# Module E

## Standard E1

**E1.1** Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 0 \\ x - z &= 1 \end{aligned}$$

**E1.2** Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_3 &= -1 \end{aligned}$$

**E1.3** Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_4 &= -1 \end{aligned}$$

**E1.4** Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\ 3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 + x_4 &= 1 \end{aligned}$$

**E1.5** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array}\right]$$

**E1.6** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array}\right]$$

**E1.7** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array}\right]$$

**E1.8** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array}\right]$$

## Standard E2

**E2.1** Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \\ 0 & 2 & 6 \end{bmatrix}$$

**E2.2** Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix}$$

**E2.3** Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{bmatrix}$$

**E2.4** Find the reduced row echelon form of the matrix below.

$$\begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix}$$

**E2.5** Find RREF $A$, where

$$A = \begin{bmatrix} 3 & -2 & 1 & 8 & -5 \\ 2 & 2 & 0 & 6 & -2 \\ -1 & 1 & 1 & -4 & 6 \end{bmatrix}$$

**E2.6** Find RREF $A$, where

$$A = \begin{bmatrix} 2 & -7 & 4 \\ 1 & -3 & 2 \\ 3 & 0 & 3 \end{bmatrix}$$

**E2.7** Find RREF $A$, where

$$A = \begin{bmatrix} 2 & -1 & 5 & 4 \\ -1 & 0 & -2 & -1 \\ 1 & 3 & -1 & -5 \end{bmatrix}$$

**E2.8** Find RREF $A$, where

$$A = \begin{bmatrix} 2 & 2 & 1 & 2 & -1 \\ 1 & 1 & 2 & 4 & 5 \\ 3 & 3 & -1 & -2 & 1 \end{bmatrix}$$

## Standard E3

**E3.1** Find the solution set for the following system of linear equations.

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 2 \end{aligned}$$

**E3.2** Find the solution set for the following system of linear equations.

$$\begin{aligned} -3x + y &= 2 \\ -8x + 2y - z &= 6 \\ 2y + 3z &= -2 \end{aligned}$$

**E3.3** Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

**E3.4** Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

**E3.5** Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\ 3x_1 + 6x_3 + x_4 &= 5 \\ -4x_1 + x_2 - 9x_3 + 2x_4 &= -7 \end{aligned}$$

**E3.6** Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 8 \\ x_1 + x_2 - x_3 + 5x_4 &= 3 \end{aligned}$$

**E3.7** Find the solution set for the following system of linear equations.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

**E3.8** Find the solution set for the following system of linear equations.

$$\begin{aligned} 3x + 2y + z &= 7 \\ x + y + z &= 1 \\ -2x + 3z &= -11 \end{aligned}$$

# Module V

## Standard V1

**V1.1** Let $V$ be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

(a) Show that scalar multiplication is associative: $a \odot (b \odot x) = (ab) \odot x$ for all scalars $a, b \in \mathbb{R}$ and $x \in V$.

(b) Explain why $V$ nonetheless isn't a vector space.

**V1.2** Let $V$ be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2 + 2x_1y_1) \\ c \odot (x_1, x_2) &= (cx_1, cx_2) \end{aligned}$$

(a) Show that the vector addition $\oplus$ is associative:
$(x_1, x_2) \oplus ((y_1, y_2) \oplus (z_1, z_2)) = ((x_1, x_2) \oplus (y_1, y_2)) \oplus (z_1, z_2)$ for all $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in V$.

(b) Explain why $V$ nonetheless isn't a vector space.

**V1.3** Let $V$ be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1 - 1, x_2 + y_2 - 1)\\ c \odot (x_1, x_2) &= (cx_1, cx_2)\end{aligned}$$

(a) Show that this vector space has an additive identity element: there exists $\vec{z} \in V$ satisfying $(x, y) \oplus \vec{z} = (x, y)$ for every $(x, y) \in V$.

(b) Explain why $V$ nonetheless isn't a vector space.

**V1.4** Let $V$ be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2)\\ c \odot (x_1, x_2) &= (0, cx_2)\end{aligned}$$

(a) Show that scalar multiplication distributes over scalar addition:
$(c + d) \odot (x_1, x_2) = c \odot (x_1, x_2) \oplus d \odot (x_1, x_2)$ for every $c, d \in \mathbb{R}$ and $(x_1, x_2) \in V$.

(b) Explain why $V$ nonetheless isn't a vector space.

**V1.5** Let $V$ be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2)\\ c \odot (x_1, x_2) &= (c^2 x_1, c^3 x_2)\end{aligned}$$

(a) Show that scalar multiplication distributes over vector addition:
$c \odot ((x_1, x_2) \oplus (y_1, y_2)) = c \odot (x_1, x_2) \oplus c \odot (y_1, y_2)$ for all $c \in \mathbb{R}$ and $(x_1, x_2), (y_1, y_2) \in V$.

(b) Explain why $V$ nonetheless isn't a vector space.

**V1.6** Let $V$ be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= \sqrt{x^2 + y^2}\\ c \odot x &= cx\end{aligned}$$

(a) Show that the vector addition $\oplus$ is associative: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ for all $x, y, z \in V$.

(b) Explain why $V$ nonetheless isn't a vector space.

**V1.7** Let $V$ be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 y_2)\\ c \odot (x_1, x_2) &= (cx_1, cx_2)\end{aligned}$$

(a) Show that there is an additive identity element: there exists an element $\vec{z} \in V$ such that $(x_1, x_2) \oplus \vec{z} = (x_1, x_2)$ for any $(x_1, x_2) \in V$.

(b) Explain why $V$ nonetheless isn't a vector space.

## Standard V2

**V2.1** Determine if $\begin{bmatrix}0\\-1\\2\\6\end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix}3\\-1\\-1\\0\end{bmatrix}$ and $\begin{bmatrix}-1\\0\\1\\2\end{bmatrix}$.

**V2.2** Determine if $\begin{bmatrix}0\\1\\-2\\1\end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix}5\\2\\-3\\2\end{bmatrix}$, $\begin{bmatrix}3\\1\\1\\0\end{bmatrix}$, and $\begin{bmatrix}8\\3\\5\\-1\end{bmatrix}$.

**V2.3** Determine if $\begin{bmatrix}0\\0\\2\end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix}-1\\-9\\15\end{bmatrix}$ and $\begin{bmatrix}1\\5\\-5\end{bmatrix}$.

**V2.4** Determine if $\begin{bmatrix}1\\4\\3\end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix}2\\3\\-1\end{bmatrix}$, $\begin{bmatrix}1\\-1\\0\end{bmatrix}$, and $\begin{bmatrix}-3\\-2\\5\end{bmatrix}$.

**V2.5** Determine if $\begin{bmatrix}1\\4\\3\end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix}3\\0\\-1\end{bmatrix}$, $\begin{bmatrix}1\\-1\\4\end{bmatrix}$, and $\begin{bmatrix}5\\1\\-6\end{bmatrix}$.

**V2.6** Determine if $\begin{bmatrix}0\\-1\\6\\-7\end{bmatrix}$ belongs to the span of the set $\left\{\begin{bmatrix}2\\0\\-1\\5\end{bmatrix}, \begin{bmatrix}4\\-1\\4\\3\end{bmatrix}\right\}$.

**V2.7** Determine if $\begin{bmatrix}4\\-1\\6\\-7\end{bmatrix}$ belongs to the span of the set $\left\{\begin{bmatrix}2\\0\\-1\\5\end{bmatrix}, \begin{bmatrix}4\\-1\\4\\3\end{bmatrix}\right\}$.

**V2.8** Determine if $\begin{bmatrix}3\\-2\\4\end{bmatrix}$ belongs to the span of the set $\left\{\begin{bmatrix}1\\2\\-3\end{bmatrix}, \begin{bmatrix}2\\4\\-6\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$.

## Standard V3

**V3.1** Determine if the vectors $\begin{bmatrix}-3\\1\\1\end{bmatrix}$, $\begin{bmatrix}5\\-1\\-2\end{bmatrix}$, $\begin{bmatrix}2\\0\\-1\end{bmatrix}$, and $\begin{bmatrix}0\\2\\-1\end{bmatrix}$ span $\mathbb{R}^3$

**V3.2** Determine if the vectors $\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\3\\6\\3\end{bmatrix}$, $\begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}$, and $\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}$ span $\mathbb{R}^4$.

**V3.3** Determine if the vectors $\begin{bmatrix}8\\21\\-7\end{bmatrix}$, $\begin{bmatrix}-3\\-8\\3\end{bmatrix}$, $\begin{bmatrix}-1\\-3\\2\end{bmatrix}$, and $\begin{bmatrix}4\\11\\-5\end{bmatrix}$ span $\mathbb{R}^3$.

**V3.4** Determine if the vectors $\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}$, $\begin{bmatrix}3\\1\\3\\6\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$, and $\begin{bmatrix}1\\2\\0\\1\end{bmatrix}$ span $\mathbb{R}^4$.

**V3.5** Determine if the vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\\0\\-3\end{bmatrix}$, $\begin{bmatrix}0\\3\\0\\-2\end{bmatrix}$, and $\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}$ span $\mathbb{R}^4$.

**V3.6** Does span $\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\end{bmatrix}, \begin{bmatrix}1\\4\\-3\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\} = \mathbb{R}^3$?

**V3.7** Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix},\begin{bmatrix}3\\12\\-9\end{bmatrix},\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}=\mathbb{R}^3$?

**V3.8** Does $\operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\\2\\1\end{bmatrix},\begin{bmatrix}-1\\3\\5\\2\\0\end{bmatrix},\begin{bmatrix}1\\0\\5\\1\\-3\end{bmatrix}\right\}=\mathbb{R}^5$?

## Standard V4

**V4.1** Consider the following two sets of Euclidean vectors.

$$W=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,x,y\text{ are integers}\right\}\qquad U=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,x=y\right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

**V4.2** Consider the following two sets of Euclidean vectors.

$$W=\left\{\begin{bmatrix}x\\y\\0\\z\end{bmatrix}\,\middle|\,x,y,z\in\mathbb{R}\right\}\qquad U=\left\{\begin{bmatrix}x\\y\\1\\z\end{bmatrix}\,\middle|\,x,y,z\in\mathbb{R}\right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^4$, and that one of the sets is not.

**V4.3** Consider the following two sets of Euclidean vectors.

$$W=\left\{\begin{bmatrix}x\\y\\z\end{bmatrix}\,\middle|\,x+y+z=1\right\}\qquad U=\left\{\begin{bmatrix}x\\y\\z\end{bmatrix}\,\middle|\,x+y+z=0\right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^3$, and that one of the sets is not.

**V4.4** Consider the following two sets of Euclidean vectors.

$$W=\left\{\begin{bmatrix}x\\y\\z\end{bmatrix}\,\middle|\,x+y=3z\right\}\qquad U=\left\{\begin{bmatrix}x\\y\\z\end{bmatrix}\,\middle|\,x+y=3,z\in\mathbb{R}\right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^3$, and that one of the sets is not.

**V4.5** Consider the following two sets of Euclidean vectors.

$$W=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,x+y=0\right\}\qquad U=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,xy=0\right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

**V4.6** Consider the following two sets of Euclidean vectors.

$$W=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,x=y\right\}\qquad U=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,|x|=|y|\right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

**V4.7** Consider the following two sets of Euclidean vectors.

$$W=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,y=2x\right\}\qquad U=\left\{\begin{bmatrix}x\\y\end{bmatrix}\,\middle|\,y=x^2\right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^2$, and that one of the sets is not.

**V4.8** Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, z = 2xy \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, z = 2x + y \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^3$, and that one of the sets is not.

## Standard V5

**V5.1** Determine if set of vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

**V5.2** Determine if the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

**V5.3** Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

**V5.4** Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

**V5.5** Determine if the set of vectors $\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

**V5.6** Determine if the set of vectors $\left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

**V5.7** Determine if the set of vectors $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

**V5.8** Determine if the set of vectors $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

## Standard V6

**V6.1** Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$.

**V6.2** Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.

**V6.3** Determine if the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$.

**V6.4** Determine if the set $\left\{\begin{bmatrix}3\\-1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\0\\2\\4\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\-1\end{bmatrix}, \begin{bmatrix}-1\\3\\0\\5\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^4$.

**V6.5** Determine if the set $\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^4$.

**V6.6** Determine if the set $\left\{\begin{bmatrix}8\\21\\-7\end{bmatrix}, \begin{bmatrix}-3\\-8\\3\end{bmatrix}, \begin{bmatrix}-1\\-3\\2\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^3$.

**V6.7** Determine if the set $\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^3$.

**V6.8** Determine if the set $\left\{\begin{bmatrix}1\\0\\2\\1\end{bmatrix}, \begin{bmatrix}3\\1\\0\\-3\end{bmatrix}, \begin{bmatrix}0\\3\\0\\-2\end{bmatrix}, \begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^4$.

## Standard V7

**V7.1** Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find a basis for $W$.

**V7.2** Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Find a basis for $W$.

**V7.3** Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Find a basis of $W$.

**V7.4** Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix}, \begin{bmatrix}2\\0\\1\\1\end{bmatrix}, \begin{bmatrix}3\\-1\\4\\-2\end{bmatrix}, \begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$. Find a basis of $W$.

**V7.5** Let $W = \operatorname{span}\left\{\begin{bmatrix}2\\0\\2\\1\end{bmatrix}, \begin{bmatrix}3\\1\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\-8\\-1\end{bmatrix}\right\}$. Find a basis for this vector space.

**V7.6** Let $W = \operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\end{bmatrix}, \begin{bmatrix}1\\4\\-3\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}$. Find a basis for this vector space.

**V7.7** Let $W = \operatorname{span}\left\{\begin{bmatrix}2\\-1\\4\\2\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\\-4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\\2\end{bmatrix}, \begin{bmatrix}1\\2\\3\\2\end{bmatrix}\right\}$. Find a basis for this vector space.

## Standard V8

**V8.1** Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find the dimension of $W$.

**V8.2** Let $W = \text{span}\left(\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}\right)$. Compute the dimension of $W$.

**V8.3** Let $W = \text{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix}, \begin{bmatrix}3\\1\\3\\6\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Compute the dimension of $W$.

**V8.4** Let $W = \text{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix}, \begin{bmatrix}2\\0\\1\\1\end{bmatrix}, \begin{bmatrix}3\\-1\\4\\-2\end{bmatrix}, \begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$. Compute the dimension of $W$.

**V8.5** Let $W = \text{span}\left\{\begin{bmatrix}2\\0\\2\\1\end{bmatrix}, \begin{bmatrix}3\\1\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\-8\\-1\end{bmatrix}\right\}$. Find the dimension of $W$.

**V8.6** Let $W = \text{span}\left\{\begin{bmatrix}2\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\end{bmatrix}, \begin{bmatrix}1\\4\\-3\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\end{bmatrix}\right\}$. Find the dimension of $W$.

**V8.7** Let $W = \text{span}\left\{\begin{bmatrix}2\\-1\\4\\2\end{bmatrix}, \begin{bmatrix}-4\\2\\-8\\-4\end{bmatrix}, \begin{bmatrix}3\\12\\-9\\2\end{bmatrix}, \begin{bmatrix}1\\2\\3\\2\end{bmatrix}\right\}$. Find the dimension of $W$.

## Standard V9

**V9.1** Find a basis for the subspace

$$W = \text{span}\left\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\right\}$$

of $\mathcal{P}^2$.

**V9.2** Find a basis for the subspace

$$W = \text{span}\left\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\right\}$$

of $\mathcal{P}^2$.

**V9.3** Find a basis for the subspace

$$W = \text{span}\left\{\begin{bmatrix}1 & -3\\2 & 2\end{bmatrix}, \begin{bmatrix}-1 & 4\\-1 & 1\end{bmatrix}, \begin{bmatrix}0 & 3\\3 & 9\end{bmatrix}, \begin{bmatrix}-1 & 0\\2 & 1\end{bmatrix}\right\}$$

of $M_{2,2}$.

**V9.4** Find a basis for the subspace

$$W = \text{span}\left\{x^3 - 3x^2 + 2x + 2, -x^3 + 4x^2 - x + 1, 3x^2 + 3x + 9, -x^3 + 2x + 1\right\}$$

of $\mathcal{P}^3$.

**V9.5** Find a basis for the subspace

$$W = \text{span}\left\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\right\}$$

of $\mathcal{P}^3$.

**V9.6** Let $W$ be the subspace of $\mathcal{P}^3$ given by

$$W = \text{span}\left(\left\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\right\}\right).$$

Find a basis for $W$.

**V9.7** Let $W = \text{span}\left\{\begin{bmatrix}2 & 0\\-2 & 0\end{bmatrix}, \begin{bmatrix}3 & 1\\3 & 6\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}, \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\right\}$. Find a basis for this vector space.

**V9.8** Let $W$ be the subspace of $\mathcal{P}^2$ given by $W = \text{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$. Find a basis for $W$.

## Standard V10

**V10.1** Find a basis for the solution space of the homogeneous system of equations

$$\begin{aligned} x + 3y + 3z + 7w &= 0 \\ x + 3y - z - w &= 0 \\ 2x + 6y + 3z + 8w &= 0 \\ x + 3y - 2z - 3w &= 0 \end{aligned}$$

**V10.2** Find a basis for the solution space of the homogeneous system of equations

$$\begin{aligned} x + 2y + 3z + w &= 0 \\ 3x - y + z + w &= 0 \\ 2x - 3y - 2z &= 0 \\ -x + 2z + 5w &= 0 \end{aligned}$$

**V10.3** Find a basis for the solution space of the homogeneous system of equations

$$\begin{aligned} x + 2y + 3z + w &= 0 \\ 3x - y + z + w &= 0 \\ 2x - 3y - 2z &= 0 \end{aligned}$$

**V10.4** Find a basis for the solution space to the system of equations

$$\begin{aligned} x + 2y - 3z &= 0 \\ 2x + y - 4z &= 0 \\ 3y - 2z &= 0 \\ x - y - z &= 0 \end{aligned}$$

**V10.5** Find a basis for the solution space to the homogeneous system of equations

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 0 \\ x_1 + x_2 - x_3 + 5x_4 &= 0 \end{aligned}$$

**V10.6** Find a basis for the solution space to the homogeneous system of equations

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 0 \\ -2x_3 - 4x_4 &= 0 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= 0 \end{aligned}$$

**V10.7** Find a basis for the solution space to the homogeneous system of equations given by

$$\begin{aligned} 3x + 2y + z &= 0 \\ x + y + z &= 0 \end{aligned}$$

**V10.8** Find a basis for the solution space to the homogeneous system of equations given by

$$\begin{aligned} 2x_1 - 2x_2 + 6x_3 - x_4 &= 0 \\ 3x_1 + 6x_3 + x_4 &= 0 \\ -4x_1 + x_2 - 9x_3 + 2x_4 &= 0 \end{aligned}$$

# Module A

## Standard A1

**A1.1** Consider the following maps of polynomials $S : \mathcal{P}^6 \to \mathcal{P}^6$ and $T : \mathcal{P}^6 \to \mathcal{P}^6$ defined by

$$S(f(x)) = f(x) + 3 \text{ and } T(f(x)) = f(x) + f(3).$$

Show that one of these maps is a linear transformation, and that the other map is not.
**A1.2** Consider the following maps of polynomials $S : \mathcal{P}^4 \to \mathcal{P}^5$ and $T : \mathcal{P}^4 \to \mathcal{P}^5$ defined by

$$S(f(x)) = xf(x) - f(1) \text{ and } T(f(x)) = xf(x) - x.$$

Show that one of these maps is a linear transformation, and that the other map is not.
**A1.3** Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = f'(x) - f''(x) \text{ and } T(f(x)) = f(x) - (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
**A1.4** Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^4$ and $T : \mathcal{P}^2 \to \mathcal{P}^4$ defined by

$$S(f(x)) = x^2 f(x) \text{ and } T(f(x)) = (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
**A1.5** Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = (f(x))^2 + 1 \text{ and } T(f(x)) = (x^2 + 1)f(x).$$

Show that one of these maps is a linear transformation, and that the other map is not.
**A1.6** Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^2$ and $T : \mathcal{P}^2 \to \mathcal{P}^2$ defined by

$$S(ax^2 + bx + c) = cx^2 + bx + a \text{ and } T(ax^2 + bx + c) = a^2x^2 + b^2x + c^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
**A1.7** Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^1$ and $T : \mathcal{P}^2 \to \mathcal{P}^1$ defined by

$$S(ax^2 + bx + c) = 2ax + b \text{ and } T(ax^2 + bx + c) = a^2x + b.$$

Show that one of these maps is a linear transformation, and that the other map is not.
**A1.8** Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^3$ and $T : \mathcal{P}^2 \to \mathcal{P}^3$ defined by

$$S(ax^2 + bx + c) = ax^3 + bx^2 + cx \text{ and } T(ax^2 + bx + c) = abcx^3.$$

Show that one of these maps is a linear transformation, and that the other map is not.

## Standard A2

**A2.1** Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.$$

(a) Write the standard matrix for $T$.

(b) Compute $T\left(\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}\right)$

**A2.2** Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 + 3x_1 \end{bmatrix}.$$

(a) Write the standard matrix for $T$.

(b) Compute $T\left(\begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}\right)$

**A2.3** Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3 \end{bmatrix}.$$

(a) Write the standard matrix for $T$.

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}\right)$

**A2.4** Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}.$$

(a) Write the standard matrix for $T$.

(b) Compute $T\left(\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\right)$

**A2.5** Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 2 & -1 & 5 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}\right)$

**A2.6** Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be the linear transformation given by the requirements

$$T(\vec{e}_1) = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 3 \end{bmatrix} \text{ and } T(\vec{e}_2) = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 3 \end{bmatrix}$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

**A2.7** Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 7x + 2y + 3z \\ 0 \end{bmatrix}.$$

(a) Write the standard matrix for $T$.

(b) Compute $T\left(\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}\right)$

**A2.8** Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 7 & 0 & 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}\right)$

**A2.9** Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -5 & 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}\right)$

**A2.10** Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be the linear transformation given by the following

$$T(\vec{e}_1) = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} \qquad T(\vec{e}_2) = \begin{bmatrix} 0 \\ 3 \\ -5 \\ 0 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right)$

**A2.11** Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - x_3 \end{bmatrix}.$$

(a) Write the standard matrix for $T$.

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}\right)$

**A2.12** Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - 5x_3 \end{bmatrix}.$$

(a) Write the standard matrix for $T$.

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}\right)$

**A2.13** Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}.$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}\right)$

**A2.14** Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation given by the following

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$

**A2.15** Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

(a) Compute $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

(b) Compute $T\left(\begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}\right)$

## Standard A3

**A3.1** Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of $T$.

**A3.2** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of $T$.

**A3.3** Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

**A3.4** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z \\ y + 3z \\ -7x + 3y + 2z \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

**A3.5** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by the standard matrix $\begin{bmatrix} 1 & 2 & 5 \\ 0 & -2 & -4 \\ -3 & 1 & -1 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

**A3.6** Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear map given by the standard matrix $\begin{bmatrix} 1 & 2 & 5 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

**A3.7** Let $T : \mathbb{R}^2 \to \mathbb{R}^5$ be the linear map given by the standard matrix $\begin{bmatrix} 3 & 1 \\ 0 & 0 \\ -6 & -2 \\ 1 & \frac{1}{3} \\ 9 & 3 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

**A3.8** Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by the standard matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ -4 & -2 & 0 & 5 \\ -2 & -1 & 0 & 0 \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of $T$.

## Standard A4

**A4.1** Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y + z \\ 2y + 3z \\ x - y - 2z \end{bmatrix}$

**A4.2** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y + z \\ 2y + 3z \\ x - y - 2z \end{bmatrix}$

(b) $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

**A4.3** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

**A4.4** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 4 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

**A4.5** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^4$ given by the standard matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 11 & -1 & 5 \end{bmatrix}$

**A4.6** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^4$ given by the standard matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 7 & -1 & 5 \end{bmatrix}$

**A4.7** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ where $S(\vec{e}_1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $S(\vec{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ where $T(\vec{e}_1) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $T(\vec{e}_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $T(\vec{e}_3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

**A4.8** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^4 \to \mathbb{R}^3$ where $S(\vec{e}_1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $S(\vec{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $S(\vec{e}_3) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, and $S(\vec{e}_4) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$,

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ where $T(\vec{e}_1) = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $T(\vec{e}_2) = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$, and $T(\vec{e}_3) = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

# Module M

## Standard M1

**M1.1** Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

**M1.2** Let

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

**M1.3** Let

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & -1 & 4 \\ 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

**M1.4** Let

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & -1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

**M1.5** Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

**M1.6** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

**M1.7** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

**M1.8** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products $AB$, $AC$, $BA$, $BC$, $CA$, $CB$ can be computed. Determine which one, and compute it.

## Standard M2

**M2.1** Determine if the matrix $\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}$ is invertible.

**M2.2** Determine if the matrix $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ is invertible.

**M2.3** Determine if the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$ is invertible.

**M2.4** Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ is invertible.

**M2.5** Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -2 & 0 & 3 \end{bmatrix}$ is invertible.

**M2.6** Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

**M2.7** Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

**M2.8** Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

## Standard M3

**M3.1** Show how to find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

**M3.2** Compute the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

**M3.3** Show how to find the inverse of the matrix $\begin{bmatrix} 6 & 0 & 1 \\ -14 & 3 & -4 \\ -23 & 4 & -6 \end{bmatrix}$.

**M3.4** Show how to find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

**M3.5** Show how to find the inverse of the matrix $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$.

**M3.6** Show how to find the inverse of the matrix $\begin{bmatrix} 1 & -4 & 5 \\ -5 & 24 & -28 \\ 1 & -5 & 6 \end{bmatrix}$.

**M3.7** Show how to find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

**M3.8** Show how to find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

**M3.9** Show how to find the inverse of the matrix $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$.

# Module G

## Standard G1

### G1.1

(a) Find $3 \times 3$ matrices $S$ and $T$ whose left multiplication represents the row operations $R_1 + 5R_3 \to R_1$ and $R_2 \leftrightarrow R_3$, respectively.

(b) If $A \in M_{3,3}$ is a matrix with $\det A = 4$, find the determinant of $STA$.

### G1.2

(a) Find $3 \times 3$ matrices $S$ and $T$ whose left multiplication represents the row operations $R_2 - 4R_3 \to R_2$ and $R_1 \leftrightarrow R_3$, respectively.

(b) If $A \in M_{3,3}$ is a matrix with $\det A = 7$, find the determinant of $STA$.

**G1.3**

(a) Find $3 \times 3$ matrices $S$ and $T$ whose left multiplication represents the row operations $3R_2 \to R_2$ and $R_3 - 2R_1 \to R_3$, respectively.

(b) If $A \in M_{3,3}$ is a matrix with $\det A = 7$, find the determinant of $STA$.

**G1.4**

(a) Find $3 \times 3$ matrices $S$ and $T$ whose left multiplication represents the row operations $3R_2 \to R_2$ and $4R_3 \to R_3$, respectively.

(b) If $A \in M_{3,3}$ is a matrix with $\det A = -2$, find the determinant of $STA$.

**G1.5**

(a) Find $3 \times 3$ matrices $S$ and $T$ whose left multiplication represents the row operations $-8R_1 \to R_1$ and $R_2 + R_1 \to R_2$, respectively.

(b) If $A \in M_{3,3}$ is a matrix with $\det A = -2$, find the determinant of $STA$.

**G1.6**

(a) Find $3 \times 3$ matrices $S$ and $T$ whose left multiplication represents the row operations $5R_2 \to R_2$ and $R_2 \leftrightarrow R_3$, respectively.

(b) If $A \in M_{3,3}$ is a matrix with $\det A = -3$, find the determinant of $STA$.

## Standard G2

**G2.1** Compute the determinant of the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$.

**G2.2** Compute the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix}$.

**G2.3** Compute the determinant of the matrix $\begin{bmatrix} 2 & 3 & 0 & 1 \\ -1 & 3 & 1 & 4 \\ 0 & 2 & 0 & 3 \\ 1 & -1 & 3 & 5 \end{bmatrix}$.

**G2.4** Compute the determinant of the matrix $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 3 & 0 & 0 & 1 \end{bmatrix}$.

**G2.5** Compute the determinant of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

**G2.6** Compute the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ 5 & 0 & -4 & 0 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

**G2.7** Compute the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

**G2.8** Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

**G2.9** Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

## Standard G3

**G3.1** Find the eigenvalues of the matrix $\begin{bmatrix} 5 & 1 \\ -24 & -6 \end{bmatrix}$.

**G3.2** Find the eigenvalues of the matrix $\begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix}$.

**G3.3** Find the eigenvalues of the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$.

**G3.4** Find the eigenvalues of the matrix $\begin{bmatrix} -2 & 5 \\ 5 & -2 \end{bmatrix}$.

**G3.5** Find the eigenvalues of the matrix $\begin{bmatrix} 10 & -8 \\ 4 & -2 \end{bmatrix}$.

**G3.6** Find the eigenvalues of the matrix $\begin{bmatrix} 6 & -4 \\ 11 & -9 \end{bmatrix}$.

**G3.7** Find the eigenvalues of the matrix $\begin{bmatrix} -6 & -11 \\ 4 & 9 \end{bmatrix}$.

**G3.8** Find the eigenvalues of the matrix $\begin{bmatrix} 1 & -5 \\ 4 & -8 \end{bmatrix}$.

**G3.9** Find the eigenvalues of the matrix $\begin{bmatrix} -2 & 2 \\ 15 & -9 \end{bmatrix}$.

**G3.10** Find the eigenvalues of the matrix $\begin{bmatrix} 10 & 2 \\ -39 & -9 \end{bmatrix}$.

**G3.11** Find the eigenvalues of the matrix $\begin{bmatrix} 8 & 2 \\ -33 & -9 \end{bmatrix}$.

## Standard G4

**G4.1** Find a basis of the eigenspace associated to the eigenvalue $-1$ for the matrix $\begin{bmatrix} 4 & -2 & -1 & 1 \\ 15 & -7 & -3 & 1 \\ -5 & 2 & 0 & 1 \\ 10 & -4 & -2 & 0 \end{bmatrix}$.

**G4.2** Find a basis of the eigenspace associated to the eigenvalue 1 for the matrix $A = \begin{bmatrix} -3 & 1 & 0 & 0 \\ -8 & 2 & 0 & -1 \\ 0 & 2 & 1 & 2 \\ 4 & 1 & 0 & 3 \end{bmatrix}$.

**G4.3** Find a basis of the eigenspace associated to the eigenvalue 1 for the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 8 & -3 & -1 \\ 2 & 21 & -8 & -3 \\ 3 & -7 & 3 & 2 \end{bmatrix}$.

**G4.4** Find a basis of the eigenspace associated to the eigenvalue 1 for the matrix
$A = \begin{bmatrix} 9 & -3 & -5 & 2 \\ 19 & -6 & -12 & 5 \\ 1 & 1 & -1 & 3 \\ -11 & 4 & 7 & -2 \end{bmatrix}$.

**G4.5** Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -4 & 4 & 0 & 0 \\ 11 & -6 & 1 & -1 \\ -9 & 5 & 1 & 3 \end{bmatrix}$.

**G4.6** Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$.

**G4.7** Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix
$A = \begin{bmatrix} 0 & -2 & -1 & 0 \\ -4 & -2 & -2 & 0 \\ 14 & 12 & 10 & 2 \\ -13 & -10 & -8 & -1 \end{bmatrix}$.

**G4.8** Find a basis of the eigenspace associated to the eigenvalue 3 for the matrix
$A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ -4 & -1 & -2 & 0 \\ 14 & 12 & 11 & 2 \\ -14 & -10 & -9 & -1 \end{bmatrix}$.

**G4.9** Find a basis of the eigenspace associated to the eigenvalue $-2$ for the matrix $A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 1 & 2 & -3 & 3 \\ 1 & 8 & -9 & 6 \\ 1 & 8 & -7 & 4 \end{bmatrix}$.

**G4.10** Find a basis of the eigenspace associated to the eigenvalue 2 for the matrix $\begin{bmatrix} -1 & 0 & 1 & 0 \\ 6 & 2 & -2 & 0 \\ -9 & 0 & 5 & 0 \\ 15 & 0 & -5 & 2 \end{bmatrix}$.