

Section E.2**Activity E.21** (~ 8 min) Consider the matrix

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & -1 & 4 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

- (a) Add row 3 to row 2 ($R_2 + R_3 \rightarrow R_2$)
- (b) Add row 2 to row 3 ($R_3 + R_2 \rightarrow R_3$)
- (c) Add -1 row 1 to row 2 ($R_2 - R_1 \rightarrow R_2$)
- (d) Add -2 row 1 to row 2 ($R_2 - 2R_1 \rightarrow R_2$)

Activity E.22 (~ 7 min) Consider the matrix

$$\begin{bmatrix} 2 & 5 & -1 \\ 3 & 5 & 1 \\ 1 & -2 & 0 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

- (a) Add row 1 to row 2 ($R_2 + R_1 \rightarrow R_2$)
- (b) Add -1 row 1 to row 2 ($R_2 - R_1 \rightarrow R_2$)
- (c) Swap rows 1 and 3 ($R_1 \leftrightarrow R_3$)
- (d) Multiply row 1 by $\frac{1}{2}$ ($\frac{1}{2}R_1 \rightarrow R_1$)
- (e) Add -1 row 3 to row 1 ($R_1 - R_3 \rightarrow R_1$)

Activity E.23 (*~5 min*) Consider the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

- (a) Add row 1 to row 2 ($R_2 + R_1 \rightarrow R_2$)
- (b) Add -1 row 1 to row 2 ($R_2 - R_1 \rightarrow R_2$)
- (c) Add -1 row 3 to row 1 ($R_1 - R_3 \rightarrow R_1$)
- (d) Add -2 row 3 to row 2 ($R_2 - 2R_3 \rightarrow R_2$)
- (e) Multiply row 3 by $\frac{1}{2}$ ($\frac{1}{2}R_3 \rightarrow R_3$)

Activity E.24 (*~10 min*) Consider the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & -1 & 1 \end{bmatrix}.$$

Perform three row operations to produce a matrix closer to RREF.

Activity E.25 (*~10 min*) Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ -1 & 3 & -5 \end{bmatrix}.$$

Compute $\text{RREF}(A)$.

Activity E.26 (*~10 min*) Consider the matrix

$$A = \begin{bmatrix} 2 & 4 & 2 & -4 \\ -2 & -4 & 1 & 1 \\ 3 & 6 & -1 & -4 \end{bmatrix}.$$

Compute $\text{RREF}(A)$.

Remark E.27 A video example of how to perform the Gauss-Jordan Elimination algorithm by hand is available at <https://youtu.be/Cq0Nxx2dhhU>.

Practicing several exercises on your own using this method is strongly recommended.

Activity E.28 (~ 10 min) Free browser-based technologies for mathematical computation are available online.

- Go to <https://octave-online.net>.
- Type `A=sym([1 3 4 ; 2 5 7])` and press **Enter** to store the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$ in the variable A .
 - The symbolic function `sym` is used to calculate precise answers rather than floating-point approximations.
 - The vertical bar in an augmented matrix does not affect row operations, so the RREF of $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$ may be computed in the same way.
- Type `rref(A)` and press **Enter** to compute the reduced row echelon form of A .

Remark E.29 We will frequently need to know the reduced row echelon form of matrices during class, so feel free to use Octave-Online.net to compute RREF efficiently.

You may alternatively use the calculator you will use during assessments. Be sure to use fractions mode to compute exact solutions rather than floating-point approximations.

Activity E.30 (~ 10 min) Consider the system of equations.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 + 3x_2 - 6x_3 &= 11 \end{aligned}$$

Part 1: Convert this to an augmented matrix and use technology to compute its reduced row echelon form:

$$\text{RREF} \left[\begin{array}{ccc|c} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{array} \right] = \left[\begin{array}{ccc|c} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{array} \right]$$

Part 2: Use the RREF matrix to write a linear system equivalent to the original system. Then find its solution set.