Name: ________________________________

**MASTERY QUIZ DAY 25** Math 237 – Linear Algebra
**Version 5** Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A3.** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

(b) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x+2y \\ x-y \\ x+4y \end{bmatrix}$

**Solution:**

(a)

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, $T$ is not injective. Since there is a zero row, $T$ is not surjective.

(b)

$$\mathrm{RREF}\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since all columns are pivot columns, $S$ is injective. Since there is a zero row, $S$ is not surjective.

□

**A4.** Let $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} a & b \\ x & y \end{bmatrix}\right) = \begin{bmatrix} a+x \\ 0 \\ b+y \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

**Solution:** Rewrite as $T'\left(\begin{bmatrix} a \\ b \\ x \\ y \end{bmatrix}\right) = \begin{bmatrix} a+x \\ 0 \\ b+y \end{bmatrix}$.

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Thus $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$ is a basis for the image, and $\left\{\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}\right\}$ is a basis for the kernel.

□

**A3:** ☐ **A4:** ☐