

## Application Activities - Module M Part 1 - Class Day 21

**Activity 21.1** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the standard matrix  $B = \begin{bmatrix} 2 & 1 & -3 \\ 5 & -3 & 4 \end{bmatrix}$  and  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  be given by the standard matrix  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 5 \\ -1 & -2 \end{bmatrix}$ .

What is the domain of the composition map  $S \circ T$ ?

- (a)  $\mathbb{R}$
  - (b)  $\mathbb{R}^2$
  - (c)  $\mathbb{R}^3$
  - (d)  $\mathbb{R}^4$
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**Activity 21.2** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the standard matrix  $B = \begin{bmatrix} 2 & 1 & -3 \\ 5 & -3 & 4 \end{bmatrix}$  and  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  be given by the standard matrix  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 5 \\ -1 & -2 \end{bmatrix}$ .

What is the codomain of the composition map  $S \circ T$ ?

- (a)  $\mathbb{R}$
  - (b)  $\mathbb{R}^2$
  - (c)  $\mathbb{R}^3$
  - (d)  $\mathbb{R}^4$
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**Activity 21.3** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the standard matrix  $B = \begin{bmatrix} 2 & 1 & -3 \\ 5 & -3 & 4 \end{bmatrix}$  and  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  be given by the standard matrix  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 5 \\ -1 & -2 \end{bmatrix}$ .

The standard matrix of  $S \circ T$  will lie in which matrix space?

- (a)  $4 \times 3$  matrices
  - (b)  $4 \times 2$  matrices
  - (c)  $3 \times 2$  matrices
  - (d)  $2 \times 3$  matrices
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(e)  $2 \times 4$  matrices

(f)  $3 \times 4$  matrices

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**Activity 21.4** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the standard matrix  $B = \begin{bmatrix} 2 & 1 & -3 \\ 5 & -3 & 4 \end{bmatrix}$  and  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  be

given by the standard matrix  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 5 \\ -1 & -2 \end{bmatrix}$ .

*Part 1:* Compute  $(S \circ T)(\mathbf{e}_1)$

*Part 2:* Compute  $(S \circ T)(\mathbf{e}_2)$

*Part 3:* Compute  $(S \circ T)(\mathbf{e}_3)$ .

*Part 4:* Find the standard matrix of  $S \circ T$ .

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**Activity 21.5** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by the matrix  $B = \begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$  and  $S : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the

matrix  $A = \begin{bmatrix} -4 & -2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ .

What is the domain of the composition map  $S \circ T$ ?

(a)  $\mathbb{R}$

(b)  $\mathbb{R}^2$

(c)  $\mathbb{R}^3$

(d)  $\mathbb{R}^4$

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**Activity 21.6** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by the matrix  $B = \begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$  and  $S : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the

matrix  $A = \begin{bmatrix} -4 & -2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ .

What is the codomain of the composition map  $S \circ T$ ?

(a)  $\mathbb{R}$

(b)  $\mathbb{R}^2$

(c)  $\mathbb{R}^3$

(d)  $\mathbb{R}^4$

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**Activity 21.7** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by the matrix  $B = \begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$  and  $S : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the

matrix  $A = \begin{bmatrix} -4 & -2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ .

The standard matrix of  $S \circ T$  will lie in which matrix space?

- (a)  $2 \times 2$  matrices
  - (b)  $2 \times 3$  matrices
  - (c)  $3 \times 2$  matrices
  - (d)  $3 \times 3$  matrices
- 

**Activity 21.8** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by the matrix  $B = \begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$  and  $S : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the

matrix  $A = \begin{bmatrix} -4 & -2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ .

Find the standard matrix of  $S \circ T$ .

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**Activity 21.9** Let  $T : \mathbb{R}^1 \rightarrow \mathbb{R}^4$  be given by the matrix  $B = \begin{bmatrix} 3 \\ -2 \\ 1 \\ -1 \end{bmatrix}$  and  $S : \mathbb{R}^4 \rightarrow \mathbb{R}^1$  be given by the

matrix  $A = [2 \ 3 \ 2 \ 5]$ .

Find the standard matrix of  $S \circ T$ .

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**Definition 21.10** We define the product of a  $m \times n$  matrix  $A$  and a  $n \times k$  matrix  $B$  to be the  $m \times k$  standard matrix (denoted  $AB$ ) of the composition map of the two corresponding linear functions.

**Fact 21.11** If  $AB$  is defined,  $BA$  need not be defined, and if it is defined, it is in general different from  $AB$ .

**Activity 21.12** Let  $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ . Compute  $AB$ .

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**Activity 21.13** Let  $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 4 \\ -1 & 3 & 5 \end{bmatrix}$  and  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ . Compute  $AX$

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**Observation 21.14** Consider the system of equations

$$\begin{aligned}3x + y - z &= 5 \\2x + 4z &= -7 \\-x + 3y + 5z &= 2\end{aligned}$$

We can interpret this as a **matrix equation**  $AX = B$  where

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 4 \\ -1 & 3 & 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$$

For this reason, we will swap out the use of Euclidean vectors  $\mathbf{x} \in \mathbb{R}^n$  and  $n \times 1$  matrices  $X$  whenever it is convenient.