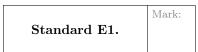
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra Fall 2017

Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{bmatrix}$$

Solution:

$$3x_1 - x_2 + x_4 = 5$$
$$-x_1 + 9x_2 + x_3 - 7x_4 = 0$$
$$x_1 - x_3 = -3$$

Standard E3.

Mark:

Solve the system of equations

$$-3x + y = 2$$
$$-8x + 2y - z = 6$$
$$2y + 3z = -2$$

Solution:

RREF
$$\left(\begin{bmatrix} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{3}{2} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The solutions are

$$\left\{ \begin{bmatrix} -1 - \frac{c}{2} \\ -1 - \frac{3c}{2} \\ c \end{bmatrix} \mid c \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} c - 1 \\ 3c - 1 \\ -2c \end{bmatrix} \mid c \in \mathbb{R} \right\}$$

Find a basis for the solution set to the system of equations

$$x + 2y - 3z = 0$$
$$2x + y - 4z = 0$$
$$3y - 2z = 0$$
$$x - y - z = 0$$

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -4 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} \frac{5}{3}a\\ \frac{2}{3}a\\ a \end{bmatrix} \middle| a \in \mathbb{R} \right\}$$

So a basis is $\left\{ \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} \right\}$ or $\left\{ \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$

- (a) Show that this scalar multiplication \odot distributes over vector addition \oplus .
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1), (x_2, y_2) \in V$ and let $c \in \mathbb{R}$.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + x_2, y_1 + y_2)$$

$$= (c^2(x_1 + x_2), c^3(y_1 + y_2))$$

$$= (c^2x_1, c^3y_1) \oplus (c^2x_2, c^3y_2)$$

$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

However, V is not a vector space, as the other distributive law fails:

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

Additional Notes/Marks