MASTERY QUIZ DAY 20   Math 237 – Linear Algebra
Version 3   Fall 2017
Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

S3.   Let $W = \text{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$. Find a basis for W.

Solution:
$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 3 & 7\\1 & 3 & -1 & -1\\2 & 6 & 3 & 8\\1 & 3 & -2 & -3\end{bmatrix}\right) = \begin{bmatrix}1 & 3 & 0 & 1\\0 & 0 & 1 & 2\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

Then a basis is $\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}\right\}$.

□

S4.   Let W be the subspace of $\mathbb{R}^{2\times 2}$ given by $W = \text{span}\left(\left\{\begin{bmatrix}2 & 0\\-2 & 0\end{bmatrix}, \begin{bmatrix}3 & 1\\3 & 6\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}, \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\right\}\right)$.
Compute the dimension of W.

Solution:
$$\text{RREF}\left(\begin{bmatrix}2 & 3 & 0 & 1\\0 & 1 & 0 & 2\\-2 & 3 & 1 & 0\\0 & 6 & 1 & 1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0 & -\frac{5}{2}\\0 & 1 & 0 & 2\\0 & 0 & 1 & -11\\0 & 0 & 0 & 0\end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

□

A1.   Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by
$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix}x_1 + 3x_3\\3x_2 - x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^4$ and $\mathbb{R}^2$.

Solution:
$$\begin{bmatrix}1 & 0 & 3 & 0\\0 & 3 & -1 & 0\end{bmatrix}$$

□

A2.   Determine if $D : \mathbb{R}^{2\times 2} \to \mathbb{R}$ given by $D\left(\begin{bmatrix}a & b\\c & d\end{bmatrix}\right) = a - 3c$ is a linear transformation or not.

S3: ☐   **S4:** ☐   **A1:** ☐   **A2:** ☐