

## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Solve a system of linear equations (including finding a basis of the solution space if it is homogeneous) by interpreting as an augmented matrix and row reducing (**Standard(s) E1, E2, E3, E4**).
- State the definition of a spanning set, and determine if a set of vectors spans a vector space or subspace (**Standard(s) V3**).
- State the definition of linear independence, and determine if a set of vectors is linearly dependent or independent (**Standard(s) S1**).
- State the definition of a basis, and determine if a set of vectors is a basis (**Standard(s) S2**).

## Readiness Assurance Resources

The following resources will help you prepare for this module.

- Review the supporting Standards listed above.

**Readiness Assurance Test**

Choose the most appropriate response for each question.

- 31) Which of the following is a solution to the system of linear equations

$$\begin{aligned}x + 3y - z &= 2 \\ 2x + 8y + 3z &= -1 \\ -x - y + 9z &= -10\end{aligned}$$

(a)  $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(c)  $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

(d)  $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

- 32) Find a basis for the solution set of the following homogeneous system of linear equations

$$\begin{aligned}x + 2y + -z - w &= 0 \\ -2x - 4y + 3z + 5w &= 0\end{aligned}$$

(a)  $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix} \right\}$

(b)  $\left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ -1 \end{bmatrix} \right\}$

(c)  $\left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$

(d)  $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 5 \end{bmatrix} \right\}$

- 33) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

- (a) It does not span and is linearly dependent
- (b) It does not span and is linearly independent
- (c) It spans but it is linearly dependent
- (d) It is a basis of  $\mathbb{R}^3$ .

- 34) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

- (a) It does not span and is linearly dependent
- (b) It does not span and is linearly independent
- (c) It spans but it is linearly dependent
- (d) It is a basis of  $\mathbb{R}^3$ .

35) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

- (a) It does not span and is linearly dependent
- (b) It does not span and is linearly independent
- (c) It spans but it is linearly dependent
- (d) It is a basis of  $\mathbb{R}^3$ .

36) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

- (a) It spans but it is linearly dependent
- (b) It is a basis of  $\mathbb{R}^3$ .
- (c) It does not span and is linearly independent
- (d) It does not span and is linearly dependent

37) Suppose  $\mathbf{v}_1, \dots, \mathbf{v}_n \in \mathbb{R}^5$  and you know that every vector in  $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  can be written uniquely as a linear combination of the vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ . What can you conclude about  $n$ ?

- (a)  $n \geq 5$
- (b)  $n \leq 5$
- (c)  $n = 5$
- (d)  $n$  could be any positive integer

38) Suppose you know that every vector in  $\mathbb{R}^5$  can be written as a linear combination of the vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ . What can you conclude about  $n$ ?

- (a)  $n = 5$
- (b)  $n$  could be any positive integer
- (c)  $n \leq 5$
- (d)  $n \geq 5$

39) Suppose you know that every vector in  $\mathbb{R}^5$  can be written uniquely as a linear combination of the vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ . What can you conclude about  $n$ ?

- (a)  $n$  could be any positive integer
- (b)  $n \leq 5$
- (c)  $n \geq 5$
- (d)  $n = 5$

- 40) Suppose you know that every vector in  $\mathbb{R}^5$  can be written uniquely as a linear combination of the vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ . What can you conclude about the set  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ ?
- (a) It does not span and is linearly dependent
  - (b) It does not span and is linearly independent
  - (c) It is a basis of  $\mathbb{R}^5$ .
  - (d) It spans but it is linearly dependent