MASTERY QUIZ DAY 6   Math 237 – Linear Algebra
Version 3   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array}\right]$$

Solution:

$$3x_1 - x_2 + x_4 = 5$$
$$-x_1 + 9x_2 + x_3 - 7x_4 = 0$$
$$x_1 - x_3 = -3$$

□

E3. Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

Solution:

$$\text{RREF}\left(\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array}\right]\right) = \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The solutions are

$$\left\{\left[\begin{array}{c} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{array}\right] \ \middle|\ c \in \mathbb{R}\right\} = \left\{\left[\begin{array}{c} c - 1 \\ 3c - 1 \\ -2c \end{array}\right] \ \middle|\ c \in \mathbb{R}\right\}$$

□

E4. Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \dfrac{5}{7} & \dfrac{3}{7} \\ 0 & 1 & \dfrac{8}{7} & \dfrac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\dfrac{5}{7}a - \dfrac{3}{7}b \\ -\dfrac{8}{7}a - \dfrac{2}{7}b \\ a \\ b \end{bmatrix} \,\middle|\, a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} -\dfrac{5}{7} \\ -\dfrac{8}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\dfrac{3}{7} \\ -\dfrac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$, or $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$.

□

E1: ☐   **E3:** ☐   **E4:** ☐   **E2:** ☐