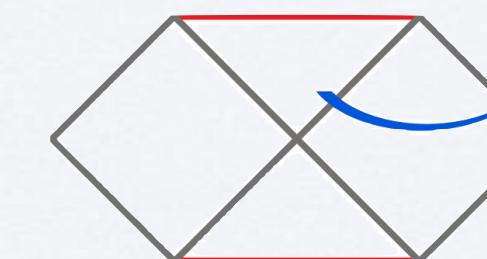


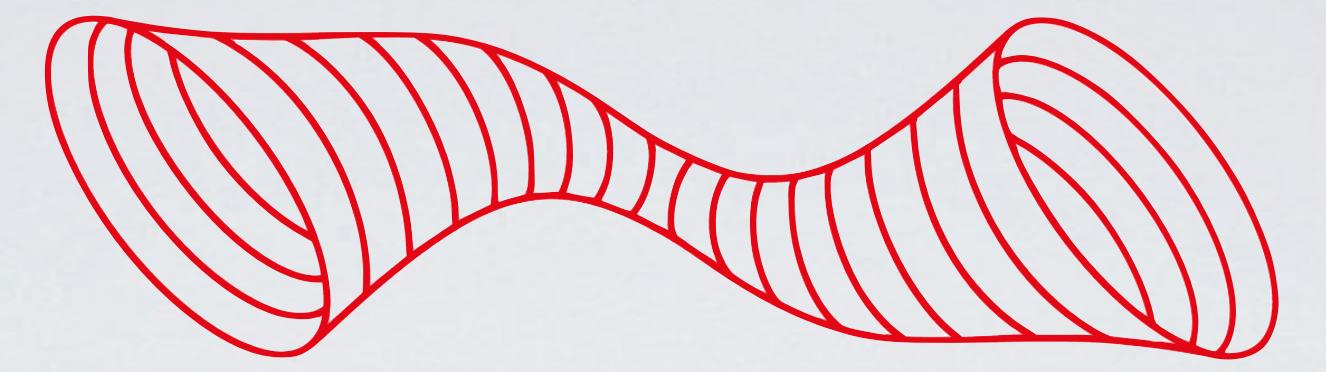
THE CENTER OF GRAVITY

GEOMETRY AND CURVED SPACES (TIMES?)

Rodrigo Panosso Macedo



<https://hyperboloid.al>

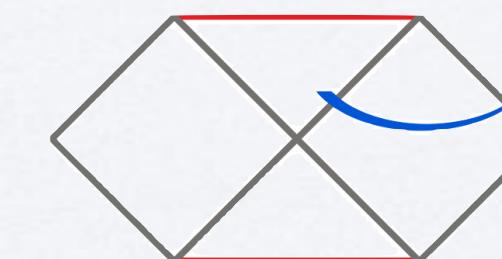


THE CENTER OF GRAVITY

GEOMETRY AND CURVED SPACES (TIMES?)

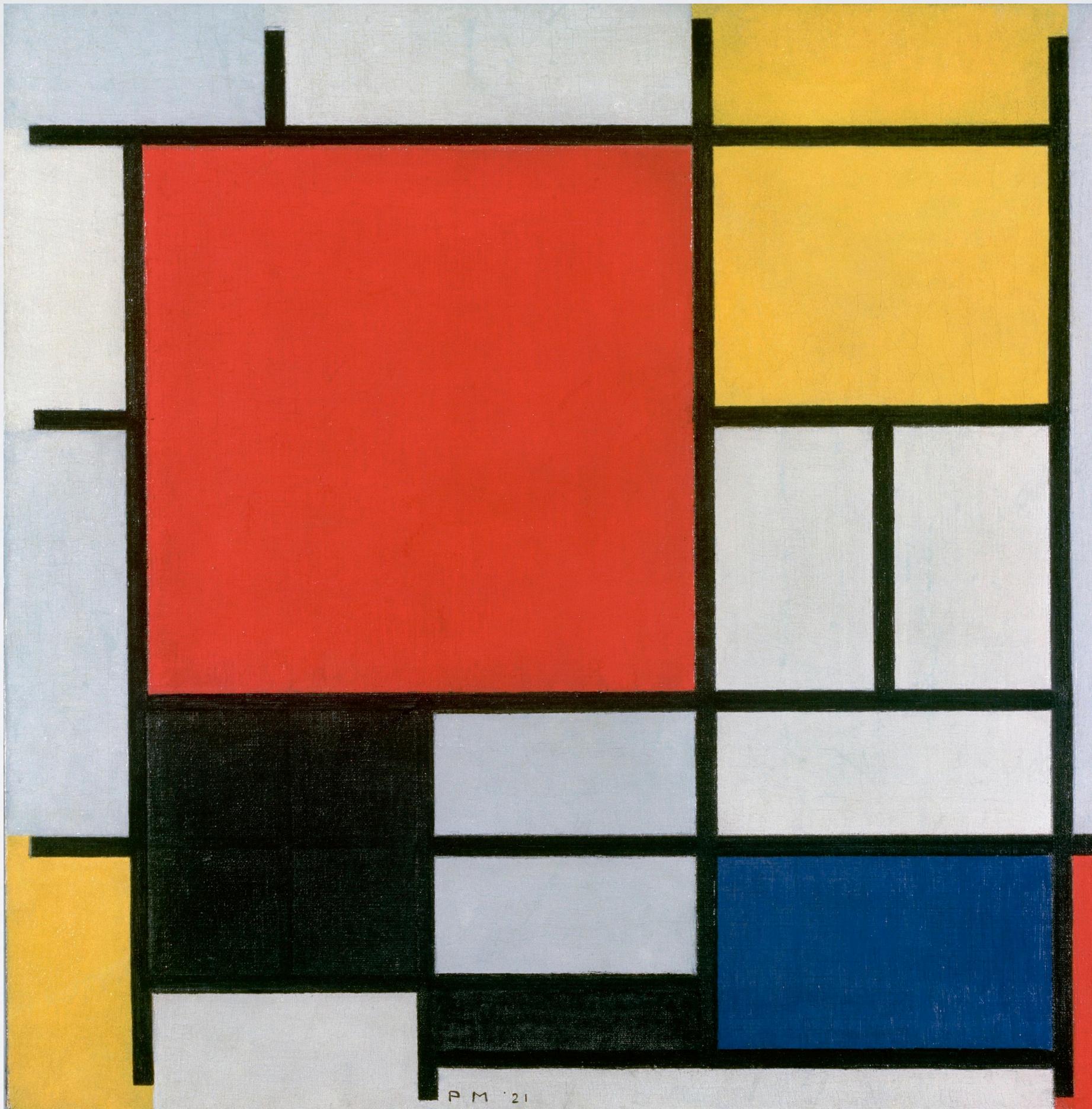
(Part I)

Rodrigo Panosso Macedo



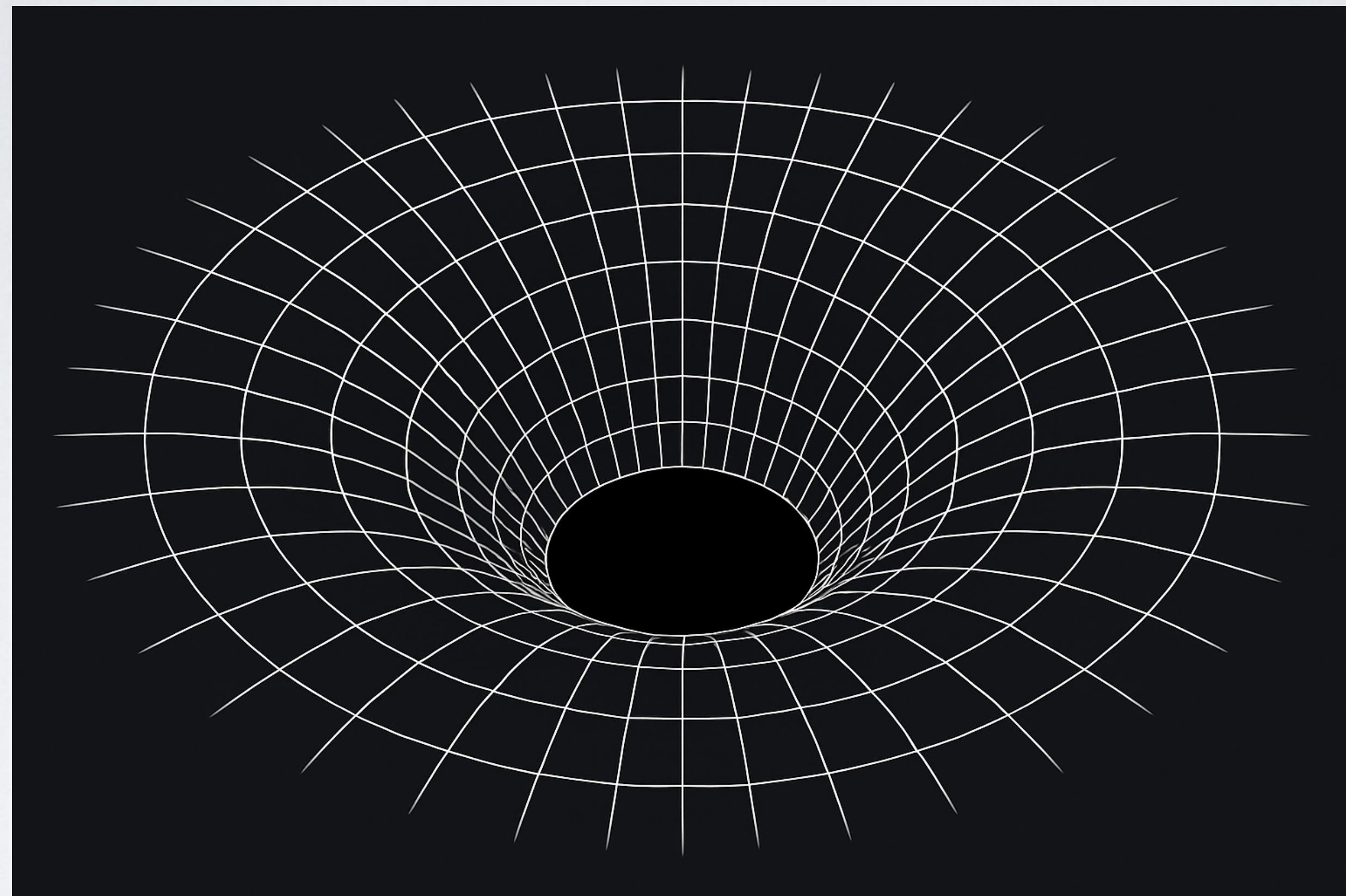
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GENERAL RELATIVITY AND GEOMETRY

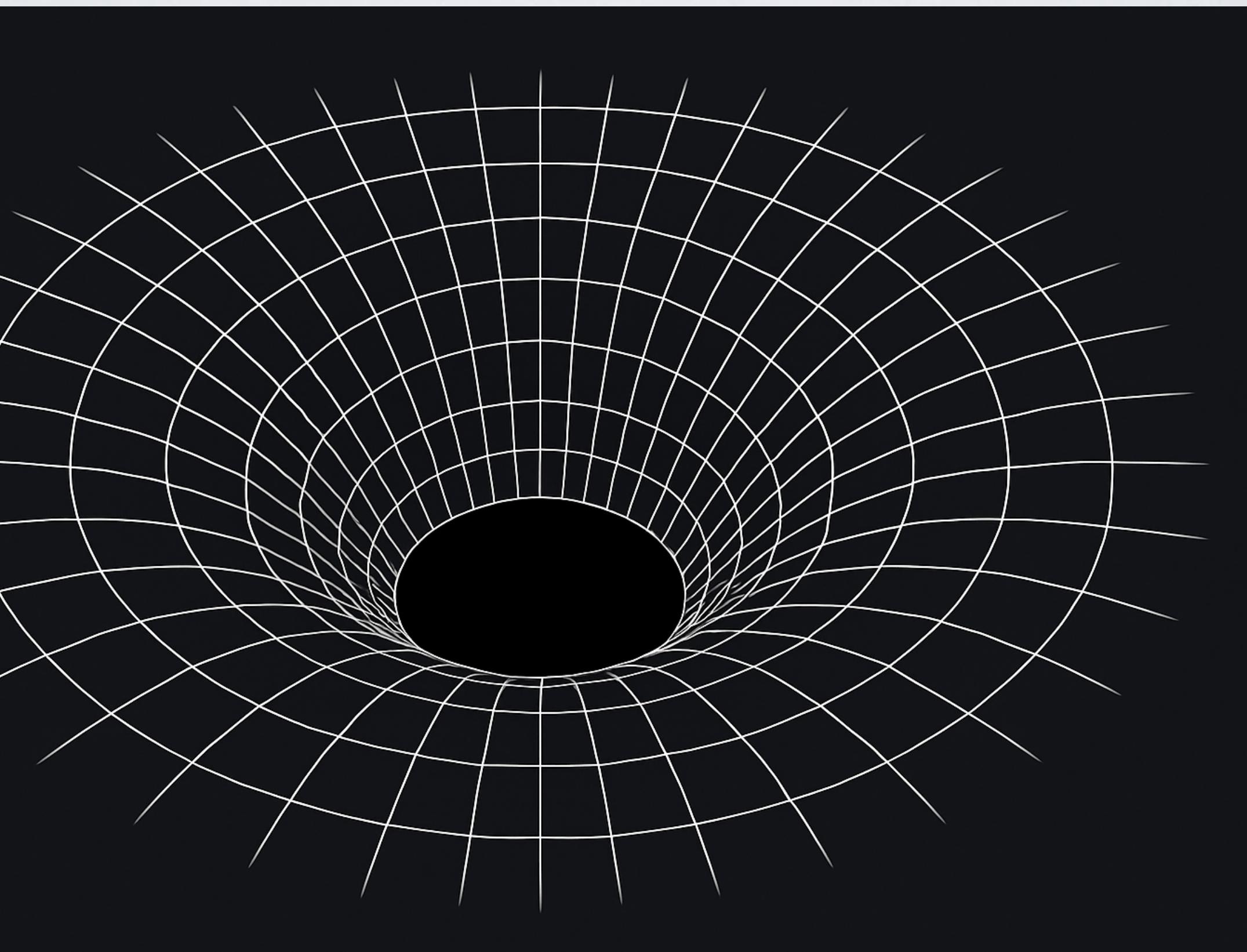
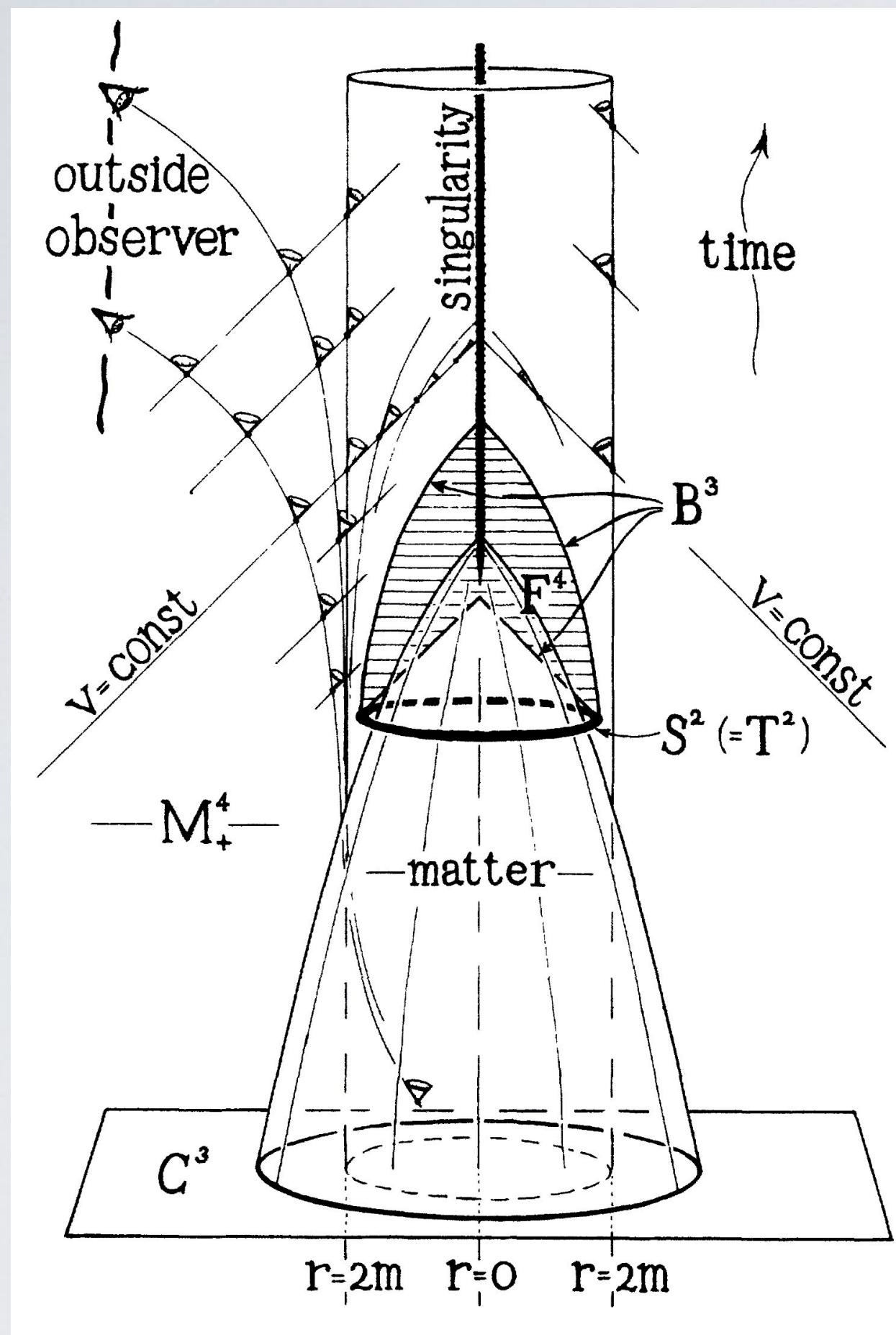


Piet Mondrian

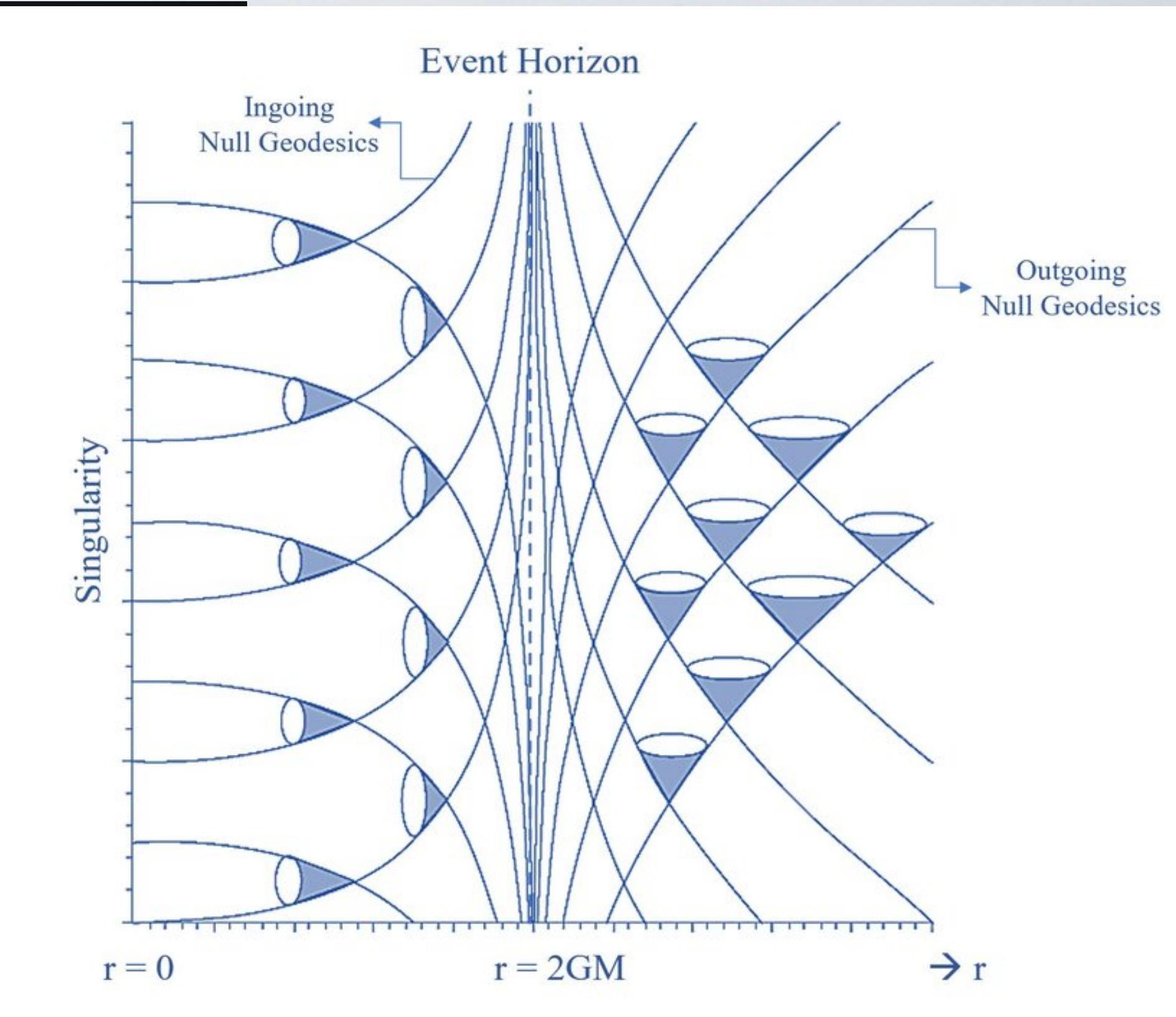
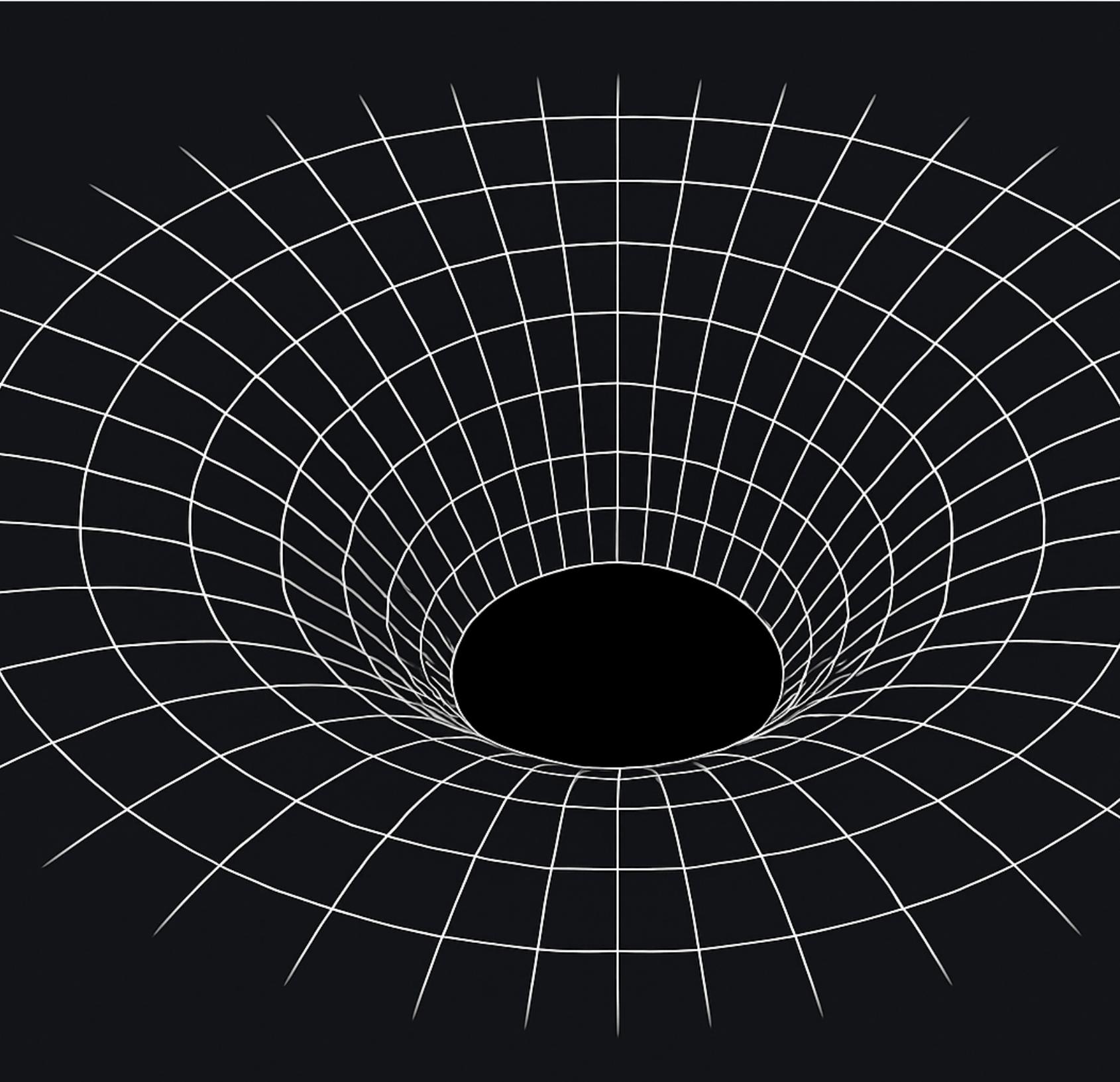
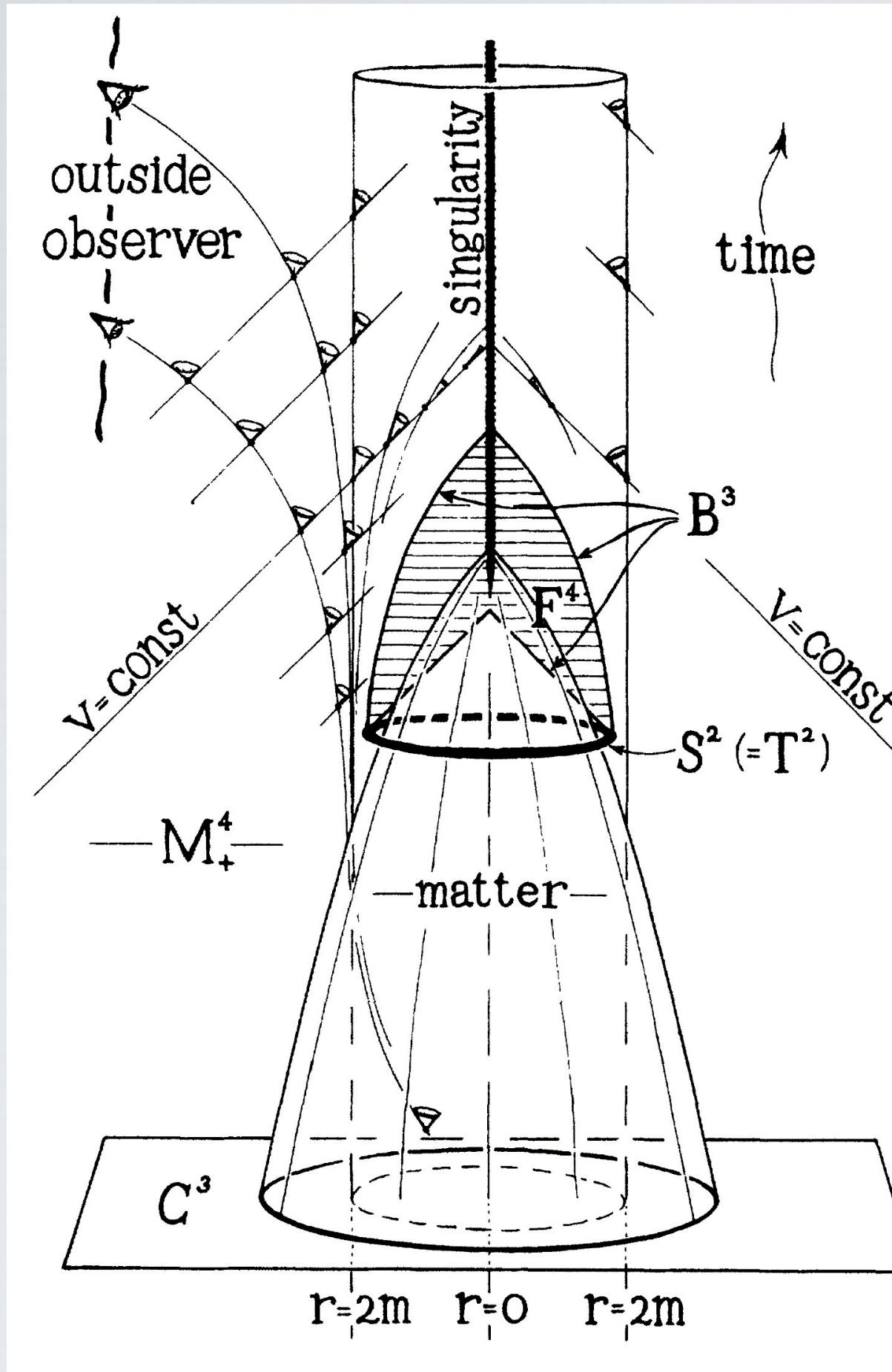
GENERAL RELATIVITY AND GEOMETRY



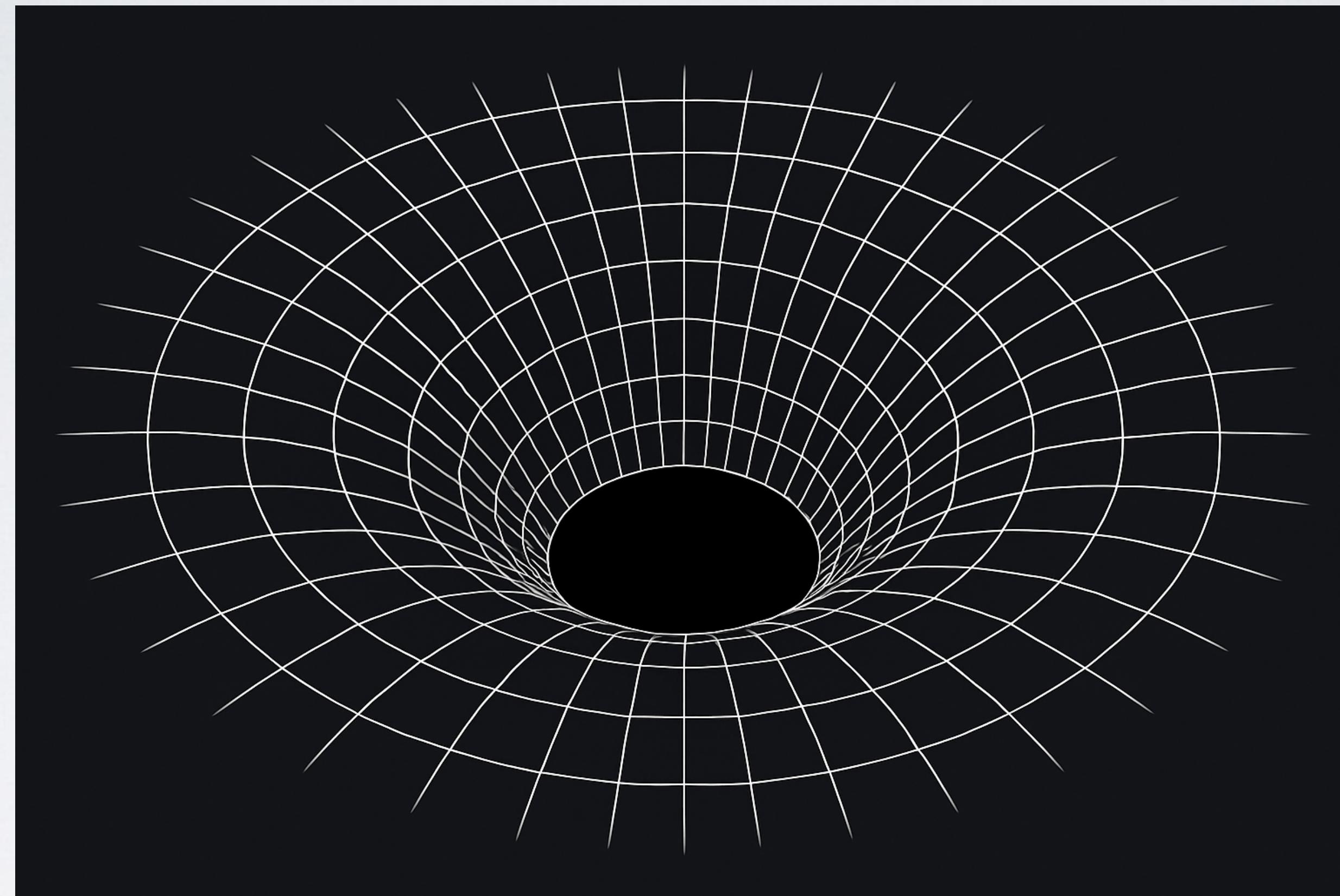
GENERAL RELATIVITY AND GEOMETRY



GENERAL RELATIVITY AND GEOMETRY



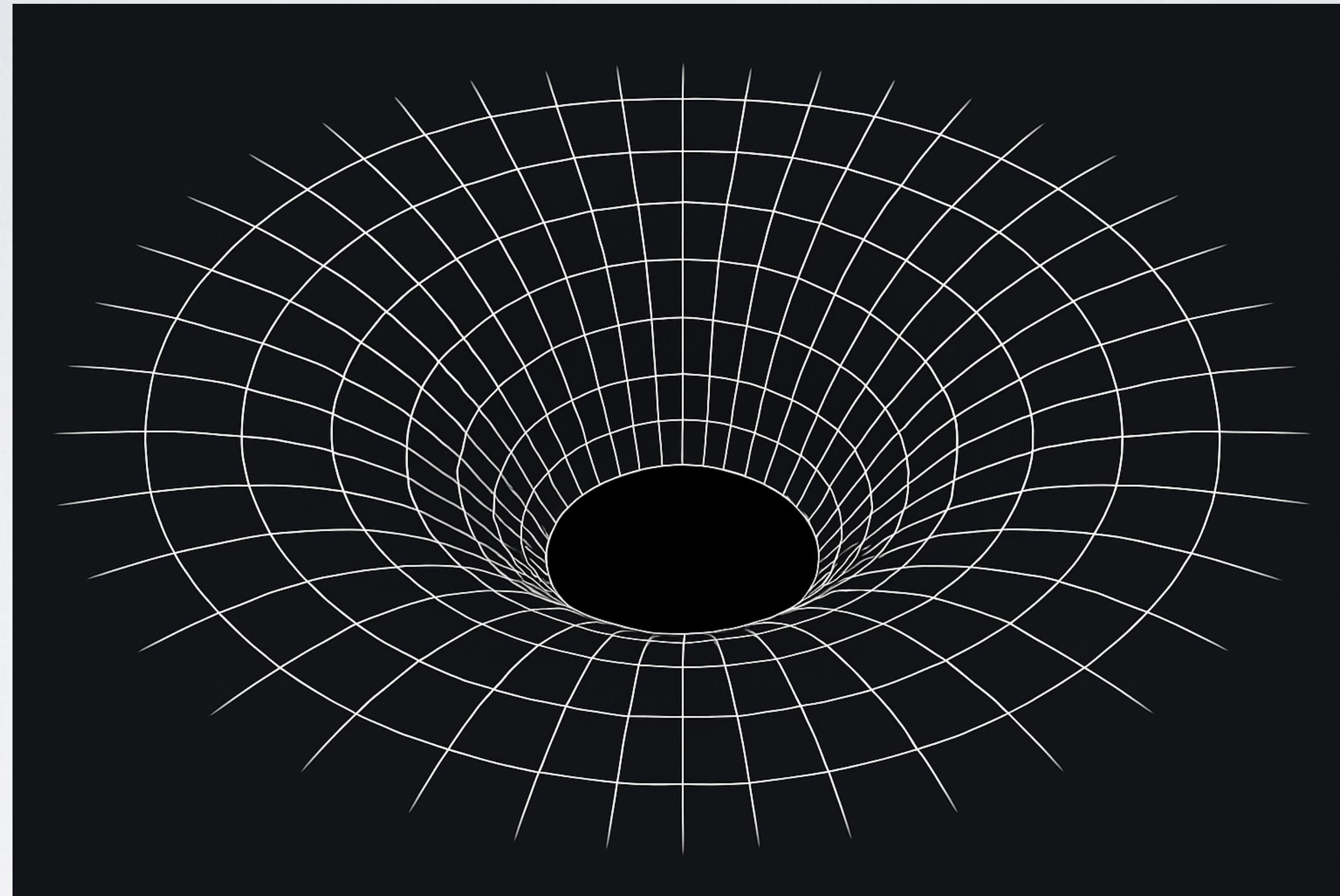
GENERAL RELATIVITY AND GEOMETRY



“Spacetime tells matter how to move; matter tells spacetime how to curve”

(John. A. Wheeler)

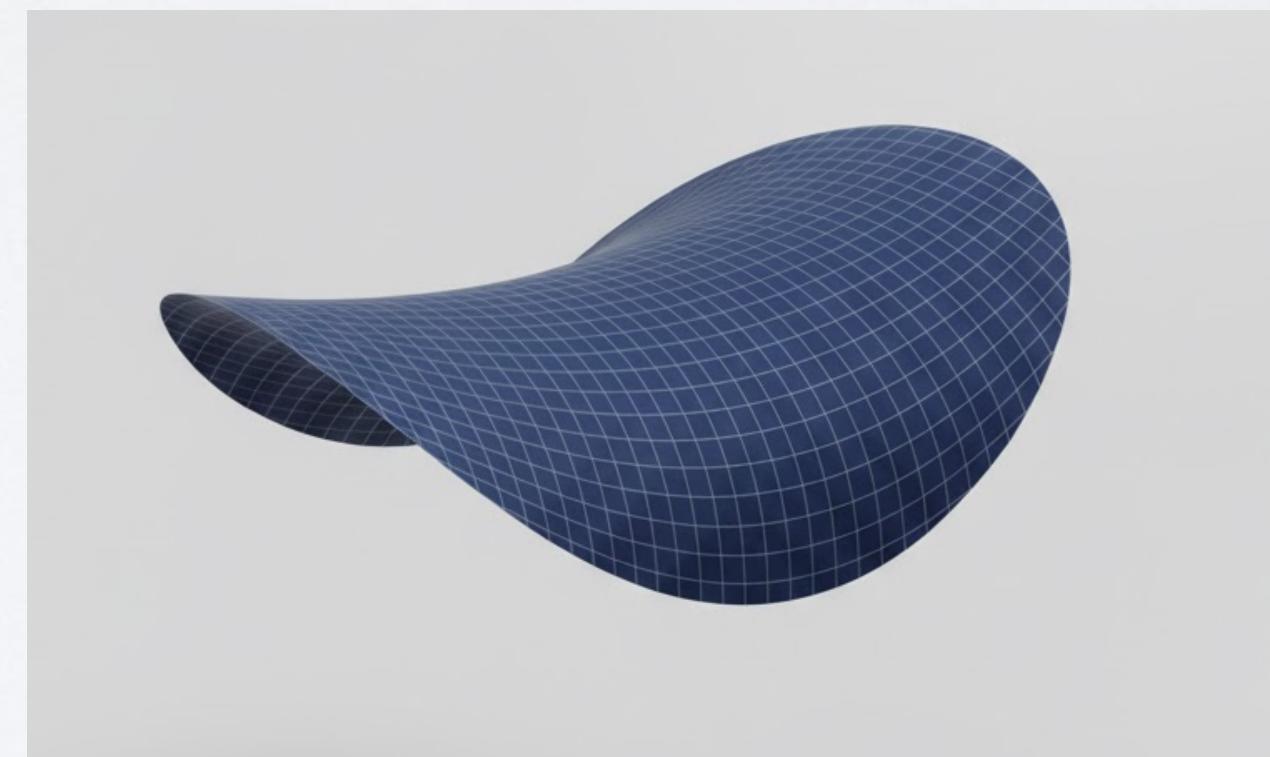
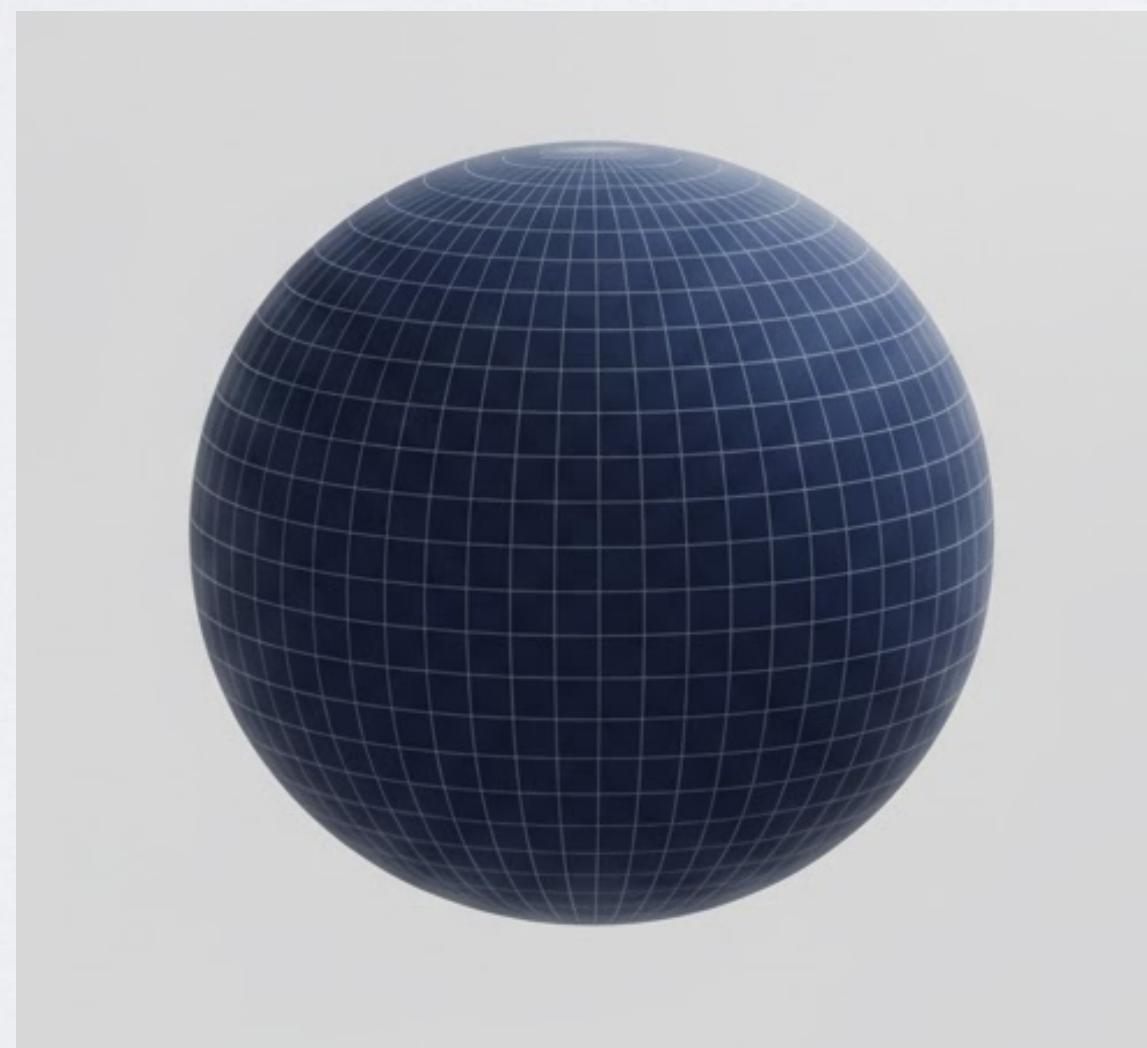
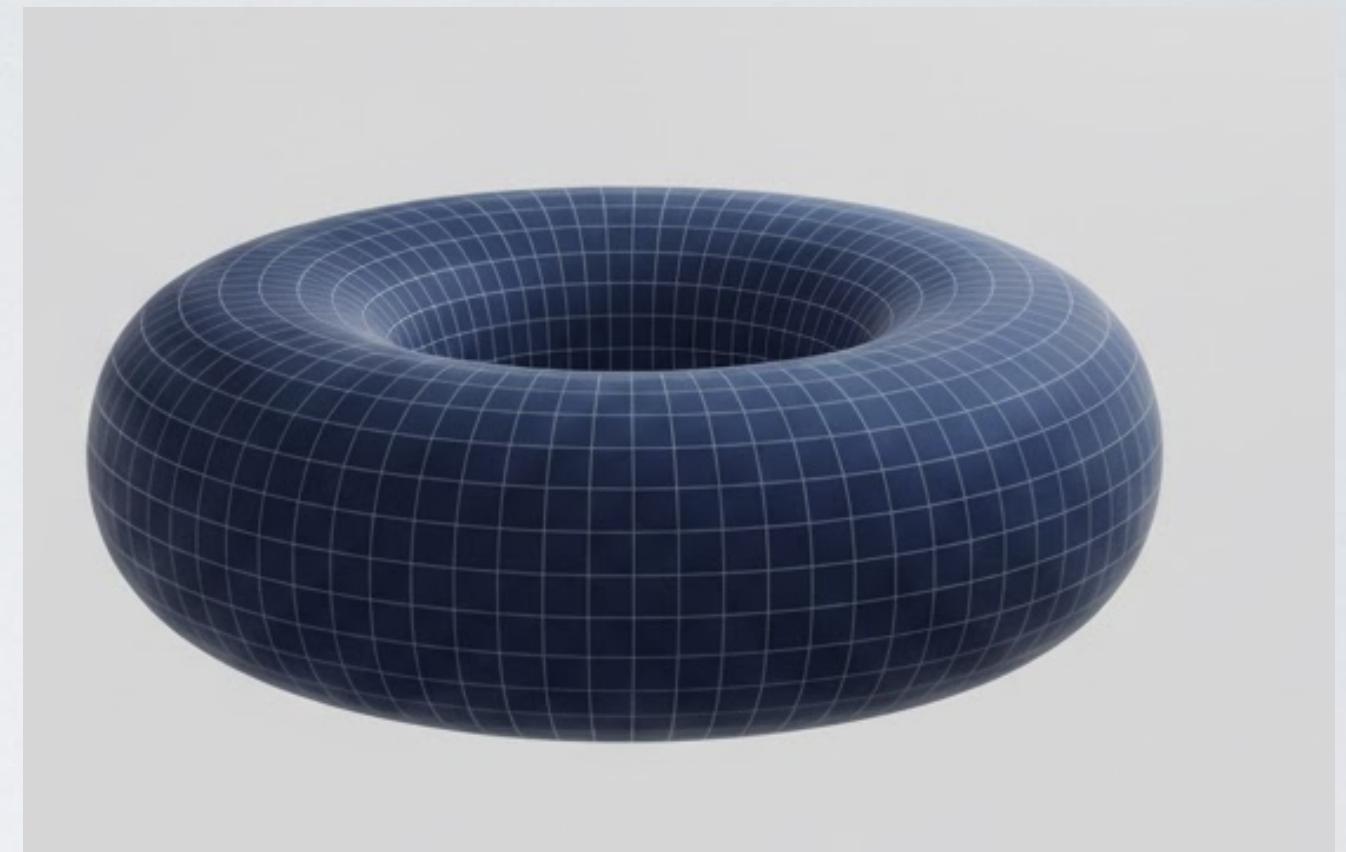
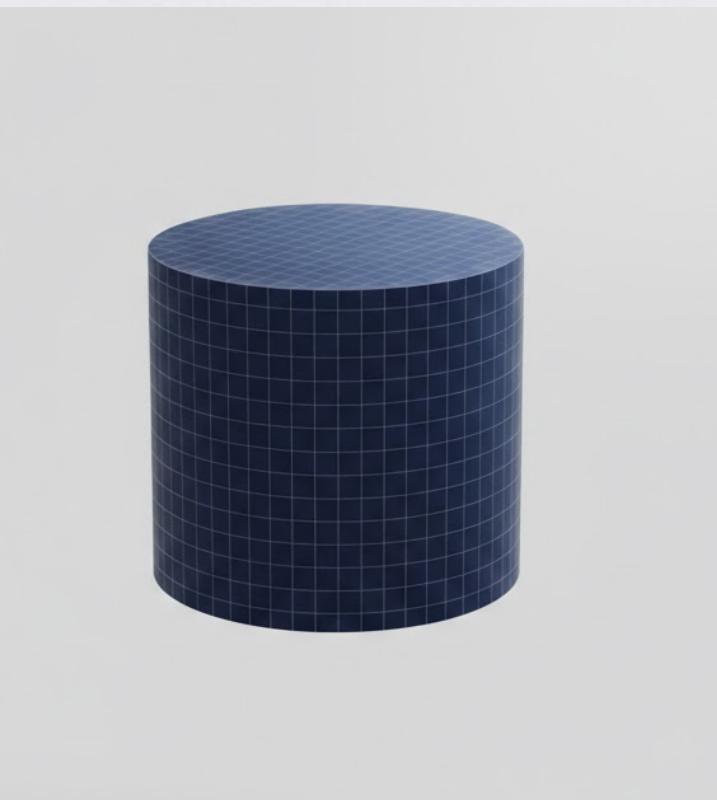
GENERAL RELATIVITY AND GEOMETRY



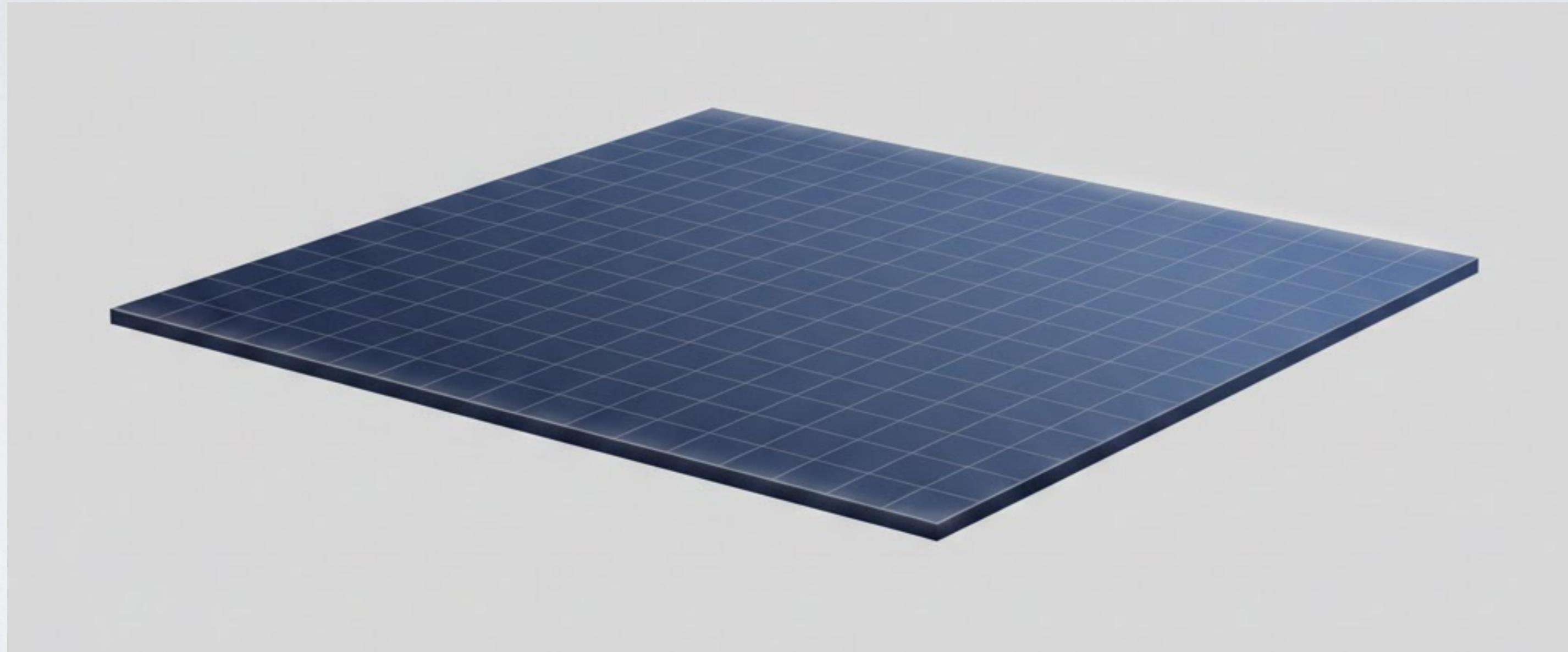
“Spacetime tells matter how to move; **matter tells spacetime how to curve**”

(John. A. Wheeler)

2D SURFACES

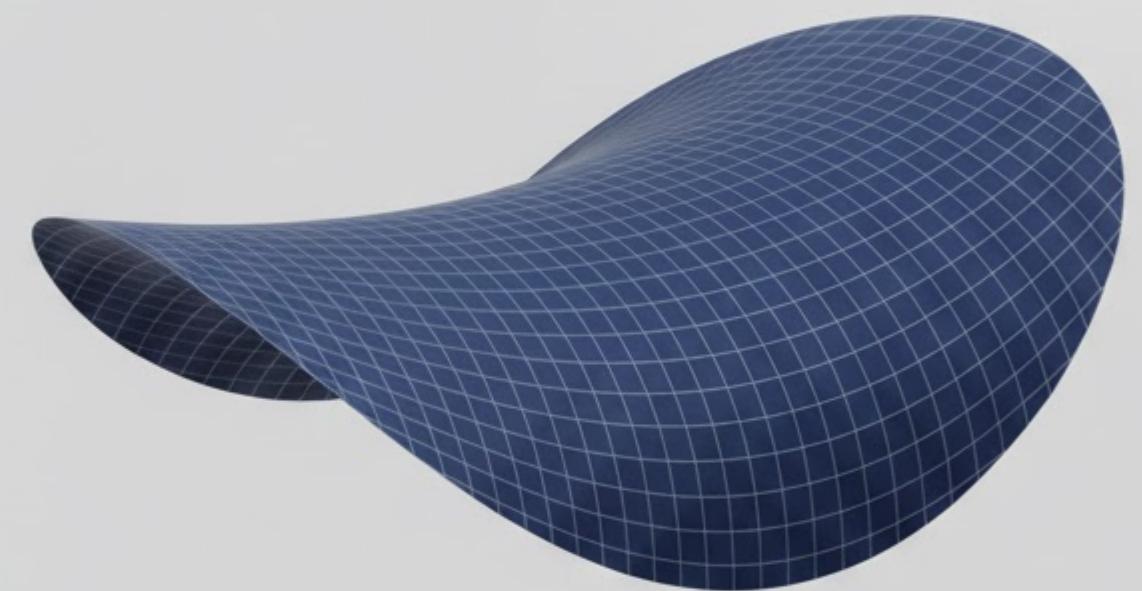
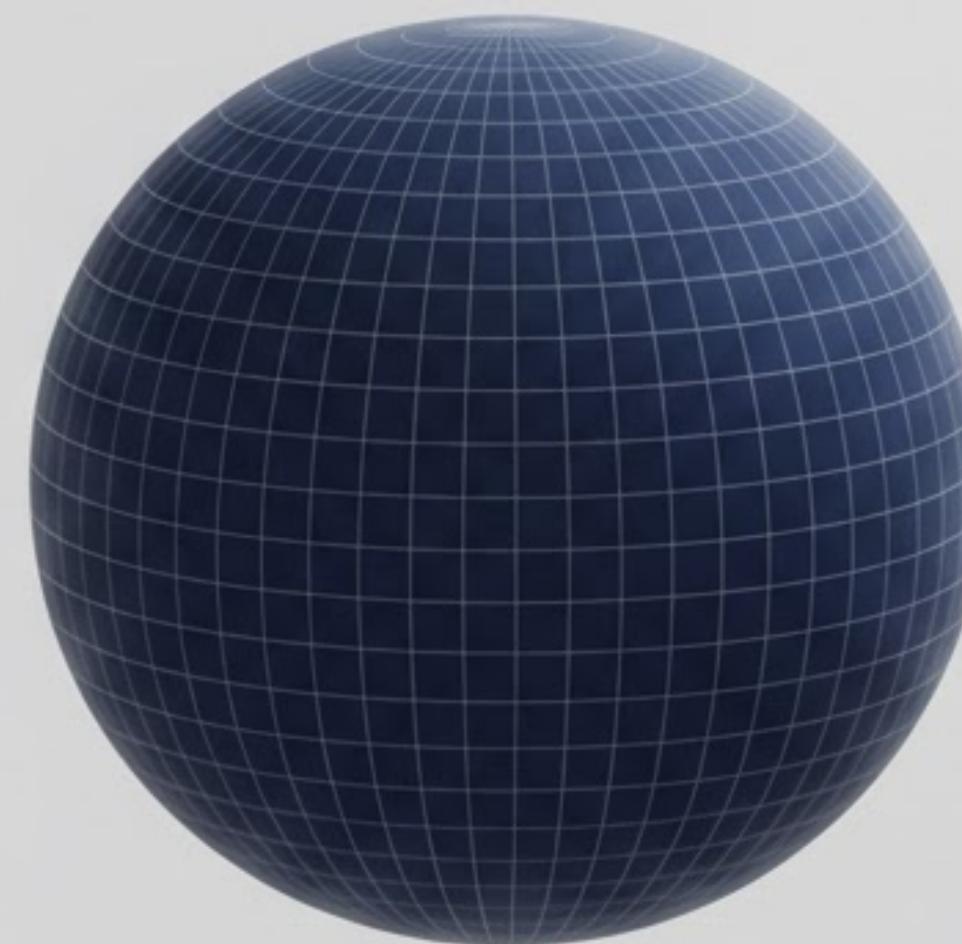
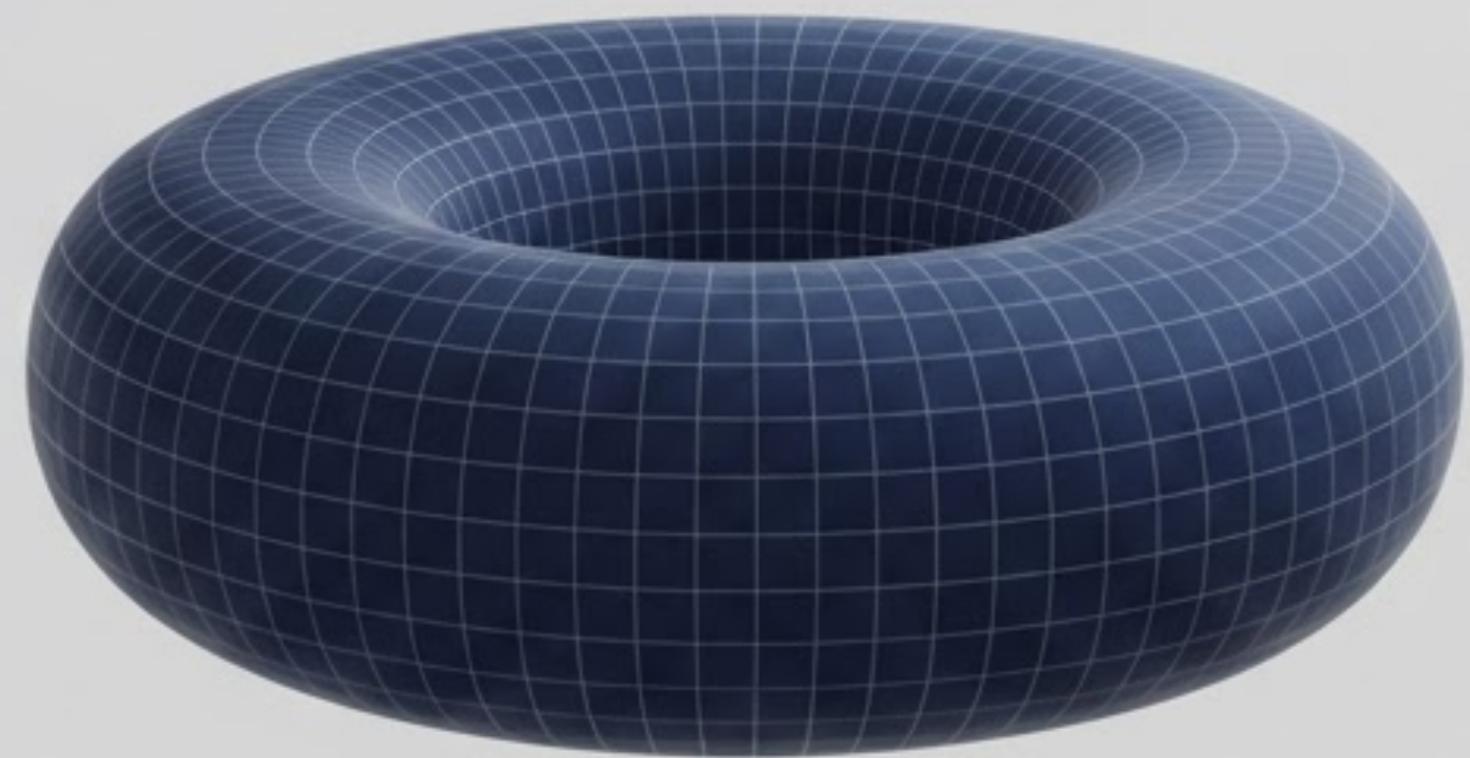


2D SURFACES



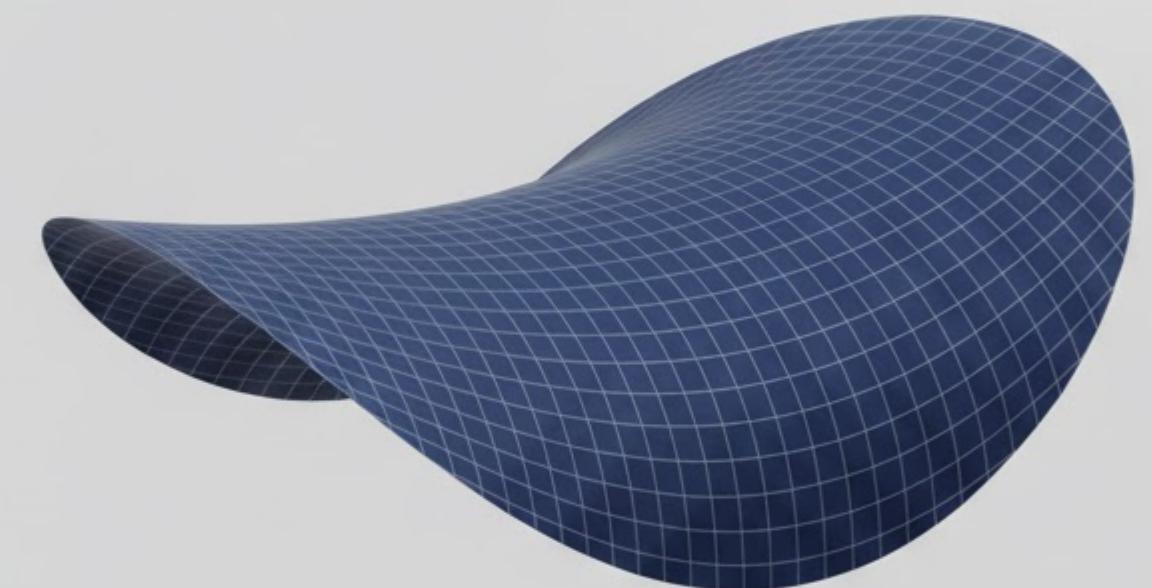
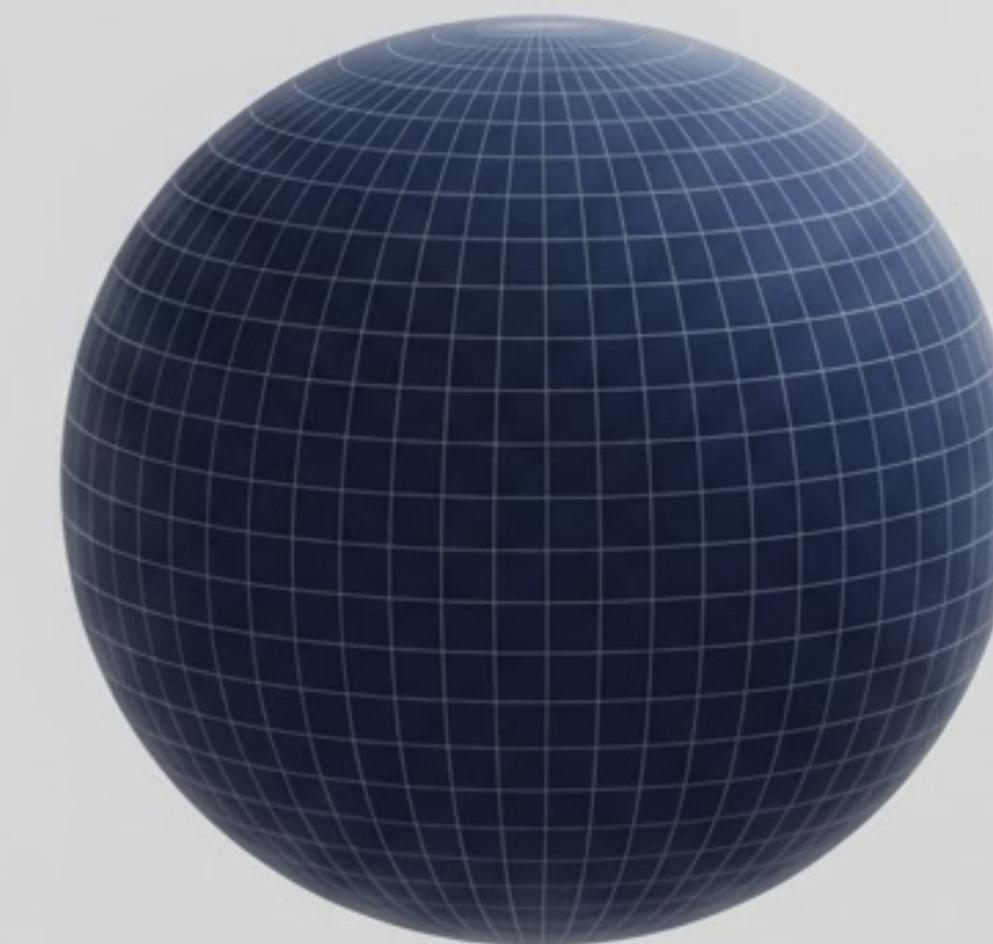
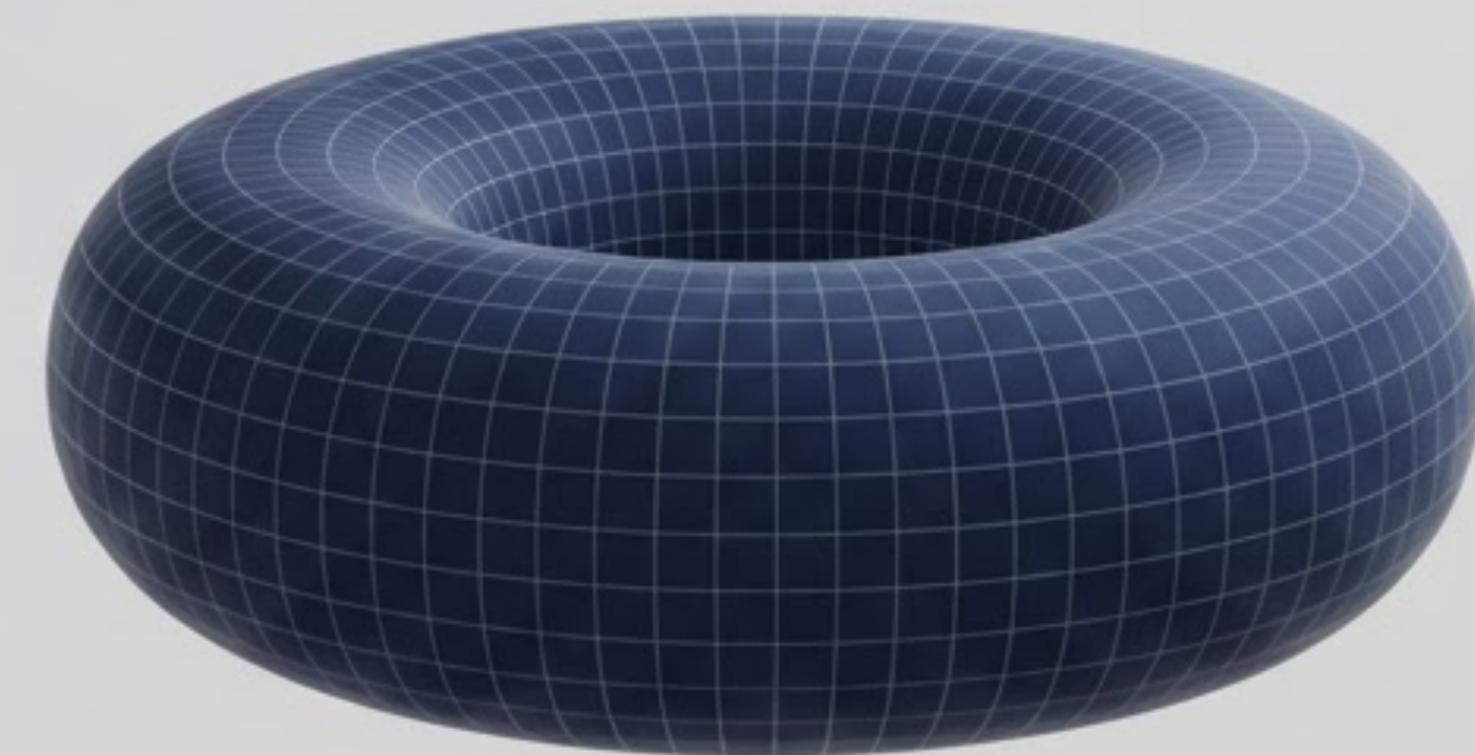
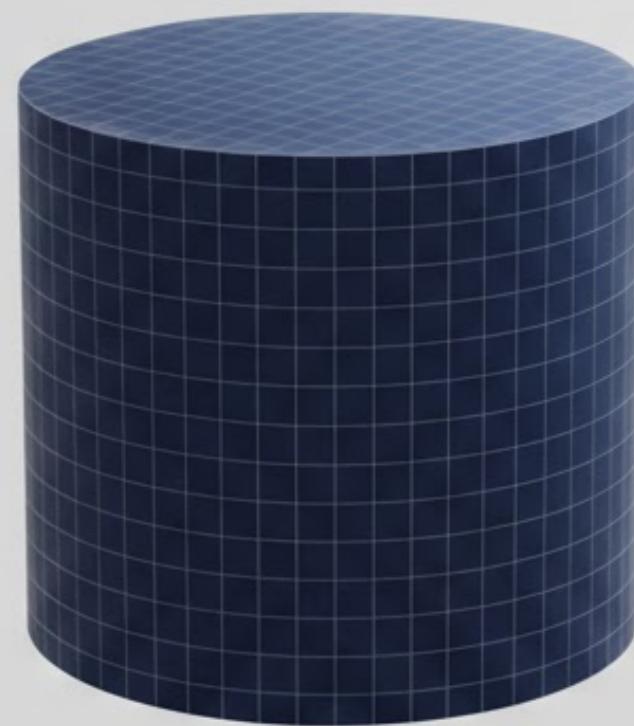
FLAT

2D SURFACES



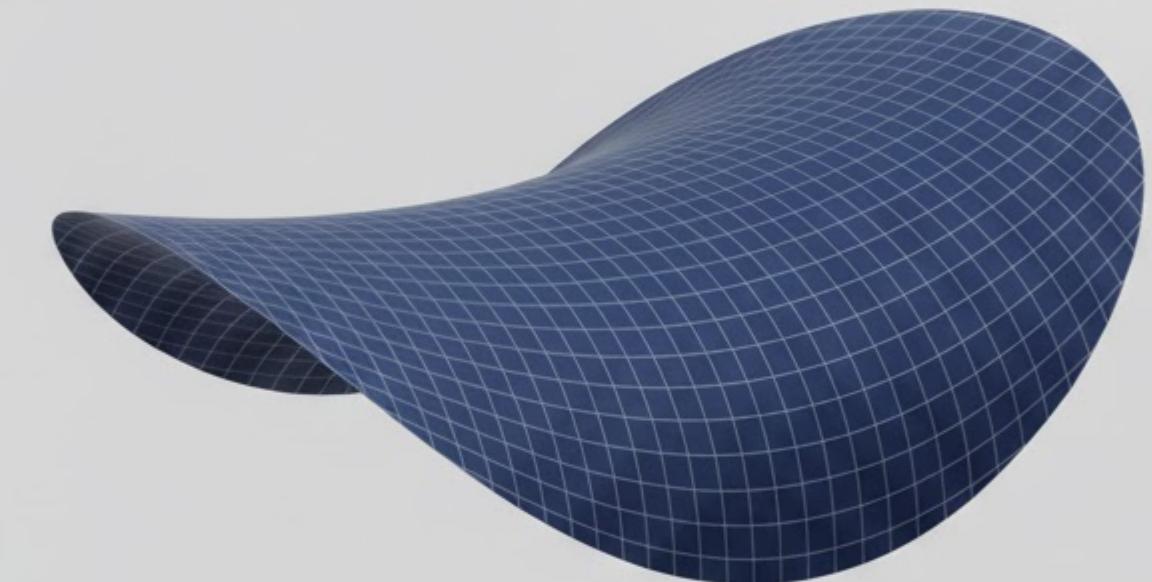
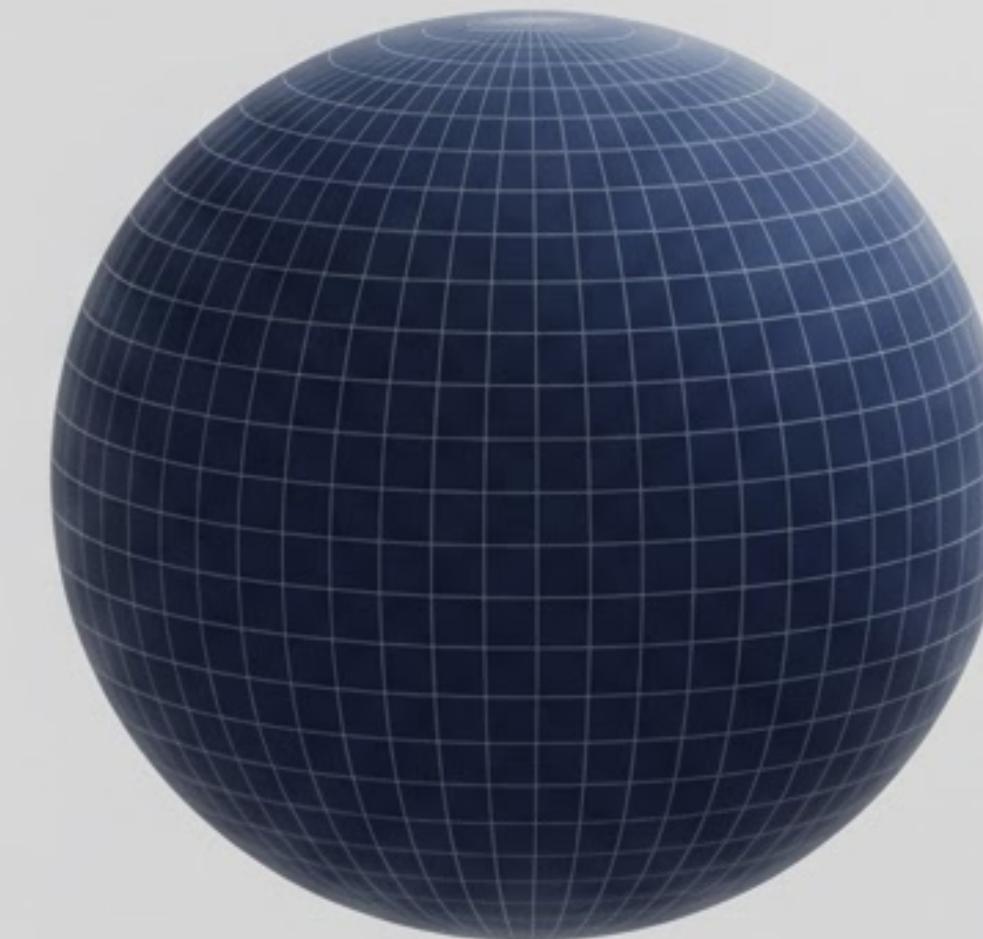
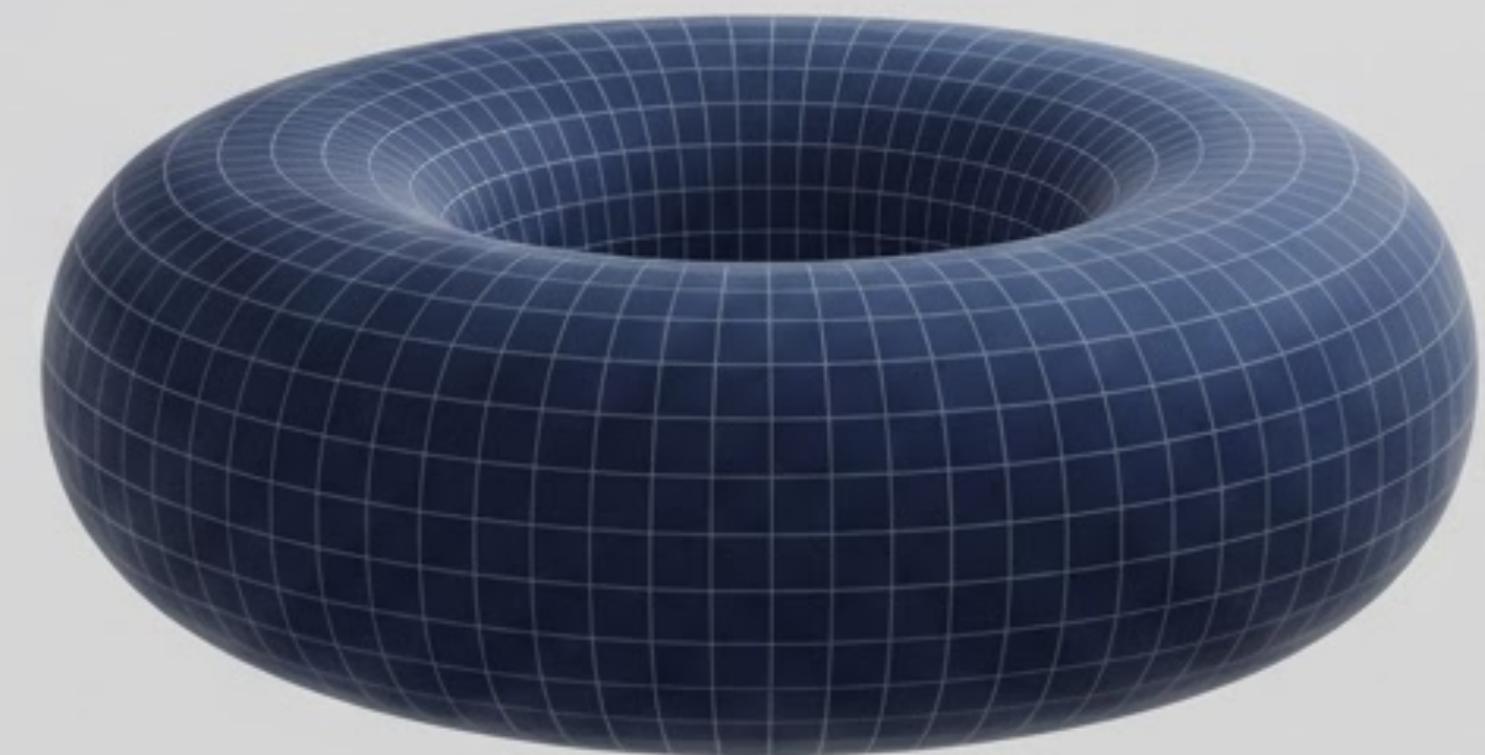
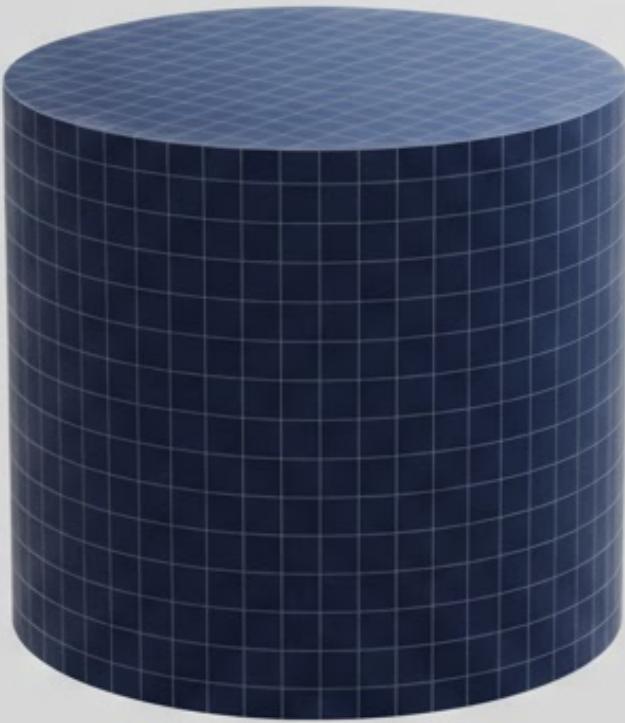
2D SURFACES **CURVED**

2D SURFACES



2D SURFACES **CURVED** WITH RESPECT TO
AN EXTERNAL **3D SPACE**

2D SURFACES



2D SURFACES **CURVED** WITH RESPECT TO
AN EXTERNAL **3D SPACE**

NON-VANISHING **EXTRINSIC** CURVATURE

FLAT X CURVATURE

FLAT X CURVATURE

- What if we can't move into the extra dimension?

FLAT X CURVATURE

- What if we can't move into the extra dimension?
(Imagine an tiny little bug in the 2D surfaced)

FLAT X CURVATURE

- What if we can't move into the extra dimension?

(Imagine an tiny little bug in the 2D surfaced)

(Imagine ourselves in our 3D volume embedded in 4D space?)

FLAT X CURVATURE

- What if we can't move into the extra dimension?

(Imagine a tiny little bug in the 2D surface)

(Imagine ourselves in our 3D volume embedded in 4D space?)

- What if the extra dimension does not exist?

FLAT X CURVATURE

- What if we can't move into the extra dimension?

(Imagine a tiny little bug in the 2D surface)

(Imagine ourselves in our 3D volume embedded in 4D space?)

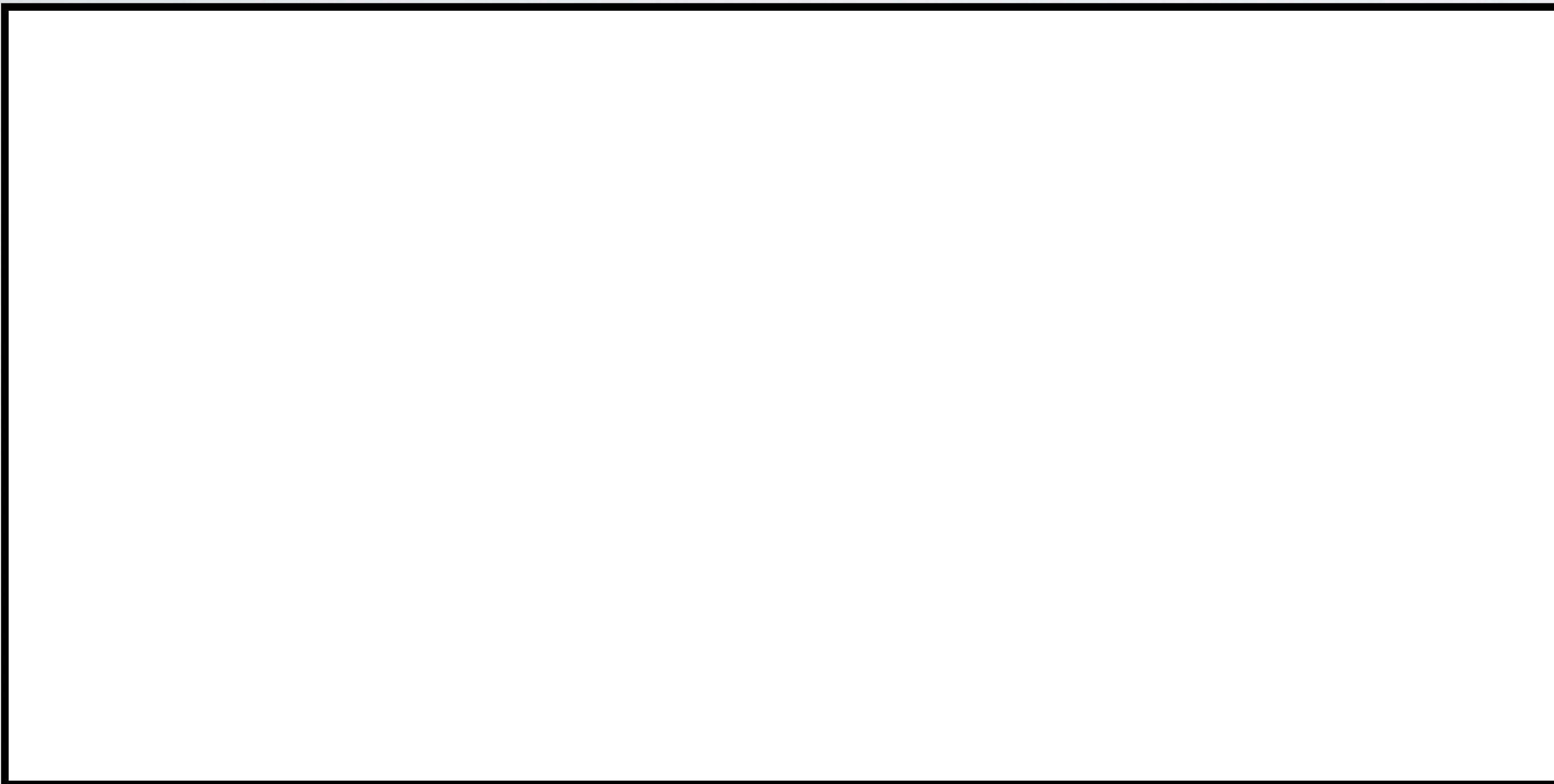
- What if the extra dimension does not exist?

(We live in 3D space. Is there a 4th space dimension?)

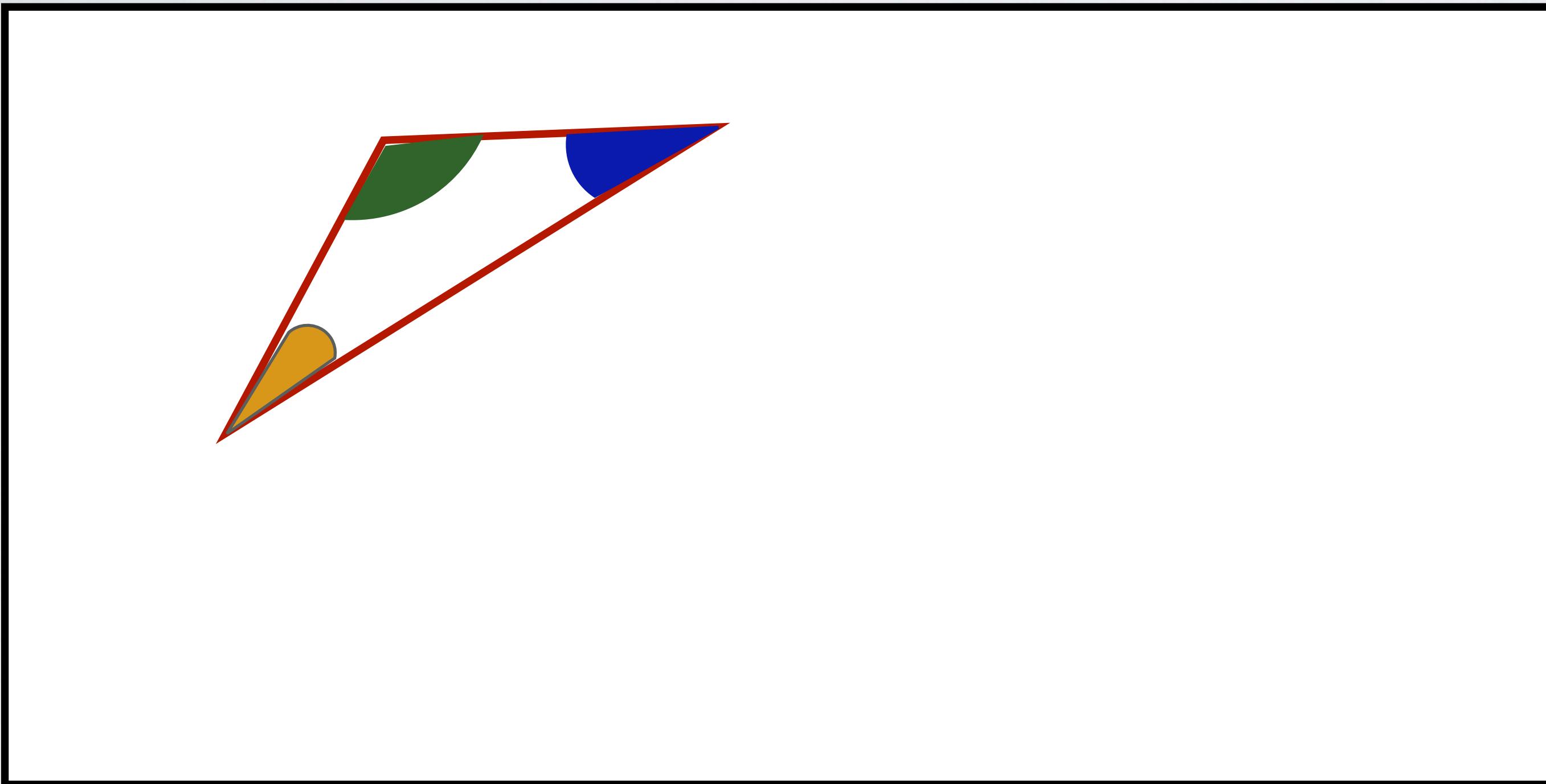
FLAT X CURVATURE

- What if we can't move into the extra dimension?
(Imagine a tiny little bug in the 2D surface)
- (Imagine ourselves in our 3D volume embedded in 4D space?)
- What if the extra dimension does not exist?
(We live in 3D space. Is there a 4th space dimension?)
- Can we tell a flat from a curved space from **intrinsic** properties?

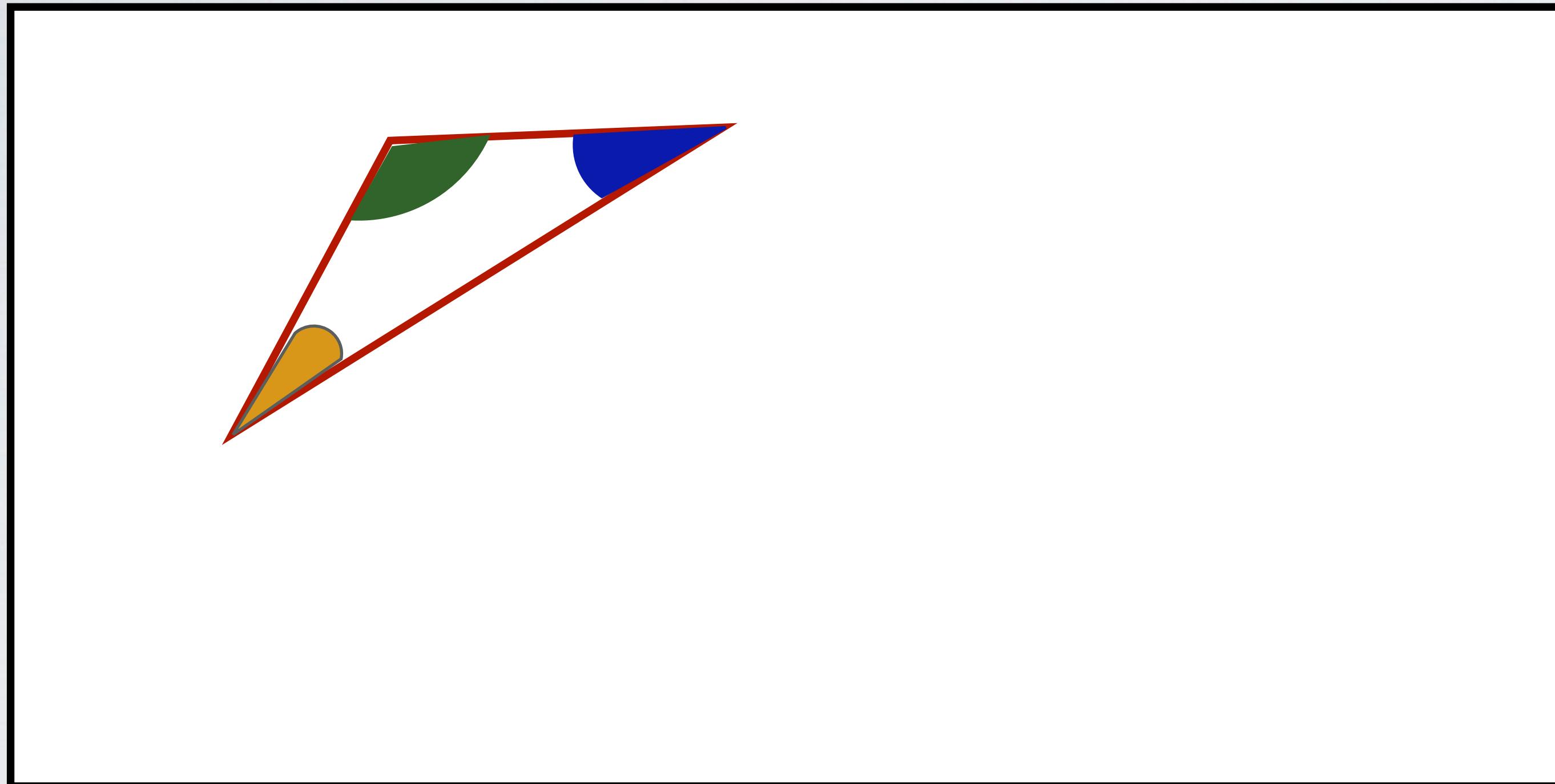
INTRINSIC CURVATURE



INTRINSIC CURVATURE

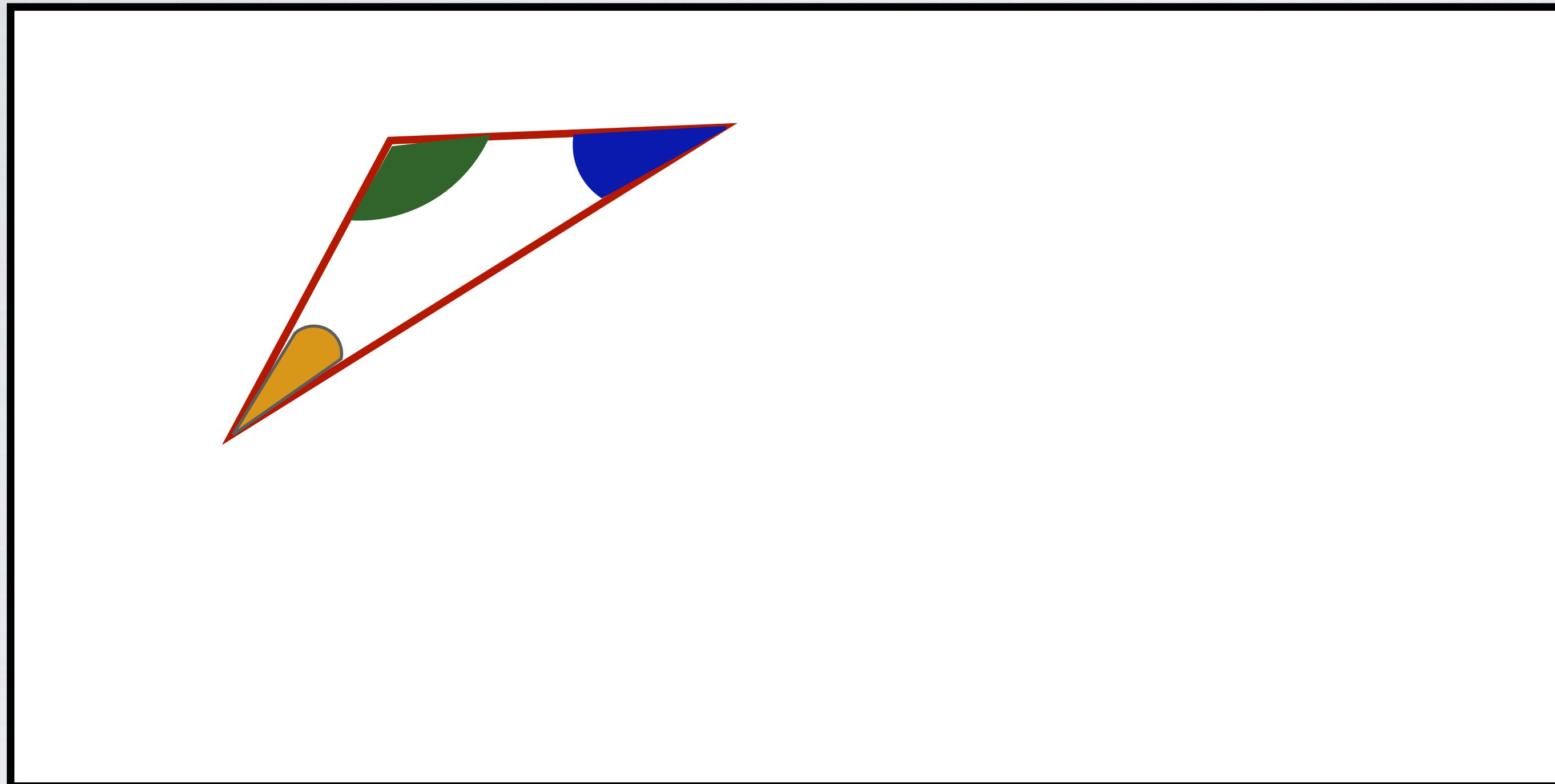


INTRINSIC CURVATURE

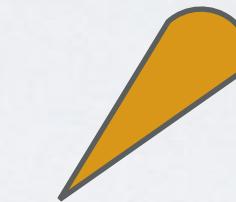


- sum of angles in a triangle

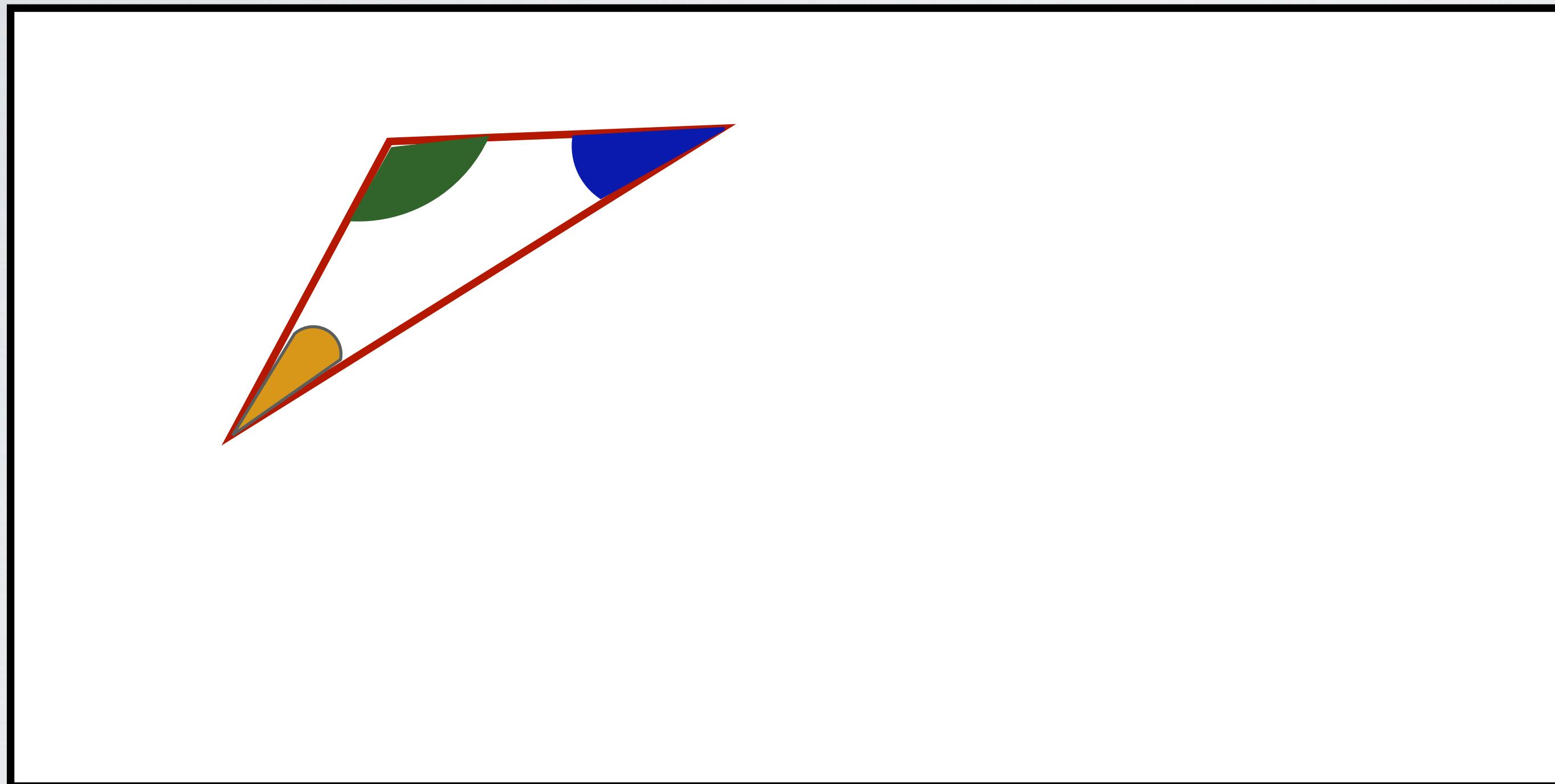
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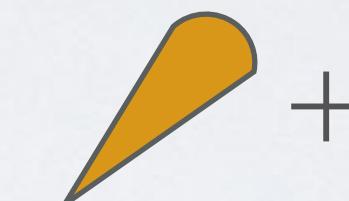
- sum of angles in a triangle



INTRINSIC CURVATURE

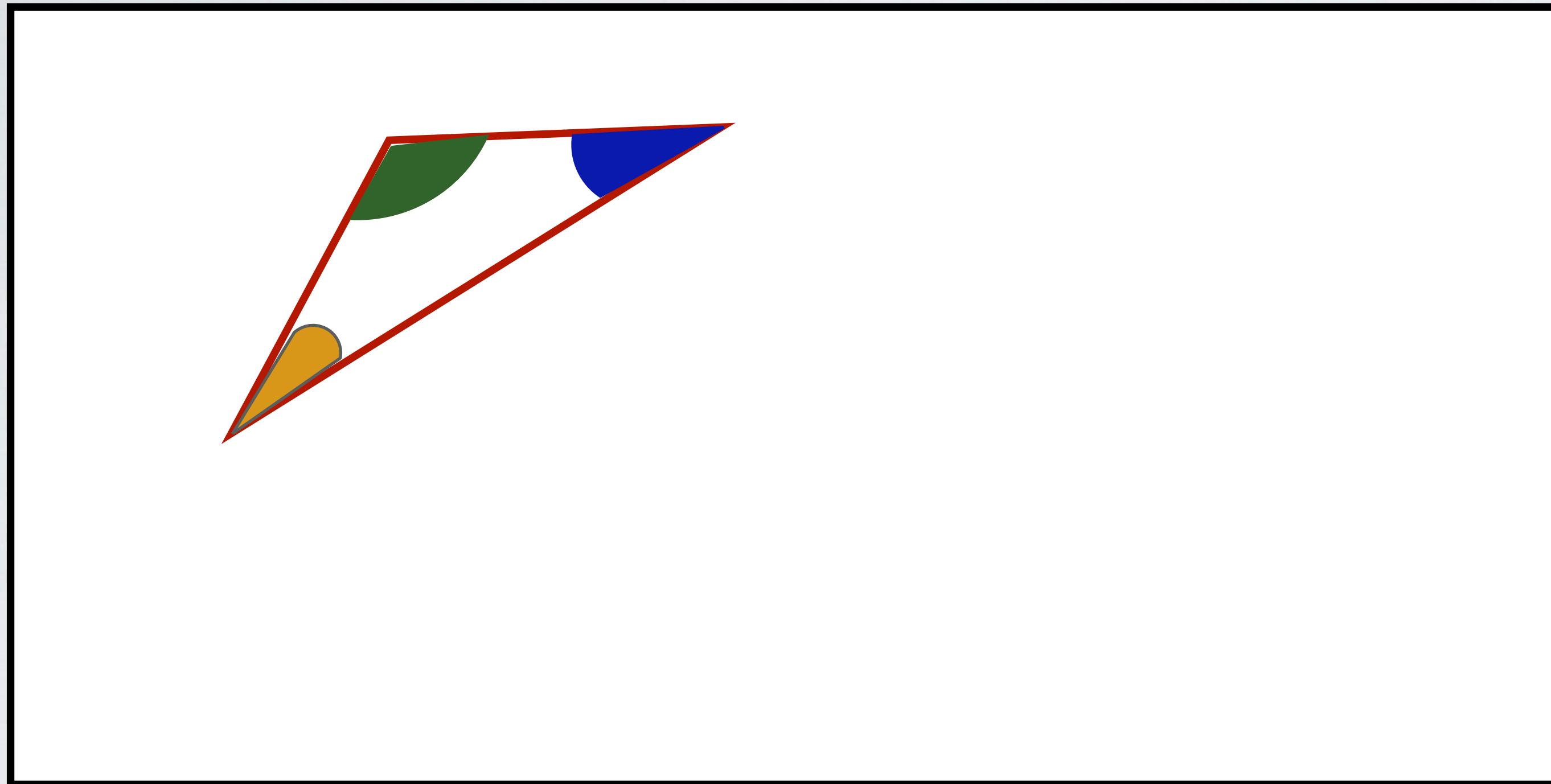


- sum of angles in a triangle

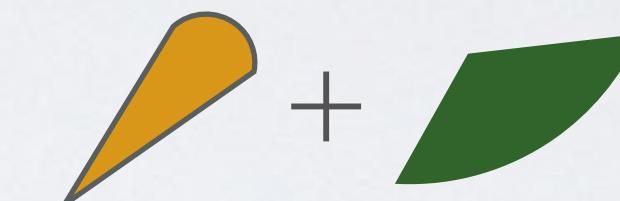


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INTRINSIC CURVATURE

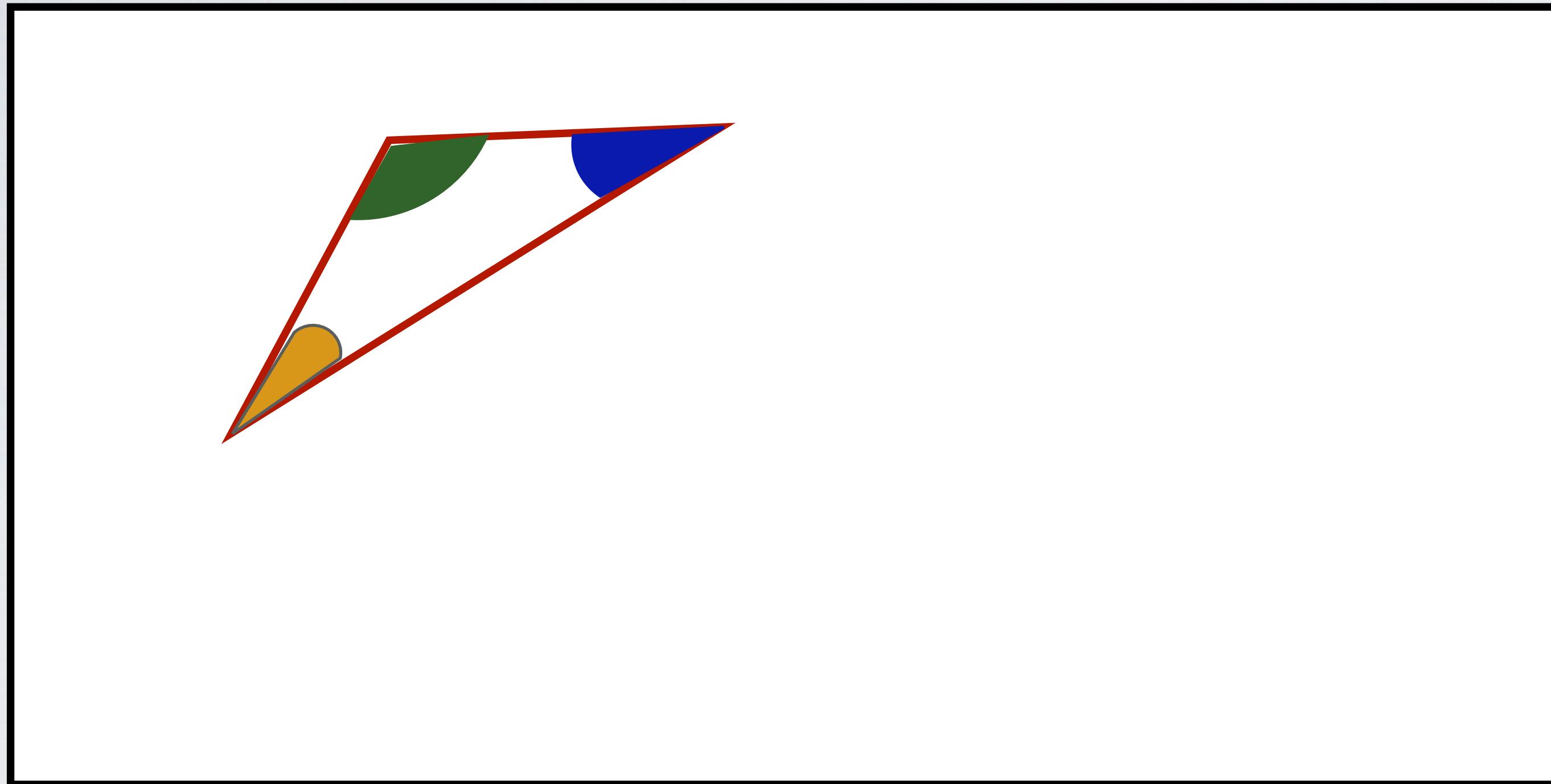


- sum of angles in a triangle

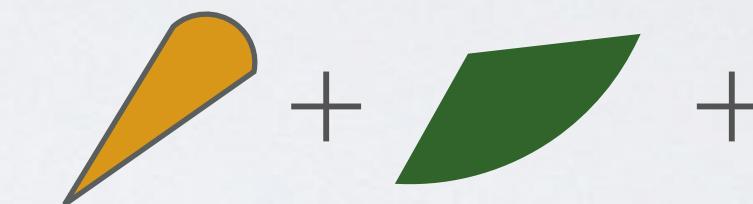


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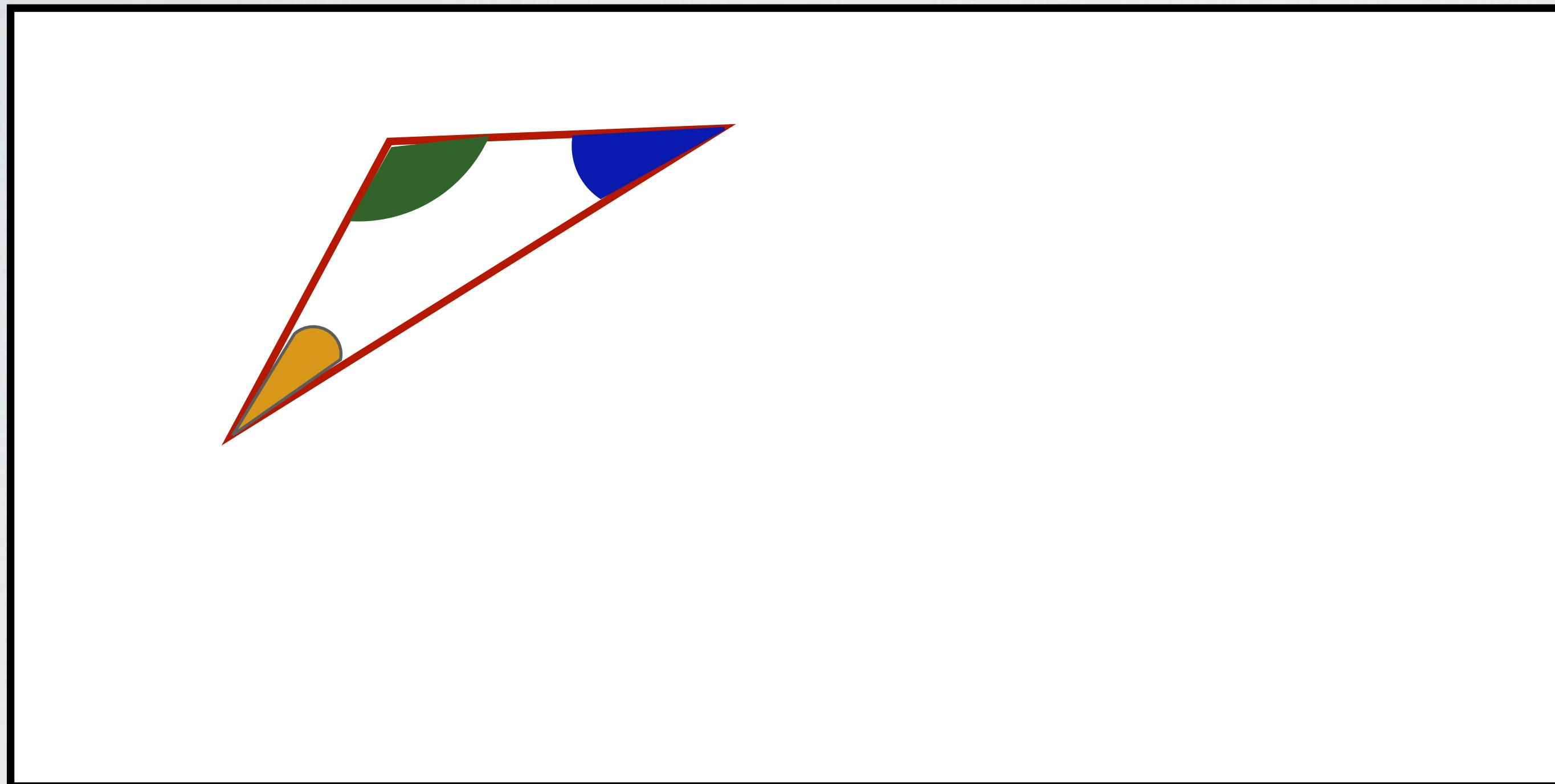
INTRINSIC CURVATURE



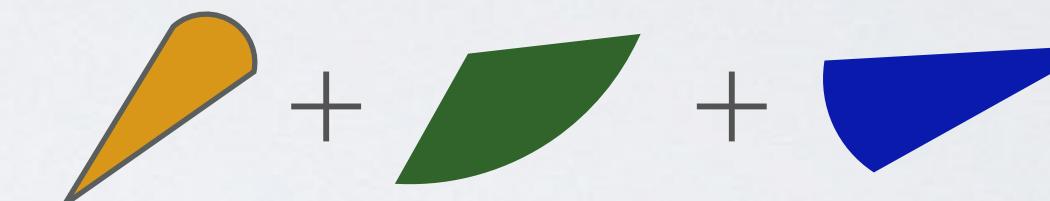
- sum of angles in a triangle



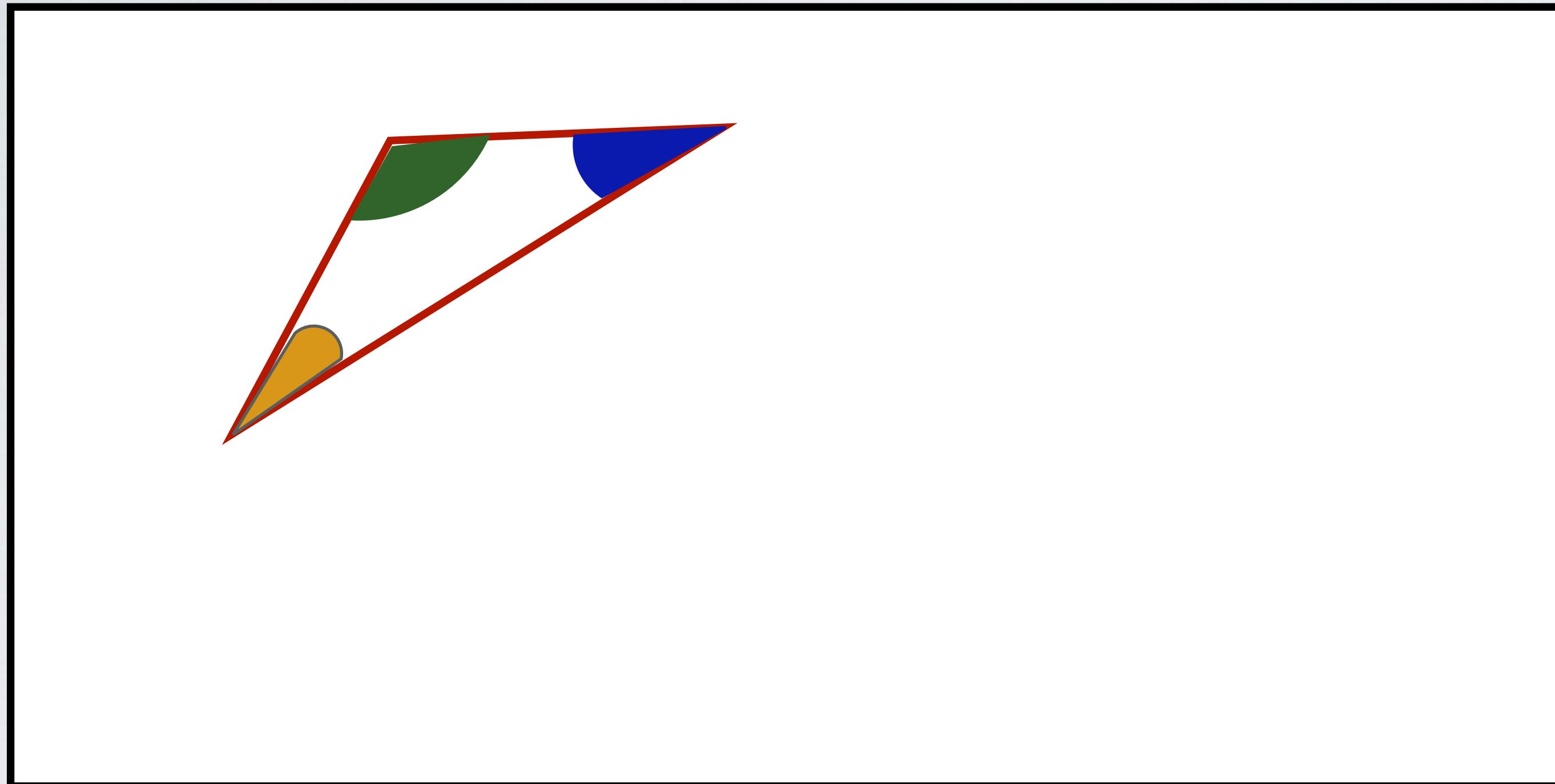
INTRINSIC CURVATURE



- sum of angles in a triangle



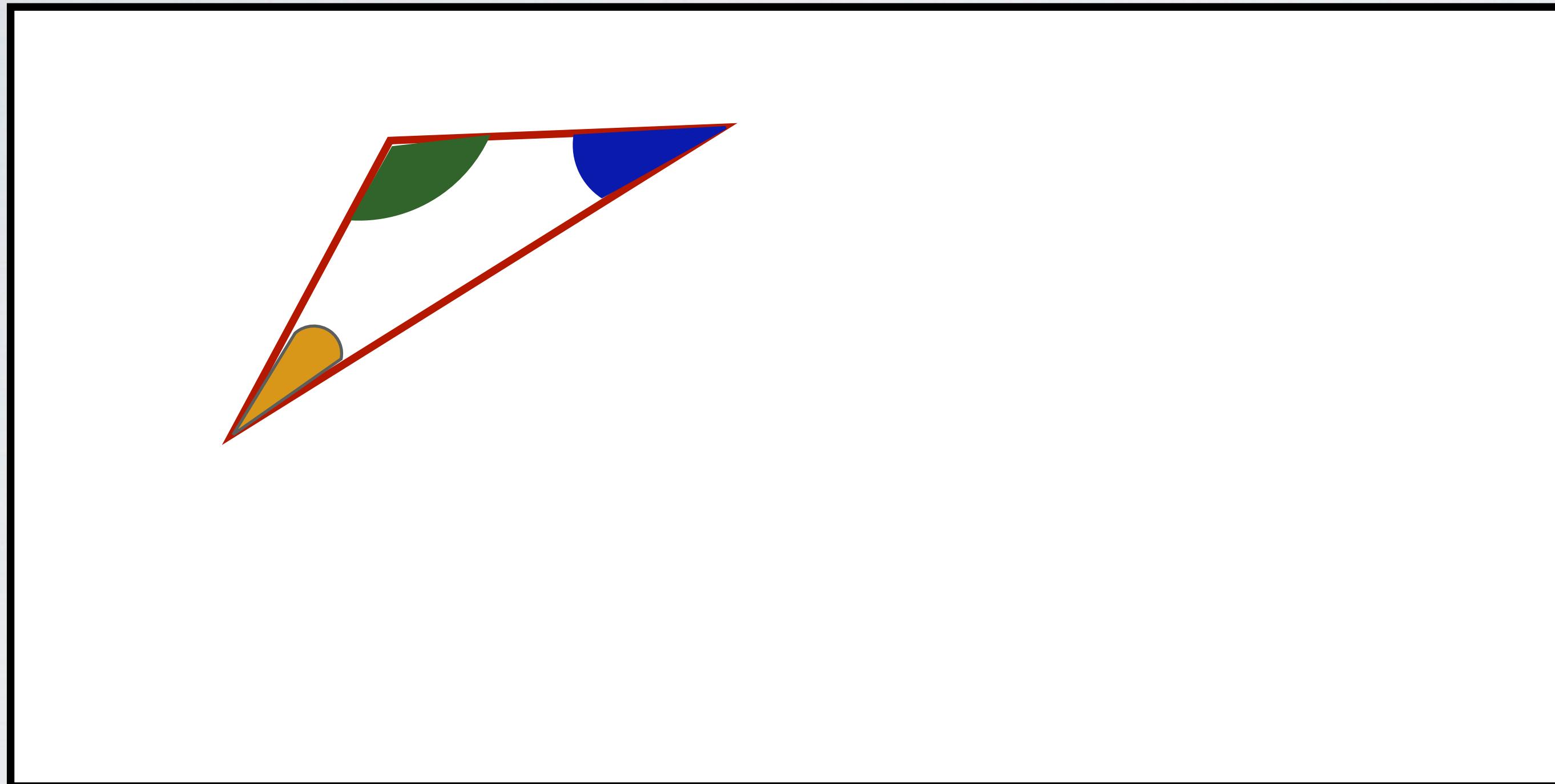
INTRINSIC CURVATURE



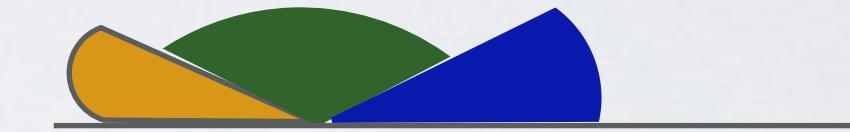
- sum of angles in a triangle

$$\text{orange angle} + \text{green angle} + \text{blue angle} = 180^\circ$$

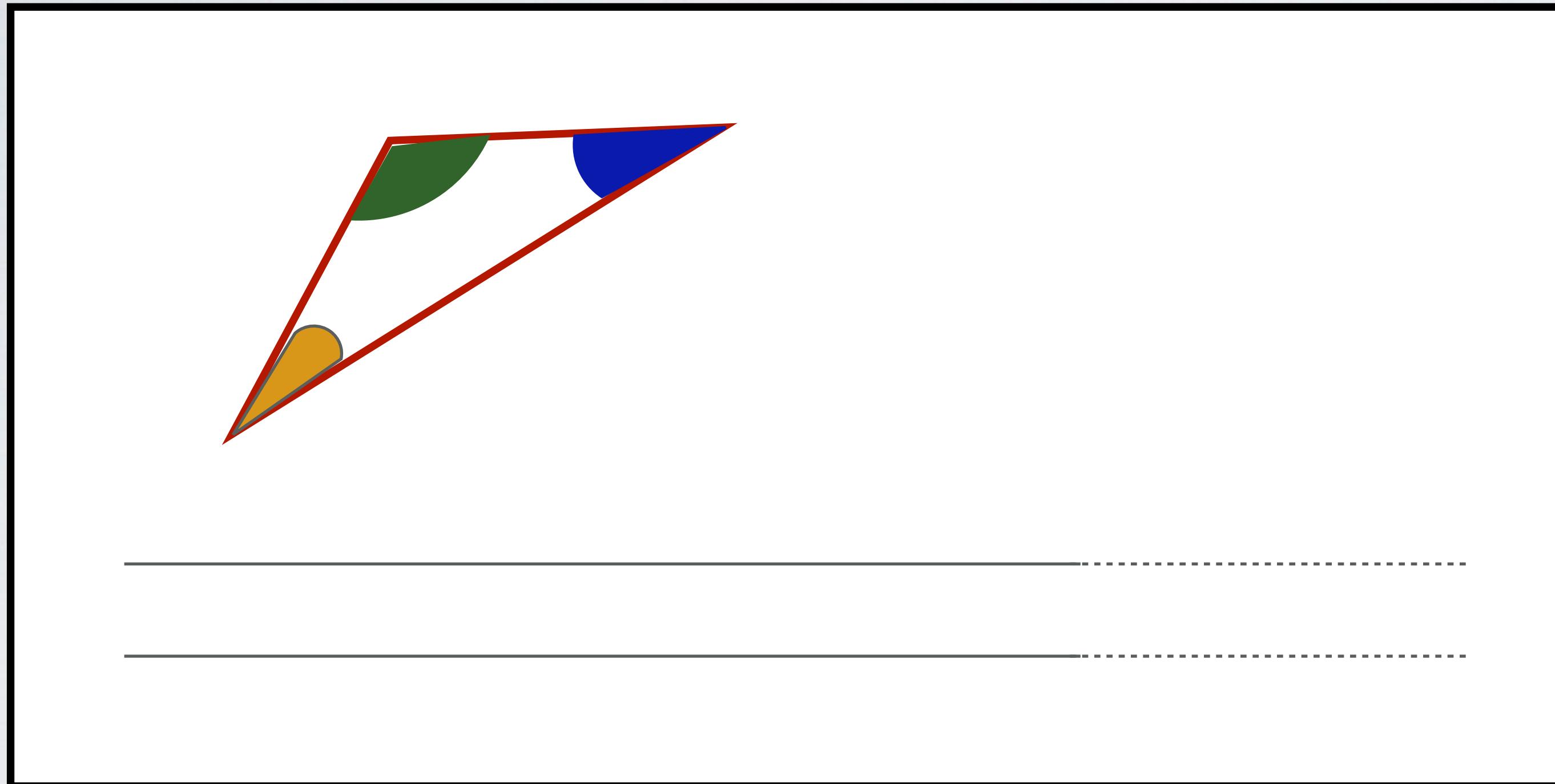
INTRINSIC CURVATURE



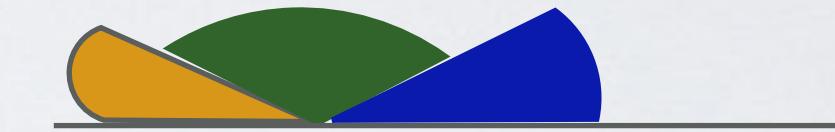
- sum of angles in a triangle



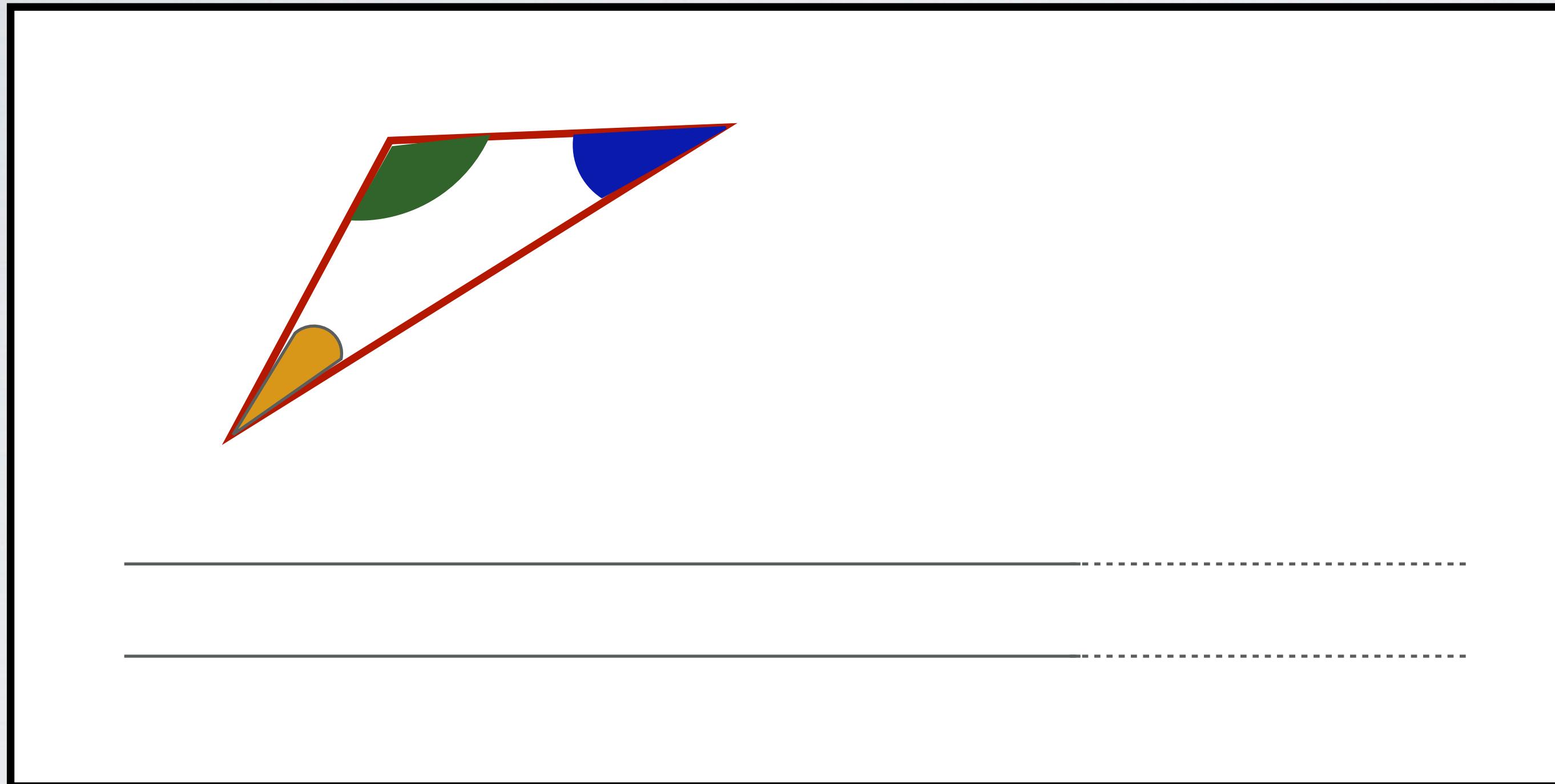
INTRINSIC CURVATURE



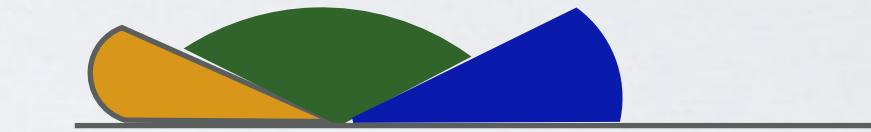
- sum of angles in a triangle



INTRINSIC CURVATURE

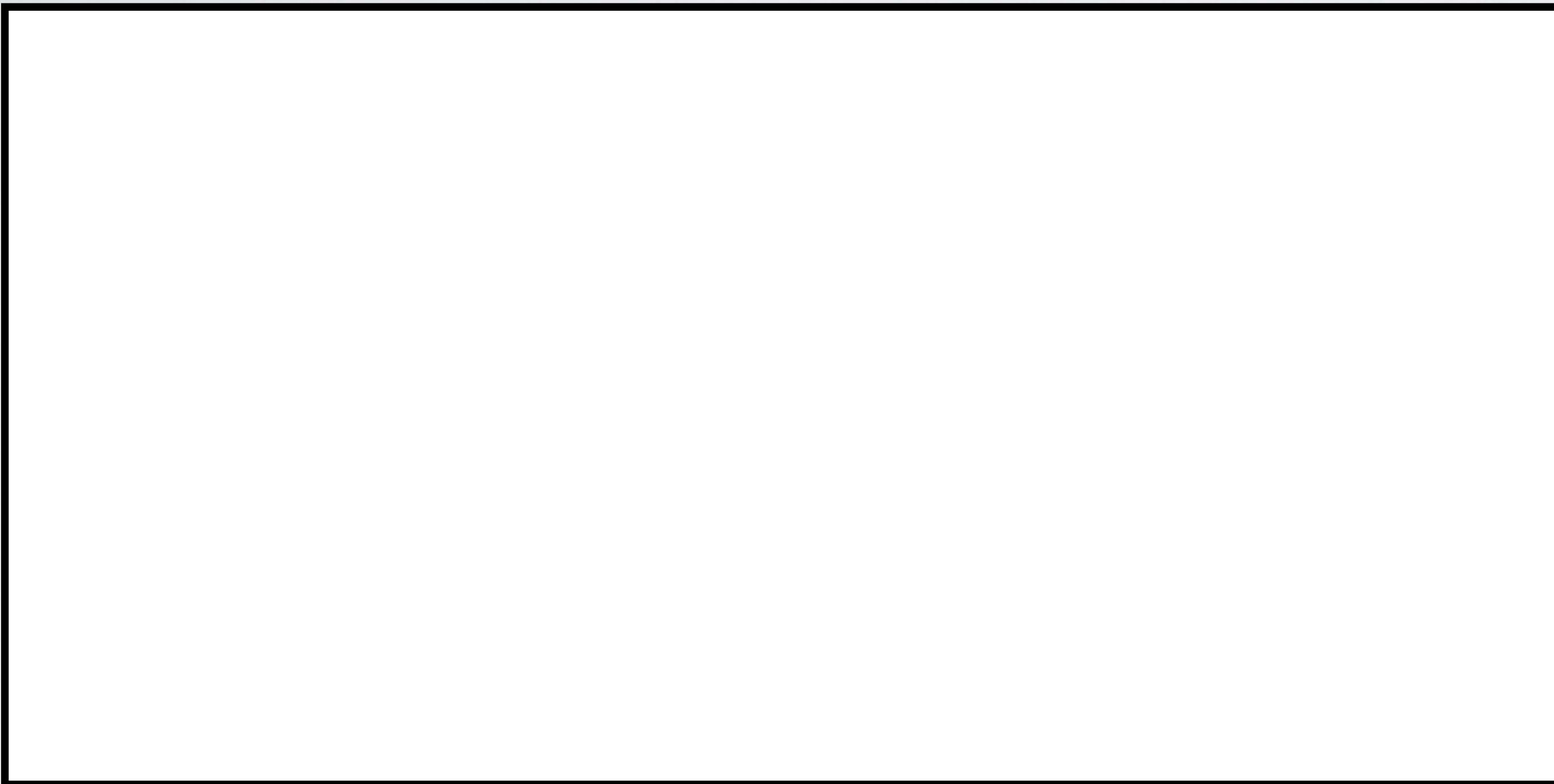


- sum of angles in a triangle



- Parallel lines stay at
same distance apart

INTRINSIC CURVATURE



INTRINSIC CURVATURE



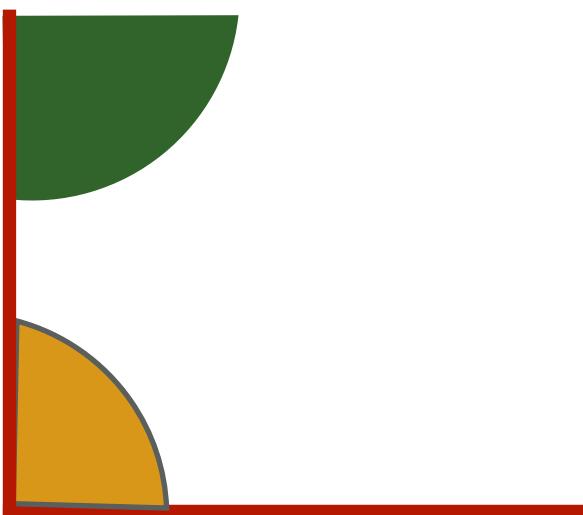
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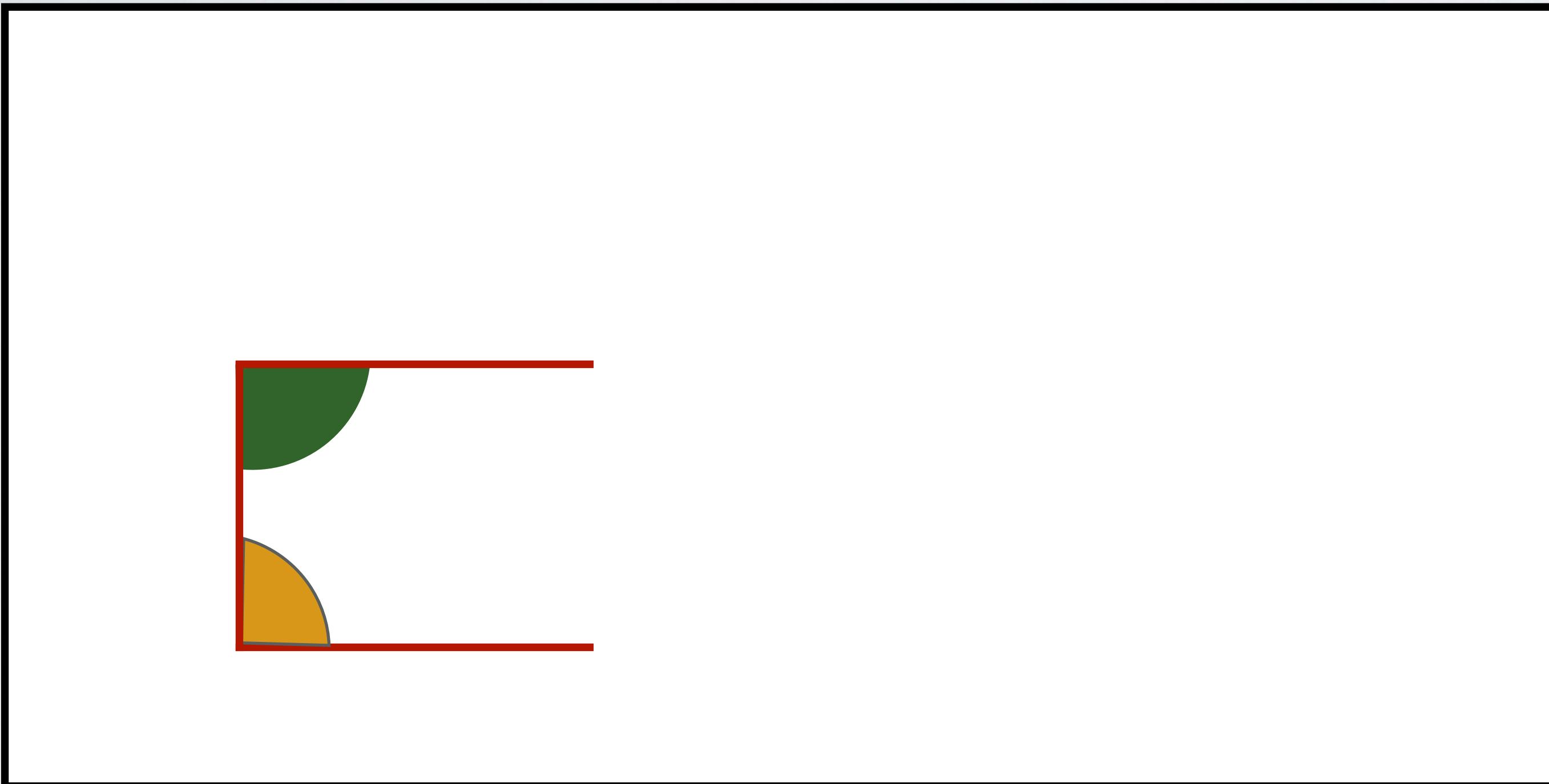
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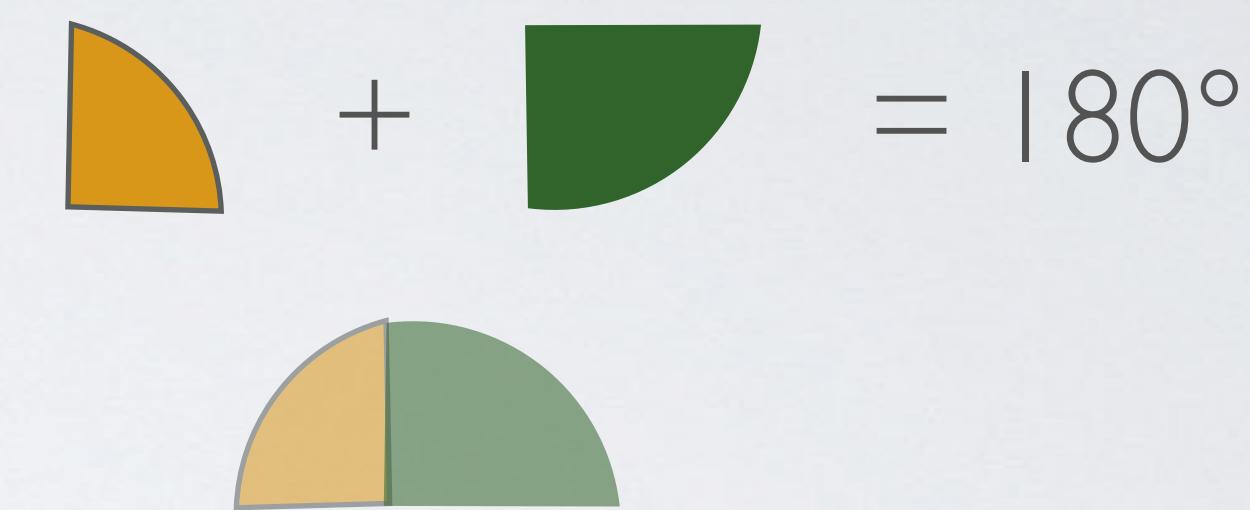
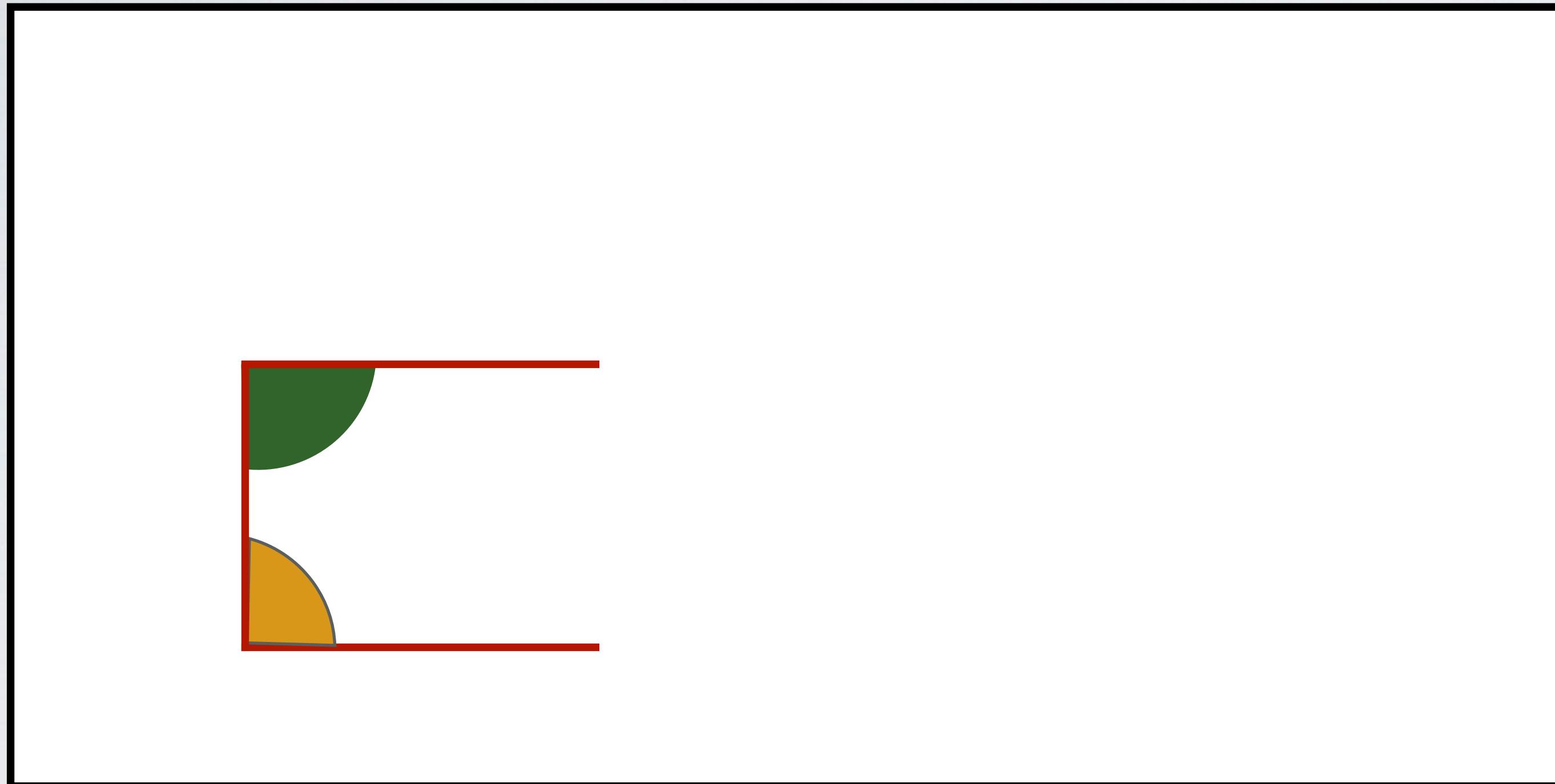
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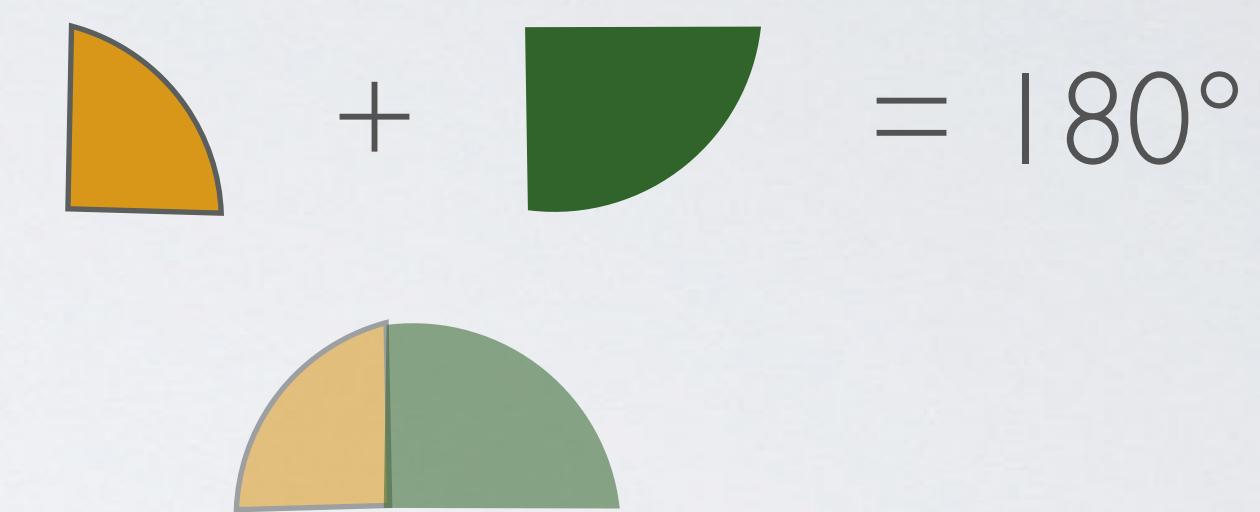
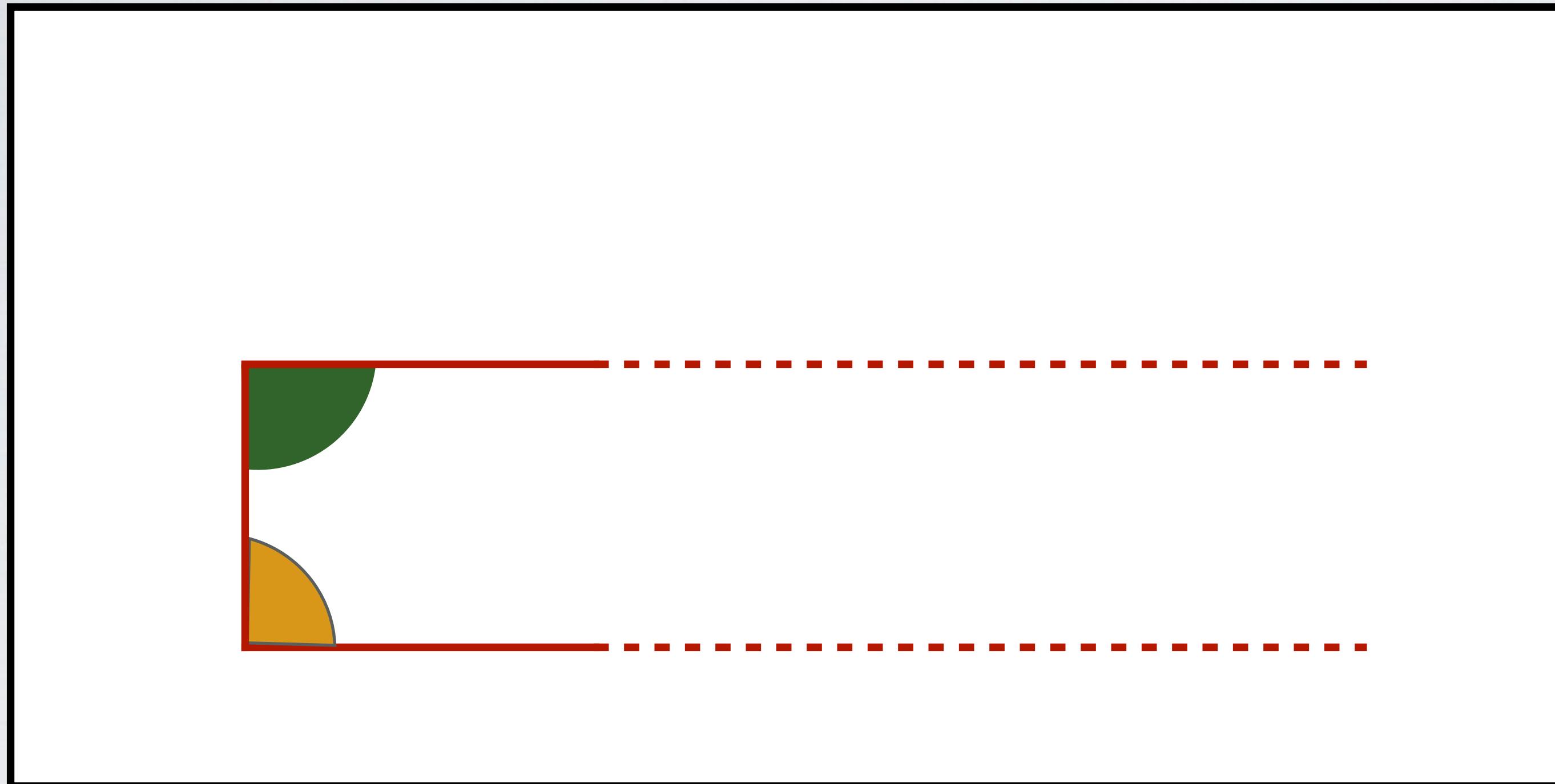
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INTRINSIC CURVATURE

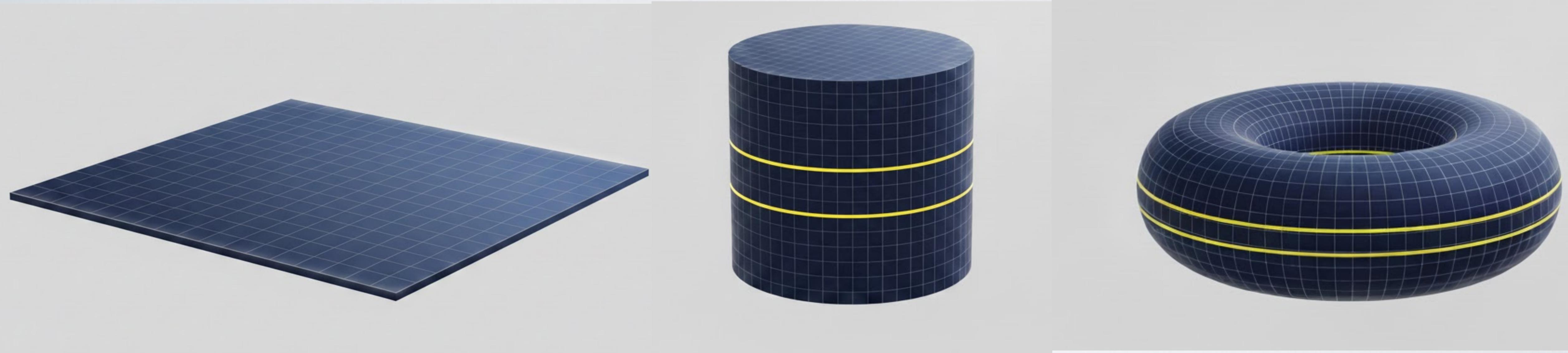


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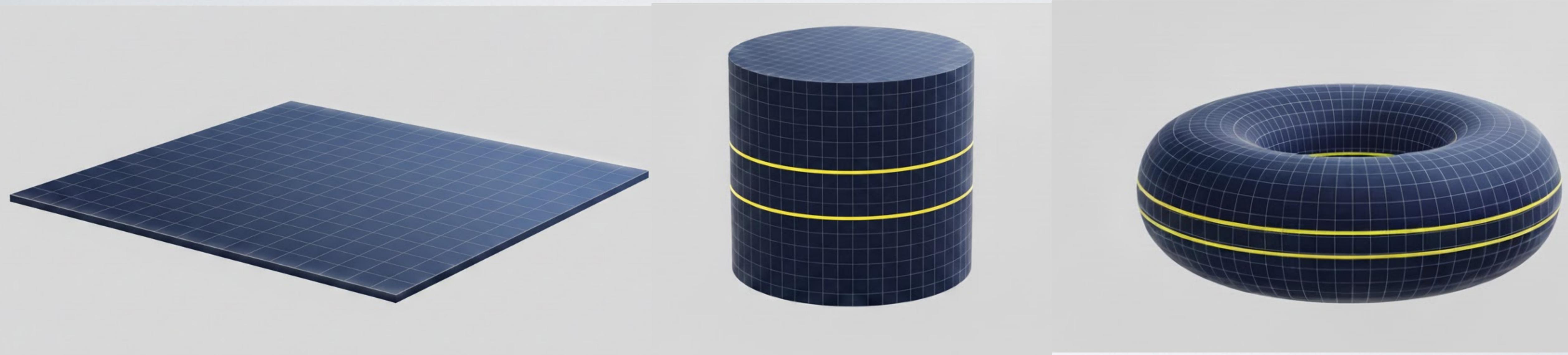
- Parallel lines stay at
same distance apart

FLAT 2D SURFACES



2D SURFACES **FLAT** WITH RESPECT TO
THEIR OWN **2D SPACE**

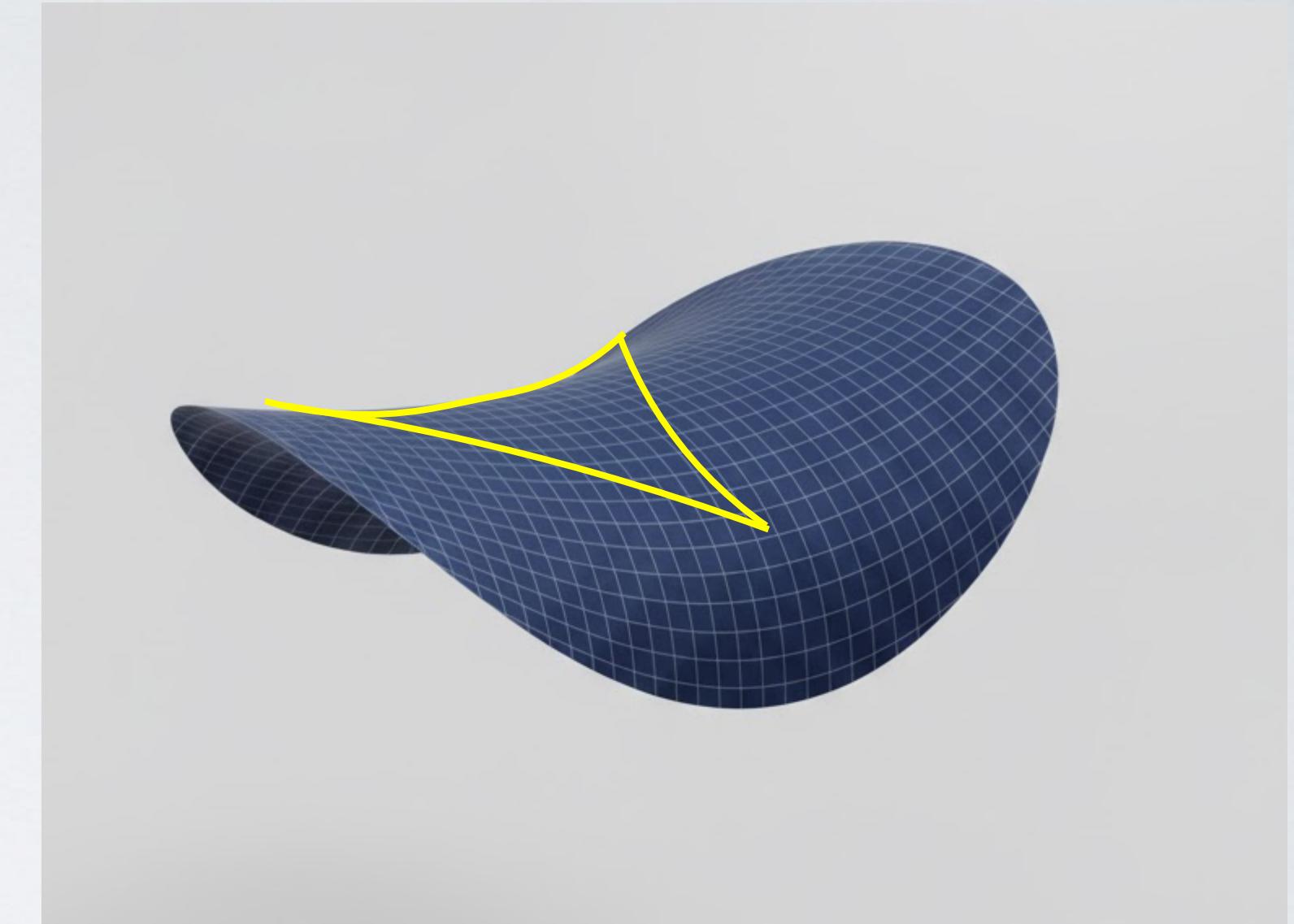
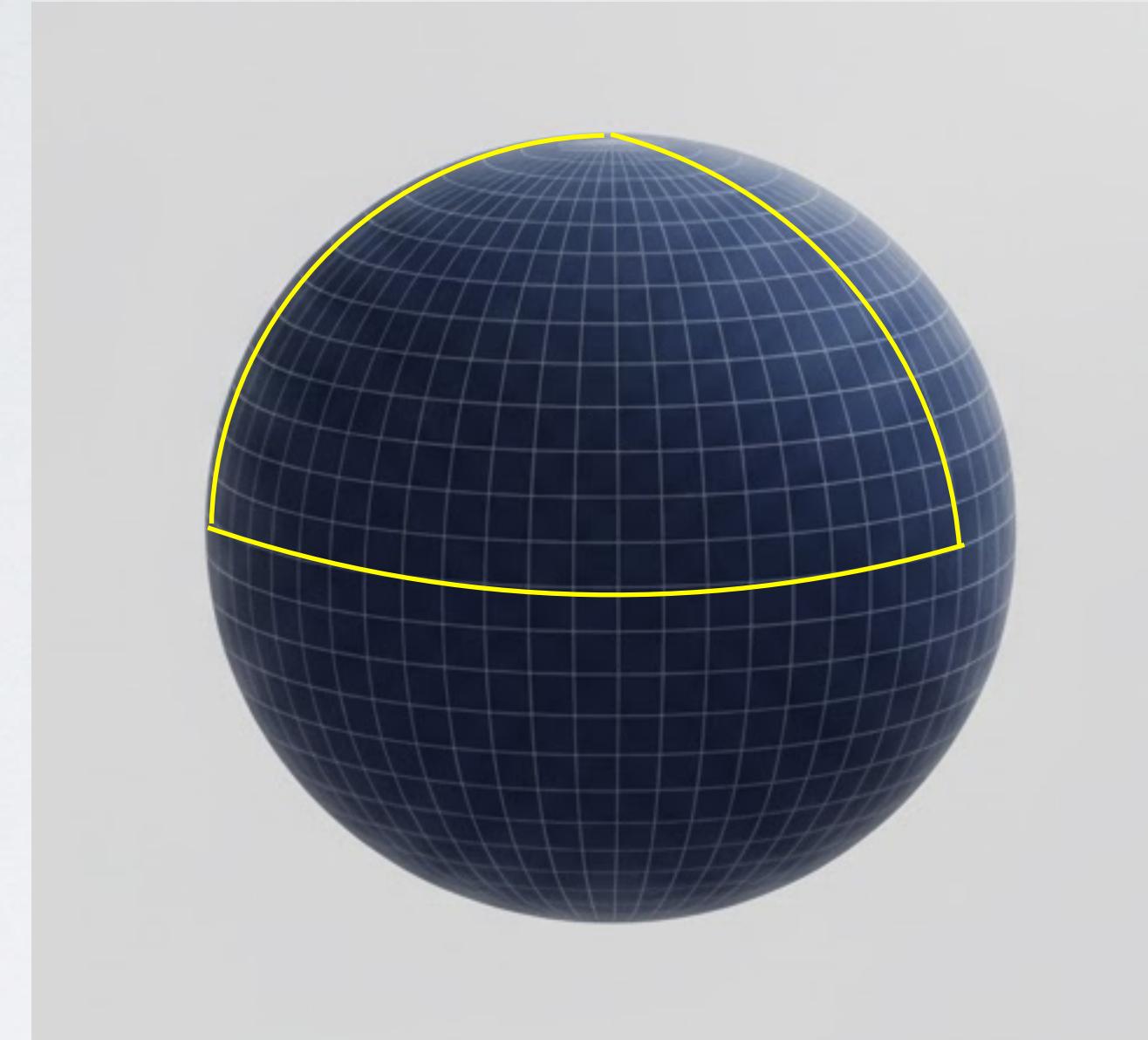
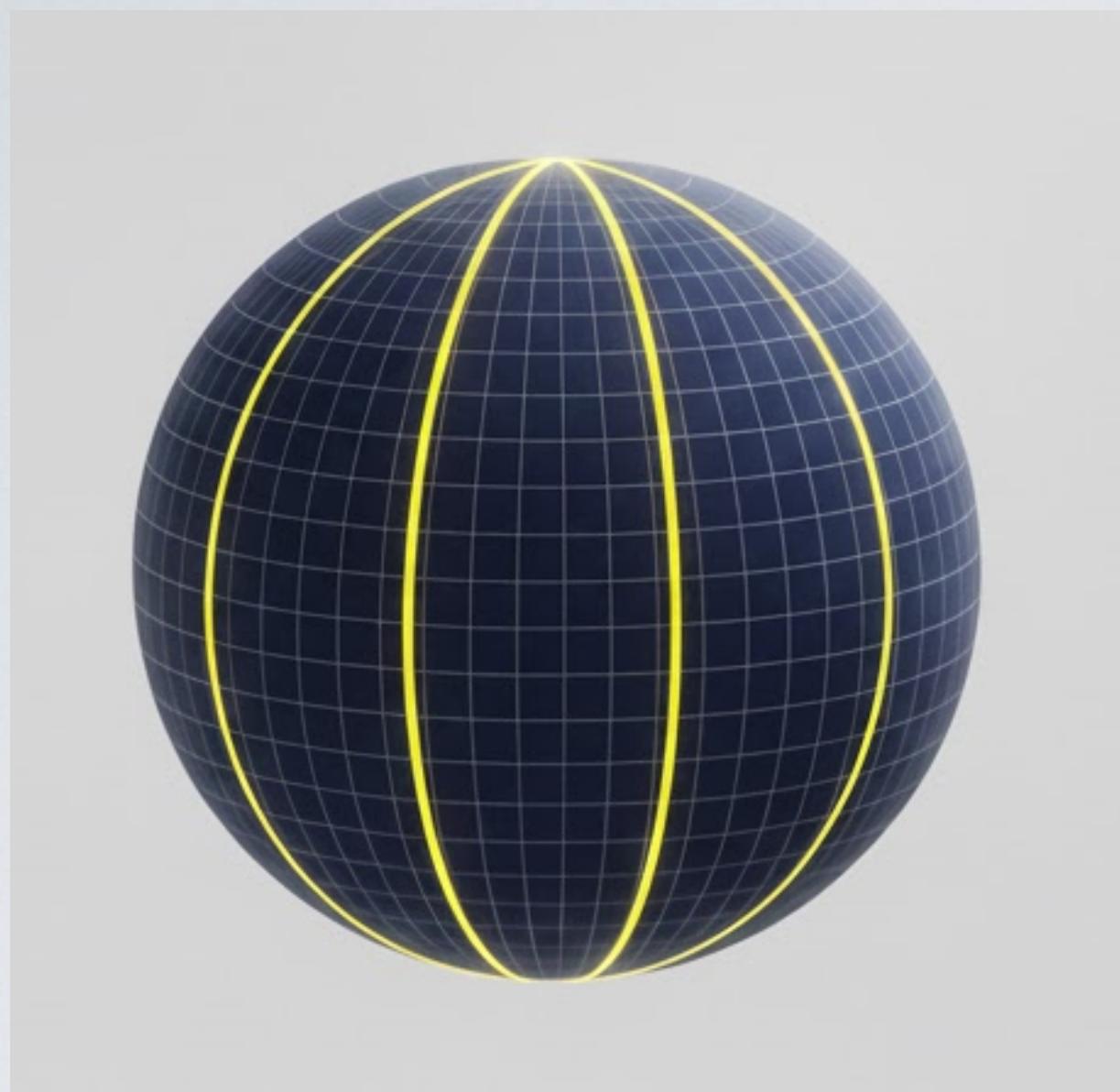
FLAT 2D SURFACES



2D SURFACES **FLAT** WITH RESPECT TO
THEIR OWN **2D SPACE**

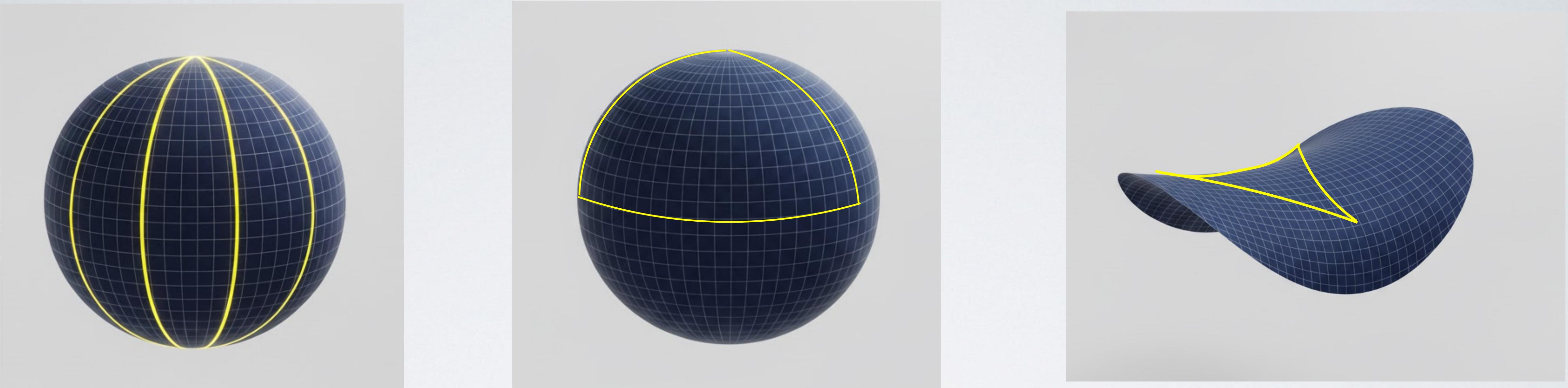
VANISHING **INTRINSIC** CURVATURE

CURVED 2D SURFACES



2D SURFACES **CURVED** WITH
RESPECTIVE TO THEIR OWN **2D SPACE**

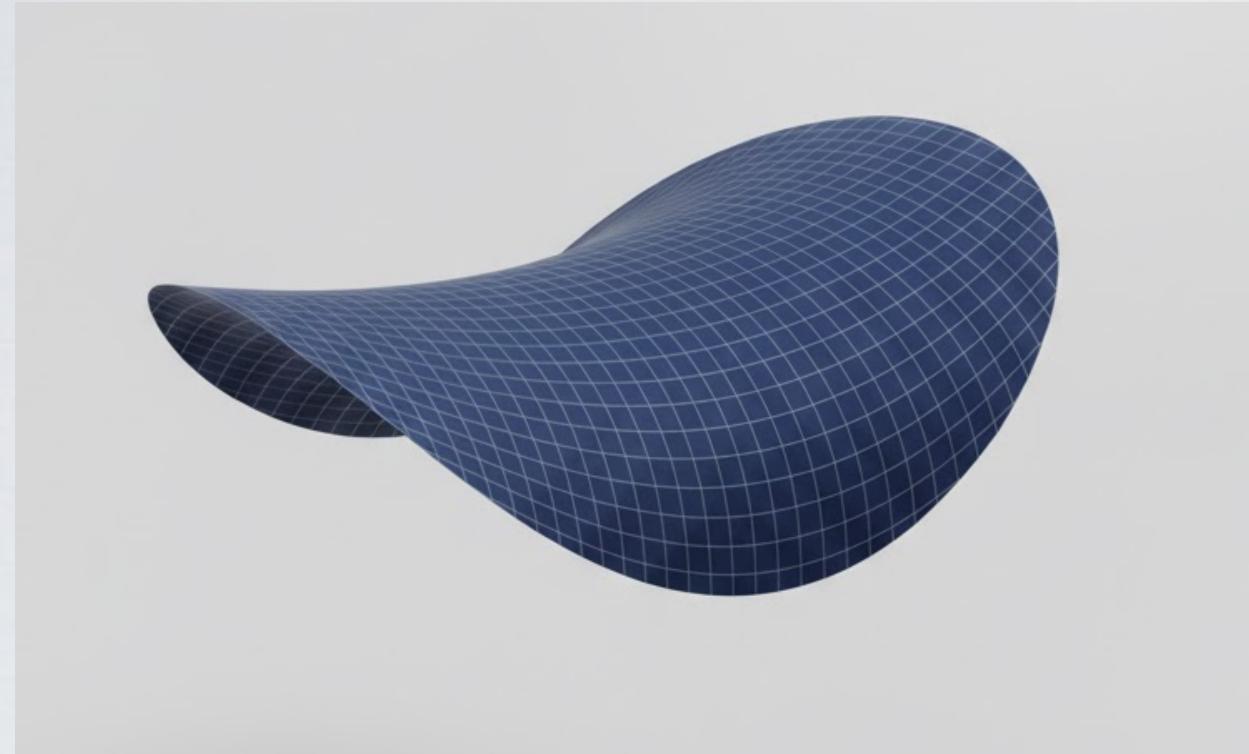
CURVED 2D SURFACES



2D SURFACES **CURVED** WITH
RESPECTIVE TO THEIR OWN **2D SPACE**

NON-VANISHING **INTRINSIC** CURVATURE

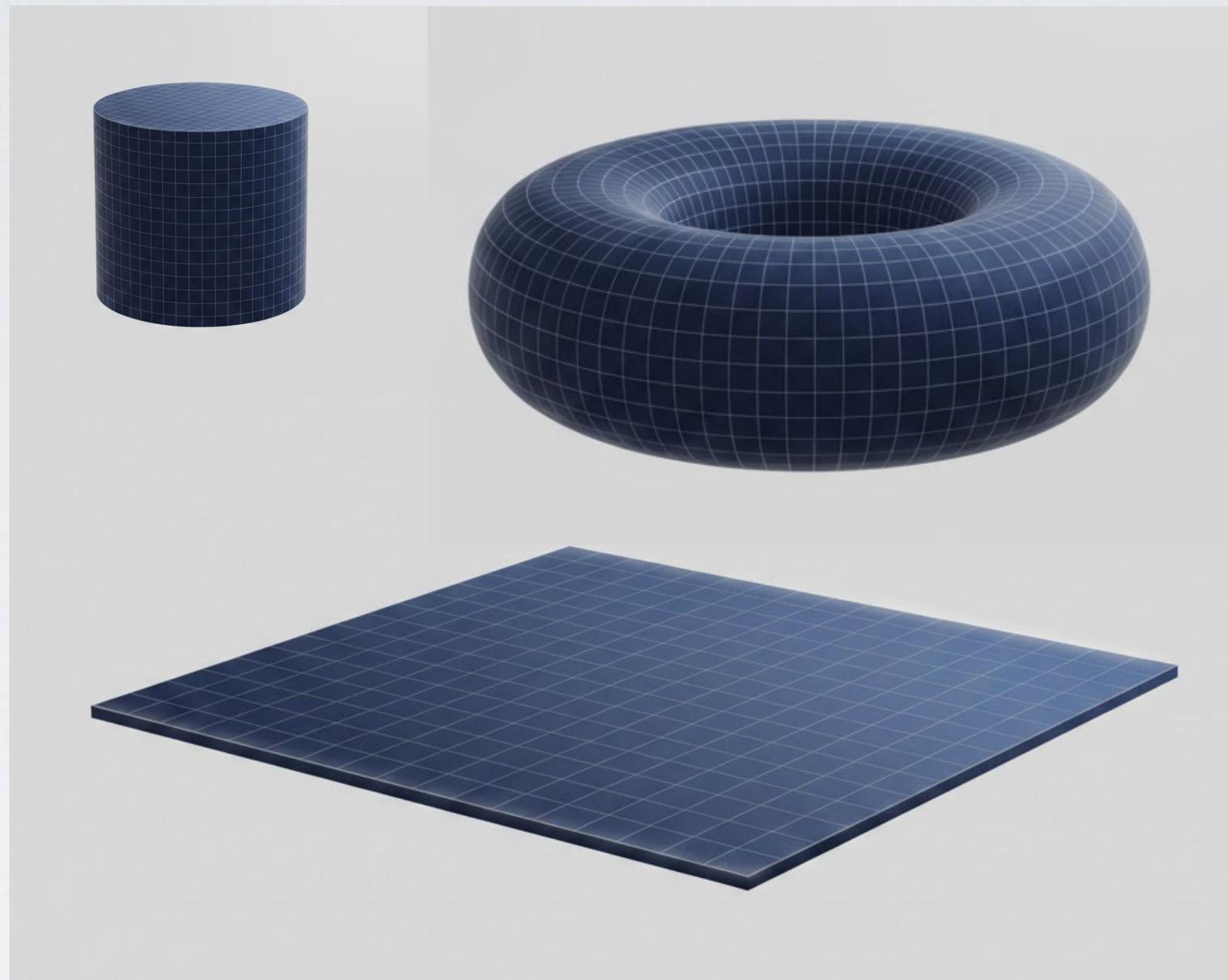
INTRINSIC CURVATURE



NEGATIVE CURVATURE

- sum of angles in a triangle

$$\text{Yellow wedge} + \text{Green wedge} + \text{Blue wedge} < 180^\circ$$



ZERO CURVATURE

- sum of angles in a triangle

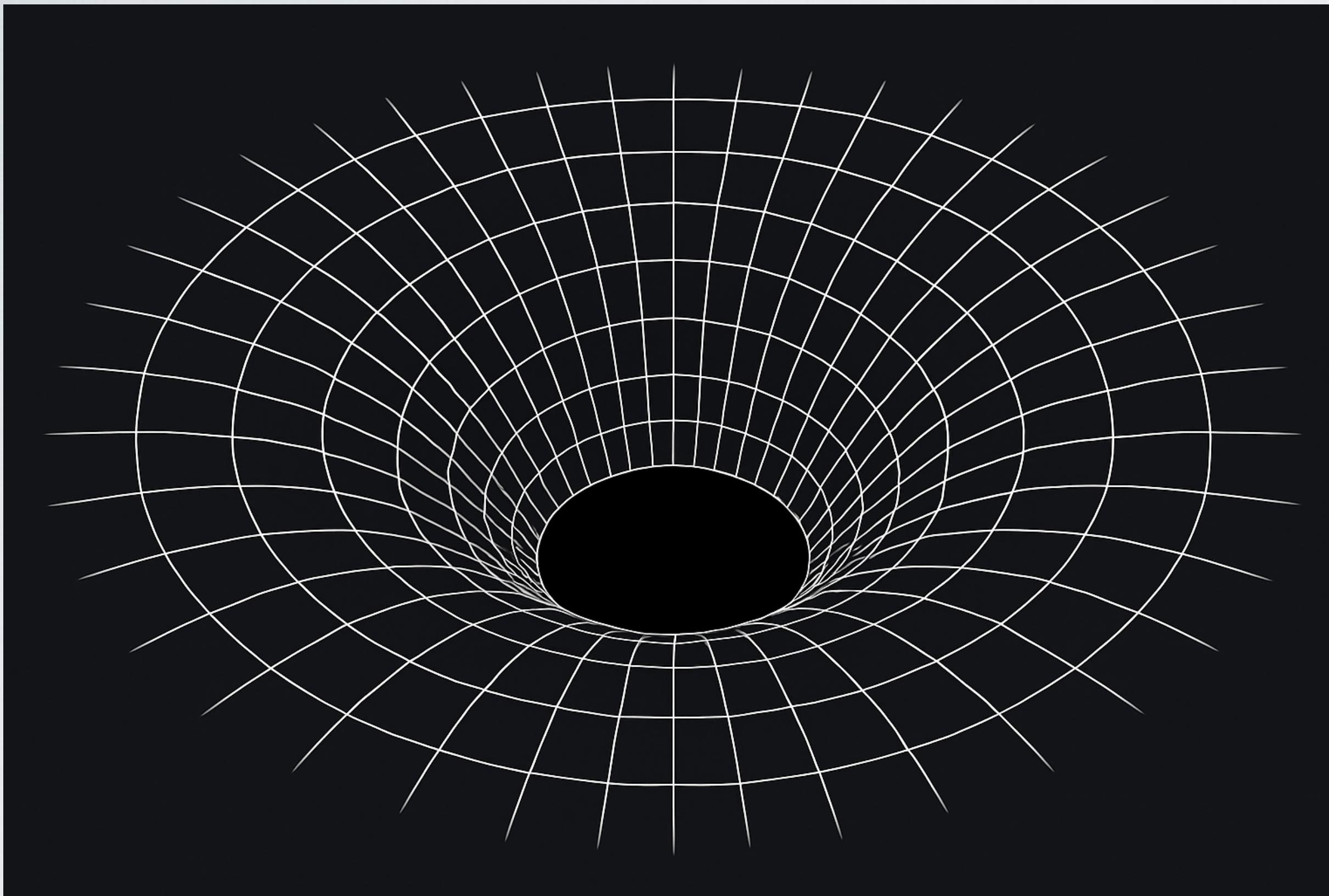
$$\text{Yellow wedge} + \text{Green wedge} + \text{Blue wedge} = 180^\circ$$

POSITIVE CURVATURE

- sum of angles in a triangle

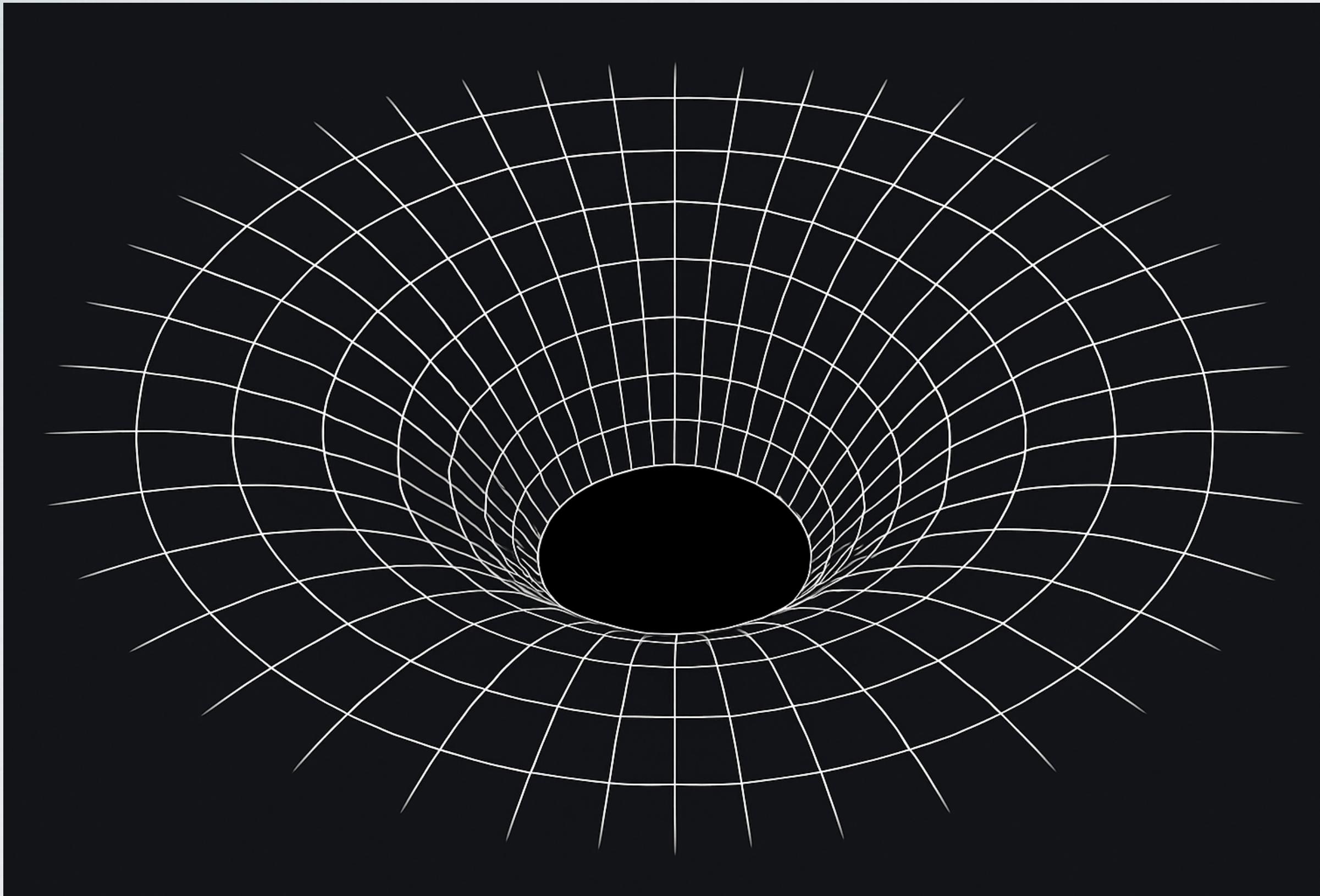
$$\text{Yellow wedge} + \text{Green wedge} + \text{Blue wedge} > 180^\circ$$

BLACK HOLE AND CURVED SPACE



2D REPRESENTATION
CURVED WITH RESPECT TO 3D SPACE

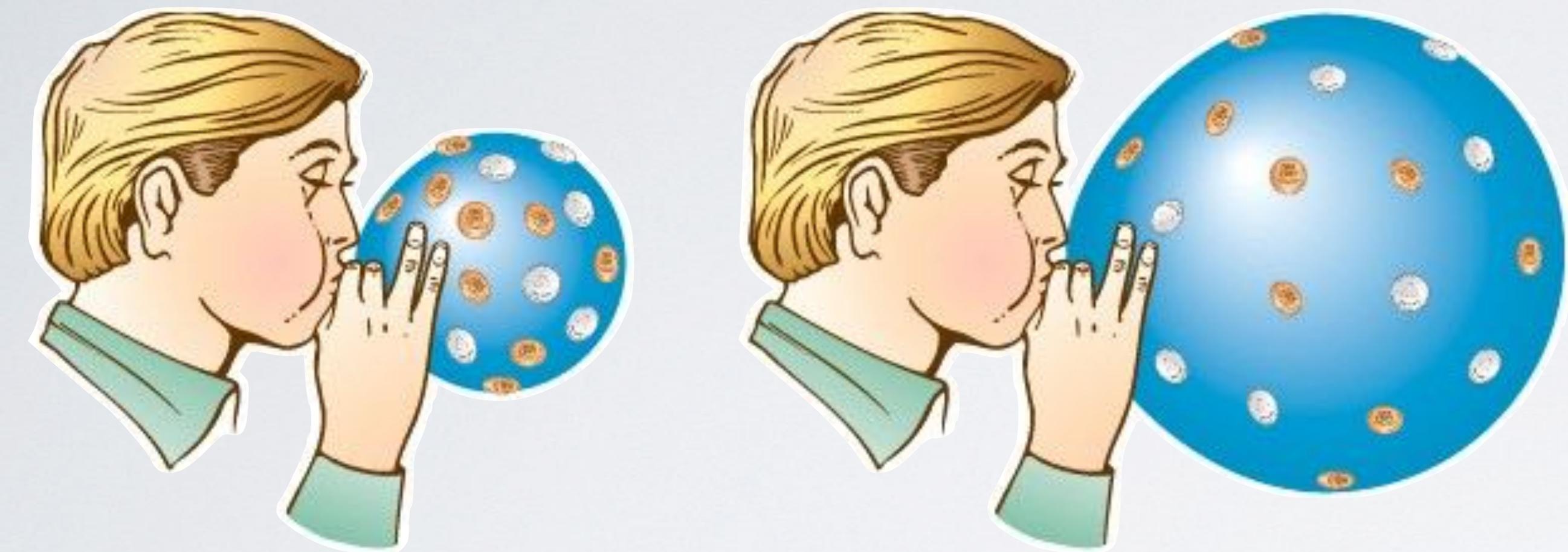
BLACK HOLE AND CURVED SPACE



2D REPRESENTATION
CURVED WITH RESPECT TO 3D SPACE

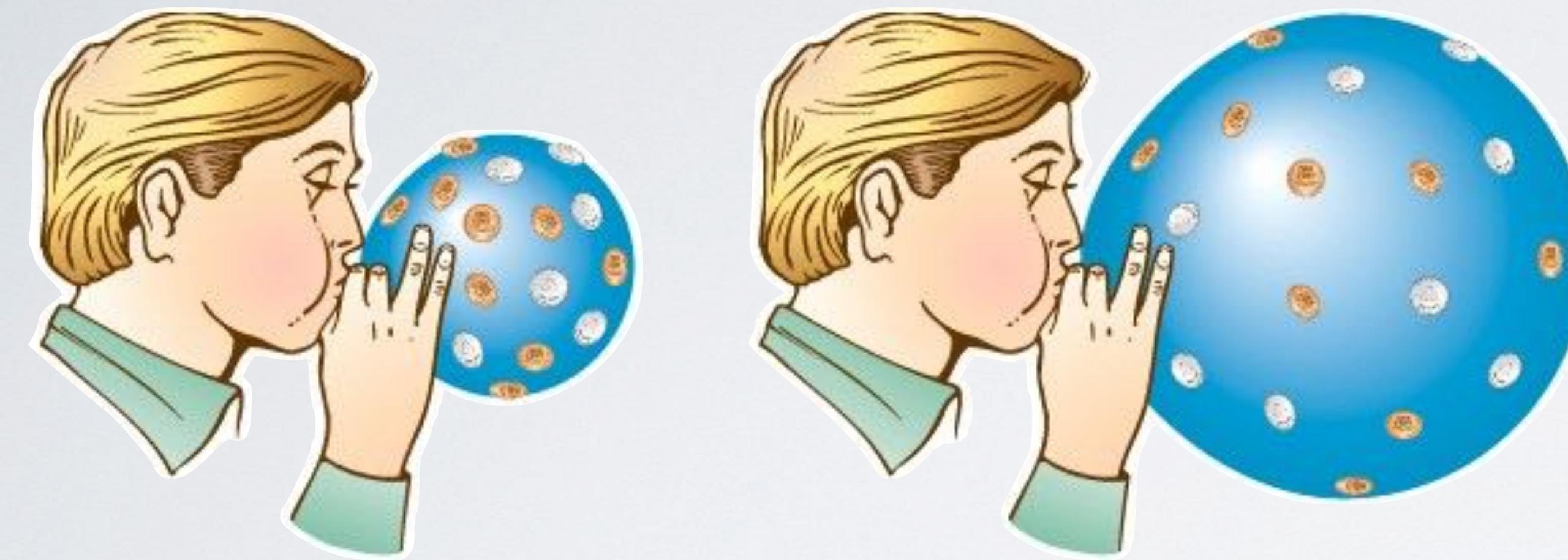
3D SURFACE **CURVED**
WITH RESPECT TO
ITS OWN 3D SPACE

UNIVERSE AND CURVED SPACE



2D REPRESENTATION
EXPANDING INTO AND WITH RESPECT
TO 3D SPACE

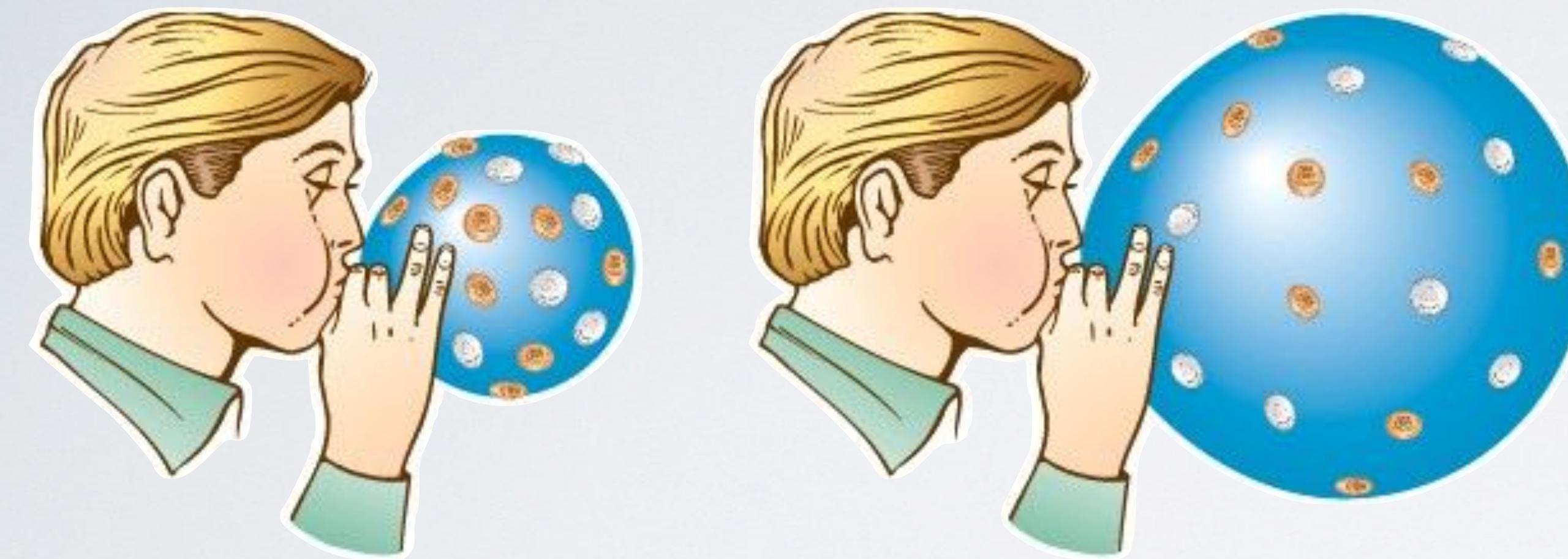
UNIVERSE AND CURVED SPACE



2D REPRESENTATION
EXPANDING INTO AND WITH RESPECT
TO 3D SPACE

3D SURFACE
EXPANDING
WITH RESPECT TO
ITS OWN 3D SPACE

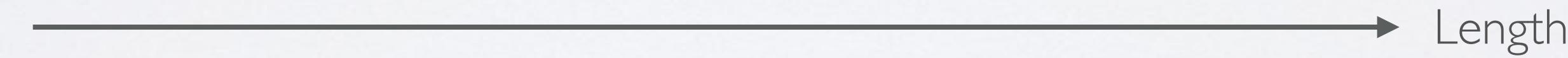
UNIVERSE AND CURVED SPACE



3D SURFACE
EXPANDING
WITH RESPECT TO
ITS OWN 3D SPACE

WHAT ABOUT TIME ?!?

SPACE INTERVALS BETWEEN TWO POINTS



(1 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(1 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(1 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(1 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(1 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



$$d = x$$

(1 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



$$d^2 = x^2$$

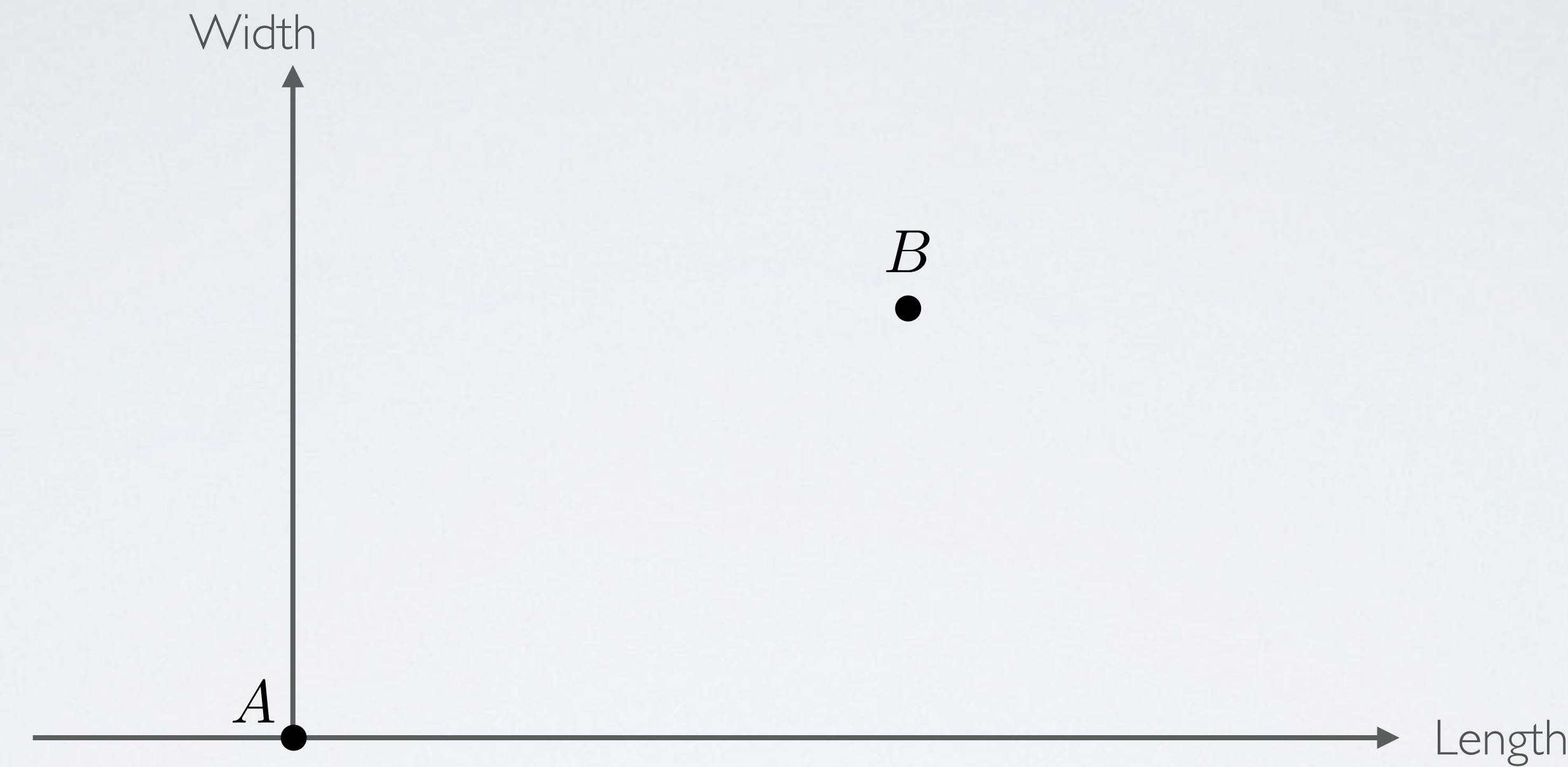
(1 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



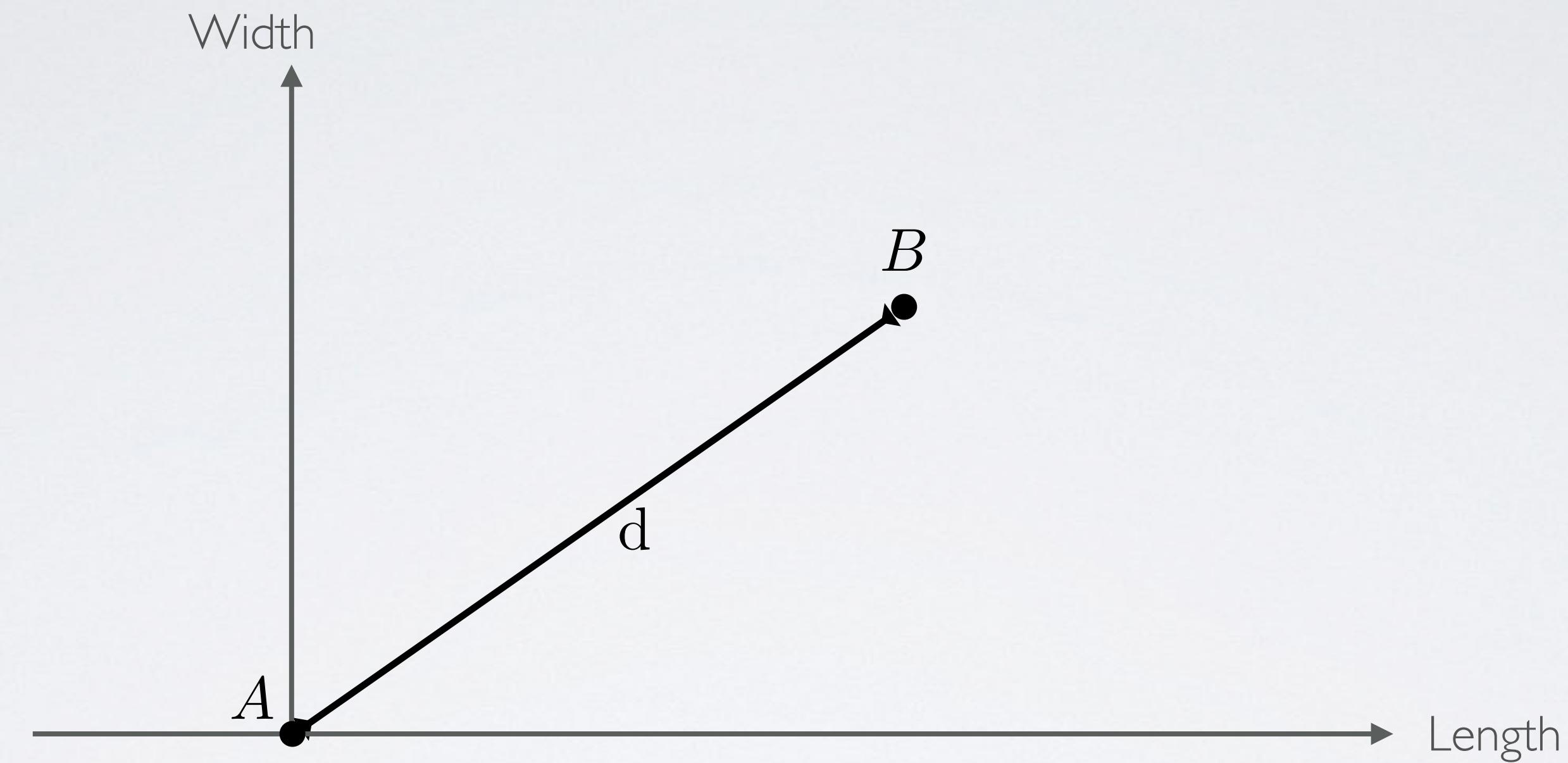
(2 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



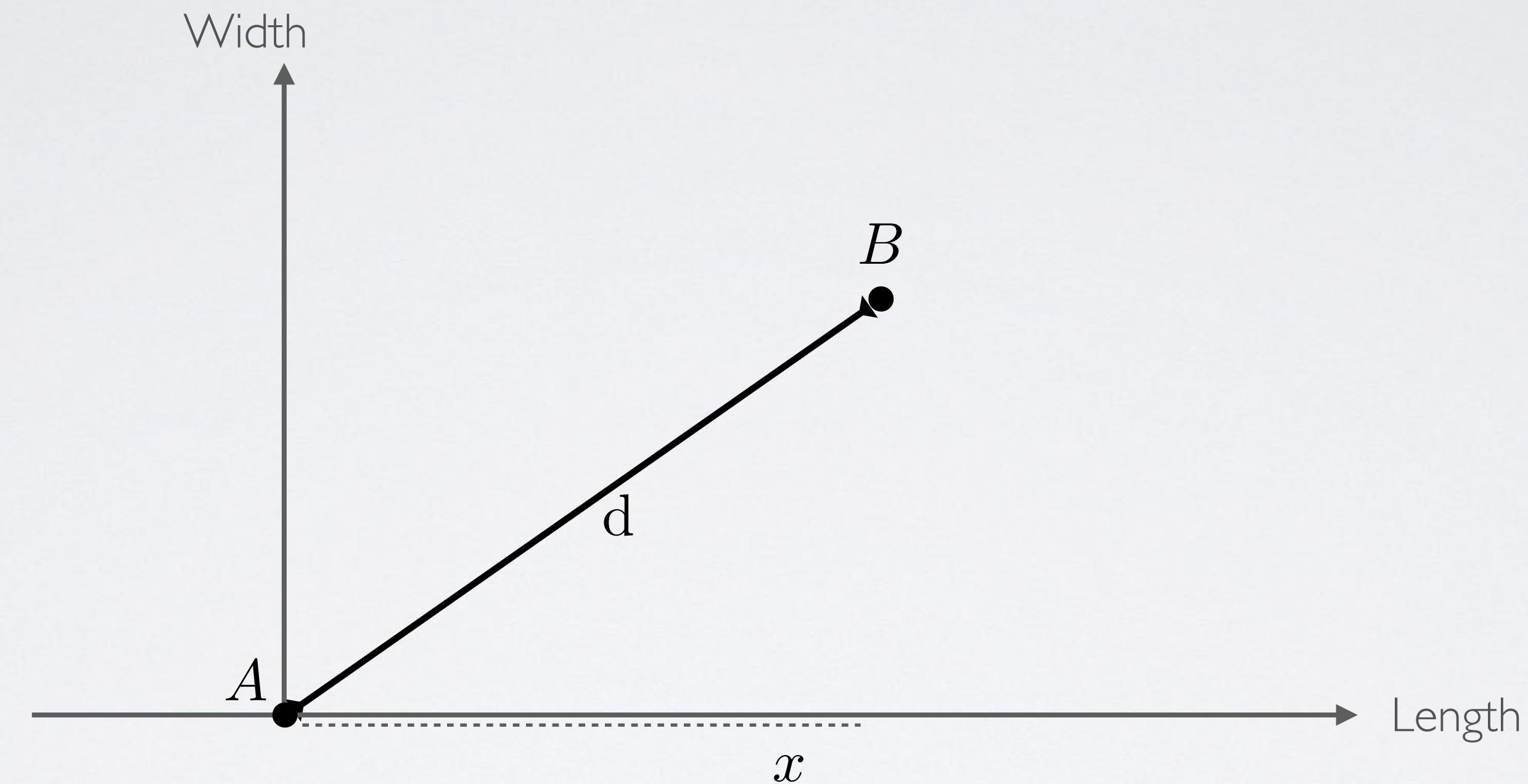
(2 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



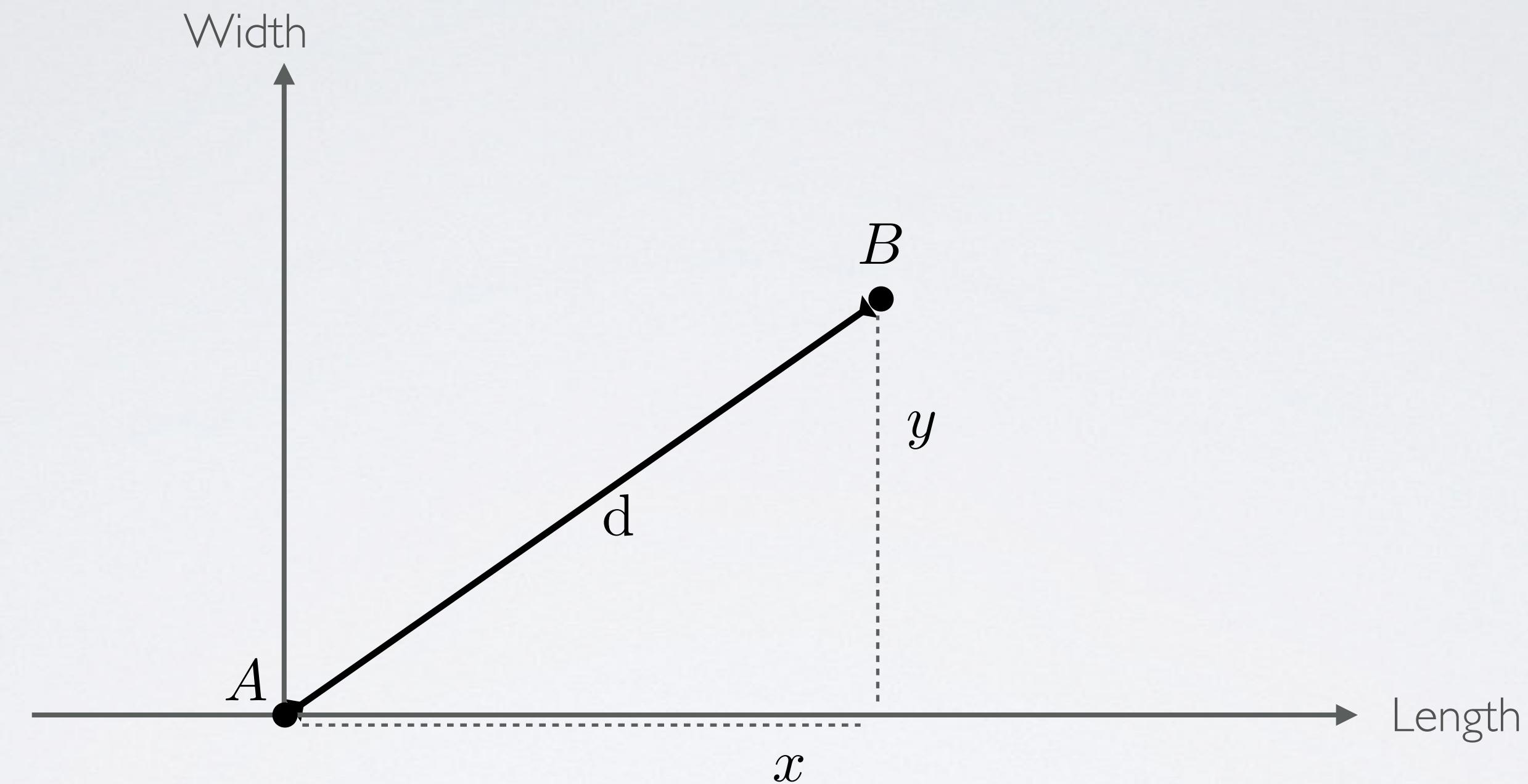
(2 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



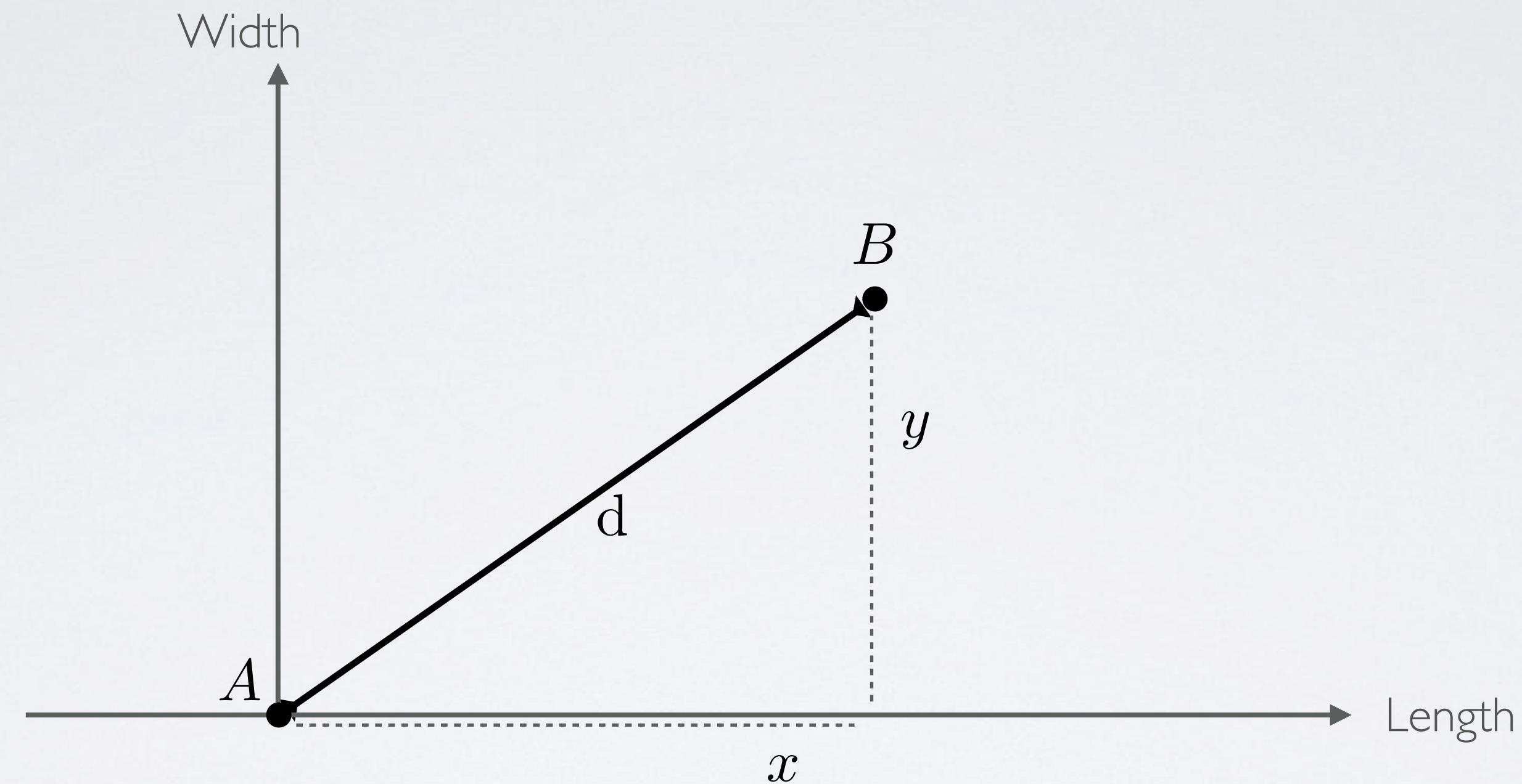
(2 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(2 Dimensional World)

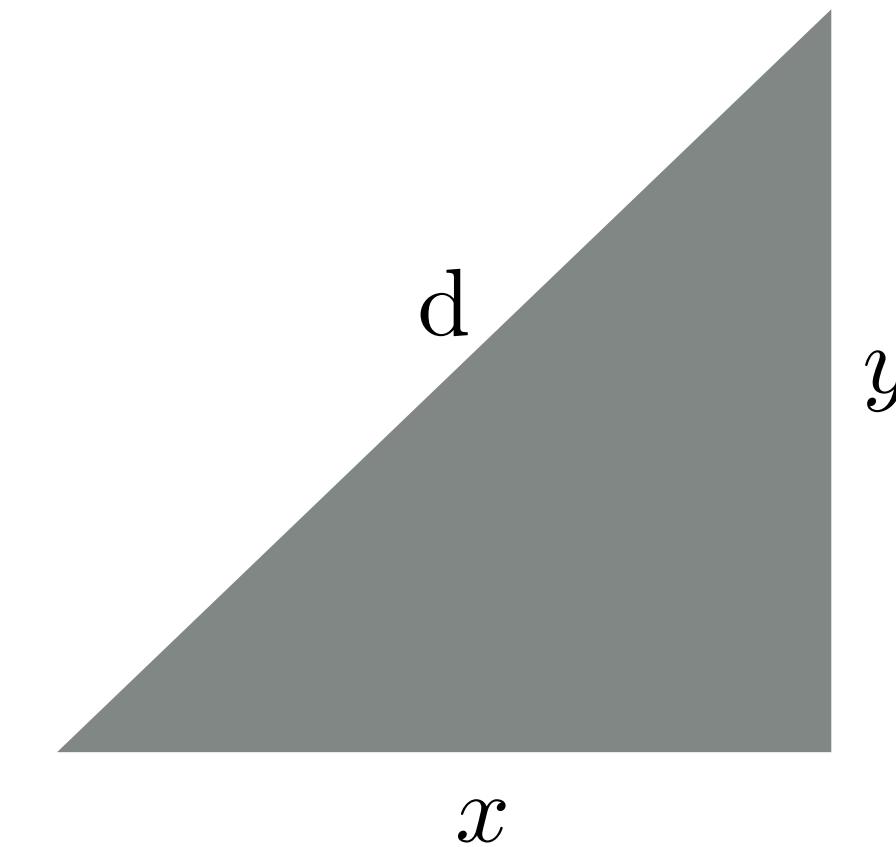
SPACE INTERVALS BETWEEN TWO POINTS



$$d^2 = x^2 + y^2$$

(2 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS

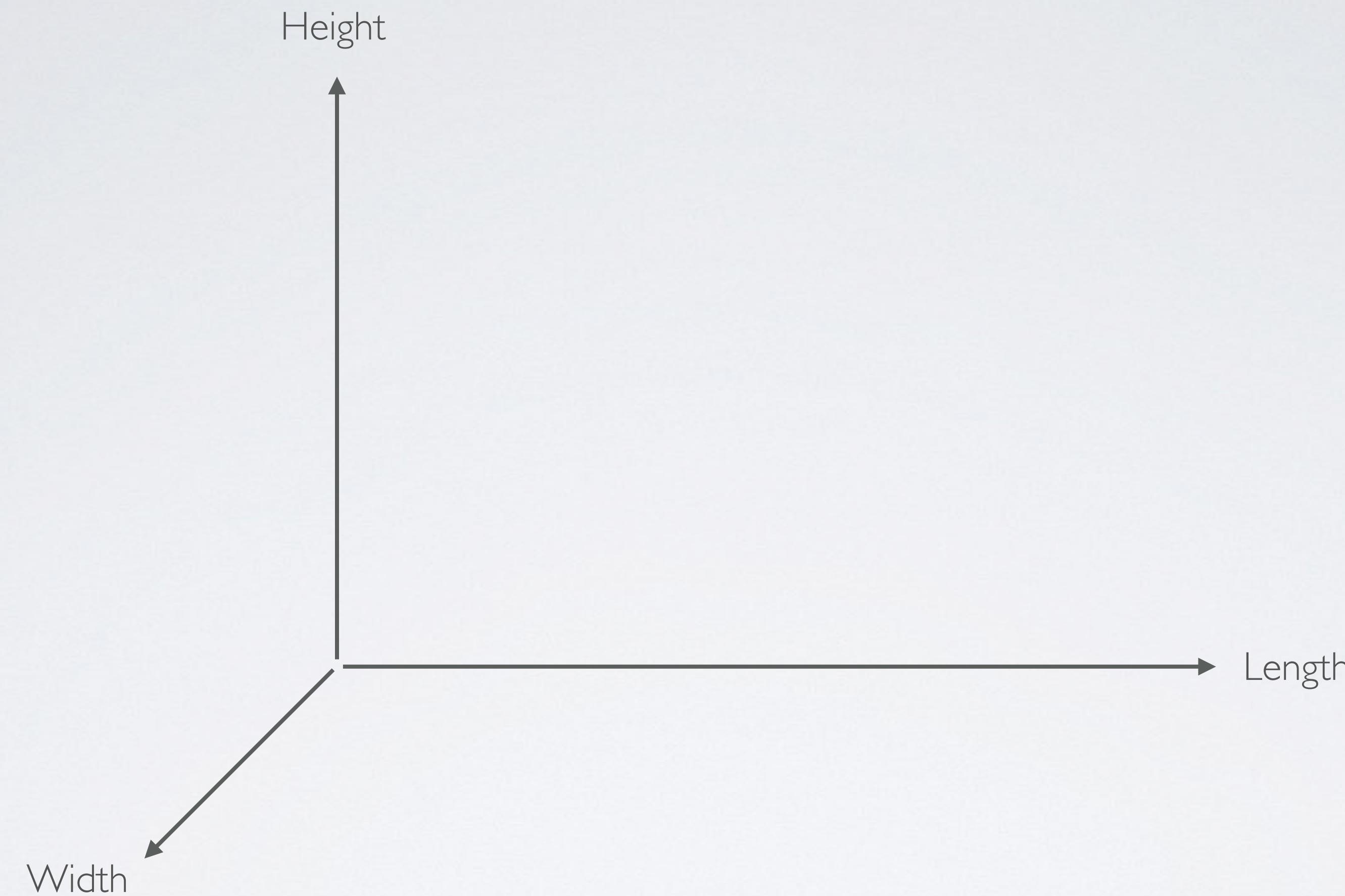


PITAGORA'S THEOREM

$$d^2 = x^2 + y^2$$

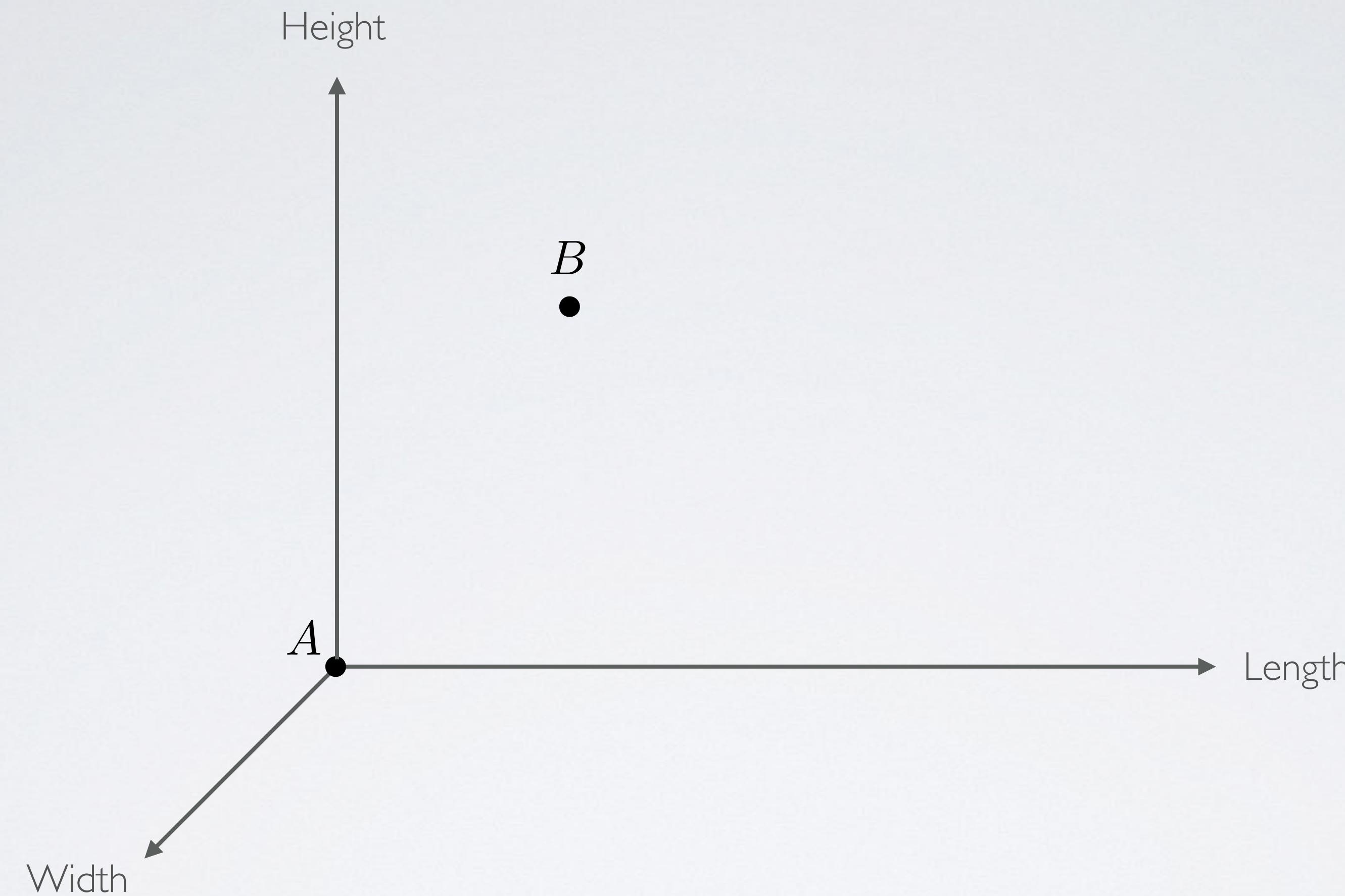
(2 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



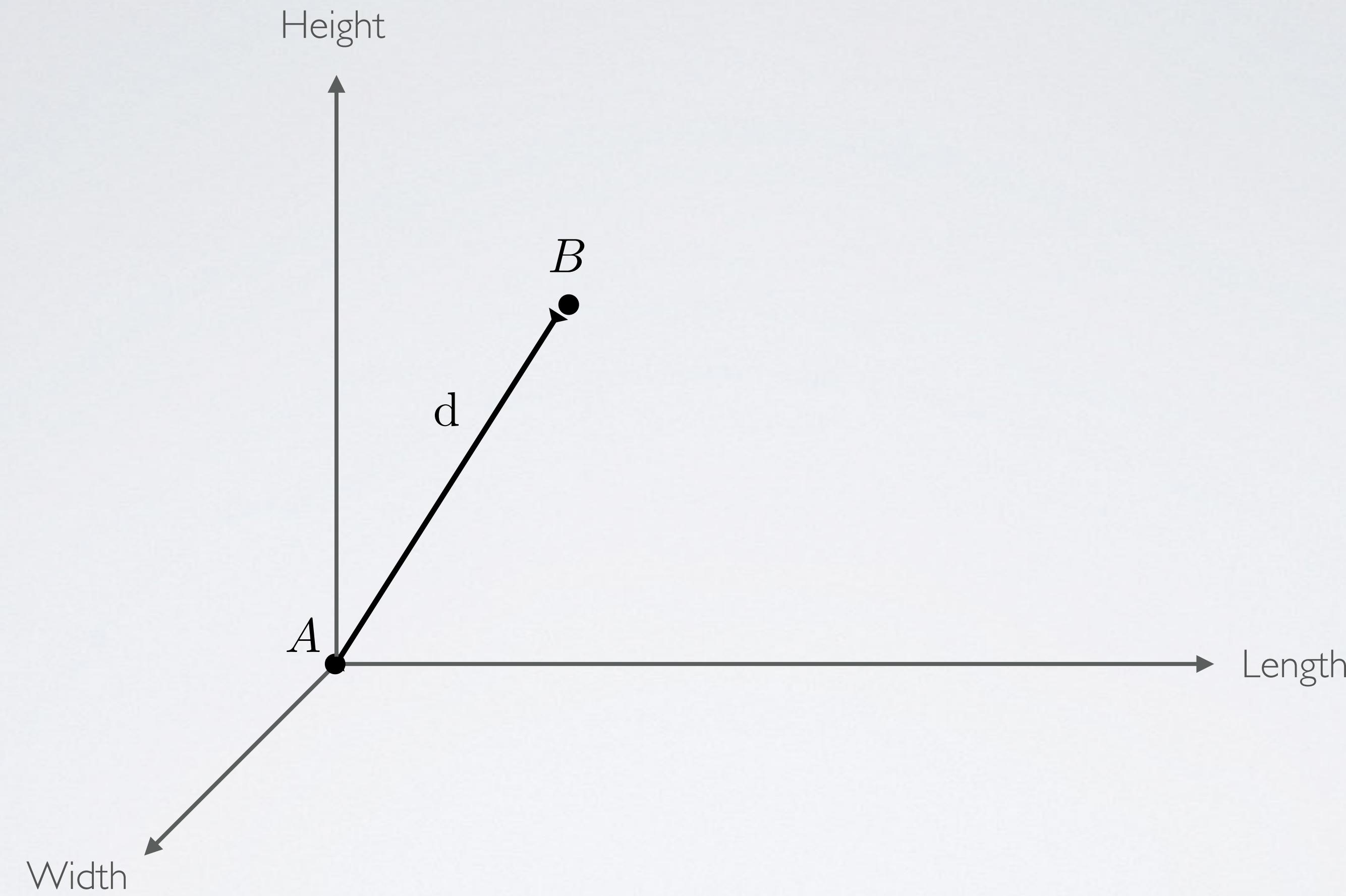
(3 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



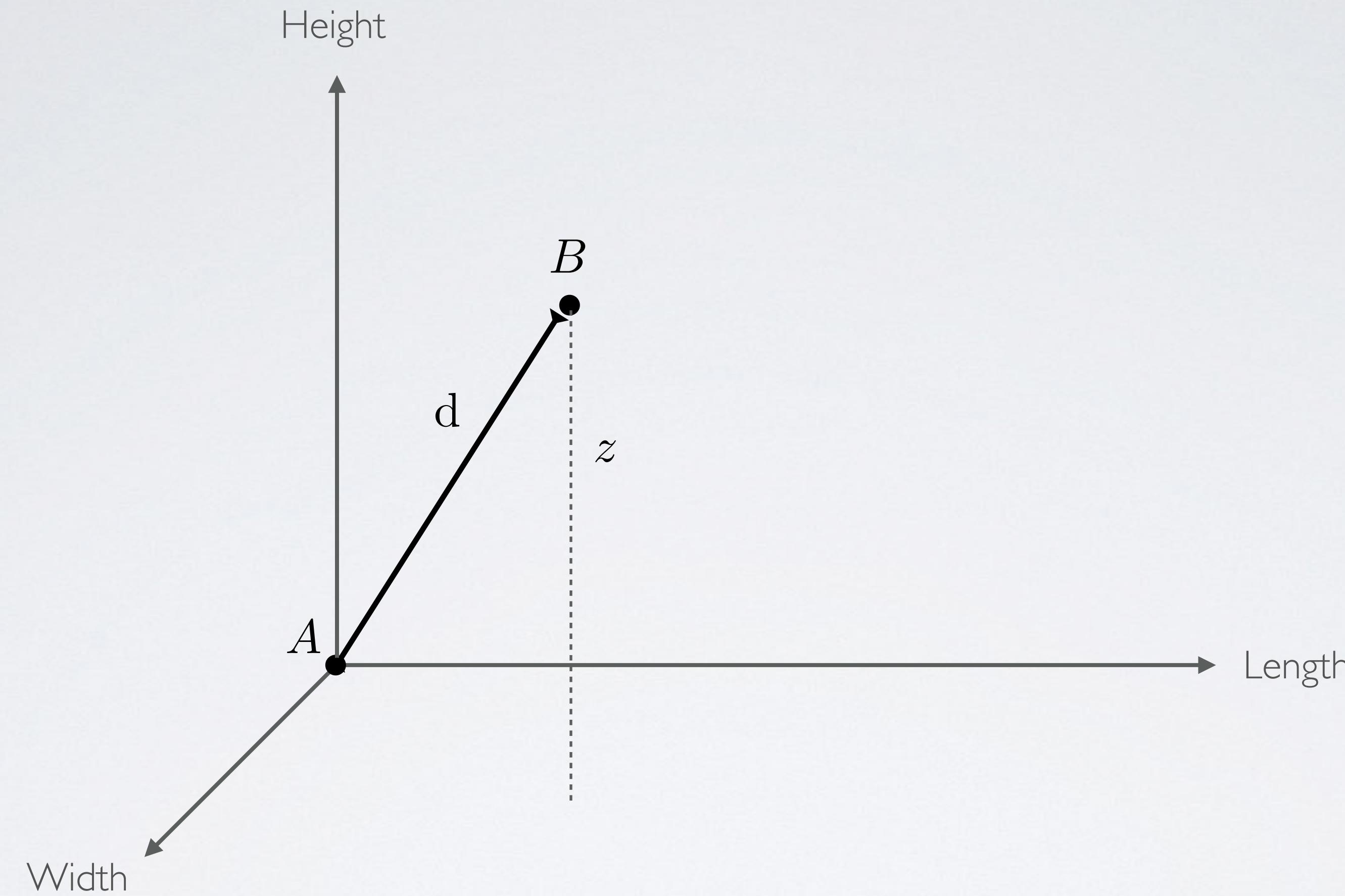
(3 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



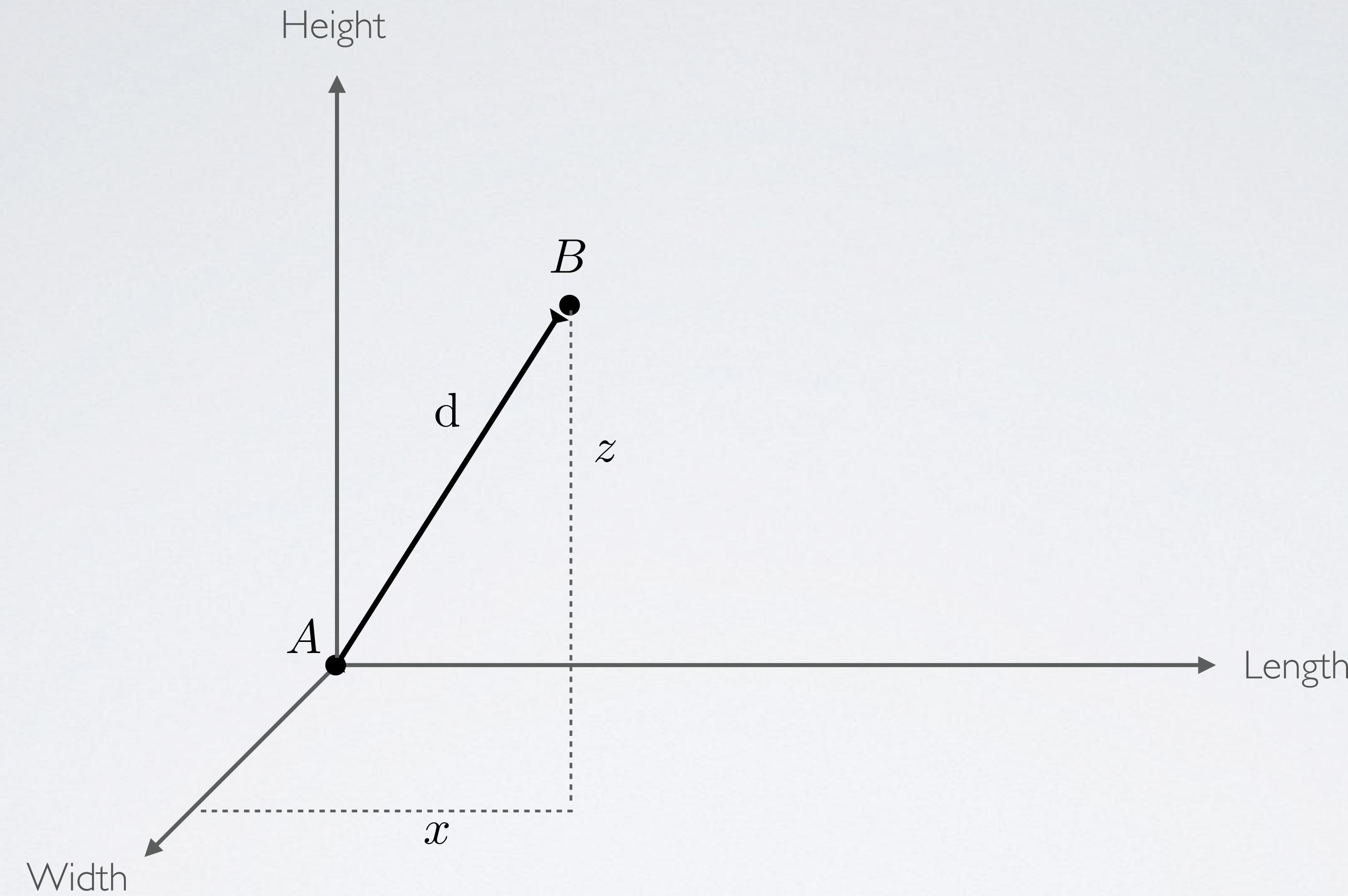
(3 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



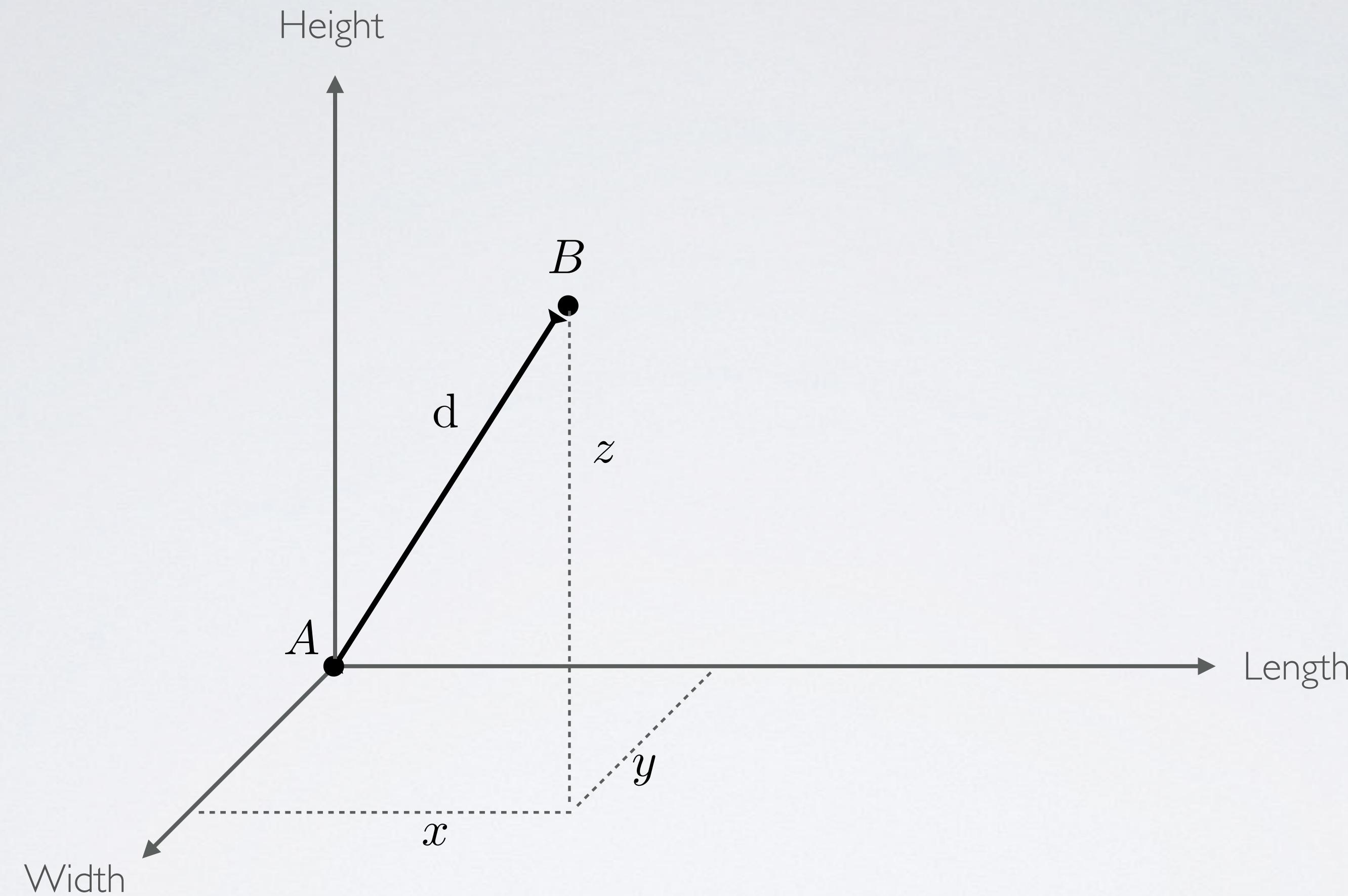
(3 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



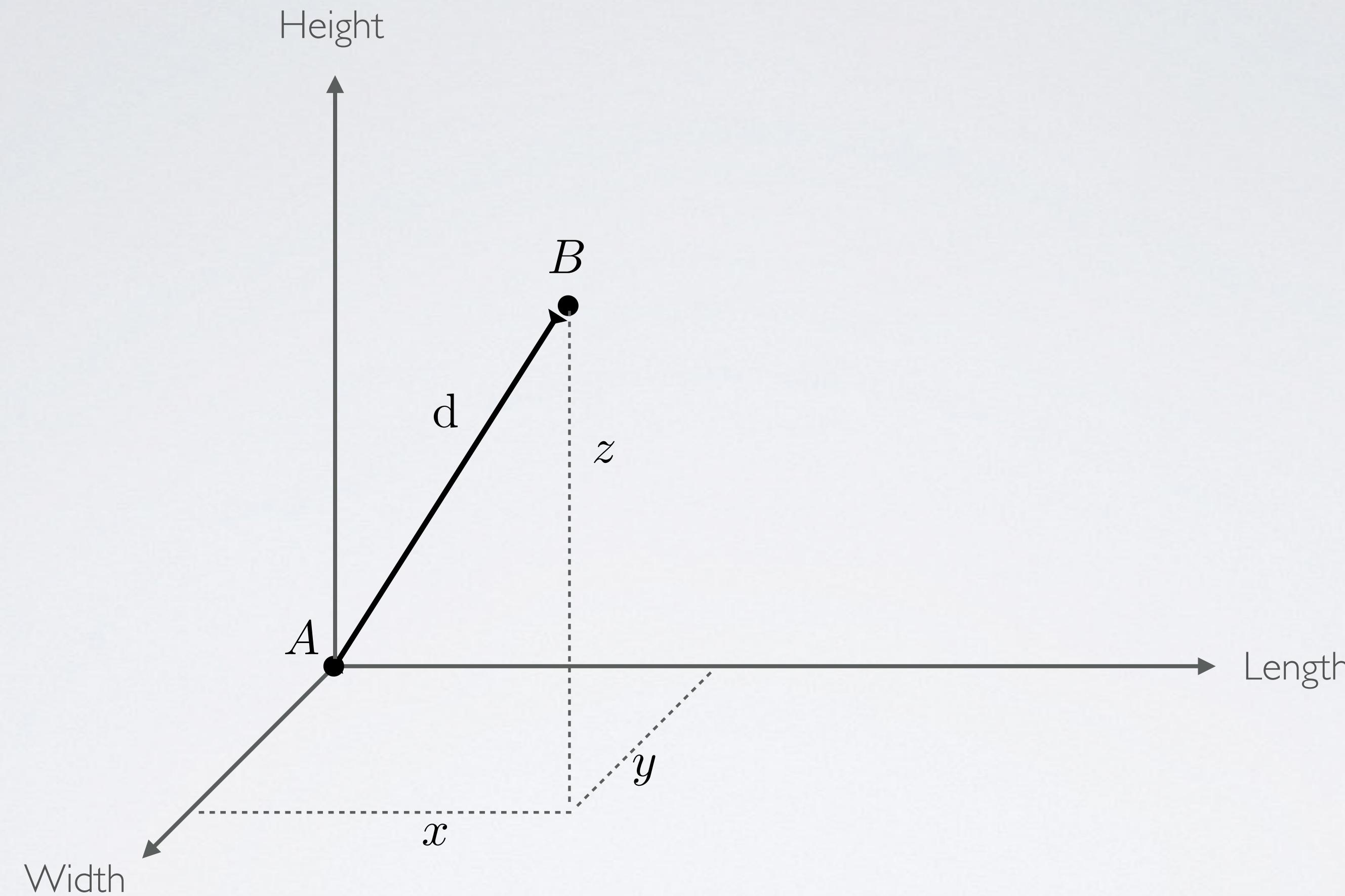
(3 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(3 Dimensional World)

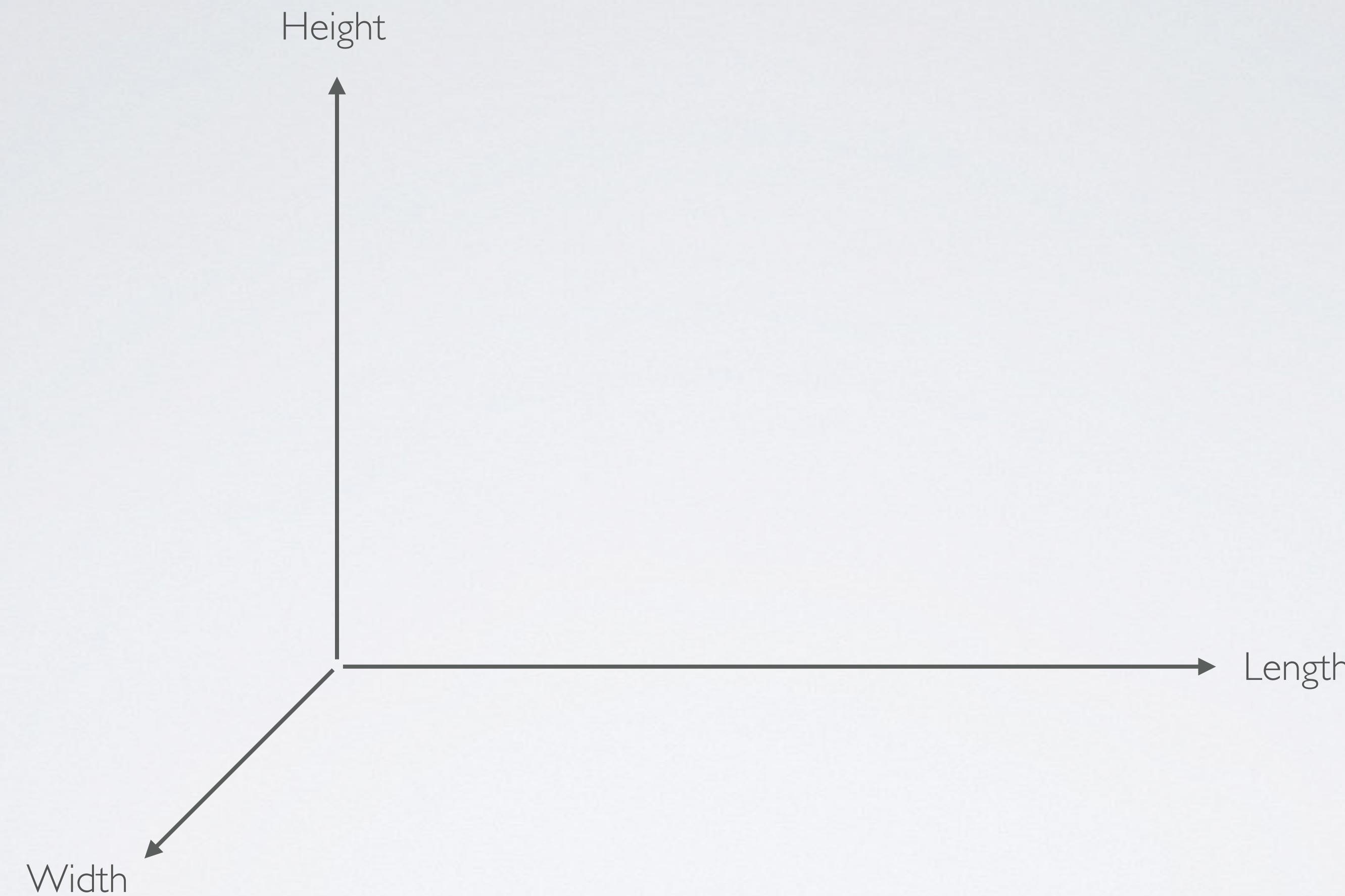
SPACE INTERVALS BETWEEN TWO POINTS



$$d^2 = x^2 + y^2 + z^2$$

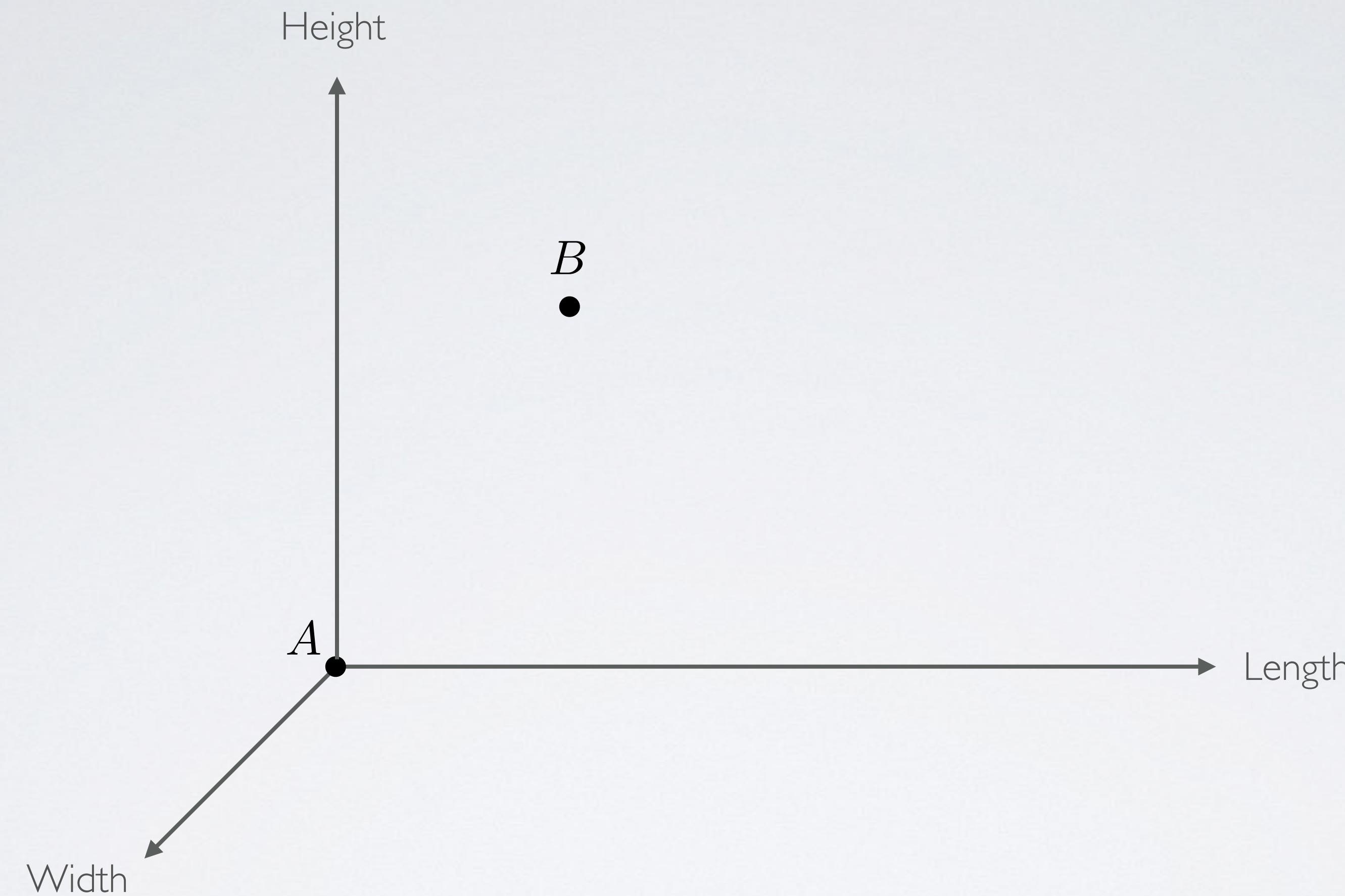
(3 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



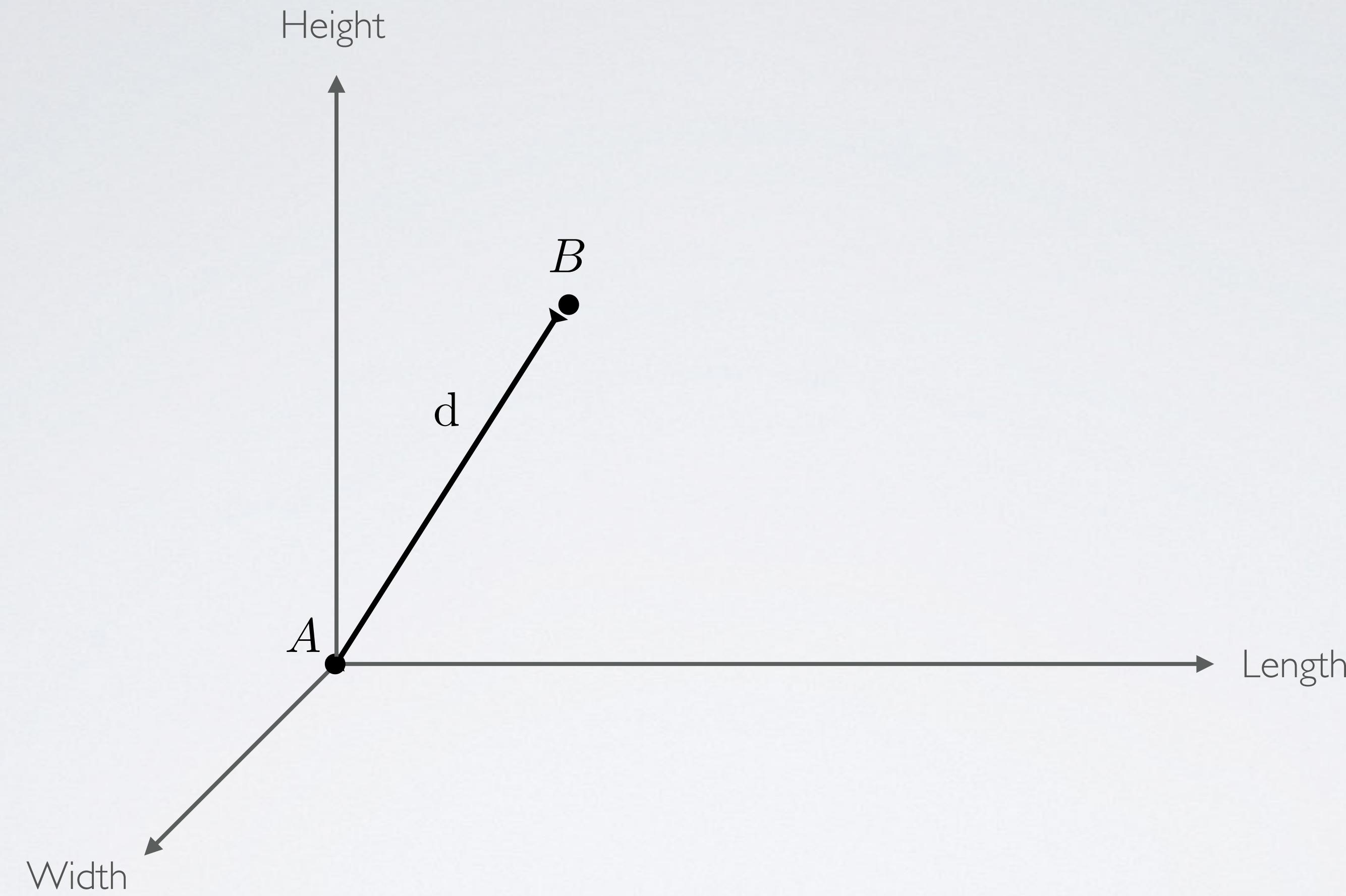
(4 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



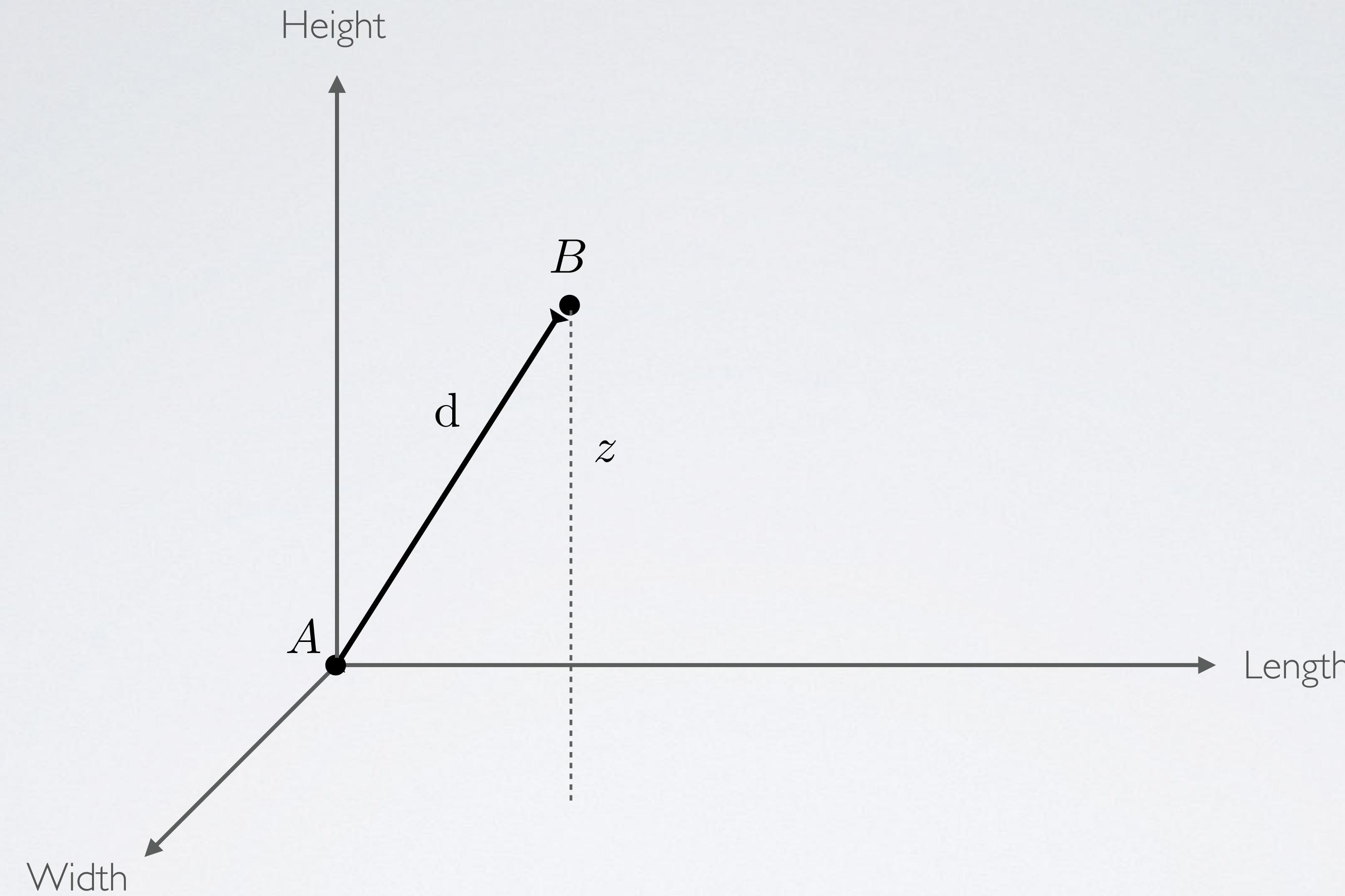
(4 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



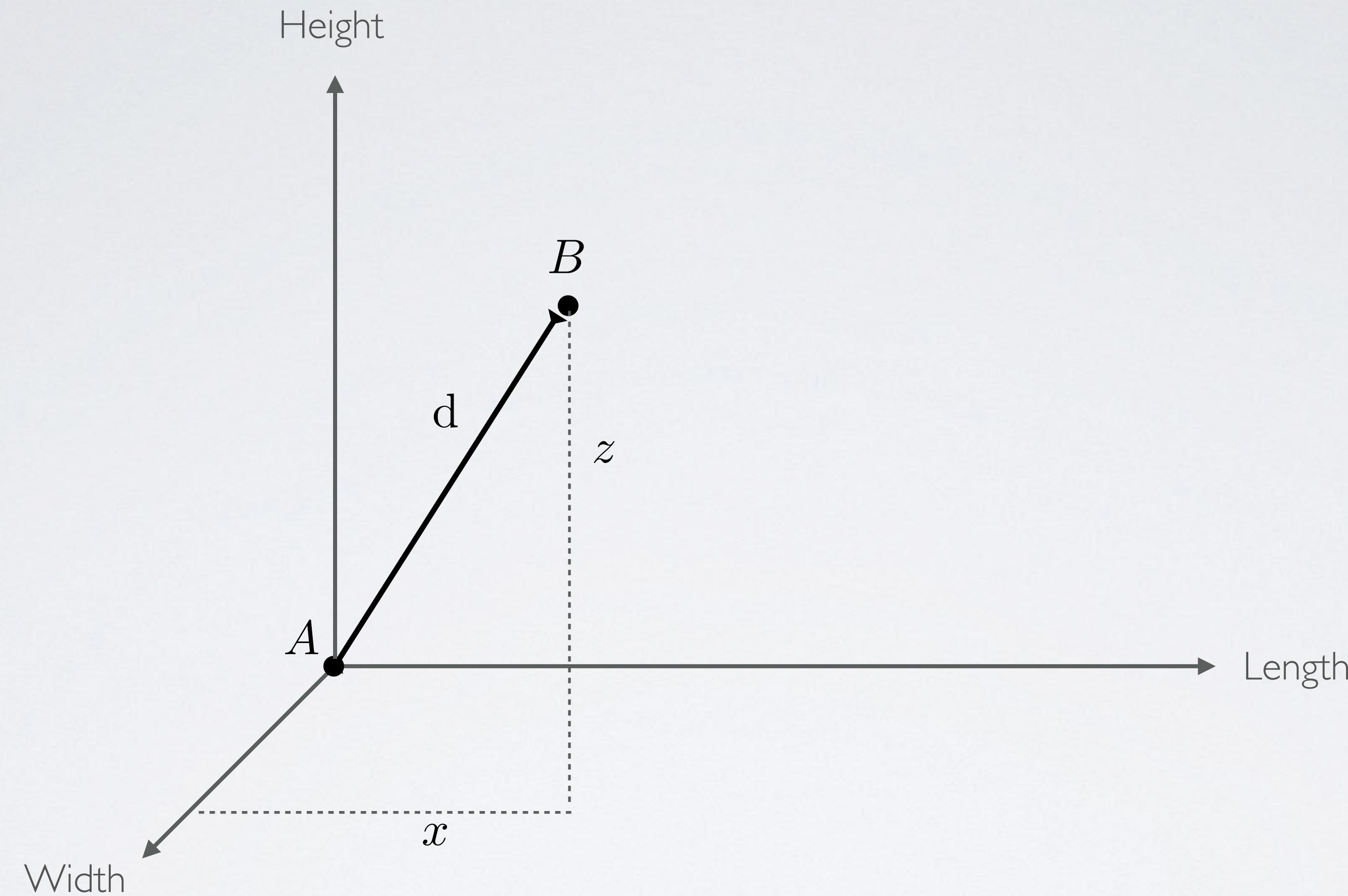
(4 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



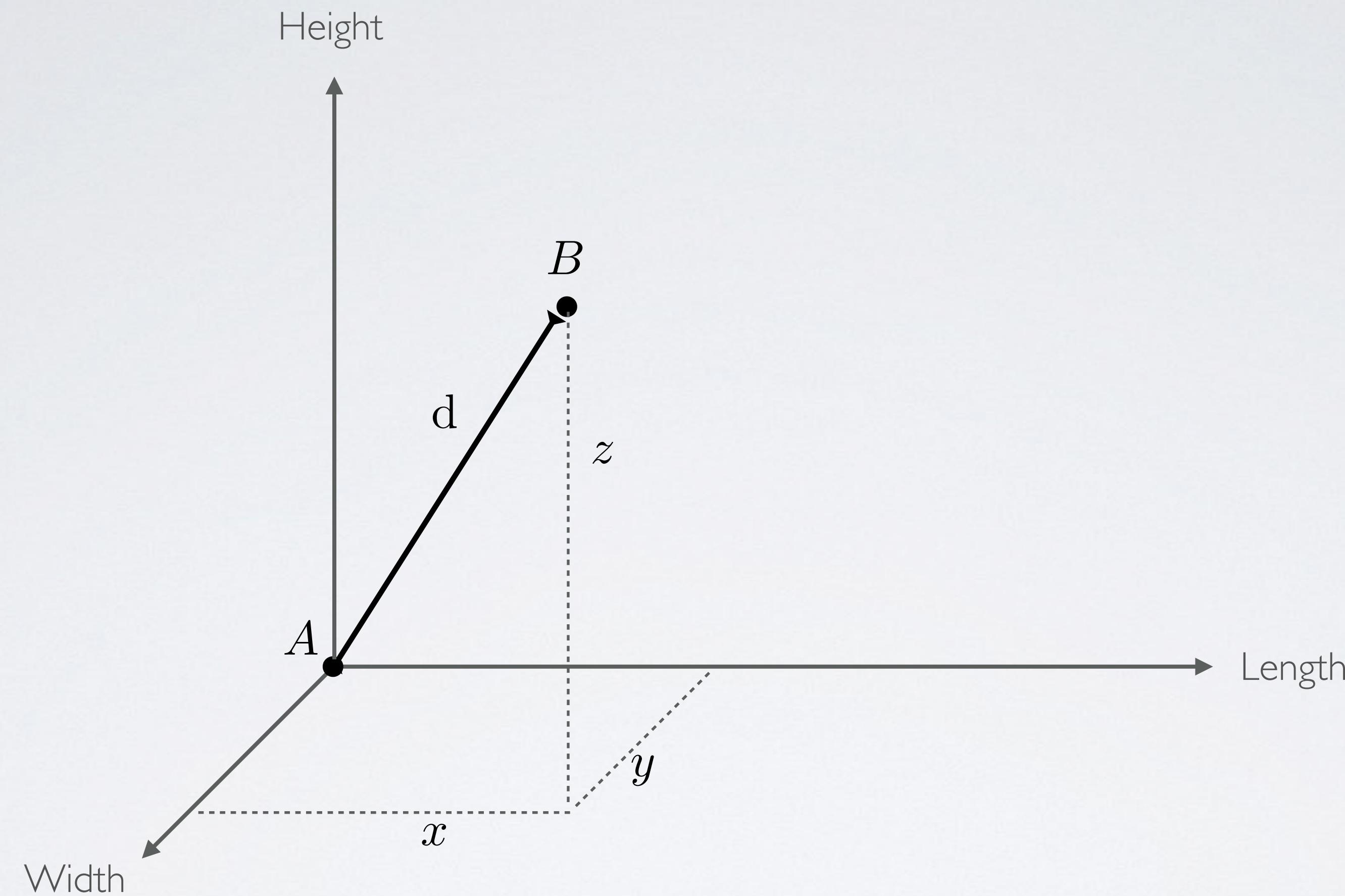
(4 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



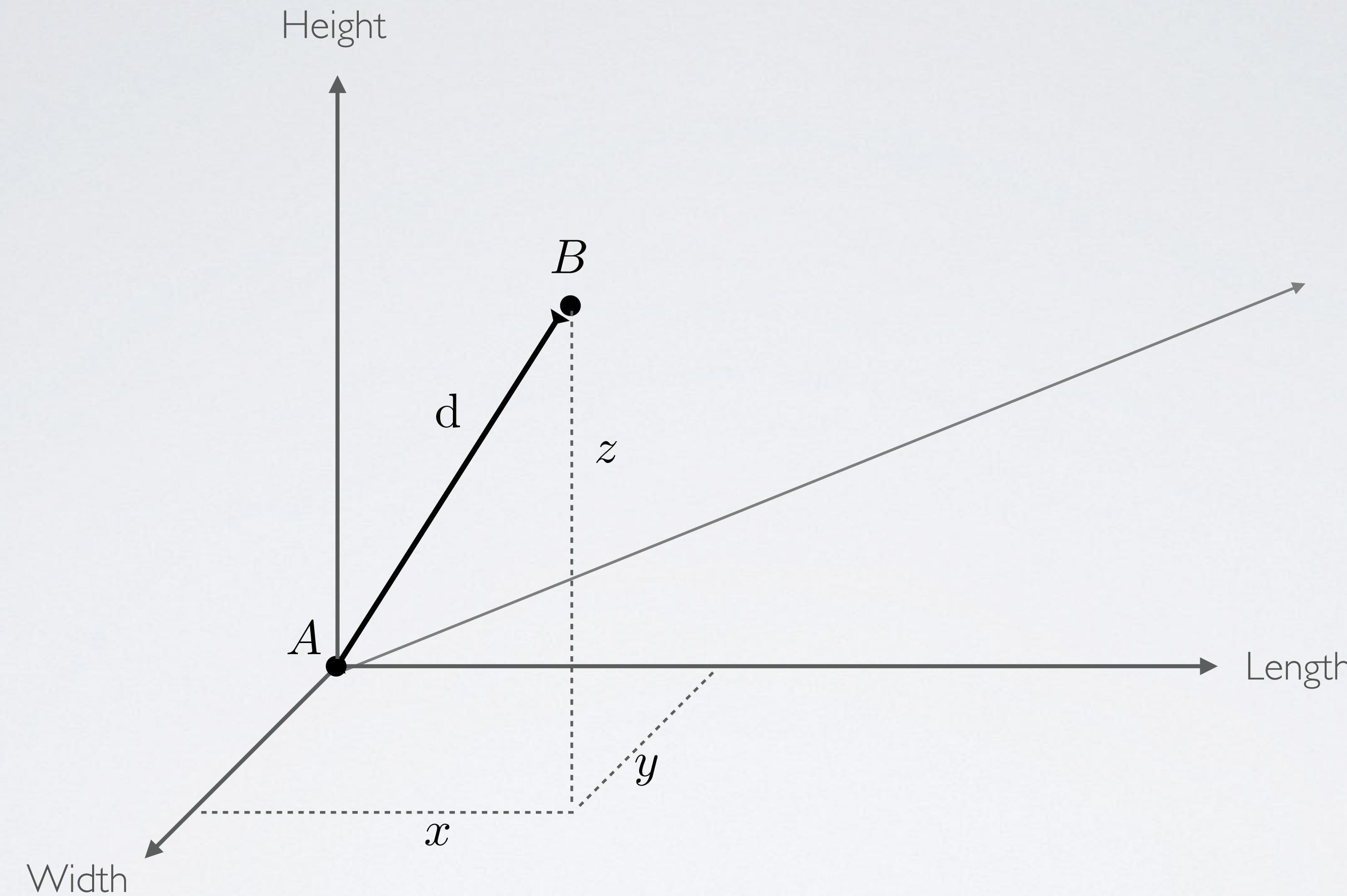
(4 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



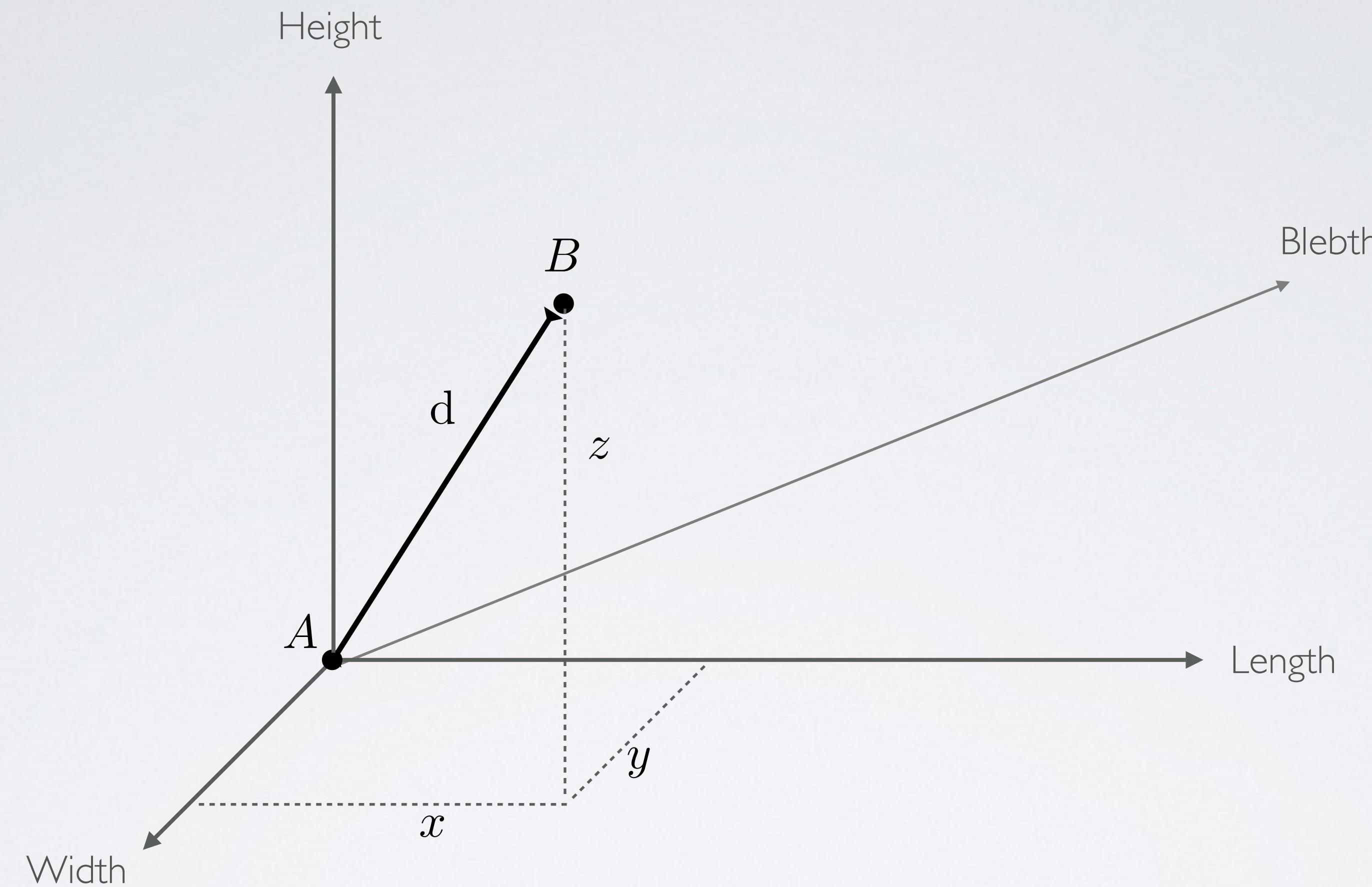
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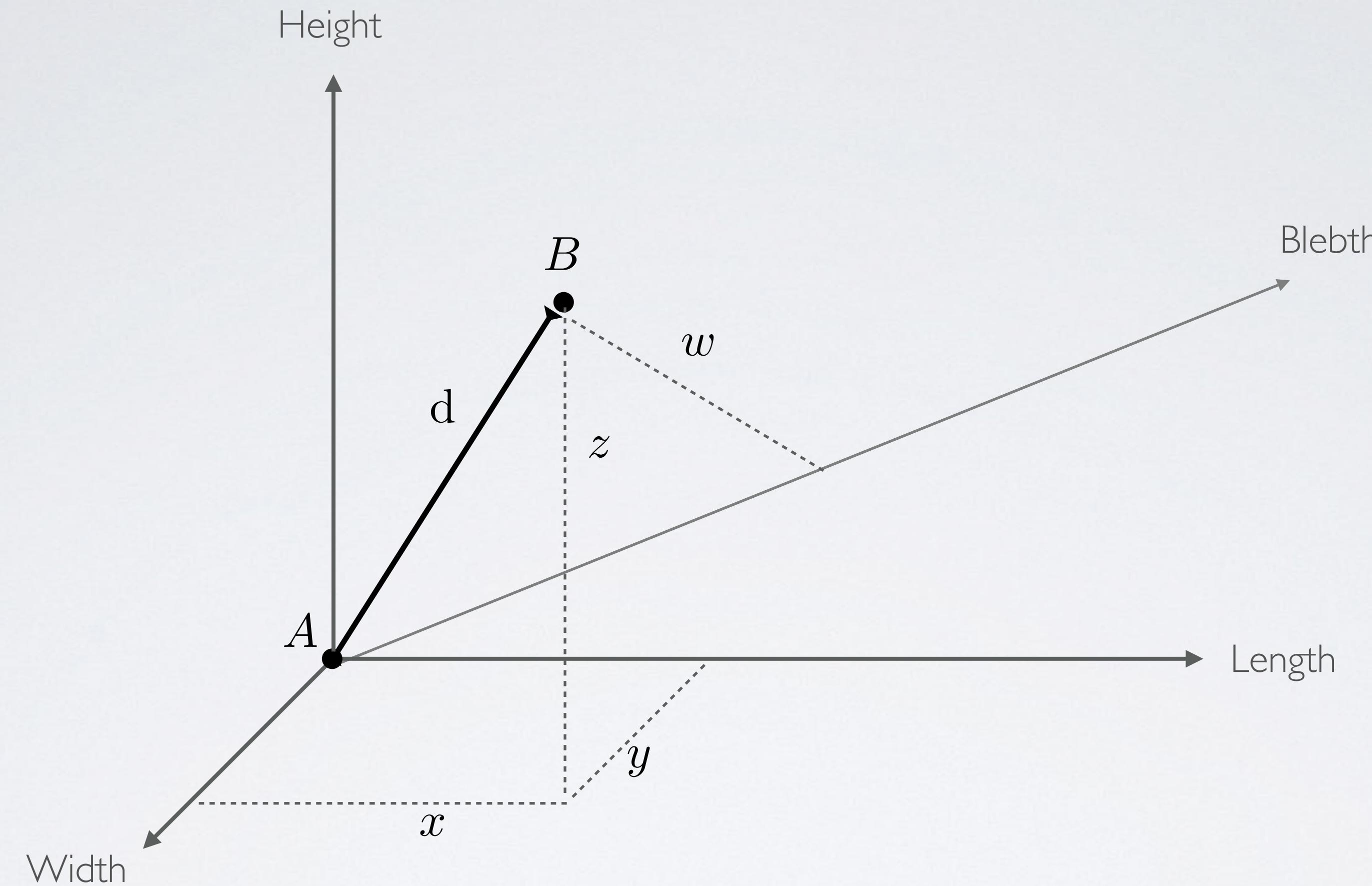
(4 Dimensional World)

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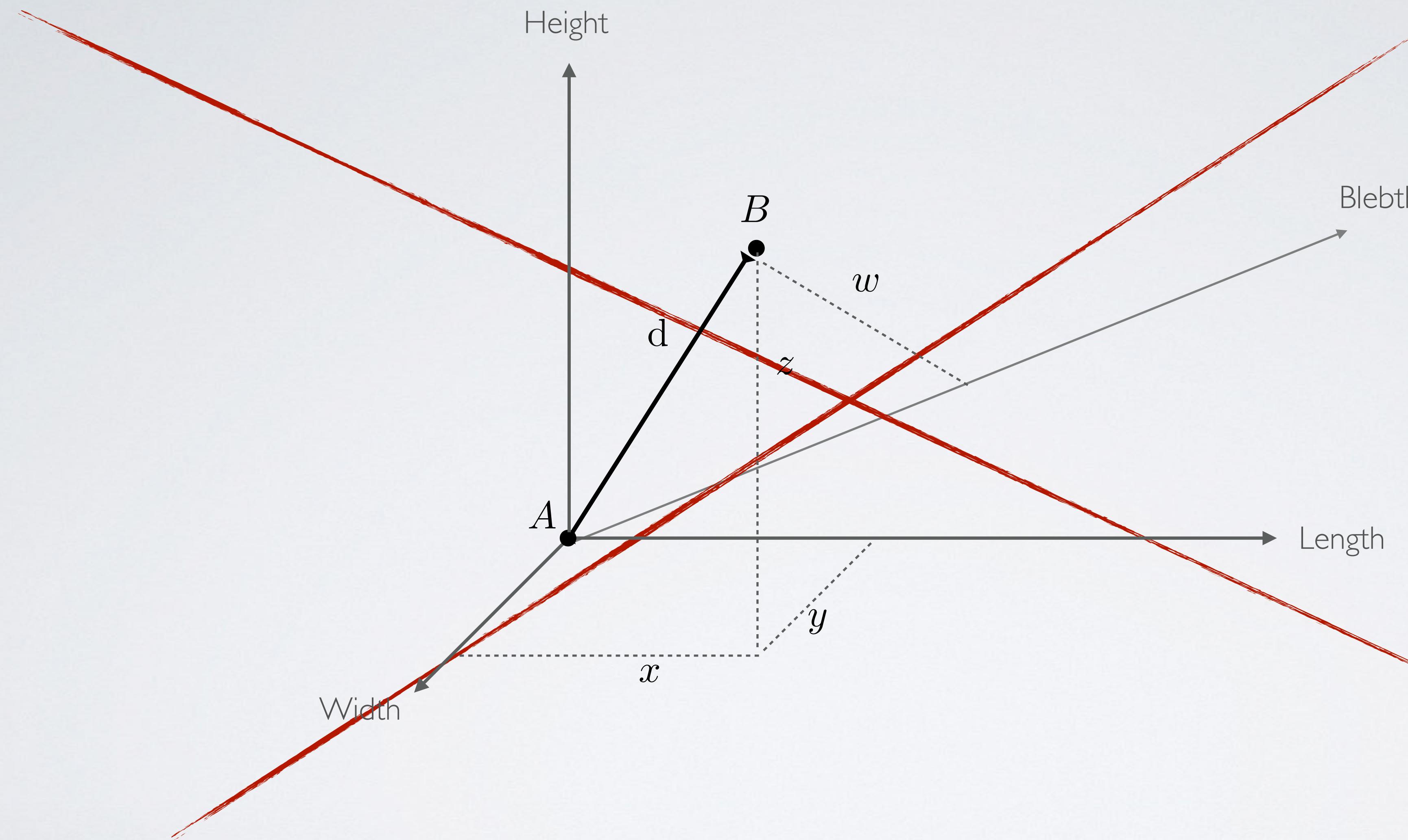
(4 Dimensional World)

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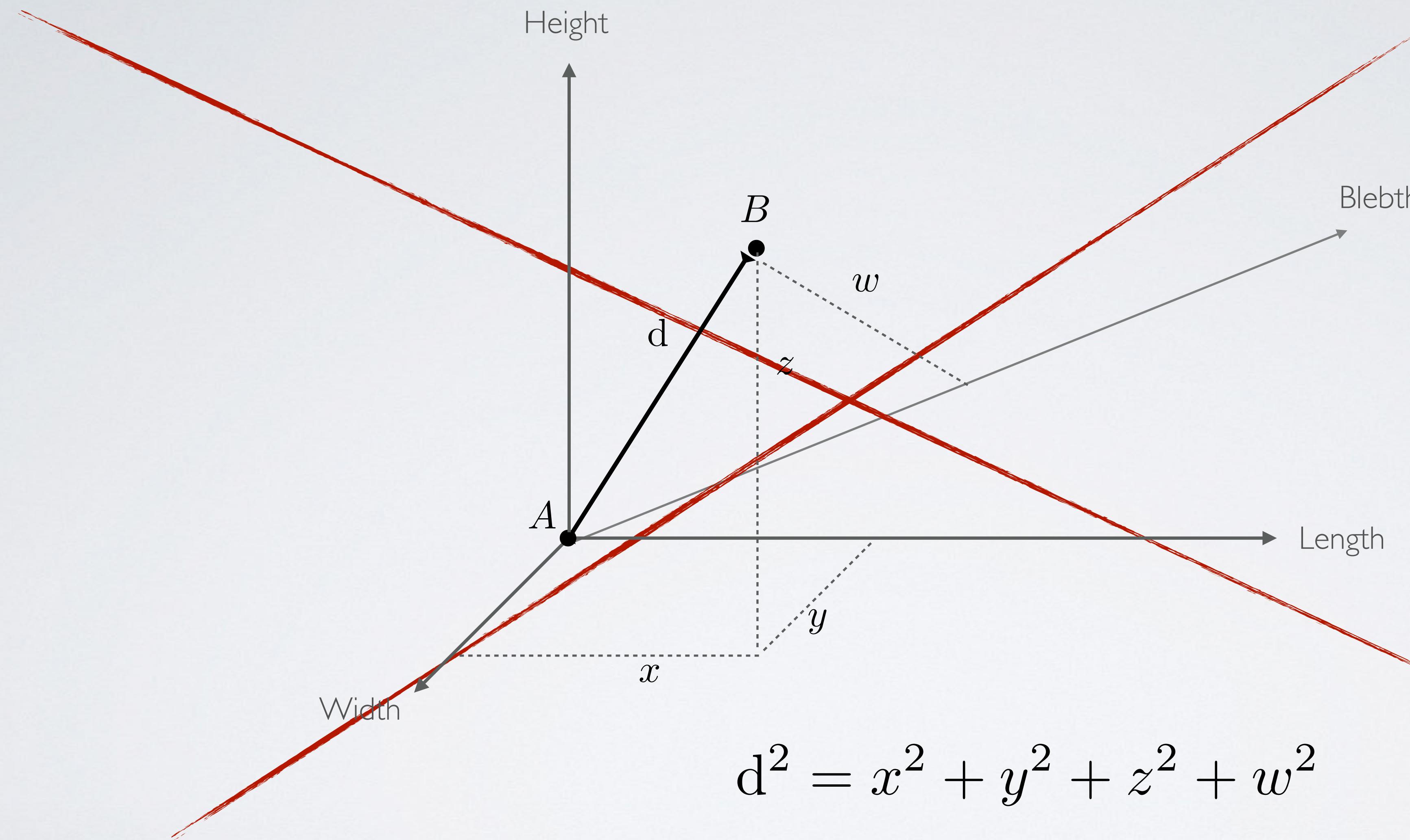
(4 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(4 Dimensional World)

SPACE INTERVALS BETWEEN TWO POINTS



(4 Dimensional World)

CURVATURE AND SPACE INTERVALS

CURVATURE AND SPACE INTERVALS

- Curvature modifies the Pitagora's Theorem*

CURVATURE AND SPACE INTERVALS

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*careful: curvature means a different Pitagora's Theorem. But a "weird Pitagora's Theorem" doesn't always mean curvature (it can be just weird parametrisation)

CURVATURE AND SPACE INTERVALS

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- Curvature generalises the Pitagora's Theorem

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CURVATURE AND SPACE INTERVALS

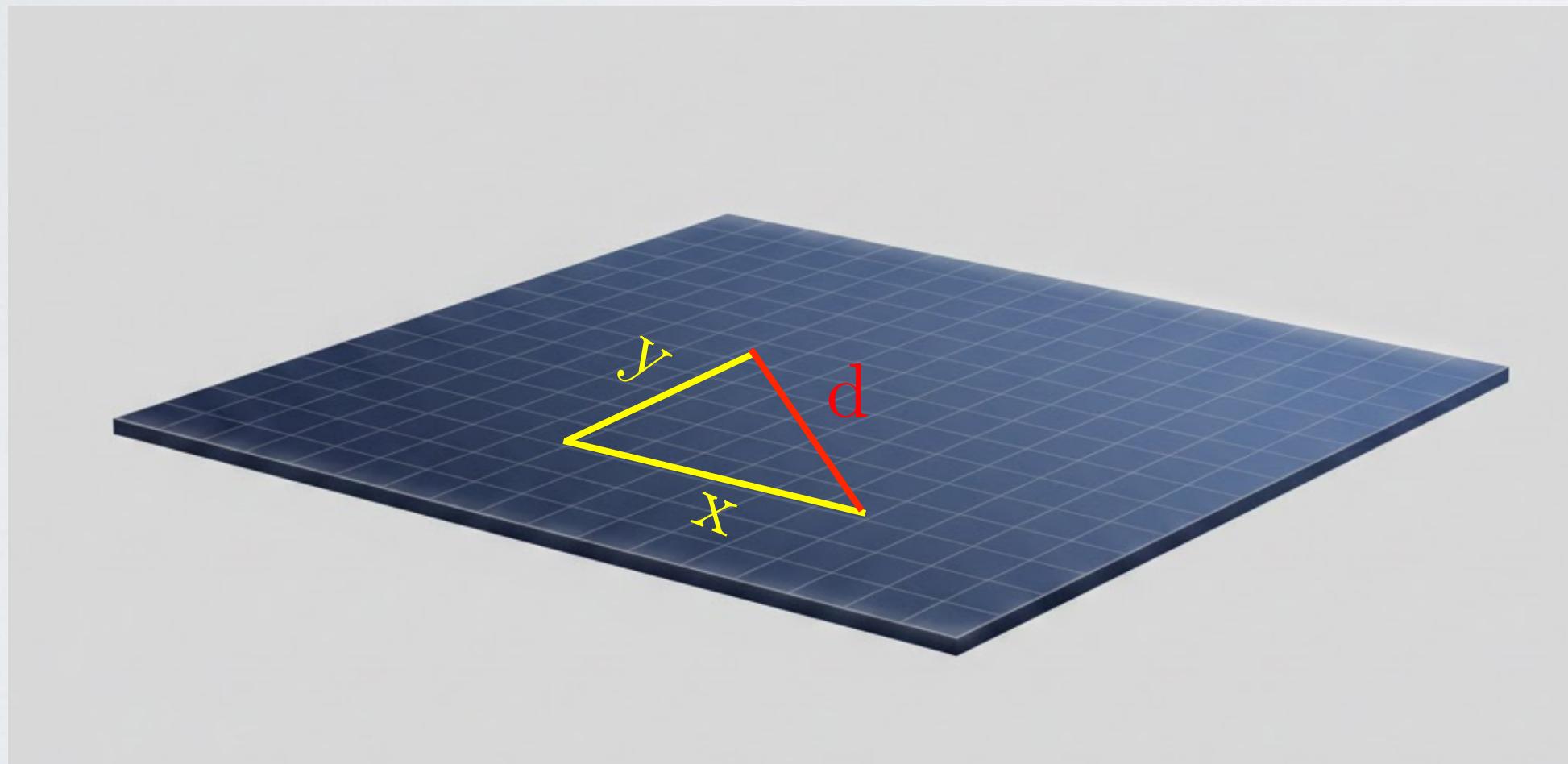
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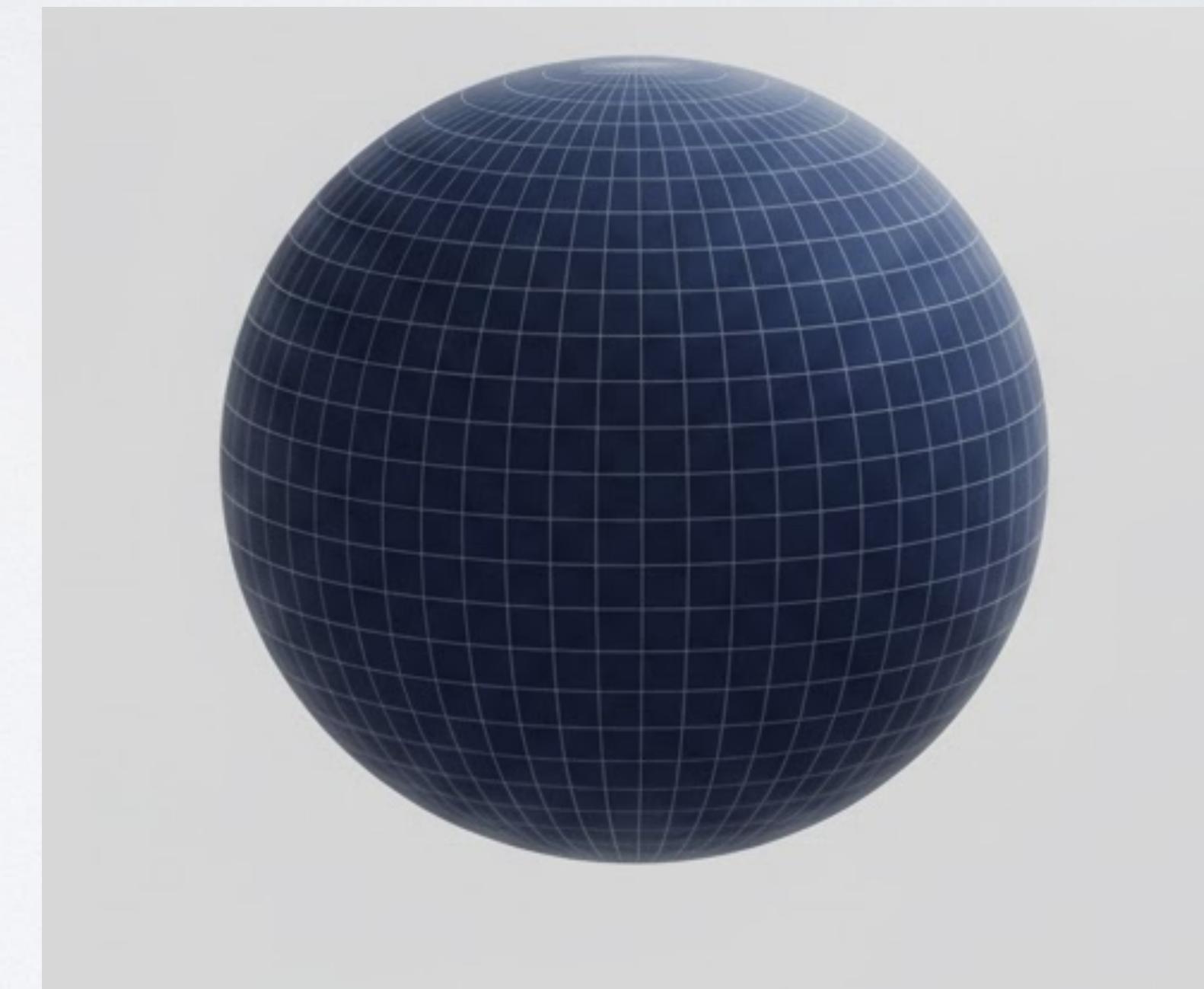
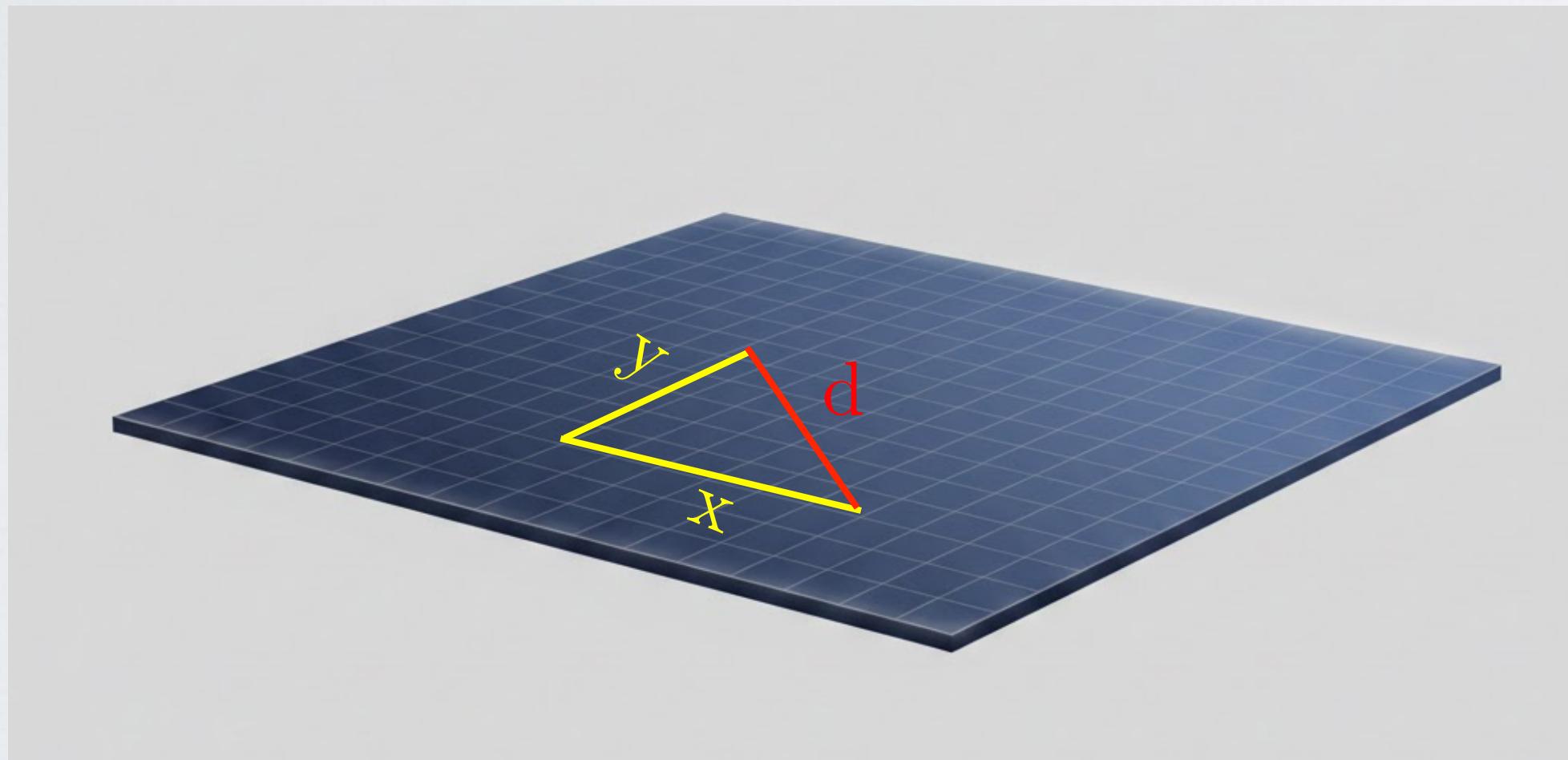


$$d^2 = x^2 + y^2$$

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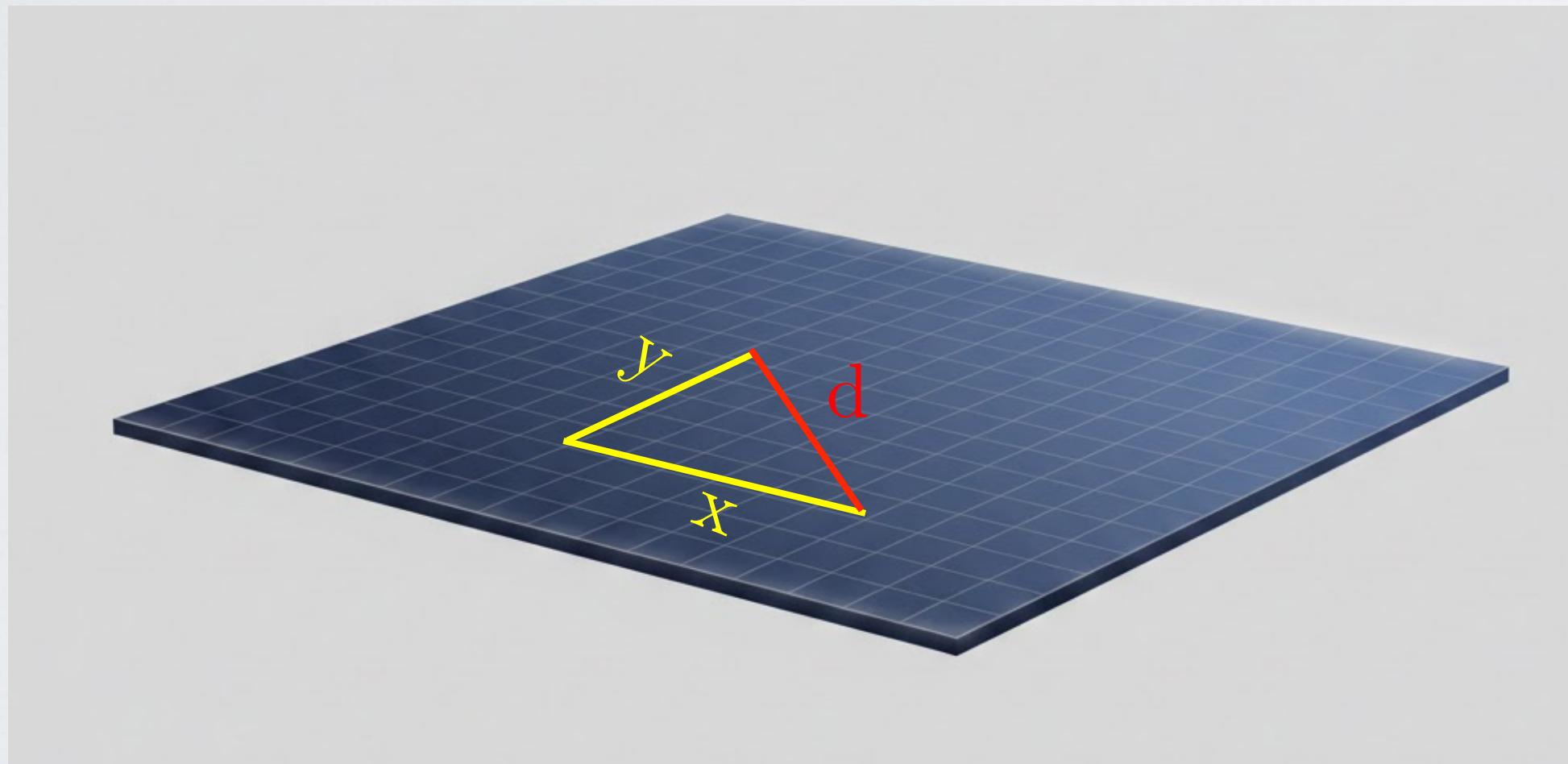


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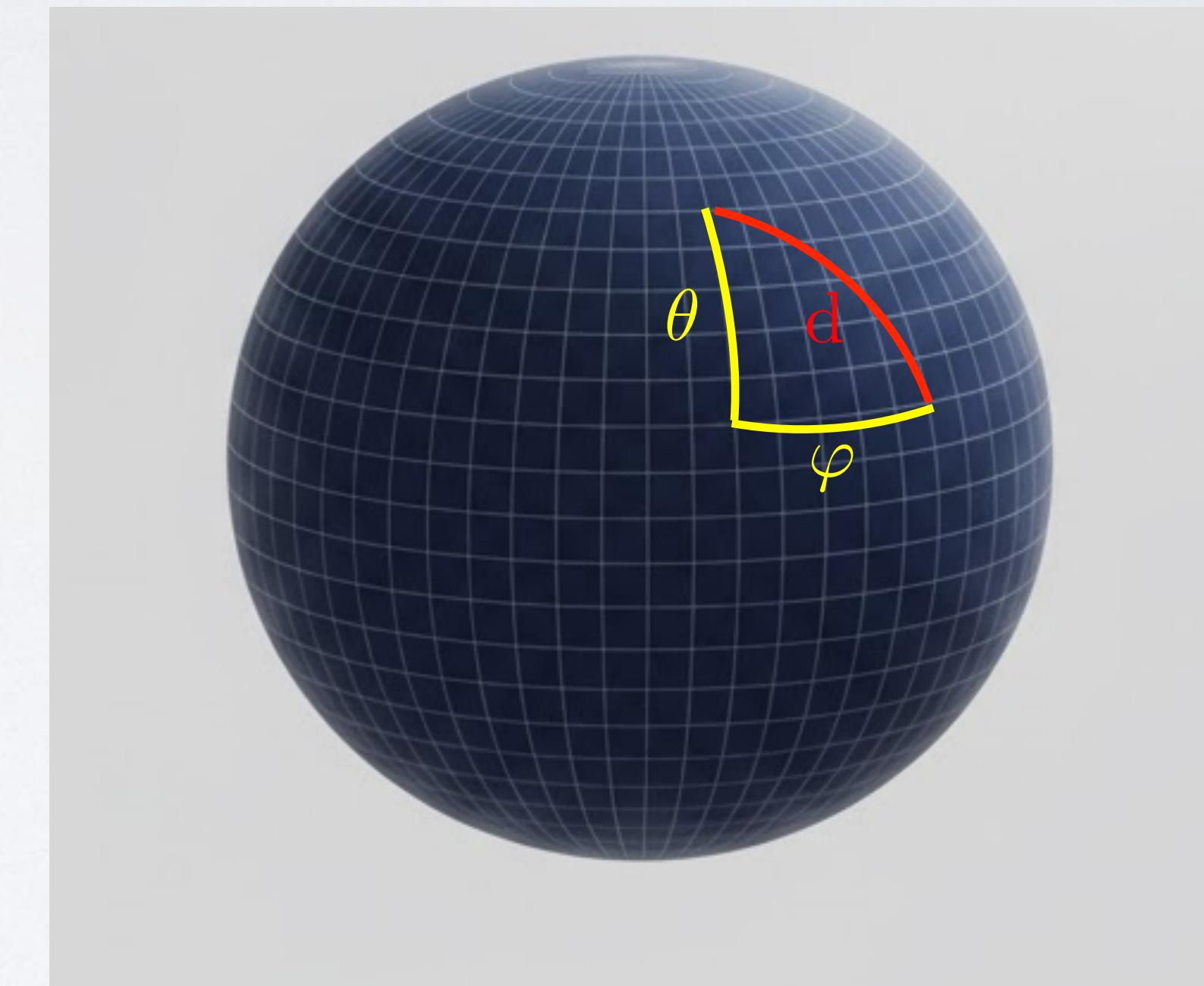
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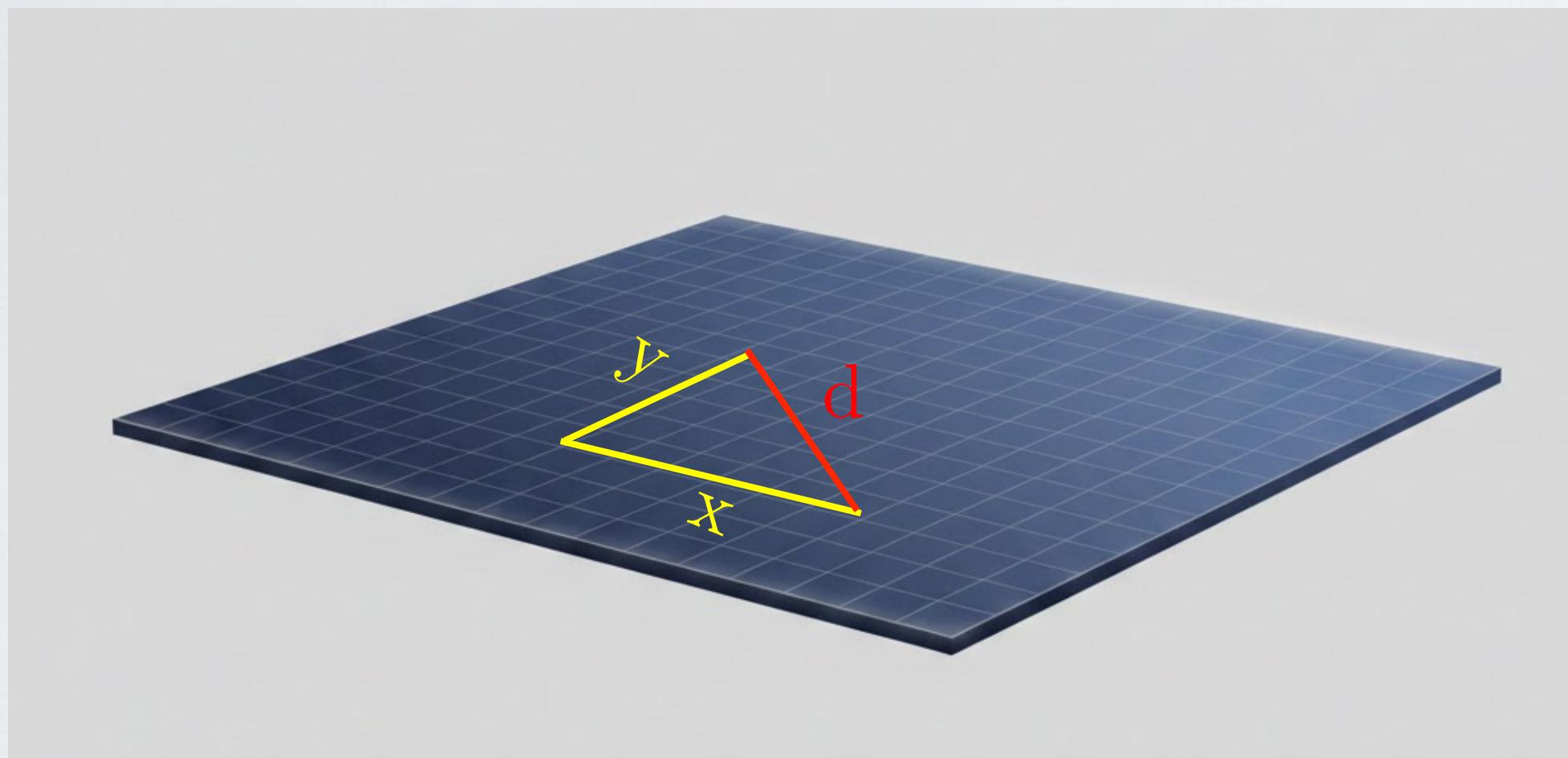


$$d^2 = \theta^2 + \sin^2 \theta \varphi^2$$

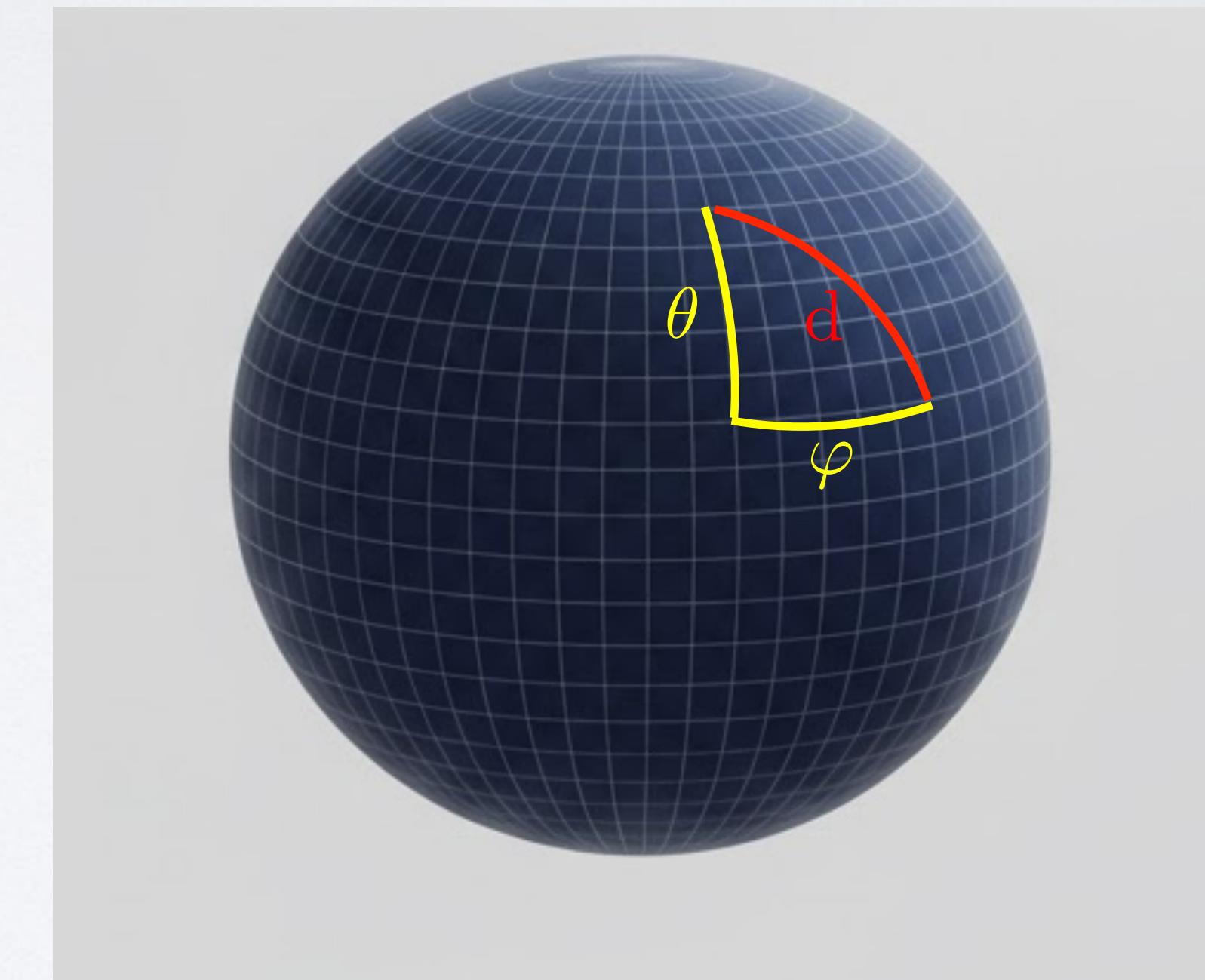
*careful: curvature means a different Pitagora's Theorem. But a "weird Pitagora's Theorem" doesn't always mean curvature (it can be just weird parametrisation)

CURVATURE AND SPACE INTERVALS

WHAT ABOUT TIME ?!?!?



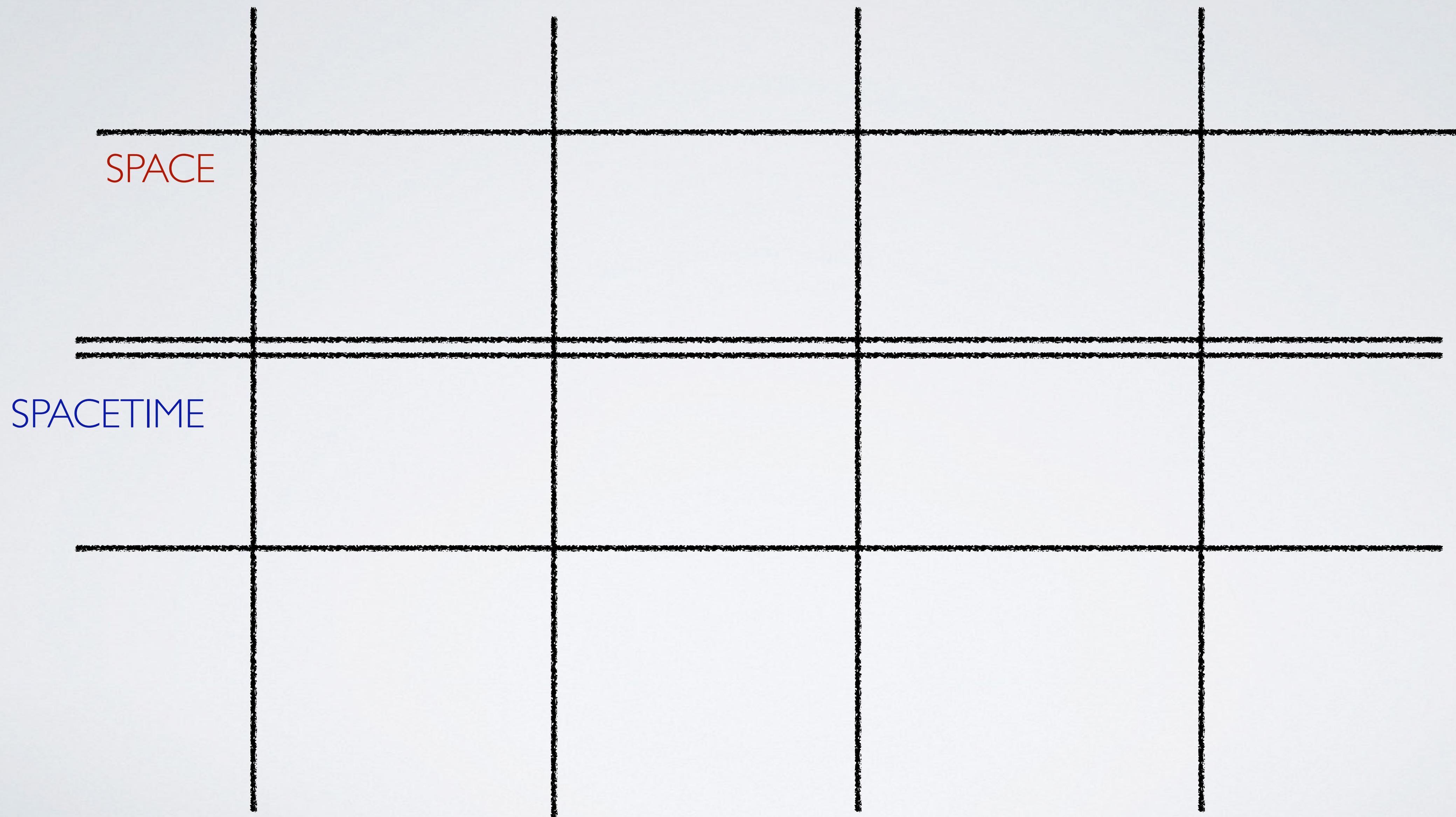
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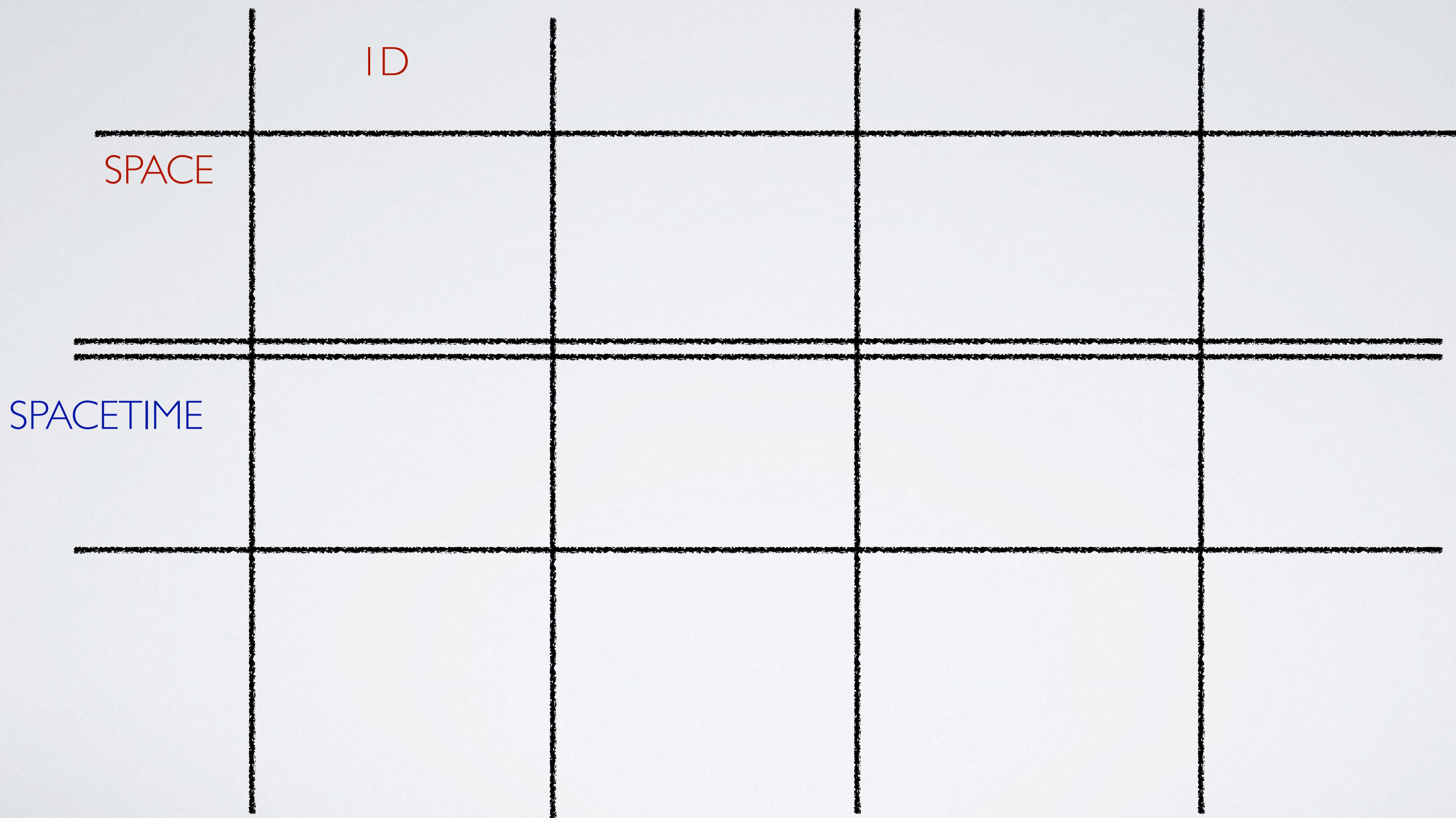
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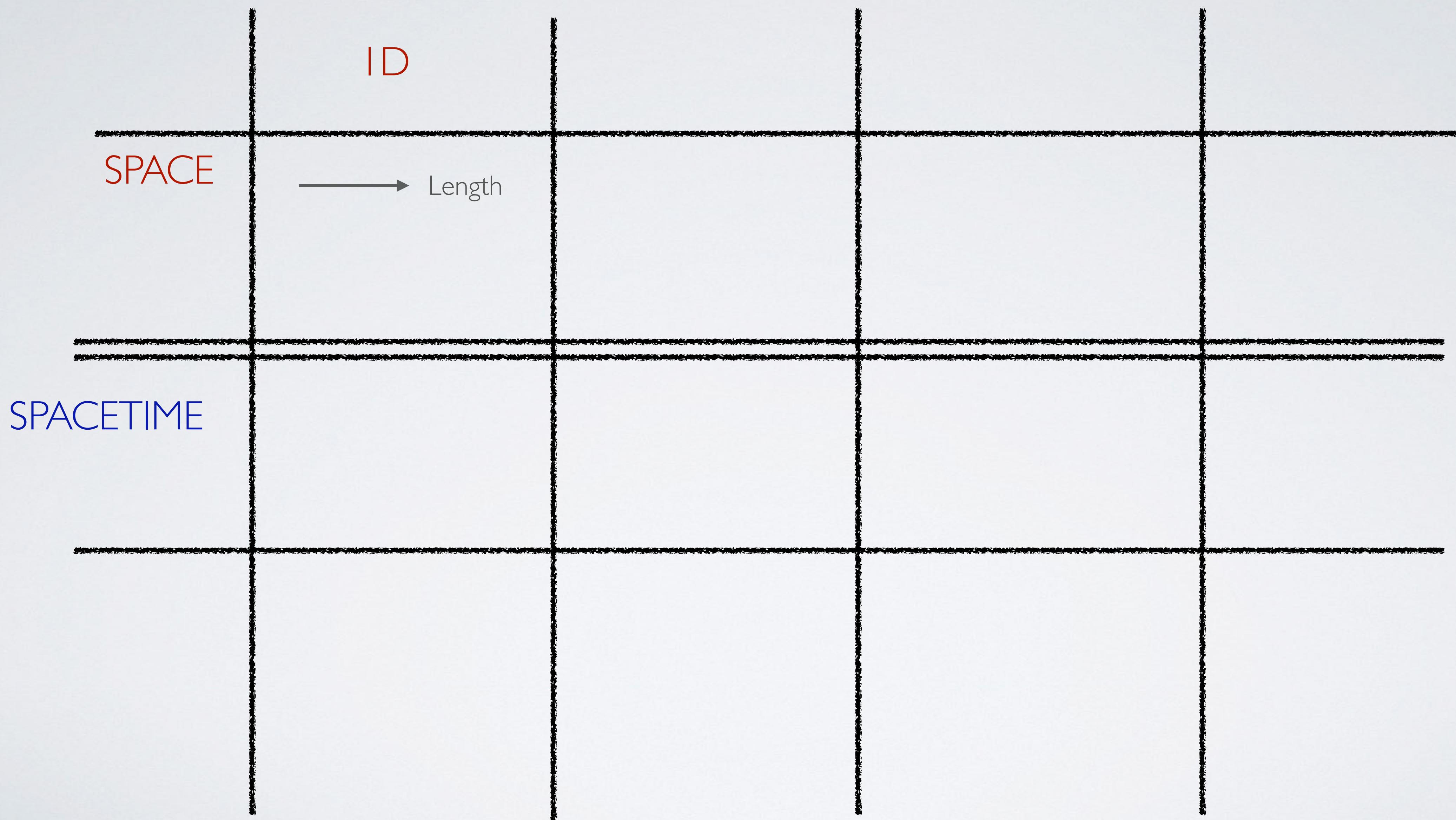
SPACETIME DIMENSIONS



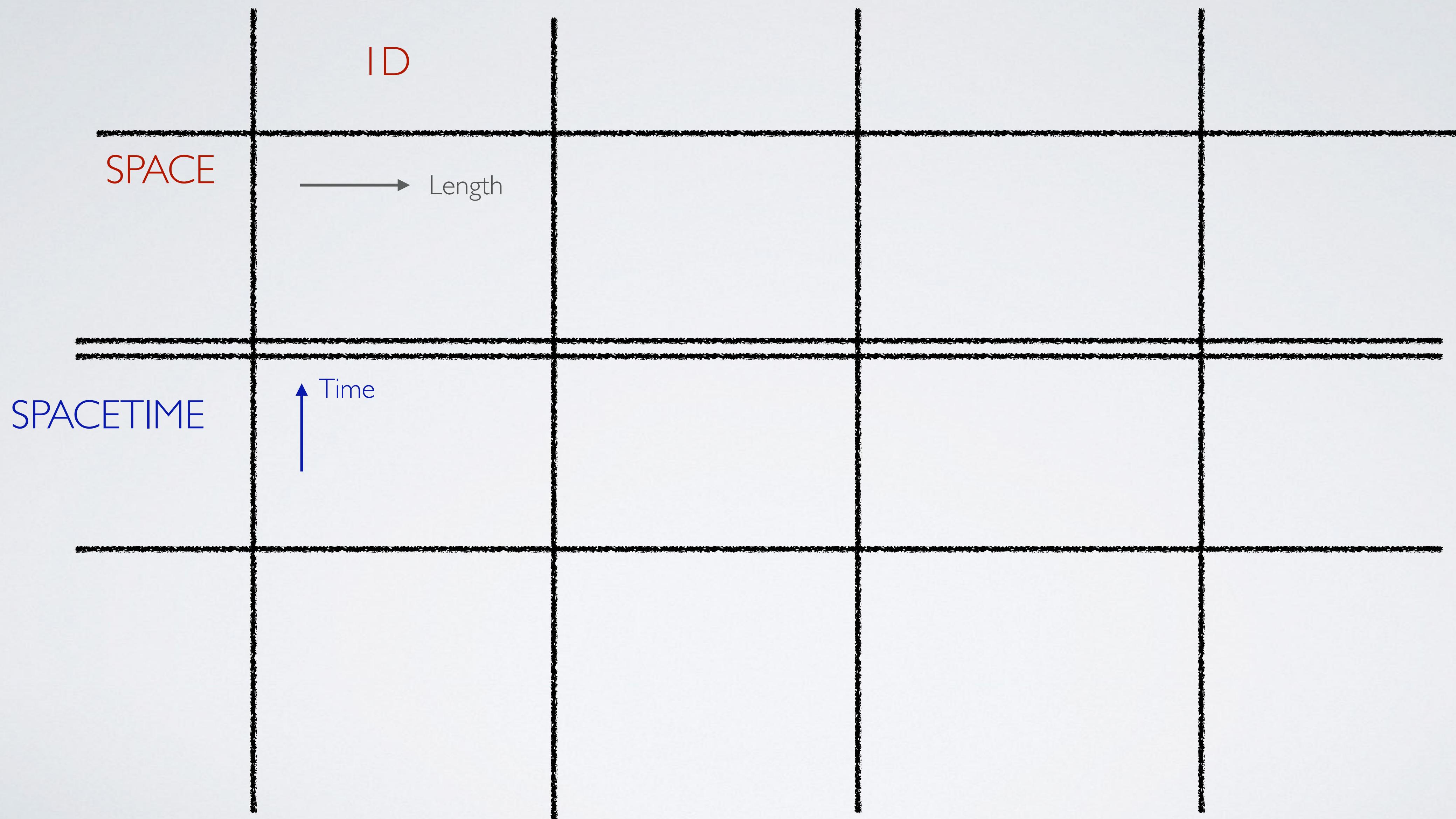
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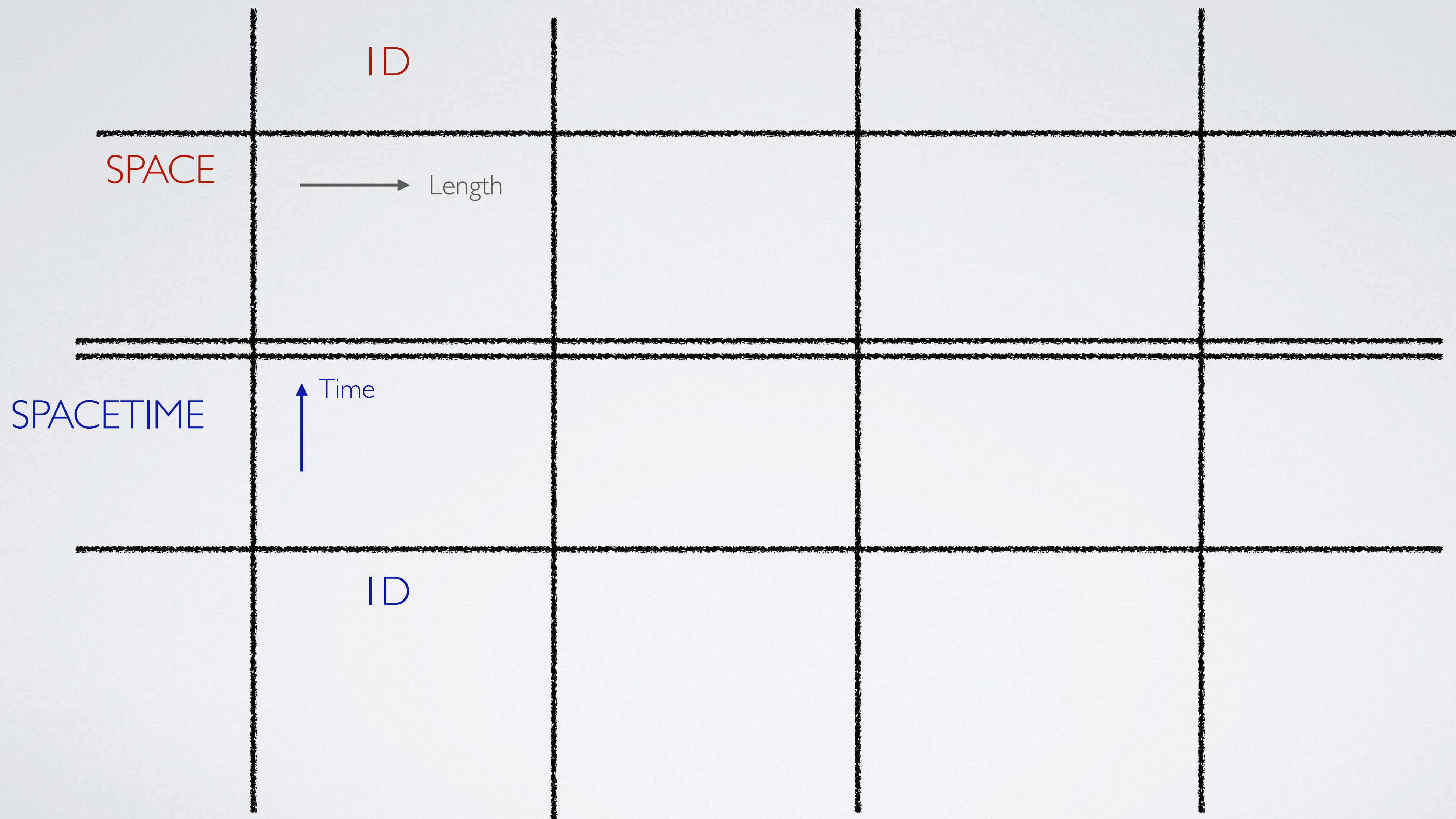
SPACETIME DIMENSIONS



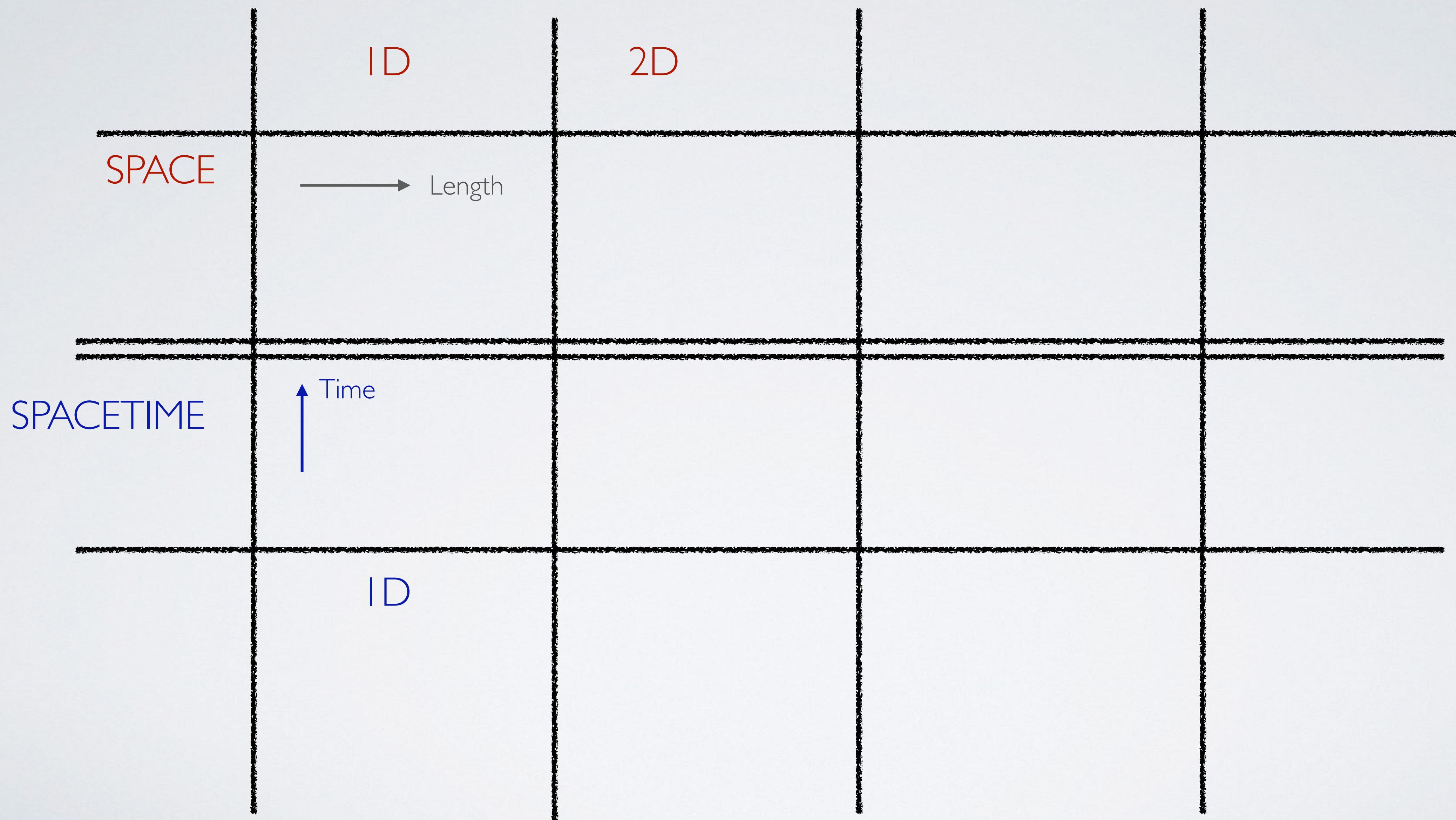
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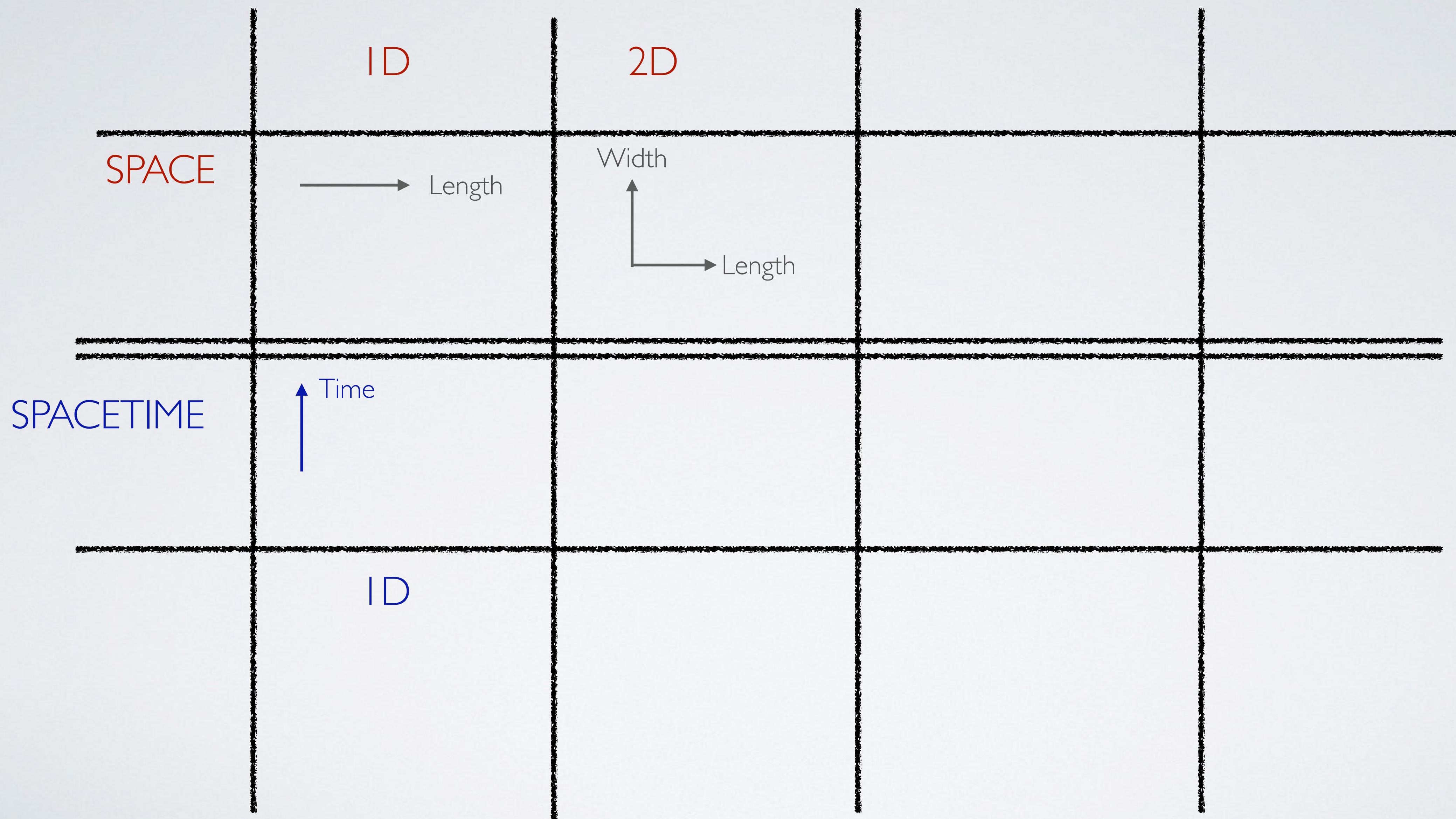
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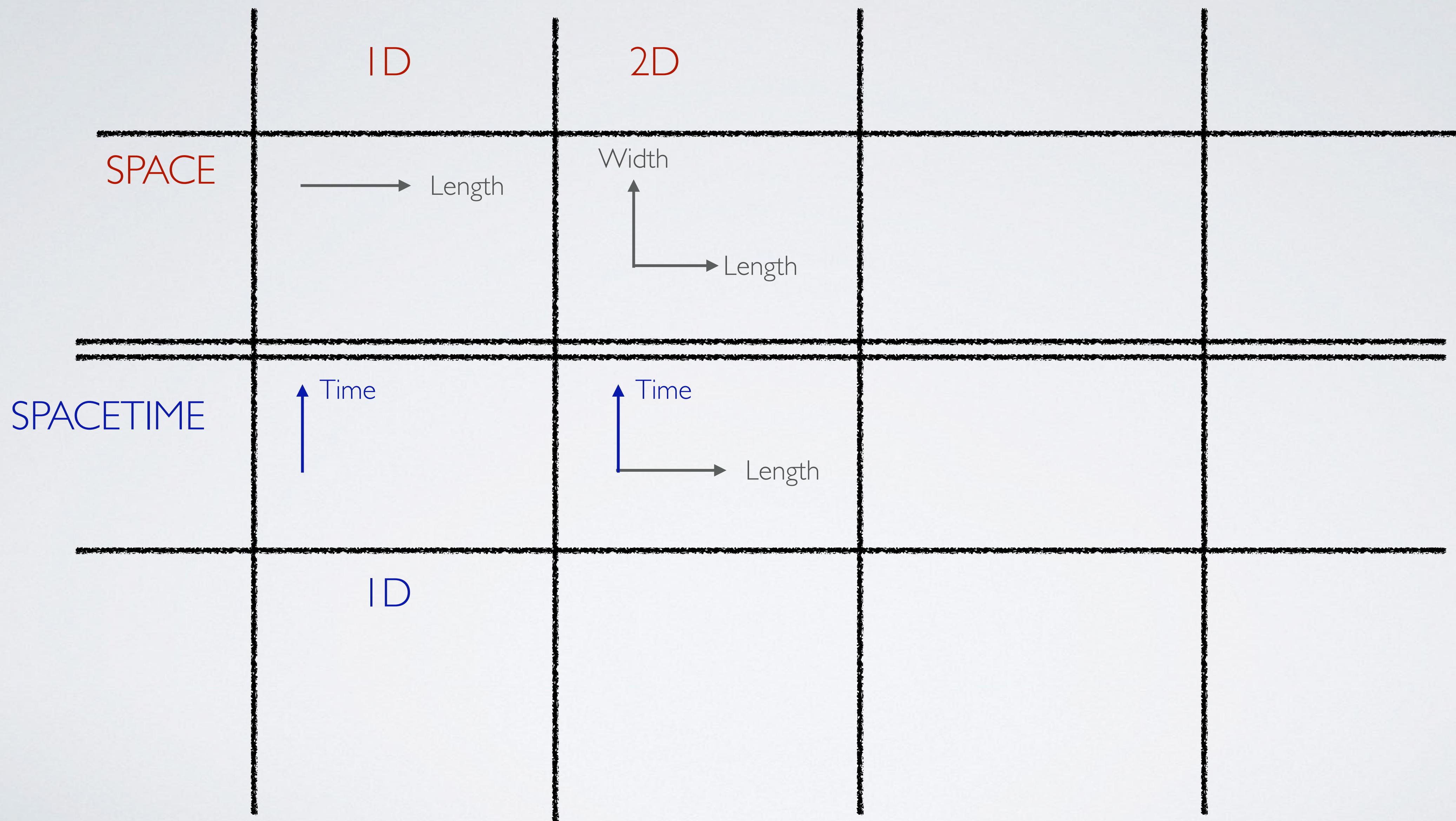
SPACETIME DIMENSIONS



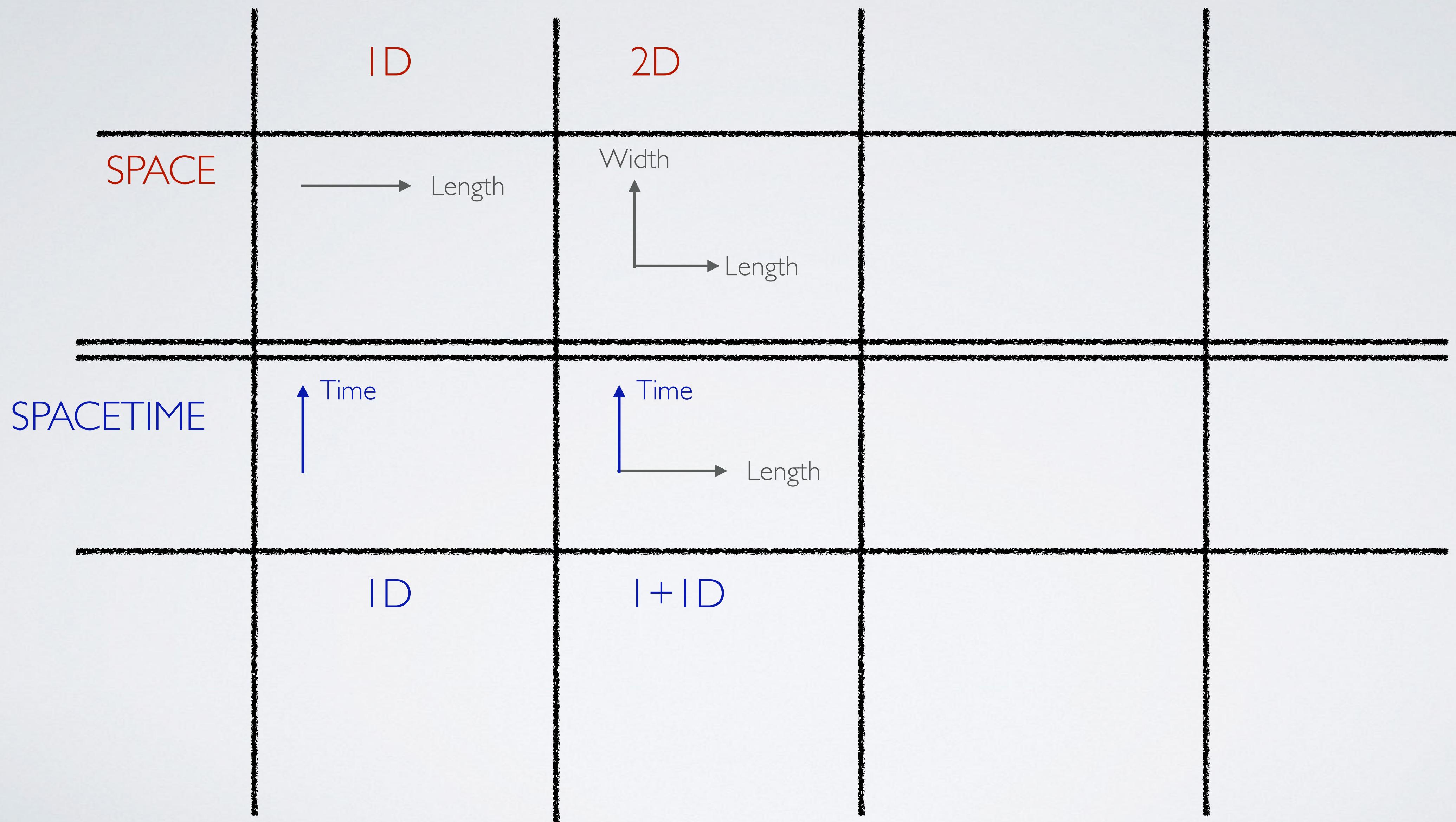
SPACETIME DIMENSIONS



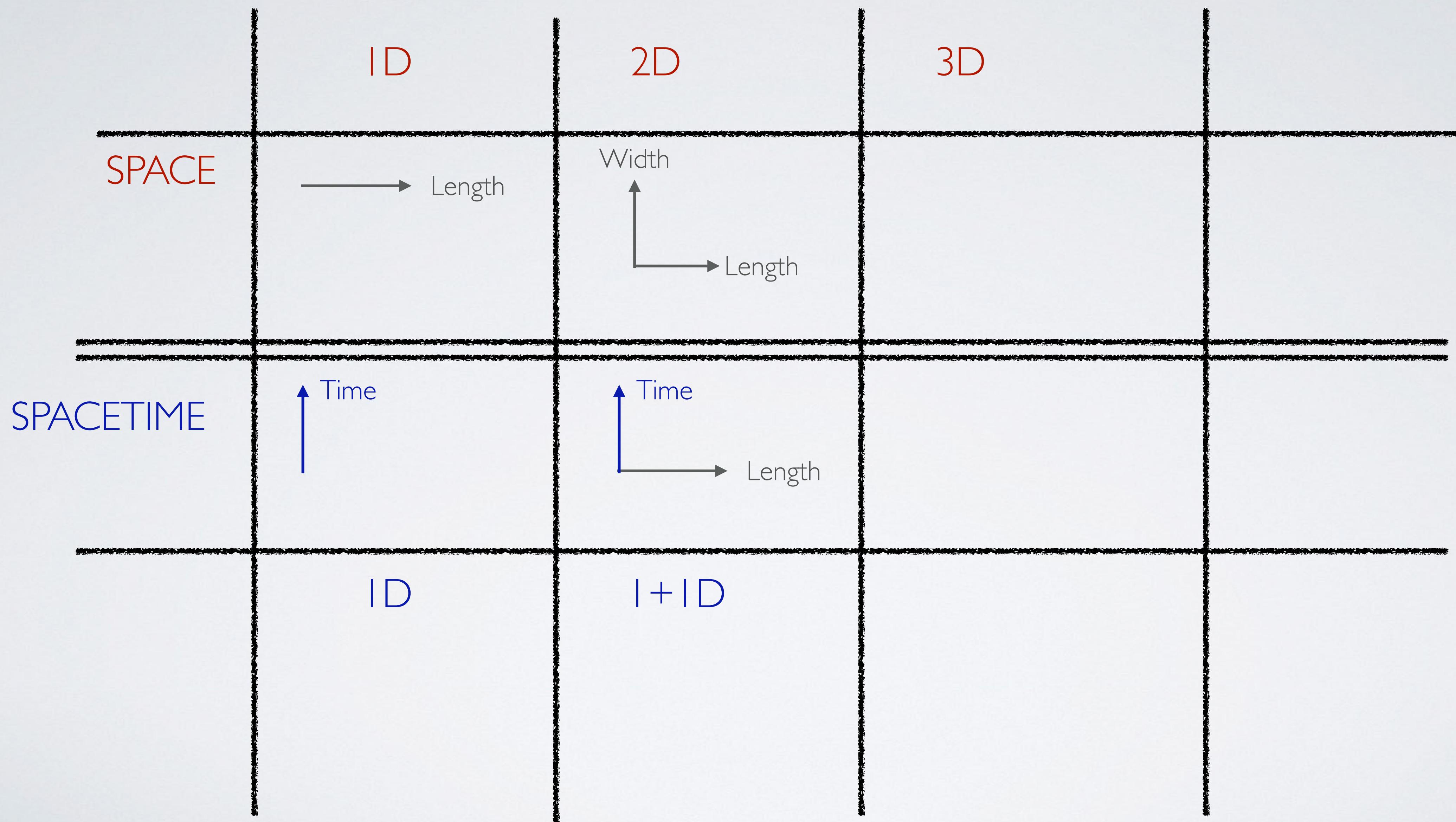
SPACETIME DIMENSIONS



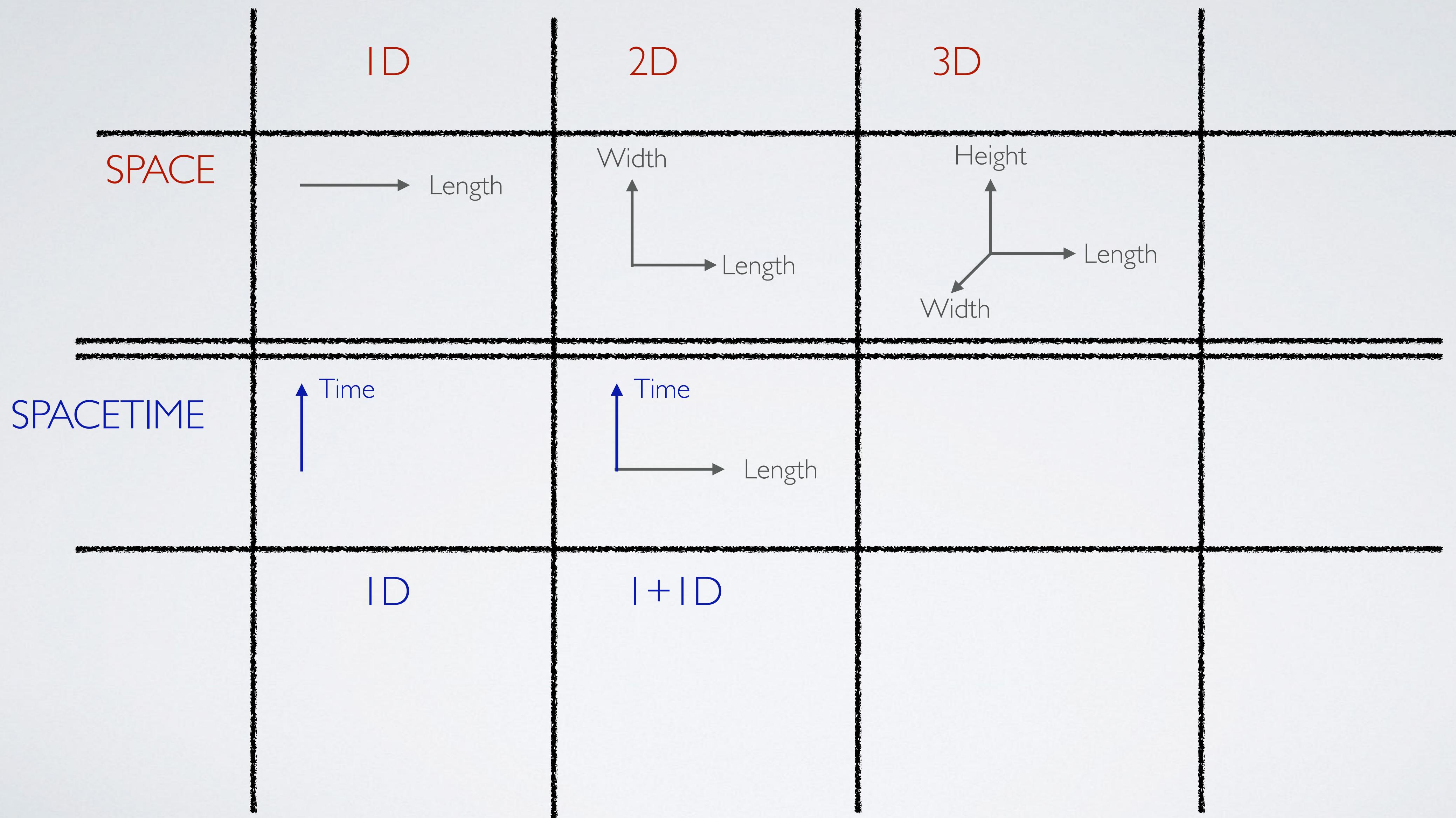
SPACETIME DIMENSIONS



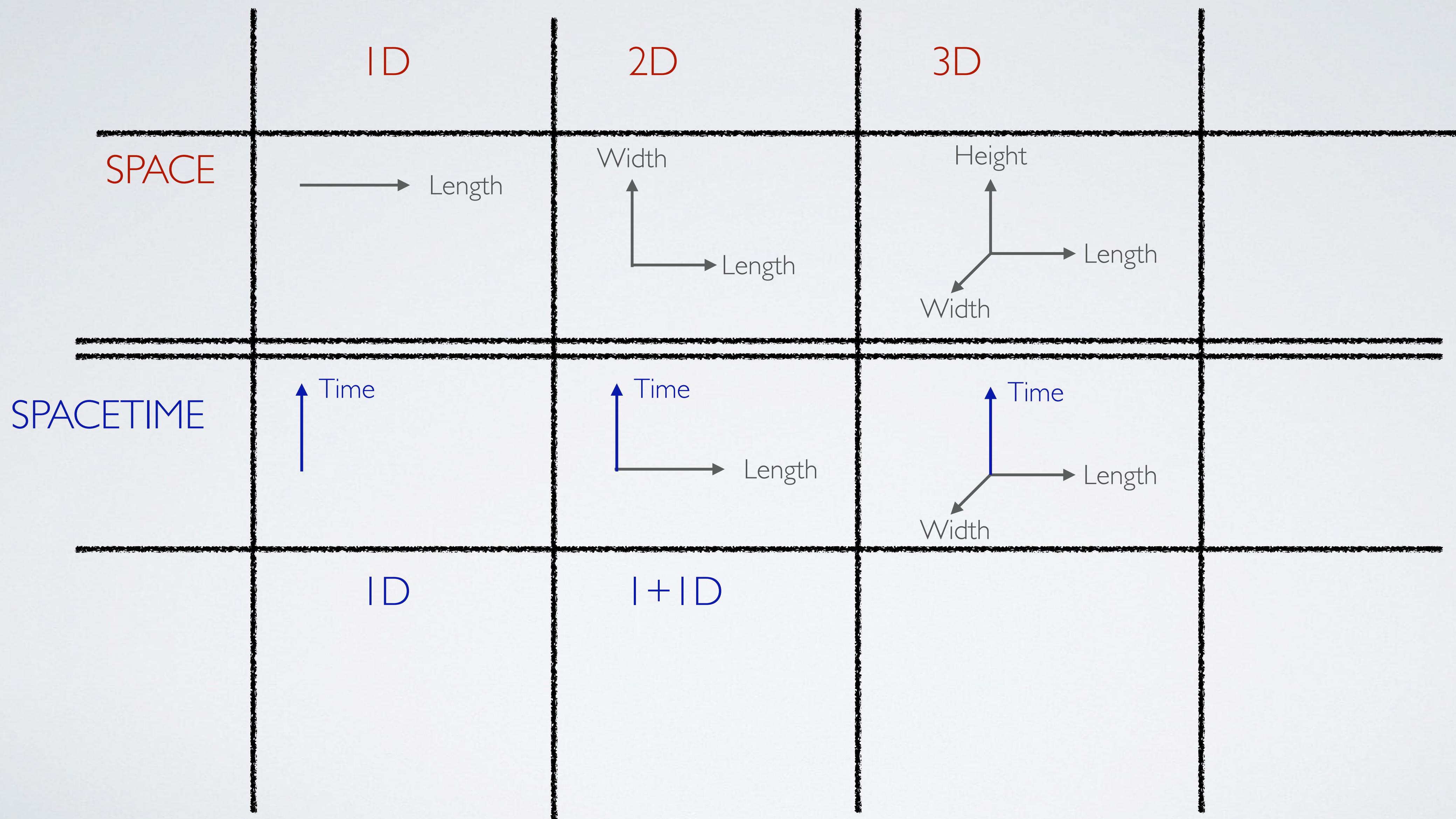
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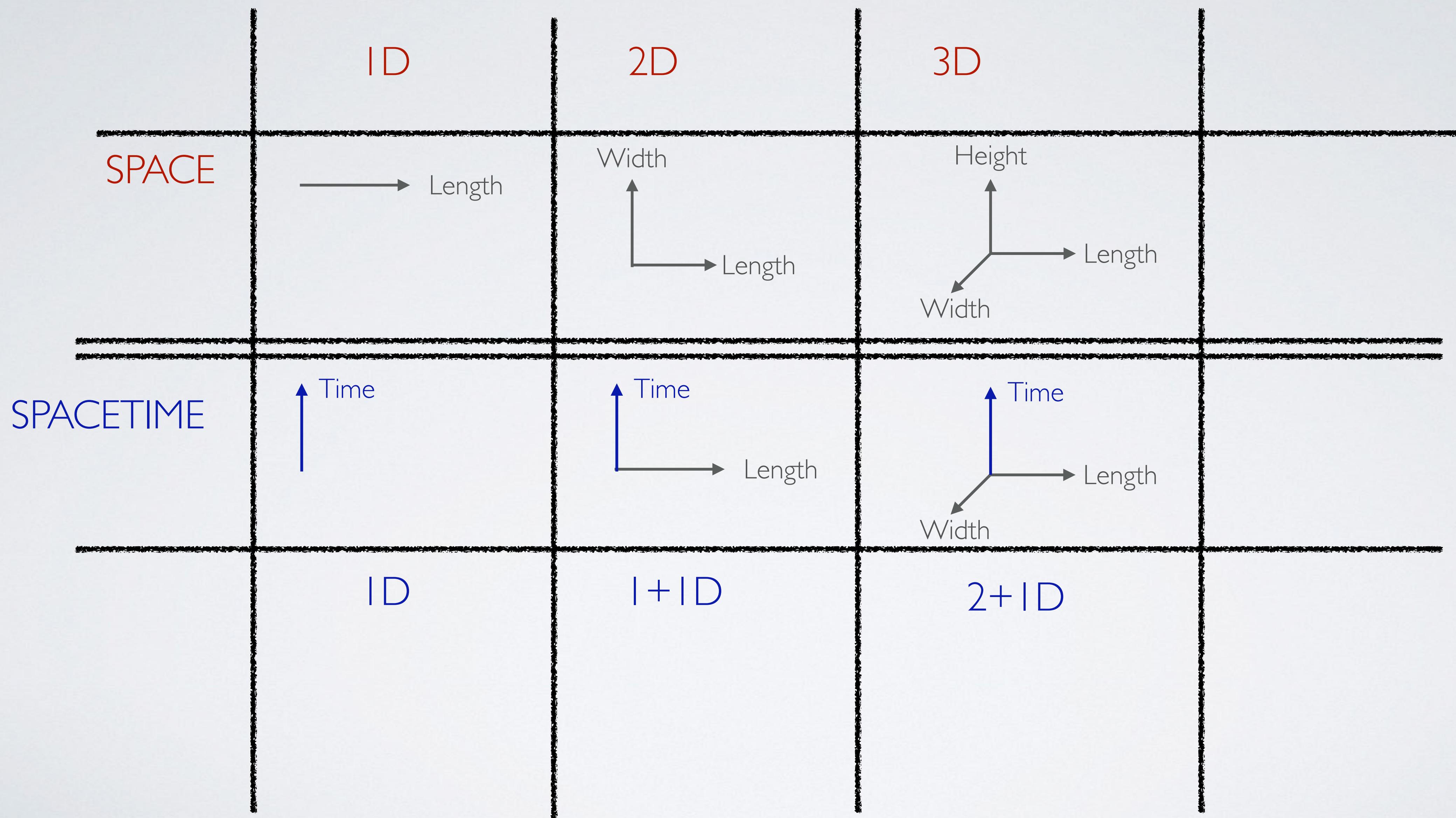
SPACETIME DIMENSIONS



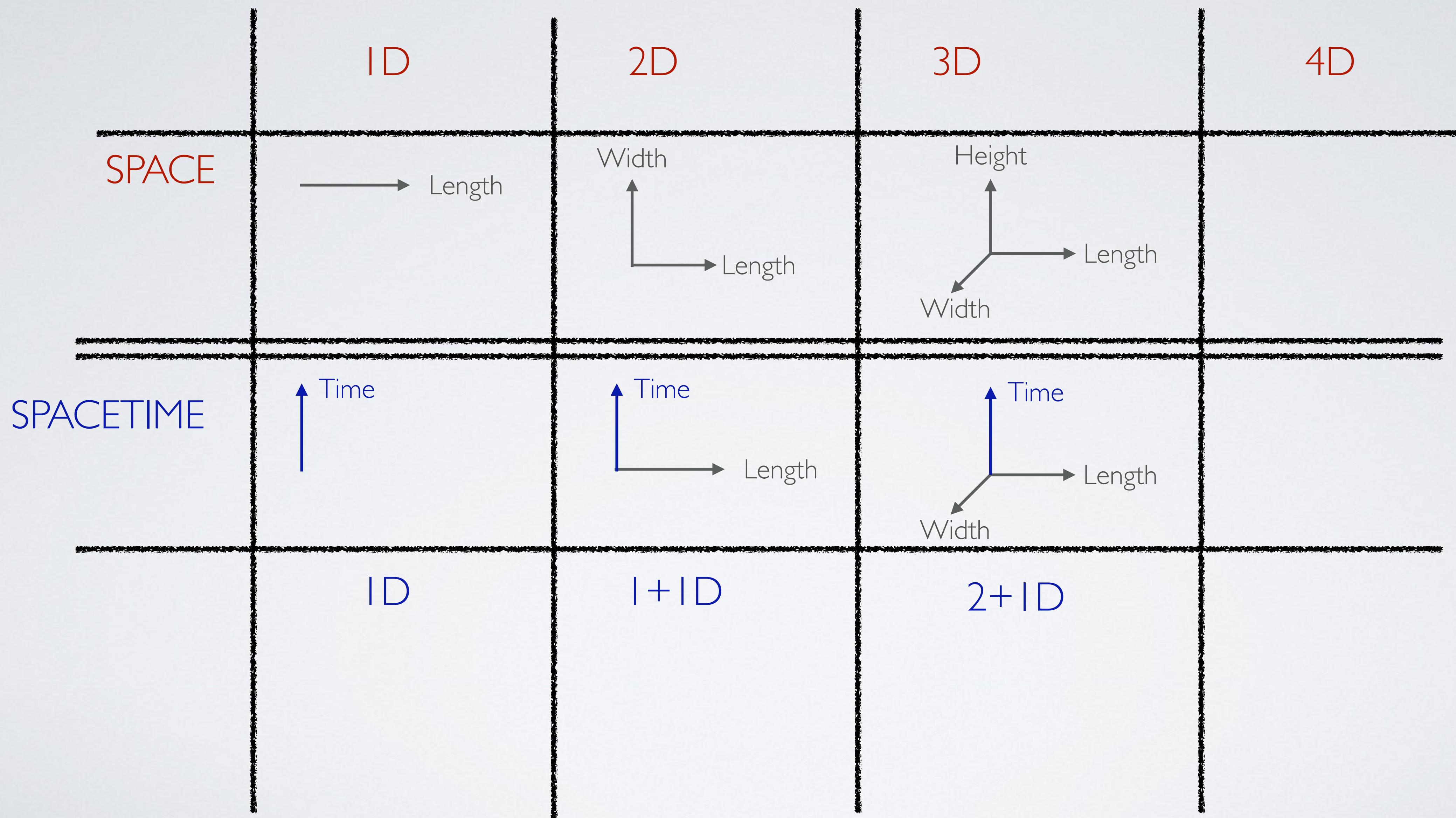
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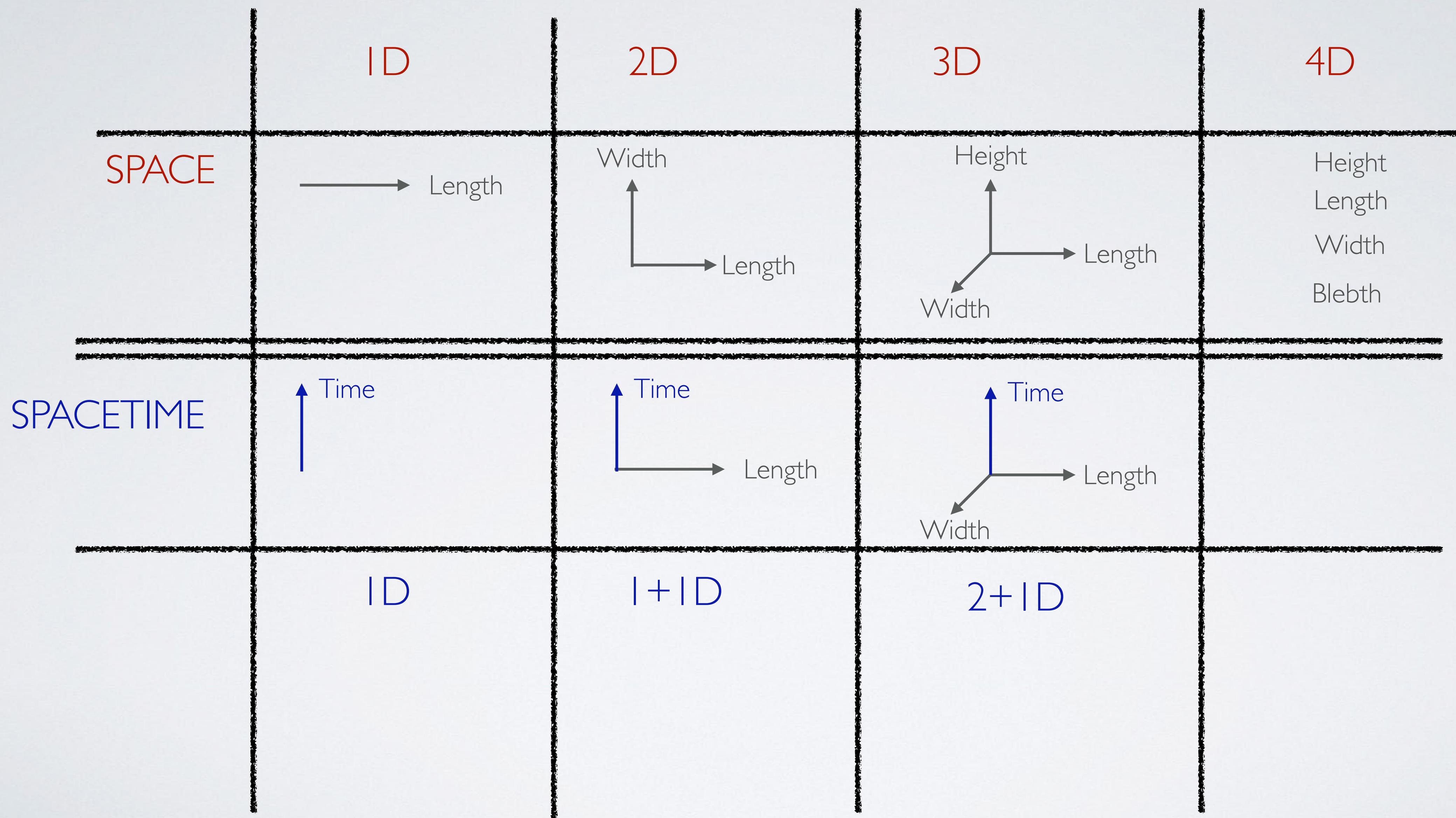
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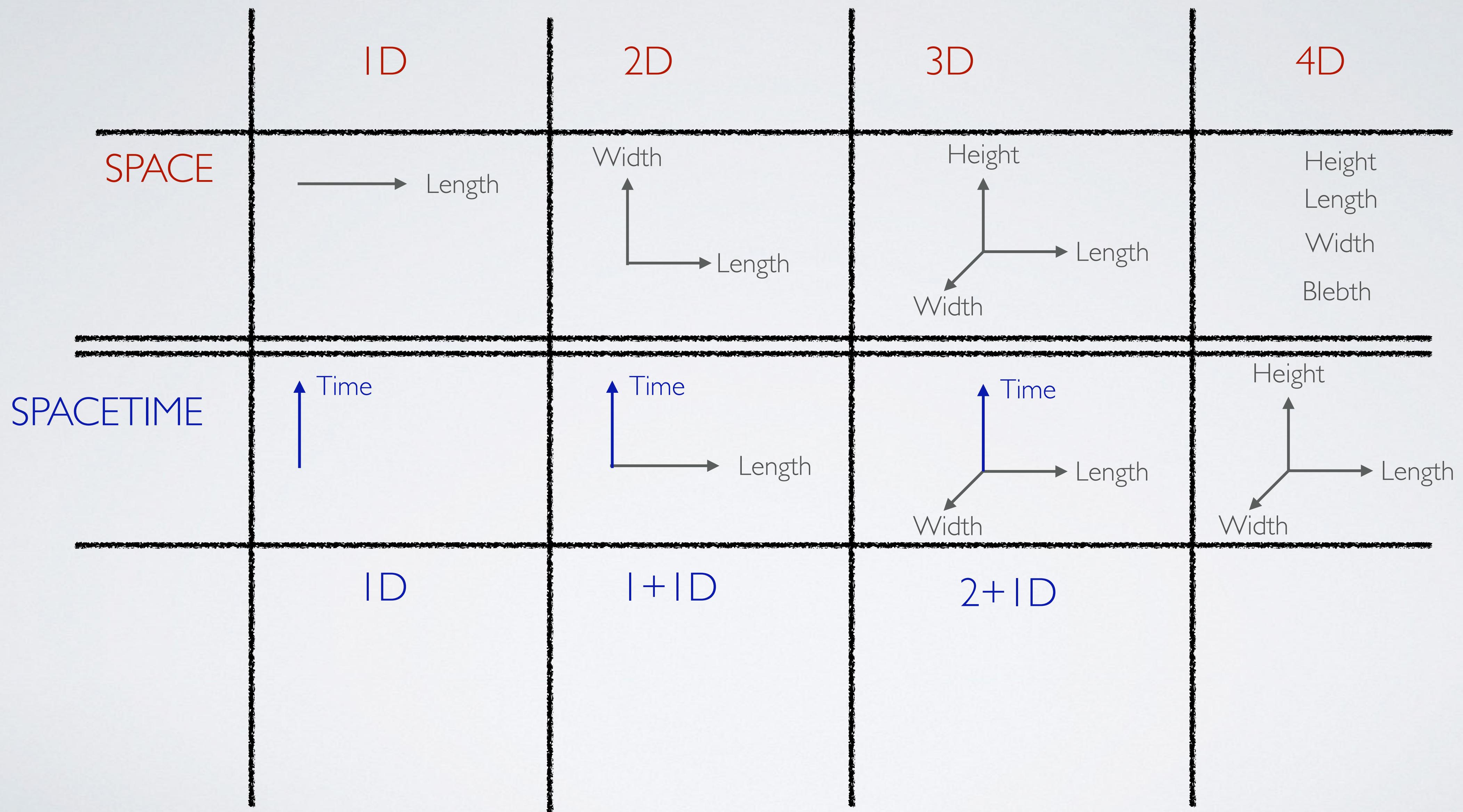
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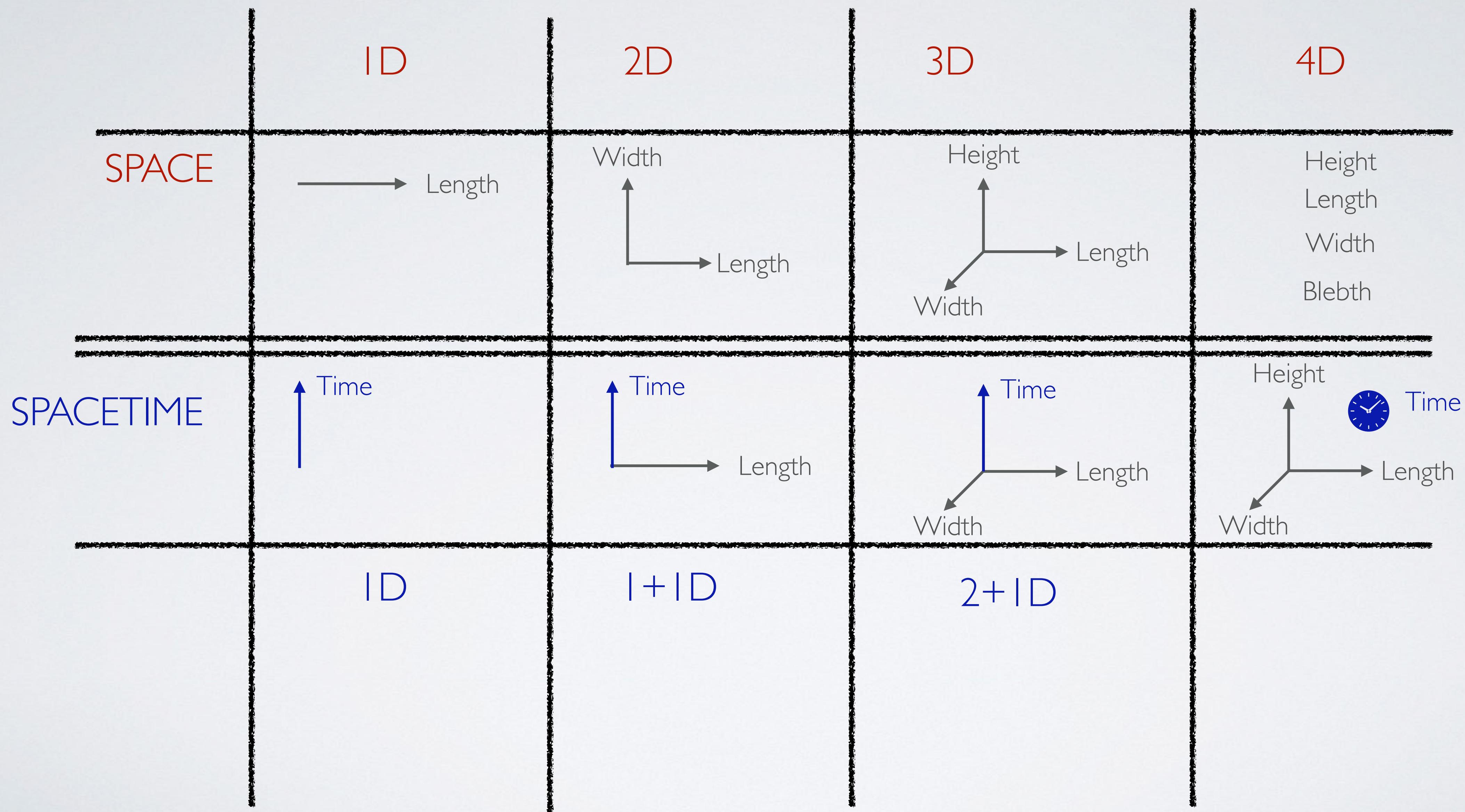
SPACETIME DIMENSIONS



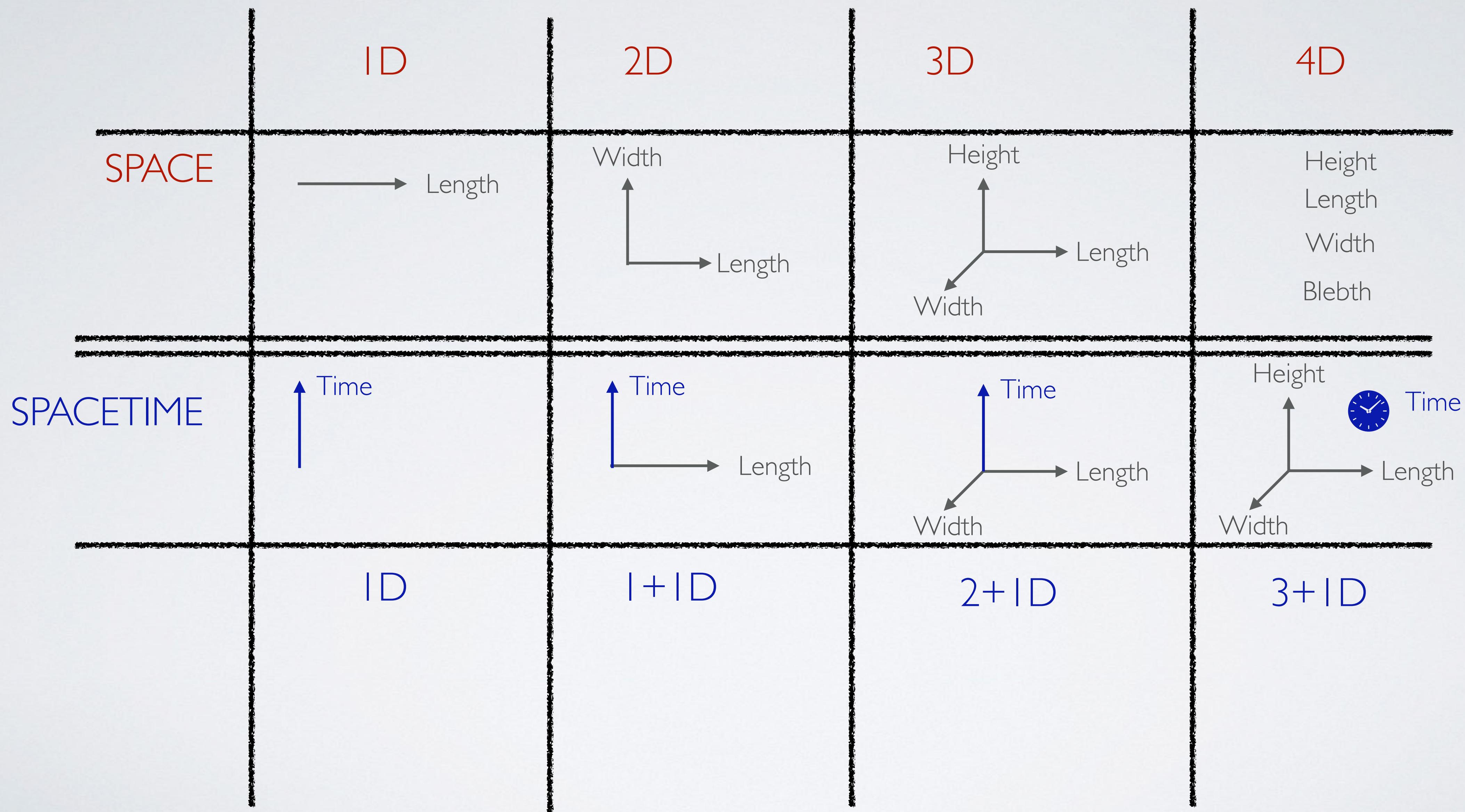
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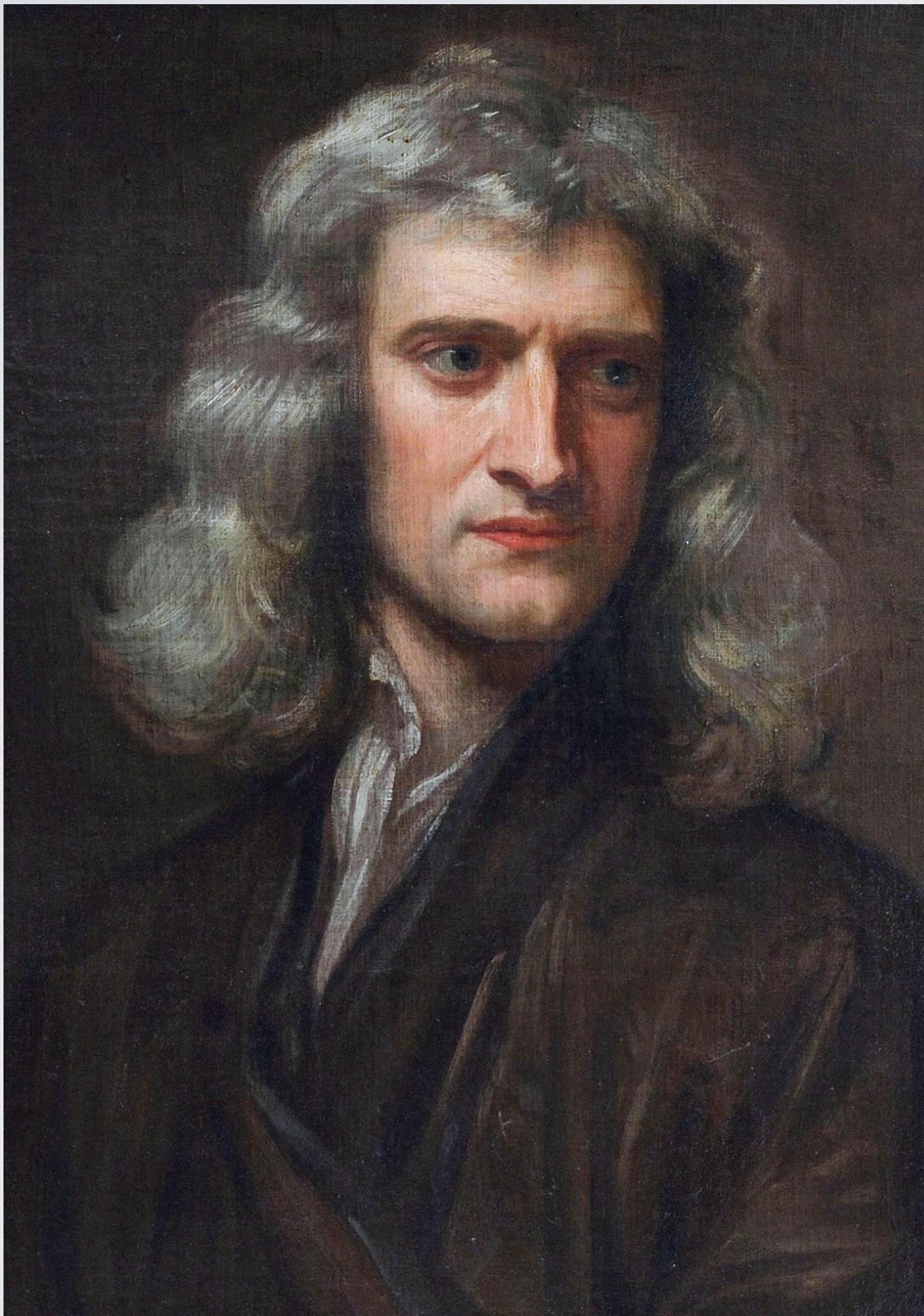
SPACETIME DIMENSIONS



SPACETIME DIMENSIONS



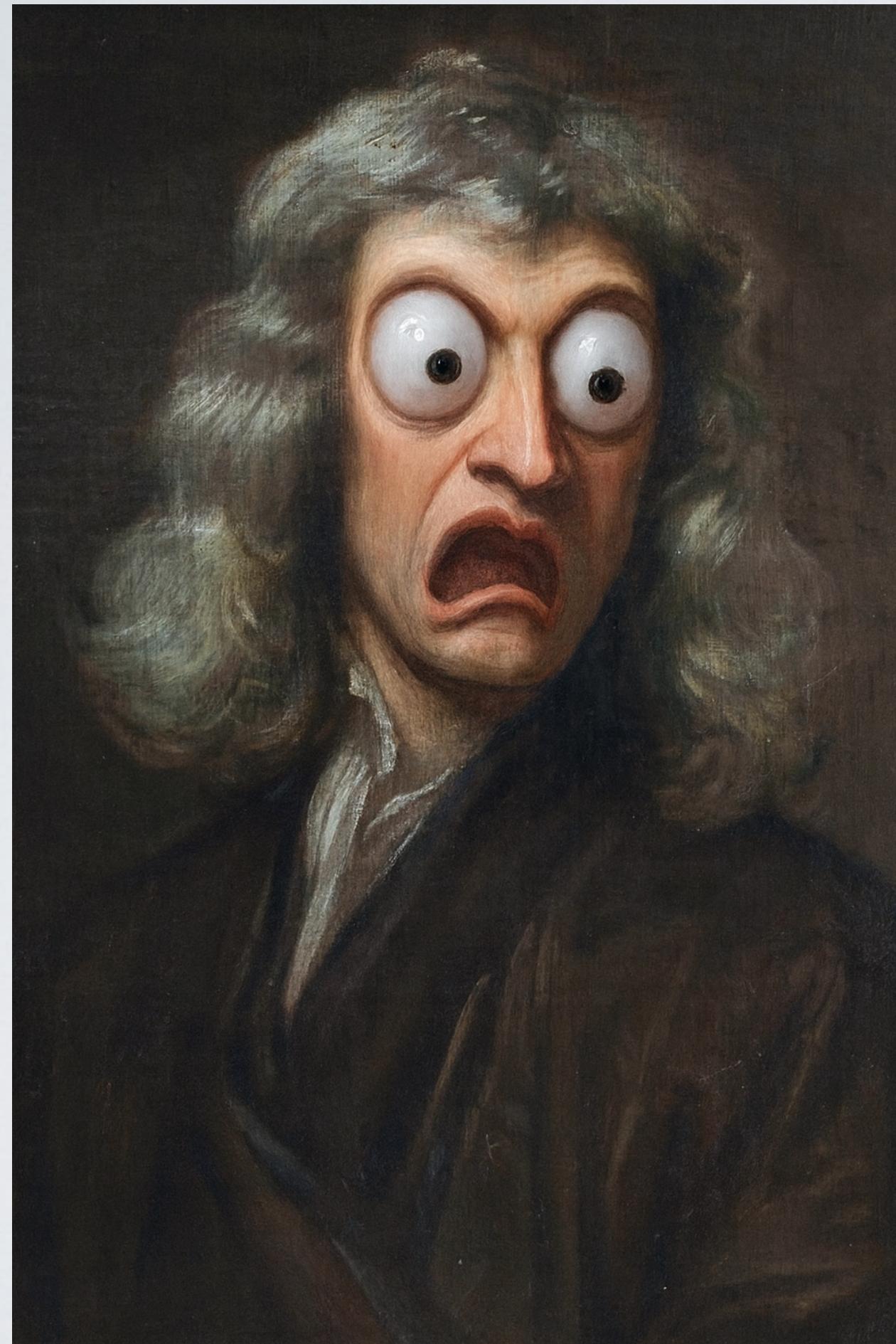
SPACE AND TIME VERSUS SPACETIME



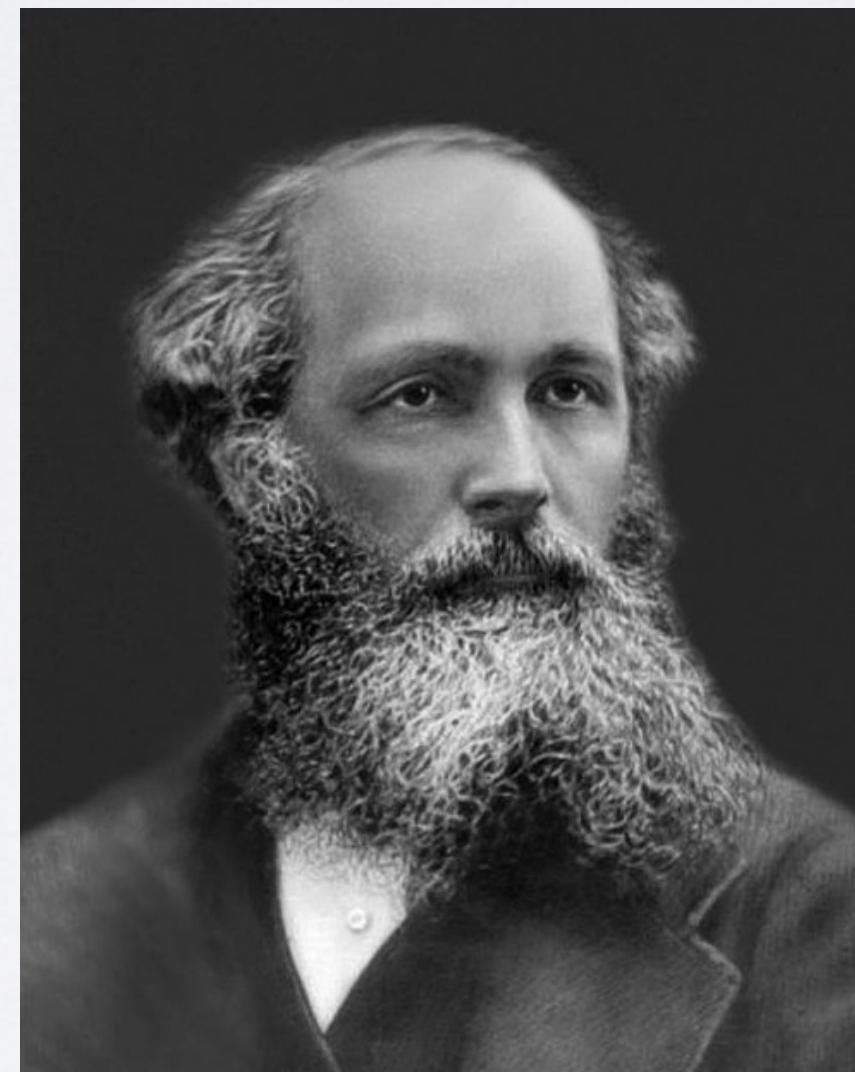
Absolute, true, and mathematical time, of itself, and from its own nature, flows equably **without relation to anything external...**

Isaac Newton (by G. Kneller)
1643-1727

SPACE AND TIME VERSUS SPACETIME



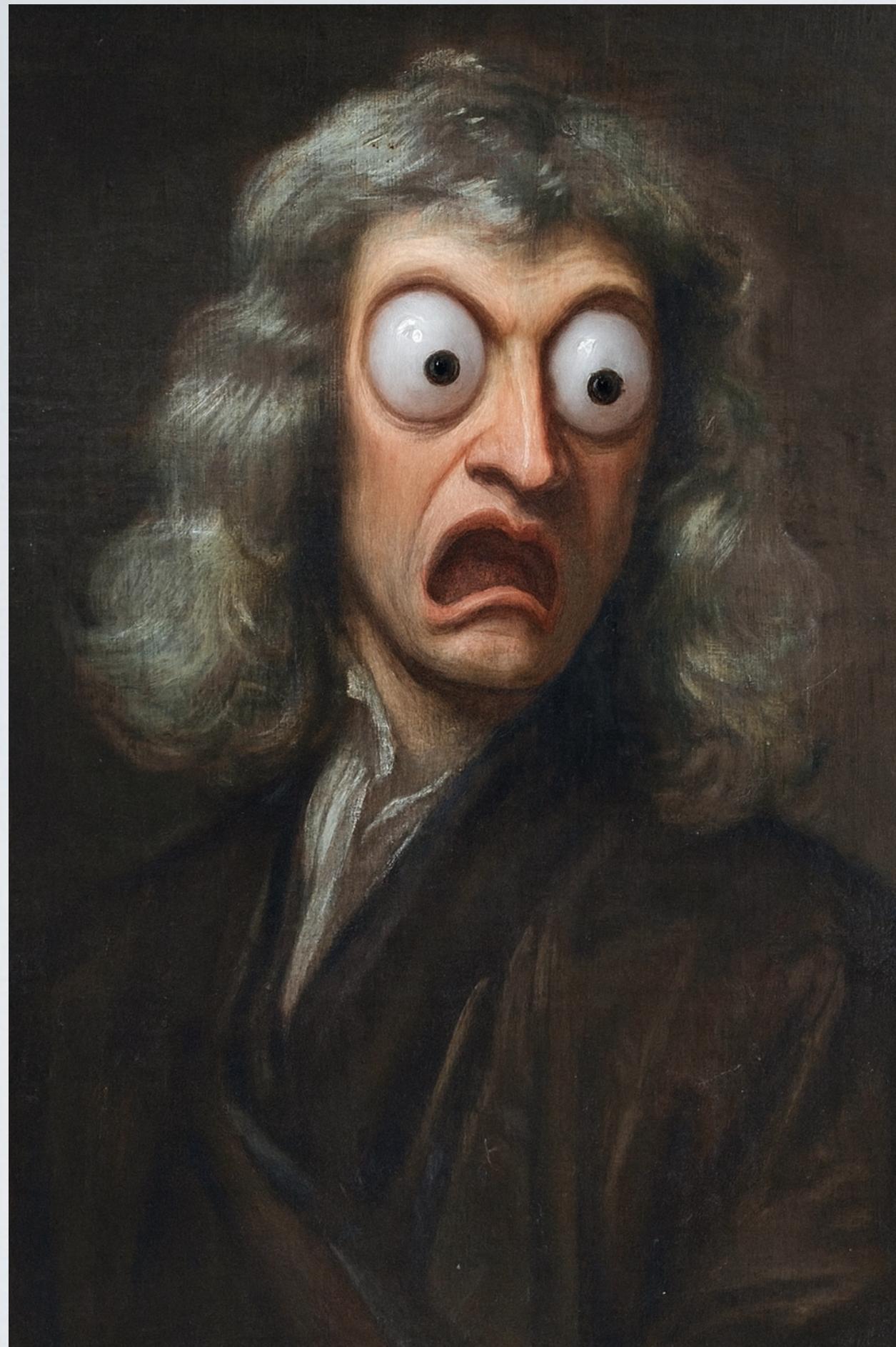
Isaac Newton (by G. Kneller)
(1643-1727)



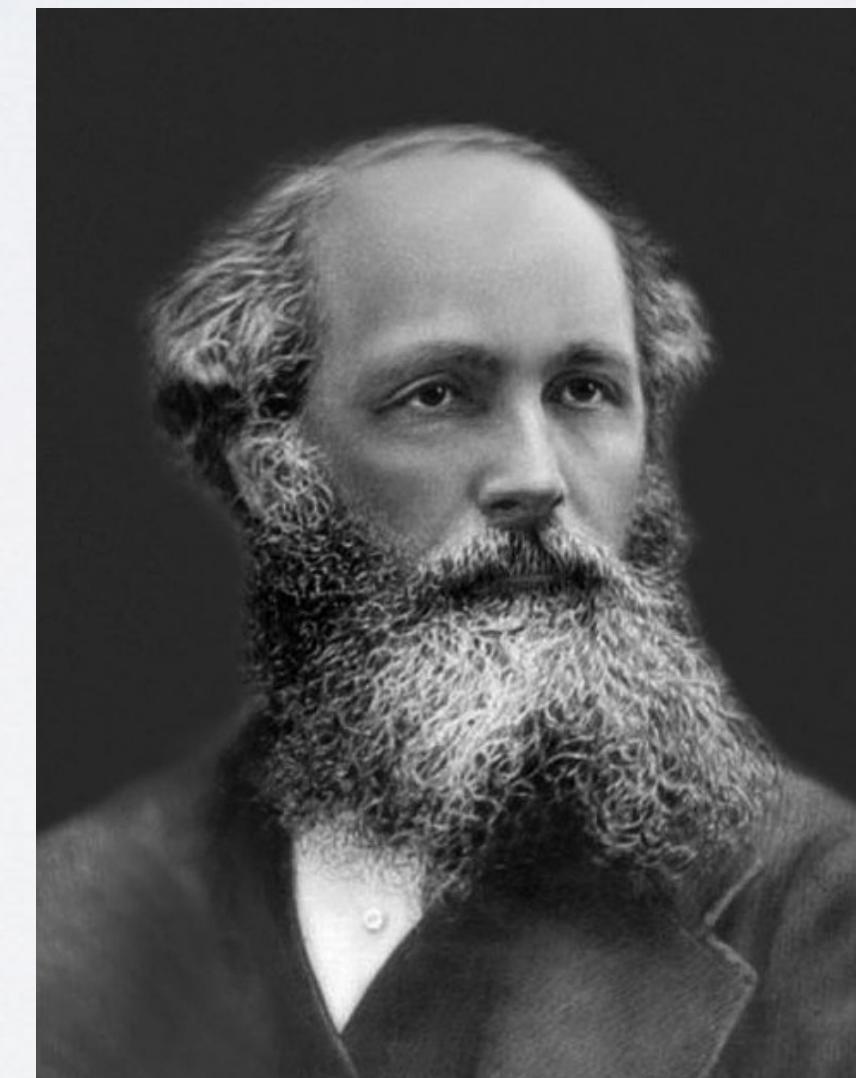
James Clerk Maxwell
(1831-1879)

Propagation of electromagnetic waves
(light) seems to be in conflict with Galileo/
Newton... (1865)

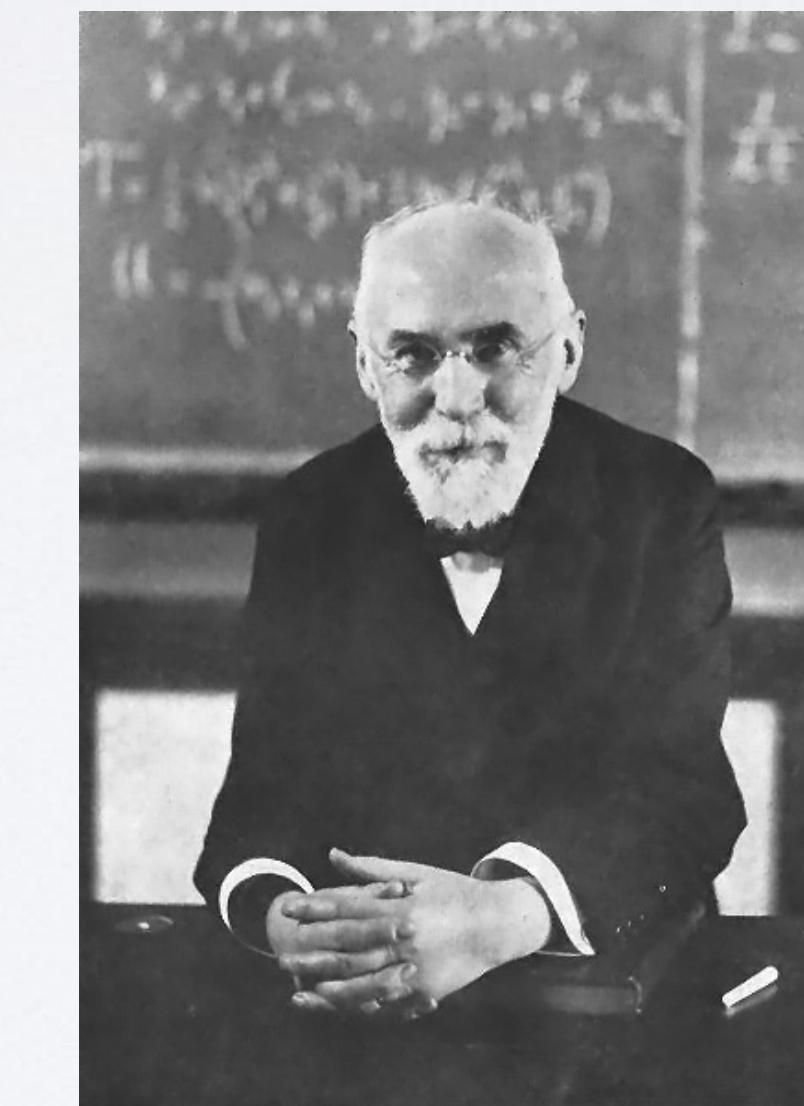
SPACE AND TIME VERSUS SPACETIME



Isaac Newton (by G. Kneller)
(1643-1727)



James Clerk Maxwell
(1831-1891)

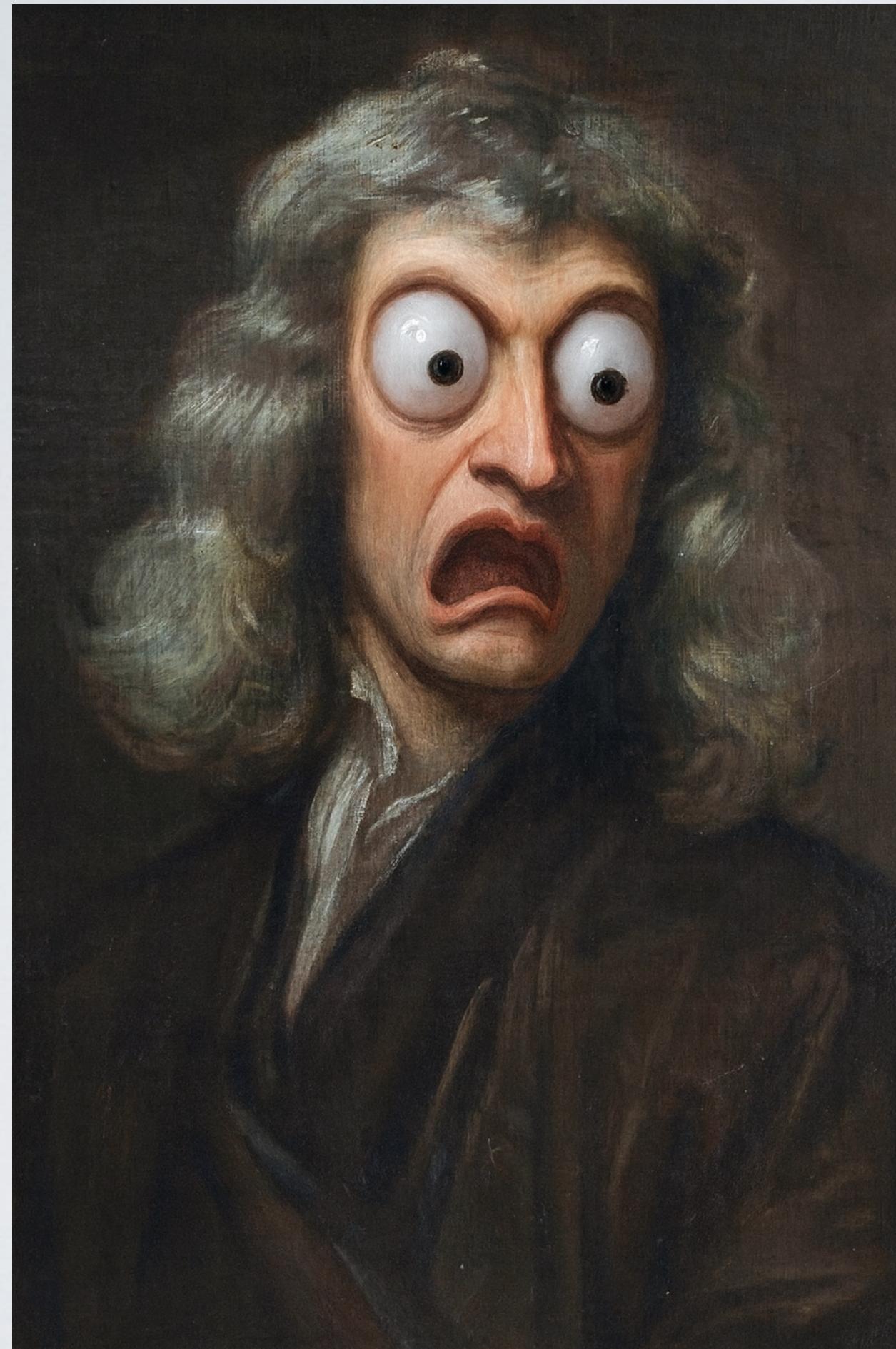


Hendrik Antoon Lorentz
(1853-1928)

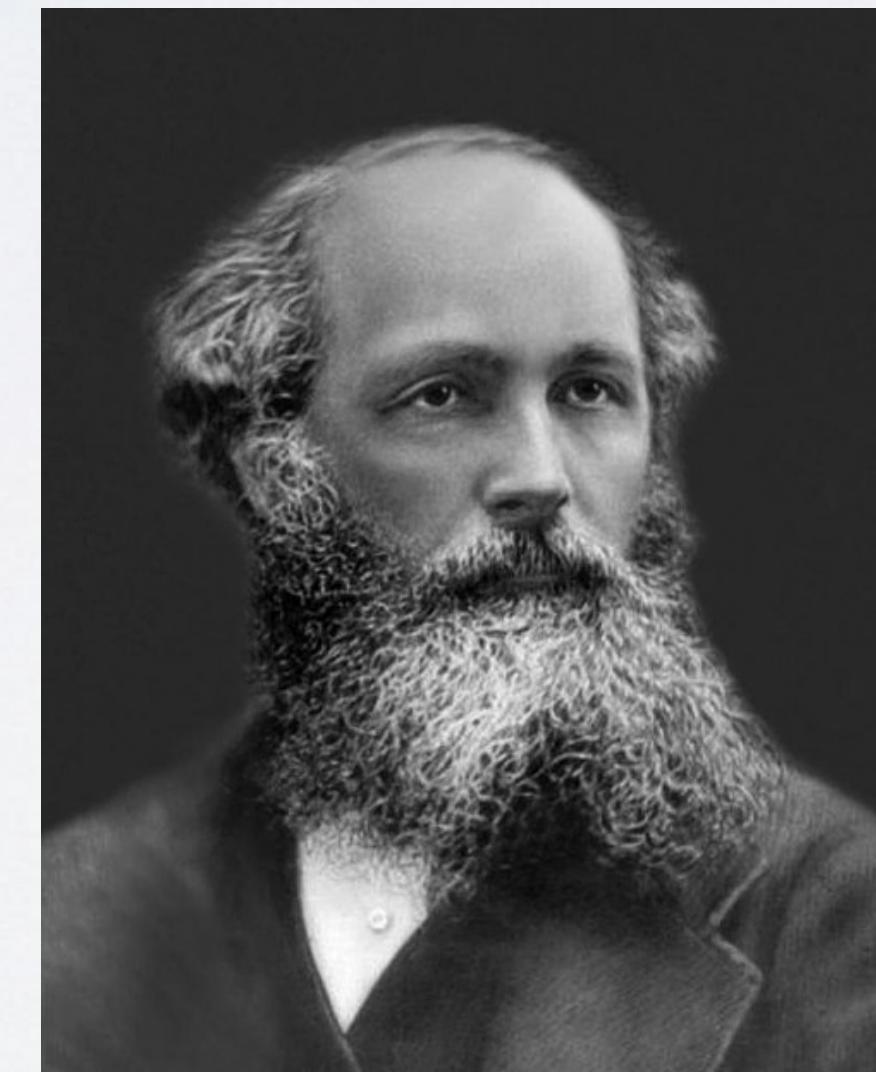
Well, mathematically everything works fine
if time could dilate, space could contract

(1892-95)

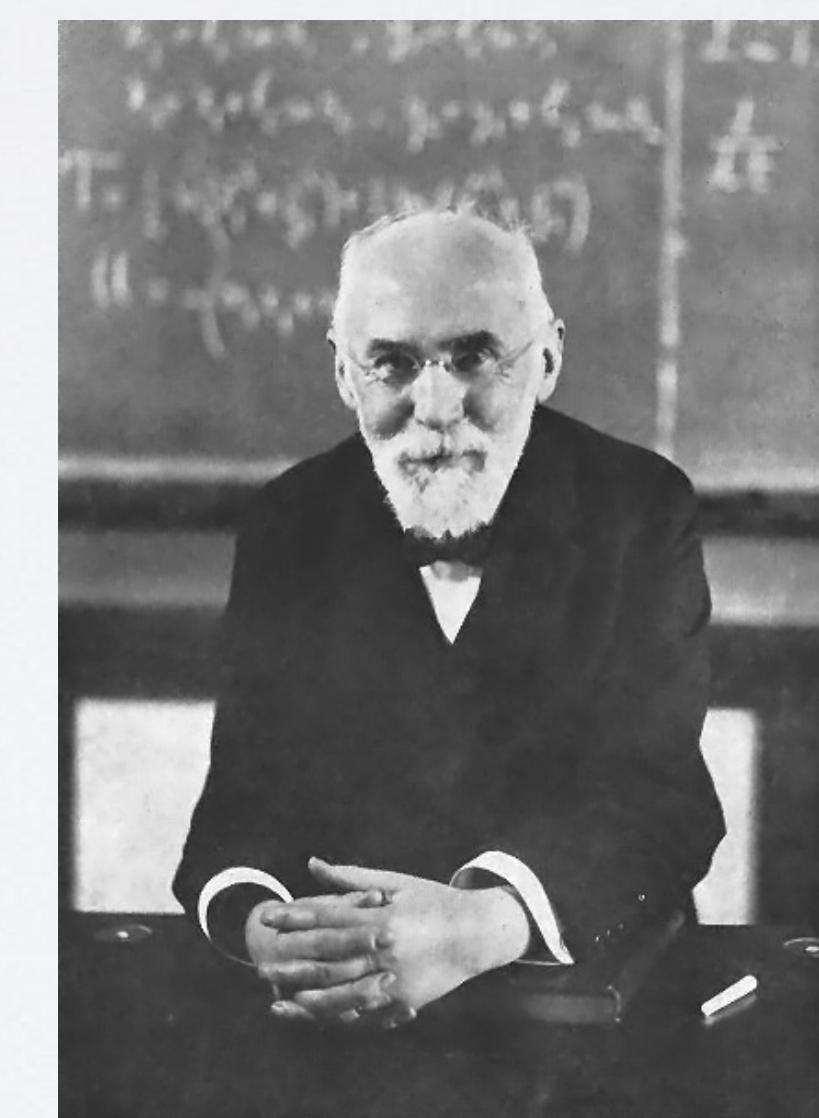
SPACE AND TIME VERSUS SPACETIME



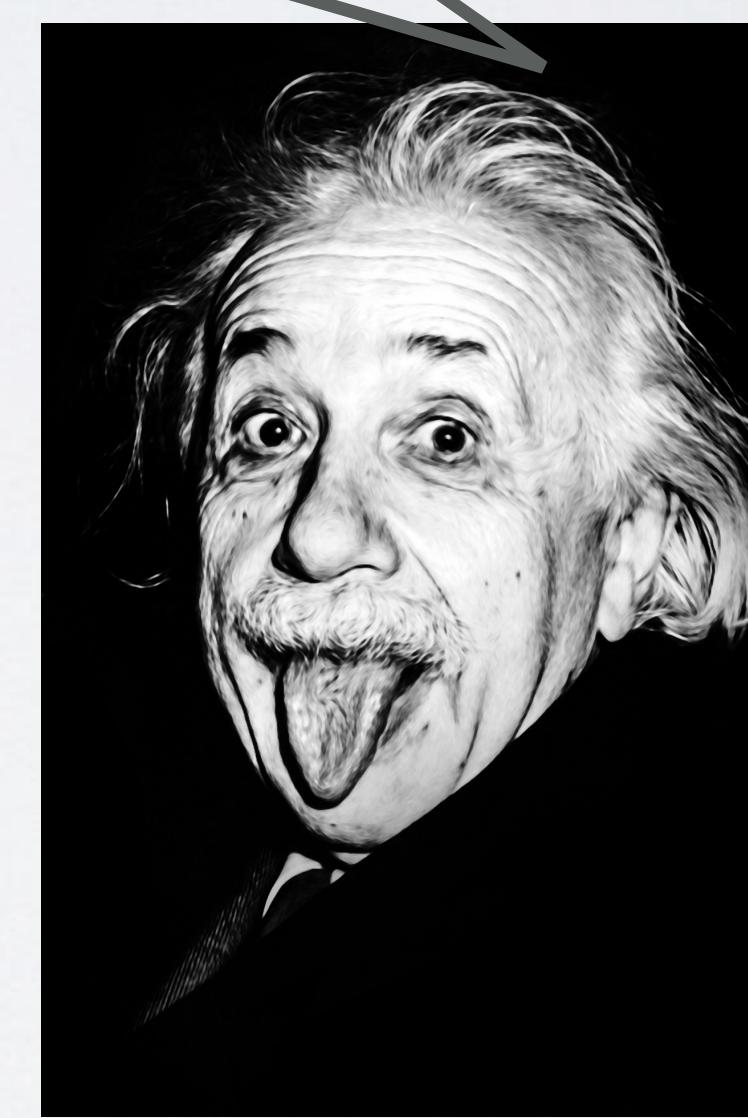
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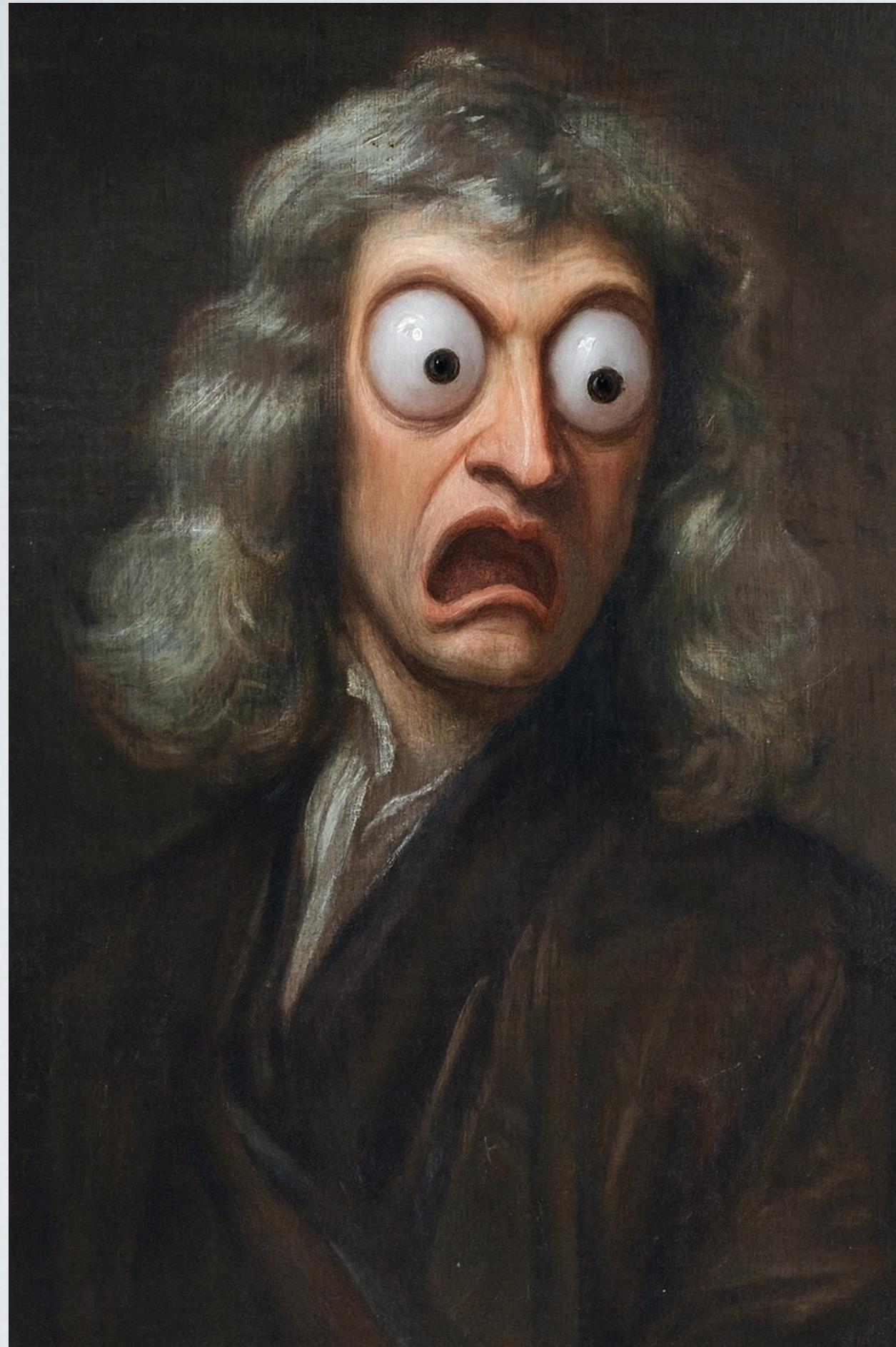
Hendrik Antoon Lorentz
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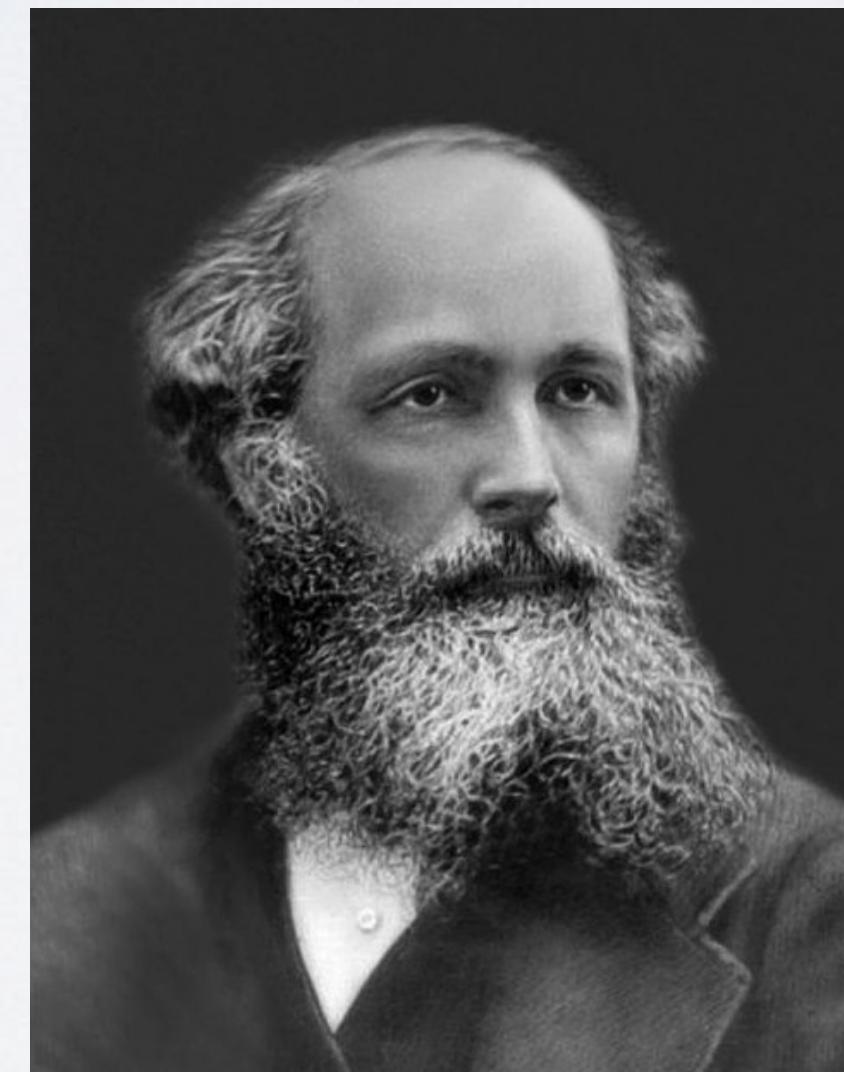
Albert Einstein
(1879-1955)

Ok guys... let's just face it: the speed of light is the **absolute** here. It is the same for all inertial observers.... Thus, **time dilation and space contraction is a physical reality** (1905)

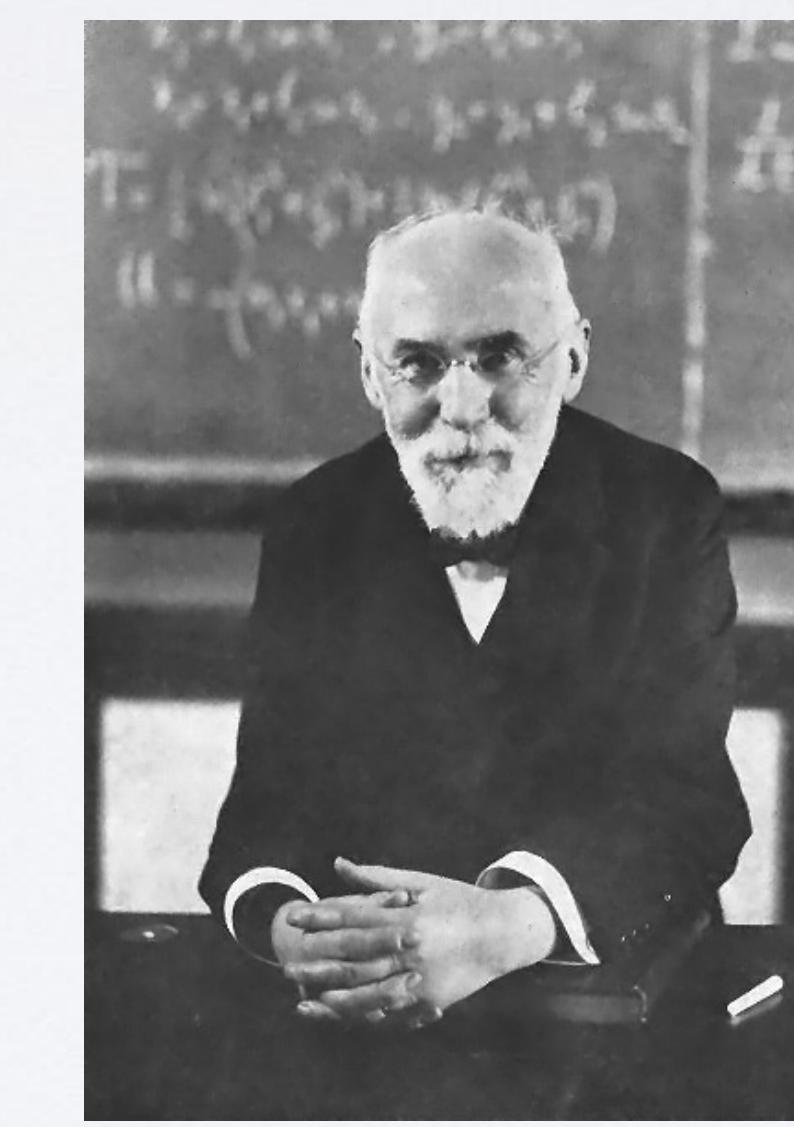
SPACE AND TIME VERSUS SPACETIME



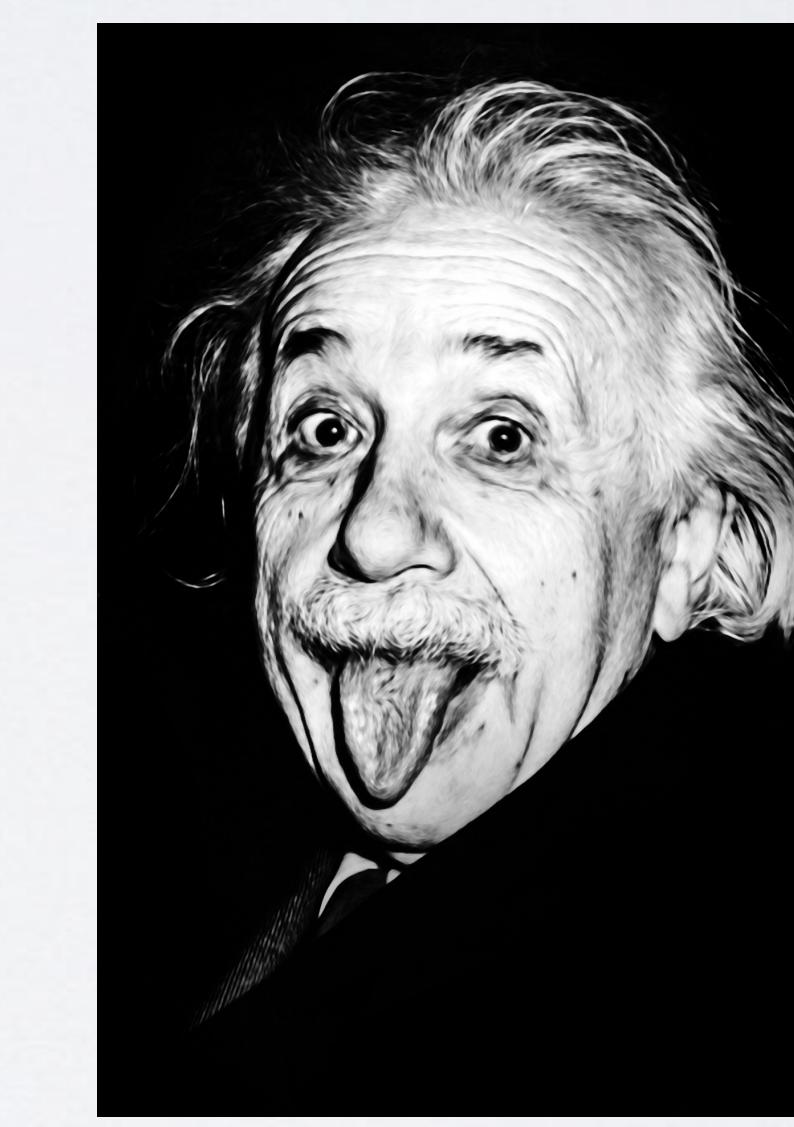
Isaac Newton (by G. Kneller)
(1643-1727)



James Clerk Maxwell
(1879-1995)



Hendrik Antoon Lorentz
(1853-1928)

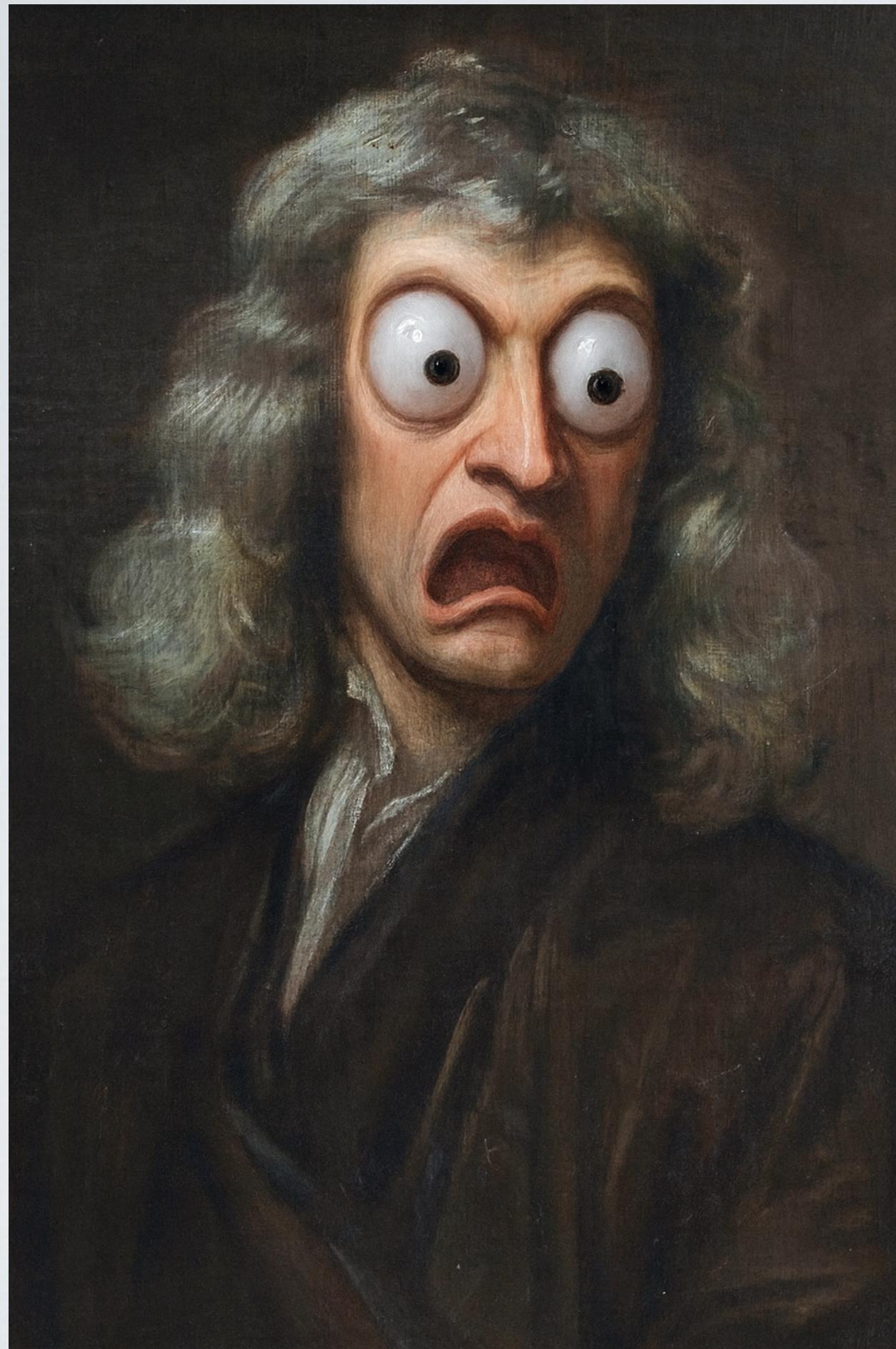


Albert Einstein
(1879-1995)

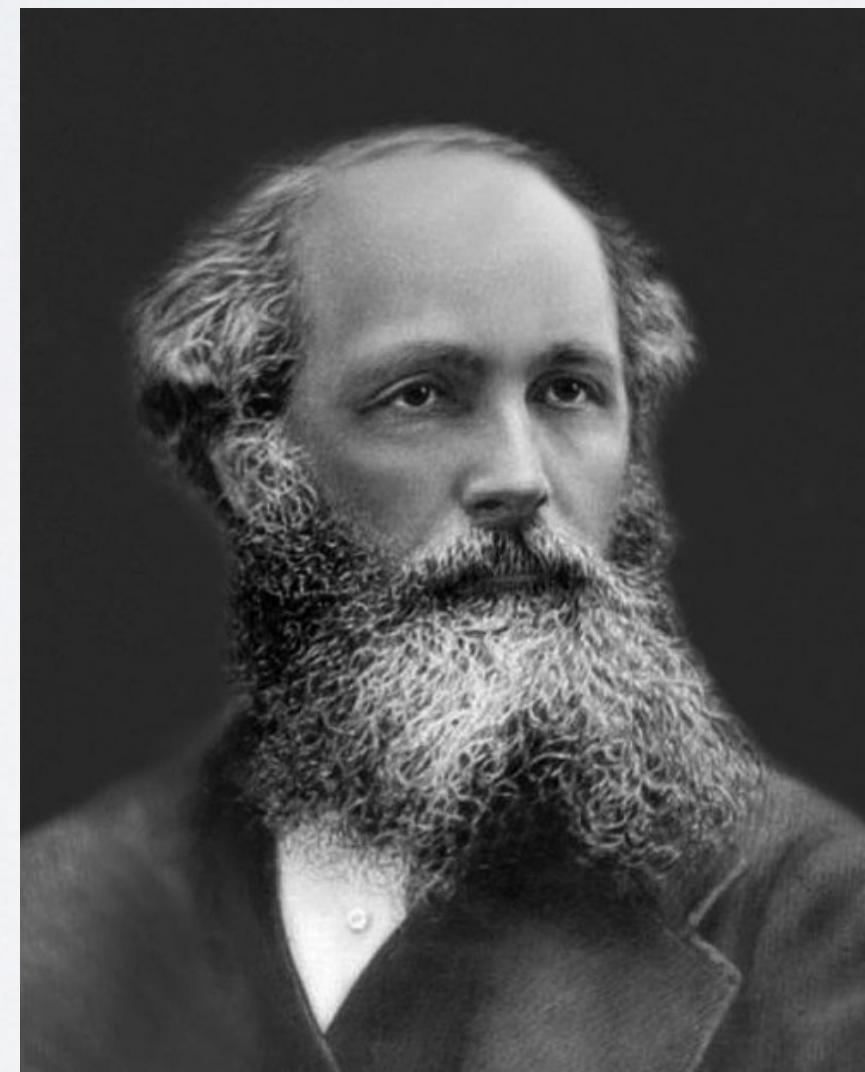


Mileva Maric
(1875-1948)

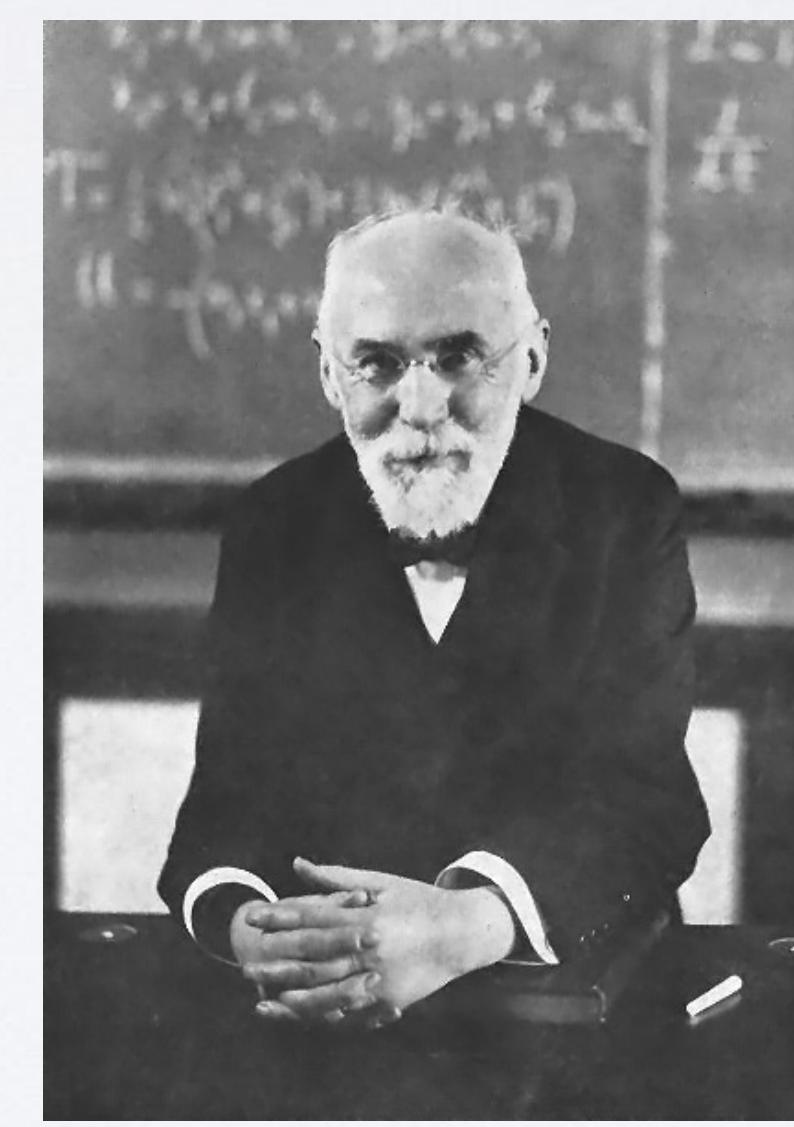
SPACE AND TIME VERSUS SPACETIME



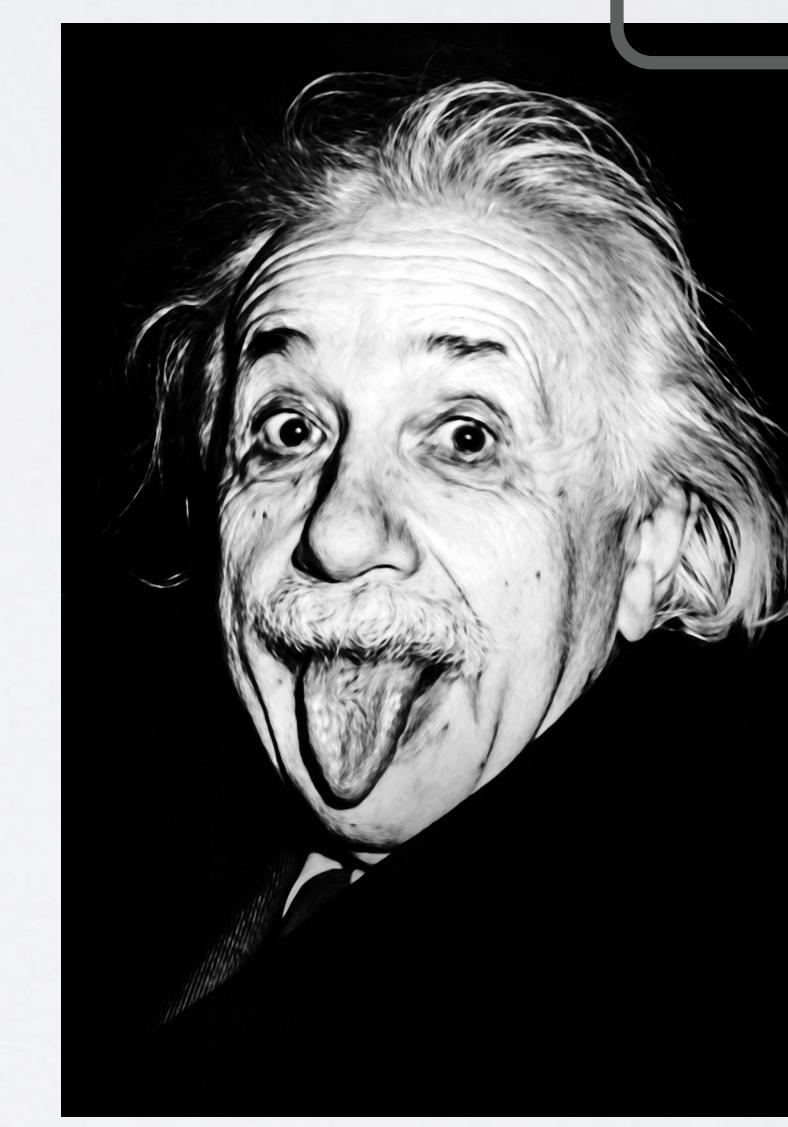
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(1643-1727)



James Clerk Maxwell
(1879-1995)



Hendrik Antoon Lorentz
(1853-1928)



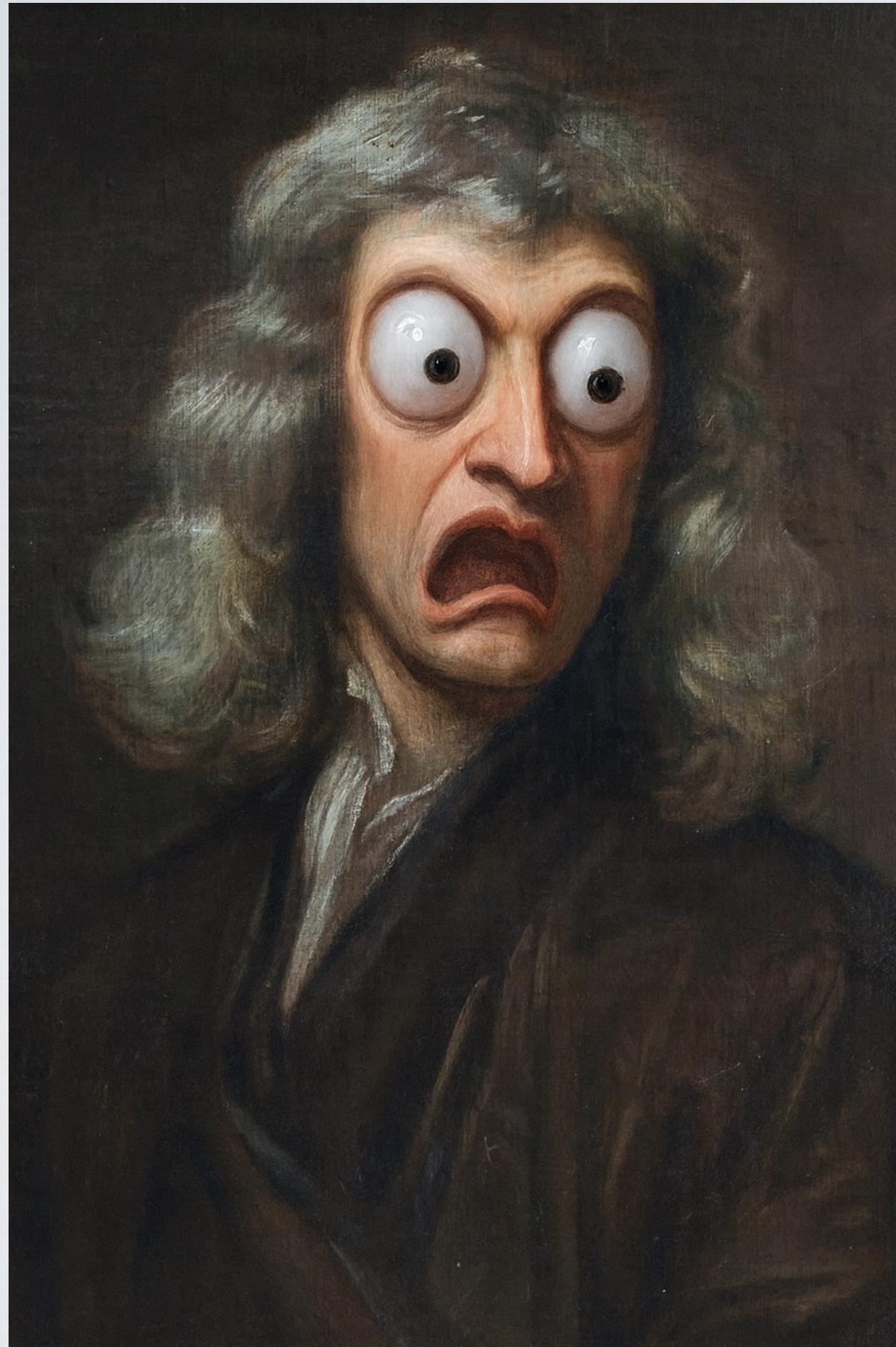
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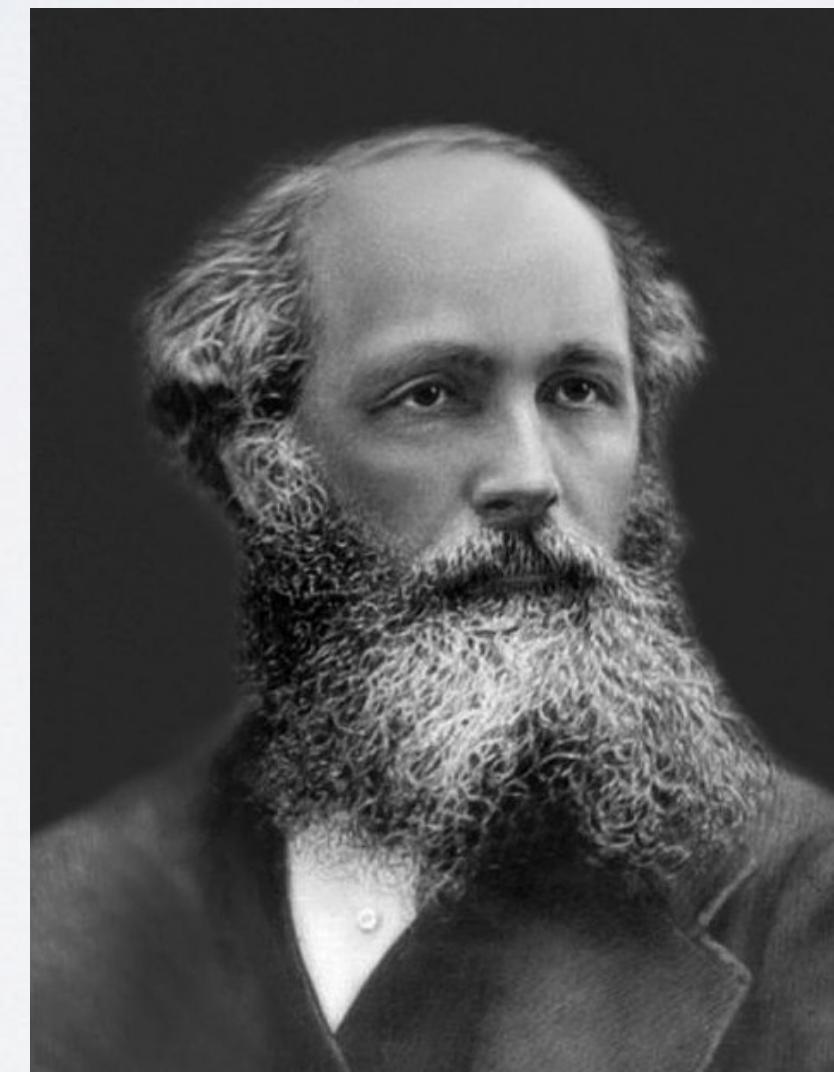
Mileva Maric
(1875-1948)

Ahan-Ahan...
Excuse me misters?!
What about my
contribution in the
intellectual discussion,
correcting Einstein's
mathematical errors while
taking care of the house and
two babies?

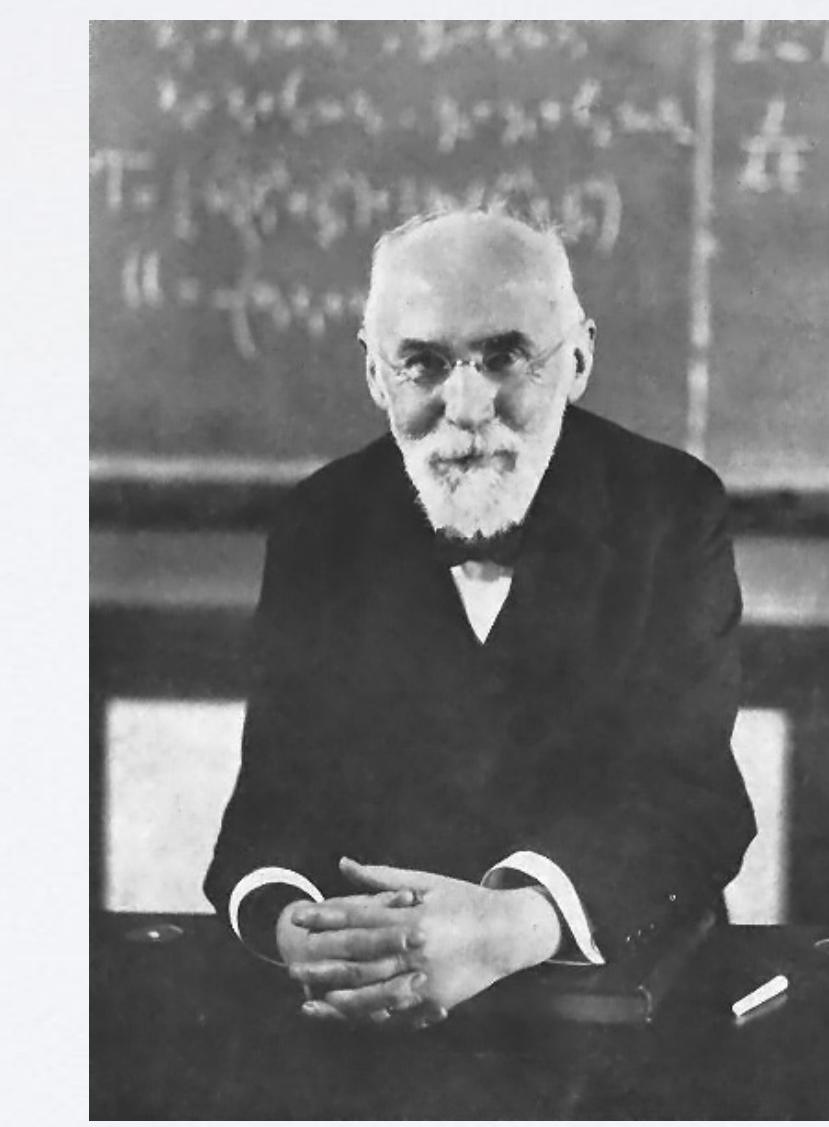
SPACE AND TIME VERSUS SPACETIME



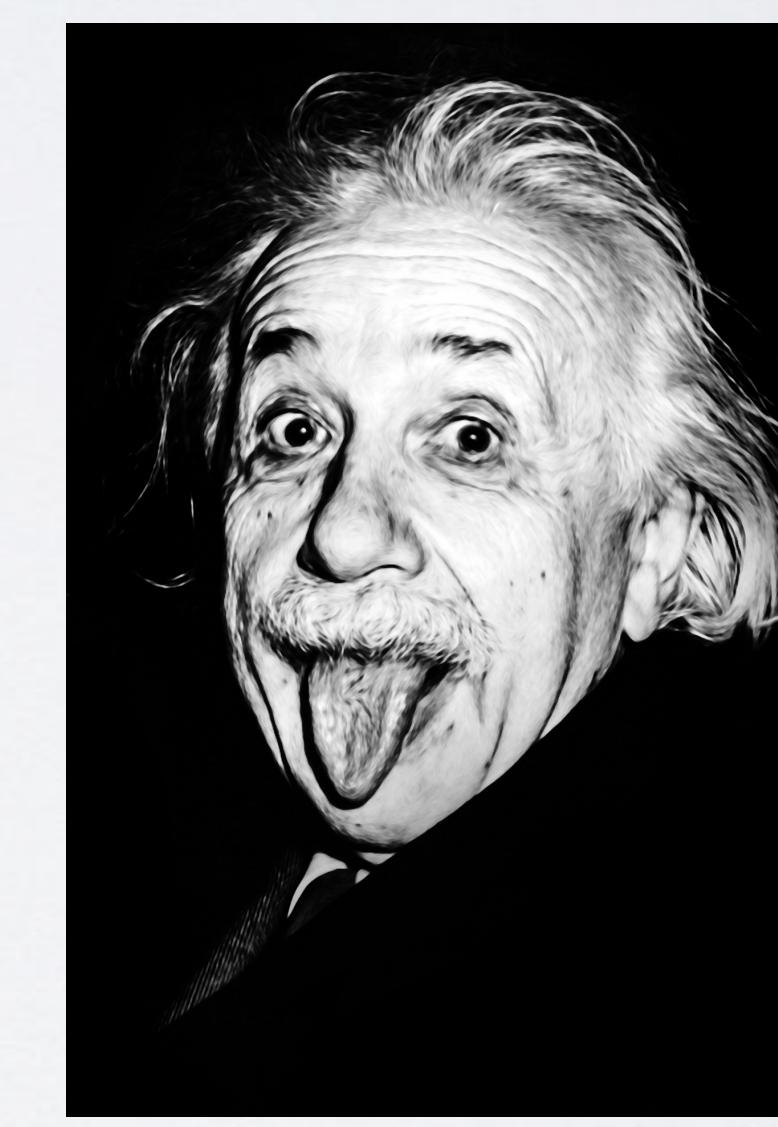
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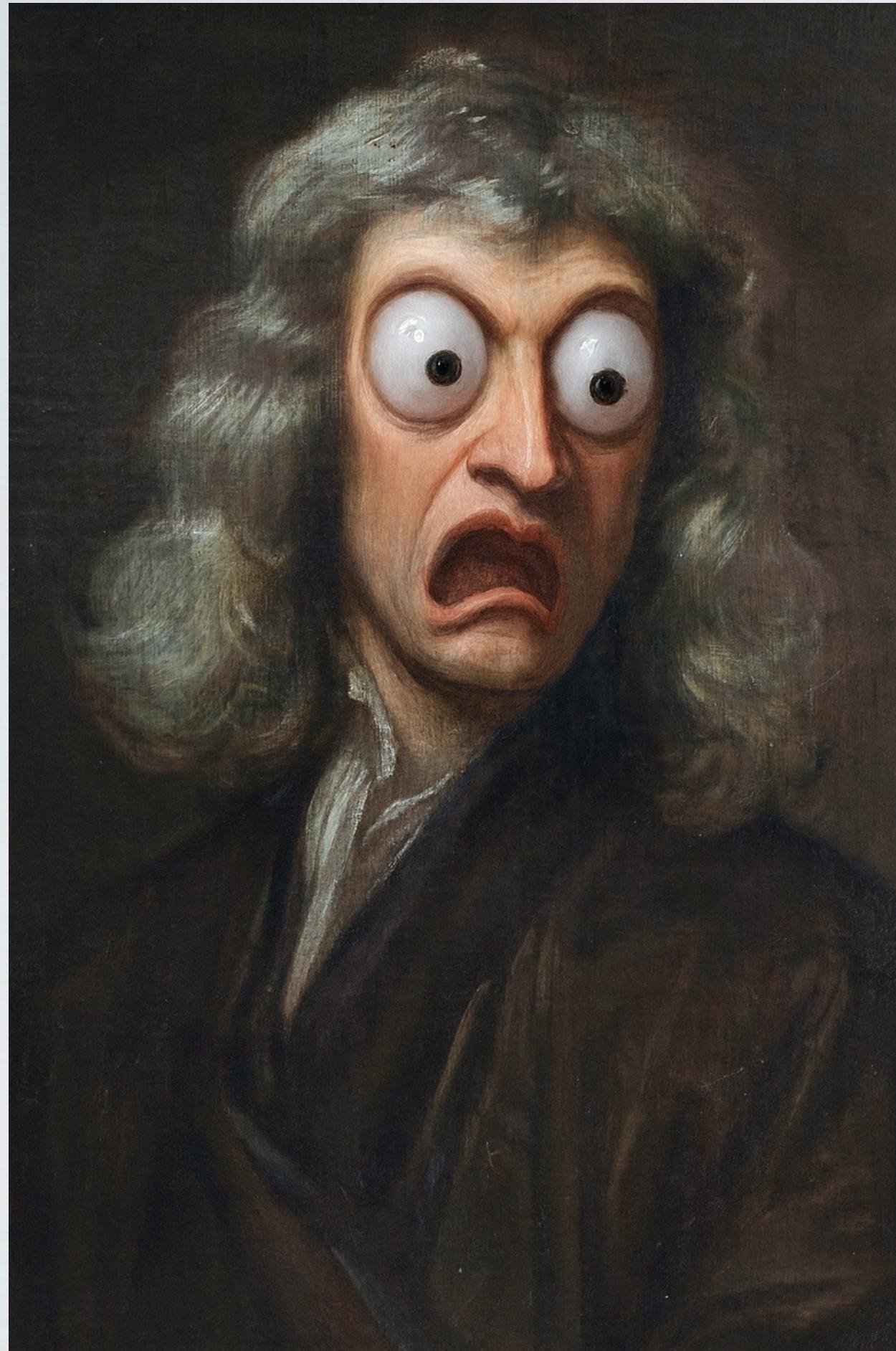


Hendrik Antoon Lorentz
(1853-1928)

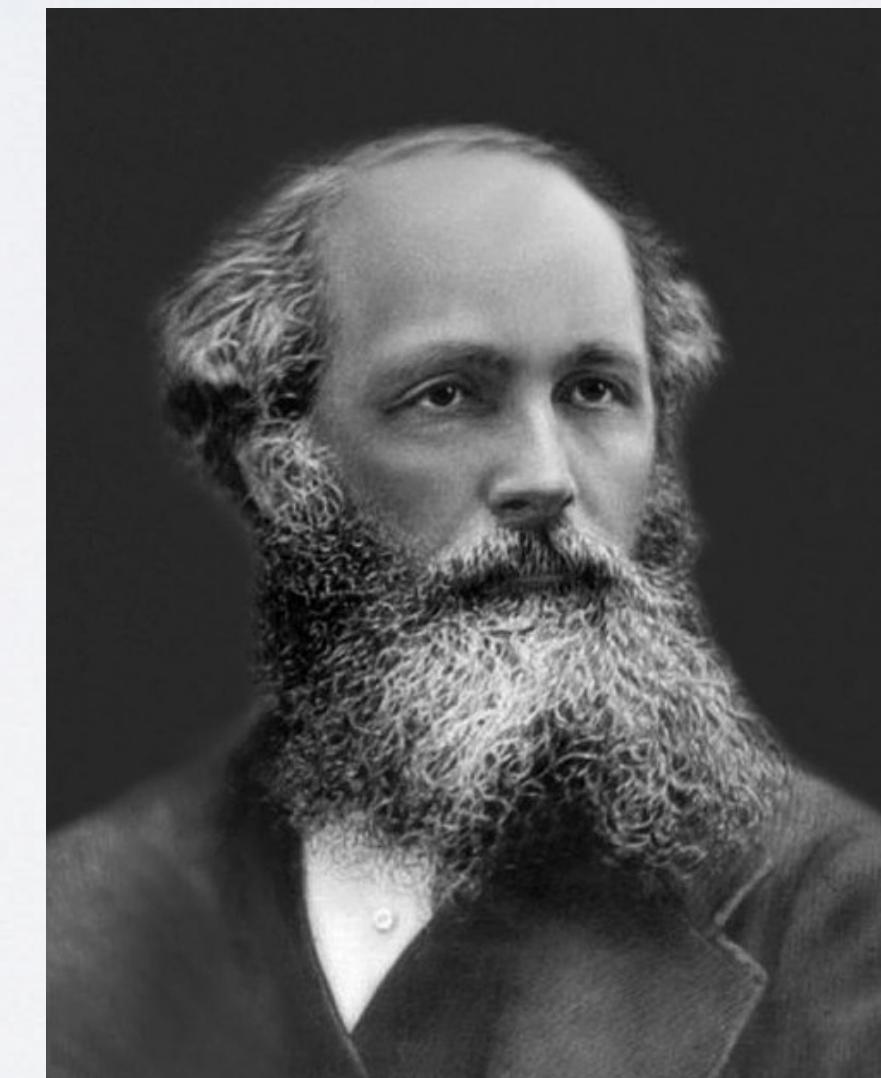


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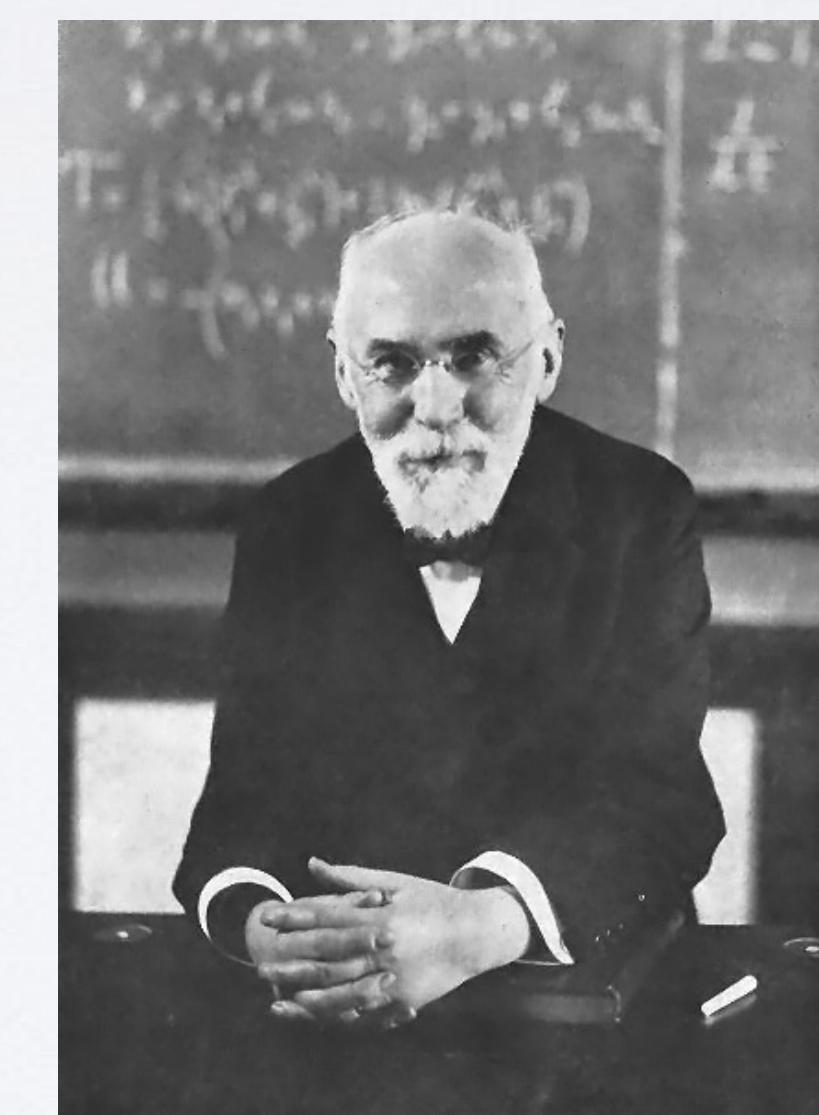
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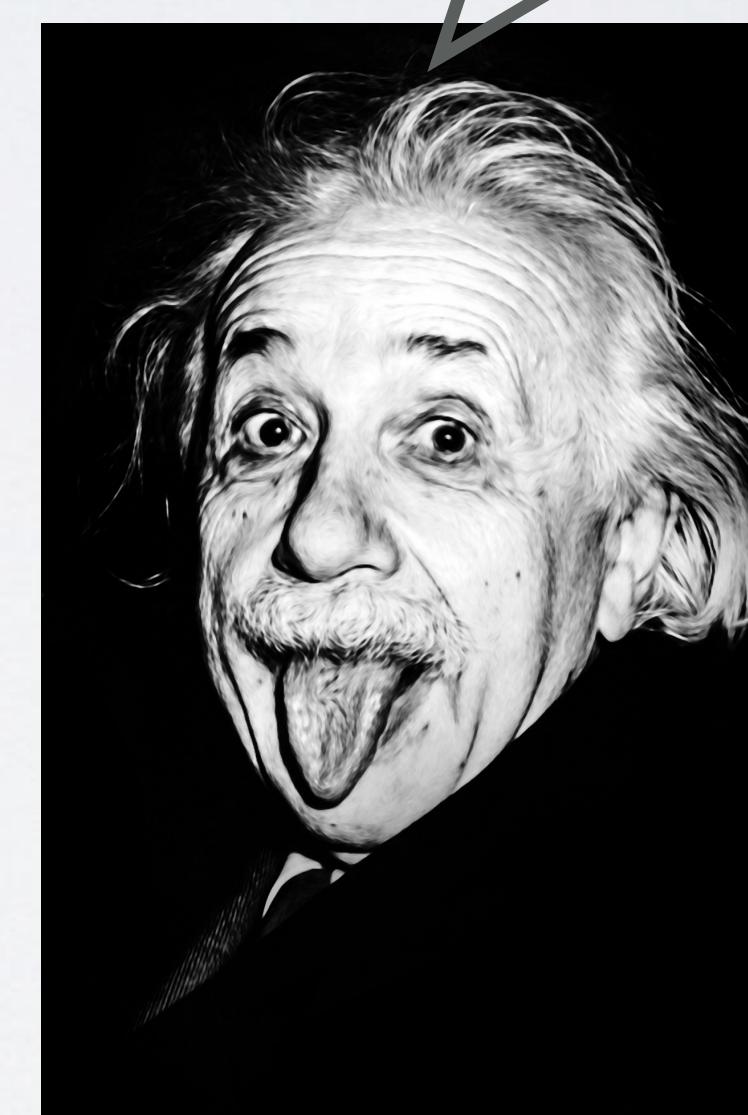
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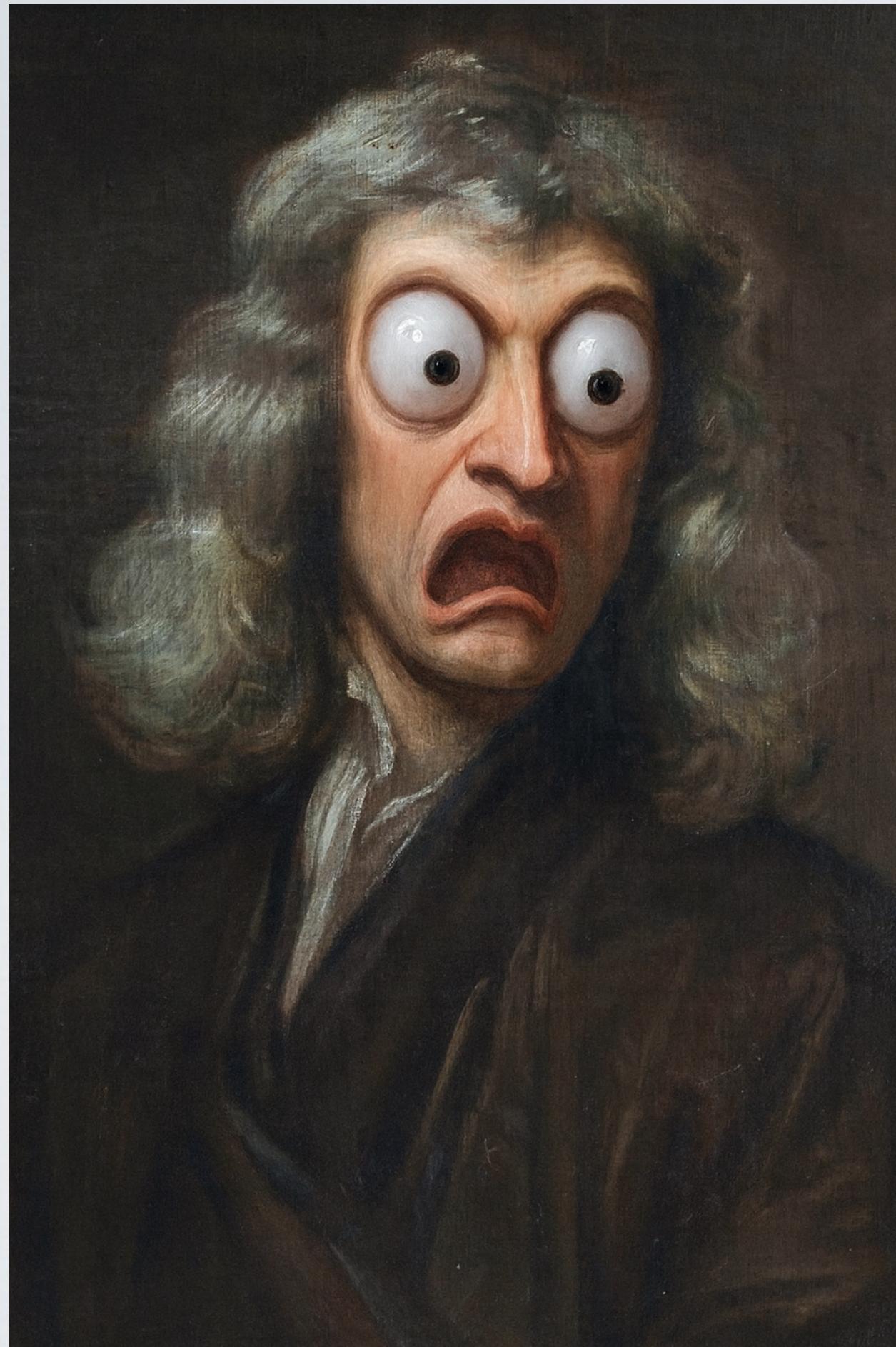


Albert Einstein
(1879-1955)

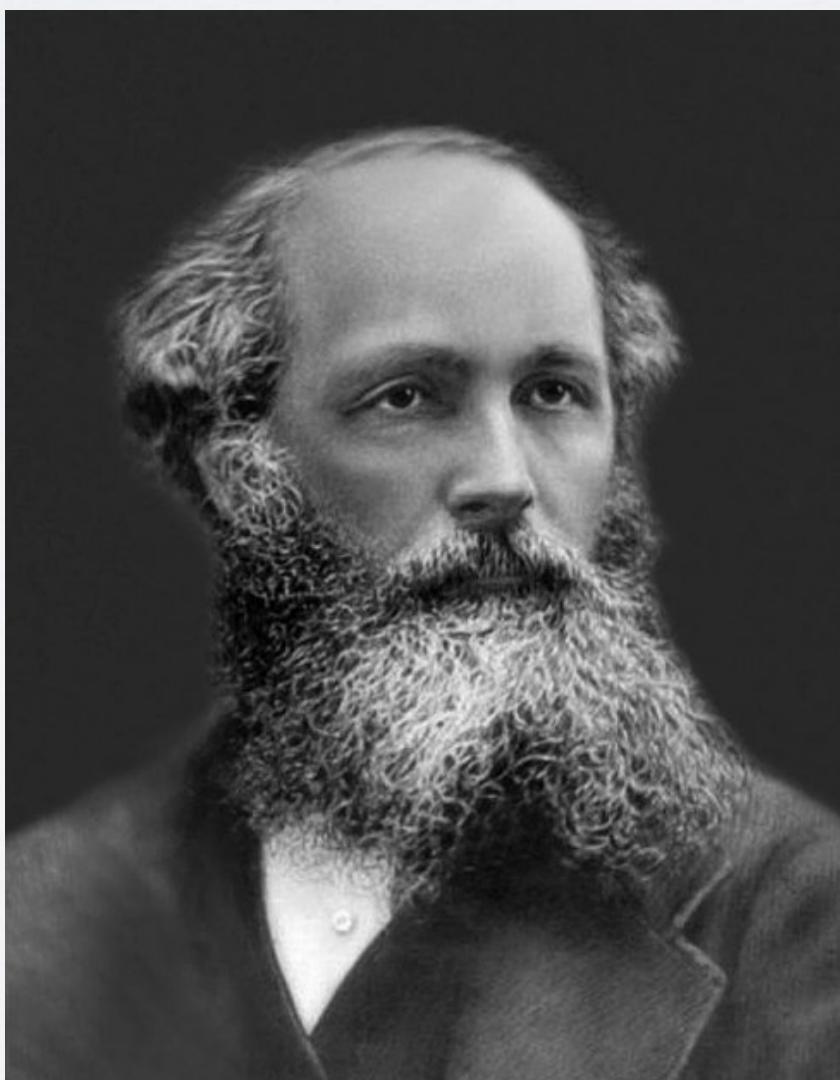
Hum... Galileo's principle of relativity is now in terms with electromagnetism. But **Newton's gravity is not...**
if only I could free fall inside an elevator, then I would remove gravity. But how can I get all these ideas together ???

(1907)

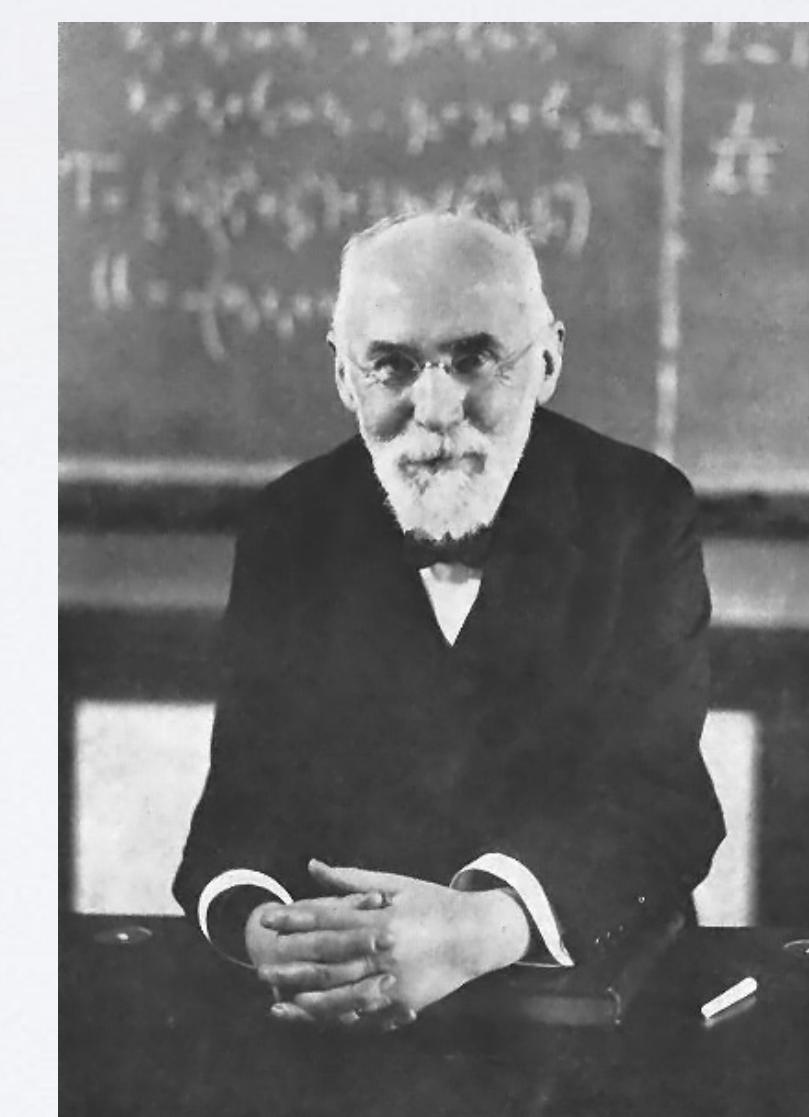
SPACE AND TIME VERSUS SPACETIME



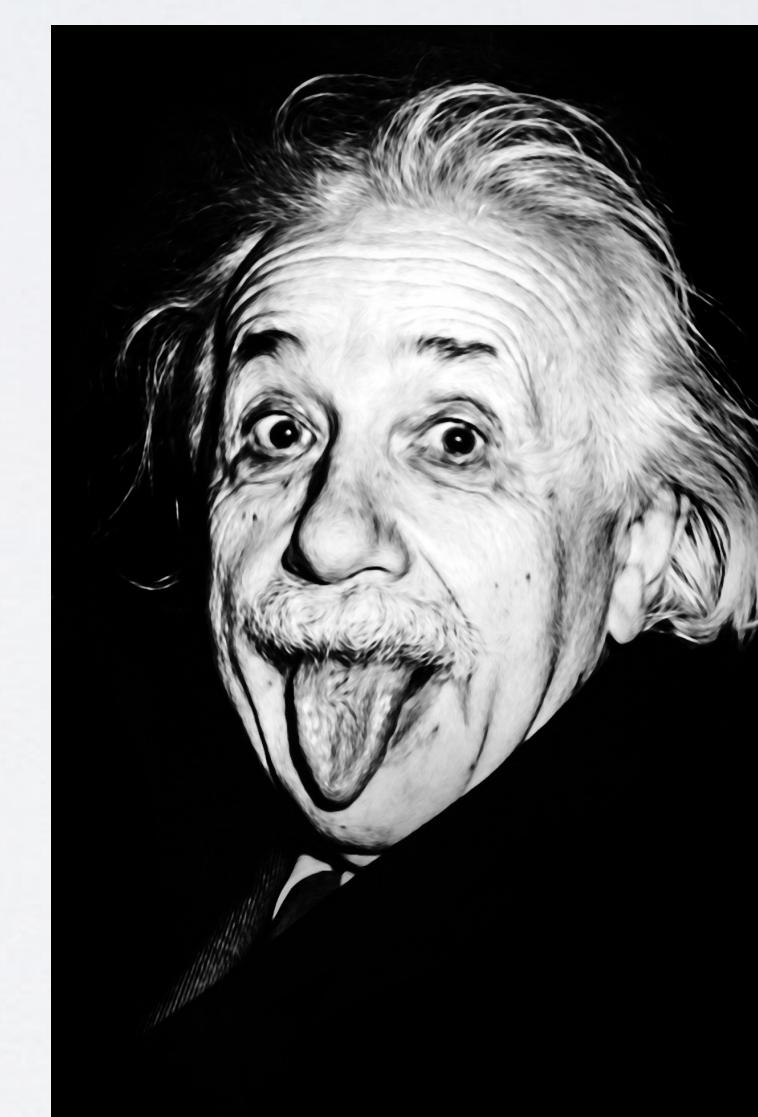
Isaac Newton (by G. Kneller)
(1643-1727)



James Clerk Maxwell
(1879-1995)



Hendrik Antoon Lorentz
(1853-1928)



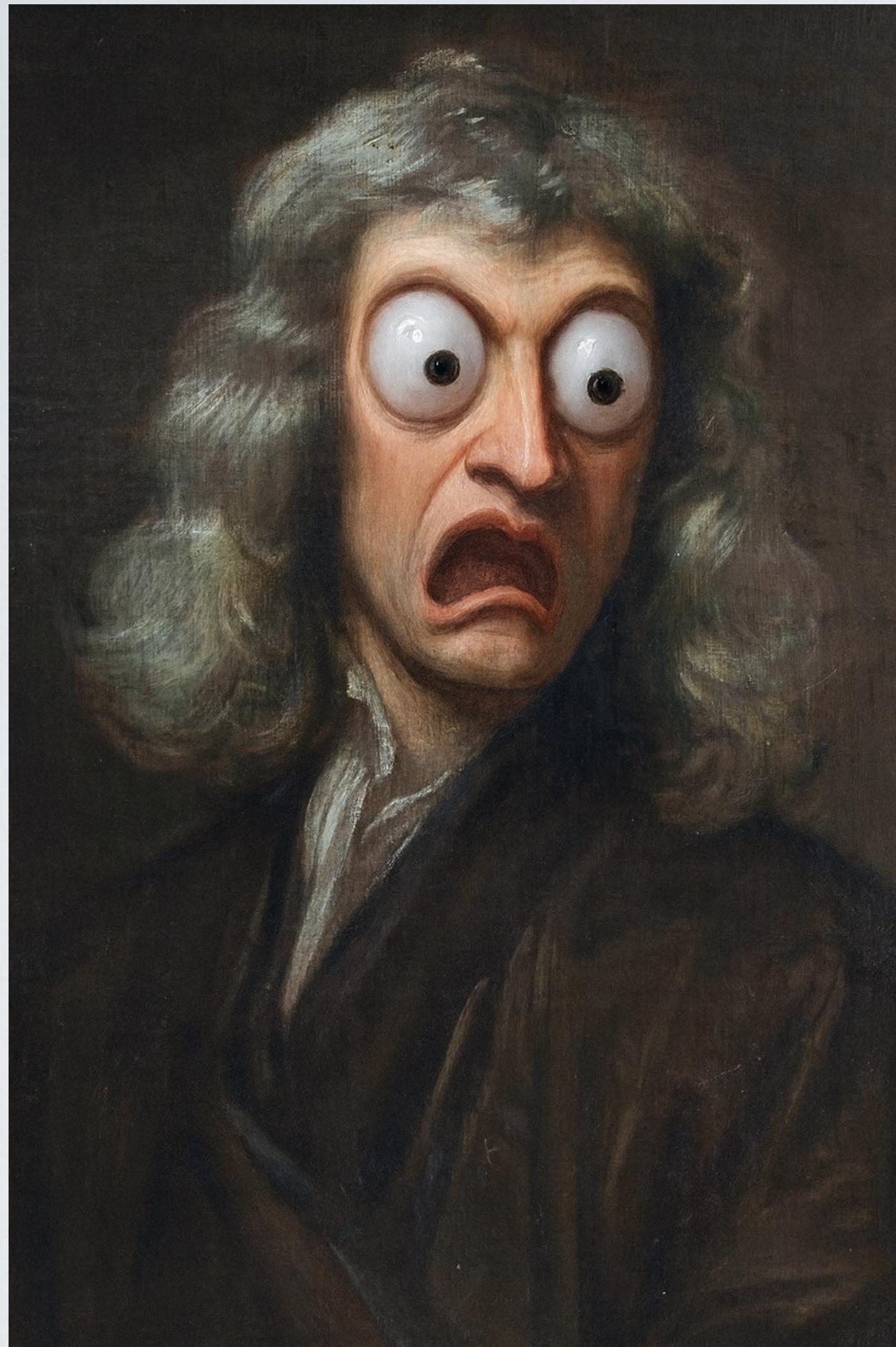
Albert Einstein
(1879-1955)



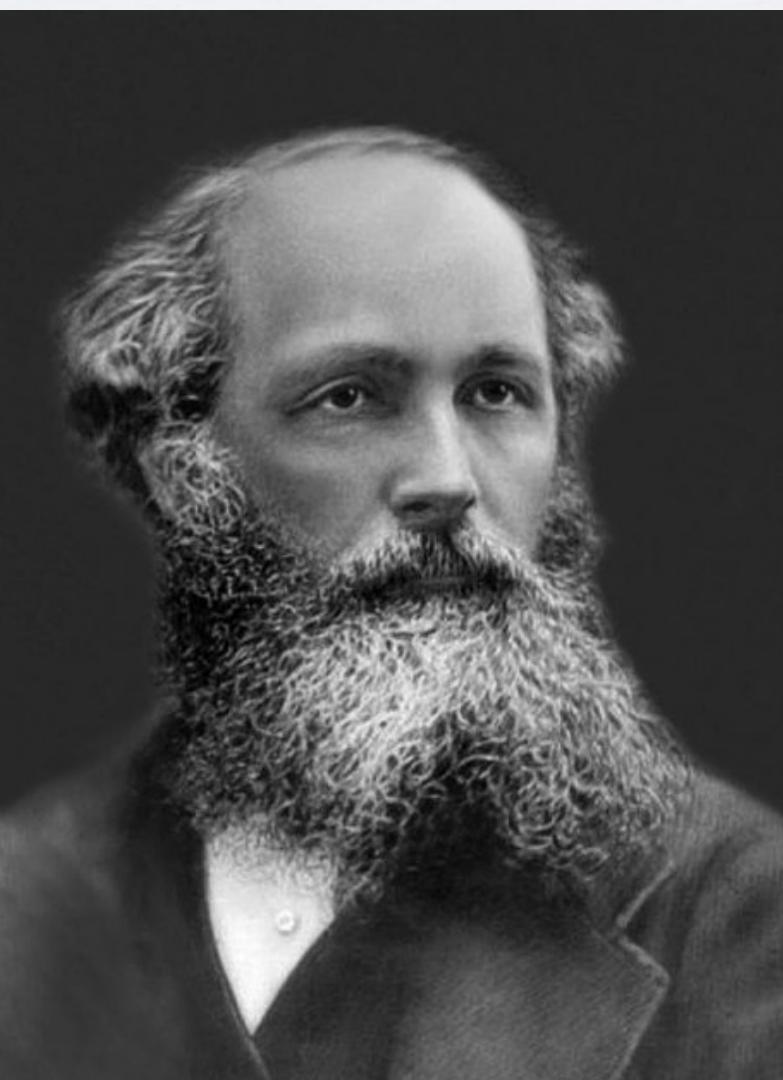
Herman Minkowski
(1864-1909)

“Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a **kind of union of the two will preserve an independent reality**” (1908)

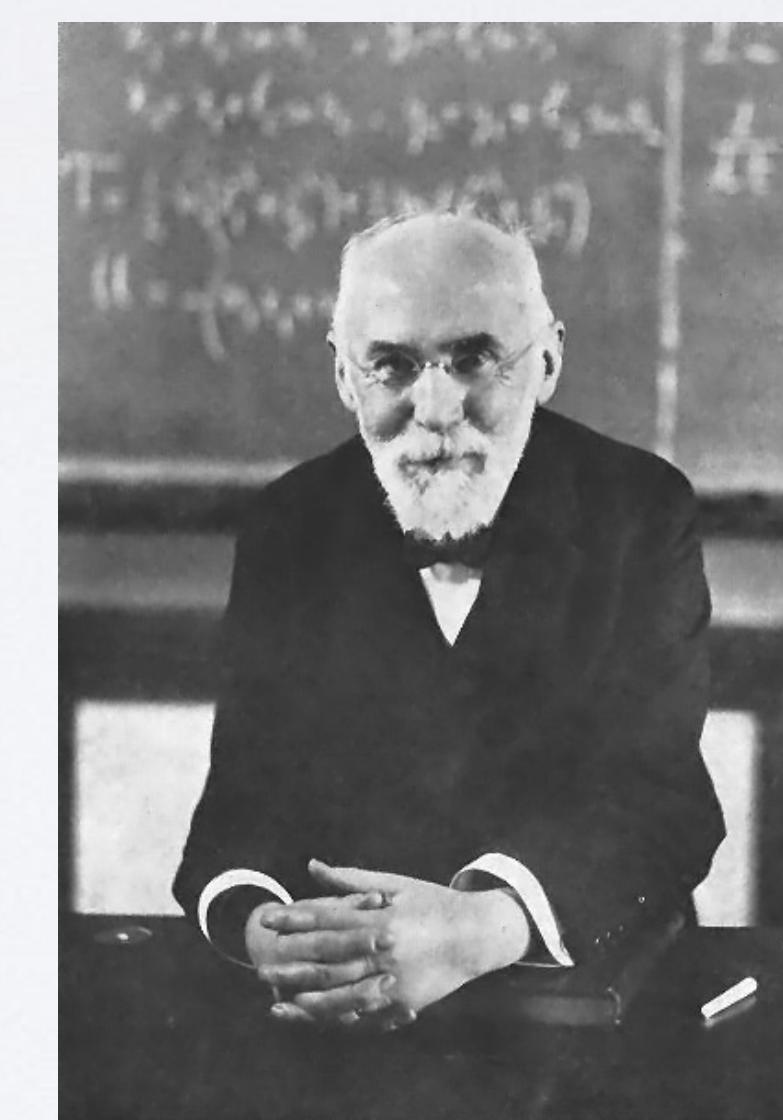
SPACE AND TIME VERSUS SPACETIME



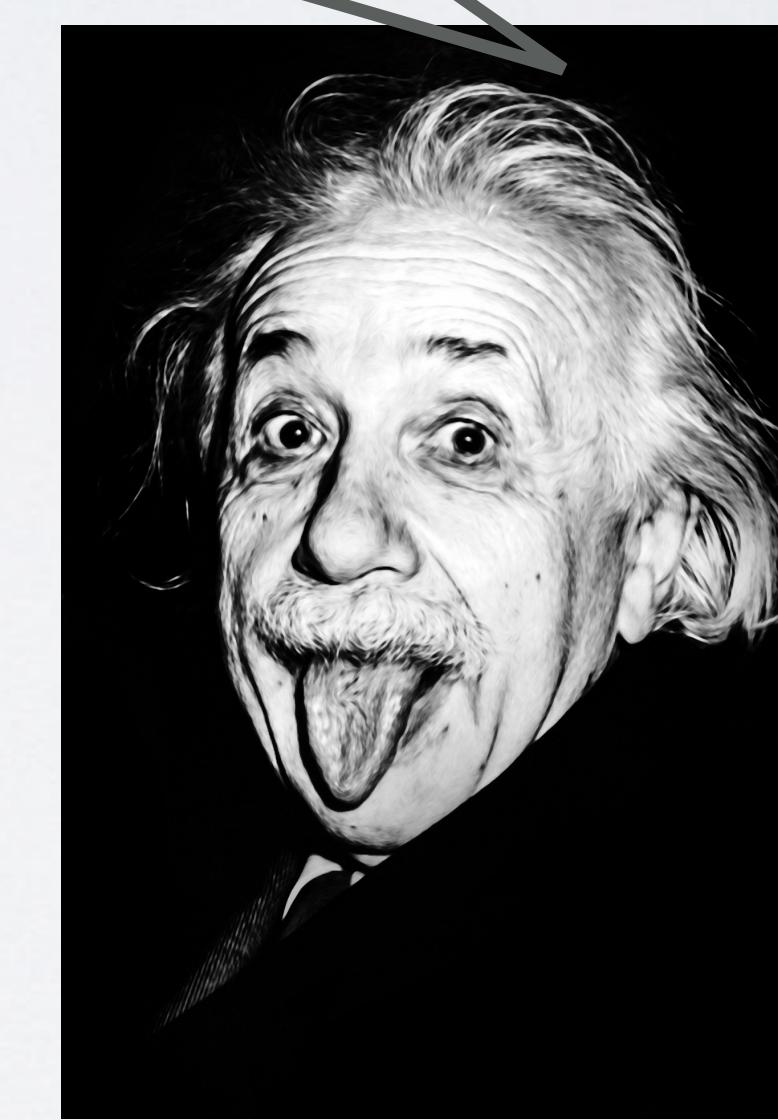
Isaac Newton (by G. Kneller)
(1643-1727)



James Clerk Maxwell
(1879-1995)



Hendrik Antoon Lorentz
(1853-1928)



Albert Einstein
(1879-1955)



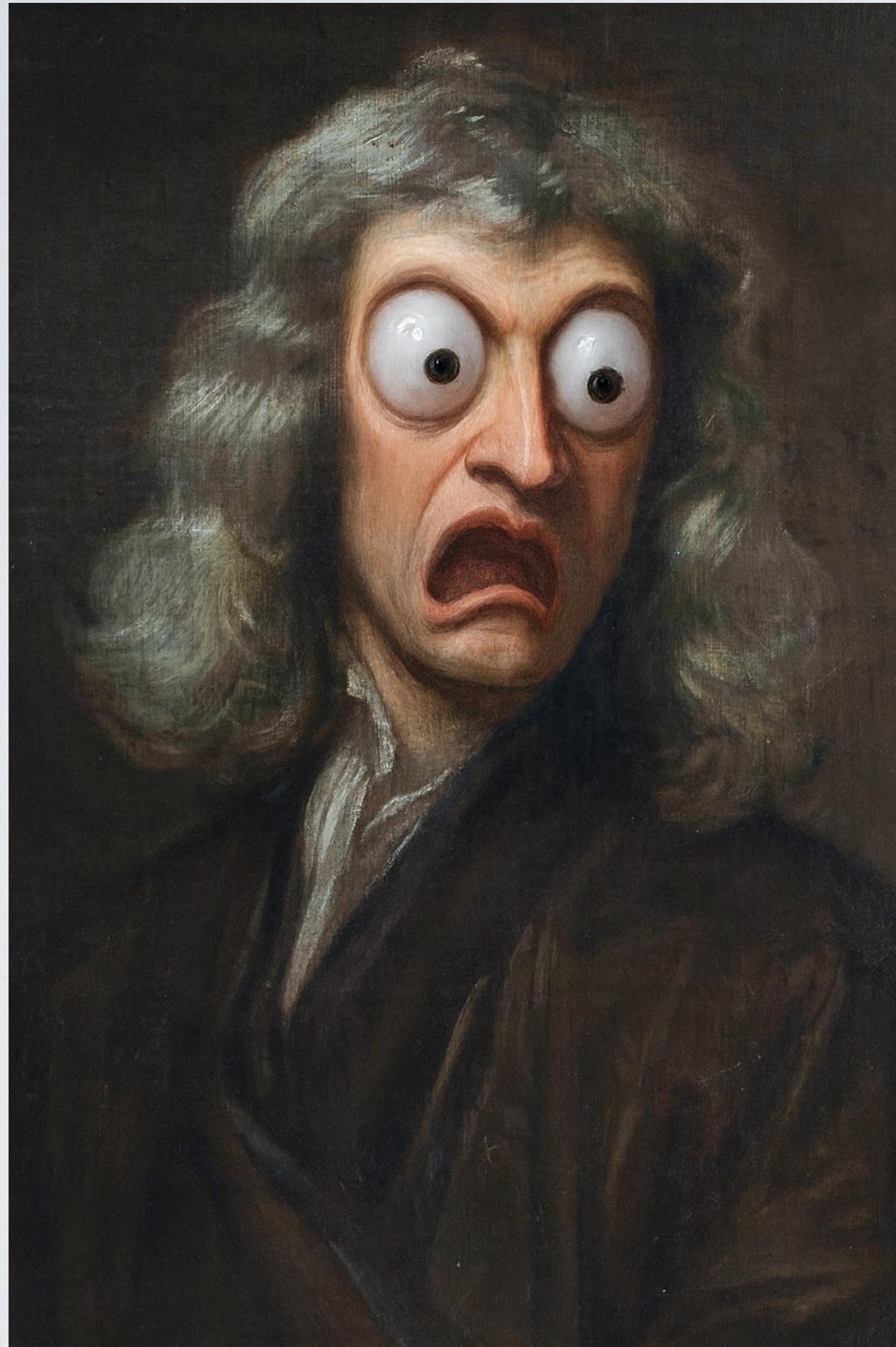
Herman Minkowski
(1864-1909)

That's amazing! Now I can use my intuition from 1907 connecting gravity with elevators with your spacetime formulation.

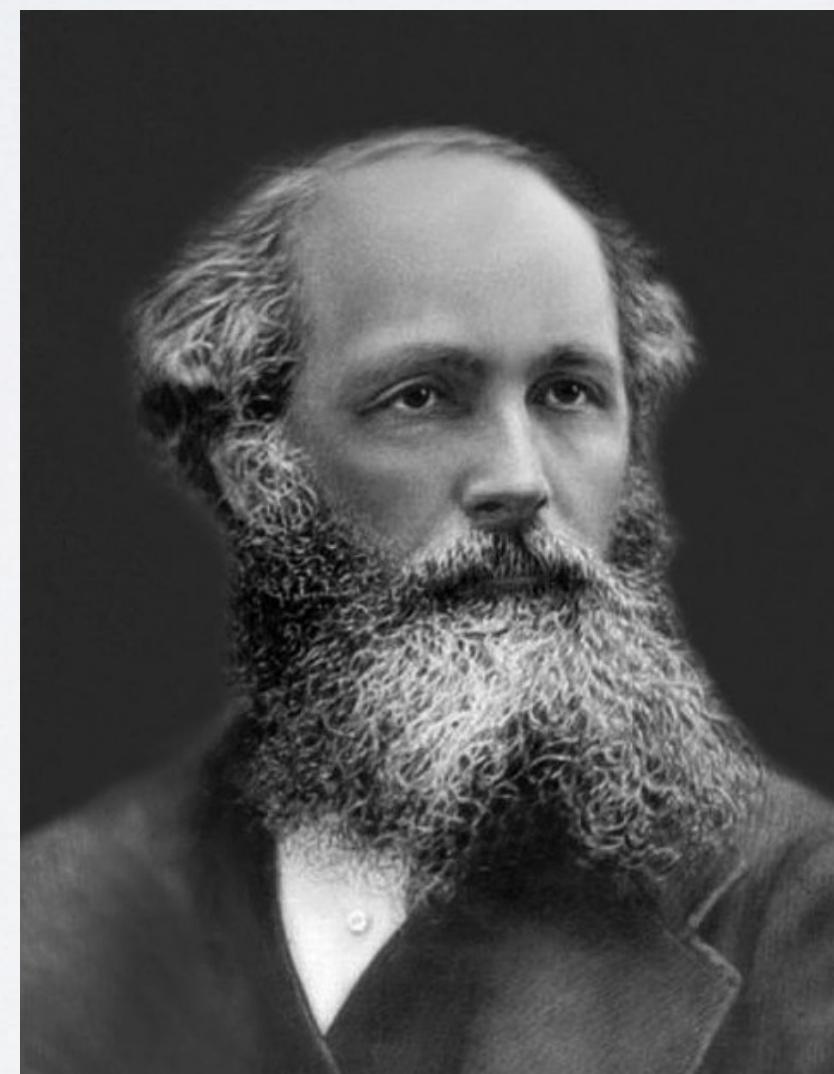
Gravity is curved spacetime

(1915)

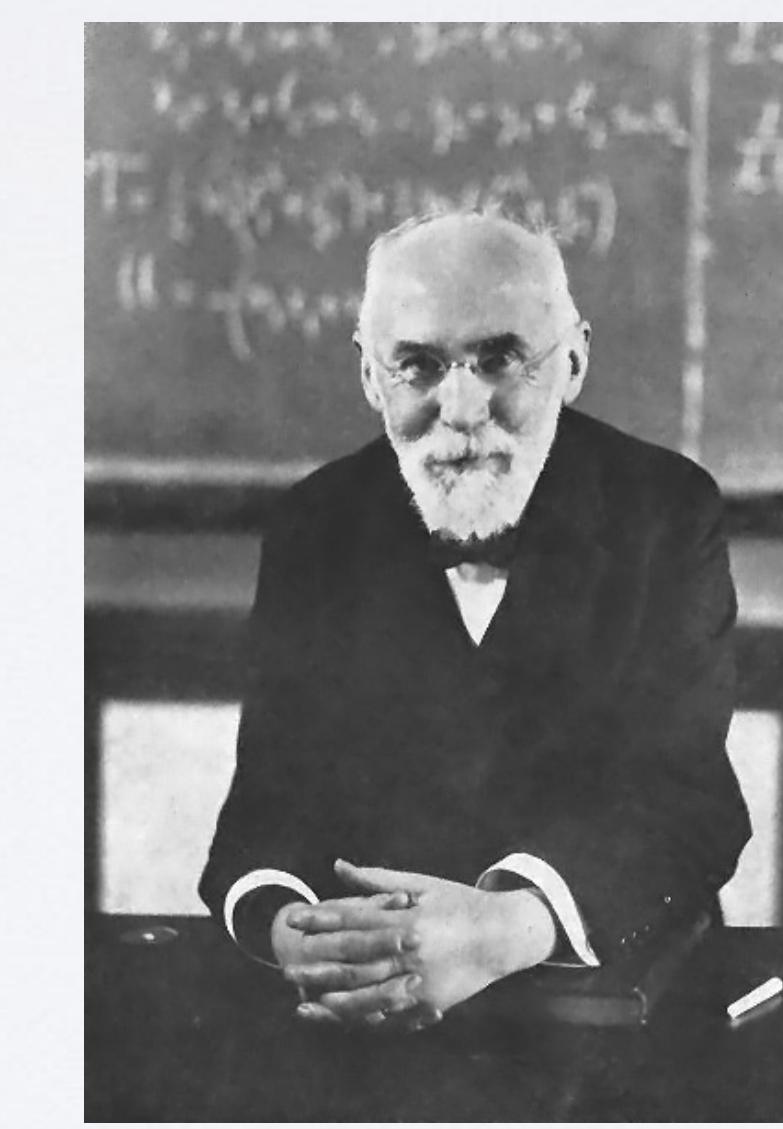
SPACE AND TIME VERSUS SPACETIME



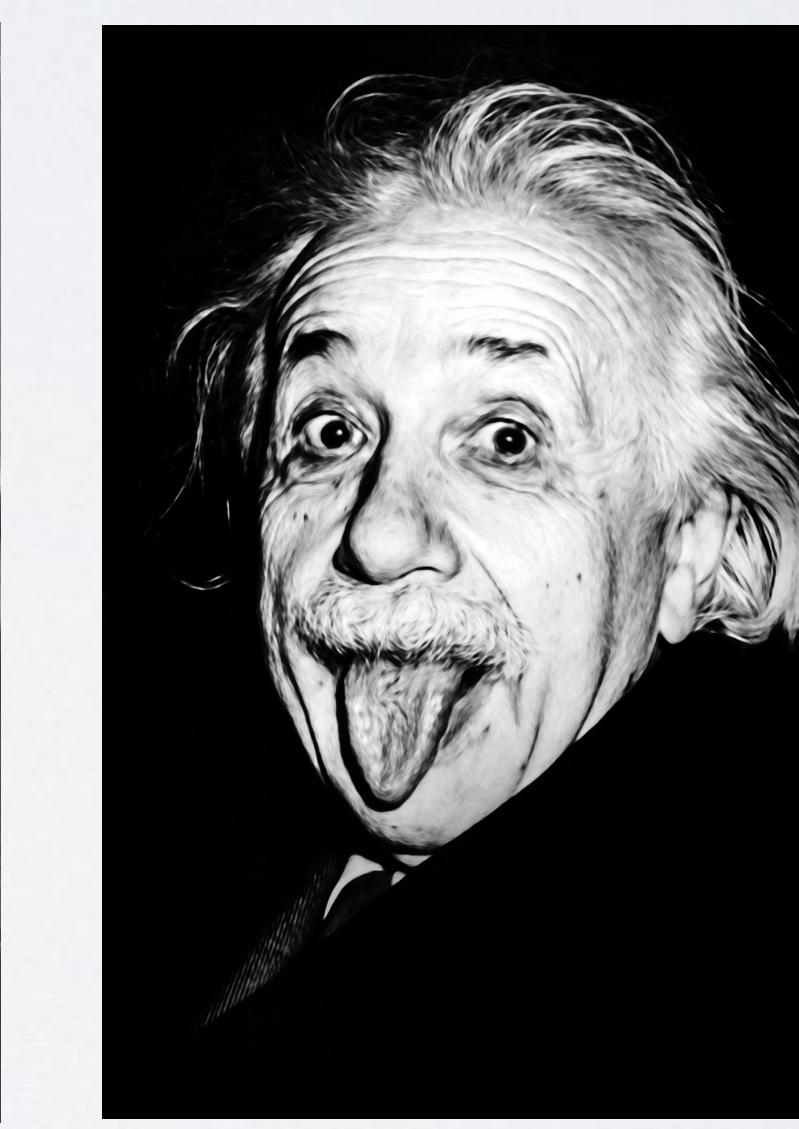
Isaac Newton (by G. Kneller)
(1643-1727)



James Clerk Maxwell
(1831-1879)



Hendrik Antoon Lorentz
(1853-1928)

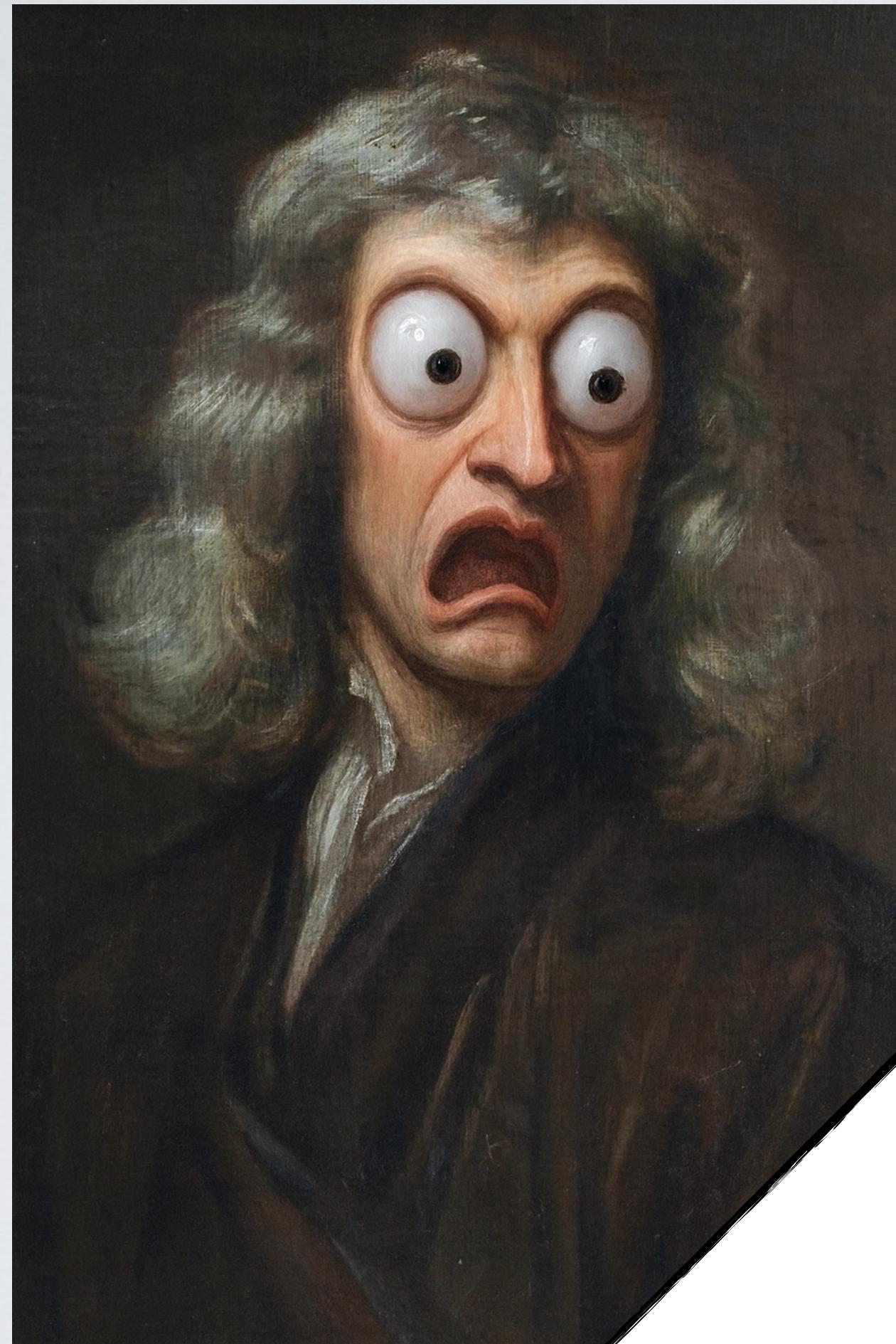


Albert Einstein
(1879-1955)



Herman Minkowski
(1864-1909)

SPACE AND TIME VERSUS spacetime

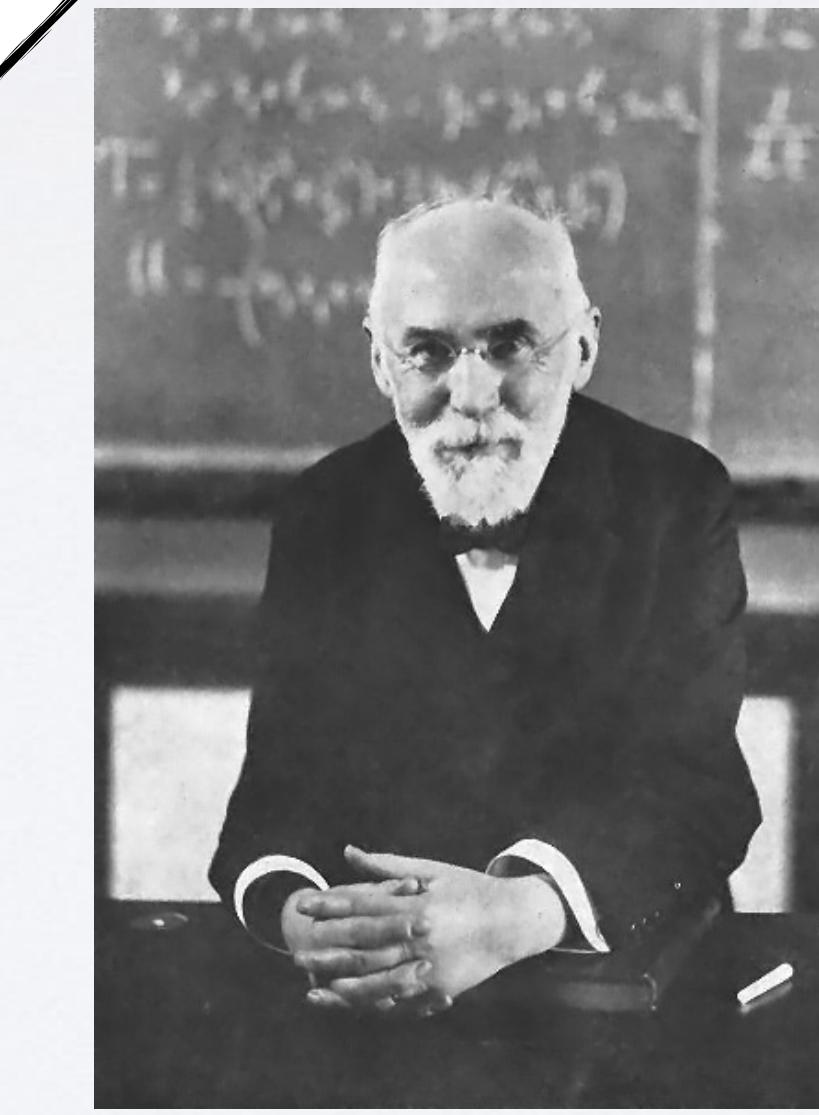


Isaac Newton (by C.
(1643-1727)

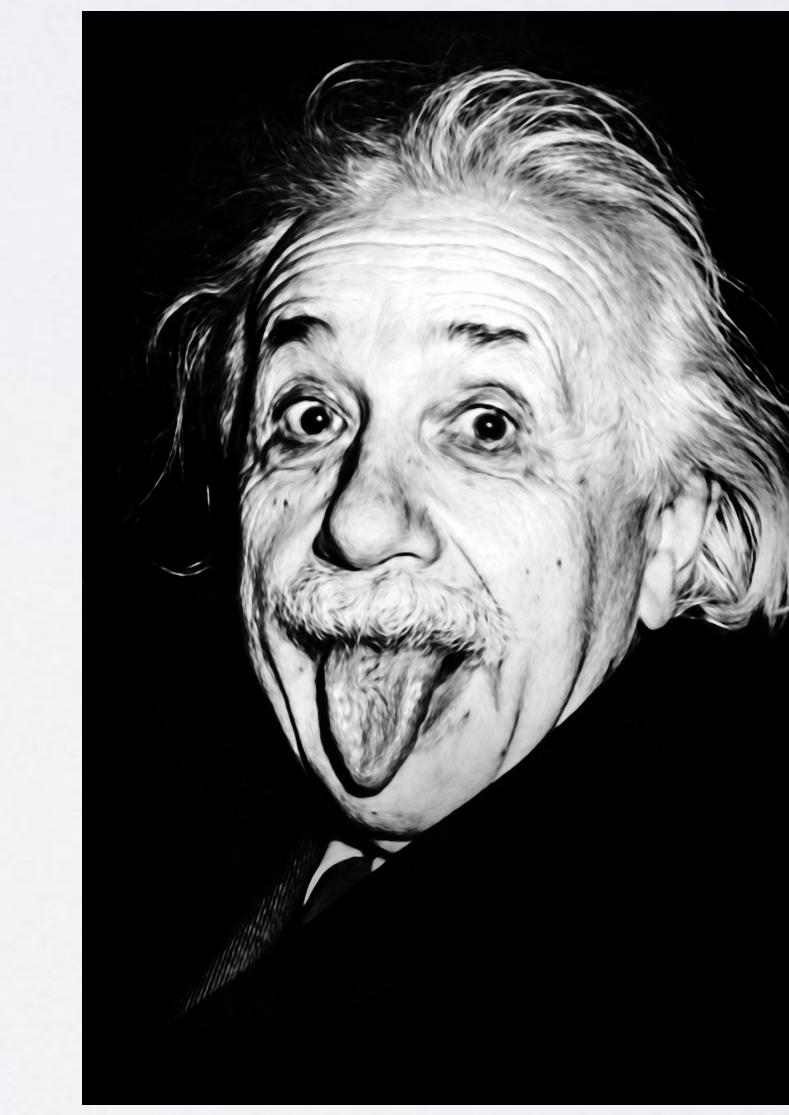
TO BE CONTINUED



James Clerk Maxwell
(1879-1995)



Hendrik Antoon Lorentz
(1853-1928)



Albert Einstein
(1879-1955)



Herman Minkowski
(1864-1909)