

Name:
J#:
Date:

Dr. Clontz

MIDTERM EXAM

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
---------------------	-------

Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array} \right]$$

Standard E2.	Mark:
---------------------	-------

Find RREF A , where

$$A = \left[\begin{array}{ccc|c} 2 & -1 & 5 & 4 \\ -1 & 0 & -2 & -1 \\ 1 & 3 & -1 & -5 \end{array} \right]$$

Standard E3.	Mark:
---------------------	-------

Solve the system of linear equations.

$$2x + y - z + w = 5$$

$$3x - y - 2w = 0$$

$$-x + 5z + 3w = -1$$

Standard E4.	Mark:
---------------------	-------

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

Standard V1.	Mark:
---------------------	-------

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$$

(a) Show that scalar multiplication **distributes scalars** over vector addition:

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

(b) Determine if V is a vector space or not. Justify your answer.

Standard V2.	Mark:
---------------------	-------

Determine if $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Standard V3.	Mark:
---------------------	-------

Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Standard V4.	Mark:
---------------------	-------

Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x + y + z = 0$ (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

Standard S1.	Mark:
---------------------	-------

Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Standard S2.	Mark:
---------------------	-------

Determine if the set $\left\{ \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 3 & 9 \end{bmatrix} \right\}$ is a basis of $M_{2,2}$ or not.

Standard S3.	Mark:
---------------------	-------

Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find a basis for W .

Standard S4.	Mark:
---------------------	-------

Let W be the subspace of \mathcal{P}_3 given by $W = \text{span} \left(\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\} \right)$. Compute the dimension of W .

Additional Notes/Marks	
-------------------------------	--