

Application Activities - Module E Part 1 - Class Day 3

Definition 3.1 A **linear equation** is an equation of the variables x_i of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b.$$

A **solution** for a linear equation is expressed in terms of the Euclidean vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$$

and must satisfy

$$a_1s_1 + a_2s_2 + \cdots + a_ns_n = b.$$

Observation 3.2 The linear equation $3x - 5y = -2$ may be graphed as a line in the xy plane.



The linear equation $x + 2y - z = 4$ may be graphed as a plane in xyz space.

Remark 3.3 In previous classes you likely assumed $x = x_1$, $y = x_2$, and $z = x_3$. However, since this course often deals with equations of four or more variables, we will almost always write our variables as x_i .

Definition 3.4 A **system of linear equations** (or a **linear system** for short) is a collection of one or more linear equations.

$$\begin{array}{ccccccc} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & = & b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & = & b_2 \\ \vdots & & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & = & b_m \end{array}$$

A **solution**

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$$

for a linear system satisfies

$$a_{i1}s_1 + a_{i2}s_2 + \cdots + a_{in}s_n = b_i$$

for $1 \leq i \leq m$ (that is, the solution satisfies all equations in the system).

Remark 3.5 When variables in a large linear system are missing, we prefer to write the system in one of the following standard forms:

Original linear system:

$$\begin{aligned}x_1 + 3x_3 &= 3 \\3x_1 - 2x_2 + 4x_3 &= 0 \\-x_2 + x_3 &= -2\end{aligned}$$

Verbose standard form:

$$\begin{aligned}1x_1 + 0x_2 + 3x_3 &= 3 \\3x_1 - 2x_2 + 4x_3 &= 0 \\0x_1 - 1x_2 + 1x_3 &= -2\end{aligned}$$

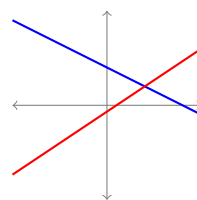
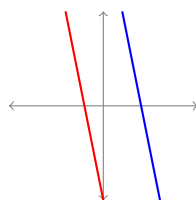
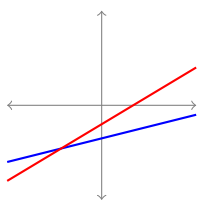
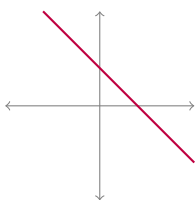
Concise standard form:

$$\begin{aligned}x_1 \quad \quad + 3x_3 &= 3 \\3x_1 - 2x_2 + 4x_3 &= 0 \\-x_2 + x_3 &= -2\end{aligned}$$

Definition 3.6 A linear system is **consistent** if there exists a solution for the system. Otherwise it is **inconsistent**.

Fact 3.7 All linear systems are either **consistent with one solution**, **consistent with infinitely-many solutions**, or **inconsistent**.

Activity 3.8 Consider the following graphs representing linear systems of two variables. Label each graph with **consistent with one solution**, **consistent with infinitely-many solutions**, or **inconsistent**.



Activity 3.9 All inconsistent linear systems contain a logical **contradiction**. Find a contradiction in this system.

$$\begin{aligned}-x_1 + 2x_2 &= 5 \\2x_1 - 4x_2 &= 6\end{aligned}$$

Activity 3.10 Consider the following consistent linear system.

$$\begin{aligned}-x_1 + 2x_2 &= -3 \\2x_1 - 4x_2 &= 6\end{aligned}$$

Part 1: Find three different solutions $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$, $\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$, $\begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$ for this system.

Part 2: Let $x_2 = a$ where a is an arbitrary real number, then find an expression for x_1 in terms of a . Use this to describe *all* solutions (the **solution set**) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ? \\ a \end{bmatrix}$ for the linear system in terms of a .

Therefore these augmented matrices are equivalent:

$$\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 1 & 4 & 5 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 4 & 2 & 6 \end{array} \right]$$

Activity 3.16 Following are six procedures used to manipulate an augmented matrix. Label the procedures that would result in an equivalent augmented matrix as **valid**, and label the procedures that would change the solution set of the corresponding linear system as **invalid**.

- | | |
|---|---|
| a) Swap two rows. | d) Multiply a row by a nonzero constant. |
| b) Swap two columns. | e) Add a constant multiple of one row to another row. |
| c) Add a constant to every term in a row. | f) Replace a column with zeros. |
-