Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$ 

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors  $\begin{bmatrix} -3\\1\\1 \end{bmatrix}$ ,  $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$ ,  $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$ , and  $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$  span  $\mathbb{R}^3$ 

**V4.** Determine if 
$$\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$$
 a subspace of  $\mathbb{R}^4$ .

**S2.** Determine if the set 
$$\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$$
 is a basis of  $\mathbb{R}^3$ 

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Version 2

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**V1.** Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$ 

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors  $\begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}$ ,  $\begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}$ ,  $\begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}$ , and  $\begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**V4.** Let W be the set of all complex numbers that are purely real (i.e of the form a+0i) or purely imaginary (i.e. of the form 0+bi). Determine if W is a subspace of  $\mathbb{C}$ .

**S2.** Determine if the set  $\left\{ \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^4$ .

Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all polynomials with the operations, for any  $f, g \in V, c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Does span 
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\4\\-3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

**V4.** Let W be the set of all complex numbers that are purely real (i.e of the form a+0i) or purely imaginary (i.e. of the form 0+bi). Determine if W is a subspace of  $\mathbb{C}$ .

**S2.** Determine if the set  $\left\{ \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^4$ .

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$ 

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors  $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$ ,  $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$ ,  $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$ , and  $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**V4.** Determine if the set of all lattice points, i.e.  $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

**S2.** Determine if the set  $\left\{ \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^4$ .

Name:
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Math 237 – Linear Algebra

Version 5

Fall 2017

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**V1.** Let V be the set of all polynomials with the operations, for any  $f, g \in V, c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors 
$$\begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}$$
,  $\begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}$ ,  $\begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}$ , and  $\begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**V4.** Determine if the set of all lattice points, i.e.  $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

**S2.** Determine if the set  $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$  is a basis of  $\mathcal{P}_3$ 

V1:

V3:

V4:

S2:

Math 237 – Linear Algebra

Version 6

Fall 2017

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**V1.** Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (0, cy_1)$ 

- (a) Show that this scalar multiplication ⊙ distributes over scalar addition.
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors  $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$ , and  $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$  span  $\mathbb{R}^3$ .

**V4.** Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=0 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

**S2.** Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathcal{P}_2$ 

V1:

V3:

V4:

S2: