

Name: \_\_\_\_\_

**SEMIFINAL**

Math 237 – Linear Algebra

**Version 1**

Fall 2017

Work any problems you wish on the provided answer sheets; take care to label the standard for each response. Show all work and justify all of your answers. Answers without work or sufficient reasoning will not receive credit. You may use a calculator, but you must show all relevant work to receive credit for a standard.

**E1.** Write a system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{ccc|c} -4 & -1 & 3 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 1 & 4 \end{array} \right]$$

**E2.** Find RREF  $A$ , where

$$A = \left[ \begin{array}{ccc|c} 2 & -1 & 5 & 4 \\ -1 & 0 & -2 & -1 \\ 1 & 3 & -1 & -5 \end{array} \right]$$

**E3.** Solve the system of equations

$$x + 3y - 4z = 5$$

$$3x + 9y + z = 2$$

**E4.** Find a basis for the solution set of the system of equations

$$x + 2y + 3z + w = 0$$

$$3x - y + z + w = 0$$

$$2x - 3y - 2z = 0$$

**V1.** Let  $V$  be the set of all real numbers with the operations, for any  $x, y \in V$ ,  $c \in \mathbb{R}$ ,

$$x \oplus y = \sqrt{x^2 + y^2}$$

$$c \odot x = cx$$

(a) Show that the vector **addition**  $\oplus$  is **associative**:  $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ .

(b) Determine if  $V$  is a vector space or not. Justify your answer.

**V2.** Determine if  $\begin{bmatrix} 4 \\ -1 \\ 6 \\ -7 \end{bmatrix}$  belongs to the span of the set  $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 4 \\ 3 \end{bmatrix} \right\}$ .

**V3.** Determine if the vectors  $\begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 6 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$  span  $\mathbb{R}^4$ .

**V4.** Determine if the set of all lattice points, i.e.  $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .

**S1.** Determine if the set of polynomials  $\{x^2 + x, x^2 + 2x - 1, x^2 + 3x - 2\}$  is linearly dependent or linearly independent

**S2.** Determine if the set  $\{x^2 + x - 1, 3x^2 - x + 1, 2x^2 - 2\}$  is a basis of  $\mathcal{P}^2$ .

**S3.** Let  $W = \text{span} \left( \left\{ \begin{bmatrix} -3 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\} \right)$ . Find a basis for  $W$ .

**S4.** Let  $W = \text{span} \left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -8 \\ -1 \end{bmatrix} \right\}$ . Find the dimension of  $W$ .

**A1.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = [x_3 + 3x_1].$$

Write the matrix for  $T$  with respect to the standard bases of  $\mathbb{R}^3$  and  $\mathbb{R}$ .

**A2.** Determine if  $D : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}$  given by  $D \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc$  is a linear transformation or not.

**A3.** Determine if each of the following linear transformations is injective (one-to-one) and/or surjective (onto).

(a)  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  given by the standard matrix  $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$ .

(b)  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  given by the standard matrix  $\begin{bmatrix} 2 & 3 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 4 & 7 & -1 & 5 \end{bmatrix}$

**A4.** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear map given by  $T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 8x - 3y - z \\ y + 3z \\ -7x + 3y + 2z \end{bmatrix}$ . Compute a basis for the kernel and a basis for the image of  $T$ .

**M1.** Let

$$A = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

Exactly one of the six products  $AB$ ,  $AC$ ,  $BA$ ,  $BC$ ,  $CA$ ,  $CB$  can be computed. Determine which one, and compute it.

**M2.** Determine if the matrix  $\begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$  is invertible.

**M3.** Find the inverse of the matrix  $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ .

**G1.** Compute the determinant of the matrix

$$\begin{bmatrix} 0 & -4 & 1 & 1 \\ -2 & 3 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -4 & 0 \end{bmatrix}.$$

**G2.** Let  $A = \begin{bmatrix} -3 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ . List the eigenvalues of  $A$  along with their algebraic multiplicities.

**G3.** Compute the eigenspace associated to the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

**G4.** Compute the geometric multiplicity of the eigenvalue 2 in the matrix  $\begin{bmatrix} -1 & 1 & 0 \\ -9 & 5 & 0 \\ 15 & -5 & 2 \end{bmatrix}$ .

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