

Name:
J#:
Date:

Dr. Clontz

MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine which of the six products AB , AC , BA , BC , CA , CB can be computed, and compute them.

Solution: AB and CA are the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

$$CA = \begin{bmatrix} 3 & 9 & 11 \\ 0 & 0 & 7 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ is invertible.

Solution: This matrix is row equivalent to the identity matrix, so it is invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}.$

□

Standard G2.	Mark:
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Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda)) \\ &= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 - \lambda \\ &= -\lambda(\lambda^2 - 2\lambda + 1) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}.$

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$.

□

Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Determine which of the six products AB , AC , BA , BC , CA , CB can be computed, and compute them.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since it is not row equivalent to the identity matrix, it is not invertible.

□

Standard M3.	Mark:
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Compute the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF}(A|I) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -11 & 37 \\ 0 & 1 & 0 & 0 & 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -11 & 37 \\ 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

□

Standard G2.	Mark:
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Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix} \\ &= (8 - \lambda) (\lambda^2 + 6\lambda - 7) + 3(-21\lambda + 21) - (-7\lambda + 7) \\ &= (\lambda - 1)((8 - \lambda)(\lambda + 7) - 63 + 7) \\ &= (\lambda - 1)(\lambda - \lambda^2) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$.

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Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Determine which of the six products AB , AC , BA , BC , CA , CB can be computed, and compute them.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

□

Standard M2.	Mark:
---------------------	-------

Determine if the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$ is invertible.

Solution:

$$\det \begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix} = -1 \det \begin{bmatrix} 2 & 7 \\ -1 & -1 \end{bmatrix} + 5 \det \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = -1(5) + 5(1) = 0$$

So it is not invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}$.

□

Standard G2.	Mark:
---------------------	-------

Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} -3 - \lambda & 1 & 0 \\ -8 & 2 - \lambda & -1 \\ 0 & 2 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) \det \begin{bmatrix} 2 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3 - \lambda \end{bmatrix} \\ &= (-3 - \lambda) ((2 - \lambda)(3 - \lambda) + 2) - (-8(3 - \lambda)) \\ &= (-3 - \lambda)(8 - 5\lambda + \lambda^2) + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 - \lambda \\ &= -\lambda(\lambda^2 - 2\lambda + 1) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

Solution: The eigenspace is the solution space of the system $(B - 2I)X = 0$.

$$\text{RREF}(B - 2I) = \text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \frac{y}{3} = 0$, or $3x = y$. Thus the eigenspace is

$$E_2 = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

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Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
---------------------	-------

Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine which of the six products AB , AC , BA , BC , CA , CB can be computed, and compute them.

Solution: AB and CA are the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

$$CA = \begin{bmatrix} 3 & 9 & 11 \\ 0 & 0 & 7 \end{bmatrix}$$

□

Standard M2.	Mark:
---------------------	-------

Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since it is not row equivalent to the identity matrix, it is not invertible.

□

Standard M3.	Mark:
---------------------	-------

Compute the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF}(A|I) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -11 & 37 \\ 0 & 1 & 0 & 0 & 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -11 & 37 \\ 0 & -1 & 4 & -14 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

□

Standard G2.	Mark:
---------------------	-------

Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$. List the eigenvalues of A along with their algebraic multiplicities.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} -3-\lambda & 1 & 0 \\ -8 & 2-\lambda & -1 \\ 0 & 2 & 3-\lambda \end{bmatrix} \\ &= (-3-\lambda) \det \begin{bmatrix} 2-\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} - (1) \det \begin{bmatrix} -8 & -1 \\ 0 & 3-\lambda \end{bmatrix} \\ &= (-3-\lambda) ((2-\lambda)(3-\lambda) + 2) - (-8(3-\lambda)) \\ &= (-3-\lambda)(8-5\lambda+\lambda^2) + 24-8\lambda \\ &= -\lambda^3 + 2\lambda^2 + 7\lambda - 24 + 24 - 8\lambda \\ &= -\lambda^3 + 2\lambda^2 - \lambda \\ &= -\lambda(\lambda^2 - 2\lambda + 1) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So A has eigenvalues 0 (with multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$.

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Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$C = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & 0 \\ 0 & -1 \\ 1 & -1 \end{bmatrix}$$

Determine which of the six products CD , CE , DC , DE , EC , ED can be computed, and compute them.

Solution:

$$EC = \begin{bmatrix} 4 & 6 \\ 0 & -1 \\ 2 & 2 \end{bmatrix}$$

$$DE = \begin{bmatrix} 6 & -1 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} 3 & -1 & 0 & 4 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & -2 & 0 & 0 \end{bmatrix}$ is invertible.

Solution: This matrix is row equivalent to the identity matrix, so it is invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}$.

□

Standard G2.	Mark:
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Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix} \\ &= (8 - \lambda) (\lambda^2 + 6\lambda - 7) + 3(-21\lambda + 21) - (-7\lambda + 7) \\ &= (\lambda - 1)((8 - \lambda)(\lambda + 7) - 63 + 7) \\ &= (\lambda - 1)(\lambda - \lambda^2) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
---------------------	-------

Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$.

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Additional Notes/Marks	
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MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
---------------------	-------

Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine which of the six products AB , AC , BA , BC , CA , CB can be computed, and compute them.

Solution: AB and CA are the only ones that can be computed, and

$$AB = \begin{bmatrix} -3 & -5 & 6 & 14 \\ 0 & 0 & 7 & 35 \end{bmatrix}$$

$$CA = \begin{bmatrix} 3 & 9 & 11 \\ 0 & 0 & 7 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ is invertible.

Solution: The determinant is 0, so it is not invertible.

□

Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

Solution:

$$\text{RREF} \left(\left[\begin{array}{cccc|cccc} 8 & 5 & 3 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 \\ 5 & -3 & 1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & -5 & 12 \\ 0 & 1 & 0 & 0 & 1 & 1 & -4 & -9 \\ 0 & 0 & 1 & 0 & -4 & -7 & 20 & 47 \\ 0 & 0 & 0 & 1 & -1 & 0 & 3 & 7 \end{array} \right]$$

So the inverse is $\begin{bmatrix} 1 & 2 & -5 & 12 \\ 1 & 1 & -4 & -9 \\ -4 & -7 & 20 & 47 \\ -1 & 0 & 3 & 7 \end{bmatrix}.$

□

Standard G2.	Mark:
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Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}.$

Solution:

$$\begin{aligned} \det(A - \lambda I) &= (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix} \\ &= (8 - \lambda) (\lambda^2 + 6\lambda - 7) + 3(-21\lambda + 21) - (-7\lambda + 7) \\ &= (\lambda - 1) ((8 - \lambda)(\lambda + 7) - 63 + 7) \\ &= (\lambda - 1)(\lambda - \lambda^2) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

□

Standard G3.	Mark:
---------------------	-------

Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}.$

Solution: The eigenspace is the solution space of the system $(B - 2I)X = 0.$

$$\text{RREF}(B - 2I) = \text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to $x - \frac{y}{3} = 0$, or $3x = y$. Thus the eigenspace is

$$E_2 = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$$

□

Additional Notes/Marks	
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