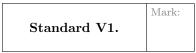
Name:	
J#:	Dr. Clontz
Date:	

## MASTERY QUIZ DAY 12

Math 237 – Linear Algebra Fall 2017

## Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot (f \oplus g) = c \odot f \oplus c \odot g$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial f.

Standard V3.

Determine if the vectors  $\begin{bmatrix} -3\\1\\1 \end{bmatrix}$ ,  $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$ ,  $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$ , and  $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$  span  $\mathbb{R}^3$ 

Solution:

$$RREF\left(\begin{bmatrix} -3 & 5 & 2 & 0\\ 1 & -1 & 0 & 2\\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5\\ 0 & 1 & 1 & 3\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span  $\mathbb{R}^3$ .

Standard V4.

Mark:

Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=0 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

**Solution:** Yes, because z = -x - y and  $a \begin{bmatrix} x_1 \\ y_1 \\ -x_1 - y_1 \end{bmatrix} + b \begin{bmatrix} x_2 \\ y_2 \\ -x_2 - y_2 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ ay_1 + by_2 \\ -(ax_1 + bx_2) - (ay_1 + by_2) \end{bmatrix}$ . Alternately, yes because W is isomorphic to  $\mathbb{R}^2$ .