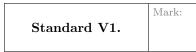
| Name: |            |
|-------|------------|
| J#:   | Dr. Clontz |
| Date: |            |

## MASTERY QUIZ DAY 12

Math 237 – Linear Algebra Fall 2017

Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$ 

- (a) Show that this scalar multiplication  $\odot$  distributes over vector addition  $\oplus$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1), (x_2, y_2) \in V$  and let  $c \in \mathbb{R}$ .

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + x_2, y_1 + y_2)$$

$$= (c^2(x_1 + x_2), c^3(y_1 + y_2))$$

$$= (c^2x_1, c^3y_1) \oplus (c^2x_2, c^3y_2)$$

$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

However, V is not a vector space, as the other distributive law fails:

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

**Solution:** 

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are zero rows, they do not span. Alternatively, by inspection  $\begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ , so the set is linearly dependent, so it spans a subspace of dimension at most 3, therefore it does not span  $\mathbb{R}^4$ .

Standard V4.

Mark:

Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  with terminal points on the plane x+y+z=1. Determine if W is a subspace of  $\mathbb{R}^3$ .

**Solution:** No, because **0** does not belong to W.