| Name: |
| J#: |
| Date: |

Dr. Clontz

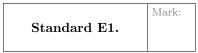**MASTERY QUIZ DAY 10**   Math 237 – Linear Algebra
Version 5   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard E1.** | Mark: |

Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_4 = -1$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -1 & 0 & 7 \\ 1 & -1 & 0 & 3 & -1 \end{array}\right]$$

□

| **Standard E3.** | Mark: |

Find the solution set for the following system of linear equations.

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 8$$
$$x_1 + x_2 - x_3 + 5x_4 = 3$$

Solution: Let $A = \begin{bmatrix} 2 & 3 & -5 & 14 & | & 8 \\ 1 & 1 & -1 & 5 & | & 3 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 1 & | & 1 \\ 0 & 1 & -3 & 4 & | & 2 \end{bmatrix}$. It follows that the solution set

is given by $\begin{bmatrix} 2 - 2a - b \\ 2 + 3a - 4b \\ a \\ b \end{bmatrix}$ for all real numbers a, b.

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| **Standard E4.** | Mark: |

Find a basis for the solution set to the homogeneous system of equations

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 0$$
$$x_1 + x_2 - x_3 + 5x_4 = 0$$

Solution: Let $A = \begin{bmatrix} 2 & 3 & -5 & 14 & | & 0 \\ 1 & 1 & -1 & 5 & | & 0 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 1 & | & 1 \\ 0 & 1 & -3 & 4 & | & 2 \end{bmatrix}$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$.

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	Mark:
Standard V1.	

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication $\odot$ distributes over scalar addition.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c + d) \odot (x_1, y_1) = (0, (c + d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

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Additional Notes/Marks	