Section V.0

Activity V.0.1 *(∼20 min)* Consider each of the following vector properties. Label each property with $\mathbb{R}^1$, $\mathbb{R}^2$, and/or $\mathbb{R}^3$ if that property holds for Euclidean vectors/scalars $\mathbf{u}, \mathbf{v}, \mathbf{w}$ of that dimension.

1. **Addition associativity.**

   $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

2. **Addition commutivity.**

   $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.

3. **Addition identity.**

   There exists some $\mathbf{z}$ where $\mathbf{v} + \mathbf{z} = \mathbf{v}$.

4. **Addition inverse.**

   There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{z}$.

5. **Addition midpoint uniqueness.**

   There exists a unique $\mathbf{m}$ where the distance from $\mathbf{u}$ to $\mathbf{m}$ equals the distance from $\mathbf{m}$ to $\mathbf{v}$.

6. **Scalar multiplication associativity.**

   $a(b\mathbf{v}) = (ab)\mathbf{v}$.

7. **Scalar multiplication identity.**

   $1\mathbf{v} = \mathbf{v}$.

8. **Scalar multiplication relativity.**

   There exists some scalar c where either $c\mathbf{v} = \mathbf{w}$ or $c\mathbf{w} = \mathbf{v}$.

9. **Scalar distribution.**

   $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$.

10. **Vector distribution.**

    $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$.

11. **Orthogonality.**

    There exists a non-zero vector $\mathbf{n}$ such that $\mathbf{n}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

12. **Bidimensionality.**

    $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ for some value of a, b.

Definition V.0.2 A **vector space** V is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ belong to V, and let a, b be scalar numbers.

- **Addition is associative.**

  $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

- **Addition is commutative.**

  $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.

- **Additive identity exists.**

  There exists some $\mathbf{z}$ where $\mathbf{v} + \mathbf{z} = \mathbf{v}$.

- **Additive inverses exist.**

  There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{z}$.

- **Scalar multiplication is associative.**

  $a(b\mathbf{v}) = (ab)\mathbf{v}$.

- 1 **is a scalar multiplicative identity.**

  $1\mathbf{v} = \mathbf{v}$.

- **Scalar multiplication distributes over vector addition.**

  $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$.

- **Scalar multiplication distributes over scalar addition.**

  $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$.

Any **Euclidean vector space** $\mathbb{R}^n$ satisfies all eight requirements regardless of the value of n, but we will also study other types of vector spaces.