

Name:
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Dr. Clontz

# MIDTERM EXAM

Math 237 – Linear Algebra

## Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard E1.</b>	Mark:
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Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

<b>Standard E2.</b>	Mark:
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Find RREF  $A$ , where

$$A = \left[ \begin{array}{cc|c} 2 & -7 & 4 \\ 1 & -3 & 2 \\ 3 & 0 & 3 \end{array} \right]$$

<b>Standard E3.</b>	Mark:
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Solve the system of linear equations.

$$2x + y - z + w = 5$$

$$3x - y - 2w = 0$$

$$-x + 5z + 3w = -1$$

<b>Standard E4.</b>	Mark:
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Find a basis for the solution set to the homogeneous system of equations

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 0$$

$$-2x_3 - 4x_4 = 0$$

$$2x_1 + 2x_2 + x_3 - 4x_4 = 0$$

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

- (a) Show that the vector **addition**  $\oplus$  is **associative**:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .
- (b) Determine if  $V$  is a vector space or not. Justify your answer.

<b>Standard V2.</b>	Mark:
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Determine if  $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 6 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ .

<b>Standard V3.</b>	Mark:
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Determine if the vectors  $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

<b>Standard V4.</b>	Mark:
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Let  $W$  be the set of all complex numbers that are purely real (i.e of the form  $a + 0i$ ) or purely imaginary (i.e. of the form  $0 + bi$ ). Determine if  $W$  is a subspace of  $\mathbb{C}$ .

<b>Standard S1.</b>	Mark:
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Determine if the set of vectors  $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$  is linearly dependent or linearly independent

<b>Standard S2.</b>	Mark:
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Determine if the set  $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^4$ .

<b>Standard S3.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

<b>Standard S4.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

<b>Additional Notes/Marks</b>	
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