FORMULA DI EULERO : sin x e cos x

Possiamo ricavare dalla formula di Eulero:

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

Proposizione: In generale posso scrivere:

$$\cos(nx) = \sum_{k=0}^{n} \binom{n}{k} \cos \frac{(n-k)\pi}{2} \cos^k x \sin^{n-k} x$$

$$\sin(nx) = \sum_{k=0}^{n} \binom{n}{k} \sin \frac{(n-k)\pi}{2} \cos^k x \sin^{n-k} x$$

dim:

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\cos nx = \frac{(e^{ix})^n + (e^{-ix})^n}{2}$$

$$= \frac{(\cos x + i \sin x)^n + (\cos x - i \sin x)^n}{2}$$

posso quindi sostituire:

$$\cos(nx) = \sum_{k=0}^{n} \binom{n}{k} \frac{\cos^k x + (i \sin x)^{n-k} + \cos^k x \, (-i \sin)^{n-k}}{2}$$

$$= \sum_{k=0}^{n} \binom{n}{k} \frac{(i)^{n-k} + (-i)^{n-k}}{2} \cos^k x \sin^{n-k} x$$

$$= \sum_{k=0}^{n} \binom{n}{k} \frac{e^{(i\frac{\pi}{2})^{n-k}} + \left(e^{(-i\frac{\pi}{2})^{n-k}}\right)}{2} \cos^k x \sin^{n-k} x$$

$$= \sum_{k=0}^{n} \binom{n}{k} \cos \frac{(n-k)\pi}{2} \cos^k x \sin^{n-k} x$$

dim:

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\sin(nx) = \frac{(e^{ix})^n - (e^{-ix})^n}{2i}$$

$$= \frac{(\cos x + i \sin x)^n - (\cos x - i \sin x)^n}{2i}$$

posso quindi sostituire:

$$\sin(nx) = \sum_{k=0}^{n} \binom{n}{k} \frac{\cos^k x + (i \sin x)^{n-k} - \cos^k x \, (-i \sin x)^{n-k}}{2i}$$

$$= \sum_{k=0}^{n} \binom{n}{k} \frac{(i)^{n-k} - (-i)^{n-k}}{2i} \cos^k x \sin^{n-k} x$$

$$= \sum_{k=0}^{n} \binom{n}{k} \frac{\left(e^{(i\frac{\pi}{2})^{n-k}}\right) - \left(e^{(-i\frac{\pi}{2})^{n-k}}\right)}{2i} \cos^k x \sin^{n-k} x$$

$$= \sum_{k=0} \binom{n}{k} \sin \frac{(n-k)\pi}{2} \cos^{k} x \sin^{n-k} x$$

CHE PORACCIO ! DIMOSTRAMELO ♡