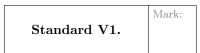
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that scalar multiplication is associative: $a \odot (b \odot x) = (ab) \odot x$.
- (b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$
$$= c (dx - 3(d - 1)) - 3(c - 1)$$
$$= cdx - 3(cd - 1)$$
$$= (cd) \odot x$$

We verify the remaining 7 properties to see that V is a vector space.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 3 = x$, so 3 is the additive identity.
- 3) $x \oplus (6-x) = x + (6-x) 3 = 3$, so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so \oplus is commutative.
- 5) Associativity shown above
- 6) $1 \odot x = x 3(1 1) = x$
- 7)

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$

= $cx - 3(c-1) + dx - 3(c-1) - 3$
= $(c \odot x) \oplus (d \odot x)$

Therefore V is a vector space.

Determine if the vectors $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$, $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$, and $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

Standard V4.

Mark:

Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of \mathcal{P}^3 .

Solution: Yes because $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$ also belongs to W. Alternately, yes because W is isomorphic to \mathbb{R}^2 .

Determine if the set $\left\{ \begin{bmatrix} 0\\1\\1\\1\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\} \text{ is a basis of } \mathbb{R}^4.$

 $RREF \begin{pmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 1 & 2 & 0 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Since this is not the identity matrix, the set is not a basis.

Additional Notes/Marks