

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 10**

Math 237 – Linear Algebra

**Version 6**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array} \right]$$

**Solution:**

$$\begin{aligned} 3x_1 - x_2 + x_4 &= 5 \\ -x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 &= -3 \end{aligned}$$

□

**E3.** Solve the system of equations

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 2 \end{aligned}$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 2 \end{array} \right] \right) = \left[ \begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

So the solution set is

$$\left\{ \begin{bmatrix} 1 - 3c \\ c \\ -1 \end{bmatrix} \mid c \in \mathbb{R} \right\}$$

□

**E4.** Find a basis for the solution set of the system of equations

$$\begin{aligned} x + 3y + 3z + 7w &= 0 \\ x + 3y - z - w &= 0 \\ 2x + 6y + 3z + 8w &= 0 \\ x + 3y - 2z - 3w &= 0 \end{aligned}$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

□

**V1.** Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c - 1) \end{aligned}$$

Determine if  $V$  is a vector space or not.

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ .

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6 - x) = x + (6 - x) - 3 = 3$ , so  $6 - x$  is the additive inverse of  $x$ .
- 4) Real addition is commutative, so  $\oplus$  is commutative.
- 5)

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\ &= c(dx - 3(d - 1)) - 3(c - 1) \\ &= cdx - 3(cd - 1) \\ &= (cd) \odot x \end{aligned}$$

6)  $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c - 1) \\ &= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned}(c+d) \odot x &= (c+d)x - 3(c+d-1) \\ &= cx - 3(c-1) + dx - 3(d-1) - 3 \\ &= (c \odot x) \oplus (d \odot x)\end{aligned}$$

Therefore  $V$  is a vector space.

□

**E1:**

**E3:**

**E4:**

**V1:**

**E2:**