

Name: _____

MIDTERM EXAM

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 0$$

$$x - z = 1$$

E2. Find RREF A , where

$$A = \left[\begin{array}{cccc|c} 2 & 2 & 1 & 2 & -1 \\ 1 & 1 & 2 & 4 & 5 \\ 3 & 3 & -1 & -2 & 1 \end{array} \right]$$

E3. Solve the following linear system.

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 5$$

$$-2x_3 - 4x_4 = 3$$

$$2x_1 + 2x_2 + x_3 - 4x_4 = -1$$

E4. Find a basis for the solution set to the homogeneous system of equations

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 0$$

$$x_1 + x_2 - x_3 + 5x_4 = 0$$

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$$

(a) Show that scalar multiplication **distributes scalars** over vector addition:

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 4 \\ 3 \end{bmatrix} \right\}$.

V3. Determine if the vectors $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ span \mathbb{R}^3

V4. Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

S2. Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}_3

S3. Let W be the subspace of \mathcal{P}_2 given by $W = \text{span}(\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\})$. Find a basis for W .

S4. Let W be the subspace of \mathcal{P}_3 given by $W = \text{span}(\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\})$. Compute the dimension of W .

E1:

E2:

E3:

E4:

V1:

V2:

V3:

V4:

S1:

S2:

S3:

S4: