

Name: _____

MIDTERM EXAM

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

E2. Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{bmatrix}$$

E3. Solve the system of equations

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 2$$

E4. Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$

$$2x + y - 4z = 0$$

$$3y - 2z = 0$$

$$x - y - z = 0$$

V1. Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}f \oplus g &= f' + g' \\c \odot f &= cf'\end{aligned}$$

(here f' denotes the derivative of f).

(a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$.

V3. Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V4. Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

S1. Determine if the vectors $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$ are linearly dependent or linearly independent

S2. Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \mathcal{P}_2

S3. Let $W = \text{span} \left(\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$. Find a basis of W .

S4. Let W be the subspace of \mathcal{P}_3 given by
 $W = \text{span} \left(\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\} \right)$. Compute the dimension of W .

E1:	<input type="text"/>	E2:	<input type="text"/>	E3:	<input type="text"/>	E4:	<input type="text"/>	V1:	<input type="text"/>	V2:	<input type="text"/>	V3:	
<input type="text"/>		V4:	<input type="text"/>	S1:	<input type="text"/>	S2:	<input type="text"/>	S3:	<input type="text"/>	S4:	<input type="text"/>		