MASTERY QUIZ DAY 28 Math 237 – Linear Algebra

Version 2 Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$C = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad\qquad D = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad\qquad E = \begin{bmatrix} 2 & 0 \\ 0 & -1 \\ 1 & -1 \end{bmatrix}$$

Determine which of the six products CD, CE, DC, DE, EC, ED can be computed, and compute them.

Solution:

$$EC = \begin{bmatrix} 4 & 6 \\ 0 & -1 \\ 2 & 2 \end{bmatrix}$$

$$DE = \begin{bmatrix} 6 & -1 \end{bmatrix}$$

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M2. Determine if the matrix $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ is invertible.

Solution: The determinant is 0, so it is not invertible.

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M3. Find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & \dfrac{1}{2} & -\dfrac{1}{2} \\ -1 & \dfrac{3}{2} & -\dfrac{3}{2} \\ 1 & -\dfrac{3}{2} & \dfrac{5}{2} \end{bmatrix}$$

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G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

Solution:

$$\det(A - \lambda I) = (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix}$$

$$= (8 - \lambda)\left(\lambda^2 + 6\lambda - 7\right) + 3\left(-21\lambda + 21\right) - \left(-7\lambda + 7\right)$$
$$= (\lambda - 1)\left((8 - \lambda)(\lambda + 7) - 63 + 7\right)$$
$$= (\lambda - 1)(\lambda - \lambda^2)$$
$$= -\lambda(\lambda - 1)^2$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

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G3. Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

Solution: The eigenspace is the solution space of the system $(B - 2I)X = 0$.

$$\text{RREF}(B - 2I) = \text{RREF}\left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\dfrac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \dfrac{y}{3} = 0$, or $3x = y$. Thus the eigenspace is

$$E_2 = \text{span}\left(\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

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M1: ☐   **M2:** ☐   **M3:** ☐   **G2:** ☐   **G3:** ☐   **G1:** ☐