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| Name: |
| J#: |
| Date: |

Dr. Clontz

MASTERY QUIZ DAY 18

Math 237 – Linear Algebra

Version 1

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| | |
|---------------------|-------|
| Standard S1. | Mark: |
|---------------------|-------|

Determine if the set of matrices $\left\{ \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -8 \\ 6 & 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

Solution:

$$\text{RREF} \left(\begin{bmatrix} 3 & 1 & 3 \\ -1 & 2 & -8 \\ 0 & -2 & 6 \\ 4 & 1 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

□

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|---------------------|-------|
| Standard S3. | Mark: |
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Let $W = \text{span} \left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$. Find a basis for this vector space.

Solution:

$$\text{RREF} \left(\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \\ 2 & -1 & -8 \\ 1 & 1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$ is a basis of W .

□

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|---------------------|-------|
| Standard S4. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left(\left(\begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -7 \end{bmatrix} \right) \right)$. Compute the dimension of W .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 1 \\ 3 & 1 & 4 & 1 \\ -3 & 1 & -2 & -7 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

□

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|---------------------|-------|
| Standard A1. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - 5x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and \mathbb{R}^2 .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -5 & 0 \end{bmatrix}$$

□

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|---------------------|-------|
| Standard A2. | Mark: |
|---------------------|-------|

Determine if the map $T : \mathcal{P} \rightarrow \mathcal{P}$ given by $T(f) = f' - f''$ is a linear transformation or not.

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| Additional Notes/Marks | |
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MASTERY QUIZ DAY 18

Math 237 – Linear Algebra

Version 2

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| | |
|--------------|-------|
| Standard S1. | Mark: |
|--------------|-------|

Determine if the set of vectors $\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ is linearly dependent or linearly independent.

Solution:

$$\text{RREF} \left(\begin{bmatrix} 3 & 1 & 3 \\ -1 & 2 & -8 \\ 0 & -2 & 6 \\ 4 & 1 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent.

□

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|--------------|-------|
| Standard S3. | Mark: |
|--------------|-------|

Let W be the subspace of \mathcal{P}^2 given by $W = \text{span}(\{-3x^2 - 8x, x^2 + 2x + 2, -x + 3\})$. Find a basis for W .

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are

pivot columns, $\{-3x^2 - 8x, x^2 + 2x + 2\}$ is a basis for W .

□

| | |
|--------------|-------|
| Standard S4. | Mark: |
|--------------|-------|

Let $W = \text{span} \left(\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$. Compute the dimension of W .

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since there are two pivot

columns, $\dim W = 2$.

□

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|---------------------|-------|
| Standard A1. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_3 + 3x_1].$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R} .

Solution:

$$\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

□

| | |
|---------------------|-------|
| Standard A2. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $T \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x+y \\ \sqrt{x} + \sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

Solution:

$$T \left(\begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

So T is not a linear transformation.

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| Additional Notes/Marks | |
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MASTERY QUIZ DAY 18

Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| | |
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| Standard S1. | Mark: |
|---------------------|-------|

Determine if the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent.

□

| | |
|---------------------|-------|
| Standard S3. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left(\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$. Find a basis for W .

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are

pivot columns, $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$ is a basis for W .

□

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|---------------------|-------|
| Standard S4. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find the dimension of W .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

□

| | |
|---------------------|-------|
| Standard A1. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 7x + 2y + 3z \\ 0 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R}^4 .

Solution:

$$\begin{bmatrix} 3 & 1 & 0 \\ -8 & 2 & -1 \\ 7 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

□

| | |
|---------------------|-------|
| Standard A2. | Mark: |
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Determine if the map $T : \mathcal{P} \rightarrow \mathcal{P}$ given by $T(f) = f' - f''$ is a linear transformation or not.

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| Additional Notes/Marks | |
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MASTERY QUIZ DAY 18

Math 237 – Linear Algebra

Version 4

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| | |
|---------------------|-------|
| Standard S1. | Mark: |
|---------------------|-------|

Determine if the set of polynomials $\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

□

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| Standard S3. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left(\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$. Find a basis for W .

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since the first two columns are

pivot columns, $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$ is a basis for W .

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| Standard S4. | Mark: |
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Let $W = \text{span} \left(\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\} \right)$. Compute the dimension of W .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has 3 pivot columns so $\dim(W) = 3$.

□

| | |
|---------------------|-------|
| Standard A1. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_2 + 3x_3].$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R} .

Solution:

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

□

| | |
|---------------------|-------|
| Standard A2. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $T \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x+y \\ \sqrt{x}+\sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

Solution:

$$T \left(\begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4T \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

So T is not a linear transformation.

□

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| Additional Notes/Marks | |
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MASTERY QUIZ DAY 18

Math 237 – Linear Algebra

Version 5

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| | |
|---------------------|-------|
| Standard S1. | Mark: |
|---------------------|-------|

Determine if the set of polynomials $\{-3x^3 - 8x^2, x^3 + 2x^2 + 2, -x^2 + 3\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

□

| | |
|---------------------|-------|
| Standard S3. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find a basis for W .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then a basis is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}$.

□

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|---------------------|-------|
| Standard S4. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left(\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$. Compute the dimension of W .

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Since there are two pivot columns, $\dim W = 2$. □

| | |
|---------------------|-------|
| Standard A1. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = [x_3 + 3x_1].$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R} .

Solution:

$$\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

□

| | |
|---------------------|-------|
| Standard A2. | Mark: |
|---------------------|-------|

Determine if the map $T : \mathcal{P}^6 \rightarrow \mathcal{P}^6$ given by $T(f) = f(x) - f(0)$ is a linear transformation or not.

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| Additional Notes/Marks | |
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MASTERY QUIZ DAY 18

Math 237 – Linear Algebra

Version 6

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

| | |
|---------------------|-------|
| Standard S1. | Mark: |
|---------------------|-------|

Determine if the set of vectors $\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$ is linearly dependent or linearly independent

Solution:

$$\text{RREF} \left(\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent.

□

| | |
|---------------------|-------|
| Standard S3. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$. Find a basis for this vector space.

Solution:

$$\text{RREF} \left(\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \\ 2 & -1 & -8 \\ 1 & 1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$ is a basis of W .

□

| | |
|---------------------|-------|
| Standard S4. | Mark: |
|---------------------|-------|

Let $W = \text{span} \left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\} \right)$. Find the dimension of W .

Solution:

$$\text{RREF} \left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This has two pivot columns, so W has dimension 2.

□

| | |
|---------------------|-------|
| Standard A1. | Mark: |
|---------------------|-------|

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_2 + 3x_3].$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R} .

Solution:

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

□

| | |
|---------------------|-------|
| Standard A2. | Mark: |
|---------------------|-------|

Determine if $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $T \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$ is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

□

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| Additional Notes/Marks | |
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