

Sample Assessment Exercises

This document contains one exercise and solution for each standard. The goal is to give you an idea of what the exercises might look like, and what the expectations for a complete solution are.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & 2 & 0 & 1 & 1 \\ -1 & -4 & 1 & -7 & 0 \\ 0 & 1 & -1 & 0 & -2 \end{array} \right]$$

Solution:

$$\begin{aligned} 3x_1 + 2x_2 &+ x_4 = 1 \\ -x_1 - 4x_2 + x_3 - 7x_4 &= 0 \\ x_2 - x_3 &= -2 \end{aligned}$$

□

E2. Put the following matrix in reduced row echelon form.

$$\left[\begin{array}{cccc} 0 & 3 & 1 & 2 \\ 1 & 2 & -1 & -3 \\ 2 & 4 & -1 & -1 \end{array} \right]$$

Solution:

$$\begin{aligned} \left[\begin{array}{cccc} 0 & 3 & 1 & 2 \\ 1 & 2 & -1 & -3 \\ 2 & 4 & -1 & -1 \end{array} \right] &\sim \left[\begin{array}{cccc} \textcircled{1} & 2 & -1 & -3 \\ 0 & 3 & 1 & 2 \\ 2 & 4 & -1 & -1 \end{array} \right] \sim \left[\begin{array}{cccc} \textcircled{1} & 2 & -1 & -3 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 1 & 5 \end{array} \right] \sim \left[\begin{array}{cccc} \textcircled{1} & 2 & -1 & -3 \\ 0 & \textcircled{1} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 5 \end{array} \right] \\ &\sim \left[\begin{array}{cccc} \textcircled{1} & 0 & -\frac{5}{3} & -\frac{13}{3} \\ 0 & \textcircled{1} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & \textcircled{1} & 5 \end{array} \right] \sim \left[\begin{array}{cccc} \textcircled{1} & 0 & -\frac{5}{3} & -\frac{13}{3} \\ 0 & \textcircled{1} & 0 & -1 \\ 0 & 0 & \textcircled{1} & 5 \end{array} \right] \sim \left[\begin{array}{cccc} \textcircled{1} & 0 & 0 & 4 \\ 0 & \textcircled{1} & 0 & -1 \\ 0 & 0 & \textcircled{1} & 5 \end{array} \right] \end{aligned}$$

□

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x + 4y + z &= 5 \\ x + 2y &= 3 \end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{ccc|c} 2 & 4 & 1 & 5 \\ 1 & 2 & 0 & 3 \end{array} \right] \right) = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

This corresponds to the system

$$\begin{aligned} x + 2y &= 3 \\ z &= -1 \end{aligned}$$

Since the y -column is a non-pivot column, it is a free variable, so we let $y = a$; then we have

$$\begin{aligned} x + 2y &= 3 \\ y &= a \\ z &= -1 \end{aligned}$$

and thus

$$x = 3 - 2a$$

$$y = a$$

$$z = -1$$

So the solution set is

$$\left\{ \begin{bmatrix} 3 - 2a \\ a \\ -1 \end{bmatrix} \middle| a \in \mathbb{R} \right\}$$

□