Name:	
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Date:	

MASTERY QUIZ DAY 18

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Determine if the set of polynomials $\{x^3 - 8x, x^3 + 2x^2 + 2, -x^2 + 3\}$ is linearly dependent or linearly independent

Solution:

$$RREF\left(\begin{bmatrix} -3 & 1 & 0\\ -8 & 2 & -1\\ 0 & 2 & 3\\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{1}{2}\\ 0 & \frac{3}{2}\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Standard S3.

$$\begin{bmatrix}
1 & 3 & 3 & 3 \\
-1 & 3 & -1 \\
3 & -3 & 1
\end{bmatrix}, \begin{bmatrix}
2 & 3 & 1 \\
0 & 1 \\
1 & 4 & -2
\end{bmatrix}, \begin{bmatrix}
1 & 1 \\
1 & 1 \\
-7 & -7
\end{bmatrix}$$
. Find a basis of W .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then
$$\left\{ \begin{bmatrix} 1\\-1\\3\\-3 \end{bmatrix}, \begin{bmatrix} 3\\-1\\4\\-2 \end{bmatrix}, \begin{bmatrix} 1\\1\\1\\-7 \end{bmatrix} \right\}$$
 is a basis for W .

Let
$$W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix},\begin{bmatrix}2\\0\\1\\1\end{bmatrix},\begin{bmatrix}3\\-1\\4\\-2\end{bmatrix},\begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$$
. Compute the dimension of W .

Mark:

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

Standard A1.

Mark:

Let $T: \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and $\mathbb{R}.$

Solution:

 $\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$

Standard A2.

Mark

Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ \sqrt{x}+\sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

Solution:

$$T\left(\begin{bmatrix}0\\4\end{bmatrix}\right) = \begin{bmatrix}4\\2\end{bmatrix} \neq \begin{bmatrix}4\\4\end{bmatrix} = 4T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

So T is not a linear transformation.

Additional Notes/Marks