Name:	
J#:	Dr. Clontz
Date:	

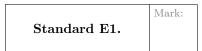**MASTERY QUIZ DAY 10**   Math 237 – Linear Algebra
Version 1   Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array}\right]$$

Solution:

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

☐

Standard E3.	Mark:

Solve the following linear system.

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 5$$
$$-2x_3 - 4x_4 = 3$$
$$2x_1 + 2x_2 + x_3 - 4x_4 = -1$$

Solution: Let $A = \left[\begin{array}{cccc|c} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{array}\right]$, so RREF $A = \left[\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$. It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction $0 = 1$).

☐

Find a basis for the solution set of the system of equations

$$x + 3y + 3z + 7w = 0$$
$$x + 3y - z - w = 0$$
$$2x + 6y + 3z + 8w = 0$$
$$x + 3y - 2z - 3w = 0$$

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

☐

Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.

(b) Determine if V is a vector space or not. Justify your answer

    Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$
$$= c(dx - 3(d - 1)) - 3(c - 1)$$
$$= cdx - 3(cd - 1)$$
$$= (cd) \odot x$$

    We verify the remaining 7 properties to see that V is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c - 1) \\
&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}$$

8)

$$\begin{aligned}
(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
&= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (d \odot x)
\end{aligned}$$

Therefore V is a vector space.

$\square$

Additional Notes/Marks	

MASTERY QUIZ DAY 10   Math 237 – Linear Algebra
Version 2   Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:

Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 0$$
$$x - z = 1$$

Solution:

$$\begin{bmatrix} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{bmatrix}$$

☐

Standard E3.	Mark:

Solve the system of equations

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 2$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

So the solution set is

$$\left\{ \begin{bmatrix} 1 - 3c \\ c \\ -1 \end{bmatrix} \,\middle|\, c \in \mathbb{R} \right\}$$

☐

Standard E4.	Mark:

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \dfrac{5}{7} & \dfrac{3}{7} \\[2mm] 0 & 1 & \dfrac{8}{7} & \dfrac{2}{7} \\[2mm] 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\dfrac{5}{7}a - \dfrac{3}{7}b \\[2mm] -\dfrac{8}{7}a - \dfrac{2}{7}b \\[2mm] a \\[1mm] b \end{bmatrix} \,\middle|\, a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} -\dfrac{5}{7} \\[2mm] -\dfrac{8}{7} \\[1mm] 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\dfrac{3}{7} \\[2mm] -\dfrac{2}{7} \\[1mm] 0 \\ 1 \end{bmatrix} \right\}$, or $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$.

□

	Mark:
Standard V1.	

Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$
$$c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$$

(a) Show that this vector space has an **additive identity** element $\mathbf{0}$ satisfying $(x, y) \oplus \mathbf{0} = (x, y)$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$; then $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so $(1, 1)$ is an additive identity element. Now we will show the other seven properties. Let $(x_1, y_1), (x_2, y_2) \in V$, and let $c, d \in \mathbb{R}$.

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) The additive identity is $(1, 1)$.

4) $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$, so $(2 - x_1, 2 - y_1)$ is the additive inverse of (x_1, y_1).

5)

$$\begin{aligned} c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= (c(dx_1 - (d - 1)) - (c - 1), c(dy_1 - (d - 1))) \\ &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1)) \\ &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1)) \\ &= (cd) \odot (x_1, y_1) \end{aligned}$$

6) $1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$

7)

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1)$$
$$= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1))$$
$$= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1)$$
$$= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1))$$
$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

8)

$$(c + d) \odot (x_1, y_1) = ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1))$$
$$= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1))$$
$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

Therefore V is a vector space.

$\square$

Additional Notes/Marks	

MASTERY QUIZ DAY 10   Math 237 – Linear Algebra
Version 3   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:

Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} -4 & -1 & 3 & | & 2 \\ 1 & 2 & -1 & | & 0 \\ -1 & 4 & 1 & | & 4 \end{bmatrix}$$

Solution:

$$-4x_1 - x_2 + 3x_3 = 2$$
$$x_1 + 2x_2 - x_3 = 0$$
$$-x_1 + 4x_2 + x_3 = 4$$

☐

Standard E3.	Mark:

Find the solution set for the following system of linear equations.

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 8$$
$$x_1 + x_2 - x_3 + 5x_4 = 3$$

Solution: Let $A = \begin{bmatrix} 2 & 3 & -5 & 14 & | & 8 \\ 1 & 1 & -1 & 5 & | & 3 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 1 & | & 1 \\ 0 & 1 & -3 & 4 & | & 2 \end{bmatrix}$. It follows that the solution set

is given by $\begin{bmatrix} 1 - 2a - b \\ 2 + 3a - 4b \\ a \\ b \end{bmatrix}$ for all real numbers a, b.

☐

Standard E4.	Mark:

Find a basis for the solution set to the homogeneous system of equations

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 0$$
$$x_1 + x_2 - x_3 + 5x_4 = 0$$

Solution: Let $A = \begin{bmatrix} 2 & 3 & -5 & 14 & | & 0 \\ 1 & 1 & -1 & 5 & | & 0 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 1 & | & 1 \\ 0 & 1 & -3 & 4 & | & 2 \end{bmatrix}$. It follows that the basis for the

solution set is given by $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$.

$\square$

	Mark:
Standard V1.	

Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$(x \oplus y) \oplus z = \sqrt{x^2 + y^2} \oplus z$$
$$= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2}$$
$$= \sqrt{x^2 + y^2 + z^2}$$
$$= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2}$$
$$= x \oplus \sqrt{y^2 + z^2}$$
$$= x \oplus (y \oplus z)$$

However, this is not a vector space, as there is no zero vector.

$\square$

Additional Notes/Marks	

MASTERY QUIZ DAY 10   Math 237 – Linear Algebra
Version 4   Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard E1.	

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array}\right]$$

Solution:

$$3x_1 - x_2 + x_4 = 5$$
$$-x_1 + 9x_2 + x_3 - 7x_4 = 0$$
$$x_1 - x_3 = -3$$

☐

	Mark:
Standard E3.	

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

$$\text{RREF}\left(\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array}\right]\right) = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\dfrac{1}{12} & 1 \\ 0 & 1 & 0 & \dfrac{7}{4} & 3 \\ 0 & 0 & 1 & \dfrac{7}{12} & 0 \end{array}\right]$$

So the solutions are

$$\left\{ \left[\begin{array}{c} 1 + a \\ 3 - 21a \\ -7a \\ 12a \end{array}\right] \,\middle|\, a \in \mathbb{R} \right\}$$

☐

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \dfrac{5}{7} & \dfrac{3}{7} \\ 0 & 1 & \dfrac{8}{7} & \dfrac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\dfrac{5}{7}a - \dfrac{3}{7}b \\ -\dfrac{8}{7}a - \dfrac{2}{7}b \\ a \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} -\dfrac{5}{7} \\ -\dfrac{8}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\dfrac{3}{7} \\ -\dfrac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$, or $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$.

☐

Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.

(b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\ &= c(dx - 3(d - 1)) - 3(c - 1) \\ &= cdx - 3(cd - 1) \\ &= (cd) \odot x \end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$
\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c - 1) \\
&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}
$$

8)

$$
\begin{aligned}
(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
&= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (d \odot x)
\end{aligned}
$$

Therefore V is a vector space.

$\square$

Additional Notes/Marks	

MASTERY QUIZ DAY 10   Math 237 – Linear Algebra
Version 5   Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show
all relevant work to receive credit for a standard.

	Mark:
Standard E1.	

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array}\right]$$

Solution:

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

☐

	Mark:
Standard E3.	

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

$$\text{RREF}\left(\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array}\right]\right) = \left[\begin{array}{ccc c|c} 1 & 0 & 0 & -\dfrac{1}{12} & 1 \\ 0 & 1 & 0 & \dfrac{7}{4} & 3 \\ 0 & 0 & 1 & \dfrac{7}{12} & 0 \end{array}\right]$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a \\ 3-21a \\ -7a \\ 12a \end{bmatrix} \,\middle|\, a \in \mathbb{R} \right\}$$

☐

<table>
<tr><td rowspan="2">**Standard E4.**</td><td>Mark:</td></tr>
<tr><td></td></tr>
</table>

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$
$$-x + 2z + 5w = 0$$

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \\ -1 & 0 & 2 & 5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} a \\ 2a \\ -2a \\ a \end{bmatrix} \middle| \ a \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \right\}$.

☐

<table>
<tr><td rowspan="2">**Standard V1.**</td><td>Mark:</td></tr>
<tr><td></td></tr>
</table>

Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$(x \oplus y) \oplus z = \sqrt{x^2 + y^2} \oplus z$$
$$= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2}$$
$$= \sqrt{x^2 + y^2 + z^2}$$
$$= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2}$$
$$= x \oplus \sqrt{y^2 + z^2}$$
$$= x \oplus (y \oplus z)$$

However, this is not a vector space, as there is no zero vector.

☐

<table>
<tr><td>**Additional Notes/Marks**</td><td></td></tr>
</table>

MASTERY QUIZ DAY 10   Math 237 – Linear Algebra
Version 6   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard E1.	

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array}\right]$$

Solution:

$$2x_1 - x_2 = 1$$
$$-x_1 + 4x_2 + x_3 = -7$$
$$x_1 + 2x_2 - x_3 = 0$$

☐

	Mark:
Standard E3.	

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

$$\text{RREF}\left(\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array}\right]\right) = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\dfrac{1}{12} & 1 \\ 0 & 1 & 0 & \dfrac{7}{4} & 3 \\ 0 & 0 & 1 & \dfrac{7}{12} & 0 \end{array}\right]$$

So the solutions are

$$\left\{ \begin{bmatrix} 1 + a \\ 3 - 21a \\ -7a \\ 12a \end{bmatrix} \;\middle|\; a \in \mathbb{R} \right\}$$

☐

	Mark:
Standard E4.	

Find a basis for the solution set of the system of equations

$$x + 3y + 3z + 7w = 0$$
$$x + 3y - z - w = 0$$
$$2x + 6y + 3z + 8w = 0$$
$$x + 3y - 2z - 3w = 0$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

☐

	Mark:
Standard V1.	

Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$(x \oplus y) \oplus z = \sqrt{x^2 + y^2} \oplus z$$
$$= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2}$$
$$= \sqrt{x^2 + y^2 + z^2}$$
$$= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2}$$
$$= x \oplus \sqrt{y^2 + z^2}$$
$$= x \oplus (y \oplus z)$$

However, this is not a vector space, as there is no zero vector.

☐

Additional Notes/Marks	