

## Application Activities - Module V Part 1 - Class Day 7

**Activity 7.1** Consider each of the following vector properties. Label each property with  $\mathbb{R}^1$ ,  $\mathbb{R}^2$ , and/or  $\mathbb{R}^3$  if that property holds for Euclidean vectors/scalars  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  of that dimension.

1. **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

2. **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

3. **Addition identity.**

There exists some  $\mathbf{0}$  where  $\mathbf{v} + \mathbf{0} = \mathbf{v}$ .

4. **Addition inverse.**

There exists some  $-\mathbf{v}$  where  $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$ .

5. **Addition midpoint uniqueness.**

There exists a unique  $\mathbf{m}$  where the distance from  $\mathbf{u}$  to  $\mathbf{m}$  equals the distance from  $\mathbf{m}$  to  $\mathbf{v}$ .

6. **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

7. **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

8. **Scalar multiplication relativity.**

There exists some scalar  $c$  where either  $c\mathbf{v} = \mathbf{w}$  or  $c\mathbf{w} = \mathbf{v}$ .

9. **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

10. **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

11. **Orthogonality.**

There exists a non-zero vector  $\mathbf{n}$  such that  $\mathbf{n}$  is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ .

12. **Bidimensionality.**

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} \text{ for some value of } a, b.$$

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**Definition 7.2** A **vector space**  $V$  is any collection of mathematical objects with associated addition and scalar multiplication operations that satisfy the following properties. Let  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  belong to  $V$ , and let  $a, b$  be scalar numbers.

- **Addition associativity.**

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}.$$

- **Addition commutativity.**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

- **Addition identity.**

There exists some  $\mathbf{0}$  where  $\mathbf{v} + \mathbf{0} = \mathbf{v}$ .

- **Addition inverse.**

There exists some  $-\mathbf{v}$  where  $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$ .

- **Scalar multiplication associativity.**

$$a(b\mathbf{v}) = (ab)\mathbf{v}.$$

- **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

- **Scalar distribution.**

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}.$$

- **Vector distribution.**

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}.$$

**Definition 7.3** The most important examples of vector spaces are the **Euclidean vector spaces**  $\mathbb{R}^n$ , but there are other examples as well.

**Activity 7.4** Consider the following set that models motion along the curve  $y = e^x$ . Let  $V = \{(x, y) : y = e^x\}$ . Let vector addition be defined by  $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 y_2)$ , and let scalar multiplication be defined by  $c \odot (x, y) = (cx, y^c)$ .

*Part 1:* Which of the vector space properties are satisfied by  $V$  paired with these operations?

- **Addition associativity.**

$$\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}.$$

- **Addition commutativity.**

$$\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}.$$

- **Addition identity.**

There exists some  $\mathbf{0}$  where  $\mathbf{v} \oplus \mathbf{0} = \mathbf{v}$ .

- **Addition inverse.**

There exists some  $-\mathbf{v}$  where  $\mathbf{v} \oplus (-\mathbf{v}) = \mathbf{0}$ .

- **Scalar multiplication associativity.**

$$a \odot (b \odot \mathbf{v}) = (ab) \odot \mathbf{v}.$$

- **Scalar multiplication identity.**

$$1\mathbf{v} = \mathbf{v}.$$

- **Scalar distribution.**

$$a \odot (\mathbf{u} \oplus \mathbf{v}) = (a \odot \mathbf{u}) \oplus (a \odot \mathbf{v}).$$

- **Vector distribution.**

$$(a + b) \odot \mathbf{v} = (a \odot \mathbf{v}) \oplus (b \odot \mathbf{v}).$$

*Part 2:* Is  $V$  a vector space?

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