Name:	
J#:	Dr. Clontz
Date:	

Math 237 – Linear Algebra Fall 2017

#### Version 1

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

# Determine if $\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 4 \\ 3 \end{bmatrix} \right\}$ .

Solution: Since

$$RREF \left( \begin{bmatrix} 2 & 4 & | & 4 \\ 0 & -1 & | & -1 \\ -1 & 4 & | & 6 \\ 5 & 3 & | & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 1 \\ 0 & 0 & | & 0 \end{bmatrix}$$

contains the contradiction 0 = 1,  $\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$  is not a linear combination of the three vectors.

Standard S1.

Determine if the set of polynomials  $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$  is linearly dependent or linearly independent

Solution:

RREF 
$$\left( \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Standard S3.

Mark:

Let W be the subspace of  $\mathcal{P}_2$  given by  $W = \text{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$ . Find a basis for W.

Solution: Let 
$$A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$
, and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since the first two columns are pivot columns,  $\left\{ -3x^2 - 8x, x^2 + 2x + 2 \right\}$  is a basis for  $W$ .

Standard S4.  $\begin{bmatrix} & & & \\ & & & \\ & & \\ & & -2 \\ 0 & & 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}. Compute the dimension of <math>W$ .

Solution:

 $RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$ 

This has 3 pivot columns so dim(W) = 3.

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Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

## Determine if $\begin{bmatrix} 0 \\ -1 \\ 6 \\ -7 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 4 \\ 3 \end{bmatrix} \right\}.$

Solution: Since

RREF 
$$\begin{pmatrix} \begin{bmatrix} 2 & 4 & 0 \\ 0 & -1 & -1 \\ -1 & 4 & 6 \\ 5 & 3 & -7 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

does not contain a contradiction,  $\begin{bmatrix} 0\\-1\\6\\-7 \end{bmatrix}$  is a linear combination of the three vectors.

Standard S1.

Mark:

Determine if the set of polynomials  $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$  is linearly dependent or linearly independent

**Solution:** 

RREF 
$$\left( \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Standard S3.

$$\begin{bmatrix}
Mark: \\
Mark: \\
\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}$$
Let  $W = \text{span}\left(\left\{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}\right\}\right)$ . Find a basis for  $W$ .

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is  $\left\{ \begin{bmatrix} 1\\1\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\3\\-2 \end{bmatrix} \right\}$ .

Standard S4.

Mark:

Let W be the subspace of  $\mathcal{P}_3$  given by  $W = \mathrm{span}\left(\left\{x^3-x^2+3x-3,2x^3+x+1,3x^3-x^2+4x-2,x^3+x^2+x-7\right\}\right)$ . Compute the dimension of W.

Solution:

$$RREF \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

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Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standar	d V2.	Mark:							
Determine if	$\begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix} $ can	be writte	en as a linear combination of the vectors	$\begin{bmatrix} 5 \\ 2 \\ -3 \\ 2 \end{bmatrix}$	,	$\begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$	, and	$\begin{bmatrix} 8 \\ 3 \\ 5 \\ -1 \end{bmatrix}$	

Solution:

$$RREF \left( \begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

 $\text{RREF}\left(\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  The system has no solution, so  $\begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}$  is not a linear combination of the three other vectors.

Mark: Standard S1.

Determine if the set of polynomials  $\{x^3 - 8x, x^3 + 2x^2 + 2, -x^2 + 3\}$  is linearly dependent or linearly independent

Solution:

$$RREF\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Standard S3.

$$\begin{bmatrix}
\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \\
\end{bmatrix}$$
Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ . Find a basis of  $W$ .

RREF 
$$\left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then 
$$\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$$
 is a basis of  $W$ .

Standard S4.

Mark:

Let W be the subspace of  $\mathcal{P}_3$  given by  $W = \text{span}\left(\left\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\right\}\right)$ . Compute the dimension of W.

**Solution:** 

$$RREF \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

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#### MASTERY QUIZ DAY 15

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V2.	Mark:

Determine if  $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$  is a linear combination of the vectors  $\begin{bmatrix} 3\\0\\-1 \end{bmatrix}$ ,  $\begin{bmatrix} 1\\-1\\4 \end{bmatrix}$ , and  $\begin{bmatrix} 5\\1\\-6 \end{bmatrix}$ .

Solution:

RREF 
$$\left( \begin{bmatrix} 3 & 1 & 5 & 1 \\ 0 & -1 & 1 & 4 \\ -1 & 4 & -6 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So  $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$  is not a linear combination of the three vectors.

Standard S1.

Mark:

Determine if the set of polynomials  $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$  is linearly dependent or linearly independent

**Solution:** 

RREF 
$$\left( \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

Standard S3.  $\begin{bmatrix}
\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \\
\end{bmatrix}$ Let  $W = \operatorname{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Find a basis of W.

RREF 
$$\begin{pmatrix} \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then 
$$\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$$
 is a basis of  $W$ .

Standard S4.

Mark:

Let  $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix},\begin{bmatrix}3\\1\\3\\6\end{bmatrix},\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$ . Compute the dimension of W.

Solution:

RREF 
$$\left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

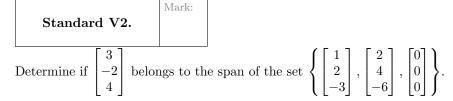
This has 3 pivot columns so  $\dim(W) = 3$ .

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Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Solution: Since

$$RREF\left(\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & -2 \\ -3 & -6 & 0 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

contains the contradiction 0 = 1,  $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$  is not a linear combination of the three vectors.

Standard S1.

Mark:

Determine if the set of vectors  $\left\{ \begin{bmatrix} -3\\8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix}, \begin{bmatrix} 0\\-1\\3 \end{bmatrix} \right\}$  is linearly dependent or linearly independent

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Standard S3.

Let W be the subspace of  $\mathcal{P}_2$  given by  $W = \text{span}\left(\left\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\right\}\right)$ . Find a basis for W.

Solution: Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since the first two columns are pivot columns,  $\left\{ -3x^2 - 8x, x^2 + 2x + 2 \right\}$  is a basis for W.

Standard S4.

Mark:

Let W be the subspace of  $M_{2,2}$  given by  $W = \operatorname{span}\left(\left\{\begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right\}\right)$ . Compute the dimension of W.

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

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#### MASTERY QUIZ DAY 15

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

### Standard V2.

Determine if  $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$  is a linear combination of the vectors  $\begin{bmatrix} 2\\3\\-1 \end{bmatrix}$ ,  $\begin{bmatrix} 1\\-1\\0 \end{bmatrix}$ , and  $\begin{bmatrix} -3\\-2\\5 \end{bmatrix}$ .

Solution:

RREF 
$$\left( \begin{bmatrix} 2 & 1 & -3 & 1 \\ 3 & -1 & -2 & 4 \\ -1 & 0 & 5 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Since this system has a solution,  $\begin{bmatrix} 1\\4\\3 \end{bmatrix}$  is a linear combination of the three vectors.

Standard S1.

Determine if the set of vectors  $\left\{ \begin{bmatrix} -3\\8\\0 \end{bmatrix}, \begin{bmatrix} 1\\2\\2 \end{bmatrix}, \begin{bmatrix} 0\\-1\\3 \end{bmatrix} \right\}$  is linearly dependent or linearly independent

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Mark:

Standard S3.  $\begin{bmatrix} & & & & \\ & &$ 

RREF 
$$\begin{pmatrix} \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then 
$$\left\{ \begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix} \right\}$$
 is a basis of  $W$ .

Standard S4.

Mark:

Let  $W = \operatorname{span}\left(\left\{\begin{bmatrix}2\\0\\-2\\0\end{bmatrix},\begin{bmatrix}3\\1\\3\\6\end{bmatrix},\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\right\}\right)$ . Compute the dimension of W.

Solution:

RREF 
$$\left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .