

## Application Activities - Module A Part 2 - Class Day 18

**Definition 18.1** Let  $T : V \rightarrow W$  be a linear transformation.  $T$  is called **injective** or **one-to-one** if  $T$  does not map two distinct values to the same place. More precisely,  $T$  is injective if  $T(\mathbf{v}) \neq T(\mathbf{w})$  whenever  $\mathbf{v} \neq \mathbf{w}$ .

**Activity 18.2** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}.$$

The standard matrix of  $T$  is thus  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ .

Is  $T$  injective?

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**Activity 18.3** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix}.$$

The standard matrix of  $T$  is thus  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ .

Is  $T$  injective?

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**Definition 18.4** Let  $T : V \rightarrow W$  be a linear transformation.  $T$  is called **surjective** or **onto** if every element of  $W$  is mapped to by an element of  $V$ . More precisely, for every  $\mathbf{w} \in W$ , there is some  $\mathbf{v} \in V$  with  $T(\mathbf{v}) = \mathbf{w}$ .

**Activity 18.5** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}.$$

The standard matrix of  $T$  is thus  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ .

Is  $T$  surjective?

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**Activity 18.6** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix}.$$

The standard matrix of  $T$  is thus  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ .

Is  $T$  surjective?

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**Definition 18.7** Let  $T : V \rightarrow W$  be a linear transformation. The **kernel** of  $T$  is an important subspace of  $V$  defined by

$$\ker T = \{\mathbf{v} \in V \mid T(\mathbf{v}) = \mathbf{0}\}$$

**Activity 18.8** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by the standard matrix  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ . Find the kernel of  $T$ .

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**Activity 18.9** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the standard matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ . Find the kernel of  $T$ .

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**Activity 18.10** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be the linear transformation given by the standard matrix  $A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ .

*Part 1:* Write a system of equations whose solution set is the kernel.

*Part 2:* Use  $\text{RREF}(A)$  to solve the system of equations and find the kernel of  $T$ .

*Part 3:* Find a basis for the kernel of  $T$ .

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**Definition 18.11** Let  $T : V \rightarrow W$  be a linear transformation. The **image** of  $T$  is an important subspace of  $W$  defined by

$$\text{Im } T = \{\mathbf{w} \in W \mid \text{there is some } v \in V \text{ with } T(\mathbf{v}) = \mathbf{w}\}$$

**Activity 18.12** Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be given by the standard matrix  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ . Find the image of  $T$ .

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**Activity 18.13** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be given by the standard matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ . Find the image of  $T$ .

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**Activity 18.14** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be the linear transformation given by the standard matrix  $A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ .

*Part 1:* Find a convenient set of vectors  $S \subseteq \mathbb{R}^2$  such that  $\text{span } S = \text{Im } T$ .

*Part 2:* Find a convenient basis for the image of  $T$ .

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**Observation 18.15** Let  $T : V \rightarrow W$  be a linear transformation with corresponding matrix  $A$ .

- If  $A$  is a matrix corresponding to  $T$ , the kernel is the solution set of the homogeneous system with coefficients given by  $A$ .
  - If  $A$  is a matrix corresponding to  $T$ , the image is the span of the columns of  $A$ .
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