

## Application Activities - Module G Part 4 - Class Day 28

**Observation 28.1** Recall from last class:

- To find the eigenvalues of a matrix  $A$ , we need to find values of  $\lambda$  such that  $A - \lambda I$  has a nontrivial kernel. Equivalently, we want values where  $A - \lambda I$  is not invertible, so we want to know the values of  $\lambda$  where  $\det(A - \lambda I) = 0$ .
- $\det(A - \lambda I)$  is a polynomial with variable  $\lambda$ , called the **characteristic polynomial** of  $A$ . Thus the roots of the characteristic polynomial of  $A$  are exactly the eigenvalues of  $A$ .
- Once an eigenvalue  $\lambda$  is found, the **eigenspace** containing all **eigenvectors**  $\mathbf{x}$  satisfying  $A\mathbf{x} = \lambda\mathbf{x}$  is given by  $\ker(A - \lambda I)$ .

**Activity 28.2** If  $A$  is a  $4 \times 4$  matrix, what is the largest number of eigenvalues  $A$  can have?

- (a) 3
  - (b) 4
  - (c) 5
  - (d) 6
  - (e) It can have infinitely many
- 

**Activity 28.3** 2 is an eigenvalue of the matrix  $A = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 0 & 1 \\ -1 & -2 & 3 \end{bmatrix}$ .

Compute the eigenspace of  $A$  associated to the eigenvalue 2 by solving for the kernel of

$$A - 2I = \begin{bmatrix} 1-2 & -2 & 1 \\ -1 & 0-2 & 1 \\ -1 & -2 & 3-2 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 1 \\ -1 & -2 & 1 \\ -1 & -2 & 1 \end{bmatrix}$$


---

**Activity 28.4** 2 is an eigenvalue of the matrix  $B = \begin{bmatrix} -3 & -9 & 5 \\ -2 & -2 & 2 \\ -7 & -13 & 9 \end{bmatrix}$ .

Compute the eigenspace of  $B$  associated to the eigenvalue 2 by solving for the kernel of  $B - 2I$ .

---

**Definition 28.5**

- The **algebraic multiplicity** of an eigenvalue is its multiplicity as a root of the characteristic polynomial.
  - The **geometric multiplicity** of an eigenvalue is the dimension of the eigenspace.
-

**Fact 28.6** The geometric multiplicity of an eigenvalue cannot exceed its algebraic multiplicity (but it *can* be different).

**Activity 28.7** Find all of the eigenvalues, along with both their algebraic and geometric multiplicities, for the matrix  $\begin{bmatrix} -3 & 1 & 2 & 1 \\ -9 & 5 & -2 & -1 \\ 31 & -17 & 6 & 3 \\ -69 & 39 & -18 & -9 \end{bmatrix}$ . Use technology to help you!

---

**Activity 28.8** Let  $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ .

*Part 1:* Find the eigenvalues of  $A$

*Part 2:* Describe what this linear transformation is doing geometrically; draw a picture.

---