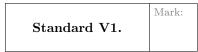
Name:	
J#:	Dr. Clontz
Date:	

### MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

### Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot (f \oplus g) = c \odot f \oplus c \odot g$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial f.

Standard V3.

Determine if the vectors  $\begin{bmatrix} -3\\1\\1 \end{bmatrix}$ ,  $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$ ,  $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$ , and  $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$  span  $\mathbb{R}^3$ 

Solution:

$$RREF\left(\begin{bmatrix} -3 & 5 & 2 & 0\\ 1 & -1 & 0 & 2\\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5\\ 0 & 1 & 1 & 3\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span  $\mathbb{R}^3$ .

# Standard V4.

Mark:

Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=0 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

## Standard S2.

Mark

Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathcal{P}_2$ 

Solution:

RREF 
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

### Additional Notes/Marks