ARJUNA

for

JEE

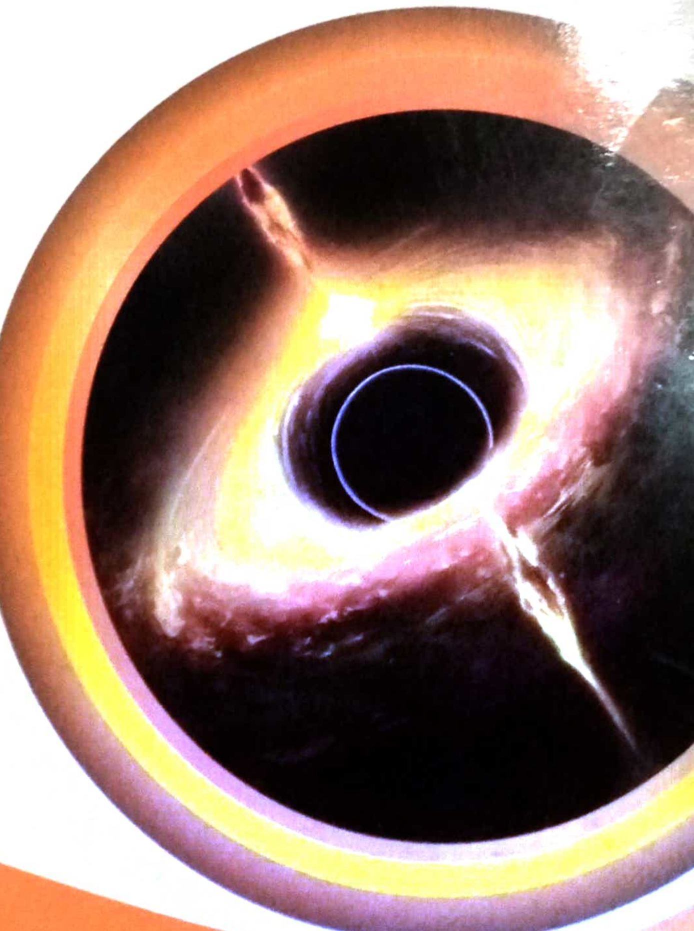
MAIN & ADVANCED

PHYSICS

FULL COURSE STUDY MATERIAL

Class XI

- Newton's Laws of Motion
- Friction
- Work, Power and Energy
- Circular Motion
- System of Particles and Centre of Mass



Module-2

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CHAPTER

6 Newton's Laws of Motion

FORCE

A force is a push or pull on a body which changes its state of motion. Body may speed up, retard or change its direction of velocity. Force is defined as an interaction between two bodies.

BASIC FORCES IN NATURE

(i) Gravitational force

(ii) Electromagnetic force

(iii) Strong nuclear force

(iv) Weak nuclear force

- Other well known forces like frictional force, elastic force, viscous force, spring force, intermolecular force are manifestations of electromagnetic forces.
- Electromagnetic force is about 10^{36} times stronger than gravitational force while weak nuclear forces are about 10^{25} times as strong as gravitational force.

  $F_N > F_E > F_W > F_G$

  $F_G : F_W : F_E : F_N :: 1 : 10^{25} : 10^{36} : 10^{38}$
- Gravitational and electromagnetic forces are central or conservative forces and obey inverse square law.
- The gravitational and the electromagnetic forces are long range forces having infinite range while strong nuclear and weak nuclear forces are very short range forces.
- The strong nuclear force is the strongest force while the gravitational force is the weakest force of nature.
- Nuclear force is a non-central force and varies inversely with some higher power of distance.
- Beta decay can be explained only on the basis of weak nuclear forces.

NEWTON'S FIRST LAW OF MOTION OR LAW OF INERTIA

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion "Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line".

Inertia: Inertia is the property of the body due to which the body oppose the change of state of rest or of uniform motion by itself. Inertia of a body is measured by mass of the body.

Inertia $\propto$ mass: Heavier the body, greater is the force required to change its state and hence greater is inertia. The reverse is also true. i.e. lighter body has less inertia.

TYPES OF INERTIA

Inertia of rest: It is the inability of a body to change by itself, its state of rest.

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.

Inertia of motion: It is the inability of a body to change by itself its state of uniform motion.

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.

Inertia of direction: It is the inability of a body to change by itself its direction of motion.

- When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud. Mudguard over the wheels stop this mud.

NEWTON'S SECOND LAW OF MOTION

Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad or \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \text{ if } m = \text{constant}$$

$$\vec{F} = \vec{v}\frac{dm}{dt}$$ (as in case of conveyor belt)

Special points:

1. This law gives magnitude, unit and dimension of force.
2. There are two types of unit of force:
   (*a*) Absolute unit
   (*b*) Gravitational or Practical unit
   In M.K.S. : Newton (N)
   In M.K.S. : kg-wt or kg-f
   In C.G.S. : Dyne (dyne)
   In C.G.S. : gm-wt or gm-f
   1 kg-wt = 9.8 N
   1 gm-wt = 980 dyne

NEWTON'S THIRD LAW OF MOTION

It states that to every action, there is an equal (in magnitude) and opposite (in direction) reaction.

If a body A exerts of force $\vec{F}$ on another body B, then B exerts a force $-\vec{F}$ on A, the two forces acting along the same line. The two forces $\vec{F}$ and $-\vec{F}$ connected by Newton's third law are called action-reaction pair. Any one may be called 'action' and the other 'reaction'.

"Action and reaction acts on different bodies hence they never cancel each other".

Notes:

(i) Action-Reaction pair acts on two different bodies.
(ii) Magnitude of force is same.
(iii) Direction of forces are in opposite direction.
(iv) For action-reaction pair there is no need of contact

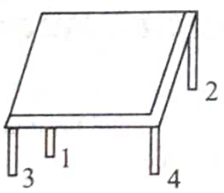
TYPES OF FORCES

Contact Forces

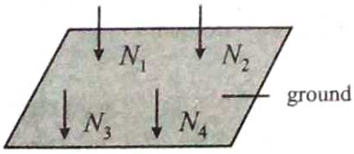
Whenever two bodies come in contact they exert forces on each other, that are called **contact forces.**

(*a*) **Normal Force (N):** It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force.

1. A table is placed on Earth as shown in figure.

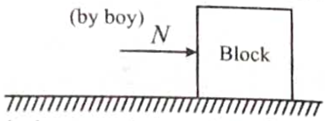


Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.

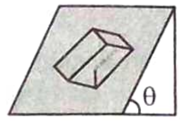


2. Now a boy pushes a block kept on a frictionless surface

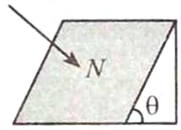
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



3. A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



Normal force exerted by block on the surface of inclined plane is shown in figure.



Force acts perpendicular to the surface

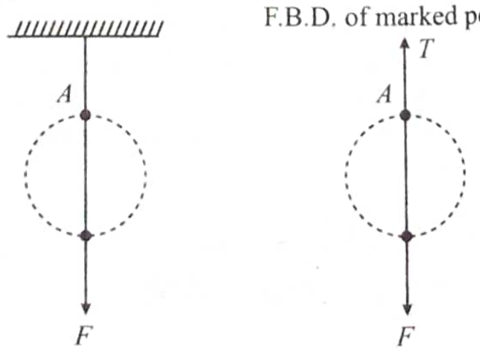
Notes:

1. Normal force acts in such a fashion that it tries to compress the body
2. Normal force is a self adjusting force, it comes in role when one surface presses the other.

(*b*) **Frictional Force (f)**: It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.

(*c*) **Tension:** Tension in a string is an electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force 'F' as shown in figure, for calculating the tension at point 'A' we draw F.B.D. of marked portion of the string; Here string is massless.

F.B.D. of marked portion



$\Rightarrow T = F$

String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature. The attraction force between two segments is equal and opposite due to newton's third law.

Conclusion:

(i) Tension always acts along the string and in such a direction that it tries to reduce the length of string

(ii) If the string is massless then the tension will be same along the string but if the string have some mass then the tension will continuously change along the string.

(iii) It is a self adjusting force.

(d) Spring Force: Every spring resists any attempt to change its length, the more you change its length the harder it resists. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is spring constant or stiffness constant (units N/m).

Non Contact Forces

These forces do not involve physical contact between two points and are also called **field forces** e.g. Gravitational force, electric force.

Weight: Weight of a body is the force with which earth attracts it. It is also defined as force of gravity or the gravitational force.

FREE BODY DIAGRAM

A free body diagram is a diagram showing the chosen body by itself, "free" of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces applied to the body by the various other bodies that interact with it.

Be careful to include all the forces acting on the body, but the equally careful not to include any force that the body exerts on any other body. In particular, the two forces in an action-reaction pair must never appear in the same free-body diagram because they never act on the same body.

[Forces that one part of body exerts on other part never included, since these cancel out by Newton's third law]

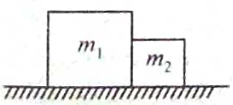
Note:

- The forces exerted on bodies of a given system by bodies situated outside are called external forces.
- The forces of interaction between bodies composing a system are called internal forces.
- A single isolated force is physically impossible.
- Whenever one force acts on a body it gives rise to another force called reaction.
- Total internal force in an isolated system is always zero.

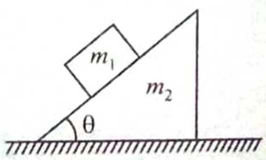
Train Your Brain

Example 1: Draw the *FBD* for the following individual parts of the systems

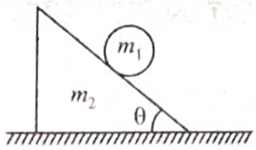
(*a*)



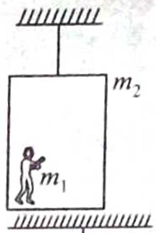
(*b*)



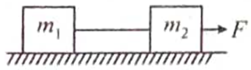
(*c*)



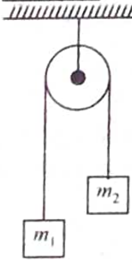
(*d*)



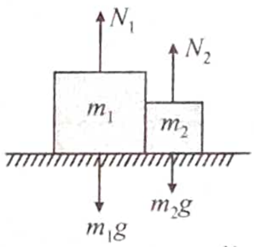
(*e*)



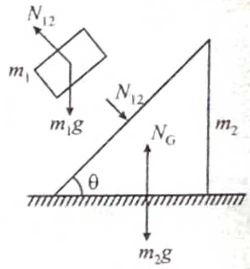
(*f*)



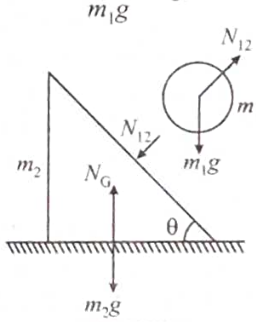
Sol. (*a*)



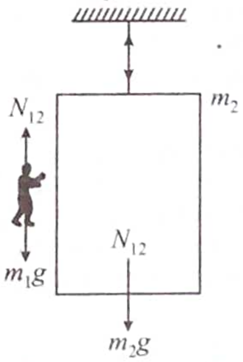
(*b*)



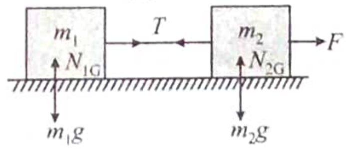
(*c*)



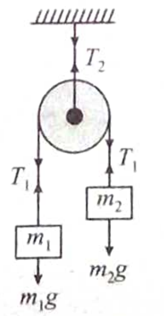
(*d*)



(*e*)



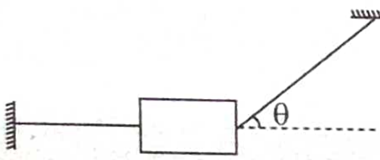
(*f*)



Concept Application

1. A cylinder of weight W is resting on a V-groove as shown in figure. Draw its free body diagram.

2. A block of mass m is attached with two strings as shown in figure. Draw the free body diagram of the block.



PROBLEM SOLVING STRATEGY

Newton's laws refer to a particle and relate the forces acting on the particle to its mass and to its acceleration. But before writing any equation from Newton's law, you should be careful about which particle you are considering. The laws are applicable to an extended body too which is nothing but collection of a large number of particles.

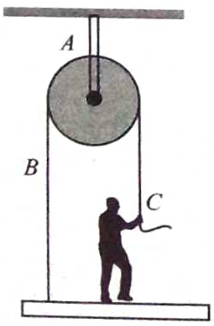
Follow the steps given below in writing the equations:

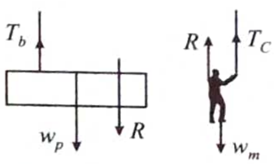
Step 1: Select the body: The first step is to decide the body on which the laws of motion are to be applied. The body may be a single particle, an extended body like a block, a combination of two blocks-one kept over another or connected by a string. The only condition is that **all the parts of the body or system must have the same acceleration.**

Step 2: Identify the forces: Once the system is decided, list down all the forces acting on the system due to all the objects in the environment such as inclined planes, strings, springs etc. However, any force applied by the system shouldn't be included in the list. You should also be clear about the nature and direction of these forces.

Step 3: Make a Free-body diagram (FBD): Make a separate diagram representing the body by a point and draw vectors representing the forces acting on the body with this point as the common origin.

This is called a free-body diagram of the body.





F.B.D. of platform F.B.D. of man

Look at the adjoining free-body diagrams for the platform and the man. Note that the force applied by the man on the rope hasn't been included in the FBD.

Once you get enough practice, you'd be able to identify and draw forces in the main diagram itself instead of making a separate one.

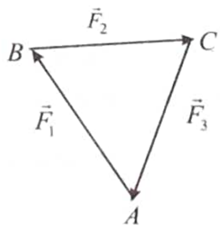
Step 4 : Select axes and Write equations: When the body is in equilibrium then choose the axis in such a manner that maximum number of force lie along the axis.

If the body is moving in a straight line with some acceleration then first find out the direction of real acceleration and choose the axis. One is along the real acceleration direction and other perpendicular to it.

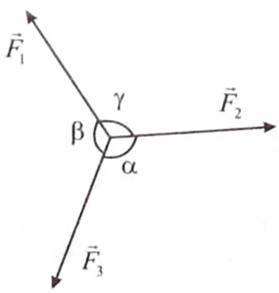
Write the equations according to the newton's second law ($F_{net} = ma$) in the corresponding axis.

EQUILIBRIUM OF CONCURRENT FORCES AND EQUILIBRIUM PROBLEMS

1. If all the forces working on a body are acting on the same point, then they are said to be concurrent.
2. A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.
3. The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.
4. Mathematically for equilibrium $\sum F_{net} = 0$ or $\sum F_x = 0$; $\sum F_y = 0; \sum F_z = 0$
5. Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.

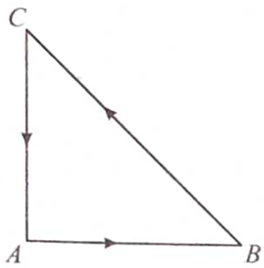


6. **Lami's Theorem:** For concurrent forces $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



Train Your Brain

Example 2: Three forces starts acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity



(*a*) $\vec{v}$ remaining unchanged

(*b*) Less than $\vec{v}$

(*c*) Greater than $\vec{v}$

(*d*) $\vec{v}$ in the direction of the largest force BC

Sol. (*a*) Given three forces are in equilibrium i.e. net force will be zero. It means the particle will move with same velocity.

Example 3: Two forces are such that the sum of their magnitudes is 18 N and their resultant is perpendicular to the smaller force and magnitude of resultant is 12. Then the magnitudes of the forces are

(*a*) 12 N, 6 N (*b*) 13 N, 5 N

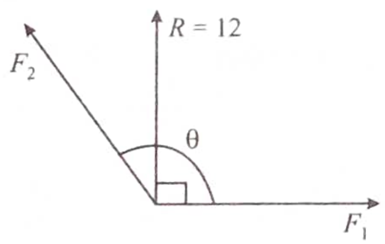
(*c*) 10 N, 8 N (*d*) 16 N, 2 N

Sol. (*b*) Let two forces are F_1 and $F_2\left(F_1 < F_2\right)$.

According to problem: $F_1 + F_2 = 18$ ...(i)

Angle between F_1 and resultant (R) is 90°

$$\therefore\ \tan 90 = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta} = \infty$$



$$F_1 + F_2\cos\theta = 0$$

$$\Rightarrow \cos\theta = -\frac{F_1}{F_2} \quad \text{..(ii)}$$

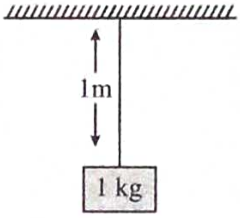
and $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$$144 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \quad \text{...(iii)}$$

By solving (i), (ii) and (iii)

we get $F_1 = 5\text{N}$ and $F_2 = 13\text{N}$

Example 4: A block of mass 1 kg is suspended by a string of mass 1 kg, length 1 m as shown in figure. ($g = 10\ \text{m/s}^2$) Calculate:

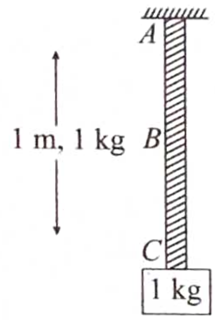


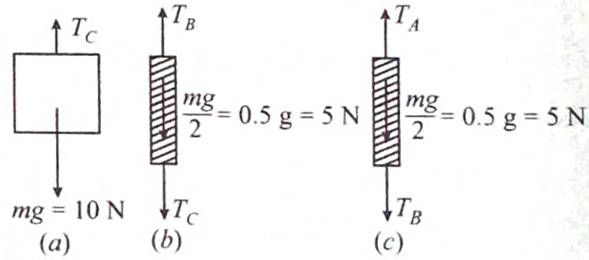
(*a*) The tension in string at its lowest point.

(*b*) The tension in string at its mid-point.

(*c*) Force exerted by support on string.

Sol.





$T_C - 10 = 0$ [Equilibrium of block]

$\Rightarrow T_C = 10\ \text{N}$

$T_B - T_C - 5 = 0$ [Equilibrium of 2]

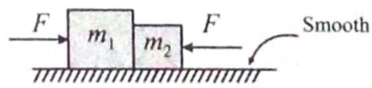
$\Rightarrow T_B - 10 - 5 = 0$

$\Rightarrow T_B = 15\ \text{N}$

$T_A - T_B - 5 = 0$ [Equilibrium of 1]

$\Rightarrow T_A = 20\ \text{N}$

Example 5: Two blocks of masses m_1 and m_2 are placed on ground as shown in figure. Two forces of magnitude F act on m_1 and m_2 in opposite directions.



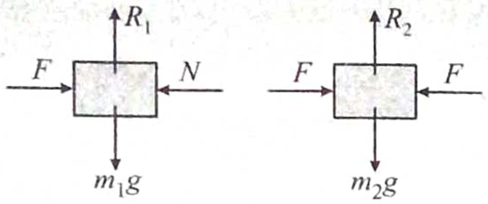
(i) Draw F.B.D. of masses m_1 and m_2.

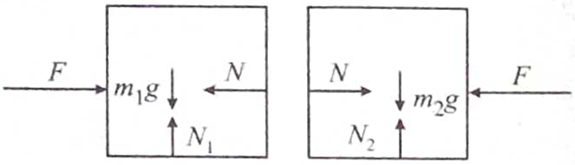
(ii) Calculate the contact force between m_1 and m_2.

(iii) What will be the value of normal force between m_1 and m_2.

(iv) Calculate force exerted by ground surface on mass m_1 and m_2

Sol. (i)





It is obvious that both blocks will have same acceleration. If we take both block as one system then.

$F - F = (m_1 + m_2)\,a$ [Newton's second law in horizontal direction]

$\Rightarrow a = 0$

Now take m_1 as a system

$F - N = m_1\,a$ [Newton's second law in horizontal direction]

$\Rightarrow F - N = 0$

$\Rightarrow F = N$

$m_1g - N_1 = 0$ [Equilibrium in vertical direction]

Now take m_2 as system

$N - F = m_2\,a$

$\Rightarrow N - F = 0$

$N = F$

$m_2g - N_2 = 0$ [Equilibrium in vertical direction]

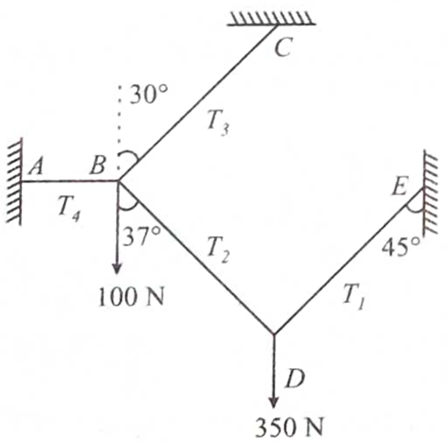
$\Rightarrow N_2 = m_2g$

(ii) $N = F$

(iii) F

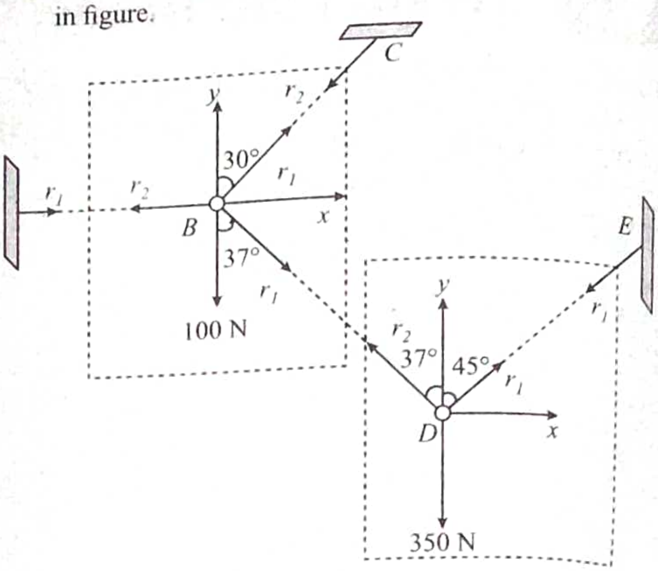
(iv) m_1g, m_2g.

Example 6: Two particles of masses 10 kg and 35 kg are connected with four strings at points B and D as shown in figure.



Determine the tensions in various segments of the string.

Sol. The free-body diagram of the whole system is shown in figure.



Analysing the equilibrium of point D:

$\sum F_x = 0$ or $T_1 \sin 45° - T_2 \sin 37° = 0$ ...(i)

and $\sum F_y = 0$ or $T_1 \cos 45° + T_2 \cos 37° = 350$ ...(ii)

From (i), we have

$$T_2 = \frac{T_1 \sin 45°}{\sin 37°}$$

Now from (ii),

$$T_1 \cos 45° + \frac{T_1 \sin 45°}{\sin 37°} \times \cos 37° = 350$$

or $\dfrac{T_1}{\sqrt{2}} + \dfrac{T_1}{\sqrt{2}} \times \dfrac{4}{3} = 350$ or $\dfrac{T_1}{\sqrt{2}}\left[1 + \dfrac{4}{3}\right] = 350$

$\Rightarrow T_1 = 150\sqrt{2}$ N

and $T_2 = \dfrac{T_1 \sin 45°}{\sin 37°} = \dfrac{150\sqrt{2} \times \left(\dfrac{1}{\sqrt{2}}\right)}{3/5} = 250$ N

Analysing the equilibrium of point B

$\sum F_x = 0$ or $T_2 \sin 37° + T_3 \sin 30° - T_4 = 0$ ...(iii)

and $\sum F_y = 0$

or $T_3 \cos 30° - T_2 \cos 37° - 100 = 0$ ...(iv)

From (iv),

$$T_3 \cos 30° - 250 \times \frac{4}{5} - 100 = 0$$

$\Rightarrow T_3 = 200\sqrt{3}$ N

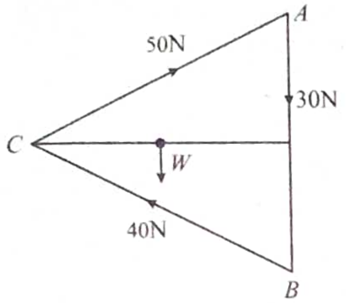
From (iii),

$T_4 = 150 + 100\sqrt{3}$

$= 50\,(3 + 2\sqrt{3})$ N

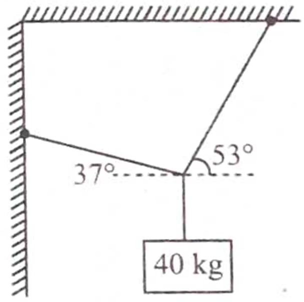
Concept Application

3. Forces of 30N, 40N, and 50N act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ of an equilateral triangle ABC. The triangle is of mass 0.5 kg and kept in a vertical plane as shown in figure, with the side AB vertical. The net vertical force acting on the triangle will be ($g = 10$ m/s^2)

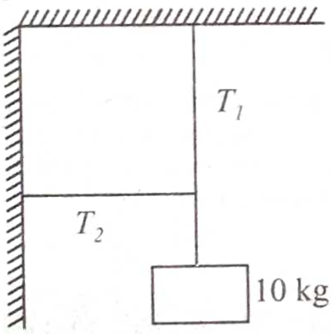


(a) 125 N upward (b) 5 N downwards
(c) 10 N upwards (d) 10 N downwards

4. The object in weighs 40 kg and hangs at rest. Find the tensions (in N) in the three cords that hold it.



5. A block of mass $m = 10$ kg is suspended with the help of three strings as shown in Fig. Find the tensions T_1 and T_2 (in N)



MOTION OF A BLOCK ON A HORIZONTAL SMOOTH SURFACE

When a body is constraint to move in a straight line. Then it is convenient to choose one axis along the direction of motion (say x-axis) and other perpendicular to it (say y-axis)

$\Sigma F_x = ma, \Sigma F_y = 0$

(a) When subjected to a horizontal pull.

(b) When subjected to a pull acting at an angle θ to the horizontal.

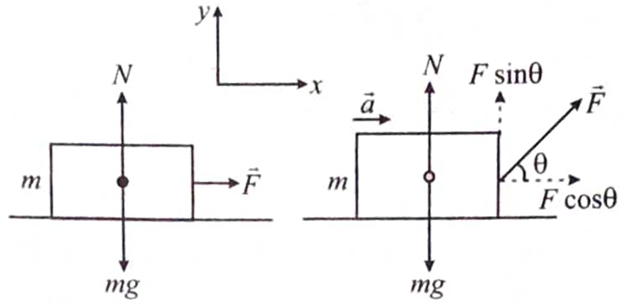


figure-I

$N - mg = 0$ (y-axis)

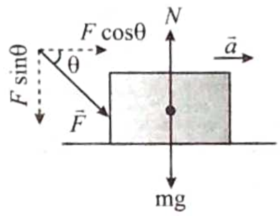
and $a = \dfrac{F}{m}$ (x-axis)

figure-II

$N - mg + F\sin\theta = 0$(y-axis)

$a = \dfrac{F\cos\theta}{m}$ (x-axis)

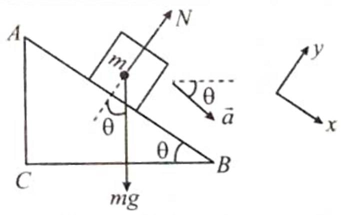
(c) When subjected to a push acting at an angle θ to the horizontal.



$N - mg - F\sin\theta = 0$ (for y-axis) and $ma = F\cos\theta$ (for y-axis)

$a = \dfrac{F\cos\theta}{m}$

MOTION OF A BODY ON A SMOOTH INCLINED PLANE

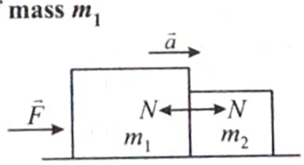


$N - mg\cos\theta = 0$ (for y-axis)

and $ma = mg\sin\theta$ (for x-axis), $a = g\sin\theta$

MOTION OF BODIES IN CONTACT

(a) **Two bodies are kept in contact and force is applied on the body of mass m_1**

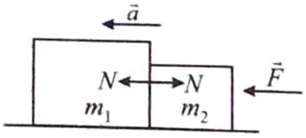


As both mass have same acceleration $a = \dfrac{F}{m_1 + m_2}$

for block 1 applying Newton's II law in horizontal direction.

$m_1 a = F - N \Rightarrow N = \dfrac{m_2 F}{m_1 + m_2}$

(*b*) **Two bodies are kept in contact and force is applied on the body of mass m_2**



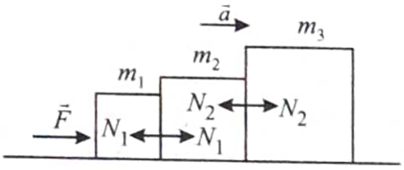
$$a = \frac{F}{m_1 + m_2}$$

for block 1 applying Newton's II law in horizontal direction.

$m_1 a = N$

$$\Rightarrow N = \frac{m_1 F}{m_1 + m_2}$$

(*c*) **Three bodies are kept in contact and force is applied on the body of mass m_1**



Since all three masses have same acceleration

$$a = \frac{F}{m_1 + m_2 + m_3} \quad \text{...(i)}$$

$m_1 a = F - N_1$ (for m_1) ...(ii)

$m_2 a = N_1 - N_2$ (for m_2) ...(iii)

$m_3 a = N_2$ (for m_3) ...(iv)

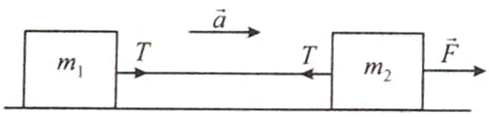
From equation (i), (ii), (iii) and (iv)

$$N_1 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

$$N_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

MOTION OF CONNECTED BODIES

(*a*) **Two bodies are connected by a string and placed on a smooth horizontal surface**



$m_1 a = T$ (for m_1) ...(i)

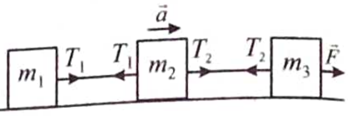
$m_2 a = F - T$ (for m_2) ...(ii)

From (i) and (ii)

$$a = \frac{F}{m_1 + m_2}$$

$$T = \frac{m_1 F}{m_1 + m_2}$$

(*b*) **When three bodies are connected through strings as shown in figure and placed on a smooth horizontal surface.**



$m_1 a = T_1$ (for m_1) ...(i)

$m_2 a = T_2 - T_1$ (for m_2) ...(ii)

$m_3 a = F - T_2$ (for m_3) ...(iii)

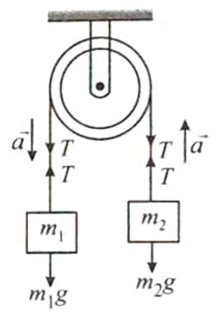
From (i), (ii) and (iii)

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

$$T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

$$T_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$$

(*c*) **When two bodies of mass m_1 and m_2 are attached at the ends of a string passing over a pulley** (neglecting the mass of pulley).



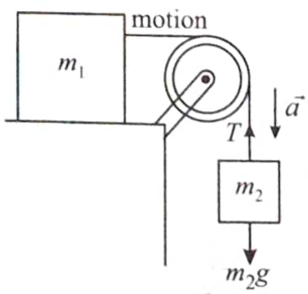
$m_1 a = m_1 g - T$ (for m_1) ...(i)

$m_2 a = T - m_2 g$ (for m_2) ...(ii)

From (i) and (ii)

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}, \; T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$$

(*d*) **When two bodies of masses m_1 and m_2 are attached at the ends of a string passing over a pulley in such a way that mass m_1 rests on a smooth horizontal table and mass m_2 is hanging vertically**



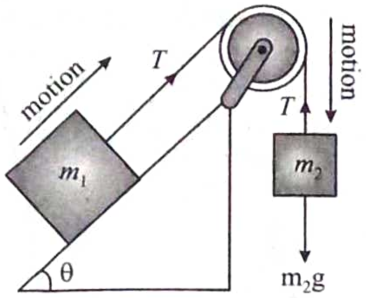
$m_1 a = T$ (for m_1) ...(i)

$m_2 a = m_2 g - T$ (for m_2) ...(ii)

From (i) and (ii)

$$a = \frac{m_2 g}{(m_1 + m_2)}, \; T = \frac{m_1 m_2 g}{(m_1 + m_2)}$$

(*e*) **If in the above case, mass m_1 is placed on a smooth inclined plane making an angle θ with horizontal**



$m_1a = T - m_1g\sin\theta$ (for m_1) ...(i)

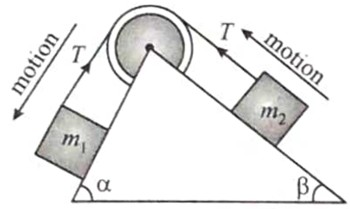
$m_2a = m_2g - T$ (for m_2) ...(ii)

From (i) and (ii)

$$a = \frac{(m_2 - m_1\sin\theta)g}{m_1 + m_2},\ T = \frac{m_1m_2g(1+\sin\theta)}{m_1+m_2}$$

If the system remains in equilibrium, then $m_1g\sin\theta = m_2g$

(*f*) **If case (*d*), masses m_1 and m_2 are placed on inclined planes making angles α and β with the horizontal respectively.**



$m_1a = m_1g\sin\alpha - T$ (for m_1) ...(i)

$m_2a = T - m_2g\sin\beta$ (for m_2) ...(ii)

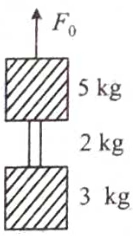
From (i) and (ii)

$\Rightarrow$ If $m_1\sin\alpha > m_2\sin\beta$ then $a = \dfrac{g(m_1\sin\alpha - m_2\sin\beta)}{(m_1+m_2)}$,

$$T = \frac{m_1m_2}{m_1+m_2}(\sin\alpha + \sin\beta)g$$

Train Your Brain

Example 7: A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s² by an external force F_0. (g = 10 m/s²)



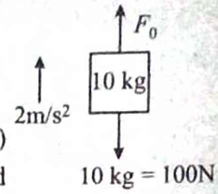
(*a*) What is F_0?

(*b*) What is the net force on the rope?

(*c*) What is the tension at middle point of the rope?

Sol. For calculating the value of F_0, consider two blocks with the rope as a system.

F.B.D. of whole system



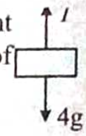
(*a*) $F_0 - 100 = 10 \times 2$

$F = 120$ N ...(i)

(*b*) According to Newton's second law, net force on rope.

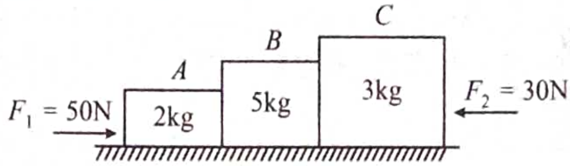
$F = ma = (2)(2) = 4$ N ...(ii)

(*c*) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.

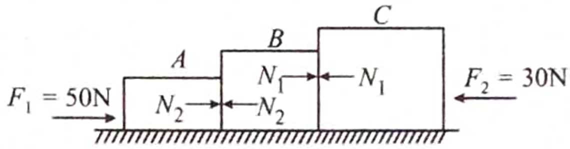


$T - 4g = 4 \times (2) = 48$N

Example 8: Find the contact force between the block and acceleration of the blocks as shown in figure.

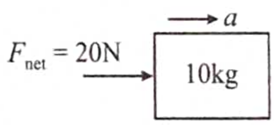


Sol. Considering all the three block as a system to find the common acceleration

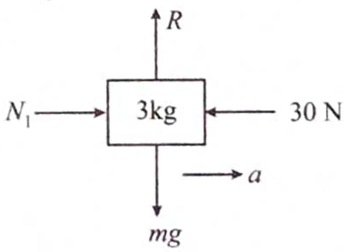


$F_{net} = 50 - 30 = 20$ N

$$a = \frac{20}{10} = 2\text{m/s}^2$$

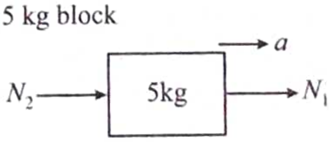


To find the contact force between B and C we draw F.B.D. of 3 kg block.



$\Sigma F_{net} = ma \Rightarrow N_1 - 30 = 3(2) \Rightarrow N_1 = 36$ N

To find contact force between A and B we draw F.B.D. of 5 kg block

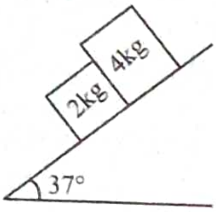


$\Rightarrow N_2 - N_1 = 5a$

$\Rightarrow N_2 = 5 \times 2 + 36$

$\Rightarrow N_2 = 46$ N

Example 9: Find out the contact force between the 2kg and 4kg block as shown in figure.



Sol. On an incline plane acceleration of the block is independent of mass. So both the blocks will move with the same acceleration (gsin 37°) so the contact force between them is zero.

Concept Application

In Q.6 to Q.11, rigid blocks are kept in contact on a frictionless surface and acted upon by force (s). In each case, find

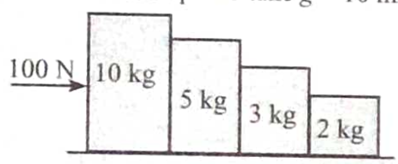
(*a*) Acceleration of each block (in m/s^2)

(*b*) Contact forces between the blocks (in N) and

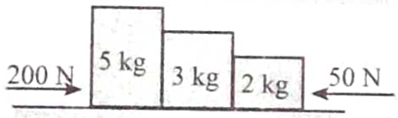
(*c*) Net force on each block (in N).

Masses of blocks are indicated in diagram. Wherever required take $g = 10\ m/s^2$.

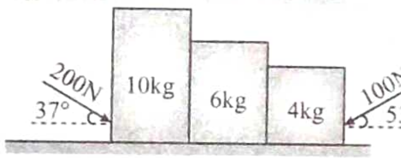
6.



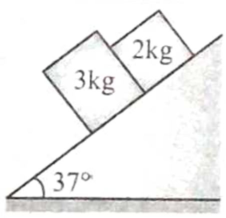
7.



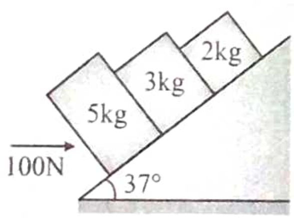
8.



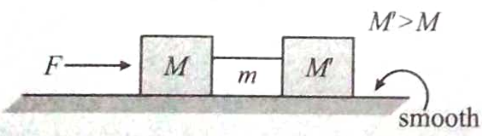
9.



10.



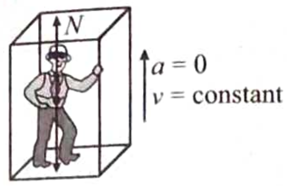
11. A constant force F is applied in horizontal direction as shown. Contact force between M and m is N and between m and M is N then find the relation between N' and N



APPARENT WEIGHT OF A BODY IN A LIFT

The weight recorded in the weighing machine is equal to the force with which your feet press the weighing machine. It is therefore equal to normal reaction.

- **The lift is at rest or moving with uniform velocity**



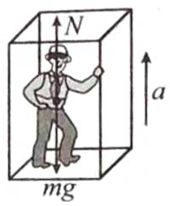
$a = 0$

$mg - N = 0$ or $N = mg$ or $W_{app.} = W_0$

$W_{app.} = N =$ reaction of supporting surface

and $W_0 = mg =$ true weight.

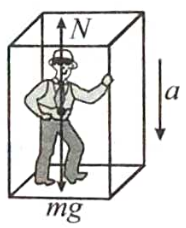
- **When the lift moves upwards with an acceleration a:**



$N - mg = ma$ or $N = m(g + a) = mg\left(1+\frac{a}{g}\right)$

$\therefore W_{app.} = W_0\left(1+\frac{a}{g}\right)$

- **When the lift moves downwards with an acceleration a:**

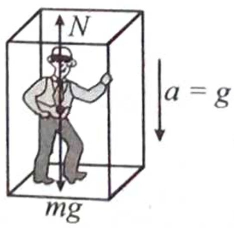


$mg - N = ma$ or $N = m(g - a) = mg\left(1-\frac{a}{g}\right)$

$\therefore W_{app.} = W_0\left(1-\frac{a}{g}\right)$

Here, if $a > g$, $W_{app.}$ will be negative. Negative apparent weight will mean that the body is pressed against the roof of the lift instead of floor.

- **When the lift falls freely, i.e., $a = g$:**



$N = m(g - g) = 0$ or $W_{app.} = 0$

CONSTRAINED MOTION

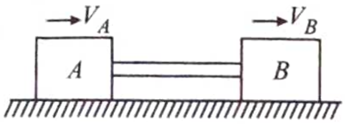
String Constraint

When two objects are connected through a string and if the string have the following properties:

(a) The length of the string remains constant i.e. inextensible string.

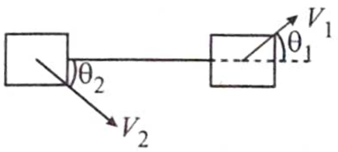
(b) String always remains tight, does not slacks.

(i) If the velocities of the block is along the string as shown, then their velocities must be same.



$V_A = V_B = V$

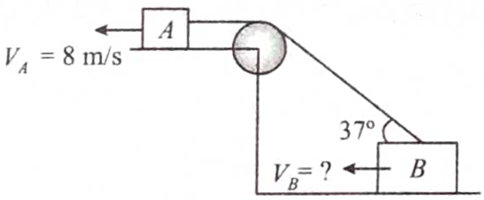
(ii) If the velocities of the blocks are not along the string as shown below, then components of velocities of the block along the string must be same as string is inextensible



$V_1\cos\theta_1 = V_2\cos\theta_2$

Train Your Brain

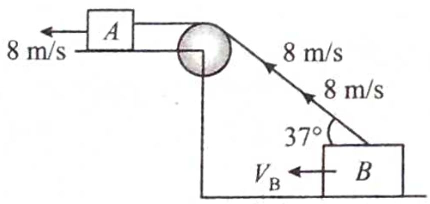
Example 10:



Sol. Block A is moving with velocity 8 ms^{-1}.

∴ velocity of every point on the string must be 8 m/s along the string.

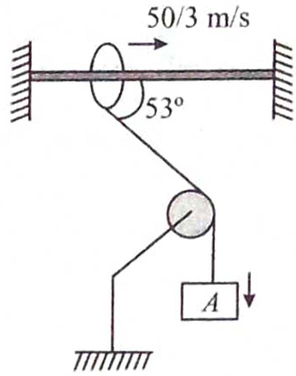
The velocity of B is v_B. Then the string will not break only when the component of v_B along string is 8 m/s.



$\Rightarrow v_B \cos 37° = 8$

$\Rightarrow v_B = \dfrac{8}{\cos 37°} = 10$ m/sec

Example 11: What is the velocity of block A in the figure as shown below.



Sol. The component of velocity of ring along string

= velocity of A

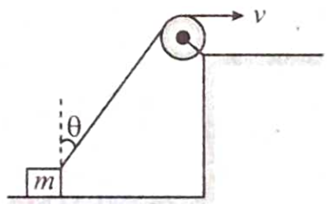
$= \dfrac{50}{3}\cos 53°$

$= v_A$

$\Rightarrow v_A = 10$ m/s

Concept Application

12. A block of dragged on smooth plane with the help of a rope which moves with velocity v. The horizontal velocity of the block is



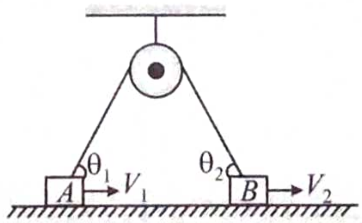
(a) v

(b) $\dfrac{v}{\sin\theta}$

(c) $v\sin\theta$

(d) $\dfrac{v}{\cos\theta}$

13. In the figure shown, blocks A and B move with velocities v_1 and v_2 along horizontal direction. Find the ratio of $\dfrac{v_1}{v_2}$.



Virtual Work Method

For more complex constrain relation, virtual work method can be used.

An ideal (massless and inextensible) string can neither possess kinetic energy nor store potential energy. So it can never gain or supply energy. Total work done by the string on block attached to it in displacing them is always zero or equivalently total power input by it at any instant is zero.

Now, $P = \vec{F}\cdot\vec{v}$

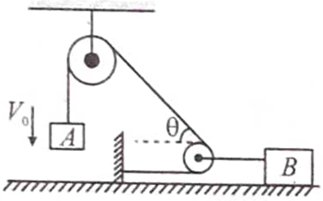
$$P_{\text{Total}} = \sum \vec{F}_i \cdot \vec{v}_i$$

$$= \sum \vec{T}_i \cdot \vec{V}_i = 0$$

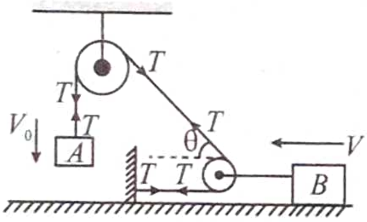
This gives us relation between velocities of bodies constrained to move when string is attached between them.

Train Your Brain

Example 12: In the figure, find the velocity of block B, if velocity of A is V_0 in downward direction?



Sol.

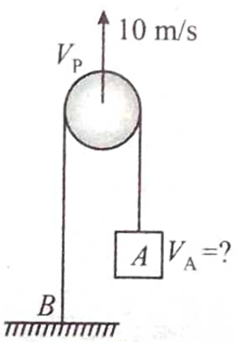


Power input = $TV_0\cos 180° + TV\cos\theta + TV\cos(0)$

$-TV_0 + TV\cos\theta + TV = 0$

$$\Rightarrow V = \frac{V_0}{1+\cos\theta}$$

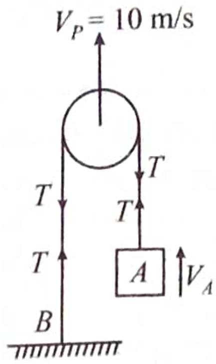
Example 13: Find $V_A = ?$



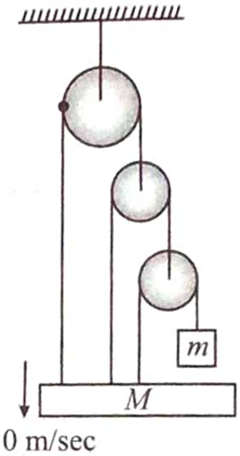
Sol. $P_{\text{total}} = P_{\text{pulley}} + P_{\text{block}}$

$= -2T \times 10 + TV_A = 0$

$V_A = 20$ m/s.



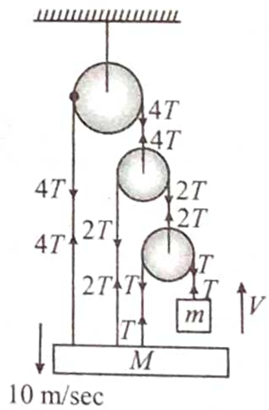
Example 14: Find the velocity of m?



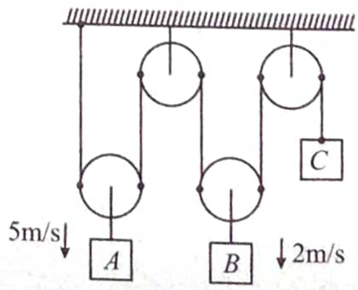
Sol. $P_{\text{input}} = P_M + P_m$

$(-7T \times 10) + TV = 0$

$\Rightarrow V = 70$ m/s



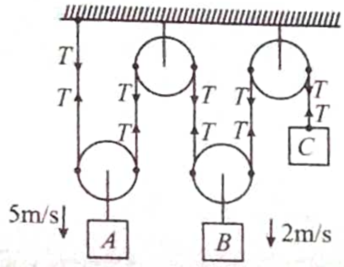
Example 15: Find velocity of block C?



Sol. $P_{input} = P_A + P_B + P_C$

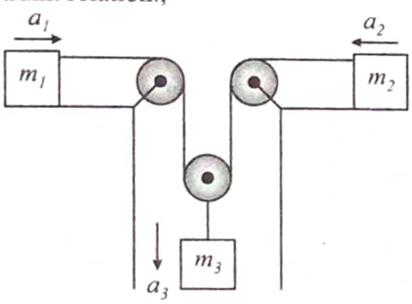
$(-2T \times 5) - (2T \times 2) + TV_C = 0$

$\Rightarrow V_C = 10 + 4 = 14$ m/s.

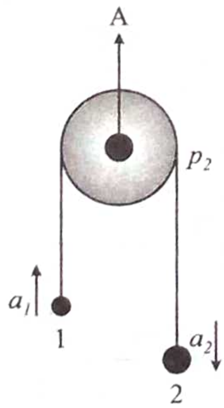


Concept Application

14. In the arrangement of three blocks as shown in fig, the string is inextensible. If the directions of accelerations are as shown in the figure, then determine the constraint relation.,



15. Two masses are connected by a string which passes over a pulley accelerating upward at a rate A as shown. If a_1 and a_2 be the acceleration of bodies 1 and 2 respectively then



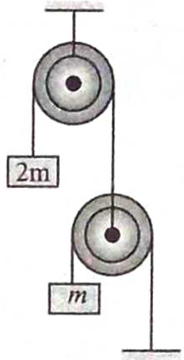
(a) $A = a_1 - a_2$

(b) $A = a_1 + a_2$

(c) $A = \frac{a_1 - a_2}{2}$

(d) $A = \frac{a_1 + a_2}{2}$

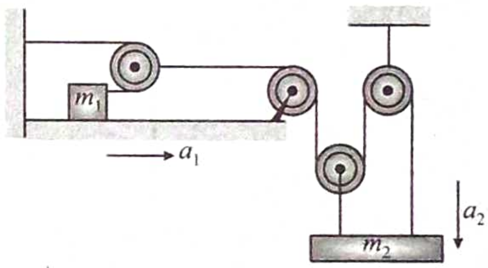
16. In the system shown in the figure, the friction and mass of rope is negligible then acceleration of the block of mass 2m is:



(a) $\frac{g}{5}$ (b) $\frac{2g}{5}$

(c) 0 (d) $\frac{5g}{2}$

17. Two blocks are arranged as shown in the figure. The relation between acceleration a_1 and a_2 is:



(a) $a_1 = a_2$ (b) $a_1 = 6a_2$

(c) $a_1 = 3a_2$ (d) $a_1 = 4a_2$

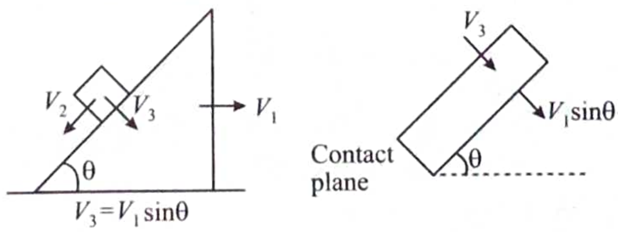
WEDGE CONSTRAINT

Conditions

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.

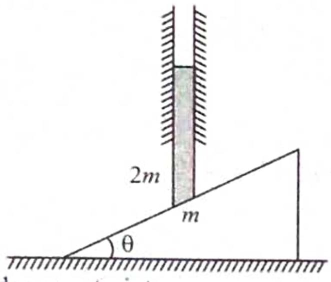


In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

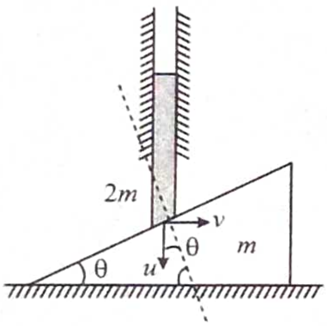
Train Your Brain

Example 16: A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



Sol. Using wedge constraint.

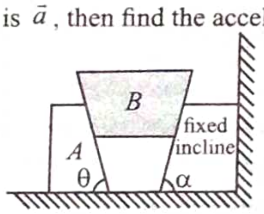
Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.



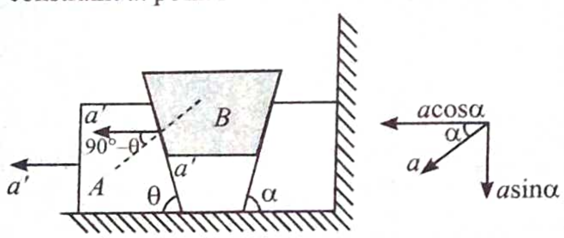
perpendicular to contact of two blocks

$$u\cos\theta = v\sin\theta \Rightarrow \frac{u}{v} = \tan\theta \Rightarrow u = v\tan\theta$$

Example 17: In the arrangement shown in figure, if the acceleration of B is $\vec{a}$, then find the acceleration of A.



Sol. Let a' be the acceleration of block A using wedge constraint at point P



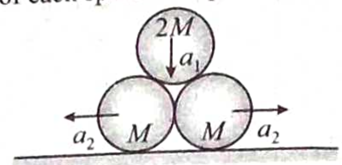
$$a\sin\alpha\cos\theta + a\cos\alpha\sin\theta = a'\cos(90° - \theta)$$

$$a\sin(\alpha + \theta) = a'\sin\theta$$

$$a' = \frac{a\sin(\theta+\alpha)}{\sin\theta}$$

Concept Application

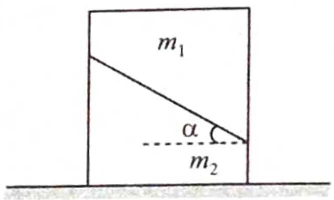
18. The relation between acceleration a_1 and a_2, if the radius of each sphere is equal to R.



(a) $a_2 = a_1\sqrt{3}$ (b) $a_1 = a_2\sqrt{3}$

(c) $a_1 = 2a_2$ (d) $a_2 = 2a_1$

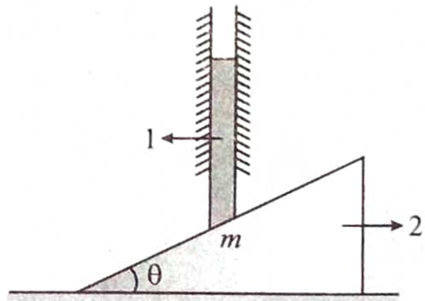
19. A block is kept on a smooth horizontal floor. It is cut along an inclined plane making an angle a with the horizontal in two parts of masses m_1 and m_2. Neglecting the friction between the surfaces, the ratio of magnitude of horizontal acceleration of m_1 and m_2 is



(a) $\frac{m_2}{m_1}\sin\alpha$ (b) $\frac{m_1}{m_2}\sin\alpha$

(c) $\frac{m_2}{m_1}$ (d) $\frac{m_1}{m_2}$

20. Acceleration of (1) w.r.t. ground is "a" downwards. Acceleration of (2) w.r.t. (1) will be :



(a) $a\sec\theta$ (b) $a\,\mathrm{cosec}\,\theta$

(c) $a\tan\theta$ (d) $a\cot\theta$

SPRING FORCE

Many spring follow Hooke's law for small extension and compression.

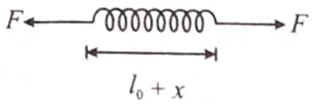
That is, the extension or compression – the increase or decrease in length from the relaxed length – is proportional to the force applied to the ends of the spring.

Hooke's Law for an ideal spring $F = k\Delta L$

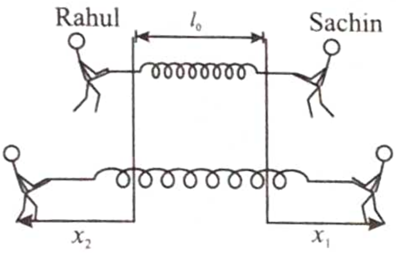
In equation F is the magnitude of the force exerted on each end of the spring and ΔL is the modulus of change in length of the spring from its relaxed length. The constant k is called the spring constant for a particular spring. The *SI* units of a spring constant are N/m.

When we say an ideal spring, we mean a spring that obeys Hooke's law and is also massless. Since we have assumed spring to be massless we know forces acting on both ends have to be equal and opposite, to have net force on spring to be zero.

Note: If we look at *FBD* of the spring we will note that force on spring must act from both ends.



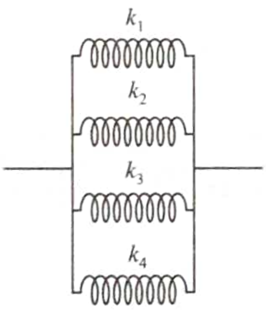
Lets say Rahul and Sachin are pulling a spring from two ends as shown. Rahul moves x_2 and Sachin moves x_1.



The force acting on Rahul and Sachin is $k(x_1 + x_2)$, Not kx_2 on Rahul and kx_1 on Sachin. Force due to spring is kx where x is defined as $|l - l_0|$, where l is present length and l_0 is natural length.

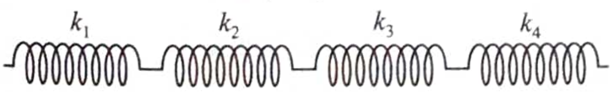
EQUIVALENT SPRING CONSTANT

(*a*) When springs are connected in parallel then we can replace them by single spring of spring constant k_e where $k_e = k_1 + k_2$.



For more than two spring $k = k_1 + k_2 + k_3 + \ldots\ldots\ldots\ldots$

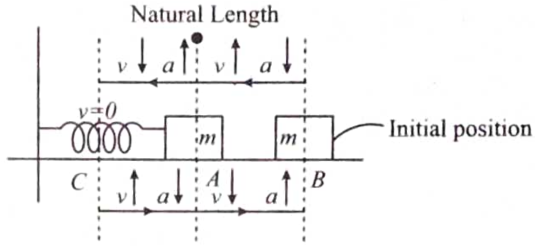
(*b*) When springs are connected in series then we can replaced them by single spring of spring constant k_e where $1/k_e = 1/k_1 + 1/k_2$. As spring constants are not equal so extensions will not be equal, but total extension y can be written as sum of two extensions $y = y_1 + y_2 + y_3 + \ldots$. for more than two springs.



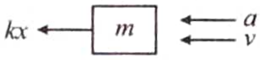
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots\ldots\ldots$$

ANALYSIS OF MOTION OF BLOCK

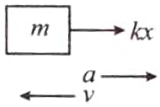
Suppose block is released from rest from position B in it the figure shown



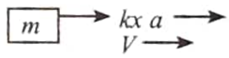
(i) From B to A speed of block increase and acceleration decreases. (due to decrease in spring force kx)



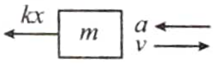
(ii) Due to inertia block crosses natural length at A. From A to C speed of the block decreases and acceleration increases. (due to increase in springforce kx)



(iii) At C the block stops momentarily at this instant and since the spring is compressed spring force is towards right and the block starts to move towards right. From C to A speed of block increases and acceleration decreases.(due to decrease in spring force kx)



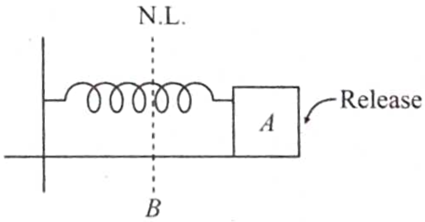
(iv) Again block crosses point A due to inertia then from A to B speed decreases and acceleration increases.



In this way block does SHM (to be explained later) if no resistive force is acting on the block.

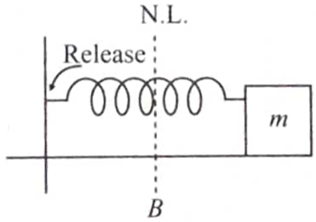
Note:

1.



When the block A is released then it take some finite time to reach at B. i.e., spring force doesn't change instantaneously

2.

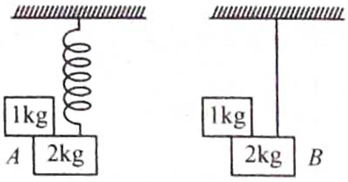


When point A of the spring is released in the above situation then the spring forces changes instantaneously and becomes zero because one end of the spring is free.

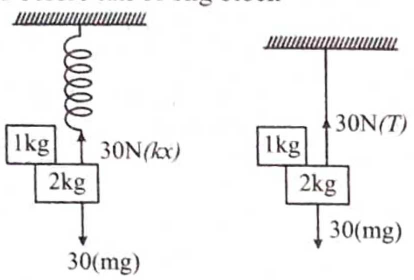
3. In string tension may change instantaneously.

Train Your Brain

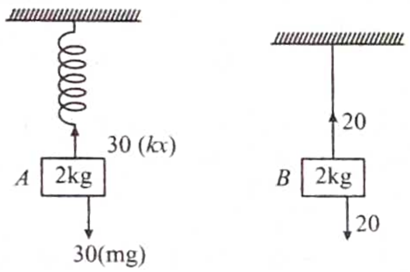
Example 18: Find out the acceleration of 2 kg block in the figures shown at the instant 1 kg block falls from 2 kg block. (at t = 0)



Sol. F.B.D.s before fall of 1kg block

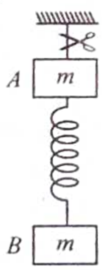


After the fall of the 1 kg block tension in string will change instantaneously but spring force (kx) doesn't change instantaneously. F.B.D.s just after the fall of 1 kg block



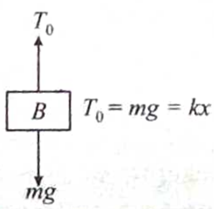
$$a_A = \frac{30-20}{2} = 5\text{m/s}^2\text{(downward)},\ a_B = 0\ \text{m/s}^2$$

Example 19: Two blocks '*A*' and '*B*' of same mass '*m*' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block '*A*' and '*B*' just after the string is cut.



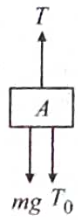
Sol. When block *A* and *B* are in equilibrium position

F.B.D of '*B*'



$T_0 = mg = kx$ ...(i)

F.B.D of '*A*'



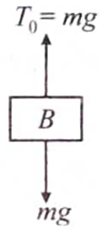
$T = mg + T_0$ ...(ii)

$T = 2\ mg$

when string is cut, tension *T* becomes zero. But spring does not change its shape instantaneously. So spring force continues to act on mass *B*.

Again draw F.B.D. of block *A* and *B* as shown in figure

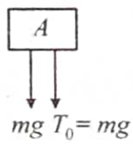
F.B.D of '*B*'



$T_0 = mg = m.a_B$

$\Rightarrow a_B = 0$

F.B.D. of '*A*'



$mg + T_0 = m.a_A$

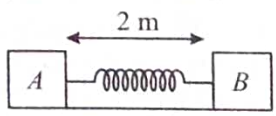
$\Rightarrow 2mg = m.a_A$

$\Rightarrow a_A = 2g$ (downwards)

Concept Application

21. Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring (in m) $(g = 10\ \text{m/s}^2)$

22. Two blocks are connected by a spring of natural length 2 m. The force (in N) constant of spring is 200 N/m. Find spring force in following situations.



(*a*) If block '*A*' and '*B*' both are displaced by 0.5 m in same direction.

(*b*) If block '*A*' and '*B*' both are displaced by 0.5 m in opposite direction.

PSEUDO FORCE

Motion in Accelerated Frames

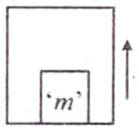
Till now we have restricted ourselves to applying Netwon's laws of motion, to describe observation that are made in an inertial frame of reference. In this section we learn how Newton's laws can be applied by an observer in a noninertial reference frame. For example consider a block kept on smooth surface of a compartment of train.

If the train accelerates, the block accelerates toward the back of the train. We may conclude based on Newton's second law $F = ma$ that a force is acting the block to cause it to accelerate, but the Newton's second law is not applicable in this non - inertial frame. So we can not relate observed acceleration with the force acting on the block.

If we still want to use Newton's second law we need to apply a pseudo force, acting in backward direction, i.e. opposite to the acceleration of noninertial reference frame. This force explains the motion of block towards the back of car. The fictitious force is equal to – ma where a is the acceleration of the noninertial reference frame. Fictitious force appears to act on an object in the same way as a real force, but real forces are always interactions between two objects. On the other hand there is no second object for a fictitious force.

Train Your Brain

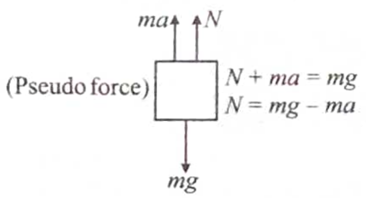
Example 20: A box is moving upward with retardation 'a' < g, find the direction and magnitude of "pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



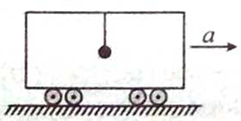
Sol. Pseudo force acts opposite to the direction of acceleration of reference frame.

Pseudo force = ma in upward direction

F.B.D of 'm' w.r.t box (non-inertial)



Example 21: Figure shows a pendulum suspended from the roof of a car that has a constant acceleration a relative to the ground. Find the deflection of the pendulum from the vertical as observed from the ground frame and from the frame attached with the car.



Sol. In inertial frame

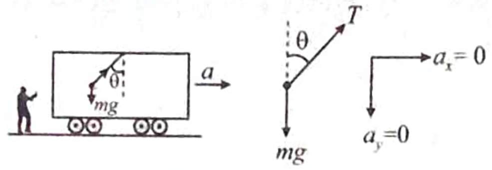


Figure represents free Body diagram of the bob w.r.t ground.

In an inertial frame the suspended bob has an acceleration a caused by the horizontal component of tension T.

$T \sin\theta = ma$ ...(i)

$T \cos\theta = mg$ ...(ii)

From equation (i) and (ii)

$$\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

In a non-inertial frame

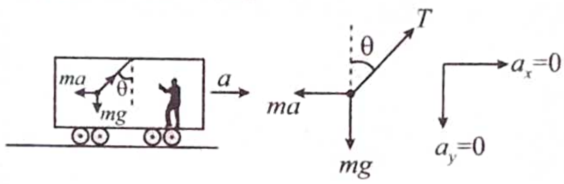


Figure represents free Body diagram of bob w.r.t car.

In the non-inertial frame of the car, the bob is in static equilibrium under the action of three forces, T, mg and ma (pseudo force)

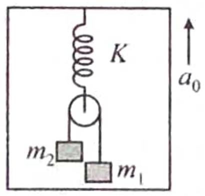
$T \sin\theta = ma$ ...(iii)

$T \cos\theta = mg$ ....(iv)

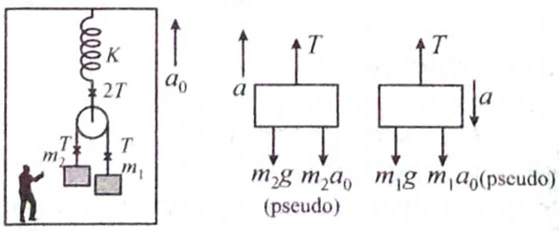
From equation (iii) and (iv)

$$\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Example 22: A pulley with two blocks system is attached to the ceiling of a lift moving upward with an acceleration a_0. Find the deformation in the spring.



Sol. Non-Inertial Frame



Let relative to the centre of pulley, m_1 accelerates downward with a and m_2 accelerates upwards with a. Applying Newton's 2nd law.

$m_1a + m_1a_0 - T = m_1a$ ...(i)

$T - m_2g - m_2a_0 = m_2a$ ...(ii)

On adding (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)(g + a_0) \quad \text{...(iii)}$$

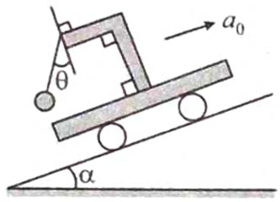
Substituting a in equation (i)

We get

$$T = \frac{2m_1m_2(g + a_0)}{m_1 + m_2}$$

$$\therefore x = \frac{F}{k} = \frac{2T}{k} = \frac{4m_1m_2(g + a_0)}{(m_1 + m_2)k}$$

Example 23: A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is (String and bob remain fixed with respect to trolley)



(a) $\theta = \tan^{-1}\alpha$ (b) $\theta = \tan^{-1}\left(\frac{a_0}{g}\right)$

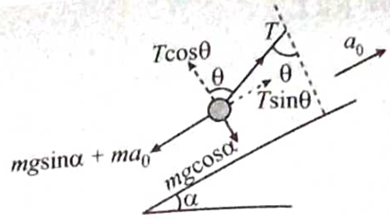
(c) $\theta = \tan^{-1}\left(\frac{g}{a_0}\right)$ (d) $\theta = \tan^{-1}\left(\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right)$

Sol. Balancing forces in the frame of trolley (non inertial frame)

$T\sin\theta = m(g\sin\alpha + a_0)$

$T\cos\theta = mg\cos\alpha$

$$\Rightarrow \tan\theta = \left(\frac{g\sin\alpha + a_0}{g\cos\alpha}\right)$$

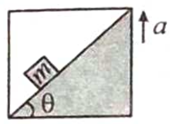


$$\theta = \tan^{-1}\left(\frac{g\sin\alpha + a_0}{g\cos\alpha}\right)$$

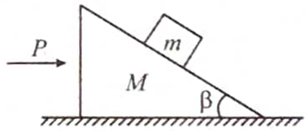
Concept Application

23. An object of mass 2 kg moving with constant velocity $10\hat{i}$ m/s is seen in a frame moving with constant velocity $10\hat{i}$ m/s. What will be the value of 'pseudo force' acting on object in this frame.

24. In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration 'a'. A block of mass 'm' is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.



25. Two wooden blocks are moving on a smooth horizontal surface such that the mass m remains stationary with respect to block of mass M as shown in the figure. The magnitude of force P is



(a) $(M + m)g\tan\beta$ (b) $g\tan\beta$

(c) $mg\cos\beta$ (d) $(M + m)g\,\text{cosec}\,\beta$

Short Notes

First Law of Motion

A body continues to be in its state of rest or of uniform motion along a straight line unless an external force is applied on it. This law is also called law of inertia.

Example: If a moving vehicle suddenly stops, then the passengers inside the vehicle bend outward.

Second Law of Motion

$$F_x = \frac{dP_x}{dt} = ma_x;\quad F_y = \frac{dP_y}{dt} = ma_y;\quad F_z = \frac{dP_z}{dt} = ma_z$$

Third Law of Motion

$\vec{F}_{AB} = -\vec{F}_{BA}$

$\vec{F}_{AB}$ = Force on A due to B

$\vec{F}_{BA}$ = Force on B due to A

Force

Force is a push or pull which changes or tries to change the state of rest, the state of uniform motion, size or shape of body.

Its SI unit is newton (N) and its dimensional formula is $[MLT^{-2}]$.

Forces can be categorised into two types:

(i) **Contact Forces:** Frictional force, tensional force, spring force, normal force etc are the contact forces.

(ii) **Distant Forces:** (Field Forces) Electrostatic force, gravitational force, magnetic force etc. are distant forces.

Weight (w)

It is a field force. It is the force with which a body is pulled towards the centre of the earth due to gravity. It has the magnitude mg, where m is the mass of the body and g is the acceleration due to gravity.

$$w = mg$$

Weighing Machine

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

Normal Reaction

It is a contact force. It is the force between two surfaces in contact, which is always perpendicular to the surfaces in contact.

Tension

Tension force always pulls a body. Tension is a reactive force. It is not an active force. Tension across a massless pulley or frictionless pulley remains constant. Rope becomes slack when tension force becomes zero.

Spring Force

$$F = -kx$$

x is displacement of the free end from its natural length or deformation of the spring where K = spring constant.

Spring Property

$K \times \ell$ = constant where ℓ = Natural length of spring

If spring is cut into two in the ratio $m : n$ then spring constant is given by $\ell_1 = \dfrac{m\ell}{m+n}$; $\ell_2 = \dfrac{n\ell}{m+n}$

$$k\ell = k_1\ell_1 = k_2\ell_2$$

For series combination of springs

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$

For parallel combination of spring

$$k_{eq} = k_1 + k_2 + k_3 \ldots$$

Spring Balance

It does not measure the weight. It measures the force exerted by the object at the hook.

Wedge Constraint

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations at contact place and they remain in contact.

Newton's Law for a System

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots$$

F_{ext} = Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and a_1, a_2, a_3 are the acceleration of the objects respectively.

Equilibrium of a Particle

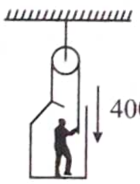
When the vector sum of the forces acting on a body is zero, then the body is said to be in equilibrium.

$$F_2 \longleftarrow \overset{O}{\bullet} \longrightarrow F_1$$

If two forces F_1 and F_2 act on a particles, then they will be in equilibrium if $F_1 + F_2 = 0$.

Solved Examples

1. A 60 kg painter is standing on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.



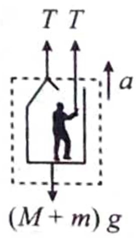
(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.

(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s. in 1s?

(iii) What force should apply now to maintain the constant speed of 1 m/s?

Sol. The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulles the rope.

(i) Applying Newton's Second Law



$$2T - (M+m)g = (M+m)a$$

$$\text{or } a = \frac{2T - (M+m)g}{M+m}$$

Here $M = 60$ kg; $m = 15$ kg; $T = 400$ N

$g = 10\ m/s^2$

$$\Rightarrow a = \frac{2(400) - (60+15)(10)}{60+15} = 0.67\ m/s^2$$

(ii) To attain a speed of 1 m/s in one second the acceleration a must be 1 m/s² Thus, the applied force is

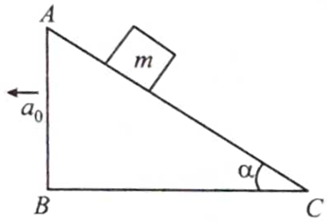
$$F = \frac{1}{2}(M+m)(g+a) = \frac{(60+15)(10+1)}{2} = 412.5 \text{ N}$$

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.

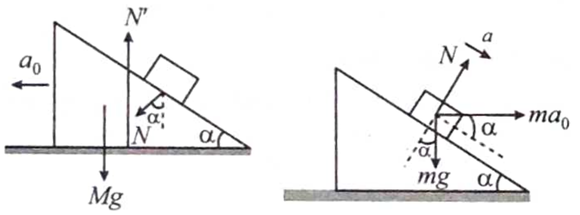
Thus, $2F - (M + m)\,g = 0$

$$\text{or } F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2} = 375 \text{ N}$$

2. All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.



Sol. Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block from the frame of the prism The forces on the block are (figure)



(i) N normal force

(ii) mg downward (gravity),

(iii) ma_0 forward (Psuedo Force)

The block slides down the plane. Components of the forces parallel to the incline give $ma_0 \cos\alpha + mg \sin\alpha = ma$

or; $a = a_0 \cos\alpha + g \sin\alpha$ ...(i)

Components of the forces perpendicular to the incline give

$N + ma_0 \sin\alpha = mg \cos\alpha$ ...(ii)

Now consider the motion of the prism from the ground frame. No pseudo force is needed as the frame used is inertial. The forces are (figure)

(i) Mg downward

(ii) N normal to the incline (by the block)

(iii) N' upward (by the horizontal surface)

Horizontal components give,

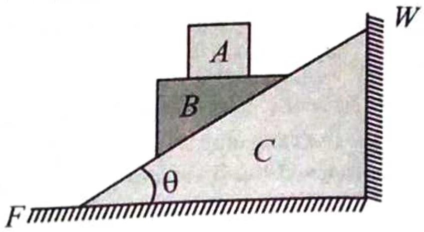
$N \sin\alpha = Ma_0$ or $N = Ma_0/\sin\alpha$ ...(iii)

Putting in (ii)

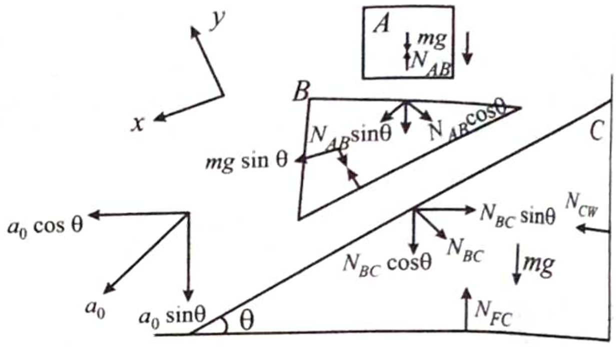
$$\frac{Ma_0}{\sin\alpha} + ma_0 \sin\alpha = mg\cos\alpha \quad \text{or} \quad a_0 = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$$

$$\text{From (i), } a = \frac{mg\sin\alpha\cos^2\alpha}{M + m\sin^2\alpha} + g\sin\alpha = \frac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$$

3. In the figure shown all the surfaces are smooth. The blocks A, B and C have the same mass m. F is floor and W is the wall. Find the magnitude of the contact forces at all the surfaces after the system is released from rest. The angle of inclination of the inclined plane with the horizontal is θ.



Sol.



$mg - N_{AB} = ma_A$

[Newton's II law for block A in vertical direction]

$mg \sin\theta + N_{AB} \sin\theta = ma_B$

[Newton's II law for block B in x direction]

$a_A = a_B \sin\theta$

[Constrained relation for contact surface between block A and B]

Solving above three equations we get

$$N_{AB} = \frac{mg\cos^2\theta}{1+\sin^2\theta}$$

$mg\cos\theta + N_{AB}\cos\theta - N_{BC} = 0$

[Equilibrium of block B in y direction]

$$\Rightarrow N_{BC} = mg\cos\theta + \frac{mg\cos^2\theta\cos\theta}{1+\sin^2\theta}$$

$$\Rightarrow N_{BC} = \frac{2mg\cos\theta}{1+\sin^2\theta}$$

$N_{BC}\sin\theta - N_{WC} = 0$

[Equilibrium of block in horizontal direction]

$$N_{WC} = \frac{2mg\sin\theta\cos\theta}{1+\sin^2\theta}$$

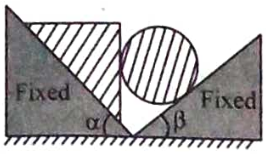
$N_{BC}\cos\theta + mg - N_{FC} = 0$

[Equilibrium of block C in vertical direction]

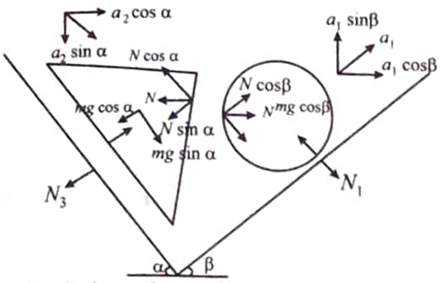
$$\Rightarrow N_{FC} = \frac{2mg\cos^2\theta}{1+\sin^2\theta} + mg$$

$$\Rightarrow N_{FC} = \frac{mg\,(2+\cos^2\theta)}{1+\sin^2\theta}$$

4. A cylinder and a wedge of same masses with a vertical face, touching each other, move along two smooth inclined planes forming the same angle α and β respectively with the horizontal. Determine the force of normal N exerted by the wedge on the cylinder, neglecting the friction between them.



Sol.



It is obvious that acceleration of cylinder is parallel to the wedge I and acceleration of triangular block is parallel to the wedge 2.

$a_2 \cos\alpha = a_1 \cos\beta$

[constrained relation between the contact surface of block and cylinder]

$N \cos\beta - m_1 g \sin\beta = m_1 a_1$

[Newton's II law for cylinder along the direction parallel to the wedge1]

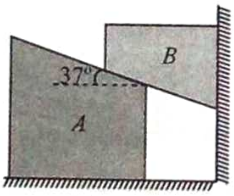
$m_2 g \sin\alpha - N\cos\alpha = m_2 a_2$

[Newton's II law for block along the direction parallel to the wedge 2]

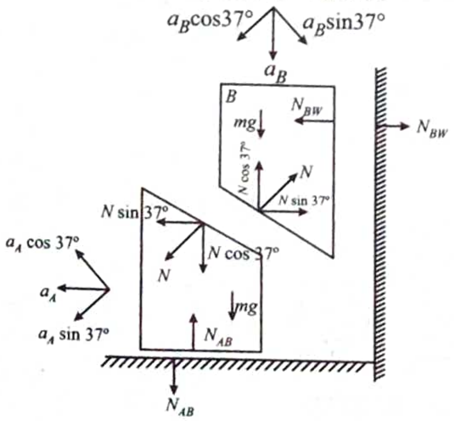
By solving equation I, II and III we get

$$N = mg\left(\frac{\sin\alpha\cos\alpha + \sin\beta\cos\beta}{\cos^2\alpha + \cos^2\beta}\right)$$

5. The masses of blocks A and B are same and equal to m. Friction is absent everywhere. Find the magnitude of normal force with which block B presses on the wall and accelerations of the blocks A and B.



Sol.



$mg - N\cos 37 = ma_B$

[Newton's II law for block B in vertical direction]

$N \sin 37 = ma_A$

[Newton's II law for block A in horizontal direction]

$a_B \cos 37 = a_A \sin 37$

[constrained relation for contact surface between block A and B]

By solving above three equations we get

$a_A = \dfrac{12g}{25}$ $a_B = \dfrac{9g}{25}$ $N = \dfrac{4mg}{5}$

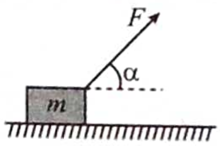
$N_{BW} = N\sin 37$

[Equilibrium of block B in horizontal direction]

$\Rightarrow N_{BW} = \dfrac{12mg}{25}$

6. At the moment $t = 0$ the force $F = at$ is applied to a small body of mass m resting on a smooth horizontal plane (a is constant).

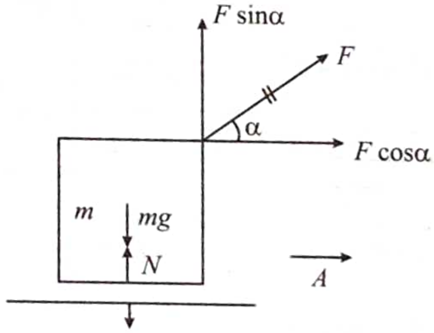
The permanent direction of this force forms an angle α with the horizontal (as shown in the figure). Find



(*a*) the velocity of the body at the moment of its breaking off the plane;

(*b*) the distance traversed by the body up to this moment.

Sol.



$mg - N - F\sin\alpha = 0$ [Equilibrium of block in vertical direction]

at breaking off the contact $N = 0$.

$\Rightarrow F\sin\alpha = mg \Rightarrow at\sin\alpha = mg$

$\Rightarrow t = \dfrac{mg}{a\sin\alpha}$

$F\cos\alpha = m A$

[Newton's second law for block in horizontal direction]

$\Rightarrow at\cos\alpha = m\dfrac{d\upsilon}{dt}$

$$\int_0^v dv = \frac{a\cos\alpha}{m}\int_0^{t=\frac{mg}{a\sin\alpha}} t\,dt \Rightarrow v = \frac{a\cos\alpha}{m}\left[\frac{t^2}{2}\right]_0^{\frac{mg}{a\sin\alpha}} \quad \text{...(i)}$$

After putting time limits.

$$v = \frac{mg^2 \cos\alpha}{2a\sin^2\alpha}$$

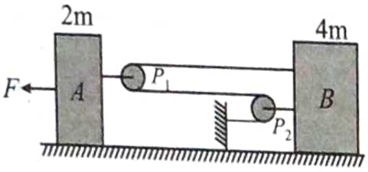
Equation (i) can be written as

$$\frac{dx}{dt} = \frac{a\cos\alpha}{2m}t^2$$

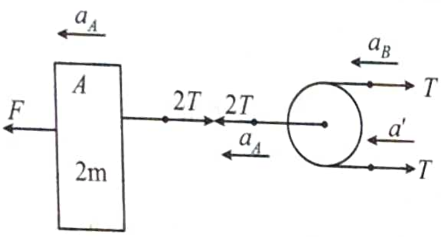
$$\int_0^x dx = \frac{a\cos\alpha}{2m}\int_0^{\frac{mg}{a\sin\alpha}} t^2\,dt = \frac{a\cos\alpha}{2m}\left[\frac{t^3}{3}\right]_0^{\frac{mg}{a\sin\alpha}}$$

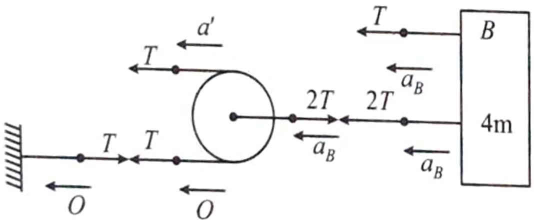
After putting limits $x = \dfrac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$

7. Calculate the acceleration of the block B in the figure, assuming the surfaces and the pulleys P_1 and P_2 are all smooth and pulleys and string are light



Sol.





$-2TV_A + 3TV_B = 0$ [constrained relation]

$3V_B = 2V_A$

From above two equations

$3a_B = 2a_A \Rightarrow a_A = \dfrac{3}{2}a_B$ ...(i)

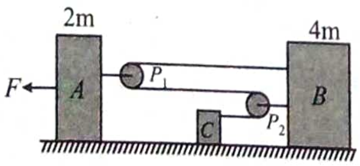
$F - 2T = 2ma_A$ [Netwon's II law for block A] ...(ii)

$3T = 4\,m\,a_B$ [Netwon's II law for block B] ...(iii)

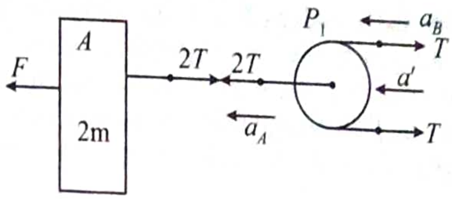
From equation I, II and III

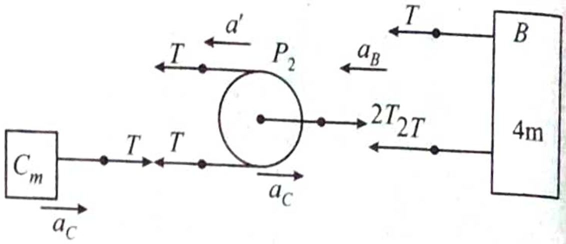
$$a_B = \frac{3F}{17m}.$$

8. In above question vertical surface is replaced by block C of mass m as shown in figure. Find the acceleration of block B.



Sol.





$-2TV_A + 3TV_B + TV_C = 0 \Rightarrow 3V_B + V_C = 2V_A$

From above equation

$3a_B + a_c = 2a_A$ ...(i) [Constant relati

$F - 2T = 2ma_A$ ...(ii) [Newton's II law for block

$3T = 4m\,a_B$ ....(iii) [Newton's II law for block

$T = ma_C$ ....(iv) [Newton's II law for block

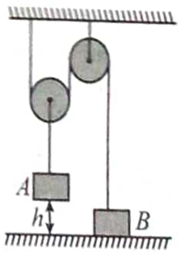
From (i), (ii), (iii) and (iv)

$$3 \times \frac{3T}{4m} + \frac{T}{m} = 2\left(\frac{F-2T}{2m}\right)$$

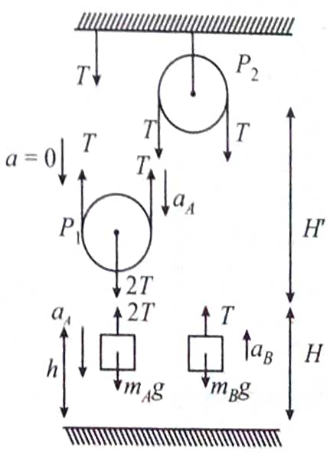
$$\frac{9T}{4} + T = F - 2T \Rightarrow T = \frac{4F}{21}$$

$$a_B = \frac{3T}{4m} \Rightarrow a_B = \frac{F}{7m}.$$

9. In the arrangement shown in figure, the mass of the bo A is $n = 4$ times that of body B. The height $h = 20$ cm. A certain instant, the body B is released and the system is in motion. What is the maximum height, the body B will up? Assume enough space above B and A sticks to groun (A and B are of small size) ($g = 10\text{ m/s}^2$)



Sol.



$m_A g - 2T = m_A a_A$ [Newton's II law for block A]

$T - m_B g = m_B a_B$ [Newton's II law for block B]

$a_B = 2a_A$ [constrained relation]

$m_A = 4m_B$ [Given in question]

From above four equations

$$a_A = \frac{g}{4} = 2.5 \text{ m/s}^2$$

$$a_B = \frac{g}{2} = 5 \text{ m/s}^2$$

$h = \frac{1}{2}a_A t^2$ [Equation of motion for block A]

$$\Rightarrow t = \sqrt{\frac{2h}{a_A}} = \sqrt{\frac{2 \times 0.2}{2.5}} \Rightarrow t = \frac{2}{5} \text{ sec.}$$

H is the distance travelled by block B in vertical direction till $\frac{2}{5}$ second

$\Rightarrow H = \frac{1}{2}a_B t^2$ [Equation of motion for block B]

$$\Rightarrow \frac{1}{2}5\left(\frac{2}{5}\right)^2$$

$H = 0.4$ m

H' is the distance travelled by block B due to gained velocity.

$v_1 = a_B t = 5 \times 0.4$

$v_1 = 2$ m/s

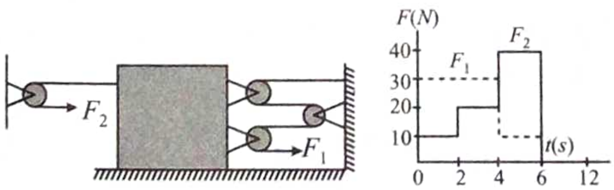
$v_2^2 = v_1^2 + 2a H'$

$0^2 = 2^2 + 2(-10) H'$

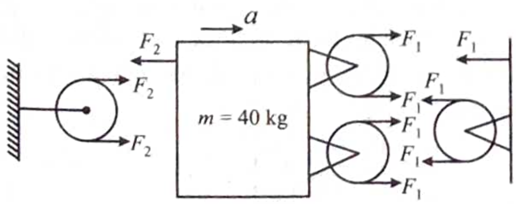
$$H' = \frac{2}{10} = 0.2 \text{ m}$$

Total distance $= H + H' = 0.6$ m $= 60$ cm

10. The 40 kg block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces F_1 and F_2. These forces vary in the manner shown in the graph. Find the velocity of the block after $t = 12$ s. Neglect friction and masses of the pulleys and cords.



Sol.



$4F_1 - F_2 = ma$ [Newtons II law for block]

$$\Rightarrow a = \frac{4F_1 - F_2}{m}$$

For $t = 0$ to 2 sec.

$F_1 = 30$N, $F_2 = 10$N

$$\Rightarrow a = \frac{4 \times 30 - 10}{40} = 2.75 \text{ m/s}^2$$

For $t = 2$ to 4 sec

$F_1 = 30$N, $F_2 = 20$N

$$\Rightarrow a = \frac{4 \times 30 - 20}{40} = 2.5 \text{ m/s}^2$$

For $t = 4$ to 6 sec.

$F_1 = 10$N, $F_2 = 40$N

$$\Rightarrow a = \frac{4 \times 10 - 40}{40} = 0 \text{ m/s}^2$$

For $t = 6$ to 12 sec

$F_1 = 0, F_2 = 0 \Rightarrow a = 0$ m/s^2

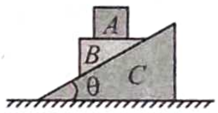
$V_{12} \to$ Velocity of block after 12 seconds

$V_{12} - V_0 = a_{0-2}(2-0) + a_{2-4}(4-2) + a_{4-6}(6-4) + a_{6-12}(12-6)$

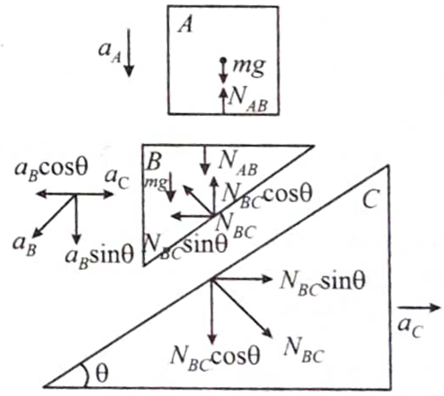
$V_{12} - 1.5 = 2.75 \times 2 + 2.5 \times 2 + 0 \times 2 + 0 \times 6$

$V_{12} = 12$ m/s

11. In the figure shown all blocks are of equal mass 'm'. All surfaces are smooth. Find the acceleration of all the blocks.



Sol.



a_A and a_C are acceleration of block A and C w.r.t ground and a_B is the acceleration of block B w.r.t. block C

$N_{BC} \sin\theta = ma_C$

[Newton's II law for block C in horizontal direction in ground frame]

$N_{AB} + mg - N_{BC}\cos\theta = ma_B \sin\theta$

[Newton's II law for block B in vertical direction in ground frame]

$N_{BC}\sin\theta = m(a_B\cos\theta - a_c)$

[Newton's II law for block B in horizontal direction in ground frame]

$mg - N_{AB} = ma_A$

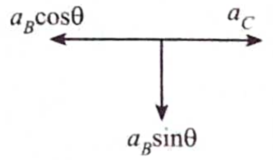
[Newton's II law for block A in vertical direction in ground frame]

$a_A = a_B \sin\theta$

[constrained relation for block A and B in vertical direction]

Now we have 5 equations and 5 unknowns.

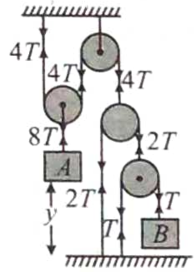
$$\Rightarrow a_A = \frac{4g\sin^2\theta}{1+3\sin^2\theta} \Rightarrow a_B = \frac{4g\sin\theta}{1+3\sin^2\theta} \Rightarrow a_C = \frac{2g\sin\theta\cos\theta}{1+3\sin^2\theta}$$



$\Rightarrow$ Acceleration of B w.r.t ground

$$= \sqrt{(a_B\cos\theta - a_C)^2 + (a_B\sin\theta)^2} = \frac{2g\sin\theta}{\sqrt{1+3\sin^2\theta}}$$

12. The vertical displacement of block A in meter is given by $y = t^2/4$ where t is in second. Calculate the downward acceleration a_B of block B.



Sol. $a_A = \dfrac{d^2y}{dt^2} = \dfrac{1}{2}$

$P_A + P_B = 0$

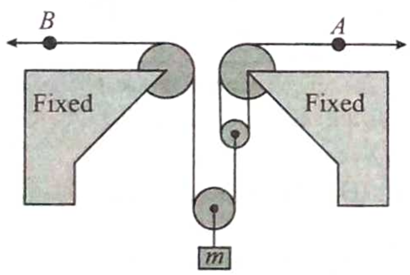
$-8TV_A + TV_B = 0$

$V_B = 8V_A$

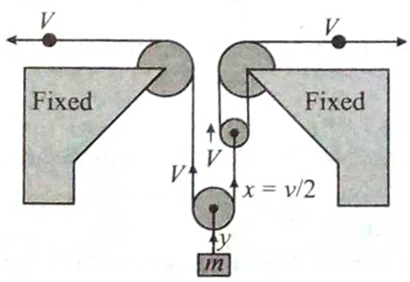
$a_B = 8a_A$ (Constrained relation)

$a_B = 4 \text{ m/s}^2$

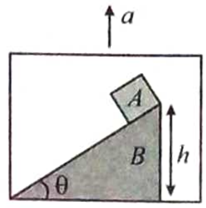
13. For the pulley system, each of the cables at A and B is given velocity of 2m/s in the direction of the arrow. Determine the upward velocity v of the load m.



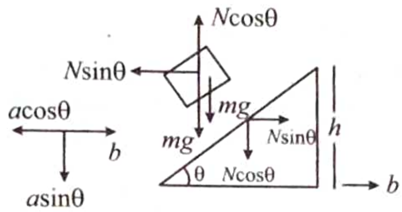
Sol. $v_m = \dfrac{v_B + {}^{v_A}\!/_2}{2} = \dfrac{2 + {}^2\!/_2}{2} = \dfrac{2+1}{2} = 1.5 \text{ m/s}$



14. A lift is moving upwards with a constant acceleration $a = g$. A small block A of mass 'm' is kept on a wedge B of the same mass 'm'. The height of the vertical face of the wedge is 'h'. A is released from the top most point of the wedge. Find the time taken by A to reach the bottom of B. All surfaces are smooth and B is also free to move.



Sol.



$N\sin\theta = mb$ ...(i)

$N\sin\theta = m(a\cos\theta - b)$ ...(ii)

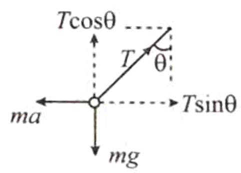
$2mg - N\cos\theta = ma\sin\theta$ ...(iii)

From (i), (ii) and (iii)

$$\Rightarrow a = \frac{4g\sin\theta}{1+\sin^2\theta} \Rightarrow h = \frac{1}{2}a\sin\theta\, t^2 \Rightarrow t = \sqrt{\frac{h(1+\sin^2\theta)}{2g\sin^2\theta}}.$$

15. A ball is suspended from the ceiling of car which is speeding up on a horizontal road with a constant acceleration a. Find the angle made by the string if the ball and string remain static with respect to car. If in the same car a block is kept on a smooth fixed incline and does not slip on the fixed incline then find the angle of inclination of incline plane, with the horizontal.

Sol.



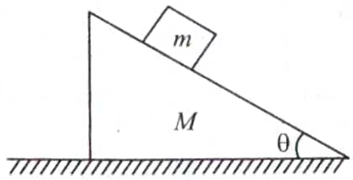
$T\cos\theta = mg$ ...(i)

$T\sin\theta = ma$ ...(ii)

By (i) and (ii)

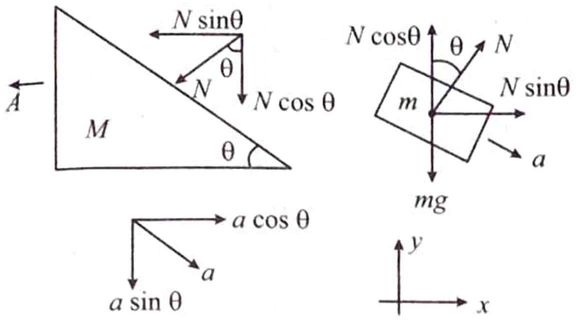
$$\tan\theta = \frac{ma}{mg} \Rightarrow \theta = \tan^{-1}\frac{a}{g}.$$

16. A block of mass m is placed on the inclined surface of a wedge as. Calculate the acceleration of the wedge and the block when the block is released. Assume all surfaces are frictionless.



Sol. Let the acceleration of wedge be A and that of block be a (w.r.f. wedge). Then acceleration of m w.r.t. ground is

$(a\cos\theta - A)\hat{i} - a\sin\theta\ \hat{j}$



For M: $N\sin\theta = MA$ ...(i)

For m: $mg - N\cos\theta = ma\sin\theta$ ...(ii)

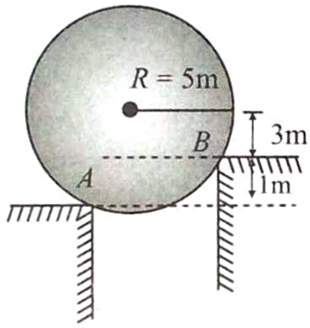
$N\sin\theta = m(a\cos\theta - A)$ ...(iii)

Solved (i), (ii) and (iii) to get

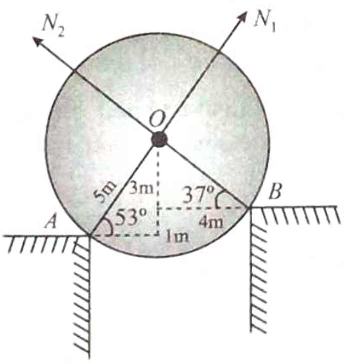
$$A = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta} \text{ and } a = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta}$$

Now acceleration of block can be found by putting the values of A and a in (i).

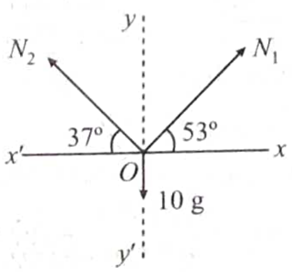
17. Find out the normal reaction at point A and B if the mass of sphere is 10 kg.



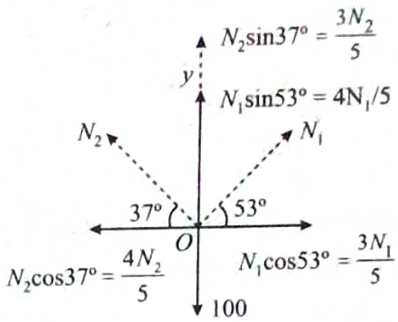
Sol.



Now F.B.D.



Now resolve the forces along x and y direction



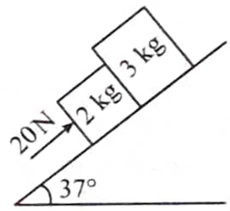
$\therefore$ The body is in equilibrium so equate the force in x and y direction

In x-direction, $\dfrac{3N_1}{5} = \dfrac{4N_2}{5}$ ...(i)

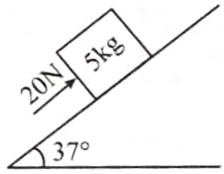
In y-direction, $\dfrac{4N_1}{5} + \dfrac{3N_2}{5} = 100$ ...(ii)

After solving above equation, $N_1 = 80$ N, $N_2 = 60$ N

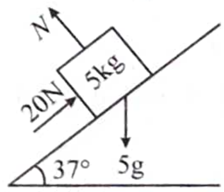
18. Find out the contact force between 2kg & 3kg block placed on the incline plane as shown in figure.



Sol. Considering both the block as a 5 kg system because both will move the same acceleration.

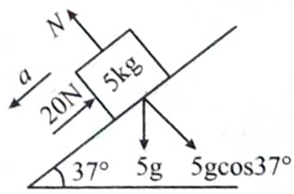


Now show forces on the 5 kg block



$\because$ Acceleration of 5 kg block is down the incline. So choose one axis down the incline and other perpendicular to it

From Newton's second Law

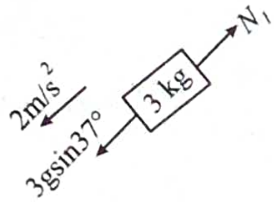


$N = 5g\cos 37°$ ...(i)

$5g\sin 37° - 20 = 5a$ ...(ii)

$30 - 20 = 5a$

$a = 2\text{m/s}^2$ (down the incline)



For contact force (N_1) between 2kg and 3kg block we draw F.B.D. of 3kg block

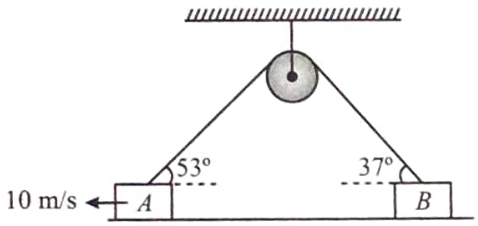
From, $F_{net} = ma$

$\Rightarrow 3g\sin 37° - N_1 = 3 \times 2$

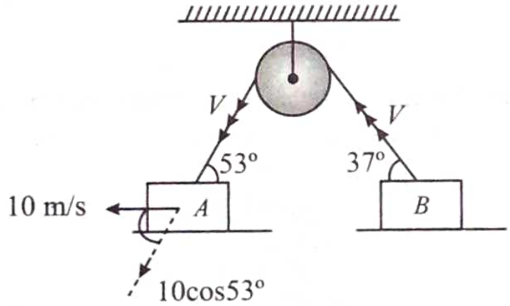
$18 - N_1 = 6$

$N_1 = 12$ N

19. Find out the velocity of block B in a pulley block system as shown in figure.



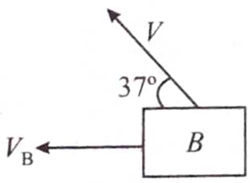
Sol. Let us assume the velocity of all the particle of string is v.



10 m/s is the velocity of block A then its component along string is v.

$\Rightarrow 10 \cos 53° = v$ ...(i)

If v_B is the velocity of block B then it component



Along string is v then

$v_B \cos 37° = v$ ...(ii)

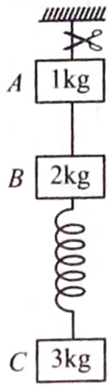
From (i) and (ii)

$v_B \cos 37° = 10 \cos 53°$

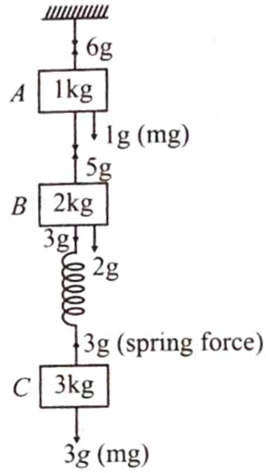
$$\Rightarrow v_B = \frac{10 \times 3/5}{4/5}$$

$$= \frac{30}{4} = \frac{15}{2} m/\text{sec}$$

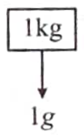
20. Find out the acceleration of 1 kg, 2 kg and 3 kg block and tension in the string between 1 kg and 2 kg block just after cutting the string as shown in figure.



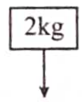
Sol. F.B.D before cutting of string



Let us assume the Tension in the string connecting blocks A and B becomes zero just after cutting the string then.



$a_1 = \frac{1g}{1} = g \text{ ms}^{-2}$

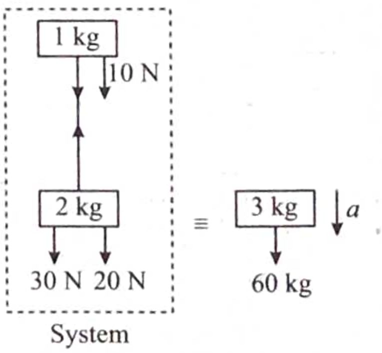


$a_2 = \frac{5g}{2} = 2.5 \text{ g ms}^{-2}$

$2g + 3g$
(weight) (spring force)

$\because a_2 > a_1$ i.e., $\therefore T \neq 0$

If $T \neq 0$ that means string is tight and both block A and B will have same acceleration. So we can take it as a system of 3 kg mass.

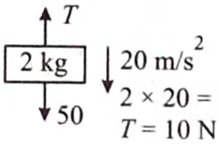


System

Total force down ward = 10 + 30 + 20 = 60 N

Total mass = 3 kg $a = \frac{60}{3} = 20 \text{ m/s}^2$

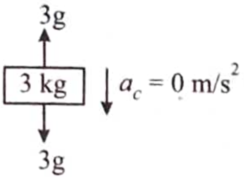
Now apply, $F_{net} = ma$ for block B.



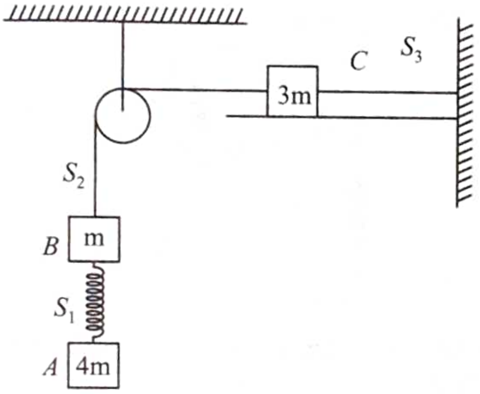
$2 \times 20 = 5g - T$

$T = 10$ N

$\because$ The spring force does not change instantaneously the F.B.D of 'C'



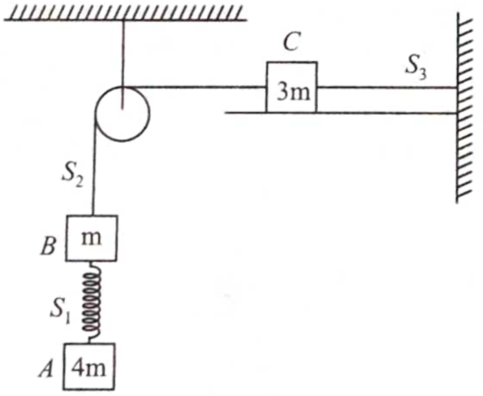
21. In following setup pulley, strings and spring are light. Initially all masses are in equilibrium and at rest.

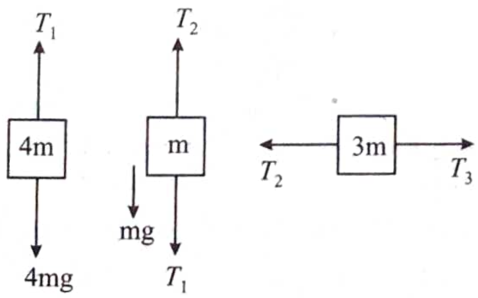


(a) Find tension in spring and tension in ropes

(b) Find acceleration of masses immediately after the string S_3 is cut.

Sol. (a)





Applying Newton's 2nd law to block A, $4mg - T_1 = 0$

Applying Newton's 2nd law to block B, $mg + T_1 - T_2 = 0$

Applying Newton's 2nd law to block C, $T_3 - T_2 = 0$

Solving, $T_1 = 4$ mg

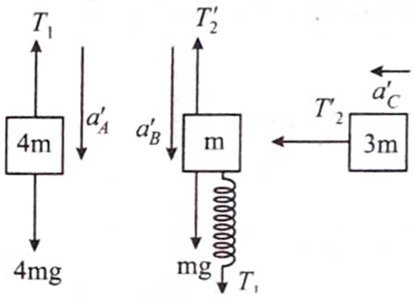
$T_2 = 5\ mg$

$T_3 = 5\ mg$

Here spring is behaving same as string except that it is stretched while string can not stretch.

(b) The most important point in this problem is that any object of finite mass can not change its position instantaneously as this requires infinite velocity.

Thus immediately after cutting the string S_3 all masses will remain at same position and force due to spring will not change. As force of spring is kx. At the same time we would like to emphasize that tension in string S_2 will change instantaneously (Tension is a self adjusting force). To maintain constraint relation, blocks B and C have same magnitude of acceleration. We can identify all the forces acting on all objects. Only tension T in string S_2 is unknown force all other forces are known. considering FBD,



We know from part [a] that tension in spring is T_1 and $T_1 = 4\ mg$

Writing Newton's Second law for A

$4mg - T_1 = 4ma'_A$

Writing Newton's Second law for B

$T_1 + mg - T'_2 = ma'_B$

Writing Newton's Second law for C

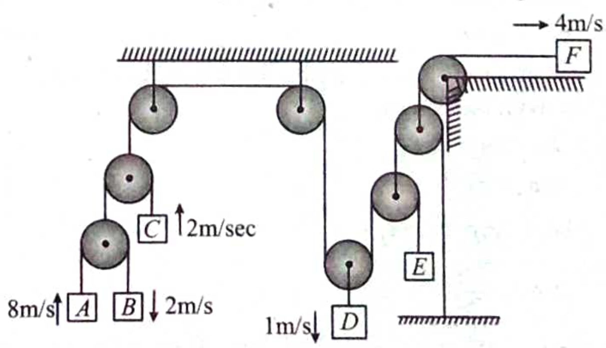
$T_2' = 3ma'_C$

Quantities which may have different value from part (a) are represented using symbol, e.g. tension in string S_2 is T_2' others which have same value as part (a) have been retained with same symbol.

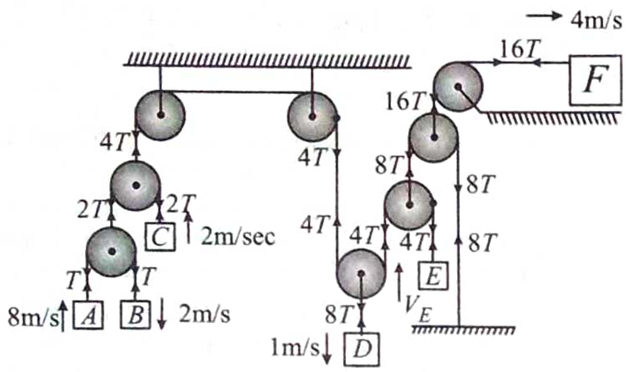
As $a_B' = a_C'$

Solving we get, $a_A' = 0$, $a_B' = a_C' = \frac{5}{4} g$

22. Find out the velocity of block E as shown in figure.



Sol.



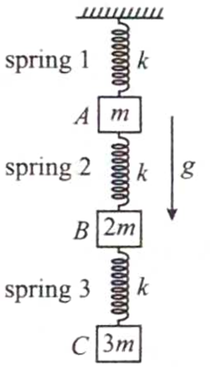
$P_{input} = P_A + P_B + P_C + P_D + P_E + P_F = 0$

$8T - 2T + (2T \times 2) - (8T \times 1) + 4TV_E - (16T \times 4) = 0$

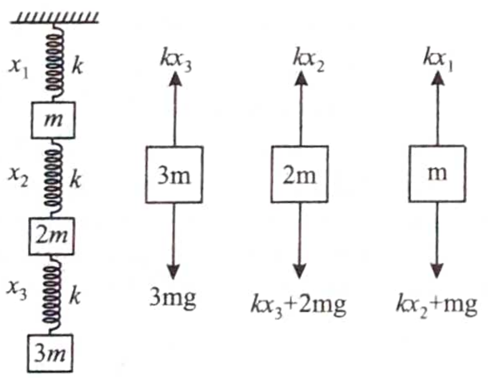
$4V_E = -8 + 2 - 4 + 8 + 64 = 62$

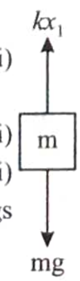
$V_E = \dfrac{62}{4} = \dfrac{31}{2}\text{ms}^{-1}$

23. The system shown in the figure is in equilibrium. Find the initial acceleration of A, B, and C just after the spring-2 is cut.



Sol.





$3mg = kx_3$ ...(i)

$2mg + kx_3 = kx_2$

$2mg + 3mg = kx_2 \Rightarrow 5mg = kx_2$ ...(ii)

$kx_1 = 6mg$ ...(iii)

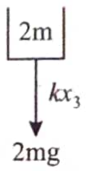
when spring 2 is cut spring force in other two springs remain unchanged.

, $kx_1 - mg = ma_3$

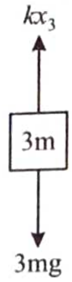
$\Rightarrow a_3 = 5g \uparrow$

$kx_3 + 2mg = 2ma_2$

$\Rightarrow a_2 = \dfrac{5g}{2} \downarrow$

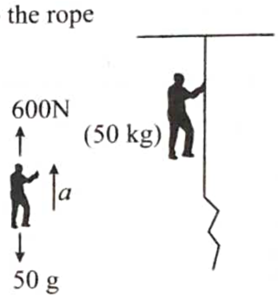


Acceleration of $3m$ will be zero.



Important Point: It is important to remember that ropes can change tension instantaneously while spring need to move to change tension, so in this example tension in spring is not changing instantaneously

24. If the breaking strength of string is 600 N then find out the maximum acceleration of the man of 50 kg with which he can climb up the rope

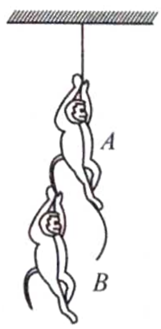


Sol. Maximum force that can be exerted on the man by the rope is 600 N.

$\Rightarrow 600 - 50\,g = 50\,a$ $(g = 10\text{ ms}^{-2})$

$a_{max} = 2\text{ m/s}^2$

25. The monkey A shown in the figure climbing on a rope while monkey B holding tail of the monkey A which is climbing on a rope. The masses of the monkeys A and B are 7 kg and 3 kg respectively. If A can tolerate a tension of 45 N in its tail, what force should monkey A apply on the rope in order to carry the monkey B with it? Take $g = 10\text{ m/s}^2$.



Sol. $w = m\,(g + a)$

$45 = 3\,(10 + a)$

$a = 5\text{ m/s}^2$

$T_{max} = (m_A + m_B)\,(g + a)$

$= 10 \times 15 = 150\text{ N}$

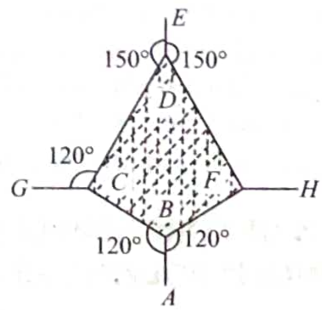
$T_{min} = (m_A + m_B)\,g = 10 \times 10 = 100\text{ N}$

Between 100 N and 150 N

Exercise-1 (Topicwise)

LAWS OF MOTION

1. A rider on horse back falls when horse starts running all of a sudden because
   (*a*) Rider is taken back
   (*b*) Rider is suddenly afraid of falling
   (*c*) Inertia of rest keeps the upper part of body at rest whereas lower part of the body moves forward with the horse
   (*d*) None of the above
2. When a train stops suddenly, passengers in the running train feel an instant jerk in the forward direction because
   (*a*) The back of seat suddenly pushes the passengers forward
   (*b*) Inertia of rest stops the train and takes the body forward
   (*c*) Upper part of the body continues to be in the state of motion whereas the lower part of the body in contact with seat remains at rest
   (*d*) Nothing can be said due to insufficient data
3. A man getting down a running bus falls forward because
   (*a*) Due to inertia of rest, road is left behind and man reaches forward
   (*b*) Due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
   (*c*) He leans forward as a matter of habit
   (*d*) Of the combined effect of all the three factors stated in (*a*), (*b*) and (*c*)
4. A boy sitting on the topmost berth in the compartment of a train which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother sitting vertically below his hands at a distance of about 2 meter. The apple will fall
   (*a*) Precisely on the hand of his brother
   (*b*) Slightly away from the hand of his brother in the direction of motion of the train
   (*c*) Slightly away from the hand of his brother in the direction opposite to the direction of motion of the train
   (*d*) None of the above
5. A force of 100 dynes acts on a mass of 5 gm for 10 sec. The velocity produced is
   (*a*) 2 cm/sec
   (*b*) 20 cm/sec
   (*c*) 200 cm/sec
   (*d*) 2000 cm/sec
6. A machine gun is mounted on a 2000 kg car on a horizontal frictionless surface. At some instant the gun fires bullets of mass 10 gm with a velocity of 500 m/sec with respect to the car. The number of bullets fired per second is ten. The average thrust on the system is
   (*a*) 550 N (*b*) 50 N
   (*c*) 250 N (*d*) 250 dyne
7. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 *cm* away from the origin
   (*a*) 5 m/s^2 (*b*) 10 m/s^2
   (*c*) 3 m/s^2 (*d*) 15 m/s^2
8. In a rocket of mass 1000 kg fuel is consumed at a rate of 40 kg/s. The velocity of the gases ejected from the rocket is 5×10^4 m/s. The thrust on the rocket is
   (*a*) 2×10^3 N (*b*) 5×10^4 N
   (*c*) 2×10^6 N (*d*) 2×10^9 N
9. A force vector applied on a mass is represented as $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ and accelerates with 1 m/s^2. What will be the mass of the body
   (*a*) $10\sqrt{2}$ kg (*b*) $2\sqrt{10}$ kg
   (*c*) 10 kg (*d*) 20 kg
10. A cricket ball of mass 250 g collides with a bat with velocity 10 m/s and returns with the same velocity within 0.01 second. The force acted on bat is
    (*a*) 25 N (*b*) 50 N (*c*) 250 N (*d*) 500 N
11. A body of mass 2 kg is moving with a velocity 8 m/s on a smooth surface. If it is to be brought to rest in 4 seconds, then the force to be applied is
    (*a*) 8 N (*b*) 4 N (*c*) 2 N (*d*) 1 N
12. The adjacent figure is the part of a horizontally stretched net. Section *AB* is stretched with a force of 10 N. The tensions in the sections *BC* and *BF* are



    (*a*) 10 N, 11 N
    (*b*) 10 N, 6 N
    (*c*) 10 N, 10 N
    (*d*) Can't calculate due to insufficient data

13. When the speed of a moving body is doubled
(a) Its acceleration is doubled
(b) Its momentum is doubled
(c) Its kinetic energy is doubled
(d) Its potential energy is doubled

14. A ball of mass m moves with speed v and it strikes normally with a wall and reflected back normally, if its time of contact with wall is t then find force exerted by ball on wall
(a) $\frac{2mv}{t}$ (b) $\frac{mv}{t}$
(c) mvt (d) $\frac{mv}{2t}$

15. A particle moves in the xy - plane under the action of a force F such that the components of its linear momentum p at any time t are $p_x = 2\cos t$, $p_y = 2\sin t$. The angle between F and p at time t is
(a) 90° (b) 0°
(c) 180° (d) 30°

16. Swimming is possible on account of
(a) First law of motion
(b) Second law of motion
(c) Third law of motion
(d) Newton's law of gravitation

17. A man is at rest in the middle of a pond on perfectly smooth ice. He can get himself to the shore by making use of Newton's
(a) First law (b) Second law
(c) Third law (d) All the laws

18. If a force of 250 N act on body, the momentum acquired is 125 kg-m/s. What is the period for which force acts on the body
(a) 0.5 sec (b) 0.2 sec (c) 0.4 sec (d) 0.25 sec

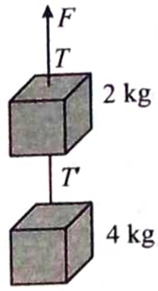
19. An aircraft is moving with a velocity of 300 ms^{-1}. If all the forces acting on it are balanced, then
(a) It still moves with the same velocity
(b) It will be just floating at the same point in space
(c) It will fall down instantaneously
(d) It will lose its velocity gradually

20. When a horse pulls a cart , the force needed to move the horse in forward direction is the force exerted by
(a) The cart on the horse (b) The ground on the horse
(c) The ground on the cart (d) The horse on the ground

APPLICATION OF FORCE/IMPULSE, STATICS AND DYNAMICS INVOLVING SINGLE SYSTEM

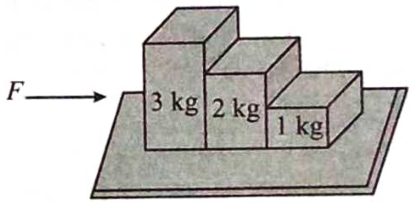
21. Two forces of magnitude F have a resultant of the same magnitude F. The angle between the two forces is
(a) 45° (b) 120° (c) 150° (d) 60°

22. Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force F applied on the upper string produces an acceleration of 2 m/s^2 in the upward direction in both the blocks. If T and T' be the tensions in the two parts of the string, then



(a) $T = 70.8$ N and $T' = 47.2$ N
(b) $T = 58.8$ N and $T' = 47.2$ N
(c) $T = 70.8$ N and $T' = 58.8$ N
(d) $T = 70.8$ N and $T' = 0$

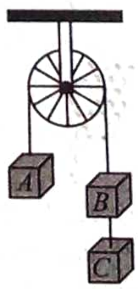
23. Consider the following statements about the blocks shown in the diagram that are being pushed by a constant force on a frictionless table



A. All blocks move with the same acceleration
B. The net force on each block is the same.
Which of these statements are/is correct.
(a) A only (b) B only
(c) Both A and B (d) Neither A nor B

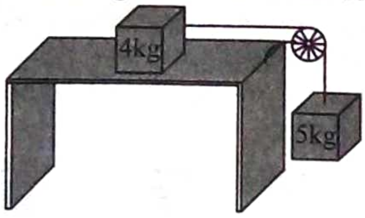
24. A rope of length L is pulled by a constant force F. What is the tension in the rope at a distance x from the end where the force is applied
(a) $\frac{FL}{x}$ (b) $\frac{F(L-x)}{L}$
(c) $\frac{FL}{L-x}$ (d) $\frac{Fx}{L-x}$

25. Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the figure The tension in the string connecting weights B and C is



(a) Zero (b) 13 N (c) 3.3 N (d) 19.6 N

26. Two masses of 4 kg and 5 kg are connected by a string passing through a frictionless pulley and are kept on a frictionless table as shown in the figure. The acceleration of 5 kg mass is



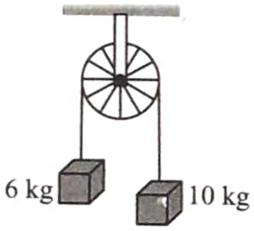
(*a*) 49 m/s^2 (*b*) 5.44 m/s^2 (*c*) 19.5 m/s^2 (*d*) 2.72 m/s^2

27. Three solids of masses m_1, m_2 and m_3 are connected with weightless string in succession and are placed on a frictionless table. If the mass m_3 is dragged with a force T, the tension in the string between m_2 and m_3 is

(*a*) $\dfrac{m_2}{m_1+m_2+m_3}T$ (*b*) $\dfrac{m_3}{m_1+m_2+m_3}T$

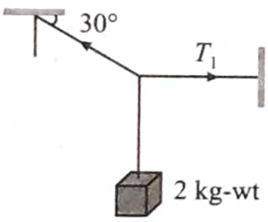
(*c*) $\dfrac{m_1+m_2}{m_1+m_2+m_3}T$ (*d*) $\dfrac{m_2+m_3}{m_1+m_2+m_3}T$

28. A light string passes over a frictionless pulley. To one of its ends a mass of 6 kg is attached. To its other end a mass of 10 kg is attached. The tension in the thread will be



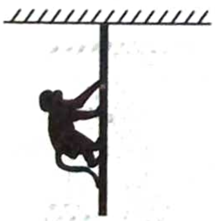
(*a*) 24.5 N (*b*) 2.45 N (*c*) 79 N (*d*) 73.5 N

29. A body of weight 2 kg is suspended as shown in the figure. The tension T_1 in the horizontal string (in kg wt) is



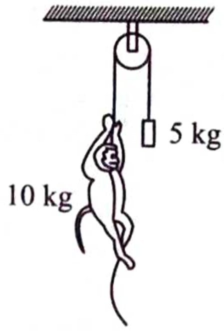
(*a*) $2/\sqrt{3}$ (*b*) $\sqrt{3}/2$ (*c*) $2\sqrt{3}$ (*d*) 2

30. A monkey of mass 40 kg climbs on a rope which can stand a maximum tension of 600 N. In which of the following cases will the rope break the monkey



(*a*) Climbs up with an acceleration of 6 ms^{-2}
(*b*) Climbs down with an acceleration of 4 ms^{-2}
(*c*) Climbs up with an uniform speed of 5 ms^{-1}
(*d*) Falls down the rope nearly freely under gravity?

31. In the figure shown acceleration of monkey relative to the rope if it exerts a force of 80 N on string will be:



(*a*) 2 m/s^2 downwards
(*b*) 4 m/s^2 upwards
(*c*) 4 m/s^2 downwards
(*d*) 8 m/s^2 downwards

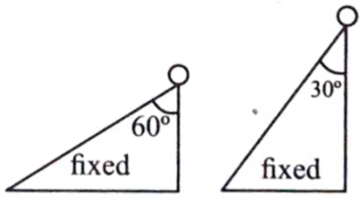
32. A person standing on the floor of an elevator drops a coin. The coin reaches the floor of the elevator in time t_1 when elevator is stationary and in time t_2 if it is moving uniformly. Then

(*a*) $t_1 = t_2$
(*b*) $t_1 > t_2$
(*c*) $t_1 > t_2$
(*d*) $t_1 < t_2$ or $t_1 > t_2$ depending upon that elevator is moving upwards or downwards

33. A block of mass M is placed on a fixed smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has a magnitude

(*a*) Mg
(*b*) $Mg/\cos\theta$
(*c*) $Mg\cos\theta$
(*d*) $Mg\tan\theta$

34. Two particles start together from a point O and slide down along straight smooth inclined planes at 30° and 60° to the vertical and in the same vertical plane as in figure. The relative acceleration of second with respect to first will be (in magnitude and direction) as



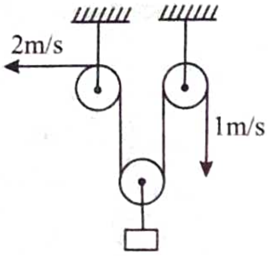
(*a*) $\dfrac{g}{2}$ in the vertical direction

(*b*) $\dfrac{g\sqrt{3}}{2}$ at 45° with vertical

(*c*) $\dfrac{g}{\sqrt{3}}$ inclined at 60° to vertical

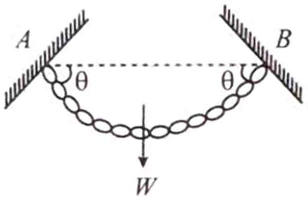
(*d*) g in the vertical direction

35. Find the velocity of the hanging block if the velocities of the free ends of the rope are as indicated in the figure.



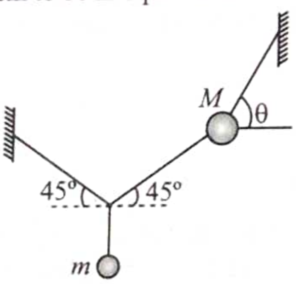
(a) 3/2 m/s ↑ (b) 3/2 m/s ↓
(c) 1/2 m/s ↑ (d) 1/2 m/s ↓

36. A flexible chain of weight W hangs between two fixed points A and B at the same level. The inclination of the chain with the horizontal at the two points of support is θ. What is the tension of the chain at the endpoint.



(a) $\frac{W}{2}$ cosec θ (b) $\frac{W}{2}$ sec θ
(c) $W \cos\theta$ (d) $\frac{W}{3}$ sin θ

37. Two masses m and M are attached with strings as shown. For the system to be in equilibrium we have

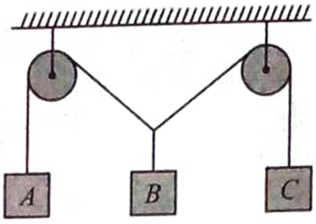


(a) $\tan\theta = 1 + \frac{2M}{m}$
(b) $\tan\theta = 1 + \frac{2m}{M}$
(c) $\tan\theta = 1 + \frac{M}{2m}$
(d) $\tan\theta = 1 + \frac{m}{2M}$

38. A balloon of gross weight w newton is falling vertically downward with a constant acceleration $a(<g)$. The magnitude of the air resistance is: (Neglecting buoyant force)

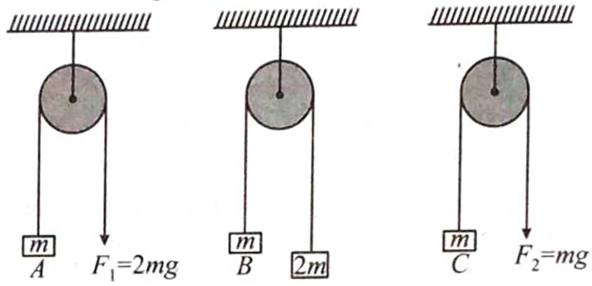
(a) w (b) $w\left(1+\frac{a}{g}\right)$
(c) $w\left(1-\frac{a}{g}\right)$ (d) $w\frac{a}{g}$

39. Three blocks A, B and C are suspended as shown in the figure. Mass of each block A and C is m. If system is in equilibrium and mass of B is M, then:



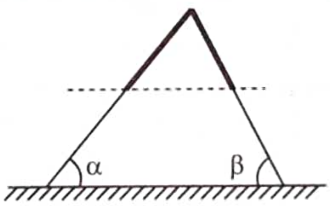
(a) $M = 2$ m (b*) $M < 2$ m
(c) $M > 2$ m (d) $M = m$

40. In the figure, the blocks A, B and C of mass m each have acceleration a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes 2 mg and mg respectively.



(a) $a_1 = a_2 = a_3$ (b*) $a_1 > a_2 > a_3$
(c) $a_1 = a_2, a_2 > a_3$ (d) $a_1 > a_2, a_2 = a_3$

41. A uniform rope of length L and mass M is placed on a smooth fixed wedge as shown. Both ends of rope are at same horizontal level. The rope is initially released from rest, then the magnitude of initial acceleration of rope is



(a) Zero (b) $M(\cos\alpha - \cos\beta)g$
(c) $M(\tan\alpha - \tan\beta)g$ (d) None of these

42. A force of magnitude F_1 acts on a particle so as to accelerate it from rest to a velocity v. The force F_1 is then replaced by another force of magnitude F_2 which decelerates it to rest.

(a) F_1 must be the equal to F_2
(b) F_1 may be equal to F_2
(c) F_1 must be unequal to F_2
(d) None of these

43. In an imaginary atmosphere, the air exerts a small force F on any particle in the direction of the particle's motion. A particle of mass m projected upward takes a time t_1 in reaching the maximum height and t_2 in the return journey to the original point. Then

(a) $t_1 < t_2$
(b) $t_1 > t_2$
(c) $t_1 = t_2$
(d) The relation between t_1 and t_2 depends on the mass of the particle.

CONSTRAINT RELATION, DYNAMICS OF MULTI SYSTEM SPRING

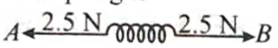
44. A man is standing at a spring platform. Reading of spring balance is 60 kg wt. If man jumps outside platform, then reading of spring balance

(*a*) First increases then decreases to zero
(*b*) Decreases
(*c*) Increases
(*d*) Remains same

45. A bird is sitting in a large closed cage which is placed on a spring balance. It records a weight of 25 N. The bird (mass m = 0.5 kg) flies upward in the cage with an acceleration of 2 m/s^2. The spring balance will now record a weight of

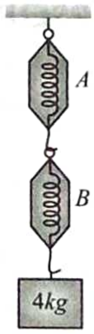
(*a*) 24 N (*b*) 25 (*c*) 26 N (*d*) 27 N

46. The tension in the spring is



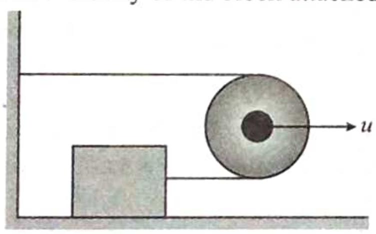
(*a*) Zero (*b*) 2.5 N
(*c*) 5 N (*d*) 10 N

47. A block of mass 4 kg is suspended through two light spring balances A and B. Then A and B will read respectively



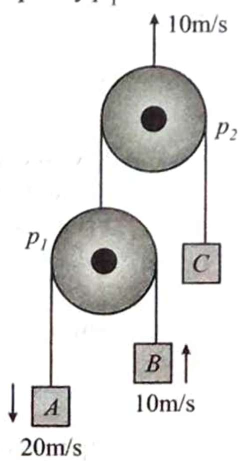
(*a*) 4 kg and zero kg
(*b*) Zero kg and 4 kg
(*c*) 4 kg and 4 kg
(*d*) 2 kg and 2 kg

48. In the figure shown, the pulley is moving with velocity u. Calculate the velocity of the block attached with string.



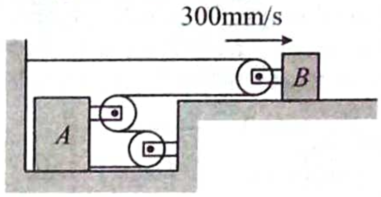
(*a*) u (*b*) $2u$ (*c*) $3u$ (*d*) $2.5\,u$

49. Velocities of blocks A, B and pulley p_1 are shown in figure. Find velocity of pulley p_1 and mass C.



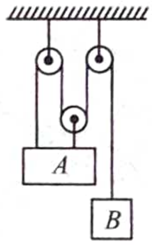
(*a*) 10 m/s (*b*) 20 m/s (*c*) 5 m/s (*d*) 25 m/s

50. If velocity of block B in the given arrangement is 300 mm/sec towards right. Then velocity of A will be



(*a*) 200 mm/sec
(*b*) 100 mm/sec
(*c*) 450 mm/sec
(*d*) 150 mm/sec

51. At a given instant, A is moving with velocity of 5 m/s upwards. What is velocity of B at the time

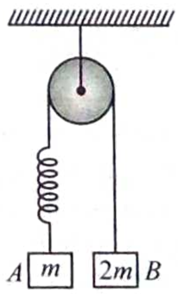


(*a*) 15 m/s↓ (*b*) 15 m/s↑
(*c*) 5 m/s ↓ (*d*) 5 m/s ↑

52. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block has a magnitude.

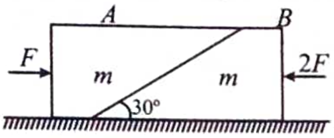
(*a*) mg
(*b*) $mg/\cos\theta$
(*c*) $mg\cos\theta$
(*d*) $mg\tan\theta$

53. In the figure a block 'A' of mass 'm' is attached at one end of a light spring and the other end of the spring is connected to another block 'B' of mass 2m through a light string. 'A' is held and B is in static equilibrium. Now A is released. The acceleration of A just after that instant is 'a'. In the next case, B is held and A is in static equilibrium. Now when B is released, its acceleration immediately after the release is 'b'. The value of a/b is:



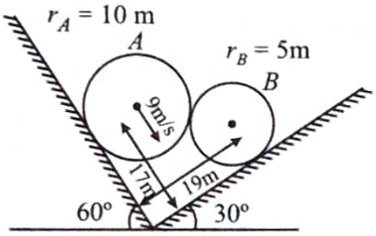
(*a*) 0 (*b*) $\frac{1}{2}$
(*c*) 2 (*d*) Undefined

54. Two blocks 'A' and 'B' each of mass 'm' are placed on a smooth horizontal surface. Two horizontal forces F and $2F$ are applied on the two blocks 'A' and 'B' respectively as shown in figure. The block A does not slide on block B. Then the normal reaction acting between the two blocks is:



(a) F (b) $F/2$ (c) $\frac{F}{\sqrt{3}}$ (d) $3F$

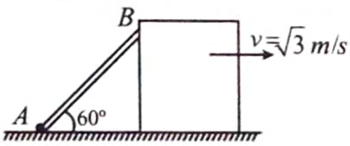
55. System is shown in the figure. Velocity of sphere A is 9 m/s. Then speed of sphere B will be



(a) 9 m/s (b) 12 m/s

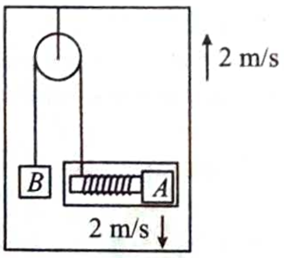
(c) $9 \times \frac{5}{4}$ m/s (d) None of these

56. A rod AB is shown in figure. End A of the rod is fixed on the ground. Block is moving with velocity $\sqrt{3}$ m/s towards right. The velocity of end B of rod when rod makes an angle of 60° with the ground is:



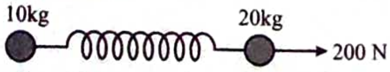
(a) $\sqrt{3}$ m/s (b) 2 m/s (c) $2\sqrt{3}$ m/s (d) 3 m/s

57. In the figure shown the velocity of lift is 2 m/s while string is winding on the motor shaft with velocity 2 m/s and block A is moving downwards with a velocity of 2 m/s, then find out the velocity of block B.



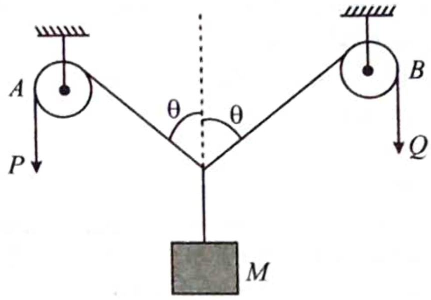
(a) 2 m/s ↑ (b) 2 m/s ↓ (c) 4 m/s ↑ (d) 6 m/s ↑

58. Two masses of 10 kg and 20 kg respectively are connected by a massless spring as shown in figure. A force of 200 N acts on the 20 kg mass at the instant when the 10 kg mass has an acceleration of 12 ms^{-2} towards right, the acceleration of the 20 kg mass is:



(a) 2 ms^{-2} (b) 4 ms^{-2} (c) 10 ms^{-2} (d) 20 ms^{-2}

59. In the arrangement shown in figure the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed.



(a) $2U \cos\theta$ (b) $U \cos\theta$

(c) $\frac{2U}{\cos\theta}$ (d) $\frac{U}{\cos\theta}$

NON INERTIAL REFERENCE FRAME AND PSEUDO FORCE

60. A person is standing in an elevator. In which situation he finds his weight less than actual when

(a) The elevator moves upward with constant acceleration

(b) The elevator moves downward with constant acceleration.

(c) The elevator moves upward with uniform velocity

(d) The elevator moves downward with uniform velocity

61. A body of mass 4 kg weighs 4.8 kg when suspended in a moving lift. The acceleration of the lift is

(a) 9.80 ms^{-2} downwards

(b) 9.80 ms^{-2} upwards

(c) 1.96 ms^{-2} downwards

(d) 1.96 ms^{-2} upwards

62. The mass of a lift is 500 kg. When it ascends with an acceleration of 2 m/s², the tension in the cable will be [g = 10 m/s²]

(a) 6000 N (b) 5000 N

(c) 4000 N (d) 50 N

63. If in a stationary lift, a man is standing with a bucket full of water, having a hole at its bottom. The rate of flow of water through this hole is R_0. If the lift starts to move up and down with same acceleration and then that rates of flow of water are R_u and R_d, then

(a) $R_0 > R_u > R_d$

(b) $R_u > R_0 > R_d$

(c) $R_d > R_0 > R_u$

(d) $R_u > R_d > R_0$

64. A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of a. What will be the angle of inclination with vertical

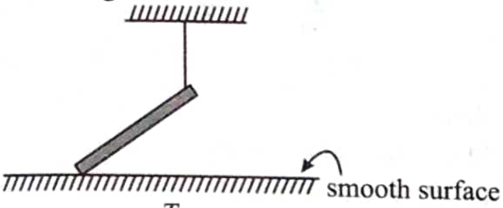
(a) $\tan^{-1}(a/g)$ (b) $\tan^{-1}(g/a)$

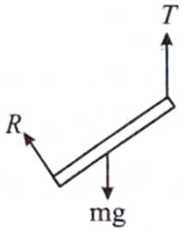
(c) $\cos^{-1}(a/g)$ (d) $\cos^{-1}(g/a)$

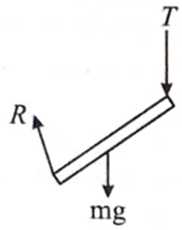
Exercise-2 (Learning Plus)

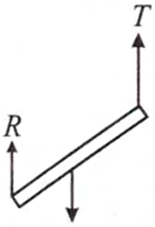
1. Let E, G and N represents the magnitude of electromagnetic, gravitational and nuclear forces between two protons at a given separation (1 fermi meter). Then
   (a) $N < E < G$ (b) $E > N > G$
   (c) $G > N > E$ (d) $N > E > G$

2. Which figure represents the correct F.B.D. of rod of mass m as shown in figure:



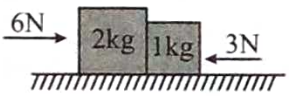
(a) 

(b) 

(c) 

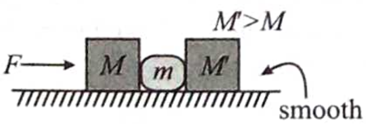
(d) None of these

3. Two forces of 6 N and 3 N are acting on the two blocks of 2 kg and 1 kg kept on frictionless floor. What is the force exerted on 2 kg block by 1 kg block?



   (a) 1 N (b) 2 N
   (c) 4 N (d) 5 N

4. A constant force F is applied in horizontal direction as shown. Contact force between M and m is N and between m and M' is N' then

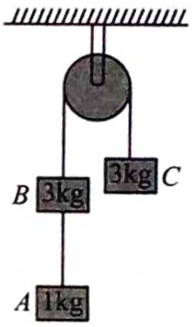


   (a) $N = N'$ (b) $N > N'$
   (c) $N' > N$ (d) Cannot be determined

5. In which of the following cases the net force is not zero?
   (a) A kite skillfully held stationary in the sky
   (b) A ball freely falling from a height
   (c) An aeroplane rising upwards at an angle of 45° with the horizontal with a constant speed
   (d) A cork floating on the surface of water

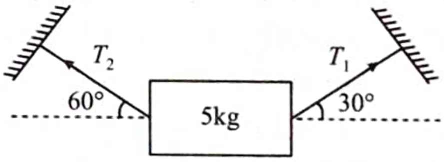
6. Two persons are holding a rope of negligible weight tightly at its ends so that it is horizontal. A 15 kg weight is attached to the rope at the mid point which now no longer remains horizontal. The minimum tension required to completely straighten the rope is
   (a) 15 kg (b) $\frac{15}{2}$ kg
   (c) 5 kg
   (d) Infinitely large (or not possible)

7. In the system shown in the figure, the acceleration of the 1 kg mass and the tension in the string connecting between A and B is



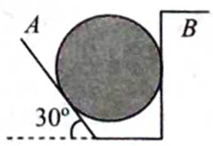
   (a) $\frac{g}{4}$ downwards, $\frac{8g}{7}$
   (b) $\frac{g}{4}$ upwards, $\frac{g}{7}$
   (c) $\frac{g}{7}$ downwards, $\frac{6}{7}$g
   (d) $\frac{g}{2}$ upwards, g

8. A body of mass 5 kg is suspended by the strings making angles 60° and 30° with the horizontal



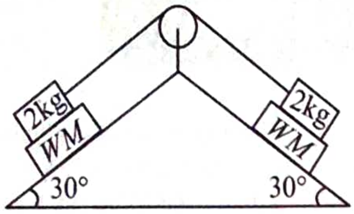
   A. $T_1 = 25$ N B. $T_2 = 25$ N
   C. $T_1 = 25\sqrt{3}$ N D. $T_2 = 25\sqrt{3}$ N
   (a) A, B (b) A, D (c) C, D (d) B, C

9. The 50 kg homogeneous smooth sphere rests on the 30° incline A and bears against the smooth vertical wall B. Calculate the contact forces at A and B.



   (a) $N_B = \frac{1000}{\sqrt{3}} N$, $N_A = \frac{500}{\sqrt{3}} N$
   (b) $N_A = \frac{1000}{\sqrt{3}} N$, $N_B = \frac{500}{\sqrt{3}} N$
   (c) $N_A = \frac{100}{\sqrt{3}} N$, $N_B = \frac{500}{\sqrt{3}} N$
   (d) $N_A = \frac{1000}{\sqrt{3}} N$, $N_B = \frac{50}{\sqrt{3}} N$

10. Find out the reading of the weighing machine in the following cases.



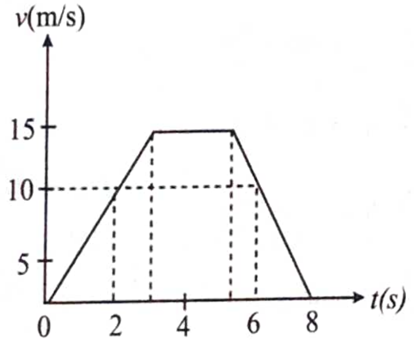
(a) $10\sqrt{3}$

(b) $10\sqrt{2}$

(c) $20\sqrt{3}$

(d) $30\sqrt{3}$

11. A particle of mass 50 gram moves on a straight line. The variation of speed with time is shown in figure. Find the force acting on the particle at $t = 2$, 4 and 6 seconds.



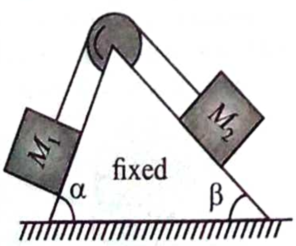
(a) 0.25 N along motion, zero, 0.25 opposite to motion

(b) 0.25 N along motion, zero, 0.25 along motion

(c) 0.25 N opposite to motion, zero, 0.25 along motion

(d) 0.25 N opposite to motion, zero, 0.25 opposite motion

12. Two masses M_1 and M_2 are attached to the ends of a light string which passes over a massless pulley attached to the top of a double inclined smooth plane of angles of inclination α and β. If $M_2 > M_1$ and $\beta > \alpha$ then the acceleration of block M_2 down the incline will be:



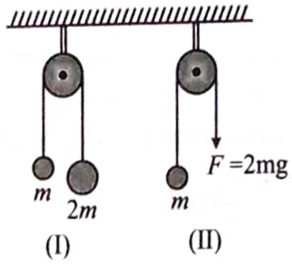
(a) $\dfrac{M_2 g(\sin\beta)}{M_1 + M_2}$

(b) $\dfrac{M_1 g(\sin\alpha)}{M_1 + M_2}$

(c) $\left[\dfrac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right]g$

(d) Zero

13. The pulley arrangements shown in figure are identical, the mass of the rope being negligible. In case-I, the mass m is lifted by attaching a mass 2m to the other end of the rope. In case-II, the mass m is lifted by pulling the other end of the rope with a constant downward force $F = 2$mg, where g is acceleration due to gravity. The acceleration of mass in case-I is



(a) Zero

(b) More than that in case-II

(c) Less than that in case-II

(d) Equal to that in case-II

14. A fireman wants to slide down a rope. The rope can bear a tension of $\frac{3}{4}th$ of the weight of the man. With what minimum acceleration should the fireman slide down:

(a) $\dfrac{g}{3}$ (b) $\dfrac{g}{6}$

(c) $\dfrac{g}{4}$ (d) $\dfrac{g}{2}$

15. A rope of mass 5 kg is moving vertically with an upwards force of 100 N acting at the upper end and a downwards force of 70 N acting at the lower end. The tension at midpoint of the rope is

(a) 100 N (b) 85 N

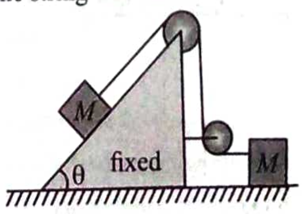
(c) 75 N (d) 105 N

16. A body is moving with a speed of 1 m/s and a force F is needed to stop it in a distance x. If the speed of the body is 3 m/s the force needed to stop it in the same distance x will be

(a) 1.5F (b) 3F

(c) 6F (d) 9F

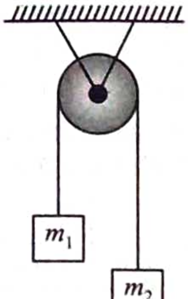
17. Two blocks, each having mass M, rest on frictionless surfaces as shown in the figure. If the pulleys are light and frictionless, and M on the incline is allowed to move down, then the tension in the string will be:



(a) $\dfrac{2}{3}Mg\sin\theta$ (b) $\dfrac{3}{2}Mg\sin\theta$

(c) $\dfrac{Mg\sin\theta}{2}$ (d) 2 $Mg\sin\theta$

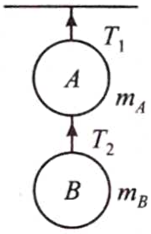
18. Two masses are hanging vertically over frictionless pulley. The acceleration of the two masses is



(a) $\frac{m_1}{m_2}g$ (b) $\frac{m_2}{m_1}g$

(c) $\left(\frac{m_2-m_1}{m_1+m_2}\right)g$ (d) $\left(\frac{m_1+m_2}{m_2-m_1}\right)g$

19. Two objects A and B of masses m_A and m_B are attached by strings as shown in fig. If they are given upward acceleration, then the ratio of tension $T_1 : T_2$ is



(a) $(m_A+m_B)/m_B$ (b) $(m_A+m_B)/m_A$

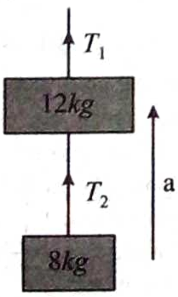
(c) $\frac{m_A+m_B}{m_A-m_B}$ (d) $\frac{m_A-m_B}{m_A+m_B}$

20. A monkey of mass 20 kg is holding a vertical rope. The rope can break when a mass of 25 kg is suspended from it. What is the maximum acceleration with which the monkey can climb up along the rope?

(a) $7\ ms^{-2}$ (b) $10\ ms^{-2}$

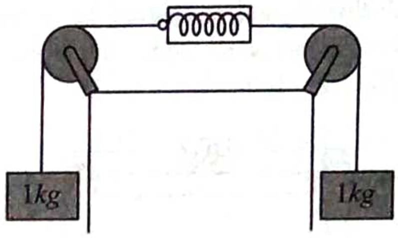
(c) $5\ ms^{-2}$ (d) $2.5\ ms^{-2}$

21. A body of mass 8 kg is hanging from another body of mass 12 kg. The combination is being pulled by a string with an acceleration of $2.2\ ms^{-2}$. The tension T_1 and T_2 will be respectively: (use $g = 9.8 m/s^2$)



(a) 200 N, 80 N
(b) 20 N, 90 N
(c) 240 N, 96 N
(d) 260 N, 96 N

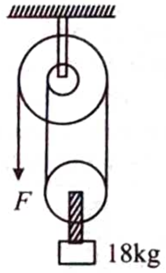
22. In the given figure, what is the reading of the spring balance?



(a) 10 N (b) 20 N

(c) 5 N (d) Zero

23. In the figure at the free end a force F is applied to keep the suspended mass of 18 kg at rest. The value of F is-



(a) 180 N (b) 90 N

(c) 60 N (d) 30 N

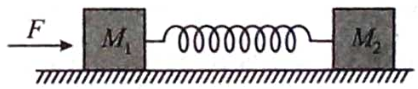
24. If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be

A. going up with increasing speed
B. going down with increasing speed
C. going up with uniform speed
D. going down with uniform speed

(a) A, D (b) A, B, C

(c) C, D (d) A, B

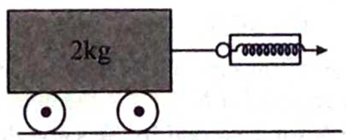
25. Two blocks of masses M_1 and M_2 are connected to each other through a light spring as shown in figure. If we push mass M_1 with force F and cause acceleration a_1 in right direction in mass M_1, what will be the magnitude of acceleration in M_2?



(a) F/M_2 (b) $F/(M_1+M_2)$

(c) a_1 (d) $(F-M_1a_1)/M_2$

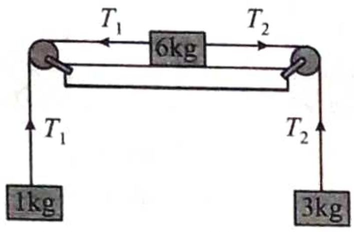
26. A massless spring balance is attached to 2 kg trolley and is used to pull the trolley along a flat surface as shown in the fig. The reading on the spring balance remains at 10 kg during the motion. The acceleration of the trolley is (Use $g = 9.8\ ms^{-2}$)



(a) $4.9\ ms^{-2}$ (b) $9.8\ ms^{-2}$

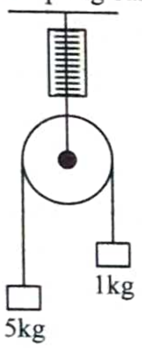
(c) $49\ ms^{-2}$ (d) $98\ ms^{-2}$

27. Three masses of 1 kg, 6 kg and 3 kg are connected to each other with threads and are placed on table as shown in figure. What is the acceleration with which the system is moving? Take $g = 10\ ms^{-2}$.



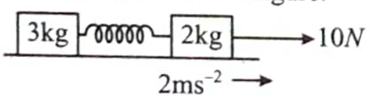
(*a*) Zero (*b*) $1\ ms^{-2}$
(*c*) $2\ ms^{-2}$ (*d*) $3\ ms^{-2}$

28. In the figure a smooth pulley of negligible weight is suspended by a spring balance. Weights of 1 kg and 5 kg are attached to the opposite ends of a string passing over the pulley and move with acceleration because of gravity. During the motion, the spring balance reads a weight of



(*a*) 6 kg
(*b*) Less than 6 kg
(*c*) More than 6 kg
(*d*) May be more or less than 6 kg

29. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is $2\ ms^{-2}$ as shown in figure.



(*a*) $3\ ms^{-2}$ (*b*) $2\ ms^{-2}$ (*c*) $0.5\ ms^{-2}$ (*d*) Zero

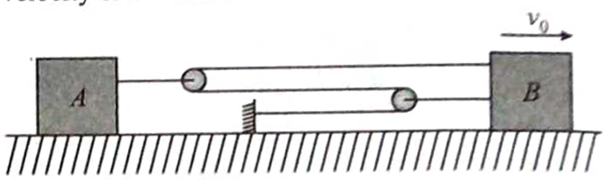
30. The ratio of the weight of a man in a stationary lift and when it is moving downward with uniform acceleration '*a*' is 3 : 2. The value of '*a*' is: (g = acceleration. due to gravity)
(*a*) (3/2) g (*b*) g
(*c*) (2/3) g (*d*) $g/3$

31. A ball weighing 10 gm hits a hard surface vertically with a speed of 5m/s and rebounds with the same speed. The ball remain in contact with the surface for 0.01 sec. The average force exerted by the surface on the ball is :
(*a*) 100 N (*b*) 10 N (*c*) 1 N (*d*) 150 N

32. A 10 kg wagon is pushed with a force of 7N for 1.5 second, then with a force of 5 N for 1.7 seconds, and then with a force of 10 N for 3 second in the same direction. What is the change in velocity brought about ?
(*a*) 9.8 m/s (b) 19.6 m/s (*c*) 4.9 m/s (d) 10 m/s

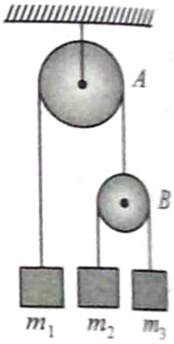
33. What will be the displacement of a block in first 0.2s if the block is kept on the floor of an elevator at rest. Suddenly elevator starts descending with an acceleration of 13 m/s^2. Take $g = 10\ m/s^2$.
(*a*) 26 cm (*b*) 6 cm (*c*) 46 cm (*d*) 20 cm

34. Block B moves to the right with a constant velocity v_0. The velocity of block A relative to B is



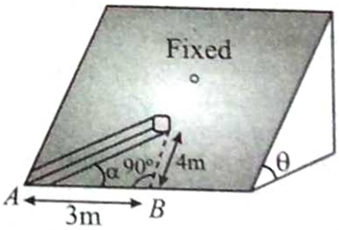
(*a*) $\frac{v_0}{2}$, towards left (*b*) $\frac{v_0}{2}$, towards right
(*c*) $\frac{3v_0}{2}$, towards left (*d*) $\frac{3v_0}{2}$, towards right

35. In the arrangement shown in figure, pulleys are massless and frictionless and threads are light and inextensible. Block of mass m_1 will remain at rest if:



(*a*) $\frac{1}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}$
(*b*) $\frac{4}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}$
(*c*) $m_1 = m_2 + m_3$
(*d*) $\frac{1}{m_3} = \frac{2}{m_2} + \frac{3}{m_1}$

36. There is an inclined surface of inclination $\theta = 30°$. A smooth groove is cut into it forming angle α with AB. A steel ball is free to slide along the groove. If the ball is released from the point O at top end of the groove, the speed when it comes to A is: [$g = 10\ m/s^2$]

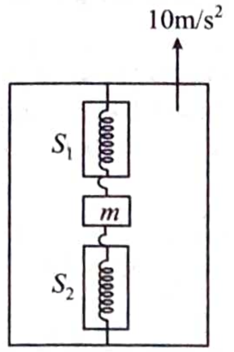


(*a*) $\sqrt{40}$ m/s (*b*) $\sqrt{20}$ m/s
(*c*) $\sqrt{10}$ m/s (*d*) $\sqrt{15}$ m/s

37. A block tied between two springs is in equilibrium. If upper spring is cut then the acceleration of the block just after cut is 6 m/s^2 downwards. Now, if instead of upper spring, lower spring is cut then the magnitude of acceleration of the block just after the cut will be : (Take $g = 10\ m/s^2$)

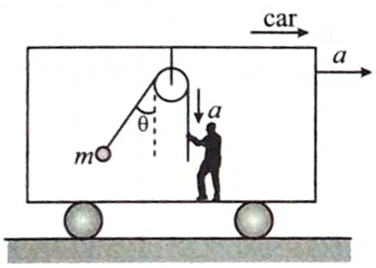
(*a*) $16\ m/s^2$
(*b*) $4\ m/s^2$
(*c*) Cannot be determined
(*d*) None of these

38. Reading shown in two spring balances S_1 and S_2 is 90 kg and 30 kg respectively when lift is accelerating upwards with acceleration 10 m/s^2. The mass is stationary with respect to lift. Then the mass of the block will be:



(*a*) 60 kg (*b*) 30 kg
(*c*) 120 kg (*d*) None of these

39. A bob is hanging over a pulley inside a car through a string . The second end of the string is in the hand of a person standing in the car. The car is moving with constant acceleration '*a*' directed horizontally as shown in figure . Other end of the string is pulled with constant acceleration '*a*' (relative to car) vertically. The tension in the string is equal to



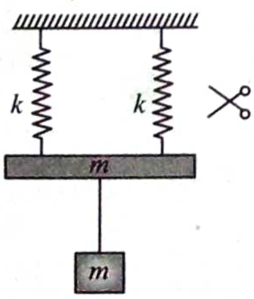
(*a*) $m\sqrt{g^2+a^2}$

(*b*) $m\sqrt{g^2+a^2}-ma$

(*c*) $m\sqrt{g^2+a^2}+ma$

(*d*) $m(g+a)$

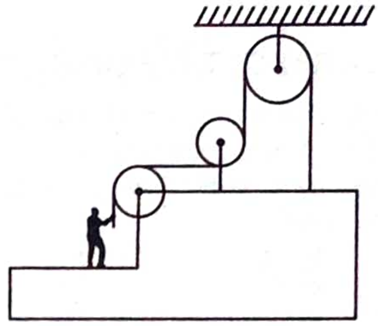
40. System shown in figure is in equilibrium. The magnitude of change in tension in the string just before and just after, when one of the spring is cut. Mass of both the blocks is same and equal to m and spring constant of both springs is *k*. (Neglect any effect of rotation)



(*a*) $\dfrac{mg}{2}$ (*b*) $\dfrac{mg}{4}$

(*c*) $\dfrac{3mg}{4}$ (*d*) $\dfrac{3mg}{2}$

41. A system is shown in the figure. A man standing on the block is pulling the rope. Velocity of the point of string in contact with the hand of the man is 2 m/s downwards. The velocity of the block will be: [Assume that the block does not rotate]



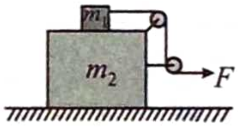
(*a*) 3 m/s

(*b*) 2 m/s

(*c*) 1/2 m/s

(*d*) 1 m/s

42. In the arrangement shown in the figure all surfaces are frictionless, pulley and string are light. The masses of the block are m_1 = 20 kg and m_2 = 30 kg. The accelerations of masses m_1 and m_2 will be if F = 180 N is applied according to figure.



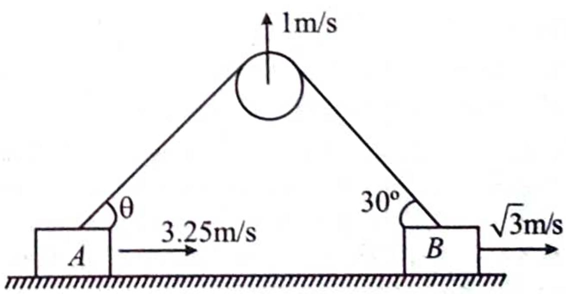
(*a*) $a_{m_1} = 9\,\text{m/s}^2,\ a_{m_2} = 0$

(*b*) $a_{m_1} = 9\text{m/s}^2,\ a_{m_2} = 9\ \text{m/s}^2$

(*c*) $a_{m_1} = 0,\ a_{m_2} = 9\,\text{m/s}^2$

(*d*) None of these

43. In the figure shown, find out the value of θ at this instant [assume string to be tight]



(*a*) $\tan^{-1}\dfrac{3}{4}$

(*b*) $\tan^{-1}\dfrac{4}{3}$

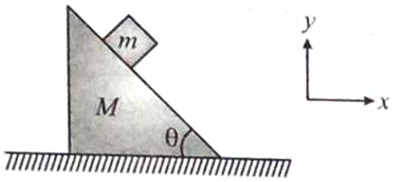
(*c*) $\tan^{-1}\dfrac{3}{8}$

(*d*) None of these

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. Consider the shown arrangement. Assume all surfaces to be smooth. If 'N' represents magnitude of normal reaction between block and wedge then acceleration of 'M' along horizontal equals:



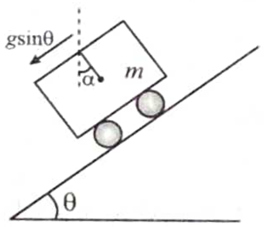
(a) $\frac{N\sin\theta}{M}$ along + ve-x-axis

(b) $\frac{N\cos\theta}{M}$ along-ve x-axis

(c) $\frac{N\sin\theta}{M}$ along – ve x-axis

(d) $\frac{N\sin\theta}{m+M}$ along –ve x-axis

2. A trolley is accelerating down an incline of angle θ with acceleration g sin θ. Which of the following is correct. (α is the constant angle made by the string with vertical).



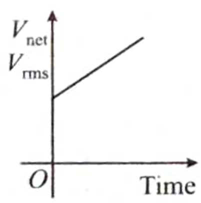
(a) $\alpha = \theta$

(b) $\alpha = 0°$

(c) Tension in the string, $T = mg$

(d) Tension in the string, $T = mg \sec\theta$

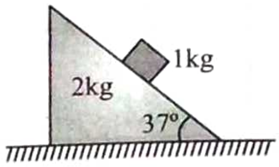
3. A particle is observed from two frames S_1 and S_2. The graph of relative velocity of S_1 with respect to S_2 is shown in figure. Let F_1 and F_2 be the pseudo forces on the particle when seen from S_1 and S_2 respectively. Which one of the following is not possible?



(a) $F_1 = 0, F_2 \neq 0$ (b) $F_1 \neq 0, F_2 = 0$

(c) $F_1 \neq 0, F_2 \neq 0$ (d) $F_1 = 0, F_2 = 0$

4. Figure shows a wedge of mass 2 kg resting on a frictionless floor. A block of mass 1 kg is kept on the wedge and the wedge is given an acceleration of 5 m/sec^2 towards right. Then:



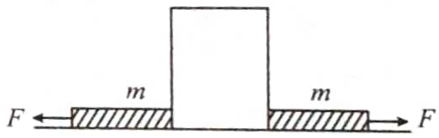
(a) Block will remain stationary w.r.t. wedge

(b) The block will have an acceleration of 1 m/sec^2 w.r.t. the wedge

(c) Normal reaction on the block is 11 N

(d) Net force acting on the wedge is 2 N

5. A heavy block kept on a frictionless surface and being pulled by two ropes of equal mass m as shown in figure. At $t = 0$, the force on the left rope is withdrawn but the force on the right end continues to act. Let F_1 and F_2 be the magnitudes of the forces by the right rope and the left rope on the block respectively.



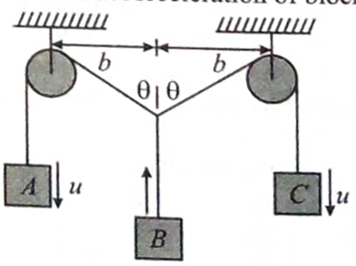
(a) $F_1 = F_2 = F$ for $t < 0$

(b) $F_1 = F_2 = F + mg$ for $t < 0$

(c) $F_1 = F, F_2 = F$ for $t > 0$

(d) $F_1 < F, F_2 = F$ for $t > 0$

6. In the figure shown the blocks A and C are pulled down with constant velocities u. Acceleration of block B is:



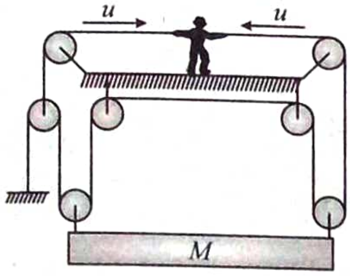
(a) $\frac{u^2}{b}\tan^2\theta\sec\theta$

(b) $\frac{u^2}{b}\tan^3\theta$

(c) $\frac{u^2}{b}\sec^2\theta\tan\theta$

(d) Zero

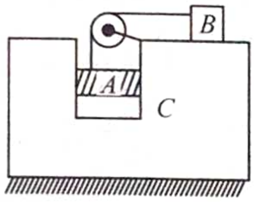
7. System is shown in the figure and man is pulling the rope from both sides with constant speed 'u'. Then the speed of the block will be (M moves vertical):



(a) $\frac{3u}{4}$ (b) $\frac{3u}{2}$

(c) $\frac{u}{4}$ (d) None of these

8. In the system shown in figure $m_A = 4m$, $m_B = 3m$ and $m_C = 8m$. Friction is absent everywhere. String is light and inextensible. If the system is released from rest find the acceleration of block B



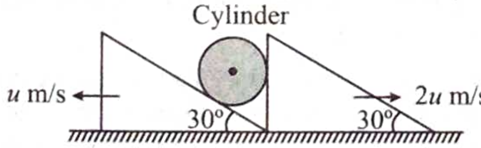
(a) $\frac{g}{8}$ (leftward)

(b) $\frac{g}{2}$ (leftward)

(c) $\frac{g}{6}$ (rightward)

(d) $\frac{g}{4}$ (rightward)

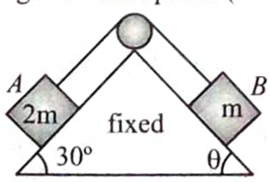
9. System is shown in the figure. Assume that cylinder remains in contact with the two wedges. The velocity of cylinder is-



(a) $\sqrt{19-4\sqrt{3}}\,\frac{u}{2}$ m/s (b) $\frac{\sqrt{13}u}{2}$ m/s

(c) $\sqrt{3}u$ m/s (d*) $\sqrt{7}u$ m/s

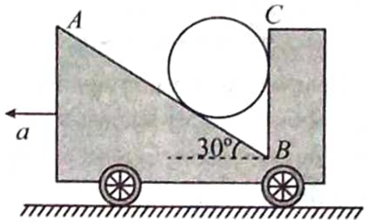
10. The value of angle θ such that the acceleration of A is g/6 downward along the incline plane. (All surfaces are smooth)



(a) $\theta = 30°$ (b) $\theta = 60°$

(c) $\theta = 45°$ (d) $\theta = 53°$

11. A cylinder rests in a supporting carriage as shown. The side AB of carriage makes an angle 30° with the horizontal and side BC is vertical. The carriage lies on a fixed horizontal surface and is being pulled towards left with an horizontal acceleration 'a'. The magnitude of normal reactions exerted by sides AB and BC of carriage on the cylinder be N_{AB} and N_{BC} respectively. Neglect friction everywhere. Then as the magnitude of acceleration 'a' of the carriage is increased, pick up the correct statement:



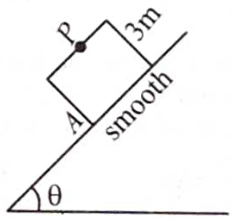
(a) N_{AB} increases and N_{BC} decreases.

(b) Both N_{AB} and N_{BC} increase.

(c) N_{AB} remains constant and N_{BC} increases.

(d) N_{AB} increases and N_{BC} remains constant.

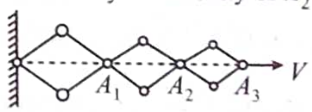
12. A cuboidal car of height 3 m is slipping on a smooth inclined plane. A bolt released from the roof of car from centre of roof (P) then distance from centre of roof where bolt hits the floor with respect to car is:



(a) 5 m (b) 4 m

(c) 3m (d) None of these

13. A hinged construction consists of three rhombus with the ratio of sides (5 : 3 : 2). Vertex A_3 moves in the horizontal direction with velocity V. Velocity of A_2 will be:



(a) 2.5 V (b) 1.5V

(c) (2/3)V (d) 0.8 V

14. Five persons A, B, C, D and E are pulling a cart of mass 100 kg on a smooth surface and cart is moving with acceleration 3 m/s^2 in east direction. When person 'A' stops pulling, it moves with acceleration 1m/s^2 in the west direction. When only person 'B' stops pulling, it moves with acceleration 24 m/s^2 in the north direction. The magnitude of acceleration of the cart when only A and B pull the cart keeping their directions same as the old directions, is:

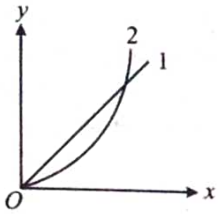
(a) 26 m/s^2

(b) $3\sqrt{71}$ m/s^2

(c) 25 m/s^2

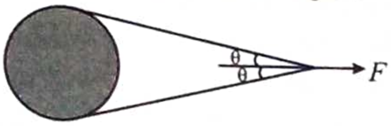
(d) 30 m/s^2

15. A particle is resting on a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time according to law $F = \alpha.t$, where α is a constant. For the figure shown which of the following statements is/are correct?



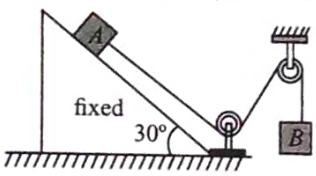
(*a*) Curve 1 shows acceleration against time.
(*b*) Curve 2 shows velocity against time.
(*c*) Curve 2 shows velocity against acceleration.
(*d*) None of these.

16. A string is wrapped round a log of wood and it is pulled with a constant force F as shown in the figure.



(*a*) Tension T in the string increases with increase in θ
(*b*) Tension T in the string decreases with increase in θ
(*c*) Tension $T > F$ if $\theta > \pi/3$
(*d*) Tension $T > F$ if $\theta > \pi/4$

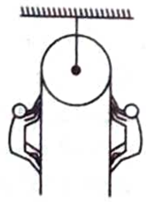
17. Two blocks A and B of equal mass m are connected through a massless string and arranged as shown in figure. Friction is absent everywhere. When the system is released from rest.



(*a*) Tension in string is $\frac{mg}{2}$ (*b*) Tension in string is $\frac{mg}{4}$

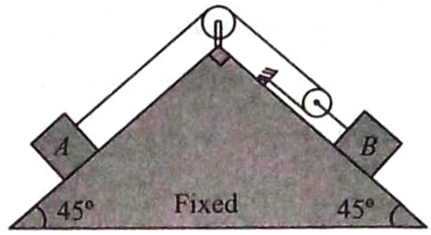
(*c*) Acceleration of A is $\frac{g}{2}$ (*d*) Acceleration of A is $\frac{3}{4}g$

18. Two men of unequal masses hold on to the two sections of a light rope passing over a smooth light pulley. Which of the following are possible?



(*a*) The lighter man is stationary while the heavier man slides with some acceleration
(*b*) The heavier man is stationary while the lighter man climbs with some acceleration
(*c*) The two men slide with the same acceleration in the same direction
(*d*) The two men move with accelerations of the same magnitude in opposite directions

19. Two blocks A and B of mass 10 kg and 40 kg are connected by an ideal string as shown in the figure. Neglect the masses of the pulleys and effect of friction.($g = 10$ m/s^2)



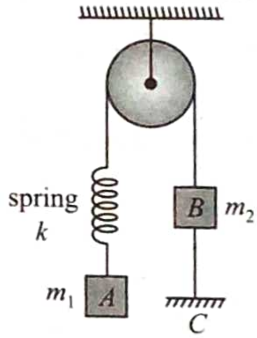
(*a*) The acceleration of block A is $\frac{5}{\sqrt{2}}$ ms^{-2}

(*b*) The acceleration of block B is $\frac{5}{2\sqrt{2}}$ ms^{-2}

(*c*) The tension in the string is $\frac{125}{\sqrt{2}}$ N

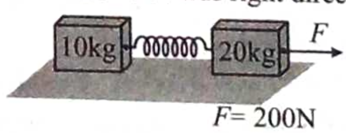
(*d*) The tension in the string is $\frac{150}{\sqrt{2}}$ N

20. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt:



(*a*) Acceleration of m_2 will be upwards
(*b*) Magnitude of acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$
(*c*) Acceleration of m_1 will be equal to zero
(*d*) Magnitude of acceleration of two blocks will be non-zero and unequal

21. Two blocks of masses 10 kg and 20 kg are connected by a light spring as shown. A force of 200 N acts on the 20 kg mass as shown. At a certain instant the acceleration of 10 kg mass is 12 ms^{-2} towards right direction.



(*a*) At that instant the 20 kg mass has an acceleration of 12 ms^{-2}
(*b*) At that instant the 20 kg mass has an acceleration of 4 ms^{-2}
(*c*) The stretching force in the spring is 120 N
(*d*) The collective system moves with a common acceleration of 30 ms^{-2} when the extension in the connecting spring is the maximum

22. A particle stays at rest as seen in a frame. We can conclude that

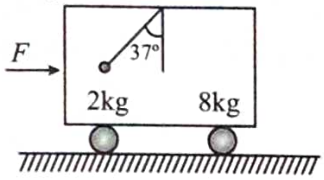
(a) The frame is inertial

(b) Resultant force on the particle is zero

(c) If the frame is inertial then the resultant force on the particle is zero

(d) If the frame is non-inertial then there is a non zero resultant force

23. A trolley of mass 8 kg is standing on a frictionless surface inside which an object of mass 2 kg is suspended. A constant force F starts acting on the trolley as a result of which the string stood at an angle of 37° from the vertical (bob at rest relative to trolley) Then :



(a) Acceleration of the trolley is 40/3 m/sec^2.

(b) Force applied is 60 N

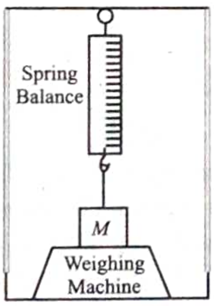
(c) Force applied is 75 N

(d) Tension in the string is 25 N

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 24 to 26): Figure shows a weighing machine kept in a lift. Lift is moving upwards with acceleration of 5 m/s^2. A block is kept on the weighing machine. Upper surface of block is attached with a spring balance. Reading shown by weighing machine and spring balance is 15 kg and 45 kg respectively.

Answer the following questions. Assume that the weighing machine can measure weight by having negligible deformation due to block, while the spring balance requires larger expansion: (take g = 10 m/s^2)



24. Mass of the object in kg and the normal force acting on the block due to weighing machine are

(a) 60 kg, 450 N

(b) 40 kg, 150 N

(c) 80 kg, 400 N

(d) 10 kg, zero

25. If lift is stopped and equilibrium is reached. Reading of weighing machine and spring balance will be

(a) 40 kg, zero (b) 10 kg, 20 kg

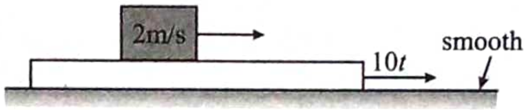
(c) 20 kg, 10 kg (d) Zero, 40 kg

26. Find the acceleration of the lift such that the weighing machine shows its true weight

(a) $\frac{45}{4}$ m/s^2 (b) $\frac{85}{4}$ m/s^2

(c) $\frac{22}{4}$ m/s^2 (d) $\frac{60}{4}$ m/s^2

Comprehension (Q. 27 to 29): A small block of mass 1 kg starts moving with constant velocity 1 m/s on a smooth long plank of mass 10 kg which is also pulled by a horizontal force $F = 10\,t\,N$ where t is in seconds and F is in newtons. (the initial velocity of the plank is zero).



27. Displacement of 1 kg block with respect to plank at the instant when both have same velocity is

(a) 4 m (b) 4 m

(c) 8/3 m (d) 2 m

28. The time ($t \neq 0$) at which displacement of block and plank with respect to ground is same will be

(a) 12 s (b) $2\sqrt{3}$ s

(c) $3\sqrt{3}$ s (d) $\sqrt{3}/2$ s

29. Relative velocity of plank with respect to block when acceleration of plank is 4 m/s^2 will be

(a) Zero (b) 10 m/s

(c) 6 m/s (d) 8 m/s

Comprehension (Q. 30 to 32): An object of mass 2 kg is placed at rest in a frame (S_1) moving with velocity $10\hat{i} + 5\hat{j}$ m/s and having acceleration $5\hat{i} + 10\hat{j}\,m/s^2$. The object is also seen by an observer standing in a frame (S_2) moving with velocity $5\hat{i} + 10\hat{j}\,m/s$

30. Calculate 'Pseudo force' acting on object. Which frame is responsible for this force.

(a) $F = -10\hat{i} - 20\hat{j}$ due to acceleration of frame S_1

(b) $F = -20\hat{i} - 20\hat{j}$ due to acceleration of frame S_1

(c) $F = -10\hat{i} - 30\hat{j}$ due to acceleration of frame S_1

(d) None of these

31. Calculate net force acting on object with respect to S_2 frame.

(a) $F = 20\hat{i} + 20\hat{j}$

(b) $F = 10\hat{i} + 20\hat{j}$

(c) $F = 5\hat{i} + 20\hat{j}$

(d) $F = 10\hat{i} + 5\hat{j}$

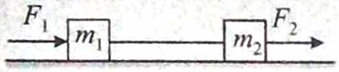
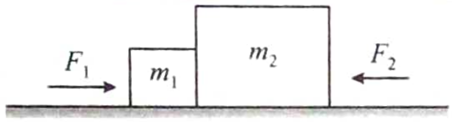
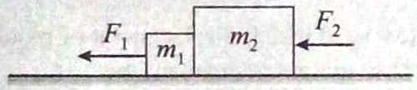
32. Calculate net force acting on object with respect of S_1 frame.

(a) 0 (b) 1

(c) 2 (d) None of these

MATCH THE COLUMN TYPE QUESTIONS

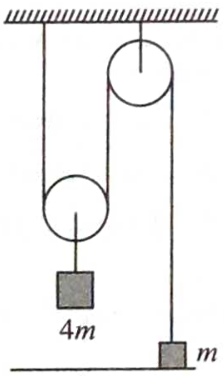
33. Column-I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on a smooth horizonta surface as shown. In each of the situations horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 respectively ac also $m_2 F_1 < m_1 F_2$. Match the statements in column-I with corresponding results in column-II.

	Column-I		Column-II
A.	 Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is	p.	$\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_1}{m_1}-\frac{F_2}{m_2}\right)$
B.	 Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is	q.	$\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_1}{m_1}+\frac{F_2}{m_2}\right)$
C.	 The magnitude of normal reaction between the blocks is	r.	$\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_2}{m_2}-\frac{F_1}{m_1}\right)$
D.	 The magnitude of normal reaction between the blocks is	s.	$m_1 m_2\left(\frac{F_1+F_2}{m_1+m_2}\right)$

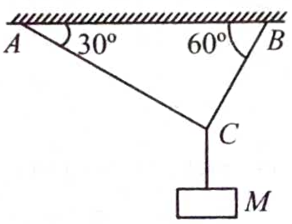
(*a*) A-(q); B-(r); C-(q); D-(r)
(*b*) A-(p); B-(q); C-(r); D-(s)
(*c*) A-(r); B-(p); C-(s); D-(q)
(*d*) A-(q); B-(r); C-(p); D-(r)

NUMERICAL TYPE QUESTIONS

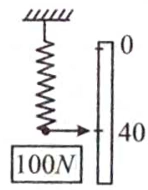
34. The mass of the body which is hanging on the rope attached to the movable pulley is four times as much as the mass of the body which is fixed to the ground. At a given instant the fixed body is released. What is its initial acceleration (in m/s^2)? (The mass of the pulleys and the ropes is negligible.)



35. If mass M is 2 kg what is the tension in string AC? (in N)

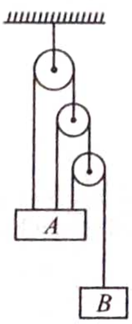


36. In ideal spring, with a pointer attached to its end, hangs next to a scale. With a 100 N weight attached and at rest, the pointer indicates '40' on the scale as shown. Using a 200 N weight instead results in 60 on the scale. Using an unknown weight 'X' instead results in '30' on the scale. Find value of 'X' (in newton).

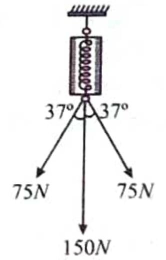


37. A block of mass 5 kg is being pulled up by a constant force of 90 N starting at ground from rest. 2 sec later, a part of the block having mass 3 kg falls off. Find the distance between the two parts when 3 kg part hits the ground.

38. If $M_B = 10$ kg, find the value of M_A (in kg), so that block A remains in equilibrium. The pulleys are ideal.



39. The scale in Figure is being pulled on by three ropes. What net force (in N) does the spring scale read?



40. A rope has a length of 12 m and a mass of 16 kg. The rope hangs from a rigid support. An operator whose mass is 80 kg slides down the rope at a constant speed of 0.8 m/s. What is the tension (in N) in the rope at a point 6 m from the top when the man has slid below this point?

41. Compute the least acceleration (in m/s^2) with which a 50 kg man can slide down a rope, if the rope can withstand a tension of 300 N.

Exercise-4 (Past Year Questions)

JEE MAIN

1. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$) **(2019)**

(a) 200 N (b) 140 N
(c) 70 N (d) 100 N

2. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is **(2019)**

(a) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(b) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(c) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

(d) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

3. A mass of 10 kg is suspended by a rope of length 4m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals
(Take $g = 10\ ms^{-2}$ and the rope to be massless) **(2020)**

(a) 75 N
(b) 90 N
(c) 100 N
(d) 70 N

4. A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2, where v is its speed. The maximum height attained by the ball is **(2020)**

(a) $\frac{1}{2K}\tan^{-1}\frac{ku^2}{g}$

(b) $\frac{1}{K}\ln\left(1+\frac{ku^2}{2g}\right)$

(c) $\frac{1}{2K}\ln\left(1+\frac{ku^2}{g}\right)$

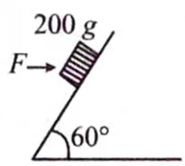
(d) $\frac{1}{K}\tan^{-1}\frac{ku^2}{2g}$

5. A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is 'x' N. The value of 'x' to the nearest integer is **(2021)**

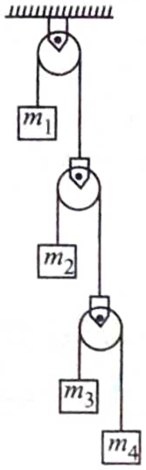
6. A body of mass 2 kg moves under a force of $(2\hat{i}+3\hat{j}+5\hat{k})N$. It starts from rest and was at the origin initially. After 4s, its new coordinates are (8, b, 20). The value of b is: (Round off to the Nearest Integer). **(2021)**

7. A force on an object of mass 100 g is $(10\hat{i}+5\hat{j})$N. The position of that object at $t = 2$s is $(a\hat{i}+b\hat{j})$m after starting from rest. The value of a/b will be ____________.**(2022)**

8. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force $F = \sqrt{x}N$ as shown in figure. The value of x = ________________.**(2022**

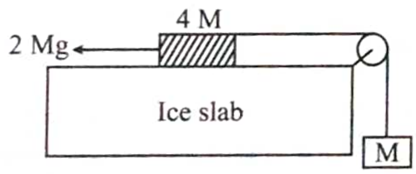


9. In the arrangement shown in figure a_1, a_2, a_3 and a_4 are the accelerations of masses m_1, m_2, m_3 and m_4 respectively. Which of the following relation is true for this arrangement? **(2022)**

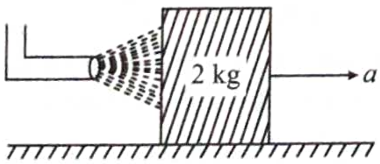


(*a*) $4a_1 + 2a_2 + a_3 + a_4 = 0$
(*b*) $a_1 + 4a_2 + 3a_3 + a_4 = 0$
(*c*) $a_1 + 4a_2 + 3a_3 + 2a_4 = 0$
(*d*) $2a_1 + 2a_2 + 3a_3 + a_4 = 0$

10. A hanging mass M is connected to a four times bigger mass by using a string pulley arrangement. as shown in the figure. The bigger mass is placed on a horizontal ice-slab and being pulled by 2 Mg force. In this situation, tension in the string is $x/5$ Mg for x = _______. Neglect mass of the string and friction of the block (bigger mass) with ice slab.
(Given g = acceleration due to gravity) **(2022)**



11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be **(2022)**

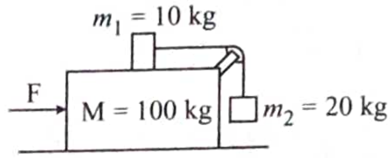


(*a*) 3 (*b*) 6
(*c*) 5 (*d*) 4

12. A block of mass M placed inside a box descends vertically with acceleration 'a'. The block exerts a force equal to one-fourth of its weight on the floor of the box. The value of 'a' will be **(2022)**

(*a*) $\frac{g}{4}$ (*b*) $\frac{g}{2}$
(*c*) $\frac{3g}{4}$ (*d*) g

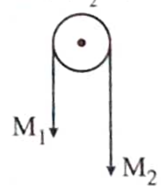
13. Three masses $M = 100$ kg, $m_1 = 10$ kg and $m_2 = 20$ kg are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of 2 ms^{-2}. The value of F is
(Take g = 10 ms^{-2}) **(2022)**



(*a*) 3360 N (*b*) 3380 N
(*c*) 3120 N (*d*) 3240 N

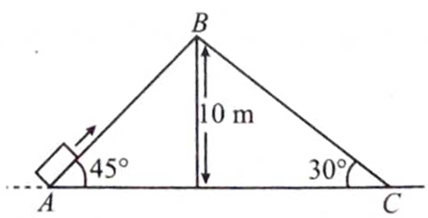
14. A monkey of mass 50 kg climbs on a rope which can withstand the tension (T) of 350 N. If monkey initially climbs down with an acceleration of 4 m/s^2 and then climbs up with an acceleration of 5 m/s^2. Choose the correct option
(Take g = 10 ms^{-2}) **(2022)**
(*a*) T = 700 N while climbing upward
(*b*) T = 350 N while going downward
(*c*) Rope will break while climbing upward
(*d*) Rope will break while going downward

15. Two masses M_1 and M_2 are tied together at the two ends of a light inextensible string that passes over a frictionless pulley. When the mass M_2 is twice that of M_1 the acceleration of the system is a_1. When the mass M_2 is thrice that of M_1. The acceleration of system is a_2. The ratio a_1/a_2 will be **(2022)**



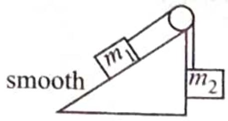
(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$
(*c*) $\frac{3}{2}$ (*d*) $\frac{1}{2}$

16. Two inclined planes are placed as shown in figure. A block is projected from the point A of inclined plane AB along its surface with velocity just sufficient to carry it to the top Point B at a height 10 m. After reaching the point B the block slide down on inclined plane BC. Time it takes to reach to the point C from point A is $t(\sqrt{2}+1)s$. The value of t is _____ (use g = 10 m/s^2) **(2022)**



17. If the shown system is in equilibrium, then force applied by inclined plane on the block m_1 is equal to

Given, $m_1 = 5$ kg, $m_2 = 3$ kg **(2022)**

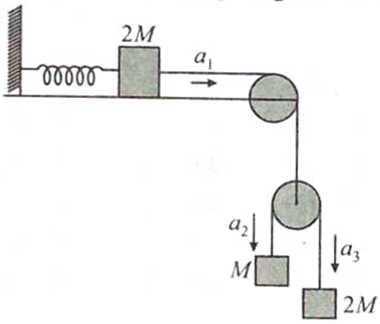


(*a*) 30 N (*b*) 50 N (*c*) 40 N (*d*) 60 N

JEE ADVANCED

18. A block of mass $2M$ is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and $2M$ using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option (s) is/are correct?

[g is the acceleration due to gravity. Neglect friction] **(2019)**



(*a*) $x_0 = \dfrac{4Mg}{k}$

(*b*) When spring achieves an extension of $\dfrac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\dfrac{M}{5k}}$.

(*c*) $a_2 - a_1 = a_1 - a_3$

(*d*) At an extension of $\dfrac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\dfrac{3g}{10}$

19. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(2021)

(*a*) The force applied on the particle is constant

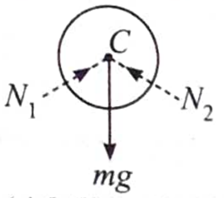
(*b*) The speed of the particle is proportional to time

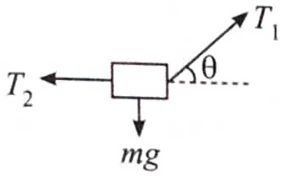
(*c*) The distance of the particle from the origin increases linearly with time

(*d*) The force is conservative

ANSWER KEY

CONCEPT APPLICATION

1. 

2. 

3. (*c*) **4.** [240, 320] **5.** (T_1 = 100 N, T_2 = 0)

6. (*a*) 5, (*b*) 50, 25, 10 (*c*) 50, 25, 15, 10 **7.** (*a*) 15, (*b*) 120, 80, (*c*) 75, 45, 30

8. (*a*) 5, (*b*) 110, 80, (*c*) 50, 30, 20 **9.** (*a*) 6 (*b*) 0, (*c*) 24, 16 **10.** (*a*) 2, (*b*) 40, 16 (*c*) 40, 24, 16 **11.** $N > N'$ **12.** (*b*)

13. $\dfrac{\cos\theta_2}{\cos\theta_1}$ **14.** $a_1 + a_2 = 2a_3$ **15.** (*c*) **16.** (*c*) **17.** (*b*) **18.** (*a*) **19.** (*c*) **20.** (*b*) **21.** $x = 1$

22. (*a*) 0 (*b*) 200 **23.** $F = 0$ **24.** $(g + a) \sin\theta$ **25.** (*a*)

EXERCISE-1 (TOPICWISE)

1. (*c*) **2.** (*c*) **3.** (*b*) **4.** (*b*) **5.** (*c*) **6.** (*b*) **7.** (*b*) **8.** (*c*) **9.** (*a*) **10.** (*d*)
11. (*b*) **12.** (*c*) **13.** (*b*) **14.** (*a*) **15.** (*a*) **16.** (*c*) **17.** (*c*) **18.** (*a*) **19.** (*a*) **20.** (*b*)
21. (*b*) **22.** (*a*) **23.** (*a*) **24.** (*b*) **25.** (*b*) **26.** (*b*) **27.** (*c*) **28.** (*d*) **29.** (*c*) **30.** (*a*)
31. (*d*) **32.** (*a*) **33.** (*c*) **34.** (*a*) **35.** (*a*) **36.** (*a*) **37.** (*a*) **38.** (*c*) **39.** (*b*) **40.** (*b*)
41. (*a*) **42.** (*b*) **43.** (*b*) **44.** (*a*) **45.** (*c*) **46.** (*c*) **47.** (*c*) **48.** (*b*) **49.** (*d*) **50.** (*a*)
51. (*b*) **52.** (*b*) **53.** (*c*) **54.** (*d*) **55.** (*b*) **56.** (*b*) **57.** (*d*) **58.** (*b*) **59.** (*d*) **60.** (*b*)
61. (*d*) **62.** (*a*) **63.** (*b*) **64.** (*a*)

EXERCISE-2 (LEARNING PLUS)

1. (*d*) **2.** (*c*) **3.** (*c*) **4.** (*b*) **5.** (*b*) **6.** (*d*) **7.** (*c*) **8.** (*b*) **9.** (*b*) **10.** (*a*)
11. (*a*) **12.** (*c*) **13.** (*c*) **14.** (*c*) **15.** (*b*) **16.** (*d*) **17.** (*c*) **18.** (*c*) **19.** (*a*) **20.** (*d*)
21. (*c*) **22.** (*a*) **23.** (*b*) **24.** (*c*) **25.** (*d*) **26.** (*c*) **27.** (*c*) **28.** (*b*) **29.** (*b*) **30.** (*d*)
31. (*b*) **32.** (*c*) **33.** (*d*) **34.** (*b*) **35.** (*b*) **36.** (*a*) **37.** (*b*) **38.** (*b*) **39.** (*c*) **40.** (*a*)
41. (*b*) **42.** (*a*) **43.** (*a*)

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*c*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*a*) **6.** (*b*) **7.** (*a*) **8.** (*b*) **9.** (*d*) **10.** (*a*)
11. (*c*) **12.** (*c*) **13.** (*d*) **14.** (*c*) **15.** (*a,b,c*) **16.** (*a,c*) **17.** (*b,d*) **18.** (*a,b,d*) **19.** (*a,b,d*) **20.** (*a,c*)
21. (*b,c*) **22.** (*c,d*) **23.** (*c,d*) **24.** (*b*) **25.** (*d*) **26.** (*a*) **27.** (*c*) **28.** (*b*) **29.** (*c*) **30.** (*a*)
31. (*b*) **32.** (*a*) **33.** (*a*) **34.** [5] **35.** [10] **36.** [50] **37.** [360] **38.** [70] **39.** [270] **40.** [880]
41. [4]

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*d*) **2.** (*b*) **3.** (*c*) **4.** (*c*) **5.** [10] **6.** [12] **7.** [2] **8.** [12] **9.** (*a*) **10.** [6]
11. (*c*) **12.** (*c*) **13.** (*a*) **14.** (*c*) **15.** (*b*) **16.** [2] **17.** (*c*)

JEE Advanced

18. (*c*) **19.** (*a,b,d*)

CHAPTER

7 Friction

FRICTION

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a force of friction. Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Friction force is of two types

1. Static friction 'f_s'
2. Kinetic friction 'f_k'

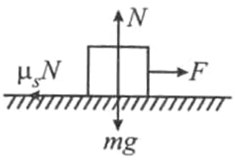
1. **Static Friction (f_s):** The static friction between two contact surfaces is given by $f_s \le \mu_s N$, where N is the normal force between the contact surfaces and μ_s is a constant which depends on the nature of the surfaces and is called 'coefficient of limiting friction'.

   Static friction acts between two objects when there is no relative motion between them. Static friction is a self adjusting force and its values satisfy the condition

   $f_s \le \mu_s N$

   μ_s = Co-efficient of limiting friction

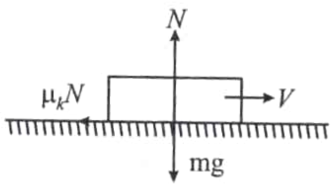
   Static friction takes its peak value ($f_{s(\max)} = \mu_s N$) when one surface is 'about to slide' on the other. Static friction in this case is called limiting friction.



2. **Kinetic Friction (f_k):** It acts on the two contact surfaces only when there is relative slipping or relative motion between two contact surfaces.

   $f_k = \mu_k N$

   The relative motion of a contact surface with respect to each other is opposed by a force given by $f_k = \mu_k N$, where N is the normal force between the contact surfaces and μ_k is a constant called **'coefficient of kinetic friction'**, which depends, largely, on the nature of the contact surfaces.

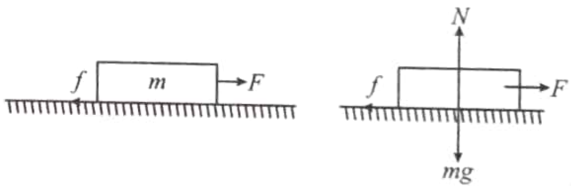


Note:

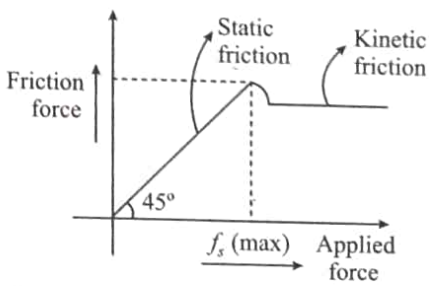
1. Value of μ_k is always less than μ_s ($\mu_k < \mu_s$) from experimental observation.
2. If only coefficient of friction (μ) is given in a problem then use $\mu_s = \mu_k = \mu$
3. Value of μ_s and μ_k is independent of surface area. It depends only on surface properties of contact surface.
4. μ_k is independent of relative speed.
5. μ_s and μ_k are properties of a given pair of surfaces i.e. for wood to wood combination μ_1, then for wood to iron μ_2 and so on.
6. Friction force acts along the surface.

Graph of frictional force versus applied force:

Consider a block of mass m kept on a rough surface and a force F is applied on the block which gradually increases from zero. Let μ_s and μ_k be coefficient of static and kinetic friction respectively.



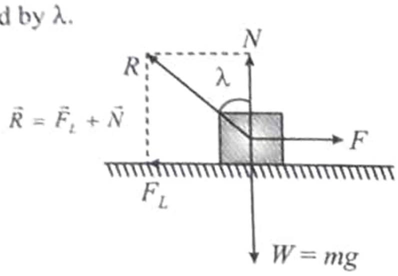
The maximum value of static friction is $\mu_s N$. As long as $F < \mu_s N$, block will not move. When F exceed $\mu_s N$, block will slip on the surface and kinetic friction $f_k = \mu_k N$ will act on the block which has a fixed value. (See graph below)



Graphical representation of variation of friction force with the force applied on a body

Angle of Friction

The angle of friction is the angle which the resultant of limiting friction F_L and normal reaction N makes with the normal reaction. It is represented by λ.



Thus from the figure.

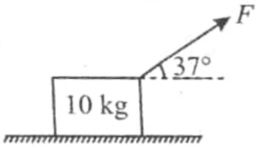
$\tan\lambda = \dfrac{F_L}{N}$

or $\tan\lambda = \mu_s$ $\qquad (\because F_L = \mu_s N)$

For smooth surfaces, $\lambda = 0$ (zero)

Train Your Brain

Example 1: A force F is applied on the block as shown in the figure. If the coefficient of static friction $\mu_s = 0.3$ and coefficient of kinetic friction is $\mu_k = 0.2$.

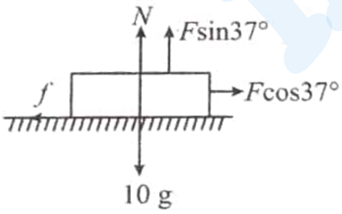


Find frictional force and acceleration of the block if

(i) $F = 10$ N

(ii) $F = 50$ N

Sol. F.B.D. of the block



(i) $F = 10$ N

Balancing forces normal to the surface

$N + 10\sin37° - 10g = 0 \quad \Rightarrow \quad N = 94$ N

$f_{s,\,\max} = \mu_s N = 0.3 \times 94 = 28.2$ N

$F\cos37° = 10 \times \dfrac{4}{5} = 8$ N

Since, $F\cos37° < \mu_s N$, block will not move and static friction $f_s = F\cos37° = 8$N.

(ii) $F = 50$ N

$N + 50\sin37° - 10\,g = 0 \quad \Rightarrow \quad N = 70$N

$f_{s,\,\max} = \mu_s N = 0.3 \times 70 = 21$N

$F\cos37° = 50 \times \dfrac{4}{5} = 40$ N

Since $F\cos37° > \mu_s N$, friction will not be able to prevent slipping.

Block will slide and kinetic friction will act on the block.

$f_k = 0.2$ N $= 0.2 \times 70 = 14$ N

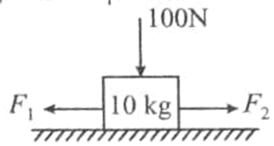
To find acceleration, we apply Newton's law along horizontal direction

$F\cos37° - f_k = ma$

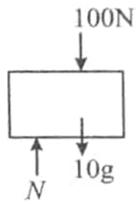
$50\cos37° - 14 = 10a$

$\Rightarrow a = \dfrac{40-14}{10} = 2.6$ m/s^2

Example 2: Find frictional force acting on the block in the following cases given that $\mu = 0.1$



Sol. F.B.D. of the block



$N = 10g + 100 = 200$ N

$f_{s,\max} = \mu_s N = 200 \times 0.1 = 20$ N

If $|F_2 - F_1| < \mu_s N$ block will not move and self adjusting static friction will act on its block and will be equal to $|F_2 - F_1|$.

If $|F_2 - F_1| > \mu_s N$

then, $f_k = \mu_k N$

Case-I: $F_1 = 0$

$F_2 = 5N \qquad fr = 5\ N$ (towards left)

$F_2 = 15N \qquad fr = 15\ N$ (towards left)

$F_2 = 25N \qquad fr = 20\ N$ (towards left)

Case-II: $F_1 = 20\ N$

$F_2 = 10$ N $\qquad fr = 10\ N$ (towards right)

$F_2 = 20\ N \qquad fr = 0$

$F_2 = 42\ N \qquad fr = 20\ N$ (towards left)

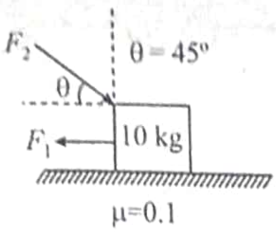
$F_2 = 50\ N \qquad fr = 20\ N$ (towards left)

Example 3: Find friction force acting on block in the following cases.

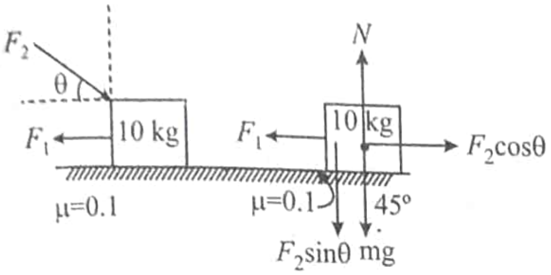
Case-I $\qquad F_1 = 0, F_2 = 5\sqrt{2}$

Case-II $\qquad F_1 = 0, F_2 = 50\sqrt{2}$

Case-III $\qquad F_1 = 5N, F_2 = 5\sqrt{2}$

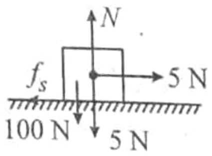


Sol.



Both of the above are same cases.

Case-I:



Along vertical:

$N - 100 - 5 = 0$

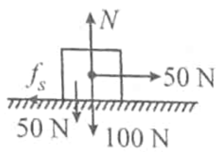
$N = 105$ N

$f_{s,\max} = 0.1 \times 105 = 10.5$ N

$f_s = 5$N

Since, applied force along horizontal is less than $f_{s,\max}$, block will not move and f_s = applied force = 5 N

Case-II:



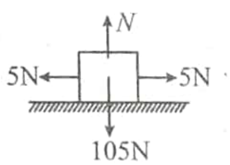
$N - 100 - 50 = 0$

$N = 150$ N

$f_{s,\max} = 0.1 \times 150 = 15$ N

so, block will skid and $f_s = 15$ N

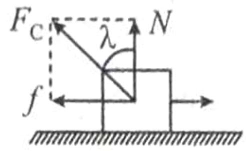
Case-III:



Since force along horizontal balance, there is no tendency to slip $f_s = 0$.

Example 4: A body of mass 400 g slides on a rough horizontal surface. If the frictional force is 3.0 N, find (a) the angle made by the contact force on the body with the vertical and (b) the magnitude of the contact force. Take $g = 10$ m/s².

Sol. Let the contact force on the block by the surface be F_c which makes an angle λ with the vertical (shown in figure)



The component of F_c perpendicular to the contact surface is the normal force N and the component of F parallel to the surface is the friction f. As the surface is horizontal, N is vertically upward. For vertical equilibrium

$N = Mg = (0.400 \text{ kg}) (10 \text{ m/s}^2) = 4.0\ N$

The frictional force is $f = 3.0\ N$

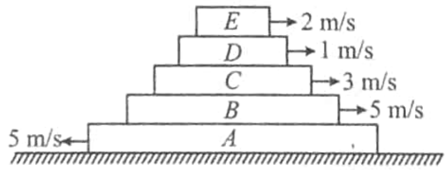
(i) $\tan\lambda = \dfrac{f}{N} = \dfrac{3}{4}$ or, $\lambda = \tan^{-1}(3/4) = 37°$

(ii) The magnitude of the contact force is

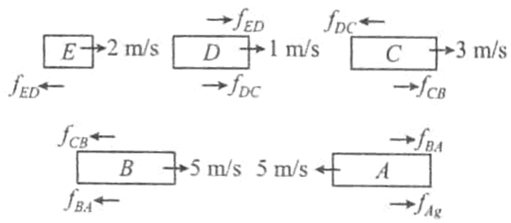
$$F = \sqrt{N^2 + f^2}$$

$$= \sqrt{(4.0\text{N})^2 + (3.0\text{ N})^2} = 5.0\text{N}$$

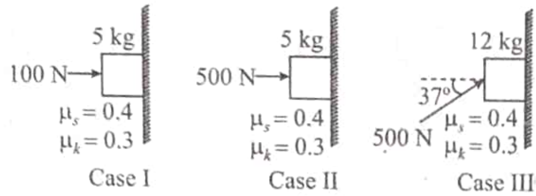
Example 5: In the given diagram find the direction of friction forces on each block and the ground (Assume all surfaces are rough and all velocities are with respect to ground).



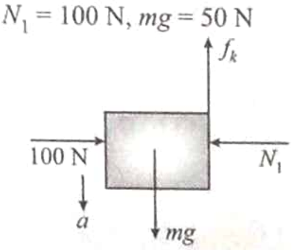
Sol. Since friction opposes relative motion, direction of friction will be as shown in below



Example 6: Determine the magnitude of frictional force and acceleration of the block in each of the following cases:



Sol. Case-I: $N_1 = 100$ N, $mg = 50$ N



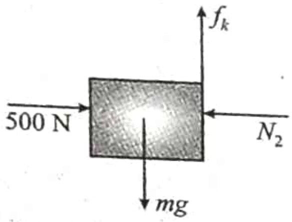
$f_l = \mu_s N_1 = 0.4 \times 100 = 40$ N, $f_k = \mu_k N_1$
$= 0.3 \times 100 = 30$ N

Here mg (driving force) is greater than maximum friction $f_l = 40$ N. Hence the block will not be able to stay at rest. It will accelerate downwards. But when it starts slipping, then kinetic friction will come into play. Now

$$a = \frac{mg - f_k}{m} = \frac{50-30}{5} = 4\ \text{ms}^{-2}$$

So in this case $f = f_k = 30$ N and $a = 4$ ms^{-2} (downwards)

Case-II: $N_2 = 500$ N, $mg = 50$ N

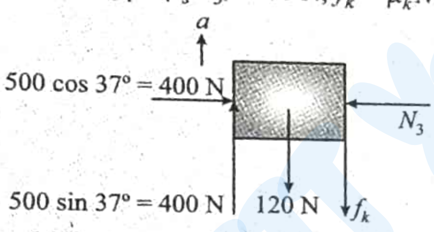


$$f_l = \mu_s N_2 = 200\ \text{N}, f_k = \mu_k N_2 = 150\ \text{N}$$

Here f_l is greater than mg (driving force), Hence, block will not move. So in this case $a = 0, f = mg = 50$ N

Case-III:

$$N_3 = 400\ \text{N}, f_l = \mu_s N_3 = 160\ \text{N}, f_k = \mu_k N_3 = 120\ \text{N}$$



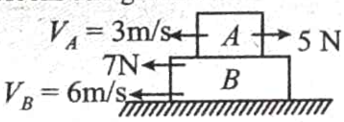
Here driving force = 300 − 120 = 180 N in upward direction; hence, friction will act downwards. Driving force is more than f_l. So the block will accelerate upwards.

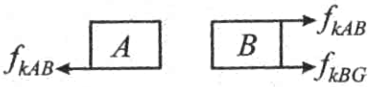
$$a = \frac{180 - f_k}{m} = \frac{180-120}{12} = 5\ \text{ms}^{-2}\ \text{(upwards)}$$

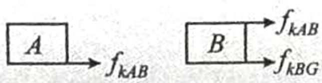
So in this case $f = f_k = 120$ N and $a = 5$ ms^{-2} (upwards)

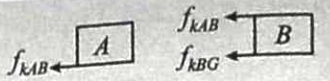
Concept Application

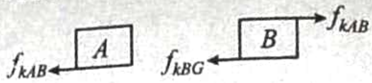
1. In the following figure, find the direction of friction on the blocks and ground .



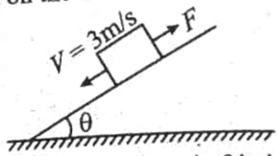
(a)


(b)


(c)


(d)


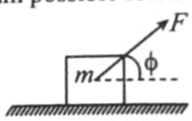
2. In the following figure, find the direction and nature of friction on the block.



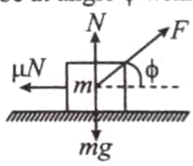
(a) Down the incline, kinetic friction.
(b) Up the incline, kinetic friction.
(c) Up the incline, static friction.
(d) Down the incline, static friction.

MINIMUM FORCE REQUIRED TO MOVE A BLOCK

A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body slide by applying the minimum possible force F.



Let the applied force F be at angle ϕ with the horizontal



N = Normal force

For vertical equilibrium,

$$N + F\sin\phi = mg \quad \text{or,} \quad N = (mg - F\sin\phi) \quad ...(i)$$

For horizontal equilibrium i.e. when the block is just about to slide,

$$F\cos\phi = \mu N \quad ...(ii)$$

Substituting for N,

$F\cos\phi = \mu(mg - F\sin\phi)$ or $F = \mu mg/(\cos\phi + \mu\sin\phi)$

for minimum $F(\cos\phi + \mu\sin\phi)$ is maximum,

$\Rightarrow$ Let $x = \cos\phi + \mu\sin\phi$

$$\frac{dx}{d\phi} = -\sin\phi + \mu\cos\phi$$

for maximum of $x, \frac{dx}{d\phi} = 0$

$\tan\phi = \mu$ and at this value of ϕ

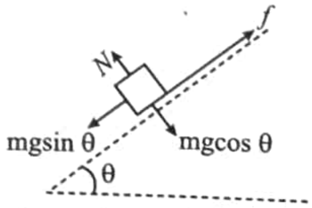
$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

MOTION ON A ROUGH INCLINED PLANE

Angle of Repose

Consider a rough inclined plane whose angle of inclination θ with ground can be changed. A block of mass m is resting on the plane. Coefficient of (static) friction between the block and plane is μ_s.

For a given angle θ, the FBD (Free body diagram) of the block is



where f is force of static friction on the block. For direction normal to the plane, we have $N = mg\cos\theta$

As θ increases, the force of gravity down the plane, mg sin θ, increases. Friction force resists the slide till it attains its maximum value.

$$f_{max} = \mu_s N = \mu_s mg\cos\theta$$

which decreases with θ (because cos θ decreases as θ increases)

Hence, beyond a critical value $\theta = \theta_c$, the blocks starts to slide down the plane. The critical angle is the one when mg sin θ is just equal to f_{max}, i.e. when

$$mg\sin\theta_c = \mu_s\, mg\cos\theta_c$$

$$\text{or } \tan\theta_c = \mu_s$$

where θ_c is called angle of repose

For $\theta < \theta_c$

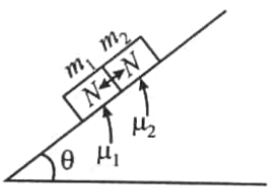
$a = 0$ and $f_s = mg\sin\theta$

For $\theta > \theta_c$

$a = g\sin\theta - \mu_k g\cos\theta$ (as the block is moving)

TWO BLOCKS ON AN INCLINED PLANE

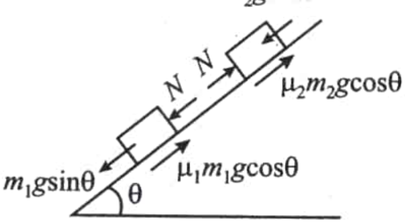
Consider two blocks having masses m_1 and m_2 placed on a rough inclined plane. μ_1 and μ_2 are the friction coefficient for m_1 and m_2 respectively. If N is the normal force between the contact surface of m_1 and m_2



Now two condition arises.

(i) If $\mu_1 > \mu_2$

Acceleration of m_2 will be greater than m_1 if they were not in contact. When in contact, block m_2 will press m_1 and both will move together with same acceleration.



$$m_1 g\sin\theta + N - \mu_1 m_1 g\cos\theta = m_1 a \quad \text{...(i)}$$

$$m_2 g\sin\theta - N - \mu_2 m_2 g\cos\theta = m_2 a \quad \text{...(ii)}$$

Adding (i) and (ii)

and solving we get

$$a = \frac{(m_1+m_2)g\sin\theta - (\mu_1 m_1 + \mu_2 m_2)g\cos\theta}{m_1+m_2}$$

we can also find N by putting value of a in equation (i).

(ii) If $\mu_1 < \mu_2$ then, blocks will move with different acceleration.

$N = 0$ because, there is no contact between the blocks.

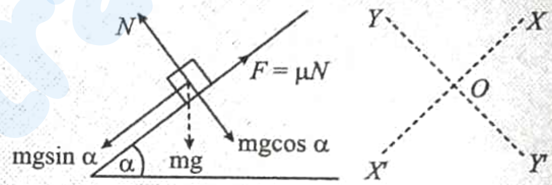
$a_1 = g\sin\theta - \mu_1 g\cos\theta$

$a_2 = g\sin\theta - \mu_2 g\cos\theta$

$\Rightarrow a_1 > a_2$

Train Your Brain

Example 7: A 20 kg box is gently placed on a rough inclined plane of inclination 30° with horizontal. The coefficient of sliding friction between the box and the plane is 0.4. Find the acceleration of the box down the incline.



Sol. In solving inclined plane problems, the X and Y directions along which the forces are to be considered, may be taken as shown. The components of weight of the box are

(i) $mg\sin\alpha$ acting down the plane and

(ii) $mg\cos\alpha$ acting perpendicular to the plane.

$N = mg\cos\alpha$

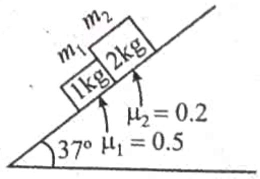
$mg\sin\alpha - \mu N = ma \Rightarrow mg\sin\alpha - \mu\, mg\cos\alpha = ma$

$a = g\sin\alpha - \mu g\cos\alpha = g(\sin\alpha - \mu\cos\alpha)$

$$= 9.8\left(\frac{1}{2} - 0.4\times\frac{\sqrt{3}}{2}\right) = 4.9\times 0.3072 = 1.505 \text{ m/s}^2$$

The box accelerates down the plane at 1.505 m/s^2.

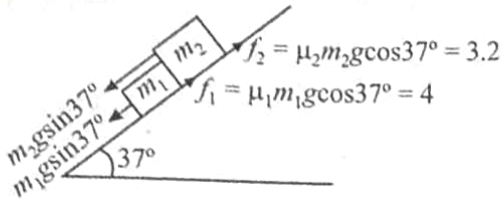
Example 8: Mass m_1 and m_2 are placed on a rough inclined plane as shown in figure. Find out the acceleration of the blocks and contact force in between these surface.



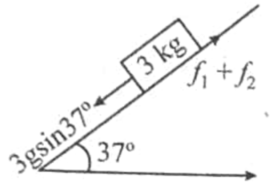
Sol. As we know if $\mu_1 > \mu_2$ both will travel together so

$a_1 = a_2 = a$

F.B.D

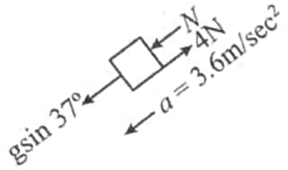


which is equivalent to



$$a = \frac{3g\sin 37^\circ - (f_1 + f_2)}{3} \Rightarrow a = \frac{18 - 7.2}{3} = 3.6\,\text{m/sec}^2$$

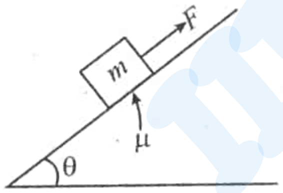
Now F.B.D of 1 kg block is



$g\sin 37^\circ + N - 4 = (1)\,a$

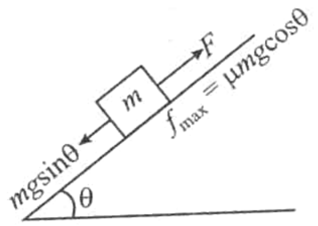
$N = 3.6 + 4 - 6 = 1.6$ Newton

Example 9:



Find out the range of force in the above situation for which m kg block does not move on the incline. ($\theta > \theta_c$)

Sol. F.B.D of the block



Block will slide upwards if

$F - mg\sin\theta > \mu mg\cos\theta$ (Friction acts down the incline)

$\Rightarrow F > \mu mg\cos\theta + mg\sin\theta$.

and block will slide downwards if

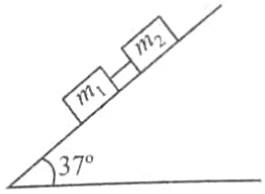
$mg\sin\theta - F > \mu mg\cos\theta$ (Friction acts up the incline)

$\Rightarrow F < mg\sin\theta - \mu mg\cos\theta$

Therefore, block will not move if

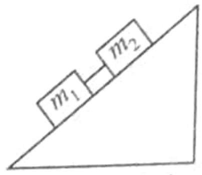
$mg\sin\theta - \mu mg\cos\theta < F < mg\sin\theta + \mu mg\cos\theta$

Example 10: Two blocks $m_1 = 4$ kg and $m_2 = 2$ kg, connected by a weightless rod on a plane having inclination of 37° as shown in figure. The coefficients of dynamic friction of m_1 and m_2 with the inclined plane are $\mu = 0.25$. Then the common acceleration of the two blocks and the tension in the rod are:



(*a*) 4 m/s², $T = 0$
(*b*) 2 m/s², $T = 5$ N
(*c*) 10 m/s², $T = 10$ N
(*d*) 15 m/s², $T = 9$N

Sol. (*a*)

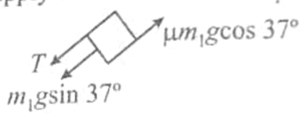


Apply Newton's law for system of m_1 and m_2

$$a = \frac{(m_1 + m_2)g\sin 37^\circ - \mu[m_1 g\cos 37^\circ + m_2 g\cos 37^\circ]}{m_1 + m_2}$$

$= g[\sin 37^\circ - \mu\cos 37^\circ]$

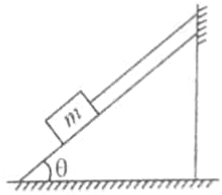
Now apply Newton's law for M_1



$m_1 g\sin 37^\circ + T - \mu m_1 g\cos 37^\circ = m_1 a = m_1 g[\sin 37^\circ - \mu\cos 37^\circ]$

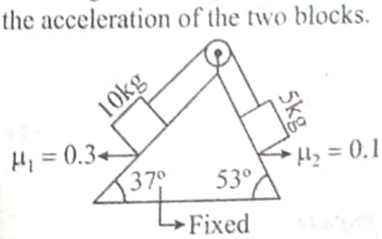
$\Rightarrow T = 0$ N and $a = 4\text{m/sec}^2$

Example 11: If $\mu = 0.9$ and $\theta = 45^\circ$, then find tension in string



Sol. As $\theta < \theta_c$ block has no tendency to move hence tension in the string will be zero.

Example 12: Two blocks of masses 5 kg and 10 kg are attached with the help of light string and placed on a rough incline as shown in the figure. Coefficients of friction are as marked in the figure. The system is released from rest. Determine the acceleration of the two blocks.



Sol. Let 10 kg block is sliding down, then acceleration of both the blocks are given by,

$$a=\frac{(10\,g\sin 37^\circ-\mu_1\times 10\,g\cos 37^\circ-5\,g\sin 53^\circ-\mu_2\times 5\,g\cos 53^\circ)}{15}=-ve$$

It means our assumed direction of motion is wrong and 5 kg block is going to slide down, if this would be the case, the direction of friction force will reverse and acceleration of blocks would be given by:

$$a_1=\frac{(5g\sin 53^\circ-\mu_2\times 5g\cos 53^\circ-\mu_1\times 10g\cos 37^\circ-10\,g\sin 37^\circ)}{15}=-ve$$

It means in this direction also there is no motion. So we can conclude that the system remains at rest and friction force is static in nature.

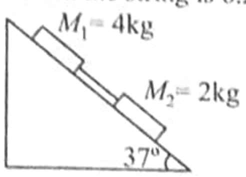
Example 13: Two blocks connected by a massless string slide down an inclined plane having angle of inclination 37°. The masses of the two blocks are $M_1=4$kg and $M_2=2$kg and the coefficients of friction 0.75 and 0.25 respectively

A. The common acceleration of the two masses is 1.3 ms^{-2}

B. The tension in the string is 14.7N

C. The common acceleration of the two masses is 2.94 ms^{-2}

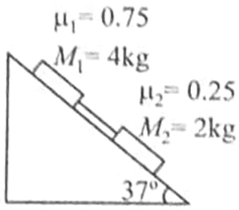
D. The tension in the string is 6.2N



(*a*) A, D (*b*) C, D

(*c*) B, D (*d*) B, C

Sol. (*a*)



Friction on M_1

$f_{1max}=\mu_1 M_1 g\cos 37$

$=0.75\times 4\times 10\times\frac{4}{5}$

$=\frac{3\times 4\times 10\times 4}{4\times 5}$

$f_{1max}=24$N

Friction on M_2

$f_{2max}=\mu_2 M_2 g\cos 37$

$=0.25\times 2\times 10\times\frac{4}{5}$

$=\frac{1}{4}\times\frac{20\times 4}{5}$

$f_{2max}=4$N

Downward force on M_1

$F_1=M_1 g\sin 37$

$F_1=4\times 10\times\frac{3}{5}$

$F_1=24$ N

Downward force on M_2

$F_2=M_2 g\sin 37$

$F_2=2\times 10\times\frac{3}{5}$

$F_2=12$ N

So resultant force on M_1 and M_2 is due to M_2

$F=F_2-f_{2max}=12-4=8$

$8=(M_1+M_2)a$

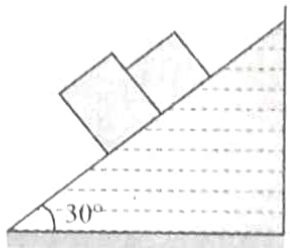
$\therefore a=\frac{8}{6}=1.3$ m/s^2

$T=M_2(g-a)\sin 37-f=2\times 8.5\times\frac{3}{5}-4$

$=10.2-4=6.2$ N

Concept Application

3. Figure shows two blocks in contact sliding down an inclined surface of inclination 30°. The friction coefficient between the block of mass 2.0 kg and the incline is $\mu_1=0.20$ and that between the block of mass 4.0 kg and the incline $\mu_2=0.30$. is Find the acceleration of 2.0 kg block. (g = 10 m/s^2).



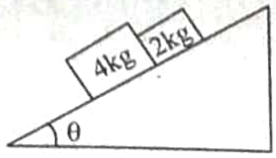
(*a*) 2.7 m/s^2

(*b*) 2.2 m/s^2

(*c*) 3.7 m/s^2

(*d*) 3.2 m/s^2

4. Figure shows two blocks in contact and sliding down the inclined surface of inclination 30°. Coefficient of friction between the block of mass 4 kg and the inclined plane is $\mu_1=0.3$ and that between the block of mass 2 kg and the inclined plane is $\mu_2=0.20$. Assume coefficients of static and kinetic friction to be equal. Find the acceleration of 2 kg block. Take g = 10 m/s^2.



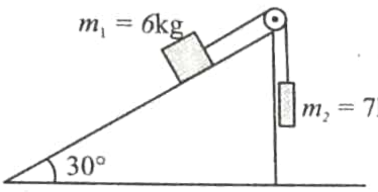
(*a*) $\frac{15-4\sqrt{3}}{3}\,m/s^2$

(*b*) $5-\sqrt{3}\ m/s^2$

(*c*) $\sqrt{15}\ m/s^2$

(*d*) $5\ m/s^2$

5. Two blocks of masses $m_1 = 6$ kg and $m_2 = 7$ kg are connected by a light string passing over a light frictionless pulley as shown in fig. The mass m_1 is at rest on the inclined plane and mass m_2 hangs vertically. If the angle of incline $\theta = 30°$. What is the magnitude and direction of the force of friction on the 6 kg block? (Take $g = 10\ cm^{-2}$).



(*a*) 40 N up the plane

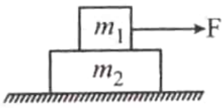
(*b*) 40 N down the plane

(*c*) 90 N up the plane

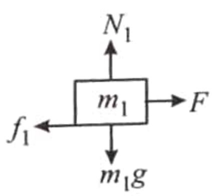
(*d*) 40 N down the plane

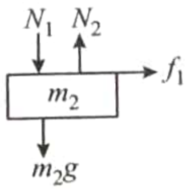
BLOCK OVER BLOCK PROBLEMS

Consider two block kept one over the other as shown in Figure where m_1 is kept on top of m_2.



Let the coefficient of friction between m_1 and m_2 is μ_1 and between m_2 and ground is $\mu_2 = 0$. To find acceleration of m_1 and m_2 we make their free-body diagrams and write Newton's second law





Along vertical

$N_1 - m_1g = 0$ and $N_1 + m_2g - N_2 = 0$

Solving the two gives.

$N_1 = m_1g$ and $N_2 = (m_1 + m_2)g$

Along horizontal

$F - f_1 = m_1a_1$ and $f_1 = m_2a_2$

We know $f_1 < \mu_1 N_1$

There are two equations and three unknowns in equation along horizontal. This problem cannot be solved unless we make some assumptions.

Case-I: If we assume both blocks are moving together then friction between them need not be equal to maximum value. Thus, taking $a_1 = a_2$, we are down to two unknowns and we can solve the equations, but we must verify our answer by checking $f_1 \le \mu_1 N_1$. If this fails, then blocks slip an each other and we consider next possibility.

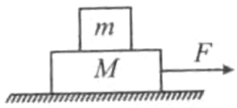
Case-II: If we assume blocks arc moving relative to each other, then friction between them must have reached maximum value (Kinetic friction)

$$a_1 \neq a_2 \quad \text{but} \quad f_1 = \mu_1 N_1$$

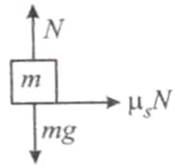
Again we get two unknowns and two equation. We can solve it.

When Force is Applied on Lower Block

Consider a block of mass m is placed on another block of mass M lying on a smooth horizontal surface. The coefficient of static friction between m and M is μ_s. The maximum force that should be applied to M so that the blocks remains at rest relative to each other then:



Draw the force diagrams of blocks at the moment when F is at its maximum value and m is about to slide relative to it.



Frictional force between m and $M = \mu_s N$

(N: normal reaction between the block)

Due to the friction. M will try to drag m towards right and hence frictional force will act on in towards right.

Let a = acceleration each block

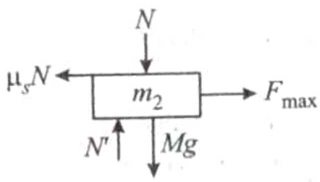
N' = normal reaction between M and the surface

From F.B.D. of m

$N = mg$

$\mu_s N = ma$

From F.B.D. of M.



$N + Mg = N'$

$F_{max} = \mu_s N = Ma$

combining these two equation, we get

$F_{max} = \mu_s(m + M)g$

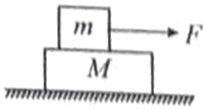
Hence $\mu_s(m + M)g$ is the critical value of force F.

If F is greater than this critical value, m begins to slip relative to M and their acceleration will be different.

If F is smaller than this critical value, m and M move together without any relative motion.

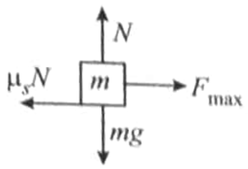
When Force is Applied on Upper Block

A block of mass m is placed on another block of mass M lying on a smooth horizontal surface. The coefficient of static friction between m and M is μ_s. The maximum force that can be applied to m so that blocks remains at rest relative to each other then:



Consider the situation when F is at its maximum value so that m is about to start slipping relative to M

Since, friction opposes relative motion. The mass m tries to drag M toward right due to friction.



From forces on m:

$F - \mu_s N = ma$ ...(i)

$N = mg$

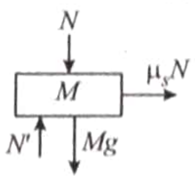
From F.B.D. of M:

$\mu_s N = Ma$

$\mu_s mg = Ma$ ...(ii)

or $a = \dfrac{\mu_s mg}{M}$

Putting the value of a in equation (i) we get,



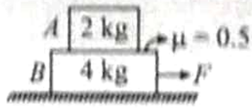
$$F_{max} = \frac{\mu_s(m+M)mg}{M}$$

If F is less than F_{max}, the blocks stick together without any relative motion.

If F is greater than F_{max} value, the blocks slide relative to each and their acceleration are different.

Train Your Brain

Example 14:

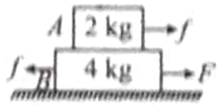


Frictionless

Find out the maximum value of F for which both the blocks will move together

Sol. In the given situation 2 kg block will move only due to friction force exerted by the 4 kg block

F.B.D.



The maximum friction force exerted on the block B is $f_{max} = \mu N$

$f_{max} = (0.5)(20) = 10$ N

So the maximum acceleration of 2 kg block is

$\boxed{2\text{ kg}} \rightarrow f_{max} = 10\text{N}$

$a_{2max} = \dfrac{10}{2} = 5$ m/s^2

a_{max} is the maximum acceleration for which both the block will move together. i.e., for $a \le 5$ ms^{-2} acceleration of both blocks will be same and we can take both the blocks as a system.

F.B.D

$\xrightarrow{a}$ 5m/s^2

$\boxed{6\text{ kg}} \rightarrow F_{max}$

$F_{max} = 6 \times 5 = 30$ N

For $0 < F < 30$

Both the block move together.

Example 15: In the above question find the acceleration of both the blocks when

(i) $F = 18$ N (ii) $F = 36$ N

Sol. (i) Since $F < 30$ both the blocks will move together

F.B.D

$\boxed{6\text{ kg}} \rightarrow F = 18\text{N}$

$a = \dfrac{18}{6} = 3$ m/s^2

(ii) $F = 36$ N

When $F > 30$ both the blocks will move separately so we treat each block independently

F.B.D of 2 kg block

$B\ \boxed{2\text{ kg}} \rightarrow f = 10\text{N}$ (Friction force)

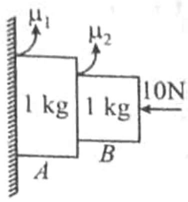
$a_B = 5$ m/s^2

F.B.D of 4 kg block

$f = 10\text{N} \leftarrow$ 4 kg $\rightarrow F = 36\text{N}$

$$a_A = \frac{36-10}{4} = \frac{26}{4}\ \text{m/s}^2$$

Example 16: Find the accelerations of blocks A and B for the following cases.

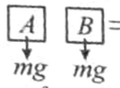


A. $\mu_1 = 0$ and $\mu_2 = 0.1$

B. $\mu_2 = 0$ and $\mu_1 = 0.1$

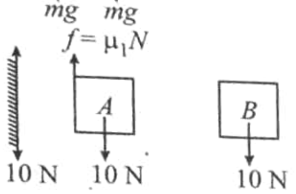
C. $\mu_1 = 0.1$ and $\mu_2 = 1.0$

Sol. (*a*) Due to gravity, both will have same acceleration. Since there is no tendency for slipping, friction will be zero.



$\Rightarrow a_A = a_B = 10\ \text{m/s}^2$

(ii)



Friction between wall and block A oppose relative motion since wall is stationary so friction opposes motion of block A. Acceleration of A will be less than g. Since $\mu_2 = 0$, block B falls freely.

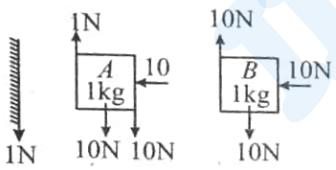
For block A,

$10 - f = 1a_A$

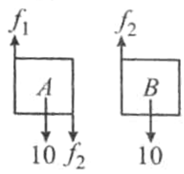
$a_A = 10 - 0.1 \times 10 = 9\ \text{m/s}^2$

$a_B = 10\ \text{m/s}^2$

(iii)



Maximum friction between wall and block A is 1 N but maximum friction available between block A and B is 10 N.



Let us assume, both move together with acceleration a.

$20 - f_1 = 2a \quad \Rightarrow \quad a = \dfrac{19}{2} = 9.5\ \text{m/s}^2$

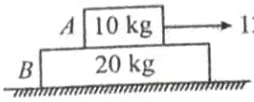
For block B: $10 - f_2 = 1 \times 9.5 \quad \Rightarrow \quad f_2 = 0.5$ N

But $f_2 < 10$ N.

So, our assumption is correct.

Thus, $a_A = a_B = 9.5\ \text{m/s}^2$.

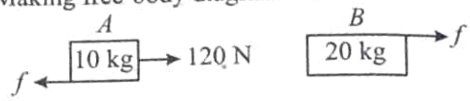
Example 17: Friction coefficient between the block is 0.5 and ground is smooth. If force of 120 N is applied on the upper block as shown in figure find acceleration of the blocks.



$f_{max} = 0.5 \times 10g = 50$ N

Let's assume they have same acceleration. The value of common acceleration is $\dfrac{120}{30} = 4\ \text{m/s}^2$.

Making free-body diagram



Newton's second law given

$f = 20 \times 4 = 80$ N

Solving we get, $a = 4\ \text{m/s}^2$ and $f = 80$ N while limiting value of friction is 50 N. hence our assumption is wrong.

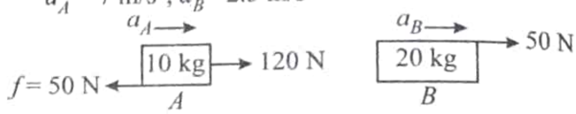
Let's say they are not moving together. Then $f_k = 50$ N.

By Newton's second law.

$120 - 50 = 10\, a_A$

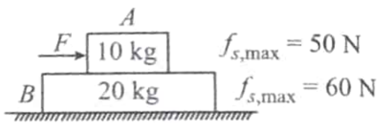
$50 = 20\, a_B$

$a_A = 7\ \text{m/s}^2$, $a_B = 2.5\ \text{m/s}^2$



This gives us the correct direction of friction.

Example 18:



Friction coefficient between block is 0.5 and between ground and 20 kg block is 0.2. Find acceleration of blocks if force $F = 80$ N is applied on 10 kg block as shown in figure

Sol. The values of limiting friction at contact surfaces ($f_{s,max}$)

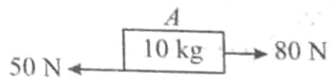
f_{max} between blocks $f_1 = 0.5 \times 10 \times 10 = 50$ N

f_{max} between 20 kg block and ground,

$$f_2 = 0.2 \times 30 \times 10 = 60 \text{ N}$$

The only driving force that the block B can experience is the one applied by the lower surface of block A on block B.

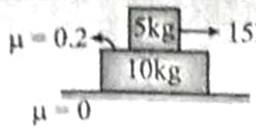
The maximum value of this force is 50 N. This is lower than the minimum force required to move block B with respect to ground $f_{required} = 0.2 \times 30 \times 10 = 60$ N. Hence only the block A will move.



$$a_A = \frac{80-50}{10} = 3\ \text{m/s}^2$$

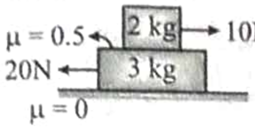
Concept Application

6. The frictional force between the blocks is:



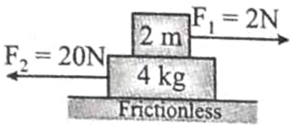
(*a*) 5 N (*b*) 10 N
(*c*) 8 N (*d*) 6 N

7. The frictional force on block of mass 2 kg is:



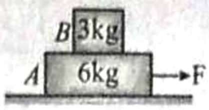
(*a*) 8 N (b) 10 N
(c) 18 N (d) 6 N

8. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = 1/2$. The force of friction acting between the two blocks is:



(*a*) 8 N (*b*) 10 N
(*c*) 6 N (*d*) 4 N

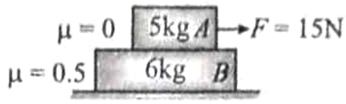
9. Two blocks *A* and *B* of masses 6 kg and 3 kg rest on a frictionless horizontal surface as shown in the figure. If coefficient of friction between *A* and *B* is 0.4, the maximum horizontal force which can make them move without separation is



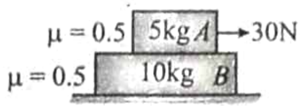
(*a*) 72 N (*b*) 40 N
(*c*) 36 N (*d*) 20 N

10. Find the accelerations and the friction forces involved:

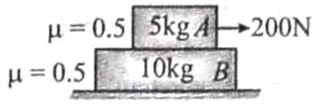
(*a*)



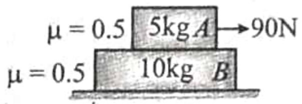
(*b*)



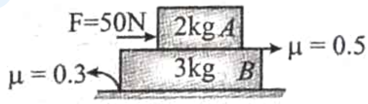
(*c*)



(*d*)



11. From the figure shown, find out acceleration of 3kg block.



(*a*) 7 m/s^2
(*b*) 10 m/s^2
(*c*) 10/3 m/s^2
(*d*) None of these

Short Notes

Friction

Friction force is of two types: (*a*) Kinetic, (*b*) Static.

Kinetic Friction

$$f_k = \mu_k \times N$$

The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.

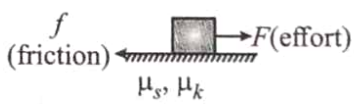
Static Friction

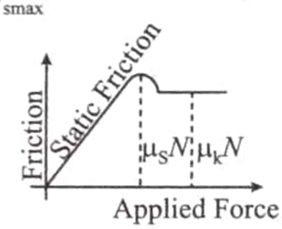
It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surfaces.

This means static friction is a variable and self adjusting force. However it has a maximum value called limiting friction.

$$f_{max} = \mu_S N$$

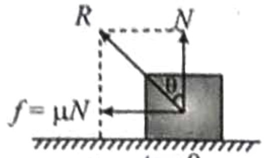
$$0 < f_S < f_{smax}$$





Angle of Friction

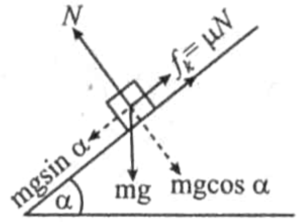
It is the angle which the resultant of the force of limiting friction and the normal reaction (*N*) makes with the direction of *N*.



$\mu_1 = \tan\theta$

Angle of Repose or Angle of Sliding

It is the minimum angle of inclination of a plane with the horizontal, such that a body placed on it, just begins to slide down.

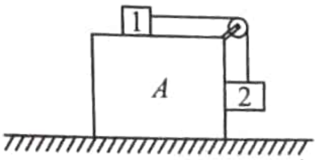


If angle of repose is α and coefficient of limiting friction μ_1, then

$$\mu_1 = \tan\alpha$$

Solved Examples

1. What is the minimum acceleration with which bar A should be shifted horizontally to keep the bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal and the coefficient of friction between the bar and the bodies equal to μ. The masses of the pulley and the threads are negligible while the friction in the pulley is absent.



Sol. Let us place the observer on A.

Since we have non-inertial frame we have pseudo forces.

For body '1' we have,

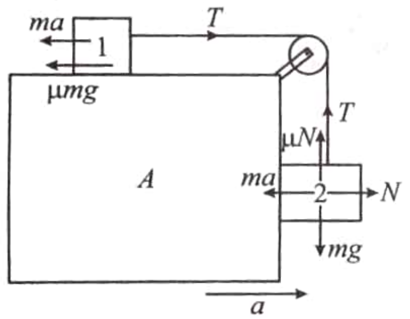
$T = ma + \mu mg$ ...(i)

For body '2' we have,

$N = ma$

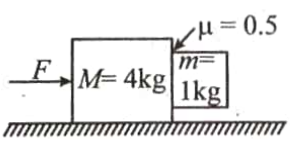
$mg - T - \mu ma = 0$

$\therefore\ mg = T + \mu ma$ ....(ii)



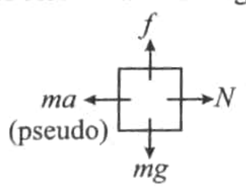
From (i) and (ii) $a_{min} = g\left(\dfrac{1-\mu}{1+\mu}\right)$

2. Find out the range of force for which smaller block is at rest with respect to bigger block.



Sol. Smaller block is at rest w.r.t. the bigger block. Let both t block travel together with acceleration a

F.B.D of smaller block w.r. to the bigger block.



$f_{max} = \mu \times N$

$N = ma$

$f = \mu\, ma$ ...(

also, $f = mg$ ...(

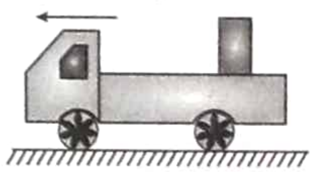
from (i) and (ii)

$a = g/\mu = 20\ \text{m/s}^2$

So $F = 20\ (M + m) = 20\ (5) = 100$ N

If $F \geq 100$ N Both will travel together

3. The rear side of a truck is open and a box of 40 kg mass placed 5 m away from the open end as shown. The coefficie of friction between the box and the surface below it is 0.1 On a straight road, the truck starts from rest and accelerat with 2 ms^{-2}. At what distance from the starting point do the box fall off the truck (i.e. distance travelled by the truck [Ignore the size of the box]



Sol. In the reference frame of the truck FBD of 40 kg block

Net force $\Rightarrow ma - \mu N \Rightarrow 40\times 2 - \dfrac{15}{100}\times 40\times 10$

$ma_{block} \Rightarrow 80 - 60 \Rightarrow a_{block} = \dfrac{20}{40} = \dfrac{1}{2}\ \text{m/s}^2$

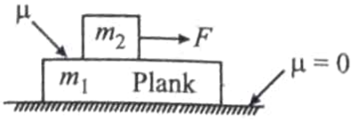
This acceleration of the block in reference frame of truck s time taken by box to fall down from truck

$$S_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2 \Rightarrow 5 = 0 + \frac{1}{2} \times \frac{1}{2} \times t^2 \Rightarrow t^2 = 20$$

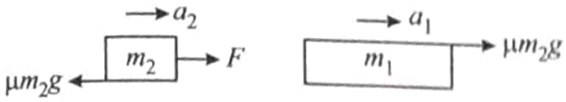
So distance moved by the truck

$$\Rightarrow \frac{1}{2} \times a_{truck} \times t^2 = \frac{1}{2} \times 2 \times (20) = 20 \text{ meter}$$

4. Mass m_2 placed on a plank of mass m_1 lying on a smooth horizontal plane. A horizontal force $F = a_0 t$ (a_0 is a constant) is applied to a bar. If acceleration of the plank and bar are a_1 and a_2 respectively and the coefficient of friction between m_1 and m_2 is μ. Then find acceleration a_2 with time t.



Sol. If $F < \mu m_2 g$ then both blocks move with common acceleration, i.e, $a_1 = a_2 = \dfrac{(a_0 t)}{m_1 + m_2}$



When $F > \mu m_2 g$, then

Equation for block of mass m_2

$$F - \mu m_2 g = m_2 a_2 \quad \Rightarrow \quad a_2 = \frac{(a_0 t - \mu m_2 g)}{m_2} \qquad \text{...(i)}$$

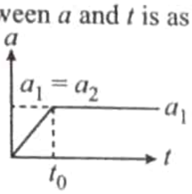
Also $\mu m_2 g = m_1 a_1$

$$a_1 = \frac{\mu m_2 g}{m_1} \qquad \text{...(ii)}$$

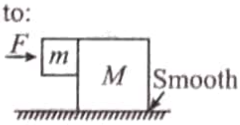
i.e., acceleration a_2 varies with time linearly, its slope positive and intercept negative.

From equation (ii) a_1 is independent of time.

So, the graph between a and t is as follow.



5. The two blocks, $m = 10$ kg and $M = 50$ kg are free to move as shown. The coefficient of static friction between the blocks is 0.5 and there is no friction between M and the ground. A minimum horizontal force F is applied to hold m against M that is equal to:



(a) 100 N (b) 50 N
(c) 240 N (d) 180 N

Sol. (c) As m would slip in vertically downward direction, then $mg = \mu N$

$$\Rightarrow N = \frac{mg}{\mu} = \frac{100}{0.5} = 200 \text{ Newton}$$

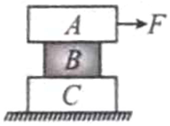
Same normal force would accelerate M, thus $a_M = \dfrac{200}{50}$

$= 4 \text{ m/s}^2$

Taking $(m + M)$ as system

$F = (m + M).\ 4 = 240$ N

6. Given $m_A = 30$ kg, $m_B = 10$ kg, $m_C = 20$ kg. Between A and B friction coefficient $\mu_1 = 0.3$, between B and C friction coefficient $\mu_2 = 0.2$ and between C and ground $\mu_3 = 0.1$. The least horizontal force F to start the motion of any part of the system of three blocks resting upon one another as shown in figure is ($g = 10$ m/s²)



(a) 60 N (b) 90 N
(c) 80 N (d) 150 N

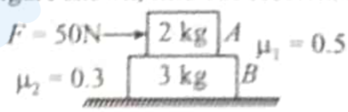
Sol. (a) Limiting friction between A and B = 90 N

Limiting friction between B and C = 80 N

Limiting friction between C and ground = 60 N

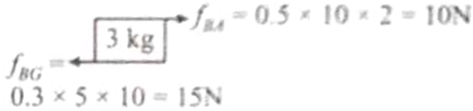
Since limiting friction is least between C and ground, slipping will occur at first between C and ground. This will occur when $F = 60$ N.

7. From the figure shown, find out acceleration of 3 kg block.



(a) 7 m/s² (b) 10 m/s²
(c) 10/3 m/s² (d) Zero

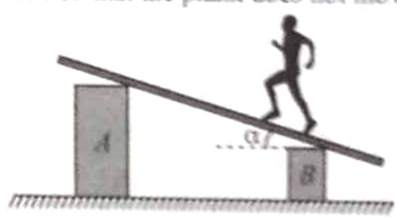
Sol. (d)



$\because f_{BA} < f_{BG}$

Hence, B will remain at rest. so $a_g = 0$

8. A plank is held at an angle α to the horizontal (Fig.) on two fixed supports A and B. The plank can slide against the supports (without friction) because of its weight Mg. With what acceleration and in what direction, a man of mass m should move so that the plank does not move.



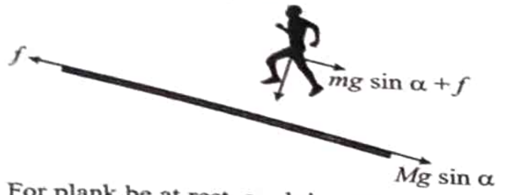
(a) $g \sin\alpha \left(1 + \dfrac{m}{M}\right)$ down the incline

(b) $g \sin\alpha \left(1 + \dfrac{M}{m}\right)$ down the incline

(c) $g \sin\alpha \left(1 + \dfrac{m}{M}\right)$ up the incline

(d) $g \sin\alpha \left(1 + \dfrac{M}{m}\right)$ up the incline

Sol. (*b*) F.B.D. of man and plank are



For plank be at rest, applying Newton's second law to plank along the incline

$Mg \sin \alpha = f$ ...(i)

and applying Newton's second law to man along the incline.

$mg \sin \alpha + f = ma$ ...(ii)

$a = g \sin \alpha \left(1+\dfrac{M}{m}\right)$ down the incline

Alternate Solution: If the friction force is taken up the incline on man, then application of Newton's second law to man and plank along incline yields.

$f + Mg \sin \alpha = 0$ ...(i)

$mg \sin \alpha - f = ma$ ...(ii)

Solving (i) and (ii)

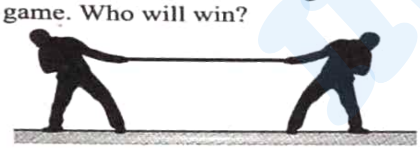
$a = g \sin \alpha \left(1+\dfrac{M}{m}\right)$ down the incline

Alternate Solution: Application of Newton's seconds law to system of man + plank along the incline yields

$mg \sin \alpha + Mg \sin \alpha = ma$

$a = g \sin \alpha \left(1+\dfrac{M}{m}\right)$ down the incline

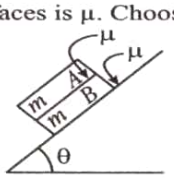
9. Two persons, pull each other through a massless rope in 'tug of war' game. Who will win?



(*a*) One whose weight is more.
(*b*) One who pulls the rope with a greater force.
(*c*) One who applies more friction force (shear force) on ground.
(*d*) One who applies more normal force (compressive force) on ground.

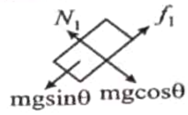
Sol. (*c*) The person applying more frictional force on ground will win because friction is resisting the slipping.

10. Two identical blocks of same masses are placed on a fixed wedge as shown in figure. Coefficient of friction between all the contact surfaces is μ. Choose the correct alternative

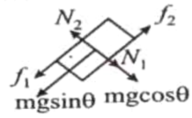


(*a*) For motion at any surface, $\theta \leq \tan^{-1}(\mu)$.
(*b*) Acceleration of block A will be more than acceleration of block B in downward direction.
(*c*) Acceleration of block A will be less than acceleration of block B in down ward direction.
(*d*) Two blocks A and B move with same acceleration.

Sol. (*d*) F.B.D. for A block



F.B.D. for B block



for block A

$mg \sin\theta - f_1 = ma$ ...(i)

for motion w.r.t. block B

$mg \sin \theta - \mu mg \cos \theta = ma$ ...(ii)

for limiting case

$a = 0$

and $a = b = 0$

$\Rightarrow mg \sin\theta = \mu mg \cos\theta$

$\mu = \tan\theta$

$\theta = \tan^{-1} \mu$

for block B

$mg\sin\theta + f_1 - f_2 = mb$

for motion w.r.t wedge

$f_2 = 2\mu mg \cos \theta$

$mg\sin\theta + f_1 - 2\mu mg \cos\theta = mb$ ...(iii)

for no relative motion between A and B block from equation (i) and (iii) : $a = b$

$2mg \sin\theta - 2 \mu mg \cos\theta = 2ma$

for limiting case $a = 0$

$\Rightarrow \theta = \tan^{-1}(\mu)$

for motion $\theta \geq \tan^{-1}(\mu)$

when block B is moving w.r.t wedge

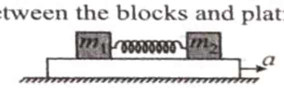
$mg\sin\theta + f_1 - 2\mu\, mg\cos\theta = mb$

But $f_1 = \mu mg \cos\theta \Rightarrow mg \sin\theta - \mu mg \cos\theta = mb$

for block A

$mg \sin\theta - \mu mg \cos \theta = ma \Rightarrow a = b$.

11. Two blocks of masses m_1 and m_2 are connected with a massless undeformed spring and placed over a plank moving with an acceleration 'a' as shown in figure. The coefficient of friction between the blocks and platform is μ.



(*a*) Spring will be stretched if $a > \mu g$.

(*b*) Spring will be compressed if $a \le \mu g$.

(*c*) Spring will neither be compressed nor be stretched only if $a \le \mu g$.

(*d*) Spring will be in its natural length under all conditions only if initial velocities of blocks are same.

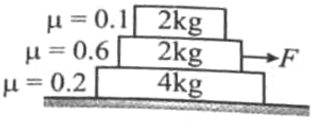
Sol. (*d*) Let the value of 'a' be increased from zero. As long as $a \le \mu g$, there shall be no relative motion between m_1 or m_2 and platform, that is, m_1 and m_2 shall move with acceleration a.

As $a > \mu g$ the acceleration of m_1 and m_2 shall become μg each.

Hence at all instants the velocity of m_1 and m_2 shall be same

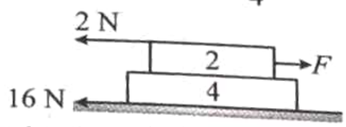
$\therefore$ The spring shall always remain in natural length.

12. In the situation shown in figure for what value of horizontal force F(in Newton), sliding between middle and lower block will start? (Take $g = 10$ m/s^2)



Sol. for uppermost block $a_{max.} = 1$ m/s^2

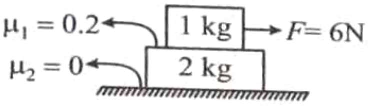
for lowermost block $a_{max} = \dfrac{(24-16)}{4} = 2$ m/s^2



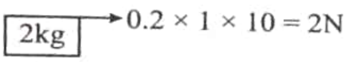
Hence sliding between middle and lower block will start only after sliding between middle and upper block has already started.

for middle block $F - 18 = 6 \times 2 \Rightarrow F = 30$ N

13. In the situation shown find the accelerations of the blocks. Also find the accelerations if the force is shifted from the upper block to the lower block.



Sol. **Case-I:** For the lower block



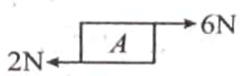
$$a_{max} = \frac{2}{2} = 1\text{m/s}^2$$

and common possible acceleration

$$= \frac{6}{(1+2)} = 2 \text{ m/s}^2$$

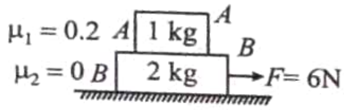
Hence, blocks move with different accelerations.

$$a_A = \frac{6-2}{1} = 4 \text{ m/s}^2$$

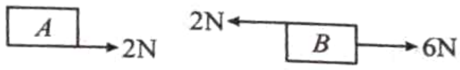


$$a_B = \frac{2}{2} = 1 \text{ m/s}^2$$

Case-II: When the force is acting on the lower block maximum possible acceleration of A



$$= \frac{2}{1} = 2 \text{ m/s}^2$$



and common acceleration of the two blocks

$$= \frac{6}{(1+2)} = 2 \text{ m/s}^2$$

Hence, both blocks move with common acceleration of $a_A = a_B = 2$ m/s^2

14. A small body was launched up an inclined plane set at an angle $\alpha = 15°$ against the horizontal. Find the coefficient of friction, if the time of the ascent of the body is $\eta = 2.0$ times less than the time of its descent.

Sol. This problem can be solved in two steps:

Step I: When the body is moving up on the inclined plane (shown in fig. *A*)

Here $N = mg \cos \alpha$

The acceleration of the body is

$$a_1 = -\left(\frac{mg\sin\alpha + \mu N}{m}\right) = -(g \sin \alpha + \mu g \cos \alpha)$$

Let body is projected with speed v_0 along the inclined plane (along the x-axis). After time t_1 body reaches at point $P(v = 0)$ (as shown in fig. *B*).

Let $OP = s$

$$s = \left(\frac{v_0 + v}{2}\right) t_1 = \frac{v_0}{2} t_1$$

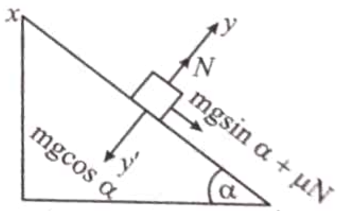


Fig. A

$$a_1 = \frac{0 - v_0}{t_1} \quad \therefore v_0 = -a_1 t_1$$

$$\therefore s = -\frac{a_1 t_1^2}{2}$$

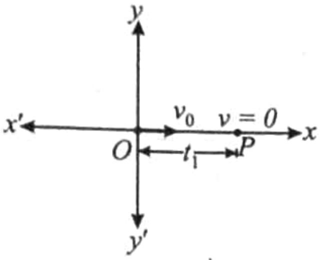


Fig. B

$$\therefore t_1 = \sqrt{-\frac{2s}{a_1}}$$

$$= \sqrt{\frac{2s}{g \sin \alpha + \mu g \cos \alpha}} \qquad \text{...(i)}$$

Step-II: When the body starts to return from point P, the acceleration is

$$a_2 = \frac{mg \sin \alpha - \mu mg \cos \alpha}{m} \text{ (In fig. } C)$$

$$= g \sin \alpha - \mu g \cos \alpha$$

$$\therefore s = \frac{1}{2} a_2 t_2^2$$

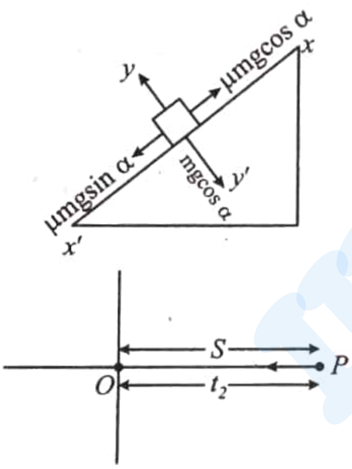


Fig. C

$$\therefore t_2 = \sqrt{\frac{2s}{a_2}} = \sqrt{\frac{2s}{g \sin \alpha - \mu g \cos \alpha}} \qquad \text{...(ii)}$$

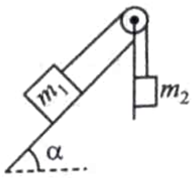
According to the problem,

$t_2 = \eta t_1$,

Putting the value of t_1 and t_2 from equations (i) and (ii), we get

$$\mu = \left[\frac{(\eta^2 - 1)}{(\eta^2 + 1)}\right] \tan \alpha = 0.16$$

15. The inclined plane of figure forms an angle $\alpha = 30°$ with the horizontal. The mass ratio $m_2/m_1 = \eta = 2/3$. The coefficient of friction between the body m_1 and the inclined plane is equal to $k = 0.10$. The masses of the pulley and the threads are negligible. Find the magnitude and the direction of acceleration of the body m_2 when the formerly stationary system of masses starts moving.

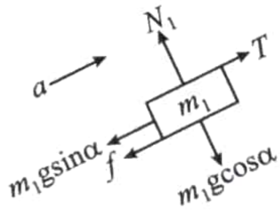


Sol. For checking the direction of friction, let us assume there is no friction. Then net force acting on the system along the string in vertically downward direction is given by

$$m_2 g - m_1 g \sin \alpha = (m_1 + m_2) a$$

$$a = \frac{\eta - \frac{1}{2}}{\eta + 1} g \Rightarrow a > 0.$$

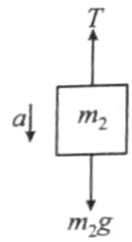
So the friction will act down the incline FBD of m_1 gives:



$$T - f - m_1 g \sin \alpha = m_1 a.$$

$$\Rightarrow T - k\, m_1 g \cos \alpha - m_1 g \sin\alpha = m_1 a \qquad \text{...(i)}$$

$\therefore$ FBD of m_2



$$m_2 g - T = m_2 a \qquad \text{...(ii)}$$

from (i) and (ii)

$$a = \frac{g(\eta - \sin \alpha - k \cos \alpha)}{(\eta + 1)}$$

Putting $\eta = 2/3$, $\alpha = 30°$ and $k = 0.1$

$a = 0.05$ g (downward for m_2)

16. A small mass slides down an inclined plane of inclination θ with the horizontal. The co-efficient of friction is $\mu = \mu_0 x$ where x is the distance through which the mass slides down and μ_0, a constant. Then find

(*a*) How much distance it will cover to get that maximum speed

(*b*) Maximum speed of particle

Sol. (*a*) Acceleration of mass at distance x

$a = g(\sin\theta - \mu_0 x \cos\theta)$

Speed is maximum, when $a = 0$

$g(\sin\theta - \mu_0 x \cos\theta) = 0$

$$x = \frac{\tan \theta}{\mu_0}$$

(*b*) $a = g(\sin\theta - \mu_0 x\cos\theta)$

$$\frac{vdv}{dx} = g(\sin\theta - \mu_0 x\cos\theta)$$

$$\int_0^v vdv = g\int(\sin\theta - \mu_0 x\cos\theta)dx$$

$$\frac{v^2}{2} = g\left[x\sin\theta - \frac{\mu_0 x^2}{2}\cos\theta\right]_0^{\frac{\tan\theta}{\mu_0}}$$

$$\frac{v^2}{2} = g\left[\frac{\tan\theta}{\mu_0}\sin\theta - \frac{\mu_0}{2}\frac{\tan^2\theta}{\mu_0^2}\cos\theta\right]$$

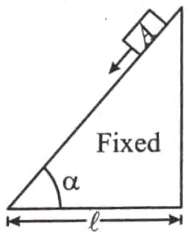
$$\frac{v^2}{2} = g\left[\frac{\sin^2\theta}{\mu_0\cos\theta} - \frac{\sin^2\theta}{2\mu_0\cos\theta}\right]$$

$$\frac{v^2}{2} = g\left[\frac{\sin^2\theta}{2\mu_0\cos\theta}\right]$$

$$v_{max}^2 = \frac{g\sin^2\theta}{\mu_0\cos\theta}$$

$$v_{max} = \sqrt{\frac{g\sin^2\theta}{\mu_0\cos\theta}}$$

17. A small body *A* starts sliding down from the top of a wedge (Fig.) whose base is equal to $\ell = 2.10$ m. The coefficient of friction between the body and the wedge-surface is $k = 0.140$. At what value of the angle α will the time of sliding be the least? What will it be equal to?



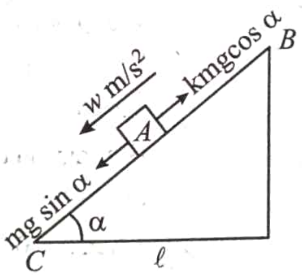
Sol. The acceleration of block is

$$w = \frac{mg\sin\alpha - k\,mg\cos\alpha}{m}$$

or $w = g\sin\alpha - kg\cos\alpha$

Let $BC = s$ $\quad\therefore \cos\alpha = \frac{\ell}{s}$

$\therefore s = \ell\sec\alpha$



$\therefore$ According to kinematics equation

$$s = \frac{1}{2}wt^2$$

$$\therefore t = \sqrt{\frac{2s}{w}} = \sqrt{\frac{2\ell\sec\alpha}{g\sin\alpha - kg\cos\alpha}} \quad \text{...(i)}$$

$$\text{or } t^2 = \frac{2\ell\sec\alpha}{g(\sin\alpha - k\cos\alpha)} = \frac{2\ell}{g(\sin\alpha\cos\alpha - k\cos^2\alpha)}$$

For being t minimum

$\sin\alpha\cos\alpha - k\cos^2\alpha$ is maximum

$$\frac{d}{d\alpha}(\sin\alpha\cos\alpha - k\cos^2\alpha) = 0$$

$$\cos^2\alpha - \sin^2\alpha + 2k\cos\alpha\sin\alpha = 0$$

$$\cos 2\alpha + k\sin 2\alpha = 0$$

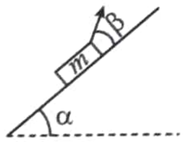
$$\tan 2\alpha = \frac{-1}{k}$$

$$\Rightarrow \alpha = \frac{1}{2}\tan^{-1}\left(-\frac{1}{k}\right)$$

After putting the values, $\alpha = 49°$

and putting the value of α, k and ℓ, we get $t_{min} = 1$ s

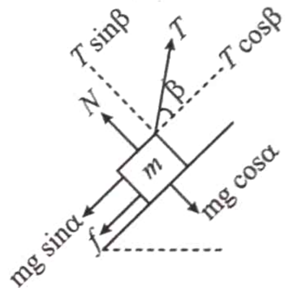
18. A bar of mass m is pulled by means of a thread up an inclined plane forming an angle α with the horizontal (fig.). The coefficient of friction is equal to k. Find the angle β which the thread must form with the inclined plane for the tension of the thread to be minimum. What is it equal to?



Sol. For limiting friction

$$N + T\sin\beta = mg\cos\alpha$$

$$\Rightarrow N = mg\cos\alpha - T\sin\beta \quad \text{...(i)}$$



and, $T\cos\beta = mg\sin\alpha + f$

$= mg\sin\alpha + k(mg\cos\alpha - T\sin\beta)$ (from (i))

$$\Rightarrow T(\cos\beta + k\sin\beta) = mg\sin\alpha + k\,mg\cos\alpha$$

$$\Rightarrow T = \frac{mg(\sin\alpha + k\cos\alpha)}{(\cos\beta + k\sin\beta)}$$

for minimum T, $\cos\beta + k\sin\beta$ should be maximum

Let $y = \cos\beta + k\sin\beta$

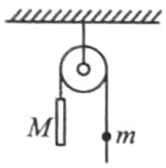
$$\frac{dy}{d\beta} = -\sin\beta + k\cos\beta = 0$$

$$\Rightarrow \tan\beta = k$$

$$\therefore T_{min} = \frac{mg(\sin\alpha + k\cos\alpha)}{\left(\frac{1}{\sqrt{1+k^2}} + \frac{k^2}{\sqrt{1+k^2}}\right)}$$

$$= \frac{mg(\sin\alpha + k\cos\alpha)}{\sqrt{1+k^2}}$$

19. In the arrangement shown in figure the mass of the rod M exceeds the mass m of the ball. The ball has an opening permitting it to slide along the thread with some friction. The mass of the pulley, mass of the string and the friction in its axle are negligible. At the initial moment the ball was located opposite the lower end of the rod. When set free, both bodies began moving with constant accelerations. Find the friction force between the ball and the thread if t seconds after the beginning of motion the ball got opposite the upper end of the rod. The rod length equals ℓ.



Sol. **Step-I:** Draw force diagram separately: In fig B, P is a point on the string

From fig. A,

$mg - f = mw_1$

$$\therefore w_1 = \frac{mg - f}{m} \qquad ...(i)$$

From figure B,

$$T = f \qquad ...(ii)$$

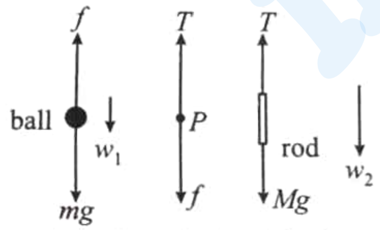


Fig. A Fig. B Fig. C

From fig. C

$Mg - T = Mw_2$

$Mg - f = Mw_2$

$$\therefore w_2 = \frac{Mg - f}{M} \qquad ...(iii)$$

Step-II: Apply kinematic relation :

$$s_{rel} = u_{rel}\,t + \frac{1}{2}\,w_{rel}\,t^2 \qquad \text{(Shown in fig. D)}$$

Here $s_{rel} = \ell$, $u_{rel} = 0$

$w_{rel} = w_2 - w_1$

$$\therefore \ell = \frac{1}{2}(w_2 - w_1)\,t^2$$

$$\therefore t = \sqrt{\frac{2\ell}{(w_2 - w_1)}}$$

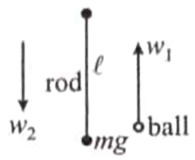
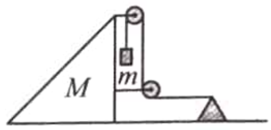


Fig. D

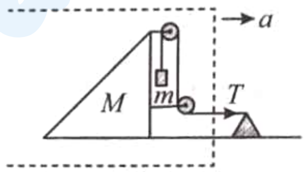
Putting the value of w_1 and w_2, w_1

$$f = \frac{2\ell\,Mm}{(M-m)t^2}$$

20. In the arrangement shown in figure the masses of the wedg M and the body m are known. The appreciable frictio exists only between the wedge and the body m, the frictio coefficient being equal to k. The masses of the pulley an the thread are negligible. Find the acceleration of the body m relative to the horizontal surface on which the wedge Slides



Sol.



Considering the system as marked in the diagram

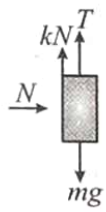
$\therefore T = (M+m)a.$

a is the common acceleration of the two masses in horizonta direction.

Now taking the body m as system.

F.B.D. for the body will be

$\therefore T = (M+m)a.$



$\therefore N = ma$ (m has acceleration a in horizontal direction)

Also $mg - kN - T = ma$ (m has acceleration a downwar w.r.t. wedge because of the constraint)

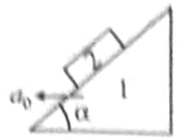
$$\text{or } a = \frac{mg}{M + 2m + km}$$

$$\vec{a}_{bg} = \vec{a}_{bw} + \vec{a}_{wg}$$

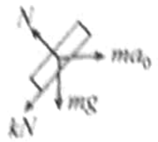
$$|\vec{a}_{bg}| = \sqrt{a^2 + a^2} = \sqrt{2}\,a$$

$$= \frac{mg/\sqrt{2}}{M + 2m + km} = \frac{g/\sqrt{2}}{2 + k + M/m}$$

21. Prism 1 with bar 2 of mass m placed on it gets a horizontal acceleration a_0 directed to the left (figure). At what maximum value of this acceleration will the bar be still stationary relative to the prism, if the coefficient of friction between them $k < \cot\alpha$?



Sol.



(a_0 is acceleration of the wedge leftward)

As a_0 increases the component of ma_0 up the up the incline increases and friction attains its max value.

Writing the force equation along the incline and perpendicular to the incline.

$ma_0 \cos\alpha - mg\sin\alpha - kN = 0$ ...(i)

$mg\cos\alpha + ma_0 \sin\alpha = N$ ...(ii)

From equation (i) and (ii),

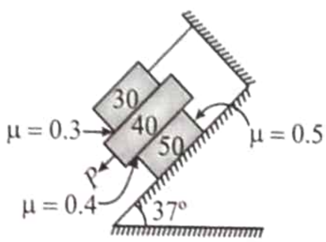
$a_0\cos\alpha - g\sin\alpha = kg\cos\alpha + ka_0\sin\alpha$

$a_0(\cos\alpha - k\sin\alpha) = g\{k\cos\alpha + \sin\alpha\}$

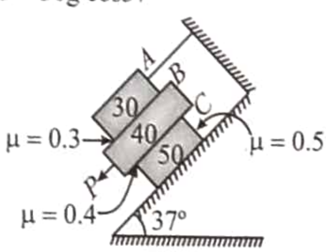
$$a_0 = \frac{g\{k\cos\alpha + \sin\alpha\}}{\{\cos\alpha - k\sin\alpha\}}$$

$= g(1 + k\cot\alpha)/(\cot\alpha - k)$

22. The three flat blocks as shown in the figure are positioned on the 37° incline and a force P parallel to the inclined plane is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The masses of three blocks in kg and coefficient of static friction for each of the three pairs of contact surfaces are shown in the figure. Determine the maximum value which force P may have before slipping take place anywhere. ($g = 10$ m/s²)



Sol. $f_{AB\max} = 0.3 \times 30g\cos37°$



$f_{AB\max} = 72$ N

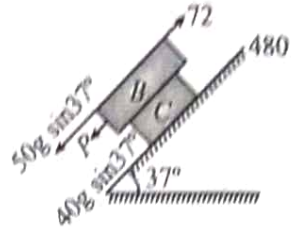
$f_{BC\max} = 0.4 \times 80g\cos37^\circ = 256$ N

$f_{C\max} = 0.5 \times 120g\cos37^\circ = 480$ N

when block 'B' is pulled two cases are possible

1. A and C remains stationary only 'B' tends to move downwards.
2. B and C both moves together and there is just slipping and A & B contact and C and wedge contact

Taking case (2) Let B and C moves together and there is just slipping between A & B and between wedge and C

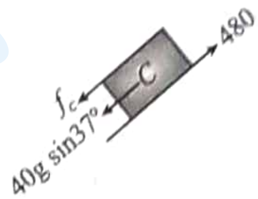


$\Rightarrow P + 40\,g\sin37^\circ + 50\,g\sin37^\circ - 72 - 480 = 0$

$\Rightarrow P = 12$ N

checking friction between B and C

$f_{BC} + 40\,g\sin37^\circ - 480 = 0$

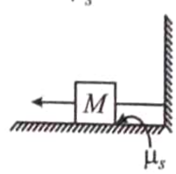


$f_{BC} = 240$

which is within limit so case is correct.

so max. force for which there will be no slipping $P_{max} = 12$ N the at this force both B and C tends to move together.

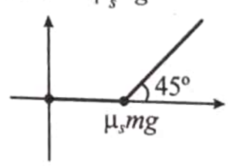
23. In the following figure force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s



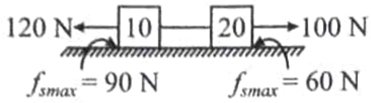
Sol. As the external force F is gradually increased from zero it is compensated by the friction and the string beares no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s mg$, the further increase in F is transferred to the string.

$T = 0$ if $F < \mu_s mg$

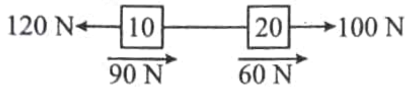
$T = F - \mu_s mg$ if $F > \mu_s mg$



24. Find the tension in the string in situation as shown in the figure below. Forces 120 N and 100 N start acting when the system is at rest.

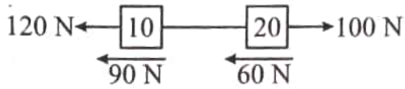


Sol. (i) Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore the assumption is not valid.



Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

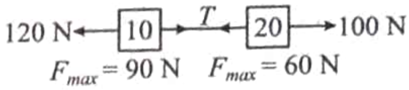
(ii) Similarly let us assume that system moves towards right.



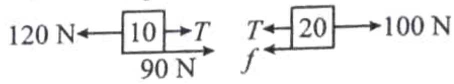
Above assumption is also not possible as net force on the system is towards left in this situation.

Hence assumption is again not valid.

Therefore it can be concluded that the system is stationary



Assuming that the 10 kg block reaches limiting friction first then using FBD's



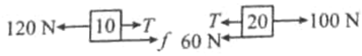
$120 = T + 90 \Rightarrow T = 30$ N

Also $T + f = 100$

$\therefore\ 30 + f = 100 \Rightarrow f = 70$ N

which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have



$T + 60 = 100\text{ N} \Rightarrow T = 40$ N

Also $f + T = 120\ N \Rightarrow f = 80$ N

This is acceptable as static friction at this surface should be less than 90 N.

Hence the tension in the string is $T = 40$ N

Exercise-1 (Topicwise)

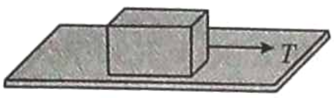
STATIC FRICTION

1. The coefficient of friction μ and the angle of friction λ are related as
   (*a*) $\sin \lambda = \mu$ (*b*) $\cos \lambda = \mu$
   (*c*) $\tan \lambda = \mu$ (*d*) $\tan \mu = \lambda$

2. A force of 98 N is required to just slide body of mass 100 kg over ice. The coefficient of static friction is
   (*a*) 0.6 (*b*) 0.4 (*c*) 0.2 (*d*) 0.1

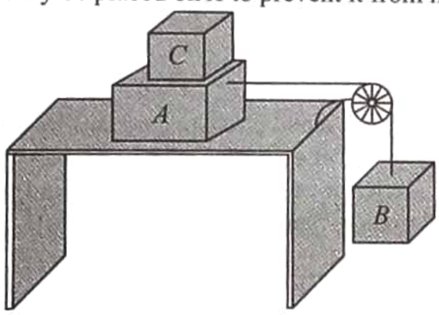
3. Maximum value of static friction is called
   (*a*) Limiting friction (*b*) Rolling friction
   (*c*) Normal reaction (*d*) Coefficient of friction

4. In the figure shown, a block of weight 10 N resting on a horizontal surface. The coefficient of static friction between the block and the surface is $\mu_s = 0.4$. A force of 3.5 N will keep the block in uniform motion, once it has been set in motion. A horizontal force of 3 N is applied to the block, then the block will



   (*a*) Move over the surface with constant velocity
   (*b*) Move having accelerated motion over the surface
   (*c*) Not move
   (*d*) First it will move with a constant velocity for some time and then will have accelerated motion

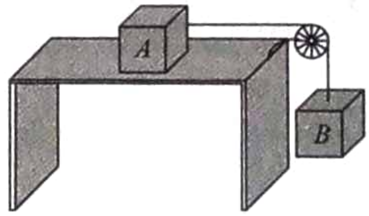
5. Two masses *A* and *B* of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown. The coefficient of static friction of *A* with table is 0.2. The minimum mass of *C* that may be placed on *A* to prevent it from moving is



   (*a*) 15 kg (*b*) 10 kg (*c*) 5 kg (*d*) 12 kg

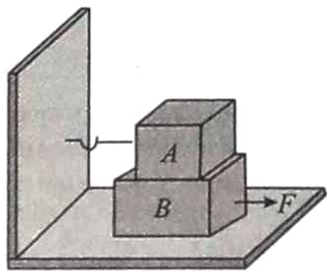
6. Which of the following statements is not true
   (*a*) The coefficient of friction between two surfaces increases as the surface in contact are made rough
   (*b*) The force of friction acts in a direction opposite to the applied force
   (*c*) Rolling friction is greater than sliding friction
   (*d*) The coefficient of friction between wood and wood is less than 1

7. The blocks *A* and *B* are arranged as shown in the figure. The pulley is frictionless. The mass of *A* is 10 kg. The coefficient of friction of *A* with the horizontal surface is 0.20. The minimum mass of *B* to start the motion will be



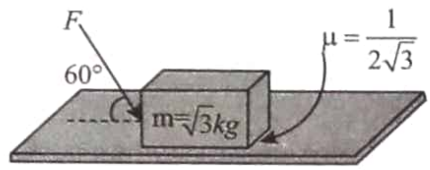
   (*a*) 2 kg (*b*) 0.2 kg
   (*c*) 5 kg (*d*) 10 kg

8. A block *A* with mass 100 kg is resting on another block *B* of mass 200 kg. As shown in figure a horizontal rope tied to a wall holds it. The coefficient of friction between *A* and *B* is 0.2 while coefficient of friction between *B* and the ground is 0.3. The minimum required force *F* to start moving *B* will be



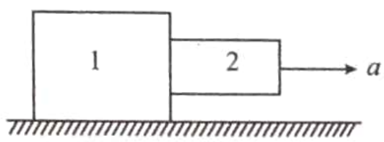
   (*a*) 900 N (*b*) 100 N
   (*c*) 1100 N (*d*) 1200 N

9. What is the maximum value of the force *F* such that the block shown in the arrangement, does not move?



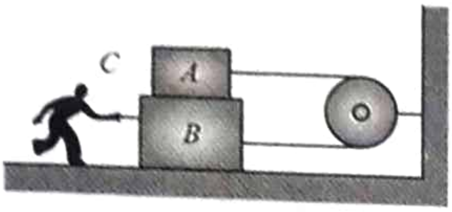
   (*a*) 20 N (*b*) 10 N
   (*c*) 12 N (*d*) 15 N

10. The coefficient of static friction between the two blocks is 0.363. What is the minimum acceleration of block 1 so that block 2 does not fall



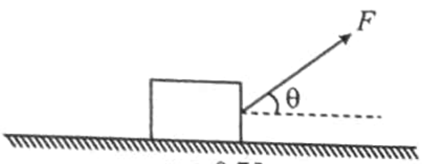
   (*a*) 6 ms^{-2} (*b*) 12 ms^{-2}
   (*c*) 18 ms^{-2} (*d*) 27 ms^{-2}

11. In the figure $m_A = m_B = m_C = 60$ kg. The coefficient of friction between C and ground is 0.5, B and ground is 0.3, A and B is 0.4. C is pulling the string with the maximum possible force without moving. Then the tension in the string connected to A will be



(a) 120 N (b) 60 N (c) 100 N (d) Zero

12. A block of mass $m = 10$ kg is to be pulled on a horizontal rough surface with the minimum force.



$\mu = 0.75$

(i) The block should be pulled at an angle = ...

(ii) The magnitude of the force F is equal to ....

(a) 45°, 30 N (b) 37°, 30 N

(c) 37°, 60 N (d) 53°, 30 N

KINETIC FRICTION

13. A car is moving along a straight horizontal road with a speed v_0. If the coefficient of friction between the tyres and the road is μ, the shortest distance in which the car can be stopped is

(a) $\dfrac{v_0^2}{2\mu g}$ (b) $\dfrac{v_0}{\mu g}$

(c) $\left(\dfrac{v_0}{\mu g}\right)^2$ (d) $\dfrac{v_0}{\mu}$

14. A body B lies on a smooth horizontal table and another body A is placed on B. The coefficient of friction between A and B is μ. What acceleration given to B will cause slipping to occur between A and B

(a) μg (b) g/μ

(c) μ/g (d) $\sqrt{\mu g}$

15. A 500 kg horse pulls a cart of mass 1500 kg along a level road with an acceleration of 1 ms^{-2}. If the coefficient of sliding friction is 0.2, then the force exerted by the horse on cart in forward direction is

(a) 3000 N (b) 4000 N

(c) 5000 N (d) 4500 N

16. A horizontal force of 129.4 N is applied on a 10 kg block which rests on a horizontal surface. If the coefficient of friction is 0.3, the acceleration should be

(a) 9.8 m/s^2 (b) 10 m/s^2

(c) 12.6 m/s^2 (d) 19.6 m/s^2

17. If μ_s, μ_k and μ_r are coefficients of static friction, sliding friction and rolling friction, then

(a) $\mu_s < \mu_k < \mu_r$ (b) $\mu_k < \mu_r < \mu_s$

(c) $\mu_r < \mu_k < \mu_s$ (d) $\mu_r = \mu_k = \mu_s$

18. The coefficient of friction between a body and the surface of an inclined plane at 45° is 0.5. If $g = 9.8$ m/s^2, the acceleration of the body downwards in m/s^2 is

(a) $\dfrac{4.9}{\sqrt{2}}$ (b) $4.9\sqrt{2}$

(c) $19.6\sqrt{2}$ (d) 4.9

19. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A body starting from the rest at top comes back to rest at the bottom if the coefficient of friction for the lower half is given by

(a) $\mu = \sin\theta$ (b) $\mu = \cot\theta$

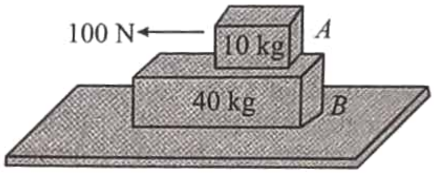
(c) $\mu = 2\cos\theta$ (d) $\mu = 2\tan\theta$

20. The time taken by a body to slide down a rough 45° inclined plane is twice that required to slide down a smooth 45° inclined plane. The coefficient of kinetic friction between the object and rough plane is given by

(a) $\dfrac{1}{3}$ (b) $\dfrac{3}{4}$ (c) $\sqrt{\dfrac{3}{4}}$ (d) $\sqrt{\dfrac{2}{3}}$

TWO AND THREE BLOCKS PROBLEMS

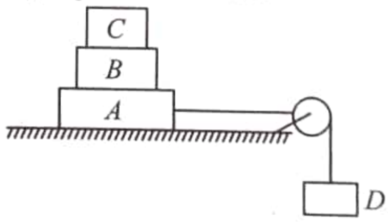
21. A 40 kg slab rests on a frictionless floor as shown in the figure. A 10 kg block rests on the top of the slab. The static coefficient of friction between the block and slab is 0.60 while that of the kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8$ m/s^2, the resulting acceleration of the slab will be



(a) 0.98 m/s^2 (b) 1.47 m/s^2

(c) 1.52 m/s^2 (d) 6.1 m/s^2

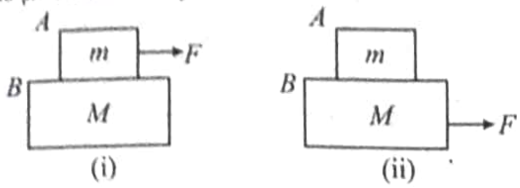
22. Three blocks A, B and C of equal masses 'm' each are placed one over the other on a frictionless table. The coefficient of friction between any two blocks is μ. Find the maximum value of mass of block D so that the blocks A, B and C move without slipping over each other.



(a) $\dfrac{3m\mu}{\mu+1}$ (b) $\dfrac{3m(1-\mu)}{\mu}$

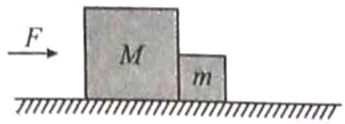
(c) $\dfrac{3m(1+\mu)}{\mu}$ (d) $\dfrac{3m\mu}{(1-\mu)}$

23. A body A rests on B and friction coefficient between A and B is μ. Block M is placed on a frictionless surface. Then



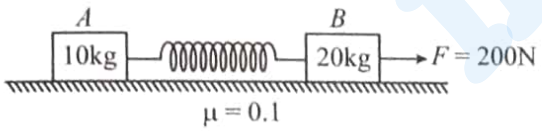
(a) The maximum possible value of F so that both bodies move together in case (i) is $\mu(M+m)g$.
(b) The maximum possible value of F so that both bodies move together in case is μmg.
(c) The maximum possible value of F so that both bodies move together in case (ii) is $\mu(M+m)g$.
(d) The maximum possible value of F so that both bodies move together in case (ii) is μmg.

24. Two blocks of masses $M = 3$ kg and $m = 2$ kg are in contact on a horizontal table. A constant horizontal force $F = 5$ N is applied to block M as shown. There is a constant frictional force of 2 N between the table and the block m but no frictional force between the table and the first block M, then the acceleration of the two blocks is



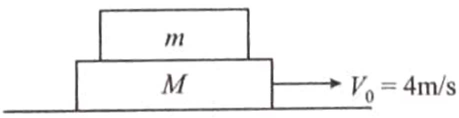
(a) $0.4\ m/s^2$
(b) $0.6\ m/s^2$
(c) $0.8\ m/s^2$
(d) $1\ m/s^2$

25. Two blocks A and B attached to each other by a massless spring, are kept on a rough horizontal surface ($\mu = 0.1$) and pulled by a force $F = 200$ N as shown in figure. If at some instant the 10 kg mass has acceleration of $12\ m/s^2$, what is the acceleration of 20 kg mass?



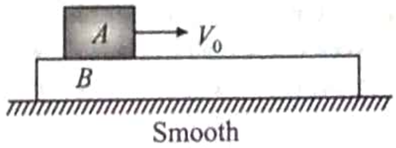
(a) $2.5\ m/s^2$
(b) $4.0\ m/s^2$
(c) $3.6\ m/s^2$
(d) $1.2\ m/s^2$

26. A stationary body of mass m is slowly lowered onto a massive platform of mass M ($M > m$) moving at a speed $V_0 = 4$ m/s as shown in fig. How far will the body slide along the platform? ($\mu = 0.2$ and $g = 10\ m/s^2$)



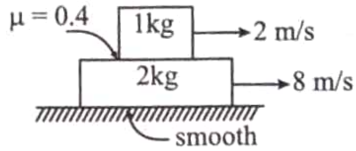
(a) 4 m (b) 6 m (c) 12 m (d) 8 m

27. A block A of mass m is placed over a plank B of mass 2 m. Plank B is placed over a smooth horizontal surface. The coefficient of friction between A and B is 0.5. Block A is given a velocity v_0 towards right. Acceleration of B relative to A is



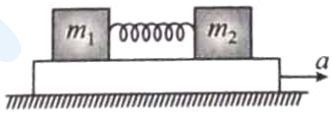
(a) $\frac{g}{2}$
(b) g
(c) $\frac{3g}{4}$
(d) Zero

28. The time when relative motion between the block will stop is



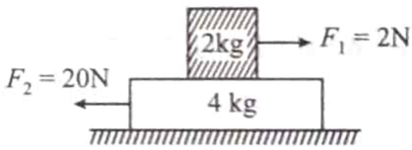
(a) 1 s
(b) 2 s
(c) 3 s
(d) 4 s

29. Two block of masses m_1 and m_2 are connected with a massless unstretched spring and placed over a plank moving with an acceleration 'a' as shown in figure. The coefficient of friction between the blocks and platform is μ.



(a) Spring will be stretched if $a > \mu g$.
(b) Spring will be compressed if $a \le \mu g$.
(c) Spring will neither be compressed nor be strecteched for $a \le \mu g$.
(d) Spring will be in its natural length under all conditions.

30. In the arrangement shown in figure, coefficient of friction between the two block is $\mu = \frac{1}{2}$. The force of friction acting between the two blocks is

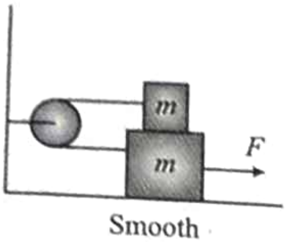


(a) 8 N (b) 10 N (c) 6 N (d) 4 N

31. Two blocks of masses m_1 and m_2 are placed in contact with each other on a horizontal platform as shown in figure. The coefficient of friction between m_1 and platform is 2μ and that between block m_2 and platform is μ. The platform moves with an acceleration a. The normal reaction between the block is

(a) Zero in all cases
(b) Zero only if $m_1 = m_2$
(c) Non zero only if $a > 2\mu g$
(d) Non zero only if $a > \mu g$

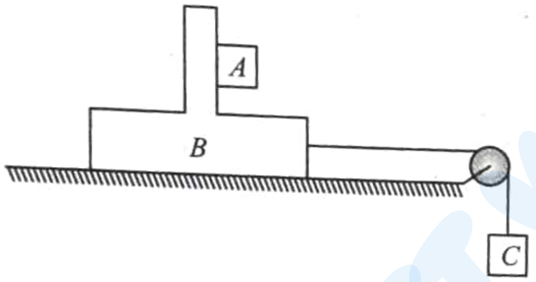
32. For the arrangement shown in figure the coefficient of friction between the two blocks is α. If both the blocks are identical are identical and moving, then the acceleration of each block is



(a) $\frac{F}{2m} - 2\alpha g$ (b) $\frac{F}{2m}$

(c) $\frac{F}{2m} - \alpha g$ (d) Zero

33. In the arrangement shown in the figure, mass of the block B and A is 2 m and m respectively. Surface between B and floor is smooth. The block B is connected to the block C by means of a string pulley system. If the whole system is released, then find the minimum value of mass of block C so that A remains stationary w.r.t. B. Coefficient of friction between A and B is μ.

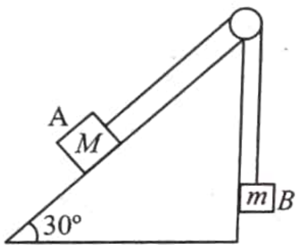


(a) $\frac{m}{\mu}$ (b) $\frac{2m+1}{\mu+1}$

(c) $\frac{3m}{\mu-1}$ (d) $\frac{6m}{\mu+1}$

INCLINED PLANE PROBLEMS

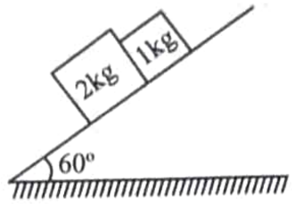
34. Block A of mass M in the system shown in the figure slides down the incline at a constant speed. The coefficient of friction between block A and the surface is $\frac{1}{3\sqrt{3}}$. The mass of block B is



(a) $M/2$ (b) $M/3$

(c) $2M/3$ (d) $M/\sqrt{3}$

35. In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is $\mu_1 = 0.5$ and $\mu_2 = 0.4$ respectively, then



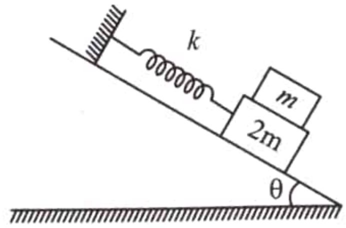
(a) Both block will move together

(b) Both block will move separately

(c) There is a non zero contact force between two blocks

(d) None of these

36. The coefficient of friction between block of mass m and $2m$ is $\mu = 2\tan\theta$. There is no friction between block of mass $2m$ and inclined plane. The maximum amplitude of two block system for which there is no relative motion between both the blocks.



(a) $g\sin\theta\sqrt{\frac{k}{m}}$ (b) $\frac{mg\sin\theta}{k}$

(c) $\frac{3mg\sin\theta}{k}$ (d) None of these

37. The force (along incline) required to just move a body up the inclined plane is double the force required to just prevent the body from sliding down the plane. The coefficient of friction is μ. The inclination θ of the plane is

(a) $\tan^{-1}(\mu)$ (b) $\tan^{-1}\left(\frac{\mu}{2}\right)$

(c) $\tan^{-1}(2\mu)$ (d) $\tan^{-1}(3\mu)$

38. A block has been placed on an inclined plane. The slope angle θ of the plane is such that the block slides down the plane at a constant speed. The coefficient of kinetic friction is equal to

(a) $\sin\theta$ (b) $\cos\theta$

(c) g (d) $\tan\theta$

39. A plank 1 m long is fixed with one end, 28 cm above the level of the other end. The top half of the plank is smooth and the bottom half is rough. When a small block of mass m is released at the top, it just reaches the bottom

(a) Coefficient of friction between the block and the rough part of plank is $\frac{7}{12}$

(b) The coefficient of friction between the block and the rough part of plank is $\frac{1}{2}$

(c) On the rough part, the reaction on the block is $\left(\frac{24}{25}\right)mg$

(d) On the rough part, the reaction on the block is $\left(\frac{28}{100}\right)mg$

40. A body is moving down inclined plane of slope 37°. The coefficient of friction between the body and plane varies as $\mu = 0.3\,x$, where x is distance traveled down the plane. The body will have maximum speed

$\left(\sin 37° = \frac{3}{5} \text{and } g = 10 m/s^2\right)$

(a) At $x = 1.16$ m
(b) At $x = 2$ m
(c) At bottom of plane
(d) At $x = 2.5$ m

41. A block of mass m is placed at rest on an inclined plane of inclination θ to the horizontal. If the coefficient of friction between the block and the plane is μ, then the total force the inclined plane exerts on the block is

(a) mg (b) $\mu mg \cos\theta$
(c) $mg \sin\theta$ (d) $\mu mg \tan\theta$

42. The force required to just move a body up the inclined plane is double the force required to just prevent the body from sliding down the plane. The coefficient of friction is x. If θ is the angle of inclination of the plane than $\tan\theta$ is equal to

(a) x (b) $3x$
(c) $2x$ (d) $0.5x$

43. A smooth inclined plane of length L having inclination θ with the horizontal is inside a lift which is moving down with a retardation a. The time taken by a body to slide down the inclined plane from rest will be

(a) $\sqrt{\frac{2L}{(g+a)\sin\theta}}$ (b) $\sqrt{\frac{2L}{(g-a)\sin\theta}}$
(c) $\sqrt{\frac{2L}{a\sin\theta}}$ (d) $\sqrt{\frac{2L}{g\sin\theta}}$

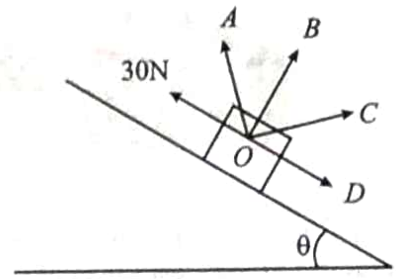
44. A block rests on a rough inclined plane making an angle of 30° with horizontal. The coefficient of static friction between the block and inclined plane is 0.8. If the frictional force on the block is 10 N, the mass of the block in kg is ($g = 10$ m/s^2)

(a) 2.0 (b) 4.0
(c) 1.6 (d) 2.5

45. A body takes time t to reach the bottom of a smooth inclined plane of angle θ with the horizontal. If the plane is made rough, time taken now is $2t$. The coefficient of friction of the rough surface is

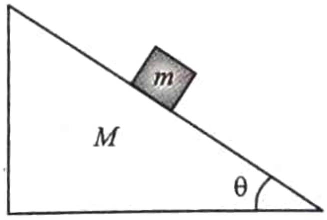
(a) $\frac{3}{4}\tan\theta$ (b) $\frac{2}{3}\tan\theta$
(c) $\frac{1}{4}\tan\theta$ (d) $\frac{1}{2}\tan\theta$

46. A body of mass 10 kg lies on a rough inclined plane of inclination $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ with the horizontal. When the force of 30 N is applied on the block parallel to and upward the plane, the total force by the plane on the block is nearly along



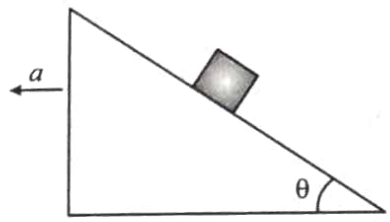
(a) OA (c) OB
(b) OC (d) OD

47. A smooth block of mass m is held stationary on a smooth wedge of mass M and inclination θ as shown in figure. If the system is released from rest, then the normal reaction between the block and the wedge is



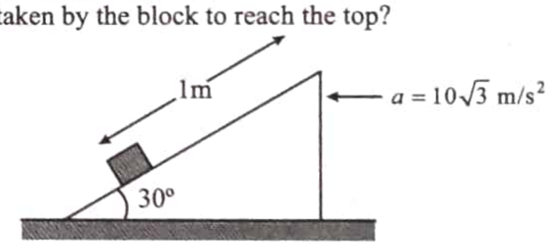
(a) $mg \cos\theta$
(b) Less than $mg \cos\theta$
(c) Greater than $mg \cos\theta$
(d) May be less or greater than $mg \cos\theta$ depending upon whether M is less or greater than m

48. A block of mass m is resting on a wedge of angle θ as shown in the figure. With what minimum acceleration a should the wedge move so that the mass m falls freely?



(a) g
(b) $g \cot\theta$
(c) $g \cos\theta$
(d) $g \tan\theta$

49. In the figure, the wedge is pushed with an acceleration of $10\sqrt{3}$ m/s^2. It is seen that the block starts climbing up on the smooth inclined face of wedge. What will be the time taken by the block to reach the top?



(a) $\frac{2}{\sqrt{5}}$ s (b) $\frac{1}{\sqrt{5}}$ s
(c) $\sqrt{5}$ s (d) $\frac{\sqrt{5}}{2}$ s

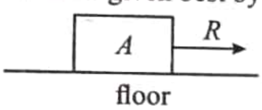
Exercise-2 (Learning Plus)

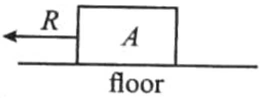
1. A block is placed on a rough floor and a horizontal force F is applied on it. The force of friction f by the floor on the block is measured for different values of F and a graph is plotted between them
   A. The graph is a straight lie of slope 45°
   B. The graph is straight line parallel to the F axis
   C. The graph is a straight line of slope 45° for small F and a straight line parallel to the F-axis for large F.
   D. There is small kink on the graph
   (*a*) C, D (*b*) A, D
   (*c*) A, B (*d*) A, C

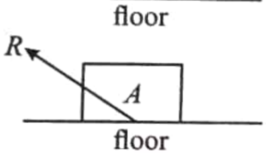
2. A monkey of mass m is climbing a rope hanging from the roof with acceleration a. The coefficient of static friction between the body of the monkey and the rope is μ. Find the direction and value of friction force on the monkey.

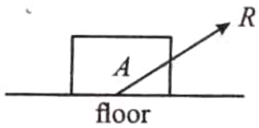
   (*a*) Upward, $F = m(g + a)$
   (*b*) Downward, $F = m(g + a)$
   (*c*) Upward, $F = mg$
   (*d*) Downward, $F = mg$

3. A box 'A' is lying on the horizontal floor of the compartment of a train running along horizontal rails from left to right. At time 't', it decelerates. Then the reaction R by the floor on the box is given best by

(*a*)


(*b*)


(*c*)


(*d*)


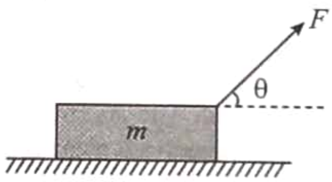
4. A body is placed on a rough inclined plane of inclination θ. As the angle θ is increased from 0° to 90° the contact force between the block and the plane.
   (*a*) Remains constant
   (*b*) First remains constant then decreases
   (*c*) First decreases then increases
   (*d*) First increases then decreases

5. A chain is lying on a rough table with a fraction $1/n$ of its length hanging down from the edge of the table. If it is just on the point of siding down from the table, then the coefficient of friction between the table and the chain is-
   (*a*) $\frac{1}{n}$ (*b*) $\frac{1}{(n-1)}$
   (*c*) $\frac{1}{(n+1)}$ (*d*) $\frac{n-1}{(n+1)}$

6. For the equilibrium of a body on an inclined plane of inclination 45°. The coefficient of static friction will be
   (*a*) Greater than one
   (*b*) Less than one
   (*c*) Zero
   (*d*) Less than zero

7. A block of mass 5 kg and surface area 2 m^2 just begins to slide down an inclined plane when the angle of inclination is 30°. Keeping mass same, the surface area of the block is doubled. The angle at which this starts sliding down is:
   (*a*) 30° (*b*) 60°
   (*c*) 15° (*d*) None of these

8. A wooden block of mass m resting on a rough horizontal table (coefficient of friction = μ) is pulled by a force F as shown in figure. The acceleration of the block moving horizontally is:



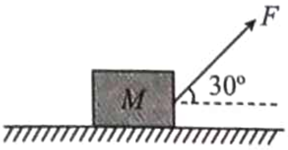
   (*a*) $\frac{F\cos\theta}{m}$
   (*b*) $\frac{\mu F\sin\theta}{M}$
   (*c*) $\frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$
   (*d*) None of these

9. A block moves down a smooth inclined plane of inclination θ. Its velocity on reaching the bottom is v. If it slides down a rough inclined plane of some inclination, its velocity on reaching the bottom is v/n, where n is a number greater than 0. The coefficient of friction is given by
   (*a*) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)$ (*b*) $\mu = \cot\theta\left(1-\frac{1}{n^2}\right)$
   (*c*) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)^{1/2}$ (*d*) $\mu = \cot\theta\left(1-\frac{1}{n^2}\right)^{1/2}$

10. Starting from rest a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The co-efficient of friction between the body and the inclined plane is:

(*a*) 0.75 (*b*) 0.33
(*c*) 0.25 (*d*) 0.80

11. A block of mass $M = 5$ kg is resting on a rough horizontal surface for which the coefficient of friction is 0.2. When a force $F = 40$ N is applied as shown in figure the acceleration of the block will be ($g = 10$ m/s^2):

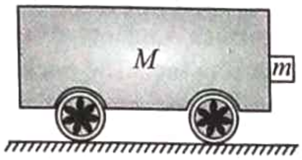


(*a*) 5.73 m/sec^2 (*b*) 8.0 m/sec^2
(*c*) 3.17 m/sec^2 (*d*) 10.0 m/sec^2

12. If the normal force is doubled the co-efficient of friction is:

(*a*) Halved (*b*) Doubled
(*c*) Tripled (*d*) Not changed

13. A cart of mass M has a block of mass m attached to it as shown in the figure. Co-efficient of friction between the block and cart is μ. What is the minimum acceleration of the cart so that the block m does not fall?



(*a*) μg (*b*) μ/g
(*c*) g/μ (*d*) None of these

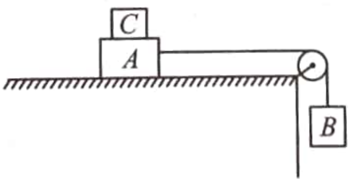
14. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6 If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is :

(*a*) 5 N (*b*) 6 N
(*c*) 10 N (*d*) 15 N

15. A block of mass 2 kg rests on a rough inclined plane making an angle of 30^0 with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is :

(*a*) 9.8 N (*b*) $0.7 \times 9.8\sqrt{3}$ N
(*c*) 9.8×7 N (*d*) 0.8×9.8 N

16. Two masses A and B of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown. The coefficient of static friction of A with table is 0.2. The minimum mass of C that may be placed on A to prevent it from moving is



(*a*) 15 kg (*b*) 10 kg
(*c*) 5 kg (*d*) 12 kg

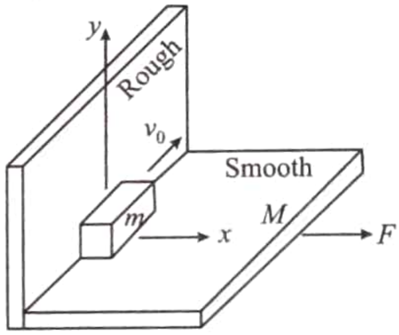
17. A 60 kg body is pushed horizontally with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficient of static friction and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is:

(*a*) 6 m/s^2
(*b*) 4.9 m/s^2
(*c*) 3.92 m/s^2
(*d*) 1 m/s^2

18. You pour a quantity of flour of volume $V = 225$ cm^3 onto a board, where it forms a conical pile. The coefficient of static friction between flour grains is $\mu_s = \sqrt{1.60}$. Find the maximum height (in cm) of the pile. (Take $\pi = 3.15$)

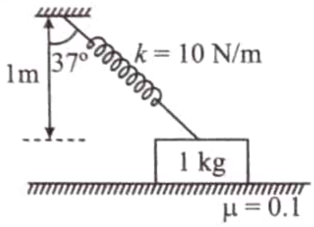
(*a*) 7 cm
(*b*) 70 cm
(*c*) 7.5 cm
(*d*) 8 cm

19. Figure shows a block placed on a bracket. Bracket is placed on a smooth floor, it is pulled by a force $F = 6$N horizontally. Block is projected with velocity v_0 relative to bracket as shown in figure. Find time in second after which it stops relative to bracket. Horizontal surface of bracket is smooth while vertical surface is rough (Given: $m = 1$kg, $M = 5$kg, $v_0 = 5$ m/s, $\mu = 0.5$) (Round off nearest integer)



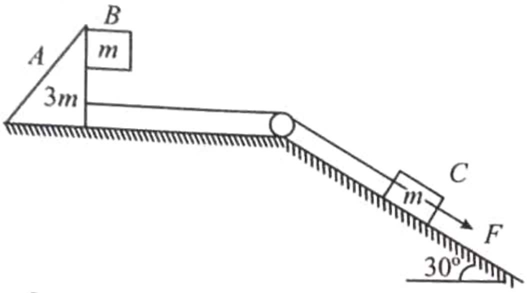
(*a*) 5 sec (*b*) 7 sec
(*c*) 15 sec (*d*) 10 sec

20. The spring shown in the figure has a natural length of 1m. What is the initial acceleration (in cm/sec^2) of the block when released?



(*a*) 700 cm/s^2
(*b*) 70 cm/s^2
(*c*) 0.7 cm/s^2
(*d*) 140 cm/s^2

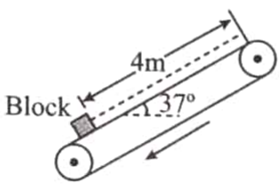
21. In the arrangement shown in the figure. The mass of wedge A and that of the block B are 3m and m respectively. Friction exists between A and B only. The mass of the block C is m. The force $F = 19.5\ \text{m} \times g$ is applied on the block C as shown in the figure. The minimum coefficient of friction (μ) between A and B so that B remains stationary with respect to wedge A will be



(a) $\frac{2}{5}$ (b) $\frac{1}{10}$

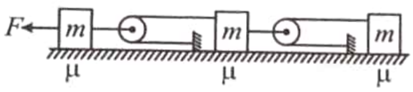
(c) $\frac{2}{5}$ (d) $\frac{1}{4}$

22. The following figure shows an accelerating conveyor belt inclined at an angle 37° above horizontal. The coefficient of friction between the belt and block is '1'. The least time in which block can reach the top, starting from rest at the bottom is:



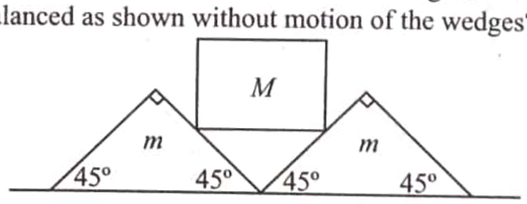
(a) 2s (b) 4s (c) 1s (d) 0.5s

23. On a table, three blocks (including the first block) are placed as shown in the figure. Mass of each block is m and coefficient of friction for each block is μ. A force F is applied on the first block so as to move the system. The minimum value of F should be



(a) $8\mu mg$ (b) $9\mu mg$ (c) $7\mu mg$ (d) $5\mu mg$

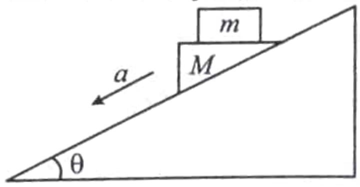
24. Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown below. Assume no friction between the cube and the wedges, but a coefficient of static friction $\mu < 1$ between the wedges and the floor. What is the largest M that can be balanced as shown without motion of the wedges?



(a) $\frac{\mu m}{\sqrt{2}}$ (b) $\frac{\mu m}{1-\mu}$

(c) $\frac{2\mu m}{1-\mu}$ (d) All M will balance

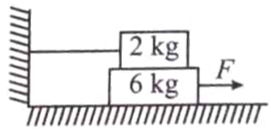
25. As shown in the figure, a wedge of mass M is placed on a smooth inclined ramp that makes an angle θ to the horizontal. An object of mass m rests on top of the wedge. The system is sliding down the ramp at acceleration a. Determine the apparent weight of the object as it slides down. Note that there is friction between the object and the wedge so that the object remains relatively at rest on the wedge.



(a) $mg \cos\theta$ (b) $mg \cos^2\theta$

(c) $mg \sin\theta \cos\theta$ (d) $mg \tan\theta$

26. If the coefficient of friction at all surfaces is 0.4, then the force required to pull out the 6 kg block with an acceleration of 1.5 m/s^2 will be



(a) 49 (b) 59

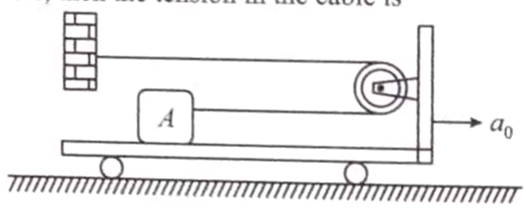
(c) 41 (d) 40

27. The rear side of a truck is open and a box of mass 20kg is placed on the truck 4 meters away from the open end $\mu = 0.15$ and $g = 10\text{m/sec}^2$. The truck starts from rest with an acceleration of 2m/sec^2 on a straight road. The box will fall off the truck when truck is at a distance from the starting point equal to:

(a) 4 m (b) 8 m

(c) 16 m (d) 32 m

28. A flat car is given an acceleration $a_0 = 2\ \text{m/s}^2$ starting from rest. A cable is connected to a crate A of weight 50 kg as shown. Neglect friction between the floor and the car wheels. Also the mass of the pulley is negligible. If coefficient of static friction is $\mu = 0.30$ between the crate and the floor of the car, then the tension in the cable is



(a) 350 N (b) 250 N

(c) 300 N (d) 400 N

29. A block is given certain upward velocity along the incline of elevation α. The time of ascent to upper point was found to be half the time of descent to initial point. The co-efficient of friction between block and incline is :

(a) $0.5 \tan\alpha$

(b) $0.3 \tan\alpha$

(c) $0.6 \tan\alpha$

(d) $0.2 \tan\alpha$

30. A plank of mass 2kg and length 1 m is placed on a horizontal floor. A small block of mass 1 kg is placed on top of the plank, at its right extreme end. The coefficient of friction between plank and floor is 0.5 and that between plank and block is 0.2. If a horizontal force = 30 N starts acting on the plank to the right, the time after which the block will fall off the plank is (g = 10 m/s^2)

(a) (2/3) s
(b) 1.5 s
(c) Block will never fall off the plank
(d) (4/3) s

31. Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can be hang from the table without sliding.

(a) $\dfrac{\mu L}{2}$

(b) $\dfrac{\mu}{2L}$

(c) $\dfrac{\mu L}{1+\mu}$

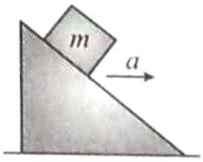
(d) $\dfrac{\mu L}{1-\mu}$

32. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bowl.

(a) $r\left(1-\dfrac{1}{\sqrt{\mu^2+1}}\right)$ (b) $r\left(1-\dfrac{1}{\sqrt{\mu+1}}\right)$

(c) $r\left(1+\dfrac{1}{\sqrt{\mu^2+1}}\right)$ (d) $r\left(1+\dfrac{1}{\sqrt{\mu-1}}\right)$

33. A block of mass 1 kg sits on an incline as shown in figure.



(i) What must be the frictional force between block and incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 m/s^2?
(ii) What is the least value μ_s can have for this to happen?

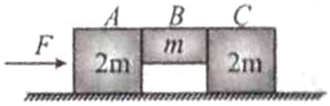
(a) 3.48, 0.8 (b) 3.48, 0.36
(c) 0.36, 3.48 (d) 0.36, 0.8

34. A body of mass 5×10^{-3} kg is launched up on a rough inclined plane making an angle of 30° with the horizontal. Find the coefficient of friction between the body and the plane if the time of ascent is half of the time of descent.

(a) $\mu = \dfrac{1}{2}$ (b) $\mu = \dfrac{3}{5}$

(c) $\mu = \dfrac{\sqrt{2}}{5}$ (d) $\mu = \dfrac{\sqrt{3}}{5}$

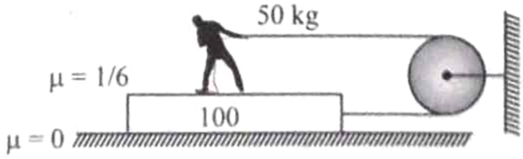
35. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from downward slipping is



(a) $\left(\dfrac{3}{2\mu}\right)mg$ (b) $\left(\dfrac{5}{2\mu}\right)mg$

(c) $\left(\dfrac{5}{2}\right)\mu mg$ (d) $\left(\dfrac{3}{2}\right)\mu mg$

36. A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man and plank move together, find force of friction acting on man.



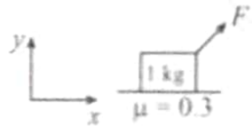
(a) $\dfrac{100}{3}$ N towards left

(b) $\dfrac{100}{3}$ N towards right

(c) $\dfrac{250}{3}$ N towards left

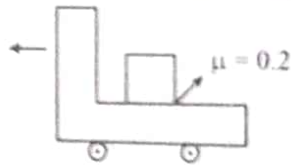
(d) $\dfrac{250}{3}$ N towards right

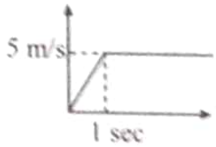
37. A force $\vec{F} = \hat{i} + 4\hat{j}$ acts on block shown. The force of friction acting on the block is

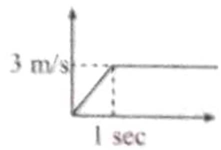


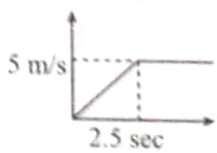
(a) $-\hat{i}$ (b) $-1.8\hat{i}$ (c) $-2.4\hat{i}$ (d) $-3\hat{i}$

38. A truck starting from rest moves with an acceleration of 5 m/s^2 for 1 sec and then moves with constant velocity. The velocity w.r.t. ground v/s time graph for block in truck is (Assume that block does not fall off the truck)



(a) 

(b) 

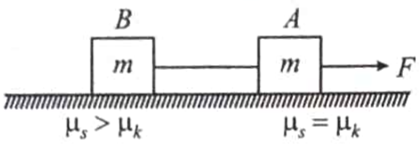
(c) 

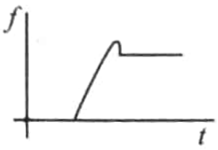
(d) None of these

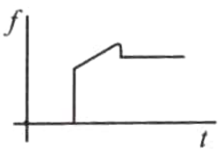
Exercise-3 (JEE Advanced Level)

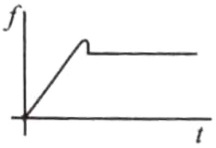
MULTIPLE CORRECT TYPE QUESTIONS

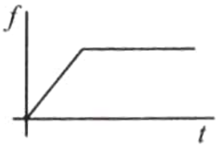
1. A force $F = t$ is applied to block A as shown in figure. The force is applied at $t = 0$ seconds when the system was at rest and string is just straight without tension. Which of the following graphs gives the friction force between B and horizontal surface as a function of time 't'.



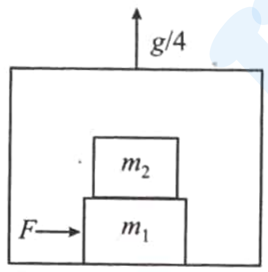
(a) 

(b) 

(c) 

(d) 

2. A plank of mass $m_1 = 8$ kg with a bar of mass $m_2 = 2$ kg placed on its rough surface, lie on a smooth floor of elevator ascending with an acceleration $g/4$. The coefficient of friction is $\mu = 1/5$ between m_1 and m_2. A horizontal force $F = 30$ N is applied to the plank. Then the acceleration of bar and the plank in the reference frame of elevator are:



(a) 3.5 m/s^2, 5 m/s^2 (b) 5 m/s^2, $\frac{50}{8}$ m/s^2

(c) 2.5 m/s^2, $\frac{25}{8}$ m/s^2 (d) 4.5 m/s^2, 4.5m/s^2

3. A man of mass m is applying a horizontal force to slide a box of mass m' on a rough horizontal surface. It is known that the man does not slide. The coefficient of friction between the shoes of the man and the floor is μ and between the box and the floor is μ'. In which of the following cases it is certainly not possible to slide the box?

(a) $\mu > \mu'$, $m < m'$

(b) $\mu < \mu'$, $m < m'$

(c) $\mu < \mu'$, $m > m'$

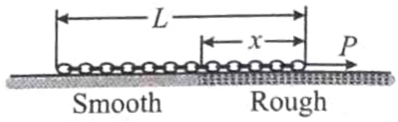
(d) $\mu > \mu'$, $m > m'$

4. A body is projected up along the rough inclined plane fr the bottom with some velocity. It travels up the incline a then returns back. If the time of ascent is t_a and time descent is t_d, then

(a) $t_a = t_d$ (b) $t_a > t_d$

(c) $t_a < t_d$ (d) Data insufficient

5. A chain of length L is placed on a horizontal plane as sho in figure. At any instant x is the length of chain on rou surface and the remaining portion lies on smooth surfa Initially $x = 0$. A horizontal force P is applied to the ch (as shown in figure). In the direction x changes from $x =$ to $x = L$, for chain to move with constant speed.



(a) The magnitude of P should increase with time

(b) The magnitude of P should decrease with time

(c) The magnitude of P should increase first and th decrease with time

(d) The magnitude of P should decrease first and th increase with time

6. The upper portion of an inclined plane of inclination α smooth and the lower portion is rough. A particle slides dov from rest from the top and just comes to rest at the foot. the ratio of the smooth length to rough length is m : n, t coefficient of friction is:

(a) $\left(\frac{m+n}{n}\right)\tan\alpha$ (b) $\left(\frac{m+n}{n}\right)\cot\alpha$

(c) $\left(\frac{m-n}{n}\right)\cot\alpha$ (d) $\frac{1}{2}$

7. A uniform rope so lies on a table that part of it lays ove The rope begins to slide when the length of hanging part 25 % of entire length. The co-efficient of friction betwee rope and table is:

(a) 0.33 (b) 0.25

(c) 0.5 (d) 0.2

8. A 1.5 kg box is initially at rest on a horizontal surface whe at t = 0 a horizontal force $\vec{F} = (1.8t)\hat{i}N$ (with t in seconds is applied to the box. The acceleration of the box as a functio of time t is given by: ($g = 10$m/s^2)

$\vec{a} = 0$ for $0 \le t \le 2.85$

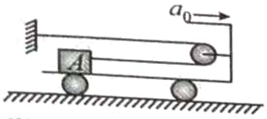
$\vec{a} = (1.2t - 2.4)\hat{i}$ m/s^2 for $t > 2.85$

The coefficient of kinetic friction between the box and th surface is:

(a) 0.12 (b) 0.24

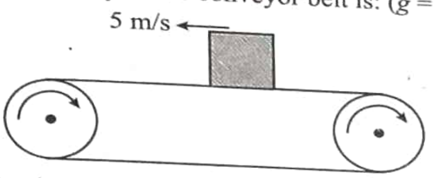
(c) 0.36 (d) 0.48

9. Starting from rest, A flat car is given a constant acceleration $a_o = 2\ m/s^2$. A cable is connected to a crate A of mass 50 kg as shown. Neglect the friction between floor and car wheels and mass of pulley. The coefficient of friction between crate and floor of the car is $\mu = 0.3$. The tension in cable is-



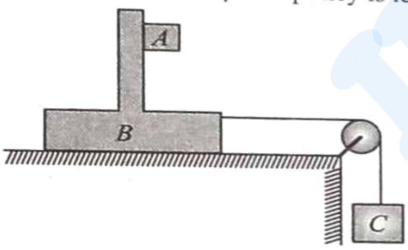
(*a*) 700 N (*b*) 350 N (*c*) 175 N (*d*) 0

10. A block lying on a long horizontal conveyor belt moving at a constant velocity receives a velocity 5 m/s at $t = 0$ sec. relative to the ground in the direction opposite to the direction of motion of the conveyor. After $t = 4$ sec, the velocity of the block becomes equal to the velocity of the belt. The coefficient of friction between the block and the belt is 0.2. Then the velocity of the conveyor belt is: ($g = 10\ m/s^2$)



(*a*) 13 m/s (*b*) –13 m/s (*c*) 3 m/s (*d*) 6 m/s

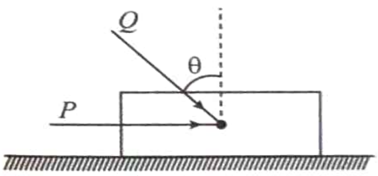
11. In the arrangement shown in the figure mass of the block B and A are 2 m, 8 m respectively. Surface between B and floor is smooth. The block B is connected to block C by means of a pulley. If the whole system is released then the minimum value of mass of the block C so that the block A remains stationary with respect to B is: (Co-efficient of friction between A and B is μ and pulley is ideal)



(*a*) $\dfrac{m}{\mu}$ (*b*) $\dfrac{2m}{\mu+1}$

(*c*) $\dfrac{10\ m}{1-\mu}$ (*d*) $\dfrac{10m}{\mu-1}$

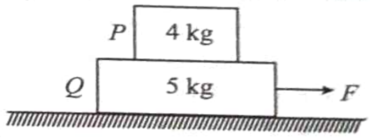
12. A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined an at an angle θ to the vertical. The minimum value of coefficient of friction between the block and the surface for which the block will remain in equilibrium is:



(*a*) $\dfrac{P+Q\sin\theta}{mg+Q\cos\theta}$ (*b*) $\dfrac{P\cos\theta+Q}{mg-Q\sin\theta}$

(*c*) $\dfrac{P+Q\cos\theta}{mg+Q\sin\theta}$ (*d*) $\dfrac{P\sin\theta-Q}{mg-Q\cos\theta}$

13. In the given figure the coefficient of friction between 4kg and 5 kg blocks is 0.2 and between 5 kg block and ground is 0.1 respectively. Choose the correct statements



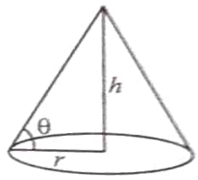
(*a*) Minimum force needed to cause system to move is 17 N

(*b*) When force is 4N static friction at all surfaces is 4N to keep system at rest

(*c*) Maximum acceleration of 4kg block is $2m/s^2$

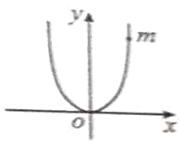
(*d*) Slipping between 4kg and 5 kg blocks start when F is > 17N

14. A worker wishes to pile a cone of sand into a circular area in his yard. The radius of the circle is r, and no sand is to spill onto the surrounding area. If μ is the static coefficient of friction between each layer of sand along the slope and the sand, the greatest volume of sand that can be stored in this manner is:



(*a*) $\mu\pi r^3$ (*b*) $\dfrac{1}{3}\mu\pi r^3$ (*c*) $2\mu\pi r^2$ (*d*) $2\mu\pi r$

15. A bead of mass m is located on a parabolic wire with its axis vertical and vertex directed downward as in figure and whose equation is $x^2 = ay$. If the coefficient of friction is μ, the highest distance above the x-axis at which the particle will be in equilibrium is



(*a*) μa (*b*) $\mu^2 a$ (*c*) $\dfrac{1}{4}\mu^2 a$ (*d*) $\dfrac{1}{2}\mu a$

16. A contact force exerted by one body on horizontal surface is equal to the normal force ($\neq 0$) between them. It can be said that:

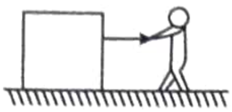
(*a*) The contact surfaces must be frictionless.

(*b*) The force of friction between the contact surfaces is zero.

(*c*) The magnitude of normal force equals that of friction.

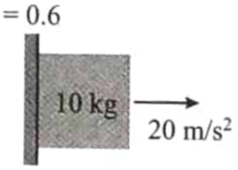
(*d*) It is possible that the bodies are rough and they do not slip on each other.

17. A man pulls a block heavier than himself with a light horizontal rope. The coefficient of friction is the same between the man and the ground, and between the block and the ground.



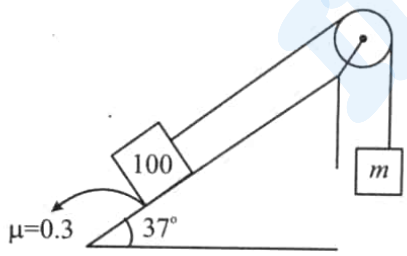
(*a*) The block will not move unless the man also moves.
(*b*) The man can move even when the block is stationary.
(*c*) If both move, the acceleration of the man is greater than the acceleration of the block.
(*d*) None of the above assertions is correct.

18. Car is accelerating with acceleration = 20 m/s^2. A box that is placed inside the car, of mass m = 10 kg is put in contact with the vertical wall as shown. The friction coefficient between the box and the wall is μ = 0.6, then which one is true:



(*a*) The acceleration of the box will be 20 m/sec^2.
(*b*) The friction force acting on the box will be 100 N.
(*c*) The contact force between the vertical wall and the box will be $100\sqrt{5}$N.
(*d*) The net contact force between the vertical wall and the box is only of electromagnetic in nature.

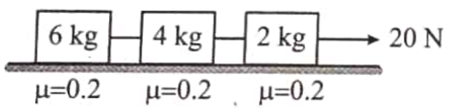
19. The value (s) of mass m for which the 100 kg block remains in static equilibrium is



(*a*) 36 kg (*b*) 37 kg
(*c*) 83 kg (*d*) 85 kg

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 20 to 22): In the arrangement shown

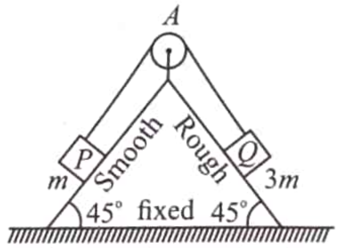


20. Tension in the string connecting 4kg and 6kg masses is
(*a*) 8N (*b*) 12N
(*c*) 6N (*d*) 4N

21. Friction force on 4 kg block is
(*a*) 4N (*b*) 6 N
(*c*) 12 N (*d*) 8 N

22. Friction force on 6 kg block is
(*a*) 12 N (*b*) 8 N
(*c*) 6 N (*d*) 4 N

Comprehension (Q. 23 to 25): A fixed wedge with both surfa inclined at 45° to the horizontal as shown in the figure. A partic P of mass m is held on the smooth plane by a light string whi passes over a smooth pulley A and attached to a particle Q of ma 3m which rests on the rough plane. The system is released fro rest. Given that the acceleration of each particle is of magnitu $\frac{g}{5\sqrt{2}}$ then

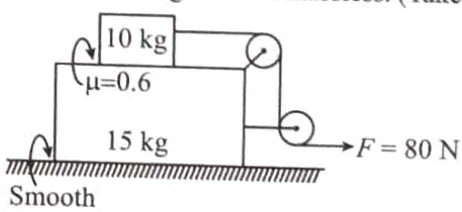


23. In the above question the coefficient of friction between (and the rough plane is:
(*a*) $\frac{4}{5}$ (*b*) $\frac{1}{5}$
(*c*) $\frac{3}{5}$ (*d*) $\frac{2}{5}$

24. The tension in the string is:
(*a*) mg (*b*) $\frac{6mg}{5\sqrt{2}}$
(*c*) $\frac{mg}{2}$ (*d*) $\frac{mg}{4}$

25. In the above question the magnitude and direction of the force exerted by the string on the pulley is :
(*a*) $\frac{6mg}{5}$ downward (*b*) $\frac{6mg}{5}$ upward
(*c*) $\frac{mg}{5}$ downward (*d*) $\frac{mg}{4}$ downward

Comprehension (Q. 26 to 27): A block of mass 15 kg is placed over a frictionless horizontal surface. Another block of mass 10 kg is placed over it, that is connected with a light string passing over two pulleys fastened to the 15 kg block. A force F = 80 N is applied horizontally to the free end of the string. Friction coefficient between two blocks is 0.6. The portion of the string between 10 kg block and the upper pulley is horizontal as shown in figure Pulley string and connecting rods are massless. (Take g = 10 m/s^2)



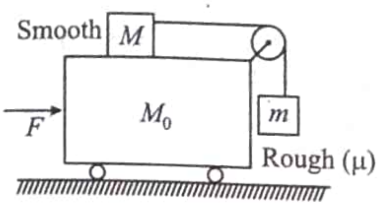
26. The magnitude of acceleration of the 10 kg block is:
(*a*) 3.2 m/s^2 (*b*) 2.0 m/s^2
(*c*) 1.6 m/s^2 (*d*) 0.8 m/s^2

27. The magnitude of acceleration of the 15 kg block is:

(*a*) 4.2 m/s^2 (*b*) 3.2 m/s^2

(*c*) 16/3 m/s^2 (*d*) 2.0 m/s^2

Comprehension (Q. 28 to 29): Imagine a situation in which the horizontal surface of block M_0 is smooth and its vertical surface is rough with a coefficient of friction μ



28. Identify the correct statement(s)

(*a*) If $F = 0$, the blocks cannot remain stationary.

(*b*) For one unique value of F, the blocks M and m remain stationary with respect to M_0.

(*c*) The limiting friction between m and M_0 is independent of F.

(*d*) There exist a value of F at which friction force is equal to zero.

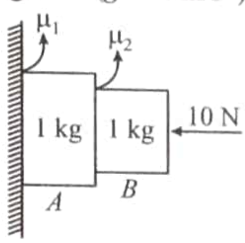
29. In above problem, choose the correct value(s) of F which the blocks M and m remain stationary with respect to M_0

(*a*) $(M_0 + M + m)\frac{g}{\mu}$ (*b*) $\frac{m(M_0 + M + m)g}{M - \mu m}$

(c) $(M_0 + M + m)\frac{mg}{M}$ (*d*) None of these

MATCH THE COLUMN TYPE QUESTIONS

30. In the given figure find the accelerations of blocks A and B for the following cases ($g = 10$ m/s^2)



	Column-I		Column-II
A.	$\mu_1 = 0$ and $\mu_2 = 0.1$	p.	$a_A = a_B = 9.5$ m/s^2
B.	$\mu_2 = 0$ and $\mu_1 = 0.1$	q.	$a_A = 9$ m/s^2, $a_B = 10$ m/s^2
C.	$\mu_1 = 0.1$ and $\mu_2 = 1.0$	r.	$a_A = a_B = g = 10$ m/s^2
D.	$\mu_1 = 1.0$ and $\mu_2 = 0.1$	s.	$a_A = 1, a_B = 9$ m/s^2

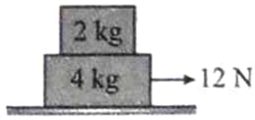
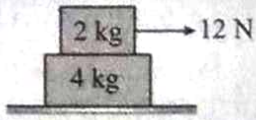
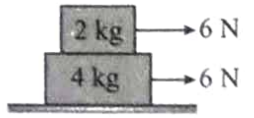
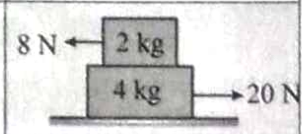
(*a*) A-(r); B-(s); C-(p); D-(q)

(*b*) A-(r); B-(q); C-(p); D-(s)

(*c*) A-(s); B-(q); C-(r); D-(s)

(*d*) A-(q); B-(s); C-(p); D-(r)

31. Column-II gives certain situations involving two blocks of mass 2 kg and 4 kg. The 4 kg block lies on a smooth horizontal table. There is sufficient friction between both the block and there is no relative motion between both the blocks in all situations. Horizontal forces act on one or both blocks as shown. Column-I gives certain statement related to figures given in column-II. Match the statements in column-I with the figure in column-II.

	Column-I		Column-II
A.	Magnitude of frictional force is maximum.	p.	
B.	Magnitude of friction force is least.	q.	
C.	Friction force on 2 kg block is towards right.	r.	
D.	Friction force on 2 kg block is towards left.	s.	

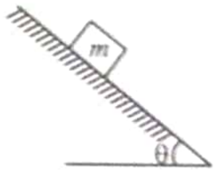
(*a*) A-(r); B-(q,r); C-(p); D-(p,s)

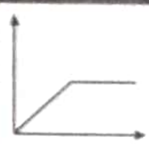
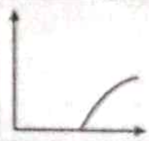
(*b*) A-(r); B-(q); C-(p); D-(s)

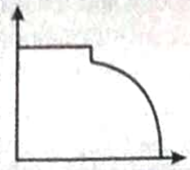
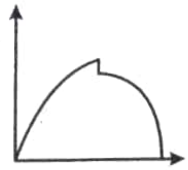
(*c*) A-(s); B-(r); C-(p,s); D-(q,r)

(*d*) A-(r); B-(q,r); C-(p); D-(s)

32. A block placed on a rough inclined plane. Angle of inclination θ of the plane as shown is varied starting from zero. The coefficient of static friction and kinetic friction between the block and the plane is μ_s and μ_k respectively ($\mu_s > \mu_k$). Column-II shows the graphs which necessarily contains θ taken on x-axis. Column-I represents the quantities taken on y-axis of Column-I. Match the quantities of Column-I with graphs of Column-II.



	Column-I		Column-II
A.	Friction force between the block and plane.	p.	
B.	Normal force between the block and the plane	q.	
C.	Total contact force between the block and the plane	r.	

D.	Acceleration of the block	s.	
		t.	

(*a*) A-(t); B-(s); C-(r); D-(q)

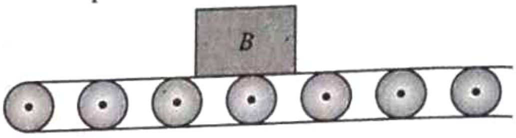
(*b*) A-(p); B-(q); C-(t); D-(s)

(*c*) A-(s); B-(q); C-(r); D-(s)

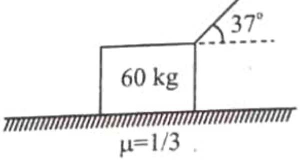
(*d*) A-(t); B-(r); C-(s); D-(q)

NUMERICAL TYPE QUESTIONS

33. The conveyor belt is moving at speed 4 m/s. If the coefficier of static friction between the conveyor and the package B c mass 10 kg is 0.2, determine the shortest time (in sec) th belt can stop so that the package does not slide on the bel



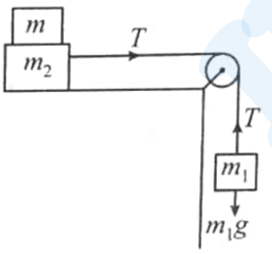
34. A force P pulls a block at a constant speed across the floo What is the magnitude of the force (in N)?



Exercise-4 (Past Year Questions)

JEE MAIN

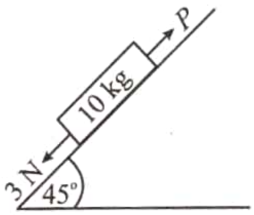
1. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is: **(2018)**



(*a*) 23.3 kg (*b*) 43.3 kg

(*c*) 10.3 kg (*d*) 18.3 kg

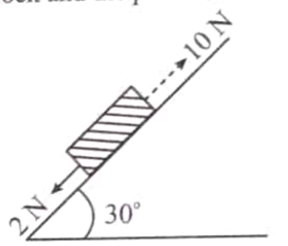
2. Block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward? (take = g =10 ms^{-2}) **(2019)**



(*a*) 32 N (*b*) 18 N

(*c*) 23 N (*d*) 25 N

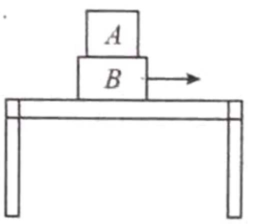
3. A block kept on a rough inclined plane, as shown in the figur remains at rest up-to a maximum force 2 N down the incline plane. The maximum external force up the inclined plane th does not move the block is 10 N. The coefficient of static frictio between the block and the plane is: **(2019**



(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{\sqrt{3}}{4}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{2}{3}$

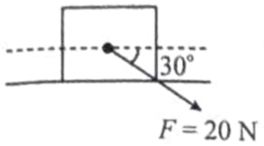
4. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ k are kept on the table as shown in figure. The coefficien of friction between A and B is 0.2 and between B and th surface of the table is also 0.2. The maximum force F tha can be applied on horizontally, so that the block A does nc slide over the block B is: (Take g = 10 m/s^2 **(2019**

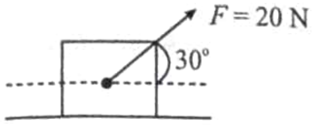


(*a*) 16 N (*b*) 40 N

(*c*) 12 N (*d*) 8 N

5. A block of mass 5 kg is (i) pushed in case (a) and (ii) pulled in case (b). by a force $F = 20$ N. Making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between th block and floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (b) and case (a) will be: (g = 10 ms^{-2}) **(2019)**

(a) 

(b) 

(a) 0 ms^{-2} (b) 0.8 ms^{-2}
(c) 0.4 ms^{-2} (d) 3.2 ms^{-2}

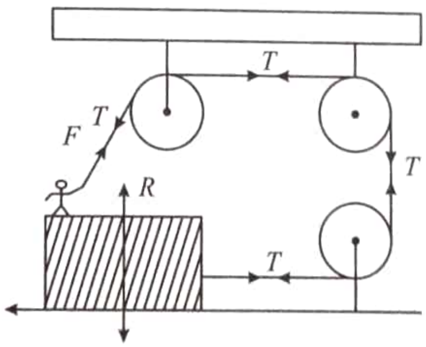
6. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$. The nearest integer to I is __________. **(2020)**

7. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is (g = 10 ms^{-2}) **(2020)**

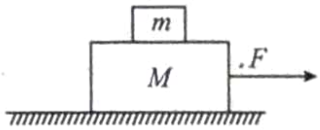
(a) 0.45 m (b) 0.80 m
(c) 0.20 m (d) 0.60 m

8. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F (in N). The value of F will be ______ (Round off to the Nearest Integer) [Take g = 10 ms^{-2}] **(2021)**

9. A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, what is the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is ______ (in N). (Round off to the Nearest Integer) [Take g = 10 ms^{-2}] **(2021)**



10. Two blocks (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is ______ N. (Round off to the Nearest Integer) [Take g as 9.8 ms^{-2}] **(2021)**



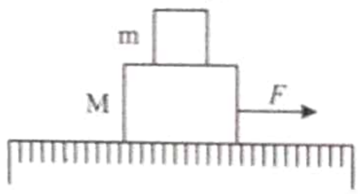
11. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is _______ m. **(2022)**

12. A disc with a flat small bottom beaker placed on it at a distance R from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity w. The coefficient of static friction between the bottom of the beaker and the surface of the disc is m. The beaker will revolve with the disc if: **(2022)**

(a) $R \le \frac{\mu g}{2\omega^2}$ (b) $R \le \frac{\mu g}{\omega^2}$
(c) $R \ge \frac{\mu g}{2\omega^2}$ (d) $R \ge \frac{\mu g}{\omega^2}$

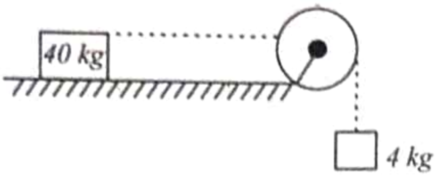
13. A curved in a level road has a radius 75 m. The maximum speed of a car turning this curved road can be 30 m/s without skidding. If radius of curved road is changed to 48m and the coefficient of friction between the tyres and the road remains same, then maximum allowed speed would be _____ m/s. **(2022)**

14. A system of two blocks of masses m = 2 kg and m = 8 kg is placed on a smooth table as shown in figure. The coefficient of static friction between two blocks is 0.5. The maximum horizontal force F that can be applied to the block of mass M so that the blocks move together will be: **(2022)**



(a) 9.8 N (b) 39.2 N
(c) 49 N (d) 78.4 N

15. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below. The coefficient of kinetic friction between the surface and block is 0.02. The acceleration of block is. (Given g = 10 ms^{-2}.) **(2022)**



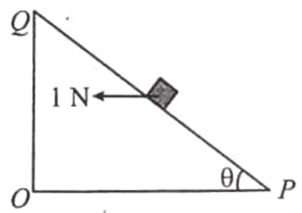
(a) $1\ ms^{-2}$ (b) $1/5\ ms^{-2}$
(c) $4/5\ ms^{-2}$ (d) $8/11\ ms^{-2}$

16. A bag is gently dropped on a conveyor belt moving at a speed of 2 m/s. The coefficient of friction between the conveyor belt and bag is 0.4 Initially, the bag slips on the belt before it stops due to friction. The distance travelled by the bag on the belt during slipping motion is: [Take $g = 10\ m/s^{-2}$] **(2022)**

(a) 2 m (b) 0.5 m
(c) 3.2 m (d) 0.8 m

JEE ADVANCED

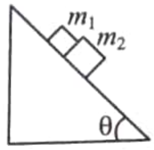
17. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N on the block through its center of mass as shown in the figure. The block remains stationary if (take $g = 10\ m/s^2$) **(2012)**



(a) $\theta = 45°$
(b) $\theta > 45°$ and a frictional force acts on the block towards P.
(c) $\theta > 45°$ and a frictional force acts on the block towards
(d) $\theta < 45°$ and a frictional force acts on the block towards

18. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$ kg , are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in Column-I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In Column-II expression for the friction on block m_2 given. Match the correct expression of the friction in Column-II with the angles given in Column-I, and choose the correct option. The acceleration due to gravity is denoted by **(2014)**

[Useful information: $\tan(5.5°) \approx 0.1$; $\tan(11.5°) \approx 0.2$; $\tan(16.5° \approx 0.3)$]



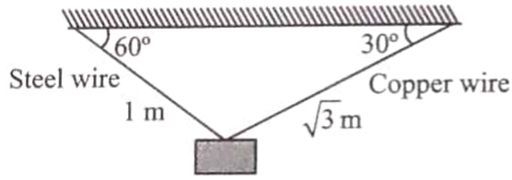
	Column-I		Column-II
A.	$\theta = 5°$	p.	$m_2 g \sin\theta$
B.	$\theta = 10°$	q.	$(m_1 + m_2)g \sin\theta$
C.	$\theta = 15°$	r.	$\mu m_2 g \cos\theta$
D.	$\theta = 20°$	s.	$\mu(m_1 + m_2)g \cos\theta$

(a) A-(t); B-(s); C-(r); D-(q)
(b) A-(p); B-(q); C-(t); D-(s)
(c) A-(s); B-(q); C-(r); D-(s)
(d) A-(q); B-(q); C-(r); D-(r)

19. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area $0.5\ cm^2$ and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is (Δl_C) and elongation in steel wire is (Δl_S)

then the ratio $\dfrac{\Delta \ell_C}{\Delta \ell_S}$ is __________. **(2019)**

[Young's modulus for copper and steel are $1 \times 10^{11}\ N/m^2$ and $2 \times 10^{11}\ N/m^2$ respectively]



ANSWER KEY

CONCEPT APPLICATION

1. (*a*) 2. (*b*) 3. (*a*) 4. (*a*) 5. (*b*) 6. (*b*) 7. (*b*) 8. (*a*) 9. (*c*)

10. (*a*) $a_A = 3\ ms^{-2}, a_B = 0, f_A = 0, f_B = 0$, (*b*) $a_A = 1\ ms^{-2}, a_B = 0, f_A = 25\ N, f_B = 25\ N$, (*c*) $a_A = 35\ ms^{-2}, a_B = 0, f_A = 25\ N, f_B = 25\ N$ (*d*) $a_A = 13\ ms^{-2}, a_B = 0, f_A = 25\ N, f_B = 25\ N$ 11. (*d*)

EXERCISE-1 (TOPICWISE)

1. (*c*)	2. (*d*)	3. (*a*)	4. (*c*)	5. (*a*)	6. (*c*)	7. (*a*)	8. (*c*)	9. (*a*)	10. (*d*)
11. (*d*)	12. (*c*)	13. (*a*)	14. (*a*)	15. (*d*)	16. (*b*)	17. (*c*)	18. (*a*)	19. (*d*)	20. (*b*)
21. (*a*)	22. (*d*)	23. (*c*)	24. (*b*)	25. (*a*)	26. (*a*)	27. (*c*)	28. (*a*)	29. (*d*)	30. (*a*)
31. (*d*)	32. (*c*)	33. (*c*)	34. (*b*)	35. (*b*)	36. (*c*)	37. (*d*)	38. (*d*)	39. (*a*)	40. (*d*)
41. (*a*)	42. (*b*)	43. (*a*)	44. (*a*)	45. (*a*)	46. (*a*)	47. (*b*)	48. (*b*)	49. (*b*)	

EXERCISE-2 (LEARNING PLUS)

1. (*a*)	2. (*a*)	3. (*c*)	4. (*b*)	5. (*b*)	6. (*a*)	7. (*a*)	8. (*c*)	9. (*a*)	10. (*a*)
11. (*a*)	12. (*d*)	13. (*c*)	14. (*a*)	15. (*a*)	16. (*a*)	17. (*d*)	18. (*a*)	19. (*d*)	20. (*b*)
21. (*d*)	22. (*a*)	23. (*c*)	24. (*c*)	25. (*b*)	26. (*a*)	27. (*c*)	28. (*a*)	29. (*c*)	30. (*a*)
31. (*c*)	32. (*a*)	33. (*b*)	34. (*d*)	35. (*b*)	36. (*a*)	37. (*a*)	38. (*a*)		

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*a*)	2. (*c*)	3. (*b*)	4. (*c*)	5. (*a*)	6. (*a*)	7. (*a*)	8. (*b*)	9. (*b*)	10. (*c*)
11. (*d*)	12. (*a*)	13. (*c*)	14. (*b*)	15. (*c*)	16. (*b,d*)	17. (*a,b,c*)	18. (*b*)	19. (*a*)	20. (*a*)
21. (*d*)	22. (*b*)	23. (*d*)	24. (*b*)	25. (*a*)	26. (*a*)	27. (*b*)	28. (*a,c,d*)	29. (*c*)	30. (*b*)
31. (*c*)	32. (*d*)	33. [0002]	34. [0200]						

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*a*)	2. (*a*)	3. (*a*)	4. (*a*)	5. (*b*)	6. [346]	7. (*c*)	8. [5 N]	9. [30]	10. [21]
11. [2]	12. (*b*)	13. [24]	14. (*c*)	15. (*d*)	16. (*b*)				

JEE Advanced

17. (*a,c*) 18. (*d*) 19. [2]

CHAPTER

8 Work, Power and Energy

WORK

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

Work is the product of the applied force and the displacement of the body in the direction of force.

$$W = F\,s\cos\theta = \vec{F}.\vec{s}$$

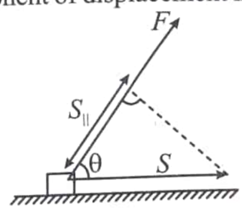
If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{s} = x\hat{i} + y\,\hat{j} + z\,\hat{k}$

Then $W = \vec{F}.\vec{s} = F_x x + F_y y + F_z z$

Work done by a force may also be written as

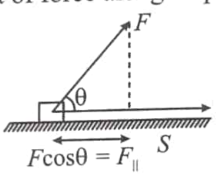
(i) $W = F(S\cos\theta) = FS_{\|}$

$= F \times$ component of displacement in the direction of force



(ii) $W = F\cos\theta S = F_{\|}S$

$=$ Component of force along displacement $\times\, S$



Note:

(i) Dimension : $M^1L^2T^{-2}$

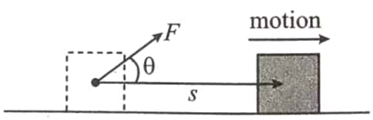
(ii) SI Unit : joule, C.G.S. Unit: erg, 1 joule $= 10^7$ erg

(iii) Positive work : $W = FS\cos\theta$

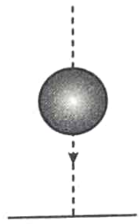
If the angle θ is acute $(\theta < 90^\circ)$ then the work is said to be positive.

The positive work signifies that the external force favours the motion of the body.

$\theta < 90^\circ$ Positive work



(a) When a body falls freely under the action of gravity $(\theta = 0^\circ)$, the work done by gravity is positive.



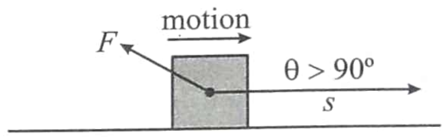
Positive work done by gravitational force

(b) When a spring is stretched, stretching force and the displacement both are in the same direction. So work done by stretching force is positive.

(iv) Negative work: If the angle θ is obtuse $(\theta > 90^\circ)$.

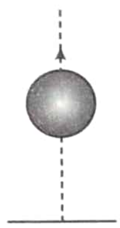
Then the work is said to be negative.

$\theta > 90^\circ$ Negative work



It signifies that the direction of force is such that it opposes the motion of the body.

(a) Work done by frictional force is negative when it opposes the motion.



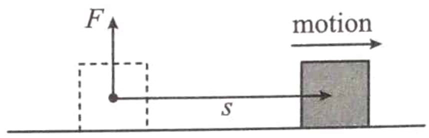
Negative work done by gravitational force

(b) Work done by air resistance on a moving car is negative.

(v) Zero work: $W = Fs\cos\theta$

Work done will be zero if $F = 0$ or $s = 0$ or $\theta = 90^\circ$

$\theta = 90^\circ$ Work done is zero



(vi) Work done by a variable force: When magnitude and direction of the force varies with position, the work done by force for infinitesimal displacement ds is $dW = \vec{F}.\vec{ds}$

The total work done for displacement from A to B is

$$W_{AB} = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)\,ds$$

In terms of rectangular components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\,,\; \vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

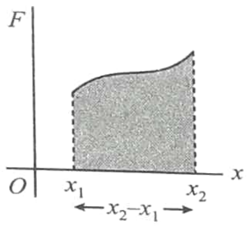
$$W_{AB} = \int_A^B \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$= \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

(vii) Work done by several Forces: When several forces act on a body then the net work done on the body is the algebraic sum of work done by individual forces.

$$W_{net} = \vec{F}_1.\vec{s}_1 + \vec{F}_2.\vec{s}_2 + \ldots\ldots \vec{F}_n.\vec{s}_n$$
$$= W_1 + W_2 + \ldots. + W_N$$

here $\vec{s}_1, \vec{s}_2, \ldots\ldots \vec{s}_n$ are the displacement of points of application of forces $\vec{F}_1, \vec{F}_2, \ldots\ldots \vec{F}_n$ respectively.



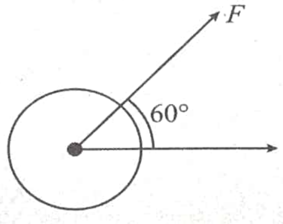
If a graph is constructed of the component $F\cos\theta$ of a variable force, then the work done by the force can be determined by measuring the area between the curve and the displacement axis.

Note: Displacement depends on reference frame so work done by a force is reference frame dependent. Work done by a force can be different in different reference frame.

Train Your Brain

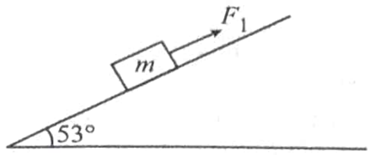
Example 1: A gardener pulls a lawn roller along the ground through a distance of 20 m. If he applies a force of 20 kg weight in a direction inclined at 60° to the ground, find the work done by him. (Take $g = 10$ m/s²)

Sol. $W_F = Fs\cos60^\circ = 200 \times 20 \times \frac{1}{2}$ J $= 2000$ J.



Example 2: A block of mass 20 kg is slowly slid up on a smooth incline of inclination 53° by a person. Calculate the work done by the person in moving the block through a distance of 4 m, if the driving force is (a) parallel to the incline and (b) in the horizontal direction. [g = 10 m/s²]

Sol.

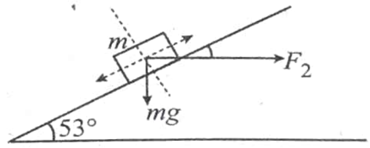


$F_1 = mg\sin53^\circ$ $\quad (\because a = 0)$

$W_1 = (mg\sin53^\circ).(4)$

$= (20 \times 10 \times \frac{4}{5})(4)$

$= 640$ J

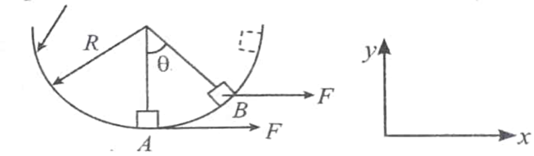


$W_{F_2} = F_2\cos\theta \times 4 = 0$

But $F_2\cos\theta = mg\sin\theta$ $\quad (\because a = 0)$

$W_2 = W_{F_2} = 4mg\sin\theta = 640$ J

Example 3: A block of mass m is taken from A to B along spherical bowl.



Sol: $S_x = R\sin\theta$

$S_y = R - R\cos\theta = R(1 - \cos\theta)$

Work Done by gravity $F = -mgS_y = -mgR(1-\cos\theta)$

Work Done by force $FS_x = FR(\sin\theta)$

Work Done by normal $= 0$

Example 4: A particle is shifted form point (0 m, 0 m, 1 m) to (1 m, 1 m, 2 m), under two forces. The forces are $\vec{F_1} = (2i + 3\hat{j} - \hat{k})N$ and $\vec{F_2} = (\hat{i} - 2\hat{j} + 2\hat{k})N$. Find work done by these combined two forces.

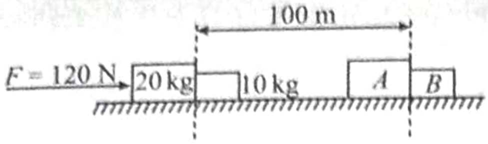
Sol. Work done by a constant force equals to dot product of the force and displacement vectors.

$W = \vec{F}\cdot\Delta\vec{r} \rightarrow W = \left(\vec{F_1} + \vec{F_2}\right)\cdot\Delta\vec{r}$

Substituting given values, we have

$W = (3\hat{i} + \hat{j} + \hat{k})\cdot(\hat{i} + \hat{j} + \hat{k}) = 3 + 1 + 1 = 5$J

Example 5:



(i) Find work done by force F on A during 100 m displacement.

(ii) Find work done by force F on B during 100 m displacement.

(iii) Find work done by normal reaction on B and A during the given displacement.

Sol. (i) $(W_F)_{\text{on } A} = F\Delta S \cos\theta$

$= 120 \times 100 \times \cos 0°$

$= 12000$ J

(ii) $(W_F)_{\text{on } B} = 0$

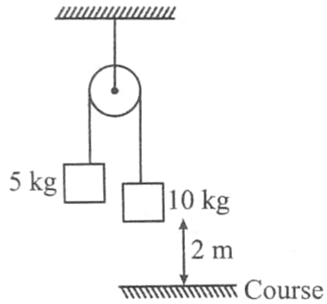
$\because$ F does not act on B

(iii) $(W_N)_{\text{on } B} = 40 \times 100 \times \cos 0° = 4000$ J

$(W_N)_{\text{on } A} = 40 \times 100 \times \cos 180° = -4000$ J

W.D. by normal reaction on system of A and B is zero. i.e. w.d. by internal reaction on a rigid system is zero.

Example 6: The system is released from rest. When 10 kg block reaches at ground then find



(i) Work done by gravity on 10 kg

(ii) Work done by gravity on 5 kg

(iii) Work done by tension on 10 kg

(iv) Work done by tension on 5 kg.

Sol. (i) $(W_g)_{10\text{ kg}} = 10\text{ g} \times 2 = 200$ J

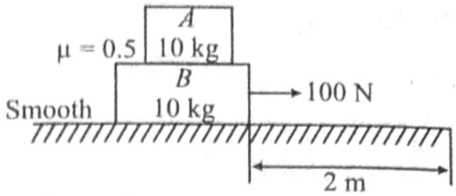
(ii) $(W_g)_{5\text{kg}} = 5\text{ g} \times 2 \times \cos 180° = -100$J

(iii) $(W_T)_{10\text{kg}} = \frac{200}{3} \times 2 \times \cos 180° = \frac{-400}{3}$ J

(iv) $(W_T)_{5\text{ kg}} = \frac{200}{3} \times 2 \times \cos 0° = \frac{400}{3}$ J

Net work done by tension is zero. Work done by internal tension i.e. (tension acting within system) on the system is always zero if the length remains constant.

Example 7: In the given figure



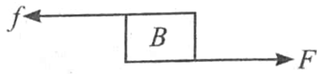
(i) Find work done by applied force during displacement 2 m.

(ii) Find work done by frictional force on B by A during the displacement.

(iii) Find work done by friction force on A by B during displacement.

Sol: (i) $W_F = 100 \times 2 \times \cos 0° = 200$ J

(ii) $f_{smax} = \mu mg = 0.5 \times 10\text{ g} = 50$ N



Assuming they move together.

$100 = 20a \Rightarrow a = 5\text{ m/s}^2$

Check Friction on A, $f = 10 \times 5 = 50$ N

$f = f_{s,max}$

$\therefore$ They move together,

hence $(W_f)_{\text{on } B} = -50 \times 2 = -100$ J

$(W_f)_{\text{on } A} = 50 \times 2 = 100$ J

Note total work done by internal static friction is zero.

(i) Work done by action reaction pair of constraint forces such as static friction, normal reaction and tension in the string is always zero when they are internal forces

(ii) When parts of a body undergo different displacements then work done by the force is given by force × displacement of point of application of the force for example when a man moves up on a stair case work done by normal reaction on him is zero.

Concept Application

1. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in a elevator. The elevator goes down with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block with respect to ground in time t will be

(a) zero

(b) $-mgvt\cos^2\theta$

(c) $-mgvt\sin^2\theta$

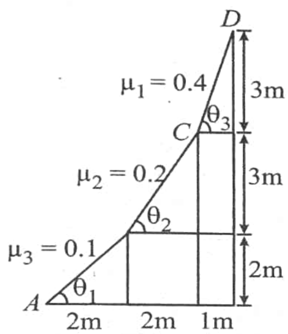
(d) $mgvt\sin 2\theta$

2. Calculate the work done against gravity by a coolie in carrying a load of mass 10 kg on his head when he moves uniformly a distance of 5 m in the (i) horizontal direction (ii) upwards vertical direction.
(Take g = 10 m/s^2)

3. A body is constrained to move in the y-direction. It is subjected to a force $F = (-2\hat{i} + 15\hat{j} + 6\hat{k})$ Newton. What is the work done by this force in moving the body through a distance of 10 m in positive y direction?

4. A block of mass 500 g slides down on a rough incline plane of inclination 53° with a uniform speed. Find the work done against the friction as the block slides through 2 m. [g = 10 m/s^2]

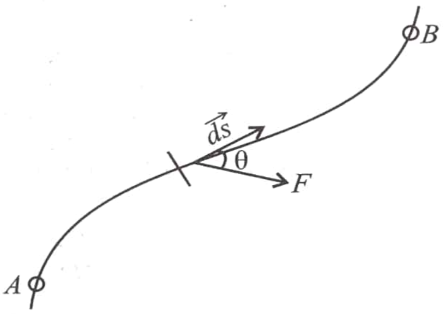
5. As shown in figure a body of mass 1 kg is shifted from A to D slowly on inclined planes by applying a force parallel to incline plane, such that the block is always in contact with the plane surfaces. Neglecting the jerk experienced at points C and B, total work done by the force is



(a) 90 J (b) 56 J (c) 180 J (d) 0 J

6. A block of mass m is pulled on a rough horizontal surface which has a friction coefficient μ. A horizontal force F is applied which is capable of moving the body uniformly with speed v. Find the work done on the block in time t by
(a) Weight of the block
(b) Normal reaction by surface on the block
(c) Friction
(d) F

Work done by Variable Force



When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement ds is given by $dw = \vec{F}.\vec{ds}$

The total work done in going from A to B is

$$W_{AB} = \int_A^B \vec{F}\cdot d\vec{s} = \int_A^B (F \cdot ds\cos\theta)$$

In terms of rectangular components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

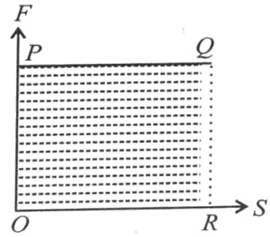
$$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W = \int_A^B F_x dx + \int_A^B F_y dy + \int_A^B F_z dz$$

Graphical Interpretation Of Work Done

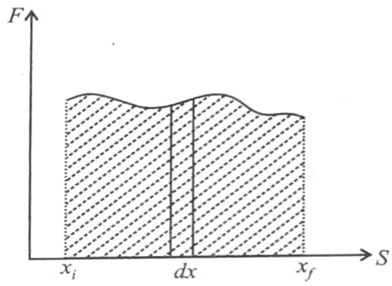
Area of F–S graph gives work, work done by constant force.

The area enclosed by the graph on displacement axis gives the amount of work done by the force



Work = FS = Area of $OPQR$

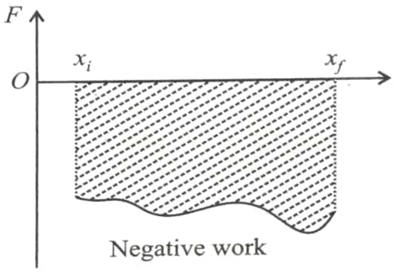
Work done by variable force



For a small displacement dx the work done will be the area of the strip of width dx

$$W = \int_{x_i}^{x_f} dw = \int_{x_i}^{x_f} F\,dx$$

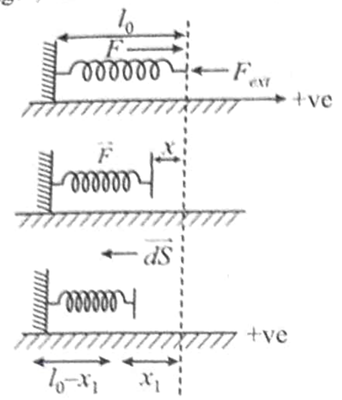
In this case work done is positive. If area lies above X-axis work done is $+ve$ if the area lies below X-axis work done is $-ve$



Negative work

Spring Force

Natural length of spring is l_0. When we **compress** spring by x_1 from natural length, then work done by spring force.



$\vec{F} = kx\hat{i}$

$d\vec{S} = (dx)(-\hat{i})$ {dx is $+ve$ as x is increasing}

$dW = \vec{F}\cdot d\vec{S}$

$$\int dw = -\int_0^{x_1} kx\,dx = -\frac{1}{2}kx_1^2$$

In general work done by the string is given by

$$W = -\frac{1}{2}k(x_f^2 - x_i^2)$$

where x_i and x_f are initial and final deformation of the spring.

$x_i = l_i - l_0$

$x_f = l_f - l_0$

Train Your Brain

Example 8: A force $\vec{F} = \left(3t\hat{i} + 5\hat{j}\right)$N acts on a body due to which its position varies as $\vec{s} = \left(2t^2\hat{i} - 5\hat{j}\right)$. Work done by this force in first two seconds is:

(*a*) 23 J (*b*) 32 J

(*c*) zero (*d*) can't be obtained

Sol. (*b*) $W = \int \vec{F}.\vec{ds}$

$= \int (3t\hat{i} + 5\hat{j}) \cdot (4t\,dt\hat{i})$

$= \int_0^2 12t^2\,dt$

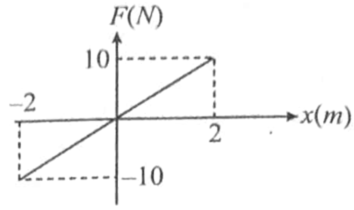
$= \dfrac{12\left[t^3\right]_0^2}{3}$

$= 32$ J

Example 9: A force F acting an a particle varies with the position x as shown in figure. Find the total work done by this force in displacing the particle from

(*a*) $x = -2$ m to $x = 0$

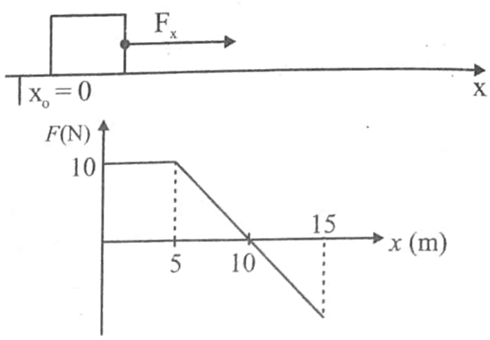
(*b*) $x = 0$ to $x = 2$ m



Sol. (*a*) $W = -\frac{1}{2}(2)(10) = -10$ J

(*b*) $W = \frac{1}{2}(2)(10) = 10$ J

Example 10: A horizontal force F is used to pull a box placed on floor. Variation in the force with position coordinate x measured along the floor is shown in the graph.

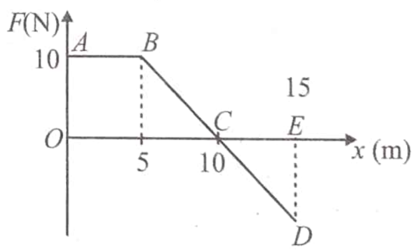


(*a*) Calculate work done by the force in moving the box from $x = 0$ m to $x = 10$ m.

(*b*) Calculate work done by the force in moving the box from $x = 10$ m to $x = 15$ m.

(*c*) Calculate work done by the force in moving the box from $x = 0$ m to $x = 15$ m.

Sol. In rectilinear motion work done by a force equals to area under the force-position graph and the position axis



(*a*) $W_{0\to10}$ = Area of trapazium $OABC$ =75 J

(*b*) $W_{10\to15}$ = –Area of triangle CDE = –25 J

(*c*) $W_{0\to15}$ = Area of trapazium $OABC$ – Area of triangle CDE = 50 J

Example 11: A spring of force constant 100 N/m is stretched upto 5 cm. Find out work done.

Sol. Spring is stretched by length $l = 5$ cm $= 0.05$ m

Work done for small displacement dx is

$dW = Fdx = kx\,dx$

Total work done for length l is

$$W = \int_0^l F\,dx = \int_0^l kx\,dx = \frac{1}{2}kl^2 = \frac{1}{2}100\,(0.05)^2 = 0.125 \text{ J}$$

Example 12: Find the work done by $\vec{F} = (xy\hat{i} + y\hat{j})N$ on particle, when it moves from origin to point (1, 1) along the curve $y = x^2$ where x and y are in meters

Sol. $W = \int F_x dx + \int F_y dy$

$W = \int xy\,dx + \int y\,dy$

$$\Rightarrow y = x^2 \Rightarrow \frac{dy}{dx} = 2x \text{ or } dy = 2x\,dx$$

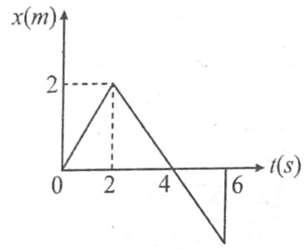
$$\Rightarrow W = \int_0^1 x^3 dx + \int_0^1 x^2\, 2x\,dx$$

$$= \frac{x^4}{4}\Big|_0^1 + \frac{2x^4}{4}\Big|_0^1 = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}\text{J}$$

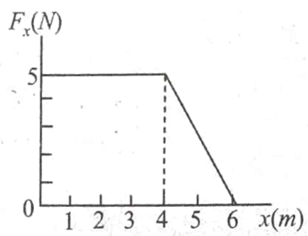
Concept Application

7. A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. Calculate the work done in joule.

8. Position-time graph of a particle of mass 2 kg is shown in figure. Total work done on the particle from $t = 0$ to $t = 4$s is



9. Force acting on a particle varies with x as shown in figure. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

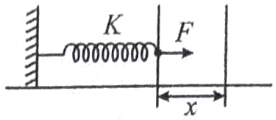


10. An object is displaced from point $A(1, 2)$ to $B(0, 1)$ by applying force $\vec{F} = x\hat{i} + 2y\hat{j}$.

Find out work done by $\vec{F}$ to move the object from point A to B.

11. A spring, which is initially in its unstretched condition, is first stretched by a length x and then again by a further length x. The work done in the first case is W_1 and in the second case is W_2. Find $\frac{W_1}{W_2}$.

12. Initially spring is relaxed. A person starts pulling the spring by applying a variable force F. Find out the work done by F to stretch it slowly to a distance by x.



(a) Work done by spring on wall is zero. Why?

(b) Work done by spring force on man is ________.

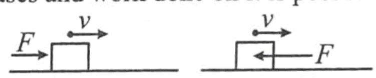
Kinetic Energy

The energy associated with a particle due to its motion is called kinetic energy of the particle, $K = \frac{1}{2}mv^2$

This is a scalar quantity. It is equal to work done by an external force in increasing its velocity from zero to v as is shown in next section.

WORK ENERGY THEOREM

When a force is applied on a particle in the direction of its velocity its speed increases and work done on it is positive



(i) (ii)

On the other hand, when force is applied opposite to its direction of motion, its speed decreases and negative work is done on it.

From this we can infer that kinetic energy increases when positive work is done on the particle and decreases when negative work is done on it.

We know, $\vec{F}_{\text{net}} = m\vec{a}$

$$W = \int \vec{F}_{\text{net}} \cdot d\vec{S} = \int m\vec{a} \cdot d\vec{S}$$

$$\int m\frac{d\vec{v}}{dt} \cdot \vec{v}dt = \int m\vec{v} \cdot d\vec{v} = m\int_i^f (v_x dv_x + v_y dv_y + v_z dv_z)$$

$$= \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)\Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$= K_f - K_i =$ change in kinetic energy

Therefore change in kinetic energy of a particle is equal to total work done and this is called work energy theorem.

Train Your Brain

Example 13: Under the action of a force, the velocity of a body charges from $(3\hat{i}+4\hat{j})\text{m/s}$ to $(-12\hat{i}+5\hat{j})\text{m/s}$. If mass of the body is 2 kg, find the total work done on it.

Sol. $K_i = \frac{1}{2}\times 2\times|3\hat{i}+4\hat{j}|^2 = 25\text{ J}$

$K_f = \frac{1}{2}\times 2\times|-12\hat{i}+5\hat{j}|^2 = 169\text{ J}$

Work done $= K_f - K_i$

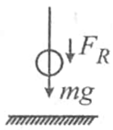
$= 169 - 25 = 144\text{ J}$

Example 14: A ball of mass $m = 0.4$ kg is thrown vertically up with a velocity of 20 m/s in vertically upward direction. It the ball rises to a maximum height of 19 m, find force air resistance that acts on it ($g = 9.8$ m/s²)

Sol. $v_i = 20$ m/s

$v_f = 0$ (at maximum height)

$\Delta K = K_f - K_i = -\frac{1}{2}\times 0.4\times 20^2 = -80\text{ J}$



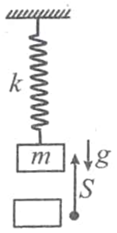
$W_{gravity} = -mg \times 19 = -76$ J

$W_{gravity} + W_R = \Delta K$

$-76 + W_R = -80$

$W_R = -4$ J

Example 15: A block of mass m is attached to a spring of spring constant k. Spring is initially relaxed. If the block is now released, find the maximum vertical displacements of the block?



Sol. At the lowest positive blocks velocity will be zero.

$K_i = 0$

$K_f = 0$

$W_{gravity} = mgS$

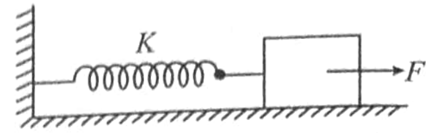
$W_{spring} = -\frac{1}{2}kS^2$

$W_{gravity} + W_{spring} = K_f - K_i$

$mgS - \frac{1}{2}kS^2 = 0$

$S = \frac{2mg}{K}$

Example 16: A block attached to a spring, pulled by a constant horizontal force, is kept on a smooth surface as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is [Given that spring does not break]

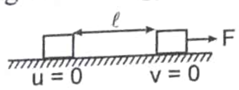


(a) $\frac{F^2}{K}$ (b) $\frac{2F^2}{K}$

(c) ∞ (d) $\frac{F^2}{2K}$

Sol. (b) Force will continue to do positive work till the velocity of the block becomes zero

Applying work energy theorem on block

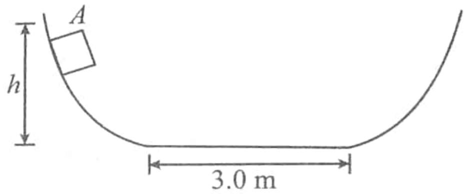


$W_F + W_S = 0$

$F\ell - \frac{1}{2}k\ell^2 = 0$

$\therefore \ell = \frac{2F}{k}$ or work done $= F\ell = \frac{2F^2}{k}$

Example 17: A small particle slides along a track with elevated ends and a flat central part, as shown in figure. The flat part has a length 3 m. the curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu = 0.2$. The particle is released at point A, which is at a height $h = 1.5$ m above the flat part of the track. The position where the particle finally come to rest is



(a) left to mid point of the flat part

(b) right to the mid point of the flat part

(c) Mid point of the flat part

(d) None of these

Sol. (c) $W_G - W_f = 0$

$mgh = \mu mg\ell$

$h = \mu\ell$

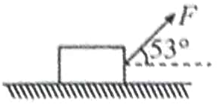
$h = (0.2)\ell$

$\Rightarrow \ell = \frac{1.5}{0.2}$

$\ell = 7.5\text{ m} = (3+3+1.5)\text{m}$

Concept Application

13. A block of mass $m = 4$ kg is dragged 2 m along a horizontal surface by a force $F = 30$ N acting at $53°$ to the horizontal. The initial speed is 3 m/s and $\mu_k = 1/8$.



(*a*) Find the change in kinetic energy of the block

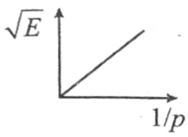
(*b*) Find its final speed

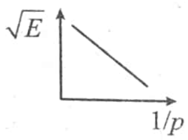
14. An open knife edge of mass 'm' is dropped from a height 'h' on a wooden floor. If the knife penetrates upto depth 'd' into the wood, the average resistance offered by the wood to the knife edge is

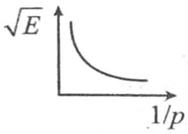
(*a*) mg (*b*) $mg\left(1-\frac{h}{d}\right)$

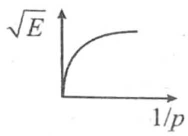
(*c*) $mg\left(1+\frac{h}{d}\right)$ (*d*) $mg\left(1+\frac{h}{d}\right)^2$

15. The graph between $\sqrt{E}$ and $\frac{1}{p}$ is (E = kinetic energy and p = momentum)

(*a*)


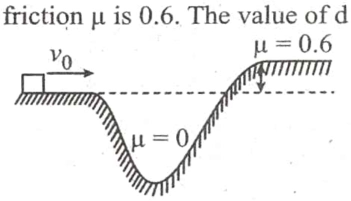
(*b*)


(*c*)


(*d*)


16. The displacement x of a body of mass 1 kg on horizontal smooth surface as a function of time t is given by $x = \frac{t^3}{3}$. Find the work done by the external agent for the first one second.

17. In the figure a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d. The block's initial speed v_0 is 6m/s, the height difference h is 1.1 m and the coefficient of kinetic friction μ is 0.6. The value of d is:

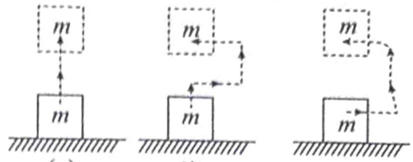


(*a*) 1.17 m (*b*) 1.71 m

(c) 7.11 m (*d*) 11.7 m

CONSERVATIVE FORCE

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and does not depend on the nature of path followed between the initial and final positions.



(*a*) (*b*) (*c*)

Consider a body of mass m being raised to a height h vertically upwards as shown in above figure. The work done is mgh. Suppose we take the body along the path as in (*b*). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result mgh once again. Any arbitrary path like the one shown in (*c*) can be broken into elementary horizontal and vertical portions. Work done along the horizontal path is zero. The work done along the vertical parts add up to mgh. Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the inertial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative Forces

(i) Gravitational force, not only due to the Earth, but due in its general form as given by the universal law of gravitation, is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

(iii) Electrostatic force between two electric charges is a conservative force.

(iv) Magnetic force between two magnetic poles is a conservative force.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and electrostatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of Conservative Forces

- Work done by or against a conservative force depends only on the initial and the final position of the body.
- Work done by or against a conservative force does not depend upon the nature of the path between initial and final position of the body.
- Work done by or against a conservative force in a round trip is zero.
- If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.
- The concept of potential energy exists only in the case of conservative forces.
- The work done by a conservative force is completely recoverable.
- Complete recoverability is an important aspect of the work done by a conservative force.

NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force etc., are non conservative forces.

Difference between Conservative and Non-conservative forces

S.No.	Conservative forces	Non-conservative forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in a round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable.	Work done in not completely recoverable.

ENERGY

A body is said to possess energy if it has the capacity to do work. When a body possessing energy does some work, part of its energy is used up. Conversely if some work is done upon an object, the object will be given some energy. Energy and work are mutually convertible.

There are various forms of energy. Heat, electricity, light, sound and chemical energy are all familiar forms. In studying mechanics, we are however concerned chiefly with mechanical energy. This type of energy is a property of configuration or position.

1. **Kinetic Energy:** Kinetic energy (K.E.), is the capacity of a body to do work by virtue of its motion.

   If a body of mass m has velocity v, its kinetic energy is equivalent to the work, which an external force would have to do to bring the body from rest up to its velocity v.

   The numerical value of the kinetic energy can be calculated from the formula, K.E. $= \frac{1}{2}mv^2 = \frac{m\vec{v}.\vec{v}}{2}$

   Since both m and v^2 are always positive, K.E. is always positive and does not depend upon the direction of motion of the body.

2. **Potential Energy:** Potential energy is energy of the body by virtue of its position. A body is capable to do work by virtue of its position, configuration or state of strain.

   Now relation between Potential energy and work done by a conservative force,

   $W_C = -\Delta U$, where ΔU is change in potential energy

   There are two common forms of potential energy, gravitational and elastic.

Important points related to Potential energy

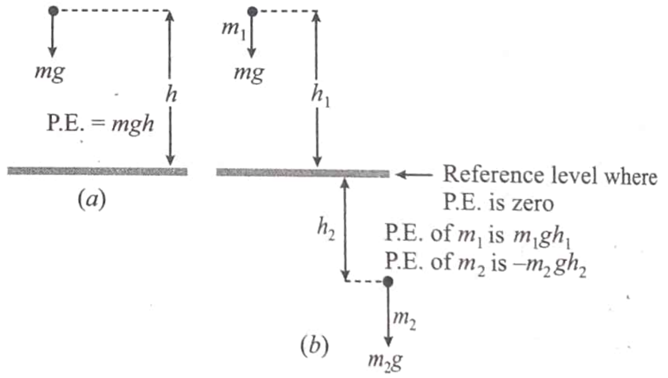
1. Potential energy is a straight function (defined only for position)
2. Potential energy of a point depends on a reference point
3. Potential energy difference between two position doesn't depend on the frame of reference.
4. Potential energy is defined only for conservative force because work done by conservative force is path independent.
5. If we define Potential energy for non conservative force then we have to define P.E. of a single point through different path which gives different value of P.E. at single point, this doesn't make any sense.

Types of Potential Energy

(a) Gravitational Potential Energy (GPE): It is possessed by virtue of height.

When an object is allowed to fall from one level to a lower level it gains speed due to gravitational pull, i.e., it gains kinetic energy. Therefore, in possessing height, a body has the ability to convert its gravitational potential energy into kinetic energy. The gravitational potential energy is equivalent to the negative of the amount of work done by the weight of the body in causing the descent. If a mass m is at a height h above a lower level the P.E. possessed by the mass is $(mg)(h)$. Since h is the height of an object above a specified level, an object below the specified level has negative potential energy.

Therefore GPE = $\pm mgh$



- The chosen level from which height is measured has no absolute position. It is important therefore to indicate clearly the zero P.E. level in any problem in which P.E. is to be calculated.
- GPE = $\pm mgh$ is applicable only when h is very small in comparison to the radius of the earth. We have discussed GPE in detail in 'GRAVITATION'.

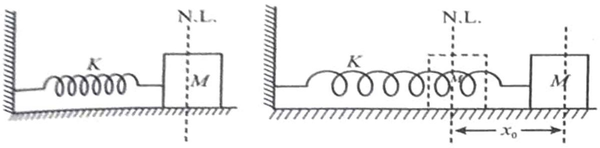
(b) Elastic Potential Energy: It is a property of stretched or compressed springs.

The end of a stretched elastic spring will begin to move if it is released. The spring. therefore possesses potential energy due to its elasticity. (i.e., due to change in its configuration)

The amount of elastic potential energy stored in a spring of natural length a and spring constant k when it is extended by a length x **(from the natural length)** is equivalent to the amount of work necessary to produce the extension.

Elastic Potential Energy $= \frac{1}{2}kx^2$

It is never negative whether the spring is extended or compressed.



Consider a spring block system as shown in the figure and let us calculate work done by spring when the block is displaced by x_0 from the natural length.

At any moment if the elongation in spring is x, then the force on the block by the spring is kx towards left. Therefore, the work done by the spring when block further displaces by dx

$dW = -kx\,dx$

$\therefore$ Total work done by the spring,

$$W = -\int_0^{x_0} kx\,dx = -\frac{1}{2}kx_0^2$$

Similarly, work done by the spring when it is given a compression x_0 is $-\frac{1}{2}kx_0^2$.

We assume zero potential energy at natural length of the spring.

Work Energy Theorem for a System

From work energy theorem, we know

$$W_{total} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Now, if we consider a system of particles,

$$W_{total} = \Delta K_1 + \Delta K_2 + \ldots. + \Delta K_n = \Delta K_{system}$$

If we split the forces into internal and external forces.

$W_{ext} + W_{int} = \Delta K$

or $W_{ext} + W_{int\text{-}C} + W_{int\text{-}NC} = \Delta K$

But $\Delta U = -W_{int\text{-}C}$

Therefore, $\Delta K + \Delta U = W_{ext} + W_{int\text{-}NC}$

This is work energy theorem for a system of particles

For an isolated system $W_{ext} = 0$.

It internal non-conservative forces are also absent

$\Delta K + \Delta U = 0$

$\Delta(K + U) = 0$

$K + U =$ constant

$K + U =$ Mechanical energy is constant.

The principle of conservation of energy may be stated as: If all forces exerted by objects in a system are conservative and no work is done on the system by an outside agent, the mechanical energy (sum of kinetic and potential energy) does not change.

Note: While solving problems involving conservation of mechanical energy, one should note the following key points:

1. One should identify the system for which mechanical energy is conserved. That means one should be able to draw a closed surface such that what ever is inside the surface is the system and whatever is outside is the environment.
2. Is friction or viscous forces (non-conservative force) absent? If they are present, mechanical energy will not be conserved.
3. The system under consideration should be isolated. The principle of mechanical energy conservation holds only for an isolated system. If external forces are present then mechanical energy will not be conserved.
4. The system changes from some initial state to some final state. One must be clear what these states are. Applying energy conservation, one should then set

   $(ME)_{initial} = (ME)_{final}$ or $(K_f - K_i) + (U_f - U_i) = 0$

External Work: Let us consider a system of objects that can interact with each other by conservative forces and with other particles (objects) outside the system. Its mechanical energy will not be conserved if external forces do work on the system. Suppose, for instance, a box is lifted by a person form the ground. The force applied by the person on the box is external if the system consist of earth and the box. The work done by this force increases the mechanical energy of earth - box system. If the velocity of the box does not change, the increase in energy appears as potential energy; the box being further away from the earth. If the box's speed also increases, the increase in energy is both potential and kinetic.

The energy equation can be written as

$\Delta K + \Delta U = W_{ext}$

where U arises from the interaction of the objects in the system with each other and K is the kinetic energy of the system.

Friction and Thermal Energy

Let us now suppose that in addition to conservative forces, a single non-conservative force due to friction acts between the bodies of an isolated system. Then

$\Delta K + \Delta U = W_f$

This equation tells us that if friction force acts, the total mechanical energy is not constant, but changes by an amount equal to work done by frictional force. We can rewrite the above equation as

$\Delta ME = ME_f - ME_f = W_f$

But frictional force is a dissipative force. The total work done by kinetic frictional force is always negative. Therefore, presence of friction diminishes the mechanical energy of the system. The lost mechanical energy in this case appears as thermal energy (heat) and is denoted by E_{th}. This is the energy associated with random motion of individual molecules, as distinct from the motion of the macroscopic body as a whole. Just as the work done by a conservative force between bodies is the negative of the potential energy gain, the work done by frictional force is negative of the thermal (heat) energy gain. Thus

$\Delta E_{th} = -W_f$ and $\Delta ME + \Delta E_{th} = 0$ (for an isolated system)

The Conservation of Energy

If we consider not only conservative forces and the force of friction but also other non-frictional non-conservative forces we can regroup the work-energy theorem

$$\Delta K + \Delta U + \Delta E_{th} = W_{NC}$$

where W_{NC} is the work done by non-frictional non-conservative forces.

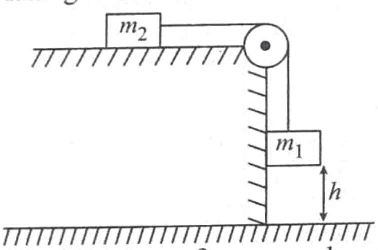
Now whatever the W_{NC} are, it is always possible to find new forms of energy whose change corresponds to negative of this work. We can then represent ΣW_{NC} by another forms of energy. As a result, we can rewrite work energy theorem as

$$\Delta K + \Delta U + \Delta E_{th} + \text{(change in other forms of energy)} = 0$$

In other words, total energy may be transformed from one form to another but the total energy for an isolated system remains constant.

Train Your Brain

Example 18: Two masses are connected by a light string which passes over a fixed pulley. The system is released from rest. Find the speed of block m_1 when it strikes the ground after falling a distance h.



Sol. Considering the system of m_1, m_2 and earth, we notice that there is no friction (non-conservative force) and system is isolated. The mechanical energy of the system is conserved. Hence $\Delta K + \Delta U = 0$

$$\Rightarrow \left[\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - 0\right] - m_1gh = 0$$

in which $v_1 = v_2 = v$

The change in U of m_1 is $(-m_1gh)$ and that of m_2 is zero.

$$\Rightarrow v = \sqrt{\frac{2m_1gh}{m_1 + m_2}}$$

Example 19: If the coefficient of kinetic friction between m_2 and horizontal surface is μ in the previous problem, find the velocity of m_1 when it strikes the ground.

Sol. In this case mechanical energy will not be conserved because work done by friction (non-conservative force) is non-zero.

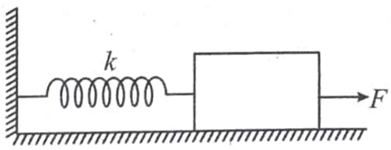
Hence, $\Delta K + \Delta U = W_{\text{friction}}$

in which $W_{\text{friction}} = -\mu Nh = -\mu m_2gh$

$$\Rightarrow \frac{1}{2}(m_1 + m_2)v^2 - m_1gh = -\mu m_2gh$$

$$\Rightarrow v = \sqrt{\frac{2(m_1 - \mu m_2)gh}{m_1 + m_2}}$$

Example 20: A block of mass m is attached to a spring of spring constant k. The other end of the spring is attached to a fixed wall (see fig.). If a force F is applied to the block as shown, find the maximum elongation of the spring



Sol. Let us consider the system consisting of the block and the spring. In this case mechanical energy will not be conserved, since external force F does some work.

Thus $\Delta K + \Delta U =$ work done by F

The length of the string will continue to increase as long as block moves to the right. Consequently, the maximum elongation of spring will occur when block comes to state of momentarily rest.

Let x_m be the maximum elongation of the spring.

Then $\Delta U_{\text{spring}} = \frac{1}{2}kx_m^2 - 0$

$\Delta K = 0 - 0 = 0$

Work done by $F = Fx_m$

Hence, $\frac{1}{2}kx_m^2 = Fx_m$

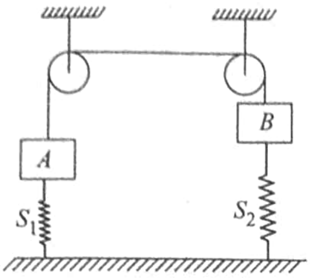
$$\Rightarrow x_m = \frac{2F}{k}$$

Notice that the block will start moving towards left after it comes to rest because force applied by the spring will be more than F at this instant.

Try your self: Let the coefficient of kinetic friction between block and surface be μ. Find the maximum elongation of spring in this case.

The answer is $x_m = 2\frac{(F - \mu mg)}{k}$

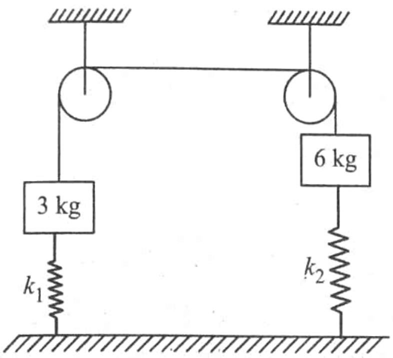
Example 21: In the figure shown below masses of blocks A and B are 3 kg and 6 kg respectively. The force constants of springs S_1 and S_2 are 160 N/m and 40 N/m respectively. Length of the light string connecting the blocks is 8 m. The system is released from rest with the springs at their natural lengths. The maximum elongation of spring S_1 will be:



(*a*) 0.294 m (*b*) 0.490 m
(*c*) 0.588 m (*d*) 0.882 m

Sol. (*a*) $\Delta K = 0 \Rightarrow \Delta U_{\text{gravity}} = -6gx + 3gx$

$$\Delta U_{\text{spring}} = \frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2$$



Applying, $\Delta K + \Delta U_{\text{gravity}} + \Delta U_{\text{spring}} = 0$

$$6g(x) - 3g(x) = \frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2$$

$$6g = (k_1 + k_2)x = \frac{6\times 9.8}{200} = \frac{3\times 9.8}{100}$$

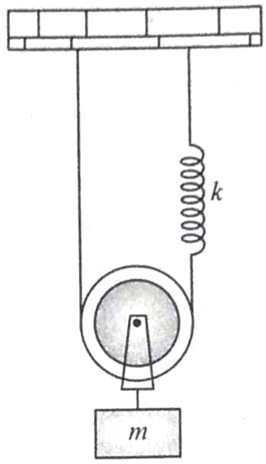
$\Rightarrow x = 0.294$ m

Concept Application

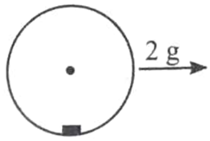
18. The potential energy of a particle of mass m free to move along x-axis is given by $U = 1/2kx^2$ for $x < 0$ and $U = 0$ for $x \geq 0$ (x denotes the x-coordinate of the particle and k is a positive constant). If the total mechanical energy of the particle is E, then its speed at $x = -\sqrt{\dfrac{2E}{k}}$ is

(*a*) Zero (*b*) $\sqrt{\dfrac{2E}{m}}$ (*c*) $\sqrt{\dfrac{E}{m}}$ (*d*) $\sqrt{\dfrac{E}{2m}}$

19. In the given figure, spring, string and pulley are massless. System released from rest when spring in its natural length. Find maximum elongation in the spring.



20. A block of mass m is placed inside a smooth hollow cylinder of radius R whose axis is kept horizontally. Initially system was at rest. Now cylinder is given constant acceleration 2 g in the horizontal direction by external agent. The maximum angular displacement of the block with the vertical is

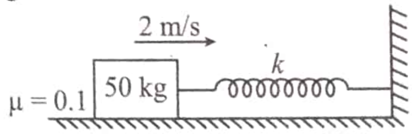


(*a*) $2\tan^{-1}2$ (*b*) $\tan^{-1}2$
(*c*) $\tan^{-1}1$ (*d*) $\tan^{-1}\left(\dfrac{1}{2}\right)$

21. A rod of length 1 m and mass 0.5 kg hinged at one end, is initially hanging vertical. The other end is now raised slowly until it makes an angle 60° with the vertical. The required work is: (use $g = 9.8$ m/s^2)

(*a*) 1.522 J (*b*) 1.225 J
(*c*) 2.125 J (*d*) 3.125 J

22. A block of mass 50 kg is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is 0.1. The block strikes a light spring of stiffness k = 100 N/m with a velocity 2 m/s. The maximum compression of the spring is



(*a*) 1 m (*b*) 2 m (*c*) 3 m (*d*) 4 m

23. A uniform string of mass 'M' and length $2a$, is placed symmetrically over a smooth and small pulley and has particles of masses 'm' and 'm'' attached to its ends; show that when the string runs off the peg its velocity is. $\left\{\dfrac{M + 2(m - m')}{M + m + m'}ag\right\}$ (Assume $m > m'$).

Conservative Force and Potential Energy

$$F_s = -\frac{\partial U}{\partial s}$$

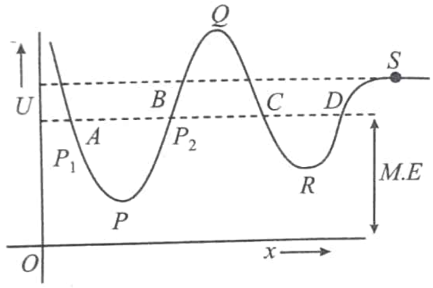
The projection of the force field , the vector F, at a given point in the direction of the displacement r equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial/\partial s$ emphasizes the fact of deriving with respect to a definite direction. So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projection F_x, F_y and F_z of the vector F on the unit vectors i, j and k. Hence, one can readily find the vector itself:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}, \text{ or } F = -\left(\frac{\partial U}{\partial x}i + \frac{\partial U}{\partial y}j + \frac{\partial U}{\partial z}k\right)$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇U. We shall use the second, more convenient, designation where ∇ ("nabla") signifies the symbolic vector or operator, $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

Potential Energy Curve

- A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve.
- Using graph, we can predict the velocity of a particle at various positions if we know the mechanical energy of the particle i.e., $\frac{1}{2}mu^2 + U = M \cdot E$
- Force on the particle is $F_{(x)} = -\frac{dU}{dx}$



Case: I On increasing x, if U increases, force is in (–) ve x direction i.e. attraction force.

Case: II On increasing x, if U decreases, force is in (+) ve x-direction i.e. repulsion force.

Different Positions of a particle

Position of Equilibrium: If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium $\frac{dU}{dx} = 0$. Points P, Q, R and S are the states of equilibrium positions.

Types of Equilibrium

- **Stable equilibrium:** When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

  Necessary conditions: $\frac{dU}{dx} = 0$, and $\frac{d^2U}{dx^2} = +ve$

  In figure P and R point shows stable equilibrium point.

- **Unstable Equilibrium:** When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

  Condition: $\frac{dU}{dx} = 0$ potential energy is maximum

  i.e. $\frac{d^2U}{dx^2} = -$ ve

  Q point in figure shows unstable equilibrium point

- **Neutral equilibrium:** In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

  In figure S is the neutral point

  Condition: $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$

Train Your Brain

Example 22: The potential energy between two atoms in a molecule is given by, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The system is in stable equilibrium when

(a) $x = 0$ (b) $x = \frac{a}{2b}$

(c) $x = \left(\frac{2a}{b}\right)^{1/6}$ (d) $x = \left(\frac{11a}{5b}\right)$

Sol. (c) Given that, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

We, know $F = -\frac{du}{dx} = (-12)\,a\,x^{-13} - (-6b)\,x^{-7} = 0$

or $\frac{6b}{x^7} = \frac{12a}{4x^{13}}$ or $x^6 = 12a/6b = 2a/b$ or $x = \left(\frac{2a}{b}\right)^{1/6}$

Example 23: The potential energy of a conservative system is given by $U = ax^2 - bx$ where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.

Sol. In a conservative field $F = -\frac{dU}{dx}$

$\therefore F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$

For equilibrium $F = 0$ or $b - 2ax = 0 \Rightarrow x = \frac{b}{2a}$

From the given equation we can see that $\frac{d^2U}{dx^2} = 2a$ (positive), i.e., U is minimum.

Therefore, $x = b/2a$ is the stable equilibrium position.

Concept Application

24. The potential energy U of a conservative system is given by $U = ax^2 - bxy^2$. Find the x and y components of the conservative force F_x and F_y.

25. The potential energy of a conservative system is given by $U = ax^2 - bx^4$ where a and b are constant. find the position where system is in stable and unstable equilibrium.

POWER

Rate of work done by a force is called power delivered by the force.

Average power $= \dfrac{\text{work done}}{\text{time}}$ or $P = \dfrac{W}{t}$

W is the amount of the work done in total time t.

Instantaneous Power $(P) = \dfrac{dW}{dt}$ or $P = \dfrac{\vec{F}.d\vec{s}}{dt}$ $(\because dW = \vec{F}.d\vec{s})$

or $P = \vec{F}.\dfrac{d\vec{s}}{dt}$ or $P = \vec{F}.\vec{v}$

Note:

(i) Dimensions: $[M^1L^2T^{-3}]$ $\left(\text{from } P = \dfrac{W}{t}\right)$

(ii) SI unit: watt $= \dfrac{\text{joule}}{\text{second}}$

(iii) Bigger Unit of power:
1 kilo watt (kW) = 1000 watt = 10^3 W
1 maga watt = 1000, 000 watt or 1MW = 10^3kW
1 horse power (h.p.) = 746 W = 550 ft. lb/s

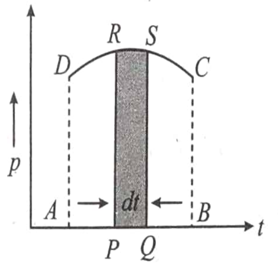
FPS Unit : $\dfrac{(\text{foot} \times \text{pound})}{\text{second}}$, 1W = 0.738 ft. lb/s

(iv) When an agent delivers power at a uniform rate, the average power is equal to the instantaneous power.

(v) The slope of work – time graph gives the instantaneous power

slope $= \tan\theta = \dfrac{dW}{dt} = P$ (instantaneous power)

(vi) Area under power – time graph gives the work done.
Area under P-t graph = Area $ABCD$ = work done from A to B.



Area under power – time graph $= \int P dt = W$

Train Your Brain

Example 24: Force acting on a particle moving in a straight line varies with the velocity v of the particle as $F = \dfrac{K}{v}$, where K is a constant. The work done by this force in time t is

(a) $\dfrac{K}{v^2}t$ (b) $2Kt$

(c) Kt (d) $\dfrac{2Kt}{v^2}$

Sol. **(c)** It can be observed that power delivered to particle by force F is

$P = Fv = K.$

The power is constant.

Hence work done by force in time t is

$\Delta W = Pt = Kt$

Example 25: What is the power of an engine which can lift 20 metric ton of coal per hour from a 20 metre deep Imine?

Sol. Mass, m = 20 metric ton = 20 × 1000 kg

Distance, S = 20 m

Time, t = 1

Hour = 3600 s

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$= \frac{\text{mg} \times \text{S}}{\text{t}}$$

$$= \frac{20 \times 1000 \times 9.8 \times 20}{3600} \text{ watt}$$

$$= 1.09 \times 10^3 \text{ W}$$

Example 26: A large family uses 8 kW of power. Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy. how large an area is needed to supply 8 kW?

Sol. If A must be the area.

Useful electrical energy produced/s

$$= \frac{20}{100}(200\text{ A})$$

= 100 40. A watts

But 40 A = 8000

or A = 800 m^2

Example 27: An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 ms^{-1}. A frictional force of 4000 N oppose its motion. Determine the minimum power delivered by the motor to the elevator. (Take g = 10 ms^{-2}).

Sol. Weight of (elevator + passenger) = mg = 1800 × 10 N = 18000 N, frictional force = 4000 N

Total downward force on the elevator (18000+ 4000) N = 22000 N

Clearly, the motor must have enough power to balance this force.

Now, power, $P = Fv$ = 2200 N × 2 ms^{-1}

$$= \frac{4000}{746} \text{ hp} = 58.98 \text{ hp}$$

Concept Application

26. An electric motor creates a tension of 4500 N in hoisting cable and reels it at the rate of 2 m/s. What is the power of electric motor?

(*a*) 9 W
(*b**) 9 KW
(*c*) 225 W
(*d*) 9000 H.P

27. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1/P_2) will be:

(*a*) 1
(*b*) 4/3
(*c*) 16/9
(*d*) None of the above

Short Notes

Work Done by constant Force

$$W = \vec{F} \cdot \vec{S}$$

Work Done by Multiple Forces

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

$$W = [\Sigma\vec{F}] \cdot \vec{S}$$

$$W = \vec{F}_1 \cdot \vec{S} + \vec{F}_2 \cdot \vec{S} + \vec{F}_3 \cdot \vec{S} + \cdots \text{ or } W = W_1 + W_2 + W_3 + \ldots$$

Work Done by Variable Force

$$W = \int dW = \int \vec{F} \cdot d\vec{s}$$

Area under the force and displacement curve gives work done.

Important cases:

(i) The force is perpendicular to velocity at all times. The work done in this case is zero. e.g., :

(*a*) A particle attached to a string whose other end is attached to a fixed point. The work done by tension in the string is zero.

(*b*) A body moving on a fixed surface. The work done by normal reaction is zero in this case.

(ii) Work done by gravity on a body near the surface of earth

$= -mg(h_f - h_i)$

$= -mg \times$ increase in height

(iii) Work done by friction on a block moving on a fixed surface

$= -\mu_k \int N(s)ds$

$= -\mu_k N \times$ distance travelled (if N is constant).

Here N is the normal reaction between the block and the surface on which it is moving.

(iv) Work done by a spring on a block

$$= -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

where x_i and x_f refer to the initial and final elongation or compression of the spring.

Relation between Momentum and Kinetic Energy

$$K = \frac{p^2}{2m} \text{ and } P = \sqrt{2mK}$$

P = linear momentum

Potential Energy

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

i.e., $U_f - U_i = -\int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} = -W_{\text{Conservative}}$

Conservative Forces

$$\vec{F} = -\frac{dU}{dr}\hat{r}$$

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

Equilibrium Conditions

Stable Equilibrium

$$F(r) = -\frac{dU}{dr} = 0; \text{and } \frac{dF}{dr} < 0; \text{ or } \frac{d^2U}{dr^2} > 0$$

Unstable Equilibrium

$$F(r) = -\frac{dU}{dr} = 0; \text{ therefore } \frac{dF}{dr} > 0; \text{ and } \frac{d^2U}{dr^2} < 0$$

Neutral Equilibrium

$$F(r) = -\frac{dU}{dr} = 0; \text{ therefore } \frac{dF}{dr} = 0; \text{ and } \frac{d^2U}{dr^2} = 0$$

Work-Energy Theorem for a Particle

$$W_{Net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Work-Energy Theorem for a System

$$\Delta K + \Delta U = W_{ext} + W_{int\text{-}NC}$$

Law of Conservation of Mechanical Energy

If the net external force acting on a system is zero, then the mechanical energy is conserved.

$$K_f + U_f = K_i + U_i$$

Power

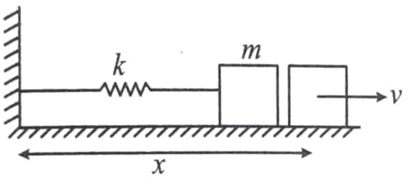
The average power delivered by an agent is given by

$$P_{avg} = \frac{W}{t}$$

$$P = \frac{d\vec{F}\cdot\vec{S}}{dt} = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\cdot\vec{v}$$

Solved Examples

1. A block of mass 'm' is pushed against a spring of spring constant 'k' fixed at one end to a wall. The block can slide on a frictionless table as shown in the figure. The natural length of the spring is L_0 and it is compressed to one-fourth of natural length and the block is released. Find its velocity as a function of its distance (x) from the wall and maximum velocity of the block. The block is not attached to the spring.



Sol. $\frac{1}{2}k\left(\frac{3L_0}{4}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}k(L_0 - x)^2$

when $x < L_0$

$$\Rightarrow v = \sqrt{\frac{k}{m}\left[\left(\frac{3L_0}{4}\right)^2 - (L_0 - x)^2\right]}$$

when $x \geq L_0$

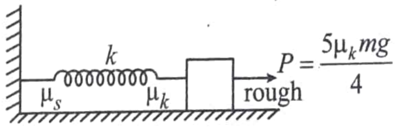
$$\frac{1}{2}K\left(\frac{3L_0}{4}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v = \frac{3L_0}{4}\sqrt{\frac{k}{m}}$$

which is also the maximum speed of the block.

Thus, $v_{max} = \frac{3L_0}{4}\sqrt{\frac{k}{m}}$

2. A block of mass m rests on a rough horizontal plane having coefficient of kinetic friction μ_k and coefficient of static friction μ_s. The spring is in its natural length, when a constant force of magnitude $P = \frac{5\mu_k mg}{4}$ starts acting on the block. The spring force F is a function of extension x as $F = kx^3$. (Where k is spring constant)

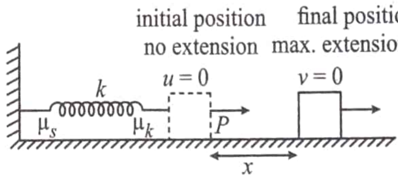


(a) Comment on the relation between μ_s and μ_k for the motion to start.

(b) Find the maximum extension in the spring (Assume the force P is sufficient to make the block move).

Sol. (a) For motion to start $\frac{5\mu_k mg}{4} > \mu_s mg$ or $5\mu_k > 4\mu_s$

(b)



At the final position of the block extension in spring is maximum and the speed of the block is $v = 0$. Hence the net work done in taking the block from initial to final position ΔW = (Work done by force P + work done by spring force F + work done by friction) = $\Delta K = 0$

$$= Px - \int_0^x Kx^3 \, dx - \mu_k mgx = \frac{5\mu_k mg}{4}x - \frac{kx^4}{4} - \mu_k mgx = 0$$

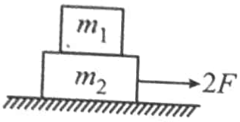
Solving we get, $x = \left(\frac{\mu_K mg}{K}\right)^{1/3}$

3. A block of mass m_1 is kept over another block of mass m_2 and the system rests on a horizontal surface (as shown in figure). A constant horizontal force $2F$ acting on the lower block produces an acceleration $\dfrac{F}{(m_1+m_2)}$ in the system, the two blocks always move together.

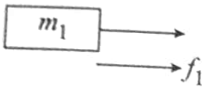
(*a*) Find the coefficient of kinetic friction between the bigger block and the horizontal surface.

(*b*) Find the frictional force acting on the smaller block.

(*c*) Find the work done by the force of friction on the smaller block by the bigger block during a displacement x of the system.

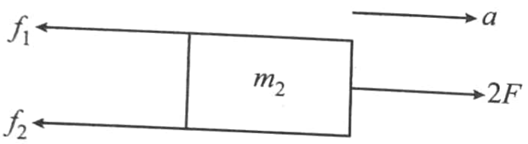


Sol.



$$a = \frac{F}{(m_1+m_2)},$$

$$f_1 = m_1 a = m_1 \frac{F}{(m_1+m_2)}$$



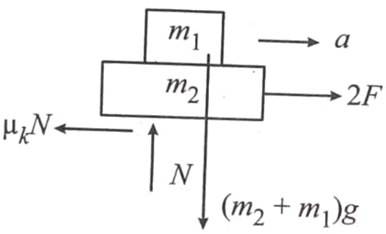
$$2F - f_1 - f_2 = m_2 a$$

$$-f_2 = -2F + f_1 + m_2 a = m_1 a + m_2 a - 2F$$

$$-f_2 = (m_2+m_1)\frac{F}{(m_1+m_2)} - 2F$$

$$= F - 2F = -F$$

$$\Rightarrow f_2 = -F$$



$$2F - \mu_K(m_2+m_1)g = (m_2+m_1)a$$

$$2F - (m_2+m_1)\frac{F}{(m_1+m_2)} = \mu_K(m_2+m_1)g$$

$$\Rightarrow \frac{F}{(m_1+m_2)g} = \mu_K$$

W = work done by friction force on smaller block

$$= f_1 x = \frac{m_1 F}{(m_2+m_1)} x$$

4. A particle of mass m approaches a region of force starting from $r = +\infty$. The potential energy function in terms of distance r from the origin is given by,

$$U(r) = \frac{K}{2a^3}(3a^2 - r^2) \text{ for, } 0 \le r \le a$$

$$= K/r \text{ for, } r \ge a$$

where $K > 0$ (Positive constant)

(*a*) Derive the force $F(r)$ and determine whether it is repulsive or attractive.

(*b*) With what velocity should the particle start at $r = \infty$ to cross over to other side of the origin.

(*c*) If the velocity of the particle at $r = \infty$ is $\sqrt{\dfrac{2K}{am}}$ towards the origin describe the motion.

Sol. (*a*) $\dfrac{du}{dr} = \dfrac{K}{2a^3}(-2r) \Rightarrow F(r) = \dfrac{K}{a^3} r$

$$\vec{F}(r) = \left(\frac{K}{a^3} r\right)\hat{r} \text{ for } 0 \le r \le a$$

$$\vec{F}(r) = \frac{K}{r^2}\hat{r} \text{ for } r \ge a$$

$\because K > 0$

$\therefore$ force is +ve

$\therefore$ force is repulsive

(*b*) $\dfrac{1}{2}mu^2 + 0 = 0 + \dfrac{K3a^2}{2a^3}$

$$u^2 = \frac{3K}{ma} \Rightarrow u = \sqrt{\frac{3K}{am}}$$

(*c*) $\dfrac{1}{2}m\dfrac{2K}{am} + 0 = 0 + \text{P.E.}$

$$\Rightarrow \text{P.E} = \frac{K}{a} \Rightarrow r = a$$ (Circular motion)

5. A single conservative force $F(x)$ acts on a 1.0 kg particle that moves along the x-axis. The potential energy $U(x)$ is given by: $U(x) = 20 + (x-2)^2$

where x is in meters. At $x = 5.0$ m the particle has a kinetic energy of 20 J.

(i) What is the mechanical energy of the system?

(ii) Make a plot of $U(x)$ as a function of x for $-10\text{m} < x < 10\text{m}$, and on the same graph draw the line that represents the mechanical energy of the system.

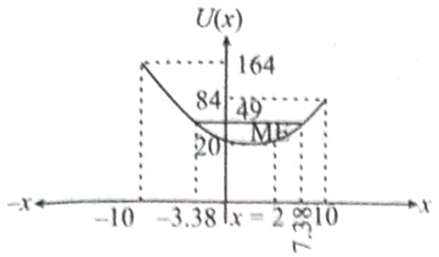
Use part (ii) to determine

(iii) The least value of x and the greatest value of x between which the particle can move.

(iv) The maximum kinetic energy of the particle and the value of x at which it occurs.

(v) Determine the equation for $F(x)$ as a function of x.

(vi) For what value of x does $F(x) = 0$?



Sol. (i) Potential energy at $x = 5.0$ m is

$U = 20 + (5 - 2)^2 = 29$ J

$\therefore$ Mechanical energy, $E = K + U = 20 + 29 = 49$ J

(ii) At $x = 10$ m, $U = 84$ J at $x = -10$ m, $U = 164$ J and at $x = 2$ m, U = minimum = 20 J

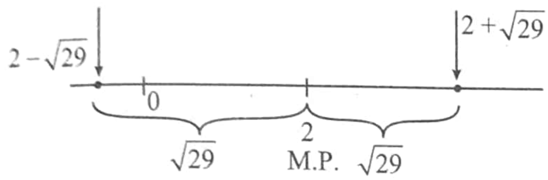
(iii) Particle will move between the points where kinetic energy becomes zero or its potentials energy is equal to its mechanical energy

Thus, $49 = 20 + (x - 2)^2$

or $(x - 2)^2 = 29$ or $x - 2 = \pm\sqrt{29} = \pm 5.38\,\text{m}$

$\therefore x = 7.38$ m and -3.38 m

So the particle will move between $x = -3.38$ m and $x = 7.38$ m.



(iv) Maximum kinetic energy is at $x = 2$ m, where the potential energy is minimum

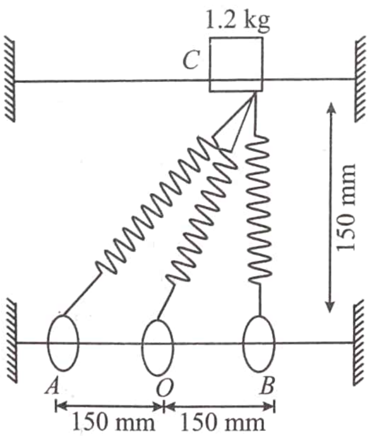
So, $K_{max} = E - U_{min} = 49\text{ J} - 20 = 29$ J

(v) $F = -\dfrac{dU}{dx} = -2(x-2) = 2(2-x)$

(vi) $F(x) = 0$ at $x = 20$ m

where potential energy is minimum (the position of stable equilibrium)

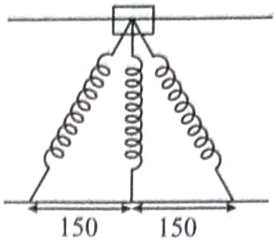
6. A 1.2 kg collar C may slide without friction along a fixed smooth horizontal rod. It is attached to three springs each of constant $k = 400$ N/m and 150 mm undeformed length. Knowing that the collar is released from rest in the position shown. Determine the maximum velocity it will reach in its motion. [Here A, O, B are fixed points.]



Sol. Velocity will be maximum when $a = 0$

For $a = 0$, $F = 0$,

This situation occurs for v_e following arrangement of springs.



Natural length is $c = 150$ mm

Now , $U_i + K_i = U_f + K_f$

$$U_i = \frac{1}{2}k\{\sqrt{5}\,c - c\}^2 + \frac{1}{2}k\{\sqrt{2}\,c - c\}^2 \text{ and } K_i = 0$$

$$U_f = 2.\,\frac{1}{2}k\{\sqrt{2}\,c - c\}^2 \text{ and } K_f = \frac{1}{2}mv^2$$

$$\therefore\ \frac{1}{2}k\{\sqrt{5}\,c - c\}^2 + \frac{1}{2}k\{\sqrt{2}\,c - c\}^2$$

$$= \frac{1}{2}mv^2 + 2.\,\frac{1}{2}k\{\sqrt{2}\,c - c\}^2$$

Solving the equation and putting the values

we have

$$v = \left[\frac{15}{2}\left\{(\sqrt{5}-1)^2 + (\sqrt{2}-1)^2\right\}\right]^{1/2} \text{ m/s} = 3.189\ \text{ms}^{-1}.$$

7. Wind entering in a wind mill with a velocity of 20 m/sec facing area of the windmill is 10 m^2 and density of air is 1.2 kg/m^3. If wind energy is converted into electrical energy with 33.3% efficiency, then find electrical power produced by the wind mill in kW.

Sol. Energy entering in the windmill $= \dfrac{1}{2}mv^2$

$$P_{in} = \frac{dE}{dt} = \left(\frac{1}{2}v^2\right)\left(\frac{dm}{dt}\right)$$

$$P_{in} = \left(\frac{1}{2}v^2\right)(\rho A V) = \frac{1}{2}\rho A V^3$$

Electrical power output

$$P_{out} = \frac{1}{3}\left(\frac{1}{2}\rho A V^3\right)$$

$$P_{out} = \frac{1}{6}\rho A V^3 = \frac{1}{6}\times 1.2\times 10\times(20)^3$$

$P_{out} = 16$ kW.

8. An engine can pull 4 coaches at a maximum speed of 20 m/s. Mass of the engine is twice the mass of every coach. Assuming resistive forces to be proportional to the weight, approximate maximum speeds of the engine when it pulls 12 and 6 coaches are (power of engine remains constant).

Sol. When 4 coaches (m each) are attached with engine ($2m$) according to question $P = K\,6mgv$ ...(i)
(constant power), (K being proportionality constant)
Since resistive force is proportional to weight
Now if 12 coaches are attached
$P = K.14mg.v_1$ ...(ii)
Since engine power is constant
So by equation (i) and (ii)

$$6Kmgv = 14Kmgv_1 \Rightarrow v_1 = \frac{6}{14} \times v$$

$$= \frac{6}{14} \times 20 = \frac{6\times 10}{7} = \frac{60}{7}$$

$= v_1 = 8.5$ m/sec

Similarly for 6 coaches $\Rightarrow K6mgv = K8mgv_2$

$$\Rightarrow v_2 = \frac{6}{8} \times 20 = \frac{3}{4} \times 20 = 15 \text{ m/sec}$$

9. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain "n" times water from the same pipe in the same time, the factor by which the power of the motor should be increased is

Sol. Power $P = \vec{F}\cdot\vec{V} = FV$

$$F = V\left(\frac{dm}{dt}\right)$$

$$= V\left\{\frac{d(\rho\times volume)}{dt}\right\} \quad \rho = \text{density}$$

$$= \rho V\left\{\frac{d(volume)}{dt}\right\}$$

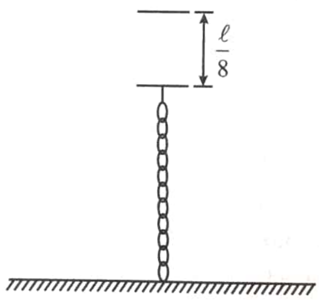
$$= \rho V(AV)$$

$$= \rho AV^2$$

$\therefore$ Power $P = \rho AV^3$ or $P \propto V^3$

10. A chain of mass M and length ℓ is held vertically such that its bottom end just touches the surface of a horizontal table. The chain is released from rest. Assume that the portion of chain on the table does not form a heap. The momentum of the portion of the chain above the table after the top end of the chain falls down by a distance $\frac{\ell}{8}$.

Sol. After the top end of chain falls down by $\frac{\ell}{8}$, the speed of chain is



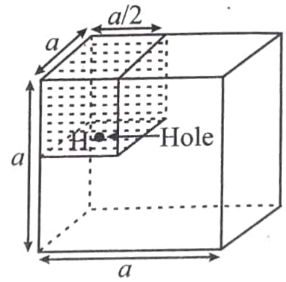
$$v = \sqrt{2g\frac{\ell}{8}} = \frac{\sqrt{g\ell}}{2}$$

The mass of chain above table is $\frac{7}{8}$ M.

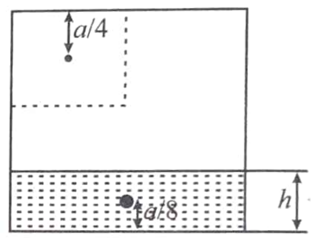
$$\therefore \text{Momentum of chain is} = \frac{7}{8}M\frac{\sqrt{g\ell}}{2}$$

$$= \frac{7}{16}M\sqrt{g\ell}$$

11. The figure shows a hollow cube of side 'a' of volume V. There is a small chamber of volume $\frac{V}{4}$ in the cube as shown. This chamber is completely filled by m kg of water. Water leaks through a hole H and spreads in the whole cube. Then the work done by gravity in this process assuming that the complete water finally lies at the bottom of the cube is:



Sol. Let h be the height of water surface, finally



Volume of water remains the same hence,

$$a^2h = a.\frac{a}{2}.\frac{a}{2};\ h = \frac{a}{4}$$

$$\therefore \text{C.M. gets lowered by } a - \left(\frac{a}{4}+\frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$$

$$\therefore \text{Work done by gravity} = mg\frac{5a}{8}$$

12. The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i} + \frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x-y plane is:

(*a*) $\frac{2K\pi}{a}$ (*b*) $\frac{K\pi}{a}$

(*c*) $\frac{K\pi}{2a}$ (*d*) 0

Sol. (*d*) Suppose $x = r\cos\theta$, $y = r\sin\theta$

The expression of force on particle now becomes

$$\vec{F} = \frac{K}{r^3}\left(r\cos\theta\hat{i} + r\sin\theta\hat{j}\right)$$

This force is in radial direction so work done by this force along given path (circle) is zero.

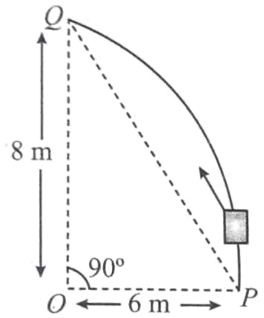
13. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is :

Sol. $E = P.t = 0.5W \times 5$ s

$= 2.5\text{ J} = \frac{1}{2}mv^2$

$\Rightarrow v = 5$ m/s

14. Consider an elliptically shaped rail PQ in the vertical plane with $OP = 6$ m and $OQ = 8$ m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, Which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



Sol. $W_F + W_g = K_f - K_i$

$18 \times 10 + 1g(-8) = K_f$

$180 - 40 = K_f$

$K_f = 100\text{J} = 10 \times 10\text{ J} \Rightarrow n = 10$

15. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$ where λ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(*a*) The force applied on the particle is constant

(*b*) The speed of the particle is proportional to time

(*c*) The distance of the particle from the origin increases linearly with time

(*d*) The force is conservative

Sol. (*a,b,d*)

$$mv\frac{dv}{dt} = \frac{dk}{dt} = \gamma t$$

$$v\,dv = \frac{\gamma}{m}t\,dt$$

$$\frac{v^2}{2} = \frac{\gamma}{m}\frac{t^2}{2} \Rightarrow v \propto t$$

$$\frac{dv}{dt} = \text{constant} \Rightarrow F = \text{constant}$$

$$\frac{dx}{dt} \propto t \Rightarrow x \propto t^2$$

16. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ where K is a positive constant, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a,0)$ and then parallel to the y-axis to the point (a, a). The total work done by the force $\vec{F}$ on the particle is

(*a*) $-2Ka^2$ (*b*) $2Ka^2$ (*c*) $-Ka^2$ (*d*) Ka^2

Sol. $dW = \vec{F}.\vec{ds}$ where $\vec{ds} = dx\hat{i} + dy\hat{j}$ and $\vec{F} = -K(y\hat{i} + x\hat{j})$

$\therefore dW = -K(ydx + xdy) = -Kd(xy)$

$$\therefore W = \int_{(0,0)}^{(a,a)} dW = -K\int_{(0,0)}^{(a,a)} d(xy) = -\{K(xy)\}_{(0,\,0)}^{(a,\,a)}$$

$W = -Ka^2$

Exercise-1 (Topicwise)

DEFINITION OF WORK

1. A man pushes wall and fails to displace it. He does
   (a) Negative work
   (b) Positive but not maximum work
   (c) No work at all
   (d) Maximum work

2. If the unit of force and length each be increased by four times, then the unit of energy is increased by
   (a) 16 times (b) 8 times
   (c) 2 times (d) 4 times

WORK DONE BY CONSTANT FORCE, VARIABLE FORCE

3. You lift a heavy book from the floor of the room and keep it in the book-shelf having a height 2 m. In this process you take 5 seconds. The work done by you will depend upon
   (a) Mass of the book and time taken
   (b) Weight of the book and height of the book-shelf
   (c) Height of the book-shelf and time taken
   (d) Mass of the book, height of the book-shelf and time taken

4. A body of mass m kg is lifted by a man to a height of one metre in 30 sec. Another man lifts the same mass to the same height in 60 sec. The work done by them are in the ratio
   (a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 4 : 1

5. The work done against gravity in taking 10 kg mass at 1m height in 1sec will be
   (a) 49 J (b) 98 J
   (c) 196 J (d) None of these

6. A particle moves from position $\vec{r}_1 = 3\hat{i} + 2\hat{j} - 6\hat{k}$ to position $\vec{r}_2 = 14\hat{i} + 13\hat{j} + 9\hat{k}$ under the action of force $4\hat{i} + \hat{j} + 3\hat{k}$ N. The work done will be
   (a) 100 J (b) 50 J (c) 200 J (d) 75 J

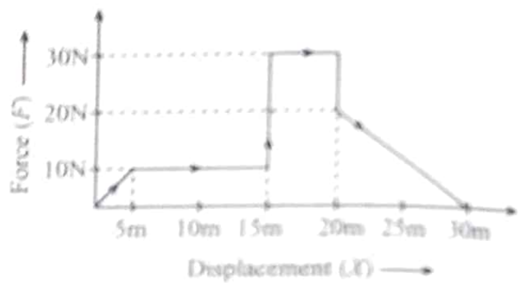
7. A force $(\vec{F}) = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes a displacement $(\vec{s}) = -4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If the work done is 6 J, then the value of 'c' is
   (a) 0 (b) 1 (c) 6 (d) 12

8. A body of mass 6kg is under a force which causes displacement in it given by $S = \frac{t^2}{4}$ metres where t is time. The work done by the force in 2 seconds is
   (a) 12 J (b) 9 J (c) 6 J (d) 3 J

9. A cord is used to lower vertically a block of mass M by a distance d with constant downward acceleration $\frac{g}{4}$. Work done by the cord on the block is
   (a) $Mg\frac{d}{4}$ (b) $3Mg\frac{d}{4}$
   (c) $-3Mg\frac{d}{4}$ (d) Mgd

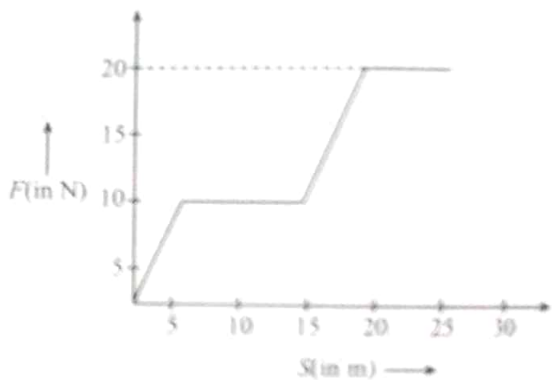
10. The force constant of a wire is k and that of another wire is 2k. When both the wires are stretched through same distance, then the work done
    (a) $W_2 = 2W_1^2$ (b) $W_2 = 2W_1$
    (c) $W_2 = W_1$ (d) $W_2 = 0.5\ W_1$

11. Given below is a graph between a variable force (F) (along y-axis) and the displacement (X) (along x-axis) of a particle is one dimension. The work done by the force in the displacement interval between 0m and 30 m is



    (a) 275 J (b) 375 J (c) 400 J (d) 300 J

12. The work done by a force acting on a body is as shown in the graph. The total work done in covering an initial distance of 20 m is



    (a) 225 J (b) 200 J (c) 400 J (d) 175 J

13. A rigid body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done by this force on the body is 25 joules, the angle which the force makes with the direction of motion of the body is
    (a) 0° (b) 30° (c) 60° (d) 90°

14. A particle moves from position $\vec{r}_1 = 6\hat{i} + 4\hat{j} - 12\hat{k}$ to position $\vec{r}_2 = 12\hat{i} + 8\hat{j} - 6\hat{k}$ under the action of force $4\hat{i} + \hat{j} + 3\hat{k}\ N$. The work done by this force will be

(a) 46 J (b) 50 J
(c) 200 J (d) 75 J

15. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is

(a) 1 : 2 : 3 (b) 1 : 4 : 9
(c) 1 : 3 : 5 (d) 1 : 5 : 3

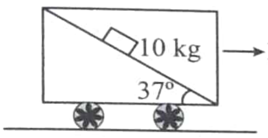
16. A force acting on a particle varies with the displacement x as $F = ax - bx^2$. Where $a = 1$ N/m and $b = 1$ N/m². The work done by his force for the first one meter (F is in newtons, x is in meters) is

(a) $\frac{1}{6}$ J (b) $\frac{2}{6}$ J
(c) $\frac{3}{6}$ J (d) None of these

17. A force $F = (3x\hat{i} + 4\hat{j})$ Newton (where x is in metres) acts on a particle which moves from a position (2 m, 3 m) to (3 m, 0 m). Then the work done is

(a) 7.5 J (b) –12 J
(c) –4.5 J (d) +4.5 J

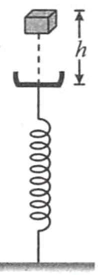
18. A block of mass 10 kg is released on a fixed wedge inside a cart which is moving with constant velocity 10 ms^{-1} towards right. There is no relative motion between block and cart. Then work done by normal reaction on block in two seconds from ground frame will be ($g = 10$ ms^{-2})



(a) 1320 J (b) 960 J
(c) 1200 J (d) 240 J

WORK ENERGY THEOREM

19. A block of mass m initially at rest is dropped from a height h on to a spring of force constant k. The maximum compression in the spring is x then



(a) $mgh = \frac{1}{2}kx^2$

(b) $mg(h+x) = \frac{1}{2}kx^2$

(c) $mgh = \frac{1}{2}k(x+h)^2$

(d) $mg(h+x) = \frac{1}{2}k(x+h)^2$

20. A block of mass 3 kg starts from rest and slides down a curved path in the shape of a quarter-circle of radius 2 m and reaches the bottom of path with a 4 m/s speed. If g is 10 m/s², the amount of work done against friction is

(a) 60 J (b) 36 J
(c) 24 J (d) 12 J

21. A truck accelerates from speed v to 2v. Work done in doing this is

(a) Three times as the work done in accelerating it from rest to v.
(b) Same as the work done in accelerating it from rest to v.
(c) Four times as the work done in accelerating it from rest to v.
(d) Less than the work done in accelerating it from rest to v.

22. If K_i and K_f are the initial and final values of kinetic energy of a body respectively, then the work done by the net force on the body is equal to

(a) $\frac{K_f K_i}{K_f - K_i}$ (b) $K_f + K_i$
(c) $\frac{K_f + K_i}{2}$ (d) $K_f - K_i$

23. A body possessing kinetic energy T moving on a rough horizontal surface is stopped in a distance y. The frictional force exerted on the body is

(a) Ty (b) $\frac{\sqrt{T}}{y}$
(c) $\frac{T}{y}$ (d) $\frac{T}{\sqrt{y}}$

24. A force of 20 N is applied on a body of mass 5 kg resting on a horizontal plane. The body gains a kinetic energy of 10 J after it moves a distance 2m. The frictional force is

(a) 10 N (b) 15 N
(c) 20 N (d) 30 N

25. The work done in pushing a block of mass 10 kg from bottom to the top of a frictionless inclined plane 5 m long and 3 m high is- ($g = 9.8$ m/sec²)

(a) 392 J (b) 294 J (c) 98 J (d) 0.98 J

26. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass during the above pulling is

(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$ (c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$

27. A particle of mass m at rest is acted upon by a force F for a time t. Its kinetic energy after an interval t is

(a) $\frac{F^2t^2}{m}$ (b) $\frac{F^2t^2}{2m}$
(c) $\frac{F^2t^2}{3m}$ (d) $\frac{Ft}{2m}$

28. A body starts from rest with uniform acceleration and acquires a velocity V in time T. The instantaneous kinetic energy of the body after any time t is proportional to:

(a) $(V/T)t$ (b) $(V^2/T)t^2$
(c) $(V^2/T^2)t$ (d) $(V^2/T^2)t^2$

29. A body of mass m accelerates uniformly from rest to a speed v_0 in time t_0. The work done on the body till any time t is

(a) $\frac{1}{2}mv_0^2\left(\frac{t^2}{t_0^2}\right)$ (b) $\frac{1}{2}mv_0^2\left(\frac{t_0}{t}\right)$

(c) $mv_0^2\left(\frac{t}{t_0}\right)$ (d) $mv_0^2\left(\frac{t}{t_0}\right)^3$

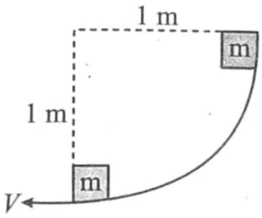
30. A body moving at 2 m/s can be stopped over a distance x. If its kinetic energy is doubled, how long will it go before coming to rest, if the retarding force remains unchanged?

(a) x (b) $2x$
(c) $4x$ (d) $8x$

31. The work done in joules in increasing the extension of a spring of stiffness 10 N/cm from 4 cm to 6 cm is :

(a) 1 (b) 10 (c) 50 (d) 100

32. A block of mass 1 kg slides down a curved track which forms one quadrant of a circle of radius 1 m as shown in figure. The speed of block at the bottom of the track is $v = 2\text{ ms}^{-1}$. The work done by the force of friction is



(a) +4 J (b) –4 J
(c) –8 J (d) +8 J

POWER

33. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg to a height of 40 m? (Take $g = 10\text{ ms}^{-2}$) :

(a) 4s (b) 5s (c) 8s (d) 10s

34. A body of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of time t, the instantaneous power delivered to the body is

(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1}$

(c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1^2}$

35. A man is riding on a cycle with velocity 7.2 km/hr up a hill having a slope 1 in 20. The total mass of the man and cycle is 100 kg. The power of the man is

(a) 200 W (b) 175 W (c) 125 W (d) 98 W

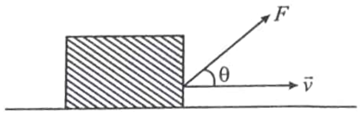
36. An electric motor creates a tension of 4500 newton in a hoisting cable and reels it in at the rate of 2 m/sec. What is the power of electric motor

(a) 15 kW (b) 9 kW
(c) 225 W (d) 9000 HP

37. A weight lifter lifts 300 kg from the ground to a height of 2 meter in 3 second. The average power generated by him is

(a) 5880 watt (b) 4410 watt
(c) 2205 watt (d) 1960 watt

38. A constant force $\vec{F}$ is acting on a body of mass m with constant velocity $\vec{v}$ as shown in the figure. The power P exerted is



(a) $F\cos\theta v$ (b) $F\cos\theta/mg$
(c) $F\,mg\cos\theta/v$ (d) $mg\sin\theta/F$

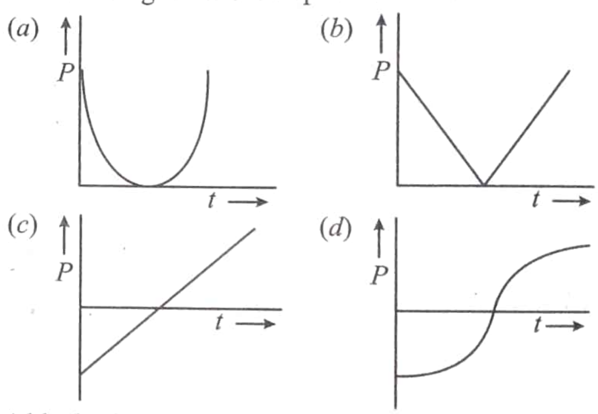
39. The power of a pump, which can pump 200 kg of water to a height of 200 m in 10 sec is $(g = 10\text{ m/s}^2)$

(a) 40 kW (b) 80 kW
(c) 400 kW (d) 960 kW

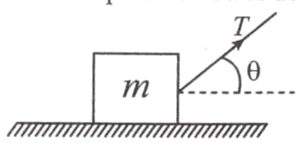
40. A force applied by an engine of a train of mass 2.05×10^6 kg changes ts velocity from 5 m/s to 25 m/s in 5 minutes. The power of the engine is

(a) 1.025 MW (b) 2.05 MW
(c) 5 MW (d) 6 MW

41. A particle is projected at $t = 0$ from a point on the ground with certain velocity at an angle with the horizontal. The power of gravitation force is plotted against time. Which of the following is the best representation?



42. A block of mass m slides along the track with kinetic friction μ. A man pulls the block through a rope which makes an angle θ with the horizontal as shown in the figure. The block moves with constant speed v. Power delivered by man is



(a) Tv (b) $Tv\cos\theta$
(c) $(T\cos\theta - \mu mg)v$ (d) Zero

IDEA OF POTENTIAL ENERGY EQUILIBRIUM

43. In which case does the potential energy decrease
(*a*) On compressing a spring
(*b*) On stretching a spring
(*c*) On moving a body against gravitational force
(*d*) On the rising of an air bubble in water

44. A 2g ball of glass is released from the edge of a hemispherical cup whose radius is 20 cm. How much work is done on the ball by the gravitational force during the ball's motion to the bottom of the cup?

(*a*) 1.96 mJ
(*b*) 3.92 mJ
(*c*) 4.90 mJ
(*d*) 5.88 mJ

45. A uniform chain of length L and mass m is kept on a smooth table. It is released from rest when the overhanging pat was n^{th} fraction of total length. Find the kinetic energy of the chain as it completely slips off the table :

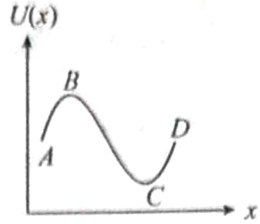
(*a*) $\frac{1}{2}mgL(1-n^2)$
(*b*) $2mgL(1-n^2)$
(*c*) $\frac{1}{2}mgL(n^2-1)$
(*d*) $2mgL(n^2-1)$

46. In the stable equilibrium position, a body has
(*a*) Maximum potential energy
(*b*) Minimum potential energy
(*c*) Minimum kinetic energy
(*d*) Zero kinetic energy

47. The potential energy of a conservative system is given by $V(x) = (x^2 - 3x)$ joule, where x is measured in metre. Then its equilibrium position is at
(*a*) 1.5 m (*b*) 2 m
(*c*) 3 m (*d*) 1 m

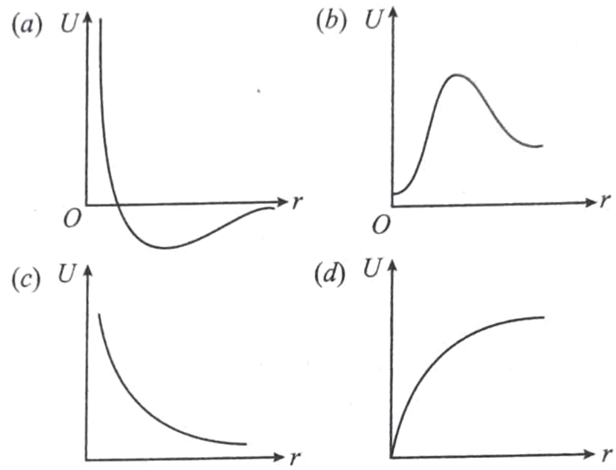
48. When a conservative force does positive work on a body
(*a*) The potential energy increases
(*b*) The potential energy decreases
(*c*) Total energy increases
(*d*) Total energy decreases

49. The potential energy of a particle varies with distance x as shown in the graph.

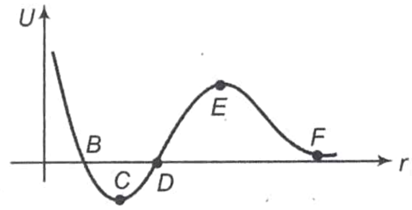


The force acting on the particle is zero at
(*a*) C
(*b*) B
(*c*) B and C
(*d*) A and D

50. The diagrams represent the potential energy U of a function of the inter-atomic distance r. Which diagram corresponds to stable molecules found in nature.



51. The given plot shows the variation of U, the potential energy of interaction between two particles with the distance separating them r.

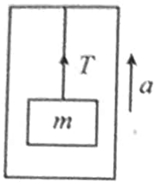


1. B and D are equilibrium points
2. C is a point of stable equilibrium
3. The force of interaction between the two particles is attractive between points C and D and repulsive between D and E
4. The force of interaction between particles is repulsive between points E and F.

Which of the above statements are correct?
(*a*) 1 and 2
(*b*) 1 and 4
(*c*) 2 and 4
(*d*) 2 and 3

Exercise-2 (Learning Plus)

1. A block of mass m is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration a. The work done by tension on the block during t seconds is ($u = 0$)

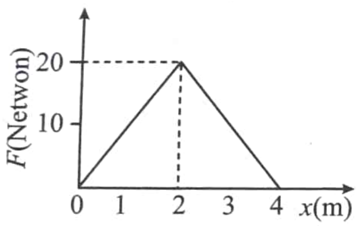


(a) $\frac{m}{2}(g+a)\,at^2$ (b) $\frac{m}{2}(g-a)at^2$

(c) $\frac{m}{2}gat^2$ (d) 0

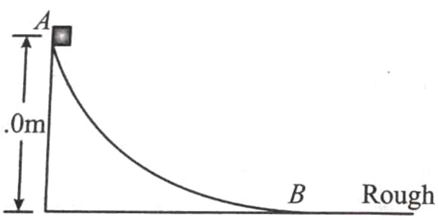
2. Two springs have their force constant as k_1 and $k_2 (k_1 > k_2)$. When they are stretched by the same force up to equilibrium
(a) No work is done by this force in case of both the springs
(b) Equal work is done by this force in case of both the springs
(c) More work is done by this force in case of second spring
(d) More work is done by this force in case of first spring

3. The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, its kinetic energy would be



(a) 50 J (b) 40 J (c) 20 J (d) 10 J

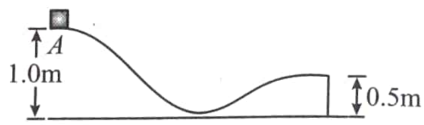
4. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface (figure). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, the distance it will move on the rough surface is



(a) 5.0 m (b) 10.0 m (c) 15.0 m (d) 20.0 m

5. A light spring of length 20 cm and force constant 2 kg/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The length h from the surface of the table at which the ball will have the maximum velocity is
(a) 20 cm (b) 15 cm
(c) 10 cm (d) 5 cm

6. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?



(a) At a horizontal distance of 1 m from the end of the track.
(b) At a horizontal distance of 2 m from the end of the track.
(c) At a horizontal distance of 3 m from the end of the track.
(d) Insufficient information

7. A block of mass m is moving with a constant acceleration 'a' on a rough horizontal plane. If the coefficient of friction between the block and plane is μ. The power delivered by the external agent at a time t from the beginning is equal to
(a) ma^2t (b) $\mu mgat$
(c) $\mu m(a + \mu g)\,gt$ (d) $m(a + \mu g)\,at$

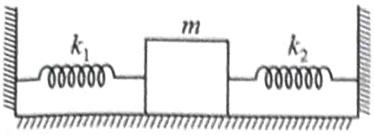
8. A particle moves with a velocity $\vec{v} = (5\hat{i} - 3\hat{j} + 6\hat{k})$ m/s under the influence of a constant force $\vec{F} = (10\hat{i} + 10\hat{j} + 20\hat{k})$ N. The instantaneous power applied to the article is
(a) 200 J/s (b) 40 J/s
(c) 140 J (d) 170 J/s

9. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1 / P_2) will be
(a) 1 (b) 4/3
(c) 16/9 (d) None of the above

10. A pump ejects 12000 kg of water at speed of 4 m/s in 40 second. Find the average rate at which the pump is working
(a) 0.24 KW (b) 2.4 W
(c) 2.4 KW (d) 24 W

11. Two springs A and B ($k_A = 2k_B$) are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in A is E, that in B is
(a) $E/2$ (b) $2E$
(c) E (d) $E/4$

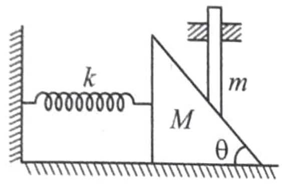
12. A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure. The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.



(a) $\sqrt{\frac{k_1+k_2}{m}}x$ (b) $\sqrt{\frac{k_1k_2}{m(k_1+k_2)}}x$

(c) $\sqrt{\frac{k_1^2k_2^2}{m(k_1^2+k_2^2)}}x$ (d) $\sqrt{\frac{k_1^3k_2^3}{m(k_1^3+k_2^3)}}$

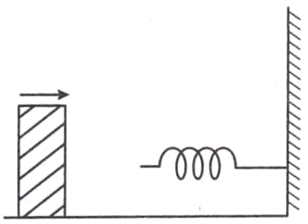
13. A wedge of mass M fitted with a spring of stiffness 'K' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is



(a) $\frac{mg^2\tan^2\theta}{2K}$ (b) $\frac{m^2g\tan^2\theta}{2K}$

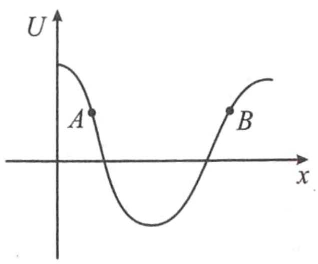
(c) $\frac{m^2g^2\tan^2\theta}{2K}$ (d) $\frac{m^2g^2\tan^2\theta}{K}$

14. A 1.0 kg block collides with a horizontal weightless spring of force constant 2.75 Nm^{-1} as shown in figure. The block compresses the spring 4.0 m from the rest position. If the coefficient of kinetic friction between the block and horizontal surface is 0.25, the speed of the block at the instant of collision is



(a) 0.4 ms^{-1} (b) 4 ms^{-1} (c) 0.8 ms^{-1} (d) 8 ms^{-1}

15. Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A and B is respectively.



(a) Positive, Positive (b) Positive, Negative

(c) Negative, Positive (d) Negative, Negative

16. The potential energy for a force field $\vec{F}$ is given by $U(x, y) = \sin(x+y)$. The force acting on the particle of mass $m\left(0, \frac{\pi}{4}\right)$ at is

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

17. A particle is taken from point A to point B under the influence of a force field. Now it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_c is the work done by non-conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change in kinetic energy, then

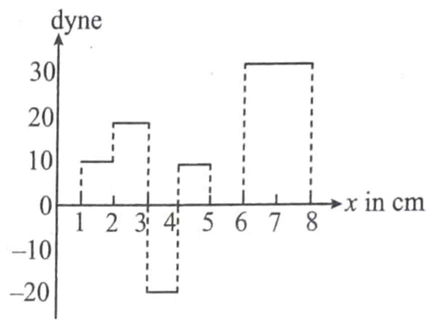
(a) $W_{nc} - \Delta U = \Delta k$

(b) $W_c = +\Delta U$

(c) $W_{nc} + W_c = \Delta k$

(d) $W_{nc} - \Delta U = -\Delta k$

18. The relationship between force and position is shown in fig (in one dimensional case). The work done in displacing a body from $x = 1$cm to $x = 5$ cm is



(a) 20 erg (b) 60 erg

(c) 70 erg (d) 700 erg

19. $F = 2x^2 - 3x - 2$. Choose correct option

(a) $x = -1/2$ is position of stable equilibrium

(b) $x = 2$ is position of stable equilibrium

(c) $x = -1/2$ is position of unstable equilibrium

(d) $x = 2$ is position of neutral equilibrium

20. The kinetic energy of a particle continuously increases with time

(a) The resultant force on the particle must be parallel to the velocity at all instants.

(b) The resultant force on the particle must be at an angle less than 90° with the velocity all the time

(c) Its height above the ground level must continuously decrease

(d) The magnitude of its linear momentum is increasing continuously

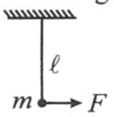
21. A box of mass m is initially at rest on a horizontal surface. A constant horizontal force of $mg/2$ is applied to the box directed to the right. The coefficient of friction of the surface changes with the distance pushed as $\mu = \mu_0 x$ where x is the distance from the initial location. For what distance is the box pushed until it comes to rest again

(a) $\frac{2}{\mu_0}$ (b) $\frac{1}{\mu_0}$ (c) $\frac{1}{2\mu_0}$ (d) $\frac{1}{4\mu_0}$

22. One end of a light rope is tied directly to the ceiling. A man of mass M initially at rest on the ground starts climbing the rope upto a height ℓ. From the time he starts at rest on the ground to the time he is hanging at rest at a height how much work was done on the man by the rope?

(a) 0
(b) $Mg\ell$
(c) $-Mg\ell$
(d) It depends on how fast the man goes up.

23. A pendulum bob of mass m is suspended at rest. A constant horizontal force $F = mg/2$ starts acting on it. The maximum angular deflection of the string is



(a) 90° (b) 53°
(c) 37° (d) None of these

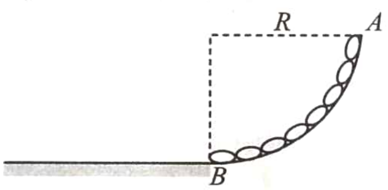
24. The potential energy for the force $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$, if the zero of the potential energy is to be chosen at the point (2, 2, 2), is

(a) $8 + xyz$ (b) $8 - xyz$
(c) $4 - xyz$ (d) $4 + xyz$

25. The upper half of an inclined plane with inclination 6 is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half given by

(a) $\tan\theta$ (b) $2\tan\theta$ (c) $2\cos\theta$ (d) $2\sin\theta$

26. A smooth chain AB of mass m rests against a surface in the form of a quarter of a circle of radius R. If it is released from rest, the velocity of the chain after it comes over the horizontal part of the surface is

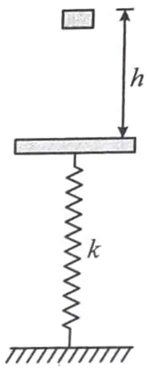


(a) $\sqrt{2gR}$
(b) $\sqrt{gR}$
(c) $\sqrt{2gR\left(1-\frac{2}{\pi}\right)}$
(d) $\sqrt{2gR(2-\pi)}$

27. A body is moving down an inclined plane of slope 37°. The coefficient of friction between the body and the plane varies as $\mu = 0.3\,x$, where x is the distance traveled down the plane by the body. The body will have maximum speed. $\left(\sin 37° = \frac{3}{5}\right)$

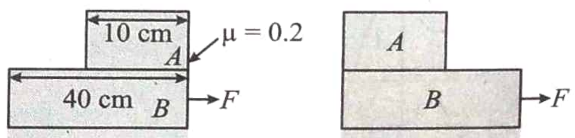
(a) At $x = 1.16$ m
(b) At $x = 2$
(c) At bottommost point of the plane
(d) At $x = 2.5$ m

28. A vertical spring is fixed to one of its end and a massless plank fitted to the other end. A block is released from a height h as shown. Spring is in relaxed position. Then choose the correct statement.



(a) The maximum compression of the spring does not depend on h
(b) The maximum kinetic energy of the block does not depend on h
(c) The compression of the spring at maximum KE of the block does not depend on h
(d) The maximum compression of the spring does not depend on k

29. A block A of mass 45 kg is placed on another block B of mass 123 kg. Now block B is displaced by external agent by 50 cm horizontally towards right. During the same time block A just reaches to the left end of block B. Initial and final positions are shown in figures. The work done on block A in ground frame is



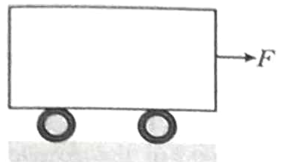
Initial Position Final Position

(a) –18 J (b) 18 J
(c) 36 J (d) –36 J

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

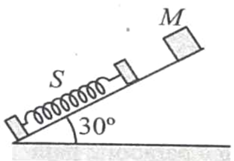
1. A car of mass m is accelerating on a level smooth road under the action of a single force F. The power delivered to the car is constant and equal to P. If the velocity of the car at an instant is v, then after travelling how much distance it becomes double?



(a) $\frac{7mv^3}{3P}$ (b) $\frac{4mv^3}{3P}$

(c) $\frac{mv^3}{P}$ (d) $\frac{18mv^3}{7P}$

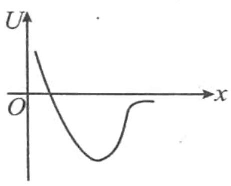
2. An ideal massless spring S can be compressed 1 m by a force of 100 N in equilibrium. The same spring is placed at the bottom of a frictionless inclined plane inclined at 30° to the horizontal. A 10 kg block M is released from rest at the top of the incline and is brought to rest momentarily after compressing the spring by 2 m. If $g = 10$ ms^{-2}, what is the speed of mass just before it touches the spring?

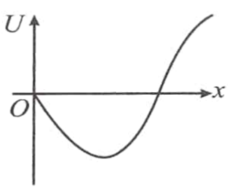


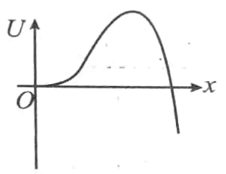
(a) $\sqrt{20}$ ms^{-1} (b) $\sqrt{30}$ ms^{-1}

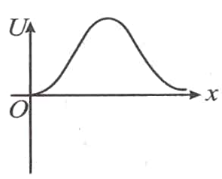
(c) $\sqrt{10}$ ms^{-1} (d) $\sqrt{40}$ ms^{-1}

3. A particle free to move along x-axis is acted upon by a force $F = -ax + bx^2$ where a and b are positive constants. For $x \geq 0$, the correct variation of potential energy function $U(x)$ is best represented by

(a) 

(b) 

(c) 

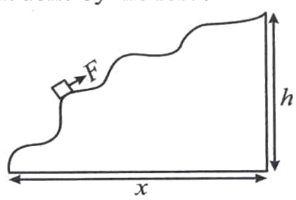
(d) 

4. Simple pendulums P_1 and P_2 have lengths $\ell_1 = 80$ cm and $\ell_2 = 100$ cm respectively. The bobs are of mass m_1 and m_2. Initially are at rest in equilibrium position. If each of the bobs is given a displacement of 2 cm, the work done cm is W_1 and W_2 respectively. Then,

(a) $W_1 > W_2$ if $m_1 = m_2$ (b) $W_1 < W_2$ if $m_1 = m_2$

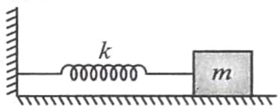
(c) $W_1 = W_2$ if $\frac{m_1}{m_2} = \frac{5}{4}$ (d) $W_1 = W_2$ if $\frac{m_1}{m_2} = \frac{4}{5}$

5. A body of mass m is slowly hauled up the rough hill by a force F which at each point is directed along a tangent to the hill. Work done by the force



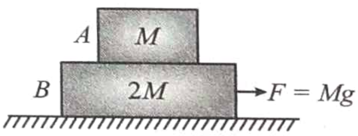
(a) Independent of shape of trajectory.
(b) Depends upon x.
(c) Depends upon h.
(d) Depends upon coefficient of friction (μ)

6. A spring block system is placed on a rough horizontal surface having coefficient of friction μ. The spring is given initial elongation and the block is released from rest. For the subsequent motion



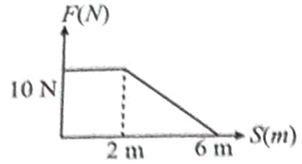
(a) Initial acceleration of block is 2 μg.
(b) Maximum compression in spring is $\mu mg/k$
(c) Minimum compression in spring is zero.
(d) Maximum speed of the block is $2\mu g\sqrt{\frac{m}{k}}$

7. In the figure shown, there is no friction between B and ground and $\mu = 2/3$ between A and B.



(a) The net work done on block A with respect to B is zero
(b) The net work done on block A with respect to ground for a displacement 'S' is $MgS/3$
(c) The net work done on block B with respect to ground for a displacement 'S' is $2MgS/3$
(d) The work done by friction with respect to ground on A and B is equal and opposite in sign.

8. A body of constant mass $m = 1$ kg moves under variable force F as shown. If at $t = 0$, $S = 0$ and velocity of the body is $\sqrt{20}$ m/s and the force is always along direction of velocity, then choose the incorrect options



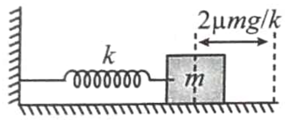
(*a*) Velocity of the particle will increase upto S = 2m and then decrease.

(*b*) The final velocity at $S = 6$ m is 10 m/s

(*c*) The final velocity at $S = 6$ m is $4\sqrt{5}$ m/s

(*d*) The acceleration is constant up to $S = 2$ m and then it is negative.

9. A block of mass 'm' is attached to one end of a massless spring of spring constant 'k'. The other end of the spring is fixed to a wall. The block can move on a horizontal rough surface. The coefficient of friction between the block and the surface is μ. The block is released when the spring has a compression $\frac{2\mu mg}{k}$ of. then choose the incorrect option(s)



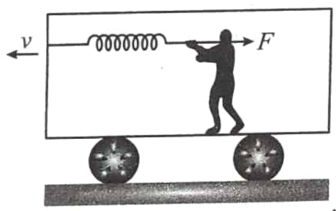
(*a*) The maximum speed of the block is $\mu g\sqrt{\frac{m}{k}}$

(*b*) The maximum speed of the block is $2\mu g\sqrt{\frac{m}{k}}$

(*c*) The block will have velocity towards left during its motion.

(*d*) The extension in the spring at the instant the velocity of block become zero for the first time after being released is $\frac{\mu mg}{k}$.

10. A man applying a force F upon a stretched spring is stationary in a compartment moving with constant speed v. The compartment covers a distance L in some time t.



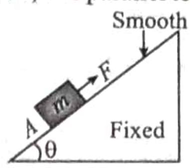
(*a*) The man acting with force F on spring does the work $w = -FL$.

(*b*) The total work performed by man on the (compartment + spring) with respect to ground is zero.

(*c*) The work done by friction acting on man with respect to ground is, $w = -FL$.

(*d*) The total work done by man with respect to ground is, $w = -FL$.

11. A force F (power P = constant) is applied on block A of mass m as shown in figure, F is parallel to the inclined plane. Then



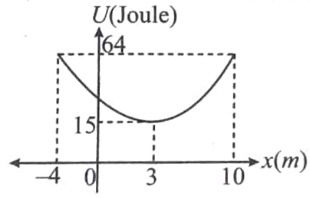
(*a*) The maximum speed of block A is $\frac{P}{mg\sin\theta}$

(*b*) The maximum speed of block A is $\frac{P}{mg\cos\theta}$

(*c*) The speed of block A first increases and then becomes constant

(*d*) Speed of block A continuously increases

12. A single conservative force $F(x)$ acts on a particle that moves along the x-axis. The graph of the potential energy with x is given. At $x = 5$ m, the particle has a kinetic energy of 50 J and its potential energy is related to position 'x' as $U = 15+(x-3)^2$ Joule, where x is in meter.



(*a*) The mechanical energy of system is 69 J.

(*b*) The mechanical energy of system is 19J.

(*c*) At $x = 3$, the kinetic energy of particle is minimum

(*d*) The maximum value of kinetic energy is 54 J.

13. A body of mass 1.0 kg moves in X-Y plane under the influence of a conservative force. Its potential energy is given by $U = 2x + 3y$ where (x, y) denote the coordinates of the body. The body is at rest at $(2, -4)$ initially. All the quantities have SI units. Therefore, the body

(*a*) Moves along a parabolic path

(*b*) Moves with a constant acceleration

(*c*) Never crosses the X axis

(*d*) Has a speed of $2\sqrt{13}$ m/s at time $t = 2$s.

14. A disc of mass 3 m and a disc of mass m are connected by a massless spring of stiffness k. The heavier disc is placed on the ground with the spring vertical and lighter disc on top. From its equilibrium position the upper disc is pushed down by a distance δ and released. Then

(*a*) If $\delta > \frac{3mg}{k}$, the lower disc will bounce up

(*b*) If $\delta = \frac{2mg}{k}$, maximum normal reaction from ground on lower disc = 6 mg

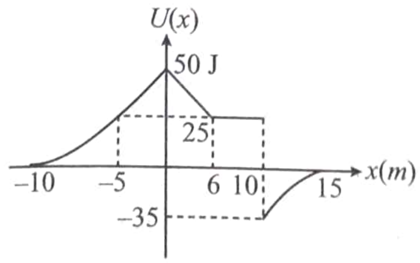
(*c*) If $\delta = \frac{2mg}{k}$, maximum normal reaction from ground on lower disc = 4 mg

(*d*) If $\delta > \frac{4mg}{k}$, the lower disc will bounce up

15. A ball of mass m is attached to the lower end of light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length, comes to rest again after descending through a distance x.
   (*a*) $x = mg/k$
   (*b*) $x = 2\ mg/k$
   (*c*) The ball will have no acceleration at the position where it has descended through $x/2$.
   (*d*) The ball will have an upward acceleration equal to g at its lowermost position.

COMPREHENSION BASED QUESTIONS

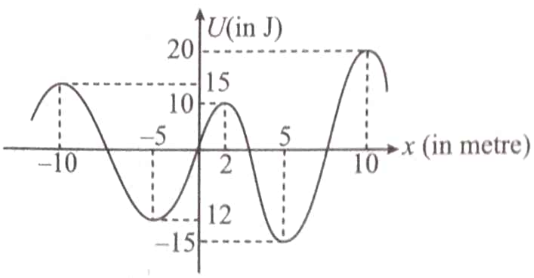
Comprehension (Q. 16 to 17): The figure shows the variation of potential energy of a particle as a function of x, the x-coordinate of the region. It has been assumed that potential energy depends only on x. For all other values of x, U is zero, i.e. for $x < -10$ and $x > 15$, $U = 0$. Based on above information answer the following questions



16. If total mechanical energy of the particle is 25 J, then it can be found in the region
   (*a*) $-10 < x < -5$ and $6 < x < 15$
   (*b*) $-10 < x < 0$ and $6 < x < 10$
   (*c*) $-5 < x < 6$
   (*d*) $-10 < x < 10$

17. If total mechanical energy of the particle is – 40 J, then it can be found in region
   (a) $x < -10$ and $x > 15$
   (b) $-10 < x < -5$ and $6 < x < 15$
   (c) $10 < x < 15$
   (d) It is not possible

Comprehension (Q. 18 to 20): In the figure the variation of potential energy of a particle of mass $m = 2$ kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



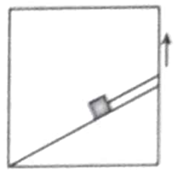
18. If the particle is released at the origin then:
   (*a*) It will move towards positive x-axis.
   (*b*) It will move towards negative x-axis.
   (*c*) It will remain stationary at the origin.
   (*d*) Its subsequent motion cannot be decided due to lack of information.

19. If the particle is released at $x = 2 + \Delta x$ where $\Delta x \to 0$ (it is positive) then its maximum speed in subsequent motion will be
   (*a*) $\sqrt{10}$ m/s (*b*) 5 m/s (*c*) $5\sqrt{2}$ (*d*) 7.5 m/s

20. $x = -5$ m and $x = 10$ m positions of the particle are respectively are of
   (*a*) Neutral and stable equilibrium.
   (*b*) Neutral and unstable equilibrium.
   (*c*) Unstable and stable equilibrium.
   (*d*) Stable and unstable equilibrium.

MATCH THE COLUMN TYPE QUESTIONS

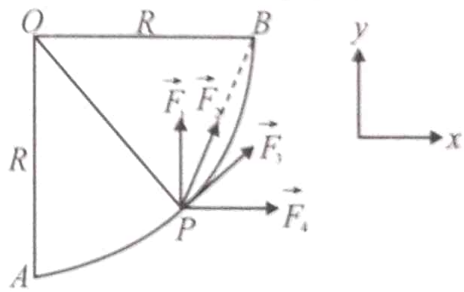
21. A block is placed on a rough wedge fixed on a lift as shown in figure. A string is also attached with the block. The whole system moves upwards. Block does not lose contact with wedge on the block. Match the following two columns regarding the work done (on the block).|



	Column-I		Column-II
A.	Work done by normal reaction	p.	Positive
B.	Work done by gravity	q.	Negative
C.	Work done by friction	r.	zero
D.	Work done by tension	s.	Can't say anything

   (*a*) A-(q); B-(r); C-(q); D-(r)
   (*b*) A-(p); B-(q); C-(r); D-(p)
   (*c*) A-(r); B-(p); C-(s); D-(q)
   (*d*) A-(q); B-(r); C-(p); D-(r)

22. AB is a quarter of a smooth horizontal circular track of radius R. A particle P of mass m moves along the track from A to B under the action of following forces:



$\vec{F_1} = F$ (always towards y-axis)

$\vec{F_2} = F$ (always towards point B)

$\vec{F_3} = F$ (always along the tangent to path AB)

$\vec{F_4} = F$ (always towards x-axis)

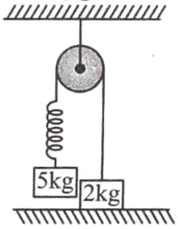
	Column-I		Column-II
A.	Work done by $\vec{F_1}$	p.	$\sqrt{2}$FR
B.	Work done by $\vec{F_2}$	q.	$\frac{1}{\sqrt{2}}$FR
C.	Work done by $\vec{F_3}$	r.	FR
D.	Work done by $\vec{F_4}$	s.	$\frac{\pi FR}{2}$
		T.	$\frac{\pi FR}{2}$

(*a*) A-(q); B-(r); C-(q); D-(r)
(*b*) A-(p); B-(q); C-(r); D-(p)
(*c*) A-(r); B-(p); C-(s); D-(r)
(*d*) A-(q); B-(r); C-(p); D-(r)

NUMERICAL TYPE QUESTIONS

23. A uniform chain of length ℓ and mass m overhangs on a rough horizontal table with its 3/4 part on the table. The friction coefficient between the table and the chain is μ. Find the magnitude of work (in joules) done by the friction during the period the chain slips off the table

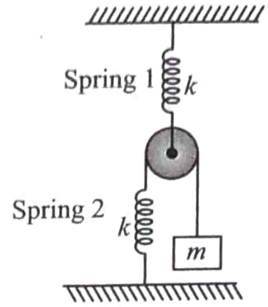
24. The system as shown in the figure is released from rest. The pulley, spring and string are ideal & friction is absent everywhere. If speed of 5 kg block when 2 kg block leaves the contact with ground is $2\sqrt{x}$ m/s, then value of x is (spring constant $k = 40$ N/m and $g = 10$ m/s^2)



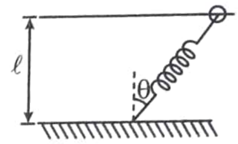
25. A spring ($k = 100$ Nm^{-1}) is suspended in vertical position having one end fixed at top and other end joined with a 2 kg block. When the spring is in non deformed shape, the block is given initial velocity 2 m/s in downward direction. The maximum elongation of the spring is $\left(\frac{\sqrt{3}+1}{n}\right)$ meter. Find n.

26. Two blocks of masses m_1 and m_2 are connected by a spring of stiffness $k = 200$ Nm^{-1}. The coefficient of friction between the blocks and the surface is μ. Find the minimum constant horizontal force F (in Newton) to be applied to m_1 in order to slide the mass m_2. (Initally spring is in its natural length). (Take $m_1 = 3$ kg, $m_2 = 5$ kg, $g = 10$ m/s^2, $\mu = 0.2$)

27. All springs, string and pulley shown in figure are light. Initially when both the springs were in their natural state, the system was released from rest. The maximum displacement of block m is $x \times \left(\frac{5mg}{k}\right)$. Calculate x.



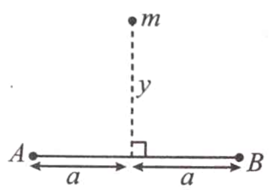
28. One end of a spring of natural length ℓ and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height ℓ (figure). Initially, the spring makes an angle of θ with the vertical when the system is released from rest. If the speed of the ring when the spring becomes vertical is $(2\ell/3)\sqrt{\frac{k}{m}}$ m/s then find the value of angle θ (in degree):



29. A particle of mass 'M' is moved rectilinearly under constant power P_0. At some instant after the start, its speed is v and at a later instant, the speed is 2 v. Neglecting friction, distance travelled (in m) by the particle as its speed increases from v to $2v$ is $7x$. Find x (Take $P_0 = 4$ watt, $M = 12$ kg, $v = 3$m/s).

30. A particle of mass 2 kg is subjected to a two dimensional conservative force given by, $F_x = -2x + 2y$, $F_y = 2x - y^2$. (x, y in m and F in N). If the particle has kinetic energy of 8/3 J at point (2,3), find the speed (in m/s) of the particle when it reaches (1, 2).

31. Potential energy of a particle of mass m, depends on distance y from line AB according to given relation $U = \frac{K}{\sqrt{y^2+a^2}}$, where K is a positive constant. A particle of mass m is projected from $y = \sqrt{3}\,a$ towards line AB (perpendicular to it) then minimum velocity so that it cannot return to its initial point is $\sqrt{\frac{K}{aNm}}$, calculate N.

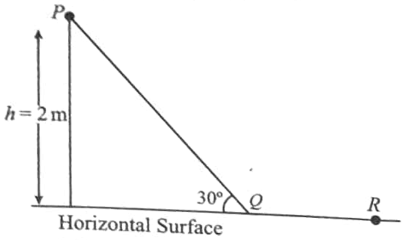


32. The potential energy (in SI units) of a particle of mass 2 kg in a conservative field is $U = 6x - 8y$. If the initial velocity of the particle is $\vec{u} = -1.5\hat{i} + 2\hat{j}$ then find the total distance (in meter) travelled by the particle in first two seconds.

Exercise-4 (Past Year Questions)

JEE MAIN

1. A point particle of mass m moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies lost by the ball over the parts PQ and QR of the track are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(= QR)$, are, respectively close to **(2016)**



(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

2. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work is done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. (Take $g = 9.8\ ms^{-2}$) **(2016)**

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg
(c) 9.89×10^{-3} kg (d) 12.89×1^{-3} kg

3. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10\ ms^{-1}$. If after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be **(2017)**

(a) 10^{-4} kg m^{-1} (b) 10^{-1} kg m^{-1} s^{-1}
(c) 10^{-3} kg m^{-1} (d) 10^{-3} kg s^{-1}

4. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be **(2017)**

(a) 9 J (b) 18 J (c) 4.5 J (d) 22 J

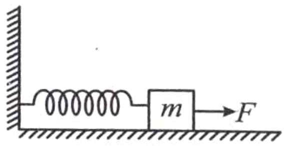
5. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is **(2018)**

(a) $\frac{k}{2a^2}$ (b) Zero
(c) $-\frac{3}{2}\frac{k}{a^2}$ (d) $-\frac{k}{4a^2}$

6. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? **(2019)**

(a) 850 J (b) 950 J
(c) 875 J (d) 900 J

7. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in an equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is **(2019)**

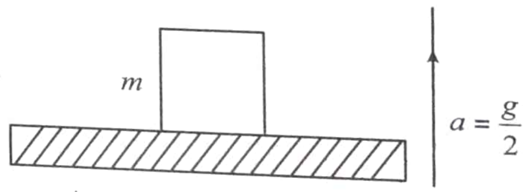


(a) $\frac{2F}{\sqrt{mk}}$ (b) $\frac{F}{\pi\sqrt{mk}}$
(c) $\frac{\pi F}{\sqrt{mk}}$ (d) $\frac{F}{\sqrt{mk}}$

8. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement? **(2019)**

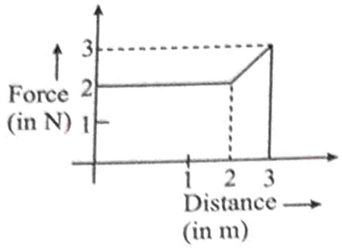
(a) 9 J (b) 12 J
(c) 10 J (d) 15 J

9. A block of mass m is kept on a platform which starts from rest with constant acceleration $g/2$ upward, as shown in figure. Work done by normal reaction on block in time t is **(2019)**



(a) $-\frac{mg^2t^2}{8}$
(b) $\frac{mg^2t^2}{8}$
(c) 0
(d) $\frac{3mg^2t^2}{8}$

10. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is **(2019)**



(a) 6.5 J (b) 2.5 J (c) 4 J (d) 5 J

11. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be **(2019)**

(a) $\frac{MgL}{n^2}$ (b) $\frac{MgL}{2n^2}$

(c) $\frac{2MgL}{n^2}$ (d) $nMgL$

12. A spring whose unstreched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be **(2019)**

(a) $\frac{1}{n^2}$ (b) n^2

(c) $\frac{1}{n}$ (d) n

13. A person of mass M is sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $\ell(\ell << L)$, is close to **(2019)**

(a) $Mg\ell$ (b) $Mg\ell(1+\theta_0^2)$

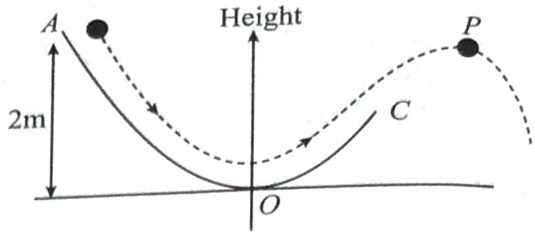
(c) $Mg\ell(1-\theta_0^2)$ (d) $Mg\ell\left(1+\frac{\theta_0^2}{2}\right)$

14. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to : (1 HP = 746 W, $g = 10$ ms^{-2}) **(2020)**

(a) 1.5 ms^{-1} (b) 1.9 ms^{-1}

(c) 2.0 ms^{-1} (d) 1.7 ms^{-1}

15. A particle ($m = 1$ kg) slides down a frictionless track (AOC) starting from rest at a point A(height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale)(Take $g = 10$ ms^{-2}) **(2020)**

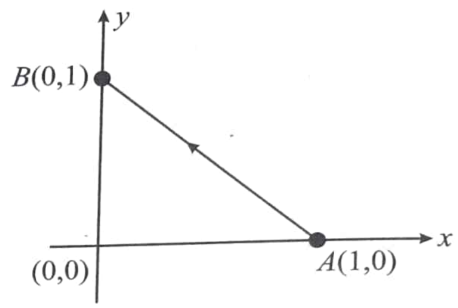


16. An elevator in a building can carry a maximum of 10 persons with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator must be at least ($g = 10$ m/s^2) **(2020)**

(a) 62360 W (b) 56300 W

(c) 48000 W (d) 66000 W

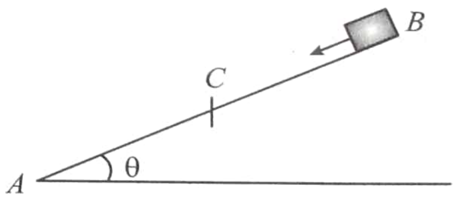
17. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point $A(1,0)$ to $B(0,1)$ along the line segment is (All quantities are in SI units) **(2020)**



(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) 2 (d) 1

18. A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section. BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k\tan\theta$. The value of k is ________. **(2020)**



19. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is ($g = 10$ ms^{-2}) ________. **(2020)**

20. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30m. What is the work done by the person during the total movement of the box? **(2020)**

(*a*) 5690 J (*b*) 3280 J
(*c*) 5250 J (*d*) 2780 J

21. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) ________. **(2020)**

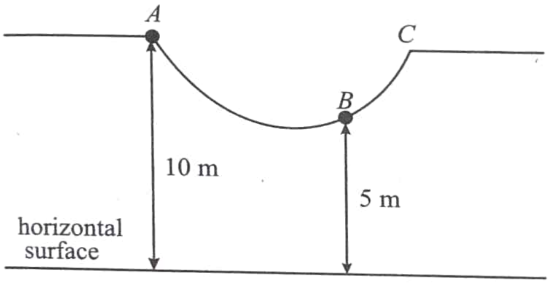
22. If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are **(2020)**

(*a*) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$ (*b*) $\left(\frac{B}{2A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$

(*c*) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{4B}$ (*d*) $\left(\frac{B}{A}\right)^{\frac{1}{6}}, 0$

23. A ball of mass 4 kg, moving with a velocity of $10 ms^{-1}$, collides with a spring of length 8 m and force constant $100 Nm^{-1}$. The length of the compressed spring is x m. The value of x, to the nearest integer, is **(2021)**

24. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. The value of 'x' to the nearest integer is (Take $g = 10 m/s^2$) **(2021)**



25. Two masses of 1 g and 4 g are moving with equal kinetic energy. The ratio of the magnitudes of their momenta is **(2021)**

(*a*) 4 : 1 (*b*) $\sqrt{2}:1$
(*c*) 1 : 2 (*d*) 1 : 16

26. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time 't' as $a_c = k^2rt^2$ where 'k' is a constant. The power delivered to the particle by the force acting on it is **(2021)**

(*a*) $2\pi mk^2r^2t$ (*b*) mk^2r^2t
(*c*) $(mk^4r^2t^5)/3$ (*d*) Zero

27. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is **(2021)**

(*a*) $\sqrt{u^2 - 2gL}$ (*b*) $\sqrt{u^2 - gL}$
(*c*) $\sqrt{2(u^2 - gL)}$ (*d*) $\sqrt{2gL}$

28. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is **(2021)**

(*a*) $(aL^2)/2 + (bL^2)/3$ (*b*) $1/2(aL^2)/2 + (bL^3)/3)$
(*c*) $aL^2 + bL^3$ (*d*) $1/2(aL^2 + bL^3)$

29. A body of mass $m = 10^{-2}$ kg is moving in a medium and experience a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10 ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be **(2021)**

(*a*) $10^{-4} kg m^{-1}$ (*b*) $10^{-1} kg m^{-1}s^{-1}$
(*c*) $10^{-3} kg m^{-1}$ (*d*) $10^{-3} kg s^{-1}$

30. A block of mass 10 kg starts sliding on a surface with an initial velocity of $9.8 ms^{-1}$. The coefficient of friction between the surface and block is 0.5. The distance covered by the block before coming to rest is [$g = 9.8 ms^{-2}$] **(2022)**

(*a*) 4.9 m (*b*) 9.8 m
(*c*) 12.5 m (*d*) 19.6 m

31. A boy ties a stone of mass 100 g to the end of a 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\frac{K}{\pi}$ rev./min, the value of K is (Assume the string is massless and unstretchable) **(2022)**

(*a*) 400 (*b*) 300
(*c*) 600 (*d*) 800

32. A 0.5 kg block moving at a speed of $12 ms^{-1}$ compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be ________ Nm^{-1}. **(2022)**

33. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in its velocity, as it reaches a position where the string is horizontal, is $\sqrt{x(u^2 - gL)}$. The value of x is **(2022)**

(*a*) 3 (*b*) 2
(*c*) 1 (*d*) 5

34. A particle of mass 500 gm is moving in a straight line with velocity $v = bx^{5/2}$. The work done by the net force during its displacement from $x = 0$ to $x = 4$ m is (Take: $b = 0.25\ \mathrm{m^{-3/2}\ s^{-1}}$) **(2022)**

(*a*) 2 J (*b*) 4 J
(*c*) 8 J (*d*) 16 J

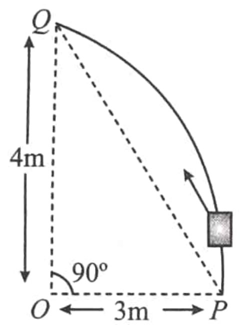
JEE ADVANCED

35. The work done on a particle of mass m by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x-y plane is **(2013)**

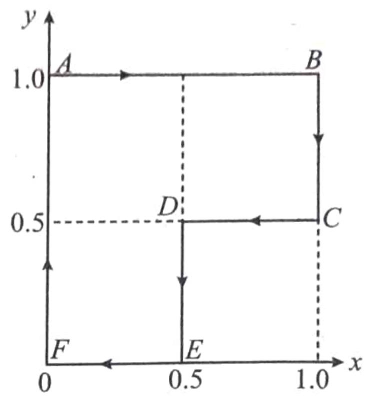
(*a*) $\frac{2K\pi}{a}$ (*b*) $\frac{K\pi}{a}$
(*c*) $\frac{K\pi}{2a}$ (*d*) 0

36. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in $\mathrm{ms^{-1}}$) of the particle is zero, the speed (in $\mathrm{ms^{-1}}$) after 5s is **(2013)**

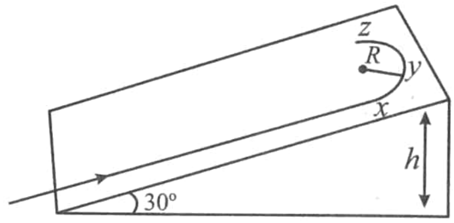
37. Consider an elliptically shaped rail PQ in the vertical plane with $OP = 3$m and $OQ = 4$m. A block of mass 1kg is pulled along the rail from P to Q with a force of 18 N which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ joules. The value of n is (take acceleration due to gravity = 10 $\mathrm{ms^{-2}}$) **(2014)**



38. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N where x and y are in meter and $\alpha = -1\ \mathrm{N/m^{-1}}$. The work done on the particle by this force $\vec{F}$ will be ____ Joule **(2019)**



39. A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only, then (g is the acceleration due to gravity) **(2020)**



(*a*) $v_0^2 - 2gh = \frac{1}{2}gR$

(*b*) $v_0^2 - 2gh = \frac{\sqrt{3}}{2}gR$

(*c*) The centripetal force required at points x and z is zero

(*d*) The centripetal force required is maximum at points x and z

ANSWER KEY

CONCEPT APPLICATION

1. (*c*) 2. (*i*) Zero (*ii*) 500J, 3. [150 J] 4. [8 J] 5. (*a*) 6. (*a*) zero (*b*) zero (*c*) $-\mu mgvt$ (*d*) $\mu mgvt$

7. [135 J] 8. [0 J] 9. [25 J] 10. [$W = -3.5$ J] 11. [3] 12. (*a*) Zero (*b*) $-\frac{1}{2}kx^2$ 13. (*a*) 32 J, (*b*) 5 m/s

14. (*c*) 15. (*c*) 16. [0.5 J] 17. (*a*) 18. (*a*) 19. $\frac{mg}{k}$ 20. (*a*) 21. (*b*) 22. (*a*)

23. $\left\{\frac{M+2(m-m')}{M+m+m'}ag\right\}$ 24. $F_x = \frac{-\partial U}{\partial x} = -2ax + by^2$, $F_y = 2bxy$ 25. $x = 0$, stable equilibrium, $x = \pm\sqrt{\frac{a}{2b}}$ unstable equilibrium

26. (*b*) 27. (*b*)

EXERCISE-1 (TOPICWISE)

1. (*c*)	2. (*a*)	3. (*b*)	4. (*b*)	5. (*b*)	6. (*a*)	7. (*c*)	8. (*d*)	9. (*c*)	10. (*b*)
11. (*b*)	12. (*b*)	13. (*c*)	14. (*a*)	15. (*c*)	16. (*a*)	17. (*c*)	18. (*b*)	19. (*b*)	20. (*b*)
21. (*a*)	22. (*d*)	23. (*c*)	24. (*b*)	25. (*b*)	26. (*d*)	27. (*b*)	28. (*d*)	29. (*a*)	30. (*b*)
31. (*a*)	32. (*c*)	33. (*c*)	34. (*d*)	35. (*d*)	36. (*b*)	37. (*d*)	38. (*a*)	39. (*a*)	40. (*b*)
41. (*c*)	42. (*b*)	43. (*d*)	44. (*b*)	45. (*a*)	46. (*b*)	47. (*a*)	48. (*b*)	49. (*c*)	50. (*a*)
51. (*c*)									

EXERCISE-2 (LEARNING PLUS)

1. (*a*)	2. (*c*)	3. (*d*)	4. (*a*)	5. (*b*)	6. (*a*)	7. (*d*)	8. (*c*)	9. (*b*)	10. (*c*)
11. (*b*)	12. (*a*)	13. (*c*)	14. (*d*)	15. (*b*)	16. (*a*)	17. (*a*)	18. (*a*)	19. (*a*)	20. (*b,d*)
21. (*b*)	22. (*b*)	23. (*d*)	24. (*b*)	25. (*b*)	26. (*c*)	27. (*d*)	28. (*c*)	29. (*b*)	

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*a*)	2. (*a*)	3. (*c*)	4. (*a,d*)	5. (*a,b,c,d*)	6. (*a,b,c,d*)	7. (*a,b,c,d*)	8. (*a,c,d*)	9. (*a,b,c,d*)	10. (*a,b,c*)
11. (*a,c*)	12. (*a,d*)	13. (*b,c,d*)	14. (*b,d*)	15. (*b,c,d*)	16. (*a*)	17. (*d*)	18. (*b*)	19. (*b*)	20. (*d*)
21. (*b*)	22. (*c*)	23. [18]	24. [2]	25. [5]	26. [11]	27. [2]	28. [53°]	29. [27]	30. [2]
31. [1]	32. [15]								

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*c*)	2. (*d*)	3. (*a*)	4. (*c*)	5. (*b*)	6. (*d*)	7. (*d*)	8. (*d*)	9. (*d*)	10. (*a*)
11. (*b*)	12. (*c*)	13. (*b*)	14. (*b*)	15. [10]	16. (*d*)	17. (*d*)	18. [3]	19. [150]	20. (*c*)
21. [18]	22. (*c*)	23. [6]	24. [10]	25. (*c*)	26. (*b*)	27. (*c*)	28. (*a*)	29. (*a*)	30. (*b*)
31. (*c*)	32. [600]	33. (*b*)	34. (*d*)						

JEE Advanced

35. (*d*)	36. [5]	37. [5]	38. [0.75]	39. (*a,d*)

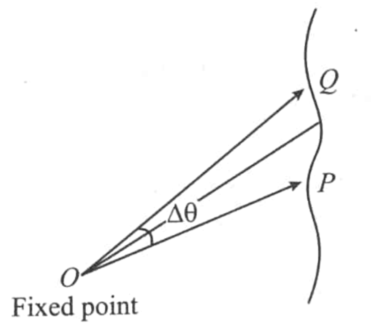
CHAPTER

9 Circular Motion

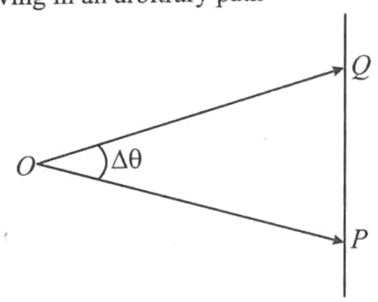
KINEMATICS OF CIRCULAR MOTION

Angular Displacement

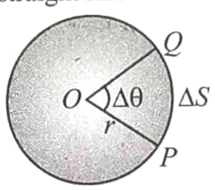
Angle subtended by position vector of a particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.



(a) Particle moving in an arbitrary path



(b) Particle moving in straight line



(c) Particle moving in circular path

(i) Infinitesimaly small Angular displacement is a vector quantity.

(ii) Its direction is perpendicular to plane of rotation and given by right hand screw rule.

Note: Clockwise angular displacement is taken as negative and anticlockwise displacement as positive.

(iii) For circular motion $\Delta S = r \times \Delta\theta$

(iv) Its unit is radian (in M.K.S)

Note: Always change degree into radian, if it occurs in numerical problems.

Note: 1 radian $= \dfrac{360^\circ}{2\pi} \Rightarrow \pi$ radian $= 180^\circ$

(v) It is a dimensionless quantity i.e. dimension $[M^0L^0T^0]$

Important Point

- It is dimensionless and its unit SI unit is radian while other units are degree or revolution 2π rad $= 360^\circ = 1$ rev
- Infinitely small angular displacement is a vector quantity but finite angular displacement is not because the addition of the small angular displacement is cummutative while for large is not.
- $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$ but $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$
- Direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point then thumb will represents the direction of angular displacement.
- Angular displacement can be different for different observers

Angular Velocity

It is defined as the rate of change of angular displacement of a body or particle moving in circular path.

(i) It is a vector quantity.

(ii) Its direction is same as that of angular displacement i.e perpendicular to plane of rotation.

Note: If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane inwards. Whereas in case of anticlockwise direction the direction will be outwards.

(iii) Its unit is Radian/sec

(iv) Its dimension is $[M^0L^0T^{-1}]$

Types of Angular Velocity

Average Angular Velocity

$$\omega_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}} = \frac{\Delta\theta}{\Delta t}$$

It is a scalar quantity because finite angular displacement is a scalar quantity.

Instantaneous Angular Velocity

The instantaneous angular velocity is defined as the angular velocity at some particular instant.

Instantaneous angular velocity

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

Note: Instantaneous angular velocity can also be called as simply angular velocity.

Important Points

- It is an axial vector with dimensions $[T^{-1}]$ and SI unit rad/s.
- For a rigid body as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about its own axis is $(2\pi/24)$ rad /hr.
- If a body makes 'n' rotations in 't' seconds then angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t}$$

If T is the period and 'f' the frequency of uniform circular motion

$$\omega_{av} = \frac{2\pi \times 1}{T} = 2\pi f$$

- If $\theta = a - bt + ct^2$ then $\omega = \frac{d\theta}{dt} = -b + 2ct$

Relation Between Linear Velocity and Angular Velocity

We have $\omega = \frac{d\theta}{dt} = \frac{d\theta}{ds} \cdot \frac{ds}{dt} = \frac{1}{r} \cdot v$

$[\because d\theta = \frac{ds}{dr}$, angle $= \frac{arc}{radius}$

and $v = \frac{ds}{dt}$ = linear velocity]

$\Rightarrow \; v = r\omega$

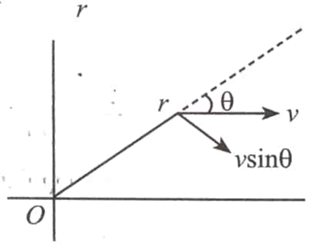
In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

Note:

(i) When a particle moves along a curved path, its linear velocity at a point is along the tangent drawn at that point

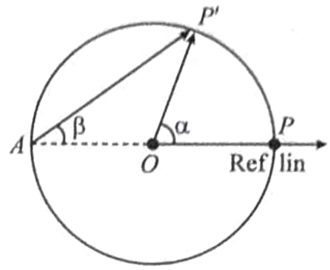
(ii) When a particle moves along curved path, its velocity has two components. One along the radius, which increases or decreases the radius and another one perpendicular to the radius, which makes the particle to revolve about the point of observation.

(iii) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v\sin\theta}{r}$



Relative Angular Velocity

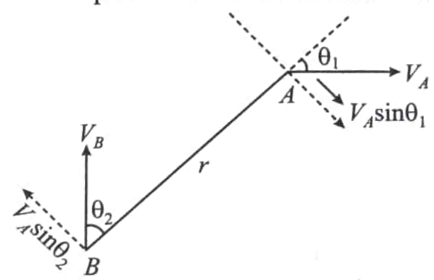
Angular velocity is defined with respect to the point from which the position vector of the moving particle is drawn Here angular velocity of the particle w.r.t. 'O' and 'A' will be different



$$\omega_{PO} = \frac{d\alpha}{dt}; \; \omega_{PA} = \frac{d\beta}{dt}$$

Definition

Relative angular velocity of a particle 'A' with respect to the other moving particle 'B' is the angular velocity of the position vector of 'A' with respect to 'B'. That means it is the rate at which position vector of 'A' with respect to 'B' rotates at that instant



$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}} \quad \text{here } V_{AB\perp}$$

= Relative velocity $\perp$ to position vector AB

$$= \frac{\text{Relative velocity of } A \text{ wr.t. B perpendicular to line } AB}{\text{Seperation between } A \text{ and } B}$$

$$(V_{AB})_\perp = V_A\sin\theta_1 + V_B\sin\theta_2$$

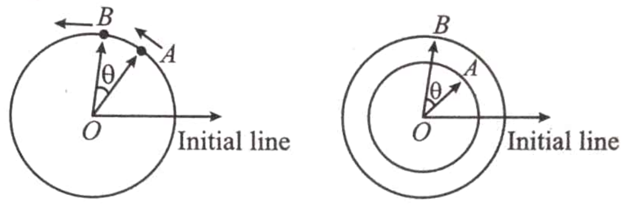
$$r_{AB} = r$$

$$\omega_{AB} = \frac{V_A\sin\theta_1 + V_B\sin\theta_2}{r}$$

Important Points

If two particles are moving on the same circle or different coplanar concentric circles in same direction with different uniform angular speed ω_A and ω_B respectively, the rate of change of angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is

$$\frac{d\theta}{dt} = \omega_B - \omega_A$$

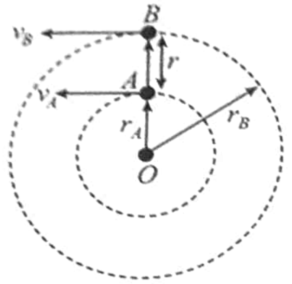


So the time taken by one to complete one revolution around O w.r.t. the other

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1T_2}{T_1 - T_2}$$

If two particles are moving on two different concentric circles with different velocities then angular velocity of B relative to A as observed by A will depend on their positions and velocities. consider the case when A and B are closest to each other moving in same direction as shown in figure. In this situation

$$v_{rel} = |\vec{v}_B - \vec{v}_A| = v_B - v_A$$
$$r_{rel} = |\vec{r}_B - \vec{r}_A| = r_B - r_A$$



$$\text{so, } \omega_{BA} = \frac{(v_{rel})_\perp}{r_{rel}} = \frac{v_B - v_A}{r_B - r_A}$$

$(v_{rel})_\perp$ = Relative velocity perpendicular to position vector

Train Your Brain

Example 1: If θ depends on time t in following way $\theta = 2t^2 + 3$ then

(i) Find out average ω upto 3 sec.

(ii) ω at 3 sec

Sol. (i) $\omega_{avg} = \dfrac{\text{Total angular displacement}}{\text{total time}} = \dfrac{\theta_f - \theta_i}{t_2 - t_1}$

$\theta_f = 2\,(3)^2 + 3 = 21$ rad

$\theta_i = 2\,(0) + 3 = 3$ rad

So, $\omega_{avg} = \dfrac{21-3}{3} = 6$ rad/sec

(ii) $\omega_{instantaneous} = \dfrac{d\theta}{dt} = 4t$

$\omega_{at\ t=3\ sec} = 4 \times 3 = 12$ rad/sec

Example 2: A particle revolving in a circular path completes first one third of circumference in 2 sec, while next one third in 1 sec. The average angular velocity of particle will be: (in rad/sec)

(a) $\dfrac{2\pi}{3}$ (b) $\dfrac{\pi}{3}$

(c) $\dfrac{4\pi}{3}$ (d) $\dfrac{5\pi}{3}$

Sol. (a) We have $\omega_{av} = \dfrac{\text{Total angular displacement}}{\text{Total time}}$

For first one third part of circle, angular displacement,

$\theta_1 = \dfrac{S_1}{r} = \dfrac{2\pi r/3}{r}$

For second one third part of circle,

$\theta_2 = \dfrac{2\pi r/3}{r} = \dfrac{2\pi}{3}$ rad

Total angular displacement,

$\theta = \theta_1 + \theta_2 = 4\pi/3$ rad

Total time = 2 + 1 = 3 sec

$\therefore\ \vec{\omega}_{av} = \dfrac{4\pi/3}{3}$ rad/s $= \dfrac{4\pi}{6} = \dfrac{2\pi}{3}$ rad/s

Example 3: The ratio of angular speeds of minute hand and hour hand of a watch is

(a) 1 : 12 (b) 6 : 1

(c) 12 : 1 (d) 1 : 6

Sol. (c) Angular speed of hour hand,

$\omega_1 = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2\pi}{12 \times 60}$ rad/sec

angular speed of minute hand,

$\omega_2 = \dfrac{2\pi}{60}$ rad/sec $\Rightarrow \dfrac{\omega_2}{\omega_1} = \dfrac{12}{1}$

Example 4: The angular displacement of a particle is given by $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$, where ω_0 and α are constant and $\omega_0 = 1$ rad/sec, $\alpha = 1.5$ rad/sec^2. The angular velocity at time, $t = 2$ sec will be (in rad/sec)

(a) 1 (b) 5

(c) 3 (d) 4

Sol. (d) We have $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$

$\Rightarrow \dfrac{d\theta}{dt} = \omega_0 + \alpha t$

This is angular velocity at time t. Now angular velocity at $t = 2$ sec will be

$\omega = \left(\dfrac{d\theta}{dt}\right)_{t=2sec} = \omega_0 + 2\alpha$

$= 1 + 2 \times 1.5 = 4$ rad/sec

Example 5: A particle moves in a circle of radius 20 cm with a linear speed of 10m/s. The angular velocity will be

(a) 50 rad/s

(b) 100 rad/s

(c) 25 rad/s

(d) 75 rad/s

Sol. (*a*) The angular velocity is

$$\omega = \frac{v}{r}$$

Hence $v = 10$ m/s, $r = 20$ cm $= 0.2$ m,

$\therefore \omega = 50$ rad/s

Example 6: Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then, which is incorrect.

(*a*) They cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed

(*b*) They cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed

(*c*) They cross at intervals of time $\frac{\pi}{30}$ if their angular velocities are oppositely directed

(*d*) They cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense

Sol. (*a*) If the angular velocities are oppositely directed, they meet at intervals of

$$\text{time } t = \frac{2\pi}{\omega_{\text{rel}}} = \frac{2\pi}{6\omega} = \frac{\pi}{3\omega}$$

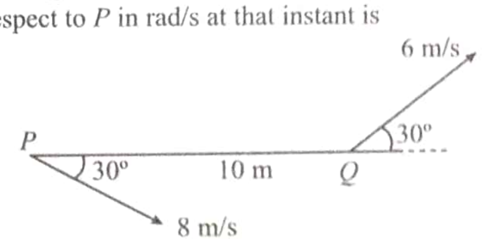
Angle subtended at the centre by the crossing points

$$\theta = \omega t = \frac{\pi}{3} = 60^\circ$$

When their angular velocities are in the same direction,

$$t' = \frac{2\pi}{\omega_{\text{rel}}} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega} \text{ and } \theta' = \frac{\pi}{2\omega} \times \omega = \frac{\pi}{2}$$

Example 7: Two moving particles P and Q are 10 m apart at a certain instant. The velocity of P is 8 m/s making 30° with the line joining P and Q and that of Q is 6 m/s making 30° with PQ in the figure. Then the angular velocity of Q with respect to P in rad/s at that instant is



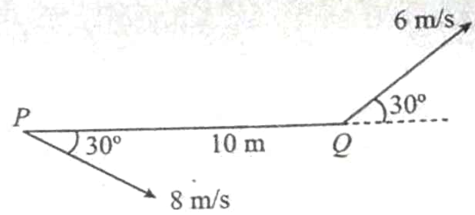
(*a*) 0

(*b*) 0.1

(*c*) 0.4

(*d*) 0.7

Sol. (*d*)



Angular velocity of Q relative to

$$P = \frac{\text{Projection of } V_{QP} \text{ perpendicular to the line } PQ}{\text{Separation between } P \text{ and } Q}$$

$$\frac{V_Q \sin\theta_2 - V_P \sin\theta_1}{PQ} = \frac{6\sin 30^\circ - (-8\sin 30^\circ)}{10}$$

$$= 7 \text{ rad/s}$$

Concept Application

1. Two racing cars of masses m_1 and m_2 are moving in circles of radii r and $2r$ respectively and their angular speeds are equal. The ratio of the time taken by cars to complete one revolution is:

(*a*) $m_1 : m_2$ (*b*) 1 : 2

(*c*) 1 : 1 (*d*) $m_1 : 2m_2$

2. A wheel is at rest. Its angular velocity increases uniformly and becomes 80 radian per second after 5 second. The total angular displacement is :

(*a*) 800 rad (*b*) 400 rad

(*c*) 200 rad (*d*) 100 rad

ANGULAR ACCELERATION

The rate of change of angular velocity is defined as angular acceleration.

1. If $\Delta\omega$ be change in angular velocity in time Δt, then angular acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

(i) It is a vector quantity

(ii) Its direction is that of change in angular velocity

(iii) Unit : rad/sec^2

(iv) Dimension: $M^0L^0T^{-2}$

2. Instantaneous Angular Acceleration:

It is the limit of average angular acceleration as Δt approaches zero, i.e.,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$$

Important Points

- It is also an axial vector with dimension $[T^{-2}]$ and unit rad/s^2
- If $\alpha = 0$, circular motion is said to be uniform.

- As $\omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$.
- i.e. second derivative of angular displacement w.r.t time gives angular acceleration.
- α is a axial vector and direction of α is along ω if ω increases and opposite to ω if ω decreases

The net acceleration is neither parallel nor perpendicular to the velocity.

We can resolve the acceleration vector into two components:

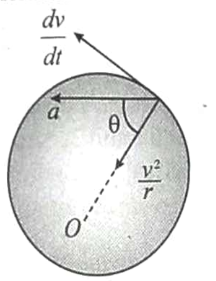
(a) **Radial Acceleration**

The component of the acceleration towards the centre is $\omega^2 r = \frac{v^2}{r}$

a_r perpendicular to the velocity

$\Rightarrow$ changes only the direction of velocity.

Acts just like the acceleration in uniform circular motion.



$$a_c = \frac{v^2}{r} = \omega^2 r \; [a_c = a_r]$$

Centripetal force $F_c = \frac{mv^2}{r} = m\omega^2 r$

(b) **Tangential Acceleration**

The component along the tangent (along the direction of motion) is $a_t = \frac{dv}{dt}$ parallel to the velocity (since it is tangent to the path)

$\Rightarrow$ changes magnitude of the velocity and acts just like one-dimensional acceleration

$\Rightarrow a_t = \frac{dv}{dt}$ where $v = \frac{ds}{dt}$ and s = length of arc

Tangential force $F_t = ma_t$

Net acceleration vector is obtained by vector addition of these two components.

The magnitude of the net acceleration

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

(c) **Relation between Angular Acceleration and Tangential Acceleration**

we know that

$v = r\omega$

Here, v is the linear speed of the particle

Differentiating again with respect to time, we have

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} \text{ or } a_t = r\alpha$$

Here, $a_t = \frac{dv}{dt}$ is the rate of change of speed (not the rate of change of velocity).

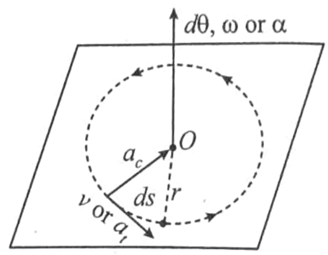
Relations Among Angular Variables

These relations are also referred as equations of rotational motion and are –

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0{}^2 + 2\alpha\theta$$



These are valid only if angular acceleration is constant and are analogous to equations of translatory motion, i.e.,

$$v = u + at; s = ut + \frac{1}{2}a^2 \text{ and } v^2 = u^2 + 2as$$

Train Your Brain

Example 8: A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5 m/s to 6 m/s in 2 s, find the angular acceleration.

Sol. The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\left(\because \text{ Here speed increases uniformly, } a_t = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}\right)$$

$$= \frac{6.0 - 5.0}{20} \text{ m/s}^2 = 0.5 \text{ m/s}^2$$

The angular acceleration is $\alpha = a_t / r$

$$= \frac{0.5 \text{ m/s}^2}{20 \text{ cm}} = 2.5 \text{rad/s}^2$$

Example 9: A particle moves in a circle of radius 20 cm. Its linear speed at any time is given by $v = 2t$ where v is in m/s and t is in seconds. Find the radial and tangential acceleration at $t = 3$ seconds and hence calculate the total acceleration at this time.

Sol. The linear speed at 3 seconds is

$$v = 2 \times 3 = 6 \text{ m/s}$$

The radial acceleration at 3 seconds

$$= \frac{v^2}{r} = \frac{6 \times 6}{0.2} = 180 \text{ m/s}^2$$

The tangential acceleration is given by

$\frac{dv}{dt} = 2$, because $v = 2t$.

$\therefore$ tangential acceleration is 2 m/s².

Net Acceleration

$$= \sqrt{a_r^2 + a_t^2} = \sqrt{(180)^2 + (2)^2} = 180.01 \text{ m/s}^2$$

Note: Is it possible for a car to move in a circular path in such a way that it has a tangential acceleration but no centripetal acceleration?

Example 10: A particle moves in a circle of radius 2 cm at a speed given by $v = 4t$, where v is in cm/s and t in seconds.

(i) Find the tangential acceleration at $t = 1$ s.

(ii) Find total acceleration at $t = 1$s.

Sol. (i) Tangential acceleration

$$a_t = \frac{dv}{dt} \text{ or } a_t = \frac{d}{dt}(4t) = 4 \text{ cm/s}^2$$

(ii) $$a_c = \frac{v^2}{R} = \frac{(4)^2}{2} = 8 \Rightarrow a = \sqrt{a_t^2 + a_c^2} = \sqrt{(4)^2 + (8)^2}$$

$$= 4\sqrt{5} \text{ cm/s}^2$$

Example 11: A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone files off horizontally and strikes the ground after traveling a horizontal distance of 10 cm. What is the magnitude of the centripetal acceleration of the stone while in circular motion?

Sol. $$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{9.8}} = 0.64 \text{ s}$$

$$v = \frac{10}{t} = 15.63 \text{ m/s}$$

$$a = \frac{v^2}{R} = 0.45 \text{ m/s}^2$$

Example 12: Find the magnitude of the acceleration of a particle moving in a circle of radius 10 cm with uniform speed completing the circle in 4 s.

Sol. The distance covered in completing the circle is $2\pi r = 2\pi \times 10$ cm. The linear speed is

$$v = \frac{2\pi r}{t} = \frac{2\pi \times 10 \text{ cm}}{4 \text{ s}} = 5\pi \text{ cm/s}.$$

The acceleration is $a_r = \frac{v^2}{r} = \frac{(5\pi)^2}{10} = 2.5\pi^2 \text{ cm/s}^2$

Example 13: A particle moves in a circle of radius 20 cm. Its linear speed is given by $v = 2t$ where t is in second and v in meter/second. Find the radial and tangential acceleration at $t = 3$ s.

Sol. The linear speed at $t = 3$s is

$v = 2t = 6$ m/s

The radial acceleration at $t = 3$s is

$$a_q = \frac{v^2}{r} = \frac{36 \text{ m}^2/\text{s}^2}{0.20 \text{ m}} = 180 \text{ m/s}^2$$

The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d(2t)}{dt} = 2 \text{ m/s}^2$$

Example 14: A particle is moving in a circular path with velocity varying with time as $v = 1.5t^2 + 2t$. If 2 cm the radius of circular path, the angular acceleration at $t = 2$ sec will be

(*a*) 4 rad/sec² (*b*) 40 rad/sec²

(*c*) 400 rad/sec² (*d*) 0.4 rad/sec²

Sol. (*c*) Given $v = 1.5\,t^2 + 2t$

Linear acceleration a

$$= \frac{dv}{dt} = 3t + 2$$

This is the linear acceleration at time t

Now angular acceleration at time t

$$\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{3t + 2}{2 \times 10^{-2}}$$

Angular acceleration at $t = 2$ sec

$$(\alpha)_{\text{at } t = 2\text{sec}} = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2 = 4 \times 10^2$$

$$= 400 \text{ rad/sec}^2$$

Hence correct answer is (*c*)

Example 15: The shaft of an electric motor starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$ during the first 2 seconds after it starts after which $\alpha = 0$. The angular velocity after 6 sec will be

(*a*) 10/3 rad/sec (*b*) 3/10 rad/sec

(*c*) 30/4 rad/sec (*d*) 4/30 rad/sec

Sol. (*a*) Given $\alpha = 3t - t^2$

$$\Rightarrow \frac{d\omega}{dt} = 3t - t^2$$

$$\Rightarrow d\omega = (3t - t^2)dt$$

$$\Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} + c$$

at $t = 0$, $\omega = 0$

$$\therefore c = 0, \omega = \frac{3t^2}{2} - \frac{t^3}{3}$$

Angular velocity at

$$t = 2 \text{ sec}, (\omega)_{t = 2 \text{ sec}} = \frac{3}{2}(4) - \frac{8}{3} = \frac{10}{3} \text{ rad/sec}$$

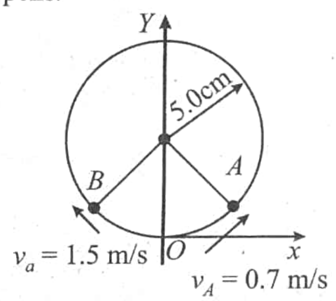
Since there is no angular acceleration after 2 sec

$\therefore$ The angular velocity after 6 sec remains the same.

Hence correct answer is (*a*)

Concept Application

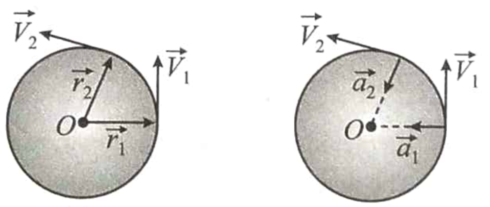
3. A grind stone starts from rest and has a constant-angular acceleration of 4.0 rad/sec^2. The angular displacement and angular velocity, after 4 sec. will respectively be
   (*a*) 32 rad, 16 rad/sec (*b*) 16rad, 32 rad/s
   (*c*) 64 rad, 32 rad/sec (*d*) 32 rad, 64rad/sec
4. A ball is fixed to the end of a string and is rotated in a horizontal circle of radius 5 m with a speed of 10 m/sec. The acceleration of the ball will be
   (*a*) 20 m/s^2 (*b*) 10 m/s^2
   (*c*) 30 m/s^2 (*d*) 40 m/s^2
5. A body of mass 2 kg lying on a smooth surface is attached to a string 3 m long and then whirled round in a horizontal circle making 60 revolution per minute. The centripetal acceleration will be
   (*a*) 118.4 m/s^2 (*b*) 1.18 m/s^2
   (*c*) 2.368 m/s^2 (*d*) 23.68 m/s^2
6. Two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds v_A = 0.7 m/s and v_B = 1.5 m/s, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



UNIFORM CIRCULAR MOTION

Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.

Uniform circular motion is an accelerated motion because in uniform circular motion the position of particle goes on changing and the position vector keeps on changing. As a result the velocity of the particle goes on changing it's direction.



(*a*) (*b*)

Speed of the particle remains constant.

The position vector and velocity vector goes on changing, though their magnitudes remain constant.

The acceleration of the particle is always towards center.

In case of uniform circular motion

- Position vector is always perpendicular to the velocity vector, i.e. $\vec{r}\cdot\vec{v} = 0$.
- Velocity vector is always perpendicular to the acceleration vector, i.e., $\vec{v}\cdot\vec{a} = 0$.
- The centripetal acceleration vector is directed opposite to the radius vector.

Non-uniform circular motion

If the speed of the particle moving in a circle is not constant, the acceleration has both the radial and the tangential components.

Examples of non uniform circular motion

A merry-go-round spinning up from rest to full speed, or a ball whirling around in a vertical circle.

In non-uniform circular motion

Speed $|\vec{v}| \neq$ constant

angular velocity $\omega \neq$ constant

If at any instant

$\Rightarrow$ v = magnitude of velocity of particle

$\Rightarrow$ r = radius of circular path

$\Rightarrow$ ω = angular velocity of a particle,

then, at that instant $v = r\omega$

DYNAMICS OF CIRCULAR MOTION

In circular motion or motion along any curved path Newton's law is applied in two perpendicular directions one along the tangent and other perpendicular to it. i.e., towards centre. The component of net force towards the centre is called centripetal force. The component of net force along the tangent is called tangential force.

tangential force $(F_t) = Ma_t = M\dfrac{dv}{dt} = M\alpha r$; where α is the angular acceleration

centripetal force $(F_c) = m\omega^2 r = \dfrac{mv^2}{r}$

Centripetal Force

Concepts: This is necessary resultant force towards the centre called the centripetal force.

$$F = \frac{mv^2}{r} = m\omega^2 r$$

(i) A body moving with constant speed in a circle is not in equilibrium.

(ii) It should be remembered that in the absence of the centripetal force the body will move in a straight line with constant speed.

(iii) It is not a new kind of force which acts on bodies. In fact, any force which is directed towards the centre may provide the necessary centripetal force.

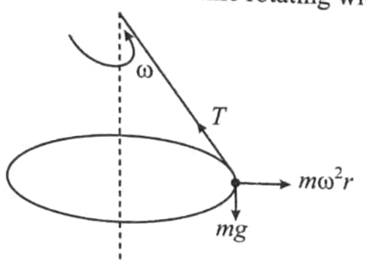
Centrifugal Force

When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer B, who is sharing the motion along the circular path (i.e., the observer B is also rotating with the body which is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.)

Its magnitude is equal to that of the centripetal force $= \dfrac{mv^2}{r}$.

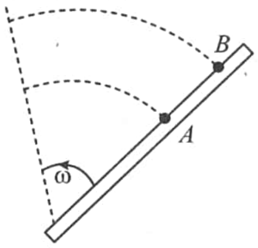
Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion in that frame)

FBD of ball w.r.t. it non inertial frame rotating with the ball.



Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyse the dynamics of a particle of mass m at a distance r from the axis of rotation, we have to assume that a force $mr\omega^2$ act radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

Note: A rod move with ω angular velocity then we conclude following for point A and B in a rod.

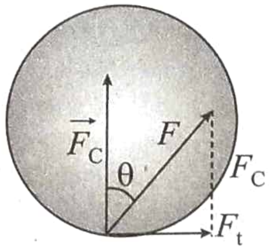


$\alpha_A = \alpha_B \quad s_B > s_A$

$\theta_A = \theta_B \quad v_B > v_A$

$\omega_A = \omega_B \quad a_{tB} > a_{tA}$

Net Force on the Particle in Non-Uniform Circular Motion



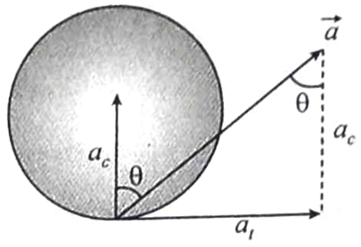
$$\vec{F} = \vec{F}_c + \vec{F}_t \Rightarrow F = \sqrt{F_c^2 + F_t^2}$$

If θ is the angle made by F with F_c

$$\text{then } \tan\theta = \frac{F_t}{F_c} \Rightarrow \theta = \tan^{-1}\left[\frac{F_t}{F_c}\right]$$

[angle between F_c and F_t is 90°]

Angle between F and F_t is $(90° - \theta)$



Net acceleration, $a = \sqrt{a_c^2 + a_t^2} = \dfrac{F_{net}}{m}$

The angle made by 'a' with a_c, $\tan\theta = \dfrac{a_t}{a_c} = \dfrac{F_t}{F_c}$

The direction of this resultant acceleration makes an angle θ with the radius.

$$\text{where } \tan\theta = \frac{\frac{dv}{dt}}{\frac{v^2}{r}}$$

Hint to Solve Numerical Problem

(i) Write down the required centripetal force

(ii) Draw the free body diagram of each component of system.

(iii) Resolve the forces acting on the rotating particle along radius and perpendicular to radius

(iv) Calculate net radial force acting towards centre of circular path.

(v) Make it equal to required centripetal force.

(vi) For remaining components see according to question.

Important Points in Uniform and non Uniform Circular Motion

- Since in both uniform and non-uniform circular motion F_c is perpendicular to velocity, so work done by centripetal force will be zero in both the cases.
- In uniform circular motion $F_t = 0$, as $a_t = 0$, so work done will be zero by tangential force.
- In non-uniform circular motion $F_t \neq 0$, so work done by tangential force is non zero.
- Rate of work done by net force in non-uniform circular motion = rate of work done by tangential force

$$\Rightarrow P = \frac{dW}{dt} = \vec{F}_t.\vec{v} = \vec{F}_t.\frac{d\vec{x}}{dt}$$

- In a circle as tangent and radius are always normal to each other, so $\vec{a}_t \perp \vec{a}_r$.

Net acceleration in case of circular motion

$a=\sqrt{a_r^2+a_t^2}$

Here it must be noted that a_t governs the magnitude of $\vec{v}$ while a_r its direction of motion, so that

$\Rightarrow$ if $a_r=0$ and $a_t=0$, $a\to 0$ motion is uniform translatory

$\Rightarrow$ if $a_r=0$ but $a_t\neq 0$, $a\to a_t$ motion is accelerated translatory

$\Rightarrow$ if $a_r\neq 0$ but $a_t=0$, $a\to a_r$ motion is uniform circular

$\Rightarrow$ if $a_r\neq 0$ and $a_t\neq 0$, $a\to\sqrt{a_r^2+a_t^2}$ motion is nonuniform circular

The centripetal force can be expressed as:

$$\vec{F}=m\vec{a}=-m\omega^2\vec{r}=-m\omega^2 r\hat{r}=-\left(\frac{mv^2}{r}\right)\hat{r}$$

(*a*) For earth's motion around the sun, the centripetal force is provided by the gravitational force.

(*b*) For electron's motion around the nucleus in hydrogen atom, centripetal force is provided by the electrostatic force.

- If the body on a circular path comes to rest, i.e., $\vec{v}\to 0$, the body will move along the radius towards the centre and if radial acceleration a_r vanishes, the body will fly off along the tangent. So a tangential velocity and a radial acceleration are must for uniform circular motion.
- As $F=\dfrac{mv^2}{r}\neq 0$, so the body is not in equilibrium and linear momentum of the particle moving on the circle is not conserved. However, as the force is central, i.e., $\tau=0$, so angular momentum is conserved.

Train Your Brain

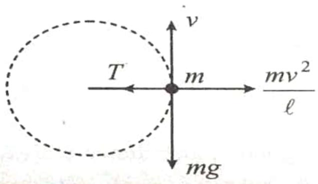
Example 16: A body of mass m is attached with a string of length ℓ. If it is whirled in a horizontal circular path with velocity v. The tension in the string will be

(*a*) mv^2 (*b*) $\dfrac{mv^2}{\ell}$

(*c*) $\dfrac{m\ell}{v^2}$ (*d*) $\dfrac{mv^2}{2\ell}$

Sol. (*b*) Required centripetal force,

$$F_c=\frac{mv^2}{\ell}$$



Here centripetal force is provided by the tension in the string

$$T=F_c=\frac{mv^2}{l}$$

Example 17: A satellite of mass m is revolving around the earth of mass M in circular orbit of radius r. The orbital velocity of the satellite will be

(*a*) $\sqrt{\dfrac{Gm}{r}}$ (*b*) $\sqrt{\dfrac{GM}{mr}}$

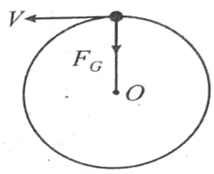
(*c*) $\sqrt{\dfrac{Gm}{Mr}}$ (*d*) $\dfrac{mv^2}{r}$

Sol. (*a*) The required centripetal force,

$F_G=\dfrac{GMm}{r^2}$ (towards the centre)

Net force towards the centre,

$$F_C=\frac{mv^2}{r}$$



(This force will provide required centripetal force)

Therefore $F_C=F_G$

$$\Rightarrow \frac{mv^2}{r}=\frac{GMm}{r^2}$$

$$\Rightarrow v=\sqrt{\frac{GM}{r}}$$

Hence correct answer is (*a*)

Note:

(i) From above example we see that orbital velocity of a body is independent to its mass

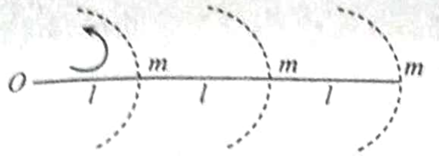
(ii) If we are asked to find out time period of above body then time period can be calculated as

$$\Rightarrow T=\frac{2\pi}{\omega}=\frac{2\pi r}{v}$$

$$T=\sqrt{\frac{r^3}{GM}}=2\pi\frac{mv^2}{3\ell}$$

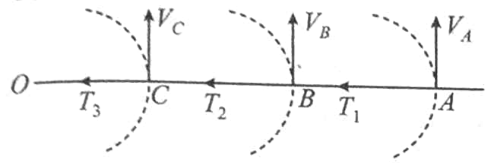
$T^2\propto r^3$ this is **Kepler's law.**

Example 18: Three identical particles are connected by three strings as shown in figure. These particles are revolving in a horizontal plane. The velocity of outer most particle is v. Then $T_1 : T_2 : T_3$ will be (Where T_1 is tension in the outer most string etc.)



(*a*) 3 : 5 : 7
(*b*) 3 : 5 : 6
(*c*) 3 : 4 : 5
(*d*) 7 : 5 : 3

Sol. (*b*) For *A*:



Required centripetal force $= \dfrac{mv_A^2}{3l} = m\omega^2(3l)$

(net force towards centre $= T_1$)

For *B*:

$T_2 - T_1 = m\omega^2(2l)$

$\Rightarrow T_2 = m\omega^2(3l + 2l) = 5\omega^2 l$

For *C*:

$T_3 - T_2 = m\omega^2 l$

$\Rightarrow T_3 = 6\omega^2 l$

Remember ω i.e. angular velocity, of all the particles is same

$\therefore \omega = \dfrac{v_A}{3l} = \dfrac{v_B}{2l} = \dfrac{v_C}{l}$

Example 19: A small block of mass 100 g moves with uniform speed in a horizontal circular groove, with vertical side walls, of radius 25 cm. If the block takes 2.0 s to complete one round, find the normal contact force by the slide wall of the groove.

Sol. The speed of the block is

$$v = \frac{2\pi \times (25\text{ cm})}{2.0\text{ s}} = 0.785\text{ m/s}$$

The acceleration of the block is

$$a = \frac{v^2}{r} = \frac{(0.785\,\text{m/s})^2}{0.25} = 2.46\,\text{m/s}^2$$

Towards the center. The only force in this direction is the normal contact force due to the side walls. Thus from Newton's second law, this force is

$$N = ma = (0.100\,\text{kg})(2.46\,\text{m/s}^2) = 0.246\,\text{N}$$

Example 20: A hemispherical bowl of radius *R* is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

Sol. Let ω be the angular speed of rotation of the bowl.

Two force are acting on the ball.

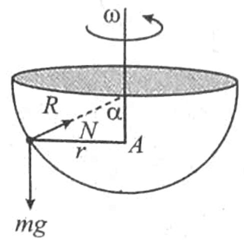
1. normal reaction *N*, 2. weight *mg*

The ball is rotating in a circle of radius r ($= R\sin\alpha$) with centre at *A* at an angular speed ω. Thus,

$N\sin\alpha = m\omega^2 r = mR\omega^2 \sin\alpha$

$N = mR\omega^2$ ...(i)

and $N\cos\alpha = mg$ ...(ii)



Dividing Eqs. (i) by (ii),

we get, $\dfrac{1}{\cos\alpha} = \dfrac{\omega^2 R}{g}$, $\therefore\ \omega = \sqrt{\dfrac{g}{R\cos\alpha}}$

Example 21: If friction is present between the surface of ball and bowl then find out the range of ω for which ball does not slip (μ is the friction coefficient)

Friction develop a range of ω for which the particle will be at rest.

Sol. When $\omega > \omega_0$

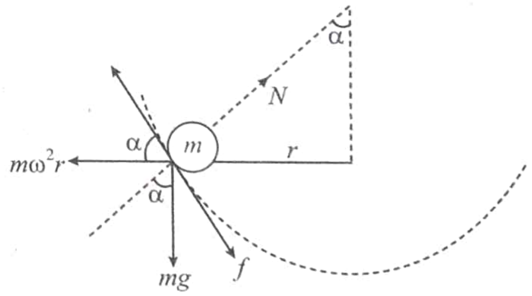
In this situation ball has a tendency to slip upwards so the friction force will act downwards. So F.B.D of ball

$N = m\omega^2 r\sin\alpha + mg\cos\alpha$

$f + mg\sin\theta = m\omega^2 r\cos\theta$ ...(ii)

$\therefore f_{\max} = \mu N = \mu(m\omega^2 r\sin\alpha + mg\cos\alpha)$

$r = R\sin\alpha$



Substituting the values of $f_{\max}$ and *r* in eq. (ii) we get

$\Rightarrow \mu\left(m\omega^2 r\sin\alpha + mg\cos\alpha\right) \geq m\omega^2 r\cos\alpha - mg\sin\alpha$

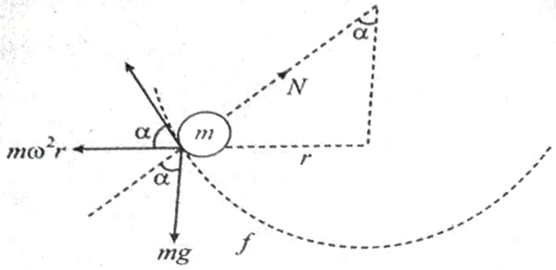
$\therefore \mu\left(m\omega^2 R\sin 2\alpha + mg\cos\alpha\right) \geq m\omega^2 R\sin\alpha\cos\alpha$

$- mg\sin\alpha$

$$\omega \leq \sqrt{\frac{\mu g\cos\alpha + g\sin\alpha}{R\sin\alpha(\cos\alpha - \mu\sin\alpha)}}$$

when $\omega < \omega_0$

In this situation ball has a tendency to slip downwards so the friction force will act upwards. So F.B.D of ball



$$\Rightarrow f + m\omega^2 r\cos\alpha = mg\sin\alpha$$

$$f = m\left(g\sin\alpha - \omega^2 r\cos\alpha\right) \qquad ...(i)$$

$$N = mg\cos\alpha + m\omega^2 r\sin\alpha \qquad ...(ii)$$

$$f_{\max} = \mu N = \mu\left(mg\cos\alpha + m\omega^2 r\sin\alpha\right)$$

for equilibrium

$$\Rightarrow \mu\left(g\cos\alpha + \omega^2 r\sin\alpha\right) \geq \left(g\sin\alpha - \omega^2 r\cos\alpha\right)$$

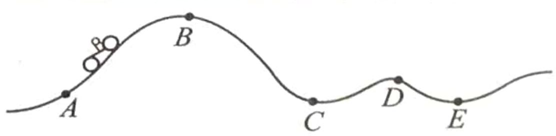
$\Rightarrow$ Substituting $r = R\sin\alpha$ then

$$\omega \geq \sqrt{\frac{g\left(\sin\alpha - \mu\cos\alpha\right)}{R\sin\alpha\left(\mu\sin\alpha + \cos\alpha\right)}}$$

Concept Application

7. A body of mass 0.1 kg is moving on circular path of diameter 1.0 m at the rate of 10 revolutions per 31.4 seconds. The centripetal force acting on the body is
   (*a*) 0.2 N (*b*) 0.4 N
   (*c*) 2 N (*d*) 4 N
8. A body of mass 4 kg is moving in a horizontal circle of radius 1 m with an angular velocity of 2 rad/s. The required centripetal force, will be
   (*a*) 16 N (*b*) 1.6 N
   (*c*) 16 Dyne (*d*) 1.6 Dyne

MOTION OF A MOTORCYCLIST ON A CURVED PATH



A cyclist having mass m move with constant speed v on a curved path as shown in figure.

We divide the motion of cyclist in four parts :

1. from A to B 2. from B to C
3. from C to D 4. from D to E

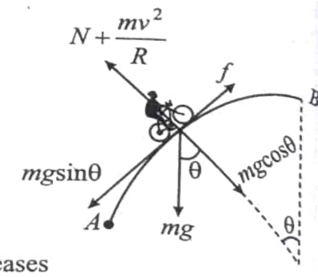
1 and 3 are same type of motion)

(A) Motion of Cyclist from A to B

$$N + \frac{mv^2}{R} = mg\cos\theta$$

$$\Rightarrow N = mg\cos\theta - \frac{mv^2}{R}$$

$f = mg\sin\theta$



(1) As cyclist move upward

$\because$ θ decreases and $\cos\theta$ increases

$\therefore$ N increases

And

$\because$ θ decreases $\sin\theta$ decreases

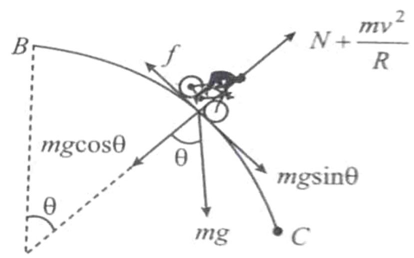
$\therefore$ friction force required to balance $mg\sin\theta$ (As cyclist is moving with constant speed) also decreases

(B) Motion of Cyclist from B to C

$$N + \frac{mv^2}{R} = mg\cos\theta$$

$$= N = mg\cos\theta - \frac{mv^2}{R}$$

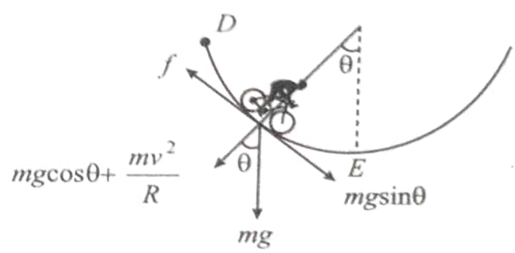
$f = mg\sin\theta$



Therefore from B to C Normal force decrease but friction force increase because θ increases.

(C) Motion of Cyclist from D to E

$$N = \frac{mv^2}{R} + mg\cos\theta, \quad f = mg\sin\theta$$



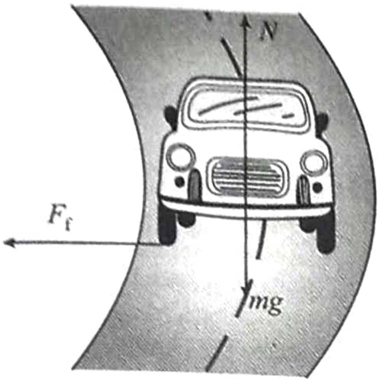
from D to E θ decreases

therefore mg cos θ increase

So N increase but f decreases

A CAR TAKING A TURN ON A LEVEL ROAD

When a car takes a turn on a level road, the portion of the turn can be approximated by an arc of a circle of radius r. If the car makes the turn at a constant speed v, then there must be some centripetal force acting on the car. This force is generated by the friction between the tyres and the road.



The maximum frictional force is $F_f = \mu_s N$, μ_s = the coefficient of static friction.

Then the maximum safe velocity v is given by

$$\frac{mv^2}{r} = \mu_s N = \mu_s mg \ (\because N = mg) \Rightarrow \boxed{v = \sqrt{\mu_s rg}} \quad \text{or} \quad \mu_s = \frac{v^2}{rg}$$

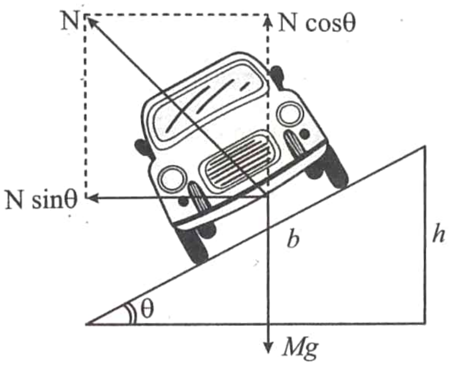
It is important to note that safe velocity is independent of the mass of the car.

Circular Turning by Banking of Tracks Only

Roads on large highways are generally banked so that a vehicle may make a safe and easier turn without depending on friction, road bend at the curved path is raised a little on the side away from the centre of the curved path.

Due to banking of the road, a component of the normal force points towards the centre of curvature of the road.

Necessary centripetal force required for circular motion is supplied by this component. The vertical component of the normal force is balanced by the weight of the vehicle.



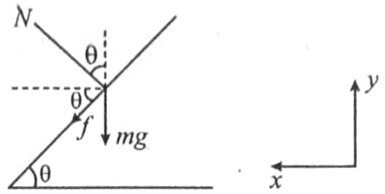
$$N \cos\theta = mg \text{ and } N \sin\theta = \frac{mv^2}{r}$$

$$\therefore \boxed{\tan\theta = \frac{v^2}{rg}} \Rightarrow v = \sqrt{rg\tan\theta}$$

For a road with angle of banking θ, the speed v at which minimum wear and tear of tyre takes place is $\boxed{v = \sqrt{rg\tan\theta}}$

Circular Turning by Banking of Tracks and Friction Both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these force, the first force, i.e., weight (mg) is fixed both in magnitude and direction.



The direction of second force i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center of these m and r are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that:

(i) Friction f will be outwards if the vehicle is at rest $v = 0$. Because in that case the component weight $mg \sin\theta$ is balanced by f.

(ii) Friction f will be inwards if $v > \sqrt{rg\tan\theta}$

(iii) Friction f will be outwards if $v < \sqrt{rg\tan\theta}$ and

(iv) Friction f will be zero if $v = \sqrt{rg\tan\theta}$

(v) For maximum safe speed (figure (ii))

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \quad ...(i)$$

$$N\cos\theta - f\sin\theta = mg \quad ...(ii)$$

As maximum value of friction

$f = \mu N$

$$\therefore \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg} \quad \therefore \boxed{v_{max} = \sqrt{\frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}}$$

$$\text{Similarty;} \boxed{v_{min} = \sqrt{\frac{rg(\mu - \tan\theta)}{(1 + \mu\tan\theta)}}}$$

+ The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle of banking for an aircraft, i,e., the angle through which it should tilt while negotiating a curve, to avoid deviation from the circular path.

- The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle at which a cyclist should lean inward, when rounding a corner: In this case, θ is the angle which the cyclist must make with the vertical to negotiate a safe turn.

Train Your Brain

Example 22: A railway track is banked for a speed v, by making the height of the outer rail (h) higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r

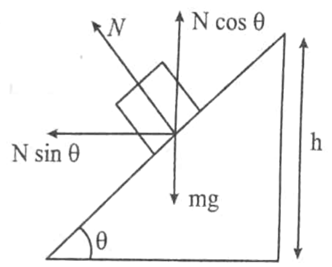
(a) $\frac{h}{d} = \frac{v^2}{rg}$

(b) $\tan\left(\sin^{-1}\frac{h}{d}\right) = \frac{v^2}{rg}$

(c) $\tan^{-1}\left(\frac{h}{d}\right) = \frac{v^2}{rg}$

(d) $\frac{h}{r} = \frac{v^2}{dg}$

Sol. (b)



$\sin\theta = \frac{h}{d}$, $\quad N\cos\theta = mg$ ...(i)

$N\sin\theta = \frac{mv^2}{r}$ ...(ii)

$\therefore \frac{(2)}{(1)} \Rightarrow \tan\theta = \frac{v^2}{rg}$

$\Rightarrow \tan\left[\sin^{-1}\frac{h}{d}\right] = \frac{v^2}{rg}$

Concept Application

9. A car moving on a horizontal road may be thrown out of the road in taking a turn :

(a) By the gravitational force.

(b) Due to lack of sufficient centripetal force.

(c) Due to friction between road and the tyre.

(d) Due to reaction of earth.

10. A curved section of a road is banked for a speed v. If there is no friction between road and tyres of the car, then:

(a) Car is more likely to slip at speeds higher than v than speeds lower than v

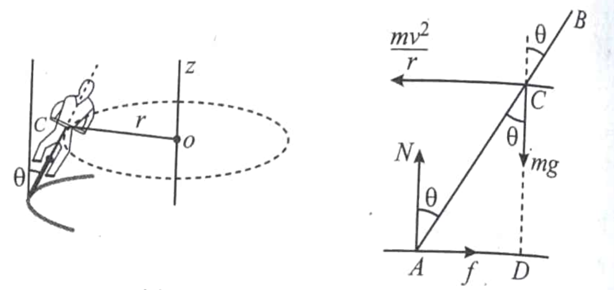
(b) Car cannot remain in static equilibrium on the curved section

(c) Car will not slip when moving with speed v

(d) None of the above

TURNING OF A CYCLIST AROUND A CORNER ON THE ROAD

Suppose a cyclist is going at a speed v on a circular horizontal road of radius r which is not banked. Consider the cycle and the rider together as the system. The centre of mass C (figure shown) of the system is going in a circle with the centre at O and radius r.



(a) (b)

Let us choose O as the origin, OC as the x-axis and vertically upward as the z-axis. This frame is rotating at an angular speed $\omega = \frac{v}{r}$ about the z-axis. In this frame the system is at rest. Since we are working from a rotating frame of reference, we will have to apply a centrifugal force on each $\omega^2 r = mv^2/r$, where m is the total mass of the system. This force will act through the centre of mass. Since the system is at rest in this frame, no other pseudo force is needed.

In the frame considered, the system is at rest. Thus, the total external force and the total external torque must be zero. Let us consider the torques of all the forces about the point A. The torques of N and f about A are zero because these forces pass through A. The torque of mg about A is $mg(AD)$ in the clockwise direction and that of $\frac{mv^2}{r}$ is $\frac{mv^2}{r}(CD)$ in the anticlockwise direction. For rotational equilibrium,

$$mg(AD) = \frac{mv^2}{r}(CD)$$

or, $\frac{AD}{CD} = \frac{v^2}{rg}$

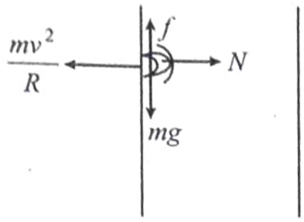
So, $\tan\theta = \frac{v^2}{rg}$, i.e.,

Angle θ should be: $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Death Well

A motor cyclist is driving in a horizontal circle on the inner surface of vertical cylinder of radius R. Friction coefficient between tyres of motorcyclist and surface of cylinder is μ. Find out the minimum velocity for which the motorcyclist can do this. v is the speed of motor cyclist and m is his mass.

$$N = \frac{mv^2}{R}$$



$$f = mg$$

$$f_{max} = \frac{\mu mv^2}{R}$$

Cyclist does not drop down when

$$f_{max} \geq mg \Rightarrow \frac{\mu mv^2}{R} \geq mg$$

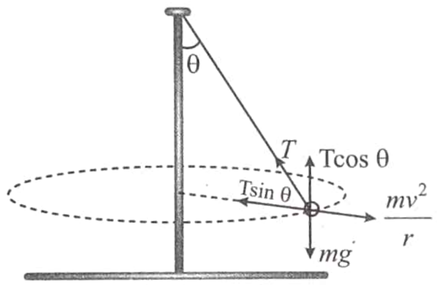
$$v \geq \sqrt{\frac{gR}{\mu}}$$

CONICAL PENDULUM

(i) If a small bob of mass m tied to a string is whirled in a horizontal circle, the string will not remain horizontal but the string becomes inclined to the vertical and sweeps a cone while the body moves on a horizontal circle with uniform speed. This arrangement is called **conical pendulum.**

(ii) In case of conical pendulum, the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force, i.e.,

$$T\cos\theta = mg \text{ and } T\sin\theta = \frac{mv^2}{r} \quad \text{or} \quad \tan\theta = \frac{v^2}{rg}$$



Also $T = m\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$

Hence $v = \sqrt{rg\tan\theta}$

i.e., $\omega = \sqrt{\frac{g\tan\theta}{r}}$

Hence, time period

$$t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

(iii) Time period t is independent of the mass of the body and depends on $L\cos\theta\ (= h)$

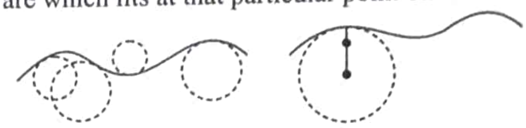
i.e., distance between point of suspension and centre of circle.

(iv) If $\theta = 90°$, the pendulum becomes horizontal and it follows that $v = \infty$, $T = \infty$ and $t = 0$

It is practically impossible.

RADIUS OF CURVATURE

Any curved path can be assume to be made of infinite circular arcs. Radius of curvature at a point is defined as the radius of that circular arc which fits at that particular point on the curve.



(a) (b)

We know that $a_c = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_C}$. This is the expression for radius of curvature.

For finding the radius of curvature first take the component of net acceleration perpendicular to the velocity and equate it to

$$R = \frac{v^2}{a_\perp} = \frac{mv^2}{F_\perp}$$

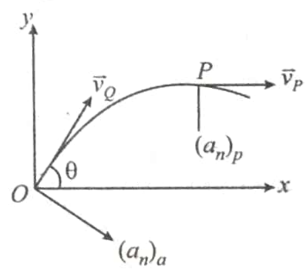
If a particle moves in a trajectory given by $y = f(x)$ then radius of curvature at any point (x, y) of the trajectory is given by

$$\Rightarrow R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Train Your Brain

Example 23: Find the ratio of the radius of curvature at the highest point of projectile to that just after its projection if the angle of projection is 30°?

Sol. If $\vec{v}_0$ is the initial velocity, $v_p = V_0\cos\theta$. Normal acceleration at $O = g\cos\theta$. Hence, if r_0 and r_p are radii of curvature at O and P, respectively. $r_0 = \frac{v_0^2}{g\cos\theta}$ and $r_P = \frac{v_0^2\cos^2\theta}{g}$



Hence, the required ratio $= \frac{r_P}{r_0} = \cos^3\theta = \frac{3\sqrt{3}}{8}$

Example 24: A stone is projected with speed u and angle of projection is θ. Find radius of curvature at $t = 0$.

(*a*) $\dfrac{u^2\cos^2\theta}{g}$ (*b*) $\dfrac{u^2}{g\sin\theta}$

(*c*) $\dfrac{u^2}{g\cos\theta}$ (*d*) $\dfrac{u^2\sin^2\theta}{g}$

Sol. (*c*) At $t = 0$

$a_\perp = g\cos\theta,$

$R = \dfrac{v^2}{a_\perp} = \dfrac{u^2}{g\cos\theta}$

Concept Application

11. A ball is projected making an angle θ with the vertical. Consider a small part of the trajectory near the highest position and take it approximately to be a circular arc. What is the radius of this circle? This radius is called the radius of curvature of the curve at the point.

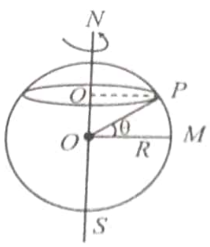
12. A particle of mass m is moving with constant velocity $\vec{v}$ on smooth horizontal surface. A constant force $\vec{F}$ starts acting on particle perpendicular to velocity v. Radius of curvature after force F start acting is:

(*a*) $\dfrac{mv^2}{F}$ (*b*) $\dfrac{mv^2}{F\cos\theta}$

(*c*) $\dfrac{mv^2}{F\sin\theta}$ (*d*) None of these

EFFECT OF EARTH'S ROTATION ON APPARENT WEIGHT

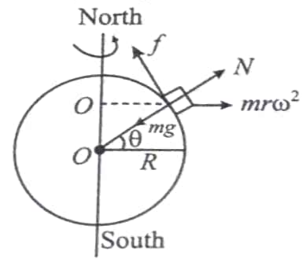
The earth rotates about its axis at an angular speed of one revolution per 24 hours. The line joining the north and the south poles is the axis of rotation.



Every point on the earth moves in a circle. A point at equator moves in a circle of radius equal to the radius of the earth and the centre of the circle is same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this. Consider a place point on the earth (figure.)

Drop a perpendicular PC from P to the axis SN. The place P rotates in a circle with the centre at C. The radius of this circle is CP. The angle between the line OM and the radius OP through P is called the latitude of the place point. We have $CP = OP\cos\theta$ or $r = R\cos\theta$ where R is the radius of the earth.

If we calculate work from the frame of reference of the earth, we shall have to assume the existence of pseudo force. In Particular; a centrifugal force $m\omega^2 r$ has to be assumed on any particle of mass m placed at P. If we consider a block of mass m at point P then this block is at rest with respect to earth. If resolve the forces along and perpendicular to the line joining the centre of earth then

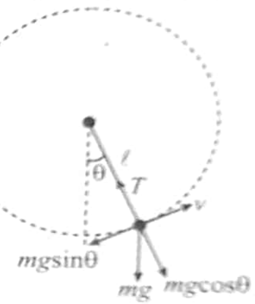


$$N + m\omega^2\cos\theta = mg \Rightarrow N = mg - mr\omega^2\cos\theta$$

$$\Rightarrow N = mg - mR\omega^2\cos^2\theta$$

MOTION IN A VERTICAL CIRCLE

Let us consider the motion of a point mass tied to a string of length ℓ and whirled in a vertical circle. If at any time the body is at angular position θ, as shown in the figure, the forces acting on it are tension T in the string along the radius towards the center and the weight of the body mg acting vertically down wards.



Applying Newton's law along radial direction

$$T - mg\cos\theta = m.a_r = \frac{mv^2}{\ell}$$

$$\text{or } T = \frac{mv^2}{\ell} + mg\cos\theta \qquad \text{...(i)}$$

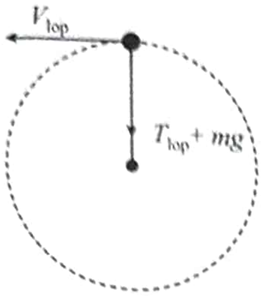
The point mass will complete the circle only and only if tension is never zero (except momentarily, if at all) if tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be similar to that of a projectile.

From equation (i), it is evident that tension decreases with increase in θ because $\cos\theta$ is a decreasing function and v decreases with height. Hence tension is minimum at the top most point. i.e. $T_{min} = T_{topmost}$.

$T > 0$ at all points. $\Rightarrow T_{min} > 0$.

However if tension is momentarily zero at highest point the body would still be able to complete the circle.

Hence condition for completing the circle (or looping the loop) is $T_{min} \geq 0$ or $T_{top} \geq 0$.



$$T_{top} + mg = \frac{mv_{top}^2}{\ell} \quad \text{...(ii)}$$

Equation (ii) could also be obtained by putting $\theta = \pi$ in equation (i).

For looping the loop, $T_{top} \geq 0$.

$$\Rightarrow \frac{mv_{top}^2}{\ell} \geq mg \Rightarrow v_{top} \geq \sqrt{g\ell} \quad \text{...(iii)}$$

Condition for looping the loop is $v_{top} \geq \sqrt{g\ell}$.

If speed at the lowest point is u, then from conservation of mechanical energy between lowest point and top most point.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_{top}^2 + mg.2\ell$$

using equation (iii) for v_{top} we get

$$u \geq \sqrt{5g\ell}$$

i.e., for looping the loop, velocity at lowest point must be $\geq \sqrt{5g\ell}$.

If velocity at lowest point is just enough for looping the loop, value of various quantities. (True for a point mass attached to a string or a mass moving on a smooth vertical circular track.)

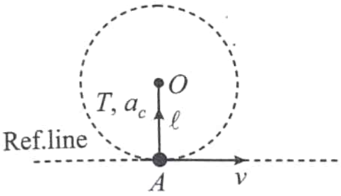


Fig. (1)

P.E. = 0

$v = \sqrt{5g\ell}$

$T - mg = \frac{mv^2}{\ell}$

could also be obtained by putting $\theta = 0$ in equation ...(i)

$\Rightarrow T = 6mg$

$a_c = 5g, a_t = 0$

$a_{net} = 5g$

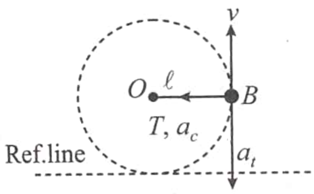


Fig. (2)

P.E. $= mg\ell$

By energy conservation,

$v = \sqrt{3g\ell}$

$T = \frac{mv^2}{\ell}$

could also be obtained by putting $\theta = 90°$ in equation ...(i)

$\Rightarrow T = 3mg$

$a_c = 3g, a_t = g$

$a_{net} = g\sqrt{10}$

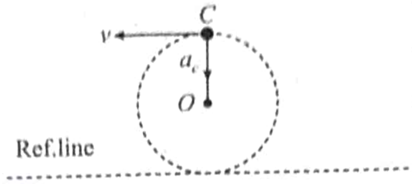


Fig. (3)

P.E. $= 2mg\ell$

By energy conservation,

$v = \sqrt{g\ell}$

$T = 0$

$a_c = g, a_t = 0$

$a_{net} = g$

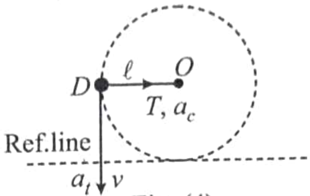


Fig. (4)

P.E. $= mg\ell$

By energy conservation,

$v = \sqrt{3g\ell}$

$T = 3mg$

$a_c = 3g, a_t = g$

$a_{net} = g$

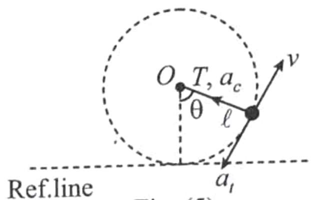


Fig. (5)

P.E. $= mg\ell\ (1 - \cos\theta)$

By energy conservation,

$v = \sqrt{g\ell(3 + 2\cos\theta)}$

$T = 3mg\ (1 + \cos\theta)$

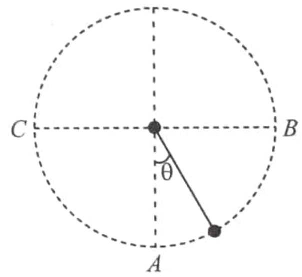
$a_c = g\ (3 + 2\cos\theta), a_t = g\sin\theta$

	A	B, D	C	P(general point)
Velocity	$\sqrt{5g\ell}$	$\sqrt{3g\ell}$	$\sqrt{g\ell}$	$\sqrt{g\ell(3+2\cos\theta)}$
Tension	$6mg$	$3mg$	0	$3mg(1+\cos\theta)$
Potential Energy	0	$mg\ell$	$2mg\ell$	$mg\ell(1-\cos\theta)$
Radial acceleration	$5g$	$3g$	g	$g(3+2\cos\theta)$
Tangential acceleration	0	g	0	$g\sin\theta$

Note: From above table we can see, $T_{bottom} - T_{top} = T_C - T_A = 6\,mg$, this difference in tension remain same even if $V > \sqrt{5g\ell}$

Condition for Oscillation or Leaving the Circle

In case of non uniform circular motion in a vertical plane if velocity of body at lowest point is lesser than $\sqrt{5g\ell}$, the particle will not complete the circle in vertical plane. In this case, the motion of the point mass which depend on 'whether tension becomes zero before speed becomes zero or vice versa.



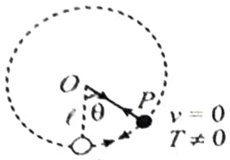
Case-I: (Speed becomes zero before tension)

In this case the ball never rises above the level of the center O i.e. the body is confined to move within C and B, ($|\theta| < 90°$) for this the speed at A, $v < \sqrt{2g\ell}$ (as proved in above example)

In this case tension cannot be zero, since a component of gravity acts radially outwards.

Hence the string will not go slack, and the ball will reverse back as soon as its speed becomes zero.

Its motion will be oscillatory motion.



(A)

For oscillation

$0 < v_\ell < \sqrt{2gl}$

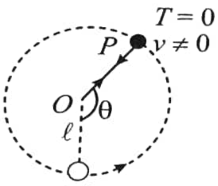
$0 < \theta < 90°$

Case-II: (Tension becomes zero before speed)

In this case the ball rises above the level of center O i.e. it goes beyond point $B(\theta > 90°)$ for this $v > \sqrt{2g\ell}$ (as proved in above example)

In this case a component of gravity will always act towards center, hence centripetal acceleration or speed will remain nonzero. Hence tension becomes zero first.

As soon as, Tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be similar to that of a projectile. In this case motion is a combination of circular and projectile motion.



(B)

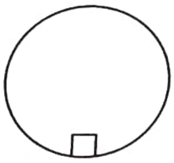
For leaving the circular path after which motion converts into projectile motion

$\sqrt{2g\ell} < v_\ell < \sqrt{5gl}$

$90° < \theta < 180°$

CONDITION FOR LOOPING THE LOOP IN SOME OTHER CASES

Case-I: A mass moving on a smooth vertical circular track.


(B)

Mass moving along a smooth vertical circular loop condition for just looping the loop, normal at highest point = 0.

By calculation similar to motion vertical circle.

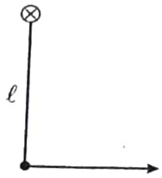
Minimum horizontal velocity at lowest point = $\sqrt{5g\ell}$

Case-II: A particle attached to a light rod rotated in vertical circle.

Condition for just looping the loop, velocity $v = 0$ at highest point (even if tension is zero, rod won't slack, and a compressive force can appear in the rod).

By energy conservation,

velocity at lowest point = $\sqrt{4g\ell}$



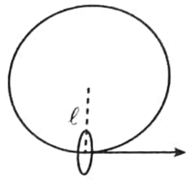
$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

Case-III: A bead attached to a ring and rotated.

Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).

By energy conservation,

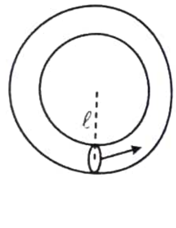
velocity at lowest point = $\sqrt{4g\ell}$



$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

Case-IV: A block rotated between smooth surfaces of a pipe.

Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).



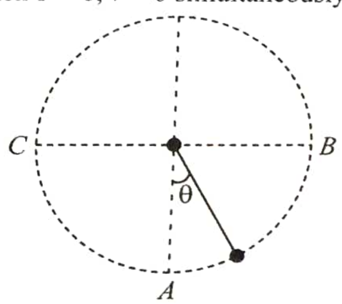
By energy conservation,

velocity at lowest point = $\sqrt{4g\ell}$

$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

Train Your Brain

Example 25: Find minimum speed at A so that the ball can reach at point B as shown in figure. Also discuss the motion of particle when $T = 0$, $v = 0$ simultaneously at $\theta = 90°$.



Sol. From energy conservation

$$\frac{1}{2}mv_A^2 + 0 = 0 + mg\ell \quad \text{(for minimum speed } v_B = 0)$$

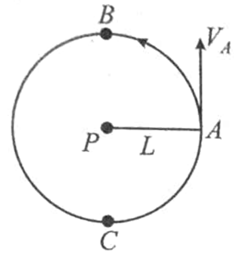
$$v_{min} = \sqrt{2g\ell}$$

at the position B, $v = 0$ and $T = 0$ (putting $v_B = 0$ or $\theta = 90°$, in equation ...(i))

ball will return back, motion is oscillatory

Example 26: If a particle of mass M is tied to a light inextensible string fixed at point P and particle is projected at A with velocity $V_A = \sqrt{4gL}$ as shown. Find:

(i) velocity at points B and C



(ii) tension in the string at B and C

Assume particle is projected in the vertical plane.

Sol. $V_B = \sqrt{2gL}$ (from energy conservation)

$V_C = \sqrt{6gL}$

$T_B = Mg$

$T_C = 7$ Mg (where $M \Rightarrow$ Mass of the particle)

Example 27: A body weighing 0.4 kg is whirled in a vertical circle with a string making 2 revolutions per second. If the radius of the circle is 1.2 m. Find the tension (i) at the top of the circle, (ii) at the bottom of the circle.

Given: $g = 10$ m s^{-2} and $\pi = 3.14$.

Sol. Mass, $m = 0.4$ kg;

time period $= \frac{1}{2}$ second, radius, r = 1.2 m

Angular velocity, $\omega = \frac{2\pi}{1/2} = 4\pi$ rad s^{-1}

$= 12.56$ rad s^{-1}.

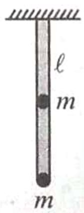
(i) At the top of the circle, $T = \frac{mv^2}{r} - mg = mr\omega^2$

$- mg = m(r\omega^2 - g)$

$= 0.4\,(1.2 \times 12.56 \times 12.56 - 9.8)\ N = 71.2$ N

(ii) At the lowest point, $T = m(r\omega^2 + g) = 80$ N

Example 28: Two point mass m are connected the light rod of length ℓ and it is free to rotate in vertical plane as shown. Calculate the minimum horizontal velocity is given to mass so that it completes the circular motion in vertical lane.



Sol. Here tension in the rod at the top most point of circle can be zero or negative for completing the loop. So velocity at the top most point is zero.

From energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}m\frac{v^2}{4} = mg(2\ell) + mg(4\ell) + 0$$

$$\Rightarrow \quad v = \sqrt{\frac{48g\ell}{5}}$$

Example 29: You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so that the cyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?

Sol. When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also act downwards.

$F_{net} = ma_c$

$$\therefore\ R + mg = \frac{mv^2}{r}$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (i) when $R = 0$.

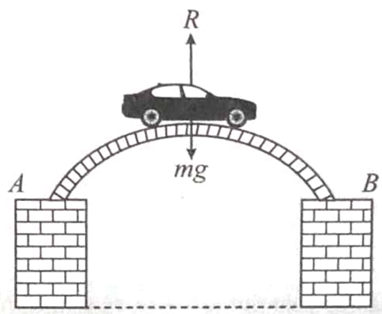
$$\therefore\ mg = \frac{mv_{min}^2}{r} \text{ or } v^2_{min} = gr$$

or $v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25}$ ms^{-1} = 15.65 ms^{-1}.

So, the minimum speed at the top, required to perform a vertical loop is 15.65 ms^{-1}.

Example 30: Prove that a motor car moving over a convex bridge is lighter than the same car resting on the same bridge.

Sol. The motion of the motor car over a convex bridge AB is the motion along the segment of a circle AB (Figure);



The centripetal force is provided by the difference of weight mg of the car and the normal reaction R of the bridge.

$$\therefore\ mg - R = \frac{mv^2}{r} \text{ or } R = mg - \frac{mv^2}{r}$$

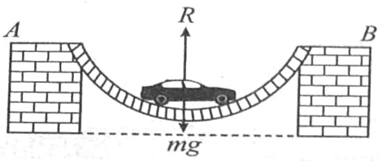
Clearly $R <$ mg, i.e., the weight of the moving car is less

than the weight of the stationary car.

Example 31: Prove that a motor car moving over a concave bridge is heavier than the same car resting on the same bridge.

Sol. The motion of the motor car over a concave bridge AB is the motion along the segment of a circle AB (Figure);

The centripetal force is provided by the difference of normal reaction R of



The bridge and weight mg of the car.

$$\therefore\ R - mg = \frac{mv^2}{r} \quad \text{or} \quad R = mg + \frac{mv^2}{r}$$

Clearly $R > mg$, i.e., the weight of the moving car is greater than the weight of the stationary car.

Concept Application

13. A weightless thread can support tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane. If $g = 10\ m/s^2$, find the maximum angular velocity of the stone.

14. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum angular displacement of the pendulum of the string with respect to the downward vertical.

Short Notes

Circular Motion

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

Radius Vector

The vector joining the centre of the circle and the centre of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction. It is directed outwards.

Frequency (n or f)

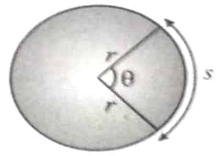
Number of revolutions described by particle per sec. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.).

Time Period (T)

It is time taken by particle to complete one revolution.

$$T = \frac{1}{n}$$

- Angle $\theta = \dfrac{\text{arc length}}{\text{radius}} = \dfrac{s}{r}$ (Unit → radian)
- Average angular velocity $\omega = \dfrac{\Delta\theta}{\Delta t}$ (a scalar) unit → rad/sec
- Instantaneous angular velocity $\omega = \dfrac{d\theta}{dt}$ (a vector) unit → rad/sec



- For uniform angular velocity $\omega = \dfrac{2\pi}{T} = 2\pi f$ or $2\pi n$
- Angular displacement $\theta = \omega t$
- Relation between ω (uniform) and v, $\omega = \dfrac{v}{r}$
- In vector from velocity $\vec{v} = \vec{\omega} \times \vec{r}$
- Acceleration $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(\vec{\omega} \times \vec{r}) = \dfrac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \dfrac{d\vec{r}}{dt}$

$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a}_t + \vec{a}_C$$

- Tangential acceleration: $a_t = \dfrac{dv}{dt} = \alpha r$

- Centripetal acceleration: $a_C = \omega v = \dfrac{v^2}{r}$

$$= \omega^2 r \quad \text{or} \quad \vec{a}_C = \omega^2 r(\hat{r})$$

- Magnitude of net acceleration:

$$a = \sqrt{a_C^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

Maximum and Minimum Speed in Circular Motion

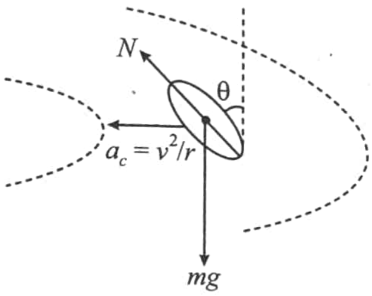
- On unbanked road: $v_{\max} = \sqrt{\mu_s Rg}$

- On banked road: $v_{\max} = \sqrt{\left(\dfrac{\mu_s + \tan\theta}{1-\mu_s \tan\theta}\right)Rg}$

$$v_{\min} = \sqrt{\frac{(\tan\theta - \mu_S)Rg}{1+\mu_S \tan\theta}} \qquad v_{\min} \le v_{car} \le v_{\max}$$

where ϕ = angle of friction = $\tan^{-1}\mu_s$; θ = angle of banking.

- Bending of cyclist: $\tan\theta = \dfrac{v^2}{rg}$



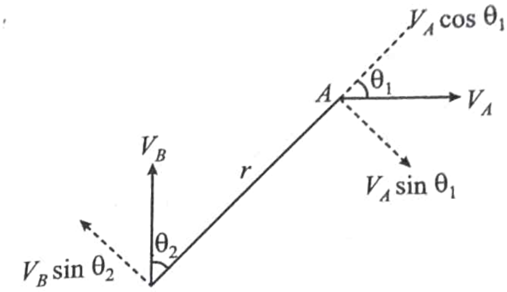
Key Tips

- Average angular velocity is a scalar physical quantity whereas instantaneous angular velocity is a vector physical quantity.
- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \quad \text{But } \vec{\theta}_1 + \vec{\theta}_2 \ne \vec{\theta}_2 + \vec{\theta}_1$$

Relative Angular Velocity

Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B.



That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$$

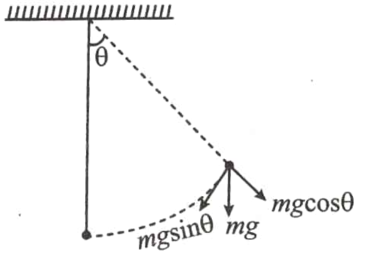
$$= \frac{\text{Relative velocity of } A \text{ w.r.t. } B \text{ perpendicular to line } AB}{\text{separation between } A \text{ and } B}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$

$$\therefore \qquad \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

Solved Examples

1. A simple pendulum is constructed by attaching a bob of mass m to a string of length L fixed at its upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is v when the string makes an angle θ with the vertical. Find the tension in the string at this instant.



Sol. The force acting on the bob are (figure)

(*a*) the tension T

(*b*) the weight mg.

As the bob moves in a vertical circle with centre at O, the radial acceleration is v^2/L towards 0. Taking the components along this radius and applying Newton's second law, we get

$$T - mg\cos\theta = mv^2/L \text{ or, } T = m(g\cos\theta + v^2/L)$$

$$|\vec{F}_{\text{nat}}| = \sqrt{(mg\sin\theta)^2 + \left(\frac{mv^2}{L}\right)^2} = m\sqrt{g^2\sin^2\theta + \frac{v^4}{L^2}}$$

2. A particle moves clockwise in a circle of radius 1 m with centre at $(x, y) = (1\text{m}, 0)$. It starts at rest at the origin at time $t = 0$. Its speed increases at the constant rate of $\left(\dfrac{\pi}{2}\right)$ m/s^2.

(*a*) How long does it take to travel halfway around the circle?

(*b*) What is the speed at that time?

(*c*) What is the net acceleration at that time?

Sol. $R = 1\text{m},$

$$a_t = \frac{dv}{dt} = \frac{\pi}{2}\ \text{m/s}^2$$

at $t = 0,\ u = 0,\ \omega_0 = 0$

$$\alpha = \frac{a_t}{R} = \frac{\pi}{2}\ \text{rad/s}^2$$

(a) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \pi = 0 + \frac{1}{2}\frac{\pi}{2}t^2 \Rightarrow t = 2\ \text{sec}$

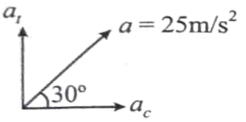
(b) $v = u + a_t t = 0 + \frac{\pi}{2} \times 2 = \pi\ \text{m/s}$

(c) $a_t = \frac{\pi}{2}\ \text{m/s}^2,\ a_c = \frac{v^2}{r} = \pi^2\ \text{m/s}^2$

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{\frac{\pi^2}{4} + \pi^4} = \frac{\pi}{2}\sqrt{1 + 4\pi^2}\ \text{m/s}^2$$

3. Figure shows the direction of total acceleration and velocity of a particle moving clockwise in a circle of radius 2.5 m at a given instant of time. At this instant if magnitude of net acceleration is 25 m/sec^2, find:
   (i) The radial acceleration,
   (ii) The speed of the particle and
   (iii) Its tangential acceleration

Sol.



(i) $a_c = a\cos 30° = 25\frac{\sqrt{3}}{2}\ \text{m/s}^2$

$$a_c = \frac{v^2}{R} \Rightarrow v^2 = a_c R = 25\frac{\sqrt{3}}{2} \times 2.5$$

(ii) $v = \left(125\frac{\sqrt{3}}{4}\right)^{1/2}\ \text{m/s}$

(iii) $a_t = a\sin 30° = \frac{25}{2}\ \text{m/s}^2$

4. Two particles A and B move anticlockwise with the same speed v in a circle of radius R and are diametrically opposite to each other. At $t = 0$, A is imparted a tangential acceleration of constant magnitude $a_t = \frac{72v^2}{25\pi R}$. Calculate the time in which A collides with B, the angle traced by A during this time, its angular velocity and radial acceleration at the time of collision.

Sol. $\omega_{0(\text{rel})} = 0,\ \theta_{\text{rel}} = \pi,\ \alpha_{\text{rel}} = \frac{72v^2}{25\pi R^2}$

$$\theta_{\text{rel}} = \omega_{0(\text{rel})}t + \frac{1}{2}\alpha_{\text{rel}}t^2$$

$$\pi = 0 + \frac{1}{2}\frac{72v^2}{25\pi R^2}t^2$$

$$t = \frac{5\pi R}{6v}\ \text{sec.}$$

Angle traced by A, $\theta = \frac{v}{R}\cdot\frac{5\pi R}{6v} + \frac{1}{2}\frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)^2$

$$= \frac{5\pi}{6} + \pi = \frac{11}{6}\pi$$

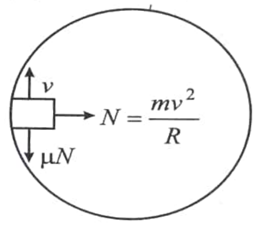
angular velocity $\omega = \omega_0 + \alpha t = \frac{v}{R} + \frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)$

$$= \frac{v}{R} + \frac{12v}{5R} = \frac{17v}{5R}$$

$$a_c = \omega^2 R = \left(\frac{17v}{5R}\right)^2 R = \frac{289v^2}{25R}$$

5. A block of mass 'm' moves on a horizontal circle against the wall of a cylindrical room of radius R. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ. The block is given an initial speed v_0. As a function of the instantaneous speed 'v' write
   (i) the normal force by the wall on the block,
   (ii) the frictional force by the wall and
   (iii) the tangential acceleration of the block.
   (iv) obtain the speed of the block after one revolution.

Sol.



(i) The normal reaction by wall on the block is $N = \frac{mv^2}{R}$

(ii) The friction force on the block by the wall is

$$f = \mu N = \frac{\mu mv^2}{R}$$

(iii) The tangential acceleration of the block $= \frac{f}{m} = \frac{\mu v^2}{R}$

(iv) $\frac{dv}{dt} = -\frac{\mu v^2}{R}$

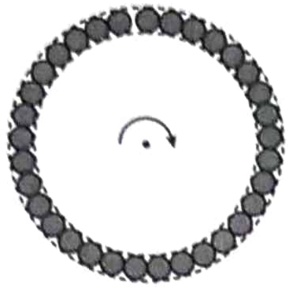
or $v\frac{dv}{ds} = -\frac{\mu v^2}{R}$

$$\Rightarrow \int_{v_0}^{v}\frac{dv}{v} = -\int_0^{2\pi R}\frac{\mu}{R}ds$$

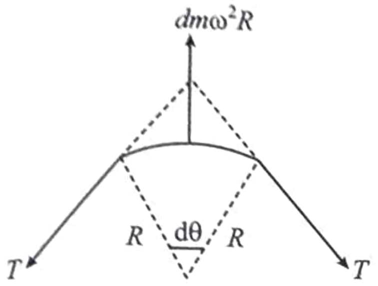
integrating we get

$$\ell n\frac{v}{v_0} = -\mu 2\pi \quad \text{or} \quad v = v_0 e^{-2\mu\pi}$$

6. A uniform metallic chain in a form of circular loop of mass m with a length ℓ rotates at the rate of n revolutions per second. Find the tension T in the chain.



Sol.



$$dm\omega^2R = 2T\sin\frac{d\theta}{2}$$

$$\Rightarrow \left(\frac{m}{\ell}Rd\theta\right)\omega^2R = Td\theta$$

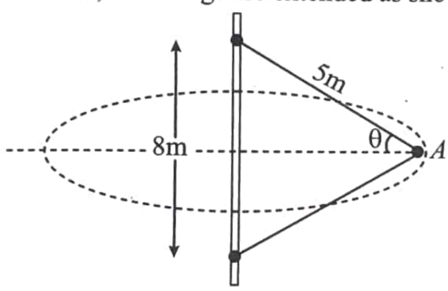
$$\therefore T = \frac{m}{\ell}\omega^2R^2 \qquad ...(i)$$

But $\omega = 2\pi n$

$\ell = 2\pi R$

$\therefore T = m\,\ell\, n^2$

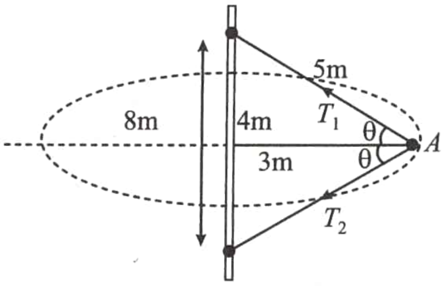
7. A 4 kg block is attached to a vertical rod by means of two strings of equal length. When the system rotates about the axis of the rod, the strings are extended as shown in figure.



(i) How many revolutions per minute must the system make in order for the tension in the upper chord to be 20 kgf?

(ii) What is the tension in the lower chord?

Sol. Centripetal acceleration



$$m\omega^2 r = T_1\cos\theta + T_2\cos\theta \qquad ...(i)$$

apply Newton law in vertical direction

$$T_1\sin\theta = mg + T_2\sin\theta \qquad ...(ii)$$

given $m = 4$ kg, $T_1 = 20\ kgf = 200$ N, $r = 3$m

$$\cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$$

Put in equation (ii)

$T_2 = 150$ N

Put in equation (i) we get

$$\omega^2 = \frac{210}{4\times 3} = \frac{35}{2}$$

$$\omega = \sqrt{\frac{35}{2}} \text{ rad/s}$$

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{35}{2}} \text{ rev/sec.}$$

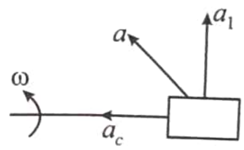
$$n = \frac{30}{\pi}\sqrt{\frac{35}{2}} \text{ rev/min.}$$

8. A block of mass m is kept on a horizontal ruler. The friction coefficient between the ruler and the block is μ. The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. (i) What can the maximum angular speed before which the block does not slip? (ii) If the angular speed of the ruler is uniformly increased from zero at a constant angular acceleration α, at what angular speed will the block slip?

Sol. (i) Ruler rotate with constant angular velocity for just slipping $m\omega^2L = \mu mg$

$$\omega = \sqrt{\frac{\mu g}{L}}$$

(ii) Angular velocity increase with constant angular acceleration α



$a_c = \omega^2L$, $a_t = \alpha L$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(\omega^2L\right)^2 + (\alpha L)^2} \qquad ...(i)$$

for just slipping $\Rightarrow ma = \mu mg$

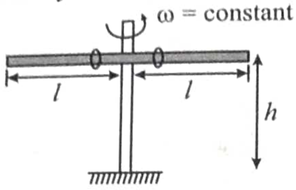
$$\Rightarrow a = \mu g \qquad ...(ii)$$

form (i) and (ii)

$$\mu g = \sqrt{\left(\omega^2L\right)^2 + (\alpha L)^2}$$

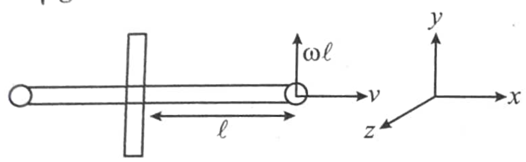
$$\omega = \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$$

9. Two identical rings which can slide along the rod are kept near the mid point of a smooth rod of length 2. The rod is rotated with constant angular velocity ω about vertical axis passing through its centre. The rod is at height 'h' from the ground. Find the distance between the points on the ground where the rings will fall after leaving the rods.



Sol. Time take by ring to fall on ground.

$$T = \sqrt{\frac{2h}{g}}$$



from centripetal force

$$m\omega^2 x = ma = mv\frac{dv}{dx}$$

$$\omega^2 x = v\frac{dv}{dx}$$

$$\int_0^{\ell} \omega^2 x\,dx = \int_0^{v} v\,dv$$

$$\omega^2\frac{\ell^2}{2} = \frac{v^2}{2}$$

$$v_x = \omega\,\ell$$

$$x = \omega\,\ell\,.T = \omega\,\ell\sqrt{\frac{2h}{g}}$$

$$v_y = \omega\,\ell$$

$$y = \omega\,\ell\cdot T = \omega\,\ell\sqrt{\frac{2h}{g}}$$

distance of one ring from center is

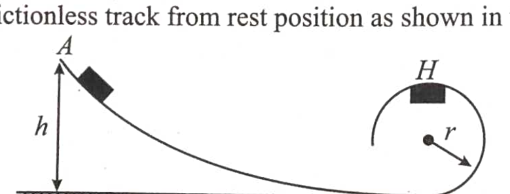
$$= \sqrt{y^2 + (x+\ell)^2}$$

distance between the point on the ground where the rings will fall after leaving the rods.

$$= 2\sqrt{y^2 + (x+\ell)^2}$$

where $x = y = \omega\,\ell\sqrt{\frac{2h}{g}}$

10. A small body of mass m is allowed to slide on an inclined frictionless track from rest position as shown in the figure.



(i) Find the minimum height h, so that body may successfully complete the loop of radius 'r'.

(ii) If h is double of that minimum height, find the resultant force on the block at position H.

Sol. (i) for complete the loop minimum velocity at lowest point is $v = \sqrt{5gr}$

from energy conservation

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}m(\sqrt{5gr})^2 = mgh \Rightarrow h = \frac{5}{2}r$$

(ii) h is double then velocity at h position is

$$mg(2h) - mg(2r) = \frac{1}{2}mv^2 \text{ (from energy conservation)}$$

$$v = \sqrt{6gr}$$

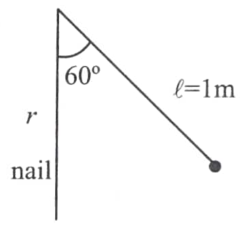
Normal reaction at highest point.

$$F_R = N + mg = \frac{m(\sqrt{6gr})^2}{r}$$

$$F_R = 6\,mg$$

11. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle of 60° from the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolution with the nail as centre. Assume the length of pendulum to be 1m.

Sol.



velocity at lowest point

$$mg\,\ell\,(1 - \cos 60°) = \frac{1}{2}mv^2 \text{ (from energy conservation)}$$

$$v = \sqrt{g\ell}$$

for completing the loop.

$$v = \sqrt{5g(\ell - r)} = \sqrt{g\ell}$$

$$r = \frac{4}{5}\text{ m}$$

12. A smooth semicircular wire-track of radius R is fixed in a vertical plane shown in fig. One end of a massless spring of natural length $(3R/4)$ is attached to the lower point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $k = mg/R$. Consider the instant when the ring is released, and (i) draw free body diagram of the ring, (ii) Determine the tangential acceleration of the ring and the normal reaction.

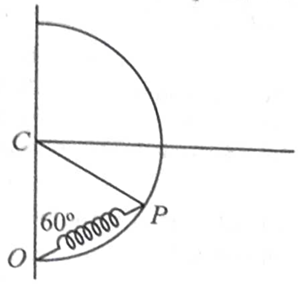
Sol. (i) $CP = CO =$ Radius of circle (R)

$\therefore\ \angle COP = \angle CPO = 60°$

$\therefore \angle OCP$ is also 60°

$\angle OCP = 60°$

Therefore, ΔOCP is an equilateral triangle.



Hence, $OP = R$

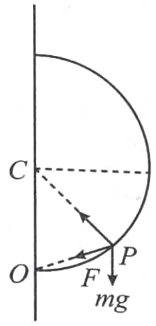
Natural length of spring is $3R/4$.

$\therefore$ Extension in the spring

$$x = R - \frac{3R}{4} = \frac{R}{4}.$$

$$\Rightarrow \text{Spring force}, F = kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$$

The free body diagram of the ring will be a shown.



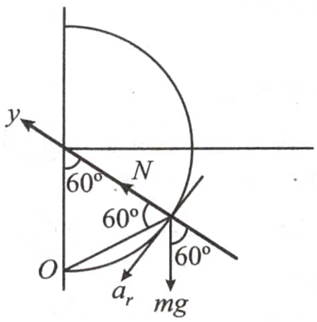
Here, $F = kx = \dfrac{mg}{4}$

and N = Normal reaction

(ii) Tangential acceleration a_r = The ring will move towards the x-axis just after the release. So, net force along x-axis:

$$F_x = F \sin 60° + mg \sin 60° = \left(\frac{mg}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$$

$$F_x = \frac{5\sqrt{3}}{8}\, mg$$



Therefore, tangential acceleration of the ring.

$$a_t = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8}g$$

$$a_t = \frac{5\sqrt{3}}{8}g$$

Normal Reaction N: Net force along y-axis on the ring just after the release will be zero.

$F_y = 0$

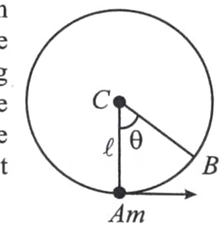
$\therefore N + F\cos 60° = mg \cos 60°$

$$\therefore N = mg\cos 60° - F\cos 60° = \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right)$$

$$= \frac{mg}{2} - \frac{mg}{8}$$

$$N = \frac{3mg}{8}$$

13. A particles of mass m is attached at one end of a light, inextensible string of length ℓ whose other end is fixed at the point C. At the lowest point the particle is given minimum velocity to complete the circular path in the vertical plane. As it moves in the circular path the tension in the string changes with θ. θ is defined in the figure. As θ varies from 0 to 2π (i.e. the particle completes one revolution) plot the variation of tension T against θ.



Sol. By Newton's law at B

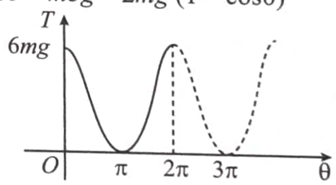
$$T - mg\cos\theta = \frac{mv^2}{\ell}$$

By energy conservation b/w A and B

$$mg\,\ell\,(1 - \cos\theta) + \frac{1}{2}mv^2 = \frac{1}{2}m(5\,\ell\,g)$$

$$mv^2 = m\,5\,\ell\,g - 2mg\,\ell\,(1 - \cos\theta)$$

$$T = mg\cos\theta + m5g - 2mg(1 - \cos\theta)$$



$= 3mg + 3mg\cos\theta$

$= 3mg(1 + \cos\theta) = 6mg\cos^2(\theta/2)$

14. A person stands on a spring balance at the equator. (i) By what percentage is the balance reading less than his true weight? (ii) If the speed of earth's rotation is increased by such an amount that the balance reading is half the true weight, what will be the length of the day in this case?

Sol. (i) at equator

$T + m\omega^2 R = mg.$

$$\%\ \frac{\Delta T}{T} = \frac{\omega^2 R}{g}$$

$$= \left(\frac{4\pi^2 \times 6400 \times 1000}{(24 \times 60 \times 60)^2 \times 9.8}\right) \times 100 = 0.65\ \%$$

(ii) $T = \frac{mg}{2}$ ...(i)

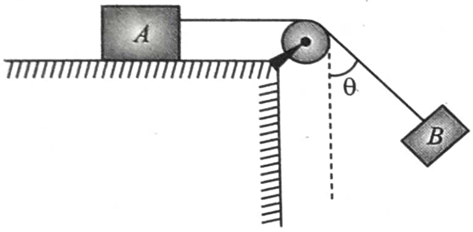
$T + m\omega^2 R = mg$ ...(ii)

from (i) and (ii)

$$\omega^2 R = g/2 \Rightarrow \omega = \sqrt{\frac{g}{2R}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2R}{g}} = 2\text{hr}$$

15. Two particles A and B each of mass m are connected by a massless string. A is placed on the rough table. The string passes over a small, smooth peg. B is left from a position making an $\angle\theta$ with the vertical. Find the minimum coefficient of friction between A and the table so that A does not slip during the motion of mass B.



Sol. Block B rotate in vertical plane. Tension is maximum in string at lowest position. When block B at lowest position and block A does not slide that means block A not slide at any position of B.

At lowest position

$$T - mg = \frac{mv^2}{\ell} \Rightarrow T = mg + \frac{mv^2}{\ell} \quad ...(i)$$

From energy conservation

$$mg\,\ell\,(1 - \cos\theta) = \frac{1}{2}mv^2 \quad ...(ii)$$

from equation (i) and (ii)

$T = mg + 2mg(1 - \cos\theta)$

$= 3mg - 2mg\cos\theta$

for no slipping.

$T = \mu mg = 3mg - 2mg\cos\theta$

$\mu_{min} = 3 - 2\cos\theta$

16. A car goes on a horizontal circular road of radius R, the speed increasing at a constant rate $\frac{dv}{dt} = a$. The friction coefficient between the road and the tyre is μ. Find the speed at which the car will skid.

Sol. Net force on car = frictional force

$= f$

$$\therefore f = m\sqrt{a^2 + \frac{v^4}{R^2}} \text{ (where m is mass of the car)} \quad ...(i)$$

For skidding to just occur

$f = \mu N = \mu mg$ ...(ii)

$\therefore$ From (i) and (ii)

$v = [R^2\{\mu^2 g^2 - a^2\}]^{1/4}$

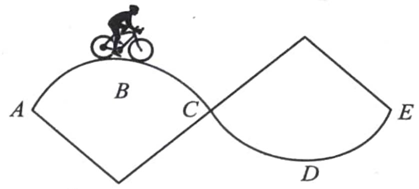
17. A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in fig. Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 km/h on the track.

(*a*) Find the normal contact force by the road on the cycle when it is at B and D.

(*b*) Find the force of friction exerted by the track on the tyres when the cycle is at B, C and D.

(*c*) Find the normal force between the road and the cycle just, before and just after the cycle crosses C.

(*d*) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed? Take $g = 10\text{m/s}^2$.



Sol. Constant speed = 18 km/hr = 5m/sec.

$m = 100$ kg, $r = 100$ m

(*a*) At B, $mg - N_B = \frac{mv^2}{r} = \frac{100\times5^2}{100} = 25$

$N_B = 975$ N **Ans.**

at D, $N_D - mg = \frac{mv^2}{r} \Rightarrow N_D = 1025$ N

(*b*) At B and D friction force act is zero.

at $C \Rightarrow f = mg\sin 45 = 100 \times 10\frac{1}{\sqrt{2}}$

($\because$ v = constant)

= 707 N

(*c*) For BC part

$$mg\cos 45 - N_{BC} = \frac{mv^2}{R} \Rightarrow N_{BC} = 682 \text{ N}$$

for CD part

$$N_{CD} - mg\cos 45 = \frac{mv^2}{R} \Rightarrow N_{CD} = 732 \text{ N}$$

(*d*) $f \le \mu N \Rightarrow \mu \ge \frac{f}{N}$

position where its maximum and N is minimum which is in part BC at C position.

$$\mu \ge \frac{mg\sin 45°}{mg\cos 45° - \frac{mv^2}{r}} \Rightarrow \mu \ge \frac{707}{682} = 1.037$$

Exercise-1 (Topicwise)

KINEMATICS OF CIRCULAR MOTION

1. If a particle moves in a circle describing equal angles in equal times, its velocity vector
   (a) Remains constant
   (b) Changes in magnitude
   (c) Changes in direction
   (d) Changes both in magnitude and direction
2. A particle moves with constant angular velocity in a circle. During the motion its
   (a) Energy is conserved
   (b) Momentum is conserved
   (c) Energy and momentum both are conserved
   (d) None of the above is conserved
3. The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is
   (a) Zero (b) $\frac{\pi}{30\sqrt{2}}$ cm/sec
   (c) $\frac{\pi}{30}$ cm/sec (d) $\frac{\pi\sqrt{2}}{30}$ cm/sec
4. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$
   (a) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (b) $-18\hat{i} - 13\hat{j} + 2\hat{k}$
   (c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$
5. A wheel is of diameter 1m. If it makes 30 revolutions/sec., then the linear speed of a point on its circumference will be.
   (a) 30π m/s (b) π m/s (c) 60π m/s (d) $\pi/2$ m/s
6. In uniform circular motion (angular momentum $L = m\vec{r} \times \vec{v}$)
   (a) Both the angular velocity and the angular momentum vary
   (b) The angular velocity varies but the angular momentum remains constant.
   (c) Both the angular velocity and the angular momentum stay constant
   (d) The angular momentum varies but the angular velocity remains constant.

CENTRIPETAL/TANGENTIAL/NET ACCELERATION

7. A body is moving in a circular path with a constant speed. It has
   (a) A constant velocity
   (b) A constant acceleration
   (c) An acceleration of constant magnitude
   (d) An acceleration which varies with time
8. Two bodies of mass 10 kg and 5 kg moving in concentric orbits of radii R and r such that their periods are the same. Then the ratio between their centripetal acceleration is
   (a) R/r (b) r/R
   (c) R^2/r^2 (d) r^2/R^2
9. A car travels north with a uniform velocity. It goes over a piece of mud which sticks to the tyre. The particles of the mud, as it leaves the ground are thrown
   (a) Vertically upwards (b) Vertically inwards
   (c) Towards north (d) Towards south
10. A stone is tied to one end of a string 50 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 s, what is the magnitude of acceleration of the stone
    (a) 493 cm/s^2 (b) 720 cm/s^2
    (c) 860 cm/s^2 (d) 990 cm/s^2
11. Two particles P and Q are located at distances r_P and r_Q respectively from the axis of a rotating disc such that $r_P > r_Q$;
    (a) Both P and Q have the same acceleration
    (b) Both P and Q do not have any acceleration
    (c) P has greater acceleration than Q
    (d) Q has greater acceleration than P
12. A string breaks if its tension exceeds 10 newtons. A stone of mass 250 gm tied to this string of length 10 cm is rotated in a horizontal circle. The maximum angular velocity of rotation can be.
    (a) 20 rad/s (b) 40 rad/s
    (c) 100 rad/s (d) 200 rad/s
13. A particle moving along a circular path due to a centripetal force having constant magnitude is an example of motion with:
    (a) Constant speed and velocity.
    (b) Variable speed and velocity.
    (c) Variable speed and constant velocity.
    (d) Constant speed and variable velocity.
14. The formula for centripetal acceleration in a circular motion is
    (a) $\vec{\alpha} \times \vec{r}$ (b) $\vec{\omega} \times \vec{v}$
    (c) $\vec{\alpha} \times \vec{v}$ (d) $\vec{\omega} \times \vec{r}$
15. A particle moves in a circular orbit under the action of a central attractive force inversely proportional to the distance 'r'. The speed of the particle is.
    (a) Proportional to r^2 (b) Independent of r
    (c) Proportional to r (d) Proportional to $1/r$

16. A particle of mass m is moving with constant velocity $\vec{v}$ on smooth horizontal surface. A constant force $\vec{F}$ starts acting on particle perpendicular to velocity v. Radius of curvature after force F start acting is:

(a) $\frac{mv^2}{F}$ (b) $\frac{mv^2}{F\cos\theta}$

(c) $\frac{mv^2}{F\sin\theta}$ (d) None of these

17. If the radii of circular paths of two particles of same masses are in the ratio of 1 : 2, then in order to have same centripetal force, their speeds should be in the ratio of:

(a) 1 : 4 (b) 4 : 1

(c) $1:\sqrt{2}$ (d) $\sqrt{2}:1$

18. A particle is moving in a horizontal circle with constant speed. It has constant

(a) Velocity (b) Acceleration

(c) Kinetic energy (d) Displacement

19. What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and halve the angular velocity ω?

(a) The centripetal acceleration remains unchanged.

(b) The centripetal acceleration is halved.

(c) The centripetal acceleration is doubled.

(d) The centripetal acceleration is quadrupled.

20. A particle is moving along a circular path. the angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}, \vec{v}, \vec{\alpha}$ and $\vec{a}_c$. Which of the following relations is not correct?

(a) $\vec{\omega} \perp \vec{v}$ (b) $\vec{\omega} \perp \vec{\alpha}$

(c) $\vec{\omega} \perp \vec{a}_c$ (d) $\vec{v} \perp \vec{a}_c$

21. A particle is moving in circular path with constant tangential acceleration, time t after the beginning of motion the direction of net acceleration is at 45° to radius vector at the instant. The angular acceleration of the particle at time 't' is proportional to:

(a) $\frac{1}{t}$ (b) $\frac{1}{t^2}$

(c) $\frac{3}{t}$ (d) t^0

22. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing it speed at the rate of 'a' m/s^2, then the resultant acceleration will be:

(a) $\sqrt{\left(\frac{v^2}{r^2}-a^2\right)}$ (b) $\sqrt{\left(\frac{v^4}{r^2}+a^2\right)}$

(c) $\sqrt{\left(\frac{v^4}{r^2}-a^2\right)}$ (d) $\sqrt{\left(\frac{v^2}{r^2}+a^2\right)}$

DYNAMICS OF CIRCULAR MOTION

23. If the overbridge is concave instead of being convex, the thrust on the road at the lowest position will be

(a) $mg+\frac{mv^2}{r}$ (b) $mg-\frac{mv^2}{r}$

(c) $\frac{m^2v^2g}{r}$ (d) $\frac{v^2g}{r}$

24. A motor cyclist moving with a velocity of 72 km/hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 meters. The acceleration due to gravity is 10 m/sec^2. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than

(a) $\theta=\tan^{-1}6$ (b) $\theta=\tan^{-1}2$

(c) $\theta=\tan^{-1}25.92$ (d) $\theta=\tan^{-1}4$

25. The force required to keep a body in uniform circular motion is

(a) Centripetal force

(b) Centrifugal force

(c) Resistance

(d) None of the above

26. The magnitude of the centripetal force acting on a body of mass m executing uniform motion in a circle of radius r with speed v is

(a) mvr (b) mv^2/r

(c) v/r^2m (d) v/rm

27. A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved

(a) 14 m/s (b) 3 m/s

(c) 3.92 m/s (d) 5 m/s

28. If a particle of mass m is moving in a horizontal circle of radius r with a centripetal force $(-k/r^2)$, the total energy is

(a) $-\frac{k}{2r}$ (b) $-\frac{k}{r}$

(c) $-\frac{2k}{r}$ (d) $-\frac{4k}{r}$

29. A car when passes through a convex bridge exerts a force on it which is equal to

(a) $Mg+\frac{Mv^2}{r}$ (b) $\frac{Mv^2}{r}$

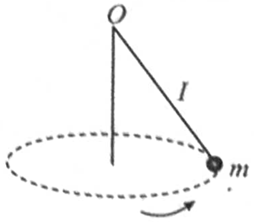
(c) Mg (d) $Mg-\frac{Mv^2}{r}$

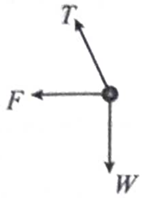
30. An unbanked curve has a radius of 60 m. The maximum speed at which a car can make a turn if the coefficient of static friction is 0.75, is

(a) 2.1 m/s (b) 14 m/s

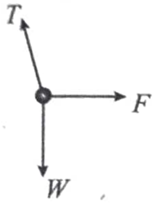
(c) 21 m/s (d) 7 m/s

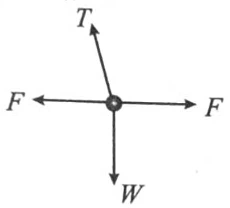
31. A point mass m is suspended from a light thread of length l, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are



(a) 

(b) 

(c) 

(d) 

32. A stone of mass 0.5 kg tied with a string of length 1 metre is moving in a circular path with a speed of 4 m/sec. The tension acting on the string in newton is

(a) 2 (b) 8 (c) 0.2 (d) 0.8

33. A particle of mass m is executing a uniform motion along a circular path of radius r. If the magnitude of its linear momentum is p, the radial force acting on the particle will be.

(a) pmr (b) rm/p
(c) mp^2/r (d) p^2/mr

34. When the road is dry and the coefficient of friction is μ, the maximum speed of a car in a circular path is 10 m/s, if the road becomes wet and $\mu' = \mu/2$. What is the maximum speed permitted?

(a) 5 m/s (b) 10 m/s
(c) $10\sqrt{2}$ m/s (d) $5\sqrt{2}$ m/s

VERTICAL CIRCULAR MOTION

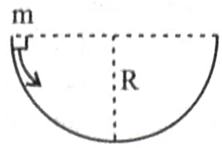
35. A small block is freely sliding down from top of a smooth inclined plane. The block reaches bottom of inclined plane then the block describes vertical circle of radius 0.5 m along smooth track. The minimum vertical height of inclined plane should be

(a) 1m (b) 1.25 m
(c) 3m (d) 2.5 m

36. A simple pendulum is oscillating with an angular amplitude 60°. If mass of bob is 50 gram, the tension in the string at mean position is ($g = 10\text{ms}^{-2}$)

(a) 0.5 N (b) 1 N
(c) 1.5 N (d) 2 N

37. A mass m slip along the wall of a semi spherical surface of radius R. The velocity at the bottom of the surface is.



(a) $\sqrt{Rg}$ (b) $\sqrt{2Rg}$
(c) $2\sqrt{\lambda Rg}$ (d) $\sqrt{\lambda Rg}$

38. A body of mass m is rotating in a vertical circle of radius 'r' with critical speed. The difference in its K.E. at the top and the bottom is

(a) 2 mgr (b) 4 mgr (b) 6 mgr (d) 3 mgr

39. A ball of mass 0.6 kg attached to a light inextensible string rotates in a vertical circle of radius 0.75m such that it has speed 5ms^{-1} when the string is horizontal. Tension in string when it is horizontal on other side is ($g - 10\text{ ms}^{-2}$)

(a) 30N (b) 26N (c) 20N (d) 6N

40. A body of mass m is rotated at uniform speed along vertical circle with help of light string. If T_1, T_2 are tensions in the string when the body is crossing highest and lowest point of vertical circle respectively then which of following expression is correct.

(a) $T_2 - T_1 = 6$ mg (b) $T_2 - T_1 = 4$mg
(c) $T_2 - T_1 = 2$ mg (d) $T_2 - T_1 =$ mg

41. A small bucket containing water is rotated in a vertical circle of radius R by means of a rope. V is velocity of bucket at highest point, Then water does not fall down if

(a) $V \le \sqrt{gR}$ (b) $V \le \sqrt{gR/2}$
(c) $V \ge \sqrt{gR}$ (d) $V \ge \sqrt{\frac{gR}{2}}$

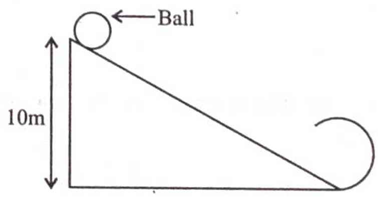
42. A sphere is suspended by a thread of length ℓ. What minimum horizontal velocity has to be imparted to the ball for it to reach the height of the suspension?

(a) $g\ell$ (b) $2g\ell$
(c) $\sqrt{g\ell}$ (d) $\sqrt{2g\ell}$

43. A ball is moving to and fro about the lowest point A of a smooth hemispherical bowl. If it is able to rise up to a height of 70 cm on either side of A, its speed at A must be? (mass of the ball 10g and take $g = 9.8\text{ m/s}^2$).

(a) 3.7 m/s (b) 5 m/s
(c) 4.8 m/s (d) 6 m/s

44. To complete the circular loop what should be the radius if initial height is 10 m?



(a) 2 m (b) 3 m (c) 4 m (d) 5 m

45. A body crosses the top most point of a vertical circle with critical speed. Its centripetal acceleration, when the string at the top point of circle/loop:

(a) 3 g (b) g
(c) 5 g (d) 6 g

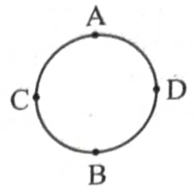
46. One end of the string of length 1.0 m is tied to a body of mass 0.5 kg. It is whirled in a vertical circle with angular frequency 4 rad/s. The tension in the string when the body is at the lower most point of its motion will be equal to (take, g = 10 m/s^2)

(a) 3 N (b) 5 N
(c) 8 N (d) 13 N

47. A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n rev/min. The total tension in the string when the stone is at the lowest point is

(a) mg (b) $m(g + \pi nr^2)$
(c) $m(g + nr)$ (d) $m\left(g + \frac{\pi^2 n^2 r}{900}\right)$

48. A stone is attached to one end of a string and rotated in a vertical circle. If string breaks at the position of maximum tension, it will break at



(a) A (b) B (c) C (d) D

49. A ball of mass 0.6 kg attached to a light inextensible string rotates in a vertical circle of radius 0.75 m such that it has speed of 5 ms^{-1} when the string is horizontal. Tension in string when it is horizontal on other side is (take, $g = 10$ ms^{-2})

(a) 30 N (b) 26 N
(c) 20 N (d) 6 N

50. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angle 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively, then

(a) $T_1 = T_2$
(b) $T_1 > T_2$
(c) $T_1 < T_2$
(d) Tension in the string always remains the same

Exercise-2 (Learning Plus)

1. Two racing cars of masses m_1 and m_2 are moving in circles of radii r and $2r$ respectively ; their angular speeds are equal. The ratio of the time taken by cars to complete one revolution is:

(a) $m_1 : m_2$ (b) 1 : 2
(c) 1 : 1 (d) $m_1 : 2m_2$

2. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be:

(a) 2π and 0 mm/s
(b) $2\sqrt{2}\pi$ and 4.44 mm/s
(c) $2\sqrt{2}\pi$ and 2π mm/s
(d) 2π and $2\sqrt{2}\pi$ mm/s

3. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1 ; in the next 2 sec, it rotates through an additional angle θ_2. The ratio of θ_2 / θ_1 is

(a) 1 (b) 2
(c) 3 (d) 5

4. The ratio of angular speed of hours hand and seconds hand of a clock is

(a) 1 : 1 (b) 1 : 60
(c) 1 : 720 (d) 3600 : 1

5. The linear and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively it will be at a distance from the axis of rotation

(a) 50 m (b) 1/2 m (c) 1 m (d) 2 m

6. The earth's radius 6400 km, makes one revolution about its own axis in 24 hours. The centripetal acceleration of a point on its equator is nearly:

(a) $340 \frac{cm}{sec^2}$ (b) $3.4 \frac{cm}{sec^2}$
(c) $34 \frac{cm}{sec^2}$ (d) $0.34 \frac{cm}{sec^2}$

7. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of particle in m/s^2 is:

(a) π^2 (b) $8\pi^2$ (c) $4\pi^2$ (d) $2\pi^2$

8. If angular velocity of a disc depends an angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at $\theta = 1$ rad is

(a) 8 rad/sec^2 (b) 10 rad/sec^2
(c) 12 rad/sec^2 (d) None

9. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal smooth surface. The maximum tension the string can withstand is 16 N. The maximum speed of revolution of the stone without breaking it, will be:

(a) 20 ms^{-1} (b) 16 ms^{-1}
(c) 14 ms^{-1} (d) 12 ms^{-1}

10. On horizontal smooth surface a mass of 2 kg is whirled in a horizontal circle by means of a string at an initial angular speed of 5 revolutions per minute. Keeping the radius constant and the tension in the string is doubled. The new angular speed is nearly:

(a) 14 rpm (b) 10 rpm

(c) 2.25 rpm (d) 7 rpm

11. A coin placed on a rotating turntable just slips if it is placed at a distance of 16 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of

(a) 1 cm (b) 2 cm

(c) 4 cm (d) 8 cm

12. In a circus, stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be:

(a) $\sqrt{2gR}$ (b) $2gR$

(c) $\sqrt{3gR}$ (d) $\sqrt{gR}$

13. A stone tied to a string is rotated in a vertical plane. If mass of the stone is m, the length of the string is r and the linear speed of the stone is v when the stone is at its lowest point, then the tension in the string at the lowest point will be:

(a) $\frac{mv^2}{r}+mg$ (b) $\frac{mv^2}{r}-mg$

(c) $\frac{mv^2}{r}$ (d) mg

14. A man is standing on a rough ($\mu = 0.5$) horizontal disc rotating with constant angular velocity of 5 rad/sec. At what distance from centre should he stand so that he does not slip on the disc?

(a) $R \leq 0.2$ m (b) $R > 0.2$ m

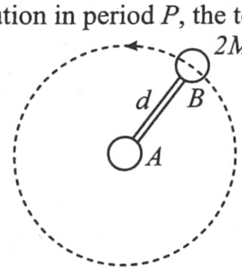
(c) $R > 0.5$ m (d) $R > 0.3$ m

15. A road is banked at an angle of 30° to the horizontal for negotiating a curve of radius $10\sqrt{3}$ m. At what velocity will a car experience no friction while negotiating the curve?

(a) 54 km/hr (b) 72 km/hr

(c) 36 km/hr (d) 18 km/hr

16. The dumbell is placed on a frictionless horizontal table. Sphere A is attached to a frictionless pivot so that B can be made to rotate about A with constant angular velocity. If B makes one revolution in period P, the tension in the rod is



(a) $\frac{4\pi^2 Md}{P^2}$ (b) $\frac{8\pi^2 Md}{P^2}$

(c) $\frac{4\pi^2 Md}{P}$ (d) $\frac{2Md}{P}$

17. A string can bear a maximum tension of 100 Newton without breaking. A body of mass 1 kg is attached to one end of 1 m length of thin string and it is revolved in a horizontal plane. The maximum linear velocity which can be imparted to the body without breaking the string, will be

(a) 10 m/s (b) 1 m/s

(c) 100 m/s (d) 1000 m/s

18. A cyclist is moving on a circular track of radius 80 m with a velocity of 72 km/hr. He has to lean from the vertical approximately through an angle

(a) $\tan^{-1}(1/4)$ (b) $\tan^{-1}(1)$

(c) $\tan^{-1}(1/2)$ (d) $\tan^{-1}(2)$

19. Consider a case in which, the driver of a car travelling at a high speed suddenly sees a wall at a distance r directly in front of him. To avoid collision,

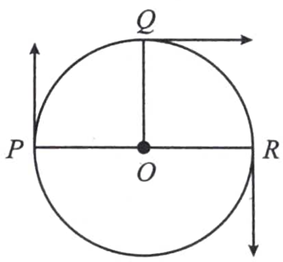
(a) He should apply brakes sharply.

(b) He should turn the car sharply.

(c) He should apply brakes and then sharply turn.

(d) None of these.

20. Three point particles P, Q, R move in a circle of radius 'r' with different but constant speeds. They start moving at $t = 0$ from their initial positions as shown in the figure. The angular velocities (in rad/sec) of P, Q and R are 5π, 2π and 3π respectively, in the same sense. The time at which they all meet is:



(a) 2/3 sec (b) 1/6 sec

(c) 1/2 sec (d) 3/2 sec

21. The velocity and acceleration vectors of a particle undergoing circular motion are $\vec{v} = 2\hat{i}$ m/s and $\vec{a} = 2\hat{i} + 4\hat{j}$ m/s^2 respectively at an instant of time. The radius of the circle is

(a) 1m (b) 2m

(c) 3m (d) 4m

22. A particle of mass m begins to slide down a fixed smooth sphere from the top with negligible initial velocity. What is its tangential acceleration when it breaks off the sphere?

(a) $\frac{2g}{3}$ (b) $\frac{\sqrt{5}g}{3}$

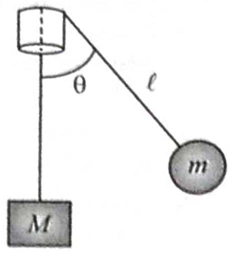
(c) g (d) $\frac{g}{3}$

23. A boy whirls a stone in a horizontal circle 1.8 m above the ground by means of a string with radius 1.2 m. while whirling the stone string was horizontal, it breaks and stone flies off horizontally, striking the ground 9.1 m away. The centripetal acceleration during the circular motion was nearly: (use $g = 9.8$ m/s^2)

(a) 94 m/s^2 (b) 141 m/s^2

(c) 188 m/s^2 (d) 282 m/s^2

24. A large mass M hangs stationary at the end of a light string that passes through a smooth fixed ring to a small mass m that moves around in a horizontal circular path. If ℓ is the length of the string from m to the top end of the tube and θ is angle between this part and vertical part of the string as shown in the figure, then time taken by m to complete one circle is equal to



(a) $2\pi\sqrt{\dfrac{\ell}{g\sin\theta}}$ (b) $2\pi\sqrt{\dfrac{\ell}{g\cos\theta}}$

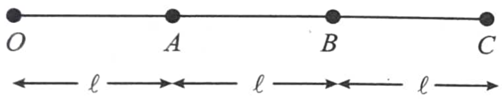
(c) $2\pi\sqrt{\dfrac{m\ell}{gM\sin\theta}}$ (d) $2\pi\sqrt{\dfrac{\ell m}{gM}}$

25. A stone of mass 1 kg tied to a light inextensible string of length $L = \dfrac{10}{3}$ m, whirling in a circular path in a vertical plane. The ratio of maximum tension in the string to the minimum tension in the string is 4, If g is taken to be 10 m/s^2, the speed of the stone at the highest point of the circle is:

(a) 10 m/s (b) $5\sqrt{2}$ m/s

(c) $10\sqrt{3}$ m/s (d) 20 m/s

26. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the speed of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is: (Assume that the string remains straight)



(a) 3 : 5 : 7 (b) 3 : 4 : 5

(c) 7 : 11 : 6 (d) 3 : 5 : 6

27. A Toy cart attached to the end of an unstretched string of length a, when revolved moves circle of radius $2a$ with a time period T. Now the toy cart is speeded up until it moves in a circle of radius 3a with a period T'. If Hook's law holds then (Assume no friction) :

(a) $T' = \sqrt{\dfrac{3}{2}}\,T$ (b) $T' = \left(\dfrac{\sqrt{3}}{2}\right)T$

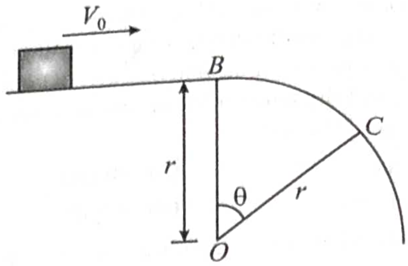
(c) $T' = \left(\dfrac{3}{2}\right)T$ (d) $T' = T$

28. A particle is projected horizontally from the top of a tower with a velocity v_0. If v be its velocity at any instant, then the radius of curvature of the path of the particle at that instant is directly proportional to:

(a) v^3 (b) v^2

(c) v (d) $1/v$

29. A small frictionless block slides with velocity $0.5\sqrt{gr}$ on the horizontal surface as shown in the Figure. The block leaves the surface at point C. The angle θ in the Figure is:



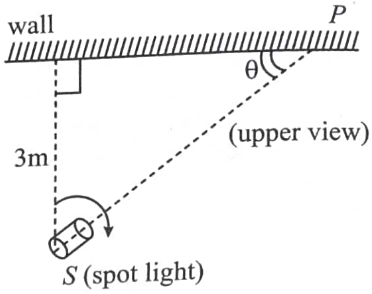
(a) $\cos^{-1}(4/9)$

(b) $\cos^{-1}(3/4)$

(c) $\cos^{-1}(1/2)$

(d) None of the above

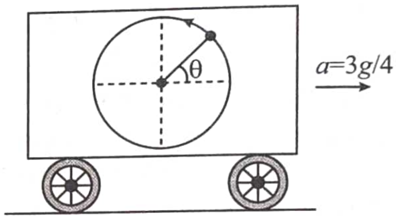
30. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when $\theta = 45°$?



(a) 0.6 m/s (b) 0.5 m/s

(c) 0.4 m/s (d) 0.3 m/s

(For Q. 31 to 33): A bus is moving with a constant acceleration $a = 3g/4$ towards right. In the bus, a ball is tied with a rope of length ℓ and is rotated in vertical circle as shown.



31. At what value of angle θ, tension in the rope will be minimum

(a) $\theta = 37°$ (b) $\theta = 53°$

(c) $\theta = 30°$ (d) $\theta = 90°$

32. At above mentioned position, find the minimum possible speed V_{min} during whole path to complete the circular motion

(a) $\sqrt{5g\ell}$ (b) $\dfrac{5}{2}\sqrt{g\ell}$

(c) $\dfrac{\sqrt{5g\ell}}{2}$ (d) $\sqrt{g\ell}$

33. For above value of V_{min} find maximum tension in the string during circular motion.

(a) 6 mg (b) $\dfrac{117}{20}mg$

(c) $\dfrac{15}{2}mg$ (d) $\dfrac{17}{2}mg$

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered s as $k = as^2$ where a is a positive constant. The total force acting on the particle is:

   (a) $2a\dfrac{s^2}{R}$ (b) $2as\left(1+\dfrac{s^2}{R^2}\right)^{1/2}$

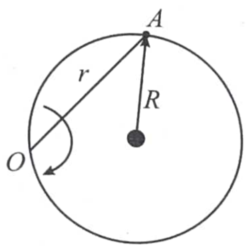
   (c) $2as$ (d) $2a\dfrac{R^2}{s}$

2. A particle moves along an arc of a circle of radius R. Its velocity depends on the distance covered s as $v = a\sqrt{s}$, where a is a constant then the angle α between the vector of the total acceleration and the vector of velocity as a function of s will be

   (a) $\tan\alpha = \dfrac{R}{2s}$ (b) $\tan\alpha = 2s/R$

   (c) $\tan\alpha = \dfrac{2R}{s}$ (d) $\tan\alpha = \dfrac{s}{2R}$

3. A particle A moves along a circle of radius R = 50 cm so that its radius vector r relative to the fixed point O (Fig.) rotates with the constant angular velocity = 0.40 rad/s. Then modulus v of the velocity of the particle, and the modulus a of its total acceleration will be



   (a) v = 0.4 m/s, = 0.4 m/s^2
   (b) v = 0.32 m/s, = 0.32 m/s^2
   (c) v = 0.32 m/s, = 0.4 m/s^2
   (d) v = 0.4 m/s, = 0.32 m/s^2

4. A sphere of mass m is suspended by a thread of length 'ℓ' is oscillating in a vertical plane, the angular amplitude being θ_0. What is the tension in the thread when it makes an angle θ with the vertical during oscillations? If the thread can support a maximum tension of 2 mg, then what can be the maximum angular amplitude of oscillation of the sphere without breaking the rope?

   (a) $3mg\cos\theta - 2mg\cos\theta_0$, $\theta_0 = 60^0$
   (b) $3mg\cos\theta + 2mg\cos\theta_0$, $\theta_0 = 60^0$
   (c) $2mg\cos\theta - 3mg\cos\theta_0$, $\theta_0 = 30^0$
   (d) $2mg\cos\theta + 3mg\cos\theta_0$, $\theta_0 = 30^0$

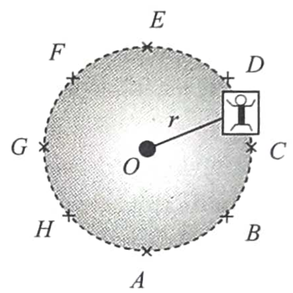
5. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen (in the plane of motion) from an inertial frame of reference.

   (a) This is not possible.
   (b) There are other forces on the particle.
   (c) The resultant of the other forces is $\dfrac{mv^2}{r}$ towards the centre.
   (d) The resultant of the other forces varies in magnitude as well as in direction.

6. Which of the following quantities may remain constant during the motion of an object along a curved path:

   (a) Speed (b) Velocity
   (c) Acceleration (d) Magnitude of acceleration

7. A machine, in an amusement park, consists of a cage at the end of one arm, hinged at O. The cage revolves along a vertical circle of radius r ($ABCDEFGH$) about its hinge O, at constant linear speed $v = \sqrt{gr}$. The cage is so attached that the man of weight 'w' standing on a weighing machine, inside the cage, is always vertical. Then which of the following is correct



   (a) The weight reading at A is greater than the weight reading at E by 2 w.
   (b) The weight reading at $G = w$.
   (c) The ratio of the weight reading at E to that at $A = 0$.
   (d) The ratio of the weight reading at A to that at $C = 2$.

8. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ to ϕ. For an angular displacement θ, $[|\theta| < \phi]$ the tension in the string and velocity of the bob are T and v respectively. The following relations hold good under the above conditions:

   (a) $T\cos\theta = Mg$
   (b) $T - Mg\cos\theta = \dfrac{Mv^2}{L}$
   (c) Tangential acceleration $= g\sin\theta$
   (d) $T = Mg\cos\theta$

9. A car of mass m attempts to go on the circular road of radius r, which banked for a speed of 36 km/hr. The friction coefficient between the tyre and the road is negligible.
(*a*) The car cannot make a turn without skidding.
(*b*) If the car turns at a speed less than 36 km/hr, it will slip down.
(*c*) If the car turns at the constant speed of 36 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$.
(*d*) If the car turns at the correct speed of 36 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$.

10. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then ($g = 9.8$ m/s^2)
(*a*) Its period of revolution is $\frac{4\pi}{7}$ sec.
(*b*) The tension in the string is double the weight of the particle
(*c*) The speed of the particle = $2.8\sqrt{3}$ m/s
(*d*) The centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

11. A car of mass M is travelling on a horizontal circular path of radius r. At an instant its speed is v and tangential acceleration is a:
(*a*) The acceleration of the car is towards the centre of the path.
(*b*) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$.
(*c*) The friction coefficient between the ground and the car is not less than a/g.
(*d*) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\frac{v^2}{rg}$.

12. A stone is projected from level ground at $t = 0$ sec such that its horizontal and vertical components of initial velocity are 10 m/s and 20 m/s respectively. Then the instant of time at which magnitude of tangential and magnitude of normal components of acceleration of stone are same is: (neglect air resistance) $g = 10$ m/s^2.
(*a*) $\frac{1}{2}$ sec (*b*) 1 sec
(*c*) 3 sec (*d*) 4 sec

COMPREHENSION BASED QUESTIONS

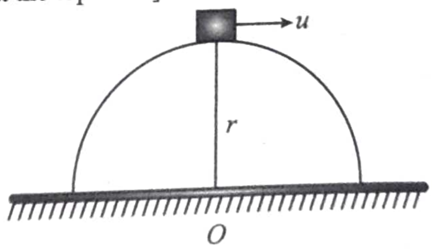
Comprehension (Q. 13 to 15): A particle undergoes uniform circular motion. The velocity and angular velocity of the particle at an instant of time is $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s and $\vec{\omega} = x\hat{i} + 6\hat{j}$ rad/sec.

13. The value of x in rad/s is
(*a*) 8 (*b*) –8
(*c*) 6 (*d*) can't be calculated

14. The radius of circle in metres is
(*a*) 1/2 m (*b*) 1 m
(*c*) 2 m (*d*) can't be calculated

15. The acceleration of particle at the given instant is
(*a*) $-50\hat{k}$ (*b*) $-42\hat{k}$
(*c*) $2\hat{i} + 3\hat{j}$ (*d*) can't be calculated

Comprehension (Q. 16 to 18): A small block of mass m is projected horizontally from the top of the smooth and fixed hemisphere of radius r with speed u as shown. For values of $u \geq u_0$, ($u_0 = \sqrt{gr}$) it does not slide on the hemisphere. [i.e. leaves the surface at the top itself]



16. For $u = 2\,u_0$, it lands at point P on ground. Find OP.
(*a*) $\sqrt{2}\,r$ (*b*) $2\,r$
(*c*) $4r$ (*d*) $2\sqrt{2}\,r$

17. For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.
(*a*) $\frac{19\,r}{9}$ (*b*) $\frac{19\,r}{27}$
(*c*) $\frac{10r}{9}$ (*d*) $\frac{10r}{27}$

18. Find its net acceleration at the instant it leaves the hemisphere.
(*a*) g/4 (*b*) g/2
(*c*) g (*d*) g/3

Comprehension (Q. 19 to 20): A particle moves with deceleration along the circle of radius R so that at any moment of time its tangential and normal accelerations are equal in moduli. At the initial moment $t = 0$ the speed of the particle equals v_0.

19. The speed of the particle as a function of the distance covered s will be
(*a*) $v = v_0\,e^{-s/R}$ (*b*) $v = v_0\,e^{s/R}$
(*c*) $v = v_0\,e^{-R/s}$ (*d*) $v = v_0\,e^{R/s}$

20. The total acceleration of the particle as function of velocity and distance covered
(*a*) $a = \sqrt{2}\frac{v^2}{R}$ (*b*) $a = \frac{v^2}{R}$
(*c*) $a = \frac{2v^2}{R}$ (*d*) $a = \frac{2\sqrt{2}\,v^2}{R}$

Comprehension (Q. 21 to 23): A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released, then:

21. The total acceleration of the sphere and the thread tension as a function of θ, the angle of deflection of the thread from the vertical will be

(*a*) $g\sqrt{1+3\cos^2\theta}$, $T = 3\,mg\cos\theta$

(*b*) $g\cos\theta$, $T = 3\,mg\cos\theta$.

(*c*) $g\sqrt{1+3\sin^2\theta}$, $T = 5\,mg\cos\theta$

(*d*) $g\sin\theta$, $T = 5\,mg\cos\theta$.

22. The angle θ between the thread and the vertical at the moment when the total acceleration vector of the sphere is directed horizontally will be

(*a*) $\cos\theta = \frac{1}{\sqrt{3}}$ (*b*) $\cos\theta = \frac{1}{3}$

(*c*) $\sin\theta = \frac{1}{\sqrt{3}}$ (*d*) $\sin\theta = \frac{1}{\sqrt{2}}$

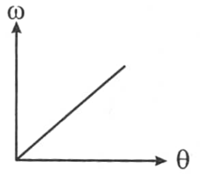
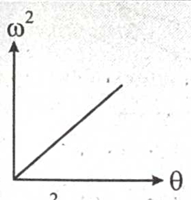
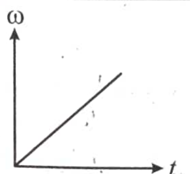
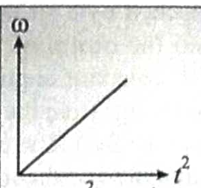
23. The thread tension at the moment when the vertical component of the sphere's velocity is maximum will be

(*a*) mg (*b*) $mg\sqrt{2}$

(*c*) $mg\sqrt{3}$ (*d*) $\frac{mg}{\sqrt{3}}$

MATCH THE COLUMN TYPE QUESTIONS

24. Each situation in column-I gives graph of a particle moving in circular path. The variables ω, θ and t represent angular speed (at any time t), angular displacement (in time t) and time respectively. Column-II gives certain resulting interpretation. Match the graphs in column-I with statements in column-II.

Column-I		Column-II	
A.	 $\omega - \theta$ graph	p.	Angular acceleration of particle is uniform
B.	 $\omega^2 - \theta$ graph	q.	Angular acceleration of particle is non-uniform
C.	 $\omega - t$ graph	r.	Angular acceleration of particle is directly proportional to t.
D.	 $\omega - t^2$ graph	s.	Angular acceleration of particle is directly proportional to θ.

(*a*) A-(q,r); B-(q); C-(p); D-(r)

(*b*) A-(q,s); B-(p); C-(p); D-(q,r)

(*c*) A-(q,s); B-(r); C-(p,r); D-(q,r)

(*d*) A-(q,s); B-(p); C-(p); D-(q,s)

25. A particle is moving with speed $v = 2t^2$ on the circumference of circle of radius R. Match the quantities given in column-I with corresponding results in column-II

Column-I		Column-II	
A.	Magnitude of tangential acceleration of particle	p.	Decreases with time.
B.	Magnitude of Centripetal acceleration of particle	q.	Increases with time
C.	Magnitude of angular speed of particle with respect to centre of circle	r.	Remains constant
D.	Angle between the total acceleration vector and centripetal acceleration vector of particle	s.	Depends on the value of radius R

(*a*) A-(q,r); B-(q,s); C-(q,s); D-(p)

(*b*) A-(q,r); B-(q,p); C-(q); D-(p,s)

(*c*) A-(q); B-(q,s); C-(q,s); D-(p,s)

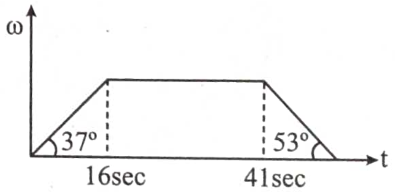
(*d*) A-(s); B-(q,s); C-(q,p); D-(p,r)

NUMERICAL TYPE QUESTIONS

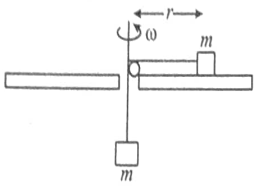
26. An electric drill starts from rest and rotates with constant angular acceleration. After it has rotated through an angle θ, the magnitude of centripetal acceleration of a point on the drill is twice the magnitude of tangential acceleration. What is the angle (in rad.)

27. Find the radius (in cm) of a rotating disc if the velocity of a point on rim is 5 times the velocity of a point located 5 m closer to the centre.

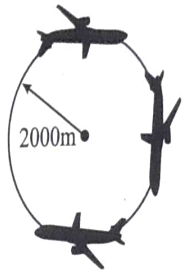
28. The angular velocity of a body moving in a circular path is shown in graph below. What is the average angular velocity (in rad/s) for the entire motion? Approximate the answer to nearest integer.



29. Two equal masses m are attached by a string. One mass lies at radial distance r from the centre of a horizontal turntable which rotates with constant angular velocity $\omega = 2$ rad s^{-1}, while the second hangs from the string inside the turntable's hollow spindle (see fig.). The coefficient of static friction between the turntable and the mass lying on it $\mu_s = 0.5$. The maximum and minimum values such that the mass lying on the turntable does not slide are r_{max}, r_{min}. Then $(r_{max} + r_{min})$ (in meter).



30. An air craft loops the vertical loop as shown with constant speed. At the top most point, the normal force exerted on the pilot by his seat is $\frac{1}{3}$ times the force exerted by the seat at the lowest point. What is the speed (in m/s) of the plane?



Exercise-4 (Past Year Questions)

JEE MAIN

1. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then, **(2018)**

(a) $T \propto R^{\frac{n}{2}+1}$ (b) $T \propto R^{(n+1)/2}$

(c) $T \propto R^{n/2}$ (d) $T \propto R^{3/2}$ for any n

2. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1$ s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10$ ms^{-2} the radius of R is: **(2019)**

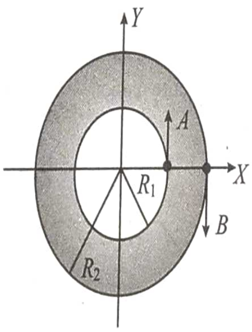
(a) 10.3 m (b) 2.8 m (c) 2.5 m (d) 5.1 m

3. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **(2019)**

(a) $10\sqrt{3}$ m/s (b) 0

(c) $10\sqrt{2}$ m/s (d) 10 m/s

4. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure: **(2019)**



The relative velocity $\vec{v_A} - \vec{v_B}$ at $t = \frac{\pi}{2\omega}$ is given by:

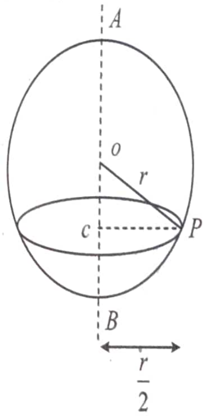
(a) $\omega(R_1 + R_2)\hat{i}$

(b) $-\omega(R_1 + R_2)\hat{i}$

(c) $\omega(R_2 - R_1)\hat{i}$

(d) $\omega(R_1 - R_2)\hat{i}$

5. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to: **(2019)**



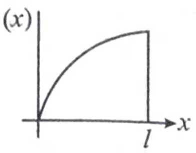
(a) $(g\sqrt{3})/r$

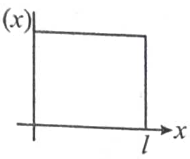
(b) $\frac{\sqrt{3}g}{2r}$

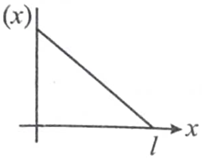
(c) $2g/r$

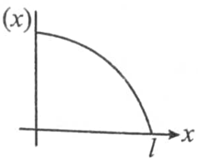
(d) $2g/(r\sqrt{3})$

6. A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a distance x from the axis, then which of the following graphs depicts it most closely? **(2019)**

(a) 

(b) 

(c) 

(d) 

7. A particle of mass m is fixed to one end of light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is: **(2020)**

(a) $\dfrac{m\ell\omega^2}{k-m\omega^2}$ (b) $\dfrac{m\ell\omega^2}{k-\omega m}$

(c) $\dfrac{m\ell\omega^2}{k+m\omega^2}$ (d) $\dfrac{m\ell\omega^2}{k+m\omega}$

8. A particle moves such that its position vector $\vec{r}(t)=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$ where ω is a constant and t is time, then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle: **(2020)**

(a) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$.

(b) $\vec{v}$ and $\vec{a}$ both are parallel to.

(c) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin.

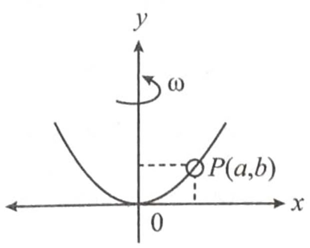
(d) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin.

9. A spring mass system (mass m, spring constant k and natural length ℓ) rests in equilibrium on a horizontal disc. The free and of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option **(2020)**

(a) $\sqrt{\dfrac{2}{3}}\left(\dfrac{m\omega^2}{k}\right)$ (b) $\dfrac{m\omega^2}{3k}$

(c) $\dfrac{2m\omega^2}{k}$ (d) $\dfrac{m\omega^2}{k}$

10. A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) **(2020)**



(a) $2\sqrt{2gC}$ (b) $2\sqrt{gC}$

(c) $\sqrt{\dfrac{2g}{C}}$ (d) $\sqrt{\dfrac{2gC}{ab}}$

11. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of **(2020)**

(a) 10^{-1} (b) 10^{-2} (c) 10^{-4} (d) 10^{-3}

12. A mosquito is moving with a velocity $\vec{v}=0.5t\hat{i}+3t\hat{j}+9\hat{k}$ m/s and accelerating in uniform conditions What will be the direction of mosquito after 2s? **(2021)**

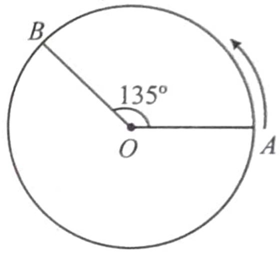
(a) $\tan^{-1}\left(\dfrac{\sqrt{85}}{6}\right)$ from y-axis

(b) $\tan^{-1}\left(\dfrac{5}{2}\right)$ from y-axis

(c) $\tan^{-1}\left(\dfrac{2}{3}\right)$ from x-axis

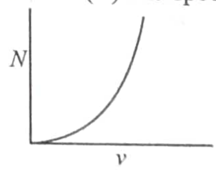
(d) $\tan^{-1}\left(\dfrac{5}{2}\right)$ from x-axis

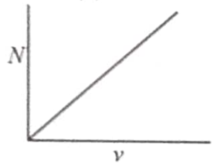
13. A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be:
(Given $\cos 135^\circ = -0.7$) **(2022)**

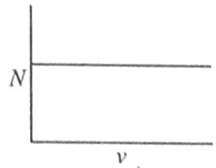


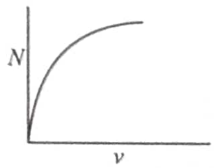
(a) 42 m (b) 47 m (c) 19 m (d) 40 m

14. A smooth circular groove has a smooth vertical wall as shown in figure. A block of mass m moves against the wall with a speed v. Which of the following curve represents the correct relation between the normal reaction on the block by the wall (N) and speed of the block (v)? **(2022)**

(a) 

(b) 

(c) 

(d) 

15. A boy ties a stone mass 100g to the end of a 2m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is K/π rev./min. The value of K is: **(2022)**

(a) 400 (b) 300 (c) 600 (d) 800

16. A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be **(2022)**

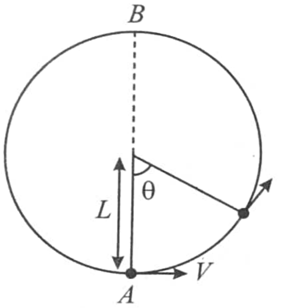
(*a*) 7.5 rad (*b*) 15 rad
(*c*) 20 rad (*d*) 30 rad

17. A stone of mass m, tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is **(2022)**

(*a*) The same throughout the motion.
(*b*) Minimum at the highest position of the circle path.
(*c*) Minimum at the lowest position of the circular path.
(*d*) Minimum when the rope is the horizontal position.

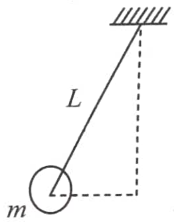
JEE ADVANCED

18. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle e at which the speed of the bob is half of that at A, satisfies **(2008)**



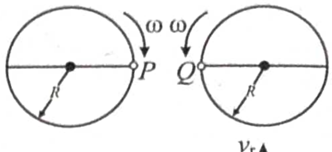
(*a*) $\theta = \frac{\pi}{4}$ (*b*) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$
(*c*) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (*d*) $\frac{3\pi}{4} < \theta < \pi$

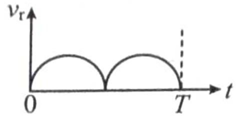
19. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. the maximum possible value of angular velocity of ball (in radian/s) is **(2011)**

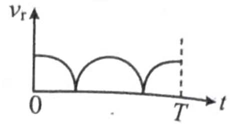


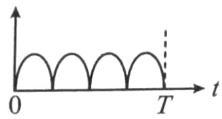
(*a*) 9 (*b*) 18
(*c*) 27 (*d*) 36

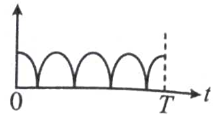
20. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by **(2012)**



(*a*)


(*b*)


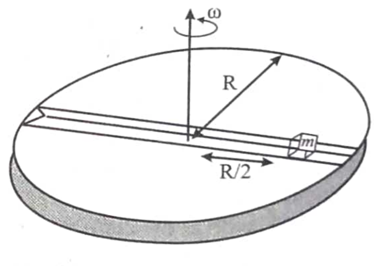
(*c*)


(*d*)


Comprehension (Q. 21 to 22): A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in inertial frame of reference is $\vec{F}_{rot} = \vec{F}_{in} + 3m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$

Where $\vec{F}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter–clockwise with a constant angular speed about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} - = (R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot. **(2016)**



21. The distance r of the block at time t is

(*a*) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (*b*) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$
(*c*) $\frac{R}{2}\cos 2\omega t$ (*d*) $\frac{R}{2}\cos \omega t$

22. The net reaction of the disc on the block is

(*a*) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\hat{k}$
(*b*) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$
(*c*) $m\omega^2 R\sin\omega t\,\hat{j} - mg\hat{k}$
(*d*) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$

ANSWER KEY

CONCEPT APPLICATION

1. (*c*)	**2.** (*c*)	**3.** (*a*)	**4.** (*a*)	**5.** (*a*)	**6.** 0.45 m/s^2	**7.** (*a*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)
11. $\frac{u^2 \sin^2 \theta}{g}$	**12.** (*a*)	**13.** [5 rad/s]	**14.** [90°]						

EXERCISE-1 (TOPICWISE)

1. (*c*)	**2.** (*a*)	**3.** (*d*)	**4.** (*b*)	**5.** (*a*)	**6.** (*c*)	**7.** (*c*)	**8.** (*a*)	**9.** (*d*)	**10.** (*a*)
11. (*c*)	**12.** (*a*)	**13.** (*d*)	**14.** (*b*)	**15.** (*b*)	**16.** (*a*)	**17.** (*c*)	**18.** (*c*)	**19.** (*a*)	**20.** (*b*)
21. (*b*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)	**25.** (*a*)	**26.** (*b*)	**27.** (*a*)	**28.** (*a*)	**29.** (*d*)	**30.** (*c*)
31. (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*d*)	**35.** (*b*)	**36.** (*b*)	**37.** (*b*)	**38.** (*a*)	**39.** (*c*)	**40.** (*c*)
41. (*c*)	**42.** (*d*)	**43.** (*a*)	**44.** (*c*)	**45.** (*b*)	**46.** (*d*)	**47.** (*d*)	**48.** (*b*)	**49.** (*c*)	**50.** (*b*)

EXERCISE-2 (LEARNING PLUS)

1. (*c*)	**2.** (*d*)	**3.** (*c*)	**4.** (*c*)	**5.** (*d*)	**6.** (*b*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*d*)
11. (*c*)	**12.** (*d*)	**13.** (*a*)	**14.** (*a*)	**15.** (*c*)	**16.** (*b*)	**17.** (*a*)	**18.** (*c*)	**19.** (*a*)	**20.** (*d*)
21. (*a*)	**22.** (*b*)	**23.** (*c*)	**24.** (*d*)	**25.** (*a*)	**26.** (*d*)	**27.** (*b*)	**28.** (*a*)	**29.** (*b*)	**30.** (*a*)
31. (*b*)	**32.** (*c*)	**33.** (*c*)							

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*b*)	**2.** (*b*)	**3.** (*d*)	**4.** (*a*)	**5.** (*b,d*)	**6.** (*a,c,d*)	**7.** (*a,b,c,d*)	**8.** (*b,c*)	**9.** (*b,d*)	**10.** (*a,b,c,d*)
11. (*b,c*)	**12.** (*b,c*)	**13.** (*b*)	**14.** (*a*)	**15.** (*a*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)	**19.** (*a*)	**20.** (*a*)
21. (*a*)	**22.** (*a*)	**23.** (*c*)	**24.** (*b*)	**25.** (*c*)	**26.** [1]	**27.** [625]	**28.** [9]	**29.** [5]	**30.** [200]

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*b*)	**2.** (*b*)	**3.** (*d*)	**4.** (*c*)	**5.** (*d*)	**6.** (*d*)	**7.** (*a*)	**8.** (*d*)	**9.** (*d*)	**10.** (*a*)
11. (*d*)	**12.** (*a*)	**13.** (*b*)	**14.** (*a*)	**15.** (*c*)	**16.** (*b*)	**17.** (*b*)			

JEE Advanced

18. (*d*)	**19.** (*d*)	**20.** (*a*)	**21.** (*b*)	**22.** (*d*)

CHAPTER

10 System of Particles and Centre of Mass

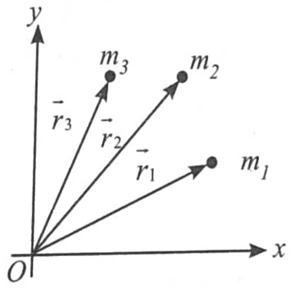
CENTRE OF MASS

In a system of extended bodies there is one special point that has some interesting and simple properties no matter how complicated the system is. This point is called the **center of mass (COM)**.

For a system of n particles whose position vectors are $\vec{r}_1, \vec{r}_2, ..., \vec{r}_n$ as shown in the figure, the position vector for the center of mass $\vec{r}_{cm}$ is defined as

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

$$\text{or} \quad \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$



where $M = \Sigma m_i$ is the total mass of the system.

The components of the above equation may be written as

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M}; \; y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M}; \; z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

Note: The location of the centre of mass is independent of the reference frame used to locate it.

The centre of mass of the system of particles depends only on the masses of the particles and the positions of the particles relative to one another.

Position of COM of Two Particle System

In case of two bodies, the ratio of distance of centre of mass from the bodies is in the inverse ratio of their masses.

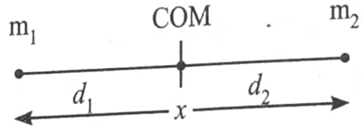
Let us choose origin at the location of center of mass in the figure shown, then

$x_1 = -d_1$

$x_2 = d_2$

$x_{cm} = 0$

$$\Rightarrow 0 = \frac{-m_1d_1 + m_2d_2}{m_1 + m_2} \; or \; m_1d_1 = m_2d_2$$



In figure, $x = d_1 + d_2$.

On solving, $m_1d_1 = m_1d_2$

$m_1d_1 = m_2 \times (x - d_2)$

$$\Rightarrow d_1 = \frac{m_2x}{m_1 + m_2} \text{ and } d_2 = \frac{m_1x}{m_1 + m_2}$$

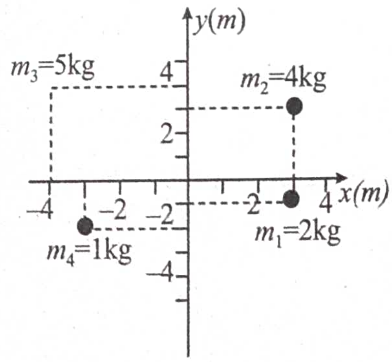
Here, d_1, d_2 are the distances of COM from m_1, m_2. Thus, COM locates nearer to heavier body.

Note: If m_1, m_2 are located at d_1, d_2 from origin then

$$x_{cm} = \frac{m_1d_1 + m_2d_2}{m_1 + m_2}$$

Train Your Brain

Example 1: Find the center of mass of the four point masses as shown in figure.



Sol. The total mass M = 12 kg, from the component equations, we have

$$x_{cm} = \frac{(2kg)(3m)+(4kg)(3m)+(5kg)(-4m)+(1kg)(-3m)}{12kg}$$

$$= -\frac{5}{12} m$$

$$y_{cm} = \frac{(2kg)(-1m)+(4kg)(3m)+(5kg)(4m)+(1kg)(-2m)}{12kg}$$

$$= \frac{28}{12} m$$

The position vector of the centre of mass is

$$\vec{r}_{cm} = -0.42\hat{i} + 2.3\hat{j} \text{ m}$$

Example 2: When 'n' number of particles of masses m,2m,3m,....nm are at distances $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, ... $x_n = n$ units respectively from origin on the x-axis, then find the distance of centre of mass of the system from origin.

(*a*) $\frac{2(n+1)}{2}$ (*b*) $\left(\frac{2n+1}{3}\right)$

(*c*) $\left(\frac{2n-1}{3}\right)$ (*d*) $\frac{n(n+1)}{2}$

Sol. Using formula of C.O.M, we have.

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$= \frac{m(1)+2m(2)+3m(3)+..........+nm(n)}{m+2m+3m+........+nm}$$

$$= \frac{m(1^2+2^2+.......+n^2)}{m(1+2+.......+n)}$$

$$\left\{\begin{array}{l} \because 1^2+2^2+....+n^2 = \frac{n(n+1)(2n+1)}{6} \\ \because 1+2+........+n = \frac{n(n+1)}{2} \end{array}\right\}$$

$$= \frac{m(n+1)(2n+1)\times \cancel{2}}{3\cancel{6}\times n(n+1)} = \frac{(2n+1)}{3}$$

Concept Application

1. Two blocks of masses 10 kg and 30 kg are placed on X-axis. The first mass is moved on the axis by a distance of 2 cm right. By what distance should the second mass be moved to keep the position of centre of mass unchanged.

   (*a*) $\frac{2}{3}$ Left (*b*) $\frac{3}{2}$ Left

   (*c*) $\frac{1}{2}$ Left (*d*) $\frac{5}{2}$ Left

2. When 'n' number of particles of masses m, 2m, 3m,.... nm are at distances $x_1 = 1, x_2 = 4, x_3 = 9, ... x_n = n^2$ units respectively from origin on the X-axis, then find the distance of their centre of mass from origin.

   (*a*) $\frac{n(n-1)}{2}$ (*b*) $\frac{n(n+1)}{2}$

   (*c*) $\frac{2n(n-1)}{3}$ (*d*) $\frac{2n(n+1)}{3}$

3. If the distance between the centers of the atoms of potassium and bromine in **KBr (potassium-bromide)** molecule is 0.282×10^{-9}m, find the centre of mass of this two particle system from potassium (mass of bromine = 80 u, and of potassium = 39 u)

   (*a*) 0.189 nm (*b*) 0.471 nm

   (*c*) 0.894 nm (*d*) 0.671 nm

CENTRE OF MASS OF CONTINUOUS MASS SYSTEMS

For continuous distribution of mass, the centre of mass is defined as

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}dm$$

where $\vec{r}$ is the position vector of the centre of mass of a small mass element dm.

The components of this equation are

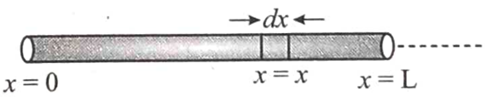
$$x_{cm} = \frac{1}{M}\int xdm, \; y_{cm} = \frac{1}{M}\int ydm \quad \text{and } z_{cm} = \frac{1}{M}\int zdm$$

Center of Mass of a Uniform Rod

Suppose a rod of mass M and length L is lying along the x-axis with its one end at $x = 0$ and the other end at $x = L$. Mass per unit length of the rod $= \frac{M}{L}$

Hence, dm, (the mass of the element dx situated at $x = x$ is) $= \frac{M}{L}$ dx

The coordinates of the element dx are (x, 0, 0). Therefore, x-coordinate of COM of the rod will be



$$x_{COM} = \frac{\int_0^L x\,dm}{\int dm} = \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M}$$

$$= \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

The y-coordinate of COM is

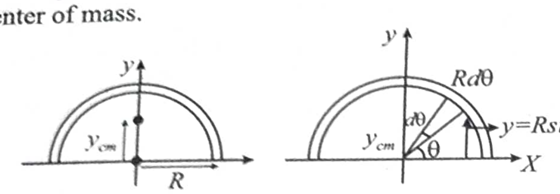
$$y_{COM} = \frac{\int y\,dm}{\int dm} = 0$$

Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$, i.e. it lies at the center of the rod.

Center of Mass of a Semicircular Ring

Figure shows the object (semi circular ring). By observation we can say that the x-coordinate of the center of mass of the ring is zero as the half ring is symmetrical about y-axis on both sides of the origin. Only we are required to find the y-coordinate of the center of mass.



To find y_{cm} we use

$$y_{cm} = \frac{1}{M}\int dm\, y \qquad \text{...}(i)$$

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width $d\theta$. If radius of the ring is R then its y coordinate will be $R\sin\theta$, here dm is given as

$$dm = \frac{M}{\pi R} \times R\, d\theta$$

So from equation $\qquad$ ...(ii)

we have

$$y_{cm} = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi R} R d\theta\,(R\sin\theta) = \frac{R}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

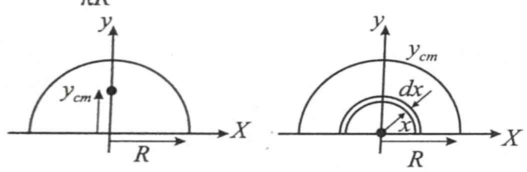
$$\frac{R}{\pi}[-\cos\theta]_0^{\pi}$$

$$y_{cm} = \frac{2R}{\pi} \qquad \text{...}(iii)$$

Center of Mass of a Semicircular Disc

Figure shows the half disc of mass M and radius R. Here, we are only required to find the y-coordinate of the center of mass of this disc as center of mass will be located on its half vertical diameter. Here to find y_{cm}, we consider a small elemental semi circular ring of mass dm of radius x on the disc (disc can be considered to be made up such thin rings of increasing radii) which will be integrated from 0 to R. Here dm is given as

$$dm = \frac{2M}{\pi R^2}(\pi x)dx$$



Now the y-coordinate of the element is taken as $\frac{2x}{\pi}$, as in previous section, we have derived that the center of mass of a semi circular ring is concentrated at $\frac{2R}{\pi}$

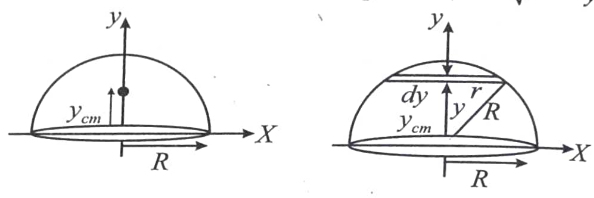
Here y_{cm} is given as

$$y_{cm} = \frac{1}{M}\int_0^R dm\,\frac{2x}{\pi} = \frac{1}{M}\int_0^R \frac{4M}{\pi R^2}x^2\,dx = \frac{4}{\pi R^2}\left[\frac{x^3}{3}\right]_0^R$$

$$\Rightarrow y_{cm} = \left(\frac{4R}{3\pi}\right)$$

Center of Mass of a Solid Hemisphere

The hemisphere is of mass M and radius R. To find its center of mass (only y-coordinate), we consider an element disc of width dy, mass dm at a distance y from the center of the hemisphere. The radius of this elemental disc will be given as, $r = \sqrt{R^2 - y^2}$



The mass dm of this disc can be given as

$$dm = \frac{3M}{2\pi R^3} \times \pi r^2\, dy = \frac{3M}{2R^3}(R^2 - y^2)\, dy$$

y_{cm} of the hemisphere is given as

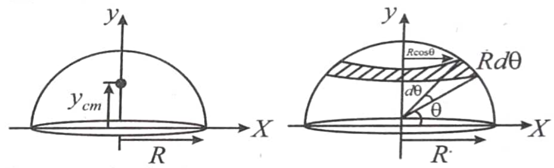
$$y_{cm} = \frac{1}{M}\int_0^R dm\, y = \frac{1}{M}\int_0^R \left[\frac{3M}{2R^3}(R^2 - y^2)\, dy\right] y$$

$$= \int_0^R \left[\frac{3M}{2R^3}(R^2 - y^2)\, dy\right] y \Rightarrow y_{cm} = \frac{3R}{8}$$

Center of Mass of a Hollow Hemisphere

A hollow hemisphere of mass M and radius R. Now we consider an elemental circular strip of angular width $d\theta$ at an angular distance θ from the base of the hemisphere. This strip will have an area.

$$dS = 2\pi R\cos\theta\, Rd\theta$$



Its mass dm is given as

$$dm = \frac{M}{2\pi R^2}\, 2\pi R\cos\theta\, Rd\theta$$

Here y - coordinate of this strip of mass dm can be taken as $R\sin\theta$. Now we can obtain the center of mass of the system as.

$$y_{cm} = \frac{1}{M}\int_0^{\frac{\pi}{2}} dm\, R\sin\theta$$

$$= \frac{1}{M}\int_0^{\frac{\pi}{2}} \left(\frac{M}{2\pi R^2} 2\pi R^2 \cos\theta\, d\theta\right) R\sin\theta$$

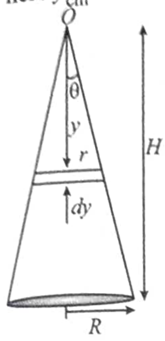
$$= R\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta\, d\theta \Rightarrow y_{cm} = \frac{R}{2}$$

Center of mass of a Solid Cone

A solid cone has mass M, height H and base radius R. Obviously the center of mass of this cone will lie somewhere on its axis, at a height less than $H/2$. To locate the center of mass we consider an elemental disc of width dy and radius r, at a distance y from the apex of the cone. Let the mass of this disc be dm, which can be given as

$$dm = \frac{3M}{\pi R^2 H} \times \pi r^2 \, dy$$

here y_{cm} can be given as

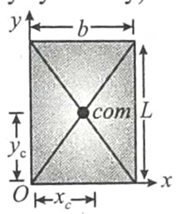


$$y_{cm} = \frac{1}{M}\int_0^H y\,dm \quad = \frac{1}{M}\int_0^H \left(\frac{3M}{\pi R^2 H}\pi\left(\frac{Ry}{H}\right)^2 dy\right) y$$

$$= \frac{3}{H^3}\int_0^H y^3\,dy = \frac{3H}{4} \text{ (from verfex)}$$

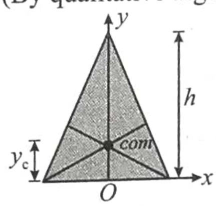
Center of Mass of Some Other Common Systems

❖ Rectangular plate (By symmetry)



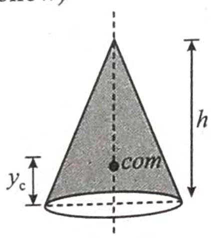
$$x_c = \frac{b}{2};\ y_c = \frac{L}{2}$$

❖ A triangular plate (By qualitative argument)



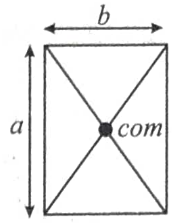
at the centroid : $y_c = \frac{h}{3}$

❖ A circular cone (hollow)

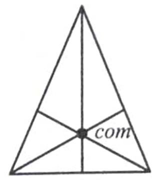


$$y_c = \frac{h}{3}$$

❖ Center of mass of rectangle $at\left(\frac{a}{2}, \frac{b}{2}\right)$



❖ Center of mass of an equilateral triangle lies at point when medians meet.



Note:

(i) Center of mass of a symmetrical body having uniform density lies at its geometrical center.

For example, center of mass of sphere (hallow or solid) lies at its center, center of mass of a ring or a disc at the center of the ring and center of mass of a uniform rod at its mid point.

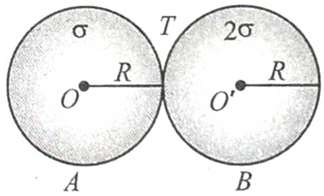
(ii) If a line or a plane divides a body into two symmetrical parts then, center of mass lies on that line or a plane.

For example, the center of a rectangular plate will lie on the diagonals. The point when diagonals meet will be its center of mass.

(iii) For locating the center of mass of a composite body, the body can be replaced by a particle placed at its center of mass.

Train Your Brain

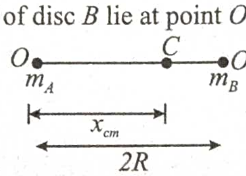
Example 3: Two circular disc having radius R and mass density σ and 2σ respectively are placed as shown in figure. Then find out the position of COM of the system (from point 'O').



Sol. Mass of disc A: $m_A = \sigma\pi R^2 = m_1$

Mass of disc B: $m_B = 2\sigma\pi R^2 = m_2$

Due to symmetry the COM of disc A lie at point O and COM of disc B lie at point O'.



We can replace two discs by two particles placed at their center.

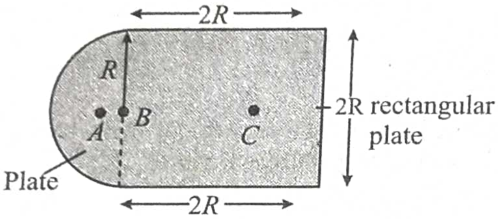
Let us choose x-axis along OO′ whth origin at O. Then,

$x_1 = 0, x_2 = 2R$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{0 + 2\sigma\pi R^2 \times 2R}{\sigma\pi R^2 + 2\sigma\pi R^2} = \frac{4R}{3}$$

So the centre of mass lie in the disc B having distance $\frac{4R}{3}$ from O.

Example 4: Find out the position of centre of mass of the object as shown in figure below heving uniform density.



Sol. We divide the above problem in two parts

(i) First find out position of centre of mass of both semicircular plate and rectangular plate separately.

(ii) Then find the position of centre of mass of given structure .

Centre of mass of semicircular disc lie at $\frac{4R}{3\pi}$

$\Rightarrow AB = \frac{4R}{3\pi}$

Centre of mass of rectangular plate lie at the centre of plate at point C

$\Rightarrow BC = R \Rightarrow$

Let us choose x-axis along AC with origin at point B.

$$x_1 = -AB = -\frac{4R}{3\pi}$$

$$x_2 = BC = R$$

$$m_1 = m_{SC} = \frac{\sigma\pi R^2}{2}$$

$$m_2 = m_R = \sigma 4R^2$$

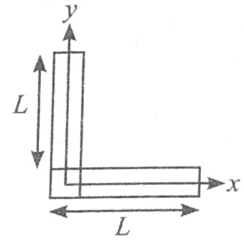
$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{\frac{-\sigma\pi R^2}{2} \times \frac{4R}{3\pi} + \sigma 4R^2 \times R}{\frac{\sigma\pi R^2}{2} + \sigma 4R^2}$$

$$= \frac{20}{24 + 3\pi}R$$

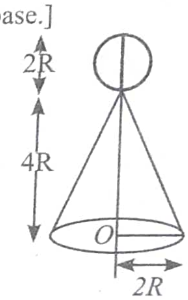
Concept Application

4. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown. can be. if the meeting point is the origin of co-ordinates



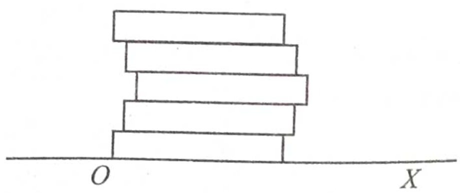
(a) (L/2, L/2)
(b) (2L/3, L/2)
(c) (L/3, L/3)
(d) (L/3, L/6)

5. A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times that of the cone, compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of h/4 from its base.]



(a) 4R from O
(b) 3R from O
(c) R from O
(d) 2R from O

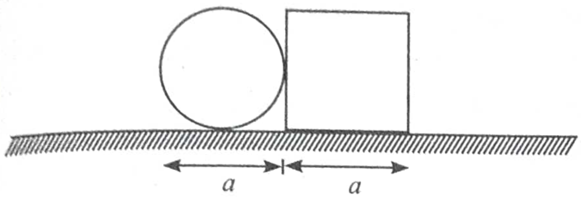
6. Five homogeneous bricks, each of length L, are arranged as shown in figure. Each brick is displaced with respect to the one in contact by L/5. Find the x-coordinate of the centre of mass relative to the origin O shown.



(a) $\frac{32L}{50}$ (b) $\frac{30L}{50}$

(c) $\frac{33L}{50}$ (d) $\frac{25L}{50}$

7. A square plate of edge 'a' and a circular disc of same diameter are placed touching each other at the midpoint of an edge of the plate as shown in figure. If mass per unit area for the two plates are same then find the distance of centre of mass of the system from the centre of the disc.



(a) $\frac{4a}{(4+\pi)}$ (b) $\frac{4a}{3+\pi}$

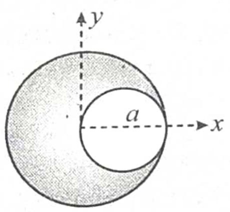
(c) $\frac{3a}{4+\pi}$ (d) $\frac{(4-\pi)a}{2(4+\pi)}$

CAVITY PROBLEMS

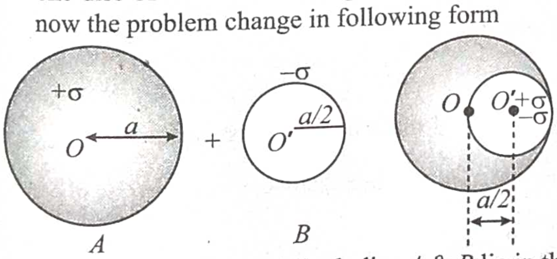
If some mass or area is removed from a rigid body then the position of centre of mass of the remaining portion is obtained by assuming that in a remaining part +m & – m mass is there. Further steps are explained by following example.

Train Your Brain

Example 5: Find the position of centre of mass of the uniform lamina as shown in figure. If the mass density of the lamina is σ.



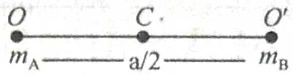
Sol. We assume that in remaining portion a disc of radius a having mass density $+\sigma$ is there then we also include one disc of a/2 radius having $-\sigma$ mass density. So now the problem change in following form



So the centre of mass of both disc A & B lie in their respective centre such as O & O'.

Now

$\Rightarrow$ C.O.M. of the lamina $= \frac{m_B\, a/2}{m_A + m_B}$



$m_A = \sigma(\pi a^2)$, $m_B = -\sigma(\pi)(a/2)^2$

$= -\sigma\,\pi\frac{a^2}{4}$

$$\Rightarrow x_{com} = \frac{-\left(\sigma\pi\frac{a^2}{4}\right)\times\frac{a}{2}}{\sigma\pi a^2 - \sigma\pi\frac{a^2}{4}}$$

$$= \frac{-4a}{24} = \frac{-a}{6}$$

x-axis is divides the body into two symmetrical parts, therefore C.O.M lies on x-axis

$\Rightarrow y_{cm} = 0$

C.O.M lie on leftward side from point O at distance of a/6.

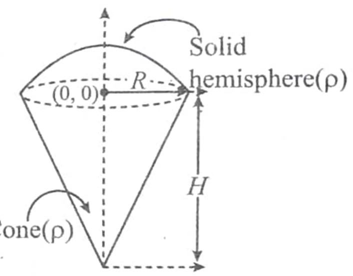
Example 6: Find the relation between R & H such that C.O.M of system is at origin.

Sol. Mass of the hemispherical Portion (M_1):

$$M_1 = \left(\frac{2}{3}\pi R^3\right)\rho$$

position of centre of mass for solid hemisphere from origin.

$y_1 = 3R/8$



Similarly; mass of solid cone:

$$M_2 = \frac{1}{3}\left(\pi R^3 H\right)\rho$$

position of com of cone. $y_2 = -\frac{H}{4}$

$$\Rightarrow y_{cm} = \frac{m_1 y_1 + m_2 y_2}{(m_1 + m_2)} = 0 \quad (\because \text{ lies at origin})$$

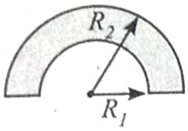
$$0 = \frac{\rho\left(\frac{2}{3}\pi R^3 \times \frac{3R}{8}\right) + \rho\cdot\left(\frac{1}{3}\pi R^2 H\right)\left(\frac{-H}{4}\right)}{\rho\cdot\left(\frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 H\right)}$$

$$\Rightarrow \rho\cdot\left(\frac{2}{3}\pi R^3\right)\frac{3R}{8} - \rho\left(\frac{1}{3}\pi R^2 H\right)\cdot\frac{H}{4} = 0$$

$\Rightarrow H = 3R$

Concept Application

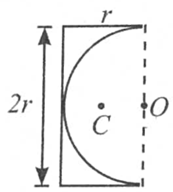
8. Find the centre of mass of an annular half disc shown in figure



(a) $\frac{4}{3\pi}\left(\frac{R_2^3-R_1^3}{R_2^2-R_1^2}\right)$ (b) $\frac{8}{3\pi}\left(\frac{R_2^3-R_1^3}{R_2^2-R_1^2}\right)$

(c) $\frac{4}{3\pi}\left(\frac{R_2^2-R_1^2}{R_2^3-R_1^3}\right)$ (d) $\frac{8}{3\pi}\left(\frac{R_2^2-R_1^2}{R_2^3-R_1^3}\right)$

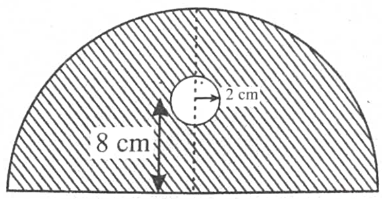
9. A semicircular portion of radius 'r' is cut from a uniform rectangular plate as shown in figure. The distance of centre of mass 'C' of remaining plate, from point 'O' is :



(a) $\frac{2r}{(3r-\pi)}$ (b) $\frac{3r}{2(4-\pi)}$

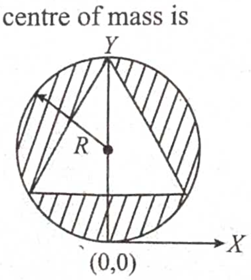
(c) $\frac{2r}{(4+\pi)}$ (d) $\frac{2r}{3(4-\pi)}$

10. In the figure shown a hole of radius 2 cm is made in a semicircular disc of radius 6π at a distance 8 cm from the centre C of the disc. The distance of the centre of mass of this system from point C is



(a) 4 cm (b) 8 cm (c) 6 cm (d) 12 cm

11. From a uniform disc of radius R, an equilateral triangle of side $\sqrt{3}R$ is cut, as shown in the figure. The new position of centre of mass is



(a) (0, 0) (b) (0, R)

(c) $\left(0, \frac{\sqrt{3}R}{2}\right)$ (d) None of these

MOTION OF THE CENTRE OF MASS

Now we can discuss the physical importance of the centre of mass concept. Consider the motion of a group of particles $m_1, m_2, \ldots\ldots, m_n$ and whose total mass is M which is a constant. From the definition of the centre of mass, we have

$$M\,\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots\ldots + m_n\vec{r}_n$$

Differentiating this equation w.r.t. time, we obtain

$$M\,\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n$$

where $\vec{v}_1 = \frac{d\vec{r}_1}{dt}$ is the velocity of the first particle, etc., and $\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt}$ is the velocity of the centre of mass.

Differentiating the above equation w.r.t. time, we obtain

$$M\,\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots\ldots + m_n\vec{a}_n$$

where $\vec{a}_1$ is the acceleration of the first particle, etc., and $\vec{a}_{cm}$ is the acceleration of the centre of mass. Now, from Newton's second law the force $\vec{F}_1$ acting on the first particle is given by $\vec{F}_1 = m_1\vec{a}_1$. Likewise, $\vec{F}_2 = m_2\vec{a}_2$, etc. We can then write the above equation as $M\,\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots + \vec{F}_n$

Among all these forces the internal forces are exerted by the particles on each other. However, from Newton's third law, these internal forces will occur in equal and opposite pairs, so that they contribute nothing to the sum. The right hand sum in the above equation is represents the sum of only the external forces acting on all the particles (system). We can then rewrite the above equation as

$$M\,\vec{a}_{cm} = \sum \vec{F}_{external}$$

This states that the centre of mass of a system of particles moves as though all the mass of the system is concentrated at the centre of mass and all the external forces were applied at that point.

One important situation is that in which $\sum \vec{F}_{external} = \vec{0}$ then $\vec{v}_{cm}$ = constant.

Note:

(i) If centre of mass is initially at rest and $\sum F_{external} = 0$ on centre of mass, then displacement of centre of Mass will be zero.

Hence shift in position of centre of mass is zero.

(ii) If ΣFext is zero on COM then v_{com} = constant but kinetic energy of system may or may not be constant because kinetic energy of the system can also change due to internal forces.

Train Your Brain

Example 7: A dog of mass 10 kg stands on a stationary boat of mass 40 kg so that he is 20 m from the shore. He then walks 8 m on the boat towards the shore and halts. How far is he from the shore now? Assume that there is no friction between the boat and water.

Sol. In the horizontal direction, there is no force on the system (dog + boat). Therefore, the centre of mass of the system does not move in the horizontal direction. Since the dog moves towards the shore, the boat moves away from the shore to keep centre of mass stationary. Let d be the distance by which the boat moves backwards and let x be the initial distance of the boat from the shore. The initial x-coordinate of the centre of mass $= \dfrac{10\times 20+40.x}{10+40} = x_i$

The final x-coordinate of the centre of mass

$$x_f = \frac{10(20-8+d)+40(x+d)}{10+40}$$

Equating the two, we get

$$d = \frac{8}{5}m$$

$\therefore$ The dog is $20 - 8 + \dfrac{8}{5} = 13.6\,m$ from the shore.

Example 8: A 75 kg man stands at the rear end of a platform of mass 25 kg and length 4m, which moves initially at 4 m/s over a frictionless surface. At $t = 0$, he walks at 2 m/s relative to the platform and then stops at the front end. During the period of walking, find the displacement of

(*a*) the platform, (*b*) the man

(*c*) the center of mass

Sol. Initially the man, the platform, and the c.o.m. have the same velocity, 4m/s. When he begins to walk forward, his increase in momentum must be compensated by a decrease in the platform's momentum. Let us say that the velocity of the platform relative to the ground while he is walking i.e.

$\vec{v}_{PG} = v_p\hat{i}$

The velocity of man relative to the ground is then

$\vec{v}_{MG} = \vec{v}_{MP} + \vec{v}_{PG} = (2 + v_P)\ \hat{i}$.

From the conservation of momentum, we have

$(75 + 25) \times 4 = 75(2 + v_p) + 25\ v_P$

Thus, the velocity of the platform is $v_P = 2.5$ m/s,

and the velocity of the man is $v_m = 4.5$ m/s.

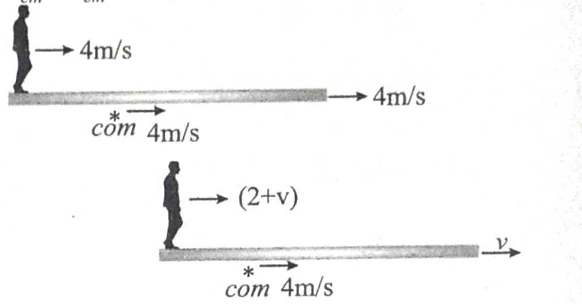
Since the velocity of man relative to platform is 2m/s, therefore, it takes $t = \dfrac{4m}{2m/s} = 2$ s for him to walk from the rear to the front.

Displacement of platform is $x_p = v_p t = (2.5)(2) = 5$m

Displacement of the man is $x_m = v_m t = (4.5)(2) = 9$m

Displacement of the center of mass

$x_{cm} = v_{cm}t = (4)(2) = 8$ m



Example 9: A particle of mass $m_1 = 4$ kg moves at $5\hat{i}$ m/s, while $m_2 = 2$ kg moves at $2\hat{i}$ m/s. Find

(*a*) the kinetic energy of center of mass

(*b*) the kinetic energy with respect to center of mass.

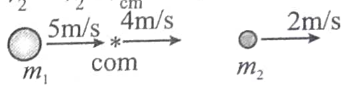
Sol. The velocity of the centre of mass is obtained as

$$v_{cm} = \frac{(4kg)(5m/s)+(2kg)(2m/s)}{6kg} = 4 \text{ m/s}$$

The following figure shows the velocities relative to the centre of mass

$v_1' = v_1 - v_{cm} = +1$ m/s

$v_2' = v_2 - v_{cm} = -2$ m/s



(a)

Laboratiory Frame

(b)

Centre of mass Frame

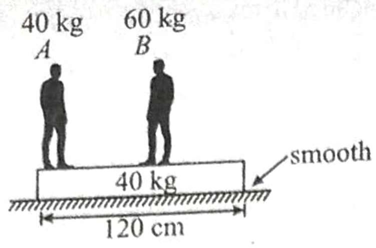
Thus,

(*a*) $K_{cm} = \dfrac{1}{2}(m_1 + m_2)v_{cm}^2 = 48$ J

(*b*) $K' = \dfrac{1}{2}m_1 v_1'^2 + \dfrac{1}{2}m_2 v_2'^2 = 6$ J

Concept Application

12. Two men 'A' and 'B' are standing on a plank. 'B' is at the middle of the plank and 'A' is at the left end of the plank. Bottom surface of the plank is smooth. System is initially at rest and masses are as shown in figure. 'A' and 'B' start moving such that the position of 'B' remains fixed with respect to ground and 'A' meets 'B'. Then the point where A meets B is located at :



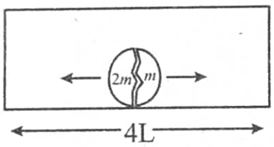
(*a*) the middle of the plank
(*b*) 30 cm from the left end of the plank
(*c*) the right end of the plank
(*d*) None of these

13. A stationary body explodes into two fragments of masses m_1 and m_2. If momentum of one fragment is p, the minimum energy of explosion is

(*a*) $\frac{p^2}{2(m_1+m_2)}$ (*b*) $\frac{p^2}{2\sqrt{m_1 m_2}}$

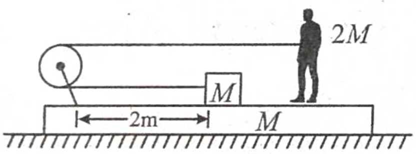
(*c*) $\frac{p^2(m_1+m_2)}{2m_1 m_2}$ (*d*) $\frac{p^2}{2(m_1-m_2)}$

14. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4L at it's centre. It explodes in two parts of mass *m* & *2m*. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



(*a*) 0 (*b*) $\frac{L}{3}$ (*c*) $\frac{L}{6}$ (*d*) $\frac{L}{12}$

15. A block of mass M is tied to one end of a massless rope. The other end of the rope is in the hands of a man of mass 2M. The block and the man are resting on a rough wedge of mass M. The whole system is resting on a smooth horizontal surface. The man starts walking towards right while holding the rope in his hands. Pulley is massless and frictionless. Find the displacement of the wedge when the block meets the pulley. Assume wedge is sufficiently long so that man does not fall down.



(*a*) 1/2 *m* towards right
(*b*) 1/2 *m* towards left
(*c*) The wedge does not move at all
(*d*) 1 *m* towards left

LINEAR MOMENTUM AND ITS CONSERVATION

When the sum of the forces on an object is zero, the equation

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

tells us that the time derivative of momentum is zero.

i.e., $\frac{d\vec{p}}{dt} = 0$

$\Rightarrow \vec{p} =$ constant

Consequently, one can state a conservation law for momentum:

When the net force on a particle is zero, its momentum is constant.

The real utility of the momentum conservation concept comes about when it is applied to a collection of particles. For a system of particles, the total momentum is simply the vector sum of the momentum of each of the particles in the system. i.e.,

$$\vec{P} = \sum_{i=1}^{N} \vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = M\vec{V}_{cm}$$

Now consider the net force on a system of particles.

There are two kinds of forces:

(i) Internal forces, resulting from the forces between the particles within the system.

(ii) External forces, arising from the forces between the particles in the system and objects outside the system.

For example, consider a system of two blocks joined together with a spring. If the system is allowed to fall freely under gravity, then the gravitational force acting on each block is the external force and the spring force acting on each block is internal.

When we calculate the net force on a system of particles by performing the vector sum, then the summation of all the internal forces is zero.

$$\sum \vec{F}_{\text{int}} = 0$$

Thus the momentum statement of the Newton's second law may be written as

$$\vec{F}_{net} = \sum\left(\vec{F}_{ext} + \vec{F}_{\text{int}}\right) = \frac{d\vec{P}}{dt}$$

$$\text{or} \sum \vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt} = M\vec{a}_{cm}$$

Now, if the summation of external forces is zero, then

$$\frac{d\vec{P}}{dt} = 0$$

Thus, $\vec{P} =$ constant $\left(\vec{v}_{cm} = \text{constant}\right)$

In the absence of a net external force, the momentum of a system is conserved.

The conservation of momentum law can be used to relate the initial motion of particles within a system to the motion of those same particles some time later. The law emphasizes the equality of momentum before and after something happens within the system.

Thus, the conservation law is usually written as

$$\vec{P}_{\text{initial}} = \vec{P}_{\text{final}} \qquad \sum_{i=1}^{N} \vec{p}_i = \sum_{f=1}^{N} \vec{p}_f$$

Train Your Brain

Example 10: The last stage of a rocket is traveling at a speed of 7600 m/s. This last stage is made up of two parts that are clamped together, namely, a rocket case with a mass of 290.0 kg and a payload capsule with a mass of 150.0 kg. When the clamp is released, a compressed spring causes the two parts to separate with a relative speed of 910.0 m/s. What are the speeds of the two parts after they have separated ? Assume that all velocities are along the same line.

Sol. We assume no external forces act on the system composed of the two parts.

So, total momentum of the system is conserved.

Now,

$(m_c + m_p) V = m_c v_c + m_p v_p$ ...(*i*)

Where,

m_c = mass of rocket case.

m_p = mass of payload capsule.

v_p = veloeity fo payload capsule.

v_c = veloeity of rocket case.

v = speed of rocket.

$\because\ v_p = v_c + v_{rel}$ ...(*ii*)

$$v_c = \frac{(m_c + m_p)v - m_p v_{rel}}{m_c + m_p}$$

So,

$$= \frac{(290+150)7600 - 150(910)}{290+150}$$

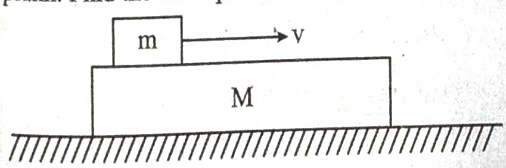
= 7290 m/s

Now, from equation (*ii*), we have

$v_p = v_c + v_{rel} = 7290 + 910$

= 8200 m/s

Example 11: A long plank of mass M, with a block of mass *m* placed on it, rests on a smooth horizontal surface . The block *m* is set in motion in the horizontal direction with velocity v. Due to friction between the plank and the block, the block slows down and moves in one piece with the plank. Find the work performed by frictional forces.



Sol. We choose the block and the plank as our system. Since friction is absent between the plank and the horizontal surface, the linear momentum in the horizontal direction will be conserved (No external force in the horizontal direction). But since there is friction between the plank and block (an internal non-conservative force), mechanical energy of the system will not be conserved . Hence

$P_i = P_f$ ...(*i*)

$\Delta KE + \Delta PE = W_{\text{friction}}$ ...(*ii*)

Let the final velocity of the block and plank be v′

From eqn (i) , we get

$mv = (m + M)\, v'$

$$\Rightarrow v' = \frac{m}{M+m} v$$ ...(*iii*)

Substituting the value of v' in eqn (ii) , we obtain

$$W_{friction} = \frac{1}{2}(m+M)v'^2 - \frac{1}{2}mv^2$$

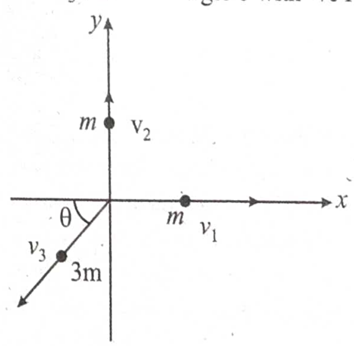
$$= -\frac{1}{2}\frac{mM}{M+m}v^2$$

Example 12: A body of mass 1kg initially at rest explodes and breaks into three fragments of masses in the ratio 1:1:3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 m/s. What is the velocity of the heavier fragment?

Sol. Internal forces can not change the momentum of the system

Let $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ be the velocities of the three fragments.

Let *x* and *y* axis point in the direction of $\vec{v}_1$ and $\vec{v}_2$ and let $\vec{v}_3$ make an angle θ with -ve X-axis.



Conserving momentum in *x* and *y* directions, we have

$mv_1 - 3mv_3 \cos\theta = 0$ ...(*i*)

$mv_2 - 3mv_3 \sin\theta = 0$ ...(*ii*)

Here, $v_1 = v_2 = 30$m/s

Solving eqn (*i*) and eqn (*ii*), we find

$\cos\theta = \sin\theta \Rightarrow \theta = 45°$

and $v_3 = \dfrac{\sqrt{2}v_1}{3} = \dfrac{\sqrt{2}}{3} \times 30 = 10\sqrt{2} m/s$

Therefore, the third fragment flies with a speed of $10\sqrt{2}$ m/s at an angle of 135° with $\vec{v}_2$

Concept Application

16. A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

(a) $\frac{mu}{m+M}$ (b) $\frac{mu}{m-M}$

(c) $\frac{2mu}{m-M}$ (d) $\frac{2mu}{m+M}$

17. A gun of mass M, fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height 'h'. The recoil velocity of the gun is

(a) $\left(\frac{2m^2gh}{M(m+M)}\right)^{\frac{1}{2}}$ (b) $\left(\frac{2m^2gh}{M(m-M)}\right)^{\frac{1}{2}}$

(c) $\left(\frac{2m^2gh}{2M(m-M)}\right)^{\frac{1}{2}}$ (d) $\left(\frac{2m^2gh}{2M(m+M)}\right)^{\frac{1}{2}}$

IMPULSE

In the previous chapter, we have learnt the concept of work which was an integral of force with respect to displacement. Now we are going to learn another concept, called impulse.

Impulse is defined as the integral of force with respect to time.

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}dt$$

Since force is a vector and time is a scalar, the result of the integral in above equation is a vector. If the force is constant (both in magnitude and direction), it may be removed from the integral so that the integral is reduced to

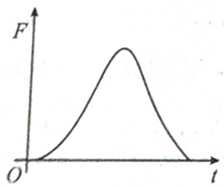
$$I = F\int_{t_i}^{t_f} dt = F(t_f - t_i) = F\Delta t$$

Graphically, the impulse is the area between the force curve and the $F = 0$ axis, as shown in figure.

The *SI* unit of impulse is *Ns*.

If more than one force are acting on a particle, then the net impulse is given by the time integral of the net force.

$$\vec{I}_{net} = \int_{t_i}^{t_f} \vec{F}_{net}dt$$



IMPULSE – MOMENTUM THEOREM

In the previous chapter, we have learnt that work done by a force brings about change in kinetic energy of a particle. Let us see what physical quantity changes due to impulse of a force.

According to Newton's second law, the net force acting on a particle is equal to the product of mass and acceleration.

$$\vec{F}_{net} = m\vec{a}$$

Since $\vec{a} = \frac{d\vec{v}}{dt}$, therefore

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt}$$

Substituting the value of net force, we get

$$\vec{I}_{net} = \int_{t_i}^{t_f}\left(m\frac{d\vec{v}}{dt}\right)dt$$

or $\vec{I}_{net} = \int_{v_i}^{v_f} m\,d\vec{v}$

Notice when we change the variable of integration from t to v, we must also change the time limits of the integral to the corresponding limits of v.

For constant mass,

$$\vec{I}_{net} = m\int_{v_i}^{v_f} d\vec{v}$$

or $\vec{I}_{net} = m(\vec{v}_f - \vec{v}_i) = m\vec{v}_f - m\vec{v}_i$

Thus $\vec{I}_{net} = \vec{p}_f - \vec{p}_i$

or $\vec{I}_{net} = \Delta\vec{p}$

The above equation shows that the net impulse of forces acting on a particle is equal to the change in momentum of the particle. This is called the **Impulse-Momentum Theorem.**

Train Your Brain

Example 13: Find the impulse due to the force $\vec{F} = a\hat{i} + bt\hat{j}$, where $a = 2$ N and b = 4 N/s, if this force acts from $t_i = 0$ to $t_f = 0.3$ s.

Sol. $\vec{I} = \int_{t_i}^{t_f}\vec{F}dt = \int_0^{0.3}\left(a\hat{i} + bt\hat{j}\right)dt$

or $\vec{I} = a\hat{i}\int_0^{0.3}dt + b\hat{j}\int_0^{0.3}tdt$

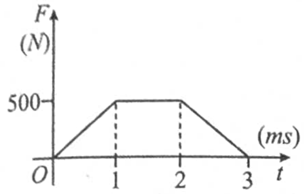
or $\vec{I} = \text{at}\hat{i}\Big|_0^{0.3} + \frac{bt^2}{2}\hat{j}\Big|_0^{0.3}$

or $\vec{I} = (2)(0.3)\hat{i} + \frac{(4)(0.3)^2\,\hat{j}}{2}$

thus $\vec{I} = \left(0.6\hat{i} + 0.18\hat{j}\right)$ Ns

Example 14: Figure shows the variation of force acting on a body with time. Calculate the impulse of this force.

Sol. Impulse of a force is the area under the graph.



Thus,

$$I = \left[\frac{1}{2}(500)(1) + (500)(2-1) + \frac{1}{2}(500)(3-2)\right] \times 10^{-3}$$

or $I = [250 + 500 + 250] \times 10^{-3}$

or $I = 1$ Ns

Example 15: A 2 kg bock is moving at a speed of 6 m/s. How large a force F is needed to stop the block in a time of 0.5 ms?

Sol. Impulse on block = Change in momentum of block

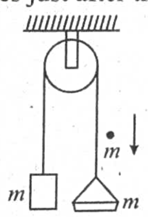
$Ft = mv_f - mv_i$

$F(5 \times 10^{-4}) = 2(0) - (2)(6)$

or $F = -2.4 \times 10^4$ N

The negative sign indicattes that the force opposes the motion.

Example 16: A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley as shown in figure. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v find the speed with which the system moves just after the collision.



Sol. Mass m strikes the pan with speed v and stick to it. Let speed of system just after collision is v_1. For pan and mass m.

Impulse = change in momentum

$-\int T\,dt = 2mv_1 - mv$

for block of mass m

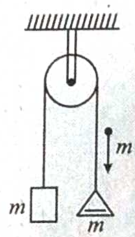
impulse = mv_1

$\int T\,dt = mv_1$

$-mv = 2mv_1 - mv$

$mv = 3mv_1$

$v_1 = v/3$



Concept Application

18. A ball falling with velocity $\vec{v}_i = \left(-0.65\hat{i} - 0.35\hat{j}\right)$ m/s is subjected to a net impulse $\vec{I} = \left(0.6\hat{i} - 0.18\hat{j}\right)$ Ns. If the ball has a mass of 275 g, calculate its velocity immediately following the impulse.

(a) $\vec{v}_f = \left(1.53\hat{i} - 0.305\hat{j}\right)$ m/s

(b) $\vec{v}_f = \left(0.305\hat{i} - 1.53\hat{j}\right)$ m/s

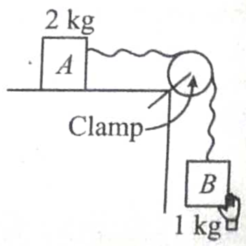
(c) $\vec{v}_f = \left(1.53\hat{i} + 0.305\hat{j}\right)$ m/s

(d) $\vec{v}_f = \left(0.305\hat{i} + 1.53\hat{j}\right)$ m/s

19. Two blocks A and B are joined by means of slacked string passing over a massless pulley as shown in diagram. The system is released from rest and it becomes taut when B false a distance 0.5 m, then

(a) Find the common velocity (m/s) of two blocks just after string become taut.

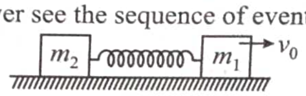
(b) Find the magnitude of impulse (Nm/s) on the pulley by the clamp during the small interval while string becomes taut.



SPRING BLOCK SYSTEMS

Analysis of Motion of Spring Block System in Ground Frame

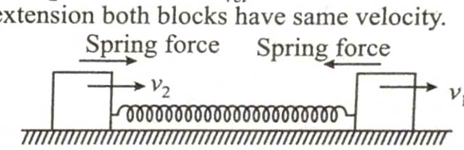
❖ Consider a two blocks system connected with a spring placed on a smooth surface. An impulse acts on frame block so that it acquires a velocity v_0. As then is no extexnal force in horizontal direction and non-conservative forces are absent, both momentum and mechanical energy are conserved. A ground observer see the sequence of events as follows.



$m_1 = m_2$

For simplicity, let us assume $m_1 = m_2 = m$

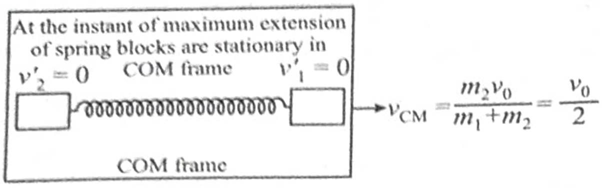
(a) Spring begins to stretch, now spring force retards m_1 but accelerates m_2. Stretch in spring continues till velocity v_1 and v_2 are equal i.e., v_{rel} is zero. At the state of maximum extension both blocks have same velocity.



In ground frame system translate forward while blocks also oscillate with respect to COM.

(*b*) Forward translation of system continues with velocity

$$v_{cm} = \frac{m_2 v_0}{m_1 + m_2} = \frac{v_0}{2}$$

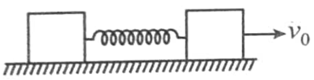


In the absence of any external force $\vec{v}_{CM}$ = const. At the instant of maximum stretch of spring both the blocks have same velocity which is equal to velocity of centre of mass.

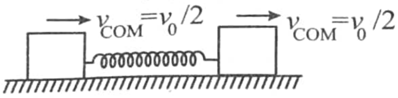
In COM frame blocks will be stationary at this moment. (Fig. ii)

(*c*) While translating forward spring begins to return to its natural length. When spring regains its natural length, velocity of left block is v_0 and right block is at rest. (Fig. iii)

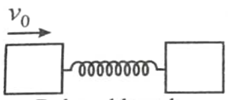
(*d*) At the instant of maximum compression velocity of both blocks is same in ground frame. (Fig. iv)



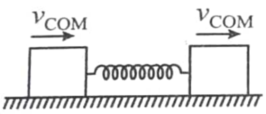
Initial state
(i)



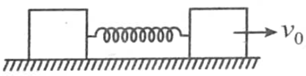
Maximum extension
(ii)



Relaxed length
(iii)



Maximum compression
(iv)



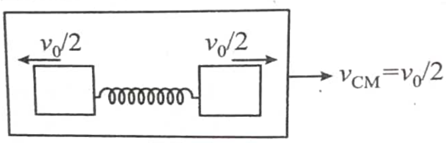
Relaxed length
(v)

In ground reference frame system moves forward with spring getting compressed and relaxed, then extended alternatively.

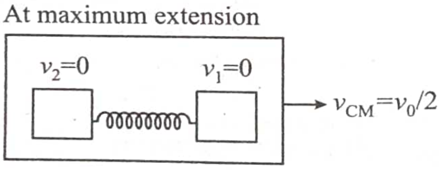
Analysis of Motion of Spring Block System in COM Frame

Now lets see what- happens in COM frame.

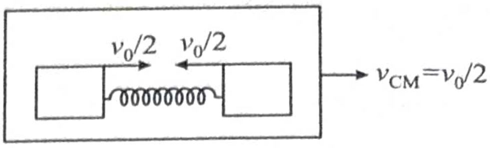
(*a*) Initial state



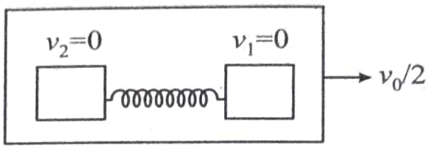
(*b*) At maximum extension



(*c*) At relaxed length of spring



(*d*) At maximum compression



In COM frame the observer sees only oscillatory motion of blocks.

At the maximum extension of spring blocks are at rest in COM frame.

or $-\left[\frac{1}{2}kx_{max}^2 - 0\right] = \Delta KE_{system/CM}$

or $-\frac{1}{2}kx_{max}^2 = \left[\frac{1}{2}m(v_0/2)^2 + \frac{1}{2}m(v_0/2)^2\right]$

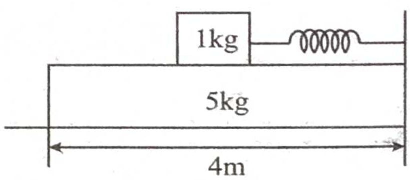
or $x_{max} = \sqrt{\frac{m}{2k}}v_0$

Note:

- COM frame is also known as zero-momentum frame because momentum is always zero in this frame.
- To solve the complex problem, COM frame is a convenient way than in ground frame.

Train Your Brain

Example 17: A plank of mass 5kg placed on a frictionless horizontal plane. Further a block of mass 1kg is placed over the plank. A massless spring of natural length 2 *m* is fixed to the plank by its one end. The other end of spring is compressed by the block by half of spring's natural length. The system is now released from the rest. What is the velocity of the plank when block leaves the plank? All the surfaces are smoth. (The stiffness constant of spring is 100 Nm^{-1})



Sol. Let the velocities of the block and the plank, when the block leaves the spring, be u and v, respectively.

By conservation of energy, $\frac{1}{2}kx^2 = \frac{1}{2}mu^2 + \frac{1}{2}Mv^2$

(where, M = mass of the plank, m = mass of the block)

$\Rightarrow 100 = u^2 + 5v^2$

By conservation of momentum,

$mu + Mv = 0 \Rightarrow u = -5v$

Solving Eqs. (i) and (ii), we get

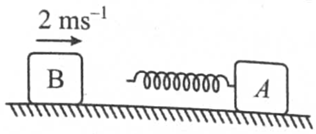
$30v^2 = 100$

$\Rightarrow v = \sqrt{\frac{10}{3}}\ \text{ms}^{-1}$

From this moment until block falls, both plank and block keep their velocity constant.

Thus, when block falls, velocity of plank $= \sqrt{10/3}\ \text{ms}^{-1}$.

Example 18: Two blocks A and B of equal mass $m = 1$ kg are lying on a smooth horizontal surface as shown in figure. A spring of force constant $k = 200\ \text{Nm}^{-1}$ is fixed at one end of block A. Block B collides with block A with velocity $v_0 = 2\ \text{ms}^{-1}$. Find the maximum compression of the spring.



Sol. At maximum compression (x_m), velocity of both the blocks is same, say v

Applying conservation of linear momentum, we have

$(m_A + m_B)v = m_B v_0$

or $(1 + 1)\, v = (1)\, v_0$

or $v = \frac{v_0}{2} = \frac{2}{2} = 1\ \text{ms}^{-1}$

Using conservation of mechanical energy, we have

$\frac{1}{2}m_B v_0^2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kx_m^2$

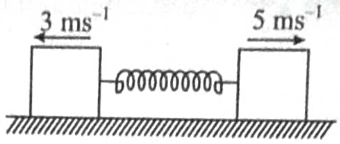
Substituting the given values in above equation, we get

$\frac{1}{2}\times(1)\times(2)^2 = \frac{1}{2}\times(1+1)\times(1)^2 + \frac{1}{2}\times(200)\times x_m^2$

or $x_m = 0.1\ \text{m} = 10\ \text{cm}$

Concept Application

20. Two identical blocks each of mass 1 kg are joined together with a compressed spring. When the system is released from rest the two blocks appear to be moving with unequal speeds in the opposite direction as shown in figure below. Choose the correct statements (s):



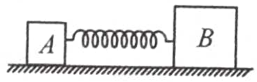
(*a*) Total momentum of the system isn't conserved.

(*b*) Whatever may be the speed of the blocks the centre of mass will remain stationary

(*c*) The centre of mass of the system is moving with a velocity of 2 ms^{-1}

(*d*) The centre of mass of the system is moving with a velocity of 1 ms^{-1}

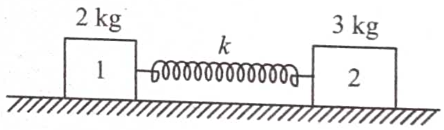
21. Two blocks A and B of mass m and 2m are connected by a massless spring of force constant k and placed on a smooth horizontal plane. The spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is



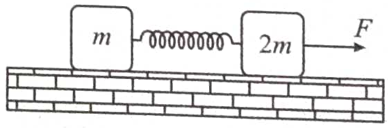
(*a*) $\left(\sqrt{\frac{3k}{2m}}\right)x$ (*b*) $\left(\sqrt{\frac{2k}{3m}}\right)x$

(*c*) $\sqrt{\frac{2kx}{m}}$ (*d*) $\sqrt{\frac{3km}{2x}}$

22. A spring of free length 15 cm is connected to the two masses as shown in the figure and compressed 5 cm. The system is released on a smooth horizontal surface. Find the speed (m/s) of each block when the spring is again at its free length: The force constant for spring is 2100 Nm^{-1}.



23. Two blocks of masses m and $2m$ are kept on a smooth horizontal surface. They are connected by an ideal spring of force constant k. Initially, the spring is unstretched. A constant force is applied to the heavier block in the direction as shown in figure. Suppose at time t, displacement of smaller block is x, then displacement of the heavier block at this moment would be



(*a*) $\frac{x}{2}$ (*b*) $\frac{Ft^2}{6m} + \frac{x}{3}$

(*c*) $\frac{x}{3}$ (*d*) $\frac{Ft^2}{4m} - \frac{x}{2}$

COLLISIONS

Collision is a brief event between objects that contact each other. The interaction between two or more objects is called a collision if there exists three identifiable stages to this interaction: before, during and after. In the before and after stage the interaction forces are zero or approaches zero asymptotically. Between these two states the interaction forces are large and often the dominating forces governing the object's motion. The magnitude of the interacting force is often unknown. Therefore, the conservation of momentum statement is useful for relating the initial velocities before the interaction to the final velocities after the interaction without requiring a detailed knowledge of the interaction forces.

Types of Collision

Collisions may be either elastic or inelastic. Linear momentum is conserved in both cases. A perfectly elastic collision is defined as one in which the total kinetic energy of the particles is also conserved.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

In an inelastic collision, the total kinetic energy of the particle changes. Some of the kinetic energy is stored as potential energy associated with a change in internal structure or state, and is not immediately recovered. Some of the energy may be used to raise the system (e.g. an atom) to a state with higher energy. Or, it may be converted into thermal energy of vibrating atoms and molecules or into light, sound or some other form of energy.

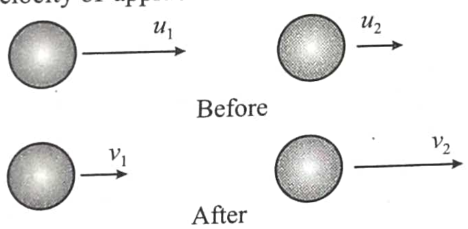
In a completely inelastic collision, the two bodies couple or stick together.

Coefficient of Restitution (e)

The elasticity of collision may be measured in terms of a dimensionless parameter called the coefficient of restitution (e).

It is defined as the ratio of velocity of separation to the velocity of approach of the two colliding bodies

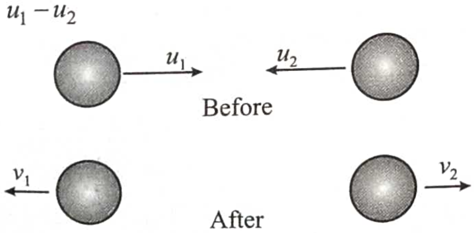
$$e = \frac{\text{Velocity of separation}}{\text{Velocity of appraoch}}$$



Velocity of separation $= v_2 - v_1$

Velocity of approach $= u_1 - u_2$

$$\therefore\ e = \frac{v_2 - v_1}{u_1 - u_2}$$



Velocity of separation $= v_1 + v_2$

Velocity of approach $= u_1 + u_2$

$$\therefore\ e = \frac{v_1 + v_2}{u_1 + u_2}$$

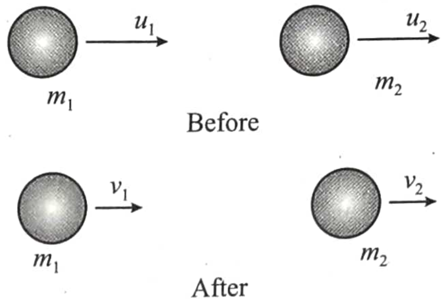
For an elastic collision: $e = 1$

For an inelastic collision: $0 < e < 1$ Partially elastic or partially inelastic

For completely inelastic collision: $e = 0$

ELASTIC COLLISION IN ONE DIMENSION

Given figure shows a one - dimensional elastic collision between two balls.



Applying momentum conservation,

$P_i = P_f$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or $m_1(u_1 - v_1) = m_2(v_2 - u_2)$

Taking e = 1, velocity of separation = velocity of approach

or $v_2 - v_1 = u_1 - u_2$

Let us consider two special cases of elastic collisions: First, when the particles have equal mass, and second when one of them, say m_2, is initially at rest.

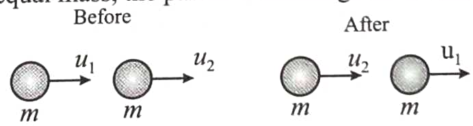
(i) Equal masses: $m_1 = m_2 = m$

The above equations become $u_1 + u_2 = v_1 + v_2$ and $v_2 - v_1 = u_1 - u_2$

Solving above equations

$v_1 = u_2$ and $v_2 = u_1$

i.e. in case of one dimensional elastic collision of particles of equal mass, the particles exchange their velocities.



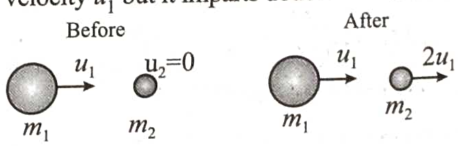
(ii) Unequal masses

$m_1 \neq m_2$. Target at Rest: $u_2 = 0$

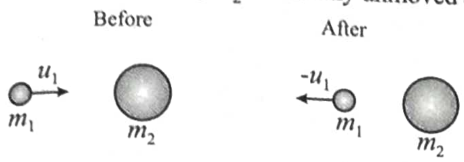
In this case, we get

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} \text{ and } v_2 = \frac{2m_1u_1}{m_1 + m_2}$$

(a) When $m_1 >> m_2$, we may ignore the mass of m_2 in comparison with m_1. This leads to $v_1 = u_1$ and $v_2 = 2u_1$, which means that m_1 maintains (approximately) its initial velocity u_1 but it imparts double this value to m_2.

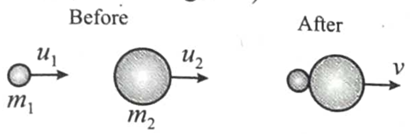


(*b*) When $m_1 << m_2$ we may ignore m_1 in comparison with m_2. We then find that $v_1 = u_1$ and $v_2 = 0$. Thus, m_1 reverses its velocity, leaving m_2 essentially unmoved as in figure.



INELASTIC COLLISION IN ONE DIMENSION

In case of inelastic collision, after collision the two bodies move with same velocity (or stick together).



If two particles of masses m_1 and m_2 moving with velocities u_1 and u_2 ($< u_1$) respectively along the same line collide "head-on" and after collision they have a common velocity v, then by conservation of linear momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\text{or } v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

The kinetic energy of the system before collision is

$$K_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

and kinetic energy after collision is

$$K_f = \frac{1}{2}(m_1 + m_2)v^2$$

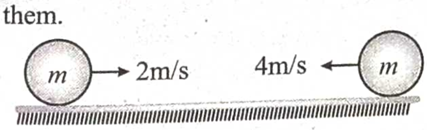
Therefore, loss in kinetic energy during the collision is

$$K_i - K_f = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

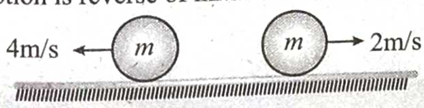
$$\frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

Train Your Brain

Example 19: Two identical balls are approaching towards each other on a straight line with velocity 2 m/s and 4 m/s respectively. Find the final velocities, after elastic collision between them.



Sol. The two velocities will be exchanged and the final motion is reverse of initial motion for both.



Example 20: Two elastic bodies *P* and *Q* having equal masses are moving along the same line with velocities of 16 m/s and 10m/s respectively. Their velocities after the elastic collision will be in m/s -

(*a*) 0 and 25 (*b*) 5 and 20

(*c*) 10 and 16 (*d*) 20 and 5

Sol. (*c*) Since collision is elastic so from linear momentum conservation

$$m \times 26 = m(v_1 + v_2);\ v_1 + v_2 = 26 \qquad ...(i)$$

From K.E. conservation

$$\frac{1}{2}m(16^2 + 10^2) = \frac{1}{2}m(v_1^2 + v_2^2);$$

$$356 = {v_1}^2 + {v_2}^2 \qquad ...(ii)$$

From equation (*i*) & (*ii*)

$v_1 = 10$ m/s

$v_2 = 16$ m/s

option (*c*) is correct

Example 21: Two solid balls of rubber *A* and *B* whose masses are 200gm and 400gm respectively, are moving in mutually opposite directions. If the velocity *A* is 0.3 m/s and both the balls come to rest after collision, then the velocity of ball *B* is-

(*a*) 0.15 ms^{-1} (*b*) –0.15 ms^{-1}

(*c*) 1.5 ms^{-1} (*d*) none of these

Sol. (*b*)

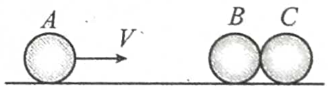
From momentum conservation

$P_1 + P_2 = 0$[Given : $m_1 = 200$ gm $m_2 = 400$ gm]

$m(0.3) + 2m.V_B = 0$

$V_B = -\frac{0.3}{2} = -0.15\ ms^{-1}$ so correct options is (*b*)

Example 22: As shown in figure *A*, *B* and *C* are identical balls *B* and *C* are at rest and, the ball *A* is moving with velocity *v* collides elastically with ball B, then after collision-



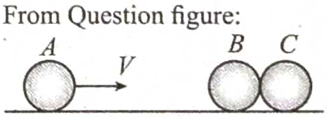
(*a*) All the three balls move with velocity *v*/2

(*b*) A comes to rest and (*B* + *C*) moves with velocit $v/\sqrt{2}$

(*c*) A moves with velocity *v* and (*B* + *C*) moves with velocity v

(*d*) *A* and *B* come to rest and *C* moves with velocity

Sol. (*d*) From Question figure:



It should follow momentum conservation

$$p_A = p_B' + p_C'$$

from option (a), $mv = m\left(\frac{v}{2}+\frac{v}{2}+\frac{v}{2}\right) = \frac{3}{2}mv$

$mv \neq \frac{3}{2}mv$ so its wrong

Option (B), $mv = m\times 0 + m\left(\frac{v}{\sqrt{2}}+\frac{v}{\sqrt{2}}\right)$

$mv \neq \sqrt{2}mv$ & its wrong

Option (c), $mv = mv + 2\,mv = 3\,mv$ (wrong)

Option (d), $mv = m\times 0 + m\times 0 + mv$

$mv = mv$; or $p_i = p_f$

So option (d) is correct option

Example 23: A sphere of mass m moving with velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, what is the ratio of velocities of two spheres after the collision?

Sol. By definition of coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

i.e. $v_2 - v_1 = eu$ ...(i)

And by conservation of momentum

$mu = mv_1 + mv_2$

or $v_2 + v_1 = u$ ...(ii)

Solving equations (i) and (ii) for v_1 and v_2, we get

$v_1 = u(1-e)/2$ and $v_2 = u(1+e)/2$

so $\frac{v_1}{v_2} = \left[\frac{1-e}{1+e}\right]$

Example 24: A large block of wood of mass M is suspended from a ceiling by two cords of length L. A bullet of mass m is fired into the block, comes quickly to rest in the block. As a result of this, the block swings through an angle θ_0. Find the velocity of the bullet (such a device is called ballistic pendulum).

Sol. During collision, momentum of the system is conserved. Hence

$mv = (M+m)V$ ...(i)

where V is the velocity of system immediately after the collision.

After the collision, mechanical energy will be conserved. If the block swings by an angle θ_0, the center of mass of the system rises by

$h = L - L\cos\theta_0 = L(1-\cos\theta_0)$

$$\Rightarrow \frac{1}{2}(M+m)V^2 = (M+m)gL(1-\cos\theta_o) \quad ...(ii)$$

$$V = \sqrt{2gL(1-\cos\theta_o)} \quad ...(iii)$$

Substituting this value in eqn (i), we get

$$v = \frac{M+m}{m}\sqrt{2gL(1-\cos\theta_o)}$$

Example 25: A moving particle of mass m makes a head on elastic collision with a particle of mass 2m which is initially at rest. Show that the colliding particle losses (8/9)th of its energy after collision.

Sol. Let u be the initial velocity of body of mass m and v_1 and v_2 be the velocities of bodies of mass m and 2m respectively after the collision. As in collision momentum is always conserved

$mu = mv_1 + 2mv_2$

i.e. $v_1 + 2v_2 = u$ or $(u - v_1) = 2v_2$ ...(i)

And as the collision in elastic

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2$$

i.e. $v_1^2 + 2v_2^2 = u^2$ or $\left(u^2 - v_1^2\right) = 2v_2^2$ ...(ii)

Dividing equation (ii) by (i), we get

$u + v_1 = v_2$

Substituting this value of v_2 in equation (i),

$u - v_1 = 2(u + v_1)$, i.e., $v_1 = -(u/3)$

$$\text{So } K_f = \frac{1}{2}mv_1^2 = \frac{1}{2}m\left(-\frac{u}{3}\right)^2 = \frac{1}{2}mu^2 \times \frac{1}{9} = \frac{1}{9}K_i$$

$\left[\text{as } K_i = \frac{1}{2}mu^2\right]$

$$\therefore \frac{K_i - K_f}{K_i} = \frac{8}{9}$$

Concept Application

24. A smooth small spherical ball of mass m, moving with velocity u collides head on with another small spherical ball of mass 3m. which was initially at rest. Two-third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) zero

25. Two balls of masses 2 kg and 4 kg are moved towards each other with velocities 4 m/s and 2 m/s, respectively, on a frictionless surface. After colliding, the 2 kg ball returns back with velocity 2 m/s. Then find

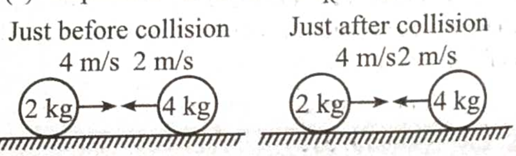
(a) Velocity (m/s) of the 4 kg ball after collision

(b) Coefficient of restitution e;

(c) Impulse of deformation J_D (Ns);

(d) Maximum potential energy (J) of deformation;

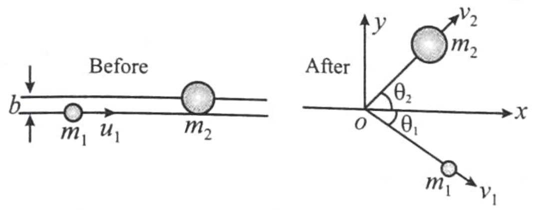
(e) Impulse of reformation J_R (Ns);



COLLISION IN TWO DIMENSIONS

In two or three dimensions (except for a completely inelastic collision) the conservation laws alone cannot tell us the motion of particles after a collision, if we know the motion before the collision. For example, for a two-dimensional elastic collision, which is simplest case, we have four unknowns, namely the two components of velocity for each of two particles after collision, but we have only three known relations between them, one for the conservation of kinetic energy and a conservation of momentum relation for each of the two dimensions. Hence we need more information than just the initial conditions. When we do not know the actual forces of interaction, as is often the case, the additional information must be obtained from experiment. It is simplest to specify the angle of recoil of one of the colliding particles.

A typical situation is shown in figure. The distance b between the initial line of motion and a line parallel to it through the center of the target particle is called the impact parameter. This is a measure of the directness of the collision, $b = 0$, corresponding to a head-on collision. The direction of motion of the incident particle m_1 after collision makes an angle θ_1 with the initial direction, and the target projectile m_2, initially at rest, moves in a direction after collision making an angle θ_2 with the initial direction of the incident projectile.



Applying the conservation of momentum, which is a vector relation, we obtain two scalar equations; for the x-component of motion we have

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

and for the y-component

$$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2$$

If the collision is an elastic one we have,

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

We have four unknowns v_1, v_2, θ_1 and θ_2 and only three equations relating them.

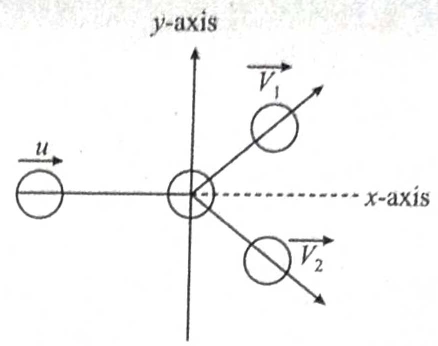
We can determine the motion after collision only if we specify a value for one of these quantities such as θ_1.

Train Your Brain

Example 26: A ball moving with a speed of 9 m/s strikes identical stationary ball such that after the collision, the direction of each ball makes an angle of 30° with the original line of motion.

(*a*) Find the speed of the balls after the collision

(*b*) Is kinetic energy conserved in the collision ?



Sol. (*a*) Choose x-axis along the direction of $\vec{u}$ and y-axis perpendicular to its direction.

Applying conservation of linear momentum along X and Y-axis , we have

$mu = mv_1\cos 30° + mv_2\cos 30°$ ...(*i*)

and $0 = mv_1\sin 30° - mv_2\sin 30°$ ...(*ii*)

After solving eqn (i) and eqn (ii) we find

$v_1 = v_2 = 3\sqrt{3}$ m/s.

(*b*) Let K_1 and K_2 be the kinetic energy before and after the collision. Then

$$K_1 = \frac{1}{2}mu^2 + 0 = \frac{81}{2}m$$

and

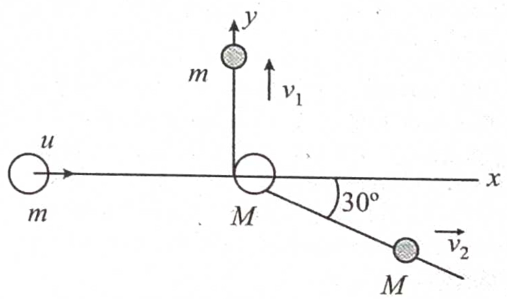
$$K_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}M\left(3\sqrt{3}\right)^2 \times 2$$

$= 27m$

Since $K_2 < K_1$, kinetic energy of the system is not conserved

Example 27: A particle of mass m having collided with a stationary particle of mass M deviated by an angle $\pi/2$ whereas particle M recoiled at an angle θ = 30° to the direction of initial motion of particle m. How much and in what way has the kinetic energy changed after the collision if $\frac{M}{m} = 5$.

Sol.



From conservation of momentum in X and Y direction, we have

$mu = Mv_2\cos 30°$ ...(*i*)

$0 = mv_1 - Mv_2\sin 30°$ ...(*ii*)

Solving eqn (*i*) and eqn (*ii*) for v_1 and v_2, we get

$$v_1 = u/\sqrt{3}, v_2 = \frac{m}{M}\frac{2u}{\sqrt{3}}$$

Let K_1 and K_2 be the initial and final kinetic energy of the particles. Then

$$K_1 = \frac{1}{2}mu^2$$

and $$K_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$= \frac{1}{2}m\left[\frac{1}{3} + \frac{4m}{3M}\right]u^2$$

The percentage loss in K.E. is

$$\frac{K_1 - K_2}{K_1} \times 100 = \frac{2}{5} \times 100 = 40\%$$

Example 28: A gas molecule having a speed of 300 m/s collides elastically with another molecule of the same mass which is initially at rest. After the collision the first molecule moves at an angle of 30° to its initial direction. Find the speed of each molecule after collision and the angle made with the incident direction by the recoiling target molecule.

Sol. This application corresponds exactly to the situation just discussed, with $m_1 = m_2$, the equations become

$$u_1 = v_1\cos\theta_1 + v_2\cos\theta_2 \quad ...(i)$$

$$v_1\sin\theta_1 = v_2\sin\theta_2 \quad ...(ii)$$

and $$u_1^2 = v_1^2 + v_2^2 \quad ...(iii)$$

Squaring and adding the first two equations, we get

$$u_1^2 + v_1^2 - 2u_1v_1\cos\theta_1 = v_2^2$$

Combining this with the third equation, we obtain

$$2v_1^2 = 2u_1v_1\cos\theta_1$$

or (since $u_1 \neq 0$) $v_1 = u_1\cos\theta_1 = 260$ m/s

From the third equation, $v_2 = 150$ m/s

and finally from the second equation,

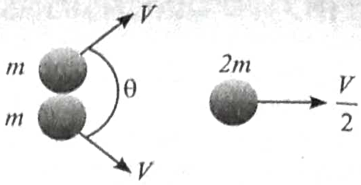
$\theta_2 = 60°$.

The molecules move apart at right angles ($\theta_1 + \theta_2 = 90°$).

Note: The student should be able to show that in an elastic collision between particles of equal mass, one of which is initially at rest, the recoiling particles always move off at right angles to one another.

Concept Application

26. After perfectly inelastic collision between two identical balls moving with same speed in different directions, the speed of the combined mass becomes half the initial speed. Find the angle between the two before collision.



(a) $\pi/6$ (b) $\pi/3$

(c) $2\pi/3$ (d) $\pi/2$

27. Three identical particles with velocities $v_0\hat{i}$, $3v_0\hat{j}$ and $5v_0\hat{k}$ collide successively with each other in such a way that they form a single particle. The velocity vector of resultant particle is

(a) $\frac{v_o}{3}(\hat{i} + \hat{j} + \hat{k})$ (b) $\frac{v_o}{3}(\hat{i} - \hat{j} + \hat{k})$

(c) $\frac{v_o}{3}(\hat{i} - 3\hat{j} + \hat{k})$ (d) $\frac{v_o}{3}(\hat{i} - 3\hat{j} + 5\hat{k})$

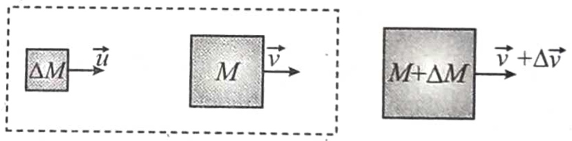
VARIABLE MASS SYSTEM

So far we have dealt with the dynamics of a system whose mass is constant. We now discuss the dynamics of a system, such as a rocket, whose mass varies. One may try to apply the following equation in such a case.

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\frac{d\vec{v}}{dt} + \vec{v}\frac{dM}{dt}$$

However, this equation is **correct** in only a few very special cases. This approach is **not correct** when mass actually enters or leaves the system.

Let us examine the motion of a body of mass M, moving with velocity **v**. Another small body of mass ΔM approaches with velocity $\vec{u}$ along the same line. We assume $\vec{u} > \vec{v}$ and that after the collision the two bodies stick together and move at velocity $(\vec{v} + \Delta\vec{v})$, as shown in the figure.



Before After

By defining the system comprising of both the bodies we can use $\vec{F}_{ext} = \frac{d\vec{P}}{dt}$, where $\vec{P}$ is the total momentum of the system of constant mass, $M + \Delta M$. The change in momentum of the system in time Δt is

$$\Delta\vec{P} = (M + \Delta M)(\vec{v} + \Delta\vec{v}) - M\vec{v} - \Delta M\vec{u}$$

$$= M\Delta\vec{v} - (\vec{u} - \vec{v})\Delta M$$

Now $(\vec{u} - \vec{v}) = \vec{v}_{rel}$ is the velocity of ΔM relative to M before the collision.

We divide both sides of this equation by Δt and take the limit as $\Delta t \to 0$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\frac{d\vec{v}}{dt} - \vec{v}_{rel}\frac{dM}{dt}$$

It is convenient to rewrite it in the form

$$M\frac{d\vec{v}}{dt} = \vec{F}_{ext} + \vec{v}_{rel}\frac{dM}{dt}$$

The last term in the above equation $\left(\vec{v}_{rel}\frac{dM}{dt}\right)$, is the rate at which momentum is being transferred into (or out of) the system by the mass that system has collected (or ejected). It can be interpreted as the force exerted on the system by the mass that leaves or joins it. This force is referred to as the reaction force.

ROCKET PROPULSION

Consider a rocket of mass M with fuel of mass Δm. Their common velocity is v relative to some inertial frame. When the rocket engines are fired, the gases are expelled backward with an exhaust velocity $\vec{v}_{ex} = -v_{ex}\hat{i}$ relative to the rocket. This is a fixed quantity determined by the design of the engine and the type of fuel. If the rocket's velocity changes to $\vec{v}+\Delta\vec{v}$ relative to the inertial frame, then the velocity of gas with respect to the frame will be

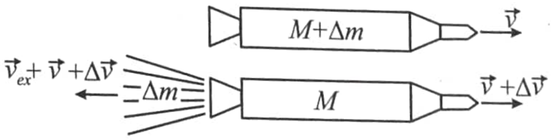
$$\vec{v}_{gas} = \vec{v}_{ex} + \vec{v} + \Delta\vec{v} = (-v_{ex} + v + \Delta v)\ \hat{i}$$

Applying the law of conservation of momentum, we get

$$(M + \Delta m)\ v$$
$$= M(v + \Delta v) + \Delta m(-v_{ex} + v + \Delta v)$$

After some cancellation we find

$$0 = M\,\Delta v + \Delta m(-v_{ex} + \Delta v)$$



If both Δv and Δm are small quantities relative to v and M, respectively, their product , $\Delta v\Delta m$, is negligible in comparison with the other terms, and we are left with

$$\Delta v = v_{ex}\frac{\Delta m}{M}$$

Since an increase in the mass of the expelled gases corresponds exactly to the loss in mass of the rocket system, we have $\Delta m = -\Delta M$. In the limit as $\Delta M \to 0$ the above equation becomes

$$dv = -v_{ex}\frac{dM}{M}$$

On integrating both sides,

$$\int_{v_i}^{v_f} dv = -\int_{M_i}^{M_f} v_{ex}\frac{dM}{M}$$

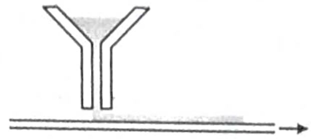
We find

$$v_f - v_i = v_{ex}\ln\frac{M_i}{M_f}$$

Train Your Brain

Example 29: A hopper releases grain at a rate dm/dt onto a conveyor belt that moves at a constant speed v. What is the power of the motor driving the belt?

Sol.



Let the system has some arbitrary length of belt whose mass we can call M. This mass of the system increases at the same rate as that the grain falls,

so $dM/dt = dm/dt$.

Since the grain falls vertically, $\vec{u} = 0$ and $\vec{v}_{rel} = \vec{u} - \vec{v} = -\vec{v}$. Since the speed is constant,

$dv/dt = 0$. Thus from equation for the system of variable mass

$$\vec{F}_{ext} - \vec{v}\frac{dm}{dt} = 0 \Rightarrow \vec{F}_{ext} = \vec{v}\frac{dm}{dt}$$

where $\vec{F}_{ext}$ is the force needed to maintain constant speed because the mass is increasing. The power required $\left(P = \vec{F}\cdot\vec{v}\right)$ is

$$P = V^2\frac{dm}{dt}$$

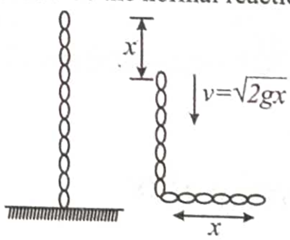
It is interesting to compare this with the rate at which the kinetic energy of the grain increases.

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}V^2\frac{dm}{dt}$$

This is only half the power input. The other half is dissipated as heat when the grain lands on the belt and slips relative to it.

Example 30: A uniform chain of mass m and length l hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the table on the chain. When half of its length has fallen on the table. The fallen part does not form heap.

Sol. Let N be the normal reaction acting on the chain.



It consists of two parts

$$N = N_{wt} + N_{thrust}$$

$$N_{wt} = \frac{m}{l}xg$$

$$N_{\text{thrust}} = v_{\text{rel}} \frac{dm}{dt}$$

If v be the instantaneous velocity of the particle falling on the floor, then

$v_{\text{rel}} = v - 0 = v$

$$\frac{dm}{dt} = \frac{dm}{dx}\frac{dx}{dt} \text{ or } \frac{dm}{dt} = v\left(\frac{m}{l}\right)$$

Thus $N_{th} = v^2\left(\frac{m}{l}\right)$, Since $v = \sqrt{2gx}$ $\therefore$ $N_{th} = \frac{2mg}{l}x$

Total reaction is

$$N = \frac{mgx}{l} + \frac{2mgx}{l},\ N = 3mg\left(\frac{x}{l}\right)$$

$\{\because$ for $x = l/2\}$

$$N = \frac{3}{2}mg$$

Concept Application

28. A wagon filled with sand has a hole so that sand leaks through the bottom at a constant rate λ. An external force $\vec{F}$ acts on the wagon in the direction of motion. Assuming instantaneous velocity of the wagon to be $\vec{v}$ and initial mass of system to be m_0, the force equation governing the motion of the wagon is :

(a) $\vec{F} = m_0\frac{d\vec{v}}{dt} + \lambda\vec{v}$ (b) $\vec{F} = m_0\frac{d\vec{v}}{dt} - \lambda\vec{v}$

(c) $\vec{F} = (m_0 - \lambda t)\frac{d\vec{v}}{dt}$ (d) $\vec{F} = (m_0 - \lambda t)\frac{d\vec{v}}{dt} + \lambda\vec{v}$

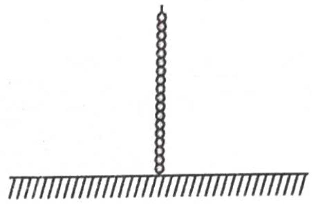
29. An open water tight railway wagon of mass 5×10^3 kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected 10^3 kg of water will be

(a) 0.5 m/s (b) 2 m/s

(c) 1 m/s (d) 1.5 m/s

30. A rocket, with an initial mass of 1000 kg, is launched vertically upwards from rest under gravity. The rocket burns fuel at the rate of 10 kg per second. The burnt matter is ejected vertically downwards with a speed of 2000 m/s relative to the rocket. Find the velocity (m/s) of the rocket after 1 min of start.

31. A uniform rope of mass m per unit length, hangs vertically from a support so that the lower end just touches the table top shown in figure. If it is released, at the time a length y of the rope has fallen, the force on the table is equivalent to the weight of a length ny of the rope. Find the value of n.



32. Sand drops from a stationary hopper at the rate of 5 kg/s on to a conveyor belt moving with a constant speed of 2 m/s. What is the force (N) required to keep the belt moving and what is the power (W) delivered by the motor, moving the belt ?

Short Notes

Centre of Mass of a System of 'N' Discrete Particles

$$r_{cm} = \frac{m_1r_1 + m_2r_2 + \cdots + m_nr_n}{m_1 + m_2 + \cdots + m_n};\ r_{cm} = \frac{\sum_{i=1}^{n} m_ir_i}{\sum_{i=1}^{n} m_i} \quad r_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_ir_i$$

Centre of Mass of a Continuous Mass Distribution

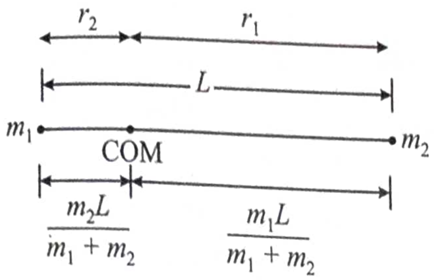
$$x_{cm} = \frac{\int xdm}{\int dm},\ y_{cm} = \frac{\int ydm}{\int dm},\ z_{cm} = \frac{\int zdm}{\int dm}$$

$$\int dm = M \text{ (mass of the body)}$$

Centre of Mass of Some Common Systems

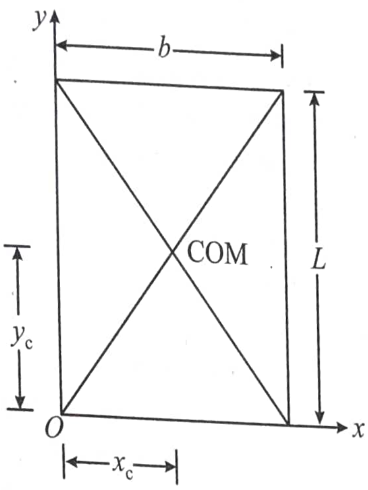
❖ **System of two point masses.**

$m_1r_1 = m_2r_2$; The centre of mass lies closer to the heavier mass.



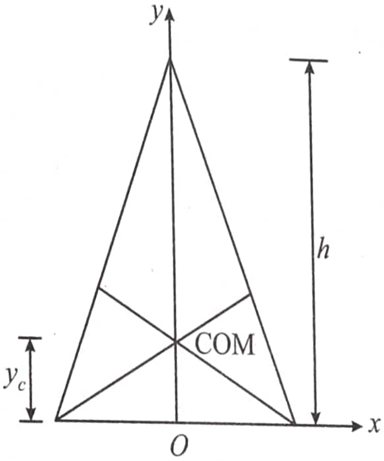
❖ **Rectangular plate**

$x_c = \frac{b}{2}$; $y_c = \frac{L}{2}$



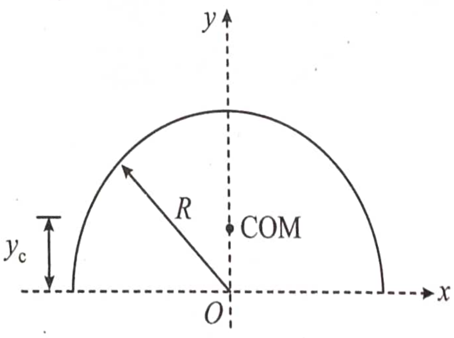
❖ **A triangular plate at the centroid**

$y_c = \frac{h}{3}$



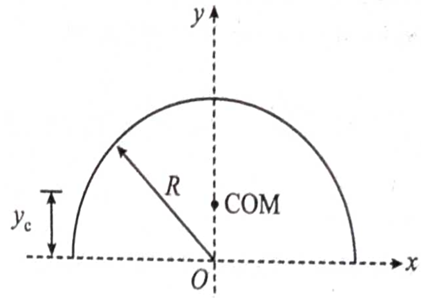
❖ **A semi-circular ring**

$y_c = \frac{2R}{\pi}$; $x_c = 0$



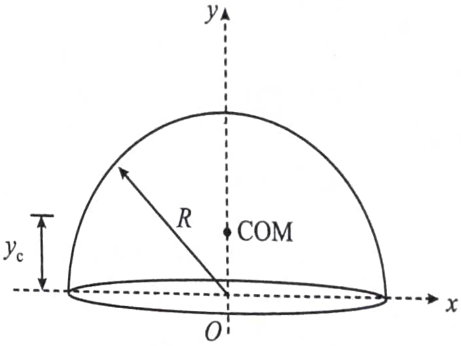
❖ **A semi-circular disc**

$y_c = \frac{4R}{3\pi}$; $x_c = 0$



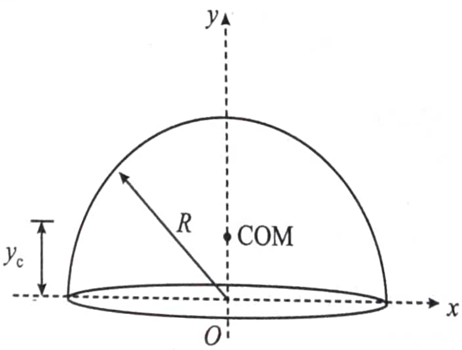
❖ **A hemispherical shell**

$y_c = \frac{R}{2}$; $x_c = 0$



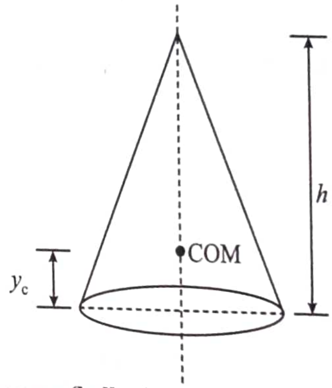
❖ **A solid hemisphere**

$y_c = \frac{3R}{8}$; $x_c = 0$



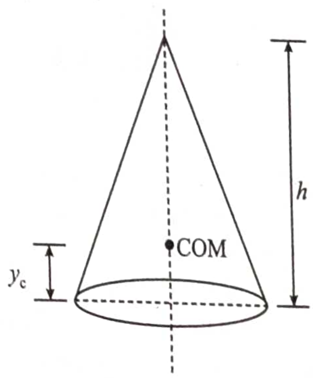
❖ **A circular cone (solid)**

$y_c = \frac{h}{4}$



❖ **A circular cone (hollow)**

$y_c = \frac{h}{3}$



Motion of Centre of Mass

Velocity of Centre of Mass of System

$$\vec{v}_{cm} = \frac{m_1\frac{d\vec{r_1}}{dt} + m_2\frac{d\vec{r_2}}{dt} + m_3\frac{d\vec{r_3}}{dt} \dots + m_n\frac{d\vec{r_n}}{dt}}{M} = \frac{m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3} \dots + m_n\vec{v_n}}{M}$$

$$\vec{P}_{sys} = M\vec{v}_{cm}$$

Acceleration of Centre of Mass of System

$$\vec{a}_{cm} = \frac{m_1\frac{d\vec{V_1}}{dt} + m_2\frac{d\vec{V_2}}{dt} + m_3\frac{d\vec{V_3}}{dt} \dots + m_n\frac{d\vec{V_n}}{dt}}{M}$$

$$= \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 \dots + m_n\vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M}$$

$$= \frac{\text{Net external force + Net internal force}}{M}$$

$$= \frac{\text{Net External Force}}{M} \qquad (\because \quad \Sigma \text{ Internal force} = 0)$$

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

Impulse

❖ Impulse of a force F on a body is defined as

$$I = \int_{t_i}^{t_f} \vec{F}\, dt = \int_{t_i}^{t_f} d\vec{P} = \Delta P$$

❖ Area under the Force vs time curve gives the impulse

❖ Impulse – momentum theorem

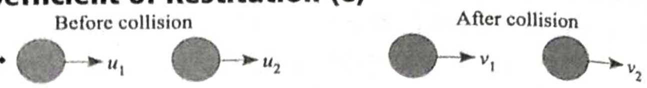
$$\vec{I} = \Delta\vec{P}$$

Principle of Conservation of Linear Momentum

❖ If, $\left(\sum F_{ext}\right)_{system} = 0 \Rightarrow (P_i)_{system} = (P_f)_{system}$

❖ $(KE)_{system} = \frac{1}{2}(m_1v_1^2 + m_2v_2^2 + \dots m_nv_n^2) \neq \frac{1}{2}MV_{com}^2$

Coefficient of Restitution (e)

❖ 

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

❖ $e = \dfrac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \dfrac{\int F_r dt}{\int F_d dt}$

$$= \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

$$V_1 = \frac{P_i + m_2e(u_2 - u_1)}{m_1 + m_2}, V_2 = \frac{P_i + m_1e(u_1 - u_2)}{m_1 + m_2}$$

❖ $e = 1$: Impulse of Reformation = Impulse of Deformation
Velocity of separation = Velocity of approach
Kinetic Energy may be conserved
Elastic collision.

❖ $e = 0$: Impulse of Reformation = 0
Velocity of separation = 0
Kinetic Energy is not conserved
Perfectly Inelastic collision.

❖ $0 < e < 1$: Impulse of Reformation < Impulse of Deformation
Velocity of seapration < Velocity of approach
Kinetic Energy is not conserved
Inelastic collision.

Variable Mass System

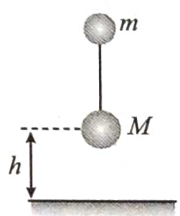
If a mass is added or ejected from a system, at rate μ kg/s and relative velocity v_{rel} (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu \mid v_{rel} \mid$.

Thrust Force (F_t)

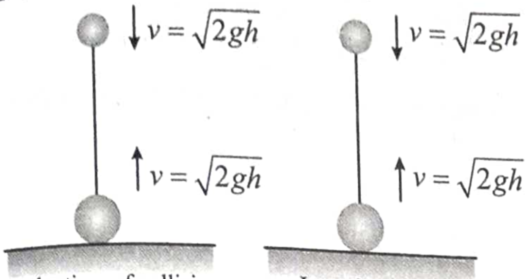
$$F_t = v_{rel}\frac{dm}{dt}$$

Solved Examples

1. A small ball of mass m is connected by an inextensible massless string of length l with an another ball of mass $M = 4$ m. They are released with zero tension in the string from a height h as shown in figure. Find the time when the string becomes taut for the first time after the mass M collides with the ground. Take all collisions to be elastic.



Sol. Just after collision velocities of both the balls will be $\sqrt{2gh}$ in opposite directions. Relative acceleration between the two balls is zero and relative velocity of approach is $2\sqrt{2gh}$. Hence they will collide after a time



At the time of collision　　Just after collision

$$t=\frac{l}{2\sqrt{2gh}} \quad ...(i)$$

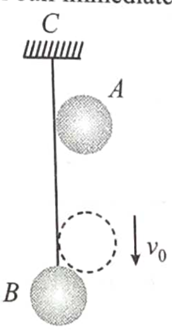
At the time of collision of two balls relative velocity of approach = $2\sqrt{2gh}$

Hence relative velocity of separation will also be $2\sqrt{2gh}$ (collision is elastic). Hence the string becomes tight after the same time

$$t=\frac{l}{2\sqrt{2gh}}$$

Hence the total time will be $2t$ or $\dfrac{l}{\sqrt{2gh}}$

2. Ball B is hanging from an inextensible cord BC. An identical ball A is released from rest when it is just touching the cord and acquires a velocity v_0 before striking ball B. Assuming perfectly elastic impact ($e = 1$) and no friction, determine the velocity of each ball immediately after impact.

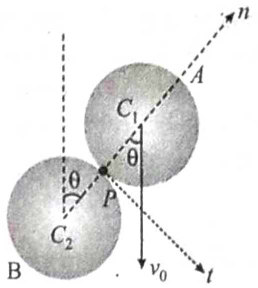


Sol. Ball A is free to move in a plane (after collision). So its velocity can be resolved in two mutually perpendicular directions. Let us resolve it along common tangent and common normal directions. Let v_t and v_n be the corresponding components of ball 'A' just ofter collision in these two directions. Ball B is attached to a vertical string. So, just after collision its velocity will be horizontal. Let it be v.

We have $\theta = 30°$

(i) Velocity component along common tangent direction remains unchanged. Hence,

$$u_t = v_t = v_0 \sin\theta = \frac{v_0}{2} \quad ...(i)$$



Just before collision

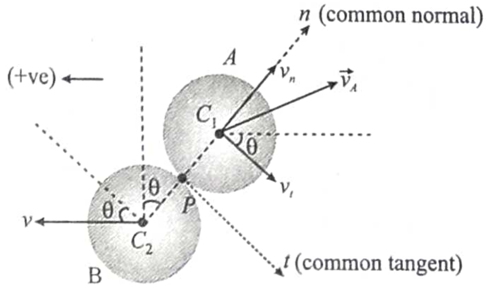
(ii) Linear momentum along horizontal will remain unchanged. Hence,

$$mv - mv_t\cos 30° - mv_n \sin 30° = 0$$

or $$v-\frac{\sqrt{3}v_t}{2}-\frac{v_n}{2}=0 \quad ...(ii)$$

(iii) $e=\dfrac{v_{sep.}}{v_{app.}}=1$ ($\because$ elastic collision)

i.e $\dfrac{v_n+v\sin\theta}{v_0\cos\theta}=1$



Just after collision

$\therefore$ $v_n + V\sin\theta = v_0\cos\theta$

or $$v_n+\frac{v}{2}=\frac{\sqrt{3}v_0}{2}\left(\text{as}\theta=30°\right). \quad ...(ii)$$

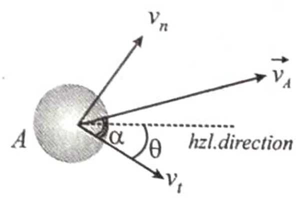
Solving Eqs. (i), (ii) and (iii), we get

$v = 0.693v_0$, $v_n = 0.52v_0$ and $v_t = 0.5\ v_0$

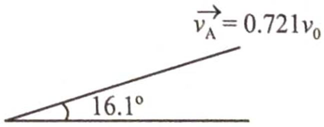
$\therefore$ Velocity of 'B' after collision $v = 0.693v_0$ (horizontally)

and Velocity of 'A' after collision $v_A=\sqrt{v_n^2+v_t^2}=0.721v_0$

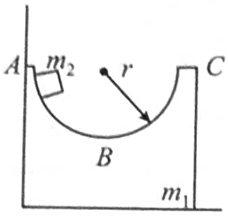
$$\alpha=\tan^{-1}\left(\frac{v_n}{v_t}\right)=46.1°$$



Hence, $\vec{v_A}$ makes an angle of (46.1° – 30°) or 16.1° with horizontal as shown



3. A wedge of mass m_1 with its upper surface hemispherical in shape, as shown in Figure, rests on a smooth horizontal, surface near the wall. A small block of mass m_2 slides without friction on the hemispherical surface of the wedge. What is the maximum velocity attained by the wedge?



Sol. As long as the block moves from A to B, the reaction on the wedge presses it to the wall. When the block reaches the lowermost position, its velocity from energy conservation is

$v = \sqrt{2gr}$

When the block moves along the right half of the wedge, during its upward journey as well as downward journey the reaction of the block on the wedge is towards right as shown in figure. Therefore during the entire motion of the block form B to C and C to B, the wedge is accelerated towards right. Thus to find the maximum velocity attained by the wedge at the instant when the block passes separated from the wall,

$P_i = P_f$

$$m_2\sqrt{2gr} = m_1v_1 + m_2v_2 \quad \ldots(i)$$

From energy conservation, $E_i = E_f$

$$m_2gr = \frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} \quad \ldots(ii)$$

On solving eqns. (i) and (ii) simultaneously, we obtain two solutions

$v_1 = 0,\ v_2 = \sqrt{2gr}$

and $v_1 = \dfrac{2m_2}{m_1+m_2}\left(\sqrt{2gr}\right)$

$$v_2 = \frac{m_2 - m_1}{m_1+m_2}\left(\sqrt{2gr}\right)$$

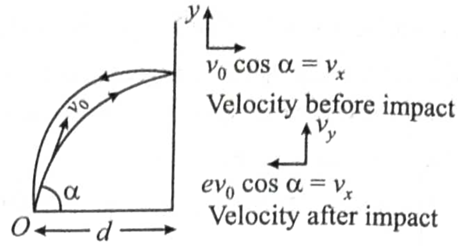
The first solution corresponds to the instant when the block reaches for the first time at point B. At this instant the block moves with velocity v_2 and the wedge is at rest. The second solution corresponds to the instant when the block has the maximum velocity

$$(v_1)_{\max} = \frac{2m_2\sqrt{2gr}}{m_1+m_2}$$

4. An inelastic ball is projected with velocity $v_0 = \sqrt{gh}$, at an angle α to the horizontal, towards a wall distant d from the point of projection. After collision the ball returns to the point of projection. What is the coefficient of restitution?

Sol. Time taken to reach the wall

$$t_1 = \frac{d}{v_0\cos\alpha}$$



Time taken to return to the point of projection after impact

$$t_2 = \frac{d}{ev_0\cos\alpha}$$

Note that x-component of velocity after impact is $ev_0 \cos\alpha$.

Total time of flight $(T) = t_1 + t_2 = \dfrac{d}{v_0\cos\alpha} + \dfrac{d}{ev_0\cos\alpha}$

$$T = \frac{d}{\sqrt{gh}\cos\alpha}\left[\frac{1+e}{e}\right].$$

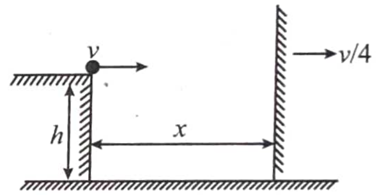
There is no change in the vertical component of the velocity after impact, therefore total time of flight remains unchanged.

$$T = \frac{2v_0\sin\alpha}{g} = \frac{2\sqrt{gh}\sin\alpha}{g} \qquad \because v_0 = \sqrt{gh}$$

$$\Rightarrow \frac{d}{\sqrt{gh}\cos\alpha}\left(\frac{1+e}{e}\right) = \frac{2\sqrt{gh}\sin\alpha}{g} \text{ or } \left(\frac{1+e}{e}\right) = \frac{h\sin 2\alpha}{d}$$

Or $e = \dfrac{d}{(h\sin 2\alpha - d)}$

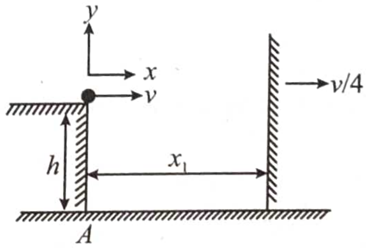
5. A particle is thrown from a height h horizontality towards a vertical wall moving away with a speed $v/4$ as if the particle returns to the point of projection after suffering two elastic collisions, one with the wall and another with the ground, find the total time of flight and initial. Separation x between the particle and the wall.



Sol. While colliding with wall x component of velocity gets changed while y-component remains same. We have

$\because e = \dfrac{v_{sep.}}{v_{app.}} = 1$ (elastic collision)

$$\Rightarrow \frac{v_1 + \frac{v}{4}}{v - \frac{v}{4}} = 1 \Rightarrow v_1 = \frac{v}{2}$$



It should be noted that time of flight will be ‘T’, and t_1 is time of ball to the ground and first collision must be occurred with wall and second with ground. Whereas if wall was moving towards the ball then, first collision must occur at ground and second with wall.

$\therefore$ Time of flight $= 2\times\sqrt{\frac{2h}{g}}$

Let the separation between point A and wall is x_1 when ball hits the wall

$$\frac{x_1}{v}+\frac{x_1}{v/2}=T=2\sqrt{\frac{2h}{g}}$$

$\therefore$ time taken by ball to cover this distance

$$t_1=\frac{x_1}{v}=\frac{2}{3}\sqrt{\frac{h}{2g}} \quad \Rightarrow x_1=\frac{2}{3}v\sqrt{\frac{h}{2g}}$$

$\therefore$ Initial separation $x = x_1-\frac{v}{4}(t_1)=x_1-\frac{x_1}{4}=\frac{3x}{4}$

$$x=\frac{3}{4}\times\left(\frac{2}{3}v\sqrt{\frac{h}{2g}}\right)\Rightarrow\frac{v}{2}\sqrt{\frac{h}{2g}}$$

i.e., $x=\frac{v}{2}\sqrt{\frac{h}{2g}}$

6. A smooth ring is kept on a smooth horizontal surface. From a point P of the ring a particle is projected at an angle α to the radius vector at P. If e is the coefficient of restitution between ‘the ring and the particle, show that the particle will return to the point of projection after two reflections if.

$$\cot^2\alpha=\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}$$

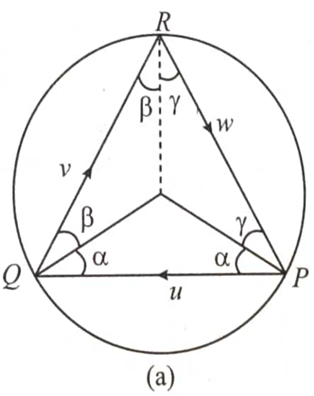
Sol. Let u be the velocity of projection at P. We can find the velocity of rebound at point Q from Fig. 4E.59 (b).

$$\tan\beta=\frac{u\sin\alpha}{eu\cos\alpha}=\frac{\tan\alpha}{e}$$

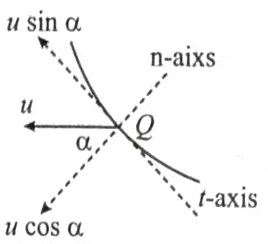
and $\quad v=\sqrt{u^2\sin^2\alpha+eu^2\cos^2\alpha}$

Similarly at R $\tan\gamma=\frac{v\sin\beta}{ev\cos\beta}=\frac{\tan\beta}{e}=\frac{\tan\alpha}{e^2}$

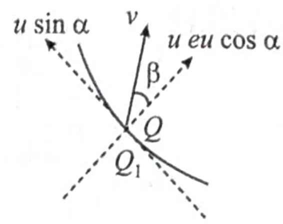
and $w=\sqrt{v^2\sin^2\alpha+ev^2\sin^2\alpha}$



(a)



Velocity components before impact



Velocity components before impact

Since the particle returns to the point of projection,

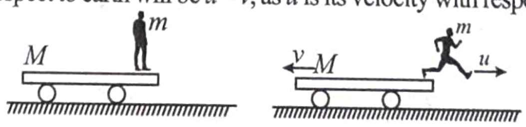
$\alpha+\beta+\gamma=\pi/2 \qquad$ or $\tan(\alpha+\beta+\gamma)=\infty$

or $\quad 1-[\tan\alpha\tan\beta+\tan\beta\tan\gamma+\tan\gamma\tan\alpha]=0$

or $\quad 1=\tan^2\alpha\left[\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}\right], \cot^2\alpha=\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}$

7. A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

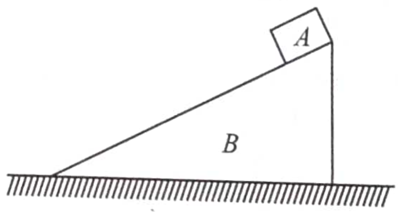
Sol. Let car attains a velocity v, and the net velocity of the child with respect to earth will be $u-v$, as u is its velocity with respect to car.



Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero, as $m(u-v)=Mv$

$$v=\frac{mu}{m+M}$$

8. A block A (mass = 4 M) is placed on the top of a wedge B of base length l (mass = 20 M) as shown in figure. When the system is released from rest. Find the distance moved by the wedge B till the block A reaches at lowest end of wedge. Assume all surfaces are frictionless.



Sol. Initial position of center of mass

$$=\frac{X_BM_B+X_AM_A}{M_B+M_B}=\frac{X_B\cdot(20M)+\ell.(4M)}{24M}=\frac{5X_B+\ell}{6}$$

Final position of center of mass

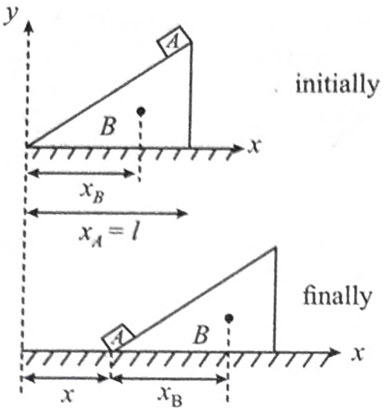
$$=\frac{(X_B+x)20M+4Mx}{24M}=\frac{5(X_B+x)+x}{6}$$

since there is no horizontal force on system center of mass initially = center of mass finally.

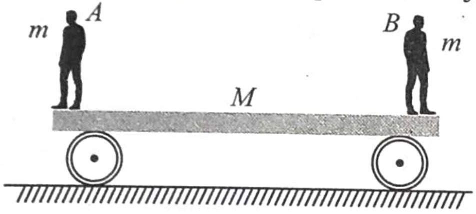
$5X_B+l=5X_B+5x+x$

$\ell=6x$

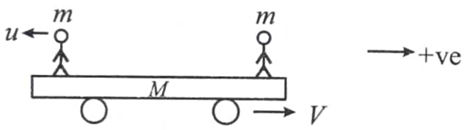
$$x=\frac{\ell}{6}$$



9. Two persons A and B, each of mass m are standing at the two ends of rail-road car of mass M. The person A jumps to the left with a horizontal speed u with respect to the car. Thereafter, the person B jumps to the right, again with the same horizontal speed u with respect to the car. Find the velocity of the car after both the persons have jumped off.



Sol. When person A jumps, let car achieve velocity V in forward direction. So, velocity of 'A' w.r.t. ground = $(u - V)$ in backward direction.

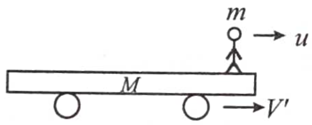


Applying momentum conservation.

$$(M+m)V - m(u-V) = 0, (M+m)V + mV = mu$$

$$V = \frac{m}{M+2m}u$$

When person B jumps, let velocity of car becomes V' in forward direction.



So velocity of 'B' w.r.t ground = $u + v$

Applying momentum conservation :

$$(m+M)V = MV' + m(u+V')$$

$$\frac{(m+M)mu}{M+2m} = (m+M)V' + mu$$

$$V' = \frac{-m^2u}{(M+m)(M+2m)}$$

$$V' = \frac{-m^2u}{(M+m)(M+2m)}$$

where '–ve' sign signifies Backward direction

10. Find the mass of the rocket as a function of time, if it moves with a constant acceleration a, in absence of external forces. The gas escapes with a constant velocity u relative to the rocket and its initial mass was m_o.

Sol. Using, $\vec{F}_{\text{net}} = \vec{V}_{rel}\left(\frac{-dm}{dt}\right)$

or $F_{\text{net}} = -u\frac{dm}{dt}$ ...(i)

and $F_{\text{net}} = ma$ ...(ii)

Solving equation (i) and (ii)

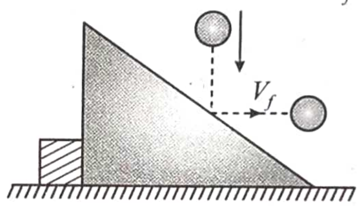
$$ma = -u\frac{dm}{dt}$$

$$\int_{m_0}^{m}\frac{dm}{m} = \int_0^t \frac{-adt}{u} = \frac{-at}{u}$$

$$\Rightarrow \{\ln(m)\}_{m_0}^{m} = -\frac{a}{u}(t)$$

or $\ln\left(\frac{m}{m_0}\right) = \frac{at}{u}$ or $\frac{m}{m_0} = e^{-at/u}$ $\Rightarrow m = m_0 e^{-at/u}$

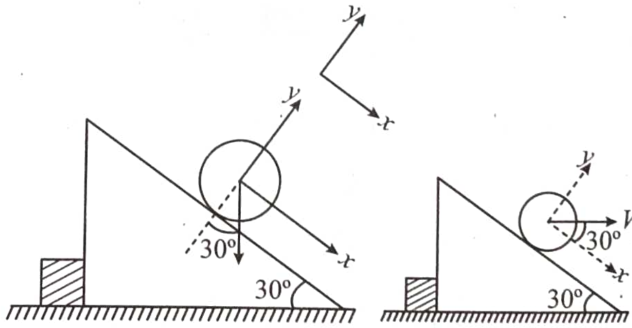
11. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f in the horizontal direction. If angle of inclination is 30°, the velocity v_f will be?



(a) 3 m/s (b) $\sqrt{3}m/s$

(c) $1/\sqrt{3}m/s$ (d) This is not possible

Sol. The momementum along the inclined plane (x-axis) remains conserved. $i.e \sum p_x = 0$

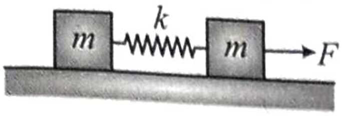


$$\Rightarrow \text{MV}_i \sin 30° = \text{M V}_f \cos 30°$$

$$\Rightarrow V_f = V_i\frac{\sin 30°}{\cos 30°} = V_i \tan 30°$$

$$\Rightarrow V_f = V_i\frac{1}{(\sqrt{3})} = 3\cdot\frac{1}{\sqrt{3}} = \sqrt{3}m/s$$

Paragraph for Question 12 to 14 : Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on the first block pulling it away from the other as shown in figure.



12. The displacement of the centre of mass at time t is

(a) $\dfrac{Ft^2}{2m}$ (b) $\dfrac{Ft^2}{3m}$

(c) $\dfrac{Ft^2}{4m}$ (d) $\dfrac{Ft^2}{m}$

Sol. (c) The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}.$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2}a_{com}t^2 = \frac{Ft^2}{4m}$$

13. If the extension of the spring is x_0 at time t, then the displacement of the right block at this instant is:

(a) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m}+x_0\right)$ (b) $-\dfrac{1}{2}\left(\dfrac{Ft^2}{2m}+x_0\right)$

(c) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m}-x_0\right)$ (d) $\left(\dfrac{Ft^2}{2m}+x_0\right)$

Sol. (a) Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m} \quad \text{or,} \quad \frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$$

or $$x_1 + x_2 = \frac{Ft^2}{2m} \qquad ...(i)$$

Further, the extension of the spring is $x_1 - x_2$. Therefore,

$$x_1 - x_2 = x_0 \qquad ...(ii)$$

From Eqs. (i) and (ii), $x_1 = \dfrac{1}{2}\left(\dfrac{Ft^2}{2m}+x_0\right)$

14. If the extension of the spring is x_0 at time t, then the displacement of the left block at this instant is:

(a) $\left(\dfrac{Ft^2}{2m}-x_0\right)$

(b) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m}+x_0\right)$

(c) $\dfrac{1}{2}\left(\dfrac{2Ft^2}{m}-x_0\right)$

(d) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m}-x_0\right)$

Sol. (d) Put the value of x_1 in equation (i). Thus,

$$x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m}-x_0\right)$$

Exercise-1 (Topicwise)

CALCULATION OF COM

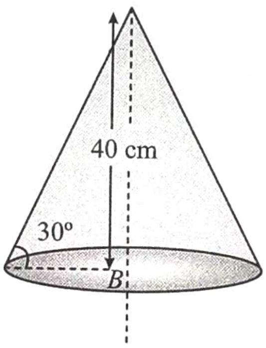
1. The centre of mass of a body:
   (a) Lies always at the geometrical centre
   (b) Lies always inside the body
   (c) Lies always outside the body
   (d) Can lie inside or outside the body

2. A body has its centre of mass at the origin. The x-coordinates of the particles
   (a) May be all positive
   (b) May be all negative
   (c) Must be all non-negative
   (d) May be positive for some particles and negative in other particles

3. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is
   (a) $= R$
   (b) $\leq R$
   (c) $> R$
   (d) $\geq R$

4. Where will the centre of mass on combining two masses m and $M(M > m)$:
   (a) Towards m
   (b) Towards M
   (c) Between m and M
   (d) Anywhere

5. A uniform solid cone of height 40 cm is shown in figure. The distance of centre of mass of the cone from point B (centre of the base) is :



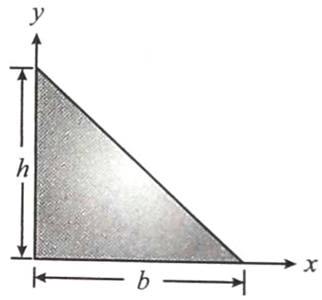
   (a) 20 cm
   (b) 10/3 cm
   (c) 20/3 cm
   (d) 10 cm

6. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å $= 10^{-10}$ m). The approximate location of the centre of mass of the molecule (distance from hydrogen atom) assuming the chlorine atom to be about 35.5 times massive as hydrogen is
   (a) 1Å
   (b) 2.5 Å
   (c) 1.24 Å
   (d) 1.5 Å

7. Centre of mass is a point
   (a) Which is geometric centre of a body
   (b) From which distance of particles are same
   (c) Where the whole mass of the body is supposed to be concentrated
   (d) Which is the origin of reference frame

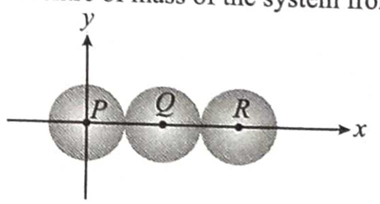
8. Choose the correct statement about the centre of mass (COM) of a system of two particles
   (a) The COM lies on the line joining the two particles midway between them
   (b) The COM lies on the line joining them at a point whose distance form each particle is inversely proportional to the mass of that particle
   (c) The COM lies on the line joining them at a point whose distance from each particle is proportional to the square of the mass of that particle
   (d) The COM is on the line joining them at a point whose distance from each particle is proportional to the mass of that particle

9. The centre of mass of triangle shown in figure has coordinates



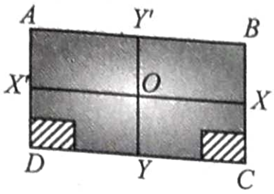
   (a) $x = \frac{h}{2}, y = \frac{b}{2}$
   (b) $x = \frac{b}{2}, y = \frac{h}{2}$
   (c) $x = \frac{b}{3}, y = \frac{h}{3}$
   (d) $x = \frac{h}{3}, y = \frac{b}{3}$

10. Three identical spheres, each of mass 1 kg are kept as shown in figure, touching each other, with their centres on a straight line. If their centres are marked P, Q, R respectively, the distance of centre of mass of the system from P (origin) is



   (a) $\frac{PQ+PR+QR}{3}$
   (b) $\frac{PQ+PR}{3}$
   (c) $\frac{PQ+QR}{3}$
   (d) $\frac{PR+QR}{3}$

11. A uniform square plate $ABCD$ has a mass of 10 kg. If two point masses of 3 kg each are placed at the corners C and D as shown in the adjoining figure, then the centre of mass shifts to a point which lies on.



(a) OC
(b) OD
(c) OY
(d) OX

DISPLACEMENT, VELOCITY, ACCELERATION OF COM

12. Two particles whose masses are 10 kg and 30 kg and their position vectors are $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}-\hat{j}-\hat{k}$ respectively would have the centre of mass at -

(a) $-\frac{(\hat{i}+\hat{j}+\hat{k})}{2}$
(b) $\frac{(\hat{i}+\hat{j}+\hat{k})}{2}$
(c) $-\frac{(\hat{i}+\hat{j}+\hat{k})}{4}$
(d) $\frac{(\hat{i}+\hat{j}+\hat{k})}{4}$

13. A bomb travelling in a parabolic path under the effect of gravity, explodes in mid air. The centre of mass of fragments will:

(a) Move vertically upwards and then downwards
(b) Move vertically downwards
(c) Move in irregular path
(d) Move in the parabolic path which the unexploded bomb would have travelled.

14. Two balls are thrown in air. The acceleration of the centre of mass of the two balls while in air (neglect air resistance)

(a) Depends on the direction of the motion of the balls
(b) Depends on the masses of the two balls
(c) Depends on the speeds of the two balls
(d) Is equal to g

15. The motion of the centre of mass of a system of two particles is unaffected by their internal forces :

(a) Irrespective of the actual directions of the internal forces
(b) Only if the are long the line joining the particles
(c) Only if they are at right angles to the line joining the particles
(d) Only if they are obliquely inclined to the line joining the particles.

16. Two objects of masses 200 gm and 500 gm posses velocities $10\hat{i}$ m/s and $3\hat{i}+5\hat{j}$ m/s respectively. The velocity of their centre of mass in m/s is :

(a) $5\hat{i}-25\hat{j}$
(b) $\frac{5}{7}\hat{i}-25\hat{j}$
(c) $5\hat{i}+\frac{25}{7}\hat{j}$
(d) $25\hat{i}-\frac{5}{7}\hat{j}$

17. 2 bodies of different masses of 2kg and 4kg are moving with velocities 20m/s and 10m/s towards each other due to mutual gravitational attraction. What is the velocity of their centre of mass?

(a) 5 m/s
(b) 6 m/s
(c) 8 m/s
(d) zero

18. Two bodies of masses 2 kg and 4 kg are moving with velocities 2 m/s and 10m/s respectively along same direction. Then the velocity of their centre of mass will be

(a) 8.1 m/s
(b) 7.3 m/s
(c) 6.4m/s
(d) 5.3 m/s

19. The two particles X and Y, initially at rest, start moving towards each other under mutual attraction. If at any instant the velocity of X is V and that of Y is $2V$, the velocity of their centre of mass will be

(a) Zero (b) V (c) $2V$ (d) $V/2$

20. Two balls A and B of masses 100 gm and 250 gm respectively are connected by a stretched spring of negligible mass and placed on a smooth table. When the balls are released simultaneously, the initial acceleration of B is 10 cm/sec^2 westward. What is the magnitude and direction of initial acceleration of the ball A ?

(a) 25 cm/sec^2 Eastward
(b) 25 cm/sec^2 Northward
(c) 25 cm/sec^2 Westward
(d) 25 cm/sec^2 Southward

LAW OF CONSERVATION OF MOMENTUM

21. Two bodies of masses m_1 and m_2 have equal kinetic energies. If p_1 and p_2 are their respective momentum, then ratio p_1 : p_2 is equal to

(a) $m_1 : m_2$
(b) $m_2 : m_1$
(c) $\sqrt{m_1} : \sqrt{m_2}$
(d) $m_1^2 : m_2^2$

22. A bullet of mass m is being fired from a stationary gun of mass M. If the velocity of the bullet is v, the velocity of the gun is-

(a) $\frac{Mv}{m+M}$
(b) $\frac{-mv}{M}$
(c) $\frac{(M+m)v}{M}$
(d) $\frac{M+m}{Mv}$

23. A bomb at rest has mass 60 kg. It explodes and a fragment of 40 kg has kinetic energy 96 joule. Then kinetic energy of other fragment is-

(a) 180 J
(b) 190 J
(c) 182 J
(d) 192 J

IMPULSE

24. A force of 50 dynes is acted on a body of mass 5 gm which is at rest for an interval of 3 sec, then impulse is-

(a) 0.16×10^{-3} N-s
(b) 0.98×10^{-3} N-s
(c) 1.5×10^{-3} N-s
(d) 2.5×10^{-3} N-s

25. The area of F-t curve is A, where 'F' is the force on one mass due to the other. If one of the colliding bodies of mass M is at rest initially, its speed just after the collision is :

(a) A/M
(b) M/A
(c) AM
(d) $\sqrt{\frac{2A}{M}}$

26. If two balls, each of mass 0.06 kg, moving in opposite directions with speed of 4 m/s, collide and rebound with the same speed, then the impulse imparted to each ball due to other (in kg-m/s) is :

(a) 0.48
(b) 0.53
(c) 0.81
(d) 0.92

COLLISIONS (OBLIQUE AND HEAD ON)

27. A block moving in air explodes in two parts then just after explosion

(a) The total momentum must be conserved
(b) The total kinetic energy of two parts must be same as that of block before explosion.
(c) The total momentum must change
(d) The total kinetic energy must not be increased

28. In head on elastic collision of two bodies of equal masses, it is not possible that

(a) The velocities are interchanged
(b) The speeds are interchanged
(c) The momenta are interchanged
(d) The faster body speeds up and the slower body slows down

29. Two identical blocks A and B, each of mass 'm' resting on smooth floor are connected by a light spring of natural length L and spring constant K, with the spring at its natural length. A third identical block 'C' (mass m) moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is (consider all collisions are elastic)

(a) $v\sqrt{\frac{m}{2k}}$
(b) $m\sqrt{\frac{v}{2k}}$
(c) $\sqrt{\frac{mv}{k}}$
(d) $\frac{mv}{2k}$

30. A ball of mass 3 kg collides with a wall with velocity 10 m/sec at an angle of 30° with the wall and after collision reflects at the same angle with the same speed. The change in momentum of ball in MKS unit is-

(a) 20
(b) 30
(c) 15
(d) 45

31. A ball of mass 'm', moving with uniform speed, collides elastically with another stationary ball. The incident ball will lose maximum kinetic energy when the mass of the stationary ball is

(a) m
(b) $2m$
(c) $4m$
(d) infinity

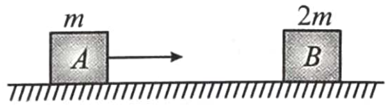
32. The coefficient of restitution e for a perfectly elastic collision is

(a) 1
(b) 0
(c) ∞
(d) -1

33. A lead ball strikes a wall and falls down, a tennis ball having the same mass and velocity strikes the wall and bounces back. Check the correct statement

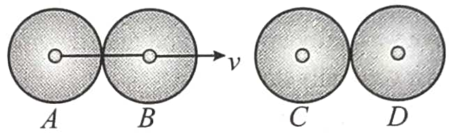
(a) The momentum of the lead ball is greater than that of the tennis ball
(b) The lead ball suffers a greater change in momentum compared with the tennis ball
(c) The tennis ball suffers a greater change in momentum as compared with the lead ball
(d) Both suffer an equal change in momentum

34. In the figure shown the block A collides head on with another block B at rest. Mass of B is twice the mass of A. The block A stops after collision. The co-efficient of restitution is:



(a) 0.5
(b) 1
(c) 0.25
(d) it is not possible

35. Two identical smooth spheres A and B are moving with same velocity and collide with similar spheres C and D, then after collision (Consider one dimensional collision and all collision are elastic)



(a) D will move with greater speed
(b) C and D will move with same velocity v
(c) C will stop and D will move with velocity v
(d) All spheres A, B, C & D will move with velocity $v/2$

36. A body of mass m having an initial velocity v, makes head on collision with a stationary body of mass M. After the collision, the body of mass m comes to rest and only the body having mass M moves. This will happen only when (consider all collision are elastic)

(a) $m >> M$
(b) $m << M$
(c) $m = M$
(d) $m = \frac{1}{2}M$

37. Two equal masses m_1 and m_2 moving along the same straight line with velocities + 3m/s and – 5m/s respectively collide elastically. Their velocities after the collision will be respectively

(a) + 4 m/s for both
(b) – 3 m/s and +5 m/s
(c) – 4 m/s and + 4 m/s
(d) – 5 m/s and + 3 m/s

38. A body falls on a surface of coefficient of restitution 0.6 from a height of 1 m. Then the body rebounds to a height of

(a) 0.6 m
(b) 0.4 m
(c) 1 m
(d) 0.36 m

39. A ball of mass m falls vertically to the ground from a height h_1 and rebound to a height h_2. The change in momentum of the ball on striking the ground is

(a) $mg(h_1 - h_2)$ (b) $m\left(\sqrt{2gh_1} + \sqrt{2gh_2}\right)$

(c) $m\sqrt{2g(h_1 + h_2)}$ (d) $m\sqrt{2g}(h_1 + h_2)$

40. Two spheres approaching each other collides elastically. Before collision the speed of A is 5 m/s and that of B is 10 m/s. Their masses are 1 kg and 0.5 kg. After collision velocities of A and B are respectively-

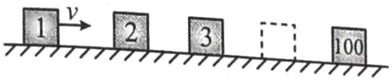
(a) 5 m/s, 10 m/s

(b) 10m/s, –5 m/s

(c) –10 m/s, –5 m/s

(d) –5 m/s, 10 m/s

41. There are hundred identical sliders equally spaced on a frictionless track as shown in the figure. Initially all the sliders are at rest. Slider 1 is pushed with velocity v towards slider 2. In a collision the sliders stick together. The final velocity of the set of hundred stucked sliders will be :



(a) $\frac{v}{99}$ (b) $\frac{v}{100}$

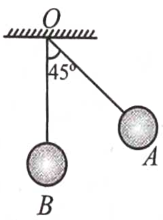
(c) zero (d) v

42. A space craft of mass M is moving with velocity V and suddenly explodes into two pieces. A part of it (of mass m) comes to rest, then the velocity of other part will be

(a) $\frac{MV}{M-m}$ (b) $\frac{MV}{M+m}$

(c) $\frac{mV}{M-m}$ (d) $\frac{(M+m)V}{m}$

43. The bob A of a simple pendulum is released when the string makes an angle of 45° with the vertical. It hits another bob B of the same material and same mass kept at rest on the table. If the collision is elastic



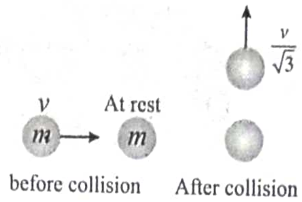
(a) Both A and B rise to the same height

(b) Both A and B come to rest at B

(c) Both A and B move with the same velocity of A

(d) A comes to rest and B moves with the velocity of A

44. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



(a) $\frac{2}{\sqrt{3}}v$ (b) $\frac{v}{\sqrt{3}}$

(c) v (d) $\sqrt{3}v$

45. A completely inelastic collision is one in which the two colliding particles

(a) Are separated after collision

(b) Remain together after collision

(c) Split into small fragments flying in all directions

(d) None of the above

46. A body of mass 2kg is moving with velocity 10 m/s towards east. Another body of same mass and same velocity moving towards north collides with former and coalesces and moves towards north-east. Its velocity is

(a) 10 m/s (b) 5 m/s

(c) 2.5 m/s (d) $5\sqrt{2}$ m/s

47. A body of mass m_1 is moving with a velocity v. It collides with another stationary body of mass m_2. They get embedded. At the point of collision, the velocity of the system

(a) Increases

(b) Decreases but does not become zero

(c) Remains same

(d) Become zero

VARIABLE MASS SYSTEMS

48. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s². The initial thrust of the blast is-

(a) 14.0×10^5 N

(b) 1.76×10^5 N

(c) 3.5×10^5 N

(d) 7.0×10^5 N

49. Fuel is consumed at the rate of 100 kg/sec in a rocket. The exhaust gases are ejected at a speed of 4.5×10^4 m/s. What is the thrust experienced by the rocket?

(a) 3×10^6 N

(b) 4.5×10^6 N

(c) 6×10^6 N

(d) 9×10^6 N

50. A 6000 kg rocket is set for vertical firing. The exhaust speed is 1000 m/sec. How much gas must be ejected each second to supply the thrust needed to give the rocket an initial upward acceleration of 20 m/sec²? (Consider $g = 9.8$ m sec^{-2} acceleration due to gravity)

(a) 92.4 kg/sec (b) 178.8 kg/sec

(c) 143.2 kg/sec (d) 47.2 kg/sec

Exercise-2 (Learning Plus)

1. A thin uniform wire is bent to form the two equal sides AB and AC of triangle ABC, where $AB = AC = 5$ cm. The third side BC, of length 6cm, is made from uniform wire of twice the density of the first. The distance of centre of mass from A is :

(a) $\frac{34}{11}$ cm (b) $\frac{11}{34}$ cm

(c) $\frac{34}{9}$ cm (d) $\frac{11}{45}$ cm

2. The centre of mass of a system of particles is at the origin. From this we conclude that

(a) The number of particles on positive x-axis is equal to the number of particles on negative x-axis

(b) The total mass of the particles on positive x-axis is same as the total mass on negative x-axis

(c) The number of particles on X-axis must be equal to the number of particles on Y-axis.

(d) If there is particle on the positive X-axis, there must be at least one particle on the negative X-axis.

3. A body of mass 1 kg moving in the x-direction, suddenly explodes into two fragments of mass 1/8 kg and 7/8 kg. An instant later, the smaller fragment is 0.14 m above the x-axis. The position of the heavier fragment is -

(a) $\frac{1}{50}$ m above x-axis (b) $\frac{1}{50}$ m below x-axis

(c) $\frac{7}{50}$ m below x-axis (d) $\frac{7}{50}$ m above x-axis

4. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is $\frac{M}{3}$,then the distance that the man moves relative to ground is:

(a) $\frac{3L}{4}$ (b) $\frac{L}{4}$

(c) $\frac{4L}{5}$ (d) $\frac{L}{3}$

5. A particle of mass $3m$ is projected from the ground at some angle with horizontal. The horizontal range is R. At the highest point of its path it breaks into two pieces m and $2m$. The smaller mass comes to rest and larger mass finally falls at a distance x from the point of projection where x is equal to

(a) $\frac{3R}{4}$ (b) $\frac{3R}{2}$

(c) $\frac{5R}{4}$ (d) $3R$

6. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speed of 2m/s and 6m/s respectively on a smooth horizontal surface. The speed of centre of mass of the system is :

(a) $\frac{10}{3}$ m/s (b) $\frac{10}{7}$ m/s (c) $\frac{11}{2}$ m/s (d) $\frac{12}{3}$ m/s

7. Two particles having mass ratio $n : 1$ are interconnected by a light inextensible string that passes over a smooth pulley. If the system is released, then the acceleration of the centre of mass of the system is:

(a) $(n-1)^2 g$ (b) $\left(\frac{n+1}{n-1}\right)^2 g$

(c) $\left(\frac{n-1}{n+1}\right)^2 g$ (d) $\left(\frac{n+1}{n-1}\right) g$

8. Internal forces in a system can change

(a) Linear momentum only

(b) Kinetic energy only

(c) Both kinetic energy and linear momentum

(d) Neither the linear momentum nor the kinetic energy of the system.

9. A man of mass 'm' climbs on a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v_{rel} (relative to rope). In what direction and with what speed (relative to ground) will the balloon move?

(a) Downwards, $\frac{mv_{rel}}{m+M}$

(b) Upwards, $\frac{Mv_{rel}}{m+M}$

(c) Downwards, $\frac{mv_{rel}}{M}$

(d) Downwards, $\frac{(M+m)v_{rel}}{M}$

10. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the compartment plus passengers system is C_2. If the passengers move about inside the compartment along the track,

(a) Both $C_1$and C_2 will move with respect to the ground

(b) Neither C_1 nor C_2 will over with respect to the ground

(c) C_1 will move but C_2 will be stationary with respect to the ground

(d) C_2 will move but C_1 will be stationary with respect to the ground

11. A shell is fired from a canon with a velocity V at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces come to rest. The speed of the other piece immediately after the explosion is

(a) $3V\cos\theta$ (b) $2V\cos\theta$

(c) $\frac{3}{2}V\cos\theta$ (d) $V\cos\theta$

12. A small sphere is moving at a constant speed in a vertical circle. Below is a list of quantities that could be used to describe some aspect of the motion of the sphere

I. Kinetic energy

II. Gravitational potential energy

III. Momentum

Which of these quantities will change as this sphere moves around the circle ?

(a) I and II only (b) I and III only

(c) III only (d) II and III only

13. A bomb at rest explodes into two parts of masses m_1 and m_2. If the momenta of the two parts be p_1 and p_2, then their kinetic energies will be in the ratio of-

(a) m_1/m_2 (b) m_2/m_1 (c) p_1/p_2 (d) p_2/p_1

14. A body of mass m collides against a wall with the velocity v and rebounds with the same speed. Its change of momentum is-

(a) $-2mv$ (b) mv (c) $-mv$ (d) 0

15. A bomb initially at rest explodes by itself into three equal mass fragments. The velocities of two fragments are $(3\hat{i}+2\hat{j})$ m/s and $(-\hat{i}-4\hat{j})$ m/s. The velocity of the third fragment is (in m/s)-

(a) $2\hat{i}+2\hat{j}$ (b) $2\hat{i}-2\hat{j}$ (c) $-2\hat{i}+2\hat{j}$ (d) $-2\hat{i}-2\hat{j}$

16. A stone of mass m_1 moving with a uniform speed v suddenly explodes on its own into two fragments. If the fragment of mass m_2 is at rest, the speed of the other fragment is-

(a) $\frac{m_1 v}{(m_1-m_2)}$ (b) $\frac{m_2 v}{(m_1-m_2)}$

(c) $\frac{m_1 v}{(m_1+m_2)}$ (d) $\frac{m_1 v}{m_2}$

17. A nucleus of mass number A originally at rest emits α-particle with speed v. The recoil speed of daughter nucleus is :

(a) $\frac{4v}{A-4}$ (b) $\frac{4v}{A+4}$

(c) $\frac{v}{A-4}$ (d) $\frac{v}{A+4}$

18. A super-ball is to bounce elastically back and forth between two rigid walls at a distance d from each other. Neglecting gravity and assuming the velocity of super-ball to be v_0 horizontally, the average force being exerted by the super-ball on each wall is :

(a) $\frac{1}{2}\frac{mv_0^2}{d}$ (b) $\frac{mv_0^2}{d}$ (c) $\frac{2mv_0^2}{d}$ (d) $\frac{4mv_0^2}{d}$

19. A force exerts an impulse I on a particle changing its speed from u to $2u$. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

(a) $\frac{3}{2}Iu$ (b) $\frac{1}{2}Iu$

(c) Iu (d) $2Iu$

20. A particle of mass $4m$ which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed 'v' each in mutually perpendicular directions. The minimum energy released in the process of explosion is :

(a) $(2/3)\,mv^2$ (b) $(3/2)\,mv^2$

(c) $(4/3)\,mv^2$ (d) $(3/4)\,mv^2$

21. A bullet of mass m moving vertically upwards instantaneously with a velocity 'u' hits the hanging block of mass 'm' and gets embedded in it. As shown in the figure the height through which block rises after the collision (assume sufficient space above block) is

(a) $u^2/2g$ (b) u^2/g

(c) $u^2/8g$ (d) $u^2/4g$

22. In an inelastic collision-

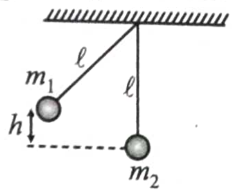
(a) Momentum is conserved but kinetic energy is not

(b) Momentum is not conserved but kinetic energy is conserved

(c) Neither momentum nor kinetic energy is conserved

(d) Both the momentum and kinetic energy are conserved

23. In the arrangement shown, the pendulum on the left is pulled aside. It is then released and allowed to collide with other pendulum which is at rest. A perfectly inelastic collision occurs and the system rises to a height $h/4$. The ratio of the masses (m_1/m_2) of the pendulum is :



(a) 1 (b) 2 (c) 3 (d) 4

24. Two perfectly elastic balls of same mass m are moving with velocities u_1 and u_2. They collide elastically n times. The kinetic energy of the system finally is:

(a) $\frac{1}{2}\frac{m}{u}u_1^2$ (b) $\frac{1}{2}\frac{m}{u}(u_1^2+u_2^2)$

(c) $\frac{1}{2}m(u_1^2+u_2^2)$ (d) $\frac{1}{2}mn(u_1^2+u_2^2)$

25. A ball hits the floor and rebounds after an inelastic collision. In this case-

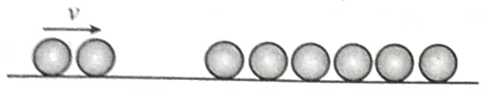
(a) The momentum of the ball just after the collision is the same as that just before the collision

(b) The mechanical energy of the ball remains the same in the collision

(c) The total momentum of the ball and the earth is conserved.

(d) The total energy of the ball and the earth is conserved

26. Six steel ball of identical size are lined up along a straight frictionless groove. Two similar balls moving with a speed v along the groove collide with this row on the extreme left hand, then (all collision are ellastic)-



Frictionless Groove

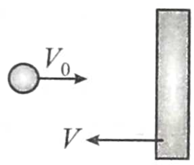
(a) All the balls will start moving to the right with speed 1/8 each

(b) All the six balls initially at rest will move on with speed $v/6$ each and two identical balls will come to rest

(c) Two balls from the extreme right end will move on with speed v each and the remaining balls will remain at rest

(d) One ball from the right end will move on with speed $2v$, the remaining balls will be at rest.

27. A particle of mass m moves with velocity V_0 = 20 m/sec towards a large wall that is moving with velocity V = 5 m/sec. towards a particle as shown. If the particle collides with the wall elastically, the speed of the particle just after the collision is:

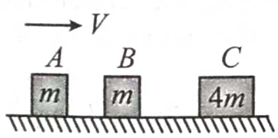


(a) 30 m/s (b) 20 m/s (c) 25 m/s (d) 22 m/s

28. A sphere of mass m moving with a constant velocity hits another stationary sphere of the same mass. If e is the coefficient of restitution, then ratio of speed of the first sphere to the speed of the second sphere after collision will be :

(a) $\left(\frac{1-e}{1+e}\right)$ (b) $\left(\frac{1+e}{1-e}\right)$ (c) $\left(\frac{e+1}{e-1}\right)$ (d) $\left(\frac{e-1}{e+1}\right)$

29. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity V to the right. Block B with mass m and block C with mass $4m$ are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



(a) $0.60V$ to the left (b) $1.4V$ to the left

(c) V to the left (d) $0.4V$ to the left

30. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of $v/3$ in the same direction. What type of collision has occured ?

(a) Inelastic

(b) Elastic

(c) Completely inelastic

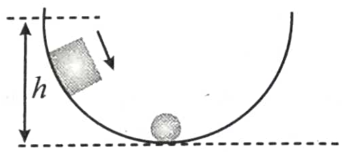
(d) None of these

31. An object of mass 5 kg and speed 10 ms^{-1} explodes into two pieces of equal mass. One piece comes to rest. The kinetic energy added to the system during the explosion is-

(a) Zero (b) 50 J

(c) 250 J (d) 500 J

32. A block of mass m starts from rest and slides down a frictionless semi-circular track from a height h as shown. When it reaches the lowest point of the track, it collides with a stationary piece of putty also having mass m. If the block and the putty stick together and continue to slide, the maximum height that the block-putty system could reach is



(a) $h/4$ (b) $h/2$

(c) h (d) Independent of h

33. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d : while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c . The values of p_d and p_c are respectively :

(a) (0.28, 0.89)

(b) (0, 0)

(c) (0, 1)

(d) (0.89, 0.28)

34. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(a) $\sqrt{2}\,v_0$ (b) $\frac{v_0}{2}$

(c) $\frac{v_0}{\sqrt{2}}$ (d) $\frac{v_0}{4}$

35. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly:

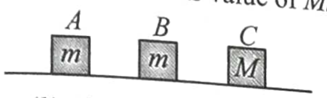
(a) 4.70×10^3 N/m^2 (b) 2.35×10^2 N/m^2

(c) 4.70×10^2 N/m^2 (d) 2.35×10^3 N/m^2

36. A particle of mass m is moving in a straight line with momentum p. Starting at time $t=0$, a force $F=kt$ acts in the same direction on the moving particle during time interval T so that its momentum changes from p to $3p$. Here k is a constant. The value of T is:

(a) $2\sqrt{\frac{k}{p}}$ (b) $2\sqrt{\frac{p}{k}}$ (c) $\sqrt{\frac{2k}{p}}$ (d) $\sqrt{\frac{2p}{k}}$

37. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically. If $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m?



(a) 5 (b) 2 (c) 4 (d) 3

38. A piece of wood of mass 0.03 kg is dropped from the top of 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is ($g = 10$ ms^{-2})

(a) 20 m (b) 30 m (c) 40 m (d) 10 m

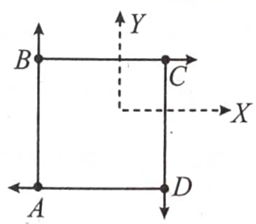
39. A particle of mass 'm' is moving with speed '$2v$' and collides with a mass '$2m$' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction. The speed of each of the moving particles will be:

(a) $v/(2\sqrt{2})$ (b) $2\sqrt{2}v$ (c) $\sqrt{2}v$ (d) $v/\sqrt{2}$

40. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by:

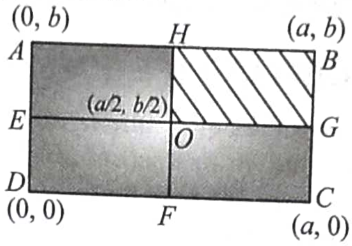
(a) $\frac{2v^2}{7g}$ (b) $\frac{v^2}{g}$ (c) $\frac{2v^2}{5g}$ (d) $\frac{v^2}{2g}$

41. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude a with directions as shown. The acceleration of the centre of mass of the particles is :



(a) $\frac{a}{5}(\hat{i}-\hat{j})$ (b) $\frac{a}{5}(\hat{i}+\hat{j})$ (c) Zero (d) $a(\hat{i}+\hat{j})$

42. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be :



(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

43. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$ undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\,m_1$ and $v_3 = 0.5\,v_1$, then v_1 is :

(a) $v_4 - \frac{v_2}{4}$ (b) $v_4 - \frac{v_2}{2}$

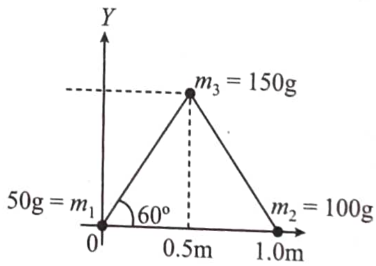
(c) $v_4 - v_2$ (d) $v_4 + v_2$

44. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(a) 1.8 kg (b) 1.2 kg

(c) 1.5 kg (d) 1.0 kg

45. Three particles of masses 50 g, 100g and 150 g are placed at the vertices of an equilateral triangle of side 1m (as shown in the figure). The (x, y) coordinates of the centre of mass will be:



(a) $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{8}\text{m}\right)$

(b) $\left(\frac{\sqrt{3}}{4}\text{m}, \frac{5}{12}\text{m}\right)$

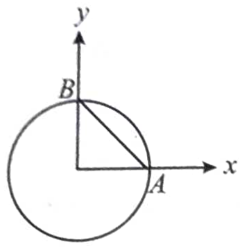
(c) $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{4}\text{m}\right)$

(d) $\left(\frac{\sqrt{3}}{8}\text{m}, \frac{7}{12}\text{m}\right)$

Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. An object comprises of a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can not be the centre of mass of the object?



(a) $(R/3, R/3)$ (b) $(R/3, R/2)$
(c) $(R/4, R/4)$ (d) $(R/\sqrt{2}, R/\sqrt{2})$

2. If the net external forces acting on a system is zero, then the centre of mass
(a) Must not move (b) Must not accelerate
(c) May move (d) May accelerate

3. An external force $\vec{F}(\vec{F} \neq 0)$ acts on a system of particles. The velocity and the acceleration of the centre of mass are found to be v_{cm} and a_{cm}, then it is possible that
(a) $v_{cm} = 0, a_{cm} = 0$
(b) $v_{cm} = 0, a_{cm} \neq 0$
(c) $v_{cm} \neq 0, a_{cm} = 0$
(d) $v_{cm} \neq 0, a_{cm} \neq 0$

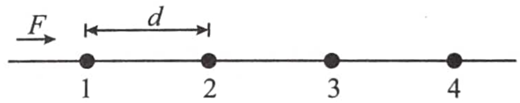
4. Two blocks A and B each of mass 'm' are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal plane. Block C also of mass m moves on the floor with a speed 'v' along the line joining A and B and collides elastically with A. Which of the following is/are correct?
(a) KE of the AB system at maximum compression of the spring is zero
(b) The KE of AB system at maximum compression is $(1/4)\, mv^2$
(c) The maximum compression of spring is $v\sqrt{m/k}$
(d) The maximum compression of spring is $v\sqrt{m/2k}$

5. In an elastic collision, in absence of external force, which of the following is/are correct?
(a) The linear momentum is conserved
(b) The potential energy is conserved in collision
(c) The final kinetic energy is less than the initial kinetic energy
(d) The final kinetic energy is equal to the initial kinetic energy

6. A block moving in air explodes in two parts, then, just after explosion (neglect change in momentum due to gravity)
(a) The total momentum of two parts must be same to the momentum of the block before explosion.
(b) That total kinetic energy of two parts must be same as that of block before explosion.
(c) The total momentum must change.
(d) The total kinetic energy must increase.

7. A set of n-identical cubical blocks lie at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surfaces of any two adjacent blocks is L. The block at one end is given a speed V towards the next one at time $t = 0$. All collisions are completely inelastic, then
(a) The last block starts moving at $t = n(n-1)\dfrac{L}{2V}$
(b) The last block starts moving at $t = (n-1)\dfrac{L}{V}$
(c) The centre of mass of the system will have a final speed v/n
(d) The centre of mass of the system will have a final speed v.

8. The figure shows a string of equally spaced beads of mass m, separated by distance d. The beads are free to slide without friction on a thin wire. A constant force F acts on the first bead initially at rest till it makes collision with the second bead. The second bead then collides with the third and so on. Suppose all collisions are elastic, then:



(a) Speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\dfrac{2Fd}{m}}$ and zero respectively.
(b) Speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\dfrac{2Fd}{m}}$ and $\dfrac{1}{2}\sqrt{\dfrac{2Fd}{m}}$ respectively.
(c) Speed of the second bead immediately after its collision with third bead is zero .
(d) The average speed of the first bead is $\dfrac{1}{2}\sqrt{\dfrac{2Fd}{m}}$

9. A shell explodes in a region of negligible gravitational field, giving out n fragments of equal mass m. Then its total
   (a) Kinetic energy is smaller than that before the explosion
   (b) Kinetic energy is greater than that before the explosion
   (c) Momentum and kinetic energy depend on n
   (d) Momentum is equal to that before the explosion.

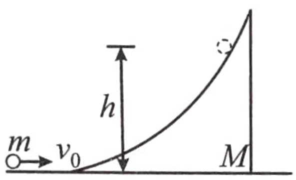
10. Two identical balls are interconnected with a massless and inextensible thread. The system is in gravity free space with the thread just taut. Each ball is imparted a velocity v, one towards the other ball and the other perpendicular to the first, at $t = 0$. Then,
   (a) The thread will become taut at $t = (L / v)$
   (b) The thread will become taut at some time $t < (L / v)$.
   (c) The thread will always remain taut for $t > (L/v)$
   (d) The kinetic energy of the system will always remain mv^2.

11. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact. Then,
   (a) The minimum kinetic energy of the system is 1 joule
   (b) The maximum elastic potential energy of the system is 2 joule.
   (c) Momentum and total kinetic energy of the system are conserved at every instant.
   (d) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

12. In an inelastic collision,
   (a) The velocity of both the particles may be same after collision.
   (b) Kinetic energy is not conserved
   (c) Linear momentum of the system is conserved.
   (d) Velocity of separation will be less than velocity of approach.

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 13 to 19): A particle of mass m moving horizontal with v_0 strikes a smooth wedge of mass M, as shown in figure. After collision, the ball starts moving up the inclined face of the wedge and rises to a height h.



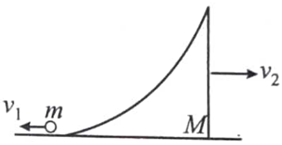
13. The final velocity of the wedge v_2 is
   (a) $\frac{mv_0}{M}$ (b) $\frac{mv_0}{M+m}$
   (c) v_0 (d) insufficient data

14. When the particle has risen to a height h on the wedge, then choose the correct alternative(s)
   (a) The particle is stationary with respect to ground
   (b) Both are stationary with respect to the centre of mass
   (c) The kinetic energy of the centre of mass remains constant
   (d) The kinetic energy with respect to centre of mass is converted into potential energy

15. The maximum height h attained by the particle is
   (a) $\left(\frac{m}{m+M}\right)\frac{v_0^2}{2g}$ (b) $\left(\frac{m}{M}\right)\frac{v_0^2}{2g}$
   (c) $\left(\frac{M}{m+M}\right)\frac{v_0^2}{2g}$ (d) None of these

16. Identify the correct statement(s) related to the situation when the particle starts moving downward.
   (a) The centre of mass of the system remains stationary
   (b) Both the particle and the wedge remain stationary with respect to centre of mass
   (c) When the particle reaches the horizontal surface its velocity relative to the wedge is v_0
   (d) None of these

17. Suppose the particle when reaches the horizontal surfaces, its velocity with respect to ground is v_1 and that of wedge is v_2. Choose the correct statement (s)

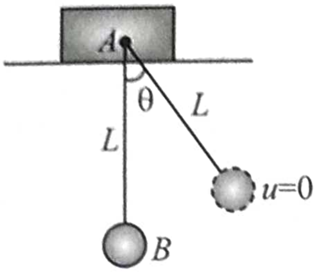


   (a) $mv_1 = Mv_2$ (b) $Mv_2 - mv_1 = mv_0$
   (c) $v_1 + v_2 = v_0$ (d) $v_1 + v_2 < v_0$

18. Choose the correct statement(s) related to particle m.
   (a) Its kinetic energy is $K_f = \left(\frac{mM}{m+M}\right)gh$
   (b) $v_1 = v_0\left(\frac{M-m}{M+m}\right)$
   (c) The ratio of its final kinetic energy to its initial kinetic energy is $\frac{K_f}{K_i} = \left(\frac{M}{m+M}\right)^2$
   (d) It moves opposite to its initial direction of motion

19. Choose the correct statement related to the wedge M.
   (a) Its kinetic energy is $K_f = \left(\frac{4m^2}{m+M}\right)gh$
   (b) $v_2 = \left(\frac{2m}{m+M}\right)v_0$
   (c) Its gain in kinetic energy is
   $$\Delta K = \left(\frac{4mM}{(m+M)^2}\right)\left(\frac{1}{2}mv_0^2\right)$$
   (d) Its velocity is more that the velocity of centre of mass

Comprehension (Q. 20 to 23): A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle θ from equilibrium position & then released.



20. The displacement of block when ball reaches the equilibrium position is

(*a*) $\frac{L\sin\theta}{2}$ (*b*) $L\sin\theta$

(*c*) L (*d*) None of these

21. Tension in string when it is vertical, is

(*a*) mg

(*b*) $mg(2-\cos\theta)$

(*c*) $mg(3-2\cos\theta)$

(*d*) $2mg(1-\cos\theta)$

22. Maximum velocity of block during subsequent motion of the system after release of ball is

(*a*) $[gl(1-\cos\theta)]^{1/2}$

(*b*) $[2gl(1-\cos\theta)]^{1/2}$

(*c*) $[gl\cos\theta]^{1/2}$

(*d*) $[gl(1-\sin\theta)]^{1/2}$

23. The displacement of centre of mass of $A+B$ system till the string becomes vertical is

(*a*) zero

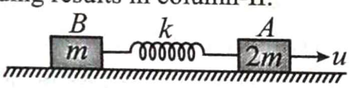
(*b*) $\frac{L}{2}(1-\cos\theta)$

(*c*) $\frac{L}{2}(1-\sin\theta)$

(*d*) None of these

MATCH THE COLUMN TYPE QUESTIONS

24. Two blocks A and B of mass m and $2m$ respectively are connected by a massless spring of spring constant k. This system lies over a smooth horizontal surface. At $t = 0$ the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column-I, certain statements are given and corresponding results are given in column-II. Match the statements in column-I corresponding results in column-II.



	Column-I		Column-II
A.	The velocity of block A	p.	can never be zero
B.	The velocity of block B	q.	may be zero at certain instants of time
C.	The kinetic energy of system of two blocks	r.	is minimum at maximum compression of spring
D.	The potential energy of spring	s.	is maximum at maximum extension of spring

(*a*) A-(p), B-(q), C-(p, r), D-(q, s)

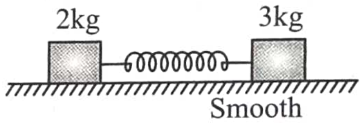
(*b*) A-(q), B-(p), C-(p, r), D-(q, s)

(*c*) A-(p), B-(q), C-(p, s), D-(q, r)

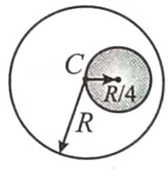
(*d*) A-(p, q), B-(q), C-(q, r), D-(q, s)

NUMERICAL TYPE QUESTIONS

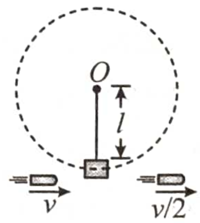
25. A 2kg mass and 3 kg mass are used to compress opposite ends of a spring ($k = 750$ N/m) by a distance of 40 cm from natural length and then is released from rest. If the speeds of the two masses as they leave the spring are v_2 and v_3, find $v_2 + v_3$ (in m/s)



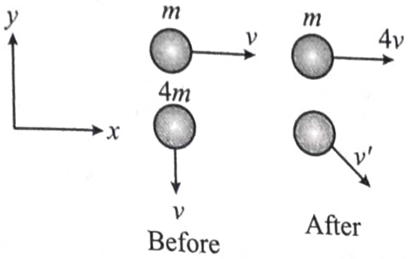
26. For years together, people thought that hell is located inside earth. Assume that hell is a spherical bubble created inside earth. It is completely evacuated to make sinful people feel suffocated. It has a radius $R/4$ and is located as shown. (C is earth's centre). What is the distance of centre of mass of the resulting body from C (in km). Assume that earth's radius is 6300 km.



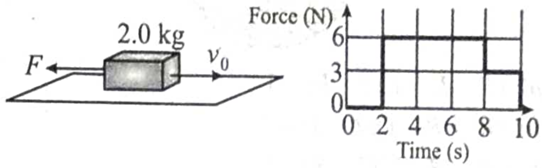
27. A bullet of mass m and velocity v passes through a pendulum bob of mass M and emerges with velocity $v/2$ (figure). The pendulum bob is at the end of a string of length l. What is the minimum value of v (in m/s) such that the pendulum bob will swing through a complete circle ? (Take : $l = 2$ m, $M = 1$kg, $m = 10$ gm.)



28. Two particles of masses m and $4m$, moving in vacuum at right angles to each other experience same force for time T simultaneously. Consequently the particle m moves with velocity $4v$ in its original direction. Find the new magnitude of the velocity v' (in m/s) of the particle $4m$. Given $v = 100$ m/s.



29. A block of mass 2kg is sliding on a smooth surface. At $t = 0$, its speed is $v_0 = 2$ m/s. At $t = 0$, a time-varying force starts acting on the block in the direction opposite to v_0. Find the speed of object (in m/s) at $t = 10$



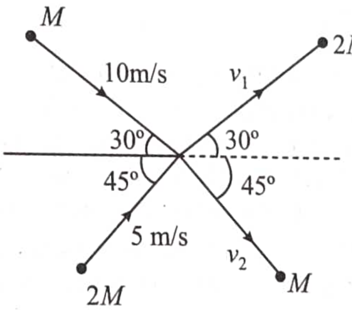
Exercise-4 (Past Year Questions)

JEE MAIN

1. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is: **(2019)**

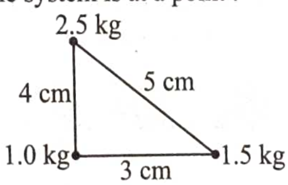
   (*a*) 0.20 ms^{-1} (*b*) 0.14 ms^{-1}
   (*c*) 0.47 ms^{-1} (*d*) 0.28 ms^{-1}

2. Two particles, of masses M and $2M$, moving as shown, with speeds of 10 m/s and 5 ms^{-1} collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2 respectively. The values of v_1 and v_2 are nearly: **(2019)**



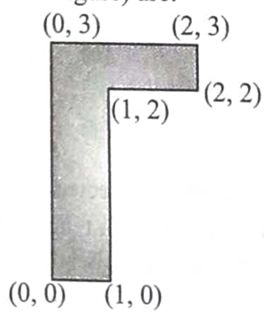
   (*a*) 3.2 m/s and 6.3 m/s (*b*) 3.2 m/s and 12.6 m/s
   (*c*) 6.5 m/s and 6.3 m/s (*d*) 6.5 m/s and 3.2 m/s

3. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point : **(2020)**



   (*a*) 0.6 cm right and 2.0 cm above 1kg mass
   (*b*) 2.0 cm right and 0.9 cm above 1 kg mass
   (*c*) 0.9 cm right and 2.0 cm above 1kg mass
   (*d*) 1.5 cm right and 1.2 cm above 1kg mass

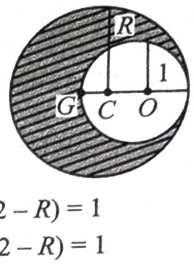
4. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are: **(2020)**



   (*a*) 1.25 m, 1.50 m)
   (*b*) (0.75 m, 0.75 m)
   (*c*) (0.75 m, 1.75 m)
   (*d*) (1m, 1.5m)

5. A body A of mass $m = 0.1$ kg has an initial velocity of $3\hat{i}$ ms^{-1}. It collides elastically with another body B of the same mass which has an initial velocity of $5\hat{j}$ ms^{-1}. After collision, A moves with a velocity $\vec{v} = 4(\hat{i} + \hat{j})$ m/s. The energy of B after colision is written as $\frac{x}{10}$ J. The value of x is_____. **(2020)**

6. As shown in figure. When a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation: **(2020)**



(a) $(R^2 - R + 1)(2 - R) = 1$
(b) $(R^2 + R - 1)(2 - R) = 1$
(c) $(R^2 - R - 1)(2 - R) = 1$
(d) $(R^2 + R - 1)(2 - R) = 1$

7. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is: **(2020)**

(a) $\sqrt{\frac{3}{4}}$ (b) $\sqrt{\frac{3}{2}}$
(c) $\sqrt{\frac{1}{2}}$ (d) $\frac{1}{2}$

8. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is **(2020)**

(a) $\frac{3}{4}mu^2$ (b) $\sqrt{\frac{2}{3}}mu^2$
(c) $\frac{1}{3}mu^2$ (d) $\frac{1}{8}mu^2$

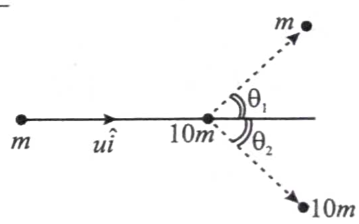
9. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \pi/3$ w.r.t. horizontal (x – axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is: **(2020)**

(a) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (b) $2\sqrt{2}\frac{u^2}{g}$
(c) $\frac{5}{8}\frac{u^2}{g}$ (d) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

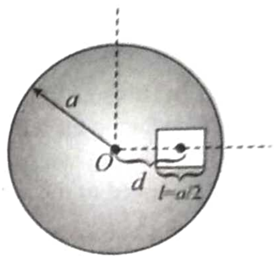
10. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$ where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at: **(2020)**

(a) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (b) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$
(c) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (d) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

11. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass $10m$ at rest and then moves with half its initial kinetic energy(see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$, then value of n is ________ **(2020)**



12. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{x}$, value of x (to the nearest integer) is ___. **(2020)**



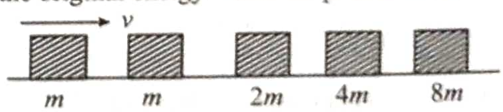
13. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass $3m$ at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by **(2020)**

(a) $v = \frac{1}{\sqrt{6}}u$ (b) $v = \frac{u}{\sqrt{3}}$
(c) $v = \sqrt{\frac{2}{3}}u$ (d) $v = \frac{u}{\sqrt{2}}$

14. A block of mass 1.9 kg is at rest at the edge of a table, of height 1m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take $g = 10$ m/s^2. Assume there is no rotational motion and loss of energy after the collision is negligible.] **(2020)**

(a) 19 J (b) 23 J (c) 20 J (d) 21 J

15. Blocks of masses m, $2m$, $4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass $8m$ starts moving, the total energy loss is $p\%$ of the original energy. Value of 'p' is close to **(2020)**



(a) 37 (b) 77 (c) 87 (d) 94

16. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its instantaneous velocity. The instantaneous acceleration of the satellite is **(2020)**

(a) $-bv^3$ (b) $-\frac{2bv^3}{M(t)}$

(c) $-\frac{bv^3}{M(t)}$ (d) $-\frac{bv^3}{2M(t)}$

17. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is ______. **(2020)**

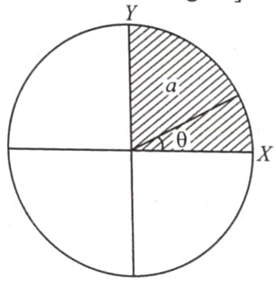
18. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i} + \hat{j})$ ms^{-1} collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i} + \sqrt{3}\hat{j})$ ms^{-1}, the angle between $\vec{V}_1$ and $\vec{V}_2$ is **(2020)**

(a) $-45°$ (b) $60°$

(c) $15°$ (d) $105°$

19. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed.
The angle between the initial velocities of the two bodies (in degree) is ____. **(2020)**

20. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is ___
(Round off to the Nearest Integer) **(2021)**
[a is an area as shown in the figure]



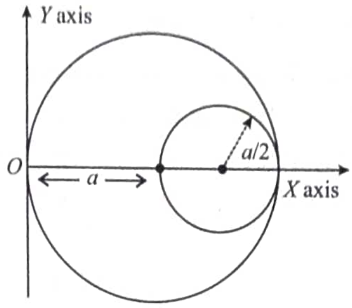
21. A ball moving with velocity 9 ms^{-1} collides with another similar stationary ball. If after the collision, both the balls move in directions making an angle of 30° with the initial direction, then their speeds after collision will be **(2021)**

(a) 5.2 ms^{-1} (b) 0.52 ms^{-1}

(c) 52 ms^{-1} (d) 26 ms^{-1}

22. A body of mass 2 kg moving with a speed of 4 m/s. makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body center of mass is $\frac{x}{10}$ m/s. The value of x is **(2021)**

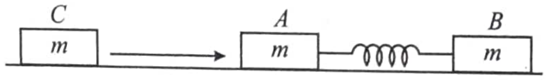
23. A circular hole of radius $a/2$ is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be **(2021)**



(a) $\frac{2}{3}a$ (b) $\frac{1}{6}a$

(c) $\frac{10}{11}a$ (d) $\frac{5}{6}a$

24. Two identical blocks A and B each of mass m resting on a smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is **(2021)**



(a) $\sqrt{\frac{mv}{2K}}$ (b) $\sqrt{\frac{m}{2K}}$

(c) $\sqrt{\frac{mv}{K}}$ (d) $v\sqrt{\frac{m}{2K}}$

25. A batsman hits back a ball of mass 0.4 kg straight in the direction of the bowler without changing its initial speed of 15 ms^{-1}. The impulse imparted to the ball is ________Ns. **(2022)**

26. Two blocks of masses 10 kg and 30 kg are placed on the same straight line with coordinates (0, 0) and (x, 0) respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is **(2022)**

(a) 4 cm towards the 10 kg block

(b) 2 cm away from the 10 kg block

(c) 2 cm towards the 10 kg block

(d) 4 cm away from the 10 kg block

27. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass? (Assume the collision to be head-on elastic collision) **(2022)**

(a) 50.0 % (b) 66.6 %

(c) 55.5 % (d) 33.3 %

28. A pendulum of length 2 m consists of a wooden bob of mass 50 g. A bullet of mass 75 g is fired towards the stationary bob with a speed v. The bullet emerges out of the bob with a speed $v/3$ and the bob just completes the vertical circle. The value of v is__ms^{-1} ($g = 10\ m/s^2$) **(2022)**

29. A body of mass M at rest explodes into three pieces, in the ratio of masses $1:1:2$. Two smaller pieces fly off perpendicular to each other with velocities of $30\ ms^{-1}$ and $40\ ms^{-1}$ respectively. The velocity of the third piece will be : **(2022)**

(a) $15\ ms^{-1}$ (b) $25\ ms^{-1}$
(c) $35\ ms^{-1}$ (d) $50\ ms^{-1}$

30. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of thesystem will be $\sqrt{x}$ m. The value of x is **(2022)**

31. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ respectively.The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector : **(2022)**

(a) $\hat{i}-2\hat{j}+\hat{k}$ (b) $-3\hat{i}-2\hat{j}+\hat{k}$
(c) $-2\hat{i}+2\hat{k}$ (d) $-2\hat{i}-\hat{j}+2\hat{k}$

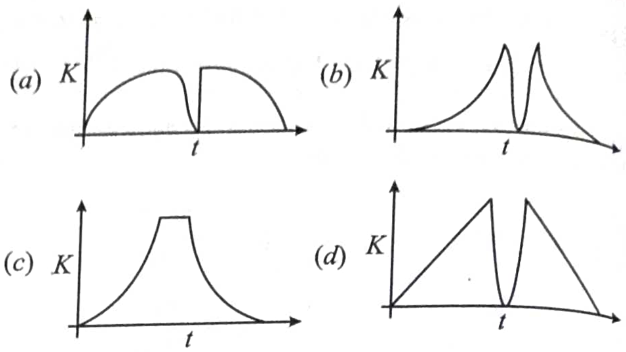
JEE ADVANCED

32. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are massless and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is: **(2013)**

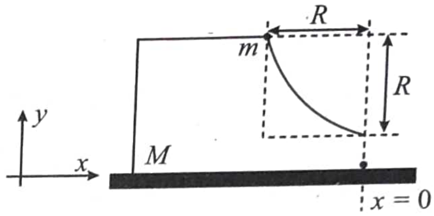
33. A particle of mass m is projected from the ground with an initial seed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is: **(2013)**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+a$ (c) $\frac{\pi}{4}-a$ (d) $-\frac{\pi}{4}$

34. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. **(2014)**



35. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$ in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and its slides down. When the mass loses contact with the block, its position is x and the velocity is v. At the instant, which of the following options is/are correct? **(2017)**



(a) The velocity of the point mass m is : $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(b) The x component of displacement of the center of mass of the block M is : $-\dfrac{mR}{M+m}$

(c) The position of the point mass is $x = -\sqrt{2}\dfrac{mR}{M+m}$

(d) The velocity of the block M is : $V = -\dfrac{m}{M}\sqrt{2gR}$

36. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is /are true? **(2017)**

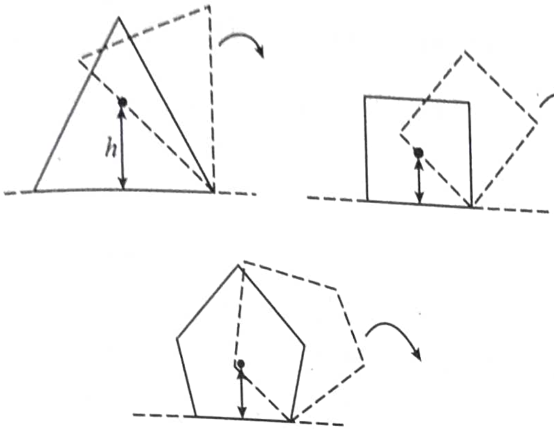
(a) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

(b) At a later time the external force F balances the resistive force

(c) The resistive force experienced by the plate is proportional to v

(d) The plate will continue to move with constant non-zero acceleration, at all times

37. Consider regular polygon with number of sides n = 3, 4, 5............. as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ.Then Δ depends on n and h as **(2017)**



(*a*) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

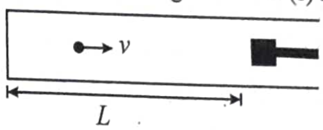
(*b*) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

(*c*) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

(*d*) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

38. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4 kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and τ = 4s. The displacement of the block, in metres, at $t = \tau$ is (Take $e^{-1} = 0.37$) **(2018)**

39. A small particle of mass m moving inside a heavy, hollow ad straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$, the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$ where dL is the infinitesimal displacement of the piston. Which of the following statement (s) is/are correct? **(2019)**



(*a*) The rate at which the particle strikes the piston is v/L

(*b*) After each collision with the piston, the particle speed increases by $2V$

(*c*) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(*d*) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

ANSWER KEY

CONCEPT APPLICATION

1. (a) 2. (b) 3. (a) 4. (d) 5. (a) 6. (c) 7. (a) 8. (a) 9. (d) 10. (b)
11. (b) 12. (c) 13. (c) 14. (b) 15. (b) 16. (a) 17. (a) 18. (c) 19. $\left[\frac{\sqrt{10}}{3}, \frac{4}{3}\sqrt{5}\right]$
20. (a) 21. (a) 22. [1.575, 1.05] 23. (d) 24. (b) 25. [1, 0.5, 8, 24, 4] 26. (c)
27. (d) 28. (d) 29. (c) 30. [1232.6] 31. [3] 32. [10, 20]

EXERCISE-1 (TOPICWISE)

1. (d) 2. (d) 3. (b) 4. (b) 5. (d) 6. (c) 7. (c) 8. (b) 9. (c) 10. (b)
11. (c) 12. (a) 13. (d) 14. (d) 15. (a) 16. (c) 17. (d) 18. (b) 19. (a) 20. (a)
21. (c) 22. (b) 23. (d) 24. (c) 25. (a) 26. (a) 27. (a) 28. (d) 29. (a) 30. (b)
31. (a) 32. (a) 33. (c) 34. (a) 35. (b) 36. (c) 37. (d) 38. (d) 39. (b) 40. (d)
41. (b) 42. (a) 43. (d) 44. (a) 45. (b) 46. (d) 47. (b) 48. (d) 49. (b) 50. (b)

EXERCISE-2 (LEARNING PLUS)

1. (a) 2. (d) 3. (b) 4. (b) 5. (c) 6. (a) 7. (c) 8. (b) 9. (a) 10. (c)
11. (b) 12. (d) 13. (b) 14. (a) 15. (c) 16. (a) 17. (a) 18. (b) 19. (b) 20. (b)
21. (c) 22. (a) 23. (a) 24. (c) 25. (c) 26. (c) 27. (a) 28. (a) 29. (a) 30. (b)
31. (c) 32. (a) 33. (d) 34. (a) 35. (d) 36. (b) 37. (c) 38. (c) 39. (b) 40. (c)
41. (a) 42. (d) 43. (c) 44. (b) 45. (c)

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (b,d) 2. (b,c) 3. (b,d) 4. (b,d) 5. (a,d) 6. (a,d) 7. (a,c) 8. (a,c) 9. (b,d) 10. (a,c)
11. (a,b,d) 12. (a,b,c,d) 13. (b) 14. (b,d) 15. (c) 16. (c) 17. (b,c) 18. (b,d) 19. (a,b,c,d)
20. (a) 21. (d) 22. (a) 23. (b) 24. (a) 25. [10] 26. [25] 27. [2000] 28. [125]
29. [19]

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (a) 2. (c) 3. (c) 4. (c) 5. [1] 6. (b) 7. (b) 8. (d) 9. (a) 10. (b)
11. [10] 12. [23] 13. (d) 14. (d) 15. (d) 16. (c) 17. [03] 18. (d) 19. [120]
20. [4] 21. (a) 22. [25] 23. (d) 24. (d) 25. [12] 26. (c) 27. (c) 28. [10] 29. (b)
30. [2] 31. (d)

JEE Advanced

32. [5] 33. (a) 34. (b) 35. (a,b) 36. (a,b,c) 37. (d) 38. [6.30] 39. (b,c)