

## Section V.2

**Activity V.17** (*~5 min*) Sketch a representation of all the vectors belonging to  $\text{span}\left\{\begin{bmatrix} 6 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix}\right\}$  in the  $xy$  plane.

**Remark V.18** Recall these definitions from last class:

- A **linear combination** of vectors is given by adding scalar multiples of those vectors, such as:

$$\begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

- The **span** of a set of vectors is the collection of all linear combinations of that set, such as:

$$\text{span}\left\{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right\} = \left\{a \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \mid a, b \in \mathbb{R}\right\}$$

**Activity V.19** (*~15 min*) The vector  $\begin{bmatrix} -1 \\ -6 \\ 1 \end{bmatrix}$  belongs to  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$  exactly when there exists a solution to the vector equation  $x_1 \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -6 \\ 1 \end{bmatrix}$ .

*Part 1:* Reinterpret this vector equation as a system of linear equations.

*Part 2:* Find its solution set, using technology to find RREF of its corresponding augmented matrix.

*Part 3:* Given this solution set, does  $\begin{bmatrix} -1 \\ -6 \\ 1 \end{bmatrix}$  belong to  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$ ?

**Fact V.20** A vector  $\vec{\mathbf{b}}$  belongs to  $\text{span}\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n\}$  if and only if the linear system corresponding to  $[\vec{\mathbf{v}}_1 \dots \vec{\mathbf{v}}_n \mid \vec{\mathbf{b}}]$  is consistent.

Put another way,  $\vec{\mathbf{b}}$  belongs to  $\text{span}\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n\}$  exactly when  $\text{RREF}[\vec{\mathbf{v}}_1 \dots \vec{\mathbf{v}}_n \mid \vec{\mathbf{b}}]$  doesn't have a row  $[0 \dots 0 \mid 1]$  representing the contradiction  $0 = 1$ .

**Activity V.21** (*~10 min*) Determine if  $\begin{bmatrix} 3 \\ -2 \\ 1 \\ 5 \end{bmatrix}$  belongs to  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \\ 2 \end{bmatrix}\right\}$  by row-reducing an appropriate matrix.

**Activity V.22** (*~5 min*) Determine if  $\begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}$  belongs to  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right\}$  by row-reducing an appropriate matrix.

**Activity V.23** (*~10 min*) Does the third-degree polynomial  $3y^3 - 2y^2 + y + 5$  in  $\mathcal{P}^3$  belong to  $\text{span}\{y^3 - 3y + 2, -y^3 - 3y^2 + 2y + 2\}$ ?

*Part 1:* Reinterpret this question as an equivalent exercise involving Euclidean vectors in  $\mathbb{R}^4$ . (Hint: What four numbers must you know to write a  $\mathcal{P}^3$  polynomial?)

*Part 2:* Solve this equivalent exercise, and use its solution to answer the original question.

**Activity V.24** (*~5 min*) Does the polynomial  $x^2 + x + 1$  belong to  $\text{span}\{x^2 - x, x + 1, x^2 - 1\}$ ?