

AI Assignment

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Question: 1

(a) Comparing Sampling strategies:-

→ Direct Sampling:-

↳ Draw sample from probability distribution.

* Strength:-

- Simple and efficiently works for known distributions.
- Easy for calculation in small and low dimension data.

* Weakness:-

- Does not work well for complex calculation (distribution) and higher dimensions data.

→ Rejection Sampling:-

↳ Sample accepts based on a probability criteria,

* Strength:-

- Use for unnormalized distributed data.
- Approximate complex distribution if good proposal (sample) chosen.

* Weakness:-

- Unefficient when multiple samples are rejected.
- Carefully choose sample (proposal) distribution.

→ Gibbs Sampling:-

↳ Iterate Samples from Conditional distribution of variables.

* Strength:-

↳ handles high dimensional distribution
↳ Very helpful as if Conditional distribution easy to compute.

* Weakness.

↳ Slow convergence for poorly chosen value.

↳ Assume conditional independent among other variables.

$$(b) P(L/T) = 0.400 \text{ — Given}$$

L - Leisure, T - Train

$$n = 100 \text{ sample people.}$$

$$N_t = 30 \text{ trains } \} \text{ Given}$$

$$N_{LT} = P(L/T) * N_t$$

$$\Rightarrow 0.400 \times 30$$

$$\Rightarrow \underline{12 \text{ train's traveller}}$$

$a \rightarrow \text{air}; b \rightarrow \text{business}$

(c) Given data:- $\begin{cases} P(a) = 0.8 \\ P(b/a) = 0.2 \end{cases}$

$$\begin{aligned} \hookrightarrow P(a \cap b) &= P(a) \times P(b/a) \quad \rightarrow \text{by Bayes' theorem.} \\ &= 0.8 \times 0.2 \\ &= \boxed{0.16} \end{aligned}$$

(d) Effect of sample size on Direct Sampling

Accuracy:-

- \hookrightarrow Large sample sizes reduce bias
- \hookrightarrow Estimate converge to true probabilities.
- \hookrightarrow Laws of large no. applies.

Precision:-

- \hookrightarrow Standard error decrease with \sqrt{n} .
- \hookrightarrow Interval of confidence became narrow.
- \hookrightarrow More reliable probability estimates.

Implication of Datasets:-

- \hookrightarrow More sample needed for rare events.
- \hookrightarrow Better estimation of conditional probabilities
- \hookrightarrow Improved reliability of model pattern analysis.

Q2^(a) Identify variables and write statements.

b = read book ; j = Access journals ; C = participate in book club

$$1. P(b \cup j) = 0.91$$

$$2. P(j|b) = 0.4$$

$$3. P(b \cap j) = 0.6$$

$$4. P(C|b) = 0.32$$

$$5. P(j \cap b) = 0.227$$

$$6. P(b \cap j) = 0.09$$

$$7. P(j|b) = 0.916$$

$$8. P(C \cap j) = 0.088$$

$$9. P(C \cup j) = 0.631$$

$$10. P(j|C) = 0.4$$

$$11. P(j) = 0.5$$

(b) non-negative :- $0 \leq P(A) \leq 1$ all events A .
 Normalize :- The sum of all possible outcomes must 1.

$$\rightarrow P(b \cup j) = P(b) + P(j) - P(b \cap j)$$

$$P(b \cap j) = P(b) + P(j) - P(b \cup j)$$

$$= P(b) + 0.5 - 0.91$$

$$P(b) \cdot P(j|b) = P(b) - 0.91 + 0.5$$

$$P(b) \cdot (0.4) = P(b) = 0.91 + 0.5$$

$$P(b) = \frac{0.41}{0.6}$$

$$P(b) = 0.683$$

(C)

B \ J \ C

 $P(B, J, C)$

0 0 0

0.09

0 0 1

0.00 0.163

0 1 0

0.139

0 1 1

0.048 0.04

1 0 0

0.411 0.279

1 0 1

0.072 0.131

1 1 0

0.012 0.225

1 1 1

0.048

(d) Check conditional independence of joint distribution
 ↳ check if B & C are conditionally indep-
 given J :- $P(B \cap C | J) = P(B | J) \cdot P(C | J)$

using joint distribution:-

$$P(B \cap C \cap J) = P(B | J) \cdot P(C | J) \cdot P(J)$$

from table

we compute

$$P(C | J) = 0.176$$

$$P(B | J) = 0.546$$

$$P(J) =$$

$$P(B \cap C | J) = 0.96 = \frac{0.176 \times 0.546}{1} <$$

que.3

(9) Bayesian Inference formulae:-

A - Adversarial perturbations

B - Backdoor attacks

M - Misclassify alarm

→ A & B are independent

$$P(A \cap B) = P(A) \times P(B)$$

$$\Rightarrow P(A/M, B) = \frac{P(M/A, B) \times P(A/B)}{P(M/B)}$$

$$\Rightarrow A \text{ independent } B \Rightarrow P(A/B) = P(A)$$

$$\text{So, that:- } P(A/M, B) \neq P(A/M)$$

(5) Probability parameters:-

⇒ Prior probabilities:-

$P(A)$ = probability of Adversarial perturbations

$P(B)$ = probability of Backdoor attacks

$P(C)$ = misclassified alarm.

→ Likelihood:-

$P(M/A)$ = misclassified given A

$P(M/B)$ = misclassified given B

$P(M/A, B)$ = misclassified given A & B.

⇒ Posterior :-

$P(A|M)$:- A given M misclassified
 $P(A|M, B)$ = probability of A given
 Misclassified & B.

(C)

1. Away effect :-

- when B is observed.
- $P(A|M, B) < P(A|M)$
- This reduction occurs because B provides an alternative of M.

2. Reasoning :-

↳ Misclassified determined by by A or B
 ↳ If B is there then A is comparatively less.

$$\Rightarrow P(A|M, B) = \frac{P(M|A, B) \times P(A)}{P(M|B)}$$