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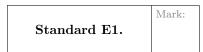
Date:

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of equations

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 2$$

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1 & 3 & -4 & 5\\ 3 & 9 & 1 & 2\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}$$

So the solution set is

$$\left\{ \begin{bmatrix} 1 - 3c \\ c \\ -1 \end{bmatrix} \middle| c \in \mathbb{R} \right\}$$

Standard E4.

Find a basis for the solution set to the homogeneous system of equations given by

$$3x + 2y + z = 0$$
$$x + y + z = 0$$

Solution: Let $A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$.

Standard V1.

Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 3 = x$, so 3 is the additive identity.

Mark:

- 3) $x \oplus (6-x) = x + (6-x) 3 = 3$, so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so \oplus is commutative.

5)

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$

$$= c (dx - 3(d - 1)) - 3(c - 1)$$

$$= cdx - 3(cd - 1)$$

$$= (cd) \odot x$$

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$

= $cx - 3(c-1) + dx - 3(c-1) - 3$
= $(c \odot x) \oplus (d \odot x)$

Therefore V is a vector space.

Additional Notes/Marks