

Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compose functions of real numbers
- Solve systems of linear equations (**Standard(s) E3**)
- Find the matrix corresponding to a linear transformation (**Standard(s) A1**)
- Determine if a linear transformation is injective and/or surjective (**Standard(s) A3**)
- Interpret the ideas of injectivity and surjectivity in multiple ways

Readiness Assurance Resources

The following resources will help you prepare for this module.

- <https://www.khanacademy.org/math/algebra2/manipulating-functions/function-composition/v/function-composition>

Readiness Assurance Test

Choose the most appropriate response for each question.

41) Let $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$. Compute the composition function $(f \circ g)(x)$.

- (a) $x^2 - 1$
- (b) $x^4 + 2x^2 - 1$
- (c) $x^4 - 4x^2 + 5$
- (d) $x^4 - x^2 - 2$

42) Suppose $f(x)$ and $g(x)$ are real-valued functions satisfying

$f(2) = 1$	$g(2) = 3$
$f(3) = 4$	$g(3) = 5$
$f(4) = 3$	$g(4) = 6$

Compute $(f \circ g)(2)$.

- (a) 2
- (b) 3
- (c) 4
- (d) 5

43) Solve the system of linear equations

$$\begin{aligned}x + 3y &= -2 \\ 2x - 7y &= 9\end{aligned}$$

- | | | | |
|---|--|--|---|
| (a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ | (b) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ | (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ | (d) $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$ |
|---|--|--|---|

44) Let a, b, c be fixed real numbers. How many solutions does the system of linear equations below have?

$$\begin{aligned}x + 2y + 3z &= a \\ y - z &= b \\ y + z &= c\end{aligned}$$

- | | | | |
|-------|-------|---------------------|---|
| (a) 0 | (b) 1 | (c) Infinitely many | (d) It depends on the values of a , b , and c . |
|-------|-------|---------------------|---|

45) What is the matrix corresponding to the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix}$?

(a) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

46) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation with associated matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$. Compute

$$T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right).$$

(a) $\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

47) Which of the following is true of the linear transformation T : ?

- (a) T is neither injective nor surjective
- (b) T is injective but not surjective
- (c) T is surjective but not injective
- (d) T is both injective and surjective

48) Which of the following is true of the linear transformation T : ?

- (a) T is neither injective nor surjective
- (b) T is injective but not surjective
- (c) T is surjective but not injective
- (d) T is both injective and surjective

49) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation with associated matrix $A \in M_{m,n}(\mathbb{R})$. Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a) T is injective
- (b) T has a non-trivial kernel
- (c) The columns of A are linearly dependent
- (d) $\text{RREF}(A)$ has a non-pivot column

50) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation with associated matrix $A \in M_{m,n}(\mathbb{R})$. Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a) T is surjective
- (b) $\text{Im } T = \mathbb{R}^m$
- (c) The columns of A span \mathbb{R}^m
- (d) $\text{RREF}(A)$ has only pivot columns