Classifying Knots

NELSON ABDIEL COLÓN VARGAS

Nel Abdiel

May 25, 2016

Outline

What Are Knots?

Outline

Outline

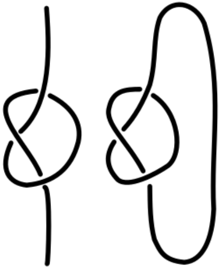
Outline

Building a mathematical knot

Take a piece of string

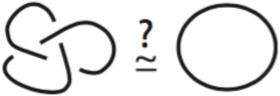
Tie as many knots as you want

Glue the ends of the string together



Given two objects, how to tell them apart?

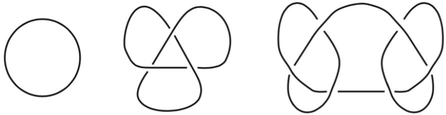
When are two knots equivalent?



Knot Theory

When can we tell the difference between Knots?

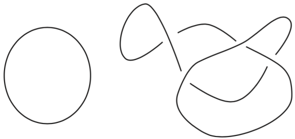
Are the knots below the same?



Knot Theory

When can we tell the difference between Knots?

How about these?



Knot Theory

Some Formalism

A knot is an injective map $h : S^1 \to \mathbb{R}^3$

- Picture in the plane (or slide) - Diagram with crossing

Knot Theory

Some Formalism

A knot is an injective map $h : S^1 \to \mathbb{R}^3$

- Picture in the plane (or slide) - Diagram with crossing

Knot Theory

Some Formalism

A knot is an injective map $h : S^1 \to \mathbb{R}^3$

- Picture in the plane (or slide) - Diagram with crossing
- Finite Number of arcs

Knot Theory

Some Formalism

A knot is an injective map $h : S^1 \to \mathbb{R}^3$

- Picture in the plane (or slide) - Diagram with crossing
- Finite Number of arcs
- Only two strands at a crossing

Knot Theory

Some Formalism

A knot is an injective map $h : S^1 \to \mathbb{R}^3$

- Picture in the plane (or slide) - Diagram with crossing
- Finite Number of arcs
- Only two strands at a crossing
- Invariant Property: $K_1 \sim K_2$ if,

Knot Theory

Some Formalism

A knot is an injective map $h : S^1 \to \mathbb{R}^3$

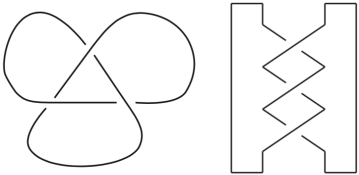
- Picture in the plane (or slide) - Diagram with crossing
- Finite Number of arcs
- Only two strands at a crossing
- Invariant Property: $K_1 \sim K_2$ if,

  There is a homeomorphism $\phi : \mathbb{R}^3 \to \mathbb{R}^3$ such that

$$\phi(K_1) = K_2.$$

Knot Theory - Diagrams

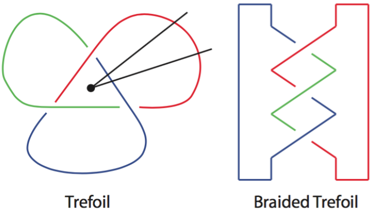
Two Knots with **similar** Diagram must be the same.



Trefoil   Braided Trefoil

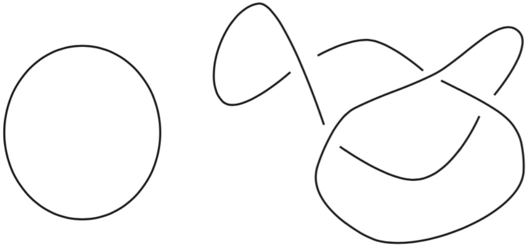
Knot Theory - Diagrams

Two Knots with **similar** Diagram must be the same.



Knot Theory - Diagrams

Even if not exactly the diagram same, they could be the same...



Invariants

- Colorability

Invariants

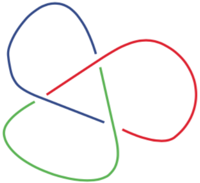
- Colorability
- Polynomial Invariants

Invariants

Colorability

K is colorable $\leftrightarrow$ each arc has one of 3 colors.

- At least two of the colors are used
- At crossing, either all different or all the same color.

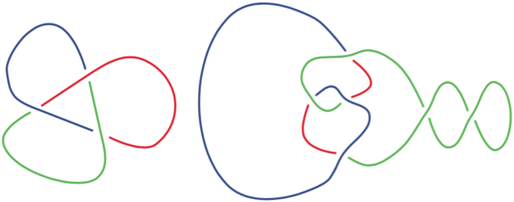


Invariants

Colorability

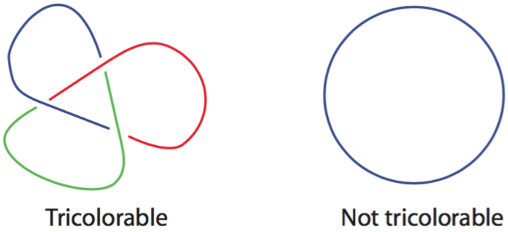
K is colorable $\leftrightarrow$ each arc has one of 3 colors.

- At least two of the colors are used
- At crossing, either all different or all the same color.



Colorability

The Unknot and Trefoil are different!



Gracias!