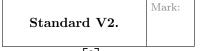
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MASTERY QUIZ DAY 15

Math 237 – Linear Algebra Fall 2017

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Determine if $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3\\0\\-1 \end{bmatrix}$, $\begin{bmatrix} 1\\-1\\4 \end{bmatrix}$, and $\begin{bmatrix} 5\\1\\-6 \end{bmatrix}$.

Solution:

RREF
$$\left(\begin{bmatrix} 3 & 1 & 5 & 1 \\ 0 & -1 & 1 & 4 \\ -1 & 4 & -6 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is not a linear combination of the three vectors.

Standard S1.

Determine if the vectors $\begin{bmatrix} 1\\1\\-1 \end{bmatrix}$, $\begin{bmatrix} 3\\-1\\1 \end{bmatrix}$, and $\begin{bmatrix} 2\\0\\-2 \end{bmatrix}$ are linearly dependent or linearly independent

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{pmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since each column is a pivot column, the vectors are linearly independent.

Standard S3.

Let W be the subspace of \mathcal{P}_3 given by $W = \text{span}\left(\left\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\right\}\right)$. Find a basis for W.

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is $\{x^3 + x^2 + 2x + 1, 3x^3 - x^2 + 3x - 2\}$.

Standard S4.

Mark:

Let W be the subspace of $M_{2,2}$ given by $W = \operatorname{span}\left(\left\{\begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

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Math 237 – Linear Algebra Fall 2017

Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Determine if $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Solution: Since

$$RREF\left(\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & -2 \\ -3 & -6 & 0 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

contains the contradiction 0 = 1, $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ is not a linear combination of the three vectors.

Standard S1.

Determine if the vectors $\begin{bmatrix} 1\\1\\-1 \end{bmatrix}$, $\begin{bmatrix} 3\\-1\\1 \end{bmatrix}$, and $\begin{bmatrix} 2\\0\\-2 \end{bmatrix}$ are linearly dependent or linearly independent

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{pmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since each column is a pivot column, the vectors are linearly independent.

Standard S3.
$$\begin{bmatrix} & & & \\ & & & & \\ & &$$

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then
$$\left\{ \begin{bmatrix} 1\\-1\\3\\-3 \end{bmatrix}, \begin{bmatrix} 3\\-1\\4\\-2 \end{bmatrix}, \begin{bmatrix} 1\\1\\1\\-7 \end{bmatrix} \right\}$$
 is a basis for W .

Standard S4.

Mark:

Let W be the subspace of $M_{2,2}$ given by $W = \operatorname{span}\left(\left\{\begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

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Math 237 – Linear Algebra Fall 2017

Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standar	d V2 .	Mark:					
Determine if	$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ can be	can be written as a linear combination of the vectors		$\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$	and	$\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$	

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} -1 & 1 & | & 0 \\ -9 & 5 & | & 0 \\ 15 & -5 & | & 2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 1 \end{bmatrix}$$

Since this system has no solution, $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ cannot be written as a linear combination of the vectors $\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$.

Determine if the set of vectors $\left\{ \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\-1 \end{bmatrix}, \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Standard S3.

Mark:

Let W be the subspace of \mathcal{P}_2 given by $W = \text{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$. Find a basis for W.

Solution: Let
$$A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$
, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are pivot columns, $\left\{ -3x^2 - 8x, x^2 + 2x + 2 \right\}$ is a basis for W .

Standard S4. $\begin{bmatrix} & & & \\ & & & \\ & & \\ & & -2 \\ 0 & & 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}. Compute the dimension of <math>W$.

Solution:

 $RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$

This has 3 pivot columns so dim(W) = 3.

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Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard V2.	

Determine if $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3\\0\\-1 \end{bmatrix}$, $\begin{bmatrix} 1\\-1\\4 \end{bmatrix}$, and $\begin{bmatrix} 5\\1\\-6 \end{bmatrix}$.

Solution:

RREF
$$\left(\begin{bmatrix} 3 & 1 & 5 & 1 \\ 0 & -1 & 1 & 4 \\ -1 & 4 & -6 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is not a linear combination of the three vectors.

Standard S1.

Determine if the set of vectors $\left\{ \begin{bmatrix} 3\\-1\\0\\4 \end{bmatrix}, \begin{bmatrix} 1\\2\\-2\\1 \end{bmatrix}, \begin{bmatrix} 3\\-8\\6\\5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

Solution:

$$RREF\left(\begin{bmatrix} 3 & 1 & 3\\ -1 & 2 & -8\\ 0 & -2 & 6\\ 4 & 1 & 5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 2\\ 0 & 1 & -3\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

Standard S3.

Let W be the subspace of \mathcal{P}_3 given by $W = \text{span}\left(\left\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\right\}\right)$. Find a basis for W.

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is $\{x^3 + x^2 + 2x + 1, 3x^3 - x^2 + 3x - 2\}$.

Standard S4.

Mark:

Let W be the subspace of \mathcal{P}_3 given by $W = \mathrm{span}\left(\left\{x^3-x^2+3x-3,2x^3+x+1,3x^3-x^2+4x-2,x^3+x^2+x-7\right\}\right)$. Compute the dimension of W.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

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Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V2.	Mark:	

Determine if $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 2\\3\\-1 \end{bmatrix}$, $\begin{bmatrix} 1\\-1\\0 \end{bmatrix}$, and $\begin{bmatrix} -3\\-2\\5 \end{bmatrix}$.

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 1 & -3 & 1 \\ 3 & -1 & -2 & 4 \\ -1 & 0 & 5 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Since this system has a solution, $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the three vectors.

Determine if the vectors $\begin{bmatrix} 1\\1\\-1 \end{bmatrix}$, $\begin{bmatrix} 3\\-1\\1 \end{bmatrix}$, and $\begin{bmatrix} 2\\0\\-2 \end{bmatrix}$ are linearly dependent or linearly independent

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{pmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since each column is a pivot column, the vectors are linearly independent.

Standard S3.

$$\begin{bmatrix}
\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \\
\end{bmatrix}$$
Let $W = \operatorname{span} \left(\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$. Find a basis of W .

RREF
$$\begin{pmatrix} \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then
$$\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$$
 is a basis of W .

Standard S4.

Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since there are two pivot columns, dim W = 2.

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Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standar	d V	72.	Mark:					
Determine if	$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$	can be	written	as a linear combination of the vectors	$\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$	and	$\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$	

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since this system has no solution, $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ cannot be written as a linear combination of the vectors $\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$.

Determine if the set of vectors $\left\{ \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\-1 \end{bmatrix}, \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Standard S3.
$$\begin{bmatrix} & & & & \\ & & & & \\ &$$

Solution: Let
$$A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$
, and compute $RREF(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are

pivot columns, $\left\{ \begin{bmatrix} -3\\-8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix} \right\}$ is a basis for W.

Standard S4.

Mark:

Let W be the subspace of $M_{2,2}$ given by $W = \operatorname{span}\left(\left\{\begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.