

Name:
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Dr. Clontz

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array} \right]$$

Solution:

$$\begin{aligned} 3x_1 - x_2 + x_4 &= 5 \\ -x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 &= -3 \end{aligned}$$

□

Standard E3.	Mark:
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Solve the system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

Solution:

$$\text{RREF} \left(\left(\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right) \right) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{array} \right)$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a \\ 3-21a \\ -7a \\ 12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

□

Standard E4.	Mark:
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Find a basis for the solution set to the homogeneous system of equations

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 0$$

$$-2x_3 - 4x_4 = 0$$

$$2x_1 + 2x_2 + x_3 - 4x_4 = 0$$

Solution: Let $A = \begin{bmatrix} 4 & 4 & 3 & -6 & | & 0 \\ 0 & 0 & -2 & -4 & | & 0 \\ 2 & 2 & 1 & -4 & | & 0 \end{bmatrix}$, so $\text{RREF } A = \begin{bmatrix} 1 & 1 & 0 & -3 & | & 0 \\ 0 & 0 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

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Standard V1.	Mark:
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Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$

$$c \odot x = cx - 3(c - 1)$$

- (a) Show that this scalar multiplication \odot is associative.
 (b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\ &= c(dx - 3(d - 1)) - 3(c - 1) \\ &= cdx - 3(cd - 1) \\ &= (cd) \odot x \end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.
- 3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .
- 4) Real addition is commutative, so \oplus is commutative.
- 5) Associativity shown above
- 6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\&= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\&= (c \odot x) \oplus (d \odot x)\end{aligned}$$

Therefore V is a vector space.

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Additional Notes/Marks	
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