

Name:
J#:
Date:

Dr. Clontz

# MASTERY QUIZ DAY 17

Math 237 – Linear Algebra

## Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$ , and  $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span  $\mathbb{R}^4$ .

□

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all polynomials of even degree. Determine if  $W$  is a subspace of the vector space of all polynomials.

**Solution:**  $W$  is closed under scalar multiplication, but not under addition. For example,  $x - x^2$  and  $x^2$  are both in  $W$ , but  $(x - x^2) + (x^2) = x \notin W$ .

□

<b>Standard S2.</b>	Mark:
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Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathbb{P}_2$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

<b>Additional Notes/Marks</b>	
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Math 237 – Linear Algebra

## Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span  $\mathbb{R}^4$ .

□

<b>Standard V4.</b>	Mark:
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Determine if the set of all lattice points, i.e.  $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

**Solution:** This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

<b>Standard S2.</b>	Mark:
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Determine if the set  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

<b>Additional Notes/Marks</b>	
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Math 237 – Linear Algebra

## Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}$ , and  $\begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are zero rows, they do not span. Alternatively, by inspection  $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ , so the set is linearly dependent, so it spans a subspace of dimension at most 3, therefore it does not span  $\mathbb{R}^4$ .

□

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all 2 by 2 matrices which are not invertible. Determine if  $W$  is a subspace of  $M_{2,2}$ .

**Solution:**  $W$  is closed under scalar multiplication, but not under addition.

□

<b>Standard S2.</b>	Mark:
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Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathbb{P}_2$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

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Math 237 – Linear Algebra

## Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
---------------------	-------

Determine if the vectors  $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$ , and  $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span  $\mathbb{R}^4$ .

□

<b>Standard V4.</b>	Mark:
---------------------	-------

Determine if the set of all lattice points, i.e.  $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

**Solution:** This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathbb{P}_2$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

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# MASTERY QUIZ DAY 17

Math 237 – Linear Algebra

## Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$ , and  $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$  span  $\mathbb{R}^3$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
---------------------	-------

Determine if the set of all lattice points, i.e.  $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

**Solution:** This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

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# MASTERY QUIZ DAY 17

Math 237 – Linear Algebra

## Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
---------------------	-------

Determine if the vectors  $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$ , and  $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$  span  $\mathbb{R}^3$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
---------------------	-------

Determine if  $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$  a subspace of  $\mathbb{R}^4$ .

**Solution:** It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from  $\mathbb{R}^3 \rightarrow \mathbb{R}^4$  given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathbb{P}_2$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.



<b>Additional Notes/Marks</b>	
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