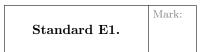
Name:	
J#:	Dr. Clontz
Date:	

## MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

## Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_4 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -1 & 0 & 7 \\ 1 & -1 & 0 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The solutions are

$$\left\{ \begin{bmatrix} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{bmatrix} \mid c \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} c - 1 \\ 3c - 1 \\ -2c \end{bmatrix} \mid c \in \mathbb{R} \right\}$$

## Standard E4. Mark:

Find a basis for the solution set of the system ...

Standard V1.
--------------

Let V be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

Determine if V is a vector space or not.

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ .

1) Real addition is associative, so  $\oplus$  is associative.

2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.

3)  $x \oplus (6-x) = x + (6-x) - 3 = 3$ , so 6-x is the additive inverse of x.

4) Real addition is commutative, so  $\oplus$  is commutative.

5)

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$

$$= c (dx - 3(d - 1)) - 3(c - 1)$$

$$= cdx - 3(cd - 1)$$

$$= (cd) \odot x$$

6) 
$$1 \odot x = x - 3(1 - 1) = x$$

7)

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$
  
=  $cx - 3(c-1) + dx - 3(c-1) - 3$   
=  $(c \odot x) \oplus (d \odot x)$ 

Therefore V is a vector space.