| Name: |  |
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## MASTERY QUIZ DAY 28

## Math 237 – Linear Algebra

## Version 5

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

**Solution:** CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 2 & 6 & 11 & 1 \\ 1 & 3 & 7 & 2 \\ -1 & -3 & -5 & 0 \end{bmatrix}$$

**M2.** Determine if the matrix  $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$  is invertible.

Solution:

RREF 
$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since it is row equivalent to the identity matrix, it is invertible.

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 6 & 0 & 1 \\ -14 & 3 & -4 \\ -23 & 4 & -6 \end{bmatrix}$ .

Solution:

$$\begin{bmatrix} 6 & 0 & 1 \\ -14 & 3 & -4 \\ -23 & 4 & -6 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 4 & -3 \\ 8 & -13 & 10 \\ 13 & -24 & 18 \end{bmatrix}$$

**G2.** Compute the eigenvalues, along with their algebraic multiplicities, of the matrix  $\begin{bmatrix} 8 & -3 & 2 \\ 23 & -9 & 5 \\ -7 & 2 & -3 \end{bmatrix}$ .

**Solution:** The eigenvalues are 0 with multiplicity 1 and -2, with algebraic multiplicity 2.

**G3.** Compute the eigenspace of the eigenvalue -1 in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

Solution:

RREF 
$$(A+I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

M1: M2: G2: G3: G1: