

Name:
J#:
Date:

Dr. Clontz

MIDTERM EXAM

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
---------------------	-------

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array} \right]$$

Standard E2.	Mark:
---------------------	-------

Put the following matrix in reduced row echelon form.

$$\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right]$$

Standard E3.	Mark:
---------------------	-------

Solve the following linear system.

$$3x + 2y + z = 7$$

$$x + y + z = 1$$

$$-2x + 3z = -11$$

Standard E4.	Mark:
---------------------	-------

Find a basis for the solution set to the homogeneous system of equations

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 0$$

$$x_1 + x_2 - x_3 + 5x_4 = 0$$

Standard V1.	Mark:
---------------------	-------

Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$

$$c \odot f = cf'$$

(here f' denotes the derivative of f).

(a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.

(b) Determine if V is a vector space or not. Justify your answer.

Standard V2.	Mark:
---------------------	-------

Determine if $\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 4 \\ 3 \end{bmatrix} \right\}$.

Standard V3.	Mark:
---------------------	-------

Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span \mathbb{R}^3 .

Standard V4.	Mark:
---------------------	-------

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Standard S1.	Mark:
---------------------	-------

Determine if the set of polynomials $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$ is linearly dependent or linearly independent

Standard S2.	Mark:
---------------------	-------

Determine if the set $\left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\}$ is a basis of $M_{2,2}$ or not.

Standard S3.	Mark:
---------------------	-------

Let W be the subspace of \mathcal{P}_3 given by
 $W = \text{span} \left(\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\} \right)$. Find a basis for W .

Standard S4.	Mark:
---------------------	-------

Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find the dimension of W .

Additional Notes/Marks	
-------------------------------	--