MASTERY QUIZ DAY 12   Math 237 – Linear Algebra
Version 5   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |

Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

(a) Show that this scalar multiplication $\odot$ is associative.

(b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$
\begin{aligned}
c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\
&= c\,(dx - 3(d - 1)) - 3(c - 1) \\
&= cdx - 3(cd - 1) \\
&= (cd) \odot x
\end{aligned}
$$

We verify the remaining 7 properties to see that V is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$
\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c - 1) \\
&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}
$$

8)

$$(c + d) \odot x = (c + d)x - 3(c + d - 1)$$
$$= cx - 3(c - 1) + dx - 3(c - 1) - 3$$
$$= (c \odot x) \oplus (d \odot x)$$

Therefore V is a vector space.

$\square$

	Mark:
Standard V3.	

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Solution: Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

$\square$

	Mark:
Standard V4.	

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

$\square$

Additional Notes/Marks	