

Name:
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Dr. Clontz

# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard E1.</b>	Mark:
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

**Solution:**

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_3 &= -1 \end{aligned}$$

□

<b>Standard E3.</b>	Mark:
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Solve the system of equations

$$\begin{aligned} -3x + y &= 2 \\ -8x + 2y - z &= 6 \\ 2y + 3z &= -2 \end{aligned}$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right] \right) = \left[ \begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The solutions are

$$\left\{ \left[ \begin{array}{c} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{array} \right] \mid c \in \mathbb{R} \right\} = \left\{ \left[ \begin{array}{c} c - 1 \\ 3c - 1 \\ -2c \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

<b>Standard E4.</b>	Mark:
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Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

$$-x + 2z + 5w = 0$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \\ -1 & 0 & 2 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} a \\ 2a \\ -2a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

So a basis for the solution set is  $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \right\}$ .

□

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all points on the line  $x + y = 2$  with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

$$c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$$

Determine if  $V$  is a vector space or not.

**Solution:**

- 1) Since real addition is associative,  $\oplus$  is associative.
- 2) Since real addition is commutative,  $\oplus$  is commutative.
- 3)  $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$ , so  $(1, 1)$  is an additive identity element.
- 4)  $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$ , so  $(2 - x_1, 2 - y_1)$  is the additive inverse of  $(x_1, y_1)$ .
- 5)

$$\begin{aligned} c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= (c(dx_1 - (d - 1)) - (c - 1), c(dy_1 - (d - 1))) \\ &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1)) \\ &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1)) \\ &= (cd) \odot (x_1, y_1) \end{aligned}$$

$$6) \ 1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$\begin{aligned} c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1) \\ &= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1)) \\ &= (cx_1 + cy_1 - 2c + 1, cx_2 + cy_2 - 2c + 1) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1)) \\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2) \end{aligned}$$

8)

$$\begin{aligned} (c + d) \odot (x_1, y_1) &= ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1)) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= c \odot (x_1, y_1) \oplus d \odot (x_1, y_1) \end{aligned}$$

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Additional Notes/Marks	
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