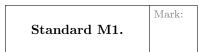
| Name: |            |
|-------|------------|
| J#:   | Dr. Clontz |
| Date: |            |

## MASTERY QUIZ DAY 28

Math 237 – Linear Algebra Fall 2017

## Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

**Solution:** AB is the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

Standard M2.

Determine if the matrix  $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$  is invertible.

Solution:

RREF 
$$\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since it is not equivalent to the identity matrix, it is not invertible.

Standard M3.

Mark:

Find the inverse of the matrix  $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}.$ 

Solution: 
$$\begin{bmatrix} 3 & 1 & 3 & 1 & 0 & 0 \\ 2 & -1 & -6 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 & -1 & -3 \\ 0 & 1 & 0 & -14 & 9 & 24 \\ 0 & 0 & 1 & 3 & -2 & -5 \end{bmatrix}.$$
 Thus the inverse is 
$$\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}.$$

Standard G2.

Mark

Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ . List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\det(A - \lambda I) = \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix}$$

$$= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix}$$

$$= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda))$$

$$= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda$$

$$= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda$$

$$= -\lambda^3 + 2\lambda^2 - \lambda$$

$$= -\lambda(\lambda^2 - 2\lambda + 1)$$

$$= -\lambda(\lambda - 1)^2$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

Standard G3.

Mark:

Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}.$ 

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to  $x - \frac{y}{3} = 0$ , or 3x = y. Thus the eigenspace is

$$E_2 = \operatorname{span}\left(\left\{ \begin{bmatrix} 1\\3\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1 \end{bmatrix} \right\}\right)$$

Additional Notes/Marks