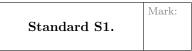
Name:	
J#:	Dr. Clontz
Date:	

## **MASTERY QUIZ DAY 18**

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Determine if the set of polynomials  $\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\}$  is linearly dependent or linearly independent

## Solution:

$$RREF\left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Mark: Standard S3.

Let W be the subspace of  $\mathcal{P}_3$  given by  $W = \text{span} \left( \left\{ x^3 + x^2 + 2x + 1, 3x^3 + 3x^2 + 6x + 3, 3x^3 - x^2 + 3x - 2, 7x^3 - x^2 + 8x - 3 \right\} \right)$ . Find a basis for W.

**Solution:** 

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is  $\{x^3 + x^2 + 2x + 1, 3x^3 - x^2 + 3x - 2\}$ .

Standard S4.

Let W be the subspace of  $M_{2,2}$  given by  $W = \operatorname{span}\left(\left\{\begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right\}\right)$ . Compute the dimension of W.

Solution:

RREF 
$$\begin{pmatrix} \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so dim(W) = 3.

Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}^4$  be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Standard A2.

Mark:

Determine if the map  $T: \mathcal{P}^3 \to \mathcal{P}^4$  given by T(f(x)) = xf(x) - f(x) is a linear transformation or not.

Additional Notes/Marks