

Name:
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Dr. Clontz

MASTERY QUIZ DAY 28

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard M1.	Mark:
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Let

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

Solution: AC is the only one that can be computed, and

$$AC = \begin{bmatrix} 9 & -2 & 14 \\ 1 & 0 & 2 \end{bmatrix}$$

□

Standard M2.	Mark:
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Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since it is row equivalent to the identity matrix, it is invertible.

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Standard M3.	Mark:
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Find the inverse of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & \frac{2}{3} & -\frac{2}{3} \\ 1 & -\frac{2}{2} & \frac{5}{2} \end{bmatrix}$$

□

Standard G2.	Mark:
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Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

Solution:

$$\begin{aligned} \det(A - \lambda I) &= (8 - \lambda) \det \begin{bmatrix} -8 - \lambda & -3 \\ 3 & 2 - \lambda \end{bmatrix} - (-3) \det \begin{bmatrix} 21 & -3 \\ -7 & 2 - \lambda \end{bmatrix} + (-1) \det \begin{bmatrix} 21 & -8 - \lambda \\ -7 & 3 \end{bmatrix} \\ &= (8 - \lambda) (\lambda^2 + 6\lambda - 7) + 3(-21\lambda + 21) - (-7\lambda + 7) \\ &= (\lambda - 1)((8 - \lambda)(\lambda + 7) - 63 + 7) \\ &= (\lambda - 1)(\lambda - \lambda^2) \\ &= -\lambda(\lambda - 1)^2 \end{aligned}$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

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Standard G3.	Mark:
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Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$.

Solution:

$$\text{RREF}(A + I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$.

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Additional Notes/Marks	
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