Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c\odot x=cx$$

- (a) Show that the vector addition  $\oplus$  is associative:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors  $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$ ,  $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$ ,  $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$ , and  $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$  span  $\mathbb{R}^4$ .



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Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (0, cy_1)$ 

- (a) Show that scalar multiplication **distributes vectors** over scalar addition:  $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Does span 
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

**V4.** Let W be the set of all complex numbers that are purely real (i.e of the form a+0i) or purely imaginary (i.e. of the form 0+bi). Determine if W is a subspace of  $\mathbb{C}$ .

**S2.** Determine if the set  $\left\{ \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^4$ .

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Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that scalar multiplication is associative:  $a \odot (b \odot x) = (ab) \odot x$ .
- (b) Determine if V is a vector space or not. Justify your answer

**V3.** Determine if the vectors  $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$ , and  $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$  span  $\mathbb{R}^3$ .

**V4.** Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

**S2.** Determine if the set  $\{x^3 - 3x^2 + 2x + 2, -x^3 + 4x^2 - x + 1, -x^3 + 2x + 1, 3x^2 + 3x + 9\}$  is a basis of  $\mathcal{P}^3$  or not.

V1:

V3:

V4:

82:

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Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c\odot x=cx$$

- (a) Show that the vector addition  $\oplus$  is associative:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors  $\begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}$ ,  $\begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}$ ,  $\begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}$ , and  $\begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**V4.** Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

**S2.** Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathcal{P}_2$ 

V1:

V3:

V4:

S2:

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Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (0, cy_1)$ 

- (a) Show that scalar multiplication **distributes vectors** over scalar addition:  $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

**V3.** Determine if the vectors 
$$\begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}$$
,  $\begin{bmatrix} 3\\3\\6\\3 \end{bmatrix}$ ,  $\begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix}$ , and  $\begin{bmatrix} 7\\-1\\8\\-3 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**V4.** Let W be the set of all polynomials of the form  $ax^3 + bx$ . Determine if W is a subspace of  $\mathcal{P}^3$ .

**S2.** Determine if the set  $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$  is a basis of  $\mathcal{P}_3$ 

V1:

**V2.** 

V4:

S2:

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Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let V be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that scalar multiplication is associative:  $a \odot (b \odot x) = (ab) \odot x$ .
- (b) Determine if V is a vector space or not. Justify your answer

**V3.** Determine if the vectors  $\begin{bmatrix} 8\\21\\-7 \end{bmatrix}$ ,  $\begin{bmatrix} -3\\-8\\3 \end{bmatrix}$ ,  $\begin{bmatrix} -1\\-3\\2 \end{bmatrix}$ , and  $\begin{bmatrix} 4\\11\\-5 \end{bmatrix}$  span  $\mathbb{R}^3$ .

**V4.** Let W be the set of all complex numbers a + bi satisfying a = 2b. Determine if W is a subspace of  $\mathbb{C}$ .

**S2.** Determine if the set  $\left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\}$  is a basis of  $M_{2,2}$  or not.

V1:

V3:

V4:

S2