

Name: _____

MASTERY QUIZ DAY 10

Math 237 – Linear Algebra

Version 3

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write an augmented matrix corresponding to the following system of linear equations.

$$\begin{aligned}x_1 + 4x_3 &= 1 \\x_2 - x_3 &= 7 \\x_1 - x_2 + 3x_3 &= -1\end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -1 & 7 \\ 1 & -1 & 3 & -1 \end{array} \right]$$

□

E3. Solve the system of equations

$$\begin{aligned}x + 3y - 4z &= 5 \\3x + 9y + z &= 2\end{aligned}$$

Solution:

$$\text{RREF} \left(\left[\begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 2 \end{array} \right] \right) = \left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

So the solution set is

$$\left\{ \left[\begin{array}{c} 1 - 3c \\ c \\ -1 \end{array} \right] \mid c \in \mathbb{R} \right\}$$

□

E4. Find a basis for the solution set to the homogeneous system of equations given by

$$\begin{aligned}2x_1 - 2x_2 + 6x_3 - x_4 &= 0 \\3x_1 + 6x_3 + x_4 &= 0 \\-4x_1 + x_2 - 9x_3 + 2x_4 &= 0\end{aligned}$$

Solution: Let $A = \left[\begin{array}{cccc|c} 2 & -2 & 6 & -1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ -4 & 1 & -9 & 2 & 0 \end{array} \right]$, so $\text{RREF } A = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$. It follows that the basis

for the solution set is given by $\left\{ \left[\begin{array}{c} -2 \\ 1 \\ 1 \\ 0 \end{array} \right] \right\}$.

□

V1. Let V be the set of all polynomials with the operations, for any $f, g \in V, c \in \mathbb{R}$,

$$\begin{aligned} f \oplus g &= f' + g' \\ c \odot f &= cf' \end{aligned}$$

(here f' denotes the derivative of f).

(a) Show that this scalar multiplication \odot distributes over vector addition \oplus .

(b) Determine if V is a vector space or not. Justify your answer.

Solution: Let $f, g \in \mathcal{P}$, and let $c \in \mathbb{R}$.

$$c \odot (f \oplus g) = c \odot (f' + g') = c(f' + g')' = cf'' + cg'' = cf' \oplus cg' = c \odot f \oplus c \odot g.$$

However, this is not a vector space, as there is no zero vector. Additionally, $1 \odot f \neq f$ for any nonzero polynomial f .

□

E1:

E3:

E4:

V1:

E2: