

Application Activities - Module V Part 1 - Class Day 7

Activity 7.1 Consider each of the following vector properties. Label each property with \mathbb{R}^1 , \mathbb{R}^2 , and/or \mathbb{R}^3 if that property holds for Euclidean vectors/scalars \mathbf{u} , \mathbf{v} , \mathbf{w} of that dimension.

1. **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

2. **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

3. **Addition identity.**

There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

4. **Addition inverse.**

There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

5. **Addition midpoint uniqueness.**

There exists a unique \mathbf{m} where the distance from \mathbf{u} to \mathbf{m} equals the distance from \mathbf{m} to \mathbf{v} .

6. **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

7. **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

8. **Scalar multiplication relativity.**

There exists some scalar c where either $c\mathbf{v} = \mathbf{w}$ or $c\mathbf{w} = \mathbf{v}$.

9. **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

10. **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

11. **Orthogonality.**

There exists a non-zero vector \mathbf{n} such that \mathbf{n} is orthogonal to both \mathbf{u} and \mathbf{v} .

12. **Bidimensionality.**

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} \text{ for some value of } a, b.$$

Definition 7.2 A **vector space** V is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ belong to V , and let a, b be scalar numbers.

• **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

• **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

• **Addition identity.**

There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

• **Addition inverse.**

There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

• **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

• **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

• **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

• **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

Definition 7.3 The most important examples of vector spaces are the **Euclidean vector spaces** \mathbb{R}^n , but there are other examples as well.

Activity 7.4 Consider the following set that models motion along the curve $y = e^x$. Let $V = \{(x, y) : y = e^x\}$. Let vector addition be defined by $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 y_2)$, and let scalar multiplication be defined by $c \odot (x, y) = (cx, y^c)$.

Part 1: Which of the vector space properties are satisfied by V paired with these operations?

- **Addition associativity.**

$$\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}.$$

- **Addition commutivity.**

$$\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}.$$

- **Addition identity.**

There exists some $\mathbf{0}$ where $\mathbf{v} \oplus \mathbf{0} = \mathbf{v}$.

- **Addition inverse.**

There exists some $-\mathbf{v}$ where $\mathbf{v} \oplus (-\mathbf{v}) = \mathbf{0}$.

- **Scalar multiplication associativity.**

$$a \odot (b \odot \mathbf{v}) = (ab) \odot \mathbf{v}.$$

- **Scalar multiplication identity.**

$$1 \odot \mathbf{v} = \mathbf{v}.$$

- **Scalar distribution.**

$$a \odot (\mathbf{u} \oplus \mathbf{v}) = (a \odot \mathbf{u}) \oplus (a \odot \mathbf{v}).$$

- **Vector distribution.**

$$(a + b) \odot \mathbf{v} = (a \odot \mathbf{v}) \oplus (b \odot \mathbf{v}).$$

Part 2: Is V a vector space?
