(KEY)

Physics

1) **4**	2) **1**	3) **3**	4) **3**	5) **3**	6) **4**	7) **1**	8) **1**	9) **1**	10) **4**
11) **2**	12) **4**	13) **1**	14) **3**	15) **1**	16) **3**	17) **3**	18) **3**	19) **2**	20) **1**
21) **1**	22) **3**	23) **2**	24) **3**	25) **3**	26) **3**	27) **3**	28) **4**	29) **2**	30) **2**
31) **3**	32) **4**	33) **1**	34) **1**	35) **2**	36) **1**	37) **1**	38) **2**	39) **1**	40) **2**
41) **2**	42) **2**	43) **1**	44) **1**	45) **2**	46) **4**	47) **3**	48) **4**	49) **1**	50) **1**
51) **4**	52) **4**	53) **3**	54) **4**	55) **3**	56) **1**	57) **4**	58) **3**	59) **2**	60) **1**

Chemistry

61) **3**	62) **2**	63) **4**	64) **3**	65) **4**	66) **2**	67) **1**	68) **4**	69) **3**	70) **4**
71) **3**	72) **3**	73) **2**	74) **1**	75) **3**	76) **2**	77) **2**	78) **4**	79) **3**	80) **2**
81) **4**	82) **3**	83) **3**	84) **3**	85) **3**	86) **4**	87) **2**	88) **3**	89) **4**	90) **3**
91) **4**	92) **1**	93) **2**	94) **3**	95) **2**	96) **4**	97) **2**	98) **4**	99) **4**	100) **3**
101) **3**	102) **1**	103) **4**	104) **3**	105) **4**	106) **4**	107) **2**	108) **1**	109) **1**	110) **1**
111) **1**	112) **4**	113) **2**	114) **1**	115) **4**	116) **1**	117) **1**	118) **3**	119) 2	120) **3**

Mathematics

121) **1**	122) **4**	123) **2**	124) **2**	125) **1**	126) **4**	127) **3**	128) **3**	129) **1**	130) **1**
131) **2**	132) **1**	133) **4**	134) **3**	135) **1**	136) **2**	137) **2**	138) **2**	139) **3**	140) **2**
141) **4**	142) **4**	143) **2**	144) **1**	145) **2**	146) **3**	147) **1**	148) **2**	149) **1**	150) **3**
151) **3**	152) **1**	153) **2**	154) **3**	155) **2**	156) **1**	157) **1**	158) **3**	159) **3**	160) **2**
161) **1**	162) **4**	163) **2**	164) **2**	165) **1**	166) **1**	167) **2**	168) **3**	169) **2**	170) **4**
171) **4**	172) **3**	173) **2**	174) **2**	175) **4**	176) **3**	177) **2**	178) **4**	179) **3**	180) **4**

Biology

181) **4**	182) **1**	183) **3**	184) **1**	185) **3**	186) **1**	187) **2**	188) **3**	189) **3**	190) **4**
191) **3**	192) **1**	193) **4**	194) **4**	195) **4**	196) **3**	197) **3**	198) **2**	199) **3**	200) **3**
201) **3**	202) **1**	203) **2**	204) **2**	205) **3**	206) **1**	207) **1**	208) **1**	209) **4**	210) **1**
211) **2**	212) **3**	213) **1**	214) **3**	215) **4**	216) **2**	217) **4**	218) **3**	219) **2**	220) **3**
221) **3**	222) **1**	223) **2**	224) **4**	225) **2**	226) **1**	227) **4**	228) **2**	229) **2**	230) **1**
231) **3**	232) **2**	233) **1**	234) **2**	235) **4**	236) **3**	237) **1**	238) **2**	239) **2**	240) **3**

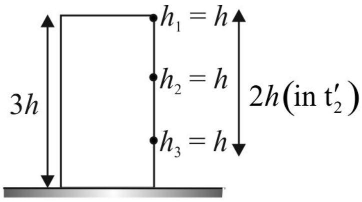
HINTS AND SOLUTIONS

SUBJECT : PHYSICS

1. **(4)** Denote : t_1 = time to cover height (downwards) h_1

t_2 = time to cover h_2

t_3 = time to cover h_3



$$S = ut + \frac{1}{2}gt^2$$

$\Rightarrow t_1 = \sqrt{\frac{2h}{g}}$ [since $u = 0$ as 'dropped',

$S = h_1 = h$ and $t = t_1$]

Denote: t_2' = (time taken to cover 2 h

distance) $\therefore 2h = \frac{1}{2}g(t_2')^2$

$$\Rightarrow t_2' = 2\sqrt{\frac{h}{g}}$$

$$\therefore t_2 = t_2' - t_1$$

$$\Rightarrow t_2 = 2\sqrt{\frac{h}{g}} - \sqrt{\frac{2h}{g}} = \sqrt{\frac{2h}{g}}\left(\sqrt{2}-1\right)$$

$$\Rightarrow t_2 = \sqrt{\frac{2h}{g}}\left(\sqrt{2}-1\right)$$

For $S = 3h$, we have: $t_3' = \sqrt{\frac{2\times 3h}{g}}$

$$t_3 = t_3' - t_2 - t_1$$

$$\Rightarrow t_3 = \sqrt{\frac{2h}{g}}\left(\sqrt{3}-\sqrt{2}+1-1\right)$$

$$\Rightarrow t_3 = \sqrt{\frac{2h}{g}}\left(\sqrt{3}-\sqrt{2}\right)$$

$$\Rightarrow t_1 : t_2 : t_3 = 1 : \left(\sqrt{2}-1\right) : \left(\sqrt{3}-\sqrt{2}\right)$$

Correct option is (4)

2. **(1)** Energy is conserved.

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}m\left[(20)^2 - v_e^2\right]$$

$$\Rightarrow v^2 = (20)^2 - (v_e)^2$$

we know : escape speed $v_e = 11.2\ km/s$

$$\Rightarrow v^2 = 272$$

$$\Rightarrow v = 16.5\ km/s$$

Correct option is (1).

3. **(3)** For perpendicular condition, $\vec{A}\cdot\vec{B} = 0$

$(\because \cos 90° = 0)$

Each vector given in choices is tried.

For vector $\vec{B}$ in choice (3): We have

$$\vec{A}\cdot\vec{B} = \left(\hat{i}A\cos\theta + \hat{j}A\sin\theta\right)\left(\hat{i}B\sin\theta - \hat{j}B\cos\theta\right)$$

$$= A\cos\theta B\sin\theta - A\sin\theta B\cos\theta$$

$$= AB\sin\theta\cos\theta - AB\sin\theta\cos\theta$$

$$= 0$$

$\vec{A}\cdot\vec{B} = 0$(condition statisfied)

Correct option is (3).

4. **(3)** Pressure $= \left(\frac{\text{Force}}{\text{Area}}\right)$

$$P = \frac{F}{A} = \frac{MLT^{-2}}{L^2} = \left[ML^{-1}T^{-2}\right] = \frac{M}{LT^2}$$

Correct option is (3)

5. **(3)** Apply Newton's 2^{nd} law:

for 2 kg block, $F - T_1 = 2a$ (1)

for 3 kg block, $T_1 - T_2 = 3a$ (2)

For 5 kg block

$T_2 = 5a \qquad (3)$

In these equations: $a = 1\ m/s^2$ (given)

$\therefore T_1 - 5a = 3a \Rightarrow T_1 = 8a$ (from eq (3) and (2))

Then from eq (1)

$\Rightarrow F - 8a = 2a$

$\Rightarrow \quad F = 10N \qquad (a = 1\ m/s^2)$

Correct option is (3).

6. **(4)** $F = (-0.1x)$ (in $Joule/metre$)

By Work Energy theorem

$W = \Delta E_K$

$$\Rightarrow \int F.dx = (E_k)_f - (E_K)_i$$

$$\Rightarrow \int_{20}^{30} -0.1x dx = (E_k)_f - \frac{1}{2}\times mu^2$$

$$\Rightarrow (-)0.1\left[\frac{x^2}{2}\right]_{20}^{30} = (E_k)_f - \frac{1}{2}\times 10\times 10^2$$

$$\Rightarrow \frac{1}{10\times 2}\left[x^2\right]_{30}^{20} = (E_k)_f - 500$$

(Where inversed limits make $-$ve to positive)

$$\Rightarrow \frac{1}{20}\times[400-900] = (E_k)_f - 500$$

$$\Rightarrow -\frac{500}{20} = (E_k)_f - 500$$

$$(E_k)_f = 500 - 25 = 475J$$

Correct option is (4).

7. **(1)** Torque: $\tau = \frac{dL}{dt} \approx \frac{\Delta L}{\Delta t} = \frac{7L-3L}{4} = \frac{4L}{4} = L$

Correct option is (1)

8. **(1)** $g = \frac{GM}{r^2} = \frac{G}{r^2}\times\frac{4}{3}\pi r^3 d = \frac{4}{3}\pi r d G$

$\Rightarrow g\ \alpha\ (d)(r)$

$\frac{g_1}{g_2} = \frac{d_1 r_1}{d_2 r_2} \Rightarrow g_1 : g_2 = r_1 d_1 ; r_2 d_2$

Correct option is (1).

9. **(1)** We know $Y = \frac{(F/A)}{(\Delta l/L)}$

$A = \pi r^2$

$$\Rightarrow Y_P = \left(\frac{F}{\pi r^2}\right)\left(\frac{L}{\Delta l}\right)$$

$$\Rightarrow Y_Q = \left[\frac{F}{\pi(r^2/4)}\right]\left[\frac{(L/3)}{\Delta l}\right]$$

$$\Rightarrow Y_Q = \frac{4}{3}\left[\frac{F}{\pi r^2)}\right]\left[\frac{L}{\Delta l}\right]$$

$$= \frac{4}{3}Y_P = \left(\frac{4}{3}\right)(Y)$$

Correct option is (1).

10. **(4)** We shall find:

Reynold number $R_e = \frac{\rho \upsilon d}{\eta}$

Where $\rho = 10^3\ kg/m^3$

$\upsilon = ?$

$d = \frac{2}{\sqrt{\pi}} cm$

$= \frac{2}{\sqrt{\pi}}\times 10^{-2} m$

$\eta = 10^{-3}\ Pa - s$

To find υ, we equate the two formulae for rate Q of flow (of fluid)

$$Q = A\upsilon = \frac{\Delta V}{\Delta t}$$

$$\Rightarrow \left(\frac{\pi d^2}{4}\right)(\upsilon) = \frac{15\,\text{litre}}{5\,\text{minute}}$$

$$\Rightarrow \left[\frac{\pi}{4}\left(\frac{2}{\sqrt{\pi}}\right)^2\times 10^{-4}\right](\upsilon)$$

$$= \left(\frac{15\times 10^{-3}}{(5\times 60)}\right)$$

$$\Rightarrow \upsilon = 0.5\,m/s$$

$$\therefore R_e = \left[\frac{(10^3)(0.5)\left(\frac{2}{\sqrt{\pi}}\times 10^{-2}\right)}{10^{-3}}\right]$$

$= \frac{1}{1.8} \times (10000) \quad \left[\text{Since} : \sqrt{\pi} = 1.8\right]$

$= 5555$

Correct option is (4).

11. **(2)** Tension is given by

$$T = \frac{2m_1 m_3}{m_1 + m_2 + m_3} \times g$$

we can take $g = 10\ m/s^2$

$$\Rightarrow T = \frac{2 \times 6 \times 6 \times 10}{6+6+6} = \frac{720}{18} = 40\,N$$

Note: If we take $g = 9.8\,m/s^2$

$$T = \frac{705.6}{18} = 39.2\,N \text{ (which is close to } 40N\text{)}$$

Correct option is (2).

12. **(4)** $W = T(2\Delta A)$

$$\Delta A = (20-8)\,cm^2$$

$$= 12 \times 10^{-4}\,m^2$$

$$\Rightarrow T = \frac{W}{2\Delta A} = \frac{3 \times 10^{-4}}{2 \times 12 \times 10^{-4}} = 0.125\ Nm^{-1}$$

Correct option is (4)

13. **(1)** $\alpha = \frac{\Delta L}{L_o \Delta T} \Rightarrow \Delta T = \frac{\Delta L}{L_o \times \alpha} = \frac{3}{100 \times 2 \times 10^{-5}}$

$$\Delta T = 1.5 \times 10^3 = 1500°C$$

Correct option is (1).

14. **(3)** Wien's law states that $\lambda T =$ constant. Where T is absolute temperature on Kelvin-scale.

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1} \Rightarrow T_2 = \frac{\lambda_1}{\lambda_2} \times T_1 = \frac{2.2 \times 10^{-6}}{4.4 \times 10^{-5}} \times 1000$$

$$\therefore T_2 = 50\,K$$

Correct option is (3).

15. **(1)** W=area under $P - V$ graph

$$= \frac{1}{2}\left[(3-1)+(2-1)\right] \times \left(2 \times 10^2 - 10^2\right)$$

$= 150$ Joule

Correct option is (1).

16. **(3)** $V_2 = \frac{P_1 V_1 T_2}{P_2 T_1}$

$$V_2 = \frac{1 \times 500 \times (273-3)}{0.5 \times (273+27)} = 900$$

Correct option is (3).

17. **(3)** Phase difference $\Delta\phi = \frac{3\pi}{6} - \frac{\pi}{6} = \frac{\pi}{3}$

Correct option is (3)

18. **(3)** According to the question

$$|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$$

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 B^2 - 2AB\cos\theta}$$

$$|\vec{A}| = |\vec{B}|$$

Squaring both side

$$A^2 + B^2 + 2AB\cos\theta =$$
$$n^2\left(A^2 + B^2 - 2AB\cos\theta\right)$$

$$A^2 + B^2 + 2AB\cos\theta =$$
$$n^2 A^2 + n^2 B^2 - n^2 \times 2AB\cos\theta$$

$$2A^2 + 2A^2\cos\theta = 2n^2 A^2 - 2n^2 A^2 \cos\theta$$

$$2A^2(1+\cos\theta) = 2n^2 A^2 (1-\cos\theta)$$

$$1 + \cos\theta = n^2 - n^2\cos\theta$$

$$1 + \cos\theta + n^2\cos\theta = n^2$$

$$\cos\theta = \frac{n^2 - 1}{n^2 + 1}$$

$$\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

Correct option is (3).

19. **(2)** $\omega = 100, K = 20$

velocity of waves $= \frac{\omega}{K} \Rightarrow \frac{100}{20} = 5\ m/s$

Correct option is (2).

20. **(1)** $\mu = \frac{0.035}{5.5}\,kg/m,\ T = 77\ N$

where μ is mass per unit length.

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{77\times 5.5}{0.035}}=110\ m/s$$

Correct option is (1).

21. **(1)** $q_1=q_2=2\times 10^{-8}C, r=1m$

Tension in the string will be equal to the force between the charge.

According to coulomb's law,

$$F=\frac{Kq_1q_2}{r^2}$$

$$=\frac{9\times 10^9\times\left(2\times 10^{-8}\right)^2}{(1)^2}$$

$$=\frac{9\times 10^9\times 4\times 10^{-16}}{1}$$

$$=36\times 10^{-7}N=3.6\times 10^{-6}N$$

Correct option is (1).

22. **(3)** According to quantisation of charge,

$q=Ne$

$$\Rightarrow N=\frac{q}{e}=\frac{1\times 10^{-7}}{1.6\times 10^{-19}}=0.625\times 10^{12}$$

$$=6.25\times 10^{13}$$

Correct option is (3).

23. **(2)** $\phi=\vec{E}\cdot\vec{A}$

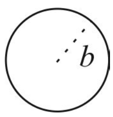
$$\phi=\left(8\hat{i}+8\hat{j}+\hat{k}\right)\cdot\left(10\hat{i}\right)$$

$$\phi=80Nm^2C^{-1}$$

Correct option is (2).

24. **(3)** Due to the conducting wire (in the form of spring), charge resides on outer sphere (of radius b) only.

$\Rightarrow$ It is equivalent to isolated sphere



$C=4\pi\in_0 b$

Correct option is (3).

25. **(3)** we know in 2 - dimensions,

$$\vec{E}=E_x\hat{i}+E_y\hat{j}$$

Given: $\vec{E}=\sqrt{3}\hat{i}-\hat{j}$

$\Rightarrow E_x=\sqrt{3}\cdot V/m$

$E_y=-1V/m$

Denote $\theta=$ Angle between $\vec{E}$ and x-axis

$$=\tan^{-1}\left(\frac{E_y}{E_x}\right)=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=-30°$$

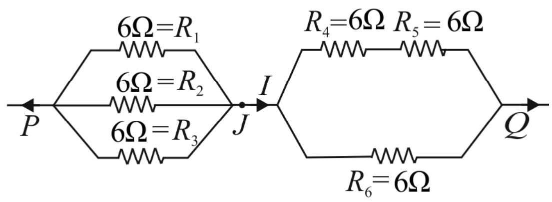
Denote $\propto=\begin{cases}\text{Angle between equipotential}\\ \text{Line and } x\text{-axis}\end{cases}$

$$=\theta+90°=-30°+90°=60°$$

$\Rightarrow$ Equipotential lives shall make 60° with x-axis.

Correct option is (3).

26. **(3)** Given part of a circuit is: (source -potential not shown)



$$\left(V_p-V_Q\right)=\left(\Delta V\right)_{PQ}$$

$$=I\left(R_{PQ}\right)$$

Where $R_{PQ}=R_{PJ}+R_{JQ}$

Here R_{PJ} is due to parallel combination of R_1, R_2 and R_3.

$$\Rightarrow\frac{1}{R_{PJ}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

$$=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$$

$$\Rightarrow\frac{1}{R_{PJ}}=\frac{1+1+1}{6}$$

$$\Rightarrow R_{PJ}=\frac{6}{3}=2\Omega$$

Then, in the network between J and Q, we have R_4 and R_5 in series and that series combination is in parallel with R_6

$$\Rightarrow \frac{1}{R_{JQ}} = \frac{1}{(R_4 + R_5)} + \left(\frac{1}{R_6}\right)$$

$$= \frac{1}{(6+6)} + \frac{1}{6}$$

$$= \frac{1}{12} + \frac{1}{6}$$

$$\Rightarrow R_{JQ} = \frac{12 \times 6}{12+6} = 4\Omega$$

$$R_{PQ} = R_{PJ} + R_{JQ}$$

$$= (2+4)$$

$$= 6\Omega$$

$$\Rightarrow (V_P - V_Q) = I(R_{PQ})$$

$$= (0.5)(6)$$

$$= 3 \text{ volt}$$

Correct option is (3).

27. **(3)** power: $P = \frac{V^2}{R}$

$$\Rightarrow \frac{\Delta P}{P} \times 100 = \frac{2 \times \Delta V}{V} \times 100\%$$

$$= 2 \times 2.5 = 5\%$$

$$\left(\because \frac{\Delta V}{V} = 2.5\right)$$

Correct option is (3).

28. **(4)** $r =$ mean radius

$$= \left(\frac{\text{inner radius + outer radius}}{2}\right)$$

$$= \frac{20+22}{2} = 21\ cm = 0.21\ m$$

Total length of toroid = circumference

$$L = 2\pi r = 2\pi \times 0.21\, m$$

$$= 0.42\pi\ m$$

$\therefore$ No. of turns per unit length

$$n = \frac{4200}{0.42\pi} = \frac{10000}{\pi} m^{-1}$$

$$B = \mu_o n I = 4\pi \times 10^{-7} \times \frac{10000}{\pi} \times 10$$

$$\Rightarrow B = 0.04T$$

we know $1\,T = 10^4$ gauss

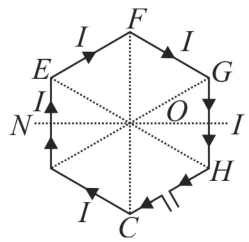
$$\Rightarrow B = 400 \text{ gauss}$$

Correct option is (4).

29. **(2)** Here $\angle EOD = 60° = \angle EOF$

$\angle EON = 30° = \angle NOD$

$OE = OD = ED = a$ (since equilateral triangle)



$$ON = OE\cos 30° = a \times \frac{\sqrt{3}}{2}$$

Total magnetic field induction at O due to current through all the six sides of hexagon is

$$B = 6 \times \frac{\mu_o}{4\pi} \times \frac{I}{a\frac{\sqrt{3}}{2}}(\sin 30° + \sin 30°)$$

$$\Rightarrow B = \frac{\sqrt{3}\mu_o I}{\pi a}$$

Correct option is (2).

30. **(2)** $F = qvB\sin\theta$,

Here q = e

$$F = 1.6 \times 10^{-19} \times 2 \times 10^7 \times 1.5 \times \sin 30^\circ$$

$$= 1.6 \times 10^{-12} \times 2 \times 1.5 \times \frac{1}{2}$$

$$\Rightarrow F = 2.4 \times 10^{-12} N$$

Correct option is (2).

31. **(3)** Denote $\delta =$ angle of dip = 30°

V = vertical component

H = Horizontal component

(of earth's magnetic field)

$$\tan\delta = \frac{V}{H}$$

$$\Rightarrow H = \frac{V}{\tan\delta} \Rightarrow \frac{0.16\times\sqrt{3}\times10^{-4}}{\frac{1}{\sqrt{3}}}$$

$= 0.16\times3\times10^{-4}$ Tesla

$\Rightarrow H = 0.48\times10^{-4}$ Tesla

Correct option is (3)

32. **(4)** $V_{ind} = BAv = B.A\times\frac{\omega}{2\pi} = B\times\pi R^2\times\frac{\omega}{2\pi}$

$$= \frac{1}{2}BR^2\omega = \frac{1}{2}\times0.05\times(2)^2\times60 = 6\text{volt}$$

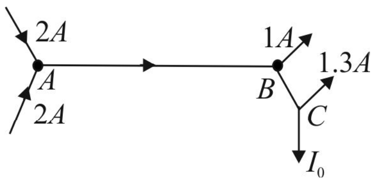
Correct option is (4).

33. **(1)** $V_{ind} = L\frac{dI}{dt} \Rightarrow 0.20 = \frac{L(5-(-5))}{0.20}$

$$\Rightarrow L = \frac{0.20\times0.20}{5+5} = 4\times10^{-3}H$$

Correct option is (1).

34. **(1)**



Apply KCL (Kirchhoff's first law,):

At junction A, $i_{AB} = 2+2 = 4A$

Reqiured current $I_0 = 4-1-1.3$

$= 1.7A$

Correct option is (1).

35. **(2)** For P = constant

$R\,\alpha\,V^2$ $\left(\text{since } P = \frac{V^2}{R}\right)$

$$\Rightarrow \therefore R_1 = \frac{R\times(110)^2}{(220)^2} = \frac{R}{4}$$

Correct option is (2).

36. **(1)** Here, $R = 15\Omega$ $\quad E_v = 120V$ $\quad f_o = 50Hz$

At resonance, $Z = R = 15\Omega$

$$I_V = \frac{E_V}{R} = \frac{120}{15} = 8A$$

L=100 $mH = 10^{-1}H$

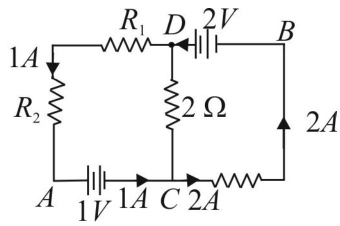
$C = 25\mu F = 25\times10^{-6}F$

$$f_o = \frac{1}{2\times\pi\sqrt{LC}} = \frac{1\times7}{2\times22\times\sqrt{10^{-1}\times25\times10^{-6}}}$$

$$f_o = \frac{7\times10^3}{44\times1.58} = 100.7Hz$$

Correct option is (1).

37. **(1)**



Given, $V_A = 0$

Path from A to B is given

Apply KVL (Kirchoff voltage law)

$V_A - 1 + 2 - 2 = V_B$

$\Rightarrow V_B = -1V$

Correct option is (1).

38. **(2)** Given $\lambda = 40$ m

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{40}J$$

Divide by $e = 1.6\times10^{-19}$ coul.

$$\Rightarrow E = \frac{6.6\times10^{-34}\times3\times10^8}{40\times1.6\times10^{-19}}eV \quad = \frac{19.8\times10^{-7}}{64}$$

$\Rightarrow E = 3.1\times10^{-8}eV$

Correct option is (2).

39. **(1)** Given Intensity $I = 2$ W/m^2

Pressure exerted by absorbed wave on the surface

$$P = \frac{I}{c} = \frac{2}{3\times10^8} = 0.667\times10^{-8}N/m^2$$

$= 6.67\times10^{-9}N/m^2 \simeq 6.67\times10^{-9}N/m^2$

Correct option is (1).

40. **(2)** Take origin at pole P to measure distances

$\Rightarrow$ Radii R_1 and R_2 are positive in the formula.

$$\left(\frac{1}{f}\right)=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1.5-1)\left(\frac{1}{(+10)}-\frac{1}{(+20)}\right)$$

$$=\left(\frac{1}{2}\right)\left(\frac{2-1}{20}\right)$$

$$=\frac{1}{40}\Rightarrow f=40cm$$

Correct option is (2).

41. **(2)** In vacuum $c=d/t$

In medium $v=\frac{5d}{T}$

$$\mu=\frac{c}{v}=\frac{T}{5t}$$

We know: sin $C=\frac{1}{\mu}$ (Where C is critical angle)

$$\Rightarrow C=\sin^{-1}\left(\frac{5t}{T}\right)$$

Correct option is (2).

42. **(2)** We know, for transformer:

$$\frac{N_s}{N_p}=\frac{I_p}{I_s}\Rightarrow\frac{2}{3}=\frac{3}{I_s}$$

$\Rightarrow I_s=4.5\ A$

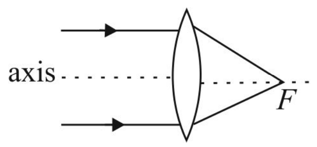
Correct option is (2).

43. **(1)** For a lens, we know

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Given $\mu=1.5,\ R_1=+20\ cm$

$R_2=-20\ cm$



Such parallel rays focus F at $L=f$

$\Rightarrow f=20cm\ =L$

Correct option is (1).

44. **(1)** Power of lens, $P=\frac{100}{f}$

$$\Rightarrow f=\frac{100}{10}=10\ cm$$

According to the lens maker formula

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Here $R_1=+10\ cm,\ R_e=-10\ cm$

$$\Rightarrow\frac{1}{10}=(\mu-1)\left[\frac{2}{10}\right]$$

$$\Rightarrow\mu=\frac{3}{2}$$

Correct option is (1).

45. **(2)** At minimum deviation

$r_1=r_2=r$

We know

$\Rightarrow r_1+r_2=A$

$\Rightarrow r+r=A$

$\Rightarrow 2r=A$

$$\Rightarrow r=\frac{A}{2}=\frac{60}{2}=30^\circ$$

Correct option is (2)..

46. **(4)** Frequency $=n$; Wavelewngth $=\lambda$; Velocity of air $=v$ and refractive index of glass slab $=\mu$. Frequency remains the same, when light changes the medium.

$\Rightarrow n'=n$

$$\mu=\frac{c}{v_2}=\frac{n\lambda_1}{n\lambda_2};\ \text{Here } c=v\ \lambda_1=\lambda$$

$$\Rightarrow\lambda_2=\frac{\lambda_1}{\mu}=\frac{\lambda_1}{\mu}=\frac{\lambda}{\mu};\ v_2=\frac{v}{\mu}$$

Correct option is (4).

47. **(3)** For an electron $\lambda=\frac{h}{\sqrt{2mE_k}}$

$$\therefore\lambda\propto\frac{1}{\sqrt{E_k}}$$

$$\therefore \frac{\lambda'}{\lambda} = \frac{\sqrt{E_k}}{\sqrt{3E_k}} \Rightarrow \lambda' = \frac{\lambda}{\sqrt{3}}$$

Correct option is (3).

48. **(4)** Width of central bright fringe $= \dfrac{2\lambda L}{a}$

The width $= \dfrac{2\times 500\times 10^{-9}\times 80\times 10^{-2}}{0.2\times 10^{-3}}$

$= 4\times 10^{-3} m$

$\Rightarrow$ Width of central bright fringe = 4 mm

Correct option is (4).

49. **(1)** Limit of resolution of telescope

$$\Delta\theta = \frac{1.22\lambda}{D} = \frac{1.22\times 6\times 10^{-5}}{254}$$

$(\because$ 1 inch $= 2.54\ cm)$

$\Rightarrow \Delta\theta = 3\times 10^{-7}\,rad$

Correct option is (1).

50. **(1)** In interference

$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ and $I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

When the widths of both the slits are equal, we have:

$I_1 \approx I_2 = I$ (say)

$\therefore I_{\max} = 4I$ and $I_{\min} = 0$

When the width of one of the (slit 2) is increased. Intensity due to that slit would increase, while that of the other (slit 1) will remain same.

$\Rightarrow I_1 = I$ and $I_2 = \eta I \quad (\eta > 1)$

$\Rightarrow I_{\max} = I\left(1+\sqrt{\eta}\right)^2 > 4I$ and

$I_{\min} = I\left(\sqrt{\eta}-1\right)^2$

But " square of any quantity " is positive.

$\Rightarrow \left(\sqrt{\eta}-1\right)^2 > 0 \Rightarrow I_{\min} > I$

$\therefore$ Intensities of both maximum and minima will increase.

Correct option is (1)

51. **(4)** Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63\times 10^{-34}\times 3\times 10^{8}}{6000\times 10^{-10}}$$

$\Rightarrow E = 3.31\times 10^{-19} J$

No. of photons emitted per second

$$n = \frac{P}{E} = \frac{25}{3.31\times 10^{-19}} = 7.55\times 10^{19}$$

Correct option is (4).

52. **(4)** K_α arises from transition from $n_2 = 2$ (*i.e.*, L shell) to $n_1 = 1$ (*i.e.*, K shell).

Ryberg formula can be used with three different levels of approximation

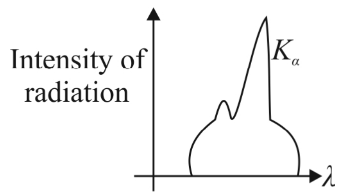
$$\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Where b is constant.

We know $c = \upsilon\lambda$

$\Rightarrow \sqrt{2}\,\alpha(Z-b)$

This is well known Mosley's law inproductive, we can take $b = 1$ as Z of target is high (here 64, 80) compared to unity.



$\Rightarrow \dfrac{1}{\lambda} \propto Z^2$

$\Rightarrow \lambda \propto \dfrac{1}{Z^2}$

$\Rightarrow \dfrac{1}{\lambda} \propto (Z)^2$

$\Rightarrow \dfrac{1}{\lambda} \propto \left(\dfrac{1}{Z^2}\right)$

$$\frac{\lambda K\alpha_1}{\lambda K\alpha_2} = \left(\frac{Z_2}{Z_1}\right)^2$$

$$= \left(\frac{80\times 80}{64\times 6}\right) = \frac{25}{16}$$

Correct option is (4).

53. **(3)** $\lambda_{K\alpha}$ = Wavelength of X-rays for transmissiom from L to K shell.

Work function is $W_o = \dfrac{hc}{\lambda_o}$

Where λ_o = threshold wavelength

$\Rightarrow W_o \propto \dfrac{1}{\lambda_o}$ (since h, c are constants)

$$\Rightarrow \frac{W_{o(T)}}{W_{o(No)}} = \frac{\lambda_{o(Na)}}{\lambda_{o(T)}} \Rightarrow \lambda_{o(T)} = \frac{\lambda_{o(Na)} \times W_{o(Na)}}{W_{o(T)}}$$

$$\lambda_{o(T)} = \frac{5460 \times 2.3}{4.5} = 2791\,\mathring{A}$$

Correct option is (3).

54. **(4)** Bohr radius of n^{th} orbit is given by:

$$r_n = \left(\frac{n^2}{Z}\right) a_o$$

Where $a_o = 0.53\ \mathring{A}$

By data ; we can write:

$$(r_{n+1}) - (r_n) = (r_{(n-1)})$$

$$\Rightarrow \frac{a_o}{Z}\left[(n+1)^2 - n^2\right] = \frac{a_o}{Z}\left[(n-1)^2\right]$$

$$\Rightarrow (n^2 + 2n + 1 - n^2) = (n^2 - 2n + 1)$$

$$\Rightarrow 2n = n^2 - 2n$$

$$\Rightarrow n^2 = 4n$$

$$\Rightarrow n = 4$$

Correct option is (4).

55. **(3)** $\dfrac{N}{N_o} = \left(\dfrac{1}{2}\right)^{\frac{t}{t_{1/2}}} \Rightarrow \dfrac{1}{4} = \left(\dfrac{1}{2}\right)^{\frac{t}{10}}$

$\Rightarrow \dfrac{t}{10} = 2 \Rightarrow t = 20$ years

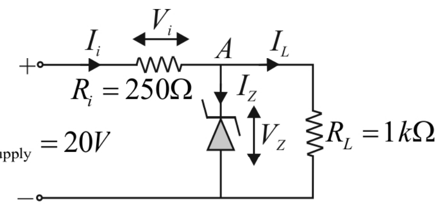
Correct option is (3).

56. **(1)** $A = 180 - 4 - 0 - 4 - 0 = 172$

$Z = 72 - 2 + 1 - 2 + 0 = 69$

Correct option is (1).

57. **(4)**



Here: Subscripts i for input side (left of Zener) and L for load side (right of Zener).

We are to find $I_i = I_L$

$$I_i = \frac{V_i}{R_i}$$

$$= \left[(V_{\text{supply}} - V_Z)/R_i\right]$$

$$= \left(\frac{20-15}{250}\right) = 20 \times 10^{-3} A = 20\,mA$$

Clearly. Zener (diode) and R_L are in parallel to each other with only wires between them.

$\Rightarrow V_L =$ Voltage drop across R_L

$= V_Z = 15\,volt$ (given)

$\Rightarrow I_L =$ Current through R_L

$$= \frac{V_L}{R_L} = \frac{15\,volt}{1k\Omega}$$

$$= \left(\frac{15}{10^3}\right) Ampere$$

$$= 15 \times 10^{-3} A = 15\,mA$$

$\therefore$ KCL at A gives us:

$$I_i = I_L + I_Z$$

$$\Rightarrow 20 = 15 + I_Z$$

$$\Rightarrow I_Z = 5\,mA$$

Correct option is (4).

58. **(3)** Diode is reversed biased, so only drift current due to minority carriers which is $20\mu A$

Potential drop across resistor

$V = 15\Omega \times 20\mu A$

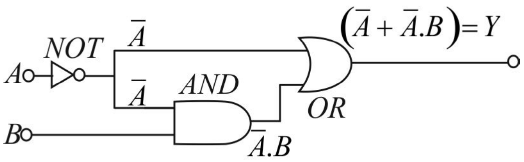
$= 300\mu v = 0.0003V$

Potential difference across the diode

$=4 - 0.0003 = 3.99 = 4V$

Correct option is (3).

59. **(2)**



$$Y=\bar{A}.B+\bar{A}=\bar{A}.(B+1)=\bar{A}.1=\bar{A}$$

Correct option is (2)

60. **(1)** Electrons in excited state atoms jump to any of the states with lower energy. (That is why more than one sectral series are possible)
Correct option is (1).

SUBJECT : CHEMISTRY

61. **(3)** Second Bohr orbit of hydrogen atom, i.e., n = 2.
Atomic number of hydrogen (Z) = 1

$$\text{Radius, } r=\frac{0.529n^2}{Z}\mathring{A}=\frac{0.529\times(2)^2}{1}\mathring{A}$$

$$=2.116\,\mathring{A}=0.2116\text{ nm}$$

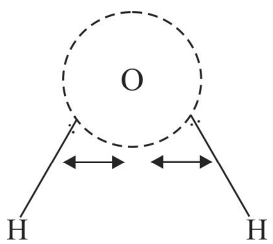
62. **(2)** $\Delta H_{combustion}=-393.5\,kJ/mol$, i.e.,1 mole of carbon on combustion release -393.5 kJ of heat.

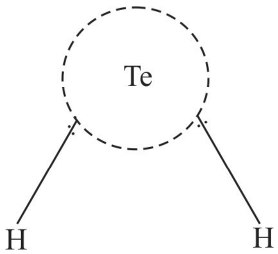
$\frac{35.2}{44}$ moles of CO_2 will release

$$\frac{-13851.2}{44}=-314.8\text{ kJ of heat.}$$

63. **(4)** Halogens have a high value of electron affinity. By gaining one electron, they acquire stable noble gas configuration.

64. **(3)** O, S, Se, Te belong to Group 16.
On moving down the group, size of atom increases. 'O' is most electronegative and lone pairs lie close to the atom electron cloud. This causes repulsion in lone pairs.





$\therefore$ Angle maximum due to $lp-bp$ repulsions.
Te has maximum size, lone pair lie far away from the atom electron cloud. Lone pair - bond pair repulsions are the least.

65. **(4)** In $AlCl_3$ molecule, central Al-atom is sp^2 hybridised.

66. **(2)** $O_2^-=(\sigma 1s)^2,(\sigma^*1s)^2,(\sigma 2s)^2,$

$(\sigma^*2s)^2,(\sigma 2p_z)^2,(\pi 2p_x)^2=$

$(\pi 2p_y)^2,(\pi^*2p_x)^2=(\pi^*2p_y)^1$

$$B.O=\frac{1}{2}(10-7)=\frac{3}{2}=1.5$$

67. **(1)** Compressibility factor; $Z=\frac{V_{obs}}{V_{calc}}$

For ideal gas, $V_{ob}=V_{cal}$; So $Z=1$

68. **(4)** $Pb(OH)_2 \rightleftharpoons Pb^{2+}+2OH^-$

S   2S

Solubility of $Pb(OH)_2$ in water is $6.7\times10^{-6}M$

$K_{sp}=(S)(2S)^2$

$K_{sp}=[Pb^{2+}][OH^-]^2$

$=(6.7\times10^{-6})(2\times6.7\times10^{-6})^2$

$=6\times10^{-6}\times144\times10^{-12}$

$=1.203\times10^{-15}M$

Solubility of $Pb(OH)_2$ in buffer

sol^n calculated as follows,

$\therefore[Pb^{2+}][OH^-]^2=K_{sp}$

$$\left[Pb^{2+}\right]\left(10^{-6}\right)^2 = 1.203\times10^{-15}$$

So, $\left[Pb^{2+}\right] = \left(1.203\times10^{-15}\right)/10^{-12}$

$$\left[Pb^{2+}\right] = 1.203\times10^{-3}\,M$$

69. **(3)** Oxidation state of I in IO_3^-

$x+(-2)3=-1 \Rightarrow x=+5$

Oxidation state of I in IO_4^-

$x+(-2)4=-1 \Rightarrow x=+7$

Oxidation state of I in KI

$+1+x=0 \Rightarrow x=-1$

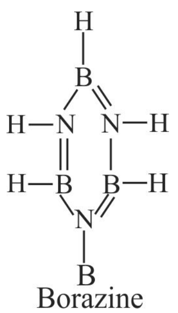
Oxidation state of I in I_2, $x=0$

Hence, the oxidation number of iodine in IO_3^-, IO_4^-, KI and I_2 are respectively $+5,+7,-1$ and 0.

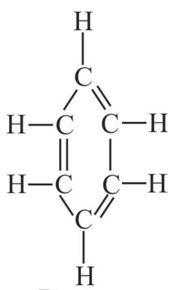
70. **(4)** When heated to about 200^0C, plaster of paris or gypsum is converted into anhydrous calcium sulphate which is known as dead burnt plaster because it does not set like plaster of paris when moistened with water.

$$\underset{\text{Plaster of paris}}{CaSO_4\cdot\frac{1}{2}H_2O} \xrightarrow{200^0C} \underset{\text{Dead burnt plaster}}{CaSO_4} + \frac{1}{2}H_2O$$

71. **(3)** Inorganic benzene is $B_3N_3H_6$. Due to its resemblance in structure and some physical and chemical properties with benzene. It is named as inorganic benzene or Borazine.



Borazine



Benzene

72. **(3)** 30 vol H_2O_2 means 1 ml of H_2O_2 aqueous solution liberates 30 ml of O_2 at STP.

73. **(2)** Nucleophiles are positive charge loving species or electron rich compounds. Lewis acids are those species which are electron deficient so nucleophiles cannot act as Lewis acids, they instead are Lewis bases.

74. **(1)** $3Na_4\left[Fe(CN)_6\right] + 4Fe^{3+}$

$\rightarrow Fe_4\left[Fe(CN)_6\right]_3 + 12Na^+$

Iron (III) hexacyanoferrate (II)
(Prussian blue)

75. **(3)** Cracking involves decomposition of higher alkanes to lower alkanes and alkenes.

76. **(2)** Green chemistry involves such reactions which reduce the use and production of hazardous or toxic chemicals to reduce pollution in environment.

77. **(2)** Anisoptropy is shown only by crystalline solids.

78. **(4)**

SCC : $a=2r$; BCC : $\sqrt{3}a=4r$; FCC : $\sqrt{2}a=4r$

79. **(3)** Some of the lattice points are unoccupied creating holes in the crystal. This defect is known as Schottky defect.

80. **(2)** Blood cells neither swell nor shrink in isotonic solution. The solutions having same osmatic pressure are called isotonic solutions.

81. **(4)** $M = \dfrac{K_b\times W_B}{\Delta T_b\times W_A\,(kg)} = \dfrac{0.52\times12.5}{0.80\times0.185}$

$$= \frac{52\times125}{0.80\times185} = 43.92\,g\,mol^{-1}$$

$(185\,g = 0.185\,kg)$

82. **(3)** Depression in freezing point is a colligative property. Thus, the compound which produces maximum ions has the least freezing point.

Sucrose $\rightarrow$ No ions

$NaCl \rightarrow Na^+ + Cl^{-1}$ (2 ions)

$CaCl_2 \rightarrow Ca^{2+} + 2Cl^-$ (3 ions)

Glucose $\rightarrow$ No ions.

Thus, 1% $CaCl_2$ has the least freezing point.

83. **(3)** $\Delta G^0 = -nFE^0$ (1)

$\Delta G^0 = -2.303\ RT\ \log K$ (2)

From eq (1) if E^0 is positive

then ΔG^0 is negative

In equation (2), ΔG^0 is negative

$\log K > 0 \Rightarrow K > 1$

84. **(3)** Due to the presence of acidic hydrogen in acetylene will give red copper acetylide ppt. with ammoniacal Cu_2Cl_2, whereas C_2H_4 will be neutral to ammoniacal Cu_2Cl_2.

85. **(3)** Due to lower value of reduction potential, alkali metals get oxidised easily. Hence, they behave as good reducing agents.

86. **(4)** According to Faraday's first law :

$$\text{(no. of moles) (x-factor)} = \frac{It}{96500}$$

$$0.1 \times 2 = \frac{1}{96500} \times 3 \times t$$

$t = 110$ min

87. **(2)** $2A \rightarrow B + C$

The rate equation of this equation may be expressed as $r = k\,[A]^0$

$r = k$

88. **(3)** $k = \frac{2.303}{10}\log\frac{100}{100-20}$

$$k = \frac{2.303}{10}\log\frac{100}{80}$$

$$k = \frac{2.303}{10}\log\frac{10}{8} = \frac{2.303}{10}[\log 10 - \log 8]$$

$$k = \frac{2.303}{10}[1 - 0.903]$$

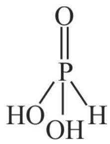
$$k = \frac{2.303}{10} \times 0.097 = 0.0223\,\text{min}^{-1}$$

89. **(4)** The difference in energy of intermediate complex and the average energy of reactants.

90. **(3)** If the concentration of adsorbate is more on the surface as compared to its concentration in the bulk phase then it is called positive adsorption.

91. **(4)** Adsorption causes decrease in surface energy which appears as heat. Thus adsorption is an exothermic process and hence lowers the activation energy of the reaction.

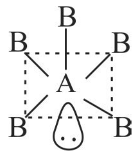
92. **(1)** Auto reduction is a process of extraction of Cu and Hg.

93. **(2)** 

H_3PO_3 is dibasic acid because it has two P - OH bonds and reducing due to the presence of one P - H bond.

94. **(3)** In AB_5, 'A' has 5 bp and 1 lp.

Hence ; the molecule is square pyramidal.



95. **(2)** $Fe_2(SO_4)_3 \xrightarrow{\Delta} Fe_2O_3 + 3SO_3$

96. **(4)** $Fe = 26 = [Ar]3d^6 4s^2$

$Fe^{3+} = [Ar]3d^5$

$\therefore n = 5$

$\mu = \sqrt{n(n+2)} = \sqrt{5(5+2)} = \sqrt{5(7)} = \sqrt{35}$

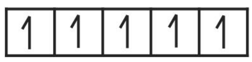
$= \sqrt{35} = 5.91 \approx 6\,B.M$

97. **(2)** The common oxidation state of lanthanoids is +3.

98. **(4)** Increase in atomic number will generally decrease basic character of oxides.

99. **(4)** Hetroleptic complexes are those which has more than one kind of ligands.

100. **(3)** The existence of two different coloured complexes is due to geometrical isomerism as cis and trans forms are present.

101. **(3)** $Fe^{3+} =$ 

CN^- is a strong ligand. Therefore pairing of electrons takes place.

$\therefore$ 

One unpaired electron is present in complex.

So, $\mu = \sqrt{n(n+2)}$

$= \sqrt{(1)(1+2)} = \sqrt{3} = 1.732\,\text{BM}$

102. (1) Rate of reaction $\propto$ No. of NO_2 groups
(EWG groups)

103. (4)

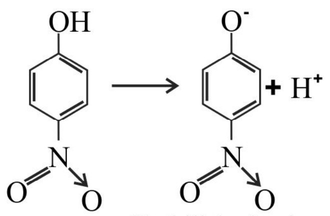
$$CH_3-CH_2-\underset{\substack{|\\CH_3}}{CH}-CH_2-Br + NH_3$$

$$CH_3-CH_2-\underset{\substack{|\\CH_3}}{CH}-CH_2NH_2$$

It is a nucleophilic substitution reaction.

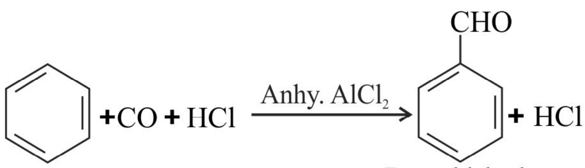
104. (3) $R-X+RO^-Na^+ \rightarrow R-O-R$
Represents Williamson ether synthesis. It involves nucleophilic substitution of the halide ion from the alkyl halide by the alkoxide ion by S_N2 mechanism.

105. (4)



(Stabilished anion because of - M and - I effect of -NO_2)

106. (4)



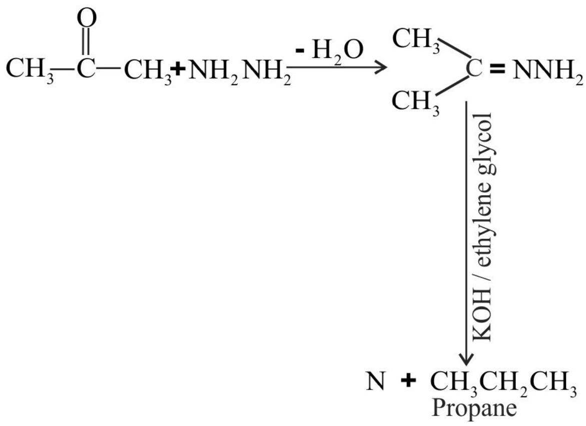
Benzaldehyde

107. (2) On treatment with an excess of I_2 in the presence of NaOH with aldehydes and ketones containing CH_3-CO- group produce a iodoform.

$$CH_3-CHO+3I_2 \xrightarrow{NaOH} CI_3CHO+3HI$$

$$\xrightarrow{NaOH} \underset{(\text{Yellw ppt})}{CHI_3} + HCOO^-Na^+$$

108. (1)



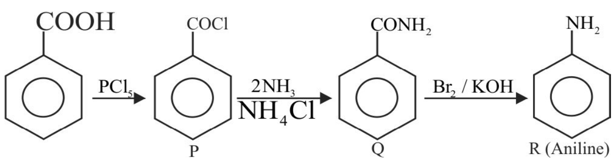
109. (1) $-Cl$ increases the polarity of $-\overset{\overset{O}{\|}}{C}-$ gorup because $-Cl$ is electron withdrawing in nature.

Therefore; the acidic strength of the compounds are;

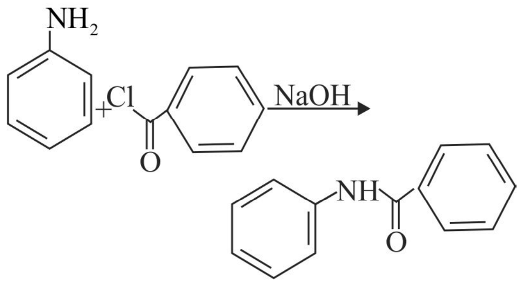
$Cl_3CCOOH > Cl_2CHCOOH$

$ClCH_2COOH > CH_3COOH$

110. (1)



111. (1)



Synthesis of amides from amines and acyl halides or anhydrides in the presence of a base in known as Schotten-Baumann reaction.

112. (4) It is structure lactose. It is made up of β-D galactose & β-D-glucose. On the left hand side is galactose & on the right hand is glucose. There is free anomeric $-OH$.

113. (2) At isoelectric point, amino acids are neutral. So, the solubility is low.

114. (1) During protein synthesis, replication of DNA occurs in a process called transcription.

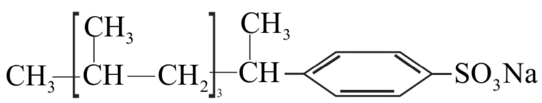
In translation RNA molecule is used to produce amino acids in ribosomes.

115. (4) Glycine and aminocaproic acid are monomers of Nylon-2-nylon-6 which is a biodegradable polymer.

116. (1) cis-polyisoprene is not a semisynthetic polymer while the other three (cellulose nitrate, cellulose acetate and vulcanised rubber) are semisynthetic polymers made from cellulose, cellulose and natural rubber respectively.

117. (1) Chloramphenicol is a broad spectrum antibiotic.

118. (3)



is an example of non - biodegradable

119. (2) E.F.M. of $CH_2O = 12+2+16 = 30$

$M.F. = n \times E.F.$

$$n = \frac{\text{Molar mass}}{\text{E.F.M.}} = \frac{180}{30} = 6$$

$\therefore M.F. = 6(CH_2O) = C_6H_{12}O_6$

120. (3) $Fe^{+3} = 3d^5 = \boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow} = 5$

$Cr^{3+} = 3d^3 = \boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}\boxed{} = 3$

$Ni^{2+} = 3d^8 = \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow} = 2$

$Cu^{2+} = 3d^9 = \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow} = 1$

SUBJECT : MATHEMATICS

121. (1) Given that, $A = \{x : x^2 = 1\} = \{-1, 1\}$ and $B = \{x : x^4 = 1\} = \{-1, 1, i, -i\}$

Now, $A - B = \phi$ and $B - A = \{-i, i\}$

$\therefore A \Delta B = (A-B) \cup (B-A) = \{-i, i\}$

122. (4) Conceptual

123. (2): For $f(x)$ to be defined, the following three inequalities must be satisfied.

(1) $\sin^{-1}(\log_2 x) \geq 0$

(2) $\cos(\sin x) \geq 0$

(3) $\left|\frac{1+x^2}{2x}\right| \leq 1$

For (1) : $x > 0, -1 \leq \log_2 x \leq 1$ and $\log_2 x \geq 0$

$\therefore$ $0 \leq \log_2 x \leq 1$ or $1 \leq x \leq 2$

For (2) : $\cos(\sin x) \geq 0$ is satified for all real x.

For (3) : $\left|\frac{1+x^2}{2x}\right| \leq 1 \Rightarrow \frac{1+|x|^2}{2|x|} - 1 \leq 0$

$$\Rightarrow \frac{(|x|-1)^2}{|x|} \leq 0 \text{ or } \frac{(|x|-1)^2}{|x|} = 0$$

$\therefore \quad |x| = 1$ or $x = \pm 1$

Hence, $f(x)$ is defined only, if $x = 1$

$\Rightarrow \quad D_f = \{1\}$

124. (2): Let $f(x) = e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$ period of

$x - [x] = 1$

$\because \quad \cos^2 \pi x = \dfrac{1 + \cos 2\pi x}{2}$

$\because$ period of $\cos^2 \pi x = 1$

$\because \cos^4 \pi x = \dfrac{1}{8}\{3 + 4\cos 2\pi x + \cos 4\pi x\}$

$\because$ Period of $\cos 2\pi x = \dfrac{2\pi}{2\pi} = 1$

$\because$ Period of $\cos 4\pi x = \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

$\therefore$ Period of $\cos^4 \pi x = LCM\left(1, \dfrac{1}{2}\right)$

$= \dfrac{LCM(1,1)}{HCF(1,2)} = \dfrac{1}{1} = 1$

$\therefore$ Period of $f(x) = 1$.

125. (1): $\log_{\cos x} \sin x + \log_{\sin x} \cos x = 2$

$\Rightarrow \quad \log_{\cos x} \sin x + \dfrac{1}{\log_{\cos x} \sin x} = 2$

$\Rightarrow \quad \left(\log_{\cos x} \sin x - 1\right)^2 = 0$

$\Rightarrow \quad \log_{\cos x} \sin x = 1$

$(\because \sin x > 0, \cos x < 0, \sin x \neq 1, \cos x \neq 1)$

$\Rightarrow \quad \sin x = \cos x$

$\Rightarrow \quad \tan x = 1, x = 2n\pi + \dfrac{\pi}{4}, n \in I$

126. (4) Sol. Since $T(1)$ is not true, which does not implies $T(2)$ is true. So the given statement is fails to satisfies induction basis so we can not assume induction hypothesis is true. Therefore it is also fails to satisfy induction step. Hence the option (4) is correct.

127. (3): $s_1 = \dfrac{1}{1-(-\alpha)} = \dfrac{1}{1+\alpha} \Rightarrow s_1 + \alpha s_1 = 1$

$\Rightarrow \alpha = \dfrac{1 - s_1}{s_1}$

Similarly, $\beta = \dfrac{1 - s_2}{s_2}$

$\therefore \quad 1 - \alpha\beta + \alpha^2\beta^2 + ... = \dfrac{1}{1-(-\alpha\beta)} = \dfrac{1}{1+\alpha\beta}$

$= \dfrac{1}{1 + \left(\dfrac{1-s_1}{s_1}\right)\left(\dfrac{1-s_2}{s_2}\right)}$

$= \dfrac{s_1 s_2}{1 - s_1 - s_2 + 2s_1 s_2}$

128. (3) We have, $x - 2 < 1$

$\Rightarrow x - 2 + 2 < 1 + 2$

$\Rightarrow x < 3$

$\therefore x \in (-\infty, 3)$

129. (1) We have

$\dfrac{|x-3|}{x-3} > 0$

Now, $\dfrac{|x-3|}{x-3} = \begin{cases} \dfrac{x-3}{x-3} = 1, & x \geq 3 \\ \dfrac{-(x-3)}{x-3} = -1, & x < 3 \end{cases}$

$\therefore \dfrac{|x-3|}{x-3} > 0$ only holds when $x \in (3, \infty)$

130. (1) Given, ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$

$$\Rightarrow \frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)2n}{(n+2)(n+1)n} = \frac{3}{5}$$

$$\Rightarrow 10(2n+1) = 3(n^2+3n+2)$$

$$\Rightarrow 3n^2 - 11n - 4 = 0$$

$$\Rightarrow n = 4$$

131. (2): Let $\left(\sqrt{2}+1\right)^6 = I + f$

where $I = \left[\left(\sqrt{2}+1\right)^6\right]$

$$0 < \sqrt{2} - 1 < 1 \Rightarrow 0 < \left(\sqrt{2}+1\right)^6 < 1 \Rightarrow 0 < G < 1$$

Now $\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$

$$= 2\left[{}^6C_0\left(\sqrt{2}\right)^6 + {}^6C_2\left(\sqrt{2}\right)^4 + {}^6C_4\left(\sqrt{2}\right)^2 + {}^6C_6\right]$$

$= 198$

$\therefore$ $I + f + G$ is an integer

$\therefore$ $f + G$ is an integer

Now $0 \le f < 1,\ 0 < G < 1 \Rightarrow 0 < f + G < 2$

$\Rightarrow f + G = 1$

$\therefore$ $I + f + G = 198 \Rightarrow I + 1 = 198$

$$\Rightarrow I = 197 \Rightarrow \left[\left(\sqrt{2}+1\right)^6\right] = 197$$

$\therefore$ Greatest integer not greater than $\left(\sqrt{2}+1\right)^6$ is 197.

132. (1): Let 'd' be the common difference of the *A.P.*

$$sum = \left(a_1^2 - a_2^2\right) + \left(a_3^2 - a_4^2\right) + ... + \left(a_{2n-1}^2 - a_{2n}^2\right)$$

$$= \left(a_1 - a_2\right)\left(a_1 + a_2\right) + \left(a_3 - a_4\right)\left(a_3 + a_4\right) + ...$$
$$+ \left(a_{2n-1} - a_{2n}\right)\left(a_{2n-1} + a_{2n}\right)$$

$$= (-d)\left(a_1 + a_2\right) + (-d)\left(a_3 + a_4\right) + ...$$
$$+ (-d)\left(a_{2n-1} + a_{2n}\right)$$

$$= -d\left[a_1 + a_2 + a_3 + a_4 + ... + a_{2n-1} + a_{2n}\right]$$

$$= (-d)\frac{2n}{2}\left(a_1 + a_{2n}\right) = -nd\left(a_1 + a_{2n}\right)$$

$$= -n\left(\frac{a_{2n} - a_1}{2n-1}\right)\left(a_1 + a_{2n}\right)$$

$$\left(\because a_{2n} = a_1 + (2n-1)d \text{ implies } d = \frac{a_{2n} - a_1}{2n-1}\right)$$

$$= \frac{n}{2n-1}\left(a_1^2 - a_{2n}^2\right).$$

133. (4): If the points are $P(0, 8/3), Q(1,3)$, $R(82,30)$, then

$$m_1 = \text{ slope of } PQ = \frac{3 - 8/3}{1-0} = \frac{1}{3};$$

$$m_2 = \text{ slope of } QR = \frac{30-3}{82-1} = \frac{1}{3}$$

Since, slope of PQ is the same as the slope of QR ,the three points P, Q, R are collinear.

134. (3) The equation of the common chord of the circles $(x-a)^2 + y^2 = a^2$ and $x^2 + (y+b)^2 = b^2$

The equation of the required circle is

$$\left\{(x-a)^2 + y^2 - a^2\right\} + \lambda(ax + by) = 0$$

$$\Rightarrow x^2 + y^2 + x(a\lambda - 2a) + \lambda by = 0 \qquad (1)$$

Since (1) is a diameter of circle (ii).

$$\therefore a\left\{-\frac{a\lambda-2a}{2}\right\}+b\left(-\frac{\lambda,b}{2}\right)=0\Rightarrow\lambda=\frac{2a^2}{a^2+b^2}$$

Putting the value of λ in (ii), the equation of the required circle is

$$\left(a^2+b^2\right)\left(x^2+y^2\right)=2ab\left(bx-ay\right)$$

135. (1) We know that, $l^2+m^2+n^2=1$

$$\Rightarrow\cos^2 60°+\cos^2 45°+\cos^2\alpha=1$$

$$\Rightarrow\frac{1}{4}+\frac{1}{2}=1-\cos^2\alpha$$

$$\Rightarrow\sin^2\alpha=\frac{3}{4}$$

136. (2): We have,

$$\lim_{h\to0}\frac{\left(a+h\right)^2\left[\sin\left(a+h\right)-\sin a\right]+\sin a\left[\left(a+h\right)^2-a^2\right]}{h}$$

$$=\lim_{h\to0}\left(a+h\right)^2\frac{\sin\left(a+h\right)-\sin a}{h}$$

$$+\left(\sin a\right)\lim_{h\to0}\frac{\left(a+h\right)^2-a^2}{h}$$

$$=a^2\left[\frac{d}{dx}\left(\sin x\right)\right]_{x=a}+\left(\sin a\right)\left[\frac{d}{dx}x^2\right]_{x=a}$$

$$=a^2\cos a+\left(\sin a\right)2a.$$

137. (2) In mathematical reasoning, while negating a statement we have to change all $\forall$ statements to $\exists$ statements and then change any one part of the statement. This question can be written as $\forall$ classmates love icecream. So, the negation of the given statement is $\exists$ some classmates who don't love icecream.

This answer is synonymous to the option there are some classmates who dont love icecream which is the correct answer.

138. (2) Conceptual

139. (3) The given numbers are 12, 23, 34, 45, 56, 67 and 78.

Here, $n=7$, and sum = 315

$$\therefore\ Mean=\frac{315}{7}=45$$

$\therefore$ Standard deviation

$$=\sqrt{\frac{(12-45)^2+(23-45)^2+(34-45)^2+(45-45)^2+(56-45)^2+(67-45)^2+(78-45)^2}{7}}$$

$$=\sqrt{\frac{(33)^2+(22)^2+(11)^2+0+(11)^2+(22)^2+(33)^2}{7}}$$

$$=\sqrt{\frac{2(1089+484+121)}{7}}=\sqrt{\frac{3388}{7}}$$

$$=\sqrt{484}=22$$

140. (2) Total no. of determinates $=2^4=16$.

Determinates with +ve and -ve values are

$$\begin{vmatrix}1&1\\0&1\end{vmatrix},\begin{vmatrix}1&0\\1&1\end{vmatrix},\begin{vmatrix}1&0\\0&1\end{vmatrix},\begin{vmatrix}1&1\\1&0\end{vmatrix},\begin{vmatrix}0&1\\1&1\end{vmatrix},\begin{vmatrix}0&1\\1&0\end{vmatrix}$$

$\therefore$ Required probability $=\frac{6}{16}=\frac{3}{8}$.

141. (4) A relation R on set A is known as void or empty relation if no element of a is related to any element of A

That is, $R=\phi$ on set $A\times A$

Let a set $A=\{1,3,5\}$

And a relation on set A is R

$$R=\{(a,b):a+b=9\}$$

Therefore $(a,b)\notin R$ and for any $a,b\in A$

$\therefore$ R is an empty set.

R is not reflexive as it does not contain (a, a) for any $a \in R$ Relation R does not contain any element of set A. So, relation R will be trivially symmetric and transitive.

142. (4)

$$(f+g)(x)=f(x)+g(x)=[x]+(x-[x])$$

$$=x\neq 0$$

$$(fg)(x)=f(x).g(x)=[x](x-[x])\neq 0$$

$$(f-g)(x)=f(x)-g(x)=[x]-(x-[x])$$

$$=2[x]-x\neq 0$$

$$(fog)(x)=f(g(x))=f(x-[x])$$

$$=[(x-[x])]=0$$

$$(\because\ x-[x]\in[0,1))$$

143. (2): The given expression

$$=\tan^{-1}\left(\frac{a_1-\frac{y}{x}}{1+a_1\cdot\frac{y}{x}}\right)+\left(\tan^{-1}a_2-\tan^{-1}a_1\right)$$

$$+\left(\tan^{-1}a_3-\tan^{-1}a_2\right)$$

$$+.....+\left(\tan^{-1}a_n-\tan^{-1}a_{n-1}\right)+\tan^{-1}\frac{1}{a_n}$$

$$=\tan^{-1}a_1-\tan^{-1}\frac{y}{x}+\tan^{-1}a_n-\tan^{-1}a_1$$

$$+\tan^{-1}\frac{1}{a_n}$$

$$=\left(\tan^{-1}a_n+\cot^{-1}a_n\right)-\tan^{-1}\frac{y}{x}$$

$$=\frac{\pi}{2}-\tan^{-1}\frac{y}{x}=\cot^{-1}\frac{y}{x}=\tan^{-1}\frac{x}{y}$$

144. (1) The resultant force is given by

$$\vec{F}=6\frac{\left(\hat{i}-2\hat{j}+2\hat{k}\right)}{\sqrt{1+4+4}}+7\frac{\left(2\hat{i}-3\hat{j}-6\hat{k}\right)}{\sqrt{4+9+36}}$$

$$=4\hat{i}-7\hat{j}-2\hat{k}$$

$\vec{d}=$ displacement

$$=\overrightarrow{PQ}=\left(5\hat{i}-\hat{j}+\hat{k}\right)-\left(2\hat{i}-\hat{j}-3\hat{k}\right)=3\hat{i}+4\hat{k}$$

$\therefore$ Work done $=\vec{F}\cdot\vec{d}=12+0-8=4$ units.

145. (2): The two rowed unit matrix is $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$\therefore\ \begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow\ \alpha^2+\beta\gamma=1\ \Rightarrow\ 1-\alpha^2-\beta\gamma=0$$

146. (3): For non-zero solution,

$$\begin{vmatrix}a^3 & (a+1)^3 & (a+2)^3\\ a & a+1 & a+2\\ 1 & 1 & 1\end{vmatrix}=0$$

$$\Rightarrow\ -\begin{vmatrix}1 & 1 & 1\\ a & a+1 & a+2\\ a^3 & (a+1)^3 & (a+2)^3\end{vmatrix}=0$$

$$\Rightarrow-(a-a-1)(a+1-a-2)(a+2-a)$$

$$\times(a+a+1+a+2)=0$$

$$\left[\because\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ x^3 & y^3 & z^3\end{vmatrix}=(x-y)(y-z)(z-x)(x+y+z)\right]$$

$$\Rightarrow-2(3a+3)=0\Rightarrow a=-1$$

147. (1): Applying $C_1\to C_1+C_2$

$$\Delta = \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

Expanding along R_2, we get

$2 + 4\sin 4\theta = 0$

$\Rightarrow \sin 4\theta = -\frac{1}{2} \Rightarrow 4\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$

148. (2) Applying $C_1 \to C_1 + C_2$

$$\begin{vmatrix} 1+\omega+\omega^2 & \omega^2 & \omega \\ \omega^2 & -\omega & \omega^2 \\ 1+\omega+\omega^2 & \omega & \omega^2 \end{vmatrix} = \begin{vmatrix} 0 & \omega^2 & \omega \\ \omega^2 & -\omega & \omega^2 \\ 0 & \omega & \omega^2 \end{vmatrix}$$

$[\because 1+\omega+\omega^2 = 0]$

Expanding along C_1 we get

$= -\omega^2(\omega^4 - \omega^2)$

$= -\omega^6 + \omega^4 = \omega - 1$ $\quad [\because \omega^3 = 1]$

149. (1): Since $x^3 - x^2 - 2004 = 0$ has three roots a, b, c

Hence $a+b+c = 1$ (1)

$ab+bc+ca = 0$ (2)

$abc = 2004$ (3)

Now, given determinant

$$\begin{vmatrix} a+c & a+b & b+c \\ a(b+c) & b(c+a) & c(a+b) \\ a^2+1 & b^2+1 & c^2+1 \end{vmatrix}$$

$$= \begin{vmatrix} 1-b & 1-c & 1-a \\ -bc & -ca & -ab \\ a^2+1 & b^2+1 & c^2+1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$= \begin{vmatrix} 2 & 1-c & 1-a \\ 0 & -ca & -ab \\ 4 & b^2+1 & c^2+1 \end{vmatrix} = -2a \begin{vmatrix} 1 & 1-c & 1-a \\ 0 & c & b \\ 2 & b^2+1 & c^2+1 \end{vmatrix}$$

Applying $R_3 \to R_3 - 2R_1$, we get

$$-2a \begin{vmatrix} 1 & 1-c & 1-a \\ 0 & c & b \\ 0 & b^2-1+2c & c^2-1+2a \end{vmatrix}$$

$= (-2a)(c^3 - c + 2ac - b^3 + b - 2bc)$

$= -2ac^3 + 2ac - 4a^2c + 2ab^3 - 2ab + 4abc \cdot$

150. (3): Given that , $f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

Continuity at $x = 0$

$LHL = \lim_{x \to 0^-} \frac{\tan x}{x} = \lim_{h \to 0} \frac{-\tan h}{-h} = 1$

$RHL = \lim_{x \to 0^+} \frac{\tan x}{x} = \lim_{h \to 0} \frac{\tan h}{h} = 1$

$\therefore \; LHL = RHL = f(0) = 1$

Hence, it is continuous at $x = 0$.

Differentiability at $x = 0$

$LHD = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$

$$= \lim_{h\to 0}\frac{\frac{\tan(-h)}{-h}-1}{-h} = \lim_{h\to 0}\frac{\tanh - h}{-h^2}$$

$$= \lim_{h\to 0}\frac{\sec^2 h - 1}{-2h} = \frac{1}{-2}\lim_{h\to 0}\frac{\tan^2 \text{h}}{h^2}\cdot h = 0$$

Similarly, $RHD = 0$

$\therefore$ $f(x)$ is differentiable at $x = 0$.

151. (3): Given that ,

$$f(x) = [x-3] + |x-4|$$

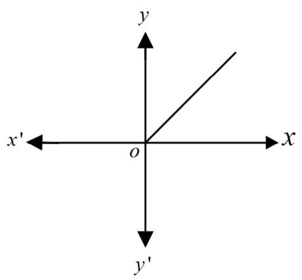
$$\therefore \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\left([x-3]+|x-4|\right)$$

$$= \lim_{h\to 0}\left([3-h-3]+|3-h-4|\right)$$

$$= \lim_{h\to 0}\left([-h]+1+h\right) = -1+1+0 = 0$$

152. (1): Given, $f(x) = x + |x|$

$$\therefore f(x) = \begin{cases} 2x, & x \ge 0 \\ 0, & x < 0 \end{cases}$$



It is clear from the graph, $f(x)$ is continuous for every value of x.

153. (2): In the neighbourhood of $x = -\frac{\pi}{6}$, we have

$$|x| = -x \text{ and } |\tan x| = -\tan x$$

$$\therefore f(x) = (-x)^{-\tan x}$$

$$\Rightarrow f(x) = e^{-\tan x\cdot\log(-x)} \Rightarrow f'(x) = (-x)^{-\tan x}$$

$$\left\{-\sec^2 x.\log(-x) - \frac{\tan x}{x}\right\}$$

$$\Rightarrow f'\left(-\frac{\pi}{6}\right) = \left(\frac{\pi}{6}\right)^{1/\sqrt{3}}\left\{-\frac{4}{3}\log\frac{\pi}{6} - \frac{2\sqrt{3}}{\pi}\right\}$$

$$\Rightarrow f'\left(-\frac{\pi}{6}\right) = \left(\frac{\pi}{6}\right)^{1/\sqrt{3}}\left\{\frac{4}{3}\log\frac{6}{\pi} - \frac{2\sqrt{3}}{\pi}\right\}$$

154. (3): We have, $f(x) = |x-1|$

$$\therefore g(x) = f(f(f(x)))$$

$$\Rightarrow g(x) = f(f(|x-1|)) \Rightarrow g(x) = f(||x-1|-1|)$$

$$\Rightarrow g(x) = \begin{cases} ||x-2|-1|, & if\ x \ge 1 \\ ||-x|-1|, & if\ x < 1 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} |x-3|, & if\ x \ge 2 \\ |2-x-1| & if\ 1 \le x < 2 \\ |x-1|, & if\ 0 \le x < 1 \\ |-x-1|, & if\ x < 0 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} x-3, & if\ x \ge 3 \\ 3-x, & if\ 2 \le x < 3 \\ x-1, & if\ 1 \le x < 2 \\ 1-x, & if\ 0 \le x < 1 \\ x+1, & if\ -1 \le x < 0 \\ -x-1, & if\ x < -1 \end{cases}$$

$$\Rightarrow g'(x) = \begin{cases} 1, & if\ x > 3 \\ -1, & if\ 2 < x < 3 \\ 1, & if\ 1 < x < 2 \\ -1, & if\ 0 < x < 1 \\ 1, & if\ -1 < x < 0 \\ -1, & if\ x < -1 \end{cases}$$

155. (2): Given curves are

$$x = t^2 + 3t - 8,\ y = 2t^2 - 2t - 5 \qquad (1)$$

when $x=2$, then $2=t^2+3t-8$

$\Rightarrow t^2+3t-10=0 \Rightarrow (t+5)(t-2)=0$

$\Rightarrow t=2,-5$ (2)

when $y=-1$, then

$-1=2t^2-2t-5$

$\Rightarrow 2t^2-2t-4=0$

$\Rightarrow (t+1)(t-2)=0$

$\Rightarrow t=-1,2$ (3)

From (2) and (3), we get $t=2$

On differentiating (1) w.r.t. t, we get

$$\frac{dx}{dt}=2t+3 \text{ and } \frac{dy}{dt}=4t-2$$

$$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{4t-2}{2t+3}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(t=2)}=\frac{8-2}{4+3}=\frac{6}{7}.$$

156. (1): Given that, $x=a(\theta+\sin\theta)$ and

$y=a(1-\cos\theta)$

$$\Rightarrow \frac{dx}{d\theta}=a(1+\cos\theta) \text{ and } \frac{dy}{d\theta}=a\sin\theta$$

$$\therefore \frac{dy}{dx}=\frac{a\sin\theta}{a(1+\cos\theta)}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}=\tan\frac{\theta}{2}$$

Now, length of sub tangent $=\left|\frac{y}{dy/dx}\right|$

$$\therefore ST=\frac{a(1-\cos\theta)}{\tan\frac{\theta}{2}}=a\frac{2\sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}=a\sin\theta$$

$\Rightarrow$ Length of sub tangent at $\theta=\frac{\pi}{2}$ is

$$ST=a\sin\frac{\pi}{2}=a$$

And length of sub normal $=\left|y\frac{dy}{dx}\right|$

$$\Rightarrow SN=a(1-\cos\theta)\tan\frac{\theta}{2}=2a\sin^2\frac{\theta}{2}\tan\frac{\theta}{2}$$

$\Rightarrow$ Length of sub normal at $\theta=\frac{\pi}{2}$ is

$$SN=a.2.\frac{1}{2}=a$$

Hence, $SN=ST$

157. (1): $y^2=4ax$ (1)

Vertex $=(0,0); m=\tan\theta$

Hence, equation of the chord is $y=(\tan\theta)x$

Put in (1), we have $\tan^2\theta(x^2)=4ax$

$\Rightarrow x=4a/\tan^2\theta$ and $y=4a/\tan\theta$

= Length of the chord

$$=\sqrt{\left(\frac{4a}{\tan^2\theta}\right)^2+\left(\frac{4a}{\tan\theta}\right)^2}=\frac{4a}{\tan^2\theta}\sqrt{1+\tan^2\theta}$$

$$=\frac{4a\cos^2\theta}{\sin^2\theta}\times\frac{1}{\cos\theta}=\frac{4a\cos\theta}{\sin^2\theta}.$$

158. (3) $f(x)=2x^3-9ax^2+12a^2x+1$

$f'(x)=6x^2-18ax+12a^2$

For maximum or minimum, $f'(x)=0$

$\Rightarrow 6x^2-18ax+12a^2=0$

$$x=\frac{18a\pm\sqrt{324a^2-288a^2}}{2\times 6}$$

$$= \frac{18a \pm \sqrt{36a^2}}{12} = \frac{18a \pm 6a}{12} = 2a, a$$

Now, $f''(x) = 12x - 18a$

At $x = 2a$.

$f''(x) = 24a - 18a$

$= 6a > 0$, min

$q = 2a$

At $x = a$,

$p = a$

$p^3 = q$

$a^3 = 2a$

$a(a^2 - 2) = 0$

$a = 0$ (or) $a = \sqrt{2}$

but $a > 0$ given

159. (3): $\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}$

$$= \int_0^1 \frac{dx}{(x + \cos\alpha)^2 + \sin^2\alpha}$$

$$= \frac{1}{\sin\alpha}\left[\tan^{-1}\left(\frac{x + \cos\alpha}{\sin\alpha}\right)\right]_0^1$$

$$= \frac{1}{\sin\alpha}\left\{\tan^{-1}\left(\frac{1 + \cos\alpha}{\sin\alpha}\right) - \tan^{-1}\left(\frac{\cos\alpha}{\sin\alpha}\right)\right\}$$

$$= \frac{1}{\sin\alpha}\left[\frac{\pi}{2} - \cot^{-1}\left(\cot\frac{\alpha}{2}\right) - \left\{\frac{\pi}{2} - \cot^{-1}(\cot\alpha)\right\}\right]$$

$$= \frac{1}{\sin\alpha}\left(-\frac{\alpha}{2} + \alpha\right) = \frac{\alpha}{2\sin\alpha}.$$

160. (2): $I_n = \int (\log x)^n \cdot 1dx$

$$= (\log x)^n \cdot x - \int n(\log x)^{n-1} \cdot \frac{1}{x} \cdot xdx$$

$$= x(\log x)^n - nI_{n-1} \Rightarrow I_n + nI_{n-1} = x(\log x)$$

161. (1): $f(x) = \int_0^x e^t(1 + t)dt = xe^x$

(on integrating by parts)

$\therefore$ Maximum value of $f(x)$ in $[1, 2]$ is

$f(2) = 2e^2$

$\therefore$ Minimum value of $f(x)$ is $f(1) = e$

$\therefore$ Difference $= 2e^2 - e$.

162. (4): $\sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27} x\,dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x\,dx$

In the first summation, put $x = -t$

$$= \sum_{n=1}^{10} \int_{2n+1}^{2n} \sin^{27}(-t)(-dt) + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x\,dx$$

$$= -\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} t\,dt + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x\,dx$$

$= 0$.

163. (2) Given that, $I = \int_{-1}^{0} \frac{dx}{x^2 + 2x + 2}$

$$= I = \int_{-1}^{0} \frac{dx}{(x+1)^2 + 1} = [\tan^{-1}(x+1)]_{-1}^{0}$$

$$= [\tan^{-1}(1) - \tan^{-1}(0)] = \frac{\pi}{4}$$

164. (2) Required area $= \int_{\frac{\pi}{2}}^{\pi} \sin^2 x dx$

$$= \int_{\frac{\pi}{2}}^{\pi} \left[\frac{1 - \cos 2x}{2}\right] dx = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} (1 - \cos 2x) dx$$

$$=\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_{\frac{\pi}{2}}^{\pi} \qquad =\frac{1}{2}\left[(\pi-0)-\left(\frac{\pi}{2}-0\right)\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{2}\right]=\frac{\pi}{4}$$

165. (1) Given, system of equation is

$$y=e^{x}(A\cos x+B\sin x)$$

$$\Rightarrow \frac{dy}{dx}=e^{x}(-A\sin x+B\cos x)+y \quad ...(1)$$

$$\Rightarrow \frac{d^2y}{dx^2}=e^{x}(-A\sin x+B\cos x)+e^{x}$$

$$[-A\cos x-B\sin x]+\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}-y\right)-y+\frac{dy}{dx} \qquad \text{[by Eq. (1)]}$$

$$\Rightarrow \frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$$

$$\Rightarrow y''-2y'+2y=0$$

This is required differential equation.

166. (1): We have,

$$dy+\{y\phi'(x)-\phi(x)\phi'(x)\}dx=0$$

$$\Rightarrow \frac{dy}{dx}+\phi'(x)\cdot y=\phi(x)\phi'(x) \qquad (1)$$

This is a linear differential equation with

$$I.F.=e^{\int \phi'(x)dx}=e^{\phi(x)}$$

Multiplying (1) by $\phi(x)$ and integrating, we get

$$ye^{\phi(x)}=\int \phi(x)\phi'(x)e^{\phi(x)}dx$$

$$\Rightarrow ye^{\phi(x)}=\int e^{\phi(x)}\phi(x)\phi'(x)\,dx$$

$$\Rightarrow ye^{\phi(x)}=\int \underbrace{\phi(x)}_{I}\underbrace{e^{\phi(x)}\phi'(x)}_{II}\,dx$$

$$\Rightarrow ye^{\phi(x)}=\phi(x)e^{\phi(x)}-\int \phi'(x)e^{\phi(x)}dx$$

$$\Rightarrow ye^{\phi(x)}=\phi(x)e^{\phi(x)}-e^{\phi(x)}+c$$

$$\Rightarrow y=\{\phi(x)-1\}+ce^{-\phi(x)}$$

Hence, (1) is the correct answer.

167. (2): Given,equation of family of curves is

$$y^2=2c\left(x+\sqrt{c}\right)$$

On differentiating w.r.t. x, we get

$$2yy_1=2c\Rightarrow c=yy_1$$

Then, $y^2=2yy_1\left(x+\sqrt{yy_1}\right)$

Then, $y^2-2yy_1x=(\sqrt{yy_1})2yy_1$

On squaring both sides, we get

$$\left(y^2-2xyy_1\right)^2=4\left(yy_1\right)^3$$

$\therefore$ The degree of above equation is 3 and order is 1.

168. (3) $y=xe^{cx}$

$$\frac{y}{x}=e^{cx}$$

taking log both the sides

$$\log\frac{y}{x}=\log^{cx}$$

$$\log\frac{y}{x}=cx\log e \quad [\because \log a^n=n\log a]$$

$$\frac{1}{x}\log\frac{y}{x}=c[\because \log e=1]$$

$$c=\frac{1}{x}\log\frac{y}{x}$$

Differentiating both the sides w.r.t.x using chain rule

$$0=\frac{-1}{x^2}\log\frac{y}{x}+\frac{\frac{1}{x}\frac{d}{dx}\left(\frac{y}{x}\right)}{\frac{y}{x}}\quad [\because c \text{ is Constant}$$

$$\frac{1}{x^2}\log\frac{y}{x}=\frac{1}{y}[\frac{x\frac{dy}{dx}-y}{x^2}]\quad \text{[using quotient rule]}$$

$$\log\frac{y}{x}=\frac{x}{y}\frac{dy}{dx}-1$$

$$\log\frac{y}{x}+1=\frac{x}{y}\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{y}{x}[\log\frac{y}{x}+1]$$

169. (2): Taking A as the origin. Let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively. Equations of lines BF and AC are

$$\vec{r}=\vec{b}+\lambda\left(\frac{\vec{b}+\vec{c}}{4}-\vec{b}\right) \text{ and } \vec{r}=\vec{0}+\mu\vec{c}$$

respectively. For the point of intersection F, we have

$$\vec{b}+\lambda\left(\frac{\vec{c}-3\vec{b}}{4}\right)=\mu\vec{c}$$

$$\Rightarrow 1-\frac{3\lambda}{4}=0 \text{ and } \frac{\lambda}{4}=\mu\Rightarrow\lambda=\frac{4}{3} \text{ and } \mu=\frac{1}{3}$$

Therefore, the position vector of $\vec{F}$ is $\vec{r}=\frac{1}{3}\vec{c}$

Now, $\overrightarrow{AF}=\frac{\vec{c}}{3}\Rightarrow\overrightarrow{AF}=\frac{1}{3}\overrightarrow{AC}$

Hence, $AF:AC=\frac{1}{3}:1=\frac{1}{3}$

170. (4): Since, the vectors $\vec{a}=2\hat{i}+\log_3 x\hat{j}+a\hat{k}$ and $\vec{b}=-3\hat{i}+a\log_3 x\hat{j}+\log_3 x\hat{k}$ are inclined at acute angle. Therefore, $\vec{a}\cdot\vec{b}>0$

$\Rightarrow -6+a(\log_3 x)^2+a\log_3 x>0$ for all $x>0$

$\Rightarrow -6+ay^2+ay>0$, where $y=\log_3 x$

$\Rightarrow ay^2+ay-6>0$ for all y

$\Rightarrow a>0$ and $a^2+24a<0\Rightarrow a>0$ and $a\in(-24,0)$

But this is not possible. Hence, the vectors $\vec{a}$ and $\vec{b}$ are not inclined at acute angle for any real value of a.

171. (4): Since, $\vec{a},\vec{b},\vec{c}$ are non-coplaner vectors,

therefore $\left[\vec{a},\vec{b},\vec{c}\right]\neq0\Rightarrow\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\neq0\Rightarrow\Delta\neq0$

where $\Delta=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$

Now, $\begin{vmatrix}a & a^2 & 1+a^3\\ b & b^2 & 1+b^3\\ c & c^2 & 1+c^3\end{vmatrix}=0$

$$\Rightarrow\begin{vmatrix}a & a^2 & 1\\ b & b^2 & 1\\ c & c^2 & 1\end{vmatrix}+\begin{vmatrix}a & a^2 & a^3\\ b & b^2 & b^3\\ c & c^2 & c^3\end{vmatrix}=0$$

$\Rightarrow\Delta(1+abc)=0\Rightarrow abc=-1\qquad[\because\Delta\neq0]$

172. (3): Let $\vec{n}_1$ and $\vec{n}_2$ be the vectors normal to the plane determined by $\hat{i},\hat{i}-\hat{j}$ and $\hat{i}+\hat{j},\hat{i}-\hat{k}$ respectively.

$\therefore\ \vec{n}_1 = \hat{i} \times (\hat{i} - \hat{j}) = -\hat{k}$

and $\vec{n}_2 = (\hat{i} + \hat{j}) \times (\hat{i} - \hat{k}) = -\hat{i} + \hat{j} - \hat{k}$

Since, $\vec{a}$ is parallel to the line of intersection of the planes determined by the given planes.

$\therefore \vec{a} \parallel (\vec{n}_1 \times \vec{n}_2)$

$\Rightarrow\ \vec{a} = \lambda(\vec{n}_1 \times \vec{n}_2) = \lambda(\hat{i} + \hat{j})$

Let θ be the angle between $\vec{a}$ and $\hat{i} + 2\hat{j} - 2\hat{k}$

$$\therefore\ \cos\theta = \frac{\lambda(\hat{i} + \hat{j}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})}{\sqrt{\lambda^2 + \lambda^2}\sqrt{1 + 4 + 4}}$$

$$= \frac{\lambda(1+2)}{3\lambda\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow\ \theta = \frac{\pi}{4}$$

173. (2) Given that,

Direction ratios of first line are 2, 2, 1

Direction ratios of second line joining the points

$(3,1,4)(7,2,12)$ and $(7-3),(2-1),(12-4)$

i.e. 4, 1, 8

Let θ be the angle between the two lines.

$$\therefore \cos\theta = \frac{2(4) + 2(1) + 1(8)}{\sqrt{(2)^2 + (2)^2 + (1)^2}\sqrt{(4)^2 + (1)^2 + 8^2}}$$

$$\left[\therefore \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right]$$

$$= \frac{8+2+8}{3\times 9} = \frac{18}{3\times 9} = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

174. (2): If $OA = a,\ OB = b$ and $OC = c,$ then centroid of $\Delta\ ABC$ is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (3, 3, 3)$

$\Rightarrow a = b = c = 9$ and hence the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \Rightarrow \frac{x}{9} + \frac{y}{9} + \frac{z}{9} = 1$.

175. (4): Radius of the sphere = Distance of $(3,6,-4)$ from the given plane = 4. Hence, the required equation of the sphere is

$$(x-3)^2 + (y-6)^2 + (z+4)^2 = (4)^2$$

$$\Rightarrow\ x^2 + y^2 + z^2 - 6x - 12y + 8z + 45 = 0$$

176. (3) Conceptual

177. (2) Given that,

$x_1 = 1, y_1 = 2, z_1 = 3$

$x_2 = -1, y_2 = 4, z_2 = 2$

and $x_3 = 3, y_3 = 1, z_3 = 1$

Equation of plane passing through these points is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ -2 & 2 & -1 \\ 2 & -1 & -2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(-4-1) - (y-2)(4+2)$$
$$+(z-3)(2-4) = 0$$

$$\Rightarrow (x-1)(-5) - (y-2)(6) + (z-3)(-2) = 0$$

$$\Rightarrow -5x + 5 - 6y + 12 - 2z + 6 = 0$$

$$\Rightarrow -5x - 6y - 2z + 23 = 0$$

$$\Rightarrow 5x + 6y + 2z - 23 = 0$$

$$\Rightarrow 5x + 6y + 2z = 23$$

178. (4) Let E_1 = Event that student know lession I

E_2 = Event that student know lession II. Now, according to the question,

$P(E_1) = 0.60,\ P(E_2) = 0.40$

$P(E_1 \cap E_2) = 0.20$

$\therefore$ Required probability = $P(E'_1 \cap E'_2)$

$P(E_1 \cup E_2)'$

$= 1 - P(E_1 \cup E_2)$

$= 1 - [P(E_1) + P(E_2) - P(E_1 \cap E_2)]$

$= 1 - [0.60 + 0.40 - 0.20]$

$= 1 - [0.80]$

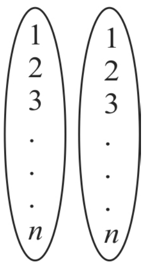
$= 0.20 \quad = \dfrac{20}{100}$

$= \dfrac{1}{5}$

179. (3): Total number of cases $= n^n$

$f : A \to A$

We next find the number of favourable cases. For the first element we have n choices. For the second element we have $(n-1)$ choices and so on.



$\therefore$ The number of favourable cases

$= n(n-1)(n-2).....2.1$

$= n!$

$\therefore$ Required probability $= \dfrac{n!}{n^n} = \dfrac{(n-1)!}{n^{n-1}}$

180. (4) We have, 2 black and 2 red balls

S = Selecting two balls

$\therefore \; n(S) = {}^4C_2$

E = Event that two balls will have no black balls
= selecting 2 red balls

$\therefore \; n(E) = {}^2C_2$

$\therefore$ Required probability = P(E)

$= \dfrac{n(E)}{n(S)}$

$= \dfrac{{}^2C_2}{{}^4C_2} = \dfrac{1}{6}$