Name: ________________________________

MASTERY QUIZ DAY 28 Math 237 – Linear Algebra
Version 2 Fall 2017

Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

M1. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 7 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 7 & 7 \\ -1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

Solution: CA is the only one that can be computed, and

$$CA = \begin{bmatrix} 3 & 9 & 11 & 1 \\ 0 & 0 & 7 & 2 \\ -2 & -6 & -5 & 0 \end{bmatrix}$$

□

M2. Determine if the matrix $\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix}$ is invertible.

Solution:

$$\text{RREF} \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 \\ 3 & 2 & -1 & 7 \\ 4 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since it is row equivalent to the identity matrix, it is invertible.

□

M3. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 2 & -1 & -6 \\ 1 & 1 & 4 \end{bmatrix}$.

Solution: $\left[\begin{array}{ccc|ccc} 3 & 1 & 3 & 1 & 0 & 0 \\ 2 & -1 & -6 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & -3 \\ 0 & 1 & 0 & -14 & 9 & 24 \\ 0 & 0 & 1 & 3 & -2 & -5 \end{array}\right]$. Thus the inverse is $\begin{bmatrix} 2 & -1 & -3 \\ -14 & 9 & 24 \\ 3 & -2 & -5 \end{bmatrix}$.

□

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 8 & -3 & -1 \\ 21 & -8 & -3 \\ -7 & 3 & 2 \end{bmatrix}$.

Solution:

$$\begin{aligned}\det(A-\lambda I) &= (8-\lambda)\det\begin{bmatrix}-8-\lambda & -3\\ 3 & 2-\lambda\end{bmatrix} - (-3)\det\begin{bmatrix}21 & -3\\ -7 & 2-\lambda\end{bmatrix} + (-1)\det\begin{bmatrix}21 & -8-\lambda\\ -7 & 3\end{bmatrix}\\ &= (8-\lambda)\left(\lambda^2+6\lambda-7\right) + 3\left(-21\lambda+21\right) - \left(-7\lambda+7\right)\\ &= (\lambda-1)\left((8-\lambda)(\lambda+7)-63+7\right)\\ &= (\lambda-1)(\lambda-\lambda^2)\\ &= -\lambda(\lambda-1)^2\end{aligned}$$

So the eigenvalues are 0 (with algebraic multiplicity 1) and 1 (with algebraic multiplicity 2).

□

G3. Compute the eigenspace of the eigenvalue -1 in the matrix $\begin{bmatrix}4 & -2 & -1\\ 15 & -7 & -3\\ -5 & 2 & 0\end{bmatrix}$.

Solution:

$$\operatorname{RREF}(A+I) = \begin{bmatrix}1 & -\frac{2}{5} & -\frac{1}{5}\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

So the eigenspace is spanned by $\begin{bmatrix}2\\5\\0\end{bmatrix}$ and $\begin{bmatrix}1\\0\\5\end{bmatrix}$.

□

M1: ☐ **M2:** ☐ **M3:** ☐ **G2:** ☐ **G3:** ☐ **G1:** ☐