

Name:
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Date:

Dr. Clontz

# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard E1.</b>	Mark:
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Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\x_1 - x_3 + x_4 &= 1\end{aligned}$$

**Solution:**

$$\left[ \begin{array}{cccc|c} 1 & 3 & -4 & 1 & 5 \\ 3 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 1 & 1 \end{array} \right]$$

□

<b>Standard E3.</b>	Mark:
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Solve the following linear system.

$$\begin{aligned}4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\-2x_3 - 4x_4 &= 3 \\2x_1 + 2x_2 + x_3 - 4x_4 &= -1\end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 4 & 4 & 3 & -6 & 5 \\ 0 & 0 & -2 & -4 & 3 \\ 2 & 2 & 1 & -4 & -1 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$ . It follows that the system is inconsistent with no solutions (since the bottom row implies the contradiction  $0 = 1$ ).

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<b>Standard E4.</b>	Mark:
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Find a basis for the solution set to the homogeneous system of equations given by

$$\begin{aligned}3x + 2y + z &= 0 \\x + y + z &= 0\end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right]$ . It follows that the basis for the solution set is given by  $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ .

□

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all points on the line  $x + y = 2$  with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$ ,

$$\begin{aligned} (x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2 - 1, y_1 + y_2 - 1) \\ c \odot (x_1, y_1) &= (cx_1 - (c - 1), cy_1 - (c - 2)) \end{aligned}$$

Determine if  $V$  is a vector space or not.

**Solution:**

- 1) Since real addition is associative,  $\oplus$  is associative.
- 2) Since real addition is commutative,  $\oplus$  is commutative.
- 3)  $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$ , so  $(1, 1)$  is an additive identity element.
- 4)  $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$ , so  $(2 - x_1, 2 - y_1)$  is the additive inverse of  $(x_1, y_1)$ .
- 5)

$$\begin{aligned} c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= (c(dx_1 - (d - 1)) - (c - 1), c(dy_1 - (d - 1))) \\ &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1)) \\ &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1)) \\ &= (cd) \odot (x_1, y_1) \end{aligned}$$

$$6) 1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$\begin{aligned} c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1) \\ &= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1)) \\ &= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1)) \\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2) \end{aligned}$$

8)

$$\begin{aligned} (c + d) \odot (x_1, y_1) &= ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1)) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= c \odot (x_1, y_1) \oplus d \odot (x_1, y_1) \end{aligned}$$

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<b>Additional Notes/Marks</b>	
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