Readiness Assurance Test

Choose the most appropriate response for each question.

31) Which of the following is a solution to the system of linear equations

$$\begin{aligned} x + 3y - z &= 2 \\ 2x + 8y + 3z &= -1 \\ -x - y + 9z &= -10 \end{aligned}$$

(a) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

32) Find a basis for the solution set of the following homogeneous system of linear equations

$$\begin{aligned} x + 2y + -z - w &= 0 \\ -2x - 4y + 3z + 5w &= 0 \end{aligned}$$

(a) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 5 \end{bmatrix} \right\}$

33) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It does not span $\mathbb{R}^3$ and is linearly dependent
(b) It spans $\mathbb{R}^3$ but it is linearly dependent
(c) It does not span $\mathbb{R}^3$ and is linearly independent
(d) It is a basis of $\mathbb{R}^3$.

34) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It does not span $\mathbb{R}^3$ and is linearly dependent
(b) It does not span $\mathbb{R}^3$ and is linearly independent
(c) It is a basis of $\mathbb{R}^3$.
(d) It spans $\mathbb{R}^3$ but it is linearly dependent

35) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It does not span $\mathbb{R}^3$ and is linearly dependent
(b) It spans $\mathbb{R}^3$ but it is linearly dependent
(c) It does not span $\mathbb{R}^3$ and is linearly independent
(d) It is a basis of $\mathbb{R}^3$.

36) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It spans $\mathbb{R}^3$ but it is linearly dependent
(b) It is a basis of $\mathbb{R}^3$.
(c) It does not span $\mathbb{R}^3$ and is linearly independent
(d) It does not span $\mathbb{R}^3$ and is linearly dependent

37) Suppose S is a set of $\mathbb{R}^5$ vectors, and you know that every vector in $\operatorname{span} S$ can be written *uniquely* as a linear combination of the vectors in S. What can you conclude about S?

(a) S has exactly 5 vectors
(b) S has at most 5 vectors
(c) S has at least 5 vectors
(d) S could have any number of vectors

38) Suppose you know that every vector in $\mathbb{R}^5$ can be written as a linear combination of the vectors in a set S. What can you conclude about S?

(a) S has exactly 5 vectors
(b) S has at most 5 vectors
(c) S has at least 5 vectors
(d) S could have any number of vectors

39) Suppose you know that every vector in $\mathbb{R}^5$ can be *uniquely* written as a linear combination of the vectors in a set S. What can you conclude about S?

(a) S has exactly 5 vectors
(b) S has at most 5 vectors
(c) S has at least 5 vectors
(d) S could have any number of vectors

40) What else can you conclude about S from the previous question?

(a) S is a basis of $\mathbb{R}^5$.

(b) S does not span $\mathbb{R}^5$ and is linearly dependent

(c) S does not span $\mathbb{R}^5$ and is linearly independent

(d) S spans $\mathbb{R}^5$ but it is linearly dependent