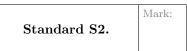
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 19

Math 237 – Linear Algebra Fall 2017

Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}_3

Solution:

$$RREF \left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

Standard A3.

Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a)
$$S: \mathbb{R}^2 \to \mathbb{R}^3$$
 given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

(b)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

Solution:

(a)

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b) $RREF \left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

Standard A4.

Mark:

Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute the kernel and image of T.

Solution:

$$RREF \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the kernel is

$$\ker(T) = \left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\} = \operatorname{span} \left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\} \right)$$

and the image is

$$\operatorname{Im}(T) = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\3\\6\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}7\\-1\\8\\-3\end{bmatrix}\right\}\right) = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\-2\end{bmatrix}\right\}\right)$$