

Name:
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Date:

Dr. Clontz

# MASTERY QUIZ DAY 8

Math 237 – Linear Algebra

## Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard E1.</b>	Mark:
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Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\x_1 - x_3 + x_4 &= 1\end{aligned}$$

**Solution:**

$$\left[ \begin{array}{cccc|c} 1 & 3 & -4 & 1 & 5 \\ 3 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 1 & 1 \end{array} \right]$$

□

<b>Standard E3.</b>	Mark:
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Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\3x_1 + 6x_3 + x_4 &= 5 \\-4x_1 + x_2 - 9x_3 + 2x_4 &= -7\end{aligned}$$

**Solution:** Let  $A = \left[ \begin{array}{cccc|c} 2 & -2 & 6 & -1 & -1 \\ 3 & 0 & 6 & 1 & 5 \\ -4 & 1 & -9 & 2 & -7 \end{array} \right]$ , so  $\text{RREF } A = \left[ \begin{array}{cccc|c} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right]$ . It follows that the

solution set is given by  $\begin{bmatrix} 2-2a \\ 3+a \\ a \\ -1 \end{bmatrix}$  for all real numbers  $a$ .

□

<b>Standard E4.</b>	Mark:
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Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$

$$2x + y - 4z = 0$$

$$3y - 2z = 0$$

$$x - y - z = 0$$

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \frac{5}{3}a \\ \frac{2}{3}a \\ \frac{2}{3}a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

$$\text{So a basis is } \left\{ \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} \right\} \text{ or } \left\{ \begin{bmatrix} 5 \\ 2 \\ 2 \\ 3 \end{bmatrix} \right\}.$$

□

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all points on the line  $x + y = 2$  with the operations, for any  $(x_1, y_1), (x_2, y_2) \in V$ ,  $c \in \mathbb{R}$ ,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

$$c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$$

(a) Show that this vector space has an additive identity element.

(b) Determine if  $V$  is a vector space or not. Justify your answer.

**Solution:** Let  $(x_1, y_1) \in V$ ; then  $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$ , so  $(1, 1)$  is an additive identity element.

Now we will show the other seven properties. Let  $(x_1, y_1), (x_2, y_2) \in V$ , and let  $c, d \in \mathbb{R}$ .

1) Since real addition is associative,  $\oplus$  is associative.

2) Since real addition is commutative,  $\oplus$  is commutative.

3) The additive identity is  $(1, 1)$ .

4)  $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$ , so  $(2 - x_1, 2 - y_1)$  is the additive inverse of  $(x_1, y_1)$ .

5)

$$\begin{aligned}
 c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1)) \\
 &= (c(dx_1 - (d - 1)) - (c - 1), c(dy_1 - (d - 1))) \\
 &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1)) \\
 &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1)) \\
 &= (cd) \odot (x_1, y_1)
 \end{aligned}$$

6)  $1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$

7)

$$\begin{aligned}
 c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1) \\
 &= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1)) \\
 &= (cx_1 + cx_2 - 2c + 1, cy_1 + cy_2 - 2c + 1) \\
 &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1)) \\
 &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)
 \end{aligned}$$

8)

$$\begin{aligned}
 (c + d) \odot (x_1, y_1) &= ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1)) \\
 &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1)) \\
 &= c \odot (x_1, y_1) \oplus d \odot (x_1, y_1)
 \end{aligned}$$

Therefore  $V$  is a vector space.

□

Additional Notes/Marks	
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