Name: ________________________________

SEMIFINAL Math 237 – Linear Algebra
Version 4 Fall 2017

Choose up to 6 problems to work. Work each problem on one of the attached pages; write the standard in the upper left corner. Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array}\right]$$

E2. Find RREF A, where

$$A = \left[\begin{array}{cccc|c} 3 & -2 & 1 & 8 & -5 \\ 2 & 2 & 0 & 6 & -2 \\ -1 & 1 & 1 & -4 & 6 \end{array}\right]$$

E3. Solve the following linear system.

$$\begin{aligned} 3x + 2y + z &= 7 \\ x + y + z &= 1 \\ -2x + 3z &= -11 \end{aligned}$$

E4. Find a basis for the solution set to the homogeneous system of equations given by

$$\begin{aligned} 3x + 2y + z &= 0 \\ x + y + z &= 0 \end{aligned}$$

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= \sqrt{x^2 + y^2} \\ c \odot x &= cx \end{aligned}$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix}$.

V3. Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span $\mathbb{R}^4$.

V4. Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of $\mathcal{P}^3$.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

S2. Determine if the set $\left\{x^3 - 3x^2 + 2x + 2, -x^3 + 4x^2 - x + 1, -x^3 + 2x + 1, 3x^2 + 3x + 9\right\}$ is a basis of $\mathcal{P}^3$ or not.

S3. Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Find a basis of W.

S4. Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find the dimension of W.

A1. Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix}x_1 + 3x_3\\3x_2 - x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^4$ and $\mathbb{R}^2$.

A2. Determine if the map $T : \mathcal{P}^3 \to \mathcal{P}^4$ given by $T(f(x)) = xf(x) - f(x)$ is a linear transformation or not.

A3. Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}3x + 2y\\x - y\\x + 4y\end{bmatrix}$

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}x + y + z\\2y + 3z\\x - y - 2z\end{bmatrix}$

A4. Let $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix}x & y\\z & w\end{bmatrix}\right) = \begin{bmatrix}8x - 3y - z + 4w\\y + 3z - 4w\\-7x + 3y + 2z - 5w\end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

M1. Let

$$A = \begin{bmatrix}1 & 3 & -1 & -1\\0 & 0 & 7 & 2\end{bmatrix} \qquad B = \begin{bmatrix}0 & 1 & 7 & 7\\-1 & -2 & 0 & 4\\0 & 0 & 1 & 5\end{bmatrix} \qquad C = \begin{bmatrix}3 & 2\\0 & 1\\-2 & -1\end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M2. Determine if the matrix $\begin{bmatrix}3 & -1 & 0 & 4\\2 & 1 & 1 & 1\\0 & 1 & 1 & -1\\1 & -2 & 0 & 3\end{bmatrix}$ is invertible.

M3. Find the inverse of the matrix $\begin{bmatrix}1 & -4 & 5\\-5 & 24 & -28\\1 & -5 & 6\end{bmatrix}$.

G1. Compute the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix}$.

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 2 & -3 & 2 \\ 8 & -9 & 5 \\ 8 & -7 & 3 \end{bmatrix}$.

G3. Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

G4. Compute the geometric multiplicity of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

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