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Dr. Clontz

MIDTERM EXAM Math 237 – Linear Algebra

Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:

Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x_1 + 4x_3 &= 1 \\ x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_3 &= -1 \end{aligned}$$

Standard E2.	Mark:

Find the reduced row echelon form of the matrix below.

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{array}\right]$$

Standard E3.	Mark:

Solve the system of linear equations.

$$\begin{aligned} 2x + y - z + w &= 5 \\ 3x - y - 2w &= 0 \\ -x + 5z + 3w &= -1 \end{aligned}$$

Standard E4.	Mark:

Find a basis for the solution set to the homogeneous system of equations

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 0 \\ -2x_3 - 4x_4 &= 0 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= 0 \end{aligned}$$

Standard V1.	Mark:

Let V be the set of all points on the line $x+y=2$ with the operations, for any $(x_1,y_1),(x_2,y_2)\in V$, $c\in\mathbb{R}$,

$$\begin{aligned}(x_1,y_1)\oplus(x_2,y_2) &= (x_1+x_2-1, y_1+y_2-1)\\ c\odot(x_1,y_1) &= (cx_1-(c-1), cy_1-(c-1))\end{aligned}$$

(a) Show that this vector space has an **additive identity** element $\mathbf{0}$ satisfying $(x,y)\oplus\mathbf{0}=(x,y)$.

(b) Determine if V is a vector space or not. Justify your answer.

Standard V2.	Mark:

Determine if $\begin{bmatrix}0\\0\\2\end{bmatrix}$ can be written as a linear combination of the vectors $\begin{bmatrix}-1\\-9\\15\end{bmatrix}$ and $\begin{bmatrix}1\\5\\-5\end{bmatrix}$.

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}1\\1\\2\\1\end{bmatrix}$, $\begin{bmatrix}3\\3\\6\\3\end{bmatrix}$, $\begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}$, and $\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}$ span $\mathbb{R}^4$.

Standard V4.	Mark:

Determine if the set of all lattice points, i.e. $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

Standard S1.	Mark:

Determine if the set of polynomials $\left\{x^2+x, x^2+2x-1, x^2+3x-2\right\}$ is linearly dependent or linearly independent

Standard S2.	Mark:

Determine if the set $\left\{x^2+x-1, 3x^2-x+1, 2x-2\right\}$ is a basis of $\mathcal{P}_2$

Standard S3.	Mark:

Let $W = \text{span}\left(\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}\right)$. Find a basis for W.

Standard S4.	Mark:

Let W be the subspace of $\mathcal{P}_3$ given by
$W = \text{span}\left(\left\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\right\}\right)$. Compute the dimension of W.

Additional Notes/Marks	