Fourth order summation-by-parts finite difference method for wave propagation in anisotropic elastic material and curvilinear coordinates with mesh refinement interface

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Abstract

We analyze

1 Introduction

2 The anisotropic elastic wave equation

We consider the anistropic elastic wave equation in three dimensional domain $\mathbf{x} \in \Omega$, $\mathbf{x} = (x_1, x_2, x_3)^T$ are Cartesian coordinates. Denote $\mathbf{u} = (u_1, u_2, u_3)^T$ to be the three dimensional displacement vector in Cartesian coordinates, then the elastic wave equation in Cartisian coordinates takes the form,

$$\begin{aligned} \rho \frac{\partial^2 \mathbf{u}}{\partial^2 t} &= \nabla \cdot \mathcal{T} + \mathbf{F}, \quad \mathbf{x} \in \Omega, \quad t \geq 0, \\ \nabla \cdot \mathcal{T} &:= \mathbf{L}\mathbf{u}, \end{aligned}$$

provided with appropriate initial and boundary conditions. Here, ρ is density, $\mathcal{T}$ is stress tensor and $\mathbf{F}$ is the force function. The spatial operator $\mathbf{L}$ is called 3×3 symmetric Kelvin-Christoffel differential operator matrix, specifically,

$$\mathbf{L}\mathbf{u} = \partial_1(A_1 \nabla \mathbf{u}) + \partial_2(A_2 \nabla \mathbf{u}) + \partial_3(A_3 \nabla \mathbf{u}),$$

with

$$\begin{aligned} A_1 \nabla \mathbf{u} &:= M^{11} \partial_1 \mathbf{u} + M^{12} \partial_2 \mathbf{u} + M^{13} \partial_3 \mathbf{u}, \\ A_2 \nabla \mathbf{u} &:= M^{21} \partial_1 \mathbf{u} + M^{22} \partial_2 \mathbf{u} + M^{23} \partial_3 \mathbf{u}, \\ A_3 \nabla \mathbf{u} &:= M^{31} \partial_1 \mathbf{u} + M^{32} \partial_2 \mathbf{u} + M^{33} \partial_3 \mathbf{u}, \end{aligned}$$

where $M^{i,j}, i = 1,2,3, j = 1,2,3$ are determined by the material properties. For example, for isotropic elastic material,

$$M^{11} = \begin{pmatrix} 2\mu + \lambda & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix}, M^{12} = \begin{pmatrix} 0 & \lambda & 0 \\ \mu & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, M^{13} = \begin{pmatrix} 0 & 0 & \lambda \\ 0 & 0 & 0 \\ \mu & 0 & 0 \end{pmatrix},$$

2.1 Energy estimate

3 Generalization to curvilinear coordinates

3.1 Boundary Condition

3.2 Energy Estimate

4 Grid refinement interface

5 Numerical Experiments