MASTERY QUIZ DAY 8 Math 237 – Linear Algebra

Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array}\right]$$

Solution:

$$2x_1 - x_2 = 1$$
$$-x_1 + 4x_2 + x_3 = -7$$
$$x_1 + 2x_2 - x_3 = 0$$

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E3. Find the solution set for the following system of linear equations.

$$2x_1 - 2x_2 + 6x_3 - x_4 = -1$$
$$3x_1 + 6x_3 + x_4 = 5$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = -7$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & -1 \\ 3 & 0 & 6 & 1 & 5 \\ -4 & 1 & -9 & 2 & -7 \end{array}\right]$, so RREF $A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$. It follows that the

solution set is given by $\begin{bmatrix} 2 - 2a \\ 3 + a \\ a \\ -1 \end{bmatrix}$ for all real numbers a.

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E4. Find a basis for the solution set of the system of equations

$$x + 3y + 3z + 7w = 0$$
$$x + 3y - z - w = 0$$
$$2x + 6y + 3z + 8w = 0$$
$$x + 3y - 2z - 3w = 0$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

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V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector addition $\oplus$ is associative.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$
\begin{aligned}
(x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\
&= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\
&= \sqrt{x^2 + y^2 + z^2} \\
&= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\
&= x \oplus \sqrt{y^2 + z^2} \\
&= x \oplus (y \oplus z)
\end{aligned}
$$

However, this is not a vector space, as there is no zero vector.

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E1: ____   **E3:** ____   **E4:** ____   **V1:** ____   **E2:** ____