Definition 7.2 A **vector space** V is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ belong to V, and let a, b be scalar numbers.

- **Addition associativity.**
  $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.
- **Addition commutivity.**
  $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.
- **Addition identity.**
  There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.
- **Addition inverse.**
  There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.
- **Scalar multiplication associativity.**
  $a(b\mathbf{v}) = (ab)\mathbf{v}$.
- **Scalar multiplication identity.**
  $1\mathbf{v} = \mathbf{v}$.
- **Scalar distribution.**
  $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$.
- **Vector distribution.**
  $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$.

Definition 7.3 The most important examples of vector spaces are the **Euclidean vector spaces** $\mathbb{R}^n$, but there are other examples as well.