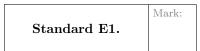
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Write a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{bmatrix}$$

Solution:

$$2x_1 - x_2 = 1$$
$$-x_1 + 4x_2 + x_3 = -7$$
$$x_1 + 2x_2 - x_3 = 0$$

Standard E3.

Solve the system of linear equations.

Mark:

$$2x + y - z + w = 5$$
$$3x - y - 2w = 0$$
$$-x + 5z + 3w = -1$$

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 1 & -1 & 0 & 5 \\ 3 & -1 & 0 & -2 & 0 \\ -1 & 0 & 5 & 0 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{12} & 1 \\ 0 & 1 & 0 & \frac{7}{4} & 3 \\ 0 & 0 & 1 & \frac{7}{12} & 0 \end{bmatrix}$$

So the solutions are

$$\left\{ \begin{bmatrix} 1+a\\ 3-21a\\ -7a\\ 12a \end{bmatrix} \mid a \in \mathbb{R} \right\}$$

Standard E4.

Mark:

Find a basis for the solution set to the homogeneous system of equations

$$4x_1 + 4x_2 + 3x_3 - 6x_4 = 0$$
$$-2x_3 - 4x_4 = 0$$
$$2x_1 + 2x_2 + x_3 - 4x_4 = 0$$

Standard V1.

Mark:

Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 3 = x$, so 3 is the additive identity.
- 3) $x \oplus (6-x) = x + (6-x) 3 = 3$, so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so \oplus is commutative.

5)

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$

$$= c (dx - 3(d - 1)) - 3(c - 1)$$

$$= cdx - 3(cd - 1)$$

$$= (cd) \odot x$$

6)
$$1 \odot x = x - 3(1 - 1) = x$$

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$

= $cx - 3(c-1) + dx - 3(c-1) - 3$
= $(c \odot x) \oplus (d \odot x)$

Therefore V is a vector space.

 ${\bf Additional\ Notes/Marks}$