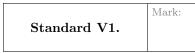
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all real numbers with the operations, for any $x,y\in V,\,c\in\mathbb{R},$

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

- (a) Show that the vector addition \oplus is associative: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$(x \oplus y) \oplus z = \sqrt{x^2 + y^2} \oplus z$$

$$= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2}$$

$$= \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2}$$

$$= x \oplus \sqrt{y^2 + z^2}$$

$$= x \oplus (y \oplus z)$$

However, this is not a vector space, as there is no zero vector.

Standard V3.

Mark:

$$\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}, \text{ and } \begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix} \text{ span } \mathbb{R}^3.$$

Solution:

$$RREF\left(\begin{bmatrix} 8 & -3 & -1 & 4\\ 21 & -8 & -3 & 11\\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1\\ 0 & 1 & 3 & -4\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

Standard V4.

Mark:

Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying x+y+z=0 (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

Standard S2.

Mark

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \mathcal{P}_2

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

Additional Notes/Marks