Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\\0\\-3\end{bmatrix}$, $\begin{bmatrix}0\\3\\0\\-2\end{bmatrix}$, and $\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 0 & -1\\0 & 1 & 3 & 1\\2 & 0 & 0 & -1\\1 & -3 & -2 & -1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Since every row contains a pivot, the vectors span $\mathbb{R}^4$.

□

Standard V4.	Mark:

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W, but $(x - x^2) + (x^2) = x \notin W$.

□

Standard S2.	Mark:

Determine if the set $\left\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\right\}$ is a basis of $\P_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra

Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span $\mathbb{R}^4$.

□

Standard V4.	Mark:

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 3 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\3\\6\\3\end{bmatrix}$, $\begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}$, and $\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 3 & 7\\1 & 3 & -1 & -1\\2 & 6 & 3 & 8\\1 & 3 & -2 & -3\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\0 & 0 & 1 & 2\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

Since there are zero rows, they do not span. Alternatively, by inspection $\begin{bmatrix}3\\3\\6\\3\end{bmatrix} = 3\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, so the set is linearly dependent, so it spans a subspace of dimension at most 3, therefore it does not span $\mathbb{R}^4$.

□

Standard V4.	Mark:

Let W be the set of all 2 by 2 matrices which are not invertible. Determine if W is a subspace of $M_{2,2}$.

Solution: W is closed under scalar multiplication, but not under addition.

□

Standard S2.	Mark:

Determine if the set $\left\{x^2+x-1, 3x^2-x+1, 2x-2\right\}$ is a basis of $\P_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\\0\\-3\end{bmatrix}$, $\begin{bmatrix}0\\3\\0\\-2\end{bmatrix}$, and $\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 0 & -1\\0 & 1 & 3 & 1\\2 & 0 & 0 & -1\\1 & -3 & -2 & -1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Since every row contains a pivot, the vectors span $\mathbb{R}^4$.

$\square$

Standard V4.	Mark:

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

$\square$

Standard S2.	Mark:

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of $¶_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra

Version 5 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span $\mathbb{R}^3$.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span $\mathbb{R}^3$.

$\square$

Standard V4.	Mark:

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

$\square$

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}\right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Additional Notes/Marks	

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MASTERY QUIZ DAY 17 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span $\mathbb{R}^3$.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span $\mathbb{R}^3$.

$\square$

Standard V4.	Mark:

Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \middle| \, x, y, z \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^4$.

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

$\square$

Standard S2.	Mark:

Determine if the set $\left\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\right\}$ is a basis of $\P_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Additional Notes/Marks	