

Categorical Model

General Principles

To model the relationship between an outcome variable in which each observation is a single choice from a set of more than two categories and one or more independent variables, we can use a *Categorical* model.

Considerations

Caution

- We have the same considerations as for [Regression for continuous variable](#).
- One way to interpret a *Categorical* model is to consider that we need to build $K - 1$ linear models, where K is the number of categories. Once we get the linear prediction for each category, we can convert these predictions to probabilities by building a simplex . To do this, we convert the regression outputs using the softmax function (see the “`jax.nn.softmax`” line in the code).
- The intercept α captures the difference in the log-odds of the outcome categories; thus, different categories need different intercepts.
- On the other hand, as we assume that the effect of each predictor on the outcome is consistent across all categories, the regression coefficients β are shared across categories.
- The relationship between the predictor variables and the log-odds of each category is modeled linearly, allowing us to interpret the effect of each predictor on the log-odds of each category.

Example

Below is an example code snippet demonstrating a Bayesian Categorical model using the Bayesian Inference (BI) package. This example is based on McElreath (2018).

Python

```
from BI import bi, jnp
import jax
# Setup device -----
m = bi('cpu')

# Import Data & Data Manipulation -----
# Import
from importlib.resources import files
data_path = files('BI.resources.data') / 'Sim data multinomial.csv'
m.data(data_path, sep=',')

# Define model -----
def model(career, income):
    a = m.dist.normal(0, 1, shape=(2,), name = 'a')
    b = m.dist.half_normal(0.5, shape=(1,), name = 'b')
    s_1 = a[0] + b * income[0]
    s_2 = a[1] + b * income[1]
    s_3 = [0] #pivot
    p = jax.nn.softmax(jnp.stack([s_1[0], s_2[0], s_3[0]]))
    m.dist.categorical(probs=p, obs=career)

# Run sampler -----
m.fit(model) # Optimize model parameters through MCMC sampling

# Summary -----
m.summary() # Get posterior distributions
```

```
jax.local_device_count 16
```

```
0%|          | 0/1000 [00:00<?, ?it/s]warmup: 0%|          | 1/1000 [00:04<1:19:27, 4.7
arviz - WARNING - Shape validation failed: input_shape: (1, 500), minimum_shape: (chains=2, c
```

	mean	sd	hdi_5.5%	hdi_94.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
a[0]	-2.09	0.27	-2.43	-1.67	0.03	0.03	85.36	78.74	NaN
a[1]	-1.56	0.16	-1.80	-1.30	0.02	0.01	101.63	28.52	NaN
b[0]	0.05	0.05	0.00	0.12	0.01	0.01	66.53	81.49	NaN

R

```
library(BayesianInference)
jax = reticulate::import('jax')

# setup platform-----
m=importBI(platform='cpu')

# import data -----
m$data(paste(system.file(package = "BayesianInference"),"/data/Sim data multinomial.csv", sep = ""))
keys <- c("income", "career")
income = unique(m$df$income)
income = income[order(income)]
values <- list(jnp$array(as.integer(income)),jnp$array( as.integer(m$df$career)))
m$data_on_model = py_dict(keys, values, convert = TRUE)

# Define model -----
model <- function(income, career){
  # Parameter prior distributions
  alpha = bi.dist.normal(0, 1, name='alpha', shape = c(2))
  beta = bi.dist.normal(0.5, name='beta')

  s_1 = alpha[0] + beta * income[0]
  s_2 = alpha[1] + beta * income[1]
  s_3 = 0 # reference category

  p = jax$nn$softmax(jnp$stack(list(s_1, s_2, s_3)))

  # Likelihood
  m$dist$categorical(probs=p[career], obs=career)
}

# Run MCMC -----
m$fit(model) # Optimize model parameters through MCMC sampling
```

```
# Summary -----
m$summary() # Get posterior distribution
```

Mathematical Details

We can model a *Categorical* model using a *CategoricalDistribution*. The multinomial distribution models the counts of outcomes falling into different categories. For an outcome variable with k categories, the multinomial likelihood function is:

$$Y_i \sim \text{Categorical}(\theta_i) \theta_i = \text{Softmax}(\phi_i) \phi_{[i,1]} = \alpha_1 + \beta_1 X_i \phi_{[i,2]} = \alpha_2 + \beta_2 X_i \dots \phi_{[i,k]} = 0 \alpha_k \sim \text{Normal}(0, 1) \beta_k \sim \text{Normal}(0, 1)$$

Where:

- Y_i is the dependent categorical variable for observation i indicating the category of the observation.
- θ_i is a vector unique to each observation, i , which gives the probability of observing i in category k .
- ϕ_i give the linear model for each of the k categories. Note that we use the softmax function to ensure that the probabilities θ_i form a simplex.
- Each element of ϕ_i is obtained by applying a linear regression model with its own respective intercept α_k and slope coefficient β_k . To ensure the model is identifiable, one category, K , is arbitrarily chosen as a reference or baseline category. The linear predictor for this reference category is set to zero. The coefficients for the other categories then represent the change in the log-odds of being in that category versus the reference category.

Reference(s)

McElreath, Richard. 2018. *Statistical Rethinking: A Bayesian course with examples in R and Stan*. Chapman; Hall/CRC.