

Name:
J#:
Date:

Dr. Clontz

## MASTERY QUIZ DAY 12

Math 237 – Linear Algebra

### Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

- (a) Show that the vector addition  $\oplus$  is associative.
- (b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $x, y, z \in \mathbb{R}$ . Then

$$\begin{aligned}
 (x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\
 &= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\
 &= \sqrt{x^2 + y^2 + z^2} \\
 &= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\
 &= x \oplus \sqrt{y^2 + z^2} \\
 &= x \oplus (y \oplus z)
 \end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

<b>Standard V3.</b>	Mark:
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Does  $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$ ?

**Solution:** Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all polynomials of the form  $ax^3 + bx$ . Determine if  $W$  is a subspace of  $\mathcal{P}^3$ .

**Solution:** Yes because  $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$  also belongs to  $W$ . Alternately, yes because  $W$  is isomorphic to  $\mathbb{R}^2$ .

□

<b>Additional Notes/Marks</b>	
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