

Name: _____

MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine which of the six products AB , AC , BA , BC , CA , CB can be computed, and compute them.

Solution: AB and CA are the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

$$CA = \begin{bmatrix} 3 & 9 & 11 \\ 0 & 0 & 7 \end{bmatrix}$$

□

M2. Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ is invertible.

Solution: This matrix is row equivalent to the identity matrix, so it is invertible.

□

M3. Find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & \frac{3}{2} & -\frac{3}{2} \\ 1 & -\frac{3}{2} & \frac{5}{2} \end{bmatrix}$$

□

G2. Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned}\det(A - \lambda I) &= \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda)) \\ &= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 - \lambda \\ &= -\lambda(\lambda^2 - 2\lambda + 1) \\ &= -\lambda(\lambda - 1)^2\end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

G3. Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

Solution: The eigenspace is the solution space of the system $(B - 2I)X = 0$.

$$\text{RREF}(B - 2I) = \text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \frac{y}{3} = 0$, or $3x = y$. Thus the eigenspace is

$$E_2 = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

□

M1: M2: M3: G2: G3: G1: