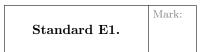
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{bmatrix}$$

Solution:

$$-4x_1 - x_2 + 3x_3 = 2$$
$$x_1 + 2x_2 - x_3 = 0$$
$$-x_1 + 4x_2 + x_3 = 4$$

Standard E3.

Mark:

Solve the following linear system.

$$3x + 2y + z = 7$$
$$x + y + z = 1$$
$$-2x + 3z = -11$$

Solution: Let $A = \begin{bmatrix} 3 & 2 & 1 & 7 \\ 1 & 1 & 1 & 1 \\ -2 & 0 & 3 & 11 \end{bmatrix}$, so RREF $A = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$. It follows that the system has exactly one solution: $\begin{bmatrix} 4 & -2 & -1 \end{bmatrix}$

Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$
$$3x - y + z + w = 0$$
$$2x - 3y - 2z = 0$$
$$-x + 2z + 5w = 0$$

Solution:

$$\operatorname{RREF}\left(\begin{bmatrix} 1 & -2 & 3 & 1\\ 3 & -1 & 1 & 1\\ 2 & -3 & -2 & 0\\ -1 & 0 & 2 & 5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -1\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} a\\2a\\-2a\\a \end{bmatrix} \middle| a \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} 1\\2\\-2\\1 \end{bmatrix} \right\}$.

Standard V1.

Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that this scalar multiplication \odot distributes over scalar addition.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c+d)\odot(x_1,y_1)=(0,(c+d)y_1)=(0,cy_1)\oplus(0,dy_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

Additional Notes/Marks