Application Activities - Module S Part 2 - Class Day 13

Activity 13.1 Last time we saw that $\left\{x^3+1, x^2+2, 4-7x, 2x^3+x\right\}$ is linearly independent. Show that it spans $\mathcal{P}^3$.

Definition 13.2 A **basis** is a linearly independent set that spans a vector space.

Observation 13.3 A basis may be thought of as a collection of building blocks for a vector space, since every vector in the space can be expressed as a unique linear combination of basis vectors.

Activity 13.4 Which of the following sets are bases for $\mathbb{R}^4$?

$$\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\} \qquad \left\{\begin{bmatrix}2\\3\\0\\-1\end{bmatrix},\begin{bmatrix}2\\0\\0\\3\end{bmatrix},\begin{bmatrix}4\\3\\0\\2\end{bmatrix},\begin{bmatrix}-3\\0\\1\\3\end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix}2\\3\\0\\-1\end{bmatrix},\begin{bmatrix}2\\0\\0\\3\end{bmatrix},\begin{bmatrix}3\\13\\7\\16\end{bmatrix},\begin{bmatrix}-1\\10\\7\\14\end{bmatrix},\begin{bmatrix}4\\3\\0\\2\end{bmatrix}\right\} \qquad \left\{\begin{bmatrix}2\\3\\0\\-1\end{bmatrix},\begin{bmatrix}4\\3\\0\\2\end{bmatrix},\begin{bmatrix}-3\\0\\1\\3\end{bmatrix},\begin{bmatrix}3\\6\\1\\5\end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix}5\\3\\0\\-1\end{bmatrix},\begin{bmatrix}-2\\1\\0\\3\end{bmatrix},\begin{bmatrix}4\\5\\1\\3\end{bmatrix}\right\}$$

Activity 13.5 If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a basis for $\mathbb{R}^4$, that means RREF$[\mathbf{v}_1\,\mathbf{v}_2\,\mathbf{v}_3\,\mathbf{v}_4]$ doesn't have a column without a pivot position, and doesn't have a row of zeros. What is RREF$[\mathbf{v}_1\,\mathbf{v}_2\,\mathbf{v}_3\,\mathbf{v}_4]$?

Fact 13.6 The set $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$ is a basis for $\mathbb{R}^n$ if and only if $m = n$ and RREF$[\mathbf{v}_1\,\dots\,\mathbf{v}_n] = \begin{bmatrix}1 & 0 & \dots & 0\\0 & 1 & \dots & 0\\\vdots & \vdots & \ddots & \vdots\\0 & 0 & \dots & 1\end{bmatrix}$.

That is, a basis for $\mathbb{R}^n$ must have exactly n vectors and its square matrix must row-reduce to the **identity matrix** containing all zeros except for a downward diagonal of ones.

Activity 13.7 Consider the set $\left\{\begin{bmatrix}2\\3\\0\\1\end{bmatrix},\begin{bmatrix}2\\0\\1\\-1\end{bmatrix},\begin{bmatrix}2\\-3\\2\\-3\end{bmatrix},\begin{bmatrix}1\\5\\-1\\0\end{bmatrix}\right\}$.

Part 1: Use RREF $\begin{bmatrix}2 & 2 & 2 & 1\\3 & 0 & -3 & 5\\0 & 1 & 2 & -1\\1 & -1 & -3 & 0\end{bmatrix}$ to identify which vector may be removed to make the set linearly independent.

Part 2: Find a basis for span $\left\{\begin{bmatrix}2\\3\\0\\1\end{bmatrix},\begin{bmatrix}2\\0\\1\\-1\end{bmatrix},\begin{bmatrix}2\\-3\\2\\-3\end{bmatrix},\begin{bmatrix}1\\5\\-1\\0\end{bmatrix}\right\}$.