

Name: \_\_\_\_\_

# **MASTERY QUIZ DAY 24**

Math 237 – Linear Algebra

## **Version 1**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A1.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_2 + 3x_3].$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

**Solution:**

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

□

**A2.** Determine if the map  $T : \mathcal{P}^6 \rightarrow \mathcal{P}^7$  given by  $T(f) = xf(x) - f(1)$  is a linear transformation or not.

**M1.** Let

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**Solution:**  $CA$  is the only one that can be computed, and

$$CA = \begin{bmatrix} 2 & 6 & 11 & 1 \\ 1 & 3 & 7 & 2 \\ -1 & -3 & -5 & 0 \end{bmatrix}$$

□

**M2.** Determine if the matrix  $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$  is invertible.

**Solution:**

$$\text{RREF} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Since it is not equivalent to the identity matrix, it is not invertible.

□

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$ .

**Solution:**  $\left[ \begin{array}{ccc|ccc} 3 & 1 & 3 & 1 & 0 & 0 \\ 2 & -1 & -6 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array} \right] \sim \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & -3 \\ 0 & 1 & 0 & -14 & 9 & 24 \\ 0 & 0 & 1 & 3 & -2 & -5 \end{array} \right]$ . Thus the inverse is  $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$ .

□

**A1:**

**A2:**

**M1:**

**M2:**

**M3:**

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# MASTERY QUIZ DAY 24

Math 237 – Linear Algebra

## Version 2

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A1.** Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - x_3 \end{bmatrix}.$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^4$  and  $\mathbb{R}^2$ .

**Solution:**

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

□

**A2.** Determine if the map  $T : \mathcal{P}^3 \rightarrow \mathcal{P}^4$  given by  $T(f(x)) = xf(x) - f(x)$  is a linear transformation or not.

**M1.** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**Solution:**  $BC$  is the only one that can be computed, and

$$BC = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

□

**M2.** Determine if the matrix  $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$  is invertible.

**Solution:** It is row equivalent to the identity matrix, so it is invertible.

□

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ .

**Solution:**  $\begin{bmatrix} 4 & -1 & -8 & | & 1 & 0 & 0 \\ 2 & 1 & 3 & | & 0 & 1 & 0 \\ 1 & 1 & 4 & | & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & | & 1 & -4 & 5 \\ 0 & 1 & 0 & | & -5 & 24 & -28 \\ 0 & 0 & 1 & | & 1 & -5 & 6 \end{bmatrix}$ . Thus the inverse is  $\begin{bmatrix} 1 & -4 & 5 \\ -5 & 24 & -28 \\ 1 & -5 & 6 \end{bmatrix}$ .

□

**A1:**

**A2:**

**M1:**

**M2:**

**M3:**

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**MASTERY QUIZ DAY 24**

Math 237 – Linear Algebra

**Version 3**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A1.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

**Solution:**

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 7 & 0 & 0 \end{bmatrix}$$

□

**A2.** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$ . Determine if  $T$  is a linear transformation.

**Solution:**

$$T\left(\begin{bmatrix} 0 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

So  $T$  is not a linear transformation.

□

**M1.** Let

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**Solution:**  $CA$  is the only one that can be computed, and

$$CA = \begin{bmatrix} 2 & 6 & 11 & 1 \\ 1 & 3 & 7 & 2 \\ -1 & -3 & -5 & 0 \end{bmatrix}$$

□

**M2.** Determine if the matrix  $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -2 & 0 & 3 \end{bmatrix}$  is invertible.

**Solution:**

$$\text{RREF} \begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This matrix is not row equivalent to the identity matrix, so it is not invertible.

□

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 1 & -4 & 5 \\ -5 & 24 & -28 \\ 1 & -5 & 6 \end{bmatrix}$ .

**Solution:**  $\left[ \begin{array}{ccc|ccc} 1 & -4 & 5 & 1 & 0 & 0 \\ -5 & 24 & -28 & 0 & 1 & 0 \\ 1 & -5 & 6 & 0 & 0 & 1 \end{array} \right] \sim \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -1 & -8 \\ 0 & 1 & 0 & 2 & 1 & 3 \\ 0 & 0 & 1 & 1 & 1 & 4 \end{array} \right]$ . Thus the inverse is  $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ .

□

**A1:**

**A2:**

**M1:**

**M2:**

**M3:**

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**MASTERY QUIZ DAY 24**

Math 237 – Linear Algebra

**Version 4**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A1.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_2 + 3x_3].$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

**Solution:**

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

□

**A2.** Determine if  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  given by  $T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

**Solution:** It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

□

**M1.** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**Solution:**  $BC$  is the only one that can be computed, and

$$BC = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

□

**M2.** Determine if the matrix  $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$  is invertible.

**Solution:**

$$\text{RREF} \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since it is row equivalent to the identity matrix, it is invertible.

□

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is  $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}$ .

□

**A1:**

**A2:**

**M1:**

**M2:**

**M3:**

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**MASTERY QUIZ DAY 24**

Math 237 – Linear Algebra

**Version 5**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A1.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  be the linear transformation given by

$$T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

**Solution:**

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 7 & 0 & 0 \end{bmatrix}$$

□

**A2.** Determine if  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  given by  $T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

**Solution:** It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

□

**M1.** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**Solution:**  $CB$  is the only one that can be computed, and

$$CB = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

□

**M2.** Determine if the matrix  $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$  is invertible.

**Solution:**

$$\text{RREF} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Since it is not equivalent to the identity matrix, it is not invertible.

□

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is  $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}$ .

□

**A1:**

**A2:**

**M1:**

**M2:**

**M3:**



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**MASTERY QUIZ DAY 24**

Math 237 – Linear Algebra

**Version 6**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A1.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}.$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

**Solution:**

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

□

**A2.** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$ . Determine if  $T$  is a linear transformation.

**Solution:**

$$T\left(\begin{bmatrix} 0 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

So  $T$  is not a linear transformation.

□

**M1.** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**Solution:**  $BC$  is the only one that can be computed, and

$$BC = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

□

**M2.** Determine if the matrix  $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$  is invertible.

**Solution:** It is row equivalent to the identity matrix, so it is invertible.

□

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$ .

**Solution:**  $\left[ \begin{array}{ccc|ccc} 3 & 1 & 3 & 1 & 0 & 0 \\ 2 & -1 & -6 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array} \right] \sim \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & -3 \\ 0 & 1 & 0 & -14 & 9 & 24 \\ 0 & 0 & 1 & 3 & -2 & -5 \end{array} \right]$ . Thus the inverse is  $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$ .

□

A1: A2: M1: M2: M3: