

Name: \_\_\_\_\_

# MASTERY QUIZ DAY 29

Math 237 – Linear Algebra

## Version 3

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ .

**Solution:**

$$\det \begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix} = -\det \begin{bmatrix} -1 & 0 & 4 \\ 1 & 1 & -1 \\ 1 & 1 & 3 \end{bmatrix} + (-2) \det \begin{bmatrix} 3 & 0 & 4 \\ 2 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix} = -1(-4) + (-2)(20) = -36$$

□

**G3.** Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system  $(B - 2I)X = 0$ .

$$\text{RREF}(B - 2I) = \text{RREF} \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to  $x - \frac{y}{3} = 0$ , or  $3x = y$ . Thus the eigenspace is

$$E_2 = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

□

**G4.** Compute the geometric multiplicity of the eigenvalue  $-1$  in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

**Solution:**

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the geometric multiplicity is 2.

□

**G1:**

**G3:**

**G4:**