Application Activities - Module S Part 3 - Class Day 14

Fact 14.1 To compute a basis for the subspace span $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$, simply remove the vectors corresponding to the non-pivot columns of RREF $[\mathbf{v}_1 \dots \mathbf{v}_m]$.

Activity 14.3 Assume $\mathbf{w}_1 \neq \mathbf{w}_2$ are distinct vectors in V, which has a basis containing a single vector: $\{\mathbf{v}\}$. Could $\{\mathbf{w}_1, \mathbf{w}_2\}$ be a basis?

Fact 14.4 All bases for a vector space are the same size.

Definition 14.5 The **dimension** of a vector space is given by the cardinality/size of any basis for the vector space.

Activity 14.6 Find the dimension of each subspace of \mathbb{R}^4 .

a)
$$\operatorname{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$
 c)
$$\operatorname{span} \left\{ \begin{bmatrix} 5 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 1 \\ 3 \end{bmatrix} \right\}$$
 e)
$$\operatorname{span} \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 1 \\ 5 \end{bmatrix} \right\}$$
 b)
$$\operatorname{span} \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 1 \\ 3 \end{bmatrix} \right\}$$
 d)
$$\operatorname{span} \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix}, \begin{bmatrix} -1 \\ 10 \\ 7 \\ 14 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \\ 2 \end{bmatrix} \right\}$$

Activity 14.7 What is the dimension of the vector space of 7th-degree (or less) polynomials \mathcal{P}^7 ?

a) 6

b) 7

c) 8

d) infinite

Activity 14.8 What is the dimension of the vector space of all polynomials \mathcal{P} ?

a) 6

b) 7

c) 8

d) infinite

Observation 14.9 Several interesting vector spaces are infinite-dimensional:

• The space of polynomials \mathcal{P} (consider the set $\{1, x, x^2, x^3, \dots\}$).

- The space of continuous functions $C(\mathbb{R})$ (which contains all polynomials, in addition to other functions like $e^x = 1 + x + x^2/2 + x^3/3 + \ldots$).
- The space of real number sequences \mathbb{R}^{∞} (consider the set $\{(1,0,0,\ldots),(0,1,0,\ldots),(0,0,1,\ldots),\ldots\}$).

Fact 14.10 Every vector space with finite dimension, that is, every vector space with a basis of the form $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ is isomorphic to a Euclidean space \mathbb{R}^n :

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_n\mathbf{v}_n \leftrightarrow \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$