

Application Activities - Module V Part 3 - Class Day 9

Fact 9.1 A vector \mathbf{b} belongs to $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ if and only if the linear system corresponding to $[\mathbf{v}_1 \dots \mathbf{v}_n \mid \mathbf{b}]$ is consistent.

Remark 9.2 To determine if \mathbf{b} belongs to $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$, find $\text{RREF}[\mathbf{v}_1 \dots \mathbf{v}_n \mid \mathbf{b}]$.

Activity 9.3 Determine if $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ belongs to $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$ by row-reducing an appropriate matrix.

Activity 9.4 Determine if $\begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}$ belongs to $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$ by row-reducing an appropriate matrix.

Observation 9.5 So far we've only discussed linear combinations of Euclidean vectors. Fortunately, many vector spaces of interest can be reinterpreted as an **isomorphic** Euclidean space \mathbb{R}^n ; that is, a Euclidean space that mirrors the behavior of the vector space exactly.

Activity 9.6 We previously checked that $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ does not belong to $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$. Does $f(x) = 3x^2 - 2x + 1$ belong to $\text{span}\{x^2 - 3, -x^2 - 3x + 2\}$?

Activity 9.7 Does the matrix $\begin{bmatrix} 6 & 3 \\ 2 & -1 \end{bmatrix}$ belong to $\text{span}\left\{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}\right\}$?

Activity 9.8 Does the complex number $2i$ belong to $\text{span}\{-3 + i, 6 - 2i\}$?

Activity 9.9 How many vectors are required to span \mathbb{R}^2 ? Sketch a drawing in the xy plane to support your guess.

Activity 9.10 How many vectors are required to span \mathbb{R}^3 ?
