

Name:
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Date:

Dr. Clontz

# MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

## Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
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Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

**Solution:**

$$\left[ \begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

□

Standard E3.	Mark:
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Solve the system of linear equations.

$$\begin{aligned}2x + y - z + w &= 5 \\3x - y - 2w &= 0 \\-x + 5z + 3w &= -1\end{aligned}$$

**Solution:**

$$\text{RREF} \left( \left[ \begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array} \right] \right) = \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{12} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{array} \right]$$

So the solutions are

$$\left\{ \left[ \begin{array}{c} 1 + a \\ 3 - 21a \\ -7a \\ 12a \end{array} \right] \mid a \in \mathbb{R} \right\}$$

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<b>Standard E4.</b>	Mark:
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Find a basis for the solution set of the system ...

<b>Standard V1.</b>	Mark:
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Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

Determine if  $V$  is a vector space or not.

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ .

- 1) Real addition is associative, so  $\oplus$  is associative.
- 2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.
- 3)  $x \oplus (6 - x) = x + (6 - x) - 3 = 3$ , so  $6 - x$  is the additive inverse of  $x$ .
- 4) Real addition is commutative, so  $\oplus$  is commutative.
- 5)

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

$$6) 1 \odot x = x - 3(1 - 1) = x$$

7)

$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\&= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\&= (c \odot x) \oplus (d \odot x)\end{aligned}$$

Therefore  $V$  is a vector space.

□