Section E.2

Activity E.21 *(~8 min)* Consider the matrix

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & -1 & 4 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

(a) Add row 3 to row 2 ($R_2 + R_3 \to R_2$)

(b) Add row 2 to row 3 ($R_3 + R_2 \to R_3$)

(c) Add -1 row 1 to row 2 ($R_2 - R_1 \to R_2$)

(d) Add -2 row 1 to row 2 ($R_2 - 2R_1 \to R_2$)

Activity E.22 *(~7 min)* Consider the matrix

$$\begin{bmatrix} 2 & 5 & -1 \\ 3 & 5 & 1 \\ 1 & -2 & 0 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

(a) Add row 1 to row 2 ($R_2 + R_1 \to R_2$)

(b) Add -1 row 1 to row 2 ($R_2 - R_1 \to R_2$)

(c) Swap rows 1 and 3 ($R_1 \leftrightarrow R_3$)

(d) Multiply row 1 by $\frac{1}{2}$ ($\frac{1}{2}R_1 \to R_1$)

(e) Add -1 row 3 to row 1 ($R_1 - R_3 \to R_1$)

Activity E.23 *(~5 min)* Consider the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

(a) Add row 1 to row 2 ($R_2 + R_1 \to R_2$)

(b) Add -1 row 1 to row 2 ($R_2 - R_1 \to R_2$)

(c) Add -1 row 3 to row 1 ($R_1 - R_3 \to R_1$)

(d) Add -2 row 3 to row 2 ($R_2 - 2R_3 \to R_2$)

(e) Multiply row 3 by $\frac{1}{2}$ ($\frac{1}{2}R_3 \to R_3$)

Activity E.24 *(~10 min)* Consider the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & -1 & 1 \end{bmatrix}.$$

Perform three row operations to produce a matrix closer to RREF.

Activity E.25 *(~10 min)* Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ -1 & 3 & -5 \end{bmatrix}.$$

Compute RREF(A).

Activity E.26 *(~10 min)* Consider the matrix

$$A = \begin{bmatrix} 2 & 4 & 2 & -4 \\ -2 & -4 & 1 & 1 \\ 3 & 6 & -1 & -4 \end{bmatrix}.$$

Compute RREF(A).

Remark E.27 A video example of how to perform the Gauss-Jordan Elimination algorithm by hand is available at `https://youtu.be/CqONxk2dhhU`.
Practicing several exercises on your own using this method is strongly recommended.

Activity E.28 *(∼10 min)* Free browser-based technologies for mathematical computation are available online.

- Go to `https://octave-online.net`.
- Type `A=sym([1 3 4 ; 2 5 7])` and press `Enter` to store the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$ in the variable A.
  - The symbolic function `sym` is used to calculate precise answers rather than floating-point approximations.
  - The vertical bar in an augmented matrix does not affect row operations, so the RREF of $\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 2 & 5 & 7 \end{array}\right]$ may be computed in the same way.
- Type `rref(A)` and press `Enter` to compute the reduced row echelon form of A.

Remark E.29 We will frequently need to know the reduced row echelon form of matrices during class, so feel free to use Octave-Online.net to compute RREF efficiently.

You may alternatively use the calculator you will use during assessments. Be sure to use fractions mode to compute exact solutions rather than floating-point approximations.

Activity E.30 *(∼10 min)* Consider the system of equations.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 + 3x_2 - \ \ 6x_3 &= 11 \end{aligned}$$

Part 1: Convert this to an augmented matrix and use technology to compute its reduced row echelon form:

$$\text{RREF}\left[\begin{array}{ccc|c} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{array}\right] = \left[\begin{array}{ccc|c} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{array}\right]$$

Part 2: Use the RREF matrix to write a linear system equivalent to the original system. Then find its solution set.