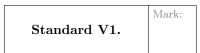
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all polynomials with the operations, for any $f,g\in V,\,c\in\mathbb{R},$

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that this scalar multiplication \odot distributes over vector addition \oplus .
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f.

Does span
$$\left\{ \begin{bmatrix} 2\\-1\\4 \end{bmatrix}, \begin{bmatrix} 3\\12\\-9 \end{bmatrix}, \begin{bmatrix} 1\\4\\-3 \end{bmatrix}, \begin{bmatrix} -4\\2\\-8 \end{bmatrix} \right\} = \mathbb{R}^3$$
?

Solution: Since

$$RREF \begin{bmatrix} 2 & 3 & 1 & -4 \\ -1 & 12 & 4 & 2 \\ 4 & -9 & -3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

has a zero row, the vectors fail to span \mathbb{R}^3 .

Standard V4.

Let W be the set of all \mathbb{R}^3 vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying x+y+z=1 (this forms a plane). Determine if W is a subspace of \mathbb{R}^3 .

Solution: No, because **0** does not belong to W.

Mark:

Standard S2.

Determine if the set $\left\{ \begin{bmatrix} 1\\1\\-1 \end{bmatrix}, \begin{bmatrix} 3\\-1\\1 \end{bmatrix}, \begin{bmatrix} 2\\0\\-2 \end{bmatrix} \right\}$ is a basis of \mathbb{R}^3

Mark:

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

Additional Notes/Marks