Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Add Euclidean vectors and multiply Euclidean vectors by scalars.

- Perform basic manipulations of augmented matrices and linear systems **(Standard(s) E1,E2,E3)**.

- Apply linear combinations and spanning sets **(Standard(s) V2,V3)**.

Readiness Assurance Resources

The following resources will help you prepare for this module.

- https://www.khanacademy.org/math/precalculus/vectors-precalc/vector-addition-subtraction/v/adding-and-subtracting-vectors

- https://www.khanacademy.org/math/precalculus/vectors-precalc/combined-vector-operations/v/combined-vector-operations-example

Readiness Assurance Test

Choose the most appropriate response for each question.

21) Simplify the following Euclidean vector expression.

$$4 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}$
(b) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
(d) $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

22) Express the following system of linear equations as an augmented matrix.

$$\begin{aligned} 2x_1 + \ x_2 + 4x_3 &= \ 0 \\ x_1 + \ x_2 + \ x_3 &= \ 1 \\ -3x_1 + 4x_2 + \ x_3 &= -7 \end{aligned}$$

(a) $\left[\begin{array}{cc|c} 2 & 1 & -3 \\ 1 & 1 & 4 \\ 4 & 1 & 1 \\ 0 & 1 & -7 \end{array}\right]$
(b) $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & -2 & 4 \\ 4 & 1 & 1 \\ 0 & 1 & -7 \end{array}\right]$
(c) $\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & 1 & 1 \\ -3 & 4 & -7 \end{array}\right]$
(d) $\left[\begin{array}{ccc|c} 2 & 1 & 4 & 0 \\ 1 & 1 & 1 & 1 \\ -3 & 4 & 1 & -7 \end{array}\right]$

23) Find RREF $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 3 & 2 & 5 \\ -2 & 0 & -2 \end{array}\right]$.

(a) $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$
(b) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 0 & 0 & 0 \end{array}\right]$
(c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$
(d) $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$

24) Solve the following system of linear equations.

$$\begin{aligned} 2x_1 + \ x_2 + 4x_3 &= \ 0 \\ x_1 + \ x_2 + \ x_3 &= \ 1 \\ -3x_1 + 4x_2 + \ x_3 &= -7 \end{aligned}$$

(a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + a \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ for all real numbers a

(c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

(d) No solutions

25) Solve the following system of linear equations.

$$x_1 + x_2 + x_3 + x_4 = 4$$
$$2x_1 + 3x_2 \quad\quad + x_4 = 0$$

(a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ 0 \\ 0 \end{bmatrix} + a \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ for all real numbers a, b

(c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \\ -5 \end{bmatrix} + a \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ for all real numbers a

(d) No solutions

26) How many vectors are required to span all of $\mathbb{R}^4$ (the space of Euclidean vectors with four components)?

(a) 3   (b) 4   (c) 5   (d) Infinitely Many

27) How many vectors are required to span all of $\mathcal{P}^3$ (the space of polynomials of degree three or less)?

(a) 3   (b) 4   (c) 5   (d) Infinitely Many

28) Which vector is a linear combination of $\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 3 \\ -7 \end{bmatrix}$ (c) $\begin{bmatrix} -5 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

29) Which vector belongs to span $\left\{ \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$?

(a) $\begin{bmatrix} 3 \\ -7 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

30) What best describes span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ in three-dimensional Euclidean space $\mathbb{R}^3$?

(a) a line   (b) a plane   (c) a sphere   (d) all of $\mathbb{R}^3$