Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 14 　　　　　　　　　　　　Math 237 – Linear Algebra
Version 4 　　　　　　　　　　　　　　　　　　　　　　　　Fall 2017
Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard V1.	

Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the vector **addition** $\oplus$ is **associative**: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$
\begin{aligned}
(x \oplus y) \oplus z &= \sqrt{x^2 + y^2} \oplus z \\
&= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2} \\
&= \sqrt{x^2 + y^2 + z^2} \\
&= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2} \\
&= x \oplus \sqrt{y^2 + z^2} \\
&= x \oplus (y \oplus z)
\end{aligned}
$$

However, this is not a vector space, as there is no zero vector.

☐

	Mark:
Standard V3.	

Determine if the vectors $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ span $\mathbb{R}^3$

Solution:

$$
\text{RREF}\left(\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Since the resulting matrix has only two pivot columns, the vectors do not span $\mathbb{R}^3$.

☐

	Mark:
Standard V4.	

Let W be the set of all complex numbers $a + bi$ satisfying $a = 2b$. Determine if W is a subspace of $\mathbb{C}$.

Solution: Yes, because $c(2b_1 + b_1 i) + d(2b_2 + b_2 i) = 2(cb_1 + db_2) + (cb_1 + db_2)i$ belongs to W. Alternately, yes because W is isomorphic to $\mathbb{R}$.

□

	Mark:
Standard S2.	

Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of $\mathcal{P}_2$

Solution:
$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
Since the resulting matrix is the identity matrix, it is a basis.

□

Additional Notes/Marks	