Stability of Planetary Orbits in Binary Star Systems: S-type orbits

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Abstract

The classical three body problem is an interesting problem to solve as it often leads to chaotic trajectories especially in the case of three body encounter. But to find conditions for stable orbits can be a difficult job. Many exoplanets are discovered in binary star systems. The habitability of these planets is determined by their dynamics. Here, I take a very simple approach to the problem, with simplistic assumptions such as the bodies being confined to a plane, and the planet having very less mass, to find some stable trajectories or orbits for these kinds of planets, by numerical solving. Later the assumptions can be relaxed one by one for a more general scenario.

1 Introduction

Many exoplanets are discovered in binary star systems. The characteristics of these planets, such as temperature, hence, availability of liquid water, and hence possibility of life or habitability for living organisms, depend on their dynamics. If they orbits are unstable, and chaotic, it is unpredictable where the planet would be at what time. It might be too near or too far, for very short or very long, from the stars causing unpredictable fluctuations in temperature and radiation exposure.

Types of orbits possible in binary star systems are planet-type (P-type) external orbits where in the planet orbits around both stars in the binary, satellite-type (S-type) internal orbits where in the planet goes around one of the two stars, libration-type (L-type) orbits around the Lagrangian equillibrium points L_4 and L_5, which are stable for stellar mass ratio $\frac{m_1}{m_1+m_2}$ and less than ≈ 0.04.

For a single star and planet system, the motion is confined to a plane. The planet moves in an elliptical orbit around the star, eccentricity and semi-major axis of the orbit depending on the energy and angular momentum values. For a 3 body system made up by two stars, primary star A and secondary star B, in elliptical motion in the center of mass frame of the two stars, we can think of the secondary star as an addition to the primary star and planet system. If the secondary star is faraway compared to the primary star, the force from it on the planet would be a minor peturbation to the force from star A and it would still move in an elliptical orbit in the frame of star A. Thus, distance of the planet from the primary star plays a crucial role on stability. The farther the planet from the primary star, the stronger the perturbation due to star B, so the orbit can eventually become unstable.

The unperturbed circular orbit is characterised by the initial transverse speed of the planet-

$$v_P = \sqrt{G\frac{m_A + m_P}{r}} \tag{1}$$

where r is the initial distance of the planet from the primary star, m_P is the mass of the planet, and m_A is the mass of the primary star. The mass ratio and eccentricity of the binary system and the initial semi-major axis of he planet orbit can be chosen to be any suitable value. For equal mass stars, $\mu = 0.5$. The smaller the μ the lower the gravitational perturbation due to the secondary star.

The stability of such an orbit is given by the average deviation $\Delta r =< r(t) - r_i >$ from the circular orbit for a given initial radius r_i. For less than 5% deviation, the orbit is stable. For more than 50% deviation, the orbit is unstable. If the orbit becomes unstable is, it might collide with one of the two stars or it escape the system. In case of a stable orbit, it will keep orbiting in a ring of some width around the unperturbed orbit.The goal is to find, for some S-type orbit of a planet in a double star system, till what critical semi-major axis a_c will the planet have a stable S-type orbit.

2 Methodology

The most simplistic approach to the problem would be a model where all the bodies are point particles, all the bodies are constrained to a plane, the planet has a negligible mass (test particle) and does not affect the orbit of the stars, and the planet starts off with a nearly circular orbit around the Star A, orbit has some distortion due to perturbation from Star B. The equations involved are Newton's equations for gravitational force on a point mass from another point mass.

$$\frac{d^2 x_{ik}}{dt^2} = \sum_{i \neq j} \frac{G m_j (x_{jk} - x_{ik})}{r_{ij}^3} \tag{2}$$

where indices i and j represent the body, index k represents the component of position or velocity. i takes 3 values, k also takes 3 values. So, there are 9 second order ordinary differential equations. These can be written as two first order differential equations each. So, there are 18 equations total.

For the binary stars, the simplest case would be equal mass and circular orbit. The transverse speed of the stars, having mass ratio 0.5, is given by

$$v_{star} = \sqrt{\frac{Gm}{4r}} \tag{3}$$

where m is mass of the star, r the radius of the orbit. Equations (1) and (3) are used to give initial conditions for solving equation (2). The initial positions are set as the Earth - Sun system for the planet and the primary star A that is, at a separation of 1 AU, while the secondary star is kept at a distance ten times the planet and star A separation. The mass of the stars is set to 1 solar mass each, the mass of the planet is set to mass of earth.

The RK4 algorithm is used to solve the coupled differential equations. First the single body orbit was simulated and compared with the results of the inbuilt scipy ODE-solver *solve_ivp*, which uses by default RK45 algorithm. Then the binary system orbit was simulated and compared. The three body orbits were simulated using RK4 only due to ease of the computation. The simulations were done for various planet-starA separations ranging from 1 AU to 2 AU and from 4 AU to 5 AU. The simulations are run till 6 time-periods of the binary star orbits. The trajectories are plotted first in 3D, then 2D. The plot of the planet orbit in the coordinate system fixed to star A are also plotted. From the trajectories, the average relative deviation from circular orbit $\Delta r/r_i$ can evaluated (I have not done it here). The stable or unstable nature of the trajectories is also apparent from the graphs.

3 Results

The trajectory of the planet in the frame of star A is a circular orbit for 1 AU radius of unperturbed orbit. For 2 AU, the orbit is still nearly circular but does not close and instead stays confined within a very thin ring around the unperturbed orbit. For gradually increasing radius, the orbits become confined in side thicker and thicker rings. After 4.6 AU, the orbit is no longer confined inside a ring. The planet trajectory is unstable, it escapes the binary star system as the trajectory becomes chaotic. In the binary star COM frame, the stable orbits look like cycloids superimposed on the star A circular orbit.

The plots are shown at the end of this report. The x and y axis are in meters. The time duration that has been simulated is 6 times the time period of the binary star orbits.

4 Conclusion

The orbit seems stable till about 4.6 AU after which it stops exhibiting a nearly circular orbit or bound orbit within a certain radius range. After this the planet moves away from the star A and is ejected from the system. An estimate of $< r(t) - r_i >$ for long enough time periods can give information about stability of the orbits. It can be evaluated by using the trajectory generated for the planet.
The trajectory of the star B (not plotted here) can also be plotted and effect on its orbit due to the planet can be observed. The mass ratio of the binary and the mass of the planets can be changed to observe effects. One can also estimate the time in which the planet starts to show unstable behaviour. One can consider orbits of general eccentricity both for the stars

and the planet. One can also study what happens if the orbit of the planet and the binary orbits do not lie in a plane.

5 Acknowledgement

I would like to acknowledge Prayush, who gave me the initial idea to work on the three body problem. I would like to express my utmost gratitude to Ankur, Koustav, Pradeeptha, Uddeepta, Manish and Swapan for always being available for discussions and being by my side through all the problems and errors through the process.

6 References

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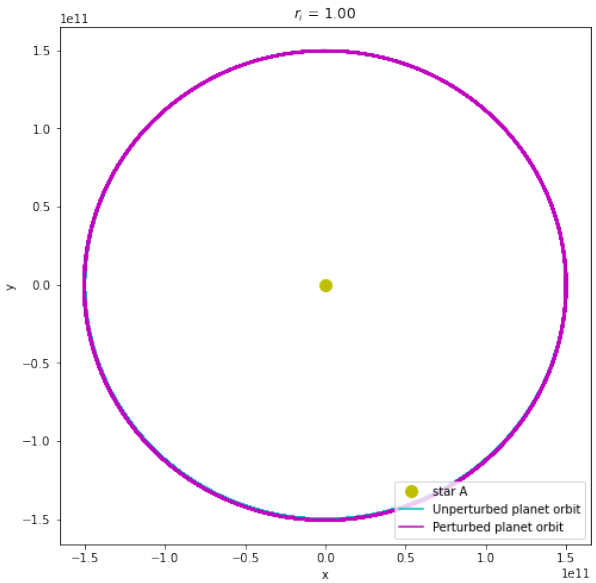
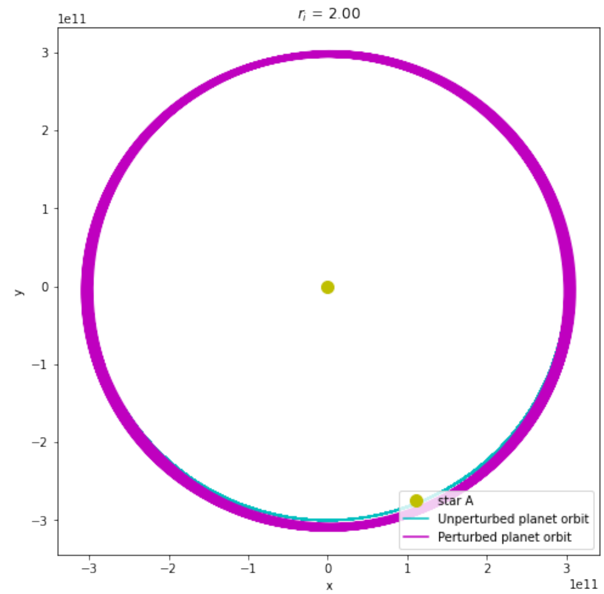
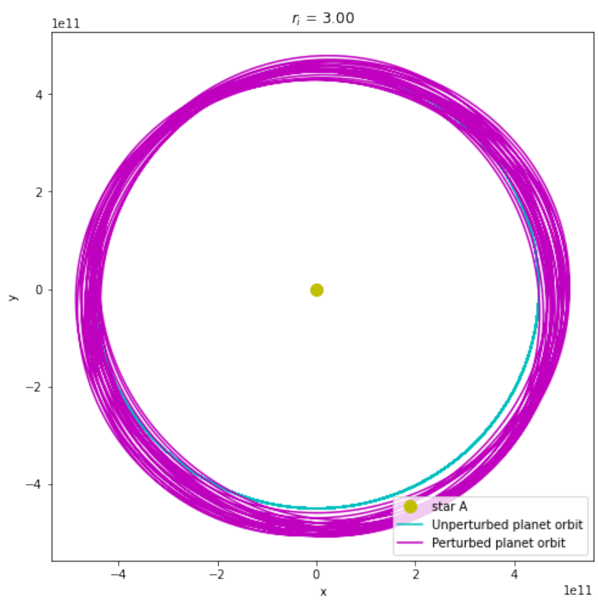


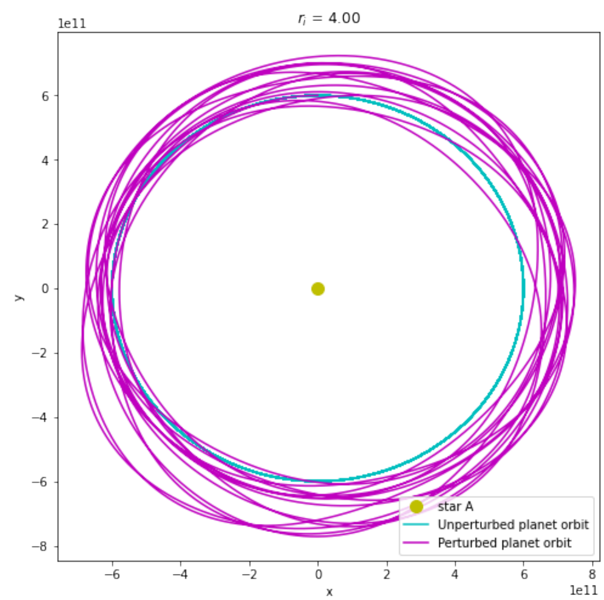
(a) 1 AU



(b) 2 AU

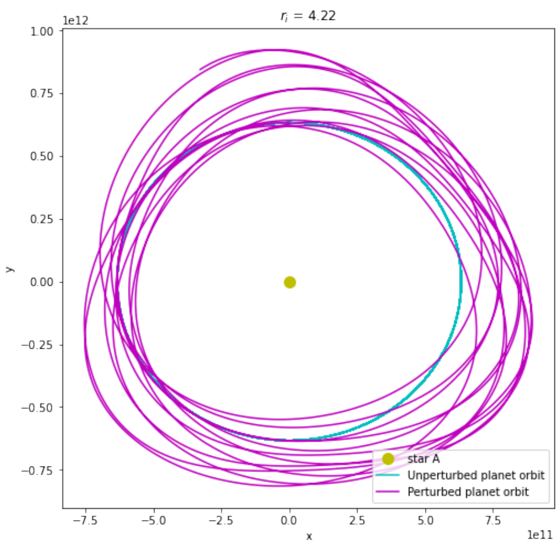


(c) 3 AU

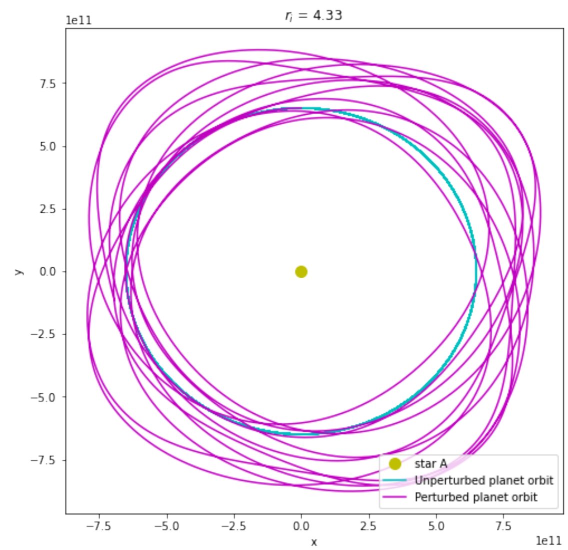


(d) 4 AU

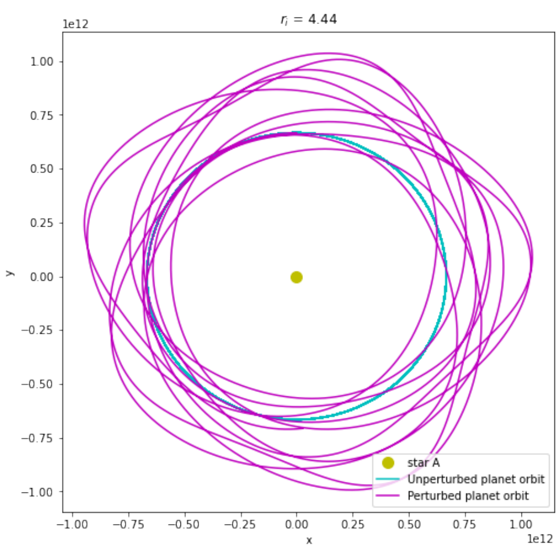
Figure 1: Trajectories of the planet in frame of star A for 1 to 4 AU radius



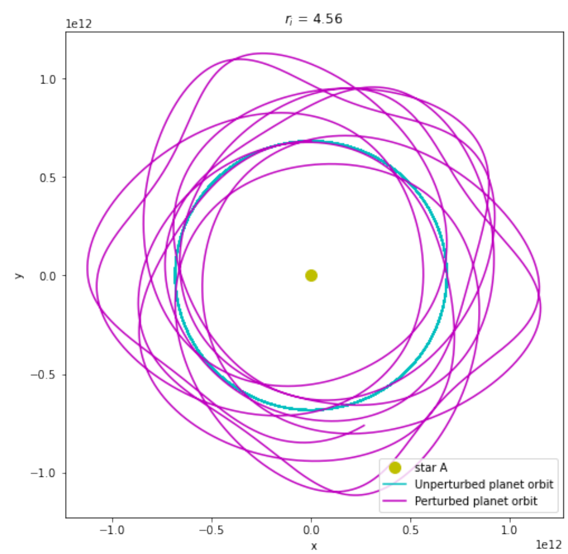
(a) 4.22 AU



(b) 4.33 AU

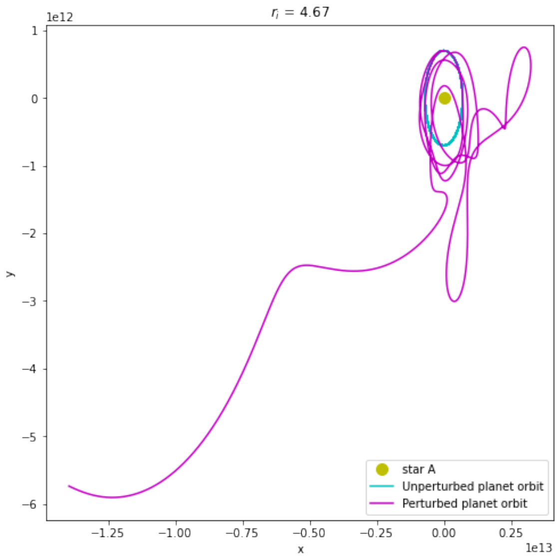


(c) 4.44 AU

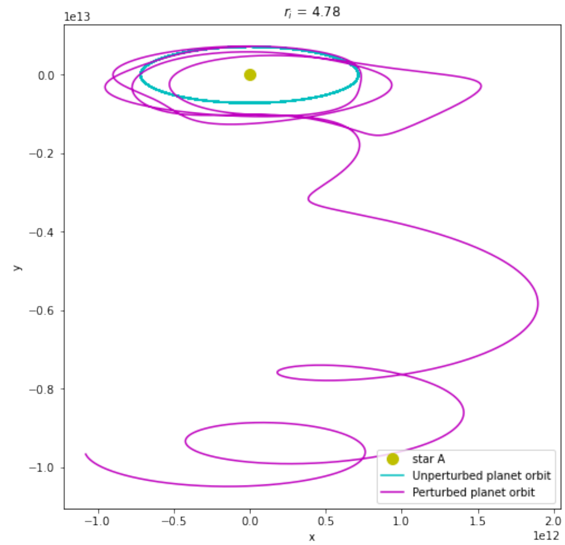


(d) 4.56 AU

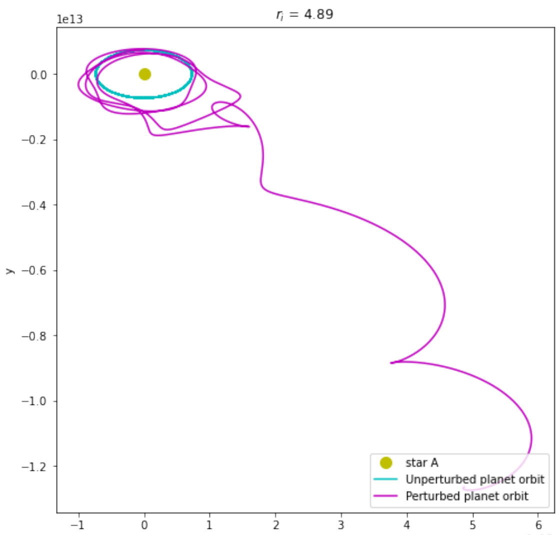
Figure 2: Trajectories of the planet in frame of star A for 4.2 to 4.6 AU radius



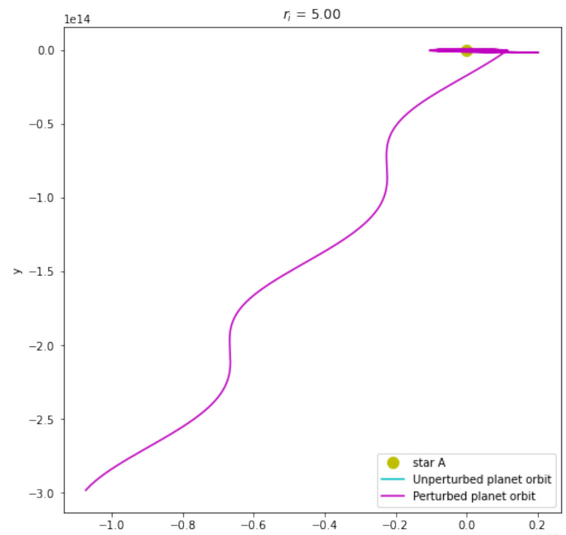
(a) 4.67 AU



(b) 4.78 AU

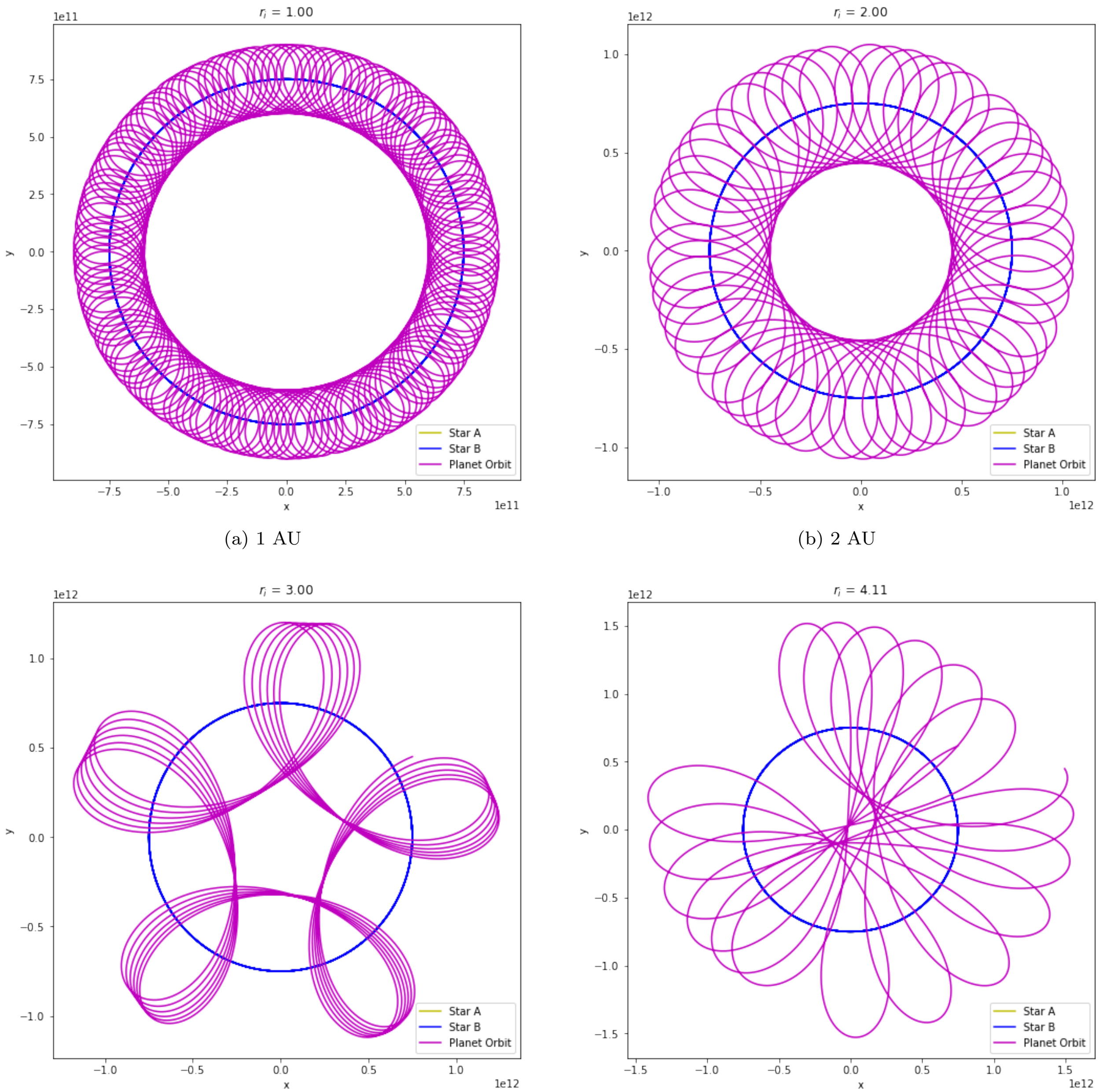


(c) 4.89 AU



(d) 5 AU

Figure 3: Unstable trajectories of the planet in frame of star A for 4.6 to 5 AU radius



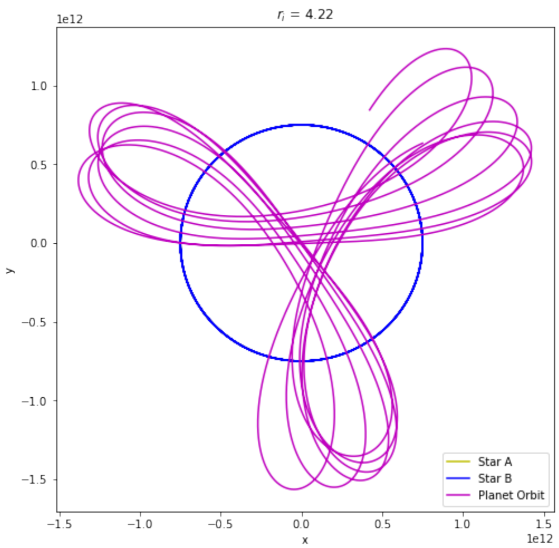
(a) 1 AU

(b) 2 AU

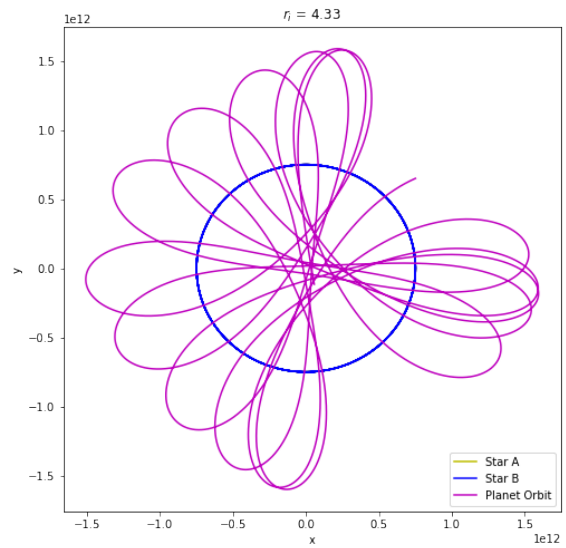
(c) 3 AU

(d) 4 AU

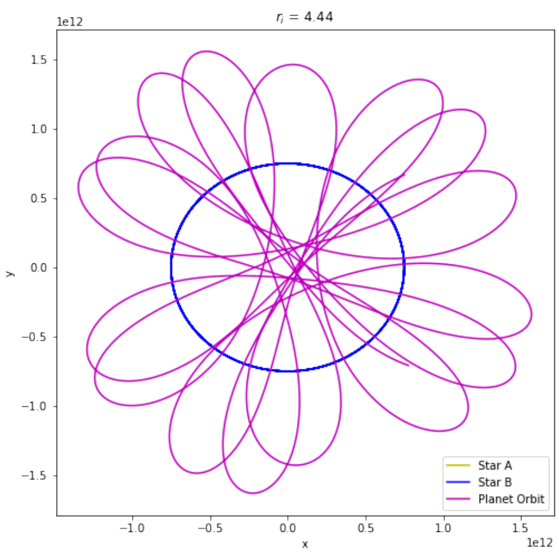
Figure 4: Trajectories of the planet, star A and star B for 1 to 4 AU radius of planet unperturbed orbit



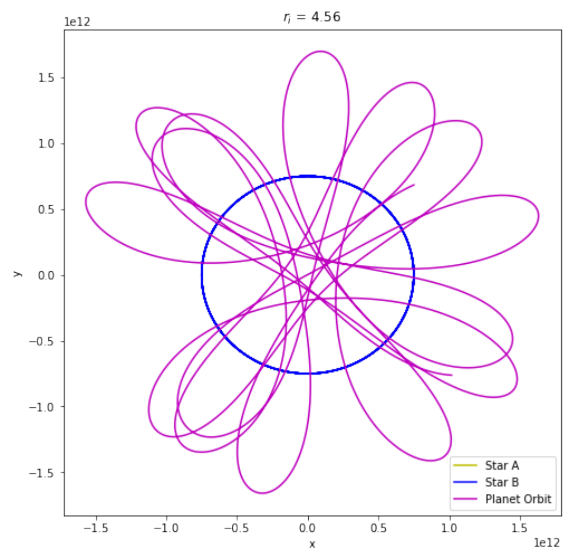
(a) 4.22 AU



(b) 4.33 AU

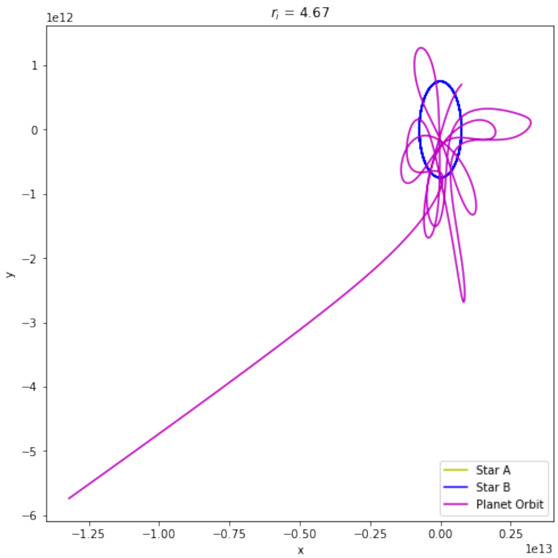


(c) 4.44 AU

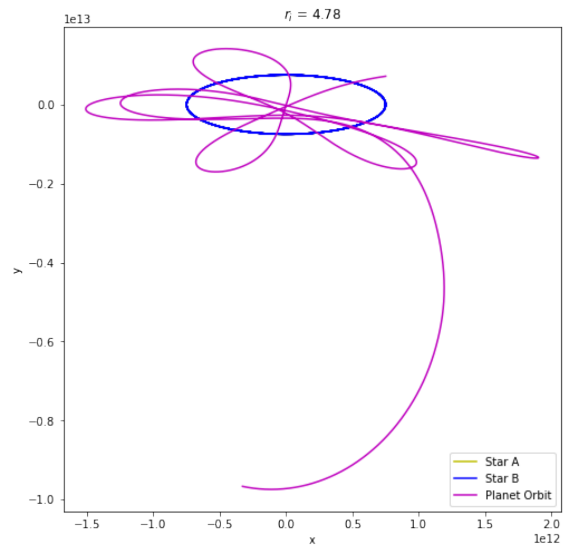


(d) 4.56 AU

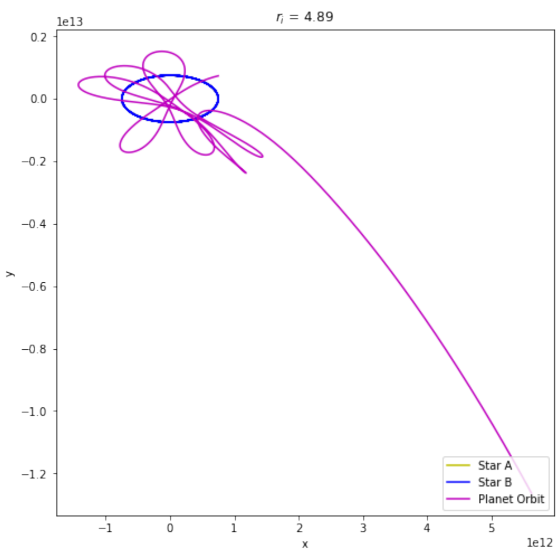
Figure 5: Trajectories of the planet, star A and star B for 4.2 to 4.6 AU radius



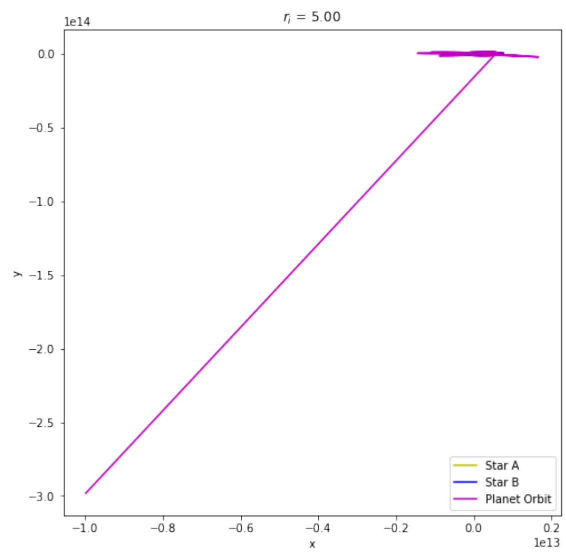
(a) 4.67 AU



(b) 4.78 AU



(c) 4.89 AU



(d) 5 AU

Figure 6: Unstable trajectories of the planet, star A and star B for 4.6 to 5 AU radius