Final Exam
(Due: 5:00pm, Friday, November 18, 2011)

Problem 1. (10 points) Suppose Box 1 contains 10 apples and 5 oranges, and Box 2 contains 7 apples and 5 ornages. One of the boxes is chosen at random (with equal probability) and an item is sellected frm the box and found to be an apple. Find the probability that the apple came from Box 1.

Problem 2. (15 points) Consider a Bayesian network with the following structure in Figure 1.

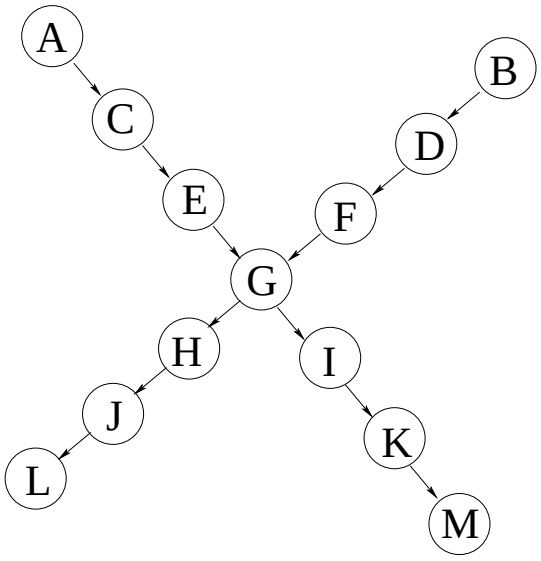

Figure 1: The Bayesian network for problem 2

Does computing $p(M|A)$ depend on:

- $p(L|J)$?
- $p(K|I)$?
- $p(D|B)$?
- $p(H|G)$?

In the network of Figure 1, if we decided not to include G in our network, but still wanted to model the joint distribution of all the other variables, what is the smallest network structure we could use?

Problem 3. (25 points) Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same with some probability $s = 0.95$ as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small probability $m = 0.05$ of a random mutation flipping the handedness.

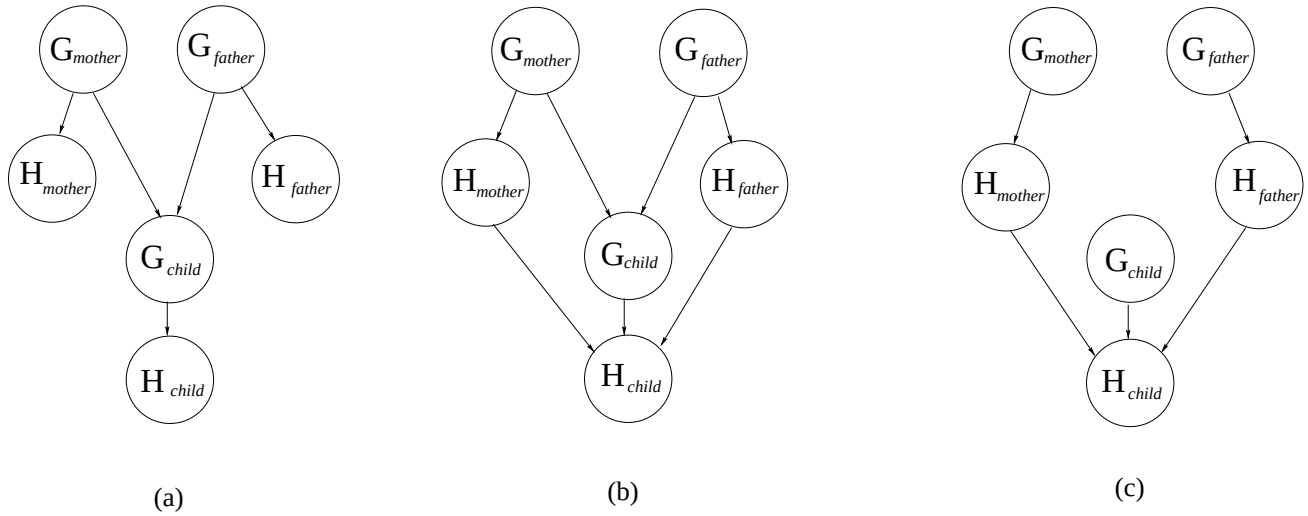

Figure 2: Three possible structures for a Bayesian network describing genetic inheritance of handedness

(a) Which of the three networks in Figure 2 claim the following?

$$P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$$

Please explain.

(b) Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness? Please explain.

(c) Which of the three networks is the best description of the hypothesis? Please explain.

(d) Write down the CPTs for the G_{child} node and H_{child} node in network (a), in terms of s and m.

(e) Suppose that $p(G_{mother} = l) = p(G_{father} = l) = 0.5$. In network (a), what is the value of $p(G_{child} = l)$? What is the value of $p(H_{child} = l)$?

(f) Again suppose that $p(G_{mother} = l) = p(G_{father} = l) = 0.5$. In network (a), suppose we observe the value of H_{child} is r, what is the most likely value of G_{mother} you can infer from this observation?

Problem 4. (25 points) Let $P(A) = \phi, P(B) = \mu, P(C) = 2\mu, P(D) = 1 - \phi - 3\mu$ denotes the probability of getting A, B, C and D grade in a class respectively. We want to estimate ϕ and μ from data.

(a) Assume in a given class, there were $a = 14$ students getting As, $b = 6$ students getting Bs, $c = 9$ students getting Cs, and $d = 10$ students getting Ds. What is the maximum likelihood estimate of ϕ and μ?

(b) Someone tells you that the number of high grades (As + Bs) is $h = 20$, and the number of low grades (Cs + Ds) is $g = 19$. What is the maximum likelihood estimate of ϕ and μ now? Please use EM algorithm to obtain the answer. Is your result a global maximum or local maximum? Please justify and explain why.

Problem 5. (25 points) Consider an HMM of a coin-tossing experiment, assume a three-state model (corresponding to three different coins) with probabilities and with all state-transition probabilities

	State 1	State 2	State 3
p(H)	0.5	0.75	0.25
p(T)	0.5	0.25	0.75

equal to $\frac{1}{3}$. (Assume initial state probabilities of $\frac{1}{3}$).

a. You observe the sequence $O = (HHHHTHTTTT)$, what state sequence is most likely? What is the probability of the observation sequence and this most likely state sequence?

b. What is the probability that the observation sequence came entirely from state 1?

c. Consider the observation $\hat{O} = (HTTHTHHTTH)$, how would your answer to parts a and b change?

d. If the state-transition probabilities were: $a_{11} = 0.9, a_{12} = 0.05, a_{13} = 0.05, a_{21} = 0.45, a_{22} = 0.1, a_{23} = 0.45, a_{31} = 0.45, a_{32} = 0.45, a_{33} = 0.1$, how would your answers to parts a-c change? What does this suggest about the type of sequence generated by the models?