Math 237 – Linear Algebra

Version 1

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A1. Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\\x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3\\3x_2 - x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and \mathbb{R}^2 .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

A2. Determine if the map $T: \mathcal{P}^6 \to \mathcal{P}^6$ given by T(f) = f(x) - f(0) is a linear transformation or not.

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CB is the only one that can be computed, and

$$CB = \begin{bmatrix} 3 & 3 & -5 & 7 \\ 4 & -4 & 12 & -12 \\ 7 & 2 & 0 & 3 \end{bmatrix}$$

A1:

M1:

Name:	

Math 237 – Linear Algebra

Version 2

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_3 + 3x_1\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R} .

Solution:

$$\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

A2. Determine if $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} e^x \\ e^y \end{bmatrix}$ is a linear transformation.

Solution: It is not linear. For example,

$$\begin{bmatrix} e^2 \\ 1 \end{bmatrix} = T \left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \neq 2T \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2e \\ 1 \end{bmatrix}$$

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CB is the only one that can be computed, and

$$CB = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

A1: M1:

Name:		

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Version 3

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive

credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A1. Let $T: \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix} x_2 + 3x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^3 and \mathbb{R} .

Solution:

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$$

A2. Determine if the map $T: \mathcal{P}^6 \to \mathcal{P}^7$ given by T(f) = xf(x) - f(1) is a linear transformation or not.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

A1:

M1:

Name:	

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Version 4

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive

credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A1. Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3\\3x_2 - 5x_3\end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and \mathbb{R}^2 .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -5 & 0 \end{bmatrix}$$

A2. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ \sqrt{x}+\sqrt{y} \end{bmatrix}$. Determine if T is a linear transformation.

Solution:

$$T\left(\begin{bmatrix}0\\4\end{bmatrix}\right) = \begin{bmatrix}4\\2\end{bmatrix} \neq \begin{bmatrix}4\\4\end{bmatrix} = 4T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

So T is not a linear transformation.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

A1: M1:

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Version 5

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive

credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A1. Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\\x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3\\3x_2 - x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and \mathbb{R}^2 .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$

A2. Determine if the map $T: \mathcal{P}^3 \to \mathcal{P}^4$ given by T(f(x)) = xf(x) - f(x) is a linear transformation or not.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

A1:

M1:

Name:	

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Version 6

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive

show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

A1. Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1\\x_2\\x_3\\x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3\\3x_2 - 5x_3 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of \mathbb{R}^4 and \mathbb{R}^2 .

Solution:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 3 & -5 & 0 \end{bmatrix}$$

A2. Determine if the map $T: \mathcal{P}^3 \to \mathcal{P}^4$ given by T(f(x)) = xf(x) - f(x) is a linear transformation or not.

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CB is the only one that can be computed, and

$$CB = \begin{bmatrix} 0 & -3 & 7 & -8 \\ 8 & 4 & -4 & 8 \\ 5 & -2 & 8 & -7 \end{bmatrix}$$

A1:

A2:

M1: