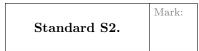
Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 19

Math 237 – Linear Algebra Fall 2017

Version 2

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.



Determine if the set $\{x^2 + x - 1, 3x^2 - x + 1, 2x - 2\}$ is a basis of \mathcal{P}_2

Solution:

RREF
$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

Standard A3.

Mark:

Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+3z \\ x-y-2z \end{bmatrix}$

(b)
$$S: \mathbb{R}^2 \to \mathbb{R}^3$$
 given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

Solution:

(a)

RREF
$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b) $RREF\left(\begin{bmatrix} 3 & 2\\ 1 & -1\\ 1 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0\\ 0 & 1\\ 0 & 0 \end{bmatrix}$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

Standard A4. Mark:

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \end{bmatrix}$$

Compute the kernel and image of T.

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Then the image is the span of

the (pivot) columns, so

$$\operatorname{Im} T = \operatorname{span} \left(\left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\} \right)$$

The kernel is the solution set of AX = 0, so

$$\ker T = \left\{ \begin{bmatrix} c \\ 3c \\ -2c \end{bmatrix} \middle| c \in \mathbb{R} \right\} = \operatorname{span} \left(\left\{ \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \right\} \right)$$