Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 18

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Determine if the set of vectors
$$\left\{ \begin{bmatrix} 3\\-1\\0\\4 \end{bmatrix}, \begin{bmatrix} 1\\2\\-2\\1 \end{bmatrix}, \begin{bmatrix} 3\\-8\\6\\5 \end{bmatrix} \right\}$$
 is linearly dependent or linearly independent.

Solution:

$$RREF \left(\begin{bmatrix} 3 & 1 & 3 \\ -1 & 2 & -8 \\ 0 & -2 & 6 \\ 4 & 1 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

Standard S3.

$$\begin{bmatrix}
Standard S3.
\end{bmatrix}$$
Let $W = \text{span}\left(\left\{\begin{bmatrix} 1\\1\\2\\1\end{bmatrix}, \begin{bmatrix} 3\\3\\6\\3\end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix} 7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find a basis for W .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is
$$\left\{ \begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix} \right\}.$$

Mark:

Let
$$W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix},\begin{bmatrix}3\\1\\3\\6\end{bmatrix},\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$$
. Compute the dimension of W .

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

Standard A1.

Mark:

Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\\x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3\\3x_2 - x_3 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and $\mathbb{R}^2.$

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

Standard A2.

Mark:

Determine if the map $T: \mathcal{P}^4 \to \mathcal{P}^3$ given by T(f) = f' - f'' is a linear transformation or not.

Additional Notes/Marks