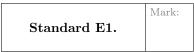
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{bmatrix}$$

Solution:

$$2x_1 - x_2 = 1$$
$$-x_1 + 4x_2 + x_3 = -7$$
$$x_1 + 2x_2 - x_3 = 0$$

Standard E3.

Mark:

Solve the system of equations

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 2$$

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1 & 3 & -4 & 5\\ 3 & 9 & 1 & 2\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}$$

So the solution set is

$$\left\{ \begin{bmatrix} 1 - 3c \\ c \\ -1 \end{bmatrix} \middle| c \in \mathbb{R} \right\}$$

Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$
$$2x + y - 4z = 0$$

$$3y - 2z = 0$$

$$x - y - z = 0$$

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \frac{5}{3}a\\ \frac{2}{3}a\\ a \end{bmatrix} \middle| a \in \mathbb{R} \right\}$$

So a basis is $\left\{ \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} \right\}$ or $\left\{ \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that this scalar multiplication \odot distributes over scalar addition.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c+d)\odot(x_1,y_1)=(0,(c+d)y_1)=(0,cy_1)\oplus(0,dy_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

Additional Notes/Marks