Math 237 – Linear Algebra Fall 2017

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all points on the parabola $y = x^2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 - x_2, y_1 + y_2 - 2x_1x_2)$$

 $c \odot (x_1, y_1) = (cx_1, c^2y_1)$

- (a) Show that scalar multiplication **distributes scalars** over vector addition: $c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors
$$\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$
, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

S2. Determine if the set $\{x^3 - 3x^2 + 2x + 2, -x^3 + 4x^2 - x + 1, -x^3 + 2x + 1, 3x^2 + 3x + 9\}$ is a basis of \mathcal{P}^3 or not.

V1:

V3:

V4:

82:

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Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all points on the parabola $y = x^2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2 + 2x_1x_2)$$

 $c \odot (x_1, y_1) = (cx_1, c^2y_1)$

- (a) Show that the vector **addition** \oplus is **associative**: $(x_1, y_1) \oplus ((x_2, y_2) \oplus (x_3, y_3)) = ((x_1, y_1) \oplus (x_2, y_2)) \oplus (x_3, y_3).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors
$$\begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}$$
, $\begin{bmatrix} 3\\3\\6\\3 \end{bmatrix}$, $\begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix}$, and $\begin{bmatrix} 7\\-1\\8\\-3 \end{bmatrix}$ span \mathbb{R}^4 .

V4. Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of \mathcal{P}^3 .

S2. Determine if the set $\left\{ \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\2 \end{bmatrix}, \begin{bmatrix} 1\\0\\-1\\0 \end{bmatrix}, \begin{bmatrix} 0\\2\\0\\-1 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4 .

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Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that scalar multiplication is associative: $a \odot (b \odot x) = (ab) \odot x$.
- (b) Determine if V is a vector space or not. Justify your answer

V3. Determine if the vectors
$$\begin{bmatrix} -3\\1\\1 \end{bmatrix}$$
, $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$, $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$, and $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$ span \mathbb{R}^3

V4. Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

S2. Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}_3

V1:

V3:

V4:

S2:

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Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that scalar multiplication **distributes vectors** over scalar addition: $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span \mathbb{R}^3 .

V4. Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

S2. Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

V1:

V3:

V4:

S2:

Name:	

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Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that scalar multiplication **distributes vectors** over scalar addition: $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Does span
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\4\\-3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

V4. Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying x+y+z=0 (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

 $\mathbf{S2.}\quad \text{Determine if the set }\left\{\begin{bmatrix}1 & -3\\2 & 2\end{bmatrix},\begin{bmatrix}-1 & 4\\-1 & 1\end{bmatrix},\begin{bmatrix}-1 & 0\\2 & 1\end{bmatrix},\begin{bmatrix}0 & 3\\3 & 9\end{bmatrix}\right\} \text{ is a basis of } M_{2,2} \text{ or not.}$

V1:

V3:

V4:

S2:

Name:

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Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that scalar multiplication **distributes vectors** over scalar addition: $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

V3. Determine if the vectors
$$\begin{bmatrix} -3\\1\\1 \end{bmatrix}$$
, $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$, $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$, and $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$ span \mathbb{R}^3



S2. Determine if the set
$$\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$$
 is a basis of \mathcal{P}_3