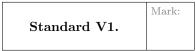
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that scalar multiplication **distributes vectors** over scalar addition: $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c+d)\odot(x_1,y_1)=(0,(c+d)y_1)=(0,cy_1)\oplus(0,dy_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

Standard V3.

Does span
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\4\\-3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

Solution: Since

RREF
$$\begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 4 & 2 \\ 4 & -9 & -3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

has a zero row, the vectors fail to span \mathbb{R}^3 .

Standard V4.

Let W be the set of all complex numbers that are purely real (i.e of the form a + 0i) or purely imaginary (i.e. of the form 0 + bi). Determine if W is a subspace of \mathbb{C} .

Solution: No, because 1 is purely real and i is purely imaginary, but the linear combination 1+i is neither.

Standard S2.

Mark:

Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}_3

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

Additional Notes/Marks