

Name:
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MASTERY QUIZ DAY 14

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
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Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.
- Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

- Real addition is associative, so \oplus is associative.
- $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.
- $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .
- Real addition is commutative, so \oplus is commutative.
- Associativity shown above
- $1 \odot x = x - 3(1 - 1) = x$
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$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}(c+d) \odot x &= (c+d)x - 3(c+d-1) \\ &= cx - 3(c-1) + dx - 3(c-1) - 3 \\ &= (c \odot x) \oplus (d \odot x)\end{aligned}$$

Therefore V is a vector space.

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Standard V3.	Mark:
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Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span \mathbb{R}^3 .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

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Standard V4.	Mark:
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Let W be the set of all complex numbers that are purely real (i.e of the form $a + 0i$) or purely imaginary (i.e. of the form $0 + bi$). Determine if W is a subspace of \mathbb{C} .

Solution: No, because 1 is purely real and i is purely imaginary, but the linear combination $1 + i$ is neither.

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Standard S2.	Mark:
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Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4 .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 1 & 2 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

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Additional Notes/Marks	
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