

## Definitions - Module V Part 1 - Class Day 7

**Definition 7.2** A **vector space**  $V$  is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  belong to  $V$ , and let  $a, b$  be scalar numbers.

- **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

- **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

- **Addition identity.**

There exists some  $\mathbf{0}$  where  $\mathbf{v} + \mathbf{0} = \mathbf{v}$ .

- **Addition inverse.**

There exists some  $-\mathbf{v}$  where  $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$ .

- **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

- **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

- **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

- **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

**Definition 7.3** The most important examples of vector spaces are the **Euclidean vector spaces**  $\mathbb{R}^n$ , but there are other examples as well.