

Application Activities - Module V Part 4 - Class Day 10

Fact 10.1 At least n vectors are required to span \mathbb{R}^n .



Activity 10.2 Choose a vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ in \mathbb{R}^3 that is not in $\text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ by ensuring $\left[\begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$. (Why does this work?)

Fact 10.3 The set $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$ fails to span all of \mathbb{R}^n exactly when $\text{RREF}[\mathbf{v}_1 \dots \mathbf{v}_m]$ has a row of zeros:

$$\left[\begin{array}{cc} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{array} \right] \sim \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Activity 10.4 Consider the set of vectors $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}$. Does $\mathbb{R}^4 = \text{span } S$?

Activity 10.5 Consider the set of third-degree polynomials

$$S = \{2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, -x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\}.$$

Does $\mathcal{P}^3 = \text{span } S$?

Definition 10.6 A subset of a vector space is called a **subspace** if it is itself a vector space.

Fact 10.7 If S is a subset of a vector space V , then $\text{span } S$ is a subspace of V .

Remark 10.8 To prove that a subset is a subspace, you need only verify that $c\mathbf{v} + d\mathbf{w}$ belongs to the subset for any choice of vectors \mathbf{v}, \mathbf{w} from the subset and any real scalars c, d .

Activity 10.9 Prove that $P = \{ax^2 + b : a, b \text{ are both real numbers}\}$ is a subspace of the vector space of all degree-two polynomials by showing that $c(a_1x^2 + b_1) + d(a_2x^2 + b_2)$ belongs to P .

Activity 10.10 Consider the subset of \mathbb{R}^2 where at least one coordinate of each vector is 0.



Find a linear combination $c\mathbf{v} + d\mathbf{w}$ that does not belong to this subset.

Fact 10.11 Suppose a subset S of V is isomorphic to another vector space W . Then S is a subspace of V .

Activity 10.12 Show that the set of 2×2 matrices

$$S = \left\{ \begin{bmatrix} a & b \\ -b & -a \end{bmatrix} : a, b \text{ are real numbers} \right\}$$

is a subspace of $\mathbb{R}^{2 \times 2}$ by identifying a Euclidean space isomorphic to S .
