Problem Set - 4

SPRING 2020

MATHEMATICS-II (MA1002)(Numerical Analysis)

Q.1 Find the solution to the following equations using Gauss-Seidel method.

$$
\begin{aligned}
12x_1 + 3x_2 - 5x_3 &= 1 \\
x_1 + 5x_2 + 3x_3 &= 28 \\
3x_1 + 7x_2 + 13x_3 &= 76
\end{aligned}
$$

take initial guess $(x_1, x_2, x_3) = (0, 1, 0)$. And conduct six iterations.

Q.2 Use the Jacobi method and Gauss-Seidel method to approximate the solution of the following linear equations correct upto 3 decimal places.

$$
\begin{aligned}
5x_1 - 2x_2 + 3x_3 &= -1 \\
-3x_1 + 9x_2 + x_3 &= 2 \\
2x_1 - x_2 - 7x_3 &= 3
\end{aligned}
$$

take initial guess $(x_1, x_2, x_3) = (0, 0, 0)$.

Q.3 The equation

$$8x^3 - 12x^2 - 2x + 3 = 0$$

has three real roots, find the intervals each of unit length containing each one of these roots.

Q.4 Perform three iterations of the bisection method to obtain the smallest positive root of the equation

$$f(x) = x^3 - 5x + 1 = 0.$$

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Q.5 Perform three iteration of the bisection method to obtain a root of the equation

$$f(x) = cos(x) - xe^x = 0.$$

Q.6 Use Newton-Raphson Method to determine x_2 for the equation

$$f(x) = x^3 - 7x^2 + 8x - 3 = 0; \quad if \quad x_0 = 5.$$

Q.7 Consider the function

$$f(x) = x^5 - x^3 + 2x^2 - 1 = 0$$

Approximate the root near 1, correct upto 8 decimal places using the Newton-Raphson Method.

Q.8 Find the root of the equation $sin(x) = 10(x - 1)$ by using fixed point iteration correct up to three decimal places.

Q.9 Approximate the positive square root of a number N by Newton-Raphson Method.Hence find first and second approximate value of $\sqrt{2}$ using initial approximate value equal to 1.5.

Q.10 Find first and second approximate value of $\sqrt[3]{7}$ by taking initial approximation as 2 using Newton-Raphson method.

Q.11 Find a root of

$$f(x) = x^4 - x - 10 = 0$$

using fixed point iteration with initial guess 1.

Q.12 The equation

$$f(x) = 3x^3 + 4x^2 + 4x + 1 = 0$$

has a root in the interval $(-1, 0)$. Determine an iteration function $\phi(x)$, such that the sequence of iterations obtained from

$$x_{k+1} = \phi(x_k) \quad , \quad x_0 = 0.5, \quad k = 0, 1, ...$$

converges to the root.