

Application Activities - Module M Part 2 - Class Day 22

Activity 22.1 Let $A = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$. Find a 3×3 matrix I such that $IA = A$, that is,

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Definition 22.2 The identity matrix I_n (or just I when n is obvious from context) is the $n \times n$ matrix

$$I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix}.$$

It has a 1 on each diagonal element and a 0 in every other position.

Fact 22.3 For any square matrix A , $IA = AI = A$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Activity 22.4 Each row operation can be interpreted as a type of matrix multiplication.

Part 1: Tweak the identity matrix slightly to create a matrix that doubles the third row of A :

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 2 & 2 & -2 \end{bmatrix}$$

Part 2: Create a matrix that swaps the second and third rows of A :

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 7 & -1 \\ 1 & 1 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

Part 3: Create a matrix that adds 5 times the third row of A to the first row:

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2+5 & 7+5 & -1-5 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Fact 22.5 If R is the result of applying a row operation to I , then RA is the result of applying the same row operation to A .

This means that for any matrix A , we can find a series of matrices R_1, \dots, R_k corresponding to the row operations such that

$$R_1 R_2 \cdots R_k A = \text{RREF}(A).$$

That is, row reduction can be thought of as the result of matrix multiplication.

Activity 22.6 Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear map with standard matrix A . Sort the following items into groups of statements about T .

- | | |
|----------------------------------------------------------------------|------------------------------------------------------|
| (a) T is injective (i.e. one-to-one) | (g) The columns of A span \mathbb{R}^m |
| (b) T is surjective (i.e. onto) | (h) The columns of A are linearly independent |
| (c) T is bijective (i.e. both injective and surjective) | (i) The columns of A are a basis of \mathbb{R}^m |
| (d) $AX = B$ has a solution for all $m \times 1$ matrices B | (j) Every column of $\text{RREF}(A)$ has a pivot |
| (e) $AX = B$ has a unique solution for all $m \times 1$ matrices B | (k) Every row of $\text{RREF}(A)$ has a pivot |
| (f) $AX = 0$ has a unique solution. | (l) $m = n$ and $\text{RREF}(A) = I$ |
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Activity 22.7 Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear map with matrix A . If T is injective, which of the following cannot be true?

- (a) A has strictly more columns than rows
 - (b) A has the same number of rows as columns (i.e. A is square)
 - (c) A has strictly more rows than columns
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Activity 22.8 Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear map with matrix A . If T is surjective, which of the following cannot be true?

- (a) A has strictly more columns than rows
 - (b) A has the same number of rows as columns (i.e. A is square)
 - (c) A has strictly more rows than columns
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Activity 22.9 Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear map with matrix A . If T is bijective, which of the following cannot be true?

- (a) A has strictly more columns than rows
 - (b) A has the same number of rows as columns (i.e. A is square)
 - (c) A has strictly more rows than columns
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