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| Name: | |
| J#: | Dr. Clontz |
| Date: | |

**MIDTERM EXAM** Math 237 – Linear Algebra
**Version 2** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard E1.** | Mark: |
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array}\right]$$

| **Standard E2.** | Mark: |
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Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{bmatrix}$$

| **Standard E3.** | Mark: |
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Solve the system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

| **Standard E4.** | Mark: |
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Find a basis for the solution set of the system of equations

$$\begin{aligned} x + 2y + 3z + w &= 0 \\ 3x - y + z + w &= 0 \\ 2x - 3y - 2z &= 0 \end{aligned}$$

| **Standard V1.** | Mark: |
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Let $V$ be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c-1)$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.

(b) Determine if $V$ is a vector space or not. Justify your answer

| **Standard V2.** | Mark: |
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Determine if $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

| **Standard V3.** | Mark: |
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Does $\operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

| **Standard V4.** | Mark: |
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Let $W$ be the set of all polynomials of even degree. Determine if $W$ is a subspace of the vector space of all polynomials.

| **Standard S1.** | Mark: |
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Determine if the set of polynomials $\left\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\right\}$ is linearly dependent or linearly independent

| **Standard S2.** | Mark: |
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Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

| **Standard S3.** | Mark: |
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Let $W = \text{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find a basis for $W$.

| **Standard S4.** | Mark: |
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Let $W = \text{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find the dimension of $W$.

| **Additional Notes/Marks** | |
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