Name:	
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Date:	

MIDTERM EXAM Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array}\right]$$

Standard E2.	Mark:

Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{bmatrix}$$

Standard E3.	Mark:

Solve the following linear system.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

Standard E4.	Mark:

Find a basis for the solution set to the homogeneous system of equations given by

$$\begin{aligned} 3x + 2y + z &= 0 \\ x + y + z &= 0 \end{aligned}$$

Standard V1.	Mark:

Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} f \oplus g &= f' + g' \\ c \odot f &= cf' \end{aligned}$$

(here f' denotes the derivative of f).

(a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.

(b) Determine if V is a vector space or not. Justify your answer.

Standard V2.	Mark:

Determine if $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Standard V3.	Mark:

Does $\operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Standard V4.	Mark:

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Standard S1.	Mark:

Determine if the set of polynomials $\left\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\right\}$ is linearly dependent or linearly independent

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 5 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.

Standard S3.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Find a basis of W.

Standard S4.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Additional Notes/Marks	