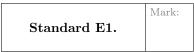
Name:	
J#:	Dr. Clontz
Date:	

### MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

### Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write an augmented matrix corresponding to the following system of linear equations.

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

Solution:

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Standard E3.

Mark:

Solve the system of linear equations.

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

RREF 
$$\left( \begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{4} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{bmatrix}$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a\\ 3-21a\\ -7a\\ 12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

# Standard E4.

Mark:

Find a basis for the solution set to the homogeneous system of equations given by

$$3x + 2y + z = 0$$
$$x + y + z = 0$$

**Solution:** Let  $A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ , so RREF  $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$ . It follows that the basis for the solution set is given by  $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ .

## Standard V1.

Mark:

Let V be the set of all points on the line x + y = 2 with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$
  
 $c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$ 

Determine if V is a vector space or not.

#### Solution:

- 1) Since real addition is associative,  $\oplus$  is associative.
- 2) Since real addition is commutative,  $\oplus$  is commutative.
- 3)  $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$ , so (1, 1) is an additive identity element.
- 4)  $(x_1, y_1) \oplus (2 x_1, 2 y_1) = (1, 1)$ , so  $(2 x_1, 2 y_1)$  is the additive inverse of  $(x_1, y_1)$ .

5)

$$\begin{split} c\odot(d\odot(x_1,y_1)) &= c\odot(dx_1-(d-1),dy_1-(d-1))\\ &= (c\left(dx_1-(d-1)\right)-(c-1),c\left(dy_1-(d-1)\right))\\ &= (cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &= (cdx_1-(cd-1),cdy_1-(cd-1))\\ &= (cd)\odot(x_1,y_1) \end{split}$$

6) 
$$1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$\begin{split} c\odot((x_1,y_1)\oplus(x_2,y_2)) &= c\odot(x_1+y_1-1,x_2+y_2-1)\\ &= (c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &= (cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &= (cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &= c\odot(x_1,y_1)\oplus c\odot(x_2,y_2) \end{split}$$

$$(c+d)\odot(x_1,y_1) = ((c+d)x_1 - (c+d-1), (c+d)y_1 - (c+d-1))$$
  
=  $(cx_1 - (c-1), cy_1 - (c-1)) \oplus (dx_1 - (d-1), dy_1 - (d-1))$   
=  $c\odot(x_1, y_1) \oplus c\odot(x_2, y_2)$ 

Therefore V is a vector space.