Application Activities - Module S Part 3 - Class Day 14

Activity 14.1 (discover that the redundant vectors are non-pivot columns)

Fact 14.2 To compute a basis for the subspace span$\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$, simply remove the vectors corresponding to the non-pivot columns of RREF$[\mathbf{v}_1 \dots \mathbf{v}_m]$.

Activity 14.3 (find ALL the bases for span S that are subsets of S)

Fact 14.4 All bases for a vector space are the same size.

Activity 14.5 Prove that if $\{\mathbf{v}\}$ is a basis for V, then $\{\mathbf{w}_1, \mathbf{w}_2\}$ is linearly dependent (assuming $\mathbf{w}_1 \neq \mathbf{w}_2$).

Fact 14.6 All bases for a vector space are the same size.

Definition 14.7 The **dimension** of a vector space is given by the cardinality/size of any basis for the vector space.

Activity 14.8 Reduce a bunch of spans to bases to find their dimension.

Activity 14.9 What is the dimension of the vector space of 7th-degree polynomials $\mathcal{P}^7$?

Activity 14.10 What is the dimension of the vector space of polynomials $\mathcal{P}$?

Observation 14.11 Several interesting vector spaces are infinite-dimensional:

- The space of polynomials $\mathcal{P}$
- The space of real number sequences $\mathbb{R}^\infty$
- The space of continuous functions $C(\mathbb{R})$

Fact 14.12 Every vector space with dimension $n < \infty$ is isomorphic to $\mathbb{R}^n$.