Module E

Standard E1

E1.1 Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 0 \\ x - z &= 1 \end{aligned}$$

E1.2 Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_3 &= -1 \end{aligned}$$

E1.3 Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_4 &= -1 \end{aligned}$$

E1.4 Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\ 3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 + x_4 &= 1 \end{aligned}$$

E1.5 Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array}\right]$$

E1.6 Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array}\right]$$

E1.7 Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array}\right]$$

E1.8 Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array}\right]$$

Standard E2

E2.1 Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{bmatrix}$$

E2.2 Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix}$$

E2.3 Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{bmatrix}$$

E2.4 Find the reduced row echelon form of the matrix below.

$$\begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix}$$

E2.5 Find RREF A, where

$$A = \begin{bmatrix} 3 & -2 & 1 & 8 & -5 \\ 2 & 2 & 0 & 6 & -2 \\ -1 & 1 & 1 & -4 & 6 \end{bmatrix}$$

E2.6 Find RREF A, where

$$A = \begin{bmatrix} 2 & -7 & 4 \\ 1 & -3 & 2 \\ 3 & 0 & 3 \end{bmatrix}$$

E2.7 Find RREF A, where

$$A = \begin{bmatrix} 2 & -1 & 5 & 4 \\ -1 & 0 & -2 & -1 \\ 1 & 3 & -1 & -5 \end{bmatrix}$$

E2.8 Find RREF A, where

$$A = \begin{bmatrix} 2 & 2 & 1 & 2 & -1 \\ 1 & 1 & 2 & 4 & 5 \\ 3 & 3 & -1 & -2 & 1 \end{bmatrix}$$

Standard E3

E3.1 Find the solution set for the following system of linear equations.

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 2 \end{aligned}$$

E3.2 Find the solution set for the following system of linear equations.

$$\begin{aligned} -3x + y &= 2 \\ -8x + 2y - z &= 6 \\ 2y + 3z &= -2 \end{aligned}$$

E3.3 Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

E3.4 Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

E3.5 Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\ 3x_1 + 6x_3 + x_4 &= 5 \\ -4x_1 + x_2 - 9x_3 + 2x_4 &= -7 \end{aligned}$$

E3.6 Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 8 \\ x_1 + x_2 - x_3 + 5x_4 &= 3 \end{aligned}$$

E3.7 Find the solution set for the following system of linear equations.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

E3.8 Find the solution set for the following system of linear equations.

$$\begin{aligned} 3x + 2y + z &= 7 \\ x + y + z &= 1 \\ -2x + 3z &= -11 \end{aligned}$$

Module V

Standard V1

V1.1 Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

(a) Show that scalar multiplication is associative: $a \odot (b \odot x) = (ab) \odot x$ for all scalars $a, b \in \mathbb{R}$ and $x \in V$.

(b) Explain why V nonetheless isn't a vector space.

V1.2 Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2 + 2x_1y_1) \\ c \odot (x_1, x_2) &= (cx_1, cx_2) \end{aligned}$$

(a) Show that the vector addition $\oplus$ is associative:
$(x_1, x_2) \oplus ((y_1, y_2) \oplus (z_1, z_2)) = ((x_1, x_2) \oplus (y_1, y_2)) \oplus (z_1, z_2)$ for all $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in V$.

(b) Explain why V nonetheless isn't a vector space.

V1.3 Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1 - 1, x_2 + y_2 - 1)\\ c \odot (x_1, x_2) &= (cx_1, cx_2)\end{aligned}$$

(a) Show that this vector space has an additive identity element: there exists $\vec{z} \in V$ satisfying
$(x, y) \oplus \vec{z} = (x, y)$ for every $(x, y) \in V$.

(b) Explain why V nonetheless isn't a vector space.

V1.4 Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2)\\ c \odot (x_1, x_2) &= (0, cx_2)\end{aligned}$$

(a) Show that scalar multiplication distributes over scalar addition:
$(c + d) \odot (x_1, x_2) = c \odot (x_1, x_2) \oplus d \odot (x_1, x_2)$ for every $c, d \in \mathbb{R}$ and $(x_1, x_2) \in V$.

(b) Explain why V nonetheless isn't a vector space.

V1.5 Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2)\\ c \odot (x_1, x_2) &= (c^2 x_1, c^3 x_2)\end{aligned}$$

(a) Show that scalar multiplication distributes over vector addition:
$c \odot ((x_1, x_2) \oplus (y_1, y_2)) = c \odot (x_1, x_2) \oplus c \odot (y_1, y_2)$ for all $c \in \mathbb{R}$ and $(x_1, x_2), (y_1, y_2) \in V$.

(b) Explain why V nonetheless isn't a vector space.

V1.6 Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= \sqrt{x^2 + y^2}\\ c \odot x &= cx\end{aligned}$$

(a) Show that the vector addition $\oplus$ is associative: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ for all $x, y, z \in V$.

(b) Explain why V nonetheless isn't a vector space.

V1.7 Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 y_2)\\ c \odot (x_1, x_2) &= (cx_1, cx_2)\end{aligned}$$

(a) Show that there is an additive identity element: there exists an element $\vec{z} \in V$ such that
$(x_1, x_2) \oplus \vec{z} = (x_1, x_2)$ for any $(x_1, x_2) \in V$.

(b) Explain why V nonetheless isn't a vector space.

Standard V2

V2.1 Determine if $\begin{bmatrix}0\\-1\\2\\6\end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix}3\\-1\\-1\\0\end{bmatrix}$ and $\begin{bmatrix}-1\\0\\1\\2\end{bmatrix}$.

V2.2 Determine if $\begin{bmatrix}0\\1\\-2\\1\end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix}5\\2\\-3\\2\end{bmatrix}$, $\begin{bmatrix}3\\1\\1\\0\end{bmatrix}$, and $\begin{bmatrix}8\\3\\5\\-1\end{bmatrix}$.

V2.3 Determine if $\begin{bmatrix}0\\0\\2\end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix}-1\\-9\\15\end{bmatrix}$ and $\begin{bmatrix}1\\5\\-5\end{bmatrix}$.

V2.4 Determine if $\begin{bmatrix}1\\4\\3\end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix}2\\3\\-1\end{bmatrix}$, $\begin{bmatrix}1\\-1\\0\end{bmatrix}$, and $\begin{bmatrix}-3\\-2\\5\end{bmatrix}$.

V2.5 Determine if $\begin{bmatrix}1\\4\\3\end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix}3\\0\\-1\end{bmatrix}$, $\begin{bmatrix}1\\-1\\4\end{bmatrix}$, and $\begin{bmatrix}5\\1\\-6\end{bmatrix}$.

V2.6 Determine if $\begin{bmatrix}0\\-1\\6\\-7\end{bmatrix}$ belongs to the span of the set $\left\{\begin{bmatrix}2\\0\\-1\\5\end{bmatrix}, \begin{bmatrix}4\\-1\\4\\3\end{bmatrix}\right\}$.

V2.7 Determine if $\begin{bmatrix}4\\-1\\6\\-7\end{bmatrix}$ belongs to the span of the set $\left\{\begin{bmatrix}2\\0\\-1\\5\end{bmatrix}, \begin{bmatrix}4\\-1\\4\\3\end{bmatrix}\right\}$.

V2.8 Determine if $\begin{bmatrix}3\\-2\\4\end{bmatrix}$ belongs to the span of the set $\left\{\begin{bmatrix}1\\2\\-3\end{bmatrix}, \begin{bmatrix}2\\4\\-6\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$.

Standard V3

V3.1 Determine if the vectors $\begin{bmatrix}-3\\1\\1\end{bmatrix}$, $\begin{bmatrix}5\\-1\\-2\end{bmatrix}$, $\begin{bmatrix}2\\0\\-1\end{bmatrix}$, and $\begin{bmatrix}0\\2\\-1\end{bmatrix}$ span $\mathbb{R}^3$

V3.2 Determine if the vectors $\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\3\\6\\3\end{bmatrix}$, $\begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}$, and $\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}$ span $\mathbb{R}^4$.

V3.3 Determine if the vectors $\begin{bmatrix}8\\21\\-7\end{bmatrix}$, $\begin{bmatrix}-3\\-8\\3\end{bmatrix}$, $\begin{bmatrix}-1\\-3\\2\end{bmatrix}$, and $\begin{bmatrix}4\\11\\-5\end{bmatrix}$ span $\mathbb{R}^3$.

V3.4 Determine if the vectors $\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}$, $\begin{bmatrix}3\\1\\3\\6\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$, and $\begin{bmatrix}1\\2\\0\\1\end{bmatrix}$ span $\mathbb{R}^4$.

V3.5 Determine if the vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\\0\\-3\end{bmatrix}$, $\begin{bmatrix}0\\3\\0\\-2\end{bmatrix}$, and $\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}$ span $\mathbb{R}^4$.

V3.6 Does span $\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\end{bmatrix}, \begin{bmatrix}1\\4\\-3\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\} = \mathbb{R}^3$?

V3.7 Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V3.8 Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5$?

Standard V4

V4.1 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, x, y \text{ are integers} \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, x = y \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

V4.2 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \middle| \, x, y, z \in \mathbb{R} \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \\ 1 \\ z \end{bmatrix} \middle| \, x, y, z \in \mathbb{R} \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^4$, and that one of the sets is not.

V4.3 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, x + y + z = 1 \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, x + y + z = 0 \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^3$, and that one of the sets is not.

V4.4 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, x + y = 3z \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, x + y = 3, z \in \mathbb{R} \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^3$, and that one of the sets is not.

V4.5 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, x + y = 0 \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, xy = 0 \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

V4.6 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, x = y \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, |x| = |y| \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

V4.7 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, y = 2x \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, y = x^2 \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

V4.8 Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, z = 2xy \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, z = 2x + y \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^3$, and that one of the sets is not.

Standard V5

V5.1 Determine if set of vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

V5.2 Determine if the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

V5.3 Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

V5.4 Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

V5.5 Determine if the set of vectors $\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

V5.6 Determine if the set of vectors $\left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

V5.7 Determine if the set of vectors $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

V5.8 Determine if the set of vectors $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

Standard V6

V6.1 Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$.

V6.2 Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.

V6.3 Determine if the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$.

V6.4 Determine if the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 5 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.

V6.5 Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^4$.

V6.6 Determine if the set $\left\{ \begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

V6.7 Determine if the set $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

V6.8 Determine if the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^4$.

Standard V7

V7.1 Let $W = \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find a basis for W.

V7.2 Let $W = \operatorname{span}\left(\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$. Find a basis for W.

V7.3 Let $W = \operatorname{span}\left(\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$. Find a basis of W.

V7.4 Let $W = \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\} \right)$. Find a basis of W.

V7.5 Let $W = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$. Find a basis for this vector space.

V7.6 Let $W = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\}$. Find a basis for this vector space.

V7.7 Let $W = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} \right\}$. Find a basis for this vector space.

Standard V8

V8.1 Let $W = \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find the dimension of W.

V8.2 Let $W = \text{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Compute the dimension of W.

V8.3 Let $W = \text{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Compute the dimension of W.

V8.4 Let $W = \text{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix}, \begin{bmatrix}2\\0\\1\\1\end{bmatrix}, \begin{bmatrix}3\\-1\\4\\-2\end{bmatrix}, \begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$. Compute the dimension of W.

V8.5 Let $W = \text{span}\left\{\begin{bmatrix}2\\0\\2\\1\end{bmatrix}, \begin{bmatrix}3\\1\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\-8\\-1\end{bmatrix}\right\}$. Find the dimension of W.

V8.6 Let $W = \text{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\end{bmatrix}, \begin{bmatrix}1\\4\\-3\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}$. Find the dimension of W.

V8.7 Let $W = \text{span}\left\{\begin{bmatrix}2\\-1\\4\\2\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\\-4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\\2\end{bmatrix}, \begin{bmatrix}1\\2\\3\\2\end{bmatrix}\right\}$. Find the dimension of W.

Standard V9

V9.1 Find a basis for the subspace

$$W = \text{span}\left\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\right\}$$

of $\mathcal{P}^2$.

V9.2 Find a basis for the subspace

$$W = \text{span}\left\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\right\}$$

of $\mathcal{P}^2$.

V9.3 Find a basis for the subspace

$$W = \text{span}\left\{\begin{bmatrix}1 & -3\\2 & 2\end{bmatrix}, \begin{bmatrix}-1 & 4\\-1 & 1\end{bmatrix}, \begin{bmatrix}0 & 3\\3 & 9\end{bmatrix}, \begin{bmatrix}-1 & 0\\2 & 1\end{bmatrix}\right\}$$

of $M_{2,2}$.

V9.4 Find a basis for the subspace

$$W = \text{span}\left\{x^3 - 3x^2 + 2x + 2, -x^3 + 4x^2 - x + 1, 3x^2 + 3x + 9, -x^3 + 2x + 1\right\}$$

of $\mathcal{P}^3$.

V9.5 Find a basis for the subspace

$$W = \text{span}\left\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\right\}$$

of $\mathcal{P}^3$.

V9.6 Let W be the subspace of $\mathcal{P}^3$ given by

$$W = \text{span}\left(\left\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\right\}\right).$$

Find a basis for W.

V9.7 Let $W = \text{span}\left\{\begin{bmatrix}2 & 0\\-2 & 0\end{bmatrix}, \begin{bmatrix}3 & 1\\3 & 6\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}, \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\right\}$. Find a basis for this vector space.

V9.8 Let W be the subspace of $\mathcal{P}^2$ given by $W = \text{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$. Find a basis for W.

Standard V10

V10.1 Find a basis for the solution space of the homogeneous system of equations

$$\begin{aligned} x+3y+3z+7w&=0\\ x+3y-z-w&=0\\ 2x+6y+3z+8w&=0\\ x+3y-2z-3w&=0 \end{aligned}$$

V10.2 Find a basis for the solution space of the homogeneous system of equations

$$\begin{aligned} x+2y+3z+w&=0\\ 3x-y+z+w&=0\\ 2x-3y-2z&=0\\ -x+2z+5w&=0 \end{aligned}$$

V10.3 Find a basis for the solution space of the homogeneous system of equations

$$\begin{aligned} x+2y+3z+w&=0\\ 3x-y+z+w&=0\\ 2x-3y-2z&=0 \end{aligned}$$

V10.4 Find a basis for the solution space to the system of equations

$$\begin{aligned} x+2y-3z&=0\\ 2x+y-4z&=0\\ 3y-2z&=0\\ x-y-z&=0 \end{aligned}$$

V10.5 Find a basis for the solution space to the homogeneous system of equations

$$\begin{aligned} 2x_1+3x_2-5x_3+14x_4&=0\\ x_1+x_2-x_3+5x_4&=0 \end{aligned}$$

V10.6 Find a basis for the solution space to the homogeneous system of equations

$$\begin{aligned} 4x_1+4x_2+3x_3-6x_4&=0\\ -2x_3-4x_4&=0\\ 2x_1+2x_2+x_3-4x_4&=0 \end{aligned}$$

V10.7 Find a basis for the solution space to the homogeneous system of equations given by

$$\begin{aligned} 3x+2y+z&=0\\ x+y+z&=0 \end{aligned}$$

V10.8 Find a basis for the solution space to the homogeneous system of equations given by

$$\begin{aligned} 2x_1-2x_2+6x_3-x_4&=0\\ 3x_1+6x_3+x_4&=0\\ -4x_1+x_2-9x_3+2x_4&=0 \end{aligned}$$

Module A

Standard A1

A1.1 Consider the following maps of polynomials $S : \mathcal{P}^6 \to \mathcal{P}^6$ and $T : \mathcal{P}^6 \to \mathcal{P}^6$ defined by

$$S(f(x)) = f(x) + 3 \text{ and } T(f(x)) = f(x) + f(3).$$

Show that one of these maps is a linear transformation, and that the other map is not.
A1.2 Consider the following maps of polynomials $S : \mathcal{P}^4 \to \mathcal{P}^5$ and $T : \mathcal{P}^4 \to \mathcal{P}^5$ defined by

$$S(f(x)) = xf(x) - f(1) \text{ and } T(f(x)) = xf(x) - x.$$

Show that one of these maps is a linear transformation, and that the other map is not.
A1.3 Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = f'(x) - f''(x) \text{ and } T(f(x)) = f(x) - (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
A1.4 Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^4$ and $T : \mathcal{P}^2 \to \mathcal{P}^4$ defined by

$$S(f(x)) = x^2 f(x) \text{ and } T(f(x)) = (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
A1.5 Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = (f(x))^2 + 1 \text{ and } T(f(x)) = (x^2 + 1)f(x).$$

Show that one of these maps is a linear transformation, and that the other map is not.
A1.6 Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^2$ and $T : \mathcal{P}^2 \to \mathcal{P}^2$ defined by

$$S(ax^2 + bx + c) = cx^2 + bx + a \text{ and } T(ax^2 + bx + c) = a^2x^2 + b^2x + c^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
A1.7 Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^1$ and $T : \mathcal{P}^2 \to \mathcal{P}^1$ defined by

$$S(ax^2 + bx + c) = 2ax + b \text{ and } T(ax^2 + bx + c) = a^2x + b.$$

Show that one of these maps is a linear transformation, and that the other map is not.
A1.8 Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^3$ and $T : \mathcal{P}^2 \to \mathcal{P}^3$ defined by

$$S(ax^2 + bx + c) = ax^3 + bx^2 + cx \text{ and } T(ax^2 + bx + c) = abcx^3.$$

Show that one of these maps is a linear transformation, and that the other map is not.

Standard A2

A2.1 Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.$$

(a) Write the standard matrix for T.

(b) Compute $T\left(\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}\right)$

A2.2 Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 + 3x_1 \end{bmatrix}.$$

(a) Write the standard matrix for T.

(b) Compute $T\left(\begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}\right)$

A2.3 Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3 \end{bmatrix}.$$

(a) Write the standard matrix for T.

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}\right)$

A2.4 Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}.$$

(a) Write the standard matrix for T.

(b) Compute $T\left(\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\right)$

A2.5 Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 2 & -1 & 5 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}\right)$

A2.6 Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be the linear transformation given by the requirements

$$T(\vec{e}_1) = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 3 \end{bmatrix} \text{ and } T(\vec{e}_2) = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 3 \end{bmatrix}$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

A2.7 Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 7x + 2y + 3z \\ 0 \end{bmatrix}.$$

(a) Write the standard matrix for T.

(b) Compute $T\left(\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}\right)$

A2.8 Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 7 & 0 & 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}\right)$

A2.9 Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -5 & 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}\right)$

A2.10 Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be the linear transformation given by the following

$$T(\vec{e}_1) = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} \qquad T(\vec{e}_2) = \begin{bmatrix} 0 \\ 3 \\ -5 \\ 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right)$

A2.11 Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - x_3 \end{bmatrix}.$$

(a) Write the standard matrix for T.

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}\right)$

A2.12 Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - 5x_3 \end{bmatrix}.$$

(a) Write the standard matrix for T.

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}\right)$

A2.13 Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}\right)$

A2.14 Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation given by the following

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$

A2.15 Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}\right)$

Standard A3

A3.1 Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T.
A3.2 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T.

A3.3 Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

A3.4 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z \\ y + 3z \\ -7x + 3y + 2z \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

A3.5 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by the standard matrix $\begin{bmatrix} 1 & 2 & 5 \\ 0 & -2 & -4 \\ -3 & 1 & -1 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

A3.6 Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear map given by the standard matrix $\begin{bmatrix} 1 & 2 & 5 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

A3.7 Let $T : \mathbb{R}^2 \to \mathbb{R}^5$ be the linear map given by the standard matrix $\begin{bmatrix} 3 & 1 \\ 0 & 0 \\ -6 & -2 \\ 1 & \frac{1}{3} \\ 9 & 3 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

A3.8 Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by the standard matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ -4 & -2 & 0 & 5 \\ -2 & -1 & 0 & 0 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

Standard A4

A4.1 Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x+2y \\ x-y \\ x+4y \end{bmatrix}$

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

A4.2 Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

(b) $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x+2y \\ x-y \\ x+4y \end{bmatrix}$

A4.3 Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4.4 Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 4 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4.5 Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^4$ given by the standard matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 11 & -1 & 5 \end{bmatrix}$

A4.6 Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^4$ given by the standard matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$.

(b) $T:\mathbb{R}^4\to\mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 7 & -1 & 5 \end{bmatrix}$

A4.7 Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S:\mathbb{R}^2\to\mathbb{R}^3$ where $S(\vec{e}_1)=\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $S(\vec{e}_2)=\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(b) $T:\mathbb{R}^3\to\mathbb{R}^2$ where $T(\vec{e}_1)=\begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $T(\vec{e}_2)=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $T(\vec{e}_3)=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

A4.8 Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S:\mathbb{R}^4\to\mathbb{R}^3$ where $S(\vec{e}_1)=\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $S(\vec{e}_2)=\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $S(\vec{e}_3)=\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, and $S(\vec{e}_4)=\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$,

(b) $T:\mathbb{R}^3\to\mathbb{R}^3$ where $T(\vec{e}_1)=\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $T(\vec{e}_2)=\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$, and $T(\vec{e}_3)=\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

Module M

Standard M1

M1.1 Let

$$A=\begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B=\begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C=\begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M1.2 Let

$$A=\begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix} \qquad B=\begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C=\begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M1.3 Let

$$A=\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad B=\begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad C=\begin{bmatrix} 0 & -1 & 4 \\ 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M1.4 Let

$$A=\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad B=\begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad C=\begin{bmatrix} 3 & -1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M1.5 Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M1.6 Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M1.7 Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M1.8 Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Standard M2

M2.1 Determine if the matrix $\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}$ is invertible.

M2.2 Determine if the matrix $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ is invertible.

M2.3 Determine if the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$ is invertible.

M2.4 Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ is invertible.

M2.5 Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -2 & 0 & 3 \end{bmatrix}$ is invertible.

M2.6 Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

M2.7 Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

M2.8 Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

Standard M3

M3.1 Show how to find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

M3.2 Compute the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

M3.3 Show how to find the inverse of the matrix $\begin{bmatrix} 6 & 0 & 1 \\ -14 & 3 & -4 \\ -23 & 4 & -6 \end{bmatrix}$.

M3.4 Show how to find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

M3.5 Show how to find the inverse of the matrix $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$.

M3.6 Show how to find the inverse of the matrix $\begin{bmatrix} 1 & -4 & 5 \\ -5 & 24 & -28 \\ 1 & -5 & 6 \end{bmatrix}$.

M3.7 Show how to find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

M3.8 Show how to find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

M3.9 Show how to find the inverse of the matrix $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$.

Module G

Standard G1

G1.1 Consider the row operation $R_1 + 5R_3 \to R_1$ applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1+5(7) & 2+5(8) & 3+5(9) \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = 4$, find the determinant of RC.

G1.2 Consider the row operation $R_2 - 4R_3 \to R_2$ applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 4-4(7) & 5-4(8) & 6-4(9) \\ 7 & 8 & 9 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = 7$, find the determinant of RC.

G1.3 Consider the row operation $R_3 - 2R_1 \to R_3$ applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7-2(1) & 8-2(2) & 9-2(3) \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = -8$, find the determinant of RC.

G1.4 Consider the row operation $4R_3 \to R_3$ applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ (4)7 & (4)8 & (4)9 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = -12$, find the determinant of RC.

G1.5 Consider the row operation $-8R_1 \to R_1$ applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} (-8)1 & (-8)2 & (-8)3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = -2$, find the determinant of RC.

G1.6 Consider the row operation $5R_2 \to R_2$ applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ (5)4 & (5)5 & (5)6 \\ 7 & 8 & 9 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = 3$, find the determinant of RC.

G1.7 Consider the row operation that swaps R_1 and R_2 applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = 3$, find the determinant of RC.

G1.8 Consider the row operation that swaps R_3 and R_2 applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = -7$, find the determinant of RC.

G1.9 Consider the row operation that swaps R_3 and R_1 applied as follows to show $A \sim B$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} = B$$

(a) Find a matrix R such that $B = RA$.

(b) If $C \in M_{3,3}$ is a matrix with $\det C = -11$, find the determinant of RC.

Standard G2

G2.1 Compute the determinant of the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$.

G2.2 Compute the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix}$.

G2.3 Compute the determinant of the matrix $\begin{bmatrix} 2 & 3 & 0 & 1 \\ -1 & 3 & 1 & 4 \\ 0 & 2 & 0 & 3 \\ 1 & -1 & 3 & 5 \end{bmatrix}$.

G2.4 Compute the determinant of the matrix $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 3 & 0 & 0 & 1 \end{bmatrix}$.

G2.5 Compute the determinant of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

G2.6 Compute the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ 5 & 0 & -4 & 0 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

G2.7 Compute the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

G2.8 Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

G2.9 Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

Standard G3

G3.1 Find the eigenvalues of the matrix $\begin{bmatrix} 5 & 1 \\ -24 & -6 \end{bmatrix}$.

G3.2 Find the eigenvalues of the matrix $\begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix}$.

G3.3 Find the eigenvalues of the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$.

G3.4 Find the eigenvalues of the matrix $\begin{bmatrix} -2 & 5 \\ 5 & -2 \end{bmatrix}$.

G3.5 Find the eigenvalues of the matrix $\begin{bmatrix} 10 & -8 \\ 4 & -2 \end{bmatrix}$.

G3.6 Find the eigenvalues of the matrix $\begin{bmatrix} 6 & -4 \\ 11 & -9 \end{bmatrix}$.

G3.7 Find the eigenvalues of the matrix $\begin{bmatrix} -6 & -11 \\ 4 & 9 \end{bmatrix}$.

G3.8 Find the eigenvalues of the matrix $\begin{bmatrix} 1 & -5 \\ 4 & -8 \end{bmatrix}$.

G3.9 Find the eigenvalues of the matrix $\begin{bmatrix} -2 & 2 \\ 15 & -9 \end{bmatrix}$.

G3.10 Find the eigenvalues of the matrix $\begin{bmatrix} 10 & 2 \\ -39 & -9 \end{bmatrix}$.

G3.11 Find the eigenvalues of the matrix $\begin{bmatrix} 8 & 2 \\ -33 & -9 \end{bmatrix}$.

Standard G4

G4.1 Find a basis of the eigenspace associated to the eigenvalue -1 for the matrix $\begin{bmatrix} 4 & -2 & -1 & 1 \\ 15 & -7 & -3 & 1 \\ -5 & 2 & 0 & 1 \\ 10 & -4 & -2 & 0 \end{bmatrix}$.

G4.2 Find a basis of the eigenspace associated to the eigenvalue 1 for the matrix $A = \begin{bmatrix} -3 & 1 & 0 & 0 \\ -8 & 2 & 0 & -1 \\ 0 & 2 & 1 & 2 \\ 4 & 1 & 0 & 3 \end{bmatrix}$.

G4.3 Find a basis of the eigenspace associated to the eigenvalue 1 for the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 8 & -3 & -1 \\ 2 & 21 & -8 & -3 \\ 3 & -7 & 3 & 2 \end{bmatrix}$.

G4.4 Find a basis of the eigenspace associated to the eigenvalue 1 for the matrix
$A = \begin{bmatrix} 9 & -3 & -5 & 2 \\ 19 & -6 & -12 & 5 \\ 1 & 1 & -1 & 3 \\ -11 & 4 & 7 & -2 \end{bmatrix}$.

G4.5 Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -4 & 4 & 0 & 0 \\ 11 & -6 & 1 & -1 \\ -9 & 5 & 1 & 3 \end{bmatrix}$.

G4.6 Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$.

G4.7 Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix
$A = \begin{bmatrix} 0 & -2 & -1 & 0 \\ -4 & -2 & -2 & 0 \\ 14 & 12 & 10 & 2 \\ -13 & -10 & -8 & -1 \end{bmatrix}$.

G4.8 Find a basis of the eigenspace associated to the eigenvalue 3 for the matrix
$A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ -4 & -1 & -2 & 0 \\ 14 & 12 & 11 & 2 \\ -14 & -10 & -9 & -1 \end{bmatrix}$.

G4.9 Find a basis of the eigenspace associated to the eigenvalue -2 for the matrix $A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 1 & 2 & -3 & 3 \\ 1 & 8 & -9 & 6 \\ 1 & 8 & -7 & 4 \end{bmatrix}$.

G4.10 Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix $\begin{bmatrix} -1 & 0 & 1 & 0 \\ 6 & 2 & -2 & 0 \\ -9 & 0 & 5 & 0 \\ 15 & 0 & -5 & 2 \end{bmatrix}$.