

## Application Activities - Module S Part 1 - Class Day 12

**Activity 12.1** In the previous module, we considered

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix}, \begin{bmatrix} -1 \\ 10 \\ 7 \\ 14 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \\ 2 \end{bmatrix} \right\}$$

and showed that  $\text{span } S \neq \mathbb{R}^4$ . Find two vectors from this set that are linear combinations of the other three vectors.

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**Definition 12.2** We say that a set of vectors is **linearly dependent** if one vector in the set belongs to the span of the others. Otherwise, we say the set is **linearly independent**.

**Activity 12.3** Suppose  $a\mathbf{v}_1 + b\mathbf{v}_2 = \mathbf{v}_3$ , so the set  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  is linearly dependent. Is the vector equation  $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$  consistent with one solution, consistent with infinitely many solutions, or inconsistent?

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**Fact 12.4** The set  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  is linearly dependent if and only if  $x_1\mathbf{v}_1 + \dots + x_n\mathbf{v}_n = \mathbf{0}$  is consistent with infinitely many solutions.

**Activity 12.5** Find

$$\text{RREF} \left[ \begin{array}{ccccc|c} 2 & 2 & 3 & -1 & 4 & 0 \\ 3 & 0 & 13 & 10 & 3 & 0 \\ 0 & 0 & 7 & 7 & 0 & 0 \\ -1 & 3 & 16 & 14 & 2 & 0 \end{array} \right]$$

and mark the part of the matrix that demonstrates that

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix}, \begin{bmatrix} -1 \\ 10 \\ 7 \\ 14 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \\ 2 \end{bmatrix} \right\}$$

is linearly dependent.

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**Fact 12.6** A set of Euclidean vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  is linearly dependent if and only if  $\text{RREF} [\mathbf{v}_1 \ \dots \ \mathbf{v}_n]$  has a column without a pivot position.

**Activity 12.7** Is the set of Euclidean vectors  $\left\{ \begin{bmatrix} -4 \\ 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 10 \\ 10 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 7 \\ 2 \\ 1 \end{bmatrix} \right\}$  linearly dependent or linearly independent?

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**Activity 12.8** Is the set of polynomials  $\{x^3 + 1, x^2 + 2, 4 - 7x, 2x^3 + x\}$  linearly dependent or linearly independent?

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