

1. 用直角坐标形式 ($x+jy$) 表示下列复数: 1) $\frac{1}{2}e^{j\pi}$; 2) $\sqrt{5}e^{j\theta\pi/6}$

1). $r = \frac{1}{2}, \theta = \pi$

$$x = r \cdot \cos \theta = \frac{1}{2} \cos \pi = -\frac{1}{2}$$

$$y = r \cdot \sin \theta = \frac{1}{2} \sin \pi = 0$$

$$\Rightarrow \frac{1}{2}e^{j\pi} \rightarrow -\frac{1}{2}$$

2). $r = \sqrt{5}, \theta' = \frac{\theta\pi}{6}$

$$x = r \cdot \cos \theta' = \sqrt{5} \cos \frac{\theta\pi}{6}$$

$$y = r \cdot \sin \theta' = \sqrt{5} \sin \frac{\theta\pi}{6}$$

$$\Rightarrow \sqrt{5}e^{j\theta\pi/6} = \sqrt{5} \cos \frac{\theta\pi}{6} + j\sqrt{5} \sin \frac{\theta\pi}{6}$$

2. 用极坐标形式 ($re^{j\theta}$) 表示下列复数: 1) $(1+j)^2$; 2) $(\sqrt{2}+j\sqrt{2})/(1+j\sqrt{3})$

1). $(1+j)^2 = 1 + 2j + j^2 = 2j$

$$r = \sqrt{0^2 + 2^2} = 2$$

$$\theta = \tan^{-1}(\frac{2}{0}) = \frac{\pi}{2}$$

$$(1+j)^2 = 2 \cdot e^{j\frac{\pi}{2}}$$

2). $\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = \frac{(\sqrt{2}+j\sqrt{2})(1-j\sqrt{3})}{(1+j\sqrt{3})(1-j\sqrt{3})} = \frac{\sqrt{2}+j\sqrt{2}-j\sqrt{6}+\sqrt{6}}{4}$

$$= \frac{\sqrt{2}+\sqrt{6}}{4} + j \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$r = \sqrt{(\frac{\sqrt{2}+\sqrt{6}}{4})^2 + (\frac{\sqrt{2}-\sqrt{6}}{4})^2} = 1$$

$$\theta = \tan^{-1}(\frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}+\sqrt{6}}) = -\frac{\pi}{12}$$

$$\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = e^{-j\frac{\pi}{12}}$$

3. 有以下函数表示的两个系统:

A) $y(t) = \cos^2(2t)x(t)$

B) $y[n] = x[n-2] - 2x[n-6]$

分别判断以上两个系统是否具有以下性质: 1) 无记忆; 2) 时不变; 3) 线性; 4) 因果; 5) 稳定。并解释原因。

A). 1). 无记忆

2). $x(t-t_0) \rightarrow \boxed{\quad} \rightarrow \cos^2(2t) x(t-t_0)$

非时不变 (时变)

$$y(t-t_0) = \cos^2(2t-2t_0) x(t-t_0)$$

3). 设 $x(t) = a x_1(t) + b x_2(t)$

$$y(t) = \cos^2(2t) [a x_1(t) + b x_2(t)] = a \cos^2(2t) x_1(t) + b \cos^2(2t) x_2(t) \\ = a y_1(t) + b y_2(t) \quad \text{线性}$$

4). 因果 5). 稳定, $\cos^2(2t) \leq 1$, 当 $x(t)$ 有界时, $y(t)$ 一定有界。

B). 1). 有记忆

2). $x[n-n_0] \rightarrow \boxed{\quad} \rightarrow x[n-n_0-2] - 2x[n-n_0-6]$

时不变

$$y[n-n_0] = x[n-n_0-2] - 2x[n-n_0-6]$$

3). 设 $x[n] = a x_1[n] + b x_2[n]$

$$y[n] = a x_1[n-2] + b x_2[n-2] - 2a x_1[n-6] - 2b x_2[n-6]$$

$$= a \{ x_1[n-2] - 2x_1[n-6] \} + b \{ x_2[n-2] - 2x_2[n-6] \} = a y_1[n] + b y_2[n] \\ \text{线性 因果}$$

4). 因果 5). 稳定