Name:	
J#:	Dr. Clontz
Date:	

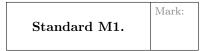
MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: BC is the only one that can be computed, and

$$BC = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

Standard M2.

Mark:

Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

Solution: It is row equivalent to the identity matrix, so it is invertible.

Standard M3.

Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$.

Solution: $\begin{bmatrix} 2 & -1 & -3 & 1 & 0 & 0 \\ -14 & 9 & 24 & 0 & 1 & 0 \\ 3 & -2 & -5 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 3 & 1 & 3 \\ 0 & 1 & 0 & 2 & -1 & -6 \\ 0 & 0 & 1 & 1 & 1 & 4 \end{bmatrix}.$ Thus the inverse is $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}.$

Mark:

Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

Solution:

$$\det(A - \lambda I) = (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix}$$

$$= (8 - \lambda) (\lambda^2 + 6\lambda - 7) + 3(-21\lambda + 21) - (-7\lambda + 7)$$

$$= (\lambda - 1) ((8 - \lambda)(\lambda + 7) - 63 + 7)$$

$$= (\lambda - 1)(\lambda - \lambda^2)$$

$$= -\lambda(\lambda - 1)^2$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

Standard G3.

Mark:

Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}.$

Solution: The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \begin{pmatrix} \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \frac{y}{3} = 0$, or 3x = y. Thus the eigenspace is

$$E_2 = \operatorname{span}\left(\left\{ \begin{bmatrix} 1\\3\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1 \end{bmatrix} \right\}\right)$$

Additional Notes/Marks