

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 10**

Math 237 – Linear Algebra

**Version 5**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

**Solution:**

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 4x_2 + x_3 &= -7 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

□

**E3.** Solve the following linear system.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$ . It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction  $0 = 1$ ).

□

**E4.** Find a basis for the solution set to the homogeneous system of equations given by

$$\begin{aligned} 3x + 2y + z &= 0 \\ x + y + z &= 0 \end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right]$ . It follows that the basis for the solution set is given by  $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ .

□

**V1.** Let  $V$  be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$\begin{aligned} (x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2, y_1 + y_2) \\ c \odot (x_1, y_1) &= (c^2 x_1, c^3 y_1) \end{aligned}$$

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1), (x_2, y_2) \in V$  and let  $c \in \mathbb{R}$ .

$$\begin{aligned} c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + x_2, y_1 + y_2) \\ &= (c^2(x_1 + x_2), c^3(y_1 + y_2)) \\ &= (c^2x_1, c^3y_1) \oplus (c^2x_2, c^3y_2) \\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2) \end{aligned}$$

However,  $V$  is not a vector space, as the other distributive law fails:

$$(c + d) \odot (x_1, y_1) = ((c + d)^2x_1, (c + d)^3y_1) \neq ((c^2 + d^2)x_1, (c^3 + d^3)y_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

□

**E1:**

**E3:**

**E4:**

**V1:**

**E2:**