

Definition 5.1 An algorithm that reduces A to $\text{RREF}(A)$ is called **Gauss-Jordan elimination**. For example:

1. Circle the top-left-most cell that (a) is below any existing pivot positions and (b) has a nonzero term either in that position or below it.
2. Turn the pivot into a 1 using only that row and rows below it.
3. Add or subtract multiples of the row with the pivot to zero out above and below the pivot.
4. Goto 1

Definition 5.3 The columns of $\text{RREF}(A)$ without a leading term represent **free variables** of the linear system modeled by A that may be set equal to arbitrary parameters. The other **bounded variables** can then be expressed in terms of those parameters to describe the solution set to the linear system modeled by A .

Definition 5.8 A **homogeneous system** is a linear system satisfying $b_i = 0$, that is, it is a linear system of the form

$$\begin{array}{cccc} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n & = & 0 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n & = & 0 \\ \vdots & & \vdots & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n & = & 0 \end{array}$$

Definition 5.10 A minimal set of Euclidean vectors generating the solution set to a homogeneous system is called a **basis** for the solution set of the homogeneous system. For example:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = a \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \text{Basis} = \left\{ \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$