Application Activities - Module V Part 1 - Class Day 7

Activity 7.1 Consider each of the following vector properties. Label each property with \mathbb{R}^1 , \mathbb{R}^2 , and/or \mathbb{R}^3 if that property holds for Euclidean vectors/scalars $\mathbf{u}, \mathbf{v}, \mathbf{w}$ of that dimension.

1. Addition associativity.

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

2. Addition commutivity.

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$
.

3. Addition identity.

There exists some **0** where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

4. Addition inverse.

There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

5. Addition midpoint uniqueness.

There exists a unique \mathbf{m} where the distance from \mathbf{u} to \mathbf{m} equals the distance from \mathbf{m} to \mathbf{v} .

6. Scalar multiplication associativity.

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

7. Scalar multiplication identity.

$$1\mathbf{v} = \mathbf{v}$$
.

8. Scalar multiplication relativity.

There exists some scalar c where either $c\mathbf{v} = \mathbf{w}$ or $c\mathbf{w} = \mathbf{v}$.

9. Scalar distribution.

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

10. Vector distribution.

$$(a+b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

11. Orthogonality.

There exists a non-zero vector \mathbf{n} such that \mathbf{n} is orthogonal to both \mathbf{u} and \mathbf{v} .

12. Bidimensionality.

 $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ for some value of a, b.

Definition 7.2 A vector space V is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ belong to V, and let a, b be scalar numbers.

• Addition associativity.

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

• Addition commutivity.

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$
.

• Addition identity.

There exists some **0** where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

• Addition inverse.

There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

• Scalar multiplication associativity.

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

• Scalar multiplication identity.

$$1\mathbf{v} = \mathbf{v}$$
.

• Scalar distribution.

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

• Vector distribution.

$$(a+b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

Definition 7.3 The most important examples of vector spaces are the **Euclidean vector spaces** \mathbb{R}^n , but there are other examples as well.

Activity 7.4 Consider the following vector space that models motion along the curve $y = e^x$. Let $V = \{(x,y): y = e^x\}$, where $(x_1,y_1) \oplus (x_2,y_2) = (x_1 + x_2, y_1y_2)$, and $c(x,y) = (cx,y^c)$. Prove that V paired with these operations satisfies all the vector space properties.

• Addition associativity.

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

• Addition commutivity.

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$
.

• Addition identity.

There exists some $\mathbf{0}$ where $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

• Addition inverse.

There exists some $-\mathbf{v}$ where $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

• Scalar multiplication associativity.

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

 \bullet Scalar multiplication identity.

$$1\mathbf{v} = \mathbf{v}$$
.

• Scalar distribution.

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

• Vector distribution.

$$(a+b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

For example, Addition Commutivity may be proven by showing

$$\mathbf{u} \oplus \mathbf{v} = (x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 y_2) = (x_2 + x_1, y_2 y_1) = (x_2, y_2) + (x_1, y_1) = \mathbf{v} \oplus \mathbf{u}$$