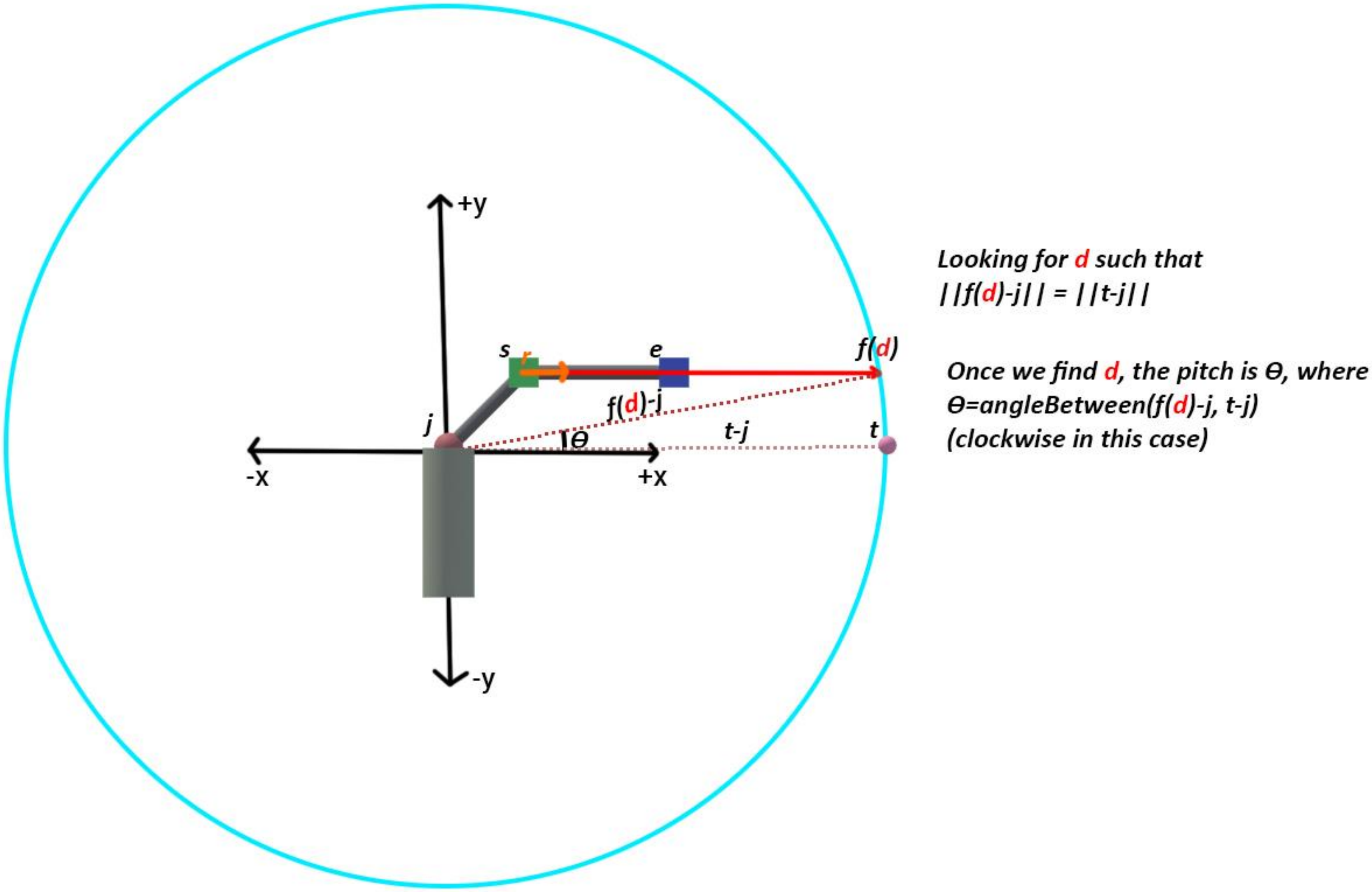
Calculating Pitch



For this document, 2D vectors (FVector2D) are bolded (**v**), while scalars (floats) are not bolded (x).

$\boldsymbol{j} = AimJoint$

$\boldsymbol{s} = BarrelStart$

$\boldsymbol{e} = BarrelEnd$

$\boldsymbol{r} = BarrelRay = (\boldsymbol{e} - \boldsymbol{s}).GetSafeNormal(\,)$

$\boldsymbol{t} = Target$

$d = BarrelRay\ distance\ we\ are\ trying\ to\ find.$

$\boldsymbol{f}(d) = A\ point\ at\ distance\ \text{"d"}\ along\ the\ Barrel\ Ray.$

$\quad = \boldsymbol{s} + \boldsymbol{r}d$

$\|\boldsymbol{v}\| = Calculates\ the\ magnitude\ (distance)\ of\ the\ 2D\ vector\ \boldsymbol{v}.$

$\quad = \boldsymbol{v}.Size(\,)$

$\quad = \sqrt{\boldsymbol{v}_x^2 + \boldsymbol{v}_y^2}$

We are trying to find the distance "d" along the BarrelRay (**r**) that we need to travel until we find a point that is as far from the AimJoint (**j**) as the target (**t**).

That is to say we are trying to find a value for "d" that solves this equation:

$\|\boldsymbol{f}(d)-\boldsymbol{j}\| = \|\boldsymbol{t}-\boldsymbol{j}\|$

$$\sqrt{(\boldsymbol{f}(d)_x-\boldsymbol{j}_x)^2+(\boldsymbol{f}(d)_y-\boldsymbol{j}_y)^2} = \sqrt{(\boldsymbol{t}_x-\boldsymbol{j}_x)^2+(\boldsymbol{t}_y-\boldsymbol{j}_y)^2}$$

Expanding the magnitude functions…

$$(\boldsymbol{f}(d)_x-\boldsymbol{j}_x)^2+(\boldsymbol{f}(d)_y-\boldsymbol{j}_y)^2 = (\boldsymbol{t}_x-\boldsymbol{j}_x)^2+(\boldsymbol{t}_y-\boldsymbol{j}_y)^2$$

Squaring both sides…

$$\boldsymbol{f}(d)_x = \boldsymbol{s}_x + \boldsymbol{r}_x d$$
$$\boldsymbol{f}(d)_y = \boldsymbol{s}_y + \boldsymbol{r}_y d$$

Recall that binomials are factored like so:

$$(a-b)^2 = (a-b)(a-b) = a^2-2ab+b^2$$
$$(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$$

$$(\boldsymbol{f}(d)_x)^2 = (\boldsymbol{s}_x+\boldsymbol{r}_x d)^2 = \boldsymbol{s}_x^2+2\boldsymbol{s}_x\boldsymbol{r}_x d+\boldsymbol{r}_x^2 d^2$$

$$\begin{aligned}(\boldsymbol{f}(d)_x-\boldsymbol{j}_x)^2 &= \boldsymbol{f}(d)_x^2-2\boldsymbol{f}(d)_x\boldsymbol{j}_x+\boldsymbol{j}_x^2\\ &= (\boldsymbol{s}_x^2+2\boldsymbol{s}_x\boldsymbol{r}_x d+\boldsymbol{r}_x^2 d^2)-2(\boldsymbol{s}_x+\boldsymbol{r}_x d)\boldsymbol{j}_x+\boldsymbol{j}_x^2\\ &= (\boldsymbol{s}_x^2+2\boldsymbol{s}_x\boldsymbol{r}_x d+\boldsymbol{r}_x^2 d^2)+(-2\boldsymbol{j}_x\boldsymbol{s}_x-2\boldsymbol{j}_x\boldsymbol{r}_x d)+\boldsymbol{j}_x^2\\ &= (\boldsymbol{j}_x^2-2\boldsymbol{j}_x\boldsymbol{s}_x+\boldsymbol{s}_x^2)+(2\boldsymbol{s}_x\boldsymbol{r}_x-2\boldsymbol{j}_x\boldsymbol{r}_x)d+\boldsymbol{r}_x^2 d^2\\ &= (\boldsymbol{j}_x-\boldsymbol{s}_x)^2+(2\boldsymbol{s}_x\boldsymbol{r}_x-2\boldsymbol{j}_x\boldsymbol{r}_x)d+\boldsymbol{r}_x^2 d^2\end{aligned}$$

$$\left(\boldsymbol{f}(d)_y-\boldsymbol{j}_y\right)^2 = \left(\boldsymbol{j}_y-\boldsymbol{s}_y\right)^2+\left(2\boldsymbol{s}_y\boldsymbol{r}_y-2\boldsymbol{j}_y\boldsymbol{r}_y\right)d+\boldsymbol{r}_y^2 d^2$$

Recall:

$$(\boldsymbol{f}(d)_x-\boldsymbol{j}_x)^2+(\boldsymbol{f}(d)_y-\boldsymbol{j}_y)^2 = (\boldsymbol{t}_x-\boldsymbol{j}_x)^2+(\boldsymbol{t}_y-\boldsymbol{j}_y)^2$$

Substituting the colored sections…

$$(\boldsymbol{j}_x-\boldsymbol{s}_x)^2+(2\boldsymbol{s}_x\boldsymbol{r}_x-2\boldsymbol{j}_x\boldsymbol{r}_x)d+\boldsymbol{r}_x^2 d^2+\left(\boldsymbol{j}_y-\boldsymbol{s}_y\right)^2+\left(2\boldsymbol{s}_y\boldsymbol{r}_y-2\boldsymbol{j}_y\boldsymbol{r}_y\right)d+\boldsymbol{r}_y^2 d^2 = (\boldsymbol{t}_x-\boldsymbol{j}_x)^2+\left(\boldsymbol{t}_y-\boldsymbol{j}_y\right)^2$$

Rearranging…

$$\left(\boldsymbol{r}_x^2+\boldsymbol{r}_y^2\right)d^2+\left((2\boldsymbol{s}_x\boldsymbol{r}_x-2\boldsymbol{j}_x\boldsymbol{r}_x)+\left(2\boldsymbol{s}_y\boldsymbol{r}_y-2\boldsymbol{j}_y\boldsymbol{r}_y\right)\right)d+\left((\boldsymbol{j}_x-\boldsymbol{s}_x)^2+\left(\boldsymbol{j}_y-\boldsymbol{s}_y\right)^2-(\boldsymbol{t}_x-\boldsymbol{j}_x)^2-\left(\boldsymbol{t}_y-\boldsymbol{j}_y\right)^2\right) = 0$$

This is a quadratic equation in the form:

$$ax^2+bx+c=0$$

Where:

$$a = \left(\boldsymbol{r}_x^2+\boldsymbol{r}_y^2\right)$$
$$b = \left((2\boldsymbol{s}_x\boldsymbol{r}_x-2\boldsymbol{j}_x\boldsymbol{r}_x)+\left(2\boldsymbol{s}_y\boldsymbol{r}_y-2\boldsymbol{j}_y\boldsymbol{r}_y\right)\right)$$
$$c = \left((\boldsymbol{j}_x-\boldsymbol{s}_x)^2+\left(\boldsymbol{j}_y-\boldsymbol{s}_y\right)^2-(\boldsymbol{t}_x-\boldsymbol{j}_x)^2-\left(\boldsymbol{t}_y-\boldsymbol{j}_y\right)^2\right)$$