

Name:
J#:
Date:

Dr. Clontz

# MIDTERM EXAM

Math 237 – Linear Algebra

## Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard E1.</b>	Mark:
---------------------	-------

Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_4 &= -1\end{aligned}$$

<b>Standard E2.</b>	Mark:
---------------------	-------

Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix}$$

<b>Standard E3.</b>	Mark:
---------------------	-------

Solve the system of equations

$$\begin{aligned} -3x + y &= 2 \\ -8x + 2y - z &= 6 \\ 2y + 3z &= -2 \end{aligned}$$

<b>Standard E4.</b>	Mark:
---------------------	-------

Find a basis for the solution set of the system of equations

$$\begin{aligned} x + 2y + 3z + w &= 0 \\ 3x - y + z + w &= 0 \\ 2x - 3y - 2z &= 0 \\ -x + 2z + 5w &= 0 \end{aligned}$$

<b>Standard V1.</b>	Mark:
---------------------	-------

Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- (a) Show that **scalar multiplication** is **associative**:  $a \odot (b \odot x) = (ab) \odot x$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer

<b>Standard V2.</b>	Mark:
---------------------	-------

Determine if  $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$ .

<b>Standard V3.</b>	Mark:
---------------------	-------

Determine if the vectors  $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$ , and  $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$  span  $\mathbb{R}^3$ .

<b>Standard V4.</b>	Mark:
---------------------	-------

Let  $W$  be the set of all complex numbers  $a + bi$  satisfying  $a = 2b$ . Determine if  $W$  is a subspace of  $\mathbb{C}$ .

<b>Standard S1.</b>	Mark:
---------------------	-------

Determine if the set of vectors  $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$  is linearly dependent or linearly independent

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$ .

<b>Standard S3.</b>	Mark:
---------------------	-------

Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\} \right)$ . Find a basis of  $W$ .

<b>Standard S4.</b>	Mark:
---------------------	-------

Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find the dimension of  $W$ .

<b>Additional Notes/Marks</b>	
-------------------------------	--