Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 18 Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.	Mark:

Determine if the set of matrices $\left\{ \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -8 \\ 6 & 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 3 & 1 & 3 \\ -1 & 2 & -8 \\ 0 & -2 & 6 \\ 4 & 1 & 5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

$\square$

Standard S3.	Mark:

Let $W = \text{span}\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$. Find a basis for this vector space.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \\ 2 & -1 & -8 \\ 1 & 1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$ is a basis of W.

$\square$

Standard S4.	Mark:

Let $W = \text{span}\left(\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\}\right)$. Compute the dimension of W.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

□

Standard A1.	Mark:

Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - 5x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^4$ and $\mathbb{R}^2$.

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -5 & 0 \end{bmatrix}$$

□

Standard A2.	Mark:

Determine if the map $T : \mathcal{P} \to \mathcal{P}$ given by $T(f) = f' - f''$ is a linear transformation or not.

Additional Notes/Marks	

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MASTERY QUIZ DAY 18 Math 237 – Linear Algebra
Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.	Mark:

Determine if the set of vectors $\left\{\begin{bmatrix}3\\-1\\0\\4\end{bmatrix}, \begin{bmatrix}1\\2\\-2\\1\end{bmatrix}, \begin{bmatrix}3\\-8\\6\\5\end{bmatrix}\right\}$ is linearly dependent or linearly independent.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix}3 & 1 & 3\\-1 & 2 & -8\\0 & -2 & 6\\4 & 1 & 5\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 2\\0 & 1 & -3\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

□

Standard S3.	Mark:

Let W be the subspace of $\mathcal{P}^2$ given by $W = \operatorname{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$. Find a basis for W.

Solution: Let $A = \begin{bmatrix}-3 & 1 & 0\\-8 & 2 & -1\\0 & 2 & 3\end{bmatrix}$, and compute $\mathrm{RREF}(A) = \begin{bmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$. Since the first two columns are pivot columns, $\left\{-3x^2 - 8x, x^2 + 2x + 2\right\}$ is a basis for W.

□

Standard S4.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution: Let $A = \begin{bmatrix}-3 & 1 & 0\\-8 & 2 & -1\\0 & 2 & 3\end{bmatrix}$, and compute $\mathrm{RREF}(A) = \begin{bmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$. Since there are two pivot columns, $\dim W = 2$.

Standard A1.	Mark:

Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 + 3x_1 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}$.

Solution:

$$\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

Standard A2.	Mark:

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

Solution:

$$T\left(\begin{bmatrix} 0 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

So T is not a linear transformation.

Additional Notes/Marks	

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MASTERY QUIZ DAY 18 Math 237 – Linear Algebra

Version 3 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.	Mark:

Determine if the set of vectors $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\2\\-1\end{bmatrix}, \begin{bmatrix}1\\3\\-2\end{bmatrix}\right\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 1 & 1\\0 & 2 & 3\\1 & -1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & -\frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

□

Standard S3.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Find a basis for W.

Solution: Let $A = \begin{bmatrix}-3 & 1 & 0\\-8 & 2 & -1\\0 & 2 & 3\end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$. Since the first two columns are pivot columns, $\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}\right\}$ is a basis for W.

□

Standard S4.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find the dimension of W.

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

$\square$

Standard A1.	Mark:

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 7x + 2y + 3z \\ 0 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^4$.

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 7 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$\square$

Standard A2.	Mark:

Determine if the map $T : \mathcal{P} \to \mathcal{P}$ given by $T(f) = f' - f''$ is a linear transformation or not.

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MASTERY QUIZ DAY 18 Math 237 – Linear Algebra

Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.	Mark:

Determine if the set of polynomials $\left\{-3x^3-8x^2, x^3+2x^2+2, -x^2+3\right\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF}\left(\begin{bmatrix}-3 & 1 & 0\\ -8 & 2 & -1\\ 0 & 0 & 0\\ 0 & 2 & 3\end{bmatrix}\right)=\begin{bmatrix}1 & 0 & \frac{1}{2}\\ 0 & 1 & \frac{3}{2}\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

□

Standard S3.	Mark:

Let $W = \text{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Find a basis for W.

Solution: Let $A=\begin{bmatrix}-3 & 1 & 0\\ -8 & 2 & -1\\ 0 & 2 & 3\end{bmatrix}$, and compute $\text{RREF}(A)=\begin{bmatrix}1 & 0 & \frac{1}{2}\\ 0 & 1 & \frac{3}{2}\\ 0 & 0 & 0\end{bmatrix}$. Since the first two columns are pivot columns, $\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}\right\}$ is a basis for W.

□

Standard S4.	Mark:

Let $W = \text{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

$\square$

Standard A1.	Mark:

Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}$.

Solution:

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

$\square$

Standard A2.	Mark:

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

Solution:

$$T\left(\begin{bmatrix} 0 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

So T is not a linear transformation.

$\square$

Additional Notes/Marks	

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MASTERY QUIZ DAY 18 Math 237 – Linear Algebra

Version 5 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.	Mark:

Determine if the set of polynomials $\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF}\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

□

Standard S3.	Mark:

Let $W = \text{span}\left(\left\{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}\right\}\right)$. Find a basis for W.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is $\left\{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}\right\}$.

□

Standard S4.	Mark:

Let $W = \text{span}\left(\left\{\begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since there are two pivot columns, $\dim W = 2$.

$\square$

Standard A1.	Mark:

Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 + 3x_1 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}$.

Solution:

$$\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

$\square$

Standard A2.	Mark:

Determine if the map $T : \mathcal{P}^6 \to \mathcal{P}^6$ given by $T(f) = f(x) - f(0)$ is a linear transformation or not.

Additional Notes/Marks	

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MASTERY QUIZ DAY 18 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.	Mark:

Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF}\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

□

Standard S3.	Mark:

Let $W = \text{span}\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$. Find a basis for this vector space.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \\ 2 & -1 & -8 \\ 1 & 1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$ is a basis of W.

□

Standard S4.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find the dimension of W.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix}1 & 3 & 3 & 7\\1 & 3 & -1 & -1\\2 & 6 & 3 & 8\\1 & 3 & -2 & -3\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\0 & 0 & 1 & 2\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

$\square$

Standard A1.	Mark:

Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix}x_2 + 3x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}$.

Solution:

$$\begin{bmatrix}0 & 1 & 3\end{bmatrix}$$

$\square$

Standard A2.	Mark:

Determine if $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}e^x\\e^y\end{bmatrix}$ is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix}e^2\\1\end{bmatrix} = T\left(\begin{bmatrix}2\\0\end{bmatrix}\right) \neq 2T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2e\\1\end{bmatrix}$$

$\square$

Additional Notes/Marks	