Name:

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Date:

MASTERY QUIZ DAY 18

Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.

Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} -3 & 1 & 0 \\ 8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Every column is a pivot column, therefore the set is linearly independent.

Standard S3. $\begin{bmatrix}
\begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix}
\end{bmatrix}$ Find a basis of W.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then $\left\{ \begin{bmatrix} 1\\-1\\3\\-3 \end{bmatrix}, \begin{bmatrix} 2\\0\\1\\1\\-7 \end{bmatrix}, \begin{bmatrix} 1\\1\\1\\-7 \end{bmatrix} \right\}$ is a basis for W.

Standard S4. $\begin{bmatrix} & & & \\ Mark: & & \\ & & \\ Let W = span \left(\left\{ \begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\3\\6\\3\\1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix}, \begin{bmatrix} 7\\-1\\8\\-3 \end{bmatrix} \right\} \right).$ Find the dimension of W.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

Standard A1.

Mark:

Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3\\3x_2 - x_3\end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and $\mathbb{R}^2.$

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

Standard A2.

Mark:

Determine if the map $T: \mathcal{P}^4 \to \mathcal{P}^3$ given by T(f) = f' - f'' is a linear transformation or not.

Additional Notes/Marks