Funzioni pari :

In una funzione pari :

- $b_K = 0$

- $a_K = \frac{2}{\pi} \int_0^{\pi} f(x) \cos(mx)\, dx$

dim ($b_K = 0$) :

$f(x) \sin(Kx) = F(x)$ dispari

$$\int_{-\pi}^{\pi} F(x)\, dx = \int_{-\pi}^{0} F(x)\, dx + \int_0^{\pi} F(x)\, dx$$

$-y = x$

$-dy = dx$

$$= - \int_{\pi}^{0} F(-x)\, dx + \int_0^{\pi} F(x)\, dx$$

$$= \int_{\pi}^{0} F(x)\, dx + \int_0^{\pi} F(x)\, dx = 0$$

dim (a_K) :

$$a_K = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(mx)\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} f(x) \cos(mx)\, dx + \int_0^{\pi} f(x) \cos(mx) dx$$

Funzione dispari:

In una dispari ho che:

- $a_K = 0$
- $b_K = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(nx) \, dx$

Dim $(a_K = 0)$:

$f(x) \cos(Kx) = F(x)$ dispari

$$\int_{-\pi}^{\pi} F(x) \, dx = \int_{-\pi}^{0} F(x) \, dx + \int_0^{\pi} F(x) \, dx$$

$-y = x$

$-dy = dx$

$$= - \int_{\pi}^{0} F(-x) \, dx + \int_0^{\pi} F(x) \, dx$$

$$= \int_{\pi}^{0} F(x) \, dx + \int_0^{\pi} F(x) \, dx = 0$$

Teorema: Se una f risulta periodica di periodo T, allora la funzione $f(nx)$ risulta periodica di periodo $\frac{T}{n}$

dim: $g(x) = f(nx)$

$$g\left(x + \frac{T}{n}\right) = f\left(n\left(x + \frac{T}{n}\right)\right)$$

$$= f(nx + T)$$

$$= f(nx)$$

$$f(nx) = g(x)$$

Esempio: Scrivere il polinomio trigonom. di $f(x) = \sin^3 x + \sin^2 x$

Sfruttando Werner:

$$\sin^2 x = \frac{1}{2}\left(1 - \cos(2nx)\right)$$

$$f(x) = \sin^2 x \sin x + \sin^2 x$$

$$= \frac{1}{2}\left(1-\cos(2x)\right)\sin x + \frac{1}{2}\left(1-\cos(2x)\right)$$

$$= \frac{1}{2}\sin x - \frac{1}{2}\cos(2x)\sin x + \frac{1}{2}\left(1-\cos(2x)\right)$$

$$f(x) = \frac{1}{2}\sin x + \frac{1}{4}\sin x - \frac{1}{4}\sin(3x) - \frac{1}{4}\cos(2x)$$

$$= \overset{0°}{\frac{1}{2}} + \overset{2^a}{\frac{3}{4}}\sin x \overset{1^a}{- \frac{1}{2}}\cos(2x) \overset{3^a}{- \frac{1}{4}}\sin(3x)$$

$\boxed{a_0 = 1}$ [0]

$a_1 = 0$, $\boxed{a_2 = -\frac{1}{2}}$ [1a] , $a_3 = 0$ ....

[1a] $\boxed{b_1 = \frac{3}{4}}$, $b_2 = 0$, $\boxed{b_3 = -\frac{1}{4}}$ [3a] , ...

Esempio : Dato la funzione:

$$f(x) = \begin{cases} 0 & -\pi < x \leq 0 \\ \sin x & 0 < x \leq \pi \end{cases}$$

calcolare la serie di Fourier

$$a_0 = \frac{1}{\pi} \int_0^{\pi} \sin x = \frac{1}{\pi} \left[-\cos x \right]_0^{\pi}$$

$$= \frac{1}{\pi} (+1 + 1) = \frac{2}{\pi}$$

$$a_k = \frac{1}{\pi} \int_0^{\pi} \sin x \cos(kx) = 0$$

$$\sin x \cos(kx) = \frac{1}{2} \left(\sin((1-k)x) + \sin((1+k)x) \right)$$

$$a_k = \frac{1}{\pi} \int_0^{\pi} \frac{1}{2} \left(\sin((1-k)x) + \sin((1+k)x) \right) dx$$

Per $k = 1$:

$$\frac{1}{2\pi} \int_0^{\pi} \sin(1-1)x + \sin((1+1)x) \, dx$$

$$\frac{1}{2\pi}\int_0^{\pi} \sin(2x)\,dx = \frac{1}{2\pi}\left[-\frac{\cos(2x)}{2}\right]_0^{\pi}$$

$$\frac{1}{2\pi}\left[-\frac{\cos(2\pi)}{2}+\frac{\cos(0)}{2}\right]=$$

$$=\frac{1}{2\pi}\left[-\frac{1}{2}+\frac{1}{2}\right]=0$$

Per $K \neq 1$ (da fare) $\frac{1}{2\pi}\frac{4}{1-4m^2}$ $K=2m$

Per b_K (da fare)

$$\begin{matrix} \frac{1}{2} & K=1 \\ 0 & K\neq 1 \end{matrix}$$