Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Solution: Since

$$RREF \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span \mathbb{R}^3 .

Standard V4.

Mark:

Let W be the set of all complex numbers that are purely real (i.e of the form a + 0i) or purely imaginary (i.e. of the form 0 + bi). Determine if W is a subspace of \mathbb{C} .

Solution: No, because 1 is purely real and i is purely imaginary, but the linear combination 1+i is neither.

Standard S2.

Determine if the set $\left\{ \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4

Solution:

$$RREF \begin{pmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 1 & 2 & 0 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

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Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.

Does span
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\4\\-3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

Solution: Since

$$RREF \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 4 & 2 \\ 4 & -9 & -3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

has a zero row, the vectors fail to span \mathbb{R}^3 .

Standard V4.

Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

Standard S2.

Mark:

Determine if the set $\{x^2+x-1,3x^2-x+1,2x-2\}$ is a basis of \mathcal{P}_2

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{pmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

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Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.	Mark:					
Determine if the vectors	$\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} - \end{bmatrix}$	$\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix},$	$\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and	$\begin{bmatrix} -1\\1\\-1\\-1\end{bmatrix}$	span \mathbb{R}^4 .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

Standard V4.

Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

Solution: No, because **0** does not belong to W.

Standard S2.

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \mathcal{P}_2

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

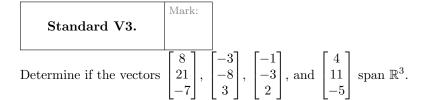
Since the resulting matrix is the identity matrix, it is a basis.

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Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Solution:

RREF
$$\left(\begin{bmatrix} 8 & -3 & -1 & 4\\ 21 & -8 & -3 & 11\\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1\\ 0 & 1 & 3 & -4\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

Mark: Standard V4.

Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of \mathcal{P}^3 .

Solution: Yes because $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$ also belongs to W. Alternately, yes because W is isomorphic to \mathbb{R}^2 .

Standard S2.

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \mathcal{P}_2

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

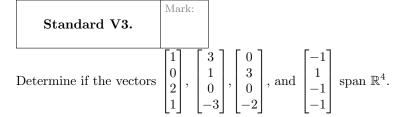
Since the resulting matrix is the identity matrix, it is a basis.

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Math 237 – Linear Algebra

Version 5

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all relevant work to receive credit for a standard.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

Mark: Standard V4.

Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

Mark: Standard S2. Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

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 ${\bf Math~237-Linear~Algebra}$

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V3.

Mark:

$$\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}, \text{ and } \begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix} \text{ span } \mathbb{R}^3.$$

Solution:

Version 6

$$RREF\left(\begin{bmatrix} 8 & -3 & -1 & 4\\ 21 & -8 & -3 & 11\\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1\\ 0 & 1 & 3 & -4\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

Standard V4.

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W, but $(x - x^2) + (x^2) = x \notin W$.

Standard S2. $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix}$ is a basis of \mathbb{R}^4

Solution:

$$RREF \begin{pmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 1 & 2 & 0 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.