

Name:
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Dr. Clontz

# MIDTERM EXAM

Math 237 – Linear Algebra

## Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard E1.</b>	Mark:
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

<b>Standard E2.</b>	Mark:
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Put the following matrix in reduced row echelon form.

$$\left[ \begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right]$$

<b>Standard E3.</b>	Mark:
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Find the solution set for the following system of linear equations.

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 8$$

$$x_1 + x_2 - x_3 + 5x_4 = 3$$

<b>Standard E4.</b>	Mark:
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Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$

$$2x + y - 4z = 0$$

$$3y - 2z = 0$$

$$x - y - z = 0$$

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$

$$c \odot x = cx - 3(c - 1)$$

- (a) Show that **scalar multiplication** is **associative**:  $a \odot (b \odot x) = (ab) \odot x$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer

<b>Standard V2.</b>	Mark:
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Determine if  $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$  is a linear combination of the vectors  $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ , and  $\begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix}$ .

<b>Standard V3.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$ , and  $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

<b>Standard V4.</b>	Mark:
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Determine if  $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$  a subspace of  $\mathbb{R}^4$ .

<b>Standard S1.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$  are linearly dependent or linearly independent

<b>Standard S2.</b>	Mark:
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Determine if the set  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$

<b>Standard S3.</b>	Mark:
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Let  $W$  be the subspace of  $\mathcal{P}_3$  given by  
 $W = \text{span} \left( \{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\} \right)$ . Find a basis for  $W$ .

<b>Standard S4.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

<b>Additional Notes/Marks</b>	
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