

Name: \_\_\_\_\_

**SEMIFINAL**

Math 237 – Linear Algebra

**Version 1**

Fall 2017

**Choose up to 6 problems to work.** Work each problem on one of the attached pages; write the standard in the upper left corner. Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 3x_2 - 4x_3 + x_4 &= 5 \\3x_1 + 9x_2 + x_3 - 7x_4 &= 0 \\x_1 - x_3 + x_4 &= 1\end{aligned}$$

**E2.** Put the following matrix in reduced row echelon form.

$$\left[ \begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right]$$

**E3.** Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\3x_1 + 6x_3 + x_4 &= 5 \\-4x_1 + x_2 - 9x_3 + 2x_4 &= -7\end{aligned}$$

**E4.** Find a basis for the solution set of the system of equations

$$\begin{aligned}x + 2y + 3z + w &= 0 \\3x - y + z + w &= 0 \\2x - 3y - 2z &= 0\end{aligned}$$

**V1.** Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

(a) Show that **scalar multiplication** is **associative**:  $a \odot (b \odot x) = (ab) \odot x$ .

(b) Determine if  $V$  is a vector space or not. Justify your answer

**V2.** Determine if  $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$  can be written as a linear combination of the vectors  $\begin{bmatrix} -1 \\ -9 \\ 15 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$ .

**V3.** Determine if the vectors  $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}$ , and  $\begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**V4.** Determine if the set of all lattice points, i.e.  $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

**S1.** Determine if the set of vectors  $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$  is linearly dependent or linearly independent

**S2.** Determine if the set  $\{2x^2 - x + 3, 2x^2 + 2, -x^2 + 4x + 1\}$  is a basis of  $\mathcal{P}^2$ .

**S3.** Let  $W$  be the subspace of  $\mathcal{P}^2$  given by  $W = \text{span}(\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\})$ . Find a basis for  $W$ .

**S4.** Let  $W = \text{span}\left(\left\{\begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix}\right\}\right)$ . Compute the dimension of  $W$ .

**A1.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 7x \end{bmatrix}.$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

**A2.** Determine if the map  $T : \mathcal{P}^6 \rightarrow \mathcal{P}^6$  given by  $T(f) = f(x) - f(0)$  is a linear transformation or not.

**A3.** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a)  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  given by the standard matrix  $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$ .

(b)  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  given by the standard matrix  $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 7 & -1 & 5 \end{bmatrix}$

**A4.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear map given by  $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z \\ y + 3z \\ -7x + 3y + 2z \end{bmatrix}$ . Compute a basis for the kernel and a basis for the image of  $T$ .

**M1.** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**M2.** Determine if the matrix  $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$  is invertible.

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$ .

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ .

**G2.** Compute the eigenvalues, along with their algebraic multiplicities, of the matrix  $\begin{bmatrix} 8 & -3 & 2 \\ 23 & -9 & 5 \\ -7 & 2 & -3 \end{bmatrix}$ .

**G3.** Find the eigenspace associated to the eigenvalue 2 in the matrix  $A = \begin{bmatrix} 0 & -2 & -1 & 0 \\ -4 & -2 & -2 & 0 \\ 14 & 12 & 10 & 2 \\ -13 & -10 & -8 & -1 \end{bmatrix}$ .

**G4.** Compute the geometric multiplicity of the eigenvalue 1 in the matrix  $A = \begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$ .

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