

Q1.

```
def visitall (G):  
    visit(v1,1)
```

```
def visit(node v, clock t):  
    v.pre=clock  
    clock++
```

```
mark v  
v.low = v.pre
```

```
for each edge v-w:  
    if v is marked:  
        v.low = min(v.low, w.pre)  
    else:  
        t = visit(w, clock)  
        clock = t  
        v.low = min(v.low, w.low)
```

```
return clock
```

No point if global variable(s)/stack(s) were used.

Points deducted for not updating clock correctly and not ensuring that $v1.pre=1$.

Q2.

(continued after the proof given in question)

Let v be one vertex to which there is no path from u . Since G is semi-connected, there must be a path from v to u .

There can be two cases:

(a) v is visited before u during DFS. Since there is path from v to u , so u will be discovered during DFS(v) and hence, u will be finished before v is finished. So, $finish(u) < finish(v)$ leading to a contradiction of the fact that $finish(u)$ is largest.

(b) u is visited before v during DFS. Since there is no path from u to v , when u is finished v will still be unexplored. Hence, v will be visited after u is finished, and therefore, $finish(v) > finish(u)$.

This also contradicts the fact that $finish(u)$ is largest.

Point deducted for using " u is the root or one of the roots of the DFS traversal" without explaining any reason.

Q3. Easy counter-example.

Q4. (a) Yes (b) No (c) $k' = |V| - 2k$ (d) Yes (since you can easily show that HalfClique is NP-complete based on the facts in Q4.)

Q5.

```

def Reduce3SAT_Clique(F):
    k = m
    Construct G in the following manner:
        for every clause (la OR lb OR lc), where la, lb, lc are literals:
            add there vertices, one for each literal
        for every pair of vertices (vi, vj):
            if vi and vj belong to the same `clause': do not add any edge
            if vi and vj correspond to a variable and its negation: do not
add any edge
            else, add an edge between vi and vj
    return (G,k)

```

- (i) Number of vertices in $G = 3m$
- (ii) Number of edges in $G \leq 9m(m-1)/2 = (3m \text{ choose } 2) - 3m$
- (iii) $k=m$
- (iv) Output of reduction : simple