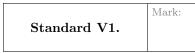
Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

Determine if V is a vector space or not.

Solution: This is not a vector space, as there is no zero vector.

Determine if the vectors $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span \mathbb{R}^4 .

Standard V4.

Let W be the set of all 2 by 2 matrices which are not invertible. Determine if W is a subspace of $M_{2,2}$.

Solution: W is closed under scalar multiplication, but not under addition.

Standard S2.
$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$
 is a basis of \mathbb{R}^3

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

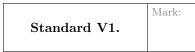
Since the resulting matrix is the identity matrix, it is a basis.

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Math 237 – Linear Algebra Fall 2017

Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

Determine if V is a vector space or not.

Solution: V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

Determine if the vectors $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$, $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$, and $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

Standard V4.

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W, but $(x - x^2) + (x^2) = x \notin W$.

Standard S2.
$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$
 is a basis of \mathbb{R}^3

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

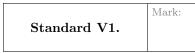
Since the resulting matrix is the identity matrix, it is a basis.

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Math 237 – Linear Algebra Fall 2017

Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

Determine if V is a vector space or not.

Solution: This is not a vector space, as there is no zero vector.

Standard V3.

Mark: $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}, \text{ and } \begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix} \text{ span } \mathbb{R}^3.$

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

Standard V4.

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W, but $(x - x^2) + (x^2) = x \notin W$.

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \P_2

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

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Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:
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Let V be the set of all points on the line x+y=2 with the operations, for any $(x_1,y_1),(x_2,y_2)\in V, c\in\mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

 $c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$

Determine if V is a vector space or not.

Solution:

- 1) Since real addition is associative, \oplus is associative.
- 2) Since real addition is commutative, \oplus is commutative.
- 3) $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so (1, 1) is an additive identity element.
- 4) $(x_1, y_1) \oplus (2 x_1, 2 y_1) = (1, 1)$, so $(2 x_1, 2 y_1)$ is the additive inverse of (x_1, y_1) .

5)

$$\begin{split} c\odot(d\odot(x_1,y_1)) &= c\odot(dx_1-(d-1),dy_1-(d-1))\\ &= (c(dx_1-(d-1))-(c-1),c(dy_1-(d-1)))\\ &= (cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &= (cdx_1-(cd-1),cdy_1-(cd-1))\\ &= (cd)\odot(x_1,y_1) \end{split}$$

6)
$$1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$\begin{split} c\odot((x_1,y_1)\oplus(x_2,y_2)) &= c\odot(x_1+y_1-1,x_2+y_2-1)\\ &= (c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &= (cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &= (cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &= c\odot(x_1,y_1)\oplus c\odot(x_2,y_2) \end{split}$$

8)

$$(c+d) \odot (x_1, y_1) = ((c+d)x_1 - (c+d-1), (c+d)y_1 - (c+d-1))$$

= $(cx_1 - (c-1), cy_1 - (c-1)) \oplus (dx_1 - (d-1), dy_1 - (d-1))$
= $c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$

Therefore V is a vector space.

Standard V3.

Determine if the vectors $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$, $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$, and $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

Standard S2.

Mark:

Determine if the set $\{x^2+x-1, 3x^2-x+1, 2x-2\}$ is a basis of \P_2

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

Additional Notes/Marks

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Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

Determine if V is a vector space or not.

Solution: V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

Standard V3. $\begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\3\\6\\3 \end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix}, \text{ and } \begin{bmatrix} 7\\-1\\8\\-3 \end{bmatrix} \text{ span } \mathbb{R}^4.$

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are zero rows, they do not span. Alternatively, by inspection $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, so the set is linearly

dependent, so it spans a subspace of dimension at most 3, therefore it does not span \mathbb{R}^4 .

Standard V4.

Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \P_2

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

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J#:

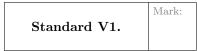
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MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$

Determine if V is a vector space or not.

Solution: V is not a vector space, as one of the distributive laws fails, namely

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

Determine if the vectors $\begin{bmatrix} 1\\0\\2\\1 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\0\\-3 \end{bmatrix}$, $\begin{bmatrix} 0\\3\\0\\-2 \end{bmatrix}$, and $\begin{bmatrix} -1\\1\\-1\\-1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 2 & 0 & 0 & -1 \\ 1 & -3 & -2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since every row contains a pivot, the vectors span \mathbb{R}^4 .

Solution: It is closed under addition and scalar multiplication, so it is a subspace. Alternatively, it is the image of the linear transformation from $\mathbb{R}^3 \to \mathbb{R}^4$ given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix}.$$

Standard S2.

Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

Additional Notes/Marks