

Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Add Euclidean vectors and multiply Euclidean vectors by scalars.
- Perform basic manipulations of augmented matrices and linear systems (**Standard(s) E1,E2,E3**).
- Apply linear combinations and spanning sets (**Standard(s) V2,V3**).

Readiness Assurance Resources

The following resources will help you prepare for this module.

- <https://www.khanacademy.org/math/precalculus/vectors-precalc/vector-addition-subtraction/v/adding-and-subtracting-vectors>
- <https://www.khanacademy.org/math/precalculus/vectors-precalc/combined-vector-operations/v/combined-vector-operations-example>

Readiness Assurance Test

Choose the most appropriate response for each question.

- 1) Simplify the following vector expression.

$$4 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

- 2) Express the following system of linear equations as an augmented matrix.

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \\ -3x_1 + 4x_2 + x_3 &= -7 \end{aligned}$$

(a) $\left[\begin{array}{ccc|c} 2 & 1 & -3 & -3 \\ 1 & 1 & 4 & 4 \\ 4 & 1 & 1 & 1 \\ 0 & 1 & -7 & -7 \end{array} \right]$

(b) $\left[\begin{array}{ccc|c} 2 & 1 & 4 & 0 \\ 1 & 1 & 1 & 1 \\ -3 & 4 & 1 & -7 \end{array} \right]$

(c) $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & -2 & 4 \\ 4 & 1 & 1 \\ 0 & 1 & -7 \end{array} \right]$

(d) $\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & 1 & 1 \\ -3 & 4 & -7 \end{array} \right]$

- 3) Find RREF $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 3 & 2 & 5 \\ -2 & 0 & -2 \end{array} \right]$.

(a) $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$

(b) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 0 & 0 & 0 \end{array} \right]$

(c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$

(d) $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$

- 4) Solve the following system of linear equations.

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \\ -3x_1 + 4x_2 + x_3 &= -7 \end{aligned}$$

(a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + a \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ for all real numbers a

(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 1 \end{bmatrix}$

(d) No solutions

5) Solve the following system of linear equations.

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 3x_2 + x_4 &= 0 \end{aligned}$$

- (a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ 0 \\ 0 \end{bmatrix} + a \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ for all real numbers a, b
- (b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \\ -5 \end{bmatrix} + a \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ for all real numbers a (d) No solutions

6) How many vectors are required to span all of \mathbb{R}^4 (the space of Euclidean vectors with four components)?

- (a) 2 (b) 3 (c) 4 (d) 5

7) How many vectors are required to span all of \mathcal{P}^4 (the space of polynomials of degree four or less)?

- (a) 2 (b) 3 (c) 4 (d) 5

8) Which vector is a linear combination of $\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$?

- (a) $\begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 3 \\ -7 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} -5 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

9) Which vector belongs to $\text{span} \left\{ \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$?

- (a) $\begin{bmatrix} 3 \\ -7 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

10) The graphical representation of $\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ in three-dimensional Euclidean space \mathbb{R}^3 would be which of the following?

(a) a line

(b) a plane

(c) a sphere

(d) all of \mathbb{R}^3