Name:

J#:

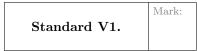
Date:

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$

Determine if V is a vector space or not.

Solution: V is not a vector space, as one of the distributive laws fails, namely

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

Determine if the vectors $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$, $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$, and $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

 Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

Standard S2.

Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.