Name: ____________________________

MASTERY QUIZ DAY 23 Math 237 – Linear Algebra
Version 3 Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^4 \to \mathbb{R}^3$ where $S(\vec{e}_1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $S(\vec{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $S(\vec{e}_3) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, and $S(\vec{e}_4) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$,

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ where $T(\vec{e}_1) = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $T(\vec{e}_2) = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$, and $T(\vec{e}_3) = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

Solution:

(a) RREF $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. The map is not injective since it has a column without pivot, but it is surjective because every row has a pivot.

(b) RREF $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$. The map is not injective since there is a column without a pivot, and it is not surjective because there is a row without a pivot.

$\square$

A4. Let $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 0 & 1 & 3 & -4 \\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus $\left\{ \begin{bmatrix} 8 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix} \right\}$ is a basis for the image, and $\left\{ \begin{bmatrix} -1 & -3 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \right\}$ is a basis for the kernel.

$\square$

A3: $\square$ **A4:** $\square$