Name: ______________________________

MASTERY QUIZ DAY 12 Math 237 – Linear Algebra
Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2 - 1, y_1 + y_2 - 1)\\ c \odot (x_1, y_1) &= (cx_1 - (c - 1), cy_1 - (c - 2))\end{aligned}$$

Determine if V is a vector space or not.

Solution:

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so $(1, 1)$ is an additive identity element.

4) $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$, so $(2 - x_1, 2 - y_1)$ is the additive inverse of (x_1, y_1).

5)

$$\begin{aligned}c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1))\\ &= (c\,(dx_1 - (d - 1)) - (c - 1), c\,(dy_1 - (d - 1)))\\ &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1))\\ &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1))\\ &= (cd) \odot (x_1, y_1)\end{aligned}$$

6) $1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1) = (x_1, y_1)$

7)

$$\begin{aligned}c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1)\\ &= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1))\\ &= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1)\\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1))\\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)\end{aligned}$$

8)

$$\begin{aligned}(c + d) \odot (x_1, y_1) &= ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1))\\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1))\\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)\end{aligned}$$

$\square$

V3. Determine if the vectors $\begin{bmatrix}-3\\1\\1\end{bmatrix}, \begin{bmatrix}5\\-1\\-2\end{bmatrix}, \begin{bmatrix}2\\0\\-1\end{bmatrix}$, and $\begin{bmatrix}0\\2\\-1\end{bmatrix}$ span $\mathbb{R}^3$

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span $\mathbb{R}^3$.

□

V4. Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W, but $(x - x^2) + (x^2) = x \notin W$.

□

V1: $\boxed{}$ **V3:** $\boxed{}$ **V4:** $\boxed{}$