the instrument.

Error: The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, has an error.

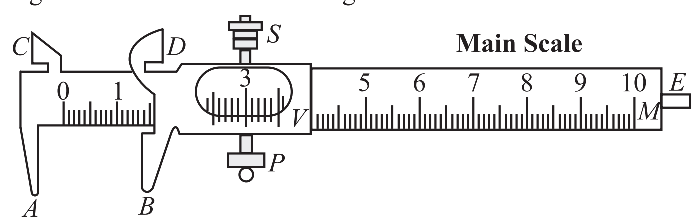
Accuracy and Precision: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

VERNIER CALLIPER

It is a device used to measure accurately upto 0.1 mm. There are two scales in the vernier calliper, vernier scale and main scale. The main scale is fixed whereas the vernier scale is movable along the main scale.

Its main parts are as follows:

Main scale: It consists of a steel metallic strip M, graduated in cm and mm at one edge and in inches and tenth of an inch at the other edge on same side. It carries fixed jaws A and C projected at right angle to the scale as shown in figure.



Vernier Scale: A vernier V slides on the strip M. It can be fixed in any position by screw S. It is graduated on both sides. The side of the vernier scale which slides over the mm side has ten divisions over a length of 9 mm, i.e., over 9 main scale divisions and the side of the vernier scale which slides over the inches side has 10 divisions over a length of 0.9 inch, i.e., over 9 main scale divisions.

Movable Jaws: The vernier scale carries jaws B and D projecting at right angle to the main scale. These are called movable jaws. When vernier scale is pushed towards A and C, then as B touches A, straight side of D will touch straight side of C. In this position, in case of an instrument free from errors, zeros of vernier scale will coincide with zeros of main scales, on both the cm and inch scales.

The object whose length or external diameter is to be measured is held between the jaws A and B, while the straight edges of C and D are used for measuring the internal diameter of a hollow object.

Metallic Strip: There is a thin metallic strip E attached to the back side of M and connected with vernier scale. When the jaws A and B touch each other, the edge of strip E touches the edge of M. When the jaws A and B are separated, E moves outwards. The strip E is used for measuring the depth of a vessel.

Determination of Least Count (Vernier Constant)

Note the value of the main scale division and count the number n of vernier scale divisions. Slide the movable jaw till the zero of vernier scale coincides with any of the mark of the main scale and find the number of divisions $(n - 1)$ on the main scale coinciding with n divisions of vernier scale. Then

$$n \text{ V.S.D.} = (n-1) \text{ M.S.D. or } 1 \text{ V.S.D.} = \left(\frac{n-1}{n}\right) \text{ M.S.D.}$$

$$1 \text{ V.C.} = \text{L.C.} = 1 \text{ M.S.D.} - 1 \text{ V.S.D.} = \left(1-\frac{n-1}{n}\right) \text{ M.S.D.}$$

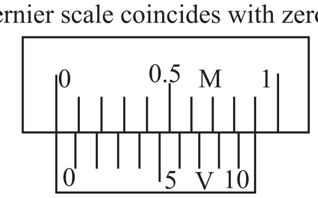
$$= \frac{1}{n} \text{ M.S.D.}$$

Determination of Zero Error and Zero Correction

For this purpose, movable jaw B is brought in contact with fixed jaw A.

One of the following situations will arise.

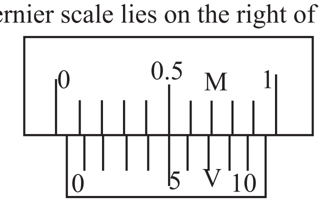
(i) Zero of Vernier scale coincides with zero of main scale



In this case, zero error and zero correction, both are nil.

Actual length = observed (measured) length.

(ii) Zero of vernier scale lies on the right of zero of main scale



Here 5[th] vernier scale division is coinciding with any main sale division and zero of vernier is ahead of N^{th} main scale division.

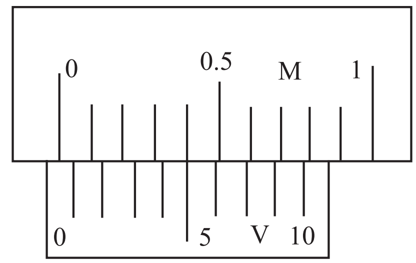
Hence, $N = 0$, $n = 5$, L.C. = 0.01 cm.

Zero error = $N + n \times$ (L.C.) = 0 + 5 × 0.01 = + 0.05 cm

Zero correction = – 0.05 cm.

Actual length will be 0.05 cm less than the observed (measured) length.

(iii) Zero of the vernier scale lies left of the main scale.



Here, 5[th] vernier scale division is coinciding with any main scale division.

In this case, zero of vernier scale lies on the right of –0.1 cm reading on main scale.

Hence, $N = -0.1$ cm, $n = 5$, L.C. = 0.01 cm

Zero error = $N + n \times$ (L.C.)

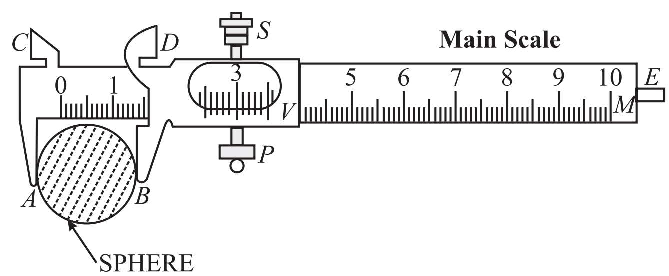
= – 0.1 + 5 × 0.01 = –0.05 cm.

Zero correction = +0.05 cm.

Actual length will be 0.05 cm more than the observed (measured) length.

Measurement Using Vernier Calliper

Let us measure the diameter of a small spherical/cylindrical body using a vernier calliper. Insert the object between the jaws as shown in the figure below.



If with the body between the jaws, the zero of vernier scale lies ahead of N^{th} division of main scale, then main scale reading (M.S.R.) = N.

If n^{th} division of vernier scale coincides with any division of main scale, then vernier scale reading (V.S.R.)

= $n \times$ (L.C.) (L.C. is least count of vernier calliper)

= $n \times$ (V.C.) (V.C. is vernier constant of vernier calliper)

Total reading,

T.R. = M.S.R. + V.S.R.

= $N + n \times$ (V.C.)

Train Your Brain

Example 32: The least count of vernier callipers is 0.1 mm. The main scale reading before the zero of the vernier scale is 10 and the zeroth division of the vernier scale coincides with the main scale division. Given that each main scale division is 1 mm, what is the measured value?

Sol. Length measured with vernier callipers

= reading before the zero of vernier scale + number of vernier divisions coinciding

with any main scale division × least count

= 10 mm + 0 × 0.1 mm = 10 mm = **1.00 cm**

Example 33: A vernier callipers has its main scale of 10 cm equally divided into 200 equal parts. Its vernier scale of 25 divisions coincides with 12 mm on the main scale. The least count of the instrument is

(*a*) 0.020 cm (*b*) 0.002 cm

(*c*) 0.010 cm (*d*) 0.001 cm

Sol. (*b*) 10 cm divided in 200 divisions, so

$$1 \text{ division} = \frac{10}{200} = 0.05 \text{ cm}.$$

Now, $25V = 24S$.

$$\Rightarrow V = \frac{24}{25}S$$

$$LC = S - V$$

$$= S - \frac{24}{25}S = \frac{1}{25}S$$

$\because$ $1S = 0.05$ cm, thus

$$\text{So, LC} = \frac{0.05}{25} = 0.002 \text{ cm}$$

Example 34: One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 10 divisions of vernier scale coincide with 8 small divisions of the main scale, the least count of the callipers is

(*a*) 0.005 cm (*b*) 0.05 cm

(*c*) 0.02 cm (*d*) 0.01 cm

Sol. (*c*) Now, $10V = 8S$

$$\Rightarrow V = \frac{8}{10}S.$$

Now, $LC = S - V$

$$= S - \frac{8}{10}S = \frac{2}{10}S = \frac{1}{5}S$$

But $1S = 0.1$ cm, thus

$$LC = \frac{0.1}{5} = 0.02 \text{ cm}$$

Concept Application

27. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), find the least count of the instrument (in min.).

28. The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found that, the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm. If 24^{th} division of the vernier scale exactly coincides with one of the main scale division, the diameter of the cylinder is

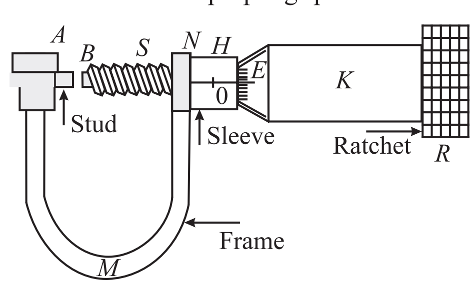
(*a*) 5.112 cm (*b*) 5.124 cm

(*c*) 5.136 cm (*d*) 5.148 cm

29. The main scale of vernier calliper is calibrated in mm and 19 divisions of main scale are equal in length to 20 divisions of vernier scale. In measuring the diameter of a cylinder by this instrument, the main scale reads 35 division and 4th division of vernier scale coincides with a M.S.D. Find LC in (cm).

SCREW GAUGE

This instrument (shown in figure) works on the principle of micrometer screw. It consists of a U-shaped frame *M*. At one end of it is fixed a small metal piece *A* of gun metal. It is called stud and it has a plane face. The other end *N* of *M* carries a cylindrical hub *H*. The hub extends few millimetre beyond the end of the frame. On the tubular hub along its axis, a line is drawn known as reference line. On the reference line graduations are in millimetre and half millimeter depending upon the pitch of the screw. This scale is called linear scale or pitch scale. A nut is threaded through the hub and the frame *N*. Through the nut moves a screw *S* made of gun metal. The front face *B* of the screw, facing the plane face *A*, is also plane. A hollow cylindrical cap *K* is capable of rotating over the hub when screw is rotated. It is attached to the right hand end of the screw. As the cap is rotated the screw either moves in or out. The bevelled surface *E* of the cap *K* is divided into 50 or 100 equal parts. It is called the circular scale or head scale. Right hand end *R* of *K* is milled for proper grip.



In most of the instrument the milled head R is not fixed to the screw head but turns it by a spring and ratchet arrangement such that when the body is just held between faces A and B, the spring yields and milled head R turns without moving in the screw.

In an accurately adjusted instrument when the faces A and B are just touching each other, the zero marks of circular scale and pitch scale exactly coincide.

Determination of Least Count of Screw Gauge

Note the value of linear (pitch) scale division. Rotate screw to bring zero mark on circular (head) scale on reference line. Note linear scale reading i.e. number of divisions of linear scale uncovered by the cap.

Now give the screw a few known number of rotations. (one rotation completed when zero of circular scale again arrives on the reference line). Again note the linear scale reading. Find difference of two readings on linear scale to find distance moved by the screw.

Then, pitch of the screw

$$= \frac{\text{Distance moved in } n \text{ rotation}}{\text{No. of full rotation } (n)}$$

Now count the total number of divisions on circular (head) scale. Then, least count is

$$\text{LC} = \frac{\text{Pitch}}{\text{Total number of divisions on the circular scale}}$$

The least count is generally 0.001 cm.

Determination of Zero Error and Zero Correction

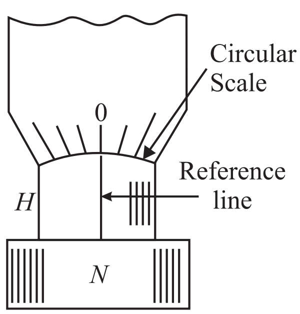
For this purpose, the screw is rotated forward till plane face B of the screw just touches the fixed plane face A of the stud and edge of cap comes on zero mark of linear scale. Screw gauge is held keeping the linear scale vertical with its zero downwards.

One of the following three situations will arise.

(i) Zero mark of circular scale comes on the reference line

In this case, zero error and zero correction, both are nil.

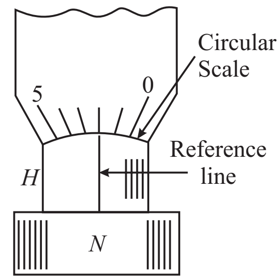
Actual thickness = Observed (measured) thickness.



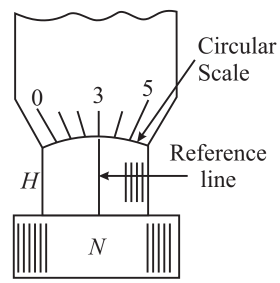
(ii) Zero mark of circular scale remains on right of reference line and does not cross it.

Here 2nd division on circular scale comes on reference line. Zero reading is already 0.02 mm. It makes zero error = + 0.02 mm and zero correction = – 0.02 mm.

Actual thickness will be 0.02 mm less than the observed (measured) thickness.



(iii) Zero mark of circular scale goes to left on reference line after crossing it. Here zero of circular scale has advanced from reference line by 3 divisions on circular scale. A backward rotation by 0.03 mm will make reading zero. It makes zero error = –0.03 mm & zero correction = + 0.03 mm.



Actual thickness will be 0.03 mm more than the observed (measured) thickness.

Measurement Using Screw Gauge

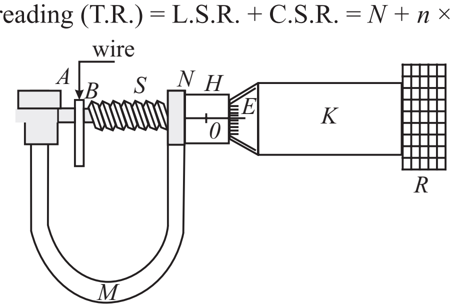
To measure diameter of a given wire using a screw gauge:

1. Determine of least count of screw gauge
2. If with the wire between plane faces A and B, the edge of the cap lies ahead of N^{th} division of linear scale, then, linear scale reading (L.S.R.) = N

   If nth division of circular scale lies over reference line, then, circular scale reading (C.S.R.) = n × (L.C.)

   (L.C. is least count of screw gauge)

   Total reading (T.R.) = L.S.R. + C.S.R. = $N + n$ × (L.C.)



Train Your Brain

Example 35: In four complete revolutions of the cap, the distance traveled on the pitch scale is 2 mm. If there are fifty divisions on the circular scale, then

(i) Calculate the pitch of the screw gauge.

(ii) Calculate the least count of the screw gauge.

Sol. Pitch of screw = Linear distance traveled in one revolution $P = \frac{2\text{ mm}}{4} = 0.5\text{ mm} = 0.05\text{ cm}$

$$\text{Least count} = \frac{\text{Pitch}}{\text{no. of divisions in circular scale}}$$

$$= \frac{0.05}{50} = 0.001\text{ cm}$$

Example 36: The pitch of a screw gauge is 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the pitch scale (or main scale) and thirty fourth division coincides with the reference line. Calculate the thickness of the metal plate.

Sol. Pitch of screw = 0.5 mm., LC = $\frac{0.5}{50}$ = 0.01 mm.

Thickness = (5 × 0.5 + 34 × 0.01) mm
= (2.5 + 0.34) = 2.84 mm

Concept Application

30. In a screw gauge, five complete rotations of the screw causes it to move a linear distance of 0.25 cm. There are 100 circular divisions. Four main scale divisions and 30 circular scale divisions is obtained as the thickness of the wire measured by this instrument. Assuming negligible zero error, find the thickness of wire (in cm).

31. The thickness of a marker measured using a screw gauge whose LC = 0.001 cm, comes out to be 0.802 cm. The percentage error in the measurement would be

(*a*) 0.125% (*b*) 2.43%
(*c*) 4.12% (*d*) 2.14%

Short Notes

Fundamental Quantity	Derived Quantity
The physical quantities which do not depend on any other physical quantities for their measurements. E.g., Mass, Length, Time Temperature, current, luminous Intensity & mole	Those quantities which can be expressed in terms of fundamental/base quantities. E.g., Angle, speed or velocity Acceleration, force etc.,

System of Units

(***a***) **FPS System:** Here length is measured in foot, mass in pounds and time in second.

(***b***) **CGS System:** In this system, L is measured in cm, M is measured in g and T is measured in sec.

(***c***) **MKS System:** In this system, L is measured in metre, M is measured in kg and T is measured in sec.

Principle of Homogeneity

According to this, the physical quantities having same dimension can be added or subtracted with each other and for a given equation, dimensions of both sides must be same.

For eg, in equation $F = A\sqrt{m} + \frac{B}{v} + C$,

all the three parts of R.H.S have same dimension as force on L.H.S.

Dimensions

The fundamental or base quantities along with their powers needed to express a physical quantity is called dimensions
E.g.: [F] = [MLT^{-2}] is dimension of force.

Usage of Dimensional Analysis

(i) To check the correctness of a given formula.

(ii) To establish relation between quantities dimensionally.

(iii) To convert the value of a quantity from one system of units to other system.

Limitations of Dimensional Analysis

(i) It does not predict the numerical value or number associated with a physical quantity in a relation

eg, $v = \frac{u}{3} + \frac{1}{5}at$ & $v = u + at$

Both are dimensionally valid.

(ii) It does not derive any relations involving trigonometric, logarithmic or exponential functions

E.g. $P = P_0 e^{-bt^2}$ cannot be derived dimensionally.

(iii) It does not give any information about dimensionally constants or nature of a quantity (vector/scalar) associated with a relation.

Significant Figure or Digits

1. Rules to find out the number of significant figures:

I Rule: All the non-zero digits are significant E.g. 1984 has 4 SF.

II Rule: All the zeros between two non-zero digits are significant. E.g. 10806 has 5 SF.

III Rule: All the zeros to the left of first non-zero digit are not significant. E.g.00108 has 3 SF.

IV Rule: If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. E.g. 0.002308 has 4 SF.

V Rule: The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. E.g. 01.080 has 4 SF.

VI Rule: The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant. E.g. m = 100 kg has 3 SF.

VII Rule: When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$, each term has 3 SF only.

2. **Rules for arithmetical operations with significant figures:**

I Rule: In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. E.g. 12.587 – 12.5 = 0.087 = 0.1 ($\because$ second term contain lesser i.e. one decimal place)

II Rule: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. E.g. 5.0 × 0.125 = 0.625 = 0.62.

Rounding Off

Rules for rounding off the numbers:

I Rule: If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. 6.87≈ 6.9

II Rule: If the digit to be rounded off is less than 5, than the preceding digit is unaffected and is left unchanged. e.g. 3.94 ≈ 3.9

III Rule: If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. 14.35 ≈ 14.4 and 14.45 ≈ 14.4

Representation of Errors

1. Mean absolute error is defined as

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n} = \sum_{i=1}^{n} \frac{\Delta a_i}{n}$$

Final result of measurement may be written as:

$$a = a_m \pm \overline{\Delta a}$$

2. **Relative Error or Fractional Error:** It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

Combination of Errors

(i) **In Sum:** If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$.

Maximum fractional error in this case is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A+B} + \frac{\Delta B}{A+B}$$

(ii) **In Difference:** If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A-B} + \frac{\Delta B}{A-B}$$

(iii) **In Product:** If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(iv) **In Division:** If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(v) **In Power:** If $Z = A^n$ then $\dfrac{\Delta Z}{Z} = n\dfrac{\Delta A}{A}$

In more general form if $Z = \dfrac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

To Find Smaller Measurements

Vernier Calliper

(i) Least count: Suppose movable Jaw is slided till the zero of vernier scale coincides with any of the mark of the main scale.

Let, n V.S.D = $(n - 1)$ MSD $\Rightarrow 1\text{VSD} = \left(\dfrac{n-1}{n}\right)$ M.S.D

$\therefore$ Vernier constant = 1 M.S.D – 1 V.S.D

$$= \left[1 - \frac{n-1}{n}\right]\text{MSD} = \frac{1}{n}\text{MSD}$$

(ii) Total reading = MSR + VSR

= MSR + n ×VC

where MSR = Main scale reading

VC = Vernier constant i.e. least count

n = n^{th} division of vernier scale coinciding with main scale.

Screw Gauge

This instrument works on the principle of micro-meter screw. It is used to measure very small (mm) measurements. It is provided with linear scale and a circular scale.

(i) Pitch of the screw gauge

$$= \frac{\text{Distance moved in } n\text{-rotation of cir-scale}}{\text{No. of full-rotation}}$$

(ii) L.C $= \dfrac{\text{Pitch}}{\text{Total number of division on the circular scale}}$

(iii) Total Reading (T.R) = L.S.R + C.S.R

L.S.R = Linear scale Reading = N where

C.S.R = Circular Scale Reading = n × L.C

If n^{th} division of circular scale coincides with the linear scale line, then

$\therefore$ Total reading = $N + n \times$ (L.C)

Solved Examples

1. $\int \frac{xdx}{\sqrt{2ax-x^2}} = a^n \sin^{-1}\left[\frac{x}{a}-1\right]$. The value of n is

   (a) 0 (b) –1

   (c) 1 (d) None of these

   Use dimensional analysis to solve the problem.

Sol. (c) $\int \frac{x\,dx}{\sqrt{2ax-x^2}} = a^n \sin^{-1}\left[\frac{x}{a}-1\right]$

Denominator $2ax - x^2$ must have the dimension of $[x]^2$

($\because$ we can add or subtract only if quantities same diemnsion)

$\therefore \left[\sqrt{2ax-x^2}\right] = [x]$

Also, dx has the dimension of $[x]$

$\therefore \frac{x\ \ dx}{\sqrt{2ax-x^2}}$ is having dimension L

Equating the dimension of L.H.S. and R.H.S. we have

$[a^n] = M^0L^1T^0$

$\{\because \sin^{-1}\left(\frac{x}{a}-1\right)$ must be dimensionless$\}$

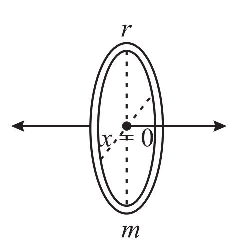
2. In the formula; $P = \frac{nRT}{V-b}\ e^{-\frac{a}{RTV}}$ find the dimensions of 'a' and 'b', where P = pressure, n = no. of moles, T = temperature, V = volume and R = universal gas constant.

Sol. $[b] = [V] = L^3$

$[a] = [RTV] = \frac{[PV]}{[n]}[V] = \frac{ML^2T^{-2}L^3}{mol}$ ($\because nRT = PV$)

$= ML^5T^{-2}mol^{-1}$.

3. A particle is performing SHM along the axis of a fixed ring. Due to gravitational force, its displacement at time t is given by $x = a \sin\omega t$.



In this equation ω is found to depend on radius of the ring (r), mass of the ring (m) and gravitational constant (G). Using dimensional analysis, find the expression of ω in terms of m, r and G.

Sol. Let $\omega = KM^ar^bG^c$ where K is a dimensionless constant

Writing the dimension of both the sides and equating them we have,

$T^{-1} = M^aL^b(M^{-1}L^3T^{-2})^c$

$= M^{a-c}L^{b+3c}T^{-2c}$

Equating the exponents,

$-2c = -1$ or $c = 1/2$;

$b + 3c = 0$ or $-3c = b = -\frac{3}{2}$;

$a - c = 0,\ c = a = +\frac{1}{2}$

Thus the required equation is $\omega = K\sqrt{\frac{Gm}{r^3}}$.

4. Using screw gauge, the observation of the diameter of a wire are 1.324, 1.326, 1.334, 1.336 cm respectively. Find the average diameter, the mean error, the relative error and % error.

Sol. Average diameter:

$$\bar{D} = \frac{\Sigma(D)}{N} = \frac{1.324+1.326+1.334+1.336}{4} = 1.330$$

$\Delta D_1 = 1.324 - 1.330 = -0.006$

$\Delta D_2 = 1.326 - 1.330 = -0.004$

$\Delta D_3 = 1.334 - 1.330 = 0.004$

$\Delta D_4 = 1.336 - 1.330 = 0.006$

Means absolute error:

$$\Delta\bar{D} = \frac{|\Delta D_1| + |\Delta D_2| + |\Delta D_3| + |\Delta D_4|}{4}$$

$$= \frac{0.006+0.004+0.004+0.006}{4} = \frac{0.020}{4} = 0.005\,\text{cm}$$

Relative error $= \frac{\Delta\bar{D}}{D} = \frac{0.005}{1.330} = 0.004$

% error $= \frac{\Delta\bar{D}}{D}\times 100 = 0.4\%$

5. If a tuning fork of frequency (f_0) 340 Hz and tolerance 1% is used in resonance column method [$v = 2f_0(\ell_2 - \ell_1)$], the first and the second resonance are measured at $\ell_1 = 24.0$ cm and $\ell_2 = 74.0$ cm. Find maximum permissible error in speed of sound.

Sol. $v = 2f_0(\ell_2 - \ell_1)$

$$\Rightarrow \left(\frac{\Delta v}{v}\right)_{max} = \frac{\Delta f_0}{f_0} + \frac{\Delta\ell_1 + \Delta\ell_2}{\ell_2 - \ell_1} = \frac{1}{100} + \frac{0.1+0.1}{74-24} = 1.4\%$$

6. Side of a cube is measured with the help of vernier calliper. Main scale reading is 10 mm and vernier scale reading is 1. It is known that 9 M.S.D. = 10 V.S.D. Mass of the cube is 2.735 g. Find density of the cube upto appropriate significant figure

Sol. Least count = 1 M.S.D. – 1 V.S.D. = 1 M.S.D. – $\frac{9}{10}$ M.S.D.

$= \frac{1}{10}$ M.S.D. $= \frac{1}{10} \times 1$ mm

Least count = 0.1 mm

Length of side of cube = M.S.R. + V.S.R. × least count

= 10 + 1 × 0.1

= 10.1 mm

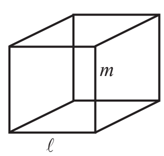
$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{2.735}{(10.1)^3} = 0.0026546$$

Using significant figures the correct answer would be 0.00265 (with 3 significant figures).

7. For a cubical block, error in measurement of sides is + 1% and error in measurement of mass is + 2%, then maximum possible error in density is

(*a*) 1% (*b*) 5% (*c*) 3% (*d*) 7%

Sol. (***b***)



$$\text{Density, } \rho = \frac{m}{V} = \frac{m}{\ell^3}$$

$$\text{Given: } \frac{\Delta m}{m} = \pm 2\% = \pm 2\times10^{-2}, \quad \frac{\Delta \ell}{\ell} = \pm 1\% = \pm 1\times10^{-2}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta\ell}{\ell} = 2\times10^{-2} + 3\times10^{-2} = 5\times10^{-2} = 5\%$$

8. The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier callipers as 1.00 cm. The least count of the meter scale and vernier callipers are 0.1 cm and 0.01 cm respectively (Obvious from readings). Maximum permissible error in area measurement is

(*a*) +0.2 cm^2 (*b*) +0.1 cm^2 (*c*) +0.3 cm^2 (*d*) Zero

Sol. (***a***) $A = \ell b = 10.0 \times 1.00 = 10.00$

$$\text{Now, } \frac{\Delta A}{A} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}$$

$$\frac{\Delta A}{10.00} = \frac{0.1}{10.0} + \frac{0.01}{1.00}$$

$$\Rightarrow \Delta A = 10.00\left(\frac{1}{100} + \frac{1}{100}\right) = 10.00\left(\frac{2}{100}\right) = \pm 0.2\,\text{cm}^2$$

9. To estimate 'g' (from $g = 4\pi^2 \frac{L}{T^2}$), error in measurement of L is + 2% and error in measurement of T is + 3%. The error in estimated 'g' will be

(*a*) +8% (*b*) +6% (*c*) +3% (*d*) +5%

Sol. (***a***) $g = 4\pi\frac{\ell}{T^2}$

$$\frac{\Delta\ell}{\ell} = 2\% = \pm 2\times10^{-2}$$

$$\frac{\Delta T}{T} = +3\% = \pm 3\times10^{-2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T} = 2\times10^{-2} + 2\times3\times10^{-2}$$

$$= 8\times10^{-2} = \pm 8\%$$

10. The mass of a ball is 1.76 kg. The mass of 25 such balls is

(*a*) 0.44×10^3 kg (*b*) 44.0 kg

(*c*) 44 kg (*d*) 44.00 kg

Sol. (***b***) $m = 1.76$ kg, $M = 25\,m = 25 \times 1.76 = 44.0$ kg

Mass of one unit has three significant figures and it is just multiplied by a pure number (magnified). So result should also have three significant figures.

11. To measure the diameter of a wire, a screw gauge is used. In a complete rotation, spindle of the screw gauge advances by 1/2 mm and its circular scale has 50 division. The main scale is graduated to 1/2mm. If the wire is put between the jaws, 4 main scale divisions are clearly visible and 10 divisions of circular scale co–inside with the reference line. The resistance of the wire is measured to be (10Ω ± 1%). Length of the wire is measured to be 10 cm using a scale of least count 1mm. Maximum permissible error in resistivity measurement is

Sol. L.C. of screw gauge $= \frac{1/2\text{ mm}}{50} = 0.01\text{mm}$

Diameter of wire as measured by the screw gauge

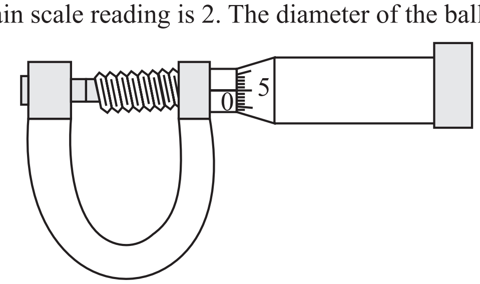
$$= 4\times\frac{1}{2} + 10\times0.01 = 2.1\text{ mm}$$

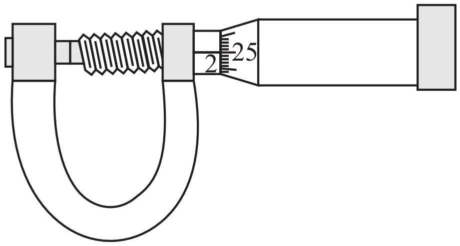
$$\text{Since, } \rho = \frac{R\pi\frac{d^2}{4}}{\ell}$$

$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{\Delta R}{R} + \frac{2\Delta d}{d} + \frac{\Delta\ell}{\ell}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{1}{100} + \frac{2\times0.01}{2.1} + \frac{1}{100} = 2.9\%$$

12. The number of circular divisions on the shown screw gauge is 50. It moves 0.5 mm on main scale for one complete rotation. Main scale reading is 2. The diameter of the ball is





(*a*) 2.25 mm (*b*) 2.20 mm
(*c*) 1.20 mm (*d*) 1.25 mm

Sol. (*c*) Least count $= \dfrac{0.5}{50} = 0.01$

Zero error = 5 × 0.01 = 0.05 mm

Actual measurement = 2 × 0.5 + 25 × $\dfrac{0.5}{50}$ – 0.05

= 1 mm + 0.25 mm – 0.05 mm = 1.20 mm.

13. A student performs an experiment for determination of $g\left(=\dfrac{4\pi^2\ell}{T^2}\right)$, $L = 1\text{m}$ and he commits an error of ΔL. For For the takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 sec. For which of the following data, the measurement of g will be most accurate ?

(*a*) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 20$
(*b*) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 50$
(*c*) $\Delta L = 0.5$, $\Delta T = 0.01$, $n = 50$
(*d*) $\Delta L = 0.1$, $\Delta T = 0.05$, $n = 50$

Sol. (*d*) $\dfrac{\Delta g}{g} = \dfrac{\Delta L}{L} + 2\dfrac{\Delta T}{T}$

In option (*d*) error in Δg is minimum and number of repetition of measurement are maximum. In this case the error in g is minimum.

14. Student I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and /or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	Length of the pendulum	Number of oscillations (n)	Total time for n oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_{I}, E_{II} and E_{III} are the percentage error in g, i.e. $\left(\dfrac{\Delta g}{g}\times 100\right)$

(*a*) $E_{\text{I}} = 0$ (*b*) E_{I} is minimum
(*c*) $E_{\text{I}} = E_{\text{II}}$ (*d*) E_{III} is maximum

Sol. (*b*) The least count of length $\Delta\ell = 0.1$ cm

The least count of length $\Delta t = 0.1$ s

% error of $g = \dfrac{\Delta g}{g}\times 100$

Now, $T = 2\pi\sqrt{\dfrac{\ell}{g}} \Rightarrow g = \dfrac{4\pi^2\ell}{T^2}$ where $T = \dfrac{1}{n}$

So, $g = \dfrac{4\pi^2\ell}{t^2}n^2$ $\left(T = \dfrac{t}{n}\right)$

$\Rightarrow \dfrac{\Delta g}{g} = \dfrac{\Delta\ell}{\ell} + 2\dfrac{\Delta t}{t}$

For student I, $\left(100\times\dfrac{\Delta g}{g}\right)_{\text{I}} = \left(\dfrac{0.1}{64.0} + \dfrac{2\times 0.1}{128.0}\right)\times 100$

$E_{\text{I}} = \dfrac{0.2}{64.0}\times 100 = \dfrac{20}{64}$

For student II,

$\left(100\times\dfrac{\Delta g}{g}\right)_{\text{II}} = \left(\dfrac{0.1}{64.0} + 2\times\dfrac{0.1}{64.0}\right)\times 100$

$E_{\text{II}} = \dfrac{0.3}{64.0}\times 100 = \dfrac{30}{64}$

For student III,

$\left(100\times\dfrac{\Delta g}{g}\right)_{\text{III}} = \left(\dfrac{0.1}{20.0} + \dfrac{0.1}{18.0}\right)\times 100 = \dfrac{19}{18}$

$E_{\text{III}} = \left(\dfrac{0.1}{20.0} + \dfrac{0.1}{18.0}\right)\times 100 = \dfrac{19}{18}$

$\Rightarrow E_{\text{I}}$ is least.

15. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(*a*) 0.9% (*b*) 2.4%
(*c*) 3.1% (*d*) 4.2%

Sol. (*c*) Least count $= \dfrac{0.5}{50} = 0.01\text{mm}$

Diameter of ball D = 2.5 mm + (20)(0.01)

$\Rightarrow D = 2.7$ mm

$$\rho = \frac{M}{\text{Vol}} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3} \Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{\Delta M}{M} + 3\frac{\Delta D}{D}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = 2\% + 3\left(\frac{0.01}{2.7}\right)\times 100\%$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = 3.1\%$$

Exercise-1 (Topicwise)

UNITS, SYSTEM OF UNITS

1. Which of the following is not the unit of time?
(*a*) Solar day (*b*) Parallactic second
(*c*) Leap year (*d*) Lunar month

2. A unit less quantity
(*a*) Never has a non zero dimension
(*b*) Always has a non zero dimension
(*c*) May have a non zero dimension
(*d*) Does not exit

3. Which of the following is not the name of a physical quantity?
(*a*) Kilogram (*b*) Impulse
(*c*) Energy (*d*) Density

4. Parsec is a unit of
(*a*) Time (*b*) Angle
(*c*) Distance (*d*) Velocity

5. Which of the following system of units is not based on the unit of mass, length and time alone
(*a*) FPS (*b*) SI
(*c*) CGS (*d*) MKS

6. In the S.I. system the unit of energy is:
(*a*) Erg (*b*) Calorie
(*c*) Joule (*d*) Electron volt

7. The SI unit of the universal gravitational constant G is
(*a*) $N\ m\ kg^{-2}$ (*b*) $N\ m^2\ kg^{-2}$
(*c*) $N\ m^2\ kg^{-1}$ (*d*) $N\ m\ kg^{-1}$

8. Surface tension has unit of
(*a*) Joule m^2 (*b*) Joule m^{-2}
(*c*) Joule m^{-1} (*d*) Joule m^3

9. The specific resistance has the unit of:
(*a*) ohm/m (*b*) ohm/m^2 (*c*) ohm m^2 (*d*) ohm m

10. The unit of magnetic moment is:
(*a*) Amp m^2 (*b*) Amp m^{-2}
(*c*) Amp m (*d*) Amp m^{-1}

11. The SI unit of the universal gas constant R is
(*a*) Erg K^{-1} mol^{-1} (*b*) Watt K^{-1} mol^{-1}
(*c*) Newton K^{-1} mol^{-1} (*d*) Joule K^{-1} mol^{-1}

12. The SI unit of Stefan's constant is
(*a*) $Ws^{-1}\ m^{-2}\ K^{-4}$ (*b*) $J\ s\ m^{-1}\ K^{-1}$
(*c*) $J\ s^{-1}\ m^{-2}\ K^{-1}$ (*d*) $W\ m^{-2}\ K^{-4}$

DIMENSION, FINDING DIMENSIONAL FORMULA

13. In SI unit the angular acceleration has unit of
(*a*) $N\ m\ kg^{-1}$ (*b*) $m\ s^{-2}$ (*c*) rad s^{-2} (*d*) $N\ kg^{-1}$

14. The angular frequency is measured in rad s^{-1}. Its exponent in length are
(*a*) – 2 (*b*) –1 (*c*) 0 (*d*) 2

15. $[M\ L\ T^{-1}]$ are the dimensions of
(*a*) Power (*b*) Momentum
(*c*) Force (*d*) Couple

16. What are the dimensions of Boltzmann's constant?
(*a*) $MLT^{-2}K^{-1}$ (*b*) $ML^2T^{-2}K^{-1}$
(*c*) M^0LT^{-2} (*d*) $M^0L^2T^{-2}K^{-1}$

17. A pair of physical quantities having the same dimensional formula is
(*a*) Angular momentum and torque
(*b*) Torque and energy
(*c*) Force and power
(*d*) Power and angular momentum

18. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(*a*) Torque (*b*) Surface tension
(*c*) Viscosity (*d*) Stress

19. The dimension of work done per unit mass per unit relative density would be equivalent to dimension of
(*a*) $(\text{Acceleration})^2$ (*b*) $(\text{Velocity})^2$
(*c*) $(\text{Force})^2$ (*d*) $(\text{Torque})^2$

20. Which of the following is dimension of intensity?
(*a*) MT^{-3} (*b*) $M^{-1}L^2T^{-2}$ (*c*) $ML^{1/2}T^{-1}$ (*d*) None

21. The dimension of $\left[\dfrac{h}{G}\right]$ where h = Planck's constant and G = gravitational constant is
(*a*) $ML^{-1}T^2$ (*b*) $M^{-1}L^3T^2$ (*c*) $M^2L^{-1}T$ (*d*) $M^3L^0T^{-1}$

22. A dimensionless quantity:
(*a*) Never has a unit (*b*) Always has a unit
(*c*) May have a unit (*d*) Does not exit

PRINCIPLE OF HOMOGENEITY OF DIMENSION

23. Force F is given in terms of time t and distance x by $F = A \sin C\,t + B \cos D\,x$ Then the dimensions of A/B and C/D are given by
(*a*) $MLT^{-2}, M^0L^0T^{-1}$ (*b*) $MLT^{-2}, M^0L^{-1}T^0$
(*c*) $M^0L^0T^0, M^0L^1T^{-1}$ (*d*) $M^0L^1T^{-1}, M^0L^0T^0$

24. The equation for the velocity of sound in a gas states that $v = \sqrt{\gamma k_b \frac{T}{m}}$. Velocity v is measured in m/s. γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. What are the units for the Boltzmann constant, k_b?
(a) kg m^2 s^{-2} K^{-1} (b) kg m^2 s^2 K
(c) kg m/s K^{-2} (d) kg m^2 s^{-2} K

25. A wave is represented by $y = a \sin (At - Bx + C)$ where A, B, C are constants and t is in seconds & x is in metre. The dimensions of A, B, C are
(a) T^{-1}, L, $M^0L^0T^0$ (b) T^{-1}, L^{-1}, $M^0L^0T^0$
(c) T, L, M (d) T^{-1}, L^{-1}, M^{-1}

26. If $v = \sqrt{\frac{\gamma P}{\rho}}$, then the dimensions of γ are (P is pressure, ρ is density and v is speed of sound has their usual dimension)
(a) $M^0L^0T^0$ (b) $M^0L^0T^{-1}$
(c) $M^1L^0T^0$ (d) $M^0L^1T^0$

27. Consider the equation $\frac{d}{dt}\left[\int \vec{F} \cdot d\vec{s}\right] = A\left[\vec{F} \cdot \vec{P}\right]$. Then dimension of A will be (where $\vec{F}$ = force, $d\vec{s}$ = small displacement, t = time and $\vec{P}$ = linear momentum)
(a) $M^oL^oT^o$ (b) $M^1L^oT^o$
(c) $M^{-1}L^oT^o$ (d) $M^oL^oT^{-1}$

28. If $F = \frac{A}{\sqrt{m}} + B$ where F = Force, m = Mass.
Then dimension of $[A \times B]$ is,
(a) $M^{5/2}L^2T^{-4}$ (b) $M^{2/5}L^2T^{-1}$
(c) $M^2L^{2/5}T^{-1}$ (d) $M^{-1}L^{2/5}T^{-2}$

29. If $v = At^3 + \frac{B}{m}$, where m = mass, v = velocity and t = time.
Then dimension of A in the given equation would be
(a) LT^{-2} (b) L^2T^{-3}
(c) L^3T^{-2} (d) LT^{-4}

APPLICATION OF DIMENSIONAL ANALYSIS

Deriving New Relation

30. The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as (where k is a dimensionless constant)
(a) $v^2 = k\lambda^{-1} g^{-1} \rho^{-1}$ (b) $v^2 = k g \lambda$
(c) $v^2 = k g \lambda \rho$ (d) $v^2 = k \lambda^3 g^{-1} \rho^{-1}$

31. Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross–sectional area of the stream (A). The expression of the force should be
(a) ρAv (b) ρAv^2
(c) $\rho^2 Av$ (d) $\rho A^2 v$

32. If velocity (v), frequency (f) and mass (m) are taken as fundamental quantity. How energy (E) may be described using above quantity.
(a) Kvf^2m (b) Kv^2f^0m
(c) Kvf^2m^0 (d) $Kv^{1/2}f^{-1}m^2$

33. If P = power delivered by a motor is dependent of force = F, velocity = v and density of material = ρ. Then power may be proportional to
(a) $F^2v\rho^{-1}$ (b) $Fv^2\rho$
(c) $Fv\rho^0$ (d) None of these

34. The velocity of a freely falling body changes as $g^p h^q$ where g = acceleration due to gravity and h is height. The value of p and q are
(a) $1, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{2}$
(c) $\frac{1}{2}, 1$ (d) 1, 1

APPLICATION OF DIMENSIONAL ANALYSIS

To Convert from One System of Unit

35. One watt-hour is equivalent to
(a) 6.3×10^3 Joule (b) 6.3×10^{-7} Joule
(c) 3.6×10^3 Joule (d) 3.6×10^{-3} Joule

36. The pressure of 10^6 dyne/cm^2 is equivalent to
(a) 10^5 N/m^2 (b) 10^6 N/m^2
(c) 10^7 N/m^2 (d) 10^8 N/m^2

37. Consider $\rho = 2$ g/cm^3. Convert it into MKS system
(a) $2 \times 10^{-3} \frac{kg}{m^3}$ (b) $2 \times 10^3 \frac{kg}{m^3}$
(c) $4 \times 10^3 \frac{kg}{m^3}$ (d) $2 \times 10^6 \frac{kg}{m^3}$

38. The density of mercury is 13600 kg m^{-3}. Its value of CGS system will be
(a) 13.6 g cm^{-3} (b) 1360 g cm^{-3}
(c) 136 g cm^{-3} (d) 1.36 g cm^{-3}

39. Force in CGS system is 20 N. Its value in SI unit will be
(a) 20×10^5 (b) 20×10^{-5}
(c) 200 N (d) 2×10^{-3} N

40. If in a system of unit mass is measured in α kg, length in β m and time in γ sec. Find the value of 100 joule in the above system.
(a) $100\ \alpha^{-1}\beta^{-2}\gamma^2$ (b) $100\ \alpha^{-2}\beta^{-1}\gamma^{-2}$
(c) $100\ \alpha\beta^{-2}\gamma$ (d) $1000\ \alpha^{-2}\beta^2\gamma^{-1}$

ERRORS IN MEASUREMENT

41. Which of the following measurements is most accurate?
(a) 9×10^{-2} m (b) 90×10^{-3} m
(c) 900×10^{-4} m (d) 0.090 m

42. A system takes 70.40 second to complete 20 oscillations. The time period of the system is:

(*a*) 3.52 s (*b*) 35.2×10 s
(*c*) 3.520 s (*d*) 3.5200 s

43. The percentage error in the measurement of mass and speed are 1% and 2% respectively. What is the percentage error in kinetic energy?

(*a*) 5% (*b*) 2.5% (*c*) 3% (*d*) 1.5%

44. Number 15462 when rounded off to numbers to three significant digits will be

(*a*) 15500 (*b*) 155
(*c*) 1546 (*d*) 150

45. Value of expression $\frac{25.2 \times 1374}{33.3}$ will be

(All the digits in this expression are significant.)

(*a*) 1040 (*b*) 1039 (*c*) 1038 (*d*) 1041

46. Value of 24.36 + 0.0623 + 256.2 will be (considering rules of significant digits)

(*a*) 280.6 (*b*) 280.8
(*c*) 280.7 (*d*) 280.6224

47. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of the kinetic energy obtained by measuring mass and speed?

(*a*) 11% (*b*) 8%
(*c*) 5% (*d*) 1%

48. The random error in the arithmetic mean of 100 observations is x; then random error in the arithmetic mean of 400 observations would be

(*a*) $4x$ (*b*) $\frac{1}{4}x$ (*c*) $2x$ (*d*) $\frac{1}{2}x$

49. What is the number of significant figures in 0.310×10^3

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 6

50. Error in the measurement of radius of a sphere is 1%. The error in the calculated value of its volume is

(*a*) 1% (*b*) 3%
(*c*) 5% (*d*) 7%

51. The mean time period of second's pendulum is 2.00 s and mean absolute error in the time period is 0.05 s. To express maximum estimate of error, the time period should be written as

(*a*) (2.00 ± 0.01) s (*b*) $(2.00 + 0.025)$ s
(*c*) (2.00 ± 0.05) s (*d*) (2.00 ± 0.10) s

52. The unit of percentage error is

(*a*) Same as that of physical quantity
(*b*) Different from that of physical quantity
(*c*) Percentage error is unit less
(*d*) Errors have got their own units which are different from that of physical quantity measured

53. The decimal equivalent of 1/20 upto three significant figures is

(*a*) 0.0500 (*b*) 0.05000
(*c*) 0.0050 (*d*) 5.0×10^{-2}

54. A thin copper wire of length l metre increases in length by 2% when heated through 10 ºC. What is the percentage increase in area when a square copper sheet of length l metre is heated through 10 ºC?

(*a*) 4% (*b*) 8%
(*c*) 16% (*d*) 32 %

55. The length and breadth of a rectangle are 20 ± 0.2 cm and 10 ± 0.1 cm. Find the percentage error in area would be

(*a*) 1% (*b*) 2%
(*c*) 3% (*d*) 4%

56. The mass rate of flow of liquid through a pipe is given by $\frac{dm}{dt} = \rho AV$, where ρ = density, A = area of cross-section and $v \rightarrow$ velocity. The readings of area, $A = 10 \pm 0.1$ m^2 and $v = 30 \pm 0.3$ m/sec. Find the percentage error in measurement of mass flow rate.

(*a*) 5% (*b*) 4%
(*c*) 8% (*d*) 3.5%

MEASURING INSTRUMENTS

57. In a vernier calliper, ten smallest divisions of the vernier scale are equal to nine smallest division on the main scale. If the smallest division on the main scale is half millimeter, then the vernier constant is:

(*a*) 0.5 mm (*b*) 0.1 mm
(*c*) 0.05 mm (*d*) 0.005 mm

58. A vernier calliper has 20 divisions on the vernier scale, which coincide with 19 on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of

(*a*) 0.5 mm (*b*) 1 mm (*c*) 2 mm (*d*) 1/4 mm

59. A vernier calliper having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then

(*a*) $n = 10, m = 0.5$ cm (*b*) $n = 9, m = 0.4$ cm
(*c*) $n = 10, m = 0.8$ cm (*d*) $n = 10, m = 0.2$ cm

60. The pitch of a screw gauge is 0.05 cm. In how many revolutions of hollow cylinder the Screw will advance 0.35 cm in the straight line?

(*a*) 7 (*b*) 10
(*c*) 15 (*d*) 14

61. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm, 1.20 mm. The percentage error in measurement is $\pm m$.71%. Find m.

(*a*) 2.20 (*b*) 2.71
(*c*) 2.85 (*d*) 3.52

Exercise-2 (Learning Plus)

1. The unit of impulse is the same as that of
(*a*) Moment force
(*b*) Linear momentum
(*c*) Rate of change of linear momentum
(*d*) Force

2. Which of the following is not the unit of energy?
(*a*) Watt-hour (*b*) Electron-volt
(*c*) N × m (*d*) kg × m/sec^2

3. If a and b are two physical quantities having different dimensions then which of the following can denote a new physical quantity?
(*a*) $a + b$ (*b*) $a - b$ (*c*) a/b (*d*) $e^{a/b}$

4. The time dependence of a physical quantity $P = P_0 \exp(-\alpha t^2)$ where α is a constant and t is time The constant α
(*a*) Will be dimensionless
(*b*) Will have dimensions of T^{-2}
(*c*) Will have dimensions as that of P
(*d*) Will have dimensions equal to the dimension of P multiplied by T^{-2}

5. Which pair of following quantities has dimensions different from each other?
(*a*) Impulse and linear momentum
(*b*) Plank's constant and angular momentum
(*c*) Moment of inertia and moment of force
(*d*) Young's modulus and pressure

6. The product of energy and time is called action. The dimensional formula for action is same as that for
(*a*) Power (*b*) Angular energy
(*c*) Force × velocity (*d*) Impulse × distance

7. What is the physical quantity whose dimensions are $[M L^2 T^{-2}]$?
(*a*) Kinetic energy (*b*) Pressure
(*c*) Momentum (*d*) Power

8. If E, M, J and G denote energy, mass, angular momentum and gravitational constant respectively, then $\dfrac{EJ^2}{M^5G^2}$ has the dimensions of
(*a*) Length (*b*) Angle (*c*) Mass (*d*) Time

9. The position of a particle at time 't' is given by the relation $x(t) = \dfrac{V_0}{\alpha}[1 - e^{-\alpha t}]$ where V_0 is a constant and $\alpha > 0$. The dimensions of V_0 and α are respectively.
(*a*) $M^0L^1T^0$ and T^{-1} (*b*) $M^0L^1T^0$ and T^{-2}
(*c*) $M^0L^1T^{-1}$ and T^{-1} (*d*) $M^0L^1T^{-1}$ and T^{-2}

10. If force (F) is given by $F = Pt^{-1} + \alpha t$, where t is time. The unit of P is same as that of
(*a*) Velocity (*b*) Displacement
(*c*) Acceleration (*d*) Momentum

11. When a wave traverses a medium, the displacement of a particle located at x at time t is given by $y = a\sin(bt - cx)$ where a, b and c are constants of the wave. The dimensions of b are the same as those of
(*a*) Wave velocity (*b*) Amplitude
(*c*) Wavelength (*d*) Wave frequency

12. In a book, the answer for a particular question is expressed as $b = \dfrac{ma}{k}\left[\sqrt{1+\dfrac{2kl}{ma}}\right]$ here m represents mass, a represents accelerations, l represents length. The unit of b should be
(*a*) m/s (*b*) m/s^2 (*c*) meter (*d*) /sec

13. $\alpha = \dfrac{F}{v^2}\sin(\beta t)$ (here v = velocity, F = force, t = time). Find the dimension of α and β
(*a*) $\alpha = [M^1L^1T^0]$, $\beta = [T^{-1}]$
(*b*) $\alpha = [M^1L^1T^{-1}]$, $\beta = [T^1]$
(*c*) $\alpha = [M^1L^1T^{-1}]$, $\beta = [T^{-1}]$
(*d*) $\alpha = [M^1L^{-1}T^0]$, $\beta = [T^{-1}]$

14. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be
(*a*) FT^2 (*b*) $F^{-1} A^2 T^{-1}$
(*c*) FA^2T (*d*) AT^2

15. If the unit of length is micrometer and the unit of time is microsecond, the unit of velocity will be
(*a*) 100 m/s (*b*) 10 m/s
(*c*) micrometers (*d*) m/s

16. In a certain system of units, 1 unit of time is 5 s, 1 unit of mass is 20 kg and unit of length is 10m. In this system, one unit of power will correspond to
(*a*) 16 watts (*b*) 1/16 watts
(*c*) 25 watts (*d*) None of these

17. If the unit of force is 1 kilonewton, the length is 1 km and time is 100 second, what will be the unit of mass?
(*a*) 1000 kg (*b*) 10 kg (*c*) 10000 kg (*d*) 100 kg

18. The units of length, velocity and force are doubled. Which of the following is the correct change in the other units?
(*a*) Unit of time is doubled
(*b*) Unit of mass is doubled
(*c*) Unit of momentum is doubled
(*d*) Unit of energy is doubled

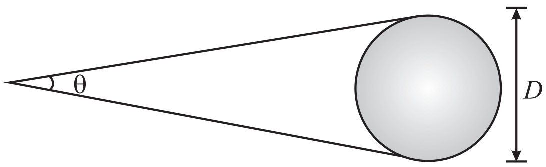
19. If the units of force and that of length are doubled, the unit of energy will be:

(*a*) 1/4 times (*b*) 1/2 times
(*c*) 2 times (*d*) 4 times

20. If the units of M, L are doubled then the unit of kinetic energy will become

(*a*) 2 times (*b*) 4 times
(*c*) 8 times (*d*) 16 times

21. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the fact that the moon is about 384000 km away to find the approximate diameter of the moon (D).



(*a*) 192000 km (*b*) 3350 km
(*c*) 1600 km (*d*) 1920 km

22. The least count of a stop watch is 0.2 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The percentage error in the time period is

(*a*) 16% (*b*) 0.8 %
(*c*) 1.8 % (*d*) 8 %

23. The dimensions of a cuboidal block measured with a vernier calliper having least count of 0.1 mm is 5 mm × 10 mm × 5 mm. The maximum percentage error in measurement of volume of the block is

(*a*) 5 % (*b*) 10 % (*c*) 15 % (*d*) 20 %

24. An experiment measures quantities x, y, z and then t is calculated from the data as $t = \frac{xy^2}{z^3}$. If percentage errors in x, y and z are respectively 1%, 3%, 2%, then percentage error in t is:

(*a*) 10 % (*b*) 4 %
(*c*) 7 % (*d*) 13 %

25. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is

(*a*) (0.34 ± 0.02) cm
(*b*) (0.17 ± 0.02) cm
(*c*) (0.17 ± 0.01) cm
(*d*) (0.34 ± 0.01) cm

26. The mass of a ball is 1.76 kg. The mass of 25 such balls is

(*a*) 0.44×10^3 kg (*b*) 44.0 kg
(*c*) 44 kg (*d*) 44.00 kg

27. Two resistors R_1 (24 ± 0.5) Ω and R_2 (8 ± 0.3) Ω are joined in series. The equivalent resistance is

(*a*) 32 ± 0.33 Ω (*b*) 32 ± 0.8 Ω
(*c*) 32 ± 0.2 Ω (*d*) 32 ± 0.5 Ω

28. The pitch of a screw gauge is 0.5 mm and there are 100 divisions on its circular scale. The instrument reads +2 divisions when nothing is put in-between its jaws. In measuring the diameter of a wire, there are 8 divisions on the main scale and 83rd division coincides with the reference line. Then the diameter of the wire is

(*a*) 4.05 mm (*b*) 4.405 mm
(*c*) 3.05 mm (*d*) 1.25 mm

29. The pitch of a screw gauge having 50 divisions on its circular scale is 1 mm. When the two jaws of the screw gauge are in contact with each other, the zero of the circular scale lies 6 division below the line of graduation. When a wire is placed between the jaws, 3 linear scale divisions are clearly visible while 31st division on the circular scale coincide with the reference line. The diameter of the wire is

(*a*) 3.62 mm (*b*) 3.50 mm
(*c*) 3.5 mm (*d*) 3.74 mm

30. The smallest division on the main scale of a vernier calipers is 1 mm, and 10 vernier divisions coincide with 9 main scale divisions. While measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.0 and 2.1 cm and the fifth division of the vernier scale coincide with a scale division. Then diameter of the sphere is

(*a*) 2.05 cm (*b*) 3.05 cm
(*c*) 2.50 cm (*d*) None of these

31. A satellite orbiting around the Earth, its orbital velocity (v_0) is found to depend on mass of Earth M, radius of earth R and universal gravitational constant G. The expression for orbital velocity is proportional to

(*a*) $G^{-1}M^1R^{-1}$ (*b*) $G^1M^1R^{-1}$
(*c*) $G^{1/2}M^{1/2}R^{-1/2}$ (*d*) None of these

32. The vernier constant of vernier calliper is 0.1 mm. and it has zero error of (– 0.05) cm. While measuring the diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The corrected diameter will be $n \times 10^{-2}$ cm. Find n.

(*a*) 2180 (*b*) 2220 (*c*) 2160 (*d*) 2200

33. In a particular system, the unit of length, mass and time are chosen to be 10 cm, 10g and 0.1s respectively. The unit of force in this system will be equivalent to

(*a*) 1/10 N (*b*) 1 N
(*c*) 10 N (*d*) 100 N

34. In sub-atomic physics, one often associates a characteristic wavelength λ with a particle of mass m. If $\hbar = \frac{h}{2\pi}$ (h being Planck's constant) and c is the speed of light, which of the following expression is most likely to be correct one ? (Use formula $E = hf$)

(*a*) $\lambda = \frac{hc}{m}$ (*b*) $\lambda = \frac{\hbar}{mc^2}$
(*c*) $\lambda = \frac{m\hbar}{c}$ (*d*) $\lambda = \frac{\hbar}{mc}$

35. If the mass, time and work are taken as fundamental physical quantities then dimensional formula of length is

(*a*) $[M^{1/2}\,T^1\,W^{-1/2}]$ (*b*) $[M^{-1/2}\,T^1\,W^{1/2}]$

(*c*) $[M^{-1}\,T^2\,W]$ (*d*) None of these

36. Given that $\ln(\alpha/p\beta) = \alpha z/k_B\theta$ where p is pressure, z is distance, k_B is Boltzmann constant and θ is temperature. The dimensions of β are (Useful formula: Energy = k_B × temperature)

(*a*) $L^0M^0T^0$ (*b*) $L^1M^{-1}T^2$

(*c*) $L^2\,M^0\,T^0$ (*d*) $L^{-1}M^1T^{-2}$

37. The dependence of g on geographical latitude at sea level is given by $g = g_0(1 + \beta\sin^2\phi)$ where ϕ is the latitude angle and β is a dimensionless constant. If Δg is the error in the measurement of g, then the error in measurement of latitude angle is

(*a*) zero (*b*) $\Delta\phi = \dfrac{\Delta g}{g_0\beta\sin(2\phi)}$

(*c*) $\Delta\phi = \dfrac{\Delta g}{g_0\beta\cos(2\phi)}$ (*d*) $\Delta\phi = \dfrac{\Delta g}{g_0}$

38. Let $y = l^2 - \dfrac{l^3}{z}$ where $l = 2.0 \pm 0.1$, $z = 1.0 \pm 0.1$, then the value of y is given by

(*a*) -4 ± 2.4 (*b*) -4 ± 1.6

(*c*) -4 ± 0.8 (*d*) None of these

39. If measured time period are $T_1 = 8.01$ s and $T_2 = 8.41$ s by a student who used stop watch having least count = 0.01 sec, then find best reported time (in sec) is

(*a*) 8.2 ± 0.2 (*b*) 8.41 ± 0.2

(*c*) 8.21 ± 0.01 (*d*) 8.41 ± 0.01

40. Assume pressure (P), length (L) and velocity (V) are fundamental quantities. The dimension of coefficient of viscosity (η) is

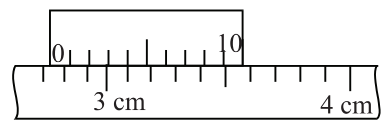
(*a*) $[PL^{-1}V]$ (*b*) $[PLV^{-1}]$ (*c*) $[P^{-2}LV^{-1}]$ (*d*) $[PL^{-1}V^{-2}]$

41. A gas bubble from an explosion under water oscillates with a period T proportional to $p^a d^b E^c$, where p is static pressure, d is the density of water, E is the total energy of the explosion. The values of c, b, a respectively will be

(*a*) $-\dfrac{5}{6}, \dfrac{1}{2}, \dfrac{1}{3}$ (*b*) $-\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{4}$

(*c*) $\dfrac{1}{3}, \dfrac{1}{2}, -\dfrac{5}{6}$ (*d*) $-\dfrac{5}{6}, \dfrac{1}{3}, \dfrac{1}{2}$

42. The diagram shows part of the vernier scale on a pair of calipers.



Which reading is correct ?

(*a*) 2.74 cm (*b*) 3.10 cm (*c*) 3.26 cm (*d*) 3.64 cm

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. Choose the correct statement(s).

(*a*) All quantities may be represented dimensionally in terms of the base quantities.

(*b*) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.

(*c*) The dimension of a base quantity in other base quantities is always zero.

(*d*) The dimension of a derived quantity is never zero in any base quantity.

2. The dimensions $ML^{-1}T^{-2}$ may correspond to

(*a*) Work done by a force

(*b*) Linear momentum

(*c*) Pressure

(*d*) Energy per unit volume

3. A student curiously picks up Resnick and Halliday and tries to understand the answers given at the end of the book using his new found knowledge of physics. He marks four answers. In which of them A has the same units as that of angular momentum ?

Useful formula $L = r \times p$, $\omega = \dfrac{2\pi}{T}$, c is velocity of light, $\tau = r \times F$, E represent energy, l represent length and f represents frequency.

(*a*) $\dfrac{1}{2}mv^2 = Af$

(*b*) $\Delta p = \dfrac{2A\omega}{v}\sqrt{1-\dfrac{v^2}{c^2}}$

(*c*) $\sin(kl) = k\sqrt{\dfrac{A^2}{2mE}}$

(*d*) $t = Al$

4. The quantity/quantities that does/do not have mass in its/their dimensions (when we take standard 7 quantities as fundamental) is/are

(*a*) Specific heat

(*b*) Latent heat

(*c*) Luminous intensity

(*d*) Mole

5. The power for the hovering helicopter depends on the factors, its linear size, the density of air and g × density of the helicopter as

$P \propto$ (linear size)^x (density of air)y (g × density of helicopter)^z

where g is acceleration due to gravity.

[Given: Power = $ML^{-2}T^{-3}$, Linear size = L, Density = ML^{-3}, g × density = $ML^{-2}T^{-2}$]

(*a*) The value of y is –1/2.

(*b*) The ratio of power output of engines of two hovering having helicopters when linear size of one helicopter is one fourth of linear size of other and all other parameters are same is 64.

(*c*) The ratio of the power output to hover a helicopter on the earth and on the imaginary planet, where g_{planet} = g/4 and density of air on imaginary planet is same as that of earth is 8.

(*d*) If helicopter is to hover at higher altitudes (like Siachin glacier) then we need less powerful engine.

6. A student taking a quiz finds on a reference sheet the two equations $\nu = 1/T$ and $v = \sqrt{T/\mu}$

(μ = mass/length, torque = $r \times F$ and rest of symbols have usual notations.)

He has forgotten what T represents in each equation. Use dimensional analysis to determine the units required for T in each equation.

(*a*) In first equation T represents tension.

(*b*) In first equation T represents time.

(*c*) In second equation T represents torque.

(*d*) In second equation T represents tension.

7. A chunk of unknown rock masses 38.254 ± 0.003 grams and has a volume of 15.0 cm^3.

(*a*) The density of the rock is 2.55 g/cm^3.

(*b*) The absolute error in density is 0.02 g/cm^3.

(*c*) The relative error in density is 0.007.

(*d*) The number of significant figure in density is 3.

8. Using screw gauge the diameter of the wire is found to be 5.00 mm. The length of wire is measured by using a scale and is found to be 50.0 cm. If mass of wire is measured as 25 g, then mark the correct statement(s) (Take π = 3.14).

(*a*) The density has to be computed upto 2 significant digits.

(*b*) The least count of scale used to measure length of wire is 1 mm.

(*c*) The density of wire is 2.5 g/cm^3

(*d*) The least count of screw gauge is 0.01 mm

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 9 to 11): Let us consider a particle P which is moving straight on the X-axis. We also know that the rate of change of its position is given by $\frac{dx}{dt}$; where x is its separation from the origin and t is time. This term $\frac{dx}{dt}$ is called the velocity of particle (v). Further the second derivation of x, with respect to time is called acceleration (a) or rate of change of velocity and represented by $\frac{d^2x}{dt^2}$ or $\frac{dv}{dt}$. If the acceleration of this particle is found to depend upon time as follows $a = At + Bt^2 + \frac{Ct}{D+t^2}$ then

9. The dimensions of A are

(*a*) LT^{-2} (*b*) LT^{-3} (*c*) LT^3 (*d*) L^2T^3

10. The dimensions of B are

(*a*) LT^{-4} (*b*) L^2T^{-3} (*c*) LT^4 (*d*) LT^{-2}

11. The dimensions of C are

(*a*) L^2T^{-2} (*b*) LT^{-2} (*c*) LT^{-1} (*d*) T^2

Comprehension (Q. 12 to 14): According to coulombs law of electrostatics there is a force between two charged particles q_1 & q_2 separated by a distance r such that $F \propto q_1$, $F \propto q_2$ & $F \propto \frac{1}{r^2}$; combining all three we get $F \propto \frac{q_1q_2}{r^2}$ or $F = \frac{kq_1q_2}{r^2}$, where k is a constant which depends on the medium and is given by $1/4\pi\varepsilon_0\varepsilon_r$ where ε_0 is absolute permittivity & ε_r is relative permittivity.

But in case of protons of a nucleus there exists another force called nuclear force; which is much higher in magnitude in comparison to electrostatic force and is given by $F = \frac{Ce^{-kr}}{r^2}$.

12. What are the dimensions of C ?

(*a*) $M^2L^3T^{-1}$ (*b*) ML^3T^{-3} (*c*) ML^3T^{-2} (*d*) ML^2T^{-3}

13. What are the dimensions of k?

(*a*) L (*b*) L^2 (*c*) L^{-3} (*d*) L^{-1}

14. What are the SI units of C?

(*a*) Nm^{-2} (*b*) Nm^2 (*c*) Nm^{-3} (*d*) Nm

Comprehension (Q. 15 to 17): The Van-der waals equation is $\left(P+\frac{a}{V^2}\right)(V-b) = nRT$, where P is pressure, V is volume and T is the temperature of the given sample of gas. R is called molar gas constant, a and b are called Van-der wall constants

15. The dimensional formula for a is same as that for

(*a*) V^2 (*b*) P (*c*) PV^2 (*d*) RT

16. Which of the following does not possess the same dimensional formula as that for nRT ?

(*a*) PV (*b*) Pb (*c*) a/V^2 (*d*) ab/V^2

17. The dimensional formula of nRT is same as that of

(*a*) Energy (*b*) Force

(*c*) Specific heat (*d*) Latent heat

Comprehension (Q. 18 to 20): Max Planck noted in 1899 the existence of a system of units based on the three fundamental constants G, c, and h. These constants are dimensionally independent in the sense that no combination is dimensionless and a length, a time, and a mass may be constructed from them. Specifically, with $\hbar \equiv \frac{h}{2\pi} = 1.05 \times 10^{-34}$ SI units in preference to h, the Planck scale is represented by ℓ_P, T_P, M_P.

18. Which of the following formula can represent length in terms of $\hbar$, c and G?

(*a*) $\sqrt{\dfrac{\hbar G}{c^3}}$ (*b*) $\sqrt{\dfrac{\hbar c}{G}}$

(*c*) $\sqrt{\dfrac{Gc}{\hbar}}$ (*d*) $\sqrt{G\hbar c}$

19. Which among the following would be the order of numerical value of a unit of mass defined in terms of $\hbar$, c and G?

(*a*) 10^{-8} kg (*b*) 10^{-10} kg

(*c*) 10^{-5} kg (*d*) 10^{-27} kg

20. The dimensional formula for acceleration in terms of these quantities would be

(*a*) $\hbar^{1/2}c^{3/2}G^{-1/2}$ (*b*) $\hbar^{3/2}c^{5/2}G^{1/2}$

(*c*) $\hbar^{-1/2}c^{7/2}G^{-1/2}$ (*d*) $\hbar^{5/2}c^{3/2}G^{1/2}$

Comprehension (Q. 21 to 23): When numbers having uncertainties or errors are used to compute other numbers, these will be uncertain. It is especially important to understand this when a number obtained from measurements is to be compared with a value obtained from theoretical prediction. Assume a student wants to verify the value of π as the ratio of circumference to diameter of a circle. The correct value of ten digits is 3.141592654. He draws a circle and measures its diameter and circumference to its nearest millimeter obtaining the values 135 mm and 424 mm, respectively. Using a calculator he finds $\pi = 3.140740741$.

21. Why does measured value not match with calculated value?

(*a*) Due to systematic error

(*b*) Due to error in calculation

(*c*) Due to random error

(*d*) Lack of precision in measuring

22. What is the value of π in the passage measured by the student?

(*a*) 3.140 (*b*) 3.141

(*c*) 3.1407 (*d*) 3.14

23. If diameter and circumference both have an error of 1%, what is the error in value of π?

(*a*) 2% (*b*) 1%

(*c*) 0.5% (*d*) 0%

MATCH THE COLUMN TYPE QUESTIONS

24. Match the following columns

	Physical quantity		Dimension		Unit
A.	Gravitational constant 'G'	p.	$M^1L^1T^{-1}$	(i)	N m
B.	Torque	q.	$M^{-1}L^3T^{-2}$	(ii)	N s
C.	Momentum	r.	$M^1 L^{-1}T^{-2}$	(iii)	N m²/kg²
D.	Pressure	s.	$M^1L^2T^{-2}$	(iv)	pascal

(*a*) A-(p)-(iii); B-(r)-(i); C-(q)-(iv); D-(r)-(ii)

(*b*) A-(q)-(iii); B-(s)-(i); C-(p)-(ii); D-(r)-(iv)

(*c*) A-(q)-(iii); B-(r)-(i); C-(r)-(ii); D-(s)-(iv)

(*d*) A-(p)-(iv); B-(s)-(ii); C-(p)-(i); D-(r)-(iii)

25. Match the following

	Physical quantity		Dimension		Unit
A.	Stefan's constant 'σ'	p.	$M^1L^1T^{-2}A^{-2}$	(i)	W/m²
B.	Wien's constant 'b'	q.	$M^1L^oT^{-3}K^{-4}$	(ii)	K.m.
C.	Coefficient of viscosity 'η'	r.	$M^1L^oT^{-3}$	(iii)	tesla .m/A
D.	Emissive power of radiation (Intensity emitted)	s.	$M^oL^1T^oK^1$	(iv)	W/m².K⁴
E.	Mutual inductance 'M'	t.	$M^1L^2T^{-2}A^{-2}$	(v)	Poise
F.	Magnetic permeability 'μ_0'	u.	$M^1L^{-1}T^{-1}$	(vi)	Henry

(*a*) A-(p)-(iv); B-(s)-(iii); C-(q)-(v); D-(r)-(i), E-(u)-(vi), F-(t)-(ii)

(*b*) A-(q)-(iii); B-(s)-(ii); C-(r)-(i); D-(u)-(v), E-(p)-(vi), F-(t)-(iv)

(*c*) A-(p)-(iv); B-(s)-(ii); C-(r)-(i); D-(u)-(v), E-(t)-(vi), F-(q)-(iii)

(*d*) A-(q)-(iv); B-(s)-(ii); C-(u)-(v); D-(r)-(i), E-(t)-(vi), F-(p)-(iii)

26. In Column-I, some physical quantities are given and some possible SI units are given in Column-II. Match the physical quantities in Column-I with the units in Column-II. Some useful formulas

$$P = \frac{F}{A},\ E = hcR,\ \lambda T = b,\ E = \sigma A T^4,\ F = \eta A\frac{v}{L}$$

(P-Pressure, F-force, v-velocity, A-Area, λ-wavelength h-Planck's constant, g-Gravitational acceleration, R-Rydberg constant, b-Wien's constant, h-Coefficient of viscosity, L-length, c-speed of light, t-time, E-energy, s-Stefan's constant and T-temperature)

	Column-I		Column-II
A.	$PAvt$	p.	$\dfrac{\text{watt second}}{\text{meter}^3}$
B.	hgR	q.	Joule
C.	$\dfrac{\sigma b^4}{A}$	r.	$\dfrac{\text{Newton}}{\text{metre}^2}$
D.	$\dfrac{\eta}{t}$	s.	$\dfrac{\text{Newton metre}}{\text{second}}$
		t.	Newton metre

(*a*) A-(q,t); B-(s,t); C-(p,t); D-(p,r)

(*b*) A-(p,t); B-(s); C-(q,t); D-(p)

(*c*) A-(q,t); B-(s); C-(q,t); D-(p,r)

(*d*) A-(t); B-(s); C-(q); D-(p,r)

27. Suppose two students are trying to make a new measurement system so that they can use it like a code measurement system and others do not understand it. Instead of taking 1 kg, 1 m and 1 sec. as basic unit they took unit of mass as α kg, the unit of length as β m and unit of time as g second. They called power in new system as SHAKTI then match the two columns.

Column-I		Column-II	
A.	1N in new system	p.	$\alpha^{-1}\,\beta^{-2}\,\gamma^{2}$
B.	1J in new system	q.	$\alpha^{-1}\,\beta^{-1}\,\gamma^{2}$
C.	1 Pascal (SI unit of pressure) in new system	r.	$\alpha^{-1}\,\beta\,\gamma^{2}$
D.	α SHAKTI in watt	s.	$\alpha^{2}\,\beta^{2}\,\gamma^{-3}$

(*a*) A-(q); B-(p); C-(r); D-(s)
(*b*) A-(p); B-(q); C-(r); D-(s)
(*c*) A-(q); B-(p); C-(s); D-(r)
(*d*) A-(p); B-(r); C-(q); D-(s)

28. Match the following

Column-I		Column-II	
A.	Latent heat constant	p.	$M^0\,L^0\,T^0$
B.	Reynold number	q.	$M\,L^2$
C.	Coefficient of friction	r.	$M\,L^0\,T^{-3}$
D.	Avogadro constant	s.	$L^2\,T^{-2}$
E.	Intensity of wave	r.	$M^0\,L^0\,T^0$
F.	Moment of inertia	s.	mol^{-1}

(*a*) A-(p); B-(s); C-(t); D-(u); E-(r); F-(q)
(*b*) A-(s); B-(t); C-(p); D-(r); E-(u); F-(q)
(*c*) A-(s); B-(p); C-(t); D-(u); E-(q); F-(r)
(*d*) A-(s); B-(p); C-(t); D-(u); E-(r); F-(q)

NUMERICAL TYPE QUESTIONS

29. Number of significant figures in 0.007 m^2 .

30. Number of significant figures in 2.64×10^{24} kg

31. Number of significant figures in 6.032 N m^{-2}

32. The velocity of sound in a gas depends on its pressure and density. The relation between velocity, pressure and density is given by $v = Kp^a D^b$, then $(a + b)$ is

33. A gas bubble, from an explosion under water, oscillates with a period proportional to $P^a d^b E^c$. Where P is the static pressure, d is the density and E is the total energy of the explosion. Find the values of $a + b + c$

34. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47$^{\text{th}}$ division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in two number of significant figures.

35. The density of a cube is measured by measuring its mass and the length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is.

36. The length of the string of a simple pendulum is measured with a metre scale to be 90.0 cm. The radius of the bob plus the length of the hook is calculated to be 2.13 cm using measurements with a slide callipers. What is the effective length of the pendulum? (This effective length is defined as the distance between the point of suspension and the center of the bob).

37. Using the approximation $(1 + x)\,n = 1 + nx$, $|x| << 1$
Find the value of $\sqrt{99}$.

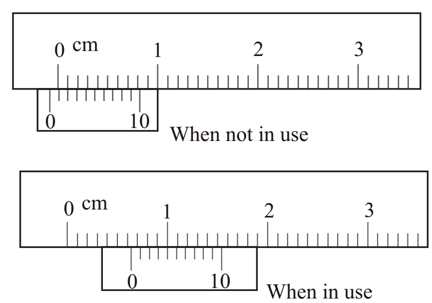
38. The time period of oscillation of a body is given by

$$T = 2\pi\sqrt{\frac{mgA}{K}}$$

K represents the kinetic energy, m mass, g acceleration due to gravity and A is unknown. If $[A] = M^x L^y T^z$, then what is the value of $x + y + z$?

39. The radius of a sphere is measured to be 5.3 ± 0.1 cm. Calculate percentage error in volume. Round off to nearest integer.

40. The main scale of a Vernier calliper reads in millimeter and its vernier is divided into 10 divisions which coincides with 9 divisions of the main scale. The length of the object for situation is found to be $\frac{12x}{10}$ mm. Find the value of x.



41. In an experiment of simple pendulum, time period measured was 50 s for 25 oscillations when the length of the simple pendulum was taken 100 cm. If the least count of stop watch is 0.1 s and that of meter scale is 0.01 cm, calculate the maximum possible percentage error (p) in the measurement of value of g. Quote100p.

Exercise-4 (Past Year Questions)

JEE MAIN

1. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass 50 g, the numerical value of density of the material is **(2019)**

(a) 40 (b) 16 (c) 640 (d) 410

2. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to **(2019)**

(a) $\sqrt{\frac{hc^5}{G}}$ (b) $\sqrt{\frac{c^3}{Gh}}$ (c) $\sqrt{\frac{Gh}{c^5}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

3. Let L, R, C and V represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{L}{RCV}$ in SI units will be **(2019)**

(a) [LA^{-2}] (b) [A^{-1}] (c) [LTA] (d) [LT^2]

4. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? **(2019)**

(a) $[M^{-2}\,L^{-2}\,T^6\,A^3]$ (b) $[M^{-1}\,L^{-2}\,T^4\,A^2]$
(c) $[M^{-3}\,L^{-2}\,T^8\,A^4]$ (d) $[M^{-2}\,L^0\,T^{-4}\,A^{-2}]$

5. If surface tension (S), Moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be **(2019)**

(a) $S^{3/2}I^{1/2}h^0$ (b) $S^{1./2}I^{1/2}h^0$
(c) $S^{1/2}I^{1/2}h^{-1}$ (d) $S^{1/2}I^{3/2}h^{-1}$

6. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale for a thin sheet are 5.5 mm and 48 respectively, the thickness of this sheet is **(2019)**

(a) 5.755 mm (b) 5.950 mm
(c) 5.725 mm (d) 5.740 mm

7. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm respectively. What will be the value of its volume in appropriate significant figures? **(2019)**

(a) 4264 ± 81 cm^3 (b) 4264 ± 81.0 cm^3
(c) 4260 ± 80 cm^3 (d) 4300 ± 80 cm^3

8. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is **(2019)**

(a) 50 (b) 200 (c) 100 (d) 500

9. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is **(2019)**

(a) 37 cm^2 (b) 37.0 cm^2
(c) 37.03 cm^2 (d) 37.030 cm^2

10. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is **(2020)**

(a) $\left[P^{\frac{1}{2}}AT^{-1}\right]$ (b) $[P^2AT^{-2}]$
(c) $\left[PA^{\frac{1}{2}}T^{-1}\right]$ (d) $[PA^{-1}T^{-2}]$

11. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be **(2020)**

(a) $FA^{-1}V^0$ (c) FA^2V^{-1}
(b) FA^2V^{-2} (d) FA^2V^{-3}

12. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is **(2020)**

(a) ML^2T^{-2} (b) MLT^{-2}
(c) $M^2L^0T^{-1}$ (d) ML^0T^{-3}

13. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and length L. The dimensional formula for x is same as that of **(2020)**

(a) Coefficient of viscosity
(b) Force constant
(c) Energy density
(d) Planck's constant

14. The quantities $x = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $y = \frac{E}{B}$ and $z = \frac{I}{CR}$ are defined where C-capacitance, R-resistance, l-length, E-electric field, B-magnetic field and ε_0, μ_0-free space permitivity and permeability respectively. Then **(2020)**

(a) Only x and y have the same dimension.
(b) Only x and z have the same dimension.
(c) x, y and z have the same dimension.
(d) Only y and z have the same dimension.

15. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is **(2020)**

(a) 4.40% (b) 3.40%
(c) 2.40% (d) 5.40%

16. If the screw on a screw–gauge is given six rotations, it moves by 3mm on the main scale. If there are 50 divisions on the circular scale, the least count of the screw gauge is **(2020)**

(*a*) 0.01 cm (*b*) 0.02 mm

(*c*) 0.001 mm (*d*) 0.001 cm

17. The least count of the main scale of a vernier calipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7^{th} division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4 VSD coincides with a main scale division. The length of the cylinder is (VSD is vernier scale division) **(2020)**

(*a*) 3.21 cm (*b*) 2.99 cm

(*c*) 3.07 cm (*d*) 3.2 cm

18. A physical quantity z depends on four observables a, b, c and d, as $\frac{a^2 b^{\frac{2}{3}}}{\sqrt{c}\, d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is **(2020)**

(*a*) 13.5% (*b*) 14.5%

(*c*) 16.5% (*d*) 12.25%

19. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm, and 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as **(2020)**

(*a*) (5.5375 ± 0.0739) mm

(*b*) (5.54 ± 0.07) mm

(*c*) (5.538 ± 0.074) mm

(*d*) (5.5375 ± 0.0740) mm

20. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)$%. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is **(2020)**

(*a*) 5010 (*b*) 5100

(*c*) 1050 (*d*) 5101

21. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that zero on the vernier scale lies between 8.5 cm and 8.6 cm, and vernier coincidence is 6, then the correct value of measurement is (least count = 0.01 cm) **(2021)**

(*a*) 8.58 cm (*b*) 8.54 cm

(*c*) 8.56 cm (*d*) 8.36 cm

22. In order to determine the Young's modulus of a wire of radius 0.2 cm (measured using a scale of least count =0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's modulus determined by this experiment? **(2021)**

(*a*) 0.14% (*b*) 9% (*c*) 1.4% (*d*) 0.9%

23. One main scale division of a vernier calipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the calipers (in mm) is **(2021)**

(*a*) $\frac{10na}{n-1}$ (*b*) $\left(\frac{n-1}{10n}\right)a$

(*c*) $\frac{10a}{n-1}$ (*d*) $\frac{10a}{n}$

24. If velocity [V], time [T] and force [F] are chosen as the base quantities, the dimensions of the mass will be **(2021)**

(*a*) $[FT^{-1}V^{-1}]$ (*b*) $[FTV^{-1}]$

(*c*) $[FT^2V]$ (*d*) $[FVT^{-1}]$

25. **Assertion (A):** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason (R): Least count $\frac{\text{Pitch}}{\text{Total divisions on circular scale}}$

In the light of the above statements, choose the most appropriate answer from the option given below **(2021)**

(*a*) A is not correct but R is correct.

(*b*) Both A and R are correct and R is correct explanation of A

(*c*) A is correct but R is not correct

(*d*) Both A and R are correct and R is not the correct explanation of A.

26. The force is given in terms of time t and displacement x by the equation $F = A\cos Bx + C\sin Dt$. The dimensional formula of AD/B is **(2021)**

(*a*) $[M^0LT^{-1}]$ (*b*) $[ML^2T^{-3}]$

(*c*) $[M^1L^1T^{-2}]$ (*d*) $[M^2L^2T^{-3}]$

27. If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is **(2021)**

(*a*) 86.4 s (*b*) 4.32 s

(*c*) 43.2 s (*d*) 8.64 s

28. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum to known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as _____. **(2021)**

29. An expression for a dimensionless quantity P is given by $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$; where α and β are constants, x is distance k is Boltzmann constant and t is the temperature. Then the dimensions of a will be **(2022)**

(*a*) $[M^0L^{-1}T^0]$ (*b*) $[ML^0T^{-2}]$
(*c*) $[MLT^{-2}]$ (*d*) $[ML^2T^2]$

30. The SI unit of a physical quantity is pascal-second. The dimensional formula of this quantity will be **(2022)**

(*a*) $[ML^{-1}T^{-1}]$
(*b*) $[ML^{-1}T^{-2}]$
(*c*) $[ML^2T^{-1}]$
(*d*) $[M^{-1}L^3T^0]$

31. In Vander Waals equation $\left[P + \frac{a}{V^2}\right][V - b] = RT$; P is pressure. V is volume, R is universal gas constant and T is temperature. The ratio of constants a/b is dimensionally equal to **(2022)**

(*a*) P/V (*b*) V/P
(*c*) PV (*d*) PV^3

32. A torque meter is calibrated to reference standards of mass, length and time each with 5% accuracy. After calibration, the measured torque with this torque meter will have net accuracy of **(2022)**

(*a*) 15% (*b*) 25%
(*c*) 75% (*d*) 5%

33. Consider the efficiency of Carnot's engine is given by $\eta = \frac{\alpha\beta}{\sin\theta}\log_e\frac{\beta x}{kT}$, where α and β are constants. If T is temperature, k is Boltzmann constant, θ is angular displacement and x has the dimensions of length. Then, choose the incorrect option. **(2022)**

(*a*) Dimension of β is same as that of force.
(*b*) Dimension of $\alpha^{-1}x$ is same as that of energy.
(*c*) Dimension of $\eta^{-1}\sin\theta$ is same of $\alpha\beta$.
(*d*) Dimension of α is same of β.

34. Match Column-I with Column-II

Column-I		Column-II	
A.	Torque	p.	Nms^{-1}
B.	Stress	q.	$J\,kg^{-1}$
C.	Latent heat	r.	Nm
D.	Power	s.	Nm^{-2}

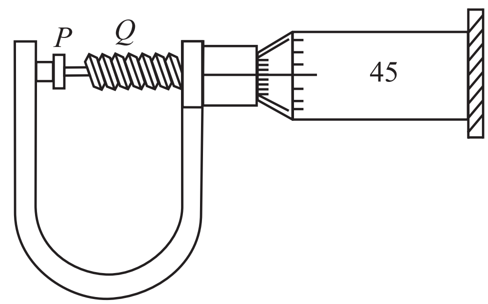
Choose the correct answer from the options given below **(2022)**

(*a*) A-(r); B-(q); C-(p); D-(s)
(*b*) A-(r); B-(s); C-(q); D-(p)
(*c*) A-(s); B-(p); C-(r); D-(q)
(*d*) A-(q); B-(r); C-(p); D-(s)

35. In a Vernier Calliper, 10 divisions of Vernier scale is equal to the 9 divisions of main scale. When both jaws of Vernier calipers touch each other, the zero of the Vernier scale is shifted to the left of zero of the main scale and 4th Vernier scale division exactly coincides with the main scale reading. One main scale division is equal to 1 mm. While measuring diameter of a spherical body, the body is held between two jaws. It is now observed that zero of the Vernier scale lies between 30 and 31 divisions of main scale reading and 6th Vernier scale division exactly, coincides with the main scale reading. The diameter of the spherical body will be **(2022)**

(*a*) 3.02 cm (*b*) 3.06 cm
(*c*) 3.10 cm (*d*) 3.20 cm

36. In an experiment to find out the diameter of wire using screw gauge, the following observations were noted? **(2022)**



A. Screw moves 0.5 mm or main scale in one complete rotation.
B. Total divisions on circular scale = 50
C. Main scale reading is 2.5 mm
D. 45^{th} division of circular scale is in the pitch line

Then the diameter of wire is

(*a*) 2.92 mm (*b*) 2.54 mm
(*c*) 2.98 mm (*d*) 3.45 mm

37. Given below are statements: One is labelled as Assertion (A) and other is labelled as Reason (R) **(2022)**

Assertion (A): Time period of oscillation of a liquid drop depends on surface tension (S). If density of the liquid is ρ and radius of the drop is r, then $T = K\sqrt{\rho r^3 / S^{3/2}}$ is dimensionally correct, where K is dimensionless.

Reason (R): Using dimensional analysis we get R.H.S having different dimension than that of time period.

In the light of above statements, choose the correct answer from the options given below.

(*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).
(*b*) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(*c*) (A) is true but (R) is false.
(*d*) (A) is false but (R) is true.

JEE ADVANCED

38. In the determination of Young's modulus $\left(Y = \dfrac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension ℓ = 0.25 mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement **(2012)**

(*a*) Due to the errors in the measurements of d and ℓ are the same.

(*b*) Due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(*c*) Due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(*d*) Due to the error in the measurement of d is four time that due to the error in the measurement of ℓ.

39. Match Column-I with Column-II and select the correct answer using the codes given below the lists: **(2013)**

	Column-I		Column-II
A.	Boltzmann constant	p.	$[ML^2T^{-1}]$
B.	Coefficient of viscosity	q.	$[ML^{-1}T^{-1}]$
C.	Planck constant	r.	$[MLT^{-3}K^{-1}]$
D.	Thermal conductivity	s.	$[ML^2T^{-2}K^{-1}]$

(*a*) A-(r); B-(p); C-(q); D-(s)

(*b*) A-(r); B-(q); C-(p); D-(s)

(*c*) A-(s); B-(q); C-(p); D-(r)

(*d*) A-(s); B-(p); C-(q); D-(r)

40. The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm. The 24th division of the vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is **(2013)**

(*a*) 5.112 cm (*b*) 5.124 cm

(*c*) 5.136 cm (*d*) 5.148 cm

41. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90º. The wavelength λ is exactly knowns and the error in θ is constant for all values of θ. As θ increases from 0º **(2013)**

(*a*) The absolute error in d remains constant.

(*b*) The absolute error in d increases.

(*c*) The fractional error in d remains constant.

(*d*) The fractional error in d decreases.

42. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer find that d is proportional to $S^{1/n}$. The value of n is **(2014)**

(*a*) $\dfrac{1}{2}$ (*b*) $\dfrac{3}{2}$

(*c*) 3 (*d*) $\dfrac{2}{3}$

43. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. The relation between M, c, G, h are as under **(2015)**

(*a*) $M \propto \sqrt{c}$ (*b*) $M \propto \sqrt{G}$

(*c*) $L \propto \sqrt{h}$ (*d*) $L \propto \sqrt{G}$

44. Consider a Vernier calipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier calipers, 5 divisions of the vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then **(2015)**

(*a*) If the pitch of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm.

(*b*) If the pitch of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.005 mm.

(*c*) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm.

(*d*) If the least count of linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.005 mm.

45. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is (are) **(2015)**

(*a*) $\mu_0 I^2 = \varepsilon_0 V^2$ (*b*) $\mu_0 I = \mu_0 V$

(*c*) $I = \varepsilon_0 c V$ (*d*) $\mu_0 c I = \varepsilon_0 V$

46. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56s, 0.57s, 0.54s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is (are) true? **(2016)**

(*a*) The error in the measurement of r is 10%

(*b*) The error in the measurement of T is 3.57%

(*c*) The error in the measurement of T is 2%

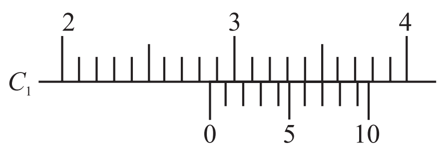
(*d*) The error in the determined value of g is 11%

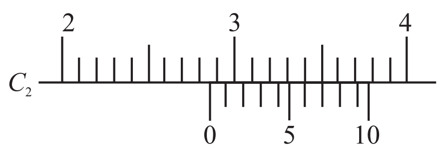
47. A length scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is (are) dimensionally correct? **(2016)**

(a) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (b) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(c) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (d) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

48. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale division. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are **(2016)**





(a) 2.87 and 2.86 (b) 2.85 and 2.82

(c) 2.87 and 2.87 (d) 2.87 and 2.83

Comprehension (Q. 49 to 50): In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while [ε_0] and [μ] stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units. **(2018)**

49. The relation between [E] and [B] is

(a) $[E] = [B]\,[L]\,[T]$

(b) $[E] = [B]\,[L]^{-1}\,[T]$

(c) $[E] = [B]\,[L]\,[T]^{-1}$

(d) $[E] = [B]\,[L]^{-1}\,[T]^{-1}$

50. The relation between [ε_0] and [μ_0] is:

(a) $[\mu_0] = [\varepsilon_0]\,[L]^2\,[T]^{-2}$

(b) $[\mu_0] = [\varepsilon_0]\,[L]^{-2}\,[T]^2$

(c) $[\mu_0] = [\varepsilon_0]^{-1}\,[L]^2\,[T]^{-2}$

(d) $[\mu_0] = [\varepsilon_0]^{-1}\,[L]^{-2}\,[T]^2$

51. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct? **(2019)**

(a) The dimension of force is L^{-3}.

(b) The dimension of energy of L^{-2}.

(c) The dimension of power is L^{-5}.

(d) The dimension of linear momentum is L^{-1}.

52. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity x as follows: [position] = [x^{α}]; [speed] = [x^{β}]; [acceleration] = [x^p]; [linear momentum] = [x^q]; [force] = [x^r]. Then **(2020)**

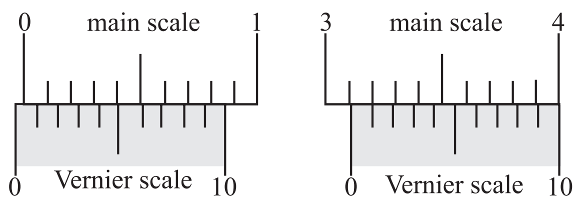
(a) $\alpha + p = 2\beta$

(b) $p + q - r = \beta$

(c) $p - q + r = \alpha$

(d) $p + q + r = \beta$

53. The smallest division on the main scale of a Vernier callipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calliper with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is **(2021)**



(a) 3.07 cm (b) 3.11 cm

(c) 3.15 cm (d) 3.17 cm

54. A physical quantity $\vec{S}$ is defined as $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity (ies)? **(2021)**

(a) $\frac{\text{Energy}}{\text{Charge} \times \text{Current}}$ (b) $\frac{\text{Force}}{\text{Length} \times \text{Time}}$

(c) $\frac{\text{Energy}}{\text{Volume}}$ (d) $\frac{\text{Power}}{\text{Area}}$

55. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and coulomb's constant $k = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^{\alpha}\,[m_e]^{\beta}\,[h]^{\gamma}\,[k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ is ________________. **(2022)**

ANSWER KEY

CONCEPT APPLICATION

1. $A = M^0L^0T^0$, $B = M^\circ L^\circ T^\circ$ 2. $A = (L^{1/2}T^{-3})$, $|B| = [LT^{-3}]$ 3. $|A| = |L|$, $|B| = M^\circ L^\circ T^2$, $|C| = M^\circ L^\circ T^\circ$ 4. $L^2 T^{-1}$

5. (c)	6. (d)	7. $m = Kc^{\frac{1}{2}}h^{\frac{1}{2}}G^{-\frac{1}{2}}$	8. (a)	9. (c)	10. (a)	11. (d)	12. (a)	13. (a)
14. (d)	15. (d)	16. (c)	17. (b)	18. (d)	19. (c)	20. (d)	21. [0.001]	22. [1.004 ± 0.001]
23. (c)	24. (d)	25. (a)	26. (c)	27. [1]	28. (b)	29. [0.005]	30. [0.215]	31. (a)

EXERCISE-1 (TOPICWISE)

1. (b)	2. (a)	3. (a)	4. (c)	5. (b)	6. (c)	7. (b)	8. (b)	9. (d)	10. (a)
11. (d)	12. (d)	13. (c)	14. (c)	15. (b)	16. (b)	17. (b)	18. (d)	19. (b)	20. (a)
21. (c)	22. (c)	23. (c)	24. (a)	25. (b)	26. (a)	27. (c)	28. (a)	29. (d)	30. (b)
31. (b)	32. (b)	33. (c)	34. (b)	35. (c)	36. (a)	37. (b)	38. (a)	39. (b)	40. (a)
41. (c)	42. (c)	43. (a)	44. (a)	45. (a)	46. (a)	47. (b)	48. (b)	49. (b)	50. (b)
51. (c)	52. (c)	53. (a)	54. (a)	55. (b)	56. (b)	57. (c)	58. (c)	59. (c)	60. (a)
61. (b)									

EXERCISE-2 (LEARNING PLUS)

1. (b)	2. (d)	3. (c)	4. (b)	5. (c)	6. (d)	7. (a)	8. (a)	9. (c)	10. (d)
11. (d)	12. (c)	13. (d)	14. (d)	15. (d)	16. (a)	17. (d)	18. (c)	19. (d)	20. (c)
21. (b)	22. (b)	23. (a)	24. (d)	25. (a)	26. (b)	27. (b)	28. (b)	29. (d)	30. (a)
31. (c)	32. (a)	33. (a)	34. (b)	35. (b)	36. (c)	37. (b)	38. (a)	39. (a)	40. (b)
41. (c)	42. (a)								

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (a,b,c)	2. (c,d)	3. (a,b,c)	4. (a,b,c,d)	5. (a,c)	6. (b,d)	7. (a,b,c,d)	8. (a,b,c,d)	9. (b)	10. (a)
11. (c)	12. (c)	13. (d)	14. (b)	15. (c)	16. (c)	17. (a)	18. (a)	19. (a)	20. (c)
21. (d)	22. (d)	23. (a)	24. (b)	25. (d)	26. (c)	27. (a)	28. (d)	29. [1]	30. [3]
31. [4]	32. [0]	33. [0]	34. [2.6]	35. [9]	36. [92.1]	37. [9.95]	38. [3]	39. [6]	40. [6]
41. [41]									

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (a)	2. (c)	3. (b)	4. (c)	5. (b)	6. (b)	7. (c)	8. (b)	9. (b)	10. (c)
11. (a)	12. (d)	13. (c)	14. (a)	15. (a)	16. (d)	17. (c)	18. (b)	19. (b)	20. (c)
21. (b)	22. (c)	23. (d)	24. (b)	25. (a)	26. (b)	27. (c)	28. [14]	29. (c)	30. (c)
31. (c)	32. (b)	33. (d)	34. (c)	35. (c)	36. (a)	37. (d)			

JEE Advanced

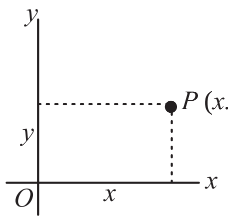
38. (a)	39. (c)	40. (b)	41. (d)	42. (c)	43. (a,c,d)	44. (b,c)	45. (a,c)	46. (a,b)	47. (b,d)
48. (b)	49. (c)	50. (d)	51. (a,b,d)	52. (a,b)	53. (d)	54. (b,d)	55. [4]		

CHAPTER

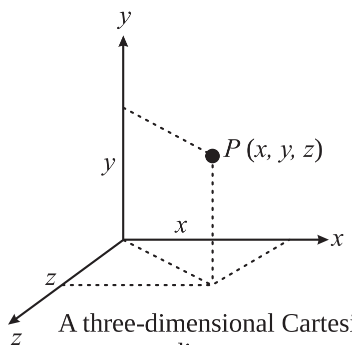
2 Vector and Calculus

LOCATION OF A POINT IN SPACE

Positions in space are designated relative to coordinate systems. The **Cartesian coordinate** system is a particularly convenient coordinate system in which positions are designated by distances (x, y, z) along three perpendicular axes that intersect at a point called the origin.



A two-dimensional Cartesian coordinate system

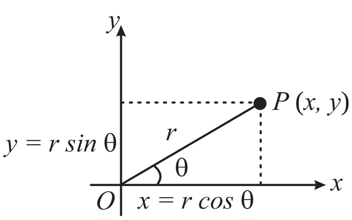


A three-dimensional Cartesian coordinate system

In a **polar coordinate** system positions in a plane are designated by a length r from the origin, and an angle θ usually measured from the positive x-axis. From simple trigonometry we see that the relationships between the polar coordinates and the Cartesian coordinates are

$$x = r\cos\theta$$

$$y = r\sin\theta$$



Relationship between polar and Cartesian coordinates

FRAME OF REFERENCE

A frame of reference is another name for the particular coordinate system with respect to which we are making observations of physical phenomena.

SCALAR AND VECTOR

(i) Scalar: A scalar quantity requires only a number for its complete description. Mass, volume, density, pressure and temperature are all examples of scalar quantities. The mathematics of scalar quantities is the ordinary algebra of numbers.

(ii) Vector: Vector quantities require both magnitude and direction for its complete description. Velocity, acceleration, force and momentum are examples of vector quantities.

Types of Vector

(i) Equal vectors: Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

(ii) Parallel vector: Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when

(*a*) Both have same direction.

(*b*) One vector is scalar (positive) non-zero multiple of another vector.

(iii) Anti-parallel vectors: Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when

(*a*) Both have opposite direction.

(*b*) One vector is scalar non-zero negative multiple of another vector.

(iv) Zero vector $(\vec{0})$: A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.

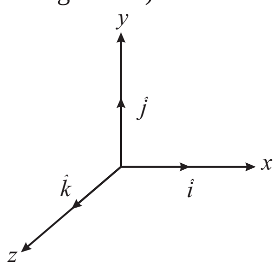
(v) Unit vector: A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap/A hat).

Since, $\hat{A} = \dfrac{\vec{A}}{A}$

$\Rightarrow \vec{A} = A\hat{A}$.

Thus, we can say that unit vector gives us the direction.

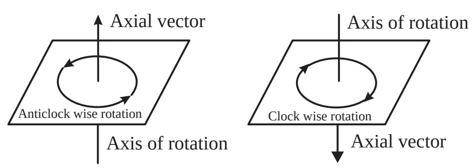
(vi) Orthogonal unit vectors: $\hat{i}$, $\hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a Right Handed Triad (It is a coordinate system such that when we Curl the fingers of right hand from x to y then we must get the direction of z along thumb).



Then $\hat{i} = \dfrac{\vec{x}}{x}$, $\hat{j} = \dfrac{\vec{y}}{y}$, $\hat{k} = \dfrac{\vec{z}}{z}$

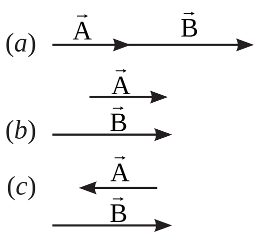
$\therefore\ \vec{x} = x\hat{i}$, $\vec{y} = y\hat{j}$, $\vec{z} = z\hat{k}$

(vii) Axial Vectors: These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are example of physical quantities of this type.



(viii) Coplanar Vectors: Three (or more) vectors are called coplanar vectors if they lie in the same plane. Two (free) vectors are always coplanar.

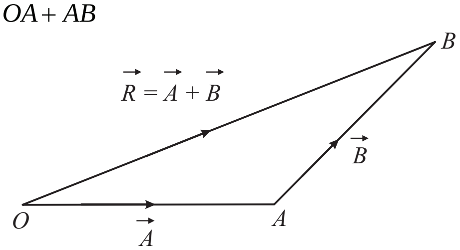
(ix) Collinear Vectors: Those vectors which are expressed along same straight line and are parallel or Anti-parallel are called collinear vectors



TRIANGLE LAW OF VECTOR ADDITION OF TWO VECTORS

If two non zero vectors are represented by the two sides of a triangle taken in same order then the resultant is given by the closing side of triangle in opposite order. i.e. $\vec{R} = \vec{A} + \vec{B}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$



(i) Magnitude of resultant vector

In ΔABN, $\cos\theta = \dfrac{AN}{B}$

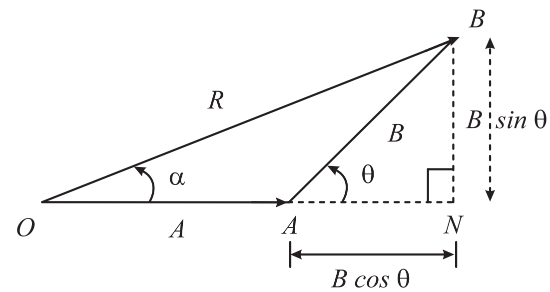
$\therefore\ AN = B\cos\theta$

$\sin\theta = \dfrac{BN}{B}$

$\therefore\ BN = \sin\theta$

In ΔOBN, we have $OB^2 = ON^2 + BN^2$

$\Rightarrow\ R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$



$\Rightarrow\ R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$\Rightarrow\ R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

$\Rightarrow\ R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow\ R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(ii) Direction of resultant vectors: If θ is angle between $\vec{A}$ and $\vec{B}$ and, $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow \alpha = \tan^{-1}\left\{\frac{B\sin\theta}{A + B\cos\theta}\right\}$$

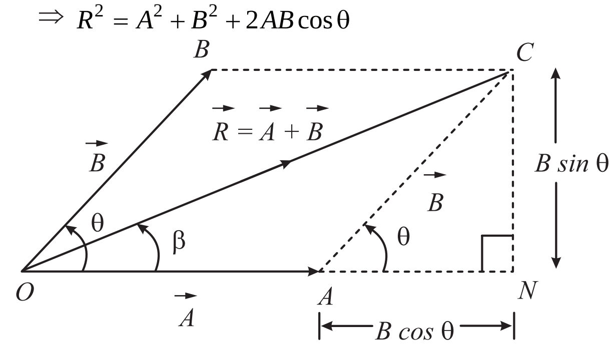
PARALLELOGRAM LAW OF VECTOR ADDITION OF TWO VECTORS

If two non zero vector are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

(i) Magnitude: Since, $R^2 = ON^2 + CN^2$

$\Rightarrow\ R^2 = (OA + AN)^2 + CN^2$

$\Rightarrow\ R^2 = A^2 + B^2 + 2AB\cos\theta$



$\therefore \ R = |\vec{R}| = |\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$

Special cases: $R = A + B$ when $\theta = 0°$

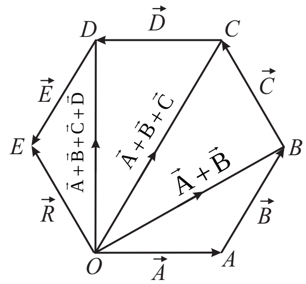
$R = A - B$ when $\theta = 180°$

$R = \sqrt{A^2+B^2}$ when $\theta = 90°$

(ii) Direction: $\tan\beta = \dfrac{CN}{ON} = \dfrac{B\sin\theta}{A+B\cos\theta}$

POLYGON LAW OF VECTOR ADDITION

If a number of non zero vectors are represented by the $(n-1)$ sides of an n-sided polygon then the resultant is given by the closing side or the n^{th} side of the polygon taken in opposite order. So,



$\vec{R} = \vec{A}+\vec{B}+\vec{C}+\vec{D}+\vec{E}$

$\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE} = \overrightarrow{OE}$

Note: Resultant of two unequal vectors can not be zero.

Resultant of three coplanar vectors may or may not be zero.

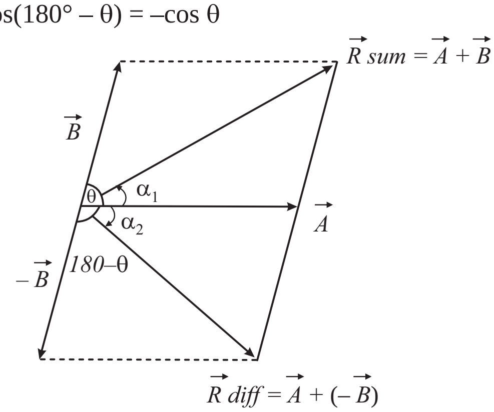
Resultant of three non coplanar vectors can not be zero.

SUBTRACTION OF VECTORS

Since $\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$ and $|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$

$\Rightarrow |\vec{A}-\vec{B}| = \sqrt{A^2+B^2+2AB\cos(180^o-\theta)}$

Since, $\cos(180° - \theta) = -\cos\theta$



$\Rightarrow |\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$

$\tan\alpha_1 = \dfrac{B\sin\theta}{A+B\cos\theta}$ and $\tan\alpha_2 = \dfrac{B\sin(180-\theta)}{A+B\cos(180-\theta)}$

But $\sin(180°-\theta) = \sin\theta$ and $\cos(180°-\theta) = -\cos\theta$

$\Rightarrow \tan\alpha_2 = \dfrac{B\sin\theta}{A-B\cos\theta}$

Train Your Brain

Example 1: There are two force vectors, one of 5 N and other of 12 N at what angle the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively

(*a*) 0°, 180° and 90° (*b*) 0°, 90° and 180°

(*c*) 0°, 90° and 90° (*d*) 180°, 0° and 90°

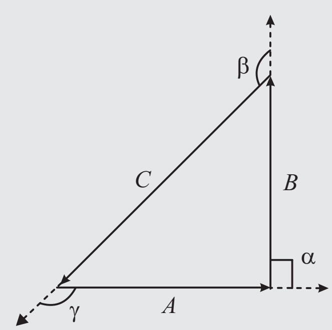
Sol. (*a*) For 17 *N* both the vector should be parallel i.e. angle between them should be zero for 7 *N* both the vectors should be antiparallel i.e. angle between them should be 180° for 13 *N* both the vectors should be perpendicular to each other i.e. angle between them should be 90°.

Example 2: Given that $\vec{A}+\vec{B}+\vec{C} = 0$. Out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

(*a*) 30°, 60°, 90° (*b*) 45°, 45°, 90°

(*c*) 45°, 60°, 90° (*d*) 90°, 135°, 135°

Sol. (*d*) From polygon law, three vectors having summation zero should form a closed polygon. (Triangle) since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. i.e. the triangle should be right angled triangle



Angle between A and B, $\alpha = 90^o$

Angle between B and C, $\beta = 135^o$

Angle between A and C, $\gamma = 135^o$

Example 3: Two vectors of 10 units and 5 units make an angle of 120° with each other. Find the magnitude and angle of resultant with vector of 10 unit magnitude.

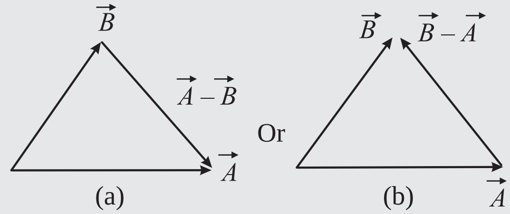
Sol. $|\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}$

$= \sqrt{100+25+2\times10\times5(-1/2)} = 5\sqrt{3}$

$\tan\alpha = \dfrac{5\sin120°}{10+5\cos120°} = \dfrac{5\sqrt{3}}{20-5} = \dfrac{5\sqrt{3}}{5\times3} = \dfrac{1}{\sqrt{3}}$

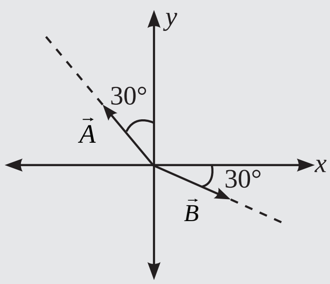
$\Rightarrow \alpha = 30°$

Note: $\vec{A}-\vec{B}$ or $\vec{B}-\vec{A}$ can also be found by making triangles as shown in figure. (*a*) and (*b*)

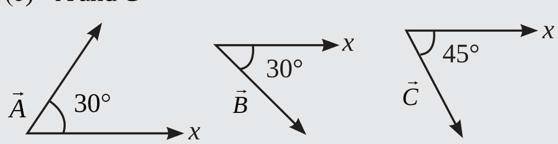


Concept Application

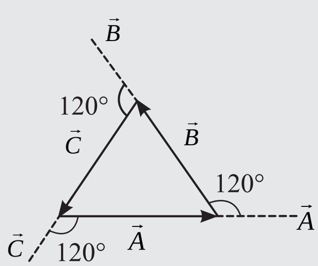
1. A vector $\vec{A}$ makes an angle 30° with the *y*-axis in anticlockwise direction. Another vector $\vec{B}$ makes an angle 30° with the *x*-axis in clockwise direction. Find angle between vectors $\vec{A}$ and $\vec{B}$.



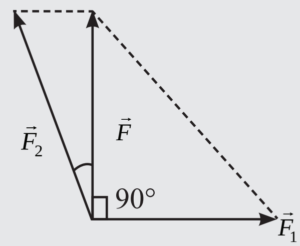
2. Three vectors $\vec{A}, \vec{B}, \vec{C}$ are shown in the figure. Find angle between
   (*a*) $\vec{A}$ and $\vec{B}$ (*b*) $\vec{B}$ and $\vec{C}$
   (*c*) $\vec{A}$ and $\vec{C}$



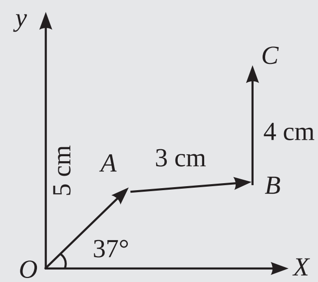
3. If $\vec{A}, \vec{B}, \vec{C}$ represents the three sides of an equilateral triangle taken in the same order then find the angle between
   (*a*) $\vec{A}$ and $\vec{B}$ (*b*) $\vec{B}$ and $\vec{C}$
   (*c*) $\vec{A}$ and $\vec{C}$



4. The sum of magnitudes of two forces acting at a point is 16 N. If their resultant is normal to the smaller force and has a magnitude of 8 N. Then the forces are



5. Find the resultant of the vectors shown in figure.



UNIT VECTOR AND ZERO VECTOR

Unit vector is a vector which has a unit magnitude and points in a particular direction. Any vector ($\vec{A}$) can be written as the product of unit vector $(\hat{A})$ in that direction and magnitude of the given vector.

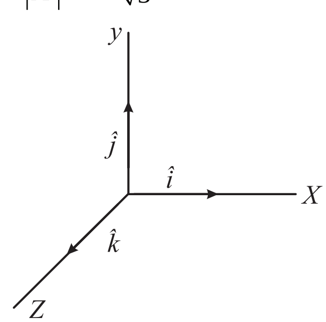
$$\vec{A} = A\,\hat{A}$$

$$\text{or}\quad \hat{A} = \frac{\vec{A}}{A}$$

A unit vector has no dimensions and unit. Unit vectors along the positive *x*, *y* and *z*-axes of a rectangular coordinate system are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively such that $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

For example: If $\vec{A} = \hat{i} + \hat{j} - \hat{k}$, then unit vector parallel to

$$\vec{A} \text{ would be } = \frac{\vec{A}}{|A|} = \frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$$



A vector of zero magnitude is called a zero or a null vector. Its direction is indeterminate.

Train Your Brain

Example 4: If the sum of two unit vectors is a unit vector, then magnitude of difference is

Sol. Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is $\vec{n}_s = \hat{n}_1 + \hat{n}_2$ or $n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta$

$= 1 + 1 + 2\cos\theta$

Since it is given that n_s is also a unit vector, therefore $1 = 1 + 1 + 2\cos\theta$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120°$

Now the difference vector is $\vec{n}_d = \hat{n}_1 - \hat{n}_2$

or $n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120°)$

$\therefore\ n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$

Example 5: If $\vec{a}_1$ and $\vec{a}_2$ are two non collinear unit vectors and if $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}$, then the value of $(\vec{a}_1 - \vec{a}_2).(2\vec{a}_1 + a_2)$ is

Sol. $a_1 = a_2 = 1$ and $a_1^2 + a_2^2 + 2a_1a_2\cos\theta\ (\sqrt{3})^2 = 3$

Or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = \frac{1}{2}$

Now $(\vec{a}_1 - \vec{a}_2)\ (2\vec{a}_1 + \vec{a}_2) = 2a_1^2 - a_2^2 - a_1a_2\cos\theta$

$= 2 - 1 - \frac{1}{2} = \frac{1}{2}$

Concept Application

6. When $\vec{A} = 3\hat{i} - 2\hat{j}, \vec{B} = \hat{i} + 2\hat{j} - \hat{k}$. Then unit vector along $\left(\vec{A} - \vec{B}\right)$ would be

(a) $\frac{1}{\sqrt{21}}\left(\hat{i} - 4\hat{j} + \hat{k}\right)$ (b) $\frac{1}{\sqrt{21}}\left(2\hat{i} - 4\hat{j} + \hat{k}\right)$

(c) $\frac{1}{\sqrt{3}}\left(2\hat{i} - 4\hat{j} + \hat{k}\right)$ (d) $\frac{1}{\sqrt{3}}\left(2\hat{i} - 4\hat{j} + \hat{k}\right)$

7. When vector $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{B} = 3\hat{i} - 3\hat{j} + \hat{k}$. The unit vectors parallel to $\vec{A} + \vec{B}$ would be

(a) $\frac{3\hat{i}}{5\sqrt{2}} - \frac{4\hat{i}}{5\sqrt{2}} + \frac{3\hat{k}}{5\sqrt{2}}$ (b) $\frac{-\hat{i}}{2\sqrt{5}} - \frac{4\hat{j}}{2\sqrt{5}} + \frac{3\hat{k}}{5\sqrt{2}}$

(c) $\frac{3\hat{i}}{2\sqrt{5}} - \frac{4\hat{j}}{2\sqrt{5}} + \frac{3\hat{k}}{2\sqrt{5}}$ (d) None of these

8. $\vec{A} = 3\hat{i} - n\hat{j} + 2\hat{k}$ for what value of n, $\frac{\vec{A}}{5}$ may be unit vector.

9. Two unit vectors are inclined at some angle so that their resultant is also a unit vectors The angle may be

(a) $3i$ (b) 60°

(c) 120° (d) 150°

10. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - j + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ and a unit vectors parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$?

(a) $\frac{9}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{1}{\sqrt{22}}\hat{k}$

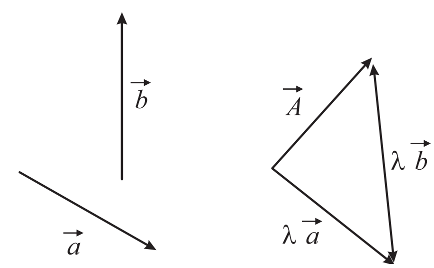
(b) $\frac{2}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$

(c) $\frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{3}{\sqrt{22}}\hat{k}$

(d) $\frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}k$

RESOLUTION OF VECTORS

If $\vec{a}$ and $\vec{b}$ be any two non-zero non parallel vectors in a plane and $\vec{A}$ be another vector in the same plane. $\vec{A}$ can be expressed as a sum of two vectors-one obtained by multiplying $\vec{a}$ by a real number and the other obtained by multiplying $\vec{b}$ by another real number.



$\vec{A} = \lambda\vec{a} + \mu\vec{b}$ (where λ and μ are real numbers)

We say that $\vec{A}$ has been resolved into two component vectors namely

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$\lambda\vec{a}$ and $\mu\vec{b}$

$\lambda\vec{a}$ and $\mu\vec{b}$ are along $\vec{a}$ and $\vec{b}$ respectively. Hence one can resolve a given vector into two component vectors along a set of two vectors – all the three lie in the same plane.

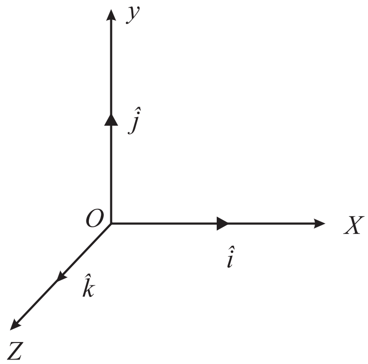
Similarly, in three dimensions, if $\vec{a}, \vec{b}$ and $\vec{c}$ are three non co-planer vectors then any vector can be expressed as a linear combinations of $\vec{a}, \vec{b}$ and $\vec{c}$

$\vec{A} = \lambda\vec{a} + \mu\vec{b} + \gamma\vec{c}$

where λ, μ and γ are real numbers

Resolution in Rectangular Components

It is convenient to resolve a general vector along axes of a rectangular coordinate system using vectors of unit magnitude, which we call as unit vectors. $\hat{i},\ \hat{j},\ \hat{k}$ are unit along x, y and z-axis as shown in figure below:



$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Resolution in two Dimension

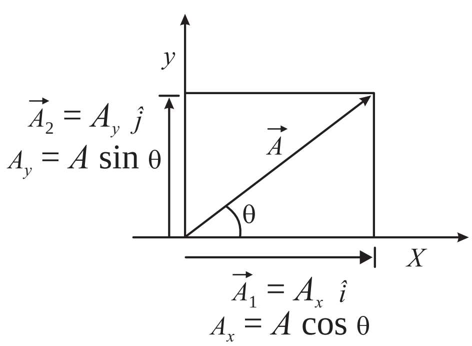
Consider a vector $\vec{A}$ that lies in xy plane as shown in figure,

$$\vec{A} = \vec{A}_1 + \vec{A}_2$$

$$\vec{A}_1 = A_x\hat{i},\ \ \vec{A}_2 = A_y\hat{j}$$

$$\Rightarrow\ \vec{A} = A_x\hat{i} + A_y\hat{j}$$

The quantities A_x and A_y are called x-and y-components of the vector $\vec{A}$.



A_x is itself not a vector but $A_x\hat{i}$ is a vector and so is $A_y\hat{j}$.

$$A_x = A\cos\theta \text{ and } A_y = A\sin\theta$$

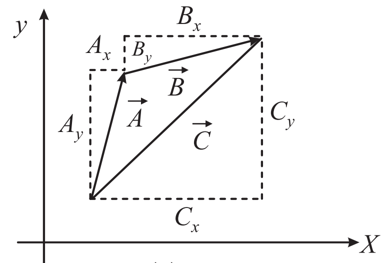
It's clear from above equation that a component of a vector can be positive, negative or zero depending on the value of θ. A vector $\vec{A}$ can be specified in a plane by two ways:

(*a*) Its magnitude A and the direction θ it makes with the x-axis; or

(*b*) its components A_x and A_y

$$A = \sqrt{A_x^2 + A_y^2}\ ,\ \theta = \tan^{-1}\frac{A_y}{A_x}$$

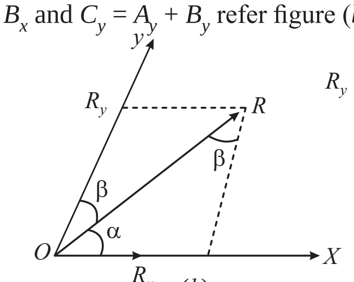
Note: If $A = A_x \Rightarrow A_y = 0$ and if $A = A_y \Rightarrow A_x = 0$ i.e., components of a vector perpendicular to itself is always zero. The rectangular components of each vector and those of the sum $\vec{C} = \vec{A} + \vec{B}$ are shown in figure.



(*a*)

We saw that $\vec{C} = \vec{A} + \vec{B}$ is equivalent to both

$C_x = A_x + B_x$ and $C_y = A_y + B_y$ refer figure (*b*)



(*b*)

Vector $\vec{R}$ has been resolved in two axes x and y not perpendicular to each other. Applying sine law in the triangle shown, we have

$$\frac{R}{\sin[180^\circ - (\alpha + \beta)]} = \frac{R_x}{\sin\beta} = \frac{R_y}{\sin\alpha}$$

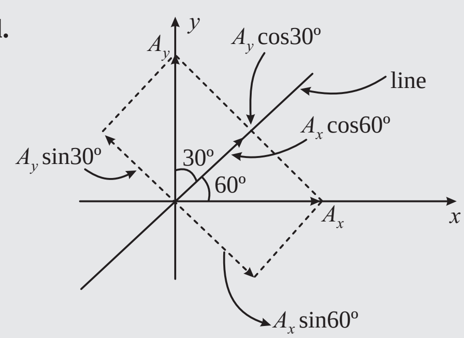
$$\text{or } R_x = \frac{R\sin\beta}{\sin(\alpha+\beta)} \text{ and } R_y = \frac{R\sin\alpha}{\sin(\alpha+\beta)}$$

If $\alpha + \beta = 90^\circ$, $R_x = R\sin\beta$ and $R_y = R\sin\alpha$

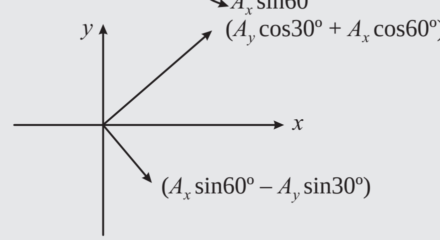
Train Your Brain

Example 6: Resolve the vector $A = A_x\hat{i} + A_y\hat{j}$ along and perpendicular to the line which make angle 60° with x-axis.

Sol.



⇒

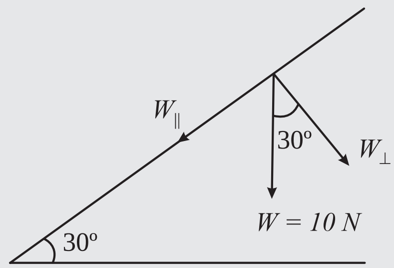


so the component along line = $|A_y\cos30^\circ + A_x\cos60^\circ|$

and perpendicular to line = $|A_x\sin60^\circ - A_y\sin30^\circ|$

Example 7: Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.

Sol. Component perpendicular to the plane



$$W_{\perp} = W\cos 30°$$

$$= (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}\ \text{N}$$

and component parallel to the plane

$$W_{\|} = W\sin 30° = (10)\left(\frac{1}{2}\right) = 5\ \text{N}$$

Example 8: Resolve horizontally and vertically a force $F = 8$ N which makes an angle of 45° with the horizontal.

Sol. Horizontal component of $\vec{F}$ is

$$F_H = F\cos 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\ \text{N}$$

and vertical component of $\vec{F}$ is

$$F_v = F\sin 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\ \text{N}$$

Concept Application

11. If two force each 5 N are simultaneously acting along x and y-axes, The magnitude and direction of resultant is

(*a*) $5\sqrt{2}, \pi/3$ (*b*) $5\sqrt{2}, \pi/4$

(*c*) $-5\sqrt{2}, \pi/3$ (*d*) $-5\sqrt{2}, \pi/4$

12. The x-component of a force of 50 N is 30 N, then what will be the y-component of the same force?

(*a*) 20 N (*b*) 30 N

(*c*) 40 N (*d*) 150 N

13. A vector $\vec{p}$, when added to the resultant of the vectors $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives a unit vector along y-axis, find the vector $\vec{p}$.

(*a*) $-5\hat{i} + 2\hat{j} - 4\hat{k}$ (*b*) $-3\hat{i} + 5\hat{j} - 3\hat{k}$

(*c*) $-5\hat{i} - 4\hat{k}$ (*d*) $-5\hat{i} + 6\hat{j} + 3\hat{k}$

14. The process of splitting a vector into two/more vectors in such a way that their combined effect is same as that of the given vector, may be called.

(*a*) Resolution of vector

(*b*) Unit vector

(*c*) Uniqueness of resolution

(*d*) None of these

15. A child pulls a rope attached to a stone with a force of 60N. The rope makes an angle of 60° to the horizontal ground. What is the effective force along the horizontal direction.

(*a*) 45.96 N (*b*) 40.3 N

(*c*) 28 N (*d*) 30 N

PROCEDURE TO SOLVE THE VECTOR EQUATION IN TWO DIMENSIONS

Consider a vector equation,

$$\vec{A} = \vec{B} + \vec{C} \quad \text{...(i)}$$

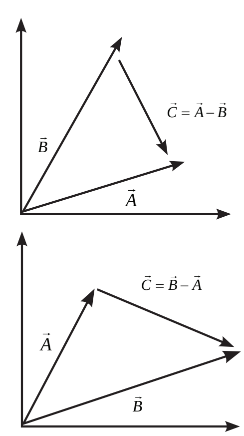
(*a*) There are 6 variables in this equation which are following:

1. Magnitude of $\vec{A}$ and its direction
2. Magnitude of $\vec{B}$ and its direction
3. Magnitude of $\vec{C}$ and its direction

(*b*) We can solve this equation if we know the value of 4 variables [**Note:** two of them must be directions]

(*c*) If we know the two direction of any two vectors then we will put them on the same side and other on the different side.

For example: If we know the directions of $\vec{A}$ and $\vec{B}$ and $\vec{C}$'s direction is unknown then we make equation as follows: $\vec{C} = \vec{A} - \vec{B}$ or $\vec{C} = \vec{B} - \vec{A}$, refer the given graphical representation.

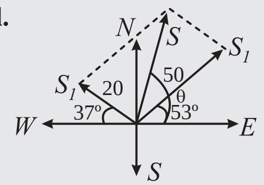


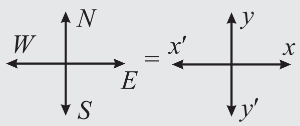
(*d*) Then we make vector diagram according to the equation and resolve the vectors to know the unknown values.

Train Your Brain

Example 9: Find the net displacement of a particle from its starting point if it undergoes two successive displacement given by $\vec{S}_1 = 20m$, 37° North of West, $\vec{S}_2 = 50m$, 53° North of East.

Sol.





$\vec{S} = \vec{S}_1 + \vec{S}_2$

$S_x = S_{1x} + S_{2x}$

$S_y = S_{1y} + S_{2y}$

$S_x = -20\cos 37° + 50\cos 53° = 14$

$S_y = 20\sin 37° + 50\sin 53° = 52$

$S = \sqrt{S_x^2 + S_y^2} = \sqrt{(14)^2 + (52)^2} = 53.85$

Angle from west-east axis (x-axis)

$$\tan\theta = \frac{S_y}{S_x} = \frac{52}{14} = \frac{26}{7}$$

$$\theta = \tan^{-1}\left(\frac{26}{7}\right)$$

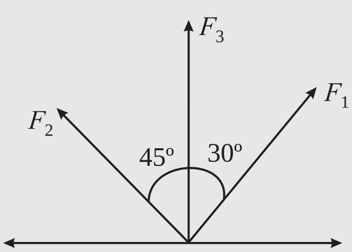
Example 10: Find the magnitude of F_1 and F_2. If F_1, F_2 make angle 30° and 45° with F_3 and magnitude of F_3 is 10 N. (given $\vec{F}_1 + \vec{F}_2 = \vec{F}_3$)

Sol. $|\vec{F}_3| = F_1\cos 30° + F_2\cos 45°$

and $F_2 \sin 45°$

$= F_1 \sin 30°$

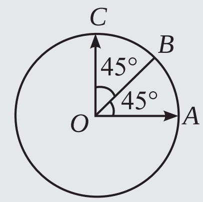
$$\Rightarrow 10 = \frac{\sqrt{3}F_1}{2} + \frac{F_2}{\sqrt{2}}, \quad \frac{F_2}{\sqrt{2}} = \frac{F_1}{2}$$



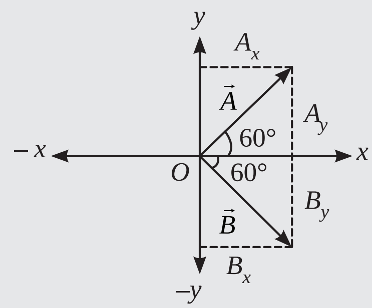
$$\Rightarrow F_1 = \frac{20}{\sqrt{3}+1} \text{ and } F_2 = \frac{20\sqrt{2}}{\sqrt{3}+1}$$

Concept Application

16. Find the resultant of the vectors $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ as shown in figure.



17. Vector $\vec{A}$ is 2 cm long and is 60° above the x-axis in the first quadrant, vector $\vec{B}$ is 2 cm long and is 60° below the x-axis in the fourth quadrant. Find $\vec{A} + \vec{B}$.



18. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$. Find a vector having the same magnitude as $\vec{B}$ but is parallel to $\vec{A}$.

19. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and equal to half of the magnitude of $\vec{B}$. Find angle between $\vec{A}$ and $\vec{B}$?

20. A car moves 40 m due east and turns towards north and moves 30 m then turns 45° east of north moves $20\sqrt{2}$m. The net displacement of car is (east is taken positive x-axis, North as positive y-axis.

(*a*) $50\hat{i} + 60\hat{j}$ (*b*) $60\hat{i} + 50\hat{j}$

(*c*) $30\hat{i} + 40\hat{j}$ (*d*) $40\hat{i} + 30\hat{j}$

21. An aeroplane is heading north east at a speed of 141.4 ms^{-1}. The northward component of its velocity is

(*a*) 141.4 ms^{-1} (*b*) 100 ms^{-1}

(*c*) zero (*d*) 50 ms^{-1}

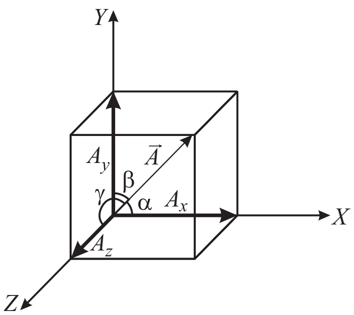
RECTANGULAR COMPONENTS IN THREE DIMENSION

A vector in three dimension can be expressed as

$$\vec{A} = A_x\hat{i} + \vec{A}_y\hat{j} + \vec{A}_z\hat{k}$$

where A_x, A_y and A_z are components of the vector along X, Y and Z axis respectively

The magnitude of the vector is given by $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



It the vector $\vec{A}$ makes angles α, β, γ with X, Y and Z axes respectively, then

$A_x = A\cos\alpha$ = Projection of a $\vec{A}$ on X-axis

$A_y = A\cos\beta$ = Projection of a $\vec{A}$ on Y-axis

$A_z = A\cos\gamma$ = Projection of a $\vec{A}$ on Z-axis

where cosα, cosβ and cosγ are called direction cosines. Now

$A_x^2 + A_y^2 + A_z^2 = A^2$

$\therefore\ A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma = A^2$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Therefore sum of squares of direction cosines is 1.

SCALAR PRODUCT OF TWO VECTORS

Definition

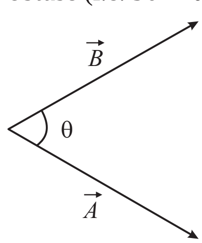
The scalar product (or dot product) of two vectors is defined as the product of the magnitude of two vectors with cosine of angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them, then their scalar product written as $\vec{A}.\vec{B}$ is defined as

$\vec{A}.\vec{B} = AB\cos\theta$

Properties

(i) It is always a scalar which is positive if an angle between the vectors is acute (i.e., < 90°) and negative if angle between them is obtuse (i.e. 90°< θ < 180°).



(ii) It is commutative, i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(iii) It is distributive, i.e. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iv) As by definition $\vec{A}.\vec{B} = AB\cos\theta$

The angle between the two vectors is given by

$\theta = \cos^{-1}\left[\dfrac{\vec{A}.\vec{B}}{AB}\right]$

(v) Scalar product of two vectors will be maximum when $\cos\theta = 1$, i.e. $\theta = 0°$, i.e., vectors are parallel $(\vec{A}.\vec{B})_{max} = AB$

(vi) Scalar product of two vectors will be zero when $|\cos\theta| = 0$, i.e. θ = 90°, $(\vec{A}.\vec{B})_{min} = 0$

i.e., if the scalar product of two nonzero vectors vanishes then the vectors are orthogonal.

(vii) The scalar product of a vector by itself is termed as self dot product and is given by

$(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2$

i.e., $A = \sqrt{\vec{A}.\vec{A}}$

(viii) In case of unit vector $\hat{n}$

$\hat{n}.\hat{n} = 1\times 1\times\cos 0 = 1$

so $\hat{n}.\hat{n} = \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ix) In case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ and

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 1\times 1\cos 90 = 0$

(x) In terms of components

$\vec{A}.\vec{B} = (\vec{i}A_x + \vec{j}A_y + \vec{k}A_z).(\vec{i}B_x + \vec{j}B_y + \vec{k}B_z)$

$= [A_xB_x + A_yB_y + A_ZB_z]$

Application

(i) **Work W:** In physics for constant force work done on a body is defined as, $W = \vec{F}.\vec{S} = FS\cos\theta$

i.e. work is the scalar product of force with displacement.

(ii) **Power P:** As $W = \vec{F}.\vec{S}$ or $\dfrac{dW}{dt} = \vec{F}.\dfrac{d\vec{S}}{dt} = \vec{F_0}\vec{V}$

i.e., power is the scalar product of force with velocity.

$\left[\text{As}\ \dfrac{dW}{dt} = P\ \text{and}\ \dfrac{d\vec{s}}{dt} = \vec{v}\right]$

(iii) To find angle between 2 vectors

(iv) To find projection of a given vector on another vector

(v) To check the orthogonal condition between vectors

Train Your Brain

Example 11: $\vec{A} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ are two vectors. The angle between them will be

(*a*) 0° (*b*) 45°

(*c*) 60° (*d*) 90°

Sol. (*d*) $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{|\vec{A}|.|\vec{B}|}$

$= \dfrac{2\times 4 + 4\times 2 - 4\times 4}{|\vec{A}|.|\vec{B}|} = 0$

$\therefore\ \theta = \cos^{-1}(0°)$

$\Rightarrow\ \theta = 90°$

Example 12: If vector $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are perpendicular to each other then value of λ be

(*a*) 25 (*b*) 26

(*c*) –26 (*d*) –25

Sol. (*b*) If $\vec{A}$ and $\vec{B}$ are perpendicular to each other then

$$\vec{A}.\vec{B} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$$

$$\text{So, } 2(-4) + 3(-6) + (-1)(-\lambda) = 0 \Rightarrow \lambda = +26$$

Example 13: A body, acted upon by a force of 50 N is displaced through a distance 10 meter in a direction making an angle of 60° with the force. The work done by the force be

(*a*) 200 J (*b*) 100 J

(*c*) 300 J (*d*) 250 J

Sol. (*d*) $W = \vec{F}.\vec{S} = FS\cos\theta$

$$= 50 \times 10 \times \cos 60° = 50 \times 10 \times \frac{1}{2} = 250\,\text{J}.$$

Example 14: A particle moves from position $3\hat{i} + 2\hat{j} - 6\hat{k}$ to $14\hat{i} + 13\hat{j} + 9\hat{k}$ due to a uniform force of $4\hat{i} + \hat{j} + 3\hat{k}\,N$. If the displacement is in meters then work done will be

(*a*) 100 J (*b*) 200 J

(*c*) 300 J (*d*) 250 J

Sol. (*a*) $S = \vec{r_2} - \vec{r_1}$

$$W = \vec{F}.\vec{S} = (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$$

$$= (4\times11 + 1\times11 + 3\times15) = 100\,\text{J}.$$

Concept Application

22. If $\vec{A} = \left(2\hat{i} + 3\hat{j}\right)$ and $\vec{B} = \left(\hat{i} - \hat{j}\right)$ then component of $\vec{A}$ perpendicular to vector $\vec{B}$ and in the same plane is

(*a*) $\frac{5}{2}\left(\hat{i} + \hat{j}\right)$ (*b*) $\frac{5}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

(*c*) $\frac{\sqrt{5}}{2}\left(\hat{i} + \hat{j}\right)$ (*d*) $\frac{\sqrt{5}}{2}\left(\hat{i} + \hat{k}\right)$

23. If $\vec{a}$ and $\vec{b}$ are two unit vector such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is.

(*a*) 120° (*b*) 90° (*c*) 60° (*d*) 45°

24. If $\vec{A} = 9\hat{i} - 7j + 5\hat{k}$ and $\vec{B} = 3\hat{i} - 2j - 6\hat{k}$ then the value of $(\vec{A} + \vec{B}).(\vec{A} - \vec{B})$

(*a*) 206 (*b*) 128 (*c*) 106 (*d*) –17

25. If $\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$, component of $\vec{B}$ along $\vec{A}$ is

(*a*) $\frac{\sqrt{14}}{38}$ (*b*) $\frac{28}{\sqrt{38}}$

(*c*) $\frac{\sqrt{28}}{38}$ (*d*) $\frac{14}{\sqrt{38}}$

26. The angle between the two vectors $-2\hat{i} + 3\hat{j} + \hat{k}$, and $\hat{i} + 2\hat{j} + 4\hat{k}$ is

(*a*) 0° (*b*) 90°

(*c*) 180° (*d*) 45°

27. If the vectors $\vec{A} = a\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = a\hat{i} - a\hat{j} + \hat{k}$ are perpendicular to each other then the positive value of 'a' is

(*a*) Zero (*b*) 1

(*c*) 2 (*d*) 3

VECTOR PRODUCT OF TWO VECTOR

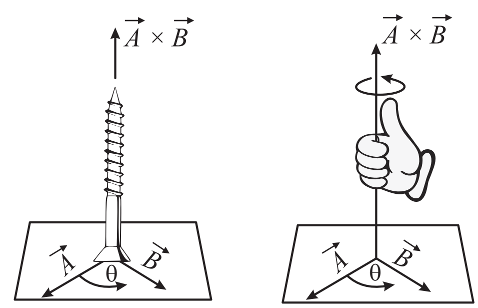
Definition

The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule.

$$\vec{C} = \vec{A} \times \vec{B}$$

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product written as $\vec{A} \times \vec{B}$ is a vector $\vec{C}$ defined by

$$\vec{C} = \vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$



The direction of $\vec{A} \times \vec{B}$ i.e. $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ and in the sense of advance of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A} \times \vec{B}$ i.e. $\vec{C}$.

Properties

(i) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(ii) Vector product of two vectors is not commutative,

i.e., $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ [but $=-\vec{B}\times\vec{A}$]

Here it is worthy to note that $|\vec{A}\times\vec{B}|=|\vec{B}\times\vec{A}|=AB\sin\theta$

i.e., in case of vector $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ magnitudes are equal but directions are opposite.

(iii) The vector product is distributive when the order of the vectors is strictly maintained, i.e.

$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

(iv) As by definition of vector product of two vectors $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$

So $|\vec{A}\times\vec{B}|=AB\sin\theta$ i.e. $\theta=\sin^{-1}\left[\dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}\right]$

(v) The vector product of two vectors will be maximum when $\sin\theta=1$, i.e., $\theta = 90°$

$[\vec{A}\times\vec{B}]_{\max}=AB\,\hat{n}$

Thus vector product is maximum if the vectors are orthogonal.

(vi) The vector product of two non-zero vectors will be minimum when $|\sin\theta|=0$, i.e., $\theta=0^o$ or $180°$

$[\vec{A}\times\vec{B}]_{\min}=0$

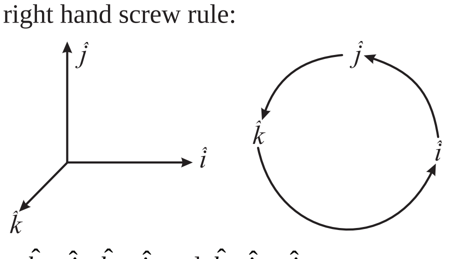
i.e. if the vector product of two non-zero vectors vanishes, if vectors are collinear.

(vii) The self cross product, i.e., product of a vector by itself vanishes, i.e., null vector $\vec{A}\times\vec{A}=AA\sin 0^o\,\hat{n}=\vec{0}$

(viii) In case of unit vector $\hat{n}\times\hat{n}=\vec{0}$.

So that $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\vec{0}$

(ix) In case of orthogonal unit vectors, $\hat{i},\hat{j},\hat{k}$ in accordance with right hand screw rule:



$\hat{i}\times\hat{j}=\hat{k},\ \ \hat{j}\times\hat{k}=\hat{i}$ and $\hat{k}\times\hat{i}=\hat{j}$

And as cross product is not commutative,

$\hat{j}\times\hat{i}=-\hat{k}$, $\hat{k}\times\hat{j}=-\hat{i}$ and $\hat{i}\times\hat{k}=-\hat{j}$

(x) In terms of components $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}$

$=\hat{i}(A_yB_z-A_zB_y)+\hat{j}(A_zB_x-A_xB_z)+\hat{k}(A_xB_y-A_yB_x)$

Application

Since vector product of two vectors is a vector, some of the vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field can be expressed as the vector product of two vectors. It is well - established in physics that:

(i) Torque $\vec{\tau}=\vec{r}\times\vec{F}$

(ii) Angular momentum $\vec{L}=\vec{r}\times\vec{p}$

(iii) Velocity $\vec{v}=\vec{\omega}\times\vec{r}$

(iv) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F}=q(\vec{v}\times\vec{B})$

(v) Torque on a dipole in a field $\overrightarrow{\tau_E}=\vec{p}\times\vec{E}$ and $\overrightarrow{\tau_B}=\vec{M}\times\vec{B}$

(vi) To find the unit vector $\perp$ to two given vectors $\vec{A},\vec{B}$

U.V $(\hat{n})=\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

Train Your Brain

Example 15: If two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $-4\hat{i}-6\hat{j}-\lambda\hat{k}$ are parallel to each other then value of λ is

(*a*) 0 (*b*) 2

(*c*) 3 (*d*) 4

Sol. (*b*) Let $\vec{A}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{B}=-4\hat{i}-6\hat{j}+\lambda\hat{k}$

$\vec{A}$ and $\vec{B}$ are parallel to each other $\dfrac{a_1}{b_1}=\dfrac{a_2}{b_2}=\dfrac{a_3}{b_3}$

i.e. $\dfrac{2}{-4}=\dfrac{3}{-6}=\dfrac{-1}{\lambda}$

$\Rightarrow \lambda=2$

Example 16: If $\vec{A}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}-2\hat{j}+4\hat{k}$ then value of $|\vec{A}\times\vec{B}|$ will be

(*a*) $8\sqrt{2}$ (*b*) $8\sqrt{3}$

(*c*) $8\sqrt{5}$ (*d*) $5\sqrt{8}$

Sol. (*b*) $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&1&2\\2&-2&4\end{vmatrix}$

$=(1\times4-2\times-2)\hat{i}+(2\times2-4\times3)\hat{j}+(3\times(-2)-1\times2)\hat{k}$

$=8\hat{i}-8\hat{j}-8\hat{k}$

$\therefore$ Magnitude of $\vec{A}\times\vec{B}$

$=|\vec{A}\times\vec{B}|=\sqrt{(8)^2+(-8)^2+(-8)^2}$

$=8\sqrt{3}$

Example 17: In above example a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ will be

(*a*) $+\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$ (*b*) $-\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

(*c*) Both (*a*) and (*b*) (*d*) None of these

Sol. (*c*) $\hat{n}=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}=\frac{8\hat{i}-8\hat{j}-8\hat{k}}{8\sqrt{3}}=\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

There are two unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$ they are $\hat{n}=\pm\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

Example 18: The angle between the vectors $\vec{A}$ and $\vec{B}$ is θ. The value of the triple product $\vec{A}.(\vec{B}\times\vec{A})$ is

(*a*) A^2B (*b*) Zero

(*c*) $A^2B\sin\theta$ (*d*) $A^2B\cos\theta$

Sol. (*b*) Let $\vec{A}.(\vec{B}\times\vec{A})=\vec{A}.\vec{C}$

Here $\vec{C}=\vec{B}\times\vec{A}$ is perpendicular to both vector $\vec{A}$ and $\vec{B}$

$\therefore\ \vec{A}.\vec{C}=0$

Concept Application

28. Find the values of x and y for which vectors $\vec{A}=6\hat{i}+x\hat{j}-2\hat{k}$ and $\vec{B}=5\hat{i}-6\hat{j}-y\hat{k}$ may be parallel

(*a*) $x=0,\ y=\frac{2}{3}$ (*b*) $x=\frac{-36}{5},\ y=\frac{5}{3}$

(*c*) $x=\frac{-15}{3},\ y=\frac{23}{5}$ (*d*) $x=\frac{36}{3},\ y=\frac{15}{4}$

29. Two sides of a triangle are given by $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+2\hat{j}+3\hat{k}$, then area of triangle is

(*a*) $\sqrt{26}$ (*b*) $\sqrt{\frac{26}{2}}$

(*c*) $\sqrt{46}$ (*d*) 26

30. When a force $\left(8\hat{i}+4\hat{j}\right)$ newton displaces a particle through $\left(3\hat{i}-3\hat{j}\right)$ metre, the power is 0.6 W. The time of action of the force is

(*a*) 20s (*b*) 7.2s

(*c*) 72s (*d*) 2s

31. Find the unit vector perpendicular to both. $\vec{A}=3\hat{i}+2\hat{j}-\hat{k}$ and $\vec{B}=\hat{i}-\hat{j}+\hat{k}$

32. If $\vec{F}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{r}=2\hat{i}-\hat{j}+\hat{k}$ find $\vec{r}\times\vec{F}$

33. The adjacent sides of a parallelogram are $\vec{A}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{B}=-2\hat{i}+4\hat{j}-\hat{k}$. What is the area of the parallelogram?

(*a*) 4 units

(*b*) 7 units

(*c*) $\sqrt{5}$ units

(*d*) $\sqrt{8}$ units

34. What is the condition for the vectors $2\hat{i}+3\hat{j}-4\hat{k}$ and $3\hat{i}-a\hat{j}+b\hat{k}$ to be parallel?

(*a*) $a=-9/2,\ b=-6$

(*b*) $a=-6,\ b=-9/2$

(*c*) $a=4,\ b=5$

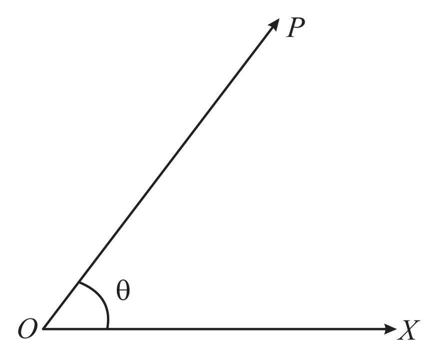
(*d*) $\sqrt{8}$ units

BASIC MATHEMATICS

Trigonometry

(i) Angle: Consider a revolving line OP. Suppose that it revolves in anticlockwise direction starting from its initial position OX.

The angle is defined as the amount of revolution that the revolving line makes with its initial position. From fig. the angle covered by the revolving line OP is given by $\theta=\angle POX$. The angle is positive, if it is traced by the revolving line in anticlockwise direction and is **negative,** if it is covered in clockwise direction.



1 right angle = 90º (degrees)

1º = 60′ (minutes)

1′ = 60″ (seconds)

In circular system

1 right angle = $\frac{\pi}{2}$ rad (radian)

One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.

1 rad = 180º/π ≈ 57º 17′ 45″ ≈ 57.3º

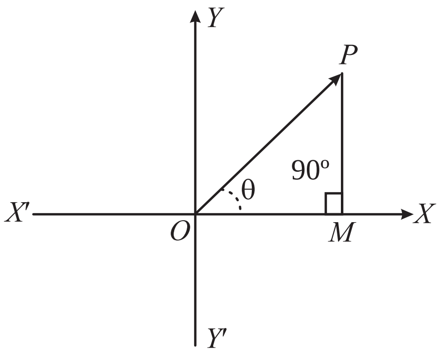
(ii) Trigonometrical Ratios (or T-ratios): Consider the two fixed lines XOX' and YOY' intersecting at right angles to each other at point O as shown in Fig. Then.

(*a*) Point O is called origin.

(*b*) $\underline{XOX'}$ and YOY' are known as X-axis and Y-axis respectively.

(*c*) Portions XOY, YOX', $X'OY'$ and $Y'OX$ are called I, II, III, and IV quadrant respectively. Consider that the revolving line OP has traced out angle θ (in I quadrant) in anticlockwise direction.

From P, drop PM perpendicular to OX. Then, side OP (in front of right angle) is called hypotenuse, side MP (in front of angle θ) is called **opposite side or perpendicular** and side OM (making angle θ with hypotenuse) is called **adjacent side or base**.



The three sides of a right angled triangle are connected to each other through six different ratios, called trigonometric ration or simply T-ratios.

These are defined as:

1. $\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{MP}{OP}$ (read as sine of angle θ)
2. $\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{OM}{OP}$ (read as cosine of angle θ)
3. $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{MP}{OM}$ (read as tangent of angle θ)
4. $\cot\theta = \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{OM}{MP}$ (read as cotangent of angle θ)
5. $\sec\theta = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{OP}{OM}$ (read as secant of angle θ)
6. $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}} = \dfrac{OP}{MP}$ (read as cosecant of angle θ)

It can be easily proved that:

1. (*a*) $\text{cosec}\ \theta = \dfrac{1}{\sin\theta}$

(*b*) $\sec\ \theta = \dfrac{1}{\cos\theta}$

(*c*) $\cot\ \theta = \dfrac{1}{\tan\theta}$

2. (*a*) $\sin^2\theta + \cos^2\theta = 1$

(*b*) $1 + \tan^2\theta = \sec^2\theta$

(*c*) $1 + \cot^2\theta = \text{cosec}^2\theta$

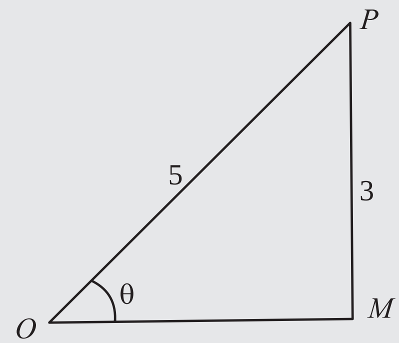
Train Your Brain

Example 19: Given sin θ = 3/5. Find all the other T-ratios, if θ lies in the first quadrant.

Sol. In Δ OMP,

sin θ = 3/5

∴ $MP = 3$ and $OP = 5$



Hence, $OM = \sqrt{(5)^2 - (3)^2}$

$= \sqrt{25-9} = \sqrt{16} = 4$

Now, $\cos\theta = \dfrac{OM}{OP} = \dfrac{4}{5}$, $\tan\theta = \dfrac{MP}{OM} = \dfrac{3}{4}$, $\cot\theta \dfrac{OM}{MP} = \dfrac{4}{3}$,

$\sec\theta = \dfrac{OP}{OM} = \dfrac{5}{4}$, $\text{cosec}\,\theta = \dfrac{OP}{MP} = \dfrac{5}{3}$

Example 20: If $\sec\theta = \dfrac{5}{3}$, find all the other T-ratios.

Sol. $\sin\theta = \dfrac{4}{5}$, $\cos\theta = \dfrac{3}{5}$, $\tan\theta = \dfrac{4}{3}$, $\cot\theta = \dfrac{3}{4}$,

$\text{cosec}\theta = \dfrac{5}{4}$

Concept Application

35. If $\cos\theta = \dfrac{4}{5}$, find the value of

(*a*) tan θ

(*b*) $\dfrac{\sin\theta - \cos\theta}{\text{cosec}\ \theta}$

36. If $\tan^2\theta = \dfrac{1}{4}$, find the value of sin θ + cos θ

37. If $\text{Sin}\,\theta = \dfrac{1}{3}$, find the value of $\dfrac{\cos^2\theta}{\tan\theta}$

(iii) *T*-Ratios of a few standard angles: The *T*-ratios of a few standard angles ranging from 0º to 180º are given in the following table

Angle (θ)	0º	30º	45º	60º	90º	120º	135º	150º	180º
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0

(iv) T-ratios of angles associated with multiples of 90º: The angles such as (90º – θ), (90º + θ), (180 – θ), (180º + θ), (270º – θ), (270º + θ), etc. are called angles allied to angle θ.

The *T*-ratios of the following allied angle as are commonly used:

1.	(*a*) sin (–θ) = – sin θ	(*b*) cos (–θ) = cosθ	(*c*) tan (–θ) = – tanθ
2.	(*a*) sin (90º – θ) = cos θ	(*b*) cos (90º – θ) = sinθ	(*c*) tan (90º – θ) = cotθ
3.	(*a*) sin (90º + θ) = cosθ	(*b*) cos (90º + θ) = – sin θ	(*c*) tan (90º + θ) = – cotθ
4.	(*a*) sin (180º – θ) = sinθ	(*b*) cos (180º – θ) = – cosθ	(*c*) tan (180º – θ) = – tanθ
5.	(*a*) sin (180º + θ) = – sin θ	(*b*) cos (180º + θ) = – cosθ	(*c*) tan (180º + θ) = tanθ
6.	(*a*) sin (270º – θ) = – cos θ	(*b*) cos (270º – θ) = – sin θ	(*c*) tan (270º – θ) = cot θ
7.	(*a*) tan (270º + θ) = – cos θ	(*b*) cos (270º + θ) = sin θ	(*c*) tan (270º + θ) = – cot θ

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Example 21: Find the value of

(i) cos (–60º) (ii) tan 210º

(iii) sin 300º (iv) cos 120º

Sol. (i) cos (–60º) = cos 60º = $\frac{1}{2}$

(ii) tan 210º = tan (180º + 30º) = tan 30º = $\frac{1}{\sqrt{3}}$

(iii) sin 300º = sin (270º + 30º) = – cos 30º = $-\frac{\sqrt{3}}{2}$

(iv) cos 120º = cos (180º – 60º) = – cos 60º = $-\frac{1}{2}$

Example 22: Find the values of the following:

(i) tan(–30º) (ii) sin 120º

(iii) sin 330º (iv) cos 150º

(v) sin 270º (vi) cos 270º

Sol. (i) $-\frac{1}{\sqrt{3}}$ (ii) $\frac{\sqrt{3}}{2}$

(iii) $-\frac{1}{2}$ (iv) $-\frac{\sqrt{3}}{2}$

(v) –1 (vi) 0

Concept Application

38. Find the value of sin 210° + cos (–30)

39. Find the value of tan 60°.

40. Find the value of $\frac{\cos 30°}{\cos 33°}$

(v) A few important trigonometric formulae:

1. Addition Formulae:

(*a*) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(*b*) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

(*c*) $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$

2. Subtraction Formulae:

(*a*) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(*b*) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(*c*) $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$

3. Multiplication Formulae:

(*a*) $\sin 2A = 2\sin A\cos A$

(*b*) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

(*c*) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

DIFFERENTIATION

(i) **Finite difference:** The finite difference between two values of a physical quantity is represented by Δ notation.

For example: Difference in two values of y is written as Δy as given in the table below

y_2	100	100	100
y_1	50	99	99.5
$\Delta y = y_2 - y_1$	50	1	0.5

(ii) **Infinitely small difference:** The infinitely small difference means very-very small difference. And this difference is represented by 'd' notation instead of 'Δ'.

For example infinitely small difference in the values of y is written as 'dy' if $y_2 = 100$ and $y_1 = 99.99999999$........

then $dy = 0.000000$..................00001

DEFINITION OF DIFFERENTIATION

Another name for differentiation is derivative. Suppose y is a function of x or $y = f(x)$

Differentiation of y with respect to x is denoted by symbol $f'(x)$ where $f'(x) = \dfrac{dy}{dx}$

dx is very small change in x and dy is corresponding very small change in y.

NOTATION: There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are these:

y'	"y prime" or "y dash"	Nice and brief but does not name the independent variable
$\dfrac{dy}{dx}$	"dy by dx"	Names the variables and used d for derivative
$\dfrac{df}{dx}$	"df by dx"	Emphasizes the function's name.
$\dfrac{d}{dx}f(x)$	"d by dx of f"	Emphasizes the idea that differentiation is an operation performed on f.
$d(f)$	"d of f"	A common operator notation.
$\dot{y}$	"y dot"	One of Newton's notations, now common for time derivatives i.e. $\dfrac{dy}{dt}$.

SLOPE OF A LINE

It is the tan of angle made by a line with the positive direction of x-axis, measured in anticlockwise direction.

Slope = $\tan\theta$

(In 1st quadrant $\tan\theta$ is +ve and 2nd quadrant $\tan\theta$ is –ve)

In figure (a)slope is positive

In figure (b)slope is negative

$\theta < 90°$ (1st quadrant)

$\theta > 90°$ (2nd quadrant)

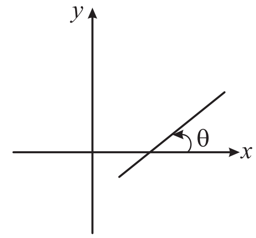


figure (a)

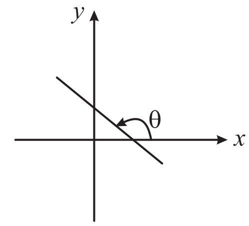
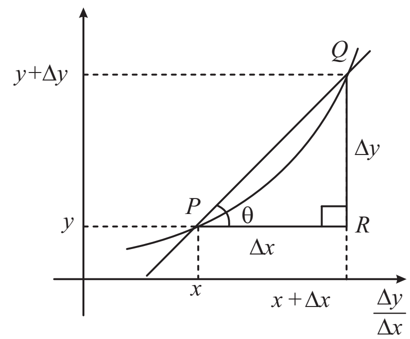


figure (b)

AVERAGE RATES OF CHANGE

Given an arbitrary function $y = f(x)$ we calculate the average rate of change of y with respect to x over the interval $(x, x + \Delta x)$ by dividing the change in value of y, i.e. $\Delta y = f(x + \Delta x) - f(x)$, by length of interval Δx over which the change occurred.



The average rate of change of y with respect to x over the interval $[x, x + \Delta x] = \dfrac{\Delta y}{\Delta x} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Geometrically, $\dfrac{\Delta y}{\Delta x} = \dfrac{QR}{PR} = \tan\theta$ = Slope of the line PQ therefore we can say that average rate of change of y with respect to x is equal to slope of the line joining P and Q.

THE DERIVATIVE OF A FUNCTION

We know that, average rate of change of y w.r.t. x is

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

If the limit of this ratio exists as $\Delta x \to 0$, then it is called the derivative of given function $f(x)$ and is denoted as

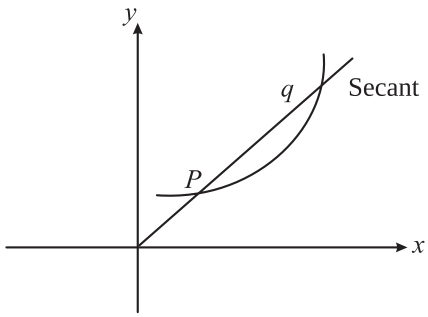
$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

GEOMETRICAL MEANING OF DIFFERENTIATION

The geometrical meaning of differentiation is very much useful in the analysis of graphs in physics. To understand the geometrical meaning of derivatives we should have knowledge of secant and tangent to a curve

Secant and tangent to a curve

A secant to a curve is a straight line, which intersects the curve at any two points.

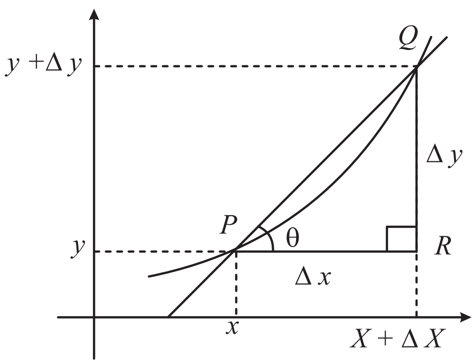


TANGENT

A tangent is a straight line, which touches the curve at a particular point. Tangent is a limiting case of secant which intersects the curve at two overlapping points.

In the figure (*a*) shown, if value of Δx is gradually reduced then the point *Q* will move nearer to the point *P*. If the process is continuously repeated (figure (*b*)) value of Δx will be infinitely small and secant *PQ* to the given curve will become a tangent at point *P*.

Therefore $\lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \frac{dy}{dx} = \tan\theta$



gure (*a*)

we can say that differentiation of *y* with respect to *x*, i.e. $\left(\frac{dy}{dx}\right)$

is equal to slope of the tangent at point *P* (*x*, *y*) or $\tan\theta = \frac{dy}{dx}$

(From fig. 1, the average rate of change of *y* from *x* to $x + \Delta x$ is identical with the slope of secant *PQ*.)

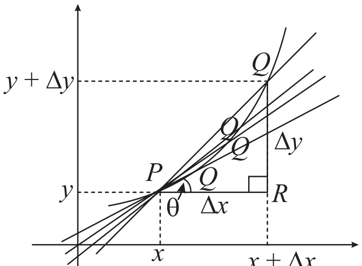


Figure (*b*)

THEOREMS OF DIFFERENTIATION

1. If *c* is constant, then $\frac{d}{dx}(c) = 0$
2. If $y = cu$, where c is a constant and *u* is a function of *x*, then
   $$\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$$
3. If $y = u \pm v \pm w$, where, *u*, *v* and *w* are functions of *x*, then
   $$\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w)$$
   $$= \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$
4. If $y = u\,v$ where *u* and *v* are function of *x*, then
   $$\frac{dy}{dx} = \frac{d}{dx}(uv)$$
   $$= u\frac{dv}{dx} + v\frac{du}{dx}$$
5. If $y = \frac{u}{v}$, where *u* and *v* are functions of *x*, then
   $$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right)$$
   $$= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$
6. If $y = x^n$ where *n* is a real number, then
   $$\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$$

Formulae for differential coefficients of trigonometric, logarithmic and exponential functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$
7. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
8. $\frac{d}{dx}(e^x) = e^x$

Train Your Brain

Example 23: Find $\frac{dy}{dx}$, when

(i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$

(iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Sol. (i) Here, $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

(ii) Here, $y = x^5 + x^4 + 7$

$$\frac{dy}{dx} = \frac{d}{dx}(x^5 + x^4 + 7) = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^4) + \frac{d}{dx}(7)$$

$= 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$

(iii) Here, $y = x^2 + 4x^{-1/2} - 3x^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$$

$$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2})$$

$$= 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3} = 2x - 2x^{-3/2} + 6x^{-3}$$

Example 24: If $3y = 4x^2 - 5$ find $\frac{dy}{dx}$

Sol. $y = \frac{4}{3}x^2 - \frac{5}{3} \Rightarrow \frac{dy}{dx} = \frac{8x}{3} - \frac{5}{3} = \frac{8x-5}{3}$

Example 25: $\sqrt{y} = x - 1$, find $\frac{dy}{dx}$

Sol. $y = x^2 - 2x + 1 \Rightarrow \frac{dy}{dx} = 2x - 2$

Concept Application

41. If acceleration $= \frac{dv}{dt}$. Find acceleration at $t = 1$ sec from $v = 3t^2 - 1$

42. $y = \sqrt{x} - 3x^2$, find $\frac{dy}{dx}$

43. $y = \frac{1}{x^2} - 2x$

44. $y = \frac{3x-5}{x^2}$ find $\frac{dy}{dx}$

45. $x = 9y^2$ find $\frac{dy}{dx}$ and $\frac{dx}{dy}$

Find $\frac{dy}{dx}$ for the following

46. $y = x^{7/2}$

47. $y = x^{-3}$

48. $y = x$

49. $y = x^5 + x^3 + 4x^{1/2} + 7$

50. $y = 5x^4 + 6x^{3/2} + 9x$

51. $y = ax^2 + bx + c$

52. $y = 3x^5 - 3x - \frac{1}{x}$

53. Given $S = t^2 + 5t + 3$, find $\frac{dS}{dt}$

54. Given $S = ut + \frac{1}{2}at^2$, where u and a are constants. Obtain the value of $\frac{dS}{dt}$.

55. The area of a blot of in k is growing such that after t seconds, its area is given by $A = (3t^2 + 7)$ cm^2. Calculate the rate of increase of area at $t = 5$ seconds.

56. The area of a circle is given by $A = \pi r^2$, where r is the radius. Calculate the rate of increase of area w.r.t. radius.

Obtain the differential coefficient (differentiation) of the following:

57. $(x - 1)(2x + 5)$

58. $(9x^3 - 8x + 7)(3x^5 + 5)$

59. If $t = \sqrt{s} - 1$, find the velocity at $t = 2$ sec

60. If $S = 3t^2$, find double differentiation of s w. r. to t.

61. Velocity of a body is given by $v = 3t^2 - 4t$, find rate of change of velocity w.r.t. to time at $t = 1$ sec.

62. If acceleration $= \frac{dv}{dt}$. Find acceleration at $t = 10$ sec from $v = 3t^2 + t$

63. $\frac{1}{2x+1}$

64. $\frac{3x+4}{4x+5}$

65. $\frac{x^2}{x^3+1}$

INTEGRATION

(i) Integration as the inverse process of differentiation: Integration is the process of finding the function, whose derivative is given. For this reason, the process of integration is the inverse process of differentiation.

Consider a function $f(x)$, whose derivative w.r.t. x is another function $f'(x)$ i.e. $\frac{d}{dx}(f(x)) = f'(x)$

If differentiation of $f(x)$ w.r.t. x is equal to $f'(x)$, then $f(x) + c$ is called the integration of $f'(x)$, where c is called the constant of integration.

Symbolically, it is written as $\int f'(x)dx = f(x) + c$

Here, $f'(x)\ dx$ is called element of integration and $\int$ is called indefinite integral operator. Let us proceed to obtain integral of x^n w.r.t. x. $\frac{d}{dx}(n^{n+1}) = (n+1)x^n$

Since the process of integration is the inverse process of differentiation,

$\int (n+1)x^n dx - x^{n+1}$ or $(n+1)\int x^n dx - x^{n+1}$

or $\int x^n dx = \frac{x^{n+1}}{n+1}$

The above formula holds for all values of n, except $n = -1$.

It is because, for $n = -1$, $\int x^n dx = \int x^{-1} dx = \int \frac{1}{x} dx$

Since $1/x$ is differential coefficient of $\log_e x$

i.e. $\frac{d}{dx}(\log_e x) = \frac{1}{x} \Rightarrow \therefore \int \frac{1}{x} dx = \log_e x$

Similarly, the formulae for integration of some other functions can be obtained if we know the differential coefficients of various functions.

(ii) Few basic formulae of integration: Following are a few basic formulae of integration:

1. $\int x^n dx = \frac{x^{x+1}}{n+1} + c$, Provided $n \neq -1$
2. $\int \sin x\, dx = -\cos x + c$ $\quad (\because \frac{d}{dx}(\cos x) = -\sin x)$
3. $\int \cos x\, dx = \sin x + c$ $\quad (\because \frac{d}{dx}(\sin x) = \cos x)$
4. $\int \frac{1}{x} dx = \log_e x + c$ $\quad (\because \frac{d}{dx}(\log_e x) = \frac{1}{x})$
5. $\int \frac{f'(x)}{f(x)} dx = \log_e f(x) + c$
6. $\int e^x\, dx = e^x + c$ $\quad (\because \frac{d}{dx}(e^x) = e^x)$

Train Your Brain

Example 26: Integrate w.r.t. x:

(i) $x^{11/2}$ (ii) x^{-7} (iii) $x^{p/q}$

Sol. (i) $\int x^{11/2} dx = \frac{x^{\frac{11}{2}+1}}{\frac{11}{2}+1} + c = \frac{2}{13} x^{13/2} + c$

(ii) $\int x^{-7} dx = \frac{x^{-7+1}}{-7+1} + c = -\frac{1}{6} x^{-6} + c$

(iii) $\int x^{\frac{p}{q}} dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} + c = \frac{q}{p+q} x^{(p+q)/q} + c$

Example 27: Find integer w. r. to x, for $\left(\frac{\sqrt{x}}{x^2}\right)$

Sol. $\int \frac{\sqrt{x}}{x} dx = \int x^{-2+\frac{1}{2}} dx = \int x^{\frac{-3}{2}} dx = \int \frac{x^{\frac{-3}{2}+1}}{-\frac{3}{2}+1}$

$= \frac{-2}{\sqrt{x}} + c$

Example 28: $\int (x+1)^2 dx$

Sol. $\int x^2 dx + 2\int x dx + 1\int dx$

$\frac{x^3}{3} + 2\frac{x^2}{2} + x + c$

Concept Application

66. $\int \left(x\sqrt{x} - \sin x^2\right) dx$	**67.** $\int \frac{6x^2+4}{x^2} dx$
68. $\int (\sin x + \cos x) dx$	**69.** $\int \left(\frac{1}{x^3} - \cos x\right) dx$
70. $\int \cos 3x\, dx$	**71.** $\left(\frac{1}{x^2} + 3\sin 2x\right) dx$

(iii) Definite integrals: When a function is integrated between a lower limit and an upper limit, it is called a definite integral. If $\frac{d}{dx}(f(x) = f'(x))$, then $\int f'(x)dx$ is called indefinite integral and $\int_a^b f'(x)dx$ is called definite integral

Here, a and b are called lower and upper limits of the variable x.

After carrying out integration, the result is evaluated between upper and lower limits as explained below:

$$\int_a^b f'(x)dx = | f(x) |_a^b = f(b) - f(a)$$

Train Your Brain

Example 29: Evaluate the integral: $\int_1^5 x^2 dx$

Sol. $\int_1^5 x^2 dx = \left|\frac{x^3}{3}\right|_1^5 = \frac{1}{3} | x^3 |_1^5 = \frac{1}{3}((5)^3 - (1)^3)$

$= \frac{1}{3}(125-1) = \frac{124}{3}$

Example 30: $\int_0^{\pi}(\sin x+\cos x)\,dx$

Sol. $\int_0^{\pi}\sin x dx+\int_0^{\pi}\cos x dx$

$\left|-\cos x\right|_0^{\pi}+\left|\sin x\right|_0^{\pi}$

$\left|-\cos\pi-\cos 0\right|+\sin\pi^0-\sin 0^0 = 2$

Concept Application

72. $\int_{x=-1}^{x=1}(ax^2+b)\,dx$

73. $\int_{x=0}^{x=2}\frac{2x}{4x^2-1}dx$

74. $\int_{x=0}^{\pi/2}\sin(3x-1)\,dx$

75. $\int_{x=A}^{B}\cos(2x-30°)\,dx$

76. $\int x^{15}dx$

77. $\int x^{-3/2}dx$

78. $\int(3x^{-7}+x^{-1})dx$

79. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx$

80. $\int\left(x+\frac{1}{x}\right)dx$

81. $\int\left(\frac{a}{x^2}+\frac{b}{x}\right)dx$ (Where a and b are constant)

82. $\int_R^{\infty}\frac{GMm}{r^2}dr$

83. $\int_{r_1}^{r_2}-k\frac{q_1q_2}{r^2}dr$

84. $\int_u^v M\upsilon d\upsilon$

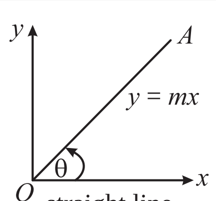
85. $\int_0^{\infty}x^{-1/2}dx$

86. $\int_0^{\pi/2}\sin x\,dx$

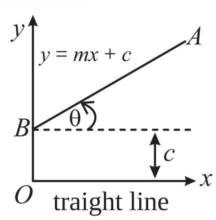
87. $\int_0^{\pi/2}\cos x\,dx$

88. $\int_{-\pi/2}^{\pi/2}\cos x\,dx$

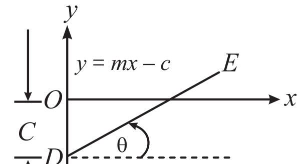
SOME STANDARD GRAPHS AND THEIR EQUATIONS



straight line



traight line



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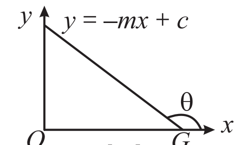
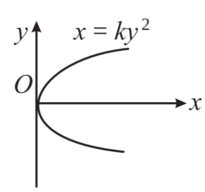
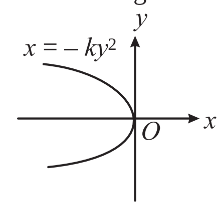


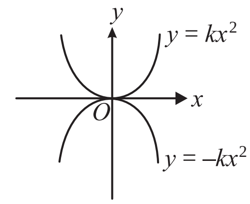
straight line



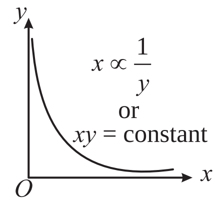
Parabola



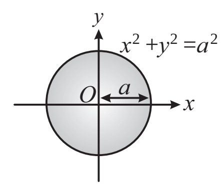
Parabola



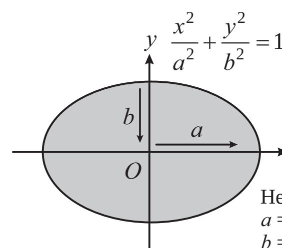
Parabola



Rectangular hyperbola

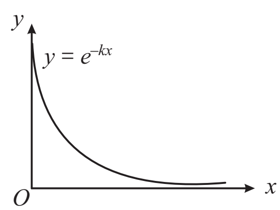


Circle



Here
a = semi major axis
b = semi minor axis
$b < a$

Ellipse



Exponential decay

Small Angle Approximation

θ is very small and it must be in radian when you are taking approximation.

$\sin\theta\simeq\theta$, $\tan\theta\simeq\theta$

$\sin\theta\simeq\tan\theta$

$\cos\theta\simeq 1$

Short Notes

Quadratic Equation

Roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\dfrac{b}{a}$

Product of roots $x_1 x_2 = \dfrac{c}{a}$

Binomial Approximation

If $x << 1$, then $(1 + x)^n \approx 1 + nx$ and $(1 - x)^n \approx 1 - nx$

Logarithm

$\log mn = \log m + \log n$

$\log m/n = \log m - \log n$

$\log m^n = n \log m$

$\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$

Componendo and Dividendo law

If $\dfrac{p}{q} = \dfrac{a}{b}$ then $\dfrac{p+q}{p-q} = \dfrac{a+b}{a-b}$

Arithmetic Progression-AP Formula

$a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)d,$

here d = common difference

Sum of n terms $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

Note:

(i) $1 + 2 + 3 + 4 + 5 \ldots + n = \dfrac{n(n+1)}{2}$

(ii) $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Geometrical Progression-GP Formula

$a, ar, ar^2, \ldots$ here, r = common ratio

Sum of n terms $S_n = \dfrac{a(1-r^n)}{1-r}$

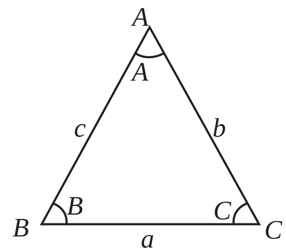
Sum of ∞ terms $S_\infty = \dfrac{a}{1-r}$

Sine law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine law

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Maxima and Minima of a Function y = f(x)

- For maximum value $\dfrac{dy}{dx} = 0$ & $\dfrac{d^2y}{dx^2} = -\text{ve}$
- For minimum value $\dfrac{dy}{dx} = 0$ & $\dfrac{d^2y}{dx^2} = +\text{ve}$

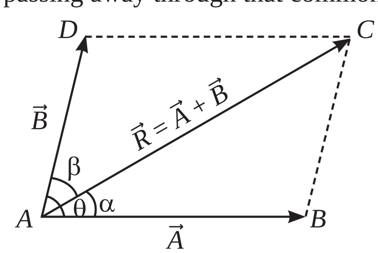
Average of a Varying Quantity

If $y = f(x)$ then $< y > = \bar{y} = \dfrac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx} = \dfrac{\int_{x_1}^{x_2} y\,dx}{x_2 - x_1}$

- To convert an angle from degree to radian, we should multiply it by $\pi/180°$ and to convert an angle from radian to degree, we should multiply it by $180°/\pi$.
- By help of differentiation, if y is given, we can find dy/dx and by help of integration, if dy/dx is given, we can find y.
- The maximum and minimum values of function $A \cos \theta + B \sin \theta$ are $\sqrt{A^2 + B^2}$ and $-\sqrt{A^2 + B^2}$ respectively.

Parallelogram Law of Vector Addition

If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the paralellogram passing away through that common point.

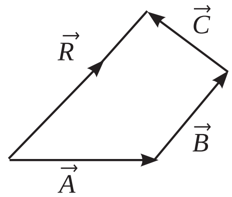


$$\overline{AB} + \overline{AD} = \overline{AC} = \overline{R} \text{ or } \vec{A} + \vec{B} = \vec{R} \Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \text{ and } \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

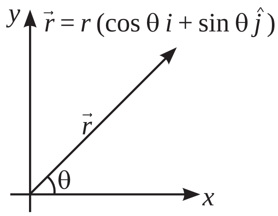
Addition of More than Two Vectors (Polygon Law)

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.



General Vector in x-y Plane

$$\vec{r} = x\hat{i} + y\hat{j} = r(\cos\theta\hat{i} + \sin\theta\hat{j})$$



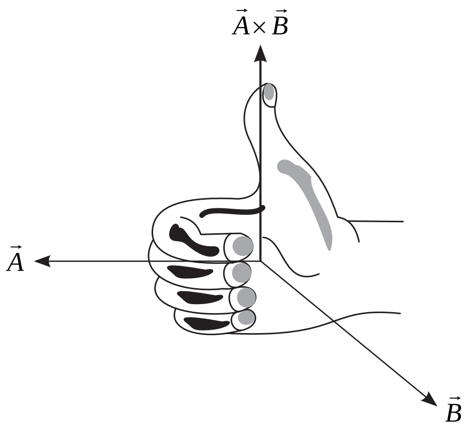
Scalar Product (Dot Product)

$$\vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \boxed{\text{Angle between two vectors} = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)}$$

e.g. work done = $\vec{F}.\vec{S}$ (where $\vec{F}$ is the Force vector and $\vec{S}$ is the displacement vector).

Cross Product (Vector Product)

$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$ where $\hat{n}$ is a vector perpendicular to $\vec{A}$ and $\vec{B}$ or their plane and its direction given by right hand thumb rule.



- ❖ $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- ❖ $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$$

Area of Parallelogram

$\overrightarrow{\text{Area}} = \left(\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta\right)\hat{n} = \vec{A}\times\vec{B}$ (where $\hat{n}$ is the unit vector normal to the plane containing $\vec{A}$ and $\vec{B}$)

Area of Triangle

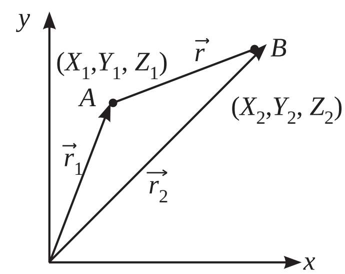
$$\text{Area} = \frac{|\vec{A}\times\vec{B}|}{2} = \frac{1}{2}AB\sin\theta$$

Differentiation of Vectors

- ❖ $\dfrac{d}{dt}(\vec{A}\cdot\vec{B}) = \dfrac{dA}{dt}\cdot\vec{B} + \vec{A}\cdot\dfrac{d\vec{B}}{dt}$
- ❖ $\dfrac{d}{dt}(\vec{A}\times\vec{B}) = \dfrac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\dfrac{d\vec{B}}{dt}$

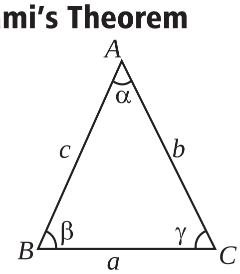
Displacement Vector

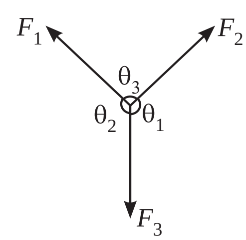
$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



Magnitude $r = |\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Lami's Theorem





$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

- ❖ A unit vector has no unit.
- ❖ Electric current is not a vector as it does not obey the law of vector addition.
- ❖ A scalar or a vector can never be divided by a vector.
- ❖ To a vector only a vector of same type can be added and the resultant is a vector of the same type.

Solved Examples

1. The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the, magnitudes of forces

(*a*) 12, 5 (*b*) 14, 4 (*c*) 5, 13 (*d*) 10, 8

Sol. **(*c*)** Let P be the smaller force and Q be the greater force then according to problem: $P + Q = 18$ ...(i)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12 \quad ...(ii)$$

$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90° = \infty$$

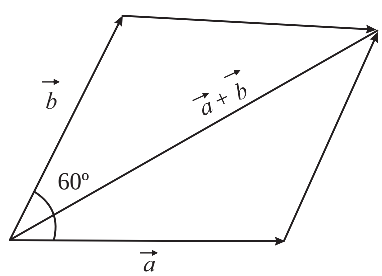
$$\therefore\ P + Q\cos\theta = 0 \quad ...(iii)$$

By solving (i), (ii) and (iii)

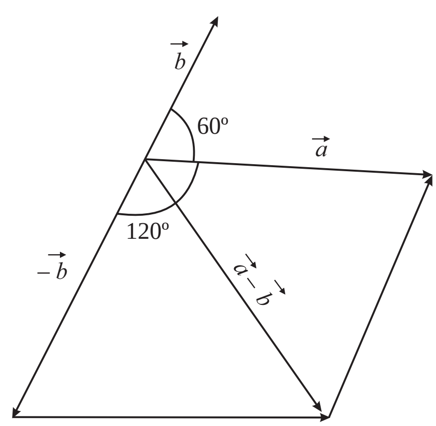
we will get $P = 5$ and $Q = 13$.

2. Two vectors of equal magnitude 2 are at an angle of 60° to each other find magnitude of their sum and difference.

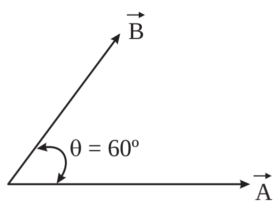
Sol. $|\vec{a} + \vec{b}| = \sqrt{2^2 + 2^2 + 2\times2\times2\cos 60°} = \sqrt{4+4+4} = 2\sqrt{3}$



$|\vec{a} - \vec{b}| = \sqrt{2^2 + 2^2 + 2\times2\times2\cos 120°} = \sqrt{4+4-4} = 2$



3. Find $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ in the diagram shown in figure. Given $A = 4$ units and $B = 3$ units.



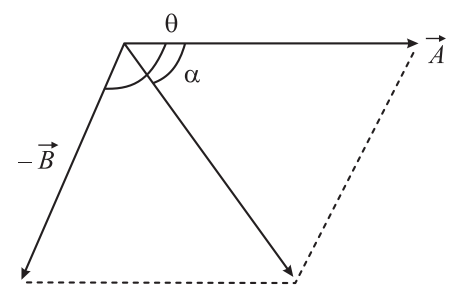
Sol. Addition: $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$= \sqrt{16 + 9 + 2\times4\times3\cos 60°} = \sqrt{37} \text{ units}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \frac{3\sin 60°}{4 + 3\cos 60°} = 0.472$$

$\therefore\ \alpha = \tan^{-1}(0.472) = 25.3°$

Thus, resultant of $\vec{A}$ and $\vec{B}$ is $\sqrt{37}$ units at angle 25.3° from $\vec{A}$ in the direction shown in figure.



$\vec{B} = \vec{A} - \vec{B}$

Subtraction: $S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{16 + 9 - 2\times4\times3\cos 60°} = \sqrt{13}\ \textit{units}$

and $\tan\theta = \dfrac{B\sin\theta}{A - B\cos\theta} = \dfrac{3\sin 60°}{4 - 3\cos 60°} = 1.04$

$\therefore\ \alpha = \tan^{-1}(1.04) = 46.1°$

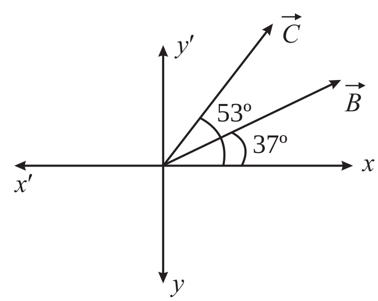
Thus, $\vec{A} - \vec{B}$ is $\sqrt{13}$ units at 46.1° from $\vec{A}$ in the direction shown in figure.

4. Find magnitude of $\vec{B}$ and direction of $\vec{A}$. If $\vec{B}$ makes angle 37° and $\vec{C}$ makes 53° with x axis and $\vec{A}$ has magnitude equal to 10 and $\vec{C}$ has 5. (given $\vec{A} + \vec{B} + \vec{C} = 0$)

Sol. $-\vec{A} = \vec{C} + \vec{B}$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

$\Rightarrow\ -\vec{A} = -A_x\hat{i} + -A_y\hat{j}$



$A_x = (B\cos 37° + C\cos 53°)$

$A_y = (B\sin 37° + C\cos 53°)$

$|\vec{A}|^2 = A_x{}^2 + A_y{}^2$

$A^2 = \left(B\times\frac{4}{5}+C\times\frac{3}{5}\right)^2 + \left(B\times\frac{3}{5}+C\times\frac{4}{5}\right)^2$

$10^2 = \left(\frac{4B}{5}+3\right)^2 + \left(\frac{3B}{5}+4\right)^2$

$\Rightarrow 100 = \frac{16}{25}B^2 + \frac{9}{25}B^2 + 25 + 2\left(\frac{3\times4}{5}+\frac{4\times3}{5}\right)B$

$\Rightarrow\ B^2 + \frac{48}{5}B - 75 = 0$

$B = 5$ (magnitude can not be negative)

and Angle made by A

$\Rightarrow A_x = \left(\frac{20}{5}+3\right) = 12$

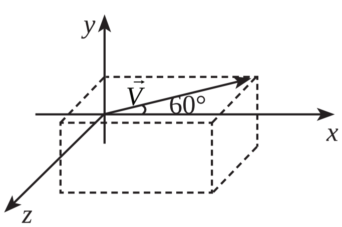
$A_y = \left(\frac{15}{5}+4\right) = 7$

$\tan\theta = \frac{A_y}{A_x} = \frac{7}{12}$

$\theta = 180° + 25° = 205°$

5. A bird moves with velocity 30 m/s in a direction making an angle of 60° with the eastern line and 60° with the vertical upward. Represent the velocity vector in rectangular form.

Sol. Let eastern line be taken as x-axis, northern as y-axis and vertical upward as z-axis.



Let the velocity $\vec{v}$ makes angle a, b and g with x, y and z axis respectively, then $a = 60°$, $g = 60°$ we have

$\cos^2 a + \cos^2 b + \cos^2 g = 1$

$\cos^2 60° + \cos^2 b + \cos^2 60° = 1$

$\cos^2 b = \frac{1}{2};\ \cos b = \frac{1}{\sqrt{2}}$

so $\vec{v} = v\cos a\hat{i} + v\cos b\hat{j} + v\cos g\hat{k}$

$= 30\left[\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}\right] = 15\hat{i} + 15\sqrt{2}\hat{j} + 15\hat{k}$

6. Find the angle that the vector $\vec{A} = 10\hat{i} - 10\sqrt{2}\hat{j} + 10\hat{k}$ makes with x, y and z axis respectively.

Sol. $|\vec{A}| = \sqrt{10^2 + (10\sqrt{2})^2 + 20^2} = \sqrt{100+200+100} = 20$

$A_x = A\cos\alpha = 10 \Rightarrow \cos\alpha = 1/2$ or $\alpha = 60°$

$A_y = A\cos\beta = -10\sqrt{2}$

$\cos\beta = -\frac{10\sqrt{2}}{20} = -\frac{1}{\sqrt{2}}$

$\Rightarrow \beta = 135°$

$A_z = A\cos r = 10$

$\Rightarrow \cos r = \frac{10}{20} = \frac{1}{2}$

$r = 60°$.

This vector makes angle of 60°, 135° and 60° with position x, y and z axis.

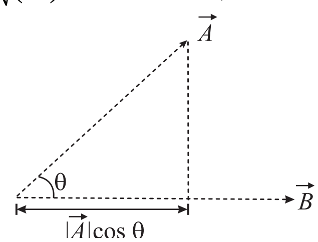
7. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$ then projection of $\vec{A}$ on $\vec{B}$ will be

(*a*) $\frac{3}{\sqrt{13}}$ (*b*) $\frac{3}{\sqrt{26}}$

(*c*) $\sqrt{\frac{3}{26}}$ (*d*) $\sqrt{\frac{3}{13}}$

Sol. (*b*) $|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4+9+1} = \sqrt{14}$

$|\vec{B}| = \sqrt{(-1)^2 + 3^2 + 4^2} = \sqrt{1+9+16} = \sqrt{26}$



$\vec{A}.\vec{B} = 2(-1) + 3\times3 + (-1)(4) = 3$

The projection of $\vec{A}$ on $\vec{B}$ $= \frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{3}{\sqrt{26}}$

8. The vectors from origin to the points A and B are $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - 2\hat{k}$ respectively. The area of the triangle OAB be

(*a*) $\frac{5}{2}\sqrt{17}$ sq.unit (*b*) $\frac{2}{5}\sqrt{17}$ sq.unit

(*c*) $\frac{3}{5}\sqrt{17}$ sq.unit (*d*) $\frac{5}{3}\sqrt{17}$ sq.unit

Sol. (*a*) Given $\overrightarrow{OA} = \vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

and $\overrightarrow{OB} = \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\therefore\ (\vec{a}\times\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$

$= (12-2)\hat{i} + (4+6)\hat{j} + (3+12)\hat{k}$

$= 10\hat{i} + 10\hat{j} + 15\hat{k}$

$\Rightarrow |\vec{a}\times\vec{b}| = \sqrt{10^2 + 10^2 + 15^2}$

$= \sqrt{425} = 5\sqrt{17}$

Area of $\Delta OAB = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{5\sqrt{17}}{2}$ sq.unit.

9. The torque of the force $\vec{F} = (2\hat{i} - 3\hat{j} + 4\hat{k})N$ acting at the point $\vec{r} = (3\hat{i} + 2\hat{j} + 3\hat{k})$ m about the origin is

(*a*) $6\hat{i} - 6\hat{j} + 12\hat{k}$ (*b*) $17\hat{i} - 6\hat{j} - 13\hat{k}$

(*c*) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (*d*) $-17\hat{i} + 6\hat{j} + 13\hat{k}$

Sol. (*b*) $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$= [(2\times4) - (3\times-3)]\,\hat{i} + [(2\times3) - (3\times4)]\,\hat{j} + [(3\times-3) - (2\times2)]\,\hat{k}$

$= 17\hat{i} - 6\hat{j} - 13\hat{k}$

10. Find the derivatives of the following:

(*a*) $(x^3 - 3x^2 + 4)\,(4x^5 + x^2 - 1)$

(*b*) $\dfrac{9x^5}{x-3}$

Sol. (*a*) Let $y = (x^3 - 3x^2 + 4)\,(4x^5 + x^2 - 1)$

$\dfrac{dy}{dx} = (x^3 - 3x^2 + 4)\,\dfrac{d}{dx}(4x^5 + x^2 - 1) + (4x^5 + x^2 - 1)$

$\dfrac{d}{dx}(x^3 - 3x^2 + 4)$

$= (x^3 - 3x^2 + 4)\left[\dfrac{d}{dx}(4x^5) + \dfrac{d}{dx}(x^2) - \dfrac{d}{dx}(1)\right]$

$+ (4x^5 + x^2 - 1)\left[\dfrac{d}{dx}(x^3) - \dfrac{d}{dx}(3x^2) + \dfrac{d}{dx}(4)\right]$

$= (x^3 - 3x^2 + 4)\left[4\dfrac{d}{dx}(x^5) + \dfrac{d}{dx}(x^2) - \dfrac{d}{dx}(1)\right]$

$+ (4x^5 + x^2 - 1)\left[\dfrac{d}{dx}(x^3) - 3\dfrac{d}{dx}(x^2) + \dfrac{d}{dx}(4)\right]$

$= (x^3 - 3x^2 + 4)\,[4 \times 5x^4 + 2x - 0] + (4x^5 + x^2 - 1)$

$[3x^2 - 3 \times 2x + 0]$

$= (x^3 - 3x^2 + 4)\,(20x^4 + 2x) + (4x^5 + x^2 - 1)\,(3x^2 - 6x)$

$= 2x(10x^3 + 1)\,(x^3 - 3x^2 + 4) + 3x(x - 2)\,(4x^5 + x^2 - 1)$

(*b*) Let $y = \dfrac{9x^5}{x-3}$

$$\frac{dy}{dx} = \frac{(x-3)\dfrac{d}{dx}(9x^5) - 9x^5\dfrac{d}{dx}(x-3)}{(x-3)^2}$$

$$= \frac{(x-3)\times 9\dfrac{d}{dx}(x^5) - 9x^5\left[\dfrac{d}{dx}(x) - \dfrac{d}{dx}(3)\right]}{(x-3)}$$

$$= \frac{(x-3)\times 9\times 5x^4 - 9x^5(1-0)}{(x-3)^2}$$

$$= \frac{45x^5 - 135x^4 - 9x^5}{(x-3)^2} = \frac{36x^5 - 135x^4}{(x-3)^2} = \frac{9x^4(4x-15)}{(x-3)^2}$$

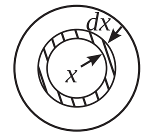
11. Evaluate $\int\left(x^2 - \cos x + \dfrac{1}{x}\right)dx$

Sol. $\int x^2 dx - \int \cos x\,dx + \int \dfrac{1}{x}dx = \dfrac{x^{2+1}}{2+1} - \sin x + \log_e x + c$

$= \dfrac{x^3}{3} - \sin x + \log_e x + c$

12. The radius of circle $r = 3x$. Find the area choosing limits of x from zero to 2 cm.

Sol.



Consider an element of radius x and width dx

Area of element, $dA = 2\pi x dx$

$\therefore$ Total area, $\int_0^A dA = 2\pi\int_0^2 x dx$

$A = 2\pi\left|\dfrac{x^2}{2}\right|_0^2 = \pi\left(2^2 - 0^2\right) = 4\pi\ \text{cm}^2$

13. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is

(*a*) 0° (*b*) 60°

(*c*) 90° (*d*) 120°

Sol. (*d*) $|\vec{A} + \vec{B}| = \vec{A} = \vec{B}$

$A^2 + B^2 + 2AB\cos\theta = A^2$

$2A^2 + 2A^2\cos\theta° = A^2$

$2A^2 + 2A^2\cos\theta° = A^2$

$2\cos\theta = -1$

$\cos\theta = -\dfrac{1}{2}$

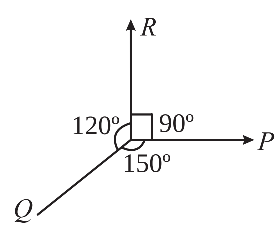
$\theta = 120°$

14. Three forces P, Q and R are acting at a point in the plane. The angle between P, Q and Q, R are 150° and 120° respectively, then for equilibrium, forces P, Q and R are in the ratio

(*a*) 1 : 2 : 3 (*b*) 1 : 2 : $\sqrt{3}$

(*c*) 3 : 2 : 1 (*d*) $\sqrt{3}$: 2 : 1

Sol. (*d*)



$$\frac{P}{\sin 120°} = \frac{Q}{\sin 90°} = \frac{R}{\sin 150°}$$

$$\frac{P}{\sqrt{3/2}} = \frac{\theta}{1} = \frac{R}{\frac{1}{2}}$$

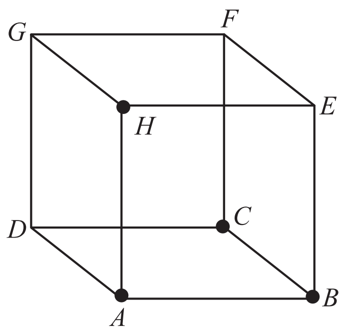
$P : Q\ \ R = \sqrt{3} : 2 : 1$

15. A hall has the dimensions 10 m × 12 m × 14 m. A fly starting at one corner ends up at a diametrically opposite corner. The magnitude of its displacement is nearly.

(*a*) 16 m (*b*) 17 m

(*c*) 18 m (*d*) 21 m

Sol. (*d*)



Start from A to F

$AC^2 = AB^2 + CB^2$

$AC^2 = 10^2 + 12^2$

$AC^2 = 100 + 144 = 244$

$AF^2 = AC^2 + CF^2 = 244 + 196$

$AF^2 = 440$

$A = 20.99 \simeq 21$

16. $y = \sin^3 x + \sin 3x$

(*a*) $3 \sin^2 x \cos x + 3 \cos 3x$

(*b*) $4 \sin^2 x \cos x + 4 \cos^3 x$

(*c*) $2 \sin^2 x \cos x + 2 \cos^3 x$

(*d*) $3 \sin^2 x \cos x + 4 \cos^3 x$

Sol. (*a*) $y = \sin^3 x + \sin^3 x$

$\Rightarrow y = y_1 + y \Rightarrow \dfrac{dy}{dx} = \dfrac{dy_1}{dx} + \dfrac{dy_2}{dx}$

$\Rightarrow y_1 \sin^3 x$

$\Rightarrow y_1 = u^3 \qquad u = \sin x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du}.\dfrac{du}{dx}$

$\Rightarrow \dfrac{dy_1}{du} = 3u^2 \qquad \dfrac{du}{dx} = \cos x$

$\Rightarrow \dfrac{dy_1}{dx} = 3\sin^2 x \cos x$

$\Rightarrow y_2 = \sin 3x$

$\Rightarrow y_2 = \sin v \Rightarrow \dfrac{dy_2}{dv} = \cos v \quad v = 3x$

$\Rightarrow \dfrac{dy_2}{dx} = 3\cos 3x \qquad \dfrac{dv}{dx} - 3$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy_1}{dx} + \dfrac{dy_2}{dx}$

$\Rightarrow \dfrac{dy}{dx} = 3\sin^2 x \cos x + 3\cos 3x$

17. $q = \sqrt{2r - r^2}$, $\dfrac{dq}{dr}$ will be

(*a*) $\dfrac{1+r}{\sqrt{2r-r^2}}$ (*b*) $\dfrac{1-r}{\sqrt{2r-r^2}}$

(*c*) $\dfrac{1-r}{\sqrt{2r+r^2}}$ (*d*) $\dfrac{1+r}{\sqrt{2r+r^2}}$

Sol. (*b*) $q = \sqrt{2r - r^2}$

$\Rightarrow q = \sqrt{u} \qquad u = 2r - r^2$

$\Rightarrow \dfrac{dq}{du} = \dfrac{1}{2\sqrt{u}} \qquad \dfrac{du}{dr} = 2 - 2r$

$\Rightarrow \dfrac{dq}{dr} = \dfrac{dq}{du}.\dfrac{du}{dr} \Rightarrow = \dfrac{1}{2\sqrt{u}}.(2-2r)$

$\Rightarrow \dfrac{1-r}{\sqrt{2r-r^2}}$

18. Two positive numbers x and y such that $x + y = 60$ and xy is maximum.

(*a*) $x = 20$ and $y = 10$ (*b*) $x = 50$ and $y = 20$

(*c*) $x = 30$ and $y = 30$ (*d*) $x = 20$ and $y = 50$

Sol. (*c*) $x + y = 60$

$\Rightarrow y = 60 - x \qquad \Rightarrow xy = 60x - x^2$

$\Rightarrow z = 60x - x^2 \qquad \Rightarrow \dfrac{dz}{dx} = 60 - 2x$

$\Rightarrow \dfrac{dz}{dx} = 0 \Rightarrow x = 30 \quad \Rightarrow \dfrac{d^2z}{dx^2} = -ve$ maximum

$\Rightarrow x = 30 \qquad y = 30$

Exercise-1 (Topicwise)

RESULTANT OF TWO VECTORS

1. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors

(*a*) Can be zero

(*b*) Cannot be zero

(*c*) Lies in the plane of $\vec{A}$ and $\vec{B}$

(*d*) Lies in the plane of $\vec{A}$ and $\vec{A}+\vec{A}$

2. The vector sum of the forces of 10 N and 6 N can be

(*a*) 2 N (*b*) 8 N

(*c*) 18 N (*d*) 20 N

3. A set of vectors taken in a given order gives a closed polygon. Then the resultant of these vectors is a

(*a*) Scalar quantity

(*b*) Pseudo vector

(*c*) Unit vector

(*d*) Null vector.

4. A car drives 6.0 km east, then 8 km north, and then 21 km west. The magnitude of the resulting displacement from origin is

(*a*) 35 km (*b*) 23 km

(*c*) 21 km (*d*) 17 km

DIFFERENCE OF TWO VECTORS

5. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same

(*a*) Magnitude

(*b*) Direction

(*c*) Magnitude as well as direction

(*d*) Neither magnitude nor direction.

6. If $\vec{P}+\vec{Q},=\vec{P}-\vec{Q},$ and θ is the angle between $\vec{P}$ and $\vec{Q}$, then

(*a*) θ = 0º (*b*) θ = 90º

(*c*) $P = 0$ (*d*) $Q = 0$

7. A particle is moving eastward with a speed of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is

(*a*) Zero

(*b*) $\dfrac{1}{\sqrt{2}}$ m/s^2 towards north-west.

(*c*) 1/2 m/s^2 towards north-west.

(*d*) 1/2 m/s^2 towards north.

8. The magnitudes of 2 forces $\vec{F}_1$ and $\vec{F}_2$ and 10 N and 8 N. The angle between them is 120°. The magnitude of their difference is

(*a*) $\sqrt{244}$ N (*b*) $\sqrt{84}$ N

(*c*) $\sqrt{164}$ N (*d*) None of these

VECTORS IN UNIT VECTOR FORM

9. The vector joining the points A (1, 1, –1) and B (2, –3, 4) and pointing from A to B is

(*a*) $-\hat{i}+4\hat{j}-5\hat{k}$ (*b*) $\hat{i}+4\hat{j}+5\hat{k}$

(*c*) $\hat{i}-4\hat{j}+5\hat{k}$ (*d*) $-\hat{i}-4\hat{j}-5\hat{k}$

10. The x and y components of a force are 2N and –3N. The force is

(*a*) $2\hat{i}-3\hat{j}$ (*b*) $2\hat{i}+3\hat{j}$

(*c*) $-2\hat{i}-3\hat{j}$ (*d*) $3\hat{i}+2\hat{j}$

11. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The values of x are

(*a*) $-\dfrac{2}{3}$ (*b*) $\dfrac{1}{3}$ (*c*) $\dfrac{2}{3}$ (*d*) 1

DOT PRODUCT

12. Three non zero vectors $\vec{A}, \vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}\,.\,\vec{B}=0$ and $\vec{A}\,.\,\vec{C}=0$. Then $\vec{A}$ can be parallel to:

(*a*) $\vec{B}$ (*b*) $\vec{C}$

(*c*) $\vec{B}\,.\,\vec{C}$ (*d*) $\vec{B}\times\vec{C}$

13. The value of scalar product of two vectors is 8 and that of vector product is $8\sqrt{3}$. The angle between them is:

(*a*) 30º (*b*) 60º (*c*) 120º (*d*) 150º

14. A particle is moving under the influence of force $\vec{F}=[\hat{i}-2\hat{j}]\,N$, is now moved from the point (x, y, z) = (2, 1, 3) m to the point (x, y, z) = (3, 2, 4) m. How much work is done by the force $\vec{F}$ during this time period? ($W = \vec{F}\,.\vec{s}$)

(*a*) 0 J (*b*) –1 J

(*c*) 1 J (*d*) None of these

15. The angle between $(\hat{i}+\hat{j}+\hat{k})$ and $(2\hat{i}+2\hat{j}-2\hat{k})$ is

(*a*) $\cos^{-1}\dfrac{1}{3}$ (*b*) $\cos^{-1}\dfrac{1}{\sqrt{3}}$

(*c*) $\text{Sin}^{-1}\dfrac{1}{3}$ (*d*) None of these

16. If $\vec{A}\,.\,\vec{B}=\vec{C}\,.\,\vec{B}$, which of the following can not be a possible case:

(*a*) $\vec{A}=\vec{C}$

(*b*) angle between $\vec{A}$ and $\vec{B}$ = 30° and angle between $\vec{B}$ and $\vec{C}$ = 150° all three vectors are of equal magnitude.

(*c*) $\vec{B}$ is a null vector

(*d*) $\vec{A}$, $\vec{B}$ and $\vec{C}$ are along *x*, *y* and *z*-axis respectively

17. The vector $5\hat{i}+2\hat{j}-\ell\hat{k}$ is perpendicular to the vector $3\hat{i}+\hat{j}+2\hat{k}$ for ℓ =

(*a*) 1 (*b*) 4.7

(*c*) 6.3 (*d*) 8.5

CROSS PRODUCT

18. What is the torque of the force $\vec{F}=(2\vec{i}-3\vec{j}+4\vec{k})\,N$ acting at the point $\vec{r}=(3\vec{i}+2\vec{j}+3\vec{k})\;m$ about the origin:

(*a*) $6\vec{i}-6\vec{j}+12\vec{k}$ (*b*) $17\vec{i}-6\vec{j}-13\vec{k}$

(*c*) $-6\vec{i}+6\vec{j}-12\vec{k}$ (*d*) $-17\vec{i}+6\vec{j}+13\vec{k}$

19. If $\vec{A}\times\vec{B}=\vec{C}$, then which of the following statements is wrong

(*a*) $\vec{C}\perp\vec{A}$ (*b*) $\vec{C}\perp\vec{B}$

(*c*) $\vec{C}\perp(\vec{A}+\vec{B})$ (*d*) $\vec{C}\perp(\vec{A}\times\vec{B})$

20. Two vectors $\vec{P}$ and $\vec{Q}$ are inclined at an angle θ to each other. Which of the following is the unit vector perpendicular to $\vec{P}$ and $\vec{Q}$?

(*a*) $\dfrac{\vec{P}\times\vec{Q}}{PQ}$ (*b*) $\dfrac{\hat{P}\times\hat{Q}}{\sin\theta}$

(*c*) $\dfrac{\hat{P}\times\hat{Q}}{PQ\sin\theta}$ (*d*) $\dfrac{\vec{P}\times\hat{Q}}{PQ\sin\theta}$

21. If $\vec{A}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{B}=-\hat{i}+\hat{j}+4\hat{k},\ \vec{C}=3\hat{i}-3\hat{j}-12\hat{k}$, then the find the angle between the vectors $\left(\vec{A}+\vec{B}+\vec{C}\right)$ and $\left(\vec{A}\times\vec{B}\right)$ in degrees.

(*a*) θ = 90° (*b*) θ = 60°

(*c*) θ = 45° (*d*) θ = 30°

22. Find a unit vector perpendicular to both the vectors $\left(2\hat{i}+3\hat{j}+\hat{k}\right)$ and $\left(\hat{i}-\hat{j}+2\hat{k}\right)$.

(*a*) $\pm\dfrac{1}{\sqrt{83}}\left(7\hat{i}-3\hat{j}-5\hat{k}\right)$ (*b*) $\pm\dfrac{1}{\sqrt{63}}\left(7\hat{i}-3\hat{j}-5\hat{k}\right)$

(*c*) $\pm\dfrac{1}{\sqrt{83}}\left(3\hat{i}-7\hat{j}-5\hat{k}\right)$ (*d*) $\pm\dfrac{1}{\sqrt{38}}\left(7\hat{i}-5\hat{j}-3\hat{k}\right)$

23. Find $\vec{A}.\vec{B}$ *if* $|\vec{A}|=2, |\vec{B}|=5,$ and $|\vec{A}\times\vec{B}|=8$

(*a*) $\vec{A}\cdot\vec{B}=\pm 6$ (*b*) $\vec{A}\cdot\vec{B}=\pm 12$

(*c*) $\vec{A}\cdot\vec{B}=\pm 8$ (*d*) $\vec{A}\cdot\vec{B}=\pm 16$

24. If $\sqrt{3}\left|\vec{A}\times\vec{B}\right|=\vec{A}.\vec{B},$ then find the angle between $\vec{A}$ and $\vec{B}$.

(*a*) 75° (*b*) 30°

(*c*) 65° (*d*) 45°

25. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along *X*, *Y* and *Z* axis respectively, then tick the wrong statement

(*a*) $\hat{i}.\hat{i}=1$ (*b*) $\hat{i}\times\hat{j}=\hat{k}$

(*c*) $\hat{i}.\hat{j}=0$ (*d*) $\hat{i}\times\hat{k}=-\hat{i}$

FUNCTION

26. $f(x)$ = cos *x* + sin *x* then $f(\pi/2)$ will be

(*a*) 2 (*b*) 1

(*c*) 3 (*d*) 0

Direction (**No. 27 to 29**)**:** Derivative of given function w.r.t. corresponding independent variable is.

27. $y=x^2+x+8$

(*a*) $\dfrac{dy}{dx}=2x-1$ (*b*) $\dfrac{dy}{dx}=-x+1$

(*c*) $\dfrac{dy}{dx}=2x+1$ (*d*) $\dfrac{dy}{dx}=x-1$

28. $s=5t^3-3t^5$

(*a*) $\dfrac{ds}{dt}=15t^2+15t^4$ (*b*) $\dfrac{ds}{dt}=15t^4+15t^2$

(*c*) $\dfrac{ds}{dt}=15t^4-15t^2$ (*d*) $\dfrac{ds}{dt}=15t^2-15t^4$

29. $y=5\sin x$

(*a*) $\dfrac{dy}{dx}=3\cos x$ (*b*) $\dfrac{dy}{dx}=5\cos x$

(*c*) $\dfrac{dy}{dx}=5\sin x$ (*d*) $\dfrac{dy}{dx}=3\sin x$

Direction (**No. 30 to 33**)**:** First derivative and second derivative of given functions w.r.t. Corresponding independent variable is.

30. $y=6x^2-10x-5x^{-2}$

(*a*) $12x - 10 + 10x^{-3}$, $12 - 30x^{-4}$

(*b*) $10x - 12 + 20x^{-3}$, $15 - 30x^{-4}$

(*c*) $12x - 10 + 15x^{-3}$, $12 - 30x^{-4}$

(*d*) $10x - 15 + 12x^{-3}$, $12 - 30x^{-4}$

31. $r=\dfrac{12}{\theta}-\dfrac{4}{\theta^3}+\dfrac{1}{\theta^4}$

(*a*) $12\theta^{-2} - 12\theta^{-4} + 4\theta^{-5}$, $24\theta^{-3} + 48\theta^{-5} + 20\theta^{-6}$

(*b*) $-12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}$, $24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

(*c*) $-6\theta^{-2} + 12\theta^{-4} - 8\theta^{-5}$, $12\theta^{-3} - 24\theta^{-5} + 10\theta^{-6}$

(*d*) $-8\theta^{-2} + 12\theta^{-4} - 6\theta^{-5}$, $24\theta^{-3} - 24\theta^{-5} + 10\theta^{-6}$

32. $\omega = 3z^7 - 7z^3 + 21z^2$

(*a*) $21z^6 + 21z^2 - 42z$, $126z^5 + 42z - 42$.
(*b*) $14z^6 - 28z^2 + 22z$, $120z^5 - 21z + 42$.
(*c*) $28z^6 - 14z^2 + 42z$, $122z^5 - 42z + 21$
(*d*) $21z^6 - 21z^2 + 42z$, $126z^5 - 42z + 42$

33. $y = \sin x + \cos x$

(*a*) $\cos x - \cos x$, $-\sin x - \sin x$
(*b*) $\sin x - \sin x$, $-\sin x - \cos x$
(*c*) $\cos x - \sin x$, $-\sin x - \cos x$
(*d*) $\sin x + \cos x$, $-\cos x - \cos x$

Direction (No. 34 to 36): Derivative of given functions w.r.t. the independent variable x is.

34. $x \sin x$

(*a*) $\sin x + x\cos x$ (*b*) $\sin x - x\cos x$
(*c*) $\cos^2 x - x\sin^2 x$ (*d*) $\sin^2 x - x\cos^2 x$

35. $y = e^x \ell nx$

(*a*) $e^x \ell nx - \dfrac{e^x}{x}$ (*b*) $e^x \ell nx - \dfrac{e^x}{x^2}$
(*c*) $e^x \ell nx + \dfrac{e^x}{x^2}$ (*d*) $e^x \ell nx + \dfrac{e^x}{x}$

36. $y = (x-1)(x^2 + x + 1)$

(*a*) $\dfrac{dy}{dx} = 3x$ (*b*) $\dfrac{dy}{dx} = 3x^2$
(*c*) $\dfrac{dy}{dx} = 2x^2$ (*d*) $\dfrac{dy}{dx} = 2x$

Direction (No. 37 to 39): Derivative of given function w.r.t. the independent variable is

37. $y = \dfrac{\sin x}{\cos x}$

(*a*) $\sec^2 x$ (*b*) $\sec x$ (*c*) $\sec^2 2x$ (*d*) $\sec^3 2x$

38. $y = \dfrac{2x+5}{3x-2}$

(*a*) $y' = \dfrac{-19}{(3x-2)^2}$ (*b*) $y' = \dfrac{19}{(3x-2)^2}$
(*c*) $y' = \dfrac{19}{(3x-2)}$ (*d*) $y' = \dfrac{-19}{(3x+2)^2}$

39. $z = \dfrac{2x+1}{x^2-1}$

(*a*) $\dfrac{-2x^2-2x+2}{(x^2+1)^2}$ (*b*) $\dfrac{-2x^2-2x-2}{(x^2-1)^2}$
(*c*) $\dfrac{-2x^2+2x+2}{(x+1)^2}$ (*d*) $\dfrac{-2x^2-2x-2}{(x^2-1)}$

Direction (No. 40 to 44): $\dfrac{dy}{dx}$ for following functions is.

40. $y = (2x+1)^5$

(*a*) $10(2x+1)^3$ (*b*) $10(2x+1)^4$
(*c*) $10(2x-1)^3$ (*d*) $10(2x-1)^4$

41. $y = (4-3x)^9$

(*a*) $-8(4-3x)^8$ (*b*) $-27(4-3x)^9$
(*c*) $-27(4+3x)^9$ (*d*) $-27(4-3x)^8$

42. $y = \left(1-\dfrac{x}{7}\right)^{-7}$

(*a*) $\left(1-\dfrac{x}{7}\right)^{8}$ (*b*) $\left(1-\dfrac{x}{7}\right)^{-8}$
(*c*) $\left(1-\dfrac{x}{7}\right)^{-5}$ (*d*) $\left(1-\dfrac{x}{7}\right)^{-4}$

43. $y = \sin(x) + \ln(x^2) + e^{2x}$

(*a*) $\cos(x) + \dfrac{2}{x} - 2e^{2x}$ (*b*) $\cos(x) + \dfrac{2}{x} + 2e^{2x}$
(*c*) $\sin(x) + \dfrac{2}{x} + 2e^{2x}$ (*d*) $\sin(x) + \dfrac{2}{x} - 2e^{2x}$

44. $y = 2\sin(\omega x + \phi)$ where ω and ϕ constants

(*a*) $2\omega\cos(\omega x + \phi)$ (*b*) $2\omega\cos(\omega x - \phi)$
(*c*) $\omega\cos(\omega x + \phi)$ (*d*) $2\omega\operatorname{cosec}(\omega x + \phi)$

DIFFERENTIATION AS A RATE MEASUREMENT

45. Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t. equation that relates dA/dt to dr/dt is.

(*a*) $\dfrac{dA}{dt} = \pi r \dfrac{dr}{dt}$ (*b*) $\dfrac{dA}{dt} = \pi r^2 \dfrac{dr}{dt}$
(*c*) $\dfrac{dA}{dt} = 2\pi r^2 \dfrac{dr}{dt}$ (*d*) $\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$

46. Maximum and minimum values of function $2x^3 - 15x^2 + 36x + 11$ is

(*a*) 39, 38 (*b*) 93, 83 (*c*) 45, 42 (*d*) 59, 58

47. $y = 2u^3$, $u = 8x - 1$. Find $\dfrac{dy}{dx}$

(*a*) $48(8x-1)^2$ (*b*) $48(8x+1)^2$
(*c*) $48(8x-1)$ (*d*) $48(8x+1)$

48. $y = \sin u$, $u = 3x + 1$. Find $\dfrac{dy}{dx}$

(*a*) $3\cos(3x-1)$ (*b*) $3\cos(3x+1)$
(*c*) $3\sin(3x-1)$ (*d*) $3\sin(3x+1)$

49. $y = \sin u, u = \cos x$. Find $\frac{dy}{dx}$.

(*a*) –cot*u* (*b*) tan*u*
(*c*) –tan*u* (*d*) cot*u*

50. $y = 3x^2 - 1, x = t^2$. Find $\frac{dy}{dx}$.

(*a*) $5t$ (*b*) $-5t$
(*c*) $3t$ (*d*) $-3t$

INTEGRATION

51. $\int (2x)\, dx$ will be

(*a*) $x^2 + C$ (*b*) $2x + C$
(*c*) $2x^2 + C$ (*d*) $-x^2 + C$

52. $\int (x^2)\, dx$ will be

(*a*) $x + C$ (*b*) $2x + C$
(*c*) $\frac{x^3}{3} + C$ (*d*) $\frac{x^2}{2} + C$

53. $\int (x^2 - 2x + 1)\, dx$ will be

(*a*) $\frac{x^3}{3} - x^2 - x + C$ (*b*) $\frac{x^3}{3} - x^2 + x + C$
(*c*) $\frac{x^3}{3} + x^2 - x + C$ (*d*) $\frac{x^3}{3} + x^2 + x + C$

54. $\int (-3x^{-4}) dx$ will be

(*a*) $x^{-3} + C$ (*b*) $x^3 + C$
(*c*) $-3x^{-3} + C$ (*d*) $3x^{-3} + C$

55. $\int (x^{-4}) dx$ will be

(*a*) $\frac{1}{3}x^{-3} + C$ (*b*) $\frac{1}{3}x^{-5} + C$
(*c*) $\frac{1}{2}x^{-2} + C$ (*d*) $-\frac{1}{3}x^{-3} + C$

56. $\int \left(\frac{5}{x^2}\right) dx$ will be

(*a*) $-\frac{5}{x} + C$ (*b*) $\frac{5}{x} + C$
(*c*) $\frac{x}{5} + C$ (*d*) $-\frac{x}{5} + C$

57. $\int \left(\frac{3}{2}\sqrt{x}\right) dx$ will be

(*a*) $\sqrt{x^2} + C$ (*b*) $3\sqrt{x^4} + C$
(*c*) $3\sqrt{x^2} + C$ (*d*) $\sqrt{x^3} + C$

58. $\int \left(\frac{3}{2\sqrt{x}}\right) dx$ will be

(*a*) $2\sqrt{x^3} + C$ (*b*) $3\sqrt{x} + C$
(*c*) $\sqrt{x^3} + C$ (*d*) $\sqrt{x^4} + C$

59. $\int \left(\frac{4}{3}\sqrt[3]{x}\right) dx$ will be

(*a*) $x^{4/3} + C$ (*b*) $x^{3/4} + C$
(*c*) $x^{2/3} + C$ (*d*) $x^{1/3} + C$

60. $\int \left(\frac{1}{3\sqrt[3]{x}}\right) dx$ will be

(*a*) $\frac{x^{\frac{3}{4}}}{2} + C$ (*b*) $\frac{x^{\frac{2}{3}}}{3} + C$
(*c*) $x^{\frac{2}{3}} + C$ (*d*) $\frac{x^{\frac{2}{3}}}{2} + C$

61. $\int \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\right) dx$ will be

(*a*) $\frac{3x^{\frac{4}{2}}}{4} + \frac{3x^{\frac{2}{4}}}{2} + C$ (*b*) $\frac{3x^{\frac{4}{2}}}{4} + \frac{3x^{\frac{2}{1}}}{2} + C$
(*c*) $\frac{3x^{\frac{3}{4}}}{4} + \frac{3x^{\frac{3}{2}}}{2} + C$ (*d*) $\frac{3x^{\frac{4}{3}}}{4} + \frac{3x^{\frac{2}{3}}}{2} + C$

62. $\int \left(\frac{1}{2}x^{-1/2}\right) dx$ will be

(*a*) $x^{2/3} + C$ (*b*) $x^{1/2} + C$
(*c*) $x^{3/1} + C$ (*d*) $x^{2/1} + C$

63. $\int \left(-\frac{1}{2}x^{-3/2}\right) dx$ will be

(*a*) $x^{-1/2} + C$ (*b*) $x^{+1/2} + C$
(*c*) $x^{-1/-2} + C$ (*d*) $x^{-2/1} + C$

64. $\int (3 \sin x)\, dx$ will be

(*a*) $+3 \cos x + C$ (*b*) $+4 \cos x + C$
(*c*) $-3 \cos x + C$ (*d*) $3 \cos x + C$

65. $\int \left(\frac{1}{3x}\right) dx$ will be

(*a*) $\frac{1}{3}\ell nx + C$ (*b*) $\frac{3}{1}\ell nx + C$
(*c*) $\frac{2}{3}\ell nx + C$ (*d*) $\frac{1}{2}\ell nx + C$

66. $\int \sin 3x\, dx$, will be (use, $u = 3x$)

(*a*) $-\cos 3x + C$ (*b*) $\frac{1}{3}\cos 3x - C$
(*c*) $-\frac{1}{2}\cos 3x - C$ (*d*) $-\frac{1}{3}\cos 3x + C$

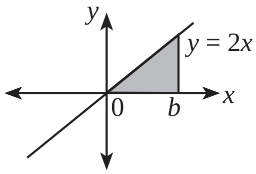
67. $\int_{-2}^{1} 5\, dx$ will be

(*a*) 15 (*b*) 16
(*c*) 17 (*d*) 18

68. $\int_{-4}^{-1} \frac{\pi}{2} d\theta$ will be

(*a*) $\frac{3\pi}{2}$ (*b*) $\frac{2\pi}{2}$ (*c*) 3π (*d*) 2π

69. Use a definite integral to find the area of the region between the given curve and the *x*-axis on the interval $[0, b]$ $y = 2x$



(*a*) b^2 (*b*) b^3

(*c*) $2b^2$ (*d*) $\frac{b^3}{3}$

70. $\int_{-2}^{+2} \left(t^2 - 1\right) dt$.

(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{2}{3}$

Exercise-2 (Learning Plus)

1. If the angle between two forces increases, the magnitude of their resultant

(*a*) decreases

(*b*) increases

(*c*) remains unchanged

(*d*) first decreases and then increases

2. Which of the following sets of displacements might be capable of bringing a car to its returning point?

(*a*) 5, 10, 30 and 50 unit

(*b*) 5, 9, 9 and 16 unit

(*c*) 40, 40, 90 and 200 unit

(*d*) 10, 20, 40 and 90 unit

3. When two vector $\vec{a}$ and $\vec{b}$ are added, the magnitude of the resultant vector is always

(*a*) greater than $(a + b)$

(*b*) less than or equal to $(a + b)$

(*c*) less than $(a + b)$

(*d*) equal to $(a + b)$

4. Vector $\vec{A}$ is of length 2 cm and is 60º above the *x*-axis in the first quadrant. Vector $\vec{B}$ is of length 2 cm and 60º below the *x*-axis in the fourth quadrant. The sum $\vec{A} + \vec{B}$ is a vector of magnitude:

(*a*) 2 along + *y*-axis (*b*) 2 along + *x*-axis

(*c*) 1 along – *x* axis (*d*) 2 along – *x* axis

5. If the resultant of two forces of magnitudes *P* and *Q* acting at a point at an angle of 60º is $\sqrt{7}Q$, then *P*/*Q* is

(*a*) 1 (*b*) 3/2

(*c*) 2 (*d*) 4

6. A man moves towards 3 m north then 4m towards east and finally 5 m towards 37º south of west. His displacement from origin is

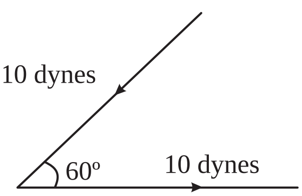
(*a*) $5\sqrt{2}$ m (*b*) 0 m

(*c*) 1 m (*d*) 12 m

7. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between the two forces is

(*a*) 150º (*b*) 90º (*c*) 60º (*d*) 120º

8. Two forces each numerically equal to 10 dynes are acting as shown in the following figure, then their resultant is:



(*a*) 10 dynes (*b*) 20 dynes

(*c*) $10\sqrt{3}$ dynes (*d*) 5 dynes

9. The vector $\vec{P}$ makes 120° with the *x*-axis and vector $\vec{Q}$ makes 30° with the *y*-axis. What is their resultant? (taking anticlockwise as positive)

(*a*) $P + Q$ (*b*) $P - Q$

(*c*) $\sqrt{P^2 + Q^2}$ (*d*) $\sqrt{P^2 - Q^2}$

10. A vector $\vec{A}$ is directed along 30^0 west of north direction and another vector $\vec{B}$ along 15^0 south of east. Their resultant cannot be in ________ direction.

(*a*) North (*b*) East

(*c*) North-East (*d*) South

11. The greatest and the least resultant of two forces acting at a point are 12 N and 8 N respectively. Find out the forces.

(*a*) 12 N and 8 N (*b*) 4 N and 8 N

(*c*) 3 N and 9 N (*d*) 10 N and 2 N

12. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(*a*) 30° (*b*) 60°

(*c*) 90° (*d*) 120°

13. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1} when it turns left through 90°. If the speed remains unchanged after turning, the change in the velocity of the car in the turning process is

(*a*) Zero

(*b*) $50\sqrt{2}$ km h^{-1} S-W direction

(*c*) $50\sqrt{2}$ km h^{-1} N-W direction

(*d*) 50 km h^{-1} due west.

14. A body is moving in circular path with constant speed v and A and B, describing angle θ, at the centre. The change of velocity may be?

(*a*) $\left(4v\sin\dfrac{\theta}{4}\right)$ (*b*) $\left(6v\sin\dfrac{\theta}{2}\right)$

(*c*) $\left(3v\sin\dfrac{\theta}{2}\right)$ (*d*) $\left(2v\sin\dfrac{\theta}{2}\right)$

15. If the angle between the unit vectors $\vec{a}$ and $\vec{b}$ is 60°, then $|\vec{a}-\vec{b}|$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 4

16. Given: $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 5\hat{i} - 6\hat{j}$. The magnitude of $\vec{A} + \vec{B}$ is

(*a*) 4 units (*b*) 10 units

(*c*) $\sqrt{58}$ units (*d*) $\sqrt{61}$ units

17. Given: $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{B} = -\hat{i} - \hat{j} + \hat{k}$. The unit vector of $\vec{A} - \vec{B}$ is

(*a*) $\dfrac{3\hat{i}+\hat{k}}{\sqrt{10}}$ (*b*) $\dfrac{3\hat{i}}{\sqrt{10}}$

(*c*) $\dfrac{\hat{k}}{\sqrt{10}}$ (*d*) $\dfrac{-3\hat{i}-\hat{k}}{\sqrt{10}}$

18. The angle that the vector $\vec{A} = 2\hat{i} + 3\hat{j}$ makes with y-axis is:

(*a*) $\tan^{-1}(3/2)$ (*b*) $\tan^{-1}(2/3)$

(*c*) $\sin^{-1}(2/3)$ (*d*) $\cos^{-1}(3/2)$

19. If $3\hat{i}+2\hat{j}+8\hat{k}$ and $2\hat{i}+x\hat{j}+\hat{k}$ are at right angles then $x =$

(*a*) 7 (*b*) –7

(*c*) 5 (*d*) –4

20. If $(\vec{a}+\vec{b})$ is perpendicular to $\vec{b}$ and $\vec{a}+2\vec{b}$ is perpendicular to $\vec{a}$, $|\vec{a}| = a$, $|\vec{b}| = b$, then

(*a*) $a = b$ (*b*) $a = 2b$

(*c*) $b = 2a$ (*d*) $a = b\sqrt{2}$

21. Two forces $\hat{i}+\hat{j}+\hat{k}$ N and $\hat{i}+2\hat{j}+3\hat{k}$ N act on a particle and displace it from (2, 3, 4) to point (5, 4, 3). Displacement is in m. Work done is

(*a*) 5 J (*b*) 4 J

(*c*) 3 J (*d*) None of these

22. If a, b, c are three unit vectors such that $a + b + c = 0$, then $a.b + b.c + c.a$ is equal to

(*a*) –1 (*b*) 3 (*c*) 0 (*d*) $-\dfrac{3}{2}$

23. A vector is not changed if

(*a*) It is displaced parallel to itself

(*b*) It is rotated through an arbitrary angle

(*c*) It is cross-multiplied by a unit vector

(*d*) It is multiplied by an arbitrary scalar

24. A vector $\vec{A}$ points vertically downward & $\vec{B}$ points towards east, then the vector product $\vec{A} \times \vec{B}$ is

(*a*) Along west (*b*) Along east

(*c*) Zero (*d*) Along south

25. Find torque of a force $2\hat{i} - \hat{j} + \hat{k}$ acting at point (1, 3, 5) about origin.

(*a*) $-8i - 9j + 7k$ (*b*) $-8i + 9j + 7k$

(*c*) $8i + 9j - 7k$ (*d*) $8i - 9j - 7k$

26. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular only if:

(*a*) $\vec{A}$ and $\vec{B}$ are perpendicular to each other and modulus of $\vec{A}$ is equal to modulus of $\vec{B}$.

(*b*) $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

(*c*) $\vec{A}$ and $\vec{B}$ are perpendicular but magnitudes are orbitrary.

(*d*) modulus of $\vec{A}$ = modulus of $\vec{B}$ and their directions are arbitrary.

27. The first derivative and second derivative of given functions w.r.t. The independent variable x will be: $y = \ell n x^2 + \sin x$

(*a*) $\dfrac{d^2y}{dx^2} = \dfrac{-2}{x^2} - \sin x,\ \dfrac{d^2y}{dx^2} = \dfrac{-2}{x^2} - \cos x$

(*b*) $\dfrac{dy}{dx} = \dfrac{2}{x} + \sin x,\ \dfrac{d^2y}{dx^2} = \dfrac{-2}{x^2} + \sin x$

(*c*) $\dfrac{dy}{dx} = \dfrac{2}{x} + \sin x,\ \dfrac{d^2y}{dx^2} = \dfrac{-2}{x^2} - \sin x$

(*d*) $\dfrac{dy}{dx} = \dfrac{2}{x} + \cos x,\ \dfrac{d^2y}{dx^2} = \dfrac{-2}{x^2} - \sin x$

Direction (**No. 28 to 32**)**:** The derivative of given functions w.r.t. the corresponding independent variable will be

28. $y = e^x \tan x$

(*a*) $e^x(\tan x + \sec^2 x)$ (*b*) $e^x(\sec x + \tan^2 x)$

(*c*) $e^x(\tan x - \sec^2 x)$ (*d*) $e^x(\sec x - \tan^2 x)$

29. $y = \left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x} + 1\right)$

(*a*) $\dfrac{dy}{dx} = 1 + 2x + \dfrac{2}{x^3} - \dfrac{1}{x^2}$ (*b*) $\dfrac{dy}{dx} = 1 + 2x + \dfrac{2}{x^3} + \dfrac{1}{x^2}$

(*c*) $\dfrac{dy}{dx} = 1 - 2x + \dfrac{2}{x^3} - \dfrac{1}{x^2}$ (*d*) $\dfrac{dy}{dx} = 1 + 2x - \dfrac{2}{x^3} + \dfrac{1}{x^2}$

30. $y = x^2 \sin x + 2x\cos x - 2\sin x$

(*a*) x^2 cosec x (*b*) x^2 sec x
(*c*) x^2 cos x (*d*) x^2 sin x

31. $y = x^2 \cos x - 2x\sin x - 2\cos x$

(*a*) $\frac{dy}{dx} = +x^2 \cos x$ (*b*) $\frac{dy}{dx} = -x^2 \cos x$
(*c*) $\frac{dy}{dx} = +x^2 \sin x$ (*d*) $\frac{dy}{dx} = -x^2 \sin x$

32. $r = (1+\sec\theta)\sin\theta$

(*a*) $\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$ (*b*) $\frac{dr}{d\theta} = \cos\theta - \sec^2\theta$
(*c*) $\frac{dr}{d\theta} = \sin\theta - \cos^2\theta$ (*d*) $\frac{dr}{d\theta} = \cos\theta - \sin^2\theta$

Direction (**No. 33 to 34**)**:** The derivative of given functions w.r.t. the respective independent variable will be

33. $y = \frac{\sin x + \cos x}{\cos x}$

(*a*) $\frac{dy}{dx} = \cos^2 x$ (*b*) $\frac{dy}{dx} = \operatorname{cosec}^2 x$
(*c*) $\frac{dy}{dx} = \sec^2 x$ (*d*) $\frac{dy}{dx} = \sin^2 x$

34. $p = \frac{\tan q}{1+\tan q}$

(*a*) $\frac{\sec^2 q}{(1-\tan q)^2}$ (*b*) $\frac{\sec^2 q}{(1+\tan q)^2}$
(*c*) $\frac{\sec^2 q}{(1-\sec q)^2}$ (*d*) $\frac{\sec^2 q}{(1-\sin q)^2}$

35. $\frac{dy}{dx}$ for following function will be: $\sin^2(x^2+1)$

(*a*) $4x \sin(x^2-1)\cos(x^2+1)$
(*b*) $4x \sin(x^2+1)\cos(x^2-1)$
(*c*) $4x \sin(x^2-1)\cos(x^2-1)$
(*d*) $4x \sin(x^2+1)\cos(x^2+1)$

36. $\int(2x^3 - 5x + 7)\,dx$ will be

(*a*) $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$ (*b*) $\frac{x^4}{2} - \frac{5x^2}{2} - 7x + C$
(*c*) $\frac{x^4}{2} - \frac{5x^2}{2} - 7x - C$ (*d*) $\frac{x^4}{2} - \frac{5x^2}{3} + 7x + C$

37. $\int(\sqrt{x} + \sqrt[3]{x})\,dx$ will be

(*a*) $\frac{4}{3}x^{3/2} + \frac{2}{3}x^{4/3} + C$ (*b*) $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$
(*c*) $\frac{3}{5}x^{3/2} + \frac{4}{2}x^{4/3} + C$ (*d*) $\frac{4}{2}x^{3/2} + \frac{2}{4}x^{4/3} + C$

38. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\,dt$ will be

(*a*) $2\sqrt{t} - \frac{4}{\sqrt{t}} - C$ (*b*) $2\sqrt{t} - \frac{3}{\sqrt{t}} - C$
(*c*) $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$ (*d*) $2\sqrt{t} - \frac{1}{\sqrt{t}} - C$

39. $\int \frac{4+\sqrt{t}}{t^3}dt$ will be

(*a*) $-2t^{-2} - \frac{2}{3}t^{-3/2} + C$ (*b*) $-2t^{-2} + \frac{2}{3}t^{-3/2} - C$
(*c*) $2t^{-2} + \frac{2}{3}t^{-3/2} - C$ (*d*) $-2t^{-2} + \frac{2}{3}t^{-3/2} - C$

40. $\int \cos\theta(\tan\theta + \sec\theta)\,d\theta$ will be

(*a*) – cosecθ + θ + C (*b*) – secθ + θ + C
(*c*) – sinθ + θ + C (*d*) – cosθ + θ + C

41. $\int_{\pi}^{2\pi} \theta\, d\theta$ will be

(*a*) $\frac{3\pi^2}{2}$ (*b*) $\frac{2\pi^2}{3}$ (*c*) $\frac{4\pi^2}{3}$ (*d*) $\frac{3\pi^2}{4}$

42. $\int \tan x\,dx$

(*a*) $\log_e \sec x$ (*b*) $\log_e \cot x$ (*c*) $\log_e \sin x$ (*d*) $\log_e \tan x$

43. $\int \frac{\sin 2x}{\sin x}dx$

(*a*) 2 sin *x* (*b*) 4 sin *x* (*c*) 2 cos *x* (*d*) 2 cot *x*

44. $\int_0^{\pi} \cos x\,dx$ will be

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

45. $\int_0^1 \frac{dx}{3x+2}$ will be

(*a*) $\frac{2}{4}\ell n\frac{2}{3}$ (*b*) $\frac{3}{1}\ell n\frac{5}{2}$ (*c*) $\frac{1}{3}\ell n\frac{2}{5}$ (*d*) $\frac{1}{3}\ell n\frac{5}{2}$

46. If $f(x) = 3x^2 - 1$, find $\int f(x)\,dx$ for limits between $x = 0$ to $x = 3$

(*a*) 24 (*b*) 35 (*c*) 65 (*d*) 40

47. $\int \frac{x}{3x^2 - 1}dx$

(*a*) $\frac{1}{3}\log e\left(3x^2 - 1\right)$ (*b*) $\frac{1}{6}\log e\left(3x^2 + 1\right)$
(*c*) $\frac{2}{3}\log e\left(3x\right)$ (*d*) $\frac{1}{6}\log e\left(3x^2 - 1\right)$

48. $\int_{-\pi/2}^{+\pi/2} \left(\text{Sin}\,x + \cos x\right)dx$

(*a*) 24 (*b*) 2 (*c*) –2 (*d*) –4

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. Which of the following is a true statement?

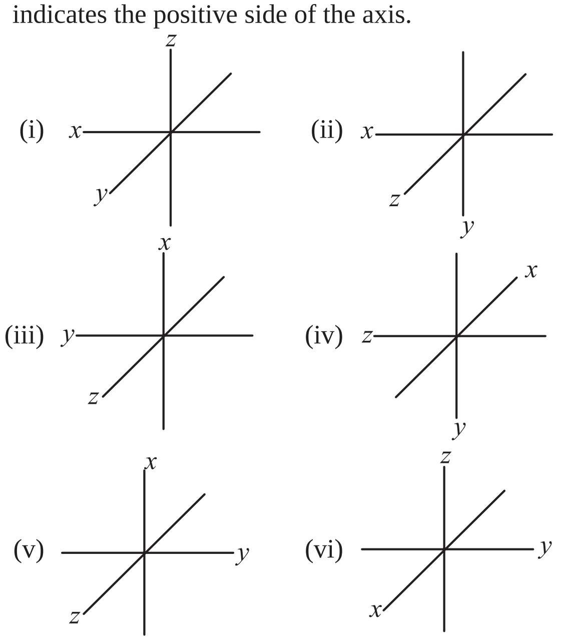
(*a*) A vector cannot be divided by another vector

(*b*) Angular displacement can either be a scalar or a vector

(*c*) Since addition of vectors is commutative therefore vector subtraction is also commutative

(*d*) The resultant of two equal forces of magnitude F acting at a point is F if the angle between the two forces is 120º.

2. Which of the arrangement of axes in figure can be labelled "right handed coordinate system"? As usual, each axis lable indicates the positive side of the axis.



(*a*) (i), (ii) (*b*) (iii) (iv)

(*c*) (vi) (*d*) (v)

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 3 to 5): A particle is moving along positive x-axis. Its position varies as $x = t^3 - 3t^2 + 12t + 20$, where x is in meters and t is in seconds.

3. Initial velocity of the particle is.

(*a*) 1 m/s (*b*) 3 m/s

(*c*) 12 m/s (*d*) 20 m/s

4. Initial acceleration of the particle is

(*a*) Zero (*b*) 1 m/s^2 (*c*) – 3m/s^2 (*d*) – 6 m/s^2

5. Velocity of the particle when its acceleration zero is

(*a*) 1 m/s (*b*) 3 m/s

(*c*) 6 m/s (*d*) 9 m/s

Comprehension (Q. 6 to 8): Two forces $\vec{F}_1 = 2\hat{i} + 2\hat{j}$ N and $\vec{F}_2 = 3\hat{j} + 4\hat{k}$ N are acting on a particle.

6. The resultant force acting on particle is:

(*a*) $2\hat{i}+5\hat{j}+4\hat{k}$ (*b*) $2\hat{i}-5\hat{j}-4\hat{k}$

(*c*) $\hat{i}-3\hat{j}-2\hat{k}$ (*d*) $\hat{i}-\hat{j}-\hat{k}$

7. The angle between $\vec{F}_1$ and $\vec{F}_2$ is

(*a*) $\theta = \cos^{-1}\left(\dfrac{3}{2\sqrt{5}}\right)$ (*b*) $\theta = \cos^{-1}\left(\dfrac{3}{5\sqrt{2}}\right)$

(*c*) $\theta = \cos^{-1}\left(\dfrac{2}{3\sqrt{5}}\right)$ (*d*) $\theta = \cos^{-1}\left(\dfrac{\sqrt{3}}{5}\right)$

8. The component of force $\vec{F}_1$ along force $\vec{F}_2$ is

(*a*) $\dfrac{5}{6}$ (*b*) $\dfrac{5}{3}$ (*c*) $\dfrac{6}{5}$ (*d*) $\dfrac{5}{2}$

MATCH THE COLUMN TYPE QUESTIONS

9. Match the integrals (given in Column-II) with the given functions (in Column-I)

Column-I		Column-II	
A.	$\int \sec x \tan x\, dx$	p.	$-\dfrac{\text{cosec}\, Kx}{K} + C$
B.	$\int \text{cosec}\, Kx \cot Kx\, dx$	q.	$-\dfrac{\cot\, Kx}{K} + C$
C.	$\int \text{cosec}^2\, Kx\, dx$	r.	$\sec x + C$
D.	$\int \cos\, Kx\, dx$	s.	$\dfrac{\sin\, Kx}{K} + C$

(*a*) A-(r); B-(p); C-(s); D-(q)

(*b*) A-(p); B-(r); C-(s); D-(q)

(*c*) A-(r); B-(s); C-(q); D-(p)

(*d*) A-(r); B-(p); C-(q); D-(s)

10. Match the statements given in Column-I with statements given in Column-II

Column-I		Column-II	
A.	if $\lvert\vec{A}+\vec{B}\rvert = \lvert\vec{A}\rvert$ then angle between $\vec{A}$ and $\vec{B}$ is	p.	90°
B.	Magnitude of resultant of two forces $\lvert\vec{F}_1\rvert$ = 8N and $\lvert\vec{F}_2\rvert = 4\ N$ may be	q.	120°

C.	Angle between $\vec{A} = 2\,\hat{i} + 2\,\hat{j}$ & $\vec{B} = 3\,\hat{k}$ is	r.	12 N
D.	Magnitude of resultant of vectors $\vec{A} = 2\,\hat{i} + \hat{j}$ and $\vec{B} = 3\hat{k}$ is	s.	$\sqrt{14}$

(*a*) A-(s); B-(q); C-(p); D-(r)
(*b*) A-(p); B-(r); C-(s); D-(q)
(*c*) A-(r); B-(s); C-(q); D-(p)
(*d*) A-(q); B-(r); C-(p); D-(s)

NUMERICAL BASED QUESTIONS

11. A boy A is standing 3 m west and 4 m north to a boy B. A starts moving along a vector $\vec{a} = 1.5\hat{i} + 2\,\hat{j}$ with a constant speed of 2 m/s for 5 s and stops. Its new position vector with respect to the boy B is $x\hat{i} + y\hat{j}$. Find xy.

12. If the vector's $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\,\hat{j} - \hat{k}$ are perpendicular to each other then find the positive value of 'a'.

13. Magnitude of resultant of two vector $\vec{A}$ and $\vec{B}$ is equal to 2. Angle between two vectors is 180º. If $|\vec{A}| = 3$ then find $|\vec{B}|$ (must be less then 2)

14. Two particles are moving with velocity $\vec{v_1} = \hat{i} - 2t\,\hat{j}$ m/s and $\vec{v_2} = 4\hat{i} + \hat{j}$ m/s respectively Time at which they are moving perpendicular to each other is.

15. Magnitude of subtraction of two vector $\vec{A}$ and $\vec{B}$ is equal to 5. Angle between both is 180º. Find the magnitude of resultant of these two vectors if $|\vec{A}| = 2$.

Exercise-4 (Past Year Questions)

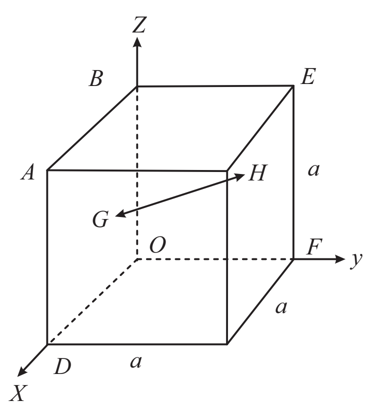
JEE MAIN

1. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is: **(2019)**

(*a*) $\cos^{-1}\left[\dfrac{n^2-1}{n^2+1}\right]$ (*b*) $\cos^{-1}\left[\dfrac{n-1}{n+1}\right]$

(*c*) $\sin^{-1}\left[\dfrac{n^2-1}{n^2+1}\right]$ (*d*) $\sin^{-1}\left[\dfrac{n-1}{n+1}\right]$

2. In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be **(2019)**



(*a*) $\frac{1}{2}a(\hat{k}-\hat{i})$ (*b*) $\frac{1}{2}a(\hat{i}-\hat{k})$ (*c*) $\frac{1}{2}a(\hat{j}-\hat{i})$ (*d*) $\frac{1}{2}a(\hat{j}-\hat{k})$

3. Two forces P and Q, of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is: **(2019)**

(*a*) 120º (*b*) 60º (*c*) 90º (*d*) 30º

4. Let $|\vec{A}_1| = 3, |\vec{A}_2| = 5$ and $|\vec{A}_1 + \vec{A}_2| = 5$. The value of $(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2)$ is: **(2019)**

(*a*) –112.5 (*b*) –106.5
(*c*) –118.5 (*d*) –99.5

5. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is **(2020)**

6. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})\,N$ acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})\,m$. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}\,N-m$. The value of x is **(2020)**

(*a*) 195 (*b*) 165
(*c*) 105 (*d*) 135

7. Statement-I: Two forces $(\vec{P} + \vec{Q})$ and $(\vec{P} - \vec{Q})$ where $\vec{P} \perp \vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement-II: In the situation given above.

$\theta_1 = 60°$ and $\theta_2 = 90°$ **(2021)**

In the light of the above statements, choose the most appropriate answer from the options given below:

(*a*) Statement-I is false but Statement-II is true.

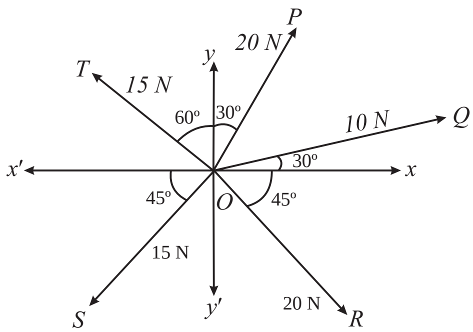
(*b*) Both Statement-I and Statement-II are true.

(*c*) Statement-I is true but Statement-II is false.

(*d*) Both Statement-I and Statement-II are false.

8. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately *N*.

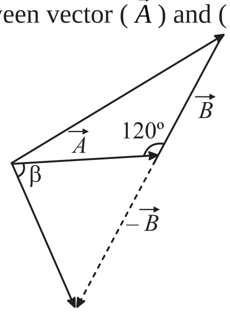
[Take $\sqrt{3} = 1.7$, $\sqrt{2} = 1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along *x*, *y* axis] **(2021)**



(*a*) $9.2\hat{i} + 5\hat{j}$ (*b*) $3\hat{i} + 15\hat{j}$

(*c*) $2.5\hat{i} - 14.5\hat{j}$ (*d*) $-1.5\hat{i} - 15.5\hat{j}$

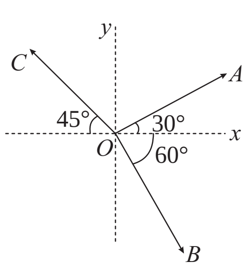
9. The angle between vector $(\vec{A})$ and $(\vec{A} - \vec{B})$ is: **(2021)**



(*a*) $\tan^{-1}\left(\dfrac{-\dfrac{B}{2}}{A - B\dfrac{\sqrt{3}}{2}}\right)$ (*b*) $\tan^{-1}\left(\dfrac{A}{0.7B}\right)$

(*c*) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A-B}\right)$ (*d*) $\tan^{-1}\left(\dfrac{B\cos\theta}{A - B\sin\theta}\right)$

10. The magnitude of vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}, + \overrightarrow{OB} - \overrightarrow{OC}$ with *x*-axis will be **(2021)**



(*a*) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$ (*b*) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$

(*c*) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$ (*d*) $\tan^{-1}\dfrac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$

11. If the projectile of $2\hat{i} + 4\hat{j} - 2\hat{k}$ on $\hat{i} + 2\hat{j} + \alpha\hat{k}$ is zero. Then, the value of a will be ______. **(2022)**

12. If $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})$m and $\vec{B} = (\hat{i} + 2\hat{j} - 2\hat{k})$m. **(2022)**

13. Which of the following relations is true for two unit vector $\hat{A}$ and $\hat{B}$ making an angle to θ each other? **(2022)**

(*a*) $|\hat{A}+\hat{B}| = |\hat{A}-\hat{B}|\tan\dfrac{\theta}{2}$ (*b*) $|\hat{A}-\hat{B}| = |\hat{A}+\hat{B}|\tan\dfrac{\theta}{2}$

(*c*) $|\hat{A}+\hat{B}| = |\hat{A}-\hat{B}|\cos\dfrac{\theta}{2}$ (*d*) $|\hat{A}-\hat{B}| = |\hat{A}+\hat{B}|\cos\dfrac{\theta}{2}$

JEE ADVANCED

14. Can the resultant of two vectors be zero **(2000)**

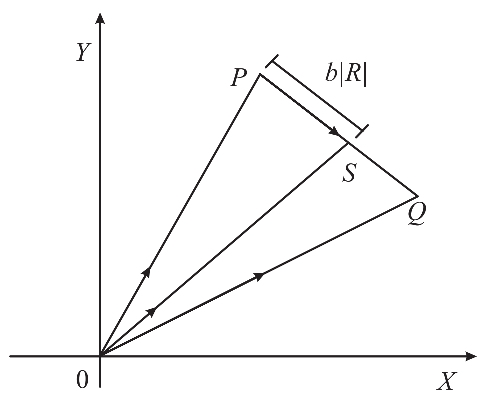
(*a*) Yes, when the 2-vectors are same in magnitude and direction.

(*b*) No

(*c*) Yes, when the 2-vectors are same in magnitude but opposite in sense.

(*d*) Yes, when the 2-vectors are same magnitude making an angle of (2π/3) with each-other

15. Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the point *P* and *S* is $b|\vec{R}|$ and $\vec{R} = \vec{Q} - \vec{P}$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is **(2017)**



(*a*) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$ (*b*) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(*c*) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (*d*) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

16. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\,\hat{i} + \sin\omega t\hat{j})$, where a is a constant and ω = π/6rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time t = τ for the first time, the value of τ, in seconds, is____ **(2018)**

ANSWER KEY

CONCEPT APPLICATION

1. [150°] **2.** (*a*) 60, (*b*) 15, (*c*) 75° **3.** (*a*) 120°, (*b*) 120°, (*c*) 120° **4.** F_1 = 6 N, F_2 = 10 N,
5. $R = 7\sqrt{2}\ cm$ and $\alpha = 45°$ **6.** (*c*) **7.** (*a*) **8.** $(-\sqrt{12})$ **9.** (*d*) **10.** (*d*) **11.** (*b*) **12.** (*c*)
13. (*a*) **14.** (*a*) **15.** (*d*) **16.** [2] **17.** $R=\sqrt{2}\,r+r$ along OB **18.** $(15i + 20j)$ **19.** [150°]
20. $(60\hat{i}+50\hat{j})$ **21.** [100 ms^{-1}] **22.** (*a*) **23.** (*c*) **24.** (*c*) **25.** (*a*) **26.** (*b*) **27.** (*c*)
28. (*b*) **29.** (*a*) **30.** [20s] **31.** $\dfrac{\hat{i}-4\hat{j}-5\hat{k}}{\sqrt{42}}$ **32.** $\hat{i}+7\hat{j}+5\hat{k}$ **33.** (*c*) **34.** (*a*)
35. (*a*) $\left(\dfrac{3}{4}\right)$, (*b*) $\left(\dfrac{-3}{25}\right)$ **36.** $\left(\dfrac{-3}{25}\right)$ **37.** $\left(\dfrac{3}{\sqrt{5}}\right)$ **38.** $\left[\dfrac{1+\sqrt{3}}{2}\right]$ **39.** $\left[-\sqrt{3}\right]$ **40.** $\left[\dfrac{1}{\sqrt{3}}\right]$ **41.** [1]
42. $\dfrac{1}{2\sqrt{x}}-6x$ **43.** $-2\left[\dfrac{1}{x^3}+1\right]$ **44.** $(-3x^2 + 10x^{-3})$ **45.** $\dfrac{dy}{dx}=\dfrac{1}{3\sqrt{x}};\dfrac{dx}{dy}=15y$ **46.** $\dfrac{7}{2}x^{5/2}$ **47.** $-3x^{-4}$ **48.** [1]
49. $5x^4 + 3x^2 + 2x^{-1/2}$ **50.** $20x^3 + 9x^{1/2} + 9$ **51.** $2ax + b$ **52.** $15x^4 - 3 + 1/x^2$ **53.** $2t + 5$ **54.** $u + at$ **55.** 30 cm^2
56. $2\pi r$ **57.** $4x + 3$ **58.** $216x^7 - 144x^5 + 105x^4 + 135x^2 - 40$ **59.** [6] **60.** [–6] **61.** [2] **62.** [6]
63. $\dfrac{2}{(2x+1)^2}$ **64.** $-\dfrac{1}{(4x+5)^2}$ **65.** $\dfrac{2x-x^4}{(x^3+1)^2}$ **66.** $\dfrac{2}{5}x^{5/2}+\dfrac{\cos x^2}{2x}+c$ **67.** $6x-\dfrac{4}{x}+c$
68. $\sin x - \cos x + C$ **69.** $-\left(\dfrac{2}{x^2}+\sin x\right)+c$ **70.** $\dfrac{\sin 3x}{3}+c$ **71.** $-\left[\dfrac{1}{x}+\dfrac{3}{2}\cos 2x\right]+c$ **72.** $\dfrac{2a}{3}+2b$ **73.** $\dfrac{1}{4}\log_e 15$,
74. $\left[-\dfrac{1}{3}\cos\left(\dfrac{3\pi}{2}-1\right)+\cos\left(\dfrac{1}{3}\right)\right]$ **75.** $\dfrac{1}{2}\left[\sin(2B-30)-\sin(2A-30)\right]$ **76.** $\dfrac{x^{16}}{16}+c$ **77.** $-2x^{-1/2} + c$
78. $\dfrac{x^{-6}}{2}+\log_e x+c$ **79.** $\dfrac{x^2}{2}+2x+\log_e x+c$ **80.** $\dfrac{x^2}{2}+\log_e x+c$ **81.** $-\dfrac{a}{x}+b\log_e x+c$ **82.** $\dfrac{GMm}{R}$
83. $kq_1 q_2\left(\dfrac{1}{r_2}-\dfrac{1}{r_1}\right)$ **84.** $\dfrac{1}{2}M(v^2-u^2)$ **85.** [∞] **86.** [1] **87.** [1] **88.** [2]

EXERCISE-1 (TOPICWISE)

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*d*) **5.** (*a*) **6.** (*d*) **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*a*)
11. (*a*) **12.** (*d*) **13.** (*b*) **14.** (*b*) **15.** (*a*) **16.** (*d*) **17.** (*d*) **18.** (*b*) **19.** (*d*) **20.** (*b*)
21. (*a*) **22.** (*a*) **23.** (*a*) **24.** (*b*) **25.** (*d*) **26.** (*b*) **27.** (*c*) **28.** (*d*) **29.** (*b*) **30.** (*a*)
31. (*b*) **32.** (*d*) **33.** (*c*) **34.** (*a*) **35.** (*d*) **36.** (*b*) **37.** (*a*) **38.** (*a*) **39.** (*b*) **40.** (*b*)
41. (*d*) **42.** (*b*) **43.** (*b*) **44.** (*a*) **45.** (*d*) **46.** (*a*) **47.** (*a*) **48.** (*b*) **49.** (*a*) **50.** (*c*)
51. (*a*) **52.** (*c*) **53.** (*b*) **54.** (*a*) **55.** (*d*) **56.** (*a*) **57.** (*d*) **58.** (*b*) **59.** (*a*) **60.** (*d*)
61. (*d*) **62.** (*b*) **63.** (*a*) **64.** (*c*) **65.** (*a*) **66.** (*d*) **67.** (*a*) **68.** (*a*) **69.** (*a*) **70.** (*b*)

EXERCISE-2 (LEARNING PLUS)

1. (*a*) **2.** (*b*) **3.** (*b*) **4.** (*b*) **5.** (*c*) **6.** (*b*) **7.** (*d*) **8.** (*a*) **9.** (*a*) **10.** (*d*)
11. (*d*) **12.** (*d*) **13.** (*b*) **14.** (*b*) **15.** (*b*) **16.** (*c*) **17.** (*a*) **18.** (*b*) **19.** (*b*) **20.** (*d*)
21. (*a*) **22.** (*d*) **23.** (*a*) **24.** (*d*) **25.** (*c*) **26.** (*d*) **27.** (*d*) **28.** (*a*) **29.** (*a*) **30.** (*c*)
31. (*d*) **32.** (*a*) **33.** (*c*) **34.** (*b*) **35.** (*d*) **36.** (*a*) **37.** (*b*) **38.** (*c*) **39.** (*a*) **40.** (*d*)
41. (*a*) **42.** (*a*) **43.** (*a*) **44.** (*a*) **45.** (*d*) **46.** (*a*) **47.** (*d*) **48.** (*c*)

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*a*, *b*, *d*) **2.** (*a*, *b*, *c*) **3.** (*c*) **4.** (*d*) **5.** (*d*) **6.** (*a*) **7.** (*b*) **8.** (*c*) **9.** (*d*) **10.** (*d*)
11. [0036] **12.** [0003] **13.** [1] **14.** [2] **15.** [1]

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*a*) **5.** (*c*) **6.** [90º] **7.** (*a*) **7.** (*b*) **8.** (*a*) **9.** (*c*)
10. (*a*) **11.** [5] **12.** [2] **13.** (*b*)

JEE Advanced

14. (*c*) **15.** (*d*) **16.** [2.00]

CHAPTER

3 Motion in a Straight Line

INTRODUCTION

A body is at rest when it does not change its position with time and is in motion if it changes its position with time in the frame of reference of the observer.

All motion is relative. There is no meaning of rest or motion without reference to the observer.

A passenger in a moving train is at rest with respect to another passenger in the same train while both are in motion with respect to observer on the ground. Therefore nothing is at absolute rest or in absolute motion.

To describe the motion of a particle, we introduce four important quantities namely position, displacement, velocity and acceleration. In general motion of a particle in three dimension these quantities are vectors which have direction as well as magnitude. But for a particle moving in a straight line, there are only two directions, distinguished by designating one as positive and one negative.

DISTANCE AND DISPLACEMENT

Distance

The length of the actual path between initial and final positions of a particle in a given interval of time is called distance covered by the particle. It is the actual length of the path covered by the body.

Characteristics of Distance

- It is a scalar quantity
- It depends on the path
- It never reduces with time
- Distance covered by a particle is always positive or zero and can never be negative
- Dimension: $[M^0 L^1 T^0]$
- Unit: In C. G. S. centimeter (cm), In S.I. system meter (m).

Displacement

The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final and initial positions.

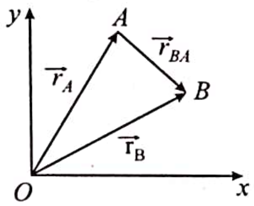
Displacement of a particle is a position vector of its final position w.r.t, initial position.

Position vector of A w.r.t. $O = \overrightarrow{OA}$

$$\Rightarrow \vec{r}_A = x_1\,\hat{i} + y_1\,\hat{j} + z_1\,\hat{k}$$

Position Vector of B w.r.t. $O = \overrightarrow{OB}$

$$\Rightarrow \vec{r}_B = x_2\,\hat{i} + y_2\,\hat{j} + z_2\,\hat{k}$$



Displacement $= \overrightarrow{AB} = \vec{r}_{BA}$

$$= (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$$

For a particle moving in a straight line, if we take x-axis along direction of motion, then displacement has only one component $S = x_2 - x_1$

Characteristics of Displacement

- It is a vector quantity.
- The displacement of a particle between any two points is equal to the shortest distance between them.
- The displacement of an object in a given time interval may be +ve, –ve or zero.
- The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, is it less than the magnitude of the displacement, i.e. Distance $\geq$ |Displacement|
- Dimension: $[M^0 L^1 T^0]$
- Unit: In C. G. S. centimeter (cm), In S.I. system meter (m).

Note: Distance is always positive but displacement may be +ve, – ve or zero.

AVERAGE SPEED AND INSTANTANEOUS SPEED

Average speed is the ratio of total distance covered by a particle in a given time interval divided by the time interval.

$$v_{av.} = \frac{\Delta s}{\Delta t} \qquad \text{(where } \Delta t = t_2 - t_1\text{)}$$

Instantaneous speed at any instant t is

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- The slope of the distance-time graph provides the value of instantaneous speed.
- The average speed is defined for a time interval while the instantaneous speed is defined at an instant. The word speed normally implies instantaneous speed.
- Average speed and instantaneous speed both are scalar quantities.
- For any moving object, the average speed can never be zero or negative, i.e., $v_{av} > 0$, as total distance covered is always +ve only.
- If a particle travels distances s_1, s_2, s_3, ..., etc., at different speeds v_1, v_2, v_3, ... etc., respectively.

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{\Sigma s_i}{\Sigma(s_i / v_i)} \quad \text{If } s_1 = s_2 = ... = s_n = s,$$

Then $\dfrac{1}{v_{av.}} = \dfrac{1}{n}\left[\dfrac{1}{v_1} + \dfrac{1}{v_2} + ...\right] = \dfrac{1}{n}\Sigma\dfrac{1}{v_i}$

Special case: If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, then

$$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

- If a particle travels at speeds $v_1, v_2, \ldots$, etc., for time intervals $t_1, t_2, \ldots$

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{v_1t_1 + v_2t_2 + ...+ v_nt_n}{t_1 + t_2 + ...t_n} = \frac{\Sigma v_it_i}{\Sigma t_i}$$

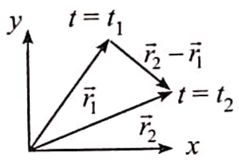
- If a particle moves for two equal intervals of time at different speeds, then $v_{av} = \dfrac{v_1 + v_2}{2}$.

AVERAGE VELOCITY

The average velocity is the ratio of displacement from time t_1 to t_2 and the time interval $t_2 - t_1$.

$$\vec{v}_{av.} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$$

In one dimension, $v_{av} = \dfrac{x_2 - x_1}{t_2 - t_1}$



INSTANTANEOUS VELOCITY

The instantaneous velocity at any instant t is

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

The magnitude of instantaneous velocity is always equal to the instantaneous speed.

Instantaneous speed

$$= |\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Instantaneous velocity is called simply velocity.

In one dimension, we can simply write velocity as

$$v = v_x = \frac{dx}{dt}$$

If an object moves along a straight line without changing its direction, then the magnitude of the average velocity is equal to the average speed, otherwise magnitude of average velocity < average speed.

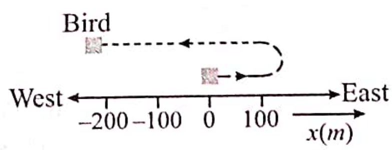
- Velocity can be $+ve$ or $-ve$ as it is a vector but speed can never be negative as it is the magnitude of velocity.
- If a body is moving with a constant velocity, then the average velocity and instantaneous velocity are equal.
- The velocity of a body is uniform, if both magnitude and direction do not change.
- If a body moves with non-uniform velocity, then magnitude of velocity may change or direction of velocity may change or both.
- A body can have non-zero speed and zero average velocity when a body completes one revolution around a circle, the average velocity is zero since the displacement is zero. But the average speed is not zero since the distance travelled $\neq 0$.
- If a body is moving with constant speed then its velocity may or may not be constant. In case of uniform circular motion though speed remains constant but velocity changes instant to instant because of change in direction.

Train Your Brain

Example 1: A bird flies towards east at 10 m/s for 100 m. It then turns around and flies at 20 m/s for 15s. Find

(*a*) Its average speed (*b*) Its average velocity

Sol. (*a*) Let us take the x axis to point eastwards A sketch of the path is shown in the figure. To find the required quantities, we need the total time interval. The first of the journey took.



$\Delta t_1 = (100 \text{ m})/(10 \text{ m/s}) = 10\text{s}$, and we are given $t_2 = 15\text{s}$ for the second part. Hence the total time interval is $\Delta t = \Delta t_1 + \Delta t_2 = 25 \text{ s}$

The bird flies 100 m east and then (20 m/s) × (15s) = 300 m west.

(a) Average speed $= \frac{\text{Distance}}{\Delta t}$

$= \frac{100\text{m} + 300\text{m}}{25\text{s}} = 16 \text{ m/s}$

(b) The net displacement is

$\Delta x = -200$ m

So, $v_{av} = \frac{\Delta x}{\Delta t} = \frac{-200\text{m}}{25\text{s}} = -8$ m/s

The negative sign means that v_{av} is directed toward the west.

Example 2: A particle moves with speed v_1 along a particular direction. After some time it turns back and reaches the starting point again travelling with speed v_2. Find (for the whole journey).

(a) Average velocity (b) Average speed

Sol. (a) Since the particle reaches the starting point again, its displacement is zero.

$\therefore$ Average velocity $= \frac{\text{Net displacement}}{\text{total time}} = 0$

(b) Let it travelled distance x while moving away as well as while moving towards the starter point.

Time taken to go away is $t_1 = \frac{x}{v_1}$

Time taken while return journey $t_2 = \frac{x}{v_2}$

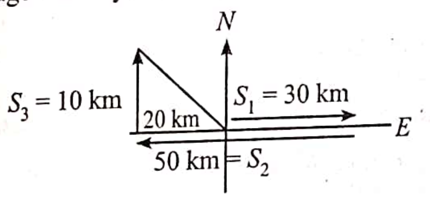
$\therefore$ Average speed $= \frac{x+x}{t_1+t_2} = \frac{2x}{\frac{x}{v_1}+\frac{x}{v_2}}$

$$v_{av} = \frac{2v_1v_2}{v_1+v_2}$$

i.e., harmonic mean of individual speeds.

Example 3: A person goes 30 km East, then he walks 50km west and then he goes 10 km N. Find Average speed and average velocity for the whole journey in 15 hrs.

Sol.



Average speed $= \frac{\text{Total Distance}}{\text{Total Time}} = \frac{30+50+10}{15}$

$= 6$ km/hr

Average velocity $= \frac{\text{Total Displacement}}{\text{Total Time}}$

$= \frac{\sqrt{(20^2+10^2)}}{15} = \frac{10\sqrt{5}}{15}$

Average velocity $= \frac{2\sqrt{5}}{3}$ km/hr

Concept Application

1. A car moves 30 km with 20 km/hr and then 30 km with 30 km/hr. Find average speed in Average velocity for the whole journey in the same straight line.
2. A person goes 20 km N, then 20 km E and then 20 km N-E, find average speed and average velocity in total time taken is 6 hr.

ACCELERATION

The rate of change of velocity of an object with time is called acceleration of the object.

Let v and v' be the velocity of the object at time t and t' respectively, then acceleration of the body is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

For one dimensional motion,

$$a = a_x = \frac{dv_x}{dt} = \frac{d\vec{v}}{dt}$$

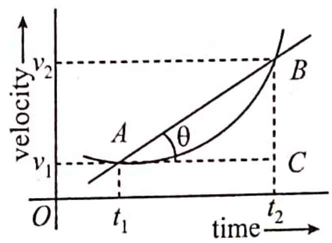
- Acceleration is a vector quantity.
- It is positive if the velocity is increasing and is negative if the velocity is decreasing.
- The negative acceleration is also called retardation or deceleration.
- **Unit:** In S.I. System m/s^2
  In C.G.S. System cm/s^2
- **Dimension :** $[M^0L^1T^{-2}]$

Average Acceleration

When an object is moving with a variable acceleration in a straight line, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken i.e.,

Average Acceleration $= \frac{\text{total change in velocity}}{\text{total time taken}}$

Suppose the velocity of a particle is v_1 at time t_1 and v_2 at time t_2.



Then, Change in velocity $= v_2 - v_1 = \Delta v$

Elapsed time in changing the velocity $= t_2 - t_1 = \Delta t$

Thus, $a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$

$\Rightarrow a_{av} = \dfrac{BC}{AC} = \tan\theta =$ the slope of chord of $v - t$ graph is average acceleration.

Note: If any body is accelerated with acceleration a_1 till time t_1 and acceleration a_2 up to time t_2 then average acceleration will $a_{av} = \dfrac{a_1t_1 + a_2t_2}{t_1 + t_2}$.

Instantaneous Acceleration

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration.

Suppose the velocity of a particle at time $t_1 = t$ is $v_1 = v$ and becomes $v = v + \Delta v$ at time $t_2 = t + \Delta t$,

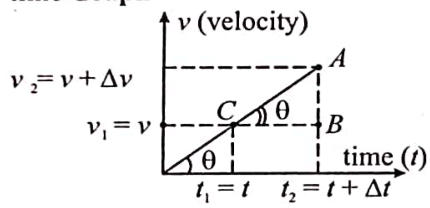
Then, $a_{av} = \dfrac{\Delta v}{\Delta t}$

If Δt approaches to zero, then the rate of change of velocity will be instantaneous acceleration.

Instantaneous acceleration $a_{\text{inst}} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Instantaneous acceleration at a point is equal to slope of tangent at that point on velocity time graph in the graph shown.

Velocity–time Graph



$$\text{Slope} = \frac{AB}{BC} = \frac{\Delta v}{\Delta t} = \text{acceleration}$$

As $v = \dfrac{dx}{dt}$ therefore $a = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$

Thus, instantaneous acceleration of an object is equal to the second derivative of the position w.r.t. time of the object at the given instant.

Note:

(i) It is not essential that when velocity is zero acceleration must be zero.

e.g. In vertical motion under gravity at the top point $v = 0$ but $a \neq 0$

(ii) If a and v are both positive or both negative, speed of a body increases. If a and v have opposite signs then speed decreases, this is called retardation.

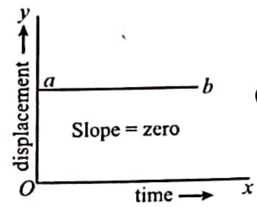
(iii) For curvilinear motion velocity may vary even if speed is constant.

DISPLACEMENT-TIME GRAPHS AND THEIR CHARACTERISTICS

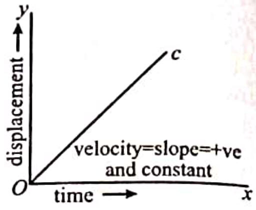
If the graph is:

- A straight line parallel to time-axis, shown by line ab, it means that the body is at rest, i.e., $v = 0$.
- A straight line inclined to x-axis (such as Oc and fg) shows that body is moving with a constant velocity.
- A straight line inclined to x-axis by an angle $> 90°$ (line fg) represent negative velocity.
- The curve is of the type Od (graph-3) whose slope decreases with time, the velocity goes on decreasing, i.e., motion is retarded.
- The curve is of the type Oe (graph-4) whose slope increases with time, the velocity goes on increasing, i.e., motion is accelerated.

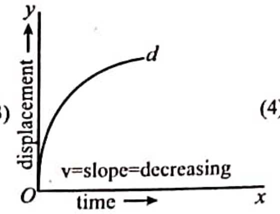
(1)



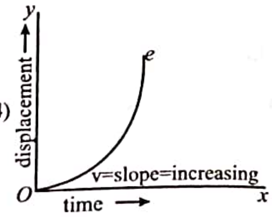
(2)



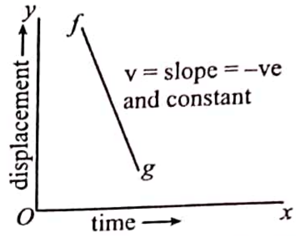
(3)



(4)



(5)



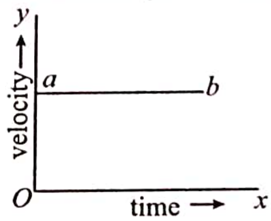
No line can ever be perpendicular to the time axis because it implies infinite velocity.

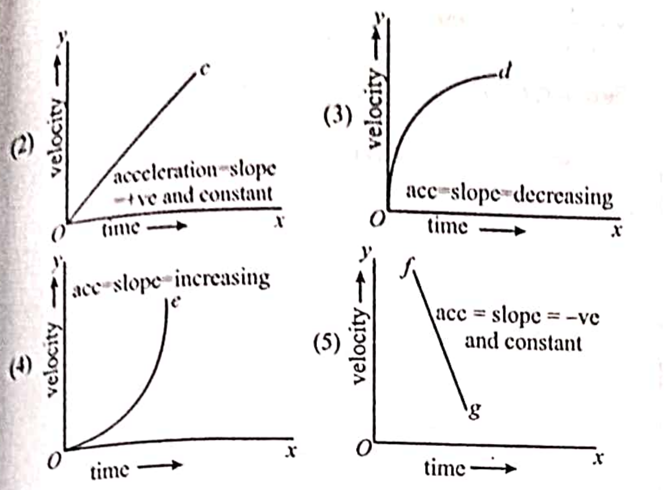
VELOCITY-TIME GRAPHS AND THEIR CHARACTERISTICS

If the graph is:

- A straight line parallel to time axis shown by line ab, it means that the body is moving with a constant velocity or acceleration (a) is zero.
- A straight line inclined to the x-axis with $+ve$ slope (line Oc) it means that the body is moving with constant acceleration.
- A straight line inclined to x-axis with negative slope it means that the body is under retardation.
- A curve like Od (graph 3) whose slope decreases with time, the acceleration goes on decreasing.
- A curve like Oe (graph 4) whose slope increases with time, the acceleration goes on increasing.

(1)





Note:
1. No velocity-time graph can ever be perpendicular to the time-axis because it implies infinite acceleration.
2. The area of velocity-time graph with time axis represents the displacement of that body.

MOTION WITH CONSTANT ACCELERATION

In many types of motion, the acceleration is either constant or approximately so. For example, near the surface of earth all objects fall vertically with constant acceleration if air resistance is neglected. Even when acceleration is not constant, we can learn something about the motion of the body by using constant acceleration results to be developed later in this section.

Let the velocity of body at $t = 0$ is u and it moves with constant acceleration a and acquires velocity v at time t.

$$\frac{dv}{dt} = a \quad \text{or} \quad dv = adt$$

$$\Rightarrow \int_u^v dv = \int_0^t adt = a\int_0^t dt$$

$$\Rightarrow v\Big|_u^v = at\Big|_0^t$$

$$\Rightarrow v - u = at \quad \text{or} \quad v = u + at \qquad \text{...(i)}$$

To find the displacement, we again integrate

Let body be at x_0 at $t = 0$ and reaches x at time t

$$v = u + at$$

$$\frac{dx}{dt} = u + at$$

$$\text{or } dx = (u + at)dt$$

$$\int_{x_0}^x dx = \int_0^t (u+at)dt = u\int_0^t dt + a\int_0^t tdt$$

$$x\Big|_{x_0}^x = ut\Big|_0^t + a\frac{t^2}{2}\Big|_0^t$$

$$x - x_0 = ut + \frac{1}{2}at^2$$

$$S = x - x_0 = ut + \frac{1}{2}at^2 \qquad \text{...(ii)}$$

We can also find, relation between velocity and displacement. By using chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{vdv}{dx}$$

$$\Rightarrow vdv = adx$$

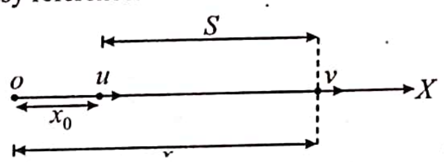
$$\int_u^v vdv = a\int_{x_0}^x dx$$

$$\frac{v^2}{2}\Big|_u^v = ax\Big|_{x_0}^x$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a(x - x_0)$$

$$\Rightarrow v^2 - u^2 = 2aS \qquad \text{...(iii)}$$

These relation are very helpful in solving the problems of motion in one dimension. All these relations are given in table below for easy reference.



Equation	Contains S	v	t
$v = u + at$	No	Yes	Yes
$s = ut + \frac{1}{2}at^2$	Yes	No	Yes
$v^2 - u^2 = 2as$	Yes	Yes	No

In simple problems in uniformly accelerated motion, two parameters are given third is to be found. Depending on convenience one can choose any one of the three relation. The following two relation are also helpful in solving problems.

Displacement of the Body in the n^{th} Second:

$$S_n = S(t = n) - s(t = n - 1)$$

$$= \left(un + \frac{1}{2}an^2\right) - \left(u(n-1 + \frac{1}{2}a(n-1)^2\right)$$

$$= u + \frac{a}{2}(2n - 1)$$

Average velocity:

$$V_{avg} = \frac{S}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{at}{2}$$

$$= u + \frac{v-u}{2} = \frac{u+v}{2}$$

$$\text{or} \quad S = \left(\frac{u+v}{2}\right)t$$

This relation is only valid for uniform acceleration.

Note:
1. These equations can be applied only when acceleration is constant.

- If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then average velocity $= \frac{v+u}{2}$.
- If a body moving with uniform acceleration has velocities u and v at two points in its path, then the velocity at the midpoint of its path $= \sqrt{\frac{u^2+v^2}{2}}$.
- In position time graph, slope is equal to velocity.
- In velocity time graph area under the curve is displacement and slope is equal to acceleration.
- In acceleration time graph area under the curve is equal to change in velocity.
- **For a body starting from rest and moving with uniform acceleration,**
  (*a*) The ratio of distances covered in first one sec, two sec, three sec, ... is :
  $1^2 : 2^2 : 3^2 : ...$, i.e., $1 : 4 : 9 : ...$
  Ratio of distances covered in
  1st, 2nd, 3rd sec, ... is $1 : 3 : 5 : ...$
  (*b*) The ratio of velocities after
  1 sec, 2 sec, 3 sec, ... is $1 : 2 : 3 : ...$

Train Your Brain

Example 4: The displacement of a particle, moving in a straight line, is given by $S = 2t^2 + 2t + 4$ where s is in metres and t in seconds. The acceleration of the particle is

(*a*) 2 m/s^2

(*b*) 4 m/s^2

(*c*) 6 m/s^2

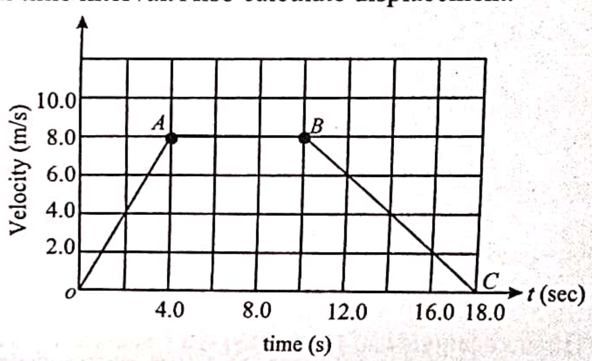
(*d*) 8 m/s^2

Sol. (*b*) Given $S = 2t^2 + 2t + 4$

$\therefore$ Velocity $(v) = \frac{dS}{dt} = 4t + 2$

Acceleration $(a) = \frac{dv}{dt} = 4(1) + 0 = 4 \text{m/s}^2$

Example 5: What is the acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.



Sol. Segment OA; $a = \frac{8-0}{4-0} = 2 \text{ m/s}^2$

Segment AB; graph horizontal i.e., slope zero i.e., $a = 0$

Segment BC; $a = \frac{0-8}{18-10} = -1 \text{ m/s}^2$

The graph is trapezium. Its area between $t = 0$ to $t = 18$s is displacement.

Area of v-t graph = displacement $= \frac{1}{2}(18+6) \times 8 = 96$ m

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for next 6 sec. and then retards uniformly to come to rest in next 8 sec.

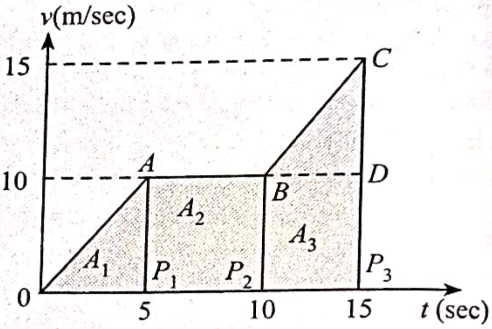
Example 6: The motion of a body is described in (v–t) graph as given under.

Find the followings:

(*a*) Max and Min acceleration

(*b*) Displacement from $t = 10$ to $t = 15$

(*c*) Av-velocity for the whole journey.



Sol. (*a*) We know slope $(v - t)$ graph gives acceleration

$$\text{Slope}_{OA} = \frac{AP_1}{OP_1} = \frac{10}{5} = 2 \text{ m/sec}^2 \text{ (Max-acceleration)}$$

$$\text{Slope}_{A\to B} = 0 \text{ m/sec}^2 \quad \text{(min-acceleration)}$$

$$\text{Slope}_{B\to C} = \frac{CD}{BD} = \frac{5}{5}$$

$= 1 \text{m/sec}^2$

(*b*) $\underset{\text{from } t=10 \text{ to } t=15 \text{ sec}}{\text{Displacement}} = \textit{Area } (v-t) \textit{graph}$

$$= \frac{1}{2}(10+15) \times 5 = 62.5 \text{ m}$$

(*c*) Average Velocity $= \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\text{Area } (v-t)\text{graph}}{t_{total}}$

$$= \frac{A_1 + A_2 + A_3}{t_{total}} = \frac{25+50+62.5}{15}$$

$$= \frac{137.5}{15} = 9.16 \text{ m/sec}$$

Example 7: How long does it take for a particle to travel 100 m if it begins from rest and accelerates at 10 m/s²? What is its velocity when it has travelled 100 m? What is the average velocity during this time.

Sol. $u = 0, a = 10 \text{ m/s}^2, S = 100 \text{ m}$

Applying $S = ut + \frac{1}{2}at^2$

we get $100 = \frac{1}{2}\times 10\times t^2$

$\Rightarrow t = \sqrt{20} = 2\sqrt{5}$ s

$v = u + at = 0 + 10\times 2\sqrt{5} = 20\sqrt{5}$ m

$v_{avg} = \frac{u+v}{2} = \frac{0+20\sqrt{5}}{2} = 10\sqrt{5}$ m/s

Example 8: A car travelling with 72 km/hr is 30 m from a barrier when the driver slams the breaks. The car hits barrier 2.0 seconds later.

(*a*) What is the car's constant deceleration before impact?

(*b*) How fast is car travelling at impact?

Sol. (*a*) $u = 72 \text{ km/hr} = 72\times\frac{5}{18} \text{ m/s} = 20 \text{ m/s}$

$S = 30$ m

$t = 2$s

$a = ?$

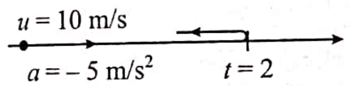
$S = ut + \frac{1}{2}at^2$

$30 = 20\times 2 + \frac{1}{2}\times a\times 2^2$

$\Rightarrow a = -\frac{10}{2} = -5 \text{ m/s}^2$

(*b*) $v = u + at = 20 + (-5)\times 2 = 10$ m/s

Example 9: A particle moving with initial velocity of 10 m/s towards East has an acceleration of 5 m/s² towards west. Find the displacement and distance travelled by the particle in first 4 seconds?

Sol. 

$v = u + at = 10 - 5t$

The direction of velocity changes after two seconds.

$S = 10\times 4 + \frac{1}{2}(-5)\times 4^2 = 0 =$ displacement

Distance travelled is not equal to displacement because during course of journey, velocity changes direction.

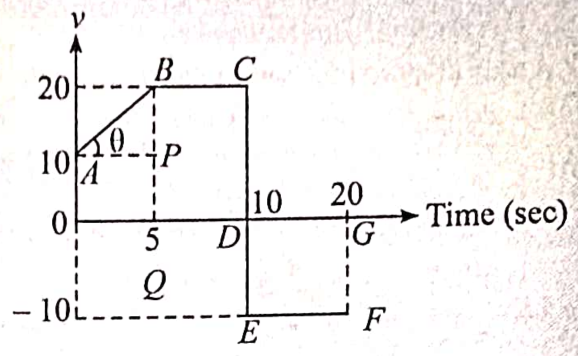
$D = S(t=2) + |S(t=4) - S(t=2)|$

$= \left(10\times 2 - \frac{1}{2}\times 5\times 2^2\right) + \left|0 - (10\times 2) - \frac{1}{2}\times 5\times 2^2\right|$

$= 10 + 10 = 20$ m

Concept Application

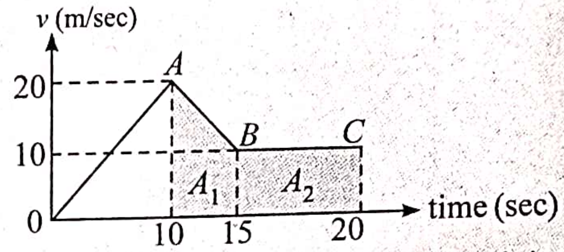
3.



(*a*) Find acceleration b/w $t = 0$ to $t = 5$

(*b*) Find total displacement b/w $t = 0$ to $t = 20$ sec.

4.



(*a*) Find the ratio of Acceleration to Retardation in the graph shown.

(*b*) Total distance covered between 10 to 20 sec.

VERTICAL MOTION UNDER GRAVITY (FREE FALL)

Motion that occurs solely under the influence of gravity is called free fall. Thus a body projected upward or downward or released from rest are all under free fall.

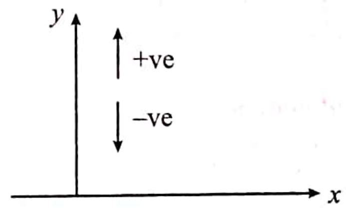
In the absence of air resistance all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

The value of the acceleration due to gravity depends on both latitude and altitude. It is approximately 9.8 m/s² near the surface of the earth. For simplicity a value of 10 m/s² used. To do calculations regarding motion under gravity, we follow a proper sign convention. If we take upward direction as positive then $a = -g$

Thus the equation of kinematics may be modified as

$$v = u - gt \quad \text{...(i)}$$

$$\Delta y = y - y_0 = ut - \frac{1}{2}gt^2 \quad \text{...(ii)}$$



$$v^2 = u^2 - 2g\,(y - y_0) \quad ...(iii)$$

y_0 = position of particle at time $t = 0$

y = position of particle at time t.

u = velocity of particle at time $t = 0$

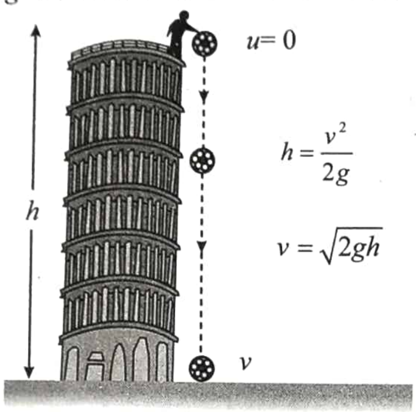
v = velocity of particle at time t.

(i) **A body dropped from some height (initial velocity zero)**

- Equation of motion: Taking initial position as origin and downward direction as negative. Here we have,

  $u = 0$ [As body starts from rest]

  $a = -g$ as acceleration is in the downward direction



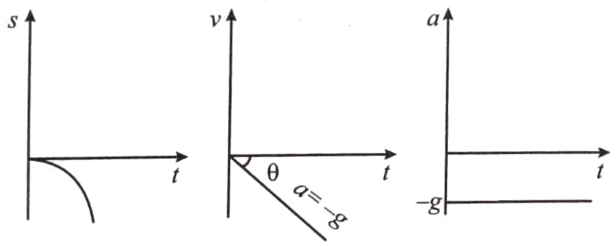
$$v = -gt \quad ...(i)$$

$$\Delta y = -h = -\frac{1}{2}gt^2 \quad ...(ii)$$

$$v^2 = 2(-g)(-h) = 2gh \quad ...(iii)$$

$$h_n = \frac{g}{2}(2n-1) = \text{Height covered in } n^{th} \text{ second.} \quad ...(iv)$$

- Graph of displacement velocity and acceleration with respect to time:



- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, $2t$, $3t$, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

- The distance covered in the nth sec, $h_n = \dfrac{1}{2}g(2n-1)$

  So distance covered in I, II, III sec, etc., will be in the ratio of 1 : 3 : 5. This is called 'Galileo's Law of odd numbers'.

(ii) **A body projected vertically downward with some initial velocity:** The initial velocity is downward and will be negative

Equation of motion: $v = -u - gt$

$$\Delta y = -h = -ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh$$

$$h_n = u + \frac{g}{2}(2n-1)$$

(iii) **A body is projected vertically upward:**

Equation of motion: Taking initial position as origi vertically up as positive

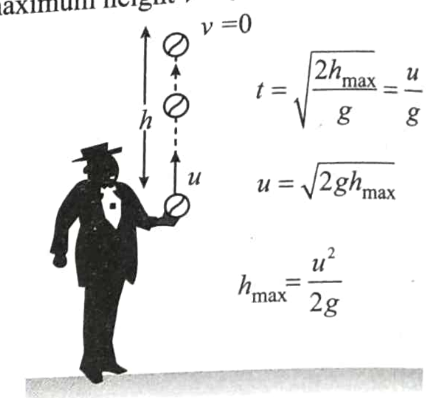
$a = -g$ [As acceleration is downwards]

So, if the body is projected with velocity u and afte t it reaches up to height h then

$$v = u - gt;\ h = ut - \frac{1}{2}gt^2;\ v^2 = u^2 - 2gh;$$

$$h_n = u - \frac{g}{2}(2n-1)$$

For maximum height $v = 0$

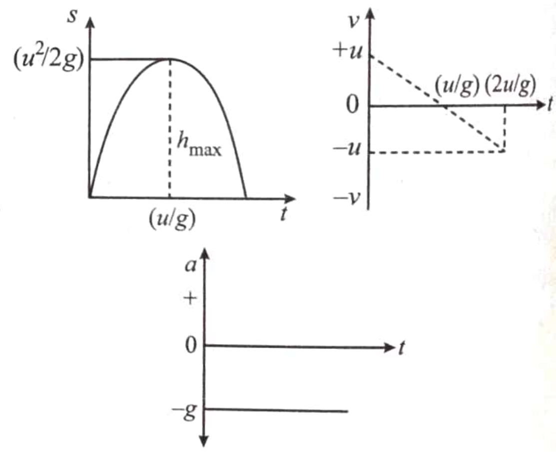


So from above equation

$u = gt$,

$h_{max} = \dfrac{1}{2}gt^2$ and $u^2 = 2gh_{max}$

- Graph of displacement, velocity anssssd accele with respect to time (for maximum height):



Important Points

(i) In case of motion under gravity for a given body, acceleration, and mechanical energy remains co while speed, velocity, momentum, kinetic energy potential energy changes.

(ii) The motion is independent of the mass of the body, any equation of motion, mass is not involved. That is heavy and light body when released from the same he reach the ground simultaneously and with same vel i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$

(iii) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of ascent (t_1) = time of descent (t_2) = u/g

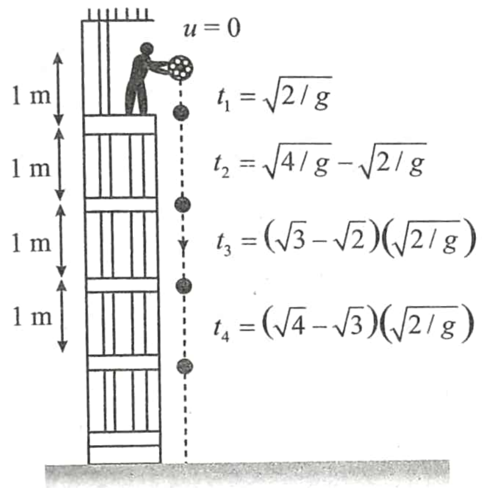
Total time of flight $T = t_1 + t_2 = \dfrac{2u}{g}$

(iv) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

Acceleration at any point on the path is same whether the body is moving in upwards or downward direction.

(v) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1 m each will then be in the ratio of the difference in the square roots of the integers i.e.

$\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3}), \ldots\ldots$



Notes:

(i) During ascent, $a = -g$, velocity becomes less positive i.e., speed decreases velocity and acceleration are in opposite direction.

(ii) During descent, $a = -g$, but now it is in the direction of velocity so it is not retardation. It makes velocity becomes more negative i.e. increases v in negative direction. Velocity and acceleration are in the same direction.

Train Your Brain

Example 10: A man is standing on the top of a building, throws a ball with speed 5 m/s in upward direction from 30 m height above the ground level. How much time does it takes to reach the ground.

Sol. $u = 5$ m/s

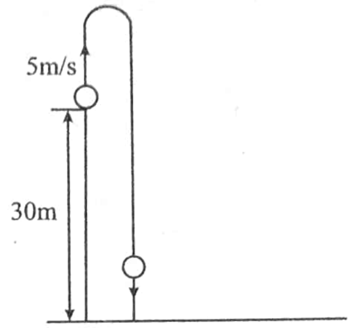
When it reaches the ground, $\Delta y = -30$ m

$\therefore$ From above equation (ii)

$$-30 = 5t - \frac{1}{2}(10)\,t^2$$

$$\Rightarrow t^2 - t - 6 = 0$$

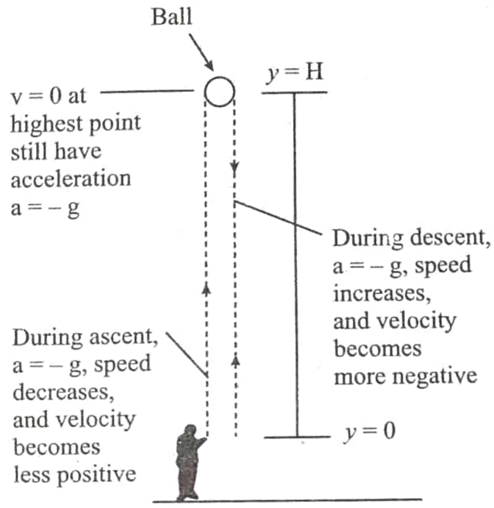
On solving, we get $t = 3$ and -2



Rejecting $t = -2$ sec, we get $t = 3$ sec

Example 11: A kid throws a ball up, with some initial speed. Comment on magnitudes and signs of acceleration and velocity of the ball.

Sol.



Example 12: If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is ($g = 10$ m/s^2)

(*a*) 11.25 m (*b*) 16.2 m
(*c*) 24.5 m (*d*) 7.62 m

Sol. (*a*) $H_{max} = \dfrac{u^2}{2g} = \dfrac{(15)^2}{2 \times 10} = 11.25\,\text{m}$

Example 13: A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is ($g = 10$ m/s^2)

(*a*) 25 m (*b*) 45 m
(*c*) 90 m (*d*) 125 m

Sol. (*b*) $h_n = \dfrac{g}{2}(2n-1) \Rightarrow h_{5\text{th}} = \dfrac{10}{2}(2 \times 5 - 1)$

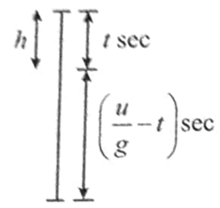
$= 45$ m

Example 14: If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(*a*) $\dfrac{1}{2}gt^2$ (*b*) $ut - \dfrac{1}{2}gt^2$
(*c*) $(u - gt)t$ (*d*) ut

Sol. (*a*) If ball is thrown with velocity u, then time of flight

$$= \frac{u}{g}$$



Velocity after $\left(\frac{u}{g}-t\right)$ sec $\quad v = u - g\left(\frac{u}{g}-t\right) = gt.$

So, distance in last 't' sec $\quad 0^2 = (gt)^2 - 2(g)h.$

$$h = \frac{1}{2}gt^2.$$

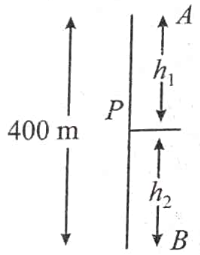
Example 15: A man drops a ball downside from the roof of a tower of height 400 m. At the same time another ball is thrown upside with a velocity 50 m/s from the foot of tower. What is the height from the foot of the tower where the two balls would meet?

(*a*) 100 meters (*b*) 320 meters
(*c*) 80 meters (*d*) 240 meters

Sol. (*c*) Let both balls meet at point P after time t.

The distance travelled by ball A

$$h_1 = \frac{1}{2}gt^2 \qquad \text{...(i)}$$



The distance travelled by ball B

$$h_2 = ut - \frac{1}{2}gt^2 \qquad \text{...(ii)}$$

By adding (i) and (ii) $h_1 + h_2 = ut = 400$

(Given $h = h_1 + h_2 = 400$)

$\therefore$ $t = 400/50 = 8$ s and $h_1 = 320$ m, $h_2 = 80$ m

Example 16: Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant

(*a*) 2.50 m (*b*) 3.75 m
(*c*) 4.00 m (*d*) 1.25 m

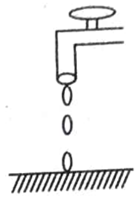
Sol. (*b*) Let the interval be t then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ ...(i)

For second drop $x = \frac{1}{2}gt^2$ ...(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and

Hence required height $h = 5 - \frac{5}{4} = 3.75m.$



Example 17: A balloon is at a height of 81 m and is ascending upwards with a velocity of 12 m/sec. A body of 2 kg weight is dropped from it. If $g = 10$ m/s^2 the body will reach the surface of the earth in

(*a*) 1.5 s (*b*) 4.025 s
(*c*) 5.4 s (*d*) 6.75 s

Sol. (*c*) As the balloon is going up we will take initial velocity of falling body = + 12 m/s,

$\Delta y = -81$ m; $a = -g = -10$ m/s^2

By applying $h = ut + \frac{1}{2}gt^2$; $-81 = 12t - \frac{1}{2}(10)t^2$

$\Rightarrow 5t^2 - 12t - 81 = 0$

$$\Rightarrow t = \frac{12 \pm \sqrt{144+1620}}{10} = \frac{12 \pm \sqrt{1764}}{10}$$

$= 5.4$ s

Example 18: A particle is dropped under gravity from rest from a height h ($g = 9.8$ m/s^2) and it travels a distance $\frac{9h}{25}$ in the last second, the height h is

(*a*) 100 m (*b*) 122.5 m
(*c*) 145 m (*d*) 167.5 m

Sol. (*b*) Distance travelled in n sec $= \frac{1}{2}gn^2 = h$ ...(i)

Distance travelled in n^{th} sec.

$$= \frac{g}{2}(2n-1) = \frac{9h}{25} \qquad \text{...(ii)}$$

Solving (i) and (ii)

We get. $h = 122.5$ m

Example 19: A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity $3u$. The height of the tower is

(*a*) $3u^2/g$ (*b*) $4u^2/g$
(*c*) $6u^2/g$ (*d*) $9u^2/g$

Sol. (*b*) For vertical downward motion we will consider initial velocity $= -u$.

By applying $v^2 = u^2 + 2gh$,

$(3u)^2 = (-u)^2 + 2gh,$

$$\Rightarrow h = \frac{4u^2}{g}$$

Example 20: A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

(*a*) 4.9 m (*b*) 9.8 m

(*c*) 19.6 m (*d*) 24.5 m

Sol. (*d*) The separation between two bodies, two second after the release of second body is given by

$$s = \frac{1}{2}g(t_1^2 - t_2^2)$$

$$= \frac{1}{2}\times 9.8\times(3^2 - 2^2) = 24.5\ m$$

Concept Application

5. A body is projected from Top of a tower vertically upward with 5 m/sec, the body lands the ground in 4 sec. Total height of tower would be

(*a*) 45 m (*b*) 60 m

(*c*) 30 m (*d*) 50 m

6. A body is thrown vertically upwards and if returns back to hand in 3 sec; its velocity of throw and max-height attained are,

(*a*) 15 m/sec, 11.25 m

(*b*) 10 m/sec, 12 m

(*c*) 30 m/sec, 22.5 m

(*d*) 20 m/sec, 30 m

7. Water drops are released from the bottom of an overhead tank at regular internal. When 1st drop touches the ground, fifth drop is just about to release. The height of overhead tank is 8m from the ground. Find the separation between 2nd and 3rd drop just when 1st drop touches the ground.

(*a*) 1.5 m (*b*) 2 m

(*c*) 2.5 m (*d*) 4.5 m

8. Two balls A & B of masses m & 5 m are dropped from the towers of height 36 m and 64 m respectively. The ratio of time taken by them to reach the ground.

(*a*) 2/3 (*b*) 7/4

(*c*) 5/2 (*d*) 3/4

9. A splash is heaved 3 sec after the stone is released into a well of depth 20 m. The velocity of sound is?

(*a*) 20 m/sec

(*b*) 40 m/sec

(*c*) 10 m/sec

(*d*) 35 m/sec

MOTION WITH VARIABLE ACCELERATION

In previous section, we studied rectilinear motion when acceleration is constant. In general acceleration can vary and depend on time, position and velocity of the particle.

Let us consider some simple cases

(i) **Acceleration only depends on time t.**

$$\frac{dv}{dt} = a(t)$$

$$\int_u^v dv = \int_0^t a(t)dt$$

$$\Rightarrow v - u = \int_0^t a(t)dt$$

$$\text{or } v = u + \int_0^t a(t)dt$$

(ii) **Acceleration only depends on position x.**

$$\frac{dv}{dt} = a(x)$$

We can use chain rule to eliminate time.

$$\frac{dv}{dx}\frac{dx}{dt} = a(x)$$

$$\Rightarrow v\frac{dv}{dx} = a(x)$$

$$\int_u^v vdv = \int_{x_0}^x a(x)dx$$

$$\text{or } \frac{v^2}{2} - \frac{u^2}{2} = \int_{x_0}^x a(x)dx$$

(iii) **Acceleration only depends on velocity.**

$$\frac{dv}{dt} = a(v)$$

$$\Rightarrow \int_u^v \frac{dv}{a(v)} = \int_0^t dt$$

$$\text{or } \int_u^v \frac{dv}{a(v)} = t$$

This gives us velocity as a function of time.

In case we want velocity as a function of position, we can use chain rule.

$$\frac{dv}{dx}\frac{dx}{dt} = a(v)$$

$$\frac{v\,dv}{a(v)} = dx$$

$$\int_u^v \frac{v\,dv}{a(v)} = \int_{x_0}^x dx = x - x_0$$

Train Your Brain

Example 21: The acceleration of a particle moving in one dimension is given by $a = 6 - 2t$. If the particle is initially at $x = 0$ and its velocity is 2 m/s, find it position and velocity at time t?

Sol. $\dfrac{dv}{dt} = 6 - 2t$

$$\int_2^v dv = \int_0^t (6-2t)dt$$

$$\Rightarrow v - 2 = (6t - t^2)|_0^t = 6t - t^2$$

$$\Rightarrow v(t) = 2 + 6t - t^2$$

To find position, we integrate velocity.

$$v = \frac{dx}{dt} = 2 + 6t - t^2$$

$$dx = (2 + 6t - t^2)dt$$

$$\int_0^x dx = \int_0^t (2+6t-t^2)dt = 2t + 3t^2 - \frac{t^3}{3}$$

$$\text{or } x(t) = 2t + 3t^2 - \frac{t^3}{3}$$

Example 22: The retardation of a car when its engine is shut off depends on its velocity as $a = -\alpha v$ where α is constant. Find the total distance travelled by the car if its initial velocity is 20 m/s and $\alpha = 0.5$/s

Sol. $\dfrac{dv}{dt} = -\alpha v$

$$\frac{dv}{dx}\left(\frac{dx}{dt}\right) = -\alpha v$$

$$\frac{vdv}{dx} = -\alpha v$$

$$\text{or } dv = -\alpha dx$$

$$\int_{20}^{0} dv = -\alpha \int_0^d dx$$

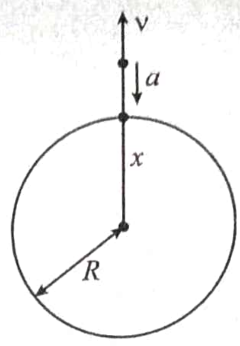
$$v|_{20}^{0} = -\alpha x|_0^d$$

$$-20 = -\alpha d$$

$$d = \frac{20}{\alpha} = \frac{20}{0.5} = 40 \text{ m}$$

Example 23: With what velocity in vertical upward direction should a body be projected from the surface of earth so that it reaches a height equal to radius of earth? The acceleration of body is given by $a = -\dfrac{GM}{x^2}$ where x is the distance from centre of earth and M is the mass of earth.

Sol. Acceleration due to gravity is nearly constant near the surface of earth. But if the height become too large its dependence on distance which can not be ignored.



$$a = \frac{dv}{dt} = -\frac{GM}{x^2}$$

$$\text{or } \frac{dv}{dx}\cdot\frac{dx}{dt} = -\frac{GM}{x^2}$$

$$vdv = = -\frac{GM}{x^2}dx$$

At the highest point, velocity is zero. Also note $x_i = R$ and $x_f = 2R$.

$$\int_u^0 vdv = -GM\int_R^{2R}\frac{dx}{x^2}$$

$$\left.\frac{v^2}{2}\right|_u^0 = -GM\int_R^{2R} x^{-2}dx = \left.\frac{GM}{x}\right|_R^{2R}$$

$$\Rightarrow -\frac{u^2}{2} = GM\left[\frac{1}{2R} - \frac{1}{R}\right]$$

$$\Rightarrow u^2 = \frac{GM}{R} \Rightarrow u = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{R^2}R}$$

$$= \sqrt{gR} = 8 \text{ km/s} \quad [\because R = 6400 \text{ km}, g = 10 \text{ m/s}^2]$$

Concept Application

10. Starting from rest at $t = 0$, a particle moves in a straight line with an acceleration a given by $a = t^3$ m/s^2 where t is in seconds. Then the velocity of particle after 4 seconds is

    (*a*) 32 m/s (*b*) 64 m/s (*c*) 128 m/s (*d*) 16 m/s

11. A particle moves in a straight line with acceleration $a = -\dfrac{1}{3v^2}$ where v is its velocity at time t. If initial velocity is 5 m/s then time t at which its velocity becomes zero is

    (*a*) 5 sec (*b*) 25 sec (*c*) 125 sec (*d*) 50 sec

12. The acceleration of a particle as a function of its position x is given $a = -2x$. If initial velocity at $x = 0$ is 20 m/s, find the position x where its velocity becomes zero.

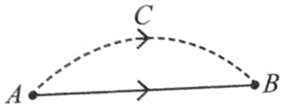
    (*a*) $10\sqrt{2}$ m (*b*) $5\sqrt{2}$ m

    (*c*) $20\sqrt{2}$ m (*d*) 20 m

Short Notes

Distance versus Displacement

Total length of path (ACB) covered by the particle is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.



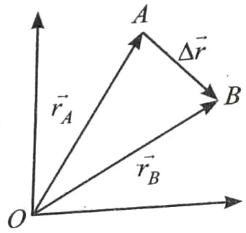
Displacement is Change of Position Vector

From ΔOAB $\Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} \Rightarrow \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

Average speed $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

For uniform motion

Average speed = | average velocity | = | instantaneous velocity|

Velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

Average Acceleration $= \dfrac{\text{Total change in velocity}}{\text{Total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Important Points About 1D Motion

- Distance $\geq$ | displacement | and Average speed $\geq$ | average velocity |
- If distance > | displacement | this implies
  (*a*) atleast at one point in path, velocity is zero.

$$\text{Displacement} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \text{Velocity} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \text{Acceleration}$$

Motion with Constant Acceleration: Equations of Motion

- **In vector form**

$\vec{v} = \vec{u} + \vec{a}t$ and

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1,\ \vec{s} = \left(\frac{\vec{u} + \vec{v}}{2}\right)t = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

$$v^2 = u^2 + 2\vec{a}.\vec{s} \text{ and } \vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n - 1)$$

($S_{nth} \rightarrow$ displacement in n^{th} second)

- **In scalar form (for one dimensional motion):**

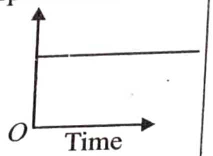
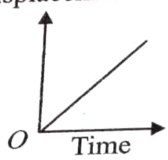
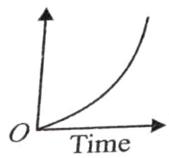
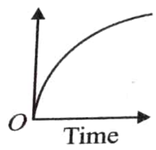
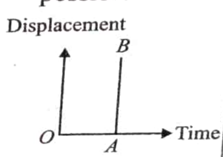
$$v = u + at \qquad s = \left(\frac{u + v}{2}\right)t = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$$

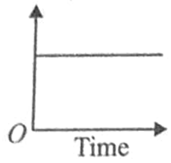
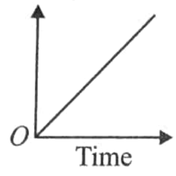
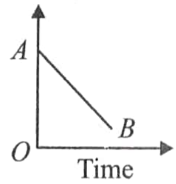
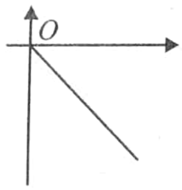
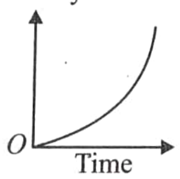
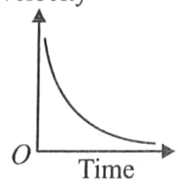
$$v^2 = u^2 + 2as \qquad s_n = u + \frac{a}{2}(2n - 1)$$

Uniform Motion

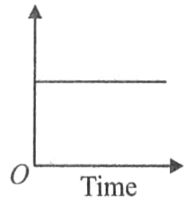
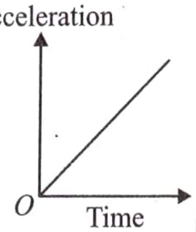
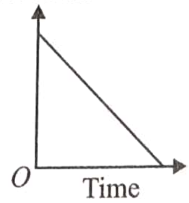
If an object is moving along the straight line covers equal distance in equal interval of time, it is said to be in uniform motion along a straight line.

Different Graphs of Motion

Displacement-Time Graph		
(*a*) For a stationary body Displacement 	(*b*) Body moving with a constant velocity Displacement 	(*c*) Body moving with a constant acceleration Displacement 
(*d*) Body moving with a constant retardation Displacement 	(*e*) Body moving with infinite velocity. But such motion of body is never possible 	

Velocity-Time Graph		
(*a*) Moving with a constant velocity Velocity  	(*b*) Moving with a constant acceleration having zero initial velocity Velocity  	(*c*) Body moving with a constant retardation and its initial retardation and its initial velocity is not zero Velocity 
(*d*) Moving with negative acceleration with zero initial velocity  	(*e*) Moving with increasing acceleration Velocity  	(*f*) Moving with decreasing retardation Velocity 

Note: Slope of velocity-time graph gives acceleration.

Acceleration-Time Graph		
(*a*) When object is moving with constant acceleration Acceleration  	(*b*) When object is moving with constant increasing acceleration Acceleration  	(*c*) When object is moving with constant decreasing acceleration Acceleration 

Motion under Gravity (No Air Resistance)

If an object is falling freely under gravity and downward direction is taken as positive, then equations of motion becomes

(i) $v = u + gt$ (ii) $h = ut + \frac{1}{2}gt^2$ (iii) $v^2 = u^2 + 2gh$

Note: If upward direction is taken as positive then g is replaced by $-g$ in above three equations.

If a body is thrown vertically up with a velocity u in the uniform gravitational field then

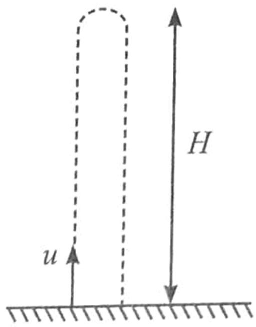
(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\frac{u}{g}$

(iii) Total time of flight = $\frac{2u}{g}$

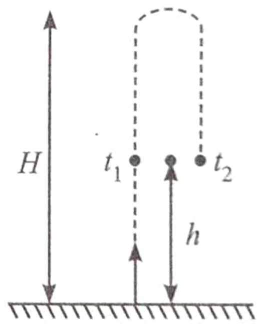
(iv) Final velocity at the point of projection = u (downwards)

(v) **Gallileo's law of odd numbers:** For a freely falling body ratio of successive distance covered in equal time interval 't'



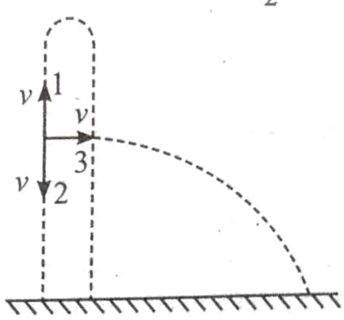
$S_1 : S_2 : S_3 : \ldots, S_n = 1 : 3 : 5 : \ldots, 2n - 1$

(vi) At any point on its path the body will have same speed for upward journey and downward journey.



(vii) If a body thrown upwards crosses a point in time t_1 and t_2 respectively then height of point $h = ½\, gt_1t_2$. Maximum height $H = \frac{1}{2}g(t_1 + t_2)^2$.

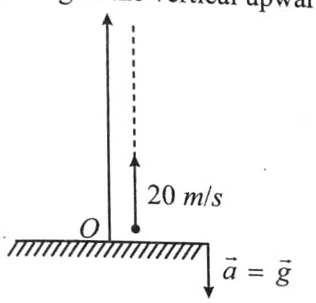
(viii) A body is thrown upward, downward and horizontally with same speed takes time t_1, t_2 and t_3 respectively to reach the ground then $t_3 = \sqrt{t_1t_2}$ and height from where the particle was thrown is $H = \frac{1}{2}gt_1t_2$.



Solved Examples

1. A stone is thrown vertical upwards from ground level with $u = 20$ m/s.
   (a) Find the maximum height attained by the stone.
   (b) time interval t after which it returns to the point of projection.
   (c) The velocity with which it strikes the ground.

Sol. Let us choose our origin O at the point of projection with +ve X-axis pointing in the vertical upwards direction.



Note that in this coordinate system, acceleration due to gravity is negative because it points in the downward direction.

Thus $a = -9.8$ m/s^2, $u = 20$ m/s

(a) At the highest point, velocity of the particle will become zero. Let h be the maximum height.

Thus $S = h$.

Using the relation,

$v^2 - u^2 = 2a\,S$

we get $\quad 0 - 20^2 = 2 \times (-9.8) \times h$

$$\Rightarrow h = \frac{400}{19.6} = 20.4\,\text{m}$$

(b) $S = 0 = 20t - \frac{1}{2}(9.8)t^2$

$$\Rightarrow t = \frac{40}{9.8} = 4.08 \text{ sec}$$

(c) Since, the particle returns to the initial position, $S = 0$.

$\Rightarrow v = 20 - 9.8 \times \frac{40}{9.8}$ (we know t from part (b))

$= -20$ m/s

Here, minus sign indicates that particle moves in the downward direction.

Note: It returns with same speed with which it was thrown.

2. If another stone was thrown from the same point after one second in the last example, with initial velocity of 25 m/s, find
   (a) The coordinates of the point where they collide
   (b) The velocities of the two stones when they collide.

Sol. (a) Let x_1 and x_2 to be the coordinates of first and second stone after time t.

$$S_1 = x_1 - 0 = 20t - \frac{1}{2}9.8t^2 \qquad \text{...(i)}$$

$$S_2 = x_2 - 0 = 25(t-1) - \frac{1}{2}9.8(t-1)^2 \qquad \text{...(ii)}$$

Please note that if the first stone is in flight for t second then second stone will be in flight for $(t - 1)$ seconds as it was thrown after 1 sec. When they collide, they are at the same place, hence

$x_1 = x_2$

$\Rightarrow 20t - 4.9t^2 = 25(t-1) - 4.9(t-1)^2$

$\Rightarrow 14.8t = 29.9 \qquad \Rightarrow t = 2.02$ sec

Substituting $t = 2.02$ in eqn (i) or eqn (ii), we find

$h = x_1 = x_2 = 20.4$ m

(b) The velocities of the stones after time t are

$$v_1 = 20 - 9.8t \qquad \text{...(iii)}$$

$$v_2 = 25 - 9.8(t-1) \qquad \text{...(iv)}$$

Substituting t = 2.02 in eqns (iii) and (iv), we find

$v_1 = 0.2$ m/s $\qquad v_2 = 15.1$ m/s

3. A body is thrown down from the top of a tower of height h with velocity 10 m/s. Simultaneously, another body is projected upward from bottom. They meet at a height $2h/3$ from the ground level. If $h = 60$ m, find the initial velocity of the lower body.

Sol. Let us choose the origin at the ground level with +ve X-axis pointing in the upward direction.

Let us refer lower and upper body 1 and 2 respectively

Then,

$a = -g$

$x_1 = 0, x_1 = 2h/3$

$x_2 = h, x_2 = 2h/3, u_2 = -10$ m/s

From eqn of motion, we have

$$x_1 = 0 + u_1 t - \frac{1}{2} \times 9.8 \times t^2 \qquad \text{...(i)}$$

$$x_2 = h - 10t - \frac{1}{2} \times 9.8 \times t^2 \qquad \text{...(ii)}$$

But, $x_1 = x_2 = 2h/3$

Hence, equating eqn (i) and (ii) we have

$u_1 t = h - 10\,t \qquad \Rightarrow t = \frac{h}{u_1 + 10}$

Putting this value in eqn (ii), we get

$$\frac{h}{3} = \frac{10h}{u_1 + 10} - 4.9\left(\frac{h}{u_1 + 10}\right)^2$$

$$\Rightarrow 20 = \frac{200}{u_1+10} - 4.9\frac{60^2}{(u_1+10)^2}$$

$$\Rightarrow (u_1+10)^2 - 10(u_1+10) - 882 = 0$$

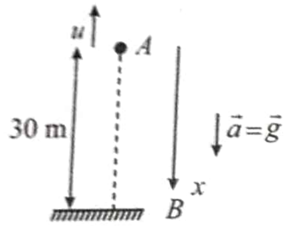
Solving this quadratic eqn, we find

$$u_1 + 10 = \frac{10 \pm \sqrt{100+3528}}{2} \qquad \Rightarrow u_1 = 30.11 \text{ m/s}$$

The other value is not possible because body is thrown upwards and is positive in the chosen coordinate system.

4. A stone is dropped from a rising balloon at a height of 30 m above the ground and it reaches the ground in 10 seconds. What was the velocity of the balloon at the moment the stone was dropped.

Sol.



Let the balloon be at point A when the stone is dropped. Let its velocity be u. The velocity of the stone will be equal to the velocity of balloon when it is dropped.

Let us choose origin at point A and take +ve X-direction to be vertically downward direction. In this case

$a = g$ and $S = 30$

Using equation, $S = ut + \frac{1}{2}at^2$

we get

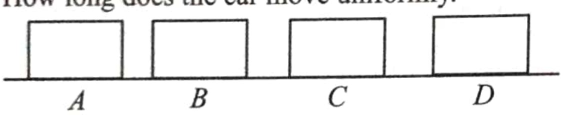
$$30 = ut + \frac{1}{2}\times 9.8 \times (10)^2$$

$$\Rightarrow u = -46 \text{ m/s}$$

minus sign indicates that the velocity is in the upward direction as we have choosen vertically downward direction to be positive.

5. A car starts moving on a straight road, first with acceleration $a = 5$ m/s^2, then uniformly, and finally decelerating at the same rate, comes to rest. The total time of motion equal 25 sec. The average velocity during that time is 72 km/hr. How long does the car move uniformly.

Sol.



Let AB, BC and CD be the displacements of the car when it accelerates, moves with constant velocity and decelerates respectively.

$$\langle v \rangle = 72 \text{ km/hr} = 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$$

Total distance travelled $= \langle v \rangle \times$ time $= 20 \times 25 = 500$ m.

From $A \to B$

$$AB = ut_{AB} + \frac{1}{2}a(t_{AB})^2$$

$$= 2.5(t_{AB})^2$$

$$v_B = 0 + at_{AB} = 5t_{AB}$$

From $B \to C$

Since velocity is constant

$$BC = v_B \times t_{BC} = 5t_{AB} \times t_{BC}$$

From $C \to D$

In this interval

$$u = v_B = 5t_{AB},\ v = 0,\ a = -5 \text{ m/s}^2$$

$$v = u + at$$

$$\Rightarrow 0 = 5t_{AB} - 5t_{CD} = 0$$

$$\Rightarrow t_{AB} = t_{CD} \text{ and}$$

$$S = ut - \frac{1}{2}at^2$$

$$\Rightarrow CD = 5t_{AB}^2 - 2.5(t_{AB})^2 = 2.5t_{AB}^2 = AB$$

$$\text{Total time} = t_{AB} + t_{BC} + t_{CD} = 25$$

$$\Rightarrow 2t_{AB} + t_{BC} = 25 \qquad \text{...(i)}$$

Also,

$$\text{Total distance} = AB + BC + CD = 500$$

$$\Rightarrow 2.5t_{AB}^2 + 5t_{AB}\,t_{BC} + 2.5t_{AB}^2 = 500 \qquad \text{...(ii)}$$

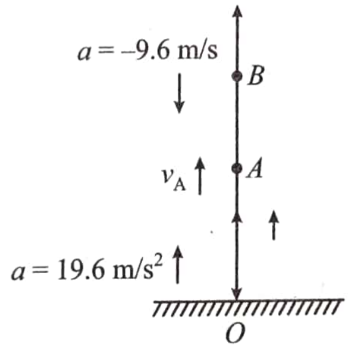
Solving eqn (i) and eqn(ii) we find $t_{BC} = 15$ sec

6. A rocket is fired vertically and ascends with constant vertical acceleration of 19.6 m/s^2 for 30 seconds. Its fuel is then all used up and it continues as a free particle.

(*a*) What is the maximum altitude reached ?

(*b*) What is total time after which it strikes the ground again?

Sol.



Let us choose the vertical upward to be the positive direction. Let A be the point at which fuel is exhausted and let point B represent the maximum altitude.

From $O \to A$

$$u = 0,\ a = 19.6 \text{ m/s}^2,\ t = 30\text{s}$$

$$OA = ut + \frac{1}{2}at^2 = 0 + 9.8 \times 900 = 8820 \text{ m}$$

$$v_A = u + at = 0 + 19.6 \times 30 = 588 \text{ m/s}$$

Form $A \to B$

$$u = v_A = 588 \text{ m/s},\ a = -9.8 \text{ m/s}^2$$

$$v = v_B = 0$$

using $v^2 - u^2 = 2as$ we get

$$0 - (588)^2 = 2 \times -9.8 \times AB$$

$$\Rightarrow AB = 17640 \text{ m}$$

Hence,

maximum altitude $= OA + AB = 26.46$ km

To find time, let us consider the path $A \to B \to O$

$a = -9.8\ \text{m/s}^2$, $s = -OA = -8820$, $u = 588$ m/s

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow -8820 = 588 \times t - 4.9t^2$$

$$\Rightarrow t^2 - 120t - 1800 = 0$$

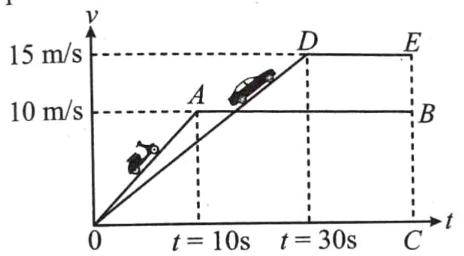
$$t = \frac{120 + \sqrt{14400 + 7200}}{2}$$

$= 133.5$ sec

So, the total time $= 30 + 133.5 = 163.5$ sec

7. A motorcycle and a car start their rectilinear motion from rest from the same place at the same time and travel in the same direction. The motorcycle accelerates at $1.0\ \text{m/s}^2$ up to a speed of 36 km/hr and the car at $0.5\ \text{m/s}^2$ up to a speed of 54 km/hr. Their velocities remain constant after that. Draw v-t graph of both. Calculate the time and distance at which the car would overtake the motorcycle.

Sol. v-t graphs for both vehicles is as below



For motorcycle

$10\ \text{m/s} = 0 + (1\text{m/s}^2)\ t$

$t = 10$s

For car

$15\ \text{m/s} = 0 + (0.5)\ \text{m/s}^2)\ t$

$t = 30$ s

Suppose after t time car over takes motorcycle.

Area under v-t graph till that time for both will be same

Area of $OABC$ = Area of $ODEC$

$$\frac{1}{2}(t + t - 10)\ 10 = \frac{1}{2}(t + t - 30)\ 15$$

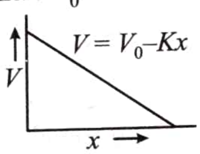
$4t - 20 = 6t - 90$

$2t = 70$

$t = 35$ sec.

Area under the graph = 300 m = distance at which car overtakes motorcycle.

8. A particle is moving along x-axis with velocity V which varies according to the law $V = V_0 - Kx$ here V_0 and K are constant. Plot acceleration vs time plot for the time interval when particle moves from $x = 0$ to $x = \dfrac{V_0}{K}$.



Sol. $V = V_0 - Kx$

$$\frac{dx}{dt} = (V_0 - Kx)$$

$$\int_0^x \frac{dx}{(V_0 - Kx)} = \int_0^t dt$$

$$x = \frac{V_0}{K}(1 - e^{-Kt})$$

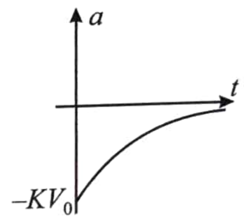
$$\therefore\ \frac{dx}{dt} = +V_0\,e^{-Kt}$$

$$a = \frac{d^2x}{dt^2} = KV_0\,e^{-Kt}$$

At $t = 0$, $a = -KV_0$

At $t = \infty$, $a = 0$

Therefore, graph is as shown



9. Two trains A and B are approaching each other on a straight track, the former with a uniform velocity of 15 m/s and the latter with 25 m/s. When they are 225 m apart brakes are simultaneously applied to both of them. The deceleration given by the brakes to the train A increases as $0.3t$ where t is time in sec. while the train B is given a uniform deceleration.

(a) What must be the minimum deceleration of B so that the trains do not collide.

(b) What is the times taken by the trains to come to stop.

Sol. $a_A = -0.3\ t$

$$\frac{dv}{dt} = -0.3\ t$$

$$\Rightarrow \int_u^v dv = -0.3\int_0^t t\,dt \Rightarrow v = 15 - \frac{0.3t^2}{2}$$

Train A stops when $v = 0$

$\Rightarrow t = 10$ sec

Now, displacement of A in 10 sec is

$$S(t) = 15t - \frac{0.3}{2} \times \frac{t^3}{3}$$

$$S = 15 \times 10 - \frac{0.3}{2} \times \frac{10^3}{3} = 100\ \text{m}$$

Train B must stop in $225 - 100 = 125$ m

$v^2 - u^2 = 2aS$

$0 - 25^2 = 2a \times 125$

$$\Rightarrow a = -\frac{625}{2 \times 225} = -2.5\ \text{m/s}^2$$

Therefore, deceleration of train B must be greater than $2.5\ \text{m/s}^2$

10. A steel ball bearing is released from the roof of a building. An observer standing in front of a window 120 cm high observes that the ball takes 0.125 sec to fall from top to the bottom of the window. The ball continues to fall and makes a completely elastic collision with side walk and reappears at the bottom of the window 2 s after passing it on the way down. How tall is the building?

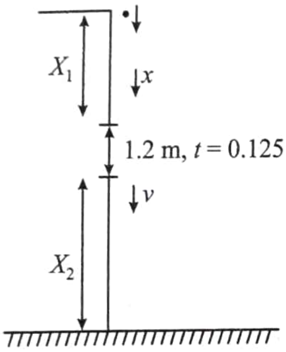
Sol. $1.2 = ut + \frac{1}{2}gt^2$

$1.2 = u \times 0.125 + \frac{1}{2} \times 10 \times (0.125)^2$

$1.2 = u \times 0.125 + 5 \times (0.125)^2$

$u = \frac{1.2 - 0.078125}{0.125} = 8.975 \text{ m/s}$

$v = 8.975 + 10 \times 0.125 = 10.225 \text{ m/s}$



$X_2 = 10.225 \times 1 + \frac{1}{2} \times 10 \times 1^2 = 15.225 \text{ m}$

$u^2 = 0 + 2 \times 10 \times X_1 \Rightarrow X_1 = \frac{u^2}{20} = \frac{(8.975)^2}{20} = 4.027 \text{ m}$

$H_{total} = X_1 + X_2 + 1.2 = 4.027 + 15.225 + 1.2 = 20.452 = 20.5 \text{ m}$

11. A car starts from rest at $t = 0$ and for the first 4 seconds of its rectilinear motion the acceleration 'a' (ms^{-2}) at time 't' (sec.) after starting is given by $a = 6 - 2t$.

(*a*) Find the maximum velocity of the car

(*b*) Find the velocity of the car after 4 seconds, and the distance travelled up to this time.

Sol. (*a*) $a = 6 - 2t = \frac{dv}{dt}$

For maximum velocity $\frac{dv}{dt} = 0$

$6 - 2t = 0$

$t = 3\text{sec}$

$\frac{dv}{dt} = 6 - 2t$

$\int_0^v dv = \int_0^t (6 - 2t)\, dt$

$v = 6t - t^2 = 18 - 9 = 9\text{m/s}$

(*b*) after 4 sec $\quad v = 6t - t^2$

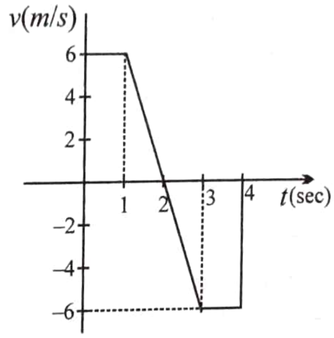
$= 6 \times 4 - 16 = 24 - 16 = 8 \text{ m/s}$

$\int_0^v dx = \int_0^t (6t - t^2)\, dt$

$x = 3t^2 - \frac{t^3}{3} = \frac{80}{3} \text{ m} = \text{distance travelled}$

[(putting $t = 4$ sec)]

12. A particle moves along a straight line along x-axis. At time $t = 0$, its position is at $x = 0$. The velocity v m/s of the object changes as a function of time t seconds as shown in the figure.



(*a*) What is x at $t = 1$ sec?

(*b*) What is the acceleration at $t = 2$ sec?

(*c*) What is x at $t = 4$ sec?

(*d*) What is the average speed between $t = 0$ and $t = 3$ sec?

Sol. (*a*) x is displacement at $t = 1$ sec.

Area under the $v - t$ curve gives displacement

From $t = 0$ to $t = 1$ sec.

$x = 6 \times 1 = 6 \text{ m}$

(*b*) Slope of the v–t curve gives acceleration from the given v–t curve

Slope at $t = 2$ sec. gives acceleration at $t = 2$ sec.

$\tan\theta = a = -\frac{6}{1} = -6 \text{ m/s}^2$

(*c*) x (at $t = 4$ sec):

Area under the curve from $t = 0$ to $t = 4$ sec

$= 6 \times 1 + \frac{1}{2} \times 6 \times 1 - \frac{1}{2} \times 6 \times 1 - 6 \times 1 = 0$

$\Rightarrow x\,(t = 4) = 0 \text{ m}$

(*d*) Average speed from $t = 0$ to $t = 3$ sec.

Displacement from $t = 0$ to $t = 2$ sec. = Area under the curve $= 6 + \frac{1}{2} \times 6 \times 1 = 9 \text{ m}$

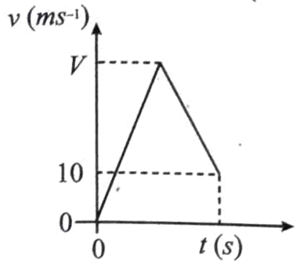
Displacement form $t = 2$ to $t = 3$ sec. $= -\frac{1}{2} \times 6 \times 1$
$= -3 \text{ m}$

Distance from $t = 0$ to $t = 3$ sec $= |9| + |-3| = 12 \text{ m}$

Average speed $= \frac{\text{Distance}}{\text{Speed}} = \frac{12}{3} = 4 \text{ m/s}$

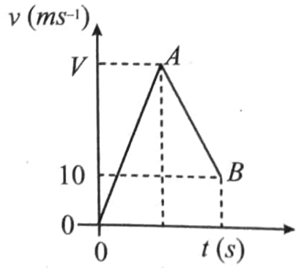
13. The figure shows the (v, t) graph for the train accelerating from rest up to a maximum speed of V ms^{-1} and then decelerating to a speed of 10 ms^{-1}. The acceleration and deceleration have the same magnitude which is equal to 0.5 m/s^2.

Show that the distance travelled is $(2V^2 - 100)$ metre.



Sol. $v^2 - u^2 = 2as$ $\qquad s = \dfrac{v^2 - u^2}{2a}$

For the motion between A and B



Distance $(s_1) = \dfrac{V^2 - 0}{2\times 0.5} = V^2$

For the motion between O and A

Distance $(s_2) = \dfrac{10^2 - V^2}{2\times(-0.5)} = V^2 - 100$

Total distance $= s_1 + s_2 = 2V^2 - 100$

14. When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elasped before he slams the breaks of the car is the reaction time. Reaction time depends on complexity of the situation and on individual. You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger. After you catch it, find the distance d travelled by the ruler. In a particular case, d was found to be 20 cm. [$g = 10$ m/s^2]

(*a*) Estimate reaction time.

(*b*) Now if you are driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after you see the need to put the brakes on.

Sol. (*a*) $0.2 = \dfrac{1}{2}\times 10t^2 \Rightarrow t^2 = \dfrac{0.4}{10} \Rightarrow t = 0.2$

Reaction time = 0.2 sec

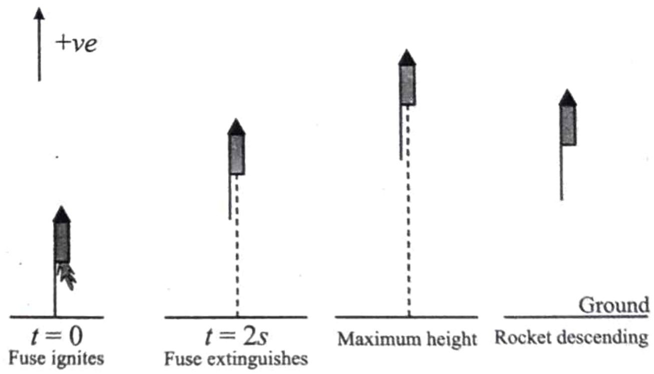
(*b*) $54 \text{ km/hr} = \dfrac{54\times 5}{18} = 15 \text{ m/s}$

Total distance = 15 × reaction time + $(v^2/2a)$

Total distance $= 0.2 \times 15 + \dfrac{15^2}{2\times 6} = 3 + \dfrac{225}{12}$

$= \dfrac{261}{12}$ m

15. A Diwali rocket is launched vertically with its fuse ignited at time $t = 0$, as shown. The engine provides constant acceleration for 2 sec. till rocket attains $V = 40$ ms^{-1}. Afterwards rocket continues to move freely under gravity.



(*a*) Draw labelled acceleration-time graph from launching till it reaches ground.

(*b*) Draw labelled velocity-time graph from launching till it reaches ground.

[Take vertically upward direction as positive]

Sol. $v = u + at$

$\Rightarrow 40 = 0 + a \times 2$

$\Rightarrow a = 20$ m/s^2

$v^2 = u^2 + 2ah$

$\Rightarrow (40)^2 = 0 + 2 \times 20h$

$\Rightarrow h = \dfrac{40\times 40}{40} = 40$ m

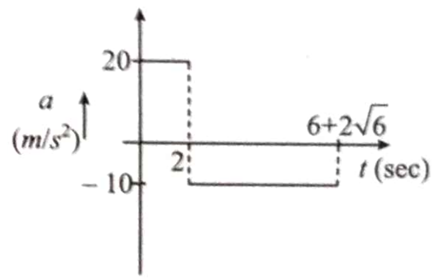
Time taken in reaching from $h = 40$ m to ground

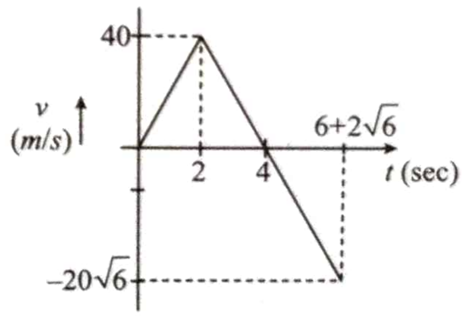
$h = ut - \dfrac{1}{2}gt^2$

$\Rightarrow -40 = 40t - \dfrac{1}{2}\times 10 \times t^2$

On solving

$t = 6 + 2\sqrt{6}$ sec





16. The force acting on a body moving in a straight line is given by $F = (3t^2 - 4t + 1)$ Newton where t is in sec. If mass of the body is 1kg and initially it was at rest at origin. Find

(*a*) Displacement between time $t = 0$ and $t = 2$ sec.

(*b*) Distance travelled between time $t = 0$ and $t = 2$ sec.

Sol. $a = F/m = 3t^2 - 4t + 1$ m/s^2

$$\therefore \int_0^v dv = \int_0^t (3t^2 - 4t + 1)dt$$

$$\Rightarrow v = (t^3 - 2t^2 + t) \text{ m/s}$$

(*a*) $\int_0^s ds = \int_0^2 v\,dt$

$$\therefore \text{Displacement } (s) = \left[\frac{t^4}{4} - \frac{2t^3}{3} + \frac{t^2}{2}\right]_0^2 = 2/3 \text{ m}$$

(*b*) Now, $v = 0 = t^3 - 2t^2 + t$

$\therefore t(t-1)^2 = 0$

i.e. $v = 0$ at $t = 0$ and $t = 1$ sec

$$\therefore \text{Distance} = \left|\int_0^1 v\,dt\right| + \left|\int_1^2 v\,dt\right| = 2/3 \text{ m}$$

[here distance = displacement, since velocity is positive $\forall\, t > 0$

17. A block of mass m is fired horizontally along a level surface that is lubricated with oil. The oil provides a viscous resistance that varies as the 3/2 power of the speed. If the initial speed of the block is v_0 at $x = 0$, find the maximum distance reached by the block. Assume no resistance to motion other than that provided by the oil.

Sol. $F = -v^{3/2}$

$$a = -\frac{1}{m}v^{3/2}$$

$$v\frac{dv}{dx} = -\frac{1}{m}v^{3/2}$$

$$\int_{v_0}^{0} v^{-\frac{1}{2}}dv = -\frac{1}{m}\int_0^d dx$$

$$2mv_0^{1/2} = d \quad \text{or} \quad d = 2mv_0^{1/2}$$

18. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s^2). It is at instantaneous rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

Sol. $\frac{vdv}{dx} = 4 - 2x$

$$\int_0^v vdv = \int_0^x (4 - 2x)dx$$

$$\Rightarrow \frac{v^2}{2} = 4x - x^2$$

when $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

$\therefore$ At $x = 4$ m, the particle will again come to rest.

Exercise-1 (Topicwise)

POSITION, DISTANCE AND DISPLACEMENT

1. A Body moves 6 m north. 8 m east and 10 m vertically upwards, what is its resultant displacement from initial position.

(a) $10\sqrt{2}$ m (b) 10 m

(c) $\frac{10}{\sqrt{2}}$m (d) 20 m

2. A man goes 10 m towards North, then 20 m towards East then displacement is

(a) 22.5 m (b) 25 m

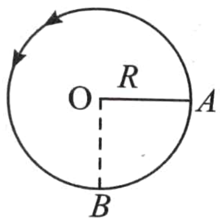
(c) 25.5 m (d) 30 m

3. An aeroplane flies from P(–4m, –5m, +8m) to Q (7m, –2m, –3m) in xyz coordinate system. The position vector of aeroplane.

(a) $11\hat{i}+3\hat{j}+11\hat{k}$ (b) $11\hat{i}-3\hat{j}+11\hat{k}$

(c) $11\hat{i}+3\hat{j}-11\hat{k}$ (d) $11\hat{i}-3\hat{j}-11\hat{k}$

4. A body moves in circular path of radius R from A to B as shown. Its displacement and distance covered are



(a) $R, \frac{3\pi R}{2}$ (b) $\sqrt{2}R, \frac{\pi R}{2}$

(c) $\sqrt{2}R, \frac{3\pi R}{2}$ (d) None of these

5. A particle covers half of the circle of radius r. Then the displacement and distance of the particle are respectively

(a) $2\pi r$, 0 (b) $2r$, πr

(c) $\frac{\pi r}{2}$, $2r$ (d) πr, r

SPEED AND VELOCITY

6. A person travels along a straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2 The average velocity is given by

(a) $v_1 v_2$ (b) $\frac{v_2^2}{v_1^2}$

(c) $\frac{v_1+v_2}{2}$ (d) $\frac{2v_1v_2}{v_1+v_2}$

7. A car travels the first half of a distance between two places at a speed of 30 km/hr and the second half of the distance at 50 km/hr. The average speed of the car for the whole journey is

(a) 42.5 km/hr (b) 40.0 km/hr

(c) 37.5 km/hr (d) 35.0 km/hr

8. A person travels along a straight road for the first half time with a velocity v_1 and the next half time with a velocity v_2. The mean velocity V of the man is

(a) $\frac{2}{V}=\frac{1}{v_1}+\frac{1}{v_2}$ (b) $V=\frac{v_1+v_2}{2}$

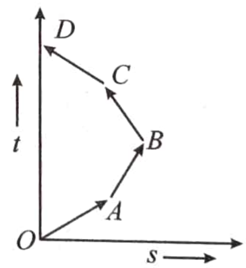
(c) $V=\sqrt{v_1v_2}$ (d) $V=\sqrt{\frac{v_1}{v_2}}$

9. If a car covers 2/5$^{\text{th}}$ of the total distance with v_1 speed and 3/5$^{\text{th}}$ distance with v_2 then average speed is

(a) $\frac{1}{2}\sqrt{v_1v_2}$ (b) $\frac{v_1+v_2}{2}$

(c) $\frac{2v_1v_2}{v_1+v_2}$ (d) $\frac{5v_1v_2}{3v_1+2v_2}$

10. Which of the following options is correct for the object having a straight line motion represented by the following graph



(a) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity

(b) Velocity of the object increases uniformly

(c) Average velocity is zero

(d) The graph shown is impossible

11. A car travels from A to B at a speed of 20 km h^{-1} and returns at a speed of 30 km h^{-1}. The average speed of the car for the whole journey is

(a) 5 km h^{-1} (b) 24 km h^{-1}

(c) 25 km h^{-1} (d) 50 km h^{-1}

12. The displacement of a body is given by $2s = gt^2$ where g is a constant. The velocity of the body at any time t is

(a) gt (b) $gt/2$

(c) $gt^2/2$ (d) $gt^3/3$

CONSTANT ACCELERATION

13. A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then

(a) $S_1 = S_2$ (b) $S_1 = S_2/3$
(c) $S_1 = S_2/2$ (d) $S_1 = S_2/4$

14. A body is moving from rest under constant acceleration and let S_1 be the displacement in the first $(p-1)$ sec and S_2 be the displacement in the first p sec. The displacement in $(p^2 - p + 1)^{th}$ sec. will be

(a) $S_1 + S_2$ (b) $S_1 S_2$
(c) $S_1 - S_2$ (d) S_1 / S_2

15. A body starts from the origin and moves along the X-axis such that the velocity at any instant is given by, $(4t^3 - 2t)$, where t is in sec and velocity is in m/s. What is the acceleration of the particle, when it is 2 m from the origin?

(a) 28 m/s^2 (b) 22 m/s^2
(c) 12 m/s^2 (d) 10 m/s^2

16. A point moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in the three successive intervals of time t_1, t_2 and t_3. Which of the following relations is correct

(a) $(v_1 - v_2):(v_2 - v_3) = (t_1 - t_2):(t_2 + t_3)$

(b) $(v_1 - v_2):(v_2 - v_3) = (t_1 + t_2):(t_2 + t_3)$

(c) $(v_1 - v_2):(v_2 - v_3) = (t_1 - t_2):(t_1 - t_3)$

(d) $(v_1 - v_2):(v_2 - v_3) = (t_1 - t_2):(t_2 + t_3)$

17. A motor car moving with a uniform speed of 20 m/sec comes to stop on the application of brakes after travelling a distance of 10 m Its acceleration is

(a) 20 m/sec^2 (b) –20 m/sec^2
(c) –40 m/sec^2 (d) +2 m/sec^2

18. Which of the following four statements is false

(a) A body can have zero velocity and still be accelerated.
(b) A body can have a constant velocity and still have a varying speed.
(c) A body can have a constant speed and still have a varying velocity.
(d) The direction of the velocity of a body can change when its acceleration is constant.

19. A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in

(a) $\frac{20}{3}$ m (b) 20 m
(c) 60 m (d) 180 m

20. The position of a particle moving along the x-axis at certain times is given below:

$t(s)$	0	1	2	3
$x(m)$	–2	0	6	16

Which of the following describes the motion correctly

(a) Uniform, accelerated
(b) Uniform, decelerated
(c) Non-uniform, accelerated
(d) There is not enough data for generalization

21. A car starts from rest and moves with uniform acceleration a on a straight road from time $t = 0$ to $t = T$. After that, constant deceleration brings it to rest. In this process the average speed of the car is

(a) $\frac{aT}{4}$ (b) $\frac{3aT}{2}$
(c) $\frac{aT}{2}$ (d) aT

22. A car, starting from rest, accelerates at constant rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then value of S

(a) $S = \frac{1}{2}ft^2$ (b) $S = \frac{1}{4}ft^2$
(c) $S = \frac{1}{72}ft^2$ (d) $S = \frac{1}{6}ft^2$

23. A body is starts from rest with acceleration 2 m/s^2 till it attains the maximum velocity then retards to rest with 3 m/s^2. If total time taken is 10 second then maximum speed attained is

(a) 12 m/s (b) 8 m/s
(c) 6 m/s (d) 4 m/s

MOTION UNDER GRAVITY

24. A stone falls from a balloon that is descending at a uniform rate of 12 m/s. The displacement of the stone from the point of release after 10 sec is

(a) 490 m
(b) 510 m
(c) 610 m
(d) 725 m

25. Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances are

(a) $a : b$ (b) $b : a$
(c) $\sqrt{a} : \sqrt{b}$ (d) $a^2 : b^2$

26. A ball P is dropped vertically and another ball Q is thrown horizontally with the same velocities from the same height and at the same time. If air resistance is neglected, then

(a) Ball P reaches the ground first
(b) Ball Q reaches the ground first
(c) Both reach the ground at the same time
(d) The respective masses of the two balls will decide the time

27. A stone dropped from the top of the tower touches the ground in 4 sec. The height of the tower is about

(*a*) 80 m (*b*) 40 m

(*c*) 20 m (*d*) 160 m

28. A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time $t/2$ sec

(*a*) At $h/2$from the ground

(*b*) At $h/4$ from the ground

(*c*) Depends upon mass and volume of the body

(*d*) At $3h/4$ from the ground

29. A body is slipping from an inclined plane of height h and length l. If the angle of inclination is θ, the time taken by the body to come from the top to the bottom of this inclined plane is

(*a*) $\sqrt{\frac{2h}{g}}$ (*b*) $\sqrt{\frac{2h}{g}}$

(*c*) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ (*d*) $\sin\theta\sqrt{\frac{2h}{g}}$

30. A man in a balloon rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is ($g = 9.8$ m/sec^2)

(*a*) 14.7 m (*b*) 19.6 m

(*c*) 9.8 m (*d*) 24.5 m

31. A rocket is fired upward from the earth's surface such that it creates an acceleration of 19.6 m/sec^2. If after 5 sec its engine is switched off, the maximum height of the rocket from earth's surface would be

(*a*) 245 (*b*) 490 m

(*c*) 980 m (*d*) 735 m

32. A ball of mass m_1 and another ball of mass m_2 are dropped from equal height. If time taken by the balls are t_1 and t_2 respectively, then

(*a*) $t_1 = \frac{t_2}{2}$ (*b*) $t_1 = t_2$

(*c*) $t_1 = 4t_2$ (*d*) $t_1 = \frac{t_2}{4}$

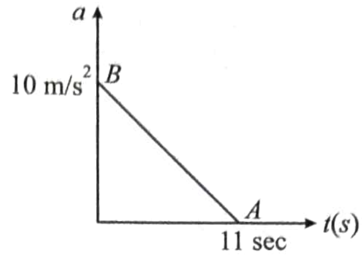
33. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same point 'O' along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of

(*a*) $m_1 : m_2 : m_3$ (*b*) $m_1 : 2m_2 : 3m_3$

(*c*) $1 : 1 : 1$ (*d*) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

VARIABLE ACCELERATION

34. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be



(*a*) 110 m/s

(*b*) 55 m/s

(*c*) 550 m/s

(*d*) 660 m/s

35. If the velocity of a particle is given by $v = (180 - 16x)^{1/2}$ m/s, then its acceleration will be

(*a*) Zero (*b*) 8 m/s^2

(*c*) -8 m/s^2 (*d*) 4 m/s^2

36. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(*a*) Go on decreasing with time

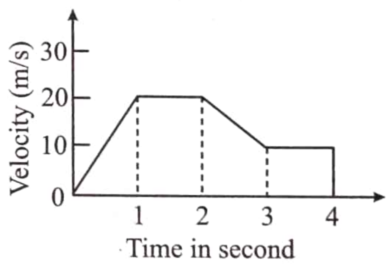
(*b*) Will be independent of α and β

(*c*) Drop to zero when $\alpha = \beta$

(*d*) Go on increasing with time

GRAPHS

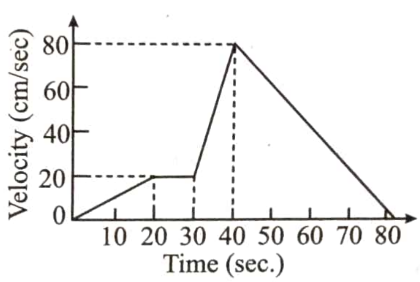
37. The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four seconds is



(*a*) 60 m (*b*) 55 m

(*c*) 25 m (*d*) 30 m

38. The $v - t$ graph of a moving object is given in figure. The maximum acceleration is



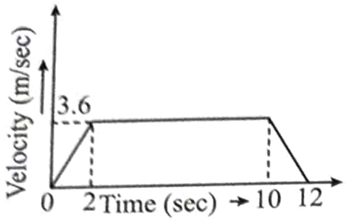
(*a*) 1 cm/sec^2

(*b*) 2 cm/sec^2

(*c*) 3 cm/sec^2

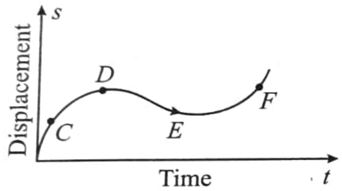
(*d*) 6 cm/sec^2

39. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers?



(a) 3.6 m
(b) 28.8 m
(c) 36.0 m
(d) Cannot be calculated from the above graph

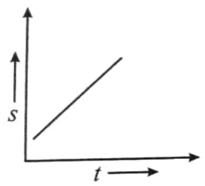
40. The displacement-time graph of moving particle is shown below

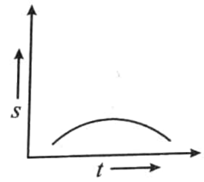


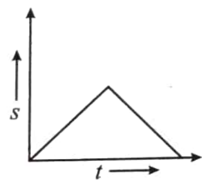
The instantaneous velocity of the particle is negative at the point

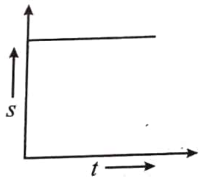
(a) D
(b) F
(c) C
(d) E

41. Which of the following graph represents uniform motion

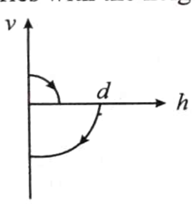
(a) 

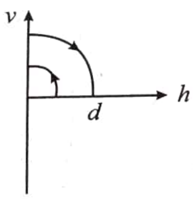
(b) 

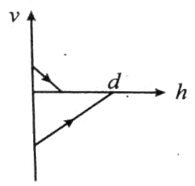
(c) 

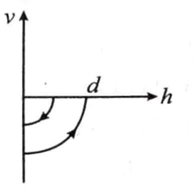
(d) 

42. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

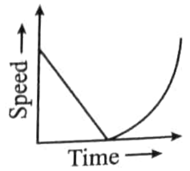
(a) 

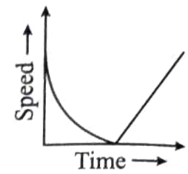
(b) 

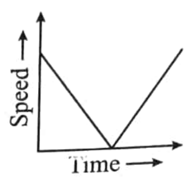
(c) 

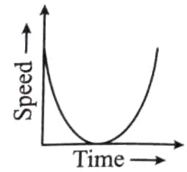
(d) 

43. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its height if the air resistance (constent) is not ignored

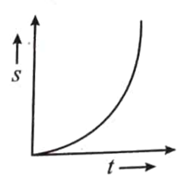
(a) 

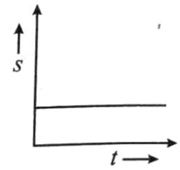
(b) 

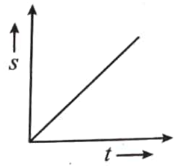
(c) 

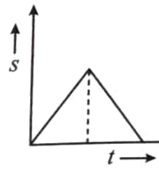
(d) 

44. Which graph represents the uniform acceleration

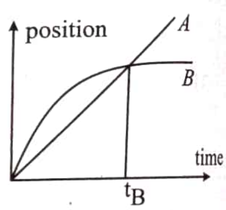
(a) 

(b) 

(c) 

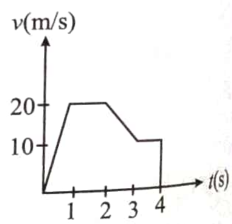
(d) 

45. The graph shows position as a function of time for two trains running on parallel tracks. Which one of the following statements is true?



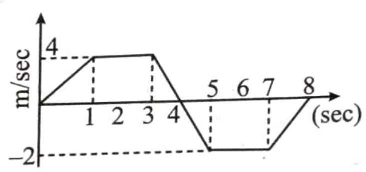
(a) At time t_B, both trains have the same velocity
(b) Both trains have the same velocity at some time after t_B
(c) Both trains have the same velocity at some time before t_B
(d) Somewhere on the graph, both trains have the same acceleration

46. The variation of velocity of a particle moving along straight line is shown in the figure. The distance travelled by the particle in 4 s is



(a) 25m
(b) 30m
(c) 55 m
(d) 60m

47. The v-t graph of a linear motion is shown in adjoining figure. The distance from origin after 8 seconds is

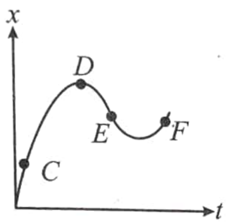


(a) 18 meters
(b) 16 meters
(c) 8 meters
(d) 6 meters

Exercise-2 (Learning Plus)

1. A car runs at constant speed on a circular track of radius 100 m taking 62.8 s on each lap. What is the average speed and average velocity on each complete lap?
(a) Velocity 10 m/s speed 10 m/s
(b) Velocity zero, speed 10 m/s
(c) Velocity zero, speed zero
(d) Velocity 10 m/s, speed zero

2. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is zero at the point



(a) C (b) D
(c) E (d) F

3. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10s is x_1, next 10 s is x_2 and in last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as
(a) 1 : 2 : 4 (b) 1 : 2 : 5
(c) 1 : 3 : 5 (d) 1 : 3 : 9

4. A body starts from rest with constant acceleration, the ratio of distances travelled by the body during 4th and 5th seconds is
(a) 7/5 (b) 7/9
(c) 7/3 (d) 3/7

5. A particle, after starting from rest, experiences, constant acceleration for 20 seconds. If it covers a distance of S_1, in first 10 seconds and distance S_2 in next 10 sec, then
(a) $S_2 = S_1/2$ (b) $S_2 = S_1$
(c) $S_2 = 2S_1$ (d) $S_2 = 3S_1$

6. A body sliding on a smooth inclined plane requires 4sec to reach the bottom after starting from rest at the top. How much time does it take to cover one fourth the distance starting from the top
(a) 1 sec (b) 2 sec
(c) 0.4 sec (d) 1.6 sec

7. A body is dropped from a height h under acceleration due to gravity g. If t_1 and t_2 are time intervals for its fall for first half and the second half distance, the relation between them is
(a) $t_1 = t_2$ (b) $t_1 = 2t_2$
(c) $t_1 = 2.414\, t_2$ (d) $t_1 = 4t_2$

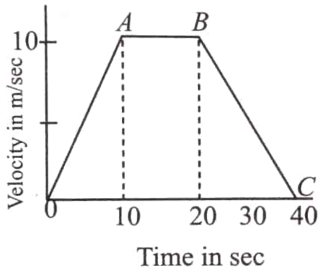
8. A body is thrown upward and reaches its maximum height. At that position
(a) Its velocity is zero and its acceleration is also zero
(b) Its velocity is zero but its acceleration is maximum
(c) Its acceleration is minimum
(d) Its velocity is zero and its acceleration is the acceleration due to gravity

9. A particle is moving so that its displacement s is given as $s = t^3 - 6t^2 + 3t + 4$ meter. Its velocity at the instant when its acceleration is zero will be-
(a) 3 m/s
(b) –12 m/s
(c) 42 m/s
(d) –9 m/s

10. The motion of a body is given by the equation
$\frac{dv(t)}{dt} = 6.0 - 3v(t)$, where $v(t)$ is speed in m/s and t in sec.
If body was at rest at $t = 0$ find the wrong option.
(a) The terminal speed is 2.0 m/s
(b) The speed varies with the time as $v(t) = 2(1 - e^{-3t})$ m/s
(c) The speed is 0.1 m/s when the acceleration is half the initial value
(d) The magnitude of the initial acceleration is 6.0 m/s^2

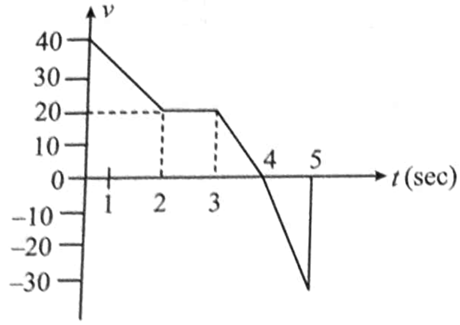
11. The displacement time graphs of two particles A and B are straight lines making angles of respectively 30° and 60° with the time axis. If the velocity of A is v_A and that of B is v_B then the value of $\frac{v_A}{v_B}$ is
(a) 1/2 (b) $1/\sqrt{3}$
(c) $\sqrt{3}$ (d) 1/3

12. The adjoining curve represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in metre/sec^2 are respectively



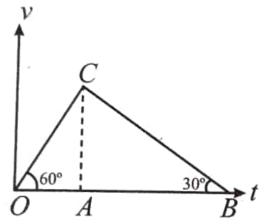
(a) 1, 0, –0.5
(b) 1, 0, 0.5
(c) 1, 1, 0.5
(d) 1, 0.5, 0

13. In the following velocity-time graph of a body, the distance and displacement travelled by the body in 5 second in meters will be



(*a*) 75,115 (*b*) 105, 75
(*c*) 45, 75 (*d*) 95, 55

14. The velocity-time graph of body is shown in figure. The ratio of the _______ during the intervals OA and AB is _______. Which of the following statement is wrong?



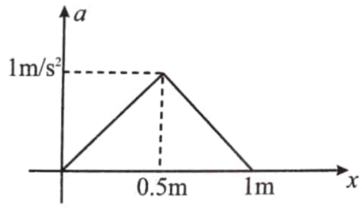
(*a*) Magnitude of average velocities, 1

(*b*) $\dfrac{OA}{OB}, \dfrac{1}{4}$

(*c*) Magnitude of average accelerations, inverse of ratio of distances covered

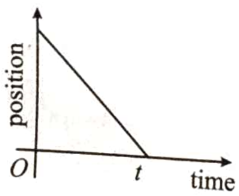
(*d*) Distance covered, 1 : 3

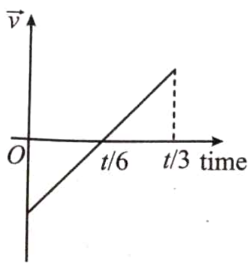
15. A body initially at rest, starts moving along x-axis in such a way so that its acceleration *vs* displacement plot is as shown in figure. The maximum velocity of particle is

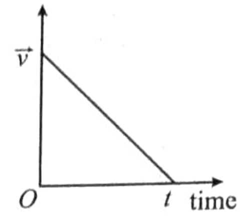


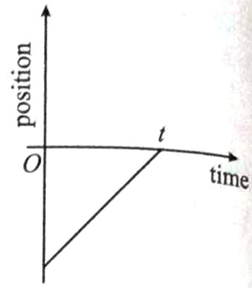
(*a*) 1 m/s
(*b*) 6 m/s
(*c*) 2 m/s
(*d*) None of these

16. For which of the following graphs the average velocity of a particle moving along a straight line for time interval $(0, t)$ must be negative.

(*a*)


(*b*)


(*c*)


(*d*)


17. Each of four particles move along x-axis. Their coordinates (in meters) as functions of time (in seconds) are given by

Particle 1 : $x(t) = 3.5 - 2.7t^3$

Particle 2 : $x(t) = 3.5 + 2.7t^3$

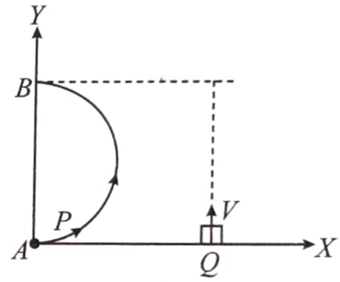
Particle 3 : $x(t) = 3.5 + 2.7t^2$

Particle 4 : $x(t) = 3.5 - 3.4t - 2.7t^2$

Which of these particles have constant acceleration?

(*a*) All four (*b*) Only 1 and 2
(*c*) Only 2 and 3 (*d*) Only 3 and 4

18. A particle P starts from origin as shown and moves along a circular path. Another particle Q crosses x-axis at the instant particle P leaves origin. Q moves with constant speed v parallel to y-axis and is all the time having y-coordinate same as that of P. When P reaches diametrically opposite at point B, its average speed is



(*a*) πV (*b*) $\dfrac{\pi V}{2}$

(*c*) $\dfrac{V}{2}$ (*d*) None of these

19. A particle is projected up from ground with initial speed v_0. Starting from time $t = 0$ to $t = t_1$,

(*a*) Distance travelled and magnitude of displacement are not equal if $t_1 < \dfrac{v_0}{g}$.

(*b*) Distance travelled and magnitude of displacement are equal if $\dfrac{v_0}{g} < t_1 < \dfrac{2v_0}{g}$

(*c*) Distance travelled and magnitude of displacement may not be equal if $0 < t_1 < \dfrac{2v_0}{g}$

(*d*) The magnitude of displacement is greater than the distance travelled if $\dfrac{v_0}{g} < t_1 < \dfrac{2v_0}{g}$

20. Two bodies P and Q have to move equal distances starting from rest. P is accelerated with 2a for first half distance then its acceleration becomes a for last half, where as Q has acceleration a for first half and acceleration $2a$ for last half, then for whole journey.

(a) Average speed of P is more than that of

(b) Average speed of both will be same

(c) Maximum speed during the journey is more for P

(d) Maximum speed during the journey is more for

21. A particle moves along the x-axis from x_i to x_f. Of the following values of the initial and final coordinates, which results in a negative displacement?

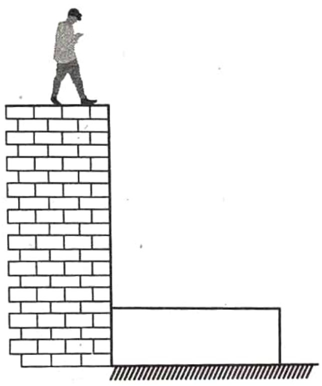
(a) $x_i = 4$ m, $x_f = 6$ m

(b) $x_i = -4$ m, $x_f = -8$ m

(c) $x_i = -4$ m, $x_f = 2$ m

(d) $x_i = -4$ m, $x_f = -2$ m

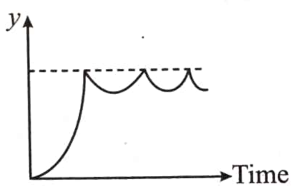
22. Suppose that a man jumps off a building 202 m high onto cushions having a total thickness of 2 m. If the cushions are crushed to a thickness of 0.5 m, what is the man's acceleration as he slows down?



(a) 10 m/s^2 (b) $\frac{4000}{3}$ m/s^2

(c) 50 m/s^2 (d) 20 m/s^2

23. The graph below describes the motion of a ball rebounding from a horizontal surface being released from a point above the surface. The quantity represented on the y-axis is the balls



(a) Displacement

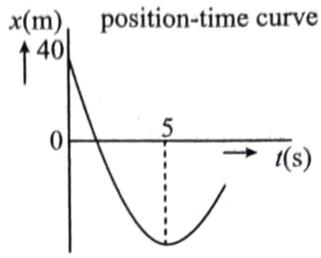
(b) Velocity

(c) Acceleration

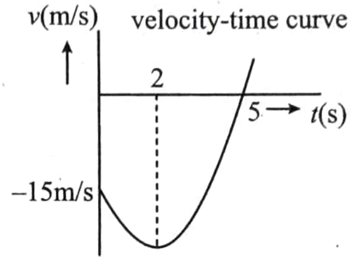
(d) Momentum

24. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in meters and t in seconds. Which of the graph does not represent the motion of the particle?

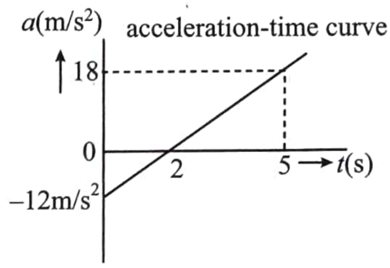
(a)



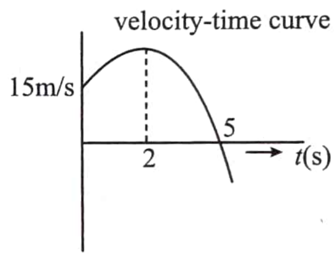
(b)



(c)



(d)



25. A Trolley is moving away from a stop with an acceleration $a = 0.2$ m/s^2. After reaching the velocity $u = 36$ km/h, it moves with a constant velocity for the time of 2 min. Then, it uniformly slows down, and stops after further travelling a distance of 100 m. Find the average speed all the way between stops.

(a) $\frac{76}{17}$ m/s (b) $\frac{208}{21}$ m/s

(c) $\frac{85}{12}$ m/s (d) $\frac{155}{19}$ m/s

26. Two cars start to move simultaneously with the same speed from point A to point B. The first moves in a straight line connecting A and B, uniformly, and the second - on the bypass road, made as a half-circle connecting the same points. The speed of the second uniformly increases so that at the end of the path its speed is doubled. Which car will arrive earlier at point B?

(a) 1st car

(b) 2nd car

(c) Both will reach simultaneously

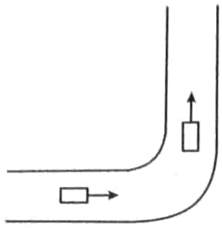
(d) Depends on values of R and v

27. A particle is dropped from rest. The particle first covers a distance x_1 in time t_1 and then a distance x_2 in further time t_2. If ratio of time $\frac{t_1}{t_2} = \frac{1}{(\sqrt{2}-1)}$, find the correct option:

(*a*) $x_1 = x_2$ (*b*) $x_1 > x_2$

(*c*) $x_1 < x_2$ (*d*) Unpredictable

28. A car is moving with 20 ms^{-1} from west to east and takes left turn in 5 sec without changing its speed. Find average acceleration of the car during this period of 5 sec.



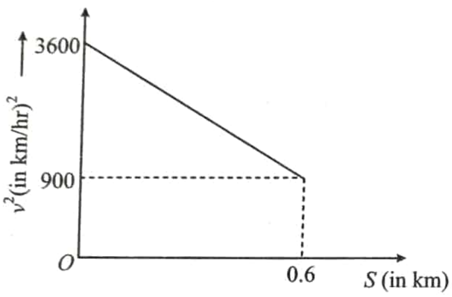
(*a*) $4\sqrt{2}$ ms^{-2} $N-E$

(*b*) $4\sqrt{2}$ ms^{-2} $N-W$

(*c*) 4 ms^{-2} $N-E$

(*d*) Zero

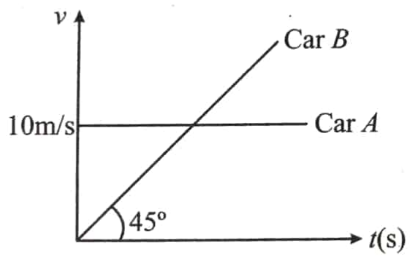
29. A graph between the square of the velocity of a particle and the distance '*S*' moved by the particle is shown in the figure. The acceleration of the particle in kilometer per hour square is



(*a*) 2250 (*b*) 225

(*c*) –2250 (*d*) –225

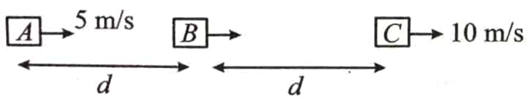
30. Initially car *A* is 10.5 m ahead of car *B*. Both start moving at time $t = 0$ in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car *B* will catch the car *A*, will be



(*a*) $t = 21$ sec (*b*) $t = 2\sqrt{5}$ s

(*c*) $t = 20$ sec (*d*) None of these

31. Three persons *A*, *B*, *C* are moving along a straight line as shown with constant and different speeds. When *B* catches *C*, the separation between *A* and *C* becomes 4*d*, then the speed of *B* is



(*a*) 15 m/s (*b*) 10 m/s

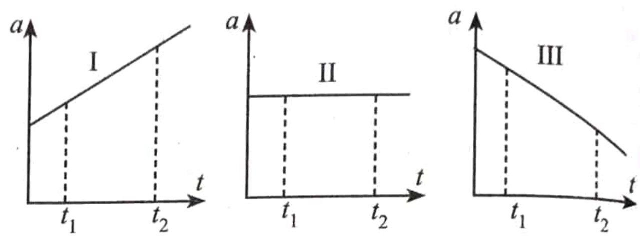
(*c*) 12.5 m/s (*d*) Not possible

32. Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. If the first ball strikes the ground with speed 20 m/s then speed of second ball when it strikes the ground is.

(*a*) 10 m/s (*b*) 20 m/s

(*c*) 40 m/s (*d*) Data insufficient

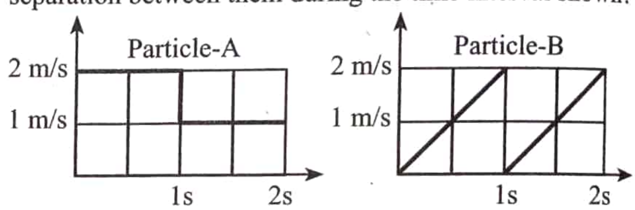
33. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?



(*a*) graph I, only (*b*) graphs I and II, only

(*c*) graphs I and III, only (*d*) graphs I, II, and

34. Two particles *A* and *B* starts from the same point and move in the positive *x*-direction. Their velocity-time relationships are shown in the following figures. What is the maximum separation between them during the time interval shown?



(*a*) 1.00 m (*b*) 1.25 m (*c*) 1.50 m (*d*) 2.00 m

35. Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. They reach the ground in 9*s* and 4*s*, respectively. The height of the tower is ($g = 10$ m/s^2)

(*a*) 90 m (*b*) 180 m (*c*) 270 m (*d*) 360 m

36. From the top of a tower, a stone is thrown up. It reaches the ground in time t_1. *A* second stone thrown down with the same speed reaches the ground in time t_2. A third stone released from rest reaches the ground in time t_3. Then:

(*a*) $t_3 = \frac{t_1 + t_2}{2}$ (*b*) $t_3 = \sqrt{t_1 t_2}$

(*c*) $\frac{1}{t_3} = \frac{1}{t_1} - \frac{1}{t_2}$ (*d*) $t_3^2 = t_1^2 - t_2^2$

37. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is [$g = 10$ m/s^2]

(*a*) 1 : 1 (*b*) $\sqrt{\frac{2}{3}}$

(*c*) $\frac{2}{3}$ (*d*) $\sqrt{\frac{3}{2}}$

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

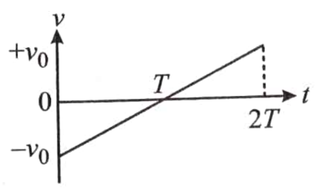
1. Mark the correct statements for a particle going on a straight line
   (*a*) If the velocity is zero at any instant, the acceleration should also be zero at that instant.
   (*b*) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.
   (*c*) If the velocity and acceleration have opposite sign, the object is slowing down.
   (*d*) If the position and velocity have opposite sign, the particle is moving towards the origin.

2. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion then which among the following is/are true
   (*a*) $|\vec{v}|$ must decrease when $\vec{a} < 0$.
   (*b*) Speed must increase when $\vec{a} > 0$.
   (*c*) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0.
   (*d*) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$.

3. A particle has initial velocity 10 m/s. It moves due to constant retarding force along the line of velocity which produces a retardation of 5 m/s^2. Then
   (*a*) The maximum displacement in the direction of initial velocity is 10 m.
   (*b*) The distance travelled in first 3 seconds is 7.5 m.
   (*c*) The distance travelled in first 3 seconds is 12.5 m.
   (*d*) The distance travelled in first 3 seconds is 17.5 m.

4. The displacement x of a particle depend on time t as $x = \alpha t^2 - \beta t^3$
   (*a*) Particle will return to its starting point after time α/β.
   (*b*) The particle will come to rest after time $\frac{2\alpha}{3\beta}$.
   (*c*) The initial velocity of the particle was zero but its initial acceleration was not zero.
   (*d*) No net force act on the particle at time $\frac{\alpha}{3\beta}$.

5. The figure shows the velocity (v) of a particle plotted against time (t)



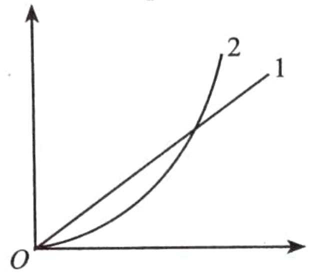
   (*a*) The particle changes its direction of motion at some point.
   (*b*) The acceleration of the article remains constant.
   (*c*) The displacement of the particle is zero.
   (*d*) The initial and final speeds of the particle are the same.

6. A particle moves with constant speed v along a regular hexagon $ABCDEF$ in the same order. Then the magnitude of the average velocity for its motion from A to
   (*a*) F is $v/5$ (*b*) D is $v/3$
   (*c*) C is $v\sqrt{3/2}$ (*d*) B is v

7. Path of a particle moving in x-y plane is $y = 3x + 4$. At some instant suppose x-component of velocity is 1 m/s and it is increasing at a rate of 1 m/s^2. Then at this instant
   (*a*) Speed of particle is $\sqrt{10}$ m/s.
   (*b*) Acceleration of particle is $\sqrt{10}$ m/s^2.
   (*c*) Velocity time graph is a straight line.
   (*d*) Acceleration-time graph is a straight line.

8. A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.
   (*a*) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$
   (*b*) The particle will come to rest at infinity.
   (*c*) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$.
   (*d*) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$.

9. A particle is resting over a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time as $F = kt$, where k is a constant. The two curves are drawn for this particle as shown.



   (*a*) Curve-1 shows acceleration versus time.
   (*b*) Curve-2 shows velocity versus time.
   (*c*) Curve-2 shows velocity versus acceleration.
   (*d*) Curve-1 shows velocity versus acceleration.

COMPREHENSION BASED QUESTIONS

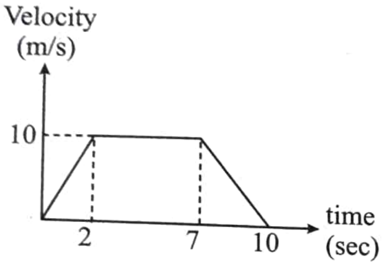
Comprehension (Q. 10 to 12): A boy is standing on a open truck. Truck is moving with an acceleration 2 m/s^2 on horizontal road. When speed of truck is 10 m/s and reaches to a electric pole, boy projected a ball with a velocity 10 m/s in vertical upward direction relative to himself (take $g = 10$ m/s^2). Neglect the height of boy and truck.

10. The distance of ball from pole where ball land is
(a) 20 m (b) 10 m (c) 30 m (d) 40 m

11. Maximum height of ball from ground is
(a) 5 m (b) 7.5 m (c) 2.5 m (d) 10 m

12. Speed of truck at the instant when boy see that ball is moving backward horizontally is
(a) 14 m/s (b) 10 m/s
(c) 12 m/s (d) Data is insufficient

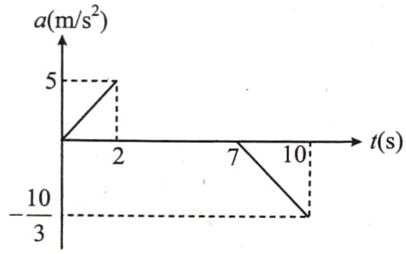
Comprehension (Q. 13 to 15): The velocity-time graph of a car moving on a straight track is given below. The car weighs 1000 kg. (Use $F = ma$)

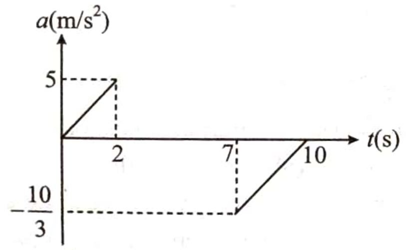


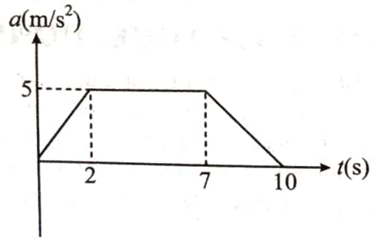
13. The distance travelled by the car during the whole motion is
(a) 50 m (b) 75 m (c) 100 m (d) 150 m

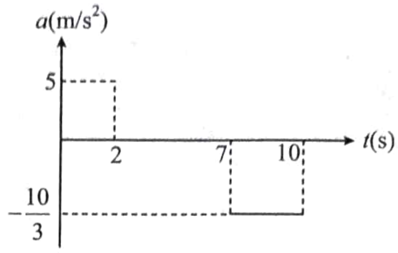
14. The braking force required to bring the car to a stop with in one second from the maximum speed is
(a) $\frac{10000}{3} N$ (b) $5000\ N$
(c) $10000\ N$ (d) $\frac{5000}{3} N$

15. Correct acceleration-time graph representing the motion of car is

(a)


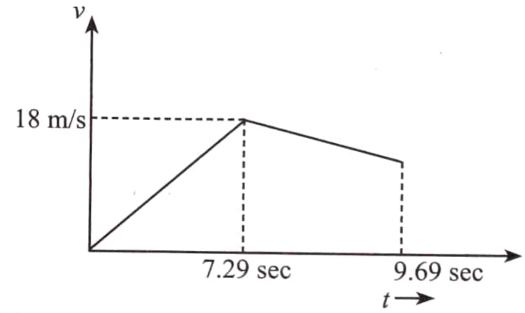
(b)


(c)


(d)


Comprehension (Q. 16 to 17): In the 2008 Olympic 100 m final, Usain Bolt broke new ground, winning in 9.69 s (unofficially 9.683 s). This was an improvement upon his own world record, and he was well ahead of second-place finisher Richard Thompson, who finished in 9.89 s. Not only was the record set without a favourable wind (+0.0 m/s), but he also visibly slowed down to celebrate before he finished and his shoelace was untied. Bolt's coach reported that, based upon the speed of Bolt's opening, he could have finished with a time of 9.52 s. After scientific analysis of Bolt's run by the Institute of Theoretical Astrophysics at the University of Oslo, Hans Eriksen and his colleagues also predicted a 9.60 s time. Considering factors such as Bolt's position, acceleration and velocity in comparison with second-place-finisher Thompson, the team estimated that Bolt could have finished in 9.55 ± 0.04 s had he not slowed to celebrate before the finishing line.

Let us also analyse the motion of Bolt. Assume that the velocity time graph of Usain Bolt is as shown below.

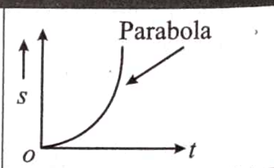
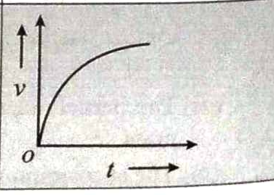


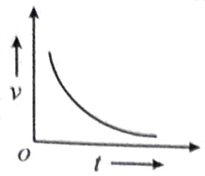
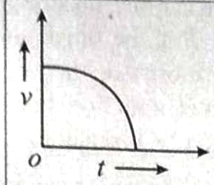
16. What was the initial acceleration of Bolt.
(a) 4.5 m/s² (b) 3.1 m/s² (c) 2.5 m/s² (d) 1.2 m/s²

17. What was the final velocity of Bolt.
(a) 10.1 m/s (b) 10.6 m/s
(c) 13.4 m/s (d) 14.6 m/s

MATCH THE COLUMN TYPE QUESTIONS

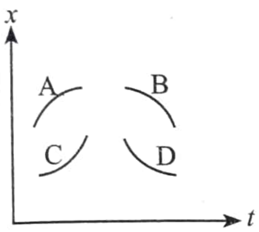
18. Match Column-I with Column-II and select the correct answer using the codes given below the lists.

Column-I		Column-II	
A.	Acceleration decreasing with time	p.	
B.	Velocity increasing with time	q.	

C.	Magnitude of acceleration increasing with time	r.	
D.	Body going farther away from the starting point with time	s.	

(*a*) A-(r); B-(p,q); C-(s); D-(p,q,r,s)
(*b*) A-(r,s); B-(r,q); C-(p,q,r,s); D-(s)
(*c*) A-(r,q); B-(q); C-(s,r); D-(p,q,r,s)
(*d*) A-(r,s); B-(p,r); C-(p,q,r,s); D-(p,q)

19. Column-I shows the position-time graph of particles moving along a straight line.

Column-I		Column-II
	p.	Acceleration $a > 0$
	q.	Acceleration $a < 0$
	r.	Speeding up
	s.	Slowing down

(*a*) A-(q,p); B-(r,p); C-(s,q); D-(r,s)
(*b*) A-(q,s); B-(q,r); C-(p,r); D-(p,s)
(*c*) A-(r,q); B-(s,r); C-(q,r); D-(p,r)
(*d*) A-(p,s); B-(r,s); C-(p,r); D-(p,q)

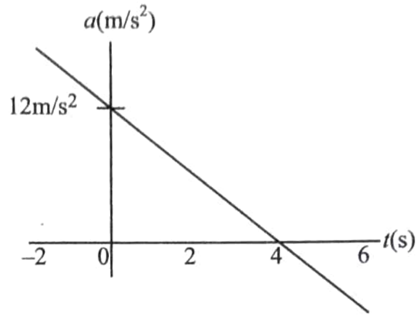
20. The position of a particle along x-axis is given by $x = (2t^3 - 21t^2 + 60t)m$. Then match the Column-I with Column-II.

Column-I		Column-II	
A.	Velocity of particle is zero	p.	2 sec
B.	Acceleration of particle is zero	q.	3 sec
C.	Acceleration of particle is negative	r.	3.5 sec
D.	Velocity of particle is towards the origin	s.	4 sec
		t.	5 sec

(*a*) A-(p,r,t); B-(r); C-(p,q); D-(q,r,s)
(*b*) A-(p,t,r); B-(r,t); C-(q); D-(s)
(*c*) A-(p,r); B-(s); C-(p,r,q); D-(r,s)
(*d*) A-(p,t); B-(r); C-(p,q); D-(q,r,s)

NUMERICAL TYPE QUESTIONS

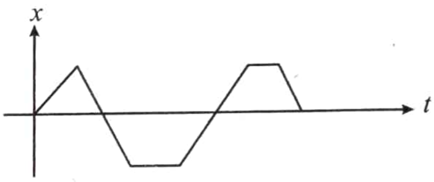
21. Figure gives the acceleration a versus time t for a particle moving along an x-axis. At $t = -2.0$ s, the particle's velocity is 7.0 m/s. What is its velocity (m /s) at $t = 6.0$ s?



22. A particle moves in xy-plane according to the equation $x = 3t$, $y = 25 - 4t$. What is the minimum distance of the particle (in m) from the origin? Both x and y are in m.

23. A rocket rises vertically up from the surface of earth so that it's distance from the earth's surface is $l = ct^2$ where c is a constant. After 10 sec. the rocket has travelled 2 km. Determine it's speed (in m/s) at that moment.

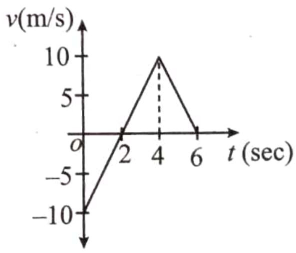
24. Consider a particle moving on a straight line with varying velocity. Its position time graph is as shown.



Find the number of times its velocity changes during motion.

25. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s^2). It is at instantaneous rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

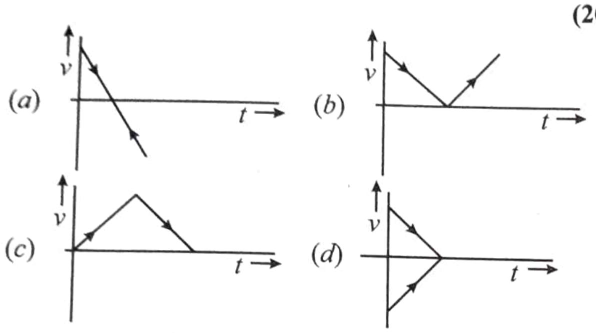
26. The figure shows the graph of velocity-time for a particle moving in a straight line. If the average speed for 6 sec is 'b' and the average acceleration from 0 sec to 4 sec is 'c' find magnitude of bc (in m^2/s^3).



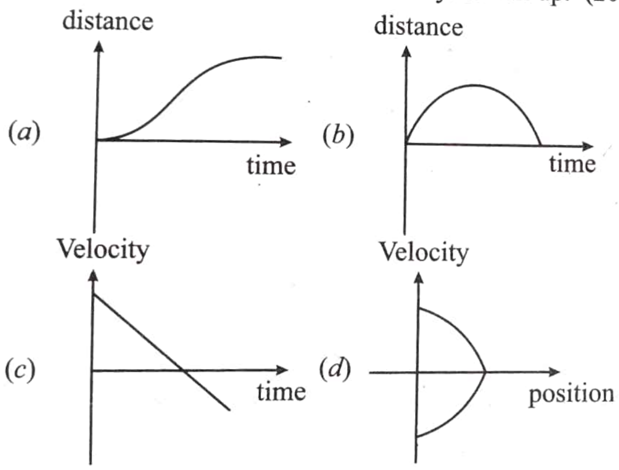
Exercise-4 (Past Year Questions)

JEE MAIN

1. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **(2017)**



2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **(2018)**



3. A particle moves from the point $(2.0\hat{i}+4.0\hat{j})$m, at $t = 0$ with an initial velocity $(5.0\hat{i}+4.0\hat{j})\,\text{ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})\,\text{ms}^{-2}$. What is the distance of the particle from the origin at time 2s? **(2019)**

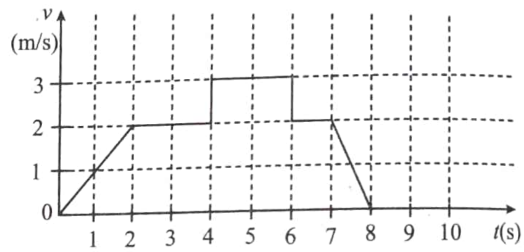
(a) 15 m (b) $20\sqrt{2}$ m
(c) 5 m (d) $10\sqrt{2}$ m

4. The position co-ordinates of a particle moving in a 3-D coordinates system is given by
$x = a\cos\omega t$
$y = a\sin\omega t$
and $z = a\omega t$
The speed of the particle is: **(2019)**
(a) $\sqrt{2}\,a\omega$
(b) $a\omega$
(c) $\sqrt{3}\,a\omega$
(d) $2a\omega$

5. In a car race on straight road, car A takes a time 't' less than car B at the finish and passes finishing point with a speed 'V' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to **(2019)**

(a) $\dfrac{2a_1a_1}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}\,t$
(c) $\sqrt{a_1a_2}\,t$ (d) $\dfrac{a_1+a_2}{2}t$

6. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5s$? **(2019)**



(a) 10 m (b) 6 m
(c) 3 m (d) 9 m

7. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be: **(2019)**

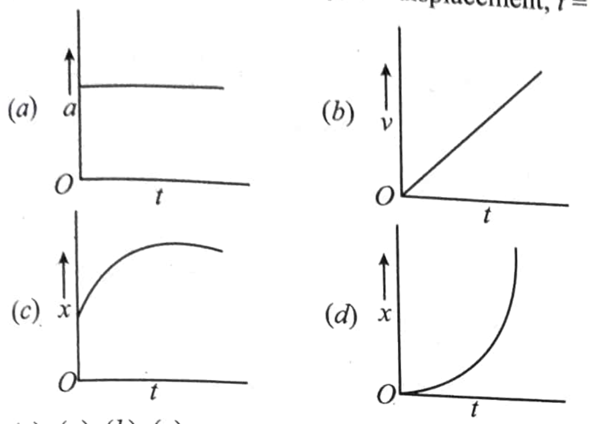
(a) $a+\dfrac{b^2}{4c}$ (b) $a+\dfrac{b^2}{c}$
(c) $a+\dfrac{b^2}{2c}$ (d) $a+\dfrac{b^2}{3c}$

8. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4-20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **(2019)**
(a) 40 (b) 100
(c) 25 (d) 50

9. A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to **(2019)**
(a) 0.4 ms^{-1}
(b) 0.1 ms^{-1}
(c) 0.3 ms^{-1}
(d) 0.7 ms^{-1}

10. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify the figure that is not correctly representing the motion qualitatively. **(2019)**

(a = acceleration, v = velocity, x = displacement, t = time)



(*a*) (*a*), (*b*), (*c*) (*b*) (*a*)

(*c*) (*c*) (*d*) (*b*), (*c*)

11. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$) **(2019)**

(*a*) $\frac{b^2\tau}{4}$ (*b*) $\frac{b^2\tau}{2}$

(*c*) $b^2\tau$ (*d*) $\frac{b^2\tau}{\sqrt{2}}$

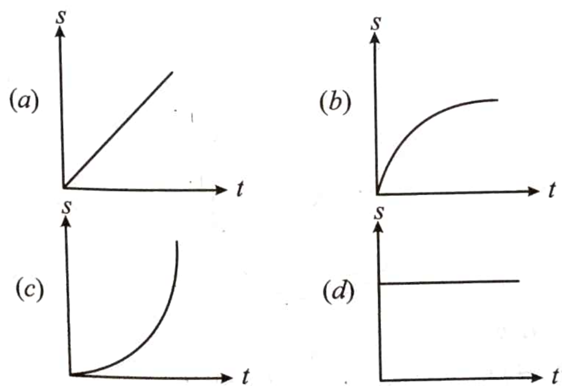
12. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is **(2019)**

13. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\,\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is **(2020)**

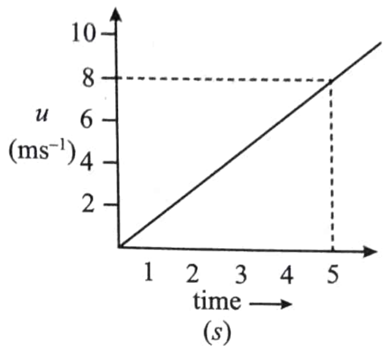
(*a*) 50 (*b*) 40

(*c*) 32 (*d*) 60

14. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) **(2020)**



15. The speed versus time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ to $t = 5$ s will be ____________. **(2020)**

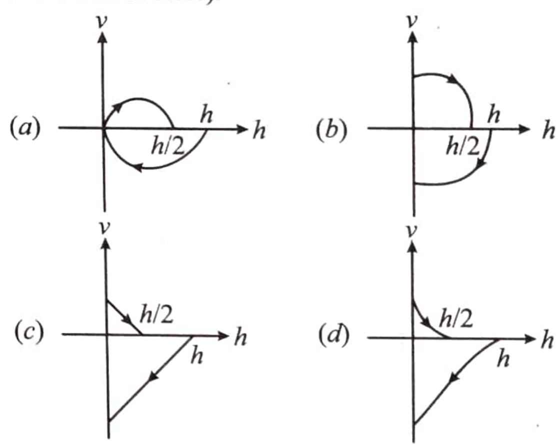


16. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ m/s, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})$ m/s^2. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively **(2020)**

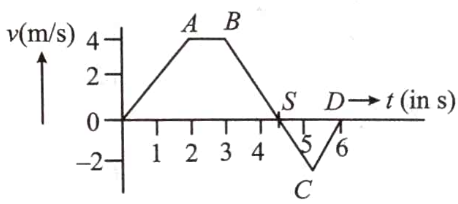
(*a*) 4 s and 52 m (*b*) 2 s and 24 m

(*c*) 5 s and 25 m (*d*) 2 s and 18 m

17. A tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $h/2$. The velocity versus height of the ball during its motion may be represented graphically by (graph are drawn schematically and on not to scale). **(2020)**



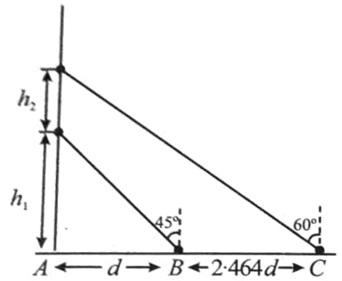
18. Th velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 second. The total distance covered by the body in 6 s is **(2020)**



(*a*) $\frac{37}{3}$ m (*b*) 11 m

(*c*) 12 m (*d*) $\frac{49}{4}$ m

19. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) **(2020)**



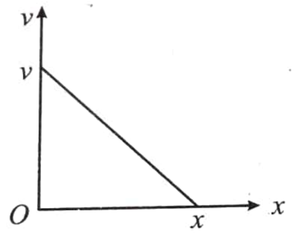
(a) d (b) $0.732\,d$
(c) $1.464\,d$ (d) $0.464\,d$

20. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] **(2020)**

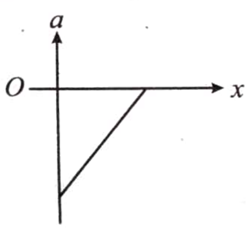
(a) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (b) $t = 1.8\sqrt{\frac{h}{g}}$

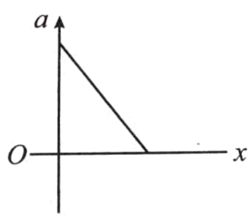
(c) $t = \sqrt{\frac{2h}{3g}}$ (d) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

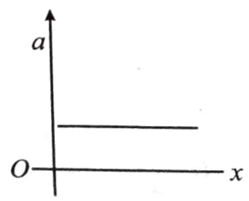
21. The velocity - displacement graph of a particle is shown in the figure. **(2021)**

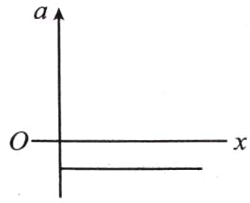


The acceleration - displacement graph of the same particle is represented by:

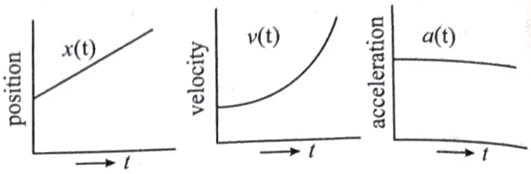
(a)


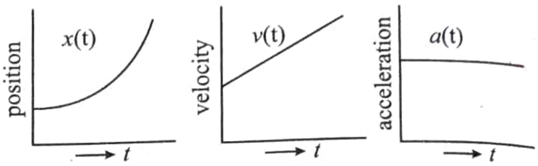
(b)


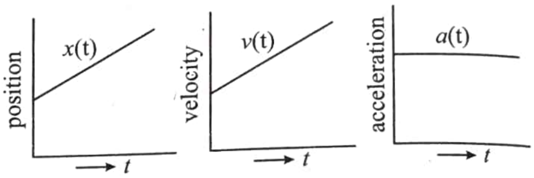
(c)


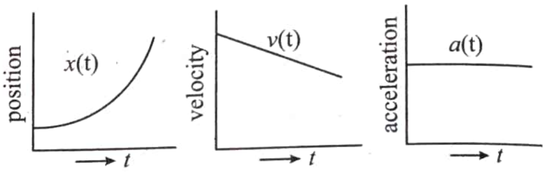
(d)


22. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by **(2021)**

(a)


(b)


(c)


(d)


23. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take $g = 10\ ms^{-2}$) **(2021)**

(a) $2.50\ ms^{-1}$ (b) $3.0\ ms^{-1}$
(c) $2.0\ ms^{-1}$ (d) $3.50\ ms^{-1}$

24. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time ($t = 1$) is: **(2021)**

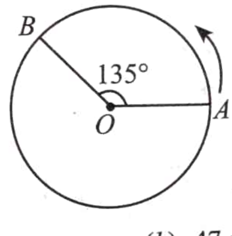
(a) $v_0 + g + f$

(b) $v_0 + \frac{g}{2} + \frac{F}{3}$

(c) $v_0 + 2g + 3F$

(d) $v_0 + \frac{g}{2} + F$

25. A person moved from A to B on a circular path a shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be:
(Given cos135° = – 0.7) **(2022)**



(a) 42 m (b) 47 m
(c) 19 m (d) 40 m

26. A car is moving with speed of 150 km/h and after applying the brake it will move 27 m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling ______ m distance. **(2022)**

27. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is______ms^{-2} at a point where its velocity is 20 ms^{-1}. **(2022)**

28. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)$ m. The value of k is ______

(use $g = 9.8$ m/s²) **(2022)**

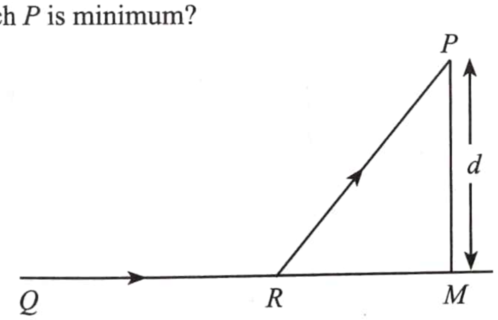
29. A ball is thrown up vertically with a certain velocity so that, it reaches a maximum height h. Find the ratio of the times in which it is at height $\frac{h}{3}$ while going up and coming down respectively. **(2022)**

(a) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (b) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{1}{3}$

JEE ADVANCED

30. A man in a car at location Q on a straight highway is moving with speed u. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM so that the time taken to reach P is minimum? **(2018)**



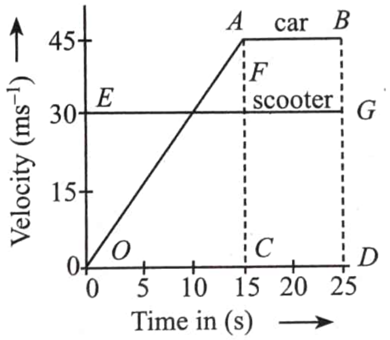
(a) $\frac{d}{\sqrt{3}}$

(b) $\frac{d}{2}$

(c) $\frac{d}{\sqrt{2}}$

(d) d

31. The velocity-time graphs of a car and a scooter are shown in the figure, (i) the difference between the distance travelled by the car and the scooter in 15s and (ii) the time at which the car will catch up with the scooter are, respectively **(2018)**



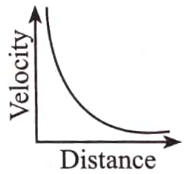
(a) 337.5 m and 25 s

(b) 225.5 m and 10 s

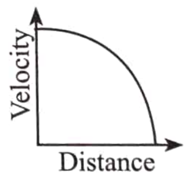
(c) 112.5 m and 22.5 s

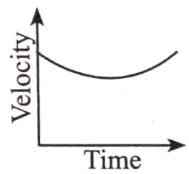
(d) 11.2.5 m and 25 s

32. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity? **(2018)**

(a) 

(b) 

(c) 

(d) 

ANSWER KEY

CONCEPT APPLICATION

1. (24 km/hr) **2.** $\frac{10}{3}(\sqrt{2}+1)$ km/hr **3.** (*a*) 2 m/sec^2, (*b*) 75 m **4.** (*a*) 1, (*b*) 50 **5.** (*b*) **6.** (*a*) **7.** (*c*)
8. (*d*) **9.** (*a*) **10.** (*b*) **11.** (*c*) **12.** (*a*)

EXERCISE-1 (TOPICWISE)

1. (*a*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)	**10.** (*c*)
11. (*b*)	**12.** (*a*)	**13.** (*b*)	**14.** (*a*)	**15.** (*b*)	**16.** (*b*)	**17.** (*b*)	**18.** (*b*)	**19.** (*d*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*a*)	**24.** (*c*)	**25.** (*c*)	**26.** (*c*)	**27.** (*a*)	**28.** (*d*)	**29.** (*c*)	**30.** (*a*)
31. (*d*)	**32.** (*b*)	**33.** (*c*)	**34.** (*b*)	**35.** (*c*)	**36.** (*d*)	**37.** (*b*)	**38.** (*d*)	**39.** (*c*)	**40.** (*d*)
41. (*a*)	**42.** (*a*)	**43.** (*c*)	**44.** (*a*)	**45.** (*c*)	**46.** (*c*)	**47.** (*a*)			

EXERCISE-2 (LEARNING PLUS)

1. (*b*)	**2.** (*b*)	**3.** (*c*)	**4.** (*b*)	**5.** (*d*)	**6.** (*b*)	**7.** (*c*)	**8.** (*d*)	**9.** (*d*)	**10.** (*c*)
11. (*d*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*a*)	**16.** (*a*)	**17.** (*d*)	**18.** (*b*)	**19.** (*c*)	**20.** (*a*)
21. (*b*)	**22.** (*b*)	**23.** (*a*)	**24.** (*d*)	**25.** (*d*)	**26.** (*a*)	**27.** (*a*)	**28.** (*b*)	**29.** (*c*)	**30.** (*a*)
31. (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*b*)	**35.** (*b*)	**36.** (*b*)	**37.** (*b*)			

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*b,c,d*)	**2.** (*c,d*)	**3.** (*a,c*)	**4.** (*a,b,c,d*)	**5.** (*a,b,c,d*)	**6.** (*a,c,d*)	**7.** (*a*,b)	**8.** (*a*,d)	**9.** (*a,b,c*)	**10.** (*a*)
11. (*a*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*d*)	**16.** (*c*)	**17.** (*b*)	**18.** (*a*)	**19.** (*b*)	**20.** (*d*)
21. [0055]	**22.** [0015]	**23.** [0400]	**24.** [0006]	**25.** [0004]	**26.** [0025]				

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*a*)	**2.** (*a*)	**3.** (*b*)	**4.** (*a*)	**5.** (*c*)	**6.** (*d*)	**7.** (*d*)	**8.** (*d*)	**9.** (*d*)	**10.** (*c*)
11. (*b*)	**12.** (*c*)	**13.** (*d*)	**14.** (*c*)	**15.** [20]	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*a*)	**20.** (*a*)
21. (*a*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)	**25.** (*b*)	**26.** [3]	**27.** [100]	**28.** [392]	**29.** (*b*)	

JEE Advanced

30. (*b*) **31.** (*c*) **32.** (*c*)

CHAPTER

4 Projectile Motion

PROJECTILE

Motion in two and three dimensions

When a particle is moving in space then its motion can be resolved in three independent motions in mutually perpendicular directions. Let these directions be x, y and z axis. The motion in these three directions is governed only by velocity and acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions.

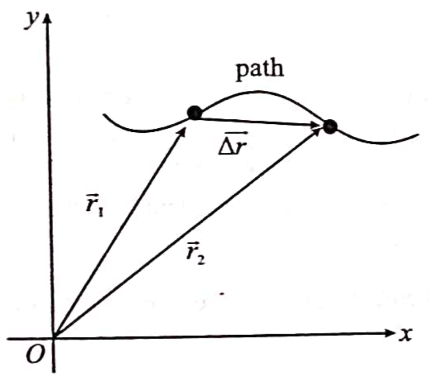
Lets say a particle is moving in space.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad \text{...(i)}$$

Gives position of particle in space.

Velocity

Differentiating $\vec{r}$ w.r.t. time (t) gives us velocity vector of particle at that time.



$$\vec{v} = \frac{d\vec{r}}{dt} \quad \text{...(ii)}$$

To write equation in a unit vector form, we substitute $\vec{r}$ from equation (i)

$$\vec{v} = \frac{d\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

This equation can be simplified some what by writing it as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \quad \text{...(iii)}$$

where the components of $\vec{v}$ are

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt} \quad \text{...(iv)}$$

Acceleration

Similarly, if we differentiate $\vec{v}$ w.r.t. time we get acceleration of particle

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \text{...(v)}$$

If the velocity changes in either magnitude or direction (or both), the particle must have an acceleration.

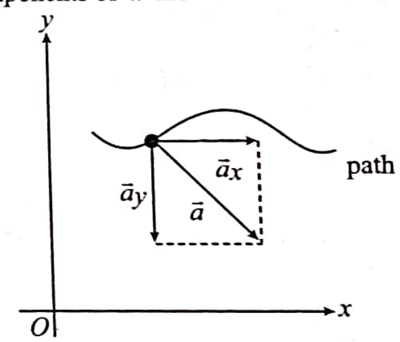
We can write equation (v) in a vector form by substituting $\vec{v}$ from equation (iii) to obtain

$$\vec{a} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

We can rewrite this as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \quad \text{...(vi)}$$

where the components of $\vec{a}$ are



$$a_x = \frac{dv_x}{dt}; a_y = \frac{dv_y}{dt}; a_z = \frac{dv_z}{dt} \quad \text{...(vii)}$$

Thus, we can find the components of $\vec{a}$ by differentiating the components of $\vec{v}$.

Figure shows an acceleration vector $\vec{a}$ and its components for a particle moving in two dimensions.

Now, collecting equations of motion related to x and y axis separately

***x*-axis**	***y*-axis**
$v_x = \frac{dx}{dt}$	$v_y = \frac{dy}{dt}$
$a_x = \frac{dv_x}{dt}$	$a_y = \frac{dv_y}{dt}$

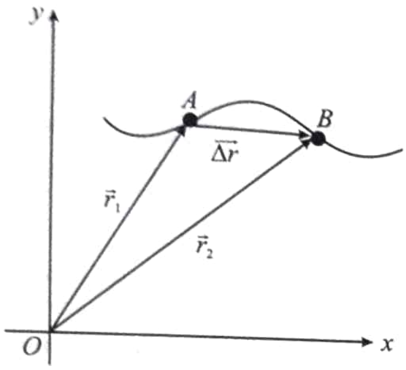
Thus we can see that motion in plane is composed of two straight line motions. These motions are completely independent of each other. Only thing connecting them is fact that they are occurring simultaneously.

Velocity is Along Tangent of Path

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position

$$v = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$



$\Delta \vec{r}$ will be along tangent at point A. when $\Delta \vec{r}$ tends to 0.

The result is the same in three dimensions.

Problem solving strategy

For solving any problem of two dimensional motion, following is standard algorithm.

1. Break the motion in two separate one dimensional motions
2. Solve both motions separately.
3. Connect the final equations thoroughly using 'time' (because it is the only common thing).

Projectile Motion

Projectile motion is the motion of an object when its thrown into the air at some angle under the action of gravity. The path followed by the particle is a curved path.

The particle which is projected is known as projectile. The path followed by the particle is called its trajectory.

It consists of two independent motions, a horizontal motion at constant speed and a vertical motion under acceleration due to gravity.

In order to deal with problems in projectile motion, one has to choose a coordinate system. Let's take horizontal as X-axis and vertical upward direction as Y-axis, then

$a_x = 0$ and $a_y = -g$; since there is only one force "mg" downward (ignore air resistance)

Equation along x-axis	**Equations along y-axis**
$v_x = u_x$ (constant)	$v_y = u_y - gt$
$S_x = u_x t$	$S_y = u_y t - \frac{1}{2}gt^2$
	$v_y^2 = u_y^2 - 2g(S_y)$

If an object is dropped from rest or projected up or down, it follows straight line path. If its initial velocity is not along the line of force it follows parabolic path which is proved mathematically in this topic later on.

PROJECTILE THROWN FROM THE GROUND LEVEL

Consider the motion of a bullet which is fired from a gun so that its initial velocity $\vec{u}$ makes an angle θ with the horizontal direction. Let us take X-axis along ground and Y-axis along vertical direction.

$\vec{u}$ can be resolved as

$u_x = u \cos\theta$ (along horizontal)

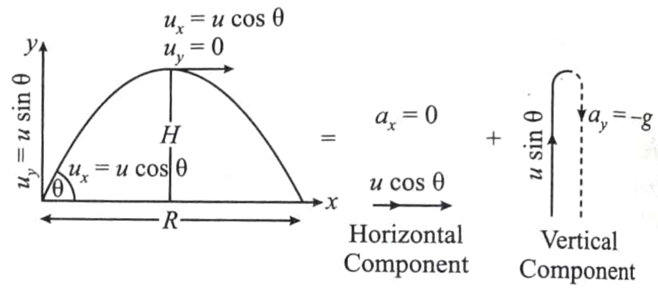
and $u_y = u \sin\theta$ (along vertical)

motion of bullet can be resolved into horizontal and vertical motion.

(i) In horizontal direction there is no acceleration so it moves with constant velocity $v_x = u_x = u\cos\theta$

So displacement along X-direction in time t is $x = u_x t$ or

$x = (u\cos\theta)\, t$ or $t = \dfrac{x}{u\cos\theta}$. ...(i)



The motion in the vertical direction is the same as that of a ball thrown upward with an initial velocity $u_y = u\sin\theta$ and acceleration $a_y = -g$ (downward).

So, at time t vertical component of velocity $v_y = u_y - gt = u\sin\theta - gt$ ...(ii)

Displacement along Y direction $y = (u\sin\theta)\,t - \frac{1}{2}gt^2$ ...(iii)

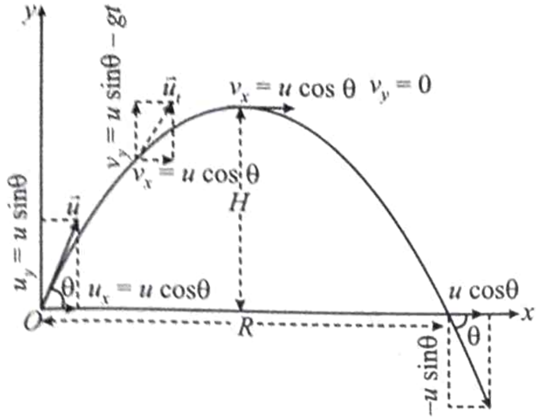
Substituting the value of t from eqn. (i) in eqn. (iii)

we get $y = (u\sin\theta)\left(\dfrac{x}{u\cos\theta}\right) - \dfrac{1}{2}g\left(\dfrac{x}{u\cos\theta}\right)^2$

or $\boxed{y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}\cdot x^2}$. **Equation of trajectory**

- This is eqn. of parabola.
- The trajectory of projectile is parabolic
- The projectile will rise to maximum height H (where $v_x = u\cos\theta$, $v_y = 0$) and then move down again to reach the ground at a distance R from origin.

Setting $x = R$ and $y = 0$ (since projectile reaches ground again)



$$0 = R\tan\theta - \frac{g}{2u^2\cos^2\theta}\cdot R^2$$

We get $R = \dfrac{2u^2\cos^2\theta}{g} \times \dfrac{\sin\theta}{\cos\theta}$

or $R = \dfrac{2u^2}{g}\cdot\sin\theta\cos\theta$

or Range $R = \dfrac{u^2\sin 2\theta}{g}$

If time for upward journey is t

at highest point $v_y = 0$ so

$0 = (u\sin\theta) - gt \;\; (v_y = u_y - gt)$

or $t = \dfrac{u\sin\theta}{g}$

$\therefore T = 2t$ (it will take same time for downward journey)

$\therefore \boxed{T = \dfrac{2u\sin\theta}{g} = \dfrac{2u_y}{g}}$ **Time of flight**

At the highest point $y = H$ and $v_y = 0$

So that $\boxed{H = \dfrac{u_y^2}{2g} = \dfrac{u^2\sin^2\theta}{2g}}$ **Maximum Height**

$[v_y^2 = u_y^2 - 2gy]$

we can also determine R as follows

$x = u_x t$

so $R = u_x \cdot T$

$= (u\cos\theta)\left(\dfrac{2u\sin\theta}{g}\right)$

or $\boxed{R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2u_x u_y}{g}}$ **Horizontal Range**

velocity at time t

$\vec{v}_t = v_{xt}\hat{i} + v_{yt}\hat{j} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$v = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$

If angle which direction of motion makes at an instant is ϕ, then

$$\tan\phi = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$\tan\phi$ is positive during its upward motion i.e. before reaching highest point and after that $\tan\phi$ is negative.

Note:

(i) Alternative eqn. of trajectory $\boxed{y = x\tan\theta\left(1 - \dfrac{x}{R}\right)}$ where $R = \dfrac{2u^2\sin\theta\cos\theta}{g}$

(ii) Vertical component of velocity $v_y = 0$, when particle is at the highest point of trajectory.

(iii) Linear momentum at highest point $= mu\cos\theta$ is in horizontal direction.

(iv) Vertical component of velocity is +ve when particle is moving up.

(v) Vertical component of velocity is –ve when particle is moving down.

(vi) Resultant velocity of particle at time t $v = \sqrt{v_x^2 + v_y^2}$ at an angle $\phi = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$.

(vii) Displacement from origin, $s = \sqrt{x^2 + y^2}$

(viii) Maximum Height and Time of Flight Depend only on Vertical Component of Initial Velocity

Train Your Brain

Example 1: A particle is projected with 20 m/s at an angle 60° with the horizontal. At what time it is moving at an angle 45° with the horizontal while moving downwards.

Sol. $u_x = 20\cos 60° = 10$ m/s

and $u_y = 20\sin 60° = 10\sqrt{3}$ m/s

At required instant, $\tan\phi = -1$

i.e. $\dfrac{u_y - gt}{u_x} = -1$

i.e. $\dfrac{10\sqrt{3} - 10t}{10} = -1$

on solving, we get $t = (\sqrt{3} + 1)$ sec

Example 2: A particle is projected in the X-Y plane with Y-axis along vertical. At 2 sec after projection the velocity of the particle makes an angle 45° with the X - axis and 4 sec after projection, it moves horizontally. Find the velocity of projection.

Sol. At $t = 2$ sec, $\tan\phi = \dfrac{u_y - 10(2)}{u_x} = 1 \;(\because \phi = 45°)$

$\Rightarrow u_y - 20 = u_x$ ...(i)

Also $\frac{1}{2}$ (time of flight) = 4 sec

$\Rightarrow \frac{1}{2}\left(\frac{2u_y}{g}\right) = 4$

$\Rightarrow u_y = 40$ m/s

$\therefore$ From equation (i), $u_x = 20$ m/s

$\therefore u = \sqrt{u_x^2 + u_y^2} = 20\sqrt{5}$ m/s

Concept Application

1. A projectile is projected in the upward direction making an angle of 60° with horizontal direction with a velocity of 150 m/sec. Then the time after which its inclination with the horizontal is 45° is-
   (*a*) 15 sec (*b*) 10.98 sec
   (*c*) 5.49 sec (*d*) 2.745 m
2. One second after projection, a stone moves at an angle 45° with horizontal. Two seconds after projection, it moves horizontally, its angle of projection is ($g = 10$ m/s²)
   (*a*) $\tan^{-1}(\sqrt{3})$ (*b*) $\tan^{-1}(4)$
   (*c*) $\tan^{-1}(3)$ (*d*) $\tan^{-1}(2)$

SPECIAL POINTS

1. **The three basic equations of motion, i.e.**

   $v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$

   For projectile motion :

   $T = \frac{2u\sin\theta}{g} \qquad R = \frac{u^2\sin 2\theta}{g} \qquad H = \frac{u^2\sin^2\theta}{2g}$

2. **In case of projectile motion,**

   The horizontal component of velocity ($u\cos\theta$), acceleration (g) and mechanical energy remains constant.

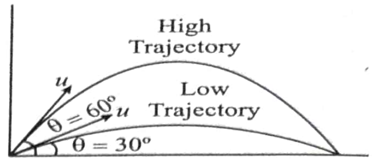
   The vertical component of velocity ($u \sin\theta$), Speed, velocity, momentum, kinetic energy and potential energy all change. Velocity and K.E. are maximum at the point of projection, while minimum (but not zero) at the highest point.

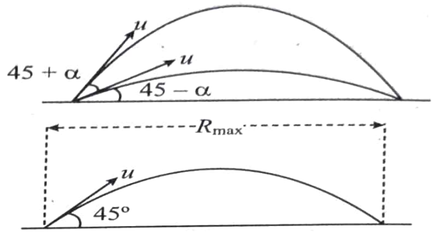
3. **If angle of projection is changed from**

   $\theta \xrightarrow{to} \theta' = (90 - \theta)$,

   then range

   $R' = \frac{u^2\sin 2\theta'}{g} = \frac{u^2\sin 2(90-\theta)}{g} = \frac{u^2\sin 2\theta}{g} = R$





So a projectile has same range for angles of projection θ and $(90 - \theta)$

But has different time of flight (T), maximum height (H) & trajectories

Range is also same for $\theta_1 = 45° - \alpha$ and $\theta_2 = 45° + \alpha$.

$\left[R = \frac{u^2\cos 2\alpha}{g}\right]$

Note: Same Range

$\alpha + \beta = 90°$

α and β are two angles of projection with same velocity.

4. **For maximum Range $R = R_{max} \Rightarrow 2\theta = 90°$**

   for $\theta = 45°$

   $R_{max} = \frac{u^2}{g}$ [For $\sin 2\theta = 1 = \sin 90°$ or $\theta = 45°$]

   When range is maximum $\Rightarrow$ Then maximum height reached is

   $H = \frac{u^2\sin^2 45}{2g}$ or $H = \frac{u^2}{4g}$

   hence maximum height reached (for R_{max}) $H = \frac{R_{max}}{4}$

If a body is projected from a place above the surface of earth, then for the maximum range, the angle of projection should be slightly less than 45°. For javelin throw and discus throw, the athlete throws the projectile at an angle slightly less than 45° to the horizontal to achieve the maximum range.

5. **For height H to be maximum**

   $H = \frac{u^2\sin^2\theta}{2g}$ = max i.e. $\sin^2\theta = 1$ (max) or for $\theta = 90°$

   So that $H_{max} = \frac{u^2}{2g}$ when projected vertically

   (i.e. at $\theta = 90°$)

   In this case, Range $R = \frac{u^2\sin(2\times 90°)}{g} = \frac{u^2\sin 180°}{g} = 0$

$H_{max} = \frac{u^2}{2g}$ (For vertical projection) and $R_{max} = \frac{u^2}{g}$ (For oblique projection with same velocity)

so $H_{max} = \frac{R_{max}}{2}$

- **If a person can throw a projectile to a maximum distance (with $\theta = 45°$) $R_{max} = \frac{u^2}{g}$.**
- **The maximum height to which he can throw the projectile (with $\theta = 90°$) $H_{max} = \frac{R_{max}}{2}$**

6. **At highest point**

Potential energy will be max and equal to $(PE)_H = mgH$

$= mg \cdot \frac{u^2 \sin^2\theta}{2g}$ or $(PE)_H = \frac{1}{2} mu^2 \sin^2 \theta$.

While K.E. will be minimum (but not zero) at the highest point as the vertical component of velocity is zero.

$$(KE)_H = \frac{1}{2} m v_H^2 = \frac{1}{2} m(u\cos\theta)^2 = \frac{1}{2} mu^2 \cos^2\theta$$

$$\text{so, } (PE)_H + (KE)_H = \frac{1}{2} mu^2 \sin^2\theta + \frac{1}{2} mu^2 \cos^2\theta = \frac{1}{2} mu^2$$

= Total M.E.

So, in projectile motion mechanical energy is conserved.

$$\left(\frac{PE}{KE}\right)_H = \frac{\frac{1}{2} mu^2 \sin^2\theta}{\frac{1}{2} mu^2 \cos^2\theta} = \tan^2\theta$$

Note: So if $\theta = 45°$ $\tan^2\theta = 1$

PE = K.E. = $\frac{1}{2}$ M.E. at highest point i.e. if a body is projected at an angle $\theta = 45°$ to the horizontal then at highest point, half of its M.E. is K.E. and half is P.E.

7. **In case of projectile motion if range R is n times the maximum height H, i.e. $R = nH$**

then $\frac{u^2 \sin 2\theta}{g} = n \cdot \frac{u^2 \sin^2\theta}{2g}$

or $2\cos\theta = \frac{n \cdot \sin\theta}{2}$

or $\tan\theta = \frac{4}{n} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{n}\right)$

8. **Weight of a body in projectile motion is zero as it is a freely falling body.**

Train Your Brain

Example 3: A projectile is projected with kinetic energy K. If it has the maximum possible horizontal range, then its kinetic energy at the highest point will be:

(a) 0.25 K (b) 0.5 K
(c) 0.75 K (d) 0.0 K

Sol. When range is maximum $\theta = 45°$

Now $u_x = u \cos 45° = u/\sqrt{2}$

At the highest point, the net velocity of projectile is u_x

$\therefore$ K.E. = $(1/2)\, m\, (u/\sqrt{2})^2$

$= (1/2)\,(1/2\, mu^2) = (1/2)\, K = 0.5\, K$

Hence correct answer is (b)

Example 4: When the angle of elevation of a gun are 60° and 30° respectively. The height it shoots are h_1 and h_2 respectively, h_1/h_2 equals to-

(a) 3/1 (b) 1/3
(c) 1/2 (d) 2/1

Sol. For angle of elevation of 60°, we have maximum height

$$h_1 = \frac{u^2 \sin^2 60°}{2g} = \frac{3u^2}{8g}$$

For angle of elevation of 30°, we have maximum height

$$h_2 = \frac{u^2 \sin^2 30°}{2g} = \frac{u^2}{8g} \Rightarrow \frac{h_1}{h_2} = \frac{3}{1}$$

Hence correct answer is (a)

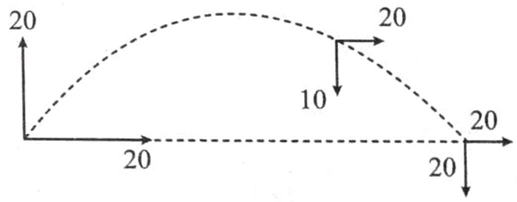
Example 5: A particle is projected from ground at angle 45° with initial velocity $20\sqrt{2}$ m/s. Find:

(a) change in velocity during in its whole journey
(b) magnitude of average velocity in a time interval from $t = 0$ to $t = 3$ s.

Sol. (a) $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$= 20\hat{i} - 20\hat{j} - (20\hat{i} + 20\hat{j})$

$= 40\hat{j}$ m/s



(b) $\vec{v}_i = 20\hat{i} + 20\hat{j}$

$\vec{v}_f = 20\hat{i} - 10\hat{j}$

$\Delta v = v_f - v_i = -30\hat{j}$

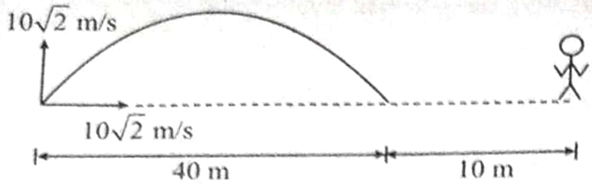
$$\Delta\vec{r} = (60\hat{i}) + (20 \times 3 - \frac{1}{2} \times 10 \times 9)\hat{j} = 60\hat{i} + 15\hat{j}$$

$$v_{avg} = \frac{\Delta r}{\Delta t}$$

$\vec{v}_{av} = 20\hat{i} + 5\hat{j}$

Example 6: The coach throws a baseball to a player with an initial speed of 20 m/s at an angle of 45° with the horizontal. At the moment the ball is thrown, the player is 50 m from the coach. At what speed and in what direction must the player run to catch the ball at the same height at which it was released?

Sol.



Time of flight

$$S_y = 10\sqrt{2}t - \frac{1}{2}\times 10t^2 = 0$$

$$t = \frac{10\sqrt{2}}{5} = 2\sqrt{2} \text{ sec}$$

Range

$$= 10\sqrt{2}\times 2\sqrt{2} = 40\ m$$

$$v = \frac{10m}{2\sqrt{2}} = \frac{5}{\sqrt{2}} ms^{-1}$$

Example 7: If T be the total time of flight of a projectile and H be the maximum height attained by it from the point of projection, then H/T will be- (u = projection velocity and θ = projection angle)

(*a*) $(1/2)\ u \sin\theta$ (*b*) $(1/4)\ u \sin\theta$
(*c*) $u \sin\theta$ (*d*) $2u \sin\theta$

Sol. (*b*) Total time of flight $= T = \dfrac{2u\sin\theta}{g}$, Maximum height attained $H = \dfrac{u^2\sin^2\theta}{2g}$

Now $\dfrac{H}{T} = \dfrac{u\sin\theta}{4}$

Example 8: If a baseball player can throw a ball at maximum distance = d over a ground, the maximum vertical height to which he can throw it, will be- (Ball have same initial speed in each case)

(*a*) $d/2$ (*b*) d
(*c*) $2d$ (*d*) $d/4$

Sol. The range $R = \dfrac{u^2\sin 2\theta}{g}$

$\therefore$ Maximum range $R_{max} = d = \dfrac{u^2}{g}$ ...(i)

Height $H = \dfrac{u^2\sin^2\theta}{2g}$

$\therefore$ Maximum height

$H_{max} = \dfrac{u^2}{2g}$ ...(ii)

From (i) & (ii), $H_{max} = d/2$

Hence correct answer is (*a*)

Example 9: A projectile can have the same range R for two angles of projections. If t_1 and t_2 be the times of flight in two cases, then the product of times of flight will be-

(*a*) $t_1t_2 \propto R$ (*b*) $t_1t_2 \propto R^2$
(*c*) $t_1t_2 \propto 1/R$ (*d*) $t_1t_2 \propto 1/R^2$

Sol. $t_1 = \dfrac{2u\sin\theta}{g}$

$t_2 = \dfrac{2u\sin(90^\circ - \theta)}{g}$

$= \dfrac{2u\cos\theta}{g}$ $[2\sin\theta\cos\theta = \sin 2\theta]$

$t_1t_2 = \dfrac{2\,u^2\sin 2\theta}{g\quad g} = \dfrac{2}{g}\cdot R$

where R is the range, Hence $t_1t_2 \propto R$

Hence correct answer is (*a*)

Example 10: A ball projected with speed 'u' at angle of projection 15° has range R. The other angle of projection at which the range with same initial speed 'u' is R–

(*a*) 45° (*b*) 35°
(*c*) 75° (*d*) 90°

Sol. Range of projectile, $R = \dfrac{u^2\sin 2\theta}{g}$

The range is same for two angles θ_1 and θ_2 provided $\theta_2 = 90^\circ - \theta_1$

At an angle θ_1, range $R_1 = \dfrac{u^2\sin 2\theta_1}{g}$

At an angle of projection θ_2,

range $R_2 = \dfrac{u^2\sin 2\theta_2}{g} = \dfrac{u^2\sin 2(90^\circ - \theta_1)}{g} = \dfrac{u^2\sin 2\theta_1}{g}$

$\Rightarrow R_1 = R_2$

$\therefore$ other angle $(\theta_2) = 90^\circ - \theta_1 = 90^\circ - 15^\circ = 75^\circ$

Hence correct answer is (*c*)

Concept Application

3. An arrow is shot into air its range is 200 metres and its time of flight is 5 seconds. If g = 10 m/sec², then the horizontal component of velocity of the arrow is-

(*a*) 12.5 m/sec (*b*) 25.0 m/sec
(*c*) 31.25 m/sec (*d*) 40 m/sec

4. A ball is projected with a velocity 80 m/s making an angle 30° with the horizontal. Its range will be-

(*a*) 433 m (*b*) 632 m
(*c*) 108 m (*d*) 565 m

5. At the top of the trajectory of a projectile the direction of its velocity and acceleration are-

(*a*) Parallel to each other.
(*b*) Inclined at an angle of 45° to the horizontal
(*c*) Perpendicular to each other
(*d*) None of the above statement is correct

6. A player kicks up a ball at an angle θ with the horizontally. The horizontal range is maximum when θ equals-

(*a*) 30° (*b*) 45°
(*c*) 60° (*d*) 90°

7. The angle of projection of a body is 25°. The other angle for which the range is the same as the first one is equal is-
(a) 30° (b) 45°
(c) 60° (d) 65°

8. At which angle with the verticle should a body be projected so as to travel the maximum horizontal range, keeping the projection velocity constant-
(a) 60° (b) 30°
(c) 45° (d) 75°

9. A body is thrown with a velocity of 9.8 m/s making an angle of 30° with the horizontal. It will hit the ground after a time-
(a) 3 s (b) 2 s
(c) 1.5 s (d) 1 s

10. The maximum range of a gun on a horizontal plane is 16 km if $g = 10$ m/sec^2 the muzzle velocity of the shell must be-
(a) 400 m/sec (b) $160\sqrt{10}$ m/sec
(c) 1600 m/sec (d) $200\sqrt{2}$ m/sec

11. The kinetic energy of a projectile at the highest point is-
(a) Zero (b) Maximum
(c) Minimum (d) Equal to total energy

12. In a projectile motion, the velocity of the projectile is perpendicular to acceleration due to gravity-
(a) For one instant only (b) Two times
(c) Three times (d) Four times

13. A cannon ball has a range R on a horizontal plane. If h and h' are the greatest heights in the two paths for which this is possible, then-
(a) $R = 4\sqrt{(hh')}$ (b) $R = \dfrac{4h}{h'}$
(c) $R = 4h\,h'$ (d) $R = \sqrt{(hh')}$

HORIZONTAL PROJECTION

In Horizontal Direction:

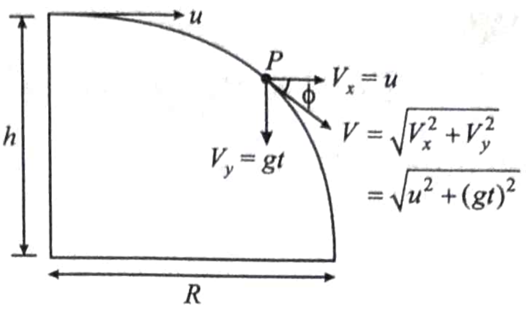
(i) Initial velocity $u_x = u$
(ii) Acceleration = 0
(iii) Horizontal velocity of particle remains same after time t horizontal velocity = $v_x = u$
(iv) Range $x = ut$

In Vertical Direction:

(i) Initial velocity $u_y = 0$
(ii) Acceleration = 'g' downward
(iii) Velocity of particle after time $t = + gt \Rightarrow gt$ (downward)
(iv) Displacement $y = (1/2)\,gt^2$ (downward)

Velocity at a general point $P(x, y)$:

$$v = \sqrt{v_x^2 + v_y^2} \qquad \tan\phi = \frac{v_y}{v_x}$$



ϕ is angle made by v with horizontal in clockwise direction

Time of flight:

$$-h = u_y t - (1/2)\,gt^2 = 0 - \frac{1}{2}gt^2$$

$$t = \pm\sqrt{\frac{2h}{g}}$$

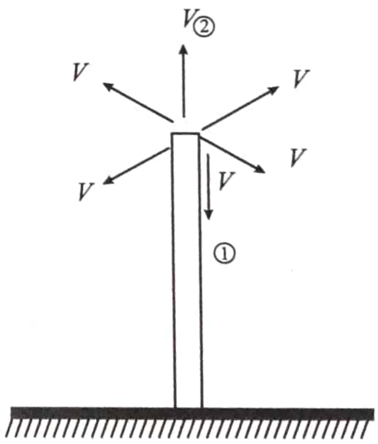
Rejecting negative time $t = \sqrt{\dfrac{2h}{g}}$ (i.e., Time can only be positive)

Range:

$$R = u_x t = u\sqrt{\frac{2h}{g}}$$

Note: If a projectile is projected with initial velocity u horizontally and another particle is dropped from same height at the same time, both the projectile would strike the ground at the same instant and both will have same vertical components of velocity but their net velocities would be different.

Relative motion of one projectile w.r.t. motion of particle dropped from same height at the same time would be in straight line joining them.



All the particles thrown with same initial velocity would strike the ground with same speed at different times irrespective of their initial direction of velocities.

(a) Time would be least for the particle thrown with velocity v downward i.e. particle (1)

(b) Time would be maximum for the particle thrown with velocity v vertically upwards i.e. particle (2)

Train Your Brain

Example 11: An aeroplane is in a level flight of 144 km/hr at an altitude of 1000 m. How far horizontally from a given target should a bomb be released from it to hit the target?

(*a*) 566 m (*b*) 671.43 m
(*c*) 471.34 m (*d*) 371.34 m

Sol. $y = (1/2)\, gt^2$ (downward)

$\Rightarrow \quad 1000 = (1/2) \times 10 \times t^2$

$\Rightarrow \quad t = 14.15$ sec

$$x = ut = \left(\frac{144 \times 5}{18}\right) \times 14.15 = 566 \text{ m}$$

Hence correct answer is (*a*)

Example 12: An aeroplane is flying in horizontal direction with a velocity 600 km/hr and at a height 1960 m. When it is vertically above the point *A*, on the ground, a body is dropped from it. The body strikes the ground at point *B*. The distance *AB* equal to

(*a*) 3.333 km (*b*) 33.33 km
(*c*) 333.3 km (*d*) 33.33 m

Sol. The velocity of plane in horizontal direction

$= (500/3)$ m/s

Initial velocity of body in vertical direction, $v_y = 0$

If t is time taken by the body to reach the earth, then from relation

$s = ut + (1/2)\, at^2$

we have

$h = (1/2)\, gt^2$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20 \text{ sec}$$

$\therefore$ Distance traversed by body in horizontal direction

$AB = v_x t$

$\Rightarrow \quad (500/3) \times 20 = 3.333$ km

Hence correct answer is (*a*)

Example 13: A bomb is dropped from an aeroplane flying horizontally with a velocity of 720 km/h at an altitude of 980 m. After what time, the bomb will hit the ground?

Sol. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 980}{9.8}}$ s $= 10\sqrt{2}$ sec $= 14.14$ sec

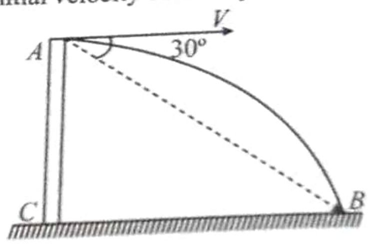
Example 14: A horizontal stream of water leaves an opening in the side of a tank. If the opening is *h* metre above the ground and the stream hits the ground *D* metre away, then what is the speed of water as it leaves the tank in terms of *g*, *h* and *D*?

Sol. The given problem is the problem of horizontal projectile. The stream of water shall follow the parabolic path.

Now, $t = \sqrt{\dfrac{2h}{g}}$; $v = \dfrac{D}{t} = D\sqrt{\dfrac{g}{2h}}$.

Concept Application

14. An object is thrown horizontally from a point '*A*' from a tower and hits the ground 3s later at *B*. The line from '*A*' to '*B*' makes an angle of 30° with the horizontal. The initial velocity of the object is : (take $g = 10$ m/s²)



(*a*) $5\sqrt{3}$ m/s (*b*) 15 m/s
(*c*) $10\sqrt{3}$ m/s (*d*) $25\sqrt{3}$ m/s

15. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45°. What is the vertical component of velocity when it strikes the ground?

(*a*) $18\sqrt{2}$ ms^{-1} (*b*) 18 ms^{-1}
(*c*) $9\sqrt{2}$ ms^{-1} (*d*) 9 ms^{-1}

16. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find : (take $g = 9.8$ m/s²)

(i) The time taken to reach the ground
(ii) The distance of the target from the hill
(iii) The velocity with which the particle hits the ground

PROJECTED FROM SOME HEIGHT AT SOME ANGLE

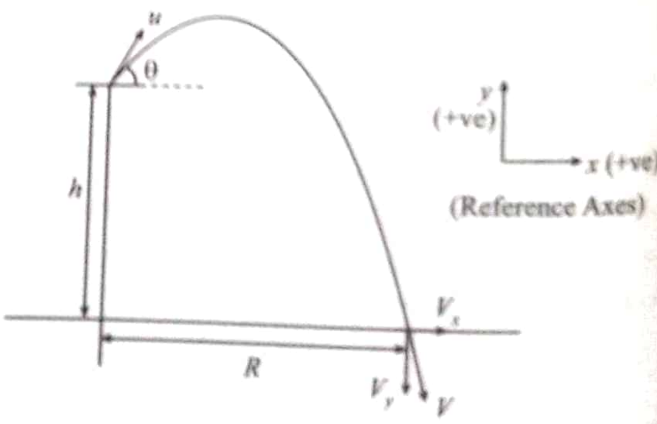
Case-I: When projected at some angle θ with the horizontal towards upward direction.

Let it takes time t_1 (time of flight) to strike the ground

$\Delta y = -h$ when $t = t_1$; here $u_y = u \sin\theta$

$$\therefore -h = (u \sin\theta)\, t_1 - \frac{1}{2} g t_1^2$$

$$\Rightarrow t_1^2 - \left(\frac{2u\sin\theta}{g}\right) t_1 - \frac{2h}{g} = 0$$



Solution of quadratic equation

$$\therefore \; t_1 = \frac{T + \sqrt{T^2 + 8h/g}}{2}$$

$$\left(\text{where } T = \frac{2u\sin\theta}{g}\right)$$

Also $R = \Delta x = u_x t_1$ (putting values of t_1 & u_x we can find R whenever required)

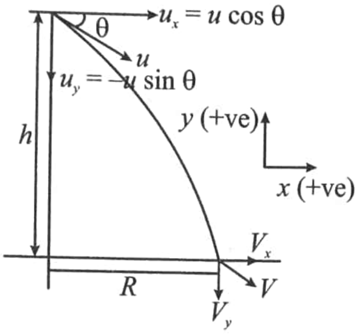
When it reaches ground $v_x = u\cos\theta$

and $v_y^{\,2} = u_y^{\,2} - 2g\,(\Delta y)$

$$\Rightarrow v_y = \sqrt{u^2\sin^2\theta - 2g(-h)} = \sqrt{u^2\sin^2\theta + 2gh}$$

$$\therefore \; v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$$

Case-II: When projected at angle θ with horizontal towards downward direction



Here $u_y = -u\sin\theta$

Thus, if it takes time t_2 to strike the ground then

$$-h = -(u\sin\theta)\,t_2 - \frac{1}{2}g\,t_2^2$$

$$\Rightarrow t_2^2 + \left(\frac{2u\sin\theta}{g}\right)t_2 - h = 0$$

$$\therefore \; t_2 = \frac{\sqrt{T^2 + 8h/g} - T}{2}$$

Also $R = u_x t_2$

Here on reaching ground, $v_x = u\cos\theta$

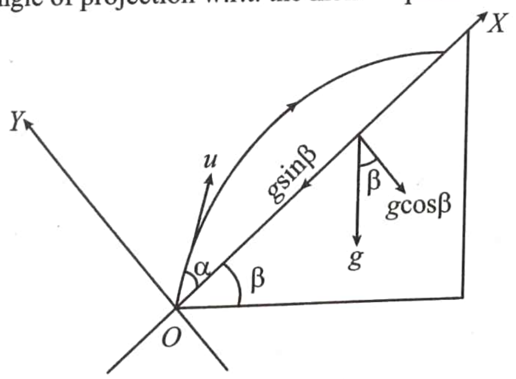
and $v_y = \sqrt{(u\sin\theta)^2 - 2g(-h)}$ (Using $v_y^2 = u_y^2 + 2a_y(\Delta y)$)

$\therefore \; v = \sqrt{v_y^2 + v_x^2} = \sqrt{u^2 + 2gh}$ $\quad a_y = -g\,;\ \Delta y = -h$

PROJECTION ON AN INCLINED PLANE

Case-I: Particle is projected up the incline

Here α is angle of projection w.r.t. the inclined plane.



X and Y axis are taken along and perpendicular to the incline as shown in the diagram.

In this case: $a_x = -g\sin\beta$

$u_x = u\cos\alpha$

$a_y = -g\cos\beta$

$u_y = u\sin\alpha$

Time of flight (T): When the particle strikes the inclined plane y becomes zero

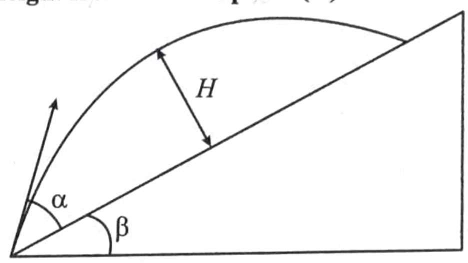
$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \; 0 = u\sin\alpha T - \frac{1}{2}g\cos\beta T^2$$

$$\Rightarrow \; T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

Maximum height from inclined plane (H):



When half of the time is elapsed y-coordinate is equal to maximum height from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \; H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_\perp^2}{2g_\perp}$$

Range along the inclined plane (*R*):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\Rightarrow \; R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \; \boxed{R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}}$$

Maximum Range on the Incline:

$$R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta} = \frac{u^2[\sin(2\alpha+\beta) - \sin\beta]}{g\cos^2\beta}$$

Range will be maximum

When $\sin(2\alpha + \beta) = 1$

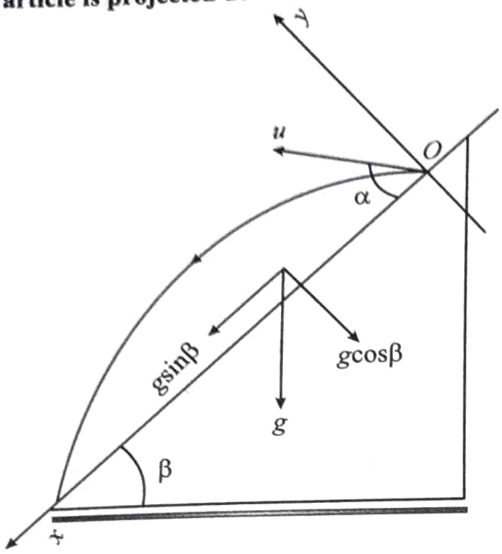
$$2\alpha + \beta = \frac{\pi}{2}$$

$$\alpha = \frac{1}{2}\left(\frac{\pi}{2} - \beta\right) = \frac{\pi}{4} - \frac{\beta}{2}$$

$$\text{Max. Range} = \frac{u^2[1 - \sin\beta]}{g\cos^2\beta}$$

$$= \frac{u^2[1 - \sin\beta]}{g(1 - \sin^2\beta)} = \frac{u^2}{g[1 + \sin\beta]}$$

Case-II: Particle is projected down the incline



In this case:

$a_x = g \sin \beta \qquad u_x = u \cos \alpha$

$a_y = -g \cos \beta \qquad u_y = u \sin \alpha$

Time of flight (*T*):

When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 = u \sin \alpha T - \frac{1}{2} g \cos \beta T^2$$

$$\Rightarrow \quad T = \frac{2u \sin \alpha}{g \cos \beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (*H*):

When half of the time is elapsed y coordinate is equal to maximum height of the projectile

$$H = u \sin \alpha \left(\frac{u \sin \alpha}{g \cos \beta}\right) - \frac{1}{2} g \sin \beta \left(\frac{u \sin \alpha}{g \cos \beta}\right)^2$$

$$H = \frac{u^2 \sin^2 \alpha}{2g \cos \beta} = \frac{u^2_\perp}{2g_\perp}$$

Range along the inclined plane (*R*):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u \cos \alpha \left(\frac{2u \sin \alpha}{g \cos \beta}\right) + \frac{1}{2} g \sin \beta \left(\frac{2u \sin \alpha}{g \cos \beta}\right)^2$$

$$\boxed{R = \frac{2u^2 \sin \alpha \cos(\alpha - \beta)}{g \cos^2 \beta}}$$

$$= \frac{u^2[\sin(2\alpha - \beta) + \sin \beta]}{g \cos^2 \beta}$$

Range will be maximum

When $\sin(2\alpha - \beta) = 1$

$$2\alpha - \beta = \frac{\pi}{2}$$

$$\alpha = \frac{1}{2}\left(\frac{\pi}{2} + \beta\right) = \frac{\pi}{4} + \frac{\beta}{2}$$

$$\text{Maximum Range} = \frac{u^2[1 + \sin \beta]}{g \cos^2 \beta}$$

$$= \frac{u^2[1 + \sin \beta]}{g(1 - \sin \beta)(1 + \sin \beta)} = \frac{u^2}{g(1 - \sin \beta)}$$

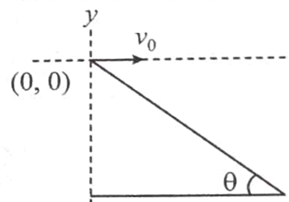
Table: Standard results for projectile motion on an inclined plane

	Up the Incline	Down the Incline
Range	$\frac{2u^2 \sin \alpha \cos(\alpha + \beta)}{g \cos^2 \beta}$	$\frac{2u^2 \sin \alpha \cos(\alpha - \beta)}{g \cos^2 \beta}$
Time of flight	$\frac{2u \sin \alpha}{g \cos \beta}$	$\frac{2u \sin \alpha}{g \cos \beta}$
Angle of projection for maximum range	$\frac{\pi}{4} - \frac{\beta}{2}$	$\frac{\pi}{4} + \frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1 + \sin \beta)}$	$\frac{u^2}{g(1 - \sin \beta)}$

Here α is the angle of projection with the incline and β is the angle of incline.

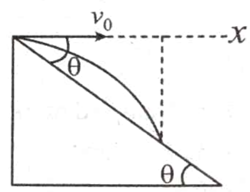
Train Your Brain

Example 15: A man standing on a hill top projects a stone horizontally with speed v_0 as shown in figure. Taking the co-ordinate system as given in the figure. Find the co-ordinates of the point where the stone will hit the hill surface.



Sol: Range of the projectile on an inclined plane (down the plane) is,

$$R = \frac{2u^2 \sin \alpha \cos(\alpha - \beta)}{g \cos^2 \beta}$$



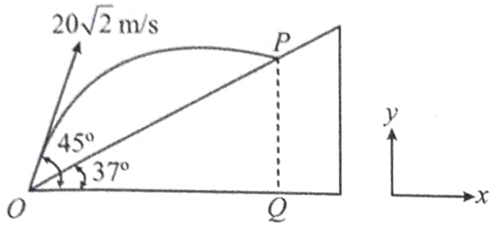
Here, $u = v_0$, $\alpha = \theta$ and $\beta = \theta$

$$\therefore \; R = \frac{2v_0^2 \sin\theta}{g \cos^2\theta}$$

Now, $x = R\cos\theta = \dfrac{2v_0^2\tan\theta}{g}$

and $y = -R\sin\theta = -\dfrac{2v_0^2\tan^2\theta}{g}$

Example 16: In the figure shown, find



(*a*) Time of flight of the projectile along the inclined plane.
(*b*) Range *OP*

Sol: (*a*) Horizontal component of initial velocity,

$u_x = 20\sqrt{2}\cos 45^\circ = 20 \text{ m/s}$

Vertical component of initial velocity

$u_y = 20\sqrt{2}\sin 45^\circ = 20 \text{ m/s}$

Let the particle strikes at P after time t, then horizontal displacement

$OQ = u_x t = 20t$

In vertical displacement, $u_y t$ or $20t$ is upwards and $\frac{1}{2}gt^2$ or $5t^2$ is downwards. But net displacement is upwards, therefore $20t$ should be greater than $5t^2$ and,

$QP = 20t - 5t^2$

In ΔOPQ,

$$\tan 37^\circ = \frac{QP}{OQ}$$

$$\text{or } \frac{3}{4} = \frac{20t - 5t^2}{20t}$$

Solving this equation, we get

$t = 1\text{s}$

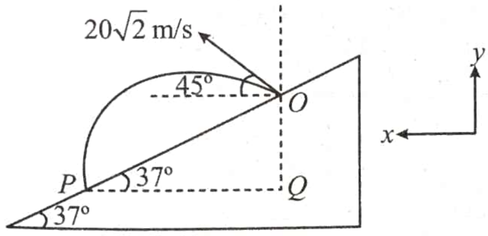
(*b*) Range $OP = OQ \sec 37^\circ$

$$= (20t)\left(\frac{5}{4}\right)$$

Substituting $t = 1s$, we have

$OP = 25$ m

Example 17: In the shown figure, find



(*a*) time of flight of the projectile along the inclined plane
(*b*) range *OP*

Sol: (*a*) Horizontal component of initial velocity,

$u_x = 20\sqrt{2}\cos 45^\circ = 20$ m/s

Vertical component of initial velocity,

$u_y = 20\sqrt{2}\sin 45^\circ = 20$ m/s

Let the particle, strikes the inclined plane at P after time t, then horizontal displacement

$QP = u_x t = 20t$

In vertical displacement, $u_y t$ or $20t$ is upwards and $\frac{1}{2}gt^2$ or $5t^2$ is downwards. But net vertical displacement is downwards. Hence $5t^2$ should be greater than $20t$ and therefore,

$OQ = 5t^2 - 20t$

In ΔOQP, $\tan 37^\circ = \dfrac{OQ}{QP}$

$$\frac{3}{4} = \frac{5t^2 - 20t}{20t}$$

Solving this equation, we get

$t = 7\text{s}$

(*b*) Range, $OP = (PQ) \sec 37^\circ$

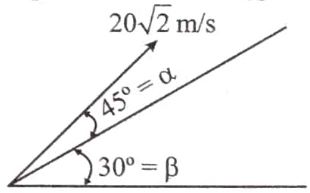
$$= (20t)\left(\frac{5}{4}\right)$$

Substituting the value of t, we get

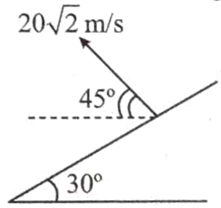
$OP = 175$ m

Concept Application

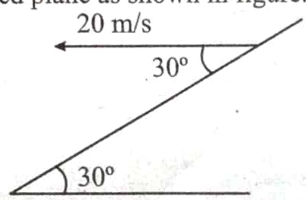
17. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10$ m/s^2)



18. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10$ m/s^2)



19. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10$ m/s^2)



Short Notes

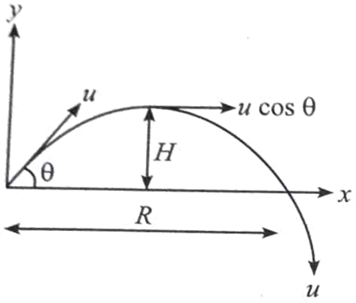
Projection Motion

Horizontal Motion of Projectile

$u \cos \theta = u_x$

$a_x = 0$

$x = u_x t = (u \cos \theta)t$



Vertical Motion of Projectile

$v_y = u_y - gt$ where $u_y = u \sin \theta$; $y = u_y t - \frac{1}{2}gt^2 = u \sin \theta t - \frac{1}{2}gt^2$

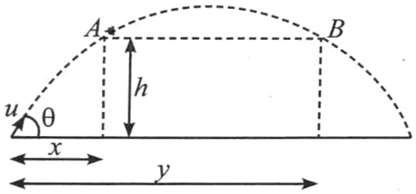
Net acceleration $= \vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

At any Instant

$v_x = u \cos \theta, \quad v_y = u \sin \theta - gt$

Ground to Ground Projectile Motion

A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then



(a) $x + y = R$

(b) $t_1 + t_2 = T$

(c) $h = ½\, g t_1 t_2$

(d) Average velocity from A to B is $u \cos \theta$

If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be $(x/2)$.

Velocity of Particle at Time t:

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ from horizontal is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

At highest point: $v_y = 0$, $v_x = u \cos \theta$

Time of flight: $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Horizontal range: $R = (u \cos \theta)T$

$$= \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

It is same for θ and $(90° - \theta)$ and maximum for $\theta = 45°$.

Maximum height: $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$

$$\frac{H}{R} = \frac{1}{4}\tan\theta$$

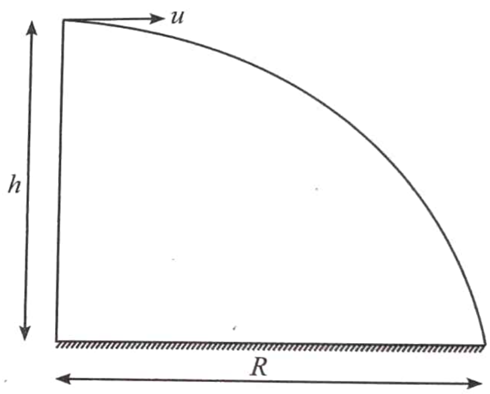
Equation of trajectory $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

Horizontal Projection from a Height h

Time of flight $T = \sqrt{\frac{2h}{g}}$

Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$

Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$



Projected from Some Height at Some Angle

Case-I: When projected at some angle θ with the horizontal towards upward direction.

time t_1 (time of flight) to strike the ground

$$\therefore \quad t_1 = \frac{T + \sqrt{T^2 + 8h/g}}{2}$$

$$\left(\text{where } T = \frac{2u\sin\theta}{g}\right)$$

$R = \Delta x = u_x t_1$

$v_x = u \cos \theta$

Case-II: When projected at angle θ with horizontal towards downward direction

time t_2 to strike the ground then

$$\therefore t_2 = \frac{\sqrt{T^2 + 8h/g} - T}{2}$$

$R = u_x t_2$

$v_x = u \cos\theta$

Projection on an Inclined Plane

Case-I: Particle is projected up the incline

Time of flight (T):

$$T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height from inclined plane (H):

$$H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2_\perp}{2g_\perp}$$

Range along the inclined plane (R):

$$R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$$

Case-II: Particle is projected down the incline

Time of flight (T):

$$T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H):

$$H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2_\perp}{2g_\perp}$$

Range along the inclined plane (R):

$$R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$

Solved Examples

1. If at an instant the velocity of a projectile be u and its inclination to the horizontal be θ, at what time interval after that instant will the particle be moving at right angles to its former direction.

Sol. At $t = 0$ $\quad u_x = u\cos\theta,\ u_y = u\sin\theta$

$\vec{u} = u\cos\theta\ \hat{i} + u\sin\theta\ \hat{j}$

Let after time 't' the velocity of projectile be to its initial velocity u

At time t

$V_x = u\cos\theta \quad V_y = u\sin\theta - gt$

$\vec{V} = u\cos\theta\ \hat{i} + (u\sin\theta - gt)\ \hat{j}$

$u \perp v$

$\vec{u}\cdot\vec{v} = 0$

$(u\cos\theta\ \hat{i} + (u\sin\theta - gt)\ \hat{j})(u\cos\theta\ \hat{i} + u\sin\theta\ \hat{j})$

$u^2\cos^2 + (u\sin\theta)^2 - gt\,u\sin\theta = 0$

$u^2(\cos^2\theta + \sin^2\theta) = gt\,u\sin\theta$

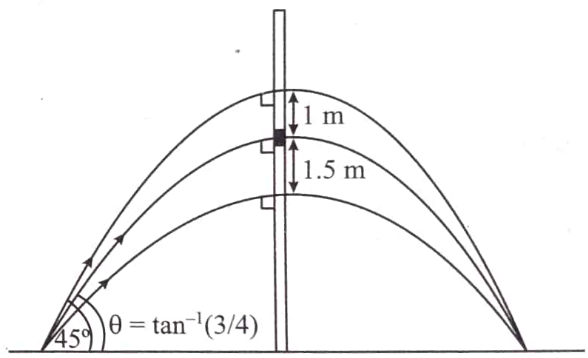
$$t = \frac{u}{g\sin\theta}$$

2. A vertical pole has a black mark at some height. A stone is projected from a fixed point on the ground.

When projected at an angle of 45° it hits the pole orthogonally 1 m above the mark. When projected with a different speed at an angle of $\tan^{-1}(3/4)$, it hits the pole orthogonally 1.5 m below the mark. Find the speed and angle of projection so that it hits the mark orthogonally to the pole.

[$g = 10$ m/sec^2]

Sol. $H = \dfrac{u^2\sin^2\theta}{2g},\ R = \dfrac{u^2\sin\theta}{g}$



So, $\dfrac{H}{R} = \left(\dfrac{\tan\theta}{4}\right)$ ...(i)

$\dfrac{H+1}{R} = \left(\dfrac{\tan 45°}{4}\right) = \dfrac{1}{4}$ ...(ii)

$\dfrac{H-1.5}{R} = \dfrac{\tan\left|\tan^{-1}(3/4)\right|}{4}$

$\dfrac{H-1.5}{R} = \dfrac{3/4}{4} = \dfrac{3}{16}$ ...(iii)

$\dfrac{H+1.5}{H-1.5} = \dfrac{4}{3} \Rightarrow \dfrac{10}{R} = \dfrac{1}{4} \Rightarrow R = 40$ m

$3H + 3 = 4H - 6$

$H = 9$ m

$\dfrac{9}{40} = \dfrac{\tan\theta}{4}$

$$\tan\theta = \frac{9}{10} \Rightarrow \theta = \tan^{-1}\left(\frac{9}{10}\right)$$

$$R = 40 \quad \tan\theta = \frac{9}{10}$$

$$\frac{u^2\sin 2\theta}{g} = 40 \quad \sin\theta = \frac{9}{\sqrt{181}}$$

Using $R = \dfrac{u^2 \cdot 2\sin\theta\cos\theta}{g}$

$$\frac{u^2 2\left(\frac{9}{\sqrt{181}}\right)\left(\frac{10}{\sqrt{181}}\right)}{10} = 40$$

$$u^2 = \frac{3620}{9} \Rightarrow u = \frac{\sqrt{3620}}{3}\text{ m/s}$$

3. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(\pi t)$ and $y(t) = b\sin(\pi t)$, where a, b $(< a)$ and π are positive constants of appropriate dimensions then

(*a*) the path of the particle is an ellipse

(*b*) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$

(*c*) the acceleration of the particle is always directed towards a focus

(*d*) the distance travelled by the particle in time interval $t = 0$ to $= \pi/2p$ is a.

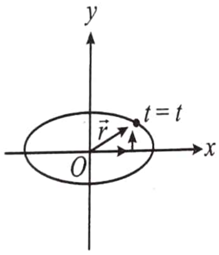
Sol. $x = a\cos pt \Rightarrow \cos(pt) = \dfrac{x}{a}$ ...(i)

$y = b\sin pt \Rightarrow \sin(pt) = \dfrac{y}{b}$ ...(ii)

Squaring and adding (i) and (ii), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$$

Therefore, path of the particle is in ellipse. Hence option (A) is correct.



From the given equations we can find

$$\frac{dx}{dt} = v_x = -a\,p\sin pt$$

$$\frac{d^2x}{dt^2} = a_x = -ap^2\cos pt$$

$$\frac{dy}{dt} = v_y = bp\cos pt$$

$$\frac{d^2y}{dt^2} = a_y = -bp^2\cos pt$$

At time $t = \pi/2p$ or $pt = \pi/2$

a_x and v_y become zero (become $\cos\pi/2 = 0$) only V_x and a_y are left, or we can say that velocity is along negative x-axis and acceleration along $-y$-axis. Hence at $t = \pi/2p$, velocity and acceleration of the particle are normal to each other. So option (B) is also correct.

At $t = t$, position of the particle

$\vec{r}(t) = x\hat{i} + y\hat{j} = a\cos\pi t\ \hat{i} + b\sin\pi t\ \hat{j}$ and acceleration of the particle is

$$\vec{a}(t) = ax\ \hat{i} + ay\ \hat{j}$$

$$= -p^2[a\cos pt\ \hat{i} + b\sin pt\ \hat{j}]$$

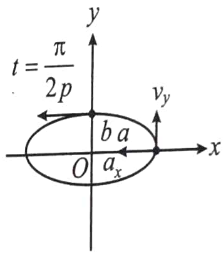
$$= -p^2[x\hat{i} + y\hat{j}] = -p^2\vec{r}(t)$$

Therefore acceleration of the particle is always directed towards origin.

Hence option (*c*) is also correct.

At $t = 0$, particle is at $(a, 0)$ and at $t = \pi/2p$, particle is at $(0, b)$. Therefore, the distance covered is one fourth of the elliptical path not a.

Hence option (*d*) is wrong.



$t = 0$

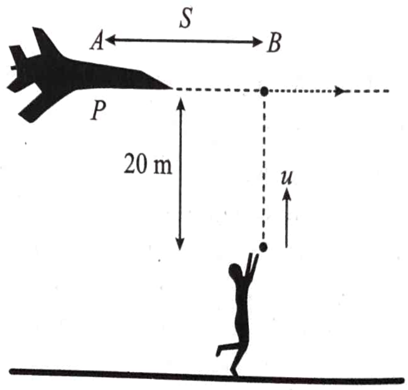
$y = 0 = v_x\, a_y$

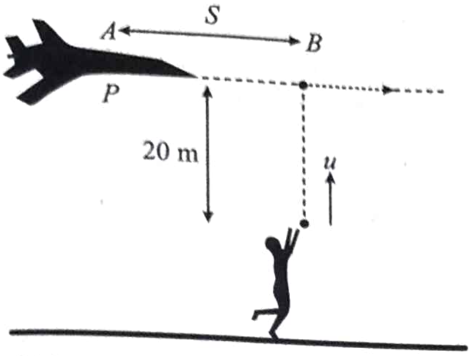
$x = a$

$v_y = bp$ and

$a_x = ap^2$

4. A toy plane P starts flying from point A along a straight horizontal line 20 m above ground level starting with zero initial velocity and acceleration 2 m/s^2 as shown. At the same instant, a man P throws a ball vertically upwards with initial velocity 'u'. Ball touches (coming to rest) the base of the plane at point B of plane's journey when it is vertically above the man. 's' is the distance of point B from point A. Just after the contact of ball with the plane, acceleration of plane increases to 4 m/s^2. Find:





(i) Initial velocity 'u' of ball.

(ii) Distance 's'.

(iii) Distance between man and plane when the man catches the ball back. ($g = 10$ m/s^2)

Sol. (i) Maximum height reached by ball = 20 m.

So, taking upward direction as positive, $v^2 = u^2 + 2as$

So, $0 = u^2 - 2 \times 10 \times 20$ or $u = 20$ m/sec Ans.

Also time taken by ball = $t = u/g = 20/10 = 2$ sec.

(for touching the plane)

(ii) Horizontal distance travelled by plane in this time t =

$s = u_x t + 1/2\ a_x t^2$

where, u_x = initial velocity of plane, a_x = acceleration of plane.

So, $s = 0 \times 2 + 1/2 \times 2 \times 2^2 = 4$ m

(iii) Man catches the ball back 2 seconds after it touches the plane.

Velocity of plane when ball touches it $\Rightarrow v_x = u_x + a_x t$

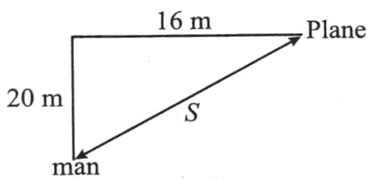
$= 0 + 2 \times 2 = 4$ m/sec.

Now, acceleration of plane becomes : $a'_x = 4$ m/sec^2

so, s'_x = horizontal distance travelled by plane after touch with ball = $u'_x + 1/2\ a'_x t^2$

$= 4 \times 2 + 1/2 \times 4 \times 4$

$= 8 + 8 = 16$ m



Final distance between man and plane =

$s = \sqrt{(20)^2 + (16)^2} = \sqrt{656}$ m

5. A particle is projected with velocity 20 m/s at an angle of 45° with horizontal. Find out the -

(*a*) horizontal Range [take $g = 10$ m/s^2]

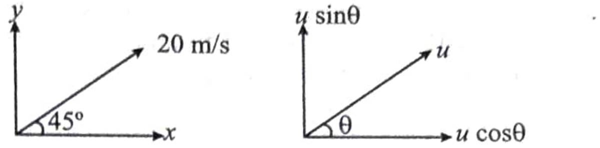
(*b*) Maximum Height

(*c*) Time of flight

(*d*) Angle for which T is maximum, $T_{max} = ?$

(*e*) Angle for which R is maximum, $R_{max} = ?$

Sol: (*a*)



Taking x axis parallel to the plane and y axis vertically upward as shown

T = Total time of flight = $\dfrac{2u_\perp}{g_\perp} = \dfrac{2 \times u \sin\theta}{g}$

where $u_\perp$ = initial velocity perpendicular to the plane.

$g_\perp$ = component of acceleration perpendicular to the plane.

$R = u_x T \left\{\because s_x = u_x t + \frac{1}{2} a_x t^2\right\}$

$= (u \cos\theta)\left(\dfrac{2u\sin\theta}{g}\right)$ where $s_x = R$, $u_x = u_\parallel = u\cos\theta$,

$t = T = \dfrac{2u\sin\theta}{g}$, $a_x = 0$

$R = \dfrac{u^2 \sin 2\theta}{g}$

Substituting values of θ, u and g.

We get, $R = \dfrac{20^2 \times \sin 90°}{10}$ $R = 40$ m

(*b*) For max height.

$S_y = u_y t + \frac{1}{2} a_y t^2$, Here $u_y = u \sin\theta$, $a_y = -g$,

$t = \dfrac{T}{2} = \dfrac{u\sin\theta}{g}$

$S_y = (u\sin\theta)\left(\dfrac{u\sin\theta}{g}\right) + \dfrac{1}{2}(-g)\left(\dfrac{u\sin\theta}{g}\right)^2 = \dfrac{u\sin^2\theta}{g}\left[1 - \dfrac{1}{2}\right]$

$H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{20^2 \times \left(\frac{1}{\sqrt{2}}\right)^2}{2 \times 10} = 10$ m

(*c*) T = Time of flight = $\dfrac{2u\sin\theta}{g} = \dfrac{2 \times 20 \times \frac{1}{\sqrt{2}}}{10} \Rightarrow T = 2\sqrt{2}$ s

$\because \Delta_y = 0 \quad \therefore V^2 - u^2 = 2a\Delta y = 0$

$\therefore v_y = -u_y = -u\sin\theta \qquad V = u + at$

$-u\sin\theta = u\sin\theta + (-g)\,t = \dfrac{2u\sin\theta}{g}$

(*d*) T_{max} = (sin θ) max $\Rightarrow \theta = 90°$

$T_{max} = \dfrac{2u}{g} = \dfrac{2 \times 20}{10} \Rightarrow T_M = 4$s

(*e*) $\theta = ?$ and $R_{max} = ?$ $R = \dfrac{u^2\sin 2\theta}{g}$

$R_{max} \Rightarrow (\sin 2\theta)_{max} = 1 \qquad 2\theta = \dfrac{\pi}{2} \Rightarrow \theta = \left(\dfrac{\pi}{4}\right)^c = 45°$

and $R_{max} = \dfrac{u^2}{g} = \dfrac{400}{10} \Rightarrow R_{max} = 40$ m

6. A body is so projected in the air that the horizontal range covered by the body is equal to the maximum vertical height attained by the body during the motion. Find the angle of projection ?

Sol. Horizontal Range $R = \dfrac{u^2 \sin 2\theta}{g}$

Vertical height $H = \dfrac{u^2 \sin^2 \theta}{2g}$

Given $R = H$

So $= \dfrac{u^2 \sin 2\theta}{g} = \dfrac{u^2 \sin^2 \theta}{2g}$

$2 \times 2 \sin \theta \cos \theta = \sin^2 \theta$

$\tan \theta = 4$

7. A projectile can have the same range R for two angles of projections at a given speed. If T_1 and T_2 be the times of flight in two cases, then find out relation between T_1, T_2 and R?

Sol. R same for θ_1 and θ_2

$\theta_2 = 90 - \theta_1$

$T = \dfrac{2u \sin\theta}{g} \quad \therefore T_1 = \dfrac{2u\sin\theta}{g}; T_2 = \dfrac{2u\sin(90-\theta)}{g}$

and, $R = \dfrac{u^2 \sin 2\theta}{g} \quad \therefore T_1 T_2 = \dfrac{2^2 \times u^2 \sin\theta\cos\theta}{g^2}$

$= \dfrac{2}{g}\left(\dfrac{u^2 \sin 2\theta}{g}\right)$

$T_1 T_2 = \dfrac{2R}{g}$

8. A player kicks a football at an angle of 45° with an initial speed of 20 m/s. A second player on the goal line 60 m away in the direction of kick starts running to receive the ball at that instant. Find the constant speed of the second player with which he should run to catch the ball before it hits the ground [$g = 10$ m/s²]

Sol. (1) $\theta_1 = 45°\quad u = 20\text{m/s}$

$T = \dfrac{2u_y}{g_y} = \dfrac{2 \times 20\frac{1}{\sqrt{2}}}{10} = 2\sqrt{2}s \qquad u_x = 20\dfrac{1}{\sqrt{2}}$

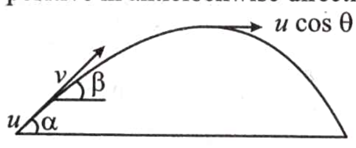
Now, $R = \left(20\dfrac{1}{\sqrt{2}}\right) \times 2\sqrt{2} = 40\text{m}$

$\Rightarrow$ The man should come (travel) 60 – 40 = 20 m

time is $2\sqrt{2}$ s & $\Rightarrow$ vel $= \dfrac{20m}{2\sqrt{2}s} = 5\sqrt{2}$ m/s

9. The direction of motion of a projectile at a certain instant is inclined at an angle α to the horizontal. After t seconds it is inclined an angle β. Find the horizontal component of velocity of projection in terms of g, t, α and β (α and β are positive in anticlockwise direction)

Sol.



Now $a_x = 0$

$\therefore u \cos \alpha = v \cos \beta$ ...(i)

Now for motion along y-axis

$a_y = -g$

$\therefore u \sin \alpha - gt = v \sin \beta$ ...(ii)

Putting the value of v

$V = \dfrac{u\cos\alpha}{\cos\beta}$ in (ii)

We have,

$u \sin \alpha - gt = \dfrac{u\cos\alpha}{\cos\beta}\sin\beta.$

or $u \sin \alpha - u \cos \alpha \tan \beta = gt$

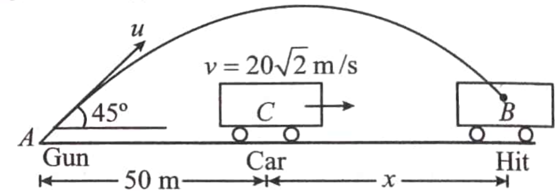
$u \{\sin \alpha - \cos \alpha \tan \beta\} = gt$

$u = \dfrac{gt}{(\sin\alpha - \cos\alpha\tan\beta)}$

horizontal component of velocity $= u \cos \alpha$

$= \dfrac{gt\cos\alpha}{(\sin\alpha - \cos\alpha\tan\beta)} = \dfrac{gt}{(\tan\alpha - \tan\beta)}$

10. A gun kept on a straight horizontal road is used to hit a car, traveling along the same road away from the gun with a uniform speed of $72 \times \sqrt{2}$ km/hour. The car is at a distance of 50 metre from the gun, when the gun is fired at an angle of 45° with the horizontal. Find (i) the distance of the car from the gun when the shell hits it, (ii) the speed of projection of the shell from the gun. [$g = 10$ m/s²]



Sol. range = 50 + x

$\dfrac{u^2}{g} = 50 + x$ ...(i)

Time taken by gun from A to B = Taken by Car from C to B

$\dfrac{2u}{\sqrt{2}g} = \dfrac{x}{20\sqrt{2}}$ ..(ii)

by solving equation (1) and (2) we have

$u = 50$ m/s

(i) Distance of car from gun $= \dfrac{u^2}{g} = \dfrac{50^2}{10} = 250$ m

(ii) Speed of projection of shell = 50 m/s

11. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find :

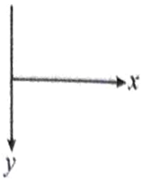
(take $g = 9.8$ m/s²)

(i) The time taken to reach the ground

(ii) The distance of the target from the foot of hill

(iii) The velocity with which the particle hits the ground

Sol. (i) $u_x = 98$ m/s



Ht = 490 m, $g = 9.8\ m/s^2$, $u_y = 0$, $a_y = g = 9.8\ m/s^2$

$s_y = u_y t + \frac{1}{2} a_y t^2$, $\therefore\ 490 = 0 + \frac{1}{2} \times 9.8\, t^2, 100 = t^2$

$t = \pm 10$

Ignoring "–ve" value, as it gives time before the time of projection, we get $t = 10$s

(ii) Distance from the hill $= u_x \times T = 98 \times 10 = 980$ m

(iii) $V = \sqrt{V_x^2 + V_y^2}$ $Vx = u_x = 98$ m/s $V_y^2 = u_y^2 + 2a_y s_y$

$V_y^2 = 0 + 2 \times 9.8 \times 490$, So, $V = \sqrt{98^2 + 2 \times 9.8 \times 490}$,

$V = 98\sqrt{2}$ m/s

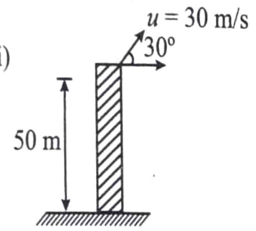
12. From the top of a tower of height 50m a ball is projected upwards with a speed of 30 m/s at an angle of 30° to the horizontal. then calculate

(i) Maximum height from the ground

(ii) At what distance from the foot of the tower does the projectile hit the ground.

(iii) Time of flight.

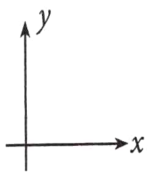
Sol. (i)



$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{30 \times 30 \times \frac{1}{2} \times \frac{1}{2}}{2 \times 10} = \frac{90}{8} = 11.25$$

H from ground $H = 50 + 11.25 = 61.25$ m.

(ii) $s_x = u_x T + \frac{1}{2} a_x T^2$, $a_x = 0 \Rightarrow s_x = u_x T$



To find T $s_y = u_y T + \frac{1}{2} a_y T^2$ Where, $s_y = -50$ = vertical displacement

$u_y = u \sin 30° = 15$ m/s, $a_y = -g = -10$ m/s^2

Substituting these values,

$-50 = 15T + \frac{1}{2}(-10)T^2$; or $T^2 - 3T - 10 = 0$; or,

$T^2 - 5T + 2T - 10 = 0$;

or, $T(T-5) + 2(T-5) = 0$; or

$(T-5)(T+2) = 0$; or, $T = 5$ or $T = -2$

$\Rightarrow T = 5$ sec

$s_x = u \cos\theta \cdot T = 30 \times \cos 30° \times T = 30 \times \frac{\sqrt{3}}{2} \times 5$

$= 75\sqrt{3}$ m

13. The equation of a projectile is $y = \sqrt{3}x - \frac{gx^2}{2}$, find the angle of projection. Also find the speed of projection. Where at t = 0, x = 0 and y = 0 also $\frac{d^2x}{dt^2} = 0$

$\frac{d^2y}{dt^2} = -g$.

Sol. $y = \sqrt{3}x - g\frac{x^2}{2}$

From the given (above) equation with the standard equation of trajectory $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

We get, $\sqrt{3} = \tan\theta \Rightarrow \theta = 60°$

$u^2\cos^2\theta = 1$ Putting $\theta = 60°$, we get $u^2 = \frac{1}{(1/2)^2}$

$\Rightarrow u = 2$m/s]

Alternate Solution

$$y = x\tan\theta - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$

In this eq. at $t = 0, x = 0, y = 0; a_x = 0; a_y = -g$

using these conditions in the given equation we get.

$$\frac{dy}{dx} = \sqrt{3} - \frac{1}{2} g\, 2x \frac{dx}{dx}$$

for $\theta = ?$, i.e. To find θ, we now find $\tan\theta = \left.\frac{dy}{dx}\right]_{at\, t=0}$

$\therefore \left.\frac{dy}{dx}\right]_{at\, t=0} = \sqrt{3} - 0$ $\{\because x = 0$ at $t = 0\}$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

$$\frac{dy}{dt} = \sqrt{3}\frac{dx}{dt} - \frac{1}{2} g \left[2x\left(\frac{dx}{dt}\right)\right]$$

$V_y = \sqrt{3}\ V_x - gx\,V_x$

At $t = 0, x = 0, V_y = u_y$ & $V_x = u_x$; $u_y = \sqrt{3}\ u_x$

$\frac{d^2y}{dt^2} = \sqrt{3}\frac{d^2x}{dt^2} - g\left[x\frac{d^2x}{dt^2} + \frac{dx}{dt} \times \frac{dx}{dt}\right]$ here $ax = \frac{d^2x}{dt^2} = 0$

$\Rightarrow \frac{d^2y}{dt^2} = \sqrt{3} \times 0 - g[0 + v_x]^2 \Rightarrow a_y = -gV_x^2$

Now, $a_y = -g \Rightarrow V_x^2 = 1$ $V_x = \pm 1$

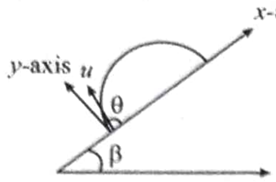
$\Rightarrow$ constant {since $V_x(t) = u_x + a_x t$}

$\therefore u_x = \pm 1 \Rightarrow u_y = 3(\pm 1)$; $u_y = \pm 3$

$\therefore$ Speed $= u = \sqrt{u_x^2 + u_y^2} = \sqrt{(\pm 1)^2 + (\pm\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4}$

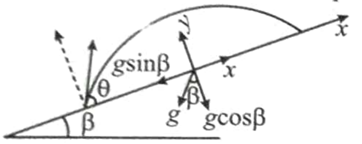
$u = 2$ m/s.

14. A particle is projected at an angle θ with an inclined plane making an angle β with the horizontal as shown in figure, speed of the particle is u, after time t find :



(a) x component of acceleration?
(b) y component of acceleration?
(c) x component of velocity?
(d) y component of velocity?
(e) x component of displacement?
(f) y component of displacement?
(g) y component of velocity when particle is at maximum distance from the incline plane?

Sol:



(a) $a_x = x$ component of acceleration $= -g\sin\beta$

(b) y – component of $acc = a_y = -g\cos\beta$

(c) Let x – component of vel $= V_x$

$V = u + at$

$\therefore V_x = u_x + a_x t = (u\cos\theta) + (-g\sin\beta)\cdot t$

i.e. $V_x = u\cos\theta - g\sin\beta . t$

(d) Let y – component of vel $= V_y$

$\therefore V_y = u_y + a_y t$

$V_y = (u\sin\theta) + (-g\cos\beta\, t)$.

i.e. $V_y = u\sin\theta - g\cos\beta\cdot t$

(e) Let x – component of displacement $= s_x$

$$s = ut + \frac{1}{2}at^2$$

$$s_x = u_x t + \frac{1}{2}a_x t^2 = (u\cos\theta)\,t + \frac{1}{2}(-g\sin\beta)\,t^2$$

i.e, $s_x = u\cos\theta\, t - \dfrac{1}{2}g\sin\beta t^2$

(f) Let $s_y = y$ – component of displacement

$$s_y = u_y t + \frac{1}{2}(-g\cos\beta)\,t^2$$

$$\Rightarrow s_y = u\sin\theta\, t - \frac{1}{2}g\cos\theta\, t^2$$

(g) $V_y = ?$ when $s_y = (s_y)_{\max}$.

$$\frac{ds}{dt} = V$$

$$\frac{d(s_y)}{dt} = V_y$$

For maximum s_y, $\dfrac{d(s_y)}{dt} = 0$ i.e., $V_y = 0$

Exercise-1 (Topicwise)

PROJECTILE MOTION: GROUND TO GROUND PROJECTION

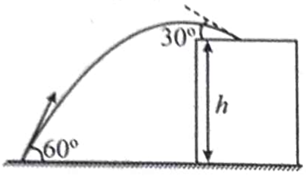
1. The point from where a ball is projected is taken as the origin of the coordinate axes. The x and y components of its displacement are given by $x = 6t$ and $y = 8t - 5t^2$. What is the velocity of projection?
   (a) 6 m s^{-1} (b) 8 m s^{-1}
   (c) 10 m s^{-1} (d) 14 m s^{-1}
2. At an instant t, the coordinates of a particle are $x = at^2$, $y = bt^2$ and $z = 0$, then its speed at the instant t will be
   (a) $t\sqrt{a^2+b^2}$ (b) $2t\sqrt{a^2+b^2}$
   (c) $\sqrt{a^2+b^2}$ (d) $2t^2\sqrt{a^2+b^2}$
3. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y varies with time t in second as:
   $x = 10\sqrt{3}t$ and $y = 10t - t^2$
   The maximum height attained by the ball is
   (a) 100 m (b) 75 m
   (c) 50 m (d) 25 m.
4. A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be
   (a) $2R$ (b) $R/2$
   (c) R (d) $4R$
5. If the initial velocity of a projectile be doubled, keeping the angle of projection same, the maximum height reached by it will
   (a) Remain the same
   (b) Be doubled
   (c) Be quadrupled
   (d) Be halved
6. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are
   (a) Perpendicular to each other
   (b) Parallel to each other
   (c) Inclined to each other at an angle of 45°
   (d) Antiparallel to each other
7. The range of a particle when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45° to the horizontal
   (a) 1.5 km (b) 3.0 km
   (c) 6.0 km (d) 0.75 km
8. A stone is projected from the ground with velocity 25 m/s. Two seconds later, it just clears a wall 5 m high. The angle of projection of the stone is ($g = 10\text{m/sec}^2$)
   (a) 30° (b) 45°
   (c) 37° (d) 60°
9. A projectile thrown with a speed v at an angle θ has a range R on the surface of earth. For same v and θ, its range on the surface of moon will be
   (a) $R/6$ (b) $6R$
   (c) $R/36$ (d) $36R$
10. A particle reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the displacement time graph is characterised by
    (a) Negative slope and zero curvature
    (b) Zero slope and negative curvature
    (c) Zero slope and positive curvature
    (d) Positive slope and zero curvature
11. If the range of a gun which fires a shell with muzzle speed V is R, then the angle of elevation of the gun is
    (a) $\cos^{-1}\left(\frac{V^2}{Rg}\right)$ (b) $\cos^{-1}\left(\frac{gR}{V^2}\right)$
    (c) $\frac{1}{2}\left(\frac{V^2}{Rg}\right)$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$
12. A ball is thrown upwards. It returns to ground describing a parabolic path. Which of the following remains constant?
    (a) Speed of the ball
    (b) Kinetic energy of the ball
    (c) Vertical component of velocity
    (d) Horizontal component of velocity.
13. A bullet is fired horizontally from a rifle at a distant target. Ignoring the effect of air resistance, which of the following is correct?

	Horizontal Acceleration	Vertical Acceleration
(a)	10 ms^{-2}	10 ms^{-2}
(b)	10 ms^{-2}	0 ms^{-2}
(c)	0 ms^{-2}	10 ms^{-2}
(d)	0 ms^{-2}	0 ms^{-2}

14. A particle is projected with a velocity of 50 m/s at 37° with horizontal. Find the co-ordinates of the particle (w.r.t. the starting point) after 2 s.
    Given, $g = 10 \text{ m/s}^2$, $\sin 37° = 0.6$ and $\cos 37° = 0.8$
    (a) (40, 80) (b) (80, 40)
    (c) (60, 80) (d) (80, 60)

15. Find the angle of projection of a projectile for which the horizontal range and maximum height are equal.
(a) 45°
(b) $\tan^{-1}(4)$
(c) $\tan^{-1}(2)$
(d) None of these

16. A stone projected at an angle of 60° from the ground level strikes at an angle of 30° on the roof of a building of height 'h'. Then the speed of projection of the stone is:



(a) $\sqrt{2gh}$ (b) $\sqrt{6gh}$
(c) $\sqrt{3gh}$ (d) $\sqrt{gh}$

17. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target in the same horizontal plane. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target.
(a) $\frac{5\pi}{36}$ rad (b) $\frac{11\pi}{36}$ rad
(c) $\frac{7\pi}{36}$ rad (d) $\frac{13\pi}{36}$ rad.

18. A projectile is thrown with a speed v at an angle θ with the vertical. Its average velocity between the instants it crosses half the maximum height is
(a) $v \sin\theta$, horizontal and in the plane of projection
(b) $v \cos\theta$, horizontal and in the plane of projection
(c) $2v \sin\theta$, horizontal and perpendicular to the plane of projection
(d) $2v \cos\theta$, vertical and in the plane of projection.

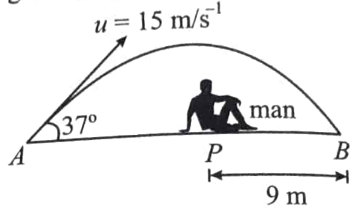
19. During projectile motion, acceleration of a particle at the highest point of its trajectory is
(a) g
(b) zero
(c) less than g
(d) dependent upon projection velocity

20. The maximum range of a projectile is 22 m. When it is thrown at an angle of 15° with the horizontal, its range will be-
(a) 22 m (b) 6 m
(c) 15 m (d) 11 m

21. The equation of projectile is $y = 16x - \frac{5x^2}{4}$. The horizontal range is:
(a) 16 m (b) 8 m
(c) 3.2 m (d) 12.8 m

22. If four balls A, B, C, D are projected with same speed at angles of 15°, 30°, 45° and 60° with the horizontal respectively, the two balls which will fall at the same place will be-
(a) (A) and (B) (b) (A) and (D)
(c) (B) and (D) (d) (A) and (C)

23. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.



(a) 3 ms^{-1} (b) 5 ms^{-1}
(c) 9 ms^{-1} (d) 12 ms^{-1}

24. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?
(a) The one with the farthest range.
(b) The one which reaches maximum height
(c) The one with the greatest initial velocity
(d) The one leaving the bat at 45° with respect to the ground.

25. A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of flight are R, H and T respectively. They are given by,
$$R = \frac{u^2 \sin 2\theta}{g}, H = \frac{u^2 \sin^2\theta}{2g} \text{ and } T = \frac{2u\sin\theta}{g}$$
Now keeping u as fixed, θ is varied from 30° to 60°. Then,
(a) R will first increase then decrease, H will increase and T will decrease
(b) R will first increase then decrease while H and T both will increase
(c) R will decrease while H and T will increase
(d) R will increase while H and T will increase

26. A point mass is projected, making an acute angle with the horizontal. If angle between velocity $\vec{v}$ and acceleration $\vec{g}$ is θ at any time t during the motion, then θ is given by
(a) $0° < \theta < 90°$
(b) $\theta = 90°$
(c) $\theta < 90°$
(d) $0° < \theta < 180°$

27. The angle of projection of a body is 15°. The other angle for which the range is the same as the first one is equal to:
(a) 30° (b) 45°
(c) 60° (d) 75°

PROJECTILE THROWN FROM SOME HEIGHT ABOVE GROUND

28. A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following

(a) Straight path (b) Circular path

(c) Parabolic path (d) Hyperbolic path

29. An aeroplane moving horizontally with a speed of 720 km/h drops a food packet, while flying at a height of 396.9 m. the time taken by a food packet to reach the ground and its horizontal range is

(Take g = 9.8 m/sec^2)

(a) 3 sec and 2000 m (b) 5 sec and 500 m

(c) 8 sec and 1500 m (d) 9 sec and 1800 m

30. A plane flying horizontally at a height of 1500 m with a velocity of 200 ms^{-1} passes directly overhead an antiaircraft gun. Then the angle with the horizontal at which the gun should be fired for the shell with a muzzle velocity of 400 ms^{-1} to hit the plane, is -

(a) 90° (b) 60°

(c) 30° (d) 45°

31. A bullet is fired horizontally from a rifle at a distant target. Ignoring the effect of air resistance, which of the following is correct? Horizontal Acceleration and Vertical Acceleration are given by:

(a) 10 ms^{-2}, 10 ms^{-2}

(b) 10 ms^{-2}, 0 ms^{-2}

(c) 0 ms^{-2}, 10 ms^{-2}

(d) 0 ms^{-2}, 0 ms^{-2}

32. One stone is projected horizontally from a 20 m high cliff with an initial speed of 10 ms^{-1}. A second stone is simultaneously dropped from that cliff. Which of the following is true?

(a) Both strike the ground with the same speed.

(b) The ball with initial speed 10 ms^{-1} reaches the ground first.

(c) Both the balls hit the ground at the different time.

(d) Both strike the ground with different speed

33. A ball is projected from top of a tower with a velocity of 5 m/s at an angle of 53° to horizontal. Its speed when it is at a height of 0.45 m from the point of projection is

(a) 2 m/s (b) 3 m/s

(c) 4 m/s (d) Data insufficient

34. An aeroplane flying at a constant velocity releases a bomb. As the bomb drops down from the aeroplane.

(a) It will always be vertically below the aeroplane

(b) It will always be vertically below the aeroplane only if the aeroplane is flying horizontally

(c) It will always be vertically below the aeroplane only if the aeroplane is flying at an angle of 45° to the horizontal.

(d) It will gradually fall behind the aeroplane if the aeroplane is flying horizontally

35. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45° with horizontal. What is the vertical component of velocity when the body strikes the ground?

(a) $18\sqrt{2}$ ms^{-1} (b) 18 ms^{-1}

(c) $9\sqrt{2}$ ms^{-1} (d) 9 ms^{-1}

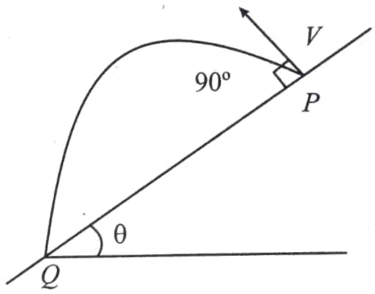
PROJECTILE MOTION ON AN INCLINED PLANE

36. Find time of flight of projectile thrown horizontally with speed 10 ms^{-1} from a long inclined plane which makes an angle of $\theta = 45°$ with the horizontal.

(a) $\sqrt{2}$ sec (b) $2\sqrt{2}$ sec

(c) 2 sec (d) None of these

37. If time taken by the projectile to reach Q is T, then $PQ =$



(a) $Tv \sin\theta$

(b) $Tv \cos\theta$

(c) $Tv \sec\theta$

(d) $Tv \tan\theta$

38. A projectile is thrown with velocity v making an angle θ with the horizontal. It just crosses the top of two poles, each of height h, after 1 second and 3 second respectively. The time of flight of the projectile is

(a) 1 s (b) 3 s

(c) 4 s (d) 7.8 s.

39. The velocity at the maximum height of a projectile is half of its initial velocity u. Its range on the horizontal plane is:

(a) $\frac{2u^2}{3g}$ (b) $\frac{\sqrt{3}u^2}{2g}$

(c) $\frac{u^2}{3g}$ (d) $\frac{u^2}{2g}$

40. A particle is projected at angle 37° with the incline plane in upward direction with speed 10 m/s. The angle of incline plane is given 53°. Then the maximum height above the incline plane attained by the particle will be

(a) 3m

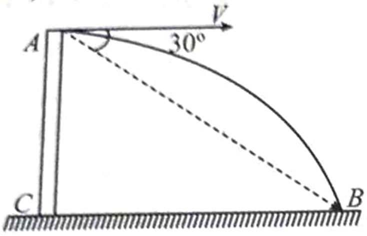
(b) 4 m

(c) 5 m

(d) Zero

Exercise-2 (Learning Plus)

1. The line from 'A' to 'B' makes an angle of 30° with the horizontal. The initial velocity of the object is : (take $g = 10$ m/s^2)



(a) $15\sqrt{3}$ m/s (b) 15 m/s
(c) $10\sqrt{3}$ m/s (d) $25/\sqrt{3}$ m/s

2. Two particles are fired from the same point, with speeds 100 m/s and 100 m/s, firing angles with horizontal = 60° and 120° respectively. The time after which their velocity vectors becomes perpendicular to each other, is

(a) 5 s (b) $5(\sqrt{3}-1)$s
(c) $5\sqrt{3}$s (d) $5\sqrt{3}/2$s

3. The equation of the path of the projectile is $y = 0.5x - 0.04x^2$. The initial speed of the projectile is

(a) 10 m/s (b) 15 m/s
(c) 12.5 m/s (d) None of these

4. A particle starts from the origin at $t = 0$ and moves in the xy plane with constant acceleration a in the y-direction. Its equation of motion is $y = bx^2$. The x-component of its velocity is

(a) variable (b) $\sqrt{\left(\frac{2a}{b}\right)}$
(c) $\frac{a}{2b}$ (d) $\sqrt{\left(\frac{a}{2b}\right)}$

5. The horizontal range of a projectiles is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of motion of the projectile, giving it a constant horizontal acceleration = $g/2$. Under the same conditions of projection, the horizontal range of the projectile will now be.

(a) $R+\frac{H}{2}$ (b) $R+H$
(c) $R+\frac{3H}{2}$ (d) $R+2H$

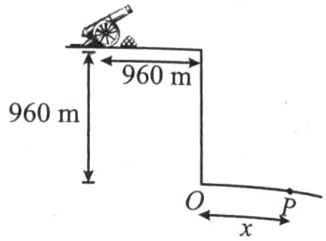
6. A projectile is fired horizontally from an inclined plane (of inclination 45° with horizontal) with speed = 50 m/s. If $g = 10$ m/s^2, the range measured along the incline is

(a) 500 m (b) $500\sqrt{2}$ m
(c) $200\sqrt{2}$ m (d) None of these

7. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?

(a) The one with the farthest range.
(b) The one which reaches maximum height.
(c) The one with the greatest initial velocity.
(d) The one leaving the bat at 45° with respect to the ground.

8. A gun if mounted on a plateau 960 m away from its edge as shown. Height of plateau is 960 m. The gun can fire shells with a velocity of 100 m/s at any angle of the following choices, what is the maximum distance $(OP)x$ from the edge of plateau where the shell of gun can reach?



(a) 480 m (b) 720 m
(c) 360 m (d) None of these

9. A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4 m from it. The height of the wall is

(a) 3/4 m (b) 2/3 m
(c) 4/3 m (d) 1/3 m

10. At $t = 0$ a particle leaves the origin with a velocity of 6 m/s in the positive y direction. Its acceleration is given by $\vec{a} = 2\hat{i} - 3\hat{j}$ m/s^2. The x and y coordinates of the particle at the instant the particle reaches maximum y coordinate are

(a) 2m, 3m
(b) 4m, 6m
(c) 1m, 3m
(d) 2m, 6m

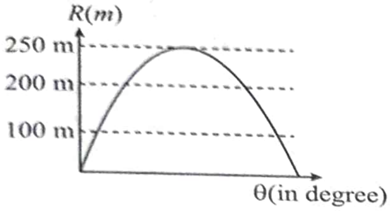
11. A particle is moving along the locus: $y = k\sqrt{x}$ $(k > 0)$ with a constant speed 'v'. At $t = 0$, it is at the origin and about to enter the first quadrant of x-y axes. At some later time $t > 0$, $v_x = v_y$. At this moment, $[a_y - a_x] =$

(a) v^2/k^2 (b) Zero
(c) $-v^2/k^2$ (d) None of these

12. Average velocity of a particle in projectile motion between its starting point and the highest point of its trajectory is: (projection speed = u, angle of projection from horizontal = θ)

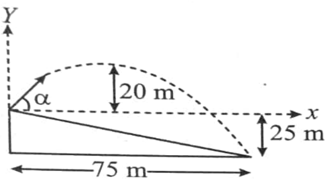
(a) $u\cos\theta$ (b) $\frac{u}{2}\sqrt{1+3\cos^2\theta}$
(c) $\frac{u}{2}\sqrt{2+\cos^2\theta}$ (d) $\frac{u}{2}\sqrt{1+\cos^2\theta}$

13. From the ground level a ball is to be shot with a certain speed. Graph shows the range R it will have versus the launch angle θ. The least speed the ball will have during its flight if θ is chosen such that the flight time is half of its maximum possible value, is equal to
(Take $g = 10$ m/s^2)



(a) 250 m/s (b) $50\sqrt{3}$ m/s
(c) 50 m/s (d) $25\sqrt{3}$ m/s

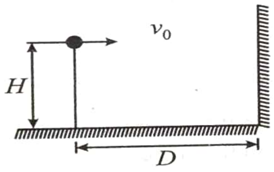
14. A ball thrown down the incline strikes at a point on the incline 25 m below the horizontal as shown in the figure. If the ball rises to a maximum height of 20 m above the point of projection, the angle of projection α (with horizontal x-axis) is



(a) $\tan^{-1}\frac{4}{3}$ (b) $\tan^{-1}\frac{3}{4}$
(c) $\tan^{-1}\frac{3}{2}$ (d) $\tan^{-1}\frac{2}{3}$

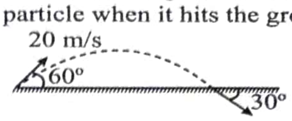
15. Trajectory of particle in a projectile motion is given as $y = x - x^2/80$. Here, x and y are in metres and considered along horizontal and vertical direction respectively ($g = 10$ m/s^2). For this projectile motion.
(a) angle of projection is 45°
(b) angle of velocity with horizontal after 4s is $\tan^{-1}(1/2)$
(c) maximum height is 80 m
(d) horizontal range is 20 m

16. A ball is projected horizontally from a table such that it collides with wall then with ground. If after collision component of velocity perpendicular to the surface is reversed in direction without change in magnitude and component of velocity parallel to the surface remains unchanged. At what distance from the wall does the ball collide with the ground. ($H = 500$ m, $v_0 = 20$ m/s, $D = 100$ m)



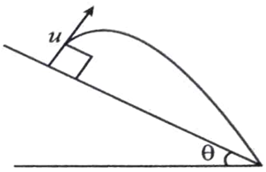
(a) 100 m (b) 200 m
(c) 50 m (d) 150 m

17. A particle is projected from a horizontal surface with a velocity of 20 m/s at an angle of 60° with the horizontal. When it hits the horizontal surface, its velocity makes an angle of 30° with the horizontal. Apart from gravity, a horizontal force acts on the particle during the motion. The speed of the particle when it hits the ground is



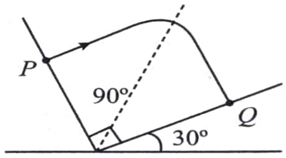
(a) 20 m/s (b) $20\sqrt{3}$ m/s (c) 10 m/s (d) $10\sqrt{3}$ m/s

18. A particle is projected perpendicularly to an inclined plane as shown in the figure. If the initial velocity of the particle is u, calculate how far from the point of projection does it hit the plane again if the distance is measured along the plane?



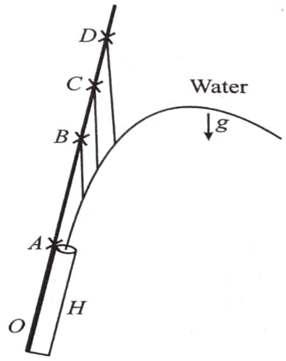
(a) $\frac{2u^2}{g}$ (b) zero
(c) $\frac{2u^2}{g}\sin\theta$ (d) $\frac{2u^2}{g}\tan\theta\sec\theta$

19. A particle is projected from point P with velocity $5\sqrt{3}$ ms^{-1} perpendicular to the surface of a hollow right angled cone as shown in figure. It collides at Q normally. The velocity with which it collides at Q will be



(a) 25 ms^{-1} (b) 15 ms^{-1} (c) $5\sqrt{3}$ ms^{-1} (d) $10\sqrt{3}$ ms^{-1}

20. A straight rod $OABCD$ is strapped on to the end of a straight hose pipe H with 'A' at the nozzle of the pipe (figure). Strings are ties to the rod at B, C and D and are allowed to drop down vertically. The lengths of the strings at B, C and D to just touch the stream of water, streaming out of the hose pipe from its nozzle, are in the ratio



(a) $\frac{1}{AB}:\frac{1}{BC}:\frac{1}{CD}$ (b) $AB^2 : AC^2 : AD^2$
(c) $\frac{1}{AB^2}:\frac{1}{AC^2}:\frac{1}{AD^2}$ (d) $\sqrt{AB}:\sqrt{AC}:\sqrt{AD}$

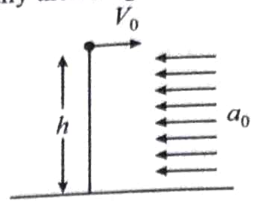
21. A particle is released from a certain height $H = 400$ m. Due to the wind the particle gathers the horizontal velocity component $v_x = a_y$ where $a = \sqrt{5}\,s^{-1}$ and y is the vertical displacement of the particle from point of release, then the horizontal drift of the particle when it strikes the ground is

(*a*) 2.67 km (*b*) 8.67 km
(*c*) 1.67 km (*d*) 5.1 km

22. A particle is projected with speed u from O along y-axis. If acceleration of the particle is $(a\hat{i} + a\hat{j})$, then equation of trajectory of the particle is :

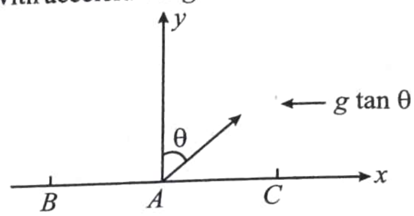
(*a*) $y = \dfrac{1}{2}\dfrac{ax^2}{u^2}$ (*b*) $(y-x)^2 = \dfrac{2xu^2}{a}$
(*c*) $(x-y)^2 = \dfrac{2yu^2}{a}$ (*d*) $x^2 + y^2 = \dfrac{2xu^2}{a}$

23. A particle is projected horizontally from a tower with velocity V_0. Wind is blowing in opposite direction and is providing a constant horizontal acceleration a_0. If particle strikes the ground vertically then height of building is



(*a*) $\dfrac{V_0^2 g}{a_0^2}$ (*b*) $\dfrac{V_0^2 g}{2a_0^2}$
(*c*) $\dfrac{2V_0^2 g}{a_0^2}$ (*d*) $\dfrac{V_0^2}{g}$

24. A ball is projected towards right from point A at an angle θ with vertical. The wind blows towards left along negative x-axis with acceleration $g \tan\theta$. The ball will return to point:



(*a*) A (*b*) B left to point A
(*c*) C right to point A (*d*) None of the above

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. The acceleration of a moving body at any 't' is given by. $\vec{a} = (4t)\hat{i} + (3t^2)\hat{j}$ m/sec². If $\vec{u} = 0$ then find the velocity (in m/s) of the particle at 4 sec.

(*a*) $16\hat{i} - 32\hat{j}$ (*b*) $32\hat{i} + 64\hat{j}$
(*c*) $4\hat{i} + 4\hat{j}$ (*d*) $16\hat{i} + 32\hat{j}$

2. A particle of mass 1 kg has a velocity of 2 m/sec. A constant force of 2N acts on the particle for 1 sec in a direction perpendicular to its initial velocity. Find the angle between velocity vector and displacement vector of the particle at the end of 1 sec:

(*a*) 45° (*b*) 90°
(*c*) 30° (*d*) None of these

3. A particle is moving in xy-plane. At certain instant, the components of its velocity and acceleration are as follows $v_x = 3$ m/s, $v_y = 4$ m/s, $a_x = 2$ m/s² and $a_y = 1$ m/s². The rate of change of speed at this moment is

(*a*) 4m/s² (*b*) 2 m/s²
(*c*) $\sqrt{3}$ m/s² (*d*) $\sqrt{5}$ m/s²

4. The figure shows the velocity and the acceleration of a point-like body at the initial moment of its motion. The direction and the absolute value of the acceleration remain constant. Find the time in seconds when the velocity reach its minimum value ?

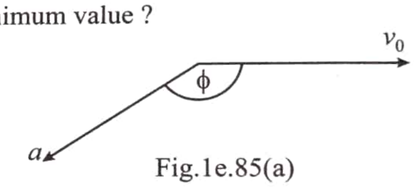
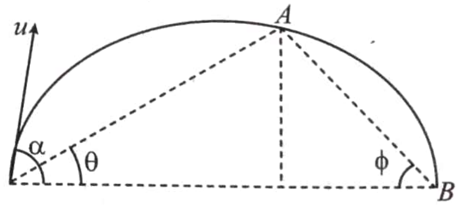


Fig.1e.85(a)

(Data: $a = 6$ m/s², $v_0 = 24$ m/s, $\phi = 143°$)

(*a*) 1.6 sec (*b*) 3.2 sec
(*c*) 4.8 ses (*d*) 6.4 sec

5. A football is kicked as shown in Figure. The angles θ and ϕ locate the point of maximum height, then find the relation between α, θ and ϕ.



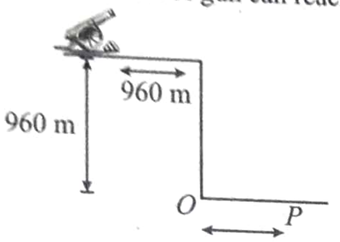
(*a*) $\cot\alpha = \cot\theta + \cot\phi$
(*b*) $\tan\alpha = \tan\theta - \tan\phi$
(*c*) $\cot\alpha = \cot\theta - \cot\phi$
(*d*) $\tan\alpha = \tan\theta + \tan\phi$

6. A ball is projected with velocity v_0 and at an angle of projection α. After what time is the ball moving at right angles to the initial direction?

(a) $\frac{v_0}{g}\operatorname{cosec}\alpha$ (b) $\frac{2v_0}{g}\operatorname{cosec}\alpha$

(c) $\frac{v_0}{g}\sec\alpha$ (d) $\frac{2v_0}{g}\sec\alpha$

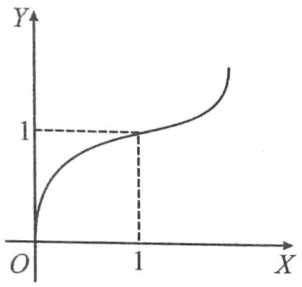
7. A gun is mounted on a plateau 960 m away from its edge as shown. Height of plateau is 960 m. The gun can fire shells with a velocity of 100 m/s at any angle. Of the following choices, what is the minimum distance (OP) x from the edge of plateau where the shell of gun can reach?



(a) 480 m (b) 720 m

(c) 360 m (d) None

8. The trajectory of a particle is as shown here and its trajectory follows the equation $y = (x - 1)^3 + 1$. Find co-ordinates of the point A on the curve such that direction of instantaneous velocity at A is same as direction of average velocity for the motion O to A:



(a) (3/2, 9/8) (b) (2,2)

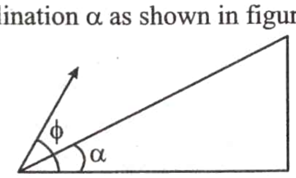
(c) (3, 9) (d) (5/2,35/8)

9. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{g(1+\sin\theta)}$

(c) $\frac{v^2}{g(1-\sin\theta)}$ (d) $\frac{v^2}{g(1+\cos\theta)}$

10. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane of inclination α as shown in figure.



(a) $\sin\alpha = \cos(\theta - \alpha)$ (b) $\cos\alpha = \sin(\theta - \alpha)$

(c) $\tan\theta = \cot(\theta - \alpha)$ (d) $\cot(\theta - \alpha) = 2\tan\alpha$

11. A ball is horizontally projected with a speed v from the top of a plane inclined at an angle 45° with the horizontal. How far from the point of projection will the ball strike the plane?

(a) $\frac{v^2}{g}$ (b) $\frac{\sqrt{2}v^2}{g}$ (c) $\frac{2v^2}{g}$ (d) $\frac{2\sqrt{2}v^2}{g}$

13. A ball is thrown upward at an angle of 30° with the horizontal and lands on the top edge of a building that is 20 m away. The top edge is 5 m above the throwing point. The initial speed of the ball in metre/second is (take $g = 10$ m/s²):

(a) $u = 40\sqrt{\frac{(4+\sqrt{3})}{13\sqrt{3}}}$ m/s

(b) $u = 40\sqrt{\frac{4-\sqrt{3}}{13\sqrt{3}}}$ m/s

(c) $u = 40\sqrt{\frac{4+\sqrt{3}}{13}}$ m/s

(d) $u = 40\frac{40}{\sqrt{\sqrt{3}(4+\sqrt{3})}}$ m/s

14. A small ball rolls of the top of a stairway horizontally with a velocity of 4.5 ms^{-1}. Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-2}, then the ball will strike the nth step where n is equal to (assume ball strike at the edge of the step).

(a) 9 (b) 10 (c) 11 (d) 12

15. Two particles are projected from the same point with the same speed in the same vertical plane at different angles with the horizontal. A frame of reference is fixed to one particle. The position vector of the other particle as observed from the frame is $\vec{r}$. Which of the following statements is/are incorrect?

(a) $\vec{r}$ is a constant vector
(b) $\vec{r}$ changes in magnitude and direction with time
(c) the magnitude of $\vec{r}$ increases linearly with time, its direction does not change
(d) the direction of $\vec{r}$ changes with time, its magnitude may or may not change depending on the angles of projection.

16. Consider a shell that has a muzzle velocity of 45 ms^{-1} fired from the tail gun of an airplane moving horizontally with a velocity of 215 ms^{-1}. The tail gun can be directed at any angle with the vertical in the plane of motion of the airplane. The shell is fired when the plane is above point A on ground, and the plane is above point B on ground when the shell hits the ground. (Assume for simplicity that the Earth is flat)

(a) Shell may hit the ground at point A.
(b) Shell may hit the ground at point B.
(c) Shell may hit a point on earth which is behind point A.
(d) Shell may hit a point on earth which is ahead of point B.

17. A projectile is projected at an angle α (> 45°) with an initial velocity u. The time t at which its component of horizontal velocity will equal the component of vertical velocity in magnitude:

(a) $t = \frac{u}{g}(\cos\alpha - \sin\alpha)$

(b) $t = \frac{u}{g}(\cos\alpha + \sin\alpha)$

(c) $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$

(d) $t = \frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

18. At what angle should a body be projected with a velocity 24 ms^{-1} just to pass over the obstacle 14 m high at a distance of 24 m. [Take g = 10 ms^{-2}]

(a) $\tan\theta = 19/5$ (b) $\tan\theta = 1$

(c) $\tan\theta = 3$ (d) $\tan\theta = 2$

19. Choose the correct alternative (s)

(a) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is $2h$.

(b) The angle of projection for a projectile motion whose range R is n times the maximum height is $\tan^{-1}(4/n)$

(c) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R\tan\theta$ where θ is the angle of projection.

(d) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two ball 1 : 1.

20. A ball is rolled off along the edge of a horizontal table with velocity 4 m/s. It hits the ground after time 0.4s. Which of the following are correct?

(a) The height of the table is 0.8 m

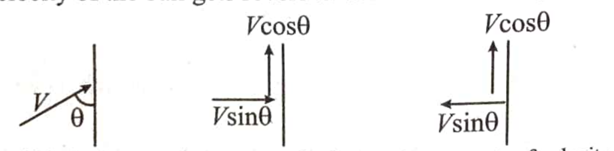
(b) It hits the ground at an angle of 60° with the vertical

(c) It covers a horizontal distance 1.6 m from the table

(d) It hits the ground with vertical velocity 4 m/s

COMPREHENSION BASED QUESTIONS

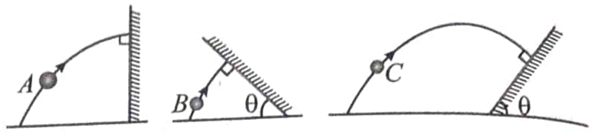
Comprehension (Q. 21 to 23): We know how by neglecting the air resistance, the problems of projectile motion can be easily solved and analysed. Now we consider the case of the collision of a ball with a wall. In this case the problem of collision can be simplified by considering the case of elastic collision only. When a ball collides with a wall we can divide its velocity into two components, one perpendicular to the wall and other parallel to the wall. If the collision is elastic then the perpendicular component of velocity of the ball gets reversed with the same magnitude.



Velocity just before collision | Components of velocity just before collision | Components of velocity just after collision

The other parallel component of velocity will remain constant if given wall is smooth.

Now let us take a problem. Three balls 'A' and 'B' & 'C' are projected from ground with same speed at same angle with the horizontal. The balls A, B and C collide with the wall during their flight in air and all three collide perpendicularly with the wall as shown in figure.



21. Which of the following relation about the maximum height H of the three balls from the ground during their motion in air is correct :

(a) $H_A = H_C > H_B$ (b) $H_A > H_B = H_C$

(c) $H_A > H_C > H_B$ (d) $H_A = H_B = H_C$

22. If the time taken by the ball A to fall back on ground is 4 seconds and that by ball B is 2 seconds. Then the time taken by the ball C to reach the inclined plane after projection will be :

(a) 6 sec. (b) 4 sec.

(c) 3 sec. (d) 5 sec.

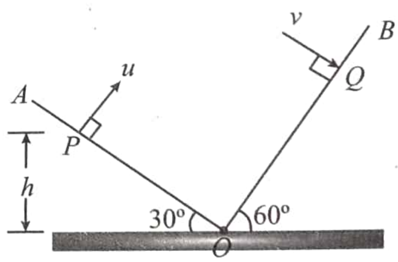
23. The maximum height attained by ball 'A' from the ground is:

(a) 10 m (b) 15 m

(c) 20 m (d) Insufficient information

Comprehension (Q. 24 to 27): Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}\ ms^{-1}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q

(Take g = 10 m/s²). Then



24. The time of flight is from P to Q is:

(a) 5 Sec. (b) 2 sec.

(c) 1 sec. (d) None of these

25. The speed with which the particle strikes the plane OB is:

(a) 10 m/s (b) 20 m/s

(c) 30 m/s (d) 40 m/s

26. The height h of point P from the ground is:

(a) $10\sqrt{3}$ m (b) 10 m

(c) 5 m (d) 20 m

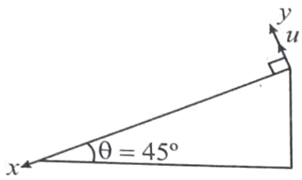
27. The distance PQ is:

(*a*) 20 m (*b*) $10\sqrt{3}$ m

(*c*) 10 m (*d*) 5 m

MATCH THE COLUMN TYPE QUESTIONS

28. An inclined plane makes an angle $\theta = 45°$ with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at $t = 0$ sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in Column-I with the results in Column-II. (g in Column-II is acceleration due to gravity.)



	Column-I		Column-II
A.	The instant of time at which velocity of stone is parallel to x-axis	p.	$\frac{2\sqrt{2}u}{g}$
B.	The instant of time at which velocity of stone makes an angle $\theta = 45°$ with positive x-axis in clockwise direction.	q.	$\frac{2u}{g}$
C.	The instant of time till which (starting from $t = 0$) component of displacement along x-axis become half the range on inclined plane is	r.	$\frac{\sqrt{2}u}{g}$
D.	Time of flight on inclined plane is	s.	$\frac{u}{\sqrt{2}g}$

(*a*) A-(r); B-(q); C-(s); D-(p)

(*b*) A-(r); B-(s); C-(q); D-(p)

(*c*) A-(r); B-(s); C-(p); D-(q)

(*d*) A-(p); B-(q); C-(r); D-(s)

29. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in Column-I with the values given in Column-II. (g in Column II is acceleration due to gravity.)

	Column-I		Column-II
A.	Horizontal component of velocity	p.	$\frac{a}{b}$
B.	Time of flight	q.	$\frac{a^2}{4b}$
C.	Maximum height	r.	$\sqrt{\frac{g}{2b}}$
D.	Horizontal range	s.	$\sqrt{\frac{2a^2}{bg}}$

(*a*) A-(q); B-(s); C-(r); D-(p)

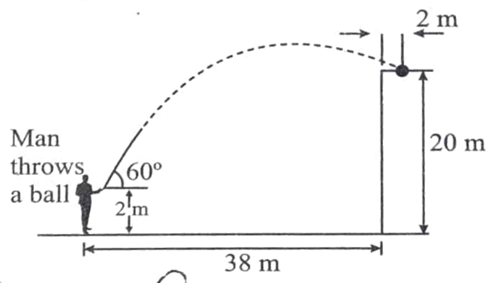
(*b*) A-(s); B-(r); C-(p); D-(q)

(*c*) A-(r); B-(s); C-(q); D-(p)

(*d*) A-(q); B-(s); C-(r); D-(p)

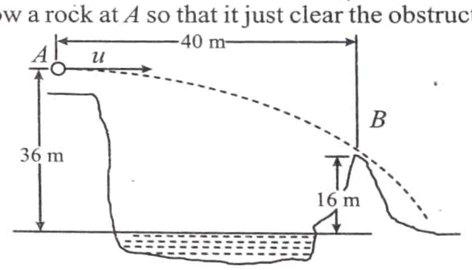
NUMERICAL TYPE QUESTIONS

30. A soft ball is thrown at an angle of $\alpha = 60°$ above the horizontal. It lands a distance $d = 2$m from the edge of a flat roof, whose height is $h = 20$ m ; the edge of the roof is $l = 38$ m from the thrower (see fig.). At what speed (in m/s) was the softball thrown?

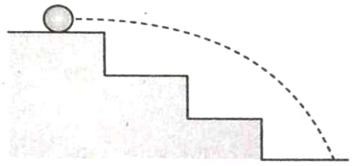


31. When a particle is projected at an angle of 15° from horizontal, it's range is found to be 3.5 m. What would be it's range (in m) if it is projected at an angle of 45° from horizontal with same speed?

32. With what minimum horizontal velocity 'u', (in m/s) can a boy throw a rock at A so that it just clear the obstruction at B



33. A staircase contains three steps each 10 cm high and 20 cm wide. What should be the minimum horizontal velocity of the ball rolling off the uppermost plane so as to hit directly the lowest plane? (in ms^{-1})



34. A particle is projected up an inclined plane of inclination β at an elevation α to the horizontal. Find the ratio between $\tan\alpha$ and $\tan\beta$, if the particle strikes the plane horizontally.

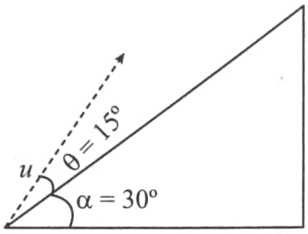
Exercise-4 (Past Year Questions)

JEE MAIN

1. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **(2019)**
   (a) $y = x^2$ + constant
   (b) $y^2 = x$ + constant
   (c) $y^2 = x^2$ + constant
   (d) xy = constant

2. Two guns A and B can fire bullets at speed 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the buyllets fired by the two guns, on the ground is: **(2019)**
   (a) 1 : 16 (b) 1 : 2
   (c) 1 : 4 (d) 1 : 8

3. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2$ ms^{-1} from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure.



   The distance from the base, at which the particle hits the plane is close to: **(2019)**
   (Take $g = 10$ ms^{-2})
   (a) 14 cm (b) 20 cm
   (c) 18 cm (d) 26cm

4. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ? **(2019)**
   (a) $R^2 = 2h_1 h_2$ (b) $R^2 = 16h_1 h_2$
   (c) $R^2 = 4h_1 h_2$ (d) $R^2 = h_1 h_2$

5. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then (g =10 ms^{-2}) **(2019)**
   (a) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ms^{-1}
   (b) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ms^{-1}
   (c) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ms^{-1}
   (d) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ms^{-1}

6. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is: **(2019)**
   (a) R/g (b) $R/4g$ (c) $2R/g$ (d) $R/2g$

7. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] **(2020)**
   (a) $t = 1.8\sqrt{\frac{h}{g}}$ (b) $t = \sqrt{\frac{2h}{3g}}$
   (c) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (d) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$

8. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by: **(2020)**
   (a) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$ (b) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$
   (c) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$ (d) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

9. A projectile is projected with velocity of 25 m/s at an angle θ with the horizonal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be [use $g = 10$ m/s^2] **(2022)**
   (a) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{4R^2}{5t^2}\right)$
   (c) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$ (d) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

10. A ball is projected from the ground with a speed 15 ms^{-1} at an angle 0 with horizontal so that its range and maximum height are equal, then 'tan θ' will be equal to **(2022)**
    (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

11. Two projectile thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is **(2022)**
    (a) $1:\sqrt{2}$ (b) 2 : 1
    (c) $\sqrt{2}:1$ (d) 1 : 2

12. A body of mass 10 kg is projected at an angle of 45° with the horizontal. The trajectory of the body is observed to pass through a point (20, 10). If T is the time of flight, then its momentum vector, at time $t = \frac{T}{\sqrt{2}}$ is [Take $g = 10\ m/s^2$] **(2022)**

(a) $100\hat{i} + (100\sqrt{2} - 200)\hat{j}$

(b) $100\sqrt{2}\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(c) $100\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(d) $100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$

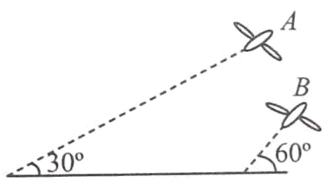
13. A ball is projected with kinetic energy E, at an angle of 60° to the horizontal. The kinetic energy of this ball at the highest point of its flight will become: **(2022)**

(a) Zero (b) $\frac{E}{2}$

(c) $\frac{E}{4}$ (d) E

JEE ADVANCED

14. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure . The speed of A is $100\sqrt{3}\ ms^{-1}$. At time $t = 0s$, an observer in A finds B at a distance of 500m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is: **(2014)**



15. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is ______ **(2018)**

16. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1 After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is______ **(2019)**

17. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1-x)$. The y component vector of the initial velocity is ________ $\hat{j}$ (Take $g = 10\ m/s^2$) **(2022)**

18. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is __________. **(2022)**

19. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be ________degree. (Mark the smallest angle possible) **(2022)]**

ANSWER KEY

CONCEPT APPLICATION

1. (c)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (a)
11. (c)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (i) 10 sec (ii) 980 m (iii) $98\sqrt{2}$ m/s			**17.** [1.69 s]	**18.** [6.31 s]
19. [53.33 m]									

EXERCISE-1 (TOPICWISE)

1. (c)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (b)
11. (d)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (d)
21. (d)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (d)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (b)
31. (c)	**32.** (d)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (c)	**37.** (d)	**38.** (c)	**39.** (b)	**40.** (a)

EXERCISE-2 (LEARNING PLUS)

1. (a)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (b)
11. (c)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (b)
21. (a)	**22.** (b)	**23.** (b)	**24.** (a)						

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (b)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (d)
11. (d)	**12.** (d)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (a,b,c,d)	**20.** (a,c)
21. (a)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (a)	**26.** (c)	27. (a)	**28.** (b)	**29.** (c)	**30.** [25]
31. [7 m]	**32.** [20 m/s]	**33.** [2 m/s]	**34.** [2]						

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (c)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (d)
11. (c)	**12.** (d)	**13.** (c)							

JEE Advanced

14. [5]	**15.** [30]	**16.** [4]	**17.** [5]	**18.** [1]	**19.** [15]

CHAPTER

5 Relative Motion

INTRODUCTION

A person travelling by train may ask his co-passenger "Has delhi come?". This statement seems to claim that Delhi is moving towards our passenger. We may say that his understanding of word is wrong as all of us know that 'actually' it is the train which is moving not the huge city Delhi.

If we observe from passenger's point of view we will realise relative to him all parts of train are at rest, which means relative to him train is at rest. Still when passenger gets down from train he is in a city, thus it must be the cities which are moving.

The point is that when a person standing on ground says that 'actually' it is the train (hence passengers sitting in train) which is moving and reaching to Delhi (which of course is stationary), he is also right. The difference here is of reference frame.

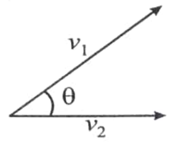
Reference Frame

Reference frame is an axis system from which motion is observed. A clock is attached to measure time. Reference frame can be stationary or moving. There are two types of reference frame.

If the velocities of two bodies are known with respect to a common frame of reference, velocity of one body relative to other can be determined. Thus, if velocities of two bodies P and Q with respect to earth are $\vec{v}_P$ and $\vec{v}_Q$ respectively, then the velocity of Q relative to P is expressed as follows: $\vec{v}_{QP} = \vec{v}_Q - \vec{v}_P$

- If two bodies P and Q are moving along the same line in the same direction with velocities v_P and v_Q relative to ground, the velocity of Q relative to P is: $\vec{v}_{QP} = \vec{v}_Q - \vec{v}_P$.
  (a) If it is positive, the direction of v_{QP} is same as that of Q and
  (b) If it is negative, the direction of v_{QP} is opposite to that of Q.
- It is worth noting here that in dealing with the motion of two bodies relative to each other, $\vec{v}_{rel.}$ is the difference of velocities of two bodies if they are moving in the same direction and is the sum of two velocities if they are moving in opposite directions.

If two bodies have velocities $\vec{v}_1$ and $\vec{v}_2$ then the relative velocity of first body with respect to second body is

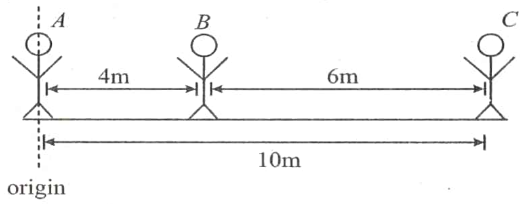


$$\vec{v}_{rel.} = \vec{v}_1 - \vec{v}_2 = \vec{v}_1 + (-\vec{v}_2)$$

$$v_{\text{relative}} = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$

Train Your Brain

Example 1: The position of three men A, B and C is shown in figure. Then find position of one man with respect to other (take +ve direction towards right and –ve towards left)



Sol. Here,

Position of B w.r.t. A is 4 m towards right. ($x_{BA} = +4$ m)

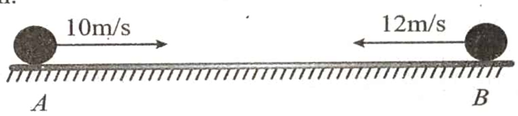
Position of C w.r.t. A is 10 m towards right. ($x_{CA} = +10$m)

Position of C w.r.t. B is 6 m towards right ($x_{CB} = +6$m)

Position of A w.r.t. B is 4 m towards left. ($x_{AB} = -4$m)

Position of A w.r.t. C is 10 m towards left. ($x_{AC} = -10$m)

Example 2: Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively as shown.



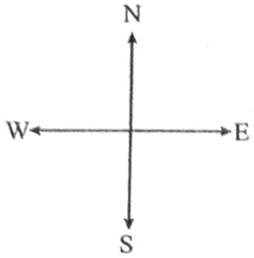
(i) Find the velocity of A with respect to B.
(ii) Find the velocity of B with respect to A

Sol. $v_A = +10$ m/s, $v_B = -12$ m/s

(i) $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s.
(ii) $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s.

Example 3: Car A has an acceleration of 2 m/s² due east and car B, 4 m/s² due north. What is the acceleration of car B with respect to car A?

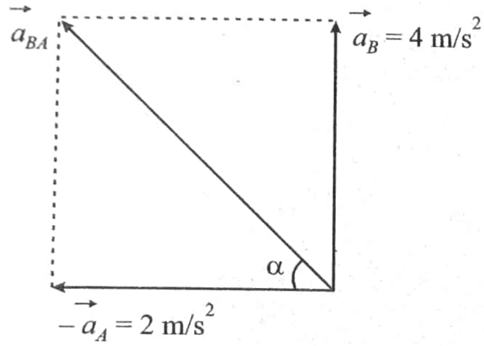
Sol.



It is a two dimensional motion. Therefore,

$\vec{a}_{BA}$ = acceleration of car B with respect to car A

$= \vec{a}_B - \vec{a}_A$



Here, $\vec{a}_B$ = acceleration of car $B = 4$ m/s² (due north)

and $\vec{a}_A$ = acceleration of car $A = 2$ m/s² (due east)

$|\vec{a}_{BA}| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5}$ m/s²

and $\alpha = \tan^{-1}\left(\dfrac{4}{2}\right) = \tan^{-1}(2)$

Thus, $\vec{a}_{BA}$ is $2\sqrt{5}$ m/s² at an angle of $\alpha = \tan^{-1}(2)$ from west towards north.

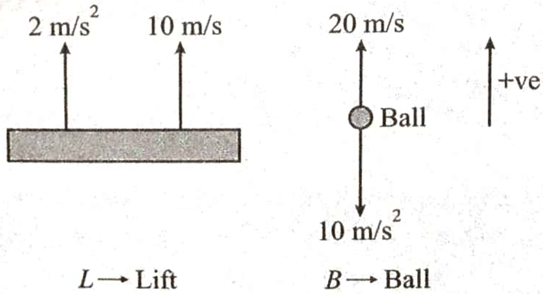
Example 4: An open lift is moving upwards with velocity 10m/s. It has an upward acceleration of 2 m/s². A ball is projected upwards with velocity 20 m/s relative to ground. Find :

(*a*) Time when ball again meets the lift.

(*b*) Displacement of lift and ball at that instant.

(*c*) Distance travelled by the ball upto that instant. Take $g = 10$ m/s²

Sol. (*a*) At the time when ball again meets the lift,



$$\therefore\ 10t + \frac{1}{2} \times 2 \times t^2 = 20\,t - \frac{1}{2} \times 10t^2$$

Solving this equation, we get

$$t = 0 \text{ and } t = \frac{5}{3}\text{ s}$$

$\therefore$ Ball will again meet the lift after $\dfrac{5}{3}$ s.

(*b*) At this instant

$$S_L = S_B = 10 \times \frac{5}{3} + \frac{1}{2} \times 2 \times \left(\frac{5}{3}\right)^2 = \frac{175}{9} = 19.4\text{m}$$

(*c*) For the ball $u\uparrow \downarrow a$ • Therefore, we will first find t_0, the time when its velocity becomes zero.

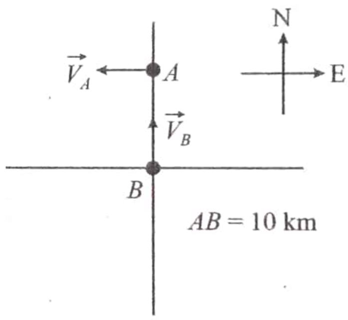
As $t\left(=\dfrac{5}{3}s\right) < t_0$ distance and displacement are equal

or $d = 19.4$ m

Concept of relative motion is more useful in two body problem in two (or three) dimensional motion. This can be understood by the following example.

Example 5: Two ships A and B are 10 km apart on a line running south to north. Ship A farther north is streaming west at 20 km/h and ship B is streaming north at 20km/h. What is their distance of closest approach and how long do they take to reach it?

Sol. Ships A and B are moving with same speed 20 km/h in the direction shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find $\vec{V}_{BA}$

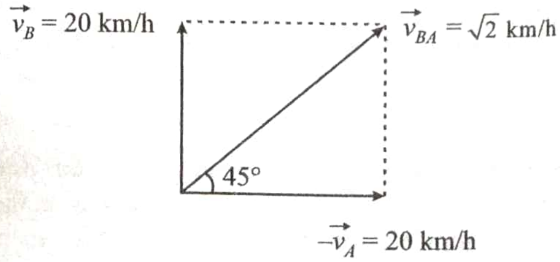


$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

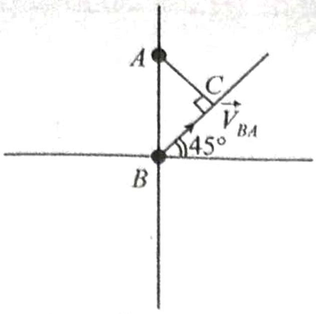
Here, $\left|\vec{V}_{BA}\right| = \sqrt{(20)^2 + (20)^2}$

$= 20\sqrt{2}$ km/h

i.e., $\vec{V}_{BA}$ is $20\sqrt{2}$ km/h at an angle of 45° from east towards north. Thus, the given problem can be simplified as:



A is at rest and B is moving with $\vec{V}_{BA}$ in the direction shown in figure.



Therefore, the minimum distance between the two is

$$S_{\min} = AC = AB \sin 45°$$

$$= 10\left(\frac{1}{\sqrt{2}}\right)\text{km} = 5\sqrt{2}\text{ km}$$

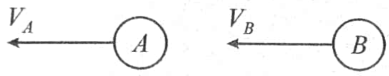
and the desired time is

$$t = \frac{BC}{|\vec{V}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} \qquad (BC = AC = 5\sqrt{2}\text{ km})$$

$$t = \frac{1}{4}h = 15\text{ min}$$

Concept Application

1. Two particles are moving along a straight line as shown. The velocity of approach between A and B is

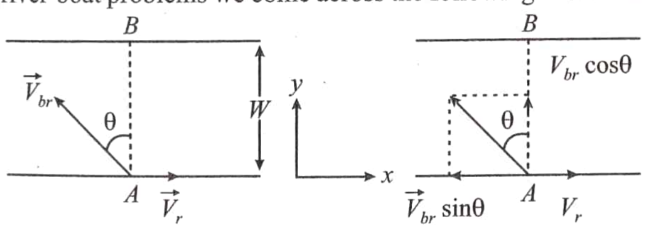


(a) $V_A + V_B$ (b) $|V_A - V_B|$
(c) $V_A - V_B$ (d) $V_B - V_A$

2. A particle is kept at rest at origin. Another particle starts from (5, 0) with a velocity of $-4\hat{i} + 3\hat{j}$. Find their closest distance of approach.

BOAT-RIVER PROBLEM

In river-boat problems we come across the following three terms :



$\vec{v}_r$ = absolute velocity of river

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water and $\vec{V}_b$ absolute velocity of boatman.

Here, it is important to note that $\vec{V}_{br}$ is the velocity of boatman with which he steers and $\vec{V}_b$ the actual velocity of boatman relative to ground.

Further, $\vec{v}_{br} = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{be}$ in the direction shown in figure. River is flowing along positive x-direction with velocity $\vec{v}_r$. Width of the river is w, then

$$\vec{v}_b = \vec{v}_{br} + \vec{v}_r$$

Therefore, $v_{brx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$

and $v_{bry} = v_{ry} + v_{bry}$

$$= 0 + v_{br}\cos\theta = v_{br}\cos\theta$$

Now, time taken by the boatman to cross the river is

$$t = \frac{W}{v_{by}} = \frac{W}{v_{br}\cos\theta}$$

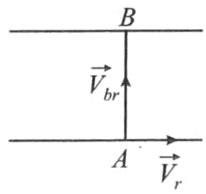
$$\text{or } t = \frac{W}{v_{br}\cos\theta}$$

Further, displacement along X-axis when he reaches on the other bank (also called drift) is:

$$\text{or } x = (v_r - v_{br}\sin\theta)\frac{W}{v_{br}\cos\theta}$$

Three special are:

(i) Condition when the boatman crosses the river in shortest interval of time



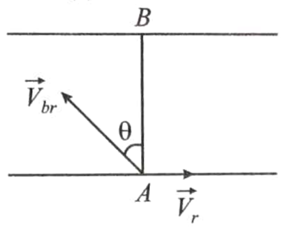
From Eq.(i) we can see that time (t) will be minimum when $\theta = 0°$, i.e., the boatman should steer his boat perpendicular to the river current.

Also, $t_{\min} = \frac{W}{v_{br}}$ as $\cos\theta = 1$

and $x = v_r \frac{W}{v_{br}}$

(ii) Condition when the boatman wants to reach point B, i.e., at a point just opposite from where he started

In this case, the drift (x) should be zero.



$\therefore x = 0$

$$\text{or } (v_r - v_{br}\sin\theta)\frac{W}{v_{br}\cos\theta} = 0$$

$$\text{or } v_r = v_{br}\sin\theta$$

or $\sin\theta = \frac{v_r}{v_{br}}$ or $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

Hence, to reach point B the boatman should row at an angle $\theta = \sin^{-1}\left(\frac{V}{V_{br}}\right)$ upstream from AB.

Time 't' taken to cross the river in this case is

$$t = \frac{W}{v_{br}\cos\theta} = \frac{W}{v_{br}\sqrt{1-\sin^2\theta}} = \frac{W}{v_{br}\sqrt{1-\frac{v_r^2}{v_{br}^2}}} = \frac{W}{\sqrt{v_{br}^2 - v_r^2}}$$

Further, since $\sin\theta$ can not be greater than 1.

So, if $v_r \geq v_{br}$, the boatman can never reach at point B. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

(iii) **Shortest path:** Path length travelled by the boatman when he reaches the opposite shore is

$$s = \sqrt{W^2 + x^2}$$

Here, W = width of river is constant. So for s to be minimum modulus of x (drift) should be minimum. Now two cases are possible.

Case-I: When $v_r < v_{br}$: In this case $x = 0$,

when $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ or $S_{\min} = W$ at $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

Case-II: When $v_r > v_{br}$: In this case x is minimum, where

$$\frac{dx}{d\theta} = 0$$

or $\frac{d}{d\theta}\left\{\frac{W}{v_{br}\cos\theta}(v_r - v_{br}\sin\theta)\right\} = 0$

or $-v_{br}\cos^2\theta - (v_r - v_{br}\sin\theta)(-\sin\theta) = 0$

or $-v_{br} + v_r\sin\theta = 0$

Or

$$\theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$$

Now, at this angle we can find $x_{\min}$ and then $s_{\min}$ which comes out to be

$$s_{\min} = W\left(\frac{v_r}{v_{br}}\right) \text{ at } \theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$$

If a man swims downstream in a river,

then the time taken by the man to cover a distance d is

$$t_1 = \frac{d}{v_b + v_w}$$

If a man swims upstream in a river, then the time taken by him to cover a distance d is

$$t_2 = \frac{d}{v_b - v_w} \qquad \text{So, } \frac{t_1}{t_2} = \frac{v_b - v_w}{v_b + v_w}$$

Train Your Brain

Example 6: A 400 m wide river is flowing at a rate of 2.0 m/s. A boat is sailing with a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.

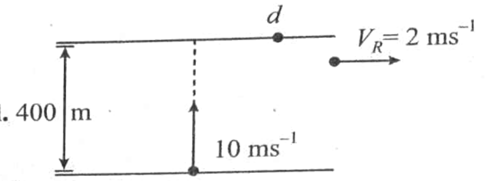
(a) Find the time taken by the boat to reach the opposite bank.

(b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.

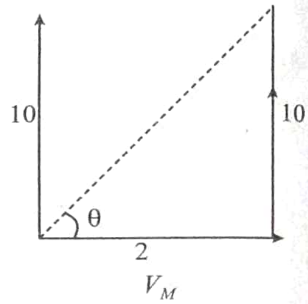
(c) In what direction does the boat actually move w.r.t. river.

(d) If he can walk at 5 ms^{-1} find the time he will take coming directly to other end.

Sol.



v_{MR} is velocity of man w.r.t. river i.e. in still river so in ground reference frame.



(a) time $= \frac{400}{10} = 40$ sec

(b) drift $= 2 \times 40 = 80$ m

(c) $\tan\theta = \frac{10}{2} = 5$

$\theta = \tan^{-1}(5)$ w.r.t. river flow.

(d) Time taken to come to directly other end.

$= 40 \text{ sec} + \frac{80}{5} = 40 + 16 = 56$ (sec)

Example 7: A man can swim at the rate of 5 km/h in still water. A 1 km wide river flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.

(a) Along what direction must the man swim?

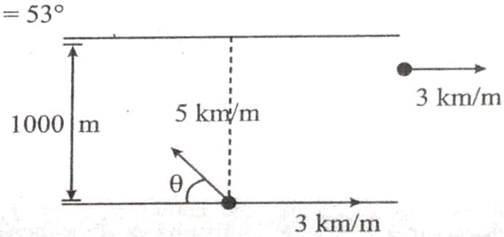
(b) What would be his resultant velocity (w.r.t. earth)?

(c) How much time will he take to cross the river?

Sol. $5\cos\theta = 3$

$\cos\theta = 3/5$

$\theta = 53°$



(a) so 180° − 53° = 127° w.r.t. river flow

(b) 4 km/h

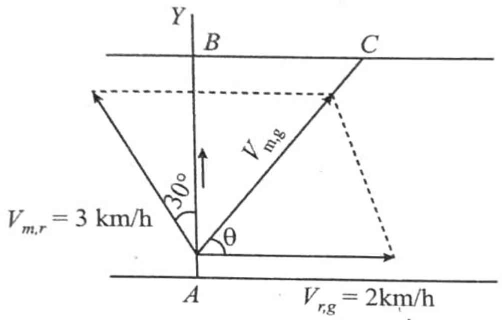
(c) time = 100/4 = 1/4 × 60 = 15 min

Example 8: A man can swim at a speed of 3 km/h in still water. He wants to cross a 500 m wide river flowing at 2 km/h. He keeps himself always at an angle of 120° with the river flow while swimming.

(a) Find the time he takes to cross the river.

(b) At what point on the opposite bank will he arrive?

Sol. The situation is shown in figure



Here $\vec{V}_{r,g}$ = velocity of the river with respect to the ground

$\vec{V}_{m,r}$ = velocity of the man with respect to the river

$\vec{V}_{m,g}$ = velocity of the man with respect to the ground.

(a) We have

$$\vec{V}_{m,g} = \vec{V}_{m,r} + \vec{V}_{r,g} \quad ...(i)$$

Hence, the velocity with respect to the ground is along *AC*.

Taking *y*-components in equation (i),

$$\vec{V}_{m,g}\ \sin\theta = 3\text{ km/h} \cos 30° + 2\text{ km/h} \cos 90° = \frac{3\sqrt{3}}{2}$$ km/h

Time taken to cross the river

$$\frac{\text{displacement along the } Y\text{-axis}}{\text{velocity along the } Y\text{-axis}} = \frac{1/2\text{km}}{3\sqrt{3}/2\text{ km/h}}$$

$$= \frac{1}{3\sqrt{3}}\text{ h}$$

(b) Taking *x*-components in equation (i),

$$\vec{V}_{m,g}\cos\theta = -3\text{km/h} \sin 30° + 2\text{ km/h} = \frac{1}{2}\text{km/h}$$

Displacement along the *X*-axis as the man crosses the river = (velocity along the *X*-axis) (time)

$$\left(\frac{1\text{km}}{2h}\right)\times\left(\frac{1}{3\sqrt{3}}h\right) = \frac{1}{6\sqrt{3}}\text{ km}$$

Example 9: A boat moves relative to water with a velocity *v* and river is flowing with 2*v*. At what angle the boat shall move with the stream to have minimum drift?

(a) 30° (b) 60°

(c) 90° (d) 120°

Sol. (d) Let boat move at angle θ to the normal as shown in figure then time to cross the river $= \frac{1}{v\cos\theta}$,

drift $x = (2v - v\sin\theta)\frac{1}{v\cos\theta}$

For *x* to be minimum, $\frac{dx}{d\theta} = 0 = 1(2\sec\theta\tan\theta - \sec^2\theta)$

or sin θ = 1/2or θ = 30° and ϕ = 90 + 30 = 120°

Concept Application

3. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is

(a) 1 (b) 3

(c) 4 (d) 2

4. A boat can go across a lake and return in time T_0 at a speed *V*. On a rough day there is uniform current at speed *v* to help the onward journey and impede the return journey. If the time taken to go across and return on the rough day be *T*, then T/T_0

(a) $1 - \frac{v^2}{V^2}$

(b) $\frac{1}{1-\frac{v^2}{V^2}}$

(c) $1 + \frac{v^2}{V^2}$

(d) $\frac{1}{1+\frac{v^2}{V^2}}$

RAIN MAN PROBLEMS

In these type of problems we again come across three terms $\vec{v}_r$, $\vec{v}_m$ and $\vec{v}_m$, Here,

$\vec{v}_r$ = velocity of rain

$\vec{v}_m$ = velocity of man (it may be velocity of cyclist or velocity of motorist also)

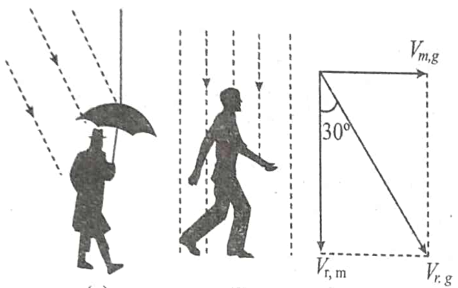
and $\vec{v}_{rm}$ = velocity of rain with respect to man.

Here, $\vec{v}_{rm}$ is the velocity of rain which appears to the man.

Train Your Brain

Example 10: A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (*a*) the road, (*b*) the moving man.

Sol. When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground. The situation when the man runs is shown in the figure



(*a*) (*b*)

Here $\vec{v}_{r,g}$ = velocity of the rain with respect to the ground

$\vec{v}_{m,g}$ = velocity of the man with respect to the ground

and $\vec{v}_{r,m}$ = velocity of the rain with respect to the man.

We have, $\vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}$ ...(i)

Taking horizontal components, equation (i) gives

$v_{r,g}\sin 30° = V_{m,g} = 10$ km/h

or, $V_{r,g} = \dfrac{10\,\text{km/h}}{\sin 30°} = 20\text{km/h}$

Taking vertical components, equation (i) gives

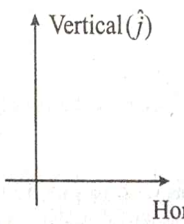
$v_{r,g}\cos 30° = v_{r,m}$ or, $v_{r,m} = (20\text{ km/h})\,\dfrac{\sqrt{3}}{2} = 10\sqrt{3}$ km/h

Example 11: To a man walking at the rate of 3 km/h the rain appears to fall vertically. When he increases his speed to 6 km/h it appears to meet him at an angle of 45° with vertical. Find the speed of rain.

Sol. Let $\hat{i}$ and $\hat{j}$ be the unit vectors in horizontal and vertical directions respectively.

Let velocity of rain

$\vec{v}_r = a\hat{i} + b\hat{j}$ ...(i)



Then speed of rain will be

$|\vec{v}_r| = \sqrt{a^2 + b^2}$ ...(ii)

In the first case $\vec{v}_m$ = velocity of man = $3\hat{i}$

$\therefore \quad \vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\hat{i} + b\hat{j}$

It seems to be in vertical direction. Hence,

$a - 3 = 0$ or $a = 3$

In the second case $\vec{v}_m = 6\hat{i}$

$\therefore \quad \vec{v}_{rm} = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$

This seems to be at 45° with vertical.

Hence, $|b| = 3$

Therefore, from Eq. (ii) speed of rain is

$|\vec{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$ km/h

Concept Application

5. It is raining vertically downwards with a velocity of 3 km h^{-1}. A man walks in the rain with a velocity of 4 kmh^{-1}. The rain drops will fall on the man with a relative velocity of ;

(*a*) 1 kmh^{-1} (*b*) 3 kmh^{-1}
(*c*) 4 kmh^{-1} (*d*) 5 kmh^{-1}

6. A man walks in rain with a velocity of 5 kmh^{-1}. The rain drops strike at him at an angle of 45° with the horizontal. Velocity of rain if it is falling vertically downward

(*a*) 5 kmh^{-1} (*b*) 4 kmh^{-1}
(*c*) 3 kmh^{-1} (*d*) 1 kmh^{-1}

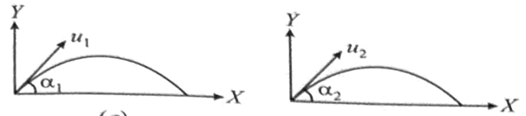
7. Raindrops are falling vertically with a velocity of 10 m/s. To a cyclist moving on a straight road the raindrops appear to be coming with a velocity of 20 m/s. The velocity of cyclist is

(*a*) 10 m/s (*b*) $10\sqrt{3}$ m/s
(*c*) 20 m/s (*d*) $20\sqrt{3}$ m/s

RELATIVE MOTION BETWEEN TWO PROJECTILES

Let us now discuss the relative motion between two projectiles or the path observed by one projectile of the other. Suppose that two particles are projected from the ground with speeds u_1 and u_2 at angles α_1 and α_2 as shown in figure (*a*) and (*b*). Acceleration of both the particles is *g* downwards. So, relative acceleration between them is zero because

$$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = \vec{g} - \vec{g} = 0$$



(a) (b)

i.e., the relative motion between the two particles is uniform.

Now

$u_{1x} = u_1 \cos\alpha_1$, $u_{2x} = u_2 \cos\alpha_2$

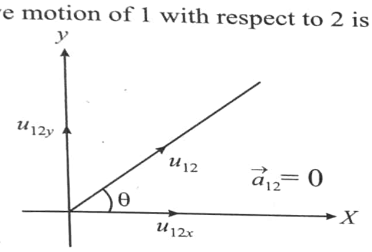
$u_{1y} = u_1 \sin\alpha_1$ and $u_{2y} = u_2 \sin\alpha_2$

Therefore, $u_{12x} = u_{1x} - u_{2x} = u_1 \cos\alpha_1 - u_2 \cos\alpha_2$

and $u_{12y} = u_{1y} - u_{2y} = u_1 \sin\alpha_1 - u_2 \sin\alpha_2$

u_{12x} and u_{12y} are the x and y components of relative velocities of 1 with respect to 2.

Hence, relative motion of 1 with respect to 2 is a straight



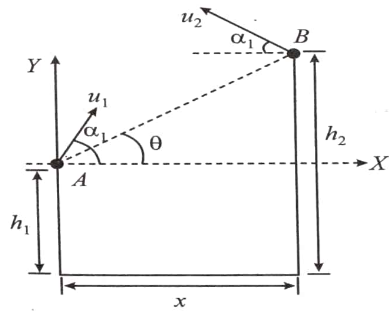
Line at an angle $\theta = \tan^{-1}\left(\frac{u_{12y}}{u_{12x}}\right)$ with positive x-axis.

Now, if $u_{12x} = 0$ or $u_1 \cos\alpha_1 = u_2 \cos\alpha_2$, the relative motion is along y-axis or in vertical direction (as $\theta = 90°$). Similarly, if $u_{12y} = 0$ or $u_1 \sin\alpha_1 = u_2 \sin\alpha_2$, the relative motion is along x-axis or in horizontal direction (as $\theta = 0°$).

Note: Relative acceleration between two projectiles is zero. Relative motion between them is uniform. Therefore, condition of collision of two particles in air is that relative velocity of one with respect to the other should be along line joining them, i.e., if two projecticles A and B collide in mid air, then $\vec{V}_{AB}$ should be along AB or $\vec{V}_{BA}$ along BA.

Condition for collision of two projectiles

Consider the situation shown in the figure. For projectiles to collide, direction of velocity of A with respect to B has to be along line AB.



Here, $v_{ABx} = u_1 \cos\alpha_1 + u_2 \cos\alpha_2$

$v_{ABy} = u_1 \sin\alpha_1 - u_2 \sin\alpha_2$

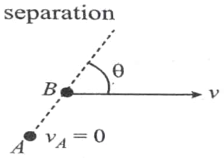
Let, direction of velocity vector of A(wrt B) is making an angle β with +ve X-axis, which is given by

$$\tan\beta = \frac{v_{ABy}}{v_{ABx}} = \frac{u_1 \sin\alpha_1 - u_2 \sin\alpha_2}{u_1 \cos\alpha_1 + u_2 \cos\alpha_2}$$

For collision to take place, $\tan\beta = \tan\theta = \dfrac{h_2 - h_1}{x}$

Train Your Brain

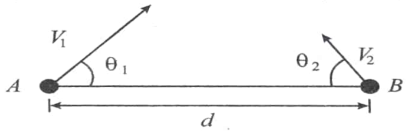
Example 12: Particle A is at rest and particle B is moving with constant velocity v as shown in the diagram at $t = 0$. Find their velocity of separation



Sol. $v_{BA} = v_B - v_A = v$

v_{sep} = component of v_{BA} along line $AB = v\cos\theta$

Example 13: Two particles A and B are moving with constant velocities v_1 and v_2. At $t = 0$, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach.

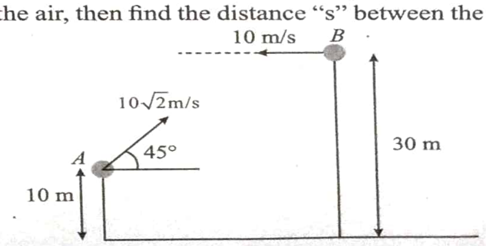


Sol. Velocity of approach is relative velocity along line AB

$v_{approach} = v_1\cos\theta_1 + v_2\cos\theta_2$

Concept Application

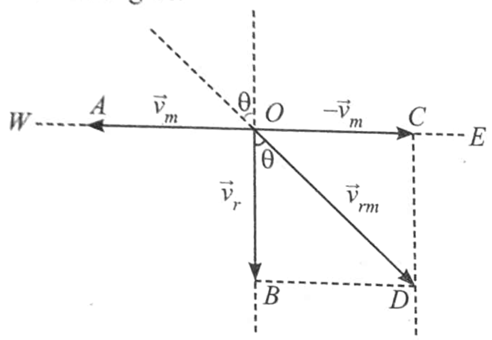
8. For two particles A and B, given that $\vec{r}_A = 2\hat{i} + 3\hat{j}$, $\vec{r}_B = 6\hat{i} + 7\hat{j}$, $\vec{v}_A = 3\hat{i} - \hat{j}$ and $\vec{v}_B = x\hat{i} - 5\hat{j}$. What is the value of x if they collide.
   (a) 1 (b) −1 (c) 2 (d) −2
9. Two projectiles are projected simultaneously from two towers as shown in figure. If the projectiles collide in the air, then find the distance "s" between the towers.



Short Notes

Relative Velocity of Rain w.r.t. the Moving Man

A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure.



The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle $OBDC$.

$$\therefore \quad v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

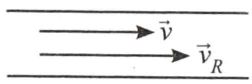
If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Swimming into the River

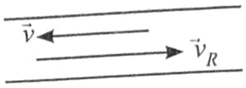
A man can swim with velocity $\vec{v}$, i.e., it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$.

If the swimming is in the direction of flow of water or along the downstream then



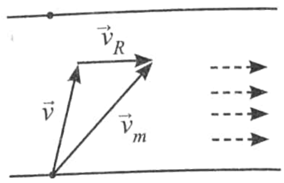
$$\vec{v}_m = \vec{v} + \vec{v}_R$$

If the swimming is in the direction opposite to the flow of water or along the upstream then

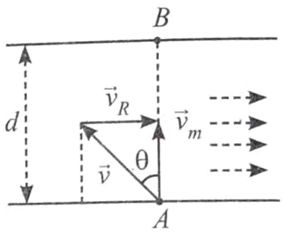


$$\vec{v}_m = \vec{v} - \vec{v}_R$$

If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ not collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming $v > v_R$)**



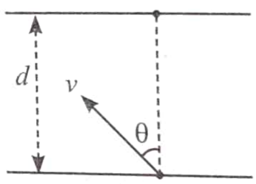
For shortest path



For minimum displacement

To reach at B, $v \sin\theta = v_R \Rightarrow \sin\theta = \dfrac{v_R}{v}$

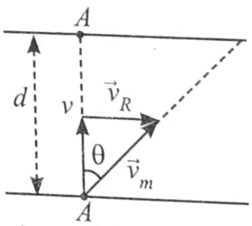
Time of crossing



$t = (d/v \cos\theta)$

Note: If $v_R > v$ then for minimum drifting $\sin\theta = \dfrac{v}{v_R}$.

For minimum time



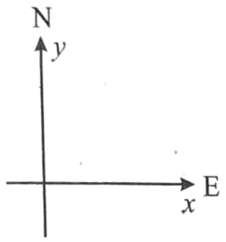
then $\boxed{t_{\min} = \dfrac{d}{v}}$

(For minimum time)

Solved Examples

1. Two parallel rail tracks run north-south. Train A moves due north with a speed of 54 km h^{-1} and train B moves due south with a speed of 90 km h^{-1}. A monkey runs on the roof of train A with a velocity of 18 km/h w.r.t. train A in a direction opposite to that of A. Calculate the (a) relative velocity of B with respect to A (b) relative velocity of ground with respect to B (c) velocity of a monkey as observed by a man standing on the ground. (d) velocity of monkey as observed by a passenger of train B.

Sol. $\vec{V}_A = 15 m/\sec\ \hat{j}$ $\vec{V}_B = 25 m/\sec\ (-\hat{j})$



(a) $\vec{V}_B - \vec{V}_A = -40\ \hat{j}$ i.e., 40 m/sec due south

= 144 km/hr due south

(b) $0 - \vec{V}_B = 25$ m/sec $\hat{j}$ i.e., 25 m/sec due north

= 90 km/hr due north

(c) $\vec{V}_{M.G} = 15\ \hat{j} - \dfrac{18\times5}{18}\ \hat{j} = 10\ \hat{j}$ i.e., 10 m/sec due north.

= 36 km/hr due north

(d) $\vec{V}_{M.B} = 10\hat{j} - (-25)\hat{j} = 35\ \hat{j}$ i.e., 35 m/sec due north.

= 126 km/hr due north

2. The driver of a train A running at 25 ms^{-1} sights a train B moving in the same direction on the same track with 15 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 1.0 ms^{-2}. What should be the minimum distance between the trains to avoid the accident.

Sol. $v_r = 25 - 15 = 10$ m/s and $a_r = -1$ m/s^2 so by $v_r^2 = u_r^2 + 2a_r S$

$$S = \frac{100}{2\times1} = 50 \text{ m}.$$

3. A particle A moves with a velocity $4\hat{i}$ and another particle B moves with a velocity $-3\hat{j}$. Find $\vec{V}_{AB}$, $\vec{V}_{BA}$ and their magnitude.

Sol. $\vec{v}_A = 4\hat{i}$, $\vec{v}_B = -3\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{V}_B = 4\hat{i} - (-3\hat{j}) = 4\hat{i} + 3\hat{j}$

$\vec{v}_{BA} = \vec{v}_B - \vec{V}_A = -3\hat{j} - 4\hat{i}$

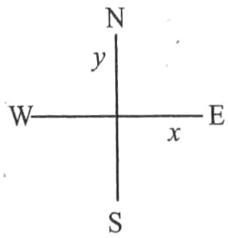
$|\vec{v}_{AB}| = \sqrt{4^2+3^2} = 5$ m/s

$|\vec{v}_{BA}| = \sqrt{3^2+4^2} = 5$ m/s

4. A ship is streaming due east at 12 ms^{-1}. A woman runs across the deck at 5 ms^{-1} (relative to ship) in a direction towards north. Calculate the velocity of the woman relative to sea.

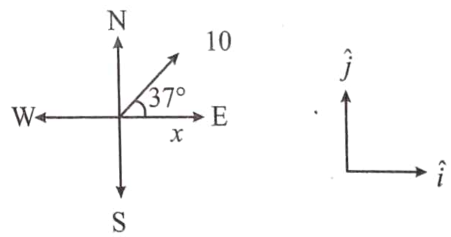
Sol. $\vec{V} = 12\hat{j} + 5\hat{j}\ |\vec{V}| = \sqrt{12^2+5^2} = 13$ m/sec

$\tan\theta = \dfrac{5}{12}$ north of east



5. A motorboat is observed to travel 10 km h^{-1} relative to the earth in the direction 37° north of east. If the velocity of the boat due to the wind only is 2 km h^{-1} westward and that due to the current only is 4 km h^{-1} southward, what is the magnitude and direction of the velocity of the boat due to its own power?

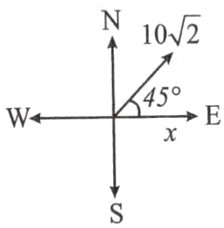
Sol. $V = 10\cos37°\,\hat{i} + 10\sin37°\,\hat{j}$



$= 10\times\dfrac{4}{5}\hat{i} + 10\times\dfrac{3}{5}\hat{j} = 8\hat{i} + 6\hat{j}$

$8\hat{i} + 6\hat{j} = V_b + V_w + V_c = V_b + (-2\hat{i}) + (-4\hat{j})$

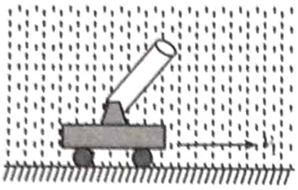
$V_b = 10\hat{i} + 10\hat{j}$



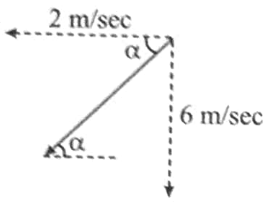
$|V_b| = 10\sqrt{2}$ km/h

$\tan\theta = \dfrac{10}{10}$ direction = 45° north of East

6. A pipe which can rotate in a vertical plane is mounted on a cart. The cart moves uniformly along a horizontal path with a speed $v_1 = 2$ m/s. At what angle α to the horizontal should the pipe be placed so that drops of rain falling vertically with a velocity v_2 = 6m/s move parallel to the axis of the pipe without touching its walls? Consider the velocity of the drops as constant due to the resistance of the air.



Sol.



$\tan \alpha = \dfrac{6}{2} = 3 \quad \Rightarrow \quad \alpha = \tan^{-1} 3$

7. To a man walking at the rate of 2 km/hour with respect to ground, the rain appears to fall vertically. When he increases his speed to 4 km/hour in same direction of his motion, rain appears to meet him at an angle of 45° with horizontal, find the real direction and speed of the rain.

Sol. $\vec{v}_m = 2\hat{i}$

$\vec{v}_r = v_x\hat{i} + v_y\hat{j}$

$\vec{v}_{r,m} = (v_x - 2)\,\hat{i} + v_y\hat{j} = v_y\hat{j}$

On comparing both sides

$\therefore\ v_x = 2$ m/sec

$\vec{v}\,'_m = 4\,\hat{i}$

$\vec{v}\,'_{r,m} = \vec{v}_r - \vec{v}\,'_m$

$= (v_x - 4)\,\hat{i} + v_y\hat{j} \; = \; -2\hat{i} + v_y\hat{j}$

$\tan 45° = \dfrac{v_y}{2}$

$v_y = 2$ m/s

so $\vec{v}_r = 2\hat{i} + 2\hat{j}$ so $\tan\theta = 1 \Rightarrow \theta = 45°$

$v_r = 2\sqrt{2}$ m/sec.

8. The driver of a car travelling at a speed of 20 m/s, wishes to overtake a truck that is moving with a constant speed of 20 m s^{-1} in the same lane. The car's maximum acceleration is 0.5 m s^{-2}. Initially the vehicles are separated by 40 m, and the car returns back into its lane after it is 40 m ahead of the truck. The car is 3 m long and the truck 17 m.

(*a*) Find the minimum time required for the car to pass the truck and return back to its lane?

(*b*) What distance does the car travel during this time?

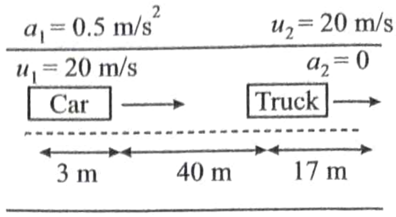
(*c*) What is the final speed of the car ?

Sol. (*a*) Displacement of car relative to truck

$x_r = 40 + 17 + 3 + 40$

$= 100$ m.

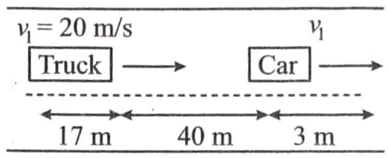
$t = 0$ (Initilly)



Relative initial velocity between car and truck

$u_r = 20 - 20 = 0$

$t = t$ (Finally)



Relative acceleration between car and truck

$a_r = 0.5 - 0 = 0.5$ m/s^2

Let required time = t .

$\therefore$ II equation of motion

$x_r = u_r\,.t + \dfrac{1}{2}\,a_r.t^2$

$\Rightarrow 100 = 0 + \dfrac{1}{2} \times 0.5 \times t^2$

$\Rightarrow t = 20$ sec.

(*b*) Distance travelled by car

$x_c = ut + \dfrac{1}{2}at^2$

$= 20 \times 20 + \dfrac{1}{2} \times 0.5 \times 20^2$

$= 500$ m

(*c*) Final speed of the car

$= u + at$

$= 20 + 0.5 \times 20$

$= 30$m/s.

9. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclists are riding in the same direction at 25 km/h at equal intervals of 30 m. At what speed an observer travel along the road in opposite direction so that whenever he meets a runner he also meets a cyclist? (neglect the size of cycle)

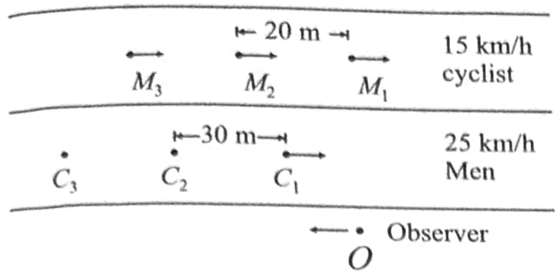
Sol. Let u = speed of observer.

Relative velocity between observer and a man .

$= u + 15$ km/h.

Relative velocity between observer and a cyclist.

$= u + 25$ km/h.



Hence, to for a man and a cyclist to meet simultaneously

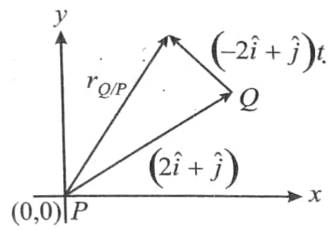
$$\frac{20\text{ m}}{(u+15)\text{ km/h}}$$

$$=\frac{30\text{ m}}{(u+25)\text{ km/h}}$$

$\Rightarrow u = 5$ km/h

10. Two particles P and Q are moving with constant velocities of $(\hat{i}+\hat{j})$ m/s and $\left(-\hat{i}+2\hat{j}\right)$ m/s respectively. At time $t=0$, P is at origin and Q is at a point with position vector $\left(2\hat{i}+\hat{j}\right)$ m. Find the equation of the trajectory of Q as observed by P.

Sol.



$\vec{r}_{QP} = \vec{r}_Q - \vec{r}_P = 2\hat{i} + \hat{j}$

$\vec{V}_{QP} = -2\hat{i} + 1\hat{j}$

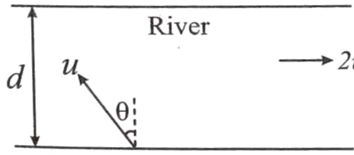
$x = 2 - 2t \quad y = 1 + t$

$\Rightarrow x = 2 - 2(y-1)$

$x + 2y = 4$

11. A boat moves relative to water with a velocity half of the river flow velocity. To minimize drifting, at what angle to the stream direction the boat must move.

Sol. Let river velocity is $2u$



time to cross river $t = \dfrac{d}{u\cos\theta}$

Drift $x = (2u - u\sin\theta)t = (2u - u\sin\theta)\dfrac{d}{u\cos\theta}$

Drift $x = (2\sec\theta - \tan\theta)d$

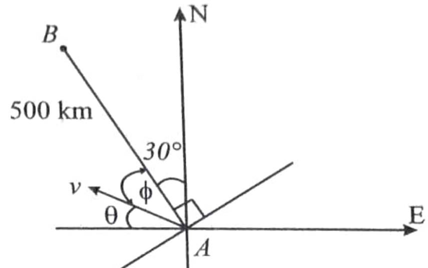
$\dfrac{dx}{d\theta} = (2\sec\theta\tan\theta - \sec^2\theta)d = 0 \Rightarrow 2\tan\theta = \sec\theta$

$\theta = 30°$

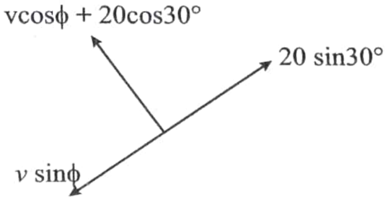
angle with stream $30° + 90° = 120°$.

12. An aeroplane has to go from a point A to another point B, 1000 km away due 30° west of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. (*a*) Find the direction in which the pilot should head the plane to reach the point B. (*b*) Find the time taken by the plane to go from A to B.

Sol.



Velocity of plane w.r.t. ground



(*a*) Velocity of plane w.r.t ground is along AB so perpendicular component (to line AB) of velocity is zero.

$v\sin\phi = 20\sin 30°$

$\sin\phi = \dfrac{10}{150} = \dfrac{1}{15}$

$\phi = \sin^{-1}\left(\dfrac{1}{15}\right)$ west of line AB

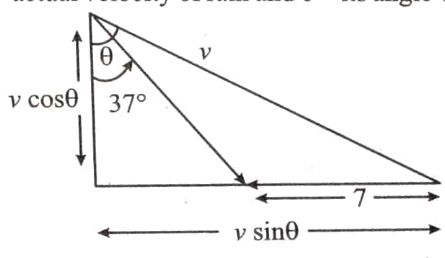
(*b*) $t = \dfrac{1000\times10^3}{v\cos\phi + 10\sqrt{3}} = \dfrac{1000\times1000}{150\times\sqrt{\dfrac{224}{225}}+10\sqrt{3}}$

$= \dfrac{1000\times1000}{10\times60(\sqrt{3}+\sqrt{224})}$ min

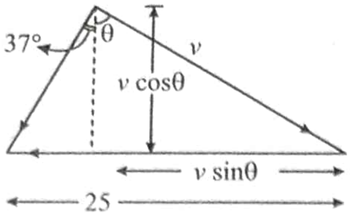
$= \dfrac{5000}{3(\sqrt{3}+\sqrt{224})}$ min ≈ 100 min

13. Rain appears to be falling at an angle of 37° with vertical to the driver of a car moving with a velocity of 7 m/sec. When he increases the velocity of the car to 25 m/sec, the rain again appears to fall at an angle 37°. with vertical. What is the actual velocity of rain and it's direction with vertical.

Sol. Let v = actual velocity of rain and θ = its angle with vertical:



(*a*)



(b)

In figure (a)

$$v \sin\theta = 7 + v \cos\theta.\tan 37^\circ = 7 + \frac{3}{4} v \cos\theta$$

$$\Rightarrow 4\, v \sin\theta - 3\, v \cos\theta = 28 \qquad ...(i)$$

In figure (b)

$$25 = v \sin\theta + v \cos\theta.\tan 37^\circ = v \sin\theta + \frac{3}{4} v \cos\theta$$

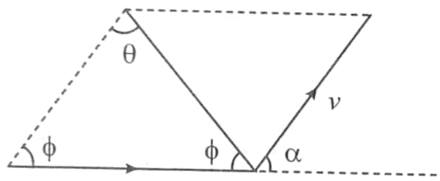
$$\Rightarrow 4\, v \sin\theta + 3\, v \cos\theta = 100 \qquad ...(ii)$$

Solvihg (i) and (ii)

$v = 20$m/s and $\theta = 53^\circ$

14. 'n' number of particles are located at the vertices of a regular polygon of 'n' sides having the edge length 'a'. They all start moving simultaneously with equal constant speed 'v' heading towards the particle next to it all the time. How long will the particles take to collide?

Sol. For a regular polygon of side n, the angle subtended by a side at the centre:



$$\theta = \frac{2\pi}{n}$$

$$\therefore \alpha = \pi - 2\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$= \theta$$

$$\Rightarrow \alpha = \theta = \frac{2\pi}{n}$$

Relative velocity of approach

$= v - v\cos\alpha = v\,(1 - \cos\alpha\,)$

Time taken for collision

$$t = \frac{a}{v(1-\cos\alpha)}$$

$$\Rightarrow t = \frac{a}{v\left(1-\cos\dfrac{2\pi}{n}\right)}$$

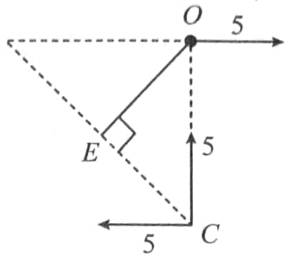
15. Two straight tracks AOB and COD meet each other at right angles at point O. A person walking at a speed of 5 km/h along AOB is at the crossing O at 12 o'clock noon. Another person walking at the same speed along COD reaches the crossing O at 1:30 PM. Find at what time the distance between them is least and what is its value?

Sol. The positions of the persons at 12:00 PM will be as shown in figure. such that

$$OC = 5 \times \frac{3}{2}\text{ km} = \frac{15}{2}\text{ km}$$

Velocity of man at C with respect to man at O will be along CE such that $\tan\theta = \frac{5}{5} = 1$

$\therefore \theta = 45^\circ$



$$\therefore \text{ least distance} = OE = OC \sin 45^\circ = \frac{15}{2\sqrt{2}}\text{ km}$$

$$\text{time taken} = \frac{CE}{5\sqrt{2}} = \frac{15}{2\sqrt{2}\times 5\sqrt{2}}$$

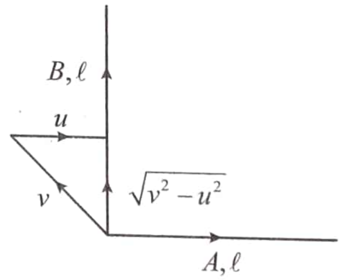
$$= \frac{3}{4}\text{ hr.}$$

So, the person will be closest at 12:45 PM

16. Two boats A and B move away from a buoy anchored at the middle of a river along the mutually perpendicular straight lines; the boat A along the river and the boat B across the river. Having moved off an equal distance from the buoy the boats returned. Find the ratio of times of motion of boats τ_A/τ_B if the velocity of each boat with respect to water is $\eta = 1.2$ times than the stream velocity.

Sol. $V_{b,r} = V,\ V_r = u$

Time taken by boat A



$$\tau_A = \frac{\ell}{v+u} + \frac{\ell}{v-u}$$

$$= \frac{2v\ell}{v^2 - u^2}$$

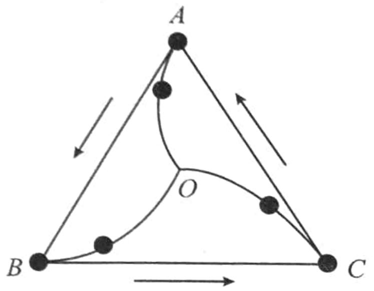
Time taken by boat B

$$\tau_B = \frac{2\ell}{\sqrt{v^2 - u^2}} = \eta$$

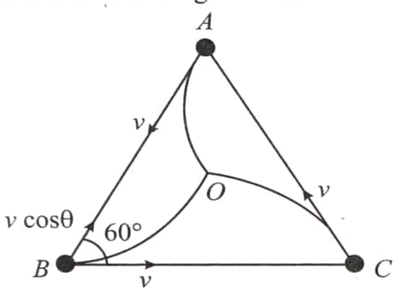
$$\text{Hence } \frac{\tau_A}{\tau_B} = \frac{\eta}{\sqrt{\eta^2 - 1}}$$

17. Three particle A, B and C situated at the vertices of an equilateral triangle starts moving simultaneously at a constant speed "v" in the direction of adjacent particle, which moves ahead in the anti-clockwise direction. If "a" be the side of the triangle, then find the time when they meet.

Sol. Here, particle "A" follows "B", "B" follows "C" and "C" follows "A". The direction of motion of each particle keeps changing as motion of each particle is always directed towards other particle. The situation after a time "t" is shown in the figure with a possible outline of path followed by the particles before they meet.



This problem appears to be complex as the path of motion is difficult to be defined. But, it has a simple solution in component analysis. Let us consider the pair "A" and "B". The initial component of velocities in the direction of line joining the initial position of the two particles is "v" and "$v\cos\theta$" as shown in the figure here:



The component velocities are directed towards each other. Now, considering the linear (one dimensional) motion in the direction of AB, the relative velocity of "A" with respect to "B" is:

$$v_{AB} = v_a - v_b$$

$$v_{AB} = v - (-v\cos\theta) = v + v\cos\theta$$

In equilateral triangle, $\theta = 60°$

$$v_{AB} = v + v\cos 60° = v + \frac{v}{2} = \frac{3v}{2}$$

The time taken to cover the displacement "a" i.e. the side of the triangle is $t = \dfrac{2a}{3v}$

18. Car A and car B start moving simultaneously in the same direction along the line joining them. Car A moves with a constant acceleration $a = 4\ \text{m/s}^2$, while car B moves with a constant velocity $v = 1$ m/s. At time $t = 0$, car A is 10 m behind car B. Find the time when car A overtakes car B.

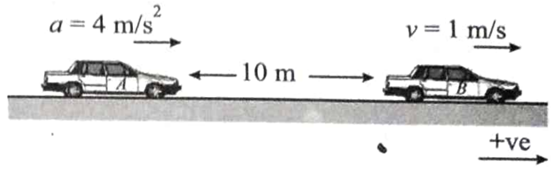
Sol. Given: $u_A = 0$, $u_B = 1$ m/s, $a_A = 4\ \text{m/s}^2$ and $a_B = 0$

Assuming car B to be at rest, we have

$$u_{AB} = u_A - u_B = 0 - 1 = -1 \text{ m/s}$$

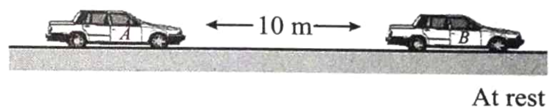
$$a_{AB} = a_A - a_B = 4 - 0 = 4\ \text{m/s}^2$$

Now, the problem can be assumed in simplified form as follow:



Substituting the proper values in equation

$u_{AB} = -1$ m/s, $a_{AB} = 4\ \text{m/s}^2$



$$S = ut + \frac{1}{2}at^2$$

we get $10 = -t + \dfrac{1}{2}(4)(t^2)$ or $2t^2 - t - 10 = 0$

Ignoring the negative value, the desired time is 2.5s.

Note: The above problem can also be solved without using the concept of relative motion as under. At the time when A overtakes B,

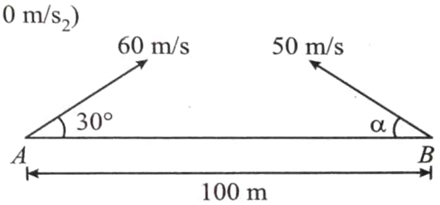
$$S_A = S_B + 10$$

$$\therefore \frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$$

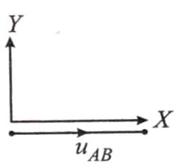
or $2t^2 - t - 10 = 0$

which on solving gives $t = 2.5$ s and -2 s, the same as we found above.

19. A particle A is projected with an initial velocity of 60 m/s. at an angle 30° to the horizontal. At the same time a second particle B is projected in opposite direction with initial speed of 50 m/s from a point ata distance of 100 m from A. If the particles collide in air, find (a) the angle of projection α of particle B, (b) time when the collision takes place and (c) the distance of P from A, where collision occurs, ($g = 10\ \text{m/s}_2$)



Sol. (a) Taking x and y directions as shown in figure.



Here, $\vec{a}_A = -g\hat{j}$, $\vec{a}_B = -g\hat{j}$

$u_{Ax} = 60 \cos 30° = 30\sqrt{3}$ m/s

$u_{Ay} = 60 \sin 30° = 30$ m/s

$u_{Bx} = -50 \cos \alpha$

and $u_{By} = 50 \sin \alpha$

Relative acceleration between the two is zero as $\vec{a}_A = \vec{a}_B$. Hence, the relative motion between the two is uniform. It can be assumed that B is at rest and A is moving with $\vec{u}_{AB}$. Hence, the two particles will collide, if $\vec{u}_{AB}$ is along AB. This is possible only when

$u_{Ay} = u_{By}$

i.e., component of relative velocity along y-axis should be zero,

or $30 = 50 \sin \alpha$

$\therefore \alpha = \sin^{-1}\left(\frac{3}{5}\right)$

(*b*) $|\vec{u}_{AB}| = u_{Ax} - u_{Bx} = (30\sqrt{3} + 50 \cos\alpha)$m/s

$= \left(30\sqrt{3} + 50 \times \frac{4}{5}\right)$ m/s

$= (30\sqrt{3} + 40)$ m/s

Therefore, time of collision is

$t = \frac{AB}{|\vec{u}_{AB}|} = \frac{100}{30\sqrt{3}+40}$ or $t = 1.09$ s

(*c*) Distance of point P from A where collision takes place is

$$S = \sqrt{(u_{Ax}t)^2 + \left(u_{Ay} - \frac{1}{2}gt^2\right)^2}$$

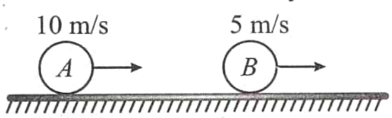
$$= \sqrt{(30\sqrt{3}\times 1.09)^2 \left(30\times 1.09 - \frac{1}{2}\times 10\times 1.09\times 1.09\right)^2}$$

$S = 62.64$ m

Exercise-1 (Topicwise)

ONE DIMENSIONAL RELATIVE MOTION

1. Two trains, each 50 m long are travelling in opposite direction with velocity 10 m/s and 15 m/s. The time of crossing is
   (a) 2s (b) 4s
   (c) $2\sqrt{3}$ s (d) $4\sqrt{3}$ s
2. A 210 meter long train is moving due North at a of 25m/s. A small bird is flying due South a little above the train with speed 5m/s. The time taken by the bird to cross the train is
   (a) 6 s (b) 7 s
   (c) 9 s (d) 10 s
3. A stone is dropped from a building, and 2 seconds later another stone is dropped. (Both a are dropped from rest.) How far apart are the two stones by the time the first one has reached a speed of 30 m/s ?
   (a) 80 m (b) 100 m
   (c) 60 m (d) 40 m
4. Two trains each of length 50 m are approaching each other on parallel rails. Their velocities are 10 m/sec and 15 m/sec. They will cross each other in
   (a) 2 sec (b) 4 sec
   (c) 10 sec (d) 6 sec
5. An object A is moving with 10 m/s and B is moving with 5 m/s in the same direction of positive x-axis. A is 100 m behind B as shown. Find time taken by A to Meet B



   (a) 18 sec (b) 16 sec
   (c) 20 sec (d) 17 sec
6. A thief is running away on a straight road with a speed of 9 ms^{-1}. A police man chases him on a jeep moving at a speed of 10 ms^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief?
   (a) 1s (b) 19s
   (c) 90s (d) 100s
7. A body is thrown up in a lift with a velocity u relative to the lift and the time of flight is found to be 't'. The acceleration with which the lift is moving up is:
   (a) $\frac{u - gt}{t}$ (b) $\frac{2u - gt}{t}$
   (c) $\frac{u + gt}{t}$ (d) $\frac{2u + gt}{t}$
8. Two cars get closer by 9 m every second while travelling in the opposite directions. They get closer by 1 m every second while travelling in the same directions. What are the speeds of the cars?
   (a) 5 ms^{-1} and 4 ms^{-1}
   (b) 4 ms^{-1} and 3 ms^{-1}
   (c) 6 ms^{-1} and 3 ms^{-1}
   (d) 6 ms^{-1} and 5 ms^{-1}

TWO DIMENSIONAL RELATIVE MOTION

9. A bird is flying towards south with a velocity 40km/hr and a train is moving with a velocity 40 km/hr towards east. What is the velocity of the bird w.r.t. an observer in train
   (a) $40\sqrt{2}$ km/hr. North – East
   (b) $40\sqrt{2}$ km/hr. South – East
   (c) $40\sqrt{2}$ km/hr. South – West
   (d) $40\sqrt{2}$ km/hr. North – West
10. A train is moving towards east and a car is along north, both with same speed. The observed direction of car to the passenger in the train is
   (a) East-north direction (b) West-north direction
   (c) South-east direction (d) None of these
11. A ship is sailing towards north at a speed of $\sqrt{2}$ m/s. The current is taking it towards East at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 1 m/s. Find the velocity of the sailor with respect to ground.
   (a) $2\hat{i} + \sqrt{2}\hat{j} + \hat{k}$ (b) $\hat{i} + \sqrt{2}\hat{j} + \hat{k}$
   (c) $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ (d) $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$
12. A helicopter is flying south with a speed of 50 kmh^{-1}. A train is moving with the same speed towards east. The relative velocity of the helicopter as seen by the passengers in the train will be towards.
   (a) North east
   (b) South east
   (c) North west
   (d) South west
13. Wind is blowing in the north direction at speed of 2 ms^{-1}, which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him:
   (a) 2 ms^{-1} south (b) 2 ms^{-1} north
   (c) 2 ms^{-1} west (d) 4 ms^{-1} south

RAIN MAN PROBLEM

14. A man is walking on a road with a velocity 3 km/hr. Suddenly rain starts falling. The velocity of rain is 10 km/hr in vertically downward direction. the relative velocity of the rain with respect to man is:

(*a*) $\sqrt{13}$ km/hr (*b*) $\sqrt{7}$ km/hr

(*c*) $\sqrt{109}$ km/hr (*d*) 13 km/hr

15. A boy is running on the plane road with velocity (v) with a long hollow tube in his hand. The water is falling vertically downwards with velocity (u). At what angle to the vertical, he must incline the tube so that the water drops enters in it without touching its side

(*a*) $\tan^{-1}\left(\frac{v}{u}\right)$ (*b*) $\sin^{-1}\left(\frac{v}{u}\right)$

(*c*) $\tan^{-1}\left(\frac{u}{v}\right)$ (*d*) $\cos^{-1}\left(\frac{v}{u}\right)$

16. A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/hr then he finds that rain drops are hitting his head vertically, then speed of rain drops with respect to moving man:

(*a*) 20 km/hr. (*b*) $10\sqrt{3}$ km/hr.

(*c*) $\frac{10}{\sqrt{3}}$ km/hr. (*d*) 10 km/hr.

17. A man walks in rain with a velocity of 5 kmh^{-1}. The rain drops strike at him at an angle of 45° with the horizontal. Velocity of rain if it is falling vertically downward -

(*a*) 5 km h^{-1} (*b*) 4 km h^{-1}

(*c*) 3 km h^{-1} (*d*) 1 km h^{-1}

18. To man running at a speed of 25 m/sec,the rain drops appear to be falling at an angle of 45° from the vertical. If the rain drops are actually falling vertically downwards, then velocity in m/sec is.

(*a*) 25 (*b*) $25\sqrt{3}$ (*c*) $25\sqrt{2}$ (*d*) 4

RIVER MAN PROBLEM

19. A swimmer crosses the river along the line making an angle of 45° with the direction of flow. Velocity of the river water is 5 m/s. Swimmer takes 6 seconds to cross the river of width 60 m. The velocity of the swimmer with respect to water will be:

(*a*) 10 m/s (*b*) 12 m/s

(*c*) $5\sqrt{5}$ m/s (*d*) $10\sqrt{2}$ m/s

20. A boat is rowed across a river (perpendicular to river flow) at the rate of 4.5 km/hr. The river flows at the rate of 6 km/hr. The velocity of boat in m/s is:

(*a*) 3.1 (*b*) 2.1 (*c*) 2.9 (*d*) 5

21. A boat is sent across a river with a velocity of 8 km/hr. If the resultant velocity of boat is 10 km/hr, then velocity of the river is

(*a*) 10 km/hr (*b*) 8 km/hr

(*c*) 6 km/hr (*d*) 4 km/hr

22. A boat is moving with velocity of $3\hat{i}+4\hat{j}$ in river and water is moving with a velocity of $-3\hat{i}-4\hat{j}$ with respect to ground. Relative velocity of boat with respect to water is:

(*a*) $-6\hat{i}-8\hat{j}$ (*b*) $6\hat{i}+8\hat{j}$

(*c*) $8\hat{i}$ (*d*) $6\hat{i}$

23. A river is flowing from W to E with a speed of 5 m/min. A man can swim in still water with a velocity 10 m/min. In which direction should the man swim so as to take the shortest possible path to go to the south.

(*a*) 30° with downstream (*b*) 60° with downstream

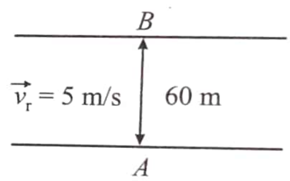
(*c*) 120° with downstream (*d*) South

24. A river is flowing from west to east at a speed of 5 meters per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction

(*a*) due north (*b*) 30° east of north

(*c*) 30° north of west (*d*) 60° east of north

25. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at a distance of 60 meter in 5 sec. His velocity in still water should be



(*a*) 12 m/s (*b*) 13 m/s

(*c*) 5 m/s (*d*) 10 m/s

26. To cross the river in shortest distance, a swimmer should swim making angle θ with the upstream. What is the ratio of the time taken to swim across in the shortest time to that in swimming across over shortest distance. [Assume speed of swimmer in still water is greater than the speed of river flow]

(*a*) cosθ (*b*) sin θ

(*c*) tan θ (*d*) cot θ

27. A man can swim with a speed of 4 km h^{-1} in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km h^{-1} and he makes his strokes normal to the river current?

(*a*) $\frac{1}{3}$hr (*b*) $\frac{1}{2}$hr

(*c*) $\frac{1}{4}$hr (*d*) 1hr

28. A swimmer's speed in the direction of flow of river is 16 km h^{-1}. Swimmer's speed against the direction of flow of river is 8 km h^{-1}. Calculate the swimmer's speed in still water and the velocity of flow of the river (in km).

(a) 2, 8
(b) 8, 4
(c) 4, 8
(d) 8, 2

29. A motorboat going down stream overcome a float at a point M which is fixed with respect to ground. 60 minutes later it turned back and after some time passed the float at a distance of 6 km from the point M. Find the velocity of the stream assuming a constant velocity for the motorboat in still water.

(a) 4 km/h
(b) 7 km/h
(c) 3 km/h
(d) 6 km/h

30. A ship goes along a river and returns in time t_0 at a speed 2 m/s. On a particular day, a uniform current at speed 1 m/s is present to help the onward journey and oppose the return journey. If the time taken to go along and return on the rough day be t, then find the value of t/t_0.

(a) $\frac{3}{4}$
(b) $\frac{4}{3}$
(c) $\frac{5}{4}$
(d) $\frac{1}{2}$

31. Two boats were going down stream with different velocities. When one overtook the other a plastic ball was dropped from one of the boats into water. Some time later both boats turned back simultaneously and went at the same speeds as before (relative to the water) towards the spot where the ball had been dropped. Which boat will reach the ball first?

(a) The boat which has greater velocity (relative to water).
(b) The boat which has lesser velocity (relative to water).
(c) Both will reach the ball simultaneously.
(d) Cannot be decided unless we know the actual values of the velocities and the time after which they turned around.

DISTANCE OF NEAREST APPROACH

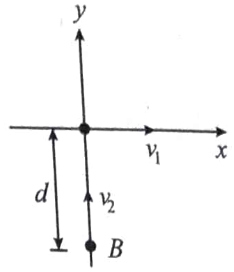
32. A body is projected vertically up at $t = 0$ with a velocity of 98 m/s. Another body is projected from the same point with same velocity after 4 seconds. Both bodies will meet after:

(a) 6 s
(b) 8 s
(c) 10 s
(d) 12 s

33. Four particles situated at the corners of a square of side 'a' move at a constant speed v. Each particle maintains a direction towards the next particle in succession. Calculate the time the particles will take to meet each other.

(a) a/v
(b) $2a/v$
(c) $a/2v$
(d) $a/3v$

34. Two particles A and B move with velocities v_1 and v_2 respectively along the x and y axis. The initial separation between them is 'd' as shown in the figure. Find the least distance between them during their motion.



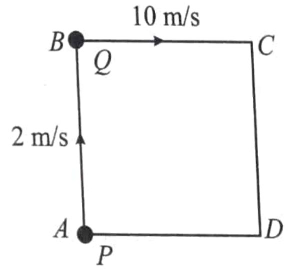
(a) $\frac{d.v_1^2}{v_1^2+v_2^2}$

(b) $\frac{d.v_2^2}{v_1^2+v_2^2}$

(c) $\frac{d.v_1}{\sqrt{v_1^2+v_2^2}}$

(d) $\frac{d.v_2}{\sqrt{v_1^2+v_2^2}}$

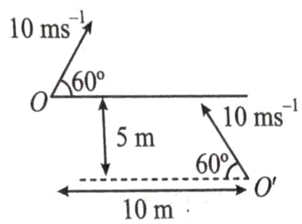
35. Two men P and Q are standing at corners A and B of square $ABCD$ of side 8 m. They start moving along the track with constant speed 2 m/s and 10 m/s respectively. Find the time when they will meet for the first time.



(a) 2 sec
(b) 3 sec
(c) 1 sec
(d) 6 sec

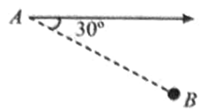
RELATIVE MOTION BETWEEN TWO PROJECTILE

36. Two particles are projected simultaneously from two points O and O' such that 10 m is the horizontal and 5 m is the vertical distance between them as shown in the figure. They are projected at the same inclination 60° to the horizontal with the same velocity 10 ms^{-1}. The time after which their separation becomes minimum is



(a) 2.5 sec
(b) 1 sec
(c) 5 sec
(d) 10 sec

37. A particle A is moving with a constant velocity of 10 m/sec. Another particle B is moving with a constant but unknown velocity. At an instant, the line joining A and B makes an angle of 30° with velocity of A. Find the minimum possible magnitude of velocity of B, if they collide after some time. (see figure)



(a) 5 m/s
(b) 4 m/s
(c) 10 m/s
(d) 6 m/s

38. Two particles are moving with velocities v_1 and v_2. Their relative velocity is the maximum, when the angle between their velocities are:

(a) Zero
(b) $\pi/4$
(c) $\pi/2$
(d) π

39. Two billiard balls are rolling on a flat table. One has velocity components $v_x = 1$m/s, $v_y = \sqrt{3}$ m/s and the other has components $v_x = 2$m/s and $v_y = 2$ m/s. If both the balls start moving from the same point, the angle between their path is

(a) 60°
(b) 45°
(c) 22.5°
(d) 15°

Exercise-2 (Learning Plus)

1. A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100s. If, the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus?

(a) 50 ms^{-1}
(b) 40 ms^{-1}
(c) 30 ms^{-1}
(d) 20 ms^{-1}

2. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car (as, seen by thief), according to thief in the car?

(a) 105 m/s
(b) 100 m/s
(c) 110 m/s
(d) 90 m/s

3. A jet airplane travelling from east to west at a speed of 500 km h^{-1} ejects out gases of combustion at a speed of 1500 km h^{-1} with respect to the jet plane. What is the velocity of the gases with respect to an observer on the ground?

(a) 1000 km h^{-1} in the direction west to east
(b) 1000 km h^{-1} in the direction east to west
(c) 2000 km h^{-1} in the direction west to east
(d) 2000 km h^{-1} in the direction east to west

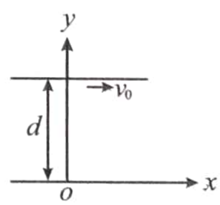
4. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is

(a) 1
(b) 3
(c) 4
(d) $\sqrt{41}$

5. When a man moves down the inclined plane with a constant speed 5ms^{-1}which makes an angle of 37° with the horizontal, he finds that the rain is falling vertically downward. When he moves up the same inclined plane with the same speed, he finds that the rain makes an angle $\theta = \tan^{-1}\left(\frac{7}{8}\right)$ with the horizontal. The speed of the rain is

(a) $\sqrt{116}$ ms^{-1}
(b) $\sqrt{32}$ ms^{-1}
(c) 5ms^{-1}
(d) $\sqrt{73}$ ms^{-1}

6. Velocity of the river with respect to ground is given by v_0. Width of the river is d. A swimmer swims (with respect to water) perpendicular to the current with acceleration $a = 2t$ (where t is time) starting from rest from the origin O at $t = 0$. The equation of trajectory of the path followed by the swimmer is

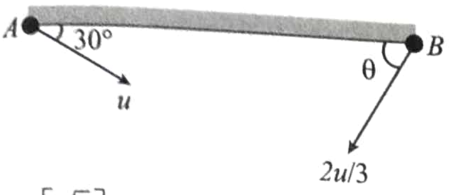


(a) $y = \dfrac{x^3}{3v_0^3}$
(b) $y = \dfrac{x^2}{2v_0^2}$
(c) $y = \dfrac{x}{v_0}$
(d) $y = \sqrt{\dfrac{x}{v_0}}$

7. A thief in a stolen car passes through a police check post at his top speed of 90 kmh^{-1}. A motorcycle cop, reacting after 2 s, accelerates from rest at 5 ms^{-2}. His top speed being 108 kmh^{-1} . Find the maximum separation between policemen and thief.

(a) 112.5 m
(b) 115 m
(c) 116.5 m
(d) None of these

8. Anoop (A) hits a ball along the ground with a speed u in a direction which makes an angle 30° with the line joining him and the fielder Babul (B). Babul runs to intercept the ball with a speed $\frac{2u}{3}$. At what angle θ should he run to intercept the ball?



(a) $\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]$ (b) $\sin^{-1}\left[\frac{2}{3}\right]$

(c) $\sin^{-1}\left[\frac{3}{4}\right]$ (d) $\sin^{-1}\left[\frac{4}{5}\right]$

9. Two particles are moving along two long straight lines, in the same plane with same speed equal to 20 cm/ s. The angle between the two lines is 60° and their intersection point is O. At a certain moment, the two particles are located at distances 3m and 4m from O and are moving towards O. Subsequently, the shortest distance between them will be

(a) 50 cm (b) $40\sqrt{2}$ cm

(c) $50\sqrt{2}$ cm (d) $50\sqrt{3}$ cm

10. Two trains are moving with velocities v_1 = 10 ms^{-1} and v_2 = 20 ms^{-1} on the same track in opposite directions. After the application of brakes if their retarding rates are a_1 = 2 ms^{-2} and a_2 = 1 ms^{-2} respectively, then the minimum distance of separation between the trains to avoid collision is

(a) 150 m (b) 225 m

(c) 450 m (d) 300 m

11. Two identical balls are shot upward one after another at an interval of 2s along the same vertical line with same initial velocity of 40 ms^{-1}. The height at which the balls collide is

(a) 50 m (b) 75 m

(c) 100 m (d) 125 m

12. A train is moving on a track at 30ms^{-1}. A ball is thrown from it perpendicular to the direction of motion with 30ms^{-1} at 45° from horizontal. Find the distance of ball from the point of projection on train to the point where it strikes the ground.

(a) 90 m (b) $90\sqrt{3}$ m

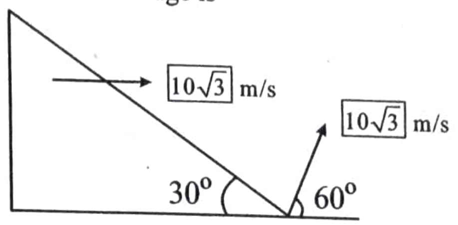
(c) 60 m (d) $60\sqrt{3}$ m

13. Two bodies were thrown simultaneously from the same point, one straight up, and the other, at an angle of θ = 30° to the horizontal. The initial velocity of each body is 20 ms^{-1}. Neglecting air resistance, the distance between the bodies at t = 1.2 later is

(a) 20 m (b) 30 m

(c) 24 m (d) 50 m

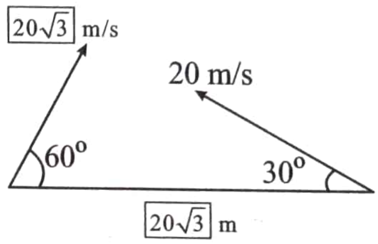
14. A particle is projected at an angle 60° with speed $10\sqrt{3}$ m/s, from the point A, as shown in the figure. At the same time the wedge is made to move with speed $10\sqrt{3}$ m/s towards right as shown in the figure. Then the time after which particle will strike with wedge is



(a) 2s (b) $2\sqrt{3}$ s

(c) $\frac{4}{\sqrt{3}}$ s (d) None of these

15. In the figure shown, the two projectiles are fired simultaneously. The minimum distance between them during their flight is



(a) 20 m (b) $10\sqrt{3}$ m

(c) 10 m (d) None of the above

16. Two trains, one travelling at 15 ms^{-1} and other at 20 ms^{-1}, are heading towards one another along a straight track. Both the drivers apply brakes simultaneously when they are 500 m apart. If each train has a retardation of 1 ms^{-2}, the separation after they stop is

(a) 192.5 m (b) 225.5 m

(c) 187.5 m (d) 155.5 m

17. Two cars are moving in same direction with speed of 30 kmh^{-1}. They are separated by a distance of 5 km. What is the speed of a car moving in opposite direction if it meets the two cars at an interval of 4 min?

(a) 60 kmh^{-1} (b) 5 kmh^{-1}

(c) 30 kmh^{-1} (d) 45 kmh^{-1}

18. Two trains A and B, 100 m and 60 m long, are moving in opposite directions on parallel tracks. The velocity of the shorter train is 3 times that of the longer one. If the trains take 4 s to cross each other, the velocities of the trains are

(a) V_A = 10 ms^{-1}, V_B = 30 ms^{-1}

(b) V_A = 2.5 ms^{-1}, V_B = 7.5 ms^{-1}

(c) V_A = 20 ms^{-1}, V_B = 60 ms^{-1}

(d) V_A = 5 ms^{-1}, V_B = 15 ms^{-1}

19. Two trains, each travelling with a speed of 37.5 kmh^{-1}, are approaching each other on the same straight track. A bird that can fly at 60 kmh^{-1} flies off from one train when they are 90 km apart and heads directly for the other train. On reaching the other train, it flies back to the first and so on. Total distance covered by the bird is

(a) 90 km (b) 54 km
(c) 36 km (d) 72 km

20. Two cars A and B are travelling in the same direction with velocities V_A and V_B ($V_A > V_B$). When the care A is a distance behind car B, the driver of the car A applies the brakes producing a uniform retardation a; there will be no collision when

(a) $s < \frac{(V_A - V_B)^2}{2a}$ (b) $s = \frac{(V_A - V_B)^2}{2a}$

(c) $s \geq \frac{(V_A - V_B)^2}{2a}$ (d) $s \leq \frac{(V_A - V_B)^2}{2a}$

21. A police party is chasing a dacoit in a jeep which is moving at a constant speed v. the dacoit is on a motorcycle. When he is at a distance x from the jeep, he accelerates from rest at a constant rate? Which of the following relations is true if the police is able to catch the dacoit?

(a) $v^2 \leq \alpha x$ (b) $v^2 \leq 2\alpha x$
(c) $v^2 \geq 2\alpha x$ (d) $v^2 \geq \alpha x$

22. Ship A is travelling with a velocity of 5 kmh^{-1} due east. A second ship is heading 30° east of north. What should be the speed of second ship if it is to remain always due north with respect to the first ship?

(a) 10 kmh^{-1} (b) 9 kmh^{-1}
(c) 8 kmh^{-1} (d) 7 kmh^{-1}

23. A river is flowing from west to east at a speed of 5 m/ min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river in the shortest time. Finally he will move in a direction

(a) $\tan^{-1}(2)$ E of N
(b) $\tan^{-1}(2)$ N of E
(c) 30° E of N
(d) 60° E of N

24. A car is moving towards east with a speed of 25 kmh. To the driver of the car, a bus appears to move towards north with a speed of $25\sqrt{3}\ kmh^{-1}$. What is the actual velocity of the bus?

(a) 50 kmh^{-1}, 30° E of N
(b) 50 kmh^{-1} 30° N of E
(c) 25 kmh^{-1} 30° E of N
(d) 25 kmh^{-1} 30° N of E

25. A swimmer wishes to cross a 500 m river flowing at 5 kmh^{-1}. His speed with respect to water is 3 kmh^{-1}. The shortest possible time to cross the river is

(a) 10 min (b) 20 min
(c) 6 min (d) 7.5 min

26. A man can swim in still water with a speed of 2 ms^{-1}. If he wants to cross a river of water current speed $\sqrt{3}\ ms^{-1}$ along shortest possible path, then in which direction should he swim?

(a) At an angle 120° to the water current
(b) At an angle 150° to the water current
(c) At an angle 90° to the water current
(d) None of these

27. Rain, driven by the wind, falls on a railway compartment with a velocity of 20 ms^{-1}, at an angle of 30° to the vertical. The train moves, along the direction of wind flow, at a speed of 108 kmh^{-1}. Determine the apparent velocity of rain for a person sitting in the train.

(a) $20\sqrt{7}\ ms^{-1}$ (b) $10\sqrt{7}\ ms^{-1}$
(c) $15\sqrt{7}\ ms^{-1}$ (d) $10\sqrt{7}\ kmh^{-1}$

28. The ratio of the distance carried away by the water current, downstream, in crossing a river, by a person, making same angle with downstream and upstream is 2: 1. The ratio of the speed of person to the water current cannot be less than

(a) 1/3 (b) 4/5
(c) 2/5 (d) 4/3

29. Twelve persons are initially at the twelve corners of a regular polygon of twelve sides of side a. Each person now moves with a uniform speed v in such a manner that 1 is always directed towards 2, 2 towards 3, 3 towards 4, and so on. The time after which they meet is

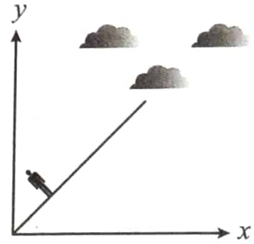
(a) $\frac{v}{a}$ (b) $\frac{2a}{v}$

(c) $\frac{2a}{v(2+\sqrt{3})}$ (d) $\frac{2a}{v(2-\sqrt{3})}$

30. A boat is moving towards east with velocity 4m/s with respect to still water and river is flowing towards north with velocity 2 m/s and the wind is blowing towards north with velocity 6 m/s. The direction of the flag blown over by the wind hoisted on the boat is:

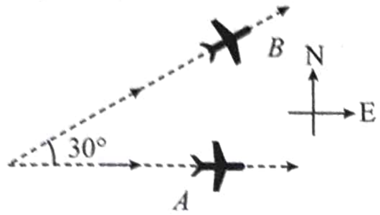
(a) North-west (b) South-east
(c) $\tan^{-1}(1/2)$ with east (d) North

31. To a man running upwards on the hill, the. rain appears to fair, vertically downwards with 4 m/s. The velocity vector of the man w.r.t. earth is $(2\hat{i} + 3\hat{j})$ m/s. If the man starts running down the hill with the same speed, then determine the relative velocity (in m/s) of the rain w.r.t. man.



(a) $2\hat{i} + 4\hat{j}$ (b) $4\hat{i} - 2\hat{j}$
(c) $2\hat{i} - 4\hat{j}$ (d) $4\hat{i} + 2\hat{j}$

32. An aeroplane A is flying horizontally due east at a speed of 400 km/hr. Passengers in A, observe another aeroplane B moving perpendicular to direction of motion at A. Aeroplane B is actually moving in a direction 30° north of east in the same horizontal plane as shown in the Figure. Determine the velocity (in m/s) of B.



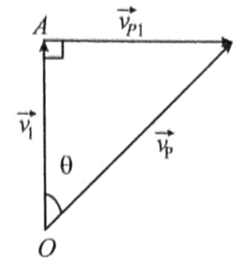
(a) $400\hat{i}+\frac{400}{\sqrt{3}}\hat{j}$

(b) $400\hat{i}+400\hat{j}$

(c) $400\sqrt{3}\hat{i}+400\hat{j}$

(d) $200\sqrt{3}\hat{i}+200\hat{j}$

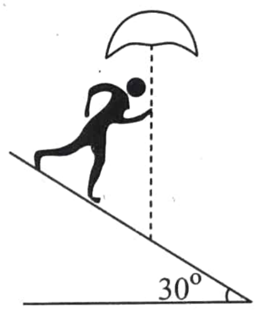
33. An airplane is observed by two persons travelling at 60 km/hour in two vehicles moving in opposite directions on a straight road. To an observer in one vehicle the plane appears' to cross the road track at right angles while to the observer in the other vehicle the angle appears to be 45°. At what angle does the plane actually cross the road track and what is its speed relative to ground?



(a) 45° (b) $\theta=\tan^{-1}(2)$

(c) $\theta=\tan^{-1}(4)$ (d) $\theta=\tan^{-1}(8)$

34. A man is coming down an incline of-angle 30°. When he walks with speed $2\sqrt{3}$ m/s he has to keep his umbrella vertical to protect himself from rain, The actual speed of rain is 3 m/s, At what angle with vertical should he keep his umbrella where he is at rest so that, he does not get drenched?



(a) 30° (b) 37°

(c) 53° (d) 60°

Exercise-3 (JEE Advanced Level)

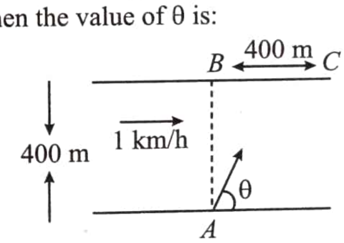
MULTIPLE CORRECT TYPE QUESTIONS

1. An object has velocity $\vec{v}_1$ w.r.t ground. An observer moving with constant velocity $\vec{v}_0$ w.r.t ground measures the velocity of the object as $\vec{v}_2$. The magnitudes of three velocities are related by
   (a) $v_0 \geq v_1+v_2$
   (b) $v_1 \leq v_2+v_0$
   (c) $v_1 \geq v_2+v_0$
   (d) All of the above

2. The minimum speed with respect to air that a particular jet aircraft must have in order to keep aloft is 300 km/hr. Suppose that at its pilot prepares to take off, the wind blows eastward at a ground speed that can vary between 0 and 30 km/hr. Ignoring any other fact, a safe procedure to follow, consistent with using up as little fuel as possible, is to:
   (a) Take off eastward at a ground speed of 320 km/hr
   (b) Take off westward at a ground speed of 320 km/hr
   (c) Take off westward at a ground speed of 300 km/hr
   (d) Take off westward at a ground speed of 280 km/hr

3. A boat moves right across a river with velocity 10 km h^{-1} relative to water. The water has a uniform speed of 5.00 km h^{-1} relative to the earth. Find the velocity of the boat' relative to an observer standing on either bank. If the width of river is 3.0 km, find the time it takes the boat to cross it.
   (a) 6 min (b) 12 min
   (c) 18 min (d) 24 min

4. A river is flowing with a speed of I km/hr. A swimmer wants to go to paint. C starting from 'A'. He swims with a speed of 5 km/hr, at an angle θ w.r.t. the river flow. If $AB = BC$ = 400 m. At what angle with river bank should swimmer swim? Then the value of θ is:



   (a) 30° (b) 37°
   (c) 53° (d) 60°

5. Ship A is located 4 km north and 3 km east of ship B. Ship A has a velocity of 20 kmh^{-1} towards the south and ship B is moving at 40 kmh^{-1} in a direction 37° north of east. X and Y-axes are along east and north directions, respectively
(a) Velocity of A relative to B is $(-32\hat{i} - 44\hat{j})$ km/h
(b) Position of A relative to B as a function of time is given by $r_{AB} = [(3-32t)\hat{i} + (4-44t)\hat{j}]$ km
(c) Velocity of A relative to B is $(32\hat{i} - 44\hat{j})$ km/h
(d) Position of A relative to B as a function of time is given by $(32t\hat{i} - 44t\hat{j})$ km

6. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself
(a) The ball will always return to him.
(b) The ball will never return to him.
(c) The ball will return to him if the cart moves with constant velocity.
(d) The ball will fall behind him if the cart moves with some positive acceleration.

7. A train is running with uniform velocity in east direction. A car is running on a road parallel to the track with uniform speed. After some time, the road becomes perpendicular to railway track. The car driver notices that initial speed of train with respect to itself was 7 m/s and later on it became 13 m/s. What can be the true speed of the driver?
(a) 12 m/s (b) 5 m/s
(c) 13 m/s (d) 7 m/s

8. State which of the following statements is/are false.
(a) If two particles are neither approaching nor separating from each other, then their relative velocity is zero.
(b) If relative velocity of particle B with respect to A is $\vec{v}_1$, relative velocity of particle C with respect to B is $\vec{v}_2$ and particle A moves with velocity $\vec{v}_0$ with respect to ground, then the velocity of C with respect to ground cannot be zero. (assuming $\vec{v}_1, \vec{v}_2$ & $\vec{v}_0$ to be non zero)
(c) Four dogs are running along a line in the same direction, such that each is running relative to the dog in front of him with equal speed. Then the rate of separation between the third and the first dog is same as that of the fourth and the second dog. (where first, second, third and fourth are taken in order)
(d) At some instant of time at a place, two particles are observed and it is found that their relative velocity is zero. Then they will remain stationary with respect to each other.

9. A block is thrown with a velocity of 2 ms^{-1} (relative to ground) on a belt, which is moving with velocity 4 ms^{-1} in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 sec of the throwing then choose the correct statements
(a) Displacement with respect to ground is zero after 2.66s and magnitude of displacement with respect to ground is 12 m after 4 sec.
(b) Magnitude of displacement with respect to ground in 4 sec is 4 m.
(c) Magnitude of displacement with respect to belt in 4 sec is 12 m.
(d) Displacement with respect to ground is zero in 8/3 sec.

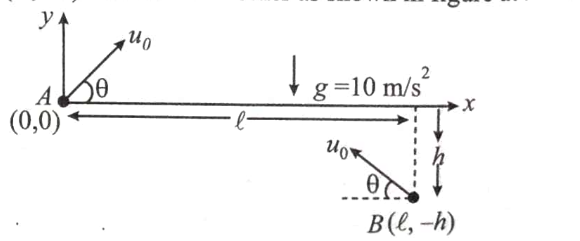
10. A river is flowing with a velocity of 2 m/s. A boat is moving downstream along the river. Velocity of the boat in still water is 3 m/s. A person standing on the boat throws a ball (w.r.t. himself) in a plane perpendicular to the direction of motion of the boat with 10 m/s at 60° with the horizontal. When the ball reaches highest point of its path.
(a) The speed of ball w.r.t. man standing on boat is 5 m/s
(b) The speed of ball w.r.t. river is 3 m/s
(c) The speed of ball w.r.t. river is 0 m/s
(d) The speed of ball w.r.t. ground is $5\sqrt{2}$ m/s

11. Two particles are projected from the same point with the same speed in the same vertical plane at different angles with the horizontal. A frame of reference is fixed to one particle. The position vector of the other particle as observed from the frame is $\vec{r}$. Which of the following statements is/are incorrect?
(a) $\vec{r}$ is a constant vector
(b) $\vec{r}$ changes in magnitude and direction with time
(c) The magnitude of $\vec{r}$ increases linearly with time, its direction does not change
(d) The direction of $\vec{r}$ changes with time, its magnitude may or may not change depending on the angles of projection.

12. Consider a shell that has a muzzle velocity of 45 ms^{-1} fired from the tail gun of an airplane moving horizontally with a velocity of 215 ms^{-1}. The tail gun can be directed at any angle with the vertical in the plane of motion of the airplane. The shell is fired when the plane is above point A on ground, and the plane is above point B on ground when the shell hits the ground. (Assume for simplicity that the Earth is flat)
(a) Shell may hit the ground at point A.
(b) Shell may hit the ground at point B.
(c) Shell may hit a point on earth which is behind point A.
(d) Shell may hit a point on earth which is ahead of point B.

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 13 to 15): Two particles 'A' and 'B' are projected in the vertical plane with same initial speed u_0 from part (0, 0) and (ℓ, $-h$) towards each other as shown in figure at $t = 0$.



13. The path of particle 'A' with respect to particle 'B' will be
   (a) Parabola
   (b) Straight line parallel to x-axis
   (c) Straight line parallel to y-axis
   (d) None of these

14. Minimum distance between particle A and B during motion will be :
   (a) ℓ (b) h
   (c) $\sqrt{\ell^2+b^2}$ (d) $\ell + h$

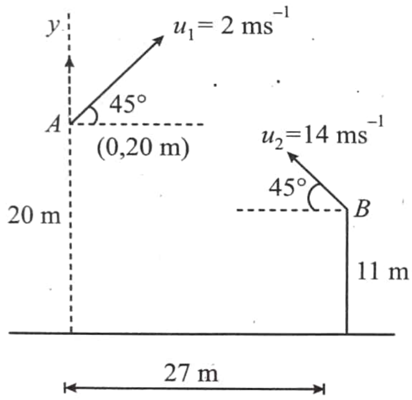
15. The time when separation between A and B is minimum is:
   (a) $\dfrac{\ell}{u_0 \cos\theta}$ (b) $\sqrt{\dfrac{2h}{g}}$
   (c) $\dfrac{\ell}{2u_0 \cos\theta}$ (d) $\dfrac{2\ell}{u_0 \cos\theta}$

Comprehension (Q. 16 to 17): A car is moving toward south with a speed of 20 ms^{-1}. A motorcyclist is moving toward east with a speed of 15 ms^{-1}. At a certain instant, the motorcyclist is due south of the car and is at a distance of 50 m from the car.

16. The Shortest distance between the motorcyclist and the car is
   (a) 20 m (b) 10 m
   (c) 40 m (d) 30 m

17. The time after which they are closest to each other
   (a) 13 s (b) 8/3 s
   (c) 1/5 s (d) 8/5 s

Comprehension (Q. 18 to 20): Two particles are thrown simultaneously from points A and B with velocities $u_1 = 2$ ms^{-1} and $u_2 = 14$ ms^{-1}, respectively, as shown in figure.



18. The relative velocity of B as seen from A in
   (a) $-8\sqrt{2}\hat{i}+6\sqrt{2}\hat{j}$ (b) $4\sqrt{2}\hat{i}+3\sqrt{3}\hat{j}$
   (c) $3\sqrt{5}\hat{i}+2\sqrt{3}\hat{j}$ (d) $3\sqrt{2}\hat{i}+4\sqrt{3}\hat{j}$

19. The direction (angle) with horizontal at which B will appear to move as seen from A is
   (a) 37° (b) 53°
   (c) 15° (d) 90°

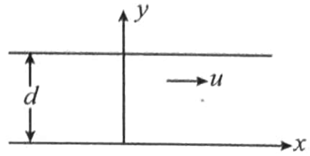
20. Minimum separation between A and B is
   (a) 3 m (b) 6 m
   (c) 12 m (d) 9 m

Comprehension (Q. 21 to 23): We know that when a boat travels in water, its net velocity w.r.t. ground is the vector sum of two velocities. First is the velocity of boat itself in river and other is the velocity of water w.r.t. ground. Mathematically:

$$\vec{v}_{\text{boat}} = \vec{v}_{\text{boat, water}} + \vec{v}_{\text{water}}$$

Now given that velocity of water w.r.t. ground in a river is u.

Width of the river is d.



A boat starting from aims perpendicular to the river with an acceleration of $a = 5t$, where t is time. The boat starts from point (1, 0) of the coordinate system in assume SI units.

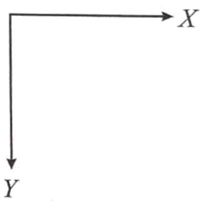
21. Obtain the total time taken to cross the river.
   (a) $(3d/5)^{1/3}$ (b) $(6d/5)^{1/3}$
   (c) $(6d/5)^{1/2}$ (d) $(2d/3)^{1/3}$

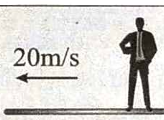
22. Find the equation of trajectory of the boat.
   (a) $x-1=\left(\dfrac{3y}{5}\right)^{1/3}$ (b) $x=u\left(\dfrac{6y}{5}\right)^{1/3}$
   (c) $x-1=u\left(\dfrac{6y}{5}\right)^{1/3}$ (d) None of these

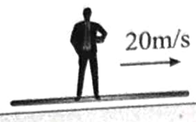
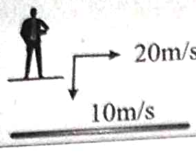
23. Find the drift of the boat when it is in the middle of the river.
   (a) $u\left(\dfrac{3d}{5}\right)^{1/3}$ (b) $u\left(\dfrac{3d}{5}\right)^{1/3}+1$
   (c) $u\left(\dfrac{6d}{5}\right)^{1/3}$ (d) None of these

MATCH THE COLUMN TYPE QUESTIONS

24. Rain is falling at velocity $10\hat{i}+10\hat{j}$ m/s. Then match the direction of velocity of rain w.r.t. man in Column-II from Column-I.



	Column-I		Column-II
A.		p.	
B.		q.	

C.		r.	↘
D.		s.	↙
		t.	←

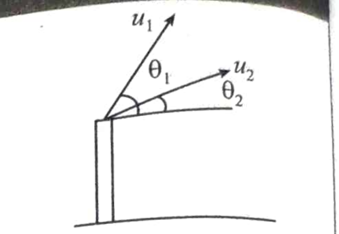
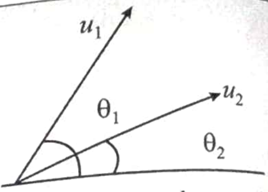
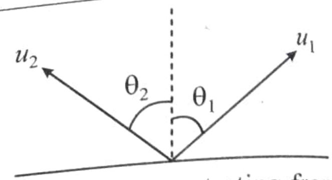
(*a*) A-(q); B-(r); C-(s); D-(t)
(*b*) A-(p); B-(s); C-(t); D-(r)
(*c*) A-(s); B-(t); C-(s); D-(r)
(*d*) A-(q); B-(s); C-(r); D-(t)

25. Match Column-I with Column-II and select the correct answer using the codes given below the lists.

	Column-I		Column-II
A.	If swimmer can swim at 5m/sec in still water and if velocity of water flow is 4m/sec then angle between direction of swimming and direction of river flow to minimize drift.	p.	53°
B.	If swimmer can swim at 5 m/sec in still water and velocity of flow is 3 m/sec then angle between direction of velocity of swimmer with respect to river and the direction of river flow if swimmer crosses the river in minimum time.	q.	127°
C.	If swimmer can swim at 4 m/sec and velocity of flow is 3m/sec then angle of resultant velocity (w.r.t. ground) with the direction of river flow if swimmer swims perpendicular to flow of river.	r.	143°
D.	Angle between direction of fluttering of flag and north if wind blows towards south west direction with a velocity $3\sqrt{2}$ m/sec. Man moves with a velocity 7m/sec along west, holding flag in his hand.	s.	90°

(*a*) A-(q); B-(r); C-(s); D-(p)
(*b*) A-(r); B-(s); C-(p); D-(q)
(*c*) A-(s); B-(q); C-(s); D-(r)
(*d*) A-(q); B-(s); C-(r); D-(q)

26. Column-I shows certain situations with certain conditions and Column-II shows the parameters in which situations of Column-I match. Which can be possible combination.

	Column-I		Column-II
A.	$u_1 = u_2; \theta_1 = \theta_2$	p.	 Two projectiles are projected from a height such that they strike ground at the same time.
B.	$u_1 > u_2; \theta_1 > \theta_2$	q.	 Two projectiles under standard ground to ground projection such that horizontal range is same.
C.	$u_1 < u_2; \theta_2 > \theta_1$	r.	 Two swimmer starting from same point on a river bank such that time of crossing is same. u_1 and u_2 are velocities relative to river.

(*a*) A-(p,s,r); B-(q,s,p); C-(r,s)
(*b*) A-(p,r); B-(q,p,s); C-(q,r)
(*c*) A-(p,s,r); B-(q,r,p); C-(q,p,s)
(*d*) A-(p,q,r); B-(q,r,s); C-(q,r,s)

27. A ball is thrown vertically upward in the air by a passenger (relative to himself) from a train that is moving as given in Column-I ($v_{ball} << v_{escape}$). Correctly match the situation as described in the Column-I, with the paths given in Column-II.

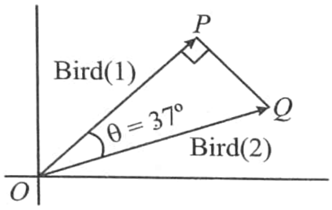
	Column-I		Column-II
A.	Train moving with constant acceleration on a slope then path of the ball as seen by the passenger.	p.	Straight line
B.	Train moving with constant acceleration on a slope then path of the ball as seen by a stationary observer outside.	q.	Parabolic

C.	Train moving with constant acceleration on horizontal ground then path of the ball as seen by the passenger.	r.	Elliptical
D.	Train moving with constant acceleration on horizontal ground then path of the ball as seen by a stationary observer outside.	s.	Hyperbolic
		t.	Circular

(*a*) A-(q); B-(q); C-(q); D-(q)
(*b*) A-(q); B-(q); C-(q); D-(s)
(*c*) A-(q); B-(s); C-(q); D-(q)
(*d*) A-(q); B-(q); C-(r); D-(q)

NUMERICAL TYPE QUESTIONS

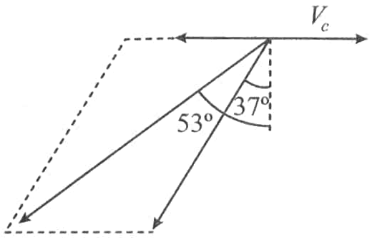
28. Two birds are at origin. They both start to fly in straight lines in different directions making an angle $\theta = 37°$ with each other at the same time as shown.



Bird(1) on reaching a point P takes a right angle turn to reach point Q simultaneously with Bird (2). If the bird (1) was flying with a constant speed of 7 km/hr then what must be the constant speed of bird (2) (in km/hr)?

29. Shore based radar indicates that a ferry boat is moving on a river with a speed of 10 m/s at an angle of 30° North of east. The instruments on the ferry boat indicate that it moves with a speed of 10 m/s at an angle of 30° North of west relative to the river. What is the true speed (in m/s) at which river flows? Round off to nearest integer.

30. A car is travelling in steady rain with constant acceleration in a straight line. When it begins to move the driver sees that the raindrops make track at an angle of 37° with the vertical on the side window. After 20 sec., the raindrops make track at an angle of 53° with vertical in same direction. Find the acceleration of the car in cm/s². Rain is falling at 3 m/s.



31. Two particles P and Q move with constant velocities $v_1 = 2\ \text{ms}^{-1}$ and $v_2 = 4\ \text{ms}^{-1}$ along two mutually perpendicular straight lines towards the intersection point O. At moment $t = 0$, the particles were located at distances $l_1 = 12$ m and $l_2 = 19$ m from O, respectively. Find the time when they are nearest and also this shortest distance (nearest integer).

Exercise-4 (Past Year Questions)

JEE MAIN

1. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in the same direction, and (ii) in the opposite direction is : **(2019)**

(*a*) $\frac{11}{5}$ (*b*) $\frac{5}{2}$
(*c*) $\frac{3}{2}$ (*d*) $\frac{25}{11}$

2. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in: **(2019)**

(*a*) 4.2 hrs.   (*b*) 2.2 hrs.
(*c*) 3.2 hrs.   (*d*) 2.6 hrs.

3. The stream of a river is flowing with a speed of 2km/h. A swimmer can swim at a speed of 4km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **(2019)**

(*a*) 60°   (*b*) 150°
(*c*) 90°   (*d*) 120°

4. A particle is moving along the x-axis with its coordinate with time t given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1s$, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v(in m/s) is **(2020)**

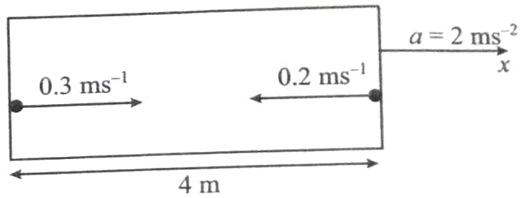
(*a*) 580   (*b*) 500
(*c*) 540   (*d*) 560

5. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45°. The value of β is close to **(2020)**
(*a*) 0.37
(*b*) 0.41
(*c*) 0.73
(*d*) 0.50

6. A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a: **(2021)**
(*a*) hyperbola
(*b*) parabola in the direction of motion of plane
(*c*) straight line vertically down the plane
(*d*) parabola in a direction opposite to the motion of plane

JEE ADVANCED

7. A rocket is moving in a gravity free space with a constant acceleration of $2ms^{-2}$ along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of $0.3ms^{-1}$ relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of $0.2ms^{-1}$ from its right end relative to the rocket. The time in seconds when the two balls hit each other is: **(2014)**



ANSWER KEY

CONCEPT APPLICATION

1. (*d*)	2. [3 m]	3. (*b*)	4. (*b*)	5. (*d*)	6. (*a*)	7. (*b*)	8. (*b*)	9. [40 m]

EXERCISE-1 (TOPICWISE)

1. (*b*)	2. (*b*)	3. (*d*)	4. (*b*)	5. (*c*)	6. (*d*)	7. (*b*)	8. (*a*)	9. (*c*)	10. (*b*)
11. (*b*)	12. (*d*)	13. (*b*)	14. (*c*)	15. (*a*)	16. (*b*)	17. (*c*)	18. (*b*)	19. (*a*)	20. (*a*)
21. (*c*)	22. (*b*)	23. (*c*)	24. (*a*)	25. (*b*)	26. (*b*)	27. (*c*)	28. (*b*)	29. (*c*)	30. (*b*)
31. (*c*)	32. (*d*)	33. (*a*)	34. (*c*)	35. (*b*)	36. (*b*)	37. (*a*)	38. (*d*)	39. (*d*)	

EXERCISE-2 (LEARNING PLUS)

1. (*d*)	2. (*a*)	3. (*a*)	4 (*b*)	5. (*b*)	6. (*a*)	7. (*a*)	8. (*c*)	9. (*d*)	10. (*b*)
11. (*b*)	12. (*a*)	13. (*c*)	14. (*a*)	15. (*b*)	16. (*c*)	17. (*d*)	18. (*c*)	19. (*d*)	20. (*a*)
21. (*c*)	22. (*a*)	23. (*b*)	24. (*a*)	25. (*a*)	26. (*b*)	27. (*b*)	28. (*a*)	29. (*d*)	30. (*a*)
31. (*d*)	32. (*a*)	33. (*b*)	34. (*b*)						

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*b*)	2. (*b*)	3. (*c*)	4. (*c*)	5. (*a,b*)	6. (*c, d*)	7. (*a, b*)	8. (*a,b,d*)	9. (*b,c,d*)	10. (*a,d*)
11. (*a,b*)	12. (*b,d*)	13. (*b*)	14. (*b*)	15. (*c*)	16. (*d*)	17. (*d*)	18. (*a*)	19. (*b*)	20. (*d*)
21. (*b*)	22. (*c*)	23. (*a*)	24. (*a*)	25. (*b*)	26. (*d*)	27. (*a*)	28. [0005]	29. [0017]	30. [0007]
31. [5]									

EXERCISE-4 (PAST YEARS QUESTIONS)

JEE Main

1. (*a*)	2. (*d*)	3. (*d*)	4. [580]	5. (*c*)	6. (*c*)

JEE Advanced

7. [2]