Name:
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Date:

Dr. Clontz

MASTERY QUIZ DAY 15
Version 3

Math 237 – Linear Algebra
Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V2.	Mark:

Determine if $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

Solution:

$$\mathrm{RREF}\left(\left[\begin{array}{cc|c} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{array}\right]\right) = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Since this system has a solution, $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$, namely

$$\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

□

Standard S1.	Mark:

Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} -3 & 1 & 0 \\ 8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Every column is a pivot column, therefore the set is linearly independent.

□

Standard S3.	Mark:

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Find a basis of W.

Solution:

$$\text{RREF}\left(\begin{bmatrix}2 & 3 & 0 & 1\\0 & 1 & 0 & 2\\-2 & 3 & 1 & 0\\0 & 6 & 1 & 1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0 & -\frac{5}{2}\\0 & 1 & 0 & 2\\0 & 0 & 1 & -11\\0 & 0 & 0 & 0\end{bmatrix}$$

Then $\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}\right\}$ is a basis of W.

$\square$

Standard S4.	Mark:

Let W be the subspace of $\mathcal{P}_3$ given by
$W = \operatorname{span}\left(\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\}\right)$. Compute the dimension of W.

Solution:

$$\text{RREF}\left(\begin{bmatrix}1 & 2 & 3 & 1\\-1 & 0 & -1 & 1\\3 & 1 & 4 & 1\\-3 & 1 & -2 & -7\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 1 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

$\square$

Additional Notes/Marks	