

Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compose functions of real numbers
- Solve systems of linear equations (**Standard(s) E.3**)
- Find the matrix corresponding to a linear transformation (**Standard(s) A.1**)
- Determine if a linear transformation is injective and/or surjective (**Standard(s) A.3**)
- Interpret the ideas of injectivity and surjectivity in multiple ways

Readiness Assurance Resources

The following resources will help you prepare for this module.

- <https://www.khanacademy.org/math/algebra2/manipulating-functions/function-composition/v/function-composition>

Readiness Assurance Test

Choose the most appropriate response for each question.

1) Let $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$. Compute the composition function $(f \circ g)(x)$.

- (a) $x^2 - 1$
- (b) $x^4 + 2x^2 - 1$
- (c) $x^4 - 4x^2 + 5$
- (d) $x^4 - x^2 - 2$

2) Suppose $f(x)$ and $g(x)$ are real-valued functions satisfying

$$\begin{array}{ll} f(2) = 1 & g(2) = 3 \\ f(3) = 4 & g(3) = 5 \\ f(4) = 3 & g(4) = 6 \end{array}$$

Compute $(f \circ g)(2)$.

- (a) 2
- (b) 3
- (c) 4
- (d) 5

3) Solve the system of linear equations

$$\begin{array}{l} x + 3y = -2 \\ 2x - 7y = 9 \end{array}$$

- (a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$

4) Let a, b, c be fixed real numbers. How many solutions does the system of linear equations below have?

$$\begin{array}{l} x + 2y + 3z = a \\ y - z = b \\ y + z = c \end{array}$$

- (a) 0 (b) 1 (c) Infinitely many (d) It depends on the values of a , b , and c .

5) What is the matrix corresponding to the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) =$

$$\begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix} ?$$

(a) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

6) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation with associated matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$. Compute

$$T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right).$$

(a) $\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

7) Which of the following is true of the linear transformation T :

- (a) T is neither injective nor surjective
- (b) T is injective but not surjective
- (c) T is surjective but not injective
- (d) T is both injective and surjective

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- (a) T is neither injective nor surjective
- (b) T is injective but not surjective
- (c) T is surjective but not injective
- (d) T is both injective and surjective

9) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation with associated matrix $A \in M_{m,n}(\mathbb{R})$. Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a) T is injective
- (b) T has a non-trivial kernel
- (c) The columns of A are linearly dependent
- (d) $\text{RREF}(A)$ has a non-pivot column

10) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation with associated matrix $A \in M_{m,n}(\mathbb{R})$. Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a) T is surjective
- (b) $\text{Im } T = \mathbb{R}^m$
- (c) The columns of A span \mathbb{R}^m
- (d) $\text{RREF}(A)$ has only pivot columns