Name:	
J#:	Dr. Clontz
Date:	

Math 237 - Linear Algebra

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Mark: Standard A1.

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 + 3x_1 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

Solution:

Version 1

 $[3 \ 0 \ 1]$ 

Standard A2.

Determine if  $D: \mathbb{R}^{2\times 2} \to \mathbb{R}$  given by  $D\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$  is a linear transformation or not.

**Solution:** D(I) = 1 but  $D(2I) = 4 \neq 2D(I)$ , so D is not linear.

Mark: Standard M1.

Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

**Solution:** CB is the only one that can be computed, and

$$CB = \begin{bmatrix} 3 & 3 & -5 & 7 \\ 4 & -4 & 12 & -12 \\ 7 & 2 & 0 & 3 \end{bmatrix}$$

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Mark: Standard A1.

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

Mark:

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

Solution:

Version 2

 $\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$ 

Standard A2.

Determine if the map  $T: \mathcal{P}^6 \to \mathcal{P}^7$  given by T(f) = xf(x) - f(1) is a linear transformation or not.

Mark: Standard M1.

Let

 $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & -1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ 

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

**Solution:** AC is the only one that can be computed, and

$$AC = \begin{bmatrix} 9 & -2 & 14 \\ 1 & 0 & 2 \end{bmatrix}$$

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Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

Solution:

Version 3

 $\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$ 

Standard A2.

Mark:

Determine if the map  $T: \mathcal{P}^3 \to \mathcal{P}^4$  given by T(f(x)) = xf(x) - f(x) is a linear transformation or not.

Standard M1.

Mark:

Let

 $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ 

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

**Solution:** AB is the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

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Version 4

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Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}^4$  be the linear transformation given by

Mark:

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 7x + 2y + 3z \\ 0 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 7 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Standard A2.

Determine if  $D: \mathbb{R}^{2\times 2} \to \mathbb{R}$  given by  $D\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a - 3c$  is a linear transformation or not.

Standard M1.

Mark:

Let

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad \qquad B = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad \qquad C = \begin{bmatrix} 3 & -1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

**Solution:** AC is the only one that can be computed, and

$$AC = \begin{bmatrix} 9 & -2 & 14 \\ 1 & 0 & 2 \end{bmatrix}$$

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Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Let  $T: \mathbb{R}^3 \to \mathbb{R}^4$  be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

Solution:

Version 5

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 7 & 0 & 0 \end{bmatrix}$$

Standard A2.

Mark:

Determine if  $T: \mathbb{R}^2 \to \mathbb{R}^2$  given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

**Solution:** It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

Standard M1.

Mark:

Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

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# Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Let  $T: \mathbb{R}^4 \to \mathbb{R}^2$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\\x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3\\3x_2 - 5x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of  $\mathbb{R}^4$  and  $\mathbb{R}^2$ .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -5 & 0 \end{bmatrix}$$

Standard A2.

Determine if  $T: \mathbb{R}^2 \to \mathbb{R}^2$  given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

Standard M1.

Mark:

Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CB is the only one that can be computed, and

$$CB = \begin{bmatrix} 3 & 3 & -5 & 7 \\ 4 & -4 & 12 & -12 \\ 7 & 2 & 0 & 3 \end{bmatrix}$$

 ${\bf Additional\ Notes/Marks}$