Name: ________________________________

MASTERY QUIZ DAY 10 Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations

$$\begin{aligned} x + 3y - 4z &= 5 \\ 3x + 9y + z &= 0 \\ x - z &= 1 \end{aligned}$$

E3. Solve the system of equations

$$\begin{aligned} x + 3y + 3z + 7w &= 0 \\ x + 3y - z - w &= 0 \\ 2x + 6y + 3z + 8w &= 0 \\ x + 3y - 2z - 3w &= 0 \end{aligned}$$

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\} = \mathrm{span}\left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}\right)$$

$\square$

E4. Find a basis for the solution set of the system ...

V1. Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c - 1) \end{aligned}$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5)

$$\begin{aligned}
c \odot (d \odot x) &= c \odot (dx - 3(d-1)) \\
&= c\,(dx - 3(d-1)) - 3(c-1) \\
&= cdx - 3(cd-1) \\
&= (cd) \odot x
\end{aligned}$$

6) $1 \odot x = x - 3(1-1) = x$

7)

$$\begin{aligned}
c \odot (x \oplus y) &= c \odot (x + y - 3) \\
&= c(x + y - 3) - 3(c-1) \\
&= cx - 3(c-1) + cy - 3(c-1) - 3 \\
&= (c \odot x) \oplus (c \odot y)
\end{aligned}$$

8)

$$\begin{aligned}
(c+d) \odot x &= (c+d)x - 3(c+d-1) \\
&= cx - 3(c-1) + dx - 3(c-1) - 3 \\
&= (c \odot x) \oplus (d \odot x)
\end{aligned}$$

Therefore V is a vector space.

$\square$

E1: $\boxed{}$ **E3:** $\boxed{}$ **E4:** $\boxed{}$ **V1:** $\boxed{}$