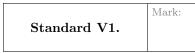
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Math 237 – Linear Algebra Fall 2017

#### Version 1

Solution:

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot (f \oplus g) = c \odot f \oplus c \odot g$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial f.

Determine if the vectors  $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$ , and  $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

$$RREF \begin{pmatrix} \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span  $\mathbb{R}^4$ .

Standard V4.	Mark:
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Let W be the set of all complex numbers that are purely real (i.e of the form a+0i) or purely imaginary (i.e. of the form 0+bi). Determine if W is a subspace of  $\mathbb{C}$ .

**Solution:** No, because 1 is purely real and i is purely imaginary, but the linear combination 1+i is neither.

Additional Notes/Marks

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Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard V1.	

Let V be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that scalar multiplication is associative:  $a \odot (b \odot x) = (ab) \odot x$ .
- (b) Determine if V is a vector space or not. Justify your answer

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ . To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$
$$= c (dx - 3(d - 1)) - 3(c - 1)$$
$$= cdx - 3(cd - 1)$$
$$= (cd) \odot x$$

We verify the remaining 7 properties to see that V is a vector space.

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6-x) = x + (6-x) 3 = 3$ , so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so  $\oplus$  is commutative.
- 5) Associativity shown above
- 6)  $1 \odot x = x 3(1 1) = x$

7)

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$
  
=  $cx - 3(c-1) + dx - 3(c-1) - 3$   
=  $(c \odot x) \oplus (d \odot x)$ 

Therefore V is a vector space.

# Standard V3.

Mark:

Does span  $\left\{ \begin{bmatrix} 2\\-1\\4\\2\\1 \end{bmatrix}, \begin{bmatrix} -1\\3\\5\\2\\0 \end{bmatrix}, \begin{bmatrix} 1\\0\\5\\1\\-3 \end{bmatrix} \right\} = \mathbb{R}^5?$ 

**Solution:** Since there are only three vectors, they cannot span  $\mathbb{R}^5$ .

### Standard V4.

Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

**Solution:** No, because **0** does not belong to W.

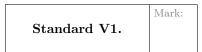
## Additional Notes/Marks

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Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$ 

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1), (x_2, y_2) \in V$  and let  $c \in \mathbb{R}$ .

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + x_2, y_1 + y_2)$$

$$= (c^2(x_1 + x_2), c^3(y_1 + y_2))$$

$$= (c^2x_1, c^3y_1) \oplus (c^2x_2, c^3y_2)$$

$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

However, V is not a vector space, as the other distributive law fails:

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

Determine if the vectors  $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$ ,  $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$ ,  $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$ , and  $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

Solution:

$$RREF \left( \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span  $\mathbb{R}^4$ .

Standard V4.	Mark:

Let W be the set of all complex numbers a+bi satisfying a=2b. Determine if W is a subspace of  $\mathbb C$ .

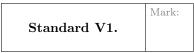
**Solution:** Yes, because  $c(2b_1 + b_1i) + d(2b_2 + b_2i) = 2(cb_1 + db_2) + (cb_1 + db_2)i$  belongs to W. Alternately, yes because W is isomorphic to  $\mathbb{R}$ .

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#### Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any  $f, g \in V$ ,  $c \in \mathbb{R}$ ,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot (f \oplus g) = c \odot f \oplus c \odot g$ .
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $f, g \in \mathcal{P}$ , and let  $c \in \mathbb{R}$ .

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally,  $1 \odot f \neq f$  for any nonzero polynomial f.

Standard V3.

Mark:  $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \text{ and } \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \text{ span } \mathbb{R}^3$ 

Solution:

$$RREF\left(\begin{bmatrix} -3 & 5 & 2 & 0\\ 1 & -1 & 0 & 2\\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5\\ 0 & 1 & 1 & 3\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span  $\mathbb{R}^3$ .

Standard V4.

Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=0 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

**Solution:** Yes, because z = -x - y and  $a \begin{bmatrix} x_1 \\ y_1 \\ -x_1 - y_1 \end{bmatrix} + b \begin{bmatrix} x_2 \\ y_2 \\ -x_2 - y_2 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ ay_1 + by_2 \\ -(ax_1 + bx_2) - (ay_1 + by_2) \end{bmatrix}$ . Alternately, yes because W is isomorphic to  $\mathbb{R}^2$ .

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Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (0, cy_1)$ 

- (a) Show that scalar multiplication **distributes vectors** over scalar addition:  $(c+d)\odot(x,y)=c\odot(x,y)\oplus d\odot(x,y).$
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1) \in V$ , and let  $c, d \in \mathbb{R}$ . Then

$$(c+d)\odot(x_1,y_1)=(0,(c+d)y_1)=(0,cy_1)\oplus(0,dy_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

However, V is not a vector space, as  $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$ .

Determine if the vectors 
$$\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$
,  $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

Solution:

RREF 
$$\left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span  $\mathbb{R}^4$ .

Standard V4.

Mark:

Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

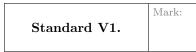
**Solution:** No, because  $\mathbf{0}$  does not belong to W.

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Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
  
 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$ 

- (a) Show that scalar multiplication **distributes scalars** over vector addition:  $c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$
- (b) Determine if V is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1), (x_2, y_2) \in V$  and let  $c \in \mathbb{R}$ .

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + x_2, y_1 + y_2)$$

$$= (c^2(x_1 + x_2), c^3(y_1 + y_2))$$

$$= (c^2x_1, c^3y_1) \oplus (c^2x_2, c^3y_2)$$

$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

However, V is not a vector space, as the other distributive law fails:

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

Standard V3.  $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \text{ and } \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \text{ span } \mathbb{R}^4.$ 

Solution:

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are zero rows, they do not span. Alternatively, by inspection  $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ , so the set is linearly dependent, so it spans a subspace of dimension at most 3, therefore it does not span  $\mathbb{R}^4$ .

Standard V4.

Mark:

Let W be the set of all  $\mathbb{R}^3$  vectors  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  satisfying x+y+z=0 (this forms a plane). Determine if W is a subspace of  $\mathbb{R}^3$ .

Additional Notes/Marks