

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 10**

Math 237 – Linear Algebra

**Version 5**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array} \right]$$

**Solution:**

$$-4x_1 - x_2 + 3x_3 = 2$$

$$x_1 + 2x_2 - x_3 = 0$$

$$-x_1 + 4x_2 + x_3 = 4$$

□

**E3.** Solve the system of linear equations.

$$2x + y - z + w = 5$$

$$3x - y - 2w = 0$$

$$-x + 5z + 3w = -1$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right] \right) = \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{4}{12} & 0 \end{array} \right]$$

So the solutions are

$$\left\{ \left[ \begin{array}{c} 1+a \\ 3-21a \\ -7a \\ 12a \end{array} \right] \mid a \in \mathbb{R} \right\}$$

□

**E4.** Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

$$-x + 2z + 5w = 0$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \\ -1 & 0 & 2 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} a \\ 2a \\ -2a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

So a basis for the solution set is  $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \right\}$ .

□

**V1.** Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$\begin{aligned} x \oplus y &= \sqrt{x^2 + y^2} \\ c \odot x &= cx \end{aligned}$$

- (a) Show that the vector **addition**  $\oplus$  is **associative**:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .  
 (b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $x, y, z \in \mathbb{R}$ . Then

$$\begin{aligned} (x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\ &= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\ &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\ &= x \oplus \sqrt{y^2 + z^2} \\ &= x \oplus (y \oplus z) \end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

**E1:**

**E3:**

**E4:**

**V1:**

**E2:**