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| Name: | |
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| Date: | |

**FINAL EXAM** Math 237 – Linear Algebra
**Version 3** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard E1.** | Mark: |
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array}\right]$$

| **Standard E2.** | Mark: |
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Find RREF $A$, where

$$A = \left[\begin{array}{cc|c} 2 & -7 & 4 \\ 1 & -3 & 2 \\ 3 & 0 & 3 \end{array}\right]$$

| **Standard E3.** | Mark: |
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Solve the following linear system.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

| **Standard E4.** | Mark: |
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Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 0 \\ x_1 + x_2 - x_3 + 5x_4 &= 0 \end{aligned}$$

| Standard V1. | Mark: |
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Let $V$ be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} f \oplus g &= f' + g' \\ c \odot f &= cf' \end{aligned}$$

(here $f'$ denotes the derivative of $f$).

(a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.

(b) Determine if $V$ is a vector space or not. Justify your answer.

| Standard V2. | Mark: |
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Determine if $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

| **Standard V3.** | Mark: |
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Determine if the vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\\0\\-3\end{bmatrix}$, $\begin{bmatrix}0\\3\\0\\-2\end{bmatrix}$, and $\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}$ span $\mathbb{R}^4$.

| **Standard V4.** | Mark: |
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Let $W$ be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix}x\\y\\z\end{bmatrix}$ satisfying $x+y+z=0$ (this forms a plane). Determine if $W$ is a subspace of $\mathbb{R}^3$.

| **Standard S1.** | Mark: |
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Determine if the set of vectors $\left\{\begin{bmatrix}-3\\8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}$ is linearly dependent or linearly independent

| **Standard S2.** | Mark: |
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Determine if the set $\left\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\right\}$ is a basis of $\mathcal{P}^3$.

| **Standard S3.** | Mark: |
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Let $W = \operatorname{span}\left\{ \begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \right\}$. Find a basis for this vector space.

| **Standard S4.** | Mark: |
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Let $W = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$. Find the dimension of $W$.

| **Standard A1.** | Mark: |
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Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 7x + 2y + 3z \\ 0 \end{bmatrix}.$$

Write the matrix for $T$ with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^4$.

| **Standard A2.** | Mark: |
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Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$. Determine if $T$ is a linear transformation.

| Standard A3. | Mark: |
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Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ where $S(\vec{e}_1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $S(\vec{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ where $T(\vec{e}_1) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $T(\vec{e}_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $T(\vec{e}_3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

| Standard A4. | Mark: |
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Let $T : \mathbb{R}^{2\times 3} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix}\right) = \begin{bmatrix} a+x \\ b+y \\ c+z \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

| **Standard M1.** | Mark: |
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Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

| **Standard M2.** | Mark: |
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Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

| **Standard M3.** | Mark: |
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Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$.

| **Standard G1.** | Mark: |
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Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

| **Standard G2.** | Mark: |
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Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & 2 \\ 15 & -5 & 5 \\ -3 & 2 & 1 \end{bmatrix}$.

| **Standard G3.** | Mark: |
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Find the eigenspace associated to the eigenvalue 2 in the matrix $A = \begin{bmatrix} 8 & -3 & 2 \\ 15 & -5 & 5 \\ -3 & 2 & 1 \end{bmatrix}$

| **Standard G4.** | Mark: |
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Compute the geometric multiplicity of the eigenvalue 1 in the matrix $A = \begin{bmatrix} 9 & -3 & 2 \\ 19 & -6 & 5 \\ -11 & 4 & -2 \end{bmatrix}$

| **Additional Notes/Marks** | |
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