Name:	
J#:	
Date:	

Dr. Clontz

MASTERY QUIZ DAY 14 Math 237 – Linear Algebra

Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication $\odot$ distributes over scalar addition.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c+d) \odot (x_1, y_1) = (0, (c+d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

□

Standard V3.	Mark:

Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\\2\\1\end{bmatrix}, \begin{bmatrix}-1\\3\\5\\2\\0\end{bmatrix}, \begin{bmatrix}1\\0\\5\\1\\-3\end{bmatrix}\right\} = \mathbb{R}^5$?

Solution: Since there are only three vectors, they cannot span $\mathbb{R}^5$.

□

Standard V4.	Mark:

Let W be the set of all complex numbers that are purely real (i.e of the form $a + 0i$) or purely imaginary (i.e. of the form $0 + bi$). Determine if W is a subspace of $\mathbb{C}$.

Solution: No, because 1 is purely real and i is purely imaginary, but the linear combination $1 + i$ is neither.

□

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Additional Notes/Marks	

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MASTERY QUIZ DAY 14 Math 237 – Linear Algebra
Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all points on the line $x+y=2$ with the operations, for any $(x_1,y_1),(x_2,y_2)\in V$, $c\in\mathbb{R}$,

$$\begin{aligned}(x_1,y_1)\oplus(x_2,y_2)&=(x_1+x_2-1,y_1+y_2-1)\\ c\odot(x_1,y_1)&=(cx_1-(c-1),cy_1-(c-2))\end{aligned}$$

(a) Show that this vector space has an additive identity element.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1,y_1)\in V$; then $(x_1,y_1)\oplus(1,1)=(x_1,y_1)$, so $(1,1)$ is an additive identity element.

Now we will show the other seven properties. Let $(x_1,y_1),(x_2,y_2)\in V$, and let $c,d\in\mathbb{R}$.

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) The additive identity is $(1,1)$.

4) $(x_1,y_1)\oplus(2-x_1,2-y_1)=(1,1)$, so $(2-x_1,2-y_1)$ is the additive inverse of (x_1,y_1).

5)

$$\begin{aligned}c\odot(d\odot(x_1,y_1))&=c\odot(dx_1-(d-1),dy_1-(d-1))\\ &=(c\,(dx_1-(d-1))-(c-1),c\,(dy_1-(d-1)))\\ &=(cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &=(cdx_1-(cd-1),cdy_1-(cd-1))\\ &=(cd)\odot(x_1,y_1)\end{aligned}$$

6) $1\odot(x_1,y_1)=(x_1-(1-1),y_1-(1-1)=(x_1,y_1)$

7)

$$\begin{aligned}c\odot((x_1,y_1)\oplus(x_2,y_2))&=c\odot(x_1+y_1-1,x_2+y_2-1)\\ &=(c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &=(cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &=(cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot(x_1,y_1) &= ((c+d)x_1-(c+d-1),(c+d)y_1-(c+d-1))\\ &= (cx_1-(c-1),cy_1-(c-1))\oplus(dx_1-(d-1),dy_1-(d-1))\\ &= c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

Therefore V is a vector space.

□

Standard V3.	Mark:

Does span $\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix},\begin{bmatrix}3\\12\\-9\end{bmatrix},\begin{bmatrix}1\\4\\-3\end{bmatrix},\begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}=\mathbb{R}^3$?

Solution: Since

$$\mathrm{RREF}\begin{bmatrix}2&3&1&-4\\-1&12&4&2\\4&-9&-3&-8\end{bmatrix}=\begin{bmatrix}1&0&0&-2\\0&1&1/3&0\\0&0&0&0\end{bmatrix}$$

has a zero row, the vectors fail to span $\mathbb{R}^3$.

□

Standard V4.	Mark:

Let W be the set of all polynomials of the form ax^3+bx. Determine if W is a subspace of $\mathcal{P}^3$.

Solution: Yes because $s(a_1x^3+b_1x)+t(a_2x^3+b_2x)=(sa_1+ta_2)x^3+(sb_1+tb_2)x$ also belongs to W. Alternately, yes because W is isomorphic to $\mathbb{R}^2$.

□

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix}1\\1\\-1\end{bmatrix},\begin{bmatrix}3\\-1\\1\end{bmatrix},\begin{bmatrix}2\\0\\-2\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix}1&3&2\\1&-1&0\\-1&1&-2\end{bmatrix}\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 14 Math 237 – Linear Algebra
Version 3 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication $\odot$ distributes over scalar addition.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c + d) \odot (x_1, y_1) = (0, (c + d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

□

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\\0\\-3\end{bmatrix}$, $\begin{bmatrix}0\\3\\0\\-2\end{bmatrix}$, and $\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 0 & -1\\0 & 1 & 3 & 1\\2 & 0 & 0 & -1\\1 & -3 & -2 & -1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Since every row contains a pivot, the vectors span $\mathbb{R}^4$.

□

Standard V4.	Mark:

Let W be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 0$ (this forms a plane). Determine if W is a subspace of $\mathbb{R}^3$.

Solution: Yes, because $z = -x - y$ and $a \begin{bmatrix} x_1 \\ y_1 \\ -x_1 - y_1 \end{bmatrix} + b \begin{bmatrix} x_2 \\ y_2 \\ -x_2 - y_2 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ ay_1 + by_2 \\ -(ax_1 + bx_2) - (ay_1 + by_2) \end{bmatrix}$.
Alternately, yes because W is isomorphic to $\mathbb{R}^2$.

$\square$

Standard S2.	Mark:

Determine if the set $\left\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\right\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Additional Notes/Marks	

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MASTERY QUIZ DAY 14 Math 237 – Linear Algebra
Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

(a) Show that this scalar multiplication $\odot$ distributes over vector addition $\oplus$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'\,' + cg'\,' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f.

□

Standard V3.	Mark:

Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\end{bmatrix}, \begin{bmatrix}1\\4\\-3\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\} = \mathbb{R}^3$?

Solution: Since

$$\text{RREF}\begin{bmatrix}2 & 3 & 1 & -4\\-1 & 12 & 4 & 2\\4 & -9 & -3 & -8\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & -2\\0 & 1 & 1/3 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

has a zero row, the vectors fail to span $\mathbb{R}^3$.

□

Standard V4.	Mark:

Let W be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 1$ (this forms a plane). Determine if W is a subspace of $\mathbb{R}^3$.

Solution: No, because $\mathbf{0}$ does not belong to W.

□

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

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MASTERY QUIZ DAY 14 Math 237 – Linear Algebra
Version 5 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2, y_1 + y_2) \\ c \odot (x_1, y_1) &= (c^2 x_1, c^3 y_1) \end{aligned}$$

(a) Show that this scalar multiplication $\odot$ distributes over vector addition $\oplus$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1), (x_2, y_2) \in V$ and let $c \in \mathbb{R}$.

$$\begin{aligned} c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + x_2, y_1 + y_2) \\ &= (c^2(x_1 + x_2), c^3(y_1 + y_2)) \\ &= (c^2 x_1, c^3 y_1) \oplus (c^2 x_2, c^3 y_2) \\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2) \end{aligned}$$

However, V is not a vector space, as the other distributive law fails:

$$(c + d) \odot (x_1, y_1) = ((c + d)^2 x_1, (c + d)^3 y_1) \neq ((c^2 + d^2) x_1, (c^3 + d^3) y_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

$\square$

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span $\mathbb{R}^3$.

Solution:

$$\text{RREF} \left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span $\mathbb{R}^3$.

$\square$

Standard V4.	Mark:

Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \;\middle|\; x, y, z \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^4$.

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 1 & 2 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

Additional Notes/Marks	

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MASTERY QUIZ DAY 14 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

(a) Show that this scalar multiplication $\odot$ is associative.

(b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d-1)) \\ &= c\,(dx - 3(d-1)) - 3(c-1) \\ &= cdx - 3(cd-1) \\ &= (cd) \odot x \end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6-x) = x + (6-x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1-1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c-1) \\ &= cx - 3(c-1) + cy - 3(c-1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot x &= (c+d)x-3(c+d-1)\\ &= cx-3(c-1)+dx-3(c-1)-3\\ &= (c\odot x)\oplus(d\odot x)\end{aligned}$$

Therefore V is a vector space.

□

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}$, $\begin{bmatrix}3\\1\\3\\6\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$, and $\begin{bmatrix}1\\2\\0\\1\end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}2&3&0&1\\0&1&0&2\\-2&3&1&0\\0&6&1&1\end{bmatrix}\right)=\begin{bmatrix}1&0&0&-\frac{5}{2}\\0&1&0&2\\0&0&1&-11\\0&0&0&0\end{bmatrix}$$

Since there is a zero row, the vectors do not span $\mathbb{R}^4$.

□

Standard V4.	Mark:

Let W be the set of all complex numbers $a+bi$ satisfying $a=2b$. Determine if W is a subspace of $\mathbb{C}$.

Solution: Yes, because $c(2b_1+b_1i)+d(2b_2+b_2i)=2(cb_1+db_2)+(cb_1+db_2)i$ belongs to W. Alternately, yes because W is isomorphic to $\mathbb{R}$.

□

Standard S2.	Mark:

Determine if the set $\left\{x^2+x-1, 3x^2-x+1, 2x-2\right\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix}1&3&2\\1&-1&0\\-1&1&-2\end{bmatrix}\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	