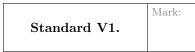
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f.

Standard V3.

Mark: $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}, \text{ and } \begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix} \text{ span } \mathbb{R}^3.$

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

Standard V4.

Let W be the set of all complex numbers that are purely real (i.e of the form a + 0i) or purely imaginary (i.e. of the form 0 + bi). Determine if W is a subspace of \mathbb{C} .

Solution: No, because 1 is purely real and i is purely imaginary, but the linear combination 1+i is neither.

Standard S2.

Mark:

Determine if the set $\left\{\begin{bmatrix}1 & -3\\ 2 & 2\end{bmatrix}, \begin{bmatrix}-1 & 4\\ -1 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix}, \begin{bmatrix}0 & 3\\ 3 & 9\end{bmatrix}\right\}$ is a basis of $M_{2,2}$ or not.

Solution:

RREF
$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -3 & 4 & 0 & 3 \\ 2 & -1 & 2 & 3 \\ 2 & 1 & 1 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

Additional Notes/Marks