

## Application Activities - Module V Part 4 - Class Day 10

**Fact 10.1** At least  $n$  vectors are required to span  $\mathbb{R}^n$ .



**Activity 10.2** Choose a vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  in  $\mathbb{R}^3$  that is not in  $\text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$  by ensuring  $\left[ \begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$ . (Why does this work?)

**Fact 10.3** The set  $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$  fails to span all of  $\mathbb{R}^n$  exactly when  $\text{RREF}[\mathbf{v}_1 \dots \mathbf{v}_m]$  has a row of zeros:

$$\left[ \begin{array}{cc} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{array} \right] \sim \left[ \begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{array} \right] \Rightarrow \left[ \begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

**Activity 10.4** Consider the set of vectors  $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}$ . Does  $\mathbb{R}^4 = \text{span } S$ ?

**Activity 10.5** Consider the set of third-degree polynomials

$$S = \{2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, -x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\}.$$

Does  $\mathcal{P}^3 = \text{span } S$ ?

**Definition 10.6** A subset of a vector space is called a **subspace** if it is itself a vector space.

**Fact 10.7** If  $S$  is a subset of a vector space  $V$ , then  $\text{span } S$  is a subspace of  $V$ .

**Remark 10.8** To prove that a subset is a subspace, you need only verify that  $c\mathbf{v} + d\mathbf{w}$  belongs to the subset for any choice of vectors  $\mathbf{v}, \mathbf{w}$  from the subset and any real scalars  $c, d$ .

**Activity 10.9** Prove that  $P = \{ax^2 + b : a, b \text{ are both real numbers}\}$  is a subspace of the vector space of all degree-two polynomials by showing that  $c(a_1x^2 + b_1) + d(a_2x^2 + b_2)$  belongs to  $P$ .

**Activity 10.10** Consider the subset of  $\mathbb{R}^2$  where at least one coordinate of each vector is 0.



Find a linear combination  $c\mathbf{v} + d\mathbf{w}$  that does not belong to this subset.

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**Fact 10.11** Suppose a subset  $S$  of  $V$  is isomorphic to another vector space  $W$ . Then  $S$  is a subspace of  $V$ .

**Activity 10.12** Show that the set of  $2 \times 2$  matrices

$$S = \left\{ \begin{bmatrix} a & b \\ -b & -a \end{bmatrix} : a, b \text{ are real numbers} \right\}$$

is a subspace of  $\mathbb{R}^{2 \times 2}$  by identifying a Euclidean space isomorphic to  $S$ .

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