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Math 237 – Linear Algebra Fall 2017

#### Version 1

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Determine which of the six products AB, AC, BA, BC, CA, CB can be computed, and compute them.

**Solution:** CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

M2. Determine if the matrix  $\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}$  is invertible.

Solution: The second column is a multiple of the first, so it is not invertible.

**M3.** Compute the inverse of the matrix  $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$ 

Solution:

$$RREF(A|I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 2 & -11 & 37 \\ 0 & 1 & 0 & 0 & 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

So the inverse is  $\begin{bmatrix} 1 & 2 & -11 & 37 \\ 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 

**M1**:

**M2**:

**M3**:

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#### Version 2

Fall 2017 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive

credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$C = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad \qquad D = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & 0 \\ 0 & -1 \\ 1 & -1 \end{bmatrix}$$

Determine which of the six products CD, CE, DC, DE, EC, ED can be computed, and compute them.

**Solution:** 

$$EC = \begin{bmatrix} 4 & 6 \\ 0 & -1 \\ 2 & 2 \end{bmatrix}$$

$$DE = \begin{bmatrix} 6 & -1 \end{bmatrix}$$

**M2.** Determine if the matrix  $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$  is invertible.

**Solution:** The determinant is 0, so it is not invertible.

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}.$ 

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{bmatrix}$$

So the inverse is  $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}.$ 

**M1**:

**M2**:

**M3**:

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# Version 3

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Determine which of the six products AB, AC, BA, BC, CA, CB can be computed, and compute them.

**Solution:** CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

**M2.** Determine if the matrix  $\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}$  is invertible.

**Solution:** The second column is a multiple of the first, so it is not invertible.

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ .

Solution:

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & \frac{3}{2} & -\frac{3}{2} \\ 1 & -\frac{3}{2} & \frac{5}{2} \end{bmatrix}$$

**M1**:

**M2**:

**M3**:

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Version 4

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$C = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad \qquad D = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad \qquad E = \begin{bmatrix} 2 & 0 \\ 0 & -1 \\ 1 & -1 \end{bmatrix}$$

Determine which of the six products CD, CE, DC, DE, EC, ED can be computed, and compute them.

Solution:

$$EC = \begin{bmatrix} 4 & 6 \\ 0 & -1 \\ 2 & 2 \end{bmatrix}$$
$$DE = \begin{bmatrix} 6 & -1 \end{bmatrix}$$

M2. Determine if the matrix  $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$  is invertible.

**Solution:** This matrix is row equivalent to the identity matrix, so it is invertible.

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 6 & 0 & 1 \\ -14 & 3 & -4 \\ -23 & 4 & -6 \end{bmatrix}$ .

Solution:

$$\begin{bmatrix} 6 & 0 & 1 \\ -14 & 3 & -4 \\ -23 & 4 & -6 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 4 & -3 \\ 8 & -13 & 10 \\ 13 & -24 & 18 \end{bmatrix}$$

M1:

M2:

M3:

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Version 5

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine which of the six products AB, AC, BA, BC, CA, CB can be computed, and compute them.

**Solution:** AB and CA are the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$
$$CA = \begin{bmatrix} 3 & 9 & 11 \\ 0 & 0 & 7 \end{bmatrix}$$

**M2.** Determine if the matrix  $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$  is invertible.

**Solution:** The determinant is 0, so it is not invertible.

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}.$ 

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{bmatrix}$$

So the inverse is  $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}.$ 

M1:

M2:

M3:

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Version 6

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine which of the six products AB, AC, BA, BC, CA, CB can be computed, and compute them.

**Solution:** AB and CA are the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

$$CA = \begin{bmatrix} 3 & 9 & 11 \\ 0 & 0 & 7 \end{bmatrix}$$

**M2.** Determine if the matrix  $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$  is invertible.

**Solution:** This matrix is row equivalent to the identity matrix, so it is invertible.

**M3.** Compute the inverse of the matrix  $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$ 

Solution:

$$RREF(A|I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 2 & -11 & 37 \\ 0 & 1 & 0 & 0 & 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

So the inverse is  $\begin{bmatrix} 1 & 2 & -11 & 37 \\ 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 

**M1**:

**M2**:

**M3**: