MASTERY QUIZ DAY 10   Math 237 – Linear Algebra
Version 1   Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard E1.	

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array}\right]$$

Solution:

$$-4x_1 - x_2 + 3x_3 = 2$$
$$x_1 + 2x_2 - x_3 = 0$$
$$-x_1 + 4x_2 + x_3 = 4$$

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	Mark:
Standard E3.	

Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

Solution:

$$\mathrm{RREF}\left(\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array}\right]\right) = \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The solutions are

$$\left\{ \begin{bmatrix} -1 - \dfrac{c}{2} \\ -1 - \dfrac{3c}{2} \\ c \end{bmatrix} \, \middle| \, c \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} c - 1 \\ 3c - 1 \\ -2c \end{bmatrix} \, \middle| \, c \in \mathbb{R} \right\}$$

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Find the solution set for the following system of linear equations.

$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$
$$3x_1 + 6x_3 + x_4 = 0$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

Solution: Let $A = \begin{bmatrix} 2 & -2 & 6 & -1 & | & 0 \\ 3 & 0 & 6 & 1 & | & 0 \\ -4 & 1 & -9 & 2 & | & 0 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 0 & | & 0 \\ 0 & 1 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \end{bmatrix}$. It follows that the basis

for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

□

Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$
$$c \odot (x_1, y_1) = (cx_1 - (c-1), cy_1 - (c-2))$$

(a) Show that this vector space has an additive identity element.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$; then $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so $(1, 1)$ is an additive identity element.
Now we will show the other seven properties. Let $(x_1, y_1), (x_2, y_2) \in V$, and let $c, d \in \mathbb{R}$.

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) The additive identity is $(1, 1)$.

4) $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$, so $(2 - x_1, 2 - y_1)$ is the additive inverse of (x_1, y_1).

5)

$$
\begin{aligned}
c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d-1), dy_1 - (d-1)) \\
&= (c(dx_1 - (d-1)) - (c-1), c(dy_1 - (d-1))) \\
&= (cdx_1 - cd + c - (c-1), cdy_1 - cd + c - (c-1)) \\
&= (cdx_1 - (cd-1), cdy_1 - (cd-1)) \\
&= (cd) \odot (x_1, y_1)
\end{aligned}
$$

6) $1 \odot (x_1, y_1) = (x_1 - (1-1), y_1 - (1-1)) = (x_1, y_1)$

7)

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1)$$
$$= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1))$$
$$= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1)$$
$$= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1))$$
$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

8)

$$(c + d) \odot (x_1, y_1) = ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1))$$
$$= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1))$$
$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

Therefore V is a vector space.

$\square$

Additional Notes/Marks	