

# Categorical Model

## General Principles

To model the relationship between an outcome variable in which each observation is a single choice from a set of more than two categories and one or more independent variables, we can use a *Categorical* model.

## Considerations

### Caution

- We have the same considerations as for [Regression for continuous variable](#).
- One way to interpret a *Categorical* model is to consider that we need to build  $K - 1$  linear models, where  $K$  is the number of categories. Once we get the linear prediction for each category, we can convert these predictions to probabilities by building a simplex . To do this, we convert the regression outputs using the softmax function (see the “`jax.nn.softmax`” line in the code).
- The intercept  $\alpha$  captures the difference in the log-odds of the outcome categories; thus, different categories need different intercepts.
- On the other hand, as we assume that the effect of each predictor on the outcome is consistent across all categories, the regression coefficients  $\beta$  are shared across categories.
- The relationship between the predictor variables and the log-odds of each category is modeled linearly, allowing us to interpret the effect of each predictor on the log-odds of each category.

## Example

Below is an example code snippet demonstrating a Bayesian Categorical model using the Bayesian Inference (BI) package. This example is based on McElreath (2018).

### Python

```
from BI import bi
import jax.numpy as jnp
import pandas as pd
import jax
# Setup device -----
m = bi('cpu')

# Import Data & Data Manipulation -----
# Import
from importlib.resources import files
data_path = files('BI.resources.data') / 'Sim data multinomial.csv'
m.data(data_path, sep=',')

# Define model -----
def model(career, income):
    a = m.dist.normal(0, 1, shape=(2,), name = 'a')
    b = m.dist.halfnormal(0.5, shape=(1,), name = 'b')
    s_1 = a[0] + b * income[0]
    s_2 = a[1] + b * income[1]
    s_3 = [0] #pivot
    p = jax.nn.softmax(jnp.stack([s_1[0], s_2[0], s_3[0]]))
    m.dist.categorical(probs=p, obs=career)

# Run sampler -----
m.fit(model) # Optimize model parameters through MCMC sampling

# Summary -----
m.summary() # Get posterior distributions
```

### R

```
library(BI)
```

```

# setup platform-----
m=importbi(platform='cpu')

# import data -----
m$data(paste(system.file(package = "BI"),"/data/Sim data multinomial.csv", sep = ''), sep=',')
keys <- c("income", "career")
income = unique(m$df$income)
income = income[order(income)]
values <- list(jnp$array(as.integer(income)),jnp$array( as.integer(m$df$career)))
m$data_on_model = py_dict(keys, values, convert = TRUE)

# Define model -----
model <- function(income, career){
  # Parameter prior distributions
  alpha = bi.dist.normal(0, 1, name='alpha', shape = c(2))
  beta = bi.dist.normal(0.5, name='beta')

  s_1 = alpha[0] + beta * income[0]
  s_2 = alpha[1] + beta * income[1]
  s_3 = 0 # reference category

  p = jax$nn$softmax(jnp$stack(list(s_1, s_2, s_3)))

  # Likelihood
  bi.dist.categorical(probs=p[career], obs=career)
}

# Run MCMC -----
m$fit(model) # Optimize model parameters through MCMC sampling

# Summary -----
m$summary() # Get posterior distribution

```

## Mathematical Details

We can model a *Categorical* model using a *Categorical distribution*. The multinomial distribution models the counts of outcomes falling into different categories. For an outcome variable with  $K$  categories, the multinomial likelihood function is:

$$Y_i \sim \text{Categorical}(\theta_i) \theta_i = \text{Softmax}(\phi_i) \phi_{[i,1]} = \alpha_1 + \beta_1 X_i \phi_{[i,2]} = \alpha_2 + \beta_2 X_i \dots \phi_{[i,k]} = 0 \alpha_k \sim \text{Normal}(0, 1) \beta_k \sim \text{Normal}(0, 1)$$

Where:

- $Y_i$  is the dependent categorical variable for observation  $i$  indicating the category of the observation.
- $\theta_i$  is a vector unique to each observation,  $i$ , which gives the probability of observing  $i$  in category  $k$ .
- $\phi_i$  give the linear model for each of the  $k$  categories. Note that we use the softmax function to ensure that the probabilities  $\theta_i$  form a simplex .
- Each element of  $\phi_i$  is obtained by applying a linear regression model with its own respective intercept  $\alpha_k$  and slope coefficient  $\beta_k$ . To ensure the model is identifiable, one category,  $K$ , is arbitrarily chosen as a reference or baseline category. The linear predictor for this reference category is set to zero. The coefficients for the other categories then represent the change in the log-odds of being in that category versus the reference category.

## Reference(s)

McElreath, Richard. 2018. *Statistical Rethinking: A Bayesian Course with Examples in r and Stan*. Chapman; Hall/CRC.