

Name: _____

MIDTERM EXAM

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

E2. Find RREF A , where

$$A = \left[\begin{array}{ccc|c} 2 & -1 & 5 & 4 \\ -1 & 0 & -2 & -1 \\ 1 & 3 & -1 & -5 \end{array} \right]$$

E3. Solve the system of equations

$$\begin{aligned} -3x + y &= 2 \\ -8x + 2y - z &= 6 \\ 2y + 3z &= -2 \end{aligned}$$

E4. Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 0 \\ -2x_3 - 4x_4 &= 0 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= 0 \end{aligned}$$

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$$

(a) Show that scalar multiplication **distributes scalars** over vector addition:

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 4 \\ 3 \end{bmatrix} \right\}$.

V3. Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V4. Let W be the set of all complex numbers $a + bi$ satisfying $a = 2b$. Determine if W is a subspace of \mathbb{C} .

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

S2. Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4 .

S3. Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find a basis for W .

S4. Let $W = \text{span} \left(\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$. Compute the dimension of W .

E1:

E2:

E3:

E4:

V1:

V2:

V3:

V4:

S1:

S2:

S3:

S4: