

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 23**

Math 237 – Linear Algebra

**Version 1**

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**A3.** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a)  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  given by the standard matrix  $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ .

(b)  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  given by the standard matrix  $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 4 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

**Solution:**

(a)  $\text{RREF} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ . Since each column is a pivot column,  $S$  is injective. Since there is no zero row,  $S$  is surjective.

(b) Since  $\dim \mathbb{R}^4 > \dim \mathbb{R}^3$ ,  $T$  is not injective.

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 4 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Since there are no zero rows,  $T$  is surjective.

□

**A4.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear map given by  $T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 8x - 3y - z \\ y + 3z \\ -7x + 3y + 2z \end{bmatrix}$ . Compute a basis for the kernel and a basis for the image of  $T$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 8 & -3 & -1 \\ 0 & 1 & 3 \\ -7 & 3 & 2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus  $\left\{ \begin{bmatrix} 8 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix} \right\}$  is a basis for the image, and  $\left\{ \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} \right\}$  is a basis for the kernel.

□

**A3:**

**A4:**