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Dr. Clontz

**MASTERY QUIZ DAY 12** Math 237 – Linear Algebra

**Version 2** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
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Let $V$ be the set of all points on the line $x+y=2$ with the operations, for any $(x_1,y_1),(x_2,y_2)\in V$, $c\in\mathbb{R}$,

$$\begin{aligned}(x_1,y_1)\oplus(x_2,y_2)&=(x_1+x_2-1,y_1+y_2-1)\\ c\odot(x_1,y_1)&=(cx_1-(c-1),cy_1-(c-2))\end{aligned}$$

Determine if $V$ is a vector space or not.

**Solution:**

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) $(x_1,y_1)\oplus(1,1)=(x_1,y_1)$, so $(1,1)$ is an additive identity element.

4) $(x_1,y_1)\oplus(2-x_1,2-y_1)=(1,1)$, so $(2-x_1,2-y_1)$ is the additive inverse of $(x_1,y_1)$.

5)

$$\begin{aligned}c\odot(d\odot(x_1,y_1))&=c\odot(dx_1-(d-1),dy_1-(d-1))\\ &=(c(dx_1-(d-1))-(c-1),c(dy_1-(d-1)))\\ &=(cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &=(cdx_1-(cd-1),cdy_1-(cd-1))\\ &=(cd)\odot(x_1,y_1)\end{aligned}$$

6) $1\odot(x_1,y_1)=(x_1-(1-1),y_1-(1-1)=(x_1,y_1)$

7)

$$\begin{aligned}c\odot((x_1,y_1)\oplus(x_2,y_2))&=c\odot(x_1+y_1-1,x_2+y_2-1)\\ &=(c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &=(cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &=(cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot(x_1,y_1)&=((c+d)x_1-(c+d-1),(c+d)y_1-(c+d-1))\\ &=(cx_1-(c-1),cy_1-(c-1))\oplus(dx_1-(d-1),dy_1-(d-1))\\ &=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

□

| **Standard V3.** | Mark: |
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Determine if the vectors $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ span $\mathbb{R}^3$

**Solution:**

$$\text{RREF}\left(\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span $\mathbb{R}^3$.

□

| **Standard V4.** | Mark: |
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Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \;\middle|\; x, y, z \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^4$.

**Solution:** It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□

| **Additional Notes/Marks** | |
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