Name:

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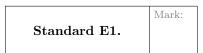
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## MASTERY QUIZ DAY 8

Math 237 – Linear Algebra

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_4 = -1$$

Solution:

Version 3

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -1 & 0 & 7 \\ 1 & -1 & 0 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of equations

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 2$$

Solution:

$$RREF\left(\begin{bmatrix}1 & 3 & -4 & 5\\ 3 & 9 & 1 & 2\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}$$

So the solution set is

$$\left\{ \begin{bmatrix} 1 - 3c \\ c \\ -1 \end{bmatrix} \middle| c \in \mathbb{R} \right\}$$

Standard E4.

Find a basis for the solution set of the system of equations

Mark:

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$

Solution:

RREF 
$$\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{5}{7} & \frac{3}{7} \\ 0 & 1 & \frac{8}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\frac{5}{7}a - \frac{3}{7}b \\ -\frac{8}{7}a - \frac{2}{7}b \\ a \\ b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is  $\left\{ \begin{bmatrix} -\frac{5}{7} \\ \frac{8}{7} \\ -\frac{1}{7} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{3}{7} \\ \frac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$ , or  $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$ .

Standard V1.

Mark:

Let V be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

Determine if V is a vector space or not.

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ .

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6-x) = x + (6-x) 3 = 3$ , so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so  $\oplus$  is commutative.

5)

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$
$$= c (dx - 3(d - 1)) - 3(c - 1)$$
$$= cdx - 3(cd - 1)$$
$$= (cd) \odot x$$

6) 
$$1 \odot x = x - 3(1 - 1) = x$$

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$
  
=  $cx - 3(c-1) + dx - 3(c-1) - 3$   
=  $(c \odot x) \oplus (d \odot x)$ 

Therefore V is a vector space.