

Name: _____

MASTERY QUIZ DAY 8

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_4 &= -1\end{aligned}$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -1 & 0 & 7 \\ 1 & -1 & 0 & 3 & -1 \end{array} \right]$$

□

E3. Solve the system of equations

$$\begin{aligned}x + 3y + 3z + 7w &= 0 \\x + 3y - z - w &= 0 \\2x + 6y + 3z + 8w &= 0 \\x + 3y - 2z - 3w &= 0\end{aligned}$$

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\} = \text{span} \left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\} \right)$$

□

E4. Find a basis for the solution set of the system ...

V1. Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

1) Real addition is associative, so \oplus is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .

4) Real addition is commutative, so \oplus is commutative.

5)

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\ &= c(dx - 3(d - 1)) - 3(c - 1) \\ &= cdx - 3(cd - 1) \\ &= (cd) \odot x \end{aligned}$$

6) $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c - 1) \\ &= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned} (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\ &= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\ &= (c \odot x) \oplus (d \odot x) \end{aligned}$$

Therefore V is a vector space.

□

E1:

E3:

E4:

V1:

E2: