Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span $\mathbb{R}^4$.

□

Standard V4.	Mark:

Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \middle|\, x, y, z \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^4$.

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Additional Notes/Marks	

Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\\2\\1\end{bmatrix}, \begin{bmatrix}-1\\3\\5\\2\\0\end{bmatrix}, \begin{bmatrix}1\\0\\5\\1\\-3\end{bmatrix}\right\} = \mathbb{R}^5$?

Solution: Since there are only three vectors, they cannot span $\mathbb{R}^5$.

□

Standard V4.	Mark:

Let W be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix}x\\y\\z\end{bmatrix}$ satisfying $x+y+z=0$ (this forms a plane). Determine if W is a subspace of $\mathbb{R}^3$.

Solution: Yes, because $z=-x-y$ and $a\begin{bmatrix}x_1\\y_1\\-x_1-y_1\end{bmatrix}+b\begin{bmatrix}x_2\\y_2\\-x_2-y_2\end{bmatrix}=\begin{bmatrix}ax_1+bx_2\\ay_1+by_2\\-(ax_1+bx_2)-(ay_1+by_2)\end{bmatrix}$. Alternately, yes because W is isomorphic to $\mathbb{R}^2$.

□

Standard S2.	Mark:

Determine if the set $\{x^2+x-1, 3x^2-x+1, 2x-2\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

Name:	
J#:	
Date:	

Dr. Clontz

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra

Version 3 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\3\\6\\3\end{bmatrix}$, $\begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}$, and $\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1 & 3 & 3 & 7\\ 1 & 3 & -1 & -1\\ 2 & 6 & 3 & 8\\ 1 & 3 & -2 & -3\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Since there are zero rows, they do not span. Alternatively, by inspection $\begin{bmatrix}3\\3\\6\\3\end{bmatrix} = 3\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, so the set is linearly dependent, so it spans a subspace of dimension at most 3, therefore it does not span $\mathbb{R}^4$.

□

Standard V4.	Mark:

Let W be the set of all complex numbers that are purely real (i.e of the form $a + 0i$) or purely imaginary (i.e. of the form $0 + bi$). Determine if W is a subspace of $\mathbb{C}$.

Solution: No, because 1 is purely real and i is purely imaginary, but the linear combination $1 + i$ is neither.

□

Standard S2.	Mark:

Determine if the set $\left\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\right\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1 & 3 & 2\\ 1 & -1 & 0\\ -1 & 1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Solution: Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

$\square$

Standard V4.	Mark:

Let W be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 1$ (this forms a plane). Determine if W is a subspace of $\mathbb{R}^3$.

Solution: No, because $\mathbf{0}$ does not belong to W.

$\square$

Standard S2.	Mark:

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Additional Notes/Marks	

Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 5 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5$?

Solution: Since there are only three vectors, they cannot span $\mathbb{R}^5$.

☐

Standard V4.	Mark:

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

☐

Standard S2.	Mark:

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

☐

Additional Notes/Marks	

Name:	
J#:	
Date:	

Dr. Clontz

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix},\begin{bmatrix}3\\12\\-9\end{bmatrix},\begin{bmatrix}1\\4\\-3\end{bmatrix},\begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}=\mathbb{R}^3$?

Solution: Since

$$\text{RREF}\begin{bmatrix}2&3&1&-4\\-1&12&4&2\\4&-9&-3&-8\end{bmatrix}=\begin{bmatrix}1&0&0&-2\\0&1&1/3&0\\0&0&0&0\end{bmatrix}$$

has a zero row, the vectors fail to span $\mathbb{R}^3$.

$\square$

Standard V4.	Mark:

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

$\square$

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix}1\\1\\-1\end{bmatrix},\begin{bmatrix}3\\-1\\1\end{bmatrix},\begin{bmatrix}2\\0\\-2\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix}1&3&2\\1&-1&0\\-1&1&-2\end{bmatrix}\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Additional Notes/Marks	