An improved quality control chart to monitor the process mean based on ranked sets

Cover page

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Abstract

In this study, we considered the design and performance of control charts using neoteric ranked set sampling (NRSS) in monitoring industrial processes. NRSS is a recently proposed sampling design, based on the traditional ranked set sampling (RSS). **NRSS differs from RSS by constituting, originally, a single set of k^2 sample units, instead of k sets of size k, where k is the final sample size. We evaluated NRSS control charts by average, median and standard deviation of run lengths, based on Monte Carlo simulation results.** NRSS control charts performed the best, compared to RSS and some of its extensions, in most simulated scenarios. The impact of imperfect ranking was also evaluated. An application on concrete strength data serves as an illustration of the proposed method.

Keywords: Quality control charts; Monte Carlo simulation; average run length; neoteric ranked set sampling.

1 Introduction

Nowadays, technological resources are widely available for the real-time monitoring of many industrial processes. Even so, it must be recognized that sampling still plays a fundamental role in statistical quality control. Factors such as high costs, time of inspection and destructive tests may limit the evaluation of a large number of items. In this context, efficient sampling designs, providing more accurate results with smaller sample sizes, are highly useful. Ranked set sampling (RSS) and its extensions have been shown as efficient alternatives to more conventional methodologies (such as simple random sampling) in industrial applications, particularly in developing statistical quality control charts.

Originally proposed in 1924 by Walter A. Shewart, statistical quality control charts (or simply control charts) constitute a relevant tool for visualizing industrial processes and identifying assignable causes of variation (Shewhart 1924; Montgomery 2009). A process is said to be under statistical control when no special or assignable causes are present. Several alternatives to the original control charts were proposed, providing greater speed in detecting out of control situations. These alternatives include: the use of additional or alternative decision rules (Koutras, Bersimis, and Maravelakis 2007); adaptive sampling schemes (Costa and De Magalhaes 2007); **nonparametric control charts** (Qiu 2018) or even the use of alternative sampling designs to the usual simple random sampling (SRS). In this study, we consider a variety of RSS based designs for constructing control charts.

Ranked set sampling is an effective sampling design when the variable of interest is expensive or difficult to measure, but it is possible ranking sample units according to some accessible and cheap criterion (Chen, Bai, and Sinha 2003; McIntyre 1952). This ranking process can be made based on an expert's judgment or an auxiliary variable, strongly correlated to the variable of interest. RSS becomes more efficient than SRS as long as a more accurate and accessible ordering criterion is available. Several studies have shown the superiority of RSS over SRS for estimation of different population parameters (see, for example, Al-Omari and Bouza (2014) and Chen's (2007)). Additionally, a large number of sampling designs derived from the original RSS were proposed, such as median ranked set sampling - MRSS (Muttlak 1997), extreme ranked set sampling - ERSS (Samawi, Ahmed, and Abu-Dayyeh 1996), double ranked set sampling - DRSS (Al-Saleh and AlKadiri 2000), among others.

Recently, RSS and its related sampling designs have been studied in the context of statistical quality control. Muttlak and Al-Sabah's (2003) developed different control charts using RSS and two of its modifications: ERSS and MRSS. The authors have shown, based on an extensive simulation study, that RSS-based control charts dominate their SRS counterpart, requiring, on average, fewer samples to detect a change in the process mean. Additionally, MRSS have showed the best performance among the three sampling designs based on ranked sets. Improved control charts for double-ranked set sampling schemes were also considered in the design of quality control charts. This class of sampling designs is characterized by the initial selection and ranking of k^3 (instead of k^2) sample units to draw a sample of size k after two ranking cycles. Double ranked set sampling control charts outperform those based on a single ordering cycle. Al-Omari and Bouza's (2014) present a bibliographic review of RSS based control charts.

Neoteric ranked set sampling (NRSS) (Zamanzade and Al-Omari 2016) is a sampling design recently originated from RSS. Technically, its fundamental difference to RSS is the constitution and ordering of a single set of k^2 sample units, instead of k sets of size k like in RSS, MRSS and ERSS. **After the ordering process, k units are chosen to compose the final sample, selected according to their specific ranks. The effect of creating a large initial set is the reduction of sample units variance, once the dispersion of order statistics decreases as the sample size increases. This reduction overcomes the covariances induced by sample units selected from the same ranked set.** In this way, it was found, for different sample sizes, correlation levels between the variable of interest and an auxiliary variable and probability distributions that NRSS overcomes RSS and SRS for estimating population mean and variance.

NRSS was firstly considered for control charts in Koyuncu and Karagöz's (2017) to monitor the mean of bivariate asymmetric distributions. The authors studied the **type I error** using different RSS designs under perfect ranking (i.e. when there are no errors in the ranking process). They considered the Type I Marshall-Olkin bivariate Weibull and bivariate lognormal distributions. They verified that the NRSS and RSS designs have **type I error** closest to 0.0027, the usual type I error for Shewart control charts.

In this paper we propose and analyze the power of control charts for monitoring the process mean based on NRSS for normal distributed processes. Its performance is evaluated by a Monte Carlo simulation study. We considered different sample sizes and processes with different shifts from statistical control. Also, we evaluated the impact of imperfect ranking by setting different correlation levels between the variable of interest and an auxiliary variable. **The effect of non normality was also assessed.** NRSS control charts were compared with SRS, RSS, MRSS and ERSS based on results already presented in the literature. Finally, an example with concrete strength data complements this study.

2 Neoteric ranked set sampling and other sampling designs based on ranked sets

The original ranked set sampling design can be described as follows:

1. Selection of k^2 units of the population using SRS, allocating them, randomly, in k sets of size k;

2. Ranking the sample units in each set according to the possible values of the variable of interest, using the pre-established ordering criterion;

3. Selection, for the final sample, of the ith judged unit in the ith set, $i = 1, 2, ..., k$.

4. Steps 1 to 3 can be replicated n times (n cycles) yielding a sample of size nk.

We denote the RSS sample by $Y_{[i]j}$ $(i = 1, 2, \ldots, k; j = 1, 2, \ldots, n)$, where $Y_{[i]j}$ represents the observation ranked in the ith position in the jth cycle. In this case, the sample units are independent, but not identically distributed random variables, as a result of the ordering process.

The usual estimator of the population mean using RSS is given by:

$$\bar{Y}_{RSS} = \frac{1}{nk} \sum_{j=1}^{n} \sum_{i=1}^{k} Y_{[i]j}, \tag{1}$$

with variance:

$$Var(\bar{Y}_{RSS}) = \frac{\sigma^2}{nk} - \frac{1}{nk^2}\sum_{j=1}^{n}\sum_{i=1}^{k}(\mu_{[i]}-\mu)^2, \tag{2}$$

where μ and σ^2 are the population mean and variance and $\mu_{[i]} = E\left[Y_{[i]j}\right]$, that is the mean of the ith order statistic from a simple random sample of size k under the perfect ranking scenario (when there are no errors in the ordering process). As alternatives to the RSS we have, for example, the median ranked set sampling (MRSS), which consists in the selection of the judged median in each set. Extreme ranked set sampling (ERSS), on the other hand, is based on the selection of the units judged as the minimum, in half of the sets, and the ones judged as the maximum, in the other half.

Similar to RSS, neoteric ranked set sampling (NRSS) is also useful when the ranking of sample units is much cheaper than obtaining their precise values (Zamanzade and Al-Omari 2016). NRSS design consists of the following steps:

1. Selection of k^2 units of the population using SRS;

2. Ranking the k^2 sample units based on the pre-established ordering criterion;

3. Selection of the $[(i-1)k+l]$-th sample unit for the final sample, $i = 1,\ldots,k$. If k is odd, then $l = \frac{k+1}{2}$; if k is even, then $l = \frac{k+2}{2}$ when i is odd and $l = \frac{k}{2}$ when i is even;

4. Again, steps 1-3 can be repeated n times, setting up n cycles and producing a final sample of size nk.

As previously stated, in NRSS the k^2 original sample units compose (and are ordered in) a single set, which induces dependence between the observations (differently from the RSS design). The variances of these variables, however, are reduced due to the greater set size, which justifies its higher efficiency. For illustration, to select a NRSS sample of size $k = 3$, we should select the positions 2, 5 and 8 from a original sample of size $k^2 = 9$, ordered in a single set; for a sample of size $k = 4$, we should select the positions 3, 6, 11 and 14 from a ordered sample of size $k^2 = 16$; and lastly, for a sample of size $k = 5$ the positions 3, 8, 13, 18 and 23 should be selected from a ordered sample of size $k^2 = 25$. These are the sample sizes considered in this study. It is possible to observe that the positions of the selected sample units are, in general, regularly spaced.

The NRSS sample is denoted by $\{Y_{[(i-1)k+l]j};\ i = 1,2,...,k;\ j = 1,2,...n\}$, in which $Y_{[(i-1)k+l]j}$ refers to the unit ranked in position $[(i-1)k+l]$ (of an initial sample of size k^2), in the jth cycle. Under perfect ranking, particularly, $Y_{[(i-1)k+l]j}$ corresponds to the $((i-1)k+l)$th order statistics in a SRS of size k^2 taken from the population.

The NRSS sample mean is an unbiased estimator for the population mean for symmetric distributions, which can be written by:

$$\bar{Y}_{NRSS} = \frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k} Y_{[(i-1)k+l]j}, \tag{3}$$

and its variance is given by:

$$Var(\bar{Y}_{NRSS}) = \frac{1}{nk^2}\sum_{i=1}^{k}\mathrm{Var}(Y_{[(i-1)k+l]}) + \frac{2}{nk^2}\sum_{1\leq i<i'\leq k}^{k}\mathrm{Cov}(Y_{[(i-1)k+l]}, Y_{[(i'-1)k+l]}). \tag{4}$$

3 Statistical quality control charts using NRSS (NRSS control charts)

Control charts for the process mean based on simple random samples of size k are defined by a central line (CL) and a pair of control limits (LCL and UCL) given by:

$$\begin{aligned} LCL &= \mu_0 - A\sqrt{Var(\bar{Y}_{SRS})} = \mu_0 - A\frac{\sigma_0}{\sqrt{k}} \\ CL &= \mu_0 \qquad\qquad\qquad\qquad\quad , \\ UCL &= \mu_0 + A\sqrt{Var(\bar{Y}_{SRS})} = \mu_0 + A\frac{\sigma_0}{\sqrt{k}} \end{aligned} \tag{5}$$

where μ_0 and σ_0 are the process mean and standard deviation under control state, $\bar{Y}_{SRS}$ the mean of a simple random sample of k units and A the amplitude parameter of the control chart. An observed sample mean beyond the control limits is an indicator of an out of control process. It is usual to consider $A = 3$, which, under normal distribution, is associated to a probability of a false alarm (a point outside the control limits for an under control process) of approximately 0.0027.

We propose control charts for the process mean using NRSS, based on the following limits:

$$\begin{aligned} LCL &= \mu_0 - A\sqrt{Var(\bar{Y}_{NRSS})} \\ CL &= \mu_0 \qquad\qquad , \\ UCL &= \mu_0 + A\sqrt{Var(\bar{Y}_{NRSS})} \end{aligned} \tag{6}$$

where μ_0 is the mean of the process under control state, and $\bar{Y}_{NRSS}$ and $Var(\bar{Y}_{NRSS})$ are defined in (3) and (4), respectively.

Our proposal constitutes an extension of the conventional SRS control charts, in such a way that the samples are periodically selected using NRSS and the control limits are based on (6). Alternatively, extensions of control charts were previously proposed for designs based on RSS. The performance of these control charts are used here as reference to NRSS control charts results.

In our study, to set the values for NRSS control limits, as described in (6), it was firstly necessary to get the values for $Var(\bar{Y}_{NRSS})$, for a process under statistical control, for each simulated scenario. Under perfect ranking, $Y_{[(i-1)k+l]}$ is equivalent to the $(i-1)k+l$ order statistic from a SRS sample of size k^2, $i = 1,2,...,k$. So, in this case we calculated $Var(\bar{Y}_{NRSS})$ based on the variances and the covariances as presented in (4), by using the properties of order statistics from the normal distribution, presented, for example, in Balakrishnan and Rao's (1998).

Under imperfect ranking, due to the ranking errors, the sampling units no longer match to order statistics. In this case, we obtained the values for $Var(\bar{Y}_{NRSS})$ by means of a preliminary simulation study. So we simulated $B = 10^6$ NRSS samples from a bivariate normal distribution for different combinations of k and ρ (the correlation between the variable of interest and the auxiliary variable). Bivariate normal distribution is very usual in several industrial applications (Montgomery 2009). Also, it is largely considered to evaluate the performance of control charts for RSS designs. Then, $Var(Y_{[(i-1)k+l]})$ and $Cov(Y_{[(i-1)k+l]}, Y_{[(i'-1)k+l]})$ were determined, respectively, by:

$$Var(Y_{[(i-1)k+l]}) = \frac{\sum_{h=1}^{B} \left(Y_{[(i-1)k+l],h} - \bar{Y}_{[(i-1)k+l]}\right)^2}{B-1}, \quad i = 1, 2, ..., k, \tag{7}$$

where

$$\bar{Y}_{[(i-1)k+l]} = \frac{\sum_{h=1}^{B} Y_{[(i-1)k+l],h}}{B}, \tag{8}$$

and

$$\begin{aligned} &Cov(Y_{[(i-1)k+l]}, Y_{[(i'-1)k+l]}) = \\ &= \frac{\sum_{h=1}^{B} \left(Y_{[(i-1)k+l],h} - \bar{Y}_{[(i-1)k+l]}\right)\left(Y_{[(i'-1)k+l],h} - \bar{Y}_{[(i'-1)k+l]}\right)}{B-1}, \end{aligned} \tag{9}$$

for $1 \leq i < i' \leq k$. Then, we replaced (7) and (9) in (4) to obtain the variances, and we used them to set the NRSS control limits under imperfect ranking.

3.1 NRSS control charts based on estimated limits

When the process parameters are unknown, we propose the estimation of μ_0 and $Var(\bar{Y}_{NRSS})$ based on the results of m independent samples of size k selected from the process in the absence of assignable causes of variation (under control process), according to (10) and (11):

$$\bar{\bar{Y}}_{NRSS} = \frac{1}{m} \sum_{p=1}^{m} \bar{Y}_{NRSSp} \tag{10}$$

$$\widehat{Var}\left(\bar{Y}_{NRSS}\right) = \frac{1}{k^2}\sum_{i=1}^{k}\widehat{Var}\left(Y_{[(i-1)k+l]}\right) + \frac{2}{k^2}\sum_{i<i'}\widehat{Cov}\left(Y_{[(i-1)k+l]}, Y_{[(i'-1)k+l]}\right), \tag{11}$$

where

$$\widehat{Var}\left(Y_{[(i-1)k+l]}\right) = \frac{1}{m-1}\sum_{p=1}^{m}\left(Y_{[(i-1)k+l]p} - \bar{Y}_{[(i-1)k+l]}\right)^2, \tag{12}$$

where $\bar{Y}_{[(i-1)k+l]} = \frac{1}{m}\sum_{p=1}^{m} Y_{[(i-1)k+l]p}$ and

$$\begin{aligned}\widehat{Cov}(Y_{[(i-1)k+l]}, Y_{[(i'-1)k+l]}) = \frac{1}{m-1}\sum_{p=1}^{m}[(&Y_{[(i-1)k+l]p} - \bar{Y}_{[(i-1)k+l]}) \\ &(Y_{[(i'-1)k+l]p} - \bar{Y}_{[(i'-1)k+l]})], 1 \leq i < i' \leq k.\end{aligned} \tag{13}$$

So, in practice the NRSS control charts for the process mean with estimated control limits are defined by substituting, in (6), μ_0 by $\bar{\bar{Y}}_{NRSS}$ and $Var(\bar{Y}_{NRSS})$ by $\widehat{Var}(\bar{Y}_{NRSS})$:

$$\begin{aligned} LCL &= \bar{\bar{Y}}_{NRSS} - A\sqrt{\widehat{Var}\left(\bar{Y}_{NRSS}\right)} \\ CL &= \bar{\bar{Y}}_{NRSS} \qquad . \\ UCL &= \bar{\bar{Y}}_{NRSS} + A\sqrt{\widehat{Var}\left(\bar{Y}_{NRSS}\right)} \end{aligned} \tag{14}$$

In order to investigate the bias of (11) in estimating (4), an additional simulation study was carried out, considering $k = 3, 4$ and 5. For each value of k, we simulated 5×10^4 replications of m samples, using NRSS, from a normal standard distribution. For m, values between 5 and 25 were set. At each step, the m simulated samples were considered to estimate (4). We found that the bias of this estimator is negligible (a relative bias lower than 0.001 was verified for all sample sizes for $m \geq 20$).

4 Run length analysis for NRSS control charts

The performance of NRSS control charts was evaluated by a Monte Carlo simulation study. We simulated samples from a bivariate normal distribution, according to:

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim \text{Normal}\left(\begin{pmatrix} 0 \\ \mu_Y \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right). \tag{15}$$

We assume $\mu_Y = \mu_0 = 0$ as the under control process mean. For the out of control scenarios, we considered $\mu_Y = \mu_0 + \delta \frac{\sigma_0}{\sqrt{k}}$, so that δ determines the shift in the process mean:

$$\delta = |\mu_Y - \mu_0| \frac{\sqrt{k}}{\sigma_0}, \tag{16}$$

such that $\delta = 0$ implies to an under control process.

As parameters settings for the simulation study we had $k = 3, 4$ and 5; $\delta = 0, 0.1, 0.2, 0.3, 0.4, 0.8, 1.2, 1.6, 2, 2.4$ and 3.2 and $\rho = 0, 0.25, 0.50, 0.75, 0.9$ and 1. To evaluate the performance of control charts we considered the average run length (ARL), defined as the average number of points in a control chart until one exceeds the control limits. Particularly, if we have an under control process, ARL_0 is the reciprocal of the false alarm error rate; for an out of control process, ARL_1 is inversely proportional to the detection probability, representing the average number of samples until the out of control state is detected. For each combination of k, and δ **we simulated 10^6 independent NRSS samples under perfect ranking and 10^7 under imperfect ranking**, for each considered correlation level (ρ value). The ARL values were calculates as the inverse of the proportion of points (sample means) beyond the control limits. In addition to that, **the simulation results were also summarized by means of standard deviation of the run length (SDRL) and median run length (MRL), since the run length distribution is quite skewed.**

The parameters for the simulation study were chosen in such a way to allow the comparison of the ARL values with those presented in other publications, referring to control charts for other sampling designs based on RSS. Moreover, it becomes evident that the considered scenarios (198 in total) comprises a great variety of processes. The sample size was limited to $k = 5$ given the context for application of sampling designs based on RSS (restrictions related to draw big samples, initial selection and ranking of k^2 - or even k^3 or more - sample units, among others). Moreover, the amplitude parameter (A) for the control limits were set, under perfect ranking, so that $ARL_0 = 370.51$. This is the ARL_0 corresponding to SRS control charts when we set $A = 3$. In this way, we could compare the ARL_1 values for NRSS control charts with those provided by other sampling designs. The double ranked set sampling designs control charts, particularly, produce low values for ARL_0 and, consequently, high false alarms rates when $A = 3$.

Table 1 present the simulated run length results for RSS based control charts. Besides NRSS, results obtained by SRS, RSS, ERSS and MRSS are also presented. In this first part of the analysis, we considered perfect ranking ($\rho = 1$), allowing to assess the maximum power provided by each design.

[Table 1 should be near here.]

Some conclusions drawn from Table 1 are the ones that follow:

- The efficiency of NRSS control charts for detecting shifts in process mean increases, as expected, for higher values of δ and k. As an illustration, for $k = 3$ and $\delta = 0.40$ we have $ARL = 120.60$ compared to $ARL = 6.41$ for $\delta = 1.20$, while for $k = 5$ and $\delta = 0.40$ we have $ARL = 102.60$ for $k = 3$ against $ARL = 60.14$ for $k = 5$;

- The NRSS control charts perform better than SRS control charts in all simulated scenarios. For example, for $k = 3$ and $\delta = 0.80$ we have $ARL = 21.25$ for NRSS control charts compared to $ARL = 71.55$ for SRS, while for $k = 5$ and $\delta = 1.60$ we have $ARL = 1.46$ for NRSS against $ARL = 12.38$ for SRS;

- The NRSS control charts dominates RSS and ERSS designs in all the simulated scenarios. For example, when compared to RSS, for $k = 3$ and $\delta = 0.80$ we have $ARL = 21.25$ for

NRSS control charts against $ARL = 35.43$ for RSS, while for $k = 5$ and $\delta = 1.60$ we have $ARL = 1.46$ for NRSS against $ARL = 2.83$ for RSS;

- The NRSS control charts overcome the MRSS competitor in all simulated scenarios. This is remarkable, once MRSS is well known by its higher efficiency in estimating the mean, compared to RSS, for symmetric distributions. Additionally, MRSS performs best under both single and double ranked set strategies for control charts for the process mean (Mehmood, Riaz, and Does 2013). When $k = 3$ and $\delta = 0.80$ it was verified $ARL = 21.25$ for NRSS control charts compared to $ARL = 29.52$ for MRSS, while when $k = 5$ and $\delta = 1.60$ we have $ARL = 1.46$ for NRSS against $ARL = 2.04$ for MRSS.

In order to summarize the performance of the different control charts designs, Figure 1 presents the geometric means of the ratios of ARL values for SRS control charts relative to the ones obtained by each of the other sampling designs, for each sample size. The ARL values for SRS control charts were, on average, 2.39 times larger than the corresponding NRSS when $k = 3$; 3 times for $k = 4$ and 3.59 times for $k = 5$. The best performance of NRSS control charts over the RSS, ERSS and MRSS counterparts becomes evident. For MRSS, for example, we have, on average, ARL 1.22 times higher than NRSS for $k = 3$; 1.25 times for $k = 4$ and 1.28 times for $k = 5$.

[Figure 1 should be near here.]

Table 2 present the simulation results under imperfect ranking by setting $A = 3$ (3-sigma limits). This is a traditional choice for Shewhart control charts. The ARL values for $\rho = 0$ are identical to the corresponding ones from SRS, once NRSS and SRS are equivalent if the ordering is completely random. Based on these results, it is possible to assess the impact of ranking errors in the performance of control charts.

[Table 2 should be near here.]

Some conclusions for Table 2 are highlighted next:

- Control charts for all RSS based designs lose performance when the correlation between the variables decreases. For example, for NRSS control charts, $k = 3$ and $\delta = 0.8$, $ARL = 21.34$ when $\rho = 1$, $ARL = 31.23$ when $\rho = 0.90$; 44.02 when rho $\rho = 0.75$ and 59.55 when $\rho = 0.50$;

- The ARL values for NRSS control charts are smaller compared to the ones provided by SRS in almost all simulated scenarios with $\delta \neq 0$. NRSS only loses in a few scenarios described by low shifts in process mean and low values for ρ;

- The ARL_0 values from NRSS control charts are around 370.4, as intended. The individual ARL_0 values varying from 365.62 when $k = 3$ and $\rho = 0.75$, to 372.04, when $k = 5$ and $\rho = 1$.

Figure 2 shows the geometric means of the ratios of ARL values for SRS control charts relative to the ones obtained by each of the RSS based designs. These results are presented for each sample size and considering the different correlation levels between the auxiliary and the variable of interest. We can notice that NRSS control charts are, in general, more efficient than all other considered sampling designs. Moreover, the superiority of NRSS control charts becomes higher when the correlation between the variables increases. For $\rho = 0.9$ and $\rho = 1$, we have, on average, higher efficiency for the NRSS control charts with $k = 4$ than for the other sampling designs taking $k = 5$, which can reflect in resource savings and lower operational effort.

[Figure 2 should be near here.]

5 Robustness analysis of the NRSS control charts for non-normally distributed processes

In order to evaluate the effect of non normality on the performance of NRSS control charts, a new simulation study was conducted. Two probability distributions were considered at this point: the skew normal and the generalized normal distributions (Azzalini 1985; Nadarajah 2005). Through these models, we were able to evaluate the impact of different levels of skewness and kurtosis on the run length results. The skew normal and the generalized normal models are briefly described in the following paragraphs.

The probability density function of a random variable with skew normal distribution is given by:

$$f(y;\varepsilon,\omega,\alpha)=\frac{2}{\omega}\phi\left(\frac{y-\varepsilon}{\omega}\right)\Phi\left(\alpha\left(\frac{y-\varepsilon}{\omega}\right)\right), \tag{17}$$

where $y\in(-\infty,\infty)$ and $\varepsilon\in(-\infty,\infty)$, $\omega>0$ and $\alpha\in(-\infty,\infty)$ are location, scale and shape parameters, respectively. Additionally, $\phi(\cdot)$ and $\Phi(\cdot)$ represent the probability density function and the cumulative distribution function of the standard normal distribution. The skew normal distribution becomes more asymmetric as $|\alpha|$ increases. When $\alpha>0$, the distribution is right skewed; left skewed if $\alpha<0$ and for $\alpha=0$ we have the normal distribution.

A random variable has generalized normal distribution if its probability density function is given by:

$$f(y;\mu,\beta,\alpha)=\frac{1}{2\,\alpha^{1/\alpha}\,\Gamma(1+1/\alpha)\,\beta}\,e^{-\frac{|y-\mu|^{\alpha}}{\alpha\beta^{\alpha}}}, \tag{18}$$

where $y\in(-\infty,\infty)$ and $\mu\in(-\infty,\infty)$, $\beta>0$ and $\alpha>0$ are location, scale and shape parameters, respectively. The generalized normal distribution is symmetric around μ and becomes the normal distribution when $\alpha=2$. In addition, for $\alpha<2$ it produces leptokurtic (fatter tails) distributions, and platykutics (thinner tails) distributions when $\alpha>2$. As particular cases of the generalized normal distribution we have, for example, the Laplace ($\alpha=1$) and uniform ($\alpha\to\infty$) distributions.

We considered four different parameter combinations for each one of the two distributions. For the skew normal model, an increasing sequence of values for α was defined ($\alpha=1,2,3$ and 5), providing distributions with different levels of skewness. Additionally, we set $\omega=1$ and, for ε, we have assigned appropriate values such that the process mean was equal to zero. For the generalized normal model, four different values for α were selected, producing two distributions with heavy tails (for $\alpha = 1$ and 1.5) and two with light tails (for $\alpha = 3$ and 4). Furthermore, for the other model parameters we set $\mu = 0$ and $\beta = 1$. In all cases, the process mean was set at zero since the objective here is to evaluate the robustness of the control charts in maintaining the average (and median) run length for an under control process ($ARL_0 = 370.4$). Also, for the sake of brevity we are only considering, at this point, the perfect ranking scenario.

For each one of the eight distributions obtained by combining the two distributions and four specific parameter settings, we have simulated 10^7 samples of sizes $k = 3, 4,$ and 5. Five different sampling designs were considered: NRSS, RSS, MRSS, ERSS and SRS. 3-sigma control limits were properly calculated as described in (6), for the NRSS control charts, and based on the expressions presented in Muttlak and Al-Sabah's (2003), for the others. Based on the simulated results, we calculated the corresponding values for *ARL*, *MRL* and *SDRL*, as we can verify in Table 3.

We can notice in Table 3 that, although all considered sample designs have their respective $ARL's$ affected by the distribution skewness, NRSS and MRSS provided, in general, the closest values to the nominal $ARL_0 = 370.4$ for the skew normal distribution. This indicates that these sampling designs are more conservative than their competitors. Table 3 points higher influence in *ARL* and *MRL* for the generalized normal if compared with the skew normal distribution in the considered simulated scenarios. This is particularly evident for $\beta = 1$ (Laplace distribution). However, we can also see that the NRSS control charts dominates all its competitors, producing, in general, ARL_0 values closer to 370.4. Our results are in agreement with those found by Koyuncu and Karagöz's (2017), who verified that NRSS control charts present lower type I error when applied to two asymmetric distributions: Type I Marshall-Olkin bivariate Weibull and bivariate lognormal.

[Table 3 should be near here.]

6 An application to real data

In order to illustrate the application of the NRSS control charts, we used a data set with 1030 observations about the concrete strength to compression (MPa) and the amount of cement (kg) used in the production of concrete blocks (Yeh 1998). Although this data was not recorded as a case of a quality control process, it serves us, under some assumptions, as a reference population, from which samples were drawn and control charts were constructed. We assumed the concrete strength as the variable of interest and the amount of cement as an auxiliary variable. Moreover, we assumed the concrete blocks strength distribution in this sample as the natural variability of an industrial process. **A square root transformation of the concrete strength was used in order to obtain a better approximation to normal distribution.**

In this application, we considered three sampling designs: SRS, RSS and NRSS; two sample sizes: $k = 3$ and $k = 5$, and processes in two different scenarios: under control ($\delta = 0$) and out of control, considering $\delta = 1.2$, as described in (16). Under each sampling design and for each sample size, we selected, with replacement, 25 samples from the original data. These samples were considered for estimating the control limits (phase 1). Afterwards, 75 new samples were selected for monitoring the process mean (phase 2). For $\delta = 0$, these 75 samples were selected with replacement from the original data; for $\delta = 1.2$, we added to the transformed strength values a normal random variable with mean $1.2\frac{\sigma_0}{\sqrt{k}}$ and standard deviation equals to 0.17 (corresponding to 11.74% of the standard deviation of the transformed concrete strength). This standard deviation value is small enough to characterize the lack of control, predominantly, due to the shift in the process mean, instead of its dispersion (variance).

Figure 3 presents (on the left) the histogram for the distribution of concrete strength, with the estimated normal distribution and Kernel density curves. The dispersion plot, on the right, indicates moderate positive linear relationship between the variables. The linear correlation coefficient is $\rho = 0.49$, which points to a moderately favourable scenario for RSS based designs.

[Figure 3 should be near here.]

Following, Figures 4 and 5 present the SRS, RSS and NRSS control charts for the process mean considering $k = 3$. In Figure 4 we have the charts when $\delta = 0$ (under control process). For the three sampling designs, it is possible to notice points randomly distributed around the central line, without any point outside the control limits. This behaviour characterizes an under control process, as expected. On the other hand, Figure 5 presents the control charts for $\delta = 1.2$ (out of control process). It is possible to observe that the NRSS control chart showed the highest number of points exceeding the control limits (7), followed by RSS (5 points outside the control limits) and SRS control charts (only 2 points outside the limits). Moreover, we notice a higher number of points below the central line in the SRS control charts compared to its contenders. This indicates higher difficulty in lack of control detection for SRS than for RSS or NRSS control charts.

[Figures 4 and 5 should be near here.]

Figures 6 and 7 present the control charts for $k = 5$, under the same three sampling designs, considering, respectively, $\delta = 0$ and $\delta = 1.2$. **Once more, it is possible to notice that NRSS control charts present satisfactory performance, showing randomness and without any point outside the control limits for an under control process, and also presenting more points exceeding the control limits (9) than RSS (6) and SRS (3) in the out of control scenario.**

[Figures 6 and 7 should be near here.]

7 Conclusion

In this paper, we considered control charts for the mean of a normal distributed process based on NRSS design. These charts were compared to their SRS and RSS based counterparts by means of a simulation study. Under perfect ranking, NRSS control charts overcome all their competitors, providing smaller ARL values for out of control process in all simulated scenarios. In addition, the NRSS control charts showed to be competitive when compared to those based on double ranked set designs. However, such sampling designs require the initial selection of k^3 sample units for, after two ordering cycles, selecting a final sample of k units. For example, the ARL for NRSS control charts were smaller in all simulated scenarios when compared to those provided by EDRSS and DERSS, and surpassed by those provided by DQRSS and QDRSS when $k = 5$. Moreover, this superiority is also verified against DRSS control charts for all considered sample sizes. When considering the DMRSS and MDRSS control charts (Abujiya and Muttlak 2004), on the other hand, these designs dominate NRSS, providing lower ARL values. However, it should be considered that double ranked set designs could be expensive, and sometimes infeasible, due to a high operational effort. Furthermore, based on its superiority over MRSS, it could be viewed, as a future work, NRSS designs based on two or more ordering cycles.

Under imperfect ranking, we have shown that the efficiency of NRSS control charts becomes smaller as the correlation between the variables decreases. This is a common fact to other designs based on RSS. Even so, the simulated ARL values for NRSS control charts are predominantly smaller (for out of control processes) than the corresponding ones reached by SRS. Additionally, it was possible to verify the superiority of the NRSS control charts with the ones provided by RSS, MRSS and ERSS in most of the simulated scenarios. **Also, NRSS was the most robust method for non-normally distributed processes.**

In an illustration with real data regarding concrete strength, the SRS, RSS and NRSS control charts presented points randomly distributed around the central line, without any points outside the control limits, when we simulated from a process under statistical control. **However, for the out of control scenarios, the NRSS control charts performed better when compared to the RSS and the usual control charts based on SRS.**

Therefore, based on these results, we recommend NRSS control charts for monitoring the process mean as an efficient alternative to SRS and to other RSS based designs. Under the operational point of view, the ranking of k^2 samples units in a single set (instead of ranking k sets of k units, as it occurs in RSS, MRSS and ERSS designs) may, eventually, become a complicating issue, if the ordering criterion is based, for example, on a visual judgment. However, this will usually not make great difference if the ordering criterion is based, for example, on an auxiliary variable.

Disclosure statement

No potential conflict of interest was reported by the authors.

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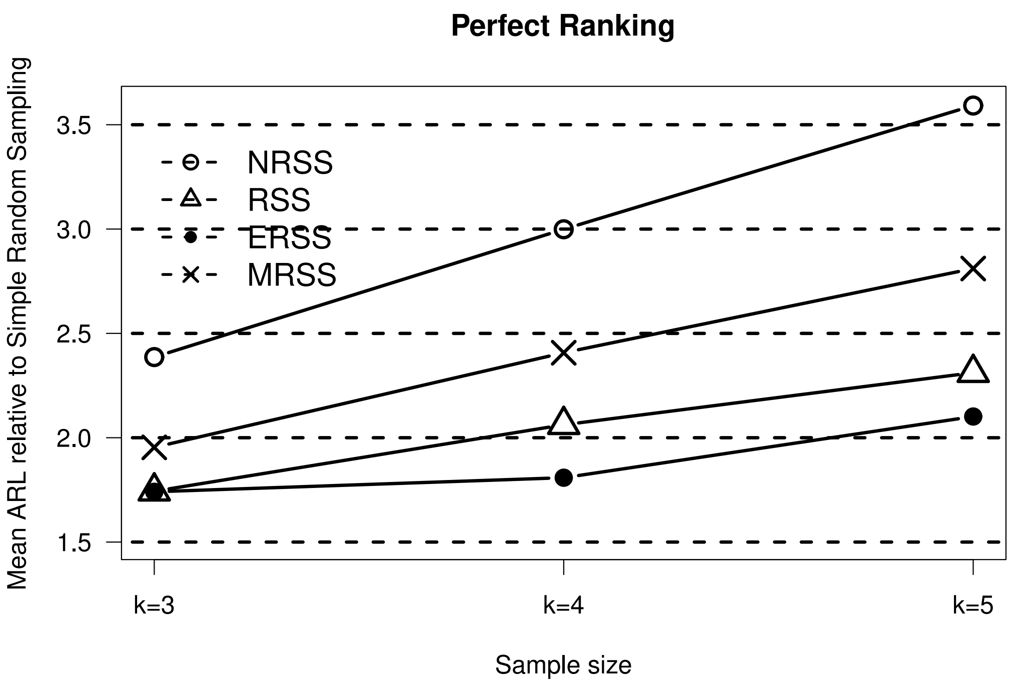

Figure 1: Average relative efficiency from control charts of designs based on RSS compared to SRS under perfect ranking. ARL from RSS, MRSS and ERSS were taken from Al-Omari and Haq (Al-Omari and Haq 2012).

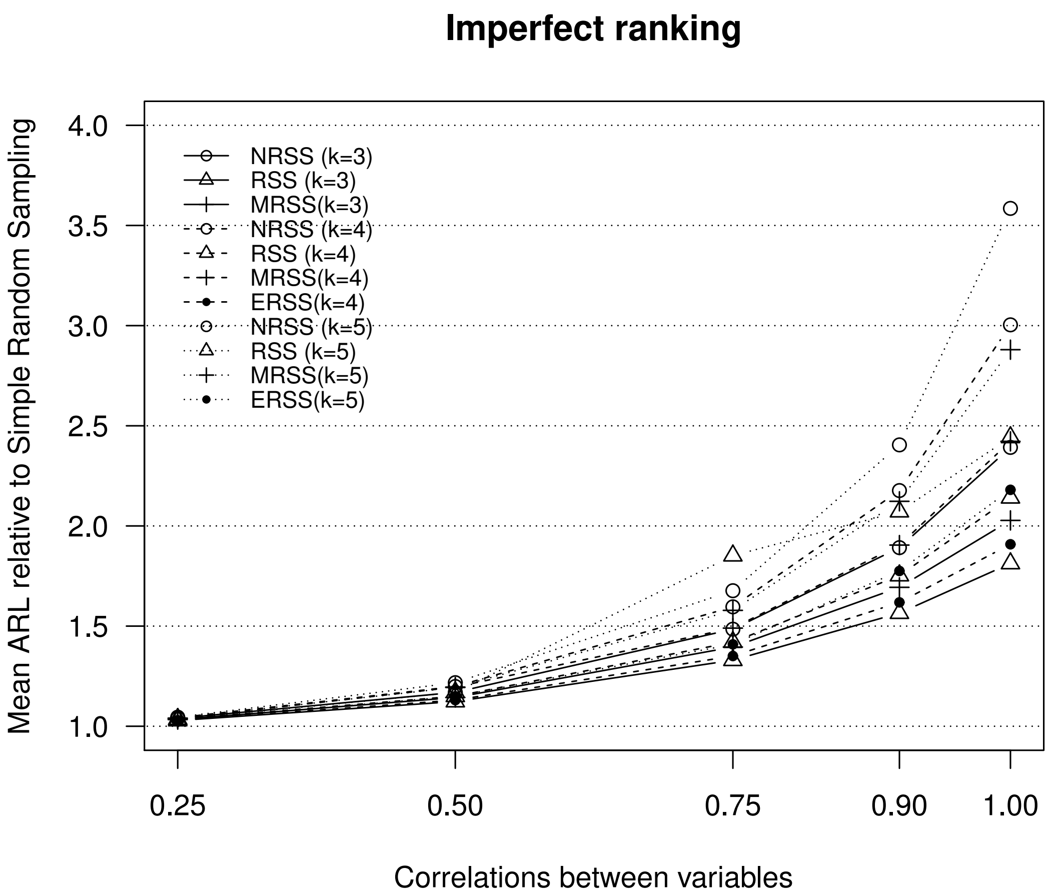

Figure 2: Average relative efficiency from control charts of designs based on RSS compared to SRS under imperfect ranking. ARL from RSS, MRSS and ERSS were taken from Al-Omari and Haq (Al-Omari and Haq 2012). For $k = 3$ RSS and ERSS provides the same sampling design.

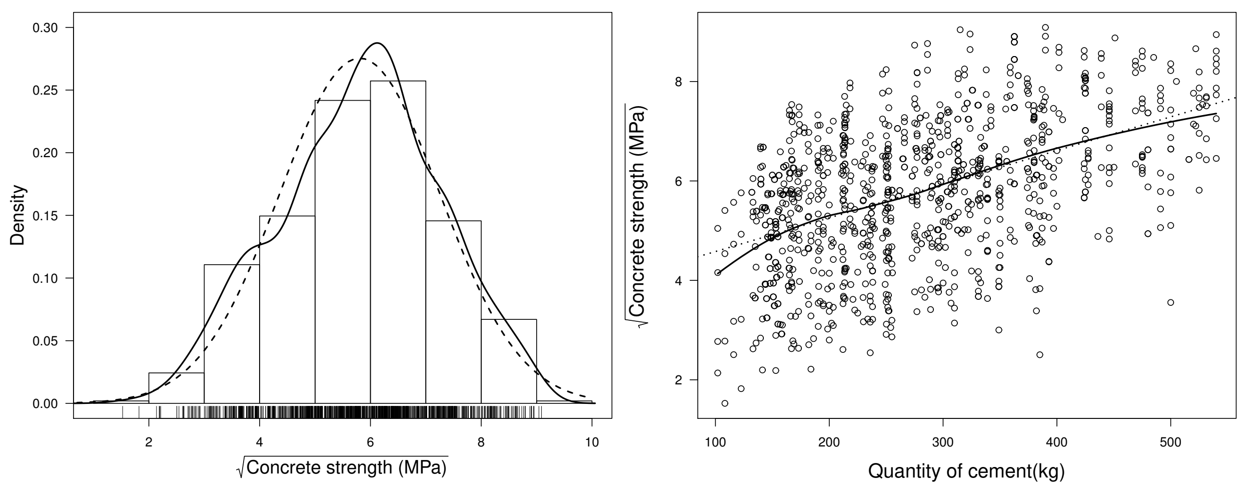

Figure 3: Histogram and scatter plot for concrete strength.

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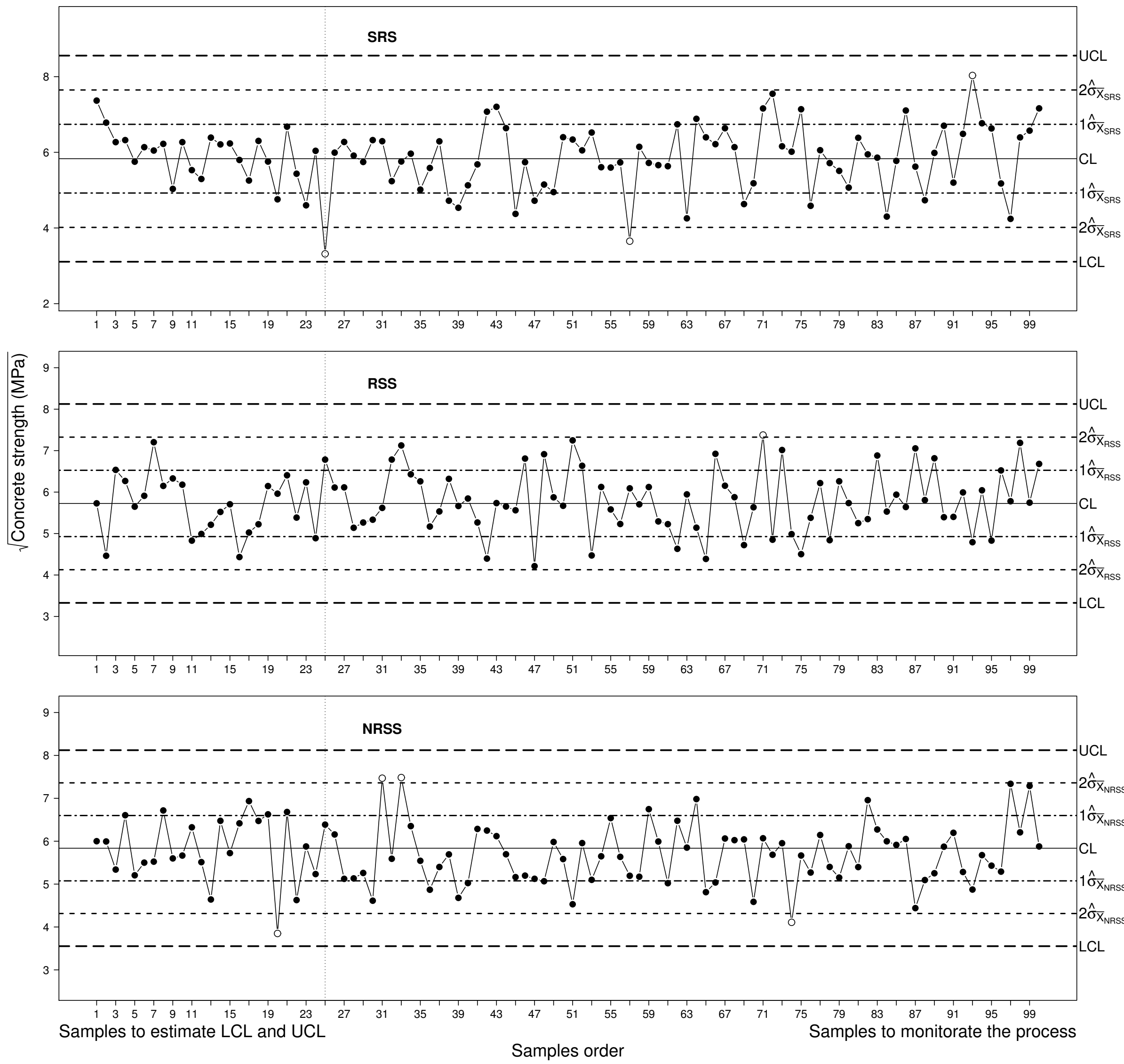

Figure 4: Control charts for concrete strength considering $k = 3$ and an under control process ($\delta = 0$)

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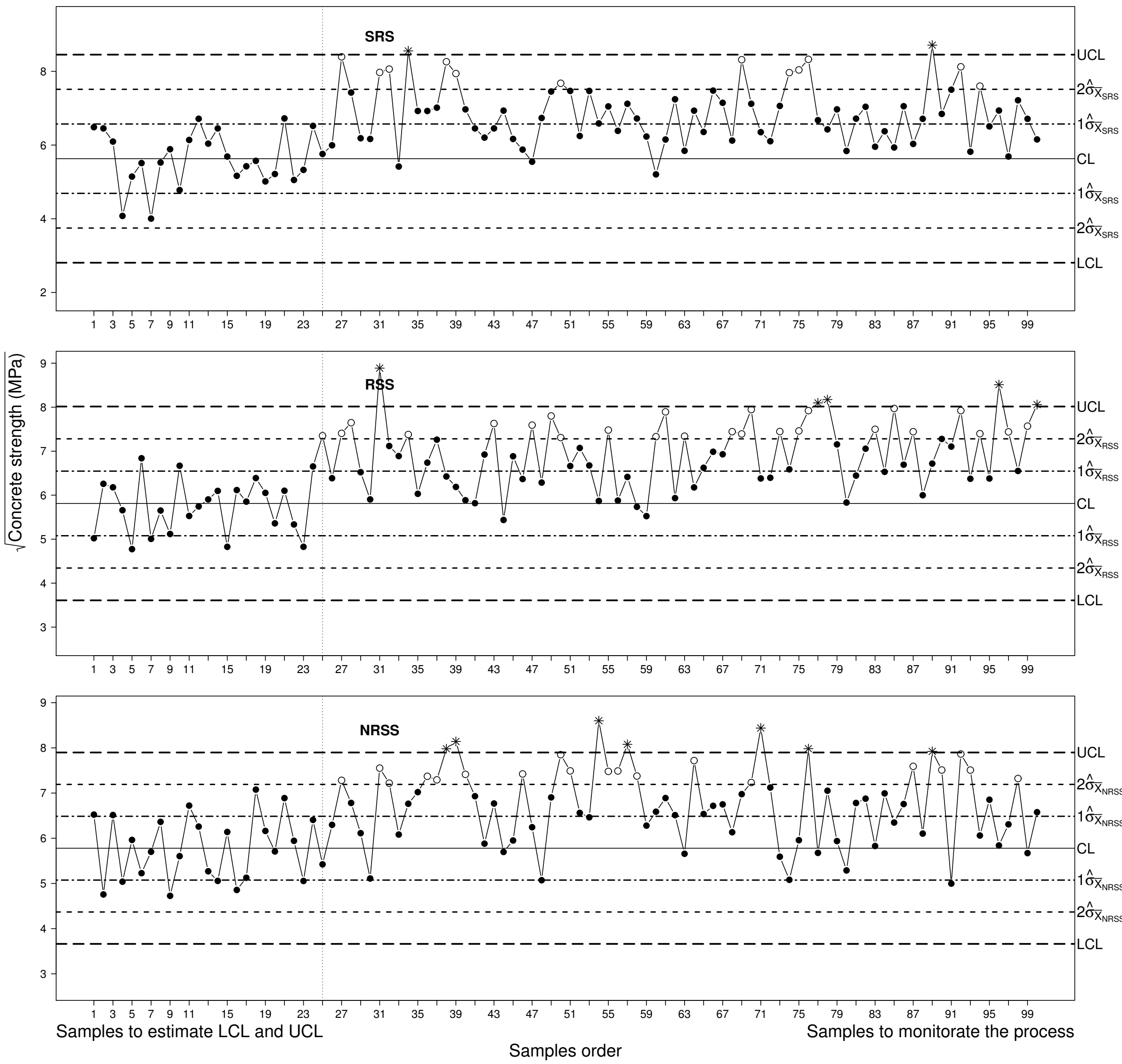

Figure 5: Control charts for concrete strength considering $k = 3$ and an out of control process ($\delta = 1.2$)

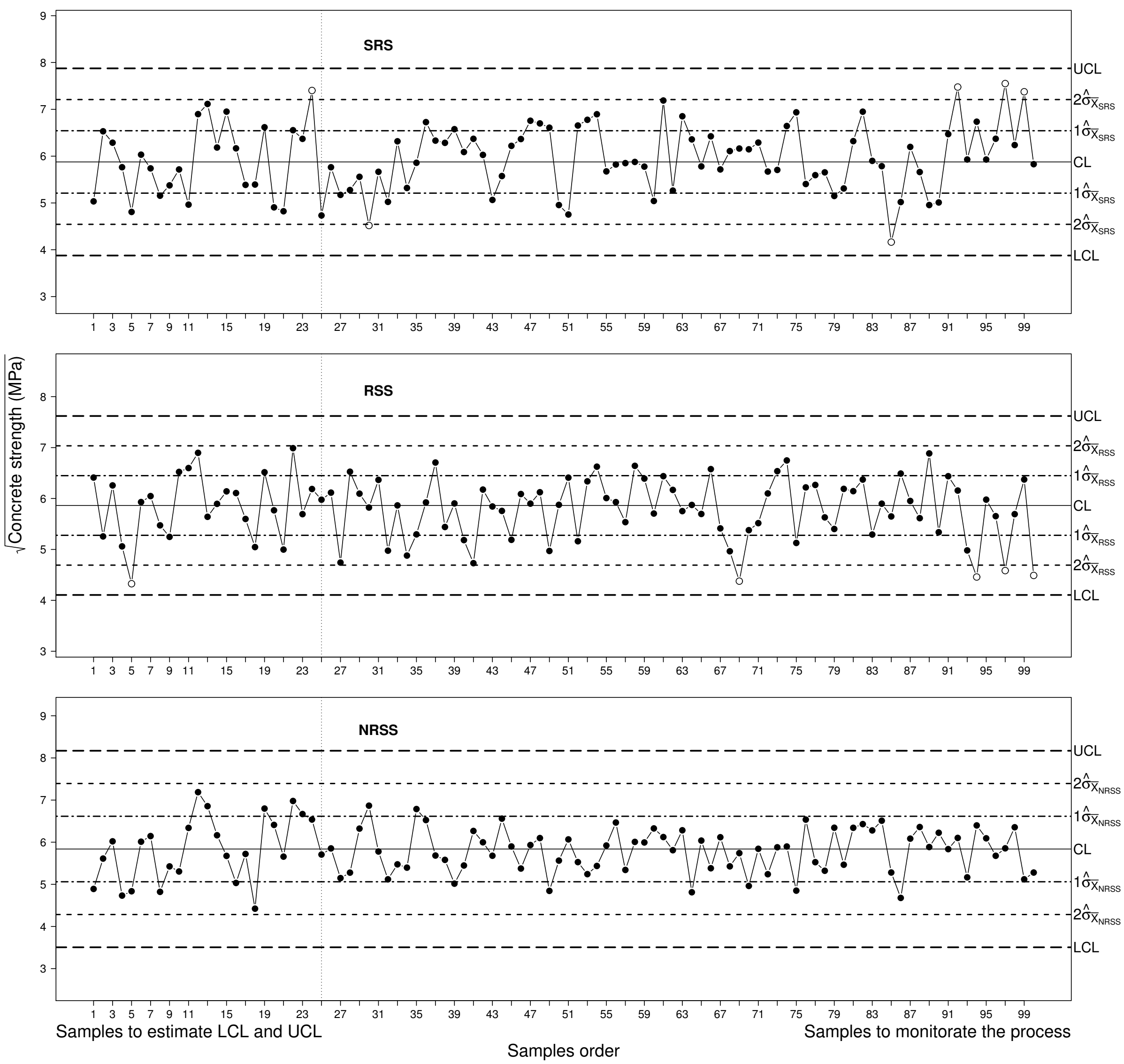

Figure 6: Control charts for concrete strength considering $k = 5$ and an under control process ($\delta = 0$)

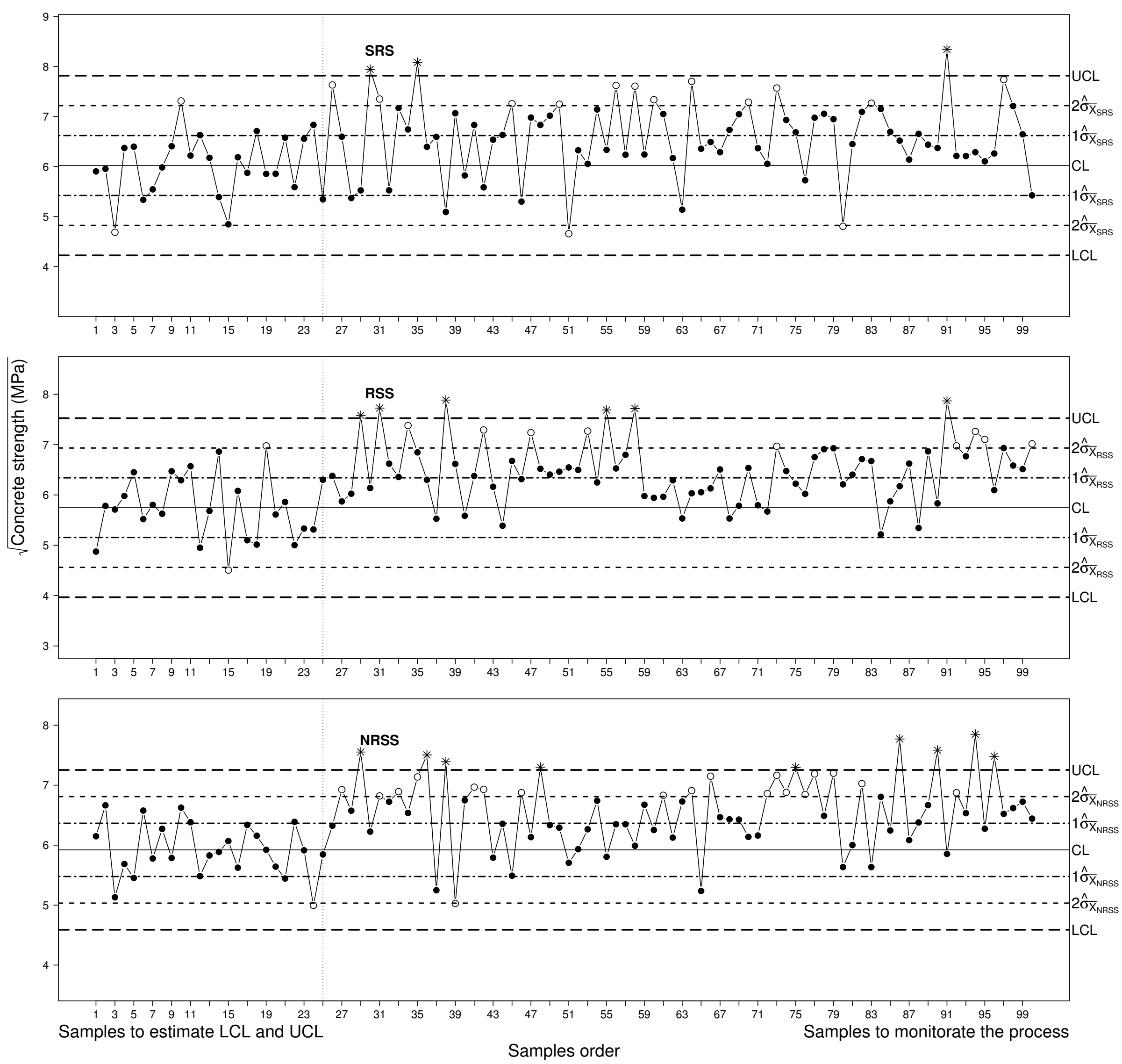

Figure 7: Control charts for concrete strength considering $k = 5$ and an out of control process ($\delta = 1.2$)

Table 1: ARL, MRL and SDRL for control charts constructed by SRS and designs based on RSS under perfect ranking

k	δ	SRS			RSS			ERSS			MRSS			NRSS		
		ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL
	0.00	370,40	257	369,90	370,51	257	370,01	370,51	257	370,01	370,51	257	370,01	370,51	257	370,01
	0.10	352,93	245	352,43	333,89	232	333,39	340,25	236	339,75	339,67	236	339,17	325,63	226	325,13
	0.20	308,43	214	307,93	266,03	185	265,53	272,18	189	271,68	265,11	184	264,61	234,03	162	233,53
	0.30	253,14	176	252,64	196,93	137	196,43	197,20	137	196,70	186,22	129	185,72	157,23	109	156,73
	0.40	200,08	139	199,58	139,43	97	138,93	137,99	96	137,49	128,12	89	127,62	102,60	71	102,10
3	0.80	71,55	50	71,05	35,43	25	34,93	35,35	25	34,85	29,52	21	29,02	21,25	15	20,74
	1.20	27,82	19	27,32	11,54	8	11,03	11,43	8	10,92	9,22	7	8,71	6,41	5	5,89
	1.60	12,38	9	11,87	4,76	3	4,23	4,75	3	4,22	3,80	3	3,26	2,76	2	2,20
	2.00	6,30	5	5,78	2,50	2	1,94	2,49	2	1,93	2,06	2	1,48	1,61	1	0,99
	2.40	3,65	3	3,11	1,61	1	0,99	1,61	1	0,99	1,40	1	0,75	1,20	1	0,49
	3.20	1,73	1	1,12	1,09	1	0,31	1,09	1	0,31	1,04	1	0,20	1,01	1	0,10
	0.00	370,40	257	369,90	370,51	257	370,01	370,51	257	370,01	370,51	257	370,01	370,51	257	370,01
	0.10	352,93	245	352,43	328,08	228	327,58	341,30	237	340,80	318,07	221	317,57	310,56	215	310,06
	0.20	308,43	214	307,93	249,81	173	249,31	266,81	185	266,31	232,45	161	231,95	210,30	146	209,80
	0.30	253,14	176	252,64	174,89	121	174,39	192,64	134	192,14	156,42	109	155,92	126,90	88	126,40
	0.40	200,08	139	199,58	119,36	83	118,86	135,85	94	135,35	100,29	70	99,79	77,86	54	77,36
4	0.80	71,55	50	71,05	27,78	19	27,28	33,69	24	33,19	21,42	15	20,91	13,89	10	13,38
	1.20	27,82	19	27,32	8,54	6	8,02	10,70	8	10,19	6,38	5	5,86	4,09	3	3,56
	1.60	12,38	9	11,87	3,55	3	3,01	4,41	3	3,88	2,73	2	2,17	1,89	1	1,30
	2.00	6,30	5	5,78	1,94	1	1,35	2,33	2	1,76	1,59	1	0,97	1,25	1	0,56
	2.40	3,65	3	3,11	1,35	1	0,69	1,53	1	0,90	1,19	1	0,48	1,06	1	0,25
	3.20	1,73	1	1,12	1,03	1	0,18	1,07	1	0,27	1,01	1	0,10	1,00	0	0,00
	0.00	370,40	257	369,90	370,51	257	370,01	370,51	257	370,01	370,51	257	370,01	370,51	257	370,01
	0.10	352,93	245	352,43	331,68	230	331,18	333,00	231	332,50	329,60	229	329,10	299,58	208	299,08
	0.20	308,43	214	307,93	244,98	170	244,48	254,77	177	254,27	223,41	155	222,91	181,06	126	180,56
	0.30	253,14	176	252,64	165,54	115	165,04	173,73	121	173,23	136,91	95	136,41	104,59	73	104,09
	0.40	200,08	139	199,58	107,88	75	107,38	117,44	82	116,94	85,20	59	84,70	60,14	42	59,64
5	0.80	71,55	50	71,05	22,53	16	22,02	26,59	19	26,09	15,56	11	15,05	9,55	7	9,04
	1.20	27,82	19	27,32	6,73	5	6,21	8,13	6	7,61	4,55	3	4,02	2,86	2	2,31
	1.60	12,38	9	11,87	2,83	2	2,28	3,38	2	2,84	2,04	2	1,46	1,46	1	0,82
	2.00	6,30	5	5,78	1,63	1	1,01	1,87	1	1,28	1,31	1	0,64	1,10	1	0,33
	2.40	3,65	3	3,11	1,21	1	0,50	1,32	1	0,65	1,08	1	0,29	1,01	1	0,10
	3.20	1,73	1	1,12	1,01	1	0,10	1,03	1	0,18	1,00	0	0,00	1,00	0	0,00

Table 2: ARL, MRL and SDRL for control charts constructed by NRSS under imperfect ranking

k	δ	$\rho = 0$			$\rho = 0.25$			$\rho = 0.50$			$\rho = 0.75$			$\rho = 0.90$			$\rho = 1.00$		
		ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL
	0.00	370.40	257	369.90	369.11	256	368.61	371.24	257	370.74	365.62	254	365.12	369.37	256	368.87	369.15	256	368.65
	0.10	352.93	245	352.43	349.32	242	348.82	349.84	243	349.34	343.41	238	342.91	335.38	233	334.88	322.07	223	321.57
	0.20	308.43	214	307.93	307.94	214	307.44	297.73	207	297.23	279.09	194	278.59	259.64	180	259.14	233.58	162	233.08
	0.30	253.14	176	252.64	248.44	172	247.94	237.54	165	237.04	211.75	147	211.25	183.29	127	182.79	155.70	108	155.20
	0.40	200.08	139	199.58	195.30	136	194.80	182.48	127	181.98	156.13	108	155.63	127.46	89	126.96	101.62	71	101.12
3	0.80	71.55	50	71.05	68.42	48	67.92	59.55	41	59.05	44.02	31	43.52	31.23	22	30.73	21.34	15	20.83
	1.20	27.82	19	27.32	26.29	18	25.79	22.04	15	21.53	15.07	11	14.56	9.92	7	9.41	6.44	5	5.92
	1.60	12.83	9	12.32	11.67	8	11.16	9.55	7	9.04	6.34	5	5.82	4.15	3	3.62	2.76	2	2.20
	2.00	6.30	5	5.78	5.92	4	5.40	4.84	3	4.31	3.26	2	2.71	2.23	2	1.66	1.61	1	0.99
	2.40	3.65	3	3.11	3.43	3	2.89	2.85	2	2.30	2.01	2	1.42	1.49	1	0.85	1.20	1	0.49
	3.20	1.73	1	1.12	1.65	1	1.04	1.45	1	0.81	1.19	1	0.48	1.06	1	0.25	1.01	1	0.10
	0.00	370.40	257	369.90	369.89	257	369.39	370.89	257	370.39	371.00	257	370.50	368.05	255	367.55	371.43	258	370.93
	0.10	352.93	245	352.43	355.25	246	354.75	345.83	240	345.33	341.05	237	340.55	333.70	231	333.20	310.75	216	310.25
	0.20	308.43	214	307.93	310.98	216	310.48	296.94	206	296.44	274.31	190	273.81	244.20	169	243.70	208.73	145	208.23
	0.30	253.14	176	252.64	246.69	171	246.19	237.34	165	236.84	206.25	143	205.75	167.96	117	167.46	127.24	88	126.74
	0.40	200.08	139	199.58	193.12	134	192.62	178.81	124	178.31	147.41	102	146.91	112.07	78	111.57	76.67	53	76.17
4	0.80	71.55	50	71.05	68.61	48	68.11	58.03	40	57.53	39.93	28	39.43	24.92	17	24.41	13.91	10	13.40
	1.20	27.82	19	27.32	26.15	18	25.65	21.15	15	20.64	13.28	9	12.77	7.66	5	7.14	4.10	3	3.57
	1.60	12.83	9	12.32	11.53	8	11.02	9.16	6	8.65	5.57	4	5.05	3.23	2	2.68	1.89	1	1.30
	2.00	6.30	5	5.78	5.86	4	5.34	4.64	3	4.11	2.89	2	2.34	1.82	1	1.22	1.25	1	0.56
	2.40	3.65	3	3.11	3.41	2	2.87	2.74	2	2.18	1.82	1	1.22	1.29	1	0.61	1.06	1	0.25
	3.20	1.73	1	1.12	1.64	1	1.02	1.42	1	0.77	1.14	1	0.40	1.02	1	0.14	1.00	1	0.00
	0.00	370.40	257	369.90	367.97	255	367.47	368.85	256	368.35	369.72	256	369.22	369.33	256	368.83	372.04	258	371.54
	0.10	352.93	245	352.43	354.70	246	354.20	345.42	240	344.92	341.92	237	341.42	328.10	228	327.60	296.14	205	295.64
	0.20	308.43	214	307.93	308.61	214	308.11	298.38	207	297.88	274.64	191	274.14	235.35	163	234.85	182.99	127	182.49
	0.30	253.14	176	252.64	249.19	173	248.69	234.05	162	233.55	197.91	137	197.41	155.22	108	154.72	104.91	73	104.41
	0.40	200.08	139	199.58	193.77	134	193.27	177.52	123	177.02	141.57	98	141.07	100.78	70	100.28	60.11	42	59.61
5	0.80	71.55	50	71.05	67.86	47	67.36	56.81	40	56.31	37.11	26	36.61	21.01	15	20.50	9.65	7	9.14
	1.20	27.82	19	27.32	26.02	18	25.52	20.68	14	20.17	12.20	9	11.69	6.34	5	5.82	2.88	2	2.33
	1.60	12.83	9	12.32	11.48	8	10.97	8.88	6	8.37	5.10	4	4.57	2.72	2	2.16	1.46	1	0.82
	2.00	6.30	5	5.78	5.83	4	5.31	4.52	3	3.99	2.67	2	2.11	1.59	1	0.97	1.10	1	0.33
	2.40	3.65	3	3.11	3.39	2	2.85	2.67	2	2.11	1.71	1	1.10	1.19	1	0.48	1.01	1	0.10
	3.20	1.73	1	1.12	1.64	1	1.02	1.40	1	0.75	1.11	1	0.35	1.01	1	0.10	1.00	1	0.00

Table 3: ARL, MRL and SDRL for control charts constructed by SRS and designs based on RSS under perfect ranking and Non-normal data

Distribution	k	α	SRS			RSS			MRSS			ERSS			NRSS		
			ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL	ARL	MRL	SDRL
SN	3	1	351.19	244	350.69	325.09	225	324.59	351.62	244	351.12	325.09	225	324.59	361.73	251	361.23
		2	281.61	195	281.11	285.26	198	284.76	346.92	241	346.42	285.26	198	284.76	348.44	242	347.94
		3	237.08	164	236.58	254.53	177	254.03	352.77	245	352.27	254.53	177	254.03	336.65	234	336.15
		5	203.99	142	203.49	232.87	162	232.37	377.92	262	377.42	232.87	162	232.37	330.02	229	329.52
	4	1	352.94	245	352.44	335.09	232	334.59	356.46	247	355.96	321.09	223	320.59	366.05	254	365.55
		2	298.32	207	297.82	300.11	208	299.61	352.44	244	351.94	237.39	165	236.89	364.65	253	364.15
		3	259.91	180	259.41	277.96	193	277.46	380.36	264	379.86	186.96	130	186.46	378.98	263	378.48
		5	227.08	158	226.58	258.42	179	257.92	423.91	294	423.41	149.55	104	149.05	391.14	271	390.64
	5	1	357.67	248	357.17	339.87	236	339.37	357.17	248	356.67	320.18	222	319.68	368.16	255	367.66
		2	310.86	216	310.36	306.96	213	306.46	340.49	236	339.99	241.84	168	241.34	364.61	253	364.11
		3	274.15	190	273.65	292.93	203	292.43	325.44	226	324.94	191.77	133	191.27	371.25	257	370.75
		5	246.36	171	245.86	274.54	190	274.04	314.66	218	314.16	152.81	106	152.31	383.72	266	383.22
GN	3	1	126.21	88	125.71	118.37	82	117.87	150.16	104	149.66	118.37	82	117.87	189.11	131	188.61
		1.5	237.37	165	236.87	232.05	161	231.55	254.48	177	253.98	232.05	161	231.55	297.96	207	297.46
		3	730.63	507	730.13	466.32	323	465.82	558.22	387	557.72	466.32	323	465.82	429.00	298	428.50
		4	1207.28	837	1206.78	520.61	361	520.11	743.17	515	742.67	520.61	361	520.11	457.01	317	456.51
	4	1	147.39	102	146.89	133.05	92	132.55	176.31	122	175.81	143.60	100	143.10	247.53	172	247.03
		1.5	257.63	179	257.13	251.53	175	251.03	280.31	194	279.81	251.46	174	250.96	328.49	228	327.99
		3	582.92	404	582.42	419.56	291	419.06	465.33	323	464.83	385.33	267	384.83	392.97	273	392.47
		4	798.23	553	797.73	437.79	304	437.29	527.04	365	526.54	371.91	258	371.41	396.34	275	395.84
	5	1	163.59	114	163.09	146.81	102	146.31	211.19	147	210.69	155.41	108	154.91	275.21	191	274.71
		1.5	273.91	190	273.41	266.41	185	265.91	298.77	207	298.27	262.38	182	261.88	345.10	239	344.60
		3	521.44	362	520.94	399.11	277	398.61	441.48	306	440.98	375.56	260	375.06	385.42	267	384.92
		4	635.37	441	634.87	413.40	287	412.90	489.72	340	489.22	377.68	262	377.18	393.67	273	393.17