Tutorial Sheet-5
Mathematics-II(MA10002)
(Numerical Analysis)

1. Prove that
$$\Delta \log f(x) = \log[1 + \frac{\Delta f(x)}{f(x)}],$$
   where Δ is the forward difference operator.

2. Show that
$$\Delta \binom{n}{x+1} = \binom{n}{x},$$
   where the forward difference operator Δ operates on n. Hence show that
$$\sum_{n=1}^{N} \binom{n}{i} = \binom{N+1}{i+1} - \binom{1}{i+1}.$$

3. For any positive integer k prove that
$$\nabla^k y_n = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{n-i},$$
   ∇ being the backward difference operator.

4. For linear interpolation, in case of equispaced tabular data, show that the error does not exceed $1/8^{\text{th}}$ of the second difference.

5. We have the following tabular values:

x	0	1	2	3
f(x)	1	0	7	28

   Find the Interpolating polynomial using

   i. Newton's Forward Interpolation Formula.
   ii. Newton's Backward Interpolation Formula.

6. From the following table find the number of students who obtained less than 45 marks :

Marks	No. of students
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

7. The population of a town in West Bengal was as given below :

$$
\begin{array}{lccccc}
\text{Year:} & 1891 & 1901 & 1911 & 1921 & 1931 \\
\text{Population (in thousands):} & 46 & 66 & 81 & 93 & 101
\end{array}
$$

Estimate the population for the year 1925.

8. Given that $f(1) = 2$, $f(2) = 4$, $f(3) = 8$, $f(4) = 16$, $f(7) = 128$. Find the value of $f(5)$ with the help of Lagrange's interpolation formula.

9. Using Lagrange's interpolation formula to express

$$
\frac{3x^2 + x + 1}{x^3 - 6x^2 + 11x - 6}
$$

as the sum of partial fraction.

10. By means of Lagrange's formula, prove that

$$
y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right].
$$

Note: $y_x = y(x)$.

11. Compute the value of following integral

$$
\int_{0.2}^{1.4} (\sin x - \ln x + e^x)\,\mathrm{d}x
$$

using trapezoidal rule with 6 equal subintervals.

12. A car laps a racetrack in 84 s. The speed of the car at each 6-s interval is determined using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table:

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?
Note: use trapezoidal formula

13. How big should the spacing h be so that the computation of

$$
\int_0^1 e^x \, \mathrm{d}x
$$

by trapezoidal rule will be correct to five decimal places.

14. Let

$$
y = ax^2 + bx + c
$$

be the equation of the parabola passing through the point $(-h, y_0), (0, y_1), (h, y_2)$. Find the area underlying the parabola bounded by the x-axis and the two ordinates $x = -h, x = h$ using Simpson's $1/3^{\mathrm{rd}}$ rule. What conclusion do you draw from the result.

15. Calculate the value of the integral

$$
\int_4^{5.2} \ln x \, \mathrm{d}x
$$

using Simpson's $1/3^{\mathrm{rd}}$ rule with 6 equal subdivisions of the interval.