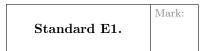
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Standard E3. Mark:

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{4} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{bmatrix}$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a\\ 3-21a\\ -7a\\ 12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

	Mark:
Standard E4.	

Find a basis for the solution set of the system ...

	Mark:
Standard V1.	

Let V be the set of all points on the line x + y = 2 with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

 $c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$

Determine if V is a vector space or not.

Solution:

- 1) Since real addition is associative, \oplus is associative.
- 2) Since real addition is commutative, \oplus is commutative.
- 3) $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so (1, 1) is an additive identity element.
- 4) $(x_1, y_1) \oplus (2 x_1, 2 y_1) = (1, 1)$, so $(2 x_1, 2 y_1)$ is the additive inverse of (x_1, y_1) .

5)

$$\begin{split} c\odot(d\odot(x_1,y_1)) &= c\odot(dx_1-(d-1),dy_1-(d-1))\\ &= (c\left(dx_1-(d-1)\right)-(c-1),c\left(dy_1-(d-1)\right))\\ &= (cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &= (cdx_1-(cd-1),cdy_1-(cd-1))\\ &= (cd)\odot(x_1,y_1) \end{split}$$

6)
$$1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1)$$

$$= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1))$$

$$= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1)$$

$$= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1))$$

$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

8)

$$(c+d) \odot (x_1, y_1) = ((c+d)x_1 - (c+d-1), (c+d)y_1 - (c+d-1))$$
$$= (cx_1 - (c-1), cy_1 - (c-1)) \oplus (dx_1 - (d-1), dy_1 - (d-1))$$
$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$