

Section V.2

Activity V.17 (*~5 min*) Sketch a representation of all the vectors belonging to $\text{span} \left\{ \begin{bmatrix} 6 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix} \right\}$ in the xy plane.

Remark V.18 Recall these definitions from last class:

- A **linear combination** of vectors is given by adding scalar multiples of those vectors, such as:

$$\begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

- The **span** of a set of vectors is the collection of all linear combinations of that set, such as:

$$\text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\} = \left\{ a \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

Activity V.19 (*~15 min*) The vector $\begin{bmatrix} -1 \\ -6 \\ 1 \end{bmatrix}$ belongs to $\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$ exactly when there exists a solution to the vector equation $x_1 \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -6 \\ 1 \end{bmatrix}$.

Part 1: Reinterpret this vector equation as a system of linear equations.

Part 2: Find its solution set, using technology to find RREF of its corresponding augmented matrix.

Part 3: Given this solution set, does $\begin{bmatrix} -1 \\ -6 \\ 1 \end{bmatrix}$ belong to $\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$?

Fact V.20 A vector $\vec{\mathbf{b}}$ belongs to $\text{span}\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n\}$ if and only if the vector equation $x_1\vec{\mathbf{v}}_1 + \dots + x_n\vec{\mathbf{v}}_n = \vec{\mathbf{b}}$ is consistent.

Quick Check V.21 The following are all equivalent statements:

- The vector $\vec{\mathbf{b}}$ belongs to $\text{span}\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n\}$.
- The vector equation $x_1\vec{\mathbf{v}}_1 + \dots + x_n\vec{\mathbf{v}}_n = \vec{\mathbf{b}}$ is consistent.
- The linear system corresponding to $[\vec{\mathbf{v}}_1 \dots \vec{\mathbf{v}}_n \mid \vec{\mathbf{b}}]$ is consistent.
- $\text{RREF}[\vec{\mathbf{v}}_1 \dots \vec{\mathbf{v}}_n \mid \vec{\mathbf{b}}]$ doesn't have a row $[0 \dots 0 \mid 1]$ representing the contradiction $0 = 1$.

Activity V.22 (*~10 min*) Determine if $\begin{bmatrix} 3 \\ -2 \\ 1 \\ 5 \end{bmatrix}$ belongs to $\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \\ 2 \end{bmatrix} \right\}$ by solving an appropriate vector equation.

Activity V.23 (*~5 min*) Determine if $\begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}$ belongs to $\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$ by solving an appropriate vector equation.

Activity V.24 (*~10 min*) Does the third-degree polynomial $3y^3 - 2y^2 + y + 5$ in \mathcal{P}^3 belong to $\text{span}\{y^3 - 3y + 2, -y^3 - 3y^2 + 2y + 2\}$?

Part 1: Reinterpret this question as a question about the solution(s) of a polynomial equation.

Part 2: Answer this equivalent question, and use its solution to answer the original question.