Definitions - Module E Part 2 - Class Day 4

Definition 4.1 The following **row operations** produce equivalent augmented matrices:

- 1. Swap two rows.
- 2. Multiply a row by a nonzero constant.
- 3. Add a constant multiple of one row to another row.

Whenever two matrices A, B are equivalent (so whenever we do any of these operations), we write $A \sim B$.

Definition 4.3 The leading term of a matrix row is its first nonzero term. A matrix is in row echelon form if all leading terms are 1, the leading term of every row is farther right than every leading term on a higher row, and all zero rows are at the bottom of the matrix. Examples:

$$\begin{bmatrix} 1 & -1 & 5 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 5 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 5 & | & 1 \\ 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 5 & | & 1 \\ 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

Definition 4.7 A matrix is in reduced row echelon form if it is in row echelon form and all terms above leading terms are 0. Examples:

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 7 \\ 0 & 0 & 1 & | & 2 \end{bmatrix} \qquad \qquad \begin{bmatrix} 1 & 0 & -2 & | & 0 \\ 0 & 1 & 3 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \qquad \qquad \begin{bmatrix} 1 & 3 & 0 & | & -2 \\ 0 & 0 & 1 & | & 7 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & | & -2 \\ 0 & 0 & 1 & | & 7 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$