最优化第十五次作业

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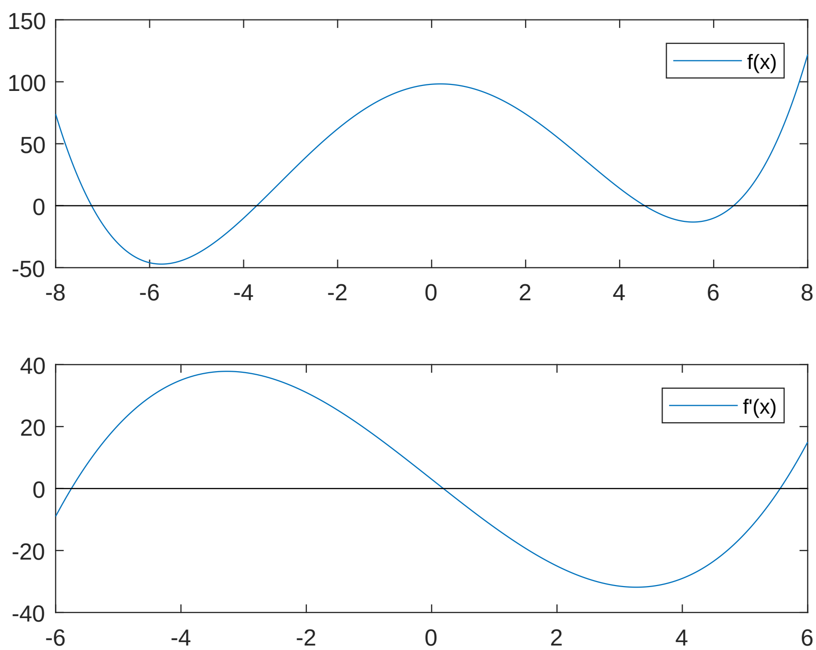
7.20 函数 $f(x)=\frac{1}{2}(\frac{1}{2}x^2-14)^2-x^2+3x$ 及其导数 $f'(x)=\frac{1}{2}x^3-16x+3$ 的图像如下:



求解 $f'(x)=0$ 得 $x_1=-5.74837, x_2=0.187707, x_3=5.56066$, 由于 $x\geq 0$, 故极小值在 $x^\star=5.56066$ 处取到, 此时无积极约束，$\mathcal{A}^\star=\emptyset$.

去掉约束条件 $x\geq 0$ 后，有两个局部极小点 $x_1=-5.74837, x_3=5.56066$，其中全局极小点在 x_1 处取到.

发现的结论: 去掉非积极约束对局部最优值没有影响，但可能会影响到全局最优解.

7.3

$$\mathcal{L}(\boldsymbol{x},\lambda)=(x_1-\frac{1}{9})^2+(x_2-2)^2+\lambda_1({x_1}^2-x_2)+\lambda_2(x_1+x_2-6)-\lambda_3x_1-\lambda_4x_2$$

$$\nabla_{\boldsymbol{x}}\mathcal{L}(\boldsymbol{x},\lambda)=\begin{bmatrix}2(x_1-\frac{9}{4})+2\lambda_1x_1+\lambda_2-\lambda_3\\2(x_2-2)-\lambda_1+\lambda_2-\lambda_4\end{bmatrix}$$

(a) **KKT 条件：**

$$\begin{aligned}2(x_1-\frac{9}{4})+2\lambda_1x_1+\lambda_2-\lambda_3&=0\\2(x_2-2)-\lambda_1+\lambda_2-\lambda_4&=0\\{x_1}^2-x_2&\le 0\\x_1+x_2-6&\le 0\\x_1&\ge 0\\x_2&\ge 0\\\lambda_1({x_1}^2-x_2)&=0\\\lambda_2(x_1+x_2-6)&=0\\\lambda_3x_1&=0\\\lambda_4x_2&=0\\\lambda_i\ge 0,\quad i&=1,2,3,4.\end{aligned}\tag{1}$$

代入 $\boldsymbol{x}^\star=(\frac{3}{2},\frac{9}{4})^T$，其满足约束条件且解得：

$$\lambda_1=\frac{1}{2},\quad\lambda_2=\lambda_3=\lambda_4=0$$

(b) 在 $\boldsymbol{x}^\star$ 处的 $\mathcal{A}^\star=\{1\}$。即在该点处 $c_1(x)=0$ 与目标函数的等高线相切

(c) 目标函数是凸函数，可行域是凸集，所以是凸规划，故局部极小点就是全局最优解.

7.4 (a) 设费用函数是 $f(x)$, 表述成优化问题为;

$$\begin{aligned} min \quad & f(\boldsymbol{x}) = 8x_1x_2 + 8x_2x_3 + 18x_1x_3 \\ \text{s.t.} \quad & x_1x_2x_3 - 1500 = 0 \\ & x_1 - 2x_2 = 0 \\ & x_1 \geq x_2 \end{aligned}$$

$$\mathcal{L}(\boldsymbol{x}, \lambda) = 8x_1x_2 + 8x_2x_3 + 18x_1x_3 + \lambda_1(x_1x_2x_3 - 1500) + \lambda_2(x_1 - 2x_2) + \lambda_3(x_2 - x_1)$$

$$\nabla_{\boldsymbol{x}}\mathcal{L}(\boldsymbol{x}, \lambda) = \begin{bmatrix} 8x_2 + 18x_3 + \lambda_1x_2x_3 + \lambda_2 - \lambda_3 \\ 8x_1 + 8x_3 + \lambda_1x_1x_3 - 2\lambda_2 + \lambda_3 \\ 8x_2 + 18x_1 + \lambda_1x_1x_2 \end{bmatrix}$$

KKT 条件:

$$\begin{aligned} 8x_2 + 18x_3 + \lambda_1x_2x_3 + \lambda_2 - \lambda_3 = 0 \\ 8x_1 + 8x_3 + \lambda_1x_1x_3 - 2\lambda_2 + \lambda_3 = 0 \\ 8x_2 + 18x_1 + \lambda_1x_1x_2 = 0 \\ x_1x_2x_3 - 1500 = 0 \\ x_1 - 2x_2 = 0 \\ \lambda_3(x_2 - x_1) = 0 \\ \lambda_3 \geq 0 \end{aligned} \tag{2}$$

由等式约束 $x_1x_2x_3 - 1500 = 0, \quad x_1 - 2x_2 = 0$ 代入原问题确定:

$$\boldsymbol{x}_2^\star = 10\sqrt[3]{\frac{33}{32}}, \qquad \boldsymbol{\lambda}^\star = (\lambda_1^\star, \lambda_2^\star, \lambda_3^\star)^T = (-\frac{22}{x_2^\star}, -8x_2^\star + -\frac{3000}{{x_2^\star}^2}, 0)^T$$

(b) 在原问题中消去 x_1, x_2 得;

$$\begin{aligned} min \quad & f(\boldsymbol{x}) = 16{x_2}^2 + \frac{33000}{x_2} \\ \text{s.t.} \quad & x_2 \geq 0 \end{aligned} \tag{3}$$

最优解为 $x_2^\star = 10\sqrt[3]{\dfrac{33}{32}} = 10.1031, \quad f(10) = 4900, \quad f(11) = 4936$, 故最优整数解为 $x_2 = 10, \quad \boldsymbol{x}^\star = (20, 10, 7.5)$

(c) 解得:

$$\boldsymbol{x}(\varepsilon) = \left[2x_2, \sqrt[3]{\frac{33000 + 22\varepsilon}{32}}, \frac{1500 + \varepsilon}{2{x_2}^2}\right]^T$$

$$h(\varepsilon) = 16(x_2(\varepsilon))^2 + \frac{33000 + 22\varepsilon}{x_2(\varepsilon)} - f(\boldsymbol{x}^\star)$$

$$h'(0) = \frac{22}{{x_2}^\star} = -\lambda_1^\star$$

$h(-150) = -332.3, \quad {\lambda_1}^\star\varepsilon = -326.6$, 两者比较接近。