Math 237 – Linear Algebra

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S1. Determine if the vectors $\begin{bmatrix} 1\\1\\-1 \end{bmatrix}$, $\begin{bmatrix} 3\\-1\\1 \end{bmatrix}$, and $\begin{bmatrix} 2\\0\\-2 \end{bmatrix}$ are linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since each column is a pivot column, the vectors are linearly independent.

S3. Let W be the subspace of \mathcal{P}_3 given by $W = \text{span}\left(\left\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\right\}\right)$. Find a basis for W.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is $\{x^3 + x^2 + 2x + 1, 3x^3 - x^2 + 3x - 2\}$.

S4. Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix},\begin{bmatrix}3\\1\\3\\6\end{bmatrix},\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R}^4 .

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

A2. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ \sqrt{x}+\sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

Solution:

$$T\left(\begin{bmatrix}0\\4\end{bmatrix}\right) = \begin{bmatrix}4\\2\end{bmatrix} \neq \begin{bmatrix}4\\4\end{bmatrix} = 4T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

So T is not a linear transformation.

S1: S3: A1: A2:

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Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\-1 \end{bmatrix}, \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

S3. Let W be the subspace of \mathcal{P}_3 given by $W = \text{span}\left(\left\{x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3\right\}\right)$. Find a basis for W.

Solution:

$$RREF \begin{pmatrix} \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is $\{x^3 + x^2 + 2x + 1, 3x^3 - x^2 + 3x - 2\}$.

 $\mathbf{S4.} \quad \text{Let } W = \text{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix},\begin{bmatrix}3\\1\\3\\6\end{bmatrix},\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right). \text{ Compute the dimension of } W.$

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and $\mathbb{R}^4.$

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

A2. Determine if the map $T: \mathcal{P}^3 \to \mathcal{P}^4$ given by T(f(x)) = xf(x) - f(x) is a linear transformation or not.

S1:

S3:

S4:

A1:

A2:

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Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S1. Determine if the set of polynomials $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

S3. Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix},\begin{bmatrix}2\\0\\1\\1\end{bmatrix},\begin{bmatrix}3\\-1\\4\\-2\end{bmatrix},\begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$. Find a basis of W.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then $\left\{ \begin{bmatrix} 1\\-1\\3\\-3 \end{bmatrix}, \begin{bmatrix} 3\\-1\\4\\-2 \end{bmatrix}, \begin{bmatrix} 1\\1\\1\\-7 \end{bmatrix} \right\}$ is a basis for W.

S4. Let $W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\-1\\3\\-3\end{bmatrix},\begin{bmatrix}2\\0\\1\\1\end{bmatrix},\begin{bmatrix}3\\-1\\4\\-2\end{bmatrix},\begin{bmatrix}1\\1\\1\\-7\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and $\mathbb{R}^4.$

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

A2. Determine if $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$ is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

S1:

S3:

S4:

A1:

A2:

Name:	

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\-1 \end{bmatrix}, \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

S3. Let $W = \text{span}\left(\left\{\begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}\right\}\right)$. Find a basis of W.

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then $\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$ is a basis of W.

S4. Let W be the subspace of \mathcal{P}_3 given by $W = \text{span} \left(\left\{ x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7 \right\} \right)$. Compute the dimension of W.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R}^4 .

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

A2. Determine if $D: M_{2,2} \to \mathbb{R}$ given by $D\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$ is a linear transformation or not.

Solution: D(I) = 1 but $D(2I) = 4 \neq 2D(I)$, so D is not linear.

S1:

S3:

S4:

A1:

A2:

Name:	

Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\-1 \end{bmatrix}, \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

S3. Let W be the subspace of \mathcal{P}_2 given by $W = \text{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$. Find a basis for W.

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are

pivot columns, $\left\{-3x^2 - 8x, x^2 + 2x + 2\right\}$ is a basis for W.

S4. Let W be the subspace of $M_{2,2}$ given by $W = \operatorname{span}\left(\left\{\begin{bmatrix}2 & 0\\ -2 & 0\end{bmatrix}, \begin{bmatrix}3 & 1\\ 3 & 6\end{bmatrix}, \begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}, \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R}^4 .

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

A2. Determine if the map $T: \mathcal{P}^4 \to \mathcal{P}^3$ given by T(f) = f' - f'' is a linear transformation or not.

S3:

S4:

A1:

A2:

Name:	

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\-1 \end{bmatrix}, \begin{bmatrix} 1\\3\\-2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

S3. Let $W = \operatorname{span}\left(\left\{\begin{bmatrix} -3\\ -8\\ 0\end{bmatrix}, \begin{bmatrix} 1\\ 2\\ 2\end{bmatrix}, \begin{bmatrix} 0\\ -1\\ 3\end{bmatrix}\right\}\right)$. Find a basis for W.

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute RREF $(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are

pivot columns, $\left\{ \begin{bmatrix} -3\\-8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix} \right\}$ is a basis for W.

S4. Let $W = \text{span}\left(\left\{\begin{bmatrix} -3\\-8\\0\end{bmatrix}, \begin{bmatrix} 1\\2\\2\end{bmatrix}, \begin{bmatrix} 0\\-1\\3\end{bmatrix}\right\}\right)$. Compute the dimension of W.

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $RREF(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since there are two pivot

columns, dim W = 2.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R}^4 .

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

A2. Determine if the map $T: \mathcal{P}^3 \to \mathcal{P}^4$ given by T(f(x)) = xf(x) - f(x) is a linear transformation or not.

S1:

S3:

A1:

A2: