

## Definitions - Module V Part 1 - Class Day 7

**Definition 7.2** A **vector space**  $V$  is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  belong to  $V$ , and let  $a, b$  be scalar numbers.

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| • <b>Addition associativity.</b><br>$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$ | • <b>Scalar multiplication associativity.</b><br>$a(b\mathbf{v}) = (ab)\mathbf{v}.$        |
| • <b>Addition commutivity.</b><br>$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$                                 | • <b>Scalar multiplication identity.</b><br>$1\mathbf{v} = \mathbf{v}.$                    |
| • <b>Addition identity.</b><br>There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}.$            | • <b>Scalar distribution.</b><br>$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$ |
| • <b>Addition inverse.</b><br>There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}.$         | • <b>Vector distribution.</b><br>$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$          |

**Definition 7.3** The most important examples of vector spaces are the **Euclidean vector spaces**  $\mathbb{R}^n$ , but there are other examples as well.