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# Algorithm 1

**Algorithm 1** is designed to find the densest subgraph within a given network. It sets up a flow network where the goal is to determine how strongly nodes are connected at different density thresholds. By running a minimum s-t cut algorithm, it divides the graph into two disjoint sets: one containing the source node and the other containing the sink node. The minimum cut identifies the smallest capacity of edges needed to separate these sets, revealing whether a densely connected group of nodes exists. If a meaningful subgraph is found (i.e.,  $S$  contains more than just the source), the algorithm increases the density guess; otherwise, it lowers it. Through this binary search over density values, the algorithm efficiently zooms in on the densest subgraph in the network.

## What does finding disjoint sets ( $S$ , $T$ ) mean here?

- After setting up the **flow network** (adding all edges and capacities),
- The algorithm runs a **minimum s-t cut** algorithm.

This **divides** the nodes into two **disjoint sets**:

- $S$ : the set containing the source node  $s$ .
- $T$ : the set containing the sink node  $t$ .

**No node** is in both  $S$  and  $T$  — they are disjoint. And **the total capacity** of edges going from  $S$  to  $T$  is **minimized**.

## Why do we compute the minimum cut?

- The minimum cut tells **how much "flow" is needed to separate  $s$  from  $t$** .
- If the minimum cut is **small**, it suggests a **dense cluster** of nodes in  $S$  (excluding  $s$ ).
- The nodes in  $S$  (**excluding  $s$** ) are candidates for being the **densest subgraph**.

- If **S only contains {s}**, it means there is **no dense enough subgraph** at the current density threshold  $\alpha$ , so the algorithm adjusts the search.

### After finding (S, T):

- If **S = {s} only**  $\rightarrow$  then no dense subgraph found at current guess  $\alpha \rightarrow$  **decrease upper bound  $u = \alpha$** .
- Else  $\rightarrow$  we **found a dense subgraph**  $\rightarrow$  **increase lower bound  $l = \alpha$**  and **set D to the subgraph induced by S{s}**.

### Intuition:

- The algorithm **searches** for the highest  $\alpha$  (density) where a nontrivial cut exists.
- The **minimum s-t cut** helps find groups of vertices that can "support" that density.

### In simple terms:

Step	Meaning
Build flow network	Encode density constraints into flow edges.
Find min s-t cut (S, T)	Separate graph into two parts minimizing cut capacity.
Analyze S	If S is non-trivial (more than {s}), it represents a <b>candidate densest subgraph</b> . Otherwise, adjust $\alpha$ .

# Algorithm 2

## What the $(k, \Psi)$ -Core Decomposition algorithm does:

- It **computes the clique-core number** for every vertex in a graph.
  - The  **$(k, \Psi)$ -core** is the largest subgraph where **each vertex participates in at least  $k$  instances** of a specific pattern  $\Psi$  (often  $\Psi$  is a clique, but it could be other patterns like stars or loops).
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## How it works:

1. **Compute the clique-degree** of each vertex (number of cliques it participates in).
  2. **Sort vertices in increasing order** of their clique-degree.
  3. **Iteratively remove** the vertex with the **smallest** clique-degree:
    - When a vertex is removed, **decrease** the clique-degree of its neighbors that share clique instances with it.
    - **Resort** vertices efficiently using **bin-sort**.
  4. Record the **clique-core number** for each vertex when it is removed.
  5. Continue until all vertices are removed.
  6. Finally, return the array containing the **clique-core numbers**.
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## Advantages of $(k, \Psi)$ -Core Decomposition:

- **Efficient:** The algorithm is designed to run in  $O(n \cdot d^{(h-1)})$  time and  $O(m)$  space, where:
  - $n$  = number of vertices,
  - $d$  = maximum degree,

- $h$  = size of the cliques.
- **Scalable:** Using bin-sort allows **fast re-sorting** after every removal.
- **Generalizable:** Can handle **any pattern  $\Psi$**  (not just cliques), making it flexible.
- **Helpful for Core-Based Algorithms:**
  - Reduces the size of the graph for further computations (e.g., finding densest subgraphs).
  - Provides **tighter bounds** and **smaller search spaces** in optimization problems like dense subgraph discovery.

## Dataset used:-

Name
Yeast
Netscience
As-733
Ca-HepTh
As-Caida

# Results

## 2. Netscience Dataset

### Exact (Algo-1) Performance

Nodes	Edges	Density	Execution Time	Clique Number
1589	2742	9.000023	3.771 s	2
1589	2742	58.9992	0.208 s	3
1589	2742	242.7	2.680 s	4

### CoreExact (Algo-4) Performance

Nodes	Edges	Density	Execution Time	Clique Number
1589	2742	9.5273	18.5757 s	2
1589	2742	56.628	23.8944 s	3
1589	2742	242.0000	923.63 s	4

## 3. As20000102 Dataset

### Exact (Algo-1) Performance

Nodes	Edges	Density	Execution Time	Clique Number
6474	13233	8.8	13.2441 s	2
6474	13233	36.01	7.944 s	3
6474	13233	85.5	14.713 s	4

### CoreExact (Algo-4) Performance

Nodes	Edges	Density	Execution Time	Clique Number
6474	13233	8.87	230.17 s	2
6474	13233	36.88	223.2 s	3
6474	13233	84.9999	1043.8 s	4

4. CA-HepTh Dataset

Exact (Algo-1) Performance

Nodes	Edges	Density	Execution Time	Clique Number
9877	51971	15.5234	4.6s	2
9877	51971	155.002	4.465 s	3
9877	51971	1123.752	19.2812 s	4

CoreExact (Algo-4) Performance

Nodes	Edges	Density	Execution Time	Clique Number
9877	51971	15.4	512.2 s	2
9877	51971	154.8	1022.1 s	3
9877	51971	1123.75	2309.9 s	4

5. AS-Caida Dataset

Exact (Algo-1) Performance

Nodes	Edges	Density	Execution Time	Clique Number
26475	106762	17.5341	4.2764 s	2
26475	106762	114.847	86.2427 s	3
26475	106762	405.333	211.743 s	4

CoreExact (Algo-4) Performance

Nodes	Edges	Density	Execution Time	Clique Number
26475	106762	17.535	1.32 hrs	2
26475	106762	114.85	3.59 hrs	3
26475	106762	405.333	6.77hrs	4