Subject	Topic	Mock Test – 05	Date
C + M + P	Complete Syllabus	CET – 12 – CT C1220230506	6th May 2023

Max. Marks: 180 **Duration: 3 Hours**

1. **This paper consists of 180 questions with 3 parts of Chemistry, Mathematics and Physics**

   - **Chemistry:** *(Q. No. 1 to 60) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Mathematics:** *(Q. No. 61 to 120) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Physics:** *(Q. No. 121 to 180) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*

2. **The OMR sheet for 200 questions is to be used**

3. **Use of calculators and log tables is prohibited**

4. **Darken the appropriate bubble using a pen in the OMR sheet provided to you. Once entered, the answer cannot be changed. Any corrections or modifications will automatically draw a penalty of 1 mark**

5. **No clarification will be entertained during the examination. Doubts in the paper can be reported to the coordinator after the exam**

6. **If the details in the OMR Sheet are not filled, If the OMR sheet is mutilated, torn, white Ink used, the circles filled and scratched, then the OMR sheet will not be graded**

   All the best!!

Useful Data

At. Wt.:

$N = 14;\ O = 16;\ H = 1;\ S = 32;\ Cl = 35.5;\ Mn = 55;\ Na = 23;\ C = 12;\ Ag = 108;\ K = 39;\ Fe = 56;\ Pb = 207$

Physical Constants:

$h = 6.626 \times 10^{-34}\,\text{Js},\ \text{N}_\text{a} = 6.022 \times 10^{23}\,\text{mol}^{-1},\ \text{c} = 2.998 \times 10^{8}\,\text{m s}^{-1},\ \text{m}_\text{e} = 9.1 \times 10^{-31}\,\text{kg},\ R = 8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$

Chemistry

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. **60 x 1 = 60**

1. A phosphorus oxide has 43.6% phosphorus (at, mass = 31).

   The empirical formula is

   Options:

   (a) P_2O_5

   (b) P_2O_3

   (c) P_4O_6

   (d) PO_2

   Sol:

Element	Percentage	Atomic ratio	Simplest ratio	Whole no ratio
P	43.6%	$\frac{43.6}{31}=1.41$	$\frac{1.41}{1.41}=1$	2
O	56.4%	$\frac{56.4}{16}=3.52$	$\frac{3.52}{1.41}=2.5$	5

   Formula $= P_2O_5$

   Ans: (a)

2. Which of the following orbital designations is not correct corresponding to quantum number?

   Options:

   (a) $n=5 \quad \ell=2 \quad \rightarrow 5d$

   (b) $n=2 \quad \ell=0 \quad \rightarrow 2s$

   (c) $n=4 \quad \ell=3 \quad \rightarrow 4f$

   (d) $n=7 \quad \ell=2 \quad \rightarrow 7p$

   Sol: When $n=7, \ \ell=2 \rightarrow 7d$

   Ans: (d)

3. Which of the following families have largest negative electron gain enthalpy values?

   Options:

   (a) Alkali metals

   (b) Noble gases

   (c) Halogens

   (d) Alkaline earth metals

   Sol: Halogens (17^{th} group) have largest negative electron gain enthalpy values.

   Ans: (c)

4. The molecule/ion having pyramidal shape is

Options:

(a) PCl_3

(b) SO_3

(c) CO_3^{2-}

(d) NH_4^+

Sol: PCl_3 is having pyramidal shape.

Ans: (a)

5. Identify a molecule which doesn't exist.

Options:

(a) C_2

(b) O_2

(c) He_2

(d) Li_2

Sol: He_2 with bond order zero does not exist

Ans: (c)

6. A real gas obeying Vander Waal's equation $\left(P+\frac{an^2}{V^2}\right)(V-b)=nRT$

will closely resemble an ideal gas if

Options:

(a) the constants a and b are large

(b) a is large and b is small

(c) a is small and b is large

(d) a and b are both small

Sol: When the constant a and b are both small.

Ans: (d)

7. A certain reaction is at equilibrium at 355k and the enthalpy change for the reaction is 213kJ. The value of $\Delta S\left(\text{in } \text{Jk}^{-1}\text{mol}^{-1}\right)$ for the reaction is

Options:

(a) 55.0

(b) 60.0

(c) 68.5

(d) 120.0

Sol: At equilibrium $\Delta G = 0$

$\therefore \Delta H = T\Delta S \quad (\because \Delta G = \Delta H - T\Delta S)$

$\therefore \Delta S = \frac{\Delta H}{T} = \frac{21.3\times1000}{355} = 60\,JK^{-1}\ mol^{-1}$

Ans: (b)

8. $H_2(g)+I_2(g) \rightleftharpoons 2HI(g) \qquad \Delta H=12.40\,kcal$

According to the reaction, heat of formation of HI will be

Options:

(a) $12.4\,kcal$

(b) $-12.4\,kcal$

(c) $-6.20\,kcal$

(d) $6.20\,kcal$

Sol: Formation reaction for $HI, \frac{1}{2}H_2(g)+\frac{1}{2}I_2(g) \rightleftharpoons HI(g)$ ΔH=6.20kcal

Ans: (d)

9. The ratio of K_p / K_c for the reaction $CO(g)+\frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$ is

Options:

(a) 1

(b) RT

(c) $(RT)^{\frac{1}{2}}$

(d) $(RT)^{-\frac{1}{2}}$

Sol: $K_p = K_c(RT)^{\Delta n}$

$\Delta n = 1-1\frac{1}{2} = -\frac{1}{2}$

$\therefore \frac{K_p}{K_c} = (RT)^{-\frac{1}{2}}$

Ans: (d)

10. Which of the following are Lewis acids?

Options:

(a) PH_3 and BCl_3

(b) $AlCl_3$ and $SiCl_4$

(c) PH_3 and $SiCl_4$

(d) BCl_3 and $AlCl_3$

Sol: BCl_3 and $AlCl_3$ are electron deficient compounds and behave like Lewis acids.

Ans: (d)

11. In the ionic equation for the reaction $IO_3^- + 6H^+ + ae^{-1} \rightarrow I^- + 3H_2O$ the value of a is

Options:

(a) 2

(b) 4

(c) 6

(d) 10

Sol: $IO_3^- + 6H^+ + 6e^{-1} \rightarrow I^- + 3H_2O$

Ans: (c)

12. Hydrolysis of phosphorous tribromide gives

Options:

(a) $POBr_3$

(b) H_3PO_3

(c) H_3PO_4

(d) P_2O_5

Sol: $PBr_3 + 3H_2O \rightarrow H_3PO_3 + 3HBr$

Ans: (b)

13. Which of the following metals dissolves in sodium hydroxide with the evolution of hydrogen?

Options:

(a) Berylium

(b) Magnesium

(c) Calcium

(d) Strontium

Sol: Berylium is amphoteric.

Ans: (a)

14. In terms of relative stability, which of the following is in general wrong

Options:

(a) tertiary free radicals are more stable than secondary

(b) secondary free radicals are more staple than primary

(c) tertiary carbocation is less stable than secondary

(d) secondary carbocation is less stable than primary

Sol: Tertiary carbocation is more stable than secondary

Ans: (c)

15. Ethylene reacts with alkaline $KMnO_4$ to give

Options:

(a) Acetaldehyde

(b) Ethylene glycol

(c) Formaldehyde

(d) Ethylene oxide

Sol: $CH_2 = CH_2 \xrightarrow{Alk.KMnO_4} \underset{OH}{\overset{|}{CH_2}} - \underset{OH}{\overset{|}{CH_2}}$

Ans: (b)

16. According to Huckel rate the aromatic compounds must have delocalised Π electrons equal to

Options:

(a) $(4n+1)$

(b) $(4n+2)$

(c) $4n$

(d) $(2n+2)$

Sol: $4n+2$

Ans: (b)

17. Ozonolysis of an organic compound $'A'$ produces acetone and propionaldehyde in equimolar mixture. Identify $'A'$ from thr following compunds.

Options:

(a) Pent $-1-$ ene

(b) $2-$ Methylpent $-1-$ ene

(c) 2 Methylpent $-2-$ ene

(d) $2-$ Methylpent $-1-$ ene

Sol: $CH_3 - \underset{CH_3}{\overset{|}{C}} = CH - CH_2 - CH_3 \xrightarrow{\text{Ozonolysis}} CH_3COCH_3 + CH_3CH_2CHO$

Ans: (c)

18. The efficiency of packing is 68% in

Options:

(a) *hcp* structure

(b) *ccp* structure

(c) *fcc* structure

(d) *bcc* structure

Sol: *bcc* structure

Ans: (d)

19. Which of the following does not belong to covalent crystalline solid?

Options:

(a) Quartz

(b) SiC

(c) Iodine

(d) Diamond

Sol: Iodine is a molecular solid whereas quartz, silicon carbide and diamond are covalent solids.

Ans: (c)

20. An element crystallizes in fcc lattice and edge length of unit cell is 400pm. of density of unit cell is 11.3g cm^{-3}, then atomic mass of the element is

Options:

(a) $215g\ \text{mol}^{-1}$

(b) $431g\ \text{mol}^{-1}$

(c) $108g\ \text{mol}^{-1}$

(d) $98.6g\ \text{mol}$

Sol: Edge length $a = 400\,pm = 400\times10^{-10}\ \text{cm}^3$

$$d = \frac{ZM}{a^3 N_A} \quad \text{or} \quad M = \frac{d\times a^3\times N_A}{Z} = \frac{11.3\times\left(400\times10^{-10}\right)^3\times6.00\times10^{23}}{4} = 108.5g\ \text{mol}^{-1}$$

Ans: (c)

21. The ratio of the value of any colligative property for $CaCl_2$ solution to that of sugar solution under equal concentration is nearly

Options:

(a) 1.0

(b) 0.33

(c) 3.0

(d) 2.5

Sol: vant Hoff factor i for $CaCl_2 = 3$

vant Hoff facto i for sugar $= 1$

$\therefore$ Ratio = 3.0

Ans: (c)

22. The freezing point of a solution containing $36g$ of a compound having empirical formula CH_2 in $1200g$ of water is found to be $-0.93°$. Molecular formula of the compound is $\left(\text{Kf} = 1.86K\,\text{kg mol}^{-1}\right)$

Options:

(a) CH_2O

(b) $C_2H_4O_2$

(c) $C_3H_6O_3$

(d) $C_4H_8O_2$

Sol: $\Delta T_f = 0-(-0.93) = 0.93$

$$M_B = \frac{Kf \times 1000 \times W_2}{\Delta T_f \times W_1} = \frac{1.86 \times 1000 \times 36}{0.93 \times 1200} = 60$$

Empirical formula mass $= 12+2+16 = 30$

$$\therefore n = \frac{60}{30} = 2$$

$\therefore$ Molecular formula = $C_2H_4O_2$

Ans: (b)

23. All form ideal solution except

Options:

(a) C_6H_6 and $C_6H_5CH_3$

(b) C_2H_5Br and C_2H_5I

(c) C_6H_5Cl and C_6H_5Br

(d) C_2H_5I and C_2H_5OH

Sol: C_2H_5I and C_2H_5OH form non-ideal solution

Ans: (d)

24. The electrode potential of a silver electrode dipped in a 0.01M solution of silver nitrate at $25°\text{C}$ $\left(E°_{Ag^+/Ag} = 0.80\text{V}\right)$

Options:

(a) 0.0741V

(b) 0.059 V

(c) 0.741V

(d) 0.859 V

Sol: $E_{Ag^+/Ag} = E°_{Ag^+/Ag} + \frac{0.059}{n}\log\left[M^+\right]$

$$= 0.80 + \frac{0.059}{1}\log 10^{-1}$$

$$0.80 - 0.059 = 0.741$$

Ans: (c)

25. In $H_2 - O_2$ fuel cell, the reaction occurring at cathode is

Options:

(a) $H^+ + OH^{-1} \rightarrow H_2O(l)$

(b) $H^+ + e^{-1} \rightarrow \frac{1}{2}H_2$

(c) $O_2 + 2H_2O + 4e^{-1} \rightarrow 4OH^{-1}$

(d) $2H_2 + O_2 \rightarrow 2H_2O(l)$

Sol: $O_2 + 2H_2O + 4e^{-1} \rightarrow 4OH^-$

Ans: (c)

26. If $\wedge_C$ for NH_4OH is $11.5\Omega^{-1}\,\text{cm}^{-2}\,\text{mol}^{-1}$, its degree of dissociation would be (given that $\lambda^\circ_{NH_4+=73.4}$ and $\lambda^\circ_{OH^-} = 197.6\Omega^{-1}\,\text{cm}^2\,\text{mol}^{-1}$)

Options:

(a) 0.0848

(b) 0.0424

(c) 0.0212

(d) 0.004

Sol: $\alpha = \frac{\wedge_c}{\wedge_c^\circ} = \frac{11.5}{73.4+197.6} = \frac{11.5}{271} = 0.0424$

Ans: (b)

27. How many hours does it take to reduce 1mol of Fe^{3+} to Fe^{2+} with $2A$ current?

Options:

(a) 35

(b) 20.0

(c) 26.8

(d) 13.4

Sol: $Fe^{3+} + e^- \rightarrow Fe^{2+}$

For $1\text{mole}, 96500C$ is required.

Change=current $\times$ time

$96500 = 2 \times \text{time}$ or $\text{time} = \frac{96500}{2} = 482505 = \frac{482505}{60\times60} = 13.4\text{hr}$

Ans: (d)

28. For the reaction: $3ClO^- \rightarrow 3Cl0_3^- + 2Cl^-$ various steps are

$Cl0^- + Cl0^- \rightarrow Cl0_2^- + Cl^-$ (slow)

$ClO_2^- + ClO^- \rightarrow ClO_3^- + Cl^-$ (fast)

The order of the reaction is

Options:

(a) 1

(b) 2

(c) 0

(d) $\frac{3}{2}$

Sol: Rate $= k\left(ClO^-\right)^2$

order $= 2$

Ans: (b)

29. If the half-life period for a reaction in A is 100mins. How long will it take $[A]$ to reach 25% of its initial concentration?

Options:

(a) $50\,\text{min}$

(b) $250\,\text{min}$

(c) $200\,\text{min}$

(d) $500\,\text{min}$

Sol: Given $t_{\frac{1}{2}} = 100\text{mins}$

$$100 \xrightarrow[1t_{\frac{1}{2}}]{} 50 \xrightarrow[2t_{\frac{1}{2}}]{} 25$$

$\therefore t_{75\%} = 2 \times t_{\frac{1}{2}} = 2 \times 100 = 200\,\text{min}$

Ans: (c)

30. A reaction having equal energies of activation for forward and reverse relations has

Options:

(a) $\Delta G = 0$

(b) $\Delta H = 0$

(c) $\Delta H = \Delta G = \Delta S = 0$

(d) $\Delta S = 0$

Sol: $\Delta H = E_f - E_b = 0$

Ans: (b)

31. A reaction has rate law expression as

$\text{Rate} = k[A]^{3/2}[B]^{-1/2}$

If concentration of both A and B are increased four times, the rate of the reaction

Options:

(a) increases 4 times

(b) decreases 4 times

(c) increases 16 times

(d) remains same

Sol: Overall order $= \frac{3}{2} - \frac{1}{2} = 1$

Rate of the reaction increases 4 times

Ans: (a)

32. The cause of Brownian movement is

Options:

(a) heat changes in the liquid state

(b) conventional currents

(c) the impact of molecules of the dispersion medium on the colloidal particles

(d) attraction forces between the colloidal particles and molecules of dispersion medium

Sol: Brownian movement is caused by collision of molecules of the dispersion medium with the colloidal particles.

Ans: (c)

33. Purification of colloids is done by

Options:

(a) heating

(b) precipitation

(c) congulation

(d) dialysis

Sol: It is done by dialysis

Ans: (d)

34. Which one of the following is not applicable to the phenomenon of adsorption?

Options:

(a) $\Delta H > O$

(b) $\Delta G < O$

(c) $\Delta S < O$

(d) $\Delta H < O$

Sol: Adsorption process is always exothermic.

$\therefore \Delta H$ cannot be greater than zero.

Ans: (a)

35. Silica is added to roasted copper ore during smelting in order to remove

Options:

(a) Cuprous oxide

(b) Cuprous sulphide

(c) Ferrous sulphide

(d) Ferrous oxide

Sol: It is ferrous oxide

$SiO_2 + FeO \rightarrow FeSiO_3$

Ans: (d)

36. In which of the following is purified by Van-Arkel method?

Options:

(a) Ga and In

(b) Zr and Ti

(c) Ag and Au

(d) Ni and Fe

Sol: Zr and Ti is purified by Van-Arkel method.

Ans: (b)

37. In the structure of diborane

Options:

(a) the $B-H$ bonds are ionic

(b) there are two three-centred two electron bond

(c) there is a $B-B$ bond

(d) there are two centred three electron bond

Sol: Diborane contains two three centred two electron bond

Ans: (b)

38. P_4O_6 is hydrolysed to

Options:

(a) $(HPO_5)_n$

(b) H_3PO_4

(c) H_3PO_3

(d) $H_4P_2O_7$

Sol: $P_4O_6 + 6H_2O \rightarrow 4H_3PO_3$

Ans: (c)

39. Oxygen exhibits positive oxidation state in

Options:

(a) CO

(b) F_2O

(c) NO

(d) N_2O

Sol: In F_2O, oxidation state of oxygen is $+2$

Ans: (b)

40. In which case order of acidic strength is not correct.

Options:

(a) $HI > HBr > HF$

(b) $HIO_4 > HBrO_4 > HClO_4$

(c) $HCl_4 > HClO_3 > HClO_2$

(d) $HF > H_2O > NH_3$

Sol: $HClO_4 > HBrO_4 > HIO_4$

It is because of higher electronegativity of chlorine

Ans: (b)

41. Which of the following hydrides of group 15 is more extensively hydrogen bonded?

Options:

(a) PH_3

(b) NH_3

(c) AsH_3

(d) SbH_3

Sol: Ammonia (NH_3)

Ans: (b)

42. A XeF_2 is isostructural with

Options:

(a) H_2O

(b) SO_2

(c) N_2O

(d) CO_2

Sol: Both XeF_2 and CO_2 are linear

Ans: (d)

43. Which of the following is most acidic?

Options:

(a) MnO_3

(b) MnO_2

(c) Mn_2O_7

(d) Mn_3O_4

Sol: Mn_2O_7

Ans: (c)

44. Which of the following is least basic?

Options:

(a) $La(OH)_3$

(b) $Lu(OH)_3$

(c) $Ce(OH)_3$

(d) $Nd(OH)_3$

Sol: $Lu(OH)_3$

Ans: (b)

45. Which of the following has maximum conductivity in aqueous solution?

Options:

(a) $Co(NH_3)_4Cl_3$

(b) $Co(NH_3)_3Cl_3$

(c) $Co(NH_3)_5Cl_3$

(d) $Co(NH_3)_6Cl_3$

Sol: $[Co(NH_3)_6]Cl_3$ maximum conductivity

Ans: (d)

46. Which of the following compounds show optical isomerism?

Options:

(a) $[Cu(NH_3)_4]^{2+}$

(b) $[ZnCl_4]^{2-}$

(c) $[Cr(C_2O_4)_3]^{3-}$

(d) $[Cr(CN)_6]^{3-}$

Sol: $[Cr(C_2O_4)_3]^{3-}$ show optical isomerism

Ans: (c)

47. Ethyl bromide can be obtained by the acting of HBr on

Options:

(a) Ethyne

(b) Ethane

(c) Propene (d) Ethanol

Sol: $C_2H_5OH + HBr \rightarrow C_2H_5Br + H_2O$

Ans: (d)

48. S_N1 mechanism of alkyl halide is favoured by

Options:

(a) Higher concentration of nucleophile

(b) Polar soluents

(c) Presence of less bulky alkyl group

(d) Strong nucleophiles

Sol: Polar solvents favour S_N1 reaction.

Ans: (b)

49. Which of the following is most acidic?

Options:

(a) CH_3CH_2OH

(b) $(CH_3)_2 CHOH$

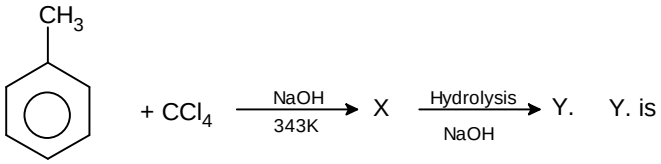
(c) $(CH_3)CHCH_2OH$

(d) CH_3OH

Sol: CH_3OH is most acidic

Ans: (d)

50. In the reaction



Options:

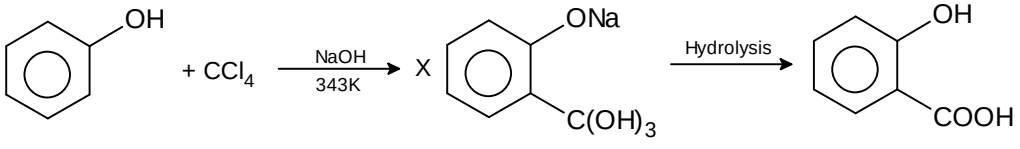
(a) Salicylaldehyde

(b) Salicylic acid

(c) $O-$Cresol

(d) Benzoic acid

Sol:



Ans: (b)

51. The major product in the reaction

$CH_3-O-CH(CH_3)_2+HI \rightarrow$ Product

Options:

(a) $ICH-O-CH(CH_3)_2$

(b) $CH_3-O-\underset{\underset{I}{|}}{C}-(CH_3)_2$

(c) $CH_3I+(CH_3)_2CHOH$

(d) $CH_3OH+(CH_3)_2CHI$

Sol: $CH_3-O-CH(CH_3)+HI \rightarrow CH_3I+(CH_3)_2CH-OH$

Ans: (c)

52. Which of the following is most reactive towards HCN ?

Options:

(a) CH_3COCH_3

(b) CH_3CHO

(c) $CH_3COC_2H_5$

(d) $HCHO$

Sol: Reactivity order: $HCHO > CH_3CHO > CH_3COCH_3 > CH_3COC_2H_5$

Ans: (d)

53. Which of the following will give cannizzaro reaction?

Options:

(a) CH_3CHO

(b) CH_3COCH_3

(c) $(CH_3)_3C-CHO$

(d) $(CH_3)_2CHCHO$

Sol: $(CH_3)_3C-CHO$ does not have $\alpha-$hydrogen. Hence undergoes cannizzaro reaction.

Ans: (c)

54. In the reaction

$CH_3CH_2COOH \xrightarrow{P,Cl_2} X \xrightarrow{alc\,KOH} Y.$ Y is

Options:

(a) $CH_2=CH-COOH$

(b) $CH_3CH_2CH_2OH$

(c) CH_3-CH_2-CHO

(d) $CH_3-\underset{\underset{OH}{|}}{CH}-COOH$

Sol: $CH_3CH_2COOH \xrightarrow{P,Cl_2} CH_3-\underset{\underset{Cl}{|}}{CH}-COOH \xrightarrow{alc\,KOH} CH_2=CH-COOH$

Ans: (a)

55. Mononitration of aniline is achieved by

Options:

(a) direct treatment with nitration mixture under reflux

(b) using fuming HNO_3

(c) acetylation followed by nitration and subsequent hydrolysis

(d) $KNO_3 + con.\ HNO_3$

Sol: $p-$Nitroaniline can be obtained by acetylation followed by nitration and hydrolysis

Ans: (c)

56. Structurally cellulose is a linear polymers of

Options:

(a) $\beta-$ glucose molecules

(b) Sucrose molecules

(c) $\alpha-$ glucose molecules

(d) Fructose molecules

Sol: Cellulose is a linear polymer of $\beta-$ glucose molecules

Ans: (a)

57. A peptide harmone is

Options:

(a) Estrone

(b) Testosterone

(c) Insulin

(d) Corticoid

Sol: Insulin is a peptide

Ans: (c)

58. Which of the following statement is not correct about DNA molecule?

Options:

(a) It has double helix structure

(b) It serves as hereditary material

(c) The two DNA strands are exactly similar

(d) Its replication is called semi-conservative mode of replication

Sol: Two DNA strands are not similar

Ans: (c)

59. Which of the following has an ester linkage?

Options:

(a) Nylon

(b) Bekelite

(c) Terylene

(d) PVC

Sol: Terylene has ester linkage

Ans: (c)

60. Which of the following is not an antibiotic?

Options:

(a) Streptomycin

(b) Valium

(c) Chloramphenicol

(d) Pencillin

Sol: Valium is tranquilizer

Ans: (b)

Mathematics

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. **60 x 1 = 60**

61. If $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$, then $\cos\alpha+\cos\beta+\cos\gamma=$

Options:

(a) 1

(b) 0

(c) -1

(d) None of these

Sol: Let $x=\cos\alpha+\cos\beta+\cos\gamma$ and $y=\sin\alpha+\sin\beta+\sin\gamma$, then

$$x^2+y^2=\left(\cos^2\alpha+\sin^2\alpha\right)+\left(\cos^2\beta+\sin^2\beta\right)+2\left\{\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)\right\}$$

$$=3+2\left(-\frac{3}{2}\right)=0$$

$\Rightarrow x=0$ and $y=0 \quad (\because x, y\in R)$

$\Rightarrow \cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$

Ans: (b)

62. The value of $\cos^2 15^\circ-\cos^2 30^\circ+\cos^2 45^\circ-\cos^2 60^\circ+\cos^2 75^\circ$ is

Options:

(a) 2

(b) 0

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

Sol: $\cos^2 15^\circ-\cos^2 30^\circ+\cos^2 45^\circ-\cos^2 60^\circ+\cos^2 75^\circ$

$$=\cos^2(90^\circ-75^\circ)-\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2-\left(\frac{1}{2}\right)^2+\cos^2 75^\circ$$

$$=\left(\sin^2 75^\circ+\cos^2 75^\circ\right)-\frac{3}{4}+\frac{1}{2}-\frac{1}{4}=\frac{1}{2}.$$

Ans: (d)

63. $\sin 10^\circ+\sin 20^\circ+\sin 30^\circ+....+\sin 360^\circ$ is equal to

Options:

(a) 0

(b) 1

(c) -1

(d) none of these

Sol: $\sin 10^\circ + \sin 20^\circ + \sin 30^\circ + ... + \sin 180^\circ + \sin(180^\circ + 10^\circ) + \sin(180^\circ + 20^\circ) + ... + \sin(180^\circ + 180^\circ) = 0$

$(\because \sin(180^\circ + \theta) = -\sin\theta)$

Ans: (a)

64. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to

Options:

(a) $\frac{24}{25}$

(b) $\frac{-24}{25}$

(c) $\frac{25}{24}$

(d) none of these

Sol: $25\cos^2\alpha + 5\cos\alpha - 12 = 0 \quad \Rightarrow \cos\alpha = -\frac{4}{5}, \frac{2}{5}$ but $\frac{\pi}{2} < \alpha < \pi$,

Therefore, $\cos\alpha = -\frac{4}{5}$ and hence $\sin\alpha = \sqrt{1 - \frac{16}{25}} = \frac{2}{5}$.

So, $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$.

Ans: (b)

65. If $\tan 3x = \cot x$ then x is

Options:

(a) $(2n+1)\frac{\pi}{8}, n \in I$

(b) $(2n+1)\frac{\pi}{6}, n \in I$

(c) $(2n+1)\frac{\pi}{4}, n \in I$

(d) None of these

Sol: $\tan 3x - \cot x \Rightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$

$\Rightarrow 3x = n\pi + \frac{\pi}{2} - x, n \in I$

$\Rightarrow 4x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{8}, n \in I$

Ans: (a)

66. The solution set of the inequation $\left(x^2+x+1\right)(2x-3)>0$ is

Options:

(a) R

(b) $\left(\frac{3}{2},\infty\right)$

(c) $\left[\frac{3}{2},\infty\right)$

(d) $\left(-\infty,\frac{3}{2}\right)$

Sol: $\left(x^2+x+1\right)(2x-3)>0 \Rightarrow 2x-3>0$

$\Rightarrow 2x>3$

$\left(\because\ x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\geq\frac{3}{4}>0\right)$

Ans: (b)

67. Complex Numbers $8+5i, -3+i$ and $-2-3i$ represent the points A, B and C respectively, then the modulus and argument of the complex number representing the centroid of the triangle ABC are

Options:

(a) $2,\frac{\pi}{4}$

(b) $\sqrt{2},\frac{\pi}{4}$

(c) $2\sqrt{2},\pi$

(d) $\sqrt{2},\frac{\pi}{2}$

Sol: Here $A\equiv(8,5)$, $B\equiv(-3,1)$ and $C\equiv(-2,-3)$. Hence, the centroid of triangle ABC has the coordinates

$\left(\frac{8-3-2}{3},\frac{5+1-3}{2}\right)=(1,1)$. This means the complex numbers representing the centroid is

$1+i=\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)=\sqrt{2}cis\frac{\pi}{4}$.

Ans: (b)

68. If $1+6+11+....+x=148$, then x is equal to

Options:

(a) 36

(b) 8

(c) 30

(d) None of these

Sol: If n is the number of terms then

$148 = S_n = \frac{n}{2}\left[2\times1+(n-1)5\right]$

$\Rightarrow 296 = n(5n-3)$

$\Rightarrow 5n^2 - 3n - 296 = 0$

$\Rightarrow n = 8$ as n cannot be negative.

$\therefore \quad x = T_8 = 1+(8-1)5 = 36$.

Ans: (a)

69. The figures $4, 5, 6, 7, 8$ are written in every possible order. The number of numbers greater than 56000 is

Options:

(a) 72

(b) 90

(c) 96

(d) 98

Sol: ${}^5P_5 - \left({}^4P_4 + {}^3P_3\right)$; as ${}^4P_4$ numbers begin with 4 and ${}^3P_3$ with 54

Ans: (b)

70. If the coefficients of x^{-7} and x^{-8} in the expansion of $\left(2+\frac{1}{3x}\right)^n$ are equal then $n =$

Options:

(a) 56

(b) 55

(c) 45

(d) 15

Sol: $T_{r+1} = {}^nC_r\, 2^{n-r}\left(\frac{1}{3x}\right)^r = {}^nC_r \frac{2^{n-r}}{3^r}x^{-r}$

So, coefficient of $x^{-r} = {}^nC_r \frac{2^{n-r}}{3^r}$

$\Rightarrow$ Coefficient of $x^{-7} = {}^nC_7 \frac{2^{n-7}}{3^7}$

and coefficient of $x^{-8} = {}^nC_8 \frac{2^{n-8}}{3^8}$

Hence, $\frac{{}^nC_7\, 2^{n-7}}{3^7} = \frac{{}^nC_8\, 2^{n-8}}{3^8}$

$\Rightarrow \frac{{}^nC_8}{{}^nC_7} = 6 \Rightarrow \frac{n-7}{8} = 6 \Rightarrow n = 55$

Ans: (b)

71. The lines $x+2y-3=0,\ 2x+y-3=0$ and the line l are concurrent. If the line l passes through the origin, then its equation is

Options:

(a) $x-y=0$

(b) $x+y=0$

(c) $x+2y=0$

(d) $2x+y=0$

Sol: The lines $x+2y-3=0$ and $2x+y-3=0$ meet in the point $(1,1)$.

Hence, the line l passes through $(0,0)$ and $(1,1)$. Its equation is $y-0=\frac{1-0}{1-0}(x-0)$

i.e., $y=x$.

Ans: (a)

72. Length of common chord of the parabolas $x^2=4y$ and $y^2=4x$ is

Options:

(a) $4\sqrt{2}$

(b) $\sqrt{2}$

(c) $2\sqrt{2}$

(d) none of these

Sol: The two paraboles meet in the points $(0,0)$ and $(4,4)$. Hence the length of their common chord is $\sqrt{(4-0)^2+(4-0)^2}=\sqrt{32}=4\sqrt{2}$.

Ans: (a)

73. The mean and S.D of $1,2,3,4,5,6$ is

Options:

(a) $\frac{7}{2},\sqrt{\frac{35}{12}}$

(b) $3,3$

(c) $\frac{7}{2},\sqrt{3}$

(d) $3,\frac{35}{12}$

Sol: Mean and S.D. of $1,2,3,....n$ is $\frac{n+1}{2}$ and $\sqrt{\frac{n^2-1}{12}}$ respectively. Put $n=6$ to get the result.

$$\text{Mean}=\frac{1+2+3+...+6}{6}=\frac{21}{6}=\frac{7}{2}$$

$$SD=\sqrt{\frac{6^2-1}{12}}=\sqrt{\frac{35}{12}}$$

Ans: (a)

74. A digit is selected at random from either of the two sets $\{1,2,3,4,5,6,7,8,9\}$ and $\{1,2,3,4,5,6,7,8,9\}$. What is the chance that the sum of the digits selected is 10?

Options:

(a) $\frac{1}{9}$

(b) $\frac{10}{81}$

(c) $\frac{10}{18}$

(d) None of these

Sol: Two digits, one from each set can be selected in $9\times9=81$ ways. Favourable outcomes are $(1,9)$, $(2,8)$, $(3,7)$, $(4,6)$, $(5,5)$, $(6,4)$, $(7,3)$, $(8,2)$ and $(9,1)$.

$\therefore$ required probability $=\frac{9}{81}=\frac{1}{9}$

Ans: (a)

75. A baised dice is tossed and the respective probabilities of various faces to show up are

Face	1	2	3	4	5	6
Probability	0.1	0.24	0.19	0.18	0.15	0.14

If an even face has turned up, then the probability that it is face 2 or face 4 is

Options:

(a) 0.42

(b) 0.75

(c) 0.25

(d) 0.33

Sol: Let E_1 : "an even number has turned up"

E_2 : "face 2 or face 4 has come up", then required probability

$=P\left(\frac{E_2}{E_1}\right)=\frac{P(E_2\cap E_1)}{P(E_1)}=\frac{P(E_2)}{P(E_1)}\qquad [E_2CE_1]$

$=\frac{0.24+0.18}{0.24+0.18+0.14}=\frac{0.42}{0.56}=\frac{3}{4}=0.75$

Ans: (b)

76. Four cards are drawn simultaneously from a deck of 52 cards. The chance that they all are the same suit is

Options:

(a) $\frac{C(13,4)}{C(52,4)}$

(b) $\frac{4C(13,4)}{C(52,4)}$

(c) $\frac{4!C(13,4)}{C(52,4)}$

(d) None of these

Sol: Note that there are four suits and each contains 13 cards.

$\therefore$ Required probability $\frac{4\times^{13}C_4}{^{52}C_4}=\frac{4C(13,4)}{C(52,4)}$

Ans: (b)

77. The probability that an event E occurs in one trial of an experiment is 0.4. Three independent trials of experiment are performed. The probability that the event E occurs atleast once is

Options:

(a) 0.784

(b) 0.904

(c) 0.936

(d) none of these

Sol: Required probability $=1-P$ (E does not occur any time)

$=1-\left(1-P(E)\right)^3=1-\left(1-0\cdot4\right)^3=1-0.216=0.784$.

Ans: (a)

78. A bag 'A' contains two white and two red balls and another bag 'B' contains 4 white and 5 red balls. A ball is drawn and is found to be red. The probability that is was drawn from bag B is

Options:

(a) $\frac{25}{52}$

(b) $\frac{1}{2}$

(c) $\frac{10}{19}$

(d) $\frac{13}{18}$

Sol: Let H_1 : "the ball is drawn from bag A"

and H_2 : "the ball is drawn from bag B"

then H_1 and H_2 are mutually exclusive and exhaustive.

Also $P(H_1)=P(H_2)=\frac{1}{2}$.

Let E: "a red ball is drawn"

Using Bayes theorem required probability $=P\left(\frac{H_2}{E}\right)$

$$=\frac{P\left(\frac{E}{H_2}\right)P(H_2)}{P\left(\frac{E}{H_1}\right)P(H_1)+P\left(\frac{E}{H_2}\right)P(H_2)}=\frac{\frac{5}{9}\times\frac{1}{2}}{\frac{2}{4}\times\frac{1}{2}+\frac{5}{9}\times\frac{1}{2}}=\frac{\frac{5}{9}}{\frac{1}{2}+\frac{5}{9}}=\frac{10}{19}.$$

Ans: (c)

79. If $P(A\cap B)=\frac{1}{2}$ and $P(A'\cap B')=\frac{1}{3}$, $P(A)=p$ and $P(B)=2p$ then value of p is

Options:

(a) $\frac{7}{18}$

(b) $\frac{1}{3}$

(c) $\frac{4}{9}$

(d) $\frac{1}{9}$

Sol: Given $P(A'\cap B')=\frac{1}{3}$

$$\Rightarrow P((A\cup B)')=\frac{1}{3}$$

$$\Rightarrow 1-P(A\cup B)=\frac{1}{3}$$

$$\Rightarrow P(A\cup B)=1-\frac{1}{3}=\frac{2}{3}$$

$$\Rightarrow P(A)+P(B)-(A\cap B)=\frac{2}{3}$$

$$\Rightarrow p+2P-\frac{1}{2}=\frac{2}{3}\Rightarrow 3p=\frac{2}{3}+\frac{1}{2}=\frac{7}{6}\Rightarrow p=\frac{7}{18}.$$

Ans: (a)

80. Let a binary operation '*' be defined on the set N of natural numbers by the rule $a*b=a+2b\forall, a, b\in N$, then

Options:

(a) '*' is commutative

(b) '*' is associative

(c) N is not closed with respect to '*'

(d) Identity element under '*' does not exist

Sol: (i) $a*b=a+2b\neq b+2a=b*a$.

(ii) $(a*b*c=a+2b)*c=a+2b+2c$ whereas

$a*(b*c)=a*(b+2c)=a+2(b-2c)=a+2b+4c$.

(iii) $a+2b\in N\forall a,b\in N$, therefore, $a*b\in N$

(iv) If $'e'$ is the identify element, then $a*e=a\Rightarrow a+2e=a\Rightarrow e=0$,

but then $e*a=0*a=0+2a=2a\neq a*e$ in general. So, identify element does not exist

Ans: (d)

81. The range of the function $f(x)=a\sin x+b\cos x$ is

Options:

(a) $[a,b]$

(b) $[a-b,a+b]$

(c) $[-(a+b),(a+b)]$

(d) $\left[-\sqrt{a^2+b^2},\sqrt{a^2+b^2}\right]$

Sol: $f(x)=a\sin x+b\cos x$

$=(r\cos\alpha)\sin x+(r\sin\alpha)\cos x$

$=r\sin(x+\alpha)$,

Where $a=r\cos\alpha, b=r\sin\alpha$.

Since $-1\leq\sin(x+\alpha)\leq 1\forall x\in R$,

Therefore, $-r\leq r\sin(x+\alpha)\leq r$

$\Rightarrow -r\leq f(x)\leq r$.

So, range of $f(x)$ is

$[-r,r]=\left[-\sqrt{a^2+b^2},\sqrt{a^2+b^2}\right]$.

Ans: (d)

82. The function $f(x)=x^2+\sin x$ is

Options:

(a) an odd function

(b) an even function

(c) neither even nor odd

(d) a constant function

Sol: $f(x)=x^2+\sin x$ is neither even nor odd as $f(-x)$ is neither equal to $f(x)$ nor to $-f(x)$.

Ans: (c)

83. Let $f:[0,\infty)\to[0,2]$ be defined by $f(x)=\frac{2x}{1+x}$, then f is

Options:

(a) one-one, but not onto

(b) onto, but not one-one

(c) both one-one onto

(d) neither one-one nor onto

Sol: Here $f(x)=\frac{2x}{1+x}$, $D_f=[0,\infty)$.

Let $x_1, x_2 \in D_f$ and $f(x_1)=f(x_2)$

$\Rightarrow \frac{2x_1}{1+x_1}=\frac{2x_2}{1+x_2}$

$\Rightarrow 2x_1(1+x_2)=2x_2(1+x_1)$

$\Rightarrow 2x_1 2x_2 \Rightarrow x_1=x_2$

Hence f is one-one. Let $y\in[0,2]$ be arbitrary, then $y=f(x)$ iff

$\Leftrightarrow y=\frac{2x}{1+x} \Leftrightarrow xy+y=2x \Leftrightarrow x(y-2)=-y \Leftrightarrow x=-\frac{y}{y-2}$

This means that $y=2$ is not the image of any element from $[0,\infty)$. So f is not onto.

Ans: (a)

84. If $f(x)=\frac{x-1}{x+1}$, then $f\left(\frac{1}{f(x)}\right)$ equals

Options:

(a) 0

(b) 1

(c) x

(d) $\frac{1}{x}$

Sol: Given $f(x)=\frac{x-1}{x+1}$, therefore,

$f\left(\frac{1}{f(x)}\right)=f\left(\frac{x+1}{x-1}\right)$

$=\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}=\frac{x+1-x+1}{x+1+x-1}$

$=\frac{2}{2x}=\frac{1}{x}$

Ans: (d)

85. The projection of the vector $\hat{i}-2\hat{j}+\hat{k}$ on the vector $4\hat{i}-4\hat{j}+7\hat{k}$ is

Options:

(a) $\frac{5}{19}\sqrt{5}$

(b) $\frac{19}{9}$

(c) $\frac{9}{19}$

(d) $\frac{1}{19}\sqrt{6}$

Sol: $\frac{\left(\hat{i}-2\hat{j}+\hat{k}\right)\cdot\left(4\hat{i}-4\hat{j}+7\hat{k}\right)}{\sqrt{4^2+(-4)^2+7^2}}=\frac{4+8+7}{\sqrt{81}}=\frac{19}{9}$

Ans: (b)

86. $\left(\hat{i}+\hat{j}\right)\times\left(\hat{j}+\hat{k}\right)\cdot\left(\hat{k}+\hat{i}\right)$ is equal to

Options:

(a) 0

(b) 1

(c) 2

(d) none of these

Sol: Required scalar triple product

$$\begin{vmatrix}1&1&0\\0&1&1\\1&0&1\end{vmatrix}=1(1-0)-1(0-1)$$

$=1+1=2$

Ans: (c)

87. If $\vec{a}+\vec{b}$ is at right angles to $\vec{b}$ and $2\vec{b}+\vec{a}$ is at right angles to $\vec{a}$ then

Options:

(a) $a=\sqrt{2}b$

(b) $a=2b$

(c) $a=b$

(d) $2a=b$

Sol: Given $\left(\vec{a}+\vec{b}\right)\cdot\vec{b}=0$ and $\left(2\vec{b}+\vec{a}\right)\cdot\vec{a}=0\Rightarrow\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}=0$

$\Rightarrow 2\vec{b}\cdot\vec{a}+\vec{a}\cdot\vec{a}=0$

$\Rightarrow 2\left(-\vec{b}\cdot\vec{b}\right)+\vec{a}\cdot\vec{a}=0$ (eliminating $\vec{a}\cdot\vec{b}$

$\Rightarrow a^2=2b^2\Rightarrow a=\sqrt{2}b$.

Ans: (a)

88. The distance of the plane $\vec{r}\cdot\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin is

Options:

(a) 1

(b) 7

(c) $\frac{1}{7}$

(d) None of these

Sol: The given equation is of the form $\vec{r}\cdot\hat{n}=p$ where $\hat{n}$ is a unit vector and $p>0$ i.e. in the normal form.

Ans: (a)

89. The plane $x-2y+z-6=0$ and the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are related as

Options:

(a) parallel to the plane

(b) at right angles to the plane

(c) lies in the plane

(d) meets the plane obliquely

Sol: Any point on the lines is $(t, 2t, 3t)$. It lies in the given plane if $t-2(2t)+3t-6=0$

i.e. if $0t=6$, which is not true for any real t. So, the line and plane do not meet i.e. the line is parallel to the plane. [Also $1\times1+(-2)(2)+(1)(3)=0$]

Ans: (a)

90. The lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-1}{3}=\frac{y-2}{4}=\frac{z-3}{5}$ are

Options:

(a) skew

(b) parallel

(c) intersecting

(d) none of these

Sol: The given lines are not parallel and both the lines pass through the point $(1, 2, 3)$.

Ans: (c)

91. The contrapositive of "If it is hot outside, then you feel thirsty"

Options:

(a) If you don't feel thirsty, then it is not hot outside

(b) If you feel thirsty then it is not hot out side

(c) If it is not hot out side, then you will not feel thirst

(d) None of these

Sol: p : It is hot outside

q : You feel thirsty $\therefore$ Given $p \to q$

Contrapositive $\sim q \to \sim p$

Ans: (a)

92. If $f(x)=(1-x)\tan\frac{\pi x}{2}$, then $\lim_{x\to 1} f(x)$ is equal to

Options:

(a) $\frac{\pi}{2}$

(b) $\frac{2}{\pi}$

(c) 0

(d) 1

Sol: $\lim_{x\to 1} f(x) = \lim_{x\to 1}(1-x)\tan\frac{\pi}{2}x$

$= \lim_{x\to 1}(h)\tan\left(\frac{\pi}{2}(1+h)\right)$

$= \lim_{x\to 1}\frac{h}{\tan\left(\frac{\pi}{2}h\right)} = \frac{2}{\pi}$

Ans: (b)

93. $\lim_{n\to\infty}\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+.......+\frac{1}{n(n+1)}\right)=$

Options:

(a) -1

(b) 1

(c) 0

(d) none of these

Sol: $= \lim_{x\to\infty}\left(\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{n(n+1)}\right)$

$= \lim_{x\to\infty}\left(\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right)$

$= \lim_{x\to\infty}\left(1-\frac{1}{n+1}=1-0=1\right)$

Ans: (b)

94. $\lim_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}=$

Options:

(a) 0

(b) $\frac{1}{3}$

(c) $-\frac{1}{3}$

(d) does not exist

Sol: $\lim_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

$\lim_{x\to 4}\frac{-\frac{1}{2\sqrt{5+x}}}{-\frac{1}{2\sqrt{5-x}}(1)}=\frac{\frac{-1}{2\times 3}}{+\frac{1}{2}}=-\frac{1}{3}$

Ans: (c)

95. If $y=\frac{\log x}{x}$, then $\frac{d^2y}{dx^2}=$

Options:

(a) $\frac{3-2\log x}{x^3}$

(b) $\frac{2\log x-3}{x^3}$

(c) $\frac{2\log x-3}{x^4}$

(d) None of these

Sol: $y_1=\frac{1-\log x}{x^2}$

$\Rightarrow y_2=\frac{x^2\left(-\frac{1}{x}\right)-(1-\log x)2x}{\left(x^2\right)^2}$

$=\frac{x(-1-2+2\log x)}{x^4}=\frac{2\log x-3}{x^3}$

Ans: (b)

96. Let $f(x)=\frac{x^3}{1-x^2}$, $x\neq 0,\pm 1$. Then derivative of $f(x)$ w.r.t x^2 is

Options:

(a) $\frac{2x}{\left(1-x^2\right)^2}$

(b) $\frac{1}{\left(1-x^2\right)^2}$

(c) $\frac{1}{\left(2+x^2\right)^2}$

(d) $\frac{1}{\left(2-x^2\right)^2}$

Sol: $f(x)=\frac{x^2}{1-x^2}$

$\Rightarrow \frac{d}{dx}(f(x))=\frac{\left(1-x^2\right)2x-x^2(-2x)}{\left(1-x^2\right)^2}=\frac{2x(1)}{\left(1-x^2\right)^2}$.

Also $\frac{d}{dx}\left(x^2\right)=2x$

Therefore, $\frac{df(x)}{d\left(x^2\right)}=\frac{1}{\left(1-x^2\right)^2}$

Ans: (b)

97. If $y=\log\left(\sqrt{x}+\sqrt{x-a}\right)$, then $\frac{dy}{dx}$ is equal to

Options:

(a) $\frac{1}{\sqrt{x}+\sqrt{x-a}}$

(b) $\frac{1}{2\sqrt{x}\sqrt{x-a}}$

(c) $\frac{1}{\sqrt{x}\sqrt{x-a}}$

(d) none of these

Sol: $\frac{dy}{dx}=\frac{d}{dx}\left\{\log\left(\sqrt{x}+\sqrt{x-a}\right)\right\}=\frac{1}{\sqrt{x}+\sqrt{x-a}}\left(\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{x-a}}\right)$

$=\frac{1}{2\sqrt{x}\sqrt{x-a}}$.

Ans: (b)

98. Differential co-efficient of $\sec\left(\tan^{-1}x\right)$ is

Options:

(a) $\frac{x}{\sqrt{1+x^2}}$

(b) $\frac{1}{\sqrt{1+x^2}}$

(c) $x\sqrt{1+x^2}$

(d) $\frac{x}{1+x^2}$

Sol: Now $\sec\left(\tan^{-1}x\right)=\sec\theta, \theta=\tan^{-1}x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\Rightarrow\sec\left(\tan^{-1}x\right)=\sec\left[\sec^{-1}\sqrt{1+x^2}\right]=\sqrt{1+x^2}$

Hence $\frac{d}{dx}\left(\sec\left(\tan^{-1}x\right)\right)$

$\frac{d}{dx}\sqrt{1+x^2}=\frac{x}{\sqrt{x^2+1}}$

Ans: (a)

99. Let $f(x)=\frac{1-\sin x}{(\pi-2x)^2}$ when $x\neq\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=k$. The value of k which makes f continuous at $\frac{\pi}{2}$ is

Options:

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

(d) none of these

Sol: $\lim\limits_{x\to\frac{\pi}{2}}f(x)=\lim\limits_{x\to\frac{\pi}{2}}\frac{1-\sin x}{(\pi-2x)^2}$

$=\lim\limits_{h\to0}\frac{1-\sin\left(\frac{\pi}{2}+h\right)}{4h^2}=\lim\limits_{h\to0}\frac{1-\cos h}{4h^2}=\frac{1}{4}\cdot\frac{1}{2}=\frac{1}{8}$

So, we must have $k=\frac{1}{8}$.

Ans: (c)

100. Let $f(x)=\begin{cases}a+x, x\geq 0\\ a-x, x<0\end{cases}$, then $f(x)$ is

Options:

(a) continuous but not derivable at 0

(b) derivable at 0

(c) not continuous at 0

(d) none of these

Sol: $f(0)=a$, $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}(a+x)=a$ and also $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}(a-x)=a$

Hence f is continuous at 0. However, $L.f'(0)=\lim\limits_{h\to 0^+}\dfrac{f(0+h)-f(0)}{h}$

$=\lim\limits_{h\to 0^-}\dfrac{a-h-a}{h}=-1$ and $Rf'(0)=\lim\limits_{h\to 0^+}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0^+}\dfrac{a+h-a}{h}=1$

Ans: (a)

101. Let $f(x)=x^{25}(1-x)^{75}$ for all $x\in[0,1]$, then $f(x)$ assumes its maximum value at

Options:

(a) 0

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{2}$

(d) $\dfrac{1}{3}$

Sol: $f'(x)=x^{25}\times 75(1-x)^{74}(-1)+(1-x)^{75}\times 25x^{24}$

$=25x^{24}(1-x)^{74}\{-3x+(1-x)\}\forall x\in R$

$\Rightarrow f'(x)=0\Rightarrow x=\dfrac{1}{4}\in(0,1)$.

Now that $f(0)=f(1)=0$ and $f\left(\dfrac{1}{4}\right)=\dfrac{3^{75}}{4^{100}}$.

So $f(x)$ is maximum at $x=\dfrac{1}{4}$.

Ans: (b)

102. Let $f(x)=\tan x-4x$, then in the interval $\left(-\dfrac{\pi}{3},\dfrac{\pi}{3}\right)$, $f(x)$ is

Options:

(a) a decreasing function

(b) an increasing function

(c) a constant function

(d) none of these

Sol: $f'(x)=\sec^2 x-4<0 in\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$.

$\left(\because x\in\left(-\frac{\pi}{3},\frac{\pi}{3}\right)\Rightarrow 1\le \sec x<2\right)\Rightarrow 1\le \sec^2 x<4$

Ans: (a)

103. The normal to the curve $y=3-x^2$ at $(1,1)$ is

Options:

(a) $x+y=0$

(b) $x+y+1=0$

(c) $x-y+1+0$

(d) $x-2y+3=0$

Sol: $y'=-2x\Rightarrow$ Slope of $\perp gt=-2$

Slope of normal $=\frac{1}{2}$

Equation of normal: $y-2=\frac{1}{2}(x-1)$

$\Rightarrow 2y-4=x-1\Rightarrow x-2y+3=0$

Ans: (d)

104. Angle of intersection of curves $y=4-x^2$ and $y=x^2$ is

Options:

(a) $\frac{\pi}{2}$

(b) $\tan^{-1}\left(\frac{4}{3}\right)$

(c) $\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$

(d) none of these

Sol: First of all we note that the two curves $y=4-x^2$ and $x^2=y$ meet where $y=4-y\Rightarrow y=2$ and hence $x^2=2\Rightarrow x=\pm\sqrt{2}$. So, the two curves meet at the points $\left(\sqrt{2},2\right),\left(-\sqrt{2},2\right)$.

We shall find the angle of intersection at $\left(\sqrt{2},2\right)$. For $x^2=y,\frac{dy}{dx}=2x$ and for $y=4-x^2,\frac{dy}{dx}=-2x$.

$m_1=2\sqrt{2}$, $m_2=-2\sqrt{2}$

$\therefore \tan\theta=\left|\frac{2\sqrt{2}+2\sqrt{2}}{1-8}\right|=\frac{4\sqrt{2}}{7}$

$\therefore \theta=\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$

Ans: (c)

105. The maximum value of $f = 4x + 3y$ subject to constraints $x \geq 0,\ y \geq 0,\ 2x + 3y \leq 18,\ x + y \geq 10$ is

Options:

(a) 35

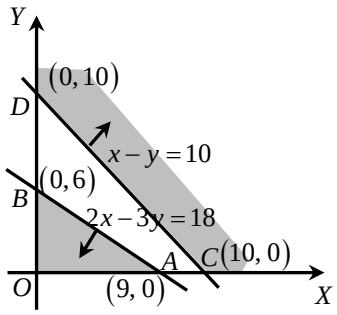
(b) 36

(c) 34

(d) none of these

Sol: Observe the adjoining figure. Graph of the inequality $2x + 3y \leq 18$ together with $x \geq 0,\ y \geq 0$ is the region bounded by the Triangle OAB (including the boundary points).

Graph of the inequality $x + y \geq 10$ together with $x \geq 0,\ y \geq 0$ is the region of whole XOY -plane except the points which lie within the triangle OCD . So feasible region is the empty set in this case. No optimum value of the objective function exists.



Ans: (d)

106. If A and B are any two sets, then $(A \cup B) - (A \cap B)$ is equal to

Options:

(a) $A - B$

(b) $B - A$

(c) $(A - B) \cup (B - A)$

(d) None of these

Sol: $A - B,\ B - A$ and $A \cap B$ are pairwise disjoint and their union is $A \cup B$.

Ans: (c)

107. $\int \frac{\cos 2x}{\cos x} dx =$

Options:

(a) $2\sin x + \log(\sec x + \tan x) + C$

(b) $2\sin x - \log(\sec x + \tan x) + C$

(c) $2\sin x - \log|\sec x + \tan x| + C$

(d) $2\sin x - \log|\sec x - \tan x| + C$

Sol: $\int \frac{\cos 2x}{\cos x} dx = \int \frac{2\cos^2 x - 1}{\cos x} dx$

$= 2\int \cos x dx - \int \sec x dx$

$= 2\sin x - \log|\sec x + \tan x| + C$

Ans: (c)

108. An anti-derivative of $\frac{x}{\cos^2 x}$ is

Options:

(a) $x\tan x + C$

(b) $\log|\cos x| + C$

(c) $x\tan x + \log|\cos x| + C$

(d) $\cot x + C$

Sol: $\int \frac{x}{\cos^2 x}dx = \int x\sec^2 x dx = x\tan x - \int \tan x dx = x\tan x + \log|\cos x| + C$

(Integrating by parts taking x as the first function)

Ans: (c)

109. $\int \frac{1}{\sqrt{1-x}}dx$ is equal to

Options:

(a) $\sqrt{1-x} + C$

(b) $-2\sqrt{1-x} + C$

(c) $2\sqrt{1-x} + C$

(d) none of these

Sol: $\int \frac{1}{\sqrt{1-x}}dx = \int (1-x)^{-1/2}dx = \frac{(1-x)^{1/2}}{\left(\frac{1}{2}\right)(-1)} = -2\sqrt{1-x} + C$

Ans: (b)

110. $\int \frac{\log x - 1}{(\log x)^2}dx$ is equal to

Options:

(a) $\frac{\log x}{x} + c$

(b) $\frac{x}{\log x} + c$

(c) $\frac{(\log x)^2 - x}{\log x}$

(d) none of these

Sol: $\int \frac{\log x - 1}{(\log x)^2}dx = \int \frac{1}{\log x}\cdot 1 dx - \int \frac{1}{(\log x)^2}dx$

$= \left(\frac{1}{\log x}\right)x - \int \frac{-1}{(\log x)^2}\frac{1}{x}\cdot x dx - \int \frac{1}{(\log x)^2}dx = \frac{x}{\log x} + C$

(Applying rule of integration by parts to the first integral taking $\frac{1}{\log x}$ as the first function)

Ans: (b)

111. $\int \frac{1}{x^3\left(x^3+1\right)^{1/3}}dx$ is equal to

Options:

(a) $-\frac{1}{2}\left(1+x^{-3}\right)^{2/3}+C$

(b) $-\left(1+x^3\right)^{2/3}+C$

(c) $-\left(1+x^{-3}\right)^{-2/3}+C$

(d) none of these

Sol: $\int \frac{1}{x^3\left(x^3+1\right)^{1/3}}dx=\int \frac{1}{x^3x\left(1+x^{-3}\right)^{1/2}}$

$=-\frac{1}{3}\int\left(1+x^{-3}\right)^{-\frac{1}{3}}\left(-3x^{-4}\right)dx=-\frac{1}{3}\frac{\left(1+x^{-3}\right)^{2/3}}{2/3}+C$.

Ans: (a)

112. The value of $\int_1^2 \frac{1}{x^2}e^{-1/x}dx$ is

Options:

(a) $\frac{1}{\sqrt{e}}+\frac{1}{e}$

(b) $\frac{1}{e}-\frac{1}{\sqrt{e}}$

(c) $\frac{1}{\sqrt{e}}-\frac{1}{e}$

(d) 0

Sol: Put $-\frac{1}{x}=t$, then $\int_1^2 \frac{1}{x^2}e^{-1/x^2}dx=\int_{-1}^{-1/2} e^t dt$

$=e^t\Big|_{-1}^{-1/2}=e^{-1/2}-e^{-1}=\frac{1}{\sqrt{e}}-\frac{1}{e}$

Ans: (c)

113. $\int_{-8}^{8}\left(\sin^{93}x+x^{295}\right)dx$ is equal to

Options:

(a) 0

(b) a number different from 0

(c) $2\left(8^{295}+1\right)$

(d) $2+8^{295}$

Sol: $f(x)=\sin^{93}x+x^{295}$ is an odd function and $\int_{-a}^{a} f(x)dx=0$ whenever $f(x)$ is an odd function.

Ans: (a)

114. $\int_0^2 \frac{dx}{\{ax+b(2-x)\}^2}$ is equal to

Options:

(a) $\frac{-1}{2ab}$

(b) $\frac{1}{2ab}$

(c) $\frac{\alpha-b}{2ab}$

(d) none of these

Sol: $\int_0^2 \frac{dx}{\{ax+b(2-x)\}^2}=\int_0^2 \{2b+(a-b)x\}^{-2}dx$

$=\left[\frac{(2b+(a-b)x)^{-1}}{(-1)(a-b)}\right]_0^2=\frac{1}{b-a}\left[\frac{1}{2a}-\frac{1}{2b}\right].$

Ans: (b)

115. If $\int_0^{\pi/2} \frac{\cos x}{4-\sin^2 x}dx=\lambda\log 3$, then λ is equal to

Options:

(a) $\frac{1}{4}$

(b) $-\frac{1}{4}$

(c) $\frac{1}{2}$

(d) none of these

Sol: Substituting $\sin x=t\Rightarrow \cos x dx=dt$, we obtain

$\int_0^{\pi/2} \frac{\cos x dx}{4-\sin^2 x}=\int_0^1 \frac{dt}{4-t^2}$

$=\frac{1}{4}\left[\log\left|\frac{2+t}{2-t}\right|\right]_0^1=\frac{1}{4}\log 3$

Ans: (a)

116. The solution of the differential equation $\cos x \sin y dx + \sin x \cos y dy = 0$ is

Options:

(a) $\frac{\sin x}{\sin y} = C$

(b) $\cos x + \cos y = C$

(c) $\sin x + \sin y = C$

(d) $\sin x \sin y = C$

Sol: Given equation is

$\cos x \sin y dx + \sin x \cos y dy = 0$

$\Rightarrow \frac{\cos x dx}{\sin x} + \frac{\cos y}{\sin y} dy = 0$

$\Rightarrow \cot x dx + \cot y dy = 0$

$\int \cot x dx + \int \cot y dy = \log c$

i.e., $\log \sin x + \log \sin y = \log c$

i.e., $\log(\sin x \sin y) = \log c$

$\Rightarrow \sin x \sin y = c$

Ans: (d)

117. The differential equation of the family of curves $y = A(x+B)^2$ after eliminating A and B is

Options:

(a) $2yy'' = y'^2$

(b) $yy'' = y'^2$

(c) $2yy'' = y' + y$

(d) $2yy'' = y' - y$

Sol: Given family of curves is $y = A(x+B)^2$ …(1)

Differentiating w.r.t x. is $y' = 2A(x+B)$ …(2)

$\frac{y}{y'} = \frac{x+B}{2}$, again differentiating w.r.t. x, we get

$\frac{y'y' - yy''}{(y')^2} = \frac{1}{2}$

$\Rightarrow 2(y')^2 - 2yy'' = (y')^2$

$\Rightarrow (y')^2 = 2yy''$.

Ans: (a)

118.If $\cos\left(2\sin^{-1}x\right)=\frac{1}{9}$ then $x=$

Options:

(a) $\frac{2}{3}$

(b) $\frac{-2}{3}$

(c) $\pm\frac{2}{3}$

(d) none of these

Sol: $\cos\left(2\sin^{-1}x\right)=\frac{1}{9}$

$\Rightarrow 1-2\sin^2\left(\sin^{-1}x\right)=\frac{1}{9}\Rightarrow 1-2x^2=\frac{1}{9}$

$\Rightarrow x=\pm\frac{2}{3}.$

Ans: (c)

119.If A and B are symmetric matrices of the same order, then

Options:

(a) AB is a symmetric matrix

(b) AB is skew-symmetric matrix

(c) $AB+BA$ is symmetric matrix

(d) $AB-BA$ is a symmetric matrix

Sol: $(AB+BA)^t=(AB)^t+(BA)^t$

$=B^tA^t+A^tB^t-BA+AB=AB+BA$

$\left(\because A^t=A \text{ and } B^t=B\right)$

Ans: (c)

120.A value of x satisfying $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}=0$ is

Options:

(a) abc

(b) $a+b+c$

(c) $-(a+b+c)$

(d) none of these

Sol: Operating $C_1 \to C_1 + C_2 + C_3$, we get $\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

Operate $R_2 \to R_2 \to R_1$ and $R_3 \to R_3 \to R_1$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

$$\Rightarrow x^2(x+a+b+c) = 0$$

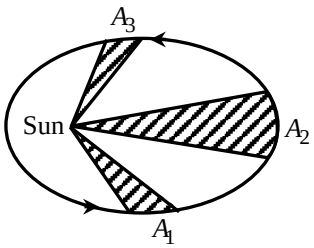
$x = 0$ or $x = -(a+b+c)$

Ans: (c)

Physics

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60

121. A planet moving around sun sweeps area A_1 in 2 days, A_2 in 3 days and A_3 in 6 days. Then the relation between A_1, A_2 and A_3 is



Options:

(a) $3A_1 = 2A_2 = A_3$

(b) $2A_1 = 3A_2 = 6A_3$

(c) $3A_1 = 2A_2 = 6A_3$

(d) $6A_1 = 3A_2 = 2A_3$

Sol: When a planet revolves around the sun, its real velocity is constant.

$$\therefore \frac{A_1}{t_1} = \frac{A_2}{t_2} = \frac{A_3}{t_3}$$

$$\frac{A_1}{2} = \frac{A_2}{3} = \frac{A_3}{6} \Rightarrow 3A_1 = 2A_2 = A_3$$

Ans: (a)

122. Identical springs of steel and copper $\left(Y_{\text{steel}} > Y_{\text{copper}}\right)$ are equally stretched. Then

Options:

(a) Less work is done on copper spring

(b) Less work is done on steel spring

(c) Equal work is done on both the springs

(d) Data is incomplete

Sol: Work done, $W = \frac{1}{2} \times F \times \Delta L$

For given F, $W \propto \Delta L$ … (i)

and $\Delta L = \frac{FL}{AY}$

As F, A and L are constants

$$\therefore \Delta L \propto \frac{1}{Y} \quad \text{… (ii)}$$

From (i) and (ii), we get

$$W \propto \frac{1}{Y}$$

Ans: (b)

123. A solid sphere is rotating in free space. If the radius of sphere is increased keeping mass same which one of the following will not be affected?

Options:

(a) Angular velocity

(b) Angular momentum

(c) Moment of inertia

(d) Rotational kinetic energy

Sol: When a solid sphere is rotated in free space, there will be no external torque, i.e., $\tau = 0$

But $\tau = \frac{dL}{dt}$ where L is the angular momentum. Hence $\tau = \frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$

Ans: (b)

124. Streamline flow is more likely for liquids with

Options:

(a) High density and low viscosity

(b) Low density and high viscosity

(c) High density and high viscosity

(d) Low density and low viscosity

Sol: Streamline flow is more likely for liquids having low density. We know that greater the coefficient of viscosity of a liquid more will be the velocity gradient, hence each of flow can be differentiated streamline flow is related with critical velocity. The critical velocity is that velocity of liquid flow up to which its flow is streamlined and above which its flow becomes turbulent. Critical velocity (V_c) varies with coefficient of viscosity (η) and density of liquid (ρ) as $(V_c) \propto \frac{\eta}{\rho}$. Hence if the density will be low and viscosity will be high, the value of critical velocity will be more.

Ans: (b)

125. $0.1\ \text{m}^3$ of water at $80°\text{C}$ is mixed with 0.3m^3 of water at $60°\text{C}$. The final temperature of the mixture is

Options:

(a) $65°\text{C}$

(b) $70°\text{C}$

(c) $60°\text{C}$

(d) $75°\text{C}$

Sol: Density of water $= 10^3\ \text{kg m}^{-3}$

Let the final temperature of the mixture be t. Assuming no heat transfer to or from container

Heat lost by water at $= 0.1\times10^3 \times s_{\text{water}} \times (80-t)$. Heat gained by water at $= 0.3\times10^3 \times s_{\text{water}} \times (t-60)$

According to principle of calorimetry, heat lost = heat gain

$0.1\times10^3 \times s_{\text{water}} \times (80-t) = 0.3\times10^3 \times s_{\text{water}} \times (t-60) \quad \Rightarrow 1\times(80-t) = 3\times(t-60) \Rightarrow t = 65°\text{C}$

Ans: (a)

126. The velocity of the molecules of a gas at temperature $120\,\text{K}$ is v. At what temperature will the velocity be $2v$?

Options:

(a) $120\,\text{K}$

(b) $240\,\text{K}$

(c) $480\,\text{K}$ (d) $1120\,\text{K}$

Sol: $v \propto \sqrt{T}$ $\therefore \frac{v'}{v} = \sqrt{\frac{T'}{T}}$

Given $v' = 2v$ or $\frac{2}{1} = \sqrt{\frac{T'}{T}}$

$\therefore T' = 4T = 4 \times 120\text{K} = 480\,\text{K}$

Ans: (c)

127. One mole of an ideal monoatomic gas is heated at a constant pressure of one atmosphere from $0°\text{C}$ to $100°\text{C}$. Then the change in the internal energy is

Options:

(a) 6.56 joules

(b) 8.32×10^2 joules

(c) 12.48×10^2 joules

(d) 20.80 joules

Sol: Change in internal energy is always equal to the heat supplied at constant volume.

i.e., $\Delta U = (\Delta Q)_V = \mu C_V \Delta T$

For monoatomic gas, $C_V = \frac{3}{2}R$ $\Rightarrow \Delta U = \mu\left(\frac{3}{2}R\right)\Delta T = 1 \times \frac{3}{2} \times 8.31 \times (100 - 0) = 12.48 \times 10^2\ \text{J}$

Ans: (c)

128. A particle is executing a simple harmonic motion of amplitude a. Its potential energy is maximum when the displacement from the position of the maximum kinetic energy is

Options:

(a) 0

(b) $\pm a$

(c) $+\frac{a}{2}$

(d) $-\frac{a}{2}$

Sol: P.E. of particle executing SHM $= \frac{1}{2}m\omega^2 x^2$

At $x = \pm a$, PE is maximum $= \frac{1}{2}m\omega^2 a^2$

Ans: (b)

129. Three sound waves of equal amplitudes have frequencies $(n-1), n, (n+1)$. They superimpose to give beats. The number of beats produced per second will be

Options:

(a) 1

(b) 4

(c) 3

(d) 2

Sol: $(n-1)$ and $(n+1)$ suppose to form frequency n and n will be at resonance.

$(n-1)$ and $n \rightarrow$ produce 1 beat. $(n+1)$ and $n \rightarrow$ produce 1 beat. Number of beats formed are '2'

Ans: (d)

130. Two charges are at a distance d apart. If a copper plate of thickness $\frac{d}{2}$ is kept between them, then effective force will be

Options:

(a) $\frac{F}{2}$

(b) $\sqrt{2}F$

(c) $2F$

(d) Zero

Sol: The dielectric constant for metal is infinity, the force between the two charges would be reduced to zero.

Ans: (d)

131. The electric field due to an extremely short dipole at distance r from it is proportional to

Options:

(a) $\frac{1}{r}$

(b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$

(d) $\frac{1}{r^4}$

Sol: Electric field due to a short dipole, $E \propto \frac{1}{r^3}$

Ans: (c)

132. A hollow conducting sphere of radius R has a charge $(+Q)$ on its surface. What is the electric potential within the sphere at a distance $r=\frac{R}{3}$ from its centre

Options:

(a) Zero

(b) $\frac{3}{4\pi\varepsilon_0}\frac{Q}{R}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

(d) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

Sol: Inside a conducting body, potential is same everywhere and equals to the potential at its surface.

Ans: (c)

133. An infinite number of charges, each of charge $1\mu C$, are placed on the $x-$axis with coordinates $x=1,2,4,8,...\infty$. If a charge of $1C$ is kept at the origin, then what is the net force acting on $1C$ charge?

Options:

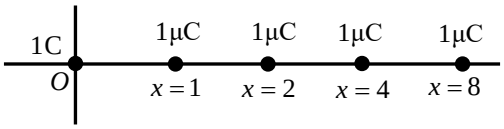
(a) $9000\,N$

(b) $12000\,N$

(c) $24000\,N$

(d) $36000\,N$

Sol: The schematic diagram of distribution of charges on $x-$axis is shown in figure below:



Total force acting on $1C$ charge is given by

$$F=\frac{1}{4\pi\varepsilon_0}\left[\frac{1\times1\times10^{-6}}{(1)^2}+\frac{1\times1\times10^{-6}}{(2)^2}+\frac{1\times1\times10^{-6}}{(4)^2}+\frac{1\times1\times10^{-6}}{(8)^2}+...\infty\right]$$

$$=\frac{10^{-6}}{4\pi\varepsilon_0}\left(\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+...\infty\right)$$

$$=9\times10^9\times10^6\left(\frac{1}{1-\frac{1}{4}}\right)$$

$$=9\times10^9\times10^{-6}\times\frac{4}{3}=9\times\frac{4}{3}\times10^3=12000\,N$$

Ans: (b)

134. A particle A has a charge q and particle B has charge $+4q$ with each of them having the mass m, when allowed to fall from rest through same potential difference. The ratio of their speeds $v_A : v_B$ will be

Options:

(a) $4:1$

(b) $1:4$

(c) $1:2$

(d) $2:1$

Sol: The energy gained by a particle of charge 'q' as it falls through a potential difference of V volts,

P.E. = change in K.E.

$qV = \frac{1}{2}mv^2$

For A: $qV = \frac{1}{2}mv_A^2$

For B: $4qV = \frac{1}{2}mv_B^2$

Ratio of speed, i.e., $v_A : v_B = 1:2$

Ans: (c)

135. Equivalent capacitance between A and B is

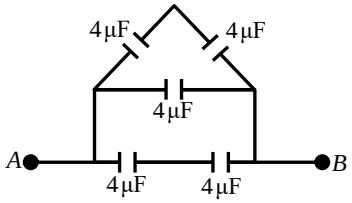
Options:

(a) $8\,\mu F$

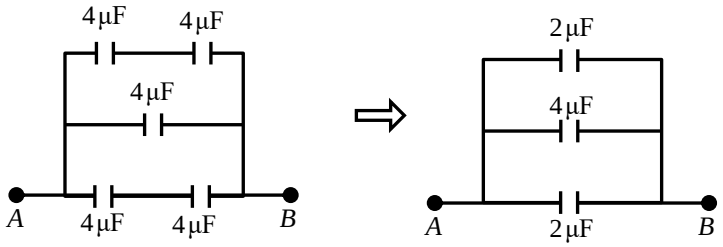
(b) $6\,\mu F$

(c) $26\,\mu F$

(d) $\frac{10}{3}\,\mu F$



Sol:



$\Rightarrow C_{AB} = 8\,\mu F$

Ans: (a)

136. A cube of side $'b'$ has a charge q at each of its vertices. The electric field at the centre of the cube is

Options:

(a) $\frac{4q}{3\pi\varepsilon_0 b^2}$

(b) $\frac{3q}{4\pi\varepsilon_0 b^2}$

(c) $\frac{2q}{\pi\varepsilon_0 b^2}$

(d) Zero

Sol: Due to symmetric charge distribution. The electric field at the centre of the cube is zero.

Ans: (d)

137. Two small similar metal spheres A and B having charges $4q$ and $-4q$, when placed at a certain distance apart, exert an electric force F on each other. When another identical uncharged sphere C, first touched with A then with B and then removed to infinity, the force of interaction between A and B for the same separation will be

Options:

(a) $\frac{F}{2}$

(b) $\frac{F}{8}$

(c) $\frac{F}{16}$

(d) $\frac{F}{32}$

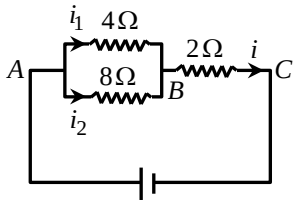
Sol: $F=\frac{1}{4\pi\varepsilon_0}\frac{(4q)(-4q)}{r^2}$

When C is touched with A, then charge on A and C each $=2q$ after that C is touched with B, charge on $B=\frac{2q+(-4q)}{2}=-q$

Now, force $F'=\frac{1}{4\pi\varepsilon_0}\frac{(2q)(-q)}{r^2}\Rightarrow F'=\frac{F}{8}$

Ans: (b)

138. In the circuit shows in figure, the current in 4Ω resistance is $1.2\,\text{A}$. What is the potential difference between B and C?



Options:

(a) 3.6 volt

(b) 6.3 volt

(c) 1.8 volt

(d) 2.4 volt

Sol: The potential difference across 4Ω resistance is given by $V = 4 \times i_1 = 4 \times 1.2 = 4.8$ volt

So, the potential across 8Ω resistance is also 4.8 volt. Current $i_2 = \frac{V}{8} = \frac{4.8}{8} = 0.6\,\text{A}$

Current in 2Ω resistance $i = i_1 + i_2$

$\therefore i = 1.2 + 0.6 = 1.8\,\text{A}$

Potential difference across 2Ω resistance, $V_{BC} = 1.8 \times 2 = 3.6$ volts

Ans: (a)

139. A cell in secondary circuit gives null deflection for $2.5\,\text{m}$ length of potentiometer having $10\,\text{m}$ length of wire. If the length of the potentiometer wire is increased by $1\,\text{m}$ without changing the cell in the primary, the position of the null point now is

Options:

(a) $3.5\,\text{m}$

(b) $3\,\text{m}$

(c) $2.75\,\text{m}$

(d) $2.0\,\text{m}$

Sol: $L \propto l$

$\frac{L_1}{L_2} = \frac{l_1}{l_2}$

$\frac{10}{11} = \frac{2.5}{l_2}$

$10 l_2 = 2.5 \times 11$

$l_2 = \frac{2.5 \times 11}{10} = 2.75\,\text{m}$

Ans: (c)

140. A piece of copper and another of germanium are cooled from room temperature to $50\,\text{K}$. The resistance of

Options:

(a) Each of them decreases

(b) Copper decreases and germanium increases

(c) Each of them increases

(d) Copper increases and germanium decreases

Sol: The resistance of metal decreases with decrease of temperature while for semiconductors, resistance increases when temperature decreases.

Ans: (b)

141. On interchanging the resistances, the balance point of a meter bridge shifts to the left by $10\,\text{cm}$. The resistance of their series combination is $1\text{k}\Omega$. How much was the resistance on the left slot before interchanging the resistances?

Options:

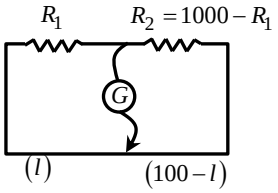
(a) $990\,\Omega$

(b) $505\,\Omega$

(c) $550\,\Omega$

(d) $910\,\Omega$

Sol: $R_1 + R_2 = 1000$



$\Rightarrow R_2 = 1000 - R_1$

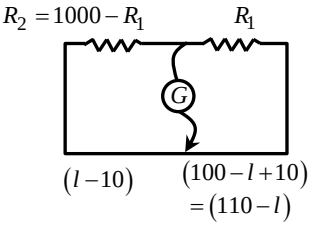
On balancing condition,

$R_l\,(100 - l) = (1000 - R_1)\,l$ … (i)

On interchanging resistance balance point shifts left by $10\,\text{cm}$

On balancing condition,

$(1000 - R_1)(110 - l) = R_1\,(l - 10)$



or, $R_1\,(l - 10) = (1000 - R_1)(110 - l)$ … (ii)

Dividing eqn. (i) by (ii)

$$\frac{100 - l}{l - 10} = \frac{l}{110 - l}$$

$\Rightarrow (100 - l)(110 - l) = l\,(l - 10)$

$\Rightarrow 11000 - 100l - 110l + l^2 = l^2 - 10l$

$\Rightarrow 11000 = 200l$ or $l = 55$

Putting the value of $'l'$ in eqn. (i), $R_1\,(100 - 55) = (1000 - R_1)55$

$\Rightarrow R_1\,(45) = (1000 - R_1)55$

$\Rightarrow R_1\,(9) = (1000 - R_1)11$

$\Rightarrow 20R_1 = 11000$

$\therefore R_1 = 550\,\Omega$

Ans: (c)

142. In circuit shown below, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3 volt. The voltage across the resistance R_4 is

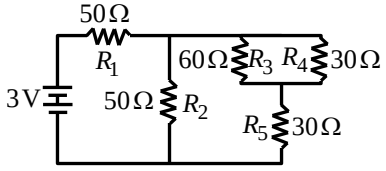
Options:

(a) $0.4\,\text{V}$

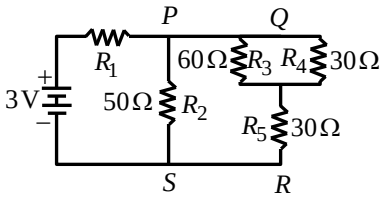
(b) $0.6\,\text{V}$

(c) $1.2\,\text{V}$

(d) $1.5\,\text{V}$



Sol:



Net resistance between Q and R, $R' = \frac{60\times 30}{60+30} + 30 = 50\,\Omega$

So, equivalent resistance, $R_{eq} = \frac{50\times 50}{50+50} + 50 = 75\,\Omega$

Current in the circuit is $I = \frac{3}{75} = \frac{1}{25}$ A. Potential drop across $R_1 = 50\times\frac{1}{25} = 2\,V$

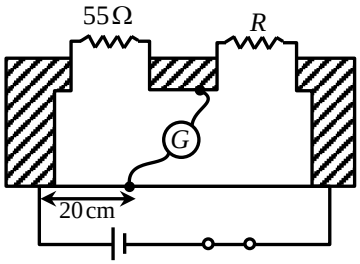
$\therefore$ Potential difference across PS or QR is same and given by $= 3-2 = 1$ volt. Current across $QR = \frac{1}{50}$ A

Potential drop across R_5 is given by $\frac{1}{50}\times 30 = 0.6\,V$

Hence potential drop across R_4 is $= 1-0.6 = 0.4\,V$

Ans: (a)

143.Shown in the figure below is a meter bridge set up with null deflection in the galvanometer.



The value of the unknown resistor R is

Options:

(a) 13.75Ω

(b) 220Ω

(c) 110Ω

(d) 55Ω

Sol: According to the condition of balancing, $\frac{55}{20} = \frac{R}{80} \Rightarrow R = 220\,\Omega$

Ans: (b)

144. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

Options:

(a) $\frac{\pi^2}{16}$

(b) $\frac{\pi^2}{8\sqrt{2}}$

(c) $\frac{\pi^2}{8}$

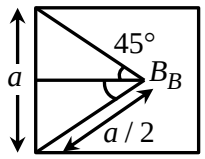
(d) $\frac{\pi^2}{16\sqrt{2}}$

Sol: **Case (a)**:

$$B_A = \frac{\mu_0}{4\pi}\frac{I}{R}\times 2\pi = \frac{\mu_0}{4\pi}\frac{I}{l/2\pi}\times 2\pi \qquad (2\pi R = l)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{l}\times(2\pi)^2$$

Case (b):



$$B_B = 4\times\frac{\mu_0}{4\pi}\frac{l}{a/2}\left[\sin 45^\circ + \sin 45^\circ\right] = 4\times\frac{\mu_0}{4\pi}\times\frac{I}{l/8}\times\frac{2}{\sqrt{2}} = \frac{\mu_0}{4\pi}\frac{I}{l}\times 32\times\sqrt{2} \qquad [4a = l]$$

$$\therefore \frac{B_B}{B_A} = \frac{\pi^2}{8\sqrt{2}}$$

Ans: (b)

145. A moving coil galvanometer has N number of turns in a coil of effective area A, it carries a current I. The magnetic field B is radial. The torque acting on the coil is

Options:

(a) NA^2B^2I

(b) $NABI^2$

(c) N^2ABI

(d) $NABI$

Sol: $\tau = MB\sin\theta \qquad \Rightarrow \tau_{\max} = NIAB,\ (\theta = 90^\circ)$

Ans: (d)

146. The distance between the wires of electric mains is $12\,\text{cm}$. These wires experience $4\,\text{mg wt}$ per unit length. The value of current flowing in each wire will be

Options:

(a) $4.85\,\text{A}$

(b) 0

(c) $4.85\times10^{-2}\,\text{A}$

(d) $4.85\times10^{-4}\,\text{A}$

Sol: $\frac{F}{l}=\frac{\mu_0 i^2}{2\pi d}=9.8\times4\times10^{-6}$ $\Rightarrow i=\sqrt{\frac{4\times10^{-6}\times9.8\times0.12}{2\times10^{-7}}}=4.85\,\text{A}$

Ans: (a)

147. A ring of radius R, made of an insulating material carries a charge Q uniformly distributed on it. If the ring rotates about the axis passing through its centre and normal to plane of the ring with constant angular speed ω, then the magnitude of the magnetic moment of the ring is

Options:

(a) $Q\omega R^2$

(b) $\frac{1}{2}Q\omega R^2$

(c) $Q\omega^2 R$

(d) $\frac{1}{2}Q\omega^2 R$

Sol: $M=iA=i\times\pi R^2$

Also, $i=\frac{Q\omega}{2\pi}\Rightarrow M=\frac{1}{2}Q\omega R^2$ $\left[\because i=\frac{Q}{t}\right]$

Ans: (b)

148. The horizontal component of the Earth's magnetic field is 3.6×10^{-5} tesla where the dip angle is $60°$. The magnitude of the Earth's magnetic field is

Options:

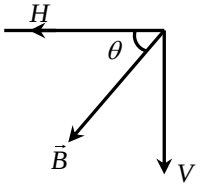
(a) 2.8×10^{-4} tesla

(b) 2.1×10^{-4} tesla

(c) 7.2×10^{-5} tesla

(d) 3.6×10^{-5} tesla

Sol: Horizontal component of earth's field, $H=B\cos\theta$



Since, $\theta=60°$

$3.6\times10^{-5}=B\times\frac{1}{2}\Rightarrow B=7.2\times10^{-5}$ Tesla

Ans: (c)

149. A deuteron of kinetic energy $50\,\text{keV}$ is describing a circular orbit of radius 0.5 metre in a plane perpendicular to the magnetic field B. The kinetic energy of the proton that describes a circular orbit of radius 0.5 metre in the same plane with the same B is

Options:

(a) $25\,\text{keV}$

(b) $50\,\text{keV}$

(c) $200\,\text{keV}$ (d) $100\,\text{keV}$

Sol: For a charged particle orbiting in a circular path in a magnetic field

$\frac{mv^2}{r} = Bqv \Rightarrow v = \frac{Bqr}{m}$ or $mv^2 = Bqvr$

Also, $E_K = \frac{1}{2}mv^2 = \frac{1}{2}Bqvr = Bq\frac{r}{2}\cdot\frac{Bqr}{m} = \frac{B^2q^2r^2}{2m}$

For deuteron, $E_1 = \frac{B^2q^2r^2}{2\times 2\,\text{m}}$

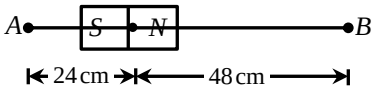
For proton, $E_2 = \frac{B^2q^2r^2}{2\,\text{m}}$

$\frac{E_1}{E_2} = \frac{1}{2} \Rightarrow \frac{50\,\text{keV}}{E_2} = \frac{1}{2} \Rightarrow E_2 = 100\,\text{keV}$

Ans: (d)

150. A bar magnet of length $3\,\text{cm}$ has points A and B along its axis at distances of $24\,\text{cm}$ and $48\,\text{cm}$ on the opposite sides. Ratio of magnetic fields at these points will be

Options:



(a) 8

(b) $\frac{1}{2\sqrt{2}}$

(c) 3

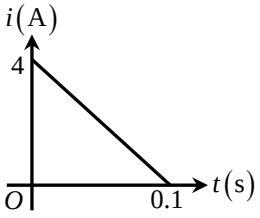
(d) 4

Sol: Both points A and B lie on axial position, $B \propto \frac{1}{d^3} \Rightarrow \frac{B_A}{B_B} = \left(\frac{d_B}{d_A}\right)^3 = \left(\frac{48}{24}\right)^3 = \frac{8}{1}$

Ans: (a)

151. In a coil of resistance $10\,\Omega$, the induced current developed by changing magnetic flux through it, is shown in figure as a function of time. The magnitude of change in flux through the coil in weber is

Options:



(a) 8

(b) 2

(c) 6

(d) 4

Sol: The charge through the coil = area of current-time $(i-t)$ graph, $q=\frac{1}{2}\times 0.1\times 4=0.2\,\text{C}$

$q=\frac{\Delta\phi}{R}$ $\quad\because$ Change in flux $(\Delta\phi)=q\times R$

$q=0.2=\frac{\Delta\phi}{10}$

$\Delta\phi=2$ Weber

Ans: (b)

152. Two coils have a mutual inductance $0.005\,\text{H}$. The current changes in the first coil according to equation $I=I_0\sin\omega t$, where $I_0=10\,\text{A}$ and $\omega=100\pi\,\text{rad s}^{-1}$. The maximum value of e.m.f. in the second coil is

Options:

(a) 2π

(b) 5π

(c) π

(d) 4π

Sol: $e=M\frac{di}{dt}=0.005\times\frac{d}{dt}(i_0\sin\omega t)$

$=0.0005\times i\omega\cos\omega t$

$\therefore\ e_{\max}=0.005\times 10\times 100\pi=5\pi$ $\quad[\because\cos\omega t=1]$

Ans: (b)

153. A coil has resistance 30 ohm and inductive reactance 20 ohm at $50\,\text{Hz}$ frequency. If an ac source, of 200 volt, $100\,\text{Hz}$, is connected across the coil, the current in the coil will be

Options:

(a) $4.0\,\text{A}$

(b) $8.0\,\text{A}$

(c) $\frac{20}{\sqrt{13}}\,\text{A}$

(d) $2.0\,\text{A}$

Sol: If $\omega=50\times 2\pi$, then $\omega L=20\,\Omega$

If $\omega'=100\times 2\pi$, then $\omega' L=40\,\Omega$

Current flowing in the coil is $I=\frac{200}{Z}=\frac{200}{\sqrt{R^2+(\omega' L)^2}}=\frac{200}{\sqrt{(30)^2+(40)^2}}$

$I=4\,\text{A}$

Ans: (a)

154. In an AC circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

Options:

(a) $P = \sqrt{2} E_0 I_0$

(b) $P = \frac{E_0 I_0}{\sqrt{2}}$

(c) $P = \text{zero}$

(d) $P = \frac{E_0 I_0}{2}$

Sol: We know that power consumed in AC circuit is given by, $P = E_{\text{rms}} \cdot I_{\text{rms}} \cos\phi$

Here, $E = E_0 \sin \omega t$

$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ which implies that the phase difference, $\phi = \frac{\pi}{2}$

$\therefore P = E_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos\frac{\pi}{2} = 0 \qquad \left[\because \cos\frac{\pi}{2} = 0\right]$

Ans: (c)

155. An AC voltage is applied to a resistance R and an inductor L in series. If R and the inductive reactance are both equal to 3Ω, the phase difference between the applied voltage and the current in the circuit is

Options:

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) Zero

Sol: The phase difference ϕ is given by $\tan\phi = \frac{X_L}{R} = \frac{3}{3} = 1 \qquad \Rightarrow \phi = \frac{\pi}{4}$

Ans: (b)

156. During the propagation of electromagnetic waves in a medium

Options:

(a) Electric energy density is double of the magnetic energy density

(b) Electric energy density is half of the magnetic energy density

(c) Electric energy density is equal to the magnetic energy density

(d) Both electric and magnetic energy densities are zero

Sol: $E_0 = cB_0$ and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Electric energy density $= \frac{1}{2}\varepsilon_0 E_0^2 = \mu_E$

Magnetic energy density $= \frac{1}{2}\frac{B_0^2}{\mu_0} = \mu_B$

Thus, $\mu_E = \mu_B$

Energy is equally divided between electric and magnetic field.

Ans: (c)

157. A ray of light passes through an equilateral prism such that the angle of incidence is equal to the angle of emergence and the latter is equal to $\frac{3}{4}$th of angle of prism. The angle of deviation is

Options:

(a) 25°

(b) 30°

(c) 45°

(d) 35°

Sol: From the figure, angle of deviation

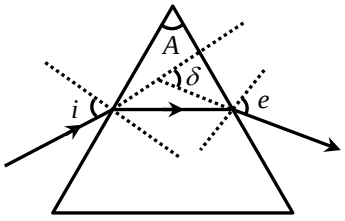
$\delta = i + e - A$

Here, $e = i$ and $e = \frac{3}{4}A$

$\therefore \delta = \frac{3}{4}A + \frac{3}{4}A - A = \frac{A}{2}$

For equilateral prism, $A = 60°$

$\therefore \delta = \frac{60°}{2} = 30°$



Ans: (b)

158. The magnifying power of a telescope is 9 . When it is adjusted for parallel rays the distance between the objective and eyepiece is 20 cm . The focal length of lenses are

Options:

(a) 10 cm, 10 cm

(b) 15 cm, 5 cm

(c) 18 cm, 2 cm

(d) 11 cm, 9 cm

Sol: M.P. $= 9 = \frac{f_0}{f_e}$

$\Rightarrow f_0 = 9 f_e$ … (1)

$f_0 + f_e = 20$ … (2)

From (1) and (2),

$f_0 = 18\,\text{cm}$ = focal length of the objective

$f_e = 2\,\text{cm}$ = focal length of the eyepiece

Ans: (c)

159. A light ray from air is incident (as shown in figure) at one end of a glass fibre (refractive index $\mu = 1.5$) making an incidence angle of $60°$ on the lateral surface, so that it undergoes a total internal reflection. How much time would it take to traverse the straight fibre of length $1\,\text{km}$?

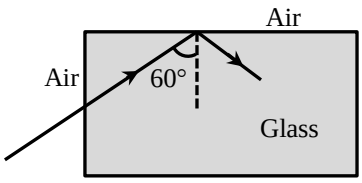
Options:

(a) $3.33\,\mu\text{s}$

(b) $6.67\,\mu\text{s}$

(c) $5.77\,\mu\text{s}$

(d) $3.85\,\mu\text{s}$



Sol: When total internal reflection just takes place from lateral surface $i = C$, i.e., $60° = C$

$$\Rightarrow \sin 60° = \sin C = \frac{1}{\mu} \Rightarrow \mu = \frac{2}{\sqrt{3}}$$

Time taken by light to traverse some distance in a medium, $t = \dfrac{\mu x}{c} = \dfrac{\frac{2}{\sqrt{3}} \times 10^3}{3 \times 10^8} = 3.85\ \mu\text{s}$

Ans: (d)

160. When plane face of plano-convex lens is silvered, it behaves as a concave mirror of focal length $30\,\text{cm}$. But when its curved surface is silvered, it behaves as a concave mirror of focal length $10\,\text{cm}$. The refractive index of lens material is

Options:

(a) 1.25

(b) 1.33

(c) 1.732 (d) 1.5

Sol: $f_1 = \dfrac{R}{2(\mu - 1)} = 30\,\text{cm};\ f_2 = \dfrac{R}{2\mu} = 10\,\text{cm}$

Solving, $\mu = 1.5$ $\left[\because \dfrac{1}{f_{eq}} = \dfrac{1}{f_1} + \dfrac{1}{f_2}\right]$

Ans: (d)

161. In double slit experiment, the angular width of the fringes is $0.20°$ for the sodium light $(\lambda = 5890\,\text{Å})$. In order to increase the angular width of the fringes by 10%, the necessary change in wavelength is

Options:

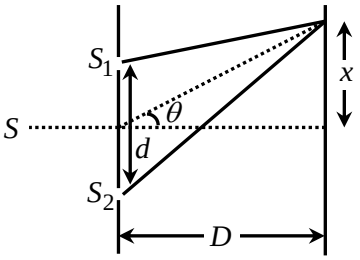
(a) Zero

(b) Increased by $6479\,\text{Å}$

(c) Decreased by $589\,\text{Å}$

(d) Increased by $589\,\text{Å}$

Sol: Let λ be wavelength of monochromatic light incident on slit S, then angular distance between two consecutive fringes, that is the angular fringe width is $\theta = \frac{\lambda}{d}$ where d is distance between coherent sources.



Give, $\frac{\Delta\theta}{\theta} = \frac{10}{100}$

So from eq. (1), $\frac{\Delta\lambda}{\lambda} = \frac{\Delta\theta}{\theta} = \frac{10}{100} = 0.1$

$\Rightarrow \Delta\lambda = 0.1\lambda = 0.1 \times 5890\,\text{Å} = 589\,\text{Å}$ (increases)

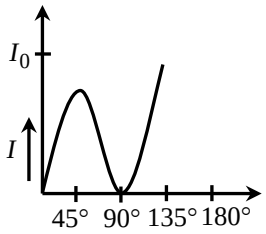
Note: Since $\theta \propto \lambda$, as θ increases, λ increases.

Ans: (d)

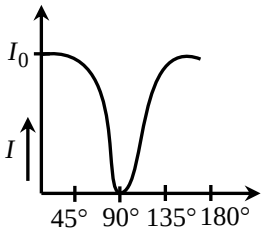
162. The graph showing the dependence of intensity of transmitted light on the angle between polariser and analyser, is

Options:

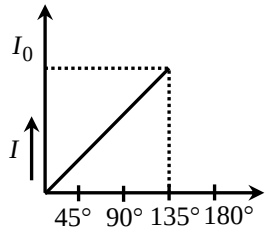
(a)



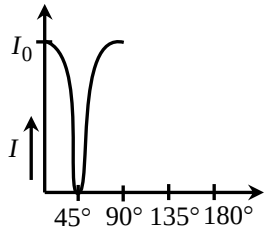
(b)



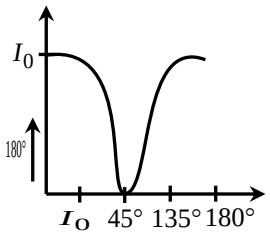
(c)



(d)



Sol: According to law of Malus, when a beam of completely plane polarised light is incident on analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of the angle (θ) between planes of transmission of analyser and polariser.



i.e., $I \propto \cos^2\theta$ and $I = I_0 \cos^2\theta$ … (i)

where I_0 = intensity of the light from polariser. From eq. (i), we note that if the transmission axes of polariser and analyser are parallel (i.e., $\theta = 0°$ or $180°$) then $I = I_0$. It means that intensity of transmitted light is maximum. When the transmission axes of polariser and analyser are perpendicular (i.e., $\theta = 90°$), then $I = I_0 \cos^2 90° = 0$. It means the intensity of transmitted light is minimum. On plotting a graph between I and θ as given by relation (i), we get the curve as shown in figure.

Ans: (b)

163. When photon of energy $4.25\,\text{eV}$ strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy $T_A\,\text{eV}$ and de Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy $4.70\,\text{eV}$ is $T_B = (T_A - 1.50)\,\text{eV}$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then

Options:

(a) The work function of A is $3.40\,\text{eV}$

(b) The work function of B is $6.75\,\text{eV}$

(c) $T_A = 2.00\,\text{eV}$

(d) $T_B = 2.75\,\text{eV}$

Sol: $K_{\max} = E - W_0$

$\therefore T_A = 4.25 - (W_0)_A$ … (i)

$T_B = (T_A - 1.5) = 4.70 - (W_0)_B$ … (ii)

Equation (i) and (ii) gives

$(W_0)_B - (W_0)_A = 1.95\,\text{eV}$

De Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2mK}} \Rightarrow \lambda \propto \dfrac{1}{\sqrt{K}}$

$\Rightarrow \dfrac{\lambda_B}{\lambda_A} = \sqrt{\dfrac{K_A}{K_B}} \Rightarrow 2 = \sqrt{\dfrac{T_A}{T_B - 1.5}} \Rightarrow T_A = 2\,\text{eV}$

From equation (i) and (ii), $W_A = 2.25\,\text{eV}$ and $W_B = 4.20\,\text{eV}$

Ans: (c)

164. Photoelectric emission is observed from a metallic surface for frequencies v_1 and v_2 of the incident light rays $(v_1 > v_2)$. If the maximum values of kinetic energy of the photoelectrons emitted in two cases are in the ratio of $1:k$, then the threshold frequency of the metallic surface is

Options:

(a) $\frac{v_1 - v_2}{k-1}$

(b) $\frac{kv_1 - v_2}{k-1}$

(c) $\frac{kv_2 - v_1}{k-1}$

(d) $\frac{v_2 - v_1}{k}$

Sol: By using $hv - hv_0 = K_{\max}$

$\Rightarrow h(v_1 - v_0) = k_1$ … (i)

and $h(v_2 - v_0) = k_2$ … (ii)

$\Rightarrow \frac{v_1 - v_0}{v_2 - v_0} = \frac{k_1}{k_2} = \frac{1}{k}$

Hence $v_0 = \frac{kv_1 - v_2}{k-1}$

Ans: (b)

165. The ratio of maximum to minimum wavelength in Balmer series is

Options:

(a) $3:4$

(b) $1:4$

(c) $5:36$

(d) $5:9$

Sol: $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For Balmer series, $n = 2$

$\frac{1}{\lambda_{\max}} = RZ^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$ and $\frac{1}{\lambda_{\min}} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

or $\frac{\frac{1}{\lambda_{\min}}}{\frac{1}{\lambda_{\max}}} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{2^2}\right)} = \frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}} = 1 - \frac{4}{9} = \frac{5}{9}$ $\Rightarrow \frac{\lambda_{\max}}{\lambda_{\min}} = \frac{5}{9}$

Ans: (d)

166. In the following atoms and molecules for the transition from $n=2$ to $n=1$, the spectral line of minimum wavelength will be produced by

Options:

(a) Hydrogen atom

(b) Deuterium atom

(c) Uni-ionized helium

(d) Di-ionized lithium

Sol: $\frac{1}{\lambda}=RZ^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$

Z is maximum for lithium. Thus, the wavelength is minimum.

Ans: (d)

167. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom $(Z=3)$ is

Options:

(a) 1.51

(b) 13.6

(c) 40.8

(d) 122.4

Sol: $E=-Z^2\times 13.6\,\text{eV}=-9\times 13.6\,\text{eV}=-122.4\,\text{eV}$

So ionisation energy $=+122.4\,\text{eV}$

Ans: (d)

168. During a negative beta decay

Options:

(a) An atomic electron is ejected

(b) An electron which is already present within the nucleus is ejected

(c) A neutron in the nucleus decays emitting an electron

(d) A part of the binding energy is converted into electron

Sol: Negative $\beta-$ decay is expressed by the equation

$n=p^+ + e^- + \bar{\nu}$

Ans: (c)

169. If T is the half-life of a radioactive material, then the fraction that would remain after a time $\frac{T}{2}$ is

Options:

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{\sqrt{2}-1}{\sqrt{2}}$

Sol: Fraction remains after n half lives

$$\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{t/T}$$

$$\therefore \frac{N}{N_0}=\left(\frac{1}{2}\right)^{\frac{T/2}{T}}=\left(\frac{1}{2}\right)^{1/2}=\frac{1}{\sqrt{2}}$$

Ans: (c)

170. The mass of a $^{7}_{3}Li$ nucleus is $0.042u$ less than the sum of masses of all its nucleons. The binding energy per nucleon of $^{7}_{3}Li$ nucleus is nearly

Options:

(a) $46\,\text{MeV}$

(b) $5.6\,\text{MeV}$

(c) $3.9\,\text{MeV}$

(d) $23\,\text{MeV}$

Sol: $\text{B.E.} = 0.042\times 931 \simeq 42\,\text{MeV}$

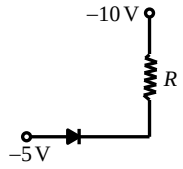
Number of nucleons in $^{7}_{3}Li$ is 7.

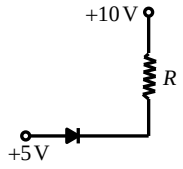
$$\therefore \text{B.E./Nucleon}=\frac{42}{7}=6\,\text{MeV}\simeq 5.6\,\text{MeV}$$

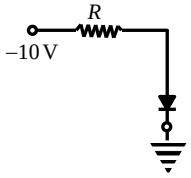
Ans: (b)

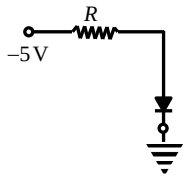
171. Which of the junction diodes shown below are forward biased?

Options:

(a) 

(b) 

(c) 

(d) 

Sol: Positive terminal is at higher potential $(-5\,\text{V})$ and negative terminal is at lower potential $-10\,\text{V}$.

Ans: (a)

172. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities?

Options:

(a) $\frac{5}{8}$

(b) $\frac{4}{5}$

(c) $\frac{5}{4}$

(d) $\frac{4}{7}$

Sol: $\frac{I_e}{I_h}=\frac{n_e eAv_e}{n_h eAv_h}\Rightarrow\frac{7}{4}=\frac{7}{5}\times\frac{v_e}{v_h}\Rightarrow\frac{v_e}{v_h}=\frac{5}{4}$

Ans: (c)

173. The reading of the ammeter for a silicon diode in the given circuit is

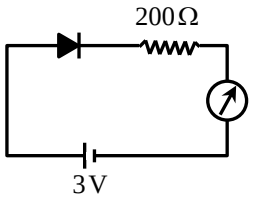
Options:

(a) 0

(b) 15 mA

(c) 11.5 mA

(d) 13.5 mA



Sol: Clearly from figure given in question, silicon diode is in forward bias.

$\therefore$ Potential barrier across diode, $\Delta V = 0.7$ volts

Current, $I = \dfrac{V - \Delta V}{R} = \dfrac{3 - 0.7}{200} = \dfrac{2.3}{200} = 11.5\,\text{mA}$

Ans: (c)

174. Two straight long conductors AOB and COD are perpendicular to each other and carry currents i_1 and i_2. The magnitude of the magnetic induction at a point P at a distance a from the point O in a direction perpendicular to the plane $ABCD$ is

Options:

(a) $\dfrac{\mu_0}{2\pi a}(i_1 + i_2)$

(b) $\dfrac{\mu_0}{2\pi a}(i_1 - i_2)$

(c) $\dfrac{\mu_0}{2\pi a}\left(i_1^2 + i_2^2\right)^{1/2}$

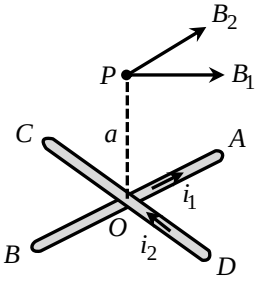
(d) $\dfrac{\mu_0}{2\pi a}\dfrac{i_1 i_2}{(i_1 + i_2)}$

Sol: At $P: B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$

$= \sqrt{\left(\dfrac{\mu_0}{4\pi}\dfrac{2i_1}{a}\right)^2 + \left(\dfrac{\mu_0}{4\pi}\dfrac{2i_2}{a}\right)^2}$

$= \dfrac{\mu_0}{2\pi a}\left(i_1^2 + i_2^2\right)^{1/2}$

Ans: (c)



175. The velocity v of a particle at time t is given by $v = at + \dfrac{b}{t + c}$, where a, b and c are constant. The dimensions of a, b and c are respectively

Options:

(a) $\left[L^2\right], [T]$ and $\left[LT^2\right]$

(b) $\left[LT^2\right], [LT]$ and $[L]$

(c) $[L], [LT]$ and $\left[T^2\right]$

(d) $\left[LT^{-2}\right], [L]$ and $[T]$

Sol: Dimension of at = dimension of velocity

$a \cdot T = LT^{-1} \qquad \Rightarrow a = LT^{-2}$

Dimension of C = dimension of t (two physical quantity of same dimension can only be added)

So, dimension of $C = T$

Dimension of $\frac{b}{t+c}$ = dimension of V

$\frac{b}{T+T} = LT^{-1} \qquad \Rightarrow b \cdot T^{-1} = LT^{-1} \qquad \Rightarrow b = L$

So answer is $\left[LT^{-2}\right], [L]$ and $[T]$

Ans: (d)

176. A ball is dropped from a bridge at a height of $176.4\,\text{m}$ over a river. After $2\,\text{s}$, a second ball is thrown straight downwards. What should be the initial velocity of the second ball so that both hit the water simultaneously?

Options:

(a) $2.45\,\text{m s}^{-1}$

(b) $49\,\text{m s}^{-1}$

(c) $14.5\,\text{m s}^{-1}$

(d) $24.5\,\text{m s}^{-1}$

Sol: If t seconds be the time taken by the first ball to reach the river then time taken by the second ball to reach the river will be $(t-2)$ second. Distance covered by first ball in t second

$= \frac{1}{2}gt^2 \Rightarrow 179.4 = \frac{1}{2} \times 9.8 \times t^2$

$\Rightarrow t = \sqrt{\frac{2 \times 176.4}{9.8}} = 6\,\text{s}$

Thus, second ball will take $4\,\text{s}$ to reach the river. If u be its initial speed, then

$176.4 = (u \times 4) + \frac{1}{2} \times 9.8 \times (4)^2 \qquad$ (using $h = ut + \frac{1}{2}gt^2$)

$\Rightarrow 4u = 176.4 - (9.8 \times 8) = 98$

$\Rightarrow u = 24.5\,\text{m s}^{-1}$

Ans: (d)

177. If the relation between the range R and time of flight T of a projectile is given as $R = 5T^2$, the value of angle of projection is

Options:

(a) 45°

(b) 15°

(c) 60° (d) 90°

Sol: Given $R = 5T^2$

$\Rightarrow \frac{u^2 \sin 2\theta}{g} = 5 \times \left(\frac{2u \sin\theta}{g}\right)^2$ $\Rightarrow \frac{\sin 2\theta}{1} = \frac{5 \times 4 \times \sin^2\theta}{g}$ $\Rightarrow 2\sin\theta\cos\theta \times g = 20 \times \sin^2\theta$

$\Rightarrow \tan\theta = 1 \quad \left(\because g = 10\,\text{m s}^{-2}\right)$

$\Rightarrow \theta = \tan^{-1}(1) = 45°$

Ans: (a)

178. A body of mass $10\,\text{kg}$ is acted upon by two perpendicular forces, $6\,\text{N}$ and $8\,\text{N}$. The resultant acceleration of the body is

Options:

(a) $1\,\text{m s}^{-2}$ at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w.r.t. $8\,\text{N}$ force

(b) $0.2\,\text{m s}^{-2}$ at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w.r.t. $8\,\text{N}$ force

(c) $1\,\text{m s}^{-2}$ at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w.r.t. $8\,\text{N}$ force

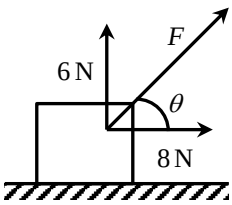
(d) $0.2\,\text{m s}^{-2}$ at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w.r.t. $8\,\text{N}$ force

Sol: Here, $m = 10\,\text{kg}$

The resultant force acting on the body is $F = \sqrt{(8\,\text{N})^2 + (6\,\text{N})^2} = 10\,\text{N}$

Let the resultant force F makes an angle θ w.r.t. $8\,\text{N}$ force.

From figure, $\tan\theta = \frac{6\,\text{N}}{8\,\text{N}} = \frac{3}{4}$



The resultant acceleration of the body is $a = \frac{F}{m} = \frac{10\,\text{N}}{10\,\text{kg}} = 1\,\text{m s}^{-2}$

The resultant acceleration is along the direction of the resultant force. Hence, the resultant acceleration of the body is $1\,\text{m s}^{-2}$ at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w.r.t. $8\,\text{N}$ force.

Ans: (a)

179. A body of mass m kg is ascending on a smooth inclined plane of inclination $\theta\left(\sin\theta=\frac{1}{x}\right)$ with constant acceleration of a $\mathrm{m\,s^{-2}}$. The final velocity of the body is $v\,\mathrm{m\,s^{-1}}$. The work done by the body during this motion is (initial velocity of the body = 0)

Options:

(a) $\frac{1}{2}mv^2(g+xa)$

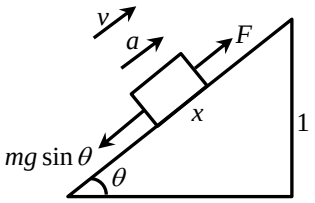
(b) $\frac{mv^2}{2}\left(\frac{g}{2}+a\right)$

(c) $\frac{2mv^2x}{a}(a+gx)$

(d) $\frac{mv^2}{2ax}(g+xa)$

Sol: $\sin\theta=\frac{1}{x}$

From free body diagram of the body



$F-mg\sin\theta=ma$

$F=m(g\sin\theta+a)=m\left(\frac{g}{x}+a\right)$ … (1)

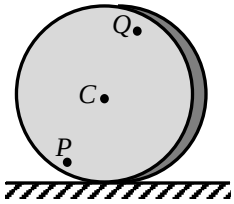
Displacement of the body till its velocity reaches v, $v^2=0+2as$ $\Rightarrow s=\frac{v^2}{2a}$

Now, work done $=Fs\cos0°=\frac{m}{x}(g+ax)\times\frac{v^2}{2a}=\frac{mv^2}{2ax}(g+ax)$

Ans: (d)

180. A disc is rolling (without slipping) on a horizontal surface. C is its centre and Q and P are two points equidistant from C. Let v_P, v_Q and v_C be the magnitude of velocities of points P, Q and C respectively, then

Options:



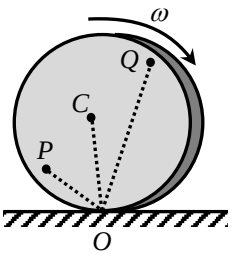
(a) $v_Q > v_C > v_P$

(b) $v_Q < v_C < v_P$

(c) $v_Q = v_P, v_C = \frac{v_P}{2}$

(d) $v_Q < v_C > v_P$

Sol: Since disc is rolling (without slipping) about point O. Hence,



$OQ > OC > OP$

$\because v=r\omega$

$\therefore v_Q > v_C > v_P$

Ans: (a)

Key Answers:

1. a	2. d	3. c	4. a	5. c	6. d	7. b	8. d	9. d	10. d
11. c	12. b	13. a	14. c	15. b	16. b	17. c	18. d	19. c	20. c
21. c	22. b	23. d	24. c	25. c	26. b	27. d	28. b	29. c	30. b
31. a	32. c	33. d	34. a	35. d	36. b	37. b	38. c	39. b	40. b
41. b	42. d	43. c	44. b	45. d	46. c	47. d	48. b	49. d	50. b
51. c	52. d	53. c	54. a	55. c	56. a	57. c	58. c	59. c	60. b
61. b	62. d	63. a	64. b	65. a	66. b	67. b	68. a	69. b	70. b
71. a	72. a	73. a	74. a	75. b	76. b	77. a	78. c	79. a	80. d
81. d	82. c	83. a	84. d	85. b	86. c	87. a	88. a	89. a	90. c
91. a	92. b	93. b	94. c	95. b	96. b	97. b	98. a	99. c	100.a
101.b	102.a	103.d	104.c	105.d	106.c	107.c	108.c	109.b	110.b
111.a	112.c	113.a	114.b	115.a	116.d	117.a	118.c	119.c	120.c
121.a	122.b	123.b	124.b	125.a	126.c	127.c	128.b	129.d	130.d
131.c	132.c	133.b	134.c	135.a	136.d	137.b	138.a	139.c	140.b
141.c	142.a	143.b	144.b	145.d	146.a	147.b	148.c	149.d	150.a
151.b	152.b	153.a	154.c	155.b	156.c	157.b	158.c	159.d	160.d
161.d	162.b	163.c	164.b	165.d	166.d	167.d	168.c	169.c	170.b
171.a	172.c	173.c	174.c	175.d	176.d	177.a	178.a	179.d	180.a