EDO Final Project

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Attachment 2: Individual Final Report

Analysis and Optimization Approach

This project aimed to optimize a drone designed for rapid pizza delivery by minimizing both total noise and time to delivery. A multi objective optimization approach was implemented using Microsoft Excel Solver. It leveraged a weighted sum of two normalized objective functions, and then a weight for each function was applied and put into one weighted objective function. The design variables were rotor tilt (deg), number of propellers (integer: 3, 4, 5, 6, 7, or 8), propeller diameter (in), angular velocity (Hz), and battery capacity (Wh).

Noise was calculated as a function of propeller tip speed and rotor count. This is shown in attachment 1 with the noise equation. Time to deliver incorporated climb, cruise, and descent segments to calculate the total time to deliver the pizza. Multiple nonlinear and implicit constraints were enforced: thrust-to-weight ratio (to ensure flight), tip speed limit (FAA constraint of < 831.3 ft/s), rotor coupling penalties (thrust reduction if spacing was negative), FAA speed limit of under 146.667 ft/s, total weight limit of less than 55 lbs, power required being less than battery capacity, and propellor sizing constraints of min and max diameters. The optimization used the Evolutionary method, with a varying weighted sum favoring time either or noise depending on the iteration. Additional nuance included logical IF statements to enforce penalties and sweep plots to explore design tradeoffs. A sweep of prop diameter and rotor tilt was used to generate a 2D sensitivity chart. Further 2DV plots, LaGrange multiplier values, and data tables are below. MATLAB was used for plotting and finding LaGrange multipliers values.

Table of Design Parameters

Parameter	Range / Value	Notes
Rotor Tilt (deg)	0 to 20	Decision variable
# of Propellers	3,4,5,6,7, or 8	Integer
Prop Diameter (in)	0.5 to 2.0	Continuous variable
Angular Velocity	200 to 800 Hz	Converted to rad/s
Battery Capacity	100 to 1000 Wh	Impacts weight and time
Distance (ft)	52,800	Fixed mission range
Height (ft)	200	Climb/descent height

Parameter	Range / Value	Notes
Gravity (ft/s^2)	32.174	Constant
Cd, Ct	Various	Aerodynamic constants

Results and Discussion

Observations

- Propeller diameter and rotor tilt had strong influence on both noise and climb performance.
- Tip speed constraint was limiting at higher RPMs and required tradeoffs with diameter.
- Logical penalties enforced for coupling spacing < 0 reduced feasible design space.
- Constraint g(x4): battery weight and thrust vs payload most frequently active.
- Sweep plots showed that increasing rotor tilt improved forward speed, but hurt vertical lift.

Limitations and Assumptions

- Noise model assumes proxy based on thrust and blade dynamics, not frequency spectrum.
- Assumes linear thrust model and ignores environmental effects (wind, turbulence).
- Solver uses local optimization and may not always return global optimum.

Induvial Results per Iteration

On the plots below the optimal value is shown to be not in the feasible space. The reason why the optimal value is not in the feasible space in the 2d plot is because we have chosen multiple other variables to be nominal, so the optimal design point is for the optimal values for all the variables. While the plot shows it for only the two variables focused on. This can be visualized by the array of 2DV charts below:

Weighted Sum Equation = 0.3* Noise + 0.7 *Time

This run was for .3 Noise weighted value and 0.7-Time weighted value, and the results are below:

The optimal design was found using Excel Solver:

Variable	Optimal Value
Rotor Tilt	2.72 deg

Variable	Optimal Value
Propeller Diameter	0.85 in
Prop Count	4
Angular Velocity	224 Hz
Battery Capacity	605 Wh

- Weighted Objective Value: **0.5459**
- Normalized Noise: **0.4929**
- Normalized Time: **0.5686**
- Total Noise: **61.6dB**
- Time to Deliver: **367.055 seconds**

Constraint Evaluation at Optimum:

Constraint	Status	Value	Limit
Tip Speed	□	600.81 ft/s	<= 831.3
Weight – Thrust <= 0	□	-12.75 lbs	Pass
Weight Limit	□	37.28 lbs	<=55 lbs
Prop diameter max/min	□	0.85 in	[0.5, 2.0]
Rotor tilt min/max	□	2.72 deg	[0, 20]
Power Required - Battery Capacity <= 0	□	-397.51 Wh	Pass
Speed Limit	□	145.74 ft/s	<= 146.667 ft/s

Figure 1: Omega vs Battery Capacity 30/70 Weighted

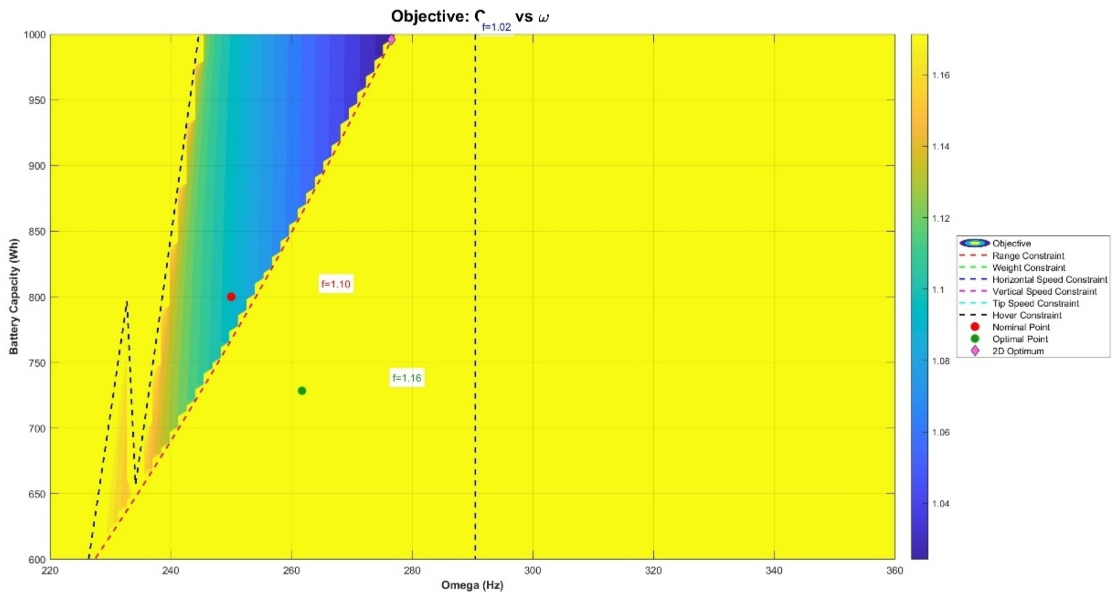


Figure 2: Propellor Diameter vs Number of Rotors 30/70 Weighted

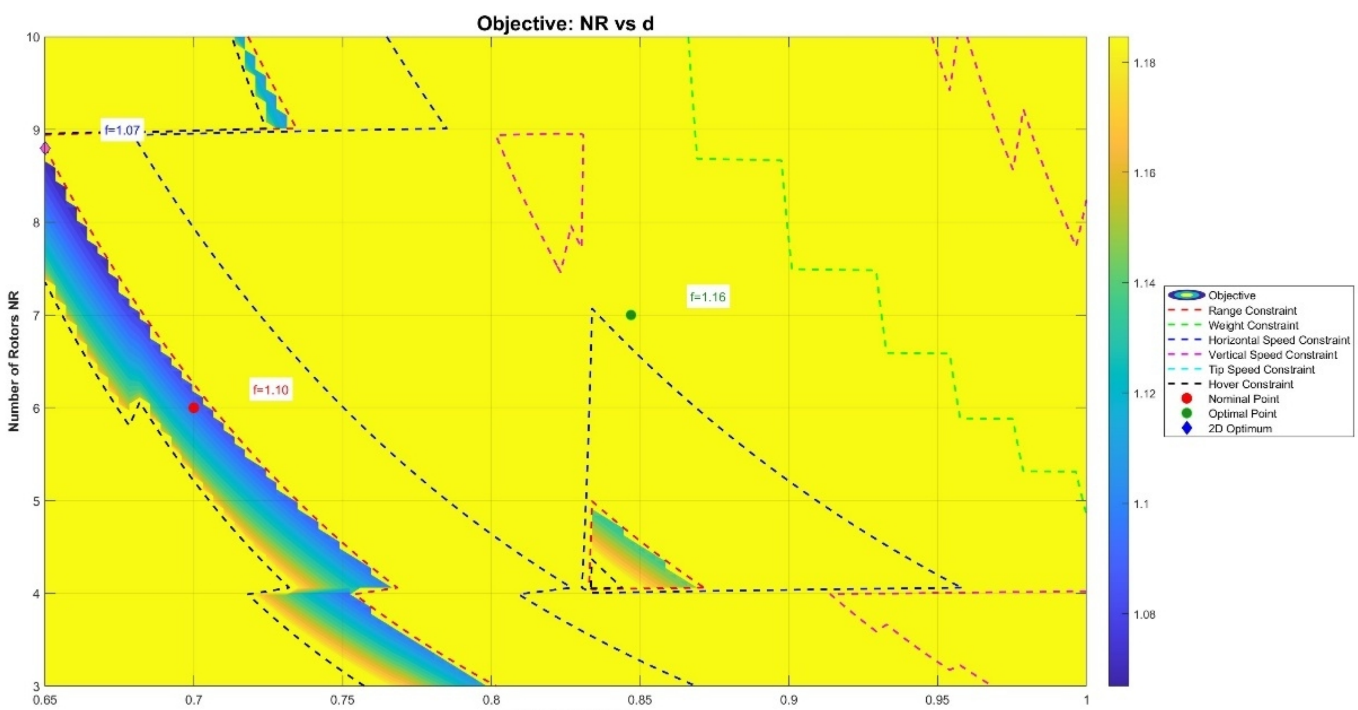


Figure 3: Tilt Angle vs Number of Rotors 30/70 Weighted

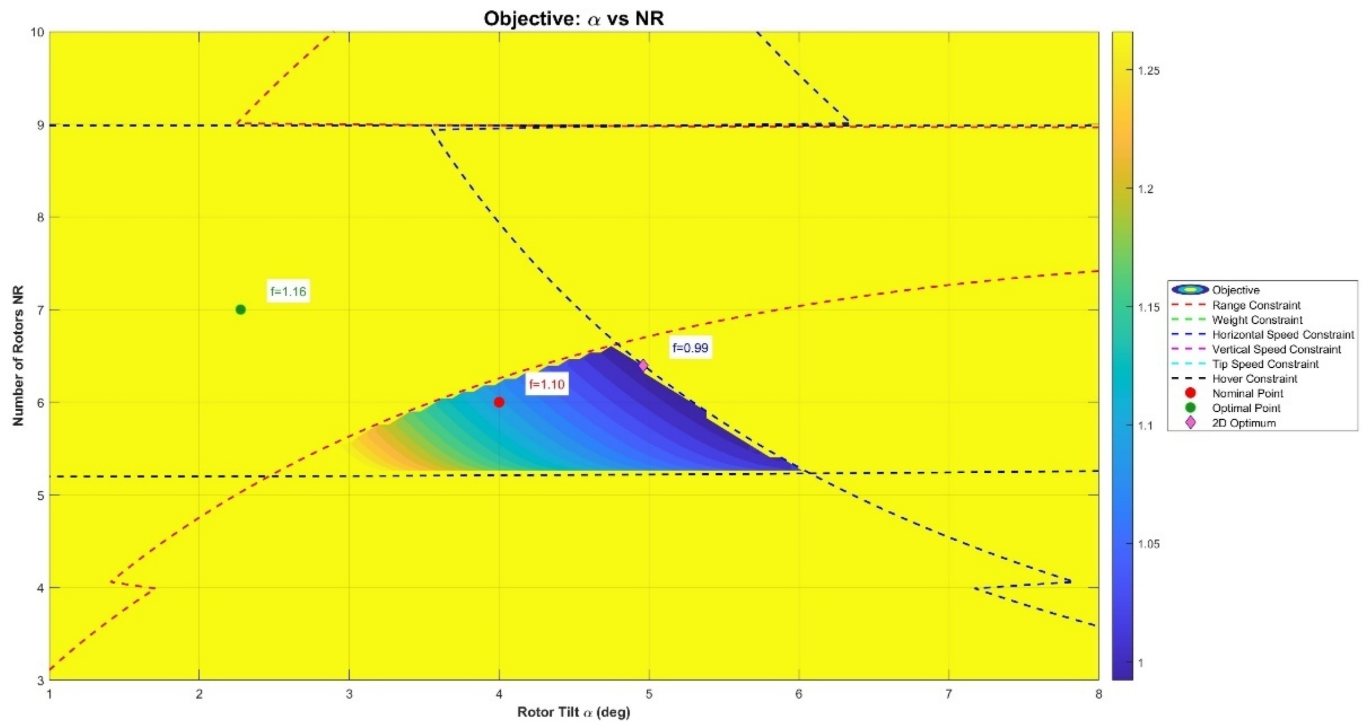
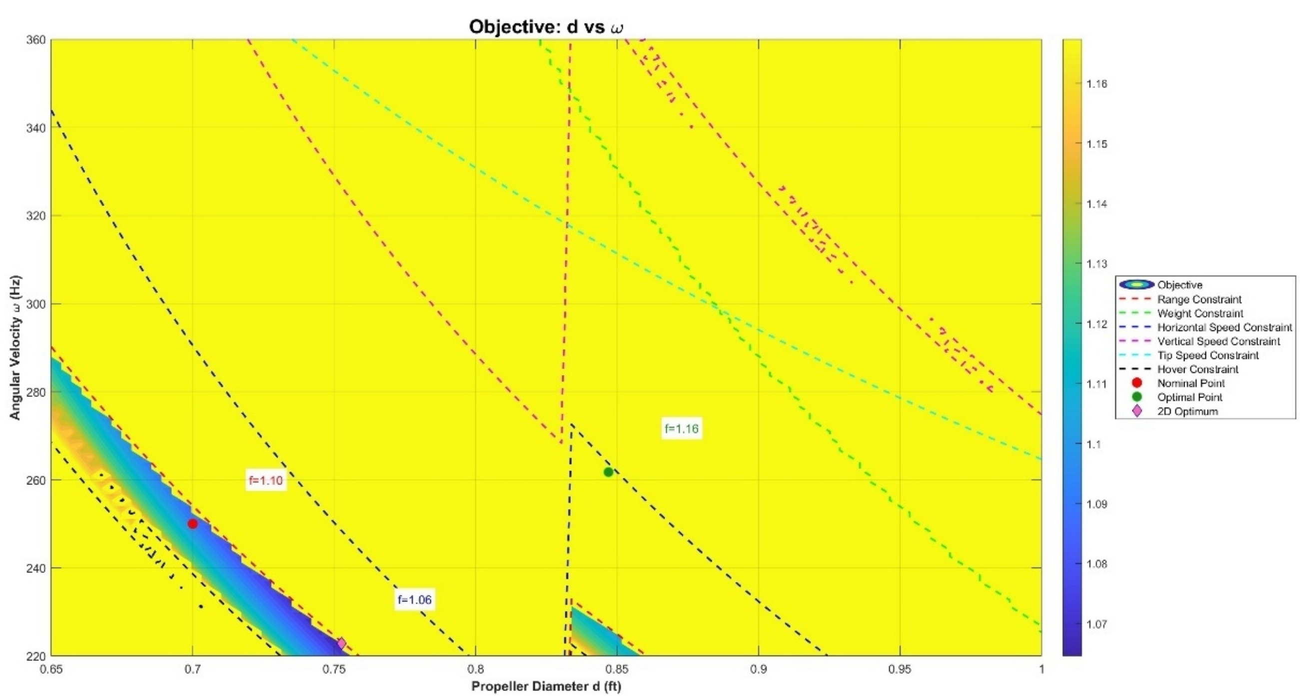


Figure 4: Propellor Diameter vs Angular Velocity 30/70 Weighted



Explain what Lagrange mean about active constraints

```
Lagrange Multipliers:
    0.0719
    0.0001
    0.7265
    0.0001
    0.0001
    0.0003
```

Each of these Figures 1-4 above present 2D slices of the weighed objective function plotted against the various design pairs. They highlight the feasible region bounded by constraints. In Figure 1, the combination of angular velocity and battery capacity shows a narrow viable region. Lower angular velocity and higher battery capacity yield better objective values, which is shown on the plot via the contour lines. Figure 2 shows that increasing the number of rotors improves performance for smaller propeller diameters but has diminishing returns for larger diameters. Figure 3 shows how rotor tilt angle interacts with rotor count, indicating that moderate tilt angles perform best in feasible zones. There is also a noticeable sweat spot for the prop diameter and tilt angle which is shown as the highlighted region in the Figure. Finally, Figure 4 demonstrates that larger prop diameters must be matched lower angular velocities to not break constraints. Across all plots, constraint boundaries significantly shape the feasible design space and in turn influence the optimal solutions.

Below are the variations of the above data for different weighted values.

Weighted Sum Equation = 0.5* Noise + 0.5 *Time

This run was for .5 Noise weighted value and 0.5-Time weighted value, and the results are below:

The optimal design was found using Excel Solver:

Variable	Optimal Value
Rotor Tilt	2.8 deg
Propeller Diameter	0.85 in
Prop Count	4
Angular Velocity	220 Hz
Battery Capacity	611 Wh

- Weighted Objective Value: **0.5307**
- Normalized Noise: **0.4896**
- Normalized Time: **0.57185**
- Total Noise: **61.45dB**
- Time to Deliver: **368.80 seconds**

Constraint Evaluation at Optimum:

Constraint	Status	Value	Limit
Tip Speed	□	591.35 ft/s	<= 831.3
Weight – Thrust <= 0	□	-11.01 lbs	Pass
Weight Limit	□	37.36 lbs	<=55 lbs
Prop diameter max/min	□	0.85 in	[0.5, 2.0]
Rotor tilt min/max	□	2.8 deg	[0, 20]
Power Required - Battery Capacity <= 0	□	-394.42 Wh	Pass
Speed Limit	□	145.14 ft/s	<= 146.667 ft/s

Figure 5: Omega vs Battery Capacity 50/50 Weighted

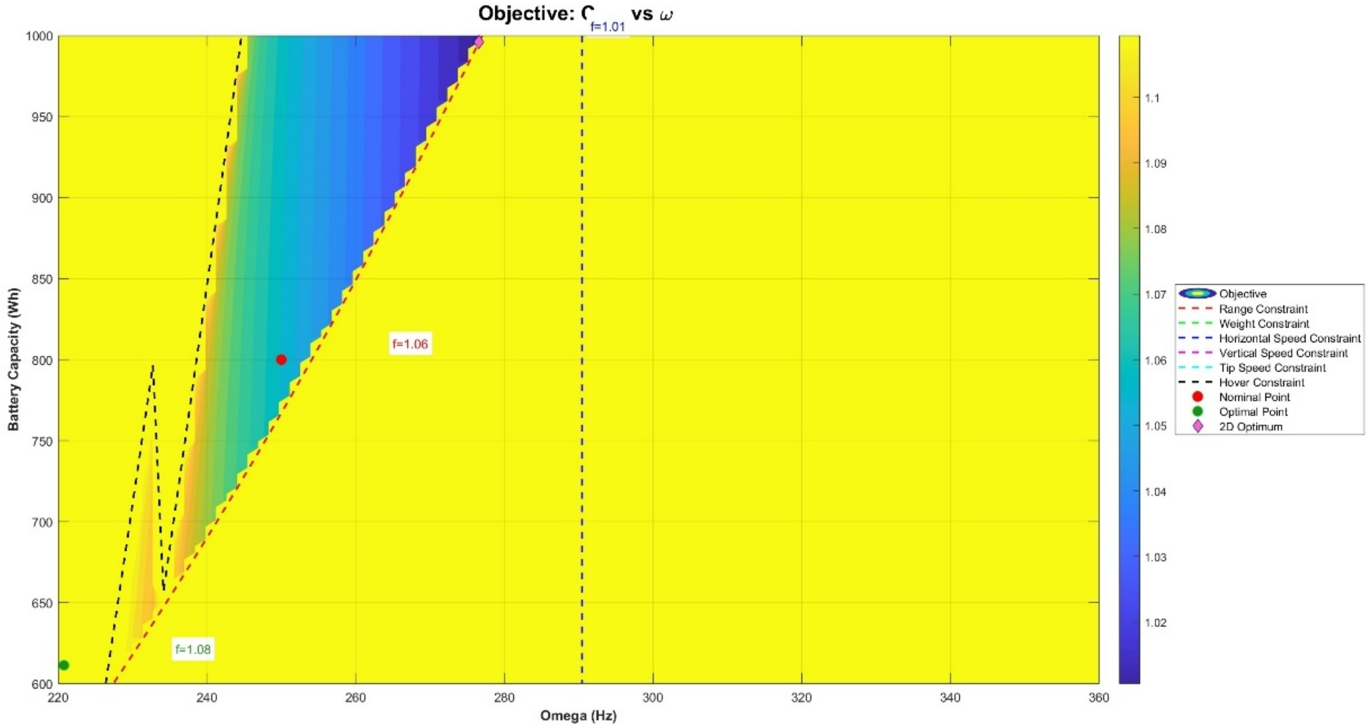


Figure 6: Propellor Diameter vs Number of Rotors 50/50 Weighted

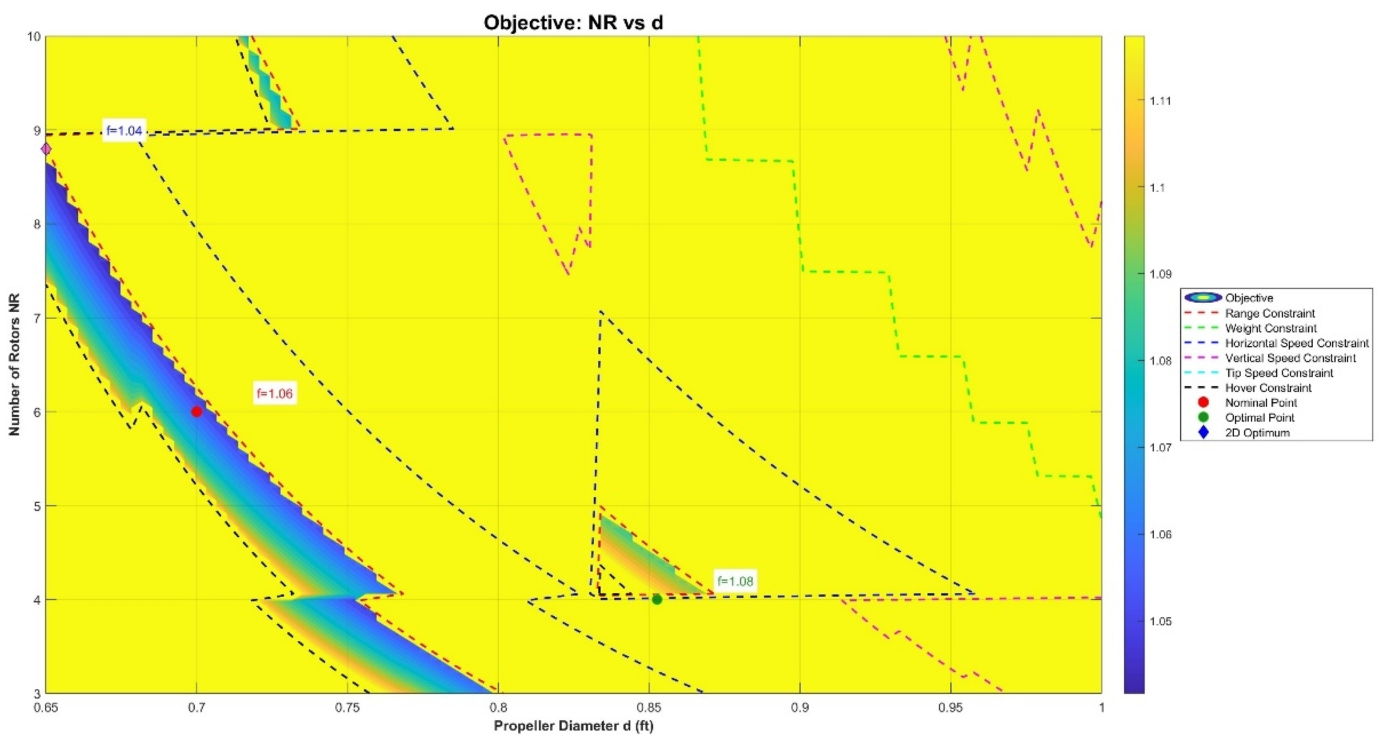


Figure 7: Tilt Angle vs Number of Rotors 50/50 Weighted

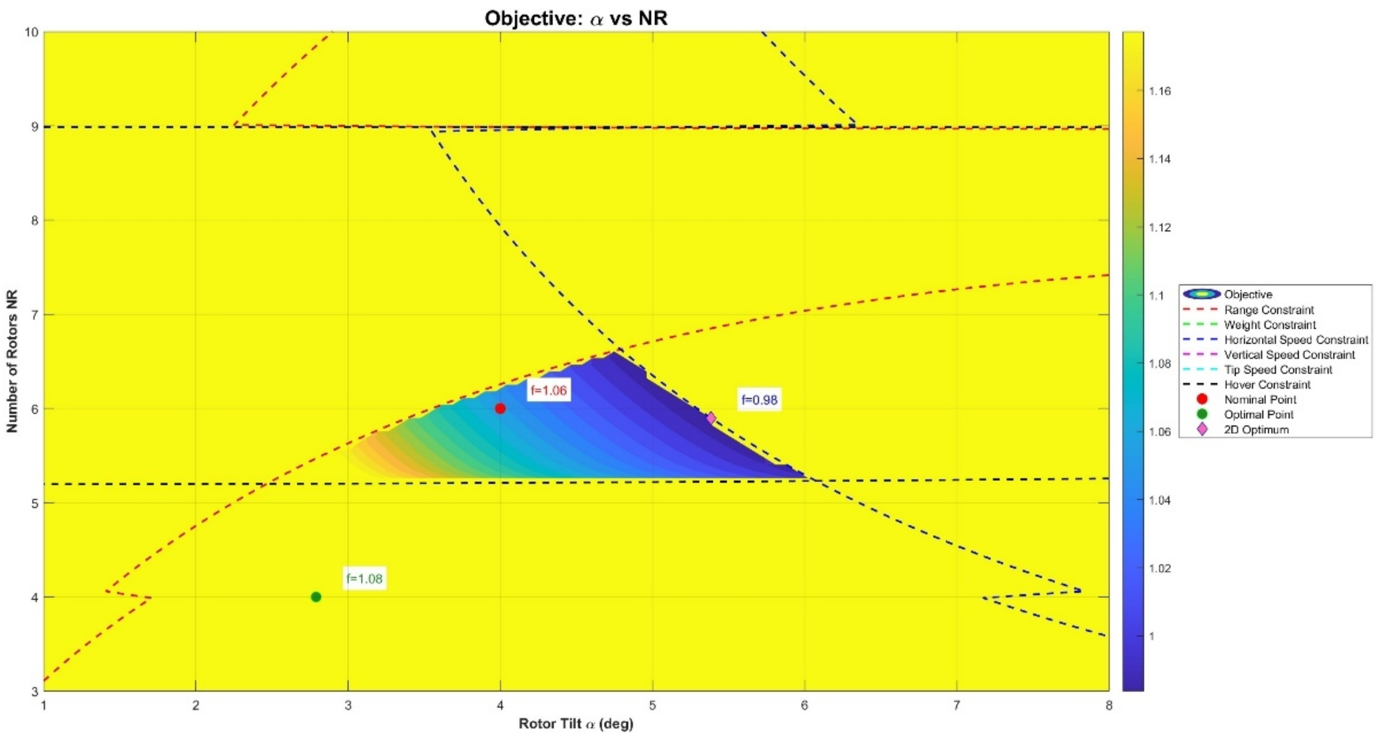
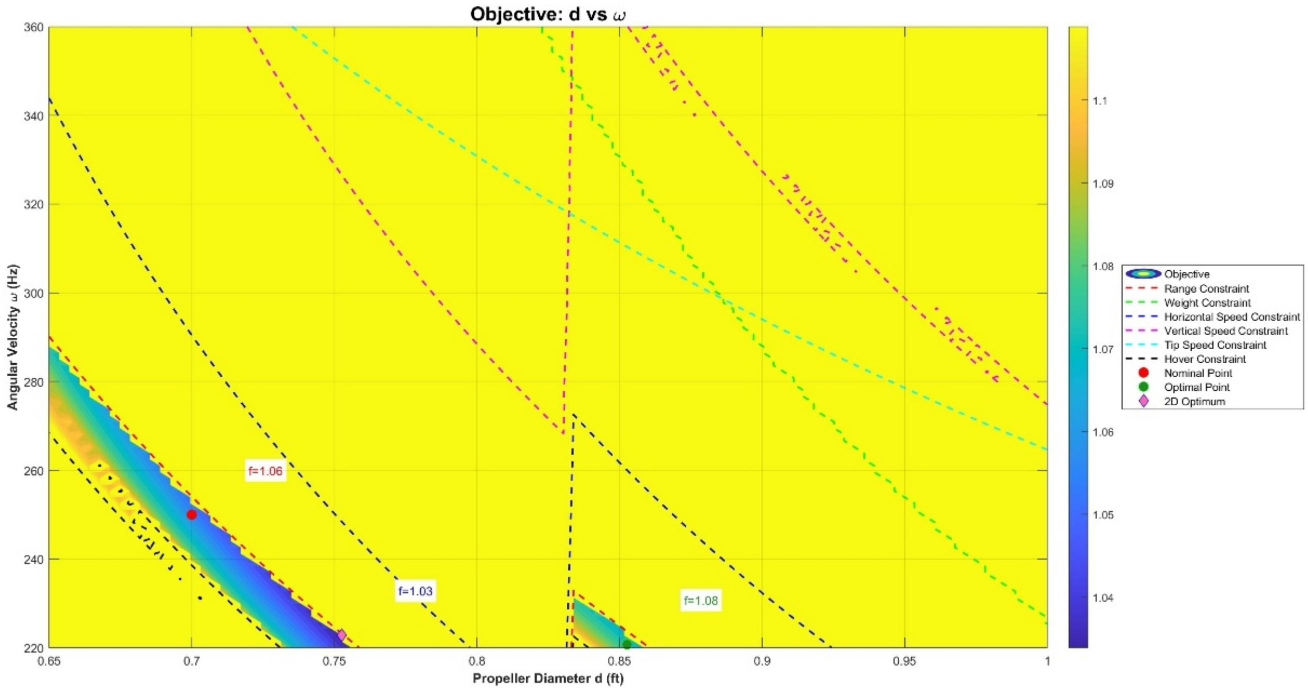


Figure 8: Propellor Diameter vs Angular Velocity 50/50 Weighted



Figures 5-8 display the updated 2D objective plots using a balanced 50/50 weighting between noise and delivery time. Each contour map reveals the regions where performance objective is minimized, while also indicating areas that violate the constraints. Like the figures 1-4 the same patterns were observed for the feasible regions and constraint equations. The main difference was where the optimal point was located and the value of the objective contours. These plots help visualize how optimal configurations shift when adjusting the relative importance of noise versus time. Below is the Lagrange Multipliers for this set of data/iteration.

```
Lagrange Multipliers:
    0.0728
    0.0001
    0.7271
    0.0001
    0.0001
    0.0001
```

Weighted Sum Equation = 0.7* Noise + 0.3 *Time

This run was for .7 Noise weighted value and 0.3-Time weighted value, and the results are below:

The optimal design was found using Excel Solver:

Variable	Optimal Value
Rotor Tilt	3.33 deg
Propeller Diameter	0.8688 in
Prop Count	3
Angular Velocity	224 Hz
Battery Capacity	580 Wh

- Weighted Objective Value: **0.4999**
- Normalized Noise: **0.4666**
- Normalized Time: **0.5766**
- Total Noise: **60.48dB**
- Time to Deliver: **371.93 seconds**

Constraint Evaluation at Optimum:

Constraint	Status	Value	Limit
Tip Speed	□	610.40 ft/s	<= 831.3
Weight – Thrust <= 0	□	-4.94 lbs	Pass
Weight Limit	□	35.21 lbs	<=55 lbs
Prop diameter max/min	□	0.87 in	[0.5, 2.0]
Rotor tilt min/max	□	3.33 deg	[0, 20]
Power Required - Battery Capacity <= 0	□	-301.61 Wh	Pass
Speed Limit	□	144.47 ft/s	<= 146.667 ft/s

Figure 9: Omega vs Battery Capacity 30/70 Weighted

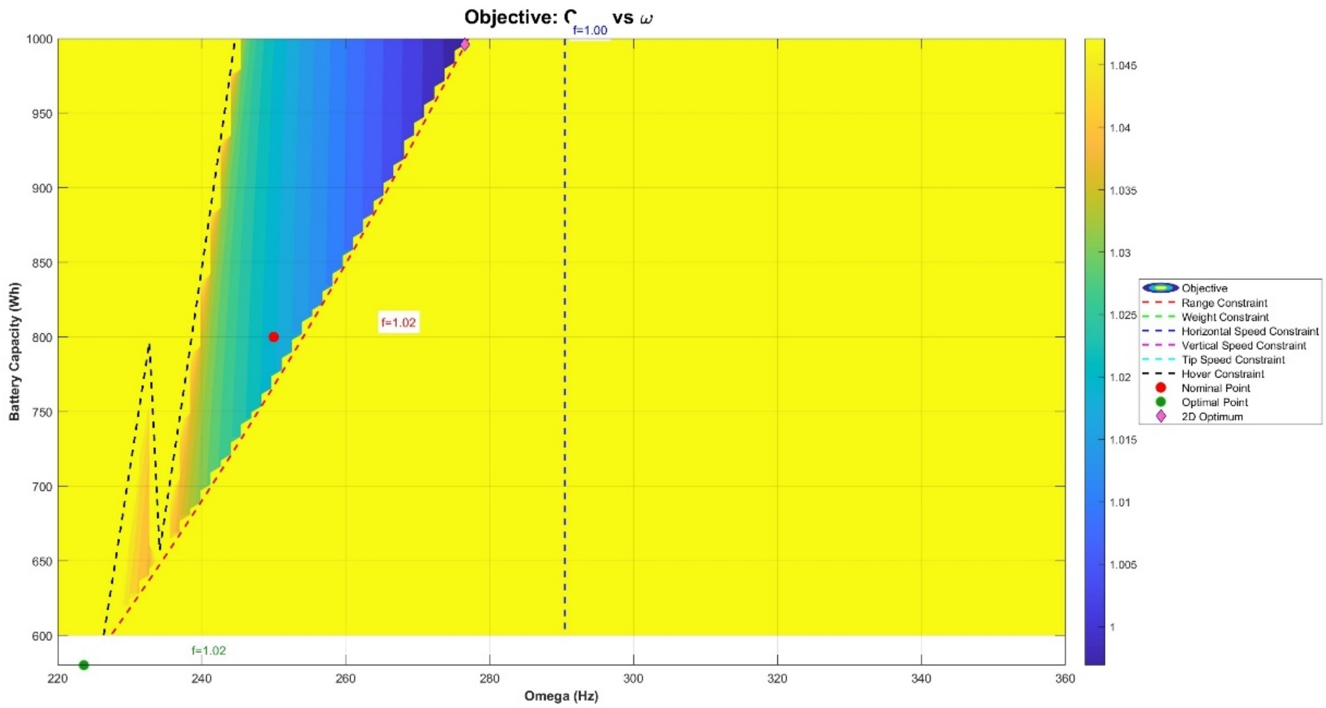


Figure 10: Propellor Diameter vs Number of Rotors 30/70 Weighted

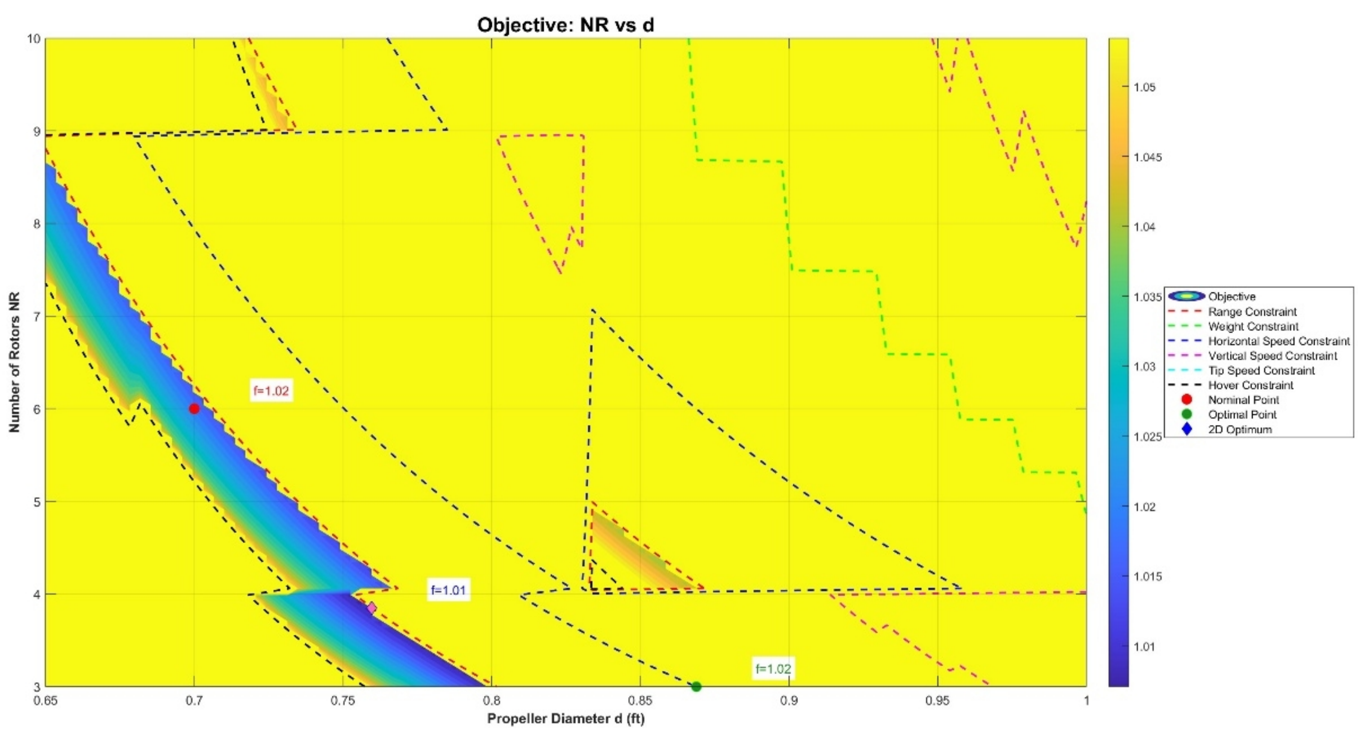


Figure 11: Tilt Angle vs Number of Rotors 30/70 Weighted

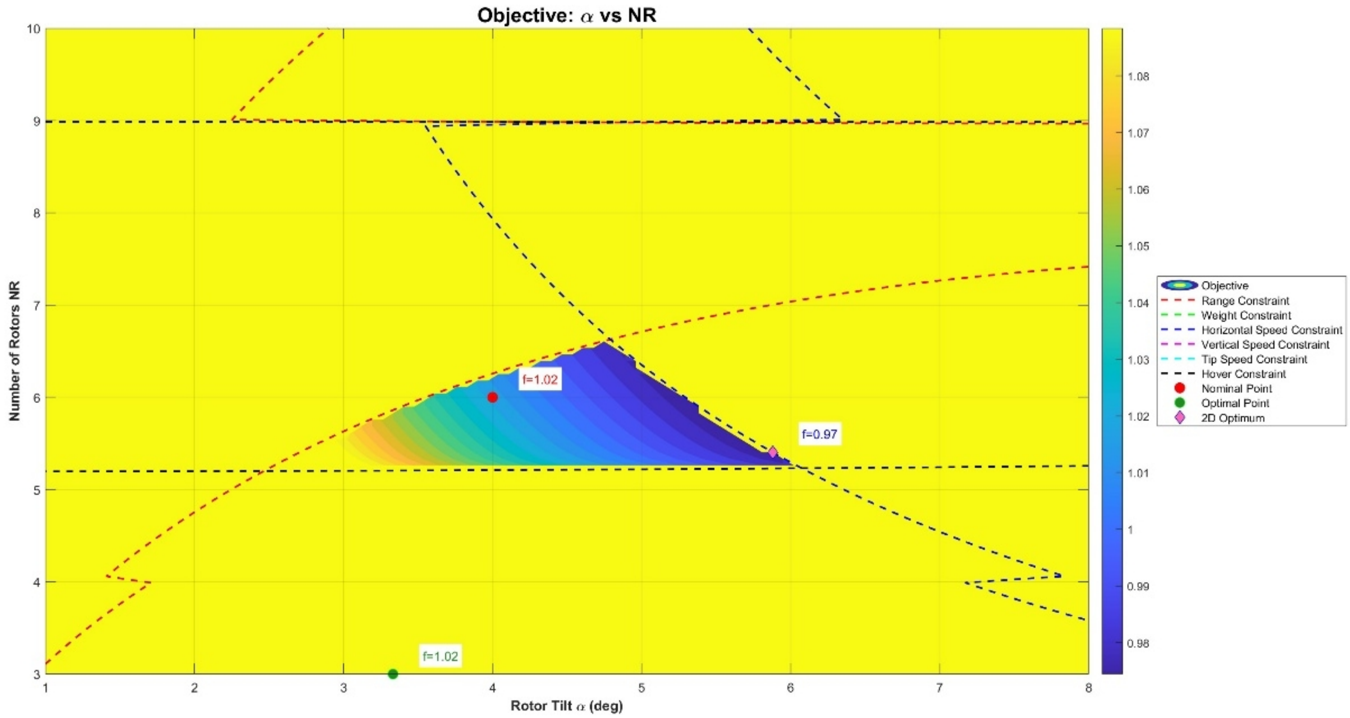
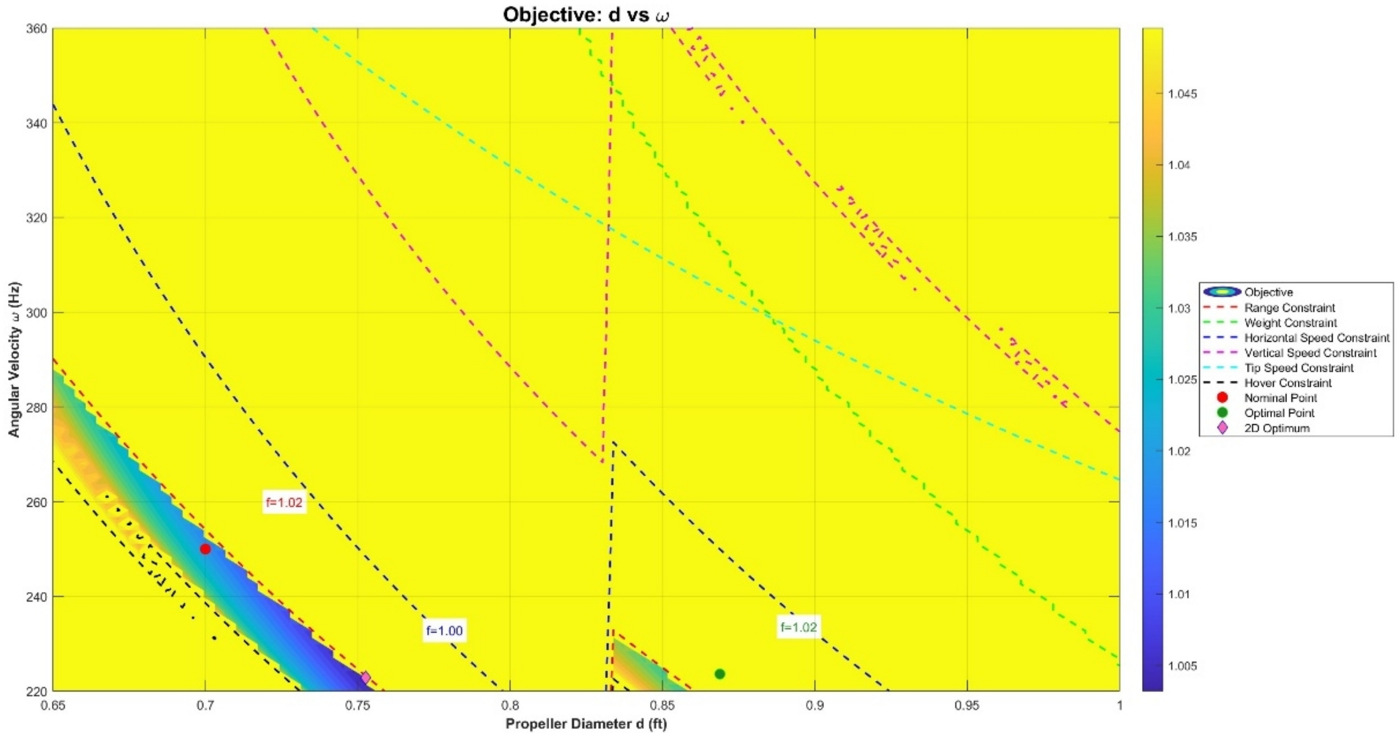


Figure 12: Propellor Diameter vs Angular Velocity 30/70 Weighted



Similar to Figures 1-8, Figures 9-12 are the examination of furthering the weight of noise in the objective function. The weight of noise was 70% and time was 30%. Comparing the 70/30 weighting , which favors lower noise levels, the optimizer went to lower number of rotors and are smaller angular velocity value. This makes logical sense because the noise equations is relient the most on those two values. They are quadratic so they have an even larger effect on the dB level. The interseting thing is that the delivery time only went up a couple seconds. This shows that we can reduce noise of the drone while only sacrificing minimal time penatlies. The lagrange values are shown below in the image. Overall, the change of the weighting of noise did not impact the time to deliver as much as expected. The main implications and realizations are below in the conclusion.

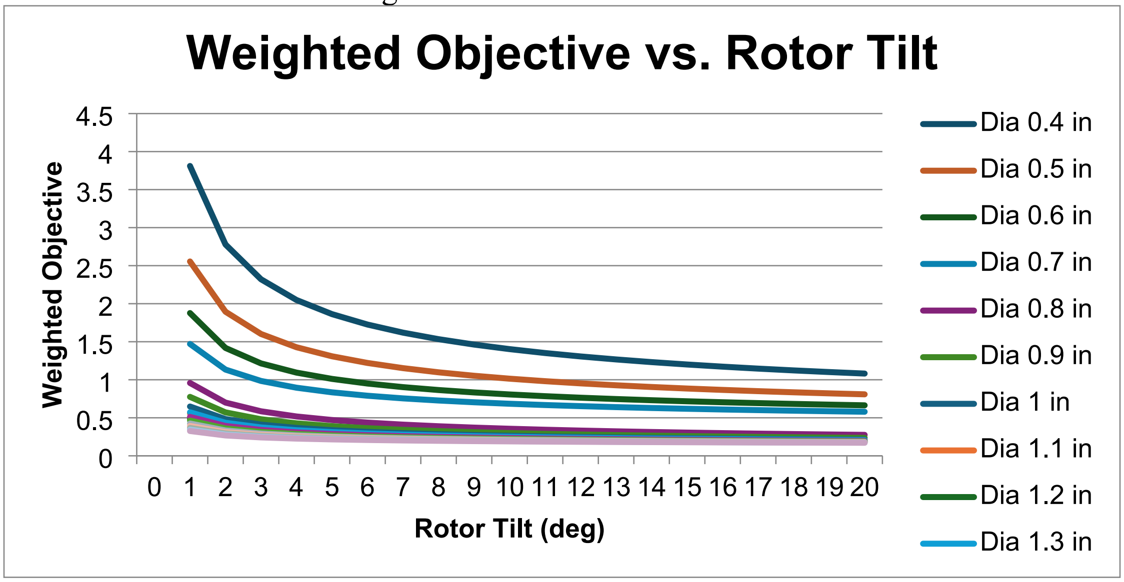
```
Lagrange Multipliers:
    0.1091
    0.0001
    0.7434
    0.0001
    0.0001
    0.0001
```

Conclusion

The drone's design was optimized to achieve a delivery time under 7 minutes while keeping noise within realistic bounds. Excel Solver successfully handled nonlinear constraints, integer and continuous variables, and logic-based penalties. Also, MATLAB was used for plotting and finding LaGrange multipliers values. The solution is physically valid, scalable, and meets mission objectives with all constraints satisfied. Further improvements could include acoustic simulation and motor thermal modeling.

Appendix

In this chart below it shows the effects of rotor tilt and diameter on the weighted objective value. It does not follow all the constraints because at certain sweeps/ranges it is not feasbile, but I wanted to show the entire range of values.



Exel Screenshots for proof of work/values.

Variables	Value	Details			Function	Normalized Function		Weighted Objective
Rotor Tilt	2.7197805	Decision variable (e.g., 0–20°)		Noise (F1)	61.59529468	0.492905288		0.545905752
Number of Propellers	4	Integer (e.g., 4, 6, 8)		Time to Delivery (F2)	367.0549277	0.568620237		
Prop Diameter	0.85354797	Decision variable (e.g., 6–20 in)						
Angular Velocity	224.056183	Decision variable (e.g., 300–1500) hz		Weight				
Battery Capacity	605.261241	Lower bound at 100 and upper bound at 1000		Noise	0.3			
Height (h)(	200	ft		Time	0.7			
Distance (D)	52800	ft farthest posssible range						
gravity (g)	32.174	ft/s^2						
Vup	66.3667259							
Tverticalflight	49.9864431	vertical thrust			Min Noise	40.73480991		
Wtotal	37.2829507	total weight			Max Noise	83		
Vforward	145.744272							
Tforward	2.37459654	thrust forward			Min Time to Del (s)	60		
Cd	0.05	interpolated			Max time (s)	600		
rho	0.00223582	density						
Area	2	Frontal Area sqft			the value is <= "☑", if the value is > 0 "✗"			
T_total	50.0428137	total thrust	g1(x): Rotor tilt	-17.2802195	☑			
Ct	0.21	thrust coefficent (varies)	g2(x): Prop dia min	-0.453547969	☑			
w_batt	7.86839613	battery weight	g3(x): Prop dia max	-1.146452031	☑			
W_prop	0.61455454	propellor weight	g4(x): weight limit	-17.71704933	☑			
W_motor	6.4	motor weight	g5(x): tip speed max	-230.50334	☑			
W_payload	14.4	pizza weight	g6(x): weight vs thrust	-12.75986306	☑			
W_empty	8	empty weight	g7(x) power required	-397.5148213	☑			
Tverticalzero	50.0428137	initial veritcal up	g8(x) speed limit	-0.922728201	☑	finding motor weight		
tip speed	600.80666	ft/s	l	2.5		torque total	6.79813501	

	A	B	C	D	E	F	G	H	I	J	K
1	**Variables**	**Value**	**Details**			**Function**	**Normalized Function**		**Weighted Objective**		
2	Rotor Tilt	2.79026422	Decision variable (e.g., 0–20°)		Noise (F1)	61.45744976	0.489648197		0.530749099		
3	Number of Propellers	4	Integer (e.g., 4, 6, 8)		Time to Delivery (F2)	368.7990008	0.571850001				
4	Prop Diameter	0.85263223	Decision variable (e.g., 6–20 in)								
5	Angular Velocity	220.765332	Decision variable (e.g., 300–1500) hz		Weight						
6	Battery Capacity	611.396434	Lower bound at 100 and upper bound at 1000		Noise	0.5					
7	Height (h)(	200	ft		Time	0.5					
8	Distance (D)	52800	ft farthest posssible range								
9	gravity (g)	32.174	ft/s^2								
10	Vup	61.5924592									
11	Tverticalflight	48.3180786	vertical thrust			Min Noise	40.73480991				
12	Wtotal	37.3620488	total weight			Max Noise	83				
13	Vforward	145.13913									
14	Tforward	2.35491844	thrust forward			Min Time to Del (s)	60				
15	Cd	0.05	interpolated			Max time (s)	600				
16	rho	0.00223582	density								
17	Area	2	Frontal Area sqft			the value is <= "☑", if the value is > 0 "✗					
18	T_total	48.3754314	total thrust	g1(x): Rotor tilt	-17.20973578	☑					
19	Ct	0.21	thrust coefficent (varies)	g2(x): Prop dia min	-0.452632228	☑					
20	w_batt	7.94815364	battery weight	g3(x): Prop dia max	-1.147367772	☑					
21	W_prop	0.6138952	propellor weight	g4(x): weight limit	-17.63795116	☑					
22	W_motor	6.4	motor weight	g5(x): tip speed max	-239.9628714	☑					
23	W_payload	14.4	pizza weight	g6(x): weight vs thrust	-11.01338253	☑					
24	W_empty	8	empty weight	g7(x) power required	-394.4260766	☑					
25	Tverticalzero	48.3754314	initial veritcal up	g8(x) speed limit	-1.527870133	☑	finding motor weight			propellor weight	
26	tip speed	591.347129	ft/s	l	2.5		torque total	6.5645767		density of plastic	(
27				w	2.5		torque per rotor	1.64114417		thickness	(

	A	B	C	D	E	F	G	H	I
1	**Variables**	**Value**	**Details**			**Function**	**Normalized Function**		**Weighted Objective**
2	Rotor Tilt	3.33205847	Decision variable (e.g., 0–20°)		Noise (F1)	60.48358617	0.466637106		0.499944622
3	Number of Propellers	3	Integer (e.g., 4, 6, 8)		Time to Delivery (F2)	371.9375649	0.577662157		
4	Prop Diameter	0.86888133	Decision variable (e.g., 6–20 in)						
5	Angular Velocity	223.618817	Decision variable (e.g., 300–1500) hz		Weight				
6	Battery Capacity	580.039565	Lower bound at 100 and upper bound at 1000		Noise	0.7			
7	Height (h)(	200	ft		Time	0.3			
8	Distance (D)	52800	ft farthest posssible range						
9	gravity (g)	32.174	ft/s^2						
10	Vup	42.4742574							
11	Tverticalflight	40.0775452	vertical thrust			Min Noise	40.73480991		
12	Wtotal	35.2097103	total weight			Max Noise	83		
13	Vforward	144.473143							
14	Tforward	2.33335642	thrust forward			Min Time to Del (s)	60		
15	Cd	0.05	interpolated			Max time (s)	600		
16	rho	0.00223582	density						
17	Area	2	Frontal Area sqft			the value is <= "☑", if the value is > 0 "✗			
18	T_total	40.145413	total thrust	g1(x): Rotor tilt	-16.66794153	☑			
19	Ct	0.21	thrust coefficent (varies)	g2(x): Prop dia min	-0.468881325	☑			
20	w_batt	7.54051434	battery weight	g3(x): Prop dia max	-1.131118675	☑			
21	W_prop	0.46919592	propellor weight	g4(x): weight limit	-19.79028975	☑			
22	W_motor	4.8	motor weight	g5(x): tip speed max	-220.9041582	☑			
23	W_payload	14.4	pizza weight	g6(x): weight vs thrust	-4.935702697	☑			
24	W_empty	8	empty weight	g7(x) power required	-301.6090512	☑			
25	Tverticalzero	40.145413	initial veritcal up	g8(x) speed limit	-2.193857067	☑	finding motor weight		
26	tip speed	610.405842	ft/s	l	2.5		torque total	5.551579	