

Name:
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Dr. Clontz

## MASTERY QUIZ DAY 14

Math 237 – Linear Algebra

### Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$\begin{aligned} f \oplus g &= f' + g' \\ c \odot f &= cf' \end{aligned}$$

(here  $f'$  denotes the derivative of  $f$ ).

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot (f \oplus g) = c \odot f \oplus c \odot g$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial  $f$ .

□

<b>Standard V3.</b>	Mark:
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Does  $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$ ?

**Solution:** Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 4 & 2 \\ 4 & -9 & -3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

has a zero row, the vectors fail to span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all complex numbers that are purely real (i.e. of the form  $a + 0i$ ) or purely imaginary (i.e. of the form  $0 + bi$ ). Determine if  $W$  is a subspace of  $\mathbb{C}$ .

**Solution:** No, because 1 is purely real and  $i$  is purely imaginary, but the linear combination  $1 + i$  is neither. □

<b>Standard S2.</b>	Mark:
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Determine if the set  $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$  is a basis of  $\mathcal{P}_3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis. □

<b>Additional Notes/Marks</b>	
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