

## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compose functions of real numbers
- Solve systems of linear equations (**Standard(s) E3**)
- Find the matrix corresponding to a linear transformation (**Standard(s) A1**)
- Determine if a linear transformation is injective and/or surjective (**Standard(s) A3**)
- Interpret the ideas of injectivity and surjectivity in multiple ways

## Readiness Assurance Resources

The following resources will help you prepare for this module.

- <https://www.khanacademy.org/math/algebra2/manipulating-functions/function-composition/v/function-composition>

**Readiness Assurance Test**

Choose the most appropriate response for each question.

- 41) Suppose  $f(x)$  and  $g(x)$  are real-valued functions satisfying

$$\begin{array}{ll} f(2) = 4 & g(2) = 4 \\ f(3) = 5 & g(3) = 5 \\ f(4) = 3 & g(4) = 2 \end{array}$$

Compute  $(f \circ g)(2)$ .

- (a) 2                      (b) 3                      (c) 4                      (d) 5

- 42) Let  $f(x) = x^2 - 2$  and  $g(x) = x^2 + 1$ . Compute the composition function  $(f \circ g)(x)$ .

- (a)  $x^2 - 1$                       (b)  $x^4 + 2x^2 - 1$                       (c)  $x^4 - 4x^2 + 5$                       (d)  $x^4 - x^2 - 2$

- 43) Solve the system of linear equations

$$\begin{array}{l} x + 3y = -2 \\ 2x - 7y = 9 \end{array}$$

- (a)  $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$                       (b)  $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$                       (c)  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$                       (d)  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

- 44) Let  $a, b, c$  be fixed real numbers. How many solutions does the system of linear equations below have?

$$\begin{array}{l} x + 2y + 3z = a \\ y - z = b \\ y + z = c \end{array}$$

- (a) 0                      (b) 1                      (c) Infinitely many                      (d) It depends on the values of  $a, b$ , and  $c$ .

- 45) What is the standard matrix corresponding to the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  given by  $T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) =$

$$\begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix} ?$$

- (a)  $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$                       (b)  $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$                       (c)  $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$                       (d)  $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

46) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be the linear transformation with standard matrix  $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$ . Compute

$$T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right).$$

(a)  $\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$

(b)  $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

(c)  $\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$

(d)  $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

47) Which of the following is true of the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 3y - 4z \\ x + y \\ 3z \end{bmatrix}?$$

- (a)  $T$  is neither injective nor surjective
- (b)  $T$  is injective but not surjective
- (c)  $T$  is surjective but not injective
- (d)  $T$  is both injective and surjective

48) Which of the following is true of the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + z \end{bmatrix}?$$

- (a)  $T$  is surjective but not injective
- (b)  $T$  is injective but not surjective
- (c)  $T$  is both injective and surjective
- (d)  $T$  is neither injective nor surjective

49) Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation with standard matrix  $A$ . Which of the following is **not** a characterization of the statement “ $T$  is injective”?

- (a) If  $T(\mathbf{v}) = T(\mathbf{w})$  for some  $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ , then  $\mathbf{v} = \mathbf{w}$ .
- (b) The columns of  $A$  are linearly independent
- (c)  $T$  has a non-trivial kernel
- (d)  $\text{RREF}(A)$  has only pivot columns

50) Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation with standard matrix  $A$ . Which of the following is **not** a characterization of the statement “ $T$  is surjective”?

- (a)  $\text{RREF}(A)$  has a pivot in every row
- (b)  $\text{RREF}(A)$  has has a pivot in every column
- (c)  $\text{Im } T = \mathbb{R}^m$
- (d) The columns of  $A$  span  $\mathbb{R}^m$