## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compose functions of real numbers
- Solve systems of linear equations **(Standard(s) E3)**
- Find the matrix corresponding to a linear transformation **(Standard(s) A1)**
- Determine if a linear transformation is injective and/or surjective **(Standard(s) A3)**
- Interpret the ideas of injectivity and surjectivity in multiple ways

### Readiness Assurance Resources

The following resources will help you prepare for this module.

- `https://www.khanacademy.org/math/algebra2/manipulating-functions/funciton-composition/v/function-composition`

## Readiness Assurance Test

Choose the most appropriate response for each question.

41) Suppose $f(x)$ and $g(x)$ are real-valued functions satisfying

$$f(2) = 4 \qquad g(2) = 4$$
$$f(3) = 5 \qquad g(3) = 5$$
$$f(4) = 3 \qquad g(4) = 2$$

Compute $(f \circ g)(2)$.

(a) 2 (b) 3 (c) 4 (d) 5

42) Let $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$. Compute the composition function $(f \circ g)(x)$.

(a) $x^2 - 1$ (b) $x^4 + 2x^2 - 1$ (c) $x^4 - 4x^2 + 5$ (d) $x^4 - x^2 - 2$

43) Solve the system of linear equations

$$\begin{aligned} x + 3y &= -2 \\ 2x - 7y &= 9 \end{aligned}$$

(a) $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

44) Let $a, b, c$ be fixed real numbers. How many solutions does the system of linear equations below have?

$$\begin{aligned} x + 2y + 3z &= a \\ y - z &= b \\ y + z &= c \end{aligned}$$

(a) 0 (b) 1 (c) Infinitely many (d) It depends on the values of $a$, $b$, and $c$.

45) What is the standard matrix corresponding to the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix}$?

(a) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

46) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation with standard matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$. Compute $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

47) Which of the following is true of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 3y - 4z \\ x + y \\ 3z \end{bmatrix}?$$

(a) $T$ is neither injective nor surjective
(b) $T$ is injective but not surjective
(c) $T$ is surjective but not injective
(d) $T$ is both injective and surjective

48) Which of the following is true of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + z \end{bmatrix}?$$

(a) $T$ is surjective but not injective
(b) $T$ is injective but not surjective
(c) $T$ is both injective and surjective
(d) $T$ is neither injective nor surjective

49) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix $A$. Which of the following is **not** a characterization of the statement "$T$ is injective"?

(a) If $T(\mathbf{v}) = T(\mathbf{w})$ for some $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$, then $\mathbf{v} = \mathbf{w}$.
(b) The columns of $A$ are linearly independent
(c) $T$ has a non-trivial kernel
(d) $\mathrm{RREF}(A)$ has only pivot columns

50) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix $A$. Which of the following is **not** a characterization of the statement "$T$ is surjective"?

(a) $\mathrm{RREF}(A)$ has a pivot in every row
(b) $\mathrm{RREF}(A)$ has has a pivot in every column
(c) $\operatorname{Im} T = \mathbb{R}^m$
(d) The columns of $A$ span $\mathbb{R}^m$