Name:	
J#:	Dr. Clontz
Date:	

Version 1

 $Math\ 237-Linear\ Algebra$

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard V1.	Mark:

Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (0, cy_1)$

- (a) Show that this scalar multiplication \odot distributes over scalar addition.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$, and let $c, d \in \mathbb{R}$. Then

$$(c+d) \odot (x_1, y_1) = (0, (c+d)y_1) = (0, cy_1) \oplus (0, dy_1) = c \odot (x_1, y_1) \oplus d \odot (x_1, y_1).$$

However, V is not a vector space, as $1 \odot (x_1, y_1) = (0, y_1) \neq (x_1, y_1)$.

Standard V3.

Determine if the vectors $\begin{bmatrix} 8\\21\\-7 \end{bmatrix}$, $\begin{bmatrix} -3\\-8\\3 \end{bmatrix}$, $\begin{bmatrix} -1\\-3\\2 \end{bmatrix}$, and $\begin{bmatrix} 4\\11\\-5 \end{bmatrix}$ span \mathbb{R}^3 .

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

Standard V4.

Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

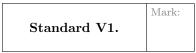
Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

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Version 2

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Let V be the set of all polynomials with the operations, for any $f, g \in V$, $c \in \mathbb{R}$,

$$f \oplus g = f' + g'$$
$$c \odot f = cf'$$

(here f' denotes the derivative of f).

- (a) Show that this scalar multiplication \odot distributes over vector addition \oplus .
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f.

Standard V3.

Mark: $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \text{ and } \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \text{ span } \mathbb{R}^3$

Solution:

$$RREF\left(\begin{bmatrix} -3 & 5 & 2 & 0\\ 1 & -1 & 0 & 2\\ 1 & -2 & -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 5\\ 0 & 1 & 1 & 3\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span \mathbb{R}^3 .

Standard V4.

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under sea both in W, but $(x - x^2) + (x^2) = 0$	but not ur	nder addition.	For example,	$x - x^2$ and x^2
Additional Notes/Marks				

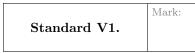
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Version 3

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

- (a) Show that the vector addition \oplus is associative.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $x, y, z \in \mathbb{R}$. Then

$$(x \oplus y) \oplus z = \sqrt{x^2 + y^2} \oplus z$$

$$= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2}$$

$$= \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{x^2 + (\sqrt{y^2 + z^2})^2}$$

$$= x \oplus \sqrt{y^2 + z^2}$$

$$= x \oplus (y \oplus z)$$

However, this is not a vector space, as there is no zero vector.

Determine if the vectors
$$\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$
, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

$$RREF\left(\begin{bmatrix} 2 & 3 & 0 & 1\\ 0 & 1 & 0 & 2\\ -2 & 3 & 1 & 0\\ 0 & 6 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2}\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & -11\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span \mathbb{R}^4 .

Standard V4.	Mark:
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Let W be the set of all 2 by 2 matrices which are not invertible. Determine if W is a subspace of $M_{2,2}$.

Solution: W is closed under scalar multiplication, but not under addition.

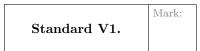
Additional Notes/Marks

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Version 4

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Let V be the set of all pairs of real numbers with the operations, for any $(x_1, y_1), (x_2, y_2) \in V, c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

 $c \odot (x_1, y_1) = (c^2 x_1, c^3 y_1)$

- (a) Show that this scalar multiplication \odot distributes over vector addition \oplus .
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1), (x_2, y_2) \in V$ and let $c \in \mathbb{R}$.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1 + x_2, y_1 + y_2)$$

$$= (c^2(x_1 + x_2), c^3(y_1 + y_2))$$

$$= (c^2x_1, c^3y_1) \oplus (c^2x_2, c^3y_2)$$

$$= c \odot (x_1, y_1) \oplus c \odot (x_2, y_2)$$

However, V is not a vector space, as the other distributive law fails:

$$(c+d)\odot(x_1,y_1)=((c+d)^2x_1,(c+d)^3y_1)\neq((c^2+d^2)x_1,(c^3+d^3)y_1)=c\odot(x_1,y_1)\oplus d\odot(x_1,y_1).$$

Standard V3.

Mark: $\begin{bmatrix} 8 \\ 21 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}, \text{ and } \begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix} \text{ span } \mathbb{R}^3.$

Solution:

RREF
$$\begin{pmatrix} \begin{bmatrix} 8 & -3 & -1 & 4 \\ 21 & -8 & -3 & 11 \\ -7 & 3 & 2 & -5 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the rank is less than 3, they do not span \mathbb{R}^3 .

Standard V4.	Mark:

Determine if the set of all lattice points, i.e. $\{(x,y) \mid x \text{ and } y \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .

Solution: This set is closed under addition, but not under scalar multiplication so it is not a subspace.

Additional Notes/Marks

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Math 237 – Linear Algebra Fall 2017

Version 5

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard V1.	

Let V be the set of all real numbers together with the operations \oplus and \odot defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$x \oplus y = x + y - 3$$
$$c \odot x = cx - 3(c - 1)$$

- (a) Show that this scalar multiplication \odot is associative.
- (b) Determine if V is a vector space or not. Justify your answer

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$. To show associativity:

$$c \odot (d \odot x) = c \odot (dx - 3(d - 1))$$
$$= c (dx - 3(d - 1)) - 3(c - 1)$$
$$= cdx - 3(cd - 1)$$
$$= (cd) \odot x$$

We verify the remaining 7 properties to see that V is a vector space.

- 1) Real addition is associative, so \oplus is associative.
- 2) $x \oplus 3 = x + 3 3 = x$, so 3 is the additive identity.
- 3) $x \oplus (6-x) = x + (6-x) 3 = 3$, so 6-x is the additive inverse of x.
- 4) Real addition is commutative, so \oplus is commutative.
- 5) Associativity shown above
- 6) $1 \odot x = x 3(1 1) = x$

7)

$$c \odot (x \oplus y) = c \odot (x + y - 3)$$

$$= c(x + y - 3) - 3(c - 1)$$

$$= cx - 3(c - 1) + cy - 3(c - 1) - 3$$

$$= (c \odot x) \oplus (c \odot y)$$

8)

$$(c+d) \odot x = (c+d)x - 3(c+d-1)$$

= $cx - 3(c-1) + dx - 3(c-1) - 3$
= $(c \odot x) \oplus (d \odot x)$

Therefore V is a vector space.

Standard V3.

Mark:

Determine if the vectors $\begin{bmatrix} -3\\1\\1 \end{bmatrix}$, $\begin{bmatrix} 5\\-1\\-2 \end{bmatrix}$, $\begin{bmatrix} 2\\0\\-1 \end{bmatrix}$, and $\begin{bmatrix} 0\\2\\-1 \end{bmatrix}$ span \mathbb{R}^3

Solution:

$$RREF \left(\begin{bmatrix} -3 & 5 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -2 & -1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix has only two pivot columns, the vectors do not span \mathbb{R}^3 .

Standard V4.

Mark:

Let W be the set of all polynomials of even degree. Determine if W is a subspace of the vector space of all polynomials.

Solution: W is closed under scalar multiplication, but not under addition. For example, $x - x^2$ and x^2 are both in W, but $(x - x^2) + (x^2) = x \notin W$.

Additional Notes/Marks

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Math 237 – Linear Algebra Fall 2017

Version 6

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

	Mark:
Standard V1.	

Let V be the set of all points on the line x + y = 2 with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1)$$

 $c \odot (x_1, y_1) = (cx_1 - (c - 1), cy_1 - (c - 2))$

- (a) Show that this vector space has an additive identity element.
- (b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $(x_1, y_1) \in V$; then $(x_1, y_1) \oplus (1, 1) = (x_1, y_1)$, so (1, 1) is an additive identity element. Now we will show the other seven properties. Let $(x_1, y_1), (x_2, y_2) \in V$, and let $c, d \in \mathbb{R}$.

- 1) Since real addition is associative, \oplus is associative.
- 2) Since real addition is commutative, \oplus is commutative.
- 3) The additive identity is (1,1).
- 4) $(x_1, y_1) \oplus (2 x_1, 2 y_1) = (1, 1)$, so $(2 x_1, 2 y_1)$ is the additive inverse of (x_1, y_1) .
- 5)

$$\begin{split} c\odot(d\odot(x_1,y_1)) &= c\odot(dx_1-(d-1),dy_1-(d-1))\\ &= (c(dx_1-(d-1))-(c-1),c(dy_1-(d-1)))\\ &= (cdx_1-cd+c-(c-1),cdy_1-cd+c-(c-1))\\ &= (cdx_1-(cd-1),cdy_1-(cd-1))\\ &= (cd)\odot(x_1,y_1) \end{split}$$

6)
$$1 \odot (x_1, y_1) = (x_1 - (1 - 1), y_1 - (1 - 1)) = (x_1, y_1)$$

7)

$$\begin{split} c\odot((x_1,y_1)\oplus(x_2,y_2)) &= c\odot(x_1+y_1-1,x_2+y_2-1)\\ &= (c(x_1+y_1-1)-(c-1),c(x_2+y_2-1)-(c-1))\\ &= (cx_1+cx_2-2c+1,cy_1+cy_2-2c+1)\\ &= (cx_1-(c-1),cy_1-(c-1))\oplus(cx_2-(c-1),cy_2-(c-1))\\ &= c\odot(x_1,y_1)\oplus c\odot(x_2,y_2) \end{split}$$

$$(c+d)\odot(x_1,y_1) = ((c+d)x_1 - (c+d-1), (c+d)y_1 - (c+d-1))$$

= $(cx_1 - (c-1), cy_1 - (c-1)) \oplus (dx_1 - (d-1), dy_1 - (d-1))$
= $c\odot(x_1, y_1) \oplus c\odot(x_2, y_2)$

Therefore V is a vector space.

Standard V3.

Determine if the vectors $\begin{bmatrix} 2\\0\\-2\\0 \end{bmatrix}$, $\begin{bmatrix} 3\\1\\3\\6 \end{bmatrix}$, $\begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}$, and $\begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}$ span \mathbb{R}^4 .

Solution:

RREF
$$\left(\begin{bmatrix} 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a zero row, the vectors do not span \mathbb{R}^4 .

Mark:

Standard V4.

Mark:

Let W be the set of all 2 by 2 matrices which are not invertible. Determine if W is a subspace of $M_{2,2}$.

Solution: W is closed under scalar multiplication, but not under addition.

Additional Notes/Marks