logistic growth, α_1 growth constant

1)

$$\frac{dx}{dt} = \alpha_1 x\,(1 - x/K) - \beta xy$$ coupling terms accounting for interactions between species B interaction constant

$$\frac{dy}{dt} = \alpha_2 y + 3\beta xy$$

- exponentional growth

x = prey (squirrels)

y = predator (Hawks)

α_1 = growth constant for prey

α_2 = growth constant for predator

K = carrying capacity for prey

B = coupling constant

2. $J = \begin{bmatrix} \frac{df}{dx} & \frac{df}{dy} \\ \frac{dg}{dx} & \frac{dg}{dy} \end{bmatrix}$

Where

$f(x,y) = \alpha_1 x \left(1 - \frac{x}{k}\right) - \beta xy \Rightarrow \alpha_1 x - \alpha_1 \frac{x^2}{k} - \beta xy$

$g(x,y) = \alpha_2 y + 3\beta xy$

$\frac{df}{dx} = \alpha_1 - 2\alpha_1 \frac{x}{k} - \beta y$

$\frac{df}{dy} = -\beta x$

$\frac{dg}{dx} = 3\beta y$

$\frac{dg}{dx} = \alpha_2 + 3\beta x$

So, $J = \begin{bmatrix} \alpha_1 - 2\alpha_1 \frac{x}{k} - \beta y & -\beta x \\ 3\beta y & \alpha_2 + 3\beta x \end{bmatrix}$

3. Finding equilibrim:

$$\frac{dx}{dt} = 0 \qquad \frac{dy}{dt} = 0$$

$$\alpha_1 x\left(1 - \frac{x}{k}\right) - Bxy = 0 \qquad \alpha_2 y + 3Bxy = 0$$

$$x\left(\alpha_1\left(1 - \frac{x}{k}\right) - By\right) = 0 \qquad y\left(\alpha_2 + 3Bx\right) = 0$$

one equilibrium: $(0,0)$

$$\alpha_1\left(1 - \frac{x}{k}\right) - By = 0 \qquad \alpha_2 + 3Bx = 0$$

$$\frac{\alpha_1\left(1 - \frac{x}{k}\right)}{k} = y \qquad x = \frac{-\alpha_2}{3B}$$

$$y = \frac{\alpha_1}{B}\left(1 - \frac{(-\alpha_2/3B)}{k}\right)$$

$$= \frac{\alpha_1}{B}\left(1 + \frac{\alpha_2}{3Bk}\right)$$

others: $\left(\frac{-\alpha_2}{3B}, \frac{\alpha_1}{B}\left(1 + \frac{\alpha_2}{3Bk}\right)\right)$