

Name:
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Date:

Dr. Clontz

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

Solution:

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 4x_2 + x_3 &= -7 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

□

Standard E3.	Mark:
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Solve the system of equations

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 2 \end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 2 \end{array} \right] \right) = \left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

So the solution set is

$$\left\{ \left[\begin{array}{c} 1 - 3c \\ c \\ -1 \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

Standard E4.	Mark:
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Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$

$$2x + y - 4z = 0$$

$$3y - 2z = 0$$

$$x - y - z = 0$$

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \frac{5}{3}a \\ \frac{2}{3}a \\ \frac{2}{3}a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

$$\text{So a basis is } \left\{ \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} \right\} \text{ or } \left\{ \begin{bmatrix} 5 \\ 2 \\ 2 \\ 3 \end{bmatrix} \right\}.$$

□

Standard V1.	Mark:
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Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication \odot distributes over scalar addition.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c + d) \odot (x_1, y_1) = (0, (c + d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

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Additional Notes/Marks	
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