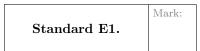
Name:	
J#:	Dr. Clontz
Date:	

## MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

## Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 0$$
$$x - z = 1$$

**Solution:** 

$$\begin{bmatrix} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{bmatrix}$$

Standard E3.

Find the solution set for the following system of linear equations.

Mark:

$$2x_1 - 2x_2 + 6x_3 - x_4 = -1$$
$$3x_1 + 6x_3 + x_4 = 5$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = -7$$

**Solution:** Let 
$$A = \begin{bmatrix} 2 & -2 & 6 & -1 & | & -1 \\ 3 & 0 & 6 & 1 & | & 5 \\ -4 & 1 & -9 & 2 & | & -7 \end{bmatrix}$$
, so RREF  $A = \begin{bmatrix} 1 & 0 & 2 & 0 & | & 2 \\ 0 & 1 & -1 & 0 & | & 3 \\ 0 & 0 & 0 & 1 & | & -1 \end{bmatrix}$ . It follows that the solution set is given by  $\begin{bmatrix} 2 - 2a \\ 3 + a \\ a \\ -1 \end{bmatrix}$  for all real numbers  $a$ .

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$

Solution:

RREF 
$$\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{5}{7} & \frac{3}{7} \\ 0 & 1 & \frac{8}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\frac{5}{7}a - \frac{3}{7}b \\ -\frac{8}{7}a - \frac{2}{7}b \\ a \\ b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is  $\left\{ \begin{bmatrix} -\frac{5}{7} \\ -\frac{8}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{3}{7} \\ \frac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$ , or  $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$ .

## Standard V1.

Mark:

Let V be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

 $\Box$ 

(here f' denotes the derivative of f).

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial f.

Additional Notes/Marks