Name: ______________________________

MASTERY QUIZ DAY 18 Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S1. Determine if the set of polynomials $\left\{x^2+x, x^2+2x-1, x^2+3x-2\right\}$ is linearly dependent or linearly independent

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 3\\ 1 & -1 & -2\end{bmatrix}\right)=\begin{bmatrix}1 & 0 & -\frac{1}{2}\\ 0 & 1 & \frac{3}{2}\\ 0 & 0 & 0\end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

□

S3. Let W be the subspace of $\mathcal{P}_2$ given by $W = \operatorname{span}\left(\left\{-3x^2-8x, x^2+2x+2, -x+3\right\}\right)$. Find a basis for W.

Solution: Let $A=\begin{bmatrix}-3 & 1 & 0\\ -8 & 2 & -1\\ 0 & 2 & 3\end{bmatrix}$, and compute $\mathrm{RREF}(A)=\begin{bmatrix}1 & 0 & \frac{1}{2}\\ 0 & 1 & \frac{3}{2}\\ 0 & 0 & 0\end{bmatrix}$. Since the first two columns are pivot columns, $\left\{-3x^2-8x, x^2+2x+2\right\}$ is a basis for W.

□

S4. Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find the dimension of W.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix}1 & 3 & 3 & 7\\ 1 & 3 & -1 & -1\\ 2 & 6 & 3 & 8\\ 1 & 3 & -2 & -3\end{bmatrix}\right)=\begin{bmatrix}1 & 3 & 0 & 1\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

□

A1. Let $T:\mathbb{R}^4\to\mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right)=\begin{bmatrix}x_1+3x_3\\3x_2-x_3\end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of $\mathbb{R}^4$ and $\mathbb{R}^2$.

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

$\square$

A2. Determine if $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$ is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) \neq 2T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

$\square$

S1:		**S3:**		**S4:**		**A1:**		**A2:**	