Version 1

Math 237 – Linear Algebra Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S2. Determine if the set
$$\left\{ \begin{bmatrix} 3\\-1\\2\\3 \end{bmatrix}, \begin{bmatrix} 2\\0\\2\\4 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\-1 \end{bmatrix}, \begin{bmatrix} -1\\3\\0\\5 \end{bmatrix} \right\}$$
 is a basis of \mathbb{R}^4 .

A3. Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a)
$$S: \mathbb{R}^2 \to \mathbb{R}^3$$
 given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

(b)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

A4. Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute the kernel and image of T.

S2:	A3:	A4:	

Version 2

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Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S2. Determine if the set
$$\left\{ \begin{bmatrix} 3\\-1\\2\\3 \end{bmatrix}, \begin{bmatrix} 2\\0\\2\\4 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\-1 \end{bmatrix}, \begin{bmatrix} -1\\3\\0\\5 \end{bmatrix} \right\}$$
 is a basis of \mathbb{R}^4 .

(a)
$$S: \mathbb{R}^2 \to \mathbb{R}^4$$
 given by the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$.

(b)
$$T: \mathbb{R}^4 \to \mathbb{R}^3$$
 given by the matrix $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 7 & -1 & 5 \end{bmatrix}$

A4. Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by $T \begin{pmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute the kernel and image of T.

Name:	

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Version 3 Fall 2017 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S2. Determine if the set $\left\{ \begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ -1 & 8 \end{bmatrix}, \begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^{2 \times 2}$.

- (a) $S: \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
- (b) $T: \mathbb{R}^4 \to \mathbb{R}^3$ given by the matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 4 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4. Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by $T \begin{pmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute the kernel and image of T.

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Version 4

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

 $\mathbf{S2.} \quad \text{Determine if the set } \left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\} \text{ is a basis of } M_{2,2} \text{ or not.}$

- (a) $S: \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
- (b) $T: \mathbb{R}^4 \to \mathbb{R}^3$ given by the matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4. Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute the kernel and image of T.

S2:	A3:	A4:	

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Version 5 Fall 2017 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S2. Determine if the set $\left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\}$ is a basis of $M_{2,2}$ or not.

A3. Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a)
$$S: \mathbb{R}^2 \to \mathbb{R}^3$$
 given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

(b)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

A4. Let $T: \mathcal{P}^3 \to \mathcal{P}^3$ be the linear transformation given by
$T(ax^3 + bx^2 + cx + d) = (a + 3b + 3c + 7d)x^3 + (a + 3b - c - d)x^2 + (2a + 6b + 3c + 8d)x + (a + 3b - 2c - 3d)x^2 + (2a + 6b + 3c + 8d)x + (a + 3b - 2c - 3d)x^2 + (2a + 6b + 3c + 8d)x + (a + 3b - 2c - 3d)x^2 + (2a + 6b + 3c + 8d)x + (a + 3b - 2c - 3d)x^2 + (2a + 6b + 3c + 8d)x + (a + 3b - 2c - 3d)x^2 + (2a + 6b + 3c + 8d)x + (a + 3b - 2c - 3d)x^2 + (a + 3b - $
Compute the kernel and image of T .

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Version 6 Fall 2017 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S2. Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x^2 - 2\}$ is a basis of \mathcal{P}^2 .

(a)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

(b)
$$S: \mathbb{R}^2 \to \mathbb{R}^3$$
 given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

A4. Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by $T \begin{pmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute the kernel and image of T.