Math 237 – Linear Algebra Fall 2017

S1:

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V2. Determine if $\begin{bmatrix} 0\\1\\-2\\1 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 5\\2\\-3\\2 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\1\\0 \end{bmatrix}$, and $\begin{bmatrix} 8\\3\\5\\-1 \end{bmatrix}$.

Solution:

$$RREF\left(\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system has no solution, so $\begin{bmatrix} 0\\1\\-2\\1 \end{bmatrix}$ is not a linear combination of the three other vectors.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\-1 \end{bmatrix}, \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Name:		

Math 237 – Linear Algebra Fall 2017

Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V2. Determine if $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$.

Solution:

$$RREF\left(\begin{bmatrix} 3 & 1 & 5 & | & 1\\ 0 & -1 & 1 & | & 4\\ -1 & 4 & -6 & | & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 2 & | & 0\\ 0 & 1 & -1 & | & 0\\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

So $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is not a linear combination of the three vectors.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 3\\-1\\0\\4 \end{bmatrix}, \begin{bmatrix} 1\\2\\-2\\1 \end{bmatrix}, \begin{bmatrix} 3\\-8\\6\\5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

Solution:

$$RREF \left(\begin{bmatrix} 3 & 1 & 3 \\ -1 & 2 & -8 \\ 0 & -2 & 6 \\ 4 & 1 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

Name:	

Math 237 – Linear Algebra Fall 2017

Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V2. Determine if $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 2\\3\\-1 \end{bmatrix}$, $\begin{bmatrix} 1\\-1\\0 \end{bmatrix}$, and $\begin{bmatrix} -3\\-2\\5 \end{bmatrix}$.

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 1 & -3 & 1 \\ 3 & -1 & -2 & 4 \\ -1 & 0 & 5 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Since this system has a solution, $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is a linear combination of the three vectors.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} -3\\8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix}, \begin{bmatrix} 0\\-1\\3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V2. Determine if $\begin{bmatrix} 0\\1\\-2\\1 \end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix} 5\\2\\-3\\2 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\1\\0 \end{bmatrix}$, and $\begin{bmatrix} 8\\3\\5\\-1 \end{bmatrix}$.

Solution:

$$RREF\left(\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system has no solution, so $\begin{bmatrix} 0\\1\\-2\\1 \end{bmatrix}$ is not a linear combination of the three other vectors.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} -3\\8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix}, \begin{bmatrix} 0\\-1\\3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

 $\mathbf{v_2}$

S1:

Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V2. Determine if
$$\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$$
 belongs to the span of the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 4 \\ 3 \end{bmatrix} \right\}$.

Solution: Since

RREF
$$\begin{pmatrix} \begin{bmatrix} 2 & 4 & | & 4 \\ 0 & -1 & | & -1 \\ -1 & 4 & | & 6 \\ 5 & 3 & | & -7 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 1 \\ 0 & 0 & | & 0 \end{bmatrix}$$

contains the contradiction 0 = 1, $\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$ is not a linear combination of the three vectors.

S1. Determine if the vectors $\begin{bmatrix} 1\\1\\-1 \end{bmatrix}$, $\begin{bmatrix} 3\\-1\\1 \end{bmatrix}$, and $\begin{bmatrix} 2\\0\\-2 \end{bmatrix}$ are linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since each column is a pivot column, the vectors are linearly independent.

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V2. Determine if $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$.

Solution:

$$RREF\left(\begin{bmatrix} 3 & 1 & 5 & | & 1\\ 0 & -1 & 1 & | & 4\\ -1 & 4 & -6 & | & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 2 & | & 0\\ 0 & 1 & -1 & | & 0\\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

So $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is not a linear combination of the three vectors.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} -3\\8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix}, \begin{bmatrix} 0\\-1\\3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.