Lecture 1 Introduction

School of Computer Engineering and Science
Shanghai University

Instructor: Jianjia Wang, Shengyu Duan

Outline: Lecture 1: Introduction

- Preliminary

  What are the signals

  What are the systems

  Application of signals and systems

- Fundamental concepts

- Types of Signals

Outline: Lecture 1: Introduction

- Preliminary

  What are the signals

  What are the systems

  Application of signals and systems

- Fundamental concepts

- Types of Signals

信号是什么？

烽火狼烟

电话

无线电

物联网

What is a Signal?

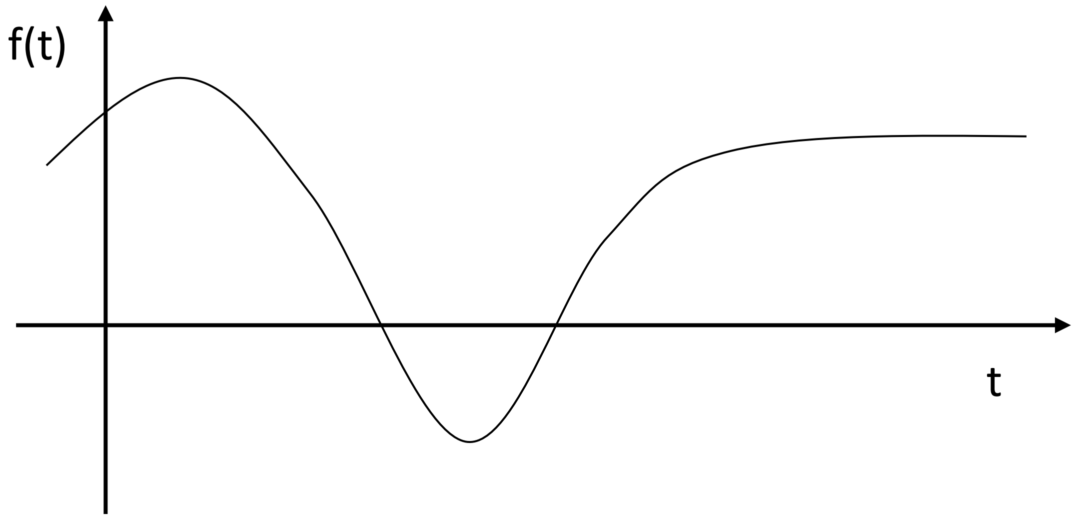
- A signal is a pattern of variation of some form
- Signals are variables that carry information

- Any kind of physical variable subject to variations represents a signal

- Examples of signal include:
  - Electrical signals
    - –Voltages and currents in a circuit
  - Acoustic signals
    - –Acoustic pressure (sound) over time
  - Mechanical signals
    - –Velocity of a car over time
  - Video signals
    - –Intensity level of a pixel (camera, video) over time

How is a Signal Represented?

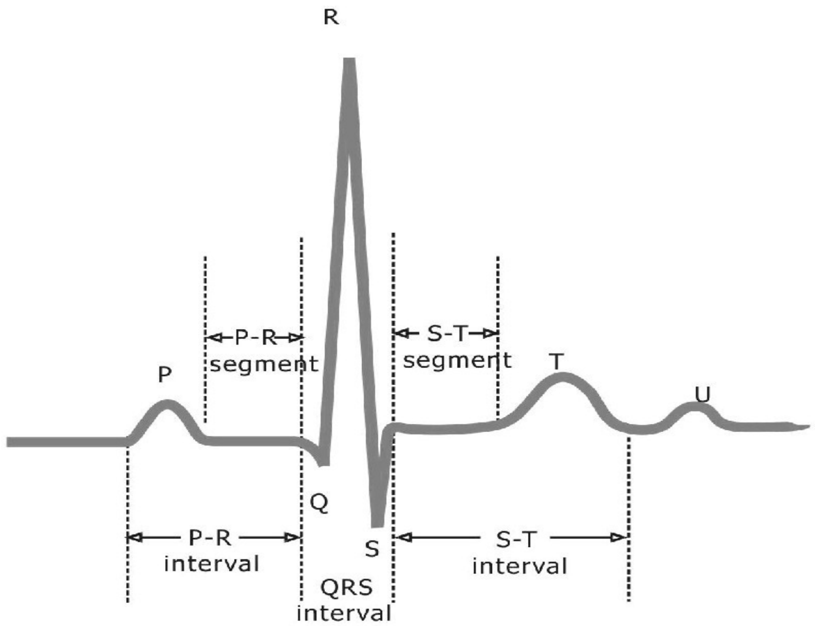
- Mathematically, signals are represented as a function of one or more **independent variables**.
- For instance a black & white video signal intensity is dependent on x, y coordinates and time t $f(x,y,t)$
- On this course, we shall be exclusively concerned with signals that are a function of a single variable: time

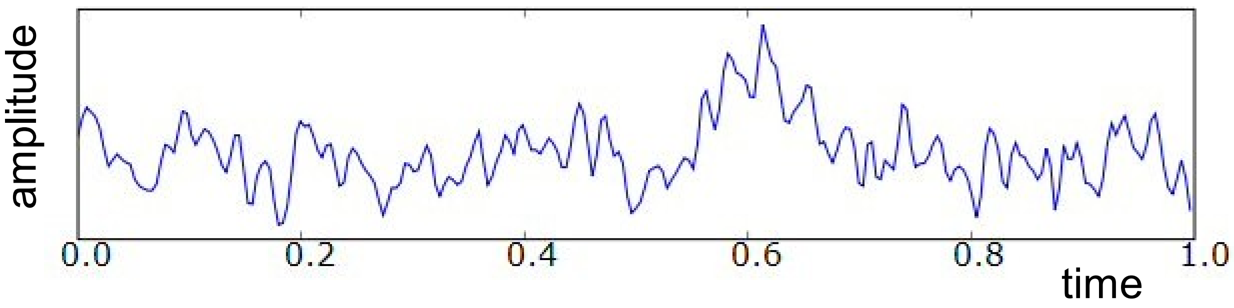


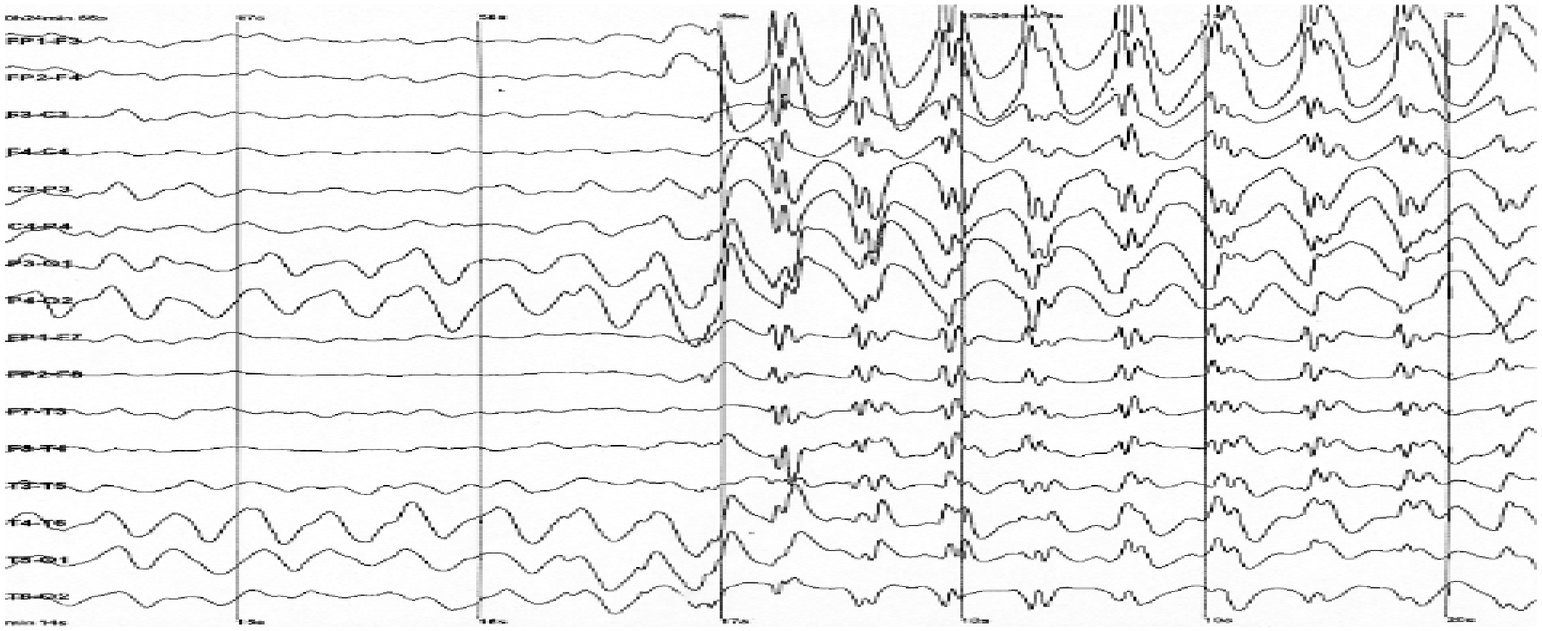
How is a Signal Represented?

- A signal is a set of information of data
  - Both the independent variable and the physical variable can be either scalars or vectors
    - Independent variable: time (t), space (x, $\mathbf{x}=[x_1,x_2]$, $\mathbf{x}=[x_1,x_2,x_3]$)
    - Signal:
    - Electrochardiography signal (EEG) 1D, voice 1D, music 1D
    - Images (2D), video sequences (2D+time), volumetric data (3D)

Example: 1D biological signals: ECG

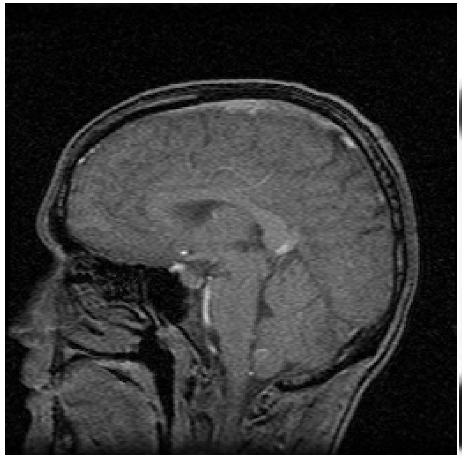




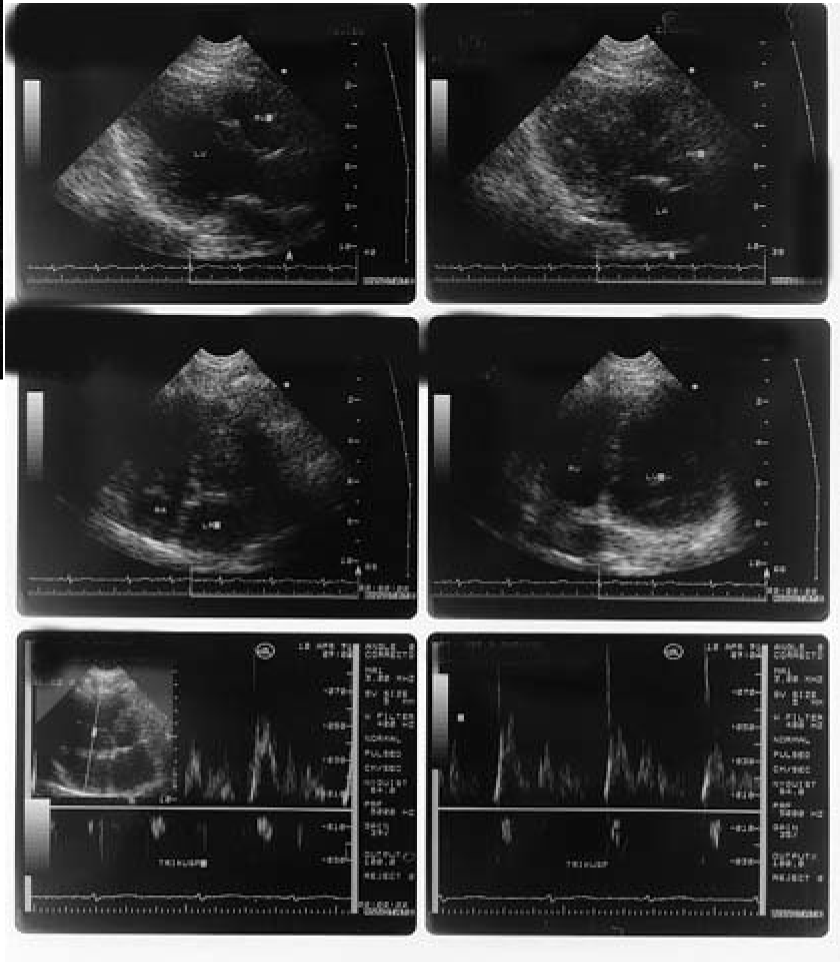


Example: 2D biological signals: MI

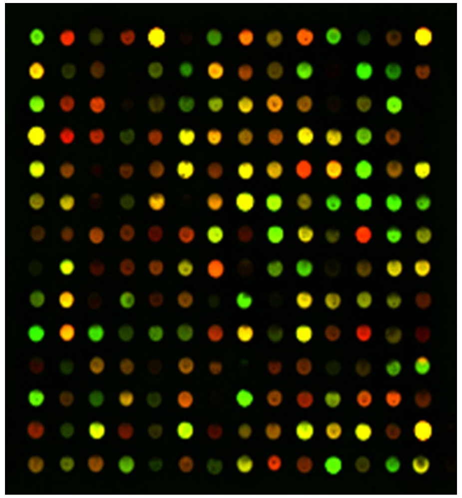
MRI

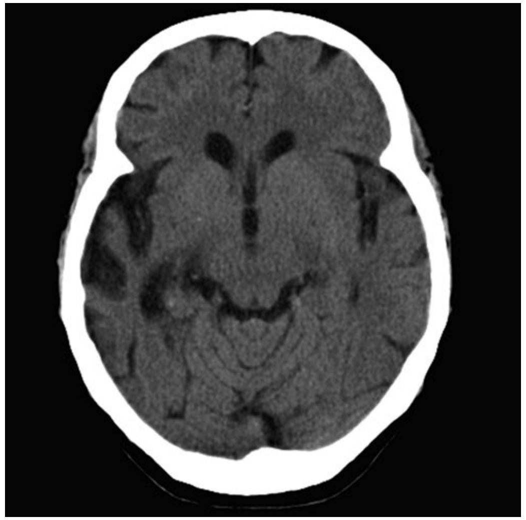


US

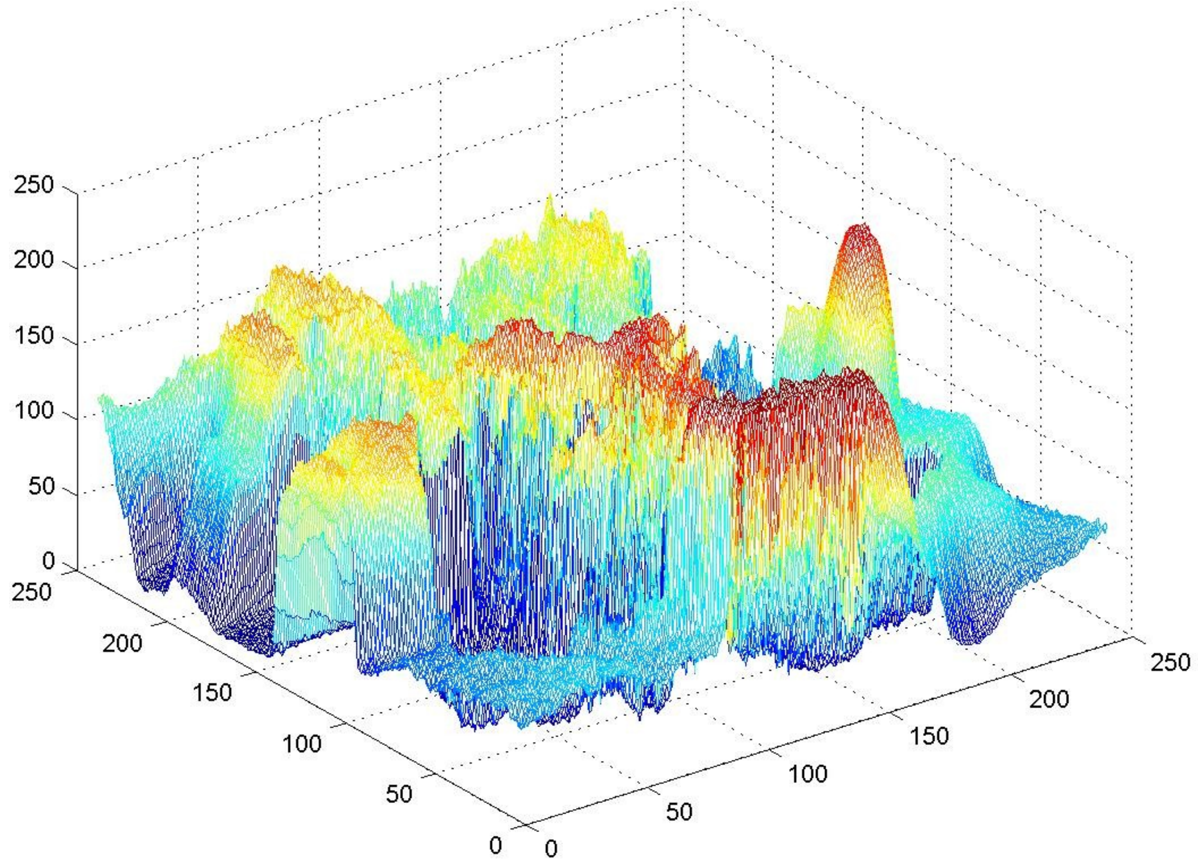


CT





Example : Natural image



Signals as functions

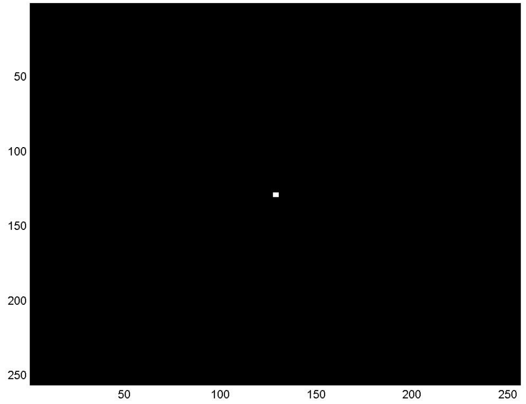
- Continuous functions of real independent variables
  - 1D: $f=f(x)$
  - 2D: $f=f(x,y)$ x,y
  - Real world signals (audio, ECG, images)
- Real valued functions of discrete variables
  - 1D: $f=f[k]$
  - 2D: $f=f[i,j]$
  - *Sampled* signals
- Discrete functions of discrete variables
  - 1D: $f^d=f^d[k]$
  - 2D: $f^d=f^d[i,j]$
  - *Sampled and quantized* signals

- Gray scale images: 2D functions
  - Domain of the functions: set of (x,y) values for which $f(x,y)$ is defined : 2D lattice $[i,j]$ defining the pixel locations
  - Set of values taken by the function : gray levels

- Digital images can be seen as functions defined over a discrete domain $\{i,j: 0<i<I, 0<j<J\}$
  - I,J: number of rows (columns) of the matrix corresponding to the image
  - $f=f[i,j]$: gray level in position $[i,j]$

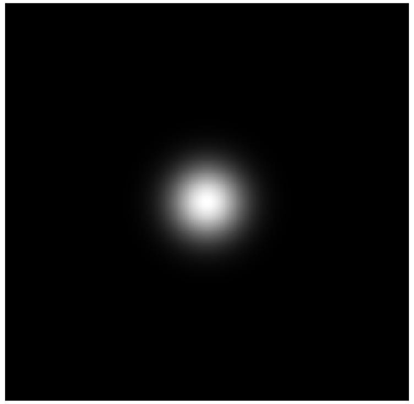
Example : function

$$\delta[i,j]=\begin{cases}1 & i=j=0\\ 0 & i,j\neq 0; i\neq j\end{cases}$$

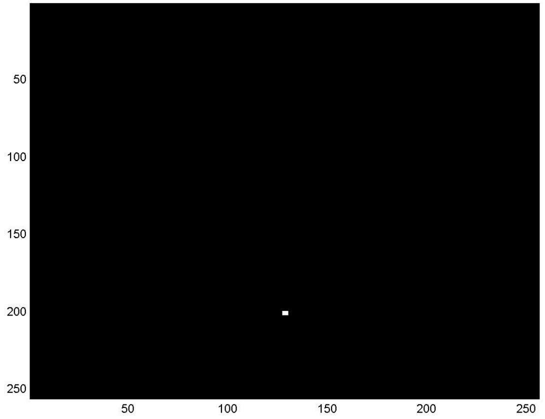


Continuous function

$$f(x,y)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{x^2+y^2}{2\sigma^2}}$$

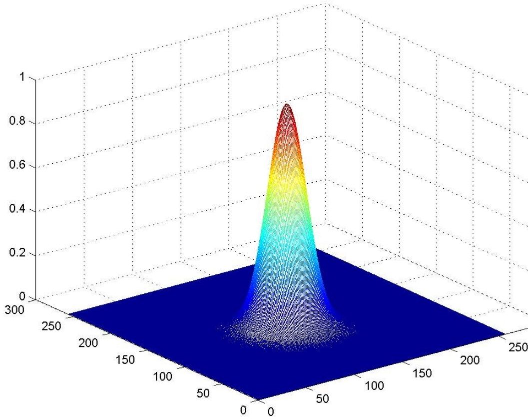


$$\delta[i,j-J]=\begin{cases}1 & i=0; j=J\\ 0 & otherwise\end{cases}$$



Discrete version

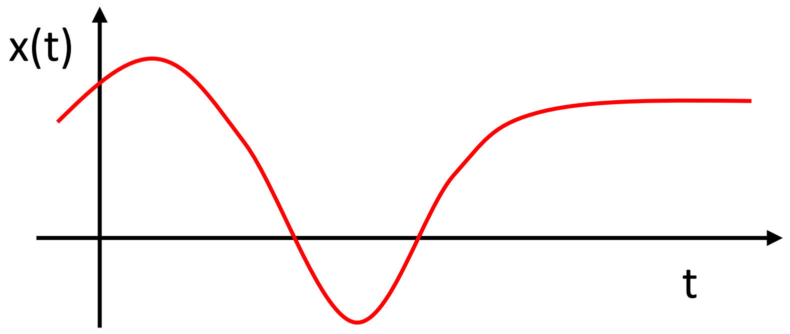
$$f[i,j]=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{i^2+j^2}{2\sigma^2}}$$

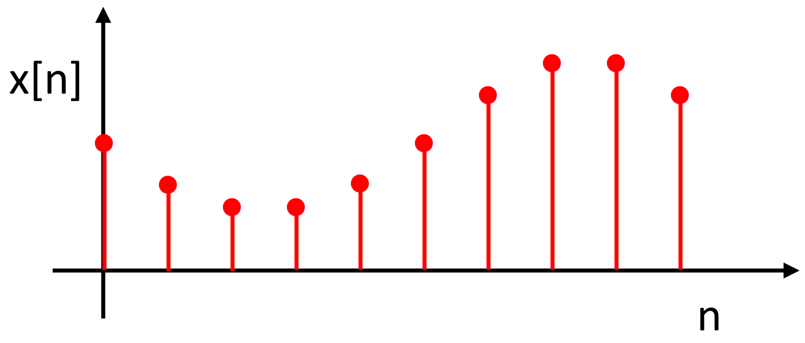


Continuous & Discrete-Time Signals

- **Continuous-Time Signals**
- Most signals in the real world are continuous time, as the scale is infinitesimally fine.
- E.g. voltage, velocity,
- Denote by $x(t)$, where the time interval may be bounded (finite) or infinite
- **Discrete-Time Signals**
- Some real world and many digital signals are discrete time, as they are sampled
- E.g. pixels, daily stock price (anything that a digital computer processes)
- Denote by x[n], where n is an integer value that varies discretely
- **Sampled continuous signal**

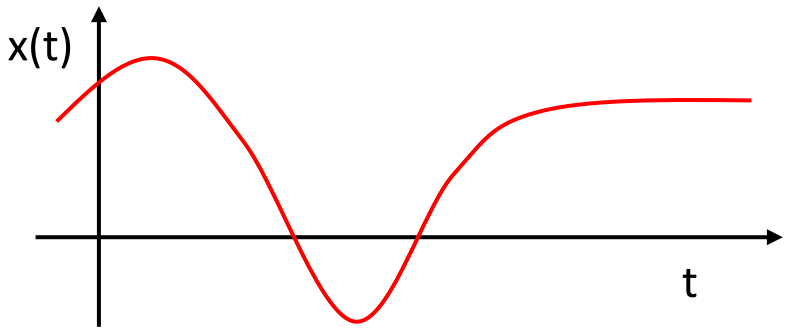
$x[n] = x(nk)$ – k is sample time



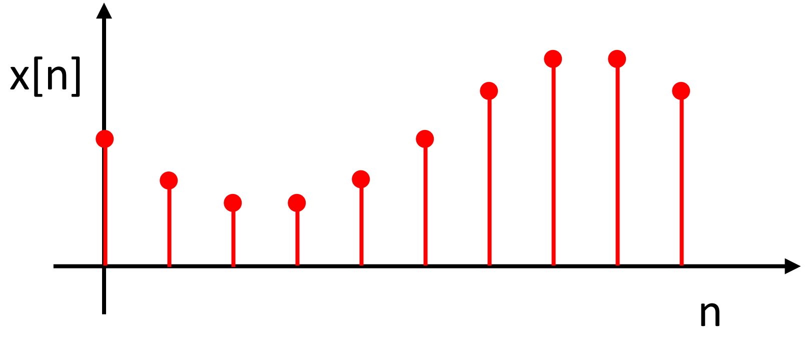


连续时间与离散时间信号

- 对连续时间信号$x(t)$， t可以是任意值（无限个数的值）；

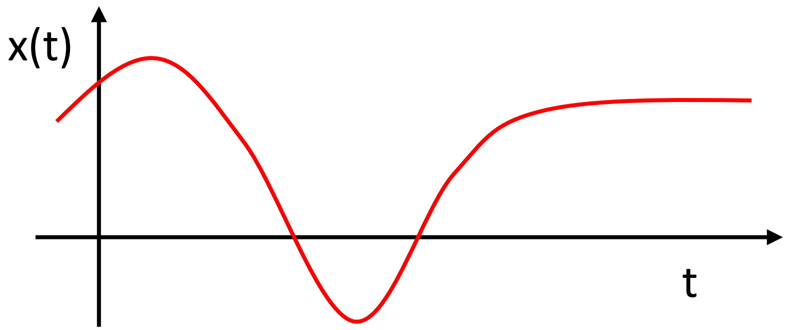


- 对离散时间信号$x[n]$， n为有限个数的值；

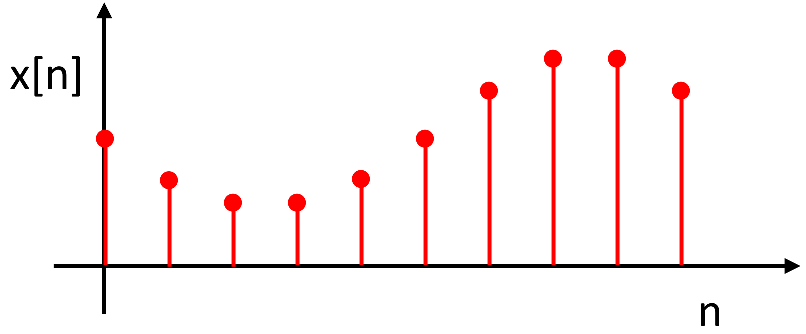


连续时间与离散时间信号

- 对连续时间信号$x(t)$， t可以是任意值（无限个数的值）；



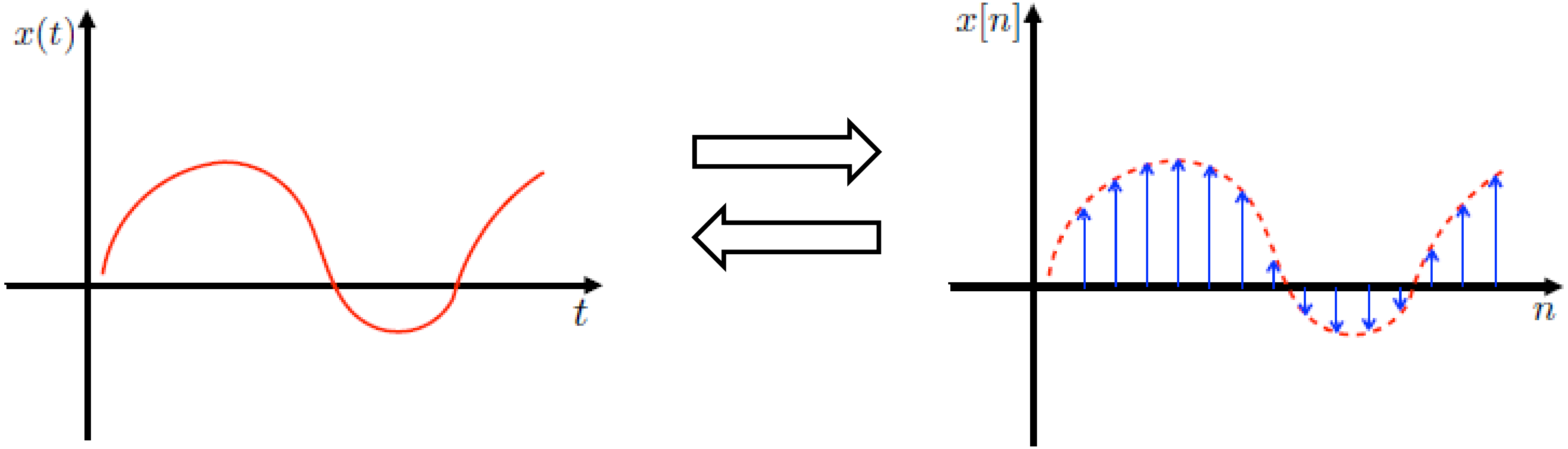
- 对离散时间信号$x[n]$， n为有限个数的值；



- 然而，连续与离散信号真的可以明确划分吗？

连续时间与离散时间信号

- 对以下连续时间信号$x(t)$，不断拉近
- 对以下离散时间信号$x[n]$，将n与n+1之间的间隔不断缩小

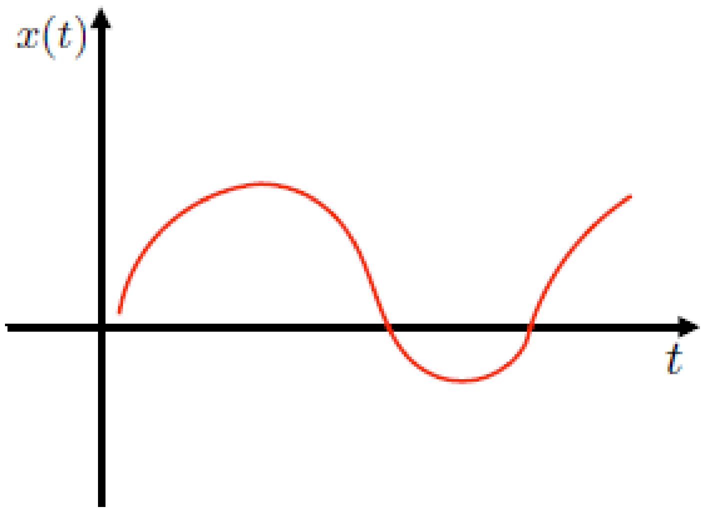


- “连续”与“离散”存在主观性：连续信号是间隔无限接近于0的离散信号。
- 上述概念帮助理解不同信号类型的傅里叶变化及采样（连续/离散时间信号无法明确划分，但有必要去划分）。

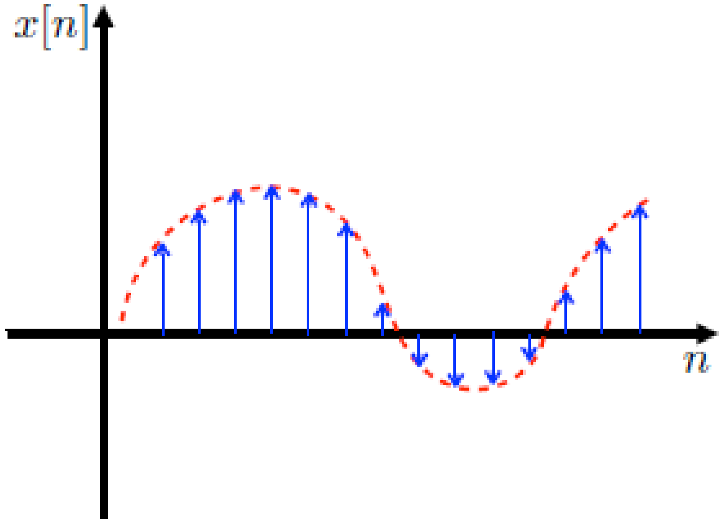
Continuous & Discrete-Time Signals

Time signals:

Continuous time signals (analog signals): for example, audio signals (FMsignals), AC voltages and currents



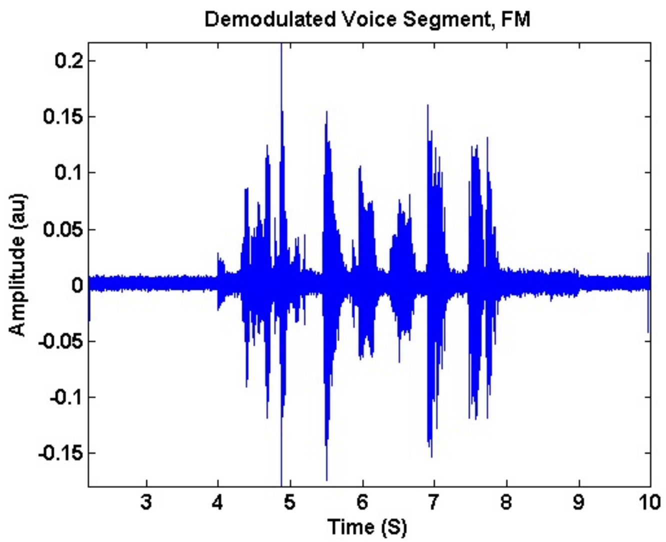
Discrete time signals (digital signals): for example, daily temperature



Note that continuous time signals and discrete time signals can be converted to each other.

Continuous & Discrete-Time Signals

1D signal (e.g., voice, music): $x(t) \text{->} x[n]$



2D signal (e.g., images): $x(s_1,s_2) \rightarrow x[n_1,n_2]$

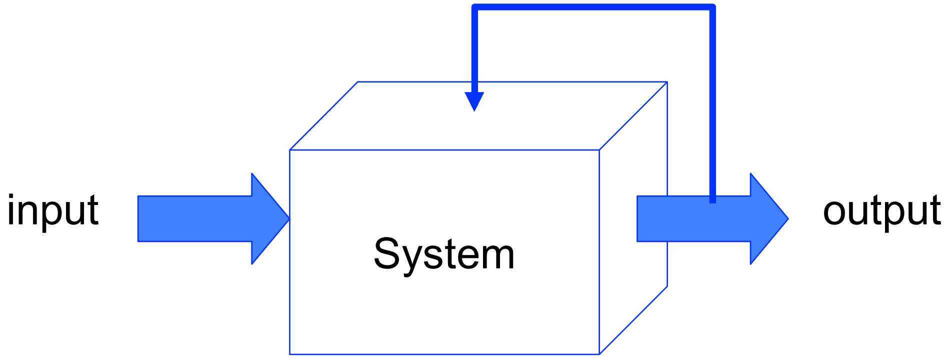
2D signal (e.g., videos: 2D spatial + time): $x(s_1,s_2,t) \rightarrow x[n_1,n_2,n]$

Signal Properties

- On this course, we shall be particularly interested in signals with certain properties:
- **Periodic signals**: a signal is periodic if it repeats itself after a fixed period T, i.e. $x(t) = x(t+T)$ for all t. A $\sin(t)$ signal is periodic.
- **Even and odd signals**: a signal is even if $x(-t) = x(t)$ (i.e. it can be reflected in the axis at zero). A signal is odd if $x(-t) = -x(t)$. Examples are $\cos(t)$ and $\sin(t)$ signals, respectively.
- **Exponential and sinusoidal signals**: a signal is (real) exponential if it can be represented as $x(t) = Ce^{at}$. A signal is (complex) exponential if it can be represented in the same form but C and a are complex numbers.
- **Step and pulse signals**: A pulse signal is one which is nearly completely zero, apart from a short spike, $d(t)$. A step signal is zero up to a certain time, and then a constant value after that time, $u(t)$.
- These properties define a large class of tractable, useful signals and will be further considered in the coming lectures

What is a System?

- **Systems process input signals to produce output signals**



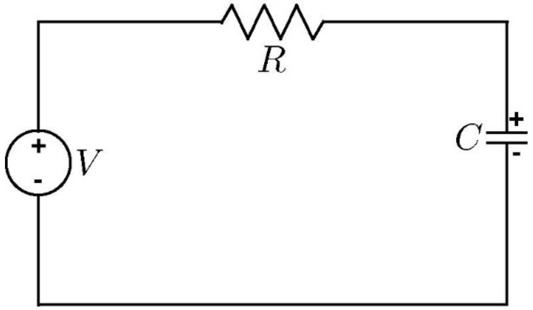
The function linking the output of the system with the input signal is called transfer function and it is typically indicated with the symbol *h(•)*

- **Examples:**

–A circuit involving a capacitor can be viewed as a system that transforms the source voltage (signal) to the voltage (signal) across the capacitor

–A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker

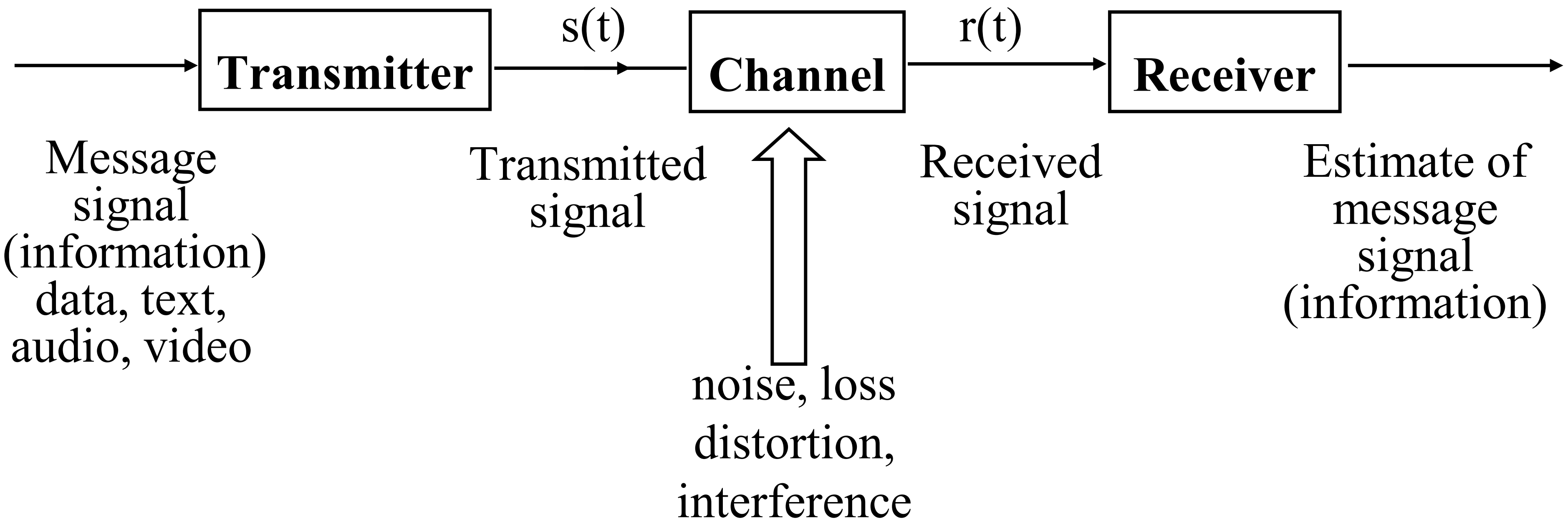
Circuit:



What is a System?

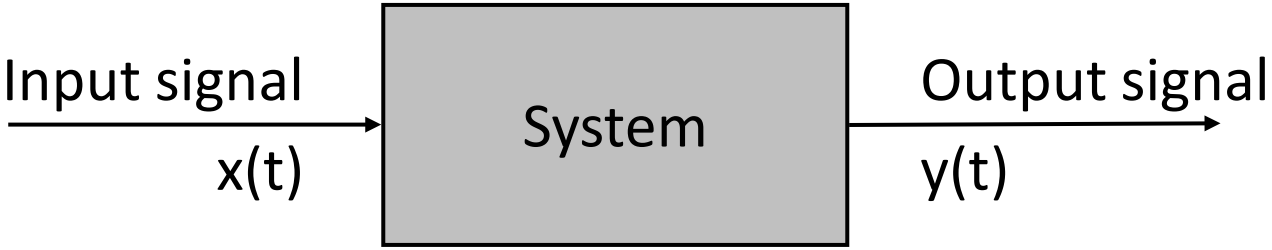
- **Communication Systems**

A communication system is generally composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver



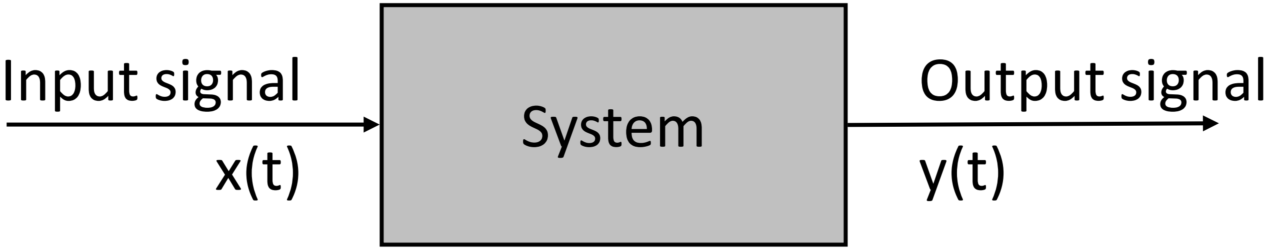
How is a system Represented?

- A system takes a signal as an input and transforms it into another signal

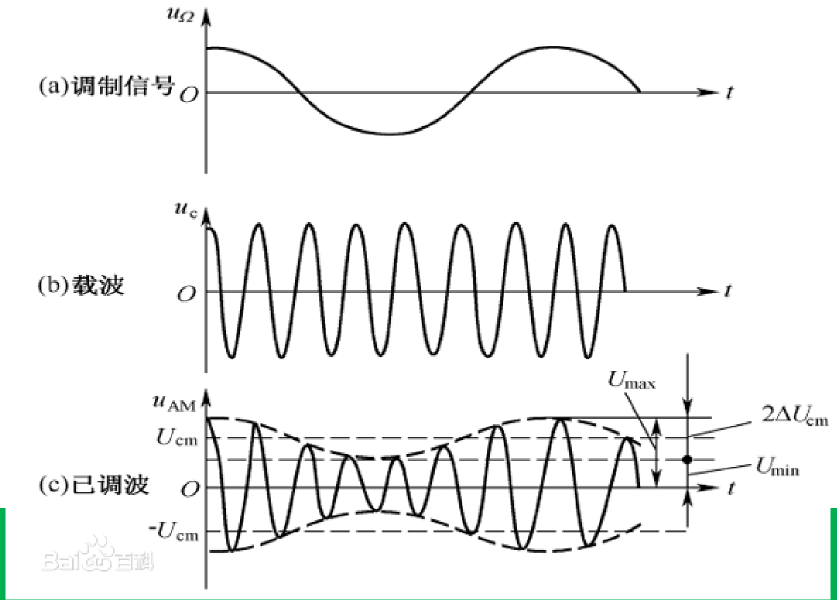


- In a very broad sense, a system can be represented as the ratio of the output signal over the input signal
  - That way, when we "multiply" the system by the input signal, we get the output signal
  - This concept will be firmed up in the coming weeks

系统



- 系统往往是以一定比例（做乘法）放大、缩小输入信号
- 例如：幅度调制（Amplitude Modulation，AM）



Continuous & Discrete-Time Mathematical Models of Systems

- **Continuous-Time Systems**
- Most continuous time systems represent how continuous signals are transformed via **differential equations**.
- E.g. circuit, car velocity

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

$$m\frac{dv(t)}{dt} + \rho v(t) = f(t)$$

First order differential equations

- **Discrete-Time Systems**
- Most discrete time systems represent how discrete signals are transformed via **difference equations**
- E.g. bank account, discrete car velocity system

$$y[n] = 1.01y[n-1] + x[n]$$

$$v[n] - \frac{m}{m+\rho\Delta} v[n-1] = \frac{\Delta}{m+\rho\Delta} f[n]$$

$$\frac{dv(n\Delta)}{dt} = \frac{v(n\Delta) - v((n-1)\Delta)}{\Delta}$$

First order difference equations

Properties of a System

- On this course, we shall be particularly interested in signals with certain properties:
- **Causal**: a system is causal if the output at a time, only depends on input values up to that time.
- **Linear**: a system is linear if the output of the scaled sum of two input signals is the equivalent scaled sum of outputs
- **Time-invariant**: a system is time invariant if the system's output is the same, given the same input signal, regardless of time.

- These properties define a large class of tractable, useful systems and will be further considered in the coming lectures

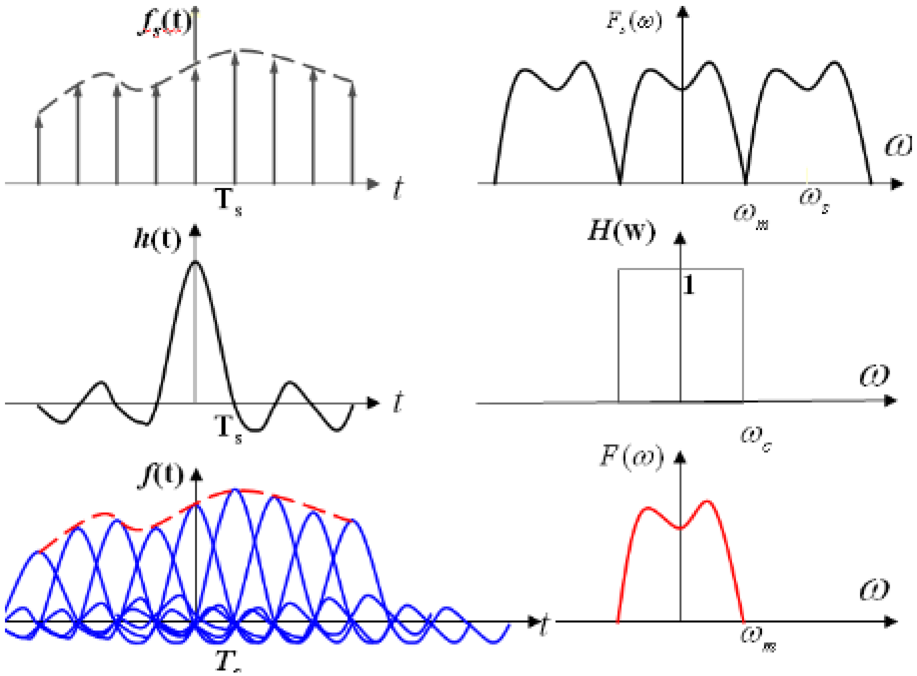
系统的因果性（Causality）

- **Causal**: a system is causal if the output at a time, only depends on input values up to that time. （系统的输出只取决于现在的输入及过去的输入）

- 具有因果性的系统才是物理可实现的系统

- 因果性的意义：判断所设计的系统是否是可以被实现的

- 例：理想低通滤波器不具有因果性，是不可能被实现的



线性（Linear）系统

- **Linear**: a system is linear if the output of the scaled sum of two input signals is the equivalent scaled sum of outputs （如果输入信号是两个信号的加权和，那么输出信号也是这两个输入信号对应输出的加权和）

- 例：如果系统对输入信号$x(t)$的输出是$y(t)$，则系统对输入信号$ax_1(t)+bx_2(t)$的输出为$ay_1(t)+by_2(t)$

- 线性系统对信号在频域表示下的处理具有重要意义。

时不变（Time-invariant）系统

- **Time-invariant**: a system is time invariant if the system's output is the same, given the same input signal, regardless of time. （系统特性不随时间发生变化）

- 例：如果系统对输入信号$x(t)$的输出是$y(t)$，则系统对输入信号$x(t-t_0)$的输出为$y(t-t_0)$

- 时不变系统对信号在频域表示下的处理具有重要意义。

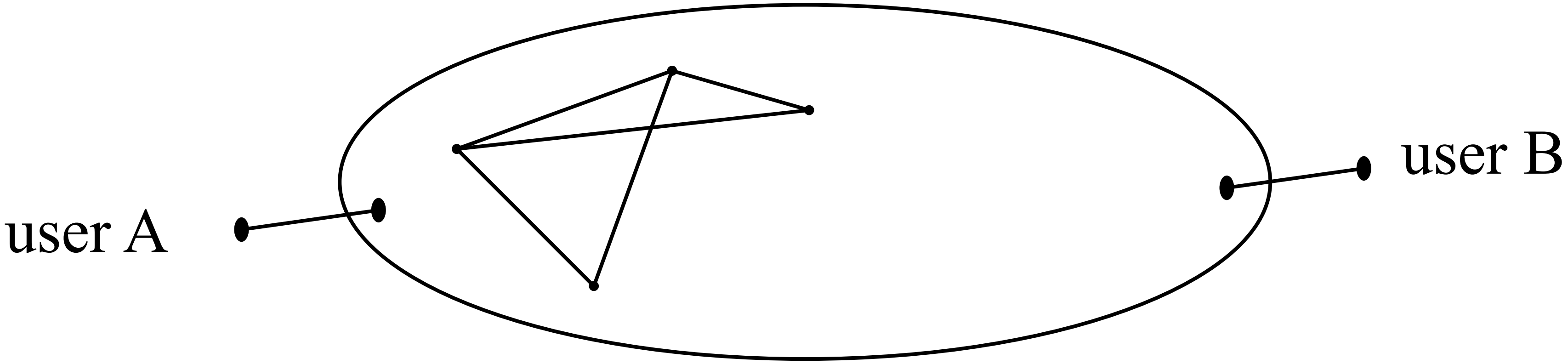
Typical Examples of Signals/Systems Concerned

- Computers

- Signal Processing Systems
  - software systems processing the signal by computation/ memory
  - examples : audio enhancement systems, picture processing systems, video compression systems, voice recognition/ synthesis systems, array signal processors, equalizers, etc.

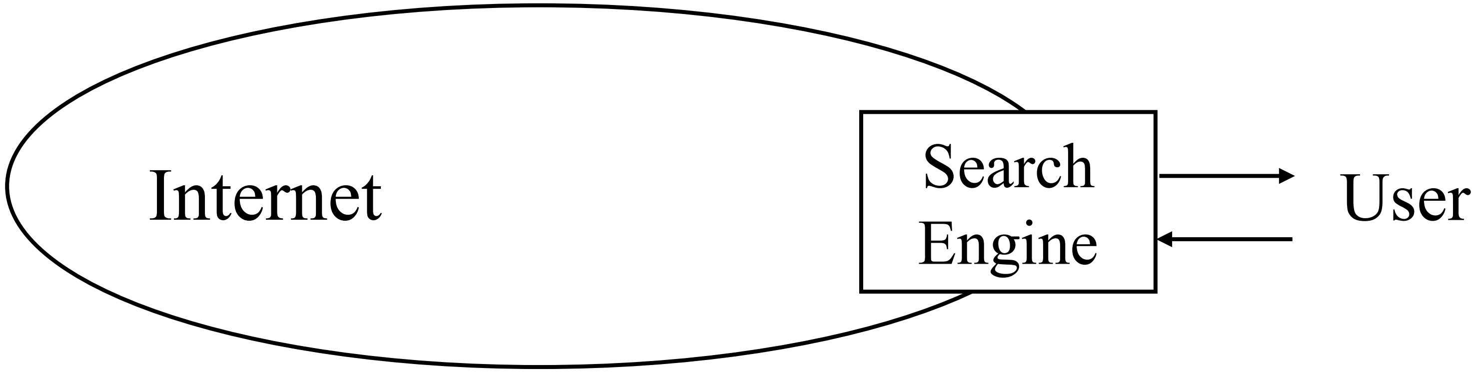
Typical Examples of Signals/Systems Concerned

- Networks



Typical Examples of Signals/Systems Concerned

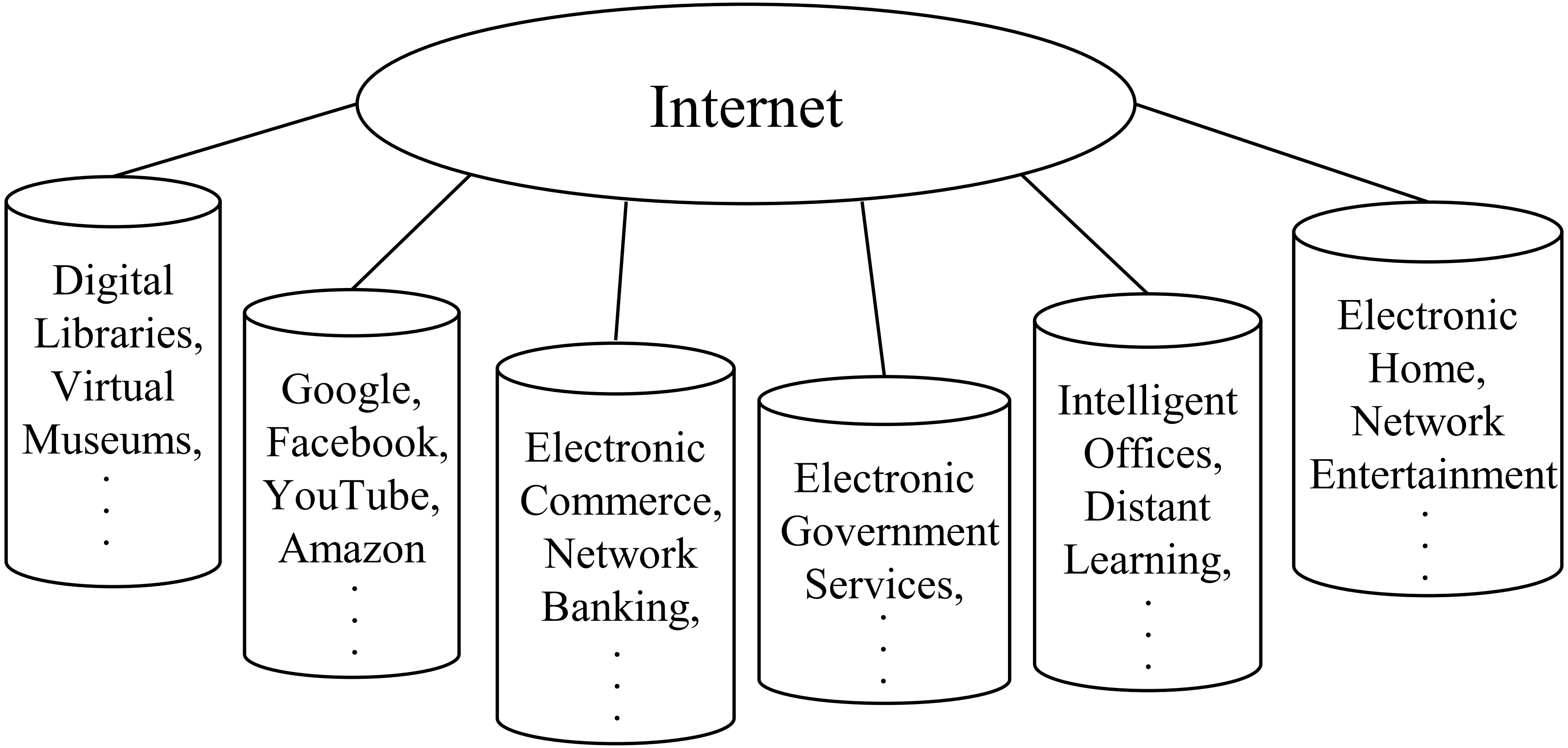
- Information Retrieval Systems



- Internet

- Other Information Systems
  - examples : remote sensing systems, biomedical signal processing systems, etc.

Internet



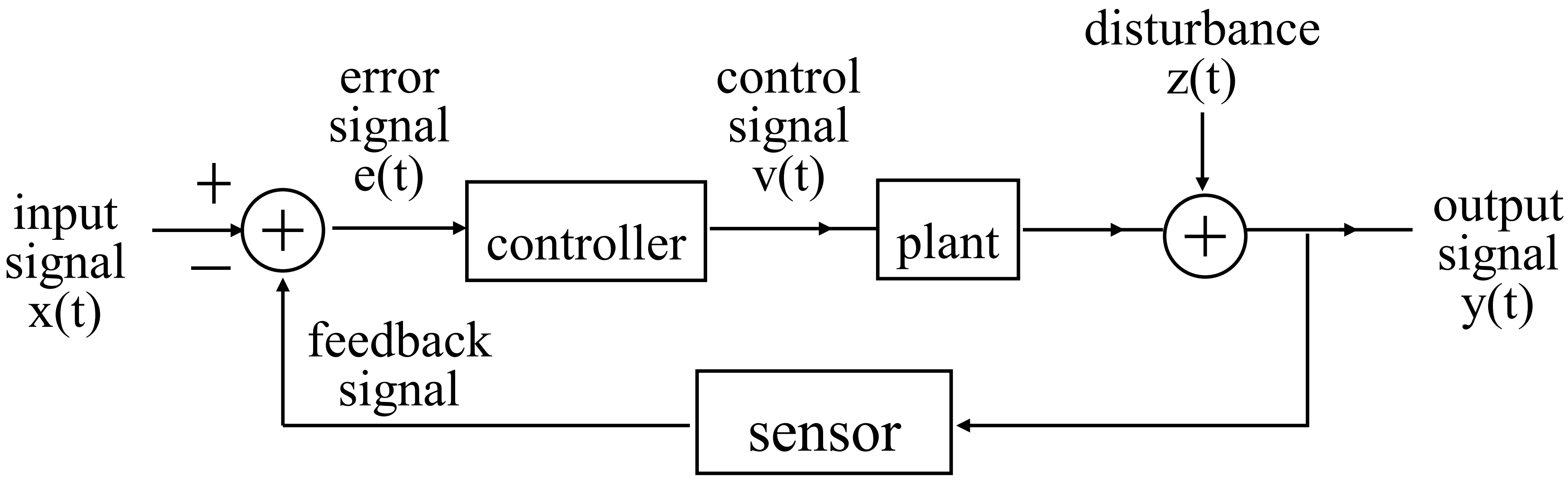
Internet
Digital Libraries, Virtual Museums,
. . .
Google, Facebook, YouTube, Amazon
. . .
Electronic Commerce, Network Banking,
. . .
Electronic Government Services,
. . .
Intelligent Offices, Distant Learning,
. . .
Electronic Home, Network Entertainment
. . .

Internet

- Network Technology Connects Everywhere Globally
- Huge Volume of Information Disseminated across the Globe in Microseconds
- Multi-media, Multi-lingual, Multi-functionality
- Cross-cultures, Cross-domains, Cross-regions
- Integrating All Knowledge Systems and Information related Activities Globally

Typical Examples of Signals/Systems Concerned

- Control Systems
  - close-loop/feedback control systems



  - example: aircraft landing systems, satellite stabilization systems, robot arm control systems, etc.

Typical Examples of Signals/Systems Concerned

- Other Systems
  - manufacturing systems, computer-aided-design systems, mechanical systems, chemical process systems, etc.

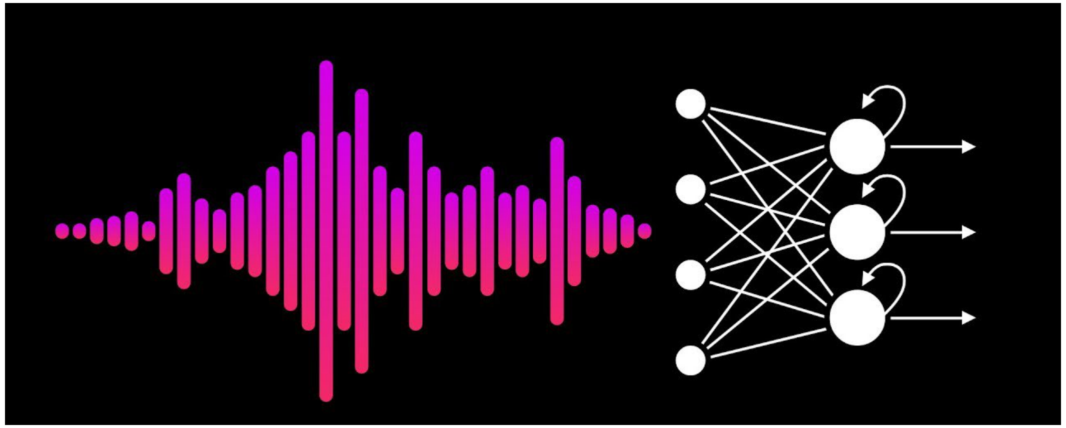
信号处理与人工智能

- 例：实现一个简单的声音识别系统，对发声者的年龄、性别进行判断

信号处理与人工智能

- 例：实现一个简单的声音识别系统，对发声者的年龄、性别进行判断

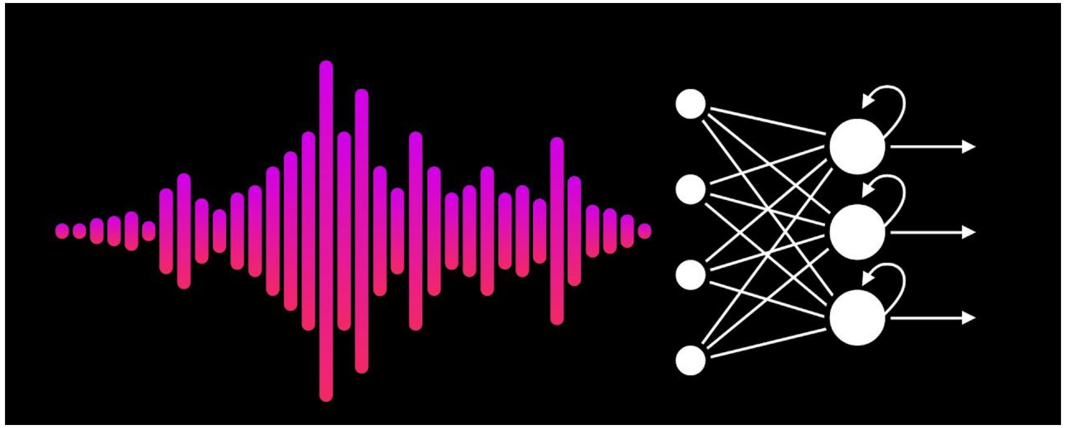
- 以人工神经网络构建分类器：
  - 每一层网络提取信号中的抽象特征



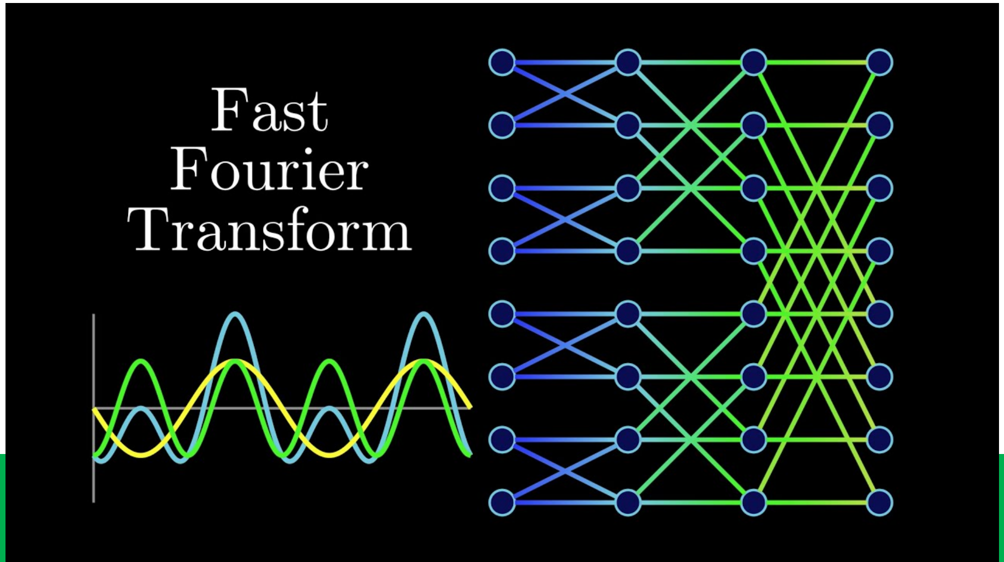
信号处理与人工智能

- 例：实现一个简单的声音识别系统，对发声者的年龄、性别进行判断

- 以人工神经网络构建分类器：
  - 每一层网络提取信号中的抽象特征



- 以传统信号处理技术构建系统：
  - 提取信号中频率特征

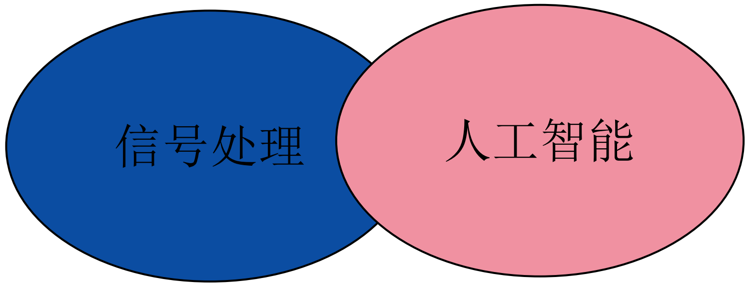


信号处理与人工智能

- 广义上来说，人工智能是抽象层面下的信号处理，以通过提取目前人类无法解释的特征来处理信号

- 信号处理可以是一种可解释的人工智能架构

- 信号处理与人工智能在未来的某一天是否会完全融合？



Outline: Lecture 1: Introduction

- Preliminary

  What are the signals

  What are the systems

  Application of signals and systems

- Fundamental concepts

- Types of Signals

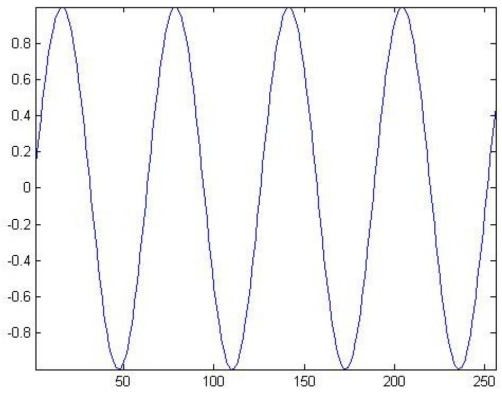
Classification of signals

- Continuous time – Discrete time
- Analog – Digital (numerical)
- Periodic – Aperiodic
- Energy – Power
- Deterministic – Random (probabilistic)
- Note
  - Such classes are not disjoint, so there are digital signals that are periodic of power type and others that are aperiodic of power type etc.
  - Any combination of single features from the different classes is possible

Continuous time – discrete time

- Continuous time signal: a signal that is specified for every real value of the independent variable
  - The independent variable is continuous, that is it takes any value on the real axis
  - E.g. The domain of the function representing the signal has the cardinality of real numbers
    - Signal ↔ f=f(t)
    - Independent variable ↔ time (t), position (x) $t \in \mathbb{R}$
    - For continuous-time signals:

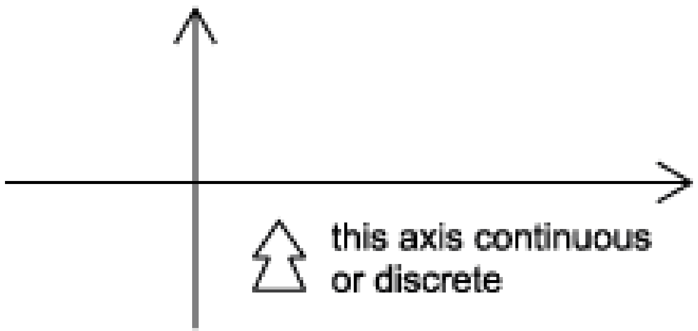
amplitude



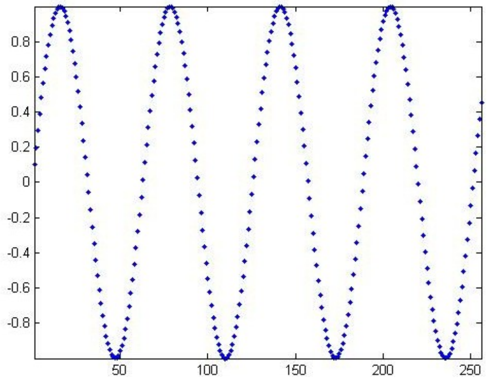
time

Continuous time – discrete time

- Discrete time signal: a signal that is specified only for *discrete values* of the independent variable
  - It is usually generated by *sampling* so it will only have values at *equally spaced* intervals along the time axis
  - E.g. The domain of the function representing the signal has the cardinality of integer numbers
    - Signal ↔ f=f[n], also called "sequence"
    - Independent variable ↔ n
    - For discrete-time functions: $t \in \mathbf{Z}$



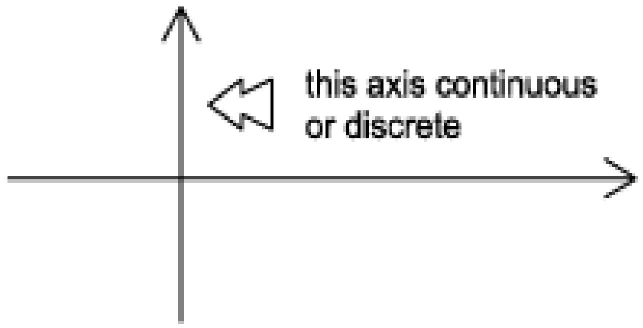
amplitude



time (discrete)

Analog - Digital

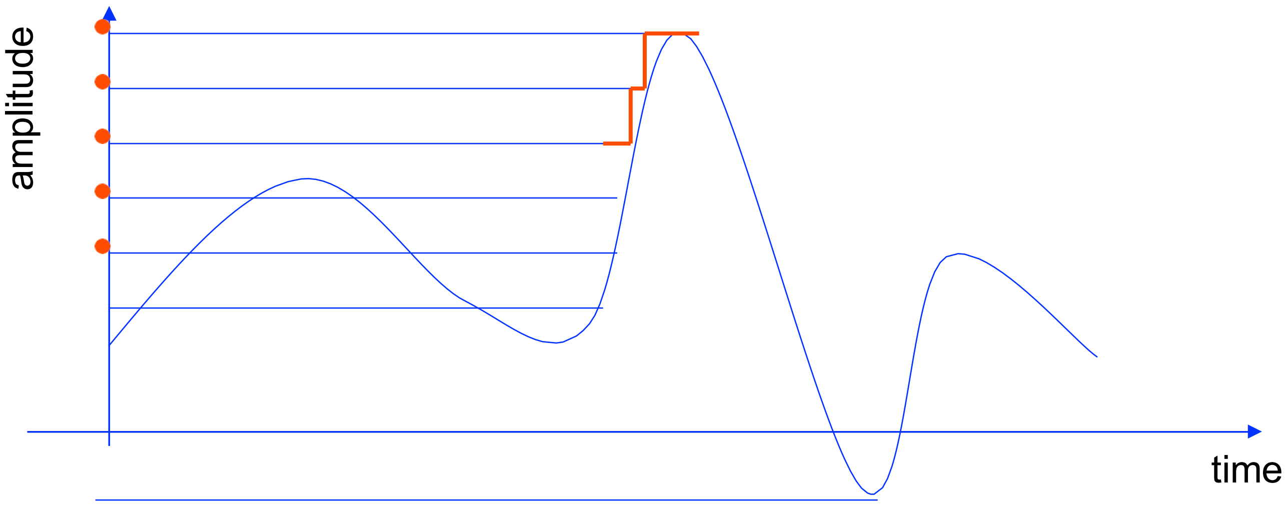
- **Analog signal**: signal whose amplitude can take on any value in a continuous range
  - The amplitude of the function f(t) (or f(x)) has the cardinality of real numbers
    - The difference between analog and digital is similar to the difference between continuous-time and discrete-time. In this case, however, the difference is with respect to the value of the function (y-axis)
  - Analog corresponds to a continuous y-axis, while digital corresponds to a discrete y-axis



- *Here we call digital what we have called quantized in the EI class*
- An analog signal can be both continuous time and discrete time

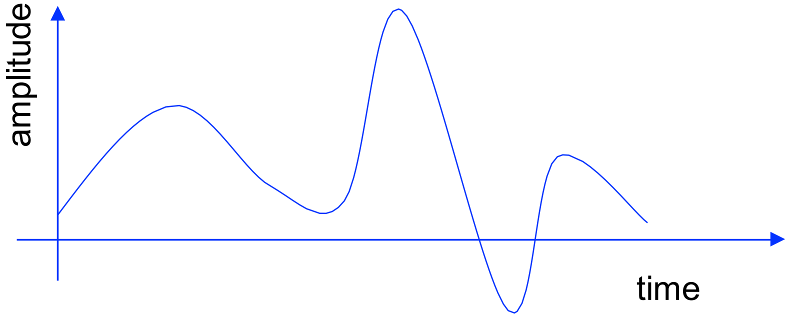
Analog - Digital

- **Digital signal**: a signal is one whose amplitude can take on only a finite number of values (thus it is quantized)
  - The amplitude of the function f() can take only a finite number of values
  - A digital signal whose amplitude can take only M different values is said to be Mary
    - Binary signals are a special case for M=2



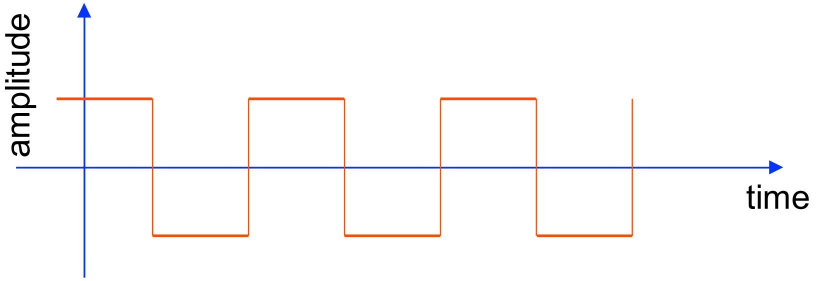
Example

– Continuous time analog

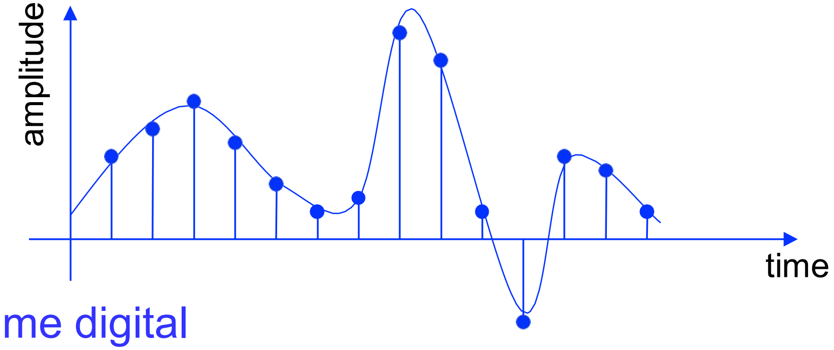


– Continuous time digital (or quantized)

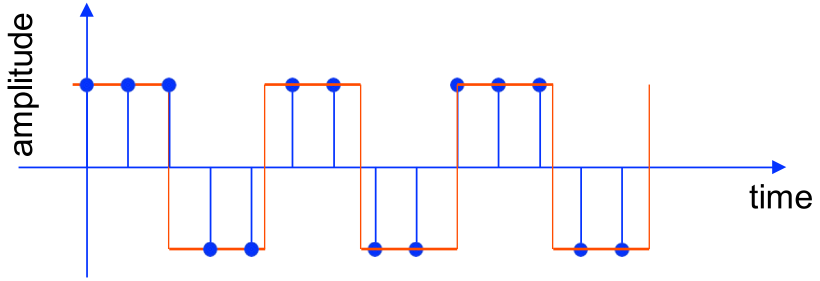
- binary sequence, where the values of the function can only be one or zero.



- Discrete time analog



- Discrete time digital
  - binary sequence, where the values of the function can only be one or zero.



Summary

Signal amplitude/ Time or space	Real	Integer
Real	Analog Continuous-time	Digital Continuous-time
Integer	Analog Discrete-time	Digital Discrete time

Periodic - Aperiodic

- A signal f(t) is *periodic* if there exists a positive constant T_0 such that

$$f(t+T_0)=f(t) \qquad \forall t$$

  - The *smallest* value of T_0 which satisfies such relation is said the *period* of the function f(t)
  - A periodic signal remains unchanged when *time-shifted* of integer multiples of the period
  - Therefore, by definition, it starts at minus infinity and lasts forever

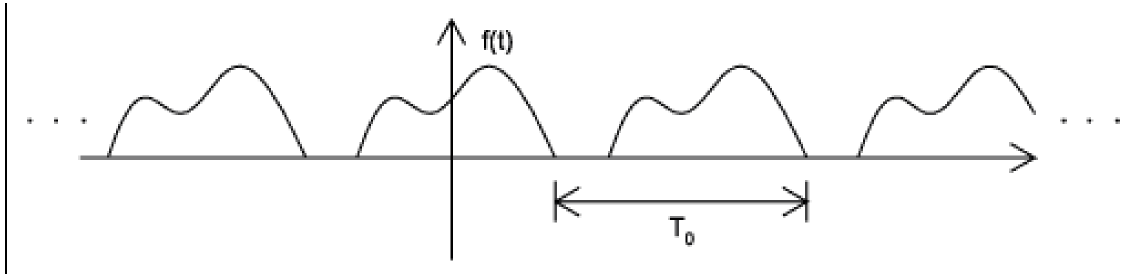
$$-\infty \leq t \leq +\infty \qquad t \in \mathbb{R}$$

$$-\infty \leq n \leq +\infty \qquad n \in \mathbf{Z}$$

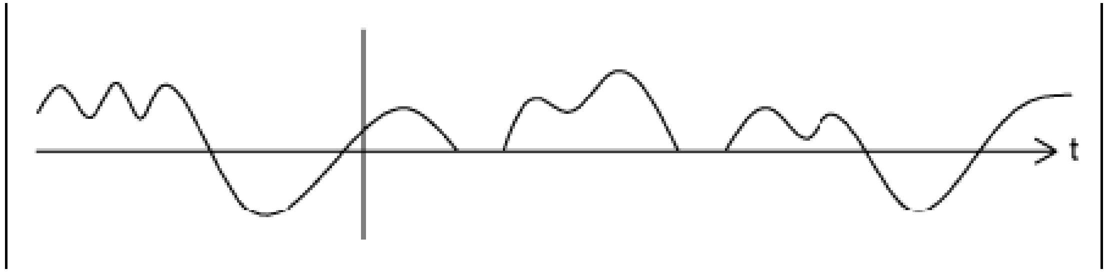
  - Periodic signals can be generated by *periodical extension*

Examples

- Periodic signal with period T_0

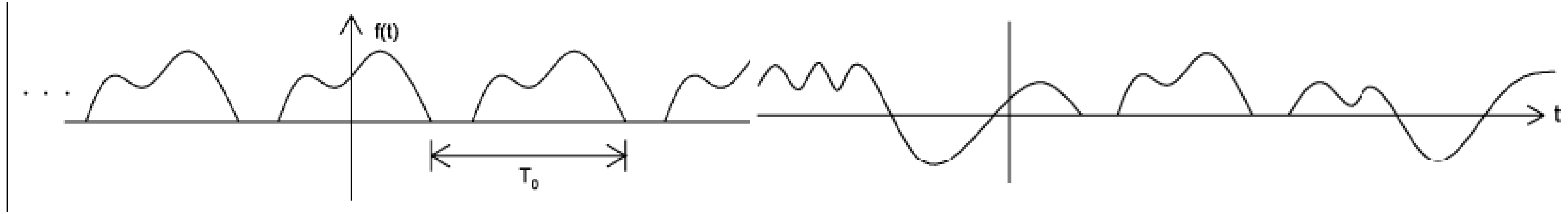


- Aperiodic signal

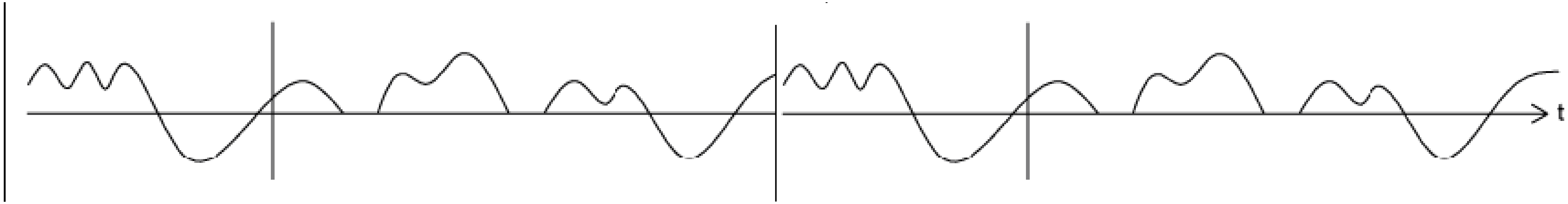


Examples

- Still periodic?



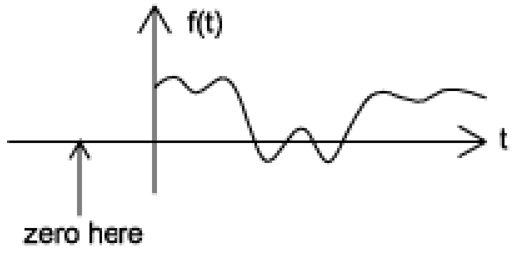
- Still an aperiodic signal?



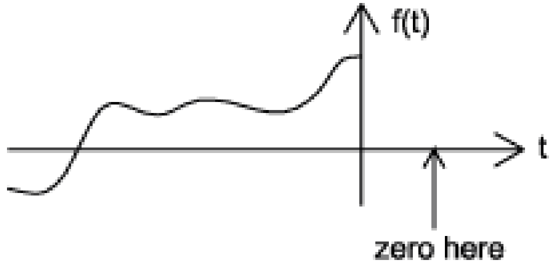
- 周期与非周期性同样具有主观性！

Causal and non-Causal signals

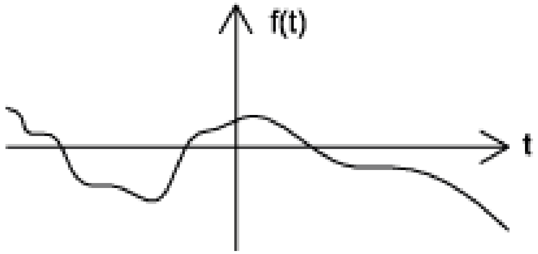
- *Causal* signals are signals that are *zero for all negative time (or spatial positions)*, while



- *Anticausal* are signals that are zero for all *positive time (or spatial positions)*.



- *Noncausal* signals are signals that have nonzero values in both positive and negative time



Causal and non-causal signals

- Causal signals

$$f(t) = 0 \qquad t < 0$$

- Anticausals signals

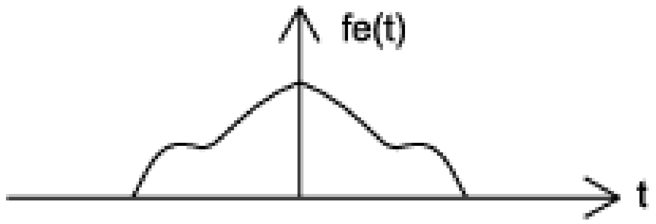
$$f(t) = 0 \qquad t \geq 0$$

- Non-causal signals

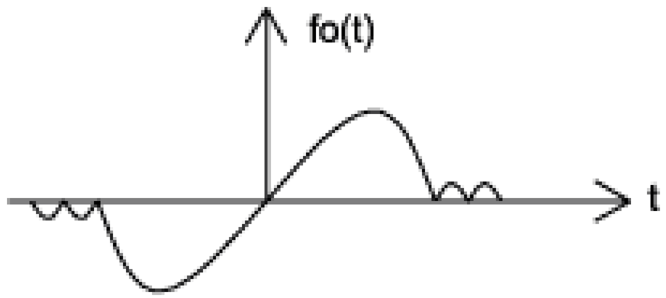
$$\exists t_1 < 0: \qquad f(t_1) = 0$$

Even and Odd signals

- An even signal is any signal f such that f (t) = f (-t). Even signals can be easily spotted as they are symmetric around the vertical axis.



- An odd signal, on the other hand, is a signal f such that f (t)= - (f (-t))



Decomposition in even and odd components

- Any signal can be written as a combination of an even and an odd signals
  - Even and odd components

重要！

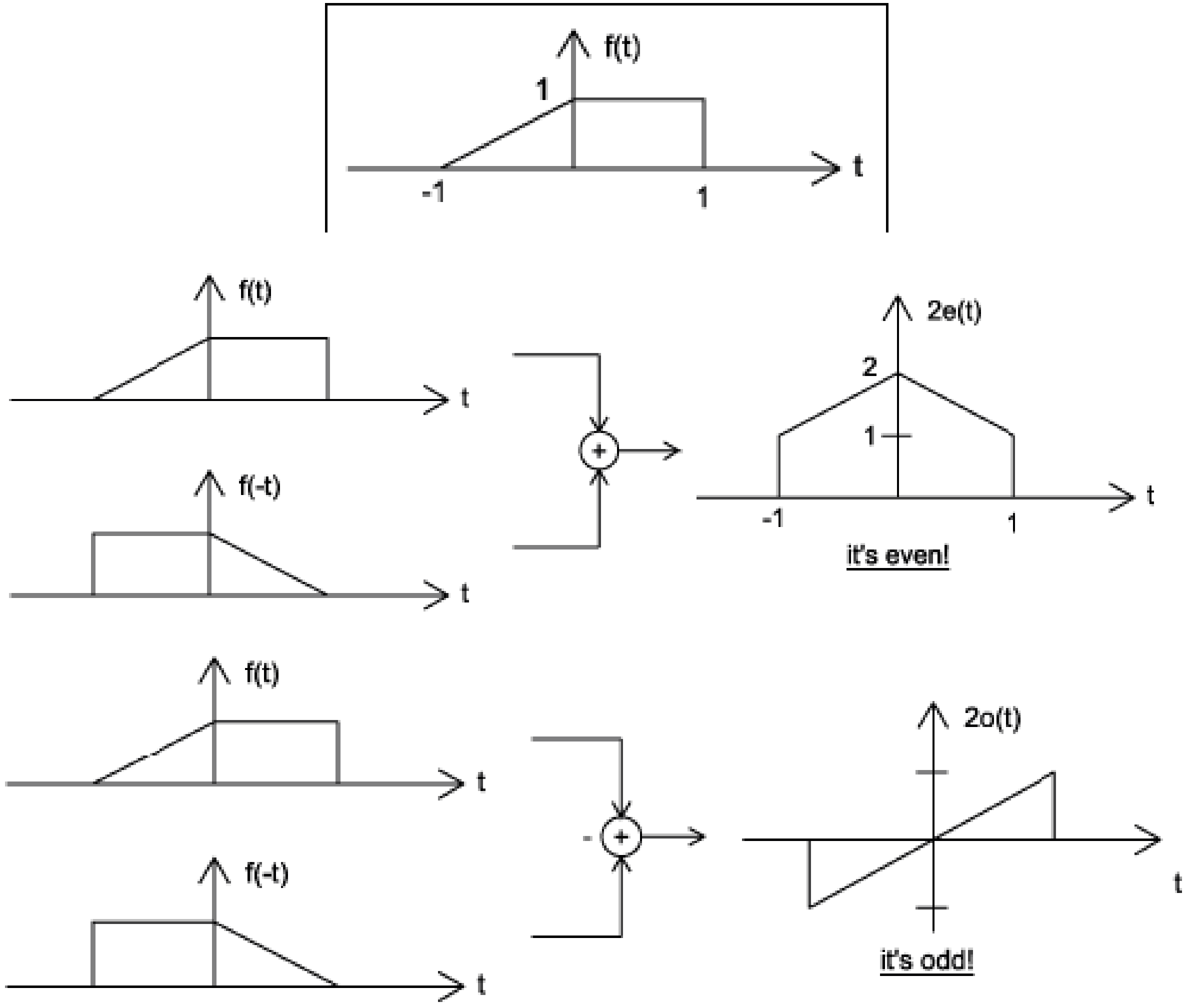
$$f(t)=\frac{1}{2}\left(f(t)+f(-t)\right)+\frac{1}{2}\left(f(t)-f(-t)\right)$$

$$f_e(t)=\frac{1}{2}\left(f(t)+f(-t)\right) \quad \text{even component}$$

$$f_o(t)=\frac{1}{2}\left(f(t)-f(-t)\right) \quad \text{odd component}$$

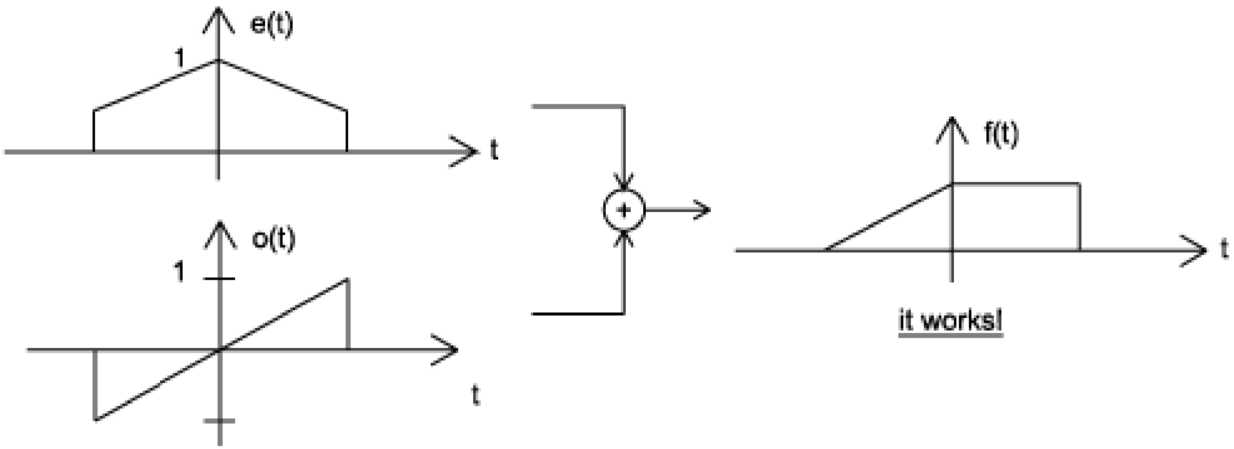
$$f(t)=f_e(t)+f_o(t)$$

Example



Example

- Proof



Some properties of even and odd functions

- even function x odd function = odd function
- odd function x odd function = even function
- even function x even function = even function
- Area

$$\int_{-a}^{a} f_e(t)\,dt = 2\int_{0}^{a} f_e(t)\,dt$$

$$\int_{-a}^{a} f_e(t)\,dt = 0$$

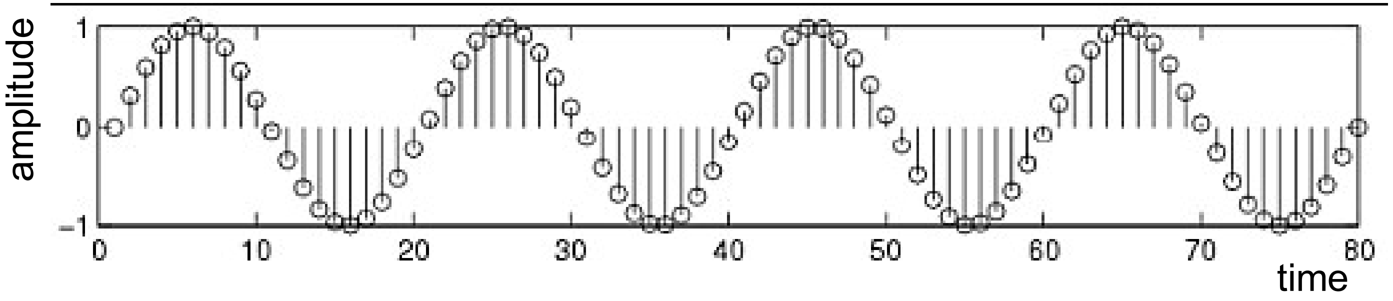
Deterministic - Probabilistic

- Deterministic signal: a signal whose *physical description* in known completely
- A deterministic signal is a signal in which each value of the signal is fixed and can be determined by a mathematical expression, rule, or table.
- Because of this the future values of the signal can be calculated from past values with complete confidence.
  - There is *no uncertainty* about its amplitude values
  - Examples: signals defined through a mathematical function or graph

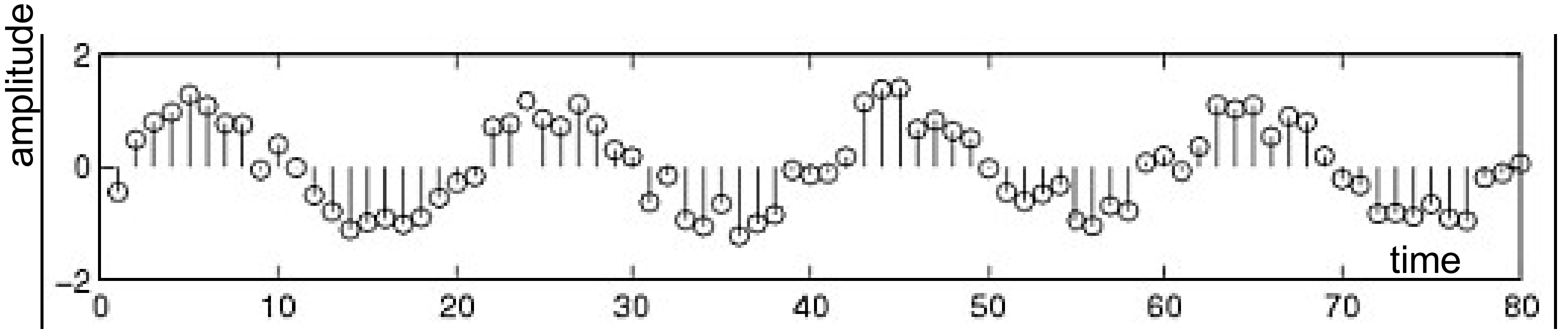
- Probabilistic (or random) signals: the amplitude values *cannot be predicted precisely* but are known only in terms of probabilistic descriptors
- The future values of a random signal cannot be accurately predicted and can usually only be guessed based on the averages of sets of signals
  - They are realization of a stochastic process for which a model could be available
  - Examples: EEG, evocated potentials, noise in CCD capture devices for digital cameras

Example

- Deterministic signal



- Random signal



Finite and Infinite length signals

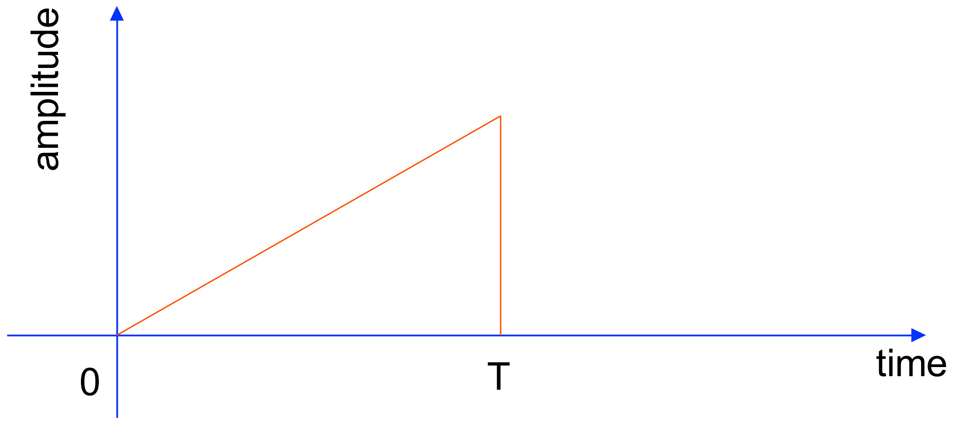
- A finite length signal is non-zero over a finite set of values of the independent variable

$$f = f(t), \forall t : t_1 \le t \le t_2$$
$$t_1 > -\infty, t_2 < +\infty$$

- An infinite length signal is non zero over an infinite set of values of the independent variable
  - For instance, a sinusoid f(t)=sin(ωt) is an infinite length signal

Size of a signal: Norms

- "Size" indicates largeness or strength.
- We will use the mathematical concept of the norm to quantify this notion for both continuous-time and discrete-time signals.
- The energy is represented by the area under the curve (of the squared signal)



Energy

- Signal energy

$$E_f = \int_{-\infty}^{+\infty} f^2(t)dt$$

$$E_f = \int_{-\infty}^{+\infty} |f(t)|^2 \, dt$$

- Generalized energy : L_p norm
  - For p=2 we get the energy (L_2 norm)

$$\|f(t)\| = \left(\int (|f(t)|)^p \, dt\right)^{1/p}$$

$$1 \le p < +\infty$$

Power

- Power
  - The power is the time average (mean) of the squared signal amplitude, that is the *mean-squared* value of f(t)

$$P_f = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} f^2(t)dt$$

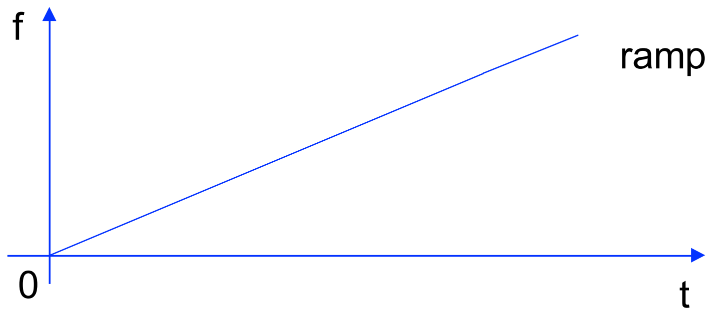
$$P_f = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} \left| f(t) \right|^2 dt$$

Power - Energy

- The square root of the power is the root mean square (*rms*) value
  - This is a very important quantity as it is the most widespread measure of similarity/dissimilarity among signals
  - It is the basis for the definition of the Signal to Noise Ratio (SNR)

$$SNR = 20\log_{10}\left(\sqrt{\frac{P_{signal}}{P_{noise}}}\right)$$

  - It is such that a constant signal whose amplitude is =rms holds the same power content of the signal itself
- There exist signals for which neither the energy nor the power are finite

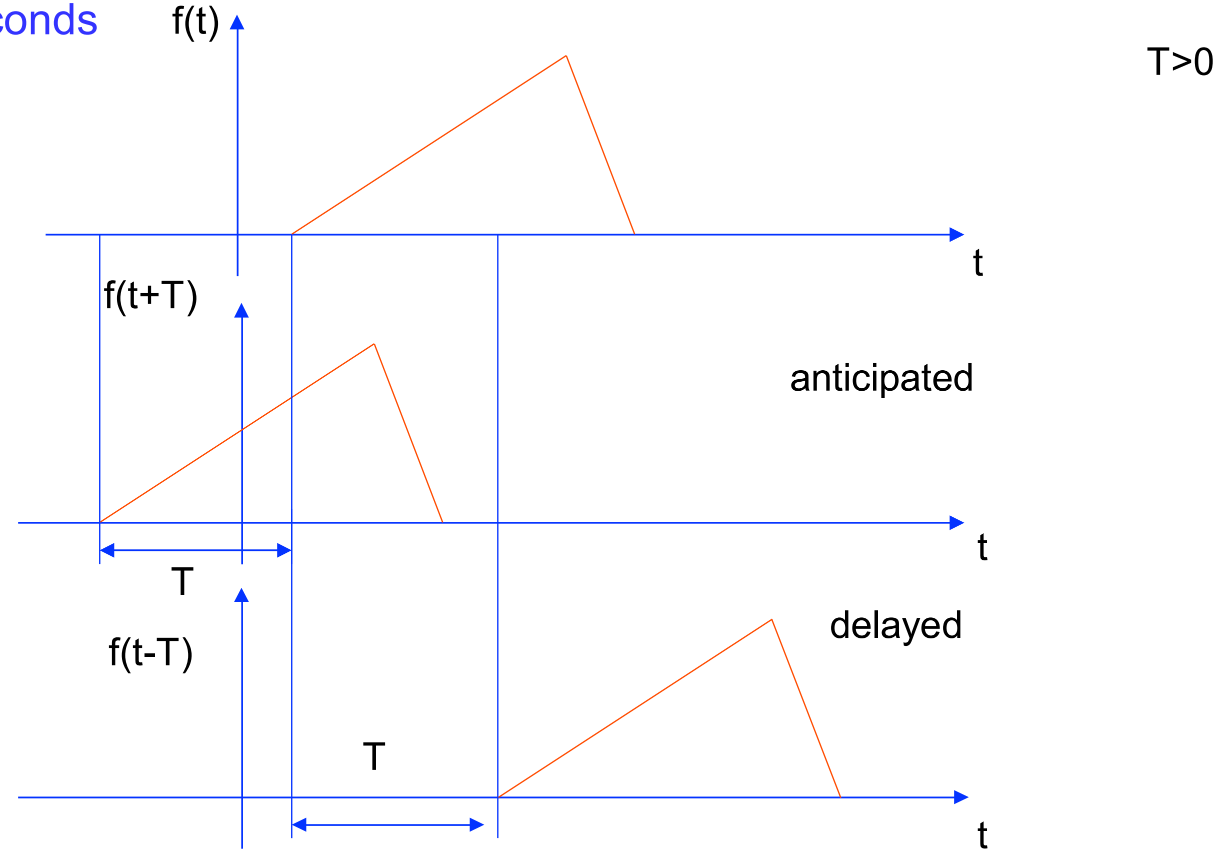


Energy and Power signals

- A signal with finite energy is an energy signal
  - Necessary condition for a signal to be of energy type is that the amplitude goes to zero as the independent variable tends to infinity
- A signal with finite and different from zero power is a power signal
  - The mean of an entity averaged over an infinite interval exists if either the entity is periodic or it has some statistical regularity
  - A power signal has infinite energy and an energy signal has zero power
  - There exist signals that are neither power nor energy, such as the ramp
- All practical signals have finite energy and thus are energy signals
  - It is impossible to generate a real power signal because this would have infinite duration and infinite energy, which is not doable.

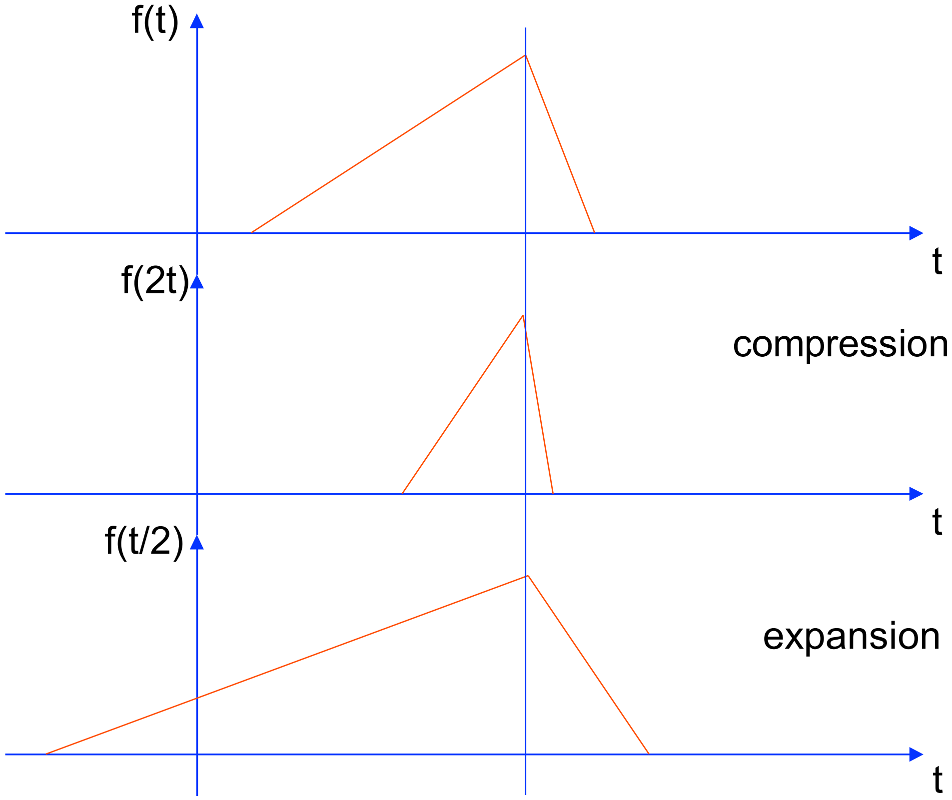
Useful signal operations: shifting, scaling, inversion

- **Shifting**: consider a signal f(t) and the same signal delayed/anticipated by T seconds



Useful signal operations: shifting, scaling, inversion

- (Time) Scaling: compression or expansion of a signal in time



compression $\varphi(t) = f(2t)$

expansion $\varphi(t) = f(t/2)$

Useful signal operations: shifting, scaling, inversion

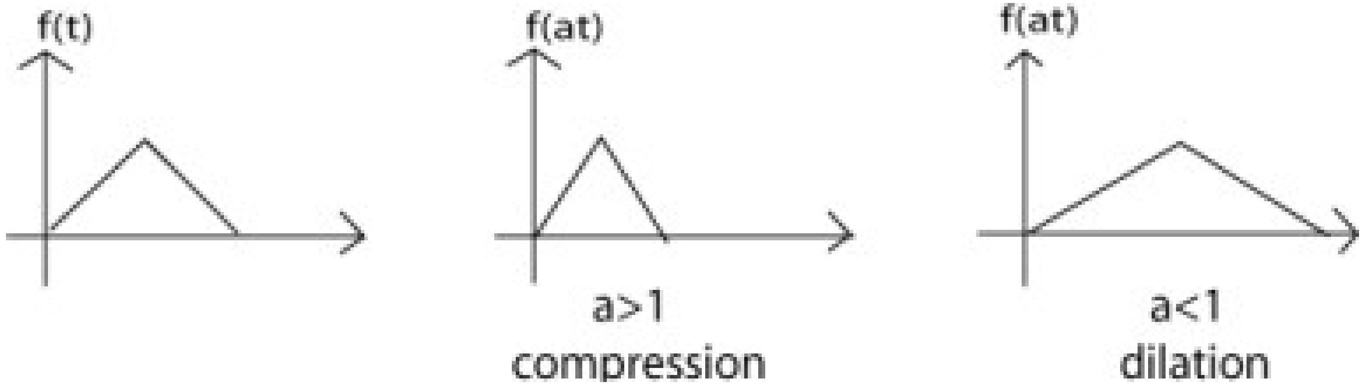
- Scaling: generalization

$$a > 1$$

$$\varphi(t) = f(at) \;\rightarrow\; \text{compressed version}$$

$$\varphi(t) = f\left(\frac{t}{a}\right) \rightarrow\; \text{dilated (or expanded) version}$$

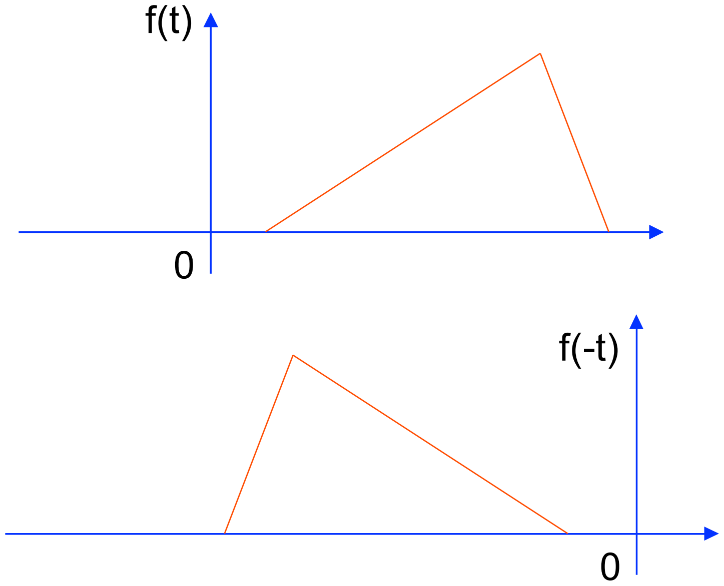
Viceversa for $a < 1$

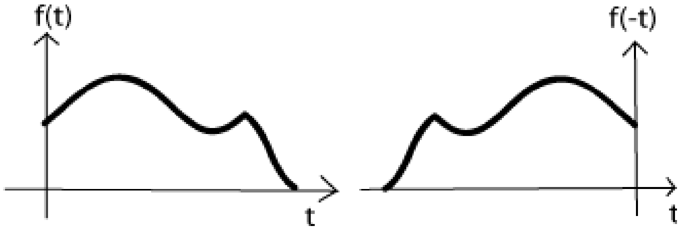


Useful signal operations: shifting, scaling, inversion

- (Time) inversion: mirror image of f(t) about the vertical axis

$$\varphi(t) = f(-t)$$





Useful signal operations: shifting, scaling, inversion

- Combined operations: $f(t) \rightarrow f(at-b)$
- Two possible sequences of operations

1. *Time shift $f(t)$ by to obtain $f(t-b)$. Now time scale the shifted signal $f(t-b)$ by a to obtain $f(at-b)$.*
2. *Time scale $f(t)$ by a to obtain $f(at)$. Now time shift $f(at)$ by b/a to obtain $f(at-b)$.*
   - *Note that you have to replace t by $(t-b/a)$ to obtain $f(at-b)$ from $f(at)$ when replacing t by the translated argument (namely $t-b/a$))*

Outline: Lecture 1: Introduction

- Preliminary
  - What are the signals
  - What are the systems
  - Application of signals and systems
- Fundamental concepts
- Types of Signals

Useful functions

- Unit step function （单位阶跃函数）
  - Useful for representing causal signals

$$u(t)=\begin{cases}1 & t\geq 0\\ 0 & t<0\end{cases}$$

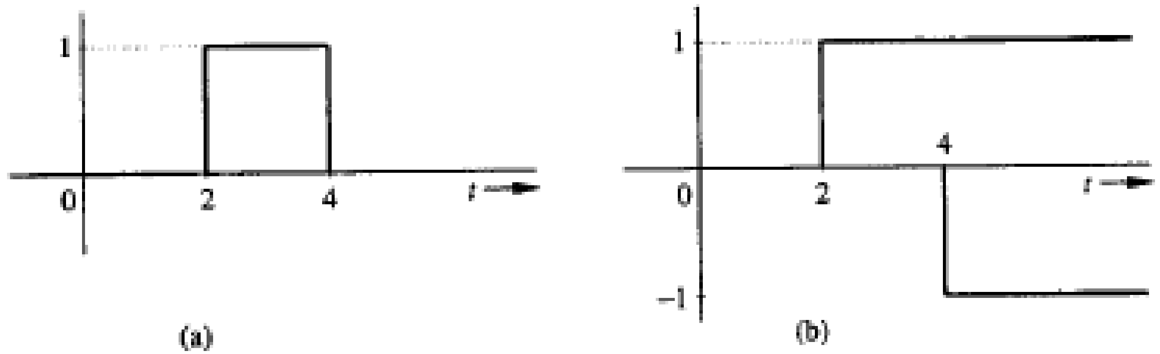
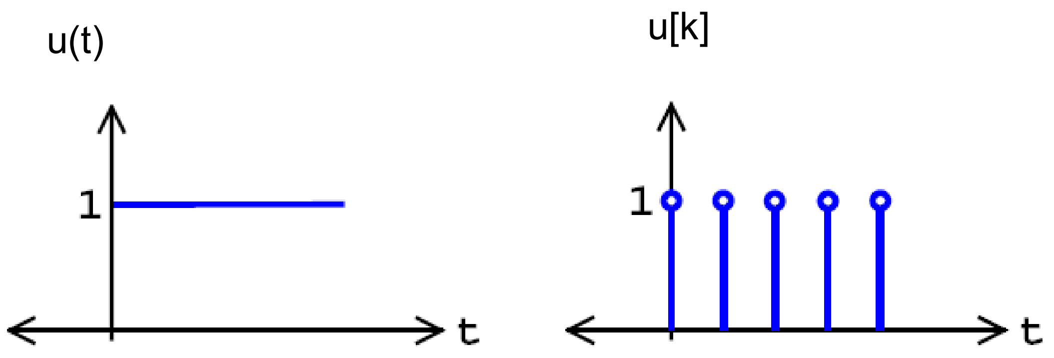


Fig. 1.15 Representation of a rectangular pulse by step functions.

$$f(t)=u(t-2)-u(t-4)$$

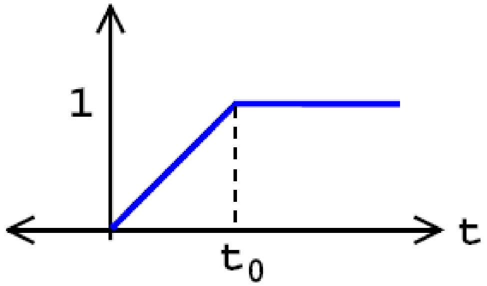
- Continuous and discrete time unit step functions



Useful functions

- Ramp function (continuous time)

$$r(t) = \begin{cases} 0 \text{ if } t < 0 \\ \frac{t}{t_0} \text{ if } 0 \leq t \leq t_0 \\ 1 \text{ if } t > t_0 \end{cases}$$



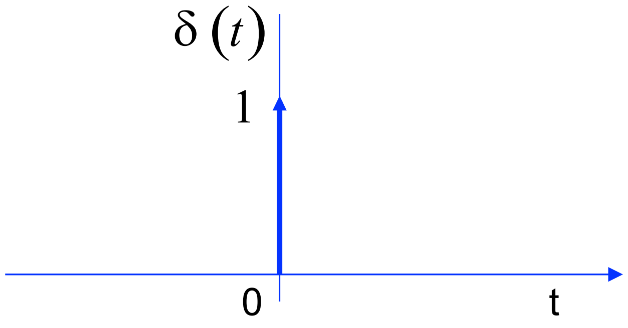
Useful functions

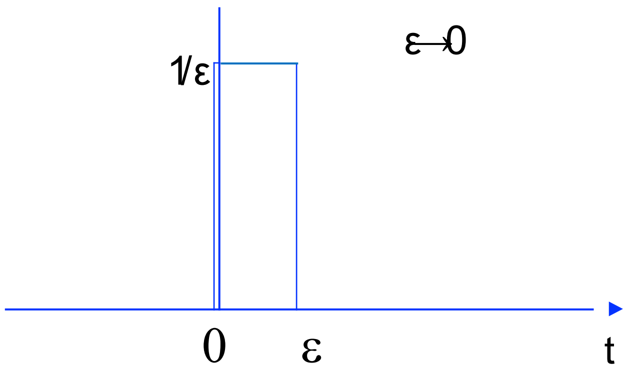
- Unit impulse function
  （单位冲激函数）

$$\delta(t)=0 \quad t\neq 0$$

$$\int_{-\infty}^{+\infty}\delta(t)dt=1$$

$$\delta_{\varepsilon}(t)=\begin{cases}1/\varepsilon, & 0\leq t\leq \varepsilon,\\ 0, & \text{其它},\end{cases}$$





Properties of the unit impulse function

- Multiplication of a function by impulse

$$\phi(t)\delta(t)=\phi(0)\delta(t)$$
$$\phi(t)\delta(t-T)=\phi(T)\delta(t-T)$$

- Sampling property of the unit function

$$\int_{-\infty}^{+\infty}\phi(t)\delta(t)dt=\int_{-\infty}^{+\infty}\phi(0)\delta(t)dt=\phi(0)\int_{-\infty}^{+\infty}\delta(t)dt=\phi(0)$$

$$\int_{-\infty}^{+\infty}\phi(t)\delta(t-T)dt=\phi(T)$$

  – The area under the curve obtained by the product of the unit impulse function shifted by T and $\varphi(t)$ is the value of the function $\varphi(t)$ for $t=T$

Properties of the unit impulse function

- The unit step function is the integral of the unit impulse function

$$\frac{du}{dt} = \delta(t)$$

$$\int_{-\infty}^{t} \delta(t)\,dt = u(t)$$

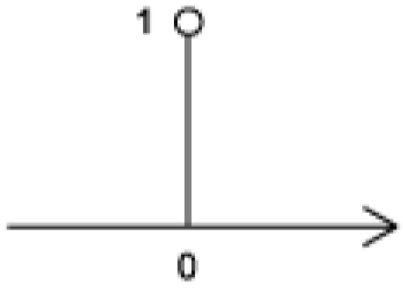
  – Thus

$$\int_{-\infty}^{t} \delta(t)\,dt = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Properties of the unit impulse function

- Discrete time impulse function

$$\delta[n] = \begin{cases} 1 \text{ if } n = 0 \\ 0 \text{ otherwise} \end{cases}$$



Unit Impulse and Unit Step Functions

- Discrete-time

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \qquad u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

  – First difference

$$\delta[n] = u[n] - u[n-1]$$

  – Running Sum

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

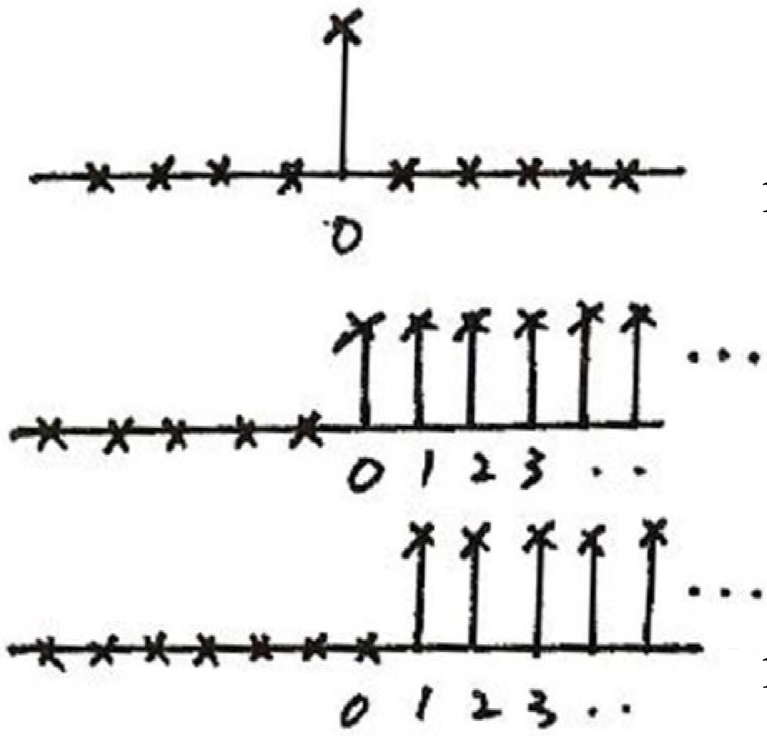
$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

  – Sampling property

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$$

Unit Impulse & Unit Step

- Discrete-time



$\delta[n]$

n

$u[n]$

n

$u[n-1]$

n

$$\delta[n] = u[n] - u[n-1]$$

Useful functions

- Continuous time complex exponential

$$f(t) = Ae^{j\omega t}$$

- Euler's relations

$$Ae^{j\omega t} = A\cos(\omega t) + j(A\sin(\omega t))$$

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-(j\omega t)}}{2}$$

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-(j\omega t)}}{2j}$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Discrete time complex exponential
  - k=nT

$$\begin{aligned} f[n] &= Be^{snT} \\ &= Be^{j\omega nT} \end{aligned}$$

Useful functions

- Exponential function e^{st}
  - Generalization of the function $e^{j\omega t}$

$$s = \sigma + j\omega$$

Therefore

$$e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t}e^{j\omega t} = e^{\sigma t}(\cos \omega t + j \sin \omega t) \qquad (1.30a)$$

If $s^* = \sigma - j\omega$ (the conjugate of s), then

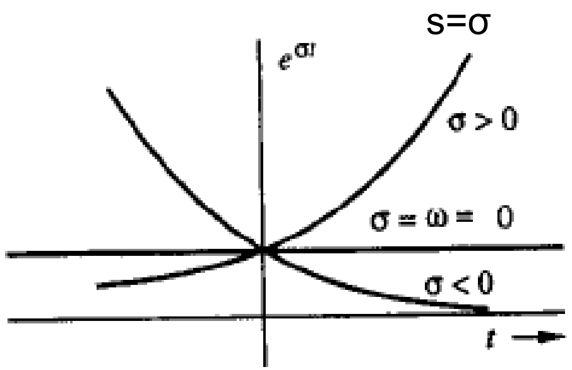
$$e^{s^* t} = e^{\sigma - j\omega} = e^{\sigma t}e^{-j\omega t} = e^{\sigma t}(\cos \omega t - j \sin \omega t) \qquad (1.30b)$$

and

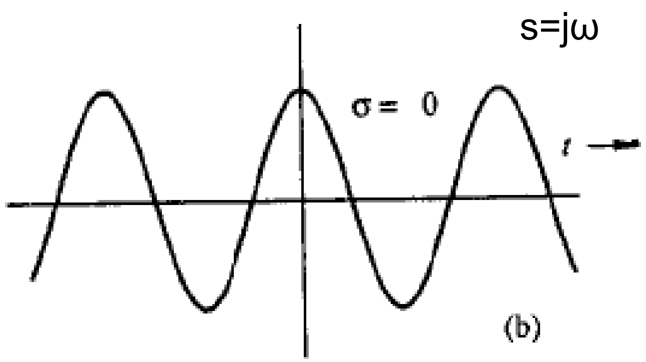
$$e^{\sigma t} \cos \omega t = \frac{1}{2}(e^{st} + e^{s^* t}) \qquad (1.30c)$$

The exponential function

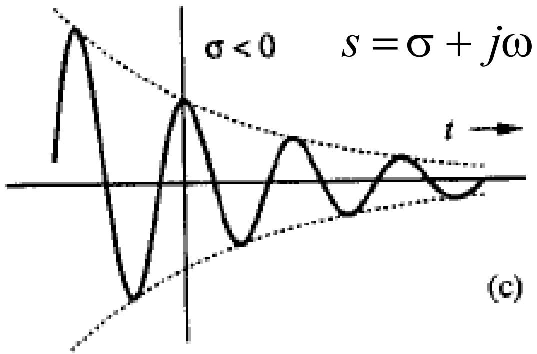
$$e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t}e^{j\omega t} = e^{\sigma t}(\cos \omega t + j \sin \omega t)$$



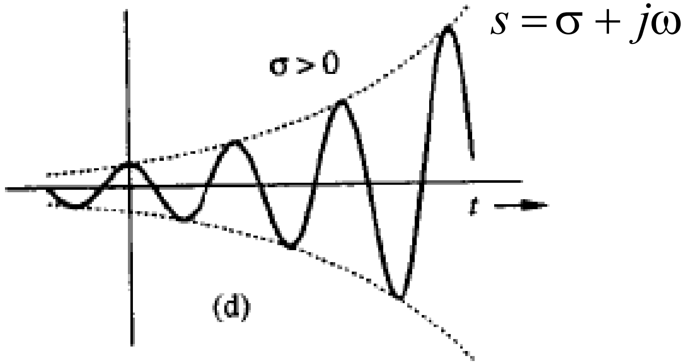
(a)



(b)

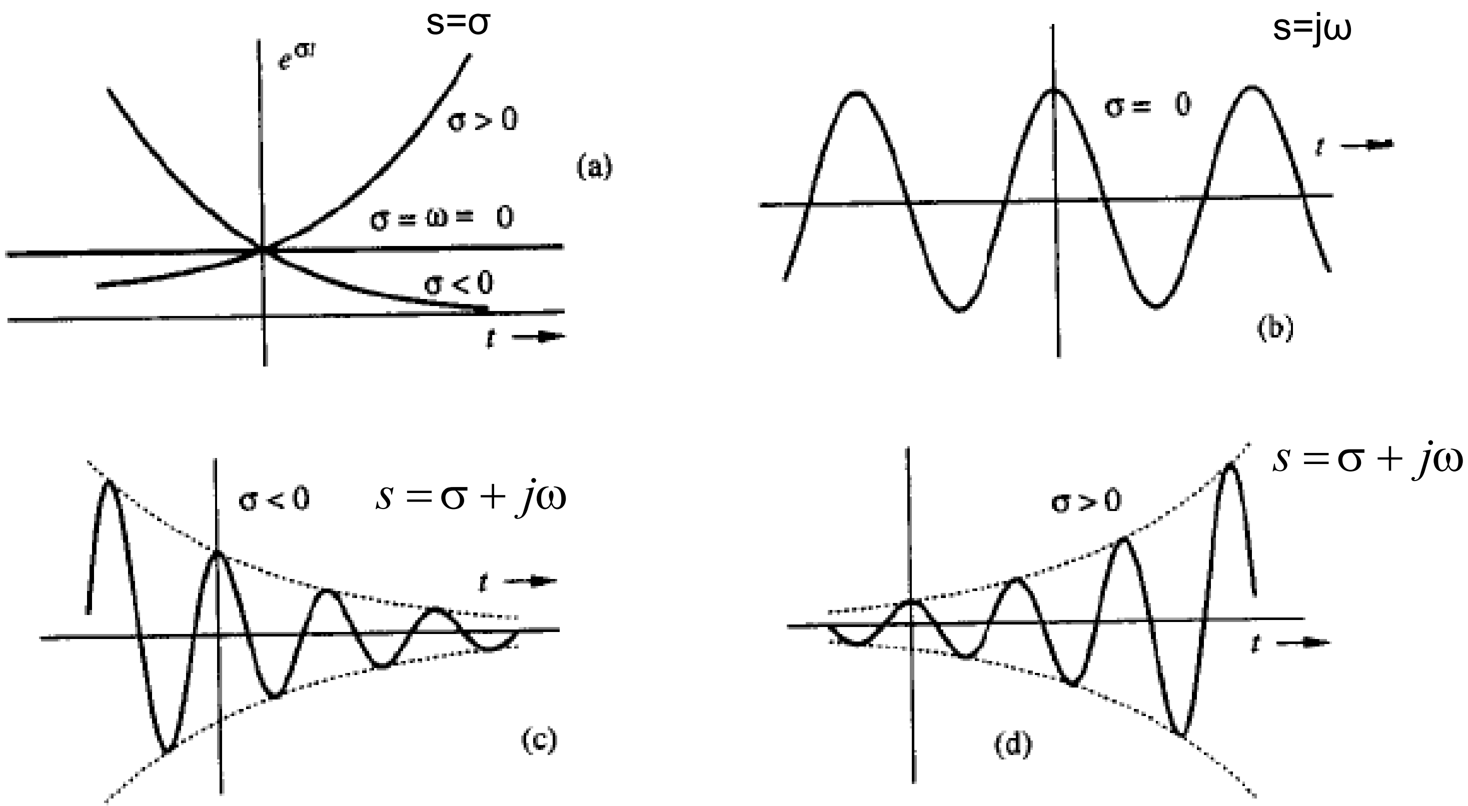


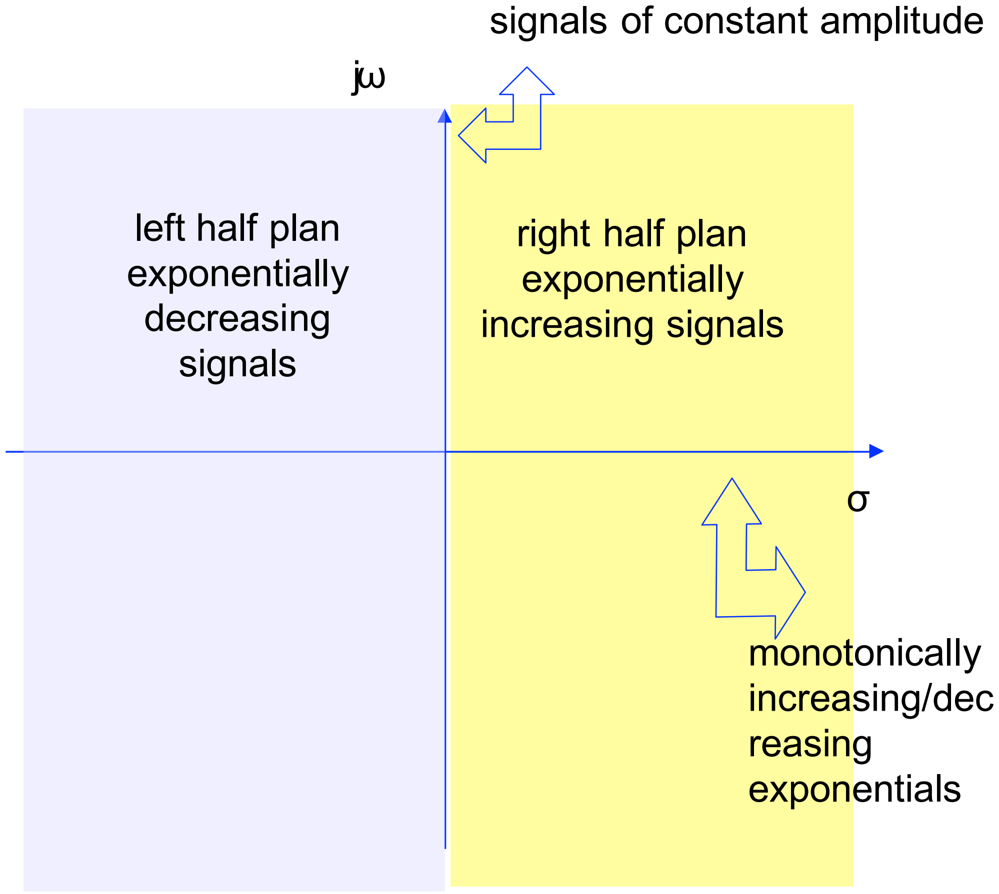
(c)



(d)

The exponential function





本章部分关键词汇中英文对照表

离散时间信号	Discrete-time signal
连续时间信号	Continuous-time signal
指数信号	Exponential signal
正弦信号	Sinusoidal signal
余弦信号	Cosinusoidal signal
阶跃函数	Step function
冲激函数	Impulse signal
斜坡函数	Ramp signal
因果系统	Casual system
模拟信号	Analog signal
数字信号	Digital signal
复数	Complex number
实数	Real number
虚数	Imaginary number

Thank you for your listening!