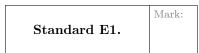
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_4 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -1 & 0 & 7 \\ 1 & -1 & 0 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{4} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{bmatrix}$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a\\3-21a\\-7a\\12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

Standard E4.

Mark:

Find the solution set for the following system of linear equations.

$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$
$$3x_1 + 6x_3 + x_4 = 0$$
$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

Solution: Let
$$A = \begin{bmatrix} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{bmatrix}$$
, so RREF $A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that this scalar multiplication \odot is associative.
- (b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$

$$= c (dx - 3(d - 1)) - 3(c - 1)$$

$$= cdx - 3(cd - 1)$$

$$= (cd) \odot x$$

We verify the remaining 7 properties to see that V is a vector space.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 3 = x$, so 3 is the additive identity.
- 3) $x \oplus (6-x) = x + (6-x) 3 = 3$, so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so \oplus is commutative.
- 5) Associativity shown above

6)
$$1 \odot x = x - 3(1 - 1) = x$$

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$

= $cx - 3(c-1) + dx - 3(c-1) - 3$
= $(c \odot x) \oplus (d \odot x)$

Therefore V is a vector space.

 ${\bf Additional\ Notes/Marks}$