Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compose functions of real numbers
- Solve systems of linear equations **(Standard(s) E3)**
- Find the matrix corresponding to a linear transformation **(Standard(s) A1)**
- Determine if a linear transformation is injective and/or surjective **(Standard(s) A3)**
- Interpret the ideas of injectivity and surjectivity in multiple ways

Readiness Assurance Resources

The following resources will help you prepare for this module.

- `https://www.khanacademy.org/math/algebra2/manipulating-functions/funciton-composition/v/function-composition`

Readiness Assurance Test

Choose the most appropriate response for each question.

41) Let $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$. Compute the composition function $(f \circ g)(x)$.

(a) $x^2 - 1$
(b) $x^4 + 2x^2 - 1$
(c) $x^4 - 4x^2 + 5$
(d) $x^4 - x^2 - 2$

42) Suppose $f(x)$ and $g(x)$ are real-valued functions satisfying

$$\begin{aligned} f(2) &= 1 & \qquad g(2) &= 3 \\ f(3) &= 4 & \qquad g(3) &= 5 \\ f(4) &= 3 & \qquad g(4) &= 6 \end{aligned}$$

Compute $(f \circ g)(2)$.

(a) 2
(b) 3
(c) 4
(d) 5

43) Solve the system of linear equations

$$\begin{aligned} x + 3y &= -2 \\ 2x - 7y &= 9 \end{aligned}$$

(a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$

44) Let a, b, c be fixed real numbers. How many solutions does the system of linear equations below have?

$$\begin{aligned} x + 2y + 3z &= a \\ y - z &= b \\ y + z &= c \end{aligned}$$

(a) 0 (b) 1 (c) Infinitely many (d) It depends on the values of a, b, and c.

45) What is the matrix corresponding to the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix}$?

(a) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

46) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation with associated matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$. Compute $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

47) Which of the following is true of the linear transformation $T :$?

(a) T is neither injective nor surjective
(b) T is injective but not surjective
(c) T is surjective but not injective
(d) T is both injective and surjective

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(a) T is neither injective nor surjective
(b) T is injective but not surjective
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49) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with associated matrix $A \in M_{m,n}(\mathbb{R})$. Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

(a) T is injective
(b) T has a non-trivial kernel
(c) The columns of A are linearly dependent
(d) RREF(A) has a non-pivot column

50) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with associated matrix $A \in M_{m,n}(\mathbb{R})$. Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

(a) T is surjective
(b) $\operatorname{Im} T = \mathbb{R}^m$
(c) The columns of A span $\mathbb{R}^m$
(d) RREF(A) has only pivot columns