K-Nearest Neighbors, a.k.a., Memory Based Learning, a.k.a., Instance Based Learning, a.k.a. Case Based Reasoning.

Alvaro Soto

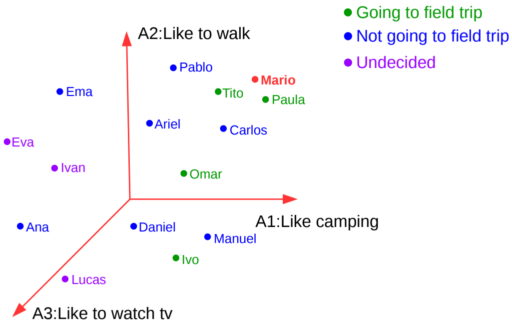
Computer Science Department (DCC), PUC

I. K-Nearest Neighbors

- The most intuitive and simple machine learning technique.
- Case based reasoning: inference about a new instance is directly based on info about known similar cases.
- A supervised machine learning approach. Supervised?
- It can be used for prediction and classification. Prediction and classification?

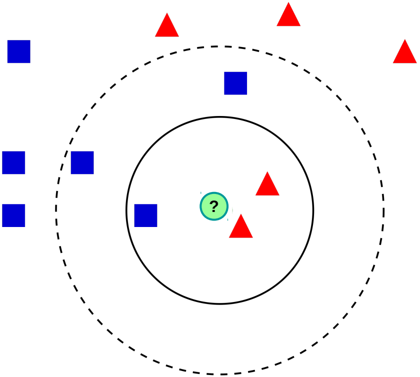
K-Nearest Neighbors

Consider K=1, Is Mario going to the field trip?



K-Nearest Neighbors: Classification

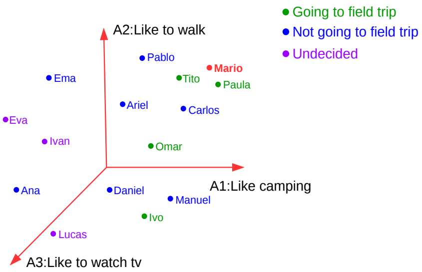
Consider K=3, What class is the instance at the center?



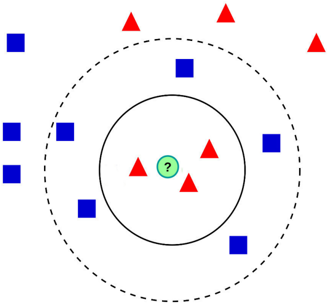
Majority Vote: choose most voted class among K neighbors.

K-Nearest Neighbors: Classification

Consider K=5, Is Mario going to the field trip?

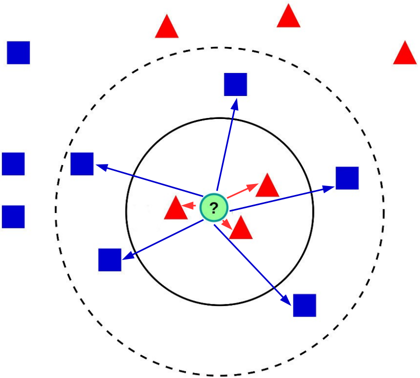


Consider K=8, What class is the instance at the center?



Any problem?, Can we improve this situation?, How?

K-Nearest Neighbors: Classification



Weighted majority vote: weight vote is proportional to the inverse of the distance to the corresponding neighbor.

II. Distance metric

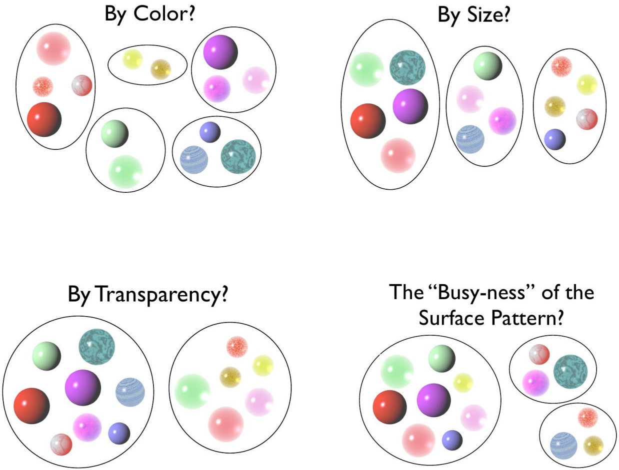
Traditional K-NNs are based on Euclidean distance.
Is this always a good choice?





Distance metric matters

Clustering Marbles



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- In general, machine learning techniques move input instances to a new space, where the underlying distance embeds semantic relations among the instances. Semantic is given by the information in the labels.
- In the this course, we will learn that deep learning is an excellent tool to build semantic distance spaces.

What About K-NN on Non Euclidean spaces?

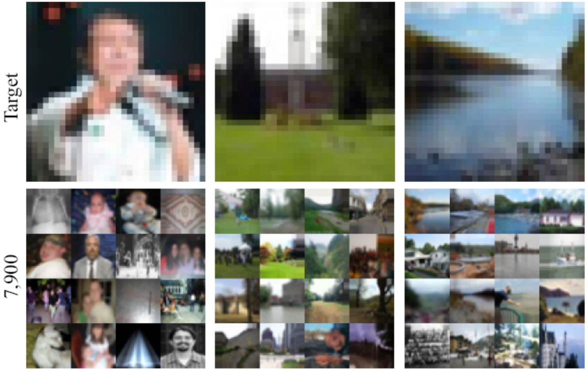
Case Study:

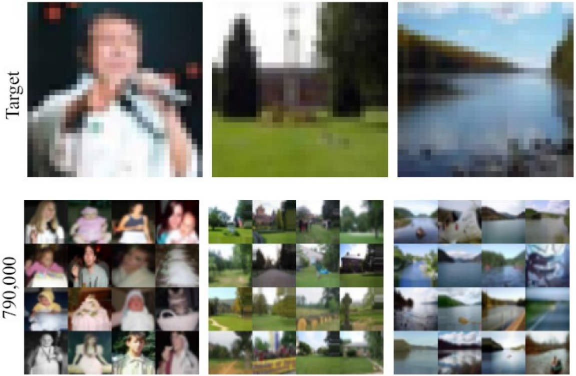
80 million tiny images: A large dataset for non-parametric object and scene recognition.

A. Torralba, R. Fergus and W. T. Freeman.
IEEE PAMI, 2006.

What can we do with 80M images?

K-NN Non Eucliden spaces




Target
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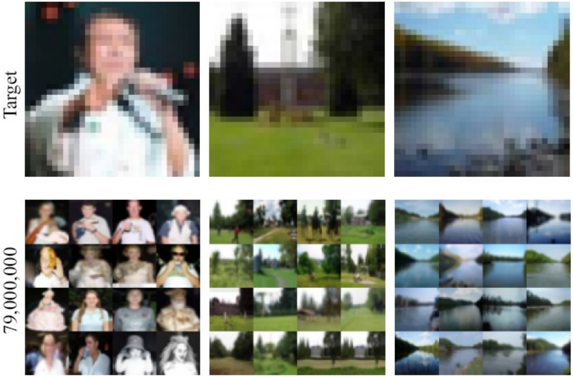
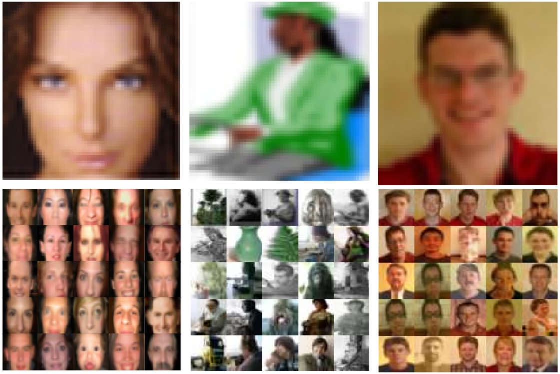
K-NN Non Eucliden spaces



Big data makes magic!



Distance metric between images.

$$D^2_{\text{ssd}} = \sum_{x,y,c} (I_1(x,y,c) - I_2(x,y,c))^2$$

$$D^2_{\text{warp}} = \min_{\theta} \sum_{x,y,c} (I_1(x,y,c) - T_\theta[I_2(x,y,c)])^2$$

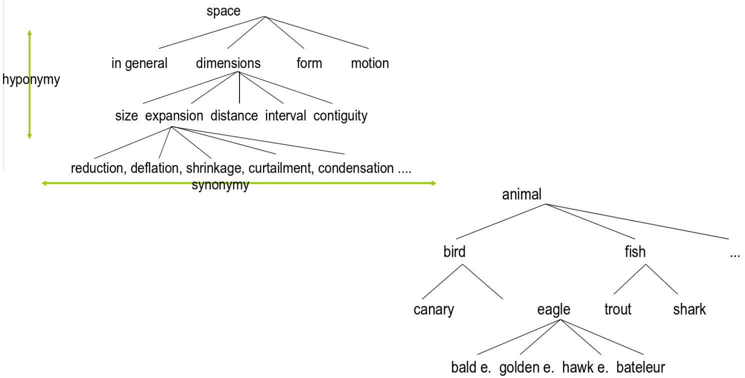
Patch horizontal mirror, translations and scaling.

$$D^2_{\text{shift}} = \min_{|D_{x,y}|\le w} \sum_{x,y,c} (I_1(x,y,c) - \hat{I}_2(x+D_x, y+D_y, c))^2$$

Individual pixel shift in 5x5 window. Tθ given By best Dwarp.

Keywords

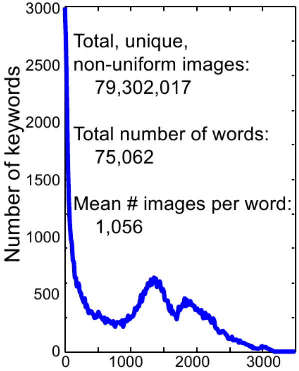
WordNet: lexical database of English words. Nouns, verbs, adjectives, and adverbs are grouped into sets of cognitive synonyms (synsets), each expressing a distinct concept. Synsets are interlinked by means of semantic and lexical relations.



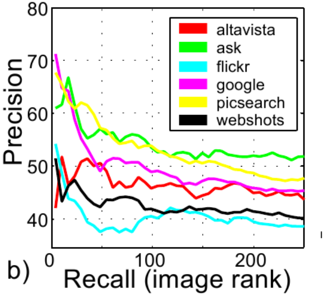
Dataset

- **Words:** They extract all non-abstract nouns from Wordnet, a total of 75.846 words.
- **Images** : For each word, they automatically download all the images provided by 7 independent image search engines: Altavista, Ask, Flickr, Cydral, Google, Picsearch and Webshots.
- Running over 8 months, they collect approx. 80M images (97.245.098 images).
- Finally, they filter out the dataset removing duplicate and uniform images (images with zero variance). Also, they remove words that do not produce enough images (around 1% of the keywords have no images). The final dataset contains 79.302.017 images from 75.062 words.
- Due to efficiency reasons, they store the images using a resolution of 32x32 pixels.

K-NN Non Eucliden spaces



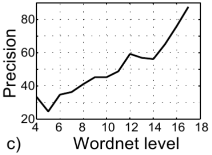
K-NN Non Eucliden spaces

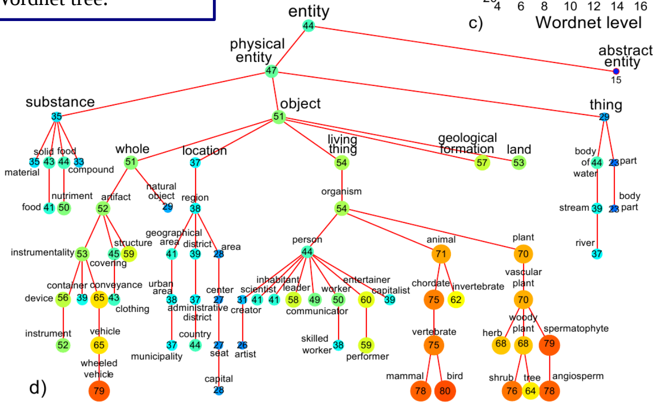


Retrieval accuracy drops after the 100th image. In average first 44 images are correct.

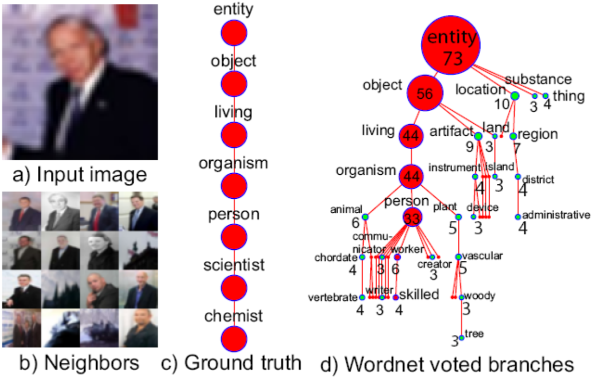
Accuracy of labeling for different nodes of a portion of the Wordnet tree.

More precision for more specific words





Object recognition results



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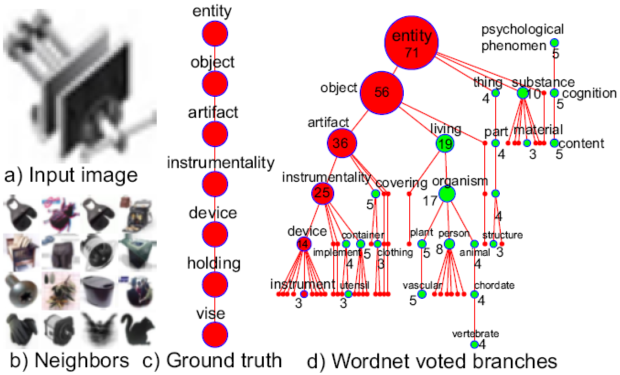

Case Study:

Scene Completion using Millions of Photographs.
J. Hays and A. Efros
SIGGRAPH 2007.

K-NN Non Eucliden spaces

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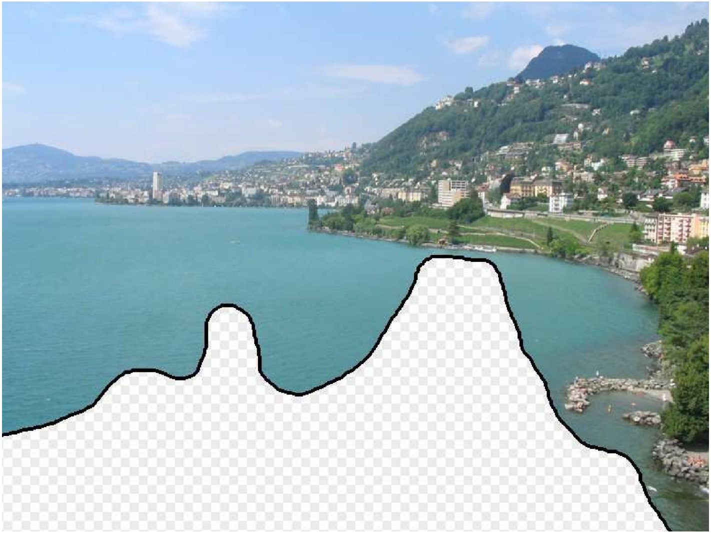
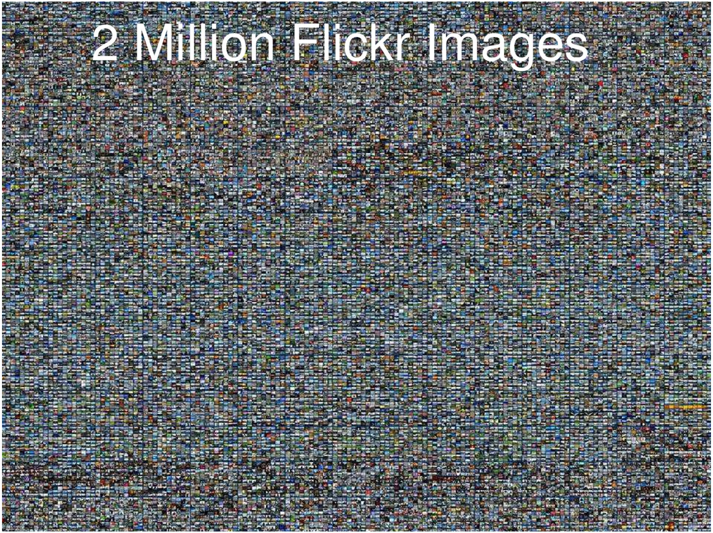
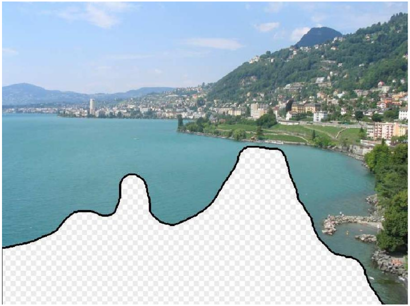


K-NN Non Eucliden spaces



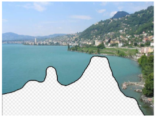
K-NN Non Eucliden spaces

Input Image



Most similar image
in Flickr dataset

K-NN Non Eucliden spaces




K-NN Non Eucliden spaces

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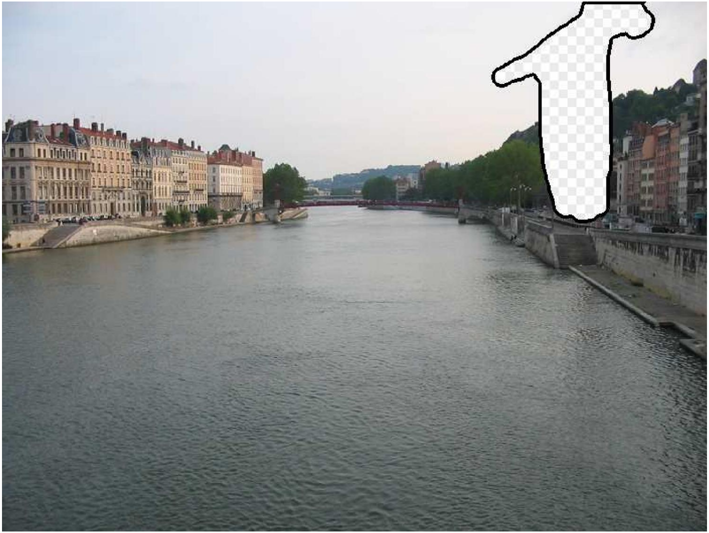
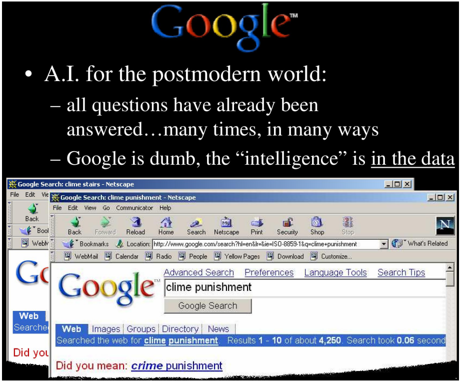


K-NN Non Eucliden spaces

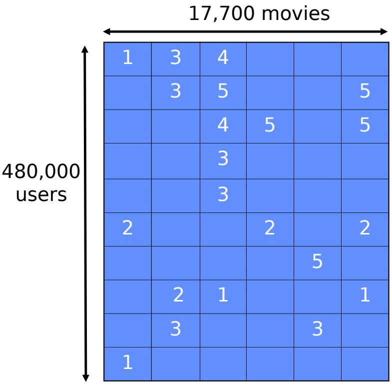
K-NN Non Eucliden spaces

A suitable distance metric ?



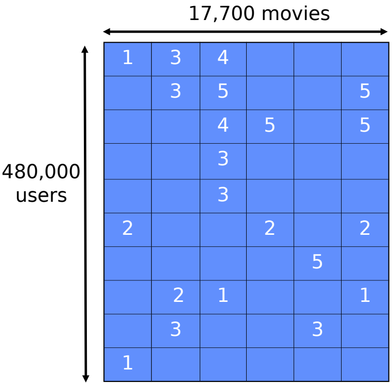
Netflix contest dataset

- 100 million ratings.
- Atributes = [user, movie-id, time-stamp, rating value].
- Generated by users between Oct. 1998 and Dec. 2005.
- Users randomly chosen among set with at least 20 ratings (small perturbations to help with anonymity).



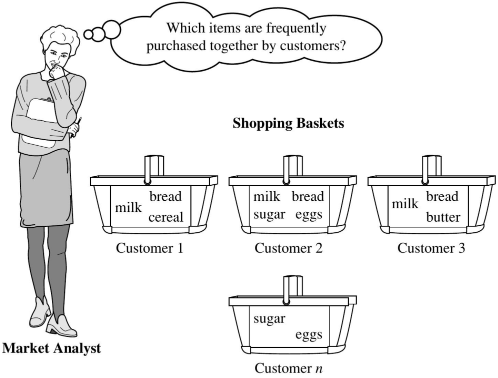
Netflix contest dataset

Problem: dataset is 99% sparse



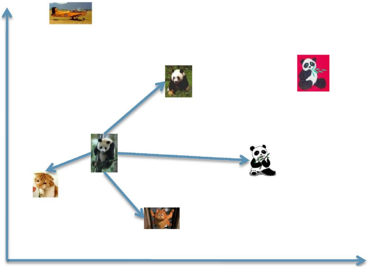
Is this a problem for a K-NN strategy?

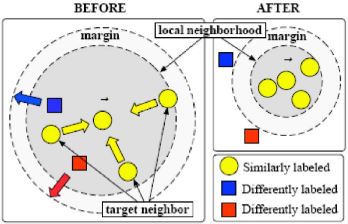
Association rules



In many cases, Euclidean distance is not a good way to estimate semantic distances between feature vectors. However, this can be valid in subspaces of the original feature space.

Learning a distance metric: Optimizing cost function





Learning a distance metric: Optimizing cost function

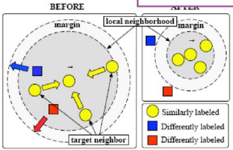
$$\varepsilon(\mathbf{L}) = \sum_{ij} \eta_{ij} \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|^2 + c \sum_{ijl} \eta_{ij}(1-y_{il}) \left[1 + \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|^2 - \|\mathbf{L}(\vec{x}_i - \vec{x}_l)\|^2\right]_+$$

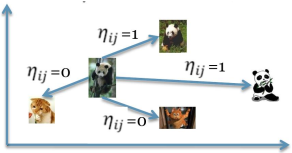
For inputs and target neighbors
It is equal to 1

$y_{il} \in \{0,1\}$ indicates if $\vec{x}_i$ and $\vec{x}_l$ has same label. So For input and neighbors having different labels, it is equal to 1

Distance between inputs and target neighbors

Distance between input and neighbors with different labels





$$\mathcal{D}(\vec{x}_i, \vec{x}_j) = (\vec{x}_i - \vec{x}_j)^\top \mathbf{M} (\vec{x}_i - \vec{x}_j)$$

$$\mathbf{M} = \mathbf{L}^\top \mathbf{L}.$$

Relevant ideas from the previous slides

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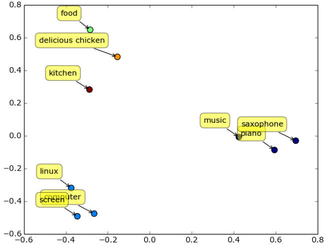
   - Finding subspaces
   - Optimizing a suitable loss function
   - And so on ...

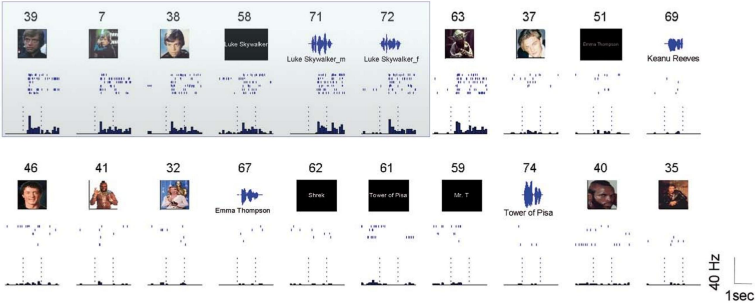
Distributed Representations

Distributed Representations

Learning a semantic space (semantic embedding, semantic distance).



III. Distributed Representations



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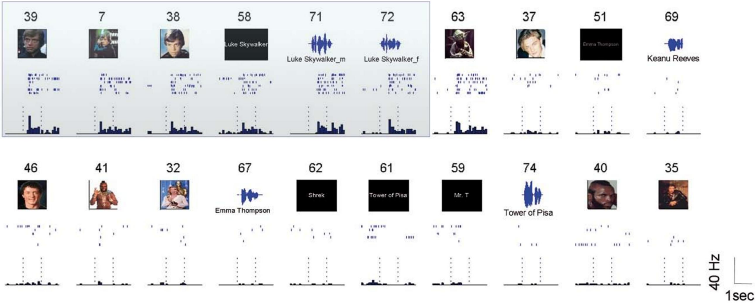
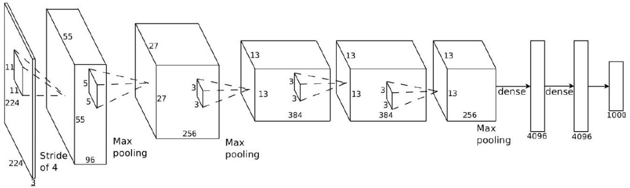


Embedding space can combine information from different sensing modalities: vision, audio, text, etc.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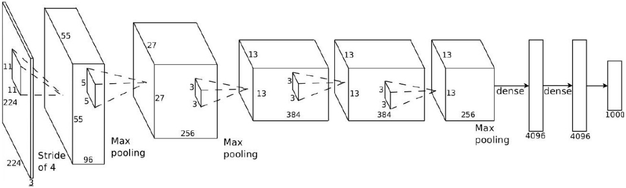
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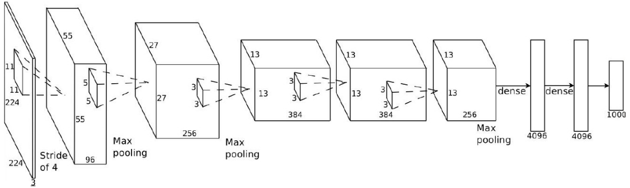

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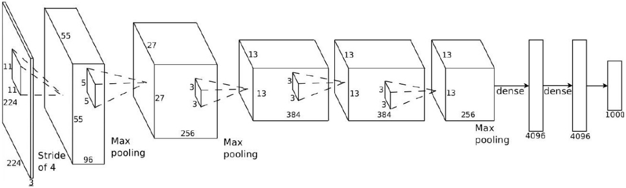
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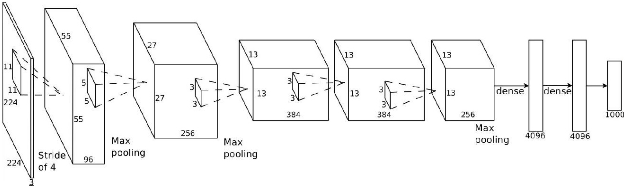
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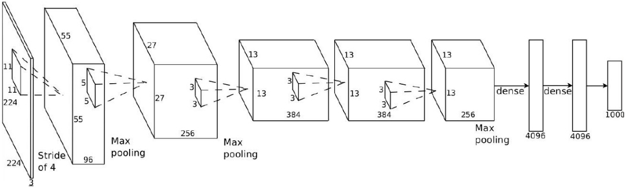
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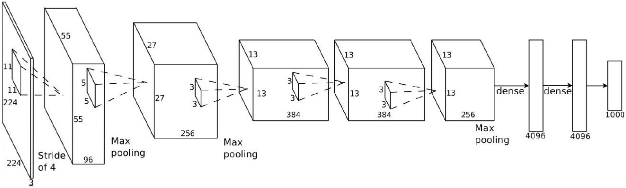
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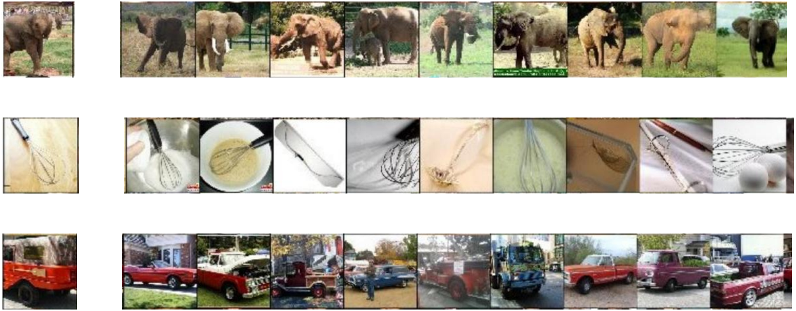

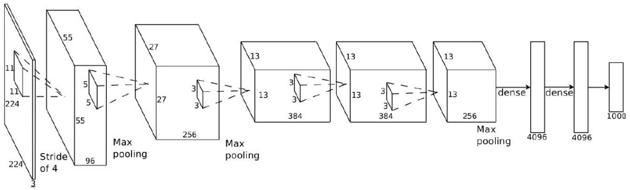

Deep learning models are excellent tools to move data to meaningful spaces, where simple Euclidean distance reveals semantic relations present in the training data (labels).

IV. Finding the NNs

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- What problem will we face when we apply that distance using a KNN strategy?
- Hint: consider a big data scenario.

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Locality Sensitive Hashing (LSH) to the rescue

Hashing

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Requirements:

- $f(\cdot)$ must avoid collisions, why?.
- $f(\cdot)$ must be quick to compute, why?.

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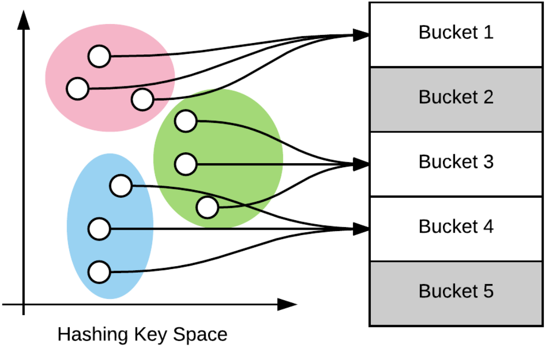

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KNNs: A powerful approach.

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- DL: "Learning an Efficent and Robust Associative Memory System"