Readiness Assurance Test

Choose the most appropriate response for each question.

31) Suppose f(x) and g(x) are real-valued functions satisfying

$$f(2) = 4$$
 $g(2) = 4$ $g(3) = 5$

f(4) = 3g(4) = 2

Compute $(f \circ g)(2)$.

(a) 3 (b) 4 (c) 5 (d) 6

32) Let $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$. Compute the composition function $(f \circ g)(x)$.

(b) $x^4 - 4x^2 + 5$ (c) $x^4 + 2x^2 - 1$ (d) $x^4 - x^2 - 2$ (a) $x^2 - 1$

33) What is the standard matrix corresponding to the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) =$

$$\begin{bmatrix} x + 2y - z \\ y + 3z \\ x + 7y \end{bmatrix}$$
?

(a) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 7 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ -1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 7 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 7 \\ -1 & 3 & 0 \end{bmatrix}$

34) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear map corresponding to the standard matrix $\begin{vmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \end{vmatrix}$. Compute

$$T\left(\begin{bmatrix}1\\-1\\3\end{bmatrix}\right).$$

(a) $\begin{bmatrix} -2 \\ 8 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ -8 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ (d) $\begin{vmatrix} 2 \\ 4 \end{vmatrix}$

35) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation corresponding to the standard matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \\ 0 & 4 \end{bmatrix}$.

Compute $T\left(\begin{bmatrix} 2\\-1\end{bmatrix}\right)$.

(a)
$$\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$
 (b)
$$\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$$

(c) $\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

- 36) Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation corresponding to the standard matrix $\begin{bmatrix} 3 & -1 & 0 & 2 \\ -2 & -4 & -1 & 1 \end{bmatrix}$. What are the domain and codomain of T?
 - (a) The domain is \mathbb{R}^2 and the codomain is \mathbb{R}^4
 - (b) The domain and codomain are both \mathbb{R}^2
 - (c) The domain is \mathbb{R}^4 and the codomain is \mathbb{R}^2
 - (d) The domain and codomain are both \mathbb{R}^4
- 37) Which of the following is true of the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 3y - 4z \\ x + y \\ 3z \end{bmatrix}?$$

- (a) T is neither injective nor surjective
- (b) T is both injective and surjective
- (c) T is injective but not surjective
- (d) T is surjective but not injective
- 38) Which of the following is true of the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + z \end{bmatrix}?$$

- (a) T is injective but not surjective
- (b) T is both injective and surjective
- (c) T is surjective but not injective
- (d) T is neither injective nor surjective
- 39) Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix A. Which of the following is **not** a characterization of the statement "T is injective"?
 - (a) If $T(\vec{\mathbf{v}}) = T(\vec{\mathbf{w}})$ for some $\vec{\mathbf{v}}, \vec{\mathbf{w}} \in \mathbb{R}^n$, then $\vec{\mathbf{v}} = \vec{\mathbf{w}}$.
 - (b) T has a non-trivial kernel
 - (c) The columns of A are linearly independent
 - (d) RREF(A) has only pivot columns
- 40) Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix A. Which of the following is **not** a characterization of the statement "T is surjective"?
 - (a) RREF(A) has a pivot in every row
 - (b) $\operatorname{Im} T = \mathbb{R}^m$
 - (c) The columns of A span \mathbb{R}^m
 - (d) RREF(A) has has a pivot in every column