

## Module X: Applications

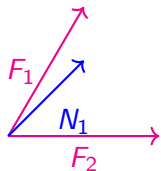
# Module X Section 1

## Observation X.1.1

Consider the truss pictured below with two fixed anchor points and a 10000 N load (assume all triangles are equilateral).



The horizontal and vertical forces must balance at each of the five intersecting nodes. For example, at the bottom left node



Apply basic trig:      thus

$$F_{1,v} = F_1 \sin(60^\circ)$$

$$F_{1,h} = F_1 \cos(60^\circ)$$

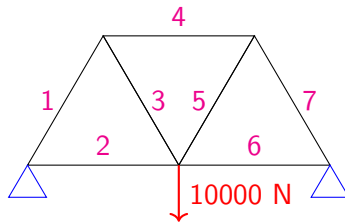
$$F_1 \sin(60^\circ) + N_{1,v} = 0$$

$$F_1 \cos(60^\circ) + N_{1,h} + F_2 = 0$$

We adhere to the convention that a compression force on a strut is positive, while a negative force represents tension.

**Activity X.1.2** ( $\sim 10$  min)

Consider the truss pictured below with two fixed anchor points and a 10000 N load (assume all triangles are equilateral).



From the bottom left node we obtained 2 equations in the four variables

- $F_1$  (compression force on strut one)
- $N_{1,v}$  and  $N_{1,h}$  (horizontal and vertical components of the normal force from the left anchor)
- $F_2$  (compression force on strut 2).

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- $F_2$  (compression force on strut 2).

*Part 1:* Determine how many total equations there will be after accounting for all of the nodes, and list all of the variables. You do not need to actually determine all of the equations.

## Activity X.1.3 (~10 min)



The resulting system is

$$N_{1,v} \quad + (\sin(60^\circ))F_1$$

$$N_{1,h} \quad + (\cos(60^\circ))F_1 + F_2$$

$$- (\sin(60^\circ))F_1 \quad - (\sin(60^\circ))F_3$$

$$- (\cos(60^\circ))F_1 \quad + (\cos(60^\circ))F_3 + F_4$$

$$(\sin(60^\circ))F_3 \quad + (\sin(60^\circ))F_5$$

$$- F_2 - (\cos(60^\circ))F_3 \quad + (\cos(60^\circ))F_5 + F_6$$

$$- (\sin(60^\circ))F_5 \quad - (\sin(60^\circ))F_7$$

$$- F_4 - (\cos(60^\circ))F_5 \quad + (\cos(60^\circ))F_7$$

$$N_{2,v}$$

$$N_{2,h}$$

$$- F_6 - (\cos(60^\circ))F_7$$

### Observation X.1.4

The determined part of the solution is

$$N_{1,v} = N_{2,v} = 5000$$

$$F_1 = F_4 = F_7 = -5882.4$$

$$F_3 = F_5 = 5882.4$$

So struts 1,4,7 are in tension, while struts 3 and 5 are compressed.

The forces on struts 2 and 6 (and the horizontal normal forces) are not strictly determined in this setting.

## Module X Section 2



**Activity X.2.1** (*~10 min*)**A \$700,000,000,000 Problem:**

In the picture below, each circle represents a webpage, and each arrow represents a link from one page to another.



Based on how these pages link to each other, write a list of the 7 webpages in order from most important to least important.

## Observation X.2.2

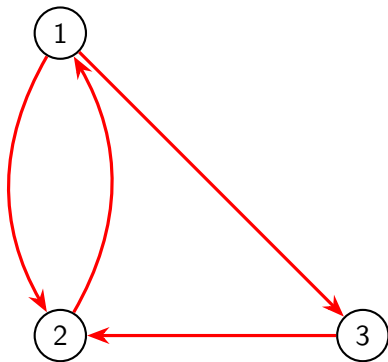
### The \$700,000,000,000 Idea:

Links are endorsements.

- ① A webpage is important if it is linked to (endorsed) by important pages.
- ② A webpage distributes its importance equally among all the pages it links to (endorses).

### Example X.2.3

Consider this small network with only three pages. Let  $x_1, x_2, x_3$  be the importance of the three pages respectively.



- 1  $x_1$  splits its endorsement in half between  $x_2$  and  $x_3$
- 2  $x_2$  sends all of its endorsement to  $x_1$
- 3  $x_3$  sends all of its endorsement to  $x_2$ .

This corresponds to the **page rank system**

$$\begin{aligned}x_2 &= x_1 \\ \frac{1}{2}x_1 + x_3 &= x_2 \\ \frac{1}{2}x_1 &= x_3\end{aligned}$$

## Example X.2.4



$$\begin{aligned} x_2 &= x_1 \\ \frac{1}{2}x_1 + x_3 &= x_2 \\ \frac{1}{2}x_1 &= x_3 \end{aligned}$$

We can summarize the left hand side of the system by putting its coefficients into a

**page rank matrix**  $A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$ , and store the right hand side of the system as

the vector  $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ .

Thus, computing the importance of pages on a network is equivalent to solving the matrix equation  $A\mathbf{x} = \mathbf{x}$ .

**Activity X.2.5** ( $\sim 5$  min)

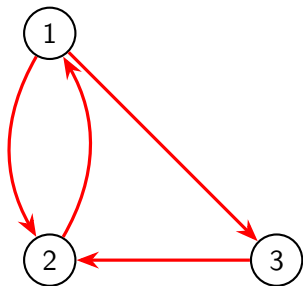
A **page rank vector** for a page rank matrix  $A$  is a vector  $\mathbf{x}$  satisfying  $A\mathbf{x} = \mathbf{x}$ . This vector describes the relative importance of webpages on the network described by  $A$ .

Thus, the \$700,000,000,000 problem is what kind of problem?

- (a) A bijection problem
- (b) A calculus problem
- (c) A determinant problem
- (d) An eigenvector problem

**Activity X.2.6** ( $\sim 10$  min)

Find a page rank vector  $\mathbf{x}$  satisfying  $A\mathbf{x} = \mathbf{x}$  (an eigenvector associated to the eigenvalue 1) for the following network's page rank matrix  $A$ .



$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$$

**Observation X.2.7**

Row-reducing  $A - I = \begin{bmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -1 & 1 \\ \frac{1}{2} & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$  yields the basic

eigenvector  $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ .

Therefore, we may conclude that pages 1 and 2 are equally important, and both pages are twice as important as page 3.



**Activity X.2.8** ( $\sim 10$  min)

Compute the  $7 \times 7$  page rank matrix for the following network.



For example, since website 1 distributes its endorsement equally between 2 and 4,

the first column is  $\begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ .



**Activity X.2.9** ( $\sim 10$  min)

Find a page rank vector for the transition matrix.



$$A = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Which webpage is most important?

**Observation X.2.10**

Since a page rank vector for the network is given by  $\mathbf{x}$ , it's reasonable to consider page 2 as the most important page.



$$\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 2.5 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$