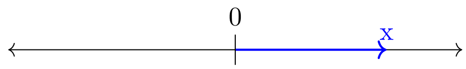
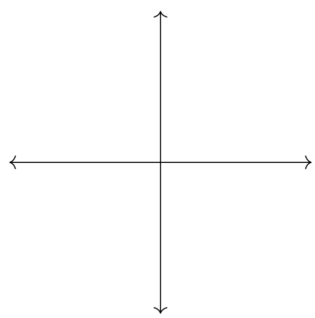
Section V.3

Observation V.3.1 Any single non-zero vector/number x in $\mathbb{R}^1$ spans $\mathbb{R}^1$, since $\mathbb{R}^1 = \{cx \mid c \in \mathbb{R}\}$.



Activity V.3.2 *(∼5 min)* How many vectors are required to span $\mathbb{R}^2$? Sketch a drawing in the xy plane to support your answer.



(a) 1

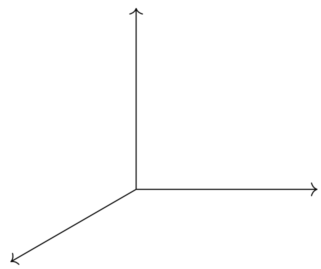
(b) 2

(c) 3

(d) 4

(e) Infinitely Many

Activity V.3.3 *(~5 min)* How many vectors are required to span $\mathbb{R}^3$?



(a) 1

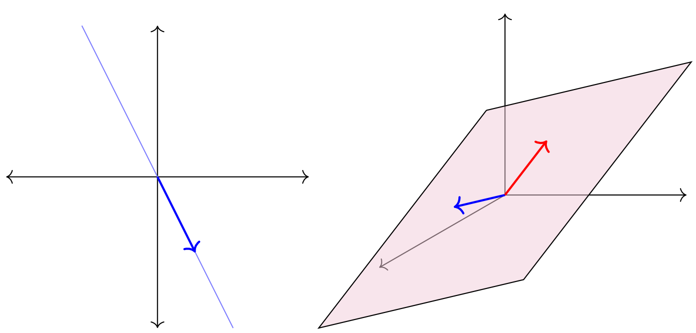
(b) 2

(c) 3

(d) 4

(e) Infinitely Many

Fact V.3.4 At least n vectors are required to span $\mathbb{R}^n$.



Activity V.3.5 *(~15 min)* Choose any vector $\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$ in $\mathbb{R}^3$ that is not in span $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ by using technology to verify that RREF $\left[\begin{array}{cc|c} 1 & -2 & ? \\ -1 & 0 & ? \\ 0 & 1 & ? \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$. (Why does this work?)

Fact V.3.6 The set $\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_m\}$ fails to span all of $\mathbb{R}^n$ exactly when RREF$[\vec{\mathbf{v}}_1 \ \dots \ \vec{\mathbf{v}}_m]$ has a non-pivot row of zeros.

$$\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & -2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \text{ for some choice of vector } \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Activity V.3.7 *(∼5 min)* Consider the set of vectors $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 13 \\ 7 \\ 16 \end{bmatrix} \right\}$. Does $\mathbb{R}^4 = \operatorname{span} S$?

Activity V.3.8 *(∼10 min)* Consider the set of third-degree polynomials

$$\begin{aligned} S = \{&2x^3 + 3x^2 - 1, 2x^3 + 3, 3x^3 + 13x^2 + 7x + 16, \\ &- x^3 + 10x^2 + 7x + 14, 4x^3 + 3x^2 + 2\}. \end{aligned}$$

Does $\mathcal{P}^3 = \operatorname{span} S$? (Hint: first rewrite the question so it is about Euclidean vectors.)

Activity V.3.9 *(∼5 min)* Consider the set of matrices

$$S = \left\{ \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right\}$$

Does $M_{2,2} = \operatorname{span} S$?

Activity V.3.10 *(∼5 min)* Let $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3 \in \mathbb{R}^7$ be three vectors, and suppose $\vec{\mathbf{w}}$ is another vector with $\vec{\mathbf{w}} \in \operatorname{span}\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$. What can you conclude about $\operatorname{span}\{\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$?

(a) $\operatorname{span}\{\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is larger than $\operatorname{span}\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$.

(b) $\operatorname{span}\{\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\} = \operatorname{span}\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$.

(c) $\operatorname{span}\{\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is smaller than $\operatorname{span}\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$.