

# Gamma-Poisson Model

## General Principles

To model the relationship between a count outcome variable and one or more independent variables with overdispersion, we can use the *Negative Binomial model*.

## Considerations

### Caution

- We have the same considerations as for the [Poisson model](#).
- Overdispersion is handled because the Gamma-Poisson model assumes that each Poisson count observation has its own rate. This is an additional parameter specified in the model (in the code, it is `log_days`).

## Example

Below is an example code snippet demonstrating a Bayesian Gamma-Poisson model using the Bayesian Inference (BI) package.

## Python

```
from BI import bi
# Setup device -----
m = bi(platform='cpu') # Import

# Import Data & Data Manipulation -----
# Import
from importlib.resources import files
```

```

data_path = files('BI.resources.data') / 'Sim dat Gamma poisson.csv'
m.data(data_path, sep=',')
m.data_to_model(['log_days', 'monastery', 'y']) # Send to model (convert to jax array)

# Define model -----
def model(log_days, monastery, y):
    a = m.dist.normal(0, 1, name = 'a', shape=(1,))
    b = m.dist.normal(0, 1, name = 'b', shape=(1,))
    phi = m.dist.exponential(1, name = 'phi', shape=(1,))
    mu = m.jnp.exp(log_days + a + b * monastery)
    Lambda = m.dist.gamma(rate = mu*phi, concentration = phi, name = 'Lambda')
    m.dist.poisson(rate = Lambda, obs=y)
# Run MCMC -----
m.fit(model) # Optimize model parameters through MCMC sampling

# Summary -----
m.summary() # Get posterior distributions

```

## R

```

library(BI)

# Setup platform-----
m=importbi(platform='cpu')

# Import data -----
m$data(paste(system.file(package = "BI"),"/data/Sim dat Gamma poisson.csv", sep = ''), sep=',')
m$data_to_model(list('log_days', 'monastery', 'y' )) # Send to model (convert to jax array)

# Define model -----
model <- function(log_days, monastery, y){
  # Parameter prior distributions
  alpha = bi.dist.normal(0, 1, name='alpha', shape=c(1))
  beta = bi.dist.normal(0, 1, name='beta', shape=c(1))
  phi = bi.dist.exponential(1, name='phi', shape=c(1))
  mu = jnp$exp(log_days + alpha + beta * monastery)
  Lambda = m.dist.gamma(rate = mu*phi, concentration = phi, name = 'Lambda')
  # Likelihood
  m$poisson(rate=Lambda, obs=y)
}

```

```
# Run MCMC -----
m$fit(model) # Optimize model parameters through MCMC sampling

# Summary -----
m$summary() # Get posterior distributions
```

## Mathematical Details

### *Bayesian model*

In the Bayesian formulation, we define each parameter with priors . We can express the Bayesian regression model accounting for prior distributions as follows:

$$Y_i \sim \text{Poisson}(\lambda_i)$$

$$\lambda_i \sim \text{Gamma}(\mu_i \phi, \phi)$$

$$\log(\mu_i) = \text{rates}_i + \alpha + \beta X_i$$

$$\alpha \sim \text{Normal}(0, 1)$$

$$\beta \sim \text{Normal}(0, 1)$$

$$\phi \sim \text{Exponential}(1)$$

Where:

- $Y_i$  is the dependent variable for observation  $i$ .
- $\lambda_i$  is the rate parameter of the Poisson distribution for observation  $i$ , assuming that each Poisson count observation has its own  $\text{rate}_i$ .
- $\mu_i$  is the mean rate parameter.
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- $\phi$  controls the level of overdispersion in the rates.
- $\alpha$  is the intercept term.
- $\beta$  is the regression coefficient.
- $X_i$  is the value of the predictor variable for observation  $i$ .

## Notes

### Note

- We can apply multiple variables similarly as in [chapter 2](#).
- We can apply interaction terms similarly as in [chapter 3](#).
- We can apply categorical variables similarly as in [chapter 4](#).

## Reference(s)