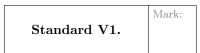
Name:	
J#:	Dr. Clontz
Date:	

## MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

## Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial f.

Standard V3.

Does span 
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\4\\-3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

Solution: Since

RREF 
$$\begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 4 & 2 \\ 4 & -9 & -3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

has a zero row, the vectors fail to span  $\mathbb{R}^3$ .

Standard V4.

Let W be the set of all complex numbers that are purely real (i.e of the form a + 0i) or purely imaginary (i.e. of the form 0 + bi). Determine if W is a subspace of  $\mathbb{C}$ .

**Solution:** No, because 1 is purely real and i is purely imaginary, but the linear combination 1+i is neither.

Standard S2.

Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathcal{P}_2$ 

Solution:

RREF 
$$\left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

Mark:

Additional Notes/Marks