

Name:
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Dr. Clontz

# MASTERY QUIZ DAY 14

Math 237 – Linear Algebra

## Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication  $\odot$  distributes over scalar addition.

(b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1) \in V$ , and let  $c, d \in \mathbb{R}$ . Then

$$(c + d) \odot (x_1, y_1) = (0, (c + d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However,  $V$  is not a vector space, as  $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$ .

□

<b>Standard V3.</b>	Mark:
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Does span  $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5$ ?

**Solution:** Since there are only three vectors, they cannot span  $\mathbb{R}^5$ .

□

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all complex numbers that are purely real (i.e of the form  $a + 0i$ ) or purely imaginary (i.e. of the form  $0 + bi$ ). Determine if  $W$  is a subspace of  $\mathbb{C}$ .

**Solution:** No, because 1 is purely real and  $i$  is purely imaginary, but the linear combination  $1 + i$  is neither.

□

<b>Standard S2.</b>	Mark:
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Determine if the set  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

<b>Additional Notes/Marks</b>	
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