MASTERY QUIZ DAY 8   Math 237 – Linear Algebra
Version 5   Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show
all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array}\right]$$

Solution:

$$2x_1 - x_2 = 1$$
$$-x_1 + 4x_2 + x_3 = -7$$
$$x_1 + 2x_2 - x_3 = 0$$

□

E3. Solve the following linear system.

$$3x + 2y + z = 7$$
$$x + y + z = 1$$
$$-2x + 3z = -11$$

Solution: Let $A = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 7 \\ 1 & 1 & 1 & 1 \\ -2 & 0 & 3 & 11 \end{array}\right]$, so RREF $A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right]$. It follows that the system has exactly
one solution: $\begin{bmatrix} 4 & -2 & -1 \end{bmatrix}$

□

E4. Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$
$$2x + y - 4z = 0$$
$$3y - 2z = 0$$
$$x - y - z = 0$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -\dfrac{5}{3} \\ 0 & 1 & -\dfrac{2}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \dfrac{5}{3}a \\ \dfrac{2}{3}a \\ a \end{bmatrix} \ \middle| \ a \in \mathbb{R} \right\}$$

So a basis is $\left\{ \begin{bmatrix} \dfrac{5}{3} \\ \dfrac{2}{3} \\ 1 \end{bmatrix} \right\}$ or $\left\{ \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} \right\}$.

□

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$$

Determine if V is a vector space or not.

Solution: V is not a vector space, as one of the distributive laws fails, namely

$$(c + d) \odot (x_1, y_1) = ((c+d)^2 x_1, (c+d)^3 y_1) \neq ((c^2 + d^2)x_1, (c^3 + d^3)y_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

□

E1: ☐　　　**E3:** ☐　　　**E4:** ☐　　　**V1:** ☐　　　**E2:** ☐