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MASTERY QUIZ DAY 18

Math 237 – Linear Algebra Fall 2017

Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ is linearly dependent or linearly independent

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Standard S3.

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Solution: Let
$$A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$
, and compute RREF $(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are

pivot columns, $\left\{ \begin{bmatrix} -3\\-8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix} \right\}$ is a basis for W.

Standard S4.
$$\begin{bmatrix} & & & \\ Mark: & & \\ & & \\ Let W = span \left(\left\{ \begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\3\\6\\3\\1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix}, \begin{bmatrix} 7\\-1\\8\\-3 \end{bmatrix} \right\} \right).$$
 Find the dimension of W .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

Standard A1.

Mark:

Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 7x + 2y + 3z \\ 0 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R}^4 .

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 7 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Standard A2.

Mark:

Determine if the map $T: \mathcal{P} \to \mathcal{P}$ given by T(f) = f' - f'' is a linear transformation or not.

Additional Notes/Marks