Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Solve a system of linear equations (including finding a basis of the solution space if it is homogeneous) by interpreting as an augmented matrix and row reducing **(Standard(s) E1, E2, E3, E4)**.
- State the definition of a spanning set, and determine if a set of vectors spans a vector space or subspace **(Standard(s) V3)**.
- State the definition of linear independence, and determine if a set of vectors is linearly dependent or independent **(Standard(s) S1)**.
- State the definition of a basis, and determine if a set of vectors is a basis **(Standard(s) S2)**.

Readiness Assurance Resources

The following resources will help you prepare for this module.

- Review the supporting Standards listed above.

Readiness Assurance Test

Choose the most appropriate response for each question.

1) Which of the following is a solution to the system of linear equations

$$\begin{aligned} x + 3y - z &= 2 \\ 2x + 8y + 3z &= -1 \\ -x - y + 9z &= -10 \end{aligned}$$

(a) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

2) Find a basis for the solution set of the following homogeneous system of linear equations

$$\begin{aligned} x + 2y + -z - w &= 0 \\ -2x - 4y + 3z + 5w &= 0 \end{aligned}$$

(a) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \end{bmatrix} \right\}$ (d) None of these are a basis.

3) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It does not span and is linearly dependent
(b) It does not span and is linearly independent
(c) It spans but it is linearly dependent
(d) It is a basis of $\mathbb{R}^3$.

4) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It does not span and is linearly dependent
(b) It does not span and is linearly independent
(c) It spans but it is linearly dependent
(d) It is a basis of $\mathbb{R}^3$.

5) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It does not span and is linearly dependent
(b) It does not span and is linearly independent
(c) It spans but it is linearly dependent
(d) It is a basis of $\mathbb{R}^3$.

6) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix} \right\} \subset \mathbb{R}^3.$$

(a) It does not span and is linearly dependent
(b) It does not span and is linearly independent
(c) It spans but it is linearly dependent
(d) It is a basis of $\mathbb{R}^3$.

7) Find a basis for the subspace of $\mathbb{R}^4$ spanned by the vectors ...

8) Suppose you know that every vector in $\mathbb{R}^5$ can be written as a linear combination of the vectors $\{\vec{v}_1, \dots, \vec{v}_n\}$. What can you conclude about n?

(a) $n \le 5$
(b) $n = 5$
(c) $n \ge 5$
(d) n could be any positive integer

9) Suppose you know that every vector in $\mathbb{R}^5$ can be written uniquely as a linear combination of the vectors $\{\vec{v}_1, \dots, \vec{v}_n\}$. What can you conclude about n?

(a) $n \le 5$
(b) $n = 5$
(c) $n \ge 5$
(d) n could be any positive integer

10) Suppose you know that every vector in $\mathbb{R}^5$ can be written uniquely as a linear combination of the vectors $\{\vec{v}_1, \dots, \vec{v}_n\}$. What can you conclude about the set $\{\vec{v}_1, \dots, \vec{v}_n\}$?

(a) It does not span and is linearly dependent
(b) It does not span and is linearly independent
(c) It spans but it is linearly dependent
(d) It is a basis of $\mathbb{R}^3$.