

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 8**

Math 237 – Linear Algebra

**Version 4**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write an augmented matrix corresponding to the following system of linear equations

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

**E3.** Solve the system of linear equations.

$$\begin{aligned}2x + y - z + w &= 5 \\3x - y - 2w &= 0 \\-x + 5z + 3w &= -1\end{aligned}$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right] \right) = \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{4}{12} & 0 \end{array} \right]$$

So the solutions are

$$\left\{ \left[ \begin{array}{c} 1+a \\ 3-21a \\ -7a \\ 12a \end{array} \right] \mid a \in \mathbb{R} \right\}$$

□

**E4.** Find a basis for the solution set of the system ...

**V1.** Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

Determine if  $V$  is a vector space or not.

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ .

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6 - x) = x + (6 - x) - 3 = 3$ , so  $6 - x$  is the additive inverse of  $x$ .
- 4) Real addition is commutative, so  $\oplus$  is commutative.

5)

$$\begin{aligned}
 c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\
 &= c(dx - 3(d - 1)) - 3(c - 1) \\
 &= cdx - 3(cd - 1) \\
 &= (cd) \odot x
 \end{aligned}$$

6)  $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned}
 c \odot (x \oplus y) &= c \odot (x + y - 3) \\
 &= c(x + y - 3) - 3(c - 1) \\
 &= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
 &= (c \odot x) \oplus (c \odot y)
 \end{aligned}$$

8)

$$\begin{aligned}
 (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
 &= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\
 &= (c \odot x) \oplus (d \odot x)
 \end{aligned}$$

Therefore  $V$  is a vector space.

□

**E1:**

**E3:**

**E4:**

**V1:**