

12. Найти производную функции по определению.

1. $f(x) = \sin 3x + 5x^2 - 4.$
2. $f(x) = \cos 4x - 8x + 7.$
3. $f(x) = \operatorname{tg} 2x - 3x + 2.$
4. $f(x) = \operatorname{ctg} 5x - x^2 + 12.$
5. $f(x) = \ln(4x) - 2x^2 - 14.$
6. $f(x) = e^{8x} + 18x + 4.$
7. $f(x) = 3^{2x} - 6x - 2.$
8. $f(x) = \sqrt{x} + 2x^3 + 2.$
9. $f(x) = \frac{4}{x} - 8x^2 + 19.$
10. $f(x) = 2\sqrt{x} + e^x - 1.$
11. $f(x) = \cos 2x + 3x^2 + 9.$
12. $f(x) = \operatorname{ctg} 3x + 8x - 5.$
13. $f(x) = 2x^2 - \sin 4x + 5.$
14. $f(x) = \operatorname{tg} x - 3x^2 + 7.$
15. $f(x) = 4\sqrt{x} - \ln(2x) - 2.$
16. $f(x) = 5^x - \frac{8}{x} + 20.$
17. $f(x) = e^{3x} + 4x^2 + 1.$
18. $f(x) = \ln(2x) - \frac{1}{x} - 6.$
19. $f(x) = 8^x + 6x^2 - 12.$
20. $f(x) = 4^x + \cos 2x.$
21. $f(x) = \ln(2x) - \sin x + 2.$
22. $f(x) = \operatorname{tg} 5x - 4x^2 + 8.$
23. $f(x) = \sin 8x - 6\sqrt{x} - 8.$
24. $f(x) = 5 - \frac{7}{x} + \cos x.$
25. $f(x) = \operatorname{ctg} 2x + e^x + 8.$
26. $f(x) = \sin 5x - 6^x - 13.$
27. $f(x) = e^{4x} + \sin 2x - 22.$
28. $f(x) = \operatorname{tg} 5x - \ln x + 18.$
29. $f(x) = \operatorname{ctg} x + \frac{5}{x} - 8.$
30. $f(x) = \operatorname{tg} 9x + \cos 5x - 9.$

13. Найти производную функции.

$$1. y = 4 \operatorname{ctg} 2x \cdot \sin x.$$

$$3. y = 3 \operatorname{arctg} x \cdot \frac{1}{x}.$$

$$5. y = 4 \cos x \cdot \arccos x.$$

$$7. y = (1 - x^2) \cdot \arcsin x.$$

$$9. y = (\sqrt{x^3} - \sqrt{x}) \ln x.$$

$$11. y = \frac{2}{x} (1 + \sqrt[3]{x}) \sqrt{x+5}.$$

$$13. y = \frac{2}{x} \ln(x+3).$$

$$15. y = 3 \operatorname{tg} 2x \cdot \operatorname{arctg} 2x.$$

$$17. y = (x^4 - 1) \cdot \operatorname{arccotg} x.$$

$$19. y = e^{2x} \sqrt{2^x}.$$

$$21. y = (e^{2x} - 4)(2x^4 + 3).$$

$$23. y = x^2 \cdot \cos x.$$

$$25. y = \log_5 x \cdot e^{2x}.$$

$$27. y = \operatorname{tg} 2x \cdot \arcsin x.$$

$$29. y = x(x^3 - \sqrt{x}) \ln x.$$

$$2. y = x^2 \ln x.$$

$$4. y = 3 \log_2 x \cdot \operatorname{arccotg} 2x.$$

$$6. y = (x^2 + 1) \cdot \operatorname{arctg} x.$$

$$8. y = \sqrt{e^x} 3^{2x}.$$

$$10. y = e^{3x} \cdot \operatorname{arctg} x.$$

$$12. y = 2 \operatorname{tg} 3x \cdot e^{2x}.$$

$$14. y = 2^x \cdot \frac{1}{x^2}.$$

$$16. y = x^3 \arccos x.$$

$$18. y = (x+1) \cdot \arcsin x.$$

$$20. y = (x^2 + \sqrt[3]{x}) \log_3 x.$$

$$22. y = 3 \sin 2x \cdot e^x.$$

$$24. y = 2^x \operatorname{arctg} x.$$

$$26. y = \sqrt[3]{x} \cdot (e^{3x} - 5).$$

$$28. y = 3^{4x} \ln 2x.$$

$$30. y = (e^{3x} + 4) \ln(2x-3).$$

14. Найти производную функции.

$$1. y = \frac{x^2}{\sqrt{x-1}}.$$

$$3. y = \frac{x^3 - 2x}{e^{2x} - 4}.$$

$$5. y = \frac{x^3 - \sin x}{\cos x}.$$

$$7. y = \frac{x^2 - 4}{\sqrt{x-2}}.$$

$$9. y = \frac{2+x^2}{\ln x}.$$

$$11. y = \frac{\ln x}{\operatorname{tg} x}.$$

$$13. y = \frac{\arcsin 2x}{1-4x^2}.$$

$$2. y = \frac{\sqrt{x-4}}{\sin x}.$$

$$4. y = \frac{\ln x}{4-x^2}.$$

$$6. y = \frac{x^2 + 1}{\operatorname{arctg} x}.$$

$$8. y = \frac{1 + \sin x}{1 - \cos x}.$$

$$10. y = \frac{\operatorname{tg} x}{\sqrt{4x+4}}.$$

$$12. y = \frac{e^x + e^{-x}}{e^x - e^{-x}}.$$

$$14. y = \frac{16-x^2}{5-\sqrt{3x}}.$$

$$15. y = \frac{x^2 + 2x + 1}{\operatorname{ctg} x}.$$

$$17. y = \operatorname{tg} x + \frac{x}{\cos x}.$$

$$19. y = \frac{\cos x}{1 + 2\sin x}.$$

$$21. y = \frac{\sqrt{x}}{\sqrt{x} + 1}.$$

$$23. y = \frac{4x^2 - 1}{\sqrt{2x + 1}}.$$

$$25. y = \frac{e^x}{2x + e^{2x}}.$$

$$27. y = \frac{x^2 - 2^x}{2x + 4}.$$

$$29. y = \frac{x^2 + 16}{1 + \log_2 x}.$$

$$16. y = \frac{\sin 2x}{\operatorname{tg} x}.$$

$$18. y = \frac{\ln x}{\sin x} + x \operatorname{ctg} x.$$

$$20. y = \frac{\operatorname{ctg} x}{\sqrt{x}}.$$

$$22. y = \frac{1 + e^x}{1 - e^x}.$$

$$24. y = \frac{x(1 - x^2)}{7x + 11}.$$

$$26. y = \frac{\ln x^2 + \cos x}{x}.$$

$$28. y = \frac{\operatorname{arccotg} x}{1 + x^2}.$$

$$30. y = \frac{2x^4 - 8}{x\sqrt{x - 1}}.$$

15. Найти производную функции.

$$1. y = \sin((3x^2 + 1)^{10}).$$

$$3. y = (\sin 3x)^3.$$

$$5. y = \frac{\ln x}{\sqrt{4x^2 + 1}}.$$

$$7. y = \frac{\sqrt{2x^2 + 3}}{x + 1}.$$

$$9. y = \frac{e^{x^2} - e^{-x^2}}{2x}.$$

$$11. y = \ln^2(x^2 + 5x + 4).$$

$$13. y = 3^{\sin\left(\frac{1}{x^2}\right)}.$$

$$15. y = \frac{1}{(\sin 2x)^2}.$$

$$17. y = \frac{\arcsin x^2}{x}.$$

$$19. y = \sqrt{\arcsin \frac{1}{x}}.$$

$$2. y = \sqrt{e^{2x} + 1}.$$

$$4. y = \sqrt[3]{4^{\cos x}}.$$

$$6. y = \sqrt{x} \cos^2 2x.$$

$$8. y = \frac{1}{4} \operatorname{tg}^4 x - \ln \cos x.$$

$$10. y = \operatorname{arccotg} \sqrt{x^2 - 1}.$$

$$12. y = \ln \cos\left(\frac{1}{x^3}\right).$$

$$14. y = \arccos\left(\frac{1}{1 - x}\right).$$

$$16. y = \sqrt{\sin(3^x)}.$$

$$18. y = \ln^2 3x - \frac{3}{x^2}.$$

$$20. y = \left(\cos \frac{1}{x}\right)^5.$$

$$21. y = \ln \left(\frac{e^x - e^{-x}}{2} \right).$$

$$23. y = (\sin 3^x)^3.$$

$$25. y = \left(\operatorname{tg} x - \frac{3}{x^3} \right)^2.$$

$$27. y = \operatorname{tg} \arccos 2x.$$

$$29. y = \sin^3(x^2 + 3x + 1).$$

$$22. y = \ln \operatorname{arctg} \sqrt{x}.$$

$$24. y = \operatorname{tg} \left(\frac{x^3 + 4}{x^2} \right).$$

$$26. y = 4^{\operatorname{ctg} \left(\frac{1}{x} \right)}.$$

$$28. y = \arcsin(e^{x^2}).$$

$$30. y = \sqrt{\frac{1}{3} \sin^3 x + 4}.$$

16. Найти производную показательно-степенной функции.

$$1. y = x^{\sin x}.$$

$$3. y = x^{\cos x}.$$

$$5. y = x^{\operatorname{ctg} x}.$$

$$7. y = x^{\arccos x}.$$

$$9. y = (\cos(5 - 4x))^{3x}.$$

$$11. y = x^{2^x}.$$

$$13. y = (\sin x)^{x^2}.$$

$$15. y = (\sin(x + 1))^x.$$

$$17. y = (2 - 3x)^{4x}.$$

$$19. y = x^{e^x}.$$

$$21. y = (\operatorname{arctg}(5 - 7x))^{4x}.$$

$$23. y = (2 - x)^{\cos(4 - 3x)}.$$

$$25. y = (1 - 4x)^{\pi^x}.$$

$$27. y = (1 - x^2)^{5 - 4x^3}.$$

$$29. y = (\operatorname{tg} 3x)^{2x - 1}.$$

$$2. y = (\sin x)^x.$$

$$4. y = (\operatorname{tg} x)^x.$$

$$6. y = x^{\operatorname{arctg} x}.$$

$$8. y = (1 - 4x)^{\sin 5x}.$$

$$10. y = (1 - 3x)^{\operatorname{tg} x}.$$

$$12. y = (1 - x)^{\operatorname{arctg}(5 - 7x)}.$$

$$14. y = x^{2^{\cos x}}.$$

$$16. y = (x - 1)^{\operatorname{tg} x}.$$

$$18. y = (\sin(3 - 4x))^{5x}.$$

$$20. y = (\operatorname{tg}(4 - 5x))^{7x}.$$

$$22. y = x^{3^x}.$$

$$24. y = (3 - 4x)^{\operatorname{tg} 5x}.$$

$$26. y = (x^2 + 1)^{4 - 5x^3}.$$

$$28. y = (\cos 2x)^{3 - 5x}.$$

$$30. y = (\arcsin x)^x.$$

17. Написать уравнения касательной и нормали к заданной в неявном виде кривой $F(x; y) = 0$, проходящих через точку $(x; y)$, координаты которой удовлетворяют приведенным условиям.

$$1. x^2 + xy + y^2 = 7 \quad (x = 1; y = 2).$$

$$2. e^x \sin y - e^{-y} \cos x + e^x = 0 \quad \left(x = \frac{\pi}{2}; y = -\frac{\pi}{2} \right).$$

$$3. e^{xy} - x^2 + y^3 = 0 \quad (x = 0; y < 0).$$

$$4. x^2 + xy^2 = 5 \quad (x = 1; y > 0).$$

5. $x^3 + y^3 - \frac{9}{2}xy = 0$ ($x=1; y=2$).
6. $x^2 + y^2 = 4$ ($x=1; y>0$).
7. $x^2 - 4y^2 = 4$ ($x=4; y>0$).
8. $x^5 + 2y^2 = 9$ ($x=1; y>0$).
9. $x^2 - 3y^4 = 22$ ($x=5; y>0$).
10. $x^2 + 3y^4 = 52$ ($x=-2; y>0$).
11. $x^8 + 5y^2 = 21$ ($x=-1; y<0$).
12. $x^3 - 3y^2 = 16$ ($x=4; y<0$).
13. $\log_2(x+1) + \sqrt{y-1} = 8$ ($x=15$).
14. $\log_3(2x+1) + \sqrt{y+1} = 7$ ($x=4$).
15. $\sqrt{x^2+9} + \sqrt{y-4} = 11$ ($x=4$).
16. $\sqrt{x^3-2} + \sqrt{3y-1} = 9$ ($x=3$).
17. $\sqrt{x-2} + \sqrt{2y^3+9} = 6$ ($x=3$).
18. $\sqrt{x^2-9} + \sqrt{y^2+9} = 9$ ($x=5$).
19. $2\sin x + 5\sin y + 7\cos y = 5$ ($x=0; y=\frac{\pi}{2}$).
20. $7\cos x + 2\sin \frac{x}{y} = 7$ ($x=0; y=\frac{\pi}{2}$).
21. $\sqrt{3\cos x + \cos y + 1} = 2$ ($x=2\pi; y=\frac{\pi}{2}$).
22. $\sqrt{3\sin y - 5\cos y + \cos x - 1} = 2$ ($x=\frac{\pi}{2}; y=\pi$).
23. $\sqrt{3\sin x + 5\cos x + 1} + 7y = 9$ ($x=\frac{\pi}{2}; y=1$).
24. $x + \sqrt{5\sin y - 3\cos y + 4} = 5$ ($x=2; y=\frac{\pi}{2}$).
25. $\sqrt{x+7} + \sqrt{5\sin y - 3\cos y + 4} = 6$ ($x=2; y=\frac{\pi}{2}$).
26. $x + \sqrt{10\sin y - 5\cos y + 4} = 6$ ($x=3; y=\pi$).
27. $\sqrt{2-x} + \sqrt{5\sin y + 3\cos y + 12} = 5$ ($x=-2; y=\pi$).
28. $\sqrt{7-3x} + \sqrt{3\sin y + 7\cos y + 8} = 2$ ($x=2; y=\pi$).
29. $y^2 + x^2y = 5$ ($x=2; y=1$).
30. $y^5 + 2x^2 = 9$ ($x=2; y=1$).

18. Найти первую $\frac{dy}{dx}$ и вторую $\frac{d^2y}{dx^2}$ производные функций, заданных параметрически.

1. $\begin{cases} x = t^2 + 1, \\ y = t^3 - 3t^2. \end{cases}$

11. $\begin{cases} x = 2t^2 + t, \\ y = 6t - 2t^2 + t^3/3. \end{cases}$

21. $\begin{cases} x = t^3 + 2t, \\ y = 2t^3 + 6t^2 + 2. \end{cases}$

2. $\begin{cases} x = 2t + t^2, \\ y = 2t^3 + 3t^2. \end{cases}$
3. $\begin{cases} x = 1 - t^2, \\ y = t^4 - 8t. \end{cases}$
4. $\begin{cases} x = 2 + 3t^2, \\ y = 3 - 2t^2 + t^3. \end{cases}$
5. $\begin{cases} x = 2t + t^2, \\ y = t^3 - 6t^2 + 5t. \end{cases}$
6. $\begin{cases} x = 3t - t^2, \\ y = t^3 - 2t^2 + t. \end{cases}$
7. $\begin{cases} x = 2 - 3t^2, \\ y = t^4 - 2t^3 - 1,5t. \end{cases}$
8. $\begin{cases} x = 3t^2 + 2t, \\ y = 2t^2 - 3t^3. \end{cases}$
9. $\begin{cases} x = 2 - 3t^2, \\ y = 3t^2 - 2t^3. \end{cases}$
10. $\begin{cases} x = 1 - t^2, \\ y = t^4 - 4t^2. \end{cases}$
12. $\begin{cases} x = t^2 - 5t, \\ y = 4t^3 - 12t. \end{cases}$
13. $\begin{cases} x = 2t - t^2, \\ y = t^4 - 16t. \end{cases}$
14. $\begin{cases} x = t - 2t^2, \\ y = t^3 + 2t^2 + 1. \end{cases}$
15. $\begin{cases} x = 3t^2 + t, \\ y = t^4 + 2t^2 + 1. \end{cases}$
16. $\begin{cases} x = t^2 + 3t, \\ y = 1 - t^2 + 2t^3. \end{cases}$
17. $\begin{cases} x = 3t - t^2, \\ y = 2 - 2t^2 - t^3. \end{cases}$
18. $\begin{cases} x = t - 3t^2, \\ y = 16t - t^4. \end{cases}$
19. $\begin{cases} x = 5t^2 + 3, \\ y = 2t^3 + t + 1. \end{cases}$
20. $\begin{cases} x = 3 - 5t^2, \\ y = 1 - 2t^2 - t^3. \end{cases}$
22. $\begin{cases} x = 4t^2 + t, \\ y = 1 - 6t^2 - 2t^3. \end{cases}$
23. $\begin{cases} x = 5t^3 + t, \\ y = 2t - 8t^3. \end{cases}$
24. $\begin{cases} x = 2t - 3t^3, \\ y = 1 - 3t + 5t^3. \end{cases}$
25. $\begin{cases} x = 3t^2 - 6t, \\ y = 6t - 3t^3. \end{cases}$
26. $\begin{cases} x = 2t - 5t^3, \\ y = 4 + 2t + 6t^3. \end{cases}$
27. $\begin{cases} x = 2t - t^3, \\ y = 3t^3 - 6t^2 + 3. \end{cases}$
28. $\begin{cases} x = 2t - 5t^5, \\ y = 3 - t + 8t^3. \end{cases}$
29. $\begin{cases} x = 1 - 4t^3, \\ y = 1 - 5t^2 + 2t^3. \end{cases}$
30. $\begin{cases} x = 6t^2 + 3t, \\ y = 2 - 5t^2 + 3t^3. \end{cases}$

11. Найти производную функции.

1. $y = (x^2 + 1)^{10}$

2. $y = \sqrt{e^{2x}}$

3. $y = (\sin 3x)^3$

4. $y = \sqrt[3]{4^{\cos x}}$

5. $y = \frac{\ln x}{\sqrt{x^2 + 1}}$

6. $y = \sqrt{x} \cos^2 x$

7. $y = \frac{\sqrt{x^2 + 1}}{x + 1}$

8. $y = \frac{1}{4} \operatorname{tg}^4 x - \ln \cos x$

9. $y = \frac{e^x - e^{-x}}{2}$

10. $y = \operatorname{arctg} \sqrt{x}$

11. $y = \ln(x^2 + 5x + 4)$

12. $y = \ln \cos \left(\frac{1}{x^3} \right)$

13. $y = 3^{\sin \left(\frac{1}{x^2} \right)}$

14. $y = \arccos \left(\frac{1}{1-x} \right)$

15. $y = \frac{1}{(\sin 2x)^2}$

16. $y = \sqrt{\sin(3^x)}$

17. $y = \frac{\arcsin(x^2)}{x}$

18. $y = \operatorname{tg} 3x - \frac{3}{x^2}$

19. $y = \sqrt[3]{\arcsin \frac{1}{x}}$

20. $y = \left(\cos \frac{1}{x} \right)^5$

21. $y = \frac{e^x - e^{-x}}{2}$

22. $y = \ln \operatorname{arctg} \sqrt{x}$

23. $y = (\sin 3^x)^3$

24. $y = \frac{x^3 + 4}{x^2}$

25. $y = \left(\operatorname{tg} x - \frac{3}{x^3} \right)^2$

26. $y = 4^{\operatorname{ctg} \left(\frac{1}{x} \right)}$

27. $y = \frac{\sin 2x}{\cos 3x}$

28. $y = \arcsin(e^{x^2})$

29. $y = \sin^3(x^2 + 3x + 1)$

30. $y = \sqrt{3 \sin x + 4}$

12. Найти производную показательно-степенной функции.

1. $y = x^{\sin x}$

2. $y = (\sin x)^x$

3. $y = x^{\cos x}$

4. $y = (\operatorname{tg} x)^x$

5. $y = x^{\operatorname{ctg} x}$

6. $y = x^{\operatorname{arctg} x}$

7. $y = x^{\arccos x}$

8. $y = (1 - 4x)^{\sin 5x}$

9. $y = (\cos(5 - 4x))^{3x}$

10. $y = (1 - 3x)^{\operatorname{tg} x}$

11. $y = x^{2^x}$

12. $y = (1 - x)^{\operatorname{arctg}(5 - 7x)}$

13. $y = (\sin x)^{x^2}$

14. $y = x^{2^{\cos x}}$

15. $y = (\sin(x + 1))^x$

16. $y = (x - 1)^{\operatorname{tg} x}$

17. $y = (2 - 3x)^{4x}$

18. $y = (\sin(3 - 4x))^{5x}$

19. $y = x^{e^x}$

20. $y = (\operatorname{tg}(4 - 5x))^{7x}$

21. $y = (\operatorname{arctg}(5 - 7x))^{4x}$

22. $y = x^{3^x}$

23. $y = (2 - x)^{\cos(4 - 3x)}$

24. $y = (3 - 4x)^{\operatorname{tg} 5x}$

25. $y = (1 - 4x)^{\pi^x}$

26. $y = (x^2 + 1)^{4 - 5x^3}$

27. $y = (1 - x^2)^{5 - 4x^3}$

28. $y = (\cos 2x)^{3 - 5x}$

29. $y = (x^2)^{\frac{1}{x^2}}$

30. $y = (x^3 + 4)^{\frac{1}{x^5}}$