Name:	
J#:	Dr. Clontz
Date:	

## MASTERY QUIZ DAY 18

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S1.

Determine if the set of vectors 
$$\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$$
 is linearly dependent or linearly independent

Solution:

RREF 
$$\left( \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

Standard S3.

$$\begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix}$$
Let  $W = \text{span} \left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$ . Find a basis for this vector space.

Solution:

$$RREF \left( \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \\ 2 & -1 & -8 \\ 1 & 1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus 
$$\left\{ \begin{bmatrix} 2\\0\\2\\1 \end{bmatrix}, \begin{bmatrix} 3\\1\\-1\\1 \end{bmatrix} \right\}$$
 is a basis of  $W$ .

Let 
$$W = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix},\begin{bmatrix}3\\3\\6\\3\end{bmatrix},\begin{bmatrix}-1\\3\\-2\end{bmatrix},\begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right)$$
. Find the dimension of  $W$ .

Solution:

$$RREF \left( \begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

Standard A1.

Mark:

Let  $T: \mathbb{R}^3 \to \mathbb{R}$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

**Solution:** 

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

Standard A2.

Mark:

Determine if  $T: \mathbb{R}^2 \to \mathbb{R}^2$  given by  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$  is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left( \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

Additional Notes/Marks