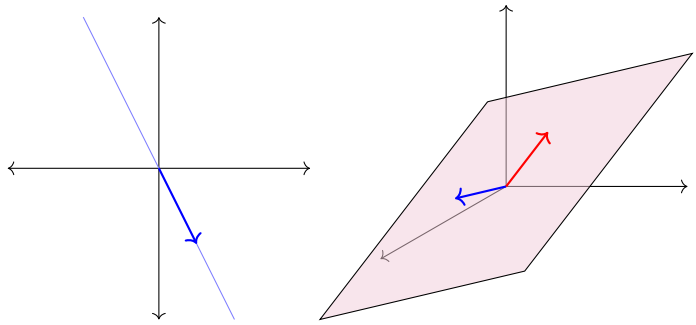
Application Activities - Module V Part 4 - Class Day 10

Fact 10.1 At least n vectors are required to span $\mathbb{R}^n$.



Activity 10.2 Choose a vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ in $\mathbb{R}^3$ that is not in span $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ by ensuring $\left[\begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$. (Why does this work?)

Fact 10.3 The set $\{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ fails to span all of $\mathbb{R}^n$ exactly when RREF$[\mathbf{v}_1 \ \ldots \ \mathbf{v}_m]$ has a row of zeros:

$$\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

Activity 10.4 Consider the set of vectors $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}$. Does $\mathbb{R}^4 = \operatorname{span} S$?

Activity 10.5 Consider the set of third-degree polynomials

$$S = \left\{ 2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, -x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2 \right\}.$$

Does $\mathcal{P}^3 = \operatorname{span} S$?

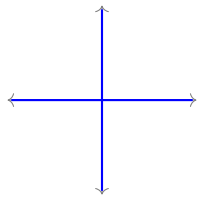
Definition 10.6 A subset of a vector space is called a **subspace** if it is itself a vector space.

Fact 10.7 If S is a subset of a vector space V, then $\operatorname{span} S$ is a subspace of V.

Remark 10.8 To prove that a subset is a subspace, you need only verify that $c\mathbf{v} + d\mathbf{w}$ belongs to the subset for any choice of vectors $\mathbf{v}, \mathbf{w}$ from the subset and any real scalars c, d.

Activity 10.9 Prove that $P = \{ax^2 + b : a, b \text{ are both real numbers}\}$ is a subspace of the vector space of all degree-two polynomials by showing that $c(a_1x^2 + b_1) + d(a_2x^2 + b_2)$ belongs to P.

Activity 10.10 Consider the subset of $\mathbb{R}^2$ where at least one coordinate of each vector is 0.



Find a linear combination $c\mathbf{v} + d\mathbf{w}$ that does not belong to this subset.

Fact 10.11 Suppose a subset S of V is isomorphic to another vector space W. Then S is a subspace of V.

Activity 10.12 Show that the set of 2×2 matrices

$$S = \left\{ \begin{bmatrix} a & b \\ -b & -a \end{bmatrix} : a, b \text{ are real numbers} \right\}$$

is a subspace of $\mathbb{R}^{2\times 2}$ by identifying a Euclidean space isomorphic to S.