最优化第十次作业

张晋 15091060

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4.7 (a)

$$\lim_{x\to\infty} \boldsymbol{x}^{(k)} = 0, \quad e_k = \frac{1}{k}, \quad \lim e_{k+1}/e_k \to 1$$

收敛阶为 1，为次线性收敛，$k = 10001$

(b)

$$\lim_{x\to\infty} \boldsymbol{x}^{(k)} = 0, \quad e_k = (\frac{1}{2})^{2^k}, \quad \lim e_{k+1}/e_k^2 \to 1$$

收敛阶为 2，为二次收敛，$k = \text{IntegerPart}\left[\log_2\left(4\log_2(10)\right)\right]+1 = 4$

(c)

$$\lim_{x\to\infty} \boldsymbol{x}^{(k)} = 0, \quad e_k = \frac{1}{k!}, \quad \lim e_{k+1}/e_k \to 0$$

收敛阶趋近于 1，为超线性收敛，$k = 8$

4.8

$$\boldsymbol{g}(\boldsymbol{x}) = \nabla f(\boldsymbol{x}) = (\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2})^T = [2(x_1 + {x_2}^2), 4(x_1 + {x_2}^2)x_2]^T \tag{1}$$

$$\boldsymbol{g}(\boldsymbol{x}^{(k)}) = (2, 0)^T \tag{2}$$

$$\boldsymbol{p}^{(k)^T}\boldsymbol{g}^{(k)} = -2 < 0 \tag{3}$$

故 $\boldsymbol{p}^k$ 是其下降方向。

由于 $f(\boldsymbol{x})$ 并非二次函数，所以使用 $\alpha_k = \dfrac{-\boldsymbol{p}^{(k)^T}\boldsymbol{g}^{(k)}}{\boldsymbol{p}^{(k)^T}\boldsymbol{G}\boldsymbol{p}^{(k)}}$ 计算 α 时会出现误差, 此时应该直接代入算得:

$$f(\boldsymbol{x}^{(k+1)}) = (\alpha^2 - \alpha + 1)^2 \tag{4}$$

故 $\alpha = 1/2$ 时，$f(\boldsymbol{x}^{(k+1)})$ 取极小值

4.11

$$\phi(\alpha) = 1 - \alpha e^{-\alpha^2}, \qquad \phi(0) = 1 \tag{5}$$

$$\phi'(\alpha) = (2\alpha^2 - 1)e^{-\alpha^2}, \qquad \phi'(0) = -1 \tag{6}$$

$$\phi(\alpha) \leq \phi(0) + \rho\phi'(0)\alpha \tag{7}$$

$$\Rightarrow \qquad \alpha \leq \sqrt{-\ln\rho} \tag{8}$$

$$\phi(\alpha) \geq \phi(0) + (1-\rho)\phi'(0)\alpha \tag{9}$$

$$\Rightarrow \qquad \alpha \geq \sqrt{-\ln(1-\rho)} \tag{10}$$

Goldstein 条件:

$$\sqrt{-\ln(1-\rho)} \leq \alpha \leq \sqrt{-\ln\rho} \tag{11}$$

Wolfe 条件:

$$(2\alpha^2 - 1)e^{-\alpha^2} \geq -\sigma \quad \& \quad \alpha \leq \sqrt{-\ln\rho} \tag{12}$$

强 Wolfe 条件:

$$-\sigma \leq (2\alpha^2 - 1)e^{-\alpha^2} \leq \sigma \quad \& \quad \alpha \leq \sqrt{-\ln\rho} \tag{13}$$

代入 σ, ρ 解得:

	$\sigma = \rho = 1/10$	$\sigma = \rho = 1/4$
Goldstein 条件	$[0.324593, 1.517427]$	$[0.536360, 1.177410]$
Wolfe 条件	$[0.650865, 1.517427]$	$[0.571578, 1.177410]$
强 Wolfe 条件	$[0.650865, 0.768257]$	$[0.571578, 0.877473]$