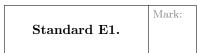
Name:	
J#:	Dr. Clontz
Date:	

## MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

## Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Solution:

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

## Standard E3.

Find the solution set for the following system of linear equations.

Mark:

$$2x_1 - 2x_2 + 6x_3 - x_4 = -1$$
$$3x_1 + 6x_3 + x_4 = 5$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = -7$$

**Solution:** Let 
$$A = \begin{bmatrix} 2 & -2 & 6 & -1 & | & -1 \\ 3 & 0 & 6 & 1 & | & 5 \\ -4 & 1 & -9 & 2 & | & -7 \end{bmatrix}$$
, so RREF  $A = \begin{bmatrix} 1 & 0 & 2 & 0 & | & 2 \\ 0 & 1 & -1 & 0 & | & 3 \\ 0 & 0 & 0 & 1 & | & -1 \end{bmatrix}$ . It follows that the solution set is given by 
$$\begin{bmatrix} 2 - 2a \\ 3 + a \\ a \\ -1 \end{bmatrix}$$
 for all real numbers  $a$ .

Find a basis for the solution set of the system of equations

$$x + 3y + 3z + 7w = 0$$
$$x + 3y - z - w = 0$$
$$2x + 6y + 3z + 8w = 0$$
$$x + 3y - 2z - 3w = 0$$

Solution:

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3\\-1\\0\\0 \end{bmatrix}, \begin{bmatrix} 1\\0\\2\\-1 \end{bmatrix} \right\}$$

Standard V1.

Mark:

Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (0, cy_1)$ 

- (a) Show that this scalar multiplication ⊙ distributes over scalar addition.
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1) \in V$ , and let  $c, d \in \mathbb{R}$ . Then

$$(c+d) \odot (x_1, y_1) = (0, (c+d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as  $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$ .

Additional Notes/Marks