

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 15**

Math 237 – Linear Algebra

**Version 3**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V2.** Determine if  $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left( \begin{array}{cc|c} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{array} \right) \right) = \left[ \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

Since this system has a solution,  $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$  and

$\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ , namely

$$\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

□

**S1.** Determine if the set of polynomials  $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$  is linearly dependent or linearly independent

**Solution:**

$$\text{RREF} \left( \left( \begin{array}{ccc} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{array} \right) \right) = \left[ \begin{array}{ccc} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{array} \right]$$

Since there is a nonpivot column, the set is linearly dependent.

□

**S3.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since the first two columns are

pivot columns,  $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$  is a basis for  $W$ .

□

**S4.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

□

**V2:**

**S1:**

**S3:**

**S4:**