Name:

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Date:

MASTERY QUIZ DAY 15

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V2.	Mark:
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Determine if $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3\\0\\-1 \end{bmatrix}$, $\begin{bmatrix} 1\\-1\\4 \end{bmatrix}$, and $\begin{bmatrix} 5\\1\\-6 \end{bmatrix}$.

Solution:

RREF
$$\left(\begin{bmatrix} 3 & 1 & 5 & 1 \\ 0 & -1 & 1 & 4 \\ -1 & 4 & -6 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$ is not a linear combination of the three vectors.

Standard S1.

Mark:

Determine if the set of polynomials $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix},\begin{bmatrix}3\\1\\3\\6\end{bmatrix},\begin{bmatrix}0\\1\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Find a basis of W.

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then
$$\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$$
 is a basis of W .

Standard S4.

Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix},\begin{bmatrix}3\\1\\3\\6\end{bmatrix},\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

Additional Notes/Marks