Name:

J#:

Date:

Dr. Clontz

**MASTERY QUIZ DAY 12**  Math 237 – Linear Algebra
**Version 1**  Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector addition $\oplus$ is associative.

(b) Determine if $V$ is a vector space or not. Justify your answer.

**Solution:** Let $x, y, z \in \mathbb{R}$. Then

$$
\begin{aligned}
(x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\
&= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\
&= \sqrt{x^2 + y^2 + z^2} \\
&= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\
&= x \oplus \sqrt{y^2 + z^2} \\
&= x \oplus (y \oplus z)
\end{aligned}
$$

However, this is not a vector space, as there is no zero vector.

☐

| **Standard V3.** | Mark: |
|---|---|

Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span $\mathbb{R}^3$.

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span $\mathbb{R}^3$.

☐

| **Standard V4.** | Mark: |

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

**Solution:** This set is closed under addition, but not under scalar multiplication so it is not a subspace.

$\square$

| **Additional Notes/Marks** | |

**MASTERY QUIZ DAY 12**                                    Math 237 – Linear Algebra
**Version 2**                                                                   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
| --- | --- |

Let $V$ be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here $f'$ denotes the derivative of $f$).

(a) Show that this scalar multiplication $\odot$ distributes over vector addition $\oplus$.

(b) Determine if $V$ is a vector space or not. Justify your answer.

**Solution:** Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'\,' + cg'\,' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial $f$.

$\square$

| **Standard V3.** | Mark: |
| --- | --- |

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

**Solution:** Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

$\square$

| **Standard V4.** | Mark: |
| --- | --- |

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

**Solution:** This set is closed under addition, but not under scalar multiplication so it is not a subspace.
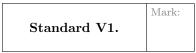
$\square$

| **Additional Notes/Marks** | |
| --- | --- |

**MASTERY QUIZ DAY 12**                                    Math 237 – Linear Algebra
**Version 3**                                                              Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show
all relevant work to receive credit for a standard.

| | Mark: |
|---|---|
| **Standard V1.** | |

Let $V$ be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and
$c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

(a) Show that this scalar multiplication $\odot$ is associative.

(b) Determine if $V$ is a vector space or not. Justify your answer

**Solution:** Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned}
c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\
&= c\,(dx - 3(d - 1)) - 3(c - 1) \\
&= cdx - 3(cd - 1) \\
&= (cd) \odot x
\end{aligned}$$

We verify the remaining 7 properties to see that $V$ is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of $x$.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c - 1) \\
&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}$$

8)

$$(c + d) \odot x = (c + d)x - 3(c + d - 1)$$
$$= cx - 3(c - 1) + dx - 3(c - 1) - 3$$
$$= (c \odot x) \oplus (d \odot x)$$

Therefore $V$ is a vector space.

☐

| | Mark: |
|---|---|
| **Standard V3.** | |

Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span $\mathbb{R}^4$.

**Solution:**

$$\text{RREF}\left( \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span $\mathbb{R}^4$.

☐

| | Mark: |
|---|---|
| **Standard V4.** | |

Let $W$ be the set of all polynomials of even degree. Determine if $W$ is a subspace of the vector space of all polynomials.
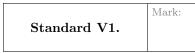
**Solution:** $W$ is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and $x^2$ are both in $W$, but $(x - x^2) + (x^2) = x \notin W$.

☐

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|---|---|
| **Additional Notes/Marks** | |

**MASTERY QUIZ DAY 12**  Math 237 – Linear Algebra
**Version 4**  Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

(a) Show that this scalar multiplication $\odot$ is associative.

(b) Determine if $V$ is a vector space or not. Justify your answer

**Solution:** Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned}
c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\
&= c(dx - 3(d - 1)) - 3(c - 1) \\
&= cdx - 3(cd - 1) \\
&= (cd) \odot x
\end{aligned}$$

We verify the remaining 7 properties to see that $V$ is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of $x$.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c - 1) \\
&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}$$

8)

$$(c + d) \odot x = (c + d)x - 3(c + d - 1)$$
$$= cx - 3(c - 1) + dx - 3(c - 1) - 3$$
$$= (c \odot x) \oplus (d \odot x)$$

Therefore $V$ is a vector space.

□

| | Mark: |
|---|---|
| **Standard V3.** | |

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

**Solution:** Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

□

| | Mark: |
|---|---|
| **Standard V4.** | |

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

**Solution:** This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

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|---|---|
| **Additional Notes/Marks** | |

**MASTERY QUIZ DAY 12**                                    Math 237 – Linear Algebra
**Version 5**                                                          Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
| --- | --- |

Let $V$ be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

(a) Show that this scalar multiplication $\odot$ is associative.

(b) Determine if $V$ is a vector space or not. Justify your answer

**Solution:** Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$
$$= c\,(dx - 3(d - 1)) - 3(c - 1)$$
$$= cdx - 3(cd - 1)$$
$$= (cd) \odot x$$

We verify the remaining 7 properties to see that $V$ is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of $x$.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$
$$= c(x + y - 3) - 3(c - 1)$$
$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$
$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c + d) \odot x = (c + d)x - 3(c + d - 1)$$
$$= cx - 3(c - 1) + dx - 3(c - 1) - 3$$
$$= (c \odot x) \oplus (d \odot x)$$

Therefore $V$ is a vector space.

$\square$

| Standard V3. | Mark: |
|---|---|

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

**Solution:** Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

$\square$

| Standard V4. | Mark: |
|---|---|

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.
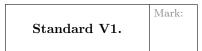
**Solution:** This set is closed under addition, but not under scalar multiplication so it is not a subspace.

$\square$

| Additional Notes/Marks | |
|---|---|

**MASTERY QUIZ DAY 12**                          Math 237 – Linear Algebra
**Version 6**                                              Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication $\odot$ distributes over scalar addition.

(b) Determine if $V$ is a vector space or not. Justify your answer.

**Solution:** Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c + d) \odot (x_1, y_1) = (0, (c + d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, $V$ is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

$\square$

| **Standard V3.** | Mark: |
|---|---|

Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span $\mathbb{R}^4$.

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span $\mathbb{R}^4$.

$\square$

| **Standard V4.** | Mark: |
|---|---|

Let $W$ be the set of all polynomials of even degree. Determine if $W$ is a subspace of the vector space of all polynomials.

**Solution:** $W$ is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and $x^2$ are both in $W$, but $(x - x^2) + (x^2) = x \notin W$.

□

| **Additional Notes/Marks** | |
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