MASTERY QUIZ DAY 14   Math 237 – Linear Algebra
Version 1   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5)

$$\begin{aligned}
c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\
&= c(dx - 3(d - 1)) - 3(c - 1) \\
&= cdx - 3(cd - 1) \\
&= (cd) \odot x
\end{aligned}$$

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c - 1) \\
&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}$$

8)

$$\begin{aligned}
(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
&= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (d \odot x)
\end{aligned}$$

Therefore V is a vector space.

$\square$

V3. Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\dfrac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span $\mathbb{R}^4$.

$\square$

V4. Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

$\square$

S2. Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

V1: [   ]   **V3:** [   ]   **V4:** [   ]   **S2:** [   ]