

Name: _____

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\x_1 - x_3 + x_4 &= 1\end{aligned}$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 3 & -4 & 1 & 5 \\ 3 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 1 & 1 \end{array} \right]$$

□

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\3x_1 + 6x_3 + x_4 &= 5 \\-4x_1 + x_2 - 9x_3 + 2x_4 &= -7\end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & -1 \\ 3 & 0 & 6 & 1 & 5 \\ -4 & 1 & -9 & 2 & -7 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right]$. It follows that the solution set is given by $\begin{bmatrix} 2 - 2a \\ 3 + a \\ a \\ -1 \end{bmatrix}$ for all real numbers a .

□

E4. Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned}2x_1 + 3x_2 - 5x_3 + 14x_4 &= 0 \\x_1 + x_2 - x_3 + 5x_4 &= 0\end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & 3 & -5 & 14 & 0 \\ 1 & 1 & -1 & 5 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$.

□

V1. Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2 - 1, y_1 + y_2 - 1) \\ c \odot (x_1, y_1) &= (cx_1 - (c - 1), cy_1 - (c - 2))\end{aligned}$$

(a) Show that this vector space has an additive identity element.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$; then $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so $(1, 1)$ is an additive identity element.

Now we will show the other seven properties. Let $(x_1, y_1), (x_2, y_2) \in V$, and let $c, d \in \mathbb{R}$.

1) Since real addition is associative, \oplus is associative.

2) Since real addition is commutative, \oplus is commutative.

3) The additive identity is $(1, 1)$.

4) $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$, so $(2 - x_1, 2 - y_1)$ is the additive inverse of (x_1, y_1) .

5)

$$\begin{aligned}c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= (c(dx_1 - (d - 1)) - (c - 1), c(dy_1 - (d - 1))) \\ &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1)) \\ &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1)) \\ &= (cd) \odot (x_1, y_1)\end{aligned}$$

6) $1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$

7)

$$\begin{aligned}c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1) \\ &= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1)) \\ &= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1)) \\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)\end{aligned}$$

8)

$$\begin{aligned}(c + d) \odot (x_1, y_1) &= ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1)) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= c \odot (x_1, y_1) \oplus d \odot (x_1, y_1)\end{aligned}$$

Therefore V is a vector space.

□

E1:

E3:

E4:

V1:

E2: