

Definitions - Module V Part 1 - Class Day 7

Definition 7.2 A **vector space** V is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ belong to V , and let a, b be scalar numbers.

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| • Addition associativity. $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$ | • Scalar multiplication associativity. $a(b\mathbf{v}) = (ab)\mathbf{v}.$ |
| • Addition commutivity. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$ | • Scalar multiplication identity. $1\mathbf{v} = \mathbf{v}.$ |
| • Addition identity. There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}.$ | • Scalar distribution. $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$ |
| • Addition inverse. There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}.$ | • Vector distribution. $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$ |

Definition 7.3 The most important examples of vector spaces are the **Euclidean vector spaces** \mathbb{R}^n , but there are other examples as well.