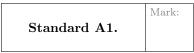**MASTERY QUIZ DAY 24**   Math 237 – Linear Algebra
Version 4   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard A1.   Mark:

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$
T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.
$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^4$.

Solution:
$$
\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 7 & 0 & 0 \end{bmatrix}
$$

□

Standard A2.   Mark:

Determine if the map $T : \mathcal{P}^3 \to \mathcal{P}^4$ given by $T(f(x)) = xf(x) - f(x)$ is a linear transformation or not.

Standard M1.   Mark:

Let

$$
A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}
$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: BC is the only one that can be computed, and

$$
BC = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}
$$

Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since it is not row equivalent to the identity matrix, it is not invertible.

Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}$.

Additional Notes/Marks	