

Name: _____

MASTERY QUIZ DAY 19

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S2. Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}_3

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

□

A3. Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by $S \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

(b) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + y + z \\ 2y + 3z \\ x - y - 2z \end{bmatrix}$

Solution:

(a)

$$\text{RREF} \left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b)

$$\text{RREF} \left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

□

A4. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear map given by $T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute the kernel and image of T .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 0 & 1 & 3 & -4 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Im}(T) = \text{span} \left(\left\{ \begin{bmatrix} 8 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix} \right\} \right)$$

$$\text{ker}(T) = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

□

S2:

A3:

A4: