

Answer sheet - 2

SPRING 2020

MATHEMATICS-II (MA10002)(Linear Algebra)

1. Ans: (b) and (c) form a basis.

2. (a) Ans: Basis :

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

$\dim V = 3$.

(b) Ans: Basis : $\{(1, 0, 1, -2), (0, 1, 2, -1)\}$, $\dim U=2$.

(c) Ans: Basis : $\{x, x^2 - \frac{1}{3}, x^3, x^4 - \frac{1}{5}\}$, $\dim U=4$.

3. Ans: $\dim U = 2$, $\dim W = 2$, $\dim U + W = 3$ and $\dim (U \cap W) = 1$.

4. (a) Ans: No.

(b) Ans: Yes.

5. Ans: $\phi(z) = \operatorname{Im}(z)$. Where $z \in \mathbb{C}$.

6. (a) $N(T)=L\{0\}$, $\dim N(T)=0$.

$R(T)=L\{3x, 2 + \frac{3}{2}x^2, 4x + x^3\}$, $\dim R(T)=3$.

(b) $N(T)=L\{(1, 1, 0)\}$, $\dim N(T)=1$.

$R(T)=L\{(\frac{1}{2}, 0), (-\frac{1}{2}, \frac{1}{2})\}$, $\dim R(T)=2$.

(c) $N(T)=$ set of all 2×2 symmetric matrices. $\dim N(T)= 3$.

$R(T)=$ set of all 2×2 skew symmetric matrix. $\dim R(T)= 1$.

7. (a) Ans: $T(x,y)=(2x-y,x-y,2x)$

(b) Ans: $T(x,y,z)=(x+2y+3z, x+3y+2z)$

(i) $T(1,1,0)=(3,4)$, $T(6,0,-1)=(3,4)$

(ii) $\operatorname{Ker}T=L\{(-5,1,1)\}$, $\operatorname{Im}(T)=L\{(1,1),(2,3),(3,2)\}$

8. (a)
$$\begin{bmatrix} 0 & 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0 & 72 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0 & 0 & 4 & 0 \\ 1 & 3 & 9 & 27 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 12 \end{bmatrix}$$

9.

10. (a) Ans: $(-\frac{1}{5}, 4, -\frac{4}{5})$

(b) Ans: $(-2c+10, c, -5, 2)$

11. (a) Ans: Rank of A = 2.

(b) Ans: Rank of A = 2.

12. (a) 2
(b) 3

13. Ans: $-\frac{1}{2}, 1, 1$

14. (a) Invertible and $A^{-1} = \begin{bmatrix} 8 & -\frac{1}{2} & -2 \\ -1 & \frac{1}{2} & 0 \\ -3 & 0 & 1 \end{bmatrix}$
(b) Not invertible.

15. $k = -4$

16. Ans : $(1-3k, -k, 5k)$, k is an integer.

17. Ans: Not possible

18. Ans: (a) $a \neq 8$
(b) $a=8, b \neq -1, 3$
(c) $a=8, b=3$ or $a=8, b=-1$