Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14   Math 237 – Linear Algebra
Version 5   Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard V1.	

Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$
\begin{aligned}
(x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\
&= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\
&= \sqrt{x^2 + y^2 + z^2} \\
&= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\
&= x \oplus \sqrt{y^2 + z^2} \\
&= x \oplus (y \oplus z)
\end{aligned}
$$

However, this is not a vector space, as there is no zero vector.

□

	Mark:
Standard V3.	

Does span $\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

Solution: Since

$$
\text{RREF} \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
$$

lacks a zero row, the vectors span $\mathbb{R}^3$.

□

	Mark:
Standard V4.	

Let W be the set of all polynomials of the form $ax^3 + bx$. Determine if W is a subspace of $\mathcal{P}^3$.

Solution: Yes because $s(a_1 x^3 + b_1 x) + t(a_2 x^3 + b_2 x) = (sa_1 + ta_2)x^3 + (sb_1 + tb_2)x$ also belongs to W. Alternately, yes because W is isomorphic to $\mathbb{R}^2$.

$\square$

	Mark:
Standard S2.	

Determine if the set $\left\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\right\}$ is a basis of $\mathcal{P}_3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

$\square$

Additional Notes/Marks	