

Name: \_\_\_\_\_

### MASTERY QUIZ DAY 14

Math 237 – Linear Algebra

#### Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**V1.** Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

(a) Show that the vector **addition**  $\oplus$  is **associative**:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .

(b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $x, y, z \in \mathbb{R}$ . Then

$$\begin{aligned}(x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\&= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\&= \sqrt{x^2 + y^2 + z^2} \\&= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\&= x \oplus \sqrt{y^2 + z^2} \\&= x \oplus (y \oplus z)\end{aligned}$$

However, this is not a vector space, as there is no zero vector.

□

**V3.** Determine if the vectors  $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span  $\mathbb{R}^4$ .

□

**V4.** Let  $W$  be the set of all complex numbers that are purely real (i.e. of the form  $a + 0i$ ) or purely imaginary (i.e. of the form  $0 + bi$ ). Determine if  $W$  is a subspace of  $\mathbb{C}$ .

**Solution:** No, because 1 is purely real and  $i$  is purely imaginary, but the linear combination  $1 + i$  is neither.

□

**S2.** Determine if the set  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

**V1:**

**V3:**

**V4:**

**S2:**