

## Application Activities - Module V Part 3 - Class Day 9

**Fact 9.1** A vector  $\mathbf{b}$  belongs to  $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  if and only if the linear system corresponding to  $[\mathbf{v}_1 \dots \mathbf{v}_n \mid \mathbf{b}]$  is consistent.

**Remark 9.2** To determine if  $\mathbf{b}$  belongs to  $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ , find  $\text{RREF}[\mathbf{v}_1 \dots \mathbf{v}_n \mid \mathbf{b}]$ .

**Activity 9.3** Determine if  $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$  belongs to  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$  by row-reducing an appropriate matrix.

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**Activity 9.4** Determine if  $\begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}$  belongs to  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$  by row-reducing an appropriate matrix.

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**Observation 9.5** So far we've only discussed linear combinations of Euclidean vectors. Fortunately, many vector spaces of interest can be reinterpreted as an **isomorphic** Euclidean space  $\mathbb{R}^n$ ; that is, a Euclidean space that mirrors the behavior of the vector space exactly.

**Activity 9.6** We previously checked that  $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$  does not belong to  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$ . Does  $f(x) = 3x^2 - 2x + 1$  belong to  $\text{span}\{x^2 - 3, -x^2 - 3x + 2\}$ ?

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**Activity 9.7** Does the matrix  $\begin{bmatrix} 6 & 3 \\ 2 & -1 \end{bmatrix}$  belong to  $\text{span}\left\{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}\right\}$ ?

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**Activity 9.8** Does the complex number  $2i$  belong to  $\text{span}\{-3 + i, 6 - 2i\}$ ?

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**Activity 9.9** How many vectors are required to span  $\mathbb{R}^2$ ? Sketch a drawing in the  $xy$  plane to support your answer.

- (a) 1
  - (b) 2
  - (c) 3
  - (d) 4
  - (e) Infinitely Many
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**Activity 9.10** How many vectors are required to span  $\mathbb{R}^3$ ?

- (a) 1
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- (b) 2
  - (c) 3
  - (d) 4
  - (e) Infinitely Many
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