Name:	

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that scalar multiplication distributes vectors over scalar addition: $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Does span
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\4\\-3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

V4. Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of \mathcal{P}^3 .

S2. Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all polynomials with the operations, for any $f, g \in V, c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors
$$\begin{bmatrix} -3\\1\\1 \end{bmatrix}$$
, $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$, $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$, and $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$ span \mathbb{R}^3

V4. Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of \mathcal{P}^3 .

S2. Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \mathcal{P}_2

V1:

V3:

V4:

S2:

Name:

Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c\odot x=cx$$

- (a) Show that the vector addition \oplus is associative: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors $\begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}$, $\begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}$, and $\begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Let W be the set of all complex numbers that are purely real (i.e of the form a + 0i) or purely imaginary (i.e. of the form 0 + bi). Determine if W is a subspace of \mathbb{C} .

S2. Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

Name:	

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that scalar multiplication distributes vectors over scalar addition: $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Does span
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

V4. Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

S2. Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \mathcal{P}_2

V1: V3:

V4:

S2:

Version 5

Math 237 – Linear Algebra Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that scalar multiplication is associative: $a \odot (b \odot x) = (ab) \odot x$.
- (b) Determine if V is a vector space or not. Justify your answer

V3. Determine if the vectors
$$\begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}$$
, $\begin{bmatrix} 3\\3\\6\\3 \end{bmatrix}$, $\begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix}$, and $\begin{bmatrix} 7\\-1\\8\\-3 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Determine if
$$\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$$
 a subspace of \mathbb{R}^4 .

S2. Determine if the set
$$\left\{ \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\}$$
 is a basis of \mathbb{R}^4 .

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$

- (a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors
$$\begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}$$
, $\begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}$, $\begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}$, and $\begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

S2. Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

V1:

V3:

V4:

S2: