ARJUNA

for

JEE

PHYSICS

Full Course Study Material **XI**

SOLUTIONS

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Email: support@pw.live

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PREFACE

A highly skilled professional team of Physics Wallah (PW) works arduously to ensure that the students receive the best content for the **JEE** exam.

From the beginning, the whole content team comprising Associate faculties, DTP operators, Proofreaders and others are involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

Faculties have adopted a new style of presenting the content in easy-to-understand language and have provided the team with their guidance and supervision throughout the creation of this Study Material.

Physics Wallah (PW) strongly believes in conceptual and fun-based learning. PW provides highly exam-oriented content to bring quality and clarity to the students.

A plethora of **JEE Study Material** is available in the market but PW professionals are continuously working to provide the supreme Study Material for our **JEE** students.

This Study Material adopts a multi-faceted approach to master and understanding the concepts by having a rich diversity of questions asked in the examination and equip the students with the knowledge for the competitive exam.

The main objective of the study material is to provide a large number of quality problems with varying cognitive levels to facilitate the teaching-learning of concepts that are presented through the book.

It has become popular among aspirants because of its easy-to-understand language and clear illustrations with diagrams, mnemonics, flow charts and tables.

Students can benefit themselves by attempting the exercise given in this Study Material for the self-assessment and also mastering the basic techniques of problem-solving.

The questions are strictly designed in accordance with the exam relevant topics that helps to develop examination temperament within the students.

Mastering the Physics Wallah (PW) study material curated by the PW team, the students can easily qualify for the exam with a Top Rank in the **JEE**.

In each chapter, for better understanding, questions have been classified according to the latest syllabus of **JEE**

- The nature and diversity of the questions help students to ace the examination
- Quality questions to strengthen the concept of the topic at the zenith level

BOOK FEATURES

- Detailed theory with Train Your Brain and Concept Application Questions
- Short Notes & Solved Miscellaneous Examples
- Topicwise, Learning Plus & Advance Level MCQs covered in exercises
- Match the Column, Comprehension Based, Numerical Type and Multicorrect questions covered for complete practice.
- JEE Past Year Questions
- Elaborated Solutions

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CHAPTER

1 Units and Measurements

Exercise-1 (Topicwise)

1. (*b*) Solar day → Time for earth to take a complete rotation on its axis

Parallactic second [1 Parsec] → It is a distance corresponding to a parallax of one second of arc.

Leap year → A leap year is year (time) containing one extra day.

Lunar Month → A lunar month is the time between two identical view moons of full moons.

1 Lunar month = 29.53059 days.

2. (*a*) Fact based

3. (*a*) Kilogram is not a physical quantity, its a unit.

4. (*c*) PARSEC is a unit of distance.

It is used in astronomical science.

5. (*b*) SI system is NOT based on unit of mass, length and time alone.

This system is based on all 7 Fundamental physical quantities and 2 supplementary physical quantities.

6. (*c*) S.I. unit of energy is Joule.

7. (*b*) From law of gravitation, $F = \frac{GM_1M_2}{R^2}$

Here M_1 and M_2 are masses

R = Distance between them M_1 and M_2

F = Force

$\Rightarrow G = \frac{FR^2}{M_1M_2} = \frac{\text{N m}^2}{\text{kg}^2}$

So, Unit of G = N m^2 kg^{-2}

8. (*b*) Surface Tension (T): Energy per unit area.

$T = \frac{E}{A} = \frac{\text{J}}{\text{m}^2}$

9. (*d*) $R = \frac{\rho l}{A} \Rightarrow \text{ohm} = \frac{\rho \text{m}}{\text{m}^2} \Rightarrow \rho = \text{ohm} \times \text{m}$

10. (*a*) $M = iA$ = Amp.m^2

Here i = current

A = cross-sectional Area

11. (*d*) $PV = nRT$

$P \rightarrow$ Pressure, $V \rightarrow$ Volume, $T \rightarrow$ Temperature

$R = \frac{PV}{nT}$

$= \frac{\text{N/m}^2 \times \text{m}^3}{\text{mol} \times \text{K}}$

$= \frac{\text{N m}}{\text{mol} \times \text{K}} = \text{Joule K}^{-1}\text{mol}^{-1}$

12. (*d*) Stefan-Constant(σ)

Unit → W/m^2 – K^4 = Wm^{-2} K^{-4}

13. (*c*) α = angular velocity/time = rad/s^2 = rad.s^{-2}

14. (*c*) Angular Frequency (f) $= \frac{1}{T} = M^oL^oT^{-1}$

So, here dimension in length is zero

15. (*b*) $P = mv$

Dimension of $[P] = [\text{MLT}^{-1}]$

16. (*b*) $E = kT \Rightarrow k = \frac{E}{T}$

Unit → J/k

Dimension $= \frac{\text{M}^1\text{L}^2\text{T}^{-2}}{\text{K}^1} = \text{M}^1\text{L}^2\text{T}^{-2}\text{K}^{-1}$

17. (*b*) Find out dimension of each physical quantity in all options.

$L = Mvr = \text{ML}^2\text{T}^{-1}$

Dimension of Torque (τ) = $\vec{r} \times \vec{F}$

(τ) = $\text{L}^1 \times \text{M}^1\text{L}^1\text{T}^{-2}$

It is also a dimension of Energy. = ML^2T^{-2}

Power = W/t = $\text{ML}^2\text{T}^{-2}/\text{T} = \text{ML}^2\text{T}^{-3}$

18. (*d*) Find dimension in all options.

Here stress = Force/Area

$= \frac{M^1L^1T^{-2}}{L^2}$

stress = $[\text{M}^1\text{L}^{-1}\text{T}^{-2}]$

19. (*b*) $\frac{\text{W}}{m = \text{RD}} = \frac{\text{MLT}^{-2}}{\text{M}} = \text{L}^2\text{T}^{-2} = (\text{Velocity})^2$

20. (*a*) $I = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = MT^{-3}$

21. (*c*) $[h] = ML^2T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

So, $\frac{h}{G} = M^2L^{-1}T$

22. (*c*) Dimensionless quantity may have a unit

Ex. Angle: Unit → Radian

Dimension → $M^0L^0T^0$

23. (*c*) All the terms in the equation must have the dimension of force

$\therefore [A \sin Ct] = MLT^{-2}$

$\Rightarrow [A]\,[M^0L^0T^0] = MLT^{-2} \Rightarrow [A] = MLT^{-2}$

Similarly, $[B] = MLT^{-2}$

$\therefore \frac{[A]}{[B]} = M^0L^0T^0$

Again $[Ct] = M^0L^0T^0 \Rightarrow [C] = T^{-1}$

$[Dx] = MLT^0 \Rightarrow [D] = L^{-1}$

$\Rightarrow \frac{[C]}{[D]} = M^0L^1T^{-1}$.

24. (*a*) $k_B = \frac{v^2m}{T} = \frac{(L^2T^{-2})(M)}{K} = ML^2T^{-2}K^{-1}$

$= kg\ m^2s^{-2}\ K^{-1}$

25. (*b*) $y = a\sin(At - Bx + C)$

Angle has no dimensions so

Dimensions of $At = M^0L^0T^0$

$\Rightarrow A = T^{-1}$

Dimensions of $Bx = M^0L^0T^0$

$\Rightarrow B = L^{-1}$

Dimensions of $C = M^0L^0T^0$

26. (*a*) Dimensions of $\sqrt{\frac{\gamma P}{\rho}}$ = Dimensions of speed.

$\Rightarrow \gamma L^2T^{-2} = L^2T^{-2}$ $\left(\frac{P}{\rho} = LT^{-1}\right)$

$\gamma = M^0L^0T^0$

27. (*c*) $A = \frac{d(\text{work})}{dt}\frac{1}{FP}$

$[A] = \frac{[\text{work}]}{[\text{time}]\frac{[\text{work}]}{[\text{displacement}]} \times [P]}$

$= \frac{L}{T}\left(\frac{1}{MLT^{-1}}\right) = \frac{1}{M}$

28. (*a*) $[F] = \frac{[A]}{M^{1/2}} \Rightarrow [A] = M^{3/2}LT^{-2}$

$[B] = [F] = MLT^{-2}$

So, $[A \times B] = M^{5/2}L^2T^{-4}$

29. (*d*) $[v] = [At^3] \Rightarrow [A] = LT^{-4}$

30. (*b*) $[v] = [k]\,[\lambda^a \rho^b g^c] \Rightarrow LT^{-1} = L^a M^b L^{-3b} L^c T^{-2c}$

$\Rightarrow LT^{-1} = M^b L^{a-3b+c} T^{-2c}$

$\Rightarrow a = ½, b = 0, c = ½$

so, $v^2 = kg\lambda$.

31. (*b*) $F \propto \rho^a v^b A^c$

$MLT^{-2} = M^aL^{-3a}L^bT^{-b}L^{2c} = M^aL^{-3a+b+2c}T^{-b}$

$\Rightarrow a = 1, b = 2, c = 1$

So, $F \propto \rho v^2 A$

32. (*b*) $E = Kv^a f^b m^c$ ...(i)

Substituting the Dimensions

$ML^2T^{-2} = [L]^a\,[T^{-1}]^a\,[T^{-1}]^b\,[M]^c$

$= [M]^c\,[L]^a\,[T]^{-a-b}$

Now comparing the powers,

For M, $1 = c$

For L, $2 = a$

For T, $-2 = -a - b \Rightarrow -2 = -2 - b$

$\Rightarrow b = 0$

$\therefore E = Kv^2f^0m$

33. (*c*) Acc. to given question

Power, $P = KF^av^b\rho^c$

$ML^2T^{-3} = [MLT^{-2}]^a \times [LT^{-1}]^b \times [ML^{-3}]^c$

$= [M]^{a+c}\,[L]^{a+b-3c}\,[T]^{-2a-b}$

Comparing the coefficients

For M, $1 = a + c$

L, $2 = a + b - 3c$

T, $-3 = -2a - b$

Solving for a, b, c we get

$a = 1, \quad b = 1, \quad c = 0$

$\therefore P \propto Fv\rho^0$

34. (*b*) $v = Kg^ph^q$

$M^0LT^{-1} = [LT^{-2}]^p\,[L]^q$

$M^0LT^{-1} = L^{p+q}\,T^{-2p}$

Comparing the coefficients,

For L, $1 = p + q$

For T, $-1 = -2p$

$\Rightarrow p = \frac{1}{2}$

Solving for q; we get $q = \frac{1}{2}$

35. (*c*) $P = \frac{W}{t}$

Watt = Joule/sec.

Joule = Watt-sec.

One watt-hour = 1 watt × 60 × 60 sec

= 3600 watt-sec

= 3600 Joule

= 3.6×10^3 Joule

36. (*a*) Given $P = 10^6$ dyne/cm^2

$$n_1u_1 = n_2u_2$$

$$n_1\left[M_1^1L_1^{-1}T_1^{-2}\right] = 10^6\left[M_2^1L_2^{-1}T_2^{-2}\right]$$

$$n_1 = 10^6\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^{-1}\left[\frac{T_2}{T_1}\right]^{-2}$$

$$= 10^6\left[\frac{1}{1000}\right]^1\left[\frac{1}{100}\right]^{-1}\left[\frac{1}{1}\right]^{-2}$$

$$= 10^6 \times \frac{10^2}{10^3} = 10^5 \text{ N/m}^2$$

37. (*b*) $\rho = 2\text{g/cm}^3$

$$n_1u_1 = n_2u_2$$

$$n_1\left[M_1^1L_1^{-3}\right] = 2\left[M_2^1L_2^{-3}\right]$$

$$n_1 = 2\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_l}\right]^{-3}$$

$$= 2\left[\frac{10^{-3}}{1}\right]^1\left[\frac{10^{-2}}{1}\right]^{-3}$$

$$= 2 \times 10^{-3} \times 10^6$$

$$= 2 \times 10^3 \text{ kg/m}^3$$

38. (*a*) $n_2 = 13600\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-3}$

$$= 13600\left[\frac{1000}{1}\right]^1\left[\frac{100}{1}\right]^{-3}$$

$$n_2 = 13.6 \text{ gcm}^{-3}$$

39. (*b*) $n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$

$$= 20\left[\frac{1\text{ gm}}{1\text{ kg}}\right]^1\left[\frac{1\text{ cm}}{1\text{ m}}\right]^1\left[\frac{1\text{ sec}}{1\text{ sec}}\right]^{-2}$$

$\{\because$ As $F = MLT^{-2}$ $a = 1, b = 1, c = -2\}$

$$= 20 \times \frac{1}{1000} \times \frac{1}{100} \times 1$$

$$= 20 \times 10^{-5} \text{ N}$$

40. (*a*) $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

$$= 100\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ sec}}{\gamma\text{ sec}}\right]^{-2}$$

$\because$ For powers we use dimension of work done $= ML^2T^{-2}$

$n_2 = 100\,\alpha^{-1}\beta^{-2}\gamma^2$ {In the new-system}

41. (*c*) Measurement 900×10^{-4} m is most accurate as significant figure is 3,

42. (*c*) 70.40s four significant figures.

Time period = 3.520 sec. (4 significant figure)

43. (*a*) $KE = \frac{1}{2}mv^2$

$$\frac{\Delta K}{K} \times 100 = 1\% + 2 \times 2\% = 5\%$$

44. (*a*) The third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore, be increased by 1. The digits to be dropped should be replaced by zeros because they appear to the left of the decimal. Thus, 15462 becomes 15500 on rounding to three significant digits.

45. (*a*) We have $\frac{25.2 \times 1374}{33.3} = 1039.7838$ .....

Out of the three numbers given in the expression 25.2 and 33.3 have 3 significant digits and 1374 has four. The answer should have three significant digits. Rounded 1039.7838 ... to three significant digits, it becomes 1040.

Thus, we write.

$$\frac{25.2 \times 1374}{33.3} = 1040$$

46. (*a*) 24.36 + 0.0623 + 256.2 = 280.6223

Rounding off to one decimal place = 280.6

47. (*b*) $\therefore\ E = \frac{1}{2}mv^2$

$\therefore$ % Error in K.E.

= % error in mass + 2 × % error in velocity

= 2 + 2 × 3 = 8 %

48. (*b*) For *n* times increase in number of observations, random error decreases by a factor of $1/n$.

49. (*b*) Number of significant figures are 3, because 10^3 is decimal multiplier.

50. (*b*) $\because\ V = \frac{4}{3}\pi r^3$

$\therefore$ % error is volume = 3 × % error in radius = 3 × 1%

51. (*c*) Mean time period $\bar{T} = 2.00$ sec

& Mean absolute error = $\Delta T = 0.05$ sec.

To express maximum estimate of error, the time period should be written as (2.00 ± 0.05) sec

52. (*c*) Fact based

53. (*a*) $\frac{1}{20} = 0.05$

$\therefore$ Decimal equivalent upto 3 significant figures is 0.0500

54. (*a*) Since percentage increase in length = 2 %

Hence, percentage increase in area of square sheet

2 × 2% = 4%

55. (*b*) $A = l \times b$

$$\Rightarrow \frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$\frac{\Delta A}{A} \times 100\% = \frac{\pm 0.2}{20} \times 100\% + \frac{\pm 0.1}{10} \times 100\% = \pm 2\%$$

56. (*b*) $\frac{dm}{dt} = \rho AV$

$$\Rightarrow \frac{dm}{dt}\times 100\% = \frac{\Delta\rho}{\rho}\times 100\% + \frac{\Delta A}{A}\times 100\% + \frac{\Delta V}{V}\times 100\%$$

$$= 0 \pm \frac{0.1}{10}\times 100\% + \frac{\pm 0.3}{30}\times 100\% = 4\%$$

57. (*c*) 1 main scale div = 0.5 mm

$10V = 9S$

$V = \frac{9}{10}S$

$S - V = S - \frac{9}{10}S = \frac{1}{10}S.$

$\therefore$ Vernier constant $= \frac{0.5 \text{ mm}}{10} = 0.05$ mm

58. (*c*) $20V = 19S$, $V = \frac{19}{20}S$

$S - V = S - \frac{19}{20}S$

$\Rightarrow$ vernier $= \frac{S}{20}$

$0.1 \text{ mm} = \frac{S}{20}$

$\therefore 1S = 20 \times 0.1 \text{ mm} = 2$ mm.

59. (*c*) 1 VC = 1 MSD – VSD

$1\text{VC} = 0.1 \text{ cm} - \frac{n}{m}$

$0.02 \text{ cm} = \frac{1}{10} - \frac{n}{m}$

$\frac{n}{m} = \frac{1}{10} - \frac{2}{100}$

$n = 10$, $m = 0.8$ cm

60. (*a*) Pitch = Linear distance moved by the screw in 1 complete rotation.

$\therefore$ No. of revolution $= \frac{\text{Total Distance moved}}{\text{Pitch}}$

$= \frac{0.35}{0.05} = 7$ rotation.

61. (*b*) Mean value of $m = \frac{m_1 + m_2 + m_3 + m_4}{4}$;

$m \rightarrow$ measurement

$= \frac{1.22 + 1.23 + 1.19 + 1.20}{4}$

$= 1.21$ mm

Absolute Deviations

1st Reading, $|\Delta m_1| = |\bar{M} - M_1| = 0.01$

2nd Reading, $|\Delta m_2| = |\bar{M} - M_2| = 0.02$

3rd Reading, $|\Delta m_3| = |\bar{M} - M_3| = 0.02$

4th Reading, $|\Delta m_4| = |\bar{M} - M_4| = 0.01$

$\therefore$ Mean $(\overline{\Delta m})$ Absolute value

$$= \frac{|\Delta M_1| + |\Delta M_2| + ... + |\Delta M_4|}{4} = 0.015 = 0.015$$

$\therefore$ Reading with Error $= \bar{m} \pm \overline{\Delta M} = 1.21 \pm 0.015$

$\therefore$ % age Error $= \frac{\pm 0.015}{1.21}\times 100\% = \pm 2.71\%$

Exercise-2 (Learning Plus)

1. (*b*) Unit of impulse $\Rightarrow$ Impulse = Force × time

$= \text{kg}\,\frac{m}{\text{sec}^2}\text{sec} = \text{kg}\,\frac{m}{\text{sec}} = mv$

The unit is same as the unit of linear momentum.

2. (*d*) Energy $W = f \times d = \text{Nm}$

$W = eV =$ electron-volt

$W = p \times t =$ Watt hour

So, kg × m/sec^2 is not the unit of energy.

3. (*c*) Only same physical quantities can be added or subtracted. Multiplication/division will give a new quantity.

So, *a*/*b* denote a new physical quantity.

4. (*b*) $P = P_0 \text{ Exp}(-\alpha t^2)$

Here Exp $(-\alpha t^2)$ is a dimensionless

So, dimension of $[\alpha t^2] = M^0L^0T^0$

So, $[\alpha] = \frac{M^oL^oT^o}{T^2}$

$[\alpha] = M^oL^oT^{-2}$

5. (*c*) By Checking the dimension in all options

Moment of Inertia $= Mr^2$

$= M^1L^2T^o$

Moment of force $= r \times F$

$= L^1 \times M^1L^1T^{-2}$

$= M^1L^2T^{-2}$

6. (*d*) Action = Energy × Time $= M^1L^2T^{-2} \times T^1$

$= M^1L^2T^{-1}$

It is same as dimension of Impulse × distance

$= MLT^{-1} \times L^1 = M^1L^2T^{-1}$

7. (*a*) $M^1L^2T^{-2}$ is a dimension of kinetic energy.

8. (*b*) $\frac{EJ^2}{M^5G^2}$ $J = mvr$, $J = [ML^2T^{-1}]$

$= [M^0L^0T^0]$

Dimension of Angle $= [M^0L^0T^0]$

9. (*c*) $x(t) = \frac{V_o}{\alpha}[1 - e^{-\alpha t}]$

Here $e^{-\alpha t}$ is dimensionless so,

$[\alpha]\,[t] = M^0L^0T^0$

$[\omega] = \frac{M^0L^0T^0}{T^{-1}} = T^{-1}$

$[\omega] = M^0L^0T^{-1}$

Here $1 - e^{-\omega t}$ is a number

$[V(t)] = \frac{V_0}{\omega}$

$[V_0] = [V^{-1}][T^{-1}]$

$[V_0] = M^0L^1T^{-1}$

10. (d) $F = Bv^{-1} + \omega t$

We have, $[F] = [Bv^{-1}]$

$[B] = \frac{[F]}{[v^{-1}]} = [F \times t] = MLT^{-2} \times T = MLT^{-1}$

We find it is same as dimension of momentum = MLT^{-1}

11. (d) $Y = a \sin(bt - cx)$

Here bt is dimensionless (Angle)

$[bt] = M^0L^0T^0$

$[b] = \frac{M^0L^0T^0}{[T^{-1}]} = M^0L^0T^{-1}$

It is a dimension of wave frequency.

12. (c) Here $\sqrt{1 + \frac{2\ell}{ma}}$ is a number.

It's a dimensionless quantity.

$\left[\frac{2k\ell}{ma}\right] = [M^0L^0T^0]$

$[k] = \frac{[m][a]}{[\ell]}$

$= \frac{M^1L^1T^{-2}}{L^1} = M^1L^0T^{-2}$

So dimension of $[b]$ is

$[b] = \left[\frac{ma}{k}\right] = \left[\frac{MLT^{-2}}{MT^{-2}}\right]$

$[b] = L$

Unit of b is metre

13. (d) $\alpha = \frac{F}{v^2}\sin(\beta t)$

Here $\sin(\beta t)$ is dimensionless.

$[\beta t] = M^0L^0T^0$

$\beta = \frac{M^0L^0T^0}{T^1} = [T^{-1}]$

$[\alpha] = \left[\frac{F}{v^2}\right] = \frac{M^1L^1T^{-2}}{[L^1T^{-1}]^2} = \frac{M^1L^1T^{-2}}{L^2T^{-2}}$

$[\alpha] = [M^1L^{-1}T^0]$

14. (d) $L \propto F^aA^bT^c$

$L = K F^aA^bT^c$ ....(i)

$M^0L^1T^0 = K[M^1L^1T^{-2}]^a [L^1T^{-2}]^b [T]^c$

$M^0L^1T^0 = K[M^a] [L^{a+b}] [T^{-2a-2b+c}]$

By comparison and solving we find

$[a = 0] [b = 1] [c = 2]$

Put these value in equation (i)

$[L = F^0A^1T^2]$

15. (d) $\because$ Velocity $= \frac{\text{Displacement}}{\text{Time taken}} = \frac{10^{-6}\,\text{m}}{10^{-6}\,\text{sec}} = \text{m/sec}$

16. (a) $n_1u_1 = n_1u_1$

$n_1\left[M_1^1L_1^2T_1^{-3}\right] = 1\left[M_2^1L_2^2T_2^{-3}\right]$

$n_1 = \left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^2\left[\frac{T_2}{T_1}\right]^{-3}$

$= \left[\frac{20}{1}\right]^1\left[\frac{10}{1}\right]^2\left[\frac{5}{1}\right]^{-3}$

$= \frac{20 \times 100}{5 \times 5 \times 5} = 16$

$n_1 = 16$

17. (c) $10^3(N) = M^1L^1T^{-2}$

$10^3 = [M]^1 [10^3]^1 [100]^{-2}$

$M = \frac{10^3}{10^3 \times (100)^{-2}} = 10000\,\text{kg}$

18. (c) In new system

Length $\rightarrow$ m $\rightarrow$ 2m

Velocity $\rightarrow$ m/sec. $\rightarrow$ 2m/sec

Force $\rightarrow$ kgm/sec^2 $\rightarrow$ 2kgm/sec^2

$\therefore$ Momentum $(P) = mv$ = kg m/sec.

$P = kg\frac{m}{\text{sec}} \times \frac{m}{m} \times \frac{\text{sec}}{\text{sec}}$

$P = kg\frac{m}{\text{sec}^2} \times \frac{m}{(\text{m/sec})}$

In new system

$P^1 = \left(2kg\frac{m}{\text{sec}^2}\right) \times \frac{(2m)}{(2\text{m/sec})}$

$P^1 = (2\text{kg m/sec}) = 2P$

So, Here unit of momentum is doubled.

19. (d) Unit of Energy = $\text{kg}\frac{m^2}{\text{sec}^2}$

$= \left(\text{kg}\frac{m}{\text{sec}^2}\right) \times (m)$

Now unit of force and length are doubled.

$= \left(2\text{kg}\frac{\text{m}}{\text{sec}^2}\right)(2\text{m}) = 4\text{kg}\frac{\text{m}^2}{\text{sec}^2}$

So, Unit of Energy is 4 times.

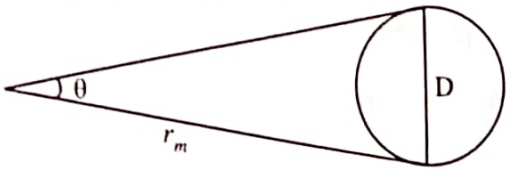
20. (*c*) $K.E. = \frac{1}{2}mv^2$

Dimension = $M^1L^2T^{-2}$

Now $M.L$ are doubled = $(2M)^1 (2L)^2 (T^{-2}) = 8\ M^1L^2T^{-2}$

So, K.E. will become 8 times.

21. (*b*) Take small angle approximation



$$\sin\theta = \frac{D}{r_m}$$

$$\sin 0.50° = \frac{D}{r_m}$$

$$0.50 \times \frac{\pi}{180} = \frac{D}{384000}$$

$$D = 0.50 \times \frac{\pi}{180} \times 384000$$

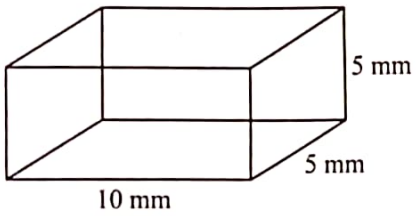
$D = 3349.33 \Rightarrow D \approx 3350$ km.

22. (*b*) $\Delta t = 0.2$ s.

$t = 25$ s

$$T = \frac{t}{N} \Rightarrow \frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{0.2}{25} = 0.8\%$$

23. (*a*)



$v = \ell bh$

$$\frac{\Delta v}{v} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

$$= \frac{0.1}{10} + \frac{0.1}{5} + \frac{0.1}{5} = \frac{0.5}{10} = \pm 5\%$$

24. (*d*) $\frac{\Delta x}{x} = 1\%$

$$\frac{\Delta y}{y} = 3\%$$

$$\frac{\Delta z}{z} = 2\%$$

$$t = \frac{xy^2}{z^3}$$

$$\frac{\Delta t}{t} = \frac{\Delta x}{x} + \frac{2\Delta y}{y} + \frac{3\Delta z}{z}$$

$= 1\% + 2\times3\% + 3 \times 2\% = 13\%$

25. (*c*) $D = (4.23 \pm 0.01)$ cm

$d = (3.89 \pm 0.01)$ cm

$\Delta t = (D - d)/2$

$$= \frac{(4.23 \pm 0.01) - (3.89 \pm 0.01)}{2}$$

$$= \frac{(4.23 - 3.89) \pm (0.01 + 0.01)}{2}$$

$= (0.34 \pm 0.02)/2$ cm

$= (0.17 \pm 0.01)$cm

26. (*b*) $m = 1.76$ kg

$M = 25$ m

$= 25 \times 1.76$

$= 44.0$ kg

Note: Mass of one unit has three significant figures and it is just multiplied by a pure number (magnified). So result should also have three significant figures.

27. (*b*) $R_1 = (24 \pm 0.5)\ \Omega$

$R_2 = (8 \pm 0.3)\ \Omega$

$R_S = R_1 + R_2 = (32 \pm 0.8)\ \Omega$

28. (*b*) $\Delta\ell = 0.5$ mm

$N = 100$ divisions

zero correction = 2 divisions

Reading = Measured value + zero correction

$$= (8 \times 0.5) \text{ mm} + (83 - 2) \times \frac{0.5}{100}.$$

$$= 4 \text{ mm} + 81 \times \frac{0.5}{100} \text{ mm}$$

$= 4.405$ mm

29. (*d*) $\Delta\ell = 1$ mm

$N = 50$ division

zero error = –6 Divisions = – 0.12 mm

Diameter = Measured value + zero correction

$$= 3 \times 1 + (6 + 31) \times \frac{1}{50}$$

$= 3 + 0.74 = 3.74$ mm

30. (*a*) $D = 2 \times 1 + 5 \times \frac{10-9}{100} = 2.05$ cm

31. (*c*) $V_0 \propto M^aR^bG^c \Rightarrow \boxed{V_0 = KM^aR^bG^c}$

Substituting the dimensions for M, R, G

$V_0 = K[M]^a [L]^b [M^{-1}L^3T^{-2}]^c$

$M^0LT^{-1} = [M]^{a-c} [L]^{b+3c} [T]^{-2c}$

Comparing the powers, we get

$a - c = 0$ ...(i)

$b + 3c = 1$ ...(ii)

$$-2c = -1 \Rightarrow C = \frac{1}{2}$$

Solving for a, b we get $a = \frac{1}{2}$, $b = \frac{1}{2}$

$$\therefore V_0 = KM^{1/2}R^{-1/2}G^{1/2} = K\sqrt{\frac{GM}{R}}$$

32. (*a*) Measured Diameter = MSR + VSR × V.C

$= 1.7 + 0.01 \times 5 = 1.75$

Corrected Reading = Measured – Error

$= 1.75 - (0.05) = 1.80$ cm

$= 180 \times 10^{-2}$ cm

$\therefore n = 180$

33. (*a*) $n_1u_1 = n_1u_2$

Let 1 dyne = nu

Where u = new unit of force

$[F] = [M^1L^1T^{-2}]$

1 gm. cm.s^{-2} = n. (10gm).(10cm).(0.1s)$^{-2}$

where 10 gm, 10 cm are 0.1 s are

new units of mass, length and time respectively

Solving the above relation we get $n = 10^{-4}$

i.e. 1 dyne = $10^{-4}u$ or 10^{-5} newton = $10^{-4}\,u$ or $u = \frac{1}{10}$ N

34. (*b*) $\lambda \,\alpha\, h^a c^b m^c$

$L \,\alpha\, (ML^2T^{-1})^a (LT^{-1})^b (m^c)$

$L \,\alpha\, m^{a+c} L^{2a+b} T^{-a-b}$

$a + c = 0; \quad 2a + b = 1; \quad -a - b = 0$

$a = -b$

$\Rightarrow 2a - a = 1; a = 1; c = 1; b = -1$

$\Rightarrow \lambda\alpha h^1c^{-1}m^{-1}$

$\lambda \,\alpha\, \frac{h}{mc}$

35. (*b*) $L = M^a T^b (ML^2 T^{-2})^C$

$a + c = 0,$

$2c = 1$

$\Rightarrow c = \frac{1}{2}, a = \frac{-1}{2}$

Also, $b = 2c = 0 \quad \Rightarrow b = 1$

36. (*c*) $\ell n\left(\frac{\alpha}{p\beta}\right) = \frac{\alpha z}{k_B\theta}$

$[\alpha\, z] = [k_B\theta] \qquad$ Also $[\alpha] = [p\beta]$

$[p\,\beta\, z] = [k\theta]$

$[\beta] = \frac{(k_\beta\theta)}{(p\,z)} = \frac{ML^2T^{-2}K^{-1}K}{ML^{-1}T^{-2}L} = L^2$

37. (*b*) $\Delta g = g_0[0 + n \sin 2\phi\, \Delta\phi]$

$\Delta\phi = \frac{\Delta g}{g_0\beta \sin 2\phi}$

38. (*a*) $y = \ell^2 - \frac{\ell^3}{z} = 2^2 - \frac{2^3}{1} = -4$

$dy = 2\ell d\ell + \frac{z \times 3\ell^2\, d\ell + \ell^3 dz}{z^2}$

$dy = 2 \times 2 \times 0.1 + \frac{3\times1\times4\times0.1+8\times0.1}{1^2}$

$d\ell = 0.4 + (1.2 + 0.8) = 2.4$

39. (*a*) $T_{avg} = \frac{T_1 + T_2}{2} = \frac{16.42}{2} = 8.21$

Mean absolute error = $\frac{|T_{avg} - T_1| + |T_{avg} - T_2|}{2}$

Hence we take error = 0.2

Reported time = $T_{avg} \pm \Delta T_{\text{mean absolute}}$

$= 8.2 \pm 0.2$

40. (*b*) $F = -\eta A \frac{dV}{dx}$

$\frac{F}{A} = -\eta \frac{dV}{dx}$

$P = \eta \frac{V}{L}$

$\eta = \frac{PV}{L} = PLV^{-1}$

41. (*c*) $T \propto P^a d^b E^c$

By dimensional analysis

$[T] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$

Solving for a, b, c we get

$a = -\frac{5}{6}, b = \frac{1}{2}, c = \frac{1}{3}$

42. (*a*) $VS = 4, MS = 2.7$

Reading = $MS + VS \times LC = 2.7 + 0.04 = 2.74$

Exercise-3 (JEE Advanced Level)

1. (*a,b,c*)

Derived quantity may have zero dimension in certain base quantity. For example acceleration which is derived quantity, has zero dimensions in mass which is base quantity.

2. (*c,d*)

By checking dimension in each option

Pressure $= \frac{F}{A} = \frac{[M^1L^1T^{-2}]}{[L^2]} = [M^1L^{-1}T^{-2}]$

Energy per unit volume $= \frac{\text{Energy}}{\text{Volume}}$

$= \frac{[M^1L^2T^{-2}]}{[L^3]} = [M^1L^{-1}T^{-2}]$

3. (*a,b,c*)

$[A] = MLT^{-1} \times L = ML^2T^{-1}$

(*a*) $[A] = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

(*b*) $MLT^{-1} = \frac{[A]T^{-1}}{LT^{-1}} \Rightarrow ML^2T^{-1}$

(c) $[kl] = L^0$

$[k] = L^{-1}$

$$\Rightarrow [A] = \left[\frac{\sqrt{2mE}}{k}\right] = \frac{(M \times ML^2T^{-2})^{1/2}}{L^{-1}} = ML^2T^{-1}$$

(d) $[\tau] = ML^2T^{-2}$

$$[A] = \frac{ML^2T^{-2}}{L} = MLT^{-2}$$

4. (a,b,c,d)

Note that the quantities specific heat and latent heat both contain a term 'energy per unit mass'. However energy itself contains mass and hence the dimensions of both these quantities do not contain mass.

5. (a,c)

$(ML^2T^{-3}) = L^x(ML^{-3})^y (ML^{-2}T^{-2})^z$

$\Rightarrow 1 = y + z \quad \Rightarrow \quad 2 = x - 3y - 2z$

$-3 = -2z$ or $z = 3/2$

and $y = -1/2 \quad \Rightarrow \quad x = 7/2$

$P \propto$ (linear size)$^{7/2}$ (density of air)$^{-1/2}$ ($g \times$ density of helicopter)$^{3/2}$

6. (b,d)

[v] = frequency $\rightarrow$ T is time

$$[v] = LT^{-1} = \left[\sqrt{\frac{T}{ML^{-1}}}\right]$$

$[T] = MLT^{-2} \Rightarrow$ tension.

7. (a,b,c,d)

$$\text{Density } \rho = \frac{\text{mass}}{\text{volume}} = \frac{38.254}{15.0} = 2.55 \text{ g/m}^2$$

No. of significant figure in density is 3.

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V} = \frac{0.003}{38.254} + \frac{0.1}{15.0}$$

$= 0.007$

$\Delta\rho = 0.007 \times 2.35 = 0.02$ g/cm^3

8. (a,b,c,d)

$$s = \frac{4m}{\pi d^2 \ell} = \frac{4 \times 25}{3.14 \times 0.5^2 \times 50}$$

$$\frac{ds}{s} = \frac{dm}{m} + \frac{2d\ell}{d} + \frac{d\ell}{\ell}$$

$$= 0 + 2 \times \frac{0.01}{5.00} + \frac{0.1}{50.0}$$

$\therefore ds = 0.02$

$\therefore s = (2.55 \pm 0.02)$ g/cc

9. (b) $A = \dfrac{a}{t} = LT^{-3}$

10. (a) $B = \dfrac{a}{t^2} = LT^{-4}$

11. (c) $c = at = [LT^{-1}]$

12. (c) $F = \dfrac{[c]}{r^2} \Rightarrow [c] = Fr^2 = [MLT^{-2}]L^2$

13. (d) kr = constant

$$k = \frac{\text{constant}}{r} = \frac{1}{L} = L^{-1}$$

14. (b) $F = \dfrac{C}{r^2} \Rightarrow C = Fr^2 = Nm^2$

15. (c) $\left(P + \dfrac{a}{V^2}\right)(V - b) = RT$

$$\left[\frac{a}{V^2}\right] = [P] \Rightarrow [a] = [PV^2]$$

16. (c) $\left[P + \dfrac{a}{V^2}\right] = [P] = \left[\dfrac{a}{V^2}\right]$

$[V - b] = [V] = [b]$

So, $[P]\,[V] = [nRT]$

LHS RHS

17. (a) $[nRT] = [PV] = [M^1 L^{-1} T^{-2}]\,[L^3] = [M^1 L^2 T^{-2}]$

Dimension of energy

18. (a) $[\hbar] = ML^2T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$[C] = LT^{-1}$

$$\sqrt{\frac{\hbar G}{C^3}} = \left[\frac{ML^2T^{-1} \times M^{-1}L^3T^{-2}}{L^3T^{-3}}\right]^{1/2} = L$$

19. (a) $\left[\dfrac{G}{\hbar}\right] = M^{-2}L^{+1}T^{-1}$

$$\left[\frac{GB}{\hbar c}\right] = M^{-2}$$

$$M = \sqrt{\frac{\hbar c}{G}} = \sqrt{\frac{1.05 \times 10^{-34} \times 3 \times 10^8}{\frac{20}{3} \times 10^{-11}}} \approx 10^{-8} \text{ kg}$$

20. (c) $[a] = LT^{-2}$

(A) $[ML^2T^{-1}]^{1/2}\,[LT^{-1}]^{3/2}\,[M^{-1}L^3T^{-2}]^{-1/2} = M^1\,(\,)$

(B) $[ML^2T^{-1}]^{3/2}\,[LT^{-1}]^{5/2}\,[M^{-1}L^3T^{-2}]^{1/2} = M^1\,(\,)$

(C) $[ML^2T^{-1}]^{-1/2}\,[LT^{-1}]^{7/2}\,[M^{-1}L^3T^{-2}]^{-1/2}$

$$= M^0 L^{-1+\frac{7}{2}-\frac{3}{2}} T^{\frac{1}{2}-\frac{7}{2}+1} = LT^{-2}$$

21. (d) When an instrument is used to measure, it has its least count. Therefore, we can measure the value to nearest correct value and not absolute value.

22. (d) Answer will have three significant digits. So it will be 3.14

23. (a) $\pi = \dfrac{C}{D}$ $C = 2\pi r$ circumference; $D = 2r$ diameter

$$\left|\frac{\Delta\pi}{\pi}\right| = \left|\frac{\Delta C}{C}\right| + \left|\frac{\Delta D}{D}\right|$$

$$\frac{\Delta\pi}{\pi} = (1\% + 1\%) = 2\%$$

24. (*b*) A→ (*q*) → (ii), B→ (*s*) → (i), C → (*p*) → (ii), D → (*r*) → (iv)

$$F = G\frac{m_1 m_2}{r^2} \Rightarrow [G] = \frac{[F][r^2]}{[m_1 m_2]}$$

$$= \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$$

[Torque] = [*f*] [*d*] = $MLT^{-2}L = ML^2T^{-2}$

[Momentum] = [*m*] [*v*] = MLT^{-1}

$$[p] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$$

25. (*d*) A→ (*q*) → (iv), B → (s) → (ii), C→ (*u*) → (*v*), D→ (*r*) → (i), E → (*t*) → (vi), F → (*p*) → (iii)

A. $U = \sigma AT^4 \Rightarrow [\sigma]\ \frac{[U]}{[A][T^4]} = \frac{ML^2T^{-3}}{L^2K^4} = MT^{-3}K^{-4}$

B. $\lambda T = b \Rightarrow [b] = [\lambda]\,[T] = LK$

C. $F = 6\pi\eta rv \Rightarrow [\eta] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{L.LT^{-1}} = ML^{-1}T^{-1}$

D. $I = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = ML^{\circ}T^{-3}$

E. Energy $= \frac{1}{2}Mi^2 \Rightarrow [M] = \frac{[E]}{[i^2]} = ML^2T^{-2}A^{-2}$

F. $\frac{[U]}{[V]} = \frac{[B^2]}{[2\mu_0]} = [\mu_0] = \frac{[B^2][V]}{[U]}$

Also, $F = qVB \Rightarrow B = \frac{F}{qv}$

$$[\mu_0] = \frac{(F)^2[V]}{[q^2v^2][U]} = MLT^{-2}A^{-2}$$

26. (*c*)

A. $PAvt = MLT^{-2}LT^{-1}T$ p. $\frac{\text{watt. second}}{m^3}$

$= ML^2T^{-2}$ $= \frac{ML^2T^{-3}.T}{L^3} = ML^{-1}T^{-2}$

B. hgR = q. ML^2T^{-2}

$= LT^{-2}\frac{ML^2T^{-2}}{LT^{-1}} = ML^2T^{-3}$

C. $\frac{\sigma b^4}{A} = \frac{E}{AT^4}.\frac{\lambda^4T^4}{A}$ r. $ML^{-1}T^{-2}$

$= \frac{E\lambda^4}{A^2} = ML^2T^{-2}\frac{L^4}{L^4}$

$= ML^2T^{-2}$

D. $\frac{\eta}{t} = \frac{FL}{Avt}$ s. ML2T–3

$= \frac{MLT^{-2}L}{L^2LT^{-1}T} = ML^{-1}T^{-2}$

t. ML^2T^{-2}

27. (*a*)

A. $1N = \frac{1kg(1m)}{1sec^2} = \frac{\left(\frac{1\text{ unit mass}}{\alpha}\right)\left(\frac{1}{\beta}\text{unit length}\right)}{\left(\frac{1}{\gamma}\text{unit time}\right)^2}$

$= \frac{\gamma^2}{\alpha\beta}$ unit force in new system.

B. $1J = \frac{1kg(1m)^2}{(1sec)^2} = \frac{\frac{1}{\alpha}\left(\frac{1}{\beta}\right)^2}{\left(\frac{1}{\gamma}\right)^2}$ unit energy $= \alpha^{-1}\beta^{-2}\gamma^2$

C. 1 pascal $= \frac{1kg}{(1m)(1sec)^2} = \frac{\frac{1}{\alpha}}{\frac{1}{\beta}\left(\frac{1}{\gamma}\right)^2}$ unit pressure

$= \alpha^{-1}\beta\gamma^2$

D. α SHAKTI $= \alpha\frac{(1\text{ unit mass})(1\text{ unit length})^2}{(1\text{ unit time})^3}$

$= \alpha\frac{\alpha kg\ \beta m}{(\gamma\ sec)^3} = \alpha^2\beta^2\gamma^{-3}$ watt

28. (*d*) Recall the dimensional formula for the quantities.

29. [1] Fact based

30. [3] Fact based

31. [4] Fact based

32. [0] Using $[v] = LT^{-1}$, $[p] = ML^{-1}T^{-2}$, $[D] = ML^{-3}$ and dimensional analysis, $a = 1/2$, $b = -1/2$

So, $a + b = 0$.

33. [0] Using [T] = T, $[p] = ML^{-1}T^{-2}$, $[d] = ML^{-3}$

$[E] = ML^2T^{-2}$ and using dimensional analysis, we get

$a = 2, b = -1, c = -1 \Rightarrow a + b + c = 0$

34. [2.6] $LC = \frac{1}{100} = 0.01\text{mm}$

Diameter, $d = 1 + 47 \times 0.01 = 1.47$ mm

CSA $= 2\pi r\,\ell = \pi d\,\ell = 2.58\text{ cm}^2$

$= 2.6$ (Rounded off 2 significant digits)

35. [9] $d = \frac{M}{V} = \frac{M}{\ell^3}$

$$\frac{\Delta d}{d}\% = \frac{\Delta M}{M}\% + \frac{3\Delta\ell}{\ell}\% = 3 + 3(2) = 9\%$$

36. [92.1]

Least count of slide calliper is 1 mm. Hence effective length of Pendulum = 90 + 2.1 = 92.1 cm

37. [9.95]

$$\sqrt{99} = (100-1)^{\frac{1}{2}}$$

$$= 10\left(1-\frac{1}{100}\right)^{\frac{1}{2}} \Rightarrow 10\left(1-\frac{1}{200}\right) = 9.95$$

38. [3] $[T] = \left[\left(\frac{mgA}{K}\right)^{1/2}\right] = \left[\frac{mgA}{(mv^2)}\right]^{1/2} = \left[\frac{A}{vT}\right]^{1/2} = \left[\frac{A}{L}\right]^{1/2}$

$\Rightarrow [A] = M^0 L T^2$

$x = 0, y = 1, T = 2$

$x + y + z = 3$

39. [6] $\frac{\Delta V}{V} = 3\frac{\Delta V}{V} = 3\frac{\Delta r}{r} = 3 \times \frac{0.1}{5.3} = \frac{3}{53} \times 100$

40. [6] Negative zero error = (10 – 2) × 0.1 mm

= 0.8 mm

Reading = 6 + 4 × 0.1 + 0.8 = 7.2 mm

$= \frac{12 \times 6}{10}$ mm

$\Rightarrow x = 6$

41. [41] $T = 2\text{sec}.\ \Delta T = \frac{0.1}{25}$

$T = 2 \pm 4 \times 10^{-3}$ sec.

Now, $g = 4\pi^2 \frac{\ell}{T^2}$

$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T} = \frac{0.01}{100} + \frac{2 \times 4 \times 10^{-3}}{2}$

$= 10^{-4} + 40 \times 10^{-4}$

$= 41 \times 10^{-4} = 41 \times 10^{-2}\% = 0.41\%$

Exercise-4 (Past Year Questions)

JEE MAIN

1. (a) $128[M_1L_1^{-3}] = n_2[M_2L_2^{-3}]$

$\Rightarrow n_2 = 128\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^{-3} = 128\left[\frac{1000}{50}\right]\left[\frac{100}{25}\right]^{-3}$

$= \frac{128 \times 20}{64} = 40$

2. (c) $t = G^a h^b c^c$

$\Rightarrow M^0L^0T^1 = (M^{-1}L^3T^{-2})^a (ML^2T^{-1})^b (LT^{-1})^c$

$\Rightarrow -a + b = 0 \Rightarrow a = b,$

$3a + 2b + c = 0 \Rightarrow c = -5a,$

$-2a - b - c = 1$

$\Rightarrow a = \frac{1}{2}; b = \frac{1}{2}; c = \frac{-5}{2}$

3. (b) Time constant $\tau = RC \Rightarrow [RC] = T$

$\Rightarrow V = L\frac{di}{dt} \Rightarrow \left[\frac{L}{V}\right] = TA^{-1}$

Thus, $\left[\frac{L}{RCV}\right] = \frac{TA^{-1}}{T} = A^{-1}$

4. (c) $X = 5YZ^2$

$Y = \frac{X}{5Z^2}$

$[Y] = \frac{[X]}{[Z^2]}$

$= \frac{A^2.M^{-1}L^{-2}.T^4}{(MA^{-1}T^{-2})^2}$

$= M^{-3}.L^{-2}.T^8.A^4$

5. (b) $p = k\,S^a l^b h^c$

where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^a(ML^2)^b(ML^2T^{-1})^c$

$a + b + c = 1$

$2b + 2c = 1$

$-2a - c = -1$

$\Rightarrow a = \frac{1}{2} \quad b = \frac{1}{2} \quad c = 0$

Thus, $p \propto S^{1/2}l^{1/2}h^0$

6. (c) Zero error = $0 + 3 \times \frac{0.5\text{mm}}{100} = 0.015$ mm

MSR = $5.5 + 48 \times \frac{0.5}{100}$

= 5.74 mm.

∴ Thickness = 5.74 – 0.015 = 5.725 mm

7. (c) $V = \pi R^2 h = \frac{\pi}{4}D^2 h = 4260\ \text{cm}^2$

$\because \frac{\Delta V}{V} = 2\frac{\Delta D}{D} + \frac{\Delta h}{h}$

$\Delta V = \left(2 \times \frac{0.1}{12.6} + \frac{0.1}{34.2}\right)V$

$= \frac{2 \times 426}{12.6} + \frac{426}{34.2}$

$= 67.61 + 12.459 = 80.075$

$\therefore V = 4260 \pm 80\ \text{cm}^3$

8. (b) Least count $= \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$\Rightarrow 5 \times 10^{-6} = \frac{10^{-3}}{N} \quad \Rightarrow N = 200$

9. (b) $A' = 7 \times A_1$

$= 7 \times 5.29$

$= 37.03$

Rounding off to 3 significant figures

$A = 37.0\ \text{cm}^2$

10. (c) Energy = Force × Distance

$\Rightarrow [\text{Energy}] = \frac{P}{T} \times \sqrt{A}$

$= \left[PA^{\frac{1}{2}}T^{-1}\right]$

$\Rightarrow$ [Young's modulus] = FA^{-1}

$\Rightarrow$ [Young's modulus] = $FA^{-1}V^0$

12. (*d*) Solar constant = $\frac{E}{AT}$

$$\frac{M^1L^2T^{-2}}{L^2T} = M^1T^{-3}$$

13. (*c*) $x = \frac{IFv^2}{WL^4}$

$$\therefore [x] = \frac{(ML^2)\times(MLT^{-2})\times(LT^{-1})^2}{(ML^2T^{-2})\times L^4}$$

$= ML^{-1}T^{-2}$

= [Energy density]

14. (*c*) $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = x$

$$c = \frac{E}{B} = y$$

$$\tau = RC = t \Rightarrow \frac{\ell}{RC} = LT^{-1}$$

$\Rightarrow [x] = [y] = [z]$

15. (*a*) $\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta g}{g} + \frac{\Delta L}{L}\right)$

$$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta L}{L}; = 2\left(\frac{1}{50}\right) + \frac{0.1}{25.0} = 4.4\,\%$$

16. (*d*) Pitch = $\frac{3}{6} = 0.5$mm

$$\text{L.C.} = \frac{0.5\text{mm}}{50} = \frac{1}{500}\text{mm} = 0.01\text{mm} = 0.001\text{ cm}$$

17. (*c*) Least count of $V.C = \left(1 - \frac{9}{10}\right) \times 1\text{mm} = 0.1$ mm

$\therefore$ Zero error = 7 × 0.1 = 0.7 mm

Positive error

Measured value = (31 + 4 × 0.1) mm = 31.4 mm

$\therefore$ Length of cylinder = 31.4 – 0.7 = 30.7 mm

= 3.07 cm

18. (*b*) $\frac{\Delta z}{z} = 2\frac{\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + 3\frac{\Delta d}{d}$

$$= 2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5 = 14.5\%$$

19. (*b*) $d_{av} = 5.5375$ mm

$\Delta d = 0.07395$ mm

$\because$ Measured data are up to two digits after decimal

$\therefore d = (5.54 \pm 0.07)$ mm

$$\therefore \%\ \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3.\left(\frac{\Delta d}{d}\right)$$

$$= 6 + 3 \times 1.5 = 10.5\% = \left(\frac{1050}{100}\right)$$

21. (*b*) Reading = $MSR + VSD \times LC$ – zero error

$$\text{Reading} = 8.5 + \frac{(0.1)\times 6}{10} - \frac{0.2}{10} = 8.54\text{cm}$$

22. (*c*) $\frac{\Delta Y}{Y} = \left(\frac{\Delta m}{m}\right) + \left(\frac{\Delta g}{g}\right) + \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta l}{l}\right) + \left(\frac{\Delta L}{L}\right)$

$$= \left(\frac{1\text{g}}{1\text{kg}}\right) + 0 + 2\left(\frac{\Delta r}{r}\right) + \left(\frac{\Delta l}{l}\right) + \left(\frac{\Delta L}{L}\right)$$

$$= \left(\frac{1\text{g}}{1\text{kg}}\right) + 2\left(\frac{0.001\text{cm}}{0.2\text{cm}}\right) + \left(\frac{0.001\text{cm}}{0.5\text{cm}}\right) + \left(\frac{0.001\text{m}}{1\text{m}}\right)$$

$$= \left(\frac{1}{1000}\right) + 2\left(\frac{1\times 10}{2\times 10^3}\right) + \left(\frac{1}{5}\times\frac{10^2}{10^3}\right) + \left(\frac{1}{10^3}\right)$$

$$= \frac{1}{1000} + \frac{1}{100} + \frac{2}{10^3} + \frac{1}{10^3}$$

$$= \frac{1+10+2+1}{1000} = \frac{14}{1000} \times 100\% = 1.4\%.$$

23. (*d*) $(n-1)\text{MSD} = m\text{VSD} \Rightarrow \text{VSD} = \frac{n-1}{n}(\text{MSD})$

Now, L.C = [1 MSD – 1VSD]cm

$$= a - \frac{n-1}{n}a = \frac{a}{n}\text{cm}$$

$$= \left(\frac{10a}{n}\right)\text{mm}$$

24. (*b*) Let $M = K\,(V)^a(F)^b(T)^c$

$\Rightarrow$ Equating dimensions of both the sides

$M^1L^0L^0 = (1)[L^1T^{-1}]^a[M^1L^1T^{-2}]^b[T^1]^c$

$M^1L^0T^0 = M^bL^{a+b}T^{-a-2b+c}$

Comparing dimensions, we get

$a = -1, b = 1, c = 1$

Thus, M = (Some Number) $(V^{-1}F^1T^1)$

$\Rightarrow [M] = [V^{-1}F^1T^1]$

25. (*a*) For the screw gauge, the least count (LC) is

$$\text{Least Count} = \frac{\text{Pitch}}{\text{total division on circular scale}}$$

In 5 revolution, distance traveled is, 5 mm

So, 1 revolution, it will travel 1 mm.

Therefore, $\text{LC} = \frac{1}{50} = 0.02\text{ mm} = 0.002\text{ cm}$

It means the mentioned reason is correct, but the assertion is incorrect for the same.

26. (*b*) $[A] = [MLT^{-2}]$

$[B] = [L^{-1}]$

$[D] = [T^{-1}]$

$$\Rightarrow \left[\frac{AD}{B}\right] = \frac{[MLT^{-2}][T^{-1}]}{[L^{-1}]} \text{ or } \left[\frac{AD}{B}\right] = [ML^2T^{-3}]$$

27. (*c*) $T = 2\pi\sqrt{\frac{l}{g}}$

$$\Rightarrow \frac{\Delta T}{T}\times 100 = \frac{1}{2}\frac{\Delta l}{l}\times 100 + \frac{1}{2}\frac{\Delta g}{g}\times 100$$

Since, $\Delta g = 0$

$$\frac{\Delta T}{T}\times 100 = \frac{1}{2}\times 0.1 = 0.05\%$$

For 1 day

$$\Delta T = \frac{0.05}{100}\times 24\times 3600$$

$\Delta T = 43.2$ sec

28. [14] $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \frac{T^2 g}{4\pi^2}$

$$E = mgl\frac{\theta^2}{2} = mg^2\frac{T^2g^2}{8\pi^2}$$

$$\Rightarrow \frac{\Delta E}{E} = 2\left(\frac{\Delta g}{g} + \frac{\Delta T}{T}\right) = (4+3) = 14\%$$

29. (*c*) $\frac{kt}{\beta x} = 1 \Rightarrow \beta = \frac{kt}{x} = \frac{ML^2T^{-2}}{L}$ $\left(\because E = \frac{1}{2}kt\right)$

As P is dimensionless

$\Rightarrow [\alpha] = [\beta] = [MLT^{-2}]$

30. (*a*) [Pascal-second] $= \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-1}$

31. (*c*) From principle of homogeinity,

$$[P] = \left[\frac{a}{v^2}\right] \text{ and } [b] = [v]$$

$$\Rightarrow \left[\frac{a}{b}\right] = [PV]$$

32. (*b*) For torque

$[\tau] = [ML^2T^{-2}]$

So, percentage error in torque $= \%\tau$

$= \%M + 2(\%L) + 2(\%T) = 25\%$

33. (*d*) Dimensions trigonometric function and logarithmic function are dimensionless quantities.

$\therefore [\eta] = [M^0L^0T^0]$

Also, dimensions of temperature, $[T] = [M^0L^0T^0K]$

Dimensions of Boltzmann constant, $[k] = [ML^2T^{-2}K^{-1}]$

Dimension of $[x] = [M^0LT^0]$

A. $[\beta] = \left[\frac{kT}{x}\right] = \left[\frac{E}{x}\right] = [MLT^{-2}] = [F]$

B. $[\alpha\beta] = [M^0L^0T^0] \Rightarrow [\alpha]^{-1} = [\beta] = \left[\frac{kT}{x}\right]$

So $[\alpha]^{-1}[x] = [kT] = [ML^2T^{-2}]$

C. $\eta \sin\theta = \alpha\beta$

So, $[\eta \sin\theta] = [\alpha\beta]$

$[\eta] = [M^0L^0T^0]$ it is dimensionless quantity

D. $[\alpha] \neq [\beta]$

34. (*c*) Torque $= F \times r_\perp$ Nm

Stress $= \frac{\text{Force}}{\text{Area}}$ N/m^2

Latent heat $= \frac{\text{Energy}}{\text{Mass}}$ $J\ kg^{-1}$

Power $= \frac{\text{Work}}{\text{Time}}$ $N\ ms^{-1}$

35. (*c*) 1 M.S.D. = 1 mm

Now, 9 M.S.D. = 10 V.S.D.

1 V.S.D. = 0.9 M.S.D. = 0.9 mm = 0.01 cm

Now, L.C. of vernier caliper = 1 – 0.9 = 0.1 mm

= 0.01 cm

zero error = –(10 – 4) × 0.1 mm = –0.6 mm

Reading = M.S.R + V.S.R = – Zero error

= 3 cm + 6 × 0.01 – [–0.06]

= 3 + 0.06 + 0.06

= 3.12 cm ~ nearest to 3.10

36. (*a*) L.C. $= \frac{0.5}{50}$ mm = 0.01 mm

$d = (2.5 + 45 \times 0.01)$ mm = 2.95 mm

37. (*d*) Let $T \propto \rho^a r^b S^c$

Using $[T] = T$, $[\rho] = ML^{-3}$, $[r] = L$, $[S] = MT^{-2}$ and dimensional analysis, we get

$a = 1/2$, $b = 3/2$, $c = -1/2$

Thus, $T = K\sqrt{\frac{\rho r^3}{S}}$

JEE ADVANCED

38. (*a*) $\Delta d = \Delta\ell = \frac{0.5}{100}$ mm

$$\text{As } Y = \frac{4MLg}{\pi\ell d^2} \Rightarrow \left(\frac{\Delta Y}{Y}\right)_{max} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta d}{d}$$

Error due to ℓ measurement $= \frac{\Delta\ell}{\ell} = \frac{0.5/100\text{ mm}}{0.25\text{ mm}}$

Error due to d measurement

$$= 2\frac{\Delta d}{d} = \frac{2\times\frac{0.5}{100}}{0.5\text{ mm}} = \frac{0.5/100}{0.25}$$

So error in Y due to ℓ measurement = error in Y due to d measurement

39. (*c*) A. $U = \frac{1}{2}kT \Rightarrow ML^2T^{-2} = [k]\,K \Rightarrow [k] = ML^2T^{-2}K^{-1}$

B. $F = \eta A\frac{dv}{dx} \Rightarrow [\eta] = \frac{MLT^{-2}L}{L^2LT^{-1}} = ML^{-1}\,T^{-1}$

C. $E = h\nu \Rightarrow ML^2T^{-2} = [h]\,T^{-1} \Rightarrow [h] = ML^2\,T^{-1}$

D. $\frac{dQ}{dt} = \frac{kA\Delta\theta}{\ell} \Rightarrow [k] = \frac{ML^2T^{-3}L}{L^2K} = MLT^{-3}\,K^{-1}$

40. (*b*) 50 *VSD* = 2.45 cm

$1\ VSD = \frac{2.45}{50}$ cm = 0.049 cm

Least count of vernier = 1*MSD* – 1 *VSD*

= 0.05 cm – 0.049 cm

= 0.001 cm

Thickness of the object = Main scale reading + vernier scale reading × least count

= 5.10 + (24) (0.001)

= 5.124 cm.

41. (*d*) $2d\sin\theta = \lambda$

$d = \frac{\lambda}{2\sin\theta}$

Differentiate

$\delta(d) = \frac{\lambda}{2}\,\delta(\text{cosec}\theta)$

$\delta(d) = \frac{-\lambda\cos\theta}{2\sin^2\theta}\delta\theta$

As θ increases, $\frac{\cos\theta}{2\sin^2\theta}$ decreases

$\Rightarrow \delta d$ decreases

Fractional error:

$$\frac{\delta(d)}{d} = \frac{\left(\frac{\lambda\cos\theta}{2\sin^2\theta}\right)\delta\theta}{\left(\frac{\lambda}{2\sin\theta}\right)} = \cos\theta\,\delta\theta$$

As θ increases cosθ decreases, so fractional error decreases.

42. (*c*) $d = k\,(\rho)^a\,(S)^b\,(f)^c$

$$\Rightarrow [L] = \left[\frac{M}{L^3}\right]^a\left[\frac{M^1L^2T^{-3}}{L^2}\right]^b\left[\frac{1}{T}\right]^c$$

So, $0 = a + b$,

$1 = -3a$

$\Rightarrow a = -\frac{1}{3}$

$\Rightarrow b = \frac{1}{3}$

So $n = 3$

43. (*a,c,d*)

$L \propto h^a c^b G^c$

Using $[h] = ML^2T^{-1}$, $[c] = LT^{-1}$, $[G] = M^{-1}L^3T^{-2}$ and dimensional analysis, we get

$a = 1/2,\ b = -3/2,\ c = 1/2$

So, $L \propto \sqrt{\frac{hG}{c^3}}$

Similar analysis gives

$M \propto \sqrt{\frac{hc}{G}}$

44. (*b,c*)

$1\text{MSD} = \frac{1}{8}\text{cm}$

Vernier constant = 1 MSD – 1 VSD

$= 1\text{MSD} - \frac{4}{5}\text{VSD}$

$= \frac{1}{5}\text{MSD} = \frac{1}{5}\times\frac{1}{8} = 0.25$ mm

One division on main scale = 2 × 0.25 mm

= 0.5 mm

LC of screw gauge $= \frac{0.5}{100} = 0.005$ mm

So, (*b*) is correct.

LC of linear scale = 2 × *VC* = 0.5 mm

Pitch = 2 × 0.5 mm = 1 mm

LC of screw gauge $= \frac{1\text{mm}}{100} = 0.01$ mm

So, (c) is correct.

45. (*a,c*)

$(BIl)c = VI \Rightarrow \mu_0 I^2 c \equiv VI \Rightarrow \mu_0 Ic = V$

$\Rightarrow \mu_0^2 I^2c^2 = V^2$

$\Rightarrow \mu_0 I^2 = \varepsilon_0 V^2 \Rightarrow \varepsilon_0 cV = I$

46. (*a,b*)

$$T = 2\pi\sqrt{\frac{7}{5}\frac{(R-r)}{g}}$$

$$\Rightarrow \ln T = \ln 2\pi\sqrt{\frac{7}{5}} + \frac{1}{2}\ln(R-r) + \frac{1}{2}\ln g$$

$$\Rightarrow \frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta R + \Delta r}{R - r}$$

So, $\frac{\Delta g}{g}\times 100 = \left(2\times\frac{0.02}{0.556} + \frac{1+1}{60-10}\right)\times 100 = 11.2\%$

and $\frac{\Delta r}{r}\times 100 = \frac{1}{10}\times 100 = 10\%$

and $\frac{\Delta T}{T}\times 100 = \frac{0.02}{0.556}\times 100 = 3.57\%$

47. (*b, d*)

$[k_B T] = [Fl]$

and $\left[\frac{q^2}{\epsilon}\right] = [Fl^2]$

48. (*d*) $R_1 = 2.8 + 0.01 \times 7 = 2.87$

$R_2 = 2.8 + (8MSD - 7VSD) = 2.8 + (8 \times 0.1 - 7 \times \frac{11}{10})$

$= 2.83$

49. (*c*) We have $\frac{E}{c} = B$

$\therefore [B] = \frac{[E]}{[c]} = [E]L^{-1}T^1$

$\Rightarrow [E] = [B][L][T]^{-1}$

50. (*d*) We have

$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$\therefore [c^2] = \left[\frac{1}{\mu_0 \epsilon_0}\right]$

$\Rightarrow L^2T^{-2} = \frac{1}{[\mu_0][\epsilon_0]}$

$\Rightarrow [\mu_0] = [\epsilon_0]^{-1}[L]^{-2}[T]^2$

51. (*a,b,d*)

Mass $= M^0L^0T^0$

$MVr = M^0L^0T^0$

$M^0 \frac{L^1}{T^1} . L^1 = M^0L^0T^0$

$L^2 = T^1$ ...(i)

Force $= M^1L^1T^{-2}$ (in *SI*)

$= M^0L^1L^{-4}$ [In new system from equation (i)]

$= L^{-3}$

Energy $= M^1L^2T^{-2}$ (In *SI*)

$= M^0L^2L^{-4}$ [In new system from equation (i)]

$= L^{-2}$

Power $= \frac{\text{Energy}}{\text{Time}}$

$= M^1L^2T^{-3}$ (in *SI*)

$= M^0L^2L^{-6}$ [In new system from equation (i)]

$= L^{-4}$

Linear momentum $= M^1L^1T^{-1}$ (in *SI*)

$= M^0L^1L^{-2}$ [In new system from equation(i)]

$= L^{-1}$

52. (*a,b*)

Postion $= [x^\alpha]$

$\Rightarrow [L] = [x^\alpha]$ ...(i)

speed $= x^\beta$

$LT^{-1} = x^\beta$

$x^\alpha T^{-1} = x^\beta$ [From (i)]

$T^{-1} = x^{\beta - \alpha}$ ...(ii)

acceleration $= x^p$

$LT^{-1} \times T^{-1} = x^p$

$x^\beta . x^{\beta - \alpha} = x^p$

$\Rightarrow 2\beta - \alpha = p$

$\alpha + p = 2\beta$

Option (a)

[Linear momentum] $= x^q$

$MLT^{-1} = x^q$

$M.x^\beta = x^q$

$M = x^{q - \beta}$ ...(iii)

[force] $= x^r$

$MLT^{-2} = x^r$

$M.X^p = x^r$

$M = x^{r-p}$ ... (iv)

From (iii) and (iv)

$q - \beta = r - p$

$\Rightarrow p + q + r = \beta$

53. (*c*) Given 10 *VSD* = 9 *MSD*

Least count = 1 *VSD* = $\frac{9}{10}$ *MSD*

Least count = 1 *MSD* – 1 *VSD* = $\left(1 - \frac{9}{10}\right)$ *MSD*

= 0.1 *MSD* = 0.1 × 0.1 cm = 0.01 cm

As '0' of V.S. lie before '0' of M.S.

Zero error = –[10 – 6] L.C.

= –4 × 0.01 cm

= –0.04 cm

Reading = 3.1 cm + 1 × LC = 3.4 cm + 1 × 0.01 cm

= 3.11 cm

True diameter = Reading – Zero error

= 3.11 – (–0.04)cm = 3.15 cm

54. (*b,d*)

$\bar{S} = [\bar{E} \times \bar{B}]\frac{1}{\mu_0}$

$\bar{S}$ is pointing vector denotes flow of energy per unit area per unit time

$\bar{S} = \frac{\text{watt}}{\text{m}^2}$

Hence (*b, d*) are correct

55. [4] $MT^{-2}I^{-1} = [IT]^\alpha[M]^\beta[ML^2T^{-1}]^\gamma[ML^3T^{-4}I^{-2}]^\delta$

$\therefore \beta + \gamma + \delta = 1$ ...(i)

$2\gamma + 3\delta = 0$ ...(ii)

$\alpha - \gamma - 4\delta = -2$ ...(iii)

$\alpha - 2\delta = -1$ ...(iv)

On solving these equations, we get

$\alpha + \beta + \gamma + \delta = 4$

CHAPTER

2 Vector and Calculus

Exercise-1 (Topicwise)

1. (*b*) Fact based

2. (*b*) $|P-Q| \le R \le |P+Q|$

If $P = 10\text{N}$ & $Q = 6\text{N}$

$4 \le R \le 16$

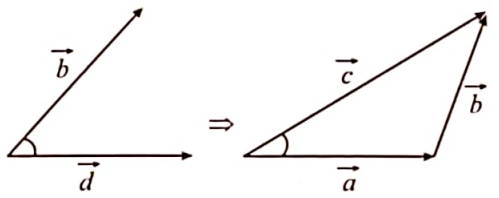
3. (*d*) Initial and final position are coincide.

4. (*d*) $\vec{S} = 6\hat{i} + 8\hat{j} - 21\hat{i}$

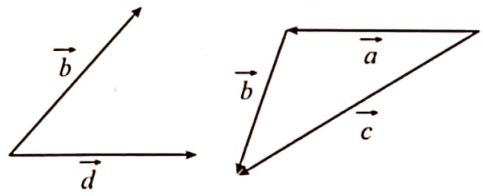
$= -15\hat{i} + 8\hat{j}$

$\therefore |\vec{S}| = 17$

5. (*a*)



Now direction one interchanged



Change in direction but angle between A & B is change so no change in magnitude.

6. (*d*) $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$

$\overline{2Q} = 0$

$\vec{Q} = 0$

$|\vec{Q}| = 0$

7. (*b*) $\vec{a}_{avg} = \dfrac{\vec{v}_f - \vec{v}_i}{t} = \dfrac{5\hat{j} - 5\hat{i}}{10} = \dfrac{1}{\sqrt{2}}\left(\dfrac{\hat{j}}{\sqrt{2}} - \dfrac{\hat{i}}{\sqrt{2}}\right)$

8. (*a*) Let $R = |F_1 - F_2|$ basically we here to find 'R'

$R^2 = |F_1 - F_2|^2$ square

$= |F_1|^2 + |F_2|^2 - 2\vec{F}_1.\vec{F}_2$

$= 164 + 80 = 244$

9. (*c*) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 2\hat{i} - 3\hat{j} + 4\hat{k} - \hat{i} - \hat{j} + \hat{k}$

$= \hat{i} - 4\hat{j} + 5\hat{k}$

10. (*a*) $2\hat{i} - 3\hat{j}$, $\vec{F} = F_x\hat{i} + F_y\hat{j}$

11. (*a*) The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is double in magnitude.

$\therefore\ 2|\hat{i} + x\hat{j} + 3\hat{k}| = |4\hat{i} + (4x-2)\hat{j} + 2\hat{k}|$

$\Rightarrow 2\sqrt{1^2 + x^2 + 3^2} = \sqrt{4^2 + (4x-2)^2 + 2^2}$

$\Rightarrow 4(10 + x^2) = 20 + 16x^2 - 16x + 4$

$\Rightarrow 40 + 4x^2 = 20 + 16x^2 - 16x + 4$

$\Rightarrow 12x^2 - 16x - 16 = 0 \Rightarrow (x-2)(3x+2) = 0$

$\therefore x = 2,\ x = -\dfrac{2}{3}$

12. (*d*) Fact based

13. (*b*) $AB\cos\theta = 8$

$AB\sin\theta = 8\sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = 60°$.

14. (*b*) The displacement is

$\vec{d} = ([3, 2, 4] - [2, 1, 3])\ m = [1, 1, 1]$ m.

Then $\vec{F}\cdot\vec{d} = [(1)(1) + (-2)(1) + (0)(1)\ \text{N.m} = -1\ J.]$

15. (*a*) $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{2+2-2}{\sqrt{3}\sqrt{12}}$

$\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

16. (*b*) $\vec{A}.\vec{B} = AB\cos 30°$

$\vec{B}.\vec{C} = BC\cos 150° = BC\cos(180° - 30°)$

$= -BC\cos 30°$

$\Rightarrow$ If $A = B = C$

$\Rightarrow \vec{A}.\vec{B} \ne \vec{B}.\vec{C}$

17. (*d*) $\vec{A}.\vec{B} = 15 + 2 - 2l = 0$

$2\ell = 17 \Rightarrow \ell = \dfrac{17}{2} = 8.5$

18. (*b*) $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$\vec{\tau} = 17\hat{i} - 6\hat{j} - 13\hat{k}$

19. (*d*) From the property of vector product, we notice that $\vec{C}$ must be perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and ($\vec{A} + \vec{B}$) vector also must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to ($\vec{A} + \vec{B}$) also but the cross product ($\vec{A} \times \vec{B}$) gives a vector $\vec{C}$ which can not be perpendicular to itself. Thus the last statement is wrong.

20. (*b*) $\vec{P} \times \vec{Q} = PQ \sin\theta$

Required unit vector $= \dfrac{\vec{P} \times \vec{Q}}{|\vec{P} \times \vec{Q}|} = \dfrac{\vec{P} \times \vec{Q}}{PQ\sin\theta} = \dfrac{\hat{P} \times \hat{Q}}{\sin\theta}$

21. (*a*) $(\vec{A} + \vec{B} + \vec{C}) = (3\hat{i} - 5\hat{k})$

$(\vec{A} \times \vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 1 & 4 \end{vmatrix} = 5\hat{i} - 7\hat{j} + 3\hat{k}$

$\cos\theta = \dfrac{15 - 15}{|3\hat{i} - 5\hat{k}||5\hat{i} - 7\hat{j} + 3\hat{k}|}$

$\cos\theta = 0$

$\theta = 90°$

22. (*a*) $\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 7\hat{i} - 3\hat{j} - 5\hat{k}$

Unit Vector $= \pm\dfrac{1}{\sqrt{83}}(7\hat{i} - 3\hat{j} - 5\hat{k})$

23. (*a*) $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$ ...(i)

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta$ ...(ii)

$\tan\theta = \dfrac{|\vec{A} \times \vec{B}|}{\vec{A} \cdot \vec{B}}$

$\vec{A} \cdot \vec{B} = \dfrac{8}{\tan\theta}$ ...(iii)

Also, from (ii)

$8 = 2 \times 5 \sin\theta$

$\sin\theta = 4/5$ then $\tan\theta = 4/3$

Now, from (iii)

$\vec{A} \cdot \vec{B} = \dfrac{8 \times 3}{4} = \pm 6$

24. (*b*) $\sqrt{3}|\vec{A}||\vec{B}|\sin\theta = |\vec{A}||\vec{B}|\cos\theta$

$\theta = 30°$

25. (*d*) Fact based

26. (*b*) $f(\pi/2) = \cos\pi/2 + \sin\pi/2 = 1$

27. (*c*) $\dfrac{dy}{dx} = 2x + 1$

28. (*d*) $\dfrac{ds}{dt} = 15t^2 - 15t^4$

29. (*b*) $\dfrac{dy}{dx} = 5\cos x + C$

30. (*a*) $\dfrac{dy}{dx} = 12x - 10 + 10x^{-3}$, $\dfrac{d^2y}{dx^2} = 12 - 30x^{-4}$

31. (*b*) $\dfrac{dr}{d\theta} = -12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}$

$\dfrac{d^2r}{d\theta^2} = 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

32. (*d*) $\dfrac{d\omega}{dz} = 21z^6 - 21z^2 + 42z$,

$\dfrac{d^2\omega}{dz^2} = 126z^5 - 42z + 42$

33. (*c*) $\dfrac{dy}{dx} = \cos x - \sin x$, $\dfrac{d^2y}{dx^2} = -\sin x - \cos x$

34. (*a*) $y = x \sin x$

$dy/dx = x \cos x + \sin x$

35. (*d*) $e^x \ell nx + \dfrac{e^x}{x}$

36. (*b*) $\dfrac{dy}{dx} = 3x^2$

37. (*a*) $\sec^2 x$

38. (*a*) $y' = \dfrac{-19}{(3x-2)^2}$

39. (*b*) $\dfrac{dz}{dx} = \dfrac{-2x^2 - 2x - 2}{(x^2 - 1)^2}$

40. (*b*) With $u = (2x + 1)$

$y = u^5 : \dfrac{dy}{dx} = \dfrac{dy}{du} = 5u^4 . 2 = 10(2x+1)^4$

41. (*d*) $\dfrac{dy}{dx} = -27(4 - 3x)^8$

42. (*b*) With $u = (1 - \left(\dfrac{x}{7}\right))$ $y = u^{-7} : \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= -7u^{-8} . \left(-\dfrac{1}{7}\right) = \left(1 - \dfrac{x}{7}\right)^{-8}$

43. (*b*) $\cos(x) + \dfrac{2}{x} + 2e^{2x}$

44. (*a*) $2\omega\cos(\omega x + \phi)$

45. (*d*) $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$

46. (*a*) For maxima and minima, $dy/dx = 0$

$6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$

At $x = 2$, $y = 39$ (max.)

At $x = 3$, $y = 38$ (min.)

47. (*a*) $\frac{dy}{dx} = 48\frac{dy/du}{dx/du} = \frac{6u^2}{\frac{1}{8}} = 48(8x-1)^2$

48. (*b*) $3\cos(3x+1)$

49. (*a*) $\frac{dy}{dx} = \cos u \cdot \frac{dy}{dx} = \cos u \cdot (-\sin x)$

50. (*c*) $6t^2 + C$

51. (*a*) $x^2 + C$

52. (*c*) $x^3/3 + C$

53. (*b*) $\frac{x^3}{3} - x^2 + x + C$

54. (*a*) $x^{-3} + C$

55. (*d*) $-1/3x^{-3} + C$

56. (*a*) $-\frac{5}{x} + C$

57. (*d*) $\sqrt{x^3} + C$

58. (*b*) $3\sqrt{x} + C$

59. (*a*) $x^{4/3} + C$

60. (*d*) $\frac{x^{\frac{2}{3}}}{2} + C$

61. (*d*) $\frac{3x^{\frac{4}{3}}}{4} + \frac{3x^{\frac{2}{3}}}{2} + C$

62. (*b*) $x^{1/2} + C$

63. (*a*) $x^{-1/2} + C$

64. (*c*) $-3\cos x + C$

65. (*a*) $\frac{1}{3}\ell nx + C$

66. (*d*) $-\frac{1}{3}\cos 3x + C$

67. (*a*) $[5x]_{-2}^{1} = (5 + 10) = 15$

68. (*a*) $\frac{\pi}{2}[\theta]_{-4}^{-1} = \frac{\pi}{2}(-1+4) = \frac{3\pi}{2}$

69. (*a*) Area $= \int_0^b 2x\, dx = b^2$

70. (*b*) $\left(\frac{t^3}{3} - t\right)_{-2}^{2} \Rightarrow \left(\frac{8}{3} - 2\right) - \left(\frac{-8}{3} + 2\right) = \frac{4}{3}$

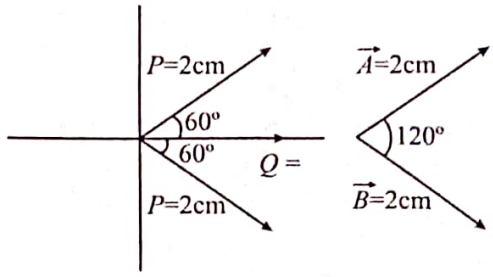
Exercise-2 (Learning Plus)

1. (*a*) θ will increase cos θ will decrease. So, magnitude of force decrease.

2. (*b*) Net displacement must be zero.

3. (*b*) $|a - b| \le R \le (a + b)$

4. (*b*)



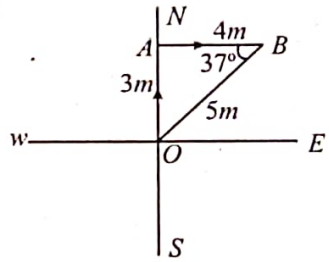
Resulting of $\bar{A}$ & $\bar{B}$ also have same unit 2 cm.

5. (*c*) $R^2 = P^2 + Q^2 + 2PQ\cos 60°$

$7Q^2 = P^2 + Q^2 + PQ \Rightarrow P^2 - 6Q^2 + PQ = 0$

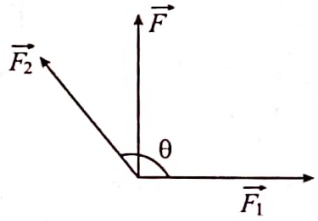
$\left(\frac{P}{Q}\right)^2 + \left(\frac{P}{Q}\right) - 6 = 0 \Rightarrow \left(\frac{P}{Q} + 3\right)\left(\frac{P}{Q} - 2\right) = 0 \Rightarrow \frac{P}{Q} = 2$

6. (*b*)



Displacement = 0

7. (*d*) $|\bar{F}_2| = |\bar{F}_2|$



$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta} \Rightarrow \alpha = 90°$

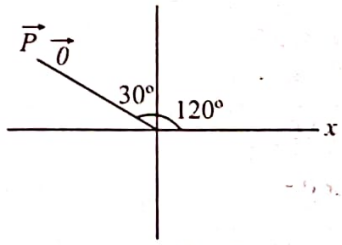
then $F_1 + F_2\cos\theta = 0 \Rightarrow \cos\theta = -\frac{1}{2}$

$\theta = 120°$

8. (*a*) $|\bar{F}_1| = |\bar{F}_2| =$ dyne $\Rightarrow \theta = 120°$

$|\bar{F}| = 2|\bar{F}_1|\cos\theta/2 \Rightarrow 10$ dyne

9. (*a*)

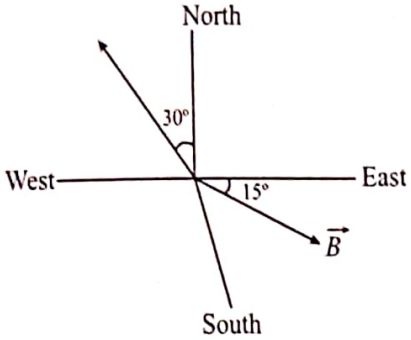


Angle between $\bar{P}$ & $\bar{\theta}$ is 0

so Resultant $= \sqrt{P^2 + \theta^2 + 2PQ\cos\theta}$

$|\bar{R}| = P + Q$

10. (*d*)



The direction of resultant must lie in between smaller angle between $\vec{A}$ and $\vec{B}$. There fore resultant in south direction is not possible.

11. (*d*) $(F_{net})_{max} = F_1 + F_2 = 12$ N

$(F_{net})_{max} = F_1 - F_2 = 8$ N

On solving

$F_1 = 10$ N and $F_2 = 2$ N

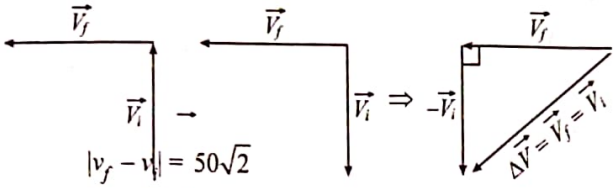
12. (*d*) $|\vec{a}+\vec{b}| = |\vec{c}|$

$1 + 1 + 2.1.1 \cos\theta = 1$

$\cos\theta = -\frac{1}{2}$

$\theta \to 120°$

13. (*b*) $\overrightarrow{\Delta_v} = \overrightarrow{v_f} - \overrightarrow{v_i}$



$|v_f - v_i| = 50\sqrt{2}$

14. (*d*) $\overrightarrow{dv} = \overrightarrow{v_2} - \overrightarrow{v_1}$

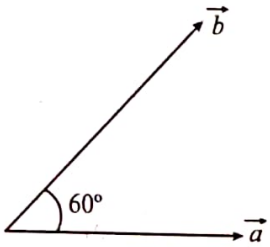
$dv = \sqrt{v_2^2 + v_1^2 - 2v_1v_2\cos\theta}$

$dv = \sqrt{2v^2(1-\cos\theta)}$

$dv = \sqrt{2v^2(2\sin^2\theta/2)}$

$dv = 2v\sin\theta/2$

15. (*b*) $|\hat{a}-\hat{b}| = \sqrt{a^2+b^2-2ab\cos\theta}$



$\theta = 60° \Rightarrow |\hat{a}-\hat{b}| = 1$

16. (*c*) $\vec{A}+\vec{B} = 7\hat{i} - 3\hat{j}$

$|A+B| = \sqrt{4a+9} = \sqrt{58}$

17. (*a*) $\vec{A}-\vec{B} = 3\hat{i} - \hat{k}$

$A - B = \frac{\vec{A}-\vec{B}}{|A-B|} = \frac{3\hat{i}+\hat{k}}{\sqrt{10}}$

18. (*b*) $\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{B} = \hat{j} \Rightarrow \vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\cos\theta = \frac{3}{\sqrt{13}} \Rightarrow \tan\theta = 2/3 \Rightarrow \theta = \tan^{-1}(2/3)$

19. (*b*) $\vec{A} = 3\hat{i} + 2\hat{j} + 8\hat{k}$

$\vec{B} = 2\hat{i} + x\hat{j} + \hat{k}$

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta \quad \theta = 90°$

$\vec{A}.\vec{B} = 0$

$6 + 2x + 8 = 0, x = -7$

20. (*d*) $\vec{a}\cdot\vec{b} + b^2 = 0$

$a^2 + 2\vec{a}\cdot\vec{b} = 0$

$a^2 - 2b^2 = 0$

21. (*a*) $\vec{F} = \vec{F_1} + \vec{F_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ N

& $\vec{d} = (5\hat{i}+4\hat{j}+3\hat{k}) - (2\hat{i}+3\hat{j}+4\hat{k}) = 3\hat{i}+\hat{j}-\hat{k}$ m

$\therefore W = \vec{F}\cdot\vec{d} = (2\hat{i}+3\hat{j}+4\hat{k})\cdot(3\hat{i}+\hat{j}-\hat{k})$

$= 6 + 3 - 4 = 5$ J

22. (*d*) if $\vec{a}+\vec{b}+\vec{c} = 0$

$(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2[(\vec{a}\cdot\vec{b}) + (\vec{b}\cdot\vec{c}) + (\vec{c}\cdot\vec{a})] = 0$

$(\vec{a}\cdot\vec{b}) + (\vec{b}\cdot\vec{c}) + (\vec{c}\cdot\vec{a}) = -3/2$

23. (*a*) By the defination of equal vector.

24. (*d*) $(\vec{A}\times\vec{B}) \perp A$

$(\vec{A}\times\vec{B}) \perp B$

25. (*c*) $\vec{\tau} = \vec{r}\times\vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 5 \\ 2 & -1 & 1 \end{vmatrix} = 8\hat{i} + 9\hat{j} - 7\hat{k}$$

26. (*d*) $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = 0$

[$\therefore$ Two vectors are perpendicular]

$A^2 - B^2 = 0$

$A = B$

Also $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are in plane of $\vec{A}$ and $\vec{B}$ whereas $\vec{A}\times\vec{B}$ is perpendicular to both of them.

27. (*d*) $\frac{dy}{dx} = \frac{2}{x} + \cos x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$

$y = \ell nx^2 + \sin x$

$\Rightarrow y = \ell nx^2 + \sin x \Rightarrow y = 2\ell nx + \sin x$

$\Rightarrow \frac{dy}{dx} = \frac{2}{x} + \cos x$

also $\frac{d^2y}{dx} = \frac{-2}{x^2} - \sin x$

28. (*a*) $e^x(\tan x + \sec^2 x)$

$y = e^x \tan x$

$\Rightarrow \frac{dy}{dx} = e^x \tan x + e^x \sec^2 x$

29. (*a*) $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

$\Rightarrow \frac{dy}{dx} = \left(1 - \frac{1}{x^2}\right)\left(x - \frac{1}{x} + 1\right) + \left(x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right)$

$\Rightarrow \frac{dy}{dx} = 1 + 2x + \frac{2}{x^3} - \frac{1}{x^2}$

30. (*c*) $x^2 \cos x$

$y = x^2 \sin x + 2x \cos x - 2 \sin x$

$\Rightarrow \frac{dy}{dx} = 2x \sin x + x^2 \cos x + 2\cos x - 2x \sin x - 2\cos x$

$\Rightarrow \frac{dy}{dx} = x^2 \cos x$

31. (*d*) $\frac{dy}{dx} = -x^2 \sin x$

$y = x^2 \cos x - 2x \sin x - 2\cos x$

$\frac{dy}{dx} = 2x\cos x - x^2 \sin x - 2\sin x - 2x\cos x + 2\sin x$

$\frac{dy}{dx} = -x^2 \sin x$

32. (*a*) $\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$

$r = (1 + \sec\theta)\sin\theta$

$\Rightarrow \frac{dx}{d\theta} = \sec\theta \tan\theta \sin\theta + (1 + \sec\theta)\cos\theta$

$\Rightarrow \frac{dr}{d\theta} = \cos\theta + \sec^2\theta$

33. (*c*) $\frac{dy}{dx} = \sec^2 x$

$y = \frac{\sin x + \cos x}{\cos x}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x(\cos x - \sin x) - (\sin x + \cos x) \times -\sin x}{\cos^2 x}$

$\frac{dy}{dx} = \frac{1}{\cos^2 x} = \sec^2 x$

34. (*b*) $p = \frac{\tan q}{1 + \tan q}$

$\Rightarrow \frac{dp}{dq} = \frac{(1 + \tan q)\sec^2 q - \tan q \sec^2 q}{(1 + \tan q)^2}$

$\Rightarrow \frac{dp}{dq} = \frac{\sec^2 q}{(1 + \tan q)^2}$

35. (*d*) $y = \sin^2(x^2 + 1)$

$\Rightarrow y = u^2 \qquad u = \sin v \quad v = x^2 + 1$

$\Rightarrow \frac{dy}{du} = 2u \qquad \frac{du}{dv} = \cos v$

$\Rightarrow \frac{dv}{dx} = 2x \quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = 2u \cos v 2x$

$\Rightarrow 4x \sin(x^2 + 1)\cos(x^2 + 1)$

36. (*a*) $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$

37. (*b*) $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

38. (*c*) $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

39. (*a*) $\left(\frac{4}{t^3} + \frac{\sqrt{t}}{t^3}\right)dt$

$\frac{4t^{-2}}{-2} - \frac{2}{3}t^{-3/2} + C = 2t^{-2} - \frac{2}{3}t^{-3/2} + C$

40. (*d*) $-\cos\theta + \theta + C$

41. (*a*) $\frac{3\pi^2}{2}$

42. (*a*) $\int \frac{\sin x}{\cos x} dx$

Let $\cos x = t$

$-\sin x \, dx = dt$

$-\int \frac{dt}{t} = -\ln t + C = -\ln(\cos x) + C = \ln(\sec x) + C$

43. (*a*) $\int \frac{\sin 2x}{\sin x} dx$

$\int \frac{2\sin x \cos x}{\sin x} dx = 2\sin x + C$

44. (*a*) $[\sin x]_0^{\pi} = 0$

45. (*d*) $\frac{1}{3}\ell n \frac{5}{2}$

46. (*a*) $F(x)\ 3x^2 - 1 = \int_0^3 (3x^2 - 1)dx$

$= 3\left|\frac{x^3}{3}\right|_0^3 - |x|_0^3 = (27 - 0) - (3 - 0) = 24$

47. (*d*) $3x^2 - 1 = p$

$x dx = \frac{dp}{6}$

$\int \frac{dp}{p.6} = \frac{1}{6}\ln(3x^2 - 1)$

48. (*b*) $[-\cos x + \sin x]_{-\pi/2}^{\pi/2} = 1 - (-1) = 2$

Exercise-3
(JEE Advanced Level)

1. (a,b,d)

2. (a,b,c)

By right handed co-ordinate system.

3. (c) $x = t^3 - 3t^2 + 12t + 20$

$v = \frac{dx}{dt} = 3t^2 - 6t + 12$

$t = 0$

$\Rightarrow v = 12$ m/s

4. (d) $a = \frac{dv}{dt} = 6t - 6$

$t = 0 \Rightarrow a = -6$ m/s^2

5. (d) $a = 0 \Rightarrow t = 1$ sec.

$v = 3t^2 - 6t + 12 = 9$ m/s

6. (a) $\vec{F}_R = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + 5\hat{j} + 4\hat{k}$

7. (b) $\cos\theta = \frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_1||\vec{F}_2|}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$

8. (c) $F_1 \cos\theta = \frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_2|} = \frac{6}{5}$

9. (d) A-(r); B-(p); C-(q); D-(s)

A. $\int \sec x \tan x \, dx = \sec x + C$

B. $\int \text{cosec } kx \cot kx \, dx = \frac{\cos kx}{k} + C$

C. $\int \text{cosec}^2 kx \, dx = -\frac{\cot kx}{k} + C$

D. $\int \cos kx \, dx = \frac{\sin kx}{k} + C$

10. (d) A-(q); B-(r); C- (p); D-(s)

A. $|\vec{A} + \vec{B}| = A^2 + B^2 + 2AB\cos\theta$

$A^2 = A^2 + A^2 + 2A^2\cos\theta$

$\Rightarrow \theta = 120$

B. $F_1 \sim F_2 \le R \le F_1 + F_2$

Here $F_1 \sim F_2 = 4$ and $F_1 + F_2 = 12$

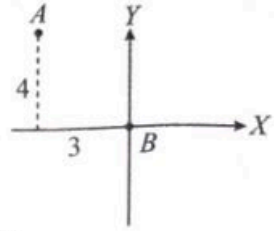
C. $\cos\theta = \frac{\vec{A} - \vec{B}}{|A| + |B|} = \frac{0}{2\sqrt{2} \times 3} = 0$

$\Rightarrow \theta = 90°$

D. $\vec{A} + \vec{B} = 2\hat{i} + \hat{j} + 3\hat{k}$

$|\vec{A} + \vec{B}| = \sqrt{2^2 + 1 + 3^2} = \sqrt{14}$

11. [0036]



$\vec{r}_f - \vec{r}_i = \vec{v}t$

$\vec{r}_f - [3(\hat{i}) + 4(\hat{j})] = 2\left(\frac{3\hat{i} + 4\hat{j}}{5}\right) \times 5$

$\vec{r}_f + 3\hat{i} - 4\hat{j} = 6\hat{i} + 8\hat{j}$

$\therefore \vec{r}_f = 3\hat{i} + 12\hat{j}$

12. [0003]

$a^2 - 2a - 3 = 0$

$a^2 - 3a + a - 3 = 0$

$a(a-3) + 1(a-3) = 0$

$\therefore a = 3$

13. [1] (Given $|A| = 3$)

$|\vec{A}| - |\vec{B}| = 2$; $|\vec{B}| > |\vec{A}|$

$|\vec{B}| = 1$

14. [2] $\vec{v}_1 \cdot \vec{v}_2 = 0 \Rightarrow t = 2$sec

15. [1] $|\vec{A}| + |\vec{B}| = 5$

$2 + |\vec{B}| = 5$

Resultant of $|\vec{A}|$ and $|\vec{B}|$

$|\vec{B}| - |\vec{A}| = 3 - 2 = 1$

Exercise-4
(Past Year Questions)

JEE MAIN

1. (a) $|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta$

$= n^2(A^2 + B^2 - 2AB\cos\theta)$

$\Rightarrow \cos\theta(1 + n^2) = \frac{2a^2(n^2 - 1)}{2a^2}$ $[A = B = a]$

$\cos\theta = \frac{n^2 - 1}{n^2 + 1}$

2. (b) $1\left(0, \frac{a}{2}, \frac{a}{2}\right)$

$2\left(\frac{a}{2}, \frac{a}{2}, 0\right)$

$\vec{r}_2 - \vec{r}_1 = \frac{a}{2}\hat{i} - \frac{a}{2}\hat{k}$

Unit vector $= \frac{\hat{i} - \hat{k}}{\sqrt{2}}$

3. (*a*) $2|\vec{P}+\vec{Q}|=|\vec{P}+2\vec{Q}|$

$\Rightarrow 13+12\cos\theta = 10+6\cos\theta$

$\cos = -\frac{1}{2}$

$\theta = 120^\circ.$

4. (*c*) $|\vec{A}_1|=3 \quad |\vec{A}_2|=5 \quad |\vec{A}_1+\vec{A}_2|=5$

$|\vec{A}_1+\vec{A}_2|=\sqrt{|\vec{A}_1|^2+|\vec{A}_2|^2+2|\vec{A}_1||\vec{A}_2|\cos\theta}$

$5=\sqrt{9+25+2\times3\times5\cos\theta}$

$\cos\theta = -\frac{9}{2\times3\times5}=-\frac{3}{10}$

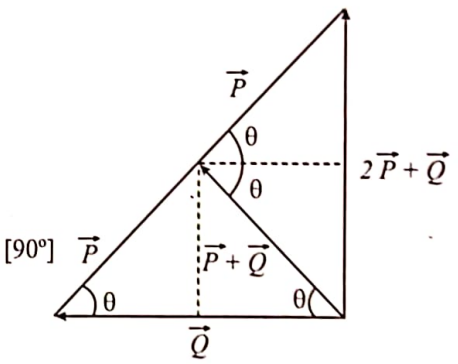
$(2\vec{A}_1+3\vec{A}_2).(3\vec{A}_1-2\vec{A}_2)$

$=6|\vec{A}_1|^2+9\vec{A}_1.\vec{A}_2-4\vec{A}_1\vec{A}_2-6|\vec{A}_2|^2$

$54+5\times3\times5\left(-\frac{3}{10}\right)-6\times25$

$=54-150-\frac{45}{2}=-118.5$

5. [90°]



So angle between $(2\vec{P}+\vec{Q})$ and $\vec{Q}$ is 90°

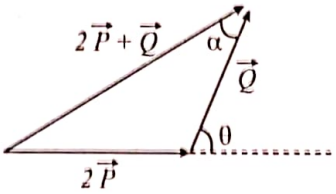
Alternate solution

$|\vec{P}+\vec{Q}|=|\vec{P}|$

$P^2+Q^2+2PQ\cos\theta=P^2$

$\Rightarrow Q+2P\cos\theta=0$

$\Rightarrow \cos\theta=-\frac{Q}{2P}$



$\tan\alpha=\frac{2P\sin\theta}{2P\cos\theta+Q}=\infty \qquad [\because 2P\cos\theta+Q=0]$

$\alpha=90^\circ$

6. (*a*) $\vec{r}=\{-3i-j+2k\}$

$\vec{F}=i+2j+3k$

$$\tau=\begin{vmatrix} i & j & k \\ -3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$=i(-3-4)-j(-9-2)+k(-6+1)$

$=-7i+11j-5k$

$=\sqrt{7^2+11^2+5^2}$

$=\sqrt{49+121+25}=\sqrt{195}$

7. (*b*) According to question the given data is two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$

$\vec{A}=\vec{P}+\vec{Q}\Rightarrow\vec{B}=\vec{P}-\vec{Q}\Rightarrow\vec{P}\perp\vec{Q}$

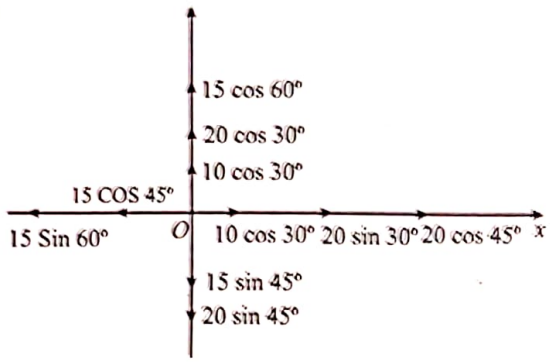
$\Rightarrow|\vec{A}|=|\vec{B}|=\sqrt{2(P^2+Q^2)(1+\cos\theta)}$

$\Rightarrow$ For $|\vec{A}+\vec{B}|=\sqrt{3(P^2+Q^2)}\Rightarrow\theta_1=60^\circ$

$\Rightarrow$ For $|\vec{A}+\vec{B}|=\sqrt{2(P^2+Q^2)}\Rightarrow\theta_2=90^\circ$

According to solution Both Statement-I and Statement-II are true

8. (*a*)



Force in x-direction,

$\vec{F_x}=\left(10\times\frac{\sqrt{3}}{2}+20\left(\frac{1}{2}\right)+20\left(\frac{1}{\sqrt{2}}\right)-15\left(\frac{1}{\sqrt{2}}\right)-15\left(\frac{\sqrt{3}}{2}\right)\right)\hat{i}$

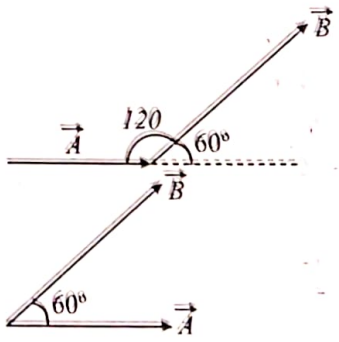
$=9.25\hat{i}$

Force in y-direction,

$\vec{F_y}=\left(15\left(\frac{1}{2}\right)+20\left(\frac{\sqrt{3}}{2}\right)+10\left(\frac{1}{2}\right)-15\left(\frac{1}{\sqrt{2}}\right)-20\left(\frac{1}{\sqrt{2}}\right)\right)\hat{j}$

$=5\hat{j}$

9. (*c*)



According to diagram,

Angle between $\vec{A}$ and $\vec{B}$,

$\theta = 60°$

Angle between $\vec{A}$ and $-\vec{B}$,

$\theta = 120°$

If angle between $\vec{A}$ and $\vec{A} - \vec{B}$ is α

then $\tan\alpha = \dfrac{|-\vec{B}|\sin\theta}{\vec{A}+|-\vec{B}|\cos\theta}$

$$= \frac{B\sin 120^\circ}{A + B\cos 120^\circ}$$

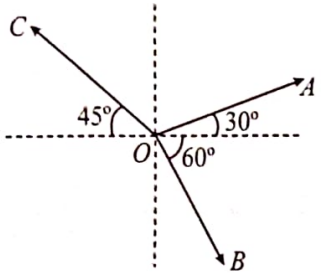
$$= \frac{B\frac{\sqrt{3}}{2}}{A - \frac{B}{2}}$$

$$\Rightarrow \tan\alpha = \frac{\sqrt{3}B}{2A - B}$$

Hence, the angle between vector $\vec{A}$ and $(\vec{A} - \vec{B})$ is

$$\tan\alpha = \frac{\sqrt{3}B}{2A - B}$$

10. (*a*)



According to diagram, let magnitude be equal to

$$\overline{OA} = \lambda\left[\cos 30^\circ\hat{i} + \sin 30\hat{j}\right] = \lambda\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

$$\overline{OB} = \lambda\left[\cos 60^\circ\hat{i} - \sin 60\hat{j}\right] = \lambda\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\overline{OC} = \lambda\left[\cos 45^\circ(-\hat{i}) + \sin 45\hat{j}\right] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$$

$$\therefore \overline{OA} + \overline{OB} - \overline{OC}$$

$$= \lambda\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)\hat{i} + \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$$\tan^{-1}\left[\frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}}\right] = \tan^{-1}\left[\frac{\sqrt{2} - \sqrt{6} - 2}{\sqrt{6} + \sqrt{2} + 2}\right]$$

$$= \tan^{-1}\left[\frac{1 - \sqrt{3} - \sqrt{2}}{\sqrt{3} + 1 + \sqrt{2}}\right]$$

11. [5] $\vec{A}\cdot\hat{B} = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$$0 = (2\hat{i} + 4\hat{j} - 2\hat{k})\cdot\left(\frac{\hat{i} + 2\hat{j} + \alpha\hat{k}}{\sqrt{1+4+\alpha^2}}\right)$$

$0 = 2 + 8 - 2\alpha$

$\alpha = 5$

12. [2] The magnitude of the component of vector A along vector B will be

$$A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|} = \frac{2+6-2}{3} = 2$$

13. (*b*) $|\vec{A} + \vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$

$= \sqrt{1 + 1 + 2\cos\theta}$

$= 2\cos\theta/2$

$|\vec{A} - \vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta}$

$= \sqrt{1 - 2\cos\theta} = 2\sin\theta/2$

$$\frac{|\vec{A} + \vec{B}|}{|\vec{A} - \vec{B}|} = \cot\theta/2 \Rightarrow |\vec{A} - \vec{B}| = |\vec{A} + \vec{B}|\tan\frac{\theta}{2}$$

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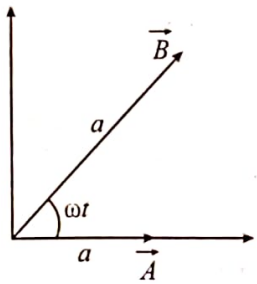
14. (*c*) Fact based

15. (*d*) $\vec{S} = \vec{P} + b\vec{R} = \vec{P} + b(\vec{Q} - \vec{P}) = \vec{P}(1-b) + b\vec{Q}$

16. [2.00]



$$|\vec{A} + \vec{B}| = 2a\cos\frac{\omega t}{2}$$

$$|\vec{A} + \vec{B}| = 2a\sin\frac{\omega t}{2}$$

So, $2a\cos\dfrac{\omega t}{2} = \sqrt{3}\left(2a\sin\dfrac{\omega t}{2}\right)$

$$\tan\frac{\omega t}{2} = \frac{\pi}{6}$$

$$\Rightarrow \omega t = \frac{\pi}{3}$$

$$\frac{\pi}{6}t = \frac{\pi}{3}$$

$t = 2.00$ sec

CHAPTER

3 Motion in a Straight Line

Exercise-1 (Topicwise)

1. (*a*) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{6^2 + 8^2 + 10^2} = 10\sqrt{2}$ m

2. (*a*) $\vec{r} = 20\hat{i} + 10\hat{j}$

$\therefore r = \sqrt{20^2 + 10^2} = 22.5$ m

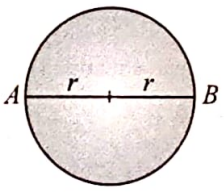
3. (*c*) $PQ = (7\hat{i} - 2\hat{j} - 3\hat{k}) - (-4\hat{i} - 5\hat{j} + 8\hat{k})$

$= 11\hat{i} + 3\hat{j} - 11\hat{k}$

4. (*c*) Displacement = $\sqrt{R^2 + R^2} = R\sqrt{2}$

Distance = $\frac{3}{4} \times 2\pi R = \frac{3}{2}\pi R$

5. (*b*)



Displacement = $2r$

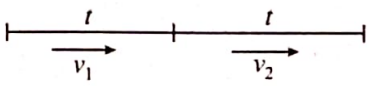
Distance = πr

6. (*d*) As the total distance is divided into two equal parts therefore distance average speed = $\frac{2v_1v_2}{v_1 + v_2}$.

7. (*c*) Distance average speed = $\frac{2v_1v_2}{v_1 + v_2}$

$= \frac{2 \times 30 \times 50}{30 + 50} = \frac{75}{2} = 37.5$ km/hr.

8. (*b*)



$V_{Arg} = \frac{S}{T} = \frac{v_1 t + v_2 t}{2t} = \frac{v_1 + v_2}{2}$

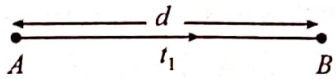
9. (*d*) Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

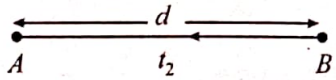
$= \frac{x}{\frac{2x/5}{v_1} + \frac{3x/5}{v_2}} = \frac{5v_1v_2}{3v_1 + 2v_2}$.

10. (*c*) From given figure, it is clear that the net displacement is zero. So average velocity will be zero.

11. (*b*) From A to B $t_1 = \frac{d}{20}$ hr

$\Rightarrow$ From B to A $t_2 = \frac{d}{30}$ hr





$\therefore$ Average Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$

$= \frac{2d}{t_1 + t_2} = \frac{2d}{\frac{d}{20} + \frac{d}{30}}$

$\Rightarrow v = 24$ km/hr

12. (*a*) $2s = gt^2 \Rightarrow s = \frac{1}{2}gt^2$

$v = \frac{ds}{dt} = gt$

13. (*b*) As $S = ut + \frac{1}{2}at^2$

$\therefore S_1 = \frac{1}{2}a(10)^2 = 50a$ ...(i)

As $v = u + at$

Velocity acquired by particle in 10 sec $v = a \times 10$

For next 10 sec, $S_2 = (10a) \times 10 + \frac{1}{2}(a) \times (10)^2$

$S_2 = 150a$ ...(ii)

From (i) and (ii) $S_1 = S_2/3$

14. (*a*) From $S = ut + \frac{1}{2}at^2$

$$S_1 = \frac{1}{2}a(P-1)^2 \text{ and } S_2 = \frac{1}{2}aP^2 \; [\text{As } u = 0]$$

From $S_n = u + \frac{a}{2}(2n-1)$

$$S_{(P^2-P+1)^{th}} = \frac{a}{2}\left[2(P^2-P+1)-1\right] = \frac{a}{2}[2P^2-2P+1]$$

It is clear that $S_{(P^2-P+1)^{th}} = S_1 + S_2$.

15. (*b*) $v = 4t^3 - 2t$ (given)

$$\therefore a = \frac{dv}{dt} = 12t^2 - 2$$

and $x = \int_0^t v\,dt = \int_0^t (4t^3 - 2t)\,dt = t^4 - t^2$

When particle is at 2m from the origin $t^4 - t^2 = 2$

$\Rightarrow t^4 - t^2 - 2 = 0 \Rightarrow (t^2-2)(t^2+1) = 0 \Rightarrow t = \sqrt{2}$ sec

Acceleration at $t = \sqrt{2}$ sec given by,

$a = 12t^2 - 2 = 12 \times 2 - 2 = 22$ m/s^2.

16. (*b*) Let u_1, u_2, u_3 and u_4 be velocities at time $t = 0$, t_1, $(t_1 + t_2)$ and $(t_1 + t_2 + t_3)$ respectively and acceleration is a then $v_1 = \frac{u_1+u_2}{2}$, $v_2 = \frac{u_2+u_3}{2}$ and $v_3 = \frac{u_3+u_4}{2}$

Also $u_4 = u_1 + at_1$, $u_3 = u_1 + a(t_1 + t_2)$

and $u_4 = u_1 + a(t_1 + t_2 + t_3)$

By solving, we get $\frac{v_1 - v_2}{v_2 - v_3} = \frac{(t_1+t_2)}{(t_2+t_3)}$.

17. (*b*) From $v^2 = u^2 + 2aS$

$\Rightarrow 0 = u^2 + 2aS$

$$\Rightarrow a = \frac{-u^2}{2S} = \frac{-(20)^2}{2\times 10} = -20 \text{ m/s}^2.$$

18. (*b*) Constant velocity means constant speed as well as same direction throughout.

19. (*d*) $S \propto u^2$ If u becomes 3 times then S will become 9 times i.e. $9 \times 20 = 180$ m.

20. (*c*) Instantaneous velocity $v = \frac{\Delta x}{\Delta t}$

By using the data from the table

$$v_1 = \frac{0-(-2)}{1} = 2 \text{ m/s}, \quad v_2 = \frac{6-0}{1} = 6 \text{ m/s}$$

$$v_3 = \frac{16-6}{1} = 10 \text{ m/s}.$$

21. (*c*) For First part,

$u = 0$, $t = T$ and acceleration $= a$

$$\therefore v = 0 + aT = aT \text{ and } S_1 = 0 + \frac{1}{2}aT^2 = \frac{1}{2}aT^2$$

For Second part,

$u = aT$, retardation $= a_1$, $v = 0$ and time taken $= T_1$ (let)

$\therefore 0 = u - a_1T_1 \Rightarrow aT = a_1T_1$

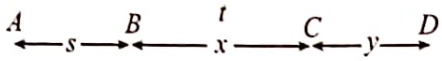
and from $v^2 = u^2 - 2aS_2$

$$\Rightarrow S_2 = \frac{u^2}{2a_1} = \frac{1}{2}\frac{a^2T^2}{a_1} \quad (v = aT)$$

$$S_2 = \frac{1}{2}aT \times T_1 \left(\text{As } a_1 = \frac{aT}{T_1}\right)$$

$$V_{Av} = \frac{S_1 + S_2}{T + T_1} = \frac{aT}{2}$$

22. (*c*) Let car starts from point A from rest and moves up to point B with acceleration f.



Velocity of car at point B, $v = \sqrt{2fS}$

[As $v^2 = u^2 + 2as$]

Car moves distance BC with this constant velocity in time t

$x = \sqrt{2fS}\,t$ ...(i) [As $s = ut$]

So the velocity of car at point C also will be $\sqrt{2fs}$ and finally car stops after covering distance y.

$$\text{Distance } CD \Rightarrow y = \frac{\left(\sqrt{2fS}\right)^2}{2(f/2)} = \frac{2fS}{f} = 2S \quad \text{...(ii)}$$

[As $v^2 = u^2 - 2as \Rightarrow s = u^2/2a$]

So, the total distance $AD = AB + BC + CD = 15S$ given

$\Rightarrow S + x + 2S = 15S$

$\Rightarrow x = 12S$

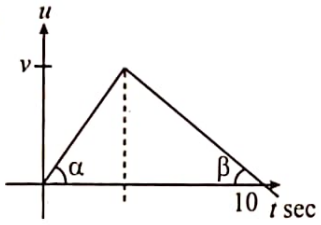
Substituting the value of x in equation (*i*) we get

$x = \sqrt{2fS}\,t \Rightarrow 12S = \sqrt{2fS}\,t$

$\Rightarrow 144S^2 = 2fS.t^2$

$$\Rightarrow S = \frac{1}{72}ft^2$$

23. (*a*)



$\tan\alpha = 2$, $\tan\beta = 3$

$$2 = \frac{v}{t}, \qquad 3 = \frac{v}{10-t}$$

$v = 2t$, $v = 30 - 3t$

$30 - 3t = 2t$

$\Rightarrow t = 6$ sec

$v = 12$ m/s

24. (*c*) $u = 12$ m/s, $g = 9.8$ m/sec^2, $t = 10$ sec

Displacement $= ut + \frac{1}{2}gt^2$

$= 12\times10 + \frac{1}{2}\times9.8\times100 = 610$ m.

25. (*c*) $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{2h/g}$

$t_a = \sqrt{\frac{2a}{g}}$ and $t_b = \sqrt{\frac{2b}{g}} \Rightarrow \frac{t_a}{t_b} = \sqrt{\frac{a}{b}}$

26. (*c*) Vertical component of velocities of both the balls are same and equal to zero. So $t = \sqrt{\frac{2h}{g}}$.

27. (*a*) $h = \frac{1}{2}gt^2 = \frac{1}{2}\times10\times(4)^2 = 80$ m.

28. (*d*) Let the body after time $t/2$ be at x from the top, then

$x = \frac{1}{2}g\frac{t^2}{4} = \frac{gt^2}{8}$ ...(i)

$h = \frac{1}{2}gt^2$ ...(ii)

Eliminate t from (i) and (ii),

we get $x = \frac{h}{4}$

$\therefore$ Height of the body from the ground $= h - \frac{h}{4} = \frac{3h}{4}$.

29. (*c*) Force down the plane $= mg\sin\theta$

$\therefore$ Acceleration down the plane $= g\sin\theta$

Since $l = 0 + \frac{1}{2}g\sin\theta t^2$

$\therefore t^2 = \frac{2l}{g\sin\theta} = \frac{2h}{g\sin^2\theta} \Rightarrow t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$.

30. (*a*) Height travelled by ball (with balloon) in 2 sec

$h_1 = \frac{1}{2}at^2 = \frac{1}{2}\times4.9\times2^2 = 9.8$ m

Velocity of the balloon after 2 sec

$v = at = 4.9\times2 = 9.8$ m/s

Now if the ball is released from the balloon then it acquire same velocity in upward direction.

Let it move up to maximum height h_2

$v^2 = u^2 - 2gh_2$

$\Rightarrow 0 = (9.8)^2 - 2\times(9.8)\times h_2$

$\therefore h_2 = 4.9$ m

Greatest height above the ground reached by the ball $= h_1 + h_2 = 9.8 + 4.9 = 14.7$ m.

31. (*d*) Given $a = 19.6$ m/s$^2 = 2g$

Resultant velocity of the rocket after 5 sec

$v = 2g \times 5 = 10g$ m/s

Height achieved after 5 sec, $h_1 = \frac{1}{2}\times2g\times25 = 245$ m

On switching off the engine it goes up to height h_2 where its velocity becomes zero.

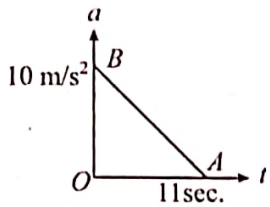
$0 = (10g)^2 - 2gh_2 \Rightarrow h_2 = 490$ m

$\therefore$ Total height of rocket $= 245 + 490 = 735$ m.

32. (*b*) The time of fall is independent of the mass.

33. (*c*) Speed of the object at reaching the ground $v = \sqrt{2gh}$.

34. (*b*) The area under acceleration time graph gives change in velocity. As acceleration is zero at the end of 11 sec



i.e. v_{max} = Area of $\Delta OAB = \frac{1}{2}\times11\times10 = 55$ m/s

35. (*c*) $v = (180 - 16x)^{1/2}$

As $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt}$

$\therefore a = \frac{1}{2}(180-16x)^{-1/2}\times(-16)\left(\frac{dx}{dt}\right)$

$= -8(180-16x)^{-1/2}\times v$

$= -8(180-16x)^{-1/2}\times(180-16x)^{1/2} = -8$ m/s^2.

36. (*d*) $x = ae^{-\alpha t} + be^{\beta t}$

Velocity $v = \frac{dx}{dt} = \frac{d}{dt}\left(ae^{-\alpha t} + be^{\beta t}\right)$

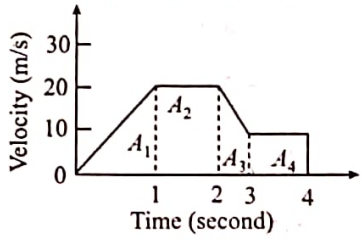
$= a.e^{-\alpha t}\left(-\alpha + be^{\beta t}.\beta\right) = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$

Acceleration $= -a\alpha e^{\alpha t}(\alpha) + b\beta e^{bt}.\beta$

$= a\alpha^2 e^{-\alpha t} + b\beta^2 e^{\beta t}$

Acceleration is positive so velocity goes on increasing with time.

37. (*b*) Distance = Area under $v - t$ graph $= A_1 + A_2 + A_3 + A_4$



$= \frac{1}{2}\times1\times20 + (20\times1) + \frac{1}{2}(20+10)\times1 + (10\times1)$

$= 10 + 20 + 15 + 10 = 55$ m.

38. (*d*) Maximum acceleration means maximum change in velocity in minimum time interval.

In time interval $t = 30$ to $t = 40$ sec

$$a = \frac{\Delta v}{\Delta t} = \frac{80-20}{40-30} = \frac{60}{10} = 6\,\text{cm/sec}^2.$$

39. (*c*) Area of trapezium $= \frac{1}{2} \times 3.6 \times (12+8) = 36.0\,\text{m}.$

40. (*d*) Slope of displacement time graph is negative only at point *E*.

41. (*a*) This graph shows uniform motion because line having a constant slope.

42. (*a*) For the given condition initial height $h = d$ and velocity of the ball is zero. When the ball moves downward its velocity increases and it will be maximum when the ball hits the ground & just after the collision it becomes half and in opposite direction. As the ball moves upward its velocity again decreases and becomes zero at height $d/2$. This explanation match with graph (*a*).

43. (*d*) For upward motion

Effective acceleration $= -(g + a)$

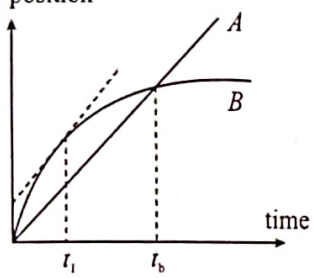
and for downward motion

Effective acceleration $= (g - a)$

But both are constants. So the slope of speed-time graph will be constant.

44. (*a*) Since slope of graph remains constant for velocity-time graph.

45. (*c*) position



Both have same velocity at some time before t_b at time t_1.

46. (*c*) From graph it is clear that velocity is always positive during its motion

So displacement = distance

Displacement = Area under *V-t* curve

$$= \frac{1}{2} \times 20 \times 1 + 20 \times 1 + \frac{1}{2} \times 1 \times 30 + 1 \times 10$$

$\Rightarrow$ 55 m

47. (*a*) Total Distance = Area

Under the curve (Position + Negative)

$$= \frac{1}{2}(4+2) + \frac{1}{2} \times (4+2) \times 2 = 18\,\text{m}$$

Exercise-2 (Learning Plus)

1. (*b*) $t = 62.8$ sec in each lap car travel a distance $2\pi R$

$2 \times 3.14 \times 100 = 628$ m

In each lap displacement of the car = 0

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{628}{62.8} = 10 \text{ m/s}$$

$$\text{Average Velocity} = \frac{\text{Total Displacement}}{\text{Total Time}} = 0$$

2. (*b*) $V_{inst} = \frac{dx}{dt}$ (slop of *x–t* graph)

At *C* $\tan\theta = +ve$ At *E* $\theta > 90°$ (–ve slope)

At *F* $\theta < 90°$ (+ve slope)

$\therefore$ At *D*, $V = 0$ as $\tan\theta = 0$

3. (*c*) Let the acceleration of the body is a and $u = 0$ then

$$x_1 = \frac{1}{2}at^2 = \frac{1}{2}a(10)^2$$

$$x_2 = \frac{1}{2}a(20)^2 - x_1$$

$$\Rightarrow x_2 = \frac{1}{2}a(20)^2 - \frac{1}{2}a(10)^2$$

$$= \frac{1}{2}a(10)(30)$$

$$\Rightarrow x_3 = \frac{1}{2}a(30)^2 - \frac{1}{2}a(20)^2$$

$$= \frac{1}{2}a(10)(50)$$

$$\therefore x_1 : x_2 : x_3 = 1:3:5$$

4. (*b*) $S_2 = \frac{1}{2} \times a \times 4,\ S_3 = \frac{1}{2} \times a \times 9$

$S_4 = \frac{1}{2} \times a \times 16,\ S_5 = \frac{1}{2} \times a \times 25$

Distance travelled by body in 4th sec $= \frac{1}{2}a(7)$

Distance travelled by body in 5th sec $= \frac{1}{2}a(9)$

Ratio = 7 : 9

5. (*d*) Let constant acceleration = *a*

$$S = \frac{1}{2}at^2$$

$$S_1 = \frac{1}{2}a \times 10^2 = 50a$$

$$S_2 = \frac{1}{2}a \times 20^2 - \frac{1}{2}a \times 10^2 = 150a$$

$$S_2 = 3S_1$$

6. (*b*) In inclined initial $u = 0$

$$S = \frac{1}{2}at^2 \text{ and } a = g\sin\theta$$

$$l = \frac{1}{2}g\sin\theta \times (4)^2 \quad ...(i)$$

$$\frac{\ell}{4} = \frac{1}{2}g\sin\theta t^2 \quad ...(ii)$$

From (i) and (ii), $t = 2$ sec

7. (*c*) $\frac{h}{2} = \frac{1}{2}gt_1^2$ ...(i)

$$h = \frac{1}{2}g(t_1 + t_2)^2 \quad ...(ii)$$

From equation (i) and (ii)

$$2t_1^2 = (t_1 + t_2)^2$$

$$\sqrt{2}t_1 = t_1 + t_2$$

$$(\sqrt{2} - 1)t_1 = t_2$$

$$t_1 = \frac{t_2}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$t_1 = \left(\sqrt{2}+1\right)t_2$$

8. (*d*) At H_{max}, $v = 0$

Acceleration constant and it is due to gravity

$|a| = g$

9. (*d*) $v = \frac{ds}{dt} = 3t^2 - 12t + 3$

$$a = \frac{dv}{dt} = 6t - 12$$

$$a = 0 \Rightarrow t = 2\text{sec}$$

$$V_{2\text{sec}} = 3(2)^2 - 12(2) + 3 = +12 - 24 + 3 = -9 \text{ m/s}$$

10. (*c*) $\frac{dv}{dt} = 6 - 3v \Rightarrow \frac{dv}{6-3v} = dt$

Integrating both sides, $\int \frac{dv}{6-3v} = \int dt$

$$\Rightarrow \frac{\log_e(6-3v)}{-3} = t + K_1$$

$$\Rightarrow \log_e(6-3v) = -3t + K_2 \quad ...(i)$$

At $t = 0$, $v = 0 \Rightarrow \log_e 6 = K_2$

Substituting the value of K_2 in equation (*i*)

$\log_e(6-3v) = -3t + \log_e 6$

$$\Rightarrow \log_e\left(\frac{6-3v}{6}\right) = -3t \Rightarrow e^{-3t} = \frac{6-3v}{6}$$

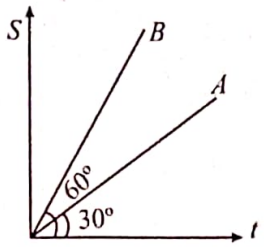
$$\Rightarrow 6 - 3v = 6e^{-3t} \Rightarrow 3v = 6(1 - e^{-3t}) \Rightarrow v = 2(1 - e^{-3t})$$

$\therefore v_{\text{terminal}} = 2$ m/s (When $t = \infty$)

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}\left[2\left(1 - e^{-3t}\right)\right] = 6e^{-3t}$

Initial acceleration = 6 m/s^2.

11. (*d*)



$$\frac{V_A}{V_B} = \frac{\tan 30°}{\tan 60°}$$

$$\therefore \frac{V_A}{V_B} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

12. (*a*) (Acceleration) = Slope $= \frac{\Delta v}{\Delta t}$

$$OA \rightarrow \frac{10}{10} = 1, \; AB \rightarrow 0 = 0$$

$$BC \rightarrow \frac{-10}{20} = -0.5$$

13. (*b*) Distance = Total Area = 105 m

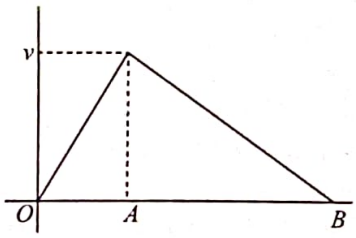
Displacement = 90 – 15 = 75 m

(+ve y axis area) – (–ve y axis area)

14. (*b*) Avg. velocity for $OA = \frac{0+v}{2}$

For $OB = \frac{0+0}{2}$ $v = 0$ constant acceleration

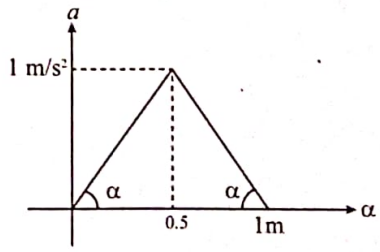
$$\frac{\text{Area } OAC}{\text{Area } ABC} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3$$



Distance covered in $(OA) = \frac{1}{2}\left(\sqrt{3}\right)\left(t_{OA}\right)^2$

Distance covered in $(AB) = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(t_{AB}\right)^2$

15. (*a*) $\int a dx = \frac{v_2^2 - v_1^2}{2} = \frac{v_2^2}{2}$



as $v_1 = 0$

$$\frac{1}{2} \times 1 \times 1 = \frac{v_2^2}{2} \Rightarrow v_2 = 1 \text{ m/s}$$

16. (a) $<\vec{v}> = \frac{\vec{s}}{t} = \frac{\Delta\vec{x}}{t}$

Slope is –ve, so velocity is –ve.

17. (d) $a = \frac{d^2x}{dt^2}$ is constant for 3 and 4

18. (b) Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{\pi R}{2R/V} = \frac{\pi V}{2}$

19. (c) At t_1, disp ≠ distance

where $0 < t_1 < \frac{2v_0}{g}$

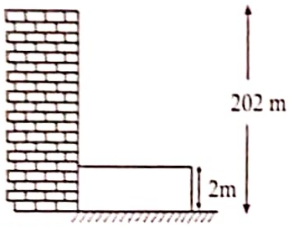
20. (a) For P : $v^2_{max} = 2 \times 2a \times \frac{L}{2} + 2 \times a \times \frac{L}{2}$

$\Rightarrow$ For Q : $v^2_{max} = 2 \times a \times \frac{L}{2} + 2 \times 2a \times \frac{L}{2}$

From velocity time graph time taken by P will be less. So average speed greater than Q

21. (b) Δx is negative for (b)

22. (b) Use basic kinematics,



$v_2 - v_0 = 2a(x - x_0)$

$v = v = \sqrt{2ax} = \sqrt{2(9.8)200} = 62.61$ m/s

$v^2 - v_0^2 = 2a(x - x_0)$

$0^2 - 62.61^2 = 2a(0.5 - 2)$

$a = 1307$ m/s$^2 \Rightarrow a = 133\ g$

23. (a) Acceleration is constant except during collision. So displacement time graph is parabolic.

24. (d) $x = t^3 - 6t^2 - 15t + 40$

$v = t^2 - 12t - 15 = 0$ at $t = 5$

$\therefore$ d is wrong.

25. (d)

$v = 36 \times \frac{5}{18} = 10$ m/s

$v = at$

$10 = 0.2 \times t \Rightarrow t = 50$

$\frac{1}{2} \times t \times 10 = 100 \Rightarrow t = 20$ sec.

Distance $= \frac{1}{2} \times (190 + 120) \times 10 = 1550$ m

$v_{av} = \frac{1550}{190} = \frac{155}{19}$ m/s

26. (a) $t_1 = \frac{2R}{v}$

For 2nd car, $v_{av} = \frac{v + 2v}{2} = \frac{\pi R}{t_2}$

$\Rightarrow t_2 = \frac{2\pi R}{3v} > t_1$

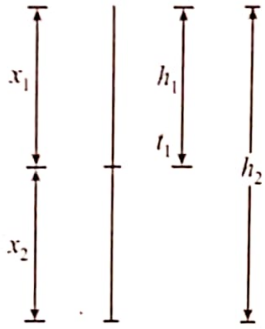
27. (a) $t_1 : t_2 = 1 : (\sqrt{2} - 1)$

So, $t_1 = 1$

and $t_1 + t_2 = \sqrt{2}$

$x_1 = h_1 = \frac{1}{2}gt_1^2 = \frac{1}{2}g$

$h_2 = \frac{1}{2}g(t_1 + t_2)^2$



$= \frac{1}{2}g \times 2 = g$

$x_2 = (h_2 - h_1) = \frac{1}{2}g$

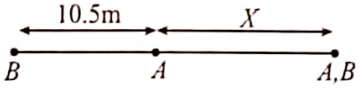
$x_1 = x_2$

28. (b) Avg. $a = \frac{20\sqrt{2}}{5} = 4\sqrt{2}\ ms^{-2} N - W$

29. (c) Since, $V_f^2 - V_i^2 = 2aS$

$a = -2250$ kmh^{-2}

30. (a)



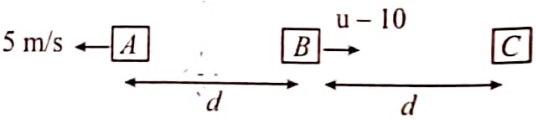
$X = 10t$

$10.5 + X = \frac{1}{2} \times 1 \times t^2$

$\Rightarrow t^2 - 20t - 21 = 0$

$\Rightarrow t = -1, 21$ sec

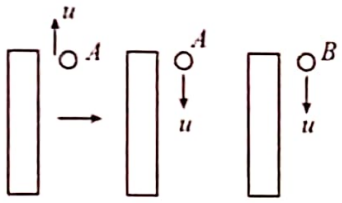
31. (c) With respect to C



$\frac{d}{u - 10} = \frac{2d}{5}$

$\Rightarrow u = 12.5$ m/s

32. (*b*)



Ball A has the same speed u when its crosses the top end of the tower while going down which is the initial speed of B.

As both have same initial velocities, so their velocities are same upon reaching the ground.

33. (*d*) All the graphs shows that speed is increasing as acceleration is positive in all three graphs for given time interval. Also we can say that area under a–t graph i.e. change in velocity is positive so velocity is increasing.

34. (*b*) $v_{AB} \geq 0$, till 1.5 sec.

$v_{AB} < 0$ afterwards, so separation is maximum at $t = 1.5$ sec,

$S_A - S_B = 2.5 - 1.25 = 1.25$ m

35. (*b*) $-h = u \times 9 - \frac{1}{2} g \times 9^2$ ...(i)

$+h = u \times 4 + \frac{1}{2} g \times (4)^2$ ...(ii)

From equation (i) and (ii)

$h = 180$ m

36. (*b*) Ist stone $h = -ut_1 + \frac{1}{2} g t_1^2$ ...(i)

2nd stone $h = ut_2 + \frac{1}{2} g t_2^2$ ...(ii)

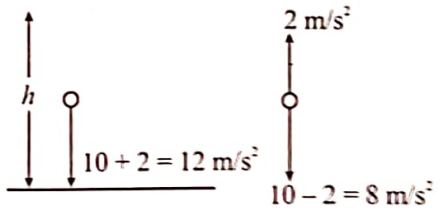
Equation (i) × t_2 and equation (ii) × t_1 and added

$h(t_1 + t_2) = g(t_1 t_2)(t_1 + t_2)$

$\Rightarrow h = \frac{1}{2} g(t_1 t_2)$

For 3rd stone $t = \sqrt{t_1 t_2}$

37. (*b*)



$h = ut - \frac{1}{2} a t_a^2 = 0 - \frac{1}{2} \times 12 t_a^2$ ...(i)

$h = ut + \frac{1}{2} a t_d^2 = 0 + \frac{1}{2} \times 8 t_d^2$ ...(ii)

From (i) and (ii)

$\frac{t_a}{t_d} = \sqrt{\frac{2}{3}}$

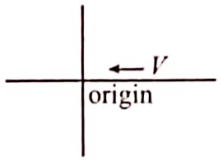
Exercise-3 (JEE Advanced Level)

1. (*b,c,d*)

(*b*) $\Rightarrow a = \frac{dv}{dt}$

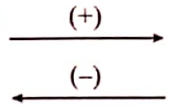
(*c*) $\frac{\vec{v}}{\vec{a}}$ Object is slowing down.

(*d*)



The particle is moving towards origin.

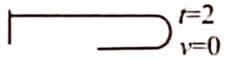
2. (*c,d*)



$\frac{\vec{v}}{\vec{a}} \cdot (v) \uparrow \frac{\vec{v}}{\vec{a}} \cdot (v) \uparrow$

$\frac{\vec{v}}{\vec{a}} \cdot (v) \downarrow \frac{\vec{v}}{\vec{a}} \cdot (v) \downarrow$

3. (*a,c*) $v = 10 - 5t$



When $v = 0$ at $t = 2$ sec.

Max displacement $= 10t - 5t^2/2$

Put $t = 2 \Rightarrow 20 - 10 = 10$ m

Distance traveled in first 3 seconds

$= 10 + \left(0 + \frac{1}{2} \times 5 \times (1)^2\right) = 12.5$ m

4. (*a,b,c,d*)

$x = \alpha t^2 - \beta t^3$

(*a*) $0 = \alpha t^2 - \beta t^3 \Rightarrow t = \frac{\alpha}{\beta}$

(*b*) $v = \frac{dx}{dt} = 2\alpha t - 3\beta t^2 \Rightarrow v = 0 \Rightarrow t = \frac{2\alpha}{3\beta}$

(*c*) $a = \frac{d^2x}{dt^2} = 2\alpha - 6\beta t$

When $t = 0 \Rightarrow a = 2\alpha$; $v = 0$

(*d*) Acceleration at $t = \frac{\alpha}{3\beta}$; $a = 0$

$\therefore$ Net force = 0

5. (*a,b,c,d*)

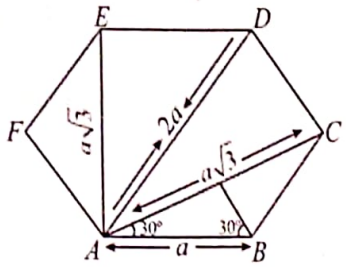
(*a*) At T (velocity changes its direction)

(*b*) slope constant

(*c*) Upper area = Lower area

(*d*) Initial speed = final speed.

6. (a,c,d)



(a) A to F

$$\text{Average velocity} = \frac{\text{Total Displacement}}{\text{Total time}}$$

$$= \frac{a}{5a/v} = \frac{v}{5}$$

(b) A to $D = \dfrac{2a}{3a/v} = \dfrac{2}{3}v$

(c) A to $C = \dfrac{a\sqrt{3}}{2a/v} = \dfrac{v\sqrt{3}}{2}$

(d) A to $B = \dfrac{a}{a/v} = v$

7. (a,b) $y = 3x + 4$

$$\therefore \frac{dy}{dt} = 3\frac{dx}{dt}$$

or $v_y = 3v_x = 3$ m/s (as $v_x = 1$ m/s)

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{1^2 + 3^2} = \sqrt{10} \text{ m/s}$$

Further, $\dfrac{dv_y}{dt} = 3 \cdot \dfrac{dv_x}{dt}$

or $a_y = 3ax = 3$ m/s^2 (as $a_x = 1$ m/s^2)

Similarly, $a = \sqrt{1^2 + 3^2} = \sqrt{10}$ m/s^2

8. (a,d) $\dfrac{-dv}{dt} = \alpha\sqrt{v}$

$$\int_{v_0}^{v} \frac{-dv}{\sqrt{v}} = \int_0^t \alpha dt$$

$$-2\sqrt{v} + 2\sqrt{v_0} = \alpha t$$

It comes to rest when $v = 0$ thus

$$\Rightarrow t\big|_{v=0} = \frac{2\sqrt{v_0}}{\alpha}$$

$$v = \left(\frac{-\alpha t + 2\sqrt{v_0}}{2}\right)^2$$

$$x = \frac{1}{4}\left(\alpha^2 \frac{t^3}{3} + 4v_0 t - 4\alpha\sqrt{v_0}\frac{t^2}{2}\right)$$

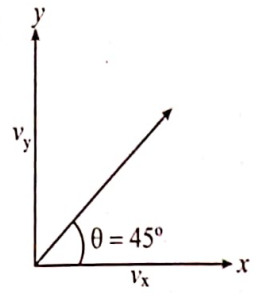
$$x\Big|_{t=\frac{2\sqrt{v_0}}{\alpha}} = \frac{2}{3}\frac{v_0^{3/2}}{\alpha}$$

9. (a,b,c)

$$F = kt \Rightarrow a = \frac{kt}{m}$$

$$v = \frac{kt^2}{2m} = \frac{k}{2m}\left(\frac{ma}{k}\right)^2 = \frac{ma^2}{2k}$$

10. (a)



Take x-axis along motion of truck and y-axis vertically upward

$\vec{v}_{\text{ball/boy}} = (10\hat{j})$ m/s

$\vec{v}_{\text{boy/g}} = (10\hat{i})$ m/s

$\vec{v}_{\text{ball/g}} = (10 + 10)$

Distance of ball from pole

$$\frac{u^2 \sin 2\theta}{g} = \frac{(10\sqrt{2})^2 \times \sin 90}{g} = 20 \text{ m}$$

11. (a) Maximum height $= \dfrac{10^2}{2g} = 5$ m

12. (c) At maximum height, velocity of ball is horizontal.

Time taken by ball to reach maximum height

$$t_H = \frac{u_y}{g} = \frac{10}{10} = 1 \text{ sec}$$

Velocity of truck is

$v = u + at$

$= 10 + 2 \times 1 = 12$ m/s

13. (b) Distance travelled = Are = $1/2 \times (10 + 5) \times 10 = 75$ m

14. (c) $ma = |-(10)(1000)\text{kg}|$

$= 10000$ N

15. (d) Fact based

16. (c) $a = \dfrac{18}{7.29} = 2.5$ m/s^2

17. (b) $100 = \dfrac{1}{2} \times 2.5(7.29)^2 + \left(\dfrac{18 + v}{2}\right) \times 2.4$

18. (a)

(a) $v > 0$ and $\dfrac{dv}{dt} < 0$ or

$v > 0$ and $\dfrac{dv}{dt} > 0$

(b) $\frac{ds}{dt} > 0$ or $v > 0$ and $\frac{dv}{dt} > 0$ or $v < 0$ and $\frac{dv}{dt} < 0$

(c) $\left|\frac{dv}{dt}\right|$ is increasing

(d) $x > 0$, and $v > 0$ or $x < 0$ and $v < 0$

19. (b)

(A) Slope is positive and decreasing. $\Rightarrow a < 0$

(B) Slope is negative and its magnitude is increasing.

(C) Slope is positive and increasing.

(D) Slope is negative and its magnitude is decreasing.

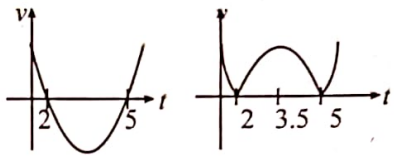
20. (d)

(A) $x = 2t^3 - 21t^2 + 60t$

$$v = \frac{dx}{dt} = 6t^2 - 42t + 60 = 0$$

$t^2 - 7t + 10 = 0$

$t = 5, 2$



(B) $a = 2t - 7 = 0$

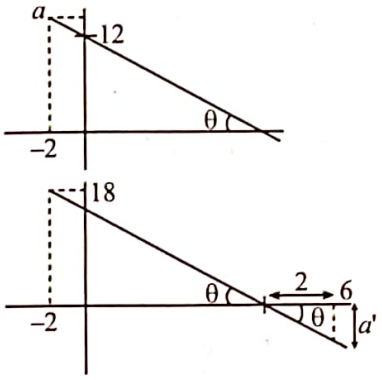
$\Rightarrow t = 3.5$ sec.

$a = -$ve at $t < 3.5$ sec.

(C) $v -$ ve from $t = 2$ to 5 sec.

21. [0055]

$\Delta V =$ area under graph



$$\tan\theta = \frac{12}{4} = 3$$

$$\frac{a}{6} = 3 \Rightarrow a' = 18$$

$$\frac{a'}{2} = \tan\theta \Rightarrow a' = 6$$

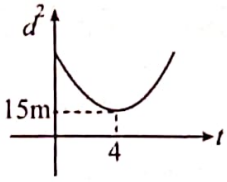
$$\text{Area} = \frac{1}{2} \times 18 \times 6 - \frac{1}{2} \times 6 \times 2$$

$V_{t=6} - 7 = 54 - 6 = 48$

$V_{t=6} = 55$ m/s

22. [0015]

$d^2 = x^2 + y^2 = (3t)^2 + (25 - 4t)^2$



$= 625 - 200\,t + 25t^2$

$d^2 = 625 - 200 \times 4 + 25 \times 16$

$= 1025 - 800$

$= 225$

$d = 15$ m

23. [0400]

$\ell = ct^2$

$2000 = c \times 10^2$

$c = 20$ m/s^2

$$v = \frac{d\ell}{dt} = 2ct$$

$= 2 \times 20 \times 10$

$= 400$ m/s

24. [0006]

In $x - t$ graphs

Every change in shape is a change in velocity

25. [0004]

$$\frac{vdv}{dx} = 4 - 2x$$

$$\int_0^v vdv = \int_0^x (4 - 2x)dx$$

$$\Rightarrow \frac{v^2}{2} = 4x - x^2$$

When $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

$\therefore$ At $x = 4$, the particle will again come to rest.

26. [0025]

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$b = \frac{\text{total area}}{\text{total time}} = \frac{10 + 20}{6} = \frac{30}{6}$$

$= 5$ m/s

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$C = \frac{10 - (-10)}{4} = \frac{20}{4}$$

$= 5$ m/s^2

$bc = (5)(5) = 25$ m^2/s^3

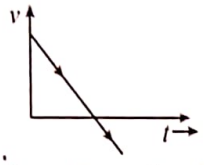
Exercise-4
(Past Year Questions)

JEE MAIN

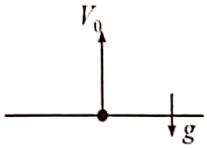
1. (*a*) Velocity at any time t is given by, $v = u + at$

$v = v_0 + (-g)t$

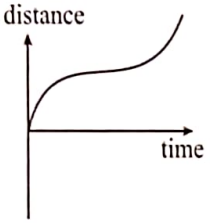
$v = v_0 - gt$



$\Rightarrow$ Straight line with negative slope



2. (*a*) In this question option (*b*) and (*d*) are the corresponding position - time graph and velocity – position graph of option (*c*) and its distance – time graph is given as



Hence incorrect graph is option (*a*)

3. (*b*) $\vec{s} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{j} + 4\hat{j})4 = 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_2 - \vec{r}_1 = 18\hat{i} + 16\hat{j}$

$\vec{r}_2 = 20\hat{i} + 20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$

4. (*a*) $v_x = \frac{dx}{dt} = -a\omega \sin \omega t$

$v_y = \frac{dy}{dt} = a\omega \cos \omega t$

$v_z = \frac{dz}{dt} = a\omega$

$\therefore\ v = \sqrt{v_x^2 + v_y^2 + v_z^2} = a\omega\sqrt{2}$

5. (*c*) $\sqrt{\frac{2\ell}{a_2}} - \sqrt{\frac{2\ell}{a_1}} = t \Rightarrow \frac{\sqrt{2\ell}}{t} = \frac{\sqrt{a_1 a_2}}{\sqrt{a_1} - \sqrt{a_2}}$

$\sqrt{2a_1\ell} - \sqrt{2a_2\ell} = v \Rightarrow \frac{\sqrt{2\ell}}{v} = \frac{1}{\sqrt{a_1} - \sqrt{a_2}}$

$\Rightarrow \frac{v}{t} = \sqrt{a_1 a_2} \Rightarrow v = (\sqrt{a_1 a_2})t$

6. (*d*) $r_{t=5}$ = area

$= \left(\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1\right)$m

$= (2 + 4 + 3)$ m $= 9$ m.

7. (*d*) $x = at + bt^2 - ct^3$

$v = \frac{dx}{dt} = a + 2bt - 3ct^2$

$a = \frac{dv}{dt} = 2b - 6ct = 0 \Rightarrow t = \frac{b}{3c}$

$v_{\left(at\, t = \frac{b}{3c}\right)} = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)$

$= a + \frac{b^2}{3c}$

8. (*d*) $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} + (-40t)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$

$|\vec{a}| = 50\,\text{m/s}^2$

9. (*d*) $m = 20$ g, $u = 1$ m/s, $v = ?$

$S = 20 \times 10^{-2}$ m

$a = \frac{-2.5 \times 10^{-2}}{20 \times 10^{-3}}\,\text{m/s}^2$

$v^2 = u^2 + 2as$

$v^2 = 1 - 2 \times \frac{2.5 \times 10^{-2}}{20 \times 10^{-3}} \times \frac{20}{100}$

$v = \frac{1}{\sqrt{2}} \simeq 0.7$m/s

10. (*c*) Given initial velocity $u = 0$ and acceleration is constant

At time t

$v = 0 + at \Rightarrow v = at$

Also $x = 0(t) + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}at^2$

Graph (*a*); (*b*) and (*d*) are correct.

11. (*b*) $v = b\sqrt{x}$

$\frac{dv}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt}$ $\qquad a = \frac{bv}{2\sqrt{x}}$

$a = \frac{b(b\sqrt{x})}{2\sqrt{x}}$

$\frac{dv}{dt} = a = \frac{b^2}{2}$

$v = \frac{b^2}{2}\tau$

12. [3] $x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$v^2 + Ax = a$

$$Ax = a - \left(\frac{at+b}{x}\right)^2$$

$$A = \frac{a(at^2+2bt+c)-(at+b)^2}{x^3}$$

$$A = \frac{ac-b^2}{x^3}$$

$A \propto x^{-3}$

13. (*d*) $S_y = u_y t + \frac{1}{2}a_y t^2$

$32 = 0 + \frac{1}{2}\times 4t^2 \Rightarrow t = 4$ sec

$S_x = u_x t + \frac{1}{2}a_x t^2 = 3\times 4 + \frac{1}{2}\times 6\times 16 = 60$ m.

14. (*c*) $\frac{1}{2}mv^2 = p\times t$

$v \propto \sqrt{t}$

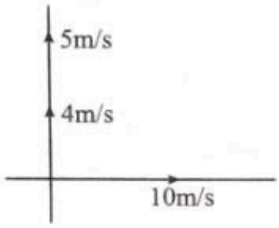
$\frac{ds}{dt} \propto \sqrt{t}$

$s \propto t^{3/2}$

15. [20] Distance travelled = Area under the *u-t* graph

$\therefore \Delta S = \frac{1}{2}\times 5\times 8 = 20$

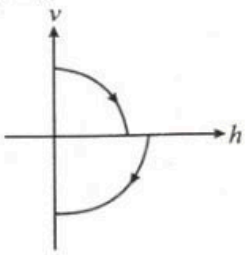
16. (*d*) At $t = t$, (20 m, y_0)



$\therefore t = \sqrt{\frac{20\times 2}{10}} = 2s$

$\therefore y_0 = 5\times 2 + \frac{1}{2}\times 4\times 2^2 = 18$ m

17. (*b*) At $H = h$, $v = 0$



At $h = 0$, $v = \sqrt{2gh}$

Also $a = -g$, throughout this motion

18. (*a*) Distance $= |A_1| + |A_2|$

$$= \frac{1}{2}\left(1+\frac{13}{3}\right)\times 4 + \frac{1}{2}\times\frac{5}{3}\times 2$$

$$= \frac{37}{3}\text{ m}$$

19. (*a*) $\frac{h_1}{d} = \tan 45^\circ$

$\frac{h_1+h_2}{d+2.464d} = \tan 30^\circ$

$\Rightarrow h_2 = d$

20. (*a*) $t_0 = \sqrt{\frac{2h}{g}}$

$\Rightarrow V_0 = \sqrt{\frac{2h}{g}}\times g = \sqrt{2gh}$

$\therefore t_1$ = time to reach top $= \frac{v_0}{g}$

$\Rightarrow H = h + h' = 2h$

$\therefore t_2$ = time of fall

$$= \sqrt{\frac{2\times(2h)}{g}} = 2\sqrt{\frac{h}{g}}$$

$\therefore$ Total time $= t_1 + t_2$

$$= (2+\sqrt{2})\sqrt{\frac{h}{g}} = 3.4\sqrt{\frac{h}{g}}$$

21. (*a*) The slope of the given *v* versus *x* graph is $m = -\frac{v_0}{x_0}$ and intercept is $c = +v_0$. Hence, varies with *x* as

$$v = -\left(\frac{\partial_0}{x_0}\right)x + v_0 \qquad \text{...(i)}$$

Where v_0 and x_0 are constants of motion. Differentiating with respect to time t_1 we have

$$\frac{dv}{dt} = -\left(\frac{2w}{x_0}\right)\frac{dx}{\dot{\alpha}t} \text{ or } a = -\left(\frac{20}{x_0}\right)y \qquad \text{...(ii)}$$

Using equation (i) in equation (ii), we get

$$a = -\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0}x + v_0\right) \Rightarrow a = \left(\frac{\partial_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus the graph of a versus *x* is a straight line having a positive slope $= \left(\frac{v_0}{x_0}\right)^2$ and negative intercept $= -\frac{v_0^2}{x_0}$.

Hence the correct choice is (*a*).

22. (*b*) Acceleration (*a*) is constant

$\therefore v \propto t$ (Straight line graph)

And $x \propto t^2$ (parabolic graph)

23. (*a*) Total distance, $d = h + 2e^2h + 2e^4h + 2e^6h + 2e^8h + ...$

$$d = h + 2e^2h\left(1 + e^2 + e^4 + e^6 + ...\right)$$

$$d = \frac{\left(1-e^2\right)k + 2e^2k}{1-e^2} = \frac{k\left(1+e^2\right)}{1-e^2}$$

Total time $= T + 2eT + 2e^2T + 2e^3T + ...$

Total time $= T + 2eT\left(1 + e + e^2 + e^3 + ...\right)$

$$= T + 2e.T\left(\frac{1}{1-e}\right)$$

Total time $= \dfrac{T(1+e)}{1-e}$

Average speed of the ball $= \dfrac{5}{1}\left(\dfrac{1+e^2}{(1+e)(1-e)}\dfrac{(1-e)}{(1+e)}\right)$

$$V_{avg} = \frac{5\left(1+e^2\right)}{(1+e)^2}$$

$\because h^1 = e^2h$

$$\frac{81}{100} = e^2 \Rightarrow e = \frac{9}{10} = 0.9$$

$$V_{avg} = \frac{5\left(1+(0.9)^2\right)}{(1+0.9)^2} = 2.50 \text{ m/sec.}$$

24. (*b*) $V = V_0 + gt + Ft^2$

$$\frac{dx}{dt} = V_0 + gt + Ft^2 \Rightarrow \int_{x=0}^{x} dx = \int_{t=0}^{t=1}\left(V_0 + gt + Ft^2\right)dt$$

$$x = \left[V_0t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_{t=0}^{t=1} \Rightarrow x = V_0 + \frac{g}{2} + \frac{F}{3}$$

25. (*b*) $d = R\theta$

$$60 = R\left(\frac{3\pi}{4}\right) \Rightarrow R = \frac{60\times 4}{3\pi} = \frac{80}{\pi}\text{m}$$

Displacement $= \sqrt{R^2 + R^2 - 2R^2\cos 135}$

$$\Rightarrow \sqrt{2R^2 - 2R^2(-0.7)} \Rightarrow \sqrt{3.4R^2}$$

$$= \sqrt{3.4\left(\frac{80}{\pi}\right)^2} = 47\text{ m}$$

26. [3] $v_f = 0, u = 150, s = 27$ m

$0^2 = 150^2 - 2a \times 27$

$v_f = 0, u = 50, s = ?$

$0^2 = 50^2 - 2as \Rightarrow s = 3$ m

27. [100]

It is given that $\dfrac{dv}{dx} = 5$

So, acceleration of particle is given by $a = v\dfrac{dv}{dx}$

$a = 100$ m/s^2 $\quad (\because v = 20$ m/s)

28. [392]

$$t_a = \frac{u}{g} = \frac{19.6}{9.8} = 2\text{s}$$

$$t_d = 6 - 2 = \sqrt{\frac{2h_{max}}{g}} \Rightarrow h_{max} = \frac{16\times 9.8}{2} = \frac{392}{5}$$

29. (*b*) At maximum height, velocity is zero so,

$0 = u^2 - 2gh$ or $u = \sqrt{2gh}$

As $s = ut + \frac{1}{2}at^2$ so, $\frac{h}{3} = \sqrt{2ght} - \frac{1}{2}gt^2$

$$gt^2 - 2\sqrt{2ght} + \frac{2h}{3} = 0 \text{ or } t = \frac{2\sqrt{2gh} \pm \sqrt{8gh - \frac{8gh}{3}}}{2g}$$

$$\frac{t_1}{t_2} = \frac{2\sqrt{2gh} - \sqrt{8gh - \frac{8gh}{3}}}{2\sqrt{2gh} + \sqrt{8gh - \frac{8gh}{3}}} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

JEE ADVANCED

30. (*a*) Let the car turn of highway at a distance 'x' from point M. So $RM = x$ and speed of car in field is v, then time taken by the car to cover the distance $QR = (QM - x)$

$$t_1 = \frac{(QM - x)}{2v}$$

Time taken to travel the distance 'PR' in field.

$$t_2 = \frac{\sqrt{d^2 + x^2}}{v}$$

So, total time elapsed to move car from Q to P

$$t = t_1 + t_2 = \left[\frac{QM - x}{2v} + \frac{\sqrt{d^2 + x^2}}{v}\right]$$

For 't' to be minimum, $\dfrac{dt}{dx} = 0$

$$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0 \Rightarrow x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$$

31. (*c*) Distance travelled in 15 sec. by both will be given by area under curve,

For car, $S_1 = \frac{1}{2}\times 15 \times 45 = 337.5$ m

For scooter, $S_2 = 15 \times 30 = 450$ m $\Rightarrow |S_1 - S_2| = 112.5$m

Suppose after time t, they will meet, then distance travelled by both of them will be equal.

32. (*c*) According to question, object is moving with constant negative acceleration i.e. $a = -$constant (c)

$$v\frac{dv}{dx} = -c \Rightarrow vdv = cdx \Rightarrow \frac{v^2}{2} = -cx + k \Rightarrow x = -\frac{v^2}{2c} + \frac{k}{c}$$

Hence, graph (*c*) represents correctly.

CHAPTER

4 Projectile Motion

Exercise-1 (Topicwise)

1. (*c*) $v_x = \frac{dx}{dt} = 6$ and $v_y = \frac{dy}{dt} = 8 - 10t = 8 - 10 \times 0 = 8$

$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10\ \text{ms}^{-1}$

2. (*b*) $V_x = 2at \Rightarrow V_y = 2bt$

$V = 2t\sqrt{a^2 + b^2}$

3. (*d*) $(Y_{max}) \Rightarrow \frac{dY}{dt} = 0$

$\Rightarrow \frac{d}{dt}(10t - t^2) = 10 - 2t \Rightarrow t = 5$

$\Rightarrow Y_{max} = 10(5) - 5^2 = 25$ m

4. (*d*) $R = \frac{u^2 \sin 2\theta}{g} \therefore R \propto u^2$. If initial velocity be doubled then range will become four times.

5. (*c*) $H = \frac{u^2 \sin^2\theta}{2g} \therefore H \propto u^2$. If initial velocity be doubled then maximum height be quadrupled, when the projectile reached on maximum height.

6. (*a*) Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

7. (*b*) $R_{15^\circ} = \frac{u^2 \sin(2 \times 15^\circ)}{g} = \frac{u^2}{2g} = 1.5$ km

$R_{45^\circ} = \frac{u^2 \sin(2 \times 45^\circ)}{g} = \frac{u^2}{g} = 1.5 \times 2 = 3$ km.

8. (*a*) For vertical upward motion $h = ut - \frac{1}{2}gt^2$

$5 = (25 \sin\theta) \times 2 - \frac{1}{2} \times 10 \times (2)^2$

$\Rightarrow 25 = 50 \sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$.

9. (*b*) Range is given by $R = \frac{u^2 \sin 2\theta}{g}$

On moon $g_m = \frac{g}{6}$. Hence $R_m = 6R$.

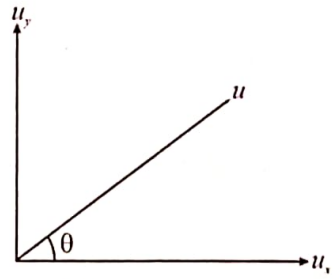
10. (*b*) $y = ut \sin\theta - \frac{gt^2}{2}$

$V_r = 0$ (zero slop)

$a_r = -g$ (negative curvature)

11. (*d*) $R = \frac{V^2 \sin 2\theta}{g} \Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$.

12. (*d*) $V = u + at$



V_y = reduced then increases

$\Rightarrow V$ = reduced then increase ($\because V_x$ is constant)

$\Rightarrow$ Speed first reduces than increases. So "*A*" is not correct

$KE = \frac{1}{2}\ m\ V^2 = \frac{m}{2}$ (speed $)^2$

$\Rightarrow$ "*B*" is not correct

V_y = changes $\Rightarrow$ "*C*" is not correct.

V_x = constt. since gravity is vertically down

$\Rightarrow$ no component of acceleration along the horizontal direction.

$\Rightarrow$ "*D*" is correct

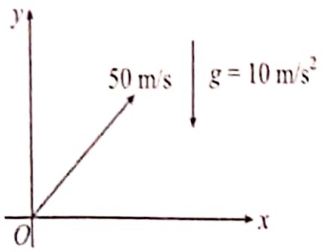
13. (*c*) In projectile motion Horizontal acceleration a_x = o & Vertical acceleration $a_y = g = 10\text{m/s}^2$

$a_x = 0$

$a_y = 10$ (down)

$\Rightarrow$ Only "*C*" is correct.

14. (*b*) In the given problem,



$u = (50\cos 37°)\,\hat{i} + (50\sin 37°)\,\hat{j} = (40\hat{i} + 30\hat{j})$ m/s

$a = (-10\hat{j})\ 2\ \text{m/s}^2$

$t = 2$s

$v = u + at = (40\hat{i} + 30\hat{j}) + (-10\hat{j})(2)$

$= (40\hat{i} + 10\hat{j})$ m/s

$s = ut + \frac{1}{2}at^2 = (40\hat{i} + 30\hat{j})(2) + \frac{1}{2}(-10\hat{j})(2)^2$

$= (80\hat{i} + 40\hat{j})$ m

Coordinates of the particle are

$x = 80$ m and $y = 40$ m

15. (*b*) Given, $R = H$

$\therefore \frac{u^2 \sin 2\alpha}{g} = \frac{u^2 \sin^2 \alpha}{2g}$ or $\sin 2\alpha = \frac{\sin^2 \alpha}{2}$ or $\sin\alpha\cos$

$\alpha = \frac{\sin^2 \alpha}{2}$

$\frac{\sin\alpha}{\cos\alpha} = 4$ or $\tan\alpha = 4$

$\therefore \alpha = \tan^{-1}(4)$

16. (*c*) Let initial and final speeds of stone be u and v.

$\therefore (v\sin 30°)^2 = (u\sin 60°)^2 - 2gh$ ...(i)

and $v\cos 30° = u\cos 60°$ ...(ii)

Solving (i) and (ii) we get

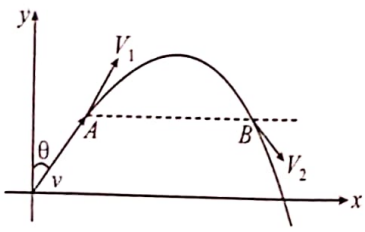
$u = \sqrt{3gh}$

17. (*d*) $\theta_1 = \frac{5\pi}{36}$

Another angle for same range is the complementary angle θ_2

Then $\theta_2 = \frac{\pi'}{2} - \left(\frac{5\pi}{36°}\right)^c = \frac{18-5}{36}\pi \Rightarrow \theta_2 = \frac{13}{36}\pi$

18. (*a*)



Avg. velocity between A & $B = \frac{\vec{V}_1 + \vec{V}_2}{2}$ ($\because$ Acceleration is constant = g)

Now, if $\vec{V}_1 = V_{1x}\hat{i} + V_{1y}\hat{j}$

Then, $\vec{V}_2 = V_{1x}\hat{i} - V_{1y}\hat{j}$ ($\because$ both A & B are at same level)

$\therefore \frac{\vec{V}_1 + \vec{V}_2}{2} = V_{1x}\hat{i} = V\sin\theta\,\hat{i}$ ($\because$ θ is from vertical)

19. (*a*) Gravitational acceleration is constant near the surface of the earth.

20. (*d*) $R = u^2 \sin 2\theta/g$

$R_{max} = u^2/g$

$22 = \frac{u^2}{g}$

for $\theta = 15°$

$R = \frac{u^2 \sin 30}{g} = 22 \times \frac{1}{2} = 11$ m

21. (*d*) $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$y = x\tan\theta - \tan\theta\frac{x^2}{R}$

Compare from eqn.

$\tan\theta = 16$

$\frac{\tan\theta}{R} = \frac{5}{4}$

$R = \frac{64}{5} = 12.8$ m

$\tan\theta = 16$

$\frac{1}{2}\frac{g}{u^2\cos^2\theta} = \frac{5}{4.2}$

$R = \frac{u^2\sin 2\theta}{g} = \frac{2g}{5} \times \frac{2\times 16}{g}$

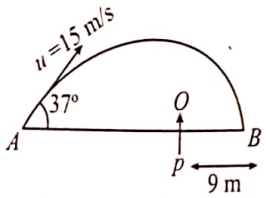
$R = 12.8$ m

22. (*c*) Range of θ and $90 - \theta$ is same

If $\theta = 30°$

So $90° - \theta = 60°$

23. (*b*)



In this process both time taken is same.

$T = \frac{2u\sin\theta}{g}$

$T = \frac{2u\sin 37°}{g} = \frac{2\times 15\times 3}{10\times 5} = 1.8$ sec

Minimum Velocity $= \frac{9}{1.8} = 5$ m/s

24. (*b*) $\therefore H_{max} = \frac{u_y^2}{2g}$

$\Rightarrow u_y = \sqrt{2gH}$

$\therefore T = \frac{2u_y}{g} = \frac{2\sqrt{2gH}}{g} = 2\sqrt{2}\sqrt{\frac{H}{g}}$

25. (*b*) As $\theta \uparrow$, H and T both increases

But $R \uparrow$ from 0° to 45° & at $\theta = 45°$ Max then decreases

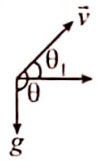
$R \uparrow$ then $\downarrow$ [θ from from 30° to 60°]

while $H \uparrow$ and $T \uparrow$.

26. (*d*) Acute Angle of velocity with horizontal possible is –90° to + 90° hence angle with g is 0° to 180°.

θ is acute

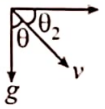
$\Rightarrow 0° \le \theta_1 < 90°$ (during the upward journey of mass)



from fig. $\theta = 90° + \theta_1$

or, $90° \le \theta < 180°$ ...(i)

During downward motion



$0° < \theta_2 < 90°$

$\theta = 90° - \theta_2$

$0° < \theta < 90°$ ...(ii)

From eq. (i) and (ii)

i.e., $0 < \theta < 90° \cup 90° \le \theta < 180°$

$\Rightarrow 0° < \theta < 180°$

27. (*d*) (i) For θ and $90° - \theta$

Range is same

$\theta = 15°$

$90° - \theta = 75°$

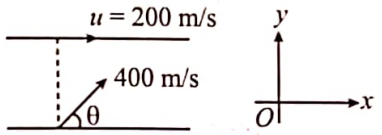
28. (*c*) Due to constant velocity along horizontal and vertical downward force of gravity stone will hit the ground following parabolic path.

29. (*d*) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 396.9}{9.8}} = 9$ sec and $u = 720$ km/hr

$= 200$ m/s

$\therefore R = u \times t = 200 \times 9 = 1800$ m

30. (*b*)



To hit 400 cos θ = 200

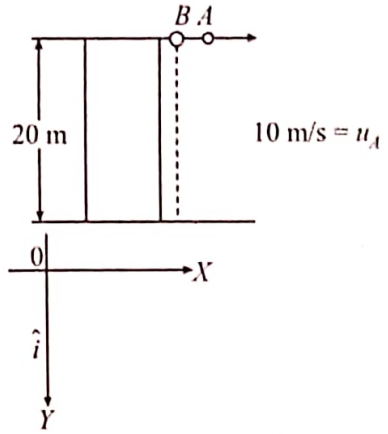
{$\because$ Both travel equal distance along horizontal, of their start and coordinates of x axis are same}

$\Rightarrow \theta = 60°$

31. (*c*) In projectile motion Horizontal acceleration $a_x = o$ and Vertical acceleration $a_y = g = 10$ m/s^2 (down)

$\Rightarrow$ only "C" is correct.

32. (*d*)



$u_B = 0$ m/s $\quad u_A = 10\hat{i}$

On reaching the ground,

Both will have same vertical velocity

$V_y^2 = u_y^2 + 2a_y s_y$

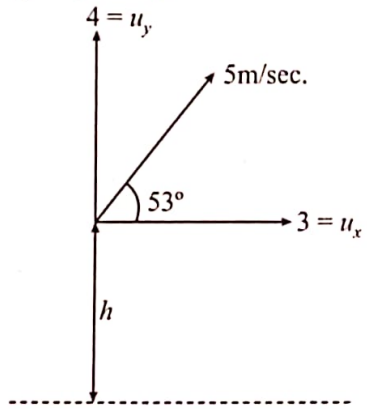
since $u_y = 0$ for both A & B

$a_y = g$ for both A & B

$s_y = 20$ m for both A & B

Thats why the time taken by both are same

33. (*c*) $v_y^2 - u_y^2 = 2\,a_y S$



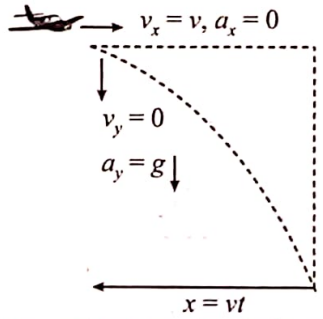
$v_y^2 - (4)^2 = -2 \times 10 \times 0.45$

$v_y^2 = 7$ m^2/s^2

$v_x = 5\cos 53° = 3$m/s (always remains same)

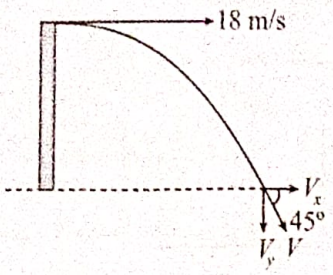
$\therefore V_{net} = \sqrt{(V_x)^2 + (V_y)^2} = \sqrt{9+7} = 4$m/s

34. (*a*)



Because horizontal velocity of plane and bomb is always same.

35. (*b*) $\tan 45° = \frac{v_y}{v_x}$



$\Rightarrow v_y = v_x = 18\text{m/s}$

36. (*c*) $T = \frac{2u \sin 45°}{g \cos 45°} = 2\text{sec}$

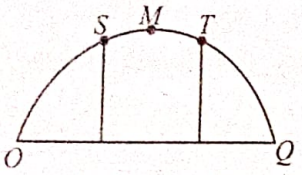
37. (*d*) $PQ = \frac{1}{2} g \sin\theta\, T^2$

$T = \frac{2v}{g\cos\theta}$

$PQ = \frac{1}{2} g \sin\theta \left(\frac{2v}{g\cos\theta}\right) T$

$PQ = Tv \tan\theta$

38. (*c*)



$t_{(OS)} = 1$ sec, $t_{(OT)} = 3$

or $t_{(ST)} = t_{(OT)} - t_{(OS)} = 3 - 1 = 2$ sec

$\therefore t_{(SM)} = \frac{1}{2} t_{(ST)} = 1$ sec.

$\therefore t_{(OM)} = t_{(OS)} + t_{(SM)} = 1 + 1 = 2$sec.

$\therefore$ Time of flight $= 2 \times 2 = 4$ sec.

39. (*b*) At maximum height $v = u\cos\theta$

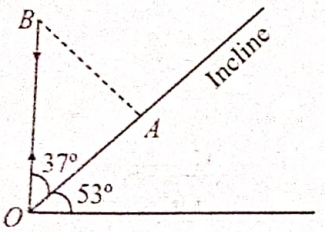
$\frac{u}{2} = v \Rightarrow \frac{u}{2} = u\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(120°)}{g}$

$= \frac{u^2 \cos 30°}{g} = \frac{\sqrt{3}u^2}{2g}$

40. (*a*) $OB = \frac{u^2}{2g} = 5\text{m}$



$\therefore AB = OB \sin 37° = 3\text{m}$

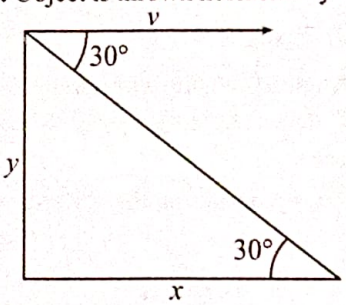
Exercise-2 (Learning Plus

1. (*a*) $AC = \frac{1}{2} gt^2 = 45$ m

$BC = 45\sqrt{3}\ m = u.t$

$u = \frac{45}{\sqrt{3}} = 15\sqrt{3}$ m/s.

Alter: Object is thrown horizontally so $u_x = v$ & u_y



from diagram

$-y = u_y t - \frac{1}{2} gt^2$

$y = \frac{1}{2} \times 10 \times (3)^2$

$y = 45\text{m}$ ...

& $\tan 30° = \frac{y}{x} \Rightarrow y\sqrt{3} = x$ ...(

& $x = vt = 3v$ ...(i

from equation (i), (ii) and (iii)

$45\sqrt{3} = 3v$

$v = 15\sqrt{3}$ m/s

2. (*b*) Let the initial velocity of particle A

$U_A = 100\ (\cos 60\hat{i} + \sin 60\hat{j})$

Initial velocity of particle B

$U_B = 100\,(-\cos 60\hat{i} + \sin 60\hat{j})$

$v_A = U_A - gt = 50\hat{i} + \left(\frac{100\sqrt{3}}{2} - 10t\right)\hat{i}$

$v_B = -50\hat{i} + (50\sqrt{3} - 10t)\hat{j}$

Let at $t = t_0$ $\vec{v}_A \perp \vec{v}_B \Rightarrow \vec{v}_A \cdot \vec{v}_B = 0$

$t_0 = 5(\sqrt{3} - 1)$

$t = 5(\sqrt{3} + 1)$ is not valid since it is greater then time o flight.

3. (*c*) $y = 0.5 - 0.04x^2$

comparing with

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta} x^2$

we get $\tan\theta = 1/2 \Rightarrow \cos\theta = 2/\sqrt{5}$

$\frac{g}{2u^2\cos^2\theta} = 0.04 \Rightarrow u = 12.5$ m/s

4. (*d*) $y = bx^2$

$$\frac{dy}{dt} = 2bx\frac{dx}{dt}$$

On differentiating again, we get

$$\frac{d^2y}{dt^2} = 2b\left[x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)\left(\frac{dx}{dt}\right)\right]$$

$$\frac{d^2y}{dt^2} = 2b\left[0 + \left(\frac{dx}{dt}\right)^2\right] \text{ as } \frac{d^2x}{dt^2} = 0$$

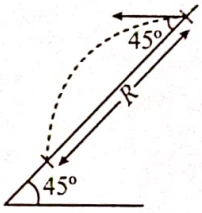
$$a = 2b\left(\frac{dx}{dt}\right)^2$$

$$v = \frac{dx}{dt} = \sqrt{\frac{a}{2b}}$$

5. (*d*) $X = R + \frac{1}{2}\left(\frac{g}{2}\right)\left[\frac{2u\sin\theta}{g}\right]^2$

$$R + \frac{g}{4}\left[\frac{4u^2\sin^2\theta}{g}\right] = R + \frac{u^2\sin^2\theta}{g} = R + 2H$$

6. (*b*) $R = u\cos 45°\, t_f + \frac{1}{2}\sin 45° + t_f^2$



$$t_f = \frac{2u\sin 45°}{g\cos 45°} = \frac{50}{\sqrt{2}}(10) + \frac{1}{2}\times\frac{10}{\sqrt{2}}\times(100)$$

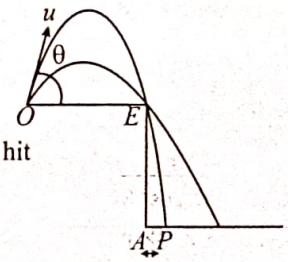
$$= t_f = \frac{2\times 50}{10} = 10 \text{ sec}$$

$R = 500\sqrt{2}$ m

7. (*b*) $T = \frac{2u\sin\theta}{g}$, $R = \frac{u^2\sin 2\theta}{g}$ and $H = \frac{u^2\sin^2\theta}{2g}$

From three equations. T & H both are max. for same value of θ. So, T will be maximum when H will be maximum.

8. (*a*) P is point on plane for shell when it passes near edge of plateau for greater angle of projection.



In region AP shell cannot hit

$x_{max} = AP$

Equation of trajectory for projectile

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

for point E $y = 0$ & $x = 960$ m

$$\Rightarrow \tan^2\theta \times \frac{10\times(960)^2}{2\times(100)^2} - (\tan\theta)x + \frac{10\times 960^2}{2\times(100)^2} = 0$$

$\Rightarrow \tan\theta = 3/4, 4/3$ for $\tan\theta = 4/3$, AP is least

Now, equation of trajectory becomes,

$$y = \frac{4}{3}x - \frac{x^2}{2000}\left(\frac{25}{9}\right)$$

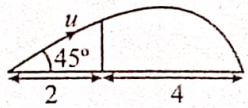
for landing point on plane (P), $y = -960$ m

$$\Rightarrow -960 = \frac{4}{3}x - \frac{x^2}{720}$$

Solving we get acceptable solution $x = 1440$ m

$AP = x - 960 = 480$ m

9. (*c*) $6 = \frac{u^2\sin 2\times 45}{10} \Rightarrow u = \sqrt{60}$



$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} = 2\times 1 - \frac{10\times 2^2}{60}$$

$$= 2 - \frac{40}{60} = \frac{4}{3} \text{ m}$$

10. (*b*) $a_y = -3$ m/s^2 and $a_x = 2$ m/s^2

It has maximum y-coordinate, when $v_y = 0 = 6 - 3(t)$

$\therefore$ $t = 2$ sec, at that instant, $y = \frac{1}{2}\times 3\times 2^2 = 6$ m

and $x = \frac{1}{2}\times 2\times 2^2 = 4$ m

11. (*c*) $v_x^2 + v_y^2 = v^2$ ...(i)

Differentiating with time

$$\frac{dy}{dx} = k\frac{1}{2\sqrt{x}} = \frac{v_y}{v_x}$$

$2v_xa_x + 2v_ya_y = 0$ ...(ii)

$x = k\frac{2}{4}$ at the given moment $v_x = v_y$

From (i) & (ii)

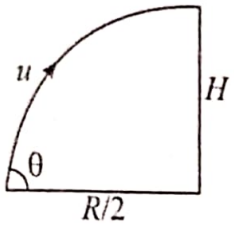
$$v_x^2\left(1 + \frac{k^2}{4x}\right) = V$$

$$2v_xa_x\left(1 + \frac{k^2}{4x}\right) + v_x^2\left[-\left(\frac{k^2}{4x^2}\right)\right]v_x = 0$$

$$\Rightarrow a_x = \frac{k^2v_x^2}{2x(k^2 + 4x)} = \frac{v^2}{2K^2}$$

$$a_y = -a_x = \frac{v^2}{2k^2} \Rightarrow a_y - a_x = -\frac{v^2}{k^2}$$

12. (*b*) $H = u^2 \sin^2\theta$ ...(i)



$$R = \frac{u^2 \sin 2\theta}{g} \quad ...(ii)$$

$$T = \frac{2u \sin\theta}{g} \quad ...(iii)$$

We know that, the average velocity

$$V_{avg} = \sqrt{\frac{4 + R^2/4}{T/2}} \quad ...(iv)$$

Putting these equation (i), (ii) and (iii) in equation (iv)

$$V_{avg} = \frac{u}{2}\sqrt{1 + 3\cos^2\theta}$$

13. (*d*) Max range $= \frac{u^2 (\sin 2\theta)}{g} \Rightarrow \frac{u^2}{g} = 250$ m

$\Rightarrow u = 50$ m/s

Max flight time is when $v_y = 50$ m/s,

$$\frac{2u}{g} = \frac{2 \times 50}{10} = 10\text{s}$$

$\frac{1}{2}$ of max flight time 5s

$$\Rightarrow \frac{2u \sin\theta}{g} = 5 = 2 \times 50 \times \sin\theta = 50 \Rightarrow \theta = 30°$$

A projectile has least speed $= u \cos\theta$

$$\Rightarrow 50 \cos 30 = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3} \text{ m/s}$$

14. (*a*) $20 = \frac{u^2 \sin^2\alpha}{2g} \Rightarrow u \sin\alpha = 20$ ...(i)

For total time of flight

$$-25 = u \sin\alpha\, t - \frac{1}{2} gt^2$$

$$5t^2 - 20t - 25 = 0$$

$$t^2 - 4t - 5 = 0$$

$$t = \frac{4 \pm \sqrt{16 + 20}}{2} = 5 \text{ sec}$$

In 5 sec horizontal displacement = 75 m

Now $u \cos\alpha \times 5 = 75 \Rightarrow u \cos\alpha = 15$ ...(ii)

From (i) and (ii)

$$\tan\alpha = \frac{20}{15} = \frac{4}{3}$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

15. (*a*) $y = x - x^2/80$

Comparing it with standard equation

$$y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$$

$\tan\theta = 1$

$$\theta = 45° \qquad \frac{g}{2u^2 \cos^2\theta} = \frac{1}{80} \qquad u = \sqrt{800}$$

$u_x = 20 \qquad u_y = 20$

$v_y = 20 - 40 = -20$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-20}{20} = -1$$

$\alpha = -45°$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{800 \times 1}{10} = 80 \text{ m}$$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{800 \times 1/2}{20} = 20 \text{ m}$$

16. (*a*) $H = \frac{1}{2} gt^2$

$$500 = \frac{1}{2} \times 0^5 \times t^2$$

$t = 10$ sec.

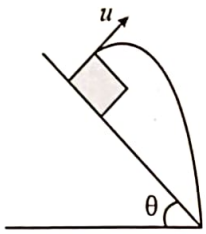
$x = 200\, xt = 200$ m

$d = 200 - D = 100$ m

17. (*b*) $\sin 30° = \frac{v_y}{v}$

$$\frac{1}{2} = \frac{(20 \sin 60°)}{v} \Rightarrow v = 20\sqrt{3}$$

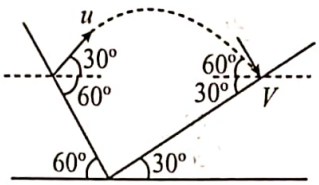
18. (*d*) $T = \frac{2u}{g \cos\theta}$



$$x = \frac{1}{2} g \sin\theta\, T^2 = \frac{1}{2} g \sin\theta \times \frac{4u^2}{g^2 \cos^2\theta}$$

$$= \frac{2u^2}{g} \tan\theta \sec\theta$$

19. (*b*)



$u \cos 30° = V \cos 60°$

$$5\sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{V \times 1}{2} \Rightarrow V = 15 \text{ m/s}$$

20. (*b*) $Ax = vt$

$$\ell = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{Ax}{v}\right)^2$$

$$\ell_1 : \ell_2 : \ell_3 = AB^2 : AC^2 : AD^2$$

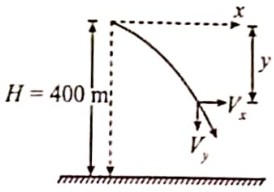
21. (*a*) $V_y = \frac{dy}{dt} = \sqrt{2gy}$ ...(i)

$V_x = \frac{dx}{dt} = ay$ ...(ii)

Dividing (i) by (ii)

$$\frac{dy}{dx} = \frac{\sqrt{2gy}}{ay} = \frac{2}{\sqrt{y}}$$

or $\sqrt{y}\,dy = 2dx$



Integrating $\int_0^{400} \sqrt{y\,dy} = 2\int_0^x dx$

$\therefore\ x = 2.67$ km

22. (*b*) $x = \frac{1}{2}at^2$ ...(i)

$y = ut + \frac{1}{2}at^2$ ...(ii)

From (i) and (ii)

$$y = u\sqrt{\frac{2x}{a}} + x$$

$$(y - x)^2 = \frac{2u^2x}{a}$$

23. (*b*) $\frac{1}{2}gt^2 = h \Rightarrow t = \sqrt{\frac{2h}{g}}$

$$v_0 - a_0t = 0 \Rightarrow \frac{v_0^2}{a_0} = \frac{2h}{g} \Rightarrow h = \frac{V_0^2 g}{2a_0^2}$$

24. (*a*) $T = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$

$$s_n = u_xT + \frac{1}{2}a_xT^2$$

$$= u\sin\theta \times \frac{2u\cos\theta}{g} + \frac{1}{2}(-g\tan\theta)\left(\frac{2u\cos\theta}{g}\right)^2$$

$$= \frac{u^2\sin 2\theta}{g} - \frac{1}{2}g\frac{\sin\theta}{\cos\theta} \times \frac{4u^2\cos^2\theta}{g^2}$$

$$= \frac{u^2\sin 2\theta}{g} - \frac{u^2\sin 2\theta}{g}$$

Hence particle returns to *A*.

Exercise-3 (JEE Advanced Level)

1. (*b*) Since $\vec{a} = (4t)\hat{i} + (3t^2)\hat{j}$ is time dependent therefore we cannot apply $\vec{v} = \vec{u} + \vec{a}t$

We can solve by applying calculus

Thus $\frac{d\vec{v}}{dt} = \vec{a}$ or $\int_0^{\vec{v}} d\vec{v} = \int_0^4 (4t\hat{i} + 3t^2\hat{j})dt$

$$= \left\{4\int_0^4 t\,dt\right\}\hat{i} + \left\{3\int_0^4 t^2\,dt\right\}\hat{j}$$

$\vec{v} = (32)\hat{i} + (64)\hat{j}$ m/sec

2. (*d*) Acceleration of the particle $a = F/m = 2$ m/sec^2

Here acceleration is constant therefore $\vec{v} = \vec{u} + \vec{a}t$

Since $\vec{u}$ and $\vec{a}t$ are perpendicular.

Hence $|\vec{v}| = \sqrt{|\vec{u}| + |\vec{a}t|^2}$, where $|\vec{u}| = 2$ m/sec;

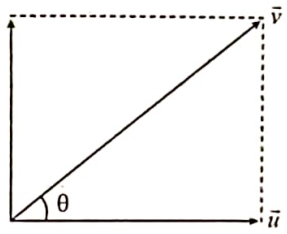
$|\vec{a}t| = 2 \times 1 = 2$ m/sec

$|\vec{v}| = 2\sqrt{2}$ m/sec or $\theta = \tan^{-1}\frac{|\vec{a}t|}{|\vec{u}|} = \tan^{-1}(1) = 45°$

Hence the velocity of the particle after one second is $2\sqrt{2}$ m/sec at an angle of 45° with its initial velocity.

For the displacement equation of motion is

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$



Or $|\vec{s}| = \sqrt{|\vec{u}t|^2 + \left|\frac{1}{2}\vec{a}t\right|^2}$

$|\vec{u}t| = 2 \times 1 = 2$;

$\left|\frac{1}{2}\vec{a}t^2\right| = \frac{1}{2} \times (2) \times (1)^2 = 1$

$|\vec{s}| = \sqrt{(2)^2 + (1)^2} = \sqrt{3}$ m

$$\tan\phi = \frac{\left|\frac{1}{2}\vec{a}t^2\right|}{|\vec{u}t|} = \frac{1}{2} \Rightarrow \phi = \tan^{-1}(1/2)$$

Displacement is at an angle $\tan^{-1}(1/2)$ with the direction of initial velocity

So the angle between velocity vector and displacement vector of the particle at the end of 1 sec will be $45° - \tan^{-1}(1/2)$

3. (*b*) $v_x = 3$ m/s, $v_y = 4$ m/s, $a_x = 2$ m/s^2, $a_y = 1$ m/s^2

$\vec{v} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 2\hat{i} + 1\hat{j}$

$\vec{v}.\vec{a} = 10$, $va\cos\theta = 10$

$$a\cos\theta = \frac{10}{v} = \frac{10}{5} = 2$$

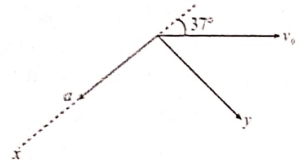
Rate of change of speed = $a\cos\theta = 2$ m/s.

4. (*b*) x - components $\quad$ y - components

$u_x = -v_0\cos 37°$ $\quad$ $u_y = v_0\sin 37°$

$a_x = a$ $\quad$ $a_y = 0$

$v_x = -\frac{4v_0}{5} + at$ $\quad$ $v_y = -\frac{3v_0}{5}$



When $v_x = 0$

$$t = \frac{4}{5}\times\frac{24}{6} = 3.2 \text{ sec}$$

5. (*d*) The equation of trajectory is

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \quad \text{...(i)}$$

$$= x\tan\alpha \times \left[1 - \frac{gx}{2u^2\cos\alpha\sin\alpha}\right]$$

Range of projectile is $R = \dfrac{2u^2\sin\alpha\cos\alpha}{g}$ ...(ii)

From equation (i) and (ii), we get, $y = x\tan\alpha\left[1 - \dfrac{x}{R}\right]$

The coordinates of A are (h cot θ, h) and range $R = h\cot\theta + h\cot\phi$. Substituting in eqn. (iii), we get

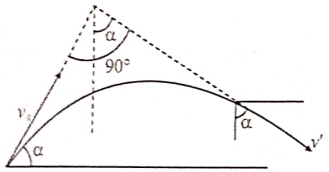
$$h = h\cot\theta\tan\alpha\left[1 - \frac{h\cot\theta}{h\cot\theta + h\cot\phi}\right]$$

or $\tan\theta = \dfrac{\tan\alpha\cot\phi}{\cot\theta + \cot\phi}$

$\tan\theta\cot\theta + \tan\theta\cot\phi = \tan\alpha\cot\phi$

$1 + \dfrac{\tan\theta}{\tan\phi} = \dfrac{\tan\alpha}{\tan\phi}$ or $\tan\theta + \tan\phi = \tan\alpha$

6. (*a*) **Method 1:** If initial velocity v_0 and velocity at time t are perpendicular, then the final velocity will be at an angle α with the vertical.



Horizontal component of velocity is unchanged throughout the motion.

Therefore $v_0\cos\alpha = v\sin\alpha$ or $v = v_0\cot\alpha$

Vertical component of velocity after time $t = -v\cos\alpha$

From the equation $v_y = v_0\sin\alpha - gt$

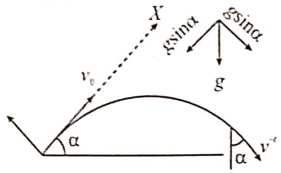
$-v\cos\alpha = v_0\sin\alpha - gt$

or $t = \dfrac{v_0\sin\alpha + v\cos\alpha}{g}$

$$= \frac{v_0\sin\alpha + v_0\cot\alpha\cos\alpha}{g}$$

$$= \frac{v_0}{g}\left[\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha}\right] = \frac{v_0}{g}\operatorname{cosec}\alpha$$

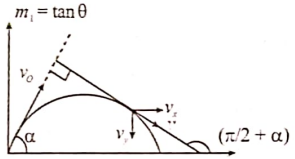
Method 2: We choose x-axis along the initial velocity. If after time t the velocity is perpendicular to initial direction, v_x must be zero after time t.



i.e., $0 = v_0 - g\sin\alpha\, t$ or $t = \dfrac{v_0}{g\sin\alpha}$

Method 3: Slope of trajectory at the point of projection,

$m_1 = \tan\theta$



Slope of trajectory after time t,

$$m_2 = \tan\alpha = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{v_y}{v_x} = \frac{v_0\sin\alpha - gt}{v_0\cos\alpha}$$

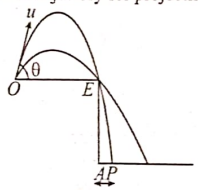
Slopes are perpendicular,

$$\left(\frac{v_0\sin\alpha - gt}{v_0\cos\alpha}\right)(\tan\alpha) = -1 \text{ or } t = \frac{v_0}{g\sin\alpha}$$

7. (*a*) P is point on plane for shell near edge of plateau for greater angle of projection. In region AP shell cannot hit

$x_{max} = AP$

Equation of trajectory for projectile



$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

For point E, $y = 0$ and $x = 960$m

$$\tan^2\theta \times \frac{10\times(960)^2}{2\times(100)^2} - (\tan\theta)\,960 + \frac{10\times(960)^2}{2\times(100)^2} = 0$$

$\Rightarrow \tan\theta = 3/4, 4/3$

For $\tan\theta = 4/3$, AP is least

Now, equation of trajectory becomes,

$$y = \frac{4}{3}x - \frac{x^2}{2000}\left(\frac{25}{9}\right)$$

For landing point on plane (P), $y = -960$ m

$$\Rightarrow -960 = \frac{4}{3}x - \frac{x^2}{720}$$

Solving we get acceptable solution $x = 1440$ m

$AP = x - 960 = 480$m

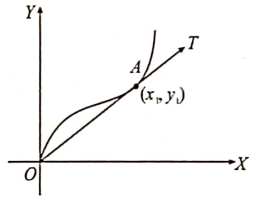
8. (*a*) Clearly A is the point such that OA is tangent to $y = (x-1)^3 + 1$ at the point A. Let point A be (x_1, y_1).

$y = (x-1)^3 + 1$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{x=x_1} = 3(x_1 - 1)^2$$

Hence equation of tangent at (x_1, y_1) is

$(y - y_1) = 3(x_1 - 1)^2(x, y_1)$



But this tangent passes through origin. Hence

$-y_1 = -3x_1(x_1 - 1)^2 \Rightarrow y_1 = 3x_1(x_1 - 1)^2$

$\Rightarrow (x_1 - 1)^3 + 1 = 3x_1(x_1 - 1)^2$

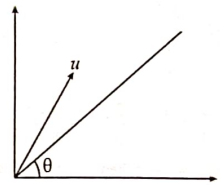
$\Rightarrow (2x_1 + 1)(x_1 - 1)^2 = 1$

$\Rightarrow 2x_1^3 - 3x_1^2 + 1 = 1 \Rightarrow x_1^2(2x_1 - 3) = 0 \Rightarrow x_1 = \dfrac{3}{2}$

$\therefore A$ is $\left(\dfrac{3}{2}, \dfrac{9}{8}\right)$

9. (*b*) On the incline plane the maximum possible range is

$$R = \frac{v^2}{g(1 + \sin\theta)}$$



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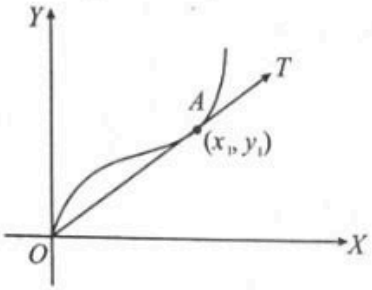

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$\Rightarrow (x_1-1)^3 + 1 = 3x_1(x_1-1)^2$

$\Rightarrow (2x_1+1)(x_1-1)^2 = 1$

$$\Rightarrow 2x_1^3 - 3x_1^2 + 1 = 1 \Rightarrow x_1^2(2x_1-3) = 0 \Rightarrow x_1 = \frac{3}{2}$$

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9. (*b*) On the incline plane the maximum possible range is

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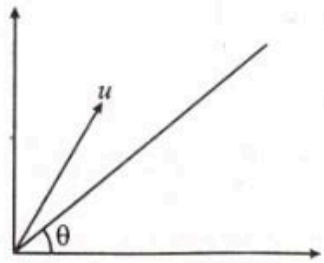
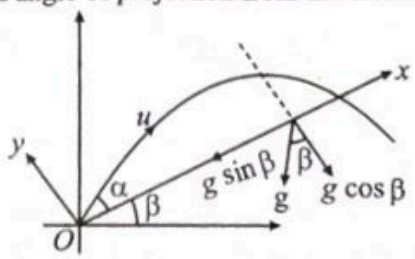


Range max = ?

Let $\theta = \beta$

And angle of projection from the inclined plane = α



$v_y = v\sin\alpha$ $v_x = \cos\alpha$ $a_x = -g\sin\beta$ $a_y = -g\cos\beta$

Range $= s_x = v_x T + \frac{1}{2}a_x T^2$

(on the inclined plane) where $T = \frac{2v_y}{g_y}$

$$\Rightarrow s_x = (v\cos\alpha)\left[\frac{2v\sin\alpha}{g\cos\beta}\right] + \frac{1}{2}[-g\sin\beta]\left[\frac{2v\sin\alpha}{g\cos\beta}\right]^2$$

$$= \frac{2v^2\sin\alpha}{g\cos^2\beta}[\cos\alpha.\cos\beta - \sin\alpha.\sin\beta]$$

$$s_x = \frac{2v^2\sin\alpha}{g\cos^2\beta}[\cos(\alpha+\beta)]$$

$$= \frac{v^2}{g\cos^2\beta}[2\sin\alpha\cos(\alpha+\beta)]$$

$$= \frac{v^2}{g\cos^2\beta}[\sin(2\alpha+\beta) + \sin(-\beta)]$$

$$s_x = \frac{v^2}{g\cos^2\beta}[\sin(2\alpha+\beta) - \sin\beta]$$

Now s_x is max

when $\sin(2\alpha+\beta)$ is max $\quad (\because \beta = \text{constant})$

$$\Rightarrow 2\alpha + \beta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\left(\frac{\pi}{2} - \beta\right)}{2}$$

i.e., when ball is projected at the angle bisector of angle formed by inclined plane and direction of net acceleration reversed.

$$\&\ (s_x)_{max} = \frac{v^2}{g\cos^2\beta}[1-\sin\beta] = \frac{v^2(1-\sin\beta)}{g(1-\sin\beta)(1-\sin\beta)}$$

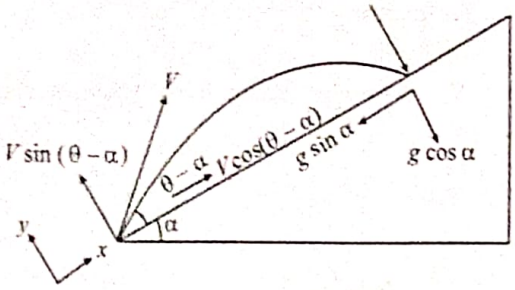
$$\text{Max. Range on an inclined plane} = \frac{v^2}{g(1+\sin\beta)}$$

$$\text{Here, } \beta = \theta \Rightarrow R_{max} = \frac{v^2}{g(1+\sin\theta)}$$

10. (*d*) Applying equation of motion perpendicular to the incline for $y = 0$.

$$0 = V\sin(\theta-\alpha)t + \frac{1}{2}(-g\cos\alpha)t^2$$

$$\Rightarrow t = 0\ \&\ \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$$



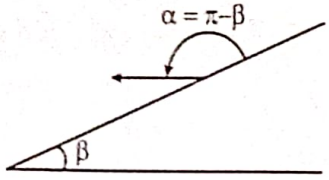
At the moment of striking the plane, as velocity is perpendicular to the inclined plane hence component of velocity along incline must be zero.

$$0 = v\cos(\theta-\alpha) + (-g\sin\alpha).\ \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$$

$$v\cos(\theta-\alpha) = \tan\alpha.\ 2V\sin(\theta-\alpha)$$

$$\cot(\theta-\alpha) = 2\tan\alpha$$

11. (*d*) $R = \dfrac{v^2}{g\cos^2\beta}[\sin(2\alpha+\beta) - \sin\beta]$



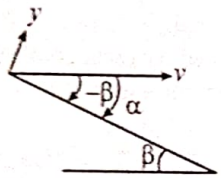
Putting $\beta = 45°$ & $\alpha = \pi - \dfrac{x}{4} = \dfrac{3\pi}{4}$

$$R = \frac{v^2}{g\left(\frac{1}{\sqrt{2}}\right)^2}\left[\sin\left(2\times\frac{3\pi}{24}+\frac{\pi}{4}\right) - \sin\frac{\pi}{4}\right]$$

$$s_x = \frac{v^2\times 2}{g}\left[-2\times\frac{1}{\sqrt{2}}\right] = -2\sqrt{2}\frac{v^2}{g}$$

(–ve sign indicates that the displacement is in –ve x direction)

$$\Rightarrow \text{Range} = 2\sqrt{2}\frac{v^2}{g}$$



Alternate II method

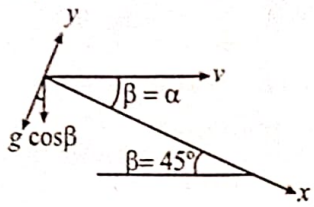
$$\beta = -\frac{\pi}{4} \text{ and } \alpha = \frac{\pi}{4}$$

$$R = \frac{v^2}{g\cos^2\beta}[\sin(2\alpha+\beta) - \sin\beta]$$

$$= \frac{v^2}{g\left(\frac{1}{\sqrt{2}}\right)^2}\left[\sin\frac{\pi}{4} - \sin\left(-\frac{\pi}{4}\right)\right]$$

$$R = \frac{2\sqrt{2}v^2}{g} \text{ (along +ve } x \text{ direction)}$$

III Method



$$v_x = v\cos\beta,\ T = \left|\frac{2v\sin\beta}{g\cos\beta}\right|,\ a_x = g\sin\beta;\ a_y = -\cos\beta$$

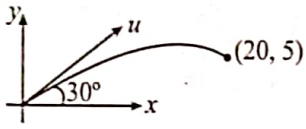
$$s_x = v_x t + \frac{1}{2}a_x t^2 = (v\cos\beta)\left[\frac{2v\sin\beta}{g\cos\beta}\right] + \frac{1}{2}(g\sin\beta)\left[\frac{2v\sin\beta}{g\cos\beta}\right]^2$$

Let, $\alpha = \beta = 45°$

$$\text{So, } s_x = \frac{v^2 2}{g}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}g\left(\frac{1}{\sqrt{2}}\right)\frac{2.2v^2}{g^2} = \frac{2v^2}{\sqrt{2}g}[1+1]$$

$$s_x = 2\sqrt{2}\frac{v^2}{g}$$

13. (*a*)

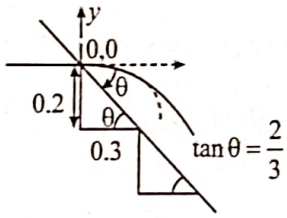


$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$5 = 20\tan 30° - \frac{1}{2}\times\frac{10\times 20^2}{u^2\cos^2 30°}$$

$$\Rightarrow u^2 = \frac{1600}{\sqrt{3}(4-\sqrt{3})} = \frac{1600}{13\sqrt{3}}(4+\sqrt{3})$$

14. (*a*) We have the point of projection as (0, 0) we have the equation of st. line (as shown in fig.)



$$y = x\tan(-\theta)$$

$$y = -\frac{2}{3}x \quad \text{....(i)}$$

Also the equation of trajectory for horizontal projection

$$y = \frac{-1}{2}g\frac{x^2}{u^2} \quad \text{....(ii)}$$

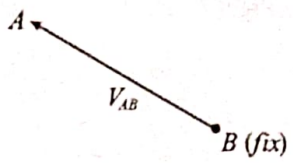
From (i) and (ii)

$$\frac{1}{2}g\frac{x^2}{u^2} = \frac{2}{3}x$$

$$\text{or } X = \frac{2}{3}\times\frac{u^2}{5} = \frac{2}{3}\times\frac{4.5\times 4.5}{5} = 3\times 0.9$$

If no. of stps be n then $n\times 0.3 = 3\times 0.9 \Rightarrow n = 9$

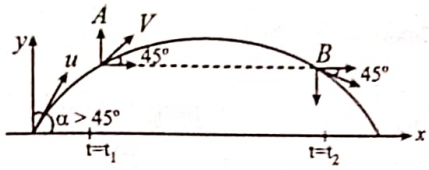
15. (*a,c,d*)



16. (*b,d*) $\vec{V}_{s/G} = V_{s/A} + \vec{V}_{A/G}$

Velocity of horizontal component of the shell will be in the direction of velocity of air plane.

17. (*b,c*)



$\Rightarrow$ Time at which $V_x = V_y$ is what we are solving

$V_x = V_y$

Now, $V_x = u\cos\alpha$

$V_y = u\sin\alpha - gt$

$\Rightarrow u\cos\alpha = u\sin\alpha - gt$ {$\because V_y = V_x$}; at $t = t_1$ (say)

$\Rightarrow t_1 = \frac{u}{g}(\sin\alpha - \cos\alpha)$

(*c*) is correct.

Also when $V_y \equiv V_x$ {i.e., when we choose 'y' axis as $-y$'}

at $t = t_2$ (say)

$-u\cos\alpha = u\sin\alpha - gt_2 \Rightarrow t_2 = \frac{u}{g}(\sin\alpha + \cos\alpha)$

(*b*) is correct.

18. (*a,b*)

$x = 24 = u\cos\theta.t$

$\Rightarrow t = \frac{24}{24\cos\theta} = \frac{1}{\cos\theta}$

$y = 14 = u\sin\theta t - \frac{1}{2}gt^2$

$\Rightarrow 14 = \frac{u\sin\theta}{\cos\theta} - \frac{5}{\cos^2\theta}$

$\Rightarrow 14 = u\tan\theta - 5\sec^2\theta$

$\Rightarrow 5\tan^2\theta - 24\tan\theta + 19 = 0 \Rightarrow \tan\theta = 1, 19/5.$

19. (*a,b,c,d*)

$h = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh}$

(*a*) $R_{max} = \frac{u^2}{g} = 2h$

(*b*) $R = nH_{max}$

$\frac{u^2\sin 2\theta}{g} = n\frac{u^2\sin^2\theta}{2g}$

$4 = n\tan\theta$

$\theta = \tan^{-1}\left(\frac{4}{n}\right)$

(*c*) $gT^2 = g \times \frac{4u^2\sin^2\theta}{g^2} = \frac{2 \times u^2\sin 2\theta}{g} \times \tan\theta$

$gT^2 = 2R\tan\theta$

(*d*) $T = \frac{2u_y}{g}, H_{max} = \frac{u_y^2}{2g}$

$\therefore$ Ratio 1 : 1

20. (*a,c,d*)

$T = \sqrt{\frac{2H}{g}} \Rightarrow 0.4 = \sqrt{\frac{2H}{g}} \Rightarrow H = 0.8m$

$R = 0.4 \times 4 = 1.6$m

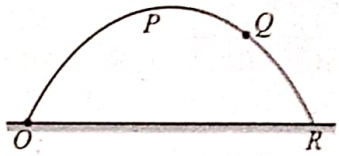
and $Uy = \sqrt{2gH} = \sqrt{2 \times 10 \times 0.8} = 4$m/s $\Rightarrow \theta = 45°$

21. (*a*) $H_A = H_C > H_B$

Obviously A just reaches its maximum height and C has crossed its maximum height which is equal to A as u and θ are same. But B is unable to reach its max. height.

22. (*c*) Time of flight of A is 4 seconds which is same as the time of flight if wall was not there.

Time taken by C to reach the inclined roof is 1 sec.



$T_{OR} = 4, T_{QR} = 1$

$\therefore T_{OQ} = T_{OR} - T_{QR} = 3$ seconds

23. (*c*) From above $= \frac{2u\sin\theta}{g} = 4$

$\therefore u\sin\theta = 20$ m/s $\Rightarrow$ vertical component is 20 m/s.

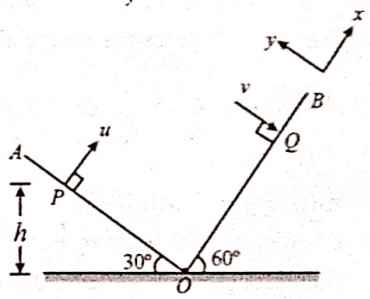
for maximum height

$v^2 = u^2 + 2as \Rightarrow 0^2 = 20^2 - 2 \times 10 \times s$

$s = 20$ m.

24. (*b*) Let us choose the x and y directions along OB and OA respectively. Then

$u_x = u = 10\sqrt{3}$ m/s, $u_y = 0$



$a_x = -g\sin 60° = -5\sqrt{3}$ m/s^2

and $a_y = -g\cos 60° = -5$ m/s^2

At point Q, x-component of velocity is zero. Hence, substituting in $v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}t$ or $t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2s$

25. (*a*) At point Q, $v = v_y = u_y + a_y t$

$\therefore\ v = 0 - (5)(b) = -10$ m/s

Here, negative sign implies that velocity of particle at Q is along negative y direction.

26. (*c*) Distance PO = |displacement of particle along y-direction|

Here, $s_y = u_y t + \frac{1}{2} a_y t^2 = 0 - \frac{1}{2}(5)(2)^2 = -10$ m

$\therefore\ PO = 10$ m

Therefore, $h = PO \sin 30° = (10)\left(\frac{1}{2}\right)$ or $h = 5$ m

27. (*a*) Distance OQ = displacement of particle along x-direction $= s_x$

Here $s_x = u_x t + \frac{1}{2} a_x t^2$

$$= (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2 = 10\sqrt{3}\ \text{m}$$

or $OQ = 10\sqrt{3}$ m

$$\therefore\ PQ = \sqrt{(PO)^2 + (OQ)^2}$$

$$= \sqrt{(10)^2 + (10\sqrt{3})^2} = \sqrt{100 + 300} = \sqrt{400}$$

$\therefore\ PQ = 20$ m

28. (*b*) A-(r); B-(s); C-(q); D-(p).

Time of flight

$$T = \frac{2u}{g\cos 45°} = \frac{2u}{g\cos 45°} = \frac{2\sqrt{2}u}{g}$$

$\therefore\ D \to p$

Velocity of stone is parallel to x-axis at half the time of flight.

$\therefore\ A \to r$

At the instant stone make 45° angle with x-axis its velocity is horizontal.

$\therefore$ The time is $= \frac{u\sin 45°}{g} = \frac{u}{\sqrt{2}g}$ $\therefore\ B \to s$

The time till its displacement along x-axis is half the range is $= \frac{1}{\sqrt{2}}T = \frac{2u}{g}$

$\therefore\ C \to q$

29. (*c*) A-(r); B-(s); C-(q); D-(p).

Equation of path is given as, $y = ax - bx^2$

Comparing it with standard equation of projectile;

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\tan\theta = a,\ \frac{g}{2u^2\cos^2\theta} = b$$

Horizontal component of velocity $= u\cos\theta = \sqrt{\frac{g}{2b}}$

Time of flight $T = \frac{2u\sin\theta}{g}$

$$= \frac{2(u\cos\theta)\tan\theta}{g}$$

$$= \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$$

Maximum height $H = \frac{u^2\sin^2\theta}{2g}$

$$= \frac{[u\cos\theta.\tan\theta]^2}{2g}$$

$$= \frac{\left[\sqrt{\frac{g}{2b}}\right]^2}{2g} = \frac{a^2}{4b}$$

Horizontal range $R = \frac{u^2\sin 2\theta}{2g}$

$$= \frac{2(u\sin\theta)(u\cos\theta)}{g}$$

$$= \frac{2\left[\sqrt{\frac{g}{2b}}.a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$$

30. [0025]

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{v^2\cos^2\theta}$$

$x = 38 + 2 = 40$

$y = 18$

$\theta = 60°$

$\Rightarrow v = 25$ m/s

31. [0007]

$$R = \frac{u^2\sin(2\theta)}{g}$$

$\therefore R \propto \sin(2\theta)$ (for same speed of projectio

$$\therefore\ \frac{R'}{R} = \frac{\sin(2\times 45°)}{\sin(2\times 15°)}$$

$$\therefore\ R' = 3.5\times\frac{1}{0.5} = 7\ \text{m}$$

32. [0020]

$$-20 = 40\tan 0 - \frac{1}{2}\frac{g\times 40^2}{u^2\cos^2 0°}$$

$$u^2 = \frac{1600}{4} = 400$$

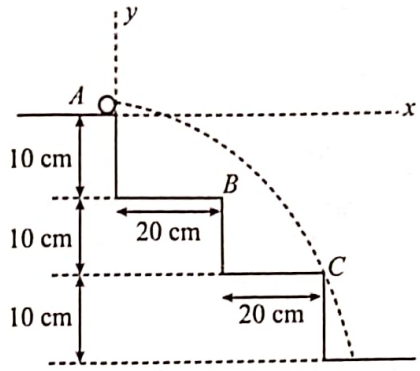
$u = 20$ m/s

33. [2] At minimum velocity, it will move just touching point E reaching the ground A is origin of reference coordinate.

If u is the minimum speed,

$x = 40, y = 20, \theta = 0°$

$$y = x\tan\theta - g\frac{x^2\sec^2\theta}{2u^2}$$



where $g = 10\ ms^{-2} = 1000\ cm\ s^{-2}$

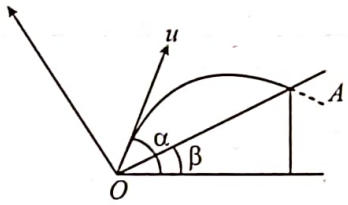
$$-20 = -\frac{800000}{2u^2}$$

$u = 200\ cm\ s^{-1} = 2\ ms^{-1}$

34. [2]

(*a*) $T = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$

Now we shall consider the motion of particle align OA.



(*b*) When the particle strikes the plane horizontally

In this case $t = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$ and $0 = u\sin\alpha - gt$

$$u\sin\alpha = g\left[\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\right]$$

$2\tan\beta = \tan\alpha \Rightarrow \tan\alpha : \tan\beta = 2$

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*c*) $\dfrac{dx}{dt} = y; \dfrac{dy}{dt} = x$

$$\frac{dx}{dy} = \frac{y}{x}$$

$\Rightarrow y^2 = x^2 + c$

2. (*a*) Area $\propto \pi(\text{Range})^2 \propto V^4$

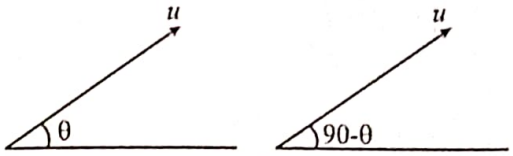
$$\therefore \frac{A_1}{A_2} = \left(\frac{1}{2}\right)^4$$

3. (*b*) $t = \dfrac{2\times 2\sin 15°}{g\cos 30°}$

$S = 2\cos 15° \times t - \dfrac{1}{2}g\sin 30°\, t^2$

Put values and solve: $S = 20$ cm

4. (*b*)



For same range angle of projection will be θ & $90 - \theta$

$$R = \frac{u^2 2\sin\theta\cos\theta}{g}$$

$$h_1 = \frac{u^2\sin^2\theta}{g}$$

$$h_2 = \frac{u^2\sin^2(90-\theta)}{g}$$

$$\frac{R^2}{h_1h_2} = 16$$

5. (*a*) Equation of trajectory is given as

$y = 2x - 9x^2$ ... (i)

Comparing with equation:

$y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}\cdot x^2$ ... (ii)

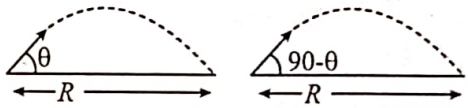
We get; $\tan\theta = 2$

$$\therefore \cos\theta = \frac{1}{\sqrt{5}}$$

Also, $\dfrac{g}{2u^2\cos^2\theta} = 9$

$$\Rightarrow \frac{10}{2\times 9\times\left(\frac{1}{\sqrt{5}}\right)^2}\times v_0^2 \Rightarrow v_0^2 = \frac{25}{9} \Rightarrow v_0 = \frac{5}{3}\ m/s$$

6. (*c*) Range will be same for time t_1 and t_2, so angles of projection will be 'θ' and '$90° - \theta$'



$$t_1 = \frac{2u\sin\theta}{g}\quad t_2 = \frac{2u\sin(90°-\theta)}{g}$$

and $R = \dfrac{u^2\sin 2\theta}{g}$

$$t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right] = \frac{2R}{g}$$

7. (*d*) $t_0 = \sqrt{\frac{2h}{g}} \Rightarrow V_0 = \sqrt{\frac{2h}{g}} \times g = \sqrt{2gh}$

$\therefore t_1$ = time to reach top $= \frac{v_0}{g} = \sqrt{\frac{2h}{g}}$

$\therefore H = h + h' = 2h$

t_2 = time of fall $= t_1 + t_2 = \sqrt{\frac{2\times(2h)}{g}} = 2\sqrt{\frac{h}{g}}$

$= \left(2+\sqrt{2}\right)\sqrt{\frac{h}{g}}$

$= 3.4\sqrt{\frac{h}{g}}$

8. (*a*) $y = \beta x - \beta x^2$

comparing with trajectory equation

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

$\Rightarrow \beta = \frac{1}{2}\frac{g}{u^2\cos^2\theta}$

$\Rightarrow u^2 = \frac{g}{2b\cos^2\theta}$

Maximum height H:

$H = \frac{u^2\sin^2\theta}{2g} = \frac{g}{2\beta^2\cos^2\theta}\cdot\frac{\sin^2\theta}{2g}$

$\Rightarrow H = \frac{\tan^2\theta}{4\beta} = \frac{\alpha^2}{4\beta}$

9. (*d*) $R = \frac{V^2(2\sin\theta\cos\theta)}{g}$

$t = \frac{V\sin\theta}{g} \Rightarrow V = \frac{gt}{\sin\theta}$

$\Rightarrow R = \frac{g^2t^2}{\sin^2\theta}\cdot\frac{2\sin\theta\cos\theta}{g}$

$\tan\theta = \frac{2gt^2}{R} = \frac{20t^2}{R}$

$\cot\theta = \frac{R}{20t^2}$

10. (*d*) $R = H$

$\frac{2v_x \times v_y}{g} = \frac{v_y^2}{2g}$

$v_x = \frac{v_y}{4}$;

$u\cos\theta = \frac{u\sin\theta}{4}$

$\tan\theta = 4$

11. (*c*) Time taken to reach maximum height

$t = \frac{u\sin\theta}{g}$

$\therefore \frac{u_1\sin\theta_1}{g} = \frac{u_2\sin\theta_2}{g}$

$\Rightarrow u_1\sin 30 = u_2\sin 45$

$\Rightarrow \frac{u_1}{u_2} = \frac{1/\sqrt{2}}{1/2} = \frac{\sqrt{2}}{1}$

12. (*d*) $y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)}$

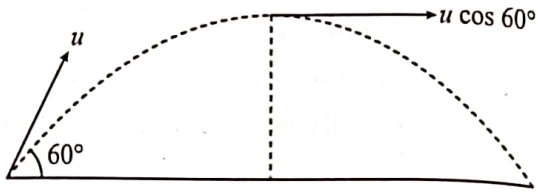
$\Rightarrow 10 = 20 - \frac{(10)(100)}{u^2}$

$T = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$

$\vec{v} = 10\sqrt{2}\,\hat{i} + \left(10\sqrt{2} - 10(2)\right]\hat{j}$

Momentum $\vec{p} = M\vec{v} = 100\sqrt{2}\,\hat{i} + \left(100\sqrt{2} - 200\right)\hat{j}$

13. (*c*)



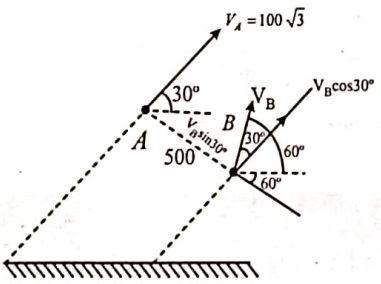
At highest point, velocity

$V = u\cos 60^\circ = \frac{u}{2}$

K.E at topmost point math $= \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{E}{4}$

JEE ADVANCED

14. [5]



For relative motion perpendicular to line of motion of A

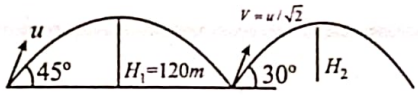
$V_A = 100\sqrt{3}$

$= V_B\cos 30^\circ$

$\Rightarrow V_B = 200$ m/s

$t_0 = \frac{500}{V_B\sin 30^\circ} = \frac{500}{200\times\frac{1}{2}} = 5$ sec

15. [30.00]



$$H_1 = \frac{u^2 \sin^2 45}{2g} = 120$$

$$\Rightarrow \frac{u^2}{4g} = 120 \quad \text{... (i)}$$

when half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \frac{u^2}{16g} \quad \text{... (ii)}$$

From (i) & (ii)

$$H_2 = \frac{H_1}{4} = 30\text{m on } 30.00$$

16. [4.00]

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$\text{Total time taken} = t_1 + t_2 + t_3 + \ldots = t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + \ldots$$

$$\text{Total time} = \frac{t_1}{1 - \frac{1}{\alpha}}$$

$$\text{Total displacement} = v_1 t_1 + v_2 t_2 + \ldots l$$

$$= v_1 t_1 + \frac{v_1}{\alpha} \cdot \frac{t_1}{\alpha} + \ldots = \frac{v_1 t_1}{1 - \frac{1}{\alpha^2}}$$

On solving

$$< v > = \frac{v_1 \alpha}{\alpha + 1}$$

$$= 0.8 v_1$$

$$\alpha = 4.00$$

17. [5] $u_x = 1$

$$y = 5x(1 - x)$$

$$\frac{dy}{dt} = 5\frac{dx}{dt} - 10x\frac{dx}{dt}$$

For initial y-component of velocity

$$u_y = \left(\frac{dy}{dt}\right)_{x=0} \Rightarrow 5(1) = 5$$

$$\vec{u}_y = 5\hat{j}$$

18. [1] Time of flight is same

$\Rightarrow$ vertical component of velocity is same

$\Rightarrow H_{max}$ is same

19. [15] $R_{max} = \frac{u^2 \sin 2(45^\circ)}{g} = \frac{u^2}{g}$

$$\frac{R}{2} = \frac{u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{1}{2}$$

$29 = 30°, 150°$

$9 = 15°, 75°$

15, 75

CHAPTER

5 Relative Motion

Exercise-1 (Topicwise)

1. (*b*) Time = $\frac{\text{Total length}}{\text{Rdative velocity}} = \frac{50+50}{10+15} = \frac{100}{25} = 4$ sec

2. (*b*) Relative velocity of bird w.r.t train= 25 + 5 = 30 m/s

time taken by the bird to cross the train

$t = \frac{210}{30} = 7$ sec

3. (*d*) At $t = 3$ sec, 1st stone will have speed of 30 m/s

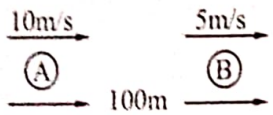
$h_1 = \frac{1}{2} \times 10 \times 9 = 45m$

$h_2 = \frac{1}{2} \times 10 \times 1^2 = 5m$

$h_1 - h_2 = 40m$

4. (*b*) Time = $\frac{\text{Total length}}{\text{Rdative velocity}} = \frac{50+50}{10+15} = \frac{100}{25} = 4$ sec

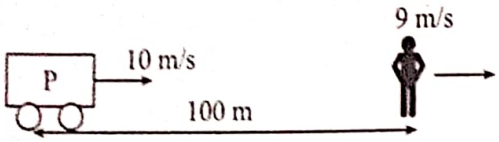
5. (*c*)



$V_{AB} = 10 - 5 = 5$ m/s

$t = \frac{100}{5} = 20$ sec

6. (*d*) $\vec{V}_{PT} = \vec{V}_P - \vec{V}_T = 10 - 9 = 1$ m/s



So time taken $t = \frac{100}{\vec{V}_{PT}} = \frac{100}{1} = 100$ sec.

7. (*b*) We have, $S_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$

$\Rightarrow 0 = ut - \frac{1}{2}(a+g)t^2$

$\Rightarrow a = \frac{2u}{t} - g = \frac{2u - gt}{t}$

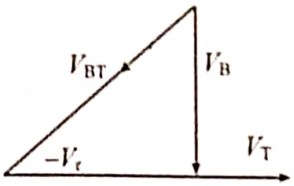
8. (*a*) Ifs speed of cars is x |m|s and y m/s then

$x + y = 9$ ...(i)

$x - y = 1$ ...(ii)

By solving (i) and (ii), $x = 5$ m/s, $y = 4$ m/s.

9. (*c*)



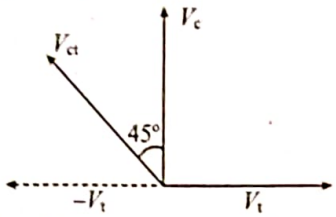
$\vec{V}_{BT} = \vec{V}_B - \vec{V}_T$

$V_{BT} = \sqrt{40^2 + 40^2} = 40\sqrt{2}$

Direction $\rightarrow SW$

10. (*b*) $\vec{v}_{ct} = \vec{v}_c - \vec{v}_t$

$\vec{v}_{ct} = \vec{v}_c + (-\vec{v}_t)$



Velocity of car w.r.t. train (v_{ct}) is towards West-North

11. (*b*) $\hat{i} + \sqrt{2}\hat{j} + \hat{k}$

12. (*d*) Let direction be such that: North = j, East = i

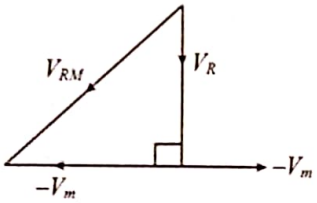
$V_{copter} = -50j$, $V_{train} = 50i$

Velocity of helicopter as seen by passengers on the train is: $V_{copter} - V_{train} = -50i - 50j$

Which is along South West direction.

13. (*b*) He must travel with the same velocity as the wind =2m/s in the direction same as wind.

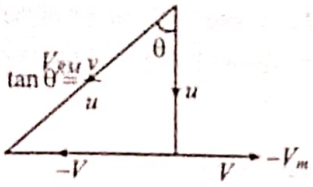
14. (*c*)



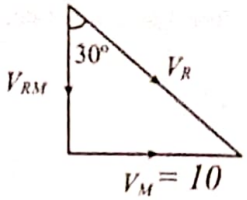
$\vec{V}_{RM} = \vec{V}_R - \vec{V}_m$

$V_{RM} = \sqrt{V_R^2 + V_m^2} = \sqrt{109}$

15. (*a*)



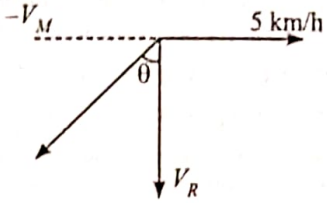
16. (*b*)



$$\tan 30° = \frac{V_m}{V_{RM}} = \frac{10}{V_{RM}}$$

$$V_{RM} = 10\sqrt{3}$$

17. (*c*)

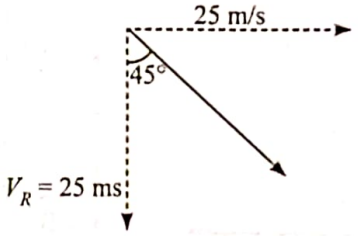


$$\tan\theta = \frac{V_M}{V_R}$$

$$\tan 45° = \frac{5 \text{ km/h}}{V_R}$$

$$V_R = 5 \text{ km/h}$$

18. (*a*)



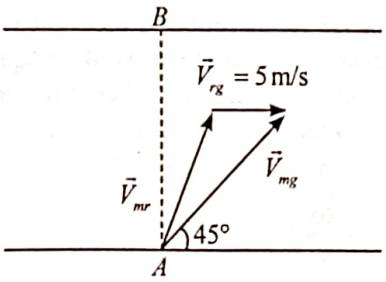
$$\tan 45° = \frac{2S}{V_R}$$

$$V_R = 25 \text{ m/s}$$

19. (*c*) Let $\vec{V}_{r,g}$ = velocity of river w.r.t. ground

$\vec{V}_{m,g}$ = velocity of swimmer w.r.t. ground

$\vec{V}_{m,r}$ = velocity of swimmer w.r.t. river



Vertical component of

$$\vec{V}_{m,g}, V_y = \frac{\text{Distance}}{\text{Time}} = \frac{60}{6} = 10 \text{ m/s}$$

Since $\vec{V}_{m,g}$ makes an angle of 45° with the direction of river, its horizontal and vertical component will be same.

$$\therefore \vec{V}_{m,g} = 10\hat{i} + 10\hat{j}$$

$$\vec{V}_{r,g} = 5\hat{i}$$

Relative velocity of swimmer w.r.t. river,

$$\vec{V}_{m,r} = \vec{V}_{m,g} - \vec{V}_{r,g}$$

$$\vec{V}_{m,r} = 10\hat{i} + 10\hat{j} - 5\hat{i} = 5\hat{i} + 10\hat{j}$$

Magnitude = $5\sqrt{5}$ m/s.

20. (*a*) As the boat's velocity is perpendicular to river velocity,

Assume that, V_B = Velocity of boat = 4.5 km/hr

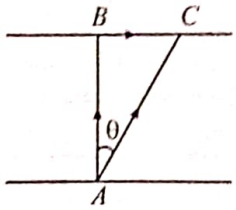
V_R = velocity of river = 6 km/hr

Resultant velocity boat^2 = $V_B^2 + V_R^2$

resultant velocity boat = $\sqrt{4.5^2 + 6^2} = 7.5$ km/hr

$$= 7.5 \times \frac{5}{18} \text{ km/hr} = 2.083 \text{ m/s} = 2.1 \text{ m/s}$$

21. (*c*) Given $\overrightarrow{AB}$ = Velocity of boat = 8 km/hr



$\overrightarrow{AC}$ = Resultant velocity of boat = 10 km/hr

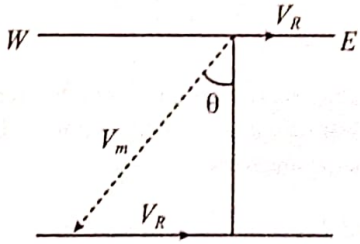
$\overrightarrow{BC}$ = Velocity of river = $\sqrt{AC^2 - AB^2}$

$= \sqrt{(10)^2 - (8)^2} = 6$ km/hr

22. (*b*) The relative velocity of boat w.r.t. water

$$= v_{boat} - v_{water} == (3\hat{i} + 4\hat{j}) - (-3\hat{i} - 4\hat{j}) = (6\hat{i} + 8\hat{j})$$

23. (*c*) For shortest possible path man should swim with an angle (90 + θ) with downstream.



From the figure

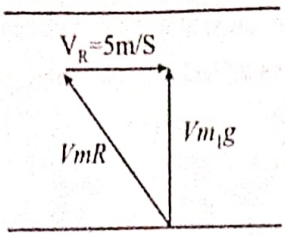
$$\sin\theta = \frac{v_r}{v_m} = \frac{5}{10} = \frac{1}{2}$$

$\therefore \theta = 30°$

So angle with downstream = 90° + 30° = 120°

24. (*a*) Due north will take him to cross in shortest time.

25. (*b*)

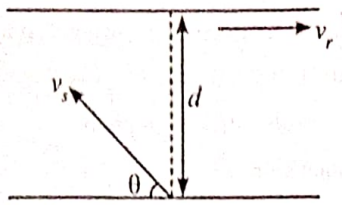


$$\frac{60m}{5} = \sqrt{v_m^2 - v_2^2}$$

$$\Rightarrow 144 + 5^2 = v_m^2$$

$$\Rightarrow v_m = 13 \text{m/s}$$

26. (*b*) The solution will be as shown in figure



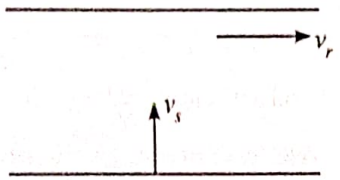
Let d be the width of river.

The velocity responsible for the swimmer to reach the other bank in shortest distance = $v_s \sin\theta$

Time taken to cross the river using shortest distance

$$t_1 = \frac{d}{v_s \sin\theta} \quad ...(i)$$

To reach the other bank in shortest possible time the velocity of swimmer should be perpendicular to flow of river.



$$\therefore \text{Time taken} = t_2 = \frac{d}{v_s} \quad ...(ii)$$

Dividing (ii) by (i); we get

$$\frac{t_2}{t_1} = \frac{d/v_s}{d/v_s \sin\theta}$$

$$\frac{t_2}{t_1} = \sin\theta$$

27. (*c*) Since he makes his strokes normal to the river current, the movement of the river will have no effect on the time of crossing the bank.

The time will be, $T = \frac{\text{Distance}}{\text{Speed}} = \frac{1}{4}\text{hr}$

28. (*b*) Let speed of swimmer and river be x and y

$x + y = 16$ ...(i)

$x - y = 8$ ...(ii)

Solving (i) and (ii)

$x = 12$ km/h

$y = 4$ km/h

29. (*c*) Let the speed of still water be y and that of the boat be x.

During the downstream journey let the boat covers a distanced d.

$(x + y) \times 1 = d$ ...(i)

During the upward journey, $(x - y) \times t = d - 6$ ...(ii)

During total time $(1 + t)$ boat travels a distance of 6km

Therefore, $y(1 + t) = 6$ ...(iii)

From equation (i), (ii) and (iii)

$y = 3$ km/h

30. (*b*) $t_0 = \frac{d}{2} + \frac{d}{2} = d$

$$t = \frac{d}{3} + \frac{d}{1} = \frac{4}{3}d \Rightarrow \frac{t}{t_0} = \frac{4}{3}$$

31. (*c*) From frame of ball, i.e., frame of water, the velocity of both boats is constant for upstream & downstream.

Let velocity of boat A w.r.t. water = $V_1 = V_{A/w}$

& velocity of boat B w.r.t water = $V_2 = V_{B/w}$

Let them move for time t_0 & then turn back by then in water frame, A has travelled $V_{A/w} \times t_0$ & B have travelled $V_{B/w} \times t_0$

Let then come back in time t_1 & t_2 respectively.

$$t_1 = \frac{V_{A/w} \times t_0}{V_{A/w}} = t_0 \Rightarrow t_2 = \frac{V_{B/w} \times t_0}{V_{B/w}} = t_0$$

Hence both are equal.

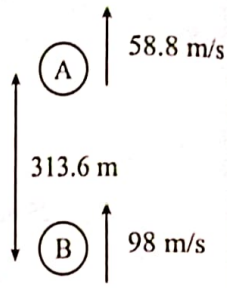
32. (*d*) At $t = 4s$

$a_{ml} = 0$

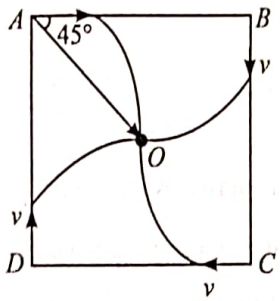
$s_{ml} = U_{nl}t$

$t = \frac{313.6}{39.2} = 8s.$

$T = t + 4 = 12s$



33. (*a*)



As the four particles have the same speed it will have to change its direction to meet each other. By the successive rotation of the shrinking squares it can be seen that they meet each other at the origin O or the centre of the square. O be the centre.

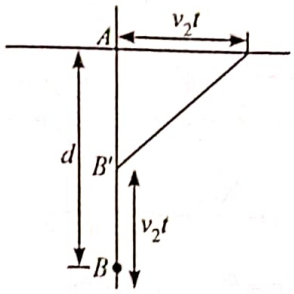
Now the net displacement = AO

$$AO = \frac{\sqrt{2}a}{2}$$

Component of velocity v along AO direction will be $v \cos 45°$

Now time taken will be, $t = \frac{AO}{v\cos(45°)} = \frac{a}{v}$

34. (*c*) $S = \sqrt{v_1^2 t^2 + (d - v_2 t)^2}$



$$\frac{ds}{dt} = 0 \qquad \text{(For max or min value)}$$

$$2v_1^2 - 2(d - v_2 t)v_2 = 0$$

$$t = \frac{v_2 d}{v_1^2 + v_2^2} \qquad \text{...(ii)}$$

Put t in equation (i)

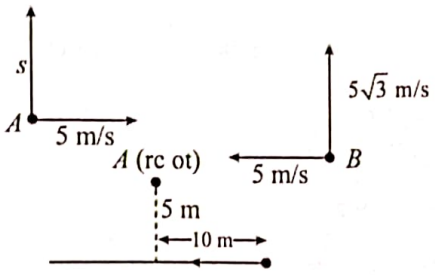
$$S_{min} = \sqrt{\frac{v_1^2 v_2^2 d^2}{(v_1^2 + v_2^2)} + \left(d - \frac{v_2 v_2 d}{v_1^2 + v_2^2}\right)^2}$$

$$S_{min} = \frac{v_1 d}{\sqrt{v_1^2 + v_2^2}}$$

35. (*b*) $V_{QP} = V_Q - V_P = 10 - 2 = 8$ m/s

$$t = \frac{\text{Distance}}{\text{Speed}} = \frac{24}{8} = 3\,\text{sec}$$

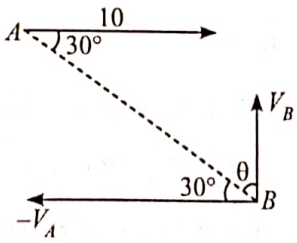
36. (*b*)



$$t = \frac{10}{10} = \text{m/s}$$

37. (*a*) V_{BA} is along AB

$\Rightarrow V_B \sin\theta = 10 \sin 30°$



$$V_B = \frac{10 \times \frac{1}{2}}{\sin\theta} = \frac{5}{\sin\theta} \Rightarrow V_{B\min} = 5$$

38. (*d*) $V_{AB} = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$

When $\theta = 180°$

Then relative velocity is maximum.

39. (*d*) Components of the relative velocity of the billiard balls are: $V_x = 1\ \text{ms}^{-1}$

$V_y = (2 - \sqrt{3})\,\text{ms}^{-1}$

$$\tan\theta = \frac{V_y}{V_x} = \frac{2-\sqrt{3}}{1} \text{ i.e. } \theta = \tan^{-1}\frac{2-\sqrt{3}}{1} = 15°$$

Exercise-2 (Learning Plus)

1. (*d*) $V_{sb} = V_s - V_s = V_s - 10$

$$V_s - 10 = \frac{1000\,\text{m}}{100\,\text{s}}$$

$V_s = 20$ m/s

2. (*a*) Speed of the police van, $v_p = 30$ km/h = 8.33 m/s

Muzzle speed of the bullet, $v_b = 150$ m/s

Speed of the thief's car, $v_t = 192$ km/h = 53.33 m/s

Resultant speed of bullet can be obtained as:

$= 150 + 8.33 = 158.33$ m/s

$v_{bt} = v_b - v_t$

$= 158.33 - 53.33 = 105$ m/s

3. (*a*) Speed of the jet airplane, $V_{jet} = 500$ km/h

Relative speed of its products of combustion with respect to the plane, $V_{smoke} - 1500$ km/h

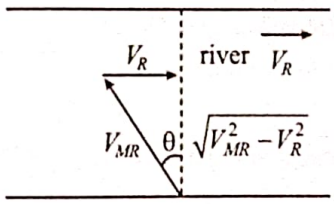
Speed of its products of combustion with respect to the ground = V'_{smoke}

Now, $V'_{smoke} = V_{smoke} + V_{jet}$

$= -1500 + 500 = -1000$ km/h

The negative sign indicates that the direction of its products of combustion is opposite to the direction of motion of the jet airplane.

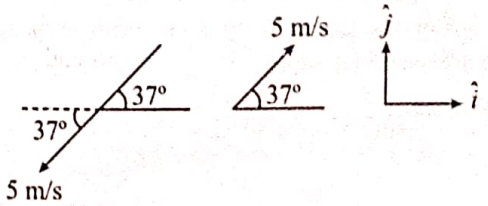
4. (*b*) 15 min = 1/4 hr.



$$t = \frac{d}{V_y} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{V_{MR}^2 - V_R^2}} = \frac{1}{4} = \frac{1}{\sqrt{5^2 - V_R^2}}$$

$\Rightarrow V_R = 3$ km/h

5. (*b*) Let velocity of rain is



$v_R = a\hat{i} - b\hat{j}$

In first case, $v_M = (-4\hat{i} - 3\hat{j})$

$\therefore v_{RM} = v_R - v_M = (a+4)\hat{i} + (-b+3)\hat{j}$

$\therefore v_{RM} = v_R - v_M = (a-4)\hat{i} + (-b-3\hat{j})$

$= -8\hat{i} + (-b-3\hat{j})$

It appears at $\theta = \tan^{-1}\left(\frac{7}{8}\right)$

$\therefore -b - 3 = -7$

$\therefore b = 4$ and speed of rain $= \sqrt{a^2+b^2} = \sqrt{32}$ m/s

6. (*a*) $\frac{dv_y}{dt} = 2t$

$\therefore v_y = t^2$ or $\frac{dy}{dt} = t^2$

or $y = \frac{t^3}{3}$ and $x = v_0 t \Rightarrow t = \frac{x}{v}$

Substituting in Eq. (i) we have,

$y = \frac{x^3}{3v_0^3}$

7. (*a*) $90 \text{ km/h} = 90 \times \frac{5}{18} = 25$ m/s

$180 \text{ km/h} = 108 \times \frac{5}{18} = 30$ m/s

At maximum separation their velocities are same.

$\therefore$ Velocity of motorcycle $= \frac{25 \text{ m}}{\text{s}}$

or at = 25

or $t = 5$ s

But thief has travelled up to 7s.

S_1 = displacement of thief

$= v_1 t_1 = 25 \times 7 = 175$ m

S_2 = displacement of motorcycle

$= \frac{1}{2} \times a_2 t_2^2 = \frac{1}{2} \times 5 \times (5)^2 = 62.5$ m

$\therefore$ Maximum separation

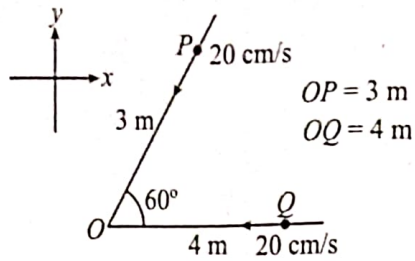
$= s_1 - s_2 = 112.5$ m Ans.

8. (*c*) Relative velocity of *A* with respect to B should be along *AB* or absolute velocity components perpendicular to *AB* should be same.

$\therefore \frac{2u}{3}\sin\theta = u \sin 30°$

$\therefore \theta = \sin^{-1}\left(\frac{3}{4}\right)$

9. (*d*) $v_Q = -20\hat{i}$



$v_p = -20 \cos 60\ \hat{i} - 20 \sin 60°\ \hat{j}$

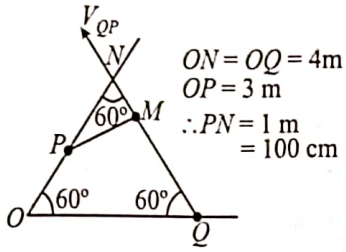
$= -10\ \hat{i} - 10\sqrt{3}\hat{j}$

Assuming *P* to be at rest,

$v_{Q'} = v_Q - v_P = -10\hat{i} + 10\sqrt{3}\hat{j}$

Now, $\therefore \tan\theta = \frac{10\sqrt{3}}{10} = \sqrt{3}$ or $\theta = 60°$

where, θ is the angle of v_{QP} from *x*-axis towards positiv *y*-axis.



Shortest distance = $PM = PN \sin 60°$

$= (100)\frac{\sqrt{3}}{2} = 50\sqrt{3}$ cm

10. (*b*) $d = d_1 + d_2$

$= \frac{v_1^2}{2a_1} + \frac{v_2^2}{2a_2}$

$= \frac{(10)^2}{2 \times 2} + \frac{(20)^2}{(2 \times 1)} = 225$ m

11. (*b*) $s_1 = s_2$

$\therefore (40)t - \frac{1}{2} \times 10 \times t^2 = (40)(t-2)$

$-\frac{1}{2} \times 10 \times (t-2)^2$

Solving this equation, we get, $t = 5$ s

Then, $s_1 = (40)(5) - \frac{1}{2} \times 10 \times (5)^2 = 75$ m

12. (*a*) Velocity of train in the direction of train is also 30 m/s. So there is no relative motion in this direction. In perpendicular direction,

$d = R = \frac{u^2 \sin 2\theta}{g}$

$= \frac{(30)^2 \sin 90°}{10} = 90$ m

13. (*c*) Relative acceleration between two is zero. Therefore, relative motion is uniform.

$u_{12} = u_1 + u_2$

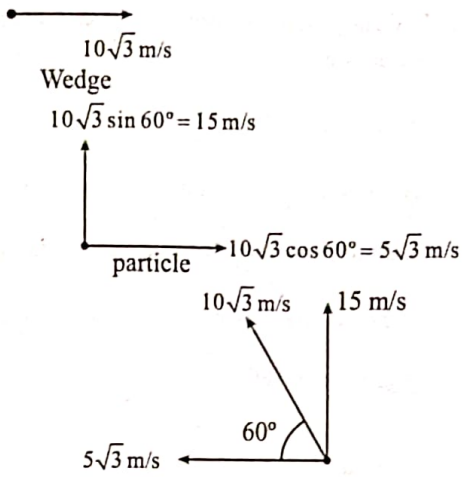
$= (20\hat{j}) - (20\cos 30\hat{i} + 20\sin 30\hat{j})$

$= (10\hat{j} - 10\sqrt{3}\hat{i})$

$\therefore |u_{12}| = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ m/s}$

$\therefore d = |u_{12}|\, t = 24 \text{ m}$

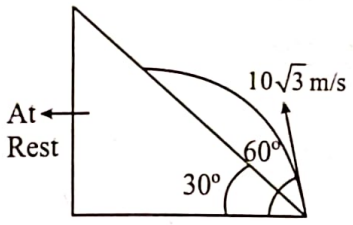
14. (*a*)



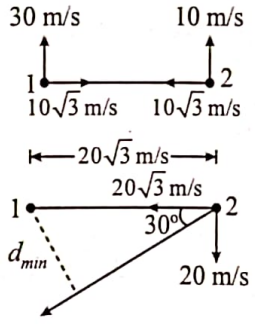
Particle with respect to wedge

$$T = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

$$= \frac{2(10\sqrt{3})\cdot\sin(60° - 30°)}{10\cos 30°} = 2s$$



15. (*b*) $a_1 = a_2 = g$ (downwards)



$\therefore a_{12} = 0$

$\therefore$ Relative motion between them is uniform.

Relative velocity v_{21}

$d_{min} = 20\sqrt{3}\sin 30°$

$= 10\sqrt{3}$m

16. (*c*) Distance travelled by first train

$$s_1 = \frac{u^2}{2a} = \frac{(15)^2}{2\times 1} = 112.5 \text{ m}$$

Distance travelled by second train

$$s_2 = \frac{(20)^2}{2\times 1} = 200 \text{ m}$$

Total distance travelled = 112.5 + 200 = 312.5 m

Distance of separation = 500 – 312.5 = 187.5 m

17. (*d*) $t = \frac{S_{rel}}{v_{rel}} \Rightarrow \frac{4}{60} = \frac{5}{30 + v_2} \Rightarrow v_2 = 45 \text{ kmh}^{-1}$

18. (*a*) $3V_A = V_B, S_{rel} = n_{rel}t \Rightarrow 100 + 60 = (V_A + V_B)\times 4$

Solve to get $V_A = 10 \text{ ms}^{-1}$ and $V_B = 30 \text{ ms}^{-1}$

19. (*d*) Relative speed of trains = 37.5 + 37.5 = 75 kmh^{-1}

Time taken by the trains to meet = 90/75 = 6/5 h

Since speed of bird = 60 kmh^{-1}, distance travelled by the bird = 60 × 6/5 = 72 km

20. (*c*) For no collision, the speed of car *A* should be reduced to v_B before the cars meet, i.e., final relative velocity of car *A* with respect to car *B* is zero, i.e., $V_r = 0$.

Here u_r = initial relative velocity = $V_A - V_B$

Relative acceleration = $a_r = -a - 0 = -a$

Let relative displacement = s_r

Then using the equation,

$v_r^2 = u_r^2 + 2a_r s_r \quad 0^2 = (V_A - V_B)^2 - 2as_r$

or $s_r = \frac{(V_A - V_B)^2}{2a}$

For no collision, $s_r \le s$ i.e., $\frac{(V_A - V_B)^2}{2a} \le s$

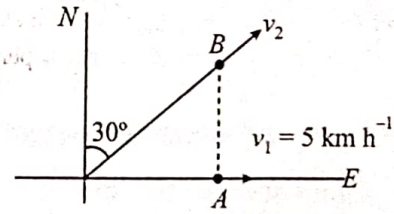
21. (*c*) If police is able to catch the dacoit after time *t*, then

$vt = x + \frac{1}{2}\alpha t^2$.

This gives $\frac{\alpha}{2}t^2 - vt + x = 0$ or $t = \frac{v \pm \sqrt{v^2 - 2\alpha x}}{\alpha}$

For *t* to be real, $v^2 \ge 2\alpha x$

22. (*a*) For *B* always to be north of *A*, the velocity components of both along east should be same.

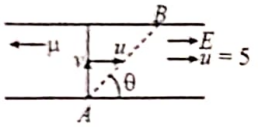


$v_2 \cos 60° = v_1$

$\Rightarrow v_2 = 10 \text{ km h}^{-1}$

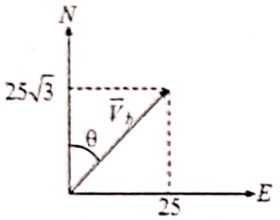
23. (*b*) Finally, he will swim along *B*.

$$\tan\theta = \frac{v}{u} = \frac{10}{5} = 2$$



$\Rightarrow \theta = \tan^{-1}(2)$ N of E.

24. (*a*)



$\vec{v}_c = 25\hat{i}, \vec{v}_{b/c} = 25\sqrt{3}\hat{j}$

$\vec{V}_{b/c} = \vec{V}_b - \vec{V}_c \Rightarrow \vec{V}_b = \vec{V}_{b/c} + \vec{V}_c \Rightarrow \vec{v}_b = 25\hat{i} + 25\sqrt{3}\hat{j}$

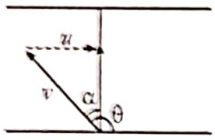
$|\vec{v}_b| = \sqrt{25^2 + (25\sqrt{3})^2} = 50 \text{ km h}^{-1}$

$\tan\theta = \frac{25}{25\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

25. (*a*) Shortest time $= \frac{d}{v} = \frac{1/2}{3} = \frac{1}{6}\text{h} = 10$ min

26. (*b*) $\sin\alpha = \frac{u}{v} = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 60° \Rightarrow \theta = 90° + \alpha = 150°$

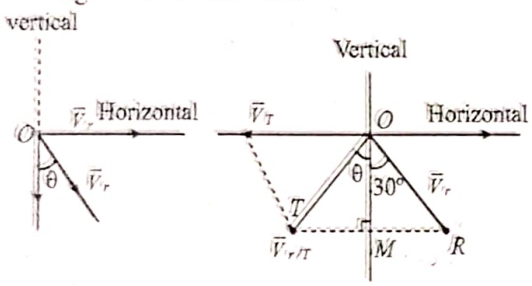


27. (*b*) Speed of train $= 108 \times \frac{5}{18} = 30 \text{ ms}^{-1}$

Let $\vec{V}_R$ and $\vec{V}_T$ represent the respective velocities of rain and train.

Now, the relative velocity of rain w.r.t. person (train) is given by $v_{R/T} = \vec{v}_R - \vec{v}_T \Rightarrow \vec{v}_R + (-\vec{v}_T)$

Let $\overline{OR}$ and $\overline{RT}$ represent the vectors, respectively, in magnitude and direction.

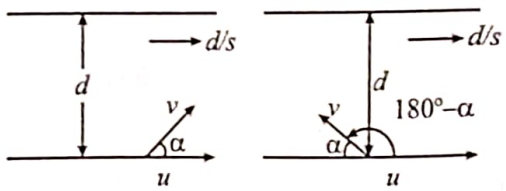


$OT^2 = OR^2 + RT^2 + 2OR \cdot RT \cos 120°$

$= 20^2 + 30^2 - 2 \times 20 \times 30 \times \frac{1}{2}$

$= 400 + 900 - 600 = 700 = \sqrt{700} \text{ ms}^{-1} = 10\sqrt{7} \text{ ms}^{-1}$

28. (*a*) Motion of the person making an angle (say α) with the downstream.



The time taken to cross the river $= \frac{d}{v\sin\alpha}$

The distance carried away downstream in the same time = speed × time.

$$x_1 = (u + v\cos\alpha)\frac{d}{v\sin\alpha} \quad \text{...(i)}$$

Motion of the person making α angle with upstream.

The time taken to cross the river is equal to $\frac{d}{v\sin\alpha}$

Distance carried away downstream in the same time

$$x_2 = [u + v\cos(180° - \alpha)]\frac{d}{v\sin\alpha}$$

$$\Rightarrow x_2 = (u - v\cos\alpha)\frac{d}{v\sin\alpha} \quad \text{...(ii)}$$

Given $\frac{(u + v\cos\alpha)\frac{d}{v\sin\alpha}}{(u - v\cos\alpha)\frac{d}{v\sin\alpha}} = \frac{2}{1}$

$\frac{(u + v\cos\alpha)}{(u - v\cos\alpha)} = \frac{2}{1} \Rightarrow 3v\cos\alpha = u$

$\Rightarrow \frac{v}{u} = \frac{\sec\alpha}{3}$

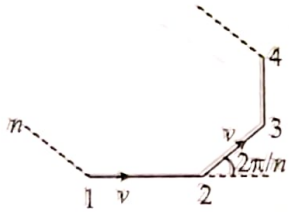
$\sec\alpha \geq 1 \Rightarrow \frac{\sec\alpha}{3} \geq \frac{1}{3}$

From equation (iii), $\frac{v}{u} \geq \frac{1}{3}$

So, v/u cannot be less than 1/3.

29. (*d*) $t = \frac{a}{v - v\cos\frac{\pi}{n}}$

Here $n = 12$



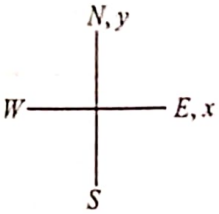
$\Rightarrow t = \frac{a}{v - v\cos\frac{2\pi}{12}} = \frac{2a}{v(2 - \sqrt{3})}$

30. (*a*) $V_{boat} = 4\hat{i} + 2\hat{j}$

$V_{wind} = 6\hat{j}$

$V_{wind/boat} = -4\hat{i} + 4\hat{j}$

$\Rightarrow$ North-West Direction



31. (*d*) $\vec{v}_{R/M} = \vec{v}_{R/G} - \vec{v}_{M/G}$

$-4j = \vec{v}_{R/G} - (2i + 3j)$

$\vec{v}_{R/G} = 2i - j$

When man starts running down $v'_{M/G} = -(2i + 3j)$

$v'_{R/M} = \vec{v}_{R/G} - \vec{v}'_{M/G}$

$= 2i - j + 2i + 3j = 4i + 2j$

32. (*a*) $\vec{v}_A = 400\ \hat{i}$ ...(i)

$\vec{v}_B = v_B \cos 30°\ \hat{i} + v_B \sin 30°\ \hat{j}$

$\vec{v}_B = \frac{\vec{v}_B\sqrt{3}}{2}\hat{i} + \frac{v_B}{2}\hat{j}$ ...(ii)

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

$\vec{v}_{B/A} = \left(\frac{v_B\sqrt{3}}{2}\hat{i} + \frac{v_B}{2}\hat{j}\right) - 400\hat{i}$

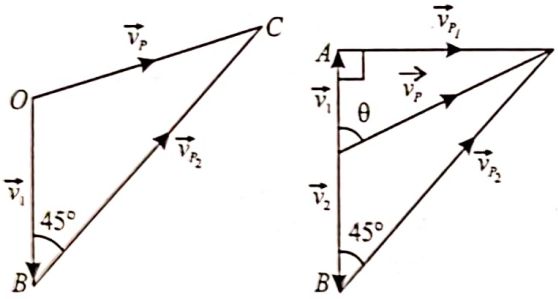
$\vec{v}_{B/A} = \left(\frac{v_B\sqrt{3}}{2} - 400\right)\hat{i} + \frac{v_B}{2}\hat{j}$

$\vec{v}_B\sqrt{3} - 800 = 0,\ v_B = \frac{800}{\sqrt{3}}$

$\vec{v}_B = 400\hat{i} + \frac{400}{\sqrt{3}}\hat{j}$

33. (*b*) Let $\vec{v}_P$, be the velocity of plane relative to the ground, at angle θ to velocity $\vec{v}_1$ of observer in car 1.

In case (i), $v_{P_1} = \vec{v}_P - \vec{v}_1$ or $\vec{v}_{P_1} = v_{P_1} + \vec{v}_1$



Vector diagram is shown in Fig. Note that according to observer in car 1 the plane crosses the road at right angles.

Similarly in case (ii)

$\vec{v}_p = v_{P_2} + \vec{v}_2$

We can combine Figures.

From the velocity diagram,

$\tan 45° = \frac{AC}{AB}$

$v_{P_1} = (v_1 + v_2)\tan 45°$

$= 120 \times 1 = 120$ km/hr

$v_p = [60^2 + 120^2]^{1/2} = 134.16$km/hr

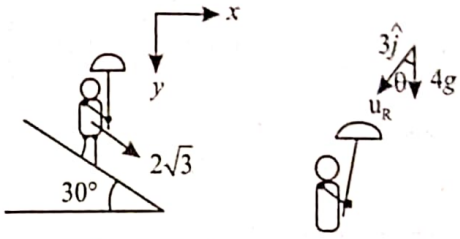
$\tan\theta = \frac{v_{P_1}}{v_1} = \frac{120}{60} = 2$

Hence $\theta = \tan^{-1} 2$

34. (*b*) $\vec{v}_{R/M} = x\hat{j} = \vec{v}_R - \vec{v}_M$

$\vec{v}_M = 2\sqrt{3}[\cos 30\hat{i} + \sin 30\hat{j}] = 3\hat{i} + \sqrt{3}\hat{j}$

$\Rightarrow \vec{v}_R = -3\hat{i} + (x - \sqrt{3})\hat{j}$



$5 = \sqrt{3^2 + (x - \sqrt{3})^2}$

$\Rightarrow 16 = (x - \sqrt{3})^2 \Rightarrow 4 + \sqrt{3} = x$

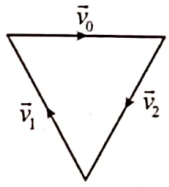
$\vec{v}_R = -3\hat{i} + 4\hat{j}$

$\tan\theta = 3/4 \Rightarrow \theta = 37°$

Exercise-3 (JEE Advanced Level)

1. (*b*) $\vec{v}_2 = \vec{v}_1 - \vec{v}_0,\ \ \vec{v}_2 + \vec{v}_0 = \vec{v}_1$

From the law of triangle, we can draw the velocity vector diagram.



From triangle property, sum of two sides ≥ third side.

So $v_1 \le v_0 + v_2$

2. (*c*) $\vec{V}_w = x\hat{i}$

$\vec{V}_A = -v_A\hat{i}$

$\vec{V}_{Aw} = (x + v_A)\hat{i}$

$\left|(\vec{V}_{Aw})\right| = x + v_A \ge 300 \Rightarrow v_A \ge 300 - x$

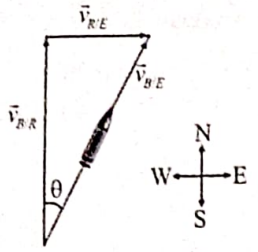
x varies between 0 to 30 km/hr means $v_A \ge 300$ km/hr in westward direction.

3. (*c*) $\vec{v}_{B/R} \rightarrow$ velocity of boat w.r.t. river

$\vec{v}_{R/E} \rightarrow$ velocity of river w.r.t. earth

$\vec{v}_{B/E} \rightarrow$ velocity of boat w.r.t. earth

$\vec{v}_{B/R} = \vec{v}_{B/E} - \vec{v}_{R/E}$ or $\vec{v}_{B/E} = \vec{v}_{B/R} + \vec{v}_{R/E}$



Hence, $\vec{v}_{B/E} = \sqrt{v_{B/R}^2 + v_{R/E}^2}$

$= \sqrt{10^2 + 5^2} = 11.2 \text{ km h}^{-1}$

The direction of $\vec{v}_{B/E}$ is

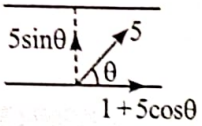
$$\theta = \tan^{-1}\left(\frac{\vec{v}_{R/E}}{\vec{v}_{B/R}}\right) = \tan^{-1}\left(\frac{5}{10}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

Note that $\vec{v}_{B/E}$ is resultant of $\vec{v}_{B/R}$ and $\vec{v}_{R/E}$ Effective velocity of boat in y-direction is $\vec{v}_{B/R}$

Hence time taken to cross the river is

$$\frac{d}{v_{B/R}} = \frac{3}{10} = 18 \text{ minute.}$$

4. (*c*) $5\sin\theta = 1 + 5\cos\theta$



$25\sin^2\theta = 1 + 25\cos^2\theta + 10\cos\theta$

$25 - 25\cos^2\theta = 1 + 25\cos^2\theta + 10\cos\theta$

$50\cos^2\theta + 10\cos\theta - 24 = 0$

$\cos\theta = 3/5$

$\theta = 53°$

5. (*a,b*) $r_B = 0$ at $t = 0$

$r_A = 3\hat{i} + 4\hat{j}$ at $t = 0$

$V_A = (-20\ \hat{j})$

$V_B = (40\cos 37°)\ \hat{i} + (40\sin 37°)\ \hat{j} = (32\hat{i} + 24\hat{j})$

$V_{AB} = (-32\hat{i} - 44\hat{j})$ km/h

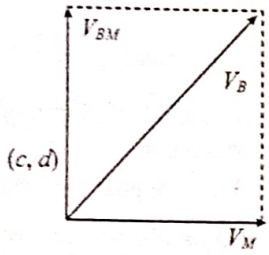
At time $t = 0$,

$r_{AB} = r_A - r_B = (3\hat{i} + 4\hat{j})$

$\therefore$ At time $t = t$,

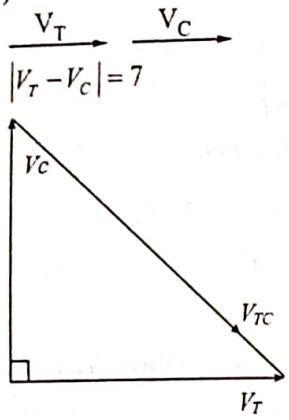
$r_{AB} = (r_{AB} \text{ at } t = 0) + v_{AB}t = (3 - 32t)\hat{i} + (4 - 44t)\hat{j}$

6. (*c,d*)



Ball will be catched by man if the cart is unaccelerated. If there is some positive accelerations, then ball will fall behind.

7. (*a,b*)

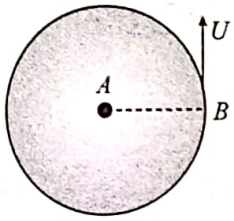


$V_T^2 + V_C = 13^2$

If $V_T = 12, V_C = 5$

If $V_T = 5,\ \ V_C = 12$

8. (*a,b,d*)



Particles are neither approaching nor separating.

But, Relative velocity is V.

$\overline{V_1} = \overline{V_B} - \overline{V_A}$ ...

$\overline{V_2} = \overline{V_C} - \overline{V_B}$ ...(

$\overline{V_0} = \overline{V_A}$ ...(

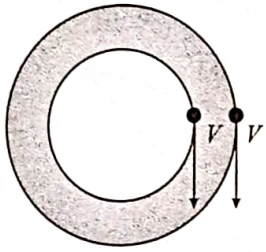
Adding (i) (ii) and (iii),

$\overline{V_c} = \overline{V_1} + \overline{V_2} + \overline{V_0}$

V_C can be zero if $\overline{V_c} = \overline{V_1} + \overline{V_2} + \overline{V_0} = 0$

Rate of separation of dog 1 w.r.t. dog 3 = 2V

Rate of separation of dog 2 w.r.t. dog 4 = 2V



At this instant $V_{rel} = 0$

They would continue to move w.r.t each other,

9. (*b,c,d*)

$V = U + at$

$-4 = 2 + a + 4$

$a = \frac{-3}{2}$ m/s^2

$S_G = V_G t + \frac{a_G t^2}{2} \Rightarrow 0 = 2t - \frac{3}{4}t^2$

$t = \frac{8}{2} = 2.66s$

Hence, option (*d*)

In 4S, $SG = 2x$

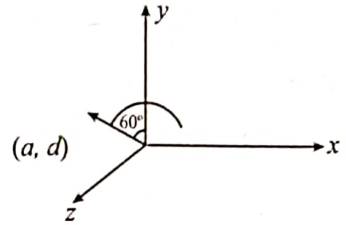
$$S_G = 2\times 4\frac{-3}{4}\times 4^2 = -4\,\text{m}$$

∴ Option (*b*)

Wrt Belt

$$S_B = U_B t + \frac{1}{2}a_B t^2 = 6\times 4 - \frac{3}{4}\times 4^2 = 12\text{m}$$

10. (*a*, *d*)



$\overline{V_R} = 2t$ (River)

$\overline{V}_{Boat} = 3t + 2t = 5t$

At $t = 0, \overline{V}_{Ball} = 5t + 5j + 5\sqrt{3}k$

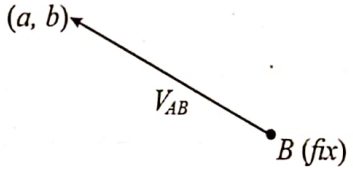
Highest Point,

$\overline{V}_{Ball} = 5t + 5j$

$\overline{V}_{Ball,\,man} = 5t$

$\left|\overline{V}_{Ball,}\right| = 5\sqrt{2}$

11. (*a*,*b*)



12. (*b*,*d*)

$\overline{V}_{s/G} = \overline{V}_{s/A} + \overline{V}_{A/G}$

Horizontal component of velocity of shell wrt ground will be in the range of 170 m/s to 260 m/s and the direction of velocity of air plane.

If air resistance is not there the shell may hit ground between *A* and *B*, or at *B*, or ahead of point *B*.

Sol. (13 & 14)

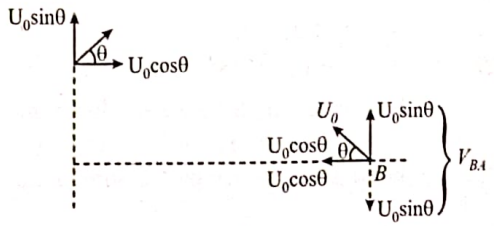
13. (*b*) **14.** (*b*)

The path of a projectile as observed by other projec tile is a straight line.

$V_A = u\cos\theta\ \hat{i} + (u\sin\theta - gt)\,\hat{j}\,.V_{AB} = (2u\cos\theta)\,\hat{i}$

$V_B = -u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}\,.$

$a_{BA} = g - g = 0$



The vertical component $u_0 \sin\theta$ will get cancelled. The relative velocity will only be horizontal which is equal to $2u_0 \cos\theta$.

Hence *B* will travel horizontally towards left w.r.t. *A* with constant speed $2u_0 \cos\theta$ and minimum distance will be *h*.

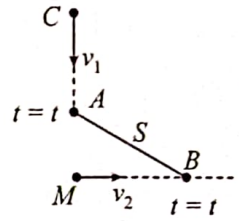
15. (*c*) Time to attain this separation will obviously be $\frac{S_{rel}}{V_{rel}} = \frac{\ell}{2u_0\cos\theta}$.

Sol. (16 to 17)

16. (*d*) **17.** (*d*)

CM = 50 m

$v_1 = 20\ \text{ms}^{-1}, v_2 = 15\ \text{ms}^{-1}, AC = v_1 t, MB = v_2 t$



$$S = \sqrt{MA^2 + MB^2} = \sqrt{(50 - v_1 t)^2 + (v_2 t)^2}$$

From $\frac{ds}{dt} = 0$ (for minima), find *t* and put *t* in *s*.

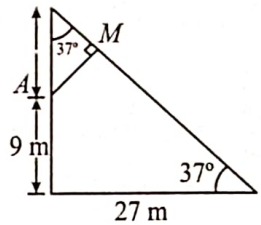
18. (a) $V_{B,A} = V_B - V_A$

$$= -14\cos 45°\hat{i} + 14\sin 45°\hat{j} - \left(2\cos 45°\hat{i}\right) - 2\sin 45\hat{j}$$

$$= -\frac{16}{\sqrt{2}}\hat{i} + \frac{12}{\sqrt{2}}\hat{j} = -8\sqrt{2}\hat{i} + 6\sqrt{2}\hat{j}$$

19. (*b*) $\tan\phi = \frac{6\sqrt{2}}{8\sqrt{2}} \Rightarrow \tan\phi = \frac{3}{4} \Rightarrow \phi = 37°$

20. (*d*)



$$\frac{x+9}{27} = \tan 37° = 3/4 \Rightarrow x = \frac{45}{4}\ \text{m}$$

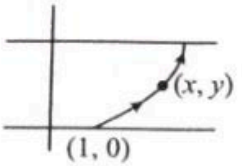
21. (*b*) $a = 5t \Rightarrow \frac{dv_y}{dt} = 5t \Rightarrow v_y = \frac{5t^2}{2}$

$$\Rightarrow \frac{dy}{dt} = \frac{5}{2}t^2 \Rightarrow y = \frac{5}{6}t^3$$

Putting $y = d$,

we get $d = \frac{5}{2}t^3 \Rightarrow t = \left(\frac{6d}{5}\right)^{1/3}$

22. (*c*) Let at any time the position coordinates of boat be (x, y).

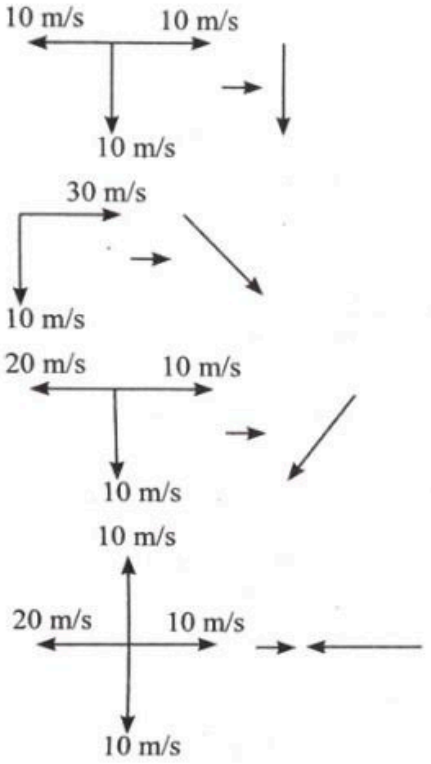


$$y = \frac{5}{6}t^3, x - 1 = ut$$

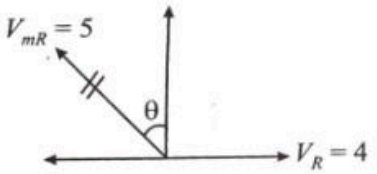
$$\Rightarrow x - 1 = u\left(\frac{6y}{5}\right)^{1/3}$$

23. (*a*) $y = d/2$, drift: $x - 1 = u(3d/5)^{1/3}$

24. (*a*)



25. (*b*)

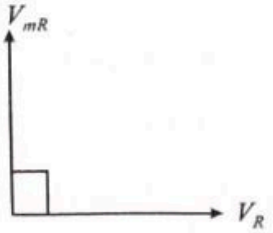


(*a*) $V_{mR} \sin\theta = V_R$

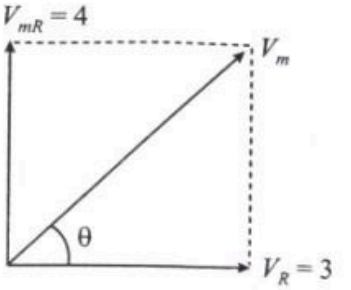
$\Rightarrow \sin\theta = \frac{4}{5} \Rightarrow \theta = 53°$

$\Rightarrow \theta + 90 = 143°$

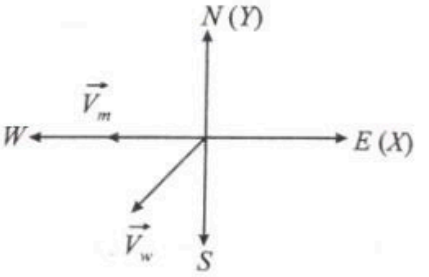
(*b*) Direction of velocity with the direction of flow = 90°

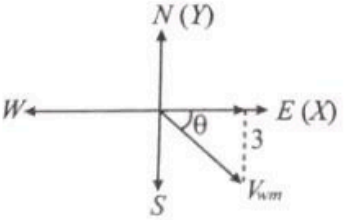


(*c*) $\tan\theta = \frac{4}{3} \Rightarrow \theta = 53°$



(*d*) $\vec{V}_w = -3\hat{i} - 3\hat{j} \Rightarrow \vec{V}_m = -7\hat{i}$





$\vec{V}_{wm} = \vec{V}_w - \vec{V}_m$

$= -3\hat{i} - 3\hat{j} + 7\hat{i} = 4\hat{i} - 3\hat{j}$

$\tan\theta = \frac{3}{4} \Rightarrow \theta = 37° \quad \Rightarrow \theta + 90 = 127°$

26. (*a*) - P, Q, R; (*b*) - Q, R, S; (*c*) - Q,R,S

P. For time period same,

$u_1 \sin\theta_1 = u_2 \sin\theta_2 \Rightarrow \frac{u_1}{u_2} = \frac{\sin\theta_2}{\sin\theta_1}$

So, $u_1 = u_2$ if $\theta_1 = \theta_2$

$u_1 > u_2$ if $\theta_2 > \theta_1$

$u_1 > u_2$ if $\theta_2 > \theta_1 \Rightarrow P \rightarrow A$

R. For time to be same

$u_1 \cos\theta_1 = u_2 \cos\theta_2$

$\frac{u_1}{u_2} = \frac{\cos\theta_2}{\cos\theta_1}$

$\Rightarrow u_1 = u_2$ if $\theta_2 = \theta_1$

$u_1 > u_2$ if $\theta_2 < \theta_1$

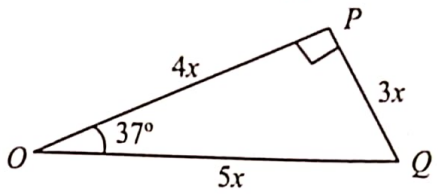
$u_1 < u_2$ if $\theta_2 > \theta_1$

27. (*a*) A-(q); B-(q); C-(q); D-(q)

In all cases, angle between velocity and net force (in the frame of observer) is in between 0° and 180° (excluding both values, in that path is straight line).

28. [0005]

Bird (a) travels $7x$ and Bird (b) travels $5x$ in same time,

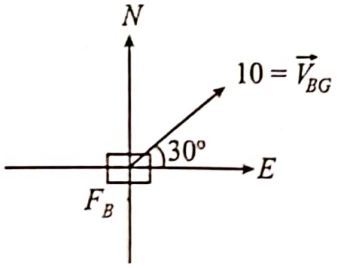


so $\frac{v_2}{v_1} = \frac{s_2}{s_1} = \frac{5x}{7x}$

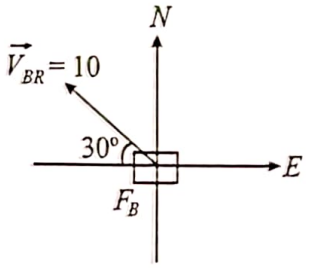
or $v_2 = \frac{5}{7}(v_1) = 5$ km/hr

29. [0017]

Ground frame:



River frame:



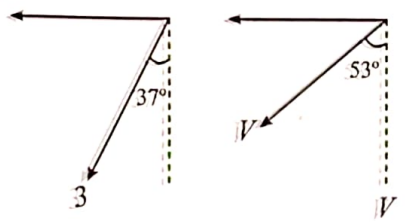
$\vec{V}_{BG} = \vec{V}_{BR} + \vec{V}_{RG}$

$\Rightarrow\ 5\sqrt{3}\hat{i} + 5\hat{j} = -5\sqrt{3}\hat{i} + 5\hat{j} + \vec{V}_{RG}$

$\therefore\ \vec{V}_{RG} = 10\sqrt{3}\hat{i}$

$\therefore\ V_{RG} = 17.3$ m/s

30. [0007]



$\frac{3V}{5} = \frac{12}{5}$

$V = 4$

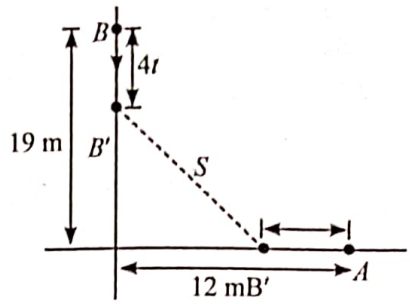
$\frac{4V}{5} = \frac{9}{5} + V_C = \frac{7}{5}$

$V_C = 0 + a_C t$

$a_C = 7$

31. [5,2]

Separation, $S = \sqrt{(12-2t)^2 + (19+4t)^2}$ ...(i)



For minimum separation $\frac{dS}{dt} = 0$

$2(12-2t)(-2) + 2(19-4t)(-4) = 0$

$t = 5$ sec ...(ii)

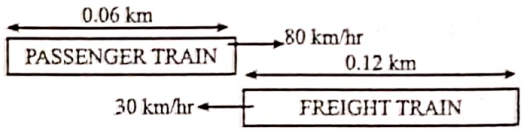
Put (ii) and (i)

$S_{min} = \sqrt{4+1} = \sqrt{5} \approx 2$ m (nearest integer)

Exercise-4 (Past Year Questions)

JEE MAIN

1. (a)



Time taken if both moving in same direction

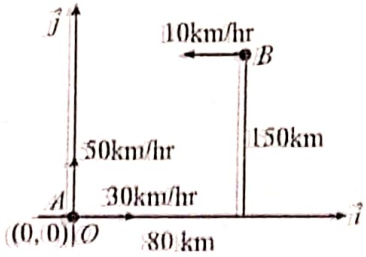
$t_1 = \frac{\text{distance}}{\text{speed}} = \frac{0.12+0.06}{80-30} = \frac{0.18}{50}$

Time taken if moving in opposite direction.

$t_2 = \frac{\text{distance}}{\text{speed}} = \frac{0.12+0.06}{80+30} = \frac{0.18}{110}$

$\frac{t_1}{t_2} = \frac{11}{5}$

2. (d) If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as:



$\vec{v}_A = (30\hat{i} + 50\hat{j})$ km/hr

$\vec{v}_B = -10\hat{i}$ km/hr

$\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = (80\hat{i} + 150\hat{j})$ km

Time after which distance between them will be minimum

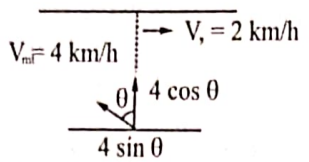
$t = -\frac{\vec{r}_{BA}\cdot\vec{v}_{BA}}{|\vec{v}_{BA}|^2}$; where $\vec{r}_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{v}_{BA} = -10\hat{i} - (30\hat{i} + 50\hat{j}) = (-40\hat{i} - 50\hat{j})$ = km/hr

$\therefore\ t = -\frac{(80\hat{i} + 150\hat{j})\cdot(-40\hat{i} - 50\hat{j})}{|-40\hat{i} + 50\hat{j}|^2}$

$= \frac{3200 + 7500}{4100} hr = \frac{10700}{4100} hr = 2.6$ hrs

3. (*d*)



For swimmer to cross the river straight

$\Rightarrow 4\sin\theta = 2$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

So, angle with direction of river flow = $90^\circ + \theta = 120^\circ$

Option (*d*)

4. [580]

$X_1 = -3t^2 + 8t + 10$

$\vec{v}_1 = (-6t + 8)\hat{i} = 2\hat{i}$

$Y_2 = 5 - 8t^3$

$\sqrt{v} = |\vec{v}_2 - \vec{v}_1| = |-24\hat{j} - 2\hat{i}|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

$v = 580$

5. (*c*) Given $\frac{v'}{v} = \tan 60^\circ$

$\frac{v'}{v(1+\beta)} = \tan 45^\circ$

From (i) and (ii)

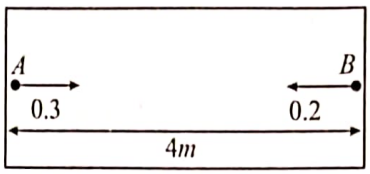
$\beta = 0.73$

6. (*c*) Relative velocity of bomb w.r.t. observer in plane = 0.

Bomb will fall down vertically So, it will move straight line w.r.t. Observer.

JEE ADVANCED

7. [2]



consider motion of two balls with respect to rocket

Maximum distance of ball A from left wall ≈ 0.02 m

so collision of two balls will take place very near to left wall

For *B*

$S = ut + \frac{1}{2}at^2$

$-4 = -0.2t - \left(\frac{1}{2}\right)2t^2$

$\Rightarrow t^2 + 0.2t - 4 = 0$

$\Rightarrow t = \frac{-0.2 \pm \sqrt{0.04 + 16}}{2} = 1.9$

Nearest integer = 2*s*

CHAPTER

6 Newton's Laws of Motion

Exercise-1 (Topicwise)

1. (*c*) From law of inertia.
2. (*c*) From law of inertia.
3. (*b*) From law of inertia
4. (*b*) Horizontal velocity of apple will remain same but due to retardation of train, velocity of train and hence velocity of boy w.r.t. ground decreases, so apple falls away from the hand of boy in the direction of motion of the train.
5. (*c*) Acceleration $a = \frac{F}{m} = \frac{100}{5} = 20 \text{ cm/s}^2$

   Now $v = at = 20 \times 10 = 200$ cm/s
6. (*b*) u = velocity of bullet

   $\frac{dm}{dt}$ = Mass thrown per second by the machine gun

   = Mass of bullet × Number ofs bullet fired per second

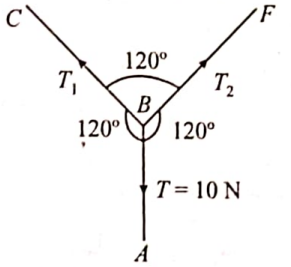
   = 10 g × 10 bullet/sec = 100 g/sec = 0.1 kg/sec

   $\therefore$ Thrust = $\frac{udm}{dt}$ = 500 × 0.1 = 50 N
7. (*b*) Force on particle at 20 *cm* away $F = kx$

   $F = 15 \times 0.2 = 3$ N [As $k = 15$ N/m]

   Acceleration = $\frac{Force}{Mass} = \frac{3}{0.3} = 10 \ m/s^2$
8. (*c*) Thrust $F = u\left(\frac{dm}{dt}\right) = 5 \times 10^4 \times 40 = 2 \times 1 \times 10^6 \ N$
9. (*a*) $m = \frac{F}{\alpha} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = \sqrt{200} = 10\sqrt{2}$ kg.
10. (*d*) Force

    $= m\left(\frac{dv}{dt}\right) = \frac{0.25 \times [(10) - (-10)]}{0.01} = 25 \times 20 = 500$ N.
11. (*b*) $F = ma = \frac{m(u - v)}{t} = \frac{2 \times (8 - 0)}{4} = 4 \ N.$
12. (*c*) By drawing the free body diagram of point *B*



    Let the tension in the section *BC* and *BF* are T_1 and T_2 respectively.

    From Lami's theorem

    $\frac{T_1}{\sin 120°} = \frac{T_2}{\sin 120°} = \frac{T}{\sin 120°}$

    $\Rightarrow T = T_1 = T_2 = 10 \ N.$
13. (*b*) $\vec{p} = m\vec{v}$
14. (*a*) For exerted by ball on wall

    = rate of change in momentum of ball

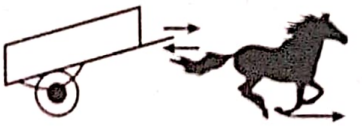
    $= \frac{mv - (-mv)}{t} = \frac{2mu}{t}.$
15. (*a*) Given that $\vec{p} = p_x\hat{i} + p_y\hat{j} = 2\cos t\hat{i} + 2\sin t\hat{j}$

    $\therefore \vec{F} = \frac{d\vec{p}}{dt} = -2\sin t\hat{i} + 2\cos t\hat{j}$

    Now, $\vec{F}.\vec{p} = 0$ i.e. angle between $\vec{F}$ *and* $\vec{p}$ is 90°.
16. (*c*) Swimming is a result of pushing water in the opposite direction of the motion.
17. (*c*)
18. (*a*) Change in momentum = Impulse

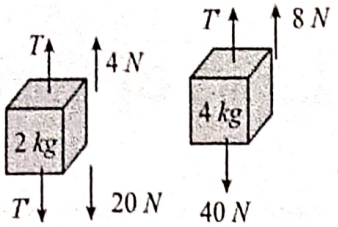
    $\Rightarrow \Delta p = F \times \Delta t \Rightarrow \Delta t = \frac{\Delta p}{F} = \frac{125}{250} = 0.5$ sec.
19. (*a*) All forces are balanced, $F = 0$, so $a = 0$. ($a = F/m$)

    If there's no acceleration, then the velocity is constant.
20. (*b*) Horse apply force on ground and reaction from ground helps horse to move forward.



21. (*b*) $F=\sqrt{(F)^2+(F)^2+2F.F\cos\theta}\Rightarrow\theta=120^\circ$.

22. (*a*) FBD of mass 2 kg FBD of mass 4 kg



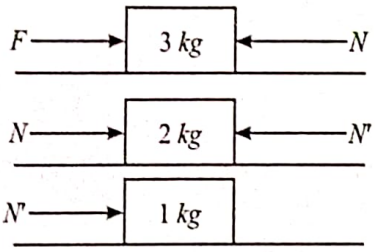
$T-T'-20=4$ ...(i)

$T'-40=8$ ...(ii)

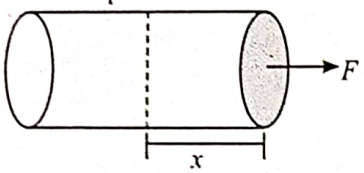
By solving (i) and (ii) $T'=47.23$ N and $T=70.8$ N

23. (*a*) Forces acting on each block is different. (as shown in FBD)

But, they will all move together as a system, therefore they will have the same acceleration



24. (*b*) Let mass of rope = m



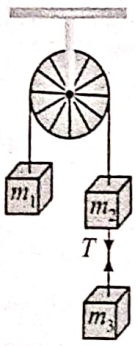
Mass of part $L-x=\frac{m}{L}(L-x)$

As this part has same acceleration

$$T=ma=\frac{M}{L}(L-x)\left(\frac{F}{m}\right)$$

$$T=\frac{(L-x)F}{L}$$

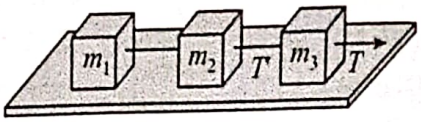
25. (*b*) Tension between m_2 and m_3 is given by



$$T=\frac{2m_1m_3}{m_1+m_2+m_3}\times g=\frac{2\times2\times2}{2+2+2}\times9.8=13\text{N}.$$

26. (*b*) $a=\frac{m_2}{m_1+m_2}\times g=\frac{5}{4+5}\times9.8=\frac{49}{9}=5.44$ m/s^2

27. (*c*)

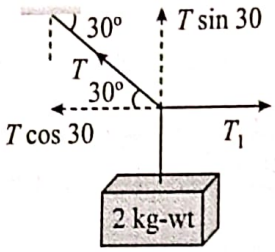


$F=ma$

$$T'=(m_1+m_2)\times\frac{T}{m_1+m_2+m_3}.$$

28. (*d*) $T=\frac{2m_1m_2}{m_1+m_2}g=\frac{2\times10\times6}{10+6}\times9.8=73.5$ N.

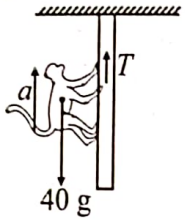
29. (*c*) $T\sin30=2$ kg wt



$\Rightarrow T=4$ kg wt

$T_1=T\cos30^\circ=4\cos30^\circ=2\sqrt{3}$.

30. (*a*) $40a=T-40g$



$T=40a+40g$

$a=6$ m/s^2

and $g=10$ m/s^2

$T=640$ N (Rope will break

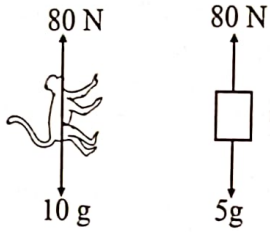
31. (*d*) For monkey $80-100=10a$

$a=-2$ m/s^2

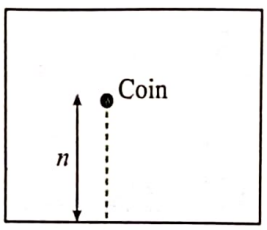
For block $80-50=5a'$

$a'=6$ m/s^2 (upward

$a_r=6+2=8$ m/s^2

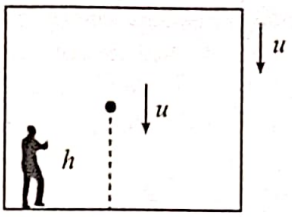


32. (*a*)



At rest

$h=\frac{1}{2}gt_1^2$ ...(i)



Relative to lift:

$u_{rel} = o\ a_{rel} = g - o = g$

$s_{rel} = u_{rel}\, t_2 + 1/2\, a_{rel}\, t_2^2$

$s_{rel} = h$

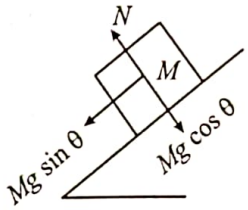
$\therefore h = \frac{1}{2} g t_2^2$ ...(ii)

By (i) and (ii),

$\frac{1}{2} g t_1^2 = \frac{1}{2} g t_2^2$

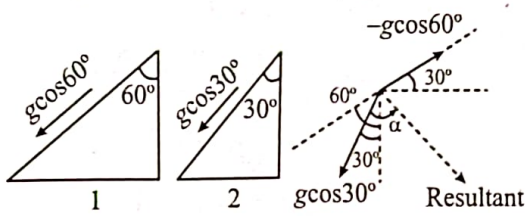
$\Rightarrow [t_1 = t_2]$

33. (*c*)



$N = Mg \cos\theta \rightarrow$ force exerted by plane on the block.

34. (*a*)



Resultant Acceleration

$$= \sqrt{\left(\frac{g}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} g\right)^2 + \frac{1}{2}.\frac{g}{2}.\frac{\sqrt{3}}{2} g \cos 150^\circ} = \frac{g}{2}$$

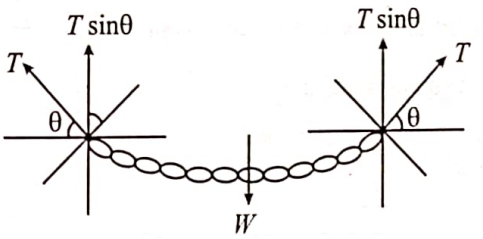
$$\tan\alpha = \frac{\frac{g}{2}.\sin 150^\circ}{\frac{\sqrt{3}g}{2} + \frac{g}{2}.\cos 150^\circ} = \frac{1}{\sqrt{3}}$$

$\alpha = 30^\circ$ i.e. Resulting acceleration is in vertical direction.

35. (*a*) From constrained $+2 - v_B - v_B + 1 = 0$

$v_B = 3/2$ m/s

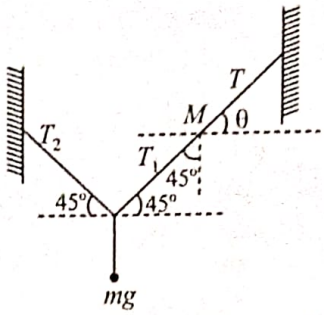
36. (*a*)



$2T \sin\theta = W$

$T = \frac{W}{2} \operatorname{cosec}\theta$

37. (*a*)



$T_1 \cos 45^\circ = T_2 \cos 45^\circ$

$\Rightarrow T_1 = T_2$

$(T_1 + T_2) \sin 45^\circ = mg$

$\sqrt{2}\, T_1 = mg$

$T_1 = \frac{mg}{\sqrt{2}}$.

$T \sin\theta = mg + \frac{T_1}{\sqrt{2}}$

$T \sin\theta = mg + \frac{mg}{2}$ ...(i)

$T \cos\theta = \frac{T_1}{\sqrt{2}} = \frac{mg}{2}$ ...(ii)

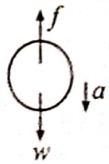
Dividing (i) and (ii)

$\tan\theta = \frac{M + m/2}{m/2} = 1 + \frac{2M}{m}$

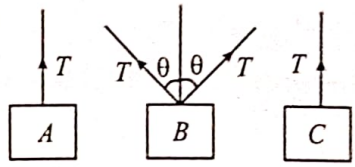
38. (*c*) $w - f = ma \qquad w - ma = g$

$w\left\{1 - \frac{m}{w} a\right\} = f \quad w\left\{1 - \frac{m}{mg} a\right\}$

$= f \quad w\left\{1 - \frac{a}{g}\right\} = f$



39. (*b*)



$T = mg$

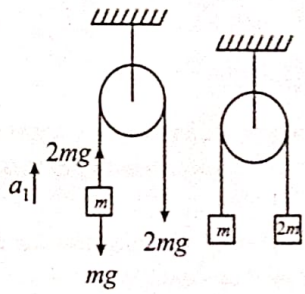
$2T \cos\theta = Mg$

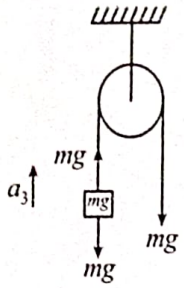
$2\, mg \cos\theta = Mg$

$\cos\theta = \frac{M}{2m} < 1$

$M < 2m$

40. (*b*)





$$a_1 = \frac{2mg - mg}{m}, a_2 = \frac{(2m-m)g}{2m+m} \quad a_3 = 0$$

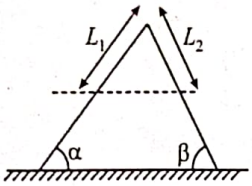
$$a_1 = g \qquad a_2 = \frac{g}{3}$$

So, $a_1 > a_2 > a_3$

41. (*a*) Let L_1 and L_2 be the portions (of length) of rope on left and right surface of wedge as shown

$\therefore$ Magnitude of acceleration of rope

$$a = \frac{\frac{M}{L}[L_1 \sin\alpha - L_2 \sin\beta]g}{M} = 0 (\because L_1 \sin\alpha = L_2 \sin\beta)$$



42. (*b*) $F_1 t_1 = F_2 t_2$

43. (*b*) Let acceleration produce by air is '*a*' than for upward motion

Net acceleration $= g - a$

so $t_1 = \sqrt{\frac{2H}{(g-a)}}$

For downward motion

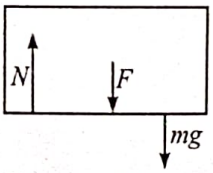
Net acceleration $= g + a$

So $t_2 = \sqrt{\frac{2H}{(g+a)}}$

Now $t_1 > t_2$

44. (*a*) For jumping he presses the spring platform, so the reading of spring balance increases first and finally it becomes zero.

45. (*c*)



$F = ma = 1\text{N}$

$N = F + mg = 26\text{ N}$

46. (*c*) 5N force will not produce any tension in spring without support of other 5N force. So here the tension in the spring will be 5N only.

47. (*c*) As the spring balances are massless therefore the reading of both balance should be equal.

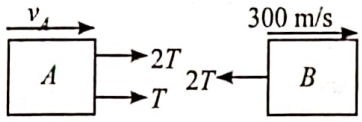
48. (*b*) If pulley covered a distance of x in any time interval then the block covered the distance of $2x$ in the same time interval. so, the velocity of the block will be double of the velocity of the pulley.

49. (*d*) $v_{P1} = \frac{-20+10}{2} = -5\text{ m/s}$

$$v_{P1} = \frac{V_{P1} + V_c}{2} \Rightarrow 10 = \frac{-5 + V_c}{2}$$

$v_c = 25\text{ m/s}$

50. (*a*) $3TV_A - 2TV_B = 0$



$$v_A = \frac{2}{3}v_B = \frac{2}{3} \times 300 = 200\text{ m/s}$$

51. (*b*) $-v_A - v_A - v_A + v_B = 0$

From constrained

$-5 - 5 - 5 + v_B = 0$

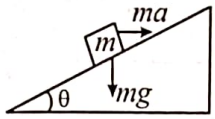
$v_B = 15\text{ m/s} \downarrow$

52. (*b*) $ma\cos\theta = mg\sin\theta$

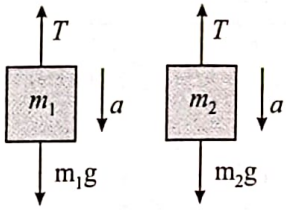
$a = g\tan\theta$

$N = mg\cos\theta + ma\sin\theta$

$$= mg\cos\theta + \frac{mg\sin^2\theta}{\cos\theta} = \frac{mg}{\cos\theta}$$



53. (*c*)



$T - m_1g = m_1a$ ...(i)

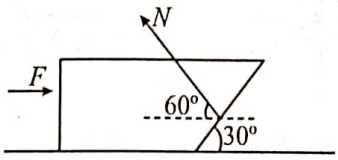
$m_2g - T = m_2a$ ...(ii)

On solving equation (i) and (ii)

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

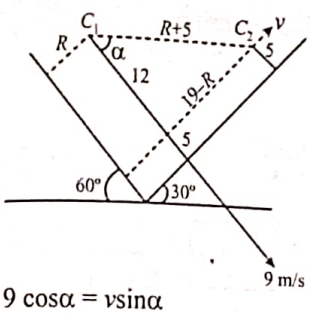
54. (*d*) Acceleration of two mass system is $a = \frac{F}{2m}$ leftward

FBD of block *A*



$N\cos 60° - F = ma = \frac{mF}{2m}$ solving $N = 3F$

55. (*b*)



$9\cos\alpha = v\sin\alpha$ ...(i)

$\frac{19-R}{12} = \tan\alpha$ ...(ii)

$(R+5)^2 = (12)^2 + (19-R)^2$ [Pythagoras theorem]

$\Rightarrow R = 10$

Hence from (i) and (ii)

$v = 12$ m/s

56. (*b*) Let $AB = \ell$, $B = (x, y)$

$\vec{v}_B = v_x\hat{i} + v_y\hat{j}$

$\vec{v}_B = \sqrt{3}\hat{i} + v_y\hat{j}$ ...(i)

$x^2 + y^2 = \ell^2$

$2x\,v_x + 2y\,v_y = 0 \Rightarrow \sqrt{3} + \frac{y}{x}v_y = 0$

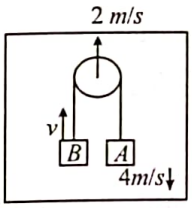
$\Rightarrow \sqrt{3} + (\tan 60^0)\,v_y = 0 \Rightarrow v_y = -1$

Hence from (i)

$\vec{v}_B = \sqrt{3}\hat{i} - \hat{j}$

Hence, $v_B = 2$ m/s

57. (*d*)

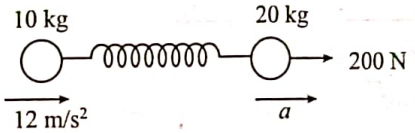


V = (velocity of B w.r.t ground)

$\frac{V-4}{2} = 2$ $V = 8$ m/s (velocity of B w.r.t ground)

$V' = 6$ m/s (velocity of B w.r.t lift)

58. (*b*)

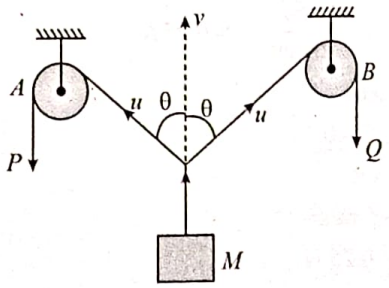


$F = m_1a_1 + m_2a_2$ [Newton's law for system]

$200 = 10 \times 12 + 20 \times a$

$a = 4$ m/s².

59. (*d*)



From constrained

The resultant velocity of the Block M is v in vertical direction.

So component of 'v' is direction of u is

$v\cos\theta = u$

$\Rightarrow v = \frac{u}{\cos\theta}$

60. (*b*) When elevator moves downward with acceleration a then $W = m(g - a)$

61. (*d*) As the apparent weight increase therefore we can say that acceleration of the lift is in upward direction.

$R = m(g + a) \Rightarrow 4.8\,g = 4\,(g + a)$

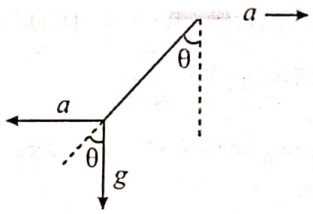
$\Rightarrow a = 0.2g = 1.96$ m/s²

62. (*a*) $T = m(g + a) = 500\,(10 + 2) = 6000$ N

63. (*b*) Rate of flow will be more when lift will move in upward direction with some acceleration because the net downward pull will be more and vice-versa.

$F_{upward} = m(g + a)$ and $F_{downward} = m(g - a)$ F_{upward}

64. (*a*) When car moves towards right with acceleration a then due to pseudo force the plumb line will tilt in backward direction making an angle q with vertical.



From the figure,

$\tan\theta = a/g$

$\theta = \tan^{-1}(a/g)$.

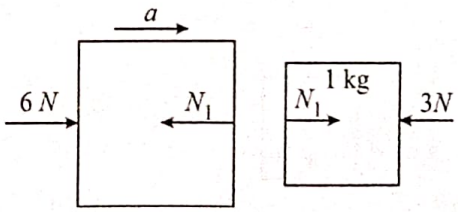
Exercise-2 (Learning Plus)

1. (*d*) Experimental fact.

2. (*c*) Force exerted by string is always along the string and of pull type.

When there is a contact between a point and a surface the normal reaction is perpendicular to the surface and of push type.

3. (*c*)



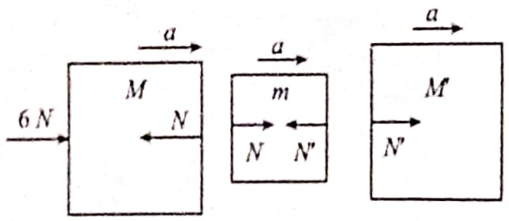
Both blocks are constrained to move with same acceleration.

$6 - N_1 = 2a$ [Newtons II law for 2 *kg* block]

$N_1 - 3 = a$ [Newtons II law for 1 *kg* block]

$\Rightarrow N_1 = 4$ Newton

4. (*b*)



$F - N = Ma$ [Newtons law for block of mass M]

$N - N' = ma$ [Newtons law for block of mass m]

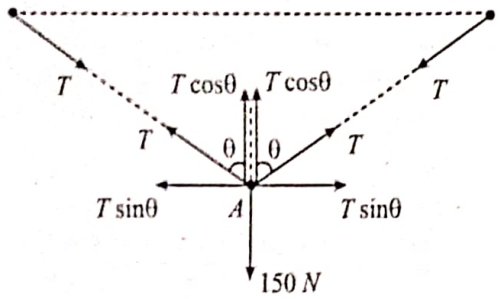
$N' = M'a$ [Newtons law for block of mass M']

$$\Rightarrow N' = M' \frac{F}{M + m + M'}$$

$$N = (m + M') \frac{F}{M + m + M'} \Rightarrow N > N'$$

5. (*b*) In free fall gravitation force acts.

6. (*d*)



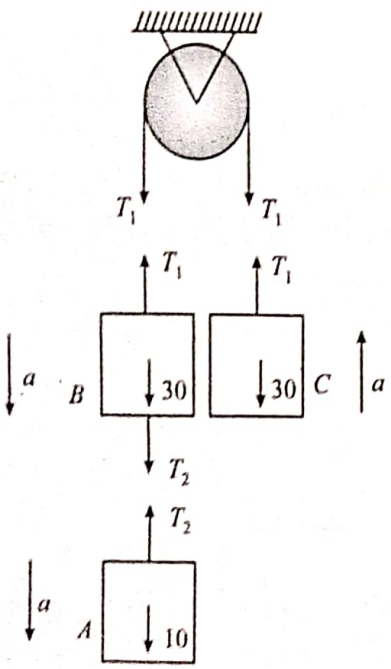
$T\cos\theta + T\cos\theta - 150 = 0$ [Equilibrium of point A]

$$2T\cos\theta = 150 \quad T = \frac{75}{\cos\theta}$$

When string become straight θ becomes 90°

$\Rightarrow T = \infty$

7. (*c*)



$10 - T_2 = 1a$ [Newton's II law for A]

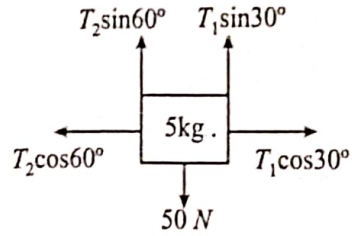
$T_2 + 30 - T_1 = 3a$ [Newton's II law for B]

$T_1 - 30 = 3a$ [Newton's II law for C]

$$\Rightarrow a = \frac{g}{7} \Rightarrow T_2 = \frac{6g}{7}$$

8. (*b*) From horizontal equilibrium

$$\frac{T_2}{2} = \frac{T_1\sqrt{3}}{2}$$



$T_2 = \sqrt{3}T_1$

From vertical equilibrium

$$\frac{T_2\sqrt{3}}{2} + \frac{T_1}{2} = 50$$

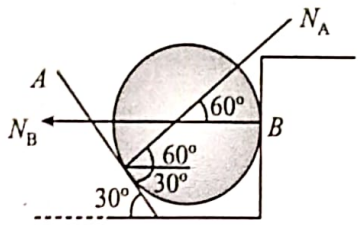
$\Rightarrow T_1 = 25N$

$T_2 = 25\sqrt{3}N$

9. (*b*) Component of force in y direction is

$N_A \sin 60° = 500$

$$N_A = \frac{1000}{\sqrt{3}}$$

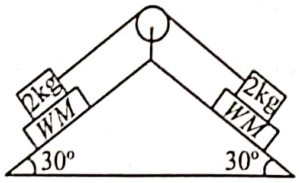


Component of force in x direction is

$N_A \cos 60° = N_B$

$$\Rightarrow N_B = \frac{500}{\sqrt{3}}$$

10. (*a*) Weighing machine always measure normal force



$N = 20\cos 30°$

$= 10\sqrt{3}$

11. (*a*) At $t = 2$ *sec*

$$\Rightarrow a = \frac{10}{2} = 5\ m/s^2$$

$$\text{So, } F = ma = \frac{50}{1000} \times 5 = 0.25\,N$$

At $t = 4$sec

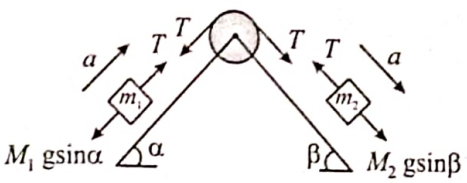
$a = 0$ so $F = 0$

At $t = 6$ sec,

$\Rightarrow a = -5\ m/s^2$

$\Rightarrow F = -0.25$ N

12. (*c*)



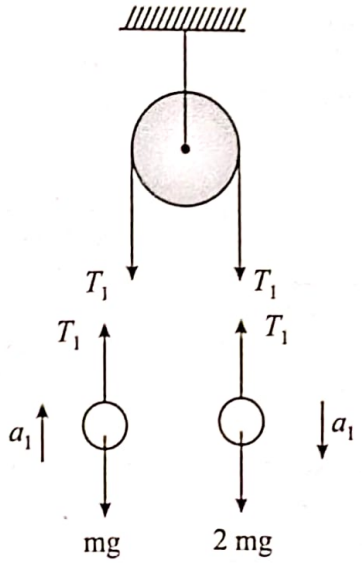
$M_2 g \sin\beta - T = M_2 a$ [Newton's II law for M_2]

$T - M_1 g \sin\alpha = M_1 a$ [Newton's II law for M_1]

By adding both equations

$$a = \left[\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right] g$$

13. (*c*) Case-I

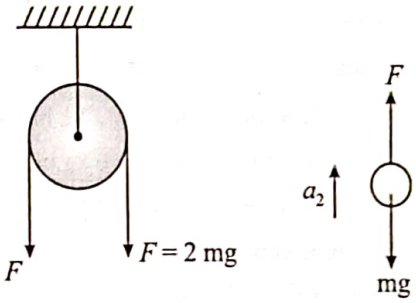


$T_1 - mg = ma_1$ [Newton's II law for m]

$2\,mg - T_1 = 2\,ma_1$ [Newton's II law for $2m$]

$\Rightarrow a_1 = \frac{g}{3}$

Case-II:



$F - mg = ma_2$ [Newton's II law for m]

$\Rightarrow 2\,mg - mg = ma_2$

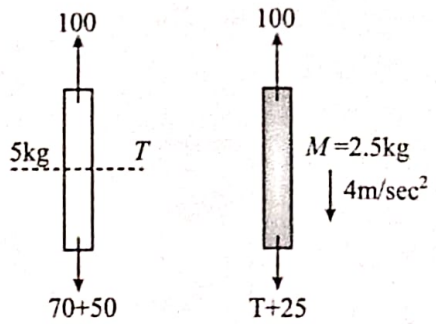
$\Rightarrow a_2 = g$

$\Rightarrow a_2 > a_1$

14. (*c*) $mg - \frac{3}{4}\,mg = ma$ [Newton's II law for man]

$\Rightarrow a = \frac{g}{4}$

15. (*b*)



$120 - 100 = 5a$ ...(i)

$a = \frac{20}{5} \Rightarrow a = 4\ \text{m/s}^2$

$T + 25 - 100 = 2.5 \times 4$ ...(ii)

$T = 85$ N

16. (*d*) $\upsilon^2 = \upsilon_0^2 + 2as$

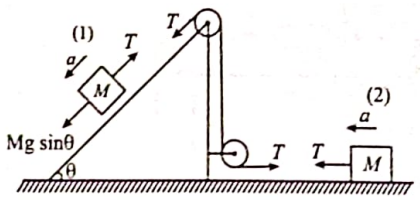
$= 1^2 + 2\frac{F}{m}x$

$x = \frac{-m}{2F}\upsilon^2 = \upsilon_0^2 + 2as$

$O^2 = 3^2 + 0 = \frac{2F^1}{m} \times 0 = 9 + \frac{2F^1}{m}\left(\frac{-m}{2F}\right)$

$\Rightarrow F_1 = 9F$

17. (*c*)



$Mg \sin\theta - T = Ma$ [Newton's II law for block 1]

$T = Ma$ [Newton's II law for block 2]

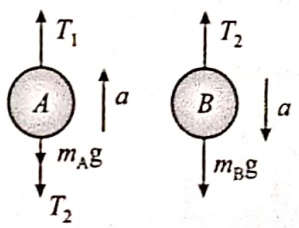
By dividing both equations

$2T = Mg \sin\theta$,

$T = \frac{Mg \sin\theta}{2}$

18. (*c*) $a = \frac{m_2 g - m_1 g}{m_1 + m_2}$

19. (*a*)



$T_1 - T_2 - m_A g = m_A a$

$T_2 - m_B g = m_B a$

$T_1 - (m_A + m_B)g = (m_A + m_B)$

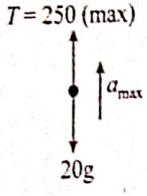
$T_2 = m_B\,(a + g)$

$\frac{T_1}{T_2} = \frac{m_A + m_B}{m_B}$

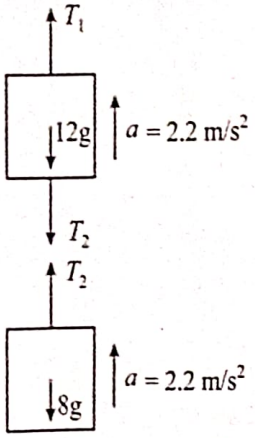
20. (*d*) $T_{max} = m_{max} g = 25 \times 10 = 250$

$250 - 200 = 20\, a_{max}$

$a_{max} = 2.5\ m/s^2$



21. (*c*)



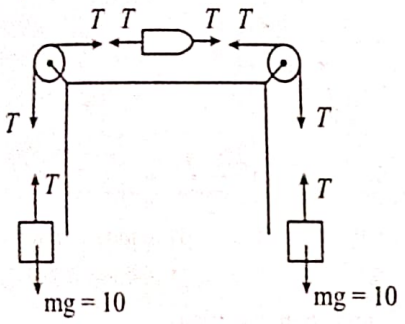
$T_2 - 8g = 8a$ [Newton's II law for 8 kg block]

$\Rightarrow T_2 = 8 \times 2.2 + 8 \times 9.8 = 96\ N$

$T_1 - 12\,g - T_2 = 12\,a \Rightarrow T_1 = 12 \times 2.2 + 12 \times 9.8 + 96$

$T_1 = 240\ N$

22. (*a*)



$T - mg = 0$ [Equilibrium of block]

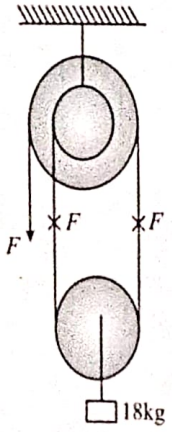
$T - 10 = 0$

$T = 10$

Reading of spring balance is same as tension in spring balance.

23. (*b*) 18 kg at rest $\Rightarrow 180 = 2F$

$F = 90\ N$



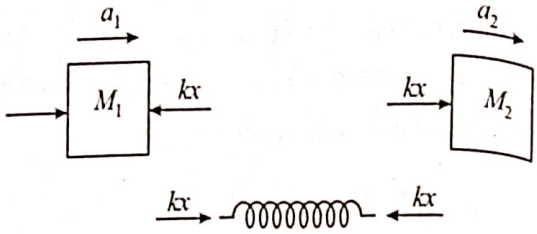
24. (*c*) $T = mg + ma$

$T = mg - ma$

$T = mg$

$a = 0$

25. (*d*)



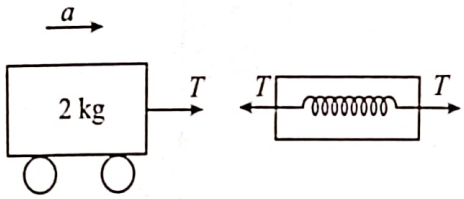
$F - kx = m_1\, a_1$ [Newton's II law for

$kx = m_2 a_2$ [Newton's II law for

By adding both equations.

$$F = m_1 a_1 + m_2 a_2 \Rightarrow a_2 = \frac{F - m_1\, a_1}{m_2}$$

26. (*c*)



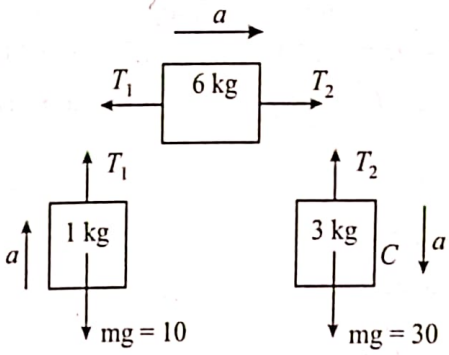
Reading of spring balance is same as tension in balance.

$\Rightarrow T = 10g = 98\ N$

$T = 2a$ [Newton's II law for 2 kg block]

$\Rightarrow a = 49\ m/s^2$

27. (*c*)



$30 - T_2 = 3a$ [Newton's II law for 3 kg block]

$T_2 - T_1 = 6a$ [Newton's II law for 6 kg block]

$T_1 - 10 = 1a$ [Newton's II law for 1 kg block]

By adding three equations

$30 - 10 = 10\,a \Rightarrow a = 2\ m/s^2$.

28. (*b*) $T = \dfrac{2m_1 m_2 g}{(m_1 + m_2)}$

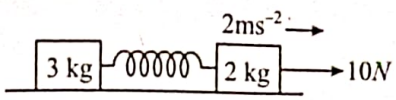
$$\Rightarrow T = \frac{2 \times 5 \times 1 \times 10}{6} = \frac{50}{3}$$

$$2T = \frac{100}{3} \approx 33.3kg$$

The spring balance reads

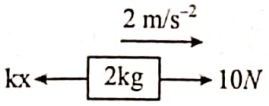
$2T = 33.33$ kg wt < 60 kg wt

29. (*b*)



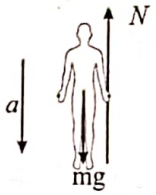
$10 - kx = 2 \times 2$

$kx = 6N$



$\therefore$ Acceleration of 3kg $= \frac{6}{3} = 2$ m/s^2

30. (*d*) Weight of man in stationary lift is mg.



$mg - N = ma$ [Newton's II law for man]

$\Rightarrow N = m(g - a)$

Weight of man in moving lift is equal to N

$$\Rightarrow \frac{mg}{m(g-a)} = \frac{3}{2} \Rightarrow a = \frac{g}{3}$$

31. (*b*) $\Delta P = 2mv$

$= 2 \times 10 \times 10^{-3} \times 5$

$= 10^{-1}$

$\Delta t = 10^{-2}$

$$F = \frac{\Delta P}{\Delta t} = \frac{10^{-1}}{10^{-2}} = 10 \text{ N}$$

32. (*c*) $v = u + a_1t_1 + a_2t_2 + a_3t_3$

33. (*d*) Maximum acceleration of block is 10 m/s^2.

$$S = \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 10 \times 0.2^2$$

$= 0.2$ m

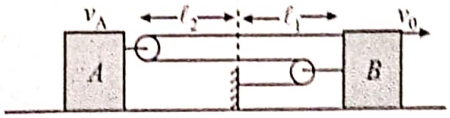
$= 20$ cm.

34. (*b*) By setting string length constant

$L = 3\ell_1 + 2\ell_2$

$\Rightarrow 3v_0 = 2v_A$

$$v_A = \frac{3}{2}v_0$$

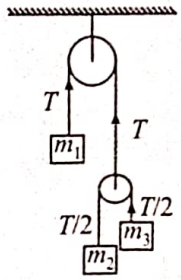


$v_{AB} = v_A - v_B$

$= \frac{v_0}{2}$ towards right.

35. (*b*) $$\frac{T}{2} = \frac{2m_2m_3}{m_2 + m_3}g$$

$$\frac{m_1g}{2} = \frac{2m_2m_3}{m_2 + m_3}g$$



$$m_1 = \frac{4m_2m_3}{m_2 + m_3}$$

$$\frac{1}{m_2} + \frac{1}{m_3} = \frac{4}{m_1}$$

36. (*a*) Length of groove $= \sqrt{3^2 + 4^2} = 5$m

acceleration along the incline $= g \sin\theta = g \sin 30^\circ = g/2$

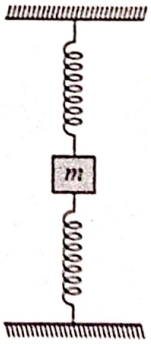
acceleration along the groove $= g/2 \cos(90 - \alpha) = g/2$

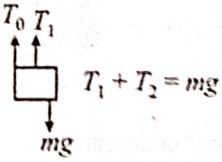
$$\sin\alpha = \frac{g}{2} \times \frac{4}{5} = 4\text{m/s}^2$$

$v^2 = 2as$

$v = \sqrt{2 \times 4 \times 5} = \sqrt{40}$ m/sec.

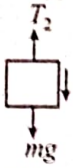
37. (*b*)





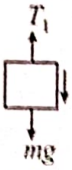
$T_1 + T_2 = mg$

If upper spring is cut



$6\,mg - T_2 = m \times 6$ ...(i)

If lower spring is cut:



$a\,mg - T_1 = ma$ ...(ii)

Adding (i) and (ii)

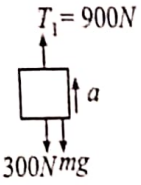
$2mg - \{T_1 + T_2\} = m(a + 6)$

$2mg - mg = m(a + 6)$

$mg = m(a + 6)$

$g = a + 6a = 4\text{m/s}^2$.

38. (*b*)

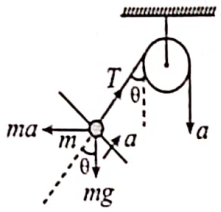


$900 - 300 - m \times 10 = ma$ $600 = m(10 + a)$

$$\frac{600}{10+a} = m$$

$$\frac{600}{10+10} = m = \frac{600}{20} = 30 \text{ kg.}$$

39. (*c*)



(Force diagram in the frame of the car)

Applying Newton's law perpendicular to string

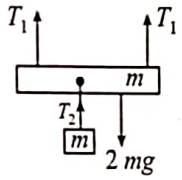
$mg \sin\theta = ma \cos\theta$

$$\tan\theta = \frac{a}{g}$$

Applying Newton's law along string

$\Rightarrow T - m\sqrt{g^2 + a^2} = ma$ $T = m\sqrt{g^2 + a^2} + ma$

40. (*a*) Before cutting the spring



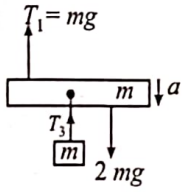
$T_2 = mg$

After cutting the spring

$2mg - mg = 2ma$

$a = g/2$

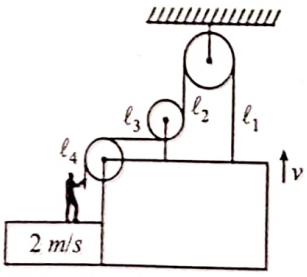
$T_3 = mg/2$



$$T_2 - T_3 = mg - \frac{mg}{2} = \frac{mg}{2}$$

41. (*b*) $\ell_1 + \ell_2 + \ell_3 + \ell_4 = C$

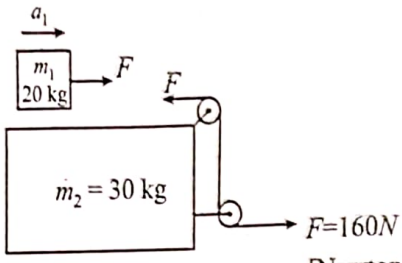
$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} + \frac{d\ell_4}{dt} = 0$$



$-v - v + 0 + v + 2 = 0$

$v = 2$m/s

42. (*a*)



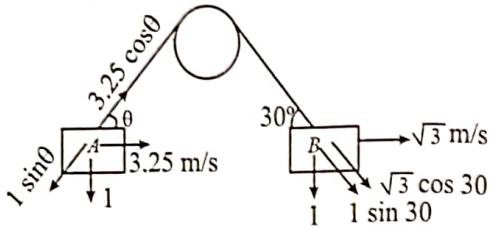
$F = m_1 a_1$ [Newton's II law for m_1]

$180 = 20a_1$

$\Rightarrow a_1 = 9$ m/s^2

Net force on m_2 is 0 therefore acceleration of m_2 is 0

43. (*a*) Solving problem in the frame of pulley



$3.25 \cos\theta - 1 \sin\theta = \sqrt{3} \cos 30 + 1 \sin 30$

$$3.25 \cos\theta - \sin\theta = \frac{3}{2} + \frac{1}{2}$$

$3.25 \cos\theta - \sin\theta = 2$

$13 \cos\theta - 4 \sin\theta = 8$

$13\sqrt{1 - \sin^2\theta} = 8 + 4 \sin\theta$

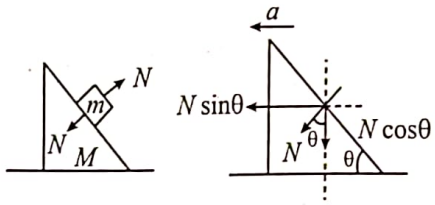
$169 - 169 \sin^2\theta = 64 + 16 \sin^2\theta + 64 \sin\theta$

$185 \sin^2\theta + 64 \sin\theta - 105 = 0$

$$\Rightarrow \sin\theta = \frac{3}{5} \Rightarrow \tan\theta = \frac{3}{4}.$$

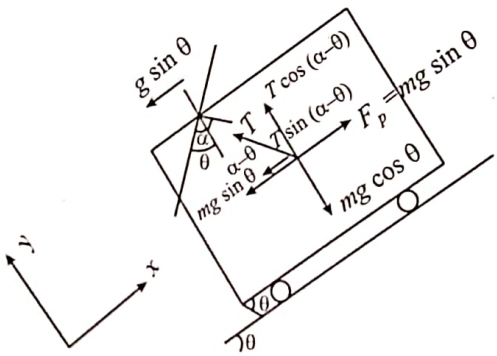
Exercise-3 (JEE Advanced Level)

1. (*c*)



$$a = \frac{N \sin\theta}{M} \text{ along } (-ve \times \text{axis})$$

2. (*a*)



F.B.D. of mass m is w.r.t. trolley

$T\sin(\alpha-\theta)+mg\sin\theta-F_p=0$

[Equilibrium of mass in x direction w.r.t. trolley]

$\Rightarrow T\sin(\alpha-\theta)+mg\sin\theta-mg\sin\theta=0$

$\Rightarrow T\sin(\alpha-\theta)=0$

since T cant be zero, $\sin(\alpha-\theta)$ must be zero

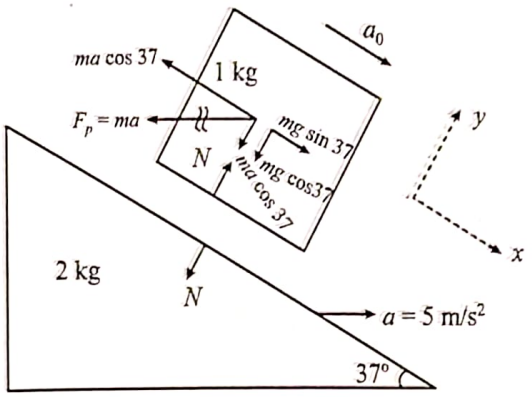
$\Rightarrow \alpha=\theta$

3. (*d*) Slope of $v_{rel}-t$ curve is Constant.

$\Rightarrow a_{rel}=\text{constant}=a_1-a_2\neq 0$

Inference that at least one reference frame is accelerating both can't be non - accelerating simultaneously.

4. (*c*)



FBD of block is shown w.r.t. wedge and FBD of wedge is shown w.r.t. ground. F_p is pseudo force.

mg sin 37 – ma cos 37 = ma_b

$\Rightarrow a_b=g\sin 37-a\cos 37=10\times\frac{3}{5}-5\times\frac{4}{5}=2\text{ m.s}^2$

w.r.t. wedge

$\Rightarrow$ Block is not stationary w.r.t. wedge

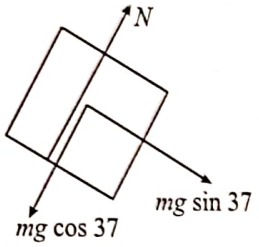
$N-ma\sin 37-mg\cos 37=0$

[Newton's II law for block]

$\Rightarrow N=1\times 10\times\frac{4}{5}+1\times 5\times\frac{3}{5}$

$N=11$ N.

Net force acting on block w.r.t. ground.



$F=\sqrt{(mg\sin 37)^2+(mg\cos 37-N)^2}$

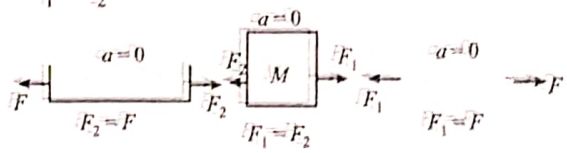
$=\sqrt{\left(10\times\frac{3}{5}\right)^2+\left(10\frac{4}{5}-11\right)^2}$

$=\sqrt{6^2+3^2}$

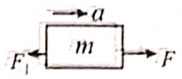
$F=3\sqrt{5}$ N.

5. (*a*) For $t<0$ As net force on system is zero, therefore acceleration of the system is zero

$\therefore F_1=F_2=F$ for $t<0$

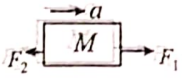


For $t>0$ system is accelerated given by $a=\frac{F}{2m+M}$



$F-F_1=ma$

$F_1=F-ma$ ...(i)

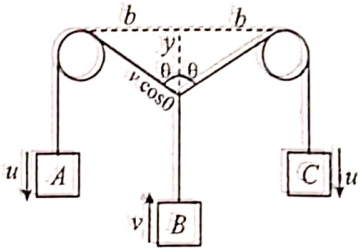


$F_1-F_2=Ma$

$F_1=F_2+Ma$ ...(ii)

$F>F_1>F_2$

6. (*b*)



$v\cos\theta=u$

$v=u\sec\theta$

$\frac{dv}{dt}=u\sec\theta\tan\theta\frac{d\theta}{dt}$ I

$\tan\theta=\frac{b}{y}$

$\sec^2\theta\frac{d\theta}{dt}=-\frac{b}{y^2}\frac{dy}{dt}$

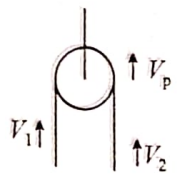
$=+\frac{b}{y^2}\cos^2\theta\frac{u}{\cos\theta}$

$=\frac{1}{b}\frac{b^2}{y^2}\cos\theta\, u$

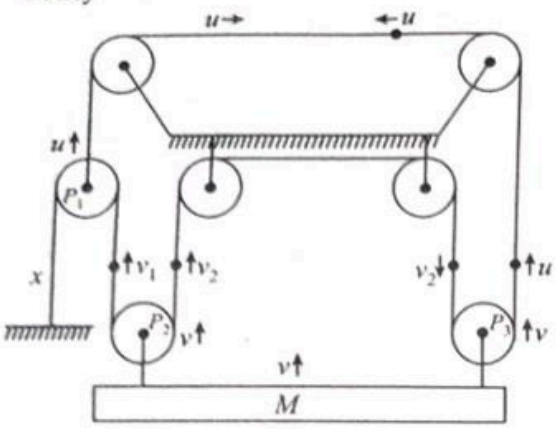
$=\frac{u\cos\theta}{b}\tan^2\theta$ II

$\Rightarrow\frac{dv}{dt}=\frac{u^2}{b}\tan^3\theta$ from I and II

7. (*a*) $\because V_p=\frac{V_1+V_2}{2}$



Pulley



$u = \dfrac{0+v_1}{2}$ ...(i)

Pulley P_2

$v = \dfrac{v_1+v_2}{2}$

$\Rightarrow 2V = V_1 + V_2$ ... (ii)

Pulley P_3 $v = \dfrac{-v_2+u}{2}$ ...(iii)

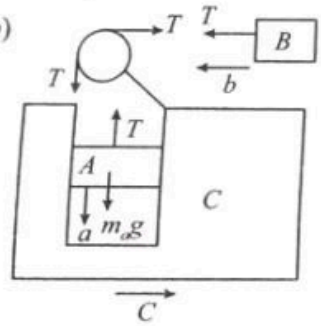
Eliminate V_1 and V_2 to get V_1 V_2

$\Rightarrow 2u + u - 2u = 2v$

$\Rightarrow 3u = 4v$

$v = \dfrac{3}{4}u$

8. (*b*)



$a = b + c$ [string constrained]

$T = m_B b$ [Newtons II law for B in horizontal direction]

$T = [m_A + m_c]\, c$ [Newton's I law for A and C in horizontal direction]

$m_A g - T = m_A a$ [Newton's II law for A in vertical direction].

$$\frac{m_A g - T}{m_A} = \frac{T}{m_B} + \frac{T}{m_A + m_C}$$

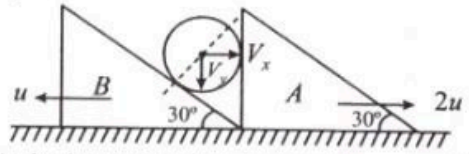
$$\frac{4mg - T}{4m} = \frac{T}{3m} + \frac{T}{12m}$$

$$\Rightarrow T = \frac{3mg}{2}$$

$$\Rightarrow b = \frac{g}{2}$$

9. (*d*) **Method-I:** As cylinder will remains in contact with wedge A

$V_x = 2u$



As it also remain in contact with wedge B

$u \sin 30° = V_y \cos 30° - V_x \sin 30°$

$$V_y = V_x \frac{\sin 30°}{\cos 30°} + \frac{U \sin 30°}{\cos 30°}$$

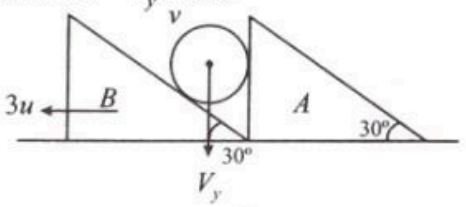
$V_y = V_x \tan 30° + u \tan 30°$

$V_y = 3u \tan 30° = \sqrt{3}\, u$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}\, u$

Method-II: In the frame of A

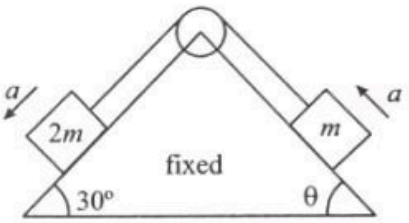
$3u \sin 30° = V_y \cos 30°$



$\Rightarrow V_y = 3u \tan 30° = \sqrt{3}\, u$

and $V_x = 2u \Rightarrow V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}\, u$

10. (*a*)



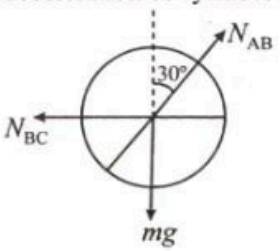
$$\text{Acceleration} = \frac{2mg \sin 30° - mg \sin\theta}{2m + m}$$

given $a = g/6$

$$\text{So } \frac{g(2\sin 30° - \sin\theta)}{3} = g/6$$

$\Rightarrow \sin\theta = 1/2 \Rightarrow \theta = 30°$

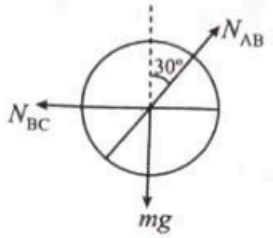
11. (*c*) The free body diagram of cylinder is as shown. Since net acceleration of cylinder is horizontal,



$N_{AB} \cos 30° = mg$ or $N_{AB} = \dfrac{2}{\sqrt{3}} mg$ ...(i)

and $N_{BC} - N_{AB} \sin 30° = ma$ or $N_{BC} = ma + N_{AB} \sin 30°$ ...(ii)

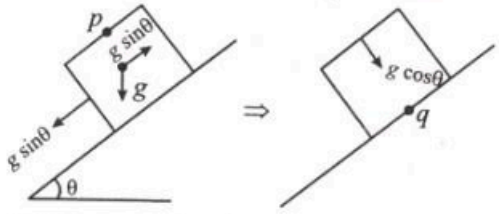
Hence N_{AB} remains constant and N_{BC} increases with increase in a.



$N_{AB}\cos 30° = mg\ N_{AB} = \frac{2}{\sqrt{3}}mg$ ...(i)

$N_{BC} - N_{AB}\sin 30° = ma\ N_{BC} = ma + N_{AB}\sin 30°$ ...(ii)

12. (*c*) Acceleration of bolt with respect to car



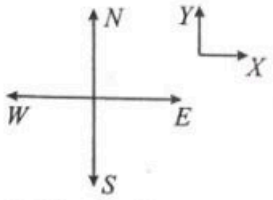
Bolt have acceleration perpendicular to inclined plane with respect to car so it will hit the surface of car at point q as shown

So distance from point $P = 3$m

13. (*d*) $v = \frac{dx}{dt} = -20\sin\theta\frac{d\theta}{dt}$

$u = \frac{dy}{dt} = -16\sin\theta\frac{d\theta}{dt} \Rightarrow u = \frac{4}{5}v = 0.8\,v$

14. (*c*)



$A + B + C + D + E = 300i$ ...(i)

$B + C + D + E = -100i$ ...(ii)

$A + C + D + E = 2400j$ ...(iii)

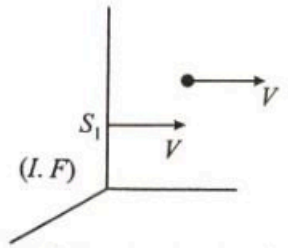
By solving

$B = 300i - 2400j$ ...(iv)

$A = 400i$ ...(v)

$\Rightarrow A + B = 700i - 2400j$

$a_{(A+B)} = \frac{A+B}{100}$



$= 7i - 24j = 25$ m/s^2

15. (*a,b,c*)

$F = \alpha t$

$a = \frac{dv}{dt} = \frac{\alpha}{m}t$ ...(i)

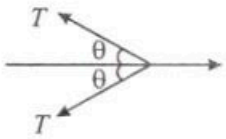
Straight line curve 1 $dv = \frac{\alpha}{m}tdt$

$v = \frac{\alpha}{m}\frac{t^2}{2}$ curve 2 ...(ii)

Divide (ii) by

$v = \frac{t}{2}a = \frac{a}{2}\times\frac{am}{\alpha} = \frac{a^2m}{2\alpha} \rightarrow$ Parabola curve 2.

16. (*a,c*)

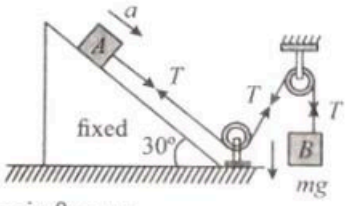


$F = 2T\cos\theta,\ T = \frac{F}{2\cos\theta}$

$\theta\uparrow \cos\theta\downarrow T\uparrow$

on increasing θ, cos θ decreases and hence T increases.

17. (*b,d*)



$T + mg\sin\theta = ma$ ...(i)

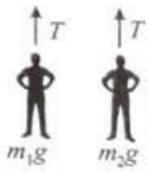
$mg - T = ma$ ...(ii)

On solving (i) and (ii)

$a = \frac{3g}{4}$

$T = \frac{3g}{4} = m\frac{mg}{4}$

18. (*a,b,d*)



(*a*) $T = m_1g < m_2g$

∴ Acceleration of m_2 is ↓

(*b*) $T = m_2g > m_1g$

∴ acceleration of m_1 is

(*c*) Masses is different

∴ Not possible

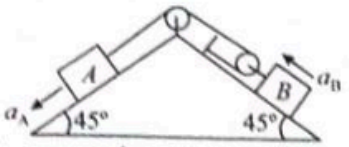
(*d*) $T - m_1g = m_1a$

$m_2g - T = m_2a$

On solving

$a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$ *Possible*

19. (a,b,d)



By string constraint

$a_A = 2a_B$ ...(i)

equation for block A,

$10 \times 10 \times \frac{1}{\sqrt{2}} - T = 10\, a_A$ ...(ii)

equation for block B,

$2T - \frac{400}{\sqrt{2}} = 40\, a_B$ ...(iii)

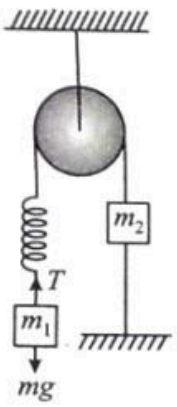
Solving equation (i), (ii) and (iii) we get

$a_A = \frac{-5}{\sqrt{2}}$ m/s^2

$a_B = \frac{-5}{2\sqrt{2}}$ m/s^2

$T = \frac{150}{\sqrt{2}}$ N

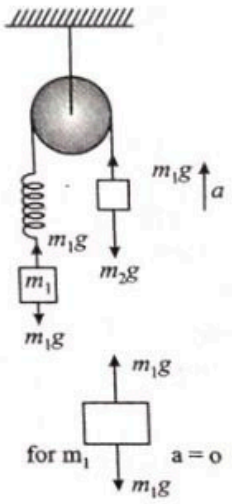
20. (a,c)



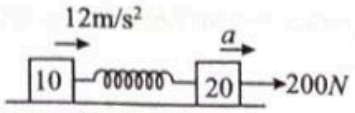
$T = m_1 g$

when thread is burnt, tension in spring remains same = $m_1 g$.

$m_1 g - m_2 g = m_2 a_1 g \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = a =$ upwards



21. (b,c)



Apply NLM on the system

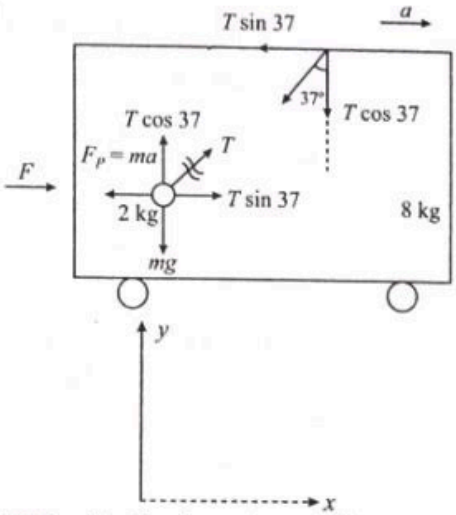
$200 = 20a + 12 \times 10 \Rightarrow \frac{80}{20} = a = 4$ m/s^2

spring Force = $10 \times 12 = 120$ N

22. (c,d)

Pseudo force depends on acceleration of frame and mass of object.

23. (c,d)



F.B.D. of trolley is w.r.t. ground

F.B.D. of suspended mass is w.r.t. Trolley.

$T\cos 37° - mg = 0$

[Equilibrium of mass in y direction w.r.t. trolley]

$\Rightarrow T = \frac{5mg}{4}$ T = 25 N

$T \sin 37° - ma = 0$

[Equilibrium of mass in x direction w.r.t. trolley]

$\Rightarrow a = \frac{T \sin 37}{m} = \frac{15}{2}$

$F - T \sin 37 = 8a$

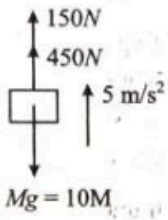
[Newton's II law for trolley in x direction w.r.t. ground]

$\Rightarrow F = 8 \times \frac{15}{2} + 25 \times \frac{3}{5}$ $F = 75$ N

24. (b) FBD of Block in ground frame:

Applying N.L. $150 + 450 - 10\,M = 5\,M$

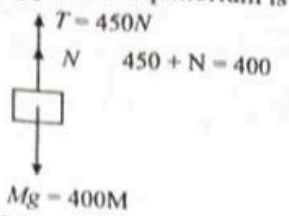
$\Rightarrow 15M = 600 \Rightarrow M = \frac{600}{15}$



$\Rightarrow M = 40\ Kg$

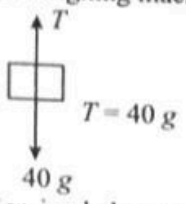
Normal on block is the reading of weighing machine i.e. 150 N.

25. (*d*) If lift is stopped and equilibrium is reached then



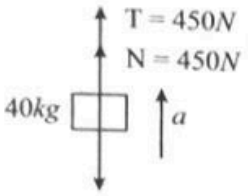
$\Rightarrow N = -50$

So block will lose the contact with weighing machine thus reading of weighing machine will be zero.



So reading of spring balance will be 40 Kg.

26. (*a*)



$$a = \frac{950-400}{40} \Rightarrow a = \frac{450}{40} = \frac{45}{4} \text{ m/s}^2$$

27. (*c*) $a_p = \frac{10t}{10} = t$

$$\therefore \frac{dv}{dt} = t \Rightarrow \int_0^v dv = \int_0^t t\,dt \Rightarrow v = \frac{t^2}{2}$$

Putting $v = 2$ we have $t = 2$ sec.

Now $\frac{dx}{dt} = \frac{t^2}{2}$ $\quad \therefore xp = \left[\frac{t^3}{6}\right]_0^2 = \frac{4}{3}$

$x_B = 2 \times 2 = 4\text{m}$

Hence relative displacement $= 4 - \frac{4}{3} = \frac{8}{3} m$

28. (*b*) From above

$$2t = \frac{t^3}{6} \Rightarrow t^2 = 12 \Rightarrow t = 2\sqrt{3} \text{ sec.}$$

29. (*c*) $a = t = 4$

$\therefore$ after 4 seconds $V_B = 2$ m/s

$V_p = \frac{4^2}{2} = 8$ m/s $\therefore V_{rel} = 8 - 2 = 6$ m/s.

30. (*a*) S_1 is accelerating frame so psuedo force act opposite to frame acceleration

F_{Pseudo} = mass of analyzing body × acceleration of frame

$= 2(-5\hat{i} - 10\hat{j}) = -10\hat{i} - 20\hat{j}$

31. (*b*) S_2 is inertial frame

$F = ma$

So $F = 10\hat{i} + 20\hat{j}$

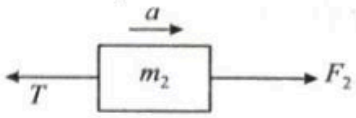
32. (*a*) With respect to S_1 frame

Net force = zero.

33. (*a*) Let a be acceleration of two block system towards right

$$\therefore a = \frac{F_2 - F_1}{m_1 + m_2}$$

The F.B.D. of m_2 is



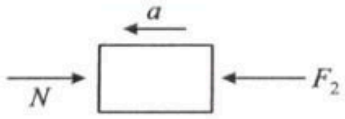
$F_2 - T = m_2 a$

Solving $T = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} + \frac{F_1}{m_1}\right)$

(*b*) Replace F_1 by $-F_1$ is result of A

$$\therefore T = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$$

(*c*) Let a be acceleration of two block system towards left



$$\therefore a = \frac{F_2 - F_1}{m_1 + m_2}$$

The FBD of m_2 is

$\therefore F_2 - N_2 = m_2 a$

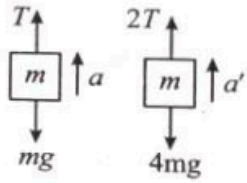
Solving $N = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right)$

(d) Replace F_1 by $-F_1$ in result of C

$$N = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$$

34. [0005]

$T - mg = ma$ ...(i)



$4mg - 2T = 4ma'$

$\Rightarrow 2mg - T = 2\,ma'$ ...(ii)

Constraint relation : $Ta - 2Ta'$

$\Rightarrow a' = a/2$

$\therefore$ From equation (ii) we get

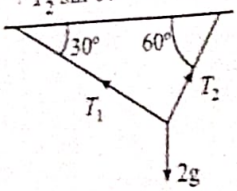
$2mg - T = 2m\,(a/2)$ ...(iii)

Solving (i) and (ii), we get

$a = \frac{g}{2} = 5$ m/s²

35. [0010]

$T_1 \sin 30° + T_2 \sin 60° = 20$



$T_1 \cos 30° = T_2 \cos 60°$

$T_2 = T_1\sqrt{3} \Rightarrow \frac{4T_1}{2} = 20$

$T_1 = 10\,N$

36. [0050]

$100 = k \times (40 - l_0)$

$200 = k \times (60 - l_0)$

$2 = \frac{60 - l_0}{40 - l_0}$

$80 - 2l_0 = 60 - l_0$

$20 = l_0 \Rightarrow 20\,k = 100 \Rightarrow k = 5$

$x = k \times (30 - 20) = 50$ N

37. [0360]

$a = \frac{90 - 50}{5} = 8$, after 2 sec

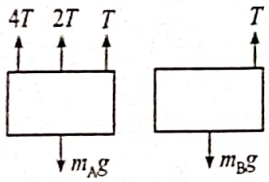
$S = 16$m, $V = 16$ m/s

3 kg part hits the ground $t = 4$ sec.

remaining 2 kg has $a = 35$

$S_{rel} = 360$

38. [0070]



$7T = m_A g, T = m_B g$

$m_A = 7m_B = 70$

39. [0270]

$F = 2 \times 75 \cos 37° + 150$

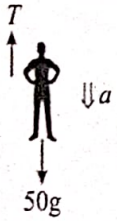
$= 150 \times \frac{4}{5} + 150 = 270$

40. [0880]

$m = 16, M = 80, x = 6, L = 12$

$T = \left(\frac{mx}{L} + M\right)g \Rightarrow \left[\frac{16 \times 6}{12} + 80\right] \times 10 = 880$ N

41. [4]



$50g - T = 50a$

$a = 10 - \frac{300}{50} = 4$ m/s^2

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*d*) $T \cos 45° = mg$

$T \sin 45° = mg$

$\Rightarrow F = mg$ 100 N.

2. (*b*) $-(g + \gamma v^2) = \frac{dv}{dt}$

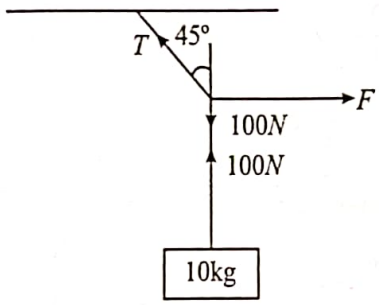
$$-gdt = \frac{g}{\gamma}\left(\frac{dv}{\frac{g}{\gamma} + v^2}\right)$$

Intergrating $0 \to t$ and $V_0 \to 0$:

$$-gt = -\sqrt{\frac{g}{\gamma}}\tan^{-1}\left(\frac{V_0}{\sqrt{\frac{g}{\gamma}}}\right)$$

$$t = \frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$$

3. (*c*)



$\frac{T}{\sqrt{2}} = 100$

$\frac{T}{\sqrt{2}} = F$

$F = 100$ N

4. (*c*)

$(mg + mkv^2)$

$|a| = g + kv^2$

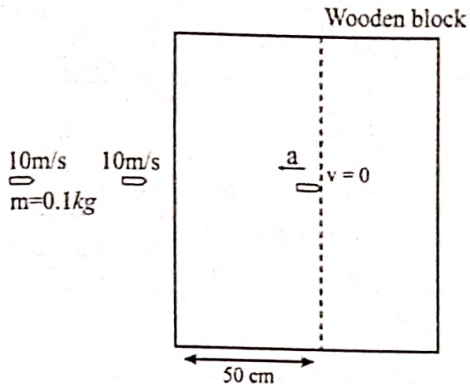
$\Rightarrow -\frac{vdv}{dh} = g + kv^2$

$\Rightarrow \int_u^0 \frac{vdv}{g + kv^2} = \int_0^{H_{max}} -dh$

On solving

$$H_{max} = \frac{1}{2K}\ln\left(1 + \frac{ku^2}{g}\right)$$

5. [10]



$M_{\text{bullet}} = 0.1\text{kg}$

$v^2 = 4^4 + 2(-a)\times 5 \Rightarrow 0 = (10)^2 - 2a\times(0.5)$

Retardation $(a) = \dfrac{1000}{2\times 5} = 100\, m/s^2$

Retarding force $(F) = ma = 0.1 \times 100$

$F_r = 10$ N

6. [12] $\vec{F} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)N$ time $= 4\,sec$ As body start from rest therefore position vector initially $\vec{r}_i = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right)$ and u (initial velocity) $= 0$ given, $r_f = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$ position vector initially $\vec{r}_i = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right)$ u (initial velocity) $= 0$ given, $r_f = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$.

Now, from second equation of motion

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \Rightarrow r_f - r_i = \frac{1}{2}\times\left(\frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}\right)\times(4)^2$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) - (0\hat{i} + 0\hat{j} + 0\hat{k}) = 8\hat{i} + 12\hat{j} + 20\hat{k}$$

$$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = 8\hat{i} + 12\hat{j} + 20\hat{k}$$

$\therefore$ The value of $b = 12$

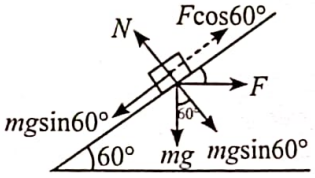
7. [2] $\vec{a} = \dfrac{\vec{F}}{m} = 100\uparrow + 50\hat{j}$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$a\hat{i} + b\hat{j} = 0 + \frac{1}{2}(100\hat{i} + 50\hat{j})\times 4 = 200\hat{i} + 100\hat{j}$$

$$\frac{a}{b} = \frac{200}{100} = 2$$

8. [12] For equilibrium of block along the inclined plane



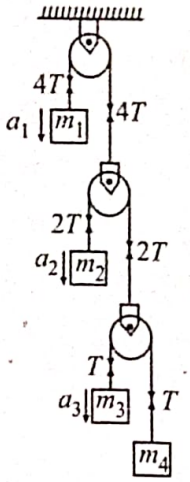
$F\cos 60° = mg\sin 60°$

$$F = 0.2\times 10\times\frac{\sqrt{3}}{2}\times\frac{2}{1} \Rightarrow F = \sqrt{12}\text{ N} \Rightarrow x = 12$$

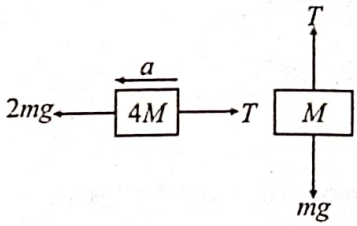
9. (a) $\sum T_i \cdot a_i = 0$

$-4Ta_1 - 2Ta_2 - Ta_3 - Ta_4 = 0$

$4a_1 + 2a_2 + a_3 + a_4 = 0$



10. [6]



for M

$ma = T - mg$ ...(i)

for 4 M

$4ma = 2mg - T$ ...(ii)

From (i) and (ii)

$a = \dfrac{g}{5}$ and $T = \dfrac{6}{5}mg$

So $x = 6$

11. (c) Force on block

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta mv}{\Delta t} = 1\text{kgs}^{-1}\times 10\text{ms}^{-1} = 10\text{kg ms}^{-1}$$

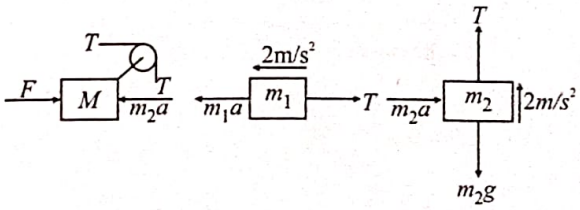
$$a = \frac{F}{m} = \frac{10}{2} = 5\text{ms}^2$$

12. (c) $W' = m(g - a)$

$$W' = mg\left(1 - \frac{a}{g}\right) = W\left(1 - \frac{a}{g}\right)$$

$$\frac{W}{4} = W\left[1 - \frac{a}{g}\right] \Rightarrow a = \frac{3g}{4}$$

13. (a)



For m_2

$20 \times 2 = T - 200$

$T = 240$ N

For m_1

$10 \times 2 = 10a - T$

$a = 26 \text{ m/s}^2$

For M

$Ma = (M + m)a + T$

$= 120 \times 26 + 240$

$= 3360 \text{ N}$

14. (*c*) $Ma = T - Mg$ (While climbing up)

$T_1 = M(a + g)$

$T_1 = 50(5 + 10)$

$= 750 \text{ N}$ (Ropewell break)

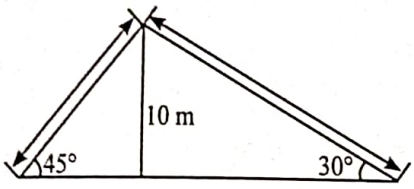
15. (*b*) $a_1 = \dfrac{2m_1g - M_1g}{2M_1 + M_1} = \dfrac{g}{3}$

$a_2 = \dfrac{3M_1g - M_1g}{3M_1 + M_1} = \dfrac{g}{2}$

$\dfrac{a_1}{a_2} = \dfrac{2}{3}$

16. [2] Acceleration between A and $B = -g\sin 45° = \dfrac{-g}{\sqrt{2}}$

Acceleration between B and $C = +g\sin 30° = \dfrac{g}{2}$



Velocity at point A.

$0 = u^2 - 2\dfrac{g}{\sqrt{2}} \times 10\sqrt{2}$

$u = 10\sqrt{2} \text{ m/s}$

From A to B

$v = u + at$

$0 = 10\sqrt{2} - \dfrac{g}{\sqrt{2}} t_{AB}$

$t_{AB} = 25$ ...(i)

From B to C

$s = ut + \dfrac{1}{2}at^2$

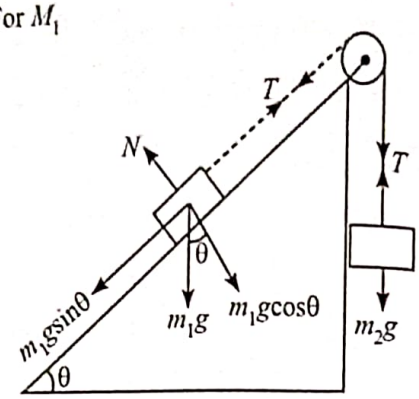
$20 = 0 + \dfrac{1}{2} \times \dfrac{g}{2} t_{BC}^2$

$t_{BC} = 2\sqrt{2}$ ...(ii)

$t_{AB} + t_{BC} = 2(\sqrt{2} + 1)$

$\Rightarrow t = 2$

17. (*c*) For M_1



$m_1g\sin\theta = T$

$m_1g\cos\theta = N$

For M_2

$T = m_2g$

Solving (i), (ii) and (iii)

$\sin\theta = \dfrac{3}{5}, N = 40$ Newton

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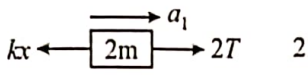
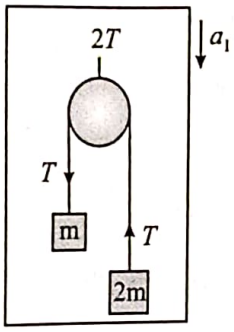
18. (*c*)



$2T - kx = 2ma_1$



$T = \dfrac{2(2m)(m)}{3m}(g - a_1)$

$= \dfrac{4m}{3}(g - a_1)$

$\dfrac{8m}{3}(g - a_1) - kx = 2ma_1$

$\dfrac{8Mg}{3} - \dfrac{8ma_1}{3} - kx = 2ma_1$

$\dfrac{8Mg}{3} - kx = \dfrac{14ma_1}{3}$

$\dfrac{8Mg - 3kx}{14m} = a_1$

$a_1 = \dfrac{8mg - 3kx}{14m}$

$\dfrac{vdv}{dx} = \left(\dfrac{8Mg}{14m} - \dfrac{3kx}{14m}\right)$

$\int vdv = \dfrac{1}{14m}\int (8Mg - 3kx)dx$

For max elongation

$$0 = \frac{1}{14m}\int_0^{x_0}(8mg - 3kx)dx$$

$$= \frac{1}{14m}\left(8Mgx_0 - \frac{3kx_0^2}{2}\right)$$

$$8Mgx_0 = \frac{3kx_0^2}{2}$$

$$x_0 = \frac{16Mg}{3k}$$

at $x = \frac{x_0}{2}$

$$\int_0^v vdv = \frac{1}{14m}\int_0^{x_0/2}(8Mg - 3kx)dx$$

$$\frac{v^2}{2} = \frac{1}{14m}\left(\frac{8Mgx_0}{2} - \frac{3kx_0^2}{2\times 4}\right)$$

$$v^2 = \frac{1}{7m}\left(\frac{8Mg}{2}\times\frac{16Mg}{3x} - \frac{3x}{8}\times\frac{16M^2g^2}{3x\times 3x}\right)$$

$$= \frac{1}{7m}\left(\frac{64M^2g^2}{3x} - \frac{2M^2g^2}{3x}\right)$$

$$v^2 = \frac{62Mg^2}{21k}$$

For acceleration $2a_1 = a_2 + a_3$ therefore

$$a_2 - a_1 = a_1 - a_3$$

$$a_1 = \frac{8Mg - 3kx_0/4}{14m}$$

$$= \frac{8g}{14} - \frac{3kx_0}{14m\times 4}$$

$$= \frac{8g}{14} - \frac{3x}{14m\times 4}\times\frac{16Mg}{3x}$$

$$= \frac{8g}{14} - \frac{4g}{14}$$

$$= \frac{4g}{14} = \frac{2g}{7}$$

OR

$$\frac{8mg}{3} - \frac{8m}{3}a_1 - kx = 2ma_1$$

$$\frac{14m}{3}a_1 = -k\left[x - \frac{8mg}{3k}\right]$$

$$a_1 = -\frac{3k}{14m}\left[x - \frac{8mg}{3k}\right] \quad ...(i)$$

that means, block $2m$ (connected with the spring) will per form SHM about $x_1 = \frac{8mg}{3k}$ therefore.

Maximum elongation in the spring $x_0 = 2x_1 = \frac{16mg}{3k}$

on comparing equation (a) with

$a = -\omega^2(x - x_0)$

$\omega = \sqrt{\frac{3k}{14m}}$ at $\left(\frac{x_0}{2}\right)$, block will be passing through its mean position therefore at mean position.

$$v_0 = A\omega = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$$

At. $\frac{x_0}{4} \Rightarrow x = \frac{A}{2}$

$$\therefore acc = -\frac{A}{2}\omega^2$$

$$= -\frac{4mg}{3k}\cdot\frac{3h}{14m} = \frac{2g}{7}$$

19. (a,b,d)

$$\frac{dk}{dt} = \gamma t \text{ as } k = \frac{1}{2}mv^2$$

$$\therefore \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$$

$$\therefore m\int_0^v vdv = \gamma\int_0^t tdt$$

$$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$$

$$v = \sqrt{\frac{\gamma}{m}}t \quad ...(i)$$

$$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}} = \text{constant}$$

since $F = ma$

$$\therefore F = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$$

CHAPTER

7 Friction

Exercise-1 (Topicwise)

1. (c) $\tan\lambda = \frac{f}{N} \Rightarrow \tan\lambda = \mu$

2. (d) $\mu = \frac{F}{R} = \frac{F}{mg} = \frac{98}{100\times 9.8} = \frac{1}{10} = 0.1$

3. (a) Fact based

4. (c) $F_1 = \mu_s R = 0.4 \times mg = 0.4 \times 10 = 4N$ i.e. minimum 4N force is required to start the motion of a body. But applied force is only 3N. So the block will not move.

5. (a) For limiting condition

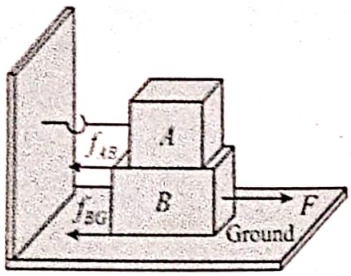
$$\mu = \frac{m_B}{m_A + m_C} \Rightarrow 0.2 = \frac{5}{10 + m_C}$$

$$\Rightarrow 2 + 0.2m_C = 5 \Rightarrow m_C = 15\ kg$$

6. (c) Rolling friction is greater than sliding friction.

7. (a) $\mu = \frac{m_B}{m_A} \Rightarrow 0.2 = \frac{m_B}{10} \Rightarrow m_B = 2kg$

8. (c)



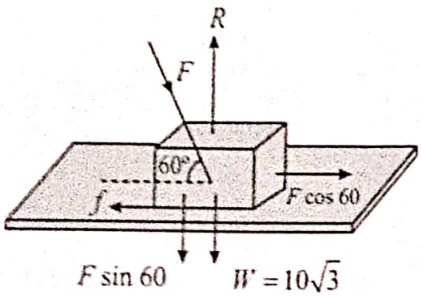
$F = f_{AB} + f_{BG}$

$= \mu_{AB} m_A g + \mu_{BG}(m_A + m_B) g$

$= 0.2 \times 100 \times 10 + 0.3\ (300) \times 10$

$= 200 + 900 = 1100$ N

9. (a)

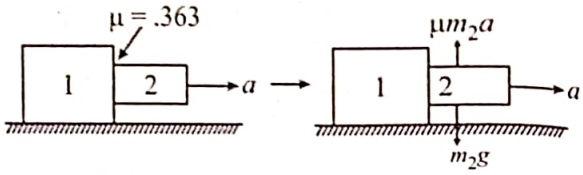


$f = \mu R$

$F\cos 60° = \mu(W + F\sin 60°)$

Substituting $\mu = \frac{1}{2\sqrt{3}}$ & $W = 10\sqrt{3}$ we get $F = 20\ N$

10. (d)



For equilibrium of 2

$\mu m_2 a = m_2 g$

$a = \frac{g}{\mu}$

$a = \frac{9.8}{0.363} = 27$ m/s

$a = 27$ m/s^2

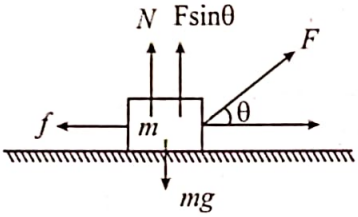
11 (d) Maximum frictional force between C and ground = 300N

Maximum frictional force between D and ground = 360N

So, man in unable to pull B.

Hence, $T = 0$ N

12. (c)



$\mu = 0.75$

$m = 10$ Kg

Normal reaction

$N = mg - F\sin\theta$

$\tan\theta = \mu = 0.75$

$\theta = 37°$

Friction $f = \mu N$

$f = \mu(mg - F\sin\theta)$

So $F\cos\theta = f = \mu mg - \mu F\sin\theta$

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

$$F = \frac{0.75 \times 10 \times 10}{\frac{4}{5} + 0.75 \times \frac{3}{5}}$$

$$F = \frac{\frac{3}{4} \times 100}{\frac{4}{5} + \frac{3}{4} \times \frac{3}{5}} = 60\,N$$

$F = 60$ N

13. (*a*) Retarding force $F = ma = \mu R = \mu mg$

$\therefore a = \mu g$

Now from equation of motion $v^2 = u^2 - 2as$

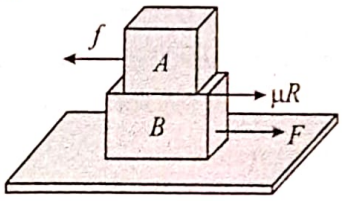
$$\Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a} = \frac{u^2}{2\mu g}.$$

14. (*a*) There is no friction between the body B and surface of the table. If the body B is pulled with force F then

$F = (m_A + m_B)\,a$

Due to this force upper body A will feel the pseudo force in a backward direction.

$F = m_A \times a$



But due to friction between A and B, body will not move. The body A will start moving when pseudo force is more than friction force.

i.e. for slipping, $m_A a = m_A g \quad \therefore a = \mu g$

15. (*d*) $F = ma + \mu mg$

$= m(a + \mu g) = 1500\,(1 + 0.2 \times 10)$

$F = 4500$ N

16. (*b*) From the relation $F - \mu mg = ma$

$$\alpha = \frac{F - \mu mg}{m} = \frac{129.4 - 0.3 \times 10 \times 9.8}{10} = 10\,m/s^2.$$

17. (*c*) Fact based

18. (*a*) $a = g\,(\sin\theta - \mu\cos\theta) = 9.8\,(\sin 45° - 0.5\cos 45°)$

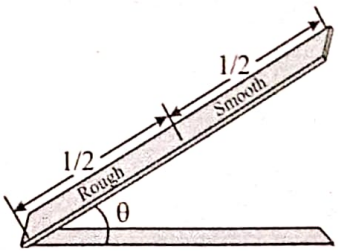
$$= \frac{4.9}{\sqrt{2}}\,m/\sec^2$$

19. (*d*) For upper half

$v^2 = u^2 + 2al/2 = 2\,(g\sin\theta)\,l/2 = gl\sin\theta$

For lower half

$$\Rightarrow 0 = u^2 + 2g(\sin\theta - \mu\cos\theta)\frac{l}{2}$$



$\Rightarrow -gl\sin\theta = gl\,(\sin\theta - \mu\cos\theta)$

$\Rightarrow \mu\cos\theta = 2\sin\theta \Rightarrow \mu = 2\tan\theta$

20. (*b*) $t = \sqrt{\frac{2s}{a}} \propto \frac{1}{\sqrt{a}}$

$$\frac{t_1}{t_2} = \sqrt{\frac{a_2}{a_1}} \Rightarrow \frac{2}{1} = \sqrt{\frac{g\sin\theta}{g\sin\theta - \mu_k g\cos\theta}} = \sqrt{\frac{1}{1 - \mu_k}}$$

as $\sin\theta = \cos\theta$ at $\theta = 45°$

on solving above equation we get $\mu_k = \frac{3}{4}$

21. (*a*) Limiting friction between block and slab $= \mu_s m_B g$

$= 0.6 \times 10 \times 9.8 = 58.8$ N

But applied force on block A is 100 N. So the block will slip over a slab.

Now kinetic friction works between block and slab

$F_k = \mu_k m_B g = 0.4 \times 10 \times 9.8 = 39.2$ N

This kinetic friction helps to move the slab

$\therefore$ Acceleration of slab.

$$= \frac{39.2}{m_B} = \frac{39.2}{40} = 0.98\,m/s^2$$

22. (*d*) Maximum acceleration of B and C can be μg. So they do not slip with each other or on A.

For system $(A + B + C)$,

$T = 3ma = \mu mg$

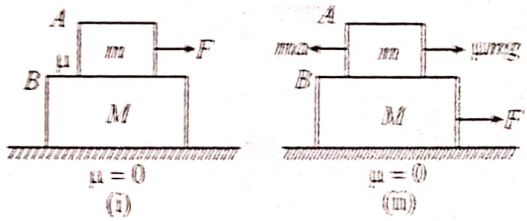
For D,

$Mg - T = ma$

$Mg - 3\mu mg = \mu mg$

$$M = \frac{3\mu m}{1 - \mu}$$

23. (*c*)



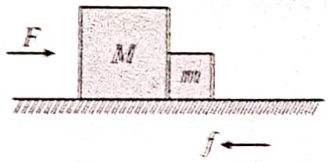
In case (ii) $F = (M + m)a$

friction on $m = \mu mg = ma$

$a = \mu g$

$F = \mu g(m + M)$

24. (*b*)



$M = 3$Kg

$m = 2$Kgss

$F = 5$N

f between m and table $= 2$N

$F - f = (M + m)a$

$5 - 2 = 5a$

$3 = 5a$

$a = 0.6 \text{ m/s}^2$

25. (*a*) $f_A = \mu m_A g$

$= 0.1 \times 10 \times 10 = 10 \text{ N}$

$f_B = \mu m_B g$

$= 0.1 \times 20 \times 10 = 20\text{N}$

Force on A

$F_A = m_A a_A = 10 \times 12 = 120\text{N}$

so total absorbed force

$F_A + f_A + f_B$

$= 120 + 10 + 20$

$= 150 \text{ N}$

Now rest of force = 200 – 150 = 50 N accelerates to the body 20 kg

$\therefore F_B = m_B a_B$

$50 = 20\, a_B$

$a_B = 2.5 \text{ m/s}^2$

26. (*a*) $f = \mu mg$; $V_0 = 4$ m/s

$a = \dfrac{\mu mg}{m}$ $v = 0$ m/s

$= 0.2 \times 10 = 2 \text{ m/s}^2$ (In opposite direction)

So, retardation $= -2\text{m/s}^2$

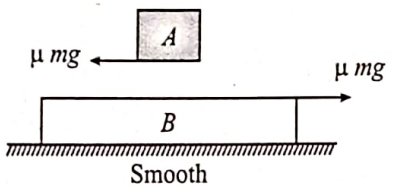
from $v^2 = u^2 + 2aS$

$S = \dfrac{v^2 - u^2}{2a}$

$S = \dfrac{-16}{-2 \times 2}$

$S = 4$ m

27. (*c*)



$f = \mu\, mg$

For B:

$a = \dfrac{\mu mg}{2m}$

$a = \dfrac{g}{4}$ (relative to ground)

For A:

$\mu mg = ma'$

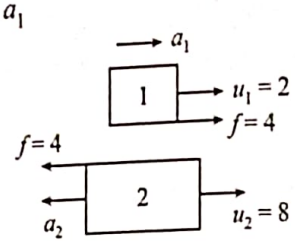
$a' = \mu g$

$\Rightarrow \quad a' = \dfrac{g}{2}$ (relative to ground in opposite direction)

Acceleration of B relative to A

$a_{\text{rel.}} = \dfrac{g}{4} + \dfrac{g}{2} = \dfrac{3g}{4}$

28. (*a*) $f = 1 \times a_1$



$a_1 = 4 \text{ m/s}^2$ ...(i)

$4 = 2a_2$

$a_2 = 2 \text{ m/s}^2$ ...(ii)

$v_r = u_r + a_r t$

$0 = -6 + 6t$

$t = 1$ sec.

29. (*d*) Force on block m_1

$f = m_1 a = \mu\, m_1 g$

$a = \mu g$

Similarly for block m_2

$a = \mu g$

So spring will be neutral length under all condition.

30. (*a*) Maximum value of force of friction is 10 N but block will move together if $\dfrac{20 - f_1}{4} = \dfrac{f_1 - 2}{2}$ which is if $f_1 = 8\text{N}$

so block move together with $f_1 = 8\text{N}$

31. (*d*) Max. friction available to m_2 is $f_{max} = \mu m_2 g$

$\therefore$ Max. acceleration which can be provided to m_2 by friction (without the help of normal section from m_1) is $a_{max} = \mu g$.

If $a > \mu g$. Normal reaction from m1 (or m_2) is non zero.

32. (*c*) $T - \alpha mg = ma$ ...(i)

$F - \alpha mg - T = ma$ ...(ii)

from equation (i) and (ii)

$a = \dfrac{F}{2m} - \alpha g$

33. (*c*) Let acceleration of system be a. Then for block A to be stationary.

$\mu ma = mg$

$a = \dfrac{g}{\mu}$

For block C we can write.

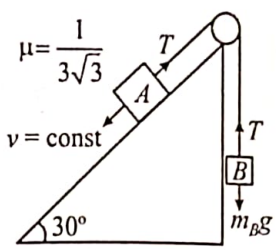
$m_c g - T = m_c \dfrac{g}{\mu}$ ...(i)

Also $T = \dfrac{3mg}{\mu}$ ...(ii)

using (i) and (ii) we have

$m_c = \dfrac{3m}{\mu - 1}$

34. (*b*)



For B, $T = m_B g$ ...(i)

For A, $T + f = mg\sin 30$

Hence $f = \mu Mg\cos 30$

$T = Mg\sin 30 - \mu Mg\cos 30$

$$T = Mg\left(\frac{1}{2} - \frac{1}{3\sqrt{3}} \times \frac{\sqrt{3}}{2}\right)$$

$$T = \frac{Mg}{2}\left(\frac{2}{3}\right) = \frac{Mg}{3} \quad ...(ii)$$

From equation (i) and (ii)

$$\frac{Mg}{3} = m_B g$$

$$m_B = \frac{M}{3}$$

35. (*b*) Acceleration of 2 kg block will be

$a_2 = g\sin\theta - 0.4\, g\cos\theta$

Acceleration of 1 kg block will be

$a_1 = g\sin\theta - 0.5\, g\cos\theta$

Since $a_2 > a_2$ both block will move separately.

36. (*c*) Let the total extension in the string is A then

$$\frac{KA - 3mg\sin\theta}{3m} = \mu g\cos\theta - g\sin\theta$$

$$\frac{KA}{3m} - g\sin\theta$$

$= \mu g\cos\theta - g\sin\theta$

$$\frac{KA}{3m} = \mu g\cos\theta$$

$$\Rightarrow \frac{KA}{3m} = 2\tan\theta\, g\cos\theta$$

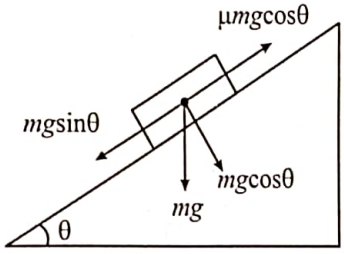
$$\frac{KA}{3m} = 2g\sin\theta$$

$$A = \frac{6mg\sin\theta}{K}$$

Maximum amplitude = A – mean position

$$= \frac{6mg\sin\theta}{K} - \frac{3mg\sin\theta}{K} = \frac{3mg\sin\theta}{K}$$

37. (*d*) Force required to prevent the body from slipping



$F = mg\sin\theta - \mu mg\cos\theta$

Force required to move the body up the plane

$F' = mg\sin\theta + \mu mg\cos\theta$

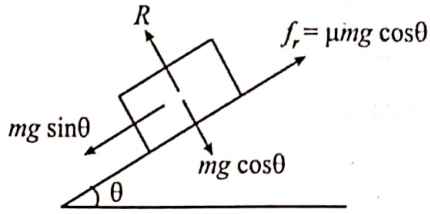
$\because$ friction is also downward along the plane

so $F' = 2F$

$mg\sin\theta + \mu mg\cos\theta = 2\, mg\sin\theta - 2\mu mg\cos\theta$

$mg\sin\theta = 3\mu\, mg\cos\theta \Rightarrow \tan\theta = 3\mu$

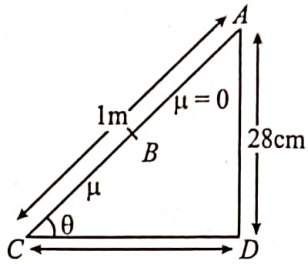
38. (*d*)



To sides down at a constant speed

$mg\sin\theta = \mu mg\cos\theta$

$\mu = \tan\theta$



39. (*a*)

$AB = 0.5$ m and $BC = 0.5$ m

For AB path $v^2 = u^2 + 2as$

$v = 0 + \sqrt{2as}$

$v = \sqrt{2 \times g\sin\theta \times 0.5}$

$v = \sqrt{g\sin\theta}$

For BC path $u = \sqrt{g\sin\theta}$

at point C, $v = 0$

retardation $a = (g\sin\theta - \mu g\cos\theta)$

from $v^2 = u^2 + 2as$

$0 = g\sin\theta + 2 \times 0.5\,(g\sin\theta - \mu g\cos\theta)$

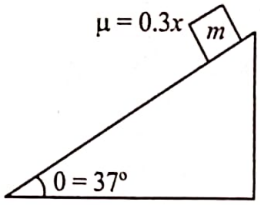
$0 = 2g\sin\theta - \mu g\cos\theta.$

$2\sin\theta = \mu\cos\theta$

$2\tan\theta = \mu$

$$\therefore \mu = 2 \times \frac{28}{96} = \frac{56}{96} = \frac{7}{12}$$

40. (*d*)



$a = g\sin\theta - \mu g\cos\theta$

$$a = 10 \times \frac{3}{5} - 0.3 \times x \times 10 \times \frac{4}{5}$$

$a = 6 - 2.4x$

$$\frac{udu}{dx} = 6 - 2.4x = 0 \quad ...(i)$$

$$\frac{du}{dx} = 6 - 2.4x = 0$$

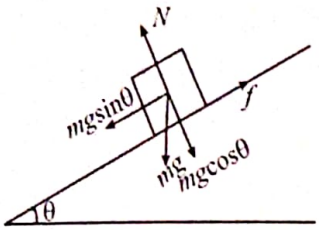
$$\int u\,du = \int (6 - 2.4x)dx$$

$$\therefore x = \frac{60}{24} = \frac{5}{2} \quad \text{[from (i)]}$$

$$\frac{u^2}{2} = 6x - \frac{2.4x^2}{2}$$

$x = 2.5$ m

41. (*a*)



As the block is at rest,

$N = mg\cos\theta$

$f = mg\sin\theta$

Force exerted by plane on block $F_{net} = \sqrt{N^2 + f^2}$

$= \sqrt{m^2g^2\cos^2\theta + m^2g^2\sin^2\theta}$

$F_{net} = mg$

42. (*b*) $F_1 = mg\sin\theta + \mu mg\cos\theta$

$F_2 = mg\sin\theta - \mu mg\cos\theta$

Given than $F_1 = 2F_2$

$\therefore (mg\sin\theta + \mu mg\cos\theta) = 2(mg\sin\theta - \mu mg\cos\theta)$

On solving, we get $\tan\theta = 3\mu$

43. (*a*) Downward retardation means upward acceleration.

$g' = g + a$

Now $t = \sqrt{\dfrac{2L}{g'\sin\theta}}$ or $t = \sqrt{\dfrac{2L}{(g+a)\sin\theta}}$

44. (*a*) We know that static frictional force $f_s = \mu sN$

$f_s = \mu_s mg\cos\theta$

$$f_s = 0.8 \times m \times \frac{10 \times \sqrt{3}}{2} = 4\sqrt{3}\text{ m}$$

As friction balances block's mass component along the incline

$f_s > mg\sin 30°$

$\therefore f = mg\sin 30° = 10$ N

$$\Rightarrow \frac{m \times 10}{2} = 10$$

$\Rightarrow m = 2$ kg

45. (*a*) We have, length of incline,

$$s = \frac{1}{2}(g\sin\theta)t^2 \quad ...(i)$$

friction force,

$f = \mu N = \mu mg\cos\theta$

$a = (\sin\theta - \mu\cos\theta)g$

$$\therefore s = \frac{1}{2}(\sin\theta - \mu\cos\theta)g \times (2t)^2 \quad ...(ii)$$

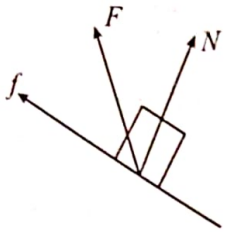
from (i) and (ii), we have,

$\sin\theta = (\sin\theta - \mu\cos\theta)4$

$$\therefore \mu = \frac{3}{4}\tan\theta$$

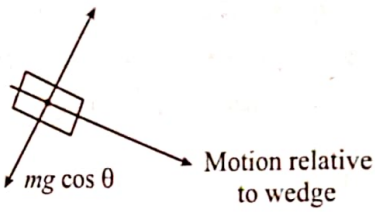
46. (*a*) $mg\sin\theta = (10)(10)(3/5) = 60$N

This 60N > 30N.Therefore friction force f will act in upward direction



$F = \sqrt{N^2 + f^2}$ = net force by plane on the block.

47. (*b*) During motion of block, a component of its acceleration comes in the direction of $mg\cos\theta$. Therefore,



$mg\cos\theta > N$

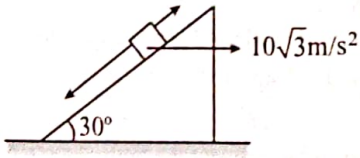
48. (*b*) When a body is in free fall, it experiences a downward acceleration of g.

Thus, the contact force between block and wedge is zero.

$a\sin\theta = g\cos\theta$

$a = g\cot\theta$

49. (*b*) $10\sqrt{3}\cos 30° = 10\sqrt{3}\dfrac{\sqrt{3}}{2} = 15\text{m/s}^2$



$g\sin 30° = 5$ m/s^2

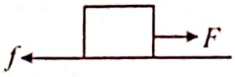
$\therefore a = 15 - 5 = 10$ m/s^2

$$\therefore s = \frac{1}{2}at^2$$

$$1 = \frac{1}{2}(10)t^2 \text{ or } t = \frac{1}{\sqrt{5}}\text{ s}$$

Exercise-2 (Learning Plus)

1. (*a*) $F < f_{smax}$



friction = F

For $F > f_{max}$

friction = constant = kinetic friction

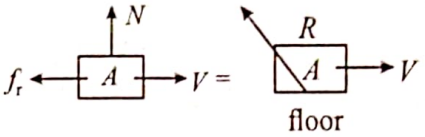
2. (*a*) Monkey is moving up due to friction force

$f - mg = ma$

$f = m(a + g)$

towards up.

3. (*c*) Floor will provide the normal force and friction force the net reaction is provide by the floor is R.



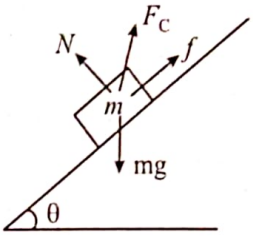
4. (*b*) $f_{max} > mg \sin\theta$

$\theta \uparrow \sin\theta \uparrow$

at this condition block remains rest when

$mg \sin\theta > f_{max}$

slipping slant



For θ < angle of repose,

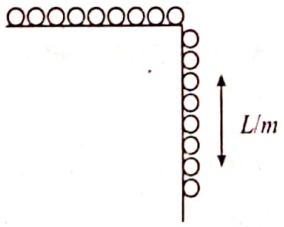
$F_C = mg$

For θ > angle of repose

as $\theta \uparrow f = \mu mg \cos\theta \downarrow$

$N = mg \cos\theta \downarrow$

5. (*b*) $\mu\lambda L\left(1 - \frac{1}{n}\right)g = \lambda\frac{L}{n}g$

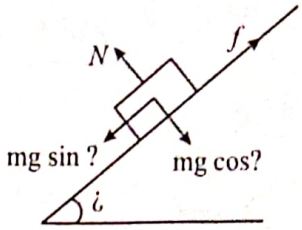


$\mu = \frac{1}{n-1}$

6. (*a*) $N = mg \cos\theta$

$f_s \le \mu N$

$mg \sin\theta \le \mu mg \cos\theta$

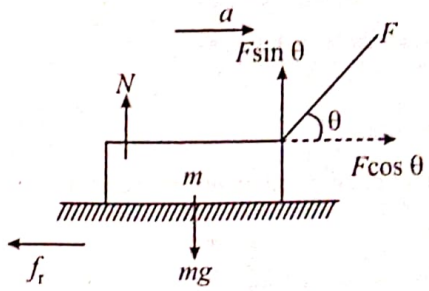


$\mu \ge 1$

7. (*a*) Friction not depend on surface Area so angle remain same.

$\therefore$ Angle = 30°

8. (*c*)



$F \sin\theta + N = mg$

or $N = mg - F \sin\theta$ ...(i)

$f_r = \mu N$ ...(ii)

$F \cos\theta - f_r = ma$ ...(iii)

on solving (i), (ii) and (iii)

$$a = \frac{F\cos\theta - \mu(mg - F\sin\theta)}{m}$$

$$a = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$$

9. (*a*) $V^2 = 2 \times g \sin\theta \times l$ ...(i)

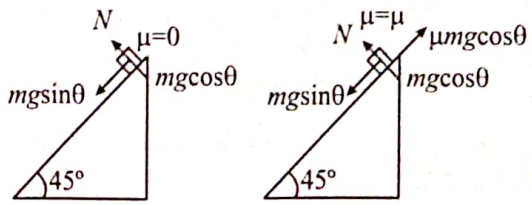
$\frac{v^2}{n^2} = 2 \times (g\sin\theta - \mu g\cos\theta)$ ...(ii)

from (i) and (ii)

$$\sin\theta\left(1 - \frac{1}{n^2}\right) = \mu\cos\theta$$

$$\mu = \tan\theta\left(1 - \frac{1}{n^2}\right)$$

10. (*a*)



Let acceleration in Ist case is a_1 and that in second case is a_2

Now, $\frac{1}{2}a_1t^2 = \frac{1}{2}a_2(2t)^2 \Rightarrow a_2 = \frac{a_1}{4}$ ...(i)

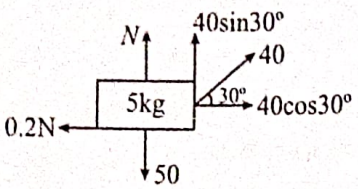
Clearly $a_1 = \frac{mg\sin\theta}{m} = g\sin\theta$ ...(ii)

and $a_2 = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = g\sin\theta - \mu g\cos\theta$ ...(iii)

From (i), (ii) and (iii),

We get $\mu = 0.75$.

11. (*a*)



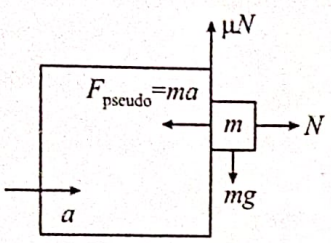
$N = 50 - 40\sin 30 = 30$

$a = \frac{40\cos 30° - 0.2\times 30}{5}$

$= 5.73$ m/sec^2

12. (*d*) μ does not depend on normal reaction.

13. (*c*)

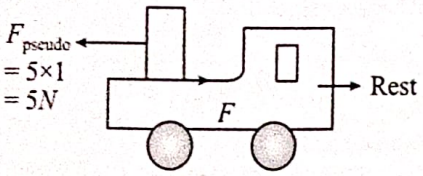


Solving from the frame of cart, we get

$N = ma$, $mg = \mu N$

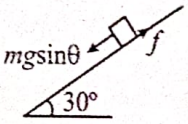
$\Rightarrow mg = \mu ma \Rightarrow a = \frac{g}{\mu}$

14. (*a*) Solving from the frame of truck



$f \le \mu mg = 6 \Rightarrow f = 5$N.

15. (*a*)



since; $\mu > \tan\theta$

the block will not slide therefore $f = mg\sin\theta$

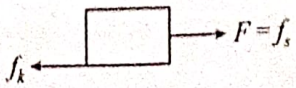
$= 2 \times 9.8 \times \frac{1}{2} = 9.8$ N.

16. (*a*) Apply Newton's law for system along the string

$m_B g = \mu(m_A + m_C) \times g$

$\Rightarrow m_C = \frac{m_B}{\mu} - m_A = \frac{5}{0.2} - 10 = 15$ kg

17. (*d*)

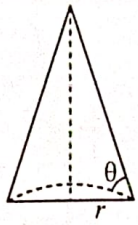


$a = \frac{f_s - f_k}{m} = \frac{(\mu_s - \mu_k)mg}{m} = (\mu_S - \mu_k)g$

$= (0.5 - 0.4)10 \quad = 1$ m/sec^2

18. (*a*) [0007 cm]

$V = \frac{1}{3}\pi r^2 h$



$\mu_s = \tan\theta = \frac{h}{r}$

$r = \frac{h}{\tan\theta} = \frac{h}{\mu_s}$

So, $V = \frac{1}{3}\pi\left(\frac{h}{\mu_s}\right)^2 h$

$\Rightarrow h = \left(\frac{3V\mu_s^2}{\pi}\right)^{1/3} = 7$ cm

19. (*d*) [10 sec]

$a_x = \frac{6}{1+5} = 1$ ms^{-2}

$N = 1 \times 1 = 1$ N

$f_r = \mu N = \frac{1}{2}$

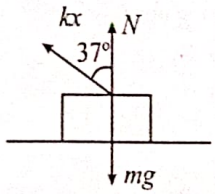
$a = \frac{f_r}{m} = \frac{1}{2}$

$0 = v_0 - \frac{1}{2} \times t$

$t = 10$ sec.

20. (*b*) [0070]

$l_{spring} = 1\sec 37° = \frac{5}{4} = 1.25$ m



$x = 0.25$ m $\qquad kx\cos 37° + N = mg$

$N = 10 - 10 \times 0.25 \times \frac{4}{5} = 8$

$kx\sin 37° - \mu N = ma$

$2.5 \times \frac{3}{5} - \mu \times 8 = 1a$

$0.7 = a$

$a = 70$ cm/s^2

21. (*d*) Block C,

$19.5\,mg + mg\sin 30° - T = ma$

$20\,mg - T = ma$ ...(i)

Block $(A + B)$,

$T = ma$ ...(ii)

from (i) and (ii)

$f = mg$

$N = ma = 4mg$

for μ_{min}, $f = f_{max} = \mu N = mg$

$\mu = \frac{1}{4}$

22. (*a*) Along the belt

$f - mg\sin 37° = ma$

maximum frictional force may be $f = \mu mg\cos 37°$

$a_{max} = g(\mu\cos 37° - \sin 37°) = 2\text{m/s}^2$

$4 = \frac{1}{2} \times 2 \times t^2, \quad t = 2\text{s}$

23. (*c*) $T > \mu mg \quad \Rightarrow \quad T' - 2T > \mu mg$

$F - 2T' > \mu mg \quad \Rightarrow \quad T' > 3\mu mg$

$F > 7\mu mg$

24. (*c*) From FBD of block,

$2N\cos 45° = Mg$ ...(i)

for wedge to remain at rest: $N\sin 45° < \mu N'$ ...(ii)

and $N = mg + N\cos 45°$ ...(iii)

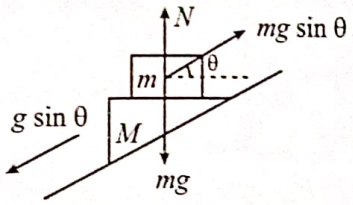
from (ii), $N = \frac{1}{\sqrt{2}} \le \mu\left(mg + N\frac{1}{\sqrt{2}}\right)$

$\frac{1}{\sqrt{2}}(1-\mu) \le \mu mg$

$\frac{Mg}{\sqrt{2}\sqrt{2}}(1-\mu) \le \mu mg$

$M \le \frac{2\mu m}{1-\mu}$

25. (*b*)



$N + mg\sin^2\theta = mg$

$N = mg\cos^2\theta$

26. (*a*) $N_1 = 2g = 2 \times 10 = 20\text{N}$

$F - \mu N_1 - \mu N_2 = 6 \times 1.5$

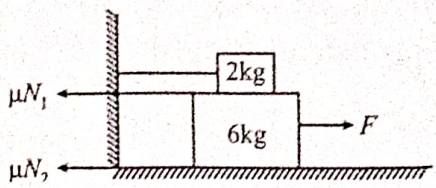
$N_2 = (6 + 2)g = 8g = 8 \times 10 = 80\text{N}$

$F = 9 + \mu N_1 + \mu N_2$

$F = 9 + 0.4 \times 2 \times 10 + 0.4 \times 8 \times 10$

$= 9 + 8 + 32$

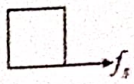
$= 49\text{ N}$



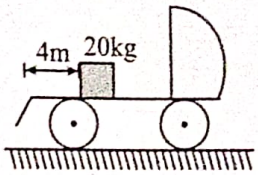
27. (*c*) $f_{s,max} = \mu\, mg$

$= 0.15 \times 20 \times 10$

$= 30\text{ N}$



$a_{block} = \frac{30}{20} = \frac{3}{2}\text{ m/s}^2$



$a_{truck} = 2\text{ m/s}^2$

$a_{rel} = \frac{1}{2}\text{ m/s}^2$

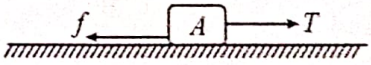
$d = \frac{1}{2}a_{rel}t^2$

$4 = \frac{1}{2} \times \frac{1}{2} \times t^2$

$t = 4$ sec.

So, distance travelled by truck $= \frac{1}{2} \times 2 \times 4 \times 4 = 16$ m

28. (*a*) $T - f = ma_A \qquad (a_A = 2a_0)$



$T = f + ma_A$

$T = \mu mg + m \times 2a_0 = 0.3 \times 50 \times 10 + 50 \times 2 \times 2 = 350\text{N}$

29. (*c*) For upward motion

$t_a = \frac{u}{g\sin\alpha + \mu g\cos\alpha}$ ...(i)

$u^2 = 2(g\sin\alpha + \mu g\cos\alpha)s$ ...(ii)

for downward motion

$t_d = \frac{v}{g\sin\alpha - \mu g\cos\alpha}$ ...(iii)

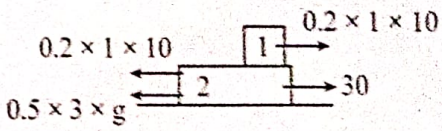
$v^2 = 2(g\sin\alpha - \mu g\cos\alpha)s$ ...(iv)

Solving equation (i), (ii), (iii) and (iv)

$\frac{t_a}{t_d} = \sqrt{\frac{\sin\alpha - \mu\cos\alpha}{\sin\alpha + \mu\cos\alpha}} = \frac{1}{2}$

$\Rightarrow \mu = 0.6\tan\alpha$

30. (*a*)



$1a_1 = 0.2 \times 1 \times 10 = 2\text{ m/s}^2$

$2a_2 = 30 - 2 - 15 = 13$

$a_2 = 6.5\ m/s^2$

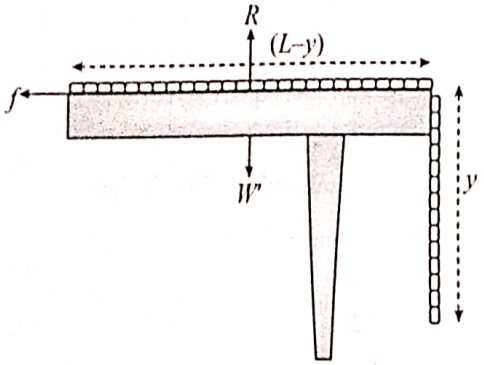
$a_{12} = -4.5\ m/s^2$

$s_{12} = \frac{1}{2} a_{12} t^2$

$-1 = \frac{-9}{4} t^2$

$t = \frac{2}{3}$ sec.

31. (*c*) Let y be the maximum length of the chain can be hold out side the table without sliding.



Length of chain on the table = $(L - y)$

Weight of part of the chain on table $W' = \frac{M}{L}(L-y)g$

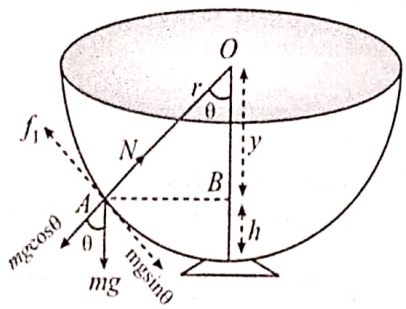
Weight of hanging part of the chain $W = \frac{M}{L} yg$

For equilibrium: limiting force of friction = weight of hanging part of the chain

$$\mu R = W \Rightarrow \mu W' = W \Rightarrow \mu \frac{M}{L}(L-y)g = \frac{M}{L} yg$$

$$\Rightarrow \mu L - \mu y = y \Rightarrow y = \frac{\mu L}{1+\mu}$$

32. (*a*) The insect will crawl up the bowl till the component of its weight along the bowl is balanced by limiting frictional force. So, resolving weight perpendicular to the bowl and along the bowl,



$N = mg\cos\theta, f_L = mg\sin\theta \Rightarrow \tan\theta = \Rightarrow \tan\theta = \mu$

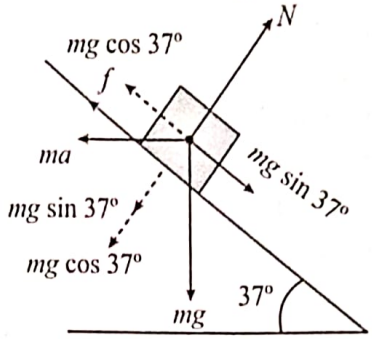
$[\because f_L = \mu N]$

$$\Rightarrow \sqrt{\frac{(r^2 - y^2)}{y}} = \mu \Rightarrow y = \frac{r}{\sqrt{1+\mu^2}}$$

$$\text{So, } h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$$

33. (*b*) $N = m(g\cos 37° + a\sin 37°) = 1(9.8 \times 0.8 + 3 \times 0.6)$
$= 9.64N$

$mg\sin 37° = ma\cos 37° + f$



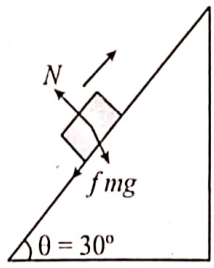
$f = 1(9.8 \times 0.6 - 3 \times 0.8) = 3.48$ N

and $f = \mu N \quad \therefore \quad \mu = \frac{f}{N} = \frac{3.48}{9.64} = 0.36$

34. (*d*) For upward motion: upward retardation

$$a_1 = \frac{\mu N + mg\sin\theta}{m}$$

$$a_1 = \mu g\cos 30° + g\sin 30° = (\sqrt{3}\mu + 1)\frac{g}{2}$$

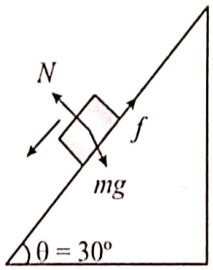


length of plane = s
time of ascent = t_1

$$\because s = \frac{1}{2} a_1 t_1^2 \quad \therefore \quad t_1 = \sqrt{\frac{2s}{a_1}} = \sqrt{\frac{4s}{(\sqrt{3}\mu + 1)g}}$$

For downward motion: downward acceleration

$$a_2 = \frac{mg\sin\theta - \mu N}{m}$$



time of descent = t_2

$$a_2 = g\sin 30° - \mu g\cos 30° = (1 - \sqrt{3}\mu)\frac{g}{2}$$

$$\Rightarrow t_2 = \sqrt{\frac{2s}{a_2}} = \sqrt{\frac{4s}{(1-\sqrt{3}\mu)g}}$$

Now according to question $2t_1 = t_2$

$$\Rightarrow 2\sqrt{\frac{4s}{(\sqrt{3}\mu + 1)}} = \sqrt{\frac{4s}{(1-\sqrt{3}\mu)g}}$$

$$\Rightarrow \frac{1-\sqrt{3}\mu}{1+\sqrt{3}\mu} = \frac{1}{4} \Rightarrow \mu = \frac{\sqrt{3}}{5}$$

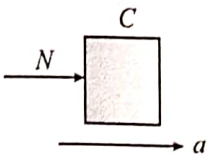
35. (*b*) Horizontal acceleration of the system is

$$a = \frac{F}{2m+m+2m} = \frac{F}{5m}$$

Let N be the normal reaction between B and C

Free body diagram of C gives

$$N = 2ma = \frac{2}{5}F$$



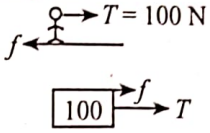
Now B will not slide downward if

$\mu N \geq m_B g$

or $\mu\left(\frac{2}{5}F\right) \geq mg$ or $F \geq \frac{5}{2\mu}mg$

So, $F_{min} = \frac{5}{2\mu}mg$

36. (*a*) Both move together



for man

$T - f = ma$

$100 - f = 50a$ ...(i)

for block

$T + f = ma$

$100 + f = 100a$ ...(ii)

from (i) and (ii)

$200 = 150a$

$a = \frac{4}{3}$

$f = 100 - \frac{200}{3} = \frac{100}{3}$ N $(\leftarrow)$

37. (*a*) Normal force,

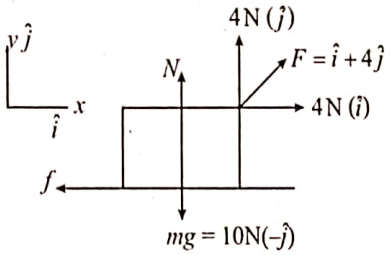
$N = (10-4)\hat{j} = 6\hat{j}$

Maximum frictional force,

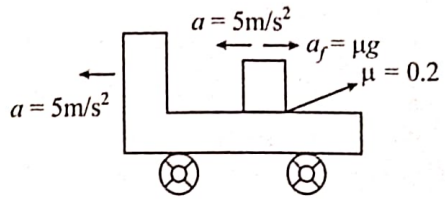
$f = \mu N(-\hat{i}) = 0.3 \times 10(-\hat{i}) = 3(-\hat{i})$

Maximum frictional force is $f > 1$N, so that the frictional force act only equal to the force in $(\hat{i})$ direction but in opposite direction;

$f = 1(-\hat{i})$



38. (*a*) We have to tell about the velocity of block with respect to ground not wrt truck. Hence whatever is the speed of truck, will be the speed of block.



Exercise-3 (JEE Advanced Level)

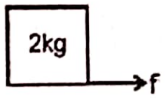
1. (*a*) When F is less than $\mu_s mg$ then tension in the string is zero.

When $\mu_s mg \leq F < \mu_s 2mg$ then friction on block B is static.

If F is further increase friction on block B is kinetic.

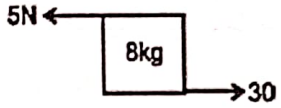
2. (*c*) Solving from the frame of elevator

$$g_{eff.} = g + \frac{g}{4} = 12.5$$



$$f = \frac{1}{5} \times 2 \times 12.5 = 5\text{N}$$

$$a_2 = \frac{5}{2} = 2.5 \text{ m/s}^2$$



$$a_1 = \frac{30-5}{8}$$

$$= \frac{25}{8} \text{ m/sec}^2$$

3. (*b*) When $\mu mg < \mu' m' g$ then man can not slide the body.

4. (*c*) Let velocity of projection be V and velocity of the block when it returns back = V'

then $V > V'$ (since some K.E. is lost to friction)

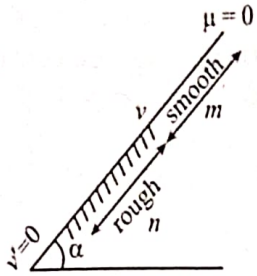
Hence average velocity during ascent > average velocity during descent

$\Rightarrow t_a < t_d$

5. (*a*) For chain to move with constant speed P needs to be equal to frictional force on the chain. As the length of chain on the rough surface increases. Hence the friction force $f_k = \mu_k N$ increases.

Hence, A is correct choice.

6. (*a*)



On smooth surface $a_1 = g \sin \alpha$

$\therefore v^2 = u^2 + 2a_1s_1$

$= 0 + 2\, g \sin \alpha . m$

On rough surface

$a_2 = g \sin \alpha - \mu g \cos \alpha$

$\therefore v'^2 = v^2 + 2a_2s_2$

$0 = 2mg \sin \alpha + 2g (\sin \alpha - \mu \cos \alpha)n$

$\Rightarrow \mu = \left[\dfrac{m+n}{n}\right] \tan \alpha$

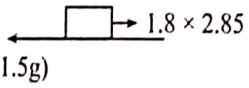
7. (*a*) Apply system equation

$\dfrac{m}{4} g = \dfrac{3m}{4} g \times \mu$

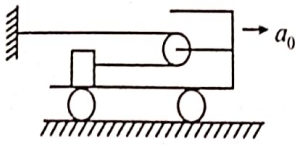
$\Rightarrow \mu = \dfrac{1}{3} = 0.33$

8. (*b*) $1.8\, t - \mu_k (15) = 1.5\,(1.2\, t - 2.4)$

for $t = 2.85$ sec.

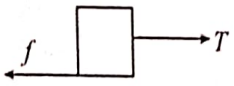


9. (*b*)



If acceleration of the car is a_0, acceleration of the block

$2a_0 = 2 \times 2 = 4$ m/s^2 (→)



$f = \mu N = 0.3 \times 50 \times 10 = 150$ N

$T - f = ma$

$\Rightarrow T - 150 = 50 \times 4$

$\Rightarrow T = 350$ N.

10. (*c*) Since there is relative motion between block and conveyor, there will be frictional force

$= \mu N = 0.2 \times m \times 10$

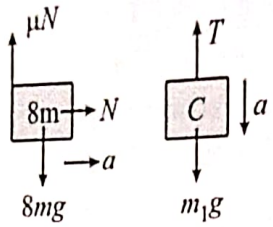
retardation $= \dfrac{f}{m} = 2$ m/sec^2

Now $v = u + at$

$\Rightarrow v = 5 + (-2)\, 4$

$= -3 = 3$ m/s opposite so the direction of initial velocity of the block.

11. (*d*) FBD of *A*



If the acceleration of '*C*' is *a*

For block '*A*' $N = 8\, ma$

$8\, mg - \mu N = 8\, ma$ ...(i)

and acceleration a can be written by the equation of system $(A + B + C)$ ...(ii)

$m_1 g = (10\, m + m_1)\, a$ ...(iii)

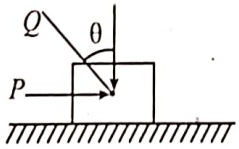
$8\, mg = \mu 8m \left(\dfrac{m_1 g}{10m + m_1}\right)$

$10\, m + m_1 = \mu\, m_1$

$10\, m = (\mu - 1)\, m_1 \Rightarrow m_1 = \dfrac{10m}{\mu - 1}$

12. (*a*) $N = mg + Q \cos \theta$

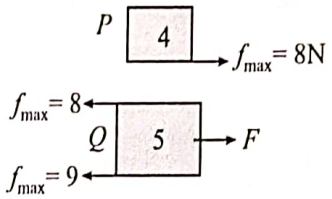
$P + Q \sin \theta \le \mu N$



$\Rightarrow \mu \ge \dfrac{P + Q \sin \theta}{N}$, $\mu \ge \dfrac{P + Q \sin \theta}{mg + Q \cos \theta}$

13. (*c*) So block '*Q*' is moving due to force while block 'P' due to friction.

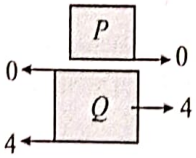
Friction direction on both blocks as shown.



First block '*Q*' will move and *P* will move with '*Q*' so by FBD taking '*P*' and '*Q*' as system

$F - 9 = 0 \quad \Rightarrow F = 9$ N

When applied force is 4 *N* then FBD



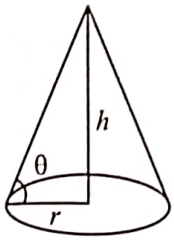
4 kg block is moving due to friction and maximum friction force is 8 N.

So acceleration $= \dfrac{8}{4} = 2$ m/s$^2 = a_{max}$.

Slipping will start at when *Q* has +ve acceleration equal to maximum acceleration of *P* i.e. 2 m/s^2.

$F - 17 = 5 \times 2 \quad \Rightarrow \quad F = 27$ N.

14. (*b*)



In limiting condition $\mu = \tan\theta = \frac{h}{r}$

$\Rightarrow h = \mu r$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2, \quad \mu r = \frac{1}{3}\pi\mu r^3$

15. (*c*) For the sliding not to occur when

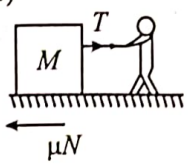
$\tan\theta \le \mu$

$\tan\theta = \frac{dy}{dx} = \frac{2x}{a} = \frac{2\sqrt{ya}}{a} = 2\sqrt{\frac{y}{a}}$

$\therefore 2\sqrt{\frac{y}{a}} \le \mu$ or $y \le \frac{a\mu^2}{4}$

16. (*b,d*) $R = \sqrt{f^2 + N^2}$

17. (*a,b,c*)

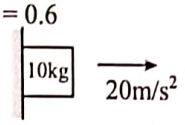


$T - \mu N = ma$

As $T \uparrow$

Man can have tendency to move

18. (*b*) $\mu = 0.6$



(*a*) Acceleration of box = 20 m/s²

(when consider as system)

Force on Box

$F = 200$ N

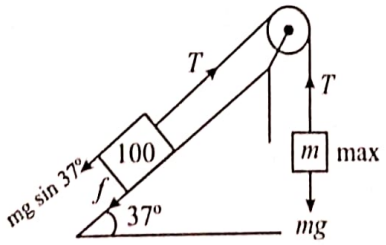
$N = 200$ N

$f_{max} = \mu N = 0.6 \times 200 = 120$ N

(*b*) $f_{required} = mg = 10 \times 10 = 100$ N

(c) $f_c = \sqrt{f^2 + N^2} = \sqrt{(100)^2 + (200)^2} = 100\sqrt{5}\,N$

19. (*a*)



For *m*,

$T = mg$ ...(i)

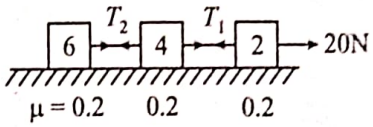
for 100 kg mass,

$T + \mu(100g\cos 37°) = 100g\sin 37°$ ...(ii)

from (i) and (ii)

$m = 36$ kg

20. (*a*)



Let the blocks does not move

then $T_1 = 20 - 4$

$T_2 = T_1 - 8 = 20 - 4 - 8 = 8$

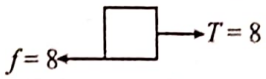
Since $T_2 <$ max possible friction force for 6 kg block

Hence it will be at rest and this assumption is right.

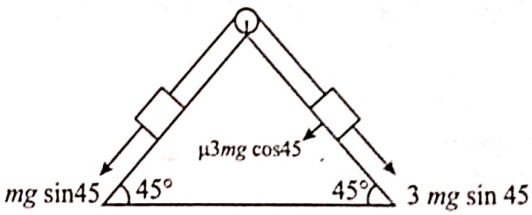
Therefore tension in the string connecting 4kg and 6 kg block = 8N

21. (*d*) Friction of 4 kg block = $\mu N = 0.2 \times 4 \times 10 = 8$N

22. (*b*) Friction force on 6 kg block = 8N



23. (*d*)



Applying Newton's law for the system of m and 3m along the length of the string

we get

$3mg\sin 45 - \mu 3mg\cos 45 - mg\sin 45 = (3m + m)a$

$\Rightarrow \mu = \frac{2}{5}$ as $a = \frac{g}{5\sqrt{2}}$

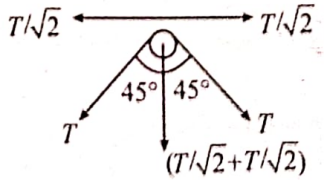
24. (*b*) Now making the F.B.D. of *m* we get

$T - mg\sin 45 = m\,a$

$\Rightarrow T = \frac{mg}{5\sqrt{2}} + \frac{mg}{\sqrt{2}}$

$T = \frac{6\,mg}{5\sqrt{2}}$

25 (*a*) Now from F.B.D. of pulley we get



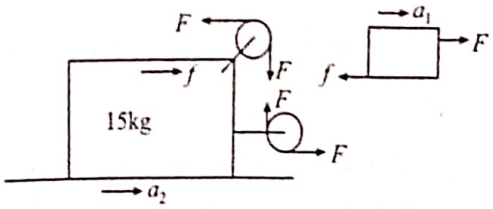
Force exerted by string on pulley

$= T\sqrt{2} = \frac{6mg}{5\sqrt{2}} \times \sqrt{2} = \frac{6mg}{5}$ (downward)

26. (*a*) **27.** (*b*)

Sol. (26 to 27)

First, let us check upto what value of F, both blocks move together. Till friction becomes limiting, they will be moving together. Using the FBDs



10kg block will not slip over the 15 kg block till acceleration of 15 kg block becomes maximum as it is created only by friction force exerted by 10 kg block on it

$a_1 > a_{2(max)}$

$\frac{F-f}{10} = \frac{f}{15}$ for limiting condition as f maximum is 60 N.

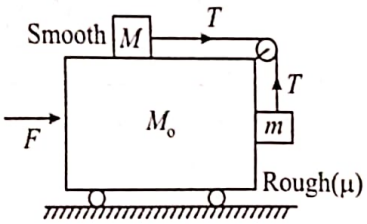
$F = 100$ N.

Therefore for $F = 80$ N, both will move together.

Their combined acceleration, by applying NLM using both as system $F = 25a$

$a = \frac{80}{25} = 3.2$ m/s^2

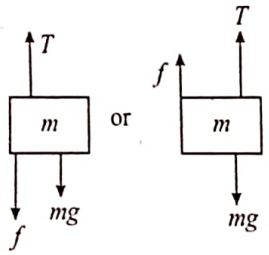
28. (*a,c,d*)



(*a*) When $F = 0$

No friction b/w m & M_0 so system will move.

(*b*) When F is applied then friction develop a range for which M and m are stationary w.r.t M_0, such that

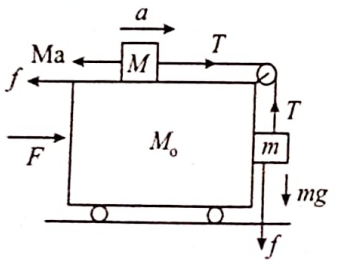


(*c*) Limiting friction between M_0 and m is $\mu\, ma$

∴ Dependent on a

(*d*) When Pseudo acts on M is equal to T, then $f = 0$

29. (*c*)



Use Pseudo concept

$T = Ma$ ...(a)

$T = f + mg$

$\Rightarrow T = \mu ma + ma$ ...(b)

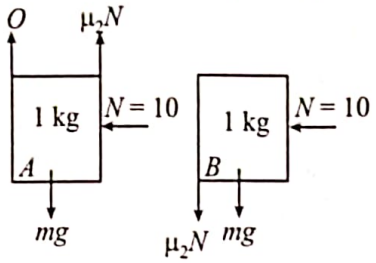
On using (*a*) and (*b*)

$Ma = \mu ma + mg$

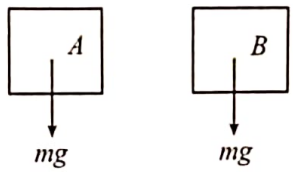
So,

$$F = (M_0 + M + m)\frac{mg}{M - \mu m}$$

30. (*b*) (i) FBD in (case (i)) $\{\mu_1 = 0, \mu_2 = 0.1\}$

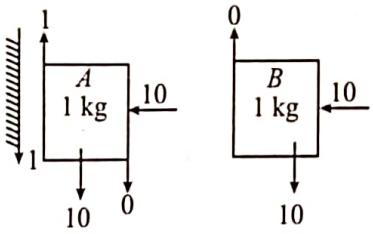


While friction's work is to oppose the relative motion and here if friction comes then relative motion will start and without friction there is no relative motion so both the block move together with same acceleration and friction will not come.



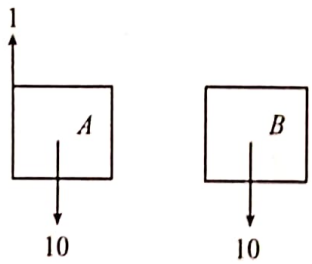
$\Rightarrow a_A = a_B = 10$ m/s^2

(ii)



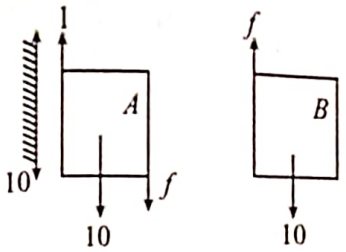
Friction between wall and block A oppose relative motion since wall is stationary so friction wants to stop block A also and maximum friction will act between wall and block while there is no friction between block.

Note: Friction between wall and block will oppose relative motion between wall and block only it will not do anything for two block motion.

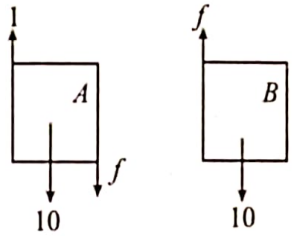


$a_A = 9$ m/s^2 ; $a_B = 10$ m/s^2

(iii)

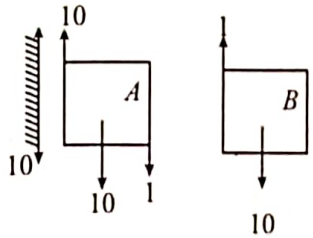


Friction between wall and block will be applied maximum equal to 1N but maximum friction available between block A and B is 10 N but if this will be there then relative motion will increase while friction is to oppose relative motion. So friction will come less than 10 so friction will be f that will be static.



by system $(20 - 1) = 2 \times a \Rightarrow a = \frac{19}{2} = 9.5 \text{ m/s}^2$

(iv)



$$a_A = \frac{11-10}{1} = 1 \text{ m/s}^2$$

$$a_B = \frac{10-1}{1} = 9 \text{ m/s}^2$$

31. (*c*) The acceleration of two block system for all cases is a $= 2 \text{ m/s}^2$

In option (p) the net force on 2 kg block is frictional force

$\therefore$ Frictional force on 2 kg block is

$f = 2 \times 2 = 4\text{N}$ towards right

In option (q) the net force on 4 kg block is frictional force

$\therefore$ Frictional force on 4 kg block is

$f = 4 \times 2 = 8\text{N}$ towards right

In option (r) the net force on 2 kg block is $2 \times 2 = 4\text{N}$

$\therefore$ Friction force f on 2 kg block is towards left.

$\therefore 6 - f = 2 \times 2$ or $f = 2\text{N}$

In option (s) the net force on 2 kg block is ma

$= 2 \times 2 = 4\text{N}$ towards right.

$\therefore$ Friction force on 2 kg block is 12N towards right.

32. (*d*) Fact based

33. [0002]

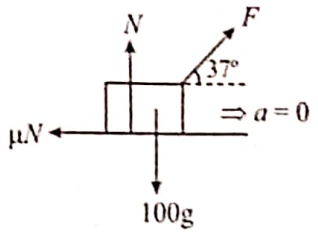
$a_{max} = \mu g = 0.2 \times 10 = 2 \text{ m/sec}^2$

$v = u + at$

$0 = 4 - 2t$

$t = 2$ sec

34. [0200]



$F \cos 37° = \mu N$ ...(i)

$F \sin 37° + N = 100\,g$ ...(ii)

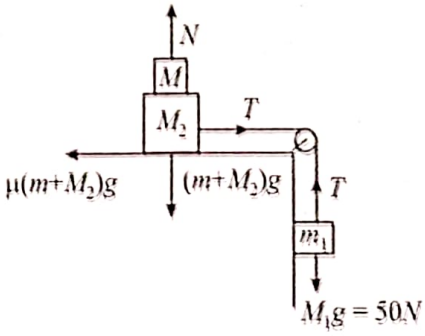
from (i) and (ii)

$$F = \frac{\mu mg}{\cos\theta + \mu \sin\theta} = 200 \text{ N}$$

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*a*)



$50 - T = 5 \times a$

$T - 0.15\,(m + 10)\,g = (10 + m)\,a$

$a = 0$ for rest

$50 = 0.15\,(m + 10)\,10$

$5 = \frac{3}{20}(m + 10)$

$\frac{100}{3} = (m + 10)$

$m = 23.3$ kg

2. (*a*) For equilibrium of the block net force be zero. Hence we can write.

$mg \sin\theta + 3 = P + \text{friction}$

$mg \sin\theta + 3 = P + \mu mg \cos\theta$

After solving, we get, $P = 32$ N

3. (*a*) $mg \sin\theta + 2 = \mu\, mg \cos\theta$ ...(*a*)

$10 - mg \sin\theta = \mu\, mg \cos\theta$ ...(*b*)

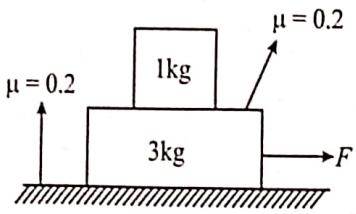
On adding (*a*) + (*b*)

$$12 = 2\,\mu mg\,\frac{\sqrt{3}}{2};\ \mu mg = \frac{12}{\sqrt{3}}$$

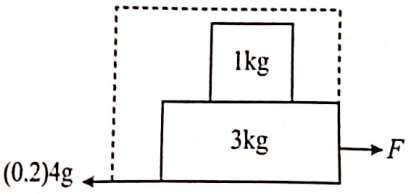
On $(a) - (b)$:

$$8 = 2mg \times \frac{1}{2};\ mg = 8;\ \mu = \frac{\sqrt{3}}{2}$$

4. (a)



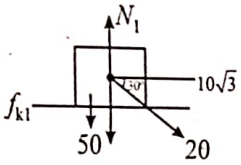
$a_{A\max} = \mu g = 2\ \text{m/s}^2$

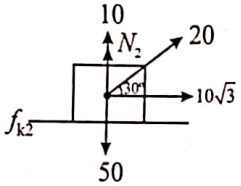


$F - 8 = 4 \times 2$

$F = 16\ \text{N}$

5. (b)





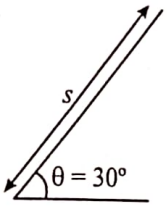
$N_1 = 60,\ N_2 = 40$

$$a_1 = \frac{10\sqrt{3} - 0.2 \times 60}{5}$$

$$a_2 = \frac{10\sqrt{3} - 0.2 \times 40}{5}$$

$a_1 - a_2 = 0.8\ \text{m/s}^2$

6. [346] For upward motion



$$\frac{1}{2}mv_0^2 = \left(mg\frac{S}{2} + \mu mg\frac{\sqrt{3}}{2}S\right)$$

$$\frac{1}{2}\frac{mv_0^2}{4} = \left(mg\frac{S}{2} - \mu mg\frac{\sqrt{3}}{2}S\right)$$

For downward

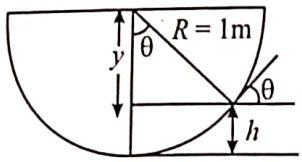
$$4 = \frac{(1+\mu\sqrt{3})}{(1-\mu\sqrt{3})}$$

$\Rightarrow 4 - 4\mu\sqrt{3} = 1 + \mu\sqrt{3}$

$\Rightarrow 3 = 5\mu\sqrt{3}$ $\quad \therefore \mu = \frac{\sqrt{3}}{5} \approx \frac{346.41}{1000}$

$I = 346$

7. (c)



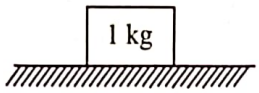
$\mu = \tan\theta$

$\Rightarrow \frac{3}{4} = \tan\theta$

$\Rightarrow \theta = 37°$

$\therefore h = R - R\cos\theta = 1 - 1 \times \frac{4}{5} = 0.2\ \text{m}$

8. [5]



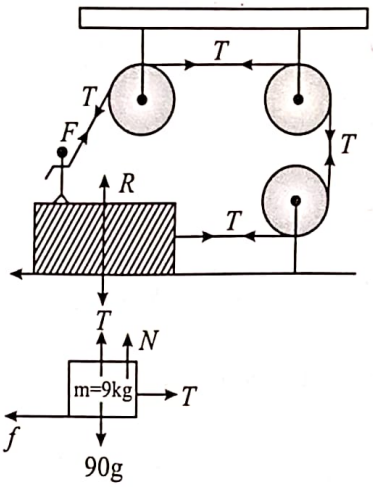
Minimum possible force

$$F = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$F_{\min} = \frac{\frac{1}{\sqrt{3} \times 1 \times 10}}{\sqrt{1+\frac{1}{3}}}$$

$F_{\min} = 5\text{N}$

9. [30]



$\therefore f = T$

$\Rightarrow \mu N = T$

$\Rightarrow \mu(90 - T) = T$

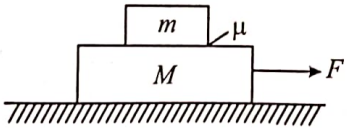
$\Rightarrow 0.5(90 - T) = T$

$\Rightarrow 90 - T = 2T$

$\Rightarrow 3T = 90$

$\Rightarrow T = 30\ N$

10. [21]



$$a = \frac{F}{M+m}$$

$$f = ma = m\frac{F}{M+m}$$

$$m\frac{F}{M+m} \le \mu mg \text{ (for no slipping)}$$

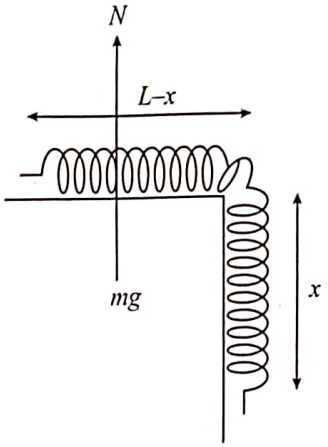
$$\Rightarrow F \le \mu(m+M)g$$

$$\therefore F_{max} = \frac{3}{7}(0.5+4.5)\times 9.8 = 21N$$

11. [2] Mass per unit length = λ

$N = mg = \lambda(L-x)g$

$f_{s,max} = \mu_s N$



$f_{s,max} = (0.5)(\lambda)(L-x)g$

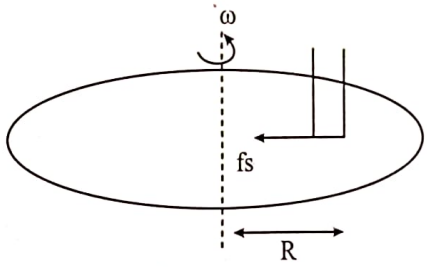
And also $f_{s,max} = m_x g$

$0.5\lambda(L-x)g = \lambda x g$

$$\frac{L-x}{2} = x$$

$$\frac{L}{2} = \frac{3x}{2} \Rightarrow x = \frac{L}{3} = \frac{6}{3} = 2 \text{ m}$$

12. (*b*) For beaker to move with discs



$f_s = m\omega^2 R$

We know that $f_s \le f_{s,max}$

$m\omega^2 R \le \mu mg$

$$R \le \frac{\mu g}{\omega^2}$$

13. [24] $f_{s\,max} = \dfrac{mv^2}{R}$

$$\mu mg = \frac{mv^2}{R}$$

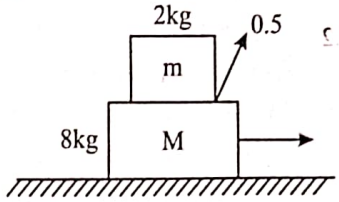
$$v = \sqrt{\mu R g}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{R_2}{R_1}}$$

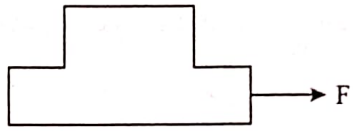
$$\frac{v_2}{30} = \sqrt{\frac{48}{75}}$$

$v_2 = 24$ m/s

14. (*c*) $(a_A)_{max} = 0.5g = 4.9$ m/s^2



For moving together



$F_{max} = m_T a_A$

$= 10 \times 4.9$

$= 49$ N

15. (*d*) For 4 kg block

$4g - T = 4a$

For 40 kg block

$T - 40g \times 0.02 = 40\,a$

Adding both equation

$40 - 8 = 44a$

$$a = \frac{32}{44} = \frac{8}{11} \text{ m/s}^2$$

16. (*b*) In frame of belt

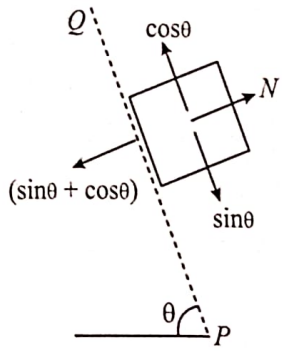
$a = \mu g = 4$ m/s^2, $v = 2$m/s, $u = 0$ m/s

$v^2 = u^2 + 2as$

$\Rightarrow s = 0.5$ m

JEE ADVANCED

17. (*a,c*)



$f = 0$, If $\sin\theta = \cos\theta \Rightarrow \theta = 45°$

f towards Q, $\sin\theta > \cos\theta \Rightarrow \theta > 45°$

f towards P, $\sin\theta < \cos\theta \Rightarrow \theta < 45°$

18. (*d*) Block will not slip if

$(m_1 + m_2)\, g \sin\theta \le \mu m_2\, g \cos\theta$

$3 \sin\theta \le \frac{3}{10}\,(b) \cos\theta$

$\tan\theta \le \frac{1}{5} \Rightarrow \boxed{\theta \le 11.5°}$

(P) $\theta = 5°$ friction is static $f = (m_1 + m_2)g \sin\theta$

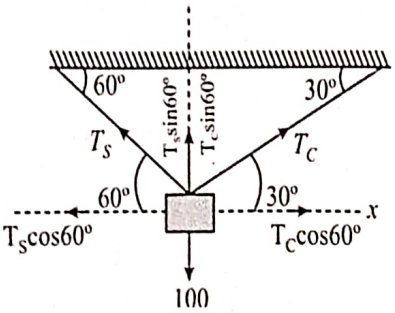
(Q) $\theta = 10°$ friction is static $f = (m_1 + m_2)g \sin\theta$

(R) $\theta = 15°$ friction is kinetic $f = \mu m_2 g \cos\theta$

(S) $\theta = 20°$ friction is kinetic $f = \mu m_2 g \cos\theta$

19. [2] Let T_S = tension in steel wire

T_C = Tension in copper wire in x direction



$T_C \cos30° = T_S \cos60°$

$T_C \times \frac{\sqrt{3}}{2} = T_S \times \frac{1}{2}$

$\sqrt{3}\, T_C = T_S$

in y direction

$T_C \sin30° + T_S \sin60° = 100$

$\frac{T_C}{2} + \frac{T_S\sqrt{3}}{2} = 100$

Solving equation (i) and (ii)

$T_C = 50\text{N}$

$T_S = 50\sqrt{3}N$

We know

$\Delta L = \frac{FL}{AY}$

$= \frac{\Delta L_C}{\Delta L_S} = \frac{T_C L_C}{A_C Y_C} \times \frac{A_S Y_S}{T_S L_S}$

On solving above equation

$\frac{\Delta L_C}{\Delta L_S} = 2$

CHAPTER

8 Work, Power and Energy

Exercise-1 (Topicwise)

1. (*c*) W = (force) (displacement) = (force) (zero) = 0

2. (*a*) Joule = (Newton) (Meter) = $\frac{4\text{ Newton}}{4} \times \frac{4\text{ Metre}}{4}$

$= \frac{\text{Joule}}{16}$

Hence, 1 Joule = 16 joule (Joule is new unit of energy)

3. (*b*) Work done = Force × displacement

= Weight of the book × Height of the book shelf

4. (*b*) Work done does not depend on time.

5. (*b*) Work done = mgh = 10 × 9.8 × 1 = 98 J

6. (*a*) $W = \vec{F}.(\vec{r}_2 - \vec{r}_1) = (4\hat{i} + \hat{j} + 3\hat{k})(11\hat{i} + 11\hat{j} + 15\hat{k})$

W = 44 + 11 + 45 = 100 Joule

7. (*c*) $W = (3\hat{i} + c\hat{j} + 2\hat{k}).(-4\hat{i} + 2\hat{j} + 3\hat{k}) = 6\text{J}$

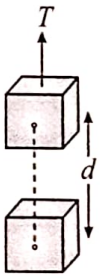
8. (*d*) $s = \frac{t^2}{4} \quad \therefore\ ds = \frac{t}{2}dt$

$$F = ma = \frac{md^2s}{dt^2} = \frac{6d^2}{dt^2}\left[\frac{t^2}{4}\right] = 3\text{N}$$

Now,

$$W = \int_0^2 F\,ds = \int_0^2 3\frac{t}{2}dt = \frac{3}{2}\left[\frac{t^2}{2}\right]_0^2 = \frac{3}{4}\left[(2)^2 - (0)^2\right] = 3\text{J}$$

9. (*c*) When the block moves vertically downward with acceleration $\frac{g}{4}$ then tension in the cord



$$T = M\left(g - \frac{g}{4}\right) = \frac{3}{4}Mg$$

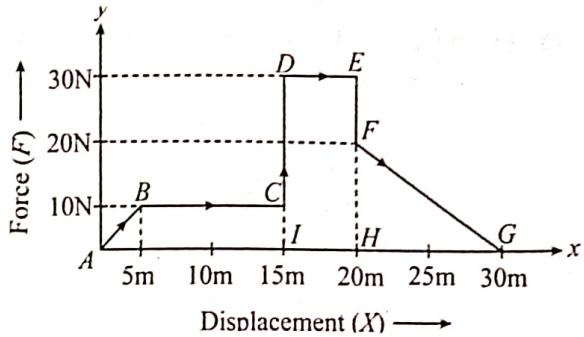
Work done by the cord = $\vec{F}.\vec{s} = Fs\cos\theta$

$$= Td\cos(180^\circ) = -\left(\frac{3\,Mg}{4}\right) \times d = -3Mg\frac{d}{4}$$

10. (*b*) $W = \frac{1}{2}kx^2$

If both wires are stretched through same distance then $W \propto k$. As $k_2 = 2k_1$ so $W_2 = 2W_1$

11. (*b*)



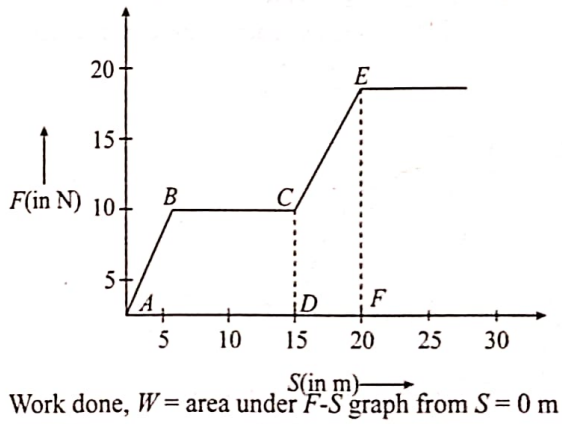
Work done = Area under the force-displacement graph

= Area of trapezium $ABCI$ + Area of rectangle $IDEH$ + Area of triangle FHG

$$= \left[\frac{1}{2}(10+15)10\right] + [5 \times 30] + \left[\frac{1}{2} \times 10 \times 20\right]$$

= 125 + 150 + 100 = 375 J.

12. (*b*)



Work done, W = area under F-S graph from S = 0 m to x = 20 m

= Area of trapezium $ABCD$ + Area of trapezium $CEFD$

$$= \frac{1}{2} \times (10+15) \times 10 + \frac{1}{2} \times (10+20) \times 5$$

= 125 + 75 = 200 J.

13. (*c*) 25 = 5 × 10 × cosθ so θ = 60°

14. (*a*) W = $\vec{F} \cdot (\vec{r}_2 - \vec{r}_1)$ = 46 J

15. (*c*) $S_1 = \frac{1}{2}g\,1^2, s_2 = \frac{1}{2}g\,2^2, S_3 = \frac{1}{2}g\,3^2$

$$S_2 - S_1 = \frac{1}{2}g\,3, S_3 - S_2 = \frac{1}{2}g\,5$$

$W_1 = (mg)\,S_1,\ W_2 = (mg)\,(S_2 - S_1)\,,\ W_3 = (mg)\,(S_3 - S_2)$

$W_1 : W_2 : W_3 = 1 : 3 : 5$

16. (*a*) $W = \int_0^1 F\,dx = \frac{1}{6}\text{J}$

17. (*c*) Let $\vec{r} = dx\,\hat{i} + dy\,\hat{j}, F = 3x\,\hat{i} + 4\hat{j}$

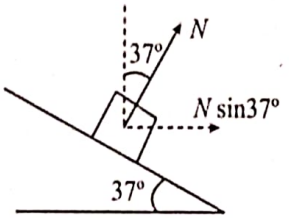
$$w = \int (3x\hat{i} + 4\hat{j})\cdot(dx\hat{i} + dy\hat{j})$$

$$= \int_{2m}^{3m} 3x dx + \int_{3m}^{0} 4dy = \left[\frac{3x^2}{2}\right]_{2m}^{3m} + [4y]_{3m}^{0}$$

$$= \left[\frac{3\times 9}{2} - \frac{3\times 2^2}{2}\right] + [0-12] = -4.5\text{J}$$

18. (*b*) Horizontal distance moved by block

$s = vt = 20$ m



Normal force on the block:

$N = mg\cos 37°$

horizontal components: $N\sin 37°$

So, work done $(W) = (N\sin 37°) \times (s)$

$W = Mg(\cos 37°)(\sin 37°) \times (20)$

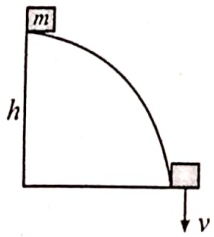
or $W = 960$ J

19. (*b*) Change in gravitational potential energy

= Elastic potential energy stored in compressed spring

$$\Rightarrow mg(h+x) = \frac{1}{2}kx^2$$

20. (*b*)



Here, mass of the block, $m = 3$kg

Initial speed of the block, $u = 0$ (as it starts from rest)

Final speed of the block, $v = 4$ m/s

Height, h (in this case the radius of quarter circle) = 2m

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2}(3\text{kg})(4\text{m/s})^2 = 24\text{ J}$$

The work done by the gravitational force is

$W_g = mgh = (3\text{ kg})\,(10\text{ m/s}^2)\,(2\text{m}) = 60$ J

If W_f is the work done by the friction, then according to work energy theorem,

$W_g + W_f = \Delta K$

or $W_f = \Delta K - W_g = 24\text{ J} - 60\text{ J} = -36$ J

As work done against friction is equal and opposite to work done by the friction,

$\therefore$ The amount of work done against friction is 36 J.

21. (*a*) Let m be mass of the truck. According to work-energy theorem

$W = K_f - K_i$

Work done in accelerating the truck from rest to speed v is

$$W_1 = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2 \quad (\because u = 0) \qquad \text{...(i)}$$

Work done in accelerating the truck from speed v to $2v$ is

$$W_2 = \frac{1}{2}m(2v)^2 - \frac{1}{2}m(v)^2$$

$$= \frac{3}{2}mv^2 = 3\left(\frac{1}{2}mv^2\right) = 3W_1 \qquad \text{(Using (i))}$$

22. (*d*) According to work-energy theorem, the work done by the net force on the body is equal to the change in its kinetic energy.

i.e., $W = K_f - K_i$.

23. (*c*) According to work energy theorem, $T = fy$ where f is the frictional force exerted on the body.

or $f = \frac{T}{y}$

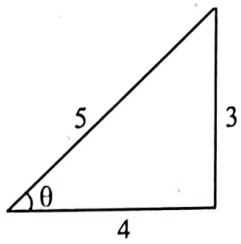
Note: One can also verify that $\frac{T}{y}$ has the dimensions of force.

i.e. $\frac{[T]}{[y]} = \frac{[ML^2T^{-2}]}{[L]} = [MLT^{-2}]$

24. (*b*) (Applied force - frictional force) × distance = Gain in kinetic energy.

$\therefore (20 - f) \times 2 = 10$ or $20 - f = 5$ or $f = 15$ N.

25. (*b*)



$w = mgh$, $\cos\theta = 4/5$

$= 10 \times 9.8 \times 3 = 294$ joule

26. (*d*) W_1 = work done by spring on first mass

W_2 = work done by spring on second mass

$W_1 = W_2 = W$ (say)

$W_1 + W_2 = U_i - U_f$

$$2W = 0 - \frac{1}{2}kx^2$$

$$W = -\frac{kx^2}{4}$$

27. (*b*) $a=\frac{F}{m}, S=\frac{1}{2}\left(\frac{F}{m}\right)t^2, W_F=FS=F\left(\frac{Ft^2}{2m}\right)$

28. (*d*) $v=0+aT,\ a=\frac{v}{T}$, velocity at time 't' $=0+at=\frac{vt}{T}$

$$K.E=\frac{1}{2}(m)\left(\frac{vt}{T}\right)^2$$

29. (*a*) $v_0=at_0$

$$\Rightarrow a=\frac{v_0}{t_0}$$

$$\therefore\ v=\frac{v_0}{t_0}\cdot t$$

$$\Rightarrow w=\Delta k=k_f-k_i$$

$$\Rightarrow \frac{1}{2}M\frac{v_0^2}{t_0^2}.t^2$$

30. (*b*) $-Fx=0-\frac{1}{2}m(v)^2$ and $-FS=0-2\left[\frac{1}{2}m(2)^2\right]$

So, $\frac{S}{x}=2, S=2x$

31. (*a*) $w=\frac{1}{2}k(x_2^2-x_1^2)$

$$=\frac{1}{2}10(6^2-4^2)=100\text{ N-cm }=1\text{ J}$$

32. (*c*) Work energy theorem.

$\Sigma W=\Delta KE$

$$\Rightarrow mgh+W_f=\frac{1}{2}mv^2-0$$

Work done by friction force

i.e., $W_f=\frac{1}{2}mv^2-mgh$

$$=\frac{1}{2}\times1\times4-1\times10\times1$$

$$W_f=-8\text{J}$$

33. (*c*) Power $=\frac{\text{Work}}{\text{Time}}$

$$\therefore\ P=\frac{mgh}{t}$$

or $t=\frac{mgh}{P}$

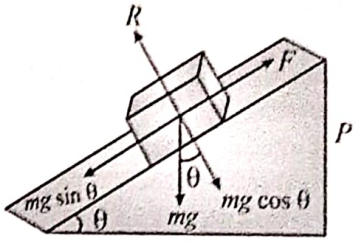
Substituting the given values, we get

$$t=\frac{200\times10\times40}{10\times10^3}=8\text{ s}$$

34. (*d*) $P=\vec{F}.\vec{v}=m\vec{a}.\vec{a}t=ma^2t$ [as $u=0$]

$$=m\left(\frac{v_1}{t_1}\right)^2t=\frac{mv_1^2t}{t_1^2}\ [\text{As } a=v_1/t_1]$$

35. (*d*) $v=7.2\frac{\text{km}}{h}=7.2\times\frac{5}{18}=2$ m/s



Slope is given 1 in 20

$$\therefore \sin\theta=\frac{1}{20}$$

When man and cycle moves up then component of weight opposes it motion i.e. $F=mg\sin\theta$

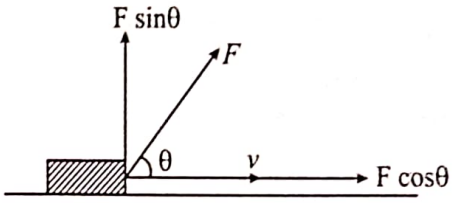
So power of the man $P=F.v=mg\sin\theta\times v$

$$=100\times9.8\times\left(\frac{1}{20}\right)\times2=98\text{ Watt}$$

36. (*b*) $P=Fv=4500\times2=9000$ W $=9$kW

37. (*d*) $P=\frac{\text{Workdone}}{\text{Time}}=\frac{mgh}{t}=\frac{300\times9.8\times2}{3}=1960$ W

38. (*a*)



Power = (component of force in the direction of velocity)

$=F\cos\theta v$

39. (*a*) $p=\frac{mgh}{t}=\frac{200\times10\times200}{10}=40$kW

40. (*b*) Power $=\frac{\text{Work done}}{\text{time}}=\frac{\frac{1}{2}m(v^2-u^2)}{t}$

$$P=\frac{1}{2}\times\frac{2.05\times10^6\times\left[(25)^2-(5^2)\right]}{5\times60}$$

$P=2.05\times10^6\ W=2.05\ MW$

41. (*c*) At any time 't' let the y-coordinate of the particle by 'y'

Work done by gravity

$W=-mg(y)$

$$P=\frac{dW}{dt}=-mg\left(\frac{dy}{dt}\right)=-mg(v_y)$$

or $P=-mg(u-gt)$

or $P=-mgu+mg^2t$

here 'u' is initial speed which is constant.

Hence the graph between 'P' vs 't' is option (*c*)

42. (*b*) $W=(T\cos\theta).s$

$$P=\frac{dW}{dt}=(T\cos\theta)\frac{ds}{dt}$$

Or $P=Tv\cos\theta$

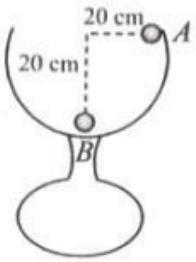
43. (*d*) In compression or extension of a spring work is done against restoring force.

In moving a body against gravity work is done against gravitational force of attraction.

It means in all three cases potential energy of the system increases.

But when the bubble rises in the direction of upthrust force then system does work so the potential energy of the system decreases.

44. (*b*) According to the conservation of energy, kinetic energy at A + potential energy at B



$\Rightarrow 0 + mgh = \frac{1}{2}mv^2 + 0$

or $v^2 = 2gh = 2 \times 9.8 \times 0.20$

($\because h$ = radius = 20 cm = 0.2m)

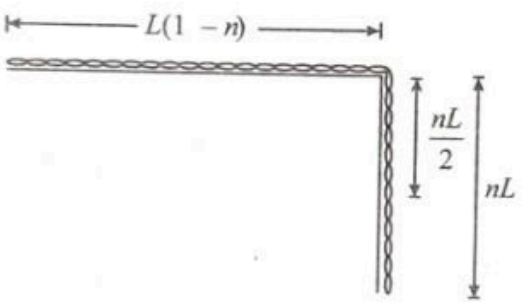
According to work - energy theorem,

Work done on the ball = change in kinetic energy

$= \frac{1}{2}mv^2 - (0)^2 = \frac{1}{2} \times \frac{2}{1000} \times 2 \times 9.8 \times 0.2$

$= 3.92 \times 10^{-3}$ J = 3.92 mJ

45. (*a*)



Total length = L

Initially held hanging length of the chain = nL

Rest length on the table = $L(1-n)$

Let the mass per unit length of the chain be μ.

Initial potential energy $= \mu nLg\frac{nL}{2}$

Final potential energy $= (\mu L)g\frac{L}{2}$

Loss in potential energy $= (\mu L)g\frac{L}{2} - \mu nLg\frac{nL}{2}$

by law of conservation of energy

$KE = \mu g\frac{L^2}{2} - \mu g\frac{L^2}{2}n^2 = mg\frac{L}{2}(1-n^2)$

$\left(\therefore \ \mu = \frac{m}{L}\right)$

46. (*b*) In the stable equilibrium, a body has minimum potential energy.

47. (*a*) Here $V(x) = (x^2 - 3x)$J

For a conservative field, force, $F = -\frac{dV}{dx}$

$\therefore F = -\frac{d}{dx}(x^2 - 3x) = -(2x-3) = -2x+3$

At equilibrium position, $F = 0$

$\therefore -2x + 3x = 0$ or $x = \frac{3}{2}m = 1.5m$

48. (*b*) Fact based

49. (*c*) $F = -\frac{dU}{dx} = 0$ at B and C

50. (*a*) Only in (*a*), U is minimum for some value of r.

51. (*c*) From the graph 'U' vs 'x'

for point 'C' $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$

Between point 'C' and 'D'

$\frac{dU}{dx} > 0$ hence $F = -\frac{dU}{dx} = -$ve

$\Leftrightarrow$ Force is attractive in nature for this region.

Exercise-2 (Learning Plus)

1. (*a*) $T = mg + ma$, $S = \frac{1}{2}at^2$

$W_T = T \times S = \frac{m(g+a)at^2}{2}$

2. (*c*) $F = k_1x_1, x_1 = \frac{F}{k_1}, W_1 = \frac{1}{2}k_1x_1^2 = \frac{F^2}{2k_1}$

similarly $W_2 = \frac{F^2}{2k_2}$ since $k_1 > k_2$, $W_1 < W_2$

3. (*d*) Area under curve $= \frac{1}{2}(d)(20) = 40$J

W = work done by resistive force $F = -40$ J

$-40 = K_f - K_i$, $K_i = 50$ J, so $K_f = 50 - 40 = 10$ J

4. (*a*) $W_G + W_f = 0 - 0$

$10 \times 1 + W_f = 0$

$10 - \mu mg\,x = 0$

$10 = (.2)(10)\,x$, $x = 5$ m

5. (*b*) Maximum velocity will be at Mean Position

Where $F_{net} = 0 \Rightarrow mg = Kx$

$1 \times 10 = 2 \times 100 \times x \Rightarrow x = 5$ cm

$\therefore h = 20 - 5 = 15$ cm

6. (*a*) $(mg)1 - mg/2 = mv^2/2$, $v = \sqrt{g}$;

$d = v\sqrt{2h/g} = \sqrt{g}\sqrt{\frac{2(0.5)}{g}} = 1$ m

7. (*d*) $V = 0 + at$, $F - \mu mg = ma$, $F = \mu mg + ma$,

$P = (\mu mg + ma)\,at$

8. (c) $P = \vec{F}.\vec{v} = 50 - 30 + 120 = 140$ J

9. (b) $P_1 = 80\,gh/15$, $P_2 = 80\,gh/20$

$$\frac{P_1}{P_2} = \frac{20}{15} = \frac{4}{3}$$

10. (c) Given $m = 12000$ kg, $v = 4$ m/sec and $t = 40$ sec

$$P_{avg} = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2}\times 12000\times 4^2}{40} = 2400\text{W} = 2.4\text{kW}$$

11. (b) $F = k_A x_A = k_B x_B$

$$E_A = \frac{1}{2}k_A x_A^2 \quad \text{...(i)}$$

dividing (i) and (ii)

$$E_B = \frac{1}{2}k_B x_B^2 \quad \text{...(ii)}$$

$$\frac{E_A}{E_B} = \left(\frac{k_A}{k_B}\right)\left(\frac{x_A}{x_B}\right)^2$$

$$\frac{E_A}{E_B} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} \Leftrightarrow E_B = 2E_A$$

12. (a) $\frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2 = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{k_1 + k_2}{m}}\,x$$

13. (c) For m, $N\cos\theta = mg$

For M, $N\sin\theta = kx$

so $\tan\theta = \frac{kx}{mg}$

so $\frac{1}{2}kx^2 = \frac{(mg\tan\theta)^2}{2k}$

14. (d) $(W.D)_{\text{by friction}} + (W.D)_{\text{by spring}} = \Delta k = k_f - k_i = 0 - k_i$

$$-0.25\times 1\times 10\times 4 - \frac{1}{2}\times 2.75\times 4^2 = -\frac{1}{2}\times 1\times v^2$$

$v = 8$ m/s

15. (b) $\left.\frac{dU}{dx}\right|_{x=A} = -ve$, $\left.\frac{dU}{dx}\right|_{x=B} = +ve$

So, F_A = positive , F_B = negative

16. (a) $\frac{\partial U}{\partial x} = \cos(x+y)$,

$$\frac{\partial U}{\partial y} = \cos(x+y)$$

$$\vec{F} = -\cos(x+y)\hat{i} - \cos(x+y)\hat{j}$$

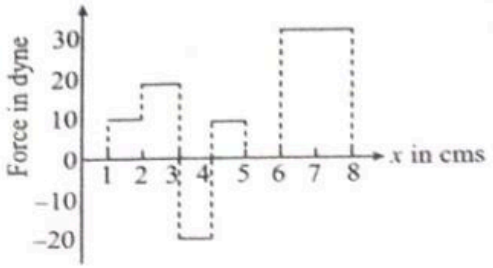
$$= -\cos\left(0+\frac{\pi}{4}\right)\hat{i} - \cos\left(0+\frac{\pi}{4}\right)\hat{j} \Rightarrow |\vec{F}| = 1$$

17. (a) From work energy theorem

$W_C + W_{nC} = \Delta K$, $W_C = -\Delta U$, $W_{nC} - \Delta U = \Delta K$

18. (a) Area under force vs displacement gives work and area above x-axis taken as positive while area below x-axis taken as negative.

$W_{net} = 10\times 1 + 20\times 1 - 20\times 1 + 10\times 1 = 20$ erg.



19. (a) $2x^2 - 3x - 2 = 0$

$$x = \frac{3\pm\sqrt{9+16}}{4} = \frac{3\pm 5}{4} \Rightarrow x = -\frac{1}{2}, 2$$

$$\frac{dF}{dx} = -\frac{d^2u}{dx^2} = 4x - 3 \Rightarrow \frac{d^2u}{dx^2} = 3 - 4x$$

$$\left(\frac{d^2u}{dx^2}\right)_{x=-\frac{1}{2}} = 3 + 4\times\frac{1}{2}$$

$= (5) > 0$ (stable)

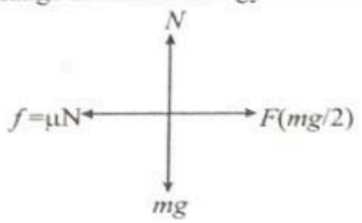
20. (b,d)

$dW_F = \vec{F}.d\vec{s} = dK > 0 \Rightarrow |\vec{F}||d\vec{s}|\cos\theta > 0$

$\Rightarrow 0 < \theta < 90^\circ$

$p = \sqrt{2m(K.E.)}$, $K.E. \uparrow$ so $p \uparrow$.

21. (b) Net change in kinetic energy = 0 ⇒ net work $W = 0$



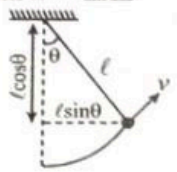
$$W = \int dW = \int F dx - \int \mu N dx$$

$$= \frac{mg}{2}x - mg\mu_0\int_0^x x dx = 0$$

$$\Rightarrow x = \frac{1}{\mu_0}$$

22. (b) Total work done on man = 0 ⇒ Work done by string = – work done by gravity = $-(-Mg\ell) = Mg\ell$

23. (d) Let at angular deflection θ its velocity be v then by work energy theorem $W = \Delta KE$



$$\frac{1}{2}mv^2 = -mg(\ell - \ell\cos\theta) + F\ell\sin\theta$$

At maximum angular deflection, $v = 0$

$$0 = -mg\ell\,(1-\cos\theta) + \frac{mg\ell}{2}\sin\theta \Rightarrow 2 - 2\cos\theta = \sin\theta$$

$$\theta = 2\tan^{-1}\frac{1}{2}$$

24. (*b*) $\therefore\ \vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$

$$\therefore\ \frac{\partial U}{\partial x} = -yz, \frac{\partial U}{\partial y} = -xz, \frac{\partial U}{\partial z} = -xy$$

Therefore $U = -xyz + C$ where C = constant As at (2, 2, 2), $U = 0$ so $C = 8$

OR

Objective question approach: Check that $U = 0$ at (2, 2, 2)

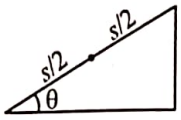
25. (*b*) Refer to figure. In the journey over the upper half of incline, $v^2 - u^2 = 2as$

$$v^2 - 0 = 2\,(g\sin\theta)\,\frac{s}{2} = (g\sin\theta).s$$

In the journey over the lower half of incline

$v^2 - u^2 = 2\ as\ 0 - g\sin\theta.s$

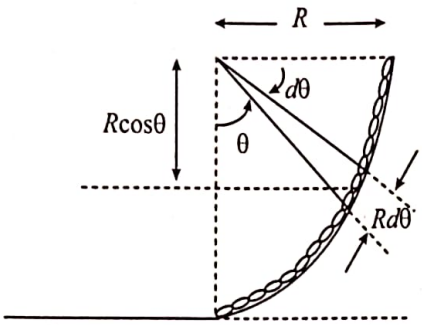
$$= 2\,g(\sin\theta - \mu\cos\theta)\,\frac{s}{2}$$



$$\Rightarrow -\sin\theta = \sin\theta - \mu\cos\theta$$

$$\Rightarrow \mu = \frac{2\sin\theta}{\cos\theta} = 2\tan\theta$$

26. (*c*)



Consider an element of '$Rd\theta$' length its mass. $(dm) = \lambda.(Rd\theta)$

or $dm = \frac{2m}{\pi}d\theta$

Change in potential energy

$dU = (dm)g.R(1-\cos\theta)$

$$\Rightarrow \int dU \int_{-\pi/2}^{0} \frac{2mgR}{\pi}(1-\cos\theta)d\theta$$

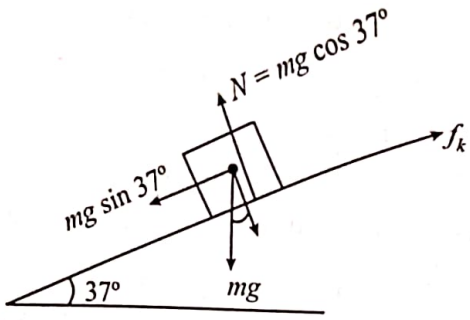
$$\Delta U = mgR\left(1-\frac{2}{\pi}\right)$$

from conservation of energy:

$$\Delta U = \Delta K \Rightarrow mgr\left(1-\frac{2}{\pi}\right) = \frac{1}{2}mv^2$$

$$v = \sqrt{2gR\left(1-\frac{2}{\pi}\right)}$$

27. (*d*)



The free body diagram of the block at any instant, is shown above.

Net force on the block:

$\Sigma F = mg\sin 37° - \mu N$

for the block to acquire maximum speed

$\Sigma F = 0$

$\Rightarrow \mu N = mg\sin 37°$

or $(0.3x).(mg\cos 37°) = mg\sin 37°$

$$\Rightarrow x = \frac{\tan 37°}{0.3}$$

$\Rightarrow x = 2.5$ m

28. (*c*) Once the block will fall on spring. It will start experiencing the spring force 'kx' upward.

If $kx > mg$, then its velocity will decrease hence kinetic energy will decrease.

If $mg > kx$, its kinetic energy will increase and it will be maximum when $kx = mg$

$\Rightarrow x = mg/k$

which is independent of 'h'

29. (*b*) Effective distance travelled by block 'A' is:

$d = 50$ cm $-$ (40 cm $-$ 10 cm) $= 20$ cm

friction force on block 'A' will be towards right.

work done by friction $= f \times d = \mu mgd$

$= 18$ J

Exercise-3 (JEE Advanced Level)

1. (*a*) We know:

$P = Fv$

at any instant 't'

$$F = m\frac{dv}{dt}$$

$$\Rightarrow P = mv\frac{dv}{dt} = mv\left(\frac{dv}{ds}\right)\times\left(\frac{ds}{dt}\right) = mv^2\frac{dv}{ds}$$

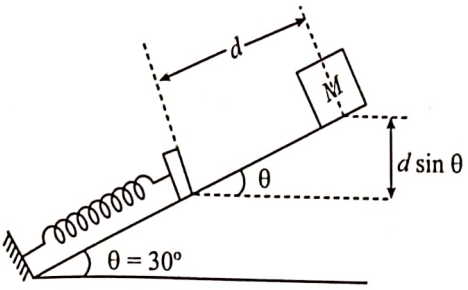
$$\text{or } \int_0^s ds = \int_v^{2v}\frac{mv^2dv}{P}$$

$$\Rightarrow s = \frac{7mv^3}{3P}$$

2. (*a*) $\because F = kx$

$\Rightarrow 100\text{ N} = k.\ (1\text{m})$ or $k = 100$ N/m

Let the block be at the distance '*d*' from the edge of spring, an inclined plane.



Using conservation of energy:

$$mg(d\sin 30°) + 0 = \frac{1}{2}mv^2 + 0$$

$\Rightarrow v = \sqrt{gd}$ (speed of bock just before striking the spring)

Now, the spring gets compressed by '2 m' and block stops.

So, from work energy theorem:

$$mg(2\sin 30°) - \frac{1}{2}kx^2 = 0 - \frac{1}{2}mv^2$$

or $mg - 200 = -50\,d$

or $d = 2$ m

hence, $v = \sqrt{gd} = \sqrt{20}$ m/s

3. (*c*) $\because F = -\dfrac{dU}{dx}$

$$\Rightarrow -ax + bx^2 = -\frac{dU}{dx}$$

or $\int dU = \int (ax - bx^2)dx$

$$\Rightarrow U(x) = \frac{ax^2}{2} - \frac{bx^3}{3} + c$$

1st derivative

$$\frac{dU}{dx} = -F = ax - bx^2 = 0$$

at $x = \dfrac{a}{b}$ and 0

2nd derivative

$$\frac{d^2U}{dx^2} = a - 2bx$$

we have $\dfrac{d^2U}{dx^2} < 0$ at $x = \dfrac{a}{b}$

4. (*a*,*d*)

With usual notation, the height through which the bob falls is $h = \ell(1 - \cos\theta) = \ell\left(2\sin^2\dfrac{\theta}{2}\right) = 2\ell\left(\dfrac{\theta^2}{4}\right)$ since θ is small. Therefore, we can write $h = \dfrac{\ell\theta^2}{2} = \dfrac{\ell}{2}\left(\dfrac{a}{\ell}\right)^2$

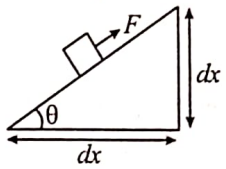
$= \dfrac{a^2}{2\ell}$. Where a = amplitude

Thus, the work done W = P.E. $= mgh = \dfrac{mga^2}{2\ell}$

$$\Rightarrow W \propto \frac{1}{\ell}$$

5. (*a*,*b*,*c*,*d*)

Work done by the force = Work done against gravity (W_g) + work done against friction (W_f)



$$W_g = \int (mg\sin\theta)\,ds$$

$$= mg\int ds\sin\theta = mg\int dh = mgh$$

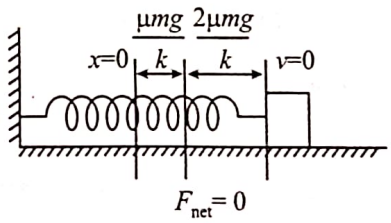
and $W_f = \int (\mu mg\cos\theta)ds$

$$= \mu mg\int ds\cos\theta = \mu mg\int dx = \mu mgx$$

6. (*a*,*b*,*c*,*d*)

For (*A*): Initial acceleration $= \dfrac{k\left(\dfrac{3\mu mg}{k}\right) - \mu mg}{m} = 2\mu g$

For (*B*, *C*) :



Therefore maximum compression

$= \dfrac{2\mu mg}{k} - \dfrac{\mu mg}{k} = \dfrac{\mu mg}{k}$ and minimum compression = 0

For (*D*) : At maximum speed $F_{net} = 0$ so by using work energy theorem

$$\frac{1}{2}mv^2 = \frac{1}{2}k\left(\frac{3\mu mg}{k}\right)^2 - \frac{1}{2}k\left(\frac{\mu mg}{k}\right)^2 - \mu mg\left(\frac{2\mu mg}{k}\right)$$

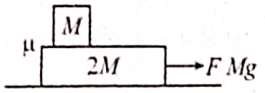
$$\Rightarrow v = 2\mu g\sqrt{m/k}$$

7. (*a*,*b*,*c*,*d*)

For the situation *A* and *B* move together and force of friction between *A* and *B* is $\dfrac{F}{3}$ i.e. $\dfrac{Mg}{3}$. Since *A* and *B* move together.

Net force acting on $A = \dfrac{Mg}{3}$.

Net force on $B = Mg - \frac{Mg}{3} = \frac{2Mg}{3}$



Since displacement is same.

8. (*a,c,d*)

velocity will continuously increase

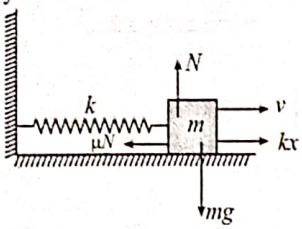
$\frac{1}{2} m (v^2 - u^2) = \Delta KE =$ work done

$\Rightarrow \frac{1}{2} \times 1.(v^2 - 20) =$ Area $= 40$

$\Rightarrow v = 10$ m/s

9. (*a,b,c,d*)

(i) Velocity will be maximum when net force = 0.



$kx = \mu N = \mu mg \Rightarrow x = \frac{\mu mg}{k}$

$W_s + W_f = \Delta K$

$\Rightarrow \frac{1}{2}k\left(\frac{2\mu mg}{k}\right)^2 - \frac{1}{2}k\left(\frac{\mu mg}{k}\right)^2 - \mu mg \times \frac{\mu mg}{k} = \frac{1}{2}mv^2$

On solving,

$v = \mu g\sqrt{\frac{m}{k}}$.

(ii) When the particle have velocity equal to zero, then let extension in spring be 'x'.

$\frac{1}{2}k\left(\frac{2\mu mg}{k}\right)^2 = \mu mg\left(\frac{2\mu mg}{k} + x\right) + \frac{1}{2}kx^2$

$2\frac{\mu^2 m^2 g^2}{k} = 2\frac{\mu^2 m^2 g^2}{k} + x\mu mg + \frac{1}{2}kx^2$

$\Rightarrow x\left[\frac{2\mu mg}{k} + x\right] = 0$

$x = 0$ (at natural length) or $x = \frac{2\mu mg}{k}$ – when compression in spring is $\frac{2\mu mg}{k}$ i.e. initially

So at natural length, velocity is zero and spring force is also zero. The block will not return or have velocity towards left.

10. (*a,b,c*)

The man acting with force F on spring does the work $w = -FL$ and work done by friction acting on car exerted by man $w = FL$

$\therefore$ total work done by man is zero.

11. (*a,c*)

$P = F.v.$

Speed will be maximum when net external force is zero i.e., $F = mg\sin\theta$

$\Rightarrow P = mgv_{max}\sin\theta \quad \therefore v_{max} = \frac{P}{mg\sin\theta}$

Hence, first increases then becomes

$v_{max} = \frac{P}{mg\sin\theta} =$ constant

12. (*a,d*)

At $x = 5$m, $KE = 50$J

$PE = 15 + (5-3)^2 = 19$ J

Mechanical energy = 69 J

$KE_{max} =$ Total Energy $- PE_{mini}$

$= 69 - 15$

$= 54$ J

13. (*b,c,d*)

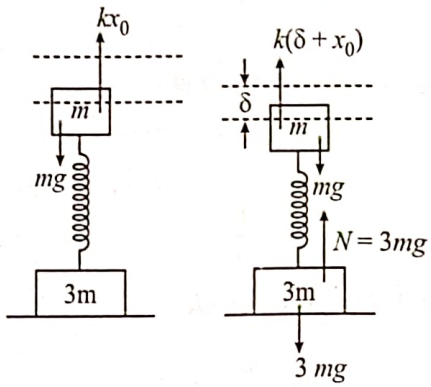
$\vec{F} = -2\hat{i} - 3\hat{j}$

$\vec{a} = -2\hat{i} - 3\hat{j}$

(*b*) Correct

(*d*) $|\vec{v}| = 2\sqrt{4+9} = 2\sqrt{13}$

14. (*b,d*)

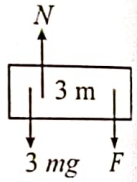


at equilibrium $kx_0 = mg \Rightarrow x_0 = \frac{mg}{k}$

After compressing it further by a distance 'δ':

$k(\delta + x_0) - mg = 3mg$

$\Rightarrow \delta = \frac{3mg}{k}$



Maximum normal reaction:

$N = F + 3mg = k\delta + 3mg = 6mg$

if $\delta > \frac{4mg}{k}$

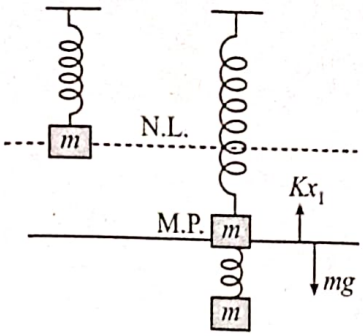
$\because N_{max} = k\delta + 3mg = 6mg$

if $N_{max} > 6mg$

the lower disc will bounce up.

for $\delta > \frac{4mg}{k}$ this situation will arise.

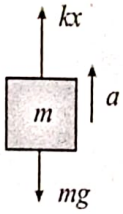
15. (*b*,*c*,*d*)



Mean position (M.P.) $x_1 = \frac{mg}{k}$

But block further move downward due to inertia. So descending through distance

$x = \frac{2mg}{k}$



If *M.P.* at $\frac{x}{2}$ $\Rightarrow$ $F_{net} = 0$;

so, $a = 0$

at lower most point

$k\left(\frac{2mg}{k}\right) - mg = ma \Rightarrow a = g$

16. (*a*) Mechanical energy

$E = K + U$

$\Rightarrow U = E - K$

for $E = 25$ J

we have $U = 25 - K$.

$\because K = \frac{1}{2}mv^2 > 0$ (positive)

$\Leftrightarrow$ 'U' will be be less than 25 J

from graph it is for the values of $-10 < x < -5$ and $6 < x < 15$.

17. (*d*) Same arguments as given above in solution (16) is mentioned, we can say it is not possible

18. (*b*) If the particle is released at the origin, it will try to go in the direction of force. Here $\frac{du}{dx}$ is positive and hence force is negative, as a result it will move towards – ve x-axis.

19. (*b*) When the particle is released at $x = 2 + \Delta x$ it will reach the point of least possible potential energy (–15 J) where it will have maximum kinetic energy.

$\therefore \frac{1}{2}mv_{max}^2 = 25 \Rightarrow v_{max} = 5$ m/s

20. (*d*) Conditions for stable equilibrium is:

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$

Condition for unstable equilibrium is:

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} < 0$

21. (*b*) A. Work done by normal force will be positive because the normal reactions vertical component is along displacement of the block i.e., upwards.

B. Gravity points downwards however block is displaced up.

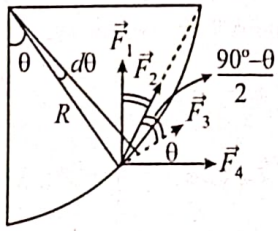
C. Friction force points along the incline but block does not move along the incline.

D. Vertical component of tension force does positive work as it is pointed upwards.

22. (*c*) For (*A*): Work done $\vec{F}_1 = FR$

For (*B*):

$dW = \vec{F} \cdot d\vec{s} = (FRd\theta)\cos\left(45 - \frac{\theta}{2}\right) = FR\left(45 - \frac{\theta}{2}\right)d\theta$



$W = \int_0^{\pi/4} FR\cos\left(45 - \frac{\theta}{2}\right)$

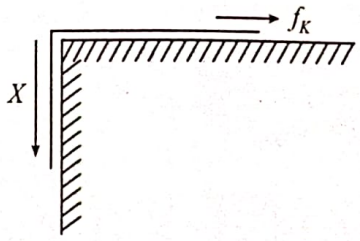
$= -2FR\left(\sin 45^\circ - \frac{\theta}{2}\right)_0^{\pi/2} = \sqrt{2}FR$

For (C) : $W = \int \vec{F} \cdot ds = F\left(\frac{\pi R}{2}\right) = \frac{\pi FR}{2}$

For (D) : $W = \int \vec{F} \cdot d\vec{s} = (F)(R) = FR$

23. [18] $f_K = \mu \frac{m}{\ell}(\ell - x)g$

$W = \int_{x=\frac{\ell}{4}}^{\ell} -\mu \frac{m(\ell - x)g}{\ell} dx$



$W = \frac{\mu mg}{\ell} \times \frac{[(\ell - x)^2]_{\frac{\ell}{4}}^{\ell}}{2}$

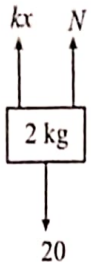
$= -\frac{\mu mg}{2\ell} \times \frac{9\ell^2}{16}$

$= -\frac{9\mu mg\ell}{32}$

24. [2] F.B.D of 2 kg block

When 2 kg block just leaves the contact

$N = 0$



$$Kx + 0 = 20 \Rightarrow x = \frac{20}{40} = \frac{1}{2}\text{ m}$$

Applying work energy theorem for the whole system

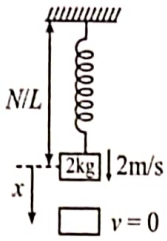
$$w_g + w_s = \Delta K$$

$$\Rightarrow 50 \times \frac{1}{2} - \frac{1}{2}(40)\left[\left(\frac{1}{2}\right)^2 - 0\right] = \frac{1}{2}(5)[v^2 - 0]$$

$$\Rightarrow v = 2\sqrt{2}\text{ m/s}$$

25. [5] At the moment, elongation is maximum, speed of the block becomes zero.

Applying W/E theorem on system

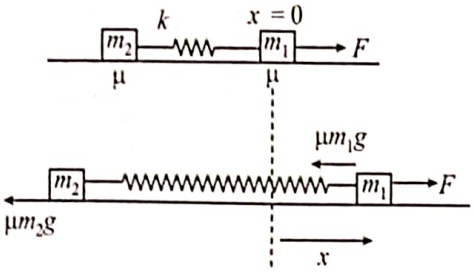


$$0 - \frac{1}{2}.mv^2 = W_g + W_s$$

$$-\frac{1}{2}\,2.4 = 2.10.x - \frac{1}{2}.100.x^2$$

Solving $x = \left(\frac{\sqrt{3}+1}{5}\right)m$

26. [11]



$$W_F + W_{Sp} + W_{fric} = \Delta k$$

$$\Rightarrow Fx - \frac{1}{2}kx^2 - \mu m_1 g x = 0$$

and $kx = \mu m_2 g$

$$\Rightarrow F - \frac{1}{2}\mu m_2 g - \mu m_1 g = 0$$

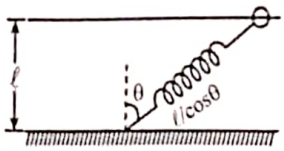
$$\Rightarrow F = \mu m_1 g + \frac{\mu m_2 g}{2}$$

27. [2] $mgx - \frac{1}{2}k\left(\frac{2x}{5}\right)^2 - \frac{1}{2}k\left(\frac{x}{5}\right)^2 = 0$

$$\Rightarrow x = \frac{10mg}{k}$$

28. [53] $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$\frac{1}{2}m\frac{k}{m}\left(\frac{2\ell}{3}\right)^2 = \frac{1}{2}kx^2$$



$$\Rightarrow x = \frac{2\ell}{3} = \frac{\ell}{\cos\theta} - \ell$$

$$\cos\theta = \frac{3}{5} \Rightarrow \theta = 53^\circ$$

29. [27] $P_0 = Fv$

$$\therefore \left(Mv\frac{dv}{dx}\right)v = P_0$$

$$\Rightarrow M\int_v^{2v} v^2 dv = P_0\int_0^x dx \Rightarrow \quad x = \frac{7Mv^3}{3P_0}$$

30. [2] $\vec{F} = F_x\hat{i} + F_y\hat{j}$

$$\vec{F}\cdot d\vec{r} = -dU$$

$$\left(F_x\hat{i} + F_y\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right) = -dU$$

$$F_x\,dx + F_y\,dy = -dU$$

$$-dU = (-2x + 2y)\,dx + (2x - y^2)dy$$

$$= -2x\,dx + 2(y\,dx + x\,dy) - y^2\,dy$$

$$U(x, y) = x^2 - 2(xy) + \frac{y^3}{3} + C$$

$$U(2, 3) = 4 - 12 + 9 + C = 1 + C$$

$$\frac{8}{3} + 1 + C = (\text{K.E}) + [1 - 4 + \frac{8}{3} + C]$$

Kinetic energy = 4 J

$$\Rightarrow \frac{1}{2}(2)v^2 = 4 \quad \Rightarrow \quad v = 2\text{m/s}$$

31. [1] $U_i + K_i = U_f + K_f$

$$\frac{K}{2a} + \frac{1}{2}mV_0^2 = \frac{K}{a}$$

$$\frac{mV_0^2}{2} = \frac{K}{2a} \Rightarrow V_0 = \sqrt{\frac{K}{am}}$$

32. [15] $F = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -[6\hat{i}] + [8]\hat{j} = -6\hat{i} + 8\hat{j}$

$\therefore\ \vec{a} = -3\hat{i} + 4\hat{j}$ has same direction as that of

$$\vec{u} = \frac{-3\hat{i} + 4\hat{j}}{2} = \left(\frac{\vec{a}}{2}\right)$$

$|\vec{a}| = 5$

$|\vec{u}| = 5/2$

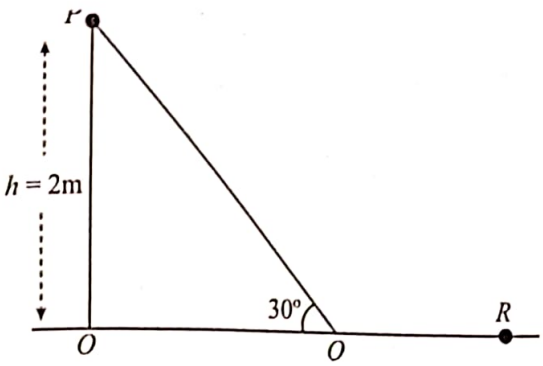
Since $\vec{u}$ and $\vec{a}$ are in same direction, particle will move along a straight line

$$\therefore S = \frac{5}{2} \times 2 + \frac{1}{2} \times 5 \times 2^2 = 5 + 10 = 15\text{m}.$$

Exercise-4 (Past Year Questions)

JEE MAIN

1. (c)



Work done by friction along PQ and QR are same

$\therefore OQ = QR$

$$QR = \frac{2}{\tan 30°} = \frac{2}{1/\sqrt{3}} = 2\sqrt{3} = 3.5m$$

$mgh = \mu mg2(QR)$

$$\mu = \frac{1}{2\sqrt{3}}$$

2. (d) $m \times 3.8 \times 10^7 \times \frac{20}{100} = 10 \times 9.8 \times 1 \times 1000$

$m = 12.89 \times 10^{-3}$ kg

3. (a) $\frac{1}{2}mv_f^2 = \frac{1}{8}mv_0^2$

$$v_f = \frac{v_0}{2} = 5\text{m/s}$$

$$(10^{-2})\frac{dv}{dt} = -kv^2$$

$$\int_{10}^{5}\frac{dv}{v^2} = -100k\int_0^{10}dt$$

$$\frac{1}{5} - \frac{1}{10} = 100k(10)$$

$k = 10^{-4}$

4. (c) $F = 6t = ma$

$\Rightarrow a = 6t$

$$\Rightarrow \frac{dv}{dt} = 6t$$

$$\int_0^v dv = \int_0^1 6t\ dt$$

$v = (3t^2)_0^1 = 3$m/s.

From work energy theorem

$$W_F = \Delta K.E = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2}(1)(9-0) = 4.5\text{J}$$

5. (b) $F = -\frac{\partial u}{\partial r} = \frac{K}{r^3}$

Since it is performing circular motion

$$F = \frac{mv^2}{r} = \frac{K}{r^3}$$

$$mv_2 = \frac{K}{r^2}$$

$$\Rightarrow K.E. = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

Total energy = $P.E. + K.E.$

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{zero}$$

6. (d) $x = 3t^2 + 5$

$v = 6t$

$\Rightarrow \Delta W = \Delta K$

$$= \frac{1}{2}(2)(30)^2 - \frac{1}{2}2(0)^2$$

$= 900$ J

7. (d) When $V_{max} \Rightarrow$ acceleration = 0

$$\Rightarrow x = \frac{F}{k}$$

Applying work energy theorem

$W_{sp} + W_F = \Delta K.E.$

$$-\frac{1}{2}kx^2 + F.x = \Delta K.E.; -\frac{1}{2}k\frac{F^2}{k^2} + \frac{F^2}{k} = \frac{1}{2}mu_{max}^2$$

$$\frac{F^2}{2k} = \frac{1}{2}mu_{max}^2;\ \frac{F}{\sqrt{mK}} = V_{max}$$

8. (d) $K.E. - 3 = \vec{F}\cdot\vec{d}$

$K.E. = 3 + (3\hat{i} - 12\hat{j})\times(4\hat{i})$

$K.E. = 3 + 12 = 15$ J

9. (d) $W = F\,s\cos\theta = m\left(g + \frac{g}{2}\right)\times\left[\frac{1}{2}\times\frac{g}{2}\times t^2\right]$

10. (a) According to work energy theorem.

Work done by force on the particle = Change in KE

Work done = Area under F-x graph

$\int F.dx$

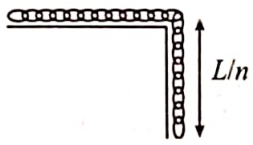
$$= 2 \times 2 + \frac{(2+3)\times 1}{2}$$

$W = KE_{final} - KE_{initial} = 6.5$

$KE_{initial} = 0$

$KE_{final} = 6.5$ J

11. (b) Mass of the hanging part = $\frac{M}{n}$



$$h_{COM} = \frac{L}{2n}$$

work done $W = mgh_{COM} = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$

12. (*c*) $k_1 = \frac{C}{\ell_1}$

$$k_2 = \frac{C}{\ell_2}$$

$$\frac{k_1}{k_2} = \frac{C\ell_2}{\ell_1 C}\ell_2 = \frac{\ell_2}{n\ell_2} = \frac{1}{n}$$

13. (*b*) Angular momentum conservation.

$$\cancel{M}V_0 L = \cancel{M}V_1(L-\ell)$$

$$V_1 = V_0\left(\frac{L}{L-\ell}\right)$$

$$w_g + w_p = \Delta KE$$

$$-mg\ell + w_p = \frac{1}{2}m\left(V_1^2 - V_0^2\right)$$

$$w_p = mg\ell + \frac{1}{2}mV_0^2\left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1-\frac{\ell}{L}\right)^{-2} - 1\right)$$

Now, $\ell << L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1+\frac{2\ell}{L}\right) - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\frac{2\ell}{L}\right)$$

$$W_p = mg\ell + mv_0^2\frac{\ell}{L}$$

here, V_0 = maximum velocity

$= \omega \times A$

$$= \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$$

$$V_0 = \theta_0\sqrt{gL}$$

so, $w_p = mg\ell + m\left(\theta_0\sqrt{gL}\right)^2\frac{\ell}{L}$

$$= mg\ell\left(1+\theta_0^2\right)$$

14. (*b*) $4000 \times V + mg \times V = P$

$$\frac{60\times 746}{4000+20000} = V$$

$V = 1.86$ m/s. ≈ 1.9 m/s.

15. [10] $KE = PE_1 \times PE_2 = mgh_1 - mgh_2$

$= 1 \times 10 \times 2 - 1 \times 10 \times 1 = 10$J

16. (*d*) Net force on motor will be

$F_m = [920 + 68\,(10)]g + 6000$

$= 22000$ N

So, required power for motor

$P_m = \vec{F}_m.\vec{v}$

$= 22000 \times 3$

$= 66000$ watt

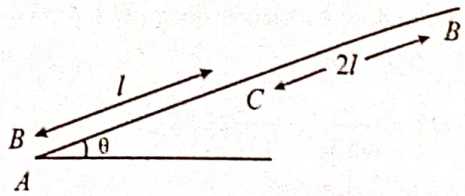
17. (*d*) $W = \int \vec{F}.d\vec{s}$

$$= \left(-x\hat{i} + y\hat{j}\right).(dx\hat{i} + dy\hat{j})$$

$$= \int_1^0 -xdx + \int_0^1 ydy$$

$$= \frac{x^2}{2}\Big|_0^1 + \frac{y^2}{2}\Big|_0^1 = \left(0+\frac{1}{2}\right)+\left(\frac{1}{2}\right) = 1\text{J}$$

18. [3]



Work done by all the forces is equal to zero.

$\therefore 0 = mg3l\sin\theta - \mu mgl\cos = \theta$

$3\sin\theta = \mu\cos\theta \Rightarrow \mu = 3\tan\theta$

$\therefore k = 3$

19. [150.00]

Initial velocity $v_0^2 = 2gh$

$\Rightarrow v_0 = \sqrt{2\times 10\times 20} = 20$ m/s

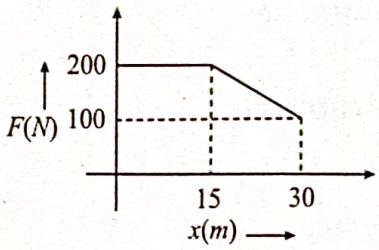
Now work done by the machine.

$$F.x = \Delta KE = \frac{1}{2}mv_0^2$$

$$F\times\frac{2}{10} = \frac{1}{2}\times\frac{15}{100}\times 400$$

$\Rightarrow F = 150$

20. (*c*)



Work = Area of F–x curve

$$= 200 \times 15 + \frac{1}{2}(200+100)\times 15$$

$= 5250$ J

21. [18.00]

$$pt = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2pt}{m}}$$

$$\Rightarrow \frac{ds}{dt} = \sqrt{\frac{2pt}{m}}$$

$$\Rightarrow \int_0^s ds = \int_0^t \sqrt{\frac{2pt}{m}} dt$$

22. (*c*) $F = \frac{dU}{dr}$

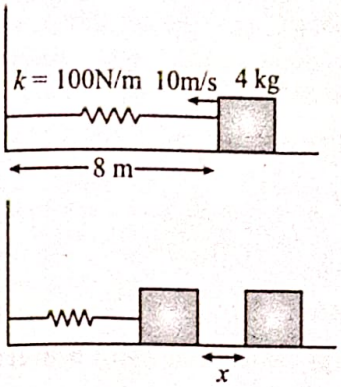
$$= -\left[\frac{6A}{r^7} - \frac{12B}{r^{13}}\right]$$

$F = 0$

$$\Rightarrow r = \left(\frac{2B}{A}\right)^{1/6}$$

$$U\left(at\, r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A^2}{4B}$$

23. [6]



If spring compressed by x

then work done by spring $= 0 - \frac{1}{2} \times 4 \times 10^2$

Applying work energy theorem,

$$-\frac{1}{2}kx^2 = -\frac{1}{2} \times 4 \times 10^2$$

$$\Rightarrow 100x^2 = 4 \times 10^2$$

$$\Rightarrow x = 2$$

$\therefore$ Final length of the spring = 8 – 2 = 6m

24. [10] By energy conservation

$$K_i + U_i = K_f + U_f$$

$$0 + 10 \times 10 \times 10 = \frac{1}{2} \times 10 \times v_B^2 + 10 \times 10 \times 5$$

$$1000 = 5v_B^2 + 500$$

$$v_B^2 = \frac{500}{5} = 100$$

$v_B = 10$m/s

$x = 10$

25. (*c*) $p = \sqrt{2Em}$

Where p denotes momentum, E denotes kinetic energy and m denotes mass.

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$$

$$\frac{p_1}{p_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

26. (*b*) Centripetal acceleration $a_c = k^2rt^2$

$v^2/r = k^2rt^2$

$v^2 = k^2r^2t^2$

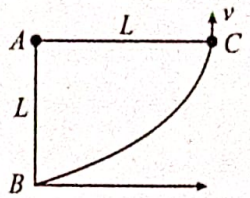
$mv^2/2 = mk^2r^2t^2/2$

Kinetic energy $= mk^2r^2t^2/2$

$$\frac{d}{dt}(K.E) = mk^2r^2t$$

Power $= mk^2r^2t$

27. (*c*)



At lowest position B, Potential Energy = 0

Kinetic Energy = $1/2\ mu^2$

Total Energy = $1/2\ mu^2$

At C, when string is horizontal,

Potential Energy = mgL

Total Energy = $1/2\ mv^2 + mgL$

Since energy is conserved,

$1/2\ mv^2 + mgL = 1/2\ mu^2$

$v^2 = u^2 - 2gL$

Since v is in the vertical direction and u is in the horizontal direction, they are mutually perpendicular to each other.

Change in velocity

$$= \sqrt{u^2 + v^2}$$

$$|\Delta\vec{v}| = \sqrt{u^2 + (u^2 - 2gL)}$$

$$|\Delta\vec{v}| = \sqrt{2(u^2 - gL)}$$

28. (*a*) $W = \int \vec{F}.\vec{ds}$

Wherein $\vec{F}$ is the variable force and $\vec{ds}$ is small displacement

$F = ax + bx^2$

Work done in displacing rubber through $dx = Fdx$

$$W = \int_0^L (ax + bx^2)dx$$

$W = aL^2/2 + bL^3/3$

29. (*a*) $1/2mv_f^2 = 1/3mv_0^2$

$v_f = v_0/2 = 5$ m/s

$F = -kv^2$ ...(i)

We know, $F = ma = m\ (dv/dt)$ ...(ii)

From (i) and (ii) we get

$m(dv/dt) = -kv^2$

$$\int_{10}^{5} \frac{dv}{v^2} = -100k\int_0^{10} dt$$

$1/5 - 1/10 = -100\,k\,(10)$

$k = 10^{-4}$ kg m^{-1}

30. (*b*) $a = -\mu g = -0.5 \times 9.8 = -4.9$ m/s^2

$$d = \frac{v^2}{2a} = \frac{9.8\times 9.8}{2(4.9)}$$

$= 9.8$ m

31. (*c*) $T = M\omega^2 R$

$T = 80$ N $\quad M = 0.1 \quad \omega = ? \quad R = 2$m

$80 = 0.1\omega^2(2)$

$\omega^2 = 400$

$\omega = 20$

$2\pi f = 20$

$$f = \frac{10}{\pi}\frac{\text{rev}}{s}$$

$$= \frac{600}{\pi}\frac{\text{rev}}{\text{min}}$$

32. [600]

$U_i + K_i = U_f + K_f$

$$\Rightarrow 0 + \frac{1}{2}m(12)^2 = \frac{1}{2}k(0.3)^2 + \frac{1}{2}m(6)^2$$

$\Rightarrow 0.5(12^2 - 6^2) = k(0.3)^2$

$k = 600$ N/m

33. (*b*) $v = \sqrt{u^2 - 2gL}$

$\Delta v = \sqrt{u^2 - v^2}$

$\Delta v = \sqrt{u^2 + v^2 - 2gL}$

$\Delta v = \sqrt{2u^2 - 2gL}$

$\Delta v = \sqrt{2(u^2 - gL)}x = 2$

34. (*d*) By work energy theorem work done by net force = ΔK.E.

$$\Rightarrow w = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$w = \frac{1}{2}\times 0.5\times(0.25)^2\times(4)^5$$

$w = 16$ J

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35. (*d*) suppose $x = r\cos\theta$, $y = r\sin\theta$

force on particle is $\frac{K}{r^3}\left(r\cos\theta\hat{i} + r\sin\theta\hat{j}\right)$

force is in radial direction so work done by this force along given path (circle) is zero.

36. [5] $E = Pt = 0.5\text{W} \times 5\text{s} = 2.5$ J $\quad = \frac{1}{2}mv^2 \Rightarrow v = 5$ m/s

37. [5] $W_F + W_g = K_f - K_i$

$18\times 5 + 1g(-4) = K_f$

$90 - 40 = K_f$

$K_f = 50\text{J} = 5\times 10\text{J}$

38. [0.75]

$\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)$

$W_{AB} = (-1\hat{i}.).(1\hat{i}) = -1$J

$$\begin{bmatrix} \vec{F} = -1i + 2\alpha x\hat{j} \\ \vec{S} = 1\hat{i} \end{bmatrix}$$

Similarly,

$W_{BC} = 1$J

$W_{CD} = 0.25$ J

$W_{DE} = 0.5$ J

$W_{EF} = W_{FA} = 0.75$ J

∴ New work in cycle = 0.75 J

39. (*a,d*) By the energy conservation (ME) between bottom point and point '*y*'

$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv_1^2$$

$\therefore v_1^2 = v_0^2 - 2gh$ ...(i)

Now at point '*y*' the centripetal force provided by the component of *mg*

$$\therefore mg\sin 30^\circ = \frac{mv_1^2}{R}$$

$$\therefore v_1^2 = \frac{gR}{2}$$

∴ from (i)

$$\frac{gR}{2} = v_0^2 - 2gh$$

At point *x* and *z* of circular path, the points are at same height but less then *h*. So the velocity more than a point *y*.

So required centripetal $= \frac{mv^2}{r}$ is more.

CHAPTER

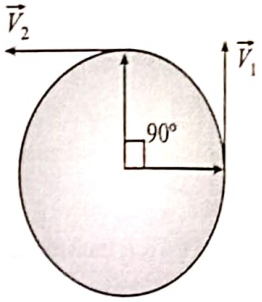
9 Circular Motion

Exercise-1 (Topicwise)

1. (c) It is always directed in a direction of tangent to circle.

2. (a) In uniform circular motion (constant angular velocity) kinetic energy remains constant but due to change in velocity of particle its momentum varies.

3. (d) In 15 second's hand rotate through 90°.



Change in velocity $|\overline{\Delta v}| = 2v\sin(\theta/2)$

$= 2(r\omega)\sin(90°/2) = 2\times1\times\frac{2\pi}{T}\times\frac{1}{\sqrt{2}}$

$= \frac{4\pi}{60\sqrt{2}} = \frac{\pi\sqrt{2}}{30}\frac{\text{cm}}{\text{sec}}$ [As $T = 60$ sec]

4. (b) $\bar{v} = \bar{\omega}\times\bar{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$.

5. (a) $v = \omega r \Rightarrow v = 30\times2\pi\times\frac{1}{2} = 30\pi$

6. (c) Direction and magnitude of Angular velocity & angular momentum stays constant.

7. (c) Centripetal acceleration $= \frac{v^2}{r} =$ constant. Direction keeps changing.

8. (a) $\frac{a_R}{a_r} = \frac{\omega_R^2\times R}{\omega_r^2\times r} = \frac{T_r^2}{T_R^2}\times\frac{R}{r} = \frac{R}{r}$ [As $T_r = T_R$]

9. (d) The particle performing circular motion flies off tangentially.

10. (a) $\alpha = 4\pi^2n^2r = 4\pi^2\left(\frac{1}{2}\right)^2\times50 = 493$ cm/s^2.

11. (c) Angular velocity of every particle of disc is same

$a_p = \omega^2r_p, a_Q = \omega^2r_Q$

$\because\ r_p > r_Q \Rightarrow a_p > a_Q$

12. (a) $T = m\omega^2x \Rightarrow \omega = \sqrt{\frac{T}{mx}} = 20$

13. (d) Centripetal force is constant in magnitude that means speed is constant and due to change in direction velocity is variable.

14. (b) Fact based

15. (b) $F = \frac{k}{r} = \frac{mv^2}{r}$, $v = \sqrt{\frac{k}{m}}$

So independent to r

16. (a) Force is perpendicular to $\bar{v}$

$R = \frac{v^2}{a_\perp} \Rightarrow R = \frac{mv^2}{F}$

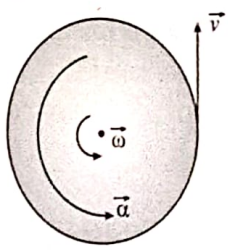
17. (c) $F_{C1} = F_{C2} \Rightarrow \frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2}$

$\frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{1}{\sqrt{2}}$

18. (c) $K = \frac{mV^2}{2}$

19. (a) $a_c = \omega v$

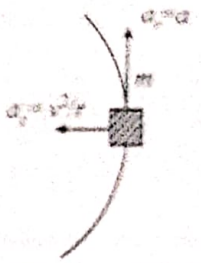
20. (b)



21. (b) $a_t = \alpha r = \omega^2r = \alpha^2t^2r$ $\{\because \alpha = \omega/t\}$

$\Rightarrow \alpha\times\frac{1}{t^2}$

22. (b) $|\vec{a}| = \sqrt{a_c^2 + a_t^2} = \sqrt{\frac{v^4}{r^2} + a^2}$



23. (a) Thrust at the lowest point of concave bridge

$= mg + \frac{mv^2}{r}$

24. (b) $v = 72$ km/hour $= 20$ m/sec

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right) = \tan^{-1}(2)$

25. (a) Centripetal force $= \frac{mv^2}{r}$

26. (b) Fact based

27. (a) $T = \frac{mv^2}{r} \Rightarrow 25 = \frac{0.25 \times v^2}{1.96} \Rightarrow v = 14$ m/s.

28. (a) $\frac{mv^2}{r} = \frac{k}{r^2}$

$\Rightarrow mv^2 = \frac{k}{r}$; K.E. $= \frac{1}{2}mv^2 = \frac{k}{2r}$ 5

P.E. $= \int F\,dr = \int \frac{k}{r^2}dr = -\frac{k}{r}$

$\therefore$ Total energy = K.E. + P.E. $= \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$.

29. (d) $F = mg - \frac{mv^2}{r}$.

30. (c) $v_{max} = \sqrt{\mu rg} = \sqrt{0.75 \times 60 \times 9.8} = 21$ m/s.

31. (c) Fact based

32. (b) $T = \frac{mv^2}{r}$

$= \frac{0.5 \times (4)^2}{1} = 8$N

33. (d) $p = mv$, and $F = mv^2/r \Rightarrow F = m\left(\frac{p}{m}\right)^2/r$

$\Rightarrow F = \frac{p^2}{mr}$

34. (d) $v_{max} = \sqrt{\mu rg}$

35. (b) $mgh = \frac{1}{2}m(5rg)$

$\Rightarrow h = \frac{5r}{2} = \frac{2.5}{2} = 1.25$m

36. (b) Tension, $T = mg + \frac{mv^2}{r} = mg + \frac{m}{l}[2gl(1 - \cos\theta)]$

$= mg + 2mg(1 - \cos\theta)$

$= mg + \frac{2mg}{2} = 2mg$

$= 2(50 \times 10^{-3})(10) = 1$ N

37. (b) According to the law of conservation of energy

$\frac{1}{2}mv^2 = mgR$, $v = \sqrt{2gR}$

38. (a) $\Delta K.E = \Delta P.E = mgh$, but $h = 2r$

39. (c) Given $v = \sqrt{3rg} = 5ms^{-1}$

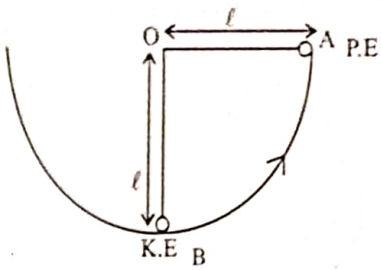
$\therefore$ Tension $T = \frac{mv^2}{r} = 20N$

40. (c) $T_1 = \frac{mv^2}{r} - mg$, $T_2 = \frac{mv^2}{r} + mg$

Then $T_2 - T_1 = 2$ mg

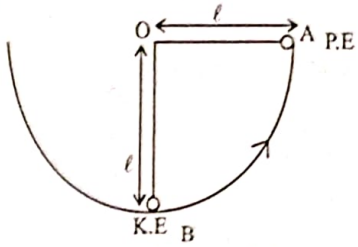
41. (c) At the highest point, $v \geq \sqrt{gR}$

42. (d)



At point A all of the K.E transformed into the potential energy, $\frac{1}{2}mv^2 = mg\ell$, $v = \sqrt{2g\ell}$

43. (a)

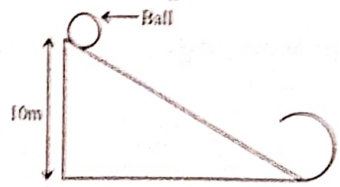


At point A total energy is in the form of P.E because all kinetic energy at B get transformed in P.E

$\frac{1}{2}mv^2 = mgh$

$\therefore v = \sqrt{2gh} \Rightarrow v = \sqrt{2 \times 9.8 \times 0.7} = 3.7$ m/s

44. (c) For such condition $h = \frac{5}{2}R$



$\therefore R = \frac{2}{5}h$, $R = 4$ m

45. (*b*) At the lowest position $v = \sqrt{gR}$

Centripetal acceleration $a_c = \frac{v^2}{R} = \frac{(\sqrt{gR})^2}{R} = g$

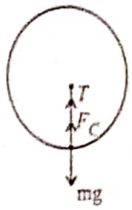
46. (*d*) $F_c = T - mg$

$T = F_c + mg$

$= m\omega^2 r + mg$

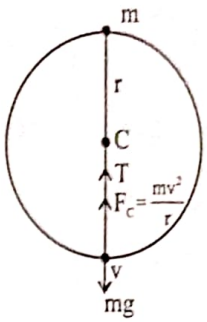
$\therefore$ Tension in the string is

$T = 0.5 \times (4)^2 \times 1 + 0.5 \times 10 = 13N$



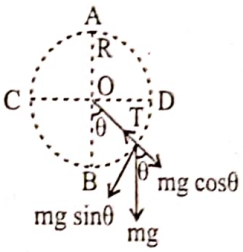
47. (*d*) Tension at lowest point is given by

$$T = \frac{mv^2}{r} + mg = mr\omega^2 + mg$$



$$= mr\left(\frac{2\pi n}{60}\right)^2 + mg = m\left(\frac{\pi^2 n^2 r}{900} + g\right)$$

48. (*b*) The body describing a vertical circle,

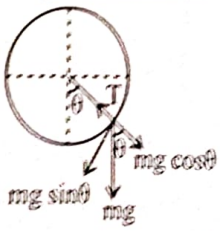


$\therefore T - mg\cos\theta = \frac{mv^2}{R} \Rightarrow T = mg\cos\theta + \frac{mv^2}{R}$

Tension is maximum when $\cos\theta = 1$ and velocity is also maximum. Both conditions are satisfied at $\theta = 0°$, i.e. at lowest point *B*.

49. (*c*) Tension in the string when it makes an angle θ with the vertical, $T = \frac{mv^2}{r} + mg\cos\theta$

When the string is horizontal, $\theta = 90°$



and $\cos 90° = 0$

$\therefore T = \frac{mv^2}{r} + mg \times 0 = \frac{mv^2}{r} = \frac{0.6 \times (5)^2}{0.75} = 20N$

50. (*b*) $T = \frac{mv^2}{r} + mg\cos\theta$

For $\theta = 30°$,

$T_1 = \frac{mv^2}{r} + mg\cos 30° = \frac{mv^2}{r} + \frac{\sqrt{3}mg}{2}$ ...(i)

For $\theta = 60°$,

$T_2 = \frac{mv^2}{r} + mg\cos 60° = \frac{mv^2}{r} + \frac{mg}{2}$ ...(ii)

From equation (i) and (ii), we can say

$\therefore T_1 > T_2$

Exercise-2 (Learning Plus)

1. (*c*) Speed $v_1 = \frac{2\pi r}{t}$,

$v_2 = \frac{2\pi r}{t}$

$\omega_1 = \frac{v_1}{r} = \frac{2\pi}{t} \Rightarrow \omega_2 = \frac{v_2}{2r} = \frac{2\pi}{t}$

$\omega_1 = \omega_2 \Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{1}$

2. (*d*) $\omega_{second} = \frac{2\pi}{T} = \frac{2\pi}{60}$ rad/sec.

$v = \omega.r = \frac{2\pi}{60} \times 0.06$ m/s $= 2\pi$ mm/s

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \sqrt{2}v = 2\sqrt{2}\,\pi$ mm/s

3. (*c*) Given, $\omega_0 = 0$, $t = 2$ sec.

$\theta = \theta_1$, next 2 sec., $\theta = \theta_2$

$\theta_1 = \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha 2^2 = 2\alpha$

$\theta_2 = \frac{1}{2}\alpha(2+2)^2 - \frac{1}{2}\alpha 2^2 = 6\alpha$

$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$

4. (*c*) $\omega_1 = \frac{2\pi}{T_1}$, $\omega_2 = \frac{2\pi}{T_2}$

$\omega_1 : \omega_2 = T_2 : T_1$

$T_1 = 12 \times 60 \times 60$ sec.

$T_2 = 60$ sec.

$\omega_1 : \omega_2 = 60 : (12 \times 60 \times 60)$

$\omega_1 : \omega_2 = 1 : 720$

5. (*d*) Given, $a = 10$m/sec$^2 \Rightarrow \alpha = 5$ rad/sec^2

$a = \alpha r$

$r = \frac{10}{5} = 2$ m

6. (*b*) $a_c = \omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4 \times 3.14^2 \times 6400 \times 10^5}{(24 \times 60 \times 60)^2}$

$\omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4 \times 3.14^2 \times 6400 \times 10^5}{(24 \times 60 \times 60)^2} = 3.4$ cm/sec^2

7. (*c*) Given $r = 25$ cm, $n = 2$

$\omega = 2\pi \times 2$ rad/s $\Rightarrow a_c = \omega^2 r$

$= (4\pi)^2 \times 0.25 = 16\pi^2 \times 0.25 = 4\pi^2$

8. (*c*) Given, $\omega = \theta^2 + 2\theta$

$\frac{d\omega}{d\theta} = 2\theta + 2 \Rightarrow \left.\frac{d\omega}{d\theta}\right|_{t=1} = 2\theta + 2 = 4$

$\alpha = \frac{\omega d\omega}{d\theta} = (\theta^2 + 2\theta).(2\theta + 2) = 12$ rad/sec^2

9. (*d*) $r = 144$ m, $m = 16$ kg, $T_{max} = 16\ N$

$T = \frac{mv^2}{r}$

$v = \sqrt{\frac{Tr}{M}} = \sqrt{\frac{16 \times 144}{16}} = 12$ m/s

10. (*d*) $T = m\omega^2 r \Rightarrow T^1 = 2T = m\omega_1^2 r$

$\omega_1 = \sqrt{2}\,\omega = \sqrt{2} \times 5 = \sqrt{50} \sim 7$ rpm

11. (*c*) For just slip $\Rightarrow \mu mg = m\omega^2 r$

Here ω is double then radius is 1/4th, $r' = 4$ cm

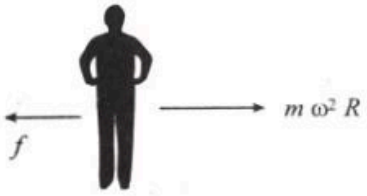
12. (*d*) For circular motion in vertical plane normal reaction is minimum at highest point and it is zero, minimum speed of motorbike is:

$mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR}$

13. (*a*) $T - mg = \frac{mv^2}{r}$ (Centripetal force at lowest point)

$T = \frac{mv^2}{r} + mg$

14. (*a*)



$\mu mg \geq \frac{mv^2}{R}$

$0.5\ mg \geq m \times (5)^2 \times R$

$\frac{0.5 \times 10}{25} \geq R$

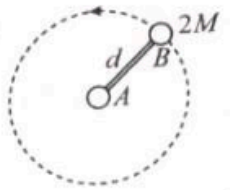
$R \leq 0.2$ m

15. (*c*) $v = \sqrt{Rg \tan\theta}$

$R = 10\sqrt{3}\ m, \theta = 30°$

$= \sqrt{10\sqrt{3} \times 10 \times \frac{1}{\sqrt{3}}} = 10$ m/sec $= 36$ km/hr

16. (*b*)



$P = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{P}$

$T = 2M\omega^2 d = \frac{8\pi^2 Md}{P^2}$

17. (*a*) The maximum bearable tension, $T = \frac{mv^2}{l}$

$T_{max} = 10$N, $m = 1$, $v = ?$, $l = 1$

$v = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{100 \times 1}{1}} = 10$ m/s

18. (*c*) Given that, $v = 72$ km/h., $r = 80$ m

We know that

$\tan\theta = \frac{v^2}{rg} = \frac{20 \times 20}{80 \times 10} = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

19. (*a*) Maximum retardation $a = \mu g$

For apply brakes sharply minimum distance require to stop.

$0 = v^2 - 2\mu gs$

$\Rightarrow s = \frac{v^2}{2\mu g}$

For taking turn minimum radius is

$\mu g = \frac{v^2}{r}, \Rightarrow r = \frac{v^2}{\mu g}$

So apply brakes sharply is safe for driver.

20. (*d*) $\omega_{QP} = 2\pi - 5\pi = -3\pi$ rad/s

$\omega_{RP} = 3\pi - 5\pi = -2\pi$ rad/s

Time when Q particle reaches at

$P = t_1 = \frac{\pi/2}{3\pi} = \frac{1}{6}$ sec.

$t_2 = \frac{5\pi/2}{3\pi} = \frac{5}{6}$ sec.

$t_3 = \frac{9\pi/2}{3\pi} = \frac{3}{2}$ sec.

Time where R particle reaches at P. $t_1 = \frac{\pi}{2\pi} = \frac{1}{2}$ sec.

$t_2 = \frac{3\pi}{2\pi} = \frac{3}{2}$ sec.

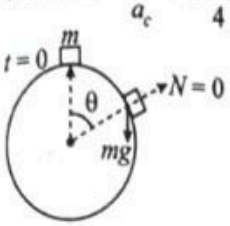
Common time to reaches at P is $\frac{3}{2}$ sec.

21. (*a*) It can be observed that component of acceleration perpendicular to velocity is

$a_c = 4\ m/s^2$

$\therefore$ Radius $= \dfrac{v^2}{a_c} = \dfrac{(2)^2}{4} = 1$ m.

22. (*b*)



At loose contact $N = 0$

$mg\cos\theta = \dfrac{mv^2}{R}$ ...(i)

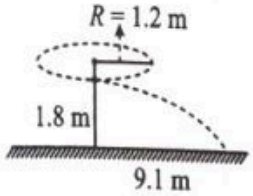
From energy conservation

$mgR(1-\cos\theta) = \dfrac{1}{2}mv^2$ ...(ii)

From (i) and (ii)

$\cos\theta = \dfrac{2}{3} \Rightarrow \sin\theta = \dfrac{\sqrt{5}}{3}$ $\left\{\because a_t = \dfrac{\sqrt{5}g}{3}\right\}$

23. (*c*)



The time taken to fall on ground $= \sqrt{\dfrac{2\times1.8}{9.8}} = \sqrt{\dfrac{36}{98}}$

Velocity at time of string breaks

$v = \dfrac{\text{distance}}{\text{time}} \Rightarrow v = 9.1\sqrt{\dfrac{98}{36}}$

Centripetal acceleration $= \dfrac{v^2}{R} = \dfrac{9.1\times9.1\times98}{1.2\times36} = 187.856$

$= 188\ m/s^2$

24. (*d*) For M to be stationary

$T = Mg$ ...(i)

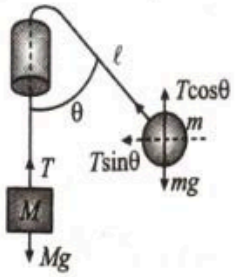
Also for mass m,

$T\cos\theta = mg$ ...(ii)

$T\sin\theta = \dfrac{mv^2}{\ell\sin\theta}$ ...(iii)

Dividing (iii) by (ii)

$\tan\theta = \dfrac{v^2}{g\ell\sin\theta} \Rightarrow v = \sqrt{\dfrac{g\ell}{\cos\theta}}.\sin\theta$



Time period $= \dfrac{2\pi R}{v} = \dfrac{2\pi\ell\sin\theta}{\sqrt{\dfrac{g\ell}{\cos\theta}}.\sin\theta}$

From (i) and (ii) $\cos\theta = \dfrac{m}{M}$

Then time period $= 2\pi\sqrt{\dfrac{\ell m}{gM}}$

25. (*a*) Maximum tension in string at lowest

$T_{max} = \dfrac{mv_{LP}^2}{L} + mg$ ...(i)

minimum tension in string at highest point.

$T_{min} = \dfrac{mv_{HP}^2}{L} - mg$ ...(ii)

from energy conservation

$\dfrac{1}{2}mv_{LP}^2 = 2mgL + \dfrac{1}{2}mv_{HP}^2$ ....(iii)

From (i) & (iii)

$T_{max} = \dfrac{1}{L}mv_{HP}^2 + 5mg$ ...(iv)

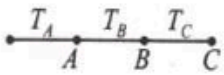
From (ii) & (iv)

$4 = \dfrac{T_{max}}{T_{min}} = \dfrac{\dfrac{mv_{HP}^2}{L} + 5mg}{\dfrac{mv_{HP}^2}{L} - mg} \Rightarrow 3mv_{HP}^2 = 9\ mgL$

$\Rightarrow V_{HP} = \sqrt{3gL} = 10$ m/s

26. (*d*) ω = const., for all three particles

$\omega = \dfrac{v}{3\ell}$



$T_C = m\omega^2\, 3\ell$

$T_B - T_C = m\omega^2 2\ell$

$T_B = 5\ m\omega^2\ell$

$T_A - T_B = m\omega^2\ell$

$T_A = 6\ m\omega^2\ell$

$T_C : T_B : T_A :: 3 : 5 : 6$

27. (*b*) $F = kx,\ T_1 = ka = m\omega^2\, 2a \Rightarrow \omega = \sqrt{\dfrac{k}{2m}}$

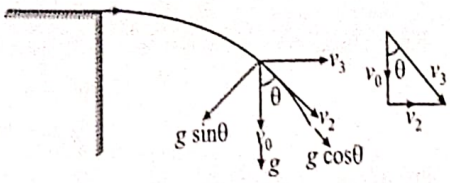
Time period $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{2m}{k}} = T$

$T_2 = 2ka = m\omega^2 3a \Rightarrow \omega = \sqrt{\dfrac{2k}{3m}}$

Time period $= 2\pi\sqrt{\dfrac{3m}{2k}} = T'$

$T' = \left(\dfrac{\sqrt{3}}{2}\right)T$

28. (*a*)



As we know: $a_C = \frac{v^2}{R}$ (centripetal acceleration)

From figure; $g\sin\theta = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{g\sin\theta}$

$\Rightarrow g.\frac{v_0}{v} = \frac{v^2}{R}$ (since $\sin\theta_i = \frac{v_0}{v}$)

$\Rightarrow R \alpha v^3$

29. (*b*) Given $v_B = 0.5\sqrt{gr}$

Assume block leave the contact at $C, N = 0$ $CN = 0$

$\frac{mv_C^2}{r} = mg\cos\theta$ ....(i)

From energy conservation

$\frac{1}{2}mv_B^2 + mgr(1-\cos\theta) = \frac{1}{2}mv_C^2$ ...(ii)

From equation (i) and (ii)

$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mgr(1-\cos\theta) = \frac{1}{2}mgr\cos\theta$

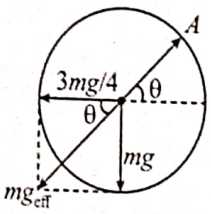
$\Rightarrow \cos\theta = \frac{3}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right)$

30. (*a*) $\omega = \frac{v_\perp}{r}$

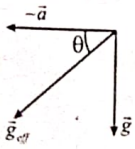
$\omega = \frac{v\sin\theta}{r}$, $v = \frac{\omega r}{\sin\theta}$

$\Rightarrow v = \frac{3\omega}{\sin\theta} = \frac{0.1\times 3\sqrt{2}}{(1/\sqrt{2})} = 0.6$ m/s

31. (*b*)



$\vec{g}_{eff} = \vec{g} - \vec{a}$

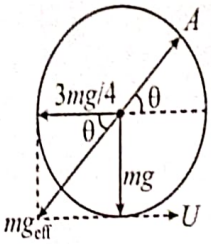


Tension would be minimum when it (tension) is along $\vec{g}_{eff}$

$\tan\theta = \frac{g}{g_{eff}} = \frac{g}{\frac{3}{4}g} = \frac{4}{3}$

$\therefore \theta = 53°$.

32. (*c*)



$V_{min} = \sqrt{\ell g_{eff}} = \sqrt{\ell\frac{5}{4}g} = \frac{\sqrt{5\ell g}}{2}$.

33. (*c*) $T_{max} = 6\,mg_{eff}$ $\left(g_{eff} = \frac{5}{4}g\right)$

$= \frac{15}{2}mg$

Exercise-3 (JEE Advanced Level)

1. (*b*) $K = \frac{1}{2}mv^2 = as^2 \Rightarrow v^2 = \frac{2as^2}{m}$

$a_C = \frac{v^2}{R} = \frac{2as^2}{mR}$

$a_t = v\frac{dv}{ds} = \frac{2as}{m}$

$a = \sqrt{\left(\frac{2as^2}{mR}\right)^2 + \left(\frac{2as}{m}\right)^2} = \frac{2as}{m}\left(1+\frac{s^2}{R^2}\right)^{1/2}$

Total force $= ma = 2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$

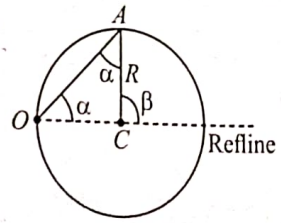
2. (*b*) $v = a\sqrt{s} \Rightarrow v^2 = a^2 s$

$a_t = v\frac{dv}{ds} = \frac{a^2}{2}$

$a_C = \frac{v^2}{R} = \frac{a^2 s}{R}$

$\tan\alpha = \frac{a_c}{a_t} = \frac{a^2 s/R}{a^2/2} = \frac{2s}{R}$

3. (*d*)



$\beta = 2\alpha \Rightarrow \frac{d\beta}{dt} = \frac{2d\alpha}{dt} = 2\times 0.4 = 0.8$ rad/s

$v_{AC} = \omega r = 0.8 \times \frac{1}{2} = 0.4$ m/s

$a_C = \omega^2 r = (0.8)^2\,\frac{1}{2} = 0.32$ m/s^2

$a = a_C = 0.32$ m/s^2 ($a_t = 0$)

4. (*a*) Apply Newton's law at angle θ

$$T - mg\cos\theta = \frac{mv^2}{\ell} \quad ...(i)$$

Apply energy conservation from θ_0 to θ

$$mg\ell(\cos\theta - \cos\theta_0) = \frac{1}{2}mv^2 \quad ...(ii)$$

From (i) and (ii)

$$T = 3mg\cos\theta - 2mg\cos\theta_0 \quad ....(iii)$$

$T_{max} = 2mg$ (Put in equation (iii))

$\Rightarrow \theta_0 = 60°$

5. (*b,d*) Fact based

6. (*a,c,d*)

In curved path, may be circular or parabolic.

In circular path speed and magnitude of acceleration are constant.

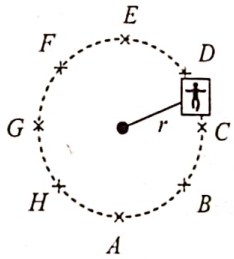
In parabolic path acceleration is constant.

7. (*a,b,c,d*)

$v = \sqrt{gr}$

$\Rightarrow$ At A, $N = mg + \frac{mv^2}{r}$

$= 2mg[v = \sqrt{gr}]$



At E, $N + \frac{mv^2}{r} = mg$

$\Rightarrow N = 0$

$\Rightarrow$ At G and C

$N = mg$

8. (*b,c*)

$$T - mg\cos\theta = \frac{mv^2}{L}$$

Tangential Acceleration = $g\sin\theta$

9. (*b,d*)

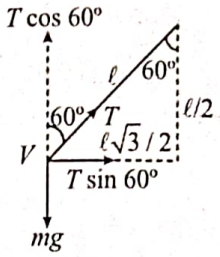
When speed of car is 36 km/hr, car can make a turn without skidding. If speed is less than 36 km/hr than tendency of slipping is downward so it will slip down. If speed is greater than 36 km/hr than tendency of slipping upward so it will slip up.

If the car's turn at correct speed 36 km/hr

$N\cos\theta = mg$

$$N\sin\theta = \frac{mv^2}{r} \Rightarrow N = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)}$$

10. (*a,b,c,d*)



$$\frac{T\sqrt{3}}{2} = \frac{mv^2}{\ell\sqrt{\frac{3}{2}}} \quad ...(i)$$

$$\frac{mv^2}{(\ell\sqrt{3}/2)} = \frac{T}{2} \quad ...(ii)$$

$$\sqrt{\frac{3\times 9.8\times 1.6}{2}} = mg$$

Hence $T = 2$ mg, So (*b*) holds

From (i) and (ii), we get

$v^2 = 3g\ell/2$

$\therefore v = 2.8\sqrt{3}$ m/s. So (*c*) hold

$a_c = v^2/r = \sqrt{3} \times g$

$= 9.8\frac{2\pi r}{v}$ m/s^2

$= 9.8\sqrt{3}$ m/s^2

$\therefore$ (*d*) holds

$$t = \frac{2\pi\sqrt{\ell\sqrt{3}/2}}{\sqrt{(3g\ell/2)}} = \sqrt{gr}$$

$t = 4\pi/7$ $\therefore$ (*a*) holds.

11. (*b,c*) $a_t = \frac{dv}{dt} = a$

Friction force on car = $m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$

Which is greater than $\frac{mv^2}{r}$

$$\mu_{min} = \frac{\sqrt{(v^2/r)^2 + a^2}}{g}$$

Therefore, it is not less than $\frac{a}{g}$ for safe turn.

12. (*b,c*)

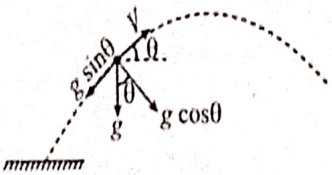
Tangential acceleration = $a_t = g\sin\theta$

Normal acceleration = $a_n = g\cos\theta$

$a_t = a_n$

$g\sin\theta = g\cos\theta$

$\Rightarrow \theta = 45°$



$\Rightarrow v_y = v_x$

$u_y - gt = u_x$

$20 - (10)t = 10$

$t = 1$ sec.

During downward motion

$a_t = a_n$

$v_y = -v_x$

$20 - 10t = -10 \Rightarrow t = 3$ sec.

13. (*b*) The angular velocity and linear velocity are mutually perpendicular

$\therefore \vec{v}\cdot\vec{\omega} = 3x + 24 = 0$ or $x = -8$

14. (*a*) The radius of circle $r = \frac{v}{\omega} = \frac{5}{10} = \frac{1}{2}$ m

15. (*a*) The acceleration of particle undergoing uniform circular motion is

$\vec{a} = \vec{\omega}\times\vec{v} = (-8\hat{i} + 6\hat{j})\times(3\hat{i} + 4\hat{j})$

$= -50\hat{k}$

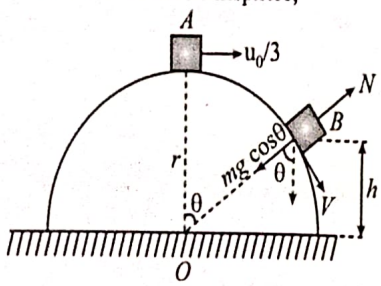
16. (*d*) $mg = \frac{mu_0^2}{r} \Rightarrow u_0 = \sqrt{gr}$

Now, along vertical

$r = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2r}{g}}$

Along horizontal; $OP = 2u_0t = 2\sqrt{2}\,r$

17. (*b*) As at B it leaves the hemisphere,



$\therefore N = 0$

$mg\cos\theta = \frac{mV^2}{r}$

$mg\frac{h}{r} = \frac{mV^2}{r}$

$mv^2 = mgh$ ... (i)

By energy conservation between A and B

$mgr + \frac{1}{2}m\left(\frac{u_0}{3}\right)^2 = mgh + \frac{1}{2}mv^2$

Put u_0 and mv^2

$\therefore h = \frac{19r}{27}$

18. (*c*) As $a_c = \frac{v^2}{r} = g\cos\theta$

$\therefore a_t = g\sin\theta$

$\therefore a_{net} = g$

Alternate Solution: When block leave only the f left is mg.

$\therefore a_{net} = g.$

19. (*a*) (i) deceleration of particles

$\left|\frac{dv}{dt}\right| = \left|v\frac{dv}{ds}\right| = \frac{v^2}{R}$

$-v\frac{dv}{ds} = \frac{v^2}{R} \Rightarrow -\frac{dv}{v} = \frac{ds}{R}$

Integrating both side

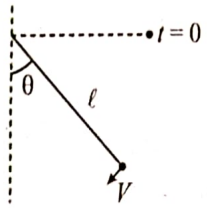
$-\int_{v_0}^{v}\frac{dv}{v} = \int_0^s\frac{ds}{R} \Rightarrow -[\log v]_{v_0}^{v} = \frac{s}{R}$

$v = v_0\,e^{-s/R}$

20. (*a*) $|a_t| = a_C$

$a = \sqrt{2}\,a_C = \sqrt{2}\,\frac{v^2}{R}$

21. (*a*)



At θ angle $a_t = g\sin\theta$

From centripetal acceleration

$T - mg\cos\theta = \frac{mv^2}{\ell}$ ...(i)

From energy conservation :

$0 + mg\ell\cos\theta = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2g\ell\cos\theta}$ ....(ii)

From (i) and (ii)

$T = 3mg\cos\theta$

$a_C = 2g\cos\theta$

$a = \sqrt{a_t^2 + a_c^2} = g\sqrt{1 + 3\cos^2\theta}$

22. (*a*) Total acceleration is directed along horizontal that means

$a_{vertical} = 0$

$\cos\theta = \frac{1}{\sqrt{3}}$

23. (*c*) Vertical component of sphere velocity is maximum when acceleration in vertical is zero that means net force in vertical direction is zero.

Net force in vertical at θ angle

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$ ...(iii)

And tension also from equation

$T = 3mg\cos\theta$ ....(iv)

From (iii) and (iv)

$$3mg\cos\theta = \frac{mg}{\cos\theta}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$T = mg\sqrt{3}$$

24. (*b*) A-(q,s); B-(p); C-(p); D-(q,r)

From graph (*a*) $\Rightarrow \omega = k\theta$ where k is positive constant

angular acceleration $= \omega\dfrac{d\omega}{d\theta} = k\theta \times k = k^2\theta$

$\therefore$ Angular acceleration is non uniform and directly proportional to θ.

$\therefore$ A-(q, s)

From graph (*b*) $\Rightarrow \omega^2 = k\theta$. Differentiating both sides with respect to θ.

$2\omega\dfrac{d\omega}{d\theta} = k$ or $\omega\dfrac{d\omega}{d\theta} = \dfrac{k}{2}$ Hence angular acceleration is uniform.

$\therefore$ B-(p)

From graph (*c*)

$\Rightarrow \omega = kt$

Angular acceleration $= \dfrac{d\omega}{dt} = k$

Hence angular acceleration is uniform $\Rightarrow$ C-(p)

From graph (*d*) $\Rightarrow \omega = kt^2$

angular acceleration $= \dfrac{d\omega}{dt} = 2kt$

Hence angular acceleration is non uniform and directly proportional to t.

$\therefore$ D-(q,r)

A. $\Rightarrow \omega = k\theta k$

$$= \omega\frac{d\omega}{d\theta} = k\theta \times k = k^2\theta$$

$\therefore \theta \therefore$ (A)-(q,s)

B. $\Rightarrow \omega^2 = k\theta.\ \theta$

$2\omega\dfrac{d\omega}{d\theta} = k$ or $\omega\dfrac{d\omega}{d\theta} = \dfrac{k}{2}$ $\therefore$ B-(p)

C. $\Rightarrow \omega = kt$

$$= \frac{d\omega}{dt} = k \Rightarrow \text{C-(p)}$$

D. $\Rightarrow \omega = kt^2$

$$= \frac{d\omega}{dt} = 2kt\ t$$

$\therefore$ D-(q,r)

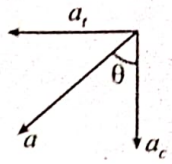
25. (*c*) A-(q); B-(q,s); C-(q,s); D-(p,s)

$v = 2t^2$

Tangential acceleration $a_t = 4t$

Centripetal acceleration $a_c = \dfrac{v^2}{R} = \dfrac{4t^4}{R}$

Angular speed $\omega = \dfrac{v}{R} = \dfrac{4t}{R}$



$$\tan\theta = \frac{a_t}{a_c} = \frac{4tR}{4t^4} = \frac{R}{t^3}$$

26. [0001]

$\omega^2 R = 2R\alpha$

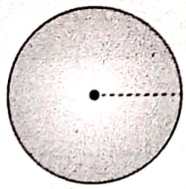
$\omega^2 = 2\alpha = 2\alpha\theta$

$\theta = 1$ rad.

27. [0625]

$v = wR$

$v' = W(R-5) = \dfrac{\omega R}{5}$



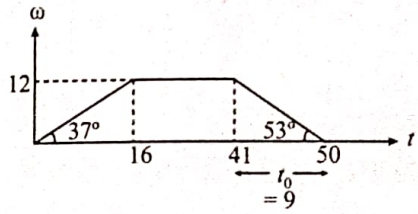
$5R - 25 = R$

$R = \dfrac{25}{4}m = 6.25$ m

28. [0009]

$$<\omega> = \frac{\int \omega\, dt}{\int dt} = \frac{\text{Area under graph}}{\text{time}}$$

$$= \frac{\frac{1}{2}\times 12[25+50]}{50} = 9 \text{ rad/s.}$$



29. [0005]

$T = mg$

$m_{r,max}\,\omega^2 = T + \mu mg$

$m_{r,min}\,\omega^2 = T - \mu mg$

$m(r_{max} + r_{min})\omega^2 = 2mg$

$\therefore r_{max} + r_{min} = \dfrac{2g}{\omega^2} = 5$ m

30. [0200]

$$N_b - mg = \frac{mv^2}{R}$$

$$N_b = mg + \frac{mv^2}{R}$$

$$N_T + mg = \frac{mv^2}{R}$$

$$N_T = \frac{mv^2}{R} - mg = \frac{1}{2}\left(mg + \frac{mv^2}{R}\right)$$

$$\frac{2}{3}\frac{mv^2}{R} = \frac{4}{3}mg$$

$$v = \sqrt{2gR} = \sqrt{20 \times 2000} = 200 \text{ m/s}$$

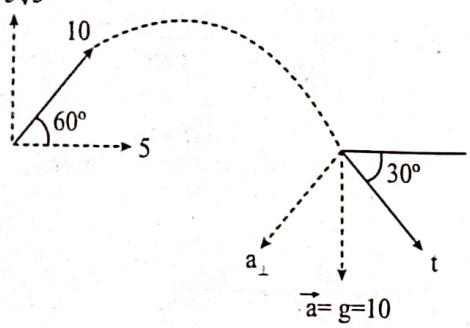
Exercise-4 (Past Year Questions)

JEE MAIN

1. (*b*) $m\omega^2 R = \text{Force} \propto \frac{1}{R^n} \Rightarrow \omega^2 \propto \frac{1}{R^{n+1}} \Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$

Time period $T = \frac{2\pi}{\omega} \propto R^{\frac{n+1}{2}}$

2. (*b*) $5\sqrt{3}$



At $t = 1$

$u_x = 5, u_y = 5\sqrt{3}$

$v_y = 5\sqrt{3} - 10; v_x = 5$

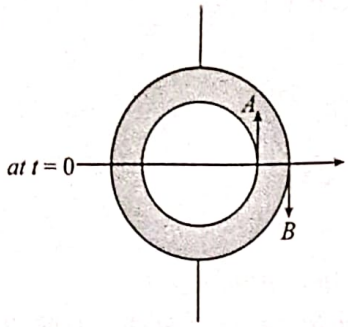
$\tan\theta = -(2 - \sqrt{3}) \Rightarrow \theta = -30°$

$$R = \frac{v^2}{a_\perp} = \frac{10^2}{(10\cos 30°)} = \frac{10}{\sqrt{3}} \times 2 = \frac{20}{\sqrt{3}} m$$

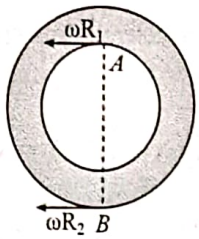
$$\frac{5^2 + (10 - 5\sqrt{3})^2}{10\cos\theta} = \frac{200 - 100\sqrt{3}}{10 \times 0.965} = 2.8 \text{ m}$$

3. (*d*) $\Delta\vec{v} = 2v\sin\left(\frac{\theta}{2}\right)$

4. (*c*)

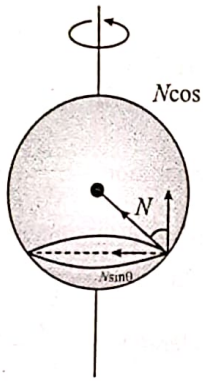


at $t = \frac{\pi}{2\omega}$



$$\vec{v}_A - \vec{v}_B = -\omega R_1\hat{i} + \omega.R_2\hat{i} = \omega(R_2 - R_1)\hat{i}$$

5. (*d*)



$$N\sin\theta = m\frac{r}{2}\omega^2 \quad ...(a)$$

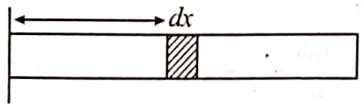
$$N\cos\theta = mg \quad ...(b)$$

$$\tan\theta = \frac{r\omega^2}{2g}$$

$$\frac{r}{2\frac{\sqrt{3}r}{2}} = \frac{r\omega^2}{2g}$$

$$\omega^2 = \frac{2g}{\sqrt{3}r}$$

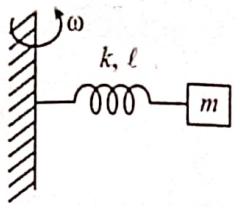
6. (*d*)



$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{\ell} dx\omega^2 x = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$

$$T = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$

7. (*a*)



$m\omega^2(\ell + x) = kx$

$\left(\frac{\ell}{x} + 1\right) = \frac{k}{m\omega^2}$

$x = \frac{\ell m\omega^2}{k - m\omega^2}$

8. (*d*) $\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \omega\left(-\sin\omega t\hat{i} + \cos\omega t\hat{j}\right)$

$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\hat{i} + \sin\omega t\hat{j})$

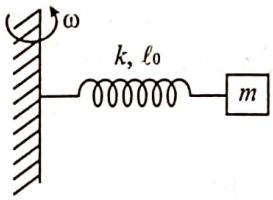
$\vec{a} = -\omega^2\vec{r}$

$\therefore$ $\vec{a}$ is antiparallel to $\vec{r}$

$\vec{v}.\vec{r} = \omega\left(-\sin\omega t \ \cos\omega t + \cos\omega t \ \sin\omega t\right) = 0$

so $\vec{v} \perp \vec{r}$

9. (*d*)



$m\omega^2(\ell_0 + x) = kx$

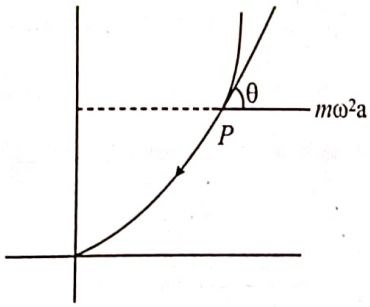
$\left(\frac{\ell_0}{x} + 1\right) = \frac{k}{m\omega^2}$

$x = \frac{\ell_0 m\omega^2}{k - m\omega^2}$

$k >> m\omega^2$

So, $\frac{x}{\ell_0}$ is equal to $\frac{m\omega^2}{k}$.

10. (*a*)



$y = 4Cx^2$

$\Rightarrow \frac{dy}{dx} = \tan\theta = 8Cx$

At P, $\tan\theta = 8Ca$

For steady circular motion

$mg\sin\theta = m\omega^2 a\cos\theta$

$\Rightarrow \tan\theta = \frac{\omega^2 a}{g}$

$\Rightarrow 8ca \times g = \omega^2 \times a$

$\Rightarrow \omega = \sqrt{8gC}$

$\Rightarrow \omega = 2\sqrt{2gC}$

11. (*d*) $a = \omega^2 \times \ell$

$= \left(\frac{2\pi}{T}\right)^2 \times \ell$

$= \left(\frac{2\pi}{60}\right)^2 \times 0.1$

$= 1.1 \times 10^{-3}$ m/s^2

12. (*a*) $\vec{v} = \left(0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}\right)$ m/s

At $t = 2s$

$\vec{v} = (2\hat{i} + 6\hat{j} + 9\hat{k})$

Direction cosine along y-axis,

$\cos\theta = \frac{6}{\sqrt{2^2 + 6^2 + 9^2}} = \frac{6}{\sqrt{121}} = \frac{6}{11}$

$\therefore \sin\theta = \frac{\sqrt{85}}{11}$

and $\tan\theta = \frac{\sqrt{85}}{6}$

$\therefore$ Mosquito make angle $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y-axis.

13. (*b*) Distance travelled = 60 m

$\Rightarrow$ Angle covered = 135°

Displacement = $2R\sin(135°/2)$

$= 2\left(\frac{60}{100} \times \frac{180}{\pi}\right)\left[\frac{1 - \cos(135°)}{2}\right]^{\frac{1}{2}}$

$= 2\left(\frac{80}{\pi}\right)(0.85)^{\frac{1}{2}}$

≈ 47 m

14. (*a*) $N = mv^2/r$

$\Rightarrow$ The graph given in option (*a*) suits the best for the above relation.

15. (*c*) $T = m\omega^2 r$

$\Rightarrow 80 = 0.1 \times \left(2\pi \times \frac{K}{\pi} \times \frac{1}{60}\right)^2 \times 2$

$\Rightarrow \frac{800}{2} = \frac{K^2}{900}$

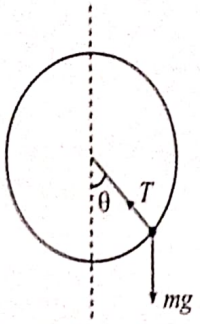
$\Rightarrow K = 30 \times 20 = 600$

16. (*b*) $\theta_1 = \frac{1}{2\alpha}(2\times1-1) = 5$ rad

$\alpha = 10$ rad/sec^2

So $\theta_2 = 12 \times \alpha(2 \times 2 - 1) = 15$ rad

17. (*b*)



At any θ:

$T - mg\cos\theta = mv^2/R$

$\Rightarrow T = mg\cos\theta + mv^2/R$

Since v is constant,

$\Rightarrow T$ will be minimum when $\cos\theta$ is minimum.

$\theta = 180°$ corresponds to $T_{minimum}$.

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18. (*d*) $\frac{1}{2}5mgl = \frac{1}{2}m\frac{5ml}{4} + mg(1-\cos\theta)$

$\cos\theta = -\frac{7}{8}$

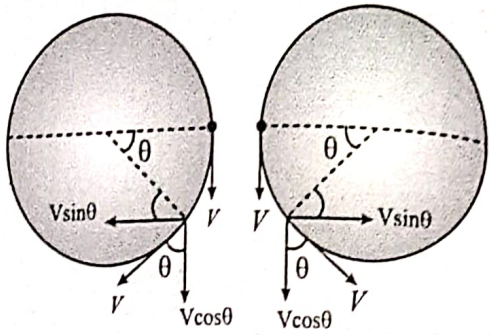
Hence, $\frac{3\pi}{4} < \theta < \pi$

19. (*d*) $T = m\omega^2 l$

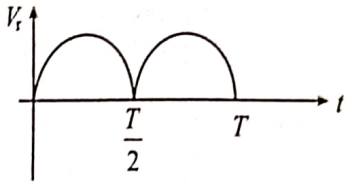
$324 = 0.5\,\omega^2\,(0.5)$

$\omega = 36$ rad/s

20. (*a*)



$v_r = |2\,v\sin\theta)| = |2v\sin\omega t)|$



21. (*b*) $a = \omega^2 r$

$$\Rightarrow \int_0^v v\,dv = \omega^2\int_{R/2}^r r\,dr \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}}$$

$$\& \int_{R/2}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \omega\int_0^t dt$$

$$\Rightarrow r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

22. (*d*) $\vec{F}_{rot} = -m\omega^2 r\hat{i} + 2mv_{rot}\omega(-\hat{j}) + m\omega^2 r\hat{i}$

$= 2mv_{rot}\omega(-\hat{j})$

$= 2m\frac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)\omega(-\hat{j})$

$\vec{F}_{net} = -\vec{F}_{rot} + mg\hat{k} = \frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$

CHAPTER

10 System of Particles and Centre of Mass

Exercise-1 (Topicwise)

1. (*d*) Centre of mass is a point which can lie within or outside the body.

2. (*d*) Both negative and positive coordinates will be required to have COM at origin.

3. (*b*) For a uniform circular distribution, COM will lie at origin. If all particles lie at a point at distance R, COM will lie at same point.

   For other distributions $0 < r_{COM} < R$.

4. (*b*) Centre of mass is nearer to heavier mass

5. (*d*) Centre of mass are $r_{cm} = \frac{h}{4} = \frac{40}{4} = 10$ cm

6. (*c*) $r = 1.27$ Å

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

Since centre of mass cannot go beyond bond length

$$r_{cm} = \frac{0 + 35.5 \times 1.27}{35.5 + 10} = \frac{35.5 \times 1.27}{36.5} = 1.24 \text{ Å}$$

7. (*c*) Fact based

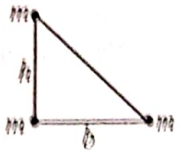
8. (*b*)

$$x_1 = \frac{m_1(0) + m_2(x_1 + x_2)}{m_1 + m_2}$$

$$\Rightarrow x_1 m_1 + x_1 m_2 = m_1 x_1 + m_2 x_2$$

$$\Rightarrow x_1 m_1 = x_2 m_2 \Rightarrow x_1 \propto \frac{1}{m_1}$$

9. (*c*) Assuming particles each of mass m, at the vertices,



$$x_{COM} = \frac{m(0) + m(0) + m(b)}{m + m + m} = \frac{b}{3}$$

$$y_{COM} = \frac{m(0) + m(h) + m(b)}{m + m + m} = \frac{h}{3}$$

10. (*b*) $x_{COM} = \frac{1(0) + 1(PQ) + 1(PR)}{1 + 1 + 1} = \frac{PQ + PR}{3}$

11. (*c*) Initially, COM is at O. On addition of masses, COM does not shift horizontally due to symmetry but shift below O in vertical direction.

12. (*a*) $\vec{r}_{COM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

13. (*d*) Fact based

14. (*d*) $a_{cm} = \frac{m_1 g + m_2 g}{m_1 + m_2} = g$

15. (*a*) Vector sum of internal forces on system is zero.

16. (*c*) $V_{cm} = \frac{M_1 V_1 + M_2 V_2}{M_1 + M_2}$

$$= \frac{200 \times 10\hat{i} + 500(3\hat{i} + 5\hat{j})}{700} = 5\hat{i} + \frac{25}{7}\hat{j}$$

17. (*d*) $V_{CM} = 0$, because internal force cannot change the velocity of centre of mass

18. (*b*) Use $\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$

19. (*a*) $V_{cm} = 0$, as the force acting is internal.

20. (*a*) $\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{COM} = 0 \Rightarrow m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$

$$100\vec{a}_1 + 250(10) = 0$$

$\therefore \vec{a}_1 = -25 \text{ cm/s}^2$, or 25 cm/s^2 eastward

21. (*c*) $P = \sqrt{2mE}$

$\therefore P \propto \sqrt{m}$ (if E = const.)

$$\therefore \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$$

22. (*b*) $0 = mv + MV \Rightarrow V = -\frac{mv}{M}$

23. (*d*) $60(0) = p_1 + p_2 \Rightarrow |p_1| = |p_2|$

$$\text{Now, } k_2 = \frac{p_2^2}{2m_2} = \frac{p_1^2}{2m_2} = \frac{2m_1 k_1}{2m_2} = \frac{m_1 k_1}{m_2}$$

$$= \frac{40}{20} \times 96 = 192 \text{ J}$$

24. (*c*) $I = F\Delta t = 50 \times 10^{-5} \times 3$

$= 1.5 \times 10^{-3}$ Ns

25. (*a*) Area of *F-t* curve = *A* = Impulse.

Impulse = $dP = A = mv - 0$

$\therefore\ v = \dfrac{A}{M}$.

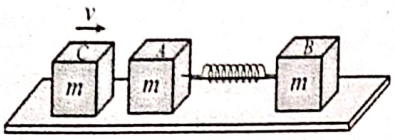
26. (*a*) $I = \Delta p = 2mv$

$= 2 \times 0.06 \times 4 = 0.48$ kg m/s

27. (*a*) Fact based

28. (*d*) Fact based

29. (*a*)



Initial momentum of the system (block *C*) = mv

After striking with *A*, the block *C* comes to rest and now both block *A* and *B* moves with velocity *V*, when compression in spring is maximum.

By the law of conservation of linear momentum

$mv = (m + m)V; \Rightarrow V = \dfrac{v}{2}$

By the law of conservation of energy

K.E. of block *C* = K.E. of system + P.E. of system

$$\frac{1}{2}mv^2 = \frac{1}{2}(2m)V^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow x = v\sqrt{\frac{m}{2k}}$$

30. (*b*) $\Delta p = 2mv\sin 30° = 2 \times 3 \times 10 \times \dfrac{1}{2} = 30$

31. (*a*) If mass = *m* first ball will stop

$\Rightarrow v = 0$

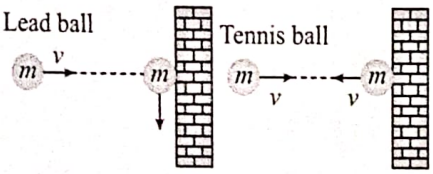
so % K.E. remaining = 0 (min) $\Rightarrow$ Loss = maximum

In other cases there will be some kinetic energy left.

(K.E. can't be negative)

32. (*a*) Fact based

33. (*c*) Change in the momentum = Final momentum – initial momentum



For lead ball $\Delta\vec{P}_{lead} = 0 - m\vec{v} = -m\vec{v}$

For tennis ball $\Delta\vec{P}_{tennis} = -m\vec{v} - m\vec{v} = -2m\vec{v}$

i.e. tennis ball suffers a greater change in momentum.

34. (*a*) $v_1 = \dfrac{(m_1 - em_2)u_1}{m_1 + m_2} + \dfrac{m_2(1+e)u_2}{m_1 + m_2}$

$$= \frac{(m - e2m)u_1}{m + 2m} + \frac{2m(1+e)\times 0}{m + 2m} = 0$$

$\Rightarrow 0 = m - e2m$

$\Rightarrow e = 1/2$

35. (*b*) It is case of head on collision between objects of same masses with one of them (*C* and *D* system) being at rest initially.

36. (*c*) Recall cases of head on elastic collision.

37. (*d*) $v_1 = +3m/s$ $\qquad$ $v_2 = -5m/s$

As $m_1 = m_2$ therefore after elastic collision velocities of masses get interchanged

i.e. velocity of mass $m_1 = -5$ m/s

and velocity of mass $m_2 = +3$m/s

38. (*d*) $h_n = he^{2n}$

$= 1 \times e^{2 \times 1}$

$= 1 \times (0.6)^2 = 0.36$ m

39. (*b*) When ball falls vertically downward from height h_1 its velocity before collision $\vec{v}_1 = \sqrt{2gh_1}$

and its velocity after collision $\vec{v}_2 = \sqrt{2gh_2}$

Change in momentum

$$\Delta\vec{P} = m(\vec{v}_2 - \vec{v}_1) = m\left(\sqrt{2gh_1} + \sqrt{2gh_2}\right)$$

(Because $\vec{v}_1$ and $\vec{v}_2$ are opposite in direction)

40. (*d*) Conservation of momentum,

$1(5) + 0.5(-10) = p'_A + p'_B$

$\Rightarrow p'_A + p'_B = 0$

$$\Rightarrow |p'_A| = |p'_B| \Rightarrow \frac{v_A}{v_B} = \frac{m_B}{m_A} = \frac{1}{2}$$

Further, directions will be opposite

So, $v_A = -5$ m/s, $v_B = 10$ m/s

41. (*b*) By conservation of linear momentum

$P_i = P_f$

$\Rightarrow mv = (100\,m)\,u$

$\Rightarrow u = v / 100$

42. (*a*) After explosion mass *m* comes at rest and let remaining $(M - m)$ mass moves with velocity *v*.

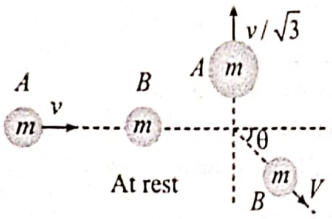
By the law of conservation of momentum

$MV = (M - m)v$

$$\Rightarrow v = \frac{MV}{(M - m)}$$

43. (*d*) Due to the same mass of *A* and *B* as well as due to elastic collision velocities of spheres get interchanged after the collision.

44. (*a*) Let mass A moves with velocity v and collides inelastically with mass B, which is at rest.



According to problem mass A moves in a perpendicular direction and let the mass B moves at angle θ with the horizontal with velocity V.

Initial horizontal momentum of system

(before collision) = mv ...(i)

Final horizontal momentum of system

(after collision) = $mV\cos\theta$ ....(ii)

From the conservation of horizontal linear momentum

$mv = mV\cos\theta$

$\Rightarrow v = V\cos\theta$ ...(iii)

Initial vertical momentum of system (before collision) is zero.

Final vertical momentum of system $\frac{mv}{\sqrt{3}} - mV\sin\theta$

From the conservation of vertical linear momentum

$\frac{mv}{\sqrt{3}} - mV\sin\theta = 0 \Rightarrow \frac{v}{\sqrt{3}} = V\sin\theta$ ...(iv)

By solving (iii) and (iv)

$$v^2 + \frac{v^2}{3} = V^2(\sin^2\theta + \cos^2\theta)$$

$$\Rightarrow \frac{4v^2}{3} = V^2 \Rightarrow V = \frac{2}{\sqrt{3}}v.$$

45. (*b*) Fact based

46. (*d*) Initial momentum = $\vec{P} = mv\hat{i} + mv\hat{j}$

$|\vec{P}| = \sqrt{2}mv$

Final momentum = $2m \times V$

By the law of conservation of momentum

$2m \times V = \sqrt{2}mv$

$$\Rightarrow V = \frac{v}{\sqrt{2}}$$

In the problem v = 10 m/s (given)

$$\therefore V = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ m/s}$$

47. (*b*) By momentum conservation before and after collision.

$m_1 v + m_2 \times 0 = (m_1 + m_2)V$

$$\Rightarrow V = \frac{m_1}{m_1 + m_2}v$$

i.e. Velocity of system is less than v.

48. (*d*) $F_t = m(g + a)$

$= (3.5 \times 10^4)(10 + 10) = 7 \times 10^5$ N

49. (*b*) $F_t = \frac{u\,dm}{dt} = (4.5\times10^4)(100\text{ kg/s}) = 4.5 \times 10^6$ N

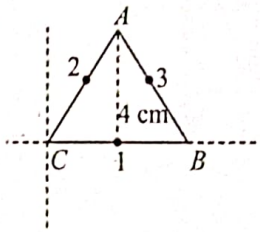
50. (*b*) $F_t = m(g + a) = \frac{u\,dm}{dt}$

$$\Rightarrow \frac{dm}{dt} = \frac{m(g+a)}{u}$$

$$= \frac{6000(9.8+20)}{1000} = 178.8\text{ kg/s}$$

Exercise-2 (Learning Plus)

1. (*a*)



$$y = \frac{m_1y_1 \times m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(6\times2\rho)4 + (5\rho)(2) + (5\rho)(2)}{12\rho + 5\rho + 5\rho} = \frac{34}{11}$$

2. (*d*) Fact based

3. (*b*) $y_{cm} = 0$

$$\Rightarrow \frac{1}{8} \times 0.14 + \frac{7}{8} \times h = 0$$

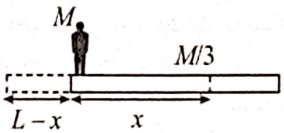
$$\therefore \frac{7h}{8} = -\frac{0.14}{8}$$

$\Rightarrow h = -0.02$ below x-axis.

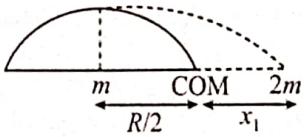
4. (*b*) Let x be the displacement of man. Then displacement of plank is $L - x$.

For centre of mass to remain stationary,

$$\frac{M}{3}(L - x) = M.x \Rightarrow x = \frac{L}{4}$$



5. (*c*) Centre of mass hits the ground at the position where original projectile would have landed.



$$m \times \frac{R}{2} = 2mx_1 \Rightarrow x_1 = \frac{R}{4}$$

$$\therefore \text{Distance} = R + \frac{R}{4} = \frac{5R}{4}$$

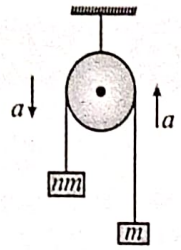
6. (a) $v_{cm} = \dfrac{1\times 2+\frac{1}{2}\times 6}{1+1/2} = \dfrac{10}{3}$ m /sec

7. (c) $a = \dfrac{(nm-m)}{nm+m}g$

$= \dfrac{(n-1)}{(n+1)}g$

$a_1 = a_2 = a$

$a_{cm} = \dfrac{nma_1 - ma_2}{(nm+m)} = \dfrac{(n-1)}{(n+1)}\times a$



$a_{cm} = \dfrac{(n-1)^2}{(n+1)^2}g$.

8. (b) Fact based

9. (a) $v_{cm} = 0$

$Mv_B + m(v_B + v_{rel}) = 0$

$\therefore v_B = -\dfrac{mv_{rel}}{m+M}$

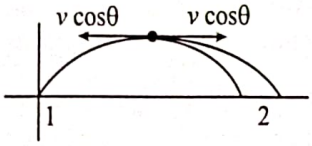
– sign means baloon moves downward

10. (c) C_1 will move but C_2 will be stationary with respect to the ground. For compartment, passengers can apply external forces, but for compartment + passengers system they will be internal forces.

11. (b) $V_{com} = V\cos\theta$

$V\cos\theta = \dfrac{-m0+mv_2}{2m}$

$\therefore v_2 = 2V\cos\theta$



12. (d) Speed is constant so $K.E. \rightarrow$ Constant

Gravitational potential energy change.

$\because$ Momentum = $m\vec{v}$

$\because$ Direction of $\vec{v}$ changes

$\therefore$ Momentum changes

13. (b) Here net force = 0, means momentum is conserved.

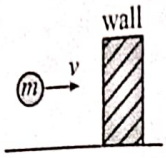
$p_i = p_f$

$0 = \vec{p}_1 + \vec{p}_2 \Rightarrow \vec{p}_1 = -\vec{p}_2$

$K.E. = \dfrac{p^2}{2m}$

$\therefore \dfrac{K_1}{K_2} = \dfrac{m_2}{m_1}$

14. (a)



Initial momentum of body = mv

and final momentum of body = $-mv$

Change in momentum = $2mv$

15. (c) $\vec{F}_{net} = 0$ then $\vec{p}$ = conserved

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

$\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

$m\vec{v}_3 = -m(\vec{v}_1 + \vec{v}_2)$

$\therefore \vec{v}_3 = -\left[(3\hat{i}+2\hat{j})+(-\hat{i}-4\hat{j})\right]$

$\vec{v}_3 = -2\hat{i}+2\hat{j}$

16. (a) $\vec{F}_{net} = 0$

then $\vec{p}$ = conserved

$p_i = p_f$

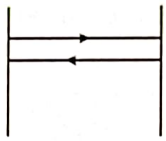
$m_1 v = m_2(0) + (m_1 - m_2)v_1$

$v_1 = \dfrac{m_1 v}{(m_1 - m_2)}$

17. (a) As $F_{net} = 0$, from momentum conservation

$(A-4)v_1 = 4v \Rightarrow v_1 = \dfrac{4v}{(A-4)}$

18. (b) Total travelled distance = $2d$, then



Time between two collisions = $\dfrac{2d}{v_0}$

So no. of collision/sec = $\dfrac{v_0}{2d}$

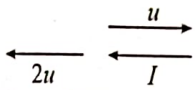
Impulse in one collision = $mv_0 - (-mv_0) = 2mv_0$

$F = 2mv_0 \times \dfrac{v_0}{2d} = \dfrac{mv_0^2}{d}$

19. (b) Impulse = change in momentum

$I = m2u + mu$

$I = 3mu$



W.D. = Change in K.E.

W.D. = $\dfrac{1}{2}m(2u)^2 - \dfrac{1}{2}mu^2$

$= \dfrac{3}{2}mu^2$

$\Rightarrow$ W.D. = $\dfrac{Iu}{2}$

20. (*b*) $mv\hat{i} + mv\hat{j} + 2m\vec{v}_3 = 0$

$$\vec{v}_3 = -\frac{(v\hat{i} + v\hat{j})}{2} = -\frac{v}{2}(\hat{i} + \hat{j})$$

$$\Rightarrow v_3 = -\frac{v}{\sqrt{2}}.$$

$$k_f = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}2m\frac{v^2}{2}.$$

$$k_f = \frac{3mv^2}{2}.$$

21. (*c*) From momentum conservation

$mu = 2mv$

$$\Rightarrow v = \frac{u}{2}$$

from energy conservation

$$\frac{1}{2} \times 2m \times \left(\frac{u}{2}\right)^2 = 2\,mgh$$

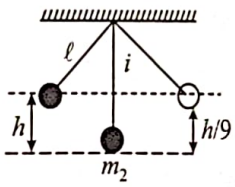
$$\Rightarrow h = \frac{u^2}{8g}$$

22. (*a*) In inelastic collision, due to collision, some fraction of mechanical energy is retained in form of deformation potential energy.

$\therefore$ Thus K.E. of particle is not conserved.

In absence of external forces momentum is conserved.

23. (*a*) $m_1\sqrt{2gh} + 0 = (m_1 + m_2)\,v$



$$v = \frac{m_1\sqrt{2gh}}{(m_1 + m_2)}$$

Also, $v^2 = 2g \times \frac{h}{4} = \frac{gh}{2}$

$$\therefore v = \sqrt{\frac{gh}{2}}$$

Now, $\sqrt{\frac{gh}{2}} = \frac{m_1\sqrt{2gh}}{m_1 + m_2}$

$\Rightarrow 2m_1 = m_1 + m_2;$

$$\therefore \frac{m_1}{m_2} = 1.$$

24. (*c*) As collisions are elastic, therefore $v_i = v_f$

So, $\Delta K = 0$

$$\Rightarrow k_f = k_i = \frac{1}{2}m\left(u_1^2 + u_2^2\right)$$

25. (*c*) In absence of external force, momentum of the system is conserved.

26. (*c*)

B collides with *C* and comes to rest while *C* moves with *v*. *C* collides with *D* and stops with *D* now moving with *v*. The process goes on and *H*, at last, moves with *v*.

A on the other hand collides with *B*(now at rest) giving it velocity *v*. At last this results in *G* moving with *v*.

27. (*a*)

$V = 5$

Vel. of Sep = Vel. of approach ($\therefore$ elastic collision)

$\therefore 20 + 5 = V - 5$

$\Rightarrow V = 30$ m/s

28. (*a*) $mu = mv_1 + mv_2$

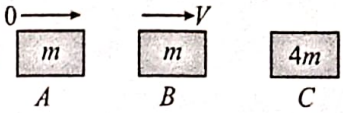
$u = v_1 + v_2$ ...(i)

$\frac{v_2 - v_1}{u} = e$ ...(ii)

As solving (i) and (ii)

$$\frac{v_1}{v_2} = \left(\frac{1-e}{1+e}\right).$$

29. (*a*) After 1st Collision



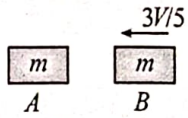
2nd Collision

Conservation of momentum:

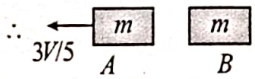
$mV = -mv' + 4mv \Rightarrow V = -v' + 4v$ ...(i)

COE: $1 = \frac{v + v'}{V} \Rightarrow V = v' + v$ ...(ii)

From (i) and (ii), $v' = \frac{3V}{5}$



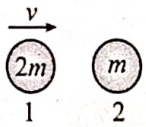
After collision of *A* and *B*.



30. (*b*) Let mass of ball 2 is *m* and mass of ball 1 is 2*m*.

$$v_1 = \frac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2}$$

$$= \frac{v}{3} \Rightarrow \frac{2mv + em(0 - v)}{3m} \Rightarrow e = 1$$



So elastic collision.

31. (*c*) $5 \times 10 = \frac{5}{2}(0) + \frac{5}{2}(v_1) \Rightarrow v_1 = 20$ m/sec

$$\Delta K = \frac{1}{2} \times \frac{5}{2}(20)^2 - \frac{1}{2} \times 5(10)^2 = 500 - 250 = 250 \text{ J}.$$

32. (*a*) $mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

By momentum conservation

$m\sqrt{2gh} + 0 = 2mv'$

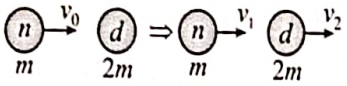
$v' = \frac{\sqrt{2gh}}{2}$

By energy conservation

$\frac{1}{2}(2m)v'^2 = 2mgh' \Rightarrow m\frac{(2gh)}{4} = 2mgh' \Rightarrow h' = \frac{h}{4}$

33. (*d*) Let initial speed of neutron is v_0 and kinetic energy is K.

1st collision



By momentum conservation

$mv_0 = mv_1 + 2mv_2 \Rightarrow v_1 + 2v_2 = v_0$

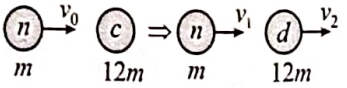
By $e = 1$ $v_2 - v_1 = v_0$

$\Rightarrow v_2 = \frac{2v_0}{3}$; $v_1 = \frac{v_0}{3}$

$$\text{Fractional loss} = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$$

$\Rightarrow P_d = \frac{8}{9} \approx .89$

2nd collision



By momentum conservation

$mv_0 = mv_1 + 12mv_2 \Rightarrow v_1 + 12v_2 = v_0$

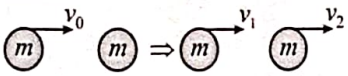
By $e = 1$: $v_2 - v_1 = v_0$

$v_2 = \frac{2v_0}{13}$; $v_1 = \frac{-11v_0}{13}$

Now fraction loss of energy

$$P_c = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2} = \frac{48}{169} \approx 0.28$$

34. (*a*) Initial



$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$

$\Rightarrow v_1^2 + v_2^2 = \frac{3}{2}v_0^2$ ...(i)

From momentum conservation

$mv_0 = m(v_1 + v_2)$

$\Rightarrow (v_1 + v_2)^2 = v_0^2 \Rightarrow v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

Using (i), $2v_1v_2 = -\frac{v_0^2}{2}$

Now, $(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$

$v_1 - v_2 = \sqrt{2}v_0$

35. (*d*) $F = \frac{dp}{dt} = 2n\,mv\cos 45°$

$$\text{Pressure} = \frac{F}{A} = \frac{2nmv\cos 45°}{\text{Area}}$$

$$= \frac{2\times10^{23}\times3.3\times10^{-27}\times10^3\times\left(\frac{1}{\sqrt{2}}\right)}{2\times10^{-4}}$$

$= 2.35 \times 10^3$ N/m^2

36. (*b*) $\frac{dp}{dt} = F = kt$

$\int_p^{3p} dp = \int_0^T kt\,dt$

$2p = \frac{kT^2}{2}$; $T = 2\sqrt{\frac{p}{k}}$

37. (*c*) Apply LMC (Linear Momentum Conservation)

$mv = (2m + M)v'$

$v' = \frac{mv}{2m + M}$

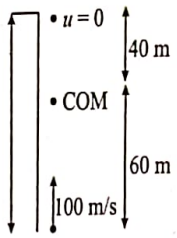
Initial energy: $K_i = \frac{1}{2}mv^2$

Final energy: $K_f = = \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2$

$K_i - K_f = \frac{5}{6}$

On solving, we get $\frac{M}{m} = 4$

38. (*c*)



Y_{cm} from ground $= \frac{0.03\times100}{0.05} = 60$ m

$V_{cm} = \frac{0.02\times100}{0.05} = 40$ m/s

$H = \frac{V_{cm}^2}{2g} = \frac{40\times40}{20} = 80$ m

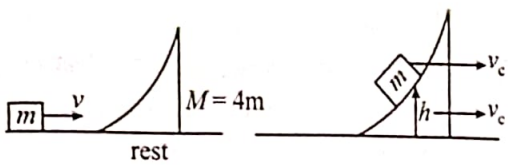
Height above building = 80 – 40 = 40 m

39. (*b*) Linear momentum conservation

$$m\,2v + 2m\,v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$$v' = 2\sqrt{2}v$$

40. (*c*)



Applying Linear momentum conservation

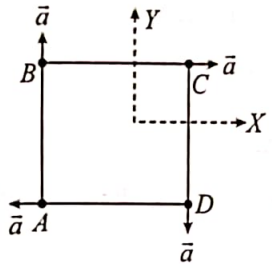
$$mv = (m + M)v_c \Rightarrow v_c = \frac{v}{5}$$

Applying work energy theorem

$$-mgh = \frac{1}{2}(m+M)v_c^2 - \frac{1}{2}mv^2$$

Solve, $h = \dfrac{2v^2}{5g}$

41. (*a*)



$$\vec{a}_A = -a\hat{i}$$

$$\vec{a}_B = a\hat{j}$$

$$\vec{a}_C = a\hat{i}$$

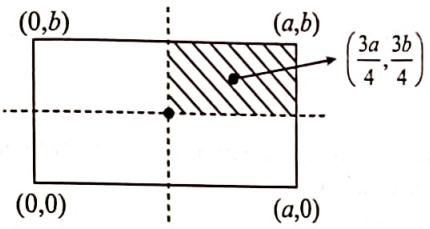
$$\vec{a}_D = -a\hat{j}$$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m} = \frac{2ma\hat{i} - 2ma\hat{j}}{10m}$$

$$= \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j} = \frac{a}{5}(\hat{i} - \hat{j})$$

42. (*d*)



$$x = \frac{M\frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}} = \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

$$y = \frac{M\frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

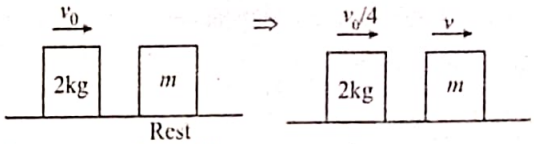
43. (*c*) Applying linear momentum conservation

$$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$$

$$\Rightarrow m_1v_1 + 0.5\,m_1v_2 = m_1(0.5\,v_1) + 0.5\,m_1v_4$$

$$\Rightarrow 0.5\,m_1v_1 = 0.5\,m_1(v_4 - v_2) \Rightarrow v_1 = v_4 - v_2$$

44. (*b*)



By conservation of linear momentum:

$$2v_0 = 2\left(\frac{v_0}{4}\right) + mv \Rightarrow 2v_0 = \frac{v_0}{2} + mv \Rightarrow \frac{3v_0}{2} = mv \quad ...(i)$$

Since collision is elastic,

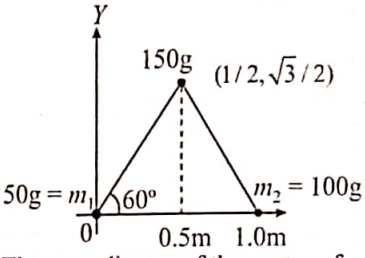
$$V_{\text{separation}} = v_{\text{approach}}$$

$$\Rightarrow v - \frac{v_0}{4} = v_0 \Rightarrow \frac{5v_0}{4} = v \quad ...(ii)$$

From (i) and (ii),

$$\frac{3v_0}{4} = m\left(\frac{5v_0}{4}\right) \Rightarrow m = \frac{6}{5} = 1.2\text{kg}$$

45. (*c*)



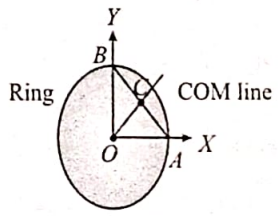
The co-ordinates of the centre of mass

$$\vec{r}_{cm} = \frac{50(0) + 150 \times \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 100 \times \hat{i}}{300}$$

$$\vec{r}_{cm} = \frac{7}{12}\hat{i} + \frac{\sqrt{3}}{4}\hat{j} = \text{Co-ordinate } \left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)\text{m}$$

Exercise-3 (JEE Advanced Level)

1. (*b,d*) Center of mass of ring is at centre and centre of mass of chord *AB* is at its mid point (*C*) So centre of mass of this combination lie at the line *OC* which makes 45° with *x* axis.



Possible combination $\left(\frac{R}{3}, \frac{R}{3}; \frac{R}{4}, \frac{R}{4}\right)$

Since $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}\right)$ does not lie on *OC*.

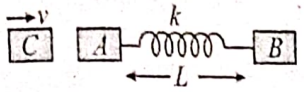
2. (*b,c*) $\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{cm} = 0$

$\Rightarrow \vec{v}_{cm} = \text{constant}$

3. (*b,d*) $\vec{F}_{ext} \neq 0 \Rightarrow \vec{a}_{cm} \neq 0$

Further, it is possible that at some instant $\vec{v}_{cm} = 0$ but $\vec{a}_{cm} \neq 0$.

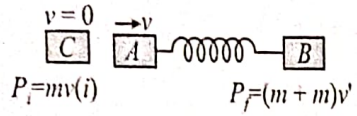
4. (*b,d*)



After collision of A with C

$C \rightarrow$ stops

$A \rightarrow$ moves with v



At maximum compression, blocks A and B move with common speed v'.

$P_i = P_f, \Rightarrow v' = v/2$

By energy conservation,

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}(2m)(v')^2 + \frac{1}{2}kx^2$$

$$\Rightarrow kx^2 = \frac{mv^2}{2} \Rightarrow x = \sqrt{\frac{m}{2k}} \times v$$

At maximum compression $k = \frac{1}{2}(m+m)v'^2$

$\Rightarrow k = mv'^2 = (mv^2)/4.$

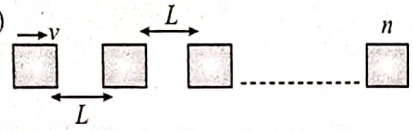
5. (*a,d*) Fact based

6. (*a,d*) It follows from the law of conservation of linear momentum that

$\vec{p}_{block} = \vec{p}_{parts}$

Also, chemical potential energy of block is converted into kinetic energy of the parts during explosion. Therefore, the kinetic energy of the system increases.

7. (*a,c*)



$mv = (nm)v' \Rightarrow v' = v/n$

$$v_{cm} = \frac{(nm)v'}{nm} = v' = \frac{v}{n}$$

Time for first collision is $t_1 = \frac{L}{V}$ (2nd block)

2nd collisions $t_2 = \frac{2\ell}{V} = 2t_1$ (3rd block)

so $t = t_1 + 2t_1 + 3t_1 + at_1$ .....$(n-1)\,t_1$.

$t = t_1\,[1 + 2 + 3$ .....$(n-1)]$

$$= \frac{(n-1)(n-1+1)t_1}{2} = \frac{n(n-1)t_1}{2}$$

so $t = \frac{L}{2V}\,n\,(n-1)$.

8. (*a,c*) $a = \frac{F}{m}$ for elastic collision $e = 1$

$v_1^2 = 0 + 2ad$

$$v_{b1}^2 = \frac{2F}{m} \cdot d \quad v_{b1} = \sqrt{\frac{2Fd}{m}}$$

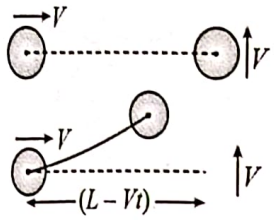
Since collision are elastic, after collision $v'_{b1} = 0$.

Similarly, speed of bead 2 after collision with bead 3 will be zero.

9. (*b,d*)

$KE_f > KE_i$ since chemical energy transforms into kinetic energy, while momentum remains conserved.

10. (*a,c*)

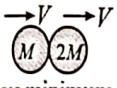


$$\sqrt{(Vt)^2 + (L - vt)^2} \leq L \Rightarrow 2V^2t^2 + L^2 - 2LVt \leq L^2$$

$$Vt - L \leq 0 \Rightarrow t \leq \frac{L}{V}$$

Kinetic energy changes where string get stretched.

11. (*a,b,d*)



For system to have minimum kinetic energy, the collision should be perfectly inelastic so that maximum KE is lost. Thus,

$$mv_0 = (m + 2m) \Rightarrow v = \frac{v_0}{3}$$

$$\Delta K = \frac{1}{2}mv_0^2 - \frac{1}{2}3m\left(\frac{v_0}{3}\right)^2 = \frac{2}{3}\left(\frac{1}{2}mv_0^2\right) = 2\text{ J}$$

So, max potential energy = $\Delta K = 2$ J

Minimum KE = 3 – 2 = 1 J

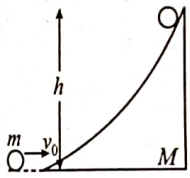
12. (*a,b,c,d*)

Inelastic collision, $0 < e < 1$

$$e = \frac{v_{sep}}{v_{app}}$$

13. (*b*) As net force in x direction is zero. So from momentum conservation.

$mv_0 = (M + m)v_2$



$$v_2 = \frac{Mv_0}{M + m}$$

14. (b,d) Velocity of center of mass, $v_{com} = \frac{Mv + mv}{M+m} = v = \frac{mv_0}{M+m}$

So both are at rest with respect to centre of mass. And kinetic energy is converted into potential energy.

15. (c) By Energy conservation,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(M+m)\left(\frac{mv_0}{M+m}\right)^2 + mgh$$

After solving

$$\Rightarrow h = \left(\frac{M}{M+m}\right)\frac{v_0^2}{2g}$$

16. (c) v_1 is the velocity of particle and v_2 is the velocity of wedge.

$(v_1 + v_2)$ = velocity of particle w.r.t. wedge

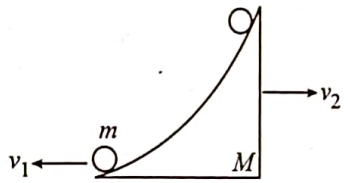
$$\Rightarrow -\left(\frac{mv_0 + M(-v_0)}{M+m}\right) + \left(\frac{mv_0 + mv_0}{M+m}\right) = v_0$$

17. (b,c) As net force in x direction is zero.

So by momentum conservation

$Mv_2 - mv_1 = mV_0$

and $v_1 + v_2 = v_0$



18. (b,d) As net force in x direction is zero.

So by momentum conservation

$Mv_2 - mv_1 = mv_0$ ...(i)

$v_1 + v_2 = v_0$ ...(ii)

By solving

$$v_1 = v_0\left(\frac{M-m}{M+m}\right)$$

$$K_f = \frac{1}{2}mv_1^2 = \frac{1}{2}m\left[\frac{M-m}{M+m}\right]^2 v_0^2$$

19. (a,b,c,d)

(a) $\because v_1 + v_2 = v_0$

$$v_2 = v_0 - v_0\left(\frac{M-m}{M+m}\right)$$

$$= \frac{(M+m)v_0 - v_0M + v_0m}{M+m} = \frac{2mv_0}{M+m}$$

$$\text{K.E.} = \frac{1}{2} \times M \times \frac{4m^2v_0^2}{(M+m)^2} \qquad \left[\because h \ \frac{M}{(m+M)}\frac{v_0^2}{2g}\right]$$

$$\therefore K.E. = \frac{4m^2}{(m+M)}gh$$

(b) $v_2 = \frac{2mv_0}{M+m}$

(c) $\Delta K.E. = k_f - k_i$

$$= \frac{1}{2}M\left(\frac{4m^2v_0^2}{(M+m)^2}\right) - 0$$

$$= \frac{4mM}{(m+M)^2}\left(\frac{1}{2}mv_0^2\right)$$

(d) $\because$ Velocity of wedge $v_2 = \frac{2mv_0}{M+m}$

Velocity of particle $v_1 = v_0\left(\frac{M-m}{M+m}\right)$

$$v_{COM} = \frac{Mv_2 + (-mv_1)}{M+m}$$

$$= \frac{mv_0}{M+m}$$

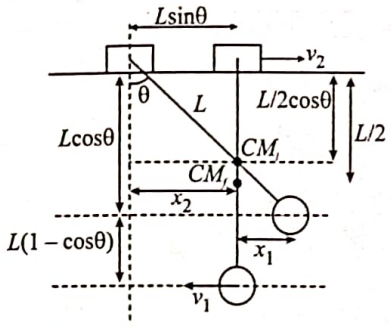
20. (a) As F_{net} in x direction = 0

$\Rightarrow mx_1 = mx_2$

[$\because F_x = 0 \Rightarrow \Delta X_{COM} = 0$, since COM is at rest initially]

$\Rightarrow x_1 = x_2$

Now $x_1 + x_2 = L\sin\theta \Rightarrow x_2 = \frac{L\sin\theta}{2}$



21. (d) $V_{CMx} = 0$ and $F_x = 0$

From momentum conservation

$mv_1 = mv_2 \Rightarrow v_1 = v_2 = v$(let)

Now energy conservation

$$mg\ell(1 - \cos\theta) = 2\left[\frac{1}{2}mv^2\right]$$

$$v^2 = g\ell(1 - \cos\theta)$$

Distance from centre of mass = $R = \frac{\ell}{2}$

$$\text{So } T = \frac{mv^2}{R} = \frac{mg\ell(1-\cos\theta)}{\ell/2}$$

$$T = 2mg(1 - \cos\theta)$$

22. (a) From previous question

$$v_{max} = v = [g\ell(1-\cos\theta)]^{1/2}$$

23. (b) Only in vertical direction displacement occurs.

[$\because f_x = 0$ always]

$$\text{So displacement} = \frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}[1 - \cos\theta]$$

24. (*a*)

A. If velocity of block A is zero, from conservation of momentum, speed of block B is $2u$. Then K.E. of block $B = \frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.

B. Since initial velocity of B is zero, it shall be zero for many other instants of time.

C. Since momentum of system is non-zero, K.E. of system cannot be zero. Also so KE of system is minimum at maximum extension of spring.

D. The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.

25. [10] From conservation of momentum, $2v_2 + 3v_3 = 0$

From conservation of energy.

$$\frac{1}{2}kx^2 = \frac{1}{2}\times 2v_2^2 + \frac{3}{2}v_3^2$$

$$\Rightarrow \frac{1}{2}\times 750(0.4)^2 = v_2^2 + \frac{3}{2}\times\left(\frac{2}{3}v_2\right)^2$$

$$375 \times 0.16 = \frac{5}{3}v_2^2$$

$$v_2 = \sqrt{225\times 0.16} = 6 \Rightarrow v_3 = \frac{2}{3}\times 6 = 4$$

Thus, $v_2 + v_3 = 10$

26. [25] $M_0 = \rho\times\frac{4}{3}\pi R^3$

$$M_c = \rho\times\frac{4}{3}\pi\left(\frac{R}{4}\right)^3 = \frac{M_0}{64}$$

$$x_c = \frac{M_0\times 0 - M_c\times\frac{R}{4}}{M_0 - M_c} = \frac{\frac{-M_0}{256}R}{M_0 - \frac{M_0}{64}} = 25 \text{ km}$$

27. [2000]

$mv = Mv_1 + Mv/2$ (From conservation of momentum)

$$\Rightarrow v_1 = \frac{mv}{2M} = \sqrt{5g\ell} \Rightarrow v = \frac{2M}{m}\sqrt{5g\ell}$$

$$v = \frac{2M}{m}\sqrt{5g\ell} = \frac{2\times 1}{0.01}\sqrt{100} = 2000 \text{ m/s}$$

28. [125]

$3mv = FT$

$4mv_x = FT = 3\,mv$

$v_x = \frac{3v}{4}$ and $v_y = v$

$$v' = \sqrt{100^2 + 75^2} = 25\sqrt{4^2 + 3^2} = 125 \text{ m/s}$$

29. [19] Area $= mv - mv_0$

$-42 = 2\,(v - 2)$

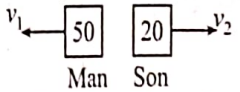
$-21 = v - 2$

$\therefore v = -19$ m/s

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*a*)



$\Rightarrow 0 = 50v_1 - 20v_2$ and $v_1 + v_2 = 0.7$

$\Rightarrow v_1 = 0.2$

2. (*c*) Along horizontal direction,

$M\times 10\cos 30° + 2M\times 5\cos 45°$

$= 2M\times v_1\cos 30° + Mv_2\cos 45°$

$$\Rightarrow 5\sqrt{3} + 5\sqrt{2} = 2v_1\frac{\sqrt{3}}{2} + \frac{v_2}{\sqrt{2}}$$

Along vertical direction,

$M\times 10\sin 30° - 2M\times 5\sin 45° = Mv_2\sin 45° - 2Mv_1\sin 30°$

$$\Rightarrow 5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1$$

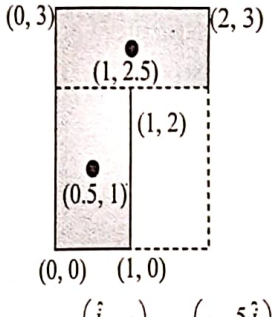
Solving $v_1 = \frac{17.5}{2.7} = 6.5$ m/s and $v_2 \approx 6.3$ m/s

3. (*c*) From 1 kg mass,

$$x_{COM} = \frac{1(0) + 2.5(0) + 1.5(3)}{1 + 2.5 + 1.5} = \frac{4.5}{5} = 0.9 \text{ cm}$$

$$y_{COM} = \frac{1(0) + 2.5(4) + 1.5(0)}{1 + 2.5 + 1.5} = 2 \text{ cm}$$

4. (*c*)



$$\bar{r}_{cm} = \frac{2\times\left(\frac{\hat{i}}{2} + \hat{j}\right) + 2\times\left(\hat{i} + \frac{5\hat{j}}{2}\right)}{2 + 2}$$

$$\Rightarrow \bar{r}_{cm} = \frac{3}{4}\hat{i} + \frac{7}{4}\hat{j}$$

5. [1] For elastic collision $KE_i = KE_f$

$$\Rightarrow \frac{1}{2}m\times 25 + \frac{1}{2}\times m\times 9 = \frac{1}{2}m\times 32 + \frac{1}{2}mv^2$$

$\Rightarrow 34 = 32 + v^2$

$\Rightarrow v^2 = 2$

So, $KE_B = \frac{1}{2}\times 0.1\times 2 = 0.1 \text{ J} = \frac{1}{10}$

$x = 1$

6. (*b*) $M_1 = \frac{4}{3}\pi R^3 \rho$

$$M_2 = \frac{4}{3}\pi(1)^3(-\rho)$$

$$X_{com} = \frac{M_1X_1 + M_2X_2}{M_1 + M_2}$$

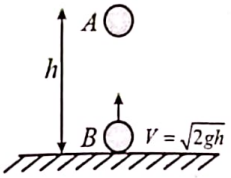
$$\Rightarrow \frac{\left[\frac{4}{3}\pi R^3\rho\right]0 + \left[\frac{4}{3}\pi(1)^3(-\rho)\right][R-1]}{\frac{4}{3}\pi R^3\rho + \frac{4}{3}\pi(1)^3(-\rho)}$$

$$\Rightarrow \frac{(R-1)}{(R^3-1)} = (2-R) \qquad (R \neq 1)$$

$$\frac{(R-1)}{(R-1)(R^2+R+1)} = 2-R$$

$$(R^2 + R + 1)(2 - R) = 1$$

7. (*b*)



Time for collision $t_1 = \frac{h}{\sqrt{2gh}}$

After t_1, $V_A = 0 - gt_1 = -\sqrt{\frac{gh}{2}}$

and $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right]$

At the time of collision $\vec{P}_i = \vec{P}_f$

$$\Rightarrow m\vec{V}_A + m\vec{V}_B = 2m\vec{V}_f$$

$$\Rightarrow -\sqrt{\frac{gh}{2}} + \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right] = 2\vec{V}_f$$

Thus, $V_f = 0$

And height from ground $= h - \frac{1}{2}gt_1^2 = h - \frac{h}{4} = \frac{3h}{4}$

So time $= \sqrt{2 \times \frac{\left(\frac{3h}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$

8. (*d*) Conserving momentum

$$mu\hat{i} + m\left(\frac{u}{2}\hat{i} + \frac{u}{2}\hat{j}\right) = 2m(v_1\hat{i} + v_2\hat{j})$$

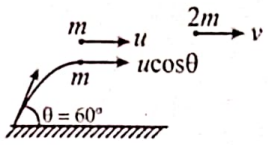
On solving $v_1 = \frac{3u}{4}$ and $v_2 = \frac{u}{4}$

Change in K.E.

$$= \left[\frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{2}\sqrt{2}\right)^2\right] - \left[\frac{1}{2}(2m)\left(\frac{9u^2}{16} + \frac{u^2}{16}\right)\right]$$

$$= \frac{3mu^2}{4} - \frac{5mu^2}{8} = \frac{mu^2}{8}$$

9. (*a*)



$p_i = p_f$

$mu + mu\cos\theta = 2mv$

$$\Rightarrow v = \frac{u(1+\cos 60°)}{2} = \frac{3}{4}u$$

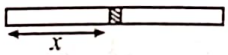
So horizontal range after collision

$$= v\sqrt{\frac{2H_{max}}{g}}$$

$$= \frac{3}{4}u\sqrt{\frac{2u^2\sin^2(60°)}{2g^2}}$$

$$= \frac{3}{4}u^2\frac{\sqrt{\frac{3}{4}}}{g} = \frac{3\sqrt{3}u^2}{8g}$$

10. (*b*)



$$x_{cm} = \frac{1}{M}\int_0^L x.dM$$

$$dM = \lambda.dx = \left(a + b\left(\frac{x}{L}\right)^2\right).dx$$

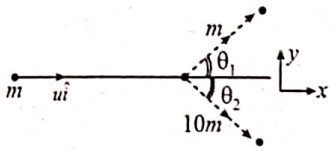
$$x_{cm} = \frac{\int xdM}{\int dM} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int_0^L x\left(a + \frac{bx^2}{L^2}\right)dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right)dx}$$

$$= \frac{a\left(\frac{x^2}{2}\right)_0^L + \frac{b}{L^2}\left(\frac{x^4}{4}\right)_0^L}{a(x)_0^L + \frac{b}{L^2}\left(\frac{x^3}{3}\right)_0^L}$$

$$= \frac{a\frac{L^2}{2} + b\frac{L^2}{4}}{aL + \frac{bL}{3}} = \frac{(2a+b)}{(3a+b)}\frac{L}{4} \times 3$$

$$= \frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$$

11. [10]



Since energy of m reduced by half

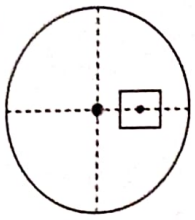
$$\therefore u_1 = \frac{u}{\sqrt{2}} \text{ and } \frac{1}{2} \times 10mv_1^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) \Rightarrow v_1 = \frac{u}{\sqrt{20}}$$

Now, momentum in y direction will remain conserved.

$\therefore mu_1\sin\theta_1 = 10mv_1\sin\theta_2$

$$\Rightarrow \frac{u}{\sqrt{2}}\sin\theta_1 = 10\frac{u}{\sqrt{20}}\sin\theta_2 \Rightarrow \sin\theta_1 = \sqrt{10}\sin\theta_2$$

12. [23]



Let the density is σ

Then original mass $m_0 = \pi a^2\sigma$

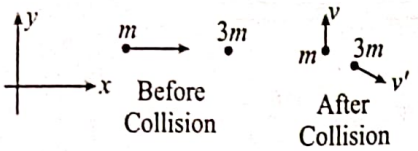
Remaining mass $m' = \left(\pi a^2 - \frac{a^2}{4}\right)\sigma = a^2\left(\frac{4\pi - 1}{4}\right)\sigma$

Removed mass $m = \frac{a^2}{4}.\sigma$

$\therefore\ \pi a^2\sigma(0) = \frac{a^2}{4}.\sigma \times \frac{a}{2} + \frac{\sigma a^2}{4}(4\pi - 1)r$

$\Rightarrow r = \frac{-a}{2(4\pi - 1)} = \frac{-a}{23.13} = \frac{-a}{23}$

13. (*d*)



Conservation of linear momentum

$3m\vec{v}' - mv\hat{j} = mu\hat{i} \Rightarrow 9v'^2 = u^2 + v^2$

For elastic collision

$\frac{1}{2}3mv'^2 + \frac{1}{2}mv^2 = \frac{1}{2}mu^2$

Eliminating v' gives

$v = \frac{u}{\sqrt{2}}$

14. (*d*) Velocity of block after collision $= \frac{0.1 \times 20}{1.9 + 0.1} = 1$ m/s

Kinetic energy just before striking the floor

$= \frac{1}{2}Mv^2 + Mgh = \frac{1}{2} \times 2 \times 1^2 + 2 \times 10 \times 1 = 21\text{J}$

15. (*d*) $\vec{P}_i = \vec{P}_f$

$\Rightarrow mv = (16\,m)v_0 \Rightarrow v_0 = \frac{v}{16}$

$\Rightarrow (\Delta KE)_{loss} = \frac{1}{2}mv^2 - \frac{1}{2}(16m)\left(\frac{v}{16}\right)^2$

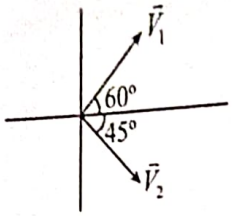
$\Rightarrow \%\ Loss = \frac{(\Delta KE)_{Loss}}{\frac{1}{2}mv^2} \times 100 = \left(1 - \frac{1}{16}\right) \times 100 \approx 94\%$

16. (*c*) $F = \frac{dp}{dt} = \left(\frac{dM}{dt}\right)v + M(t)\frac{dv}{dt} = bv^3 + M(t)a$

$\Rightarrow a = -\frac{bv^3}{M(t)}$

17. [03] $x = \frac{3R}{8} = 3cm$

18. (*d*)



$m_2 = m$ and $m_1 = 2m$

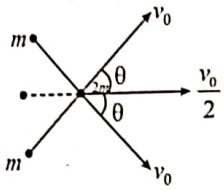
$\vec{p}_i = \vec{p}_f \Rightarrow 2m\left(\sqrt{3}\hat{i} + \hat{j}\right) = 2m\left(\hat{i} + \sqrt{3}\hat{j}\right) + m\vec{V}_2$

$\Rightarrow \vec{V}_2 = \left(2\sqrt{3} - 2\right)\hat{i} - \hat{j}\left(2\sqrt{3} - 2\right)$

$\Rightarrow$ Angle $\theta = 105°$

19. [120]

$mv_0 \times \cos\theta \times 2 = 2m\left(\frac{v_0}{2}\right)$



$\Rightarrow \cos\theta = \frac{1}{2}$

$\theta = 60°$

$\therefore 2\theta = 120°$

20. [4] $dm = \sigma\frac{1}{2}R \times Rd\theta$

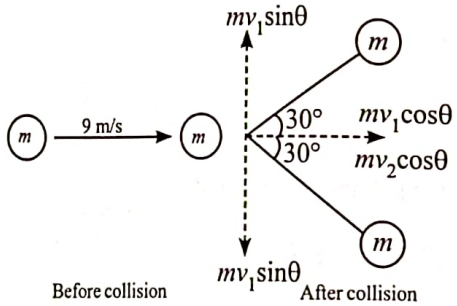
$dm = \frac{\sigma R^2 d\theta}{2}$

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^{\pi/2} \frac{\sigma R^2}{2} d\theta \left(\frac{2R}{3}\cos\theta\right)}{\int_0^{\pi/2} \frac{\sigma R^2}{2} d\theta}$$

$$= \frac{2R}{3}\frac{\int_0^{\pi/2} \cos\theta d\theta}{\int_0^{\pi/2} d\theta} = \frac{2R}{3}\left(\frac{2}{\pi}\right) = \frac{4R}{3\pi}$$

Thus, $x = 4$.

21. (*a*)



Conservation of momentum:

Vertical: $mv_1 \sin\theta = mv_2\sin\theta \Rightarrow v_1 = v_2 = v$

Horizontal: $mv_1\cos\theta = mv_2\sin\theta$

$\Rightarrow 2v\cos 30° = 9 \Rightarrow v = \frac{9}{\sqrt{3}} = 3\sqrt{3} = 5.2\,\text{ms}^{-1}$

22. [25] According to conservation of momentum principle

$P_i = P_f \Rightarrow 2 \times 4 = 2 \times 1 + m_2 \times v_2$

$\Rightarrow m_2 v_2 = 6$ ...(i)

From the coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - 1}{4} \Rightarrow v_2 = 5 \text{ m/s}$$

So, $m_2 = 1.2$ kg

$$\text{Now, } v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{2\times1 + 1.2\times5}{2+1.2} = \frac{8}{3.2} = \frac{25}{10}$$

$\therefore x = 25$

23. (*d*) $$x_{COM} = \frac{(\sigma\pi^2 a)a - \sigma\pi\left(\frac{a}{2}\right)^2 \cdot \left(\frac{3a}{2}\right)}{\sigma\pi^2 a - \sigma\pi\left(\frac{a}{2}\right)^2} = \frac{a - \frac{3a}{8}}{1 - \frac{1}{4}} = \frac{5a}{6}$$

24. (*d*) Velocity of A after collision with C, $v_A = v$

At maximum compression, $v'_A = v'_B = V$

So, from momentum conservation,

$$mv = (m+m)V \Rightarrow V = \frac{v}{2}$$

Now, $E_i = E_f$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2$$

$$\Rightarrow kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow x = \sqrt{\frac{m}{2k}}\, v$$

25. [12] Impulse = change in momentum

$= m[v - (-v)] = 2mv = 2 \times 0.4 \times 15 = 12$

26. (*c*) For position of COM to remain unchanged,

$m_1 x_1 = m_2 x_2$.

$\Rightarrow 10 \times 6 = 30 \times x_2$

$\Rightarrow x_2 = 2$ cm towards 10 kg block.

27. (*c*) For a head on elastic collision

$$v_2 = \frac{mu_1}{m+5m} + \frac{mu_1}{m+5m} = \frac{u_1}{3}$$

Initial kinetic energy of first mass: $K = \frac{1}{2}mu_1^2$

Final kinetic energy of second mass

$$K' = \frac{1}{2}5m\left(\frac{u_1}{3}\right)^2 = \frac{5}{9}\left(\frac{1}{2}mu_1^2\right) = \frac{5}{9}K$$

$$\text{Percentage of energy transferred} = \frac{\frac{5}{9}K}{K} \times 100 = 55.5\%$$

28. [10] For the bob to complete the vertical circular motion,

$v_{bob} = \sqrt{5gl} = \sqrt{5\times10\times2} = 10$ m/s

From the law of conservation of momentum.

$$75 \times v = 75 \times \frac{v}{3} + 50 \times 10 \Rightarrow v = 10 \text{ m/s}$$

29. (*b*) After explosion, let $m_1 = M/4$ (M = initial mass)

$v_1 = 30\hat{i}$ m/s

$m_2 = M/4$

$v_2 = 40\hat{j}$ m/s

$m_3 = M - (M/4 + M/4) = M/2$

$v_3 = v$

Since momentum is conserved,

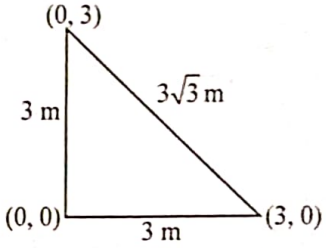
$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$, $\vec{P}_3 = -(\vec{P}_1 + \vec{P}_2)$

As $\vec{P}_1$ & $\vec{P}_2$ are perpendicular,

$$\text{so } P_3 = \sqrt{P_1^2 + P_2^2} = \frac{50M}{4} = \frac{M}{2}v$$

so, $v = 25$

30. [2]



$$\vec{r}_{com} = \frac{m(0\hat{i} + 0\hat{j}) + m(3\hat{i}) + m(3\hat{j})}{3m}$$

$\vec{r}_{com} = \hat{i} + \hat{j}$

$|\vec{r}_{com}| = \sqrt{2} = \sqrt{x} \Rightarrow x = 2$

31. (*d*) The position vector centre of mass

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{r} = \frac{1(\hat{i} + 2\hat{j} + \hat{k}) + 3(-3\hat{i} - 2\hat{j} + \hat{k})}{1+3} = -2\hat{i} - \hat{j} + \hat{k}$$

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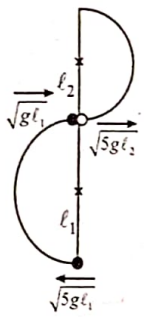
32. [5]

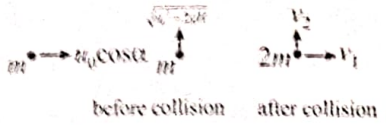


To complete the vertical circle

$\sqrt{g\ell_1} = \sqrt{5g\ell_2}$

$$\Rightarrow \frac{\ell_1}{\ell_2} = 5$$

33. (a) At the highest point



Applying momentum conservation in horizontal direction

$$v_1 = \frac{u_0 \cos\alpha}{2}$$

Applying momentum conservation in vertical direction

$$v_2 = \frac{u_0 \cos\alpha}{2}$$

$$\left(H = \frac{u_0^2 \sin^2\alpha}{2g}\right)$$

Thus, $\theta = 45°$ (as $v_1 = v_2$)

34. (b) $t = 0$.

(Before collision) t

$v = gt$

$$K = \frac{1}{2} m g^2 t^2$$

$K \propto t^2$: parabolic graph

then during collision kinetic energy first decreases to elastic potential energy and then increases.

Most appropriate graph is B.

35. (a,b)

Using conservation of linear momentum $V = \frac{mv}{M}$

Conservation of energy principle:

$$\Rightarrow mgR = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2;$$

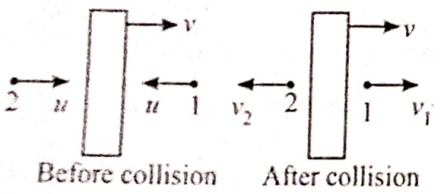
$$\Rightarrow mgR = \frac{1}{2}mv^2\left(1 + \frac{m}{M}\right)$$

$$\Rightarrow v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$$

$$\Delta X_{COM} = 0 \Rightarrow \frac{Mr + m(r+R)}{M+m} = 0$$

$$\Rightarrow r = -\frac{-mR}{M+m}$$

36. (a,b,c)



Molecule 1: From definition of coefficient of restitution,

$$1 = \frac{v_1 - v}{v + u}$$

$$\Rightarrow v_1 = u + 2v$$

$$\Rightarrow \Delta v_1 = v_1 - (-u) = 2u + 2v$$

Molecule 2:

$$1 = \frac{v + v_2}{u - v}$$

$$\Rightarrow v_2 = u - 2v$$

$$\Delta v_2 = 2u - 2v$$

Now, force on leading face

$$F_1 = \frac{dp_1}{dt} = \rho A(u+v)2(u+v) = 2\rho A(u+v)^2 \leftarrow$$

Force on trailing face

$$F_2 = \frac{dp_2}{dt} = \rho A(u-v)2(u-v) = 2\rho A(u-v)^2 \rightarrow$$

So, $F_{res} = F_1 - F_2 = 2\rho A[(u+v)^2 - (u-v)^2] = 8\rho Auv \leftarrow$

$$\Rightarrow P = \frac{F_{res}}{A} = 8\rho uv \propto uv$$

Net force on plate,

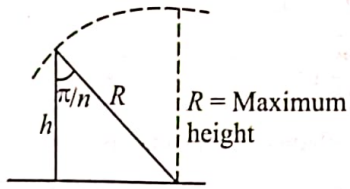
$$F_{net} = F - F_{res}$$

$$m\frac{dv}{dt} = F - 8\rho Auv$$

After long time v will be sufficient so $F = 8\rho A\alpha uv$

After that v = constant, i.e. plate will achieve terminal velocity.

37. (d)

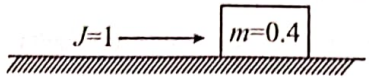


$$\cos\left(\frac{\pi}{n}\right) = \frac{h}{R}$$

$$\Delta = R - h = \frac{h}{\cos\left(\frac{\pi}{n}\right)} - h$$

$$= h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$$

38. [6.30]



$$v = v_0 e^{-t/\tau}$$

where, $v_0 = \frac{J}{m} = 2.5\,m/s$

$$\frac{dx}{dt} = v_0 e^{-t/\tau}$$

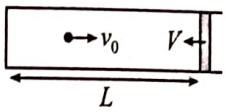
$$\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$$

$$x = v_0 \left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}} \right]_0^\tau$$

$$x = 2.5(-4)\left(e^{-1} - e^0\right)$$

$$x = 2.5\,(-4)\,(0.37 - 1) \Rightarrow x = 6.30$$

39. *(b,c)*



Average rate of collision $= \frac{2L}{v}$

Speed of particle after collision $= 2V + v_0$

Change in speed after each collision $= (2V + v_0) - v_0$

$= 2V$

No. of collision per unit time (frequency) $= \frac{v}{2L}$

Change in speed in dt time $= 2V \times$ number of collision in dt time

$$\Rightarrow dv = 2V\left(\frac{v}{2L}\right).\frac{dL}{V}$$

$$dv = \frac{vdL}{L}$$

Now, $dv = -\frac{vdL}{L}$ {as L decrease}

$$\int_{v_0}^{v} \frac{dv}{v} = -\int_{L_0}^{L_0/2} \frac{dL}{L} \Rightarrow [\ln v]_{v_0}^{v} = -[\ln L]_{L}^{L_0/2}$$

$$\Rightarrow v = 2v_0$$

$$\Rightarrow KE_{L_0} = \frac{1}{2}mv_0^2 \text{ and } KE_{L_0/2} = \frac{1}{2}m(2v_0)^2$$

$$\therefore \frac{KE_{L_0/2}}{KE_0} = 4$$

CHAPTER

11 Rotational Motion

Exercise-1 (Topicwise)

1. (*b*) $\omega = 3000$ rpm (given) $= 6000\,\pi$ rad/min

Radian it has turns in 1 sec $= \dfrac{6000\pi}{60} = 100\,\pi$

2. (*c*) $\omega = \dfrac{2\pi}{60} = \dfrac{\pi}{30}$ rad / s

3. (*a*) $T = \dfrac{2\pi}{\omega}$ if time period same then ω will be constant

$a_C = \omega^2 r;\ \dfrac{a_1}{a_2} = \dfrac{r_1}{r_2}$

4. (*d*) $\alpha = \dfrac{\omega}{t} = \dfrac{60}{5} = 12$ rad/s^2

Angular disp. $\theta = \omega_o t + \dfrac{1}{2}\alpha t^2$

$= \dfrac{1}{2} \times 12 \times 5^2 = 150$ rad

5. (*c*) $\because v = \omega r$ since ω is same for both so,

$v_1 = \omega r$

$v_2 = \omega\,(2r)$

$\dfrac{v_1}{v_2} = \dfrac{\omega r}{\omega(2r)} = \dfrac{1}{2}$; (here x and y are displacement of particle A and B is same time)

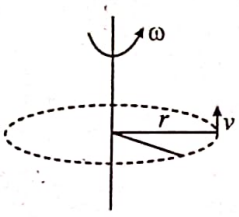
6. (*d*) Relation between linear acceleration (a_t) and angular acceleration (α) is:

$a_t = \alpha \times R$

so, $R = \dfrac{a_t}{\alpha} = \dfrac{10}{5} = 2$m

7. (*a*) $v = \omega r$

r is perpendicular distance of particle from rotational axis.



8. (*d*) $\vec{v} = \vec{\omega} \times \vec{r}$

from above we get $\omega = \dfrac{v}{r}$ but $\omega = \dfrac{d\theta}{dt}$ is not depend on distance (r) from axis of rotation.

9. (*b*) $I = 2[5(0.2)^2 + 2(0.4)^2]$

10. (*b*) Fact Based

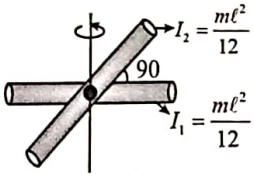
11. (*c*) As disc is lying in the x-z plane, so applying perpendicular axis theorem:

$I_x + I_z = I_y$

$30 + I_z = 40$

$\Rightarrow I_z = 40 - 30 = 10$ kg m^2

12. (*b*)

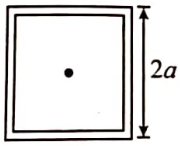


So $I = I_1 + I_2 = I = I_1 + I_2 = \dfrac{m\ell^2}{12} + \dfrac{m\ell^2}{12} = \dfrac{m\ell^2}{6}$

13. (*d*) Disk $\rightarrow \dfrac{ma^2}{2}$

Ring $\rightarrow ma^2$

Square lamina $\rightarrow \dfrac{2ma^2}{3}$



Four rods forming a square of side

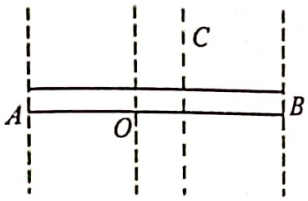
$$2a = 4\left(\frac{\left(\frac{m}{4}\right)(2a)^2}{12} + \left(\frac{m}{4}\right)a^2\right) = \frac{4ma^2}{3}$$

14. (*b*) $I_z = I_x + I_y$

$I_z = 2I$

15. (*a*)

$$I = \frac{2Mr^2}{2} = Mr^2$$

16. (*a*) 

Density increases linearly $I_A > I_B$ because maximum mass is away axis of rotation.

17. (*b*) M.I. $= mr^2 = 4 \times 1^2 = 4$ kg m^2.

18. (*d*) Since, $I_z = I_x + I_y$

$= mr_x^2 + mr_y^2$

$= m(r_x^2 + r_y^2)$

For particle 1, $= m(3^2 + 2^2) = 26$ unit

For particle 2, $m[1^2 + (-1)^2] = 2 \times (2) = 4$ unit

For particle 3, $m[1^2 + 1^2] = 2 \times (2) = 4$ unit

For particle 4, $m(0) = 0$

Total inertia $= 26 + 4 + 4 = 34$ unit.

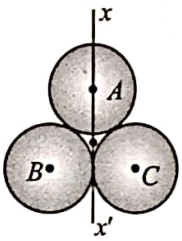
19. (*c*) Given, $I_{\text{solid sphere}} = I_{\text{hallow sphere}}$

$$\Rightarrow \frac{2}{5}Mr_1^2 = \frac{2}{3}Mr_2^2$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{5}{3}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{5} : \sqrt{3}$$

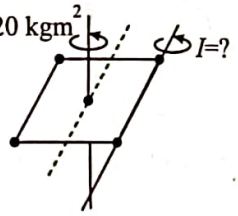
20. (*c*) $I = I_A + I_B + I_C$



$$I_A = \frac{2}{5}MR^2$$

$$I_B = I_C = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$\Rightarrow I = \frac{2}{5}MR^2 + \frac{7}{5}MR^2 + \frac{7}{5}MR^2;\ I = \frac{16}{5}MR^2$$

21. (*c*) I=20 kgm^2 

as, $I = \frac{ML^2}{6} = 20 \Rightarrow ML^2 = 120$

$$I' = I_C + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$$

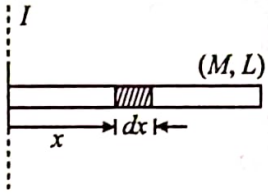
$$\Rightarrow I' = \frac{ML^2}{3} = \frac{120}{3} = 40 \text{ kg m}^2$$

22. (*a*) M inertia about yy' axis are

$I' = I_1 + I_2 + I_3 = 2I_o + I_3 \quad (\because I_1 = I_2 = I)$

Applying parallel axis theorem

$$I' = \frac{MR^2}{2} + 2\left(\frac{MR^2}{2} + MR^2\right) = \frac{7MR^2}{2} = 7/2\ PQ^2.$$

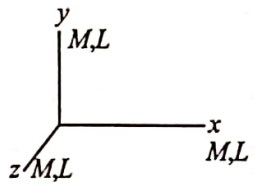
23. (*c*) 

Mass density $(\lambda) = \frac{M}{L}$

$$dm = \lambda dx = \frac{M}{L}(dx)$$

$$\int dI = \int x^2 dm = \int_0^L \frac{x^2 M}{L} dx$$

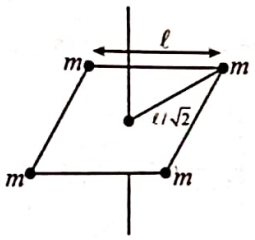
$$I = \frac{ML^2}{3}$$

24. (*b*) 

$I = I_1 + I_2 + I_3$

$$= \frac{ML^2}{3} + \frac{ML^2}{3} + 0 = \frac{2}{3}ML^2$$

25. (*a*) $I = m \times \left(\frac{\ell}{\sqrt{2}}\right)^2 \times 4$

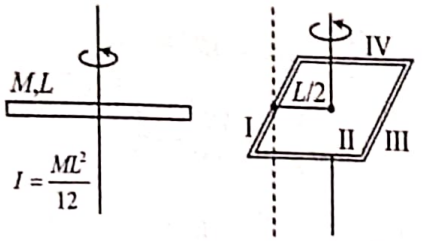


also, $I = MK^2$

where $M = 4$m

so, $m \times \frac{\ell^2}{2} \times 4 = 4m \times K^2 \quad \therefore \quad K = \frac{\ell}{\sqrt{2}}$

26. (*d*)

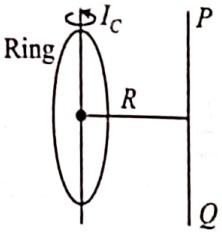


for one rad, $I_1 = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{4ML^2}{12}$

$I_{total} = 4I_1 = 4 \times \frac{4ML^2}{12} = \frac{16ML^2}{12}$

so, $I_{total} = 16(I)$

27. (*c*)



Applying parallel axis theorem

$I_{PQ} = I_C + MR^2$

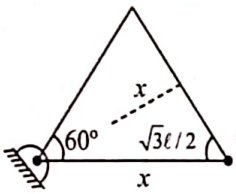
$\Rightarrow I_{PQ} = \frac{MR^2}{2} + M(R)^2 = \frac{3}{2}MR^2$

28. (*d*) $I = \Sigma m_i r_i^2$ (r_i perpendicular distance)

$= m(a)^2 + m(a)^2 + m(2a)^2$

$= 6\,ma^2$

29. (*c*) $I = \frac{m\ell^2}{3} \times 2 + \frac{m\ell^2}{12} + m \times \left(\frac{\sqrt{3}\ell}{2}\right)^2$



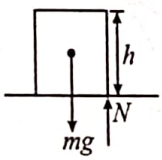
$= \frac{8m\ell^2 + m\ell^2 + 9m\ell^2}{12}$

$= \frac{3}{2}m\ell^2 = 3mk^2$; $\ell \cos 30° = x$

$k = \frac{\ell}{\sqrt{2}}$

30. (*d*) Torque can be taken about any point in space.

31. (*b*)



For toppling

$f\frac{h}{2} > N \times \frac{a}{2}$

$\mu > \frac{a}{h}$

32. (*b*) $\vec{\tau} = \vec{r} \times \vec{F} = (2\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + 2\hat{j} - 4\hat{k})$

$\vec{\tau} = 12\hat{i} + 14\hat{j} + 16\hat{k}$

$|\vec{\tau}| = \sqrt{12^2 + 14^2 + 16^2} = \sqrt{596} = 96$ N-m

33. (*d*) $\frac{dL}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\alpha$

and $\alpha = I\alpha$

34. (*c*) $\vec{\tau} = \text{const} = I\vec{\alpha}$

$\Leftrightarrow \vec{a} = \text{const}$

35. (*a*) given, $\omega_0 = 20$ rad/sec

$\omega = 0$

$I = 50$ kg-m^2

$t = 10$ sec

$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 20}{10} = -2$ rad/sec^2

and $\tau = I\alpha = 50 \times 2 = 100$ kg-m^2/s^2

$= 100$ N-m

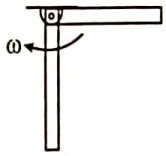
36. (*a*) $\tau = I\alpha; \left[I' = (2m)\left(\frac{r}{2}\right)^2 = \frac{mr^2}{2} = \frac{I}{2}\right]$

$\tau' = I'\alpha.$

$\frac{\tau'}{\tau} = \frac{I'\alpha}{I\alpha} = \frac{I'}{I} = \frac{1}{2}; \tau' = \frac{\tau}{2}$

37. (*d*) $\frac{1}{2}I\omega^2 = \frac{1}{2} \times mr^2\omega^2$

38. (*b*)



By work energy theorem

$mg\frac{L}{2} = \frac{1}{2}\frac{mL^2}{3}\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{L}}$

39. (*c*) as, $KE = \frac{L^2}{2I}$

if L = constant, then $KE \propto \frac{1}{I}$

as $I_A > I_B$

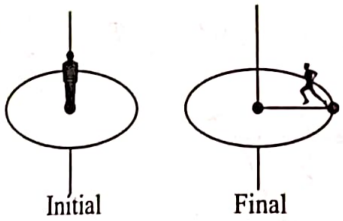
so $(KE)_A < (KE)_B$

40. (*c*) $I\omega$ = constant

$Mr^2\omega = (Mr^2 + 2mr^2)\omega'$

$\omega' = \frac{\omega M}{M + 2m}$

41. (*a*)



Applying conservation of angular momentum :

$I_1\omega_1 = I_2\omega_2$

$$\Rightarrow I_1 \times \frac{2\pi}{T_1} = I_2\omega_2 \Rightarrow 100 \times \frac{2\pi}{10} = [100 + 50 \times (2)^2] \times \omega_2$$

on solving, $\omega_2 = \frac{2\pi}{30}$ rad/sec

42. (*b*) $mv\frac{R}{2} = I\omega$

$$I = \left[mR^2 + \frac{mR^2}{4}\right] + \frac{mR^2}{4} = \frac{3mR^2}{2}$$

$$mv\frac{R}{2} = \frac{3mR^2}{2}\omega, \quad \omega = \frac{v}{3R}$$

43. (*c*) From $L = I\omega$, L = constant (because $\tau_{ext} = 0$)
Due to drop the wax on disc moment of inertia of its will increase so ω will decrease.

44. (*b*) $\tau_{ext} = 0$, So $L = I\omega$ = constant when girl moves from edge towards centre I will decrease, and 'ω' will increase.

45. (*a*) $\tau_{ext} = 0$;
from conservation of angular momentum
$L = I\omega$ = constant
$I_1\omega_1 = (I_1 + I_2)\,\omega_2$

$$\left(\omega_1 = \frac{600 \times 2\pi}{60} = 20\pi, \omega_2 = 400\frac{400 \times 2\pi}{60} = \frac{40}{3\pi}\right)$$

$$I_P \times 20\pi = (I_P + I_Q)\,\frac{40}{3\pi}$$

$$6 \times 20\pi = (I_P + I_Q)\,\frac{40}{3\pi} \qquad \text{(Given } I_P = 6\text{)}$$

After solving that we get $I_2 = 3$ kg m^2

46. (*b*) $f = \dfrac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2}$

where $v = \omega r$ and $I = \frac{2}{5}mR^2$

47. (*d*) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
where $\omega = v/r$, $I = 2/5\ mR^2$

48. (*a*) given, $E = \frac{1}{2}I\omega^2$

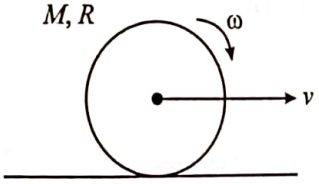
$$\bar{\tau}_{ext} = 0 = \frac{d\bar{L}}{dt} \Rightarrow \bar{L} = \text{const} \Leftrightarrow \bar{\omega} = \text{const}$$

So, $E \propto I$ only

$$\Rightarrow \frac{E_1}{E_2} = \frac{I_1}{I_2} = \frac{m_1k_1^2}{m_2k_2^2}$$

$$\frac{E_1}{E_2} = \frac{mk^2}{\frac{m}{2}.(2k)^2} = 0.5$$

49. (*b*)



$$KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Pure rolling $v = R\omega$

$$\Rightarrow KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\left(\frac{v^2}{R^2}\right)$$

$$KE_{total} = mv^2 = 2E$$

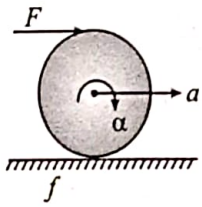
50. (*a*) Solid cylinder has less moment of inclined so it mass passes less rotational energy as compare to translational energy so it will reach first.

51. (*c*) Because sphere has maximum translational energy first decrease in Potential energy.

52. (*b*) Final speed only depend's on initial height but time will depend on length and inclination of plane.

53. (*b*) Angular moment about lowest point is conserved because torque due to friction about point of contact will be zero.

54. (*c*)



$F - f = Ma$ ...(i)

$FR + fR = I\alpha$ ...(ii)

$a = \alpha R$

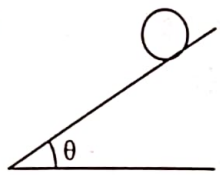
Putting $I = MR^2$ and solving we get $fR = 0$.

55. (*a*) Fraction $= \dfrac{K_{Rotation}}{K_{Total}} = \dfrac{\frac{K^2}{R^2}}{1 + \frac{K^2}{R^2}}$

For disc, $\frac{K^2}{R^2} = \frac{1}{2}$

So, fraction $= \dfrac{\frac{1}{2}}{1 + \frac{1}{2}} = 1:3$

56. (*b*)



for sliding $a = g\sin\theta$

$$\Rightarrow L = 0 + \frac{1}{2}(g\sin\theta)t_1^2 \qquad \text{...(i)}$$

for rolling $\therefore \quad a' = \dfrac{g\sin\theta}{\left(1 + \frac{k^2}{R^2}\right)}$

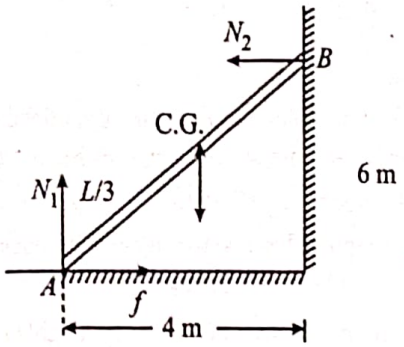
$a' = (g\sin\theta)/2$

$$\Rightarrow L = 0 + \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_2^2 \quad ...(ii)$$

from equation (i) and (ii) we have

$t_1 : t_2 = 1 : \sqrt{2}$

57. (*a*)



Translational at equilibrium:

$\Sigma\vec{F} = 0$ *i.e.*, $\Sigma F_x = 0$, $\Sigma F_y = 0$

So, $N_2 = f$ ...(i)

$N_1 = Mg$ ...(ii)

At rotational equilibrium:

$$\Rightarrow (Mg\cos\theta).\frac{L}{3} = (N_2\sin\theta)L$$

$$\Rightarrow N_2 = \frac{Mg}{3}.\cot\theta$$

$$\text{or } N_2 = \frac{500}{3}\times\frac{4}{6} \quad \Rightarrow \quad N_2 \approx 111 \text{ N}$$

Exercise-2 (Learning Plus)

1. (*b*) $\omega_0 = 3000$ rad/min

$\omega_0 = \frac{3000}{60}$ rad/sec = (50 rad/sec)

$t = 10$ sec

$\omega_f = 0$

$\omega_f = \omega_0 + \alpha t$

$\theta = 50 - \alpha\,(10)$

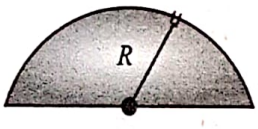
$\alpha = 5$ rad/sec^2

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = (50)(10) + \frac{1}{2}(-10)(10)^2$

$\theta = 500 - 250 = 250$ rad

2. (*a*)



$I = \int dmr^2$

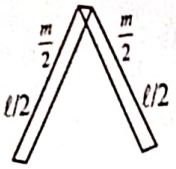
$I = r^2\int dm = r^2 m = mr^2$

3. (*a*) $\sigma_B > \sigma_A$

$I_B > I_A$

so, If the axes are parallel $I_A < I_B$

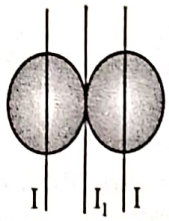
4. (*c*)



$I_0 = I_1 + I_2$

$$I_0 = \frac{(m/2)\left(\frac{\ell}{2}\right)^2}{3} + \frac{(m/2)\left(\frac{\ell}{2}\right)^2}{3} = \frac{m\ell^2}{12}$$

5. (*b*)



Moment of inertia about

diameter of sphere $I = \frac{2}{5}mr^2$

Moment of inertia about tangent at their common point

$$I_1 = \left(\frac{2}{5}mr^2 + mr^2\right)\times 2 = \frac{14}{5}mr^2 \; I_1 = 7I$$

6. (*d*) Moment of inertia of disc

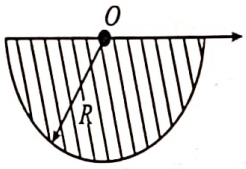
about diameter $I = \frac{mr^2}{4} = 2$,

$mr^2 = 8$

Moment of inertia about the axis through a point on rim.

$I_1 = \frac{mr^2}{4} + mr^2 = 10 = 10$ units

7. (*b*)



$I = \frac{MR^2}{2}$

(passing through *O*)

8. (*d*) M.O.I. about C.O.M. is Minimum

$I = I_{C.M.} + Mx_0^2$

$I = 2x^2 - 12x + 27$

$\therefore \frac{dI}{dx} = 4x - 12 = 0$

$\Rightarrow x = 3$

9. (*b*) $\tau = I\alpha$

τ = constant $\Rightarrow \omega$ = increases

10. (*d*) $\omega = \omega_0 + \alpha t$

$100 = 10 + \alpha(15) \Rightarrow \alpha = 6$ rad/sec^2

$\tau = I\alpha \Rightarrow 60$ Nm

11. (*d*) $\tau = I\alpha$

$2 = I \times 2$

$\Rightarrow I = 1$ kgm^2

$I = MR^2$

$1 = M(2)^2$

$M = \frac{1}{4}kg$

12. (*a*) $\tau = I\alpha$

$\tau_A = \tau_B$

$I_A \alpha_A = I_B \alpha_B$

$I_A < I_B$

$\alpha_A > \alpha_B$

$\omega_A > \omega_B$

13. (*c*) $F = 4\hat{i} - 10\hat{j}$

$\vec{r} = (-5\hat{i} - 3\hat{j})$

$\tau = \vec{r} \times \vec{F}$

$= (-5\hat{i} - 3\hat{j}) \times (4\hat{i} - 10\hat{j})$

$= 50\hat{k} + 12\hat{k} = 62\hat{k}$

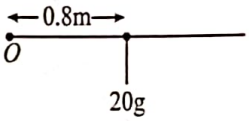
14. (*b*) Torque about O

$\tau = F \times 40 + F \times 80 - (F \times 20 + F \times 60)$

In clockwise direction

$\tau = F \times 40$

15. (*a*) Initial velocity of each point on the rod is zero so angular velocity of rod is zero.



Torque about O

$\tau = I\alpha$

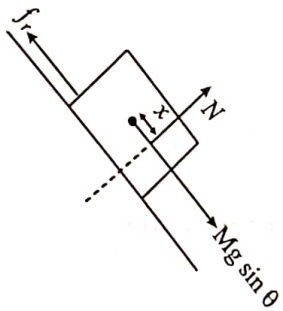
$20g(0.8) = \frac{m\ell^2}{3}\alpha \Rightarrow 20g(0.8) = \frac{20(1.6)^2}{3}\alpha$

$\Rightarrow \frac{3g}{3.2} = \alpha =$ angular acceleration

$\Rightarrow \alpha = \frac{15\,g}{16}$

16. (*c*) Beam is not at rotational equilibrium, so force exerted by the rod (beam) decrease

17. (*d*)

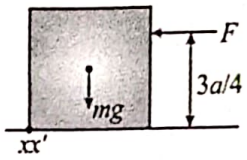


$f_r = Mg\sin\theta = \mu Mg\cos\theta$

$f_r \frac{a}{2} = N.x = \tau_N$

$\boxed{Mg\frac{a}{2}\sin\theta = \tau_N}$

18. (*a*)



For topling about edge xx'

$F_{min}\frac{3a}{4} = mg\frac{a}{2}$

$F_{min} = \frac{2mg}{3}$

19. (*d*) $\frac{1}{2}I\omega^2 = 1000$

$\omega = 10$ rad/sec

$2\pi f = 10 \Rightarrow f = \frac{5}{\pi}$ rad/sec $= \frac{300}{\pi}$ rad/min

20. (*a*) $I = I_{cm} + MR^2$

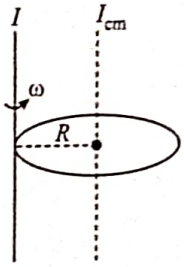
$= (MR^2) + MR^2$

$I = 2MR^2$;

$\omega = 2\pi n$

$KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times (2MR^2) \times (2\pi n)^2$

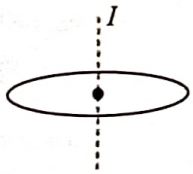
or $KE = 4\pi^2 n^2 MR^2$



21. (*d*) $I_{disc} = \frac{MR^2}{2}$ (about centre)

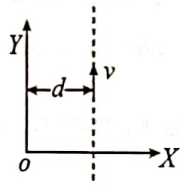
$KE = \frac{1}{2}I\omega^2 \Rightarrow K \propto \omega^2$

for constant 'I'



22. (*c*) $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{4A_0 - A_0}{4} = \left(\frac{3A_0}{4}\right)$

23. (*b*)



$\Rightarrow L = (mvd) =$ constant (because v = const. and d = const.)

24. (*c*) external torque $\vec{\tau}_{ext} = 0$

$I_1\omega_1 = I_2\omega_2$

when he stretches his arms mass redistributes

so $I_1 < I_2$

then $(\omega_1 > \omega_2)$

so, (L = constant)

25. (*c*) $\frac{1}{2}I\omega^2 = 10 \Rightarrow \frac{5}{2}\omega^2 = 10 \Rightarrow \omega = 2\,\text{rad/sec}$

Angular Momentum

$L = I\omega = 5 \times 2 = 10$ joule-sec.

26. (*b*) $I_1\omega_1 = I_2\omega_2$

$M R_1^2 \omega_1 = M R_2^2 \omega_2$

$\frac{R_1}{R_2} = \sqrt{\frac{\omega_2}{\omega_1}} = \frac{3}{1}$

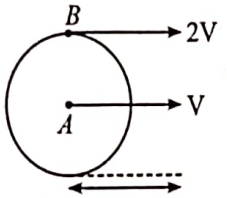
27. (*b*) $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \left(\frac{mR^2}{2}\right)\frac{v^2}{R^2} = \frac{1}{4}mv^2$

Total $KE = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

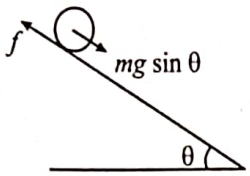
$\frac{1}{4}mv^2 + \frac{1}{2}mv^2 \Rightarrow \frac{3}{4}mv^2$

Ratio $= \frac{1}{3}$

28. (*b*) When A point travels ℓ distance then B point 2ℓ so, 2ℓ length of string passes through the hand of the boy.



29. (*a*)



$mg\sin\theta - f = ma$

$a = \left[\frac{mg\sin\theta - f}{m}\right]$ ...(i)

a is same for each body.

$f.R = I\alpha$

$\alpha = \frac{f.R}{mk^2}$

For solid sphere $k^2 = \frac{2}{5}R^2$ is minimum there fore α is maximum hence, *K.E.* for solid sphere will be max at bottom.

30. (*a*) Due to linear velocity body will move forward before pure rolling.

31. (*a*) Disk: $\frac{1}{2}I\omega^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}\frac{MR^2}{2}\frac{v_1^2}{R^2} + \frac{1}{2}mv_1^2 = \frac{3}{4}mv_1^2$

Ring: $= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mR^2 \times \frac{v_2^2}{R^2} + \frac{1}{2}mv_2^2 = mv_2^2$

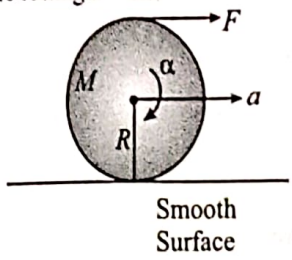
$\Rightarrow \frac{3}{4}mv_1^2 = mv_2^2$

$\Rightarrow \frac{v_1}{v_2} = \left(\frac{4}{3}\right)^{1/2}$

32. (*a*) $F = Ma$

$\Rightarrow a = \frac{F}{M}$

For pure rolling $a = \alpha R$



$F \times R = I\alpha$

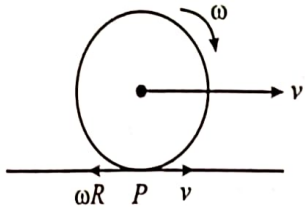
$\alpha = \frac{FR}{I}$

$\Rightarrow a = \frac{FR.R}{I} = \frac{F}{M}$

$\Leftrightarrow I = MR^2$

MR^2 is the moment of inertia of thin pipe.

33. (*d*)



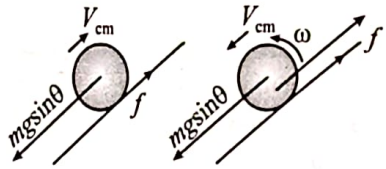
Point 'P' velocity

will be '$(v - \omega R)$' (forward) if $v > \omega R$

and '$(\omega R - v)$' backward if $v < \omega R$

So, the direction of friction will depend on the relative motion of point 'P' on the sphere with respect to surface.

34. (*c*)



Pure rolling $\uparrow V_{cm} = R\omega\uparrow$

$V_{cm} = R\omega\downarrow$

35. (*d*) As the inclined plane is smooth, the sphere can never roll rather it will just slip down.

Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

36. (*b*) Fact based

37. (*c*) $I = \frac{m\ell^2}{12} + \frac{mR^2}{4}$

or $I = \frac{m}{4}\left(\frac{\ell^2}{3} + R^2\right)$ ...(*a*)

Also $m = \pi R^2 \ell \rho$

$\Rightarrow R^2 = \frac{m}{\pi\ell\rho}$ Put in equation (*a*)

$I = \frac{m}{4}\left(\frac{\ell^2}{3} + \frac{m}{\pi\ell\rho}\right)$

For maxima and minima

$$\frac{dI}{d\ell}=\frac{m}{4}\left(\frac{2\ell}{3}-\frac{m}{\pi\ell^2\rho}\right)=0$$

$$\Rightarrow \frac{2\ell}{3}=\frac{m}{\pi\ell^2\rho}$$

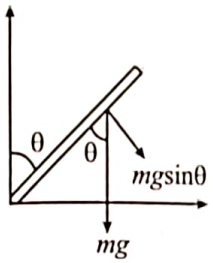
$$\Rightarrow \frac{2\ell}{3}=\frac{\pi R^2\ell\rho}{\pi\ell^2\rho}$$

or $\frac{2\ell}{3}=\frac{R^2}{\ell}$

$$\Rightarrow \frac{\ell^2}{R^2}=\frac{3}{2}$$

or $\frac{\ell}{R}=\sqrt{\frac{3}{2}}$

38. (*c*)

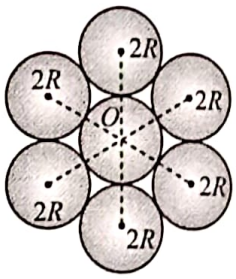


Taking torque about pivot $\tau = I\alpha$

$$mg\sin\theta\,\frac{\ell}{2}=\frac{m\ell^2}{3}\alpha$$

$$\alpha=\frac{3g}{2\ell}\sin\theta$$

39. (*c*)



$$I_0=I_{cm}+md^2$$

$$=\frac{7MR^2}{2}+(6M\times(2R)^2)=\frac{55MR^2}{2}$$

$$I_p=I_0+md^2$$

$$=\frac{55MR^2}{2}+7M(3R)^2=\frac{181}{2}MR^2$$

40. (*b*) $I=\frac{MR^2}{2}+2\left(\frac{MR^2}{4}+MR^2\right)$

$$=\frac{MR^2}{2}+\frac{MR^2}{2}+2MR^2=3MR^2$$

41. (*a*) $2.5 = 1\times 5\sin\theta$

$$\sin\theta=0.5=\frac{1}{2}\Rightarrow \theta=0=\frac{\pi}{6}$$

42. (*c*) Just before collision speed of m

$$v=\sqrt{2gL(1-\cos\theta_0)}$$

Just after collision speed of M

$$v_1=\sqrt{2gL(1-\cos\theta_1)}$$

And $v_1=\left(\frac{M-m}{M+m}\right);\ \frac{v_1}{v}=\frac{M-m}{M+m}$

$$\sqrt{\frac{1-\cos\theta_1}{1-\cos\theta_0}}=\frac{M-m}{M+m}$$

$$\frac{\sin(\theta_1/2)}{\sin(\theta_0/2)}=\frac{M-m}{M+m}\qquad[\because 1-\cos 2\theta=2\sin^2\theta]$$

$$\frac{\theta_1}{\theta_0}=\frac{M-m}{M+m}$$

$$M\theta_1+m\theta_1=M\theta_0-m\theta_0$$

$$M=m\left[\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right]$$

43. (*d*) $I=I_{CM}+Mx^2$

$$I_{CM}=\frac{2}{5}MR^2$$

$$I=\frac{2}{5}MR^2+Mx^2$$

44. (*d*) Kinetic energy $KE=\frac{1}{2}I\omega^2=k\theta^2$

$$\Rightarrow \omega^2=\frac{2k\theta^2}{I}$$

$$\Rightarrow \omega=\sqrt{\frac{2k}{I}}\theta\qquad\text{...(i)}$$

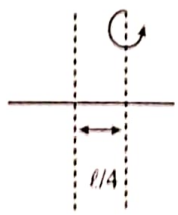
Differentiate (i) wrt time →

$$\frac{d\omega}{dt}=\alpha=\sqrt{\frac{2k}{I}}\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \alpha=\sqrt{\frac{2k}{I}}\cdot\sqrt{\frac{2k}{I}}\theta\qquad\text{(by (i))}$$

$$\Rightarrow \alpha=\frac{2k}{I}\theta$$

45. (*a*)

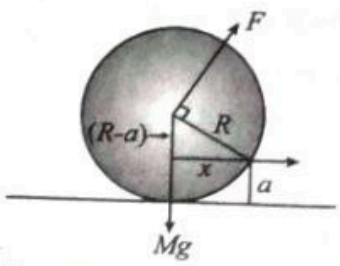


$$\frac{M\ell^2}{12}+M\left(\frac{\ell}{4}\right)^2=MK^2$$

$$\frac{\ell^2}{12}+\frac{\ell^2}{16}=K^2$$

$$K=\sqrt{\frac{7}{48}}\ell$$

46. (*a*)

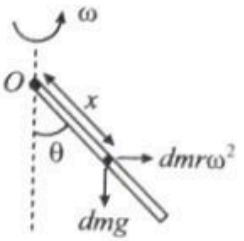


To step up, $F \times R \geq Mg \times x$

$$\Rightarrow F_{\min} = \frac{Mg}{R} \times \sqrt{R^2 - (R-a)^2}$$

$$= Mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

47. (*d*)

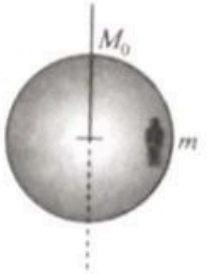


From a rotating frame rod will appear in equilibrium. Net torque about suspension point must be zero.

$$\int_0^L \left(\frac{M}{L}dx\right)(x\sin\theta)\omega^2 x\cos\theta = Mg\frac{L}{2}\sin\theta$$

$$\Rightarrow \cos\theta = \frac{3g}{2l\omega^2}$$

48. [09.00]



$M_0 = 200$ kg

$m = 80$ kg

$$I = (I_M + I_m) = \left(\frac{M_0R^2}{2} + mR^2\right)$$

$\therefore I_1\omega_1 = I_2\omega_2$

$$\therefore \omega_2 = \left(\frac{M_0R^2}{2} + mR^2\right) \times \frac{5}{\frac{M_0R^2}{2}} \quad (\because \omega_1 = 5 \text{ rpm})$$

$$= \frac{5R^2}{R^2} \times \frac{(80+100)}{100}$$

$\omega_2 = 9$ rpm

49. [20.00]

$I_1\omega_1 = I_2\omega_2$

$$\omega_2 = \frac{\left(\frac{MR^2}{2}\right)\omega_1}{\left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)} = \frac{MR^2}{2}\frac{\omega_1 \times 8}{MR^2 \times 5} = \frac{4\omega_1}{5}$$

$$\therefore \left(\frac{\Delta E}{E} \times 100\right) = p\%$$

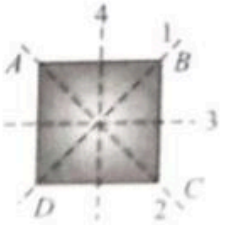
$$\Rightarrow \left(\frac{\frac{1}{2}\cdot\frac{MR^2}{2}\omega_1^2 - \frac{1}{2}\frac{5MR^2}{8}\frac{16}{25}\omega_1^2}{\frac{1}{2}\frac{MR^2}{2} \times \omega_1^2}\right) \times 100$$

$$= \frac{1}{2}\frac{MR^2}{2}\omega_1^2 \frac{\left(1 - \frac{4}{5}\right)}{\frac{1}{2}\frac{MR^2}{2} \times \omega_1^2} \times 100 = 20\%$$

$\Rightarrow p = 20$

Exercise-3 (JEE Advanced Level)

1. (*a,b,c*)



Using perpendicular theorem

$I_0 = I_4 + I_3$ $\quad I_3 = I_4$

$I_0 = I_1 + I_2$ $\quad I_2 = I_1$

$I_3 = I_2$

so, $(I_0 = I_1 + I_3)$

2. (*a,d*)

(*a*) For no slipping

$\mu\, mg\cos\theta \geq mg\sin\theta$ ...(*a*)

For toppling

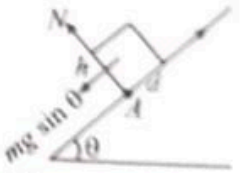
$mg\sin\theta\frac{h}{2} \geq mg\cos\theta\frac{a}{2}$ ...(*b*)

for minimum μ (by dividing)

$\mu\frac{2}{a} = \frac{2}{h}$

$\mu_{\min} = \frac{a}{h}$

(*b*)



If $f > mg\sin\theta$

$\mu\, mg\cos\theta > mg\sin\theta$

($\mu > \tan\theta$) block will topple before sliding

torque about point A $\tau_A = 0$

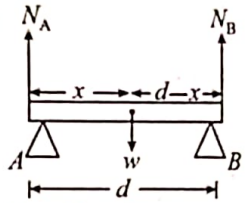
$\sin\theta\left(h/2\right) = mg\cos\theta\frac{a}{2}$

$\tan\theta = \left(a/h\right)$

$\mu > \left(a/h\right)$

If $\mu > \tan\theta$ (block will slide)

3. (*b*,*c*)



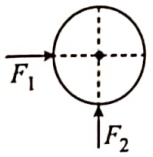
$N_A + N_B = W$

$W(d-x) = N_A \cdot d$

4. (*b*,*c*,*d*)

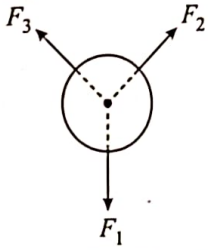
Body is in equilibrium when $\tau_{net} = 0$ and $F_{net} = 0$.

(*a*)



Here, line of actions of F_1 and F_2 pass through COM but net force on body is not zero. So, body is not in equilibrium.

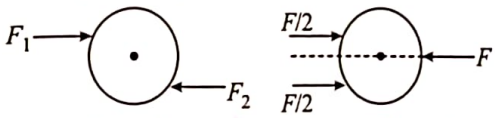
(*b*)



If $F_1 = F_2 = F_3 = F$, $F_{net} = 0$ and $\tau_{net} = 0$.

$\Rightarrow$ Body is in equilibrium.

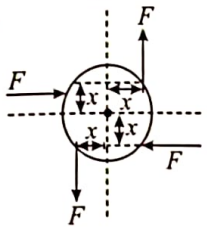
(*c*)



$F_{net} = 0$ $\qquad$ $F_{net} = 0$

but $\tau_{net} \neq 0$ $\qquad$ $\tau_{net} = 0$

(*d*)

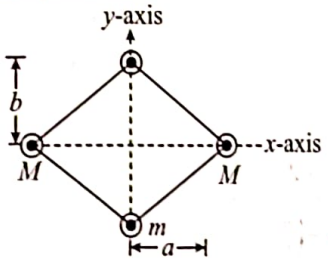


5. (*a*,*b*,*c*)

(*a*) $KE = \frac{1}{2} I\omega^2$

I depends on m

$\therefore$ KE depends on m



(*b*) $I_{y_{axis}} = 2Ma^2$

$K.E. = \frac{1}{2} \times 2Ma^2\omega^2$

$= Ma^2\omega^2$

$I_{z_{axis}} = 2\left(ma^2 + mb^2\right)$

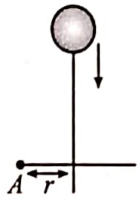
$K.E. = \frac{1}{2} I\omega^2 \; I\omega^2$

$= (Ma^2 + mb^2)\omega^2$

6. (*b*,*c*) External force will act at hinge so linear momentum of system will not remain const. but torque of external force is zero about hinge so $\vec{L}$ = const., collision is elastic so $K.E$ = const.

7. (*a*,*c*,*d*)

τ is constant(mgr)



8. (*a*,*c*,*d*)

$L = I\omega$ = constant. If I changes (due change in R), ω changes.

9. (*a*) If bicycle is accelerating on a horizontal plane then friction on front wheel will be backward and on rear wheel it will be in forward direction.

10. (*a*,*c*,*d*)

As the ring is rolling, thus $v_A = 0 \Rightarrow u = R\omega$

Thus $v_B = u + R\omega = 2u$

Hence, $0 < v < 2u$

Also, let P makes an angle θ with the horizontal.

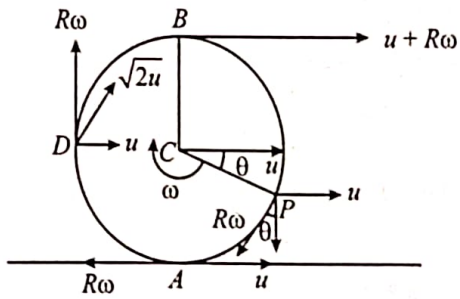
$v_P = \sqrt{u^2 + (R\omega)^2 + 2u(R\omega)\cos(\theta + 90)}$

$\Rightarrow u = \sqrt{u^2 + u^2 + 2u(u)(-\sin\theta)}$

$\Rightarrow \theta = 30°$

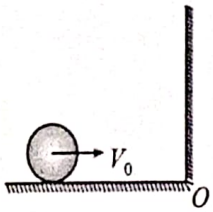
$v_D = \sqrt{u^2 + (R\omega)^2} = \sqrt{2u}$

Thus for $v = \sqrt{2u}$, CP must be horizontal.



11. (*a*,*b*,*d*)

(*a*) Change in angular momentum.



$= L_f - L_i$

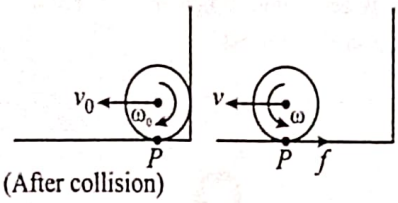
$= (I\omega - mv_0R) - (I\omega + mv_0R)$

$= -2mv_0R$

(*b*) Impulse = Change in momentum

$= -2mv_0R$

(*d*)



(After collision)

Conserving angular momentum about the point *P*

$I\omega_0 - mv_0R = I\omega + mvR$

$\Rightarrow \frac{2}{5}mR^2 \times \frac{v_0}{R} - mv_0R = \frac{2}{5}mR^2 \times \frac{v}{R} + mvR$

$\Rightarrow v = -\frac{3v_0}{7}$

12. (*b*) Let the angular speed of disc when the balls reach the end be ω. From conservation of angular momentum

$\frac{1}{2}mR^2\omega_0 = \frac{1}{2}mR^2\omega + \frac{m}{2}R^2\omega + \frac{m}{2}R^2\omega$

or $\omega = \frac{\omega_0}{3}$

13. (*c*) The angular speed of the disc just after the balls leave the disc is

$\omega = \frac{\omega_0}{3}$

Let the speed of each ball just after they leave the disc be *v*.

From conservation of energy

$\frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_0^2$

$= \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 + \frac{1}{2}\left(\frac{m}{2}\right)v^2 + \frac{1}{2}\left(\frac{m}{2}\right)v^2$

Solving we get

$v = \frac{2R\omega_0}{3}$

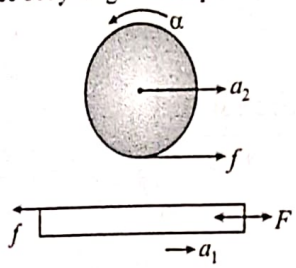
14. (*d*) Work done by all forces equal $K_f - K_i = \frac{1}{2}\left(\frac{m}{2}\right)v^2$

$= \frac{mR^2\omega_0^2}{9}$

Sol. (15 to 17)

15. (*d*) **16.** (*c*) **17.** (*a*)

The free body diagrams of plank and disc are



Applying Newton's second law

$F - f = Ma_1$ ...(i)

$f = Ma_2$ ...(ii)

$fR = \frac{1}{2}MR^2\alpha$ ...(iii)

From equation (ii) and (iii)

$a_2 = \frac{R\alpha}{2}$

From constraint $a_1 = a_2 + R\alpha$

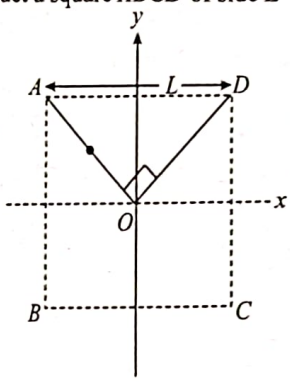
$\therefore a_1 = 3a_2$ ...(iv)

Solving we get $a_1 = \frac{3F}{4M}$ and $\alpha = \frac{F}{2MR}$

If sphere moves by *x* the plank moves by $L + x$. The from equation (iv)

$L + x = 3x$ or $x = \frac{L}{2}$

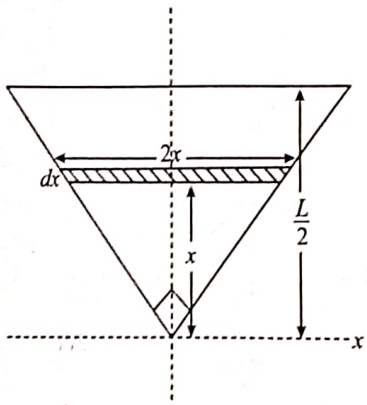
18. (*c*) Construct a square *ABCD* of side *L*



Moment of inertia of triangular plate about *z*-axis passing through *O* will be

$I_z = \frac{1}{4}\left[\frac{(4M)L^2}{6}\right] = \frac{ML^2}{6}$

19. (*a*)



MOI of elemental strip,

$dI = dmx^2$

$dm = \sigma \times dA = \sigma \times 2xdx$

Net MOI about x-axis

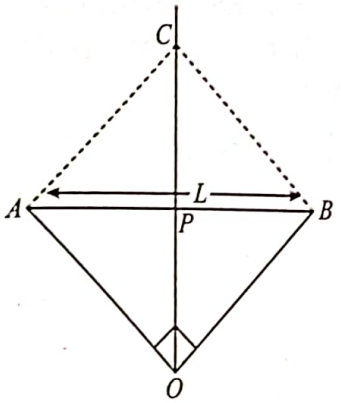
$$I_x = \int dI = \int_0^{L/2} \sigma 2x^2 dx$$

$$I_x = 2\sigma\left[\frac{x^4}{4}\right]_0^{L/2} = 2\sigma\frac{L^4}{2^4 \times 4} = \frac{\sigma L^4}{32}$$

$$\sigma = \frac{M}{A}, \text{ where } A = \frac{1}{2} \times L \times \left(\frac{L}{2}\right)$$

$$\Rightarrow \sigma = \frac{4M}{L^2} \text{ Thus, } I_x = \frac{4M}{L^2} \times \frac{L^4}{32} = \frac{ML^2}{8}$$

20. (*c*) Construct a square of radius $\frac{L}{\sqrt{2}}$



Moment of inertia of triangular plate about AB

$$I_{AB} = \frac{1}{2}\left[\frac{1}{2}\frac{(2M)\left(\frac{L}{\sqrt{2}}\right)^2}{6}\right] = \frac{1}{2}\left[\frac{ML^2}{12}\right] = \frac{ML^2}{24}$$

21. (*c*) Using perpendicular axis theorem similarly about y-axis or CO

$$I_y = I_{AB} = \frac{ML^2}{24}$$

22. (*b*) Torque due to friction is zero about A in all cases. So angular momentum will be conserved in all cases.

For A: $v_0 > R\omega_0$ (opposite direction)

So friction reduces v. So friction is left, KE is reduced as v is reduced.

For B: $v_0 > R\omega_0$ (same direction)

Friction acts left, KE is reduced.

For C: Friction is toward right. KE is reduced

For D: Friction is toward left. KE is reduced

23. [5] Angular momentum conservation

$L_i = L_f$

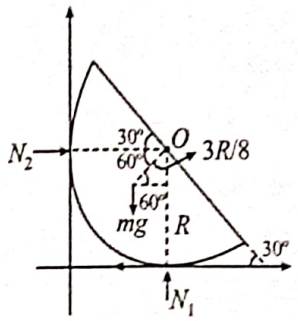
$$m_{\text{John}} V_{\text{John}} \left(\frac{d}{2}\right) + I_{\text{MGR}}\, \omega_{\text{MGR}} = I_{\text{system}}\, \omega$$

$(30)(5)(2) + (500)(5.6) = [500 + 30(2)^2]\omega$

$\omega = 5$ rad/s.

24. [6] Torque about O,

$$fR = mg \times \frac{3R}{8}\cos 60° \Rightarrow f = \frac{3mg}{16}$$



$N_2 = f$

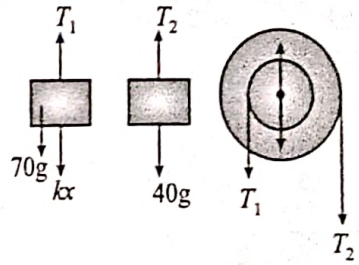
$N_1 = mg$

$f = \frac{3mg}{16}$

$f \le \mu N_1$

$\mu \ge \frac{3}{16} \Rightarrow 32\mu = 6$

25. [100]



$T_1 = 70g + kx$ ...(i)

$T_2 = 40g$ ...(ii)

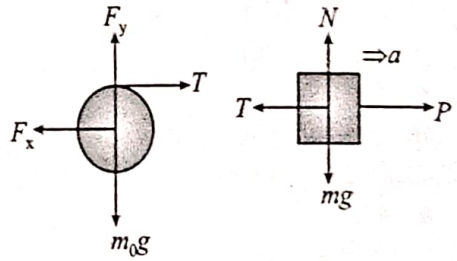
$T_1(0.3) = T_2(0.6)$ ...(iii)

$\Rightarrow 70g + kx = 80g$

$kx = 10g$

$\therefore x = 1$m

26. [4]



$P - T = ma$ ...(i)

$TR = I_C\alpha$ ...(ii)

$a = \alpha R$ ...(iii)

$$P = \frac{I_C}{R}\alpha + m\alpha R \Rightarrow \alpha = \frac{P}{\left(\frac{I_C}{R} + mR\right)}$$

$\therefore \alpha = 4$ rad/s^2

27. [10]

$\omega = \alpha t$; $10 = \alpha \times 10$; $\alpha = 1$ rad/s^2

$\tau_{net} = 100 \times 10 - \tau_f$

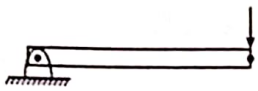
$I\alpha = 100 \times 10 - \tau_f$

$\left(\frac{1}{2}\times 10\times 10^2\right)(1) = 1000 - \tau_f \Rightarrow \tau_f = 500$

$\alpha_f = \frac{\tau_f}{I} = -1 \text{ rad/s}^2$

$0 = \omega_0 + \alpha_f t = 10 - 1 \times t \Rightarrow t = 10 \text{ sec.}$

28. [2] $Fl = \frac{ml^2}{3}\alpha_A \Rightarrow \alpha_A = \frac{3F}{ml}$



$F\frac{l}{2} = \frac{ml^2}{12}\alpha_B$

$\Rightarrow \alpha_B = \frac{6F}{ml}$

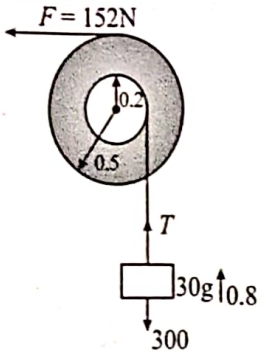
Now, $\theta_A = \frac{1}{2}\alpha_A \times 3^2 = \frac{27F}{2ml}$

$\theta_B = \frac{1}{2}\times\alpha_B \times t^2 = \frac{1}{2}\times\frac{6F}{ml}t^2 = \frac{27F}{2ml} \Rightarrow t^2 = \frac{9}{2}$

$\Rightarrow t = \frac{3}{\sqrt{2}}$ sec. $\approx$ 2sec.

29. [2.8]

$152 \times 0.5 - T \times 0.2 = I\alpha$



$T - 300 = 0.8 \times 30$

$T = 324$

$76 - 324 \times 0.2 = I\alpha$

$0.2\alpha = 0.8 \quad (\alpha = r_1\alpha)$

$\alpha = 4$

$11.2 = I \times 4$

$I = 2.8$ kg-m^2.

30. [55]

$I_{sys} = \frac{mr^2}{2} + \left[\frac{mr^2}{2} + m(2r)^2\right]\times 6$

$= \frac{55}{2}mr^2 = 55$

31. [5] $mg \times 2R = \frac{1}{2}mv^2 + \frac{1}{2}\times mR^2 \times \left(\frac{v}{R}\right)^2$

$\Rightarrow v = \sqrt{2gR} = 5$ m/s

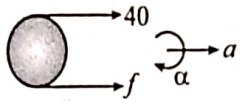
Exercise-4 (Past Year Questions)

JEE MAIN

1. (*b*) $40 + f = m(R\alpha)$ ...(i)

$40 \times R - f \times R = mR^2\alpha$

$40 - f = mR\alpha$ ...(ii)



From (i) and (ii)

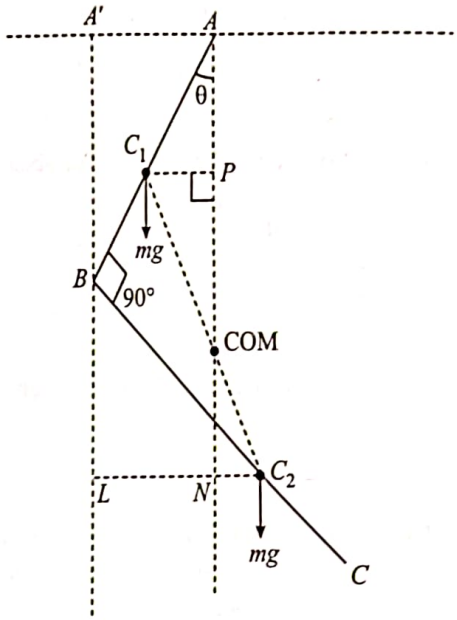
$\alpha = \frac{40}{mR} = 16 \text{ rad/s}^2$

2. (*d*) Lets consider mass of each rod is m.

For stable equilibrium the torque about point O should be zero.

$mg\frac{a}{2}\sin\theta = mg\left(\frac{a}{2}\cos\theta - a\sin\theta\right)$

$\tan\theta = \frac{1}{3}$



$\Rightarrow \tan^{-1}\left(\frac{1}{3}\right)$

3. (*b*) $mg\frac{\ell}{2}\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 \quad \left(h = \frac{\ell}{2}\sin 30°\right)$

$\Rightarrow \omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{30}$

4. (*a*) $I = 2\left[\frac{2}{5}MR^2 + M4R^2\right] + M\frac{4R^2}{12}$

$= MR^2\left[\frac{1}{3} + \frac{4}{5} + 8\right] = \frac{137}{15}MR^2$

5. (c) $v=\sqrt{5^2+2gh}=\sqrt{5^2+2\times10\times10}=\sqrt{225}$
$= 15$ m/s

$L = rmv = 20 \times (20 \times 10^{-3}\text{ kg}) \times (15)$
$= 6$ kg m²/sec

6. (d) $F-f=ma$

$fR=\frac{1}{2}mR^2\alpha$

$a=\alpha R$

From above equations,

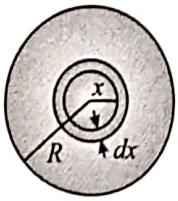
$F=\frac{3}{2}mR\alpha$

and $\alpha=\frac{2F}{3mR}$

7. (d) $d\tau=(\mu dN)x$

$=\mu\left(\frac{F}{\pi R^2}\times2\pi x dx\right)x$

$\tau=\int_0^R d\tau=\frac{2\mu F}{R^2}\int_0^R x^2dx=\frac{2\mu FR}{3}$



8. (d) Applying angular momentum conservation, about axis of rotation

$L_i=L_f$

$\frac{ML^2}{12}\omega_0=\left(\frac{ML^2}{12}+m\left(\frac{L}{2}\right)^2\times2\right)\omega$

$\Rightarrow \omega=\frac{M\omega_0}{M+6m}$

9. (d) $M=\int_0^R\rho_0 r(2\pi r dr)=\frac{\rho_0\times2\pi\times R^3}{3}$

$\underset{\text{(MOI about COM)}}{I_0}=\int_0^R\rho_0 r(2\pi r dr)\times r^2=\frac{\rho_0\times2\pi R^5}{5}$

by parallel axis theorem

$I=I_0+MR^2$

$=\frac{\rho_0\times2\pi R^5}{5}+\frac{\rho_0\times2\pi R^3}{3}\times R^2=\rho_0 2\pi R^5\times\frac{8}{15}$

$=MR^2\times\frac{8}{5}$

10. (b) $\alpha=\frac{\Delta\omega}{\Delta t}=\frac{25\times2\pi}{5}=10\pi\text{ rad/sec}^2$

$\tau=\left(\frac{5}{4}MR^2\right)\alpha=\frac{5}{4}\times5\times10^{-3}\times(10^{-2})^2\times10\pi$

$=1.9625\times10^{-5}$ Nm $=2.0\times10^{-5}$ Nm

11. (d) $I_1=\frac{\left(\frac{7M}{8}\right)(2R)^2}{2}=\left(\frac{7}{16}\times4\right)MR^2=\frac{7}{4}MR^2$

$I_2=\frac{2}{5}\left(\frac{M}{8}\right)R_1^2=\frac{2}{5}\left(\frac{M}{8}\right)\frac{R^2}{4}=\frac{MR^2}{80}$

$\frac{4}{3}\pi R^3=8\left(\frac{4}{3}\pi R_1^3\right)$

$R^3=8R_1^3$

$R=2R_1$

$\therefore\frac{I_1}{I_2}=\frac{7/4MR^2}{\frac{MR^2}{80}}=\frac{7}{4}\times80=140$

12. (c) Angular impulse = change in angular momentum

$\tau\,\Delta t=\Delta L$

$mg\frac{\ell}{2}\times.01=\frac{m\ell^2}{3}\omega$

$\omega=\frac{3g\times0.01}{2\ell}$

$=\frac{3\times10\times0.01}{2\times0.3}$

$=\frac{1}{2}=0.5$ rad/s

time taken by rod to hit the ground

$t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times5}{10}}=1$ sec.

in this time angle rotate by rod

$\theta=\omega t=0.5\times1=0.5$ radian

13. (b) $\frac{1}{2}\left(m+\frac{I}{R^2}\right)v^2=mgh$

if radius of gyration is k, then $I=mk^2$

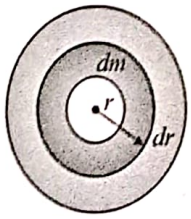
$h=\frac{\left(1+\frac{k^2}{R^2}\right)v^2}{2g}$, $\frac{k_{ring}}{R_{ring}}=1$ and

$\frac{k_{\text{solid cylinder}}}{R_{\text{solid cylinder}}}=\frac{1}{\sqrt{2}}$

$\frac{k_{\text{solid sphere}}}{R_{\text{solid sphere}}}=\sqrt{\frac{2}{5}}$

$h_1:h_2:h_3::(1+1):\left(1+\frac{1}{2}\right):\left(1+\frac{2}{5}\right)::20:15:14$

14. (d)



$dI=(dm)r^2$

$= (\sigma\, dA) r^2$

$= \left(\frac{\sigma_0}{r} 2\pi r dr\right) r^2$

$= (\sigma_0\, 2\pi) r^2 dr$

$= I = \int dI = \int_a^b \sigma_0 2\pi r^2 dr$

$\sigma_0 2\pi \left(\frac{b^3 - a^3}{3}\right)$

$m = \int dm = \int \sigma dA$

$= \sigma_0 2\pi \int_a^b dr$

$m = \sigma_0 2\pi\,(b - a)$

Radius of gyration

$$k = \sqrt{\frac{I}{m}} = \sqrt{\frac{(b^3 - a^3)}{3(b-a)}}$$

$$k = \sqrt{\left(\frac{a^2 + b^2 + ab}{3}\right)}$$

15. (*d*) $E_i = \frac{1}{2} I_1 \times \omega_1^2 + \frac{1}{2}\frac{I_1}{2} \times \frac{\omega_1^2}{4}$

$= \frac{I_1 \omega_1^2}{2}\left(\frac{9}{8}\right) = \frac{9}{16} I_1 \omega_1^2$

Conserving angular momentum

$I_1\omega_1 + \frac{I_1\omega_1}{4} = \frac{3I_1}{2}\omega$

$\frac{5}{4} I_1 \omega_1 = \frac{3I_1}{2}\omega$

$\omega = \frac{5}{6}\omega_1$

$E_f = \frac{1}{2} \times \frac{3I_1}{2} \times \frac{25}{36}\omega_1^2$

$= \frac{25}{48} I_1 \omega_1^2$

$E_f - E_i = I_1\omega_1^2\left(\frac{25}{48} - \frac{9}{16}\right) = \frac{-2}{48} I_1 \omega_1^2$

$= \frac{-I_1\omega_1^2}{24}$

16. (*d*) $mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$v = \omega r$ (no slipping)

$mgh = \frac{1}{2} m\omega^2 r^2 + \frac{1}{2}\frac{mr^2}{2}\omega^2$

$mgh = \frac{3}{4} m\omega^2 r^2$

$\omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$

17. [50.00]

For no toppling

$F\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}$

$\mu\frac{a}{2} + \mu b \le \frac{a}{2}$ $\qquad (F = \mu mg)$

$0.2a + 0.4b \le 0.5a$

$0.4b \le 0.3a$

$b \le \frac{3a}{4}$

$b \le 0.75a$ (in limiting case)

$\therefore \left(100\frac{b}{a}\right)_{max} = 50.00$

18. (*b*) $\sigma = A + Br$

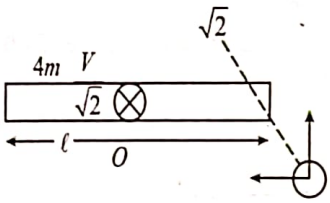
$\int dm = \int (A + Br) 2\pi r dr$

$I = \int dm\, r^2$

$= \int_0^a (A + Br) 2\pi r^3 dr$

$= 2\pi\left(A\frac{a^4}{4} + B\frac{a^5}{5}\right) = 2\pi a^4\left(\frac{A}{4} + \frac{Ba}{5}\right)$

19. (*b*)



Angular momentum conserved about O

$L_{oi} = L_{of}$

$\frac{mv}{\sqrt{2}} \times \frac{\ell}{2} = \left[\frac{4m\ell^2}{12} + \frac{m\ell^2}{4}\right] \times \omega$

$\omega = \frac{6v}{7\sqrt{2}\ell} = \frac{3\sqrt{2}v}{7\ell}$

20. (*b*) $K_i + U_i = K_f + U_f$

$0 + 0 = \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2 + \frac{1}{2} I\omega^2 - m_1 gh + m_2 gh$

$(m_1 - m_2) gh = \frac{1}{2} m_2 (\omega R)^2 + \frac{1}{2} m_1 (\omega R)^2 + \frac{1}{2} I\omega^2$

$$\sqrt{\frac{2(m_1 - m_2)gh}{\left(m_1 + m_2 + \frac{I}{R^2}\right)R^2}} = \omega$$

$$\frac{1}{R}\sqrt{\frac{2(m_1 - m_2)gh}{\left(m_1 + m_2 + \frac{I}{R^2}\right)}}$$

21. (a) $(I_1+I_2)\omega = I_1\omega_1 + I_2\omega_2$

$$\omega = \frac{0.1\times10+0.2\times5}{0.1+0.2} = \frac{20}{3}\text{ rad/s}$$

$$\text{K.E.} = \frac{1}{2}(I_1+I_2)\omega^2$$

$$= \frac{20}{3}\text{J}$$

22. (c) Angular momentum conserved about O

$$mvl = \left(\frac{Ml^2}{3}+ml^2\right)\omega \Rightarrow \omega = \frac{3mv}{3ml+Ml} = \frac{18}{5}\text{ rad/s}$$

Energy conservation

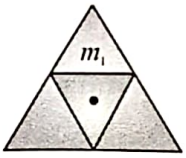
$$\frac{1}{2}\left(\frac{Ml^2}{3}+ml^2\right)\omega = \left(Mg\frac{l}{2}+mgl\right)(1-\cos\theta)$$

$$\Rightarrow \frac{1}{2}\left(\frac{5}{3}\right)\times\frac{324}{25} = (10+10)(1-\cos\theta)$$

$$\Rightarrow 20(1-\cos\theta) = \frac{324}{30}$$

$$\cos\theta = 1-\frac{324}{600} = \frac{276}{600} = 0.46 \Leftrightarrow \theta = 63°$$

23. [11]



Let I_0 be MI of lamina is about 'G' perpendicular to plane

$\therefore I_0 = kma^2$ ...(i)

Then MI of $(m_1) = \frac{km}{4}\frac{a^2}{4} = \frac{I_0}{16}$

Let moment of inertia of one triangular part (side) be I_1 about 'G' then

From symmetry

$$3I_1 + \frac{I_0}{16} = I_0 \Rightarrow 3I_1 = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

$$\Rightarrow I_1 = \frac{15I_0}{16}$$

So, moment of inertia of remaining portion

$$I'' = I_0 - \frac{15I_0}{16}$$

$$\Rightarrow I'' = \frac{11}{16}I_0$$

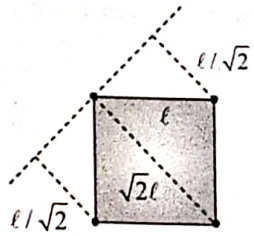
24. (d) $I\omega = 4I\omega'$

$$\Rightarrow \omega' = \frac{\omega}{4}$$

$$(\Delta K)_{\text{LOSS}} = \frac{1}{2}I\omega^2 - \frac{1}{2}(4I)\left(\frac{\omega}{4}\right)^2$$

$$\Rightarrow \text{Fractional loss} = \frac{(\Delta K)_{\text{LOSS}}}{K_1}\left(K_1 = \frac{1}{2}I\omega^2\right) = \frac{3}{4}$$

25. (a)



$$I = m\left(\frac{\ell^2}{2}\right)\times2 + m\times(\sqrt{2}\ell)^2 = 3m\ell^2$$

$\therefore L = I\omega = 3\,m\ell^2\omega$

26. (d) Acceleration of a body rolling down an inclined plane is, $a = \frac{g\sin\theta}{\left(1+\frac{I}{mR^2}\right)}$

Ring: $I_R = mR^2, a_R = g\sin\theta/2$

Disc: $I_D = \frac{mR^2}{2}, a_D = \frac{2}{3}g\sin\theta$

Solid cylinder: $I_{SC} = \frac{mR^2}{2}, a_{SC} = \frac{2}{3}g\sin\theta$

Solid sphere: $I_{SS} = \frac{2}{5}mR^2, a_{SS} = \frac{5}{7}g\sin\theta$

$$S = ut + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2S}{a}}$$

$\therefore t \propto \frac{1}{\sqrt{a}}$ For least time, a should be the highest.

Solid sphere will take minimum time.

27. [728]

$$\omega_f = 2460\times\frac{2\pi}{60} = 82\pi$$

$$\omega_i = \frac{900\times2\pi}{60} = 30\pi$$

$$\alpha = \frac{\omega_f-\omega_i}{t} = \frac{82\pi-30\pi}{26} = 2\pi\text{ rad/sec}^2$$

$$\theta = \frac{\omega_f^2-\omega_i^2}{2\alpha}$$

$$= \frac{(82\pi+30\pi)(82\pi-30\pi)}{2\times2\pi} = \frac{(112\times52)\pi^2}{4\pi} = 2\pi n$$

$$\text{No. of revolution } (n) = \frac{(112\times13)\pi}{2\pi} = 728$$

28. [3] Acceleration of a body rolling down an inclined plane is,

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta,\ b = 3$$

29. (*d*) According to question the given data is,

$\vec{F} = 4\hat{i} - 3\hat{j}$

$\vec{r}_1 = 5\hat{i} + 5\sqrt{3}\hat{j}$,$\vec{r}_2 = -5\hat{i} + 5\sqrt{3}\hat{j}$

Torque about 'O' we know that, $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}_O = \vec{r}_1 \times \vec{F} = (-15 - 20\sqrt{3})\hat{k} = (15 + 20\sqrt{3})(-\hat{k})$

Torque about 'Q'

$\vec{\tau}_Q = \vec{r}_2 \times \vec{F} = (-15 + 20\sqrt{3})\hat{k} = (15 - 20\sqrt{3})(-\hat{k})$

30. (*a*) $P_i = 0.01 \times u + 0 = P_f = 6 \times v$

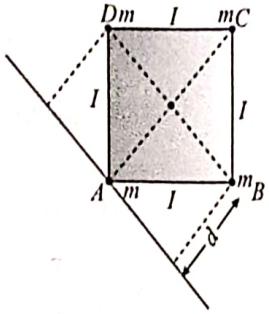
$v = \frac{0.01u}{6}$

using energy conservation

$\frac{1}{2} \times 6 \times \left(\frac{u}{600}\right)^2 = 6 \times 9.8 \times 9.8 \times 10^{-2}$

$\Rightarrow u = 6 \times 98 \times \sqrt{2} = 588\sqrt{2}$ m/s

31. (*d*) $AC = l\sqrt{2}$



$d = \frac{l\sqrt{2}}{2}$

$\Rightarrow d = \frac{l}{\sqrt{2}}$

$\Rightarrow d = \frac{l}{\sqrt{2}}$

Moment of inertia about the axis passing through A:

$I = m(O)^2 + m(d)^2 + m(d)^2 + M(AC)^2$

$\Rightarrow I = O + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m(l\sqrt{2})^2$

$\Rightarrow I = \frac{ml^2}{2} + \frac{ml^2}{2} + 2ml^2 \Rightarrow I = 3ml^2$

32. [6] The weight acts at 50 cm mark.

Balancing torque about knife edge,

$(20 \times 10^{-3})g(30 \times 10^{-2}) = mg \times (10 \times 10^{-2})$

$\Rightarrow mg = 6 \times 10^{-2}$ kg

$\Rightarrow x = 6$

33. (*b*) $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5}MR^2 \times \frac{v^2}{R^2} = \frac{Mv^2}{5}$

$K_T = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = \frac{Mv^2}{5} + \frac{1}{2}Mv^2 = \frac{7Mv^2}{10}$

Thus, $\frac{K_R}{K_T} = \frac{2}{7}$

34. [8] $I_1 = \frac{ML^2}{3}$

Now, $2\pi r = L$

$\Rightarrow r = \frac{L}{2\pi}$

$I_2 = \frac{1}{2}Mr^2 = \frac{1}{2}\frac{ML^2}{4\pi^2}$

So, $\frac{I_1}{I_2} = \frac{8\pi^2}{3} \Rightarrow x = 8$

35. (*c*) $L = I_{cm}\omega + Mv_{cm}R$

$= \left(\frac{2}{3}R^2\right)\omega + (1)(R\omega)R = \frac{5}{3}R^2\omega$

$\Rightarrow a = 5$

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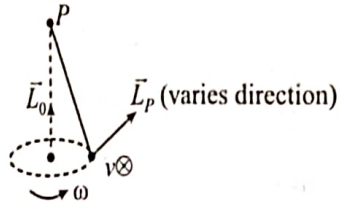
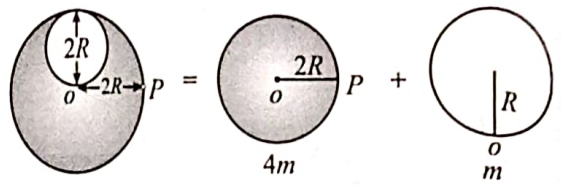
36. (*c*) L_0 remains constant in magnitude and direction but L_P changes its direction continuously hence L_P is variable



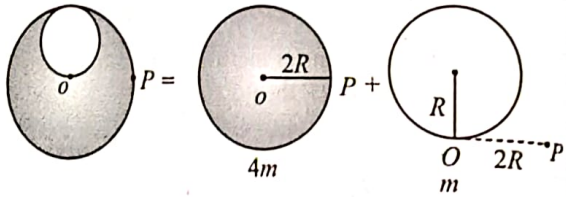
37. [3]



$I_0 = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2$

$= mR^2\left[8 - \frac{3}{2}\right]$

$= \frac{13}{2}mR^2$



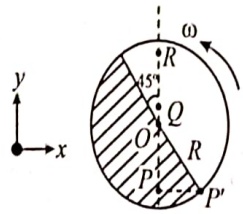
$I_P = \frac{3}{2}(4m)(2R)^2 - \left[\frac{mR^2}{2} + m[(2R)^2 + R^2]\right]$

$= 24mR^2 - \frac{11}{2}mR^2$

$= \frac{37}{2}mR^2$

$= \frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$

38. (*c*)



$\frac{1}{8}$th rotation $\Rightarrow \theta = \frac{\pi}{4} = 45°$

have x-cordinate of

$P = V_x \times t$

$= \omega R \times \frac{\pi/4}{\omega}$

$= \frac{\pi}{4} R$

Possible

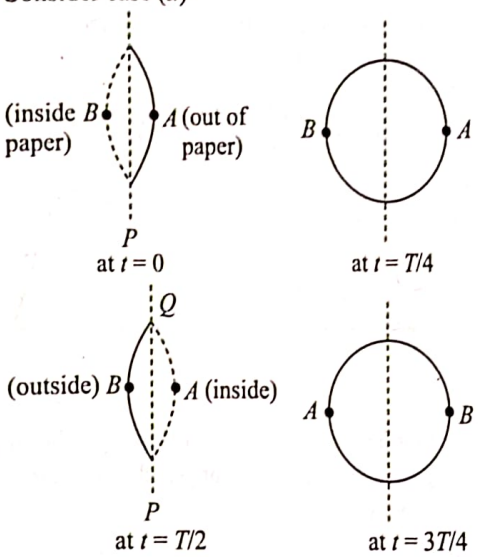
$X_{max} = PP' = R\sin 45°$

$= \frac{R}{\sqrt{2}} < \frac{\pi}{4} R$

Thus, P lands on unshaded region.

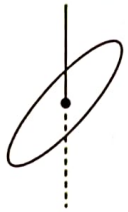
Q will definitely land up in unshaded region.

39. (*a*) Consider case (*a*)



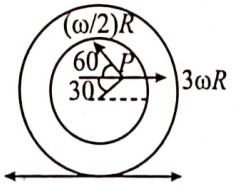
Hence axis is vertical.

For case (*b*)



40. (*d*) Angular velocity of rigid body about any axis which are parallel to each other is same . So angular velocity is ω.

41. (*a*,*b*) $V_0 = 3\omega R\,\hat{i}$ (From pure rolling condition)



$$v_P = \left(3\omega R - \frac{\omega R}{2}\cos 60°\right)\hat{i} + \frac{\omega R}{2}\sin\ 60\hat{k}$$

$$= \frac{11\omega R}{4}\hat{i} + \frac{\sqrt{3}\omega R}{4}\hat{k}$$

42. (*d*) $I_P > I_Q$

$$a = \frac{g\sin\theta}{1 + I/MR^2}$$

Hence $a_P < a_Q$

$t_P > t_Q$

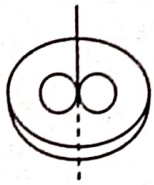
$v_P < v_Q$

And as $\omega = v/R$, So $\omega_P < \omega_Q$

43. [8] Angular momentum conservation

$I_1\omega_1 = I_2\omega_2$

$$\frac{MR^2}{2}\omega_1 = \left[\frac{MR^2}{2} + 2(mr^2 + mr^2)\right]\omega_2$$



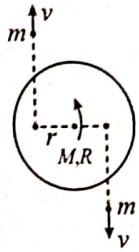
$$\Rightarrow \frac{50(0.4)^2}{2} \times 10$$

$$= \left[\frac{50(0.4)^2}{2} + 2\left\{(6.25)\left(0.2^2 + 0.2^2\right)\right\}\right]\omega_2$$

$40 = [4 + 1]\,\omega_2 \quad \Rightarrow \omega_2 = 8$ rad/s

44. [4] Applying conservation of angular momentum.

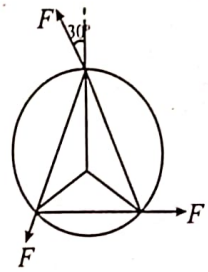
$$2mvr - \frac{MR^2}{2}\omega = 0$$



$$\omega = \frac{4mvr}{MR^2};\ \omega = \frac{(4)(5\times10^{-2})(9)\left(\frac{1}{4}\right)}{45\times10^{-2}\times\frac{1}{4}}$$

$\omega = 4$ rad/s

45. [2]



$$\omega = \frac{\int \tau dt}{I} = \frac{\int_0^t 3F\sin 30° R\ dt}{I}$$

$$= \frac{3(0.5)\ (0.5)\ (0.5)\ (1)}{\left(\frac{1.5(0.5)^2}{2}\right)} = 2 \text{ rad/s}$$

46. [7] $\left[\frac{3}{2}\right]\frac{1}{2}m(3)^2 + mg(30) = \frac{3}{2}\frac{1}{2}mv_2^2 + mg(27)$

$$\frac{3}{4}.9 + 300 = \frac{3}{4}v_2^2 + 270$$

$$\frac{27}{4} + 30 = \frac{3}{4}v_2^2$$

$$\Rightarrow v_2^2 = 9 + 40 \Rightarrow v_2 = 7$$

47. (*d*) Apply the law of conservation of angular momentum.

$$MR^2\omega = \left(MR^2 + \frac{M}{8}\frac{9R^2}{25} + \frac{Md^2}{8}\right)\frac{8\omega}{9}$$

$$R^2 = \left(\frac{200R^2 + 9R^2 + 25d^2}{8\times 25}\right)\frac{8}{9}$$

$$225\,R^2 - 209\,R^2 = 25\,d^2$$

$$d^2 = \frac{16R^2}{25}$$

$$d = \frac{4R}{5}$$

48. [6] Sphere A:

MOI of elemental shell

$$dI = \frac{2}{3}dmx^2 = \frac{2}{3}(\rho(x).4\pi x^2dx)x^2$$

$$= \frac{2}{3}\left(\frac{kx}{R}.4\pi x^2dx\right)x^2$$

$$= \frac{8\pi k}{3R}x^5dx$$

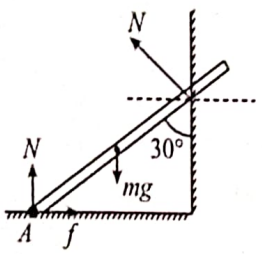
$$I_A = \int dI = \frac{8\pi k}{3R}.\frac{R^6}{6} = \frac{4\pi kR^5}{9}$$

Similarly for sphere B:

$$I_B = \frac{4\pi kR^5}{15}$$

$$\frac{I_B}{I_A} = \frac{9}{15} = \frac{6}{10} \Rightarrow n = 6$$

49. (*d*)



From vertical equilibrium condition,

$mg = N + N\cos 60$

$$N = \frac{2mg}{3}$$

Taking torque about A.

$$mg\times\frac{1}{2}\cos 60 = \frac{2mg}{3}\times\left[\frac{h}{\cos 30}\right]$$

$$\frac{h}{\ell} = \frac{3\sqrt{3}}{16}$$

and $f = N\sin 60 = \frac{2mg}{3}\times\frac{\sqrt{3}}{2} = \frac{16\sqrt{3}}{3}N$

50. (*a,b,d*) $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2\hat{i} + 2\beta t\,\hat{j}$$

So, $(\vec{r})_{t=1} = \frac{10}{3}\hat{i} + 5\hat{j}$

$(\vec{v})_{t=1} = 10\hat{i} + 10\hat{j}$

Therefore, $(\vec{p})_{t=1} = \hat{i} + \hat{j}$

Now, $\vec{L} = \vec{r}\times\vec{p} = \left(\frac{5}{3}\right)\hat{k}$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m(6\alpha t\hat{i} + 2\beta\hat{j})$$

$$\Rightarrow (\vec{F})_{t=1} = 2\hat{i} + \hat{j}$$

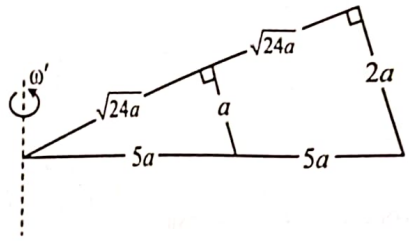
$$\vec{\tau} = \vec{r}\times\vec{F} = -\left(\frac{20}{3}\right)\hat{k}$$

51. (*a,d*) As the discs are rolling without slipping

$$\therefore \omega'\times 5a = \omega a \Rightarrow \omega' = \frac{\omega}{5}$$

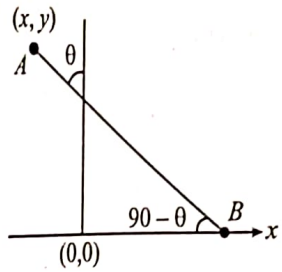
Angular momentum of system about CM through an axis along rod is

$$= \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2}{2}\omega$$



52. (*a*) Option (*a*) is correct. In this case the applied force passes through the point Q and hence, the torque is zero about Q.

53. (*b,c,d*)



$$x = -\frac{L}{2}\sin\theta$$

$y = L\cos\theta$

$$\frac{y^2}{L^2} + \frac{x^2}{\left(\frac{L}{2}\right)^2} = 1$$

Path of A is ellipse

Torque about point of contact

$$mg\frac{L}{2}\sin\theta = I\alpha$$

hence torque $\propto \sin\theta$

At angle q, $y_{cm} = \frac{L}{2}\cos\theta$

When vertical, $y_{cm} = \frac{L}{2}$

So, $\Delta y_{cm} = \frac{L}{2}(1-\cos\theta)$

The midpoint will fall vertically downwards

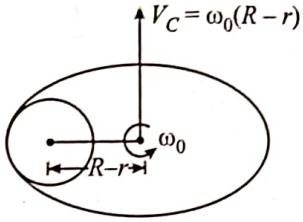
54. (Bonus)

The total kinetic energy is

$$K = \frac{1}{2}Mv_c^2 + \frac{1}{2}I_c\omega^2$$

$$= \frac{1}{2}M\omega_0^2(R-r)^2 + \frac{1}{2}MR^2\omega_0^2$$

$$= \frac{1}{2}M\omega_0^2R^2\left(1-\frac{r}{R}\right)^2 + \frac{1}{2}M\omega_0^2R^2$$

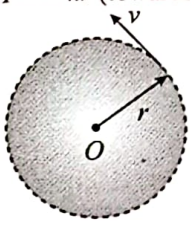


55. (a) $\mu M\omega^2_{min}(R-r) = Mg$

$$\therefore \omega_{min} = \sqrt{\frac{g}{\mu(R-r)}}$$

56. (b,c) $V = \frac{kr^2}{2}$

$F = -kr$ (towards centre) $\left[F = -\frac{dV}{dr}\right]$



At $r = R$,

$kR = \frac{mv^2}{R}$ [Centripetal force]

$$\Rightarrow v = \sqrt{\frac{kR^2}{m}} = \sqrt{\frac{k}{m}}R$$

and $L = m\sqrt{\frac{k}{m}}R^2 = \sqrt{mK}\,R$

57. (a,c) $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ [At $t = 0, v = 0, \vec{\tau} = \vec{0}$]

Since $\alpha = 1$ and $\beta = 1$, $\vec{F} = t\hat{i} + \hat{j}$

$$\Rightarrow m\frac{d\vec{v}}{dt} = t\hat{i} + \hat{j}$$

On integrating

$m\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$ [$m = 1$kg]

$\Rightarrow \frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$ [$\vec{r} = 0$ at $t = 0$]

On integrating, $\vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$.

At $t = 1$ sec, $\vec{\tau} = (\vec{r}\times\vec{F}) = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right)\times(\hat{i}+\hat{j})$

$\vec{\tau} = -\frac{1}{3}\hat{k}$; $\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

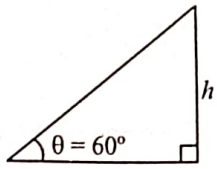
At $t = 1, \vec{v} = \left(\frac{1}{2}\hat{i} + \hat{j}\right) = \frac{1}{2}(\hat{i} + 2\hat{j})$ m/sec

At $t = 1$, $\vec{s} = \vec{r}_1 - \vec{r}_0 = \left[\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right] = [\vec{0}]$

$$\Rightarrow \vec{s} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$$

$$|\vec{s}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)^2} \Rightarrow \frac{\sqrt{10}}{6}\text{ m}$$

58. [0.75]



$$a_c = \frac{g\sin\theta}{1+\frac{I_c}{MR^2}}$$

$$a_{ring} = \frac{g\sin\theta}{2}$$

$$a_{disc} = \frac{2g\sin\theta}{3}$$

$$\frac{h}{\sin\theta} = \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2 \Rightarrow t_1 = \sqrt{\frac{4h}{g\sin^2\theta}} = \sqrt{\frac{16h}{3g}}$$

$$\Rightarrow \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\sqrt{h}\left[\frac{4}{\sqrt{3}} - 2\right] = 2-\sqrt{3}$$

$$\sqrt{h} = \frac{(2-\sqrt{3})\sqrt{3}}{(4-2\sqrt{3})} = \frac{\sqrt{3}}{2} \Rightarrow h = \frac{3}{4} = 0.75m$$

59. (a) (P) $\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = \alpha\hat{i} + \beta\hat{j}\ \{\text{constant}\}$$

$$\vec{a} = \frac{\overline{dv}}{dt} = 0$$

$\vec{P} = m\vec{v}$ (remain constant)

$K = \frac{1}{2}mv^2$ (remain constant)

$$\vec{F} = \left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{i}\right] = 0$$

$\Rightarrow U \rightarrow$ constant

$E = K + U \rightarrow$ constant

$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = 0$

$\vec{L} =$ constant

$(Q)\ \vec{r} = \alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = -\alpha\omega\sin(\omega t)\hat{i} + \beta\omega\cos(\omega t)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = -\alpha\omega^2\cos(\omega t)\hat{i} - \beta\omega^2\sin(\omega t)\hat{j}$

$= -\omega^2\left[\alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}\right]$

$a = -\omega^2\vec{r}$

$\vec{\tau} = \vec{r} \times \vec{F} = 0$ { $\vec{r}$ and $\vec{F}$ are parallel} $\Rightarrow L =$ constant

$\Delta U = -\int \vec{F}.dr = +\int_0^r m\omega^2.r.dr$

$U \propto r^2$

$r = \sqrt{\alpha^2\cos^2(\omega t) + \beta^2\sin^2(\omega t)}$

r is a function of time (t)

U depends on r hence it will change with time

Total energy remain constant because force is central.

$(R)\vec{r}(t) = \alpha\left(\cos\omega t\hat{i} + \sin(\omega t)\hat{j}\right)$

$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \alpha\left[-\omega\sin(\omega t)\hat{i} + \omega\cos(\omega t)\hat{j}\right]$

$|\vec{v}| = \alpha\omega$ (Speed remains constant)

$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \alpha\left[-\omega^2\cos(\omega t)\hat{i} - \omega^2\sin(\omega t)\hat{j}\right]$

$= -\alpha\omega^2\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right]$

$\vec{\alpha}(t) = -\omega^2(\vec{r})$

$\vec{\tau} = \vec{F} \times \vec{r} = 0 \quad \Rightarrow \quad L =$ constant

$|\vec{r}| = \alpha$ (remain constant)

Force is central in nature and distance from fixed point is constant.

Potential energy remains constant

Kinetic energy is also constant (speed is constant)

$(S)\ \vec{r} = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \alpha t\hat{i} + \beta t\hat{j}$ (speed of particle depends on 't')

$\vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}$ {constant}

$\vec{F} = m\vec{a}$ {constant}

$\Delta U = -\int \vec{F}.d\vec{r} = -m\int_0^t \beta\hat{j}.(\alpha\hat{i} + \beta t\hat{j})dt$

$U = \frac{-m\beta^2t^2}{2}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\alpha^2 + \beta^2t^2\right)$

$E = K + U = \frac{1}{2}m\alpha^2$ (remain constant)

60. (a,c,d)

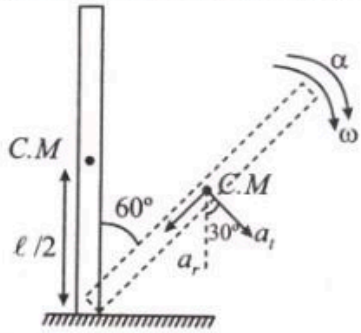
We can treat contact point as hinged.

Applying work energy theorem

$W_g = \Delta K.E.$

$mg\frac{\ell}{4} = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{2\ell}}$

Radial acceleration of C.M. of rod $= \left(\frac{\ell}{2}\right)\omega^2 = \frac{3g}{4}$



Using $\tau = I\alpha$ about contact point

$\frac{mg\ell}{2}\sin 60° = \frac{m\ell^2}{3}\alpha$

$\Rightarrow \alpha = \frac{3\sqrt{3}}{4\ell}g$

Net vertical acceleration of $C.M.$ of rod

$a_v = a_r\cos 60° + a_t\cos 30°$

$= \omega^2\frac{\ell}{2}\cos 60° + \alpha\frac{\ell}{2}\cos 30°$

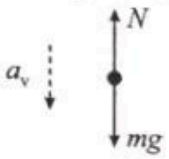
$= \left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha\frac{\ell}{2}\right)\cos 30°$

$= \frac{3g}{8} + \frac{3\sqrt{3}}{4\ell}g\left(\frac{\ell}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3g}{8} + \frac{9}{16}g = \frac{15}{16}g$

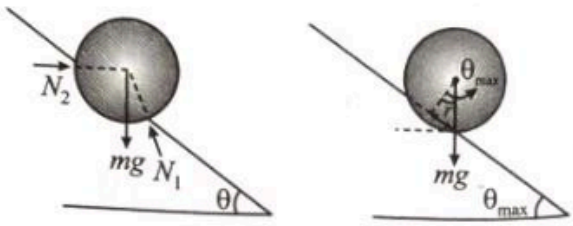
Applying $F_{net} = ma$ in vertical direction on rod as system

$mg - N = ma_v = m\left(\frac{15}{16}g\right)$



$\Rightarrow N = \frac{mg}{16}$

61. (a)



For θ_{max}, the football is about to roll, then $N_2 = 0$ and all forces (Mg and N_1) must pass through contact point

$\therefore \cos(90° - \theta_{max}) = \frac{r}{R} \Rightarrow \sin\theta_{max} = \frac{r}{R}$

62. (*b*) For no slipping at the ground, the center of roller moves with speed

$V_{centre} = \omega R$ (R is radius of roller)

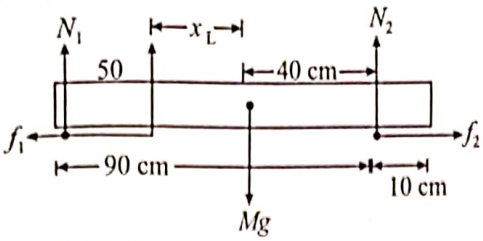
$\therefore$ Velocity of scale $= (V_{center} + \omega r)$ [r is radius of axle]

It is given that $V_{center} \cdot t = 50$ cm

$\therefore$ Distance moved by scale $= (V_{center} + \omega r)t$

$$= \left(V_{center} + \frac{V_{center} r}{R}\right)t = \frac{3V_{center}}{2} \cdot t = 75 \text{ cm} \quad \left(r = \frac{R}{2}\right)$$

63. [25.60]



Initially $N_1 + N_2 = mg$

Torque about center is zero that is

$N_1(50) = 40N_2$

$\Rightarrow 5N_1 = 4N_2$

Solving above equations we get

$N_1 = \frac{4Mg}{9}$ and $N_2 = \frac{5Mg}{9}$

Using definitions of kinetic and static frictions,

$f_{K_1} = 0.32N_1$, $f_{K_2} = 0.32N_2$ and

$f_{L_1} = 0.4N_1$, $f_{L_2} = 0.4N_2$

Suppose x_L is the distance of left finger from centre when right finger starts moving. From rotational equilibrium condition

$N_1 x_L = N_2(40)$

and $f_{K_1} = f_{L_2} \Rightarrow 0.32N_1 = 0.40N_2$

$\Rightarrow 4N_1 = 5N_2$

Therefore, $N_1 x_L = \frac{4N_1}{5}(40)$ or $x_L = 32$

Now x_R = distance when right finger stops and left finger starts moving

Again, $N_1 x_L = N_2(x_R)$

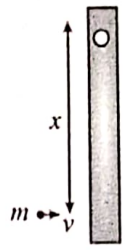
and $f_{L_1} = f_{K_2} \Rightarrow 0.4N_1 = 0.32N_2$

$\Rightarrow 5N_1 = 4N_2$

So, $\frac{4N_2}{5}(32) = N_2 x_R$

$\Rightarrow x_R = \frac{128}{5} = 25.6\,cm$

64. (*a,c,d*)



By the angular momentum conservation about the suspension point.

$$mvx = \left(\frac{mL^2}{3} + mx^2\right)\omega$$

$$\therefore \omega = \frac{mvx}{\frac{mL^2}{3} + mx^2} = \frac{2vx}{L^2 + 3x}$$

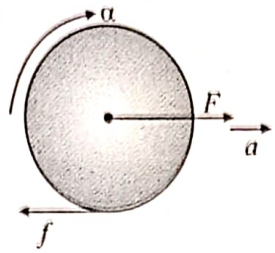
For maximum $\omega \Rightarrow \frac{d\omega}{dx} = 0$

$\therefore x_M = \frac{L}{\sqrt{3}}$

Putting the above value in ω expression, we get

$$\omega = \frac{V}{2L}\sqrt{3}$$

65. (*b,d*)



$F - f = ma_C$

$fR = I_C\alpha$

$a_C - \alpha R = 0$

$F - I_C\frac{\alpha}{R} = ma_C$

$$\Rightarrow a_C = \frac{F}{\frac{I_C}{R^2} + m}$$

So, (*a*) is incorrect

$$f = \frac{I_C\alpha}{R} = \frac{I_C}{R^2}a_C = \frac{I_C}{R^2}\frac{F}{\left[\frac{I_C}{R^2} + m\right]} = \frac{F}{\left[1 + \frac{mR^2}{I_C}\right]}$$

Thin walled hollow cylinder

$I_C = mR^2 \Rightarrow a_C = \frac{F}{2m}$

So, (*d*) is correct

Now, $fR = I_C\alpha = \frac{I_C a_C}{R}$

$$\Rightarrow f = \frac{I_C a_C}{R^2} \le \mu mg \Rightarrow a_C \le \frac{\mu mgR^2}{I_C} = \mu g$$

For solid cylinder $I_C = \frac{mR^2}{2}$

$a_C \le 2\mu g$

$(a_C)_{max} = 2\mu g$

So, (*b*) is correct.

66. [49] $L_s = L_{disc} + L_{rod}$

$L_{disc} = \vec{r} \times M\vec{v}_{cm} + I_{cm}4\Omega,\ L_{rod} = \frac{Ma^2}{3}\Omega$

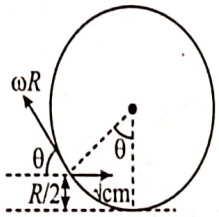
$\Rightarrow L_s = \frac{3a}{4} \times \frac{3a}{4} \times M\Omega + \frac{Ma^2}{32} \times 4\Omega + \frac{Ma^2}{3}\Omega$

$= \frac{49}{48}Ma^2\Omega$

67. [0.52]

$\theta = \frac{1}{2}\alpha t^2$

$\theta = \frac{1}{2} \times \frac{2}{3} \times \pi = \frac{\pi}{3} = 60°$



At $t = 0, \omega = 0$

at $t = \sqrt{\pi}, \omega = \alpha t = \frac{2}{3}\sqrt{\pi} \Rightarrow v = \omega r = \frac{2}{3}\sqrt{\pi}$

Now, $v_y = v\sin 60 = \frac{\sqrt{3}}{2}v$

So, $h = \frac{v_y^2}{2g} = \frac{\frac{3}{4}v^2}{2g} = \frac{\frac{3}{4} \times \frac{4}{9}\pi}{2g} = \frac{3\pi}{9 \times 2g} = \frac{\pi}{6g}$

Maximum height from plane, $H = \frac{R}{2} + h$

$H = \frac{1}{2} + \frac{\pi}{6 \times 10}$

$\Rightarrow x = \frac{\pi}{6}; x = 0.52$

68. [2.86]

Translation motion of COM:

$5 + 1 - 1 - f = 1a$

$5 - f = a$ ...(i)

Rotational motion :

$f1 - 2(1(0.5)) = \frac{2}{5}Mr^2\alpha$

$\Rightarrow f - 1 = \frac{2}{5}a \Rightarrow f = 1 + \frac{2}{5}a$ ...(ii)

From equations (i) and (ii),

$5 - a = 1 + \frac{2}{5}a$

$\Rightarrow 4 = \frac{7a}{5} \Rightarrow a = \frac{20}{7} = 2.86\ m/s^2$

69. [3] The line of action of the force passes through the origin. Therefore, the torque about the origin will be zero and as a result, the angular momentum about the origin will remain conserved. Thus,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & \frac{2}{\pi} \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0.5 \\ v_x & v_y & \frac{2}{\pi} \end{vmatrix}$$

Equating coefficient of $\hat{k}$,

$[1 + 2] = [xv_y - yv_x]$

Thus,

$xv_y - yv_x = 3$

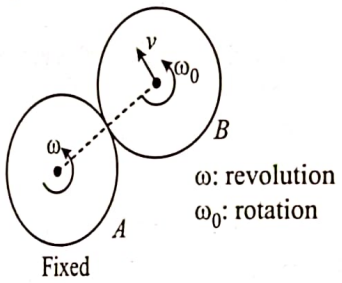
70. (*b*) If the linear velocity of center of mass of disc *B* is *v*, then

$v = \omega(2R)$

Further, since the disc *B* is rolling without slipping,

$v = \omega_0 R$: no slipping

$\therefore \omega_0 = 2\omega$



$\vec{L} = m\vec{r} \times \vec{v}_c + I_c\omega_0$

$= M^2Rv + \frac{1}{2}MR^2\omega_0$

$= 4MR^2\omega + \frac{1}{2}MR^2(2\omega) = 5MR^2\omega$

$\therefore n = 5$

CHAPTER

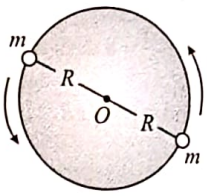
12 Gravitation

Exercise-1 (Topicwise)

1. (*d*) $F \propto \frac{1}{r^2}$. If r becomes double then F reduces to $\frac{F}{4}$

2. (*c*) Centripetal force provided by the gravitational force of attraction between two particles

i.e. $\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$



3. (*d*) Fact based

4. (*a*) Fact based

5. (*d*) Fact based

6. (*c*) $F = \frac{G \times m \times m}{(2R)^2} = \frac{G \times \left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2} = \frac{4}{9}\pi^2 \rho^2 R^4$

$\therefore F \propto R^4$

7. (*d*) Time period of simple pendulum $T = 2\pi\sqrt{\frac{l}{g'}}$

In artificial satellite $g' = 0$

$\therefore T =$ infinite.

8. (*b*) $g = \frac{GMx}{R^3}$

9. (*b*) Because acceleration due to gravity increases

10. (*b*) We know that $g = \frac{GM_e}{R_e^2}$

On the planet $g_p = \frac{GM_e/7}{R_e^2/4} = \frac{4g}{7} = \frac{4}{7}g$

Hence weight on the planet $= 700 \times \frac{4}{7} = 400$ gm wt

11. (*c*) For the condition of weightlessness at equator

$\omega = \sqrt{\frac{g}{R_e}}$

$\therefore \omega = \sqrt{\frac{1}{640 \times 10^3}} = \frac{1}{800}$ rad/s

12. (*c*) Value of g decreases when we go from poles to equator.

13. (*b*) $g = \frac{GM_e}{R_e^2}$. If radius shrinks to half of its present value then g will becomes four times.

14. (*c*) Acceleration due to gravity at poles is independent of the angular speed of earth

15. (*d*) Range of projectile $R = \frac{u^2 \sin 2\theta}{g}$

if u and θ are constant then $R \propto \frac{1}{g}$

$\frac{R_m}{R_e} = \frac{g_e}{g_m} \Rightarrow \frac{R_m}{R_e} = \frac{1}{0.2} \Rightarrow R_m = \frac{R_e}{0.2} \Rightarrow R_m = 5R_e$

16. (*a*) $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

17. (*a*) $g = \frac{4}{3}\pi G \rho R$

$\Rightarrow g \propto \rho R \Rightarrow \frac{g_e}{g_m} = \frac{\rho_e}{\rho_m} \times \frac{R_e}{R_m}$

$\Rightarrow \frac{6}{1} = \frac{5}{3} \times \frac{R_e}{R_m} \Rightarrow R_m = \frac{5}{18}R_e$

18. (*c*) $\Delta U = \frac{mgh}{1 + h/R}$

Substituting $R = 5h$

We get $\Delta U = \frac{mgh}{1+1/5} = \frac{5}{6}mgh$

19. (*a*) $I = \frac{-dV}{dx}$

If $V = 0$ then gravitational field is necessarily zero.

20. (*a*) $U = -\frac{GMm}{r}$

$\Rightarrow 7.79\times10^{28} = \frac{6.67\times10^{-11}\times7.4\times10^{22}\times6\times10^{24}}{r}$

$\Rightarrow r = 3.8\times10^{8}$ m

21. (*c*) $\frac{G\times100}{x^2} = \frac{G\times10000}{(1-x)^2} \Rightarrow \frac{10}{x} = \frac{100}{1-x} \Rightarrow x = \frac{1}{11}$ m

22. (*d*) $\Delta U = U_2 - U_1 = \frac{mgh}{1+\frac{h}{R_e}} = \frac{mgR_e}{1+\frac{R_e}{R_e}} = \frac{mgR_e}{2}$

$\Rightarrow U_2 - (-mgR_e) = \frac{mgR_e}{2} \Rightarrow U_2 = -\frac{1}{2}mgR_e$

23. (*a*) If body is projected with velocity $v(v < v_e)$ then height up to which it will rise, $h = \frac{R}{\frac{v_e^2}{v^2}-1}$

$v = \frac{v_e}{2}$ (given)

$\therefore h = \frac{R}{\left(\frac{v_e}{v_e/2}\right)^2 - 1} = \frac{R}{4-1} = \frac{R}{3}$

24. (*d*) Change in potential energy in displacing a body from r_1 to r_2 is given by

$\Delta U = GMm\left[\frac{1}{r_1}-\frac{1}{r_2}\right] = GMm\left(\frac{1}{2R}-\frac{1}{3R}\right) = \frac{GMm}{6R}$

25. (*a*) $v_e = \sqrt{\frac{2GM}{R}} = R\sqrt{\frac{8}{3}\pi G\rho}$

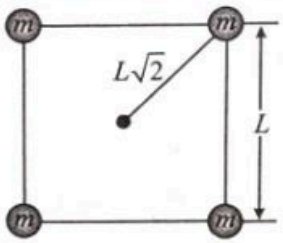
$\therefore v_e \propto R$ if ρ = constant

Since the planet having double radius in comparison to earth therefore the escape velocity becomes twice i.e. 22 km/s.

26. (*b*) If missile launched with escape velocity then it will escape from the gravitational field and at infinity its total energy becomes zero.

But if the velocity of projection is less than escape velocity then sum of energies will be negative. This shows that attractive force is working on the satellite.

27. (*a*) Potential at the centre due to single mass = $\frac{-GM}{L/\sqrt{2}}$



Potential at the centre due to all four masses

$= -4\frac{GM}{L/\sqrt{2}} = -4\sqrt{2}\frac{GM}{L} = -\sqrt{32}\times\frac{GM}{L}$

28. (*c*) Fact based

29. (*b*) $v_e = R\sqrt{\frac{8}{3}G\pi\rho}$ $\therefore v_e \propto R\sqrt{\rho}$

30. (*b*) Angular momentum is conserved in central field.

31. (*d*) $v_0 = \sqrt{\frac{GM}{r}}$

32. (*b*) K.E. required for satellite to escape from earth's gravitational field $\frac{1}{2}mv_e^2 = \frac{1}{2}m\left(\sqrt{\frac{2GM}{R}}\right)^2 = \frac{GMm}{R}$

K.E. required for satellite to move in circular orbit

$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{R}}\right)^2 = \frac{GMm}{2R}$

The ratio between these two energies = 2

33. (*a*) Escape velocity is same for all angles of projection.

34. (*d*) Fact based

35. (*b*) Time period of communication satellite T_c = 1 day

Time period of another satellite = T_s

$\frac{T_s}{T_c} = \left(\frac{r_s}{r_c}\right)^{3/2} = (4)^{3/2}$

$\Rightarrow T_s = T_s\times(4)^{3/2} = 8$ days

36. (*b*) $v_e = \sqrt{2gR}$ and $v_0 = \sqrt{gR}$

$\therefore \sqrt{2}v_0 = v_e$

37. (*b*) $T \propto r^{3/2}$. If r becomes double then time period will becomes $(2)^{3/2}$ times.

So new time period will be $24\times2\sqrt{2}$ hr i.e.

$T = 48\sqrt{2}$ hours

38. (*c*) Fact based

39. (*c*) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2}$

$\Rightarrow T_2 = 8T_1$

40. (*b*) Fact based

41. (*d*) $T^2 \propto R^3 \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

42. (*a*) Angular momentum = Mass × Orbital velocity × Radius

$= m\times\left(\sqrt{\frac{GM}{R_0}}\right)\times R_0 = m\sqrt{GMR_0}$

43. (*c*) Areal velocity of the planet remains constant. If the areas A and B are equal then $t_1 = t_2$.

44. (*a*) We know $T \propto R^{3/2}$

$\Rightarrow T_{planet} = 5^{\frac{3}{2}}\,T_{earth} = 5^{\frac{3}{2}}\times1$ year

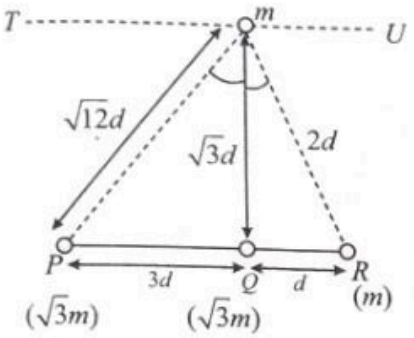
Exercise-2 (Learning Plus)

1. (*b*) $F=\frac{GM_sM_e}{R^2}$

$F'=\frac{GM_sM_e}{(3R)^2}=\frac{F}{9}$

2. (*c*) In horizontal direction

Net force $=\frac{G\sqrt{3}\,mm}{12d^2}\cos30°-\frac{Gm^2}{4d^2}\cos60°$



$=\frac{Gm^2}{8d^2}-\frac{Gm^2}{8d^2}=0$

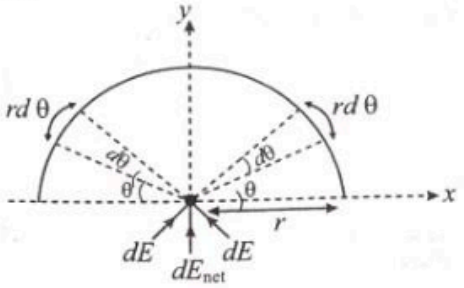
in vertical direction

Net force $=\frac{G\sqrt{3}\,m^2}{12d^2}\cos60°+\frac{G\sqrt{3}\,m^2}{3d^2}+\frac{Gm^2}{4d^2}\cos30°$

$=\frac{\sqrt{3}\,Gm^2}{24d^2}+\frac{\sqrt{3}Gm^2}{3d^2}+\frac{\sqrt{3}Gm^2}{8d^2}$

$=\frac{\sqrt{3}Gm^2}{d^2}\left[\frac{1+8+3}{24}\right]=\frac{\sqrt{3}Gm^2}{2d^2}$ along SQ

3. (*d*) $dE_{net}=2dE\sin\theta$



$=2\frac{Gdm}{r^2}\sin\theta$

$=2G.\frac{\lambda rd\theta}{r^2}\sin\theta$

$=\frac{2G\lambda}{r}\sin\theta\,d\theta$

$E_{net}=\int dE_{net}$

$=\int_0^{\pi/2}\frac{2G\lambda}{r}\sin\theta\,d\theta$

$=\frac{2G\lambda}{r}$

$\lambda=\frac{m}{\ell}$ and $r=\frac{\ell}{\pi}$

$E_{net}=\frac{2Gm\pi}{\ell^2}$

Along + y axis

4. (*d*) For point '*A*':

For any point outside, the shells acts as point situated at centre.

So, $F_A=\frac{G(M_1+M_2)}{p^2}m$

For point '*B*':

There will be no force by shell B.

So, $F_B=\frac{GM_1m}{q^2}$

For point '*C*':

There will be no gravitational field.

So, $F_C=0$

5. (*c*) $g=\frac{GM}{R^2}=\frac{G\frac{4}{3}\pi R^3\rho}{R^2}$

$\rho=\frac{3g}{4\pi RG}$

6. (*d*) $\frac{g}{4}=\frac{g}{\left(1+\frac{h}{R_e}\right)^2}$

$2=1+\frac{h}{R_e}$

$h=R_e$

7. (*b*) $g'=\frac{g}{\left(1+\frac{h}{R}\right)^2}=\frac{g}{\left(1+\frac{1}{2}\right)}=\frac{4g}{9}$

decrease $=g-g'$

$=g-\frac{4g}{9}=\frac{5g}{9}$

8. (*c*) Due to rotation of earth

$g_{eff}=g-\omega^2R_e\cos^2\theta$

$g\downarrow\therefore$ weight $\downarrow$

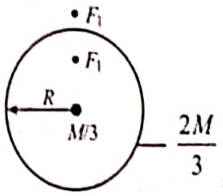
9. (*c*) $g_h=\frac{GMr}{R^3}=$ constant $\Rightarrow d\propto\frac{1}{r}$

10. (*b*) $g = \frac{GM_e}{R_e^2} = \frac{GM_e'}{(5R_e)^2}$

$$\frac{\frac{4}{3}\pi R_e^3 \rho}{R_e^2} = \frac{\frac{4}{3}\pi (5R_e)^3 \rho'}{(5R_e)^2}$$

$\rho = 5\rho'$

11. (*b*) $g' = \frac{G(2M_e)}{(2R_e)^2} = \frac{g}{2}$



$T = 2\pi\sqrt{\frac{l}{g}} = 2$

New time period $T' = 2\pi\sqrt{\frac{l}{g/2}} = 2\sqrt{2}$

12. (*c*) Weight of the body depends on gravitational force acting on it. Gravitational force on a body from surface of earth first decreases, becomes zero and then increases. Thus weight varies accordingly.

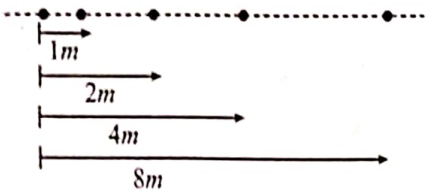
13. (*b*) $dv = -Edr = \frac{k}{r}\,dr$

Integrating both sides

$[v]_{v_i}^{v} = k\,[\ell n\, r]_{d_i}^{r}$

$v - v_i = k\,\ell n\,\frac{r}{d_i}$; $v = v_i + k\,\ell n\,\frac{r}{d_i}$

14. (*d*)



$v = v_1 + v_2 + v_3 + v_4 + ...$

$= -\frac{Gm}{1} - \frac{Gm}{2} - \frac{Gm}{4} - \frac{Gm}{8} - \frac{Gm}{16} - \frac{Gm}{32} - .....$

$= -Gm\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right) \rightarrow$ *G.P.* of infinite series

$= -Gm\left(\frac{1}{1 - 1/2}\right)$

$v = -Gm\,(2) = -2Gm$

15. (*c*) Initial total energy = Initial kinetic energy + initial potential energy

$= \frac{1}{2}m\,(0)^2 + \left(-\frac{GMm}{R_0}\right) = -\frac{GMm}{R_0}$

Total energy, when it reaches the surface of earth

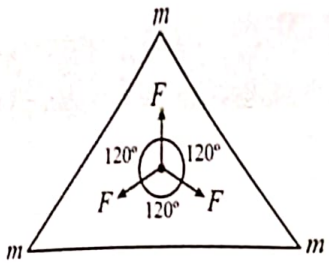
$= \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$

Applying energy conservation,

$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R_0}$

$v = \sqrt{2GM\left\{\frac{1}{R} - \frac{1}{R_0}\right\}}$

16. (*d*)



Due to geometry net force is zero.

17. (*b*) Initial potential energy

$= -\left\{\frac{Gm^2}{a} + \frac{Gm^2}{a} + \frac{Gm^2}{a}\right\} = \frac{-3Gm^2}{a}$

Final $= \frac{-3Gm^2}{2 \times a} = -\frac{3}{2}\frac{Gm^2}{a}$

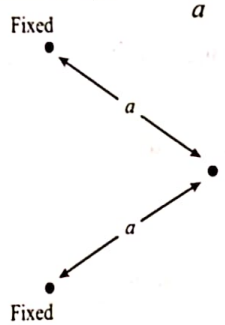
Work done on system = Final potential energy – intial potential energy

$= -\frac{3}{2}\frac{Gm^2}{a} - \left\{-\frac{3Gm^2}{a}\right\}$

$= \frac{3}{2}\frac{Gm^2}{a}$

18. (*b*) Initial kinetic energy = 0

Initial potential energy $= -\frac{Gm^2}{a} - \frac{Gm^2}{a}$



$= -\frac{2Gm^2}{a}$

Total initial energy $= -\frac{2Gm^2}{a}$

Now, kinetic energy $= \frac{1}{2}mv^2$

Potential energy $= -\frac{Gm^2}{a/2} - \frac{Gm^2}{a/2} = -\frac{4Gm^2}{a}$

Total energy $= \frac{1}{2}mv^2 - \frac{4Gm^2}{a} - \frac{4Gm^2}{a} = \frac{1}{2}mv^2$

$\sqrt{\frac{4Gm}{a}} = v$

$v = 2\sqrt{\frac{Gm}{a}}$

19. (*d*) Fact based

20. (*a*) $\frac{1}{2}mv'^2 = 2 \times \frac{1}{2}mv^2$

$v' = \sqrt{2}\,v_0$

$\therefore$ so escape.

21. (a) $g_A = \dfrac{G\frac{4}{3}\pi R_A^3\rho}{R_A^2}$; $g_B = \dfrac{G\frac{4}{3}\pi R_B^3\rho}{R_B^2}$

$R_A = 2R_B$

$\Rightarrow g_A = 2g_B$

$V_{es} = \sqrt{2gR}$

$(V_{es})_A = \sqrt{2g_A R_A} = 2\sqrt{2g_B R_B}$

$(V_{es})_B = \sqrt{2g_B R_B}$

$\dfrac{v_A}{v_B} = 2$

22. (c) $V_e = \sqrt{\dfrac{2GM}{R}}$

$V = KV_e = K\sqrt{\dfrac{2GM}{R}}$

Initial total energy $= \dfrac{1}{2}mv^2 - \dfrac{GMm}{R}$

$= \dfrac{1}{2}m.K^2\dfrac{2GM}{R} - \dfrac{GMm}{R}$

Final velocity = 0

Final total energy $= -\dfrac{GMm}{x}$

Applying energy conservation:

$\dfrac{1}{2}mk^2.\dfrac{2GM}{R} - \dfrac{GMm}{R} = 0 - \dfrac{GMm}{x}$

$\dfrac{1}{x} = \dfrac{1}{R} - \dfrac{k^2}{R}$

$x = \dfrac{R}{1-k^2}$

23. (d) Fact based

24. (b) $W = \dfrac{GmM}{R} - \dfrac{GmM}{nR+R}$

$= \dfrac{GMm}{R}\left[\dfrac{n}{n+1}\right]$

$= mgR\left(\dfrac{n}{n+1}\right)$

25. (d) $\dfrac{GM_P m}{R_P} = 54 \Rightarrow \dfrac{GM_P}{R_P}\times 3 = 54$

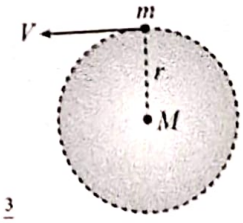
$\dfrac{GM_P}{R_P} = 18$

$v_e = \sqrt{\dfrac{2GM_P}{R_P}} = \sqrt{2\times 18} = 6\,m/\sec$

26. (c) $\dfrac{1}{2}mv^2 = \dfrac{GM_e m}{Re+h} = \dfrac{gRe^2 m}{Re+4Re}$

$\dfrac{1}{2}mv^2 = \dfrac{mgR_e}{5}$

27. (b) $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$

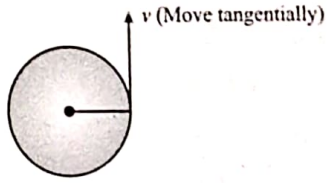


$v = \sqrt{\dfrac{GM}{r^2}}$

$T = \dfrac{2\pi r}{v} = \dfrac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}} = \dfrac{2\pi r^{\frac{3}{2}}}{\sqrt{G\rho\times\frac{4}{3}\pi r^3}}$

$T \propto \dfrac{1}{\sqrt{\rho}}$

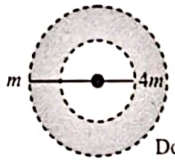
28. (b) Fact based



29. (c) $T = \dfrac{2\pi}{\omega_{rel.}} = \dfrac{2\pi}{2\omega} = \dfrac{2\pi\times 24\times hr.}{2\times 2\pi}$

$T = 12$ hr.

30. (a)



Double star system

$\dfrac{k_1}{k_2} = \dfrac{(1/2)I_1\omega^2}{(1/2)I_2\omega^2} = \dfrac{m_2}{m_1}$

31. (d) $P.E. = -\dfrac{Gm_1m_2}{r}$

$T.E. = -\dfrac{Gm_1m_2}{2r}$

$K.E. = +\dfrac{Gm_1m_2}{2r}$

32. (a) According to kepler's law applying angular momentum conservation $m_1v_1r_1 = m_2v_2r_2$ V_{max} is (a) ans.

33. (c) $\dfrac{1}{2}mv^2 = mgh = \dfrac{mGM}{R^2}\times 90$ ...(a)

$\dfrac{1}{2}mv^2 = \dfrac{mG\left(\frac{1}{10}M\right)}{\left(\frac{R}{3}\right)^2}\times G_1$ ...(b)

from (a) & (b)

$m\dfrac{GM}{R^2}\times 90 = \dfrac{9}{10}\dfrac{mGM}{R^2}\times h_1 \Rightarrow h_1 = 100m$

34. (*a*) $E=-\frac{Gm_1m_2}{2r}$ $\frac{E_B}{E_A}=\frac{v_A}{v_B}=\frac{1.4}{1}$

35. (*c*) $T\propto r^{3/2}\left[\omega=\frac{2\pi}{T}\right]$

$$\omega\propto\frac{1}{r^{3/2}}$$

$$\left(\frac{\omega}{2\omega}\right)=\left(\frac{R_1}{r}\right)^{3/2}$$

$$\frac{R_1}{r}=\frac{1}{(2)^{2/3}}$$

$$R_1=\frac{r}{(4)^{1/3}}$$

36. (b) Fact based

37. (*c*) $v=\sqrt{\frac{Gm}{r}}=\sqrt{\frac{Gm}{R+h}}$

$$v_1=\sqrt{\frac{Gm}{R+\frac{R}{2}}}=\sqrt{\frac{2}{3}}v$$

38. (*c*) Kepler's second law

$$\frac{dA}{dt}=\frac{L}{2M}=\text{constant}$$

39. (*a*) $\frac{A}{T}=\frac{L}{2m}$

$$L=\frac{2mA}{T}$$

40. (*a*) Net force is towards centre

$$F_{net}=F_2+\frac{2F_1}{\sqrt{2}}$$

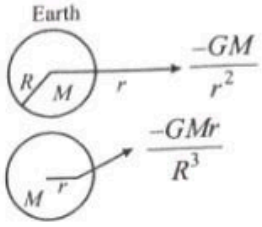
$$=F_2+\sqrt{2}F_1$$

$$\left.\begin{aligned}F_2&=\frac{Gm^2}{(2r)^2}=\frac{Gm^2}{4r^2}\\F_1&=\frac{Gm^2}{(\sqrt{2}r)^2}=\frac{Gm^2}{2r^2}\end{aligned}\right\}\rightarrow F_{net}=\frac{Gm^2}{4r^2}+\frac{\sqrt{2}Gm^2}{2r^2}=\frac{mv^2}{r}$$

$$\Rightarrow v=\sqrt{\frac{Gm}{4r}(1+2\sqrt{2})}$$

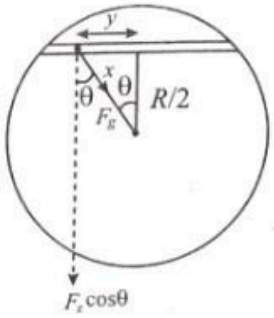
Exercise-3 (JEE Advanced Level)

1. (*a,d*)



2. (*b,c*) $F_g=\frac{GMmx}{R^3}$

pressing force $=F_g\cos\theta=\frac{GMmx\cos\theta}{R^3}$



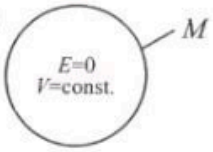
$$=\frac{GMm}{2R^2}=\text{constant.}$$

$$a=\frac{Fg\sin\theta}{m}$$

$$=\frac{GMx\sin\theta}{R^3}$$

$$a=\frac{GMy}{R^3}=\frac{GM}{R^3}\sqrt{x^2-\left(\frac{R^2}{4}\right)}$$

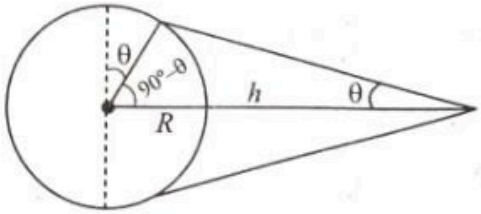
3. (*b,c,d*)



4. (*a,c*) $\sin\theta=\frac{R}{R+h}$

Area covered $=R^2 2\pi\,(1-\cos(90-\theta))$

$=2\pi R^2\,(1-\sin\theta)$



Area escaped $=4\pi R^2-2\pi R^2\,(1-\sin\theta)$

$=2\pi R^2\,(1+\sin\theta)$

5. (a,c,d) inside a uniform spherical shell

$E_{in} = 0$

$V_{in} = \text{constant} = \dfrac{Gm}{R}$

6. (b,c) $\omega_S = \dfrac{2\pi}{1.5}$,

$\omega_E = \dfrac{2\pi}{24}$

$\omega_{\text{west to east}} = 2\pi\left[\dfrac{1}{1.5} - \dfrac{1}{24}\right]$

$T_{\text{west to east}} = \dfrac{2\pi}{\omega_{\text{west to east}}} = 1.6 \text{ hours}$

Similarly

$\omega_{\text{east to west}} = 2\pi\left[\dfrac{1}{1.5} + \dfrac{1}{24}\right]$

$T_{\text{east to west}} = \dfrac{24}{17}$ hours

7. (a,d) In case of earth the gravitational field is zero at infinity as well as the centre and the potential is minimum at the centre.

8. (b,d) Theory

9. (a,d) $T = \dfrac{2\pi}{\sqrt{GM_e}} r^{3/2}$

$g = \dfrac{GM_e}{R_e^2}$ and $T_{\text{min.}}$ at $r_{\text{min.}} = R$

$\Rightarrow T_{\text{min.}} = \dfrac{2\pi R^{3/2}}{\sqrt{gR_e^2}} = 2\pi\sqrt{\dfrac{R}{g}}$

10. (c)

11. (a) $T^2 = \left(\dfrac{4\pi^2}{GM}\right)R^3$

$R = \left(\dfrac{GM}{4\pi^2}\right)^{1/3} T^{2/3}$

$\log R = \dfrac{2}{3}\log T + \dfrac{1}{3}\log\left(\dfrac{GM}{4\pi^2}\right)$

$y = mx + c$

(c) Slope $= m = \dfrac{2}{3}$

intercept $c = \dfrac{1}{3}\log\left(\dfrac{GM}{4\pi^2}\right) = 6$

$\log \dfrac{\left(\dfrac{20}{3}\times 10^{-11}\right)M}{4\times 10} = 18$

$M = 6 \times 10^{29}$ Kg

$T^2 \propto R^3$

12. (a) $\left(\dfrac{R_A}{R_B}\right)^3 = \left(\dfrac{\omega_B}{\omega_A}\right)^2$

$\left(\dfrac{R}{4R}\right)^3 = \left(\dfrac{\omega_B}{\omega_A}\right)^2 \Rightarrow \left(\dfrac{\omega_B}{\omega_A}\right) = \left(\dfrac{1}{8}\right)$

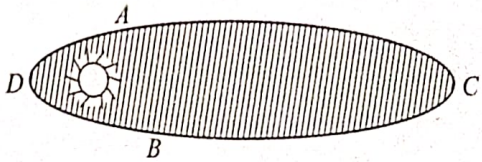
$\omega_{rel} = 8\omega_0 - \omega_0 = 7\omega_0$

$\theta_{rel} = (\omega_{rel})\, t$

$2\pi = (7\omega_0)\, t$

$t = \dfrac{2\pi}{7\omega_0}$

13. (b)



$t_1 > t_2$ because area ACB is greater than ADB.

14. (c) Fact based

15. (a) p, w; (b) r, u; (c) q, v; (d) p, t

I	II	III
A	p	w
B	r	u
C	q	v
D	p	t

$P.E. = -\dfrac{GMm}{r} \Rightarrow K.E. = \dfrac{1}{2}mV^2$

Total energy $= -\dfrac{GMm}{r} + \dfrac{1}{2}mV^2$

$T.E. = 0$ if $-\dfrac{GMm}{r} + \dfrac{1}{2}mV^2 = 0 \Rightarrow v = \sqrt{\dfrac{2GM}{r}}$

For $v < \sqrt{\dfrac{2GM}{r}}$, $T.E.$ is –ve

for $v > \sqrt{\dfrac{2GM}{r}}$, $T.E.$ is +ve

If V is $\sqrt{\dfrac{GM}{r}}$ i.e. equal to orbital velocity, path is circular.

If $T.E.$ is negative, path is elliptical.

If $T.E.$ is zero, path is parabolic.

If $T.E.$ is positive, path is hyperbolic.

16. (a) p, r (b) p, r (c) q, r (d) p, r

(a) At centre of thin spherical shell $V \neq 0, E = 0$.

(b) At centre of solid sphere $V \neq 0, E = 0$.

(c) At centre of spherical cavity inside solid sphere $V \neq 0, E \neq 0$.

(d) At centre of two point masses $V \neq 0, E = 0$.

17. [0008] $v = \sqrt{\dfrac{GM}{R}} = \dfrac{11.2}{\sqrt{2}} \simeq 8$ km/sec.

18. [0004] $\Delta V = 4J/kg$

$$g = \Delta V/\Delta x = \frac{4}{10}$$

$$F = m \times g = 2 \times \frac{4}{10}$$

$$\text{work} = 2 \times \frac{4}{10} \times 5 = 4J$$

19. [0023] $T = \dfrac{2\pi(a)^{3/2}}{\sqrt{GM}}$ so $M = \dfrac{4\pi^2(a)^3}{GT^2}$

Putting values we get

$M = 2 \times 10^{21}$ kg

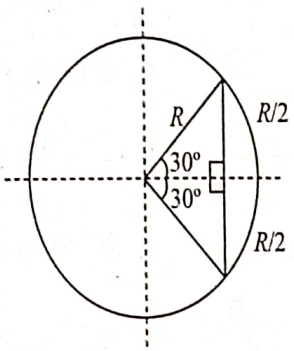
20. [837.33 sec.] The particle will perform SHM.

$$\left(\omega = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{g}{R}}\right)$$

$$\Rightarrow \omega\sqrt{A^2 - \frac{R^2}{4}} = \sqrt{\frac{3gR}{4}}$$

$$\Rightarrow \sqrt{\frac{g}{R}}\sqrt{A^2 - \frac{R^2}{4}} = \sqrt{\frac{3gR}{4}}$$

$$= A^2 - \frac{R^2}{4} = \frac{3}{4}R^2$$



$$A^2 = R^2$$

$$A = R$$

$$t = \frac{\pi}{3\omega} = \frac{\pi}{3}\sqrt{\frac{R}{g}}$$

$$= \frac{3.14}{3} \times 800 \Rightarrow 837.33 \text{ sec.}$$

21. [3000 sec.] $T = \dfrac{2\pi R}{V} = 2\pi\sqrt{\dfrac{R^3}{GM}} = \sqrt{\dfrac{3\pi}{G\rho}}$

$= 3 \times 10^3$ sec $= 3000$ sec.

22. [8] $\dfrac{-GM \times 3M}{d} + \dfrac{1}{2}\mu V_{rel}^2 = 0$

$$\frac{+3GM^2}{d} = \frac{1}{2} \times \frac{3M^2}{4M} \times V_{rel}^2$$

$$\Rightarrow V_{rel}^2 = \frac{8GM}{d}$$

$$V_{rel} = \sqrt{\frac{8GM}{d}}$$

$$\Rightarrow \eta = 8$$

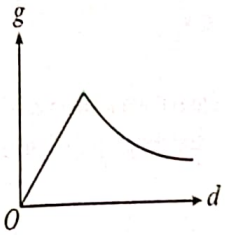
Exercise-4 (Past Year Questions)

JEE MAIN

1. (*b*) $g = \dfrac{GMx}{R^3}$ inside the Earth (straight line)

$g = \dfrac{GM}{r^2}$ outside the Earth

where M is mass of Earth



2. (*d*) $\Delta V = \sqrt{\dfrac{2GM}{R}} - \sqrt{\dfrac{GM}{R}} = (\sqrt{2} - 1)\sqrt{gR}$

3. (*b*) Initially, kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\dfrac{GM_e}{r}$

By conservation of *M.E.*,

$$\frac{-GM_e m}{r} + KE = 0$$

$$KE = \frac{GM_e m}{r} = mv^2$$

4. (*c*) Based on Kepler's law

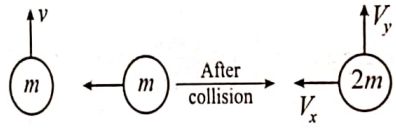
$$\frac{dA}{dt} = \frac{L}{2m}$$

5. (*b*) $KE_A = \dfrac{1}{2}m\left(\dfrac{GM}{R}\right)$

$$KE_B = \frac{1}{2}(2m)\left(\frac{GM}{2R}\right)$$

$$\Rightarrow \frac{KE_A}{KE_B} = 1$$

6. (*b*)



$$2mv_x = mv$$

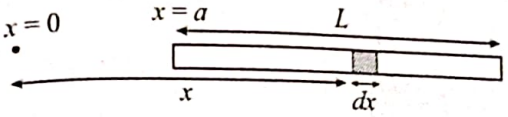
$$\Rightarrow v_x = \frac{v}{2}$$

$$v_y = \frac{V}{2}$$

$$V_{net} = \sqrt{V_{x2} + V_{y2}} = \frac{V}{\sqrt{2}}$$

So, path will be elliptical.

7. (*d*)



Mass of element = $dm = (A + Bx^2)dx$

Field due to element at $x = 0$

$$dE = \frac{G(dm)}{x^2} = \left(\frac{GA}{x^2} + GB\right)dx$$

Total field

$$E = GA\int_a^{a+L} \frac{1}{x^2}dx + GB\int_a^{a+L} dx$$

$$= G\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

So, force = mE

$$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

8. (*d*) $\frac{1}{2}mv^2 + \frac{2(-GMm)}{r} = 0$

$$V^2 = \frac{4GM}{r} = \frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{2 \times 10^{11}}$$

$$V = 20\sqrt{2} \times 10^4 \, m/s$$

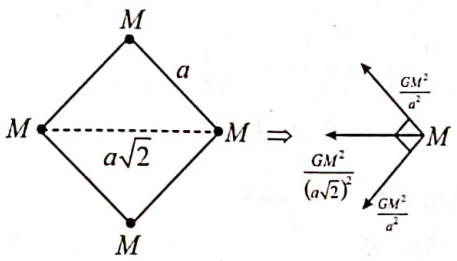
$= 2.828 \times 10^5$ m/s

9. (*d*) $m = \int_0^R \rho 4\pi r^2 dr$

$m = 4\pi KR$; $v \propto \sqrt{4\pi K}$

$$\frac{T}{R} = \frac{2\pi}{\sqrt{4\pi K}}$$

10. (*c*)



Net force on particle towards centre of circle is

$$F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2}$$

$$= \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force. Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

$$r = \frac{a}{\sqrt{2}}, F_C = \frac{mv^2}{r}$$

$$\frac{mv^2}{a/\sqrt{2}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}} + 1\right)$$

$$v^2 = \frac{GM}{a}(1.35)$$

$$v = 1.16\sqrt{\frac{GM}{a}}$$

11. (*b*) $f_g = \frac{mv^2}{r}$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$V = \sqrt{\frac{GMm}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(8 \times 10^{22})}{2.02 \times 10^6}}$$

$V = 1.625 \times 10^3$

$$T = \frac{2\pi r}{V}$$

$n \times T = 24 \times 60 \times 60$

$$n = n\left[\frac{2\pi(2.02 \times 10^6)}{1.625 \times 10^3}\right] = 24 \times 3600$$

$$n = \frac{24 \times 3600 \times 1.625 \times 10^3}{2\pi(2.02 \times 10^6)}$$

$n = 11$

12. (*b*) minimum energy required (E) = – (Potential energy of object at surface of earth)

$$= -\left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

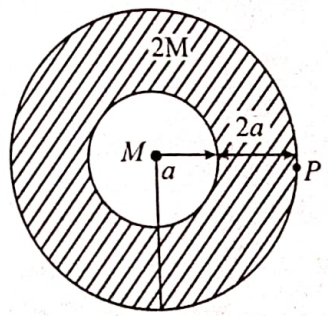
Now $M_{earth} = 64\, M_{moon}$

$$\rho.\frac{4}{3}\pi R_e^3 = 64.\frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$$

Now $\frac{E_{moon}}{E_{earth}} = \frac{M_{moon}}{M_{earth}} \cdot \frac{R_{earth}}{R_{moon}} = \frac{1}{64} \times \frac{4}{1}$

$$\Rightarrow E_{moon} = \frac{E}{16}$$

13. (*b*)



we use gauss's Law for gravitation

$g \cdot 4\pi r^2 = (\text{Mass enclosed})\, 4\pi G$

$$g = \frac{3M4\pi G}{4\pi(3a)^2} = \frac{MG}{3a^2}$$

14. (*b*) Since mass of the object remains same

$\therefore$ Weight of object will be proportional to 'g' (acceleration due to gravity)

Given

$$\frac{W_{earth}}{W_{planet}} = \frac{9}{4} = \frac{g_{earth}}{g_{planet}}$$

Also, $g_{surface} = \frac{GM}{R^2}$ (M is mass planet, G is universal gravitational constant, R is radius of planet)

$$\because \frac{9}{4} = \frac{GM_{earth}R^2_{planet}}{GM_{planet}R^2_{earth}} = \frac{M_{earth}}{M_{planet}} \times \frac{R^2_{planet}}{R^2_{earth}} = 9\frac{R^2_{planet}}{R^2_{earth}}$$

$$\therefore R_{planet} = \frac{R_{earth}}{2} = \frac{R}{2}$$

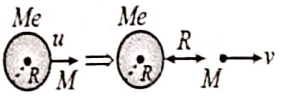
15. (*d*) $\frac{GM}{(R+h)^2} = \frac{GM}{2R^2}$

$R + h = \sqrt{2}R$

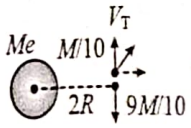
$h = (\sqrt{2} - 1)R$

$\simeq 2.6 \times 10^6$ m

16. (*b*) $\frac{-GMeM}{R} + \frac{1}{2}Mu^2 = \frac{-GMeM}{2R} + \frac{1}{2}Mv^2$



$$v = \sqrt{u^2 - \frac{GM_e}{R}}$$



Vt → Transverse velocity of rocket

Vt → Radial velocity of rocket

$$\frac{M}{10}V_T = \frac{9M}{10}\sqrt{\frac{GM_e}{2R}}$$

$$\frac{M}{10}V_r = M\sqrt{u^2 - \frac{GM_e}{R}}$$

Kinetic energy

$$= \frac{1}{2}\frac{M}{10}\left(V_T^2 + V_r^2\right) = \frac{M}{20}\left(81\frac{GM_e}{2R} + 100u^2 - 100\frac{GM_e}{R}\right)$$

$$= \frac{M}{20}\left(100u^2 - \frac{119GM_e}{2R}\right)$$

$$= 5M\left(u^2 - \frac{119GM_e}{200R}\right)$$

17. (*c*) at pole, weight

$m = 19.6$ kg

at equator

$$= 196 - (19.6)\left[\frac{2\pi}{24 \times 3600}\right]^2 \times 6400 \times 10^3 = 195.33_N$$

18. (*a*) $3 = \frac{Gm_2}{r^2}$

$$2 = \frac{Gm_1}{1^2}$$

$$\therefore \frac{3}{2} = \frac{1}{4}\frac{m_2}{m_1}$$

$$\frac{m_1}{m_2} = \frac{1}{6}$$

19. [16] $KE_i + PE_i = KE_f + PE_f$

$$\frac{1}{2}mu_0^2 + \left(-\frac{GMm}{10R}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$

$$v^2 = u_0^2 + \frac{2GM}{R}\left[1 - \frac{1}{10}\right]$$

$$v = \sqrt{u_0^2 + \frac{9}{5}\frac{GM}{R}}$$

$$= \sqrt{12^2 + \left(\frac{9}{5}\right)\frac{(11.2)^2}{2}}$$

$$= \sqrt{144 + 0.9(11.2)^2} = \sqrt{256.896}$$

$= 16.028$ km/s $\simeq 16$

20. (*d*) $v_e = \sqrt{\frac{2GM}{R}}$

$$\therefore \frac{v_1}{v_2} = \frac{\sqrt{\frac{2GM}{R}}}{\sqrt{\frac{2GM/2}{R/2}}} = 1 = \frac{n}{4}$$

$\Rightarrow n = 4$

21. (*a*) $\frac{GM}{(h+R)^2} = \frac{GM}{R^2}\left(1 - \frac{h}{R}\right)$

$R^3 = (h + R)^2 (R - h)$

Solving $h = \frac{\sqrt{5} - 1}{2}R$

22. (*b*) $\frac{dM}{dV} = \frac{K}{r}$

$$dM = \frac{K}{r} \times dV$$

$$dM = \frac{K}{r} \times 4\pi r^2 dr$$

$$M = \int_0^R \left(\frac{K}{r}\right) 4\pi r^2 dr$$

$M = 2\pi KR^2$

$\because \; m\omega^2 R = \dfrac{GMm}{R^2}$

$\omega^2 R = \dfrac{GM}{R^2}$

$\omega^2 R = 2G\pi K;\ \omega^2 = \dfrac{2G\pi K}{R}$

$\omega^2 \propto \dfrac{1}{R};\ T^2 \propto R$

23. (*c*) $\rho(r) = \rho_0\left(1 - \dfrac{r^2}{R^2}\right)$

$\dfrac{dm}{dV} = \rho_0\left(1 - \dfrac{r^2}{R^2}\right)$

$dm = \rho_0\left(1 - \dfrac{r^2}{R^2}\right)dV$

$\therefore\ m = \int_0^r \rho_0\left(1 - \dfrac{r^2}{R^2}\right) \times 4\pi r^2 dr$

$= 4\pi\rho_0\left(\dfrac{r^3}{3} - \dfrac{r^5}{5R^2}\right)$

$\therefore\ E = \dfrac{Gm}{r^2} = \dfrac{G4\pi\rho_0}{r^2}\left[\dfrac{r^3}{3} - \dfrac{r^5}{5R^2}\right] = G4\pi\rho_0\left[\dfrac{r}{3} - \dfrac{r^3}{5R^2}\right]$

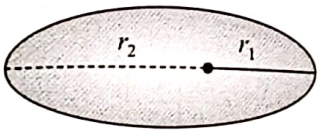
$\therefore\ \dfrac{dE}{dr} = 0 \Rightarrow r = \sqrt{\dfrac{5}{9}}R$

24. (*a*) Total energy $E = \dfrac{-GMm}{2a}$

$\therefore\ \dfrac{1}{2}m \times \left(\dfrac{2}{3}\dfrac{GM}{R}\right) - \dfrac{GMm}{R} = \dfrac{-GMm}{2a}$

$\Rightarrow a = 2R$

Major axis = $4R$



$r_1 = R_2 \Rightarrow r_2 = 3R$

25. (*d*) $v_0 = \sqrt{\dfrac{GM}{R}}$

$v_e = \sqrt{\dfrac{2GM}{R}}$

$\dfrac{v_0}{v_e} = \dfrac{1}{\sqrt{2}}$

26. (*c*) $\int_0^v dV = -\int_\infty^x E.dx$

$V = -A\int_\infty^x \dfrac{xdx}{(x^2 + a^2)^{3/2}}$

$V = \dfrac{A}{(x^2 + a^2)^{1/2}}$

27. (*a*) $g_{equator} = g - \omega^2 R$

$g_h = g_{pole}\left(1 - \dfrac{2h}{R}\right) = g\left(1 - \dfrac{2h}{R}\right) = g - g\left(\dfrac{2h}{R}\right)$

$\because\ g_h = g_{equator}$

$g - g\left(\dfrac{2h}{R}\right) = g - \omega^2 R$

$g\left(\dfrac{2h}{R}\right) = \omega^2 R$

$h = \dfrac{\omega^2 R^2}{2g}$

28. (*a*) $g_h = g_1 = g_s\left(\dfrac{R}{R+h}\right)^2 = g_s\left(\dfrac{R}{R + \frac{R}{2}}\right)^2 = g_s\left(\dfrac{R}{\frac{3R}{2}}\right)^2 = \dfrac{4}{9}g_s$

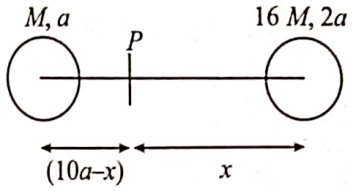
$\because\ g_d = g_1 = g_s\left(1 - \dfrac{d}{R}\right)$

$g_h = g_d$

$\dfrac{4}{9}g_s = g_s\left(1 - \dfrac{d}{R}\right)$

$d = \dfrac{5R}{9}$

29. (*a*)



Velocity should be given so as to reach a point where field is zero.

$\dfrac{16M}{x^2} = \dfrac{M}{(10a - x)^2}$

$x = 8a$

By *CoE*

$-\dfrac{GMm}{8a} - \dfrac{16GMm}{2a} + \dfrac{1}{2}mv^2 = -\dfrac{16GMm}{8a} - \dfrac{GMm}{2a} + 0$

$\dfrac{1}{2}mv^2 = \dfrac{45GMm}{8a}$

$v = \dfrac{3}{2}\sqrt{\dfrac{5GM}{a}}$

30. (*a*) $\dfrac{V_{min}}{V_{max}} = \dfrac{(1-e)}{(1+e)} = \dfrac{1}{6}$

$\dfrac{r_{min}}{r_{max}} = \dfrac{(1-e)}{(1+e)} = \dfrac{1}{6}$

31.,[64] Escape velocity of a planetary system is given by,

$V_{es} = \sqrt{\dfrac{2CM}{R}}$ Where, M is the mass of the planet, R is the radius of the planet

$\Rightarrow V_{es}\sqrt{R}$ = const

$\therefore\ V_{es}\cdot\sqrt{R} = 10V_{es}\sqrt{R'} \Rightarrow R' = \frac{R}{100} = 64$ km

Hence, Thus, the radius in kilometres to which the present radius of the earth (R = 6400 km) is to be compressed so that the escape velocity is increased to ten times is 64 km.

32. [3] According to question the given data is $\frac{x}{5}\frac{GM^2}{R}$ ...(i)

Energy given = $U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right) = \frac{3}{5}\frac{GM^2}{R} \quad ...(ii)$$

Comparing eq.(i) to eq.(ii) we get $x = 3$

33. (*b*) As mass 'M' remains same

$$\Rightarrow g' = \frac{g}{3}$$

Now, $g' = g\left(\frac{R}{R+h}\right)^2$

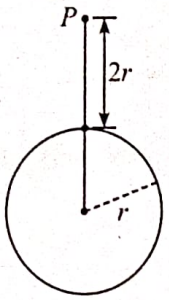
$$\Rightarrow \frac{g}{3} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{\sqrt{3}} \Rightarrow h = 4685 \text{ km}$$

34. (*d*) $T^2 \propto R^3$

If $R_2 = 3R_1$

$$\Rightarrow \frac{T_2}{T_1} = (3)^{\frac{3}{2}} \Rightarrow T_2 = 3\sqrt{3}T_1 = 3\sqrt{3} \text{ year}$$

35. (*d*)



$$g = \frac{GM}{r^2}$$

Now, $g' = \frac{GM}{(3r)^2} = \frac{g}{9}$

36. [2] $\frac{v_1}{v_2} = \frac{\sqrt{\frac{GM}{R_1}}}{\sqrt{\frac{GM}{R_2}}} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$

37. (*c*) Since $T^2 \propto R^3$

$$\Rightarrow \frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3} \text{ or } r_A^3 = 4r_B^3$$

JEE ADVANCED

38. (*b,d*) $V_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2.G\rho.\frac{4}{3}\pi R^3}{R}} = \sqrt{\frac{4G\rho}{3}}R$

$V_{es} \propto R$

Surface area of $P = A = 4\pi R_P^2$

Surface area of $Q = 4A = 4\pi R_Q^2$

$\Rightarrow R_Q = 2R_P$

mass R is $M_R = M_P + M_Q$

$$\rho\frac{4}{3}\pi R_R^3 = \rho\frac{4}{3}\pi R_P^3 + \rho\frac{4}{3}\pi R_Q^3$$

$$\Rightarrow R_R^3 = R_P^3 + R_Q^3$$

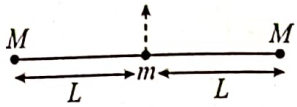
$$= 9R_P^3$$

$R_R = 9^{1/3}R_P \Rightarrow R_R > R_Q > R_P$

Therefore $V_R > V_Q > V_P$

$$\frac{V_R}{V_P} = 9^{1/3} \text{ and } \frac{V_P}{V_Q} = \frac{1}{2}$$

39. (*b,d*) $-\frac{GM.2m}{L} + \frac{1}{2}mv^2 = 0 + 0$



$$\Rightarrow v = \sqrt{\frac{4GM}{L}}$$

40. (*b*) Given, $R_{planet} = \frac{R_{earth}}{10}$ and

density, $\rho = \frac{M_{earth}}{\frac{4}{3}\pi R_{earth}^3} = \frac{M_{Planet}}{\frac{4}{3}R_{planet}^3}$

$$\Rightarrow M_{planet} = \frac{M_{earth}}{10^3}$$

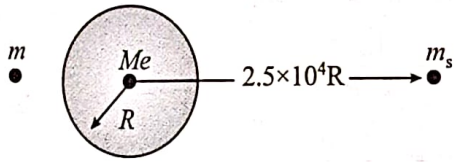
$$g_{surface\ of\ planet} = \frac{GM_{planet}}{R_{planet}^2} = \frac{GM_e.10^2}{10^3.R_e^2}$$

$$= \frac{GM_e}{10R_e^2} = \frac{g_{surface\ of\ earth}}{10}$$

$g_{depth\ of\ planet} = g_{surface\ of\ planet}\left(\frac{x}{R}\right)$ where x = distance from centre of planet

$$T = \int_{4R/5}^{R} \lambda\, dx\, g\left(\frac{x}{R}\right) = \frac{\lambda g}{R}\left[\frac{x^2}{2}\right]_{4R/5}^{R} = 108N$$

41. (*c*)



Given $\sqrt{\frac{2GM_e}{R}} = 11.2 km/s$

$$\frac{1}{2}mv^2 - \frac{GmM_e}{R} - \frac{GM_s m}{2.5\times10^4} \geq 0$$

for $v = v_e$

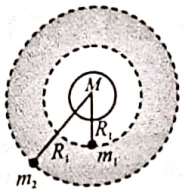
$$v_e^2 = \frac{2GM_e}{R} + \frac{2GM_s}{2.5\times10^4 R_e}$$

$$= \frac{2GM_e}{R} + \frac{6\times10^5 GM_e}{2.5\times10^4 R}$$

$$= \frac{GM_e}{R}(2+24)$$

$$= \sqrt{\frac{26GM_e}{R}} = 40.4 km/\sec.$$

42. (*b*)



$$\left.\begin{array}{l}\frac{m_1}{m_2}=2\\ \frac{R_1}{R_2}=4\end{array}\right\}given$$

$$\frac{GMm_1}{R_1^2} = \frac{m_1 v_1^2}{R_1}$$

$$v_1^2 = \frac{GM}{R_1},\ v_2^2 = \frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2} = \frac{R_2}{R_1} = 4$$

(A) $\frac{v_1}{v_2} = 2$

(B) $L = mvR$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_2 v_2 R_2} = 2\times 2\frac{1}{4} = 1$$

(C) $K = \frac{1}{2}mv^2$

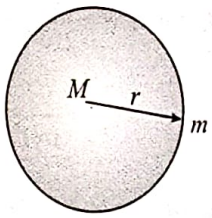
$$\frac{K_1}{K_2} = \frac{m_1 v_1^2}{m_2 v_2^2} = 2\times(2)^2 = 8$$

(D) $T = 2\pi R/V$

$$\frac{T_1}{T_2} = \frac{R_1}{v_1}\times\frac{v_2}{R_2} = \frac{R_1}{R_2}\times\frac{v_2}{v_1} = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

43. (*d*) Let total mass included in a sphere of radius r be M. For a particle of mass M,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$



$$\Rightarrow \frac{GMm}{r} = 2K$$

$$\Rightarrow M = \frac{2Kr}{GM}$$

$$\therefore dM = \frac{2Kdr}{GM}$$

$$\Rightarrow (4\pi r^2 dr)\rho = \frac{2Kdr}{GM}$$

$$\Rightarrow \rho = \frac{K}{2\pi r^2 GM}$$

$$\therefore n = \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G}$$

44. [9] Circular orbits

$$T = 2\pi\sqrt{\frac{R^3}{GM_S}}$$

Binary stars

$$nT = 2\pi\sqrt{\frac{(9R)^3}{G(3M_S+6M_S)}}$$

$$n\times 2\pi\sqrt{\frac{R^3}{GM_S}} = 9\times 2\pi\sqrt{\frac{R^3}{GM_S}}$$

45. [2.3] Given $M_B = 2M_A$

As radius is $\Rightarrow$ value same $\Rightarrow \rho_B = 2\rho_A$

Now after transfer of mass

$$M'_A = \rho_A \frac{4}{3}\pi\left(\frac{R^3}{8}\right) = \frac{M_A}{8}$$

and $M'_B = 2M_A + \frac{7M_A}{8} = \frac{23M_A}{8}$

Now volume of mass lost by star A = Volume added to Star B

$$\Rightarrow \frac{4}{3}\pi\left(R^3 - \left(\frac{R}{2}\right)^3\right) = \frac{4}{3}\pi\left(R_2^3 - R^3\right)$$

$$\Rightarrow R^2 = 15^{\frac{1}{3}}\frac{R}{2}$$

Now, $V \propto \sqrt{\frac{M}{R}}$

$$\Rightarrow \frac{V_B}{V_A} = \sqrt{\frac{M_B}{R_B}\times\frac{R_A}{M_A}} = \sqrt{\frac{23M_A}{8\times 15^{\frac{1}{3}}\frac{R}{2}}\times\frac{R}{\frac{M_A}{8}\times 2}}$$

$$\Rightarrow \frac{V_B}{V_A} = \sqrt{\frac{10\times 2.3}{15^{\frac{1}{3}}}}$$

$$\Rightarrow n = 2.3$$

CHAPTER

13 Mechanical Properties of Solids

Exercise-1 (Topicwise)

1. (*d*) An elastic rod will change its length when it is polled along its length by a force acting at one end or rotates about an axis at one end, but when the rod freely falls vertically under gravity there is no deforming force acting on it.

2. (*c*) $l = \frac{FL}{YA} \Rightarrow l \propto \frac{1}{A}$

3. (*b*)

Force same ⇒ stress same

4. (*c*) $l = \frac{FL}{AY} \Rightarrow l \propto \frac{L}{d^2} \Rightarrow \frac{l_1}{l_2} = \frac{L_1}{L_2} \times \left(\frac{d_2}{d_1}\right)^2 = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$

5. (*c*) $l = \frac{mgL}{AY} = \frac{1 \times 10 \times 1.1}{1.1 \times 10^{11} \times 10^{-6}} \text{m} = 0.1 \text{mm}$

6. (*a*) Because due to increase in temperature intermolecular forces decreases.

7. (*c*) Breaking Force ∝ Area of cross section of wire (πr^2). If radius of wire is double then breaking force will become four times.

8. (*a*) In the region *OA*, stress ∝ strain i.e. Hooke's law hold good.

9. (*a*) Force $\propto \frac{1}{\text{area of cross section}}$

$\therefore \frac{F_1}{F_2} = \frac{A_2}{A_1}$

Given, $F_1 = 400$ kg-wt, and $A_2 = 2A_1$

$\therefore \frac{F_2}{400} = \frac{2A_1}{A_1}$ or $F_2 = 400 \times 2 = 800$ kg-wt.

10. (*d*) As stress is shown on *x*-axis and strain on *y*-axis

So we can say that $Y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$

So elasticity of wire *P* is minimum and of wire *R* is maximum

11. (*b*) $\frac{F}{A} = Y \times \frac{\ell}{L}$ $F = \frac{YA}{L}\ell = K\ell$ $K = \frac{YA}{L}$

12. (*b*) Since $T_B > T_A$

$\Rightarrow \Delta L_B > \Delta L_A \Rightarrow (L_B)_i < (L_A)_i$

13. (*b*) Internal force in the wire due to strain in providing the necessary centripetal force

$\Rightarrow F = \frac{YA\Delta L}{L} = m\omega^2 L$

$\Rightarrow Y = \frac{m\omega^2 L^2}{A\Delta L}$

14. (*a*) Young's modulus is independent of the dimensions of material

15. (*c*) $Y = \frac{F \times L}{A \times \Delta L} \Rightarrow F = \frac{YA\Delta L}{L}$

$\therefore F = \frac{11.9 \times 10^{12} \times 5 \times 10^{-6} \times 5 \times 10^{-4}}{4}$

$= \frac{297.5 \times 10^2}{4} = 74.375 \times 10^2 \text{N}$

$F = 7.4 \times 10^{+3} \text{N}$

16. (*d*) $Y = \frac{F \times L}{A \times \Delta L} = \frac{500 \times 10 \times 0.7}{10 \times 10^{-6} \times 0.5 \times 10^{-2}}$

$= \frac{3500}{5 \times 10^{-8}} = 7 \times 10^{10} \text{N/m}^2$

17. (*b*) $F = \frac{YA\Delta L}{L}$ $F \propto \frac{r^2}{L}$

$\therefore \frac{F_B}{F_A} = \frac{r_B^2}{(r_A)^2} \times \frac{L_A}{L_B} = \left(\frac{1}{5}\right)^2 \times \frac{3}{4} = \frac{3}{4} \times \frac{1}{25} = 0.03$

18. (*d*) $\Delta\ell \propto \frac{\ell}{r^2}$

19. (*c*) $V = \pi r^2 L$

$dV = \pi r^2 dL - 2\pi r L dr$

20. (*d*) $Y = \frac{k}{r_0} = \frac{7}{3 \times 10^{-10}} = 2.33 \times 10^{10} \text{ N/m}^2$

21. (*a*) $Y = \frac{F.L}{A.\Delta\ell}$

for same Δl. wire A required more load.

So $Y_A > Y_B$

22. (*b*) Stress $= \frac{F}{A} = \frac{Mg}{A}$

$= \frac{A \times L \times \rho \times g}{A} = L\rho g \ (\rho = D)$

Stress = LDg.

$Strain = \frac{\Delta L}{L} = \frac{l}{L/2} = \frac{2l}{L}$

$Y = \frac{LDg}{2l} \times L \Rightarrow l = \frac{L^2 Dg}{2Y}$

23. (*b*) As we know $Y = \frac{FL}{A \times \Delta L}$

$F = \frac{YA \times \Delta L}{L} = \frac{YA(2\pi R - 2\pi r)}{2\pi r}$

$\therefore \ F = YA\left(\frac{R-r}{r}\right)$

24. (*d*) $Y = \frac{F \times L}{A \times \Delta L} \Rightarrow F = \frac{Y \times A \times \Delta L}{L}$

$= \frac{2 \times 10^{11} \times 100 \times (3L - L)}{L}$

$\therefore \ F = 2 \times 10^{11} \times 100 \times 2 = 4 \times 10^{13}$ N

25. (*b*) Fact based

26. (*c*) Fact based

27. (*d*) $Y = 2\eta\,(1 + \sigma)l$.

$2.4\,\eta = 2\eta\,(1 + \sigma) \Rightarrow 1.2 = 1 + \sigma \Rightarrow \sigma = 0.2$

28. (*c*) Fact based

29. (*c*) $L = 3$ cm ; $\Delta L = 0.3$ cm; $\sigma = 0.26$

longitudinal strain $\frac{\Delta L}{L} = \frac{0.3}{3} = 0.1$

$\sigma = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$

The Lateral Strain = σ × Longitudinal Strain

= 0.26 × 0.1 = 0.026

30. (*d*) $Y = \frac{9kn}{n + 3k}$

31. (*d*) $Y = 3k\,(1 - 2\sigma)$

32. (*d*) $Y = 2G\,(1 + \sigma)$

33. (*c*) $\frac{\Delta V}{V} = (1 - 2\sigma)\frac{\Delta L}{L} \Rightarrow \frac{\Delta V}{V} = (1 - 2 \times 0.30)3 \times 10^{-4}$

$\therefore \frac{\Delta V}{V} = 0.4 \times 3 \times 10^{-4} = 1.2 \times 10^{-4} \Rightarrow \frac{\Delta V}{V} = 0.01 \times 10^{-2}$

$\therefore \frac{\Delta V}{V} \times 100 = 1.2 \times 10^{-4} \times 100 = 0.012\%$

34. (*c*) Isothermal elasticity $K_i = P$

35. (*c*) Adiabatic elasticity $K\alpha = \gamma P$

36. (*d*) Fact based

37. (*a*) $B = \frac{F/A}{\Delta V/V} = \frac{P}{\frac{\Delta V}{V}} = \frac{\rho g h}{\frac{0.1}{100}}$

38. (*a*) Change in pressure, ΔP = 501 atm – 1 atm

= 500 atm = 500 × 1.01 × 10^5 Pa

Let p' be the density of water at the given depth. If V and V are volume of certain mass M of ocean water at surface and at a given depth,

As $B = \frac{\Delta P \times V}{\Delta V}, \frac{\Delta V}{V} = \frac{\Delta P}{B}$

or $\frac{\Delta V}{V} = \frac{(500 \times 1.01 \times 10^5)}{2.0 \times 10^9} = 2.525 \times 10^{-2}$

Further, $\frac{\Delta V}{V} = \frac{(M/\rho) - (M/\rho')}{(M/\rho)} = 1 - \frac{\rho}{\rho'}$

(as volume = mass/density)

$\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}$ or $\rho' = \frac{\rho}{1 - \frac{\Delta V}{V}}$

$\rho' = \frac{1.0 \times 10^3}{1 - 2.525 \times 10^{-2}} = 1.025 \times 10^3$ kg/m^3

39. (*d*) $B = \frac{P}{\frac{-\Delta V}{V}} \Rightarrow B = \frac{h\rho g}{0.4 \times 10^{-2}}$

$h = \frac{9 \times 10^8 \times 0.4 \times 10^{-2}}{10^3 \times 10} = 36 \times 10^1$ m

40. (*d*) $B = \frac{P}{\frac{\Delta V}{V}} \Rightarrow \frac{1}{B} = \frac{\left(\frac{\Delta V}{V}\right)}{P} \Rightarrow C = \frac{\left(\frac{\Delta V}{V}\right)}{P}$

41. (*d*) Decrease in volume $\Delta V = \frac{0.004}{100} \times 200 = 0.008$

$\Delta P = K \times \frac{\Delta V}{V} = 2100 \times 10^6 \times \frac{0.008}{200} = 84$ kPa

42. (*b*) Poisson's ratio $\sigma = \frac{-\Delta r}{r} \times \frac{l}{\Delta l} \Rightarrow \frac{\Delta r}{r} = -\sigma\frac{\Delta l}{l}$ ...(i)

Volume = Area × length

$V = \pi r^2 l$

$\therefore \log V = \log \pi + 2\log r + \log l$

$\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + \frac{\Delta l}{l}$

$\frac{\Delta V}{V} = 2\left(\frac{-\sigma \Delta l}{l}\right) + \frac{\Delta l}{l}$ (from equation 1)

$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta l}{l}(1 - 2\sigma)$

(Poisson's ratio is constant for metal or material)

$\Rightarrow \frac{\Delta V}{V} \propto \frac{\Delta l}{l}$

43. (*b*) $\frac{\text{Elastic energy}}{\text{Volume}} = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta L}{L}$ ...(i)

$Y = \frac{F}{A} \times \frac{L}{\Delta L}$

$\Delta L = \frac{FL}{AY}$ ...(ii)

Put the value of (ii) in (i) equation

$\Rightarrow \frac{1}{2} \times \frac{F}{A} \times \frac{FL}{AYL} \Rightarrow \frac{F^2}{2A^2Y}$

$\frac{U}{V} = \frac{1}{2} \times \text{stress} \times \text{strain}$

$\frac{U}{AL} = \frac{1}{2} \times \frac{F}{A} \times \frac{FL}{AYL} \Rightarrow U = \frac{F^2L}{2AY}$

44. (*a*) Area of hysteresis loop gives the energy loss in the process of stretching and unstretching of rubber band and this loss will appear in the form of heating.

45. (*b*) $v = \frac{1}{2}kx^2 = \frac{1}{2} \times mg \times x = \frac{1}{2} \times 200 \times \frac{1}{1000} = 0.1\,J$

46. (*b*) $W = \frac{1}{2} \times F \times l = \frac{1}{2}mgl = \frac{1}{2} \times 10 \times 10 \times 1 \times 10^{-3} = 0.05\,J$

47. (*d*) Due to tension, intermolecular distance between atoms is increased and therefore potential energy of the wire is increased and with the removal of force interatomic distance is reduced and so is the potential energy. This change in potential energy appears as heat in the wire and thereby increases the temperature.

48. (*a*) Increase in energy $= \frac{1}{2} \times 20 \times 1 \times 10^{-3} = 0.01\,J$

49. (*a*) $\frac{\tau}{r} = \frac{\eta\alpha}{L} \Rightarrow E = \frac{\tau^2}{4\eta} \times \text{Volume}$

Exercise-2 (Learning Plus)

1. (*c*) $\frac{T}{A} = \gamma \frac{\Delta l}{l}$

Tension is different

Hence strain is different.

2. (*d*) $2T(\Delta\theta/2) = N\cos 45° \Rightarrow \quad T = mg/2\pi$

$\Delta l = \frac{T(2\pi R)}{aY}$

3. (*a*) $d = 4$ mm

$Y = 9 \times 10^{10}$ N/m^2

$\frac{F}{A} = Y\frac{\Delta\ell}{\ell}$

$F = AY\frac{\Delta\ell}{\ell} = \pi(2\times10^{-3})^2 \times 9 \times 10^9 \times \frac{1}{100}$

$= \pi \times 4 \times 10^{-6} \times 9 \times 10^7 = 360\,\pi$N

4. (*a*) $\Delta\ell = \frac{F\ell_0}{Ay} \Rightarrow \frac{\ell_0}{A} = \frac{\ell_0'}{A'}$

$\frac{0.6}{15} = \frac{0.4}{A'} \Rightarrow A' = 10$ cm^2

Stress $= \frac{F}{A} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{A_2}{A_1} = \frac{10}{15}$

$\frac{\Delta r}{r} = -\sigma\frac{\Delta\ell}{\ell}$

$\Delta r = -\sigma\frac{\Delta\ell}{\ell} \times r = -\sigma\delta\ell\frac{\sqrt{A}}{\ell}$

$\Delta r_{\text{left}} = \frac{\sqrt{15}}{0.6} = \sqrt{\frac{1500}{0.36}}$

$\Delta r_{\text{right}} = \frac{\sqrt{10}}{0.4} = \sqrt{\frac{1000}{0.16}}$

5. (*b*) $\frac{\Delta\ell_{\text{steel}}}{\Delta\ell_{\text{brass}}} = \frac{\frac{3Mg}{A_s y_s}\ell_s}{\frac{2Mg}{A_c y_c}\ell_c} = \frac{3}{2}\frac{\ell_s}{\ell_c}\frac{A_c}{A_s}\frac{y_c}{y_s} = \frac{3}{2}\frac{a}{b^2c}$

6. (*d*) $Y = \frac{yA\ell}{L} \Rightarrow l = \frac{FL}{yA}\ell \propto \frac{L}{A} \propto \frac{L}{d^2}$

$\therefore$ As F and Y are constant

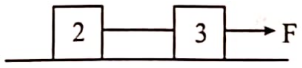
The ratio of $\frac{L}{d^2}$ is maximum for case (*d*)

7. (*d*) Stress $= \frac{F}{A}$ $\quad F = ma$

$a = \frac{1}{3}g \quad F = \frac{4}{3}mg$

Stress $= \frac{4\,mg}{3A}$

8. (*d*)



$F = 5a$

$a = \frac{F}{5}$

$T = 2a\,\frac{2F}{5} = \sigma_b A$

$= 2.5 \times 10^9 \times 0.04 \times 10^{-6} = 100$ N

$F = 250$ N

9. (*a*) $T = YA\frac{\Delta l}{l} \Rightarrow \Delta l = \frac{Tl}{YA}$

$\frac{\Delta l_1}{\Delta l_2} \times \frac{T_1}{T_2} \times \frac{l_1}{l_2} \times \frac{Y_2}{Y_1} \times \frac{A_2}{A_1} = \frac{3mg}{2mg} \times \frac{2}{1} \times \frac{1}{3} \times \frac{1}{4} = 1:4$

10. (*c*) Tension in both the rods is same $= W$

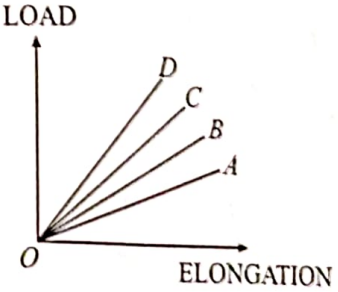
$P.E. = \frac{1}{2}\frac{W^2L}{AY}$

$\therefore$ Total $P.E. = \frac{1}{2} \times \frac{W^2L}{\pi R^2 Y} + \frac{1}{2} \times \frac{W^2L}{4\pi R^2 Y} = \frac{5W^2L}{8\pi R^2 Y}$

11. (c) $\frac{F/A}{\Delta\ell/\ell} = Y$

$\frac{F}{\Delta\ell} = \frac{Y\pi r^2}{\ell}$

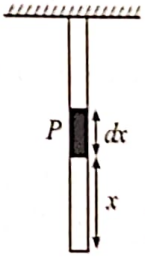
$\Rightarrow \frac{F\ell}{Y\pi} \times \frac{1}{\Delta\ell} = r^2$



Y & ℓ are same for all then

For same load $r \alpha \frac{1}{\sqrt{\Delta\ell}}$

12. (b) Tension at point P in the rod is $T = \frac{m}{\ell} \times g$



Extension of width dx length of rod is

$= \frac{Tdx}{Y\pi r^2} = \frac{mg}{Y\pi r^2 \ell} \times dx$

Total extension in rod $= \int_0^{\ell} \frac{mg}{Y\pi r^2 \ell} \times dx = \frac{mg\ell}{2\pi r^2 Y}$

13. (a) $\Delta l = \frac{F\ell}{AY}\frac{\Delta\ell}{(F/A)} = \frac{\ell}{Y}$ = slope of curve

$\frac{\ell}{Y} = \frac{(4-2)\times 10^{-3}}{4000\times 10^3}$

Given $l = 1$ m

$Y = \frac{4000\times 10^3}{2\times 10^{-3}} = 2 \times 10^9$ N/m^2

14. (a) Tension in wire-1 is same as the tension in wire-2.

15. (d) $\frac{F\times l}{A\times Y} + \frac{2F\times l}{A\times Y} = \frac{3Fl}{Ay}$

16. (d) $\eta = \frac{\tau L}{\theta\pi r^4}$ So, $\theta = \frac{\tau L}{\eta\pi r^4}$

17. (b) Angle of shear $\phi = \frac{r\theta}{L} = \frac{4\times 10^{-1}}{100} \times 30^\circ = 0.12^\circ$

18. (a) Strain $= \frac{\Delta l}{l} = \frac{5\times 10^7}{2\times 10^{11}} = 2.5\times 10^{-4}$

Volume $= V = Al$

$\Delta V = A\Delta l + l\Delta A$

$\frac{\Delta V}{v} = \frac{\Delta l}{l} + \frac{\Delta A}{A}$ $\quad 2\times 10^{-4} = 2.5\times 10^{-4} + \frac{\Delta A}{A}$

$\frac{\Delta A}{A} = 0.5\times 10^{-4} = 2\frac{\Delta r}{r}$

$\left|\frac{\Delta r}{r}\right| = 0.25\times 10^{-4}$

19. (c) $\frac{\Delta V}{V} = \frac{\Delta P}{B} = \frac{1\times 10^5}{1.25\times 10^{11}} = 8\times 10^{-7}$

20. (c) On heaving volume of substance increases while mass of the substance remains the same. Hence the density will decrease

21. (b) $B = \frac{\Delta p}{\Delta V/V}$ $l. \Rightarrow \frac{1}{B} \propto \frac{\Delta V}{V}$ [Δp = constant]

22. (d) $K = \frac{AY}{\ell}$, $K' = \frac{4AY}{\ell/2} = 8K$

$\frac{U}{2} = \frac{\frac{1}{2}\times 8K\times\Delta\ell^2}{\frac{1}{2}\times K\times\Delta\ell^2} \Rightarrow U = 16\,J$

23. (d) $V = \frac{1}{2}\left(\frac{YA}{L}\right)l^2$

$\therefore V \propto l^2$

$\frac{V_2}{V_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{10}{2}\right)^2 = 25 \Rightarrow V_2 = 25V_1$

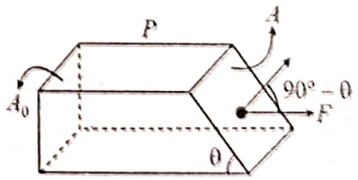
i.e. potential energy of the spring will be 25 V

24. (a) $W = \frac{1}{2}Fl$

$\therefore W \propto l$ (F is constant)

$\frac{W_1}{W_2} = \frac{l_1}{l_2} = \frac{l}{2l} = \frac{1}{2}$

25. (a) Normal force $F_n = F\sin\theta$



Area $S = \frac{S_0}{\sin\theta}$

$\therefore \sigma_n = \frac{F_n}{S} = \frac{F\sin\theta}{\frac{S_0}{\sin\theta}} = \frac{F}{S_0}\sin^2\theta \Rightarrow \frac{F_{max}\sin^2\theta}{S_0} = \sigma_0$

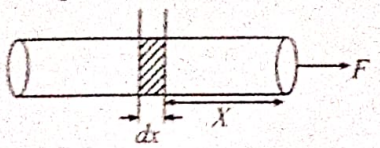
$\therefore F_{max} = \frac{\sigma_0 S_0}{\sin^2\theta}$

26. (c) At P_2, stress $= S_2 = \frac{F}{A}$

At P_1, stress $= S_1 = \frac{F\cos 60^\circ}{\frac{A}{\cos 60^\circ}} = \frac{F}{4A}$

$\Rightarrow \frac{S_1}{S_2} = \frac{1}{4}$

27. (c)



$a = \frac{F}{m}$

$T(x) = \frac{m}{L}(L-x)\times\frac{F}{m}$

Stress $= \frac{F}{A} = \frac{F(L-x)}{LA}$

Strain $= \frac{\text{Stress}}{Y} = \frac{F(L-x)}{YAL}$

$\frac{dL}{dx} = \frac{F(L-x)}{YAL}$

$\int dL = \frac{F}{YAL}\int_0^L (L-x)dx \Rightarrow \Delta L = \frac{F}{YAL}\times\frac{L^2}{2} = \frac{FL}{2YA}$

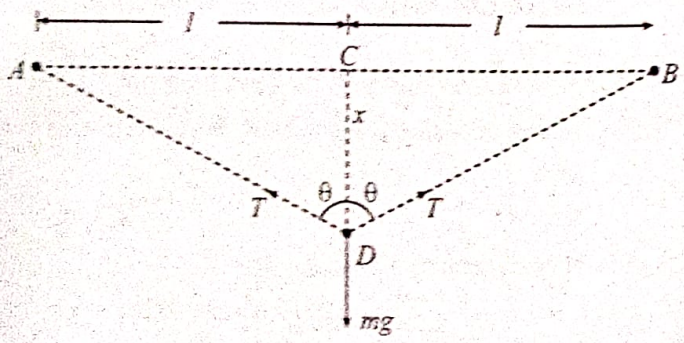
28. (b) $T_{(x)} = \frac{m\omega^2}{2l}(l^2-x^2), T_A = \frac{m\omega^2 l}{2}, T_B = \frac{3m\omega^2 l}{8}$

29. (a) Given $m = 100\ g = 0.100$ kg

Refer figure, let x be the depression at the mid-point i.e., $CD = x$

In figure, $AC = CB = l = 0.5$ m

$AD = BD = (l^2+x^2)^{1/2}$



Increase in length, $\Delta l = AD + DB - AB = 2AD - AB$

$= 2(l^2+x^2)^{1/2} - 2l = 2l\left(1+\frac{x^2}{l^2}\right)^{1/2} - 2l$

$= 2l\left[1+\frac{x^2}{2l^2}\right] - 2l = \frac{x^2}{l}$

Strain $= x^2/2l^2$

If T is the tension in the wire, then

$2T\cos\theta = mg$ or $T = \frac{mg}{2\cos\theta}$

Here,

$\cos\theta = \frac{x}{(l^2+x^2)^{1/2}} = \frac{x}{l\left(1+\frac{x^2}{l^2}\right)^{1/2}} = \frac{x}{l\left(1+\frac{1}{2}\frac{x^2}{l^2}\right)}$

As $x \ll 1$, so $1 \gg \frac{1}{2}\frac{x^2}{l^2}$ and $1+\frac{1}{2}\frac{x^2}{l^2} \approx 1$

$\therefore \cos\theta = \frac{x}{l}$

Hence, $T = \frac{mg}{2(x/l)} = \frac{mgl}{2x}$

Stress $= \frac{T}{A} = \frac{mgl}{2Ax}$

$Y = \frac{\text{stress}}{\text{strain}} = \frac{mgl}{2Ax}\times\frac{2l^2}{x^2} = \frac{mgl^3}{Ax^3}$

$\therefore x = l\left[\frac{mg}{YA}\right]^{1/3} = 0.5\left[\frac{0.1\times 10}{2\times 10^{11}\times 0.5\times 10^{-6}}\right]^{1/3}$

$= 1.074\times 10^{-2}$ m $= 1.074$ m

30. (b) As, $U = \frac{1}{2}F\Delta L = \frac{F^2L}{2AY} = \frac{F^2L}{2\pi r^2 Y}$

$\Rightarrow U \propto \frac{L}{r^2}$ (F and Y are constant)

$r_A = 2r_B$

$L_A = 3L_B \Rightarrow \frac{U_A}{U_B} = \left[\frac{L_A}{L_B}\right]\times\left[\frac{r_B}{r_A}\right]^2 = (3)\times\left(\frac{1}{2}\right)^2 = \frac{3}{4}$

31. (c) $e = e_1 + e_2$ but $e = \frac{F(2\ell)}{AY_{eq}}, e_1 = \frac{F\ell}{AY_1}, e_2 = \frac{F\ell}{AY_2}$

$\frac{F(2\ell)}{AY_{eq}} = \frac{F\ell}{AY_1} + \frac{F\ell}{AY_2}$

$\frac{2}{Y_{eq}} = \frac{1}{Y_1} + \frac{1}{Y_2} \Rightarrow Y_{eq} = \frac{2Y_1Y_2}{Y_1+Y_2}$

32. (d) Total increase in length, $e = e_1 + e_2$

$e_1 + e_2 = 2.7$ mm

We know $e = \frac{FL}{AY}$

As F, A, L are some for both the wires.

So $e \alpha \frac{1}{Y} \Rightarrow \frac{e_1}{e_2} = \frac{Y_2}{Y_1} = \frac{2\times 10^{11}}{7\times 10^{11}} = \frac{20}{7}, e_1 = \frac{20}{7}e_2$

Substituting in $e_1 + e_2 = 2.7$ mm

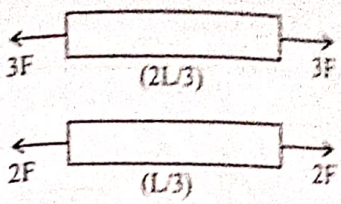
$\frac{20}{7}e_1 + e_2 = 2.7$ mm

$\frac{27e_2}{7} = 2.7$ mm

$e_2 = 0.7$ mm

$e_1 = \frac{20}{7}e_2 = \frac{20}{7}\times 0.7 = 2.0$ mm

33. (*d*)



net elongation of the rod is

$$l = \frac{3F\left(\frac{2L}{3}\right)}{AY} + \frac{2F\left(\frac{L}{3}\right)}{AY}$$

$$l = \frac{8FL}{3AY}$$

34. (*d*) As, Bulk modulus,

$$B = -\frac{P}{\Delta V/V}$$

Given, $\frac{\Delta V}{V} = -\frac{0.1}{100}, B = 9\times10^8\ \text{Nm}^{-2}$

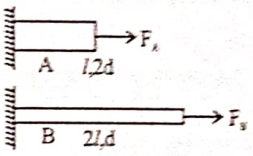
$P = hdg = h \times 1000 \times 10$

$\therefore \frac{B\Delta V}{V} = P = hdg$

$\Rightarrow h = \frac{B\Delta V}{Vdg}$

$$h = \frac{9\times10^8\times0.1}{1000\times100\times10} = 90\,\text{m}$$

35. (*d*) Let Young's modulus of each wire is Y and each wire is stretched by length Δl.



For wire A, from Hooke's law

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F_A/a_A}{\Delta l/l} = \frac{F_A\times l}{(a_A)\Delta l}$$

Where, a_A = area of cross-section of A.

$$\pi r_A^2 = \pi\left(\frac{2d}{2}\right)^2 = \pi d^2$$

$$Y = \frac{F_A l}{\pi d^2 \Delta l} \quad \text{...(i)}$$

Similarly for wire B, from Hooke's law

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F_B/a_B}{\Delta l/2l} = \frac{F_B 2l}{(a_B)(\Delta l)}$$

Where, a_B = are of a cross-section of B

$$\Rightarrow \pi r_B^2 = r\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

$$Y = \frac{F_B(2l)}{\frac{\pi d^2}{4}(\Delta l)} = \frac{(F_B)(8l)}{\pi d^2 \Delta l} \quad \text{...(ii)}$$

From Eqs. (*i*) and (*ii*), we get

$$\frac{(F_A)l}{(\pi d^2)(\Delta l)} = \frac{(F_B)(8l)}{\pi d^2 \Delta l}$$

$$\Rightarrow \frac{F_A}{F_B} = \frac{8l}{l}$$

$F_A : F_B = 8:1$

Exercise-3 (JEE Advanced Level)

1. (*a,b,c*)

Stress in wire $B = \frac{mg}{3\pi r_B^2}$

Stress in wire $A = \frac{4mg}{3\pi r_A^2}$

If $\frac{mg}{3\pi r_B^2} = \frac{4mg}{3\pi r_A^2}$ either wire will break.

2. (*a,c,d*)

Gravitational Potential Energy $U_G = Mgl$

Elastic Potential Energy $U_e = \frac{1}{2}$

Stress × Strain × Volume

$$= \frac{1}{2}\frac{F}{A}\frac{l}{l_0}\times V \left| \begin{array}{l} F = mg \\ V = A\,l_0 \end{array} \right. = \frac{1}{2}mgl$$

Heat Produced $= U_e = \frac{1}{2}Mgl$

3. (*b,c*)

Strain → Same

Stress $= \frac{F}{A}$ = constant

$F \propto A \Rightarrow F \propto r^2$

Energy $= \frac{1}{2}$ stress × strain × volume

$\propto$ Area

$\propto r^2$

4. (*a,d*)

$U = \frac{1}{2}kx^2$. It is a parabola symmetric about U-axis.

At $x = 0.2$ mm, $U = 0.2$ J (from the figure)

$\therefore\ 0.2 = \frac{1}{2}k(2\times10^{-4})^2 \Rightarrow k = 10^7\ \text{Nm}^{-1}$

$$k = \frac{YA}{L} \Rightarrow \frac{A}{L} = \frac{k}{Y} = \frac{10^7}{2\times10^{11}} = 5\times10^{-5} \quad \text{...(i)}$$

AL = Volume = $200\times10^{-6}\ \text{m}^3$

On solving Eqs. (*i*) and (*ii*), we get

$A = 10^{-4}\ \text{m}^2$ and $L = 2$ m

5. (c) $K = \frac{AY}{\ell} = \frac{4\times10^{-4}\times2\times10^{10}}{4} = 2\times10^{6}$

$\omega = \sqrt{\frac{K}{m}} = 100$

6. (d) $W(h+x) = 1/2\, kx^2$

$100(0.99+x) = \frac{1}{2}\times2\times10^{6}\times x^2$

$10^4 x^2 - x - 0.99 = 0$

$100x(100x-1) + 0.99(100x-1) = 0$

$x = \frac{1}{100}$ m = 1 cm

7. (a) $x = \frac{PL}{AY}$

$\sigma = \frac{P}{A} = \frac{xY}{L} = \frac{10^{-2}\times2\times10^{10}}{4}$

$= 5\times10^{7}$ N/m^2

8. (c) $K_1 = 10^6, K_2 = 2\times10^6$

$K_{eq} = \frac{2\times10^6\times10^6}{3\times10^6} = \frac{2}{3}\times10^6$

$\omega = \sqrt{\frac{2\times10^6}{3\times600}} = \frac{100}{3}$

9. (b) Total weight = $1000 + w$

Weight on each rod = $\frac{1000+w}{4}$

Stress = $\frac{1000+w}{4\times4\times10^{-4}} = 9\times10^6$

$\Rightarrow w = 14400 - 1000 = 13400$ N

No. of persons are = $\frac{1340}{50} = 26$

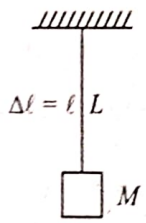
10. (d) Young's modulus is a material constant.

11. (b) On crossing the yield region, the material will experience the breaking stress and further elongation causes reduction in stress and breaking of the wire.

12. (a) Stress-strain curve flattens on crossing elastic region.

13. (a) A-(p); B-(q), C-(r), D-(q)

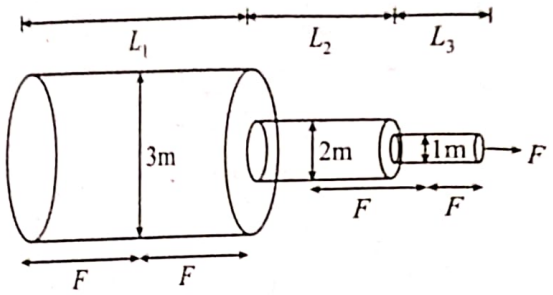
Loss in $PE = Mg\ell$



Elastic $PE = \frac{1}{2}Kx^2 = \frac{1}{2}\frac{Mg}{A}\times\frac{\ell}{L}\times AL = MgL/2$

Heat = $MgL - Mg\,L/2 = Mg\,L/2$

14. (b) A-(p); B-(r); C-(q); D-(r, s)



$Y_1 = \frac{FJ}{A.\Delta l} = \frac{F\times L_1}{\frac{9}{4}\pi.\Delta L_1} = \frac{4FL_1}{9\pi\Delta L_1}$

$Y_2 = \frac{F.L_2}{\pi(1)^2\cdot\Delta L_2} = \frac{F.L_2}{\pi\cdot\Delta L_2}$

$Y_3 = \frac{F.L_3}{\pi\left(\frac{1}{4}\right)\cdot\Delta L_3} = \frac{4F.L_3}{\pi\Delta L_3}$

If $9L_2 = 4L_1$ and $\Delta L_1 = \Delta L_2 \Rightarrow Y_1 = Y_2$

If $L_2 < 4L_3$ and $\Delta L_2 = \Delta L_3 \Rightarrow Y_3 > Y_2$

If ΔL_1 and ΔL_2 and $L_1 = L_2 \Rightarrow Y_2 > Y_1$

If $L_2 = L_3$ and $\Delta L_2 = \Delta L_3 \Rightarrow Y_3 = 4Y_2 \Rightarrow Y_3 > Y_2$

15. (c) A-(p,r); B-(p,s); C-(q,s) D-(q,r)

For A and B, the rod is in equilibrium and hence internal restoring force developed per unit area across any cross-section is same and thus stress is uniform. But in C and D, the case is opposite.

16. [0010]

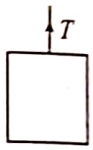
Stress = $\frac{\text{Force}}{\text{Area}} = \frac{mg}{A} = \frac{(\rho A\ell)g}{A} = \rho lg$

$8\times10^8 = (8\times10^3)l\,(10)$

17. [0030]

$T - mg = ma$

$T = mg + ma$



$\frac{800(g+a)}{4\times10^{-4}} = \frac{T}{A} = \frac{1}{3}\times2.4\times10^8 = 8\times10^7$

$(g+a) = \frac{8\times10^3}{200} = 40$

$a = 30$ m/s^2

18. [8]

$\frac{F}{A_1} = y\frac{\Delta l_1}{l_1}$

$\frac{F}{A_2} = y\frac{\Delta\ell_2}{\ell}$

$\Delta\ell_1 + \Delta\ell_2 = 10$ mm

$\frac{F\ell}{A_1 y} + \frac{F\ell}{4A_1 y} = 10$ mm

$\frac{F\ell}{A_1 y} + \frac{F\ell}{4A_1 y} = 10$ mm $\Rightarrow \frac{F\ell}{A_1 y} = 8$ mm.

19. [200]

$$P_0 + H\rho g = 3.7 \times 10^6$$

$$H \times 1.8 \times 10^3 \times 10 = 3.6 \times 10^6$$

$$H = \frac{36}{18} \times 10^2 = 200$$

20. [5] $Mg = PA$; A is sectional area

$$Alpg = 10^5 A$$

$$\ell = \frac{10^5}{2\times10^3\times10} = 5 \text{ m}$$

21. [11] We know that

$$\frac{F}{A} = Y\frac{\Delta l}{l} \Rightarrow \frac{mg}{A} = Y\frac{\Delta l}{l}$$

$$\text{i.e. } m = \frac{A}{g}Y\frac{\Delta l}{l}$$

$$= \frac{\pi(0.6\times10^{-3})^2\times2\times10^{11}\times1.6\times10^{-3}}{10\times3.2}$$

$$= \frac{0.36\pi}{10}\times10^2 = 3.6\pi \approx 11.3$$

$\therefore$ Closest mass = 11 kg

22. [0008]

The stress is zero at the free ends and maximum at the axis. Therefore, the rod will rupture at the middle.

Let us consider the element of rod at a distance 5 from the axis, the mass of element $dm = \rho A dx$

Applying Newton's second law, $F - (F + dF) = (dm)a_n$

or $-dF = (\rho A dx)\omega^2 x$

where ω is the rotation speed.

$$F = -\int dF = -(\rho A dx)\omega^2 x = -\rho A\omega^2\frac{x^2}{2} + C$$

at $x = \frac{l}{2}, F = 0$

$$\therefore\ C = \frac{\rho A\omega^2 l^2}{8}$$

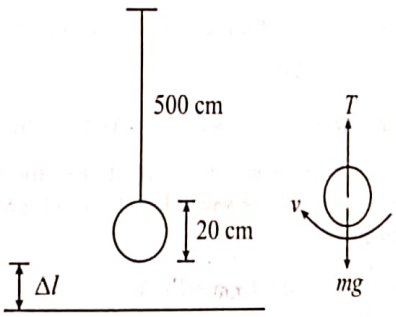
at $x = 0, F = \rho\frac{A\omega^2 l^2}{8}$

Rupture of rod will occur when Stress = σ

$$\sigma = \frac{\rho\omega^2 l^2}{8} \Rightarrow \omega = \sqrt{\frac{8\sigma}{\rho l^2}} \text{ or } v = \frac{1}{2\pi}\sqrt{\frac{8\sigma}{\rho l^2}}$$

23. [0031]

$\Delta l = 521 - 20 = 1$ cm $= 0.01$ cm



$$T - mg = \frac{mv^2}{R}$$

$$\therefore\ T = m\left(g + \frac{v^2}{R}\right) = m\left(g + \frac{v^2}{l}\right)$$

$$\Delta l = \frac{Tl}{AY} = \frac{m\left(g + \frac{v^2}{l}\right)l}{(\pi d^2/4)Y} = \frac{mgl + mv^2}{(\pi d^2/4)Y}$$

$$\therefore\ v = \sqrt{\frac{\pi d^2\Delta l Y}{4m} - gl}$$

$$= \sqrt{\frac{(3.14)(4\times10^{-3})^2(0.01)(2\times10^{11})}{4\times25} - 9.8\times5}$$

$$\approx 31 \text{ m/s}$$

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*c*) $\text{Stress} = \frac{\text{Force}}{\text{area}} = \frac{mg}{A} = \frac{\text{volume}\times\text{density}\times g}{\text{Area}}$

$$\text{Stress} = \frac{L^3\rho g}{L^2}$$

Stress $\propto L$

2. (*b*) [Bulk Modulus = $\frac{\text{volumetric stress}}{\text{volumetric strain}}$]

$$K = \frac{mg}{a\left(\frac{dV}{V}\right)}$$

$$\frac{dV}{V} = \frac{mg}{Ka} \quad \text{...(i)}$$

Volume of sphere $\rightarrow V = \frac{4}{3}\pi R^3$

Fractional change in volume $\frac{dV}{V} = \frac{3dr}{r}$ ...(ii)

Using eq. (*i*) and (*ii*) $\frac{3dr}{r} = \frac{mg}{Ka}$

$$\frac{dr}{r} = \frac{mg}{3Ka}$$

3. (*a*) In first case: $\frac{\frac{mg}{A}}{\left(\frac{\Delta\ell_1}{\ell}\right)} = Y \Rightarrow \Delta\ell_1\frac{mg}{\left(\frac{YA}{\ell}\right)}$

In second case: $\frac{\frac{mg-B}{A}}{\left(\frac{\Delta\ell_1}{\ell}\right)} = Y \Rightarrow \Delta\ell_2\frac{mg-B}{\left(\frac{YA}{\ell}\right)} = \frac{\left(\frac{3mg}{4}\right)}{\left(\frac{YA}{\ell}\right)}$

$$\Rightarrow \Delta\ell_2 = \frac{3}{4}\Delta\ell_1 = 3\text{ mm}$$

4. (*c*) Energy of catapult $= \frac{1}{2} \times \left(\frac{\Delta \ell}{\ell}\right)^2 \times Y \times A \times \ell$

$= \text{Kinetic energy of the ball} = \frac{1}{2} mv^2$ therefore,

$= \frac{1}{2} \times \left(\frac{20}{42}\right)^2 \times Y \times \pi \times 3^2 \times 10^{-6} \times 42 \times 10^{-2}$

$= \frac{1}{2} \times 2 \times 10^{-2} \times (20)^2$

$Y = 3 \times 10^6 \text{ Nm}^{-2}$

5. (*d*) Tensile stress in wire will be $= \frac{\text{Tensile force}}{\text{Cross-section area}}$

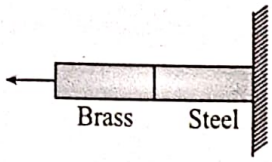
$= \frac{mg}{\pi R^2} = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} \text{ Nm}^{-2} = 3.1 \times 10^6 \text{ Nm}^{-2}$

6. (*a*) $\frac{F}{A} = \text{stress}$; $\frac{400 \times 4}{\pi d^2} = 379 \times 10^6$

$d^2 = \frac{1600}{\pi \times 379 \times 10^6} = 1.34 \times 10^{-6}$

$d = \sqrt{1.34} \times 10^{-3} = 1.15 \times 10^{-3} \text{ m}$

7. (*b*)



$k_1 = \frac{y_1 A_1}{\ell_1} = \frac{120 \times 10^9 \times A}{1}$

$k_2 = \frac{y_2 A_2}{\ell_2} = \frac{60 \times 10^9 \times A}{1}$

$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{120 \times 60}{180} \times 10^9 \times A$

$k_{eq} = 40 \times 10^9 \times A$

$F = k_{eq}(x)$

$F = (40 \times 10^9) A. (0.2 \times 10^{-3})$

$\frac{F}{A} = 8 \times 10^6 \text{ N/m}^2$

No option is matching. Here question must be bonus.

8. (*b*) Given, $\frac{Y_A}{Y_B} = \frac{7}{4}$

$L_A = 2 \text{ m } A_A = \pi R^2$

$L_B = 1.5 \text{ m } A_B = \pi (2 \text{ mm})^2$

$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$

Given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B} \Rightarrow \frac{(\pi R^2)\left(\frac{7}{4} Y_B\right)}{2} = \frac{\pi (2 \text{ mm})^2 . Y_B}{1.5}$

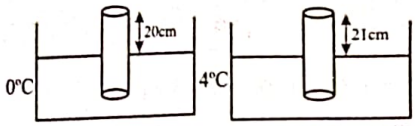
$R = 1.74 \text{ mm}$

9. [60] $V = 2\alpha_2 + \alpha_1$

$= 10 \times 10^{-6} + 5 \times 10^{-5}$

$= 60 \times 10^{-6}/°\text{C}$

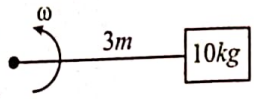
10. (*d*)



$mg = A(80)\, \rho_{0°c}\, g$; $mg = A(79)\, \rho_{4°c}\, g$

$\frac{\rho_{4°C}}{\rho_{0°C}} = \frac{80}{79} = 1.01$

11. [4]



$\frac{T}{A} = \sigma$ ...(*a*)

$T = m\omega^2 \ell$ ...(*b*)

Solving, $\omega = 4$ rad/s

12. (*d*) $\frac{du}{dv} = \frac{1}{2} \text{stress} \times \frac{\text{stress}}{y} = \frac{1}{2} \frac{F^2}{A^2 y}$

$\frac{du}{dv} \propto \frac{1}{d^4}$

$\frac{\left(\frac{du}{dv}\right)_1}{\left(\frac{du}{dv}\right)_2} = \frac{d_2^4}{d_1^4} = \frac{1}{4}$

$\frac{d_1}{d_2} = (4)^{1/4}$; $\frac{d_1}{d_2} = \sqrt{2} : 1$

13. (*b*) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$; $k = \frac{YA}{L}$

$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{YA}{mL}}$

14. [20] We know that, energy conservation

According to energy conservation

$\frac{1}{2} \cdot \frac{YA}{L} \cdot x^2 = \frac{1}{2} mv^2$

Putting all values in above equation we get,

$\frac{0.5 \times 10^9 \times 10^{-6} \times (0.04)^2}{0.1} = \frac{20}{1000} v^2$

$\therefore v^2 = 400$

$v = 20$ m/s

The velocity of the projected stone is 20 m/s.

15. [05] We know that,

Elastic energy $= \frac{Y}{2} (\text{strain})^2 \times \text{Area} \times \text{length}$

As the tracks won't be allowed to expand linearly, the rise in temperature would lead to development thermal stress in track.

$\frac{(\text{Stress})}{y} = \alpha \Delta T$ Or $\sigma = Y \alpha \Delta T$

Energy stored per unit volume $= \frac{1}{2}\frac{\sigma}{Y}$

$\Rightarrow$ Energy stored per unit length $= \frac{A\sigma^2}{2Y}$

$= \frac{A}{2} \times Y\alpha^2 \Delta T^2$

$= \frac{10^{-2} \times 10^{11} \times 10^{-10} \times 100}{2} = 5$ J/m

16. [25] $y = \frac{\text{stress}}{\text{strain}} = 2.0 \times 10^{10}$

Energy density $= \frac{1}{2}$ stress × strain

$= \frac{1}{2}(\text{strain})^2 y = \frac{1}{2}(5 \times 10^{-4})^2 \times 20 \times 10^{10}$

$= 25 \times 10^2 \times 10$

$= 25 \frac{\text{kJ}}{\text{m}^3} = 25$

17. [05] $\frac{F}{A} = Y \times \frac{z_1}{L}$

$\Rightarrow \frac{F}{\pi r^2} = Y \times \frac{5}{L}$ ...(i)

$\frac{4F}{\pi \cdot r^2 \times 16} = Y \times \frac{x_2}{4L}$ ...(ii)

$x_2 = 5$

18. (*a*) $\sigma_b = \frac{F}{A}$

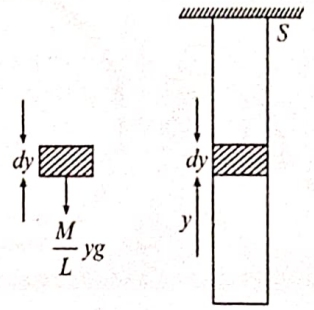
$\Rightarrow \sigma_b = \frac{10 \times 10^3 g}{2.5 \times 10^{-4}}$ ...(i)

$\sigma_b = \frac{25 \times 10^3 g}{A}$ ...(ii)

$\frac{10}{2.5 \times 10^{-4}} = \frac{25}{A}$

$\Rightarrow A = \frac{2.5 \times 10^{-4} \times 25}{100} = 6.25 \times 10^{-6} = 6.25 \times 10^{-4}$

19. [25] Let dl be small elongation due to $\frac{MY}{L}g$



Using $\frac{F}{A} = Y\frac{\Delta l}{l} \Rightarrow \frac{MYg}{AL} = Y \cdot \frac{dl}{dy}$

$\therefore \int dl = \frac{Mg}{YAL}\int_0^L y\,dy \Rightarrow \Delta l = \frac{MgL}{2AY}$

$\Delta l = \frac{MgL}{2AY}$; $\Delta l = \frac{20 \times 10 \times 20}{2 \times 0.4 \times 2 \times 10^{11}} = 25 \times 10^{-9}$ m

JEE ADVANCED

20. (*c*) $Y = \frac{\left(\frac{F}{A}\right)}{\frac{\Delta \ell_1}{L}}$ ...(i)

$Y = \frac{\left(\frac{F}{4A}\right)}{\frac{\Delta \ell_2}{2L}}$ ...(ii)

$\frac{\Delta \ell_1}{\Delta \ell_2} = 2$

21. [2.4]

$\frac{dV}{V} = \frac{3da}{a} \Rightarrow B = -V\frac{dP}{dV} = \frac{-V(\rho gh)}{dV} = \frac{-\rho gh}{3da}a$

$70 \times 10^9 = \frac{1 \times 5000 \times 10^3 \times 10 \times 1}{3 \times da}$

$da = \Delta a = \frac{5}{21} \times 10^{-2} m = 2.38$ mm ≈ 2.4 mm

CHAPTER

14 Mechanical Properties of Fluids

Exercise-1 (Topicwise)

1. (*a*) $F = mg$

$F = 10$ N

2. (*c*) Volume of ice $= \frac{M}{\rho}$, volume of water $= \frac{M}{\sigma}$.

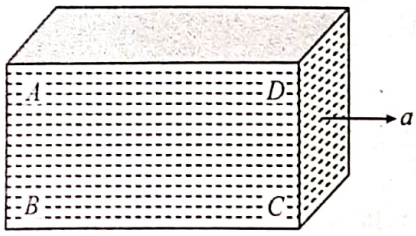
$\therefore$ Change in volume $= \frac{M}{\rho} - \frac{M}{\sigma} = M\left(\frac{1}{\rho} - \frac{1}{\sigma}\right)$

3. (*c*) $P_1V_1 = P_2V_2$

$\Rightarrow (P_0 + h\rho g)\, V = P_0 \times 3V$

$\Rightarrow h\rho g = 2P_0 \Rightarrow h = \frac{2 \times 75 \times 13.6 \times g}{\frac{13.6}{10} \times g} = 15m$

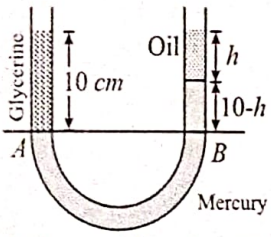
4. (*a*)



Due to acceleration towards right, there will be a pseudo force in a left direction. So the pressure will be more on rear side (Points *A* and *B*) in comparison with front side (Point *D* and *C*).

5. (*c*) A torque is acting on the wall of the dam trying to make it topple. The bottom is made very broad so that the dam will be stable.

6. (*d*)



At the condition of equilibrium

Pressure at point *A* = Pressure at point *B*

$P_A = P_B \Rightarrow 10 \times 1.3 \times g$

$= h \times 0.8 \times g + (10 - h) \times 13.6 \times g$

By solving we get $h = 9.7$ cm

7. (*a*) At same depth pressure is same

So ratio $P_1 : P_2 = 1 : 1$.

8. (*a*) $\frac{m_1 g}{A_1} = \frac{m_2 g}{A_2}$

Solving, $m_2 = 3.75$ kg.

9. (*a*) $F = [\rho g h \times A]$

$= (1000)\,(10)\,(6)\,(10)\,(8)$.

10. (*b*) Let specific gravities of concrete and saw dust are ρ_1 and ρ_2 respectively.

According to principle of floatation weight of whole sphere = upthrust on the sphere

$$\frac{4}{3}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g$$

$$\Rightarrow R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$$

$$\Rightarrow R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2) \Rightarrow \frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1}$$

$$\Rightarrow \frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1}$$

$$\Rightarrow \frac{(R^3 - r^3)\rho_1}{r^2\rho_2} = \left(\frac{1 - \rho_2}{\rho_1 - 1}\right)\frac{\rho_1}{\rho_2}$$

$$\Rightarrow \frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1 - 0.3}{2.4 - 1}\right) \times \frac{2.4}{0.3} = 4$$

11. (*d*) Apparent weight

$= V(\rho - \sigma)g = l \times b \times h \times (5 - 1) \times g$

$= 5 \times 5 \times 5 \times 4 \times g$ Dyne $= 4 \times 5 \times 5 \times 5$ *gf*.

12. (*a*) Fraction of volume immersed in the liquid $V_{in} = \left(\frac{\rho}{\sigma}\right)V$

i.e. it depends upon the densities of the block and liquid. So there will be no change in it if system moves upward or downward with constant velocity or some acceleration.

13. (*c*) Mass of Boat Before $= M + M$

Volume of tank $= V_1 = \frac{M + M}{\rho_w}$

$m \rightarrow$ Balls mass

After one Ball drop:

volume $V_2 = \frac{M}{\rho_B} + \frac{M}{\rho_w}$

$\rho_{Ball} > \rho_w$

$v_2 < v_1$

14. (*c*) Buoyant force on fish due to water will be equal and opposite to the force on water by fish. These two forces are internal forces for the fish bucket system.

Hence, they will not affect the weight the boy carries.

15. (*b*) $P = h\rho g = 6.4 \times 900 \times 10$

$= 3600 \times N/m^2$

$\therefore \quad F = P \cdot A$

$F = 3600 \times 2 \times 10^{-3}$

$= 7.2$ N

16. (*b*) Apparent weight $(W_{app.}) = W - V\rho_\ell g$

Since, $W_{app.\ (Ram)} > W_{app.\ (Shyam)}$

$\Rightarrow W_{(Ram)} > W_{(Shyam)}$

Therefore, from given passage shyam has more fat than Ram.

17. (*a*) $mg = 60$ ...(i)

$mg - \rho_l vg = 40$ ...(ii)

$$\frac{mg - \rho_\ell vg}{mg} = \frac{2}{3} = \frac{\rho_0}{\rho_\ell} = 3$$

where ρ_0 = density of the block and ρ_1 = density of the liquid.

18. (*b*) Ice is lighter than water. When ice melts, the volume occupied by water is less than that of ice, due to which the level of water goes down.

19. (*b*) $W - v \times 1 \times g = W_1$

$W - v \times x \times g = W_2$

$\Rightarrow W - (W - W_1) \times x = W_2$

$$x = \frac{W - W_2}{W - W_1}$$

20. (*b*) For streamline flow, Reynold's number $N_R \propto \frac{r\rho}{\eta}$ should be less. For less value of N_R, radius and density should be small and viscosity should be high.

21. (*a*) $d_A = 2$ cm and $d_B = 4$ cm $\therefore r_A = 1$ cm and $r_B = 2$ cm

From equation of continuity, av = constant

$$\therefore \frac{v_A}{v_B} = \frac{\alpha_B}{\alpha_A} = \frac{\pi(r_B)^2}{\pi(r_A)^2} = \left(\frac{2}{1}\right)^2 \Rightarrow v_A = 4v_B$$

22. (*c*) If the liquid is incompressible then mass of liquid entering through left end, should be equal to mass of liquid coming out from the right end.

$\therefore \quad M = m_1 + m_2 \Rightarrow Av_1 = Av_2 + 1.5\,A.v$

$\Rightarrow A \times 3 = A \times 1.5 + 1.5\,A.v \Rightarrow v = 1$ m/s

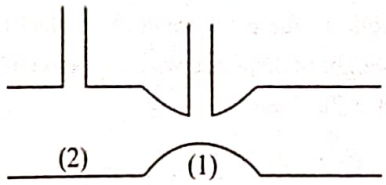
23. (*d*) As cross-section areas of both the tubes A and C are same and tube is horizontal. Hence according to equation of continuity $v_A = v_C$ and therefore according to Bernoulli's theorem $P_A = P_C$ i.e. height of liquid is same in both the tubes A and C.

24. (*b*) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s

25. (*a*) When air moves faster, the pressure of air decreases so the pressure on the top of the wing is less then the pressure of the bottom wing. The difference in pressure creates the force on the wing which lifts the wing upward.

26. (*a*) Bernoulli's equation.

27. (*a*) From continuity equation, velocity at larger cross-section is more than that at smaller cross-section.



Hence; $P_1 < P_2$

28. (*a*) Av= constant

$A \downarrow v \uparrow$

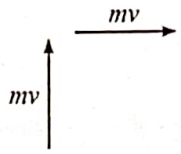
$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

$v \uparrow P \downarrow$

29. (*d*) Force exerted by the water on the corner

= change in momentum in 1 sec

$= \sqrt{2}\,mv$



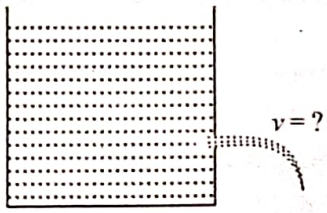
$= \sqrt{2}\,\rho vL$

30. (*b*) By Bernaulie's Theorem $\left(P_0 + \frac{F}{A}\right) + \frac{1}{2}\rho v^2 + \rho gh = k$

$$P_0 + \frac{10 \times 10}{1000 \times 10^{-4}} + 1000 \times 10 \times \frac{50}{100}$$

$$= P_0 + \frac{1}{2} \times 1000 \times v^2$$

$$6000 = \frac{1}{2} \times 1000 \times v^2$$



$v = \sqrt{12} = 3.4$ m/s

31. (*a*) At critical temp. It Become zero.

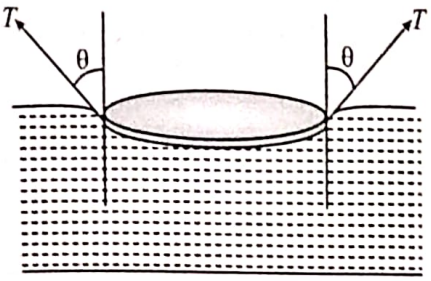
32. (*d*) Soap helps to lower the surface tension of solution, thus soap get stick to the dust particles and grease and these are removed by action of water.

33. (*a*) Surface tension = weight

34. (*b*) Fact based

35. (*d*) $T = T_0(1 - \alpha t)$

36. (*c*)



Weight of metal disc = total upward force

= upthrust force + force due to surface tension

= weight of displaced water + $T\cos\theta\,(2\pi r)$

$= W + 2\pi r T\cos\theta$

37. (*a*) $T = \dfrac{F}{2l} = \dfrac{2\times10^{-2}}{2\times10\times10^{-2}} = 0.1$ N/m

38. (*a*) When two droplets merge with each other, their surface energy decreases.

$W = T(\Delta A)$= (negative) i.e. energy is released.

39. (*d*) $W = 8\pi T\left(R_2^2 - R_1^2\right) = 8\pi T[(2r)^2 - (r)^2] = 24\pi r^2 T$

40. (*c*) Work done to increase the diameter of bubble from d to D

$W = 2\pi\left(D^2 - d^2\right)T = 2\pi\left[(2D)^2 - (D)^2\right]T = 6\pi D^2 T$

41. (*c*) $W = 8\pi T(r_2^2 - r_1^2) = 8\pi T\left[\left(\dfrac{2}{\sqrt{\pi}}\right)^2 - \left(\dfrac{1}{\sqrt{\pi}}\right)^2\right]$

$\therefore\ W = 8\times\pi\times30\times\dfrac{3}{\pi} = 720\ erg$

42. (*b*)

By equating volume: $\dfrac{4}{3}\pi R^3 = 8\times\dfrac{4}{3}\pi r^3$

get $r = R/2$.

Now pressure difference in $A = \dfrac{4\sigma}{R}$

and that in $B = \dfrac{4\sigma}{R/2} = 2\times$ pressure difference in A.

43. (*d*) Fact based

44. (*a*) Fact based

45. (*c*) This happens due to viscosity.

46. (*b*) Fact based

47. (*b*) Fact based

48. (*b*) Meniscus – canvex

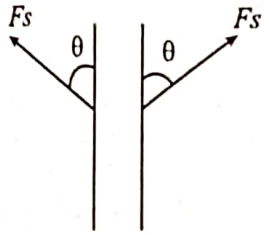
49. (*c*) Angle of contact is acute.

50. (*b*) Water will rise to a height more than h when downward force (mg_{eff}) becomes lesser than mg.

so in a lift accelerating downwards, g_{eff} is $(g - a_0)$. Hence capillary rise is more.

On the poles g_{eff} is even more than g. Hence the capillary will even drop.

51. (*a*) Since the contact angle in both cases remains the same.



$F_S\cos\theta = Mg \Rightarrow T\times 2\pi R\cos\theta = Mg$ ...(i)

After doubling the radius

$T\times 2\pi(2R)\cos\theta = M'g$ ...(ii)

$= M' = 2M.$

52. (*d*) In the satellite, g_{eff} becomes zero but the surface tension still prevails. Hence the water will experience only surface Tension force which will push it fully outward.

53. (*c*) $P_{\text{inside}} = P_o - \dfrac{4T}{r}$

54. (*c*) Since $\Delta P \propto \dfrac{1}{R}$

55. (*b*) Fact based

56. (*c*) $P_1V_1 + P_2V_2 = PV$

$\dfrac{4T}{r_1}\times\dfrac{4}{3}\pi r_1^3 + \dfrac{4T}{r_2}\times\dfrac{4}{3}\pi r_2^3 = \dfrac{4T}{R}\times\dfrac{4}{3}\pi R^3$

$r_1^2 + r_2^2 = R^2$

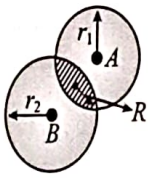
$3^2 + 4^2 = R^2 \quad R = 5cm$

57. (*a*) $\Delta P = \dfrac{4T}{r} \Rightarrow \dfrac{\Delta P_1}{\Delta P_2} = 4$

$\therefore\ \dfrac{r_2}{r_1} = 4$ and $\dfrac{V_1}{V_2} = \left(\dfrac{r_1}{r_2}\right)^3 = \dfrac{1}{64}$

58. (*c*) When charge is given to a soap bubble (whether positive or negative), these charges experience repulsive forces due to the other charges. Hence they tend to move out. Hence the size of bubble increases.

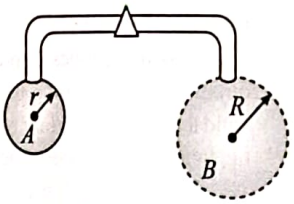
59. (*d*)



Equating pressures on the shaded portion:

$\dfrac{4\sigma}{r_1} - \dfrac{4\sigma}{r_2} = \dfrac{4\sigma}{R} \Rightarrow$ get $R = \dfrac{r_2 r_1}{r_2 - r_1}$

60. (*c*)



initially

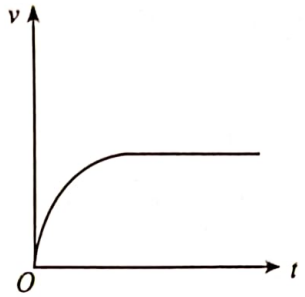
$P_A = P_0 + \frac{4\sigma}{r}$; $P_B = P_0 + \frac{4\sigma}{R}$ {P_0 = atmospheric pressure}.

Clearly $P_A > P_B$; so air will flow from A to B.

As r decreases; pressure will become more and hence more flow of air from A to B.

Ultimately bubble A collapses and B becomes bigger in size.

61. (*d*) The speed increases and become constant. Therefore the graph that best represents the velocity as function of time is graph given below.



62. (*b*) $v_T \propto r^2$ ...(i)

Let r be the radius of small rain drops and R the radius of large drop. Equating the volumes, we have

$$\frac{4}{3}\pi R^3 = 2\left(\frac{4}{3}\pi r^3\right)$$

$$R = (2)^{1/3} . r,$$

$$\text{or } \frac{R}{r} = (2)^{1/3}$$

$$\frac{v_T'}{v_T} = \left(\frac{R}{r}\right)^2 = (2)^{2/3}$$

$$v_T' = (2)^{2/3} v_T = (2)^{2/3}(1.0)\text{m/s} = 1.587 \text{ m/s}$$

63. (*a*) Net force on the ball = downward force – upward force

$$= \frac{mg}{2}$$

$$\frac{4}{3}\pi r^3(\rho - \sigma)g - 6\pi\eta r v = \frac{mg}{2}$$

$$\frac{4}{3}\pi r^3(\rho - \sigma)g - 6\pi\eta r v = \frac{1}{2}\left(\frac{4}{3}\pi r^3 \rho\right)g$$

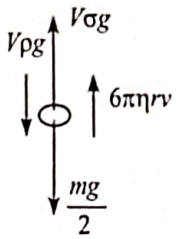
$$r^2(\rho - \sigma)g - \frac{9}{2}\eta v = \frac{1}{2}r^2\rho g$$

$$\frac{9}{2}v\eta = r^2(\rho - \sigma)g - \frac{1}{2}r^2\rho g$$

$$= \frac{1}{2}r^2\rho g - r^2\sigma g$$

$$\frac{9}{2}v\eta = \frac{1}{2}r^2 g(\rho - 2\sigma)$$

$$\Rightarrow v = \frac{r^2 g}{9\eta}(\rho - 2\sigma)$$



64. (*a*) Velocity first increases and acquires a constant value known as terminal velocity.

65. (*b*) $2\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 2^{1/3} . r$

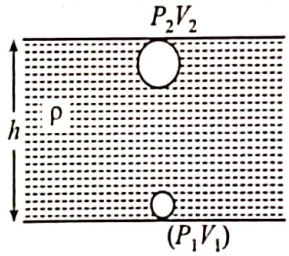
$v \propto r^2$

$v \propto 4^{1/3}$

Exercise-2 (Learning Plus)

1. (*d*) $\Delta v = v_f - v_i = \frac{m}{y} - \frac{m}{x}$.

2. (*b*) According to Boyle's law, pressure and volume are inversely proportional to each other i.e. $P \propto \frac{1}{V}$



$\Rightarrow P_1V_1 = P_2V_2$

$\Rightarrow (P_0 + h\rho_w g)V_1 = P_0V_2$ [$\therefore h = 47.6 \times 10^2$ cm]

$$\Rightarrow V_2 = \left(1 + \frac{h\rho_w g}{P_0}\right)V_1$$

$$\Rightarrow V_2 = \left(1 + \frac{47.6 \times 10^2 \times 10 \times 1000}{70 \times 13.6 \times 1000 \times 10}\right)V_1$$

$\Rightarrow V_2 = (1 + 5)\ 50 \text{ cm}^3 = 300 \text{ cm}^3$

[As $P_2 = P_0 = 70$ cm of $Hg = 70 \times 13.6 \times 1000$]

3. (*d*) O(zero) all the forces passes through O

$\therefore$ no torque.

4. (*c*) $rg(H - h)$

Because pressure varies with height.

5. (*c*) Given $m = 12$ kg, $A = 800$ cm^2, $\rho = 1000$ kg/m^3

$P = \rho g h$

$$\frac{mg}{A} = \rho g h$$

$$1 \times 980 \times h = \frac{12 \times 1000}{800} \times 980$$

$$h = \frac{1200}{80} = 15 \text{ cm}$$

6. (c) $[36-\rho_1 v_1]g = [48-\rho_1 v_2]g$

$$\left[36-\rho_1\left(\frac{36}{9}\right)\right]g = \left[48-\rho_1\left(\frac{48}{\rho_0}\right)\right]g \Rightarrow \rho_0 = 3.$$

7. (c) As, weight = Buoyant force

$mg = [100 \times 6 \times 0.6\, g] + (100 \times 1 \times 4)g$

$\therefore\ m = 760$ gm.

8. (c) $\therefore$ Volume where metal is present

$= \dfrac{9.8}{7800} = 1.256 \times 10^{-3}$

Buoyancy $= v\rho g = 1.5$ g

$\Rightarrow v \times 1000 = 1.5\ ;\ v = 1.5 \times 10^{-3}$

Fraction of volume

$$= \frac{1.5\times10^{-3} - 1.256\times10^{-3}}{1.5\times10^{-3}} \times 100 = 16\%$$

9. (a) $\sigma \times \dfrac{4}{3}\pi(R^3 - r^3)g = 1 \times \dfrac{4}{3}\pi R^3 g$

$$\frac{R}{r} = \left(\frac{\sigma}{\sigma-1}\right)^{1/3}$$

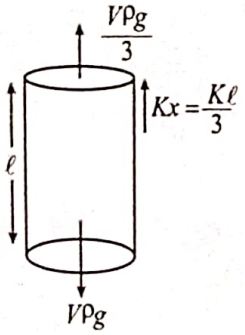
10. (b) Volume $= \dfrac{0.5}{500} = 10^{-3}\ m^3$

Buoyancy $= \rho V g = 1000 \times 10^{-3} \times 10 = 10\ N$

$m = 1$ kg

If float = 2.5 kg, Reading = 1 + 1.5 = 2.5 kg

11. (b)



$V = A.\ell.$

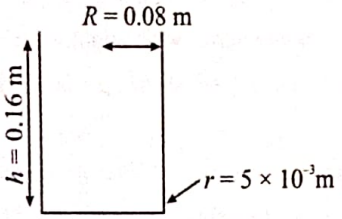
Now $\dfrac{A\ell\rho g}{3} + \dfrac{K\ell}{3} = A\ell\rho \times g$

$K = 2\rho A g$

12. (c) $R = vt$

$$= \sqrt{2gD}\sqrt{\frac{2(H-D)}{g}} = 2\sqrt{D(H-D)}.$$

13. (a)



$A_1 v_1 = A_2 v_2$

$\pi R^2\, dh/dt = \pi r^2 v$ ...(i)

$v = \sqrt{2gh}$ ...(ii)

From equation (ii) put the value of v in equation (i)

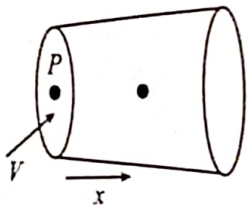
$\pi R^2\, dh/dt = \pi r^2 \sqrt{2gh}$

$$\Rightarrow \int \frac{R^2 dh}{r^2\sqrt{2gh}} = \int dt \Rightarrow \frac{R^2}{r^2\sqrt{2g}}\int_h^0 \frac{dh}{\sqrt{h}} = \int_0^t dt$$

On solving

$t = 46.26$ second.

14. (a)



$A = ax + b$

Continuity equation $bV = (ax + b)\, V_2$

By bernowlli equation $= P_2 + \dfrac{1}{2}\rho v_2^2 =$ constant

$P_2 = \text{Constant} - \dfrac{1}{2}\rho v_2^2$

$P_2 = \text{Constant} - \dfrac{1}{2}\rho\dfrac{b^2V^2}{(ax+b)^2}$

$P_2 = \text{Constant} - \dfrac{C_1}{(ax+b)^2}$

Where C_1 = Constant

15. (b) $A_1V_1 = A_2V_2$

$0.02 \times 2 = 0.01 \times V_2$

$V_2 = 4$ m/sec.

$P_1 + \dfrac{1}{2}\rho V_1^2 = P_2 + \dfrac{1}{2}\rho V_2^2$

$4 \times 10^4 + \dfrac{1}{2} \times 1000 \times 2^2 = P_2 + \dfrac{1}{2} \times 1000 \times 4^2$

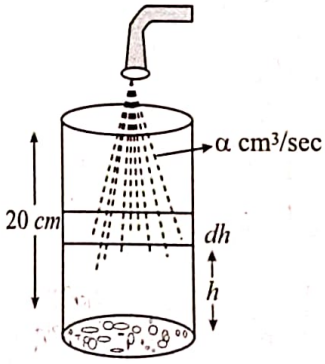
$\Rightarrow P_2 = 3.4 \times 10^4$ N/m^2

16. (b) $\sqrt{2\times20\times10^{-2}\times10} = 2$ m/sec.

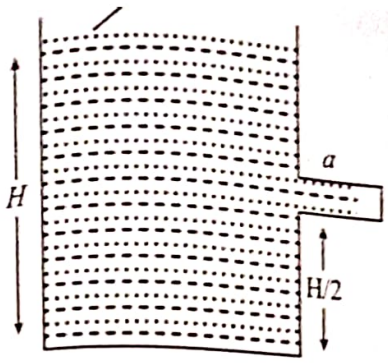
17. (d) Inlet = outlet

$\alpha\, dt = a\sqrt{2gh}\, dt$

$$h = \frac{\alpha^2}{2ga^2} = \frac{(100)^2}{2\times(1000)(1)} = 5 \text{ cm}$$



18. (c)



Force = $\rho a\left(\sqrt{2gh/2}\right)^2$

Acceleration = $\dfrac{\rho agh}{\rho Na.H} = g/N$

19. (a) Change in momentum is/sec.

$\sqrt{2}\,\rho Av^2 = 565.7$ N.

20. (c) Energy required in one second is the power

$10^{-1} = A.V.$

$\Rightarrow\ 10^{-1} = 10^{-2} \times V$

$\Rightarrow\ V = 10$ m/sec.

$mgh + \dfrac{1}{2}mV^2 = P$

Here m = mass in one second

$P = \rho\, AVgh + \dfrac{1}{2}\rho AV^3$

$P = \rho AV[10 \times 10 + 50]$

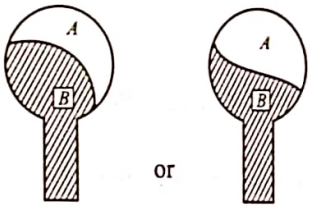
$= 15\,K$ watt

21. (d) $A_1V_1 = A_2V_2$

$\pi(1 \times 10^{-2})^2 \times 3$

$= 100 \times \pi\ \dfrac{(0.05\times10^{-2})^2}{4} \times V_2 \Rightarrow V_2 = 48$ m/sec.

22. (b) After the portion A is punctured' the thread has 2 options as shown in the figures.



(i) (ii)

Clearly, due to surface tension, the soap film wants to minimize the surface area which is happening in option (ii).

Hence the thread will become concave towards A.

23. (c) $n \times \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$ ...(i) {∵ volumes are equal

and $\Delta A = -[4\pi R^2 - n.4\pi r^2]$

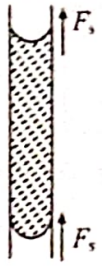
where $W = (\Delta A) \times T$.

$= -4\pi[n^{2/3}r^2 - n.r^2] \times T = 4\pi r^2 T.\, n^{2/3}\,[n^{1/3} - 1]$.

Now $R^2 = n^{2/3}.\, r^2$; so $W = 4\pi R^2 T[n^{1/3} - 1]$.

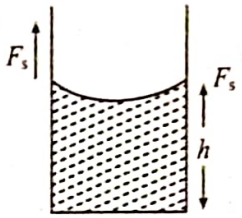
24. (a) When the capillary rise is 'h' that means the force of surface tension (F) is supporting the height 'h' of liquid level.

Now if the whole capillary is taken out the liquid tries to come out due to gravity from the bottom point.



But force of surface tension 'F' now becomes $2F$ in the upward direction. Hence $2F$ can support a maximum of '$2h$' height even if ℓ is very high. So 'h' will be $2h$ if $\ell > h$ & will be $h + \ell$ only if ℓ is lesser than h.

25. (b)

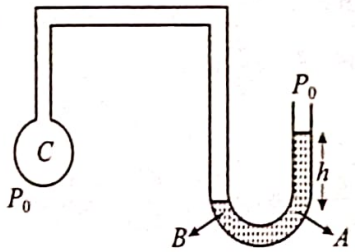


By balancing forces

$T \times (2\ell) \times (\cos\theta) = d\,x\,\ell\,h\,g$

we get $h = \dfrac{2T\cos\theta}{xdg}$.

26. (d)



P_A has to be equal to P_B,

$P_A = P_0 + \rho gh$ ...(i)

Now $P_C - P_0 = \dfrac{4\sigma}{r}$

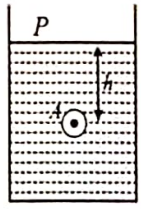
∵ soap bubble has 2 films and

$P_C = P_B$ ∵ same air is filled

$\Rightarrow P_0 + \dfrac{4\sigma}{r} = P_0 + \rho gh$ ...(ii)

get $\sigma = \dfrac{\rho ghr}{4}$

27. (b)

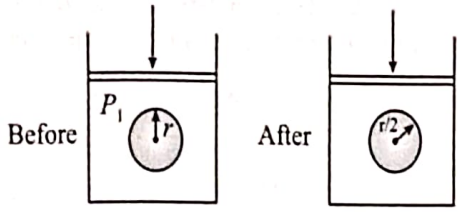


$P_{\text{inside bubble}} - P_A = \dfrac{2T}{r}$

and $P_A = P_{atm} + \rho g d$.

$\Rightarrow P_{\text{inside bubble}} = P + \rho g d + \frac{2T}{r}$

28. (*a*)



Lets say, initially, the pressure due to air inside the bubble is P_{air}.

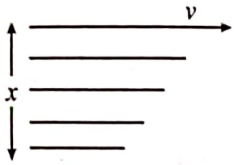
$\Rightarrow P_{air} - P_1 = \frac{4T}{r}$ ...(i)

Finally, the radius becomes half ; so volume becomes $\frac{1}{8}$th and hence pressure becomes $8P_{air}$.

So, $8P_{air} - P_2 = \frac{4T}{r/2}$ ...(ii)

Solving (*i*) and (*ii*)

29. (*c*)



$800 = \eta A.\frac{1.5}{x}$; $2400 = \eta A \frac{v}{x}$

$v = 4.5$ cm/sec.

30. (*d*) $F_R = K.\pi r^2 \cdot v^2 = \rho \frac{4}{3}\pi r^3 g$

$v \alpha \sqrt{r}$

31. (*d*) $v_T = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - \sigma) = \frac{2}{9}\frac{(0.003)^2 \times 10}{1.260}(1260)$

$v_T = 0.02$ m/sec.

$\therefore$ Time $= \frac{0.1}{0.02} = 5$ sec.

32. (*c*) Density of air trapped will increase thus block will oscillate with a very small amplitude in opposite phase.

33. (*c*) $P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$

$P_A - P_B = \frac{1}{2}\rho(v_B^2 - v_A^2)$

$= \frac{1}{2}\rho(16-4) = 6\rho$

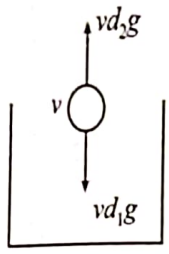
$P_A + \rho g h_1 + \rho g(h_2 - h_1) + \frac{1}{2}\rho v_C^2 = P_B + \rho g h_2 + \frac{1}{2}\rho v_D^2$

$6\rho + \frac{1}{2}\rho v_C^2 = \frac{1}{2}\rho v_D^2$

$v_c = 4$ m/s

$v_D = \sqrt{28}$ m/s

34. (*b*) $u = \sqrt{2gh}$



Retardation $vd_1 = a\, vd_2 g - vd_1 g$

$a = -\left(\frac{d_2 - d_1}{d_1}\right)g$

$0 = \sqrt{2gh} - \left(\frac{d_2 - d_1}{d_2}\right)gt$

$t = \sqrt{\frac{2h}{g}}\frac{d_1}{(d_2 - d_1)}$

35. (*a*) Force of buoyancy = upward force-downward force = mg [condition of flotation]

Upward force

$= \frac{2}{3}\pi R^2(2R)\rho g + P_0 \pi R^2 + \frac{1}{2}\pi R^2(2R - R) \times \rho g$

$= \frac{\pi R^2 \rho g}{3}\left[5 + \frac{3P_0}{R\rho g}\right]$

36. (*a*) volume of cylinder = V

$\rho\left(\frac{3}{4}V\right)g = mg$

$V = \frac{4m}{3p}$

$mg - \frac{\rho}{4}Vg_{eff} - mA = mA/3$

$g_{eff} = g - A$

$mg - \frac{\rho}{4}\left(\frac{4m}{3\rho}\right)(g - A) - mA = mA/3$

or $A = \frac{2g}{3}$

37. (*d*) $\frac{4}{3}\pi(r_1^3 + r_2^3)(\sigma - \rho)g = 6\pi\eta(r_1 + r_2)v$

$\frac{4}{3}\pi r_2^3(\sigma - \rho)g = T + 6\pi\eta r_1 v$

$T = \frac{4}{3}\pi\frac{(r_2^4 - r_1^4)(\sigma - \rho)g}{(r_1 + r_2)}$

38. (*b*) $a = 4$ m (side)

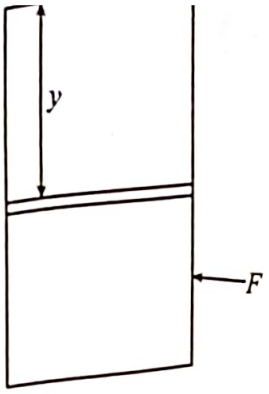
Force acting on any elementary strip at a distance y from O

$dF = (\rho g y)(a dy)$

Torque about O

$d\tau = y(dF)$; $d\tau = (\rho g y^2 a)\, dy$

Net torque $\tau = \int_0^a (\rho g y^2)\, dy = \rho g a \left.\frac{y^3}{3}\right|_0^a$



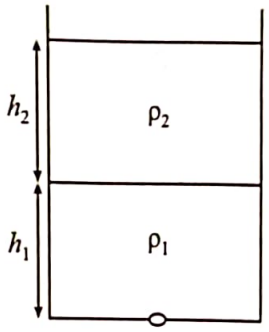
$= \frac{256\rho g}{3}$

Torque due to applied force $\tau = F \times 3$ anticlockwise

For equilibrium $\Rightarrow F \times 3 = \frac{256\rho g}{3}$

$\Rightarrow F = \frac{256}{9}\rho g$

39. (*c*) Applying Bernoullis theorem for point 1 and 2



$\rho_1 g h_1 + \rho_2 g h_2 + \frac{1}{2}\rho v^2 + P_0$

$= 0 + \frac{1}{2}\rho_1 v'^2 + p_0 (v << v')$

$\rho_1 g h_1 + \rho_2 g h_2 = \frac{1}{2}\rho_1 v_1^2$

$v_1 = \sqrt{2g\left[h_1 + h_2\left(\frac{\rho_2}{\rho_1}\right)\right]}$

40. (*a*) Excess of pressure inside the first bubble $= \frac{4T}{r}$. Excess of pressure in the second $= \frac{4T}{R}$

The excess pressure on the two sides of the separating film $= 4T\left(\frac{1}{r} - \frac{1}{R}\right)$

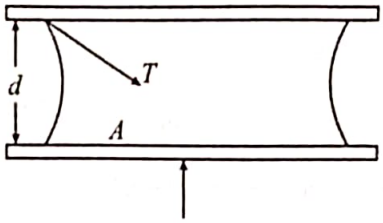
If R' is the radius of the film, then excess pressure inside the film $= \frac{4T}{R'}$

$\frac{1}{R'} = \frac{1}{r} - \frac{1}{R}$

$R' = \frac{Rr}{R-r}$

$R' = \frac{Rr}{R-r}$

41. (*a*) Let R be the radius of circular layer of water. Then $\pi R^2 d\rho = m$



Pressure at $A = P_0 - \frac{2T}{d}$ (meniscus is cylindrical in shape) Pressure between the plates is less than the atmospheric pressure and so the plates are pressed together

F = Force of attraction = $\Delta P \times$ area

$F = \frac{2T}{d} \times \pi R^2 = \frac{2T}{d} \times \frac{m}{d.\rho} = \frac{2Tm}{d^2\rho}$

$= \frac{2 \times 0.2 \times 10^{-3} \times 0.07}{0.01^2 \times 10^{-4} \times 1000} = 2.8$ N

Exercise-3 (JEE Advanced Level)

1. (*a,c*) Surface tension = Surface energy per unit area

r = Radius of each small drop

R = Radius of big drop

$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $R = n^{1/3}$

Initial surface energy $= E_i = n \times 4\pi r^2 \times T = nE$

Final surface energy

$= E_f = 4\pi R^2 \times T = 4\pi r^2 n^{2/3} T$

$\gamma = n^{2/3} E$

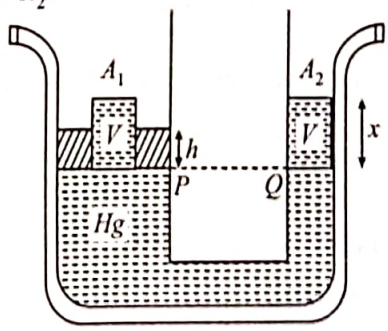
Energy released $= E_i - E_f = E(n - n^{2/3})$

2. (*a,d*) Let V_0 be the volume of iron cube of density ρ_{iron}, $V_0 = a_0^3$

If the liquid does not submerge the cube then by the laws of floatation.

$V_0 \rho_{iron} g = V \rho_{Hg} g + V \rho_{liq} g$

$V = A_2 x$



Equating pressure at P and Q, $h\rho_{liquid} = x\rho_{Hg}$

$x = \frac{h\rho_{liq}}{\rho_{Hg}}$

$$V_0\rho_{iron} = A_2\rho_{Hg}\frac{h\rho_{liq}}{\rho_{Hg}} + V_0^{2/3}h\rho_{liq}$$

$$= h\rho_{liq}\left(A_2 + V_0^{2/3}\right)$$

Height of liquid column, $h = \dfrac{V_0\rho_{iron}}{\rho_{liq}\left(A_2 + V_0^{2/3}\right)}$

If the liquid submerge the cube,

Let V' be the volume of cube inside mercury by the law of floatation.

$$V'\rho_{Hg}g + (V_0 - V')\rho_{liq}g = V_0\rho_{iron}g$$

$$V' = \frac{V_0\left(\rho_{iron} - \rho_{liquid}\right)}{\left(\rho_{Hg} - \rho_{liquid}\right)} = A_2x'$$

Equating pressures $x'\rho_{Hg} = h\,\rho_{liquid}$

Required height of liquid column

$$h = \frac{V_0\left(\rho_{iron} - \rho_{liquid}\right)\rho_{Hg}}{A_2\left(\rho_{Hg} - \rho_{liquid}\right)\rho_{liq}}$$

3. (*b*,*c*)

Initial pressure at the bottom

$= \rho g \times 2H + 2\rho \times g \times H = 4\rho gH$

Final density of the homogenous mixture

$$= \frac{\rho \times A \times 2H + 2\rho \times 2A \times H}{A \times 2H + 2A \times H} = \frac{3}{2}\rho$$

Final pressure $= \dfrac{3}{2}\rho \times g \times 3H = \dfrac{9}{2}\rho gH$

4. (*a*,*c*) In a static fluid, pressure remains same at the same level, i.e., pressure do not vary with x-coordinate.

Hence (*c*).

5. (*a*,*c*,*d*)

$P = r(2h)g$

$\dfrac{F}{A_2} = r(2h)g$

$F_{base} = 2h\,r\,g\,A_2$

$F_{wall} = h\,r\,g\,[A_2 - A_1]$, at the level x

6. (*c*,*d*) Let completely submerged in water, then

$F_b = 1000 > mg\ (920)$ So, not possible

Let complete in oil

$F_b = (0.6)\,(d)\,(1000 + (a)\,(6)\,(100) = 840$

$F_b < mg$ So, not possible

So, let 'x' part in oil and remaining in water

$920 = [(a)\,(10 - x) + (0.6)\,(x)]\ 100$

$9.2 = 10 - x + 0.6\,x$

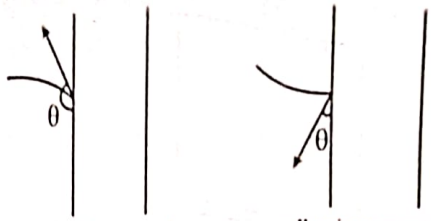
$0.4\,x = 0.8$

$x = 2$ cm.

7. (*b*,*c*) PV = constant

(Assumed isothermal process)

8. (*a*,*b*,*c*) Force of cohesion keeps the molecules of a material bounded together and does not let them stick to the solid as force of adhesion is lesser.



Cohesion more than adhesion adhesion more than cohesion

9. (*a*,*b*,*d*)

Nature of liquid and material tube determine whether force of cohesion is more or force of adhesion is more.

The inner radius also determines the rise of capillary as

$h = \dfrac{2T\cos\theta}{r\rho g}$ depends on radius r.

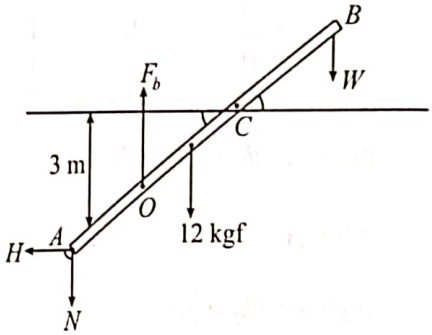
If the length is not sufficient rise will be depends length also.

10. (*a*,*d*) When ever two drops coalesce to make a bigger drop. Surface area is reduced, hence energy is released.

11. (*b*) Mass per unit length of rod is 2 kg.

Mass of submerged portion of rod is 10 kg and its volume is $\dfrac{10}{500}m^2$.

Buoyant force $= F_B = \dfrac{10}{500} \times 1000 = 20\ kgf$



Let N and H be the vertical and horizontal reaches of the hinge on die rod. Considering horizontal and vertical translation equilibrium of rod.

$N + 12 + W = 20$, where W is the weight to be attached at end

$H = 0$

Considering the rotational equilibrium of rod about A

$20 \times g \times 25\cos\theta + 12 \times g \times 3\cos\theta + W \times 6\cos\theta = 0$

$6W = 50g - 36g = 14g$

$W = \dfrac{7}{3}g = \dfrac{7}{3}$kgf

12. (*a*) $N = 8 - \dfrac{7}{3} = \dfrac{17}{3}$kgf

13. (*a*) As, $dm = A\,\rho_W\,v\,dt$

$$\Rightarrow \frac{dm}{dt} = A\rho_W v \Rightarrow \frac{dm}{dt} = v\rho_W\pi\frac{D^2}{4}$$

where 'D' is the diameter of stream.

14. (*d*) $V_1A_1 = V_2A_2$

$$\frac{\pi v_0 D_0^2}{4} = \frac{\pi v D^2}{4} \Rightarrow D = D_0\sqrt{\frac{v_0}{v}}$$

15. (*b*) $v = \sqrt{2gh} = \sqrt{2g(b+x)}$.

16. (*a*) Applying continuity equation at points with diameter D_0 & D:

$$= \sqrt{2gb}.\left[\frac{\pi.D_0^2}{4}\right] = \sqrt{2g(b+x)}\left[\frac{\pi D^2}{4}\right]$$

$$\Rightarrow D = D_0\left[\frac{b}{b+x}\right]^{1/4}$$

17. (*b*) Solving the preceding formula for the tank height h gives:

$h = x(D/D_0)^4 / (1-(D/D_0)^4) = xD^4 / (D_0^4 - D^4)$

substituting the given parameter values gives

$h = (0.3)(0.009^4) / (0.01^4 - 0.009^4) = 0.57$ m

So the height of the water above the tap is 0.57 m or 57 cm.

One way of measuring a person's body fat content is by "weighing" them under water. This works because fat tends to float on water as it is less dense than water. On the other hand muscle and bone tend to sink as they are more dense. Knowing your "weight" under water as well as your real weight out of water, the percentage of your body's volume that is made up of fat can easily be estimated. This is only an estimate since it assumes that your body is made up of only two substances, fat (low density) and everything else (high density). The "weight" is measured by spring balance both inside and outside the water. Quotes are placed around weight to indicate that the measurement read on the scale is not your true weight, i.e. the force applied to your body by gravity, but a measurement of the net downward force on the scale.

18. (*a*) For (*a*), (*b*), and (*c*): The change in apparent weight is equal to change in buoyancy force hence there is no change in the position of block w. r. t water surface.

For (*d*) : As air pressure increases, according to Pascal's law this change in pressure is transmitted to each part of water and hence buoyancy force remains same and hence fraction of block above the water remains same.

19. (*d*) A-(p); B-(q); C-(t); D-(s)

Pressure varies with height $\Rightarrow P = \rho gh$

and is horizontal with acceleration $\Rightarrow P = \rho\ell a$

so on (*a*) ρgh part is zero while average force of ρax is

$$\left[\frac{0+\rho\ell a}{2}\right][\ell^2]$$

$$= \frac{\ell\rho a}{2}(\ell^2) = \frac{(\rho\ell^3)}{2}a = \frac{ma}{2}$$

In (*b*) $\rho\ell a$ part is zero while average force of ρgx is

$$\left[\frac{0+\rho g\ell}{2}\right][\ell^2] = \frac{\rho g}{2}(\ell^3)$$

$$= \frac{\rho(\ell^3)}{2}(g) = \frac{mg}{2}$$

Similarly for other part.

20. (*d*) A-(p); B-(q); C-(r); D-(s)

A. On *ABCD* avg pressure $= \left[\frac{0+\rho_1 gh}{2}\right]$

So $F = \left[\frac{\rho_1 gh}{2}\right][\ell h] = \frac{\rho_1 gh^2\ell}{2}$

B. No contact of ρ_2 and not any pressure on *ABCD* due to ρ_2

C. On *CDEF* due to ρ_1, at every point pressure is $\rho_1 gh$ so average is also $\rho_1 gh$

so $F = (\rho_1 gh)(h\ell) = \rho_1 gh^2\ell$

D. On *CDEF* force due to liquid of density ρ_2 is $\frac{[\rho_2 gh^2\ell]}{2}$

21. [50] $1 = \sqrt{2gh}\,t \Rightarrow h = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2h}{g}} \Rightarrow 1 = \sqrt{2g(1-h)} \times \sqrt{\frac{2h}{g}}$$

$\Rightarrow h = 0.5$ m $= 50$ cm

22. [6×10^9]

$A_1v_1 = A_2v_2$

$3 \times 30 = N \times 3 \times 10^{-7} \times 0.05$

$$\frac{3\times10^8}{0.05} = N \Rightarrow N = 6 \times 10^9$$

23. [800]

In both cases, Weight = Buoyant force

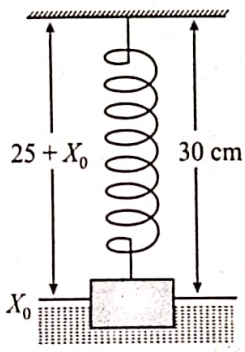
Initially, $\rho_b Vg = \rho_w\left(\frac{2}{3}V\right)g \Rightarrow \rho_b = \frac{2}{3}\rho_w$

After wards, $\rho_b Vg = \rho_{oil}\left(\frac{5V}{6}\right)g$

$$\Rightarrow \frac{2}{3}\rho_w = \rho_{oil} \times \frac{5}{6}$$

$$\Rightarrow \rho_{oil} = \frac{4}{5}\rho_w = \frac{4}{5} \times 100 = 800 \text{ kg/m}^3.$$

24. [5] $30 - (25 + X_0) = 5 - X_0$



$$V = \frac{32}{0.4} = 80 \text{ cc}$$

$A = 16\ \text{cm}^2$

$kX_0 + 10^3 \times 16 \times 10^{-4} \times X_0 \times 10 = 32 \times 10^{-3} \times 10$

$X_0\,(48 + 16) = 32 \times 10^{-2};\ X_0 = \dfrac{32}{64}\ \text{cm} = 5\ \text{mm}$

25. [400]

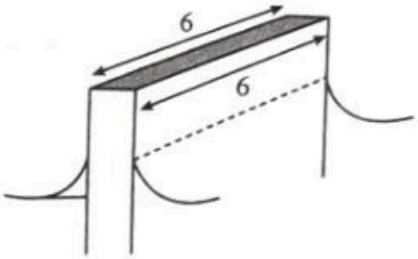
$M = 0.6\text{kg} = 600\ \text{gm}$

$mg + 600\,g = 1000\,g$

$m = 1000 - 600 = 400\ \text{gm}$

26. [0073]

App. wt. = weight in air



$mg - B + F_{\text{surface}} = mg$

$\Rightarrow B = F_{\text{surface}}$

$10^3 \times 3 \times 1.5 \times 0.2 \times 10^{-6} \times 10$

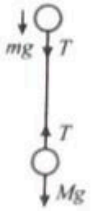
$= S \times \dfrac{6}{100} \times 2 + S \times \dfrac{0.2}{100} \times 2$

$S = \dfrac{9 \times 10^{-3}}{12.4} \times 100 = \dfrac{9000}{124}\ \text{N/m}$

$= 72.59\ \text{m N/m} = 73$

27. [8] $Mg - T - 6\pi\eta r_1 v = 0$

$mg + T - 6\pi\eta r_2 v = 0$



$\dfrac{\frac{4}{3}\pi\left(r_1^3 + r_2^3\right) \times \rho g}{6\pi\eta(r_1 + r_2)} = v$

$v = \dfrac{2}{9}(r_1^2 - r_1 r_2 + r_2^2)\ \dfrac{\rho g}{\eta}$

$T = Mg - 6\pi\eta r_1 \times \dfrac{2}{9}(r_1^2 - r_1 r_2 + r_2^2)\ \dfrac{\rho g}{\eta}$

$= \dfrac{4}{3}\pi r_1^3 \times \rho g - \dfrac{4\pi}{3}\rho g\,[r_1^3 - r_1^2 r_2 + r_2^2 r_1]$

$= \dfrac{4}{3}\pi g r\,[r_1^2 r_2 - r_2^2 r_1]$

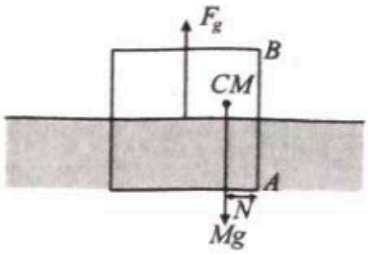
$= \dfrac{4}{3}\pi r g r_1^2 \left[r_2 - \dfrac{r_2^2}{r_1}\right]$

$\dfrac{dT}{dr_2} = 0 \Rightarrow \dfrac{4}{3}\pi g r_1^2 \left[1 - \dfrac{2r_2}{r_1}\right] = 0$

$r_1 = 2r_2$

$\dfrac{M}{m} = 8$

28. [8] The cube will topple when torque of buoyancy exceeds, the torque due to gravity about the side passing through point A.



$F_B \cdot \dfrac{a}{2} \geq Mg \geq N \Rightarrow \dfrac{a}{8} \geq N$

Since the cube just begin to topple when water reaches height $\dfrac{a}{2}$. Hence $N = \dfrac{a}{8}$

29. (*a*) [10] Let the dimension of the block perpendicular to the given figure be h. Consider a strip of with M on the wall of the block m contact with water 1 Hydrostatic force on the strip is $dF = \rho g h(b\,dh)$Torque of this force about rotation axis through A is $d\tau = (z - h)\,dF$

$\tau = \rho g b\left[z\int_0^z h\,dh - \int_0^z h^2\,dh\right] = \rho g b\left[\dfrac{z^3}{2} - \dfrac{z^3}{3}\right]$

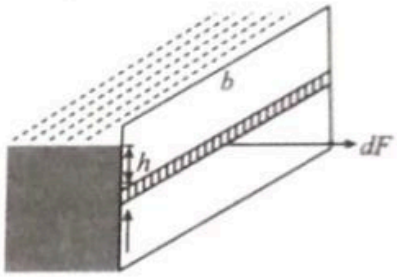
$\rho g b\dfrac{z^3}{6} = \dfrac{\rho g b}{6}\left(\dfrac{3}{4}x\right)^3 = \dfrac{9}{128}\rho g b x^3$

$\tau_g = Mg.\dfrac{y}{2} = (xyb)(2.5\rho)\,g\,\dfrac{y}{2} = \dfrac{5}{4}\rho g b x y^2$

Torque of weight concrete block is

$\therefore \dfrac{5}{4}\rho g b x y^2 = \dfrac{9}{18}\rho g b x^3 \Rightarrow \left(\dfrac{y}{x}\right)^2 = \dfrac{9}{32 \times 5}$

$\Rightarrow \dfrac{y}{x} = \dfrac{3}{4\sqrt{10}}$



(*b*) [7] When waler is present below the block, there will be an upward force due to water pressure equal to

$F_0 = (\rho g z)(by) = \dfrac{3}{4}\rho g b x y$

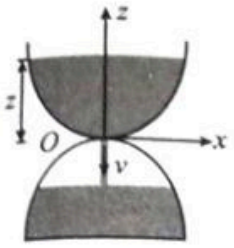
The force may be assumed to be effectively acting along the central line

$\therefore$ In critical case $\tau = \tau_g - F\dfrac{y}{2}$

$\dfrac{9}{128}\rho g b x^3 = \dfrac{5}{4}\rho g b x y^2 - \dfrac{3}{8}\rho g b x y^2$

$\Rightarrow \dfrac{9}{128}x^2 = \dfrac{7}{8}y^2 \Rightarrow \dfrac{y}{x} = \dfrac{3}{4\sqrt{7}}$

30. [4]



Let us choose a co-ordinate system as shown. When height of water surface above O is z, the speed of efflux is $v = \sqrt{2gz}$

If area of orifice is A_0, the flow rale is $\frac{dQ}{dt} = \pi x^2 \left|\frac{dz}{dt}\right|$

$\therefore -\pi x^2 \frac{dz}{dt} = \sqrt{2gz}.A_0$

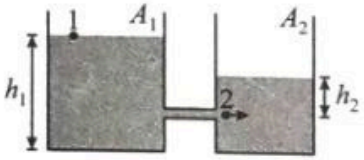
Let x be the radius of cross section at height z. If water level falls by dz in interval dt then

It is desired that a constant $= v_0$ (say)

$\therefore \sqrt{2gz}A_0 = \pi x^2 \cdot v_0$

$\therefore z = kx^4$ where $k = \frac{\pi^2 v_0^2}{2gA_0^2}$

31. [2]



Applying Bernoulli's equation along a streamline between point I and 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2$$

$$\frac{P_0}{\rho g} + 0 + h_1 = \frac{P_0 + \rho g h_2}{\rho g} + \frac{V_2^2}{2g} + 0$$

$$\therefore h_1 = h_2 + \frac{V_2^2}{2g} \Rightarrow V_2 = \sqrt{2g(h_1 - h_2)}$$

[P_0 Atmospheric Pressure]

$h_1 - h_2 = h \therefore V_2 = \sqrt{2gh}$

Now, h_1 is decreasing, h_2 is increasing and h is decreasing.

$\therefore -dh_1 + dh_2 = -dh$

And $Q = -A_1 \frac{dh_1}{dt} = A_2 \frac{dh_2}{dt}$

From (1) and (2) $-dh_1 - \frac{A_1}{A_2} dh_1 = -dh \therefore dh_1 = \frac{A_2 dh}{A_1 + A_2}$

$$\therefore Qdt = -\frac{A_1 A_2 dh}{A_1 + A_2} \Rightarrow A_0\sqrt{2gh}dt = -\frac{A_1 A_2}{A_1 + A_2} dh$$

$$\int_0^t dt = -\frac{A_1 A_2}{A_0(A_1 + A_2)\sqrt{2g}} \int_h^0 \frac{dh}{\sqrt{h}}$$

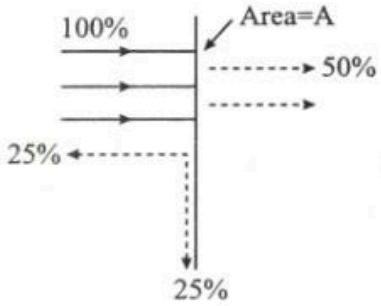
$$t = \frac{2A_1 A_2}{\sqrt{2g}A_0(A_1 + A_2)}\sqrt{h} = \sqrt{\frac{2}{g}}\frac{A_1 A_2}{A_0(A_1 + A_2)}\sqrt{h_1 - h_2}$$

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*b*) $F = \frac{1}{4} \times \rho A v^2 + \frac{1}{4} 2\rho A v^2$

$$\Rightarrow \frac{F}{A} = \frac{\frac{3}{4}\rho A v^2}{A} = \frac{3}{4}\rho v^2$$



2. (*b*) $A\rho g x = F_{\text{restoring}}$

$\pi r^2 \rho g x = m\omega^2 x$

$$\therefore \quad \omega = \sqrt{\frac{\pi r^2 \rho g}{\rho v}}$$

$$= r\sqrt{\frac{\pi g}{v}} = 2.5 \times 10^{-2}\sqrt{\frac{3.14 \times 10}{310 \times 10^{-6}}}$$

$= 2.5 \times 10^{-2} \times 10^2 \sqrt{10}$

$\omega = 2.5 \times \sqrt{10}$

$$\therefore \quad f = \frac{2.5 \times \sqrt{10}}{2\pi} = 1.25 \text{ rad/sec}$$

3. (*a*) $y = \frac{\omega^2 x^2}{2g} = \frac{(2 \times 2\pi)^2 \times (0.05)^2}{2g}$

$= 25 \times 8 \times 10^{-4} = 2$ cm

4. (*a*) $\frac{dV}{dt} = \phi - a\sqrt{2hg} = 0$ (for maximum height)

$$h = \frac{\phi^2}{2ga^2} = \frac{10^{-8}}{2 \times 9.8 \times 10^{-8}} = 5.1 \text{ cm}$$

5. (*b*) $\frac{dV}{dt} = Av \Rightarrow \frac{dV}{dt} = A\sqrt{2gh}$

$$\Rightarrow \frac{0.74}{60} = (3.14)\left(\frac{2}{100}\right)^2 \sqrt{2(9.8)h} \Rightarrow h = 4.92 \text{ m}$$

6. (*c*) In Ist situation

$V_b \rho_b g = V_s \rho_w g$

$$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad \text{...(i)}$$

Here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water

& Let ρ_o is density of oil finally in equilibrium condition

$$V_b\rho_b g = \frac{V_b}{2}\rho_0 g + \frac{V_b}{2}\rho_w g$$

$$2\rho_b = \rho_0 + \rho_w$$

$$\Rightarrow \frac{\rho_0}{\rho_w} = \frac{3}{5} = 0.6$$

7. (*c*) Reynolds Number $= \dfrac{\rho v d}{\eta}$

Volume flow rate $= v \times \pi r^2$

$$v = \frac{100\times10^{-3}}{60} \times \frac{1}{\pi\times25\times10^{-4}}$$

$$v = \frac{2}{3\pi} \text{ m/s}$$

Reynolds Number $= \dfrac{10^3\times2\times10\times10^{-2}}{10^{-3}\times3\pi} \cong 2\times10^4$

order 10^4

8. (*c*) $P_0 + \rho g d_1 = P_1$

$P_0 + \rho g d_2 = P_2$

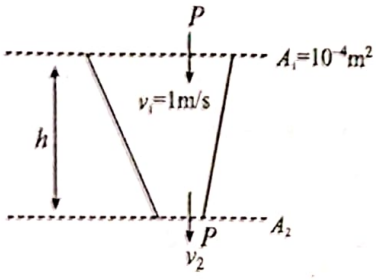
$\rho g(d_2 - d_1) = P_2 - P_1$

$10^3 \times 10\,(d_2 - d_1) = 3.03 \times 10^6$

$d_2 - d_1 = 303$ m

$\simeq 300$ m

9. (*b*)



$A_1v_1 = A_2v_2$

$10^{-4} \times 1 = A_2v_2$

$A_2v_2 = 10^{-4}$ ...(*a*)

$$P + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g h = P$$

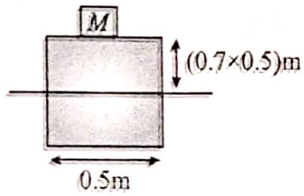
$$v_2^2 = v_1^2 + 2gh$$

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{1+2\times10\times0.15}$$

$$\frac{10^{-4}}{A_2} = 2$$

$A_2 = 5\times10^{-5}$ m^2

10. (*b*)



$M = \rho_L\,[0.5 \times 0.5 \times 0.35]$

$= 10^3[\,0.0875]$

$M = 87.5$ kg

11. (*a*) Using equation of continuity

$A_1V_1 = A_2V_2$

$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

12. (*d*) $\rho = \rho_0\left(1 - \dfrac{r^2}{R^2}\right) \quad 0, r \le R$

$mg = B$

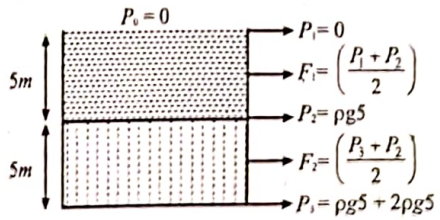
$$\int_0^R \rho\left(4\pi r^2 dr\right)\times g = \rho_L \frac{4}{3}\pi R^3 \times g$$

$$\int_0^R \rho_0\left(1 - \frac{r^2}{R^2}\right)4\pi r^2 dr = \rho_L \frac{4}{3}\pi R^3$$

$$\int_0^R \rho_0 4\pi\left(r^2 - \frac{r^4}{R^2}\right)dr = \rho_0 4\pi\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R = \rho_L \frac{4}{3}\pi R^3$$

$$\frac{2}{5}\rho_0 = \rho_L$$

13. (*c*)



$$\frac{F_1}{F_2} = \frac{1}{4}$$

14. (*a*) Using equation of continuity

$40\,V_A = 20\,V_B$

$\Rightarrow 2V_A = V_B$

Using Bernoullies equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$\Rightarrow P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

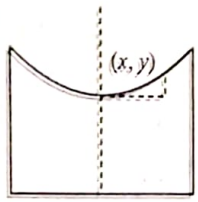
$$\Rightarrow \Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$$

$$\Rightarrow \Delta P = 500\times\frac{3V_B^2}{4}$$

$$\Rightarrow V_B = \sqrt{\frac{(\Delta P)\times4}{1500}} = \sqrt{\frac{(700)\times4}{1500}}\,m/s$$

Volume flow rate $= 20 \times 100 \times V_B = 2732$ cm^3/s

15. (*d*)



$$y = \frac{\omega^2 x^2}{2g}$$

$x = R$

$\Rightarrow y = \dfrac{25\omega^2}{2g}$

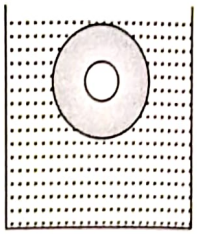
16. (*b*) |Change in energy| $= U_i - U_f$

$= (dsx_1)g\,\dfrac{x_1}{2} + (dsx_2)g\,\dfrac{x_2}{2}$

$- 2 \times \left\{ds\left(\dfrac{x_1 + x_2}{2}\right)\right\} g\left(\dfrac{x_1 + x_2}{4}\right)$

$= \dfrac{1}{4} gds(x_2 - x_1)^2$

17. (*d*) $\dfrac{4}{3}\pi R^3 \times \rho_w \times g = \dfrac{4}{3}\pi (R^3 - r^3)\rho_m \times g$



$\Rightarrow R^3 = (R^3 - r^3) \times \dfrac{27}{8}$

$\Rightarrow r = \left(\dfrac{19}{27}\right)^{\frac{1}{3}} R \approx \dfrac{8}{9} R$

18. (*a*) $P + \dfrac{1}{2}\rho v^2 = \dfrac{P}{2} + \dfrac{1}{2}\rho V^2$

$\Rightarrow V = \sqrt{\dfrac{P}{\rho} + v^2}$

19. (*d*) $V = kt = \dfrac{4}{3}\pi R^3$

$R = \left(\dfrac{3k}{4\pi}\right)^{1/3}, \quad P_{in} = P_{atm} + \dfrac{4T}{R}$

20. (*c*) Height of liquid rise in capillary tube

$h = \dfrac{2T\cos\theta_C}{\rho r g}$

$\Rightarrow h \propto \dfrac{1}{r}$

when radius becomes double height become half

$\therefore h' = \dfrac{h}{2}$

Now, $M = \pi r^2 h \times \rho$

and $M' = \pi (2r)^2 (h/2) \times \rho = 2M$

21. (*d*) We have

$V_T = \dfrac{2}{9}\dfrac{r^2}{\eta}(\rho_0 - \rho_\ell) g \Rightarrow v_T \propto r^2$

Since mass of the sphere will be same

$\therefore \rho\dfrac{4}{3}\pi R^3 = 27.\dfrac{4}{3}\pi r^3 \rho \Rightarrow r = \dfrac{R}{3}$

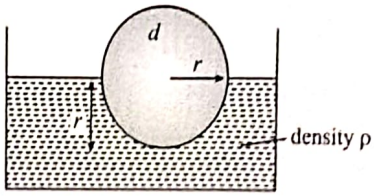
$\therefore \dfrac{v_1}{v_2} = \dfrac{R^2}{r^2} = 9$

22. (*c*) $h = \dfrac{2S_1\cos\theta_1}{r_1\rho_1 g}$

$h = \dfrac{2S_1\cos\theta_2}{r_2\rho_2 g}$

$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{2}{5}$

23. (*b*)



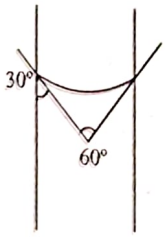
$dVg = \rho\left(\dfrac{V}{2}\right)g + T(2\pi r)$

$\Rightarrow d.\dfrac{4}{3}\pi r^3 g = \rho.\dfrac{2}{3}\pi r^3 g + 2\pi r T$

$\Rightarrow \dfrac{2}{3} r^2 g(2d - \rho) = 2T$

$\Rightarrow r = \sqrt{\dfrac{3T}{(2d - \rho)g}}$

24. (*b*) Angle of contact = 30°



$h = \dfrac{2T\cos\theta}{r\rho g} = \dfrac{2 \times 0.05 \times \left(\dfrac{\sqrt{3}}{2}\right)}{0.15 \times 10^{-3} \times 667 \times 10} = 0.087$ m

25. [101] $h = \dfrac{2T\cos\theta}{\rho g r}$

$\Rightarrow T = \dfrac{\rho g r \times h}{2 \times \cos\theta} = \dfrac{900 \times 10 \times 15 \times 15}{100 \times 1000 \times 100 \times 2}$

$\Rightarrow T = 0.10125$ Nm^{-1}

$\Rightarrow T = 101.25 \approx 101$ milli Newton m^{-1}

26. (*d*) $P_1 = P_0 + \dfrac{4T}{r_1} \Rightarrow P_2 = P_0 + \dfrac{4T}{r_2}$

$\dfrac{r_1}{r_2} = 2 \Rightarrow \dfrac{V_1}{V_2} = 8$

27. (*d*) $\dfrac{4}{3}\pi r^3 \rho_w g - mg = ma$

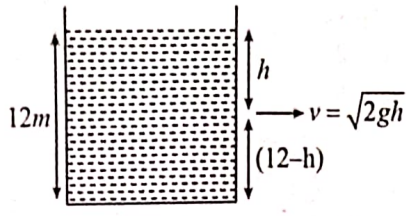
$\Rightarrow m = \dfrac{\dfrac{4}{3}\pi r^3 \rho_w g}{g + a} = 4.15$ gm

28. (*a*) $v_T \propto r^2$

$\sqrt{2gh} \propto r^2$

$\Rightarrow h \propto r^4$

29. [6]



The velocity is given as

$v = \sqrt{2gh}$

Let the hole was made at height below the water level

Then, $H - h = \frac{1}{2}gt^2$

By using the equation

$v = \frac{d}{t} \Rightarrow R = v \times t$

By substituting the time from the above equation we get

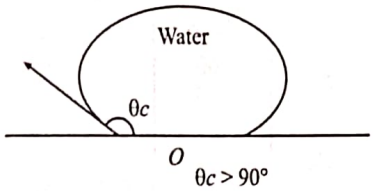
$R = \sqrt{2gh} \times \sqrt{\frac{(12-h)\times 2}{g}}$

$\sqrt{4h(12-h)} = R$

For maximum R

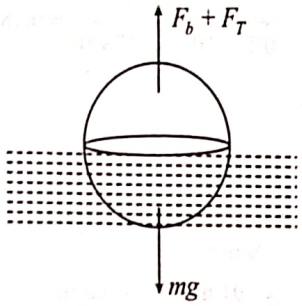
$\frac{dR}{dh} = 0 \quad \Rightarrow h = 6m$

30. (*a*)



For water oil interface

31. (*a*)



Buoyant force + surface tension = mg

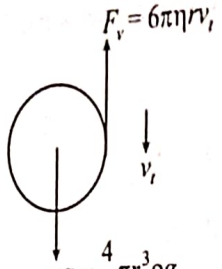
$\sigma\frac{V}{2}g + 2\pi RT = \rho Vg$

$2\pi RT = \frac{(2\rho-\sigma)}{2}\frac{4}{3}\pi R^3 g; \left[V = \frac{4}{3}\pi R^3\right]$

$R^3 = \frac{3T}{(2\rho-\sigma)g} \Rightarrow R = \sqrt{\frac{3\times 7.5\times 10^{-2}\,N-m^{-1}}{(2\rho-\sigma)\times 10}}$

$R = \frac{3}{20\sqrt{(2\rho-\sigma)}}\text{m} = \frac{15}{\sqrt{2\rho-\sigma}}\text{cm}$

32. (*d*)



$mg = \frac{4}{3}\pi r^3 \rho g$

$6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g$

$v_t = \frac{4}{3}\times\frac{\pi r^3 \rho g}{6\pi\eta r}$

$= \frac{2\times 10^{-12}\times 10^3\times 10}{9\times 1.8\times 10^{-5}} = 123.4\times 10^{-6}\, m/s$

33. (*c*) Initial surface energy = TA

Where T is surface tension and A is surface area

$U_i = \left(\frac{75\times 10^{-5}}{10^{-2}}\frac{N}{m}\right)\times\left[4\pi\left(1\times 10^{-2}\right)^2\right]$

$= 75\times 10^{-3}\times 4\pi\times 10^{-4} = 942\times 10^{-7}\, J$

To get final radius of drops by volume conservation

$\frac{4}{3}\pi R^3 = 729\left(\frac{4}{3}\pi r^3\right)$

R = Initial radius

r = final radius

$r = \frac{R}{(729)^{1/3}} = \frac{R}{9} = \frac{1}{9}\text{cm}$

Final surface energy

$U_f = 729[TA]$

$= 729\left[\frac{75\times 10^{-5}}{10^{-2}}\frac{N}{m}\right]\times\left[4\pi\left(\frac{1}{9}\times 10^{-2}\right)^2\right]$

$= 729\left[75\times 10^{-3}\times\frac{4\pi\times 10^{-4}}{81}\right] = 9\left[942\times 10^{-7}\, J\right]$

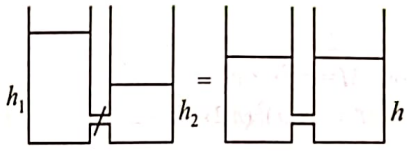
Gain in surface energy

$\Delta U = 9\times 942\times 10^{-7} - 942\times 10^{-7}$

$= 8\times 942\times 10^{-7}\text{ J} = 7536\times 10^{-7}\text{ J}$

$= 7.5\times 10^{-4}\text{J}$

34. (*b*)



$h = \frac{h_1 + h_2}{2}$

Now, $W = U_i - U_f$

$W = (\rho A h_1)g\frac{h_1}{2} + (\rho A h_2)g\frac{h_2}{2} - \rho A(h_1 + h_2)g$

$\left(\frac{h_1 + h_2}{4}\right)$

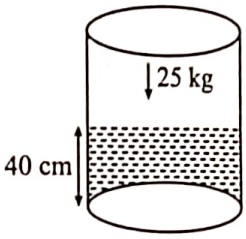
$$W = \frac{\rho A g}{2}\left[h_1^2 + h_2^2 - \frac{(h_1+h_2)^2}{2}\right]$$

$W = 1\text{J}$

35. (*b*) $Fv = mg - FB$

$$= mg - \left(\frac{m}{d_1} \times d_2\right)g = mg\left(1 - \frac{d_2}{d_1}\right)$$

36. [300]



$$P_0 + \frac{250}{0.5} + \rho g\left(40 \times 10^{-2}\right) = P_0 + \frac{1}{2}\rho v^2$$

$$500 + \frac{1000 \times 10 \times 40}{100} = \frac{1}{2} \times 1000 \times v^2$$

$V = 3$ m/s

$V = 300$ cm/s

37. [20] Speed after falling through height h Should be equal to terminal velocity

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2(d-\rho)g}{\eta}$$

$$\sqrt{2gh} = \frac{2}{9}\frac{10^{-8}(10000-1000)\times 10}{10^{-5}}$$

$$= \frac{2}{9} \times 10^{-8}\frac{9 \times 10^4}{10^{-5}} = 20$$

$2 \times 10 \times h = 400$

$h = 20$ m

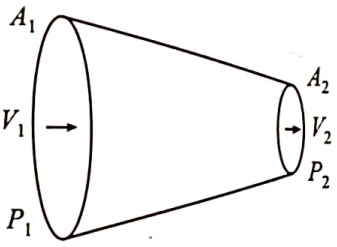
38. [100] $F = \eta A \frac{\Delta v_x}{\Delta y}$

$$\frac{F}{A} = \eta \frac{\Delta v_x}{\Delta y}$$

$$\Rightarrow 10^{-3} = 10^{-2} \times \frac{36 \times 1000}{h \times 3600}$$

$$\Rightarrow h = 10^{-2} \times \frac{36 \times 1000}{10^{-3} \times 3600} = 100\text{m}$$

39. [24]



$$A_2 = \frac{A_1}{2}$$

$P_1 - P_2 = 4500\,Pa$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$A_1 V_1 = A_2 V_2$

$$4500 = \frac{1}{2} \times 750 \times 3V_1^2$$

$V_1 = 2$ m/s

$= A_1 V_1 = 24 \times 10^{-3}\ \text{m}^3\,\text{s}^{-1}$

40. [363]

From continuity equation $av_1 = \frac{a}{2}v_2$

$v_2 = 2v_1$

From Bernoulli's theorem,

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$4100 = 800\left[\left(\frac{4v_1^2 - v_1^2}{2}\right) + 10 \times (0-1)\right]$$

$$\frac{41}{8} + 10 = \frac{3v_1^2}{2}$$

$$v_1 = \sqrt{\frac{121}{4 \times 3} \times \frac{3}{3}}$$

$$v_1 = \frac{\sqrt{363}}{6}\text{m/s}$$

$X = 363$

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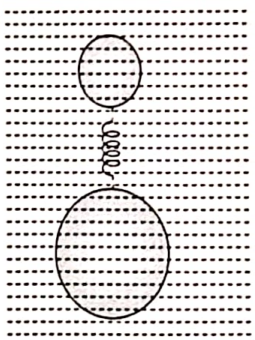
41. (*a,d*)



On small sphere

$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g \quad \text{...(i)}$$

on second sphere (large)

$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx \quad \text{...(ii)}$$

by equation (i) and (ii)

$$x = \frac{4\pi R^3 \rho g}{3k}$$

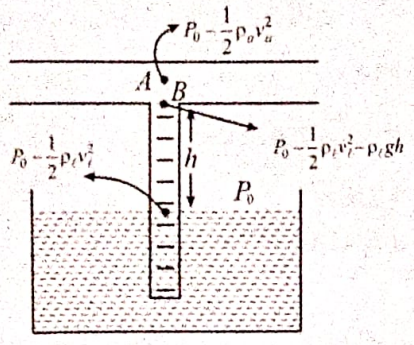
42. (*c*) $A_1 V_1 = A_2 V_2$

$A_1 = 400\,A_2$

$400\,(5 \times 10^{-3}) = V_2$

$\Rightarrow \boxed{V_2 = 2\ m/s}$

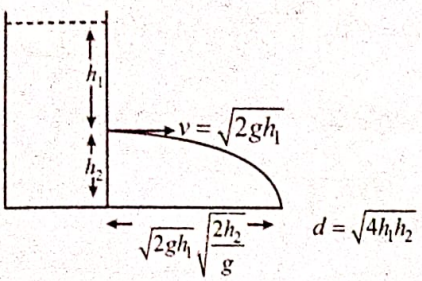
43. (*a*) Pressure at A and B will be same



$$P_0 - \frac{1}{2}\rho_a v_a^2 = P_0 - \frac{1}{2}\rho_\ell v_\ell^2 - \rho_\ell gh$$

$$v_\ell = \sqrt{\frac{\rho_a}{\rho_\ell} v_a - 2gh}$$

44. (*c*) When lift is at rest:



(P) $g_{eff} > g$ $d = \sqrt{4h_1h_2} = 1.2$ m

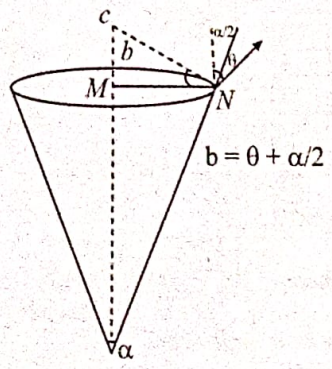
(Q) $g_{eff} < g$ $d = \sqrt{4h_1h_2} = 1.2$ m

(R) $g_{eff} = g$ $d = \sqrt{4h_1h_2} = 1.2$ m

(S) $g_{eff} = o$ No water leaks out of the jar.

(*c*) P—1; Q—1; R—1; S—4

45. (*d*)



ΔCMN

$$\cos\left(\theta + \frac{\alpha}{2}\right) = \frac{b}{R};\ P_o - \frac{2s}{R} + h\rho g = P_o$$

$$h\rho g = \frac{2s}{R} \qquad h = \frac{2s}{\rho g R}$$

$$h = \frac{2s}{\rho g b} \times \cos(\theta + \alpha/2)$$

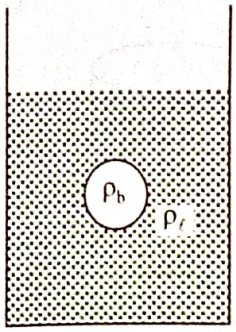
46. (*a,d*) Consider a body of density ρ_b kept in density ρ_ℓ whose viscosity is h and terminal velocity V. Then

$$\vec{F}_{viscous} + \vec{F}_{mg} + \vec{F}_{buoyancy} = 0$$

$$\vec{F}_{viscous} + \rho_b \frac{4}{3}\pi R^3(-\hat{j}) + \rho\ell \frac{4}{3}\pi R^3(\hat{j}) = 0$$

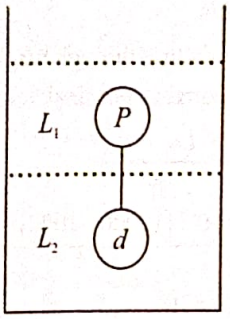
$$\therefore\ \vec{F}_{viscous} = (\rho_b - \rho_\ell)\frac{4}{3}\pi R^3(\hat{j})$$

$$\Rightarrow 6\pi\eta RV = (\rho_b - \rho_\ell)\frac{4}{3}\pi R^3$$



$\therefore$ If $\rho_b > \rho_1$ then $\vec{F}_{viscous} \uparrow V \propto \frac{1}{\eta}$ &

if $\rho_b < \rho_\ell$ $\vec{F}_{viscous} \downarrow$



As per given diagram we can say

$\sigma_2 > \sigma_1$; $\rho_1 < \sigma_1$ & $\rho_2 > \sigma_2$

$\Rightarrow \rho_2 > \sigma_2 < \sigma_1 > \rho_1$

$\therefore$ If we put P in L_2 where $|\vec{V}_P| \propto \frac{1}{\eta_2}$ when $\rho_1 < \sigma_2$

$\therefore \vec{F}_{viscous} \uparrow \ \therefore \vec{V}_P \downarrow$

$\therefore$ If we put Q in L_1 where $|\vec{V}_Q| \propto \frac{1}{\eta_1}$ when $\rho_2 < \sigma_1$

$\therefore \vec{F}_{viscous} \uparrow$

$\therefore \vec{V}_P \downarrow$

$$\Rightarrow \frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}\ \&\ \vec{V}_P \cdot \vec{V}_Q < 0$$

47. [3] $V_T = \frac{2r^2 g(\sigma - \rho)}{9\eta}$

$$\therefore \frac{V_P}{V_Q} = \frac{1^2(8 - 0.8)}{3 \times \left(\frac{1}{2}\right)^2 (8 - 1.6) \times \frac{1}{2}} = 3$$

48. [6] $R = K^{1/3} r$

$\Delta U = S.K.4\pi r^2 - S.4\pi R^2$

$$\Delta U = 4\pi S\left[K.\frac{R^2}{K^{2/3}} - R^2\right]$$

$= 0.1 \times 10^{-4}[K^{1/3} - 1] = 10^{-3}$

$K^{1/3} - 1 = 10^2$

$K^{1/3} = 101 = (10^\alpha)^{1/3}\ \alpha = 6$

49. (*a*,*c*) $\frac{2\sigma}{R} = \rho g h$ [R = Radius of meniscus]

$h = \frac{2\sigma}{R\rho g}$ $\quad R = \frac{r}{\cos\theta}$

[r = radius of capillary, θ = contact angle]

$h = \frac{2\sigma\cos\theta}{r\rho g}$

(*a*) for given material, θ = constant

$h \propto \frac{1}{r}$

(*b*) h depend on σ

(*c*) if lift is going up with constant acceleration,

$g_{off} = (g + a)$

$h = \frac{2\sigma\cos\theta}{r\rho(g+a)}$ It means h decreases.

(*d*) h is proportional to $\cos\theta$ Not θ

50. (*a*,*c*,*d*) Viscous force is given by $F = -\eta A\frac{dv}{dy}$ since h is very small therefore, magnitude of viscous force is given by

$F = \eta A\frac{Dv}{\Delta y}$

$\therefore F = \frac{\eta A u_0}{h} \Rightarrow F \propto \eta\ \&\ F \propto u_0;\ F \propto \frac{1}{h}, F \propto A$

Since plate is moving with constant velocity, same force must be acting on the floor.

51. (*a*,*c*,*d*) $h = \frac{2T\cos\theta}{\rho g R}$; $h_1 = \frac{2\times 0.075\times\cos 0^\circ}{1000\times 10\times 0.2\times 10^{-3}}$

$\Rightarrow h_1 = 75$ mm (in T_1) [If we assume entire tube of T_1]

$\Rightarrow h_2 = \frac{2\times 0.075\times\cos 60^\circ}{1000\times 10\times 0.2\times 10^{-3}} = 37.5$ mm (in T_2)

[If we assume entire tube of T_2]

Option (*a*): Since contact angles are different so correction in the height of water column raised in the tube will be different in both the cases, so option (*a*) is correct

Option (*b*): If joint is 5 cm is above water surface, then lets say water crosses the joint by height h, then:

$\Rightarrow P_0 - \frac{2T}{r} + \rho g h + \rho g \times 5\times 10^{-2} = P_0$

$\Rightarrow \cos\theta = \frac{R}{r},\ r = \frac{R}{\cos\theta}$

$\Rightarrow \rho g(h + 5\times 10^{-2}) = \frac{2T\cos\theta}{R}$

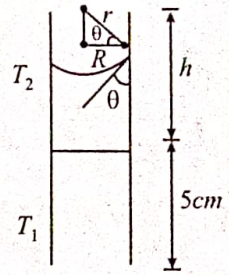
$\Rightarrow h = \frac{2\times 0.075\times\cos 60}{0.2\times 10^{-3}\times 1000\times 10} - 5\times 10^{-2}$

$\Rightarrow h$ = –ve, not possible, so liquid will not cross the interface, but angle of contact at the interface will change, to balance the pressure,

So option (*b*) is wrong.

Option (*c*): If interface is 8 cm above water then water will not even reach the interface, and water will rise till 7.5 cm only in T_1, so option (*c*) is right.

Option (*d*): If interface is 5 cm above the water in vessel, then water in capillary will not even reach the interface. Water will reach only till 3.75 cm, so option (*d*) is right.



52. (*b*) $h_1 + h_2 = 0.29 \times 2 + 0.1$

$h_1 + h_2 = 0.68$ ...(*a*)

$\Rightarrow P_0 + \rho_k g(0.1) + \rho_w g(h_1 - 0.1)$

[ρ_k = density of kerosene and ρ_w = density of water]

$-\rho_w g h_2 = P_0$

$\Rightarrow \rho_k g(0.1) + \rho_w g h_1 - \rho_w g \times (0.1) = \rho_w g h_2$

$\Rightarrow 800 \times 10 \times 0.1 + 1000 \times 10 \times h_1$

$-1000 \times 10 \times 0.1 = 1000 \times 10 \times h_2$

$\Rightarrow 10000\,(h_1 - h_2) = 200$

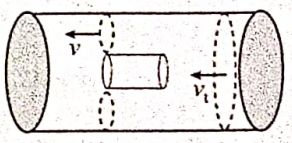
$\Rightarrow h_1 - h_2 = 0.02$ ...(*b*)

$\Rightarrow h_1 = 0.35$

$\Rightarrow h_2 = 0.33$

So, $\frac{h_1}{h_2} = \frac{35}{33}$

53. [9]



with respect to train

Applying Bernoulli's equation

$P_0 + \frac{1}{2}\rho v_t^2 = P + \frac{1}{2}\rho v^2$

$P_0 - P = \frac{1}{2}\rho\left(v^2 - v_t^2\right)$ ...(i)

From equation of continuity

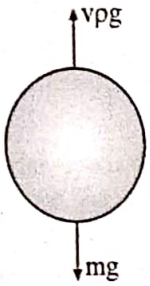
Also, $4S_t v_t = v \times 3S_t \Rightarrow v = \frac{4}{3}v_t$ ...(ii)

From (i) and (ii)

$$P_0 - P = \frac{1}{2}\rho\left(\frac{16}{9}v_t^2 - v_t^2\right) = \frac{1}{2}\rho\frac{7v_t^2}{9}$$

$\therefore\ N = 9$

54. [4]



$480 \times g = v\rho_1 g$

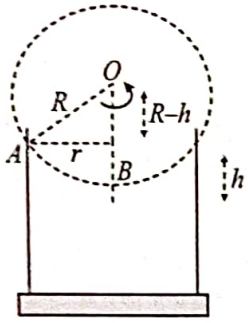
$(480 - N)g = v\rho_2 g$

$$\frac{480 - N}{480} = \frac{\rho_2}{\rho_1}$$

$$\left(1 - \frac{N}{480}\right) = \frac{e^{-h_2/h_0}}{e^{-h_1/h_0}} = e^{\frac{h_1 - h_2}{h_0}} = e^{\frac{50}{6000}}$$

$$1 - \frac{N}{480} = 1 - \frac{50}{6000} \Rightarrow N = \frac{50 \times 480}{6000} = 4$$

55. (a,d)



In ΔOAB

$R^2 = (R - h)^2 + r^2$

$R^2 = R^2 - 2hR + h^2 + r^2$

$\Rightarrow\ 2hR = h^2 + r^2$

$$\Rightarrow R = \frac{h^2 + r^2}{2h}$$

Now considering equation of surface

$$y = y_0 + \frac{\omega^2 r^2}{2g}$$

$$h = \frac{\omega^2 r^2}{2g}$$

Now using: $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{1}{v} + \frac{4}{3(H - h)} = \frac{1 - 4/3}{-R}$$

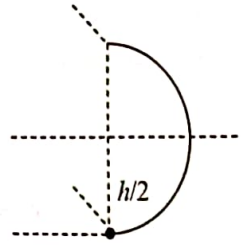
$$\Rightarrow \frac{1}{v} = \frac{1}{3R} - \frac{4}{3H}$$

$$\Rightarrow \frac{1}{v} = \frac{2h}{3r^2} - \frac{4}{3H}$$

$$\Rightarrow \frac{1}{v} = -\frac{4}{3H}\left[1 - \frac{\omega^2 H}{4g}\right]$$

$$\Rightarrow v = \frac{3H}{4}\left[1 + \frac{\omega^2 H}{4g}\right]^{-1}$$

56. [3.74]



Pressure at the bottom of disc = pressure due to surface tension

$$\rho g h = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$R_1 >>> R_2$

So $\frac{1}{R_1} <<< \frac{1}{R_2}$ and $R_2 = h/2$

$$\therefore\ \rho g h = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = T\left(0 + \frac{1}{h/2}\right)$$

$$h^2 = \frac{2T}{\rho g}$$

$$h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{2 \times 0.07}{10^3 \times 10}} = \sqrt{\frac{14 \times 100}{10^4 \times 100}}$$

$h = \sqrt{14}mm = 3.741$

57. (a,b,c) $n_1 >> (n_1 - n_2) = \Delta n$

$$p_1 = \frac{n_1 RT}{N_A} \quad p_2 = \frac{n_2 RT}{N_A}$$

$F = (n_1 - n_2)k_B TS = \Delta n k_B TS$ ...(a)

$$V = \frac{\Delta n k_B TS}{\beta}$$

Force balance ⇒ Pressure × Area = Total number of molecules × βv

$\Delta n k_B TS = \Delta n_1 S \beta v$

$\Rightarrow n_1 \beta v \ell = \Delta n k_B T$ ...(b)

Total number of molecules/sec $= \frac{(n_1 v\, dt)S}{dt}$

$$= n_1 v S = \frac{\Delta n k_B T v S}{\beta v \ell}$$

$$= \left(\frac{\Delta n}{\ell}\right)\left(\frac{k_B T}{\beta}\right)S \quad ...(c)$$

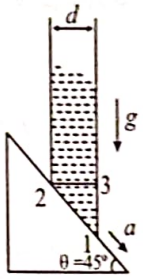
As Δn will decrease with time therefore rate of molecules getting transfer decreases with time.

58. (a,c) $\therefore\ P_1 - P_3 = \rho\left(g - \frac{a}{\sqrt{2}}\right)d$

$$P_2 - P_3 = \rho\frac{a}{\sqrt{2}}d$$

$$\therefore\ P_1 - P_2 = \rho d\left[g - \frac{2a}{\sqrt{2}}\right]$$

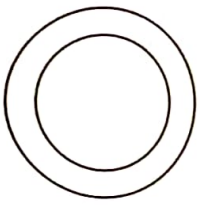
$$\therefore\ \frac{P_1 - P_2}{\rho g d} = \left[1 - \sqrt{2}\frac{a}{g}\right] = \beta$$



$\therefore$ if $\beta = 0,\ a = \frac{g}{\sqrt{2}}$ ...(A)

$\beta = \frac{\sqrt{2}-1}{2},\ a = \frac{g}{2}$ ...(C)

59. (c,d)



$$P_{gas} = P_a + \frac{4S}{r}$$

PV^{γ} = constant [adiabatic process]

$$\left(Pa_1 + \frac{4S}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right)^{5/3} = \left(P_{a_2} + \frac{4S}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)^{5/3}$$

$$\frac{r_1^3}{r_2^3} = \left(\frac{P_{a_2} + \frac{4S}{r_2}}{P_{a_1} \frac{4S}{r_1}}\right)$$

$P^{1-\gamma}T^{\gamma}$ = constant

$$\left(P_{a_2} + \frac{4S}{r_2}\right)^{1-5/3} T_2^{5/3} = \left(P_{a_1} + \frac{4S}{r_1}\right)^{1-5/3} T_1^{5/3}$$

$$\left(\frac{T_2}{T_1}\right)^{5/2} = \left(\frac{P_{a_2} + \frac{4S}{r_2}}{P_{a_1} + \frac{4s}{r_1}}\right)$$

$$\left(\frac{T_2}{T_1}\right)^{5/3} = \left(\frac{P_{a_1} + \frac{4S}{r_1}}{P_{a_2} + \frac{4S}{r_2}}\right)$$

CHAPTER

15 Thermal Properties of Matter

Exercise-1 (Topicwise)

1. (*c*) Not Reflected and Not Refracted.

2. (*d*) $\frac{dQ}{dt} = eA\sigma T^4$

So, $\frac{dQ}{dt} \propto A$

$\propto e$ (nature of surface)

$\propto T$ (temperature)

But independent of mass.

3. (*d*) $\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}\left(T^4 - T_0^4\right)$

4. (*a*) Thermal stress = $Y\alpha\Delta\theta$

$= 1.2 \times 10^{11} \times 1.1 \times 10^{-5} \times (20 - 10)$

$= 1.32 \times 10^7$ N/m^2

5. (*a*) Heat gain = Heat loss

$5(T_f - 10) = 10\,(40 - T_f) \Rightarrow T_f = 30°C$

6. (*a*) Water equivalent = $m \times c = 400 \times 0.1 = 40$g

7. (*c*) $T_A = T_B$

$T_C = T_D$

$T_A \neq T_C$

$\therefore T_B \neq T_D$

8. (*c*) Heat required = $m \times 80 + m \times 1 \times 100 + m \times 536$

$= m(716) = 1 \times 716 = 716$ Calories

9. (*c*) Pt = heat lost in time t = mL

$L = \frac{pt}{M}$

10. (*b*) Resultant temperature is 0°C while ice will not melt.

11. (*d*) $\frac{Q_1}{Q_2} = \frac{S_1}{S_2} \times \frac{\rho_2}{\rho_1} = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$

12. (*b*) $100 \times 420 \times (730 - 80) = m \times 4200 \times (80 - 15)$

$\frac{4200}{42} = m$

13. (*a*) Slope of graph is greater in the solid state i.e., temperature is rising faster, hence lower heat capacity. The transition from solid to liquid state takes lesser time, hence latent heat is smaller.

14. (*b*) Fact based

15. (*c*) $\frac{R_1}{R_2} = \frac{\frac{\ell_1}{K_1A_1}}{\frac{\ell_2}{K_2A_2}} = \frac{\frac{\ell}{K\pi(2r)^2}}{\frac{2\ell}{K\pi(3r)^2}} = \frac{9}{8}$

$\because I = \frac{\Delta T}{R}$

$\Rightarrow I \propto \frac{1}{R}$

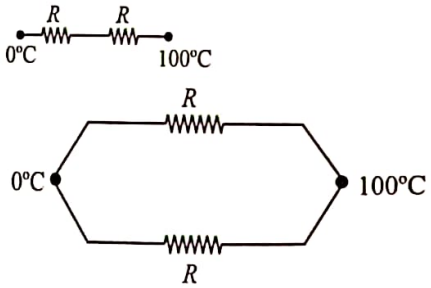
so $\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{8}{9}$

16. (*d*) Utensil should have low thermal resistance

$\left(R = \frac{\ell}{KA}\right)$,

and low specific heat so that heat loss is less

17. (*b*)



$\frac{Q_1}{t_1} = i_{H_1} = \frac{100 - 0}{2R} = \frac{50}{R}$

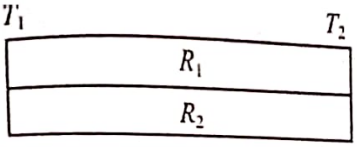
$i_{H_2} = \frac{100}{R/2} = \frac{200}{R} = \frac{Q_2}{t_2}$

$Q_1 = Q_2 = 10$ cal.

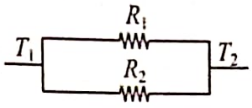
$\frac{50}{R} \times (2) = \frac{200}{R} \times t_2$

$t_2 = \frac{1}{2}$ min.

18. (*c*)



Equivalent thermal circuit



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{K_{eq} \times 2A}{\ell} = \frac{KA}{\ell} + \frac{2KA}{\ell} \Rightarrow K_{eq} = \frac{3}{2}K$$

19. (*d*) Because steady state has been reached.

20. (*d*) $\frac{Q}{t} = \frac{KA\Delta\theta}{l} \Rightarrow \frac{K_A}{K_B} = \frac{A_B}{A_A} = \left(\frac{r_B}{r_B}\right)^2 = \frac{1}{4}$

$$\Rightarrow K_A = \frac{K_B}{4}$$

21. (*c*) Temperature of interface: $\theta = \frac{K_1\theta_1 l_2 + K_2\theta_2 l_1}{K_1 l_2 + K_2 l_1}$

$$= \frac{K \times 0 \times 2 + 3K \times 100 \times 1}{K \times 2 + 3K \times 1} = \frac{300K}{5K} = 60°C$$

22. (*a*) The velocity of heat radiation in vacuum is equal to that of light.

23. (*c*) Fact based

24. (*d*) According to Kirchoff's law in spectroscopy. If a substance emit certain wavelengths at high temperature, it absorbs the same wavelength at comparatively lower temperature.

25. (*b*) $\lambda_{m2} = \frac{T_1}{T_2} \times \lambda_{m1} = \frac{2000}{3000} \times \lambda_{m1} = \frac{2}{3}\lambda_{m1} = \frac{2}{3}\lambda_m$

26. (*c*) $E \propto T^4$ (Stefan's law)

27. (*a*) $\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 \Rightarrow \frac{E}{E_2} = \left(\frac{273+0}{273+273}\right)^4$

$$\Rightarrow E_2 = 16\,E$$

28. (*a*) $E \propto T^4 \Rightarrow \frac{E_1}{E_2} = \frac{T^4}{T^4} \times 2^4 \Rightarrow E_2 = \frac{E}{16}$

29. (*c*) Total energy radiated from a body $Q = A\varepsilon\sigma T^4 t$

$\Rightarrow Q \propto AT^4 \propto r^2T^4$ ($\because A = 4\pi r^2$)

$$\Rightarrow \frac{Q_P}{Q_Q} = \left(\frac{r_P}{r_Q}\right)^2 \left(\frac{T_P}{T_Q}\right)^4 = \left(\frac{8}{2}\right)^2 \left\{\frac{(273+127)}{(273+527)}\right\}^4 = 1$$

30. (*a*) According to Wein's law $\lambda_m T$ = constant

$$\Rightarrow \lambda_{m1}T_1 = \lambda_{m2}T_2 \Rightarrow T_2 = \frac{\lambda_{m1}}{\lambda_{m2}}T_1 = \frac{\lambda_0}{3\lambda_0/4} \times T_1 = \frac{4}{3}T_1$$

Now $P \propto T^4 \Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{4/3T_1}{T_1}\right)^4 = \frac{256}{81}$$

31. (*a*) From Stefan's law $E = \sigma T^4$

$$T^4 = \frac{E}{\sigma} = \frac{6.3 \times 10^7}{5.7 \times 10^8} = 1.105 \times 10^{15} = 0.1105 \times 10^{16}$$

$T = 0.58 \times 10^4\,K = 5.8 \times 10^3\,K$

32. (*b*) According to Prevost theory every body radiate heat at all temperature (except 0*K*) and also absorbs heat from surroundings.

$\because T_A > T$

$\Rightarrow$ Object *A* emits radiations more than the radiations it absorbs.

And $T_B < T$

$\Rightarrow$ Object *B* absorbs more radiations than it emits.

After a certain time all bodies attains a common temperature

33. (*a*) For small difference of temperature, it is the special case of Stefan's law.

34. (*b*) Liquid having more specific heat has slow rate of cooling because for equal masses rate of cooling $\frac{d\theta}{dt} \propto \frac{1}{c}$.

35. (*c*) $\frac{60-50}{10} = K\left(\frac{60+50}{2} - 25\right)$ ...(i)

$\frac{50-\theta}{10} = K\left(\frac{50+\theta}{2} - 25\right)$ ...(ii)

On dividing, we get

$$\frac{10}{50-\theta} = \frac{60}{\theta}$$

$\Rightarrow \theta = 42.85°C$

36. (*d*) Power radiated = $e_r\sigma T^4 A$ or Power radiated $\propto$ (surface area) $(T)^4$. The radius is halved, hence, surface area will becomes times. Temperature is doubled, therefore, T_4 becomes 16 times.

New power = $(450)\left(\frac{1}{4}\right)(16) = 1800$ W.

$\therefore$ The correct option is (*d*).

37. (*b*) $Q \propto AT^4$ and $\lambda_m T$ = Constant.

Hence, $Q \propto \frac{A}{(\lambda_m)^4}$ or $Q \propto \frac{r^2}{(\lambda_m)^4}$

$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4} = \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

$$= 0.05 : 0.0625 : 0.0576$$

i.e. Q_B is maximum.

38. (*a*) In the equation, $-\frac{dT}{dt} = \frac{eA\sigma}{mc}(T^4 - T_0^4)$

Only mass is different. Therefore, $-\frac{dT}{dt} \propto \frac{1}{m}$

Mass of hollow sphere is less. So, hollow sphere will cool at a faster rate initially.

39. (*d*) $-\frac{dT}{dt}=\frac{eA\sigma}{mc}(T^4-T_0^4)$

$m_1=3m_2$

$\therefore R_1=(3)^{\frac{1}{3}}R_2$

$\left(-\frac{dT}{dt}\right)\propto\frac{A}{m}\propto\frac{R^2}{R^3}\propto\frac{1}{R}$

$\therefore \frac{(-dT/dt)_1}{(-dT/dt)_2}=\frac{R_2}{R_1}=\left(\frac{1}{3}\right)^{\frac{1}{3}}$

$\therefore$ The correct option is (*d*).

40. (*d*) Fact based

41. (*c*) $\frac{T_1-T_2}{t}=K\left[\frac{T_1-T_2}{2}-T_s\right]$

In the first 10 minute

$\frac{62-50}{10}=K\left[\frac{62+50}{2}-T_s\right]$

$\Rightarrow 1.2=K[56-T_s]$ ...(i)

Similarly, in the second 10 minute

$\frac{50-42}{10}=K\left[\frac{50+42}{2}-T_s\right]$

$\Rightarrow 0.8=K[46-T_s]$ ...(ii)

Solving equation (*i*) and (*ii*), we obtain

$T_s=26°C$

42. (*c*) $\frac{12-6}{5}=k\left(\frac{12+6}{2}\right)\Rightarrow k=\frac{2}{15}$

$\frac{6-3}{t}=k\left(\frac{6+3}{2}\right)\Rightarrow t=5$ minutes

43. (*c*) Fact based

44. (*a*) By using formula $\frac{80-50}{75-50}=\frac{T-0}{100-0}\Rightarrow T=120°C$

Exercise-2 (Learning Plus)

1. (*c*) Given $L=1$ mm, $\Delta L=6\times10^{-5}$ mm

$\alpha=12\times10^{-6}\,K^{-1}$

then, $\Delta L=L\alpha\,\Delta T$

6×10^{-5} mm $=(1$ mm$)(12\times10^{-6})\,\Delta T$

$\Delta T=5°C$

2. (*a*) Given, $L=25$ cm, $A=0.8\times10^{-4}$ cm^2

$\Delta T=10°C$, $\alpha=10^{-5}\,°C^{-1}$, $Y=2\times10^{10}\,N^2$

then $\frac{\Delta L}{L}=\alpha\Delta T=\frac{F}{AY}$

$F=\alpha AY\Delta T$

$=(10^{-5})(0.8\times10^{-4})\times(2\times10^{10})\times10$

$=160$ N

3. (*c*) $I=MR^2$

$dI=2MRdR$

$=2MR\,[R\alpha\Delta T]=2\alpha I\Delta T$

4. (*b*) $F=AY\frac{\Delta L}{L}=AY\alpha\Delta T$

$f=K\sqrt{\frac{F}{\mu}}=K\sqrt{\frac{AY\alpha\Delta T}{\rho A}}$

$\Rightarrow f\,\alpha\sqrt{\frac{Y\alpha}{\rho}}$

5. (*b*) Given $L=20$ cm, $\Delta L_1=0.075$ cm, $\Delta L_2=0.045$ cm

$\Delta L=L\,\alpha\,\Delta T$

$0.075=20\,\alpha_1\,(100)$

$0.045=20\,\alpha_2\,(100)$

Let for 3rd rod $20=L_1+L_2$

So $\Delta L_3=\Delta L_1+\Delta L_2$

$\Rightarrow 0.06=L_1\alpha_1\,100+(20-L_1)\,\alpha_2 100$

$L_1=10$ cm.

6. (*a*) Given, f = coefficient of cubical expansion

$\rho_{Sphere}=\rho_l'$

$\Rightarrow \frac{266.5}{\frac{4}{3}\pi\left(\frac{7}{2}\right)^3}=\frac{1.527}{1+35f}$

$\Rightarrow f=8.3\times10^{-4}/°C$

7. (*c*) $\because \gamma_m<\gamma_{Al}$ $\quad \rho_m>>\rho_{ac}$

$\Delta V_m<\Delta V_{Al}$

So completely Immersed

$\Delta\rho_m<\Delta\rho_{Al}$ $\quad$ So $W_2>W_1$

[$\because$ Displaced mass of alchol is less]

8. (*c*) $P\Delta V=nR\Delta T$ $\quad \Delta V=\frac{nR}{P}\Delta T$

$\Delta V=\frac{V}{T}\Delta T$

So, $\gamma=\frac{1}{T}$

9. (*c*) $\Delta L=\Delta L_1+\Delta L_2$

$(3L)\alpha_{net}\,\Delta t=L\alpha\Delta t+(2L)\,(2\alpha)\,\Delta t$

$\alpha_{net}=\frac{\alpha+4\alpha}{3}=\frac{5\alpha}{3}$

10. (*a*) On heating the expansion will take place hence both the distances will increase.

11. (*d*) At 0°C

$V_{ox}=20A$; $\quad V_{oy}=30A$

Now at time T y read 120°C

So, $V'_{oy}=A(120)=30A\,(1+\gamma_m T)$

and $V'_{ox}=Ah=20A\,(1+\gamma_m\,T)$

Dividing $\frac{120}{h}=\frac{30}{20}$

$\Rightarrow h=80.$

12. (*d*) $\frac{dH}{dt} = mS_A \frac{d\theta}{dt} = mS_A \tan\frac{\pi}{3} = mS_A\sqrt{3}$

$\frac{dH}{dt} = mS_B \frac{d\theta}{dt} = ms_B \times 1$

$\frac{dH}{dt} = \sqrt{3}\, mS_C \frac{d\theta}{dt} = \sqrt{3}\, mS_C \times \frac{1}{\sqrt{3}}$

13. (*d*) Heat is required to raise temperature of

(Calorimeter + Ice to vapour) $_{\text{at 0 to 100°C}}$

$= (10 \times 100 + \{10 \times 80 + 10 \times 1 \times 100 + 10 \times 540\})$

$= 8200$ Cal.

14. (*c*) $\frac{1}{2} \times 70 \times 10^{-3} \times 250^2 = m \times L = m \times 80 \times 4.2 \times 10^3$

$m = \frac{5}{768}$ kg

15. (*a*) Using Energy conservation

The energy loss due to potential energy goes into increasing the temperature of ice.

$\frac{m}{5}(L) = mgh \Rightarrow h = \frac{L}{5g}$

16. (*b*) x = fraction of water vaporized

1-x-fraction of water fixed

$x\, H_{\text{vaporization}} = (1-x)\, H_{\text{fusion}}$

$\frac{x}{1-x} = \frac{\lambda}{1}$

$x = \lambda - \lambda x$

$x = \frac{\lambda}{1+\lambda}$

17. (*d*) From the data given, $S_A\rho_A(8V) = (12V)\,\rho_B S_B$

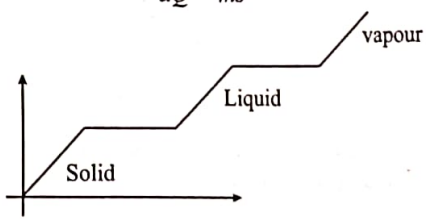
$\frac{s_A}{s_B} = \frac{12\rho_B}{8\rho_A} = \frac{3}{2} \times \frac{2000}{1500} = 2$

18. (*b*) $\Delta Q = \int mCdT$

$\Delta Q = \int_0^{20} mKT^3 dT = \left[\frac{mkT^4}{4}\right]_0^{20}$

$\Delta Q = 4\, mk \times 10^4$

19. (*d*) $\therefore dQ = msdT \Rightarrow \frac{dT}{dQ} = \frac{1}{ms}$



20. (*c*) $Q_{\text{melt}} = 5 \times 0.5 \times 10 + 5 \times 80 = 25 + 400 = 425$ cal.

$Q_{\text{av.}} = 20 \times 1 \times 30 = 600$ cal $> 425 \Rightarrow$ ice will melt

$425 + 5 \times 1 \times (T - 0) = 20 \times 1\,(30 - T)$

$425 + 5T = 600 - 20T$

$25T = 175$

$T = 7°C$

21. (*c*) For small temperature difference, Stefan's law can written as

$\Delta u = e\sigma A[(T + \Delta T)^4 - T^4]$

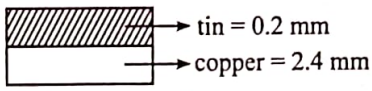
or $\Delta u = e\sigma AT^4 \left[\left[1 + \frac{\Delta T}{T}\right]^4 - 1\right]$

or $\Delta u = e\sigma AT^4 \times 4 \times \frac{\Delta T}{T}$

or $\Delta u \propto \Delta T$

Hence Newton's law of cooling is a special case of stefan's law.

22. (*c*)



(both plates in series)

So, $R_{\text{net}} = R_1 + R_L = \frac{l_1}{k_1A_1} + \frac{l_2}{k_2A_2}$

So, $\frac{Q}{L} = \frac{T_1 - T_2}{R_{\text{net}}} = \frac{T_1 - T_2}{\frac{l_1}{k_1A_1} + \frac{l_2}{k_2A_2}}$

$\frac{Q}{t} = \frac{700 - 100}{\left(\frac{2.4 \times 10^{-1}}{0.9 \times 400}\right) + \left(\frac{0.2 \times 10^{-1}}{0.15 \times 400}\right)}$

Now, as ($Q = m \times L_f$ and given ; $L_f = 540$ cal/gm)

$m = \left[\frac{600}{\frac{1}{400}\left(\frac{0.24}{0.9} + \frac{0.02}{0.15}\right)}\right] \times \frac{3600}{540}$ ($t = 1$ hr $= 3600$ sec)

$m = 4000$ kg

Thus, option (*c*) is correct.

23. (*a*) The heat current in the bottom of pot is due to temperature difference at the lower & upper surface.

$i_p = K_{\text{steel}} A.\frac{dT}{dx} = \frac{m}{t} \cdot L_v$

$50.2 \times 0.15 \times \frac{(x - 100)}{1.2 \times 10^{-2}} = \frac{0.44}{5 \times 60} \times 2.25 \times 10^6$

[Let x be temperature of surface in contact with stove]

$x = 105.25°C$

24. (*a*) We know that

$i = K(\pi R^2)\frac{dT}{dx}, i \propto \frac{R^2}{\ell}$

25. (*d*) $i = KA\frac{dT}{dx}$

$\Rightarrow \frac{dT}{dx} \propto \frac{1}{K}$

$\therefore$ i and A are same for both the layers.

$i = -KA\,(dT/dx)$

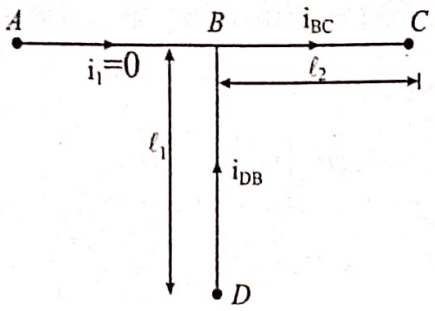
i and A are constant hence slope

$dT/dx = -i/(KA)$ is $-ve$ but

Slope $\propto (1/K)$

Hence in air slope will be more $-ve$ due to very less conductivity.

26. (*b*)

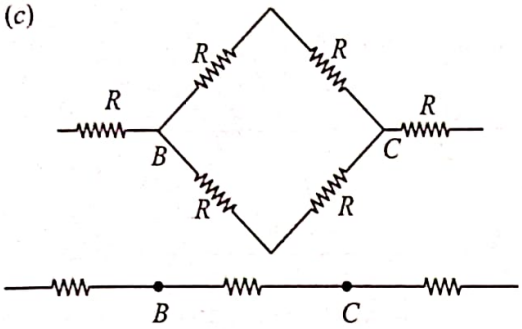


$i_{BC} = i_{DB}$

$$\Rightarrow \frac{kA(90-20)}{\ell_1} = \frac{kA(20-0)}{\ell_2}$$

$$\frac{\ell_1}{\ell_2} = \frac{7}{2}$$

27. (*c*)



$$T_C - 20 = T_B - T_C = T_A - T_B = \frac{200-20}{3} = 60$$

$T_C = 80$

So, $T_B = 80 + 60 = 140$ °C

28. (*c*) $i = -kA\, dT/dx$

Slope $dT/dx = -\, i/kA$ is $-ve$ but due to radiation loss because of not lagged, as we move ahead current i will be less. Hence slope will be more $-ve$ to less $-ve$.

29. (*a*) $T_P = \dfrac{100+0}{2} = 50°$

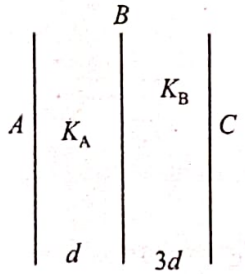
$$T_Q = \frac{30+60}{2} = 45°$$

As $T_P > T_Q$ so flow is from P to Q.

30. (*a*) Slope $dT/dX = -i/kA$ is less $-ve$ for 1st layer Hence 1st layer should have larger k.

So $k_1 > k_2$

31. (*a*)



$$\frac{A-B}{d/K_A} = \frac{B-C}{3d/K_B} = \frac{A-C}{\left(\dfrac{d}{K_A} + \dfrac{3d}{K_B}\right)}$$

32. (*c*) Initially $i = \dfrac{dm}{dt} . L_f = k\pi R^2 . \dfrac{100}{\ell}$

Hence $\dfrac{dm}{dt} \propto \dfrac{kR^2}{\ell}$

From given condition

$$\frac{\dfrac{dm_2}{dt}}{\dfrac{dm_1}{dt}} = \frac{\dfrac{k}{4}\left(\dfrac{(2R)^2}{\ell/2}\right)}{\dfrac{kR^2}{\ell}}$$

$$\frac{\dfrac{dm_2}{dt}}{0.1} = 2 \Rightarrow \frac{dm_2}{dt} = 0.2$$

33. (*a*) Req. is same for both the rods and same temperature same difference so $i_1 = i_2$

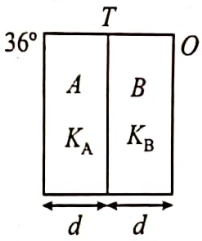
34. (*b*) $\left(-\dfrac{dT}{dt}\right) = \dfrac{\sigma eA}{mS}[T^4 - T_S^4]$

Rate of temperature fall will be maximum when $(T^4 - T_S^4)$ has mass value i.e. T has max. value

$$\left(-\frac{dT}{dt}\right)_{max} = \frac{\sigma eA}{mS}[500^4 - 300^4]$$

Put all values & get answer.

35. (*b*)



$K_A = 2K_B = 2K$

$$\left(\frac{36-T}{d}\right) K_A A = \left(\frac{T-0}{d}\right) K_B A$$

$(36 - T)\, 2K = TK$

$$T = \frac{72}{3} = 24$$

ΔT = temp diff = 36 – 24 = 12

36. (*a*) We know that, $\lambda_{max} \propto \dfrac{1}{T}$

$$\frac{\lambda_{1max}}{\lambda_{2max}} = \frac{T_{S_2}}{T_{S_1}} \Rightarrow \frac{T_{S_2}}{T_{S_1}} = \frac{3}{4}$$

37. (*b*) Initially the temperature of the substance increases and then phase change from ice to water occurs & this process continues.

38. (*d*) Area $= \int y dx = \int \dfrac{dE}{d\lambda} \times d\lambda = \int dE$

Area $(a) = E = \sigma T^4 = \sigma\left(\dfrac{b}{\lambda}\right)^4$

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4 \Rightarrow \frac{1}{9} = \left(\frac{\lambda_2}{\lambda_1}\right)^4 \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{3}$$

39. (*b*) Using formula

$P = \sigma eAT^4$

$P_P = \varepsilon_P \sigma (a) \theta_P^4$ and $P_Q = \varepsilon_Q \sigma A \theta_Q^4$

Now $P_P = P_Q$

$$\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4} \theta_Q = \theta_P$$

40. (*a*) $E_{273} = eA\,(273 + 273)^4 = E$(Given)

$E_0 = eA\,(273 + 0)^4$

$$E_0 = \frac{E}{16}$$

41. (*d*) The correct option is *c* 1500 J/°C

Formula used : $\frac{dQ}{dt} = k\Delta\theta, Q = \frac{dQ}{dt} \times t, Q = C\Delta Q$

Given: $\frac{dQ}{dt} = 10$ W of electric heater, $\Delta\theta = 30°C$, After heater is switched off its takes 1 min from 35.1°C to 34.9°C.

At 50°C,

$$\frac{dQ}{dt} = k(50-20)$$

$10 = k(50 - 20)$

$$\therefore\ k = \frac{1}{3}$$

At an average temperature of 35°C,

$$\frac{dQ}{dt} = \frac{1}{3}(35-20) = 5 \text{ J/s}$$

Heat lost in 1 minute $= \frac{dQ}{dt} \times 60 = 300$ J

Fall in temperature is 0.2°C = $\Delta\theta$

$$\therefore\ C = \frac{Q}{\Delta\theta} = \frac{300}{0.2} = 1500 \text{ J/°C}$$

42. (*a*) $w_0 - w_1 = V \times d_\ell \times g$ ...(i)

$w_0 - w_2 = V' \times d'_\ell \times g$ ...(ii)

Also, $V = V(1 + \beta\Delta T)$ ...(iii)

and $d_\ell = d'_\ell\,(1 + \gamma_\ell \Delta T)$ ...(iv)

From (ii), (iii) and (iv)

$$w_0 - w_2 = \frac{V(1+\beta\Delta T)\times d_\ell}{1+\gamma_\ell \Delta T} \times g \quad \text{...(v)}$$

Dividing (i) and (v), we get

$$\frac{w_0 - w_1}{w_0 - w_2} = \frac{Vd_\ell g(1+\gamma_\ell \Delta T)}{V(1+\beta\Delta T)d_\ell g}$$

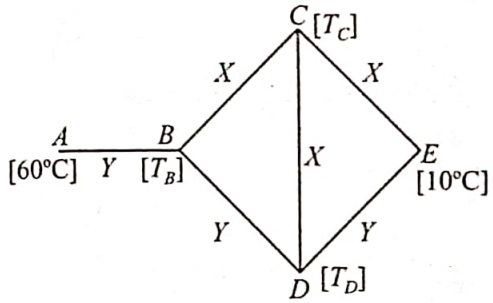
$$\Rightarrow \frac{w_0 - w_1}{w_0 - w_2} = \frac{1+\gamma_\ell \Delta T}{1+\beta\Delta T} \Rightarrow \frac{w_0 - w_1}{w_0 - w_2} = \frac{1+\gamma_\ell(t_2 - t_1)}{1+\beta(t_2 - t_1)}$$

$$\Rightarrow (w_0 - w_1)[1+\beta(t_2 - t_1) = (w_0 - w_2)[1+\gamma_\ell(t_2 - t_1)]$$

$$\Rightarrow \gamma_\ell = \frac{w_2 - w_1}{(w_0 - w_2)(t_2 - t_1)} + \frac{\beta(w_0 - w_1)}{(w_0 - w_2)}$$

43. (*a*) $K_X = 0.92$ cal/sec-cm-°C

$K_Y = 0.46$ cal/sec-cm-°C



Note This Step: The heat flow through *AB* is divided into two path *BC* and *BD*. Symmetry shows that no heat will flow through *CD*. Therefore

$$\frac{K_Y A(60 - T_B)}{\ell} = \frac{K_X A(T_B - 10)}{2\ell} + \frac{K_Y A(T_B - 10)}{2\ell}$$

On solving the above equation, we get

$T_B = 30°C$

As *C* is a point at the middle of *BE* therefore temperature at *C* is 20°*C*.

44. (*c*) Rate of heat supply to the disc,

$$\frac{dH}{dt} = kA\frac{\Delta T}{\Delta l}$$

$$\text{or, } mc\frac{dT}{dt} = 10 \times 0.004 \times \frac{400 - T}{0.4}$$

$$0.4 \times 600 \times \frac{dT}{dt} = \frac{1}{10} \times (400 - T)$$

$$\Rightarrow 2400\int_{300}^{350} \frac{dT}{400 - T} = \int_0^t dt$$

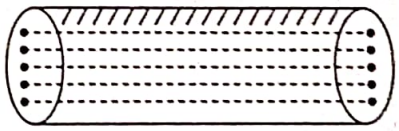
$\Rightarrow t = 2400.\ \ln(2) = 1663.2$ s.

45. (*d*) Heat required to melt the ice

$\Delta Q = mL$

$$\frac{\Delta Q}{dt} = k2\pi Lx\frac{\Delta T}{dx}$$

$$\frac{mL}{dt} = \frac{100\pi kLx}{dx}$$



$$\Rightarrow mL\int_{r_1}^{r_2} \frac{dx}{x} = 100k\pi L\int_0^t dt$$

$$\Rightarrow mL\ln\left(\frac{r_2}{r_1}\right) = 100k\pi Lt$$

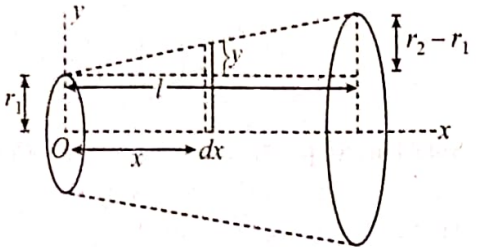
$$\Rightarrow t = \frac{mL}{100\pi kl}\ln\left(\frac{r_2}{r_1}\right)$$

46. (*c*) $\frac{r_2 - r_1}{l} = \frac{y}{x}$ (Similar triangles)

$$\Rightarrow y = \left(\frac{r_2 - r_1}{l}\right)x$$

Thermal resistance of the element,

$$dR = \frac{dx}{k \cdot \pi (r_1 + y)^2}$$



$$\int_0^R dR = \int_0^l \frac{dx}{\pi k\left(r_1 + \frac{r_2 - r_1}{l} \cdot x\right)^2} \Rightarrow R = \frac{l}{\pi k r_1 r_2}$$

$$\therefore \text{Heat current} = \frac{T_1 - T_2}{R} = \frac{(T_1 - T_2) \cdot \pi k r_1 r_2}{l}$$

47. (*a*) The rate of cooling of a body is given by

$$\frac{dT}{dt} = \frac{A\varepsilon\sigma}{C}(T^4 - T_0^4)$$

$T \rightarrow$ temperature of body, $T_0 \rightarrow$ temperature of surroundings

Now $C = mS$

$S \rightarrow$ specific heat capacity of substance and $m \rightarrow$ mass of substance

$\therefore C = \rho VS$

$V \rightarrow$ volumen of substance.

$$\therefore -\frac{dT}{dt} = \frac{A\varepsilon\sigma}{\rho VS}(T^4 - T_0^4)$$

Under identical conditions, $-\frac{dT}{dt} \propto \frac{A}{V}$

For cube; $A = 6 \times 10 \times 10 = 600\ \text{cm}^2$

$V = 1000\ \text{cm}^3$

$$\therefore \frac{A}{V} = \frac{6}{10}$$

For sphere; $A = 4\pi(10)^2, V = \frac{4}{3}\pi(10)^3$

$$\therefore \frac{A}{V} = \frac{3 \times 4\pi(10)^2}{4\pi(10)^3} = \frac{3}{10}$$

For cylinder,

$A = 2\pi(4)^2 + 2\pi(4) \times 10 = 32\pi + 80\pi = 112\pi$

$V = \pi(4)^2 10 = 160\pi$

$$\therefore \frac{A}{V} = \frac{112}{160} = \frac{7}{10}$$

As $\frac{A}{V}$ is maximum for cylinder. Therefore cylinder will cool at faster rate.

Exercise-3 (JEE Advanced Level)

1. (*d*) Loss(copper) = gain (water + beaker)

$m_{CH}s_{CH}(T_{CH} - T) = m_W s_W (T - T_W) + m_b s_b (T - T_W)$

Hence final temperature can be calculated.

2. (*a,b,c*)

Heat required to melt ice

Heat given by the water = $m \times 10 \times 1 = 10$ m

Heat required to melt ice > Heat given by water so complete ice will not melt.

3. (*a,b*)

$P_1 = P_2$

$\sigma e_A A T_A^4 = \sigma e_B A T_B^4$

$$\frac{T_A}{T_B} = \left(\frac{0.81}{0.01}\right)^{1/4} = 3$$

$$T_B = \frac{T_A}{3} = = 1934\ K$$

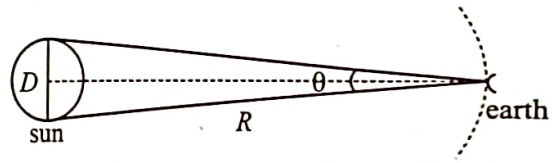
$\lambda_B = \lambda + 1 = 1.5\ \mu\text{m}$

$T_A \lambda_A = T_B \lambda_B$

$T_A \lambda = T_B (\lambda + 1)$

$$\lambda = \frac{1}{2}\ \mu\text{m}$$

4. (*b,c*)



Let the diameter of the sun be D and its distance from the earth be R.

$$\frac{D}{R} = \theta$$

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\frac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4.$$

At distance R, this radiation falls on an area $4\pi R^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore.

$$s = \frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2$$

Thus, $s \propto T^4$ and $s \propto \theta^2$

5. (*d*) Increase in tension of wire = $YA\ \alpha\Delta\theta$

$= 8 \times 10^{-6} \times 2.2 \times 10^{11} \times 10^{-2} \times 10^{-4} \times 5$

$= 8.8$ N

6. (*a,b,c*)

Good Absorbers are good emitters.

7. (*c*) $F = YA\ \alpha\Delta t$

$= 2 \times 10^{11} \times 3 \times 10^{-6} \times 10^{-5} \times (20 - 10)$

$= 60$ N

8. (*d*) $L_2 = l_2(1 + \alpha_2\Delta\theta)1.$ and $L_1 = l_1(1 + \alpha_1\Delta\theta)$

$\Rightarrow (L_2 - L_1) = (l_2 - l_1) + \Delta\theta\,(l_2\alpha_2 - l_1\alpha_1)$

Now $(L_2 - L_1) = (l_2 - l_1)$ so, $l_2\alpha_2 - l_1\alpha_1 = 0$

9. (*c,d*)

(*a*) Heat absorption is surface phenomenon hence wooden (Black surface) absorbs more. (wrong)

(*b*) After long time both will have temperature of surroundings. (wrong)

(*c*) Because metal is better conductor it feels hotter.

(*d*) Because emission depend on surface (i.e. more for black surface)

10. (*a,b,c,d*)

On heating, every dimension increases or due to thermal expansion, the distance between any two point increase.

11. (*b,c*)

Strain $\rightarrow$ Same

$$\text{Stress} = \frac{F}{A} = \text{constant}$$

$F \propto A \Rightarrow F \propto r^2$

$$\text{Energy} = \frac{1}{2}\text{ stress} \times \text{strain} \times \text{volume}$$

$\propto$ Area

$\propto r^2$

12. (*a,c,d*)

(*a*) % rise in area $= \beta\Delta T = 2(\alpha\Delta T) = 2 \times 0.2 = 0.4\%$

(*c*) % rise is volume $= 3\,\alpha\Delta T = 3 \times 0.2 = 0.6\%$

(*d*) $\alpha = \dfrac{0.2}{80 \times 100} = 0.25 \times 10^{-4}/°C$

13. (*b*) In steady state $\left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer1}} = \left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer4}}$

$$\Rightarrow \frac{0.06 \times A \times (30-25)}{1.5 \times 10^{-2}} = \frac{0.10 \times A \times \Delta T}{3.5 \times 10^{-2}} \Rightarrow \Delta T = 7°C$$

$T_3 = (-10 + 7)°C = -3°C$

14. (*a*) $\left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer1}} = \left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer3}}$

$$\Rightarrow \frac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \frac{0.04 \times A \times \Delta T}{2.8 \times 10^{-2}} \Rightarrow \Delta T = 14°C$$

$T_3 = (-3 + 14)°C = 11°C$

15. (*a*) $\left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer1}} = \left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer2}}$

$$\Rightarrow \frac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \frac{K_2 \times A \times 14}{1.4 \times 10^{-2}} \Rightarrow K_2 = 0.02\text{ W/mK}$$

16. (*b*) We have $\theta - \theta_s = (\theta_0 - \theta_s)\,e^{-kt}$

Where θ_0 = Initial temperature of body = 40°C

θ = temperature of body after time t.

Since body cools from 40°C to 38°C in 10 min, we have

$38 - 30 = (40 - 30)\,e^{-k\,10}$ ...(*a*)

Let after 10 min, The body temp. be θ

$\theta - 30 = (38 - 30)\,e^{-k\,10}$ ...(*b*)

$\dfrac{(1)}{(2)}$ gives $\dfrac{8}{\theta - 30} = \dfrac{10}{8}$, $\theta - 30 = 6.4$

$\Rightarrow \theta = 36.4$ °C

17. (*a*) Temperature decreases exponentially.

18. (*c*) During heating process from 38°C to 40°C in 10 min. The body will lose heat in the surrounding which will be exactly equal to the heat lost when it is cooled from 40°C to 38°C in 10 min, which is equal to ms $\Delta\theta = 2 \times 2 = 4$ J.

During heating process heat required by the body $= ms\Delta\theta = 4$ J.

$\therefore$ Total heat required = 8 J.

19. (*a,b*)

(*a*) Emitted energy is very less for longer and shorter wave length.

(*b*) From figure at λ_m intensity is maximum

(*c*) Area under the curve shows amount of energy emitted.

20. (*a,b,c,d*)

When $T\uparrow$ curve shifts towards shorter wavelength hence curve spreads i.e. Area increases.

21. (*b*) $\lambda_m \propto \dfrac{1}{T}$, $T' > T$ So, option *B* is correct.

22. (*a*) In 40 min. temperature of water has come down by 40°C.

Therefore rate $P = \dfrac{mS\Delta T}{t} = \dfrac{0.60 \times 4200 \times 40}{40 \times 60} = 42.0$ W

23. (*c*) Sample of ice has been receiving heat at constant rate P from water. Its temperature has increased by 30°C in time 60 min.

Therefore $\dfrac{m_i\, s_i \Delta T_i}{P} = 60$ min.

$$\Rightarrow m = \frac{(60 \times 60s) \times (42W)}{(2100J/kg).(30°C)} = 2.4\text{ kg}$$

24. (*b*) Thermal equilibrium reaches after 60 min. Ice conversion takes place for 20 min. During this time water at 0°C continues to give heat at rate P.

$$m \times L_f = P \times (20 \times 60\text{s}) \Rightarrow m = \frac{42 \times 20 \times 60}{3.3 \times 10^5}$$

kg = 0.15 kg

Sol. (25 to 27)

25. (*d*) 26. (*b*) 27. (*a*)

As steam has comparatively large amount of heat to provide in the form of latent heat we check what amount of heat is required by the water and ice to go up to 100°C, that is

$(m_i L + m_i S_w \Delta T) + m_w . S_w . \Delta T$

$= [(200 \times 80) + (200 \times 1 \times 100)] + (200 \times 1 \times 45)$

$= 45{,}000$ cal.

That is given by m mass of steam, then

$m_s.L = 45{,}000$

$$m_s = \frac{45{,}000}{540} = \frac{500}{6} = 83.3 \text{ gm}$$

Therefore 83.3 gm steam converts into water of 100°C.

Total water = 200 + 200 + 83.3 = 483.3 gm

Steam left = 16.7 gm.

28. (*a*) A→(*p*,*s*); B→(*t*); C→(*q*,*r*); D→(*t*)

(*a*) Initially more heat will enter through section A due to temperature difference and no heat will flow through section B because initially there is no temperature difference.

(*b*) At steady state rate of heat flow $\left(\frac{dQ}{dt}\right)$ is same for all sections

(*c*) At steady state $\frac{dQ}{dt} = kA\left|\frac{dT}{dx}\right|$ or $\left|\frac{dT}{dx}\right| = \frac{1}{kA}\left(\frac{dQ}{dt}\right)$

$\left|\frac{dT}{dx}\right|$ is inversely proportional to area of cross-section.

Hence is maximum at B and minimum at A.

(*d*) At steady state heat accumulation = 0

So $\frac{dT}{dt} = 0$ for any section.

29. (*b*) A→(*r*); B→(*q*); C→(*p*); D→(*s*)

(A) Coefficient of linear expansion

$L = L_0(1 + \alpha\Delta T) \Rightarrow \Delta L_0 = L_0\alpha\Delta T$

(B) Fact

(C) Heat capacity, $s = \frac{\Delta Q}{\Delta T} = \text{Cal}/°C$

(D) Specific heat $= \frac{1}{m}\frac{\Delta Q}{\Delta T} = Cal/g°C$

30. (*c*) A→(*s*); B→(*p*); C→(*r*); D→(*s*)

(A) Steffan's constant

$\because E = \sigma T^4$

$$\Rightarrow [\sigma] - \frac{[E]}{[T^4]} = \frac{[ML^2T^{-2}]}{[\theta^4]}$$

$[\sigma] = [ML2T^{-2}\theta^{-4}]$

(B) Weins constant

$\because \lambda T = b$

$\Rightarrow [b] = [\lambda][T] = [L][\theta]$

(C) Emissive $= \frac{\text{Energy}}{\text{Time} \times \text{Area}}$

$$= \frac{[E]}{[t][A]} = \frac{ML^2T^{-2}}{[T][L^2]}$$

(D) Thermal resistance $= \frac{\text{Temperature difference}}{\text{Rate of Heat a flow}}$

$$\Rightarrow \frac{\Delta T}{(\Delta Q/\Delta T)} = \frac{[\theta]}{[ML^2T^{-2}]/[\theta]} = [M^{-1}L^{-2}T^3\theta]$$

31. (*a*) A→(*r*,*s*,*t*); B→(*p*); C→(*q*); D→(*r*,*s*)

(A) The height of the liquid depends on the thermal co-efficient of the liquid and container.

(B) When heat is supplied to ice at 0°C, the internal energy must be increases.

(C) The rate of heat loss from a body or increasing temperature of surrounding must be decreases.

(D) On increasing temperature, density of water may increase and decrease.

32. [0023]

$$\alpha_1 = \frac{0.065}{30 \times 100}$$

$$\alpha_2 = \frac{0.035}{30 \times 100}$$

$\Delta l = l_1 a_1 \Delta T + (30 - l_1)\,\alpha_2\,\Delta T$

$$0.058 = l_1 \times \frac{0.065}{3000} + (30 - l_1) \times \frac{0.035}{3000}$$

$1.74 = 0.065\,l_1 + 1.05 - 0.035l,$

$0.69 = +0.03l_1$

$l_1 = 23$ cm ; $l_2 = 7$ cm

33. [0200]

$\Delta\ell = \alpha\ell\Delta T$

$\frac{\Delta\ell}{\ell} = \alpha\Delta T$

$$\sigma_b = \frac{T}{A} = \frac{y\Delta\ell}{\ell} = y\alpha\Delta T \Rightarrow \Delta T = \frac{\sigma_b}{y\alpha}$$

$$= \frac{4 \times 10^8}{2 \times 10^{11} \times 10^{-5}} = 200°\text{C}$$

34. [0001]

$\Delta V = \Delta V_m - \Delta V_{pt} = V(\gamma_2 - \gamma_1)\Delta T$

$\frac{\Delta V}{V} = (18 - 2.7) \times 10^{-5}\,\Delta T = (15.3 \times 10^{-5})\,\Delta T$

$$\Delta T = \frac{(\Delta V/V)}{15.3 \times 10^{-5}} = 1$$

35. [1000]

$$\frac{dE}{dt} = \sigma AT^4$$

$$\because T^4 = \frac{(5.67\pi \times 10^2)}{4\pi(0.05)^2(5.67 \times 10^{-8})} = 10^{12}$$

$A = 4\pi r^2$

$\Rightarrow T = 1000$ K

36. [0001]

$P = eA\sigma T^4 \Rightarrow P = (2\pi rl).\sigma T^4$

$$\Rightarrow 64W = \left(2\pi r \times \frac{10}{3\pi} \times 10^{-2}\right) \times (6 \times 10^{-8}) \times (2000)^4$$

$$64 = \frac{2}{3} r \times 10^3 \times 96$$

$$\Rightarrow r = \frac{64}{64 \times 10^3}\text{m} = 10^{-3}\text{m} = 1\text{mm}$$

37. [0060]

$140 \times 1(80 - T) = 10 \times 80 \times 2 + 20 \times 1\,(T - 0)$

$140 \times 80 - 1600 = 160\,T$

$T = 60°C$

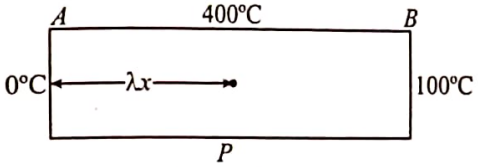
38. [600]

By conservation of heat energy

$m_A S_A \Delta T_A = m_w S_w \Delta T_w$

$$m_A = \frac{m_w S_w \Delta T_w}{S_A \Delta T_A} = \frac{(60)(4200)(10)}{140 \times 30} = 600 \text{ kg}$$

39. [9]



For heat flow from p to 0

$$L_f \frac{dm_1}{dt} = \frac{KA400}{\lambda x}$$

For heat flow from P to B

$$L_{vap} \frac{dm_2}{dt} = \frac{KA300}{10x - \lambda x} \quad ...(i) \left[\text{Given} \frac{dm_1}{dt} = \frac{dm_2}{dt}\right]$$

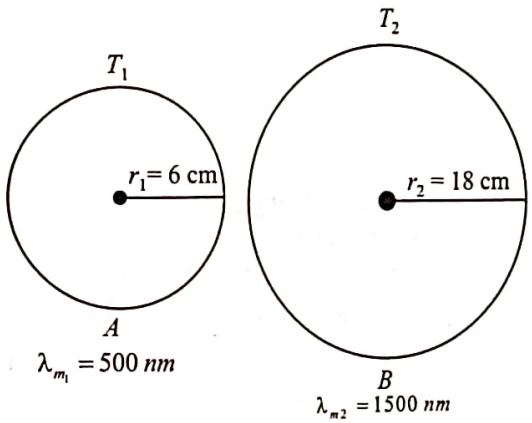
On solving (i) and (ii), we get $\lambda = 9$

40. [8] Heat supplied = Heat used in converting m grams of ice from $-5°C$ to $0°C$ + Heat used in converting 1 gram of ice at $0°C$ to water at $0°C$

$$\Rightarrow 420 = m \times \frac{2100}{1000} \times 5 + \frac{1 \times 3.36 \times 10^5}{1000}$$

$$\Rightarrow 420 = m \times 10.5 + 336 \therefore m = \frac{84}{10.5}$$

41. [9]



$$\frac{\text{Rate of total energy radiated by } A}{\text{Rate of total energy radiated by } B}$$

$$= \frac{\sigma T_1^4 (4\pi r_1^2)}{\sigma T_2^4 (4\pi r_2^2)} = \left(\frac{T_1}{T_2}\right)^4 \times \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^4 \left(\frac{r_1}{r_2}\right)^2 \left[\because \frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} \text{ by Wein's law}\right]$$

$$= \left(\frac{1500}{500}\right)^4 \left(\frac{6}{18}\right)^2 = 9$$

42. [1.71]

Here, $K = K_0(1 + \alpha x)$

$$Q = \frac{kAdT}{dx}$$

$$\Rightarrow dx = \frac{kA}{Q} dT$$

$$\Rightarrow \frac{dx}{1 + \alpha x} = \frac{kA}{Q} dT$$

$$\Rightarrow \int_0^L \frac{dx}{(1 + \alpha x)} = \frac{kA}{Q} \int_0^{100} dT$$

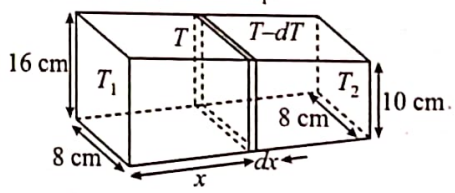
$$\Rightarrow \ln(1 + \alpha L) = \frac{kA}{Q} \times 100 = \frac{100 \times 100 \times 10^{-4}}{1}$$

$\Rightarrow 1 + \alpha L = e$

$\Rightarrow 1 + \alpha L = 2.71$

$\Rightarrow L = 1.71$ m

43. [30] In order to find thermal current, consider a small section of the slab parallel to the end faces at distance x from the face at temperature T_1.



If $-dT$ be the fall in temperature across the section, then the thermal current through the section will be

$$i = -\frac{kAdT}{dx}$$

Since the slab tapers uniformly from height 16 cm to 10 cm,

So, the height h of section under consideration will be given by

$$h = 10 + \frac{(40 - x)}{40}(16 - 10)$$

$$= 10 + \frac{(40 - x)}{40}(6)$$

$$= 10 + \frac{(40 - x)}{20} 3$$

$$= 10 + \frac{120}{20} - \frac{3x}{20} = 16 - \frac{3x}{20}$$

$$= 16 - \frac{3x}{20}$$

And area of the section $A = hb$ [$b \rightarrow$ breadth]

$$= \left(16 - \frac{3x}{20}\right) 8 = 128 - \frac{6}{5}x$$

$$\therefore i = -k\left[128 - \frac{6}{5}x\right]\frac{dT}{dx}$$

$$\text{or, } \frac{idx}{128 - \frac{6}{5}x} = -kdT$$

Now at $x = 0$

$T = 50°C$ and $x = 40$ cm

$T = 25°C$

$$\therefore \int_0^{40} \frac{i\,dx}{\left(128 - \frac{6}{5}x\right)} = -K\int_{50}^{25} dT$$

$$\frac{i}{-6/5}\left[\ln\left(128 - \frac{6}{5}x\right)\right]_0^{40} = -k[25 - 50]$$

$$\text{or, } i = \frac{k \times 15 \times 6}{5[-\ln(80) + \ln(128)]}$$

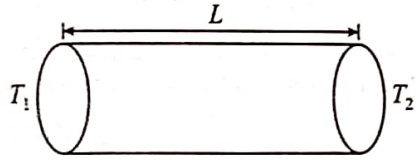
$$\text{or, } i = \frac{k \times 30}{\ln(128/80)} = \frac{30k}{\ln(8/5)}$$

$$\therefore i = \frac{30k}{\ln\left(\frac{8}{5}\right)} \text{ J/s}$$

44. [1] At steady state, $Q = kA\frac{dT}{dx}$

$$\Rightarrow dx = \frac{\alpha A}{Q}\frac{dT}{T} \Rightarrow \int_0^2 dx = \frac{\alpha A}{Q}\ln\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow Q = \frac{\alpha A}{L}\ln\left(\frac{T_2}{T_1}\right) \quad \text{...(i)}$$



$$\text{Now, } \int_0^x dx = \frac{\alpha A}{Q}\ln\left(\frac{T}{T_1}\right)$$

$$\Rightarrow x = \frac{\alpha AL}{\alpha A\ln\left(\frac{T_2}{T_1}\right)} \times \ln\left(\frac{T}{T_1}\right)$$

$$\Rightarrow n\left(\frac{T}{T_1}\right) = \frac{x}{L}\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{T_2}{T_1}\right)^{x/L} \Rightarrow T = T_1\left(\frac{T_2}{T_1}\right)^{x/L}$$

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*a*) $\frac{dl}{l} = 2\frac{dT}{T}$

$$\alpha d\theta = 2\frac{dT}{T}$$

$$\therefore 2 \times \frac{12}{24 \times 60 \times 60} = \alpha(40 - \theta)$$

$$\& \; 2 \times \frac{4}{24 \times 60 \times 60} = \alpha(\theta - 20)$$

$$\Rightarrow \theta = 25° \; \& \; \alpha = 1.85 \times 10^{-5}/°C$$

2. (*d*) Heat given = Heat taken

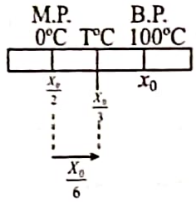
$(100)(0.1)(T - 75) = (100)(0.1)(45) + (170)(a)(45)$

$10(T - 75) = 450 + 7650 = 8100$

$T - 75 = 810$

$T = 885$ °C

3. (*a*) $t = \frac{x_t - x_0}{x_{100} - x_0}100°C$



$$\frac{\frac{x_0}{2} - \frac{x_0}{3}}{x - \frac{x_0}{3}}100°C = 25°C$$

4. (*d*) $0.1 \times 400 \times (500 - T) = 0.05 \times 4200 \times (T - 30) + 800$ $(T - 30)$

$\Rightarrow 40(500 - T) = (T - 30)(2100 + 800)$

$\Rightarrow 20000 - 40T = 2900T - 30 \times 2900$

$\Rightarrow 20000 + 30 \times 2900 = T(2940) \Rightarrow T = 36.4°C$

$$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100 \approx 20\%$$

5. (*c*) $100 \times S_A \times [100 - 90] = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5\,S_B$

$S_A = \frac{3}{4}S_B$

Now, $100 \times S_A \times [100 - T] = 50 \times S_B\,(T - 50)$

$2 \times \left(\frac{3}{4}\right)(100 - T) = (T - 50)$

$300 - 3T = 2T - 100$

$400 = 5T$

$T = 80°C$

6. (*c*) Heat loss = Heat gain

$192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4) + 240 \times 4200 \times (21.5 - 8.4)$

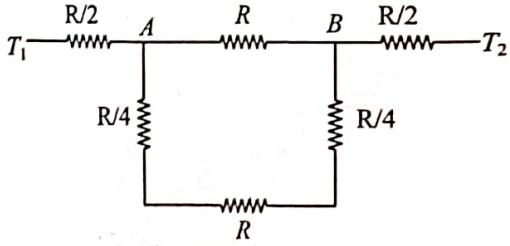
$\Rightarrow S = 916$ J/kg °K

7. (*d*) $m \times 0.5 \times 20 + (m - 20) \times 80 = 50 \times 1 \times 40$

$\Rightarrow 90m - 1600 = 2000$

$\Rightarrow 90m = 3600 \Rightarrow m = 40$ gm

8. (*a*)



$$T_A - T_B = \frac{T_1 - T_2}{\frac{8R}{5}} \times \frac{3R}{5} = \frac{3}{8} \times 120 = 45°C$$

9. (*d*) Equivalent thermal resistance, $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

$$\frac{K\pi(2R)^2}{L}=\frac{K_1\pi R^2}{L}+\frac{K_2\pi\left[(2R)^2-R^2\right]}{L}$$

$$\Rightarrow 4K=K_1+3K_2 \Rightarrow K=\frac{K_1+3K_2}{4}$$

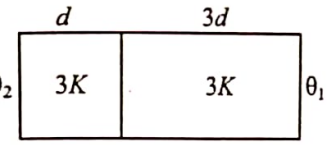
10. (*a*) $\frac{\Delta Q}{\Delta T}=\frac{kA}{\ell}(T_2-T_1) \Rightarrow \frac{1}{A}\left(\frac{\Delta Q}{\Delta T}\right)=\frac{k}{\ell}(T_2-T_1)$

11. (*d*) By law of conservation of energy

$$\frac{1}{2}kx^2=(m_1s_1+m_2s_2)\Delta T$$

$$\Delta T=\frac{16\times10^{-2}}{4384}=3.65\times10^{-5}K$$

12. (*b*)



Let the temperature of interface be "θ" $i_1=i_2$ (Steady state conduction)

$$\frac{3KA(\theta_2-\theta)}{d}=\frac{KA(\theta-\theta_1)}{3d}$$

$$\theta=\frac{9\theta_2}{10}+\frac{\theta_1}{10}$$

13. (*a*) $-ms\frac{dT}{dt}=e\sigma A\left(T^4-T_0^4\right)$

$$-\frac{dT}{dt}=\frac{e\sigma A}{ms}\left(T^4-T_0^4\right)$$

$$-\frac{dT}{dt}=\frac{4e\sigma AT_0^3}{ms}(\Delta T)$$

$$T=T_0+(T_i-T_0)e^{-kt}$$

Where $k=\frac{4e\sigma AT_0^3}{ms}$

$$k=\frac{4e\sigma AT_0^3}{\rho vs} \Rightarrow \left|\frac{dT}{dt}\right|\propto K$$

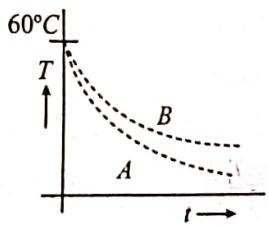
$$\therefore \left|\frac{dT}{dt}\right|\propto\frac{1}{\rho s}$$

$\rho_A s_A=2000\times8\times10^2=16\times10^5$

$\rho_B s_B=4000\times10^3=4\times10^6$

$\rho_A s_A<\rho_B s_B$

$$\left|\frac{dT}{dt}\right|_A>\left|\frac{dT}{dt}\right|_B$$



14. (*c*) Suppose '*m*' gram of water evaporates then, heat required

$\Delta Q_{req}=mL_v$

Mass that converts into ice = $(150-m)$

So, heat released in this process

$\Delta Q_{rel}=(150-m)L_f$

Now, $\Delta Q_{rel}=\Delta Q_{req}$

$(150-m)L_f=mL_v$

$m(L_f+L_v)=150L_f$

$$m=\frac{150L_f}{L_f+L_v} \Rightarrow m=20\text{ g}$$

15. (*c*) Heat lost = Heat gain

$\Rightarrow M_2\times1\times50=M_1\times0.5\times10+M_1L_f$

$$\Rightarrow L_f=\frac{50M_2-5M_1}{M_1}=\frac{50M_2}{M_1}-5$$

16. [40] $M_{ice}L_f+M_{ice}(40-0)C_W=M_{steam}L_v+M_{steam}(100-40)C_w$

$\Rightarrow 200[80+40(a)]=M(540+60(a)]$

$\Rightarrow 200(120)=M(600)$

$M=40$ gm

17. [50] $1\theta_1+2\theta_2=(1+2)60$

$\theta_1+2\theta_2=180$ ...(*a*)

$0\times\theta_1+1\times\theta_2+2\times\theta_3=(1+2)30$

$\theta_2+2\theta_3=90$ ...(*b*)

$2\times\theta_1+0\times\theta_2+1\times\theta_3=(2+1)60$

$2\theta_1+\theta_3=180$ ...(*c*)

and $\theta_1+\theta_2+\theta_3=(1+1+1)\theta$

From (*a*) + (*b*) + (*c*) ...(*d*)

$3\theta_1+3\theta_2+3\theta_3=450 \Rightarrow \theta_1+\theta_2+\theta_3=150$

From (*d*) equation $150=3\theta \Rightarrow \theta=50°C$

18. (*a*) $m[540+(100-31)]=200\times[31-25]$

$$m=\frac{1200}{609}\approx2\text{ gm}$$

19. (*a*) $160=nC_P50$ ...(i)

$240=nC_V100$ ...(*ii*)

From (*i*) and (*ii*),

$$\frac{160}{240}=\frac{C_P}{C_v}\times\frac{1}{2}$$

$$\Rightarrow \gamma=\frac{C_P}{C_v}=1+\frac{2}{f}=\frac{4}{3}$$

$\Rightarrow f=6$

20. (*d*) $MS\Delta T=mL$

$$\Rightarrow m=\frac{MS\Delta T}{L}$$

$$=\frac{(200)(4200)(25)}{3.4\times10^5}\text{ gm}$$

$=61.7$ gm.

21. (c) $\frac{\Delta T}{\Delta t} = -b(T_{av} - T_S)$

$\therefore \frac{50-40}{5} = -b[45-20]$

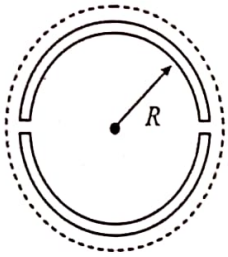
and, $\frac{40-T_3}{5} = -b\left[\frac{40+T_3}{2} - 20\right]$

$\Rightarrow T_3 = \frac{200}{6} = 33°C$

22. (a) $K_1(100-70) = K_2(50)$

$K_3(20) = K_2(50)$

23. (d)



$L \Rightarrow S$

$\Delta L = L \propto \Delta T$

$\frac{F}{A} = \frac{\Delta L}{L} Y$

$F = \alpha \Delta T\, YS$

So, $T = 2F$

$T = 2\alpha \Delta T\, YS$

24. (c) Due to thermal expansion, $\frac{\Delta v}{v} = 3\alpha\Delta T$

Due to External pressure, $\frac{\Delta v}{v} = \frac{P}{K}$

Equating both we get

$3\alpha\Delta T = \frac{P}{K}$

$\Delta T = \frac{P}{3\alpha K}$

25. (a) $\Delta\ell_1 = \Delta\ell_2$

$\ell\alpha_1 \Delta T_1 = = \ell\alpha_2 \Delta T_2$

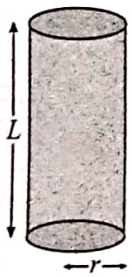
$\frac{\alpha_1}{\alpha_2} = \frac{\Delta T_1}{\Delta T_2}$; $\frac{4}{3} = \frac{T-30}{180-30}$

$T = 230°C$

26. (a) $\Delta L = L \propto \Delta T$

Strain $= \frac{\Delta L}{L} = \alpha\Delta T$; $Y = \frac{F}{A\,\alpha\,\Delta T}$

27. (b)



Length of cylinder remains unchanged

So $\left(\frac{F}{A}\right)_{\text{Compressive}} = \left(\frac{F}{A}\right)_{\text{Thermal}}$

$\frac{F}{\pi r^2} = Y\alpha T$ (α is linear coefficient of expansion)

$\therefore \alpha = \frac{F}{YT\pi r^2}$

$\therefore$ The coefficient of volume expansion $\gamma = 3\alpha$

$\therefore \gamma = 3\frac{F}{YT\pi r^2}$

28. (b) $Mg = \left(\frac{Ay}{\ell}\right)\Delta\ell$

$\frac{\Delta\ell}{\ell} = \alpha\Delta T$

$Mg = (Ay)\alpha\Delta T = 2\pi$

It is closest to 9

29. [20.00]

$V_0 = 500$ cc

$V_b = V_0 + V_0\gamma_{\text{beaker}}\Delta T$

And for Mercury

$V_m' = V_m + V_m\gamma_m\Delta T$

Unfilled volume $(V_0 - V_m) = (V_b - V'_m)$

$\Rightarrow V_0\gamma_{\text{beaker}} = V_m - \gamma_M$

$\therefore V_m = \frac{500\times6\times10^{-6}}{1.5\times10^{-4}} \Rightarrow V_m = 20$ cc

30. (c) $\frac{\Delta v}{v} = \frac{\Delta p}{B} = \frac{1}{20}$

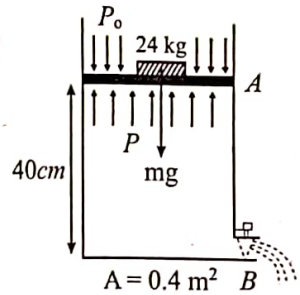
$\frac{\Delta I}{I} = \frac{1}{3}\frac{\Delta p}{B} = \frac{1}{60}$

$\frac{\Delta I}{I}\times100 = \frac{5}{3} = 1.67\%$

31. (d) $(L_1 + L_2)\alpha_{eq} \times \Delta T = L_1\alpha_1\Delta T + L_1\alpha_2\Delta T$

$\Rightarrow \alpha_{eq} = \frac{L_1\alpha_1 + L_2\alpha_2}{(L_1 + L_2)}$

32. [3]



Force at point A,

$mg + p_0A = pA \Rightarrow p = p_0 + \frac{mg}{A}$

Given, area of $A = 0.4\,\text{m}^2 = 0.4\times10^4\,\text{cm}^2$ and area of $B = 1\,\text{cm}^2$

Applying continuity equation

$AV_1 = av$ [V_1 = Velocity at point A]

$\Rightarrow V_1 = \frac{a}{A}v$

As $A \gg$ a so $\frac{a}{A}$ very small

$\therefore V_1 << eV$

So we can neglect V_1 by assuming $V_1 = 0$

On applying Bernoulli's theorem at points A and B,

$$p_0 + \frac{mg}{A} + \frac{\rho V_1^2}{2} + \rho gh = p_0 + \frac{\rho v^2}{2} + 0$$

$$\Rightarrow p_0 + \frac{mg}{A} + 0 + \rho gh = p_0 + \frac{\rho v^2}{2}$$

$$\Rightarrow \frac{mg}{A} + \rho gh = \frac{\rho v^2}{2}$$

$$\Rightarrow \frac{24 \times 10}{0.4} + 1000 \times 10 \times 0.4 = \frac{1000 \times v^2}{2} \Rightarrow v \simeq 3 \text{m/s}$$

33. [25] We know that,

Volume of film = Area of film × thickness

Also, we know: Volume of film = Volume of 100 spherical drops

Hence, the thickness of the film;

34. (*d*) $\Delta\ell = 6.241 - 6.230 = 0.011$ cm

$\Delta\ell = \ell.\ \alpha\Delta\theta$

$0.011 = 6.230 \times 1.4 \times 10^{-5}(\theta - 27)$

$\theta \approx 153.11$ nearest is 152.7°C.

35. (*b*) Increase in volume $\Delta V = \gamma V_0 \Delta T$

$y = 3\alpha$

So $\Delta V = (3\alpha)\ V_0 \Delta T$

Total surface area = $6a^2$.

Where a is side length $24 = 6a^2$ $a = 2$m

Volume $V_0 = (2)^3 = 8\text{m}^3$

$\Delta V = (3 \times 5 \times 10)(8) \times 10 = 1.2 \times 10^5 \text{cm}^3$

36. (*c*) Heat given by block to get 0°C temperature

$\Delta Q = 5 \times (0.39 \times 10^3) \times (500 - 0) = 975 \times 10^3$J

Heat absorbed by ice to melt m mass

$\Delta Q_2 = m \times (335 \times 10^3)$ J

$\Delta Q_1 = \Delta Q_2$

$m \times (335 \times 10^3) = 975$x1 O'

$$m = \frac{975}{335} = 2.910\text{kg}$$

37. [90] Energy released by water = 0.3 × 25 × 4200 = 31500 J

let m kg ice melts $m \times 3.5 \times 10^5 = 31500$

$$m = \frac{31500 \times 10^{-5}}{3.5}$$

$m = 0.09$ kg = 90 gm

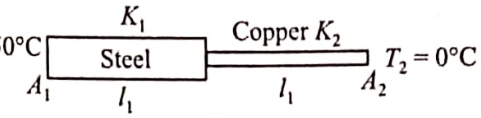
$x = 90$

38. [31] Heat rejected = $mL + mSAT$

$= (50 \times 540) + 50\ (1)\ (100 - 20)$

$= 31000$ Cal $= 31 \times 10^3$ Cal

39. (*c*) $T_1 = 450°C$



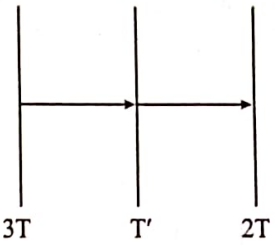
$$\frac{d\theta}{dt} = \frac{K_1 A_1}{l_1}(T_1 - T) = \frac{K_2 A_2}{l_2}(T - T_2)$$

$$\frac{450 - T}{T - 0} = \frac{K_2 A_2 l_1}{K_1 A_1 l_2} = 9 \times \frac{1}{2} \times 2$$

$\Rightarrow 450 - T = 9T \Rightarrow T = 45°C$

JEE ADVANCED

40. (*c*) In steady state energy absorbed by middle plate is equal to energy released by middle plate.



$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$

$(3T)^4 - (T')^4 = (T')^4 - (2T)^4$

$(2T')^4 = (16 + 81)\ T^4$

$$T' = \left(\frac{97}{2}\right)^{1/4} T$$

41. (*a*) Given $H_1 = H_{II}$

$$\frac{k_1 A \Delta T}{\ell + \ell} \times t_1 = k_2 \frac{(A + A)\Delta T}{\ell} \times t_2$$

$$t_2 = \frac{k_1}{k_2} \times \frac{t_1}{4}$$

Configuration-I: $\frac{\ell + \ell}{k_1} = \frac{\ell}{k} + \frac{\ell}{2k} \Rightarrow k_1 = \frac{4k}{3}$

Configuration-II: $k_2(A + A) = kA + 2kA$

$$\Rightarrow k_2 = \frac{3k}{2}$$

$$t_2 = \frac{4k/3}{3k/2} \times \frac{9}{4} = 2\text{sec}$$

42. (*a*) In steady state, $I\pi R^2 = \sigma(T^4 - T_0^4)\ 4\pi R^2$

$\Rightarrow I = \sigma\ (T^4 - T_0^4)\ 4$

$\Rightarrow T^4 - T_0^4 = 40 \times 10^8$

$\Rightarrow T^4 - 81 \times 10^8 = 40 \times 10^8$

$\Rightarrow T^4 = 121 \times 10^8 \Rightarrow T \approx 330\ K$

43. (*b*) $P_{\text{device}} - P_{\text{cooler}} = \frac{ms\Delta T}{\Delta t}$

$$3000 - P = \frac{120 \times 4.2 \times 10^3 \times 20}{3 \times 60 \times 60}$$

$\Rightarrow P = 2067\ W$

Hence, (*b*).

44. (c,d)

As $\lambda_m T = \text{constant}$

& $P = \dfrac{V^2}{R}$

Hence, (c, d)

45. [9] $1 = \log_2\left\{\dfrac{eA\sigma(487+273)^4}{P_0}\right\}$

& $x = \log_2\left\{\dfrac{eA\sigma(2767+273)^4}{P_0}\right\}$

$\therefore x = 9$

46. (a) $\dfrac{T_Q - 10}{\frac{R}{2}} = \dfrac{400 - T_Q}{R} \Rightarrow T_Q = 140$

$\dfrac{dl}{dx} = \alpha\Delta T$

$\int_0^{\Delta l} dl = \int_0^1 \alpha(130x)\,dx \Rightarrow \Delta l = 0.78\,\text{mm}$

Hence, (a)

47. (b)

(a) Since the temperature of the body remains same, therefore heat radiated by the body is same as before.

$\left(W_1 = \sigma a T^4 = \sigma a (310)^4\right)$

(b) $W \propto$ Area

If exposed area decreases, energy radiated also decreases.

(c) $\lambda_m T = b \Rightarrow T\uparrow, \lambda_m\downarrow$

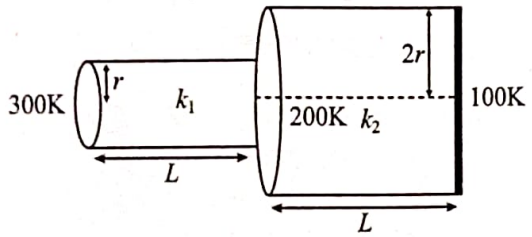
(d) $W_1 = \sigma a T^4 = \sigma a (310)^4$

$\sigma T_0^4 = 460 Wm^{-2}$

$\sigma a (310)^4 > 460\ \text{Wm}^{-2}$

48. [4.00]

We have in steady state,



$$\left(\frac{200-300}{\frac{L}{k_1\pi r^2}}\right) + \left(\frac{200-100}{\frac{L}{k_2\pi(2r)^2}}\right) = 0$$

$$\Rightarrow \frac{k_1\pi r^2 \times 100}{L} = \frac{100 k_2 \pi \times 4r^2}{L}$$

$$\Rightarrow \frac{k_1}{k_2} = 4$$

49. (a) $P = \dfrac{dQ}{dt}\ T_{(t)} = T_0(1+\beta t^{1/4})$

$$\frac{dQ}{dt} = \boxed{ms}\frac{dT}{dt} \Rightarrow S = \frac{P}{\left(\frac{dT}{dt}\right)}$$

$$\frac{dT}{dt} = T_0\left[0 + \beta\frac{1}{4}t^{-3/4}\right] = \frac{\beta T_0}{4}t^{-3/4}$$

$$S = \frac{P}{(dT/dt)} = \frac{4P}{\beta T_0}t^{3/4}$$

$$S = \frac{4P}{\beta}\left[\frac{t^{3/4}}{T_0}\right]$$

$$\frac{T(t)}{T_0} = (1+\beta t^{1/4})$$

$$\beta t^{1/4} = \frac{T(t)}{T_0} - 1 = \frac{T(t)-T_0}{T_0}$$

$$t^{3/4} = \left(\frac{T(t)-T_0}{\beta . T_0}\right)^3$$

$$\Rightarrow S = \frac{4P}{T_0\beta}\left[\frac{T(t)-T_0}{\beta . T_0}\right]^3 = \frac{4P}{\beta^4 T_0^4}[T(t)-T_0]^3$$

50. [270.00]

Let m = mass of calorimeter,

x = specific heat of calorimeter

s = specific heat of liquid

L = latent heat of liquid

First 5 g of liquid at 30° is poured to calorimeter at 110°C

$\therefore m \times x \times (110 - 80) = 5 \times s \times (80 \times 30) + 5\,L$

$\Rightarrow mx \times 30 = 250\,s + 5\,L$ ...(i)

Now, 80 g of liquid at 30° is poured into calorimeter at 80°C, the equilibrium temperature reaches to 50°C.

$\therefore m \times x \times (80 - 30) = 80 \times s \times (50 - 30)$

$\Rightarrow mx \times 30 = 1600\,s$ ...(ii)

From (i) & (ii)

$250\,s + 5\,L = 1600\,s \Rightarrow 5L = 1350\,s$

$\Rightarrow \dfrac{L}{s} = 270$

51. (b,c,d)

$A = 64\ \text{mm}^2$, $T = 2500\ K$ (A = surface area of filament, T = temperature of filament, d is distance of bulb from observer, R_e = radius of pupil of eye)

Point source $d = 100$ m

$R_e = 3$ mm

(a) $P = \sigma A e T^4$

$= 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4$

($e = 1$ black body)

$= 141.75$ w

Option (a) is wrong

(*b*) Power reaching to the eye

$$= \frac{P}{4\pi d^2} \times \left(\pi R_e^2\right)$$

$$= \frac{141.75}{4\pi \times (100)^2} \times \pi \times \left(3 \times 10^{-3}\right)^2$$

$$= 3.189375 \times 10^{-8} \text{W}$$

Option (*b*) is correct

(*c*) $\lambda_m T = b$

$\lambda_m \times 2500 = 2.9 \times 10^{-3} \Rightarrow \lambda_m = 1.16 \times 10^{-6} = 1160$ nm

Option (*c*) is correct

(*d*) Power received by one eye of observer $= \left(\frac{hc}{\lambda}\right) \times N$

N = Number of photons entering into eye per second

$$\Rightarrow 3.189375 \times 10^{-8} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1740 \times 10^{-9}} \times N$$

$$\Rightarrow N = 2.79 \times 10^{11}$$

Option (*d*) is correct.

52. [8.33]

$$\frac{dQ}{dt} = \sigma e A\left(T^4 - T_0^4\right) \quad \text{...(i)}$$

$$\frac{dQ}{Adt} = e\sigma\left(T_0 + \Delta T\right)^4 - T_0^4\right) = \sigma T_4^0\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$= e\sigma T_0^4\left[\left(1 + 4\frac{\Delta T}{T_0}\right) - 1\right]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3 \cdot 4\Delta T \quad \text{...(ii)}$$

Now from equation

$$ms\frac{dT}{dt} = \sigma e T\left(T^4 - T_0^4\right)$$

$$\frac{dT}{dt} = \frac{\sigma e A}{ms}\left[\left(T_0 + \Delta T\right)^4 - T_0^4\right]$$

$$= \frac{\sigma e A}{ms} T_0^4 \times \left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$\frac{dT}{dt} = \frac{\sigma e A}{ms} T_0^4 . 4\Delta T$$

$$\frac{dT}{dt} = e\Delta T; \left(K = \frac{4\sigma e A T_0^3}{ms}\right)$$

$$\Rightarrow 4\sigma e A T_0^3 = \frac{K}{A}(ms)$$

From eqn.

$$\frac{dQ}{Adt} = e\sigma T_0^3 . 4\Delta T$$

$700 = (K/A)(ms)\,\Delta T$

$$\therefore \Delta T = \frac{700 \times 5 \times 10^{-2}}{10^{-3} \times 4200} = \frac{50}{6} = \frac{25}{3}$$

$\Delta T = 8.33$

CHAPTER

16 Kinetic Theory of Gases and Thermodynamics

Exercise-1 (Topicwise)

1. (a) $V \propto T \Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{200}{V_2} = \frac{(273+20)}{(273-20)} = \frac{293}{253}$

$V_2 = \frac{200 \times 253}{293} = 172.6\, ml$

2. (b) Momentum $= M\vec{V}_{av}$, As $\vec{V}_{av} = 0$ (in equilibrium)

$\therefore$ Momentum = 0

3. (c) Mean free path $\lambda = \frac{1}{\sqrt{2}\pi d^2 n} \Rightarrow \lambda \propto d^{-2}$

4. (b) Self explanatory

5. (b) The collision of molecules of ideal gas is elastic collision

6. (a) $\frac{Pm}{\rho} = nRT$

slope of T_1 > slope of T_2

$\therefore T_1 > T_2$

7. (c) One molecule has some single value of speed which is equal average speed and rms speed of the gas

$\therefore v_a = v_{rms}$.

8. (b) $v_{av} \alpha \frac{1}{\sqrt{M_0}}$

$\therefore$ Oxygen molecule hits the wall with smaller average speed

9. (c) According to the Dalton's law of partial pressures, the total pressure will be $P_1 + P_2 + P_3$

10. (b) $\frac{PV}{T} = R(\text{constant}) \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\Rightarrow \frac{200 \times V}{(273+22)} = \frac{P_2 \times 1.02V}{(273+42)}$ $(V_2 = V + 0.02V)$

$\Rightarrow P_2 = \frac{200 \times 317}{275 \times 1.02} = 209\, kPa$

11. (a) $v_{rms} \propto \sqrt{\frac{3RT}{M}} \Rightarrow \frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^2 = \frac{1}{4}$

$T_2 = \frac{T_1}{4} = \frac{(273+327)}{4} = 150K = -123°C$

12. (a) $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow M = \frac{3RT}{v_{rms}^2}$ $\therefore M = \frac{3 \times 8.3 \times 300}{(1930)^2}$

$= 2 \times 10^{-3}$ kg = 2 gm $\Rightarrow$ Gas is hydrogen

13. (c) $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

14. (a) $\frac{T_A}{M_A} = 4\frac{T_B}{M_B} \Rightarrow \sqrt{\frac{T_A}{M_A}} = 2\sqrt{\frac{T_B}{M_B}}$

$\Rightarrow \sqrt{\frac{3RT_A}{M_A}} = 2\sqrt{\frac{3RT}{M_B}} \Rightarrow C_A = 2C_B \Rightarrow \frac{C_A}{C_B} = 2$

15. (c) $V \propto T \Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{V}{2V} = \frac{(273+27)}{T_2} = \frac{300}{T_2}$

$T_2 = 600\, K = 327°C$

16. (a) As initial and final state are same (isothermal)

$\therefore T_i = T_f$

As v_{rms}, p_{av} and K_{av} depends on temperature

$\therefore$ All are equal.

17. (d) $dQ = dW + dU$

$dQ = PdV + dU$

$dQ = nRdT + dU$

$dQ = \frac{2dU}{f} + dU$

$\frac{dU}{dQ} = \frac{1}{\left(\frac{2}{f}+1\right)}$

$\frac{dU}{dQ} = \frac{5}{7}$

18. (c) $s = \frac{Q}{m\Delta T}$

For changing state

T = const or $\Delta T = 0$

$\therefore s = \infty$ (infinite)

19. (a) $\frac{C_P}{C_V} = \gamma = 1 + \frac{2}{f}$

20. (a) When C_P and C_V are given with calorie and R with Joule then $C_P - C_V = R/J$

21. (c) As compare to gas solid expand very less.
$\therefore$ C_p is slightly greater then C_v.

22. (c) A stationary rigid body can vibrate in 3-direction, which are mutually perpendicular to each other. So degree of freedom will be 3.

23. (d) $$\gamma_{mixture} = \frac{\frac{\mu_1\gamma_1}{\gamma_1 - 1} + \frac{\mu_2\gamma_2}{\gamma_2 - 1}}{\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}}$$

μ_1 = moles of helium $= \frac{16}{4} = 4$

μ_2 = moles of oxygen $= \frac{16}{32} = \frac{1}{2}$

$$\gamma_{mix} = \frac{\frac{4 \times 5/3}{\frac{5}{3} - 1} + \frac{1/2 \times 7/5}{\frac{7}{5} - 1}}{\frac{4}{\frac{5}{3} - 1} + \frac{1/2}{\frac{7}{5} - 1}} = 1.62$$

24. (d) $P = \frac{2}{3} \times$ (Energy per unit volume) $= \frac{2}{3}\frac{E}{V}$

$\Rightarrow PV = \frac{2}{3}E$

25. (d) Average kinetic energy $\propto$ temperature

$\Rightarrow \frac{E_1}{E_2} = \frac{T_1}{T_2} \Rightarrow \frac{100}{E_2} = \frac{300}{450} \Rightarrow E_2 = 150\,J$

26. (b) Translational kinetic energy of all molecules of diatomic gas will also be $\frac{3}{2}PV$, because all the gases have same translational degree of freedom (c)

27. (b) The cyclic process 1 is clockwise where as process 2 is anticlockwise. Clockwise area represents positive work and anticlockwise area represents negative work. Since negative area (b) > positive area (a), hence net work done is negative.

28. (a) $T \propto P$ or $\frac{P}{T}$ = constant

As $\frac{P}{T} = \frac{nR}{V}$ = constant or V = constant

$\therefore$ $W = 0$.

29. (d) Work done = Area under curve $= \frac{6P_1 \times 3V_1}{2} = 9P_1V_1$

30. (a) As Volume decreases
$\therefore$ Pressure of the gas in the cylinder increases

31. (c) $W.D. = \pi \times$ Pressure Radius $\times$ Volume Radius (area of ellipse)

$$W = \pi\left(\frac{P_2 - P_1}{2}\right)\left(\frac{V_2 - V_1}{2}\right) = \frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$$

32. (a) $\Delta Q = \Delta U + \Delta W$ and $\Delta W = P\Delta V$

33. (b) In process AB

T = constant, $P < \frac{1}{V} \Rightarrow PT, V\downarrow$

$\Delta W = -ve$. or $\Delta Q - ve$

$\therefore$ Heat is rejected out of the system.

34. (b) $\Delta Q = \Delta W$ (T = constant)
If heat is supplied then $\Delta W = +ve$

35. (b) $B \to A$
$\Delta Q = 0$
$0 = -30 + \Delta U_{BA}$
$\Delta U_{BA} = 30$ J
$\therefore$ $\Delta U_{AB} = -\Delta U_{BA} = -30\,J$

36. (d) $\Delta U = \Delta Q - \Delta W$ is same in both methods as it is a state function

37. (d) Heat given $\Delta Q = 20$ cal $= 20 \times 4.2 = 84\,J$.
Work done $\Delta W = -50\,J$ [As process is anticlockwise]
By first law of thermodynamics
$\Rightarrow \Delta U = \Delta Q - \Delta W = 84 - (-50) = 134\,J$

38. (b) $dQ = dU + dW$

39. (c) In isothermal process temperature remains constant.

40. (b) $L \to M$ P = constant $V \alpha T$.
$M \to N$ T = constant.

41. (c) Fact based

42. (a) $PV = nRT$
Differentiating both sides
$PdV + VdP = 0$ (isothermal process)
$\Rightarrow \frac{-dP}{(dV/V)} = P$

43. (c) Isothermal $P \propto \frac{1}{V}$
Adiabatic $P \propto \frac{1}{V^\gamma}$
Also, slope of adiabatic is more as compare to isothermal

44. (c) Gas cylinder suddenly explodes is an irreversible adiabatic change and work done against expansion reduces the temperature.

45. (b) $B = \frac{VdP}{dV} = -\frac{(-PdV)}{dV}$ (for isothermal process)
$B = P$

46. (b) In a refrigerator, the heat dissipated in the atmosphere is more than that taken from the cooling chamber, therefore the room is heated if the door of a refrigerator is kept open.

47. (b) $\eta = 1 - \frac{T_2}{T_1} \Rightarrow -\frac{30}{100} = 1 - \frac{350}{T_1}$

$\Rightarrow \frac{350}{T_1} = 1 - \frac{30}{100} = \frac{70}{100} = \frac{7}{10} \Rightarrow T_1 = 500K = 227°C$

48. (*a*) $T_2 = 0°C = 273\ K,\ T_1 = 100°C = 373\ K$

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$= \left(1 - \frac{273}{373}\right) \times 100 = 26.81\%$$

49. (*a*) By adjoining graph $W_{AB} = 0$ and

$W_{BC} = 8 \times 10^4\ [5 - 2] \times 10^{-3} = 240$ J

$\therefore\ W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240$ J

Now, $\Delta Q_{AC} = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800$ J

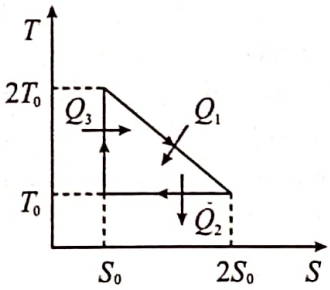
From first law of thermodynamics

$\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC}$

$\Rightarrow 800 = \Delta U_{AC} + 240$

$\Rightarrow \Delta U_{AC} = 560$ J.

50. (*a*) $Q_1 = T_0S_0 + \frac{1}{2}T_0S_0 = \frac{3}{2}T_0S_0$



$Q_2 = T_0S_0$ and $Q_3 = 0$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$$

Exercise-2 (Learning Plus)

1. (*c*) For adiabatic expansion

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = (32)^{\frac{7}{5}-1} = 4$$

$T_1 = 4T_2$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4}$$

$\eta = 0.75$

$\eta = 75\%$

2. (*a*) For a diatomic gas,

$$C_p = \frac{7}{2}R$$

Since gas undergoes isobaric process

$\Rightarrow \Delta Q = nC_p\Delta T$

Also, $\Delta W = nR\Delta T = 10\ J$ (given)

$$\therefore\ \Delta Q = n\frac{7}{2}R\Delta T = \frac{7}{2}(nR\Delta T) = 35\ J$$

3. (*a*) $\Delta Q = nC_v\,\Delta T = n\frac{3}{2}R\Delta T$

$$= \left(\frac{67.2}{22.4}\right)\left(\frac{3}{2} \times 8.31\right)(20)$$

$\approx 748\ J$

4. (*c*) $v = \frac{V_{av}}{\lambda}$

$$\lambda = \frac{RT}{\sqrt{2}\pi\sigma^2 N_A P}$$

$\sigma = 2 \times .3 \times 10^{-9}$

$$P = \frac{RT}{V}$$

$$\Rightarrow \lambda = \frac{V}{\sqrt{2}\pi\sigma^2 N_A} \Rightarrow V_{av} = \sqrt{\frac{8}{3\pi}} \times V_{rms}$$

$$\therefore\ v = \frac{200 \times \sqrt{2}\pi \times \sigma^2 N_A}{25 \times 10^{-3}} \times \sqrt{\frac{8}{3\pi}}$$

$= 17.68 \times 10^8$/sec.

$= .1768 \times 10^{10}$/sec. $\sim 10^{10}$

5. (*c*) $W = nR\Delta T$

$\Delta Q = (C_v + nR)\Delta T$

$$\frac{w}{\Delta Q} = \frac{nR}{C_v + nR}$$

6. (*a*) $$\gamma_{mixture} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{V_1} + n_2C_{V_2}} = \frac{n_1\frac{\gamma_1 R}{\gamma_1 - 1} + n_2\frac{\gamma_2 R}{\gamma_2 - 1}}{\frac{n_1R}{\gamma_1 - 1} + \frac{n_2R}{\gamma_2 - 1}}$$

or rearranging we get

$$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}; \quad \frac{5}{\gamma_{mix} - 1} = \frac{3}{\frac{1}{3}} + \frac{2}{\frac{2}{3}}$$

$$\frac{n_1 + n_2}{\gamma_{mix} - 1} = 9 + 3 = 12 \Rightarrow \gamma_{mixure} = \frac{17}{12} = 1 + \frac{5}{12}; \gamma_{mix} = 1.42$$

7. (*b*) $\tau \propto \frac{1}{\sqrt{T}}$

8. (*d*) $$\gamma mix = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{v_1} + n_2C_{v_2}}$$

$$= \frac{n\left(\frac{5}{2}R\right) + 2n\left(\frac{7}{2}R\right)}{n\left(\frac{3}{2}R\right) + 2n\left(\frac{5}{2}R\right)} = \frac{5 + 14}{3 + 10} = \frac{19}{13}$$

9. (*d*) Molar heat capacity of A at constant volume $= \frac{5R}{2}$

Molar heat capacity of B at constant volume $= \frac{7R}{2}$

Dividing both, $\frac{(C_V)_A}{(C_V)_B} = \frac{5}{7}$

10. (*a*) Mean free time τ

$$= \frac{1}{\sqrt{2}n\pi d^2 V_{avg}} = \frac{1}{\sqrt{2}n\pi(2r)^2\sqrt{\frac{8RT}{\pi M_o}}} = \frac{\sqrt{M_o}}{16nr^2\sqrt{\pi RT}}$$

$$\tau \propto \frac{\sqrt{M_o}}{r^2} \Rightarrow \frac{\tau_{Ar}}{\tau_{Xe}} = \left(\frac{r_{Xe}}{r_{Ar}}\right)^2 \times \sqrt{\frac{M_{o_{Ar}}}{M_{o_{Xe}}}}$$

$$= \left(\frac{0.1}{0.07}\right)^2 \times \sqrt{\frac{40}{140}} = \left(\frac{10}{7}\right)^2 \times \sqrt{\frac{2}{7}} = 1.07$$

11. (*c*) $V_{av} = \sqrt{\frac{8KT}{\pi m}}$, as T = constant $\therefore V_{av}$ = constant

12. (*c*) $V_{av} = \sqrt{\frac{8RT}{\pi M_0}} = v$

For nitrogen $V_{av} = \sqrt{\frac{8R \times 2T}{\pi M_0/2}} = 2.v$

13. (*b*) $V_{av} \alpha \frac{1}{\sqrt{M_0}}$

$\therefore$ oxygen molecule hits the wall with smaller average speed

14. (*a*) As translation K.E. is $= \frac{3}{2}nRT$

$E = \frac{3}{2}PV$

where E = total translational K.E.

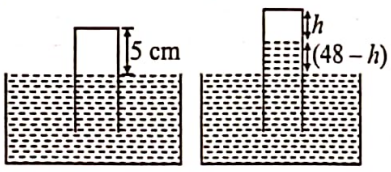
15. (*c*) Gas has different specific heat for different processes

$\therefore$ Gas has infinite number of specific heats.

16. (*c*) Initially

P_1 = 76 cm of Mercury

$V_1 = 5A$ ('A' is cross-section area of tube)



Finally,

$P_2 = 76 - (48 - h) = (28 + h)$ cm of Mercury

and $V_2 = hA$

Since temperature remains constant

$\Rightarrow P_1V_1 = P_2V_2$

$76 \times 5A = (28 + h) \times (hA)$

$380 = 28h + h^2$

$\Rightarrow h^2 + 28h - 380 = 0$

$\Rightarrow h = 10$ cm $= 0.1$ m

17. (*a*) $P + 50 = 75$ cm of Hg

$\Rightarrow P = 25$ cm of Hg

$\frac{10^5}{75} \times 25 = 33.3$ kPa

18. (*d*) $W_{net} = W_{1-2} + W_{2\to3} + W_{3-1}$

$10 = W_{1-2} + 0 - 20$

$W_{1-2} = 30J$

$\Delta U_{1-2} = 0$

$\therefore \Delta Q_{1-2} = \Delta W_{1-2} + \Delta U_{1\to2} = 30$ J

19. (*a*) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ $\left(\gamma = \frac{5}{3} \text{ for 'He'}\right)$

$\Rightarrow T_1(5.6)^{5/3-1} = T_2(0.7)^{5/3-1}$

$\Rightarrow T_2 = 4T_1$

At STP, 1 mol. of gass is occupied 22.4 litre. So 5.6 litre gas. has 1/4 mol.

Work done $W = \frac{nR(T_1 - T_2)}{(\gamma - 1)} = \frac{9}{8}RT$

20. (*a*) A-(p,r,t); B-(p,r); C-(q,s); D-(r,t)

A. $A \to B \Rightarrow V\downarrow P$ const. $\Rightarrow T\downarrow U\downarrow$ (p), (r), (t)

$B \to C \Rightarrow dW = 0; P\downarrow T\downarrow$

B. $dQ = dU + dW$ (p), (r)

$C \to D \Rightarrow V\uparrow \Rightarrow T\uparrow$

$dU \Rightarrow +ve$

C. $dW = +ve$ (q), (s)

D. $D \to A \Rightarrow dw \Rightarrow -ve$ (r), (t)

$dQ \Rightarrow -ve$

$dU = 0$

21. (*a*) $AB \to$ isothermal

$P_A V_A = P_B V_B$ ...(i)

$BC \to$ Adiabatic

$P_B V_B^{\gamma} = P_C V_C^{\gamma}$ ...(ii)

$CD \to$ Isothermal

$P_C V_C = P_D V_D$ ...(iii)

$DA \to$ Adiabatic

$P_D V_D^{\gamma} = P_A V_A^{\gamma}$ ...(iv)

From (i), (ii), (iii) and (iv)

$\frac{V_B}{V_C} = \frac{V_A}{V_D}$

22. (*a*) For adiabatic

$TV^{\gamma-1} = C\ (\gamma > 1)$ ...(i)

For isothermal. T = const ...(ii)

From (i) and (ii)

$T_2 < T_1$

23. (*c*) For isothermal

$PV = C$ or $P_1 \alpha \frac{1}{V_1}$ ...(i)

For adiabatic

$PV^{\gamma} = C,\ P_2 \alpha \frac{1}{V_2^{\gamma}}$ ...(ii)

From (i) and (ii)

$P_1 > P_2$

24. (*a*) $\frac{dP}{P} = -\gamma\frac{dV}{V}$ (For adiabatic)

$0.5 = -1.4\frac{dV}{V}$

$\therefore$ Volume decrease by 0.36%

25. (*d*) XY Adiabatic compression

YZ Isothermal Expansion

ZX Compression at constant pressure

26. (*a*) As $\Delta Q = \Delta U + W$

$\Delta U = -W$ (given) or $\Delta Q = 0$

$\therefore$ Process is adiabatic

27. (*c*) $p - p_0 = \frac{4S}{r}$

$\Rightarrow dp = \frac{4S}{r^2}dr$

$$C = C_V + \frac{dW}{ndT} = C_V + \left[R + \frac{16S\pi}{3}\frac{rdr}{ndT}\right] = \frac{7R}{2}$$

28. (*d*) $P_2 = 2P_1, V_2 = 4V, n = 1$

$$C = C_V + \frac{PdV}{dT}$$

$dW = PdV =$ Area

$$= \frac{1}{2}\left[(P_1 + P_2)(V_2 - V_1)\right] = \frac{1}{2}(3P_1 \times 3V_1) = \frac{9}{2}P_1V_1$$

$$dT = T_2 - T_1 = \frac{P_2V_2}{R} - \frac{P_1V_1}{R} = \frac{2P_1 \times 4V_1}{R} - \frac{P_1V_1}{R} = \frac{7P_1V_1}{R}$$

$$C = \frac{5}{2}R + \frac{9}{2} \times \frac{P_1V_1R}{7P_1V_1} = \frac{5}{2}R + \frac{9R}{14} = \frac{44R}{14} = \frac{22R}{7}$$

29. (*c*) The molecule contains 1 mole of helium and one mole of oxygen 4 gms of helium (monoatomic weight and molecular weight) corresponds to 1 mole 32 gms of oxygen (diatomic molecular weight 32) corresponds to one mole.

$$2C_v = \left(1 \times \frac{3}{2} + 1 \times \frac{5}{2}\right)R = 4R$$

$$\therefore 2C_p = \left(1 \times \frac{5}{2} + 1 \times \frac{7}{2}\right)R = 6R$$

$$\frac{C_p}{C_v} = \frac{6R}{4R} = 1.5$$

30. (*d*) $kx_0 = PA$ [x_0 is initial compression]

$$100x_0 = \left(\frac{nRT}{V}\right) \times 10^{-2}$$

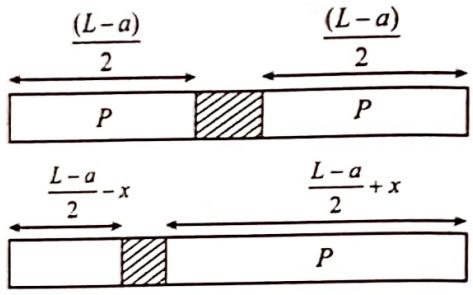
$$x_0 = \frac{1}{100} \times \frac{1 \times 8.3 \times 100}{0.83} \times \frac{1}{100} = 0.1$$

$x_1 = 0.2$ (Total compression)

$$W_{gas} = \frac{1}{2} \times 100 \times (0.2)^2 - \frac{1}{2} \times 100 \times (0.1)^2$$

$$= \frac{100}{2} \times \frac{1}{100}[4 - 1] = 1.5$$

31. (*a*)



$P_1V_1 = P_2V_2$

$$P\left(\frac{L-a}{2}\right)A = P_2\left(\frac{L-a}{2} - x\right)A$$

$$P_2 = \frac{P(L-a)}{(L-a-2x)}$$

$$p\left(\frac{L-a}{2}\right)A = \left(\frac{L-a}{2} + x\right)AP_2'$$

$$P_2' = \frac{P(L-a)}{(L-a+2x)}$$

$$P_2 - P_2' = \frac{P(L-a)}{[(L-a)-(2x)]} - \frac{P(L-a)}{[(L-a)+2x]}$$

$$\Delta P = \frac{4P(L-a)}{[(L-a)^2 - 4x^2]}$$

$$F = \Delta PA = \frac{4PA(L-a)}{[(L-a)^2 - 4x^2]}$$

$$\omega = \sqrt{\frac{4P}{L\rho a}} = 2\sqrt{\frac{P}{L\rho a}}$$

$$T = \frac{2\pi}{\omega} = \pi\left(\frac{L\rho a}{P}\right)^{1/2}$$

32. (*a*) $PV = nRT$

$$\Rightarrow P_0 + (1 - \alpha V^2)V = nRT \Rightarrow T = \frac{(P_0 + (1-\alpha)V^2)V}{nR}$$

$$\frac{dT}{dV} = \frac{P_0}{nR} + \frac{3V^2(1-\alpha)}{nR} = 0 \quad \text{...(i)}$$

$$\Rightarrow P_0 = 3(\alpha - 1)V^2 \Rightarrow V^2 = \frac{P_0}{3(\alpha - 1)}$$

$$0 = P_0 + (1-\alpha)\frac{P_0}{3(\alpha-1)}$$ [From equation (i)]

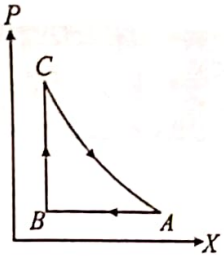
$$\Rightarrow P_0 - \frac{P_0}{3} = \frac{2P_0}{3}$$

33. (*b*) For the process AB: $U\rho$ constant (hyperbola)

$U = \frac{3}{2}RT$ (monatomic ideal gas)

$\frac{3}{2}\rho RT =$ constant

Comparing it with ideal gas equation $P = \left(\frac{1}{M}\right)\rho RT \Rightarrow P$ is constant



$P-V$ graph for the cycle is Thus, work done is positive.

34. (*c*) For isothermal process $V_f = 2V_0$

$\therefore P_f = P_0/2$

For isobaric process

$V_f = V_0/2, T_f = \frac{V_0}{2\times 2V_0}T_0.$

$= \frac{T_0}{4}$

For $P \propto V$ process

$P-V$ must be straight line

$T \propto V^2 \Rightarrow V-T$ must be parabolic

$P^2 \propto T \Rightarrow P-T$ must be parabolic

35. (*d*) As $\gamma > 1$

for $TV^{\gamma-1} = \text{constant}$ $\frac{dT}{dV} < 0$ & $\frac{dT^2}{dV^2} > 0$ and

for $TV^{\gamma} = KP^{\gamma-1}$ $\frac{dT}{dP} > 0 \frac{d^2T}{dP^2} < 0$

36. (*b*) $\Delta Q = dU + PdV$

$\Rightarrow \frac{\alpha}{T}dT = C_v dT + PdV$

$\Rightarrow \frac{\alpha}{T}dT = C_v dT + \frac{RT}{V}dV$

$\Rightarrow C = VT^{\frac{1}{\gamma-1}}e^{\alpha/RT}$

37. (*b*) Let initial pressure of the three samples be P_A, P_B and P_C.

$P_A(V)^{3/2} = (2V)^{3/2}P$

$P_B = P$

$P_C V = P(2V)$

$P_A : P_B : P_C = (2)^{3/2} : 1 : 2$

38. (*d*) $0.6 = \frac{\text{workdone}}{Q_{\text{input}}} = \frac{Q_{\text{input}} - Q_{\text{reject}}}{Q_{\text{input}}} = 1 - \frac{Q_r}{Q_i}$

$0.6 = 1 - \frac{20}{Q_i}$

$Q_i = 50$ J

$W = Q_i - Q_r = 30$ J

39. (*c*) $\Delta U = \frac{f}{2}nR(T_2 - T_1)$

$= \frac{3}{2}(P_f V_f - P_i V_i)$

$= \frac{3}{2}(10^5 \times 4 - 10^5 \times 6) = -3\times 10^5$ J

40. (*a*) The equation of process $A \rightarrow B$ is

$P = -\left(\frac{2P_0}{V_0}\right)V + 5P_0$

$T = -\left(\frac{2P_0}{nRV_0}\right)V^2 + \left(\frac{5P_0}{nR}\right)V$

$\frac{dT}{dV} = -\left(\frac{4P_0}{nRV_0}\right)V + \frac{5P_0}{nR}$

Now, $C = C_V + \frac{pdV}{ndT}$

$C = \frac{3R}{2} + R\left(\frac{5P_0V_0 - 2P_0V}{5P_0V_0 - 4P_0V}\right)$

$C = \frac{R}{2}\left(\frac{25V_0 - 16V}{5V_0 - 4V}\right)$

Hence the heat is absorbed by the gas

From V_0 to $\frac{25}{16}V_0$

Hence the process changes from endothermic to exothermic at the volume $\frac{25}{16}V_0$

41. (*b*) Since the ideal gas in each chamber undergoes an isothermal process

$\frac{dP}{dV} = -\left(\frac{P}{V}\right)$

$dP = -\left(\frac{P}{V}\right)dV$

When the piston is slightly displaced from equilibrium position by a distance 'X' the net restoring force acting on the piston is

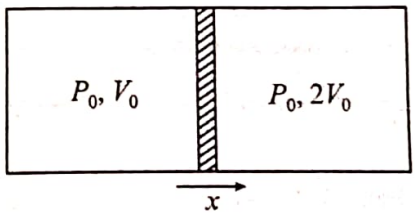
$F = \left(\frac{P_0}{2V_0} + \frac{P_0}{V_0}\right)A^2X$

$F = \left(\frac{3P_0}{2V_0}\right)A^2X$

Now $\frac{md^2X}{dt^2} = -F \Rightarrow \frac{md^2X}{dt^2} = -\left(\frac{3P_0}{2V_0}\right)A^2X$

$\Rightarrow \frac{d^2X}{dt^2} = -\left(\frac{3P_0A^2}{2mV_0}\right)X$

Time period $T = 2\pi\sqrt{\frac{2mV_0}{3P_0A^2}}$



42. (*b*) For process $T_A = 300$ K, $T_B = 600$ K

$W = nR\Delta T$

$= nR(T_B - T_A)$

$= 300nR = 600R$

$Q = nC_p\Delta T = 2\times\frac{5}{2}R(300) = 1500R.$

For process BC

$W = nRT\ln\frac{V_f}{V_i} = nRT\ln\frac{P_i}{P_f} = nRT\ln 2 = 1200R\ln 2$

$Q = W = 1200\text{ R ln } 2$

For process CA $W = \int PdV = \int_{600}^{300}\frac{K}{T}\frac{2nRT}{K}dT$.

$= -2nR(300)$

$= -1200R$

$Q = nC_v\Delta T + W$

$= 2\times\frac{3}{2}R(-300) - 1200R$

$= -900R - 1200R = -2100R$

$\eta = \frac{600R + 1200R\ln 2 - 1200R}{1500R + 1200R\ln 2}$

$= 1 - \frac{21}{12\ln 2 + 15} \Rightarrow x = 7$

43. (*a*) After opening of at equilibrium temperature and pressure of whole gas is T_1 and P_1

$n_1 = \frac{1\times V}{RT}, n_2 = \frac{0.5\times V\times 4}{RT},$

$n_1 + n_2 = n$

$\frac{V}{RT} + \frac{V\times 4}{2RT} = \frac{5VP_1}{RT_1}$

$\frac{3V}{RT} = \frac{5VP_1}{RT_1}; \frac{P_1}{T_1} = \frac{0.6}{T}$

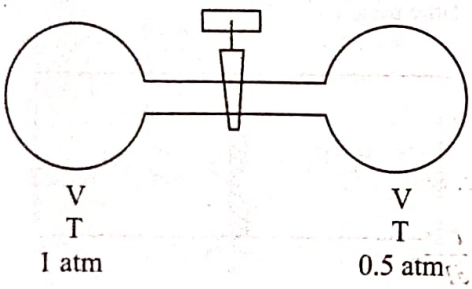
$\Delta Q = 0, \Delta W = 0$

$\therefore \Delta U = 0 n_1C_vT + n_2C_vT = (n_1 + n_2)C_vT_1$

$T_1 = T$

$\frac{P_1}{R} = \frac{0.6}{T}$

$P_1 = 0.60$ atm



Exercise-3 (JEE Advanced Level)

1. (*c,d*)

By energy conservation, energy loss by one molecule is equal to gain by other.

2. (*a,b,c*)

Avg. momentum/mol $\propto \sum V_x^2$

$\sum V_x^2$ is same at NTP

$(K.E.)_{avg} \propto T$

$(K.E.)/vol. \propto T$

3. (*b,d*)

$v_{rms} = 1.73\sqrt{\frac{KT}{m}}$

so v_{rms} does not change

$\frac{P_1}{n_1} = \frac{P_2}{n_2} \Rightarrow \frac{n_1}{n_2} = \frac{1}{2}$

4. (*a,d*)

$n_1 = \frac{7}{28}; n_2 = \frac{11}{44}$

So $n_1 + n_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$m_0 = \frac{18kg}{(1/2)} = 36kg$

$C_{V\,mix} = \frac{\frac{1}{4}\times\frac{5}{2}R + \frac{1}{4}\times 3R}{\frac{1}{4} + \frac{1}{4}} = \frac{11}{4}R$

$C_{P\,mix} = \frac{11}{4}R + R = \frac{15}{4}R$

$\gamma = \frac{C_P}{C_V} = \frac{15}{11} = \frac{45}{33} \cong \frac{47}{35}$

5. (*a,b,c,d*)

Specific heat of a substance can be finite, infinite, zero and negative.

6. (*a,b*)

$C_P > C_V$ and $C_P - C_V = 2$

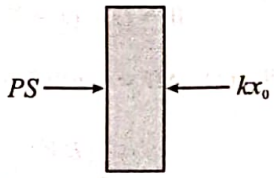
$\therefore$ Option A and B is correct.

7. (*a,d*)

For equilibrium of piston

$PS = Kx_0$

$P = \frac{Kx_0}{S}$



For piston

$W_{all} = KE_2 - KE_1$

$W_{gas} - \frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$W_{gas} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{positive}$$

$\Delta Q = 0$

$\Delta Q = \Delta U + W$

$\Delta U = -W = \text{negative}$

As internal energy of gas decreases

$\therefore$ Temperature of gas decreases.

8. (c,d)

As the process is carried out suddenly it may be adiabatic and as the conductivity is good enough then may be isothermal.

9. (b,d)

$\Delta W_A = P_1 \Delta V$

$\Delta W_D = P_2 \Delta V \{P_1 > P_2\}$

$\Delta Q_A = \Delta U_A + \Delta W_A$

$\Delta Q_D = \Delta U_D + \Delta W_D$

$\Delta Q_A - \Delta Q_D = \Delta U_A - \Delta U_D + \Delta W_A - \Delta W_D$

$\Delta Q_A - \Delta Q_D = \Delta W_A - \Delta W_D \qquad (\therefore \Delta U_A = \Delta U_D)$

$Q_A > Q_D$

$$W_B = PdV = \int_{v_1}^{v_2} \frac{k}{v} dV = k\ell n \frac{V_2}{V_1}$$

$$W_C = k\ell n \frac{V_2}{V_1}$$

hence $W_B - W_C = 0 \Rightarrow Q_B > Q_C$

$Q_A > Q_B > Q_C > Q_D$

10. (b,d)

In cyclic process, $\Delta U = 0$

$\Delta Q = \Delta W$ as $\Delta W = +ve$

$\therefore \Delta Q = +ve$

or Net heat energy has been supplied to the system in process CA

$\Delta W = 0 \qquad \Delta U = -ve$ (As T = decreases)

$\therefore$ Heat energy is rejected out by system

Temperature at C is maximum

11. (a,c)

In cyclic process $\Delta U_1 + \Delta U_2 = 0$

$\Delta U_{Net} = 0$

$\Delta Q - \Delta W = 0$

12. (a,b)

$$\Delta Q = \Delta U + \Delta W \Rightarrow 25 = \frac{nfR\Delta T}{2} + 0$$

$$25 = \frac{1 \times f \times 25 \times 2}{2 \times 3}$$

$f = 3$ (monoatomic)

13. (a,b)

$A \to B$ constant pressure

$B \to C$ T = constant

$C \to D$ constant Volume

$D \to A$ T = constant

14. (a,b,c,d)

$\because U = nRT$, where f, n, R are constants. Also temperature T is same at A & B.

$\therefore U_A = U_B$

Also, $\Delta W_{AB} = nRT_0 = nRT_0 = nRT_0 \ell n4 = P_0V_0 \ell n4$

15. (b,c)

Slope of x > slope of y

During expansion, $W_y > W_x$

$U_y > U_x \Rightarrow C_{v_2} > C_{v_1}$

$f_2 > f_1$

16. (a,b,d)

$$\eta = \frac{0/P}{1/P} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}.$$

17. (a,b)

$$\gamma = \frac{dV}{VdT} = \frac{2}{T} \Rightarrow \int \frac{dV}{V} = \frac{2}{T}$$

$V = KT^2$ i.e $PV^{1/2}$ = constant

$$C = C_V + \frac{R}{\left(1 - \frac{1}{2}\right)} \Rightarrow C = C_V + 2R = C_p + R$$

18. (a,b,c,d)

(a) For isothermal process $Q = W$.

Curve 3 corresponds to isothermal process, and Curve 1 and 2 corresponds to isobaric processes.

(b) For isobaric process

$W = p\Delta V = R\Delta T \ (n = 1)$

$$Q = C_p \Delta T = \left(\frac{f+2}{2}\right) R\Delta T$$

(Where f = degrees of freedom)

$$\frac{W}{Q} = \frac{2}{f+2}$$

For curve 1, $\frac{2}{f+2} = \frac{W_1}{Q_1} = \frac{20}{80} = \frac{1}{4} \Rightarrow f = 6$

(polyatomic gas)

For curve 2, $\frac{2}{f+2} = \frac{W_2}{Q_2} = \frac{32}{80} = \frac{2}{5} \Rightarrow f = 3$

(monoatomic gas)

19. (b,c)

Ideal gas $PV = RT \Rightarrow V = \frac{RT}{P}$

Adiabatic : PV^γ = constt. $\Rightarrow P\left(\frac{RT}{P}\right)^\gamma$ = constt.

$\Rightarrow P^{1-\gamma}T^\gamma$ = constt. (c say) $\Rightarrow P = c \cdot T^{\frac{\gamma}{\gamma-1}}$

$\therefore$ Slope : $\frac{dP}{dt} = \frac{c\gamma}{\gamma - 1} T^{\frac{1}{\gamma-1}} > 0$ (not constt.)

$$\frac{d^2P}{dT^2} > 0$$

20. (a,d)

For isothermal process PV = constant and for adiabatic process PV^{γ} = constant where $\gamma > 1$

First we assuming that BC is isothermal and CA is adiabatic

$$\Rightarrow P_0V_0 = 3P_0V_C \Rightarrow V_C = \frac{V_0}{3} \qquad \text{[For process } BC\text{]}$$

$$3P_0\left(\frac{V_0}{3}\right)^{\gamma} = \frac{V_0P_0}{2} \Rightarrow \gamma = \frac{\ln 6}{\ln 3} \qquad \text{[For process } CA\text{]}$$

Now we are assuming that process BC is adiabatic and AC is isothermal

$$3P_0V_C = \frac{P_0V_0}{2} \Rightarrow V_C = \frac{V_0}{6} \qquad \text{[For process } CA\text{]}$$

$$3P_0\left(\frac{V_0}{6}\right)^{\gamma} = P_0V_0^{\gamma} \Rightarrow 6^{\gamma} = 3 \qquad \text{[For process } BC\text{]}$$

$\Rightarrow$ [not possible]

Process AC is adiabatic and $V = \frac{ln6}{ln3}$

21. (a,c,d)

(a) Process AB : PT const.

$\Rightarrow nRT^2$ = const.

$$W = \int_A^B PdV = \int_A^B \frac{const.}{T}dV$$

$$\Rightarrow \frac{dV}{dT} = \frac{2nRT}{const.}$$

$$\Rightarrow W = \int_{300}^{100} \frac{const.}{T} \cdot \frac{2nRT}{const.}dt$$

$$\Rightarrow \frac{P_A}{P_B} = \frac{T_B}{T_A} \Rightarrow \frac{1}{3} = \frac{T_B}{T_A}$$

$$\Rightarrow T_B = \frac{300}{3} = 100$$

$W = 2nR\,(100 - 300)$

$W_{AB} = -400\,nR$

(b) Process CA

$$\frac{P}{T} = C \Rightarrow V = C$$

$$\therefore \frac{T_A}{T_C} = \frac{P_A}{P_B} = \frac{1}{3}$$

$\Rightarrow T_C = 3T_A$

$\Delta U = nC_v(T_A - T_C)$

$= 1 \times \frac{3}{2} \times R(300 - 900)$

$\Delta U = -900\,R$

(c) Process CA

$Q = nC_p\Delta T$

$= 1 \times \frac{5}{2} R \times (900 - 100)$

$Q = 2000\,R$

22. (c) Heat given: $\Delta Q = n_1C_{V_1}\Delta T \rightarrow$ For gas A

[As V = constant $\therefore dW = 0$]

& for Gas B and $\Delta Q = n_2C_{V_2}\Delta T$

($\because$ For same heat given, temperature rises by same value for both the gases.)

$\Rightarrow n_1C_{V_1} = n_2C_{V_2}$ ...(a)

Also, $(\Delta P_B)V = n_2R\Delta T$ and $(\Delta P_A)V = n_1R\Delta T$

$$\Rightarrow \frac{n_1}{n_2} = \frac{\Delta P_A}{\Delta P_B} = \frac{2.5}{1.5} = \frac{5}{3}$$

$$\Rightarrow n_1 = \frac{5}{3}n_2$$

Substituting in (a)

$$\frac{5}{3}n_2\,C_{V_1} = n_2C_{V_2}$$

$$\Rightarrow \frac{C_{V_2}}{C_{V_1}} = \frac{5}{3} = \frac{\left(\frac{5}{2}R\right)}{\left(\frac{3}{2}R\right)}$$

Hence Gas B is diatomic and Gas A is monoatomic.

23. (d) Since $n_1 = \frac{5}{3}n_2$. Therefore $\frac{125}{M_A} = \frac{5}{3}\left(\frac{60}{M_B}\right)$

(From experiment 1 : W_A = 125 gm & w_B = 60 gm)

$\Rightarrow 5M_B = 4M_A$

The above relation holds for the pair–Gas A : Ar and Gas B : O_2.

24. (b) No. of molecules in 'A' $= nN_A = \frac{125}{40}N_A = 3.125\,N_A$

(Since $n = \frac{125}{40}$ for Ar)

25. (c) Internal energy at any temperature T

$= nC_VT$

$= \left(\frac{125}{40}\right)\left(\frac{3R}{2}\right)(300)$[$\because C_V$ for mono atomic gas $= \frac{3R}{2}$]

$\Rightarrow U_i = 2812.5$ cal.

26. (b) Let initial temperature and volume be T_0 and V_0. Since the process is adiabatic, the final temperature and volume is $TV^{\gamma-1} = T_0V_0^{\gamma-1}$

($\gamma = \frac{5}{3}$ for mono atomic gas)

$$\therefore T = T_0\left(\frac{V_0}{V_0/8}\right)^{\frac{2}{3}} = 4T_0$$

$\therefore$ Percentage increase in temperature of gas is

$$\frac{\Delta T}{T_0} \times 100 = \frac{3T_0}{T_0} \times 100 = 300\%$$

27. (c) Adiabatic bulk modulus $B = -V\frac{dP}{dV} = \gamma P = \gamma\frac{nRT}{V}$

$$\therefore \frac{B_i}{B_f} = \frac{T_0}{V_0} \times \frac{V}{T} = \frac{T_0}{V_0} \times \frac{V_0/8}{4T_0} = \frac{1}{32}$$

28. (*b*) For adiabatic process $dQ = 0$

$\therefore dU + dW = 0$ or $\frac{dW}{dU} = -1$

29. (*a*)

(*a*) If $P = 2V^2$, from an ideal gas equation $PV = nRT$ we get

$2V^3 = nRT$

$\therefore$ With increase in volume

(*i*) Temperature increases implies $dU = +ve$

(*ii*) $dW = +ve$

Hence $dQ = dU + dW = +ve$

(*b*) If $PV^2 =$ constant, from an ideal gas equation $PV = nRT$ we get $VT = K$ (constant)

Hence with increase in volume, temperature decreases

Now $dQ = dU + PdV = nC_v dT - \frac{PV}{T} dT$

$[\because dV = -\frac{V}{T} dT]$

$= nC_v dT - \frac{PV}{T} dT = n(C_v - R)\, dT$

$\therefore$ With increase in volume $dT = -ve$

and since $C_v > R$ for monoatomic gas. Hence $dQ = -ve$

with increases in temperature $dV = -ve$,

$\therefore W = -ve$

(*c*) $dQ = nC\, dT = nC_v\, dT + PdV$

$\Rightarrow n\,(C_v + 2R)\, dT = nC_v dT + PdV$

$\therefore 2nRdT = PdV$

$\therefore \frac{dV}{dT} = +ve$

Hence with increase in temperature volume increases and vice versa.

$\therefore dQ = dU + dW = +ve$

(*d*) $dQ = nC\, dT = nC_v\, dT + PdV$

or $n\,(C_v - 2R)dT = nC_v dT + PdV$

or $-2nRdT = PdV$

$\therefore \frac{dV}{dT} = -ve$

$\therefore$ With increase in volume temperature decreases.

Also $dQ = n(C_v - 2R)dT$

For expantion $dT = -ve$ but $C_v < 2R$ for monoatomic gas. Therefore $dQ = +ve$

with increase in temperature $dV = -ve$,

$\therefore W = -ve$

30. (*d*)

(*a*) $PV = nRT$

$P = (nRT)\frac{1}{V} = (\text{constant})\frac{1}{V}, P \propto \frac{1}{V}$

$T =$ constant i.e. isothermal process

As $\frac{1}{V}$ decreases or V increases

$\therefore \Delta W =$ positive

and $\Delta Q = \Delta U + \Delta W = \Delta W > 0\ (\Delta U = 0)$

(*b*) $\Delta Q = 0$ and $V =$ increases

$\therefore \Delta W =$ positive

(*c*) $PV = nRT \quad V \propto T\ (P =$ constant$)$

As volume increases, T also increases

i.e., $\Delta U > 0$ and $\Delta W > 0$So $\Delta Q > 0$

(*d*) For cyclic process $\Delta U = 0$

$\Delta W < 0$ (anticlockwise)

$\Delta Q = \Delta U + \Delta W < 0$

31. [0075]

$J\Delta V = \frac{f}{2} nR\Delta T = \frac{5}{2}(P_2V_2 - P_1V_1) = 63\ J$

$mgx + \frac{1}{2}kx^2 + P_0Ax = W_{gas}$

$W_{gas} = 12\ J \Rightarrow \Delta Q = 75\ J$

32. [40] $PV = \frac{m}{M} RT$

$m = \frac{PVM}{RT}$

$= \frac{500 \times 10^3 \times 5000 \times 10^{-6} \times 40 \times 10^{-3}}{\frac{25}{3} \times 300}$

$= \frac{4}{100}$ kg $= 40$ gm

33. [0020]

$P_i = P_0 + \frac{kx}{A} = 2 \times 10^5\ P$

$P_f = 2 \times 10^5 + 10^6\, x$

$V_i = 1A$ (A: cross section)

$V_f = (1 + x)A$

$\because \frac{PV}{T} =$ const.

$\Rightarrow \frac{2 \times 10^5 \times 1A}{T_0} = \frac{2 \times 10^5 + 10^6 x}{2T_0} \times (1 + x)A$

$2.4 = \frac{(2 + 10x)(1 + x)}{2}$

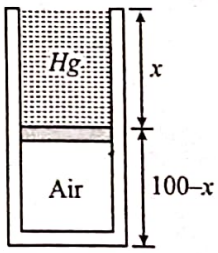
$2.4 = (1 + 5x)(1 + x)$

$2.4 = 1 + 5x^2 + 6x$

$5x^2 + 6x - 1.4 = 0$

$x = \frac{-6 + \sqrt{36 + 4 \times 5 \times 1.4}}{10} = 0.2$ m $= 20$ cm

34. [76]

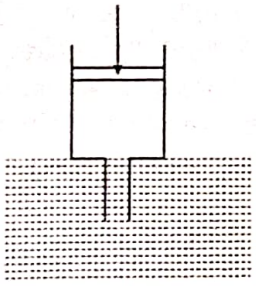


$P_1V_1 = P_2V_2$

$(76)(100) = (76 + x)(100 - x)$ or $x = 24$ cm

gas column length $= 76$ cm

35. [0075]



$$\frac{P_0V_0}{n_0} = \frac{P_1V_1}{n_1}$$

$$\frac{10^5 \times A_1h_0}{n_0} = \frac{10^5 + 5\times10^3\times10\times0.2\times V_1}{0.4n_0}$$

$$= \frac{10^5\times4\times10^{-4}\times h_0}{n_0}$$

$$= \frac{(10^5 + 0.2\times10\times5\times10^3)V_1}{0.4n_0}$$

$V_1 = 4 \times 10^{-4}\,[h_0 - h] + 0.2 \times 1 \times 10^{-4}$

$h_0 = 1.1 \times 10^5 \times [4 \times 10^{-4}\,(h_0 - h) + 2 \times 10^{-5}]$

$= 1.1\,[40h_0 - 40h + 2]$

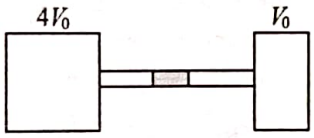
$16h_0 = 44h_0 - 44h + 2.2$

$44h = 28h_0 + 2.2$

$44h = 28 \times 1.1 + 2.2 = 33$

$h = 0.75$ m

36. [0125]



$$n_1 = \frac{P_0\times V_0}{R\times300};\ n_2 = \frac{P_0\times4V_0}{R\times300};\ n = \frac{5P_0V_0}{300R}$$

$$n_1' = \frac{P\times V_0}{R\times300};\ n_2' = \frac{P\times4V_0}{400R};\ n = \frac{P\times4V_0}{400R}$$

$$\Rightarrow \frac{5}{4}P_0 = P;\quad P = 125\ kPa$$

37. [0005]

Temperature is constant

$$\Rightarrow \Delta E = 0,\ dW = nRT\frac{dv}{v} = nRT\frac{Adx}{AL/2}$$

$Q = \Delta E + W$

$$\Rightarrow \frac{dW}{dt} = \frac{nRT}{L/2}\frac{dx}{dt}$$

$$\frac{dQ}{dt} = \frac{dW}{dt} \Rightarrow k\frac{1}{900}\frac{\Delta T}{L} = \frac{2nRT}{L}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \frac{dx}{dt} = \frac{k\times27}{900nRT}$$

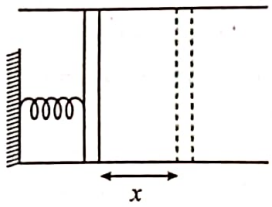
$$= \frac{415.5\times27\times2}{900\times0.5\times8.31\times300} = \frac{1}{200}\ \text{m/s} = 5\ \text{mm/s}$$

38. [0100]

$P_2V_2 = P_1V_1$

$V_1 = 20000$cc. $\quad P_1 = 10^5\ Pa$

$V_2 = Ax \quad P_2 = P_0 + \frac{kx}{A}$



$$\Rightarrow 10^5\times20000\times10^{-6} = \left(10^5 + \frac{1000x}{100\times10^{-4}}\right)10^{-4}\times100x$$

$\Rightarrow 2000 = 1000\,[x\,(1 + x)]$

$x^2 + x - 2 = 0$

$$x = \frac{-1\pm\sqrt{1+8}}{2} = 1\ \text{m}$$

$x = 100$ cm.]

39. [0000]

$P_0V_0 = nRT_a$

$(P_0 + \Delta P)\,(V_0 + \Delta V) = nRT_b$

$(P_0 + \Delta P)\,(V_0 - \Delta V) = nRT_b$

$\Delta V = 0$

40. [217]

$P_0H_0A = P_ih_iA$; (T-constant)

$\Rightarrow P_0H_0 = P_ih_i$ ...(i)

$\frac{P_i}{T_i} = \frac{P_f}{P_f}$; (volume is same finally)

$$\frac{P_0 + m_1g/A}{P_0 + (m_1 + m_2)g/A} = \frac{273+20}{273+\theta}$$

$$\Rightarrow \frac{10^5 + \frac{200}{100\times10^{-4}}}{10^5 + \frac{1000}{100\times10^{-4}}} \Rightarrow \theta = 217°$$

41. [0126]

$3n = n_1 + n_2 + n_3$

$$3\frac{P_0V}{RT_0} = \frac{PV}{R(2T_0)} + \frac{PV}{R(3T_0)} + \frac{PV}{RT_0}$$

$$3P_0 = \frac{P}{2} + \frac{P}{3} + P$$

$$\Rightarrow \frac{3P + 2P + 6P}{6} = \frac{11P}{6}$$

$P = \frac{18}{11}P_0 = 18 \times 77/11$

$= 126$ cm of Hg

Exercise-4
(Past Year Questions)

JEE MAIN

1. (c) Mean free path is independent of temperature and relaxation time decreases as temperature increases.

2. (b) $E = 3 \times \frac{5}{2}RT + 5 \times \frac{3}{2} \times RT = 15RT$

3. (a) $U = \frac{nfRT}{2}$

$f = 6$ (for triatomic)

$\Rightarrow U = 3RT$

4. (c) For Monatomic Triatomic

$\frac{C_v}{C_p} = \frac{5}{3}$ $\frac{C_v}{C_p} = \frac{4}{3}$

For Diatomic

Rigid $\frac{C_v}{C_p} = \frac{7}{5}$

Non Rigid $\frac{C_v}{C_p} = \frac{9}{7}$

5. [40.93]

V_{rms} for $N_2 = \sqrt{\frac{3KT}{28}} = \sqrt{\frac{3 \times K \times 573}{28}}$

v_{rms} for $H_2 = \sqrt{\frac{3KT_1}{2}}$

v_{rms} for $N_2 = V_{rms}$ for H_2

$\therefore \frac{3KT_1}{2} = \frac{3K \times 573}{28} \Rightarrow T_1 = \frac{573}{14}K$

$T_1 = 40.93$ kelvin

6. (d) $PV = NkT$

$E = \frac{3}{2}kT$

$N = \frac{3PV}{2E}$ $(P = hrg)$

$= 4 \times 10^{18}$

7. (c) $\tau \propto \frac{1}{\sqrt{T}}$

$\because \ell_{mean} = \frac{1}{\sqrt{2}\pi n D^2}$ and $V \propto \sqrt{T}$

$\therefore \tau_{mean} = \frac{\ell_{mean}}{v} \Rightarrow T \propto \frac{1}{\sqrt{T}}$

8. (d) $f = 5$

$\therefore U = \frac{5}{2}RT$

and $\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$

9. (a) $P_1 = 1$ atm, $T_1 = 273K$

$P_1V_1^\gamma = P_2V_2^\gamma$

$P_2 = P_1\left[\frac{V_1}{V_2}\right]^\gamma = 1 \text{ atm}\left(\frac{1}{3}\right)^{1.4}$

Now work done $\frac{P_1V_1 - P_2V_2}{\gamma - 1} = 88.7\ J$

Closest an s is $90.5\ J$

10. (b) Relaxation time $(\tau) \propto \frac{V}{\sqrt{T}}$ and $T \propto \frac{1}{V^{\gamma-1}}$

$\tau \propto V^{1+\frac{1-\gamma}{2}}$

$\tau \propto V^{\frac{1+\gamma}{2}}$

$\frac{\tau_f}{\tau_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}}$

$\frac{\tau_f}{\tau_i} = (2)^{\frac{1+\gamma}{2}}$

11. (b) Let, Q_H : Heat input to Ist engine

Q_L : Heat rejected from Ist engine

Q'_L : Heat rejected from IInd engine

Work done by Ist engine = work done by IInd engine

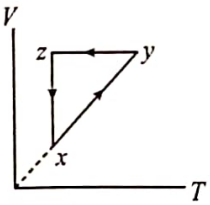
$Q_H - Q_L = Q_L - Q'_L$

$2Q_L = Q_H + Q'_L$

$2 = \frac{T_1}{T} + \frac{T_2}{T}$

$T = \frac{T_1 + T_2}{2}$

12. (a)



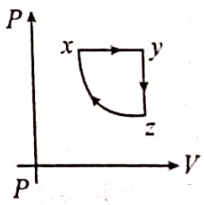
From the corresponding V-T graph

Process $xy \rightarrow$ Isobaric expansion,

Process $yz \rightarrow$ Isochoric (Pressure decreases)

Process $z - x \rightarrow$ Isothermal compression

Therefore, corresponding PV graph is



13. (a) For process 3 – 1, volume is constant

$PV = nRT$; as P increases; T increases

For PV^γ = constant $\Rightarrow TV^{\gamma-1}$ = Constant

For process 2 – 3; pressure is constant

$V = kT$

14. [266.67]

$$\left(0.1\times\frac{3}{2}R\times 200\right)+\left(0.05\times\frac{3}{2}R\times 400\right)=0.15\times\frac{3}{2}RT_f$$

$$\Rightarrow (20+20)=0.15\,T_f$$

$$\Rightarrow T_f=\frac{40}{15}\times 100=266.67$$

15. (*a*) $PV^{\gamma}=$ constant

$$V\propto\frac{1}{\rho}$$

$$\frac{P}{\rho^{\gamma}}=\text{constant}$$

16. (*d*) For all process

$$E_{AB}=E_{AC}=E_{AD}$$

$$W_{AB}>0$$

$$W_{AC}=0$$

$$W_{AD}<0$$

17. [19] $W=2P_0V_0$

$$Q_+=2P_0V_0\times\frac{3}{2}+\frac{5}{2}\times 3P_0V_0$$

$$\eta=\frac{2P_0V_0}{3P_0V_0+\frac{15}{2}P_0V_0}=\frac{4}{21}$$

$$\eta=19.04\%$$

18. [32] $\rho_A=\rho_B$

$$r_B=4r_A$$

$$\Delta l_a=2\text{ mm}$$

$$\Delta l_B=4\text{ mm}$$

$$Y=\frac{\text{stress}}{\text{strain}}=\frac{F/A}{\Delta\ell/\ell}$$

$$\Rightarrow Y=\frac{\Delta\ell}{\ell}=\frac{F}{A}$$

$$\Rightarrow \ell=\frac{AY\Delta\ell}{F}$$

$$\Rightarrow \frac{\ell_a}{\ell_b}=\frac{\pi r_a^2\times Y\times\Delta\ell_a\times F}{\pi r_b^2\times Y\times\Delta\ell_b\times F}$$

$$\Rightarrow \frac{\ell_a}{\ell_b}=\frac{r_a^2\times\Delta\ell_a}{r_b^2\times\Delta\ell_b}=\frac{r_b^2\times 2}{(4r_a)^2\times 4}=\frac{r_d^2}{16r^2\times 2}$$

$$\Rightarrow \frac{\ell_a}{\ell_b}=\frac{1}{32}$$

$$\therefore x=32$$

19. [113]

$$n=0.60=1=\frac{T_L}{T_H}$$

$$\frac{T_L}{T_H}=0.4$$

$$\Rightarrow T_L=0.4\times 400=160\,K=-113°\text{C}$$

20. (*a*) Efficiency $\eta=1-\frac{T_L}{T_H}$

$$\eta_1=1-\frac{147+273}{447+273}=1-\frac{420}{720}$$

$$\eta_1=\frac{300}{720}$$

$$\eta_2=1-\frac{47+273}{947+273}=1-\frac{320}{1220}$$

$$\eta_2=\frac{900}{1220}$$

$$\frac{\eta_1}{\eta_2}=\frac{300}{720}\times\frac{1220}{900}=\frac{122}{72\times 3}$$

$$\frac{\eta_1}{\eta_2}=0.56$$

21. (*b*) $\frac{C_P}{C_v}=1+\frac{2}{f}=1.4\Rightarrow f=5$

By conservation of energy

$$\frac{f}{2}nR\Delta T=\frac{1}{2}[M]v^2$$

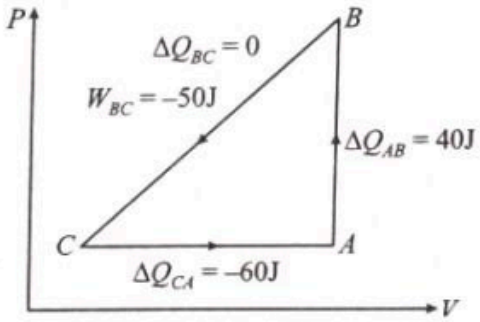
$$\Delta T=\frac{mv^2}{fR}=\frac{Mv^2}{5R}$$

22. (*a*) $\eta=\left[1-\frac{T_L}{T_H}\right]\times 100\%$

$$T_L=0^{\circ}\text{C}=273\text{ K},\ T_n=373\text{ K}$$

$$\eta=26.809\%$$

23. (*b*)



$$\Delta Q_{\text{cycle}}=40-60=\Delta W$$

$$\Delta W=-20\text{J}=W_{BC}+W_{CA}$$

$$W_{CA}=-20\text{J}-W_B=-20-(-50)=30\text{J}$$

24. (*c*) Constant entropy means process is adiabatic

$$PV^{\gamma}=\text{constant}$$

$$V_2=\frac{V_1}{8}$$

$$P_1V_1^{\gamma}=P_2V_2^{\gamma}$$

$$P_1V_1^{\gamma}=P_2\left(\frac{V_1}{8}\right)^{5/3}$$

$$P_1V_1^{5/3}=\frac{P_2V_1^{5/3}}{32}$$

$$P_2=32\,P_1$$

25. (*a*) Heat Transfer

A. By Newton's law of colling $\frac{d\theta}{dt} \propto \Delta T$

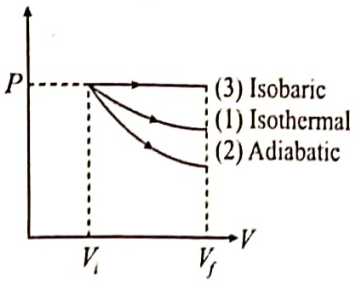
B. $H = \frac{d\theta}{dt} = \sigma e T^4 \Rightarrow \frac{H_P}{H_Q} = \left(\frac{T_P}{T_Q}\right)^4 = \left(\frac{283}{293}\right)^4$

$H_P : H_Q = 1(1.03)^4 = 1:(1.03)^4 = 1:1.15$

C. $\eta = 1 - \frac{100}{400} = \frac{3}{4} = 75\%$

D. is wrong as $\frac{d\theta}{dt} \propto \Delta T$

26. (*d*)



Area under curve is work

$W_2 < W_1 < W_3$

27. (*b*) Molar heat capacity at constant volume $C_V = \frac{fR}{2}$

where f is degree of freedom.

Molar heat capacity at constant pressure can be written as

$C_P = R + C_V = R + \frac{fR}{2} = \left(1 + \frac{f}{2}\right)R$

So $\frac{C_P}{C_V} = 1 + \frac{2}{f}$

28. [4] $PV^\gamma = \text{const}, \because d = \frac{m}{v}$

$\Rightarrow P\left(\frac{m}{d}\right)^\gamma = \text{const}$

$\frac{P}{d^\gamma} = \text{const}$

$\frac{P_1}{P_2} = \left(\frac{d_1}{d_2}\right)^\gamma = \left(\frac{1}{32}\right)^{7/5} = \frac{1}{128}$

$\frac{T_1}{T_2} = \frac{P_1 V_1}{P_2 V_2} = 32$

29. [3] $(C_V)_{mix} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$

$= \frac{1 \cdot \frac{3R}{2} + 3 \cdot \frac{5R}{2}}{1+3}$

$= \frac{9R}{4} = \frac{a^2}{4}R$

$a = 3$

30. [16] $(T_2)T_{sink} = 200K$

$(T_1)T_{Reservoir} = 527 + 273 = 800K$

$W = 12000KJ = 12 \times 10^6 J$

$\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} = 1 - \frac{200}{800} = \frac{12 \times 10^6}{Q_1}$

$\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} = 1 - \frac{200}{800} = \frac{12 \times 10^6}{Q_1}$

$\frac{3}{4} = \frac{12 \times 10^6}{Q_1} = Q_1 = 16 \times 10^6$ J

31. [1400]

$Q = nC_p\Delta T = n \cdot \left(\frac{\gamma R}{\gamma - 1}\right)\Delta T$

or $Q = \frac{\gamma}{\gamma - 1}W = \frac{1.4}{0.4} \times 400 = 1400J$

32. (*b*) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$T \to 2T,\ M \to \frac{M}{2}$ then $V'_{rms} = 2\sqrt{\frac{3RT}{M}} \Rightarrow V'_{rms} = 2V_{rms}$

33. (*b*) For a quasi-static process the change in internal energy of an ideal gas is $\Delta U = nC_V\Delta T = n \times \frac{3R}{2} \times \Delta T$

[molar heat capacity at constant volume for mono atomic gas $= \frac{3R}{2}$]

$\Delta U = 7 \times \frac{3}{2} \times 8.3 \times 40 = 3486J$

34. (*c*)

(*a*) $V_{Rms} = \sqrt{\frac{3RT}{M_w}} \Rightarrow V_{Rms}$ is same

(*b*) $\frac{P_1}{P_2} = \frac{N_1}{N_2}$ [*a* & *b* only are correct]

35. (*c*) No of moles of H_2 = 8 moles

No of moles of O_2 = 4 moles

Total moles = 12 moles

At STP 1 mole occupy = 22.4 $\ell \times 10^3$ cm^3

12 moles will occupy 12 × 22.4 × 10^3 cm^3

≈ 26.8 × 10^4 cm^3

36. (*b*) $PV = nRT$

$n = \frac{100 \times 10^3 \times 2000 \times 10^{-6}}{\frac{25}{3} \times 300}$

$n = 80 \times 10^{-3}$

$n_1 + n_2 = 0.08$

$n_1 \times 2 + n_2 \times 32 = 0.76$

$(0.08 - n_2)2 + n_2(32) = 0.76$

$n_2 = 0.02$

$n_1 = 0.06$

$\frac{n_1}{n_2} = \frac{3}{1}$

37. (*a*) $PV = nRT$

$\frac{V}{T} = \frac{nR}{P}$

$\frac{nR}{P_1} < \frac{nR}{P_2}$

$P_2 < P_{1t}$

38. (*b*) $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$

39. (*b*) $v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_{mp} = \sqrt{\frac{2RT}{M}}$

40. [250]

$PV = nRT$

$\Delta P.V. = nR\Delta T$

$\frac{\Delta P}{P} = \frac{\Delta T}{T} = \frac{0.4}{100}$

$T = \frac{100 \times 1}{0.4} = 250\,K$

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41. (*d*) $\frac{V_{Rms_{He}}}{V_{Rms_{Ar}}} = \frac{\sqrt{\frac{3RT}{m_{He}}}}{\sqrt{\frac{3RT}{m_{Ar}}}} = \sqrt{\frac{m_{Ar}}{m_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$

42. (*d*) $\Delta Q = nC_P \Delta T$

$= 2\left(\frac{f}{2}R + R\right)\Delta T$

$= 2\left[\frac{3}{2}R + R\right] \times 5$

$= 2 \times \frac{5}{2} \times 8.31 \times 5 = 208\,J$

43. (*d*) $P_1 = \frac{\rho_1 RT}{M_1}$ ...(i)

$P_2 = \frac{\rho_2 RT}{M_2}$ ...(ii)

By (i) and (ii)

$\frac{\rho_1}{\rho_2} = \frac{8}{9}$

44. (*a,b,c,d*)

$q = mCT$

$\frac{dq}{dt} = mc\frac{dT}{dt}$

R = rate of absorption of heat $= \frac{dq}{dt} \propto C$

(i) in $0 - 100\,K$

C increases, so R increases but not linearly

(ii) $\Delta q = mC\Delta T$ as C is more in (400 K – 500 K) then (0 – 100 K) so heat is increasing.

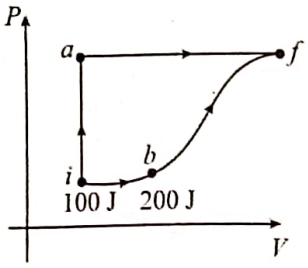
(iii) C remains constant so there no change in R from (400 K – 500 K)

(iv) C is increases so R is increases in range (200 K – 300 K)

45. [2] $W_{ibf} = 150$ J

$W_{iaf} = 200$ J

$Q_{iaf} = 500$ J So $U_{iaf} = 300$ J



So $U_f = 400\,J$

$U_{ib} = 100\,J$

$Q_{ib} = 100 + 50 = 150\,J$

$Q_{ibf} = 300 + 150 = 450\,J$

So the required ratio $\frac{Q_{bf}}{Q_{ib}} = \frac{450 - 150}{150} = 2$

46. (*d*) Let final temperature of gases is T

Heat rejected by gas in lower compartment ($nC_v\Delta T$)

$= 2 \times \frac{3}{2}R(700 - T)$

Heat received by gas in above compartment ($nC_P\Delta T$)

$= 2 \times \frac{7}{2}R(T - 400)$

Equating above

$2100 - 3T = 7T - 2800$

$\Rightarrow T = 490\,K$

47. (*d*) $\Delta W_1 + \Delta U_1 = \Delta Q_1$

$\Delta W_2 + \Delta U_2 = \Delta Q_2$

$\Delta Q_1 + \Delta Q_2 = 0$

$\frac{7}{2}R(T - 400) = \frac{5}{2}R(700 - T)$

$\Rightarrow T = \frac{6300}{12} = 525\,K$

So $\Delta W_1 + \Delta W_2 = 2.R.(525 - 400) + 2R(525 - 700)$

$= +250R - 350R$

$= -100R$

48. (*d*) In first process, $\Delta U = W = \frac{P_iV_i - P_fV_f}{\lambda - 1} = 112.5J$

In two-step process, $\Delta Q = \Delta U + (W_1 + W_2)$

$= \Delta U + P_i(V_f - V_i)$

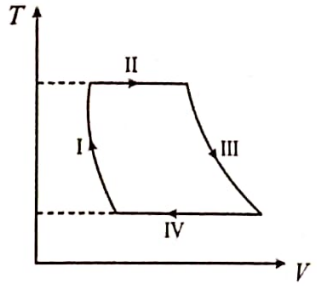
$= 112.5 + 700 = 812.5J$

49. (*a*) For Isobaric process $W = -P(V_2 - V_1)$, graph (*P*)

50. (*d*) For Isochoric (*S*) $W = 0$, graph (*S*)

51. (*d*) For Adiabatic process (*Q*) $W = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$ graph (*Q*)

52. (*b,c,d*)



(*a*) Process I is not isochoric, *V* is decreasing

(*b*) Process II is isothermal expansion

$\Delta U = 0,\ W > 0$

$\Delta Q > 0$

(*c*) Process IV is isothermal compression.

$\Delta U = 0,\ W < 0$

$\Delta Q < 0$

(*d*) Process I and III are NOT isobaric because in isobaric process $T \propto V$ hence isobaric *T-V* graph will be linear.

53. [900]

$V_i = V,\ V_f = 8V$

For adiabatic process $\left\{\gamma = \frac{5}{3}\right.$ for monoatomic process

$T_1V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$

$100(V)^{2/3} = T_2(8V)^{2/3} \Rightarrow T_2 = 25K$

$\Delta U = nc_v\Delta T = 1\left(\frac{fR}{2}\right)[100-25] = 12\times 75 = 900$ Joule

54. (*c*) Process - I is an adiabatic process

$\Delta Q = \Delta U + W \quad \Delta Q = 0$

$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperature of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding

Process - II is an isobaric process

(Pressure remain constant)

$W = P\,\Delta\,V = 3P_0[3V_0 - V_0] = 6P_0V_0$

Process - III is an isochoric process

(Volume remain constant)

$\Delta Q = \Delta U = W$

$W = 0$

$\Delta Q = \Delta U$

Process - IV is an isothermal process

(Temperature remains constant)

$\Delta Q = \Delta U + W$

$\Delta U = 0$

55. (*a,b*)

From graph

Process $1 \to 2$ is isobaric with $P = \frac{RT_0}{V_0}$

Process $2 \to 3$ is isochoric with $V = 2V_0$

Process $3 \to 4$ is isobaric with $P = \frac{RT_0}{2V_0}$

Process $4 \to 1$ is isochoric with $V = V_0$

Work in cycle $= \frac{RT_0}{V_0}.V_0 - \frac{RT_0}{2V_0}.V_0 = \frac{RT_0}{2}$

$Q_{1-2} = nC_p\Delta T = n.\frac{5R}{2}.T_0$

$Q_{2-3} = nC_V\Delta T = n.\frac{3R}{2}.T_0$

$\therefore \left|\frac{Q_{1-2}}{Q_{2-3}}\right| = \frac{5}{3}$

$Q_{3-4} = nC_p\Delta T = n.\frac{5R}{2}.\frac{T_0}{2}$

$\therefore \left|\frac{Q_{1-2}}{Q_{3-4}}\right| = 2$

56. (*a,c,d*)

$n_1 = 5$ moles $C_{V_1} = \frac{3R}{2}\ P_0V_0T_0$

$n_2 = 1$ mole $C_{V_2} = \frac{5R}{2}$

$(C_v)_m = \frac{n_1C_{V_1} + n_2C_{v2}}{n_1 + n_2} = \frac{5\times\frac{3R}{2} + 1\times\frac{5R}{2}}{6} = \frac{5R}{3}$

Similarly, $(C_p)_m = \frac{5R}{3} + R = \frac{8R}{3}$

$\gamma_{mix} = \frac{(c_p)_m}{(c_v)_m} = \frac{8}{5}$

(*a*) $P_0V_0^{\gamma} = P\left(\frac{V_0}{4}\right)^{\gamma}$

$\Rightarrow P = P(d)^{8/5} = 9.2P_0$ which is between $9P_0$ and $10P_0$

(*b*) Average K.E. $= 5 \times \frac{3}{2}RT + 1 \times \frac{5RT}{2} = 10RT$

To calculate *T*

$\frac{P_0V_0}{T_0} = 9.2P_0 \times \frac{V_0}{4\times T}$

so $T = \frac{9.2}{4}T_0$

Now average K.E. $= 10R \times 9.2\ \frac{T_0}{4} = 23RT_0$

(*c*) $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$= \frac{P_0V_0 - 9.2P_0 \times \frac{V_0}{4}}{3/5} = -13RT_0$

57. (*c*) (I) Degree of freedom $f = 3$

Work done in any process = Area under P–V graph

$\Rightarrow$ Work done in $1 \to 2 \to 3 = P_0V_0$

$= \frac{RT_0}{3} \Rightarrow (Q)$

(II) Change in internal energy $1 \to 2 \to 3$

$= \Delta U = nC_v \Delta T$

$= \frac{f}{2} nR\Delta T = \frac{f}{2}(P_fV_f - P_iV_i)$

$= \frac{3}{2}\left(\frac{3P_0}{2}2V_0 - P_0V_0\right)$

$= 3P_0V_0$

$= \Delta U = RT_0 \Rightarrow (R)$

(III) Heat absorbed in $1 \to 2 \to 3$

for any process, 1st law of thermodynamics

$\Delta Q = \Delta W + W$

$\Delta Q = RT_0 + \frac{RT_0}{3}$

$\Delta Q = \frac{4RT_0}{3} \Rightarrow (S)$

(IV) Heat absorbed in process $1 \to 2$

$\Delta Q = \Delta U + W$

$= \frac{f}{2}(P_fV_f - P_0V_0) + W$

$= \frac{3}{2}(P_02V_0 - P_0V_0) + P_0V_0 = \frac{5}{2}P_0V_0 = \frac{5}{2}\left(\frac{RT_0}{3}\right)$

$\Delta Q = \frac{5RT}{6} \Rightarrow (U)$

58. (*d*) Process $1 \to 2$ is isothermal (temperature constant)

Process $2 \to 3$ is isochoric (volume constant)

(I) Work done in $1 \to 2 \to 3$

$W = W_{1\to2} + W_{2\to3}$

$= nRT \ln\left(\frac{V_f}{V_i}\right) + W_{2\to3} = \frac{RT_0}{3} \ln\left(\frac{2V_0}{V_0}\right) + 0$

$W = \frac{RT_0}{3} \ln 2 \Rightarrow (P)$

(II) ΔU in $1 \to 2 \to 3$

$\Delta U = \frac{f}{2} nR(T_f - T_i) = \frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right)$

$= \frac{3}{2}R\left(\frac{2T_0}{3}\right)$

$\Delta U = RT_0 \Rightarrow (R)$

(III) For any system, first law of thermodynamics

for $1 \to 2 \to 3$

$\Delta Q = \Delta U + W$

$\Delta Q = RT_0 + \frac{RT_0}{3} \text{kn } 2$

$\Delta Q = \frac{RT_0}{3}(3 + \ln 2) \Rightarrow (T)$

(IV) For process $1 \to 2$ (isothermal)

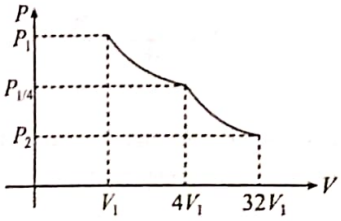
$\Delta Q = \Delta U + W$

$= \frac{f}{2} nR(T_f - T_i) + nRT \ln (V_f/V_i)$

$= 0 + R\left(\frac{T_0}{3}\right)\ln\left(\frac{2V_0}{V_0}\right)$

$\Delta Q = \frac{RT_0}{3} \ln 2 \Rightarrow (P)$

59. [1.77 to 1.78]



$\frac{P_1}{4}(4V_1)^{5/3} = P_2(32V_1)^{5/3}$

$P_2 = \frac{P_1}{4}\left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$

$W_{adi} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1V_1 - \frac{P_1}{128}(32V_1)}{\frac{5}{3} - 1}$

$= \frac{P_1V_1(3/4)}{2/3} = \frac{9}{8}P_1V_1$

$W_{iso} = P_1V_1 \ln\left(\frac{4V_1}{V_1}\right) = 2P_1V_1 \ln 2$

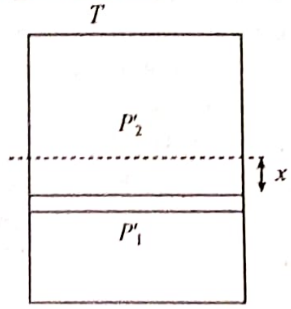
$\frac{W_{iso}}{W_{adio}} = \frac{2P_1V_1 \ln 2}{\frac{9}{8}P_1V_1} = \frac{16}{9}\ln 2 = f \ln 2$

$f = \frac{16}{9} = 1.7778 \approx 1.78$

60. [6] Assuming temperature remains constant at 300 K

From $P_1V_1 = P_2V_2$

$\frac{P_1\left(\frac{V_0}{2}\right)}{T} = \frac{P_1'\left(\frac{V_0}{2} - Ax\right)}{T}$



$(p_1' - P_2')A = mg$

$\left[\frac{P_1\left(\frac{V_0}{2}\right)}{\frac{V_0}{2} - Ax} - \frac{P_2\left(\frac{V_0}{2}\right)}{\frac{V_0}{2} + Ax}\right]A = mg$

$$nRT\left[\frac{1}{4-x}-\frac{1}{4+x}\right]=mg$$

$$(0.1)(8.3)\left[\frac{4+x-4+x}{16-x^2}\right]=mg$$

$$3\left(\frac{2x}{16-x^2}\right)=1$$

$6x = 16 - x^2$

$x^2 + 6x - 16 = 0$

$x = 2$

Distance = 4 + 2 = 6m

61. [2.05]

$$W = (\Delta P)_{avg} \times 4\pi R^2 a$$

$$=\left|\frac{dP}{2}.4\pi R^2 a\right|$$

(for small change $(\Delta P)_{avg}$ <P> arithmetic mean)

$$= PV^{\gamma} = c \Rightarrow dP = -\gamma\frac{P}{V}dV = -\frac{\gamma P_0}{V}4\pi R^2 a$$

$$=\frac{\gamma P_0}{2V}\times 4\pi R^2 a\times 4\pi R^2 a = \frac{\gamma P_0}{2\times 4\pi R^3}4\pi R^2 a\times 4\pi R^2 a$$

$$=\left(4pRP\times a^2\right)\frac{3\gamma}{2}$$

$\therefore\ x \simeq 2.05$

62. [9] $\sigma A T^4 = -ms\frac{dT}{dt}$

$$\int_{200}^{100}\frac{dT}{T^4}=\int_0^{t_1}kdt$$

$$\left.\frac{1}{3T^3}\right|_{200}^{100}=kt_1$$

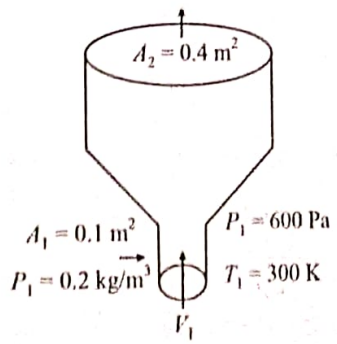
$$\frac{1}{3}\left(\frac{1}{100^3}-\frac{1}{200^3}\right)=kt_1$$

$$\left.\frac{1}{3T^3}\right|_{200}^{50}=kt_2$$

$$\frac{1}{3}\left(\frac{1}{50^3}-\frac{1}{200^3}\right)=kt_2$$

$$\frac{t_2}{t_1}=\left(\frac{200^3-50^3}{200^3-100^3}\right)\frac{100^3}{50^3}=9$$

63. (*b*)



$$\frac{dm}{dt}=\rho_1 A_1 V_1 = 0.8\,\text{kg/s} \text{ or } V_1=\frac{0.8}{0.2\times 0.1}=40\,\text{m/s}$$

$g = 10\ m/s^2$

$\gamma = 2,\ P^{1-\gamma}T^{\gamma} = \text{constant}$

Gas undergoes adiabatic expansion,

$$\frac{P_2}{P_1}=\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{1-\gamma}}$$

$$P_2=\left(\frac{300}{150}\right)^{\frac{2}{-1}}\times 600$$

$$P_2=\frac{600}{4}=150\,\text{Pa}$$

Now $\rho=\frac{PM}{RT}\Rightarrow \rho\propto\frac{P}{T}$

$$\frac{\rho_1}{\rho_2}=\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)=\left(\frac{150}{600}\right)\left(\frac{300}{150}\right)=\frac{1}{2}$$

$$\rho_2=\frac{\rho_1}{2}=0.1\,\text{kg/m}^3$$

Now $\rho_2 A_2 V_2 = 0.8 \Rightarrow V_2=\frac{0.8}{0.1\times 0.4}=20\,\text{m/s}$

Now work done W on gas = $\Delta K + \Delta U$ + Internal energy

$$P_1 A_1\Delta x_1 - P_2 A_2\Delta x_2 = \frac{1}{2}\Delta m V_2^2 - \frac{1}{2}\Delta m V_1^2 + \Delta mgh + \frac{f}{2}\left(P_2\Delta V_2 - P_1\Delta V_1\right)$$

$$\Rightarrow 2P_1\frac{\Delta V_1}{\Delta m}-2P_2\frac{\Delta V_2}{\Delta m}=\frac{V_2^2-V_1^2}{2}+gh$$

$$\Rightarrow \frac{2\times 600}{0.2}-\frac{2\times 150}{0.1}=\frac{20^2-40^2}{2}+10h$$

$h = 360$ m

64. (*c*)

(I) $\Delta U = \Delta Q - \Delta W$

$$=\left\{\left(10^{-3}\times 2250\right)-\frac{10^5\left(10^{-3}-10^{-6}\right)}{10^3}\right\}\text{kJ}$$

$= (2.25 - 0.00999)$ kJ $= (2.1501)$ kJ

(II) $U = nCV\Delta T = \frac{5}{2}nR\Delta T$

$$=\frac{5}{2}\cdot(0.2)(8)(1500-500)J = 4\text{ kJ}$$

(III) $P_1V_2^{\gamma} = P_2V_2^{\gamma}$

$$\Rightarrow 2\left(\frac{1}{3}\right)^{5/3}=P_2\left(\frac{1}{24}\right)^{5/3}\Rightarrow P_2 = 64\text{kPa}$$

$$\Delta U = nC_v\Delta T = \frac{3}{2}\cdot\left(P_2V_2 - P_1V_1\right)$$

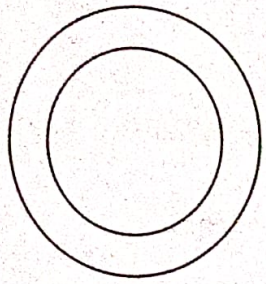
$$=\frac{3}{2}\left(64\times\frac{1}{24}-2\times\frac{1}{3}\right)kJ = 3\text{ kJ}$$

(IV) $\Delta U = nC_V\Delta T$

$$= n\cdot\frac{7}{2}R\Delta T$$

$$= \frac{7}{9}\Delta Q = 7\text{ kJ}$$

65. (*c*,*d*)



$$P_{gas} = P_a + \frac{4S}{r}$$

PV^{γ} = constant [adiabatic process]

$$\left(Pa_1 + \frac{4S}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right)^{5/3} = \left(P_{a_2} + \frac{4S}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)^{5/3}$$

$$\frac{r_1^3}{r_2^3} = \left(\frac{P_{a_2} + \frac{4S}{r_2}}{P_{a_1} + \frac{4S}{r_1}}\right)$$

$p^{1-\gamma}T^{\gamma}$ = constant

$$\left(P_{a_2} + \frac{4S}{r_2}\right)^{1-5/3} T_2^{5/3} = \left(P_{a_1} + \frac{4S}{r_1}\right)^{1-5/3} T_1^{5/3}$$

$$\left(\frac{T_2}{T_1}\right)^{5/3} = \left(\frac{P_{a_1} + \frac{4S}{r_1}}{P_{a_2} + \frac{4S}{r_2}}\right)^{-2/3}$$

$$\left(\frac{T_2}{T_1}\right)^{5/2} = \left(\frac{P_{a_2} + \frac{4S}{r_2}}{P_{a_1} + \frac{4S}{r_1}}\right)$$

D is correct.

66. (*b*,*c*,*d*)

For adiabatic process ($A \to B$)

$$P_A V_A^{\gamma} = P_B V_B^{\gamma}$$

$$10^5 \times (0.8)^{\frac{5}{3}} = 3\times 10^5 (V_B)^{\frac{5}{3}}$$

$$\Rightarrow V_B = 0.8 \times \left(\frac{1}{3}\right)^{0.6} = 0.4$$

Work done in process $A \to B$

$$W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1}$$

$\Rightarrow W_{AB} = 0\ 60$ kJ =

$\Rightarrow |W_{AB}| = 60$ kJ

Work done $B \to C$ in process (Isothermal process)

$$W_{BC} = nRT\ell n\frac{V_C}{V_B} = P_B V_B \ell n\frac{V_C}{V_B}$$

$$\Rightarrow W_{BC} = 3\times 10^5 \times 0.4\ell n\frac{0.8}{0.4}$$

$\Rightarrow W_{BC} = 84$ kJ

Work done in process $C \to A$

$W_{CA} = P\Delta V = 0$ $\quad (\therefore V = 0)$

So total work done in the process

$A \to B \to C$

$$W_{ABC} = W_{AB} + W_{BC} + W_{CA}$$
$$= 60 + 84 + 0$$

$W_{ABC} = 24$ kJ

CHAPTER

17 Simple Harmonic Motion

Exercise-1 (Topicwise)

1. (*a*) For S.H.M. $F = -kx$.

$\therefore$ Force = Mass × Acceleration $\propto -x$

$\Rightarrow F = -Akx$; where A and k are positive constants.

2. (*d*) Standard equation of S.H.M. $\frac{d^2y}{dt^2} = -\omega^2 y$, is not satisfied by $y = a \tan \omega t$.

3. (*c*) At centre $v_{max} = a\omega = a.\frac{2\pi}{T} = \frac{0.2 \times 2\pi}{0.01} = 40\pi$

4. (*b*) $v_{max} = a\omega = a\frac{2\pi}{T} = 3 \times \frac{2\pi}{6} = \pi$ cm/s

5. (*b*) $\frac{v_{max}}{A_{max}} = \frac{a\omega}{a\omega^2} = \frac{1}{\omega}$

6. (*b*) $a_{max} = \omega^2 A$

$(1.57)^2 = \omega^2 (1)$

$\omega = 1.57$ rad/s^2

$\frac{2\pi}{T} = 1.57$

$T = 4$ sec

7. (*a*) Maximum velocity = $a\omega = 16$

Maximum acceleration = $\omega^2 a = 24$

$\Rightarrow a = \frac{(a\omega)^2}{\omega^2 a} = \frac{16 \times 16}{24} = \frac{32}{3} m$

8. (*b*) Given, $v = \pi$ cm/sec, $x = 1$ cm and $\omega = \pi$ s^{-1}

Using $v = \omega\sqrt{a^2 - x^2} \Rightarrow \pi = \pi\sqrt{a^2 - 1}$

$\Rightarrow 1 = a^2 - 1 \Rightarrow a = \sqrt{2}$ cm

9. (*a*) Comparing given equation with standard equation, $x = a \cos(\omega t + \phi)$ we get, $a = 0.01$ and $\omega = \pi$

$\Rightarrow 2\pi n = \pi$

$\Rightarrow n = 0.5$ Hz

10. (*a*) $y = 5 \sin \pi(t + 4) \Rightarrow y = 5 \sin(\pi t + 4\pi)$ ...(i)

The standard equation is $y = A\sin(\omega t + \phi)$ ...(ii)

Comparing equation (i) & (ii)

$A = 5$ m, $\omega = \pi$, $T = \frac{2\pi}{\omega} = 2$ sec.

11. (*d*) From the given equation, $a = 5$ and $\omega = 4$

$\therefore v = \omega\sqrt{a^2 - y^2} = 4\sqrt{(5)^2 - (3)^2} = 16$

12. (*a*) Velocity of a particle executing S.H.M. is given by

$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi}{T}\sqrt{\frac{3A^2}{4}}$

$= \frac{\pi A\sqrt{3}}{T}$

13. (*a*) At mean position velocity is maximum

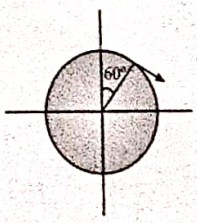
i.e., $v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$

$\therefore v = \omega\sqrt{a^2 - y^2} \Rightarrow 8 = 4\sqrt{4^2 - y^2}$

$\Rightarrow y = 2\sqrt{3}$ cm

14. (*a*) Velocity is maximum at mean. To come back to mean the particle has to move $\left(\pi - \frac{\pi}{3}\right) = \frac{2\pi}{3}$.

Hence $t = \frac{2\pi}{3.2\pi} = \frac{1}{3}$ sec.



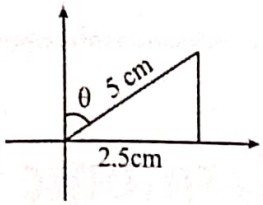
15. (*a*) Speed maximum when $x = 0$

$\left(\cos\left(\omega t + \frac{\pi}{4}\right)\right) = 0$

$\omega t + \frac{\pi}{4} = \frac{\pi}{2}$

$t = \frac{\pi}{4\omega}$

16. (*d*) $A \sin\theta = x$



So $\theta = \pi/6$

17. (*c*) $y = a\cos\omega t$

$$\frac{a}{2} = a\cos\omega t$$

$$\omega t = \frac{\pi}{3}$$

$$\frac{2\pi}{24}t = \frac{\pi}{3}$$

$t = 4$ sec.

18. (*c*) In the particle moves from extreme to half of amplitude then let the time taken is t

$y = A\sin(\omega t + \phi)$

$y = A\sin\left(\omega t + \frac{\pi}{2}\right)$ (As particle starts from extreme)

$$\frac{A}{2} = A\sin\left(\frac{2\pi}{4}t + \frac{\pi}{2}\right)$$

$$\sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) = \sin\frac{5\pi}{6}$$

$$\frac{\pi}{2}t + \frac{\pi}{2} = \frac{5\pi}{6}$$

$$\frac{\pi}{2}t = \frac{2\pi}{6}$$

$$t = \frac{2}{3}\text{sec}$$

19. (*d*) Kinetic energy $T = \frac{1}{2}m\omega^2(a^2 - x^2)$ and potential energy, $V = \frac{1}{2}m\omega^2 x^2$

$$\therefore \frac{T}{V} = \frac{a^2 - x^2}{x^2}$$

20. (*a*) $E = \frac{1}{2}ma^2\omega^2 = \frac{1}{2}ma^2\left(\frac{4\pi^2}{T^2}\right) \Rightarrow E \propto \frac{a^2}{T^2}$

21. (*d*) In simple harmonic motion, energy changes from kinetic to potential and potential to kinetic but the sum of two always remains constant.

22. (*a*) Since maximum value of $\cos^2\omega t$ is 1.

$\therefore K_{max} = K_0\cos^2\omega t - K_0$

Also $K_{max} = PE_{max} = K_0$

23. (*a*) At mean position, the kinetic energy is maximum.

24. (*d*) Given spring system has parallel combination, so

$k_{eq} = k_1 + k_2$ and time period $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

25. (*b*) Equivalent spring constant for two spring in parallel $K'_{eq} = K + K = 2K$

Now, the two spring is connected in series with another spring

$$K_{eq} = \frac{K'_{eq} \times K}{K'_{eq} + K} = \frac{2K \times K}{2K + K} = \frac{2}{3}K$$

Now using the formula for time period, we have

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{3m}{2K}}$$

26. (*c*) In this case springs are in parallel, so $k_{eq} = k_1 + k_2$ and

$$\omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$$

27. (*c*) Slope of F-x curve gives K

$$\text{slope} = \frac{13.5}{1.5}F = -Kx \Rightarrow K = 9$$

$$\omega^2 = \frac{K}{m} = 9, \quad \omega = 3, \quad T = \frac{2\pi}{3}$$

28. (*b*) $k_{eq} = 2k + k + \frac{2k \times 2k}{2k + 2k} = 4k$

So, frequency, $f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{M}} = \frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

29. (*c*) $f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}}$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m_2}}$$

$f_2 = \frac{f_1}{2}$ or $m_2 = 4m_1$; $\quad m_2 - m_1 = 3$ kg

30. (*d*) Maximum velocity $= a\omega = a\sqrt{\frac{K}{m}}$

Given that $a_1\sqrt{\frac{K_1}{m}} = a_2\sqrt{\frac{K_2}{m}}$

$$\Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{K_2}{K_1}}$$

31. (*c*) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \frac{n_S}{n_P} = \sqrt{\frac{k_S}{k_P}} \Rightarrow \frac{n_s}{n_p} = \sqrt{\frac{\left(\frac{k}{2}\right)}{2k}} = \frac{1}{2}$

32. (*a*) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \frac{n}{n'} = \sqrt{\frac{k}{m} \times \frac{m'}{K'}} = \sqrt{\frac{k}{m} \times \frac{2m}{2K}} = 1$

$\Rightarrow n' = n$

33. (*c*) $\frac{1}{k_{eff}} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \ldots$

$= \frac{1}{k}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right] = \frac{1}{k}\left(\frac{1}{1-1/2}\right) = \frac{2}{k}$

{By using sum of infinite geometrical progression $a + \frac{a}{r} + \frac{a}{r^2} + \ldots\infty$, sum (S) $= \frac{a}{1-r}$}

$\therefore k_{eff} = \frac{k}{2}$

34. (*c*) Given elastic energies are equal i.e., $\frac{1}{2}k_1x_1^2 = \frac{1}{2}k_2x_2^2$

$\Rightarrow \frac{k_1}{k_2} = \left(\frac{x_2}{x_1}\right)^2$ and using $F = kx$

$\Rightarrow \frac{F_1}{F_2} = \frac{k_1x_1}{k_2x_2} = \frac{k_1}{k_2} \times \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{k_1}{k_2}}$

35. (*b*) $F = kx \Rightarrow mg = kx \Rightarrow m \propto kx$

Hence $\frac{m_1}{m_2} = \frac{k_1}{k_2} \times \frac{x_1}{x_2} \Rightarrow \frac{4}{6} = \frac{k}{k/2} \times \frac{1}{x_2}$

$\Rightarrow x_2 = 3$ cm

36. (*a*) With mass m_2 alone, the extension of the spring l is given as

$m_2g = kl$ ...(i)

With mass $(m_1 + m_2)$, the extension 'l' is given by

$(m_1 + m_2)g = k(l + \Delta l)$ ...(ii)

The increase in extension is Δl which is the amplitude of vibration. Subtracting (i) from (ii), we get

$m_1g = k\Delta l$ or $\Delta l = \frac{m_1g}{k}$

37. (*c*) If amplitude is large motion will not remain simple harmonic.

38. (*b*) When a little mercury is drained off, the position of c.g. of ball falls (w.r.t. fixed and) so that effective length of pendulum increases hence T increase

39. (*d*) $T = 2\pi\sqrt{\frac{\ell}{g}}$, As it does not depend on amplitude

$\therefore$ % change in time period is 0% Hence option (*d*) is correct.

40. (*b*) As we know $g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_{earth}}{g_{planet}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \quad \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$

$\Rightarrow T_p = 2\sqrt{2}$ sec

41. (*c*) $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{g + g/4}} = \frac{2}{\sqrt{5}}T.$

42. (*a*) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \sqrt{\frac{l}{g}} =$ constant

$\Rightarrow l \propto g \Rightarrow l_m = \frac{l_e}{6} \Rightarrow l_m = \frac{1}{6}$m

43. (*d*) $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

$g_{(effective)} = 0$

So $T = \infty$

44. (*d*) Frequency $n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_1}{l_2} = \frac{n_2^2}{n_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$

45. (*a*) Given time for both are same

$9T_1 = 7T_2$

$9 \times 2\pi\sqrt{\frac{\ell_1}{g}} = 7 \times 2\pi\sqrt{\frac{\ell_2}{g}}$

$\Rightarrow 9\sqrt{\ell_1} = 7\sqrt{\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{49}{81}$

46. (*b*) At B, the velocity is maximum using conservation of mechanical energy

$\Delta PE = \Delta KE \Rightarrow mgH = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gH}$

47. (*c*) According to the principle of conservation of energy,

$\frac{1}{2}mv^2 = mgh$ or $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1}$

$= 1.4$ m/s

Exercise-2 (Learning Plus)

1. (*a*) Comparing $F = -kx$

with $F = -cx^{1/3}$

$\Rightarrow kx = cx^{1/3}$

$\Rightarrow c = kx^{2/3}$

As x increases c also increases.

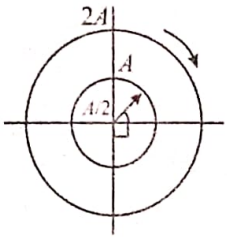
2. (*d*) $a = -\omega^2 x$

$= -\omega^2 A \sin \omega t$

$$\langle a \rangle = \frac{-\omega^2 A\int_0^T (\sin \omega t)dt}{\int_0^T dt}$$

$$= \frac{-\omega^2 A\left[\frac{-\cos \omega t}{\omega}\right]_0^T}{T - 0} = 0$$

3. (*c*) Let particle A be the particle shown with initial phase 135° and B be the particle at extreme. Hence the phase difference between them is 135°.



4. (*c*) $v = \omega\sqrt{A^2 - x^2}$

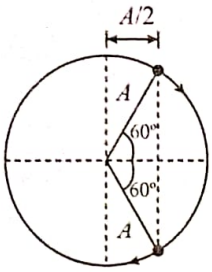
$(50\pi)^2 = (10\pi)^2 (10^2 - x^2)$

$\Rightarrow x = \pm\sqrt{75} = \pm 5\sqrt{3}$

So, separation between points is

$\therefore \Delta x = 2 \times 5\sqrt{3} = 10\sqrt{3} = 17.32$ cm.

5. (*b*) Consider SHM as projection of uniform circular motion. From figure the phase difference between two particles is 120°.



6. (*b*) $x = a\sin\omega t$ with $\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$ rad / s

$$x = a\sin\frac{\pi t}{4}$$

In 2s (which is equal to $T/4$), one amplitude will be covered. In 1st and 2nd,

$$x = a\sin\frac{\pi}{4} = \frac{a}{\sqrt{2}}$$

$$\text{Required ratio} = \frac{\frac{a}{\sqrt{2}}}{a - \frac{a}{\sqrt{2}}} = \frac{1}{\sqrt{2} - 1}$$

7. (*b*) Velocity $v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow v^2 = \omega^2 A^2 - \omega^2 x^2$ ...(i)

Acceleration $a = -\omega^2 x \Rightarrow a^2 = \omega^4 x^2$ ...(ii)

From (i) and (ii):

$v^2 = \omega^2 A^2 - a^2/\omega^2$

$\Rightarrow v^2 + a^2/\omega^2 = \omega^2 A^2$

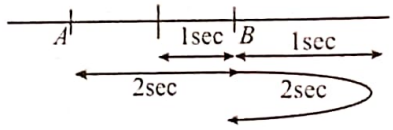
$$\Rightarrow \frac{v^2}{\omega^2 A^2} + \frac{a^2}{\omega^4 A^2} = 1$$

$\Rightarrow v^2 = -a^2\left(\frac{1}{\omega^2}\right) + 1$ its straight line with –ve slope and +*ve* intercept

8. (*c*) $\frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2)$

or $x = \frac{A}{\sqrt{2}}$.

9. (*b*)



Hence $\frac{T}{4} = 2$ sec. $T = 8$ sec.

10. (*a*) $x = A\cos\omega t$

$$\text{K.E.} = \frac{1}{2}k(A^2 - x^2)$$

$$= \frac{1}{2}kA^2\sin^2\omega t$$

$$= \frac{1}{2}kA^2\frac{(1 - \cos 2\omega t)}{2}$$

$$= \frac{kA^2}{4}(1 - \cos 2\omega t)$$

Frequency of *K.E.* will double of acceleration.

11. (*a*) Total Energy of S.H.M. remains constant so average energy = Total energy

12. (*c*) $E_1 = \frac{1}{2}Kx^2$

$E_2 = \frac{1}{2}Ky^2$

$E = \frac{1}{2}K(x + y)^2$

$\Rightarrow E = \frac{1}{2}K(x^2 + y^2 + 2xy)$

$= E_1 + E_2 + Kxy$

$= E_1 + E_2 + K\sqrt{\frac{2E_1}{K}}\sqrt{\frac{2E_2}{K}}$

$= E_1 + E_2 + 2\sqrt{E_1E_2}$

13. (*a*) $x = A\sin\omega t$ if $t = 1$s then $x = 0$

On differentiating above term, we gets

$$v = A\omega\cos\omega t \Rightarrow 0.25 = A \times \frac{2\pi}{6}\cos\left(\frac{\pi}{3}(t - 1)\right)$$

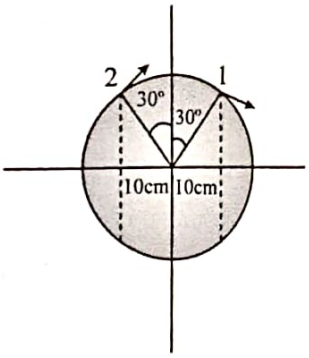
At $t = 2$ sec.

$$\Rightarrow 0.25 = A \times \frac{2\pi}{6} \times \frac{1}{2} \Rightarrow A = \frac{3}{2\pi}$$

14. (*a*) As the range of motion is 4 cm, the amplitude of motion is +2 cm. 10.5 s is equal to 10.5 time period of simple harmonic motion; so we have to find the height and position of cork after one-half of time period at

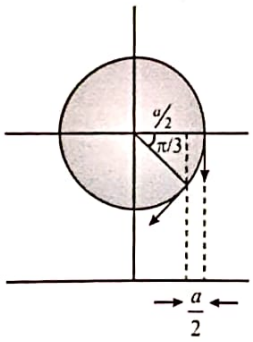
$T/2 = 0.5$ s. As at $t = 10$ s, the particle is at its lowest position, after half a time period the cork would be at its maximum height and velocity of cork at extreme position is zero.

15. (*c*)



Particle 1 and 2 are as shown and their phase difference is 60°.

16. (*c*)



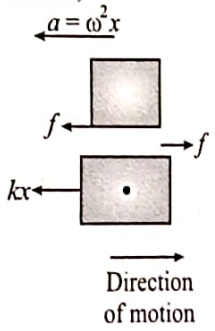
$$\frac{a}{2} = a\cos\omega t$$

$$\Rightarrow \theta = \frac{\pi}{3} = \omega t$$

$$t = \frac{T}{6}$$

$$\therefore\ v_{avg} = \frac{a/2}{T/6} = \frac{3a}{T}$$

17. (*b*) Let A be the maximum amplitude of oscillation for the required situation. Then $f = m\omega^2 x$ (from free body diagram of blocks)



Let A be the maximum amplitude of oscillation then,

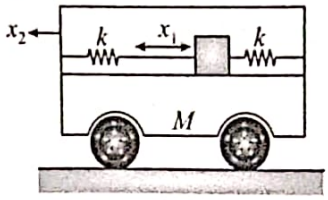
For required condition, $f = m\omega^2 A$

for no slipping $f < f_L$

i.e., $m\omega^2 A < \mu_s mg$

$$\Rightarrow A < \frac{0.6 \times g}{\frac{4\pi^2}{4}} = 0.6 \text{ m} \Rightarrow A_{max} = 0.6 \text{ m}$$

18. (*c*) Let displacement of block is x_1 and of cart is x_2 as shown



by linear momentum conservation

$$mv_1 = Mv_2 \Rightarrow v_2 = \frac{mv_1}{M} \text{ so, } x_2 = \frac{mx_1}{M}$$

For block Force equation can be written as

$$F = 2k(x_1 + x_2) = m\omega^2 x_1$$

$$\Rightarrow 2k\left(x_1 + \frac{m}{M}x_1\right) = m\omega^2 x_1 \Rightarrow \omega^2 = 2k\left(\frac{M+m}{Mm}\right)$$

$$\text{So, } T = 2\pi\sqrt{\frac{Mm}{2k(M+m)}}$$

19. (*c*) $T = 2\pi\sqrt{\frac{M}{K}}$ When the rubber ribbon slacks it will not exert any force.

20. (*c*) $T = \frac{25}{50} = \frac{1}{2}s \Rightarrow \omega = \frac{2\pi}{T} = 4\pi$ rad/s

Spring constant $k = m\omega^2 = 5 \times (4\pi\omega t)^2 = 80\pi^2$ N.m

Force required to stretch the spring by 5 cm is

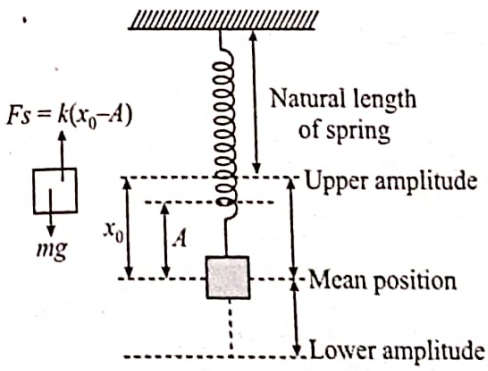
$F = kx = 80\pi^2 \times 0.05$ N $= 4\pi^2$

21. (*a*) $\omega = \sqrt{\frac{K}{m}}$ and $Ke = mg$ (at mean position)

or $\omega = \sqrt{\frac{g}{e}}$

22. (*a*) The spring is never compressed. Hence spring shall exert least force on the block when the block is at topmost position.

At equilibrium position $kx_0 = mg \Rightarrow x_0 = mg/k$.



$$F_{max} = k(x_0 - A) = k\left[\frac{mg}{k} - A\right]$$

$$= mg - 4\frac{\pi^2}{T^2}mA = mg - kA = mg - m\omega^2 A$$

23. (*d*) $kx = mg\sin 30°$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g\sin 30°}{x}} = \sqrt{\frac{5\times 100}{2.5}} = 14.14 \text{ rad/s}$$

24. (*d*) Total *M.E.*= *T.E.* at *M.P.* + Total oscillation energy

$9 = 5 + 4$

Total oscillation energy $= \frac{1}{2}Ka^2 = 4$

$$\Rightarrow K = 8\times 10^4 \quad \Rightarrow T = 2\pi\sqrt{\frac{M}{K}} \Rightarrow T = \frac{\pi}{100}$$

25. (*d*) $t = n\frac{T}{2}$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{3\times 24}} = \frac{\pi}{3}$$

26. (*a*) In equilibrium, let us say deformations (elongations) in springs are x_{01} and x_{02}. Then

$mg + k_1x_{01} = k_2x_{02}$

Let the block be displaced down by x; then elongation in spring 1 reduces by x and in spring 2 it increases by x. In this situation, the net force acting on the block towards equilibrium position is

$F = k_2(x + x_{02}) - mg - k_1(x_{01} - x)$

$= (k_2 + k_1)x$ (using equilibrium equation)

So, the angular frequency of SHM is

$$\sqrt{\frac{k_1 + k_2}{m}} = 10\sqrt{2}\text{rad/s}$$

27. (*d*) $I = \frac{2}{5}mR^2 = \frac{2}{5}\times 25\times (0.2)^2 = \frac{2}{5}$

$$\tau = c\theta \quad \Rightarrow \quad c = \frac{\tau}{\theta} = \frac{0.1}{1} = 0.1$$

$$T = 2\pi\sqrt{\frac{I}{c}} = 2\pi\sqrt{\frac{2}{5\times 0.1}} = 2\pi\times 2 = 4\pi \text{ sec}$$

28. (*a*) Small pendulum: $2\pi \rightarrow$ T

Big pendulum: $2\pi \rightarrow \frac{5T}{4}$

Now, for big pendulum, we have

$$T \rightarrow \frac{2\pi\times 4}{5} = \frac{8\pi}{5} = 1.6\pi$$

So, phase difference is $= 2\pi - 1.6\pi = 0.4\pi$

Now, smaller pendulum meet,

$$n = \frac{2\pi}{0.4\pi} = 5 \text{ oscillations}$$

29. (*d*) Maximum amplitude

So time to travel angle α

$x = x_0\sin\omega t$

$\alpha\ell = \beta\ell\sin\omega t$

$$t = \frac{1}{\omega}\sin^{-1}\frac{\alpha}{\beta}$$

Now period of pendulum $= \frac{1}{2}$ (actual time period +2(time to travel angle α)

$$= \frac{1}{2}\cdot 2\pi\sqrt{\frac{\ell}{g}} + 2\sqrt{\frac{\ell}{g}}\sin^{-1}\frac{\alpha}{\beta}$$

$$= 2\sqrt{\frac{\ell}{g}}\left[\pi/2 + \sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$$

Also if time taken to travel from right extreme position to wall is t then

$-\alpha = \beta\cos\omega t$

$$\Rightarrow \cos\omega t = -\frac{\alpha}{\beta}$$

$$t = \frac{1}{\omega}\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

$$T = 2t = \frac{2}{\omega}\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

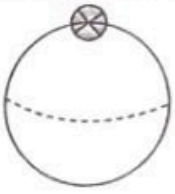
$$T = 2\sqrt{\frac{\ell}{g}}\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

30. (*a*) $T = 2$ sec.

$$T = 2\pi\sqrt{\frac{I}{mgd}} \text{ s}$$

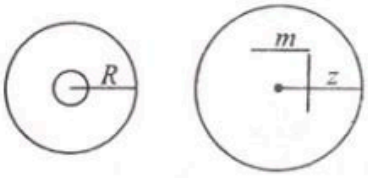
Time period of a second's pendulum is two seconds.

second pendulum



$$2 = 2\pi\sqrt{\frac{2mR^2}{mgR}} \quad \Rightarrow \quad R = 0.5 \text{ m}$$

31. (*c*)



$$T = 2\pi\sqrt{\frac{I}{mgz}} \Rightarrow I = \frac{mR^2 + 2mz^2}{2}$$

$$T = \sqrt{\frac{2\pi}{g}}\sqrt{m\frac{R^2}{2} + 2mz}$$

$\frac{mR^2}{2} = 2mz$ for minimum T

$$z = \frac{R}{\sqrt{2}}$$

32. (c) The centripetal acceleration on the bob as it oscillates (acting along the radius of circle) = v^2/R. This will act horizontally towards the centre of circular path. The total acceleration acting on the pendulum bob is therefore $a = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$

The frequency of oscillation will therefore be

$$n = \frac{1}{2\pi}\sqrt{\frac{a}{l}} = \frac{1}{2\pi}\sqrt{\frac{\left(g^2 + \frac{v^4}{R^2}\right)^{1/2}}{l}}$$

33. (b) $y = A \sin(\omega t + \phi)$ and $x = A \sin(\omega t + \phi)$ then $y = x$ so path is straight line.

34. (b) $x = C \sin \omega t + D \sin(\omega t + \pi/2)$

$$A_r = \sqrt{C^2 + D^2 + 2CD\cos\frac{\pi}{2}} \quad A_r = \sqrt{C^2 + D^2}$$

35. (b) $x = A \sin \omega t$, $y = A \cos \omega t$ or $x^2 + y^2 = A^2$ Thus the motion of the particle is on a circle.

36. (b) $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$

$$y = 2\cos\left(\frac{t}{2}\right)\left[\sin\left(\frac{2001t}{2}\right) + \sin\left(\frac{1999t}{2}\right)\right]$$

$y = \sin(1001t) + \cos(1000t) + \sin(1000t) + \cos(1999t)$

$y = \sin(1001t) + \sqrt{2}\sin(1000t + \pi/4) + \cos(1999t)$

So the given expression is composed by three equation of S.H.M.

37. (c) $a_1 = 1, a_2 = 1$

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta} \Rightarrow a = \sqrt{3}$$

Exercise-3 (JEE Advanced Level)

1. (a,d)

$$x = x_0 \sin^2 \omega t = \frac{x_0(1 - \cos 2\omega t)}{2}$$

2. (b,c,d)

$$u = 5x^2 - 20x \Rightarrow F = \frac{-du}{dx} = -10(x - 2)$$

M.P. at $x = 2$ m

3. (a,b,d)

In S.H.M direction of v and a may be same or opposite and also direction of y and v may be same but direction of a and y never same because $a = -\omega^2 y$ (always opposite to y).

4. (a,b,c)

$x = 5\sin(4\pi t + \tan^{-1} 4/3)$

5. (a,c) Fact based

6. (b,d)

$x = 3\sin 100t + 4(1 + \cos 100t)$

$$x = 5\sin\left(100t + \tan^{-1}\frac{4}{3}\right) + 4$$

M.P. is at 4 with $A = 5$

7. (a,b,c)

$$v = \omega A \Rightarrow \omega = \frac{10}{2.5} = 4$$

(a) $T = \frac{2\pi}{\omega} = \frac{\pi}{2} = 1.57$

(b) $a = \omega^2 A = 40$

(c) $v = \omega\sqrt{A^2 - x^2} = 2\sqrt{21}$

8. (b,d)

Acceleration $a = \omega^2 x$

Maximum acceleration $= \omega^2 A = \left(\frac{2\pi}{T}\right)^2 \frac{2.5}{100}$

When block and platform are separated

$a_{max} = g = 10$

$$\frac{4\pi^2}{T^2} \cdot \frac{1}{40} = 10$$

$$\Rightarrow T^2 = \frac{\pi^2}{100} \quad T = p/10 \text{ sec}$$

9. (b,c)

Given $y = (\sin \omega t + \cos \omega t) y = \sqrt{2}\sin(\omega t + \pi/4)$

So $A = \sqrt{2}$ m and at $t = 0$ $y = 1$ m

10. (a,c)

$$v = \sqrt{\frac{900}{3}}\sqrt{(2^2 - 1^2)} = 10\sqrt{3} \times \sqrt{3} = 30 \text{ m/s}$$

For C.M. $3 \times 30 = 9 \times v \Rightarrow v = 10$ m/s

$$10 = \sqrt{\frac{900}{9}}\sqrt{(A^2 - 1^2)} \Rightarrow A = \sqrt{2} \text{ m}$$

11. (a) $\omega = \sqrt{\frac{K}{m}} = \sqrt{200}$ rad/s

12. (c) Maximum extension the spring from natural position is x.

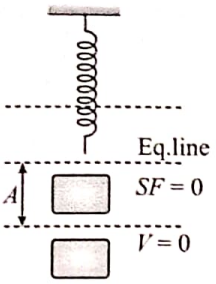
Then $mg + ma = kx$

$$\Rightarrow x = \frac{2(10 + 5)}{400} = 7.5 \text{ cm}$$

Extension of the spring when it is stretched to equilibrium line is x'.

$mg = kx'$

$$\Rightarrow x' = \frac{2 \times 10}{400} = 5 \text{ cm}$$



Therefore amplitude $A = x - x' = 2.5$ cm

13. (*d*) If upward direction is taken as positive at $t = 0$,

$x = -A$

Using $x = A \sin(\omega t + \phi)$

$-A = A \sin\phi$

$\phi = \frac{3\pi}{2}$

14. (*b,c,d*)

Option '*A*' is wrong because it is not given $\omega_1 = \omega_2$

In option '*B*' the resultant displacement is given as

$$x = \left[2A_1 \cos\frac{(\omega_1 - \omega_2)}{2}t\right]\cos\left(\frac{\omega_1 + \omega_2}{2}\right)t$$

In option '*C*' the resultant amplitude is zero.

In option '*D*' the resultant amplitude is $A_1 + A_2 = 2A$.

15. (*a,b,c*)

In option (*a*) the resultant displacement '*r*' is given as

$r = [A_1^2 + A_3^2]^{1/2} \cos\omega_1 t$ along line $y = \frac{A_3}{A_1}x$

$r = [A_1^2 + A_3^2]^{1/2} \cos\omega_1 t = \frac{A_3}{A_1}x$

In option (*b*) $x = A_1 \text{cox}\omega_1 t$

$y = A_1 \cos\left(\omega_1 t + \frac{3\pi}{2}\right)$ or $y = A_1 \sin\omega_1 t$

Thus, $x^2 + y^2 = A_1^2$

$V_x = \frac{dx}{dt} = -A_1\omega_1 \sin\omega_1 t$

$V_y = \frac{dy}{dt} = A_1\omega_1 \cos\omega_1 t$

at $t = 0$, $V_x = 0$, $V_y = +$ve hence the motion is in counter clockwise direction

In option (*c*) $\frac{x^2}{A_1^2} + \frac{y^2}{A_3^2} = 1$

$x = A_1 \cos\omega_1 t$

$y = A_3 \cos(\omega_1 t + \pi/2) = -A_3 \sin\omega_1 t$

$V_x = -A_1\omega_1 \sin\omega_1 t$ and $V_y = -A_3\omega_1 \cos\omega_1 t$

Velocity at $t = 0$ $V_x = 0$, $V_y = -ve$ at $x = A_1$, $y = 0$ so motion is clockwise

In option (*d*) the motion is SHM along a straight line.

16. (*b*) Let $\omega_1 = \omega_2 = \omega$

The resultant SHM is given as

$x = A \sin(\omega t + \alpha)$

Where $\tan\alpha = \frac{A_2 \sin\delta_2}{A_1 + A_2 \cos\delta_2}$.

17. (*a*) A-(r); B-(p); C-(q); D-(s)

$V_{max} = A\omega$

$\Rightarrow A = \frac{V_{max}}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20$ m

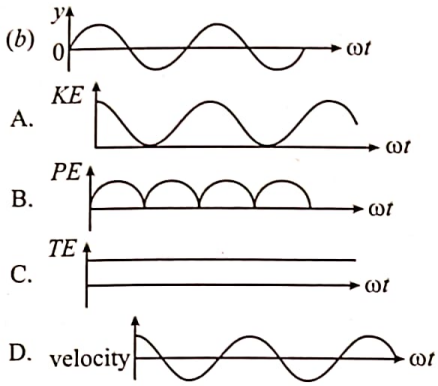
$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2} = 0.2$ kg

At $t = 0.1$, acc. is maximum

$\Rightarrow a_{max} = -\omega^2 A = -\left(\frac{2\pi}{0.2}\right)^2 \times 0.2 = -200$ m/s^2

Maximum energy $= \frac{1}{2}mV_{max}^2 = 4$ J

18. (*b*)



19. [0.25 m]

Using energy conservation, we have

$mgx = \frac{1}{2}kx$

$x = \frac{2mg}{k}$ ...(i)

Putting the values in equation (i), we get

$x = \frac{2 \times 50 \times 10^{-3} \times 9.8}{4}$

$x = 0.245$ m

20. [0.248 m]

Using formula of acceleration in SHM, piston to separate, we have

$a_{max} \geq g$

So, $\omega_A^2 \geq g$

$A \geq \frac{gT^2}{4\pi^2}$ ...(i) $\left\{\because \omega = \frac{2\pi}{T}\right\}$

Putting the values in (i), we get

$A \geq \frac{(9.8)(1)^2}{4\pi^2}$

$A \geq 0.248$ m

21. [0.2 kg]

Using formula of velocity in SHM, we have

$V_{max} = A\omega$

$\omega = \frac{56}{16}$

Now, $\omega = \sqrt{\frac{k}{m}}$

$m = \frac{k}{\omega^2} = \frac{2.45 \times 16 \times 16}{56 \times 56}$

$m = 0.2$ kg

22. [0375]

$$T = 2\pi\sqrt{\frac{I}{mg\,\ell/2}} = 2\pi\sqrt{\frac{\frac{m\ell^2}{3}}{mg\ell/2}} = \pi$$

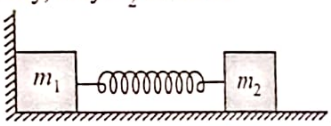
$\frac{2\ell}{30} = \frac{1}{4} \Rightarrow \ell = \frac{15}{4}$ m = 3.75 m = 375 cm

23. [3] $T = \pi\sqrt{\frac{\mu}{k_1}} + \pi\sqrt{\frac{m}{k_2}} + \frac{60}{120} \times 2$

$= \pi\sqrt{\frac{0.2}{1.8}} + \pi\sqrt{\frac{0.2}{3.2}} + 1$

$= 3$ sec

24. [1] Initially, only m_2 oscillates



$\Rightarrow t = \frac{T}{2} = \pi\sqrt{\frac{m_2}{k}} = \frac{\pi}{5\sqrt{2}}$

Next both m_1 and m_2 oscillate about their *CM*.

$t' = \frac{T'}{2} = \pi\sqrt{\frac{\mu}{k}} = \pi\sqrt{\frac{m_1 m_2}{(m_1+m_2)k}} = \frac{\pi}{10\sqrt{2}}$

$m_2 = \frac{k}{50} = 3$ kg

$\frac{m_1 \times 3}{(3+m_1)} \times 150 = \frac{1}{200}$

$4m_1 + 3 - m_1 = 0 \Rightarrow m_1 = 1$ kg

Exercise-4 (Past Year Questions)

JEE MAIN

1. (*d*) Initial velocity is

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} \quad ...(i)$$

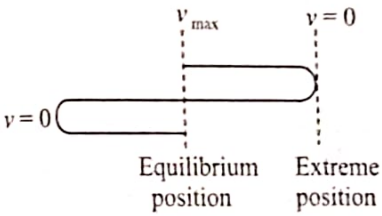
Final velocity is

$$(3v)^2 = \omega^2\left(A^2 - \frac{4A^2}{9}\right) \quad ...(ii)$$

On solving (i) and (ii) we get

$A' = \frac{7A}{3}$

2. (*b*)



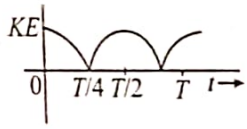
Time taken to reach the extreme position from equilibrium position is $\frac{T}{4}$. Velocity is maximum at equilibrium position and zero at extreme position.

$V = A\,\omega \cos \omega t$

K.E. $= \frac{1}{2}mv^2$

(m is the mass of particle and v is the velocity of particle

$K.E. = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$



Hence, graph of K.E. v/s time is square cos function

3. (*a*) Time period of SHM is given by

$T = 2\pi\sqrt{\frac{m}{k}}$

Frequency $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$

Where, m = mass of one atom

$m = \frac{108}{6.02 \times 10^{23}} \times 10^{-3}$ kg

$\frac{1}{2\pi}\sqrt{\frac{k}{108 \times 10^{-3}} \times 6.02 \times 10^{23}} = 10^{12}$

On solving $k = 7.1$ N/m

4. (*a*) Angular frequency of pendulum

$\omega \propto \sqrt{\frac{g_{eff}}{\ell}}$

$\therefore \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$

$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$

[ω_s = angular frequency of support]

$$\frac{\Delta\omega}{\omega}\frac{1}{2}\times\frac{\Delta g}{g}$$

$$=\frac{1}{2}\times\frac{2A\omega^2}{100}\times 100$$

$\Rightarrow \Delta\omega = 10^{-3}$ rad/s

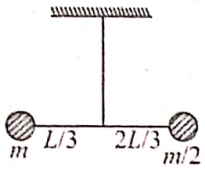
5. (*a*) $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

$\Rightarrow y = 10\sin(3\pi t + \phi)$

$\Rightarrow A = 10$ cm

$\Rightarrow T = \frac{2}{3}$ sec

6. (*c*)



$\Omega = \sqrt{\frac{k}{I}}$; $\omega = \theta_0 \times \Omega$

$T = m\Omega^2\frac{\ell}{3}$ $\{\because T = m\Omega^2 r_1\}$

$T = m\omega^2\frac{\ell}{3}\theta_0^2$ $\left\{I = m\frac{\ell^2}{3}\text{ and }\omega = \sqrt{\frac{k}{I}}\right\}$ s

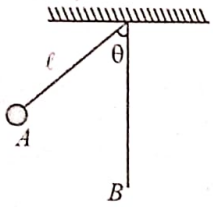
$=\frac{\theta_0^2 k}{\ell}$

7. (*a*) Maximum kinetic energy at lowest point *B* is given by

$k = mg\ell(1-\cos\theta)$

Where θ = angular amp.

$k_1 = mg\ell(1-\cos\theta)$



$k_2 = mg(2\ell)(1-\cos\theta)$

$k_2 = 2k_1$

8. (*d*) $K = \frac{1}{2}mv^2$; $U = \frac{1}{2}kx^2 = \frac{1}{2}m^2x^2$

$$\therefore \frac{K}{U} = \frac{v^2}{\omega^2x^2} = \left(\frac{\cos(\omega t)}{\sin(\omega t)}\right)^2$$

$$= \cot^2\left(\frac{\pi}{90}\times 210\right)$$

$$= \cot^2\left(2\pi + \frac{\pi}{3}\right)$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

9. (*d*) $\because g = \frac{GM}{R^2}$

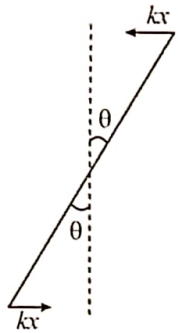
$$\frac{g_p}{g_e} = \frac{M_p}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also, $T \propto \frac{1}{\sqrt{g}}$

$$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$\Rightarrow T_p = 2\sqrt{3}$ s

10. (*c*)



Torque on rod at displacement θ from mean position θ is very small.

$x = \frac{L}{2}\theta$

$$\tau = 2kx\frac{L}{2} = 2k\frac{L_2}{4}\theta = \frac{kL^2}{2}\theta$$

Now, $\tau = i\alpha$

$$\frac{kL^2}{2}\theta = \frac{mL^2}{12}\alpha;\quad \alpha = \frac{6k}{m}\theta$$

$$\tau = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

11. (*c*) $f = \frac{1}{2\pi}\sqrt{\frac{C}{\frac{ML^2}{3}}}$ & $0.8f = \frac{1}{2\pi}\sqrt{\frac{C}{\left(\frac{ML^2}{3}+\frac{mL^2}{2}\right)}}$

$$\Rightarrow \frac{25}{16} = \frac{\frac{ML^2}{3}+\frac{mL^2}{2}}{\frac{ML^2}{3}}$$

$$\Rightarrow \frac{25}{16} = 1 + \frac{3m}{2M}$$

$$\Rightarrow \frac{9}{16} = \frac{3m}{2M}$$

$$\Rightarrow \frac{m}{M} = \frac{3}{8} = 0.37$$

12. (c) $|v_4| = |a_4|$

$$\Rightarrow \left(\omega\sqrt{A^2 - X^2}\right) = \left(\omega^2 x\right)$$

$$\Rightarrow \left(\omega\sqrt{25-16} = \omega^2 \times 4\right)$$

$$\Rightarrow \omega = \frac{3}{4}$$

$$T = \frac{2\pi}{\omega} = 2\pi\frac{4}{3} = \frac{8\pi}{3}$$

13. (b) $A = A_0 e^{-\gamma t}$

$A = \frac{A_0}{2}$ (after 10 oscillations)

$\because$ After 2 seconds

$$\frac{A_0}{2} = A_0 e^{-\gamma(2)}$$

$$2 = e^{2\gamma} \quad \ell n2 = 2\gamma \quad \gamma = \frac{\ell n2}{2}$$

$$\because A = A_0 e^{-\gamma t}$$

$$\ell n\frac{A_0}{A} = \gamma t$$

$$\ell n1000 = \frac{\ell n2}{2}t$$

$$2\left(\frac{3\ell n10}{\ell n2}\right) = t$$

$$\frac{6\ell n10}{\ell n2} = t$$

$t = 19.931$ sec

$t \approx 20$ sec

14. (a) For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{eff}}}$

Situation 1: when pendulum is in air $\rightarrow g_{eff} = g$

Situation 2: when pendulum is in liquid $\rightarrow g_{eff} = g$

$$\left(1 - \frac{\rho_{liquid}}{\rho_{body}}\right) = g\left(1 - \frac{1}{16}\right) = \frac{15g}{16}$$

$$\text{So, } \frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}} \Rightarrow T' = \frac{4T}{\sqrt{15}}$$

15. (b) $A = A_0 e^{-0.1t} = \frac{A_0}{2}$

$\ln 2 = 0.693$

$t = 10 \ln 2 = 6.93 \approx 7$ sec

16. (c) At $= \frac{3T}{4}$, particle is at mean position

$\Rightarrow$ Force = 0

At $t = T$, particle is at extreme position

$\Rightarrow$ Acceleration is maximum

At $t = \frac{T}{4}$, particle is at mean position

$\Rightarrow$ Speed is maximum

At $t = \frac{T}{2}$, particle is at extreme position

$\Rightarrow$ Speed = 0 $\Rightarrow$ K.E. = 0 $\Rightarrow$ P.E. is maximum.

17. (a) $\omega = \sqrt{\frac{k}{m}}$

$$\Rightarrow \omega \propto \frac{1}{\sqrt{m}}$$

$$\Rightarrow \frac{\omega'}{\omega} = \sqrt{\frac{m}{m'}}$$

$$\Rightarrow \frac{\omega'}{\omega} = \sqrt{\frac{m}{m/2}}$$

$$\Rightarrow \omega' = \sqrt{2}\omega$$

$\because v_{max} = v'_{max}$

$A'\omega' = A\omega$

$$\Rightarrow A' = A\frac{\omega}{\omega'}$$

$$\Rightarrow A' = A\frac{\omega}{\sqrt{2}\omega}$$

$$A' = \frac{A}{\sqrt{2}}$$

$$fA = \frac{A}{\sqrt{2}}$$

$$f = \frac{1}{\sqrt{2}}$$

18. (b) $T = 2\pi\sqrt{\frac{I}{Mgd}}$

$$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}}$$

$$I_1 = MR^2 + MR^2 = 2MR^2$$

$$I_2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{2MR^2}{3MR^2/2}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

19. (c) $y(t) = \frac{y_0}{2}(1 - \cos 2\omega t)$

Amplitude $\frac{Mg}{k} = \frac{y_0}{2}$

$$\Rightarrow \frac{k}{M} = \frac{2g}{y_0}$$

Angular frequency $2\omega = \sqrt{\frac{k}{m}}$

$$\Rightarrow \omega = \frac{1}{2}\sqrt{\frac{k}{m}} = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$$

20. (*d*) General equation of SHM

$$x = A\sin\left(\omega' t \pm \phi\right)$$

We know, $\omega = \frac{2\pi}{T}$

Given, $T = \frac{\pi}{\omega}$

$$\therefore\ \omega' = \frac{2\pi}{\frac{\pi}{\omega}} = 2\omega$$

$\therefore$ Equation becomes,

$$x = a\sin(2\omega t \pm \phi)$$

Here coefficient of t is 2ω.

You can see only option (*D*) has coefficient 2ω.

21. (*b*) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 2\pi\left(\frac{l}{g}\right) \Rightarrow g = 2\pi\frac{l}{T^2}$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{1\times10^{-3}}{1\times10^{-2}} + \frac{2\times1}{100}$$

$$\Rightarrow \frac{\Delta g}{g} = 0.02 + 0.01 = 0.03$$

$$\Rightarrow 100\times\frac{\Delta g}{g} = 0.03\times100 = 3\%$$

$$\Rightarrow \frac{\Delta g}{g}\times100 = 3\%$$

22. (*c*) $A = A_o e^{\frac{-b}{2m}t} \Rightarrow 6 = 12e^{\frac{-b}{2\times1}\times120}$

$$\Rightarrow 6 = 12e^{-b\times60}$$

$$\Rightarrow \frac{1}{2} = e^{-60b}$$

$$\Rightarrow \ln(2) = 60b$$

$$\Rightarrow b = \frac{\ln(2)}{60} = 1.16\times10^2\ \text{Kg/s}$$

23. (*d*) $\because V_{max} = A\omega$

Given, $\omega_1 A_1 = \omega_2 A_2$

We know that $\omega = \sqrt{\frac{K}{m}}$

$$\therefore \sqrt{\frac{k_1}{m}}A_1 = \sqrt{\frac{k_2}{m}}A_2$$

$$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

24. (*b*) $A = A_0 e^{-\frac{bt}{2m}} \Rightarrow \frac{bt}{2m} = \ln 2 = 0.693$

$$\Rightarrow t = \frac{2m}{b}\times0.693 \quad \Rightarrow \quad t = 2\times\frac{500}{20}\times0.693$$

$$\Rightarrow t = 50\times0.693 = 34.6\ \text{sec.}$$

25. (*b*) When lift is stationary $T = 2\pi\sqrt{\frac{L}{g}}$

A pseudo force will act downwards when lift is moving upwards.

$$\therefore\ g_{eff} = g + \frac{g}{2} = \frac{3g}{2}$$

$\therefore$ New time period

$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}}$$

$$T' = 2\pi\sqrt{\frac{2L}{3g}} \Rightarrow \therefore T' = \sqrt{\frac{2}{3}}T$$

26. (*d*) $v^2 = \omega^2(A^2 - x^2)$

$$A^2 = x_1^2 + \frac{v_1^2}{\omega^2} = x_2^2 + \frac{v_2^2}{\omega^2}$$

$$\omega^2 = \frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}$$

$$T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

27. (*b*) $K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

$$= \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$$

$$= \frac{1}{2}m\omega^2\left(\frac{3A^2}{4}\right)$$

$$K = \frac{3}{4}\left(\frac{1}{2}m\omega^2 A^2\right)$$

28. (*a*) $T_1 = 2\pi\sqrt{\frac{3m}{2k}}$

$$T_2 = 2\pi\sqrt{\frac{m}{3k}}$$

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{3m}{2k}}}{2\pi\sqrt{\frac{m}{3k}}} = \frac{3}{\sqrt{2}}$$

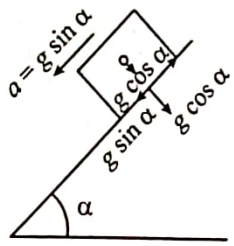
29. [16] $\frac{1}{2}kA^2 = \frac{p^2}{2m}$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = \frac{m_2}{m_1} = \frac{1024}{900}$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{32}{30} = \frac{16}{15} = \frac{16}{16-1}$$

$\therefore \alpha = 16$

30. (*a*) $g_{eff} = g\cos\alpha$



$$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

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31. (*a*) Time of flight for projectile

$$T = \frac{2u\sin\theta}{g} = 1 \text{ sec.}$$

$$\frac{2u\sin 45}{g} = 1 \text{ sec.}$$

$$u = \frac{g}{\sqrt{2}}$$

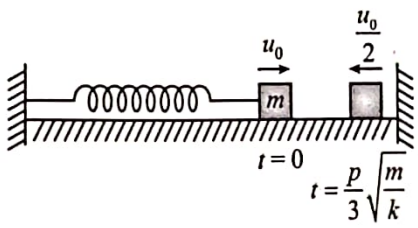
$u = \sqrt{50}$ m/s

32. (*a*,*d*) $x = A\sin\omega t$

$$v = A\omega\cos\omega t = \frac{\omega A}{2}$$

$$\Rightarrow \cos\omega t = \frac{1}{2}$$

$$\omega t = \frac{\pi}{3} \Rightarrow t = \frac{2\pi}{3} = \frac{\pi}{3}\sqrt{\frac{m}{k}}$$



for (*C*) time $= \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{2}\sqrt{\frac{m}{k}}$

$$= \frac{5\pi}{6}\sqrt{\frac{m}{k}}$$

for (*D*) time $= \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{k}}$

$$= \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

33. (*a*,*b*,*d*)

Case I: $\omega' = \sqrt{\frac{k}{M+m}}$,

$$\therefore MA\sqrt{\frac{k}{M}} = (M+m)A'\sqrt{\frac{k}{M+m}}$$

$$\Rightarrow A' = \sqrt{\frac{M}{M+m}}A$$

Now $E' = \frac{1}{2}(M+m)\omega'^2 A'^2 = \frac{1}{2}\frac{kA^2M}{M+m}$

$$\Rightarrow v' = A\sqrt{\frac{k}{M+m}}$$

Case II: amplitude will not change.

$$E' = \frac{1}{2}kA^2$$

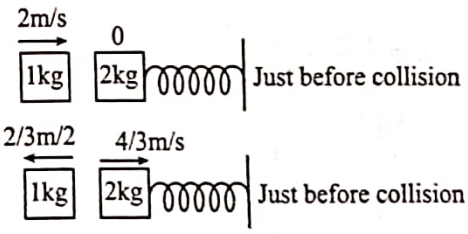
$$\therefore v' = A\sqrt{\frac{k}{M+m}}$$

$\therefore$ (*a*,*b*,*d*)

34. [2.09]

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \text{sec}$$

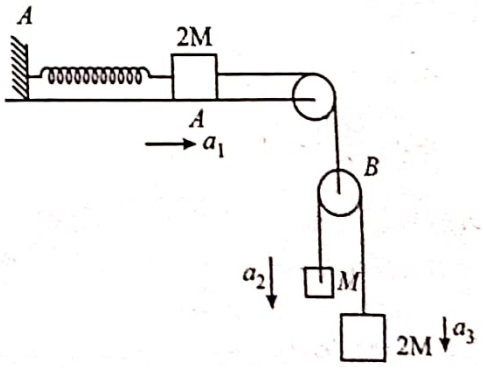
Block returns to original position in $\frac{T}{2} = \pi$sec



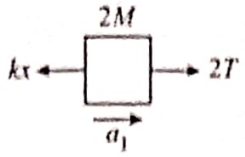
$$d = \frac{2}{3}(\pi) = \frac{2}{3}(3.14) = 2.0933\text{m}$$

sd = 2.09 m

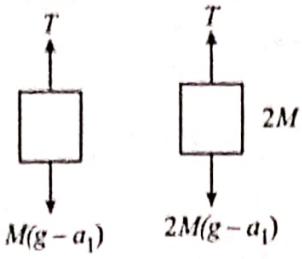
35. (*c*)



FBD of A:



FBDS of other blocks in frame of pulley:



In the frame of pulley B,

the hanging masses have accelerations:

$M \to (a_2 - a)$, $2M \to (a_3 - a_1)$: downward.

$\therefore (a_2 - a_1) = -(a_3 - a_1)$ [constant]

Assuming that the extension of the spring is x

We consider the FBD of A:

$2Ma_1 = 2T - kx$

Upward acceleration of block M w.r.t. the pulley B = Downward acceleration of block $2M$ w.r.t. the pulley

$$\frac{T - M(g - a_1)}{M} = \frac{2M(g - a_1) - T}{2M}$$

$$\Rightarrow T = \frac{4M}{3}(g - a_1) \quad \text{...(ii)}$$

substituting in equation (i) we get

$$2M.a_1 = \frac{8M}{3}(g - a_1) - kx$$

$$a_1 = \frac{8Mg - 3kx_0/4}{14m}$$

$$= \frac{8g}{14} - \frac{3kx_0}{14m \times 4}$$

$$= \frac{8g}{14} - \frac{3x}{14m \times 4} \times \frac{16Mg}{3x}$$

$$= \frac{8g}{14} - \frac{4g}{14}$$

$$= \frac{8g}{14} - \frac{2g}{7}$$

or

$$\frac{14m}{3} a_1 = -k\left[x - \frac{8mg}{3k}\right]$$

$$a_1 = -\frac{3k}{14m}\left[x - \frac{8mg}{3k}\right] \quad \text{...(iii)}$$

that means, block 2m (connected with the spring) will perform SHM about $x_1 = \frac{8mg}{3k}$

Therefore, maximum elongation in the spring $x_0 = 2x_1$

$$= \frac{16mg}{3k}$$

On comparing equation (iii) with

$a = -\omega^2(x - x_0)$ we get

$$\omega = \sqrt{\frac{3k}{14m}}$$

Now, at $\left(\frac{x_0}{2}\right)$, block will be passing through its mean position. Therefore speed is

$$= A\omega = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$$

At, $\frac{x_0}{4}$ from equation (iii)

$$a = \frac{-3k}{14m}\left[\frac{4mg}{3k} - \frac{8mg}{3k}\right]$$

$$= \frac{2g}{7}$$

36. [6] Time period of oscillation, will be given as

$$T = \sqrt{\frac{m}{k_1}} + \pi\sqrt{\frac{m}{k_2}} = \pi\sqrt{\frac{0.1}{0.009}} + \pi\sqrt{\frac{0.1}{0.016}}$$

$$= \frac{\pi}{0.3} + \frac{\pi}{0.4} \Rightarrow T = 5.83\pi \approx 6\pi$$

Thus, $n = 6$.

CHAPTER

18 Waves

Exercise-1 (Topicwise)

1. (*a*) $(V_{Particle})_{maximum} = A\omega$

$V_{wave} = f\lambda$

Now, $A\omega = f \times \lambda$

$A(2\pi f) = f \times \lambda \Rightarrow A = \lambda/2\pi$

2. (*c*) $0.25 = 2.5\lambda$

$\lambda = 0.1$ m

$k = \frac{2\pi}{0.1}$

We know $V = \lambda f$

$330 = 0.1f \Rightarrow f = 3300 \Rightarrow \omega = 2\pi \times 3300$

$y = A\sin(\omega t - kx)$

$y = 0.05\sin(3300 \times 2\pi t - \frac{2\pi}{0.1})$

$y = 0.05 \sin 2\pi(3300t - 10x)$

3. (*a*) As $\frac{5\lambda}{2} = 20 \Rightarrow \lambda = 8$ cm

$k = \frac{2\pi}{\lambda} = \frac{314}{4}$

$\omega = kv \frac{2\pi}{8\times10^{-2}} \times 350 = 27475$

$\therefore\ y = 0.05 \sin\left(\frac{314}{4}x - 27475t\right)$

4. (*a*) $V = f\lambda$

5. (*c*) Path difference $\Delta = \frac{\lambda}{2\pi}\times\phi \Rightarrow 1 = \frac{\lambda}{2\pi}\times\frac{\pi}{2} \Rightarrow \lambda = 4$ m

Hence $v = n\lambda = 120 \times 4 = 480$ m/s

6. (*a*) Recall standard equation.

7. (*c*) The given equation representing a wave travelling along $-y$ direction (because '+' sign is given between t term and x term). On comparing it with $x = A \sin(\omega t + ky)$

We get $k = \frac{2\pi}{\lambda} = 12.56 \Rightarrow \lambda = \frac{2\times3.14}{12.56} = 0.5$ m

8. (*d*) Comparing with standard wave equation

$y = a\sin\frac{2\pi}{\lambda}(vt - x)$,

we get, v = 200 m/s

9. (*d*) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{1\times10^{-3}}} = 100$ m/s

$\therefore t = \frac{1}{100} = 0.01$ s

10. (*a*) $y = A\sin(\omega t - kx)$

$y = 0.1\sin\left(\frac{8\pi}{70}t - \frac{8\pi}{7\times20}\right)x$

$V = \frac{\omega}{k} = 2$ m/s

11. (*d*) Maximum particle velocity is

$(v_P)_{max} = a\omega \Rightarrow (v_P)_{max} = a \times 2\pi\omega$

Now for wave velocity,

$v = \frac{\omega}{k} = \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{t} = \lambda f$

$\Rightarrow (v_P)_{max} = 3v$

$\Rightarrow 2 \times 2\pi f = 3\lambda f \Rightarrow \lambda = \frac{2\pi a}{3}$

12. (*a*) $v_{max} = A\omega = A \times 2\pi f$

$= 60\pi$ cm/s

13. (*a*) Fact based

14. (*d*) $I = 2\pi^2A^2f^2\rho V$

$I = 2\,(3.14)^2\,(10^{-8})^2 \times (2 \times 10^3)^2\,(1.3)\,(340)$

$= 3.5 \times 10^{-6}$ wm^{-2}

15. (*d*) $y = \frac{1}{\sqrt{a}}\sin\omega t \pm \frac{1}{\sqrt{b}}\sin\left(\omega t + \frac{\pi}{2}\right)$

Here phase difference $= \frac{\pi}{2}$

$\therefore$ The resultant amplitude

$\sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2} = \sqrt{\frac{1}{a} + \frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$

16. (c) $A=\sqrt{a_1^2+a_2^2+2a_1a_2\cos 90^\circ}$

$=\sqrt{a_1^2+a_2^2}$

17. (b) Ration of intensities of two waves is

$$\frac{I_1}{I_2}=\frac{A_1^2}{A_2^2}=\frac{1}{16}\Rightarrow\frac{A_1}{A_2}=\frac{1}{4}$$

Thus the ratio of their amplitudes is $\frac{1}{4}$.

18. (c) Resultant amplitude depends on ϕ

$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\varphi}$

19. (c) Resultant amplitude depends on ϕ

$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\varphi}$

Frequency of the resulting wave is same.

20. (d) The waves will pass through each other without any change in their shape.

21. (a) Recall the standard equation.

22. (d) The direction of waves must be opposite and frequencies will be same then by superposition, standing wave formation takes place.

23. (c) $\lambda=\frac{\lambda}{2}$

$\phi=\frac{2\pi}{\lambda}\times\frac{\lambda}{2}=\pi$

24. (a) $f\propto\sqrt{\frac{T}{\mu}}\propto\sqrt{\frac{T}{\rho A}}$

$\frac{f_1}{f_2}=\sqrt{\frac{T_1}{\rho A_1}}\times\sqrt{\frac{\rho A_2}{T_2}}$

$f_2=2f_1$ and $T_1=2$ kg wt

$\frac{A_1}{A_2}=\left(\frac{4}{3}\right)^2\Rightarrow\frac{1}{2}=\sqrt{\frac{2\text{ kg}-\text{wt}}{T_2}\times\frac{9}{16}}$

$T_2=\frac{9}{2}=4.5\text{ kg}-\text{wt}$

25. (a) $T_1P_1^2=T_2P_2^2$

$0.75\times 64=4\times 0.75\times P_2^2$

$P_2=4$

26. (d) $n\propto\frac{1}{l}\Rightarrow\frac{n_2}{n_1}=\frac{l_1}{l_2}\Rightarrow n_2=\frac{l_1}{l_2}n_1=\frac{1\times 256}{1/4}=1024\text{ Hz}$

27. (c) $f=\frac{4V}{2L}\Rightarrow\lambda=\frac{v}{f}=\frac{L}{2}=40$ cm

28. (d) $y=0.021\sin(x+30t)\Rightarrow v=\frac{\omega}{k}=\frac{30}{1}=30\text{ m/s}$

Using, $v=\sqrt{\frac{T}{m}}\Rightarrow 30=\sqrt{\frac{T}{1.3\times 10^{-4}}}\Rightarrow T=0.117$ N

29. (c) $n\propto\sqrt{T}$

30. (d) $n=\frac{1}{2l}\sqrt{\frac{T}{m}}\Rightarrow n\propto\frac{\sqrt{T}}{l}$

$$\Rightarrow\frac{T_2}{T_1}=\left(\frac{n_2}{n_1}\right)^2\left(\frac{l_2}{l_1}\right)^2=(2)^2\left(\frac{3}{4}\right)^2=\frac{9}{4}$$

31. (a) The time taken by the stone to reach the lake

$$t_1=\sqrt{\left(\frac{2h}{g}\right)}=\sqrt{\left(\frac{2\times 500}{10}\right)}=10\text{ sec}$$

$\left(\text{Using } h=ut+\frac{1}{2}gt^2\right)$

Now time taken by sound from lake to the man

$t_2=\frac{h}{v}=\frac{500}{340}\approx 1.5$ sec

$\Rightarrow$ Total time $=t_1+t_2=10+1.5=11.5$ sec.

32. (b) When medium changes, velocity and wavelength changes but frequency remains constant.

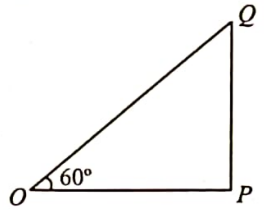
33. (a) $f_1\lambda_1=f_2\lambda_2$

$(300)(a)=(f_2)(1.5)$

$200\text{ Hz}=f_2$

34. (c) $OQ=8\times 330=2640$ m

$\therefore PQ=OQ\sin 60^\circ$



$PQ=2640\times\frac{\sqrt{3}}{2}=2286$ m

35. (d) $\lambda=\frac{v}{n}=\frac{352}{384}$; during 1 vibration of fork sound will travel $\frac{352}{384}$m during 36 vibration of fork sound will travel $\frac{352}{384}\times 36=33$ m

36. (c) $I\propto a^2\propto\frac{1}{d^2}\Rightarrow a\propto\frac{1}{d}$

37. (c) Velocity of sound in gas $v=\sqrt{\frac{\gamma RT}{M}}\Rightarrow v\propto\sqrt{\frac{\gamma T}{M}}$

$$\Rightarrow\frac{v_{N_2}}{v_{He}}=\sqrt{\frac{\gamma_{N_2}}{\gamma_{He}}\times\frac{M_{He}}{M_{H_2}}}=\sqrt{\frac{\frac{7}{5}R\times 4}{\frac{5}{3}R\times 28}}=\frac{\sqrt{3}}{5}$$

38. (b) $v\propto\sqrt{T}\Rightarrow\frac{v_2}{v_1}=\sqrt{\frac{T_2}{T_1}}\Rightarrow 2=\sqrt{\frac{T_2}{(273+0)}}$

$\Rightarrow T_2=273\times 4=1092$ K $=819^\circ$C

39. (*b*) By using $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T}$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{T+600}{T}} = \sqrt{3} \Rightarrow T = 300\,K = 27°C$$

40. (*a*) The quality of sound depends upon the number of harmonics present. Due to different number of harmonics present in two sounds, the shape of the resultant wave is also different

41. (*c*) Loudness depends upon intensity while pitch depends upon frequency.

42. (*d*) $I \propto \frac{1}{r^2} \Rightarrow \frac{\Delta I}{I} = -2\frac{\Delta r}{r} = -2 \times 2 = -4\%$

Hence intensity is decreased by 4%.

43. (*a*) Intensity $\propto \frac{1}{(\text{Distance})^2}$

$$\Rightarrow \frac{I_1}{I_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

44. (*d*) $\frac{P}{4\pi r^2} = I$ for an isotropic point sound source.

$\Rightarrow P = I.4\pi r^2$

$= (0.008\ w/m^2)\ (4.\pi.10^2) = 10.048 \cong 10$ watt.

45. (*d*) $A_{max} = \sqrt{A_2 + A_2} = A\sqrt{2}$, frequency will remain same i.e. ω

46. (*a*) Phase difference is 2π means constrictive interference so resultant amplitude will be maximum.

47. (*d*) This is a case of destructive interference

48. (*c*) Time interval between two consecutive beats

$$T = \frac{1}{n_1 - n_2} = \frac{1}{260-256} = \frac{1}{4}\text{sec so}, t = \frac{1}{16} = \frac{T}{4}\text{sec}$$

By using time difference $= \frac{T}{2\pi} \times$Phase difference

$$\Rightarrow = \frac{T}{4} = \frac{T}{2\pi} \times \phi \Rightarrow \phi = \frac{\pi}{2}$$

49. (*c*) The tuning fork whose frequency is being tested produces 2 beats with oscillator at 514 Hz, therefore, frequency of tuning fork may either be 512 or 516. With oscillator frequency 510 it gives 6 beats/sec, therefore frequency of tuning fork may be either 516 or 504. Therefore, the actual frequency is 516 Hz which gives 2 beats/sec with 514 Hz and 6 beats/sec with 510 Hz.

50. (*a*) Comparing given equation with standard equation

$y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$ gives us $\frac{2\pi}{\lambda} = \frac{\pi}{15}$

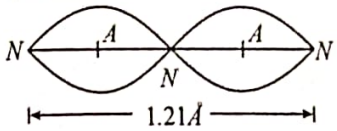
$\Rightarrow \lambda = 30$

Distance between nearest node and antinodes

$$= \frac{\lambda}{4} = \frac{30}{4} = 7.5$$

51. (*d*) Particles have kinetic energy maximum at mean position

52. (*a*) $\lambda = 1.21Å$



53. (*a*) Fundamental frequency of open pipe

$$n_1 = \frac{v}{2l} = \frac{350}{2 \times 0.5} = 350\text{ Hz.}$$

54. (*c*) First overtone of closed organ pipe $n_1 = \frac{3v}{4l_1}$

Third overtone of open organ pipe $n_2 = \frac{4v}{2l_2}$

$$n_1 = n_2\text{ (Given)} \Rightarrow \frac{3v}{4l_1} = \frac{4v}{2l_2} \Rightarrow \frac{l_1}{l_2} = \frac{3}{8}$$

55. (*c*) $n_1 - n_2 = 10$ ...(i)

Using $n_1, = \frac{v}{4l_1}$ and $n_2 = \frac{v}{4l_2}$

$\Rightarrow \frac{n_1}{n_2} = \frac{l_2}{l_1} = \frac{26}{25}$ ...(ii)

After solving these equation $n_1 = 260$ Hz, $n_2 = 250$ Hz

56. (*c*) For closed organ pipe $n_1 : n_2 : n_3 ..... = 1 : 3 : 5 : .....$

57. (*d*) Fact based

58. (*b*) $n' = n\left(\frac{v}{v - v_0}\right) = 450\left(\frac{340}{340-34}\right) = 500$ cycles/sec

59. (*c*) $n' = n\left(\frac{v}{v - v_S}\right) = 1200 \times \left(\frac{350}{350-50}\right) = 1400$ cps

60. (*c*) Since there is no relative motion between the listener and source, hence actual frequency will be heard by listener.

61. (*c*) By using $n' = \left(\frac{v}{v - v_s}\right) \Rightarrow 2n = n\left(\frac{v}{v - v_S}\right)$

$$\Rightarrow v_S = \frac{v}{2}$$

Exercise-2 (Learning Plus)

1. (*c*) Clear from the figure

2. (*d*) $v = \lambda f$

$10 = \lambda 100$

$\lambda = \frac{1}{10}$ m

$$\Delta\phi = \frac{2\pi}{1} \times 10 \times \frac{2.5}{100} = \frac{\pi}{2}$$

3. (*d*) Path difference is λ between *B* and *G*.

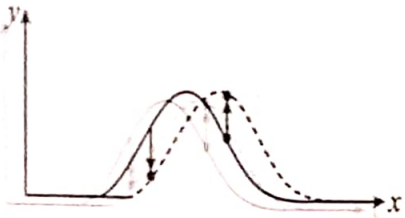
4. (a) $V_{AB} = \sqrt{\frac{6.4g}{10\times10^{-3}}} = \sqrt{6400} = 80\ \frac{m}{sec}$

$V_{CD} = \sqrt{\frac{3.2g}{8\times10^{-3}}} = \sqrt{4000} = 20\sqrt{10}\ \frac{m}{sec}$

$V_{DE} = \sqrt{\frac{1.6g}{10\times10^{-3}}} = \sqrt{1600} = 40$ m/s

5. (d) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{YA\alpha\Delta T}{\mu}}$ put values $V = 70$ m/s

6. (b)



Dotted shape shows pulse position after a short time interval. Direction of the velocities are decided according to direction of displacements of the particles.

At $x = 1.5$ slope is $+ve$

At $x = 2.5$ slope is $-ve$

7. (a) $y = \frac{10}{\pi}\sin\left(200\pi t - \frac{\pi x}{17}\right)$

Comparing with

$y = A\sin(\omega t - kx)$

$\omega = 2000\pi = \frac{2\pi}{T}$

$\Rightarrow T = 10^{-3}$ sec

Maximum velocity $= A\omega$

$\Rightarrow \frac{10}{\pi}\times 2000\pi\times10^{-2} = 200$ m/s

8. (c) Wavelength at lower end is 0.08 m

Velocity of wave on string $= \sqrt{\frac{T}{\mu}}$.

$\mu = \frac{15}{10} = 1.5$

Tension at lower end $\Rightarrow T = 50$ N

Tension at upper end

$\Rightarrow T = 200$ N

$v_{lower} = \sqrt{\frac{50}{1.5}}$

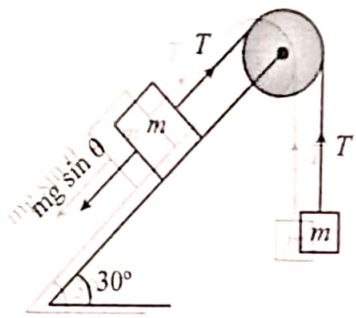
$v_{upper} = \sqrt{\frac{200}{1.5}}$

Frequency is constant hence $\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$

$\frac{\sqrt{50}}{0.08} = \frac{\sqrt{200}}{\lambda}$

$\Rightarrow \lambda = 2\times0.08 = 0.16$ m.

9. (c) $100 = \sqrt{\frac{T}{10^{-2}}} \Rightarrow T = 100$ N



$T = mg\sin 30° = 100$

$m = 20$ kg

10. (c) As $\langle P\rangle = 2\pi^2 f^2 A^2 \mu v$ put values

$90 = 2\times10\times f^2\times25\times10^{-4}\times4\times10^{-2}\sqrt{\frac{100}{4\times10^{-2}}}$

$\Rightarrow f = 30$ Hz

11. (c) $f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

$f_{n+1} - f_n = \frac{1}{2L}\sqrt{\frac{T}{\mu}}(n+1-n)$

$70 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

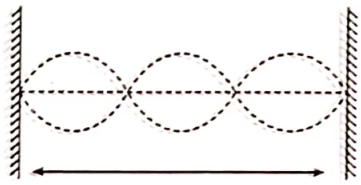
12. (b) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{A\rho}} = \sqrt{\frac{T}{\pi r^2\rho}}$

$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}\cdot\frac{\rho_2}{\rho_1}\cdot\frac{r_2^2}{r_1^2}} = \sqrt{2\times\frac{1}{2}\times\frac{1}{4}} = \frac{1}{2}$

13. (b) $L = \frac{3\lambda}{2} = 1$

$\Rightarrow \lambda = \frac{2}{3}$

$\therefore v = \frac{2}{3}\times300 = 200$ m/s



14. (a) Let f_0 = frequency of tuning fork

Then, $f_0 = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}}$ (m = mass per unit length of wire)

$= \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$

Solving this, we get $M = 25$ kg

In the first case, frequency corresponds to fifth harmonic while in the second case it corresponds to third harmonic

15. (*a*) $n = \frac{V}{\lambda} = \frac{21}{15\times10^{-3}} = \frac{210}{15}$

$V_{max} = A\,\omega = 5\times10^{-3}\times\frac{210}{15}\times2\pi$

$= 70\times2\times\frac{22}{7}\times10^{-3} = .44\text{m/sec.}$

16. (*b*) Time to reach sound wave $= \frac{544}{340}$

Time to reach bullet $= \frac{544}{(340-20)} = \frac{544}{320}$

$\Delta t = 544\left[\frac{1}{320}-\frac{1}{340}\right] = 544\times\frac{20}{320\times340} = 0.1$ sec

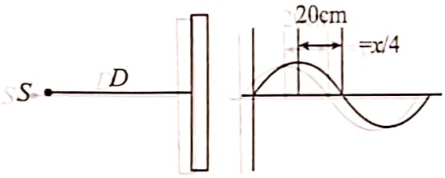
17. (*a*) According to the given data

$V = 336$ m/s,

$\lambda/4$ = distance between maximum and minimum intensity

$= (20\text{ cm}) \Rightarrow \lambda = 80$ cm

$\Rightarrow$ n = frequency $= \frac{V}{\lambda} = \frac{336}{80\times10^{-2}} = 420$ Hz



18. (*a*) $v = \sqrt{\frac{\gamma RT}{M}}$

γ for monoatomic = 1.67

γ for triatomic = 1.3

$= \sqrt{\frac{\gamma_1}{M_1}\times\frac{M_2}{\gamma_2}}$

$= \sqrt{\frac{1.67\times1.8\times10^{-2}}{1.3\times2.02\times10^{-2}}} = 1.067$

19. (*d*) $\left(\frac{d}{v}-\frac{d}{u}\right) = t,\ d\left(\frac{1}{v}-\frac{1}{u}\right) = t \Rightarrow d = \left(\frac{uv}{u-v}\right)t$

20. (*a*) The speed of sound in air is $v = \sqrt{\frac{\gamma RT}{M}}$

$\frac{\gamma}{M}$ of H_2 is greatest in the given gases, hence speed of sound in H_2 shall be maximum.

21. (*d*) $(90-40) = 10\log\frac{I_1}{I_2}$

$5 = \log\frac{I_1}{I_2} \Rightarrow \frac{I_1}{I_2} = 10^5$

22. (*d*) $\Delta x = 12 = \frac{\lambda}{2}$, $\lambda = 24$ cm

$f = \frac{v}{\lambda} = \frac{330}{24\times10^{-2}} = 1375$ Hz

23. (*b*) $\frac{I_1}{I_2} = \frac{4}{1} = 4$

$I_{max} = \left(\sqrt{4}+\sqrt{1}\right)^2 = 9,\ I_{max} = \left(\sqrt{4}-\sqrt{1}\right)^2 = 1$

$\Delta L = 10\log\frac{I_{max}}{I_{min}}, = 10\log\frac{9}{1} = 20\log 3,$

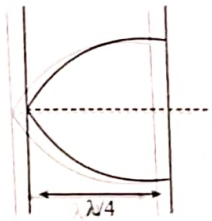
24. (*b*) 11th minima from the current minima, $\Delta x = 10\lambda$

$\lambda = \frac{2\times5}{10} = 1$ cm

25. (*d*) **For displacement**$\rightarrow$ Phase change π at close end

For pressure $\rightarrow$ No phase change at close end

26. (*a*) $\lambda = \frac{330}{600} = \frac{1}{2}$



$\Rightarrow \frac{\lambda}{4} = \frac{1}{8} \Rightarrow 0.125$ m

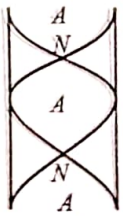
27. (*c*) $n_1 : n_2 : n_3 = 1:2:3$

$\frac{V}{\lambda_1}:\frac{V}{\lambda_2}:\frac{V}{\lambda_3} = 1:2:3$

$\lambda_1:\lambda_2:\lambda_3 = 1:\frac{1}{2}:\frac{1}{3}$

28. (*b*) In an open pipe the ends are points of displacement antinodes and hence pressure node. The midpoint (for fundamental mode) is a point of displacement node and hence pressure antinode. (variation of pressure is maximum at pressure antinode and zero at pressure - node).

29. (*b*)



First overtone (second harmonic)

$L = \lambda$

Since there is no displacement of particles at nodes the pressure is maximum there.

Hence, in open organ pipe the pressure vibration is maximum at a distance $L/4$ from either end inside the tube

30. (*b*) Second overtone of open pipe = $\frac{3V}{2\ell_1}$

Second overtone of closed pipe = $\frac{5V}{4\ell_2}$

Since, these frequency are same

$\therefore \frac{3V}{2\ell_1} = \frac{5V}{4\ell_2}$

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{4\times3}{2\times5} = \frac{6}{5}$

Now, the ratio of fundamental frequencies: $\frac{f_1}{f_2} = \frac{\frac{V}{2\ell_1}}{\frac{V}{4\ell_2}}$

$\Rightarrow \frac{2\ell_2}{\ell_1} = 10:6 = 5:3$

31. (*b*) $\frac{\lambda}{4} = \ell_1 + e$ ...(i)

$\frac{3\lambda}{4} = \ell_2 + e$ ...(ii)

From (i) and (ii) $e = 2$ cm

32. (*b*) Let Δl be the end correction.

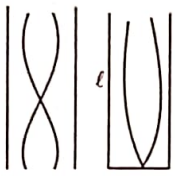
Given that, fundamental tone for a length 0.1 m = first overtone for the length 0.35 cm.

$\frac{v}{4(0.1+\Delta l)} = \frac{3v}{4(0.35+\Delta l)}$

Solving this equation, we get

$\Delta\ell = 0.025$ m $= 2.5$ cm

33. (*a*) $\eta_1 = \frac{V}{2\ell}$ $\eta_2 = \frac{V}{4\ell}$



No. of beat heard $n_1 - n_2 = \frac{V}{4\ell} = 4$

If length of pipes are doubled. no of beats heard

$n_1' - n_2' = \frac{V}{8\ell} = \frac{4}{2} = 2$

34. (*d*) $1750 = \frac{350}{\lambda}$

$\Rightarrow \lambda = \frac{1}{5} = 20$ cm

Should be raised by $\frac{\lambda}{2} = 10$ cm.

35. (*b*) For the first resonant length

$\upsilon = \frac{v}{4l_1} = \frac{v}{4\times18}$ (in winter)

For second resonant length

$\upsilon' = \frac{v'}{4l_2} = \frac{3v'}{4x}$ (in summer)

$\therefore \frac{v}{4\times18} = \frac{3v'}{4x}$ $(x = 3\times18\times v'/v)$

$x = 54\times\frac{v'}{v}$ cm

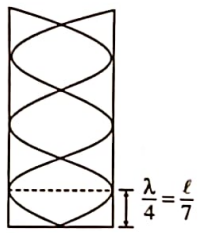
$v' > v$ because velocity of light is greater in summer as compared to winter

$(v \propto \sqrt{T})$

$\therefore x = 54$ cm

36. (*b*) $\frac{7\lambda}{4} = \ell$

$\frac{\lambda}{4} = \frac{\ell}{7}$



$\therefore$ Amplitude $= a$

37. (*d*) 280 ± 10

$f_s = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$\Rightarrow T\uparrow, v\uparrow, f_s\uparrow \Rightarrow \therefore 290$ Hz

38. (*a*) With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4.

Frequency of tuning fork

= Third harmonic frequency of closed pipe + 4

$= 3\left(\frac{v}{4\ell}\right) + 4 = 3\left(\frac{340}{4\times0.75}\right) + 4 = 344$ Hz

39. (*b*) $\frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 6 \Rightarrow \frac{v}{1} - \frac{v}{1.02} = 6$

$v = \frac{6\times1.02}{0.02} = 306 \approx 300$ m/s.

40. (*c*) $v_s - v_2 \rightarrow$ Relative velocity

$\Rightarrow t_1 = \frac{a}{v_s - v_2}$

41. (c) $f' = \left(\frac{v+gt}{v}\right)f$

$f' = f + \frac{gt}{v}.t$

$\Rightarrow f' = 1000 + \frac{g \times 1000}{v}.t$

$t = 30$ s.

$f' = 2000$ Hz

$\Rightarrow v = 300$ m/s.

42. (a) $V_{sound/round} = V_{sound} + V_{medium}$

$V_{sg} = V_s + V_m = c + u$

frequency does not depends on velocity of medium received by observer

$\lambda = \frac{c+u}{f}$

Exercise-3 (JEE Advanced Level)

1. (a,d) Compare with $y = A\cos(\omega t - kx)$

2. (a,c) $V_{max} = A\omega = \frac{v}{10}$

$\Rightarrow 10^{-3}\, 2\pi f = \frac{10}{10}$

$\Rightarrow f = \frac{1000}{2\pi}$ Hz

Now $V = f\lambda$ find λ

3. (a,b,c,d)

Given that, $y = 10^{-4}\sin(60t + 2x)$

Compare it with $y = A\sin(\omega t + kx)$

(a) $v = \frac{\omega}{k} = \frac{60}{2} = 30$ m/s –ve direction

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$

(c) $f = \frac{\omega}{2\pi} = \frac{30}{\pi}$ Hz

(d) $A = 10^{-4}$ m

4. (c) for $x = 4$ m

$y = 2\sin(8\pi - 100\pi t + \frac{\pi}{3})$

$= -2\sin(100\pi t - \frac{\pi}{3})$

$\therefore\ 0 = -2\sin(100\pi t - \frac{\pi}{3})$

$t = \frac{1}{300}$ s.

5. (a,b,d)

$y = \cos(500t - 70x)$

$v = \frac{500}{70} = \frac{50}{7}$ m/s

$f = \frac{\omega}{2\pi} = \frac{500}{2\pi} = \frac{250}{\pi}$

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{70} \times 10^{-2}$ cm

$\lambda = \frac{20\pi}{7}$ cm.

6. (c,d) $\frac{\partial y}{\partial t} = -v\frac{\partial y}{\partial x}$

7. (c,d) $A_{min} = |3A - A| = 2A$

$A_{max} = A + 3A = 4A$

Distance between maximum and adjacent minima

$= \frac{\lambda}{4} = \frac{1}{4}\left(\frac{v}{f}\right)$

8. (a,b,c,d)

Compare with $y = A\sin Kx\cos\omega t$

9. (c,d) $V_w = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{\rho\pi r^2}}$

$V'_w = \sqrt{\frac{2T}{2\rho\pi(2r)^2}} = \frac{1}{2}V_w$

$f'_A = \frac{v_w}{2L}$

$f' = \frac{n}{2(2\ell)} \times v_w' = \frac{n}{4}f_A$ = 3rd overtone

10. (a,c)

$\int_0^L \frac{1}{2}\mu.dx(2A\sin kx)^2$

$\propto \mu A^2\omega^2 L$

$\propto \mu A^2(2\pi)^2 f^2 L$

$\propto n^2$

11. (c,d)

For stable interference, phase difference should not vary with time.

Hence waves should have same frequency and a constant phase difference.

12. (a,d) $P_o = B\,k\,S_0$, $B_1 k_1 S_1 = B_2 k_2 S_2$ here $B_1 = B_2$ $\because$ of same medium

$\frac{S_1}{S_2} = \frac{k_2}{k_1} = \frac{\lambda_1}{\lambda_2}$ = 2Power = $\frac{P_o^2}{2\rho v}$

P_0 is same for both waves

ρ is same for both waves

v is same for both waves

So, $P_1 = P_2$

13. (*b*,*d*)

Making hole at $\frac{\ell}{3}$ length from close end. Pipe start behaving as closed pipe of length $\frac{\ell}{3}$

So new fundamental frequency $n' = \frac{V}{4\ell/3}$

Original fundamental frequency $n = \frac{V}{4\ell} n' = 3\,n$

So option *B* and *D* are correct Ans *B*, *D*

14. (*b*,*d*)

Fundamental frequency of wave in wire. $n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

Here beat frequency $= n_1 - n_2 = \frac{1}{2\ell\sqrt{\mu}}\left(\sqrt{T_1} - \sqrt{T_2}\right)$

Beat frequency remain same

T_2 could be increased or T_1 could be decreased

15. (*a*,*d*)

At closed end displacement node and pressure antinode are formed

16. (*c*,*d*)

High pitch means apparent frequency is increased in reflected wave so there is relative motion between girl and reflection and it is of approaching nature so C and D is correct.

17. (*a*,*c*,*d*)

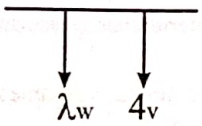
For observer O_1, $\lambda_1 = \frac{V - V_s}{f}$

$$= \frac{V - V/5}{f} = \frac{4V}{5f}$$

For O_2, there is change of medium hence at the surface of water, keeping frequency unchanged

$$\frac{V}{\lambda_a} = \frac{4V}{\lambda_w}$$

$$\Rightarrow \lambda_w = 4\lambda_a = \frac{16V}{5f}$$



$$f'' = \frac{\text{velo. of wave relative to observer}}{\lambda_w}$$

$$= \frac{4V + \frac{V}{5}}{\lambda_w}$$

$$= \frac{21V}{5} \cdot \frac{5f}{16V} = \frac{21f}{16}$$

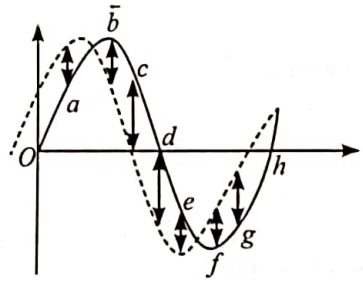
18. (*a*,*d*) **19.** (*c*) **20.** (*b*,*c*)

21. (*c*,*d*)

$$\frac{\partial y}{\partial t} = -v_w \frac{\partial y}{\partial x}$$

v_w = Negative

For question 18 to 21



22. (*a*,*b*,*c*)

$D\,x = pR = nl$

$\lambda = \frac{\pi R}{n}$

$n = 1\lambda = \pi R$

$n = 2\lambda = \frac{\pi R}{2}$

$n = 3\lambda = \frac{\pi R}{3}$

$n = 4\lambda = \frac{\pi R}{4}$

23. (*a*,*c*,*d*)

$\Delta x = \frac{3}{4}(2\pi r) - \frac{2\pi r}{4} = \pi r = (2n+1)\frac{\lambda}{2}$

$\lambda = \frac{2\pi r}{2n+1}$

$n = 0\lambda = 2\pi r$

$n = 1\lambda = \frac{2\pi r}{3}$

$n = 2\lambda = \frac{2\pi r}{5}$

24. (*b*) $I_{max} = 4I_0 \cos^2 \frac{\Delta\varphi}{2}$, $\Delta\phi = \frac{\pi}{2} = 4I_0 \cos^2\left(\frac{\pi}{4}\right), = 2I_0$

25. (*a*) $\Delta x = \pi R = n\lambda$

$\lambda = \frac{\pi R}{n}$ for λ max $n = 1$

$\lambda = \pi R$

26. (*b*) $\Delta x = \pi R = (2n+1)\frac{\pi}{2}$

$\lambda = \frac{2\pi R}{(2n+1)}$

for $\lambda_{max} n = 0$

$\lambda = 2\pi R\ \lambda = \frac{\pi R}{n}$

27. (*a*) $\xi = (0.1 \text{ mm}) \cos \frac{2\pi}{80} (y + 1 \text{ cm}) \cos 2\pi\, 7(400)\, t$

end correction is 1 cm., so at $y = -1$ cm.

$\xi = (0.1 \text{ mm}) \cos \frac{2\pi}{80} (-1 \text{ cm} + 1 \text{ cm}) = (0.1 \text{ mm})$

$\cos(0) =$ Antinode

So upper end is open.

At lower end $y = 99$ cm

$\xi = (0.1 \text{ mm}) \cos \frac{2\pi}{80} (99 + 1) = 0.01 \cos \frac{5\pi}{2} = 0$

$\Rightarrow$ Node

Tube is closed at lower end

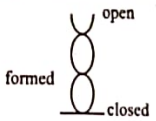
So tube is open closed.

28. (*b*) $\frac{2\pi}{80} = \frac{2\pi}{\lambda}$ So $\lambda = 80$

and effective length of air column = 99 + 1 = 100 cm

So $\frac{\ell}{\lambda} = \frac{5}{4} \Rightarrow l = 5\frac{\lambda}{4}$,

so five half loops will be formed



$\ell = 5\left(\frac{\lambda}{4}\right)$ so second overtone.

29. (*a*) $P_{ex} = -B\frac{d\xi}{dx}$

$= (5 \times 10^5) \times (0.1 \times 10^{-3}) \frac{2\pi}{80} \sin \frac{2\pi}{80} (y + 1 \text{ cm}) \cos 2\pi(400)\, t$

$= (125\, \pi \text{ N/m}^2) \sin \frac{2\pi}{80} (y + 1 \text{ cm}) \cos 2\pi(400t)$

30. (*a*) End correction = (0.3) d = 1 cm

$d = \frac{10}{3}$ cm

Volume of tube $= \left(\pi\frac{d^2}{4}\right)\ell = \frac{\pi}{4}\left(\frac{10}{3}\right)^2 \times 100 \text{ cm}^3$

(Take l = 0.99 m ≈ 1 m)

$= \frac{10\pi}{36}$ lit

Moles $= \frac{10\pi}{36 \times 22.4}$ moles

(22.4 lt. contains 1 mole $\frac{10\pi}{36}$ lt contains $\frac{10\pi}{36 \times 22.4}$ mole)

31. (*b*) Use $x = 0$; $t = 0$ for y and particle velocity $\frac{\partial y}{\partial t}$. Like for (*a*), $y = 0$ at $x = 0$ and $t = 0$. $\frac{\partial y}{\partial t} > 0$ i.e. positive therefore it matches with (*R*).

32. (*c*)

A. $y = 4\sin(5x - 4t) + 3\cos(4t - 5x + \pi/6)$ is super position of two coherent waves moving in positive direction, so their equivalent will be an another travelling wave.

B. $y = 10\cos\left(t - \frac{x}{330}\right)\sin(100)\left(t - \frac{x}{330}\right)$ lets check at any point, say at $x = 0$,

$y = (10\cos t)\sin(100\,t)$ at any point amplitude is changing sinusoidally. So this is equation of beats.

C. $y = 10\sin(2\pi x - 120t) + 10\cos(120t + 2\pi x) =$ superposition of two coherent waves travelling in opposite direction.

$\Rightarrow$ Equation of standing waves.

D. $y = 10\sin(2\pi x - 120\,t) + 8\cos(118t - 59/30\pi x) =$ superposition of two waves whose frequencies are slightly different ($\omega_1 = 120$, $\omega_2 = 118$)

$\Rightarrow$ Equation of Beats.

33. (*b*) Find all speeds w.r.t. wind and apply doppler effect.

A. The wavelength of the waves coming towards the observer from source $= (V - V_W + V_S)/f$

B. The wavelength of the waves incident on the wall $= (V + V_W - V_S)/f$

C. The wavelength of the waves coming towards observer from the wall $= (V - V_W - V_D)/f_r$

D. Frequency of the waves (as detected by O) coming from wall after reflection $= (V - V_W - V_O)f_r/(V - V_W - V_D)$

34. [0080]

$\Delta t = \frac{L}{v} = 0.1$

$v = 40 \text{ m/s} = \sqrt{\frac{T}{\mu}}$

$T = \frac{0.2}{4} \times 16000 = 80$ N

35. [0006]

$nf_0 = 720 \quad f_0 = 120, n = 6$

$(n + 1)f_0 = 840$

36. [0007]

$340 - V_m = \frac{3015}{9}$

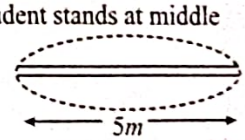
$340 + V_T = \frac{855}{5} \times 2 = 342$

$V_T = 2$ m/s, $V_m = 5$ m/s

$V_{rel} = 7$ m/s

37. [0004]

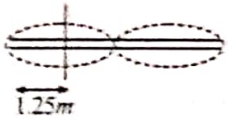
When student stands at middle



$\frac{\lambda}{2} = 5$ m, $\lambda = 10$ m

Wave velocity $v = \nu\lambda = 20$ m/s

When student stands at 1.25 m



$\lambda/4 = 1.25$ m ; $\lambda = 5$m

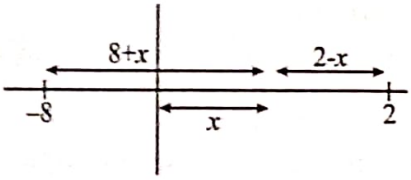
$$\nu = \frac{v}{\lambda} = \frac{20}{5} = 4 \text{ Hz}$$

38. [005]

$$\Delta\phi = k\Delta x = \frac{2\pi}{\lambda}[10-5] = \frac{2\pi \times 5}{2} = 5\pi$$

39. [0004]

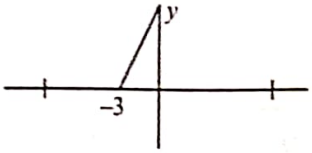
$$t = \frac{8+x}{c} = \frac{2-x}{c} \Rightarrow -6 = 2x$$



$x = -3$ m

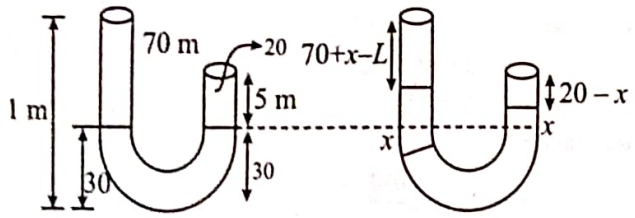
$$\Rightarrow t = \frac{5}{c}$$

$$5\sqrt{3^2 + y^2}$$



$\Rightarrow y = 4$

40. [0060] $2x \times \rho \times g = L \times \frac{\rho}{2} g$



$\Rightarrow L = 4x$

Frequency same

$\Rightarrow \lambda$ = same

$$20 - x = \frac{\lambda}{4}$$

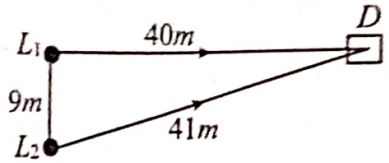
$$70 + 3x = \frac{5\lambda}{4}$$

$70 - 3x = 100 - 5x$

$2x = 30; x = 15$

$L = 4x = 60$ cm

41. [0330]



$\Delta x = 1$ m

$n\lambda = 1$

$$f = \frac{C}{\lambda} = C \times n ; n = 1; f = 330 \text{ Hz}$$

42. [100]

$\Delta X = BP - AP$

$(2.5 + x) - (2.5 - x)$

$\Delta X = 2x$

For maxima, $\Delta X = n\lambda$

$2x = n\lambda$ $(n = 1)$

$$x = \frac{\lambda}{2} = \frac{2}{2} = 1\text{m} \quad \left[\lambda = \frac{v}{f}\right]$$

$x = 100$ cm

Exercise-4 (Past Year Questions)

JEE MAIN

1. (c) $v = \sqrt{\frac{T}{\mu}} = \sqrt{xg}$

$$\frac{dx}{dt} = \sqrt{xg}$$

$$\int_0^L \frac{dx}{\sqrt{x}} = \sqrt{g}\int_0^t dt$$

$t = 2\sqrt{2}s$

2. (c) $V = \frac{\omega}{k} = \frac{450}{9} = 50\,\text{ms}^{-1}$

$$V = \sqrt{\frac{T}{\mu}}$$

$T = V^2\mu = 50^2 \times 5 \times 10^{-3} = 12.5$ N

3. (d) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8}{5 \times 10^{-3}}} = 40$ m/s

$$\lambda = \frac{V}{f} = \frac{40}{100}\text{m} = 0.4\text{m}$$

Separation between 2 successive node $= \frac{\lambda}{2} = 20$ cm

4. (*b*) $f_1 = f_0\left[\frac{340}{340-34}\right] \Rightarrow f_2 = f_0\left[\frac{340}{340-17}\right]$

$$\frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$$

5. (*a*) $50t + 2x = \text{constant}$

$$50 + \frac{2dx}{dt} = 0$$

$$V = \frac{dx}{dt} = -25 \text{ m/s}$$

6. (*d*) $f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$

$$240 = \frac{3}{2\times 2}V$$

$V = 320$ m/s

$$f_1 = \frac{1}{2L}V = \frac{1}{2\times 2}\times 320 = 80 \text{ Hz}$$

7. (*d*) $\therefore\ \mu_1 = \frac{\rho\pi r^2 L}{L} = \mu$ and $\mu = \frac{\rho 4\pi r^2 L}{L} = 4\mu$

Tension in both are the same $= T$

Let the speed of the wave in the wires be V_1 and V_2

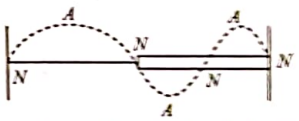
$$V_1 = \sqrt{\frac{T}{\mu}} = V,\ V_2 = \sqrt{\frac{T}{4\mu}} = \frac{V}{2}$$

So fundamental frequencies in both wires are

$$f_1 = \frac{V_1}{2L} = \frac{V}{2L} \text{ and } f_2 = \frac{V_2}{2L} = \frac{V}{4L}$$

Frequency at which both resonate is L.C.M. of both frequencies i.e., $\frac{V}{2L}$.

Hence no. of loops in wires are 1 and 2 respectively.



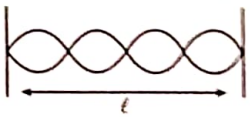
So, ratio of no. of antinodes is 1 : 2.

8. (*d*) $f_{beat} = 11 - 9 = 2$ Hz

$\therefore$ Time period of oscillation of amplitude $= \frac{1}{f_{beat}} = \frac{1}{2}$

Although the graph of oscillation is not given, the equation of envelope is given by option (*d*)

9. (*b*) 4th harmonic



$$4\frac{\lambda}{2} = \ell$$

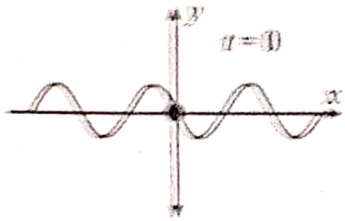
$$2\lambda = \ell$$

From equation $\frac{2\pi}{\lambda} = 0.157$

$\lambda = 40$

$\ell = 2\lambda = 80$ m

10. (*c*)



$y = A\sin(kx - \omega t + \phi)$

At $x = 0, t = 0, y = 0$ and slope is negative

$\Rightarrow \phi = \pi$

11. (*a*) $v \propto \sqrt{T}$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \quad \Rightarrow \frac{v}{(v/2)} = \sqrt{\frac{2.06\times 10^4}{T}}$$

$$\Rightarrow T = \frac{2.06\times 10^4}{4}\text{N} = 0.515\times 10^4\text{N} = 5.15\times 10^3\text{N}$$

12. (*b*) Fundamental frequency $= 490 - 420 = 70$ Hz

$$\text{Fundamental frequency} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$70 = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow 70 = \frac{1}{2\ell}\sqrt{\frac{540}{6\times 10^{-3}}}$$

$$\Rightarrow \ell = \frac{1}{2\times 70}\sqrt{90\times 10^{3}} = \frac{300}{140}$$

$$\Rightarrow \ell \approx 2.14 \text{ m}$$

13. (*b*) $f = \frac{v}{2L}$

$$\Rightarrow f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad \frac{f_x}{f_z} = \sqrt{\frac{T_x}{T_z}}$$

$$\Rightarrow \frac{450}{300} = \sqrt{\frac{T_X}{T_Z}}$$

$$\Rightarrow \frac{T_X}{T_Z} = 2.25$$

14. (*c*) $\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$

$\lambda \propto \sqrt{T}$

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\lambda_2 = \lambda_1\sqrt{\frac{T_2}{T_1}}$$

$$\lambda_2 = (6 \text{ cm})\sqrt{\frac{2g + 6g}{2g}}$$

$$\Rightarrow \lambda_2 = 12 \text{ cm}$$

15. (c) $\left(\frac{\lambda}{2}\right)\times n = 1.5 \Rightarrow \lambda = \frac{3}{n}$, n is odd integer

$\lambda \times m = 5 \Rightarrow \lambda = \frac{5}{m}$, m is integer

So, $\frac{3}{n} = \frac{5}{m} \Rightarrow m = \frac{5n}{3}$

$n = 1, 3, 5, 7, 9, 11, 13, 15, ...$

$m = \frac{5}{3}, 5, \frac{25}{3}, \frac{35}{3}, 15, \frac{55}{3}, \frac{65}{3}, 25, ...$

So, $\lambda = \frac{5}{n} = \frac{3}{3}, \frac{3}{9}, \frac{3}{15}, ... = 1, \frac{1}{3}, \frac{1}{5}, ...$

or, $\lambda = \frac{5}{m} = \frac{5}{5}, \frac{5}{15}, \frac{5}{25}, ... = 1, \frac{1}{3}, \frac{1}{5}, ...$

16. (d) $L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$

$\lambda = 2L$

$f = \frac{V}{2L}$

$\frac{L}{2} = \frac{L}{4} \Rightarrow \lambda = 2L$

$f' = \frac{V}{2L} = f$

17. (a) Doppler effect in light (speed of observer is not very small compare to speed of light)

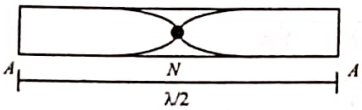
$$f^1 = \sqrt{\frac{1+V/C}{1-V/C}} f_{Source} = \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}}(10\,GHz) = 17.3 \text{ GHz}$$

18. (d) Velocity of wave $= \sqrt{\frac{Y}{\rho}}$

$$= \sqrt{\frac{9.27\times10^{10}}{2.7\times10^{3}}} = \sqrt{3.433\times10^{7}} = 103 \times \sqrt{34.33}$$

$v_\omega = 5.85 \times 10^3$ m/sec.

Since rod is clamped at middle fundamental wave shape is as follow



$\frac{\lambda}{2} = L$

$\lambda = 2L$

$L = 60$ cm $= 0.65$ m (given)

$\lambda = 1.2$ m

$v = f\lambda$

$$f = \frac{v}{\lambda} = \frac{5.85\times10^{3}}{1.2} = 4.88 \times 10^3 \text{ Hz} \simeq 5 \text{ KHz}$$

19. (a) $f = \frac{2}{2\ell}v_s = \frac{330}{0.5} = 660$ Hz

$$\therefore f' = f\left(\frac{v_s + v}{v_s}\right) = (660)\left(\frac{330+\frac{50}{18}}{330}\right)$$

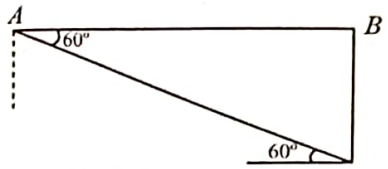
$$= 660\left(1+\frac{50}{18\times330}\right) = 666 \text{ Hz.}$$

20. (a) $\frac{250\text{kHz}}{1.5\text{kHz}} = 13.33$

$\therefore$ Possible hormones

1, 3, 5, 7, 9, 11, 13 i.e. 6

21. (d)



$V_P \rightarrow$ Speed of plane

$V \rightarrow$ Speed of sound

$V\cos60° = V_P$

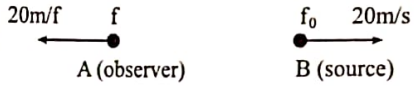
$V_P = \frac{V}{2}$

22. (d) $\lambda_1 = 4(11+e)\frac{v}{512} \Rightarrow \lambda_2 = 4(27+e) = \frac{v}{256}$

$\frac{11+e}{27+e} = \frac{1}{2}$

$\Rightarrow 22 + 2e = 27 + e \Rightarrow e = 5$

23. (a)



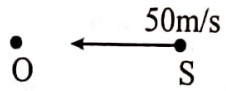
$f = \frac{v+v_0}{v-v_s}f_0$

(v_0 and v_s is taken $\oplus$ when approaching each other)

$2000 = \frac{340+(-20)}{340-(-20)}f_0$

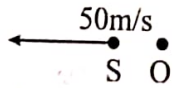
$f_0 = 2250$ Hz.

24. (c)



$f_{app} = \left(\frac{V-0}{V-50}\right)f_{source}$

$f_{source} = \frac{1000\times300}{350}$



$f_{app} = \left(\frac{V}{V+50}\right)\cdot f_{source}$

$= \frac{350}{400}\times1000\times\frac{300}{350} = 750$Hz

25. (*c*) Speed of wave from wave equation

$$v = -\frac{(\text{coeffecient of } t)}{(\text{coeffecient of } x)}$$

$$v = -\frac{1000}{(-3)} = \frac{1000}{3}$$

Since speed of wave $\propto \sqrt{T}$

$$\text{So } = \frac{\frac{1000}{3}}{336} = \sqrt{\frac{273}{T}} \Rightarrow T = 277.41 \text{ K}$$

$T = 4.41°$ C

26. (*c*) $v = 2f(l_2 - l_1)$

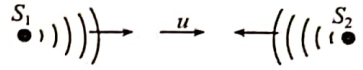
$v = 2 \times 480 \times (70 - 30) \times 10^{-2}$

$v = 960 \times 40 \times 10^{-2}$

$v = 38400 \times 10^{-2}$ m/s

$v = 384$ m/s

27. (*a*) $f = 660$ Hz, $v = 330$ m/s



$$f_1 = f\left(\frac{v-u}{v}\right) \Rightarrow f_2 = f\left(\frac{v+u}{v}\right)$$

$$f_2 - f_1 = \frac{f}{v}[v + u - (v - u)]$$

$$10 = f_2 - f_1 = \frac{f}{v}[2u]$$

$u = 2.5$ m/s

28. (*c*) Loudness of sound is given by

$$dB = 10 \log \frac{I}{I_0}$$

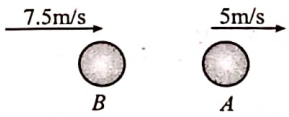
(I is intensity of sound, I_0 is reference intensity of sound)

$$\therefore\ 120 = 10 \log\left(\frac{I}{I_0}\right) \Rightarrow I = 1 \text{ W/m}^2$$

$$\text{Also } I = \frac{P}{4\pi r^2} = \frac{2}{4\pi r^2}$$

$$\therefore\ r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{1}{2\pi}} m = 0.399 \approx 40 \text{ cm}$$

29. (*b*)



$f_0 = 500$ Hz

Frequency received by $A \Rightarrow \left(\frac{1500-5}{1500-7.5}\right) f_0 = f_1$

and frequency received by B again = 1500

(*b*) (*a*)

7.5 m/s ⟶ ⟶ 5 m/sec

$$f_2 = \left(\frac{1500+7.5}{1500+5}\right) \times \left(\frac{1500-5}{1500-7.5}\right) f_0 = 502 \text{ Hz}$$

30. (*b*) $f = 480 = \frac{300 - v_1}{300} \times 500$

$$\frac{1440}{5} = 300 - v_1$$

$$v_1 = \frac{60}{5} = 12 \text{ m/s}$$

$$530 = \frac{300 + v_2}{300} \times 500$$

$1590 = 1500 + 5v_2$

$5v_2 = 90$

$v_2 = 18$ m/s

31. (*a*) $\overset{V}{s_1} \longrightarrow$ observer $\overset{V}{s_2} \longrightarrow$

$$f_0\left(\frac{C}{C-V}\right) - f_0\left(\frac{C}{C+V}\right) = 2$$

$$V = \frac{1}{4} \text{ m/s}$$

32. [106] $V = \sqrt{\frac{B}{\rho}}$

$$\frac{V_{pipe}}{V_{air}} = \frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}} = \frac{1}{\sqrt{2}}$$

$$V_{pipe} = \frac{V_{air}}{\sqrt{2}}$$

$$f_n = \frac{(n+1)V_{pipe}}{2\ell}$$

$$f_1 - f_0 = \frac{V_{pipe}}{2\ell} = \frac{300}{2\sqrt{2}}$$

= 105.75 Hz (If $\sqrt{2} = 1.41$)

= 106.05 Hz (If $\sqrt{2} = 1.414$)

33. (*b*) $\frac{330 + v}{330 - v} \times 420 = 490$

$$\Rightarrow \frac{330 + v}{330 - v} = \frac{7}{6}$$

$\Rightarrow 13\,v = 330$

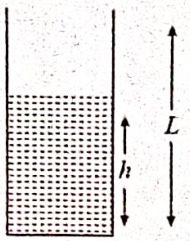
$$\Rightarrow v = \frac{330}{13} = 25.38 \text{ m/s} = 91 \text{ km/h}$$

34. (*b*) $\frac{345 + u}{345 - v} \times 440 = 480$

$\Rightarrow 11 \times 345 + 11v = 12 \times 345 - 12v$

$$\Rightarrow v = \frac{345}{23} = 15 \text{m/s} = 54 \text{ km/h}$$

35. (*a*)



$$L - h_1 = n\frac{\lambda}{2} + \frac{\lambda}{4}$$

$$L - h_2 = (n-1)\frac{\lambda}{2} + \frac{\lambda}{4}$$

$$\Rightarrow \frac{\lambda}{2} = h_2 - h_1 = 24.5 - 17.0 = 7.5 \text{ cm}$$

$$\Rightarrow \lambda = 15 \text{ cm}$$

$$u = f\lambda$$

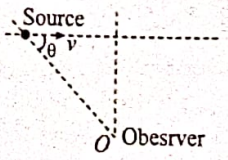
$$f = 2200 \text{ Hz}$$

36. (*a*) $\because \Delta P_0 = B\frac{\omega}{v} \times S_0 \Rightarrow S = \frac{\Delta P \times v}{B\omega} = \frac{\Delta P \times v}{\rho v^2 \times 2\pi v}$

$$\Rightarrow S \propto \frac{\Delta P}{\rho v f}$$

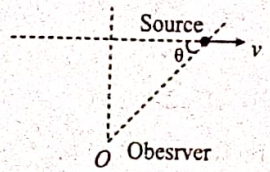
$$\Rightarrow S = \frac{\Delta P}{\rho v f} = \frac{10}{1 \times 300 \times 1000} = \frac{1}{30} \text{ mm} \approx \frac{3}{100} \text{ mm}$$

37. (*a*) While approaching



$$\nu = \nu_0\left(\frac{c}{c - v\cos\theta}\right)$$

While receding



$$\nu = \nu_0\left(\frac{c}{c + v\cos\theta}\right)$$

38. [4] Both piper vibrate in first overtone.

$$f_c = f_o \Rightarrow \frac{3V_c}{4L} = \frac{2V_o}{2l_o} \Rightarrow \frac{3V_c}{4L} = \frac{V_o}{l_o}$$

$$\therefore \frac{V_o}{V_c} = \frac{3l_o}{4L}$$

Also, $\frac{V_o}{V_c} = \sqrt{\frac{\rho_1}{\rho_2}}$

Then, $\sqrt{\frac{\rho_1}{\rho_2}} = \frac{3l_0}{4L} \Rightarrow l_0 = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$

39. (*d*) $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

$$\phi = \frac{\pi}{2}$$

$$A = \sqrt{5^2 + 3^2 + 0} = \sqrt{34}$$

40. (*c*) $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$$I_P = I + 9I + 0 = 10I$$

$$I_Q = I + 9I - 6I = 4I$$

$$\frac{I_P}{I_Q} = \frac{5}{2}$$

41. (*b*) $\lambda = \frac{V}{f} = \frac{336}{400} = 0.84 \text{ m}$

For I resonance, $\frac{\lambda}{4} = \ell + e$

$$\frac{84}{4} = 20 + e \Rightarrow e = 1 \text{cm}$$

For III resonance, $\frac{5\lambda}{4} = \ell_2 + e$

$$\ell_2 = 104 \text{ cm}$$

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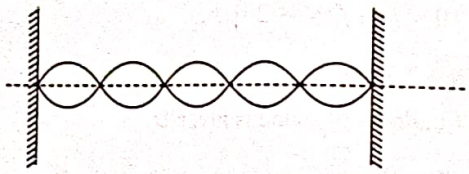
42. (*b,c*)

(*a*) There are 5 complete loops.



Total number of nodes = 6

(*b*) $\omega = 628 \text{ sec}^{-1}$

$$k = 62.8 \text{ m}^{-1} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{1}{10}$$

$$v_w = \frac{\omega}{k} = \frac{628}{62.8} = 10 \text{ ms}^{-1}$$

$$L = \frac{5\lambda}{2} = 0.25$$

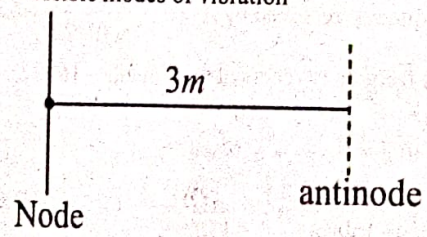
(*c*) 2A = 0.01 = maximum amplitude of antinode

(*d*) $f = \frac{v}{2\ell} = \frac{10}{2 \times 0.25} = 20$ Hz.

43. (*a,c,d*)

$V = 100$ m/s

Possible modes of vibration



$$\ell = (2n+1)\frac{\lambda}{4}$$

$$\lambda = \frac{12}{(2n+1)}\ \text{m}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{12/(2n+1)} = \frac{(2n+1)\pi}{6}$$

$$\omega = vk = 100\,(2n+1)\,\frac{\pi}{6} = \frac{(2n+1)50\pi}{3}$$

If $n = 0$ $\quad k = \frac{\pi}{6} \quad \omega = \frac{50\pi}{3}$

$n = 1$ $\quad k = \frac{5\pi}{6} \quad \omega = \frac{250\pi}{3}$

$n = 7$ $\quad k = \frac{5\pi}{2} \quad \omega = 250\,\pi$

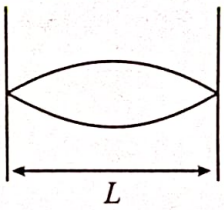
44. (*b*,*d*)

$V = \sqrt{\frac{T}{\mu}}$, so speed at any position will be same for both pulses, therefore time taken by both pulses will be same.

$\lambda f = v \Rightarrow \lambda = \frac{V}{f} \Rightarrow \lambda \propto V$, since when pulse 1 reaches at A, speed decreases therefore λ decreases.

At mid point, magnitude of velocity is same, but direction will be opposite. Hence velocity will be in opposite direction.

45. (*a*) For fundamental mode



$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$

$$f = \frac{V}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

For string (*a*)

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \Rightarrow (P)$$

For string (*b*)

$$f = \frac{1}{2L}\sqrt{\frac{T}{2\mu}} = \frac{f_0}{\sqrt{2}} \Rightarrow (R)$$

For string (*c*)

$$f = \frac{1}{2L}\sqrt{\frac{T}{3\mu}} = \frac{f_0}{\sqrt{3}} \Rightarrow (S)$$

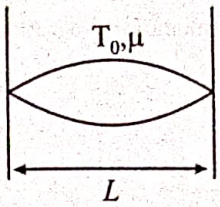
For string (*d*)

$$f = \frac{1}{2L}\sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \Rightarrow (Q)$$

46. (*a*) For string (*a*)

Length of string $= L_0$

It is vibrating in 1st harmonic i.e. fundamental mode.



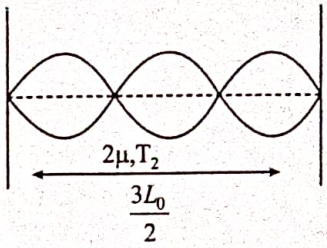
$$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow (P)$$

For string (*b*)

Length of string $= \frac{3L_0}{2}$

It is vibrating in IIIrd harmonic but frequency is still f_0.

$$f_0 = \frac{3v}{2L}$$



$$f_0 = \frac{3}{2\left(\frac{3L_0}{2}\right)}\sqrt{\frac{T_2}{2\mu}}$$

$$\Rightarrow f_0 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

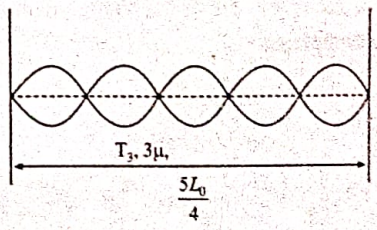
$$\Rightarrow T_2 = \frac{T_0}{2} \Rightarrow (Q)$$

For string (*c*)

Length of string $= \frac{5L_0}{4}$

It is vibrating in 5th harmonic but frequency is still f_0.

$$f_0 = \frac{5V}{2L}$$



$$\Rightarrow f_0 = \frac{5}{2\left(\frac{5L_0}{4}\right)}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \frac{2}{L_0}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

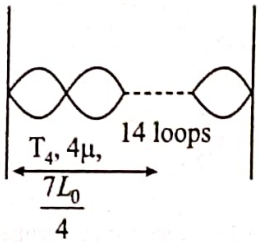
$$T_3 = \frac{3T_0}{16} \Rightarrow (T)$$

For string (*d*)

Length of string = $\frac{7L_0}{4}$

It is vibrating in 14th harmonic but frequency is still f_0.

$$f_0 = \frac{14v}{2L}$$



$$\Rightarrow f_0 = \frac{14v}{2\left(\frac{7L_0}{4}\right)}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \frac{4}{L_0}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow (U)$$

47. (*a*)

A. $\frac{\lambda}{4} = L, \lambda = 4L,$

Sound waves are longitudinal waves

B. $\frac{\lambda}{2} = L, \lambda = 2L$

Sound waves are longitudinal waves

C. $\frac{\lambda}{2} = L, \lambda = 2L$

String waves are transverse waves

D. $\lambda = L$

String waves are transverse waves

48. (*b,d*)

At open end phase of pressure wave charge by π so compression returns as rare fraction. While at closed end phase of pressure wave does not change so compression return as compression.

49. (*b*) $\frac{V}{4(\ell + e)} = f$

$$\Rightarrow \ell + e = \frac{V}{4f}$$

$$\Rightarrow \ell = \frac{V}{4f} - e$$

Here $e = (0.6)r$

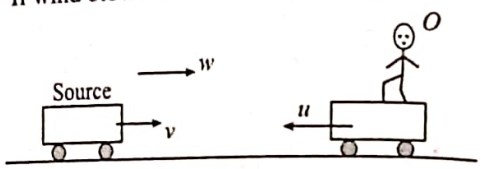
$= (0.6)(b) = 1.2$ cm

So $\ell = \frac{336 \times 10^2}{4 \times 512} - 1.2$

$= 15.2$ cm

50. (*a,b*)

If wind blows from source to observer



$$f_2 = \left(\frac{(v+w)+u}{(v+w)-u}\right)f_1$$

$$\Rightarrow f_2 > f_1$$

51. (*d*) $f = \frac{1}{4\ell}\sqrt{\frac{\gamma RT}{M}}$ & $\frac{\Delta f}{f} = \frac{\Delta \ell}{\ell}$

(*a*) $M = 20 \times 10^{-3}$ $f = 320$ Hz $\Delta f = \pm 4.5$ Hz

Not possible

(*b*) $M = 20 \times 10^{-3}$ $f = 253$ Hz $\Delta f = \pm 3.6$ Hz

Not possible

(*c*) $M = 32 \times 10^{-3}$ $f = 237$ Hz $\Delta f = \pm 3.4$ Hz

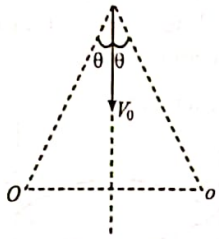
Not possible

(*d*) $M = 36 \times 10^{-3}$ $f = 242.8$ Hz $\Delta f = \pm 3.5$ Hz

Possible

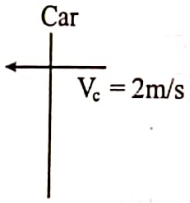
52. (*a,b,c*)

$$\beta = \frac{c + V_0\cos\theta}{c}f_1 - \frac{c + V_0\cos\theta}{c}f_2$$



$$= \frac{c + V_0\cos\theta}{c}(f_1 - f_2)$$

53. [6]



Frequency observed at car, $f_1 = f_0\left(\frac{V + V_C}{V}\right)$

Frequency of reflected sound as observed at the source

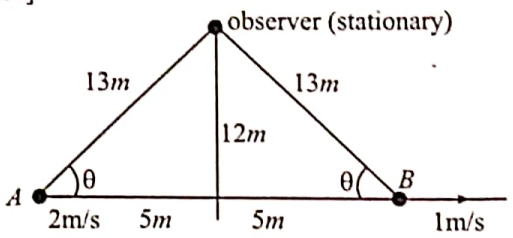
$$f_2 = f_1\left(\frac{V}{V - V_C}\right) = f_0\left(\frac{V + V_C}{V - V_C}\right)$$

Beat frequency $= f_2 - f_0$

$$= f_0\left[\frac{V + V_C}{V - V_C} - 1\right] = f_0\left[\frac{2V_C}{V - V_C}\right]$$

$$= 492 \times \frac{2 \times 2}{328} = 6 \text{ beat/s}$$

54. [5.00]



$$\cos\theta = \frac{5}{13}$$

$$f_A = 1430\left[\frac{330}{330-2\cos\theta}\right] = 1430\left[\frac{1}{1-\frac{2\cos\theta}{330}}\right]$$

$$= 1430\left[1+\frac{2\cos\theta}{330}\right] \quad \text{(By bionomial expansion)}$$

$$f_B = 1430\left[\frac{330}{330-1\cos\theta}\right] = 1430\left[1-\frac{\cos\theta}{330}\right]$$

$$\Delta f = f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

$$= 13\left(\frac{5}{13}\right) = 5.00 \text{ Hz}$$

55. (a,b,c)

Let n_1 harmonic is corresponding to 50.7 cm & n_2 harmonic is corresponding 83.9 cm.

Since both one consecutive harmonics.

$\therefore$ Their difference $= \frac{\lambda}{2}$

$\therefore \frac{\lambda}{2} = (83.9 - 50.7)$ cm

$\frac{\lambda}{2} = 33.2$ cm

$\lambda = 66.4$ cm

$\therefore \frac{\lambda}{4} = 16.6$ cm

Length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ & 50.7 cm must be an odd multiple of this length 16.6 × 3 = 49.8 cm. Therefore 50.7 is 3rd harmonic

If end correction is e, then

$e + 50.7 = \frac{3\lambda}{4}$

$e = 49.8 - 50.7 = -0.9$ cm

Speed of sound, $v = f\lambda$

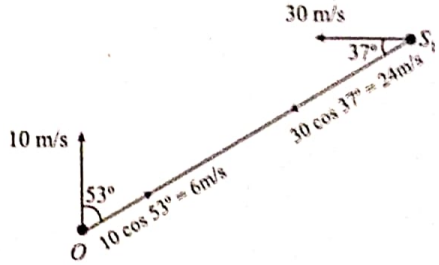
$\therefore v = 500 \times 66.4$ cm/sec $= 332.00$ m/s

56. [8] Frequency observed by O from S_2

$$F_2 = \frac{330+10}{330}\times 120 = \frac{340}{330}\times 120 = 123.63\text{Hz}$$

Frequency observed by O from S_1

$$f_1 = \frac{330+6}{330-24}\times 120 = \frac{336}{306}\times 120 \approx 131.76\text{Hz}$$



Beat frequency = 131.76 – 123.63 ≈ 8 Hz

57. (0.62 to 0.63)

$$f \propto \frac{1}{\ell_1} \Rightarrow f = \frac{k}{\ell_1} \quad \text{...(i)}$$

($\ell_1 \Rightarrow$ initial length of pipe)

$$\left(\frac{V}{V-V_T}\right)f = \frac{k}{\ell_2} \quad \text{...(ii)}$$

{V_T speed of tuning fork, $\ell_1 \rightarrow$ new length of pipe}

(i) ÷ (ii)

$$\frac{V-V_T}{V} = \frac{\ell_2}{\ell_1}$$

$$\frac{\ell_2}{\ell_1} - 1 = \frac{V-V_T}{V} - 1$$

$$\frac{\ell_2-\ell_1}{\ell_1} = \frac{V_T}{V}$$

$$\frac{\ell_2-\ell_1}{\ell_1}\times 100 = \frac{-2}{320}\times 100 = -0.625$$

Therefore smallest value of percentage change required in the length of pipe is 0.625.

58. (a,d)

$$f = f_s\left(\frac{v}{v-u}\right)$$

(a) $f = f_0\left(\frac{v}{v-0.8v}\right) = 5f_0$

(b) $f = 2f_0\left(\frac{v}{v-0.8v}\right) = 10f_0$

(c) $f = 0.5f_0\left(\frac{v}{v-0.8v}\right) = 2.5f_0$

(d) $f = 1.5f_0\left(\frac{v}{v-0.5v}\right) = 3f_0$

close

All odd harmonics are available in closed pipe therefore Correct Ans (A,D)