

Name:
J#:
Date:

Dr. Clontz

MASTERY QUIZ DAY 12

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
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Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

- (a) Show that the vector addition \oplus is associative.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$\begin{aligned}
 (x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\
 &= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\
 &= \sqrt{x^2 + y^2 + z^2} \\
 &= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\
 &= x \oplus \sqrt{y^2 + z^2} \\
 &= x \oplus (y \oplus z)
 \end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

Standard V3.	Mark:
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Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span \mathbb{R}^3 .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

□

Standard V4.	Mark:
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Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

Additional Notes/Marks	
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MASTERY QUIZ DAY 12

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
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Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$

$$c \odot f = cf'$$

(here f' denotes the derivative of f).

(a) Show that this scalar multiplication \odot distributes over vector addition \oplus .

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f .

□

Standard V3.	Mark:
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Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Solution: Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span \mathbb{R}^3 .

□

Standard V4.	Mark:
---------------------	-------

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

□

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Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
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Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- Show that this scalar multiplication \odot is associative.
- Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

- Real addition is associative, so \oplus is associative.
- $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.
- $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .
- Real addition is commutative, so \oplus is commutative.
- Associativity shown above
- $1 \odot x = x - 3(1 - 1) = x$
-

$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}
 (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
 &= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
 &= (c \odot x) \oplus (d \odot x)
 \end{aligned}$$

Therefore V is a vector space.

□

Standard V3.	Mark:
---------------------	-------

Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

□

Standard V4.	Mark:
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Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W , but $(x - x^2) + (x^2) = x \notin W$.

□

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Math 237 – Linear Algebra

Version 4

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Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
---------------------	-------

Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- Show that this scalar multiplication \odot is associative.
- Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

- Real addition is associative, so \oplus is associative.
- $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.
- $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .
- Real addition is commutative, so \oplus is commutative.
- Associativity shown above
- $1 \odot x = x - 3(1 - 1) = x$
-

$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}
 (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
 &= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
 &= (c \odot x) \oplus (d \odot x)
 \end{aligned}$$

Therefore V is a vector space.

□

Standard V3.	Mark:
---------------------	-------

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Solution: Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span \mathbb{R}^3 .

□

Standard V4.	Mark:
---------------------	-------

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

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Math 237 – Linear Algebra

Version 5

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Standard V1.	Mark:
---------------------	-------

Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

- Show that this scalar multiplication \odot is associative.
- Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

- Real addition is associative, so \oplus is associative.
- $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.
- $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .
- Real addition is commutative, so \oplus is commutative.
- Associativity shown above
- $1 \odot x = x - 3(1 - 1) = x$
-

$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}
 (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
 &= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
 &= (c \odot x) \oplus (d \odot x)
 \end{aligned}$$

Therefore V is a vector space.

□

Standard V3.	Mark:
---------------------	-------

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Solution: Since

$$\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span \mathbb{R}^3 .

□

Standard V4.	Mark:
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Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

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Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
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Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication \odot distributes over scalar addition.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c + d) \odot (x_1, y_1) = (0, (c + d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

□

Standard V3.	Mark:
---------------------	-------

Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

□

Standard V4.	Mark:
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Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W , but $(x - x^2) + (x^2) = x \notin W$.

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