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| Name: |
| J#: |
| Date: |

Dr. Clontz

MASTERY QUIZ DAY 6

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| | |
|---------------------|-------|
| Standard E1. | Mark: |
|---------------------|-------|

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array} \right]$$

Solution:

$$\begin{aligned} 3x_1 - x_2 + x_4 &= 5 \\ -x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 &= -3 \end{aligned}$$

□

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|---------------------|-------|
| Standard E3. | Mark: |
|---------------------|-------|

Solve the system of equations

$$\begin{aligned} -3x + y &= 2 \\ -8x + 2y - z &= 6 \\ 2y + 3z &= -2 \end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right] \right) = \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The solutions are

$$\left\{ \left[\begin{array}{c} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{array} \right] \mid c \in \mathbb{R} \right\} = \left\{ \left[\begin{array}{c} c - 1 \\ 3c - 1 \\ -2c \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

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| Standard E4. | Mark: |
|---------------------|-------|

Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 0 \\ x_1 + x_2 - x_3 + 5x_4 &= 0 \end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & 3 & -5 & 14 & 0 \\ 1 & 1 & -1 & 5 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \left[\begin{array}{c} -2 \\ 3 \\ 1 \\ 0 \end{array} \right], \left[\begin{array}{c} -1 \\ -4 \\ 0 \\ 1 \end{array} \right] \right\}$.

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| Additional Notes/Marks | |
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Math 237 – Linear Algebra

Version 2

Fall 2017

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| | |
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| Standard E1. | Mark: |
|---------------------|-------|

Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

□

| | |
|---------------------|-------|
| Standard E3. | Mark: |
|---------------------|-------|

Solve the system of equations

$$\begin{aligned}-3x + y &= 2 \\-8x + 2y - z &= 6 \\2y + 3z &= -2\end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right] \right) = \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The solutions are

$$\left\{ \left[\begin{array}{c} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{array} \right] \mid c \in \mathbb{R} \right\} = \left\{ \left[\begin{array}{c} c - 1 \\ 3c - 1 \\ -2c \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

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| Standard E4. | Mark: |
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Find a basis for the solution set to the homogeneous system of equations given by

$$\begin{aligned} 3x + 2y + z &= 0 \\ x + y + z &= 0 \end{aligned}$$

Solution: Let $A = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$.

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Math 237 – Linear Algebra

Version 3

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| | |
|---------------------|-------|
| Standard E1. | Mark: |
|---------------------|-------|

Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

□

| | |
|---------------------|-------|
| Standard E3. | Mark: |
|---------------------|-------|

Solve the system of linear equations.

$$\begin{aligned}2x + y - z + w &= 5 \\3x - y - 2w &= 0 \\-x + 5z + 3w &= -1\end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right] \right) = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{4}{12} & 0 \end{array} \right]$$

So the solutions are

$$\left\{ \left[\begin{array}{c} 1 + a \\ 3 - 21a \\ -7a \\ 12a \end{array} \right] \mid a \in \mathbb{R} \right\}$$

□

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|---------------------|-------|
| Standard E4. | Mark: |
|---------------------|-------|

Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 0 \\ x_1 + x_2 - x_3 + 5x_4 &= 0 \end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & 3 & -5 & 14 & 0 \\ 1 & 1 & -1 & 5 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \left[\begin{array}{c} -2 \\ 3 \\ 1 \\ 0 \end{array} \right], \left[\begin{array}{c} -1 \\ -4 \\ 0 \\ 1 \end{array} \right] \right\}$.

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Math 237 – Linear Algebra

Version 4

Fall 2017

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| | |
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| Standard E1. | Mark: |
|---------------------|-------|

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array} \right]$$

Solution:

$$-4x_1 - x_2 + 3x_3 = 2$$

$$x_1 + 2x_2 - x_3 = 0$$

$$-x_1 + 4x_2 + x_3 = 4$$

□

| | |
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| Standard E3. | Mark: |
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Solve the following linear system.

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 5$$

$$-2x_3 - 4x_4 = 3$$

$$2x_1 + 2x_2 + x_3 - 4x_4 = -1$$

Solution: Let $A = \left[\begin{array}{cccc|c} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$. It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction $0 = 1$).

□

| | |
|---------------------|-------|
| Standard E4. | Mark: |
|---------------------|-------|

Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$

$$2x + y - 4z = 0$$

$$3y - 2z = 0$$

$$x - y - z = 0$$

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \frac{5}{3}a \\ \frac{2}{3}a \\ \frac{3}{3}a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

$$\text{So a basis is } \left\{ \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{3}{3} \\ 1 \end{bmatrix} \right\} \text{ or } \left\{ \begin{bmatrix} 5 \\ 2 \\ 3 \\ 3 \end{bmatrix} \right\}.$$

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Math 237 – Linear Algebra

Version 5

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| | |
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| Standard E1. | Mark: |
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Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

□

| | |
|---------------------|-------|
| Standard E3. | Mark: |
|---------------------|-------|

Solve the system of equations

$$\begin{aligned}-3x + y &= 2 \\-8x + 2y - z &= 6 \\2y + 3z &= -2\end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right] \right) = \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The solutions are

$$\left\{ \left[\begin{array}{c} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{array} \right] \mid c \in \mathbb{R} \right\} = \left\{ \left[\begin{array}{c} c - 1 \\ 3c - 1 \\ -2c \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

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|---------------------|-------|
| Standard E4. | Mark: |
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Find the solution set for the following system of linear equations.

$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$

$$3x_1 + 6x_3 + x_4 = 0$$

$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

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Math 237 – Linear Algebra

Version 6

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| Standard E1. | Mark: |
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Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\x_1 - x_3 + x_4 &= 1\end{aligned}$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 3 & -4 & 1 & 5 \\ 3 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 1 & 1 \end{array} \right]$$

□

| | |
|---------------------|-------|
| Standard E3. | Mark: |
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Solve the following linear system.

$$\begin{aligned}4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\-2x_3 - 4x_4 &= 3 \\2x_1 + 2x_2 + x_3 - 4x_4 &= -1\end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$. It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction $0 = 1$).

□

| | |
|---------------------|-------|
| Standard E4. | Mark: |
|---------------------|-------|

Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 0 \\ -2x_3 - 4x_4 &= 0 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= 0 \end{aligned}$$

Solution: Let $A = \begin{bmatrix} 4 & 4 & 3 & -6 & | & 0 \\ 0 & 0 & -2 & -4 & | & 0 \\ 2 & 2 & 1 & -4 & | & 0 \end{bmatrix}$, so $\text{RREF } A = \begin{bmatrix} 1 & 1 & 0 & -3 & | & 0 \\ 0 & 0 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

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