Application Activities - Module G Part 4 - Class Day 28

Observation 28.1 Recall from last class:

- To find the eigenvalues of a matrix A, we need to find values of λ such that $A - \lambda I$ has a nontrivial kernel. Equivalently, we want values where $A - \lambda I$ is not invertible, so we want to know the values of λ where $\det(A - \lambda I) = 0$.
- $\det(A - \lambda I)$ is a polynomial with variable λ, called the **characteristic polynomial** of A. Thus the roots of the characteristic polynomial of A are exactly the eigenvalues of A.
- Once an eigenvalue λ is found, the **eigenspace** containing all **eigenvectors** $\mathbf{x}$ satisfying $A\mathbf{x} = \lambda\mathbf{x}$ is given by $\ker(A - \lambda I)$.

Activity 28.2 If A is a 4×4 matrix, what is the largest number of eigenvalues A can have?

(a) 3

(b) 4

(c) 5

(d) 6

(e) It can have infinitely many

Activity 28.3 2 is an eigenvalue of the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 0 & 1 \\ -1 & -2 & 3 \end{bmatrix}$.

Compute the eigenspace of A associated to the eigenvalue 2 by solving for the kernel of

$$A - 2I = \begin{bmatrix} 1-2 & -2 & 1 \\ -1 & 0-2 & 1 \\ -1 & -2 & 3-2 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 1 \\ -1 & -2 & 1 \\ -1 & -2 & 1 \end{bmatrix}$$

Activity 28.4 2 is an eigenvalue of the matrix $B = \begin{bmatrix} -3 & -9 & 5 \\ -2 & -2 & 2 \\ -7 & -13 & 9 \end{bmatrix}$.

Compute the eigenspace of B associated to the eigenvalue 2 by solving for the kernel of $B - 2I$.

Definition 28.5

- The **algebraic multiplicity** of an eigenvalue is its multiplicity as a root of the characteristic polynomial.
- The **geometric multiplicity** of an eigenvalue is the dimension of the eigenspace.

Fact 28.6 The geometric multiplicity of an eigenvalue cannot exceed its algebraic multiplicity (but it *can* be different).

Activity 28.7 Find all of the eigenvalues, along with both their algebraic and geometric multiplicities, for the matrix $\begin{bmatrix} -3 & 1 & 2 & 1 \\ -9 & 5 & -2 & -1 \\ 31 & -17 & 6 & 3 \\ -69 & 39 & -18 & -9 \end{bmatrix}$. Use technology to help you!

Activity 28.8 Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.
Part 1: Find the eigenvalues of A
Part 2: Describe what this linear transformation is doing geometrically; draw a picture.
