Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Calculate the area of a parallelogram.
- Find the matrix corresponding to a linear transformation of Euclidean spaces **(Standard(s) A1)**.
- Recall and use the definition of a linear transformation **(Standard(s) A2)**.
- Find all roots of quadratic polynomials (including complex ones), and be able to use the rational root theorem to find all rational roots of a higher degree polynomial.
- Interpret the statement "A is an invertible matrix" in many equivalent ways in different contexts.

Readiness Assurance Resources

The following resources will help you prepare for this module.

- Finding the area of a parallelogram: `https://www.khanacademy.org/math/basic-geo/basic-geo-area-and-perime parallelogram-area/a/area-of-parallelogram`
- Factoring quadratics: `https://www.khanacademy.org/math/algebra2/polynomial-functions/factoring-polynom v/factoring-polynomials-1`
- Finding complex roots of quadratics: `https://www.khanacademy.org/math/algebra2/polynomial-functions/quadratic-equations-with-complex-numbers/v/complex-roots-from-the-quadratic-formula`
- Finding all roots of polynomials: `https://www.khanacademy.org/math/algebra2/polynomial-functions/finding-zeros-of-polynomials/v/finding-roots-or-zeros-of-polynomial-1`
- The Rational Root Theorem: `https://artofproblemsolving.com/wiki/index.php?title=Rational_Root_Theorem`

Readiness Assurance Test

Choose the most appropriate response for each question.

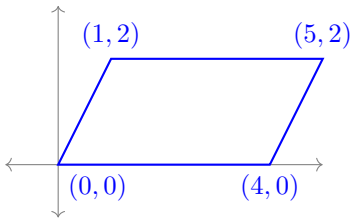
1) Find the area of the parallelogram with vertices $(0,0)$, $(4,0)$, $(5,2)$, and $(1,2)$.

(a) 8

(b) 10

(c) 12

(d) 14



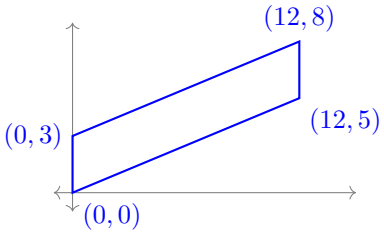
2) Find the area of the parallelogram with vertices $(0,0)$, $(12,5)$, $(12,8)$, and $(0,3)$.

(a) 36

(b) 54

(c) 72

(d) 96



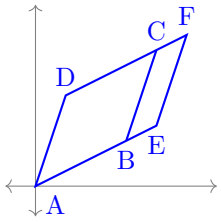
3) The parallelogram ABCD has area 6. If AE is 50% longer than AB, what is the area of the parallelogram AEFD?

(a) 18

(b) 15

(c) 12

(d) 9



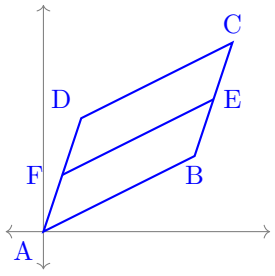
4) The parallelogram ABCD has area 6. If AD is twice as long as AF, what is the area of the parallelogram ABEF?

(a) 1

(b) 2

(c) 3

(d) 4



5) Let $T : \mathbb{R}^2 \to \mathbb{R}$ be a linear transformation. Which of the following is equal to $2T\left(\begin{bmatrix} a+b \\ a+b \end{bmatrix}\right)$?

(a) $T\left(\begin{bmatrix} a \\ a \end{bmatrix}\right) + T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) + T\left(\begin{bmatrix} b \\ a \end{bmatrix}\right) + T\left(\begin{bmatrix} b \\ b \end{bmatrix}\right)$

(b) $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) + T\left(\begin{bmatrix} b \\ a \end{bmatrix}\right)$

(c) $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right)$

(d) $2T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right)$

6) Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation with standard matrix A. Which of the following is equivalent to the statement "A is an invertible matrix"?

(a) A is a square matrix

(b) The matrix equation $AX = B$ has no solution for some $n \times 1$ matrix B.

(c) RREF(A) has a column without a pivot

(d) T is both injective and surjective

7) What is the matrix corresponding to the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y - z \\ y + z \\ x + 7z \end{bmatrix}?$$

(a) $\begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ -1 & 1 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ 1 & 7 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 7 \\ -1 & 0 & 0 \end{bmatrix}$

8) How many distinct real roots does the polynomial $x^4 + 3x^3 + x^2 - 3x - 2$ have? (Hint: all the roots are rational.)

(a) 4 (b) 3 (c) 2 (d) 1

9) Which of the following is a root of the polynomial $x^2 - 4x + 13$?

(a) $2 - 3i$ (b) $3 + 4i$ (c) $4 - 5i$ (d) $5 + 6i$

10) Which of the following conditions imply that the quadratic polynomial $ax^2 + bx + c$ has no real roots?

(a) $b^2 - 4ac < 0$

(b) $a^2 + 4bc < 0$

(c) $ac - 4b^2 < 0$

(d) $ab + 4c^2 < 0$