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4. 判断以下信号是否具有周期性。若有周期性, 计算最小周期  $T$ 。

1)  $x(t) = e^{j(\pi t - 1)}$ ; 2)  $x[n] = \cos(\frac{\pi}{8}n - \pi)$ ; 3)  $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

1).  $x(t) = e^{j(\pi t - 1)} = \cos(\pi t - 1) + j\sin(\pi t - 1)$

$$\Rightarrow T = \frac{2\pi}{\pi} = 2$$

2). 若有周期性, 则  $T = \frac{2\pi}{\frac{\pi}{8}} = 16\pi$

 $T$  不为整数, 因此无周期性

3).  $x_1[n] = 2\cos(\frac{\pi}{4}n) \Rightarrow T_1 = \frac{2\pi}{\pi/4} \cdot m_1 = 8m_1$

$x_2[n] = \sin(\frac{\pi}{8}n) \Rightarrow T_2 = \frac{2\pi}{\pi/8} \cdot m_2 = 16m_2$

$x_3[n] = -2\cos(\frac{\pi}{2}n + \frac{\pi}{6}) \Rightarrow T_3 = \frac{2\pi}{\pi/2} \cdot m_3 = 4m_3$

5. 计算以下卷积结果:

1)  $y[n] = x[n] * h[n]$ ,  $x[n] = u[n-2] - u[n-6]$ ,  $h[n] = u[n] - u[n-9]$ ;

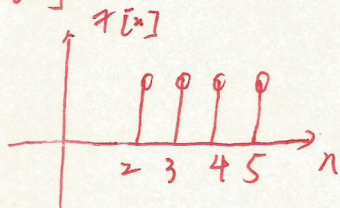
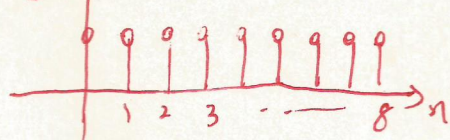
2)  $y(t) = x(t) * h(t)$ ,  $x(t) = u(t-3) - u(t-6)$ ,  $h(t) = e^{-2t}u(t)$

若有周期性, 则:

存在整数  $m_1, m_2, m_3$ , 使得:

$$T = 8m_1 = 16m_2 = 4m_3$$

$$\Rightarrow \begin{cases} m_1 = 2 \\ m_2 = 1 \\ m_3 = 4 \end{cases} \Rightarrow T = 16$$

1).  $x[n]$ : $h[n]$ :

① 当  $n < 2$  时:  $y[n] = 0$

② 当  $2 \leq n < 5$  时:

$$y[n] = \sum_{k=2}^n 1 \times 1 = n - 2 + 1 = n - 1$$

③ 当  $5 \leq n < 6$  时:

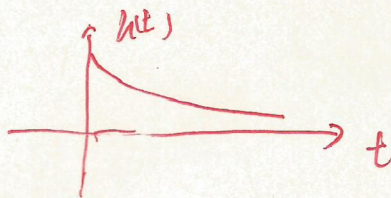
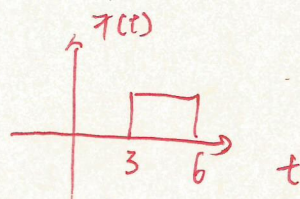
$$y[n] = \sum_{k=2}^5 1 \times 1 = 4$$

④ 当  $6 \leq n < 13$  时:

$$y[n] = \sum_{k=n-8}^5 1 \times 1 = 5 - n + 8 + 1 = 14 - n$$

⑤ 当  $n \geq 13$  时:  $y[n] = 0$

2).



① 当  $t < 3$  时,  $y(t) = 0$ ;

② 当  $3 \leq t < 6$  时,

$$\begin{aligned}
 y(t) &= \int_3^t e^{-2(t-\tau)} d\tau \\
 &= e^{-2t} \int_3^t e^{2\tau} d\tau \\
 &= e^{-2t} \cdot \frac{1}{2} (e^{2t} - e^6) \\
 &= \frac{1}{2} (1 - e^{-2(t-3)})
 \end{aligned}$$

③ 当  $t \geq 6$  时,

$$\begin{aligned}
 y(t) &= \int_3^6 e^{-2(t-\tau)} d\tau \\
 &= \frac{1}{2} e^{-2t} (e^{12} - e^6)
 \end{aligned}$$