Southern University of Science And Technology
School of Life Sciences
BIO210 Biostatistics
(Autumn, 2022)

Assignment 6
Due on 16th Dec, 11 p.m.

1. **Gas Mileage**: Li Lei and Han Meimei want to investigate if the gas mileages of two brands of gasoline are the same or not. They plan to select cars and perform experiments to compare the mean mileages of Brand A and Brand B.

   1.1) **(2.5 points)** Li Lei randomly selected many cars, assign half of them to use Brand A and the other half to use Brand B. He needs to use which of the following test for the investigation:

   (A) One-sample $\boldsymbol{t}-$test
   (B) Paired $\boldsymbol{t}-$test
   (C) Independent two-sample $\boldsymbol{t}-$test
   (D) $\boldsymbol{Z}-$test

   1.2) **(2.5 points)** Han Meimei decides to use a different strategy. She randomly selects many cars and has each car use Brand A and Brand B. Then she compares the differences in mileage for each car. She needs to use which of the following test for the investigation:

   (A) One-sample $\boldsymbol{t}-$test
   (B) Paired $\boldsymbol{t}-$test
   (C) Independent two-sample $\boldsymbol{t}-$test
   (D) $\boldsymbol{Z}-$test

2. **Comparing Packing Machines**: In a packing plant, a machine packs cartons with jars. It is supposed that a new machine will pack faster on the average than the machine currently used. To test that hypothesis, the times it takes each machine to pack ten cartons are recorded. The results, in seconds, are shown in the tables:

New machine				
42.1	41.3	42.4	43.2	41.8
41.0	41.8	42.8	42.3	42.7

Old machine				
42.7	43.8	42.5	43.1	44.0
43.6	43.3	43.5	41.7	44.1

You want to perform a hypothesis testing, at the significance level of $\alpha = 0.05$, to see if the new machine packs faster or slower than the old machine.

2.1) **(2 points)** Write the null ($\boldsymbol{H_0}$) and alternative ($\boldsymbol{H_1}$) hypotheses.

$\boldsymbol{H_0}$: There is no difference in the packing efficiency between old and new machines.

$\boldsymbol{H_1}$: There is some difference in the packing efficiency between old and new machines.

2.2) **(3 points)** In you own words, explain what is the meaning of selecting a significance level of $\alpha = 0.05$.

This is an open question. You get full score as along as it is reasonable. For example, this is an acceptable answer: when we set $\alpha = 0.05$, we want to only risk 5% of the chance to make a Type I error

2.3) **(5 points)** Compute the sample statistics: sample means and sample variances of the new and old machines.

$\bar{x}_{new} = 42.14$, $s^2_{new} = 0.467$

$\bar{x}_{old} = 43.23$, $s^2_{old} = 0.562$

2.4) **(2 points)** What statistical test should you use, and what assumptions should you check?

Since we are comparing means from two independent samples, we should use two-sample $\boldsymbol{t}-$test for independent samples. We should check if the samples are randomly and independently selected. We also need to make sure the test statistic follow a $\boldsymbol{t}$ distribution. Since the sample sizes are small (both are 10), we should make sure the population distributions follow normal distributions. We also need to check the variances of the two populations are the same or not.

2.5) **(5 points)** Assume all the assumptions are met, and the variances of the two machines are equal. Compute the test statistic and $p-$value.

Assume equal variance:

$$s_p = \frac{(n_{new}-1)s^2_{new} + (n_{old}-1)s^2_{old}}{n_{new}+n_{old}-2} = \frac{9\times 0.467 + 9\times 0.562}{9+9} = 0.515$$

Therefore, the test statistic is:

$$\boldsymbol{t} = \frac{\bar{x}_{new} - \bar{x}_{old}}{\sqrt{s_p\left(\frac{1}{n_{new}} + \frac{1}{n_{old}}\right)}} = \frac{42.14 - 43.23}{\sqrt{0.515 \times \left(\frac{1}{10} + \frac{1}{10}\right)}} = -3.40$$

So, the two-sided $p-$value is $2 \times P(\boldsymbol{t} \leqslant -3.40) = 0.0032$

2.6) **(3 points)** The $p-$value you get from question **2.5)** is p, which of the following statement is true:

(A) The null hypothesis is rejected, but it still has a probability of p to be true.

(B) At the significance level of $\alpha = 0.05$, the null hypothesis is wrong.

(C) The null hypothesis is rejected, but there is a chance that we made a wrong decision. The probability of the null hypothesis is true is p.

(D) If the null hypothesis were true, the probability of observing the difference in our current samples or more extreme is small.

3. **(5 points)** Analysis of variance (ANOVA) is a statistical method of comparing which of the following of several populations.

(A) Standard deviations

(B) Variances

(C) Means

(D) Proportions

4. **(5 points)** In an ANOVA, if the true means of k populations are equal, then the ratio of the Mean Square Between and the Mean Square Within should be:

(A) < 0

(B) $\leqslant 1$

(C) > 1

(D) ≈ 0.5

5. **(5 points)** In a study, subjects are randomly assigned to one of three groups: control, experimental A, or experimental B. After treatment, the mean scores for the three groups are compared using a classical ANOVA test, and the significance level was set at 0.05 in advance. The p-value of the ANOVA test is 0.2. Which of the following is NOT correct:

(A) Before the ANOVA test, we should check the population distributions of the scores follow normal distributions, and the variances are the same.

(B) Since $p > 0.05$, the null hypothesis should NOT be rejected.

(C) Since $p > 0.05$, we should perform two-sample $t-$tests for every possible pairs of groups to find out whether they are different.

(D) We should check if samples are randomly and independently selected before the ANOVA test.

6. **(5 points)** What would happen if instead of using an ANOVA to compare 10 groups, you performed multiple $t-$tests?

(A) Nothing, there is no difference between using an ANOVA and using a $t-$test.

(B) Making multiple comparisons with a $t-$test increases the probability of making a Type I error.

(C) Nothing serious, except that making multiple comparisons with a t-test requires more computation than doing a single ANOVA.

(D) None of the above.

7. **(5 points)** What is the purpose of a *post hoc* test in ANOVA?

(A) Find out which two groups have different group means.

(B) Set the critical value for the F test.

(C) Help to reject the null hypothesis.

(D) None of the above.

8. **Fisheries**: A fisheries researcher wishes to test for a difference in mean weights of a single species of fish caught by fishermen in three different lakes in Nova Scotia. The significance level for the test is 0.05.

8.1) **(5 points)** State the null and alternative hypotheses.

8.2) **(10 points)** Complete the following ANOVA table:

Source of variation	d.f.	SS	MS	F	$p-$value
Between	2	17.04	8.52	5.39	0.02888
Within	9	14.19	1.58		
Total	11	31.23	×		

8.3) **(5 points)** What is the $p-$value of the ANOVA and will you reject the null hypothesis?

$p = 0.02888 < 0.05$, so we reject the null hypothesis.

9. Tar Content Comparisons: We want to see whether the tar contents (in milligrams) for three different brands of cigarettes are different. The lab took six samples from each of the three brands and wanted to compare the mean of the tar content from them. The data is shown below:

Sample	Brand A	Brand B	Brand C
1	10.21	11.32	11.60
2	10.25	11.20	11.90
3	10.24	11.40	11.80
4	9.80	10.50	12.30
5	9.77	10.68	12.20
6	9.73	10.90	12.20

9.1) **(5 points)** Write the null and alternative hypotheses.

$\boldsymbol{H_0}$: The mean tar content from the three brands is the same.

$\boldsymbol{H_1}$: The mean tar content from the three brands is not the same.

9.2) **(10 points)** Construct an ANOVA table and see if the null hypothesis should be rejected at the significance level $\alpha = 0.05$.

Source of variation	**d.f.**	**SS**	**MS**	$\boldsymbol{F}$	$\boldsymbol{p-}$value
Between	2	12	6	65.46	3.89×10^{-8}
Within	15	1.375	0.092		
Total	17	13.375	×		

9.3) **(15 points)** Use $\boldsymbol{t}-$tests with Fisher's Least Significant Difference (LSD) to perform *post hoc* tests to figure out which two brands are different from each other in terms of mean tar content. You can simply fill out the following table:

Comparison	$\boldsymbol{t}$	p	$p.adj$ (Bonferroni)
A vs. B	-5.72	4.05×10^{-5}	1.21×10^{-4}
A vs. C	-11.44	8.26×10^{-9}	2.48×10^{-8}
B vs. C	-5.72	4.05×10^{-5}	1.21×10^{-4}