Name: ____________________

MASTERY QUIZ DAY 28 Math 237 – Linear Algebra
Version 2 Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CB is the only one that can be computed, and

$$CB = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

□

M2. Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

Solution: It is row equivalent to the identity matrix, so it is invertible.

□

M3. Compute the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF}(A|I) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -11 & 37 \\ 0 & 1 & 0 & 0 & 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -11 & 37 \\ 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

□

G2. Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned}
\det(A-\lambda I) &= \det\begin{bmatrix} -3-\lambda & 1 & 0 \\ -8 & 2-\lambda & -1 \\ 0 & 2 & 3-\lambda \end{bmatrix} \\
&= (-3-\lambda)\det\begin{bmatrix} 2-\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} - (1)\det\begin{bmatrix} -8 & -1 \\ 0 & 3-\lambda \end{bmatrix} \\
&= (-3-\lambda)\left((2-\lambda)(3-\lambda)+2\right) - (-8(3-\lambda)) \\
&= (-3-\lambda)(8-5\lambda+\lambda^2) + 24 - 8\lambda \\
&= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\
&= -\lambda^3 + 2\lambda^2 - \lambda \\
&= -\lambda(\lambda^2 - 2\lambda + 1) \\
&= -\lambda(\lambda-1)^2
\end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

G3. Find the eigenspace associated to the eigenvalue 2 in the matrix $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$.

Solution: The eigenspace is spanned by $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$.

□

M1: ☐ **M2:** ☐ **M3:** ☐ **G2:** ☐ **G3:** ☐ **G1:** ☐