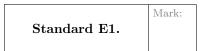
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{bmatrix}$$

Solution:

$$x_1 + 4x_3 = 1$$
$$x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -1$$

Standard E3.

Mark:

Solve the following linear system.

$$3x + 2y + z = 7$$
$$x + y + z = 1$$
$$-2x + 3z = -11$$

Solution: Let $A = \begin{bmatrix} 3 & 2 & 1 & 7 \\ 1 & 1 & 1 & 1 \\ -2 & 0 & 3 & 11 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$. It follows that the system has exactly one solution: $\begin{bmatrix} 4 & -2 & -1 \end{bmatrix}$

Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$
$$2x + y - 4z = 0$$
$$3y - 2z = 0$$
$$x - y - z = 0$$

Solution:

$$RREF \left(\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \frac{5}{3}a\\ \frac{2}{3}a\\ \frac{1}{3}a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

So a basis is
$$\left\{ \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} \right\}$$
 or $\left\{ \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$

Determine if V is a vector space or not.

Solution: V is not a vector space, as one of the distributive laws fails, namely

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

Additional Notes/Marks