

Name:
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Date:

Dr. Clontz

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard E1.	Mark:
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Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -1 & 0 & 1 & 5 \\ -1 & 9 & 1 & -7 & 0 \\ 1 & 0 & -1 & 0 & -3 \end{array} \right]$$

Solution:

$$\begin{aligned} 3x_1 - x_2 + x_4 &= 5 \\ -x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\ x_1 - x_3 &= -3 \end{aligned}$$

□

Standard E3.	Mark:
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Solve the system of equations

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 2 \end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 2 \end{array} \right] \right) = \left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

So the solution set is

$$\left\{ \left[\begin{array}{c} 1 - 3c \\ c \\ -1 \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

Standard E4.	Mark:
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Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

$$-x + 2z + 5w = 0$$

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \\ -1 & 0 & 2 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} a \\ 2a \\ -2a \\ a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \right\}$.

□

Standard V1.	Mark:
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Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

$$c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$$

Determine if V is a vector space or not.

Solution:

- 1) Since real addition is associative, \oplus is associative.
- 2) Since real addition is commutative, \oplus is commutative.
- 3) $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so $(1, 1)$ is an additive identity element.
- 4) $(x_1, y_1) \oplus (2 - x_1, 2 - y_1) = (1, 1)$, so $(2 - x_1, 2 - y_1)$ is the additive inverse of (x_1, y_1) .
- 5)

$$\begin{aligned} c \odot (d \odot (x_1, y_1)) &= c \odot (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= (c(dx_1 - (d - 1)) - (c - 1), c(dy_1 - (d - 1))) \\ &= (cdx_1 - cd + c - (c - 1), cdy_1 - cd + c - (c - 1)) \\ &= (cdx_1 - (cd - 1), cdy_1 - (cd - 1)) \\ &= (cd) \odot (x_1, y_1) \end{aligned}$$

$$6) \ 1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$\begin{aligned} c \odot ((x_1, y_1) \oplus (x_2, y_2)) &= c \odot (x_1 + y_1 - 1, x_2 + y_2 - 1) \\ &= (c(x_1 + y_1 - 1) - (c - 1), c(x_2 + y_2 - 1) - (c - 1)) \\ &= (cx_1 + cy_1 - 2c + 1, cx_2 + cy_2 - 2c + 1) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (cx_2 - (c - 1), cy_2 - (c - 1)) \\ &= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2) \end{aligned}$$

8)

$$\begin{aligned} (c + d) \odot (x_1, y_1) &= ((c + d)x_1 - (c + d - 1), (c + d)y_1 - (c + d - 1)) \\ &= (cx_1 - (c - 1), cy_1 - (c - 1)) \oplus (dx_1 - (d - 1), dy_1 - (d - 1)) \\ &= c \odot (x_1, y_1) \oplus d \odot (x_1, y_1) \end{aligned}$$

□