Name:	

## **MASTERY QUIZ DAY 23**

Math 237 – Linear Algebra Fall 2017

Version 5

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A3.** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

$$\text{(a)} \ \ S:\mathbb{R}^4\to\mathbb{R}^3 \text{ where } S(\vec{e}_1)=\begin{bmatrix}2\\1\\0\end{bmatrix}, \ S(\vec{e}_2)=\begin{bmatrix}1\\2\\1\end{bmatrix}, \ S(\vec{e}_3)=\begin{bmatrix}0\\-1\\0\end{bmatrix}, \ \text{and} \ S(\vec{e}_4)=\begin{bmatrix}3\\2\\1\end{bmatrix},$$

(b) 
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 where  $T(\vec{e_1}) = \begin{bmatrix} 2\\2\\1 \end{bmatrix}$ ,  $T(\vec{e_2}) = \begin{bmatrix} 1\\0\\4 \end{bmatrix}$ , and  $T(\vec{e_3}) = \begin{bmatrix} 1\\2\\-3 \end{bmatrix}$ .

Solution:

- (a) RREF  $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ . The map is not injective since it has a column without pivot, but it is surjective because every row has a pivot.
- (b) RREF  $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ . The map is not injective since there is a column without a pivot, and it is not surjective because there is a row without a pivot.

**A4.** Let  $T: \mathbb{R}^3 \to \mathbb{R}^3$  be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T.

Solution: Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $RREF(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Then a basis for the image is its columns,

$$\left\{ \begin{bmatrix} -3\\-8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix} \right\}$$

And the kernel is the solution set of AX = 0, so a basis would be

$$\left\{ \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$$

A3:

A4: