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| Name: |
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Dr. Clontz

MASTERY QUIZ DAY 12

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

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| Standard V1. | Mark: |
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Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned}x \oplus y &= x + y - 3 \\c \odot x &= cx - 3(c - 1)\end{aligned}$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.
- 3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x .
- 4) Real addition is commutative, so \oplus is commutative.
- 5)

$$\begin{aligned}c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\&= c(dx - 3(d - 1)) - 3(c - 1) \\&= cdx - 3(cd - 1) \\&= (cd) \odot x\end{aligned}$$

$$6) 1 \odot x = x - 3(1 - 1) = x$$

7)

$$\begin{aligned}c \odot (x \oplus y) &= c \odot (x + y - 3) \\&= c(x + y - 3) - 3(c - 1) \\&= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\&= (c \odot x) \oplus (c \odot y)\end{aligned}$$

8)

$$\begin{aligned}(c + d) \odot x &= (c + d)x - 3(c + d - 1) \\&= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\&= (c \odot x) \oplus (d \odot x)\end{aligned}$$

Therefore V is a vector space.

□

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| Standard V3. | Mark: |
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Determine if the vectors $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$ span \mathbb{R}^3 .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

□

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| Standard V4. | Mark: |
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Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W , but $(x - x^2) + (x^2) = x \notin W$.

□

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| Additional Notes/Marks | |
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