$$e^{ix} = \cos x + i \sin x$$

Per $z \in \mathbb{C}$:

$$e^{z} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$$

con $z = ix$:

$$\begin{aligned}
e^{ix} &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \dots \\
&= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \dots \\
& \qquad i^2 = -1 \\
&= \Big(\underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots}_{\cos x}\Big) + i\Big(\underbrace{x - \frac{x^3}{3!} + \dots}_{\sin x}\Big) \\
&= \cos x + i \sin x \quad \square
\end{aligned}$$