Section S.1

Activity S.1.1 *(∼10 min)* Consider the two sets

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\} \qquad T = \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -11 \end{bmatrix} \right\}$$

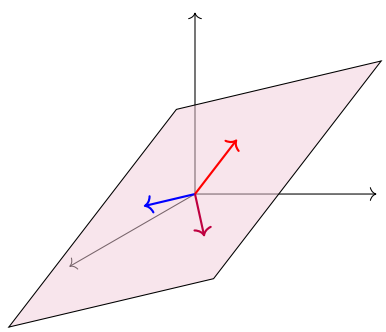
Which of the following is true?

(A) $\operatorname{span} S$ is bigger than $\operatorname{span} T$.

(B) $\operatorname{span} S$ and $\operatorname{span} T$ are the same size.

(C) $\operatorname{span} S$ is smaller than $\operatorname{span} T$.

Definition S.1.2 We say that a set of vectors is **linearly dependent** if one vector in the set belongs to the span of the others. Otherwise, we say the set is **linearly independent**.



You can think of linearly dependent sets as containing a redundant vector, in the sense that you can drop a vector out without reducing the span of the set. In the above image, all three vectors lay on the same planar subspace, but only two vectors are needed to span the plane, so the set is linearly dependent.

Activity S.1.3 *(∼10 min)* Let $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$ be vectors in $\mathbb{R}^n$. Suppose $3\vec{\mathbf{u}} - 5\vec{\mathbf{v}} = \vec{\mathbf{w}}$, so the set $\{\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}\}$ is linearly dependent. Which of the following is true of the vector equation $x\vec{\mathbf{u}} + y\vec{\mathbf{v}} + z\vec{\mathbf{w}} = \vec{\mathbf{0}}$?

(A) It is consistent with one solution

(B) It is consistent with infinitely many solutions

(C) It is inconsistent.

Fact S.1.4 For any vector space, the set $\{\vec{\mathbf{v}}_1, \dots \vec{\mathbf{v}}_n\}$ is linearly dependent if and only if $x_1\vec{\mathbf{v}}_1 + \cdots + x_n\vec{\mathbf{v}}_n = \vec{\mathbf{z}}$ is consistent with infinitely many solutions.

Activity S.1.5 *(∼10 min)* Find

$$\text{RREF}\left[\begin{array}{ccccc|c} 2 & 2 & 3 & -1 & 4 & 0 \\ 3 & 0 & 13 & 10 & 3 & 0 \\ 0 & 0 & 7 & 7 & 0 & 0 \\ -1 & 3 & 16 & 14 & 2 & 0 \end{array}\right]$$

and mark the part of the matrix that demonstrates that

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix}, \begin{bmatrix} -1 \\ 10 \\ 7 \\ 14 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \\ 2 \end{bmatrix} \right\}$$

is linearly dependent (the part that shows its linear system has infinitely many solutions).

Fact S.1.6 A set of Euclidean vectors $\{\vec{\mathbf{v}}_1, \dots \vec{\mathbf{v}}_n\}$ is linearly dependent if and only if RREF $\begin{bmatrix} \vec{\mathbf{v}}_1 & \dots & \vec{\mathbf{v}}_n \end{bmatrix}$ has a column without a pivot position.

Activity S.1.7 *(∼5 min)* Is the set of Euclidean vectors $\left\{ \begin{bmatrix} -4 \\ 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 10 \\ 10 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 7 \\ 2 \\ 1 \end{bmatrix} \right\}$ linearly dependent or linearly independent?

Activity S.1.8 *(∼10 min)* Is the set of polynomials $\left\{x^3 + 1, x^2 + 2x, x^2 + 7x + 4\right\}$ linearly dependent or linearly independent?

Activity S.1.9 *(∼5 min)* What is the largest number of vectors in $\mathbb{R}^4$ that can form a linearly independent set?

(a) 3

(b) 4

(c) 5

(d) You can have infinitely many vectors and still be linearly independent.

Activity S.1.10 *(∼5 min)* What is the largest number of vectors in

$$\mathcal{P}^4 = \left\{ ax^4 + bx^3 + cx^2 + dx + e \,\middle|\, a, b, c, d, e \in \mathbb{R} \right\}$$

that can form a linearly independent set?

(a) 3

(b) 4

(c) 5

(d) You can have infinitely many vectors and still be linearly independent.

Activity S.1.11 *(∼5 min)* What is the largest number of vectors in

$$\mathcal{P} = \{ f(x) \mid f(x) \text{ is any polynomial} \}$$

that can form a linearly independent set?

(a) 3

(b) 4

(c) 5

(d) You can have infinitely many vectors and still be linearly independent.