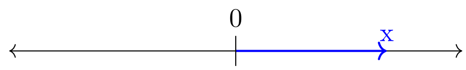
Section V.3

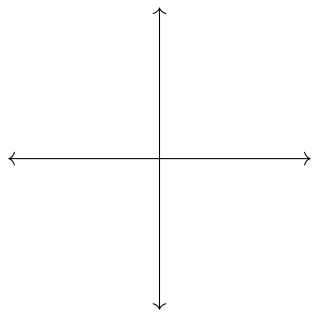
Activity V.25 *(∼5 min)* Does the polynomial $x^2 + x + 1$ belong to $\operatorname{span}\{x^2 - x, x + 1, x^2 - 1\}$?

Activity V.26 *(∼5 min)* Does the matrix $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$ belong to $\operatorname{span}\left\{\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 2 & 2 \end{bmatrix}\right\}$?
Part 1: Reinterpret this question as a question about the solution(s) of a matrix equation.
Part 2: Answer this equivalent question, and use its solution to answer the original question.

Observation V.27 Any single non-zero vector/number x in $\mathbb{R}^1$ spans $\mathbb{R}^1$, since $\mathbb{R}^1 = \{cx \mid c \in \mathbb{R}\}$.



Activity V.28 *(∼5 min)* How many vectors are required to span $\mathbb{R}^2$? Sketch a drawing in the xy plane to support your answer.



(a) 1

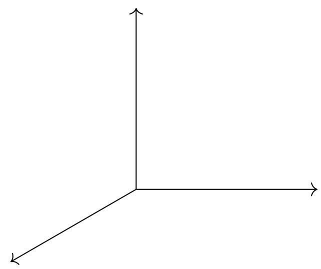
(b) 2

(c) 3

(d) 4

(e) Infinitely Many

Activity V.29 *(∼5 min)* How many vectors are required to span $\mathbb{R}^3$?



(a) 1

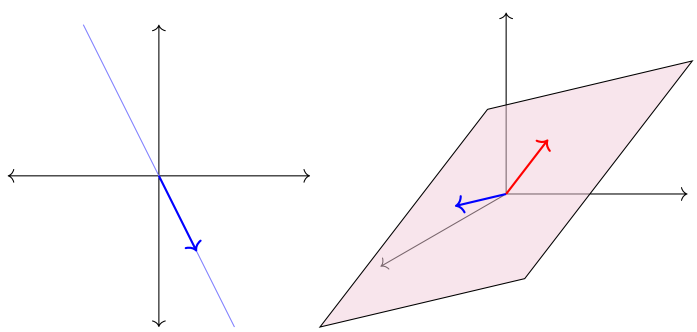
(b) 2

(c) 3

(d) 4

(e) Infinitely Many

Fact V.30 At least n vectors are required to span $\mathbb{R}^n$.



Activity V.31 *(∼15 min)* Choose any vector $\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$ in $\mathbb{R}^3$ that is not in span $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ by using technology to verify that RREF $\left[\begin{array}{cc|c} 1 & -2 & ? \\ -1 & 0 & ? \\ 0 & 1 & ? \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$. (Why does this work?)

Fact V.32 The set $\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_m\}$ fails to span all of $\mathbb{R}^n$ exactly when the vector equation

$$x_1\vec{\mathbf{v}}_1 + \cdots x_m\vec{\mathbf{v}}_m = \vec{\mathbf{w}}$$

is inconsistent for **some** vector $\vec{\mathbf{w}}$.
Note that this happens exactly when RREF$[\vec{\mathbf{v}}_1 \dots \vec{\mathbf{v}}_m]$ has a non-pivot row of zeros.

$$\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \text{ for some choice of vector } \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Activity V.33 *(∼5 min)* Consider the set of vectors $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}$. Does $\mathbb{R}^4 = \operatorname{span} S$?
Part 1: Rewrite this as a question about the solutions to a vector equation.
Part 2: Answer your new question, and use this to answer the original question.

Activity V.34 *(∼10 min)* Consider the set of third-degree polynomials

$$\begin{aligned} S = \{&2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, \\ &-x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\}. \end{aligned}$$

Does $\mathcal{P}^3 = \operatorname{span} S$?
Part 1: Rewrite this as a question about the solutions to a polynomial equation.
Part 2: Answer your new question, and use this to answer the original question.