

**Section E.2****Activity E.21** ( $\sim 8$  min) Consider the matrix

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & -1 & 4 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

- (a) Add row 3 to row 2 ( $R_2 + R_3 \rightarrow R_2$ )
- (b) Add row 2 to row 3 ( $R_3 + R_2 \rightarrow R_3$ )
- (c) Add -1 row 1 to row 2 ( $R_2 - R_1 \rightarrow R_2$ )
- (d) Add -2 row 1 to row 2 ( $R_2 - 2R_1 \rightarrow R_2$ )

**Activity E.22** ( $\sim 7$  min) Consider the matrix

$$\begin{bmatrix} 2 & 5 & -1 \\ 3 & 5 & 1 \\ 1 & -2 & 0 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

- (a) Add row 1 to row 2 ( $R_2 + R_1 \rightarrow R_2$ )
- (b) Add -1 row 1 to row 2 ( $R_2 - R_1 \rightarrow R_2$ )
- (c) Swap rows 1 and 3 ( $R_1 \leftrightarrow R_3$ )
- (d) Multiply row 1 by  $\frac{1}{2}$  ( $\frac{1}{2}R_1 \rightarrow R_1$ )
- (e) Add -1 row 3 to row 1 ( $R_1 - R_3 \rightarrow R_1$ )

**Activity E.23** ( $\sim 5$  min) Consider the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

- (a) Add row 1 to row 2 ( $R_2 + R_1 \rightarrow R_2$ )
- (b) Add -1 row 1 to row 2 ( $R_2 - R_1 \rightarrow R_2$ )
- (c) Add -1 row 3 to row 1 ( $R_1 - R_3 \rightarrow R_1$ )
- (d) Add -2 row 3 to row 2 ( $R_2 - 2R_3 \rightarrow R_2$ )
- (e) Multiply row 3 by  $\frac{1}{2}$  ( $\frac{1}{2}R_3 \rightarrow R_3$ )

**Activity E.24** ( $\sim 10$  min) Consider the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & -1 & 1 \end{bmatrix}.$$

Perform three row operations to produce a matrix closer to RREF.

**Activity E.25** ( $\sim 10$  min) Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ -1 & 3 & -5 \end{bmatrix}.$$

Compute  $\text{RREF}(A)$ .

**Activity E.26** ( $\sim 10$  min) Consider the matrix

$$A = \begin{bmatrix} 2 & 4 & 2 & -4 \\ -2 & -4 & 1 & 1 \\ 3 & 6 & -1 & -4 \end{bmatrix}.$$

Compute  $\text{RREF}(A)$ .

**Remark E.27** A video example of how to perform the Gauss-Jordan Elimination algorithm by hand is available at <https://youtu.be/Cq0Nxxk2dhhU>.

Practicing several exercises on your own using this method is strongly recommended.

**Activity E.28** (*~10 min*) Free browser-based technologies for mathematical computation are available online.

- Go to <https://octave-online.net>.
- Type `A=sym([1 3 4 ; 2 5 7])` and press **Enter** to store the matrix  $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$  in the variable  $A$ .
  - The symbolic function `sym` is used to calculate precise answers rather than floating-point approximations.
  - The vertical bar in an augmented matrix does not affect row operations, so the RREF of  $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$  may be computed in the same way.
- Type `rref(A)` and press **Enter** to compute the reduced row echelon form of  $A$ .

**Remark E.29** We will frequently need to know the reduced row echelon form of matrices during class, so feel free to use Octave-Online.net to compute RREF efficiently.

You may alternatively use the calculator you will use during assessments. Be sure to use fractions mode to compute exact solutions rather than floating-point approximations.

**Activity E.30** (*~10 min*) Consider the system of equations.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 + 3x_2 - 6x_3 &= 11 \end{aligned}$$

*Part 1:* Convert this to an augmented matrix and use technology to compute its reduced row echelon form:

$$\text{RREF} \left[ \begin{array}{ccc|c} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{array} \right] = \left[ \begin{array}{ccc|c} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{array} \right]$$

*Part 2:* Use the RREF matrix to write a linear system equivalent to the original system. Then find its solution set.