

Section V.3

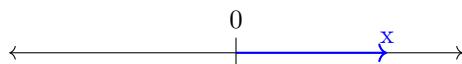
Activity V.25 (*~5 min*) Does the polynomial $x^2 + x + 1$ belong to $\text{span}\{x^2 - x, x + 1, x^2 - 1\}$?

Activity V.26 (*~5 min*) Does the matrix $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$ belong to $\text{span}\left\{\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 2 & 2 \end{bmatrix}\right\}$?

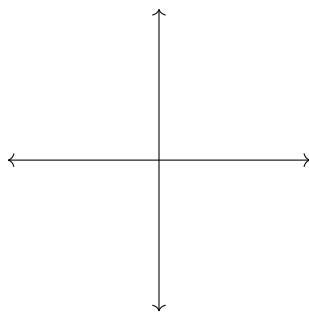
Part 1: Reinterpret this question as a question about the solution(s) of a matrix equation.

Part 2: Answer this equivalent question, and use its solution to answer the original question.

Observation V.27 Any single non-zero vector/number x in \mathbb{R}^1 spans \mathbb{R}^1 , since $\mathbb{R}^1 = \{cx \mid c \in \mathbb{R}\}$.

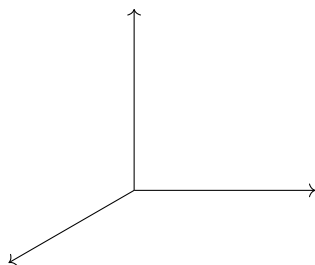


Activity V.28 (*~5 min*) How many vectors are required to span \mathbb{R}^2 ? Sketch a drawing in the xy plane to support your answer.



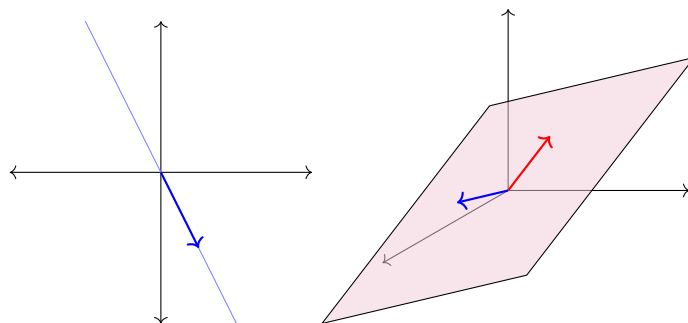
- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) Infinitely Many

Activity V.29 (*~5 min*) How many vectors are required to span \mathbb{R}^3 ?



- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) Infinitely Many

Fact V.30 At least n vectors are required to span \mathbb{R}^n .



Activity V.31 (*~15 min*) Choose any vector $\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$ in \mathbb{R}^3 that is not in $\text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ by using technology to verify that $\text{RREF} \left[\begin{array}{cc|c} 1 & -2 & ? \\ -1 & 0 & ? \\ 0 & 1 & ? \end{array} \right] = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$. (Why does this work?)

Fact V.32 The set $\{\vec{v}_1, \dots, \vec{v}_m\}$ fails to span all of \mathbb{R}^n exactly when the vector equation

$$x_1 \vec{v}_1 + \dots + x_m \vec{v}_m = \vec{w}$$

is inconsistent for **some** vector \vec{w} .

Note that this happens exactly when $\text{RREF}[\vec{v}_1 \dots \vec{v}_m]$ has a non-pivot row of zeros.

$$\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \text{ for some choice of vector } \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Activity V.33 (~ 5 min) Consider the set of vectors $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}$. Does

$\mathbb{R}^4 = \text{span } S$?

Part 1: Rewrite this as a question about the solutions to a vector equation.

Part 2: Answer your new question, and use this to answer the original question.

Activity V.34 (~ 10 min) Consider the set of third-degree polynomials

$$S = \{2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, \\ -x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\}.$$

Does $\mathcal{P}^3 = \text{span } S$?

Part 1: Rewrite this as a question about the solutions to a polynomial equation.

Part 2: Answer your new question, and use this to answer the original question.