Real-time Game Physics

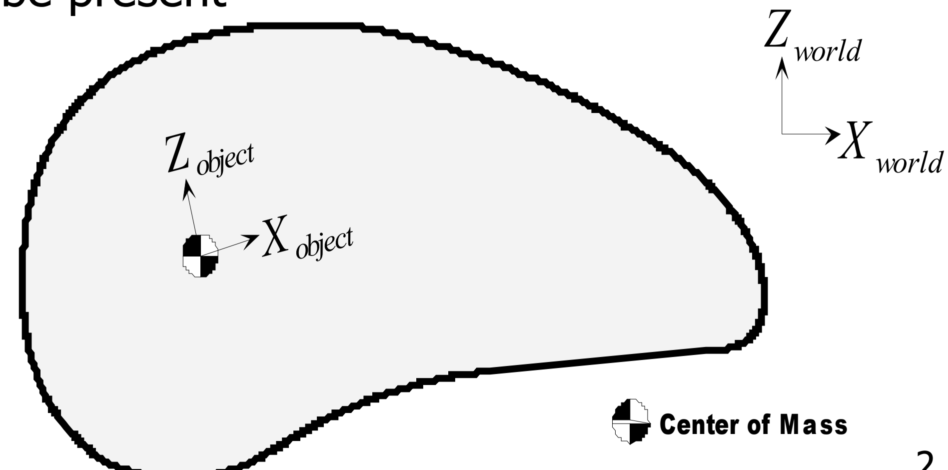
Generalized Rigid Bodies

Generalized Rigid Bodies

- Key Differences from Particles
  - Not necessarily spherical in shape
  - Position, **p**, represents object's center-of-mass location

$$p = \frac{1}{mass} \iiint \wp\, r\, dxdydz \,(\wp : densité)$$

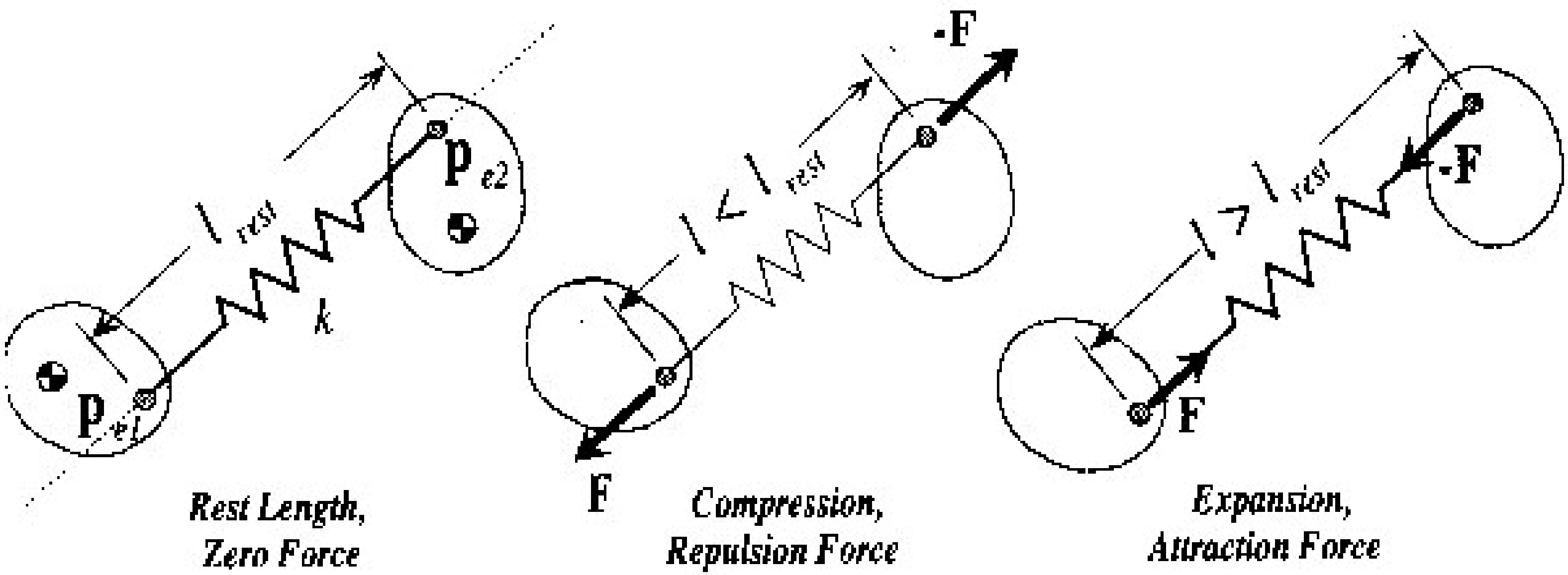
  - Surface may not be perfectly smooth
    - Friction forces may be present
  - translational motion
  - rotational motion



Generalized Rigid Bodies Translational motion

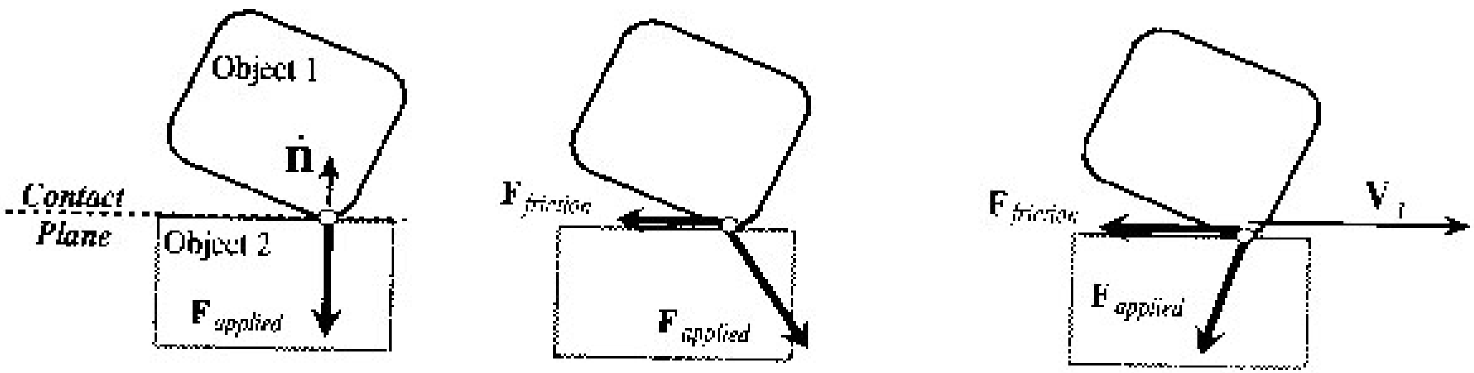
- Linear spring

$$F_{spring} = k\left(l - l_{rest}\right)\hat{d}$$



Generalized Rigid Bodies Translational motion

- Surface Friction



$\mathbf{F}_{applied}$ parallel to $\dot{\mathbf{n}}$
No relative velocity in contact plane
$\mathbf{F}_{friction} = <0,0,0>$

$\mathbf{F}_{applied}$ not parallel to $\dot{\mathbf{n}}$
No relative velocity in contact plane
$\mathbf{F}_{friction}$ given by Eq. 24

$\mathbf{V}_t$ is nonzero
$\mathbf{F}_{friction}$ given by Eq. 25

$$F_{friction} = \frac{-F_t}{|F_t|} min(\wp_s |F_n|, |F_t|) \qquad F_{friction} = \frac{-V_t}{|V_t|} \wp_d |F_n|$$

Generalized Rigid Bodies Translational motion

- Aerodynamic Drag
  - Trough a fluid, like air or water
  - Acts in the opposite of the velocity

$$F_{friction} = -\frac{1}{2}\wp_{fluid}|V|^2 C_D S_{ref}\frac{V}{|V|}$$

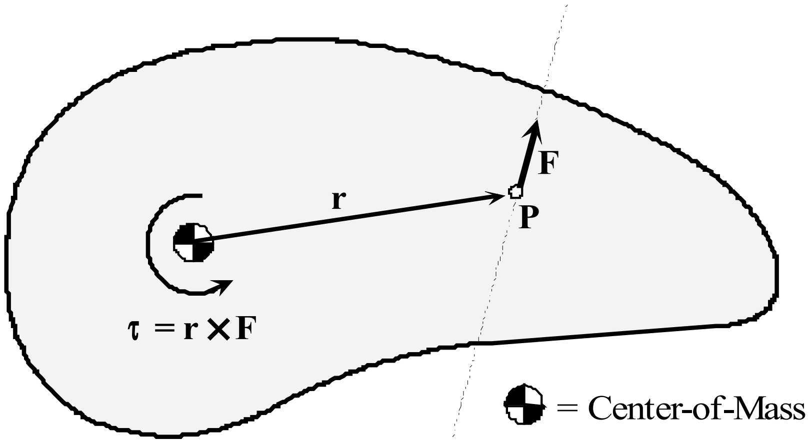
$$\wp_f : Mass\ density\ of\ the\ fluid$$

$$C_D : drag\ coefficient$$

$$S_{ref} : Representative\ front-projected\ area$$

Generalized Rigid Bodies Rotational Motion

- Torque
  - Analogous to a force
  - Causes rotational acceleration
    - Cause a change in angular momentum
  - Torque is the result of a force (friction, collision response, spring, etc.)



Generalized Rigid Bodies Rotational motion

- Angular Kinematics
  - Center-of-mass
  - Orientation, 3x3 matrix $\mathbf{R}$ or quaternion, q
  - Angular velocity, $\boldsymbol{\omega}$
  - Inertia tensor, $\mathbf{J}$ (3x3 matrix, distribution of mass in the volume)
  - Angular momentum, $\mathbf{L}=\mathbf{J}\boldsymbol{\omega}$

Generalized Rigid Bodies – Numerical Simulation

- Using Finite Difference Integrators
  - Translational components of state $<m\mathbf{V}, \mathbf{p}>$ are the same
  - $\mathbf{S}$ and $d\mathbf{S}/dt$ are expanded to include angular momentum and orientation, and their derivatives
  - Be careful about coordinate system representation for $\mathbf{J}$, $\mathbf{R}$, etc.
  - Otherwise, integration step is identical to the translation only case
- Additional Post-integration Steps
  - Adjust orientation for consistency
    - Adjust updated $\mathbf{R}$ to ensure it is orthogonal
    - Normalize q
  - Update angular velocity, $\boldsymbol{\omega}$