Name: ________________________________

MASTERY QUIZ DAY 14 Math 237 – Linear Algebra
Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all points on the line $x+y=2$ with the operations, for any $(x_1,y_1),(x_2,y_2)\in V$, $c\in\mathbb{R}$,

$$\begin{aligned}(x_1,y_1)\oplus(x_2,y_2)&=(x_1+x_2-1,y_1+y_2-1)\\ c\odot(x_1,y_1)&=(cx_1-(c-1),cy_1-(c-1))\end{aligned}$$

(a) Show that this vector space has an **additive identity** element $\mathbf{0}$ satisfying $(x,y)\oplus\mathbf{0}=(x,y)$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1,y_1)\in V$; then $(x_1,y_1)\oplus(1,1)=(x_1,y_1)$, so $(1,1)$ is an additive identity element.

Now we will show the other seven properties. Let $(x_1,y_1),(x_2,y_2)\in V$, and let $c,d\in\mathbb{R}$.

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) The additive identity is $(1,1)$.

4) $(x_1,y_1)\oplus(2-x_1,2-y_1)=(1,1)$, so $(2-x_1,2-y_1)$ is the additive inverse of (x_1,y_1).

5)

$$\begin{aligned}c\odot(d\odot(x_1,y_1))&=c\odot(dx_1-(d-1),dy_1-(d-1))\\&=(c(dx_1-(d-1))-(c-1),c(dy_1-(d-1)))\\&=(cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\&=(cdx_1-(cd-1),cdy_1-(cd-1))\\&=(cd)\odot(x_1,y_1)\end{aligned}$$

6) $1\odot(x_1,y_1)=(x_1-(1-1),y_1-(1-1)=(x_1,y_1)$

7)

$$\begin{aligned}c\odot((x_1,y_1)\oplus(x_2,y_2))&=c\odot(x_1+y_1-1,x_2+y_2-1)\\&=(c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\&=(cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\&=(cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\&=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot(x_1,y_1)&=((c+d)x_1-(c+d-1),(c+d)y_1-(c+d-1))\\&=(cx_1-(c-1),cy_1-(c-1))\oplus(dx_1-(d-1),dy_1-(d-1))\\&=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

Therefore V is a vector space.

□

V3. Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix},\begin{bmatrix}3\\12\\-9\end{bmatrix},\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}=\mathbb{R}^3$?

Solution: Since

$$\text{RREF}\begin{bmatrix}2 & 3 & 1 & -4\\ -1 & 12 & 2 & 2\\ 4 & -9 & 3 & -8\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & -2\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

$\square$

V4. Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x-x^2$ and x^2 are both in W, but $(x-x^2)+(x^2)=x\notin W$.

$\square$

S2. Determine if the set $\left\{x^2+x-1, 3x^2-x+1, 2x^2-2\right\}$ is a basis of $\mathcal{P}^2$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\ 1 & -1 & 0\\ -1 & 1 & -2\end{bmatrix}\right)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

V1: $\boxed{}$ **V3:** $\boxed{}$ **V4:** $\boxed{}$ **S2:** $\boxed{}$