

Name:
J#:
Date:

Dr. Clontz

## MASTERY QUIZ DAY 17

Math 237 – Linear Algebra

### Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
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Does span  $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5?$

**Solution:** Since there are only three vectors, they cannot span  $\mathbb{R}^5$ .

□

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying  $x + y + z = 0$  (this forms a plane). Determine if  $W$  is a subspace of  $\mathbb{R}^3$ .

**Solution:** Yes, because  $z = -x - y$  and  $a \begin{bmatrix} x_1 \\ y_1 \\ -x_1 - y_1 \end{bmatrix} + b \begin{bmatrix} x_2 \\ y_2 \\ -x_2 - y_2 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ ay_1 + by_2 \\ -(ax_1 + bx_2) - (ay_1 + by_2) \end{bmatrix}$ .

Alternately, yes because  $W$  is isomorphic to  $\mathbb{R}^2$ .

□

<b>Standard S2.</b>	Mark:
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Determine if the set  $\left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\}$  is a basis of  $M_{2,2}$  or not.

**Solution:**

$$\text{RREF} \begin{bmatrix} 1 & -1 & -1 & 0 \\ -3 & 4 & 0 & 3 \\ 2 & -1 & 2 & 3 \\ 2 & 1 & 1 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

<b>Additional Notes/Marks</b>	
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Math 237 – Linear Algebra

## Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ , and  $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
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Determine if  $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$  a subspace of  $\mathbb{R}^4$ .

**Solution:** It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from  $\mathbb{R}^3 \rightarrow \mathbb{R}^4$  given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

<b>Additional Notes/Marks</b>	
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## MASTERY QUIZ DAY 17

Math 237 – Linear Algebra

### Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
---------------------	-------

Determine if the vectors  $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ , and  $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
---------------------	-------

Determine if  $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$  a subspace of  $\mathbb{R}^4$ .

**Solution:** It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from  $\mathbb{R}^3 \rightarrow \mathbb{R}^4$  given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\}$  is a basis of  $M_{2,2}$  or not.

**Solution:**

$$\text{RREF} \begin{bmatrix} 1 & -1 & -1 & 0 \\ -3 & 4 & 0 & 3 \\ 2 & -1 & 2 & 3 \\ 2 & 1 & 1 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

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Math 237 – Linear Algebra

## Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
---------------------	-------

Determine if the vectors  $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$ , and  $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span  $\mathbb{R}^4$ .

□

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all polynomials of even degree. Determine if  $W$  is a subspace of the vector space of all polynomials.

**Solution:**  $W$  is closed under scalar multiplication, but not under addition. For example,  $x - x^2$  and  $x^2$  are both in  $W$ , but  $(x - x^2) + (x^2) = x \notin W$ .

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$  is a basis of  $\mathcal{P}_2$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

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Math 237 – Linear Algebra

## Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
---------------------	-------

Determine if the vectors  $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ , and  $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
---------------------	-------

Let  $W$  be the set of all polynomials of the form  $ax^3 + bx$ . Determine if  $W$  is a subspace of  $\mathcal{P}^3$ .

**Solution:** Yes because  $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$  also belongs to  $W$ . Alternately, yes because  $W$  is isomorphic to  $\mathbb{R}^2$ .

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$  is a basis of  $\mathcal{P}_3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

□

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## Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V3.</b>	Mark:
---------------------	-------

Determine if the vectors  $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$ , and  $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$  span  $\mathbb{R}^3$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span  $\mathbb{R}^3$ .

□

<b>Standard V4.</b>	Mark:
---------------------	-------

Let  $W$  be the set of all polynomials of even degree. Determine if  $W$  is a subspace of the vector space of all polynomials.

**Solution:**  $W$  is closed under scalar multiplication, but not under addition. For example,  $x - x^2$  and  $x^2$  are both in  $W$ , but  $(x - x^2) + (x^2) = x \notin W$ .

□

<b>Standard S2.</b>	Mark:
---------------------	-------

Determine if the set  $\left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\}$  is a basis of  $M_{2,2}$  or not.

**Solution:**

$$\text{RREF} \begin{bmatrix} 1 & -1 & -1 & 0 \\ -3 & 4 & 0 & 3 \\ 2 & -1 & 2 & 3 \\ 2 & 1 & 1 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

<b>Additional Notes/Marks</b>	
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