

Name:
J#:
Date:

Dr. Clontz

MIDTERM EXAM

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
---------------------	-------

Write an augmented matrix corresponding to the following linear system.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_4 &= -1\end{aligned}$$

Standard E2.	Mark:
---------------------	-------

Find the reduced row echelon form of the matrix below.

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right]$$

Standard E3.	Mark:
---------------------	-------

Find the solution set for the following system of linear equations.

$$2x_1 + 3x_2 - 5x_3 + 14x_4 = 8$$

$$x_1 + x_2 - x_3 + 5x_4 = 3$$

Standard E4.	Mark:
---------------------	-------

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

$$-x + 2z + 5w = 0$$

Standard V1.	Mark:
---------------------	-------

Let V be the set of all points on the line $x + y = 4$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 2, y_1 + y_2 - 2)$$

$$c \odot (x_1, y_1) = (cx_1 - 2(c - 1), cy_1 - 2(c - 1))$$

- (a) Show that this vector space has an **additive identity** element $\mathbf{0}$ satisfying $(x, y) \oplus \mathbf{0} = (x, y)$.
- (b) Determine if V is a vector space or not. Justify your answer.

Standard V2.	Mark:
---------------------	-------

Determine if $\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Standard V3.	Mark:
---------------------	-------

Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ 10 \end{bmatrix}$ span \mathbb{R}^3 .

Standard V4.	Mark:
---------------------	-------

Determine if $\left\{ \begin{bmatrix} x \\ y \\ 1 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ a subspace of \mathbb{R}^4 .

Standard S1.	Mark:
---------------------	-------

Determine if the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Standard S2.	Mark:
---------------------	-------

Determine if the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 4 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^4 .

Standard S3.	Mark:
---------------------	-------

Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\} \right)$. Find a basis of W .

Standard S4.	Mark:
---------------------	-------

Let $W = \text{span} \left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \\ 0 \end{bmatrix} \right\}$. Find the dimension of W .

Additional Notes/Marks	
-------------------------------	--