Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

## Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Determine if the set of polynomials  $\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\}$  is linearly dependent or linearly independent

## Solution:

$$RREF\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Standard S3.

$$\begin{bmatrix}
\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}
\end{bmatrix}$$
Let  $W = \text{span}\left(\left\{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}\right\}\right)$ . Find a basis for  $W$ .

Solution:

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is 
$$\left\{ \begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix} \right\}$$
.

Mark:

Let 
$$W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix},\begin{bmatrix}2\\0\\1\\1\end{bmatrix},\begin{bmatrix}3\\-1\\4\\-2\end{bmatrix},\begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$$
. Compute the dimension of  $W$ .

Solution:

$$RREF \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

## Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}.$ 

Solution:

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

Standard A2.

Mark:

Determine if  $T: \mathbb{R}^2 \to \mathbb{R}^2$  given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.

Mark:

Determine if the set of polynomials  $\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\}$  is linearly dependent or linearly independent

Solution:

$$RREF\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Standard S3.

Mark:

Let W be the subspace of  $\mathcal{P}_2$  given by  $W = \text{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$ . Find a basis for W.

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $RREF(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since the first two columns are

pivot columns,  $\{-3x^2 - 8x, x^2 + 2x + 2\}$  is a basis for W.

Standard S4.

Let  $W = \operatorname{span}\left(\left\{\begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}\right\}\right)$ . Compute the dimension of W.

Solution: Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since there are two pivot columns, dim W = 2.

Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

Solution:

 $\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$ 

Standard A2.

Mark:

Determine if the map  $T: \mathcal{P}^4 \to \mathcal{P}^3$  given by T(f) = f' - f'' is a linear transformation or not.

Name:

J#:

Date:

## MASTERY QUIZ DAY 18

Math 237 – Linear Algebra Fall 2017

Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.

Mark:

Determine if the set of matrices  $\left\{ \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -8 \\ 6 & 5 \end{bmatrix} \right\}$  is linearly dependent or linearly independent.

Solution:

$$RREF \left( \begin{bmatrix} 3 & 1 & 3 \\ -1 & 2 & -8 \\ 0 & -2 & 6 \\ 4 & 1 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

Standard S3.  $\begin{bmatrix} & & & & \\ & &$ 

Solution:

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is  $\left\{ \begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix} \right\}$ .

Standard S4.  $\begin{bmatrix} & & & \\ & & & & \\ & &$ 

Solution: Let 
$$A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$
, and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since there are two pivot columns, dim  $W = 2$ .

Standard A1.

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

Mark:

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

Solution:

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

Standard A2.

Mark:

Determine if  $T: \mathbb{R}^2 \to \mathbb{R}^2$  given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

**Solution:** It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

Additional Notes/Marks

Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

Version 4 Fall 2017 Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.

Determine if the set of vectors 
$$\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$$
 is linearly dependent or linearly independent

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Standard S3.

$$\begin{bmatrix}
1 & 3 & 3 & 3 \\
-1 & 3 & -1 & 1 \\
3 & -3 & 1 & -2 & -2
\end{bmatrix}, \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -7 & -7 & -2 & -7 & -7 \end{bmatrix}$$
Find a basis of  $W$ .

Solution:

$$RREF \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then 
$$\left\{ \begin{bmatrix} 1\\-1\\3\\-3 \end{bmatrix}, \begin{bmatrix} 3\\-1\\4\\-2 \end{bmatrix}, \begin{bmatrix} 1\\1\\1\\-7 \end{bmatrix} \right\}$$
 is a basis for  $W$ .

Let 
$$W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix},\begin{bmatrix}3\\3\\6\\3\end{bmatrix},\begin{bmatrix}3\\-1\\3\\-2\end{bmatrix},\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$$
. Find the dimension of  $W$ .

Solution:

$$RREF \begin{pmatrix} \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

# Standard A1. Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}^4$  be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4.$ 

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

## Standard A2.

Mark:

Determine if  $T: \mathbb{R}^2 \to \mathbb{R}^2$  given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

**Solution:** It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Determine if the set of polynomials  $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$  is linearly dependent or linearly independent

## Solution:

RREF 
$$\left( \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Let  $W = \operatorname{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Find a basis of W.

**Solution:** 

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then 
$$\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$$
 is a basis of  $W$ .

Standard S4.

Let W be the subspace of  $\mathcal{P}_3$  given by  $W = \text{span}\left(\left\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\right\}\right)$ . Compute the dimension of W.

Solution:

$$RREF \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}.$ 

Solution:

 $\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$ 

Standard A2.

Mark:

Determine if  $D: M_{2,2} \to \mathbb{R}$  given by  $D\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$  is a linear transformation or not.

**Solution:** D(I) = 1 but  $D(2I) = 4 \neq 2D(I)$ , so D is not linear.

Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

## Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Determine if the set of polynomials  $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$  is linearly dependent or linearly independent

## Solution:

RREF 
$$\left( \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Standard S3.

$$\begin{bmatrix}
\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \\
\end{bmatrix}$$
Let  $W = \operatorname{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Find a basis of  $W$ .

Solution:

RREF 
$$\begin{pmatrix} \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then 
$$\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$$
 is a basis of  $W$ .

Let 
$$W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix},\begin{bmatrix}3\\3\\6\\3\end{bmatrix},\begin{bmatrix}-1\\3\\-2\end{bmatrix},\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$$
. Find the dimension of  $W$ .

Solution:

$$RREF \begin{pmatrix} \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

Mark:

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

Solution:

 $\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$ 

Standard A2.

Let  $T: \mathbb{R}^2 \to \mathbb{R}^2$  be given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ \sqrt{x}+\sqrt{y} \end{bmatrix}$ . Determine if T is a linear transformation.

**Solution:** 

$$T\left(\begin{bmatrix}0\\4\end{bmatrix}\right) = \begin{bmatrix}4\\2\end{bmatrix} \neq \begin{bmatrix}4\\4\end{bmatrix} = 4T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

So T is not a linear transformation.