Name:	
J#:	Dr. Clontz
Date:	

MASTERY QUIZ DAY 19 Math 237 – Linear Algebra
Version 1 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S2.	Mark:

Determine if the set $\left\{x^2+x-1, 3x^2-x+1, 2x-2\right\}$ is a basis of $\mathcal{P}_2$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

□

Standard A3.	Mark:

Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

Solution:

(a) $\det \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = 1$, so the matrix is invertible, hence S is bijective.

(b) Since $\dim \mathbb{R}^4 > \dim \mathbb{R}^3$, T is not injective.

$$\text{RREF}\left(\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -5 & -\frac{5}{2} \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are only two pivot columns, T is not surjective.

□

Standard A4.	Mark:

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x + y \\ -8x + 2y - z \\ 2y + 3z \end{bmatrix}$$

Compute the kernel and image of T.

Solution: Let $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$, and compute $\mathrm{RREF}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$. Then the image is the span of the (pivot) columns, so

$$\mathrm{Im}\, T = \mathrm{span}\left(\left\{\begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}\right\}\right)$$

The kernel is the solution set of $AX = 0$, so

$$\ker T = \left\{\begin{bmatrix} c \\ 3c \\ -2c \end{bmatrix} \;\middle|\; c \in \mathbb{R}\right\} = \mathrm{span}\left(\left\{\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}\right\}\right)$$

□

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Version 2 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S2.	Mark:

Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of $\mathcal{P}_3$

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

□

Standard A3.	Mark:

Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y + z \\ 2y + 3z \\ x - y - 2z \end{bmatrix}$

(b) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

Solution:

(a)

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b)

$$\mathrm{RREF}\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

□

Standard A4.	Mark:

Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} 8x - 3y - z + 4w \\ y + 3z - 4w \\ -7x + 3y + 2z - 5w \end{bmatrix}$. Compute the kernel and image of T.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 8 & -3 & -1 & 4 \\ 0 & 1 & 3 & -4 \\ -7 & 3 & 2 & -5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\operatorname{Im}(T) = \operatorname{span}\left(\left\{\begin{bmatrix} 8 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}\right\}\right)$$

$$\ker(T) = \operatorname{span}\left(\left\{\begin{bmatrix} 1 \\ 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 0 \\ 1 \end{bmatrix}\right\}\right)$$

$\square$

Additional Notes/Marks	

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MASTERY QUIZ DAY 19 Math 237 – Linear Algebra
Version 3 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the resulting matrix is the identity matrix, it is a basis.

$\square$

Standard A3.	Mark:

Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + y + z \\ 2y + 3z \\ x - y - 2z \end{bmatrix}$

(b) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

Solution:

(a)

$$\text{RREF}\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b)

$$\text{RREF}\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

$\square$

Standard A4.	Mark:

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+3y+3z+7w \\ x+3y-z-w \\ 2x+6y+3z+8w \\ x+3y-2z-3w \end{bmatrix}$$

Compute the kernel and image of T.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the kernel is

$$\ker(T) = \left\{ \begin{bmatrix} -3a-b \\ a \\ -2b \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\} = \text{span}\left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}\right)$$

and the image is

$$\text{Im}(T) = \text{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\}\right) = \text{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}\right)$$

$\square$

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MASTERY QUIZ DAY 19 Math 237 – Linear Algebra
Version 4 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S2.	Mark:

Determine if the set $\{x^3 - x, x^2 + x + 1, x^3 - x^2 + 2, 2x^2 - 1\}$ is a basis of $\mathcal{P}_3$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the resulting matrix is not the identity matrix, it is not a basis.

□

Standard A3.	Mark:

Determine if the following linear maps are injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y \\ x - y \\ x + 4y \end{bmatrix}$

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y + z \\ 2y + 3z \\ x - y - 2z \end{bmatrix}$

Solution:

(a)

$$\text{RREF}\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b)

$$\text{RREF}\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

□

Standard A4.	Mark:

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+3y+3z+7w \\ x+3y-z-w \\ 2x+6y+3z+8w \\ x+3y-2z-3w \end{bmatrix}$$

Compute the kernel and image of T.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the kernel is

$$\ker(T) = \left\{ \begin{bmatrix} -3a-b \\ a \\ -2b \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\} = \mathrm{span}\left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}\right)$$

and the image is

$$\mathrm{Im}(T) = \mathrm{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\}\right) = \mathrm{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}\right)$$

□

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MASTERY QUIZ DAY 19 Math 237 – Linear Algebra
Version 5 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S2.	Mark:

Determine if the set $\left\{\begin{bmatrix}0\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\0\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\\-1\end{bmatrix}\right\}$ is a basis of $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}0 & 1 & 1 & 0\\1 & -1 & 0 & 2\\1 & 0 & -1 & 0\\1 & 2 & 0 & -1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0 & 1\\0 & 1 & 0 & -1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 0\end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

Standard A3.	Mark:

Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}x+y+z\\2y+3z\\x-y-2z\end{bmatrix}$

(b) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}3x+2y\\x-y\\x+4y\end{bmatrix}$

Solution:

(a)

$$\text{RREF}\left(\begin{bmatrix}1 & 1 & 1\\0 & 2 & 3\\1 & -1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & -\frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b)

$$\mathrm{RREF}\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

□

Standard A4.	Mark:

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute the kernel and image of T.

Solution:

$$\mathrm{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the kernel is

$$\ker(T) = \left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\} = \mathrm{span}\left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}\right)$$

and the image is

$$\mathrm{Im}(T) = \mathrm{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\}\right) = \mathrm{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}\right)$$

□

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MASTERY QUIZ DAY 19 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

Standard S2.	Mark:

Determine if the set $\left\{ \begin{bmatrix}0\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\0\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\\-1\end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.

Solution:

$$\text{RREF}\left(\begin{bmatrix}0 & 1 & 1 & 0\\1 & -1 & 0 & 2\\1 & 0 & -1 & 0\\1 & 2 & 0 & -1\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0 & 1\\0 & 1 & 0 & -1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 0\end{bmatrix}$$

Since this is not the identity matrix, the set is not a basis.

□

Standard A3.	Mark:

Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}x+y+z\\2y+3z\\x-y-2z\end{bmatrix}$

(b) $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by $S\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}3x+2y\\x-y\\x+4y\end{bmatrix}$

Solution:

(a)

$$\text{RREF}\left(\begin{bmatrix}1 & 1 & 1\\0 & 2 & 3\\1 & -1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & -\frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$$

Since there is a nonpivot column, T is not injective. Since there is a zero row, T is not surjective.

(b)

$$\text{RREF}\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 1 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since all columns are pivot columns, S is injective. Since there is a zero row, S is not surjective.

□

Standard A4.	Mark:

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 3z + 7w \\ x + 3y - z - w \\ 2x + 6y + 3z + 8w \\ x + 3y - 2z - 3w \end{bmatrix}$$

Compute the kernel and image of T.

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the kernel is

$$\ker(T) = \left\{ \begin{bmatrix} -3a - b \\ a \\ -2b \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\} = \text{span}\left(\left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}\right)$$

and the image is

$$\text{Im}(T) = \text{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 8 \\ -3 \end{bmatrix} \right\}\right) = \text{span}\left(\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ -2 \end{bmatrix} \right\}\right)$$

□

Additional Notes/Marks	