Definitions - Module E Part 1 - Class Day 3

Definition 3.1 A **linear equation** is an equation of the variables x_i of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b.$$

A **solution** for a linear equation is expressed in terms of the Euclidean vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$$

and must satisfy

$$a_1s_1 + a_2s_2 + \cdots + a_ns_n = b.$$

Definition 3.4 A **system of linear equations** (or a **linear system** for short) is a collection of one or more linear equations.

$$\begin{alignedat}{4} a_{11}x_1 + {} & a_{12}x_2 + \ldots + {} & a_{1n}x_n & = b_1 \\ a_{21}x_1 + {} & a_{22}x_2 + \ldots + {} & a_{2n}x_n & = b_2 \\ \vdots \quad & \quad \vdots & \vdots \quad & \quad \vdots \\ a_{m1}x_1 + {} & a_{m2}x_2 + \ldots + {} & a_{mn}x_n & = b_m \end{alignedat}$$

A **solution**

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$$

for a linear system satisfies

$$a_{i1}s_1 + a_{i2}s_2 + \cdots + a_{in}s_n = b_i$$

for $1 \leq i \leq m$ (that is, the solution satisfies all equations in the system).

Definition 3.6 A linear system is **consistent** if there exists a solution for the system. Otherwise it is **inconsistent**.

Definition 3.14 A system of m linear equations with n variables is often represented by writing its coefficients and constants in an **augmented matrix**.

$$\begin{alignedat}{4} a_{11}x_1 + {} & a_{12}x_2 + \ldots + {} & a_{1n}x_n & = b_1 \\ a_{21}x_1 + {} & a_{22}x_2 + \ldots + {} & a_{2n}x_n & = b_2 \\ \vdots \quad & \quad \vdots & \vdots \quad & \quad \vdots \\ a_{m1}x_1 + {} & a_{m2}x_2 + \ldots + {} & a_{mn}x_n & = b_m \end{alignedat}$$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

Definition 3.15 Two systems of linear equations (and their corresponding augmented matrices) are said to be **equivalent** if they have the same solution set.
For example, both of these systems have a single solution: $(x_1, x_2) = (1, 1)$.

$$\begin{aligned} 3x_1 - 2x_2 &= 1 \\ x_1 + 4x_2 &= 5 \end{aligned} \qquad\qquad \begin{aligned} 3x_1 - 2x_2 &= 1 \\ 4x_1 + 2x_2 &= 6 \end{aligned}$$

Therefore these augmented matrices are equivalent:

$$\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 1 & 4 & 5 \end{array}\right] \qquad\qquad \left[\begin{array}{cc|c} 3 & -2 & 1 \\ 4 & 2 & 6 \end{array}\right]$$