Readiness Assurance Test

Choose the most appropriate response for each question.

31) Suppose $f(x)$ and $g(x)$ are real-valued functions satisfying

$$\begin{array}{ll} f(2)=4 & g(2)=4 \\ f(3)=5 & g(3)=5 \\ f(4)=3 & g(4)=2 \end{array}$$

Compute $(f \circ g)(2)$.

(a) 3 (b) 4 (c) 5 (d) 6

32) Let $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$. Compute the composition function $(f \circ g)(x)$.

(a) $x^2 - 1$ (b) $x^4 - 4x^2 + 5$ (c) $x^4 + 2x^2 - 1$ (d) $x^4 - x^2 - 2$

33) What is the standard matrix corresponding to the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+2y-z \\ y+3z \\ x+7y \end{bmatrix}$?

(a) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

34) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear map corresponding to the standard matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix}$. Compute $T\left(\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} -2 \\ 8 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ -8 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

35) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation corresponding to the standard matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$.

Compute $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$

36) Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation corresponding to the standard matrix $\begin{bmatrix} 3 & -1 & 0 & 2 \\ -2 & -4 & -1 & 1 \end{bmatrix}$. What are the domain and codomain of T?

(a) The domain is $\mathbb{R}^2$ and the codomain is $\mathbb{R}^4$
(b) The domain and codomain are both $\mathbb{R}^2$
(c) The domain is $\mathbb{R}^4$ and the codomain is $\mathbb{R}^2$
(d) The domain and codomain are both $\mathbb{R}^4$

37) Which of the following is true of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 3y - 4z \\ x + y \\ 3z \end{bmatrix}?$$

(a) T is neither injective nor surjective
(b) T is both injective and surjective
(c) T is injective but not surjective
(d) T is surjective but not injective

38) Which of the following is true of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + z \end{bmatrix}?$$

(a) T is injective but not surjective
(b) T is both injective and surjective
(c) T is surjective but not injective
(d) T is neither injective nor surjective

39) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix A. Which of the following is **not** a characterization of the statement "T is injective"?

(a) If $T(\vec{\mathbf{v}}) = T(\vec{\mathbf{w}})$ for some $\vec{\mathbf{v}}, \vec{\mathbf{w}} \in \mathbb{R}^n$, then $\vec{\mathbf{v}} = \vec{\mathbf{w}}$.
(b) T has a non-trivial kernel, i.e. $\ker T \neq \left\{\vec{\mathbf{0}}\right\}$
(c) The columns of A are linearly independent
(d) $\mathrm{RREF}(A)$ has a pivot in every column

40) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix A. Which of the following is **not** a characterization of the statement "T is surjective"?

(a) $\mathrm{RREF}(A)$ has a pivot in every row
(b) $\operatorname{Im} T = \mathbb{R}^m$
(c) The columns of A span $\mathbb{R}^m$
(d) $\mathrm{RREF}(A)$ has a pivot in every column