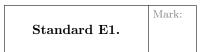
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 3x_2 - 4x_3 + x_4 = 5$$
$$3x_1 + 9x_2 + x_3 - 7x_4 = 0$$
$$x_1 - x_3 + x_4 = 1$$

Solution:

$$\begin{bmatrix} 1 & 3 & -4 & 1 & 5 \\ 3 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 1 & 1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{4} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{bmatrix}$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a\\3-21a\\-7a\\12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

Standard E4.

Mark:

Find the solution set for the following system of linear equations.

$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$
$$3x_1 + 6x_3 + x_4 = 0$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

Solution: Let $A = \begin{bmatrix} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all points on the line x + y = 2 with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

 $c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$

Determine if V is a vector space or not.

Solution:

- 1) Since real addition is associative, \oplus is associative.
- 2) Since real addition is commutative, \oplus is commutative.
- 3) $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so (1, 1) is an additive identity element.
- 4) $(x_1, y_1) \oplus (2 x_1, 2 y_1) = (1, 1)$, so $(2 x_1, 2 y_1)$ is the additive inverse of (x_1, y_1) .

5)

$$c \odot (d \odot (x_1, y_1)) = c \odot (dx_1 - (d-1), dy_1 - (d-1))$$

$$= (c (dx_1 - (d-1)) - (c-1), c (dy_1 - (d-1)))$$

$$= (cdx_1 - cd + c - (c-1), cdy_1 - cd + c - (c-1))$$

$$= (cdx_1 - (cd-1), cdy_1 - (cd-1))$$

$$= (cd) \odot (x_1, y_1)$$

6)
$$1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

$$\begin{split} c\odot((x_1,y_1)\oplus(x_2,y_2))&=c\odot(x_1+y_1-1,x_2+y_2-1)\\ &=(c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &=(cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &=(cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2) \end{split}$$

8)

$$(c+d)\odot(x_1,y_1) = ((c+d)x_1 - (c+d-1), (c+d)y_1 - (c+d-1))$$

= $(cx_1 - (c-1), cy_1 - (c-1)) \oplus (dx_1 - (d-1), dy_1 - (d-1))$
= $c\odot(x_1, y_1) \oplus c\odot(x_2, y_2)$

Therefore V is a vector space.

 ${\bf Additional\ Notes/Marks}$