

## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compose functions of real numbers
- Solve systems of linear equations (**Standard(s) E.3**)
- Find the matrix corresponding to a linear transformation (**Standard(s) A.1**)
- Determine if a linear transformation is injective and/or surjective (**Standard(s) A.3**)
- Interpret the ideas of injectivity and surjectivity in multiple ways

## Readiness Assurance Resources

The following resources will help you prepare for this module.

- <https://www.khanacademy.org/math/algebra2/manipulating-functions/function-composition/v/function-composition>

## Readiness Assurance Test

Choose the most appropriate response for each question.

1) Let  $f(x) = x^2 - 2$  and  $g(x) = x^2 + 1$ . Compute the composition function  $(f \circ g)(x)$ .

- (a)  $x^2 - 1$
- (b)  $x^4 + 2x^2 - 1$
- (c)  $x^4 - 4x^2 + 5$
- (d)  $x^4 - x^2 - 2$

2) Suppose  $f(x)$  and  $g(x)$  are real-valued functions satisfying

$$\begin{array}{ll} f(2) = 1 & g(2) = 3 \\ f(3) = 4 & g(3) = 5 \\ f(4) = 3 & g(4) = 6 \end{array}$$

Compute  $(f \circ g)(2)$ .

- (a) 2
- (b) 3
- (c) 4
- (d) 5

3) Solve the system of linear equations

$$\begin{array}{l} x + 3y = -2 \\ 2x - 7y = 9 \end{array}$$

- (a)  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- (b)  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$
- (c)  $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$
- (d)  $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$

4) Let  $a, b, c$  be fixed real numbers. How many solutions does the system of linear equations below have?

$$\begin{array}{l} x + 2y + 3z = a \\ y - z = b \\ y + z = c \end{array}$$

- (a) 0
- (b) 1
- (c) Infinitely many
- (d) It depends on the values of  $a$ ,  $b$ , and  $c$ .

5) What is the matrix corresponding to the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  given by  $T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) =$

$$\begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix} ?$$

(a)  $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$

(c)  $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$

(d)  $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

6) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be the linear transformation with associated matrix  $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$ . Compute

$T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right).$

(a)  $\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$

(c)  $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

(d)  $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

7) Which of the following is true of the linear transformation  $T : ?$

- (a)  $T$  is neither injective nor surjective
- (b)  $T$  is injective but not surjective
- (c)  $T$  is surjective but not injective
- (d)  $T$  is both injective and surjective

8) Which of the following is true of the linear transformation  $T : ?$

- (a)  $T$  is neither injective nor surjective
- (b)  $T$  is injective but not surjective
- (c)  $T$  is surjective but not injective
- (d)  $T$  is both injective and surjective

9) Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation with associated matrix  $A \in M_{m,n}(\mathbb{R})$ . Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a)  $T$  is injective
- (b)  $T$  has a non-trivial kernel
- (c) The columns of  $A$  are linearly dependent
- (d)  $\text{RREF}(A)$  has a non-pivot column

10) Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation with associated matrix  $A \in M_{m,n}(\mathbb{R})$ . Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a)  $T$  is surjective
- (b)  $\text{Im } T = \mathbb{R}^m$
- (c) The columns of  $A$  span  $\mathbb{R}^m$
- (d)  $\text{RREF}(A)$  has only pivot columns