Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

#### Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{bmatrix}$$

Solution:

$$3x_1 - x_2 + x_4 = 5$$
$$-x_1 + 9x_2 + x_3 - 7x_4 = 0$$
$$x_1 - x_3 = -3$$

Standard E3.

Mark:

Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The solutions are

$$\left\{ \begin{bmatrix} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{bmatrix} \mid c \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} c - 1 \\ 3c - 1 \\ -2c \end{bmatrix} \mid c \in \mathbb{R} \right\}$$

# Standard E4. Mark:

Find a basis for the solution set to the homogeneous system of equations

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 0$$
$$x_1 + x_2 - x_3 + 5x_4 = 0$$

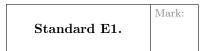
Solution: Let 
$$A = \begin{bmatrix} 2 & 3 & -5 & 14 & 0 \\ 1 & 1 & -1 & 5 & 0 \end{bmatrix}$$
, so RREF  $A = \begin{bmatrix} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{bmatrix}$ . It follows that the basis for the solution set is given by  $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$ .

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### Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The solutions are

$$\left\{ \begin{bmatrix} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{bmatrix} \mid c \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} c - 1 \\ 3c - 1 \\ -2c \end{bmatrix} \mid c \in \mathbb{R} \right\}$$

### Standard E4. Mark:

Find a basis for the solution set to the homogeneous system of equations given by

$$3x + 2y + z = 0$$
$$x + y + z = 0$$

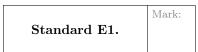
**Solution:** Let 
$$A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$
, so RREF  $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$ . It follows that the basis for the solution set is given by  $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ .

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### Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

RREF 
$$\left( \begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{4} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{bmatrix}$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a\\ 3-21a\\ -7a\\ 12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

# Standard E4. Mark:

Find a basis for the solution set to the homogeneous system of equations

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 0$$
$$x_1 + x_2 - x_3 + 5x_4 = 0$$

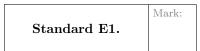
Solution: Let 
$$A = \begin{bmatrix} 2 & 3 & -5 & 14 & 0 \\ 1 & 1 & -1 & 5 & 0 \end{bmatrix}$$
, so RREF  $A = \begin{bmatrix} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{bmatrix}$ . It follows that the basis for the solution set is given by  $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$ .

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Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{bmatrix}$$

Solution:

$$-4x_1 - x_2 + 3x_3 = 2$$
$$x_1 + 2x_2 - x_3 = 0$$
$$-x_1 + 4x_2 + x_3 = 4$$

Standard E3.

Mark:

Solve the following linear system.

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 5$$
$$-2x_3 - 4x_4 = 3$$
$$2x_1 + 2x_2 + x_3 - 4x_4 = -1$$

**Solution:** Let  $A = \begin{bmatrix} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{bmatrix}$ , so RREF  $A = \begin{bmatrix} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ . It follows that the system

is inconsistent with no solutions (since the bottom row implies the contradiction 0 = 1).

Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$

$$2x + y - 4z = 0$$

$$3y - 2z = 0$$

$$x - y - z = 0$$

Solution:

RREF 
$$\begin{pmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \frac{5}{3}a\\ \frac{2}{3}a\\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

So a basis is 
$$\left\{ \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} \right\}$$
 or  $\left\{ \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} \right\}$ .

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### Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The solutions are

$$\left\{ \begin{bmatrix} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{bmatrix} \mid c \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} c - 1 \\ 3c - 1 \\ -2c \end{bmatrix} \mid c \in \mathbb{R} \right\}$$

### Standard E4.

Mark:

Find the solution set for the following system of linear equations.

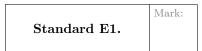
$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$
$$3x_1 + 6x_3 + x_4 = 0$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

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### Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 3x_2 - 4x_3 + x_4 = 5$$
$$3x_1 + 9x_2 + x_3 - 7x_4 = 0$$
$$x_1 - x_3 + x_4 = 1$$

Solution:

$$\begin{bmatrix} 1 & 3 & -4 & 1 & 5 \\ 3 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 1 & 1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the following linear system.

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 5$$
$$-2x_3 - 4x_4 = 3$$
$$2x_1 + 2x_2 + x_3 - 4x_4 = -1$$

**Solution:** Let  $A = \begin{bmatrix} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{bmatrix}$ , so RREF  $A = \begin{bmatrix} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ . It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction 0 = 1).

### Standard E4. Mark:

Find a basis for the solution set to the homogeneous system of equations

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 0$$
$$-2x_3 - 4x_4 = 0$$
$$2x_1 + 2x_2 + x_3 - 4x_4 = 0$$

Additional Notes/Marks