Name:	
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Date:	

Dr. Clontz

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra

Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}$, and $\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix}1&3&3&7\\1&3&-1&-1\\2&6&3&8\\1&3&-2&-3\end{bmatrix}\right)=\begin{bmatrix}1&3&0&1\\0&0&1&2\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

Since there are zero rows, they do not span. Alternatively, by inspection $\begin{bmatrix}3\\3\\6\\3\end{bmatrix}=3\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, so the set is linearly dependent, so it spans a subspace of dimension at most 3, therefore it does not span $\mathbb{R}^4$.

$\square$

Standard V4.	Mark:

Let W be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix}x\\y\\z\end{bmatrix}$ satisfying $x+y+z=1$ (this forms a plane). Determine if W is a subspace of $\mathbb{R}^3$.

Solution: No, because $\mathbf{0}$ does not belong to W.

$\square$

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix}0\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\0\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\\-1\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 1 & 2 & 0 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra

Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\\2\\1\end{bmatrix},\begin{bmatrix}-1\\3\\5\\2\\0\end{bmatrix},\begin{bmatrix}1\\0\\5\\1\\-3\end{bmatrix}\right\}=\mathbb{R}^5$?

Solution: Since there are only three vectors, they cannot span $\mathbb{R}^5$.

□

Standard V4.	Mark:

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x-x^2$ and x^2 are both in W, but $(x-x^2)+(x^2)=x\notin W$.

□

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix}1\\1\\-1\end{bmatrix},\begin{bmatrix}3\\-1\\1\end{bmatrix},\begin{bmatrix}2\\0\\-2\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1&3&2\\1&-1&0\\-1&1&-2\end{bmatrix}\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 3 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Solution: Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

□

Standard V4.	Mark:

Let W be the set of all complex numbers $a + bi$ satisfying $a = 2b$. Determine if W is a subspace of $\mathbb{C}$.

Solution: Yes, because $c(2b_1 + b_1 i) + d(2b_2 + b_2 i) = 2(cb_1 + db_2) + (cb_1 + db_2)i$ belongs to W. Alternately, yes because W is isomorphic to $\mathbb{R}$.

□

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra

Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\\0\\-3\end{bmatrix}$, $\begin{bmatrix}0\\3\\0\\-2\end{bmatrix}$, and $\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 0 & -1\\0 & 1 & 3 & 1\\2 & 0 & 0 & -1\\1 & -3 & -2 & -1\end{bmatrix}\right)=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Since every row contains a pivot, the vectors span $\mathbb{R}^4$.

$\square$

Standard V4.	Mark:

Let W be the set of all complex numbers $a+bi$ satisfying $a=2b$. Determine if W is a subspace of $\mathbb{C}$.

Solution: Yes, because $c(2b_1+b_1i)+d(2b_2+b_2i)=2(cb_1+db_2)+(cb_1+db_2)i$ belongs to W. Alternately, yes because W is isomorphic to $\mathbb{R}$.

$\square$

Standard S2.	Mark:

Determine if the set $\left\{x^2+x-1, 3x^2-x+1, 2x-2\right\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right)=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 5 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}-3\\1\\1\end{bmatrix}, \begin{bmatrix}5\\-1\\-2\end{bmatrix}, \begin{bmatrix}2\\0\\-1\end{bmatrix}$, and $\begin{bmatrix}0\\2\\-1\end{bmatrix}$ span $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix}-3 & 5 & 2 & 0\\1 & -1 & 0 & 2\\1 & -2 & -1 & -1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 1 & 5\\0 & 1 & 1 & 3\\0 & 0 & 0 & 0\end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span $\mathbb{R}^3$.

☐

Standard V4.	Mark:

Let W be the set of all complex numbers $a + bi$ satisfying $a = 2b$. Determine if W is a subspace of $\mathbb{C}$.

Solution: Yes, because $c(2b_1 + b_1 i) + d(2b_2 + b_2 i) = 2(cb_1 + db_2) + (cb_1 + db_2)i$ belongs to W. Alternately, yes because W is isomorphic to $\mathbb{R}$.

☐

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix}1\\1\\-1\end{bmatrix}, \begin{bmatrix}3\\-1\\1\end{bmatrix}, \begin{bmatrix}2\\0\\-2\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

☐

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5$?

Solution: Since there are only three vectors, they cannot span $\mathbb{R}^5$.

□

Standard V4.	Mark:

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W, but $(x - x^2) + (x^2) = x \notin W$.

□

Standard S2.	Mark:

Determine if the set $\left\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\right\}$ is a basis of $\mathcal{P}_3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

□

Additional Notes/Marks	