

Name: \_\_\_\_\_

**MASTERY QUIZ DAY 8**

Math 237 – Linear Algebra

**Version 4**

Fall 2017

Show all work. Answers without work will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write an augmented matrix corresponding to the following system of linear equations.

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 0$$

$$x - z = 1$$

**Solution:**

$$\left[ \begin{array}{ccc|c} 1 & 3 & -4 & 5 \\ 3 & 9 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{array} \right]$$

□

**E3.** Solve the system of equations

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 2$$

**E4.** Find a basis for the solution set of the system ...

**V1.** Let  $V$  be the set of all real numbers together with the operations  $\oplus$  and  $\odot$  defined by, for any  $x, y \in V$  and  $c \in \mathbb{R}$ ,

$$x \oplus y = x + y - 3$$

$$c \odot x = cx - 3(c - 1)$$

Determine if  $V$  is a vector space or not.

**Solution:** Let  $x, y \in V$ ,  $c, d \in \mathbb{R}$ .

1) Real addition is associative, so  $\oplus$  is associative.

2)  $x \oplus 3 = x + 3 - 3 = x$ , so 3 is the additive identity.

3)  $x \oplus (6 - x) = x + (6 - x) - 3 = 3$ , so  $6 - x$  is the additive inverse of  $x$ .

4) Real addition is commutative, so  $\oplus$  is commutative.

5)

$$\begin{aligned} c \odot (d \odot x) &= c \odot (dx - 3(d - 1)) \\ &= c(dx - 3(d - 1)) - 3(c - 1) \\ &= cdx - 3(cd - 1) \\ &= (cd) \odot x \end{aligned}$$

6)  $1 \odot x = x - 3(1 - 1) = x$

7)

$$\begin{aligned}
 c \odot (x \oplus y) &= c \odot (x + y - 3) \\
 &= c(x + y - 3) - 3(c - 1) \\
 &= cx - 3(c - 1) + cy - 3(c - 1) - 3 \\
 &= (c \odot x) \oplus (c \odot y)
 \end{aligned}$$

8)

$$\begin{aligned}
 (c + d) \odot x &= (c + d)x - 3(c + d - 1) \\
 &= cx - 3(c - 1) + dx - 3(d - 1) - 3 \\
 &= (c \odot x) \oplus (d \odot x)
 \end{aligned}$$

Therefore  $V$  is a vector space.

□

**E1:**

**E3:**

**E4:**

**V1:**

**E2:**