

Name:
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Dr. Clontz

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
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Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$$

Determine if V is a vector space or not.

Solution: V is not a vector space, as one of the distributive laws fails, namely

$$(c + d) \odot (x_1, y_1) = ((c + d)^2 x_1, (c + d)^3 y_1) \neq ((c^2 + d^2) x_1, (c^3 + d^3) y_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

□

Standard V3.	Mark:
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Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

□

Standard V4.	Mark:
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Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ a subspace of \mathbb{R}^4 .

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \rightarrow \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□

Standard S2.	Mark:
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Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

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Additional Notes/Marks	
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