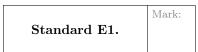
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra Fall 2017

Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_4 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -1 & 0 & 7 \\ 1 & -1 & 0 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Find the solution set for the following system of linear equations.

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 8$$
$$x_1 + x_2 - x_3 + 5x_4 = 3$$

Solution: Let $A = \begin{bmatrix} 2 & 3 & -5 & 14 & 8 \\ 1 & 1 & -1 & 5 & 3 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -3 & 4 & 2 \end{bmatrix}$. It follows that the solution set is given by $\begin{bmatrix} 2-2a-b \\ 2+3a-4b \\ a \\ b \end{bmatrix}$ for all real numbers a,b.

Standard E4.

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$

Solution:

RREF
$$\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{5}{7} & \frac{3}{7} \\ 0 & 1 & \frac{8}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\frac{5}{7}a - \frac{3}{7}b \\ -\frac{8}{7}a - \frac{2}{7}b \\ a \\ b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} -\frac{5}{7} \\ -\frac{8}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{3}{7} \\ -\frac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$, or $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all points on the line x + y = 2 with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

 $c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$

Determine if V is a vector space or not.

Solution:

- 1) Since real addition is associative, \oplus is associative.
- 2) Since real addition is commutative, \oplus is commutative.
- 3) $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so (1, 1) is an additive identity element.
- 4) $(x_1, y_1) \oplus (2 x_1, 2 y_1) = (1, 1)$, so $(2 x_1, 2 y_1)$ is the additive inverse of (x_1, y_1) .

5)

$$\begin{split} c\odot(d\odot(x_1,y_1)) &= c\odot(dx_1-(d-1),dy_1-(d-1))\\ &= (c(dx_1-(d-1))-(c-1),c(dy_1-(d-1)))\\ &= (cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &= (cdx_1-(cd-1),cdy_1-(cd-1))\\ &= (cd)\odot(x_1,y_1) \end{split}$$

6)
$$1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

$$\begin{split} c\odot((x_1,y_1)\oplus(x_2,y_2))&=c\odot(x_1+y_1-1,x_2+y_2-1)\\ &=(c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &=(cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &=(cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &=c\odot(x_1,y_1)\oplus c\odot(x_2,y_2) \end{split}$$

$$(c+d)\odot(x_1,y_1) = ((c+d)x_1 - (c+d-1), (c+d)y_1 - (c+d-1))$$
$$= (cx_1 - (c-1), cy_1 - (c-1)) \oplus (dx_1 - (d-1), dy_1 - (d-1))$$
$$= c\odot(x_1, y_1) \oplus c\odot(x_2, y_2)$$

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