Name:	
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Date:	

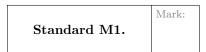
MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 3 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 2 & 6 & 11 & 1 \\ 1 & 3 & 7 & 2 \\ -1 & -3 & -5 & 0 \end{bmatrix}$$

Standard M2.

Mark:

Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is invertible.

Solution: It is row equivalent to the identity matrix, so it is invertible.

Standard M3.

Mark

Find the inverse of the matrix $\begin{bmatrix} 4 & -1 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}.$

Solution: $\begin{bmatrix} 4 & -1 & -8 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -4 & 5 \\ 0 & 1 & 0 & -5 & 24 & -28 \\ 0 & 0 & 1 & 1 & -5 & 6 \end{bmatrix} .$ Thus the inverse is $\begin{bmatrix} 1 & -4 & 5 \\ -5 & 24 & -28 \\ 1 & -5 & 6 \end{bmatrix} .$

Mark:

Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\det(A - \lambda I) = \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix}$$

$$= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix}$$

$$= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda))$$

$$= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda$$

$$= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda$$

$$= -\lambda^3 + 2\lambda^2 - \lambda$$

$$= -\lambda(\lambda^2 - 2\lambda + 1)$$

$$= -\lambda(\lambda - 1)^2$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

Standard G3.

Mark

Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}.$

Solution: The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \frac{y}{3} = 0$, or 3x = y. Thus the eigenspace is

$$E_2 = \operatorname{span}\left(\left\{\begin{bmatrix}1\\3\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}\right)$$

Additional Notes/Marks