

Name: _____

FINAL EXAM

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

E2. Find RREF A , where

$$A = \left[\begin{array}{cc|c} 2 & -7 & 4 \\ 1 & -3 & 2 \\ 3 & 0 & 3 \end{array} \right]$$

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\3x_1 + 6x_3 + x_4 &= 5 \\-4x_1 + x_2 - 9x_3 + 2x_4 &= -7\end{aligned}$$

E4. Find a basis for the solution set to the system of equations

$$\begin{aligned}x + 2y - 3z &= 0 \\2x + y - 4z &= 0 \\3y - 2z &= 0 \\x - y - z &= 0\end{aligned}$$

V1. Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}f \oplus g &= f' + g' \\c \odot f &= cf'\end{aligned}$$

(here f' denotes the derivative of f).

(a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$.

V3. Determine if the vectors $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of \mathcal{P}^3 .

S1. Determine if the set of polynomials $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$ is linearly dependent or linearly independent

S2. Determine if the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 5 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4 .

S3. Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find a basis for W .

S4. Let $W = \text{span} \left(\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$. Compute the dimension of W .

A1. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R}^4 .

A2. Determine if $D : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}$ given by $D \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc$ is a linear transformation or not.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + y + z \\ 2y + 3z \\ x - y - 2z \end{bmatrix}$

(b) $S : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by $S \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

A4. Let $T : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^3$ be the linear map given by $T \left(\begin{bmatrix} x & y \\ z & w \end{bmatrix} \right) = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T .

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

M2. Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

M3. Find the inverse of the matrix $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$.

G1. Compute the determinant of the matrix $\begin{bmatrix} 2 & 3 & 0 & 1 \\ -1 & 3 & 1 & 4 \\ 0 & 2 & 0 & 3 \\ 1 & -1 & 3 & 5 \end{bmatrix}$.

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 9 & -3 & 2 \\ 19 & -6 & 5 \\ -5 & 2 & 0 \end{bmatrix}$.

G3. Find the eigenspace associated to the eigenvalue 2 in the matrix $A = \begin{bmatrix} 0 & -2 & -1 & 0 \\ -4 & -2 & -2 & 0 \\ 14 & 12 & 10 & 2 \\ -13 & -10 & -8 & -1 \end{bmatrix}$.

G4. Compute the geometric multiplicity of the eigenvalue 2 in the matrix $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -4 & 4 & 0 & 0 \\ 11 & -6 & 1 & -1 \\ -9 & 5 & 1 & 3 \end{bmatrix}$.

E1:

E2:

E3:

E4:

V1:

V2:

V3:

V4:

S1:

S2:

S3:

S4:

A1:

A2:

A3:

A4:

M1:

M2:

M3:

G1:

G2:

G3:

G4: