| | |
|---|---|
| Name: | |
| J#: | |
| Date: | |

Dr. Clontz

**MASTERY QUIZ DAY 14** Math 237 – Linear Algebra
**Version 1** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that scalar multiplication **distributes vectors** over scalar addition: $(c + d) \odot (x, y) = c \odot (x, y) \oplus d \odot (x, y)$.

(b) Determine if $V$ is a vector space or not. Justify your answer.

**Solution:** Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c + d) \odot (x_1, y_1) = (0, (c + d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, $V$ is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

□

| **Standard V3.** | Mark: |
|---|---|

Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span $\mathbb{R}^4$.

**Solution:**

$$\mathrm{RREF}\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span $\mathbb{R}^4$.

□

| **Standard V4.** | Mark: |
|---|---|

Let $W$ be the set of all polynomials of the form $ax^3 + bx$. Determine if $W$ is a subspace of $\mathcal{P}^3$.

**Solution:** Yes because $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$ also belongs to $W$. Alternately, yes because $W$ is isomorphic to $\mathbb{R}^2$.

$\square$

| **Standard S2.** | Mark: |
|---|---|

Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

**Solution:**

$$\text{RREF}\left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

| **Additional Notes/Marks** | |
|---|---|

| | |
|---|---|
| Name: | |
| J#: | Dr. Clontz |
| Date: | |

**MASTERY QUIZ DAY 14** Math 237 – Linear Algebra
**Version 2** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here $f'$ denotes the derivative of $f$).

(a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.

(b) Determine if $V$ is a vector space or not. Justify your answer.

**Solution:** Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'\,' + cg'\,' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial $f$.

□

| **Standard V3.** | Mark: |
|---|---|

Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span $\mathbb{R}^3$.

**Solution:**

$$\text{RREF}\left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span $\mathbb{R}^3$.

□

| **Standard V4.** | Mark: |
|---|---|

Let $W$ be the set of all polynomials of the form $ax^3 + bx$. Determine if $W$ is a subspace of $\mathcal{P}^3$.

**Solution:** Yes because $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$ also belongs to $W$. Alternately, yes because $W$ is isomorphic to $\mathbb{R}^2$.

$\square$

| **Standard S2.** | Mark: |
| --- | --- |

Determine if the set $\left\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\right\}$ is a basis of $\mathcal{P}_2$

**Solution:**

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

| **Additional Notes/Marks** | |
| --- | --- |

| | |
|---|---|
| Name: | |
| J#: | |
| Date: | |

Dr. Clontz

**MASTERY QUIZ DAY 14**
**Version 3**

Math 237 – Linear Algebra
Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.

(b) Determine if $V$ is a vector space or not. Justify your answer

**Solution:** Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d-1)) \\ &= c\,(dx - 3(d-1)) - 3(c-1) \\ &= cdx - 3(cd-1) \\ &= (cd) \odot x \end{aligned}$$

We verify the remaining 7 properties to see that $V$ is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of $x$.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c-1) \\ &= cx - 3(c-1) + cy - 3(c-1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot x &= (c+d)x - 3(c+d-1)\\ &= cx - 3(c-1) + dx - 3(c-1) - 3\\ &= (c\odot x)\oplus(d\odot x)\end{aligned}$$

Therefore $V$ is a vector space.

□

| **Standard V3.** | Mark: |
|---|---|

Does span $\left\{\begin{bmatrix}2\\-1\\4\\2\\1\end{bmatrix}, \begin{bmatrix}-1\\3\\5\\2\\0\end{bmatrix}, \begin{bmatrix}1\\0\\5\\1\\-3\end{bmatrix}\right\} = \mathbb{R}^5$?

**Solution:** Since there are only three vectors, they cannot span $\mathbb{R}^5$.

□

| **Standard V4.** | Mark: |
|---|---|

Let $W$ be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix}x\\y\\z\end{bmatrix}$ satisfying $x+y+z=0$ (this forms a plane). Determine if $W$ is a subspace of $\mathbb{R}^3$.

**Solution:** Yes, because $z=-x-y$ and $a\begin{bmatrix}x_1\\y_1\\-x_1-y_1\end{bmatrix}+b\begin{bmatrix}x_2\\y_2\\-x_2-y_2\end{bmatrix}=\begin{bmatrix}ax_1+bx_2\\ay_1+by_2\\-(ax_1+bx_2)-(ay_1+by_2)\end{bmatrix}$.
Alternately, yes because $W$ is isomorphic to $\mathbb{R}^2$.

□

| **Standard S2.** | Mark: |
|---|---|

Determine if the set $\left\{\begin{bmatrix}1\\1\\-1\end{bmatrix}, \begin{bmatrix}3\\-1\\1\end{bmatrix}, \begin{bmatrix}2\\0\\-2\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^3$

**Solution:**

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

| **Additional Notes/Marks** | |
|---|---|

| |
|---|
| Name: |
| J#: |
| Date: |

Dr. Clontz

**MASTERY QUIZ DAY 14** Math 237 – Linear Algebra
**Version 4** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here $f'$ denotes the derivative of $f$).

(a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.

(b) Determine if $V$ is a vector space or not. Justify your answer.

**Solution:** Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'\,' + cg'\,' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial $f$.

$\square$

| **Standard V3.** | Mark: |
|---|---|

Determine if the vectors $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ span $\mathbb{R}^3$

**Solution:**

$$\text{RREF}\left(\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span $\mathbb{R}^3$.

$\square$

| **Standard V4.** | Mark: |
|---|---|

Let $W$ be the set of all polynomials of the form $ax^3 + bx$. Determine if $W$ is a subspace of $\mathcal{P}^3$.

**Solution:** Yes because $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$ also belongs to $W$. Alternately, yes because $W$ is isomorphic to $\mathbb{R}^2$.

$\square$

| **Standard S2.** | Mark: |
|---|---|

Determine if the set $\left\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\right\}$ is a basis of $\mathcal{P}_2$

**Solution:**

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

| **Additional Notes/Marks** | |
|---|---|

| | |
|---|---|
| Name: | |
| J#: | Dr. Clontz |
| Date: | |

**MASTERY QUIZ DAY 14** Math 237 – Linear Algebra
**Version 5** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if $V$ is a vector space or not. Justify your answer.

**Solution:** Let $x, y, z \in \mathbb{R}$. Then

$$\begin{aligned}
(x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\
&= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\
&= \sqrt{x^2 + y^2 + z^2} \\
&= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\
&= x \oplus \sqrt{y^2 + z^2} \\
&= x \oplus (y \oplus z)
\end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

| **Standard V3.** | Mark: |
|---|---|

Does $\operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5$?

**Solution:** Since there are only three vectors, they cannot span $\mathbb{R}^5$.

□

| **Standard V4.** | Mark: |
|---|---|

Let $W$ be the set of all polynomials of the form $ax^3 + bx$. Determine if $W$ is a subspace of $\mathcal{P}^3$.

**Solution:** Yes because $s(a_1x^3 + b_1x) + t(a_2x^3 + b_2x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$ also belongs to $W$. Alternately, yes because $W$ is isomorphic to $\mathbb{R}^2$.

$\square$

| **Standard S2.** | Mark: |
|---|---|

Determine if the set $\left\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\right\}$ is a basis of $\mathcal{P}_3$

**Solution:**

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

$\square$

| **Additional Notes/Marks** | |
|---|---|

| | |
|---|---|
| Name: | |
| J#: | |
| Date: | |

Dr. Clontz

**MASTERY QUIZ DAY 14** Math 237 – Linear Algebra

**Version 6** Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| **Standard V1.** | Mark: |
|---|---|

Let $V$ be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.

(b) Determine if $V$ is a vector space or not. Justify your answer

**Solution:** Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d-1)) \\ &= c\,(dx - 3(d-1)) - 3(c-1) \\ &= cdx - 3(cd-1) \\ &= (cd) \odot x \end{aligned}$$

We verify the remaining 7 properties to see that $V$ is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of $x$.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c-1) \\ &= cx - 3(c-1) + cy - 3(c-1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot x &= (c+d)x - 3(c+d-1)\\ &= cx - 3(c-1) + dx - 3(c-1) - 3\\ &= (c\odot x)\oplus(d\odot x)\end{aligned}$$

Therefore $V$ is a vector space.

□

| **Standard V3.** | Mark: |
|---|---|

Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix},\begin{bmatrix}3\\12\\-9\end{bmatrix},\begin{bmatrix}1\\4\\-3\end{bmatrix},\begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}=\mathbb{R}^3$?

**Solution:** Since

$$\mathrm{RREF}\begin{bmatrix}2&3&1&-4\\-1&12&4&2\\4&-9&-3&-8\end{bmatrix}=\begin{bmatrix}1&0&0&-2\\0&1&1/3&0\\0&0&0&0\end{bmatrix}$$

has a zero row, the vectors fail to span $\mathbb{R}^3$.

□

| **Standard V4.** | Mark: |
|---|---|

Let $W$ be the set of all complex numbers $a+bi$ satisfying $a=2b$. Determine if $W$ is a subspace of $\mathbb{C}$.

**Solution:** Yes, because $c(2b_1+b_1i)+d(2b_2+b_2i)=2(cb_1+db_2)+(cb_1+db_2)i$ belongs to $W$. Alternately, yes because $W$ is isomorphic to $\mathbb{R}$.

□

| **Standard S2.** | Mark: |
|---|---|

Determine if the set $\left\{x^2+x-1, 3x^2-x+1, 2x-2\right\}$ is a basis of $\mathcal{P}_2$

**Solution:**

$$\mathrm{RREF}\left(\begin{bmatrix}1&3&2\\1&-1&0\\-1&1&-2\end{bmatrix}\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

| **Additional Notes/Marks** | |
|---|---|