Position and Velocity of SHM

$x = A\sin(\omega t + \Phi_0)$

$v = A\omega\cos(\omega t + \Phi_0)$

$v = \omega\sqrt{A^2 - x^2}$

a) $x = 0 \longrightarrow v_{max} = A\omega$

b) $x = \frac{A}{2} \longrightarrow v = \frac{\sqrt{3}A\omega}{2}$

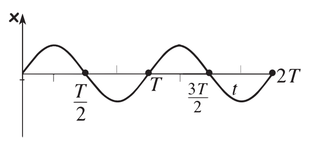
c) $x = \frac{A}{\sqrt{2}} \longrightarrow v = \frac{A\omega}{\sqrt{2}}$

d) $x = A \longrightarrow v = 0$

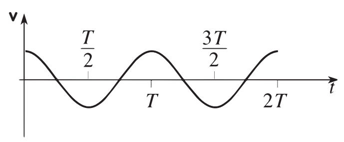
Graphical Representation of Position and Velocity

Start from mean position, $\Phi_0 = 0$

$x = A\sin(\omega t)$

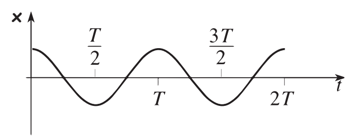


$v = A\omega\cos(\omega t)$

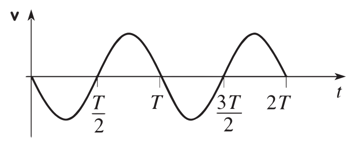


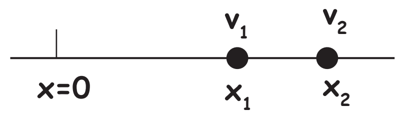
Start from extreme position $\Phi_0 = \frac{\pi}{2}$

$x = A\cos(\omega t)$



$v = -A\omega\sin(\omega t)$





$A = \sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}$ $\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$

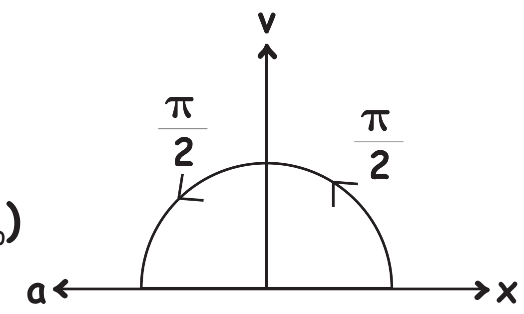
Acceleration of SHM

$x = A\sin(\omega t + \Phi_0)$

$v = A\omega\cos(\omega t + \Phi_0)$

$a = -A\omega^2\sin(\omega t + \Phi_0)$

$a_{max} = -A\omega^2$



Phase difference between x and v= $\frac{\pi}{2}$

Phase difference between v and a= $\frac{\pi}{2}$

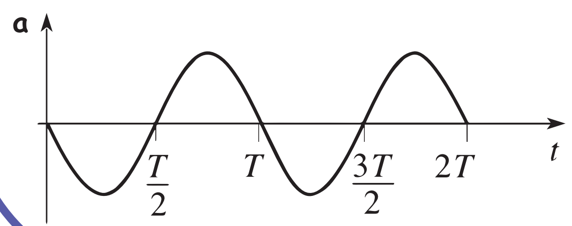
Phase difference between x and a= π

OSCILLATIONS 01

Graphical Representation of Acceleration

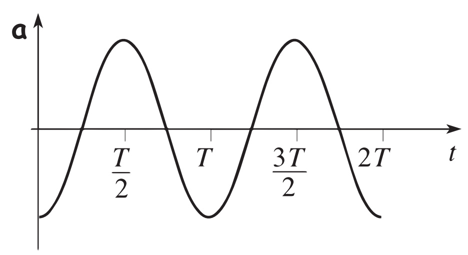
Start from mean position, $\Phi_0 = 0$

$a = -A\omega^2\sin(\omega t)$



Start from extreme position, $\Phi_0 = \frac{\pi}{2}$

$a = -A\omega^2\cos(\omega t)$



Calculation of Time period and amplitude

$v_{max} = A\omega$ $a_{max} = A\omega^2$

$\omega = \frac{a_{max}}{v_{max}}$ $A = \frac{v_{max}^2}{a_{max}}$

$\frac{2\pi}{T} = \frac{a_{max}}{v_{max}}$

$T = 2\pi\frac{v_{max}}{a_{max}}$

PHYSICS WALLAH

OSCILLATIONS 02

Energy of SHM

$$K.E=\frac{1}{2}mv^2=\frac{1}{2}m\omega^2(A^2-x^2)$$

KE=0 $\quad KE_{max}=\frac{1}{2}m\omega^2A^2 \quad$ KE=0

x=-A $\quad$ x=0 $\quad$ x=A

$$P.E=\frac{1}{2}m\omega^2x^2$$

$PE_{max}=\frac{1}{2}m\omega^2A^2 \quad$ PE=0 $\quad PE_{max}=\frac{1}{2}m\omega^2A^2$

x=-A $\quad$ x=0 $\quad$ x=A

Total mechanical energy, $E=\frac{1}{2}m\omega^2A^2$=Constant

1) $x=\frac{A}{\sqrt{2}}$

$K.E=P.E=\frac{E}{2}$

2) $x=\frac{A}{2}$

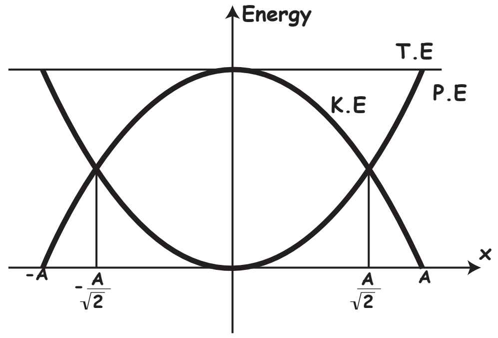
$K.E=\frac{3E}{4}$, $P.E=\frac{E}{4}$

3) x=0

K.E=E $\quad$ P.E=0

4) x=A

K.E=0 $\quad$ P.E=E

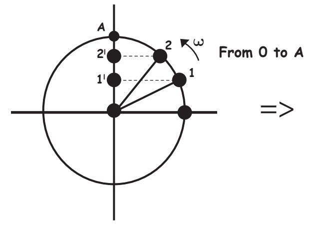


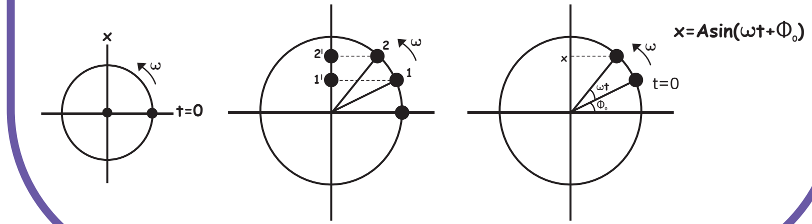
Total mechanical energy(E) $=\frac{1}{2}m\omega^2A^2$

Note: In SHM, if particle oscillates with frequency ω, then the K.E & P.E oscillate with 2ω

PROJECTION OF CIRCULAR MOTION

Projection/shadow of uniform circular motion on y axis is SHM





Two particles executing SHM meet at $x=\frac{\sqrt{3}A}{2}$

for particle A

$x=\frac{\sqrt{3}A}{2}$, t=0 => $\frac{\sqrt{3}A}{2}=A\sin\Phi_A$

$\Phi_A=60^\circ=\frac{\pi}{3}$

for particle B

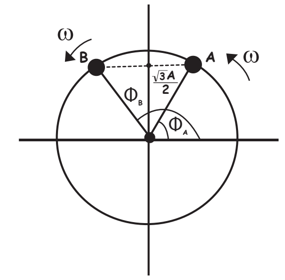
$x=\frac{\sqrt{3}A}{2}$, t=0 => $\frac{\sqrt{3}A}{2}=A\sin\Phi_B$

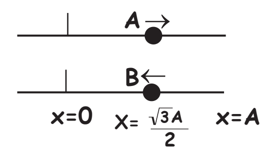
$\Phi_B=90^\circ+30^\circ$

$\frac{\pi}{2}+\frac{\pi}{6}=\frac{2\pi}{3}$

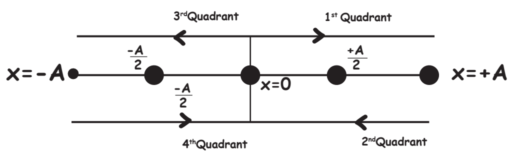
Phase difference between particles

$\Phi=\frac{2\pi}{3}-\frac{\pi}{3}=\frac{\pi}{3}=60^\circ$





INITIAL PHASE FROM POSITION & DIRECTION



Eg: Particle is at $x=\frac{A}{2}$ [t=0] and moves towards A

x=0 $\quad \frac{A}{2} \quad$ x=A $\quad$ 1st Quadrant

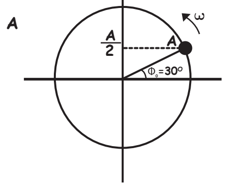
1st Quadrant

1) Particle is at $x=\frac{A}{2}$ [t=0] and moves towards A

$x=\frac{A}{2}$, t=0 ⇒ $\frac{A}{2}=A\sin\Phi_0$

$\Phi_0=30^\circ=\frac{\pi}{6}$

$x=A\sin(\omega t+\frac{\pi}{6})$

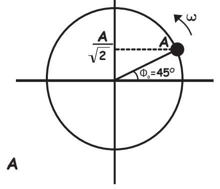


2) Particle is at $x=\frac{A}{\sqrt{2}}$ [t=0] and moves towards A

$x=\frac{A}{\sqrt{2}}$, t=0 ⇒ $\frac{A}{\sqrt{2}}=A\sin\Phi_0$

$\Phi_0=45^\circ=\frac{\pi}{4}$

$x=A\sin(\omega t+\frac{\pi}{4})$

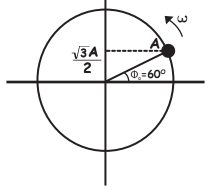


3) Particle is at $x=\frac{\sqrt{3}A}{2}$ [t=0] and moves towards A

$x=\frac{\sqrt{3}A}{2}$, t=0 ⇒ $\frac{\sqrt{3}A}{2}=A\sin\Phi_0$

$\Phi_0=60^\circ=\frac{\pi}{3}$

$x=A\sin(\omega t+\frac{\pi}{3})$



x=0 $\quad \frac{A}{2} \quad \frac{A}{\sqrt{2}} \quad \frac{\sqrt{3}A}{2} \quad$ x=A

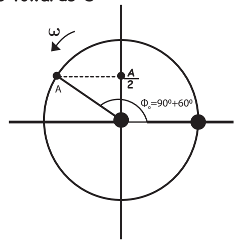
2nd Quadrant

1) Particle is at $x=\frac{A}{2}$ [t=0] and moves towards O

$x=\frac{A}{2}$, t=0 ⇒ $\frac{A}{2}=A\cos\Phi_0$

$\Phi_0=90^\circ+60^\circ=\frac{\pi}{2}+\frac{\pi}{3}=\frac{5\pi}{6}$

$x=A\cos(\omega t+\frac{5\pi}{6})$

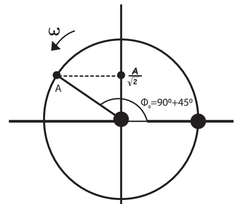


2) Particle is at $x=\frac{A}{\sqrt{2}}$ [t=0] and moves towards O

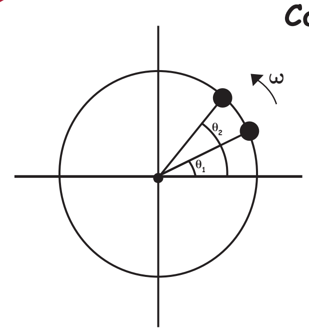
$x=\frac{A}{\sqrt{2}}$, t=0 ⇒ $\frac{A}{\sqrt{2}}=A\cos\Phi_0$

$\Phi_0=90^\circ+45^\circ=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

$x=A\cos(\omega t+\frac{3\pi}{4})$



Calculation of time



$$\Delta t=\frac{\Delta\theta}{\omega}=\left(\frac{\theta_1-\theta_2}{2\pi}\right)T$$

x=0 $\xrightarrow{T/4}$ x=A

x=0 $\xrightarrow{T/12}$ $\frac{A}{2}$ $\xrightarrow{T/6}$ x=A

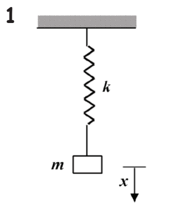
x=0 $\xrightarrow{T/8}$ $\frac{A}{\sqrt{2}}$ $\xrightarrow{T/8}$ x=A

x=0 $\xrightarrow{T/6}$ $\frac{\sqrt{3}A}{2}$ $\xrightarrow{T/12}$ x=A

OSCILLATIONS 03

TIME PERIOD OF S.H.M

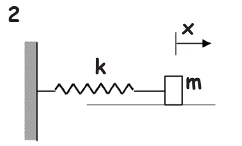
1



$F=-kx$

$T=2\pi\sqrt{\frac{m}{k}}$

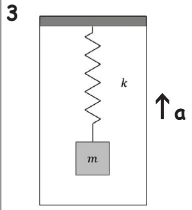
2



$F=-kx$

$T=2\pi\sqrt{\frac{m}{k}}$

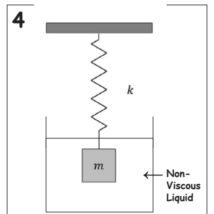
3



$F=-kx$

$T=2\pi\sqrt{\frac{m}{k}}$

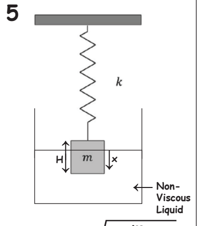
4



$F=-kx$

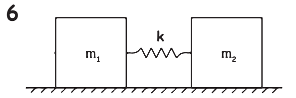
$T=2\pi\sqrt{\frac{m}{k}}$

5



$T=2\pi\sqrt{\frac{m}{k+\rho Hg}}$

6



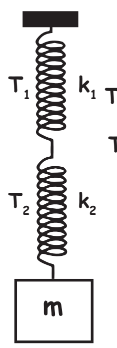
$F=-kx$

$T=2\pi\sqrt{\frac{m_{red}}{k}}$

$m_{red}=\frac{m_1 m_2}{m_1+m_2}$

COMBINATIONS OF SPRINGS

1) Series



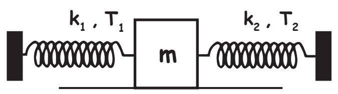
$k_{eq}=\frac{k_1 k_2}{k_1+k_2}$

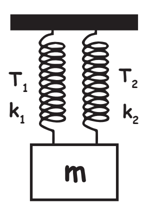
$T_{series}=2\pi\sqrt{\frac{m}{k_{eq}}}$

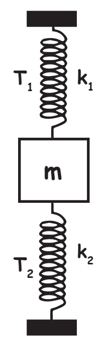
$T^2_{series}=T_1^2+T_2^2$

$T_{series}=\sqrt{T_1^2+T_2^2}$

2) Parallel





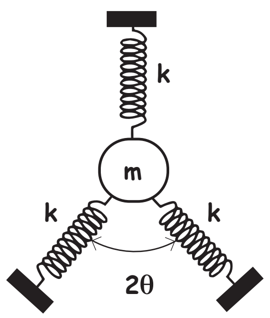


$k_{eq}=k_1+k_2$

$T_{parallel}=2\pi\sqrt{\frac{m}{k_{eq}}}$

$\frac{1}{T^2_{parallel}}=\frac{1}{T_1^2}+\frac{1}{T_2^2}$

SPECIAL CASE



$k_{eq}=k+2k\cos^2\theta$

$T=2\pi\sqrt{\frac{m}{k_{eq}}}$

eg:

if $\theta=120^\circ$ or $\theta=60^\circ$

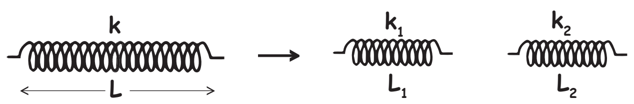
$k_{eq}=k+\frac{2k}{4}=\frac{3k}{2}$

$T=2\pi\sqrt{\frac{2m}{3k}}$

CUTTING OF SPRINGS

$k\propto\frac{1}{L}$

$k_1L_1=k_2L_2=kL$

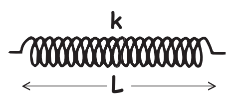


$L=L_1+L_2$

$k_1=\frac{kL}{L_1}$ $\quad k_2=\frac{kL}{L_2}$

$k_1=\frac{k(L_1+L_2)}{L_1}$ $\quad k_2=\frac{k(L_1+L_2)}{L_2}$

eg:



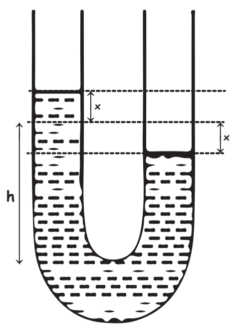
$L_1:L_2:L_3=1:2:3$

$k_1=\frac{k(L_1+L_2+L_3)}{L_1}=\frac{6k}{1}=6k$

$k_2=\frac{6k}{2}=3k$

$k_3=\frac{6k}{3}=2k$

OSCILLATION OF LIQUID COLUMN



$T=2\pi\sqrt{\frac{m}{2\rho gA}}$

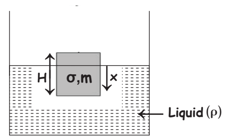
$T=2\pi\sqrt{\frac{h}{g}}$

m=mass of liquid

ρ=Density of liquid

A=Area of U-tube

OSCILLATION OF FLOATING BODY



$T=2\pi\sqrt{\frac{m}{\rho gA}}$

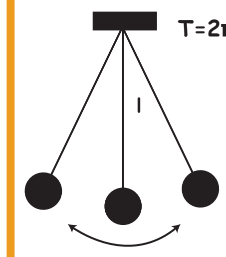
$T=2\pi\sqrt{\frac{\sigma H}{\rho g}}$

m=mass of body

σ=Density of Body

ρ=Density of liquid

SIMPLE PENDULUM



$T=2\pi\sqrt{\frac{l}{g}}$

- Second's pendulum

  T=2 second

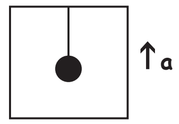
  l=1 meter

Concept of $g_{effective}$

$T=2\pi\sqrt{\frac{l}{g_{eff}}}$

Pendulum in lift

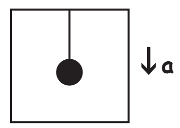
Case 1-Moving with constant upward acceleration 'a'



$g_{eff}=g+a$

$T=2\pi\sqrt{\frac{l}{g+a}}$

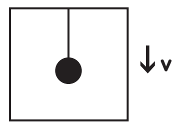
Case 2-Moving with constant downward acceleration 'a'



$g_{eff}=g-a$

$T=2\pi\sqrt{\frac{l}{g-a}}$

Case 3-Moving with constant velocity



$a=0$

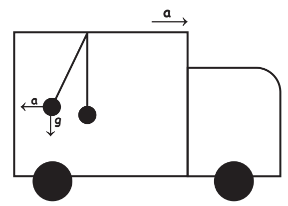
$T=2\pi\sqrt{\frac{l}{g}}$

Case 4-Free fall

$a=g$

$g_{eff}=g-a=g-g=0 \quad T\to\infty$

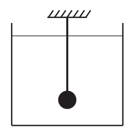
Pendulum in a truck moving with constant acceleration



$g_{eff}=\sqrt{g^2+a^2}$

$T=2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

Pendulum in Water



σ=Density of Bob

ρ=Density of liquid

$g_{eff}=g\left(1-\frac{\rho}{\sigma}\right)$

$T=2\pi\sqrt{\frac{l}{g\left(1-\frac{\rho}{\sigma}\right)}}$

DIFFERENTIAL EQUATION OF S.H.M

$ma=-kx$

$m\frac{d^2x}{dt^2}=-kx$

$\frac{d^2x}{dt^2}=\frac{-k}{m}x$

$\frac{d^2x}{dt^2}+\frac{k}{m}x=0$

$\frac{d^2x}{dt^2}+\omega^2x=0$

where, $\omega^2=\frac{k}{m}$ $\quad \omega=\sqrt{\frac{k}{m}}$

Solving,

$x=A\sin(\omega t+\Phi_0)$

A=Amplitude of SHM

Φ_0=Initial phase angle

Eg: $\left[\begin{matrix} m=4\text{ Kg} \\ K=320\text{ N/m}\end{matrix}\right]$

$4\frac{d^2x}{dt^2}+320x=0$

$\frac{d^2x}{dt^2}+80x=0$

$\omega^2=80 \quad \omega=\sqrt{80}$

$\frac{2\pi}{T}=\sqrt{80}$

$T=\frac{2\pi}{\sqrt{80}}$

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