

Name:
J#:
Date:

Dr. Clontz

# MASTERY QUIZ DAY 20

Math 237 – Linear Algebra

## Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard S3.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}$ .

□

<b>Standard S4.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

□

<b>Standard A1.</b>	Mark:
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Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  be the linear transformation given by

$$T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \\ 0 \end{bmatrix}$$

. Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

**Solution:**

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

□

<b>Standard A2.</b>	Mark:
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Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by  $T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$ . Determine if  $T$  is a linear transformation.

**Solution:**

$$T \left( \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

So  $T$  is not a linear transformation.

□

<b>Additional Notes/Marks</b>	
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## MASTERY QUIZ DAY 20

Math 237 – Linear Algebra

### Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard S3.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since the first two columns are

pivot columns,  $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$  is a basis for  $W$ .

□

<b>Standard S4.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so  $W$  has dimension 2.

□

<b>Standard A1.</b>	Mark:
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Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_2 + 3x_3]$$

. Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

**Solution:**

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

□

<b>Standard A2.</b>	Mark:
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Determine if  $D : M_{2,2} \rightarrow \mathbb{R}$  given by  $D \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc$  is a linear transformation or not.

**Solution:**  $D(I) = 1$  but  $D(2I) = 4 \neq 2D(I)$ , so  $D$  is not linear.

□

<b>Additional Notes/Marks</b>	
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### MASTERY QUIZ DAY 20

Math 237 – Linear Algebra

#### Version 3

Fall 2017

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<b>Standard S3.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\} \right)$ . Find a basis of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then  $\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\}$  is a basis for  $W$ .

□

<b>Standard S4.</b>	Mark:
---------------------	-------

Let  $W = \text{span} \left( \left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since there are two pivot columns,  $\dim W = 2$ .

□

<b>Standard A1.</b>	Mark:
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Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - x_3 \end{bmatrix}$$

. Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^4$  and  $\mathbb{R}^2$ .

**Solution:**

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

□

<b>Standard A2.</b>	Mark:
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Determine if the map  $T : \mathcal{P}^3 \rightarrow \mathcal{P}^4$  given by  $T(f(x)) = xf(x) - f(x)$  is a linear transformation or not.

<b>Additional Notes/Marks</b>	
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# MASTERY QUIZ DAY 20

Math 237 – Linear Algebra

## Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard S3.</b>	Mark:
---------------------	-------

Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}$ .

□

<b>Standard S4.</b>	Mark:
---------------------	-------

Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Compute the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

□

<b>Standard A1.</b>	Mark:
---------------------	-------

Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_2 + 3x_3]$$

. Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

**Solution:**

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

□

<b>Standard A2.</b>	Mark:
---------------------	-------

Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by  $T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$ . Determine if  $T$  is a linear transformation.

**Solution:**

$$T \left( \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

So  $T$  is not a linear transformation.

□

<b>Additional Notes/Marks</b>	
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# MASTERY QUIZ DAY 20

Math 237 – Linear Algebra

## Version 5

Fall 2017

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<b>Standard S3.</b>	Mark:
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Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$ . Find a basis of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then  $\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$  is a basis of  $W$ .

□

<b>Standard S4.</b>	Mark:
---------------------	-------

Let  $W = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$ . Find the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so  $W$  has dimension 2.

□

<b>Standard A1.</b>	Mark:
---------------------	-------

Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - x_3 \end{bmatrix}$$

. Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^4$  and  $\mathbb{R}^2$ .

**Solution:**

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

□

<b>Standard A2.</b>	Mark:
---------------------	-------

Determine if  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  given by  $T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

**Solution:** It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

□

<b>Additional Notes/Marks</b>	
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## MASTERY QUIZ DAY 20

Math 237 – Linear Algebra

### Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

<b>Standard S3.</b>	Mark:
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Let  $W$  be the subspace of  $\mathcal{P}_2$  given by  $W = \text{span}(\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\})$ . Find a basis for  $W$ .

**Solution:** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ , and compute  $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$ . Since the first two columns are

pivot columns,  $\{-3x^2 - 8x, x^2 + 2x + 2\}$  is a basis for  $W$ .

□

<b>Standard S4.</b>	Mark:
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Let  $W$  be the subspace of  $\mathcal{P}_3$  given by

$W = \text{span}(\{x^3 - x^2 + 3x - 3, 2x^3 + x + 1, 3x^3 - x^2 + 4x - 2, x^3 + x^2 + x - 7\})$ . Compute the dimension of  $W$ .

**Solution:**

$$\text{RREF} \left( \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so  $\dim(W) = 3$ .

□

<b>Standard A1.</b>	Mark:
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Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - x_3 \end{bmatrix}$$

. Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^4$  and  $\mathbb{R}^2$ .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

□

Standard A2.	Mark:
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Determine if the map  $T : \mathcal{P}^3 \rightarrow \mathcal{P}^4$  given by  $T(f(x)) = xf(x) - f(x)$  is a linear transformation or not.

Additional Notes/Marks	
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