

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 10**

Math 237 – Linear Algebra

**Version 2**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array} \right]$$

**Solution:**

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 4x_2 + x_3 &= -7 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

□

**E3.** Solve the following linear system.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$ . It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction  $0 = 1$ ).

□

**E4.** Find a basis for the solution set of the system of equations

$$\begin{aligned} x + 3y + 3z + 7w &= 0 \\ x + 3y - z - w &= 0 \\ 2x + 6y + 3z + 8w &= 0 \\ x + 3y - 2z - 3w &= 0 \end{aligned}$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

□

**V1.** Let  $V$  be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$\begin{aligned} f \oplus g &= f' + g' \\ c \odot f &= cf' \end{aligned}$$

(here  $f'$  denotes the derivative of  $f$ ).

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot (f \oplus g) = c \odot f \oplus c \odot g$ .  
 (b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial  $f$ .

□

**E1:**

**E3:**

**E4:**

**V1:**

**E2:**