Section V.3

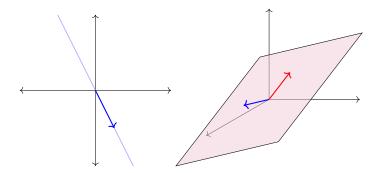
Activity V.3.1 (~ 5 min) How many vectors are required to span \mathbb{R}^2 ? Sketch a drawing in the xy plane to support your answer.

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) Infinitely Many

Activity V.3.2 (~ 5 min) How many vectors are required to span \mathbb{R}^3 ?

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) Infinitely Many

Fact V.3.3 At least n vectors are required to span \mathbb{R}^n .



Activity V.3.4 (~15 min) Choose a vector $\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$ in \mathbb{R}^3 that is not in span $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ by using CoCalc to verify that RREF $\begin{bmatrix} 1 & -2 & | ? \\ -1 & 0 & | ? \\ 0 & 1 & | ? \end{bmatrix} = \begin{bmatrix} 1 & 0 & | 0 \\ 0 & 1 & | 0 \\ 0 & 0 & | 1 \end{bmatrix}$. (Why does this work?)

Fact V.3.5 The set $\{\vec{\mathbf{v}}_1,\ldots,\vec{\mathbf{v}}_m\}$ fails to span all of \mathbb{R}^n exactly when $\text{RREF}[\vec{\mathbf{v}}_1\ldots\vec{\mathbf{v}}_m]$ has a row of zeros:

$$\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
for some choice of vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Activity V.3.6 (~5 min) Consider the set of vectors $S = \left\{ \begin{bmatrix} 2\\3\\0\\-1 \end{bmatrix}, \begin{bmatrix} 1\\-4\\3\\0 \end{bmatrix}, \begin{bmatrix} 2\\0\\0\\3 \end{bmatrix}, \begin{bmatrix} 0\\3\\5\\7 \end{bmatrix}, \begin{bmatrix} 3\\13\\7\\16 \end{bmatrix} \right\}$. Does $\mathbb{R}^4 = \operatorname{span} S$?

Activity V.3.7 ($\sim 10 \ min$) Consider the set of third-degree polynomials

$$S = \{2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, -x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\}.$$

Does $\mathcal{P}^3 = \operatorname{span} S$? (Hint: first rewrite the question so it is about Euclidean vectors.)

Activity V.3.8 (~ 5 min) Consider the set of matrices

$$S = \left\{ \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right\}$$

Does $M_{2,2} = \operatorname{span} S$?

Activity V.3.9 (~5 min) Let $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3 \in \mathbb{R}^7$ be three vectors, and suppose $\vec{\mathbf{w}}$ is another vector with $\vec{\mathbf{w}} \in \operatorname{span} \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$. What can you conclude about $\operatorname{span} \{\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$?

- (a) span $\{\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is larger than span $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$.
- (b) span $\{\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\} = \text{span} \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}.$
- (c) span $\{\vec{\mathbf{v}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is smaller than span $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$.