Name: ______________________________

MASTERY QUIZ DAY 10 Math 237 – Linear Algebra
Version 5 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array}\right]$$

Solution:

$$\begin{aligned} 3x_1 - x_2 + x_4 &= 5 \\ -x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 &= -3 \end{aligned}$$

□

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 + 14x_4 &= 8 \\ x_1 + x_2 - x_3 + 5x_4 &= 3 \end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & 3 & -5 & 14 & 8 \\ 1 & 1 & -1 & 5 & 3 \end{array}\right]$, so RREF $A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array}\right]$. It follows that the solution set is given by $\begin{bmatrix} 2-2a-b \\ 2+3a-4b \\ a \\ b \end{bmatrix}$ for all real numbers a, b.

□

E4. Find a basis for the solution set of the system of equations

$$\begin{aligned} x + 3y + 3z + 7w &= 0 \\ x + 3y - z - w &= 0 \\ 2x + 6y + 3z + 8w &= 0 \\ x + 3y - 2z - 3w &= 0 \end{aligned}$$

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix}1 & 3 & 3 & 7\\ 1 & 3 & -1 & -1\\ 2 & 6 & 3 & 8\\ 1 & 3 & -2 & -3\end{bmatrix}\right)=\begin{bmatrix}1 & 3 & 0 & 1\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Then the solution set is

$$\left\{\begin{bmatrix}-3a-b\\ a\\ -2b\\ b\end{bmatrix} \;\middle|\; a,b\in\mathbb{R}\right\}$$

So a basis for the solution set is

$$\left\{\begin{bmatrix}3\\ -1\\ 0\\ 0\end{bmatrix},\begin{bmatrix}1\\ 0\\ 2\\ -1\end{bmatrix}\right\}$$

$\square$

V1. Let V be the set of all points on the line $x+y=2$ with the operations, for any $(x_1,y_1),(x_2,y_2)\in V$, $c\in\mathbb{R}$,

$$\begin{aligned}(x_1,y_1)\oplus(x_2,y_2)&=(x_1+x_2-1,y_1+y_2-1)\\ c\odot(x_1,y_1)&=(cx_1-(c-1),cy_1-(c-2))\end{aligned}$$

Determine if V is a vector space or not.

Solution:

1) Since real addition is associative, $\oplus$ is associative.

2) Since real addition is commutative, $\oplus$ is commutative.

3) $(x_1,y_1)\oplus(1,1)=(x_1,y_1)$, so $(1,1)$ is an additive identity element.

4) $(x_1,y_1)\oplus(2-x_1,2-y_1)=(1,1)$, so $(2-x_1,2-y_1)$ is the additive inverse of (x_1,y_1).

5)

$$\begin{aligned}c\odot(d\odot(x_1,y_1))&=c\odot(dx_1-(d-1),dy_1-(d-1))\\ &=(c\,(dx_1-(d-1))-(c-1),c\,(dy_1-(d-1)))\\ &=(cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &=(cdx_1-(cd-1),cdy_1-(cd-1))\\ &=(cd)\odot(x_1,y_1)\end{aligned}$$

6) $1\odot(x_1,y_1)=(x_1-(1-1),y_1-(1-1)=(x_1,y_1)$

7)

$$\begin{aligned}c\odot((x_1,y_1)\oplus(x_2,y_2))&=c\odot(x_1+y_1-1,x_2+y_2-1)\\ &=(c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &=(cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &=(cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot(x_1,y_1) &= ((c+d)x_1-(c+d-1),(c+d)y_1-(c+d-1))\\ &= (cx_1-(c-1),cy_1-(c-1))\oplus(dx_1-(d-1),dy_1-(d-1))\\ &= c\odot(x_1,y_1)\oplus c\odot(x_2,y_2)\end{aligned}$$

Therefore V is a vector space.

$\square$

E1:   **E3:**   **E4:**   **V1:**   **E2:**