

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 10**

Math 237 – Linear Algebra

**Version 6**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 0$$

$$x - z = 1$$

**Solution:**

$$\left[ \begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{array} \right]$$

□

**E3.** Solve the system of equations

$$-3x + y = 2$$

$$-8x + 2y - z = 6$$

$$2y + 3z = -2$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right] \right) = \left[ \begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The solutions are

$$\left\{ \left[ \begin{array}{c} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{array} \right] \mid c \in \mathbb{R} \right\} = \left\{ \left[ \begin{array}{c} c - 1 \\ 3c - 1 \\ -2c \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

**E4.** Find a basis for the solution set to the homogeneous system of equations

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 0$$

$$-2x_3 - 4x_4 = 0$$

$$2x_1 + 2x_2 + x_3 - 4x_4 = 0$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 4 & 4 & 3 & -6 & 0 \\ 0 & 0 & -2 & -4 & 0 \\ 2 & 2 & 1 & -4 & 0 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$ . It follows that the basis for

the solution set is given by  $\left\{ \left[ \begin{array}{c} -1 \\ 1 \\ 0 \\ 0 \end{array} \right], \left[ \begin{array}{c} 3 \\ 0 \\ -2 \\ 1 \end{array} \right] \right\}$ .

□

**V1.** Let  $V$  be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$\begin{aligned}(x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2, y_1 + y_2) \\ c \odot (x_1, y_1) &= (0, cy_1)\end{aligned}$$

Determine if  $V$  is a vector space or not.

**Solution:**  $V$  is not a vector space, as  $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$ .

□

**E1:**

**E3:**

**E4:**

**V1:**

**E2:**