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| Name: |
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| Date: |

Dr. Clontz

MASTERY QUIZ DAY 12

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

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| Standard V1. | Mark: |
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Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.
- Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

- Real addition is associative, so \oplus is associative.
- $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.
- $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .
- Real addition is commutative, so \oplus is commutative.
- Associativity shown above
- $1 \odot x = x - 3(1 - 1) = x$
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$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

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$$\begin{aligned}
 (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
 &= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
 &= (c \odot x) \oplus (d \odot x)
 \end{aligned}$$

Therefore V is a vector space.

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| Standard V3. | Mark: |
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Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^5?$

Solution: Since there are only three vectors, they cannot span \mathbb{R}^5 .

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| Standard V4. | Mark: |
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Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 1$ (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

Solution: No, because $\mathbf{0}$ does not belong to W .

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| Additional Notes/Marks | |
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