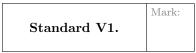
Name:	
J#:	Dr. Clontz
Date:	

## MASTERY QUIZ DAY 12

Math 237 – Linear Algebra Fall 2017

## Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that this scalar multiplication  $\odot$  is associative.
- (b) Determine if V is a vector space or not. Justify your answer

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ . To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$
$$= c (dx - 3(d - 1)) - 3(c - 1)$$
$$= cdx - 3(cd - 1)$$
$$= (cd) \odot x$$

We verify the remaining 7 properties to see that V is a vector space.

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6-x) = x + (6-x) 3 = 3$ , so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so  $\oplus$  is commutative.
- 5) Associativity shown above
- 6)  $1 \odot x = x 3(1 1) = x$

7)

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$
  
=  $cx - 3(c-1) + dx - 3(c-1) - 3$   
=  $(c \odot x) \oplus (d \odot x)$ 

Therefore V is a vector space.

Standard V3.  $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} = \mathbb{R}^3?$ 

Solution: Since

RREF 
$$\begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 2 & 2 \\ 4 & -9 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

lacks a zero row, the vectors span  $\mathbb{R}^3$ .

Standard V4.

Determine if the set of all lattice points, i.e.  $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

Additional Notes/Marks