

Name: \_\_\_\_\_

# **MASTERY QUIZ DAY 15**

Math 237 – Linear Algebra

## **Version 1**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V2.** Determine if  $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$  belongs to the span of the set  $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ .

**Solution:** Since

$$\text{RREF} \left( \left[ \begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & -2 \\ -3 & -6 & 0 & 4 \end{array} \right] \right) = \left[ \begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

contains the contradiction  $0 = 1$ ,  $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$  is not a linear combination of the three vectors.

□

**S1.** Determine if the vectors  $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$  are linearly dependent or linearly independent

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{array} \right] \right) = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Since each column is a pivot column, the vectors are linearly independent.

□

**S3.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Find a basis of  $W$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{array} \right] \right) = \left[ \begin{array}{cccc} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Then  $\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$  is a basis of  $W$ .

□

**S4.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so  $W$  has dimension 2.

□

**V2:**

**S1:**

**S3:**

**S4:**

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**MASTERY QUIZ DAY 15**

Math 237 – Linear Algebra

**Version 2**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V2.** Determine if  $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$  is a linear combination of the vectors  $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ , and  $\begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc|c} 3 & 1 & 5 & 1 \\ 0 & -1 & 1 & 4 \\ -1 & 4 & -6 & 3 \end{array} \right] \right) = \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

So  $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$  is not a linear combination of the three vectors.

□

**S1.** Determine if the set of polynomials  $\{x^3 - 8x, x^3 + 2x^2 + 2, -x^2 + 3\}$  is linearly dependent or linearly independent

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{array} \right] \right) = \left[ \begin{array}{ccc} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

This has a non pivot column, therefore the set is linearly dependent.

□

**S3.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Find a basis of  $W$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{array} \right] \right) = \left[ \begin{array}{cccc} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Then  $\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$  is a basis of  $W$ .

□

**S4.** Let  $W$  be the subspace of  $\mathcal{P}_3$  given by  
 $W = \text{span}(\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\})$ . Compute the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

□

**V2:**

**S1:**

**S3:**

**S4:**

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# **MASTERY QUIZ DAY 15**

Math 237 – Linear Algebra

## **Version 3**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V2.** Determine if  $\begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 5 \\ 2 \\ -3 \\ 2 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ , and  $\begin{bmatrix} 8 \\ 3 \\ 5 \\ -1 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc|c} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{array} \right] \right) = \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

The system has no solution, so  $\begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}$  is not a linear combination of the three other vectors.

□

**S1.** Determine if the set of polynomials  $\{x^3 - 8x, x^3 + 2x^2 + 2, -x^2 + 3\}$  is linearly dependent or linearly independent

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{array} \right] \right) = \left[ \begin{array}{ccc} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

This has a non pivot column, therefore the set is linearly dependent.

□

**S3.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since the first two columns are

pivot columns,  $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$  is a basis for  $W$ .

□

**S4.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since there are two pivot columns,  $\dim W = 2$ .

□

**V2:**

**S1:**

**S3:**

**S4:**

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# **MASTERY QUIZ DAY 15**

Math 237 – Linear Algebra

## **Version 4**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V2.** Determine if  $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cc|c} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{array} \right] \right) = \left[ \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

Since this system has a solution,  $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$  and

$\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ , namely

$$\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

□

**S1.** Determine if the set of vectors  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \right\}$  is linearly dependent or linearly independent

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

□

**S3.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}$ .

□

**S4.** Let  $W$  be the subspace of  $M_{2,2}$  given by  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \right\} \right)$ .  
Compute the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

□

**V2:**

**S1:**

**S3:**

**S4:**



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**MASTERY QUIZ DAY 15**

Math 237 – Linear Algebra

**Version 5**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V2.** Determine if  $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cc|c} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{array} \right] \right) = \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Since this system has no solution,  $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$  cannot be written as a linear combination of the vectors  $\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$ .

□

**S1.** Determine if the vectors  $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$  are linearly dependent or linearly independent

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{array} \right] \right) = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Since each column is a pivot column, the vectors are linearly independent.

□

**S3.** Let  $W$  be the subspace of  $\mathcal{P}_3$  given by

$W = \text{span}(\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\})$ . Find a basis for  $W$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{array} \right] \right) = \left[ \begin{array}{cccc} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Then a basis is  $\{x^3 + x^2 + 2x + 1, 3x^3 - x^2 + 3x - 2\}$ .

□

**S4.** Let  $W$  be the subspace of  $\mathcal{P}_3$  given by

$W = \text{span}(\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\})$ . Compute the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

□

**V2:**

**S1:**

**S3:**

**S4:**

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**MASTERY QUIZ DAY 15**

Math 237 – Linear Algebra

**Version 6**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V2.** Determine if  $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ .

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cc|c} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{array} \right] \right) = \left[ \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

Since this system has a solution,  $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$  and

$\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ , namely

$$\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

□

**S1.** Determine if the set of polynomials  $\{x^3 - 8x, x^3 + 2x^2 + 2, -x^2 + 3\}$  is linearly dependent or linearly independent

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{ccc} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{array} \right] \right) = \left[ \begin{array}{ccc} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

This has a non pivot column, therefore the set is linearly dependent.

□

**S3.** Let  $W$  be the subspace of  $\mathcal{P}_3$  given by

$W = \text{span}(\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\})$ . Find a basis for  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is  $\{x^3 + x^2 + 2x + 1, 3x^3 - x^2 + 3x - 2\}$ .

□

**S4.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since there are two pivot columns,  $\dim W = 2$ .

□

**V2:**

**S1:**

**S3:**

**S4:**