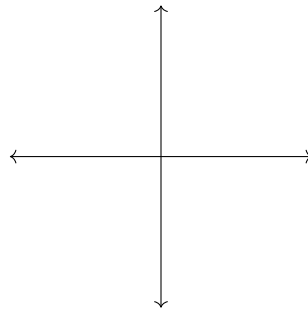


### Section V.3

**Observation V.3.1** Any single non-zero vector/number  $x$  in  $\mathbb{R}^1$  spans  $\mathbb{R}^1$ , since  $\mathbb{R}^1 = \{cx \mid c \in \mathbb{R}\}$ .



**Activity V.3.2** ( $\sim 5$  min) How many vectors are required to span  $\mathbb{R}^2$ ? Sketch a drawing in the  $xy$  plane to support your answer.



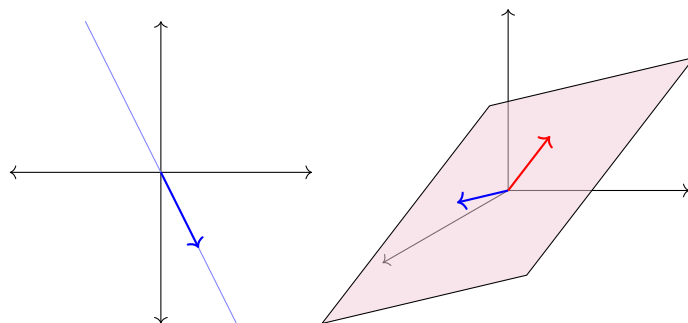
- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) Infinitely Many

**Activity V.3.3** (*~5 min*) How many vectors are required to span  $\mathbb{R}^3$ ?



- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) Infinitely Many

**Fact V.3.4** At least  $n$  vectors are required to span  $\mathbb{R}^n$ .



**Activity V.3.5** (*~15 min*) Choose any vector  $\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$  in  $\mathbb{R}^3$  that is not in  $\text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$  by using technology to verify that  $\text{RREF} \left[ \begin{array}{cc|c} 1 & -2 & ? \\ -1 & 0 & ? \\ 0 & 1 & ? \end{array} \right] = \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$ . (Why does this work?)

**Fact V.3.6** The set  $\{\vec{v}_1, \dots, \vec{v}_m\}$  fails to span all of  $\mathbb{R}^n$  exactly when  $\text{RREF}[\vec{v}_1 \dots \vec{v}_m]$  has a non-pivot row of zeros.

$$\begin{aligned} & \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \\ \Rightarrow & \left[ \begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \text{ for some choice of vector } \begin{bmatrix} a \\ b \\ c \end{bmatrix} \end{aligned}$$

**Activity V.3.7** (*~5 min*) Consider the set of vectors  $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}$ . Does  $\mathbb{R}^4 = \text{span } S$ ?

**Activity V.3.8** (*~10 min*) Consider the set of third-degree polynomials

$$S = \{2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, -x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\}.$$

Does  $\mathcal{P}^3 = \text{span } S$ ? (Hint: first rewrite the question so it is about Euclidean vectors.)

**Activity V.3.9** (*~5 min*) Consider the set of matrices

$$S = \left\{ \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right\}$$

Does  $M_{2,2} = \text{span } S$ ?

**Activity V.3.10** (*~5 min*) Let  $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^7$  be three vectors, and suppose  $\vec{w}$  is another vector with  $\vec{w} \in \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ . What can you conclude about  $\text{span}\{\vec{w}, \vec{v}_1, \vec{v}_2, \vec{v}_3\}$ ?

- (a)  $\text{span}\{\vec{w}, \vec{v}_1, \vec{v}_2, \vec{v}_3\}$  is larger than  $\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ .
- (b)  $\text{span}\{\vec{w}, \vec{v}_1, \vec{v}_2, \vec{v}_3\} = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ .
- (c)  $\text{span}\{\vec{w}, \vec{v}_1, \vec{v}_2, \vec{v}_3\}$  is smaller than  $\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ .