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| Name: |
| J#:   |
| Date: |

Dr. Clontz

# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

|              |       |
|--------------|-------|
| Standard E1. | Mark: |
|--------------|-------|

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

**Solution:**

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_3 &= -1 \end{aligned}$$

□

|              |       |
|--------------|-------|
| Standard E3. | Mark: |
|--------------|-------|

Solve the following linear system.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$ . It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction  $0 = 1$ ).

□

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| <b>Standard E4.</b> | Mark: |
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Find a basis for the solution set of the system of equations

$$\begin{aligned}x + 3y + 3z + 7w &= 0 \\x + 3y - z - w &= 0 \\2x + 6y + 3z + 8w &= 0 \\x + 3y - 2z - 3w &= 0\end{aligned}$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

□

|                     |       |
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| <b>Standard V1.</b> | Mark: |
|---------------------|-------|

Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- (a) Show that **scalar multiplication** is **associative**:  $a \odot (b \odot x) = (ab) \odot x$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ . To show associativity:

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

We verify the remaining 7 properties to see that  $V$  is a vector space.

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6 - x) = x + (6 - x) - 3 = 3$ , so  $6 - x$  is the additive inverse of  $x$ .

4) Real addition is commutative, so  $\oplus$  is commutative.

5) Associativity shown above

6)  $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c - 1) \\ &= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned} (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\ &= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\ &= (c \odot x) \oplus (d \odot x) \end{aligned}$$

Therefore  $V$  is a vector space.

□

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| Additional Notes/Marks |  |
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# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

|                     |       |
|---------------------|-------|
| <b>Standard E1.</b> | Mark: |
|---------------------|-------|

Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 0$$

$$x - z = 1$$

**Solution:**

$$\left[ \begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{array} \right]$$

□

|                     |       |
|---------------------|-------|
| <b>Standard E3.</b> | Mark: |
|---------------------|-------|

Solve the system of equations

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 2$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 2 \end{array} \right] \right) = \left[ \begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

So the solution set is

$$\left\{ \left[ \begin{array}{c} 1 - 3c \\ c \\ -1 \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

|                     |       |
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| <b>Standard E4.</b> | Mark: |
|---------------------|-------|

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{5}{7} & \frac{3}{7} \\ 0 & 1 & \frac{8}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\frac{5}{7}a - \frac{3}{7}b \\ \frac{8}{7}a + \frac{2}{7}b \\ a \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is  $\left\{ \begin{bmatrix} -\frac{5}{7} \\ \frac{8}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{3}{7} \\ \frac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$ , or  $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$ .

□

|                     |       |
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| <b>Standard V1.</b> | Mark: |
|---------------------|-------|

Let  $V$  be the set of all points on the line  $x + y = 2$  with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$\begin{aligned} (x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2 - 1, y_1 + y_2 - 1) \\ c \odot (x_1, y_1) &= (cx_1 - (c - 1), cy_1 - (c - 2)) \end{aligned}$$

- Show that this vector space has an **additive identity** element  $\mathbf{0}$  satisfying  $(x, y) \oplus \mathbf{0} = (x, y)$ .
- Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1) \in V$ ; then  $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$ , so  $(1, 1)$  is an additive identity element.

Now we will show the other seven properties. Let  $(x_1, y_1), (x_2, y_2) \in V$ , and let  $c, d \in \mathbb{R}$ .

- Since real addition is associative,  $\oplus$  is associative.
- Since real addition is commutative,  $\oplus$  is commutative.
- The additive identity is  $(1, 1)$ .
- $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$ , so  $(2 - x_1, 2 - y_1)$  is the additive inverse of  $(x_1, y_1)$ .
- 

$$\begin{aligned} c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= (c(dx_1 - (d - 1)) - (c - 1), c(dy_1 - (d - 1)) - (c - 1)) \\ &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1)) \\ &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1)) \\ &= (cd) \odot (x_1, y_1) \end{aligned}$$

$$6) 1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$\begin{aligned}
c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1) \\
&= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1)) \\
&= (cx_1 + cy_1 - 2c + 1, cx_2 + cy_2 - 2c + 1) \\
&= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1)) \\
&= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)
\end{aligned}$$

8)

$$\begin{aligned}
(c + d) \odot (x_1, y_1) &= ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1)) \\
&= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1)) \\
&= c \odot (x_1, y_1) \oplus d \odot (x_1, y_1)
\end{aligned}$$

Therefore  $V$  is a vector space.

□

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| Additional Notes/Marks |  |
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Dr. Clontz

# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

|                     |       |
|---------------------|-------|
| <b>Standard E1.</b> | Mark: |
|---------------------|-------|

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array} \right]$$

**Solution:**

$$-4x_1 - x_2 + 3x_3 = 2$$

$$x_1 + 2x_2 - x_3 = 0$$

$$-x_1 + 4x_2 + x_3 = 4$$

□

|                     |       |
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| <b>Standard E3.</b> | Mark: |
|---------------------|-------|

Find the solution set for the following system of linear equations.

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 8$$

$$x_1 + x_2 - x_3 + 5x_4 = 3$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 2 & 3 & -5 & 14 & 8 \\ 1 & 1 & -1 & 5 & 3 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array} \right]$ . It follows that the solution set

is given by  $\begin{bmatrix} 1 - 2a - b \\ 2 + 3a - 4b \\ a \\ b \end{bmatrix}$  for all real numbers  $a, b$ .

□

|                     |       |
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| <b>Standard E4.</b> | Mark: |
|---------------------|-------|

Find a basis for the solution set to the homogeneous system of equations

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 0$$

$$x_1 + x_2 - x_3 + 5x_4 = 0$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 2 & 3 & -5 & 14 & 0 \\ 1 & 1 & -1 & 5 & 0 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array} \right]$ . It follows that the basis for the solution set is given by  $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$ .

□

|                     |       |
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| <b>Standard V1.</b> | Mark: |
|---------------------|-------|

Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$\begin{aligned} x \oplus y &= \sqrt{x^2 + y^2} \\ c \odot x &= cx \end{aligned}$$

- (a) Show that the vector **addition**  $\oplus$  is **associative**:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $x, y, z \in \mathbb{R}$ . Then

$$\begin{aligned} (x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\ &= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\ &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\ &= x \oplus \sqrt{y^2 + z^2} \\ &= x \oplus (y \oplus z) \end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

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| <b>Additional Notes/Marks</b> |  |
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# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

|              |       |
|--------------|-------|
| Standard E1. | Mark: |
|--------------|-------|

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array} \right]$$

**Solution:**

$$\begin{aligned} 3x_1 - x_2 + x_4 &= 5 \\ -x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 &= -3 \end{aligned}$$

□

|              |       |
|--------------|-------|
| Standard E3. | Mark: |
|--------------|-------|

Solve the system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

**Solution:**

$$\text{RREF} \left( \left( \begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right) \right) = \left( \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{array} \right)$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a \\ 3-21a \\ -7a \\ 12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

□

|                     |       |
|---------------------|-------|
| <b>Standard E4.</b> | Mark: |
|---------------------|-------|

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{5}{7} & \frac{3}{7} \\ 0 & 1 & \frac{8}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\frac{5}{7}a - \frac{3}{7}b \\ -\frac{8}{7}a - \frac{2}{7}b \\ a \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is  $\left\{ \begin{bmatrix} -\frac{5}{7} \\ -\frac{8}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{3}{7} \\ -\frac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$ , or  $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$ .

□

|                     |       |
|---------------------|-------|
| <b>Standard V1.</b> | Mark: |
|---------------------|-------|

Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$

$$c \odot x = cx - 3(c - 1)$$

- (a) Show that **scalar multiplication** is **associative**:  $a \odot (b \odot x) = (ab) \odot x$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ . To show associativity:

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\ &= c(dx - 3(d - 1)) - 3(c - 1) \\ &= cdx - 3(cd - 1) \\ &= (cd) \odot x \end{aligned}$$

We verify the remaining 7 properties to see that  $V$  is a vector space.

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.

3)  $x \oplus (6 - x) = x + (6 - x) - 3 = 3$ , so  $6 - x$  is the additive inverse of  $x$ .

4) Real addition is commutative, so  $\oplus$  is commutative.

5) Associativity shown above

6)  $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c - 1) \\ &= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned} (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\ &= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\ &= (c \odot x) \oplus (d \odot x) \end{aligned}$$

Therefore  $V$  is a vector space.

□

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| Additional Notes/Marks |  |
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Dr. Clontz

# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

|              |       |
|--------------|-------|
| Standard E1. | Mark: |
|--------------|-------|

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

**Solution:**

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_3 &= -1 \end{aligned}$$

□

|              |       |
|--------------|-------|
| Standard E3. | Mark: |
|--------------|-------|

Solve the system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right] \right) = \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{array} \right]$$

So the solutions are

$$\left\{ \left[ \begin{array}{c} 1+a \\ 3-21a \\ -7a \\ 12a \end{array} \right] \mid a \in \mathbb{R} \right\}$$

□

|                     |       |
|---------------------|-------|
| <b>Standard E4.</b> | Mark: |
|---------------------|-------|

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

$$-x + 2z + 5w = 0$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \\ -1 & 0 & 2 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} a \\ 2a \\ -2a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

So a basis for the solution set is  $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \right\}$ .

□

|                     |       |
|---------------------|-------|
| <b>Standard V1.</b> | Mark: |
|---------------------|-------|

Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

(a) Show that the vector **addition**  $\oplus$  is **associative**:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .

(b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $x, y, z \in \mathbb{R}$ . Then

$$\begin{aligned} (x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\ &= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\ &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\ &= x \oplus \sqrt{y^2 + z^2} \\ &= x \oplus (y \oplus z) \end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

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| <b>Additional Notes/Marks</b> |  |
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# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

|              |       |
|--------------|-------|
| Standard E1. | Mark: |
|--------------|-------|

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

**Solution:**

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 4x_2 + x_3 &= -7 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

□

|              |       |
|--------------|-------|
| Standard E3. | Mark: |
|--------------|-------|

Solve the system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right] \right) = \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{array} \right]$$

So the solutions are

$$\left\{ \left[ \begin{array}{c} 1+a \\ 3-21a \\ -7a \\ 12a \end{array} \right] \mid a \in \mathbb{R} \right\}$$

□

|                     |       |
|---------------------|-------|
| <b>Standard E4.</b> | Mark: |
|---------------------|-------|

Find a basis for the solution set of the system of equations

$$x + 3y + 3z + 7w = 0$$

$$x + 3y - z - w = 0$$

$$2x + 6y + 3z + 8w = 0$$

$$x + 3y - 2z - 3w = 0$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

□

|                     |       |
|---------------------|-------|
| <b>Standard V1.</b> | Mark: |
|---------------------|-------|

Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

(a) Show that the vector **addition**  $\oplus$  is **associative**:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .

(b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $x, y, z \in \mathbb{R}$ . Then

$$\begin{aligned} (x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\ &= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\ &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\ &= x \oplus \sqrt{y^2 + z^2} \\ &= x \oplus (y \oplus z) \end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

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| <b>Additional Notes/Marks</b> |  |
|-------------------------------|--|