

Name: _____

SEMIFINAL

Math 237 – Linear Algebra

Version 5

Fall 2017

Choose up to 6 problems to work. Work each problem on one of the attached pages; write the standard in the upper left corner. Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

E2. Put the following matrix in reduced row echelon form.

$$\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2 \\ -8 & 2 & -1 & 6 \\ 0 & 2 & 3 & -2 \end{array} \right]$$

E3. Find the solution set for the following system of linear equations.

$$\begin{aligned}2x_1 - 2x_2 + 6x_3 - x_4 &= -1 \\3x_1 + 6x_3 + x_4 &= 5 \\-4x_1 + x_2 - 9x_3 + 2x_4 &= -7\end{aligned}$$

E4. Find a basis for the solution set of the system of equations

$$\begin{aligned}x + 3y + 3z + 7w &= 0 \\x + 3y - z - w &= 0 \\2x + 6y + 3z + 8w &= 0 \\x + 3y - 2z - 3w &= 0\end{aligned}$$

V1. Let V be the set of all points on the line $x + y = 2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned}(x_1, y_1) \oplus (x_2, y_2) &= (x_1 + x_2 - 1, y_1 + y_2 - 1) \\c \odot (x_1, y_1) &= (cx_1 - (c - 1), cy_1 - (c - 1))\end{aligned}$$

(a) Show that this vector space has an **additive identity** element $\mathbf{0}$ satisfying $(x, y) \oplus \mathbf{0} = (x, y)$.

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix}$.

V3. Does $\text{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\} = \mathbb{R}^3$?

V4. Determine if $\left\{ \begin{bmatrix} x \\ y \\ 0 \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ a subspace of \mathbb{R}^4 .

S1. Determine if the set of vectors $\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

S2. Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of \mathcal{P}^3 .

S3. Let W be the subspace of \mathcal{P}^2 given by $W = \text{span}(\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\})$. Find a basis for W .

S4. Let $W = \text{span}\left(\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right\}\right)$. Compute the dimension of W .

A1. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = [x_2 + 3x_3].$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R} .

A2. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T .

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB , AC , BA , BC , CA , CB can be computed. Determine which one, and compute it.

M2. Determine if the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 0 \\ -1 & -1 & 5 \end{bmatrix}$ is invertible.

M3. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

G1. Compute the determinant of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

G3. Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

G4. Compute the geometric multiplicity of the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

Standard: _____



Standard: _____



Standard: _____



Standard: _____



Standard: _____



Standard: _____

