

Name: _____

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_4 &= -1\end{aligned}$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -1 & 0 & 7 \\ 1 & -1 & 0 & 3 & -1 \end{array} \right]$$

□

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 + 3x_2 - 5x_3 + 14x_4 &= 8 \\x_1 + x_2 - x_3 + 5x_4 &= 3\end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & 3 & -5 & 14 & 8 \\ 1 & 1 & -1 & 5 & 3 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array} \right]$. It follows that the solution set is given by $\begin{bmatrix} 2 - 2a - b \\ 2 + 3a - 4b \\ a \\ b \end{bmatrix}$ for all real numbers a, b .

□

E4. Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 - 2x_2 + 6x_3 - x_4 &= 0 \\3x_1 + 6x_3 + x_4 &= 0 \\-4x_1 + x_2 - 9x_3 + 2x_4 &= 0\end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

□

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

Determine if V is a vector space or not.

Solution: This is not a vector space, as there is no zero vector.

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E1:

E3:

E4:

V1:

E2: