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Dr. Clontz

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

Solution:

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 4x_2 + x_3 &= -7 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

□

Standard E3.	Mark:
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Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\ 3x_1 + 6x_3 + x_4 &= 5 \\ -4x_1 + x_2 - 9x_3 + 2x_4 &= -7 \end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & -1 \\ 3 & 0 & 6 & 1 & 5 \\ -4 & 1 & -9 & 2 & -7 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right]$. It follows that the

solution set is given by $\begin{bmatrix} 2 - 2a \\ 3 + a \\ a \\ -1 \end{bmatrix}$ for all real numbers a .

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Standard E4.	Mark:
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Find a basis for the solution set to the homogeneous system of equations given by

$$3x + 2y + z = 0$$

$$x + y + z = 0$$

Solution: Let $A = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$.

□

Standard V1.	Mark:
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Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$$

(a) Show that this scalar multiplication \odot distributes over vector addition \oplus .

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1), (x_2, y_2) \in V$ and let $c \in \mathbb{R}$.

$$\begin{aligned} c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + x_2, y_1 + y_2) \\ &= (c^2(x_1 + x_2), c^3(y_1 + y_2)) \\ &= (c^2 x_1, c^3 y_1) \oplus (c^2 x_2, c^3 y_2) \\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2) \end{aligned}$$

However, V is not a vector space, as the other distributive law fails:

$$(c + d) \odot (x_1, y_1) = ((c + d)^2 x_1, (c + d)^3 y_1) \neq ((c^2 + d^2) x_1, (c^3 + d^3) y_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

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Additional Notes/Marks	
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