

Section E.2

Activity E.2.1 (*~10 min*)

- Go to <http://www.cocalc.com> and create an account.
- Create a project titled “Linear Algebra Team X” with your appropriate team number. Add all team members as collaborators.
- Open the project and click on “New”
- Give it an appropriate name such as “Class E2 workbook”. Make a new Jupyter notebook.
- Click on “Kernel” and make sure “Octave” is selected.
- Type $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 7 \end{bmatrix}$ to store the matrix $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 7 \end{bmatrix}$ in the variable A ; hold shift when you press enter.
- Type `rref(A)` to compute the reduced row echelon form of A .

Remark E.2.2 If you need to find the reduced row echelon form of a matrix during class, you should feel free to use CoCalc/Octave.

You can change a cell from “Code” to “Markdown” or “Raw” to put comments around your calculations such as Activity numbers.

Activity E.2.3 (*~8 min*) Consider the system of equations.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 + 3x_2 - 6x_3 &= 11 \end{aligned}$$

Convert this to an augmented matrix, use CoCalc to compute the reduced row echelon form, and convert back to a simpler system of equations to solve this system. Write your solution on your whiteboard.

Activity E.2.4 (*~7 min*) Consider our system of equations from above.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 &\quad - 3x_3 = 1 \end{aligned}$$

Convert this to an augmented matrix, use CoCalc to compute the reduced row echelon form, and convert back to a simpler system of equations to solve this system. Write your solution on your whiteboard.

Activity E.2.5 (*~10 min*) Consider the following matrix.

$$A = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 4 & 8 & 0 \end{array} \right]$$

Part 1: Find $\text{RREF}(A)$ (Use CoCalc).

Part 2: How many solutions does the corresponding linear system have?

Activity E.2.6 (*~10 min*) Consider the (simpler) system from the previous problem:

$$\begin{aligned} x_1 + 2x_2 &= 4 \\ x_3 &= -1 \end{aligned}$$

Part 1: Let $x_1 = a$ and write the solution set in the form $\left\{ \begin{bmatrix} a \\ ? \\ ? \end{bmatrix} \mid a \in \mathbb{R} \right\}$

Part 2: Let $x_2 = b$ and write the solution set in the form $\left\{ \begin{bmatrix} ? \\ b \\ ? \end{bmatrix} \mid b \in \mathbb{R} \right\}$

Part 3: Which of these was easier? What features of the RREF matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

cause this?

Definition E.2.7 If a matrix is in reduced row echelon form, a **pivot** is an entry satisfying

1. It is 1
2. Everything else in the same row but to the left of it is zero
3. Everything else in the same column is zero.

For example, the pivots are circled in

$$\left[\begin{array}{ccc|c} \textcircled{1} & 2 & 0 & 4 \\ 0 & 0 & \textcircled{1} & -1 \end{array} \right]$$

Activity E.2.8 (*~5 min*) Circle the pivots in each matrix below.

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Definition E.2.9 The pivots in a matrix correspond to **bound variables** in the system of equations. The remaining variables are called **free variables**.

To efficiently solve a system in RREF form, assign letters to free variables and solve for the bound variables.

Activity E.2.10 (*~10 min*) Find the solution set for the system

$$2x_1 - 2x_2 - 6x_3 + x_4 - x_5 = 3$$

$$-x_1 + x_2 + 3x_3 - x_4 + 2x_5 = -3$$

$$x_1 - 2x_2 - x_3 + x_4 + x_5 = 2$$

by assigning letters to the free variables and solving for the bounded variables.

Observation E.2.11 The solution set to the system

$$2x_1 - 2x_2 - 6x_3 + x_4 - x_5 = 3$$

$$-x_1 + x_2 + 3x_3 - x_4 + 2x_5 = -3$$

$$x_1 - 2x_2 - x_3 + x_4 + x_5 = 2$$

is

$$\left\{ \left[\begin{array}{c} 1 + 5a + 2b \\ 1 + 2a + 3b \\ a \\ 3 + 3b \\ b \end{array} \right] \middle| a, b \in \mathbb{R} \right\}.$$

Remark E.2.12 You should always use set-builder notation to describe the solution set of a linear system.