Assignment 2

ZHU Cai
Student No.: 09902510R
Department of Applied mathematics

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This assignment is programed and calculated using R-2.10.1.

Question 1

(a) From the definition, the rate of return, r, can be calculate as $r = \frac{X_1 - X_0}{X_0}$:

$$X_0 = 1e6 + 0.5 \times u \tag{1}$$
$$X_1 = 0.6 \times 4e6 + 0.4 \times u = 2.4e6 + 0.4 \times u \tag{2}$$

Thus, the rate of return

$$r = \frac{1.4e6 - 0.1u}{1e6 + 0.5u} \tag{3}$$

(b) There is a simple answer: we can make the rate of return 4e6 in probability of 1, so we should buy 4e6 pieces of insurance. If it does not rain, the investor will get 4e6, and if it does rain, the investor will also get 4e6 from those insurance. Thus, there is no risk. Now, lets do some calculation.
The variance of return can be definited as follows:

$$var = 0.6 \times (4e6 - 2.4e6 - 0.4u)^2 + 0.4 \times (u - 2.4e6 - 0.4u)^2 \tag{4}$$

```
> variance <- function(x) {
+     fun <- 0.6 * (4 - 2.4 - 0.4 * x)^2 + 0.4 * (x - 2.4 - 0.4 *
+         x)^2
+     return(fun)
+ }
> result <- optimize(variance, interval = c(3, 6))
> result
```

```
$minimum
[1] 4

$objective
[1] 0
```

From the result, we can see we should buy 4000000 units of insurance.

Question 3

For (a)

```
> rm(list = ls(all = TRUE))
> a <- c(1, 1, 0, -1, 1, 2, 1, -1, 0, 1, 2, -1, 1, 1, 1, 0)
> A <- matrix(a, nrow = 4, ncol = 4, byrow = TRUE)
> B <- c(0, 0, 0, 1)
> A
```

```
     [,1] [,2] [,3] [,4]
[1,]    1    1    0   -1
[2,]    1    2    1   -1
[3,]    0    1    2   -1
[4,]    1    1    1    0
```

```
> solve(A, B)
```

```
[1]  1.0 -0.5  0.5  0.5
```

For (b)

```
> rm(list = ls(all = TRUE))
> d <- c(1, 1, 0, -0.4, -1, 1, 2, 1, -0.8, -1, 0, 1, 2, -0.8, -1,
+     0.4, 0.8, 0.8, 0, 0, 1, 1, 1, 0, 0)
> D <- matrix(d, nrow = 5, ncol = 5, byrow = TRUE)
> E <- c(0, 0, 0, 0.6, 1)
> D
```

```
     [,1] [,2] [,3] [,4] [,5]
[1,]  1.0  1.0  0.0 -0.4   -1
[2,]  1.0  2.0  1.0 -0.8   -1
[3,]  0.0  1.0  2.0 -0.8   -1
[4,]  0.4  0.8  0.8  0.0    0
[5,]  1.0  1.0  1.0  0.0    0
```

```
> solve(D, E)

[1]  5.000000e-01 -2.220446e-16  5.000000e-01  1.250000e+00  2.775558e-16
```

For (c)

```
> rm(list = ls(all = TRUE))
> cov <- matrix(c(1, 1, 0, 1, 2, 1, 0, 1, 2), nrow = 3, ncol = 3,
+     byrow = TRUE)
> Return <- c(0.4, 0.8, 0.8)
> omega1 <- solve(cov, Return)
> omega1/sum(omega1)

[1] 0.5 0.0 0.5

> omega2 <- solve(cov, (Return - 0.1))
> omega2/sum(omega2)

[1] 0.3333333 0.1666667 0.5000000
```

Thus, (a)$w_1 = 1, w_2 = -0.5, w_3 = 0.5, \mu = 0.5$;
(b) $w_1 = 0.5, w_2 = 0, w_3 = 0.5, \lambda = 1.25, \mu = 0$
(c) when $\lambda = 1, \mu = 0$, we have $w_1 = 0.5, w_2 = 0, w_3 = 0.5$ and when $r_f = 0.1$, we have $w_1 = 0.33, w_2 = 0.17, w_3 = 0.5$

Question 4

```
> rm(list = ls(all = TRUE))
> data <- read.csv("C:/Documents and Settings/zc/Desktop/mywork/course-IS/assig
> stocks <- data[, 2:16]
> Ocal <- function(data) {
+     T <- 22
+     n <- length(data)
+     K <- n - T
+     O <- rep(0, length = K)
+     for (j in 1:K) {
+         O[j] <- (data[(j + T)] - data[j])/data[j]
+     }
+     return(O)
+ }
> returns <- apply(stocks, 2, Ocal)
> dim(returns)
```

```
[1] 106  15
```

```
> r <- apply(returns, 2, mean)
> COV <- cov(returns)
> N <- ncol(COV)
> N
```

```
[1] 15
```

```
> vector <- rep(-1, length = N)
> A1 <- cbind(COV, vector)
> dim(A1)
```

```
[1] 15 16
```

```
> vector <- c(rep(1, length = N), 0)
> A2 <- rbind(A1, vector)
> dim(A2)
```

```
[1] 16 16
```

```
> B1 <- c(rep(0, length = N), 1)
> omega1 <- solve(A2, B1)
> round(omega1, 4)
```

```
 [1] -0.3703 -0.0470 -0.0546  0.3686  0.3723  0.4399 -0.0149  0.1366  0.4735
[10] -0.2853  0.0319 -0.0566  0.0249  0.0080 -0.0269  0.0001
```

```
> e <- rep(1, length = N) * (-1)
> r <- -1 * r
> C1 <- cbind(COV, r, e)
> r <- c(r, 0, 0)
> e <- c(e, 0, 0)
> C2 <- rbind(C1, r, e)
> dim(C2)
```

```
[1] 17 17
```

```
> B2 <- c(rep(0, length = N), 0.008, 1)
> omega2 <- solve(C2, B2)
> round(omega2, 4)
```

```
 [1]  0.2404  0.2038  0.1021 -0.2564 -0.2264 -0.5614 -0.1026 -0.1339 -0.2909
[10]  0.1355 -0.0636  0.0645 -0.0197 -0.1193  0.0277 -0.0013 -0.0001
```

Thus, omega1 is for question (a), and omega2 is for question (b).

Question 7

(a) The Captical market line is as follows:

$$r_i = r_f + \frac{\sigma_i}{\sigma_M}(r_M - r_f) \tag{5}$$

that is,

$$r_i = 0.03 + \frac{0.17}{0.32}\sigma \tag{6}$$

(b) We know $r_i = 0.3$, from (a), we can calculate $\sigma = 0.51$
suppose 100α percent of money is invested into the risk free asset, we have the follwoing equation:

$$30 \times X_0 = 3 \times \alpha X_0 + 20 \times (1 - \alpha)X_0 \tag{7}$$

where $X_0 = 1000$, so $\alpha = -0.59$, which means we should borrow $\alpha \times 1000 = 590$ at risk free rate, and invest these borrowed money into the risk asset.
(c) The return rate $r = 0.5 \times 3 + 0.5 \times 20 = 11.5$, so we can expect to have $1000 \times (1 + r) = 1115$ at the end of year.

Question 8

From Question 7, we could know that there is a linear relationship between r_i and β_i. The equation is as follows:

$$r = r_f + (r_M - r_f) \times \beta \tag{8}$$

where r_f is the risk free return rate, and r_M is the rate of return on the market potrfolio.

```
> return <- c(0.07, 0.13)
> beta <- c(0.6, 1.5)
> summary(lm(return ~ beta))

Call:
lm(formula = return ~ beta)

Residuals:
ALL 2 residuals are 0: no residual degrees of freedom!

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)  0.03000          NA      NA       NA
beta         0.06667          NA      NA       NA

Residual standard error: NaN on 0 degrees of freedom
Multiple R-squared:     1,        Adjusted R-squared:   NaN
F-statistic:   NaN on 1 and 0 DF,  p-value: NA
```

from the result, we have the security market line:

$$r = 0.03 + 0.067 \times \beta \tag{9}$$

Thus, when β is 2, r is 0.16.