Name:	
J#:	
Date:	

Dr. Clontz

MASTERY QUIZ DAY 10 Math 237 – Linear Algebra
Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array}\right]$$

Solution:

$$\begin{aligned} -4x_1 - x_2 + 3x_3 &= 2 \\ x_1 + 2x_2 - x_3 &= 0 \\ -x_1 + 4x_2 + x_3 &= 4 \end{aligned}$$

□

Standard E3.	Mark:

Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\ 3x_1 + 6x_3 + x_4 &= 5 \\ -4x_1 + x_2 - 9x_3 + 2x_4 &= -7 \end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & -1 \\ 3 & 0 & 6 & 1 & 5 \\ -4 & 1 & -9 & 2 & -7 \end{array}\right]$, so RREF $A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$. It follows that the solution set is given by $\left[\begin{array}{c} 2-2a \\ 3+a \\ a \\ -1 \end{array}\right]$ for all real numbers a.

□

Standard E4.	Mark:

Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 0 \\ -2x_3 - 4x_4 &= 0 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= 0 \end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 4 & 4 & 3 & -6 & 0 \\ 0 & 0 & -2 & -4 & 0 \\ 2 & 2 & 1 & -4 & 0 \end{array}\right]$, so RREF $A = \left[\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

□

Standard V1.	Mark:

Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2 - 1, y_1 + y_2 - 1) \\ c \odot (x_1, y_1) &= (cx_1 - (c-1), cy_1 - (c-2)) \end{aligned}$$

(a) Show that this vector space has an additive identity element.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$; then $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so $(1, 1)$ is an additive identity element.
Now we will show the other seven properties. Let $(x_1, y_1), (x_2, y_2) \in V$, and let $c, d \in \mathbb{R}$.

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) The additive identity is $(1, 1)$.

4) $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$, so $(2 - x_1, 2 - y_1)$ is the additive inverse of (x_1, y_1).

5)

$$\begin{aligned} c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d-1), dy_1 - (d-1)) \\ &= (c(dx_1 - (d-1)) - (c-1), c(dy_1 - (d-1))) \\ &= (cdx_1 - cd + c - (c-1), cdy_1 - cd + c - (c-1)) \\ &= (cdx_1 - (cd-1), cdy_1 - (cd-1)) \\ &= (cd) \odot (x_1, y_1) \end{aligned}$$

6) $1 \odot (x_1, y_1) = (x_1 - (1-1), y_1 - (1-1) = (x_1, y_1)$

7)

$$\begin{aligned}
c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1) \\
&= (c(x_1 + y_1 - 1) - (c-1), c(x_2 + y_2 - 1) - (c-1)) \\
&= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1) \\
&= (cx_1 - (c-1), cy_1 - (c-1)) \oplus (cx_2 - (c-1), cy_2 - (c-1)) \\
&= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)
\end{aligned}$$

8)

$$\begin{aligned}
(c+d) \odot (x_1, y_1) &= ((c+d)x_1 - (c+d-1), (c+d)y_1 - (c+d-1)) \\
&= (cx_1 - (c-1), cy_1 - (c-1)) \oplus (dx_1 - (d-1), dy_1 - (d-1)) \\
&= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)
\end{aligned}$$

Therefore V is a vector space.

$\square$

Additional Notes/Marks	