

Name: _____

MASTERY QUIZ DAY 6

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

□

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 + 3x_2 - 5x_3 + 14x_4 &= 8 \\x_1 + x_2 - x_3 + 5x_4 &= 3\end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & 3 & -5 & 14 & 8 \\ 1 & 1 & -1 & 5 & 3 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{array} \right]$. It follows that the solution set is given by $\begin{bmatrix} 2 - 2a - b \\ 2 + 3a - 4b \\ a \\ b \end{bmatrix}$ for all real numbers a, b .

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E4. Find a basis for the solution set of the system of equations

$$\begin{aligned}x + 2y + 3z + w &= 0 \\3x - y + z + w &= 0 \\2x - 3y - 2z &= 0 \\-x + 2z + 5w &= 0\end{aligned}$$

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \\ -1 & 0 & 2 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} a \\ 2a \\ -2a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \right\}$.

□

E1:

E3:

E4:

E2: