Name: ______________________________

FINAL EXAM Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\ 3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 + x_4 &= 1 \end{aligned}$$

E2. Put the following matrix in reduced row echelon form.

$$\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array}\right]$$

E3. Solve the following linear system.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

E4. Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 0 \\ -2x_3 - 4x_4 &= 0 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= 0 \end{aligned}$$

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 5 \\ 2 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 8 \\ 3 \\ 5 \\ -1 \end{bmatrix}$.

V3. Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span $\mathbb{R}^4$.

V4. Let W be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 1$ (this forms a plane). Determine if W is a subspace of $\mathbb{R}^3$.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

S2. Determine if the set $\left\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\right\}$ is a basis of $\mathcal{P}^3$.

S3. Let $W = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$. Find a basis for this vector space.

S4. Let $W = \operatorname{span}\left\{2x^2 - x + 3, 2x^2 + 2, -x^2 + 4x + 1\right\}$. Find the dimension of W.

A1. Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - 5x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^4$ and $\mathbb{R}^2$.

A2. Determine if the map $T : \mathcal{P}^6 \to \mathcal{P}^7$ given by $T(f) = xf(x) - f(1)$ is a linear transformation or not.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ where $S(\vec{e}_1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $S(\vec{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ where $T(\vec{e}_1) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $T(\vec{e}_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $T(\vec{e}_3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

A4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T.

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M2. Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ is invertible.

M3. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

G1. Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 9 & -3 & 2 \\ 19 & -6 & 5 \\ -11 & 4 & -2 \end{bmatrix}$.

G3. Find the eigenspace associated to the eigenvalue 2 in the matrix $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$.

G4. Compute the geometric multiplicity of the eigenvalue 2 in the matrix $A = \begin{bmatrix} 8 & -3 & 2 \\ 15 & -5 & 5 \\ -3 & 2 & 1 \end{bmatrix}$

E1:

E2:

E3:

E4:

V1:

V2:

V3:

V4:

S1:

S2:

S3:

S4:

A1:

A2:

A3:

A4:

M1:

M2:

M3:

G1:

G2:

G3:

G4: