Application Activities - Module G Part 4 - Class Day 28

Observation 28.1 Recall from last class:

- To find the eigenvalues of a matrix A, we need to find values of λ such that $A - \lambda I$ has a nontrivial kernel. Equivalently, we want values where $A - \lambda I$ is not invertible, so we want to know the values of λ where $\det(A - \lambda I) = 0$.
- $\det(A - \lambda I)$ is a polynomial with variable λ, called the **characteristic polynomial** of A. Thus the roots of the characteristic polynomial of A are exactly the eigenvalues of A.
- Once an eigenvalue λ is found, the **eigenspace** containing all **eigenvectors** $\mathbf{x}$ satisfying $A\mathbf{x} = \lambda\mathbf{x}$ is given by $\ker(A - \lambda I)$.

Activity 28.2 Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.
Part 1: Compute the eigenvalues of A.
Part 2: Sketch a picture of the transformation of the unit square. What about this picture reveals that A has no eigenvectors?

Activity 28.3 If A is a 4×4 matrix, what is the largest number of eigenvalues A can have?

(a) 3

(b) 4

(c) 5

(d) 6

(e) It can have infinitely many

Observation 28.4 An $n \times n$ matrix may have anywhere between 0 and n real-valued eigenvalues. But if complex eigenvalues are included, then every $n \times n$ matrix has n eigenvalues (counting algebraic multiplicites).

Activity 28.5 The matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 0 & 1 \\ -1 & -2 & 3 \end{bmatrix}$ has characteristic polynomial $-\lambda(\lambda - 2)^2$.
Find the dimension of the eigenspace of A associated to the eigenvalue 2 (the dimension of the kernel of $A - 2I$).

Activity 28.6 The matrix $B = \begin{bmatrix} -3 & -9 & 5 \\ -2 & -2 & 2 \\ -7 & -13 & 9 \end{bmatrix}$ has characteristc polynomial $-\lambda(\lambda - 2)^2$.
Find the dimension of the eigenspace of B associated to the eigenvalue 2 (the dimension of the kernel of $B - 2I$).

Definition 28.7 While the **algebraic multiplicity** of an eigenvalue is its multiplicity as a root of the characteristic polynomial, the **geometric multiplicity** of an eigenvalue is the dimension of its eigenspace.

Fact 28.8 As we've seen, the geometric multiplicity may be different than its algebraic multiplicity, but it cannot exceed it.

Activity 28.9 Consider the 4×4 matrix

$$\begin{bmatrix} -3 & 1 & 2 & 1 \\ -9 & 5 & -2 & -1 \\ 31 & -17 & 6 & 3 \\ -69 & 39 & -18 & -9 \end{bmatrix}$$

Part 1: Use technology (e.g. Wolfram Alpha) to find its characteristic polynomial.
Part 2: Find the algebraic and geometric multiplicities for both eigenvalues.