Application Activities - Module V Part 1 - Class Day 7

Activity 7.1 Consider each of the following vector properties. Label each property with $\mathbb{R}^1$, $\mathbb{R}^2$, and/or $\mathbb{R}^3$ if that property holds for Euclidean vectors/scalars $\mathbf{u}, \mathbf{v}, \mathbf{w}$ of that dimension.

1. **Addition associativity.**

   $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

2. **Addition commutivity.**

   $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.

3. **Addition identity.**

   There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

4. **Addition inverse.**

   There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

5. **Addition midpoint uniqueness.**

   There exists a unique $\mathbf{m}$ where the distance from $\mathbf{u}$ to $\mathbf{m}$ equals the distance from $\mathbf{m}$ to $\mathbf{v}$.

6. **Scalar multiplication associativity.**

   $a(b\mathbf{v}) = (ab)\mathbf{v}$.

7. **Scalar multiplication identity.**

   $1\mathbf{v} = \mathbf{v}$.

8. **Scalar multiplication relativity.**

   There exists some scalar c where either $c\mathbf{v} = \mathbf{w}$ or $c\mathbf{w} = \mathbf{v}$.

9. **Scalar distribution.**

   $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$.

10. **Vector distribution.**

    $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$.

11. **Orthogonality.**

    There exists a non-zero vector $\mathbf{n}$ such that $\mathbf{n}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

12. **Bidimensionality.**

    $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ for some value of a, b.

Definition 7.2 A **vector space** V is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ belong to V, and let a, b be scalar numbers.

- **Addition associativity.**

  $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

- **Addition commutivity.**

  $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.

- **Addition identity.**

  There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

- **Addition inverse.**

  There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

- **Scalar multiplication associativity.**

  $a(b\mathbf{v}) = (ab)\mathbf{v}$.

- **Scalar multiplication identity.**

  $1\mathbf{v} = \mathbf{v}$.

- **Scalar distribution.**

  $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$.

- **Vector distribution.**

  $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$.

Definition 7.3 The most important examples of vector spaces are the **Euclidean vector spaces** $\mathbb{R}^n$, but there are other examples as well.

Activity 7.4 Consider the following set that models motion along the curve $y = e^x$. Let $V = \{(x, y) : y = e^x\}$. Let vector addition be defined by $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 y_2)$, and let scalar multiplication be defined by $c \odot (x, y) = (cx, y^c)$.
Part 1: Which of the vector space properties are satisfied by V paired with these operations?

- **Addition associativity.**
  $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$.
- **Addition commutivity.**
  $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$.
- **Addition identity.**
  There exists some $\mathbf{0}$ where $\mathbf{v} \oplus \mathbf{0} = \mathbf{v}$.
- **Addition inverse.**
  There exists some $-\mathbf{v}$ where $\mathbf{v} \oplus (-\mathbf{v}) = \mathbf{0}$.
- **Scalar multiplication associativity.**
  $a \odot (b \odot \mathbf{v}) = (ab) \odot \mathbf{v}$.
- **Scalar multiplication identity.**
  $1 \odot \mathbf{v} = \mathbf{v}$.
- **Scalar distribution.**
  $a \odot (\mathbf{u} \oplus \mathbf{v}) = (a \odot \mathbf{u}) \oplus (a \odot \mathbf{v})$.
- **Vector distribution.**
  $(a + b) \odot \mathbf{v} = (a \odot \mathbf{v}) \oplus (b \odot \mathbf{v})$.

Part 2: Is V a vector space?