Name:	
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Dr. Clontz

MASTERY QUIZ DAY 14 Math 237 – Linear Algebra
Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$.

(b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d-1)) \\ &= c\,(dx - 3(d-1)) - 3(c-1) \\ &= cdx - 3(cd-1) \\ &= (cd) \odot x \end{aligned}$$

We verify the remaining 7 properties to see that V is a vector space.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6-x) = x + (6-x) - 3 = 3$, so $6-x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5) Associativity shown above

6) $1 \odot x = x - 3(1-1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c-1) \\ &= cx - 3(c-1) + cy - 3(c-1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot x &= (c+d)x - 3(c+d-1)\\ &= cx - 3(c-1) + dx - 3(c-1) - 3\\ &= (c\odot x)\oplus(d\odot x)\end{aligned}$$

Therefore V is a vector space.

□

Standard V3.	Mark:

Determine if the vectors $\begin{bmatrix}8\\21\\-7\end{bmatrix}$, $\begin{bmatrix}-3\\-8\\3\end{bmatrix}$, $\begin{bmatrix}-1\\-3\\2\end{bmatrix}$, and $\begin{bmatrix}4\\11\\-5\end{bmatrix}$ span $\mathbb{R}^3$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}8 & -3 & -1 & 4\\21 & -8 & -3 & 11\\-7 & 3 & 2 & -5\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 1 & -1\\0 & 1 & 3 & -4\\0 & 0 & 0 & 0\end{bmatrix}$$

Since the rank is less than 3, they do not span $\mathbb{R}^3$.

□

Standard V4.	Mark:

Let W be the set of all $\mathbb{R}^3$ vectors $\begin{bmatrix}x\\y\\z\end{bmatrix}$ satisfying $x+y+z=1$ (this forms a plane). Determine if W is a subspace of $\mathbb{R}^3$.

Solution: No, because $\mathbf{0}$ does not belong to W.

□

Standard S2.	Mark:

Determine if the set $\left\{x^3-3x^2+2x+2, -x^3+4x^2-x+1, -x^3+2x+1, 3x^2+3x+9\right\}$ is a basis of $\mathcal{P}^3$ or not.

Solution:

$$\text{RREF}\begin{bmatrix}1 & -1 & -1 & 0\\-3 & 4 & 0 & 3\\2 & -1 & 2 & 3\\2 & 1 & 1 & 9\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 3\\0 & 1 & 0 & 3\\0 & 0 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

Additional Notes/Marks	