

Section V.0

Activity V.0.1 (*~20 min*) Consider each of the following vector properties. Label each property with \mathbb{R}^1 , \mathbb{R}^2 , and/or \mathbb{R}^3 if that property holds for Euclidean vectors/scalars $\mathbf{u}, \mathbf{v}, \mathbf{w}$ of that dimension.

1. **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

2. **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

3. **Addition identity.**

There exists some \mathbf{z} where $\mathbf{v} + \mathbf{z} = \mathbf{v}$.

4. **Addition inverse.**

There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{z}$.

5. **Addition midpoint uniqueness.**

There exists a unique \mathbf{m} where the distance from \mathbf{u} to \mathbf{m} equals the distance from \mathbf{m} to \mathbf{v} .

6. **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

7. **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

8. **Scalar multiplication relativity.**

There exists some scalar c where either $c\mathbf{v} = \mathbf{w}$ or $c\mathbf{w} = \mathbf{v}$.

9. **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

10. **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

11. **Orthogonality.**

There exists a non-zero vector \mathbf{n} such that \mathbf{n} is orthogonal to both \mathbf{u} and \mathbf{v} .

12. **Bidimensionality.**

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} \text{ for some value of } a, b.$$

Definition V.0.2 A **vector space** V is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ belong to V , and let a, b be scalar numbers.

- **Addition is associative.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

- **Addition is commutative.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

- **Additive identity exists.**

There exists some \mathbf{z} where $\mathbf{v} + \mathbf{z} = \mathbf{v}$.

- **Additive inverses exist.**

There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{z}$.

- **Scalar multiplication is associative.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

- **1 is a scalar multiplicative identity.**

$$1\mathbf{v} = \mathbf{v}.$$

- **Scalar multiplication distributes over vector addition.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

- **Scalar multiplication distributes over scalar addition.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

Any **Euclidean vector space** \mathbb{R}^n satisfies all eight requirements regardless of the value of n , but we will also study other types of vector spaces.