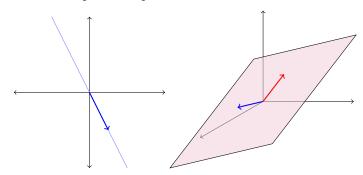
## Application Activities - Module V Part 4 - Class Day 10

**Fact 10.1** At least n vectors are required to span  $\mathbb{R}^n$ .



**Activity 10.2** Choose a vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  in  $\mathbb{R}^3$  that is not in span  $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$  by ensuring  $\begin{bmatrix} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{bmatrix} \sim$ 

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
. (Why does this work?)

Fact 10.3 The set  $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$  fails to span all of  $\mathbb{R}^n$  exactly when RREF $[\mathbf{v}_1 \dots \mathbf{v}_m]$  has a row of zeros:

$$\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\textbf{Activity 10.4 Consider the set of vectors } S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}. \text{ Does } \mathbb{R}^4 = \operatorname{span} S?$$

Activity 10.5 Consider the set of third-degree polynomials

$$S = \left\{2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, -x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\right\}.$$

Does  $\mathcal{P}^3 = \operatorname{span} S$ ?

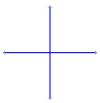
**Definition 10.6** A subset of a vector space is called a **subspace** if it is itself a vector space.

**Fact 10.7** If S is a subset of a vector space V, then span S is a subspace of V.

**Remark 10.8** To prove that a subset is a subspace, you need only verify that  $c\mathbf{v} + d\mathbf{w}$  belongs to the subset for any choice of vectors  $\mathbf{v}$ ,  $\mathbf{w}$  from the subset and any real scalars c, d.

**Activity 10.9** Prove that  $P = \{ax^2 + b : a, b \text{ are both real numbers}\}$  is a subspace of the vector space of all degree-two polynomials by showing that  $c(a_1x^2 + b_1) + d(a_2x^2 + b_2)$  belongs to P.

**Activity 10.10** Consider the subset of  $\mathbb{R}^2$  where at least one coordinate of each vector is 0.



Find a linear combination  $c\mathbf{v} + d\mathbf{w}$  that does not belong to this subset.

Fact 10.11 Suppose a subset S of V is isomorphic to another vector space W. Then S is a subspace of V.

Activity 10.12 Show that the set of  $2 \times 2$  matrices

$$S = \left\{ \begin{bmatrix} a & b \\ -b & -a \end{bmatrix} : a, b \text{ are real numbers} \right\}$$

is a subspace of  $\mathbb{R}^{2\times 2}$  by selecting a Euclidean space isomorphic to S.