Name:	
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Dr. Clontz

MASTERY QUIZ DAY 19 Math 237 – Linear Algebra
Version 3 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix}1 & -3\\2 & 2\end{bmatrix}, \begin{bmatrix}-1 & 4\\-1 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\2 & 1\end{bmatrix}, \begin{bmatrix}0 & 3\\3 & 9\end{bmatrix}\right\}$ is a basis of $M_{2,2}$ or not.

Solution:

$$\text{RREF}\begin{bmatrix}1 & -1 & -1 & 0\\-3 & 4 & 0 & 3\\2 & -1 & 2 & 3\\2 & 1 & 1 & 9\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 3\\0 & 1 & 0 & 3\\0 & 0 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

Standard A3.	Mark:

Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the matrix $\begin{bmatrix}2 & 3 & -1 & -2\\0 & 1 & 3 & 1\\2 & 1 & -7 & -4\end{bmatrix}$

Solution:

(a) RREF $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$. Since each column is a pivot column, S is injective. Since there is no zero row, S is surjective.

(b) Since $\dim \mathbb{R}^4 > \dim \mathbb{R}^3$, T is not injective.

$$\text{RREF}\left(\begin{bmatrix}2 & 3 & -1 & -2\\0 & 1 & 3 & 1\\2 & 1 & -7 & -4\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & -5 & -\frac{5}{2}\\0 & 1 & 3 & 1\\0 & 0 & 0 & 0\end{bmatrix}$$

Since there are only two pivot columns, T is not surjective.

□

Standard A4.	Mark:

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+3y+3z+7w \\ x+3y-z-w \\ 2x+6y+3z+8w \\ x+3y-2z-3w \end{bmatrix}$$

Compute the kernel and image of T.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the kernel is

$$\ker(T) = \left\{ \begin{bmatrix} -3a-b \\ a \\ -2b \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\} = \operatorname{span}\left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}\right)$$

and the image is

$$\mathrm{Im}(T) = \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\}\right) = \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}\right)$$

$\square$

Additional Notes/Marks	