Name: ________________________________

MASTERY QUIZ DAY 10 Math 237 – Linear Algebra
Version 6 Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned} x+3y-4z&=5\\ 3x+9y+z&=0\\ x-z&=1 \end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & 5\\ 3 & 9 & 1 & 0\\ 1 & 0 & -1 & 1 \end{array}\right]$$

□

E3. Solve the system of equations

$$\begin{aligned} -3x+y&=2\\ -8x+2y-z&=6\\ 2y+3z&=-2 \end{aligned}$$

Solution:

$$\mathrm{RREF}\left(\left[\begin{array}{ccc|c} -3 & 1 & 0 & 2\\ -8 & 2 & -1 & 6\\ 0 & 2 & 3 & -2 \end{array}\right]\right) = \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1\\ 0 & 1 & \frac{3}{2} & -1\\ 0 & 0 & 0 & 0 \end{array}\right]$$

The solutions are

$$\left\{\begin{bmatrix} -1-\frac{c}{2}\\ -1-\frac{3c}{2}\\ c \end{bmatrix} \middle| \, c\in\mathbb{R}\right\} = \left\{\begin{bmatrix} c-1\\ 3c-1\\ -2c \end{bmatrix} \middle| \, c\in\mathbb{R}\right\}$$

□

E4. Find a basis for the solution set of the system of equations

$$\begin{aligned} x+2y+3z+w&=0\\ 3x-y+z+w&=0\\ 2x-3y-2z&=0 \end{aligned}$$

Solution:

$$\text{RREF}\left(\begin{bmatrix} 1 & -2 & 3 & 1 \\ 3 & -1 & 1 & 1 \\ 2 & -3 & -2 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & \frac{5}{7} & \frac{3}{7} \\ 0 & 1 & \frac{8}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the solution set is

$$\left\{ \begin{bmatrix} -\frac{5}{7}a - \frac{3}{7}b \\ -\frac{8}{7}a - \frac{2}{7}b \\ a \\ b \end{bmatrix} \,\middle|\, a, b \in \mathbb{R} \right\}$$

So a basis for the solution set is $\left\{ \begin{bmatrix} -\frac{5}{7} \\ -\frac{8}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{3}{7} \\ -\frac{2}{7} \\ 0 \\ 1 \end{bmatrix} \right\}$, or $\left\{ \begin{bmatrix} 5 \\ 8 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -7 \end{bmatrix} \right\}$.

□

V1. Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y - 3 \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

Determine if V is a vector space or not.

Solution: Let $x, y \in V$, $c, d \in \mathbb{R}$.

1) Real addition is associative, so $\oplus$ is associative.

2) $x \oplus 3 = x + 3 - 3 = x$, so 3 is the additive identity.

3) $x \oplus (6 - x) = x + (6 - x) - 3 = 3$, so $6 - x$ is the additive inverse of x.

4) Real addition is commutative, so $\oplus$ is commutative.

5)

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d-1)) \\ &= c\,(dx - 3(d-1)) - 3(c-1) \\ &= cdx - 3(cd - 1) \\ &= (cd) \odot x \end{aligned}$$

6) $1 \odot x = x - 3(1-1) = x$

7)

$$\begin{aligned} c \odot (x \oplus y) &= c \odot (x + y - 3) \\ &= c(x + y - 3) - 3(c-1) \\ &= cx - 3(c-1) + cy - 3(c-1) - 3 \\ &= (c \odot x) \oplus (c \odot y) \end{aligned}$$

8)

$$\begin{aligned}(c+d)\odot x &= (c+d)x - 3(c+d-1)\\ &= cx - 3(c-1) + dx - 3(c-1) - 3\\ &= (c\odot x)\oplus(d\odot x)\end{aligned}$$

Therefore V is a vector space.

$\square$

E1:		E3:		E4:		V1:		E2:	