Application Activities - Module V Part 3 - Class Day 9

Fact 9.1 A vector $\mathbf{b}$ belongs to $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ if and only if the linear system corresponding to $[\mathbf{v}_1 \ \dots \ \mathbf{v}_n \,|\, \mathbf{b}]$ is consistent.

Remark 9.2 To determine if $\mathbf{b}$ belongs to $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$, find $\text{RREF}[\mathbf{v}_1 \ \dots \ \mathbf{v}_n \,|\, \mathbf{b}]$.

Activity 9.3 Determine if $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ belongs to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$ by row-reducing an appropriate matrix.

Activity 9.4 Determine if $\begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}$ belongs to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$ by row-reducing an appropriate matrix.

Observation 9.5 So far we've only discussed linear combinations of Euclidean vectors. Fortunately, many vector spaces of interest can be reinterpreted as an **isomorphic** Euclidean space $\mathbb{R}^n$; that is, a Euclidean space that mirrors the behavior of the vector space exactly.

Activity 9.6 We previously checked that $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ does not belong to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$. Does $f(x) = 3x^2 - 2x + 1$ belong to $\text{span}\{x^2 - 3, -x^2 - 3x + 2\}$?

Activity 9.7 Does the matrix $\begin{bmatrix} 6 & 3 \\ 2 & -1 \end{bmatrix}$ belong to span $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \right\}$?

Activity 9.8 Does the complex number $2i$ belong to $\text{span}\{-3 + i, 6 - 2i\}$?

Activity 9.9 How many vectors are required to span $\mathbb{R}^2$? Sketch a drawing in the xy plane to support your answer.

(a) 1

(b) 2

(c) 3

(d) 4

(e) Infinitely Many

Activity 9.10 How many vectors are required to span $\mathbb{R}^3$?

(a) 1

(b) 2

(c) 3

(d) 4

(e) Infinitely Many