Module P: Applications of Linear Algebra

Module P Section 1

Definition P.1.1

In geology, a **phase** is any physically separable material in the system, such as various minerals or liquids.

A **component** is a chemical compound necessary to make up the phases; for historical reasons these are usually oxides such as Calcium Oxide (CaO) or Silicone Dioxide $(\mathrm{SiO_2})$.

In a typical problem, a geologist knows how to build each phase from the components, and is interested in determining reactions among the different phases.

Activity P.1.2 *(∼5 min)*

Consider the 3 components $\mathbf{c}_1 = \mathrm{CaO}$, $\mathbf{c}_2 = \mathrm{MgO}$, and $\mathbf{c}_3 = \mathrm{SiO}_2$, and the 5 phases

$$\mathbf{p}_1 = \mathrm{Ca_3MgSi_2O_8} \qquad \mathbf{p}_2 = \mathrm{CaMgSiO_4} \qquad \mathbf{p}_3 = \mathrm{CaSiO_3}$$
$$\mathbf{p}_4 = \mathrm{CaMgSi_2O_6} \qquad \mathbf{p}_5 = \mathrm{Ca_2MgSi_2O_7}$$

Geologists will know

$$\mathbf{p}_1 = 3\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_2 = \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3 \qquad \mathbf{p}_3 = \mathbf{c}_1 + 0\mathbf{c}_2 + \mathbf{c}_3$$
$$\mathbf{p}_4 = \mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_5 = 2\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3$$

or more compactly,

$$\mathbf{p}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \mathbf{p}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{p}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{p}_4 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{p}_5 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}.$$

Determine if the 5 phases are linearly dependent or linearly independent.

Activity P.1.3 *(∼15 min)*

Recall our five phases:

$$\mathbf{p}_1 = 3\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_2 = \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3 \qquad \mathbf{p}_3 = \mathbf{c}_1 + 0\mathbf{c}_2 + \mathbf{c}_3$$
$$\mathbf{p}_4 = \mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_5 = 2\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3$$

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Geologists want to find chemical reactions among the 5 phases; that is, they want to find numbers x_1, x_2, x_3, x_4, x_5 such that

$$x_1\mathbf{p}_1 + x_2\mathbf{p}_2 + x_3\mathbf{p}_3 + x_4\mathbf{p}_4 + x_5\mathbf{p}_5 = 0.$$

Activity P.1.3 *(∼15 min)*
Recall our five phases:

$$\mathbf{p}_1 = 3\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_2 = \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3 \qquad \mathbf{p}_3 = \mathbf{c}_1 + 0\mathbf{c}_2 + \mathbf{c}_3$$
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$$\mathbf{p}_1 = \begin{bmatrix}3\\1\\2\end{bmatrix}, \mathbf{p}_2 = \begin{bmatrix}1\\1\\1\end{bmatrix}, \mathbf{p}_3 = \begin{bmatrix}1\\0\\1\end{bmatrix}, \mathbf{p}_4 = \begin{bmatrix}1\\1\\2\end{bmatrix}, \mathbf{p}_5 = \begin{bmatrix}2\\1\\2\end{bmatrix}.$$

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$$x_1\mathbf{p}_1 + x_2\mathbf{p}_2 + x_3\mathbf{p}_3 + x_4\mathbf{p}_4 + x_5\mathbf{p}_5 = 0.$$

Part 1: Set up a system of equations that gives these chemical equations.

Activity P.1.3 *(∼15 min)*
Recall our five phases:

$$\mathbf{p}_1 = 3\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_2 = \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3 \qquad \mathbf{p}_3 = \mathbf{c}_1 + 0\mathbf{c}_2 + \mathbf{c}_3$$
$$\mathbf{p}_4 = \mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_5 = 2\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3$$

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Geologists want to find chemical reactions among the 5 phases; that is, they want to find numbers x_1, x_2, x_3, x_4, x_5 such that

$$x_1\mathbf{p}_1 + x_2\mathbf{p}_2 + x_3\mathbf{p}_3 + x_4\mathbf{p}_4 + x_5\mathbf{p}_5 = 0.$$

Part 1: Set up a system of equations that gives these chemical equations.
Part 2: Find a basis for the solution set.

Activity P.1.3 *(∼15 min)*

Recall our five phases:

$$\mathbf{p}_1 = 3\mathbf{c}_1 + \mathbf{c}_2 + 2\mathbf{c}_3 \qquad \mathbf{p}_2 = \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3 \qquad \mathbf{p}_3 = \mathbf{c}_1 + 0\mathbf{c}_2 + \mathbf{c}_3$$
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Geologists want to find chemical reactions among the 5 phases; that is, they want to find numbers x_1, x_2, x_3, x_4, x_5 such that

$$x_1\mathbf{p}_1 + x_2\mathbf{p}_2 + x_3\mathbf{p}_3 + x_4\mathbf{p}_4 + x_5\mathbf{p}_5 = 0.$$

Part 1: Set up a system of equations that gives these chemical equations.
Part 2: Find a basis for the solution set.
Part 3: Interpret each basis vector as a chemical equation.

Activity P.1.4 *(∼10 min)*

We found two basis vector $\begin{bmatrix} -1 \\ 2 \\ 2 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, corresponding to two chemical equations

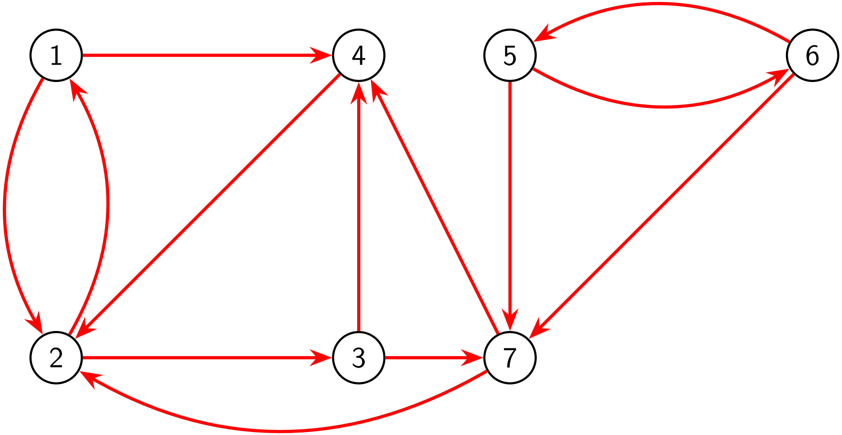
$$\begin{array}{cc} 2\mathbf{p}_2 + 2\mathbf{p}_3 = \mathbf{p}_1 + \mathbf{p}_4 & 2\mathrm{CaMgSiO}_4 + 2\mathrm{CaSiO}_3 = \mathrm{Ca}_3\mathrm{MgSi}_2\mathrm{O}_8 + \mathrm{CaMgSi}_2\mathrm{O}_6 \\ \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{p}_5 & \mathrm{CaMgSiO}_4 + \mathrm{CaSiO}_3 = \mathrm{Ca}_2\mathrm{MgSi}_2\mathrm{O}_7 \end{array}$$

Find a chemical equation among the five phases that does not involve $\mathbf{p}_2 = \mathrm{CaMgSiO}_4$.

Module P Section 2

Activity P.2.1 *(∼10 min)*
A $700,000,000,000 Problem:

In the picture below, each circle represents a webpage, and each arrow represents a link from one page to another.



Based on how these pages link to each other, write a list of the 7 webpages in order from most imporant to least important.

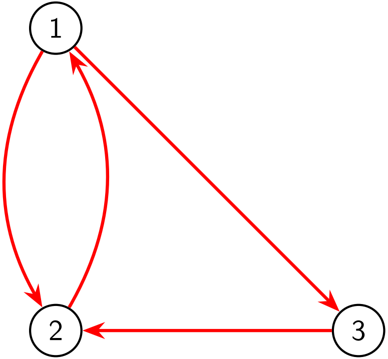
Observation P.2.2
The $700,000,000,000 Idea:

Links are endorsements.

1. A webpage is important if it is linked to (endorsed) by important pages.
2. A webpage distributes its importance equally among all the pages it links to (endorses).

Example P.2.3

Consider this small network with only three pages. Let x_1, x_2, x_3 be the importance of the three pages respectively.

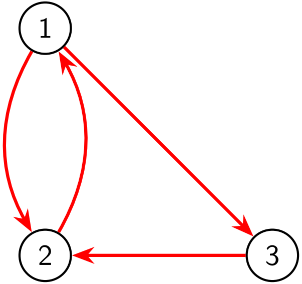


1. x_1 splits its endorsement in half between x_2 and x_3
2. x_2 sends all of its endorsement to x_1
3. x_3 sends all of its endorsement to x_2.

This corresponds to the **page rank system**

$$\begin{aligned} x_2 \quad &= x_1 \\ \frac{1}{2}x_1 \quad + x_3 &= x_2 \\ \frac{1}{2}x_1 \quad &= x_3 \end{aligned}$$

Observation P.2.4



$$\begin{aligned} x_2 \qquad\qquad &= x_1 \\ \frac{1}{2}x_1 \qquad + x_3 &= x_2 \\ \frac{1}{2}x_1 \qquad\qquad &= x_3 \end{aligned}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

By writing this linear system in terms of matrix multiplication, we obtain the **page rank matrix** $A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$ and page rank vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

Thus, computing the imporance of pages on a network is equivalent to solving the matrix equation $A\mathbf{x} = 1\mathbf{x}$.

Activity P.2.5 *(∼5 min)*
Thus, our \$700,000,000,000 problem is what kind of problem?

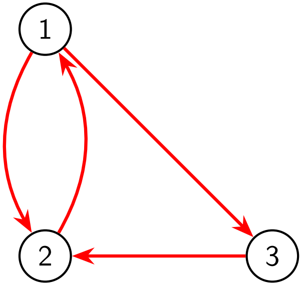
$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(a) An antiderivative problem
(b) A bijection problem
(c) A cofactoring problem
(d) A determinant problem
(e) An eigenvector problem

Activity P.2.6 *(~10 min)*

Find a page rank vector $\mathbf{x}$ satisfying $A\mathbf{x} = 1\mathbf{x}$ for the following network's page rank matrix A.

That is, find the eigenspace associated with $\lambda = 1$ for the matrix A, and choose a vector from that eigenspace.

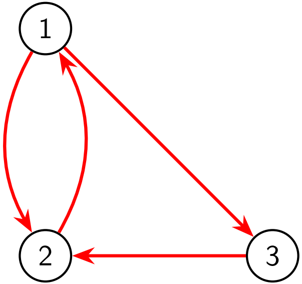


$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Observation P.2.7

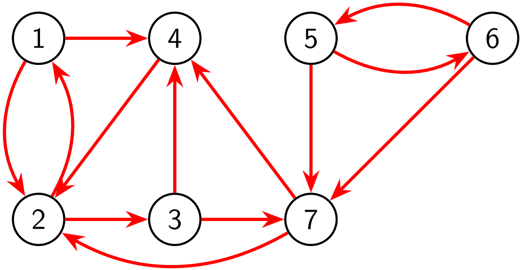
Row-reducing $A - I = \begin{bmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -1 & 1 \\ \frac{1}{2} & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ yields the basic eigenvector $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$.

Therefore, we may conclude that pages 1 and 2 are equally important, and both pages are twice as important as page 3.



Activity P.2.8 *(∼5 min)*

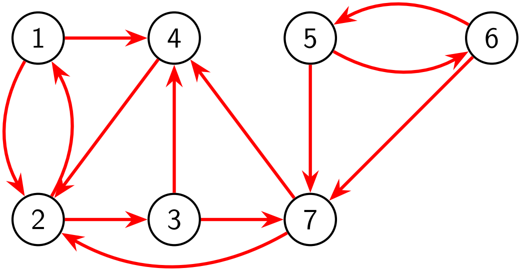
Compute the 7×7 page rank matrix for the following network.



For example, since website 1 distributes its endorsement equally between 2 and 4, the first column is $\begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Activity P.2.9 *(∼10 min)*

Find a page rank vector for the given page rank matrix.

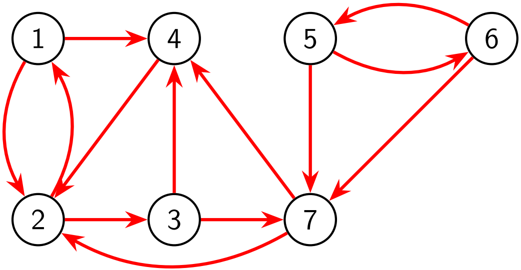


$$A = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Which webpage is most important?

Observation P.2.10

Since a page rank vector for the network is given by $\mathbf{x}$, it's reasonable to consider page 2 as the most important page.



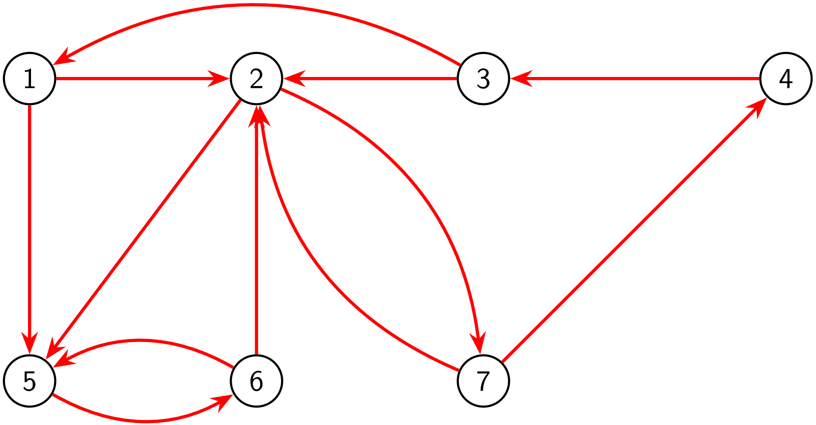
$$\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 2.5 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Based upon this page rank vector, here is a complete ranking of all seven pages from most important to least important:

$$2, 4, 1, 3, 7, 5, 6$$

Activity P.2.11 *(∼10 min)*

Given the following diagram, use a page rank vector to rank the pages 1 through 7 in order from most important to least important.



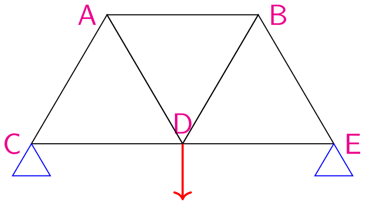
Module P Section 3

Example P.3.1

In engineering, a **truss** is a structure designed from several beams of material called **struts**, assembled to behave as a single object.

Activity P.3.2 *(∼5 min)*

Consider the representation of a simple truss pictured below. All of the seven struts are of equal length, affixed to two anchor points applying a normal force to nodes C and E, and with a $10000N$ load applied to the node given by D.



Which of the following must hold for the truss to be stable?

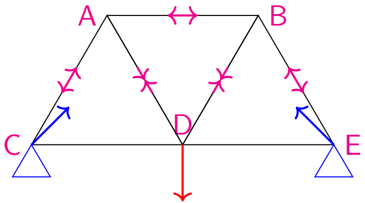
a) All of the struts will experience compression.

b) All of the struts will experience tension.

c) Some of the struts will be compressed, but others will be tensioned.

Observation P.3.3

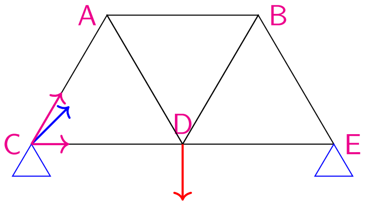
Since the forces must balance at each node for the truss to be stable, some of the struts will be compressed, while others will be tensioned.



By finding vector equations that must hold at each node, we may determine many of the forces at play.

Remark P.3.4

For example, at the bottom left node there are 3 forces acting.



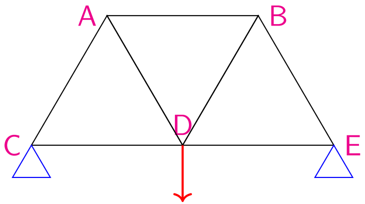
Let $\mathbf{F}_{CA}$ be the force on C given by the compression/tension of the strut CA, let $\mathbf{F}_{CD}$ be defined similarly, and let $\mathbf{N}_C$ be the normal force of the anchor point on C.

For the truss to be stable, we must have

$$\mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{N}_C = \mathbf{0}.$$

Activity P.3.5 *(∼10 min)*

Using the conventions of the previous slide, and where $\mathbf{L}$ represents the load vector on node D, find four more vector equations that must be satisfied for each of the other four nodes of the truss.



$$A : \ ?$$

$$B : \ ?$$

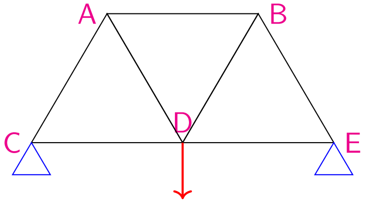
$$C : \mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{N}_C = \mathbf{0}$$

$$D : \ ?$$

$$E : \ ?$$

Remark P.3.6

The five vector equations may be written as follows.



$$A : \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F}_{AB} = \mathbf{0}$$

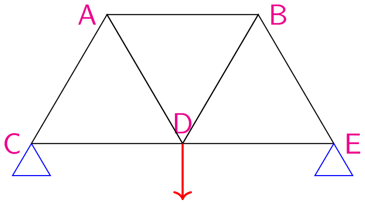
$$B : \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_{BE} = \mathbf{0}$$

$$C : \mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{N}_{C} = \mathbf{0}$$

$$D : \mathbf{F}_{DC} + \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DE} + \mathbf{L} = \mathbf{0}$$

$$E : \mathbf{F}_{EB} + \mathbf{F}_{ED} + \mathbf{N}_{E} = \mathbf{0}$$

Observation P.3.7



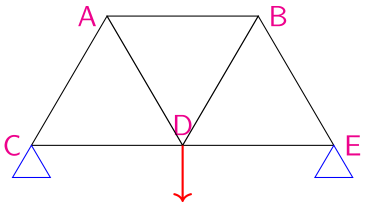
Each vector may be treated as an $\mathbb{R}^2$ vector by decomposing its magnitude into vertical and horizontal components. Note that $\mathbf{F}_{CA}$ must have the same magnitude (and opposite direction) as $\mathbf{F}_{AC}$.

$$\mathbf{F}_{CA} = x \begin{bmatrix} \cos(60^\circ) \\ \sin(60^\circ) \end{bmatrix} = x \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}$$

$$\mathbf{F}_{AC} = x \begin{bmatrix} \cos(-120^\circ) \\ \sin(-120^\circ) \end{bmatrix} = x \begin{bmatrix} -1/2 \\ -\sqrt{3}/2 \end{bmatrix}$$

Activity P.3.8 *(∼5 min)*

To write a linear system that models the truss under consideration, how many variables will be required?



a) 7: 5 from the nodes, 2 from the anchors

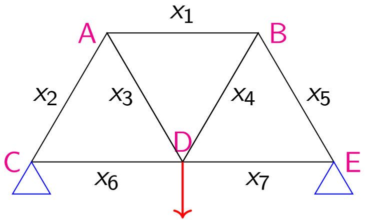
b) 10: 7 from the struts, 2 from the anchors, 1 from the load

c) 11: 7 from the struts, 4 from the anchors

d) 13: 5 from the nodes, 7 from the struts, 1 from the load

Observation P.3.9

Since the angles for each strut are known, one variable may be used to represent each.



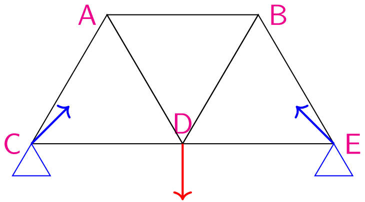
For example:

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA} = x_1 \begin{bmatrix} \cos(0) \\ \sin(0) \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathbf{F}_{BE} = -\mathbf{F}_{EB} = x_5 \begin{bmatrix} \cos(-60^\circ) \\ \sin(-60^\circ) \end{bmatrix} = x_5 \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix}$$

Observation P.3.10

Since the angle of the normal forces for each anchor point are unknown, two variables may be used to represent each.



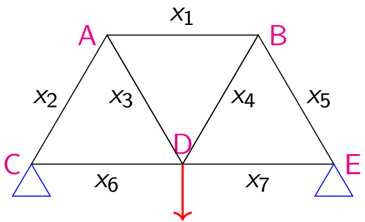
$$\mathbf{N}_C = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \mathbf{N}_D = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

The load vector is constant.

$$\mathbf{L} = \begin{bmatrix} 0 \\ -10000 \end{bmatrix}$$

Remark P.3.11

Each of the five vector equations found previously represent two linear equations: one for the horizontal component and one for the vertical.



$$C : \mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{N}_C = \mathbf{0}$$

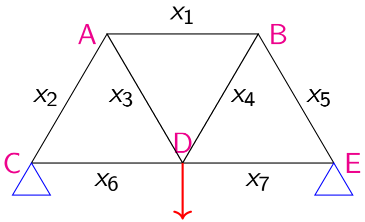
$$\Leftrightarrow x_2 \begin{bmatrix} \cos(60^\circ) \\ \sin(60^\circ) \end{bmatrix} + x_6 \begin{bmatrix} \cos(0^\circ) \\ \sin(0^\circ) \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using the approximation $\sqrt{3}/2 \approx 0.866$, we have

$$\Leftrightarrow x_2 \begin{bmatrix} 0.5 \\ 0.866 \end{bmatrix} + x_6 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Activity P.3.12 *(∼10 min)*

Expand the vector equation given below using sine and cosine of appropriate angles, then compute each component (approximating $\sqrt{3}/2 \approx 0.866$).



$$D : \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC} + \mathbf{F}_{DE} = -\mathbf{L}$$

$$\Leftrightarrow x_3 \begin{bmatrix} \cos(\,?\,) \\ \sin(\,?\,) \end{bmatrix} + x_4 \begin{bmatrix} \cos(\,?\,) \\ \sin(\,?\,) \end{bmatrix} + x_6 \begin{bmatrix} \cos(\,?\,) \\ \sin(\,?\,) \end{bmatrix} + x_7 \begin{bmatrix} \cos(\,?\,) \\ \sin(\,?\,) \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\Leftrightarrow x_3 \begin{bmatrix} ? \\ ? \end{bmatrix} + x_4 \begin{bmatrix} ? \\ ? \end{bmatrix} + x_6 \begin{bmatrix} ? \\ ? \end{bmatrix} + x_7 \begin{bmatrix} ? \\ ? \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

Observation P.3.13

The full augmented matrix given by the ten equations in this linear system is given below, where the elevent columns correspond to $x_1, \dots, x_7, y_1, y_2, z_1, z_2$, and the ten rows correspond to the horizontal and vertical components of the forces acting at $A, \dots, E$.

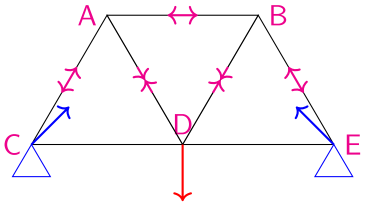
$$\left[\begin{array}{ccccccccccc|c}
1 & -0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.866 & -0.866 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & -0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.866 & -0.866 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0.866 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -0.5 & 0.5 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.866 & 0.866 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10000 \\
0 & 0 & 0 & 0 & -0.5 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.866 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]$$

Observation P.3.14

This matrix row-reduces to the following.

$$\sim \left[\begin{array}{ccccccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5773.7 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5773.7 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5773.7 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5773.7 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -5773.7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 2886.8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 2886.8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 5000 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 5000 \end{array}\right]$$

Observation P.3.15



Thus we know the truss must satisfy the following conditions.

$$x_1 = x_2 = x_5 = -5882.4$$
$$x_3 = x_4 = 5882.4$$
$$x_6 = x_7 = 2886.8 + z_1$$
$$y_1 = -z_1$$
$$y_2 = z_2 = 5000$$

In particular, the negative x_1, x_2, x_5 represent tension (forces pointing into the nodes), and the postive x_3, x_4 represent compression (forces pointing out of the nodes). The vertical normal forces $y_2 + z_2$ counteract the 10000 load.