

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 8**

Math 237 – Linear Algebra

**Version 2**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

**Solution:**

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 4x_2 + x_3 &= -7 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

□

**E3.** Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\ 3x_1 + 6x_3 + x_4 &= 5 \\ -4x_1 + x_2 - 9x_3 + 2x_4 &= -7 \end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 2 & -2 & 6 & -1 & -1 \\ 3 & 0 & 6 & 1 & 5 \\ -4 & 1 & -9 & 2 & -7 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right]$ . It follows that the

solution set is given by  $\begin{bmatrix} 2 - 2a \\ 3 + a \\ a \\ -1 \end{bmatrix}$  for all real numbers  $a$ .

□

**E4.** Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 + 6x_3 - x_4 &= 0 \\ 3x_1 + 6x_3 + x_4 &= 0 \\ -4x_1 + x_2 - 9x_3 + 2x_4 &= 0 \end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$ . It follows that the basis

for the solution set is given by  $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ .

□

**V1.** Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

Determine if  $V$  is a vector space or not.

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ .

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6 - x) = x + (6 - x) - 3 = 3$ , so  $6 - x$  is the additive inverse of  $x$ .
- 4) Real addition is commutative, so  $\oplus$  is commutative.
- 5)

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

$$6) 1 \odot x = x - 3(1 - 1) = x$$

7)

$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\&= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\&= (c \odot x) \oplus (d \odot x)\end{aligned}$$

Therefore  $V$  is a vector space.

□

**E1:**

**E3:**

**E4:**

**V1:**

**E2:**