

## Section E.2

**Remark E.21** In practice, if we simply need to convert a matrix into reduced row echelon form, we use technology to do so.

However, it is also important to understand the **Gauss-Jordan elimination** algorithm that a computer or calculator uses to convert a matrix (augmented or not) into reduced row echelon form. Understanding this algorithm will help us better understand how to interpret the results in many applications we use it for in Module V.

**Activity E.22** ( $\sim 8$  min) Consider the matrix

$$\begin{bmatrix} 2 & 6 & -1 & 6 \\ 1 & 3 & -1 & 2 \\ -1 & -3 & 2 & 0 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

- (a) Add row 3 to row 2 ( $R_2 + R_3 \rightarrow R_2$ )
- (b) Add row 2 to row 3 ( $R_3 + R_2 \rightarrow R_3$ )
- (c) Swap row 1 to row 2 ( $R_1 \leftrightarrow R_2$ )
- (d) Add -2 row 2 to row 1 ( $R_1 - 2R_2 \rightarrow R_1$ )

**Activity E.23** ( $\sim 7$  min) Consider the matrix

$$\begin{bmatrix} \textcircled{1} & 3 & -1 & 2 \\ 2 & 6 & -1 & 6 \\ -1 & -3 & 2 & 0 \end{bmatrix}.$$

Which row operation is the best choice for the next move in converting to RREF?

- (a) Add row 1 to row 3 ( $R_3 + R_1 \rightarrow R_3$ )
- (b) Add -2 row 1 to row 2 ( $R_2 - 2R_1 \rightarrow R_2$ )
- (c) Add 2 row 2 to row 3 ( $R_3 + 2R_2 \rightarrow R_3$ )
- (d) Add 2 row 3 to row 2 ( $R_2 + 2R_3 \rightarrow R_2$ )

**Activity E.24** ( $\sim 5$  min) Consider the matrix

$$\begin{bmatrix} \textcircled{1} & 3 & -1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

Which row operation is the best choice for the next move in converting to RREF?

- (a) Add row 1 to row 2 ( $R_2 + R_1 \rightarrow R_2$ )
- (b) Add -1 row 3 to row 2 ( $R_2 - R_3 \rightarrow R_2$ )
- (c) Add -1 row 2 to row 3 ( $R_3 - R_2 \rightarrow R_3$ )
- (d) Add row 2 to row 1 ( $R_1 + R_2 \rightarrow R_1$ )

**Activity E.25** ( $\sim 10$  min) Consider the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & -1 & 1 \end{bmatrix}.$$

*Part 1:* Perform three row operations to produce a matrix closer to RREF.

*Part 2:* Finish putting it in RREF.

**Activity E.26** ( $\sim 10$  min) Consider the matrix

$$A = \begin{bmatrix} 2 & 3 & 2 & 3 \\ -2 & 1 & 6 & 1 \\ -1 & -3 & -4 & 1 \end{bmatrix}.$$

Compute  $\text{RREF}(A)$ .

**Activity E.27** ( $\sim 10$  min) Consider the matrix

$$A = \begin{bmatrix} 2 & 4 & 2 & -4 \\ -2 & -4 & 1 & 1 \\ 3 & 6 & -1 & -4 \end{bmatrix}.$$

Compute  $\text{RREF}(A)$ .

**Remark E.28** A video example of how to perform the Gauss-Jordan Elimination algorithm by hand is available at <https://youtu.be/Cq0Nxx2dhhU>.

Practicing several exercises on your own using this method is strongly recommended.

**Activity E.29** (*~10 min*) Free browser-based technologies for mathematical computation are available online.

- Go to <https://octave-online.net>.
- Type `A=sym([1 3 4 ; 2 5 7])` and press **Enter** to store the matrix  $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$  in the variable  $A$ .
  - The symbolic function `sym` is used to calculate precise answers rather than floating-point approximations.
  - The vertical bar in an augmented matrix does not affect row operations, so the RREF of  $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$  may be computed in the same way.
- Type `rref(A)` and press **Enter** to compute the reduced row echelon form of  $A$ .

**Remark E.30** We will frequently need to know the reduced row echelon form of matrices during class, so feel free to use Octave-Online.net to compute RREF efficiently.

You may alternatively use the calculator you will use during assessments. Be sure to use fractions mode to compute exact solutions rather than floating-point approximations.