Math 237 – Linear Algebra Fall 2017

Version 1 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

- (a) $S: \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
- (b) $T: \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 4 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 8x - 3y - z \\ y + 3z \\ -7x + 3y + 2z \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

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Version 2

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

(b)
$$S: \mathbb{R}^2 \to \mathbb{R}^3$$
 given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

A4. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z \\ y + 3z \\ -7x + 3y + 2z \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

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Version 3 Fall 2017 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a)
$$S: \mathbb{R}^4 \to \mathbb{R}^3$$
 where $S(\vec{e}_1) = \begin{bmatrix} 2\\1\\0 \end{bmatrix}$, $S(\vec{e}_2) = \begin{bmatrix} 1\\2\\1 \end{bmatrix}$, $S(\vec{e}_3) = \begin{bmatrix} 0\\-1\\0 \end{bmatrix}$, and $S(\vec{e}_4) = \begin{bmatrix} 3\\2\\1 \end{bmatrix}$,

(b)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 where $T(\vec{e_1}) = \begin{bmatrix} 2\\2\\1 \end{bmatrix}$, $T(\vec{e_2}) = \begin{bmatrix} 1\\0\\4 \end{bmatrix}$, and $T(\vec{e_3}) = \begin{bmatrix} 1\\2\\-3 \end{bmatrix}$.

A4. Let $T: \mathbb{R}^{2 \times 2} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

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Version 4

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

- (a) $S: \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
- (b) $T: \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 4 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4. Let $T: \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} a & b \\ x & y \end{bmatrix}\right) = \begin{bmatrix} a+x \\ 0 \\ b+y \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

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Version 5 Fall 2017 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a)
$$S: \mathbb{R}^4 \to \mathbb{R}^3$$
 where $S(\vec{e}_1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, S(\vec{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, S(\vec{e}_3) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \text{ and } S(\vec{e}_4) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix},$

(b)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 where $T(\vec{e_1}) = \begin{bmatrix} 2\\2\\1 \end{bmatrix}$, $T(\vec{e_2}) = \begin{bmatrix} 1\\0\\4 \end{bmatrix}$, and $T(\vec{e_3}) = \begin{bmatrix} 1\\2\\-3 \end{bmatrix}$.

A4. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T.

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Version 6 Fall 2017 Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

- (a) $S: \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
- (b) $T: \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4. Let $T: \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} a & b \\ x & y \end{bmatrix}\right) = \begin{bmatrix} a+x \\ 0 \\ b+y \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.