

Name: _____

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all points on the parabola $y = x^2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2 + 2x_1x_2)$$

$$c \odot (x_1, y_1) = (cx_1, c^2y_1)$$

(a) Show that the vector **addition** \oplus is **associative**:

$$(x_1, y_1) \oplus ((x_2, y_2) \oplus (x_3, y_3)) = ((x_1, y_1) \oplus (x_2, y_2)) \oplus (x_3, y_3).$$

(b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Let W be the set of all complex numbers that are purely real (i.e of the form $a + 0i$) or purely imaginary (i.e. of the form $0 + bi$). Determine if W is a subspace of \mathbb{C} .

S2. Determine if the set $\{2x^2 - x + 3, 2x^2 + 2, -x^2 + 4x + 1\}$ is a basis of \mathcal{P}^2 .

V1: ☐

V3: ☐

V4: ☐

S2: ☐

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Version 2

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V1. Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}f \oplus g &= f' + g' \\c \odot f &= cf'\end{aligned}$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V4. Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 1$ (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

S2. Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4 .

V1: ☐

V3: ☐

V4: ☐

S2: ☐

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Math 237 – Linear Algebra

Version 3

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Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- (a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.
- (b) Determine if V is a vector space or not. Justify your answer

V3. Determine if the vectors $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 1$ (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

S2. Determine if the set $\{x^3 - 3x^2 + 2x + 2, -x^3 + 4x^2 - x + 1, -x^3 + 2x + 1, 3x^2 + 3x + 9\}$ is a basis of \mathcal{P}^3 or not.

V1:

V3:

V4:

S2:

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Math 237 – Linear Algebra

Version 4

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Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2 - 1, y_1 + y_2 - 1) \\ c \odot (x_1, y_1) &= (cx_1 - (c - 1), cy_1 - (c - 1))\end{aligned}$$

- (a) Show that this vector space has an **additive identity** element $\mathbf{0}$ satisfying $(x, y) \oplus \mathbf{0} = (x, y)$.
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V4. Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

S2. Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x^2 - 2\}$ is a basis of \mathcal{P}^2 .

V1: ☐

V3: ☐

V4: ☐

S2: ☐

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Math 237 – Linear Algebra

Version 5

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V1. Let V be the set of all points on the parabola $y = x^2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 - x_2, y_1 + y_2 - 2x_1x_2)$$

$$c \odot (x_1, y_1) = (cx_1, c^2y_1)$$

(a) Show that scalar multiplication **distributes scalars** over vector addition:

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

(b) Determine if V is a vector space or not. Justify your answer.

V3. Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V4. Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ a subspace of \mathbb{R}^4 .

S2. Determine if the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 5 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4 .

V1: ☐

V3: ☐

V4: ☐

S2: ☐

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MASTERY QUIZ DAY 14

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$$

(a) Show that scalar multiplication **distributes scalars** over vector addition:

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

(b) Determine if V is a vector space or not. Justify your answer.

V3. Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V4. Let W be the set of all complex numbers $a + bi$ satisfying $a = 2b$. Determine if W is a subspace of \mathbb{C} .

S2. Determine if the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3 .

V1: ☐

V3: ☐

V4: ☐

S2: ☐