Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

Solution: 55.

**G3.** Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to  $x - \frac{y}{3} = 0$ , or 3x = y. Thus the eigenspace is

$$E_2 = \operatorname{span}\left(\left\{ \begin{bmatrix} 1\\3\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1 \end{bmatrix} \right\}\right)$$

**G4.** Compute the geometric multiplicity of the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the geometric multiplicity is 2.

**G1**:

**G3**:

**G4**:

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Version 2

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}.$ 

Solution: -1.

**G3.** Compute the eigenspace of the eigenvalue -1 in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

Solution:

RREF 
$$(A+I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

**G4.** Compute the geometric multiplicity of the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the geometric multiplicity is 2.

G1:

G3:

G4:

Name:	

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Version 3

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$ 

Solution: 2

**G3.** Compute the eigenspace of the eigenvalue -1 in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

Solution:

RREF 
$$(A+I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

**G4.** Compute the geometric multiplicity of the eigenvalue -1 in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

Solution:

RREF 
$$(A+I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the geometric multiplicity is 2.

G1: G3: G4:

Name:	

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Version 4

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix  $\begin{bmatrix} 3 & -1 & 0 & 7 \\ 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$ 

Solution: 2

**G3.** Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to  $x - \frac{y}{3} = 0$ , or 3x = y. Thus the eigenspace is

$$E_2 = \operatorname{span}\left(\left\{\begin{bmatrix}1\\3\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}\right)$$

**G4.** Compute the geometric multiplicity of the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the geometric multiplicity is 2.

G1: G3:

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Version 5

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

Solution: -55.

**G3.** Compute the eigenspace of the eigenvalue -1 in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

Solution:

RREF 
$$(A+I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the eigenspace is spanned by  $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ .

**G4.** Compute the geometric multiplicity of the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the geometric multiplicity is 2.

G1:

G3:

G4:

Name:	

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Version 6

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**G1.** Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

Solution: -55.

**G3.** Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**Solution:** The eigenspace is the solution space of the system (B-2I)X=0.

$$RREF(B-2I) = RREF \left( \begin{bmatrix} -3 & 1 & 0 \\ -9 & 3 & 0 \\ 15 & -5 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the system simplifies to  $x - \frac{y}{3} = 0$ , or 3x = y. Thus the eigenspace is

$$E_2 = \operatorname{span}\left(\left\{ \begin{bmatrix} 1\\3\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1 \end{bmatrix} \right\}\right)$$

**G4.** Compute the geometric multiplicity of the eigenvalue -1 in the matrix  $\begin{bmatrix} 4 & -2 & -1 \\ 15 & -7 & -3 \\ -5 & 2 & 0 \end{bmatrix}$ .

Solution:

RREF 
$$(A+I) = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the geometric multiplicity is 2.

G1: G3: