

Name: _____

MASTERY QUIZ DAY 24

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A1. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and \mathbb{R}^2 .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

□

A2. Determine if the map $T : \mathcal{P}^6 \rightarrow \mathcal{P}^6$ given by $T(f) = xf(x) - f(1)$ is a linear transformation or not.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine which of the six products AB , AC , BA , BC , CA , CB can be computed, and compute them.

Solution: AB and CA are the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

$$CA = \begin{bmatrix} 3 & 9 & 11 \\ 0 & 0 & 7 \end{bmatrix}$$

□

M2. Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

Solution: The determinant is 2, so it is invertible.

□

M3. Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}.$

□

A1:

A2:

M1:

M2:

M3: