

## Section V.0

**Activity V.0.1** (*~20 min*) Consider each of the following vector properties. Label each property with  $\mathbb{R}^1$ ,  $\mathbb{R}^2$ , and/or  $\mathbb{R}^3$  if that property holds for Euclidean vectors/scalars  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  of that dimension.

1. **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

2. **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

3. **Addition identity.**

There exists some  $\mathbf{z}$  where  $\mathbf{v} + \mathbf{z} = \mathbf{v}$ .

4. **Addition inverse.**

There exists some  $-\mathbf{v}$  where  $\mathbf{v} + (-\mathbf{v}) = \mathbf{z}$ .

5. **Addition midpoint uniqueness.**

There exists a unique  $\mathbf{m}$  where the distance from  $\mathbf{u}$  to  $\mathbf{m}$  equals the distance from  $\mathbf{m}$  to  $\mathbf{v}$ .

6. **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

7. **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

8. **Scalar multiplication relativity.**

There exists some scalar  $c$  where either  $c\mathbf{v} = \mathbf{w}$  or  $c\mathbf{w} = \mathbf{v}$ .

9. **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

10. **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

11. **Orthogonality.**

There exists a non-zero vector  $\mathbf{n}$  such that  $\mathbf{n}$  is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ .

12. **Bidimensionality.**

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} \text{ for some value of } a, b.$$

**Definition V.0.2** A **vector space**  $V$  is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  belong to  $V$ , and let  $a, b$  be scalar numbers.

- **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

- **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

- **Addition identity.**

There exists some  $\mathbf{z}$  where  $\mathbf{v} + \mathbf{z} = \mathbf{v}$ .

- **Addition inverse.**

There exists some  $-\mathbf{v}$  where  $\mathbf{v} + (-\mathbf{v}) = \mathbf{z}$ .

- **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

- **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

- **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

- **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

Any **Euclidean vector space**  $\mathbb{R}^n$  satisfies all eight requirements regardless of the value of  $n$ , but we will also study other types of vector spaces.