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SEMIFINAL   Math 237 – Linear Algebra
Version 5   Fall 2017
Choose up to 6 problems to work. Work each problem on one of the attached pages; write the standard in the lower left corner. Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 0$$
$$x - z = 1$$

E2. Find RREF A, where

$$A = \begin{bmatrix} 2 & 2 & 1 & 2 & | & -1 \\ 1 & 1 & 2 & 4 & | & 5 \\ 3 & 3 & -1 & -2 & | & 1 \end{bmatrix}$$

E3. Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

E4. Find a basis for the solution set to the homogeneous system of equations given by

$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$
$$3x_1 + 6x_3 + x_4 = 0$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

V3. Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span $\mathbb{R}^4$.

V4. Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of $\mathcal{P}^3$.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

S2. Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.

S3. Let W be the subspace of $\mathcal{P}^3$ given by
$W = \text{span} \left(\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\} \right)$. Find a basis for W.

S4. Let W be the subspace of $\mathbb{R}^{2\times 2}$ given by $W = \text{span} \left(\left\{ \begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \right\} \right)$.
Compute the dimension of W.

A1. Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_2 + 3x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}$.

A2. Determine if the map $T : \mathcal{P}^3 \to \mathcal{P}^4$ given by $T(f(x)) = xf(x) - f(x)$ is a linear transformation or not.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^4$ given by the standard matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 11 & -1 & 5 \end{bmatrix}$

A4. Let $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be the linear map given by $T \left(\begin{bmatrix} a & b \\ x & y \end{bmatrix} \right) = \begin{bmatrix} a + x \\ 0 \\ b + y \end{bmatrix}$. Compute a basis for the
kernel and a basis for the image of T.

M1. Let

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad\qquad B = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad\qquad C = \begin{bmatrix} 3 & -1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M2. Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

M3. Find the inverse of the matrix $\begin{bmatrix} 6 & 0 & 1 \\ -14 & 3 & -4 \\ -23 & 4 & -6 \end{bmatrix}$.

G1. Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

G3. Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

G4. Compute the geometric multiplicity of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

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