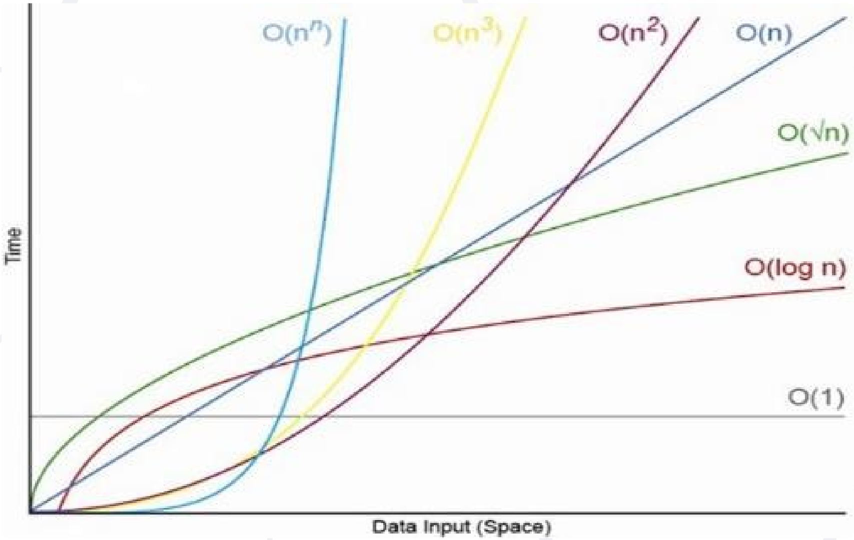
Algorithms and Complexity

Analyzing Algorithm Complexity. Asymptotic Notation



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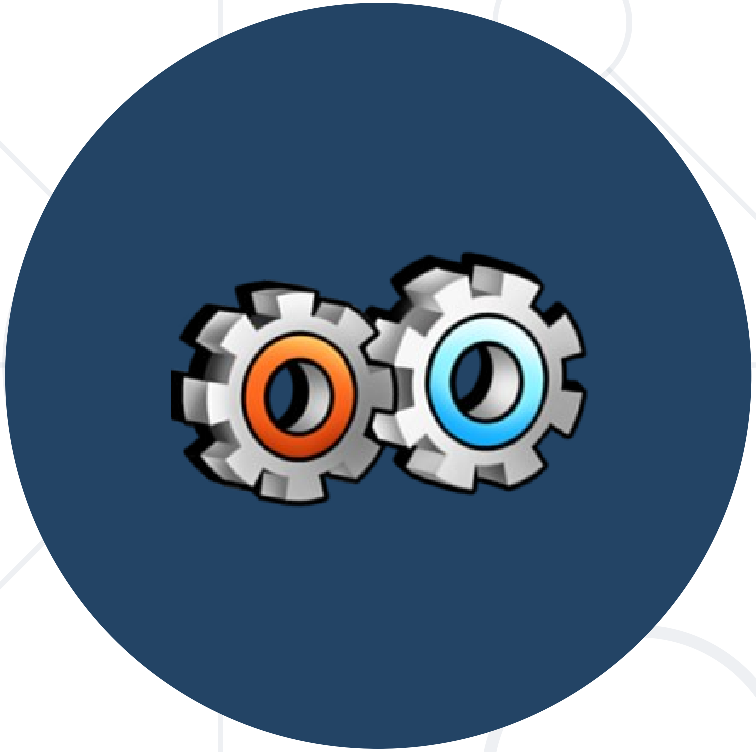


Overview

What is an Algorithm?

- The term "**algorithm**" means "**a sequence of steps**"
  - Derived from **Muḥammad Al-Khwārizmī'**, a Persia mathematician and astronomer
    - He described an algorithm for solving quadratic equations in 825

> **"In mathematics and computer science, an algorithm is a step-by-step procedure for calculations. An algorithm is an effective method expressed as a finite list of well-defined instructions for calculating a function."**
>
> ***-- Wikipedia***

Algorithms in Computer Science

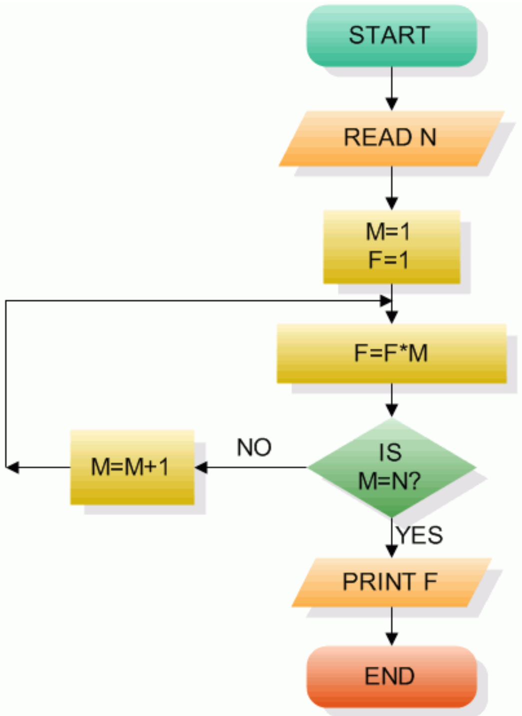
- Algorithms are fundamental in programming
  - **Imperative** (traditional, algorithmic) programming means to **describe in formal steps** how to do something
  - **Algorithm** == sequence of operations (steps)
    - Can include branches (conditional blocks) and repeated logic (loops)
- **Algorithmic thinking** (mathematical thinking, logical thinking, engineering thinking)
  - Ability to decompose the problems into formal sequences of steps (algorithms)

Pseudocode and Flowcharts

- **Algorithms** can be expressed as **pseudocode**, through **flowcharts** or **program code**

```
BFS(node)
{
  queue ← node
  while queue not empty
    v ← queue
    print v
    for each child c of v
      queue ← c
}
```

Pseudo-code



Flowchart

```
public void DFS(Node node)
{
  Print(node.Name);
  for (int i = 0; i < node.
    Children.Count; i++)
  {
    if (!visited[node.Id])
      DFS(node.Children[i]);
  }
  visited[node.Id] = true;
}
```

Source code

Some Algorithms in Programming

- Sorting and searching
- Combinatorial algorithms
  - Recursive algorithms
- Dynamic programming
- Graph algorithms
  - DFS and BFS traversals
- Other algorithms
  - Greedy algorithms, computational geometry, randomized algorithms, parallel algorithms, genetic algorithms

Asymptotic Notation

Algorithm Analysis

- **Why should we analyze algorithms?**
  - Predict the **resources** the algorithm will need
    - Computational **time** (CPU consumption)
    - **Memory** space (RAM consumption)
    - Communication **bandwidth** consumption
  - The expected **running time** of an algorithm is:
    - The total number of **primitive operations** executed (machine independent steps)
    - Also known as **algorithm complexity**

Algorithmic Complexity

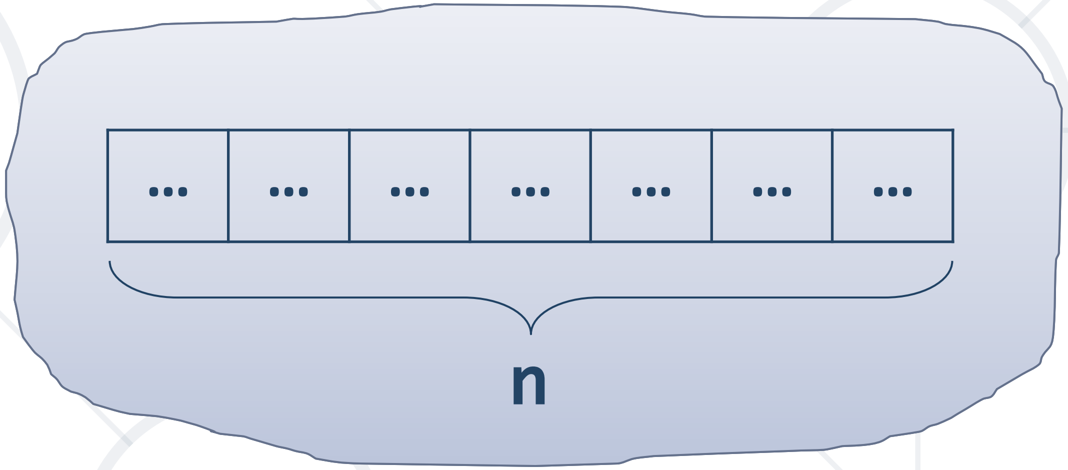
- **What to measure?**
  - CPU time
  - Memory consumption
  - Number of steps
  - Number of particular operations
    - Number of disk operations
    - Number of network packets
  - Asymptotic complexity

Time Complexity

- **Worst-case**
  - An upper bound on the running time for any input of given size
  - Typically, algorithms performance is measured for their worst case
- **Average-case**
  - The running time averaged over all possible inputs
  - Used to measure algorithms that are repeated many times
- **Best-case**
  - The lower bound on the running time (the optimal case)

Time Complexity: Example

- Sequential search in a list of size **n**
  - Worst-case:
    - **n** comparisons
  - Best-case:
    - **1** comparison
  - Average-case:
    - **n/2** comparisons
- The algorithm runs in **linear time**
  - Linear number of operations



Algorithms Complexity

- **Algorithm complexity** - rough estimation of the **number of steps** of a given computation, depending on the **size of the input**
  - Measured with asymptotic notation
    - **`O(g)`** where **g** is a function of the size of the input data
- Examples:
  - Linear complexity **$O(n)$**
    - All elements are processed once (or constant number of times)
  - Quadratic complexity **$O(n^2)$**
    - Each of the elements is processed **n** times

Asymptotic Notation: Definition

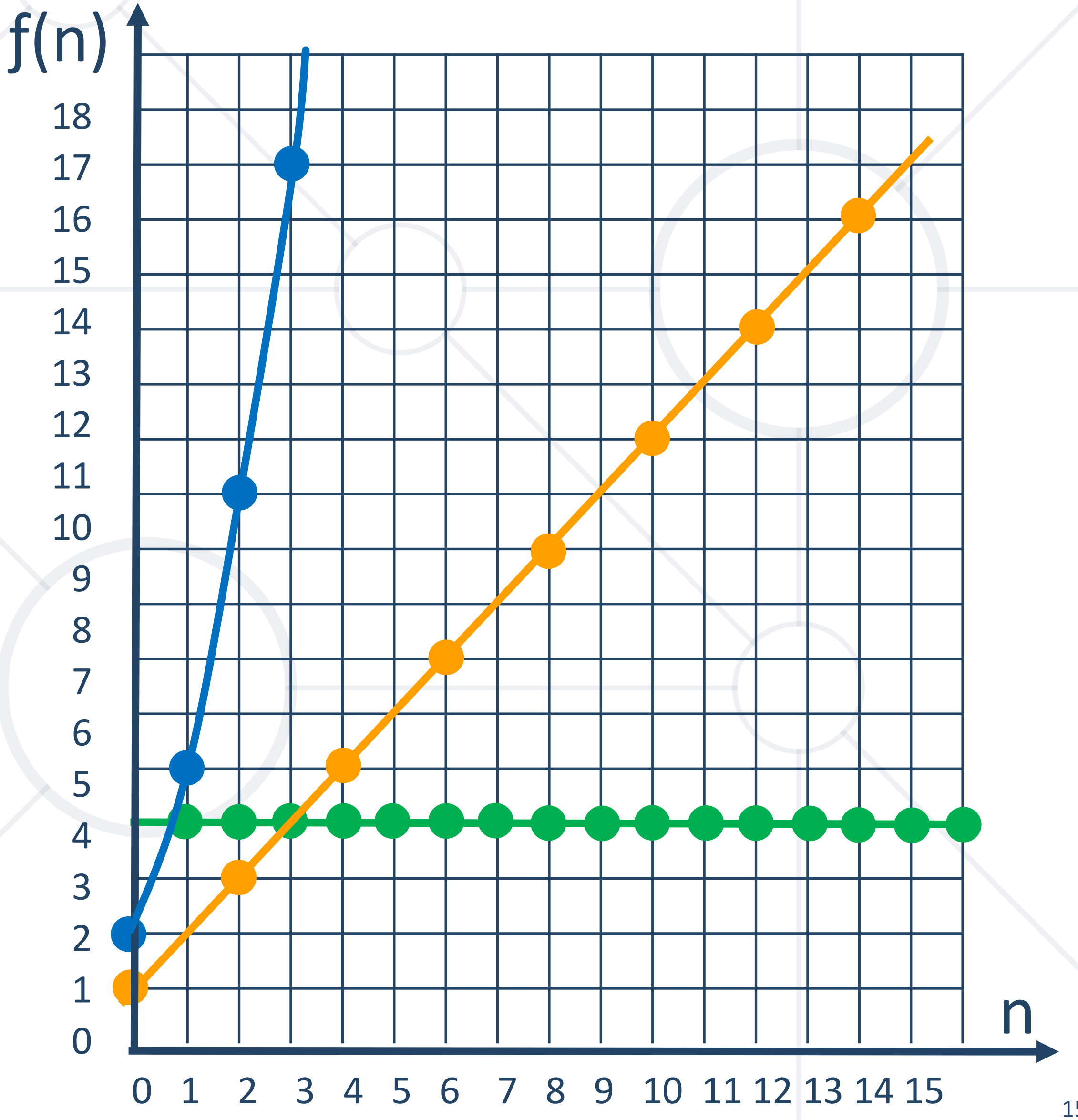
- Asymptotic upper bound
  - O-notation **(Big O notation)**
- For a given function $g(n)$, we denote by $O(g(n))$ the set of functions that are different than $g(n)$ by a constant

$O(g(n)) = \{f(n)$: there exist positive constants c and n_0 such that $f(n) <= c*g(n)$ for all $n >= n_0\}$

- Examples:
  - $3 * n^2 + n/2 + 12 \in O(n^2)$
  - $4*n*\log_2(3*n+1) + 2*n-1 \in O(n * \log n)$

Functions Growth Rate

- **O(n)** means a function grows linearly when n increases
  - E.g. $f(n)=n+1$
- **O(n^2)** means a function grows exponentially when n increases
  - E.g. $f(n)=n^2+2n+2$
- **O(1)** means a function does not grow when n changes
  - E.g. $f(n)=4$



Asymptotic Notation: Examples

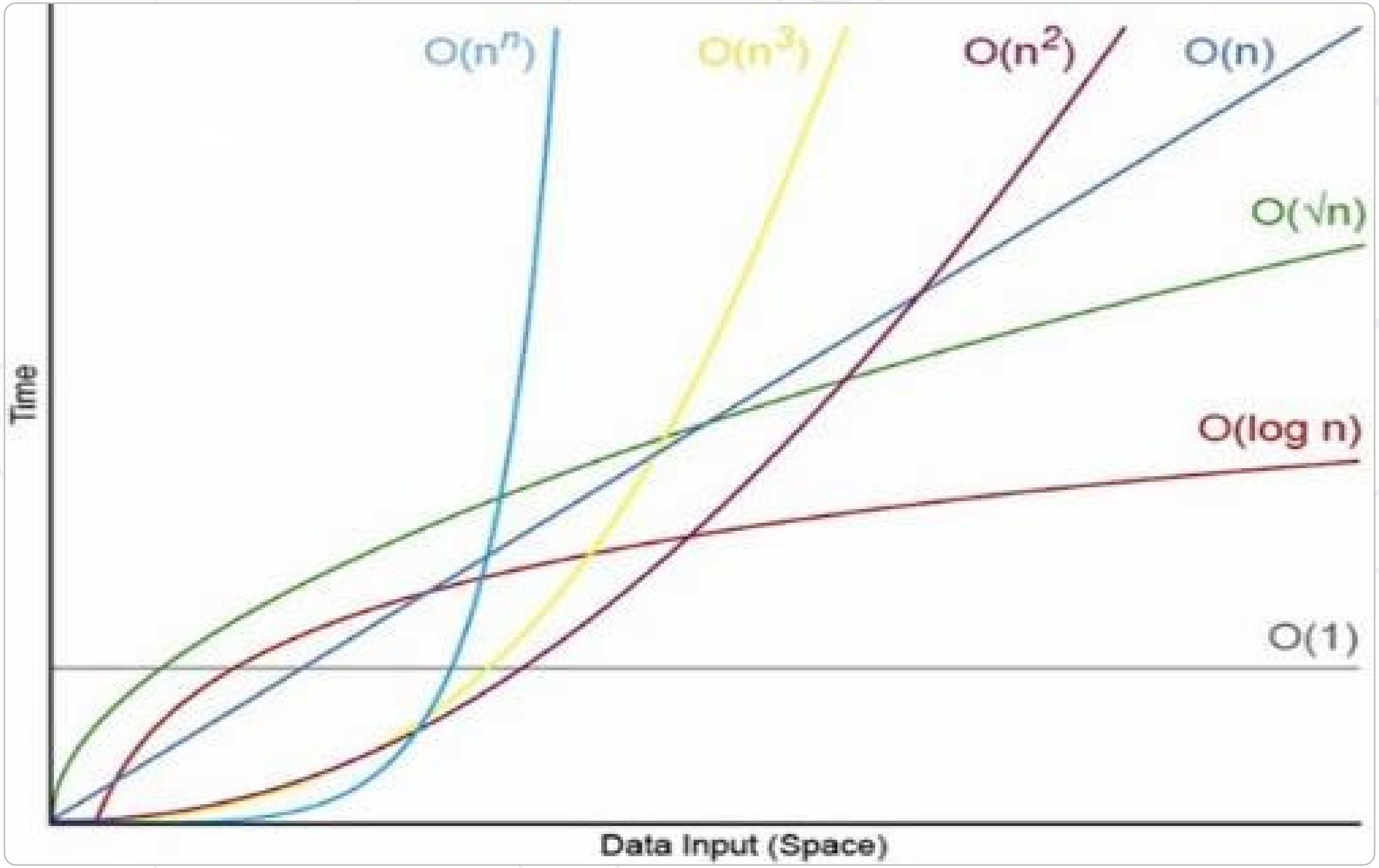
Positive examples:

$$10n \in O(n),$$
$$10n \in O(n^2),$$
$$10n \in O(n^4)$$
$$10n \in O(3n^4 - 10n^2 + 7)$$
$$10n+3 \in O(n)$$
$$4n^2 - 5n + 2 \in O(n^2)$$
$$4n^3 + 5n^2 + 5 \in O(n^3)$$
$$\sqrt{n} \in O(n)$$
$$\log n \in O(\sqrt{n})$$

Negative examples:

$$2n \notin O(1)$$
$$4n^2 - 5n + 2 \notin O(n)$$
$$5n + 1 \notin O(\sqrt{n})$$
$$\sqrt{n^3} \notin O(n)$$

Asymptotic Functions



Typical Complexities

Complexity	Notation	Description
constant	O(1)	Constant number of operations, not depending on the input data size, e.g. n = 1 000 000 → 1-2 operations
logarithmic	O(log n)	Number of operations proportional to $\log_2(n)$ where n is the size of the input data, e.g. n = 1 000 000 000 → 30 operations
linear	O(n)	Number of operations proportional to the input data size, e. g. n = 10 000 → 5 000 operations

Typical Complexities (2)

Complexity	Notation	Description
quadratic	$O(n^2)$	Number of operations proportional to the square of the size of the input data, e.g. n = 500 → 250 000 operations
cubic	$O(n^3)$	Number of operations propor-tional to the cube of the size of the input data, e.g. n = 200 → 8 000 000 operations
exponential	$O(2^n)$, $O(k^n)$, $O(n!)$	Exponential number of operations, fast growing, e.g. n = 20 → 1 048 576 operations

Function Values

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Function	*Value*				
	$n = 1$	$n = 2$	$n = 10$	$n = 100$	$n = 1000$
5	5	5	5	5	5
$\log n$	0	1	3,32	6,64	9,96
n	1	2	10	100	1000
$n \log n$	0	2	33,2	664	9966
n^2	1	4	100	10000	10^6
n^3	1	8	1000	10^6	10^9
2^n	2	4	1024	10^{30}	10^{300}
$n!$	1	2	3628800	10^{157}	10^{2567}
n^n	1	4	10^{10}	10^{200}	10^{3000}

Time Complexity and Program Speed

Complexity	10	20	50	100	1 000	10 000	100 000
O(1)	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
O(log(n))	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
O(n)	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
O(n*log(n))	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
$O(n^2)$	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	2 s	3-4 min
$O(n^3)$	< 1 s	< 1 s	< 1 s	< 1 s	20 s	5 hours	231 days
$O(2^n)$	< 1 s	< 1 s	260 days	hangs	hangs	hangs	hangs
O(n!)	< 1 s	hangs	hangs	hangs	hangs	hangs	hangs
$O(n^n)$	3-4 min	hangs	hangs	hangs	hangs	hangs	hangs

$O(n)$

$O(n+m)$

$O(n^2)$

$O(n!)$

$O(n*m)$

$f(x)$

Examples

Complexity Examples

```
int FindMaxElement(int[] array)
{
  int max = array[0];
  for (int i = 1; i < array.length; i++)
  {
    if (array[i] > max)
      max = array[i];
  }
  return max;
}
```

- Runs in **O(n)** where **n** is the size of the array
- The number of elementary steps is **~ n**

Complexity Examples (2)

```
long FindInversions(int[] array)
{
  long inversions = 0;
  for (int i = 0; i < array.Length; i++)
    for (int j = i + 1; j < array.Length; i++)
      if (array[i] > array[j])
        inversions++;
  return inversions;
}
```

- Runs in **$O(n^2)$** where **n** is the size of the array
- The number of elementary steps is **~ n * (n+1) / 2**

Complexity Examples (3)

```
decimal Sum3(int n)
{
    decimal sum = 0;
    for (int a = 0; a < n; a++)
        for (int b = 0; b < n; b++)
            for (int c = 0; c < n; c++)
                sum += a * b * c;
    return sum;
}
```

- Runs in cubic time $O(n^3)$
- The number of elementary steps is $\sim n^3$

Complexity Examples (4)

```
decimal SpecialCalculation(int n)
{
    decimal sum = 0;
    for (int a = 0; a < n; a++)
        for (int b = 0; b < n; b++)
            if (a == b)
              for (int c = 0; c < n; c++)
                sum += a * b * c;
    return sum;
}
```

- Runs in quadratic time **$O(n^2)$** – think why!
- The number of elementary steps is **$\sim n^2$**

Summary

- **Algorithms** are sequences of steps for calculating / doing something
- **Algorithm complexity** is a rough estimation of the **number of steps** performed by given computation
  - Can be **logarithmic linear**, **n log n**, **square** (n^2), **cubic** (n^3), **exponential**, etc.
  - Complexity predicts the speed of given code before its execution

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Questions?

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Resources

1. "Fundamentals of Computer Programming with C#“ → "Data Structures and Algorithm Complexity" → pages 787-798
   - https://introprogramming.info/wp-content/uploads/2018/07/CSharp-Principles-Book-Nakov-v2018.pdf

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