

Name:
J#:
Date:

Dr. Clontz

MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

Solution: AB is the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Since it is not equivalent to the identity matrix, it is not invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}.$

□

Standard G2.	Mark:
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Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}.$

Solution:

$$\begin{aligned} \det(A - \lambda I) &= (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix} \\ &= (8 - \lambda) (\lambda^2 + 6\lambda - 7) + 3(-21\lambda + 21) - (-7\lambda + 7) \\ &= (\lambda - 1)((8 - \lambda)(\lambda + 7) - 63 + 7) \\ &= (\lambda - 1)(\lambda - \lambda^2) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}.$

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}.$

□

Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
---------------------	-------

Let

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 2 & 6 & 11 & 1 \\ 1 & 3 & 7 & 2 \\ -1 & -3 & -5 & 0 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -2 & 0 & 3 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This matrix is not row equivalent to the identity matrix, so it is not invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$.

Solution: $\left[\begin{array}{ccc|ccc} 2 & -1 & -3 & 1 & 0 & 0 \\ -14 & 9 & 24 & 0 & 1 & 0 \\ 3 & -2 & -5 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3 \\ 0 & 1 & 0 & 2 & -1 & -6 \\ 0 & 0 & 1 & 1 & 1 & 4 \end{array} \right]$. Thus the inverse is $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

□

Standard G2.	Mark:
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Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned}
 \det(A - \lambda I) &= \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix} \\
 &= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix} \\
 &= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda)) \\
 &= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda \\
 &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\
 &= -\lambda^3 + 2\lambda^2 - \lambda \\
 &= -\lambda(\lambda^2 - 2\lambda + 1) \\
 &= -\lambda(\lambda - 1)^2
 \end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$.

□

Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
---------------------	-------

Let

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 2 & 6 & 11 & 1 \\ 1 & 3 & 7 & 2 \\ -1 & -3 & -5 & 0 \end{bmatrix}$$

□

Standard M2.	Mark:
---------------------	-------

Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

Solution: It is row equivalent to the identity matrix, so it is invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$.

Solution: $\left[\begin{array}{ccc|ccc} 4 & -1 & -8 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & 5 \\ 0 & 1 & 0 & -5 & 24 & -28 \\ 0 & 0 & 1 & 1 & -5 & 6 \end{array} \right]$. Thus the inverse is $\begin{bmatrix} 1 & -4 & 5 \\ -5 & 24 & -28 \\ 1 & -5 & 6 \end{bmatrix}$.

□

Standard G2.	Mark:
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Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned}
 \det(A - \lambda I) &= \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix} \\
 &= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix} \\
 &= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda)) \\
 &= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda \\
 &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\
 &= -\lambda^3 + 2\lambda^2 - \lambda \\
 &= -\lambda(\lambda^2 - 2\lambda + 1) \\
 &= -\lambda(\lambda - 1)^2
 \end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

Solution: The eigenspace is the solution space of the system $(B - 2I)X = 0$.

$$\text{RREF}(B - 2I) = \text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \frac{y}{3} = 0$, or $3x = y$. Thus the eigenspace is

$$E_2 = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

□

Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

Solution: BC is the only one that can be computed, and

$$BC = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ is invertible.

Solution: This matrix is row equivalent to the identity matrix, so it is invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & \frac{2}{3} & -\frac{2}{3} \\ 1 & -\frac{2}{2} & \frac{5}{2} \end{bmatrix}$$

□

Standard G2.	Mark:
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Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda)) \\ &= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 - \lambda \\ &= -\lambda(\lambda^2 - 2\lambda + 1) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$.

□

Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

Solution: AB is the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

□

Standard M2.	Mark:
---------------------	-------

Determine if the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Since it is not equivalent to the identity matrix, it is not invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & \frac{2}{3} & -\frac{2}{3} \\ 1 & -\frac{2}{2} & \frac{5}{2} \end{bmatrix}$$

□

Standard G2.	Mark:
---------------------	-------

Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda)) \\ &= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 - \lambda \\ &= -\lambda(\lambda^2 - 2\lambda + 1) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

Solution: The eigenspace is the solution space of the system $(B - 2I)X = 0$.

$$\text{RREF}(B - 2I) = \text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \frac{y}{3} = 0$, or $3x = y$. Thus the eigenspace is

$$E_2 = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$



Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

□

Standard M2.	Mark:
---------------------	-------

Determine if the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Since it is not equivalent to the identity matrix, it is not invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$.

Solution: $\left[\begin{array}{ccc|ccc} 4 & -1 & -8 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & 5 \\ 0 & 1 & 0 & -5 & 24 & -28 \\ 0 & 0 & 1 & 1 & -5 & 6 \end{array} \right]$. Thus the inverse is $\left[\begin{array}{ccc} 1 & -4 & 5 \\ -5 & 24 & -28 \\ 1 & -5 & 6 \end{array} \right]$. □

Standard G2.	Mark:
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Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

Solution:

$$\begin{aligned}
 \det(A - \lambda I) &= (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix} \\
 &= (8 - \lambda) (\lambda^2 + 6\lambda - 7) + 3(-21\lambda + 21) - (-7\lambda + 7) \\
 &= (\lambda - 1)((8 - \lambda)(\lambda + 7) - 63 + 7) \\
 &= (\lambda - 1)(\lambda - \lambda^2) \\
 &= -\lambda(\lambda - 1)^2
 \end{aligned}$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2). □

Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$. □

Additional Notes/Marks	
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