Name:	
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Date:	

MASTERY QUIZ DAY 15 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V2.	Mark:

Determine if $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

Solution:

$$\mathrm{RREF}\left(\left[\begin{array}{cc|c} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{array}\right]\right) = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Since this system has a solution, $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$, namely

$$\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

□

Standard S1.	Mark:

Determine if the set of polynomials $\left\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\right\}$ is linearly dependent or linearly independent

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

□

Standard S3.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Find a basis for W.

Solution: Let $A = \begin{bmatrix}-3 & 1 & 0\\-8 & 2 & -1\\0 & 2 & 3\end{bmatrix}$, and compute $\operatorname{RREF}(A) = \begin{bmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$. Since the first two columns are pivot columns, $\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}\right\}$ is a basis for W.

$\square$

Standard S4.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix}, \begin{bmatrix}2\\0\\1\\1\end{bmatrix}, \begin{bmatrix}3\\-1\\4\\-2\end{bmatrix}, \begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1 & 2 & 3 & 1\\-1 & 0 & -1 & 1\\3 & 1 & 4 & 1\\-3 & 1 & -2 & -7\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 1 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

$\square$

Additional Notes/Marks	