

Name: _____

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 0$$

$$x - z = 1$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{array} \right]$$

□

E3. Solve the following linear system.

$$3x + 2y + z = 7$$

$$x + y + z = 1$$

$$-2x + 3z = -11$$

Solution: Let $A = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 7 \\ 1 & 1 & 1 & 1 \\ -2 & 0 & 3 & 11 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array} \right]$. It follows that the system has exactly one solution: $\begin{bmatrix} 4 & -2 & -1 \end{bmatrix}$

□

E4. Find the solution set for the following system of linear equations.

$$2x_1 - 2x_2 + 6x_3 - x_4 = 0$$

$$3x_1 + 6x_3 + x_4 = 0$$

$$-4x_1 + x_2 - 9x_3 + 2x_4 = 0$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$. It follows that the basis for the solution set is given by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

□

V1. Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} f \oplus g &= f' + g' \\ c \odot f &= cf' \end{aligned}$$

(here f' denotes the derivative of f). Determine if V is a vector space or not.

Solution: This is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f .

□

E1:

E3:

E4:

V1:

E2: