

## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Add Euclidean vectors and multiply Euclidean vectors by scalars.
- Add complex numbers and multiply complex numbers by scalars.
- Add polynomials and multiply polynomials by scalars.
- Perform basic manipulations of augmented matrices and linear systems (**Standard(s) E1,E2,E3**).

## Readiness Assurance Resources

The following resources will help you prepare for this module.

- <https://www.khanacademy.org/math/precalculus/vectors-prec calc/vector-addition-subtraction/v/adding-and-subtracting-vectors>
- <https://www.khanacademy.org/math/precalculus/vectors-prec calc/combined-vector-operations/v/combined-vector-operations-example>
- <https://www.khanacademy.org/math/precalculus/imaginary-and-complex-numbers/adding-and-subtracting-v/adding-complex-numbers>
- <https://www.khanacademy.org/math/algebra/introduction-to-polynomial-expressions/adding-and-subtracting-polynomials-1>

**Readiness Assurance Test**

Choose the most appropriate response for each question.

- 11) Simplify the following Euclidean vector expression.

$$2 \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

(a)  $\begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$

(b)  $\begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$

(c)  $\begin{bmatrix} 6 \\ -8 \\ -3 \end{bmatrix}$

(d)  $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

- 12) Simplify the following Euclidean vector expression.

$$2 \left( \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \right)$$

(a)  $\begin{bmatrix} 6 \\ -8 \\ -3 \end{bmatrix}$

(b)  $\begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$

(c)  $\begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$

(d)  $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

- 13) Simplify the complex number expression  $-4(3 - 2i) + 2(5 + i)$ .

(a)  $-2 + 10i$

(b)  $3 - 7i$

(c)  $4 + i$

(d)  $-1 - 5i$

- 14) Which of these complex numbers might be represented by the following Euclidean vector plotted on the complex plane (where the horizontal axis gives the real part and the vertical axis gives the imaginary part)?



(a)  $5 + i$

(b)  $-3 - 9i$

(c)  $-2 + 3i$

(d)  $4i$

- 15) Simplify  $3f(x) - 2g(x)$  where  $f(x) = 7 - x^2$  and  $g(x) = 2x^3 + x - 1$ .

(a)  $-4x^3 - 3x^2 - 2x + 23$

(b)  $x^3 + 4x - 5$

(c)  $3x^3 + 5x^2 - 3x + 17$

(d)  $-x^3 + 19x^2 - 4$

- 16) Express the following system of linear equations as an augmented matrix.

$$\begin{aligned} x_1 + 2x_2 - x_4 &= 3 \\ x_3 + 4x_4 &= -2 \end{aligned}$$

(a) 
$$\left[ \begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 4 \\ -1 & 4 & 3 \\ -2 & 3 & 3 \end{array} \right]$$

(b) 
$$\left[ \begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -1 & 3 \\ 3 & 0 & 0 \\ 0 & 1 & 4 \\ 4 & -2 & -2 \end{array} \right]$$

(c) 
$$\left[ \begin{array}{cccc|c} 1 & 2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 4 & -2 \end{array} \right]$$

(d) 
$$\left[ \begin{array}{cccc|c} 1 & 2 & 1 & 4 & 3 \\ -2 & 1 & 3 & 4 & 5 \end{array} \right]$$

17) Which of the following matrices is equivalent to the following matrix?

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 4 & -1 & 2 \\ 2 & 3 & 2 & 3 \end{array} \right]$$

(Hint: The correct answer was obtained from a single row operation.)

(a) 
$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 4 & -1 & 2 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

(b) 
$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 4 & -1 & 2 \\ 0 & -1 & -4 & 5 \end{array} \right]$$

(c) 
$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 1 & 3 & 4 & 3 \\ 2 & 3 & 2 & 3 \end{array} \right]$$

(d) 
$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & 4 \\ 2 & 3 & 2 & 3 \end{array} \right]$$

18) Find RREF  $\left[ \begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 4 & -1 \\ 2 & 3 & 2 \end{array} \right]$ .

(a) 
$$\left[ \begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array} \right]$$

(b) 
$$\left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

(c) 
$$\left[ \begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 0 & 0 & 0 \end{array} \right]$$

(d) 
$$\left[ \begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$$

19) Solve the following system of linear equations.

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \\ -3x_1 + 4x_2 + x_3 &= -7 \end{aligned}$$

(a) 
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 1 \end{bmatrix}$$

(c) 
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + a \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \text{ for all real numbers } a$$

(b) 
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

(d) No solutions

20) Solve the following system of linear equations.

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 0 \\ x_1 + x_2 + x_3 &= 0 \end{aligned}$$

$$(a) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -5 \end{bmatrix}$$

$$(b) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$(c) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = a \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} \text{ for all real numbers } a$$

(d) No solutions