

# Zero-Inflated Models

## General Principles

Zero-Inflated Regression models are used when the outcome variable is a count variable with an excess of zero counts. These models combine a count model (e.g., Poisson or Negative Binomial) with a separate model for predicting the probability of excess zeros.

## Example

Below is an example code snippet demonstrating Bayesian Zero-Inflated Poisson regression using the Bayesian Inference (BI) package. The data represent the production of books in a monastery ( $y$ ), which is affected by the number of days that individuals work, as well as the number of days individuals drink. This example is based on McElreath (2018).

## Python

```
from BI import bi, jnp

# Setup device -----
m = bi('cpu', rand_seed = False) # Fixing seed for reproducibility

# Simulated data-----
prob_drink = 0.2 # 20% of days
rate_work = 1    # average 1 manuscript per day

# Sample one year of production
N = 365
drink = m.dist.binomial(1, prob_drink, shape = (N,), sample = True)
y = (1 - drink) * m.dist.poisson(rate_work, shape = (N,), sample = True)

# Setup device-----
```

```

m.data_on_model = dict(
    y=jnp.array(y)
)

# Define model -----
def model(y):
    al = m.dist.normal(1, 0.5, name='al')
    ap = m.dist.normal(-1.5, 1, name='ap')
    p = m.link.inv_logit(ap)
    lambda_ = jnp.exp(al)
    m.dist.zero_inflated_poisson(p, lambda_, obs=y)

# Run MCMC -----
m.fit(model)

# Summary -----
m.summary()

```

jax.local\_device\_count 16

0%| | 0/1000 [00:00<?, ?it/s]warmup: 0%| | 1/1000 [00:00<16:21, 1.02it/s]  
arviz - WARNING - Shape validation failed: input\_shape: (1, 500), minimum\_shape: (chains=2, 0)

	mean	sd	hdi_5.5%	hdi_94.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
al	0.15	0.09	0.03	0.30	0.01	0.00	185.03	264.67	NaN
ap	-0.74	0.23	-1.08	-0.34	0.02	0.01	179.61	283.56	NaN

## R

```

library(BayesianInference)

# setup platform-----
m=importBI(platform='cpu')

# Simulate data -----
prob_drink = 0.2 # 20% of days
rate_work = 1   # average 1 manuscript per day
# sample one year of production

```

```

N = as.integer(365)
drink = bi.dist.binomial(total_count = as.integer(1), probs = prob_drink, shape = c(N), sample = T)
y = (1 - drink) * bi.dist.poisson(rate_work, shape = c(N), sample = T)
data = list()
data$y = y
m$data_on_model = data

# Define model -----
model <- function(y){
  al = bi.dist.normal(1, 0.5, name='al', shape=c(1))
  ap = bi.dist.normal(-1, 1, name='ap', shape=c(1))
  p = m$link$inv_logit(ap)
  lambda_ = jnp$exp(al)
  bi.dist.zero_inflated_poisson(p, lambda_, obs=y)
}

# Run MCMC -----
m$fit(model) # Optimize model parameters through MCMC sampling

# Summary -----
m$summary() # Get posterior distribution

```

## Julia

```

using BayesianInference

# Setup device-----
m = importBI(platform="cpu")

# Simulated data -----
prob_drink = 0.2
rate_work = 1

# Sample one year of production
N = 365

# Note: Use lowercase 'true' for booleans in Julia
# 'drink' will be a Python/JAX object
drink = m.dist.binomial(1, prob_drink, shape=(N,), sample=true)

```

```

# Math works automatically because 'drink' is a Py object
# and we taught Julia how to handle Py arithmetic in the previous step
y = (1 - drink) * m.dist.poisson(rate_work, shape=(N,), sample=true)

# Send data to BI class object -----
m.data_on_model = pydict(Dict("y" =>y))

# Define model -----
@BI function model(y)
    al = m.dist.normal(1, 0.5, name="al")
    ap = m.dist.normal(-1.5, 1, name="ap")
    p = m.link.inv_logit(ap)
    lambda_ = jnp.exp(al)
    m.dist.zero_inflated_poisson(p, lambda_, obs=y)
end

# Run mcmc -----
m.fit(model) # Optimize model parameters through MCMC sampling

# Summary -----
m.summary() # Get posterior distributions

```

## Mathematical Details

We model the relationship between the independent variable  $X$  and the count outcome variable  $Y$ . The model assumes that the data-generating process has two states: a “zero” state that generates only zero counts, and a “Poisson” state that generates counts from a Poisson distribution.

The overall model has two main linear models ( $\pi_i$  and  $\lambda_i$ ):

$$Y_i \sim \text{ZIPoisson}(\pi_i, \lambda_i)$$

Where the mass probability mass function  $\text{ZIPoisson}(\pi_i, \lambda_i)$  is given by:

$$P(Y_i | \pi_i, \lambda_i) = \begin{cases} \pi_i + (1 - \pi_i) \times \text{Poisson}(0 | \lambda_i) & \text{if } Y_i = 0 \\ (1 - \pi_i) \times \text{Poisson}(Y_i | \lambda_i) & \text{if } Y_i > 0 \end{cases}$$

1.  $\pi_i$  A logistic regression model to predict the probability of a structural zero.

2.  $\lambda_i$  is a Poisson regression conditional on the outcome not resulting from a structural zero.

Jointly these two parameters allow us to estimate the probability of getting a zero ( $Y_i = 0$ ) as the sum of two processes: the probability of a structural zero ( $\pi_i$ ) plus the probability of not being a structural zero ( $1 - \pi_i$ ) and then getting a zero from the Poisson distribution anyway.

The parameters  $\pi_i$  and  $\lambda_i$  can be modeled as functions of an independent variable  $X_i$ :

$$\text{logit}(\pi_i) = \alpha_\pi + \beta_\pi X_i$$

$$\log(\lambda_i) = \alpha_\lambda + \beta_\lambda X_i$$

We can assign prior distributions to the model parameters with weakly informative Normal priors:

$$\alpha_\pi \sim \text{Normal}(0, 1)$$

$$\beta_\pi \sim \text{Normal}(0, 1)$$

$$\alpha_\lambda \sim \text{Normal}(0, 1)$$

$$\beta_\lambda \sim \text{Normal}(0, 1)$$

## Reference(s)

McElreath, Richard. 2018. *Statistical Rethinking: A Bayesian course with examples in R and Stan*. Chapman; Hall/CRC.