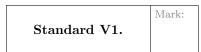
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot (f \oplus g) = c \odot f \oplus c \odot g$.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f.

Standard V3.

Determine if the vectors $\begin{bmatrix} -3\\1\\1 \end{bmatrix}$, $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$, $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$, and $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$ span \mathbb{R}^3

Solution:

$$RREF\left(\begin{bmatrix} -3 & 5 & 2 & 0\\ 1 & -1 & 0 & 2\\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5\\ 0 & 1 & 1 & 3\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span \mathbb{R}^3 .

Determine if
$$\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x,y,z \in \mathbb{R} \right\}$$
 a subspace of \mathbb{R}^4 .

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

Standard S2.

Mark:

Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}_3

Solution:

$$RREF \left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

Additional Notes/Marks