Section E.2

Activity E.21 *(∼15 min)* Recall that a matrix is in **reduced row echelon form** (**RREF**) if

1. The leading term (first nonzero term) of each nonzero row is a 1. Call these terms **pivots**.
2. Each pivot is to the right of every higher pivot.
3. Each term above or below a pivot is zero.
4. All rows of zeroes are at the bottom of the matrix.

(A)
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(C)
$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

(E)
$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 7 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(B)
$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(D)
$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(F)
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

For each matrix, circle the leading terms, and label it as RREF or not RREF. For the ones not in RREF, find their RREF.

Remark E.22 In practice, if we simply need to convert a matrix into reduced row echelon form, we use technology to do so.
However, it is also important to understand the **Gauss-Jordan elimination** algorithm that a computer or calculator uses to convert a matrix (augmented or not) into reduced row echelon form. Understanding this algorithm will help us better understand how to interpret the results in many applications we use it for in Module V.

Activity E.23 *(∼8 min)* Consider the matrix

$$\begin{bmatrix} 2 & 6 & -1 & 6 \\ 1 & 3 & -1 & 2 \\ -1 & -3 & 2 & 0 \end{bmatrix}.$$

Which row operation is the best choice for the first move in converting to RREF?

(a) Add row 3 to row 2 ($R_2 + R_3 \to R_2$)

(b) Add row 2 to row 3 ($R_3 + R_2 \to R_3$)

(c) Swap row 1 to row 2 ($R_1 \leftrightarrow R_2$)

(d) Add -2 row 2 to row 1 ($R_1 - 2R_2 \to R_1$)

Activity E.24 *(∼7 min)* Consider the matrix

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 2 & 6 & -1 & 6 \\ -1 & -3 & 2 & 0 \end{bmatrix}.$$

Which row operation is the best choice for the next move in converting to RREF?

(a) Add row 1 to row 3 ($R_3 + R_1 \to R_3$)

(b) Add -2 row 1 to row 2 ($R_2 - 2R_1 \to R_2$)

(c) Add 2 row 2 to row 3 ($R_3 + 2R_2 \to R_3$)

(d) Add 2 row 3 to row 2 ($R_2 + 2R_3 \to R_2$)

Activity E.25 *(∼5 min)* Consider the matrix

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

Which row operation is the best choice for the next move in converting to RREF?

(a) Add row 1 to row 2 ($R_2 + R_1 \to R_2$)

(b) Add -1 row 3 to row 2 ($R_2 - R_3 \to R_2$)

(c) Add -1 row 2 to row 3 ($R_3 - R_2 \to R_3$)

(d) Add row 2 to row 1 ($R_1 + R_2 \to R_1$)

Activity E.26 *(∼10 min)* Consider the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & -1 & 1 \end{bmatrix}.$$

Part 1: Perform three row operations to produce a matrix closer to RREF.
Part 2: Finish putting it in RREF.

Activity E.27 *(∼10 min)* Consider the matrix

$$A = \begin{bmatrix} 2 & 3 & 2 & 3 \\ -2 & 1 & 6 & 1 \\ -1 & -3 & -4 & 1 \end{bmatrix}.$$

Compute RREF(A).

Activity E.28 *(∼10 min)* Consider the matrix

$$A = \begin{bmatrix} 2 & 4 & 2 & -4 \\ -2 & -4 & 1 & 1 \\ 3 & 6 & -1 & -4 \end{bmatrix}.$$

Compute RREF(A).

Remark E.29 A video example of how to perform the Gauss-Jordan Elimination algorithm by hand is available at `https://youtu.be/Cq0Nxk2dhhU`.
Practicing several exercises on your own using this method is strongly recommended.

Activity E.30 *(∼10 min)* Free browser-based technologies for mathematical computation are available online.

- Go to `https://octave-online.net`.
- Type `A=sym([1 3 4 ; 2 5 7])` and press `Enter` to store the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$ in the variable A.
  - The symbolic function `sym` is used to calculate precise answers rather than floating-point approximations.
  - The vertical bar in an augmented matrix does not affect row operations, so the RREF of $\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 2 & 5 & 7 \end{array}\right]$ may be computed in the same way.
- Type `rref(A)` and press `Enter` to compute the reduced row echelon form of A.