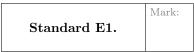
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J#:	
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MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{bmatrix}$$

Solution:

$$-4x_1 - x_2 + 3x_3 = 2$$
$$x_1 + 2x_2 - x_3 = 0$$
$$-x_1 + 4x_2 + x_3 = 4$$

Standard E3.

Mark:

Solve the system of equations

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 2$$

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix}1 & 3 & -4 & 5\\ 3 & 9 & 1 & 2\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}$$

So the solution set is

$$\left\{ \begin{bmatrix} 1 - 3c \\ c \\ -1 \end{bmatrix} \middle| c \in \mathbb{R} \right\}$$

Mark:

Find a basis for the solution set to the homogeneous system of equations

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 0$$
$$-2x_3 - 4x_4 = 0$$
$$2x_1 + 2x_2 + x_3 - 4x_4 = 0$$

Solution: Let
$$A = \begin{bmatrix} 4 & 4 & 3 & -6 & 0 \\ 0 & 0 & -2 & -4 & 0 \\ 2 & 2 & 1 & -4 & 0 \end{bmatrix}$$
, so RREF $A = \begin{bmatrix} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that this scalar multiplication ⊙ is associative.
- (b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$

$$= c (dx - 3(d - 1)) - 3(c - 1)$$

$$= cdx - 3(cd - 1)$$

$$= (cd) \odot x$$

We verify the remaining 7 properties to see that V is a vector space.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 3 = x$, so 3 is the additive identity.
- 3) $x \oplus (6-x) = x + (6-x) 3 = 3$, so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so \oplus is commutative.
- 5) Associativity shown above
- 6) $1 \odot x = x 3(1 1) = x$

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$

= $cx - 3(c-1) + dx - 3(c-1) - 3$
= $(c \odot x) \oplus (d \odot x)$

Therefore V is a vector space.

 ${\bf Additional\ Notes/Marks}$