

Name: _____

MIDTERM EXAM

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array} \right]$$

E2. Put the following matrix in reduced row echelon form.

$$\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right]$$

E3. Solve the following linear system.

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 5$$

$$-2x_3 - 4x_4 = 3$$

$$2x_1 + 2x_2 + x_3 - 4x_4 = -1$$

E4. Find a basis for the solution set to the homogeneous system of equations

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 0$$

$$-2x_3 - 4x_4 = 0$$

$$2x_1 + 2x_2 + x_3 - 4x_4 = 0$$

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

- (a) Show that the vector **addition** \oplus is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.
- (b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$.

V3. Determine if the vectors $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Let W be the set of all complex numbers that are purely real (i.e. of the form $a + 0i$) or purely imaginary (i.e. of the form $0 + bi$). Determine if W is a subspace of \mathbb{C} .

S1. Determine if the set of matrices $\left\{ \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -8 \\ 6 & 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

S2. Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}_3

S3. Let W be the subspace of \mathcal{P}_3 given by
 $W = \text{span}(\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\})$. Find a basis for W .

S4. Let $W = \text{span}\left(\left\{\begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}\right\}\right)$. Compute the dimension of W .

E1:	<input type="text"/>	E2:	<input type="text"/>	E3:	<input type="text"/>	E4:	<input type="text"/>	V1:	<input type="text"/>	V2:	<input type="text"/>	V3:	
<input type="text"/>		V4:	<input type="text"/>	S1:	<input type="text"/>	S2:	<input type="text"/>	S3:	<input type="text"/>	S4:	<input type="text"/>		