Section V.2

Activity V.17 *(∼5 min)* Sketch a representation of all the vectors belonging to span $\left\{\begin{bmatrix}6\\-4\end{bmatrix}, \begin{bmatrix}-3\\2\end{bmatrix}\right\}$ in the xy plane.

Remark V.18 Recall these definitions from last class:

- A **linear combination** of vectors is given by adding scalar multiples of those vectors, such as:

$$\begin{bmatrix}3\\0\\5\end{bmatrix} = 2\begin{bmatrix}1\\-1\\2\end{bmatrix} + 1\begin{bmatrix}1\\2\\1\end{bmatrix}$$

- The **span** of a set of vectors is the collection of all linear combinations of that set, such as:

$$\operatorname{span}\left\{\begin{bmatrix}1\\-1\\2\end{bmatrix}, \begin{bmatrix}1\\2\\1\end{bmatrix}\right\} = \left\{a\begin{bmatrix}1\\-1\\2\end{bmatrix} + b\begin{bmatrix}1\\2\\1\end{bmatrix} \middle| a, b \in \mathbb{R}\right\}$$

Activity V.19 *(∼15 min)* The vector $\begin{bmatrix}-1\\-6\\1\end{bmatrix}$ belongs to span $\left\{\begin{bmatrix}1\\0\\-3\end{bmatrix}, \begin{bmatrix}-1\\-3\\2\end{bmatrix}\right\}$ exactly when there exists a solution to the vector equation $x_1\begin{bmatrix}1\\0\\-3\end{bmatrix} + x_2\begin{bmatrix}-1\\-3\\2\end{bmatrix} = \begin{bmatrix}-1\\-6\\1\end{bmatrix}$.
Part 1: Reinterpret this vector equation as a system of linear equations.
Part 2: Find its solution set, using technology to find RREF of its corresponding augmented matrix.
Part 3: Given this solution set, does $\begin{bmatrix}-1\\-6\\1\end{bmatrix}$ belong to span $\left\{\begin{bmatrix}1\\0\\-3\end{bmatrix}, \begin{bmatrix}-1\\-3\\2\end{bmatrix}\right\}$?

Fact V.20 A vector $\vec{\mathbf{b}}$ belongs to $\operatorname{span}\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n\}$ if and only if the vector equation $x_1\vec{\mathbf{v}}_1 + \cdots + x_n\vec{\mathbf{v}}_{\mathbf{n}} = \vec{\mathbf{b}}$ is consistent.

Quick Check V.21 The following are all equivalent statements:

- The vector $\vec{\mathbf{b}}$ belongs to $\operatorname{span}\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n\}$.
- The vector equation $x_1\vec{\mathbf{v}}_1 + \cdots + x_n\vec{\mathbf{v}}_{\mathbf{n}} = \vec{\mathbf{b}}$ is consistent.
- The linear system corresponding to $[\vec{\mathbf{v}}_1 \dots \vec{\mathbf{v}}_n \,|\, \vec{\mathbf{b}}]$ is consistent.
- $\mathrm{RREF}[\vec{\mathbf{v}}_1 \dots \vec{\mathbf{v}}_n \,|\, \vec{\mathbf{b}}]$ doesn't have a row $[0 \cdots 0 \,|\, 1]$ representing the contradiction $0 = 1$.

Activity V.22 *(∼10 min)* Determine if $\begin{bmatrix} 3 \\ -2 \\ 1 \\ 5 \end{bmatrix}$ belongs to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \\ 2 \end{bmatrix} \right\}$ by solving an appropriate vector equation.

Activity V.23 *(∼5 min)* Determine if $\begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}$ belongs to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$ by solving an appropriate vector equation.

Activity V.24 *(∼10 min)* Does the third-degree polynomial $3y^3 - 2y^2 + y + 5$ in $\mathcal{P}^3$ belong to span$\{y^3 - 3y + 2, -y^3 - 3y^2 + 2y + 2\}$?
Part 1: Reinterpret this question as a question about the solution(s) of a polynomial equation.
Part 2: Answer this equivalent question, and use its solution to answer the original question.