Name: ______________________________

SEMIFINAL Math 237 – Linear Algebra
Version 1 Fall 2017

Choose up to 6 problems to work. Work each problem on one of the attached pages; write the standard in the lower left corner. Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

E1. Write a system of linear equations corresponding to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & 1 \\ -1 & 4 & 1 & -7 \\ 1 & 2 & -1 & 0 \end{array}\right]$$

E2. Find RREF A, where

$$A = \left[\begin{array}{ccc|c} 2 & -1 & 5 & 4 \\ -1 & 0 & -2 & -1 \\ 1 & 3 & -1 & -5 \end{array}\right]$$

E3. Solve the following linear system.

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ -2x_3 - 4x_4 &= 3 \\ 2x_1 + 2x_2 + x_3 - 4x_4 &= -1 \end{aligned}$$

E4. Find a basis for the solution set of the system of equations

$$\begin{aligned} x + 3y + 3z + 7w &= 0 \\ x + 3y - z - w &= 0 \\ 2x + 6y + 3z + 8w &= 0 \\ x + 3y - 2z - 3w &= 0 \end{aligned}$$

V1. Let V be the set of all points on the parabola $y = x^2$ with the operations, for any $(x_1, y_1), (x_2, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2 + 2x_1x_2)$$
$$c \odot (x_1, y_1) = (cx_1, c^2y_1)$$

(a) Show that the vector **addition** $\oplus$ is **associative**:
$(x_1, y_1) \oplus ((x_2, y_2) \oplus (x_3, y_3)) = ((x_1, y_1) \oplus (x_2, y_2)) \oplus (x_3, y_3)$.

(b) Determine if V is a vector space or not. Justify your answer.

V2. Determine if $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ belongs to the span of the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

V3. Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ span $\mathbb{R}^4$.

V4. Let W be the set of all complex numbers that are purely real (i.e of the form $a + 0i$) or purely imaginary (i.e. of the form $0 + bi$). Determine if W is a subspace of $\mathbb{C}$.

S1. Determine if the set of polynomials $\{x^2+x, x^2+2x-1, x^2+3x-2\}$ is linearly dependent or linearly independent

S2. Determine if the set $\{x^3-x, x^2+x+1, x^3-x^2+2, 2x^2-1\}$ is a basis of $\mathcal{P}^3$.

S3. Let $W = \operatorname{span}\left\{\begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right\}$. Find a basis for this vector space.

S4. Let $W = \operatorname{span}\{2x^2-x+3, 2x^2+2, -x^2+4x+1\}$. Find the dimension of W.

A1. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -3x+y \\ -8x+2y-z \\ 7x+2y+3z \\ 0 \end{bmatrix}.$$

Write the matrix for T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^4$.

A2. Determine if the map $T : \mathcal{P}^6 \to \mathcal{P}^7$ given by $T(f) = xf(x) - f(1)$ is a linear transformation or not.

A3. Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by the standard matrix $\begin{bmatrix} 2 & 3 & -1 & -2 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & -7 & -4 \end{bmatrix}$

A4. Let $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be the linear map given by $T\left(\begin{bmatrix} a & b \\ x & y \end{bmatrix}\right) = \begin{bmatrix} a+x \\ 0 \\ b+y \end{bmatrix}$. Compute a basis for the kernel and a basis for the image of T.

M1. Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Exactly one of the six products AB, AC, BA, BC, CA, CB can be computed. Determine which one, and compute it.

M2. Determine if the matrix $\begin{bmatrix} 1 & 3 & 3 & 7 \\ 1 & 3 & -1 & -1 \\ 2 & 6 & 3 & 8 \\ 1 & 3 & -2 & -3 \end{bmatrix}$ is invertible.

M3. Find the inverse of the matrix $\begin{bmatrix} 8 & 5 & 3 & 0 \\ 3 & 2 & 1 & 1 \\ 5 & -3 & 1 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$.

G1. Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

G2. Compute the eigenvalues, along with their algebraic multiplicities, of the matrix $\begin{bmatrix} 3 & -3 & 2 \\ 108 & -9 & 5 \\ 10 & -7 & 3 \end{bmatrix}$.

G3. Compute the eigenspace associated to the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

G4. Compute the geometric multiplicity of the eigenvalue 2 in the matrix $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$.

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