

Name: _____

MASTERY QUIZ DAY 12

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

- (a) Show that the vector addition \oplus is associative.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$\begin{aligned}(x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\&= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\&= \sqrt{x^2 + y^2 + z^2} \\&= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\&= x \oplus \sqrt{y^2 + z^2} \\&= x \oplus (y \oplus z)\end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

V3. Determine if the vectors $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ span \mathbb{R}^3

Solution:

$$\text{RREF} \left(\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span \mathbb{R}^3 .

□

V4. Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

V1:

V3:

V4: