

CHAPTER 2

Units and Measurements

PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationships.

Fundamental Quantities

A set of physical quantities which are completely independent of each other but all other physical quantities can be expressed in terms of these physical quantities is called set of Fundamental Quantities.

The Fundamental Quantities that are currently being accepted by the scientific community are mass, time, length, current, temperature, luminous intensity and amount of substance.

Derived Physical Quantities

The physical quantities that can be expressed in terms of fundamental physical quantities are called derived physical quantities. E.g. speed = distance/time.

MEASUREMENT

Measurement is the comparison of a quantity with a standard of the same physical quantity. In the past different countries followed different standards.

UNITS

All physical quantities are measured with respect to standard magnitude of the same physical quantity and these standards are called UNITS. e.g. second, meter, kilogram, etc.

Four basic properties of units are:

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

SYSTEM OF UNITS

1. **FPS or British Engineering system:** In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

2. **CGS or Gaussian system:** In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).

3. **MKS system:** In this system also the fundamental quantities are length, mass and time but their fundamental units are meter (m), kilogram (kg) and second (s) respectively.

Table: Units of some physical quantities in different systems

Type of physical Quantity	Physical Quantity	System		
		CGS	MKS	FPS
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s

4. **International system (SI) of units:** This system is modification over the MKS system. Besides the three base units of MKS system four other fundamental and two supplementary units are also included in this system.

Table: SI base quantities and their units

S. No.	Physical quantity	unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Electric current	ampere	A
6.	Luminous Intensity	candela	cd
7.	Amount of substance	mole	mol

Supplementary Units

In SI, two supplementary units are also defined viz.

- (i) radian (rad) for plane angle and
- (ii) steradian (sr) for solid angle.

Quantity	SI Unit	
	Name	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

- (i) **Radian:** Radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.
- (ii) **Steradian:** Steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere which is equal in area to the square of the radius of the sphere.

Physical Quantity (SI Unit)	Definition
Length (m)	The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit m s^{-1} , where the second is defined in terms of the caesium frequency $\Delta\nu_{\text{Cs}}$.
Mass (kg)	The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.62607015 \times 10^{-34}$ when expressed in the unit J s , which is equal to $\text{kg m}^2 \text{s}^{-1}$, where the metre and the second are defined in terms of c and $\Delta\nu_{\text{Cs}}$.
Time (s)	The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{\text{Cs}}$, the unperturbed ground state hyperfine transition frequency of the caesium-133 atom, to be 9192631770 when expressed in the unit Hz , which is equal to s^{-1} .
Electric Current (A)	The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602176634 \times 10^{-19}$ when expressed in the unit C , which is equal to A s , where the second is defined in terms of $\Delta\nu_{\text{Cs}}$.
Thermodynamic Temperature (K)	The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380649×10^{-23} when expressed in the unit J K^{-1} , which is equal to $\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$, where the kilogram, metre and second are defined in terms of h , c and $\Delta\nu_{\text{Cs}}$.
Amount of substance (mole)	The mole, symbol mol, is the SI unit of amount of substance. One mole substance contains exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_{A} , when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n , of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

Physical Quantity (SI Unit)	Definition
Luminous Intensity (cd)	The candela, symbol cd, is the SI unit of luminous intensity in given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12} \text{ Hz}$, K_{cd} , to be 683 when expressed in the unit lm W^{-1} , which is equal to cd sr W^{-1} , or $\text{cd sr kg}^{-1} \text{m}^{-2} \text{s}^3$, where the kilogram, metre and second are defined in terms of h , c and $\Delta\nu_{\text{Cs}}$.

DIMENSIONS OF PHYSICAL QUANTITIES

All the physical quantities of interest can be derived from the base quantities. The power (exponent) of base quantity that enters into the expression of a physical quantity, is called the dimension of the quantity in that base. To make it clear, consider the physical quantity force.

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$= \text{Mass} \times \frac{\text{Length} / \text{Time}}{\text{Time}} = \text{Mass} \times \text{Length} \times (\text{Time})^{-2}$$

So the dimensions of force are 1 in mass, 1 in length and -2 in time. Thus

$$[\text{Force}] = \text{MLT}^{-2}$$

Similarly, energy has dimensional formula given by

$$[\text{Energy}] = \text{ML}^2\text{T}^{-2}$$

i.e. energy has dimensions 1 in mass, 2 in length and -2 in time.

Such an expression for a physical quantity in terms of base quantities is called dimensional formula

Physical quantity can be further of four types:

1. Dimensionless constant i.e. $1, 2, 3, \pi$ etc.
2. Dimensionless variable i.e. angle θ etc.
3. Dimensional constant i.e. G, h etc.
4. Dimensional variable i.e. F, v , etc.

The variables inside any mathematical function are always dimensionless. Eg., in $\log x$, $\sin x$, e^x etc, x is dimensionless.

ADVANCED LEARNING

Table: Units and dimensions of some physical quantities

Quantity	SI Unit	Dimension
Density	kg/m^3	M/L^3
Force	newton (N)	ML/T^2
Work	joule (J) (= N m)	ML^2/T^2
Energy	joule (J)	ML^2/T^2
Power	Watt (W) (= J/s)	ML^2/T^3
Momentum	kg m/s	ML/T
Gravitational constant	$\text{N m}^2/\text{kg}^2$	L^3/MT^2
Angular velocity	radian/s	T^{-1}
Angular acceleration	radian/ s^2	T^{-2}
Angular momentum	$\text{kg m}^2/\text{s}$	ML^2/T

Quantity	SI Unit	Dimension
Moment of inertia	kg m ²	ML ²
Torque	N m	ML ² /T ²
Angular frequency	radian/s	T ⁻¹
Frequency	hertz (Hz)	T ⁻¹
Period	s	T
Surface Tension	N/m	M/T ²
Coefficient of viscosity	N s/m ²	M/LT
Wavelength	m	L
Intensity of wave	W/m ²	M/T ³
Temperature	Kelvin (K)	K
Specific heat capacity	J/kg K	L ² /T ² K
Stefan's constant	W/m ² K ⁴	M/T ³ K ⁴
Heat	J	ML ² /T ²
Thermal conductivity	W/m-K	ML/T ³ K
Current density	A/m ²	I/L ²
Electrical conductivity	1/Ω m (= mho/m)	I ² T ³ /ML ³
Electric dipole moment	C m	LIT
Electric field	V/m (=N/C)	ML/IT ³
Potential (voltage)	Volt (V) (=J/C)	ML ² /IT ³
Electric flux	V m	ML ³ /IT ³
Capacitance	Farad (F)	I ² T ⁴ /ML ²
Electromotive force	Volt (V)	ML ² /IT ³
Resistance	ohm (Ω)	ML ² /I ² T ³
Permittivity of space	C ² /N m ²	I ² T ⁴ /ML ³
Permeability of space	N/A ²	ML/I ² T ²
Magnetic field	Tesla (T) (= Wb/m ²)	M/IT ²
Magnetic flux	Weber (Wb)	ML ² /IT ²
Magnetic dipole moment	A m ²	IL ²
Inductance	Henry (H)	ML ² /I ² T ²

Dimensional Equation

Whenever the dimension of a physical quantity is equated with its dimensional formula, we get a dimensional equation.

Principle of Homogeneity

The magnitude of a physical quantity may be added or subtracted from each other only if they have the same dimension. Also the dimension on both sides of an equation must be same. This is called as principle of homogeneity.



Train Your Brain

Example 1: The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a , b , c and d .

Sol. The equation contains five terms. All of them should have the same dimensions. Since $[x] = \text{length}$, each of the remaining four must have the dimension of length.

Thus, $[a] = \text{length} = L$

$$[bt] = L, \quad \text{or} \quad [b] = [LT^{-1}]$$

$$[ct^2] = L, \quad \text{or} \quad [c] = [LT^{-2}]$$

$$\text{and} \quad [dt^3] = L \quad \text{or} \quad [d] = [LT^{-3}]$$

Example 2: Calculate the dimensional formula of energy

from the equation $E = \frac{1}{2}mv^2$.

Sol. Dimensionally, $E = \text{mass} \times (\text{velocity})^2$, since $\frac{1}{2}$ is a number and has no dimension.

$$\text{or, } [E] = [M] \times \left[\frac{L}{T} \right]^2 = [ML^2T^{-2}].$$

Example 3: Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ where s is the distance travelled by the particle. Determine dimensions of α .

$$\text{Sol. } K = \alpha s^2 \Rightarrow \alpha = \frac{K}{s^2}$$

$$[\alpha] = \left[\frac{(ML^2T^{-2})}{(L^2)} \right]$$

$$[\alpha] = [M^1 L^0 T^{-2}]$$

$$[\alpha] = [M T^{-2}]$$

Example 4: The position of a particle at time t , is given by

the equation, $x(t) = \frac{v_0}{\alpha} (1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively.

$$(a) [M^0 L^1 T^0] \text{ and } [T^{-1}] \quad (b) [M^0 L^1 T^{-1}] \text{ and } [T]$$

$$(c) [M^0 L^1 T^{-1}] \text{ and } [T^{-1}] \quad (d) [M^1 L^1 T^{-1}] \text{ and } [LT^{-2}]$$

Sol. $[\alpha] [t] = [M^0 L^0 T^0]$ and $[v_0] = [x] [\alpha]$

$$[\alpha] = [M^0 L^0 T^{-1}] = [M^0 L^1 T^{-1}]$$

Example 5: If $\frac{v + A\sqrt{s}}{Bt^2} = F$; find the dimension of A and B where $F = \text{force}$, $v = \text{velocity}$, $s = \text{displacement}$ and $t = \text{time}$

Sol. Using principle of homogeneity; $[A\sqrt{s}] = [v]$

$$\left[AL^{\frac{1}{2}} \right] = [LT^{-1}]$$

$$\therefore [A] = \left[L^{\frac{1}{2}} T^{-1} \right]$$

$$\text{Also, we can write, } B = \frac{v + A\sqrt{s}}{Ft^2}$$

$$[B] = \frac{[LT^{-1}]}{[(MLT^{-2})(T^2)]} \Rightarrow [B] = [M^{-1} L^0 T^{-1}]$$



Concept Application

1. If $\frac{v}{A} - Bu = at$, find the dimension of A and B where u, v = velocity, a = acceleration, t = time.
2. If $\frac{v}{t^2} = A\sqrt{s} - B$ where v is velocity and s is displacement. Find the dimension of A and B .
3. If $A \sin\left(\frac{B}{t^2} + C\right)$ is equation of displacement of a body as a function of time t . Find dimensions of A, B, C .
4. If displacement, $y = \frac{A}{t^2} - Bt^3$ where t is time. Find dimension of $[A \times B]$.

DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

Checking the Dimensional Consistency of Equations

It is based on principle of homogeneity, which states that a given physical relation is dimensionally correct if the dimensions of the various terms on either side of the relation are the same.

Remark:

- ❖ Powers are dimensionless
- ❖ The input and output of all trigonometric and logarithmic, functions must be dimensionless. Eg., $\sin\theta, e^\theta, \cos\theta, \log\theta$ give dimensionless value and in above expression θ is dimensionless
- ❖ We can add or subtract quantity having same dimensions only.



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Example 6: Check the accuracy of the relation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

for a simple pendulum using dimensional analysis.

Sol. From principle of homogeneity of dimension, the dimensions of LHS = The dimension of RHS

$$\text{Now, } [T] = [M^0 L^0 T^1]$$

The dimensions of

$$\text{RHS} = \left(\frac{\text{dimension of length}}{\text{dimension of acceleration}} \right)^{1/2}$$

($\because 2\pi$ is a dimensionless constant)

$$[\text{RHS}] = \left[\frac{L}{LT^{-2}} \right]^{1/2} = [T^2]^{1/2} = [T] = [M^0 L^0 T^1] = [\text{LHS}].$$

So, equation is correct.

Example 7: Check whether the given relation $\frac{Fv^2}{t} = \text{KE}$ (Kinetic Energy) is dimensionally correct? Here F = force, v = velocity and t = time?

$$\text{Sol. } [\text{LHS}] = \left[\frac{Fv^2}{t} \right] = \frac{[MLT^{-2}] \times [L^2 T^{-2}]}{[T]} = [ML^3 T^{-5}]$$

$$[\text{RHS}] = [\text{KE}] = \left[\frac{1}{2}mv^2 \right] = [M \times L^2 T^{-2}] = [ML^2 T^{-2}]$$

$\therefore [\text{LHS}] \neq [\text{RHS}]$, so the given relation is incorrect dimensionally.



Concept Application

5. Consider the following equation: $a = qvbt^2 + c\left(\frac{t}{x}\right)^2$, where a, b, c are constants (not necessarily dimensionless) and q, v, x and t represent charge, velocity, distance and time respectively. For the equation to be dimensionally correct,

$$(a) \left[\frac{a}{c} \right]^{\frac{1}{2}} = [LT^{-1}]$$

$$(b) \left[\frac{a}{c} \right]^{\frac{1}{2}} = [L^{-1}T^{-1}]$$

$$(c) \left[\frac{b}{c} \right] = [L^{-3}A^{-1}]$$

$$(d) \left[\frac{a}{b} \right] = [LTA]$$

6. Consider the statements below.
 - (i) A dimensionally consistent equation is a physically correct equation.
 - (ii) A dimensionally consistent equation may or may not be correct.
 - (iii) A dimensionally inconsistent equation is an incorrect equation.
 - (iv) A dimensionally inconsistent equation may or may not be incorrect.

The correct statement(s) is/are

- (a) (i), (iii) and (iv)
- (b) (ii) and (iv)
- (c) only (iii)
- (d) (ii) and (iii)

Deducing Relation Among the Physical Quantities

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors by using principle of homogeneity.



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Example 8: Find an expression for the time period T of a simple pendulum. The time period T may depend upon (i) mass m of the bob of the pendulum, (ii) length ℓ of pendulum, (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Sol. Let (i) $T \propto m^a$ (ii) $T \propto \ell^b$ (iii) $T \propto g^c$

Combining all the three factors, we get

$$T \propto m^a \ell^b g^c \quad \text{or} \quad T = K m^a \ell^b g^c \quad \dots(i)$$

where K is a dimensionless constant of proportionality.

Writing down the dimensions on either side of equation (i), we get

$$[T] = [M^a][L^b][LT^{-2}]^c = [M^a L^{b+c} T^{-2c}]$$

Comparing dimensions, $a = 0$, $b + c = 0$, $-2c = 1$

$$\therefore a = 0, c = -1/2, b = 1/2$$

From equation (i) $T = K m^0 \ell^{1/2} g^{-1/2}$

$$\text{or} \quad T = K \left(\frac{\ell}{g} \right)^{1/2} = K \sqrt{\frac{\ell}{g}}$$

The value of K , as found by experiment or mathematical investigation, comes out to be 2π .

$$\therefore T = 2\pi \sqrt{\frac{\ell}{g}}$$

Example 9: When a solid sphere moves through a liquid, the liquid opposes the motion with a force F . The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Sol. Suppose the formula is $F = k \eta^a r^b v^c$

$$\begin{aligned} \text{Then, } [MLT^{-2}] &= [ML^{-1} T^{-1}]^a [L]^b \left[\frac{L}{T} \right]^c \\ &= [M^a L^{-a+b+c} T^{-a-c}] \end{aligned}$$

Equating the exponents of M , L and T from both sides,

$$a = 1$$

$$-a + b + c = 1$$

$$-a - c = -2$$

Solving these, $a = 1$, $b = 1$ and $c = 1$

Thus, the formula for F is $F = k\eta r v$.

Example 10: If P is the pressure of a gas and ρ is its density, then find the dimension of velocity in terms of P and ρ .

$$(a) P^{1/2} \rho^{-1/2}$$

$$(b) P^{1/2} \rho^{1/2}$$

$$(c) P^{-1/2} \rho^{1/2}$$

$$(d) P^{-1/2} \rho^{-1/2}$$

Sol. Method - I

$$[P] = [ML^{-1}T^{-2}] \quad \dots(i)$$

$$[\rho] = [ML^{-3}]$$

...(ii)

Dividing eq. (i) by (ii)

$$[P\rho^{-1}] = [L^2T^{-2}]$$

$$\Rightarrow [LT^{-1}] = [P^{1/2}\rho^{-1/2}]$$

$$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$$

Method - II

$$v \propto P^a \rho^b$$

$$v = k P^a \rho^b$$

$$[LT^{-1}] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b$$

$$\Rightarrow a = \frac{1}{2}, b = -\frac{1}{2} \quad (\text{Equating dimensions})$$

$$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$$

Example 11: Find relationship between speed (v) of sound in a medium, the elastic constant (E) and the density of the medium (ρ).

Sol. Let the speed depends upon elastic constant and density according to the relation

$$v \propto E^a \rho^b$$

$$\text{or} \quad v = K E^a \rho^b \quad \dots(i)$$

where K is a dimensionless constant of proportionality.

Considering dimensions of the quantities

$$[v] = M^0 L T^{-1}$$

$$[E] = \frac{[\text{stress}]}{[\text{strain}]} = \frac{[\text{force}]/[\text{area}]}{[\Delta\ell]/[\ell]} = \frac{[M^1 L^1 T^{-2}]/[L^2]}{[L^1]/[L^1]} = [M^1 L^{-1} T^{-2}]$$

$$\therefore [E^a] = [M^a L^{-a} T^{-2a}]$$

$$[\rho] = [\text{mass}]/[\text{volume}] = [M]/[L^3] = [M^1 L^{-3} T^0]$$

$$\therefore [\rho^b] = [M^b L^{-3b} T^0]$$

Equating the dimensions of the LHS and RHS quantities of equation (i), we get

$$[M^0 L^1 T^{-1}] = [M^a L^{-a} T^{-2a}] \times [M^b L^{-3b} T^0]$$

$$\text{or } [M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}]$$

Comparing the individual dimensions of M , L and T

$$a + b = 0, \quad \dots(ii)$$

$$-a - 3b = 1, \text{ and} \quad \dots(iii)$$

$$-2a = -1 \quad \dots(iv)$$

Solving we get

$$a = \frac{1}{2}, b = -\frac{1}{2}$$

Therefore the required relation is

$$v = K \sqrt{\frac{E}{\rho}}$$

Example 12: Pressure (P) acting due to a fluid kept in a container depends on, weight of liquid (w), Area of cross-section of container (A) and density of fluid (ρ). Establish a formula of pressure (P).

Sol. According to the question

$$P \propto w^x A^y \rho^z$$

$$P = K[w^x A^y \rho^z]$$

Now writing the dimension of each quantity on either side.

$$[ML^{-1}T^{-2}] = K [MLT^{-2}]^x [L^2]^y [ML^{-3}]^z$$

$$[ML^{-1}T^{-2}] = K [M]^{x+z} [L]^{x+2y-3z} [T]^{-2x}$$

Now comparing the powers

$$\text{For } M, \quad 1 = x + z \quad \dots(i)$$

$$L, \quad -1 = x + 2y - 3z \quad \dots(ii)$$

$$T, \quad -2 = -2x \quad \dots(iii)$$

\Rightarrow Solving we get, $x = 1, y = -1, z = 0$

$$\therefore P = KwA^{-1}\rho^0$$

Concept Application

- If c is the velocity of light, h is Planck's constant and G is Gravitational constant are taken as fundamental quantities, then the dimensional formula of mass is
- Taking frequency f , velocity (v) and Density (ρ) to be the fundamental quantities then the dimensional formula for momentum will be
 - $(\rho v^4 f^{-3})$
 - $(\rho v^3 f^{-1})$
 - $(\rho v f^2)$
 - $(\rho^2 v^2 f^2)$
- If momentum (P), mass (M) and time (T) are chosen as fundamental quantities the dimensional formula for length is _____.
 - $[P^1 T^1 M^1]$
 - $[P^1 T^1 M^2]$
 - $[P^1 T^1 M^{-1}]$
 - $[P^2 T^2 M^1]$
- For the equation $F = A^a v^b d^c$ where F is force, A is area, v is velocity and d is density, with the dimensional analysis gives the following values for exponents.
 - $a = 1, b = 2, c = 1$
 - $a = 2, b = 1, c = 1$
 - $a = 1, b = 1, c = 2$
 - $a = 0, b = 1, c = 1$

Conversion From One System of Units to Other

It is based on the fact that,

$$\text{Numerical value} \times \text{unit} = \text{constant}$$

So on changing unit, numerical value will also change. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1 u_1 = n_2 u_2$$

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

LIMITATIONS OF DIMENSIONAL ANALYSIS

- It supplies no information about dimensionless constants and the nature (vector and scalar) of physical quantities.
- This method is applicable only if relation is of product type. It fails in the case of exponential and trigonometric relations.
- It does not predict numerical correctness of formula.

Train Your Brain

Example 13: Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is $[F] = [M^1 L^1 T^{-2}]$

Therefore if n_1, u_1 and n_2, u_2 corresponds to SI and CGS numerical value and unit respectively, then

$$\begin{aligned} n_2 &= n_1 \left[\frac{M_1}{M_2} \right]^1 \left[\frac{L_1}{L_2} \right]^1 \left[\frac{T_1}{T_2} \right]^{-2} \\ &= 1 \left[\frac{\text{kg}}{\text{g}} \right] \left[\frac{\text{m}}{\text{cm}} \right] \left[\frac{\text{s}}{\text{s}} \right]^{-2} = 1 \times (1000)(100)(1) = 10^5 \end{aligned}$$

Example 14: A calorie is a unit of heat or energy and it equals about 4.2 J, where $1 \text{ J} = 1 \text{ kg m}^2/\text{s}^2$. Suppose we employ a system of units in which the unit of mass equals $\alpha \text{ kg}$, the unit of length equals $\beta \text{ metre}$, the unit of time is $\gamma \text{ second}$. Show that a calorie has a magnitude $4.2 \alpha^{-1} \beta^{-2} \gamma^2$ in terms of the new units.

Sol. $1 \text{ cal} = 4.2 \text{ kg m}^2 \text{ s}^{-2}$

SI	New system
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1 \text{ kg}$	$M_2 = \alpha \text{ kg}$
$L_1 = 1 \text{ m}$	$L_2 = \beta \text{ metre}$
$T_1 = 1 \text{ s}$	$T_2 = \gamma \text{ second}$

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^a L^b T^c]$,

We find that $a = 1, b = 2, c = -2$

$$\begin{aligned} \text{Now, } n_2 &= n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c \\ &= 4.2 \left[\frac{1 \text{ kg}}{\alpha \text{ kg}} \right]^1 \left[\frac{1 \text{ m}}{\beta \text{ m}} \right]^2 \left[\frac{1 \text{ s}}{\gamma \text{ s}} \right]^{-2} = 4.2 \alpha^{-1} \beta^{-2} \gamma^2 \end{aligned}$$

Example 15: Young's modulus of steel is $19 \times 10^{10} \text{ N/m}^2$. Express it in dyne/cm^2 . Here dyne is the CGS unit of force.

Sol. The unit of Young's modulus is N/m^2 .

This suggest that it has dimensions of $\frac{\text{Force}}{(\text{Distance})^2}$.

$$\text{Thus, } [Y] = \left[\frac{F}{L^2} \right] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}].$$

N/m^2 is in SI units, so, $1 \text{ N/m}^2 = (1 \text{ kg})(1\text{m})^{-1} (1\text{s})^{-2}$
 and $1 \text{ dyne/cm}^2 = (1\text{g})(1\text{cm})^{-1} (1\text{s})^{-2}$ so, $\frac{1 \text{ N/m}^2}{1 \text{ dyne/cm}^2}$
 $= \left(\frac{1\text{kg}}{1\text{g}}\right) \left(\frac{1\text{m}}{1\text{cm}}\right)^{-1} \left(\frac{1\text{s}}{1\text{s}}\right)^{-2} = 1000 \times \frac{1}{100} \times 1 = 10$
 or, $1 \text{ N/m}^2 = 10 \text{ dyne/cm}^2$
 or, $19 \times 10^{10} \text{ N/m}^2 = 19 \times 10^{11} \text{ dyne/cm}^2$.

Example 16: The dimensional formula for viscosity of fluids is $[\eta] = [\text{M}^1 \text{L}^{-1} \text{T}^{-1}]$. Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $[\eta] = [\text{M}^1 \text{L}^{-1} \text{T}^{-1}]$
 $1 \text{ CGS units} = \text{g cm}^{-1} \text{s}^{-1}$
 $1 \text{ SI units} = \text{kg m}^{-1} \text{s}^{-1}$
 $= (1000 \text{ g})(100 \text{ cm})^{-1} \text{s}^{-1} = 10 \text{ g cm}^{-1} \text{s}^{-1}$
 Thus, 1 Poiseuille = 10 poise



Concept Application

- If minute is the unit of time, 10 ms^{-2} is the unit of acceleration and 100 kg is the unit of mass, the new unit of work in joule is
 (a) 10^5 (b) 10^6
 (c) 6×10^6 (d) 36×10^6
- The magnitude of force is 100 N. What will be its value if the units of mass and time are doubled and that of length is halved?
 (a) 25 (b) 100
 (c) 200 (d) 400
- The value of universal gravitational constant G in CGS system is $6.67 \times 10^{-8} \text{ dyne cm}^2 \text{g}^{-2}$. Its value in SI system is
 (a) $6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$ (b) $6.67 \times 10^{-5} \text{ Nm}^2 \text{kg}^{-2}$
 (c) $6.67 \times 10^{-10} \text{ Nm}^2 \text{kg}^{-2}$ (d) $6.67 \times 10^{-9} \text{ Nm}^2 \text{kg}^{-2}$
- Which equation cannot be derived using dimensional analysis from the following? (k is a dimensionless constant and x does not necessarily represent distance. Remaining variables follow standard meaning)
 (a) $x = 2at^2$ (b) $x = k \sin(\omega t)$
 (c) $x = \frac{r^2 \rho g}{\eta}$ (d) All of these

MEASUREMENT OF LENGTH

You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for lengths from 10^{-3} m to 10^2 m . A vernier callipers is used for lengths to an accuracy of 10^{-4} m . A screw gauge and a spherometer can be used to measure lengths as less as 10^{-5} m . To measure lengths beyond these ranges, we make use of some special indirect methods.

Range of Lengths

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-15} m of the proton to the size of the order of 10^{26} m of the extent of the observable universe.

We also use certain special length units for short and large lengths. These are

$$1 \text{ fermi} = 1 \text{ f} = 10^{-15} \text{ m}$$

1 angstrom = $1 \text{ \AA} = 10^{-10} \text{ m}$ (It is used mainly in measuring wavelength of light)

1 astronomical unit = 1 AU (average distance of the Sun from the Earth) = $1.496 \times 10^{11} \text{ m}$

1 light year = $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$ (distance that light travels with velocity of $3 \times 10^8 \text{ m s}^{-1}$ in 1 year)

1 parsec = $3.08 \times 10^{16} \text{ m}$ (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)

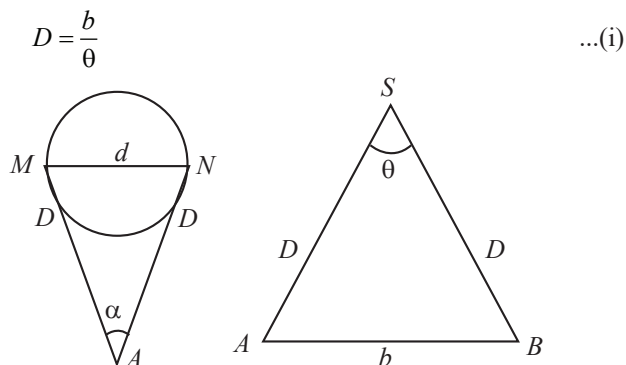
MEASUREMENT OF LARGE DISTANCES

Parallax Method

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method. When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called parallax.

The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes. To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance $AB = b$ at the same time as shown in figure. We measure the angle between the two directions along which the planet is viewed at these two points. The $\angle ASB$ in figure represented by symbol θ is called the parallax angle or parallactic angle.

As the planet is very far away, $\frac{b}{D} \ll 1$ and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius $AS = BS$ so that $AB = b = D \theta$ where θ is in radians.



$$D = \frac{b}{\theta} \quad \dots(i)$$

Having determined D , we can employ a similar method to determine the size or angular diameter of the planet. If d is the diameter of the planet and α the angular size of the planet (the angle subtended by d at the earth), we have

$$\alpha = d/D \quad \dots(ii)$$

The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through the telescope. Since D is known, the diameter d of the planet can be determined using equation (ii).

ESTIMATION OF VERY SMALL DISTANCES

Size of a Molecule

To measure a very small size, like that of a molecule (10^{-8} m to 10^{-10} m), we have to adopt special methods. We cannot use a screw gauge or similar instruments. Even a microscope has certain limitations. An optical microscope uses visible light to 'look' at the system under investigation. As light has wave like features, the resolution to which an optical microscope can be used is the wavelength of light.

For visible light the range of wavelengths is from about 4000 Å to 7000 Å (1 angstrom = $1 \text{ Å} = 10^{-10}$ m). Hence an optical microscope cannot resolve particles with sizes smaller than this. Instead of visible light, we can use an electron beam. Electron beams can be focused by properly designed electric and magnetic fields. The resolution of such an electron microscope is limited finally by the fact that electrons can also behave as waves.

The wavelength of an electron can be as small as a fraction of an angstrom. Such electron microscopes with a resolution of 0.6 Å have been built. They can almost resolve atoms and molecules in a material. In recent times, tunneling microscopy has been developed in which again the limit of resolution is better than an angstrom. It is possible to estimate the sizes of molecules.

MEASUREMENT OF MASS

Mass is a basic property of matter. It does not depend on the temperature, pressure or location of the object in space. The SI unit of mass is kilogram (kg). It is defined by taking the fixed numerical value of the Planck Constant h to be $6.62607015 \times 10^{-34}$ when expressed in the unit of $J \cdot s$ which is equal to $kg \cdot m^2 \cdot s^{-1}$, where the metre and the second are defined in terms of C and $\Delta \nu_{Cs}$.

While dealing with atoms and molecules, the kilogram is an inconvenient unit. In this case, there is an important standard unit of mass, called the **unified atomic mass unit** (u), which has been established for expressing the mass of atoms as

$$1 \text{ unified atomic mass unit} = 1u$$

$$= \left(\frac{1}{12} \right) \text{ of the mass of an atom of carbon-12 isotope } ({}^{12}_6C)$$

including the mass of electrons

$$= 1.66 \times 10^{-27} \text{ kg}$$

Mass of commonly available objects can be determined by a common balance like the one used in a grocery shop. Large masses in the universe like planets, stars, etc., based on Newton's law of gravitation can be measured by using gravitational method

(See Chapter 8). For measurement of small masses of atomic/sub-atomic particles etc., we make use of mass spectrograph in which radius of the trajectory is proportional to the mass of a charged particle moving in uniform electric and magnetic field.

Range of Masses

The masses of the objects, we come across in the universe, vary over a very wide range. These may vary from tiny mass of the order of 10^{-30} kg of an electron to the huge mass of about 10^{55} kg of the known universe. Table 2.4 gives the range and order of the typical masses of various objects.

Table: Range and order of lengths

Size of object or distance	Length (m)
Size of a proton	10^{-15}
Size of atomic nucleus	10^{-14}
Size of hydrogen atom	10^{-10}
Length of typical virus	10^{-8}
Wavelength of light	10^{-7}
Size of red blood corpuscle	10^{-5}
Thickness of a paper	10^{-4}
Height of the Mount Everest above sea level	10^4
Radius of the Earth	10^7
Distance of moon from the Earth	10^8
Distance of the Sun from the Earth	10^{11}
Distance of Pluto from the Sun	10^{13}
Size of our galaxy	10^{21}
Distance to Andromeda galaxy	10^{22}
Distance to the boundary of observable universe	10^{26}

Table: Range and order of masses

Size of object or distance	Length (m)
Electron	10^{-30}
Proton	10^{-27}
Uranium atom	10^{-25}
Red blood cell	10^{-13}
Dust particle	10^{-9}
Rain drop	10^{-6}
Mosquito	10^{-5}
Grape	10^{-3}
Human	10^2
Automobile	10^3
Boeing 747 aircraft	10^8
Moon	10^{23}
Earth	10^{25}
Sun	10^{30}
Milky way galaxy	10^{41}
Observable Universe	10^{55}

MEASUREMENT OF TIME

To measure any time interval we need a clock. We now use an **atomic standard of time**, which is based on the periodic vibrations produced in a cesium atom. This is the basis of the **caesium clock**, sometimes called **atomic clock**, used in the national standards. Such standards are available in many laboratories. In the caesium atomic clock, the second is taken as the time needed for 9,192,631,770 vibrations of the radiation corresponding to the transition between the two hyperfine levels of the ground state of caesium-133 atom. The vibrations of the caesium atom regulate the rate of this caesium atomic clock just as the vibrations of a balance wheel regulate an ordinary wristwatch or the vibrations of a small quartz crystal regulate a quartz wristwatch.

The caesium atomic clocks are very accurate. In principle they provide portable standard. The national standard of time interval 'second' as well as the frequency is maintained through four caesium atomic clocks. A caesium atomic clock is used at the National Physical Laboratory (NPL), New Delhi to maintain the Indian standard of time.

In our country, the NPL has the responsibility of maintenance and improvement of physical standards, including that of time, frequency, etc. Note that the Indian Standard Time (IST) is linked to this set of atomic clocks. The efficient caesium atomic clocks are so accurate that they impart the uncertainty in time realisation as $\pm 1 \times 10^{-15}$, i.e. 1 part in 10^{15} . This implies that the uncertainty gained over time by such a device is less than 1 part in 10^{15} ; they lose or gain no more than 32 μ s in one year. In view of the tremendous accuracy in time measurement, the SI unit of length has been expressed in terms the path length light travels in certain interval of time (1/299,792,458 of a second) (Table 2.1).

The time interval of events that we come across in the universe vary over a very wide range. Table 2.5 gives the range and order of some typical time intervals.

You may notice that there is an interesting coincidence between the numbers appearing in Tables 2.3 and 2.5. Note that the ratio of the longest and shortest lengths of objects in our universe is about 1041. Interestingly enough, the ratio of the longest and shortest time intervals associated with the events and objects in our universe is also about 10^{41} . This number, 10^{41} comes up again in Table 2.4, which lists typical masses of objects. The ratio of the largest and smallest masses of the objects in our universe is about $(10^{41})^2$. Is this a curious coincidence between these large numbers purely accidental?

ORDER OF MAGNITUDE

If a number P can be expressed as

$$P = A \times 10^x$$

where $0.5 \leq A < 5$, then x is called order of magnitude of the number.

SI Prefixes: The magnitudes of physical quantities vary over a wide range. The mass of an electron is 9.1×10^{-31} kg and that of our earth is about 6×10^{24} kg. Standard prefixes for certain power of 10. Table shows these prefixes

Power of 10	Prefix	Symbol
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deca	da
-1	deci	d
-2	centi	c
-3	milli	m
-6	micro	μ
-9	nano	n
-12	pico	p
-15	femto	f



Train Your Brain

Example 17: The Sun's angular diameter is measured to be 1920". The distance D of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun?

Sol. Sun's angular diameter α .

$$= 1920''$$

$$= 1920 \times 4.85 \times 10^{-6} \text{ rad} = 9.31 \times 10^{-3} \text{ rad}$$

$$\text{Sun's diameter } d = \alpha D$$

$$= (9.31 \times 10^{-3}) \times (1.496 \times 10^{11}) \text{ m} = 1.39 \times 10^9 \text{ m}$$

Example 18: The moon is observed from two diametrically opposite points on the earth with angle subtended $= 1^\circ 54'$ given diameter of earth $= 1.276 \times 10^7$ m. Find the distance of moon from the earth.

Sol. From parallax method,

$$\theta = \frac{b}{D}$$

$$\text{We have, } \theta = 1^\circ 54' = 60' + 54' = 114'$$

$$\text{Also } 1' = 2.91 \times 10^{-4} \text{ rad}$$

$$\therefore \theta = 114 \times 2.91 \times 10^{-4} = 0.033 \text{ rad}$$

$$\therefore D = \frac{1.2760 \times 10^7}{0.033} = 3.8 \times 10^8 \text{ m}$$

Example 19: If the size of a nucleus (in the range of 10^{-15} to 10^{-11} m) is scaled up to the tip of a sharp pin, what roughly is the size of an atom? Assume tip of the pin to be in the range 10^{-5} m to 10^{-4} m.)

Sol. The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10^{-5} m and 10^{-4} m.

Thus we are scaling up by a factor of 10^{10} . An atom roughly of size 10^{-10} m will be scaled up to a size of 1 m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.



Concept Application

15. Considering the distance between sun and moon to be 15×10^{10} m, the angular diameter of the sun as observed from the moon is close to (Take the radius of sun to be 7×10^8 m.)
- (a) 214 rad (b) 107 rad
(c) 0.005 rad (d) 0.009 rad
16. The mass of an object is 75.2×10^4 kg. The order of magnitude of the mass is
- (a) 4 (b) 5
(c) 6 (d) 7

ERROR ANALYSIS IN EXPERIMENTS

Significant Figures or Digits

The significant figures (SF) in a measurement are the figures or digits that are known with certainty plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

1. Rules to find out the number of significant figures:

I - Rule: All the non-zero digits are significant. E.g. 1984 has 4 SF.

II - Rule: All the zeros between two non-zero digits are significant. E.g. 10806 has 5 SF.

III - Rule: All the zeros to the left of first non-zero digit are not significant. E.g. 00108 has 3 SF.

IV - Rule: If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. E.g. 0.002308 has 4 SF.

V - Rule: The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. E.g. 01.080 has 4 SF.

VI - Rule: The trailing zeros in a number without a decimal point are not significant. E.g. 010100 has 3 SF.

VII - Rule: When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$, each term has 3 SF only. (Note: It has 3 significant figure in each expression.)

2. Rules for arithmetical operations with significant figures:

I - Rule: In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. E.g. $12.587 - 12.5 = 0.087 = 0.1$ (\because second term contain lesser i.e. one decimal place)

II - Rule: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. E.g. $5.0 \times 0.125 = 0.625 = 0.62$

To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in **scientific notation** (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number a is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).

Note: The change in the unit of measurement of a quantity does not affect the number of SF. For example in $2.308 \text{ cm} = 23.08 \text{ mm} = 0.02308 \text{ m} = 23080 \mu\text{m}$ each term has 4 SF.

ROUNDING OFF

To represent the result of any computation containing more than one uncertain digit, it is rounded off to appropriate number of significant figures.

Rules for rounding off the numbers:

I - Rule: If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. $6.87 \approx 6.9$

II - Rule: If the digit to be rounded off is less than 5, then the preceding digit is unaffected and is left unchanged. e.g. $3.94 \approx 3.9$

III - Rule: If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. $14.35 \approx 14.4$ and $14.45 \approx 14.4$



Train Your Brain

Example 20: Write down the number of significant figures in the following.

- (i) 165 (ii) 2.05
(iii) 34.000 m (iv) 0.005
(v) 0.02340 N m^{-1} (vi) 26900

Sol. (i) 3 SF (following rule I)
(ii) 3 SF (following rules I and II)
(iii) 5 SF (following rules I and V)
(iv) 1 SF (following rules I and IV)
(v) 4 SF (following rules I, IV and V)
(vi) 3 SF (see rule VI)

Example 21: The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Sol. Length (ℓ) = 4.234 m, Breadth (b) = 1.005 m
Thickness (t) = 2.01 cm = $2.01 \times 10^{-2} \text{ m}$
Therefore, area of the sheet = $2(\ell \times b + b \times t + t \times \ell)$

$$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) \text{ m}^2$$

$$= 2(4.3604739) \text{ m}^2 = 8.720978 \text{ m}^2$$

Since area can contain a maximum of 3 SF (Rule II of arithmetic operations) therefore, rounding off, we get

$$\text{Area} = 8.72 \text{ m}^2$$

$$\text{Like wise volume} = \ell \times b \times t$$

$$= 4.234 \times 1.005 \times 0.0201 \text{ m}^3 = 0.0855289 \text{ m}^3$$

Since volume can contain 3 SF, therefore, rounding off,

$$\text{Volume} = 0.0855 \text{ m}^3$$

Example 22: Find the number of significant figures in each

- (i) 36.72 (ii) 0.003303

Sol. (i) It has four significant figures. All non-zero digits are significant.

- (ii) Here first three zeros are insignificant but zero between 3's is significant. So it has four significant figures.

Example 23: Round off following values to four significant figures.

- (i) 36.879 (ii) 1.0084
(iii) 11.115 (iv) 11.1250
(v) 11.1251

Sol. The following values can be rounded off to four significant figures as follows:

- (i) $36.879 \approx 36.88$ ($\because 9 > 5 \therefore 7$ is increased by one i.e. I Rule)
(ii) $1.0084 \approx 1.008$ ($\because 4 < 5 \therefore 8$ is left unchanged i.e. II Rule)
(iii) $11.115 \approx 11.12$ (\because last 1 is odd it is increased by one i.e. III Rule)
(iv) $11.1250 \approx 11.12$ ($\because 2$ is even it is left unchanged i.e. III Rule)
(v) $11.1251 \approx 11.13$ ($\because 51 > 50 \therefore 2$ is increased by one i.e. I Rule)



Concept Application

- The number of significant figures in 0.0006032 is
(a) 7 (b) 4
(c) 5 (d) 2
- The radius of disc is 1.2 cm, its area according to idea of significant figures is _____
(a) 4.5216 cm^2 (b) 4.521 cm^2
(c) 4.52 cm^2 (d) 4.5 cm^2
- The number of significant figures in $5.69 \times 10^{15} \text{ kg}$ is
(a) 1 (b) 2
(c) 3 (d) 4
- When 57.986 is rounded off to 4 significant figures, then it becomes _____.
(a) 58 (b) 57
(c) 57.90 (d) 57.99

ERRORS IN MEASUREMENT

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Classification of Errors

Errors may arise from different sources and are usually classified as follows:

Systematic or controllable errors: Systematic errors are the errors whose causes are known. They can be either positive or negative. Systematic errors can further be classified into three categories:

- Instrumental errors:** These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.
- Environmental errors:** These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic and electrostatic fields.
- Observational errors:** These errors arise due to improper setting of the apparatus or carelessness in taking observations. This can be reduced by proper setting of the instrument before we start using it.
- Personal Error:** This type of error may creep if the experimenter is not careful enough while performing the experiment and noting down the readings.

Random errors: These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely they can not be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random errors can be reduced by repeating the observation a large number of times and taking the arithmetic mean of all the observations. This mean value would be very close to the most accurate reading.

Note: If the number of observations is made n times then the random error reduces to $\left(\frac{1}{n}\right)$ times. E.g. If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$

REPRESENTATION OF ERRORS

Errors can be expressed in the following ways:

- Mean Absolute Error:** The mean of measurements of a physical quantity (a_1, a_2, \dots, a_n) is expressed as

$$a_m = \frac{a_1 + a_2 + \dots + a_n}{n} = \sum_{i=1}^n \frac{a_i}{n}$$

a_m is taken as the true value of a quantity if the same is not known.

The absolute error in the measurements are expressed as

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

.....

$$\Delta a_n = a_m - a_n$$

Mean absolute error is defined as

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n} = \sum_{i=1}^n \frac{\Delta a_i}{n}$$

Final result of measurement may be written as:

$$a = a_m \pm \overline{\Delta a}$$

2. Relative Error or Fractional Error: It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. Percentage Error = $\frac{\overline{\Delta a}}{a_m} \times 100\%$



Train Your Brain

Example 24: The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. Find (i) mean time period (ii) absolute error in each observation and percentage error.

Sol. (i) Mean time period is given by

$$\bar{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.62\text{s}$$

(ii) The absolute error in each observation is

$$2.62 - 2.63 = -0.01, 2.62 - 2.56$$

$$= 0.06, 2.62 - 2.42$$

$$= 0.20, 2.62 - 2.71$$

$$= -0.09, 2.62 - 2.80$$

$$= -0.18$$

$$\text{Mean absolute error, } \overline{\Delta T} = \frac{\sum |\Delta T|}{5}$$

$$= \frac{0.01 + 0.06 + 0.2 + 0.09 + 0.18}{5} = 0.11\text{sec}$$

\therefore Percentage error is

$$= \frac{\overline{\Delta T}}{\bar{T}} \times 100 = \frac{0.11}{2.62} \times 100 = 4.2\%$$

Example 25: In an experiment the values of refractive indices of glass material recorded as 1.56, 1.53, 1.54, 1.45, 1.44 and 1.43 in repeated measurements. Find

(i) Mean value of μ

(ii) Mean absolute error

(iii) Relative error

(iv) Percentage error

Sol. (i) Mean value of μ

$$\bar{\mu} = \frac{\mu_1 + \mu_2 + \dots + \mu_n}{n}$$

$$\bar{\mu} = \frac{1.56 + 1.54 + 1.53 + 1.45 + 1.44 + 1.43}{6} = 1.491 \approx 1.50$$

(ii) Absolute error $|\Delta \mu|$

$$|\Delta \mu_1| = |\bar{\mu} - \mu_1| = 0.06$$

$$|\Delta \mu_2| = |\bar{\mu} - \mu_2| = 0.04$$

$$|\Delta \mu_3| = |\bar{\mu} - \mu_3| = 0.03$$

$$|\Delta \mu_4| = |\bar{\mu} - \mu_4| = 0.05$$

$$|\Delta \mu_5| = |\bar{\mu} - \mu_5| = 0.06$$

$$|\Delta \mu_6| = |\bar{\mu} - \mu_6| = 0.07$$

\therefore mean absolute error

$$|\Delta \bar{\mu}| = \frac{|\Delta \mu_1| + |\Delta \mu_2| + \dots + |\Delta \mu_6|}{6}$$

$$= \frac{0.06 + 0.04 + 0.03 + 0.05 + 0.06 + 0.07}{6}$$

$$= 0.051$$

$$\therefore \text{Reading} = \bar{\mu} \pm |\Delta \bar{\mu}| = 1.50 \pm 0.05$$

$$(iii) \text{ Relative error} = \pm \frac{0.05}{1.50} = \pm 0.033$$

$$(iv) \text{ Relative percentage error} = \text{Relative error} \times 100\%$$

$$= \pm 0.033 \times 100\%$$

$$= \pm 3.3\% \text{ (approx)}$$



Concept Application

21. The diameter of a wire as measured by a screw gauge was found to be 1.004 cm, 1.000 cm. Find the absolute error in first reading.

22. The diameter of a thick wire measured by a screw-gauge.

$$D_1 = 1.006 \text{ cm}, D_2 = 1.004 \text{ cm}, D_3 = 1.002 \text{ cm}$$

Find mean absolute error and write the reading.

COMBINATION OF ERRORS

(i) **In Sum:** If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$.

Maximum fractional error in this case is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A+B} + \frac{\Delta B}{A+B}$$

i.e. when two physical quantities are added then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

(ii) **In Difference:** If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A - B} + \frac{\Delta B}{A - B}$$

(iii) **In Product:** If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where $\Delta Z/Z$ is known as fractional error.

(iv) **In Division:** If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(v) **In Power:** If $Z = A^n$ then $\frac{\Delta Z}{Z} = n \frac{\Delta A}{A}$

In more general form if $Z = \frac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x \frac{\Delta A}{A} + y \frac{\Delta B}{B} + q \frac{\Delta C}{C}$$

Applications:

1. For a simple pendulum, $T \propto l^{1/2}$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta l}{l}$$

2. For a sphere, area and volume are given as

$$A = 4\pi r^2, V = \frac{4}{3}\pi r^3$$

So, relative errors in them are given as

$$\frac{\Delta A}{A} = 2 \frac{\Delta r}{r} \text{ and } \frac{\Delta V}{V} = 3 \frac{\Delta r}{r}$$

3. When two resistors R_1 and R_2 are connected

(i) In series $R_s = R_1 + R_2 \Rightarrow \Delta R_s = \Delta R_1 + \Delta R_2$

$$\Rightarrow \frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

(ii) In parallel,

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$



Train Your Brain

Example 26: In an experiment of simple pendulum, the errors in the measurement of length of the pendulum (L) and time period (T) are 3% and 2% respectively. The maximum percentage error in the value of L/T^2 is

- (a) 5% (b) 7%
(c) 8% (d) 1%

Sol. Maximum percentage in the value of L/T^2

$$= \frac{\Delta L}{L} \times 100\% + 2 \frac{\Delta T}{T} \times 100\% \\ = 3 + 2 \times 2 = 7\%$$

Example 27: If $X = \frac{A^2 \sqrt{B}}{C}$, then

$$(a) \Delta X = \Delta A + \Delta B + \Delta C$$

$$(b) \frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$$

$$(c) \frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$$

$$(d) \frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$$

Sol. $\therefore X = A^2 B^{1/2} C$

$$\therefore \frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$$

Example 28: A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity?

Sol. Given distance, $s = (13.8 \pm 0.2)$ m

and time $t = (4.0 \pm 0.3)$ s

$$\text{Velocity } v = \frac{s}{t} = \frac{13.8}{4.0} = 3.45 \text{ ms}^{-1} = 3.5 \text{ ms}^{-1}$$

$$\frac{\Delta v}{v} = \pm \left(\frac{\Delta s}{s} + \frac{\Delta t}{t} \right) = \pm \left(\frac{0.2}{13.8} + \frac{0.3}{4.0} \right) = \pm \left(\frac{0.8 + 4.14}{13.8 \times 4.0} \right)$$

$$\therefore \Delta v = \pm 0.0895 \times v = \pm 0.0895 \times 3.45 = \pm 0.3087 \\ = \pm 0.31$$

Hence $v = (3.5 \pm 0.31) \text{ ms}^{-1}$

Percentage error in velocity

$$= \frac{\Delta v}{v} \times 100 = \pm 0.0895 \times 100 = \pm 8.95\% = \pm 9\%$$

Example 29: The heat generated in a circuit is given by $Q = I^2 R t$, where I is current, R is resistance and t is time. If the percentage errors in measuring I , R and t are 2%, 1% and 1% respectively, then the maximum error in measuring heat will be

- (a) 2% (b) 3% (c) 4% (d) 6%

Sol. The percentage error in heat is given as

$$\frac{\Delta Q}{Q} \% = 2 \frac{\Delta I}{I} \% + \frac{\Delta R}{R} \% + \frac{\Delta t}{t} \%$$

Substituting the values, maximum possible percentage error is

$$= 2 \times 2\% + 1 \times 1\% + 1 \times 1\% = 6\%$$

Example 30: Given: Resistance, $R_1 = (8 \pm 0.4) \Omega$ and Resistance, $R_2 = (8 \pm 0.6) \Omega$. What is the net resistance when R_1 and R_2 are connected in series?

- (a) $(16 \pm 0.4) \Omega$ (b) $(16 \pm 0.6) \Omega$
(c) $(16 \pm 1.0) \Omega$ (d) $(16 \pm 0.2) \Omega$

Sol. $R_1 = (8 \pm 0.4) \Omega$, $R_2 = (8 \pm 0.6) \Omega$

$$R_s = R_1 + R_2 = (16 \pm 1.0) \Omega$$

Example 31: The following observations were taken for determining surface tension of water by capillary tube method: Diameter of capillary, $D = 1.25 \times 10^{-2}$ m and rise of water in capillary, $h = 1.45 \times 10^{-2}$ m.

Taking $g = 9.80 \text{ ms}^{-2}$ and using the relation

$T = (rgh/2) \times 10^3 \text{ Nm}^{-1}$, what is the possible error in surface tension T ?

- (a) 2.4 % (b) 15 %
(c) 1.6 % (d) 0.15 %

Sol. Given $T = (rgh/2) \times 10^3 \text{ Nm}^{-1}$,

$D = 1.25 \times 10^{-2}$ m, $h = 1.45 \times 10^{-2}$ m,

$g = 9.80 \text{ ms}^{-2}$

$$\frac{\delta T}{T} = \frac{\delta r}{r} + \frac{\delta h}{h} + \frac{\delta g}{g} \Rightarrow \frac{\delta T}{T} = \frac{0.01}{1.25} + \frac{0.01}{1.45} + \frac{0.01}{9.8}$$

after applying the above values in this relation we get $\delta T \% = 1.6\%$



Concept Application

23. If the length of cylinder is measured to be 4.28 cm with an error of 0.01 cm. The percentage error in the measured length is approximately.
(a) 0.4% (b) 0.5% (c) 0.2% (d) 0.1%
24. The pressure on square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in measurement of force and length are respectively 4% and 2% then the maximum error in measurement of pressure is
(a) 1% (b) 2% (c) 6% (d) 8%
25. The length and breadth of rectangular object are 25.2 cm and 16.8 cm respectively and have been measured to an accuracy of 0.1 cm. Relative error and percentage error in the area of the object are
(a) 0.01 and 1% (b) 0.02 and 2%
(c) 0.03 and 3% (d) 0.04 and 4%
26. The error in the measurement of length of a simple pendulum is 0.1% and error in the time period is 2%. The possible maximum error in the quantity having dimensional formula LT^{-2} is
(a) 1.1% (b) 2.1%
(c) 4.1% (d) 6.1%

MEASURING INSTRUMENTS

Measurement is an important aspect of physics. Whenever we want to know about a physical quantity, we take its measurement first of all. Instruments used in measurement are called measuring instruments.

Least Count: The least value of a quantity, which the instrument can measure accurately, is called the least count of the instrument.

Error: The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, has an error.

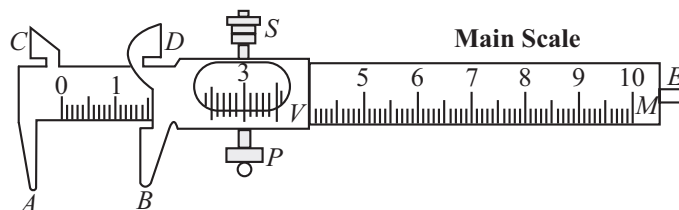
Accuracy and Precision: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

VERNIER CALLIPERS

It is a device used to measure accurately upto 0.1 mm. There are two scales in the vernier callipers, vernier scale and main scale. The main scale is fixed whereas the vernier scale is movable along the main scale.

Its main parts are as follows:

Main scale: It consists of a steel metallic strip M , graduated in cm and mm at one edge and in inches and tenth of an inch at the other edge on same side. It carries fixed jaws A and C projected at right angle to the scale as shown in figure.



Vernier Scale: A vernier V slides on the strip M . It can be fixed in any position by screw S . It is graduated on both sides. The side of the vernier scale which slides over the mm side has ten divisions over a length of 9 mm, i.e., over 9 main scale divisions and the side of the vernier scale which slides over the inches side has 10 divisions over a length of 0.9 inch, i.e., over 9 main scale divisions.

Movable Jaws: The vernier scale carries jaws B and D projected at right angle to the main scale. These are called movable jaws. When vernier scale is pushed towards A and C , then as B touches A , straight side of D will touch straight side of C . In this position, in case of an instrument free from errors, zeros of vernier scale will coincide with zeros of main scales, on both the cm and inch scales.

The object whose length or external diameter is to be measured is held between the jaws A and B , while the straight edges of C and D are used for measuring the internal diameter of a hollow object.

Metallic Strip: There is a thin metallic strip E attached to the back side of M and connected with vernier scale. When the jaws A and B touch each other, the edge of strip E touches the edge of M . When the jaws A and B are separated, E moves outwards. The strip E is used for measuring the depth of a vessel.

Determination of Least Count (Vernier Constant)

Note the value of the main scale division and count the number n of vernier scale divisions. Slide the movable jaw till the zero of vernier scale coincides with any of the mark of the main scale and find the number of divisions $(n - 1)$ on the main scale coinciding with n divisions of vernier scale. Then

$$n \text{ V.S.D.} = (n - 1) \text{ M.S.D. or } 1 \text{ V.S.D.} = \left(\frac{n-1}{n} \right) \text{ M.S.D.}$$

$$1 \text{ V.C.} = \text{L.C.} = 1 \text{ M.S.D.} - 1 \text{ V.S.D.} = \left(1 - \frac{n-1}{n} \right) \text{ M.S.D.}$$

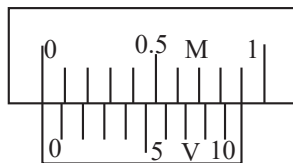
$$= \frac{1}{n} \text{ M.S.D.}$$

Determination of Zero Error and Zero Correction

For this purpose, movable jaw B is brought in contact with fixed jaw A .

One of the following situations will arise.

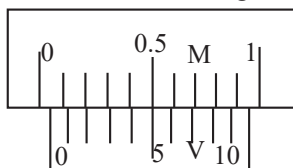
- (i) Zero of Vernier scale coincides with zero of main scale



In this case, zero error and zero correction, both are nil.

Actual length = observed (measured) length.

- (ii) Zero of vernier scale lies on the right of zero of main scale



Here 5th vernier scale division is coinciding with any main scale division and zero of vernier is ahead of N^{th} main scale division.

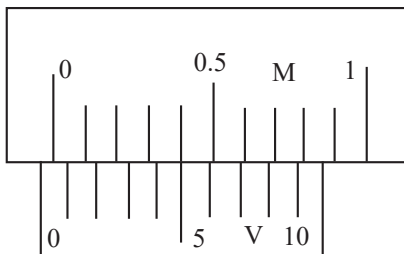
Hence, $N = 0$, $n = 5$, $\text{L.C.} = 0.01 \text{ cm}$.

Zero error = $N + n \times (\text{L.C.}) = 0 + 5 \times 0.01 = +0.05 \text{ cm}$

Zero correction = -0.05 cm .

Actual length will be 0.05 cm less than the observed (measured) length.

- (iii) Zero of the vernier scale lies left of the main scale.



Here, 5th vernier scale division is coinciding with any main scale division.

In this case, zero of vernier scale lies on the right of -0.1 cm reading on main scale.

Hence, $N = -0.1 \text{ cm}$, $n = 5$, $\text{L.C.} = 0.01 \text{ cm}$

Zero error = $N + n \times (\text{L.C.})$

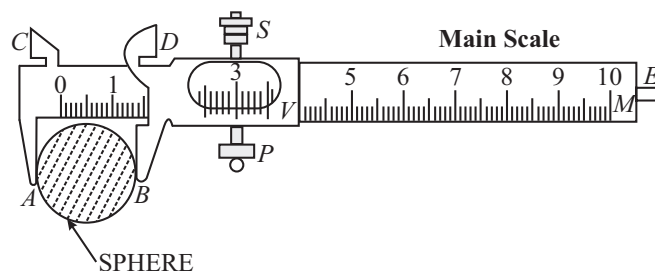
$$= -0.1 + 5 \times 0.01 = -0.05 \text{ cm.}$$

Zero correction = $+0.05 \text{ cm}$.

Actual length will be 0.05 cm more than the observed (measured) length.

Measurement Using Vernier Calliper

Let us measure the diameter of a small spherical/cylindrical body using a vernier calliper. Insert the object between the jaws as shown in the figure below.



If with the body between the jaws, the zero of vernier scale lies ahead of N^{th} division of main scale, then main scale reading (M.S.R.) = N .

If n^{th} division of vernier scale coincides with any division of main scale, then vernier scale reading (V.S.R.)

$$= n \times (\text{L.C.}) \text{ (L.C. is least count of vernier calliper)}$$

$$= n \times (\text{V.C.}) \text{ (V.C. is vernier constant of vernier calliper)}$$

Total reading,

$$\text{T.R.} = \text{M.S.R.} + \text{V.S.R.}$$

$$= N + n \times (\text{V.C.})$$



Train Your Brain

Example 32: The least count of vernier callipers is 0.1 mm. The main scale reading before the zero of the vernier scale is 10 and the zeroth division of the vernier scale coincides with the main scale division. Given that each main scale division is 1 mm, what is the measured value?

Sol. Length measured with vernier callipers

= reading before the zero of vernier scale + number of vernier divisions coinciding

with any main scale division \times least count

$$= 10 \text{ mm} + 0 \times 0.1 \text{ mm} = 10 \text{ mm} = 1.00 \text{ cm}$$

Example 33: A vernier callipers has its main scale of 10 cm equally divided into 200 equal parts. Its vernier scale of 25 divisions coincides with 12 mm on the main scale. The least count of the instrument is

(a) 0.020 cm

(b) 0.002 cm

(c) 0.010 cm

(d) 0.001 cm

Sol. 10 cm divided in 200 divisions, so

$$1 \text{ division} = \frac{10}{200} = 0.05 \text{ cm.}$$

Now, $25V = 24S$.

$$\Rightarrow V = \frac{24}{25}S$$

$$LC = S - V$$

$$= S - \frac{24}{25}S = \frac{1}{25}S$$

$\therefore 1S = 0.05 \text{ cm}$, thus

$$\text{So, } LC = \frac{0.05}{25} = 0.002 \text{ cm}$$

Example 34: One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 10 divisions of vernier scale coincide with 8 small divisions of the main scale, the least count of the callipers is

- (a) 0.005 cm (b) 0.05 cm
(c) 0.02 cm (d) 0.01 cm

Sol. Now, $10V = 8S$

$$\Rightarrow V = \frac{8}{10}S$$

$$\text{Now, } LC = S - V$$

$$= S - \frac{8}{10}S = \frac{2}{10}S = \frac{1}{5}S$$

But $1S = 0.1 \text{ cm}$, thus

$$LC = \frac{0.1}{5} = 0.02 \text{ cm}$$



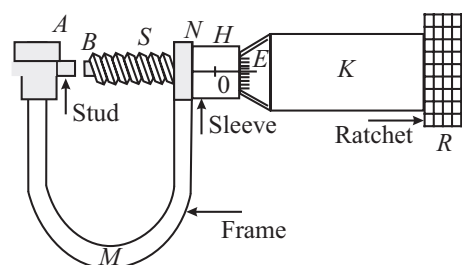
Concept Application

27. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree ($= 0.5^\circ$), find the least count of the instrument (in min.).
28. The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found that, the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm. If 24th division of the vernier scale exactly coincides with one of the main scale division, the diameter of the cylinder is

(a) 5.112 cm (b) 5.124 cm
(c) 5.136 cm (d) 5.148 cm
29. The main scale of vernier calliper is calibrated in mm and 19 divisions of main scale are equal in length to 20 divisions of vernier scale. In measuring the diameter of a cylinder by this instrument, the main scale reads 35 division and 4th division of vernier scale coincides with a M.S.D. Find LC in (cm).

SCREW GAUGE

This instrument (shown in figure) works on the principle of micrometer screw. It consists of a U-shaped frame M . At one end of it is fixed a small metal piece A of gun metal. It is called stud and it has a plane face. The other end N of M carries a cylindrical hub H . The hub extends few millimetre beyond the end of the frame. On the tubular hub along its axis, a line is drawn known as reference line. On the reference line graduations are in millimetre and half millimetre depending upon the pitch of the screw. This scale is called linear scale or pitch scale. A nut is threaded through the hub and the frame N . Through the nut moves a screw S made of gun metal. The front face B of the screw, facing the plane face A , is also plane. A hollow cylindrical cap K is capable of rotating over the hub when screw is rotated. It is attached to the right hand end of the screw. As the cap is rotated the screw either moves in or out. The bevelled surface E of the cap K is divided into 50 or 100 equal parts. It is called the circular scale or head scale. Right hand end R of K is milled for proper grip.



In most of the instrument the milled head R is not fixed to the screw head but turns it by a spring and ratchet arrangement such that when the body is just held between faces A and B , the spring yields and milled head R turns without moving in the screw.

In an accurately adjusted instrument when the faces A and B are just touching each other, the zero marks of circular scale and pitch scale exactly coincide.

Determination of Least Count of Screw Gauge

Note the value of linear (pitch) scale division. Rotate screw to bring zero mark on circular (head) scale on reference line. Note linear scale reading i.e. number of divisions of linear scale uncovered by the cap.

Now give the screw a few known number of rotations. (one rotation completed when zero of circular scale again arrives on the reference line). Again note the linear scale reading. Find difference of two readings on linear scale to find distance moved by the screw.

Then, pitch of the screw

$$= \frac{\text{Distance moved in } n \text{ rotation}}{\text{No. of full rotation } (n)}$$

Now count the total number of divisions on circular (head) scale. Then, least count is

$$LC = \frac{\text{Pitch}}{\text{Total number of divisions on the circular scale}}$$

The least count is generally 0.001 cm.

Determination of Zero Error and Zero Correction

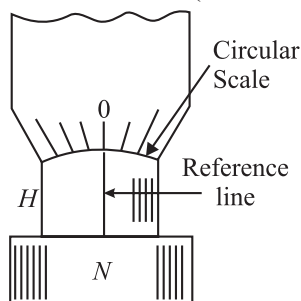
For this purpose, the screw is rotated forward till plane face B of the screw just touches the fixed plane face A of the stud and edge of cap comes on zero mark of linear scale. Screw gauge is held keeping the linear scale vertical with its zero downwards.

One of the following three situations will arise.

(i) Zero mark of circular scale comes on the reference line

In this case, zero error and zero correction, both are nil.

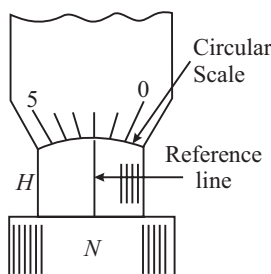
Actual thickness = Observed (measured) thickness.



(ii) Zero mark of circular scale remains on right of reference line and does not cross it.

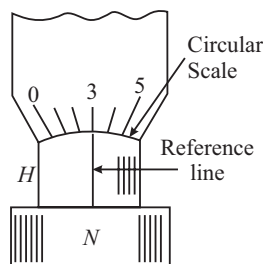
Here 2nd division on circular scale comes on reference line. Zero reading is already 0.02 mm. It makes zero error = + 0.02 mm and zero correction = - 0.02 mm.

Actual thickness will be 0.02 mm less than the observed (measured) thickness.



(iii) Zero mark of circular scale goes to left on reference line after crossing it.

Here zero of circular scale has advanced from reference line by 3 divisions on circular scale. A backward rotation by 0.03 mm will make reading zero. It makes zero error = -0.03 mm & zero correction = + 0.03 mm.



Actual thickness will be 0.03 mm more than the observed (measured) thickness.

Measurement Using Screw Gauge

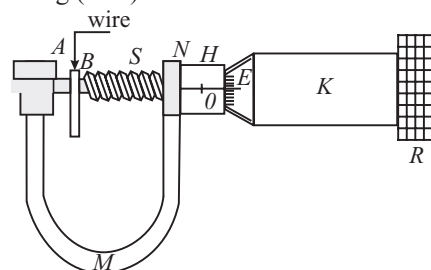
To measure diameter of a given wire using a screw gauge:

1. Determine of least count of screw gauge
2. If with the wire between plane faces A and B , the edge of the cap lies ahead of N^{th} division of linear scale, then, linear scale reading (L.S.R.) = N

If n^{th} division of circular scale lies over reference line, then, circular scale reading (C.S.R.) = $n \times (\text{L.C.})$

(L.C. is least count of screw gauge)

Total reading (T.R.) = L.S.R. + C.S.R. = $N + n \times (\text{L.C.})$



Train Your Brain

Example 35: In four complete revolutions of the cap, the distance traveled on the pitch scale is 2 mm. If there are fifty divisions on the circular scale, then

- (i) Calculate the pitch of the screw gauge.
- (ii) Calculate the least count of the screw gauge.

Sol. Pitch of screw = Linear distance traveled in one

$$\text{revolution } P = \frac{2 \text{ mm}}{4} = 0.5 \text{ mm} = 0.05 \text{ cm}$$

$$\begin{aligned} \text{Least count} &= \frac{\text{Pitch}}{\text{no. of divisions in circular scale}} \\ &= \frac{0.05}{50} = 0.001 \text{ cm} \end{aligned}$$

Example 36: The pitch of a screw gauge is 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the pitch scale (or main scale) and thirty fourth division coincides with the reference line. Calculate the thickness of the metal plate.

Sol. Pitch of screw = 0.5 mm., LC = $\frac{0.5}{50} = 0.01 \text{ mm}$.

$$\begin{aligned} \text{Thickness} &= (5 \times 0.5 + 34 \times 0.01) \text{ mm} \\ &= (2.5 + 0.34) = 2.84 \text{ mm} \end{aligned}$$

Concept Application

30. In a screw gauge, five complete rotations of the screw causes it to move a linear distance of 0.25 cm. There are 100 circular divisions. Four main scale divisions and 30 circular scale divisions is obtained as the thickness of the wire measured by this instrument. Assuming negligible zero error, find the thickness of wire (in cm).
31. The thickness of a marker measured using a screw gauge whose LC = 0.001 cm, comes out to be 0.802 cm. The percentage error in the measurement would be
 - (a) 0.125%
 - (b) 2.43%
 - (c) 4.12%
 - (d) 2.14%

AARAMBH (SOLVED EXAMPLES)

1. $\int \frac{x dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1} \left[\frac{x}{a} - 1 \right]$. The value of n is

- (a) 0 (b) -1
(c) 1 (d) None of these

Use dimensional analysis to solve the problem.

(JEE Arjuna Physics M-1)

Sol. $\int \frac{x dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1} \left[\frac{x}{a} - 1 \right]$

Denominator $2ax - x^2$ must have the dimension of $[x]^2$
(\because we can add or subtract only if quantities same dimension)

$$\therefore [\sqrt{2ax - x^2}] = [x]$$

Also, dx has the dimension of $[x]$

$$\therefore \frac{x dx}{\sqrt{2ax - x^2}} \text{ is having dimension } L$$

Equating the dimension of L.H.S. and R.H.S. we have

$$[a^n] = M^0 L^1 T^0$$

$$\left\{ \because \sin^{-1} \left(\frac{x}{a} - 1 \right) \text{ must be dimensionless} \right\}$$

Therefore, option (c) is the correct answer.

2. In the formula; $P = \frac{nRT}{V - b} e^{-\frac{a}{RTV}}$ find the dimensions of 'b' and 'a' respectively, where P = pressure, n = no. of moles, T = temperature, V = volume and R = universal gas constant.

(a) $[L^3]$, $[MLT^{-2}]$ (b) $[M^2L]$, $[ML^5T^{-2}]$
(c) $[L^3]$, $[ML^5T^{-2}mol^{-1}]$ (d) $[M^2L]$, $[MLT^{-2}]$

(JEE Arjuna Physics M-1)

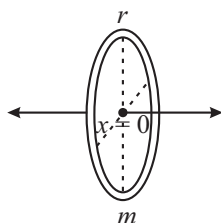
Sol. $[b] = [V] = [L^3]$

$$[a] = [RTV] = \frac{[PV]}{[n]} [V] = \frac{ML^2T^{-2}L^3}{mol} \quad (\because nRT = PV)$$

$$= ML^5T^{-2}mol^{-1}.$$

Therefore, option (c) is the correct answer.

3. A particle is performing SHM along the axis of a fixed ring. Due to gravitational force, its displacement at time t is given by $x = a \sin \omega t$.



In this equation ω is found to depend on radius of the ring (r), mass of the ring (m) and gravitational constant (G).

Using dimensional analysis, find the expression of ω in terms of m , r and G .

(a) $\sqrt{\frac{GM}{r}}$ (b) $\sqrt{\frac{GM}{r^3}}$ (c) $\sqrt{\frac{G}{Mr^2}}$ (d) $\sqrt{\frac{Gr^3}{M}}$

(JEE Arjuna Physics M-1)

Sol. Let $\omega = KM^a r^b G^c$ where K is a dimensionless constant

Writing the dimension of both the sides and equating them we have,

$$T^{-1} = M^a L^b (M^{-1} L^3 T^{-2})^c$$

$$= M^{a-c} L^{b+3c} T^{-2c}$$

Equating the exponents,

$$-2c = -1 \text{ or } c = 1/2;$$

$$b + 3c = 0 \text{ or } -3c = b = -\frac{3}{2};$$

$$a - c = 0, c = a = +\frac{1}{2}$$

$$\text{Thus the required equation is } \omega = K \sqrt{\frac{Gm}{r^3}}.$$

Therefore, option (b) is the correct answer.

4. Using screw gauge, the observation of the diameter of a wire are 1.324, 1.326, 1.334, 1.336 cm respectively. The value of diameter of wire along with its percentage uncertainty will be

- (a) $1.33 \pm 1\%$ (b) $1.30 \pm 0.4\%$
(c) $1.30 \pm 1\%$ (d) $1.33 \pm 0.4\%$

(JEE Arjuna Physics M-1)

Sol. Average diameter:

$$\bar{D} = \frac{\Sigma(D)}{N} = \frac{1.324 + 1.326 + 1.334 + 1.336}{4} = 1.330$$

$$\Delta D_1 = 1.324 - 1.330 = -0.006$$

$$\Delta D_2 = 1.326 - 1.330 = -0.004$$

$$\Delta D_3 = 1.334 - 1.330 = 0.004$$

$$\Delta D_4 = 1.336 - 1.330 = 0.006$$

Mean absolute error:

$$\Delta \bar{D} = \frac{|\Delta D_1| + |\Delta D_2| + |\Delta D_3| + |\Delta D_4|}{4}$$

$$= \frac{0.006 + 0.004 + 0.004 + 0.006}{4} = \frac{0.020}{4} = 0.005 \text{ cm}$$

$$\text{Relative error} = \frac{\Delta \bar{D}}{\bar{D}} = \frac{0.005}{1.330} = 0.004$$

$$\% \text{ error} = \frac{\Delta \bar{D}}{\bar{D}} \times 100 = 0.4\%$$

Therefore, option (d) is the correct answer.

5. If a tuning fork of frequency (f_0) 340 Hz and tolerance 1% is used in resonance column method [$v = 2f_0 (\ell_2 - \ell_1)$], the first and the second resonance are measured at $\ell_1 = 24.0$ cm and $\ell_2 = 74.0$ cm. Find maximum permissible error in speed of sound.

- (a) 1.4% (b) 2.8% (c) 0.5% (d) 0.7%

(JEE Arjuna Physics M-1)

Sol. $v = 2f_0(\ell_2 - \ell_1)$

$$\Rightarrow \left(\frac{\Delta v}{v} \right)_{\max} = \frac{\Delta f_0}{f_0} + \frac{\Delta \ell_1 + \Delta \ell_2}{\ell_2 - \ell_1} = \frac{1}{100} + \frac{0.1 + 0.1}{74 - 24} = 1.4\%$$

Therefore, option (a) is the correct answer.

6. Side of a cube is measured with the help of vernier calliper. Main scale reading is 10 mm and vernier scale reading is 1. It is known that 9 M.S.D. = 10 V.S.D. Mass of the cube is 2.735 g. Find density of the cube upto appropriate significant figure

- (a) 0.0027 g/mm³ (b) 0.00265 g/mm³
(c) 0.003 g/mm³ (d) 0.002655 g/mm³

(JEE Arjuna Physics M-1)

Sol. Least count = 1 M.S.D. - 1 V.S.D. = 1 M.S.D. - $\frac{9}{10}$ M.S.D.

$$= \frac{1}{10} \text{ M.S.D.} = \frac{1}{10} \times 1 \text{ mm}$$

Least count = 0.1 mm

Length of side of cube = M.S.R. + V.S.R. \times least count
= 10 + 1 \times 0.1 = 10.1 mm

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{2.735}{(10.1)^3} = 0.0026546 \text{ g/mm}^3$$

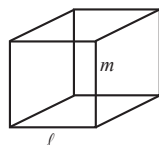
Using significant figures the correct answer would be 0.00265 (with 3 significant figures).

Therefore, option (b) is the correct answer.

7. For a cubical block, error in measurement of sides is + 1% and error in measurement of mass is + 2%, then maximum possible error in density is
(a) 1% (b) 5% (c) 3% (d) 7%

(JEE Arjuna Physics M-1)

Sol.



$$\text{Density, } \rho = \frac{m}{V} = \frac{m}{\ell^3}$$

$$\text{Given: } \frac{\Delta m}{m} = \pm 2\% = \pm 2 \times 10^{-2}, \quad \frac{\Delta \ell}{\ell} = \pm 1\% = \pm 1 \times 10^{-2}$$

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 3 \frac{\Delta \ell}{\ell} = 2 \times 10^{-2} + 3 \times 10^{-2} = 5 \times 10^{-2} = 5\%$$

Therefore, option (b) is the correct answer.

8. The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier callipers as 1.00 cm. The least count of the meter scale and vernier callipers are 0.1 cm and 0.01 cm respectively (Obvious from readings). Maximum permissible error in area measurement is
(a) +0.2 cm² (b) +0.1 cm² (c) +0.3 cm² (d) Zero

(JEE Arjuna Physics M-1)

Sol. $A = \ell b = 10.0 \times 1.00 = 10.00$

$$\text{Now, } \frac{\Delta A}{A} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}$$

$$\frac{\Delta A}{10.00} = \frac{0.1}{10.0} + \frac{0.01}{1.00}$$

$$\Rightarrow \Delta A = 10.00 \left(\frac{1}{100} + \frac{1}{100} \right) = 10.00 \left(\frac{2}{100} \right) = \pm 0.2 \text{ cm}^2$$

Therefore, option (a) is the correct answer.

9. To estimate 'g' (from $g = 4\pi^2 \frac{L}{T^2}$), error in measurement of L is + 2% and error in measurement of T is + 3%. The error in estimated 'g' will be

- (a) +8% (b) +6% (c) +3% (d) +5%

(JEE Arjuna Physics M-1)

Sol. $g = 4\pi^2 \frac{\ell}{T^2}$

$$\frac{\Delta \ell}{\ell} = 2\% = \pm 2 \times 10^{-2}$$

$$\frac{\Delta T}{T} = +3\% = \pm 3 \times 10^{-2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T} = 2 \times 10^{-2} + 2 \times 3 \times 10^{-2} = 8 \times 10^{-2} = \pm 8\%$$

Therefore, option (a) is the correct answer.

10. The mass of a ball is 1.76 kg. The mass of 25 such balls is
(a) 0.44×10^3 kg (b) 44.0 kg
(c) 44 kg (d) 44.00 kg

(JEE Arjuna Physics M-1)

Sol. $m = 1.76$ kg, $M = 25 m = 25 \times 1.76 = 44.0$ kg

Mass of one unit has three significant figures and it is just multiplied by a pure number (magnified). So result should also have three significant figures.

Therefore, option (b) is the correct answer.

11. To measure the diameter of a wire, a screw gauge is used. In a complete rotation, spindle of the screw gauge advances by 1/2 mm and its circular scale has 50 division. The main scale is graduated to 1/2 mm. If the wire is put between the jaws, 4 main scale divisions are clearly visible and 10 divisions of circular scale co-inside with the reference line. The resistance of the wire is measured to be $(10\Omega \pm 1\%)$. Length of the wire is measured to be 10 cm using a scale of least count 1 mm. Maximum permissible error in resistivity measurement is
(a) 4% (b) 2% (c) 2.95% (d) 1.03%

(JEE Arjuna Physics M-1)

Sol. L.C. of screw gauge = $\frac{1/2 \text{ mm}}{50} = 0.01 \text{ mm}$

Diameter of wire as measured by the screw gauge

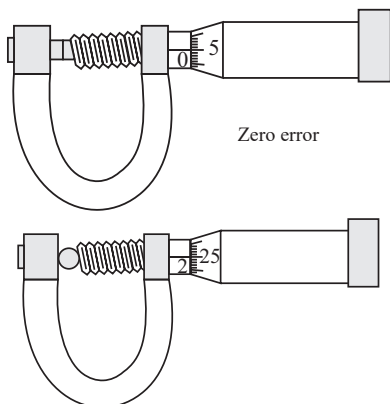
$$= 4 \times \frac{1}{2} + 10 \times 0.01 = 2.1 \text{ mm}$$

$$\text{Since, } \rho = \frac{R\pi d^2}{\ell} \Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{\Delta R}{R} + \frac{2\Delta d}{d} + \frac{\Delta\ell}{\ell}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{1}{100} + \frac{2 \times 0.01}{2.1} + \frac{1}{100} = 2.95\%$$

Therefore, option (c) is the correct answer.

12. The number of circular divisions on the shown screw gauge is 50. It moves 0.5 mm on main scale for one complete rotation. Main scale reading is 2. The diameter of the ball is



- (a) 2.25 mm (b) 2.20 mm (c) 1.20 mm (d) 1.25 mm

(JEE Arjuna Physics M-1)

Sol. Least count = $\frac{0.5}{50} = 0.01$

Zero error = $5 \times 0.01 = 0.05$ mm

Actual measurement = $2 \times 0.5 + 25 \times \frac{0.5}{50} - 0.05$

= $1 \text{ mm} + 0.25 \text{ mm} - 0.05 \text{ mm} = 1.20 \text{ mm}$.

Therefore, option (c) is the correct answer.

13. A student performs an experiment for determination of $g \left(= \frac{4\pi^2 L}{T^2} \right)$, $L = 1 \text{ m}$ and he commits an error of ΔL . In

measuring n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 sec. For which of the following data, the measurement of g will be most accurate ?

- (a) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 20$
 (b) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 50$
 (c) $\Delta L = 0.5$, $\Delta T = 0.01$, $n = 50$
 (d) $\Delta L = 0.1$, $\Delta T = 0.05$, $n = 50$

(JEE Arjuna Physics M-1)

Sol. $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2 \frac{\Delta T}{T}$

In option (d) error in Δg is minimum and number of repetition of measurement are maximum. In this case the error in g is minimum.

Therefore, option (d) is the correct answer.

14. Student I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and record time

for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	Length of the pendulum	Number of oscillations (n)	Total time for n oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I , E_{II} and E_{III} are the percentage error in g , i.e. $\left(\frac{\Delta g}{g} \times 100 \right)$

- (a) $E_I = 0$ (b) E_I is minimum
 (c) $E_I = E_{II}$ (d) E_{III} is maximum

(JEE Arjuna Physics M-1)

Sol. The least count of length $\Delta\ell = 0.1$ cm

The least count of length $\Delta t = 0.1$ s

% error of $g = \frac{\Delta g}{g} \times 100$

Now, $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2\ell}{T^2}$ where $T = \frac{t}{n}$

So, $g = \frac{4\pi^2\ell}{t^2} n^2$ $\left(T = \frac{t}{n} \right)$

$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + 2 \frac{\Delta t}{t}$

For student I, $\left(100 \times \frac{\Delta g}{g} \right)_I = \left(\frac{0.1}{64.0} + \frac{2 \times 0.1}{128.0} \right) \times 100$

$E_I = \frac{0.2}{64.0} \times 100 = \frac{20}{64}$

For student II,

$\left(100 \times \frac{\Delta g}{g} \right)_{II} = \left(\frac{0.1}{64.0} + 2 \times \frac{0.1}{64.0} \right) \times 100$

$E_{II} = \frac{0.3}{64.0} \times 100 = \frac{30}{64}$

For student III,

$\left(100 \times \frac{\Delta g}{g} \right)_{III} = \left(\frac{0.1}{20.0} + \frac{0.1}{18.0} \right) \times 100 = \frac{19}{18}$

$E_{III} = \left(\frac{0.1}{20.0} + \frac{0.1}{18.0} \right) \times 100 = \frac{19}{18} \Rightarrow E_I$ is least.

Therefore, option (b) is the correct answer.

15. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is

2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

- (a) 0.9% (b) 2.4% (c) 3.1% (d) 4.2%

(JEE Arjuna Physics M-1)

Sol. Least count = $\frac{0.5}{50} = 0.01 \text{ mm}$

Diameter of ball $D = 2.5 \text{ mm} + (20)(0.01)$

$\Rightarrow D = 2.7 \text{ mm}$

$$\rho = \frac{M}{\text{Vol}} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3} \Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{\Delta M}{M} + 3\frac{\Delta D}{D}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = 2\% + 3\left(\frac{0.01}{2.7}\right) \times 100\%$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = 3.1\%$$

Therefore, option (c) is the correct answer.

SCHOOL LEVEL PROBLEMS

SINGLE CORRECT TYPE QUESTIONS

1. Which among the following is the supplementary unit—

- (a) Mass (b) Time
(c) Solid angle (d) Luminosity

(JEE Arjuna Physics M-1)

2. The number of significant digits in 1559.00 is

- (a) 6 (b) 5 (c) 3 (d) 4

(JEE Arjuna Physics M-1)

3. Joule second is the unit of

- (a) Force (b) Angular momentum
(c) Energy (d) Power

(JEE Arjuna Physics M-1)

4. What is the number 75.66852 rounded off to 5 significant digits?

- (a) 75.67 (b) 75.669
(c) 75.668 (d) 75.667

(JEE Arjuna Physics M-1)

5. The division of energy by time is X . The dimensional formula of X is same as that of

- (a) power (b) electric field
(c) momentum (d) torque

(JEE Arjuna Physics M-1)

6. 1 kWh =

- (a) 1000 W (b) $36 \times 10^5 \text{ J}$
(c) 1000 J (d) 3600 J

(JEE Arjuna Physics M-1)

7. Density of wood is 0.5 gm/cc in the CGS system of units. The corresponding value in MKS units is:

- (a) 500 (b) 5
(c) 0.5 (d) 5000

(JEE Arjuna Physics M-1)

8. Which of the following is a derived unit?

- (a) Unit of mass (b) Unit of area
(c) Unit of time (d) Unit of current

(JEE Arjuna Physics M-1)

9. Newton/meter² is the unit of:

- (a) Energy (b) Momentum
(c) Force (d) Pressure

(JEE Arjuna Physics M-1)

10. Dimensional formula of heat energy is

- (a) ML^2T^{-2} (b) MLT^{-1}
(c) $\text{M}^0\text{L}^0\text{T}^{-2}$ (d) None of these

(JEE Arjuna Physics M-1)

11. **Assertion:** If the units of force and length are doubled, the unit of energy will become four times.

Reason: The unit of energy is independent of the units of force and length.

- (a) Assertion and reason both are true and reason is correct explanation for assertion.
(b) Assertion and reason both are true but reason is not correct explanation for assertion.
(c) Assertion is true but reason is false.
(d) Assertion is false but reason is true.

(JEE Arjuna Physics M-1)

12. **Assertion:** Dimensional constants are the quantities whose values are constant.

Reason: Dimensional constants are dimensionless.

- (a) Assertion and reason both are true and reason is correct explanation for assertion.
(b) Assertion and reason both are true but reason is not correct explanation for assertion.
(c) Assertion is true but reason is false.
(d) Assertion is false but reason is true.

(JEE Arjuna Physics M-1)

VERY SHORT ANSWER TYPE QUESTIONS

13. What is the number of significant figures in 0.06070?
(JEE Arjuna Physics M-1)
14. If $x = a + bt + ct^2$, where x is in meter and t in seconds, what is the unit of c ?
(JEE Arjuna Physics M-1)
15. Will the dimensions of a physical quantity be the same, whatever be the units in which it is measured? Why?
(JEE Arjuna Physics M-1)
16. What do you mean by the term measurement?
(JEE Arjuna Physics M-1)
17. Define dimensions of a physical quantity.
(JEE Arjuna Physics M-1)
18. S.I units are coherent. Explain.
(JEE Arjuna Physics M-1)
19. What are the limitations of dimensional analysis? (Any two)
(JEE Arjuna Physics M-1)

SHORT ANSWER TYPE QUESTIONS

20. 5.74 g of a substance occupies 1.2 cm^3 . Express its density to correct significant figures.
(JEE Arjuna Physics M-1)
21. Derive the dimensional formula of:
(a) Angular velocity
(b) Angular momentum
(JEE Arjuna Physics M-1)

LONG ANSWER TYPE QUESTIONS

22. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm, respectively. Give the area and volume of the sheet to correct significant figures.
(JEE Arjuna Physics M-1)
23. The frequency of vibration of a string depends on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.
(JEE Arjuna Physics M-1)

CASE STUDY BASED QUESTIONS

24. The nature of physical quantity is described by its dimension. All the physical quantities can be expressed in terms of some combination of seven fundamental units. The dimensions of a physical quantity are thus the powers to which the base quantities are raised to represent that quantity.
If a given physical quantity depends on a^{th} power of mass, b^{th} power of length, c^{th} power of time etc., then its dimensions are expressed as $[M^a L^b T^c]$.

(JEE Arjuna Physics M-1)

- (i) The dimension of planck's constant equal to that of
(a) Energy (b) Momentum
(c) angular momentum (d) power
- (ii) If force (F), velocity (V) and time (T) are taken as fundamental units, then dimension of mass are
(a) $[FVT^{-2}]$ (b) $[FV^{-1}T^{-1}]$
(c) $[FV^{-1}T]$ (d) $[FVT^{-1}]$
- (iii) Which of the following sets have different dimensions
(a) Pressure and Young's modulus
(b) Emf and potential difference
(c) Heat and work done
(d) Dipole moment and electric flux
- (iv) The dimension of $a \times b$ in the relations:-
 $P = \frac{b-x^2}{at}$, where P is power, x is distance and t is time.
(a) $[M^{-1}L^2T^2]$ (b) $[M^{-1}L^0T^2]$
(c) $[M^0L^2T^0]$ (d) $[ML^2T^{-1}]$
- (v) Find the value of 100 J on a system which has 20 cm, 250 g and half minute as fundamental units of length, mass and time.
(a) 9×10^{-6} new units (b) 2.16×10^6 new units
(c) 9×10^6 new units (d) 100 new units

(JEE Arjuna Physics M-1)

25. The Vander Waal's equation is $\left(P + \frac{a}{V^2}\right)(V - b) = RT$
Where P is pressure, V is volume, T is absolute temperature of given sample of a gas, R is called molar gas constant, a and b are Vander waal's constant.

Now answer the following

(JEE Arjuna Physics M-1)

- (i) The dimensional formula for b is same as for
(a) V (b) PV^2 (c) RT (d) P
- (ii) The dimensional formula for a is same as for
(a) V^2 (b) P (c) PV^2 (d) RT
- (iii) The dimensional formula of $\frac{ab}{RT}$ is
(a) $[ML^5T^{-2}]$ (b) $[M^0L^3T^0]$
(c) $[ML^{-1}T^{-2}]$ (d) $[M^0L^6T^0]$
- (iv) The dimensional formula for RT is same as for
(a) Energy (b) force
(c) Latent heat (d) Specific heat
- (v) The dimensional formula for RT is not same as that for
(a) $\frac{ab}{V^2}$ (b) Pb
(c) $\frac{a}{V^2}$ (d) PV

UNITS, SYSTEM OF UNITS

1. Which of the following is not the unit of time?
 (a) Solar day (b) Parallaxic second
 (c) Leap year (d) Lunar month
(JEE Arjuna Physics M-1)
2. A unitless quantity
 (a) Never has a non zero dimension
 (b) Always has a non zero dimension
 (c) May have a non zero dimension
 (d) Does not exist **(JEE Arjuna Physics M-1)**
3. Which of the following is not the name of a physical quantity?
 (a) Kilogram (b) Impulse
 (c) Energy (d) Density
(JEE Arjuna Physics M-1)
4. Parsec is a unit of
 (a) Time (b) Angle
 (c) Distance (d) Velocity
(JEE Arjuna Physics M-1)
5. Which of the following system of units is not based on the unit of mass, length and time alone?
 (a) FPS (b) SI
 (c) CGS (d) MKS
(JEE Arjuna Physics M-1)
6. In the S.I. system the unit of energy is:
 (a) Erg (b) Calorie
 (c) Joule (d) Electron volt
(JEE Arjuna Physics M-1)
7. The SI unit of the universal gravitational constant G is
 (a) N m kg^{-2} (b) $\text{N m}^2 \text{kg}^{-2}$
 (c) $\text{N m}^2 \text{kg}^{-1}$ (d) N m kg^{-1}
(JEE Arjuna Physics M-1)
8. Surface tension has unit of
 (a) Joule m^2 (b) Joule m^{-2}
 (c) Joule m^{-1} (d) Joule m^3
(JEE Arjuna Physics M-1)
9. The specific resistance has the unit of:
 (a) ohm/m (b) ohm/m^2
 (c) ohm m^2 (d) ohm m
(JEE Arjuna Physics M-1)
10. The unit of magnetic moment is:
 (a) Amp m^2 (b) Amp m^{-2}
 (c) Amp m (d) Amp m^{-1}
(JEE Arjuna Physics M-1)

11. The SI unit of the universal gas constant R is
 (a) $\text{Erg K}^{-1} \text{mol}^{-1}$ (b) $\text{Watt K}^{-1} \text{mol}^{-1}$
 (c) $\text{Newton K}^{-1} \text{mol}^{-1}$ (d) $\text{Joule K}^{-1} \text{mol}^{-1}$
(JEE Arjuna Physics M-1)
12. The SI unit of Stefan's constant is
 (a) $\text{Ws}^{-1} \text{m}^{-2} \text{K}^{-4}$ (b) $\text{J s m}^{-1} \text{K}^{-1}$
 (c) $\text{J s}^{-1} \text{m}^{-2} \text{K}^{-1}$ (d) $\text{W m}^{-2} \text{K}^{-4}$
(JEE Arjuna Physics M-1)

DIMENSIONS OF PHYSICAL QUANTITIES

13. In SI units the angular acceleration has unit of
 (a) N m kg^{-1} (b) m s^{-2} (c) rad s^{-2} (d) N kg^{-1}
(JEE Arjuna Physics M-1)
14. The angular frequency is measured in rad s^{-1} . Its exponent in length are
 (a) -2 (b) -1 (c) 0 (d) 2
(JEE Arjuna Physics M-1)
15. $[MLT^{-1}]$ are the dimensions of
 (a) Power (b) Momentum
 (c) Force (d) Couple
(JEE Arjuna Physics M-1)
16. What are the dimensions of Boltzmann's constant?
 (a) $\text{MLT}^{-2}\text{K}^{-1}$ (b) $\text{ML}^2\text{T}^{-2}\text{K}^{-1}$
 (c) M^0LT^{-2} (d) $\text{M}^0\text{L}^2\text{T}^{-2}\text{K}^{-1}$
(JEE Arjuna Physics M-1)
17. The pair of physical quantities having the same dimensional formula among the following is
 (a) Angular momentum and torque
 (b) Torque and energy
 (c) Force and power
 (d) Power and angular momentum
(JEE Arjuna Physics M-1)
18. Which one of the following has the dimensions of $\text{ML}^{-1}\text{T}^{-2}$?
 (a) Torque (b) Surface tension
 (c) Viscosity (d) Stress
(JEE Arjuna Physics M-1)
19. The dimensions of work done per unit mass per unit relative density would be equivalent to the dimensions of
 (a) $(\text{Acceleration})^2$ (b) $(\text{Velocity})^2$
 (c) $(\text{Force})^2$ (d) $(\text{Torque})^2$
(JEE Arjuna Physics M-1)
20. Which of the following is dimensional formula of intensity?
 (a) MT^{-3} (b) $\text{M}^{-1}\text{L}^2\text{T}^{-2}$ (c) $\text{ML}^{1/2}\text{T}^{-1}$ (d) None
(JEE Arjuna Physics M-1)

21. The dimensions of $\left[\frac{h}{G}\right]$, where h = Planck's constant and G = gravitational constant, is
 (a) $ML^{-1}T^2$ (b) $M^{-1}L^3T^2$ (c) $M^2L^{-1}T$ (d) $M^3L^0T^{-1}$
 (JEE Arjuna Physics M-1)
22. A dimensionless quantity:
 (a) Never has a unit (b) Always has a unit
 (c) May have a unit (d) Does not exist
 (JEE Arjuna Physics M-1)

DIMENSIONAL ANALYSIS AND ITS APPLICATION

Principle of Homogeneity

23. Force F is given in terms of time t and distance x by $F = A \sin Ct + B \cos Dx$. Then the dimensions of A/B and C/D are given by
 (a) MLT^{-2} , $M^0L^0T^{-1}$ (b) MLT^{-2} , $M^0L^{-1}T^0$
 (c) $M^0L^0T^0$, $M^0L^1T^{-1}$ (d) $M^0L^1T^{-1}$, $M^0L^0T^0$
 (JEE Arjuna Physics M-1)
24. The velocity of sound in a gas is given by $v = \sqrt{\gamma k_b \frac{T}{m}}$.
 Velocity v is measured in m/s, γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. What are the units for the Boltzmann constant, k_b ?
 (a) $kgm^2s^{-2}K^{-1}$ (b) kgm^2s^2K
 (c) $kgms^{-1}K^{-2}$ (d) $kgm^2s^{-2}K$
 (JEE Arjuna Physics M-1)
25. A wave is represented by $y = a \sin (At - Bx + C)$ where A , B , C are constants, t is in seconds x and y are in metre. The dimensions of A , B and C are
 (a) T^{-1} , L , $M^0L^0T^0$ (b) T^{-1} , L^{-1} , $M^0L^0T^0$
 (c) T , L , M (d) T^{-1} , L^{-1} , M^{-1}
 (JEE Arjuna Physics M-1)
26. If $v = \sqrt{\frac{\gamma P}{\rho}}$, then the dimensions of γ are (P is pressure, ρ is density and v is speed of sound)
 (a) $M^0L^0T^0$ (b) $M^0L^0T^{-1}$
 (c) $M^1L^0T^0$ (d) $M^0L^1T^0$
 (JEE Arjuna Physics M-1)
27. Consider the equation $\frac{d}{dt} \left[\int \vec{F} \cdot d\vec{s} \right] = A [\vec{F} \cdot \vec{P}]$. Then dimensions of A will be (\vec{F} = force, $d\vec{s}$ = small displacement, t = time and \vec{P} = linear momentum)
 (a) $M^0L^0T^0$ (b) $M^1L^0T^0$
 (c) $M^{-1}L^0T^0$ (d) $M^0L^0T^{-1}$
 (JEE Arjuna Physics M-1)

28. If $F = \frac{A}{\sqrt{m}} + B$ where F = Force, m = Mass.

Then dimensions of $A \times B$ is,

- (a) $M^{5/2}L^2T^{-4}$ (b) $M^{2/5}L^2T^{-1}$
 (c) $M^2L^{2/5}T^{-1}$ (d) $M^{-1}L^{2/5}T^{-2}$

(JEE Arjuna Physics M-1)

29. If $v = At^3 + \frac{B}{m}$, where m = mass, v = velocity and t = time.

Then dimensions of A in the given equation would be

- (a) LT^{-2} (b) L^2T^{-3} (c) L^3T^{-2} (d) LT^{-4}

(JEE Arjuna Physics M-1)

Deducing Relation Among Various Physical Quantities

30. The velocity of water waves may depend on their wavelength λ , the density of water ρ and the acceleration due to gravity g . The method of dimensions gives the relation between these quantities as (where k is a dimensionless constant)
 (a) $v^2 = k\lambda^{-1}g^{-1}\rho^{-1}$ (b) $v^2 = k g \lambda$
 (c) $v^2 = k g \lambda \rho$ (d) $v^2 = k \lambda^3 g^{-1} \rho^{-1}$
 (JEE Arjuna Physics M-1)
31. Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross-sectional area of the stream (A). The expression of the force should be
 (a) ρAv (b) ρAv^2
 (c) $\rho^2 Av$ (d) $\rho A^2 v$
 (JEE Arjuna Physics M-1)
32. If velocity (v), frequency (f) and mass (m) are taken as fundamental quantities, how energy (E) may be described using above quantity.
 (a) Kvf^2m (b) Kv^2f^0m
 (c) Kvf^2m^0 (d) $Kv^{1/2}f^1m^2$
 (JEE Arjuna Physics M-1)
33. If the power, P delivered by a motor is dependent on force F , velocity v and density of material ρ . Then power may be proportional to
 (a) $F^2v\rho^{-1}$ (b) $Fv^2\rho$
 (c) $Fv\rho^0$ (d) None of these
 (JEE Arjuna Physics M-1)
34. The velocity of a freely falling body changes as $g^p h^q$ where g = acceleration due to gravity and h is height. The value of p and q are
 (a) $1, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{2}$
 (c) $\frac{1}{2}, 1$ (d) $1, 1$
 (JEE Arjuna Physics M-1)

Conversion from One System of Units to Other

35. One watt-hour is equivalent to
(a) 6.3×10^3 Joule (b) 6.3×10^{-7} Joule
(c) 3.6×10^3 Joule (d) 3.6×10^{-3} Joule
(JEE Arjuna Physics M-1)
36. The pressure of 10^6 dyne/cm² is equivalent to
(a) 10^5 N/m² (b) 10^6 N/m²
(c) 10^7 N/m² (d) 10^8 N/m²
(JEE Arjuna Physics M-1)
37. Consider $\rho = 2$ g/cm³. Convert it into MKS system
(a) $2 \times 10^{-3} \frac{\text{kg}}{\text{m}^3}$ (b) $2 \times 10^3 \frac{\text{kg}}{\text{m}^3}$
(c) $4 \times 10^3 \frac{\text{kg}}{\text{m}^3}$ (d) $2 \times 10^6 \frac{\text{kg}}{\text{m}^3}$
(JEE Arjuna Physics M-1)
38. The density of mercury is 13600 kg m^{-3} . Its value of CGS system will be
(a) 13.6 g cm^{-3} (b) 1360 g cm^{-3}
(c) 136 g cm^{-3} (d) 1.36 g cm^{-3}
(JEE Arjuna Physics M-1)
39. Force in CGS system is 20. Its value in SI units will be
(a) 20×10^5 (b) 20×10^{-5}
(c) 200 (d) 2×10^{-3}
(JEE Arjuna Physics M-1)
40. If in a system of measurements unit of mass is α kg, unit of length is β m and that of time is γ sec. Find the value of 100 joule in this system.
(a) $100 \alpha^{-1} \beta^{-2} \gamma^2$ (b) $100 \alpha^{-2} \beta^{-1} \gamma^{-2}$
(c) $100 \alpha \beta^{-2} \gamma$ (d) $1000 \alpha^{-2} \beta^2 \gamma^{-1}$
(JEE Arjuna Physics M-1)

ERRORS IN MEASUREMENT

41. Which of the following measurements is most precise?
(a) 9×10^{-2} m (b) 90×10^{-3} m
(c) 900×10^{-4} m (d) 0.090 m
(JEE Arjuna Physics M-1)
42. A system takes 70.40 seconds to complete 20 oscillations. The time period of the system is:
(a) 3.52 s (b) 35.2×10 s
(c) 3.520 s (d) 3.5200 s
(JEE Arjuna Physics M-1)
43. The percentage error in the measurement of mass and speed are 1% and 2% respectively. What is the percentage error in kinetic energy?
(a) 5% (b) 2.5% (c) 3% (d) 1.5%
(JEE Arjuna Physics M-1)

44. Number 15462 when rounded off to three significant digits will be
(a) 15500 (b) 155 (c) 1546 (d) 150
(JEE Arjuna Physics M-1)
45. Value of expression $\frac{25.2 \times 1374}{33.3}$ will be
(All the digits in the expression are significant)
(a) 1040 (b) 1039 (c) 1038 (d) 1041
(JEE Arjuna Physics M-1)
46. Value of $24.36 + 0.0623 + 256.2$ will be (considering rules of significant digits)
(a) 280.6 (b) 280.8 (c) 280.7 (d) 280.6224
(JEE Arjuna Physics M-1)
47. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of momentum obtained by measuring mass and speed?
(a) 11% (b) 8% (c) 5% (d) 1%
(JEE Arjuna Physics M-1)
48. The random error in the arithmetic mean of 100 observations is x . Then random error in the arithmetic mean of 400 observations would be
(a) $4x$ (b) $\frac{1}{4}x$ (c) $2x$ (d) $\frac{1}{2}x$
(JEE Arjuna Physics M-1)
49. What is the number of significant figures in 0.310×10^3
(a) 2 (b) 3 (c) 4 (d) 6
(JEE Arjuna Physics M-1)
50. Error in the measurement of radius of a sphere is 1%. The error in the calculated value of its volume is
(a) 1% (b) 3% (c) 5% (d) 7%
(JEE Arjuna Physics M-1)
51. The mean time period of second's pendulum is 2.00 s and mean absolute error in the time period is 0.05 s. To express maximum estimate of error, the time period should be written as
(a) $(2.00 \pm 0.01)\text{s}$ (b) $(2.00 + 0.025)\text{s}$
(c) $(2.00 \pm 0.05)\text{s}$ (d) $(2.00 \pm 0.10)\text{s}$
(JEE Arjuna Physics M-1)
52. The unit of percentage error is
(a) Same as that of physical quantity
(b) Different from that of physical quantity
(c) Percentage error is unit less
(d) Errors have got their own units which are different from that of physical quantity measured
(JEE Arjuna Physics M-1)
53. The decimal equivalent of $1/20$ upto three significant figures is
(a) 0.0500 (b) 0.05000
(c) 0.0050 (d) 5.0×10^{-2}
(JEE Arjuna Physics M-1)

54. A thin copper wire of length l metre increases in length by 2% when heated through 10°C . What is the percentage increase in area when a square copper sheet of length l metre is heated through 10°C ?

(a) 4% (b) 8% (c) 16% (d) 32%

(JEE Arjuna Physics M-1)

55. The length and breadth of a rectangle are 20 ± 0.2 cm and 10 ± 0.1 cm. The percentage error in area would be

(a) 1% (b) 2% (c) 3% (d) 4%

(JEE Arjuna Physics M-1)

56. The mass rate of flow of liquid through a pipe is given by

$\frac{dm}{dt} = \rho Av$, where ρ = density, A = area of cross-section and v = velocity. The readings of area, $A = 10 \pm 0.1$ m² and $v = 30 \pm 0.3$ m/sec. Find the percentage error in measurement of mass flow rate.

(a) 5% (b) 2% (c) 8% (d) 3.5%

MEASURING INSTRUMENTS

(JEE Arjuna Physics M-1)

57. In a vernier callipers, ten divisions of the vernier scale is equal to the length of nine division on the main scale. If the smallest division on the main scale is half millimeter, then the vernier constant is:

(a) 0.5 mm (b) 0.1 mm
(c) 0.05 mm (d) 0.005 mm

(JEE Arjuna Physics M-1)

58. A vernier callipers has 20 divisions on the vernier scale, which coincide with 19 divisions on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of

(a) 0.5 mm (b) 1 mm
(c) 2 mm (d) 1/4 mm

(JEE ARJUNA PHYSICS M-1)

59. A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then

(a) $n = 10, m = 0.5$ cm (b) $n = 9, m = 0.4$ cm
(c) $n = 10, m = 0.8$ cm (d) $n = 10, m = 0.2$ cm

(JEE Arjuna Physics M-1)

60. The pitch of a screw gauge is 0.05 cm. In how many revolutions of hollow cylinder the screw will advance 0.35 cm in the straight line?

(a) 7 (b) 10 (c) 15 (d) 14

(JEE Arjuna Physics M-1)

61. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm, 1.20 mm. The percentage error in measurement.

(a) 2.20% (b) 1.24% (c) 2.85% (d) 3.52%

(JEE Arjuna Physics M-1)

PRABAL (JEE MAIN LEVEL)

1. The unit of impulse is the same as that of

(a) Moment of force
(b) Linear momentum
(c) Rate of change of linear momentum
(d) Force

(JEE Arjuna Physics M-1)

2. Which of the following is not the unit of energy?

(a) Watt-hour (b) Electron-volt
(c) N \times m (d) kg \times m/sec²

3. If a and b are two physical quantities having different dimensions then which of the following can denote a new physical quantity?

(a) $a + b$ (b) $a - b$
(c) a/b (d) $e^{a/b}$

(JEE Arjuna Physics M-1)

4. The time dependence of a physical quantity

$P = P_0 \exp(-\alpha t^2)$ where α is a constant and t is time. The constant α

(a) will be dimensionless

(b) will have dimensions of T^{-2}

(c) will have dimensions as that of P

(d) will have dimensions equal to the dimension of P multiplied by T^{-2}

(JEE Arjuna Physics M-1)

5. Which pair of following quantities has dimensions different from each other?

(a) Impulse and linear momentum
(b) Planck's constant and angular momentum
(c) Moment of inertia and moment of force
(d) Young's modulus and pressure

(JEE Arjuna Physics M-1)

6. The product of energy and time is called action. The dimensional formula for action is same as that for

(a) Power (b) Angular energy
(c) Force \times velocity (d) Impulse \times distance

(JEE Arjuna Physics M-1)

7. What is the physical quantity whose dimensions are $[M L^2 T^{-2}]$?

(a) Kinetic energy (b) Pressure
(c) Momentum (d) Power

(JEE Arjuna Physics M-1)

8. If E , M , J and G denote energy, mass, angular momentum and gravitational constant respectively, then $\frac{EJ^2}{M^5 G^2}$ has the dimensions of

(a) Length (b) Angle
(c) Mass (d) Time

(JEE Arjuna Physics M-1)

9. The position of a particle at time ' t ' is given by the relation $x(t) = \frac{V_0}{\alpha}[1 - e^{-\alpha t}]$ where V_0 is a constant and $\alpha > 0$. The dimensions of V_0 and α are respectively.

(a) $M^0 L^1 T^0$ and T^{-1} (b) $M^0 L^1 T^0$ and T^{-2}
(c) $M^0 L^1 T^{-1}$ and T^{-1} (d) $M^0 L^1 T^{-1}$ and T^{-2}

(JEE Arjuna Physics M-1)

10. If force (F) is given by $F = Pt^{-1} + \alpha t$, where t is time. The unit of P is same as that of

(a) Velocity (b) Displacement
(c) Acceleration (d) Momentum

(JEE Arjuna Physics M-1)

11. When a wave traverses a medium, the displacement of a particle located at x at time t is given by $y = a \sin(bt - cx)$ where a , b and c are constants of the wave. The dimensions of b are the same as those of

(a) Wave velocity (b) Amplitude
(c) Wavelength (d) Wave frequency

(JEE Arjuna Physics M-1)

12. In a book, the answer for a particular question is expressed as $b = \frac{ma}{k} \left[\sqrt{1 + \frac{2kl}{ma}} \right]$ here m represents mass, a represents accelerations, l represents length. The unit of b should be

(a) m/s (b) m/s²
(c) meter (d) /sec

(JEE Arjuna Physics M-1)

13. $\alpha = \frac{F}{v^2} \sin(\beta t)$ (here v = velocity, F = force, t = time). Find the dimensions of α and β

(a) $\alpha = [M^1 L^1 T^0]$, $\beta = [T^{-1}]$
(b) $\alpha = [M^1 L^1 T^{-1}]$, $\beta = [T^1]$
(c) $\alpha = [M^1 L^1 T^{-1}]$, $\beta = [T^{-1}]$
(d) $\alpha = [M^1 L^{-1} T^0]$, $\beta = [T^{-1}]$

(JEE Arjuna Physics M-1)

14. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be

(a) FT^2 (b) $F^{-1} A^2 T^{-1}$ (c) $FA^2 T$ (d) AT^2

(JEE Arjuna Physics M-1)

15. If the unit of length is micrometer and the unit of time is microsecond, the unit of velocity will be

(a) 100 m/s (b) 10 m/s (c) 10^{-6} m/s (d) 1 m/s

(JEE Arjuna Physics M-1)

16. In a certain system of units, unit of time is 5 s, unit of mass is 20 kg and unit of length is 10m. In this system, one unit of power will be equal to

(a) 16 watts (b) 1/16 watts
(c) 25 watts (d) None of these

(JEE Arjuna Physics M-1)

17. If the unit of force is 1 kN, the unit of length is 1 km and the unit of time is 100 s, what will be the unit of mass?

(a) 1000 kg (b) 10 kg (c) 10000 kg (d) 100 kg

(JEE Arjuna Physics M-1)

18. The units of length, velocity and force are doubled. Which of the following is the correct change in the other units?

(a) Unit of time is doubled
(b) Unit of mass is doubled
(c) Unit of momentum is doubled
(d) Unit of energy is doubled

(JEE Arjuna Physics M-1)

19. If the units of force and that of length are doubled, the unit of energy will become

(a) 1/4 times (b) 1/2 times
(c) 2 times (d) 4 times

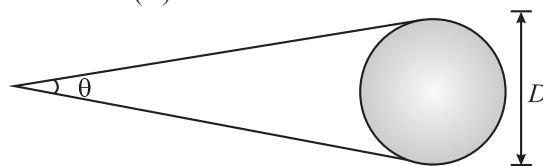
(JEE Arjuna Physics M-1)

20. If the units of mass and length are doubled then the unit of kinetic energy will become

(a) 2 times (b) 4 times (c) 8 times (d) 16 times

(JEE Arjuna Physics M-1)

21. The angle subtended by the moon's diameter at a point on the earth is about 0.50° . Use this and the fact that the moon is about 384000 km away to find the approximate diameter of the moon (D).



(a) 192000 km (b) 3350 km
(c) 1600 km (d) 1920 km

(JEE Arjuna Physics M-1)

22. The least count of a stop watch is 0.2 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The percentage error in the time period is

(a) 16% (b) 0.8% (c) 1.8% (d) 8%

(JEE Arjuna Physics M-1)

23. The dimensions of a cuboidal block measured with a vernier callipers having least count of 0.1 mm is 5 mm \times 10 mm \times 5 mm. The maximum percentage error in measurement of volume of the block is

(a) 5% (b) 10% (c) 15% (d) 20%

(JEE Arjuna Physics M-1)

24. In an experiment the quantities x, y and z are measured. Then t is calculated from the data as $t = \frac{xy^2}{z^3}$. If percentage errors in x, y and z are respectively 1%, 3%, 2%, then percentage error in t is:

- (a) 10 % (b) 4 %
(c) 7 % (d) 13 %

(JEE Arjuna Physics M-1)

25. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is

- (a) (0.34 ± 0.02) cm (b) (0.17 ± 0.02) cm
(c) (0.17 ± 0.01) cm (d) (0.34 ± 0.01) cm

(JEE Arjuna Physics M-1)

26. The mass of a ball is 1.76 kg. The mass of 25 such balls is

- (a) 0.44×10^3 kg (b) 44.0 kg
(c) 44 kg (d) 44.00 kg

(JEE Arjuna Physics M-1)

27. Two resistors R_1 (24 ± 0.5) Ω and R_2 (8 ± 0.3) Ω are joined in series. The equivalent resistance is

- (a) 32 ± 0.33 Ω (b) 32 ± 0.8 Ω
(c) 32 ± 0.2 Ω (d) 32 ± 0.5 Ω

(JEE Arjuna Physics M-1)

28. The pitch of a screw gauge is 0.5 mm and there are 100 divisions on its circular scale. The instrument reads +2 divisions when nothing is put in-between its jaws. In measuring the diameter of a wire, there are 8 divisions on the main scale and 83rd division coincides with the reference line. Then the diameter of the wire is

- (a) 4.05 mm (b) 4.405 mm
(c) 3.05 mm (d) 1.25 mm

(JEE Arjuna Physics M-1)

29. The pitch of a screw gauge having 50 divisions on its circular scale is 1 mm. When the two jaws of the screw gauge are in contact with each other, the zero of the circular scale lies 6 divisions below the line of graduation. When a wire is placed between the jaws, 3 linear scale divisions are clearly visible while 31st division on the circular scale coincides with the reference line. The diameter of the wire is

- (a) 3.62 mm (b) 3.50 mm
(c) 3.5 mm (d) 3.74 mm

(JEE Arjuna Physics M-1)

30. The smallest division on the main scale of a vernier callipers is 1 mm, and 10 vernier divisions coincide with 9 main scale divisions. While measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.0 and 2.1 cm and the fifth division of the vernier scale coincides with a mark on main scale. Then diameter of the sphere is

- (a) 2.05 cm (b) 3.05 cm
(c) 2.50 cm (d) None of these

(JEE Arjuna Physics M-1)

31. For a satellite orbiting around the Earth, its orbital velocity (v_0) is found to depend on mass of Earth M , radius of earth R

and universal gravitational constant G . The orbital velocity is proportional to

- (a) $G^{-1}M^1R^{-1}$ (b) $G^1M^1R^{-1}$
(c) $G^{1/2}M^{1/2}R^{-1/2}$ (d) None of these

(JEE Arjuna Physics M-1)

32. The vernier constant of a vernier callipers is 0.1 mm and it has zero error of -0.05 cm. While measuring the diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The correct diameter will be $n \times 10^{-2}$ cm. Find n .

- (a) 180 (b) 220 (c) 160 (d) 200

(JEE Arjuna Physics M-1)

33. In a particular system, the unit of length, mass and time are chosen to be 10 cm, 10g and 0.1s respectively. The unit of force in this system will be equal to

- (a) 1/10 N (b) 1 N (c) 10 N (d) 100 N

(JEE Arjuna Physics M-1)

34. In sub-atomic physics, one often associates a characteristic wavelength λ with a particle of mass m . If $\hbar = \frac{h}{2\pi}$ (h being Planck's constant) and c is the speed of light, which of the following expression is most likely to be correct one? (Use formula $E = hf$)

- (a) $\lambda = \frac{hc}{m}$ (b) $\lambda = \frac{\hbar}{mc^2}$
(c) $\lambda = \frac{m\hbar}{c}$ (d) $\lambda = \frac{\hbar}{mc}$

(JEE Arjuna Physics M-1)

35. If the mass, time and energy are taken as fundamental physical quantities then dimensional formula of length is

- (a) $[M^{1/2} T^1 E^{-1/2}]$ (b) $[M^{-1/2} T^1 E^{1/2}]$
(c) $[M^{-1} T^2 E]$ (d) None of these

(JEE Arjuna Physics M-1)

36. Given that $\ln(\alpha/p\beta) = \alpha z/k_B\theta$ where p is pressure, z is distance, k_B is Boltzmann constant and θ is temperature. The dimensions of β are (Useful formula: Energy = $k_B \times$ temperature)

- (a) $L^0 M^0 T^0$ (b) $L^1 M^{-1} T^2$
(c) $L^2 M^0 T^0$ (d) $L^{-1} M^1 T^{-2}$

(JEE Arjuna Physics M-1)

37. The dependence of g on geographical latitude at sea level is given by $g = g_0(1 + \beta \sin^2 \phi)$ where ϕ is the latitude angle and β is a dimensionless constant. If Δg is the error in the measurement of g , then the error in measurement of latitude angle is

- (a) zero (b) $\Delta \phi = \frac{\Delta g}{g_0 \beta \sin(2\phi)}$
(c) $\Delta \phi = \frac{\Delta g}{g_0 \beta \cos(2\phi)}$ (d) $\Delta \phi = \frac{\Delta g}{g_0}$

(JEE Arjuna Physics M-1)

38. Let $y = l^2 - \frac{l^3}{z}$ where $l = 2.0 \pm 0.1$, $z = 1.0 \pm 0.1$, then the value of y is given by
 (a) -4 ± 1.6 (b) -4 ± 2.4
 (c) -4 ± 0.8 (d) None of these

(JEE Arjuna Physics M-1)

39. The time periods of a pendulum are measured to be $T_1 = 8.01$ s and $T_2 = 8.41$ s by a student who used stop watch having least count = 0.01 sec, then the best reported measurement of time period (in sec) is
 (a) 8.2 ± 0.2 (b) 8.41 ± 0.2
 (c) 8.21 ± 0.01 (d) 8.41 ± 0.01

(JEE Arjuna Physics M-1)

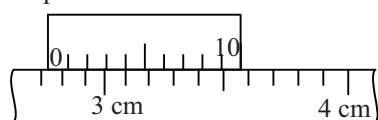
40. Assume pressure (P), length (L) and velocity (V) as fundamental quantities. The dimensions of coefficient of viscosity (η) are
 (a) $[PL^{-1}V]$ (b) $[PLV^{-1}]$
 (c) $[P^{-2}LV^{-1}]$ (d) $[PL^{-1}V^{-2}]$

(JEE Arjuna Physics M-1)

41. A gas bubble from an explosion under water oscillates with a period T proportional to $p^a d^b E^c$, where p is static pressure, d is the density of water, E is the total energy of the explosion. The values of c, b, a respectively will be
 (a) $-\frac{5}{6}, \frac{1}{2}, \frac{1}{3}$ (b) $-\frac{1}{3}, \frac{1}{3}, \frac{1}{4}$
 (c) $\frac{1}{3}, \frac{1}{2}, -\frac{5}{6}$ (d) $-\frac{5}{6}, \frac{1}{3}, \frac{1}{2}$

(JEE Arjuna Physics M-1)

42. The diagram shows part of the vernier scale of a pair of vernier callipers.



Which reading is correct ?

- (a) 2.74 cm (b) 3.10 cm (c) 3.26 cm (d) 3.64 cm

(JEE Arjuna Physics M-1)

INTEGER TYPE QUESTIONS

43. Number of significant figures in 0.007 m^2 are ____.

(JEE Arjuna Physics M-1)

44. Number of significant figures in 2.64×10^{24} are ____.

(JEE Arjuna Physics M-1)

45. Number of significant figures in 6.032 N are ____.

(JEE Arjuna Physics M-1)

46. The velocity of sound (v) in a gas depends on its pressure (p) and density (D). If relation between velocity, pressure and density is given by $v = Kp^a D^b$, then $(a + b)$ is ____.

(JEE Arjuna Physics M-1)

47. The density of a cube is calculate D by measuring its mass and the length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the value of density is ____%.

(JEE Arjuna Physics M-1)

48. The time period of oscillation of a body is given by

$$T = 2\pi \sqrt{\frac{mgA}{K}}$$

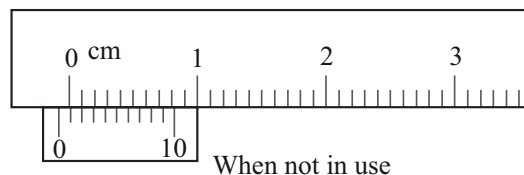
K represents kinetic energy, m represents mass, g represents acceleration due to gravity and A is unknown. If $[A] = M^x L^y T^z$, then what is the value of $x + y + z$?

(JEE Arjuna Physics M-1)

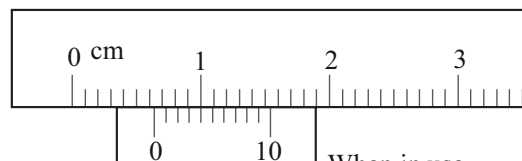
49. The radius of a sphere is measured to be 5.3 ± 0.1 cm. Calculate percentage error in volume. Round off to nearest integer.

(JEE Arjuna Physics M-1)

50. The main scale of a Vernier callipers reads in millimeter and its vernier scale is divided into 10 divisions which coincide with 9 divisions of the main scale. The length of the object for the situation shown below is found to be $\frac{12x}{10}$ mm. Find the value of x .



When not in use



When in use

(JEE Arjuna Physics M-1)

MULTIPLE CORRECT TYPE QUESTIONS

- Choose the correct statement(s).
 - All quantities may be represented dimensionally in terms of the base quantities.
 - A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.
 - The dimension of a base quantity in other base quantities is always zero.
 - The dimension of a derived quantity is never zero in any base quantity.

(JEE Arjuna Physics M-1)

- The dimensions $[ML^{-1}T^{-2}]$ may correspond to
 - Work done by a force
 - Linear momentum
 - Pressure
 - Energy per unit volume

(JEE Arjuna Physics M-1)

- A student curiously picks up Resnick and Halliday and tries to understand the answers given at the end of the book using his new found knowledge of physics. He marks four answers. In which of the following quantity A has the same units as that of angular momentum ?

Useful formula, $\vec{L} = \vec{r} \times \vec{p}$, $\omega = \frac{2\pi}{T}$, $\vec{\tau} = \vec{r} \times \vec{F}$, E represent energy, l represents length c represents velocity of light, f represents frequency and t represents time.

- $\frac{1}{2}mv^2 = Af$
- $\Delta p = \frac{2A\omega}{v} \sqrt{1 - \frac{v^2}{c^2}}$
- $\sin(kl) = k \sqrt{\frac{A^2}{2mE}}$
- $t = Al$

(JEE Arjuna Physics M-1)

- The quantity/quantities that does/do not have mass in its/their dimensions (when we take standard 7 quantities as fundamental) is/are
 - Specific heat
 - Latent heat
 - Luminous intensity
 - Mole

(JEE Arjuna Physics M-1)

- The power output for a hovering helicopter depends on its linear size, the density of air and $g \times$ density of the helicopter as $P \propto (\text{linear size})^x (\text{density of air})^y (g \times \text{density of helicopter})^z$ where g is acceleration due to gravity.
[Given: $[\text{Power}] = [ML^2T^{-3}]$, $[\text{Linear size}] = [L]$, $[\text{Density}] = [ML^{-3}]$, $[g \times \text{density}] = [ML^{-2}T^{-2}]$]
 - The value of y is $-1/2$.
 - The ratio of power output of engines of two hovering helicopters when linear size of one helicopter is one fourth of the other keeping all other parameters same, is 64.

- The ratio of the power output to hover a helicopter on the earth and on an imaginary planet, where $g_{\text{planet}} = g/4$ and density of air on imaginary planet is same as that of earth, is 8.
- If helicopter is to hover at higher altitudes then we need less powerful engine.

(JEE Arjuna Physics M-1)

- A student taking a quiz finds on a reference sheet the two equations $v = 1/T$ and $v = \sqrt{T/\mu}$

(μ = mass/length, torque = $\vec{r} \times \vec{F}$ and rest of symbols have usual notations.)

He has forgotten what T represents in each equation. Use dimensional analysis to determine the units of T in each equation.

- In first equation T represents tension.
- In first equation T represents time.
- In second equation T represents torque.
- In second equation T represents tension.

(JEE Arjuna Physics M-1)

- A chunk of unknown rock has mass 38.254 ± 0.003 grams and has a volume of 15.0 cm^3 .

- The density of the rock is 2.55 g/cm^3 .
- The absolute error in density is 0.02 g/cm^3 .
- The relative error in density is 0.007.
- The number of significant figure 3 in density are 3.

(JEE Arjuna Physics M-1)

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 8 to 10): Let us consider a particle P which is moving straight on the X -axis. We also know that the rate of change of its position is given by $\frac{dx}{dt}$; where x is its separation

from the origin and t is time. This term $\frac{dx}{dt}$ is called the velocity

of particle (v). Further the second derivation of x with respect to time is called acceleration (a) or rate of change of velocity and is represented by $\frac{d^2x}{dt^2}$ or $\frac{dv}{dt}$. If the acceleration of this particle is

found to depend upon time as follows $a = At + Bt^2 + \frac{Ct}{D+t^2}$ then

- The dimensions of A are

- LT^{-2}
- LT^{-3}
- LT^3
- L^2T^3

(JEE Arjuna Physics M-1)

- The dimensions of B are

- LT^{-4}
- L^2T^{-3}
- LT^4
- LT^{-2}

(JEE Arjuna Physics M-1)

- The dimensions of C are

- L^2T^{-2}
- LT^{-2}
- LT^{-1}
- T^2

(JEE Arjuna Physics M-1)

Comprehension (Q. 11 to 13): According to Coulomb's law of electrostatics there is a force between two charged particles q_1 & q_2 separated by a distance r such that $F \propto q_1$, $F \propto q_2$ & $F \propto \frac{1}{r^2}$;

combining all three we get $F \propto \frac{q_1 q_2}{r^2}$ or $F = \frac{k q_1 q_2}{r^2}$, where k is a constant which depends on the medium and is given by $1/4\pi\epsilon_0\epsilon_r$ where ϵ_0 is absolute permittivity & ϵ_r is relative permittivity.

But in case of protons of a nucleus there exists another force called nuclear force which is much higher in magnitude in comparison to electrostatic force and is given by $F = \frac{C e^{-kr}}{r^2}$.

11. What are the dimensions of C ?

- (a) $M^2 L^3 T^{-1}$ (b) $M L^3 T^{-3}$ (c) $M L^3 T^{-2}$ (d) $M L^2 T^{-3}$

(JEE Arjuna Physics M-1)

12. What are the dimensions of k ?

- (a) L (b) L^2 (c) L^{-3} (d) L^{-1}

(JEE Arjuna Physics M-1)

13. What are the SI units of C ?

- (a) Nm^{-2} (b) Nm^2 (c) Nm^{-3} (d) Nm

Comprehension (Q. 14 to 16): When numbers having uncertainties or errors are used to compute other numbers, these will be uncertain too. It is especially important to understand it when a number obtained from measurements is to be compared with a value obtained from theoretical prediction. Assume a student wants to verify the value of π as the ratio of circumference to diameter of a circle. The correct value of ten digits is 3.141592654. He draws a circle and measures its diameter and circumference to its nearest millimeter obtaining the values 135 mm and 424 mm, respectively. Using a calculator he finds $\pi = 3.140740741$.

14. Why does measured value not match with calculated value?

- (a) Due to systematic error
(b) Due to error in calculation
(c) Due to random error
(d) Lack of precision in measuring

(JEE Arjuna Physics M-1)

15. What is the value of π in the passage measured by the student?

- (a) 3.140 (b) 3.141 (c) 3.1407 (d) 3.14

(JEE Arjuna Physics M-1)

16. If diameter and circumference both have an error of 1%, what is the error in value of π ?

- (a) 2% (b) 1% (c) 0.5% (d) 0%

(JEE Arjuna Physics M-1)

MATCH THE COLUMN TYPE QUESTIONS

17. Match the following columns:

Physical quantity	Dimensions	Unit
A. Gravitational constant 'G'	p. $[M^1 L^1 T^{-1}]$	(i) N m
B. Torque	q. $[M^{-1} L^3 T^{-2}]$	(ii) N s

C. Momentum	r. $[M^1 L^{-1} T^{-2}]$	(iii) N m ² /kg ²
D. Pressure	s. $[M^1 L^2 T^{-2}]$	(iv) pascal

- (a) A-(p)-(iii); B-(r)-(i); C-(q)-(iv); D-(r)-(ii)
(b) A-(q)-(iii); B-(s)-(i); C-(p)-(ii); D-(r)-(iv)
(c) A-(q)-(iii); B-(r)-(i); C-(r)-(ii); D-(s)-(iv)
(d) A-(p)-(iv); B-(s)-(ii); C-(p)-(i); D-(r)-(iii)

(JEE Arjuna Physics M-1)

18. Match the following:

Physical quantity	Dimension	Unit
A. Stefan's constant ' σ '	p. $[M^1 L^1 T^{-2} A^{-2}]$	(i) W/m ²
B. Wien's constant ' b '	q. $[M^1 L^0 T^{-3} K^{-4}]$	(ii) K.m.
C. Coefficient of viscosity ' η '	r. $[M^1 L^0 T^{-3}]$	(iii) tesla .m/A
D. Emissive power of radiation (Intensity emitted)	s. $[M^0 L^1 T^0 K^{-1}]$	(iv) Wm ⁻² K ⁻⁴
E. Mutual inductance 'M'	t. $[M^1 L^2 T^{-2} A^{-2}]$	(v) Poise
F. Magnetic permeability ' μ_0 '	u. $[M^1 L^{-1} T^{-1}]$	(vi) Henry

- (a) A-(p)-(iv); B-(s)-(iii); C-(q)-(v); D-(r)-(i); E-(u)-(vi); F-(t)-(ii)
(b) A-(q)-(iii); B-(s)-(ii); C-(r)-(i); D-(u)-(v); E-(p)-(vi); F-(t)-(iv)
(c) A-(p)-(iv); B-(s)-(ii); C-(r)-(i); D-(u)-(v); E-(t)-(vi); F-(q)-(iii)
(d) A-(q)-(iv); B-(s)-(ii); C-(u)-(v); D-(r)-(i); E-(t)-(vi); F-(p)-(iii)

(JEE Arjuna Physics M-1)

19. In Column-I, some physical quantities are given and some SI units are given in Column-II. Match the physical quantities in Column-I with the units in Column-II. Some useful formulae are given below.

$$P = \frac{F}{A}, E = hcR, \lambda T = b, \frac{E}{t} = \sigma AT^4, F = \eta A \frac{v}{L}$$

(P-Pressure, F-force, v-velocity, A-Area, λ -wavelength, h -Planck's constant, g -Gravitational acceleration, R -Rydberg's constant, b -Wien's constant, η -Coefficient of viscosity, L -length, c -speed of light, t -time, E -energy, σ -Stefan's constant and T -temperature)

Column-I	Column-II
A. $PAvt$	p. $\frac{\text{watt second}}{\text{meter}^3}$
B. hgR	q. joule
C. $\frac{\sigma b^4}{A}$	r. $\frac{\text{newton}}{\text{metre}^2}$
D. $\frac{\eta}{t}$	s. $\frac{\text{newton metre}}{\text{second}}$
	t. newton metre

- (a) A-(q,t); B-(s,t); C-(p,t); D-(p,r)
(b) A-(p,t); B-(s); C-(q,t); D-(p)

(c) A-(q,t); B-(s); C-(s); D-(p,r)

(d) A-(t); B-(s); C-(q); D-(p,r)

(JEE Arjuna Physics M-1)

20. Suppose two students are trying to make a new measurement system so that they can use it like a code measurement system and others do not understand it. Instead of taking 1 kg, 1 m and 1 s. as basic unit they took unit of mass as α kg, the unit of length as β m and unit of time as γ second. They called power in new system as SHAKTI, then match the two columns.

Column-I		Column-II	
A.	1N in new system	p.	$\alpha^{-1} \beta^{-2} \gamma^2$
B.	1J in new system	q.	$\alpha^{-1} \beta^{-1} \gamma^2$
C.	1 Pascal (SI unit of pressure) in new system	r.	$\alpha^{-1} \beta \gamma^2$
D.	α SHAKTI in watt	s.	$\alpha^2 \beta^2 \gamma^3$

(a) A-(q); B-(p); C-(r); D-(s)

(b) A-(p); B-(q); C-(r); D-(s)

(c) A-(q); B-(p); C-(s); D-(r)

(d) A-(p); B-(r); C-(q); D-(s)

(JEE Arjuna Physics M-1)

NUMERICAL TYPE QUESTIONS

Answer should be rounded off upto two decimal places

21. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm²) of the wire.

(JEE Arjuna Physics M-1)

22. The length of the string of a simple pendulum is measured with a metre scale to be 90.01 cm. The radius of the bob plus the length of the hook is measured to be 2.132 cm using a screw gauge. What is the effective length of the pendulum? (This effective length is defined as the distance between the point of suspension and the center of the bob).

(JEE Arjuna Physics M-1)

23. In an experiment of simple pendulum, time period measured was 50 s for 25 oscillations when the length of the simple pendulum was taken as 100 cm. If the least count of stop watch is 0.1 s and that of meter scale is 0.01 cm, then the maximum possible percentage error in the calculated value of g is _____ %.

(JEE Arjuna Physics M-1)

INTEGER TYPE QUESTIONS

24. A gas bubble, from an explosion under water, oscillates with a period proportional to $P^a d^b E^c$, where P is the static pressure, d is the density and E is the total energy of the explosion.

Find the value of $a + b + c$.

(JEE Arjuna Physics M-1)

25. An artificial satellite is revolving around a planet of mass M and radius R , in a circular orbit of radius r . From Kepler's third law for the period of a satellite around a common central body, the cube of the radius of the orbit r is proportional to square of time period of revolution T . This means that $r^3 = kT^2$. Here k depends on mass of planet M , its radius R and universal gravitational constant G . If $k \propto G^x$, find x .

(JEE Arjuna Physics M-1)

26. A quantity A appears in an equation.

$$\text{Pressure} = \frac{Ae^{-Af/kT}}{B}$$

T represent temperature, ' f ' frequency & ' k ' boltzmann constant. The dimensions of B are given as, $[B] = [M^a L^b T^c]$.

Then find the value of $a + b + c$?

(JEE Arjuna Physics M-1)

SINGLE CORRECT TYPE QUESTIONS

27. An unknown quantity ' α ' is expressed as $\alpha =$

$$\frac{2ma}{\beta} \log \left(1 + \frac{2\beta\ell}{ma} \right), \text{ where } m = \text{mass, } a = \text{acceleration,}$$

ℓ = length. The unit of α should be

- (a) s^{-1} (b) m/s (c) m/s^2 (d) meter

(JEE Arjuna Physics M-1)

28. The volume V of water passing through a uniform tube during t seconds is related to the cross sectional area A of the tube and velocity v of water by relation

$$V = A^a v^b t^c$$

Which one of the following will be true ?

- (a) $\alpha > \beta = \gamma$ (b) $\alpha = \beta = \gamma$
(c) $\beta = \gamma$ (d) $\alpha \neq \beta \neq \gamma$

(JEE Arjuna Physics M-1)

29. A projectile is to be projected towards enemy territory at the same horizontal level. The initial velocity of the projectile is known to be 100 ± 1 m/s. Initial angle of the projectile is known to be $45^\circ \pm 1^\circ$ with horizontal. What is the possible range of the projectile?

- (a) $990 \text{ m} \leq R \leq 1010 \text{ m}$ (b) $980 \text{ m} \leq R \leq 1020 \text{ m}$
(c) $970 \text{ m} \leq R \leq 1030 \text{ m}$ (d) $930 \text{ m} \leq R \leq 970 \text{ m}$

(JEE Arjuna Physics M-1)

30. The dependence of g on geographical latitude at sea level is given by $g = g_0(1 + \beta \sin^2 \phi)$ where ϕ is the latitude angle and β is a dimensionless constant. If Δg is the error in the measurement of g then the error in measurement of latitude angle is

- (a) zero (b) $\Delta \phi = \frac{\Delta g}{g_0 \beta \sin(2\phi)}$
(c) $\Delta \phi = \frac{\Delta g}{g_0 \beta \cos(2\phi)}$ (d) $\Delta \phi = \frac{\Delta g}{g_0}$

(JEE Arjuna Physics M-1)

PYQ's (PAST YEAR QUESTIONS)

UNITS, SYSTEM OF UNITS

1. Electric field in a certain region is given by

$$\vec{E} = \left(\frac{A}{x^2} \hat{i} + \frac{B}{y^3} \hat{j} \right). \text{ The SI unit of } A \text{ and } B \text{ are:}$$

[30 Jan, 2023 (Shift-I)]

- (a) $Nm^3C^{-1}; Nm^2C^{-1}$
 (b) $Nm^2C^{-1}; Nm^3C^{-1}$
 (c) $Nm^3C; Nm^2C$
 (d) $Nm^2C; Nm^3C$

(JEE Arjuna Physics M-1)

2. Match List-I with List-II. [27 Aug, 2021 (Shift-II)]

List-I		List-II	
A.	R_H (Rydberg constant)	I.	$kg\ m^{-1}\ s^{-1}$
B.	h (Planck's constant)	II.	$kg\ m^2\ s^{-1}$
C.	μ_B (Magnetic field energy density)	III.	m^{-1}
D.	η (coefficient of viscosity)	IV.	$kg\ m^{-1}\ s^{-2}$

- (a) A-II, B-III, C-IV, D-I (b) A-III, B-II, C-I, D-IV
 (c) A-III, B-II, C-IV, D-I (d) A-IV, B-II, C-I, D-III

(JEE Arjuna Physics M-1)

3. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be:

[27 Aug, 2021 (Shift-I)]

- (a) Joule (b) Newton
 (c) Ohm (d) Mho

(JEE Arjuna Physics M-1)

4. A physical quantity \vec{S} is defined as $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$, where \vec{E} is electric field, \vec{B} is magnetic field and μ_0 is the permeability of free space. The dimensions of \vec{S} are the same as the dimensions of which of the following quantity (ies)? [JEE Adv, 2021]

- (a) $\frac{\text{Energy}}{\text{Charge} \times \text{Current}}$ (b) $\frac{\text{Force}}{\text{Length} \times \text{Time}}$
 (c) $\frac{\text{Energy}}{\text{Volume}}$ (d) $\frac{\text{Power}}{\text{Area}}$

(JEE Arjuna Physics M-1)

5. The density of a material in SI units is $128\ kg\ m^{-3}$. In certain units in which the unit of length is 25 cm and the unit of mass 50 g, the numerical value of density of the material is

[10 Jan, 2019 (Shift-I)]

- (a) 40 (b) 16 (c) 640 (d) 410

(JEE Arjuna Physics M-1)

DIMENSION, FINDING DIMENSIONAL FORMULA

6. Match List-I with List-II

List-I		List-II	
A.	Spring constant	I.	(T^{-1})
B.	Angular speed	II.	(MT^{-2})
C.	Angular momentum	III.	(ML^2)
D.	Moment of Inertia	IV.	(ML^2T^{-1})

Choose the correct answer from the options given below:

[12 April, 2023 (Shift-I)]

- (a) A-II, B-I, C-IV, D-III (b) A-IV, B-I, C-III, D-II
 (c) A-II, B-III, C-I, D-IV (d) A-I, B-III, C-II, D-IV

(JEE Arjuna Physics M-1)

7. Match List-I with List-II

List-I		List-II	
A.	Planck's constant (h)	I.	$[M^1 L^2 T^{-2}]$
B.	Stopping potential (V_s)	II.	$[M^1 L^1 T^{-1}]$
C.	Work function (ϕ)	III.	$[M^1 L^2 T^{-1}]$
D.	Momentum (p)	IV.	$[M^1 L^2 T^{-3} A^{-1}]$

[24 Jan, 2023 (Shift-I)]

- (a) A-III, B-I, C-II, D-IV (b) A-III, B-IV, C-I, D-II
 (c) A-II, B-IV, C-III, D-I (d) A-I, B-III, C-IV, D-II

(JEE Arjuna Physics M-1)

8. Which of the following physical quantities have the same dimensions? [25 July, 2022 (Shift-I)]

- (a) Electric displacement (\vec{D}) and surface charge density
 (b) Displacement current and electric field
 (c) Current density and surface charge density
 (d) Electric potential and energy

(JEE Arjuna Physics M-1)

9. Match List-I with List-II. [27 July, 2021 (Shift-II)]

List-I		List-II	
A.	Capacitance, C	I.	$M^1 L^1 T^{-3} A^{-1}$
B.	Permittivity of free space, ϵ_0	II.	$M^{-1} L^{-3} T^4 A^2$
C.	Permeability of free space, μ_0	III.	$M^{-1} L^{-2} T^4 A^2$
D.	Electric field, E	IV.	$M^1 L^1 T^{-2} A^{-2}$

Choose the correct answer from the options given below:

- (a) A-III, B-II, C-IV, D-I
 (b) A-III, B-IV, C-II, D-I

- (c) A-IV, B-II, C-III, D-I
(d) A-IV, B-III, C-II, D-I

(JEE Arjuna Physics M-1)

10. If surface tension (S), Moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be

[8 April, 2019 (Shift-II)]

- (a) $S^{3/2} I^{1/2} h^0$ (b) $S^{1/2} I^{1/2} h^0$
(c) $S^{1/2} I^{1/2} h^{-1}$ (d) $S^{1/2} I^{3/2} h^{-1}$

(JEE Arjuna Physics M-1)

APPLICATION OF DIMENSIONAL ANALYSIS

11. The frequency (ν) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as: $\nu = r^a \rho^b s^c$. The values of a , b and c respectively are [24 Jan, 2023 (Shift-II)]

- (a) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$
(c) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$

(JEE Arjuna Physics M-1)

12. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G , Planck's constant h and the speed of light c , as $Y = c^\alpha h^\beta G^\gamma$. Which of the following is the correct option?

[JEE Adv, 2023]

- (a) $\alpha = 7, \beta = -1, \gamma = -2$ (b) $\alpha = -7, \beta = -1, \gamma = -2$
(c) $\alpha = 7, \beta = -1, \gamma = 2$ (d) $\alpha = -7, \beta = 1, \gamma = -2$

(JEE Arjuna Physics M-1)

13. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e , electron mass m_e , Planck's constant h , and coulomb's constant

$$k = \frac{1}{4\pi\epsilon_0}, \text{ where } \epsilon_0 \text{ is the permittivity of vacuum. In}$$

terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$. The value of $\alpha + \beta + \gamma + \delta$ is [JEE Adv, 2022]

(JEE Arjuna Physics M-1)

14. If time (t), velocity (v), and angular momentum (ℓ) are taken as the fundamental units. Then the dimension of mass (m) in terms of t , v , and ℓ is: [20 July, 2021 (Shift-II)]

- (a) $[t^{-2} v^{-1} \ell^1]$ (b) $[t^{-1} v^1 \ell^{-2}]$
(c) $[t^{-1} v^{-2} \ell^1]$ (d) $[t^1 v^2 \ell^{-1}]$

(JEE Arjuna Physics M-1)

15. In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the

temperature. If α and β are constants, dimensions of α will be: [26 Feb, 2021 (Shift-I)]

- (a) $[MLT^{-1}]$ (b) $[M^0 L T^0]$ (c) $[M^2 L T^{-2}]$ (d) $[MLT^{-2}]$

(JEE Arjuna Physics M-1)

16. Dimensional formula for thermal conductivity is (here K denotes the temperature): [04 Sep, 2020 (Shift-I)]

- (a) $MLT^{-3} K^{-1}$ (b) $MLT^{-2} K^{-2}$
(c) $MLT^{-2} K$ (d) $MLT^{-3} K$

(JEE Arjuna Physics M-1)

17. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to: [9 Jan, 2019 (Shift-II)]

- (a) $\sqrt{\frac{hc^5}{G}}$ (b) $\sqrt{\frac{c^3}{Gh}}$ (c) $\sqrt{\frac{Gh}{c^5}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

(JEE Arjuna Physics M-1)

18. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L , which of the following statement(s) is/are correct? [JEE Adv, 2019]

- (a) The dimension of force is L^{-3} .
(b) The dimension of energy of L^{-2} .
(c) The dimension of power is L^{-5} .
(d) The dimension of linear momentum is L^{-1} .

(JEE Arjuna Physics M-1)

ERRORS IN MEASUREMENT

19. Two resistances are given as $R_1 = (10 \pm 0.5)\Omega$ and $R_2 = (15 \pm 0.5)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is [6 Apr, 2023 (Shift-I)]

- (a) 6.33 (b) 2.33 (c) 4.33 (d) 5.33

(JEE Arjuna Physics M-1)

20. If $Z = \frac{A^2 B^3}{C^4}$, then the relative error in Z will be: [25 June, 2022 (Shift-I)]

- (a) $\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$ (b) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}$
(c) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$ (d) $\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{\Delta C}{C}$

(JEE Arjuna Physics M-1)

21. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2 , without any significant error, his observation are as follows: [1 Sept, 2021 (Shift-II)]

Physical Quantity	Least count of the Equipment used for measurement	Observed value
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Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then the fractional error in the measurement of Y is:

- (a) 0.083 (b) 0.0155
(c) 0.0083 (d) 0.155

(JEE Arjuna Physics M-1)

22. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1 s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is:

[8 Jan, 2020 (Shift-II)]

- (a) 4.40% (b) 3.40%
(c) 2.40% (d) 5.40%

(JEE Arjuna Physics M-1)

23. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to:

[8 April, 2019 (Shift-II)]

- (a) 0.7% (b) 3.5% (c) 6.8% (d) 0.2%

(JEE Arjuna Physics M-1)

MEASURING INSTRUMENTS

24. Given below are two statements:

Statements-I: Astronomical unit (Au). Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements-II: $\text{Au} < \text{Parsec (Pc)} < \text{ly}$

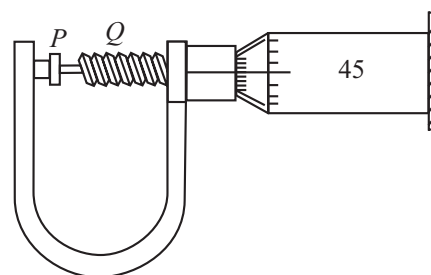
In the light of the above statements. Choose the most appropriate answer from the options given below:

[11 Apr, 2023 (Shift-I)]

- (a) Both Statements-I and Statements-II are correct
(b) Statements-I is correct but Statements-II is incorrect
(c) Both Statements-I and Statements-II are incorrect
(d) Statements-I is incorrect but statements-II is correct

(JEE Arjuna Physics M-1)

25. In an experiment to find out the diameter of wire using screw gauge, the following observations were noted?



- A. Screw moves 0.5 mm or main scale in one complete rotation.
B. Total divisions on circular scale = 50
C. Main scale reading is 2.5 mm
D. 45th division of circular scale is in the pitch line

Then the diameter of wire is [29 July, 2022 (Shift-I)]

- (a) 2.92 mm (b) 2.54 mm
(c) 2.98 mm (d) 3.45 mm

(JEE Arjuna Physics M-1)

26. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that zero on the vernier scale lies between 8.5 cm and 8.6 cm. and vernier coincidence is 6, then the correct value of measurement is _____ (least count = 0.01 cm)

[17 March, 2021 (Shift-I)]

- (a) 8.58 cm (b) 8.54 cm
(c) 8.56 cm (d) 8.36 cm

(JEE Arjuna Physics M-1)

27. **Assertion A:** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R: Least Count = $\frac{\text{Pitch}}{\text{Total divisions on circular scale}}$

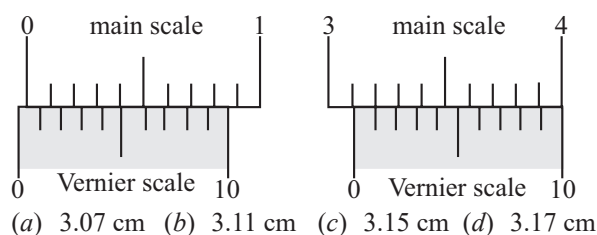
In the light of the above statement, choose the most appropriate answer from the options given below:

[27 July, 2021 (Shift-I)]

- (a) A is correct but R is not correct.
(b) A is not correct but R is correct.
(c) Both A and R are correct and R is NOT the correct explanation of A.
(d) Both A and R are correct and R is the correct explanation of A.

(JEE Arjuna Physics M-1)

28. The smallest division on the main scale of a Vernier callipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calliper with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is [JEE Adv, 2021]



(JEE Arjuna Physics M-1)

29. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as [3 Sep, 2020 (Shift-I)]

- (a) 2.121 cm (b) 2.123 cm
(c) 2.124 cm (d) 2.125 cm

(JEE Arjuna Physics M-1)

30. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is

[12 Jan, 2019 (Shift-I)]

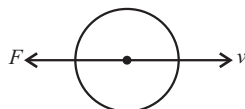
- (a) 50 (b) 200 (c) 100 (d) 500

(JEE Arjuna Physics M-1)

PW CHALLENGERS

SINGLE CORRECT TYPE QUESTIONS

1. In a certain fluid, measurements indicate that drag force F is proportional to velocity of the object v in power of $\frac{3}{2}$:
 $F \propto v^{\frac{3}{2}}$



If two spheres of radii r_1 and r_2 are moving with equal velocity through same fluid they will experience drag force

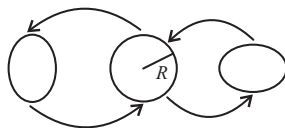
F_1 and F_2 respectively. If $r_2 = 4r_1$ find $\frac{F_2}{F_1}$.

(The density of fluid is ρ and coefficient of viscosity is η)

- (a) 2 (b) 4 (c) 6 (d) 8

(JEE Arjuna Physics M-1)

2. A small droplet of water floating at a space station usually has a spherical shape due to surface tension which changes to ellipsoidal shape with some regular time interval T



Find ratio $\frac{T_2}{T_1}$ of two period of oscillation T_1 and T_2 for water

droplets with radii r_1 and r_2 respectively where $r_2 = 2r_1$. Assume no artificial gravity is present at the space station.

- (a) 2 (b) 4
(c) $\sqrt{2}$ (d) $2\sqrt{2}$

(JEE Arjuna Physics M-1)

3. British physicist G.I Taylor was able to estimate equivalent energy of E of first atomic bomb by looking at pictures that

were published in a popular magazine. From the pictures it was clearly established that after time $t = 0.006\text{s}$, radius of the cloud was $r \simeq 80\text{ m}$. If the density of air $\rho = 1.2\text{ kg/m}^3$, the energy estimated by Taylor was



- (a) 10^{14} J (b) 10^{10} J
(c) 10^{16} J (d) 10^8 J

(JEE Arjuna Physics M-1)

4. A certain drone is able to hover if the mechanical power input is P_1 . If an exact replica is made where all linear dimensions have been doubled, then new power P_2 of the mechanical output is



- (a) $2^{5/2}P_1$ (b) $2^{3/2}P_1$ (c) $2^{7/2}P_1$ (d) $2^{1/2}P_1$

(JEE Arjuna Physics M-1)

5. In cosmology, geometrized system of units (GU) is used where we assume

$G = c = 1$

Angular momentum L_{GU} in geometrized units is measured in m^2 . For conversion between geometrized units and SI units for angular momentum we need a conversion factor such

that following relation is true

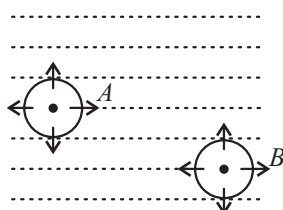
$$L_{SI} = c^x G^y L_{GU}$$

Find the value of x and y

- (a) 3, -1 (b) -1, 3 (c) 2, -3 (d) -2, 3

(JEE Arjuna Physics M-1)

6. Two identical amounts of explosive are burst under water at two different location A and B where hydrostatic pressure are P_A and P_B respectively ($P_B = 1.3 P_A$). A hollow space occurring due to under water explosion expands and shrinks with some regular period of time T which does not depend on size of the hollow bubble. Assuming that sizes of hollow spaces are much smaller than depths of explosion A and B , determine ratio x which is defined as $x = \frac{T_B}{T_A}$ where T_B and T_A are periods of oscillations of two bubbles occurring at location A and B respectively.



- (a) 0.2 (b) 0.5
(c) 0.8 (d) 0.4

(JEE Arjuna Physics M-1)

7. The pitch of a screw gauge is 1 mm and its cap is divided into 100 divisions. When nothing is placed between its studs, the zero marking of the circular scale is getting 4 divisions above the reference line. When a wire is placed between its studs, the main scale reading is 2 mm and 56th division of circular scale coincides with the reference line. If the length of the wire is 10.5 cm, then find the curved surface area (in cm²) of the wire in true significant figures.

- (a) 8.50 cm² (b) 8.54 cm²
(c) 8.58 cm² (d) 9.00 cm²

(JEE Arjuna Physics M-1)

8. Suppose that you measure three independent variables as $x = 10.0 \pm 0.2$, $y = 7.0 \pm 0.1$, $\theta = 40.0 \pm 0.3^\circ$ and use these values to compute

$$q = \frac{x+2}{x+y \cos 4\theta}$$

What should be your answer for q and its uncertainty?

- (a) 3.5 ± 0.1 (b) 2.5 ± 0.1
(c) 3.5 ± 0.3 (d) 2.5 ± 0.3

(JEE Arjuna Physics M-1)

9. The Atwood machine consists of two masses M and m (with $M > m$) attached to the ends of a light string that passes over a light, frictionless pulley. When the masses are released, the mass M is easily shown to accelerate down with an acceleration

$$a = g \frac{M-m}{M+m}$$

Suppose that M and m are measured as $M = 100 \pm 1$ and $m = 50 \pm 1$, both in grams. Find a and the uncertainty δa .

- (a) 3.3 ± 0.1 (b) 3.3 ± 0.2 (c) 3.3 ± 0.3 (d) 3.3 ± 0.01

(JEE Arjuna Physics M-1)

10. If a narrow collimated beam of monoenergetic γ -ray of intensity I_0 is incident on a thin sheet of material of thickness 'x', the intensity of the emerging beam is given by

$$I = I_0 \exp(-\mu x)$$

where μ is a quantity known as the linear attenuation coefficient. The following values are obtained for γ -rays of energy 1 MeV incident on lead:

$$I = (0.926 \pm 0.010) \times 10^{10} \text{ } \gamma\text{-rays m}^{-2}\text{s}^{-1}$$

$$I_0 = (2.026 \pm 0.012) \times 10^{10} \text{ } \gamma\text{-rays m}^{-2}\text{s}^{-1}$$

$$x = (10.00 \pm 0.02) \text{ mm}$$

Find out μ and corresponding absolute error in μ .

- (a) 75.3 ± 1.5 (b) 76.3 ± 1.6
(c) 74.3 ± 1.2 (d) 78.3 ± 1.9

Answer Key



CONCEPT APPLICATION

1. $[A] = [M^0 L^0 T^0]$, $[B] = [M^0 L^0 T^0]$
2. $[A] = [L^{1/2} T^{-3}]$, $[B] = [L T^{-3}]$
3. $[A] = [L]$, $[B] = M^0 L^0 T^2$, $[C] = [M^0 L^0 T^0]$
4. $[L^2 T^{-1}]$
5. (c)
6. (d)
7. $m = K c \frac{1}{2} h^{\frac{1}{2}} G^{-\frac{1}{2}}$
8. (a)
9. (c)
10. (a)
11. (d)
12. (d)
13. (a)
14. (d)
15. (d)
16. (c)
17. (b)
18. (d)
19. (c)
20. (d)
21. [0.002]
22. $[1.004 \pm 0.001]$
23. (c)
24. (d)
25. (a)
26. (c)
27. [1]
28. (b)
29. [0.005]
30. [0.215]
31. (a)

SCHOOL LEVEL PROBLEMS

1. (c)
2. (a)
3. (b)
4. (c)
5. (a)
6. (b)
7. (a)
8. (b)
9. (d)
10. (a)
11. (c)
12. (c)
24. (i) (c), (ii) (c), (iii) (d), (iv) (a), (v) (c)
25. (i) (a), (ii) (c), (iii) (d), (iv) (a), (v) (c)

PRARAMBH (TOPICWISE)

1. (b)
2. (a)
3. (a)
4. (c)
5. (b)
6. (c)
7. (b)
8. (b)
9. (d)
10. (a)
11. (d)
12. (d)
13. (c)
14. (c)
15. (b)
16. (b)
17. (b)
18. (d)
19. (b)
20. (a)
21. (c)
22. (c)
23. (c)
24. (a)
25. (b)
26. (a)
27. (c)
28. (a)
29. (d)
30. (b)
31. (b)
32. (b)
33. (c)
34. (b)
35. (c)
36. (a)
37. (b)
38. (a)
39. (b)
40. (a)
41. (c)
42. (c)
43. (a)
44. (a)
45. (a)
46. (a)
47. (c)
48. (b)
49. (b)
50. (b)
51. (c)
52. (c)
53. (a)
54. (a)
55. (b)
56. (b)
57. (c)
58. (c)
59. (c)
60. (a)
61. (b)

PRABAL (JEE MAIN LEVEL)

1. (b)
2. (d)
3. (c)
4. (b)
5. (c)
6. (d)
7. (a)
8. (b)
9. (c)
10. (d)
11. (d)
12. (c)
13. (d)
14. (d)
15. (d)
16. (a)
17. (c)
18. (c)
19. (d)
20. (c)
21. (b)
22. (b)
23. (a)
24. (d)
25. (c)
26. (b)
27. (b)
28. (b)
29. (d)
30. (a)
31. (c)
32. (a)
33. (a)
34. (d)
35. (b)
36. (c)
37. (b)
38. (a)
39. (a)
40. (b)
41. (c)
42. (a)
43. [1]
44. [3]
45. [4]
46. [0]
47. [9]
48. [3]
49. [6]
50. [6]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (a, b, c)
2. (c, d)
3. (a, b, c)
4. (a, b, c, d)
5. (a, c)
6. (b, d)
7. (a, b, c, d)
8. (b)
9. (a)
10. (c)
11. (c)
12. (d)
13. (b)
14. (d)
15. (d)
16. (a)
17. (b)
18. (d)
19. (c)
20. (a)
21. [2.59]
22. [92.14]
23. [0.41]
24. [0]
25. [1]
26. [4]
27. (d)
28. (c)
29. (b)
30. (b)

PYQ's (PAST YEAR QUESTIONS)

1. (b)
2. (c)
3. (c)
4. (b, d)
5. (a)
6. (a)
7. (b)
8. (a)
9. (a)
10. (b)
11. (a)
12. (a)
13. [4]
14. (c)
15. (b)
16. (a)
17. (c)
18. (a, b, d)
19. (c)
20. (c)
21. (b)
22. (a)
23. (c)
24. (b)
25. (a)
26. (b)
27. (b)
28. (c)
29. (c)
30. (b)

PW CHALLENGERS

1. (d)
2. (d)
3. (a)
4. (c)
5. (a)
6. (c)
7. (c)
8. (c)
9. (a)
10. (d)