Name: ______________________________

SEMIFINAL Math 237 – Linear Algebra
Version 1 Fall 2017

Choose up to 6 problems to work. Work each problem on one of the attached pages; write the standard in the lower left corner. Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array}\right]$$

E2. Find RREF A, where

$$A = \left[\begin{array}{cccc|c} 3 & -2 & 1 & 8 & -5 \\ 2 & 2 & 0 & 6 & -2 \\ -1 & 1 & 1 & -4 & 6 \end{array}\right]$$

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 8 \\ x_1 + x_2 - x_3 + 5x_4 &= 3 \end{aligned}$$

E4. Find a basis for the solution set to the system of equations

$$\begin{aligned} x + 2y - 3z &= 0 \\ 2x + y - 4z &= 0 \\ 3y - 2z &= 0 \\ x - y - z &= 0 \end{aligned}$$

V1. Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c - 1) \end{aligned}$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.

(b) Determine if V is a vector space or not. Justify your answer

V2. Determine if $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

V3. Does $\operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5$?

V4. Let W be the set of all complex numbers $a + bi$ satisfying $a = 2b$. Determine if W is a subspace of $\mathbb{C}$.

S1. Determine if the set of polynomials $\left\{x^2+x, x^2+2x-1, x^2+3x-2\right\}$ is linearly dependent or linearly independent

S2. Determine if the set $\left\{\begin{bmatrix}1 & -3\\ 2 & 2\end{bmatrix}, \begin{bmatrix}-1 & 4\\ -1 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix}, \begin{bmatrix}0 & 3\\ 3 & 9\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^{2\times 2}$ or not.

S3. Let $W = \operatorname{span}\left\{\begin{bmatrix}2 & 0\\ -2 & 0\end{bmatrix}, \begin{bmatrix}3 & 1\\ 3 & 6\end{bmatrix}, \begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}, \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\right\}$. Find a basis for this vector space.

S4. Let W be the subspace of $\mathbb{R}^{2\times 2}$ given by $W = \operatorname{span}\left(\left\{\begin{bmatrix}2 & 0\\ -2 & 0\end{bmatrix}, \begin{bmatrix}3 & 1\\ 3 & 6\end{bmatrix}, \begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}, \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\right\}\right)$. Compute the dimension of W.

A1. Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}\right) = \begin{bmatrix}x_1 + 3x_3\\ 3x_2 - 5x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^4$ and $\mathbb{R}^2$.

A2. Determine if the map $T : \mathcal{P}^3 \to \mathcal{P}^4$ given by $T(f(x)) = xf(x) - f(x)$ is a linear transformation or not.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^4$ given by the standard matrix $\begin{bmatrix}2 & 1\\ 1 & 2\\ 0 & 1\\ 3 & -3\end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix}2 & 3 & -1 & 1\\ -1 & 1 & 1 & 1\\ 4 & 7 & -1 & 5\end{bmatrix}$

A4. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix}x\\ y\\ z\\ w\end{bmatrix}\right) = \begin{bmatrix}x + 3y + 3z + 7w\\ x + 3y - z - w\\ 2x + 6y + 3z + 8w\\ x + 3y - 2z - 3w\end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T.

M1. Let

$$A = \begin{bmatrix}3\\ 5\\ -1\end{bmatrix} \qquad B = \begin{bmatrix}2 & -1\\ 0 & 4\\ 3 & 1\end{bmatrix} \qquad C = \begin{bmatrix}1 & -1 & 3 & -3\\ 2 & 1 & -1 & 2\end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M2. Determine if the matrix $\begin{bmatrix}3 & -1 & 0 & 4\\ 2 & 1 & 1 & 1\\ 0 & 1 & 1 & -1\\ 1 & -2 & 0 & 3\end{bmatrix}$ is invertible.

M3. Find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

G1. Compute the determinant of the matrix $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 3 & -3 & 2 \\ 108 & -9 & 5 \\ 10 & -7 & 3 \end{bmatrix}$.

G3. Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

G4. Compute the geometric multiplicity of the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

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