

# Poisson Model

## General Principles

To model the relationship between a count outcome variable—e.g., counts of events occurring in a fixed interval of time or space—and one or more independent variables, we can use the *Poisson model*.

This is a special shape of the binomial distribution; it is useful because it models binomial events for which the number of trials  $n$  is unknown or uncountably large.

## Considerations

### Caution

- We have the same considerations as for [Regression for a continuous variable](#).
- We have the second link function :  $\log$ . The  $\log$  link ensures that  $\mu$  is always positive.
- To invert the log link function and linearly model the relationship between the predictor variables and the log of the mean rate parameter, we can apply the exponential function (see comment in code).

## Example

Below is an example code snippet demonstrating a Bayesian Poisson model using the Bayesian Inference (BI) package. Data consist of:

- 1) A continuous dependent variable  $total\_tools$ , which represents the number of tools produced by a civilization.
- 2) A continuous independent variable  $population$  representing population size.

3) A categorical independent variable  $cid$  representing different civilizations.

The goal is to estimate the production of tools based on population size, accounting for each civilization.

## Python

```
from BI import bi
import jax.numpy as jnp
# Setup device-----
m = bi(platform='cpu')

# import data -----
# Import
from importlib.resources import files
data_path = files('BI.resources.data') / 'Kline.csv'
m.data(data_path, sep=';')
m.scale(['population'])
m.df["cid"] = (m.df.contact == "high").astype(int)
#m.data_to_model(['total_tools', 'population', 'cid'])

def model(cid, population, total_tools):
    a = m.dist.normal(3, 0.5, shape=(2,), name='a')
    b = m.dist.normal(0, 0.2, shape=(2,), name='b')
    l = jnp.exp(a[cid] + b[cid]*population)
    m.poisson(l, obs=total_tools)

# Run sampler -----
m.fit(model)

# Diagnostic -----
m.summary()
```

## R

```
library(BI)

# Setup platform-----
m=importbi(platform='cpu')

# import data -----
```

```

m$data(paste(system.file(package = "BI"), "/data/Kline.csv", sep = ''), sep=';')
m$scale(list('population'))# Scale
m$df["cid"] = as.integer(ifelse(m$df$contact == "high", 1, 0)) # Manipulate
m$data_to_model(list('total_tools', 'population', 'cid')) # Send to model (convert to jax array)

# Define model -----
model <- function(total_tools, population, cid){
  # Parameter prior distributions
  alpha = bi.dist.normal(3, 0.5, name='alpha', shape = c(2))
  beta = bi.dist.normal(0, 0.2, name='beta', shape = c(2))
  l = jnp$exp(alpha[cid] + beta[cid]*population)
  # Likelihood
  m$poisson(l, obs=total_tools)
}

# Run MCMC -----
m$run(model) # Optimize model parameters through MCMC sampling

# Summary -----
m$summary() # Get posterior distribution

```

## Mathematical Details

### *Frequentist formulation*

We model the relationship between the predictor variable ( $X_i$ ) and the count outcome variable ( $Y_i$ ) using the following equation:

$$\log(\lambda_i) = \alpha + \beta X_i$$

Where:

- $\lambda_i$  is the mean rate parameter of the Poisson distribution (expected count) for observation  $i$ , modeled as the exponential function of the linear combination of predictors.
- $\log(\lambda_i)$  is the log of the mean rate parameter for observation  $i$ , ensuring it is positive.
- $\beta$  is the regression coefficient.
- $\alpha$  is the intercept term.
- $X_i$  is the value of the independent variable for observation  $i$ .

### **Bayesian formulation**

In the Bayesian formulation, we define each parameter with priors . We can express the Bayesian regression model accounting for prior distributions as follows:

$$Y \sim Poisson(\lambda_i)$$

$$\log(\lambda_i) = \alpha + \beta X_i$$

$$\alpha \sim Normal(0, 1)$$

$$\beta \sim Normal(0, 1)$$

Where:

- $Y_i$  is the dependent variable for observation  $i$ .
- $\lambda_i$  is the mean rate parameter of the Poisson distribution for observation  $i$ , modeled as the exponential function of the linear combination of predictors.
- $\log(\lambda_i)$  is the log of the mean rate parameter for observation  $i$ .
- $\alpha$  and  $\beta$  are the prior distributions for the intercept and the regression coefficients, respectively.
- $\lambda_i$  is the mean rate parameter of the Poisson distribution, modeled as the exponential function of the linear combination of predictors.
- $X_i$  is the value of the independent variable for observation  $i$ .

### **Notes**

**i** Note

- We can apply multiple variables similarly to [chapter 2](#).
- We can apply interaction terms similarly to [chapter 3](#).
- We can apply categorical variables similarly to [chapter 4](#).

**Reference(s)**

McElreath (2018)

McElreath, Richard. 2018. *Statistical Rethinking: A Bayesian Course with Examples in r and Stan*. Chapman; Hall/CRC.