

## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compose functions of real numbers
- Solve systems of linear equations (**Standard(s) E3**)
- Find the matrix corresponding to a linear transformation (**Standard(s) A1**)
- Determine if a linear transformation is injective and/or surjective (**Standard(s) A3**)
- Interpret the ideas of injectivity and surjectivity in multiple ways

## Readiness Assurance Resources

The following resources will help you prepare for this module.

- <https://www.khanacademy.org/math/algebra2/manipulating-functions/function-composition/v/function-composition>

**Readiness Assurance Test**

Choose the most appropriate response for each question.

41) Let  $f(x) = x^2 - 2$  and  $g(x) = x^2 + 1$ . Compute the composition function  $(f \circ g)(x)$ .

- (a)  $x^2 - 1$
- (b)  $x^4 + 2x^2 - 1$
- (c)  $x^4 - 4x^2 + 5$
- (d)  $x^4 - x^2 - 2$

42) Suppose  $f(x)$  and  $g(x)$  are real-valued functions satisfying

$f(2) = 1$	$g(2) = 3$
$f(3) = 4$	$g(3) = 5$
$f(4) = 3$	$g(4) = 6$

Compute  $(f \circ g)(2)$ .

- (a) 2
- (b) 3
- (c) 4
- (d) 5

43) Solve the system of linear equations

$$\begin{aligned}x + 3y &= -2 \\ 2x - 7y &= 9\end{aligned}$$

- |   |  |  |   |
|---|--|--|---|
| (a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ | (b) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ | (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ | (d) $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$ |
|---|--|--|---|

44) Let  $a, b, c$  be fixed real numbers. How many solutions does the system of linear equations below have?

$$\begin{aligned}x + 2y + 3z &= a \\ y - z &= b \\ y + z &= c\end{aligned}$$

- |       |       |                     |   |
|-------|-------|---------------------|---|
| (a) 0 | (b) 1 | (c) Infinitely many | (d) It depends on the values of $a$ , $b$ , and $c$ . |
|-------|-------|---------------------|---|

45) What is the matrix corresponding to the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  given by  $T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix}$ ?

(a) 
$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$$

(b) 
$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$$

(c) 
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$$

(d) 
$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$$

46) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be the linear transformation with associated matrix  $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$ . Compute

$$T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right).$$

(a) 
$$\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$$

(b) 
$$\begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$$

(c) 
$$\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

(d) 
$$\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$$

47) Which of the following is true of the linear transformation  $T : ?$

- (a)  $T$  is neither injective nor surjective
- (b)  $T$  is injective but not surjective
- (c)  $T$  is surjective but not injective
- (d)  $T$  is both injective and surjective

48) Which of the following is true of the linear transformation  $T : ?$

- (a)  $T$  is neither injective nor surjective
- (b)  $T$  is injective but not surjective
- (c)  $T$  is surjective but not injective
- (d)  $T$  is both injective and surjective

49) Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation with associated matrix  $A$ . Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a)  $T$  is injective
- (b)  $T$  has a non-trivial kernel
- (c) The columns of  $A$  are linearly dependent
- (d)  $\text{RREF}(A)$  has a non-pivot column

50) Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation with associated matrix  $A$ . Three of the four answer choices are equivalent to each other; which one is not equivalent to the other three?

- (a)  $T$  is surjective
- (b)  $\text{Im } T = \mathbb{R}^m$
- (c) The columns of  $A$  span  $\mathbb{R}^m$
- (d)  $\text{RREF}(A)$  has only pivot columns