MASTERY QUIZ DAY 10   Math 237 – Linear Algebra
Version 6   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1.   Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array}\right]$$

Solution:

$$2x_1 - x_2 = 1$$
$$-x_1 + 4x_2 + x_3 = -7$$
$$x_1 + 2x_2 - x_3 = 0$$

$\square$

E3.   Solve the system of equations

$$x + 3y - 4z = 5$$
$$3x + 9y + z = 2$$

Solution:

$$\text{RREF}\left(\left[\begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 2 \end{array}\right]\right) = \left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

So the solution set is

$$\left\{ \begin{bmatrix} 1 - 3c \\ c \\ -1 \end{bmatrix} \,\middle|\, c \in \mathbb{R} \right\}$$

$\square$

E4.   Find the solution set for the following system of linear equations.

$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$
$$3x_1 + 6x_3 + x_4 = 0$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{array}\right]$, so RREF $A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$. It follows that the basis

for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

$\square$

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c \odot (x_1, y_1) = (0, cy_1)$$

(a) Show that this scalar multiplication $\odot$ distributes over scalar addition.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c + d) \odot (x_1, y_1) = (0, (c+d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

$\square$

E1: [ ]   E3: [ ]   E4: [ ]   V1: [ ]   E2: [ ]