

Name: \_\_\_\_\_

# MASTERY QUIZ DAY 29

Math 237 – Linear Algebra

## Version 1

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix}$ .

**Solution:**

$$\begin{aligned} \det \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} &= 2 \det \begin{bmatrix} 3 & 0 & -1 \\ 1 & 3 & 1 \\ -3 & -2 & -1 \end{bmatrix} - (-1) \det \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix} \\ &= 2 \left( 3 \det \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} + (-1) \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} \right) + \left( 1 \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} - 3 \det \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix} \right) \\ &= 2 (3(-1) + (-1)(7)) + ((1)(7) - 3(-3)) \\ &= 2(-10) + 16 \\ &= -4 \end{aligned}$$

□

**G3.** Compute the eigenspace of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

□

**G4.** Compute the geometric multiplicity of the eigenvalue  $2$  in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system  $(B - 2I)X = 0$ .

$$\text{RREF}(B - 2I) = \text{RREF} \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the geometric multiplicity is 2.

□

**G1:**

**G3:**

**G4:**

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# **MASTERY QUIZ DAY 29**

Math 237 – Linear Algebra

## **Version 2**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix}$ .

**Solution:**

$$\begin{aligned} \det \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} &= 2 \det \begin{bmatrix} 3 & 0 & -1 \\ 1 & 3 & 1 \\ -3 & -2 & -1 \end{bmatrix} - (-1) \det \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix} \\ &= 2 \left( 3 \det \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} + (-1) \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} \right) + \left( 1 \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} - 3 \det \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix} \right) \\ &= 2 (3(-1) + (-1)(7)) + ((1)(7) - 3(-3)) \\ &= 2(-10) + 16 \\ &= -4 \end{aligned}$$

□

**G3.** Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system  $(B - 2I)X = 0$ .

$$\text{RREF}(B - 2I) = \text{RREF} \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to  $x - \frac{y}{3} = 0$ , or  $3x = y$ . Thus the eigenspace is

$$E_2 = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

□

**G4.** Compute the geometric multiplicity of the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system  $(B - 2I)X = 0$ .

$$\text{RREF}(B - 2I) = \text{RREF} \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the geometric multiplicity is 2.

□

G1:

G3:

G4:

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 29**

Math 237 – Linear Algebra

**Version 3**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ .

**Solution:** 2

□

**G3.** Compute the eigenspace of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

□

**G4.** Compute the geometric multiplicity of the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system  $(B - 2I)X = 0$ .

$$\text{RREF}(B - 2I) = \text{RREF}\left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the geometric multiplicity is 2.

□

**G1:**

**G3:**

**G4:**

Name: \_\_\_\_\_

# **MASTERY QUIZ DAY 29**

Math 237 – Linear Algebra

## **Version 4**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix}$ .

**Solution:**

$$\begin{aligned} \det \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} &= 2 \det \begin{bmatrix} 3 & 0 & -1 \\ 1 & 3 & 1 \\ -3 & -2 & -1 \end{bmatrix} - (-1) \det \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix} \\ &= 2 \left( 3 \det \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} + (-1) \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} \right) + \left( 1 \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} - 3 \det \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix} \right) \\ &= 2(3(-1) + (-1)(7)) + ((1)(7) - 3(-3)) \\ &= 2(-10) + 16 \\ &= -4 \end{aligned}$$

□

**G3.** Compute the eigenspace of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

□

**G4.** Compute the geometric multiplicity of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the geometric multiplicity is 2.

□

**G1:**

**G3:**

**G4:**

Name: \_\_\_\_\_

# MASTERY QUIZ DAY 29

Math 237 – Linear Algebra

## Version 5

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix}$ .

**Solution:**

$$\begin{aligned} \det \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} &= 2 \det \begin{bmatrix} 3 & 0 & -1 \\ 1 & 3 & 1 \\ -3 & -2 & -1 \end{bmatrix} - (-1) \det \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix} \\ &= 2 \left( 3 \det \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} + (-1) \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} \right) + \left( 1 \det \begin{bmatrix} 1 & 3 \\ -3 & -2 \end{bmatrix} - 3 \det \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix} \right) \\ &= 2(3(-1) + (-1)(7)) + ((1)(7) - 3(-3)) \\ &= 2(-10) + 16 \\ &= -4 \end{aligned}$$

□

**G3.** Compute the eigenspace of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

□

**G4.** Compute the geometric multiplicity of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the geometric multiplicity is 2.

□

**G1:**

**G3:**

**G4:**

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 29**

Math 237 – Linear Algebra

**Version 6**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ .

**Solution:** 2

□

**G3.** Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system  $(B - 2I)X = 0$ .

$$\text{RREF}(B - 2I) = \text{RREF} \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to  $x - \frac{y}{3} = 0$ , or  $3x = y$ . Thus the eigenspace is

$$E_2 = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

□

**G4.** Compute the geometric multiplicity of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the geometric multiplicity is 2.

□

**G1:**

**G3:**

**G4:**