Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Add Euclidean vectors and multiply Euclidean vectors by scalars.
- Perform basic manipulations of augmented matrices and linear systems **(Standard(s) E1,E2,E3)**.
- Apply linear combinations and spanning sets **(Standard(s) V2,V3)**.

Readiness Assurance Resources

The following resources will help you prepare for this module.

- `https://www.khanacademy.org/math/precalculus/vectors-precalc/vector-addition-subtraction/v/adding-and-subtracting-vectors`
- `https://www.khanacademy.org/math/precalculus/vectors-precalc/combined-vector-operations/v/combined-vector-operations-example`

Readiness Assurance Test

Choose the most appropriate response for each question.

1) Simplify the following vector expression.

$$4\begin{bmatrix}1\\2\\3\end{bmatrix} - 2\begin{bmatrix}1\\2\\3\end{bmatrix}$$

(a) $\begin{bmatrix}2\\4\\6\end{bmatrix}$ (b) $\begin{bmatrix}1\\-2\\-4\end{bmatrix}$ (c) $\begin{bmatrix}0\\2\\0\end{bmatrix}$ (d) $\begin{bmatrix}1\\2\\3\end{bmatrix}$

2) Express the following system of linear equations as an augmented matrix.

$$\begin{aligned} 2x_1 + \ x_2 + 4x_3 &= \ 0 \\ x_1 + \ x_2 + \ x_3 &= \ 1 \\ -3x_1 + 4x_2 + \ x_3 &= -7 \end{aligned}$$

(a) $\left[\begin{array}{cc|c}2&1&-3\\1&1&4\\4&1&1\\0&1&-7\end{array}\right]$ (b) $\left[\begin{array}{ccc|c}2&1&4&0\\1&1&1&1\\-3&4&1&-7\end{array}\right]$ (c) $\left[\begin{array}{cc|c}1&1&1\\1&-2&4\\4&1&1\\0&1&-7\end{array}\right]$ (d) $\left[\begin{array}{cc|c}2&1&4\\1&1&1\\-3&4&-7\end{array}\right]$

3) Find RREF $\left[\begin{array}{cc|c}1&2&3\\3&2&5\\-2&0&-2\end{array}\right]$.

(a) $\left[\begin{array}{cc|c}1&0&1\\0&1&1\\0&0&0\end{array}\right]$ (b) $\left[\begin{array}{cc|c}1&2&3\\1&3&4\\0&0&0\end{array}\right]$ (c) $\left[\begin{array}{cc|c}1&2&3\\0&1&1\\0&0&0\end{array}\right]$ (d) $\left[\begin{array}{cc|c}1&0&0\\0&1&0\\0&0&1\end{array}\right]$

4) Solve the following system of linear equations.

$$\begin{aligned} 2x_1 + \ x_2 + 4x_3 &= \ 0 \\ x_1 + \ x_2 + \ x_3 &= \ 1 \\ -3x_1 + 4x_2 + \ x_3 &= -7 \end{aligned}$$

(a) $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}2\\0\\-1\end{bmatrix}$

(b) $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\-6\\1\end{bmatrix}$

(c) $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\-2\\1\end{bmatrix} + a\begin{bmatrix}1\\0\\3\end{bmatrix}$ for all real numbers a

(d) No solutions

5) Solve the following system of linear equations.

$$\begin{aligned} x_1 + \ x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 3x_2 \qquad + x_4 &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \\ -5 \end{bmatrix} + a \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ for all real numbers a

(c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ 0 \\ 0 \end{bmatrix} + a \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ for all real numbers a, b

(d) No solutions

6) How many vectors are required to span all of $\mathbb{R}^4$ (the space of Euclidean vectors with four components)?

(a) 2 (b) 3 (c) 4 (d) 5

7) How many vectors are required to span all of $\mathcal{P}^4$ (the space of polynomials of degree four or less)?

(a) 2 (b) 3 (c) 4 (d) 5

8) Which vector is a linear combination of $\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 3 \\ -7 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} -5 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

9) Which vector belongs to span $\left\{ \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$?

(a) $\begin{bmatrix} 3 \\ -7 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

10) The graphical representation of span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ in three-dimensional Euclidean space $\mathbb{R}^3$ would be which of the following?

(a) a line (b) a plane (c) a sphere (d) all of $\mathbb{R}^3$