Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 12   Math 237 – Linear Algebra
Version 3   Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5)
$$
\begin{aligned}
c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\
&= c\,(dx - 3(d - 1)) - 3(c - 1) \\
&= cdx - 3(cd - 1) \\
&= (cd) \odot x
\end{aligned}
$$

6) $1 \odot x = x - 3(1 - 1) = x$

7)
$$
\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c - 1) \\
&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}
$$

8)
$$
\begin{aligned}
(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
&= cx - 3(c - 1) + dx - 3(c - 1) - 3 \\
&= (c \odot x) \oplus (d \odot x)
\end{aligned}
$$

Therefore V is a vector space.

□

Standard V3.

Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span $\mathbb{R}^4$.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span $\mathbb{R}^4$.

□

Standard V4.

Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \ \middle| \ x, y, z \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^4$.

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

□