Section E.2

Activity E.2.1 *(∼10 min)*

- Go to `http://www.cocalc.com` and create an account.
- Create a project titled "Linear Algebra Team X" with your appropriate team number. Add all team members as collaborators.
- Open the project and click on "New"
- Give it an appropriate name such as "Class E2 workbook". Make a new Jupyter notebook.
- Click on "Kernel" and make sure "Octave" is selected.
- Type `A=[1 3 4 ; 2 5 7]` to store the matrix $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 7 \end{bmatrix}$ in the variable A; hold shift when you press enter.
- Type `rref(A)` to compute the reduced row echelon form of A.

Remark E.2.2 If you need to find the reduced row echelon form of a matrix during class, you should feel free to use CoCalc/Octave.

You can change a cell from "Code" to "Markdown" or "Raw" to put comments around your calculations such as Activity numbers.

Activity E.2.3 *(∼8 min)* Consider the system of equations.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 + 3x_2 - 6x_3 &= 11 \end{aligned}$$

Convert this to an augmented matrix, use CoCalc to compute the reduced row echelon form, and convert back to a simpler system of equations to solve this system. Write your solution on your whiteboard.

Activity E.2.4 *(∼7 min)* Consider our system of equations from above.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 \qquad\quad - 3x_3 &= 1 \end{aligned}$$

Convert this to an augmented matrix, use CoCalc to compute the reduced row echelon form, and convert back to a simpler system of equations to solve this system. Write your solution on your whiteboard.

Activity E.2.5 *(∼10 min)* Consider the following matrix.

$$A = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 4 & 8 & 0 \end{array}\right]$$

Part 1: Find RREF(A) (Use CoCalc).
Part 2: How many solutions does the corresponding linear system have?

Activity E.2.6 *(∼10 min)* Consider the (simpler) system from the previous problem:

$$\begin{aligned} x_1 + 2x_2 \quad &= 4 \\ x_3 &= -1 \end{aligned}$$

Part 1: Let $x_1 = a$ and write the solution set in the form $\left\{ \begin{bmatrix} a \\ ? \\ ? \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$

Part 2: Let $x_2 = b$ and write the solution set in the form $\left\{ \begin{bmatrix} ? \\ b \\ ? \end{bmatrix} \middle| \, b \in \mathbb{R} \right\}$

Part 3: Which of these was easier? What features of the RREF matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

cause this?

Definition E.2.7 If a matrix is in reduced row echelon form, a **pivot** is an entry satisfying

1. It is 1
2. Everything else in the same row but to the left of it is zero
3. Everything else in the same column is zero.

For example, the pivots are circled in

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Activity E.2.8 *(~5 min)* Circle the pivots in each matrix below.

$$\left[\begin{array}{cccc|c}1&1&0&0&2\\0&0&1&0&0\\0&0&0&1&1\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1&0&1&2\\0&1&0&1\end{array}\right]$$

$$\left[\begin{array}{cccc|c}1&1&0&1&0\\0&0&1&1&0\\0&0&0&0&0\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1&0&0&2\\0&1&1&0\\0&0&0&0\end{array}\right]$$

Definition E.2.9 The pivots in a matrix correspond to **bound variables** in the system of equations. The remaining variables are called **free variables**.
To efficiently solve a system in RREF form, assign letters to free variables and solve for the bound variables.

Activity E.2.10 *(~10 min)* Find the solution set for the system

$$\begin{alignedat}{6}2x_1&-2x_2&-6x_3&+x_4&-x_5&=&3\\-x_1&+x_2&+3x_3&-x_4&+2x_5&=&-3\\x_1&-2x_2&-x_3&+x_4&+x_5&=&2\end{alignedat}$$

by assigning letters to the free variables and solving for the bounded variables.

Observation E.2.11 The solution set to the system

$$\begin{alignedat}{6}2x_1&-2x_2&-6x_3&+x_4&-x_5&=&3\\-x_1&+x_2&+3x_3&-x_4&+2x_5&=&-3\\x_1&-2x_2&-x_3&+x_4&+x_5&=&2\end{alignedat}$$

is

$$\left\{\begin{bmatrix}1+5a+2b\\1+2a+3b\\a\\3+3b\\b\end{bmatrix}\,\middle|\, a,b\in\mathbb{R}\right\}.$$

Remark E.2.12 You should always use set-builder notation to describe the solution set of a linear system.