Subject	Topic	Mock Test – 01	Date
C + M + P	Complete Syllabus	CET – 12 – CT	

Max. Marks: 180 **Duration: 3 Hours**

1. **This paper consists of 180 questions with 3 parts of Chemistry, Mathematics and Physics**

   - **Chemistry:** *(Q. No. 1 to 60) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Mathematics:** *(Q. No. 61 to 120) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Physics:** *(Q. No. 121 to 180) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*

2. **The OMR sheet for 200 questions is to be used**

3. **Use of calculators and log tables is prohibited**

4. **Darken the appropriate bubble using a pen in the OMR sheet provided to you. Once entered, the answer cannot be changed. Any corrections or modifications will automatically draw a penalty of 1 mark**

5. **No clarification will be entertained during the examination. Doubts in the paper can be reported to the coordinator after the exam**

6. **If the details in the OMR Sheet are not filled, If the OMR sheet is mutilated, torn, white Ink used, the circles filled and scratched, then the OMR sheet will not be graded**

   All the best!!

Useful Data

At. Wt.:

$N = 14;\ O = 16;\ H = 1;\ S = 32;\ Cl = 35.5;\ Mn = 55;\ Na = 23;\ C = 12;\ Ag = 108;\ K = 39;\ Fe = 56;\ Pb = 207$

Physical Constants:

$h = 6.626 \times 10^{-34}\,\text{Js},\ \text{N}_\text{a} = 6.022 \times 10^{23}\,\text{mol}^{-1},\ \text{c} = 2.998 \times 10^{8}\,\text{m s}^{-1},\ \text{m}_\text{e} = 9.1 \times 10^{-31}\,\text{kg},\ R = 8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$

Chemistry

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60

1. Among the following pairs of compounds, the one that illustrates the law of multiple proportions is

   Options:

   (a) NH_3 and NCl_3

   (b) H_2S and SO_2

   (c) MnO and Mn_2O_3

   (d) CS_2 and $FeSO_4$

   Sol: For the same amount of oxygen weight ratio of copper is $1:2$ therefore law of multiple proportion.

   Ans: (c)

2. In the reaction, $3Cl_2 + 6NaOH \longrightarrow NaClO_3 + 5NaCl + 3H_2O$

   The element which loses as well as gains electrons is

   Options:

   (a) Na
   (b) Cl

   (c) O

   (d) H

   Sol: In the reaction

   $3Cl_2 + 6NaOH \longrightarrow NaClO_3 + 5NaCl + 3H_2O$

   O.N. of Cl increases from zero in Cl_2 to $+5$ in $NaClO_3$ and decreases from zero in Cl_2 to -1 to $NaCl$.

   Ans: (b)

3. Critical temperature of H_2O is greater than that of O_2 because H_2O molecules have:

   Options:
   (a) Greater dipole moment

   (b) V – shape

   (c) Lesser number of electrons

   (d) It has only sigma bonds

   Sol: H – bond is forming, interaction is more

   Ans: (a)

4. The uncertainty in the position of an electron moving with a velocity of $3.0\times10^2\,\text{m/s}$ accurate upto 0.011% will be $\left(m = 9.1\times10^{-31}\text{Kg}\right)$

   Options:

   (a) $80\times10^{-4}\,\text{m}$

   (b) $40\times10^{-3}\,\text{m}$

   (c) $1.75\times10^{-3}\,\text{m}$

   (d) $1.75\times10^{-5}\,\text{m}$

Sol: $\Delta x = \frac{h}{4\pi\Delta p} = \frac{h}{4\pi m\Delta v} = \frac{6.6\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times3.0\times10^{2}\times\frac{0.011}{100}} = 1.75\times10^{-3}\text{m}$

Ans: (c)

5. The pair of species having same percentage of carbon is

Options:

(a) C_2H_5OH and CH_3OCH_3

(b) CH_3COOH and $HCOOH$

(c) $HCOOCH_3$ and C_3H_7OH

(d) $C_6H_{12}O_6$ and C_6H_5OH

Sol: % of C in $C_2H_5OH = \frac{24\times100}{46} = 52\%$

% of C in $CH_3OCH_3 = \frac{24\times100}{46} = 52\%$

Ans: (a)

6. Which of the following properties show gradual decrease with increase in atomic number across a period in the Periodic Table?

Options:

(a) Electron affinity

(b) Ionisation potential

(c) Electronegativity

(d) Size of atom

Sol: Size of atom decreases with increase in atomic number across the period in Periodic Table.

Ans: (d)

7. For the reaction $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$.

Which of the following is correct?

Options:

(a) $\Delta H = \Delta U$

(b) $\Delta H > \Delta U$

(c) $\Delta H < \Delta U$

(d) $\Delta H = 2\Delta U$

Sol: $\Delta H = \Delta U + \Delta nRT$

$\Delta n = (2-4) = -2$

$\therefore \Delta H = \Delta U - 2RT$

Or $\Delta U = \Delta H + 2RT$

Ans: (c)

8. Which of the following is NOT a state function?

Options:

(a) Internal energy

(b) Enthalpy

(c) Work

(d) Entropy

Sol: Work is not a state function.

Ans: (c)

9. Given the reaction between two gases represented by A_2 and B_2 to give the compound $AB_{(g)}$

$A_{2(g)} + B_{2(g)} \rightleftharpoons 2AB_{(g)}$

At equilibrium, the concentration of $A_2 = 3.0\times10^{-3}M$, of $B_2 = 4.2\times10^{-3}M$, of $AB = 2.8\times10^{-3}M$. If the reaction takes place in a sealed vessel at $527°C$, then the value of K_c will be

Options:

(a) 2.0

(b) 1.9

(c) 0.62

(d) 4.5

Sol: $A_{2(g)} + B_{2(g)} \rightleftharpoons 2AB_g$

$$K_c = \frac{[AB]^2}{[A_2][B_2]} = \frac{\left(2.8\times10^{-3}\right)^2}{\left(3.0\times10^{-3}\right)\left(4.2\times10^{-3}\right)} = \frac{2.8\times2.8}{3.0\times4.2} = 0.62$$

Ans: (c)

10. pH Value of which one of the following is not equal to one?

Options:

(a) 0.1 M CH_3COOH

(b) 0.1 M HNO_3

(c) 0.05 M H_2SO_4

(d) 50 cm^3 0.4 M $HCl + 50$ cm^3 0.2 M $NaOH$

Sol: CH_3COOH is a weak acid, it does not dissociate completely.

$\therefore \quad pH \neq 1$

Ans: (a)

11. Which of the following aqueous solution will have a pH less than 7.0 ?

Options:

(a) KNO_3

(b) $NaOH$

(c) $FeCl_3$

(d) $NaCN$

Sol: Fe^{3+} ions are hydrolysis to develop acidic nature.

Ans: (c)

12. Which of the following is incorrect?

Options:

(a) Hydrogen > Deuterium > Tritium (% relative abundance)

(b) Hydrogen > Deuterium > Tritium (melting point)

(c) Hydrogen < Deuterium < Tritium (boiling point)

(d) Hydrogen < Deuterium < Tritium (dissociation energy)

Sol: $\underset{13.90\,K}{\text{Hydrogen}} < \underset{18.50\,K}{\text{Deuterium}} < \underset{20.60\,K}{\text{Tritium}}$ (Melting point)

Ans: (b)

13. Which of the following is incorrect regarding properties of lithium?

Options:

(a) The melting point and boiling point of Li are comparatively high

(b) Li is much softer than the other group I meals

(c) Li forms a nitride Li_3N unlike group I metals

(d) The ion of Li and its compounds are more covalent

Sol: Actually Li is harder than other alkali metals.

Ans: (b)

14. Which is not the use of orthoboric acid?

Options:

(a) As an antiseptic and eye wash

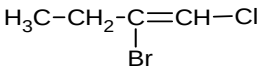
(b) In glass industry

(c) In glazes for pottery

(d) In borax- bead test

Sol: Borax on heating gives B_2O_3 and $NaBO_2$ which is glassy mass and used for borax-bead test.

Ans: (d)

15. IUPAC name of  is

Options:

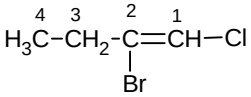
(a) 2-Bromo-1-chlorobut-1-ene

(b) 1-chloro-2-bromobut-1-ene

(c) 3-chloro-2-bromobut-1-ene

(d) 3-Bromo-4-chlorobut-3-ene

Sol:



Ans: (a)

16. 2-butyne is reduced to trans-but-2-ene using

Options:

(a) $H_2 | Ni$

(b) $H_2 | Pd - C$

(c) Na in liq. NH_3

(d) Zn in dil. HCl

Sol: 2 butyne $\xrightarrow{Na/lqNH_3}$ trans – but – 2 – ene

Ans: (c)

17. Minamata disease is because of pollution of

Options:

(a) Organic waste going into drinking water

(b) Oil spill in water

(c) Industrial waste containing mercury into fishing water

(d) Arsenic going into the atmosphere

Sol: Minamata disease is due to pollution of industrial waste containing mercury into fishing water

Ans: (c)

18. Monoclinic sulphur has the unit cell dimensions

Options:

(a) $a = b \neq c,\ \alpha = \beta = \gamma = 90°$

(b) $a \neq b \neq c,\ \alpha = \gamma = 90°,\ \beta \neq 90°$

(c) $a = b = c,\ \alpha = \beta = \gamma \neq 90°$

(d) $a = b = c,\ \alpha = \beta = \gamma = 90°$

Sol: $a \neq b \neq c,\ \alpha = \gamma = 90°,\ \beta \neq 90°$

Ans: (b)

19. An FCC lattice contains an $'A'$ type atom at the corner and $'B'$ type atom at the face centre. If four $'A'$ type corner atoms are missing, the formula of the compound is

Options:

(a) A_4B

(b) AB_6

(c) AB_4

(d) A_2B_3

Sol: The remaining four corner A atoms contribute $4 \times \frac{1}{8} = \frac{1}{2}$ atom of A to the unit cell. The six face centre atoms contribute $6 \times \frac{1}{2} = 3B$ atoms to unit cell.

Formula $= A_{1/2}B_3 = AB_6$

Ans: (b)

20. Germanium doped with arsenic gives rise to a

Options:

(a) $p-$ type semiconductor

(b) Hole (electron vacancy)

(c) $n-$ type semiconductor

(d) Rectifier

Sol: $n-$ type semiconductor

Ans: (c)

21. A solution containing $1.8g$ of a compound (empirical formula CH_2O) in $40\,g$ of water is observed to freeze at $-0.465°C$. The molecular formula of the compound is (K_f of water $=1.86\,kg$ K mol^{-1}

Options:

(a) $C_2H_4O_2$

(b) C_3H_6

(c) $C_4H_8O_4$

(d) $C_6H_{12}O_6$

Sol: $M_2=\dfrac{1000K_f w_2}{w_1\times\Delta T_f}=\dfrac{1000\times1.86\times1.8}{40\times0.465}=180$

E.F. mass $CH_2O=30 \quad \therefore\ n=\dfrac{180}{30}=6$

$\therefore$ Molecular formula $=6\times CH_2O=C_6H_{12}O_6$

Ans: (d)

22. What is the amount of urea dissolved per litre if its aqueous solution is isotonic with 20% cane sugar solution? (mol. wt. of urea $=60$)

Options:

(a) 200 g / L

(b) 35.08 g / L

(c) 17.54 g / L

(d) 16.7 g / L

Sol:

Isonic solution means

$\pi_1=\pi_2\Rightarrow\dfrac{w_1}{m_1V_1}=\dfrac{w_2}{m_2V_2}$

$V_1=1\text{Lit} \qquad V_2=100\,\text{ml}=0.1\text{L}$

$\dfrac{w_1}{60\times1}=\dfrac{20}{342\times0.1}=35.08=w_1$

Ans: (b)

23. A salt dissolves in water if:

Options:

(a) Lattice energy $<$ hydration energy

(b) Ionic product $<$ solubility product

(c) Ions may form hydrogen bonds with water

(d) All of the above

Sol: All of the above

Ans: (d)

24. An electric current of $0.5F$ is passed through 1 litre of 1M $CuSO_4$ solution. After the completion of electrolysis the molarity of the resulting solution will be:

Options:

(a) 0.75 M

(b) 0.60 M

(c) 0.50 M

(d) 0.90 M

Sol: $0.5F$ deposits 0.5 equivalent of $Cu = 0.25$ mol of Cu.

$1 - 0.25 = 0.75$ mol in litre $= 0.75\,(\text{M})$

Ans: (a)

25. The Standard Reduction Potential values of Ag, Cu, Co and Zn electrodes are 0.799, 0.337, -0.277 and -0.762 V respectively. Which of the following cells will have maximum cell emf?

Options:

(a) $Zn_{(s)} / Zn^{+2}{}_{(aq)} // Co^{+2}{}_{(aq)} / Co_{(s)}$

(b) $Zn_{(s)} / Zn^{+2}{}_{(aq)} // Ag^{+}{}_{(aq)} / Ag_{(s)}$

(c) $Cu_{(s)} / Cu^{+2}{}_{(aq)} // Ag^{+}{}_{(aq)} / Ag_{(s)}$

(d) $Zn_{(s)} / Zn^{+2}{}_{(aq)} // Cu^{+2}{}_{(aq)} / Cu_{(s)}$

Sol: $0.799 - (-0.762)$

Ans: (b)

26. The molar conductance of 0.1M solution of a weak acid HA is $1.4\,\text{S cm}^2\,\text{mol}^{-1}$. The molar conductance of HA at infinite dilution is $140\,\text{S cm}^2\text{mol}^{-1}$. Calculate the pH of $0.1\,\text{M}$ solution of HA.

Options:

(a) 4

(b) 2

(c) 3

(d) 8

Sol: The degree of dissociation of $HA = \alpha = \frac{\Lambda m}{\Lambda_m^0} = \frac{1.4}{140} = 0.01$

$[H^+] = C\alpha = 0.01 \times 0.1 = 0.001M$

$pH = -\log(0.001) = 3$

Ans: (c)

27. For the reaction $A + 2B \rightarrow 3C$, the rate of reaction at a given instant can be represented by

Options:

(a) $+\frac{d[A]}{dt} = +\frac{1}{2}\frac{d[B]}{dt} = +\frac{1}{3}\frac{d[C]}{dt}$

(b) $\frac{d[A]}{dt} = +\frac{1}{2}\frac{d[B]}{dt} = -\frac{1}{3}\frac{d[C]}{dt}$

(c) $-\frac{d[A]}{dt} = -\frac{1}{2}\frac{d[B]}{dt} = +\frac{1}{3}\frac{d[C]}{dt}$

(d) $-\frac{d[A]}{dt} = +\frac{2d[B]}{dt} + \frac{3d[C]}{dt}$

Sol: For reaction $A + 2B \rightarrow C$, Rate $= -\frac{d[A]}{dt} = -\frac{1}{2}\frac{d[B]}{dt} = +\frac{1}{3}\frac{d[C]}{dt}$.

Ans: (c)

28. The rate constant of a reaction is $2.1 \times 10^{-2}\,\text{mol}^{-2}L^2\,\text{min}^{-1}$. The order of reaction is

Options:

(a) Zero

(b) 1

(c) 2

(d) 3

Sol: For 3rd order, $\frac{dx}{dt} = k[conc.]^3$, i.e., n = 3

Unit of rate constant = $\left(\text{mol}\,L^{-1}\right)^{1-n} s^{-1} = \left(\text{mol}\,L^{-1}\right)^{-2} s^{-1} = \text{mol}^{-2}L^2 s^{-1}$

Ans: (d)

29. If the activation energy for the forward reaction is $150\,\text{kJ mol}^{-1}$ and that of the reverse reaction is $260\,\text{kJ mol}^{-1}$, what is the enthalpy change for the reaction?

Options:

(a) $410\,\text{kJ mol}^{-1}$

(b) $-110\,\text{kJ mol}^{-1}$

(c) $110\,\text{kJ mol}^{-1}$

(d) $-410\,\text{kJ mol}^{-1}$

Sol: $\Delta H = E_f - E_b = 150 - 260\,\text{kJ} = -110\,\text{kJ}$

Ans: (b)

30. Which of the following statements is incorrect regarding physisorption?

Options:

(a) It occurs because of van der Waals forces

(b) More easily liquefiable gases are adsorbed readily

(c) Under high pressure it results into multi molecular layer on adsorbent surface

(d) Enthalpy of adsorption $\left(\Delta H_{\text{adsorption}}\right)$ is high and positive

Sol: Physical adsorption is an exothermic process (i.e., $\Delta H = -\text{ve}$) but its value is quite low because the attraction of gas molecules and solid surface is due to weak van der Waals forces.

Ans: (d)

31. In Bredig's arc method some alkali is added because

Options:

(a) It increases electrical conductance

(b) To obtain molecular colloid

(c) To obtain colloidal particles of same size

(d) To stabilise the sol.

Sol: Lyophobic sols are unstable. To make the gold sol stable, a little alkali is added

Ans: (d)

32. Lyophobic sols are unstable. To make the gold sol stable, a little alkali is added

Options:

(a) Na_3PO_4

(b) Na_2SO_4

(c) Na_2S

(d) $NaCl$

Sol: Flocculating value $\propto \frac{1}{\text{Flocculating power}}$

Ans: (d)

33. The common impurity present in bauxite is

Options:

(a) CuO

(b) ZnO

(c) Fe_2O_3

(d) Cr_2O_3

Sol: Impurity in bauxite is Fe_2O_3

Ans: (c)

34. Which metal has the highest melting point?

Options:

(a) Tungsten

(b) Platinum

(c) Silver

(d) Gold

Sol: Melting point of tungsten is $3410°C$. Melting points of Pt, Ag and Au are $1769°C, 960°C$ and $1063°C$ respectively.

Ans: (a)

35. Which is colourless in water?

Options:

(a) Ti^{4+}

(b) V^{3+}

(c) Cr^{3+}

(d) Ti^{3+}

Sol: $Ti^{4+}\left(3d^0\right)$ has no d-electron. Hence there is no d-d transition of electron and Ti^{4+} ion is colourless.

Ans: (a)

36. Which of the following is not a consequence of the Lanthanide contraction?

Options:

(a) $5d$ Series elements have a higher IE_1 than $3d$ or $4d$ series

(b) Irregularity in the ionization enthalpy of $3d$ series

(c) Zr and Hf occurs together in the earth crust in their minerals

(d) Zr and Hf have a comparable size

Sol: Irregularity in the ionization enthalpy of $3d$ series is not due to lanthanide contraction.

Ans: (b)

37. Which of the following has the highest molar conductivity in solution?

Options:

(a) $\left[Pt\left(NH_3\right)_6\right]Cl_4$

(b) $\left[Pt\left(NH_3\right)_5 Cl\right]Cl_3$

(c) $\left[Pt\left(NH_3\right)_4 Cl_2\right]Cl_2$

(d) $\left[Pt\left(NH_3\right)_3 Cl_3\right]Cl$

Sol:

More number of ions is available in aqueous solution.

Ans: (a)

38. The IUPAC name of $\left[Cr\left(NH_3\right)_5 Cl\right]SO_4$ is:

Options:

(a) pentaaminechloridochromium sulphate

(b) pentaamminechloridochromium(III) sulphate

(c) chloridopentaamminechromium(III) sulphate

(d) pentaaminochloridochromium(II) sulphate

Sol: IUPAC name of $\left[Cr\left(NH_3\right)_5 Cl\right]SO_4$ is pentamminechloridochromium(III) sulphate

Ans: (b)

39. Which of the following ligand has lowest Δ_0 value?

Options:

(a) en

(b) F^-

(c) ox

(d) CN^-

Sol: Crystal field splitting ability of ligands as per spectrochemical series is

$$I^- < Br^- < SCN^- < S^{-2} < F^- < OH^- < ox < H_2O < NSC^- < EDTA^{-4} < NH_3 < en < CN^- < CO$$

Ans: (b)

40. Which of the following is NOT True for S_N1 reaction?

Options:

(a) The rate of the reaction does not depend upon the molar concentration of the nucleophile

(b) $1°-$ alkyl halides generally react through S_N1 reaction

(c) Favoured by polar solvents

(d) $3°-$ alkyl halides generally react through S_N1 reaction

Sol: 1° – alkyl halides generally react through S_N1 reaction

Ans: (b)

41. The arrangement of following compounds

(i) Bromomethane (ii) Bromoform

(iii) Chloromethane (iv) Dibromomethane

In the increasing order of their boiling point is

Options:

(a) $(i)<(ii)<(iii)<(iv)$

(b) $(ii)<(iii)<(i)<(iv)$

(c) $(iv)<(iii)<(i)<(ii)$

(d) $(iii)<(i)<(iv)<(ii)$

Sol: (i) CH_3Br (ii) $CHBr_3$ (iii) CH_3Cl (iv) CH_2Br_2

Ans: (d)

42. Propane nitrile may be prepared by heating:

Options:

(a) Ethyl chloride with KCN

(b) Propyl alcohol with KCN

(c) Propyl chloride with KCN

(d) Propane with KCN

Sol: $CH_3CH_2Cl + KCN \rightarrow \underset{\text{Propane nitrile}}{CH_3CH_2CN}$

Ans: (a)

43. Which one of the following alcohols undergoes acid catalysed dehydration to alkene readily?

Options:

(a) $(CH_3)_2CHCH_2OH$

(b) $(CH_3)_3COH$

(c) $CH_3CHOHCH_2CH_3$

(d) $CH_3CH_2CH_2CH_2OH$

Sol: Alcohol which forms the more stable carbocation undergoes dehydration most readily. Since *tert*-butyl alcohol forms more stable *tert*-butyl carbocation, therefore, it undergoes dehydration most readily and n-butyl alcohol undergoes dehydration slowly.

Ans: (b)

44. Phenol reacts with bromine in water to give

Options:

(a) $m-$Bromophenol

(b) $2,\ 4,\ 6-$Tribromophenol

(c) $p-$Bromophenol

(d) Mixture of ortho and para-bromophenol

Sol: Phenols reacts with bromine in water to give $2,\ 4,\ 6-$Tribromophenol

Ans: (b)

45. The compound which does not react with Lucas reagent is

Options:

(a) $n-$Butyl alcohol

(b) $sec-$Butyl alcohol

(c) Isobutyl alcohol

(d) $tert-$Butyl alcohol

Sol: Primary alcohols such as *n*-butyl alcohol do not react with Lucas reagent.

Ans: (a)

46. Anisole on treatment with CH_3Cl in presence of anhydrous $AlCl_3$ gives

Options:

(a) Toluene

(b) o-Chloroanisole

(c) $p-$ Chloroanisole

(d) $o-$ and $p-$ methylanisoles

Sol: $-CH_3$ group in anisole is $o,p-$ directing. Therefore, anisole on Friedel Craft reaction gives a mixture of $o-$ and $p-$ methylanisoles.

Ans: (d)

47. One mole of a symmetrical alkene on ozonolysis gives two moles of an acetaldehyde. The alkene is

Options:

(a) $2-$ Butene

(b) Ethene

(c) Propene

(d) $1-$ Butene

Sol: $2-$ butane on ozonolysis gives two moles of acetaldehyde with molecular mass 44u.

Ans: (a)

48. The final product (Y) in the following sequence of chemical reaction is

$$CH_3OH \xrightarrow[300°C]{Cu} X \xrightarrow{NaOH} Y + CH_3OH$$

Options:

(a) an alkene

(b) a carboxylic acid

(c) an aldehyde

(d) sodium salt of carboxylic acid

Sol: $CH_3OH \xrightarrow[300°C]{Cu} \underset{(X)}{HCHO} \xrightarrow{NaOH} \underset{(Y)}{HCOONa} + CH_3OH$

Ans: (d)

49. In presence of dry HCl gas, CH_3CHO condenses with C_2H_5OH to give

Options:

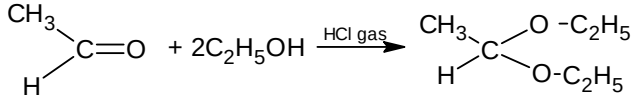
(a) aldol

(b) paraldehyde

(c) ethyl acetate

(d) acetal

Sol:



Ans: (d)

50. Ethanoyl chloride cannot be obtained by treating ethanoic acid with:

Options:

(a) $SOCl_2$

(b) $CHCl_3$

(c) PCl_3

(d) PCl_5

Sol: $R-COOH+PCl_5 \longrightarrow R-COCl+POCl_3+HCl$

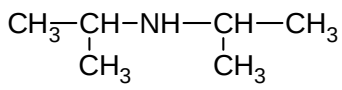
$3R-COOH+PCl_3 \longrightarrow 3R-COCl+H_3PO_3$

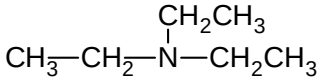
$R-COOH+SOCl_2 \longrightarrow R-COCl+SO_2+HCl$

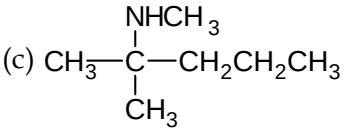
Ans: (b)

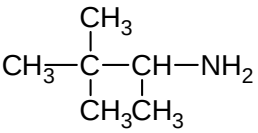
51. The amine that reacts with Hinsberg's reagent to give the product soluble in alkali

Options:

(a) 

(b) 

(c) 

(d) 

Sol: Primary amines with Hinsberg's reagent give N -alkylbenzene sulphonamide which is soluble in alkali. Secondary amines on reaction with Hinsberg's reagent gives N,N -dialkylbenzene sulphonamide which does not contain any hydrogen atom attached to N −atom, it is not acidic and hence insoluble in alkali. Tertiary amines do not react with Hinsberg's reagent.

Ans: (d)

52. The bad smelling substance formed by the action of alcoholic caustic potash on chloroform and aniline is

Options:

(a) Nitrobenzene

(b) Phenyl isocyanide

(c) Phenyl cyanide

(d) Phenyl isocyanate

Sol:

$$C_6H_5NH_2 + CHCl_3 + 3KOH \xrightarrow[\text{(alc)}]{\Delta} \underset{\text{Phenyl isocyanide}}{C_6H_5NC} + 3KCl + 3H_2O$$

Ans: (b)

53. Aniline on heating with conc. H_2SO_4 at $460K$ gives:

Options:

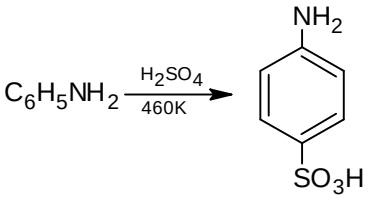
(a) Aniline sulphate

(b) Benzene sulphonic acid

(c) Sulphanilic acid

(d) Sulphonic acid

Sol:



Ans: (c)

54. The presence of primary alcoholic group in glucose can be confirmed by

Options:

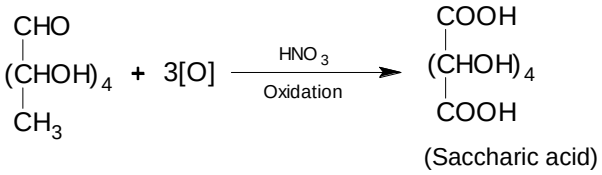
(a) Oxidation of glucose with mild oxidising agent

(b) Acetylation of glucose with acetic anhydride

(c) Oxidation of glucose with nitric acid

(d) Prolonged heating of glucose with HI

Sol: Strong oxidising agents like nitric acid oxidise both the terminal groups $(-CHO$ and $-CH_2OH)$ of glucose to give the dibasic acid, saccharic acid (also known as glucaric acid). This indicates the presence of a primary alcoholic $(-OH)$ group in glucose.



Ans: (c)

55. What type of sugar molecule is present in RNA?

Options:

(a) $D-3-$ Deoxyribose

(b) $D-$ Ribose

(c) $D-2-$ Deoxyribose

(d) $D-$ Glucopyranose

Sol: RNA contains $D-$ Ribose sugar moiety

Ans: (b)

56. Cheilosis and digestive disorders are due to the deficiency of

Options:

(a) Vitamin A

(b) Riboflavin

(c) Thiamine

(d) Ascorbic acid

Sol: Due to the deficiency of vitamin B_2 or G (Riboflavin), Cheilosis (i.e., cracking of lips and corners of the mouth) and digestive disorders occur.

Ans: (b)

57. Neoprene is

Options:

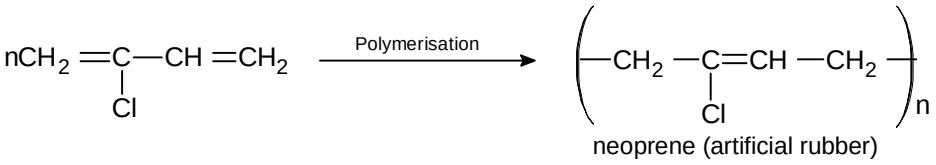
(a) A monomer of rubber

(b) Synthetic rubber

(c) Natural rubber

(d) Vulcanized rubber

Sol: Neoprene is synthetic rubber. It is a polymer of chloroprene ($2-$chlorobuta $-1,\ 3-$ diene)



neoprene (artificial rubber)

Ans: (b)

58. Which of the following is a synthetic rubber?

Options:

(a) Dacron

(b) Buna-S

(c) Polythene

(d) Novolac

Sol: Buna-S is a synthetic rubber

Ans: (b)

59. Which cleansing agent gets precipitated in hard water?

Options:

(a) Sodium lauryl sulphate

(b) Cetyl trimethyl ammonium bromide

(c) Sodium stearate

(d) Sodium dodecyl benzene sulphonate

Sol:

Sodium stearate being soap, gets precipitated in hard water.

Ans: (c)

60. Anti-histamine among the following is

Options:

(a) Brompheniramine

(b) Amoxicillin

(c) Morphine

(d) Chloroxylenol

Sol: Antihistamine – Brompheniramine

Ans: (a)

Mathematics

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60

61. For any two real numbers θ and ϕ, we define $\theta R\phi$, if and only if $\sec^2\theta - \tan^2\phi = 1$. The relation R is

Options:

(a) reflexive but not transitive

(b) symmetric but not reflexive

(c) both reflexive and symmetric but not transitive

(d) an equivalence relation

Sol: Given relation is defined as $\theta R\phi$ such that $\sec^2\theta - \tan^2\phi = 1$

Reflexive When $\theta R\theta$, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad 1 = 1$, which is true.

Thus, it is reflexive

Symmetric

When $\theta R\phi$, $\sec^2\theta - \tan^2\phi = 1$

$\Rightarrow \quad \left(1+\tan^2\theta\right) - \left(\sec^2\phi - 1\right) = 1$

$\Rightarrow \quad 2 + \tan^2\theta - \sec^2\phi = 1$

$\Rightarrow \quad \sec^2\phi - \tan^2\theta = 1 \quad \Rightarrow \quad \phi R\theta$

Thus, it is symmetric.

Transitive

When $\theta R\phi$ and $\phi R\psi$, then

$\sec^2\theta - \tan^2\phi = 1$ and $\sec^2\phi - \tan^2\psi = 1$

Now, if $\theta R\psi$, then

$\sec^2\theta - \tan^2\psi = 1 \quad \Rightarrow \quad \sec^2\theta - \tan^2\psi + 1 = 1 + 1$

Here $\sec^2\theta - \tan^2\psi + 1 = \sec^2\theta - \tan^2\psi + \sec^2\phi - \tan^2\phi = 1 + 1$

$\therefore \quad \theta R\phi$ and $\phi R\psi \quad \Rightarrow \quad \theta R\psi$

Thus, it is transitive. Hence, it is an equivalence relation.

Ans: (d)

62. If $A = \{x, y, z\}$ and $B = \{a, b, c, d\}$. Then, which one of the following is not a relation from A to B ?

Options:

(a) $\{(x,a), (x,c)\}$

(b) $\{(y,c), (y,d)\}$

(c) $\{(z,a), (z,d)\}$

(d) $\{(z,b), (y,b), (a,d)\}$

Sol: $\{(z,b),(y,b),(a,d)\}$ is not a relation from A to B because $a \notin A$.

Ans: (d)

63. If $A=\{1,3,5,7\}$ and $B=\{1,2,3,4,5,6,7,8\}$ then, the number of one-one function from A into B is

Options:

(a) 1340

(b) 1860

(c) 1430

(d) 1680

Sol: Given, $A=\{1,3,5,7\}$ and $B=\{1,2,3,4,5,6,7,8\}$

Here, $n(A)=4$ and $n(B)=8$

$\therefore$ Number of one-one function from A into $B = {}^{8}P_4 = 8\cdot7\cdot6\cdot5 = 1680$

Ans: (d)

64. The range of the function $f(x)=x^2+2x+2$ is

Options:

(a) $(1,\infty)$

(b) $(2,\infty)$

(c) $(0,\infty)$

(d) $[1,\infty)$

Sol: Given, $f(x)=x^2+2x+2 = x^2+2x+1+1=(x+1)^2+1\geq 1$

So, the range of $f(x)$ is $[1,\infty)$

Ans: (d)

65. If $f(x)=4x^3+3x^2+3x+4$, then $x^3 f\left(\frac{1}{x}\right)$ is equal to

Options:

(a) $f(-x)$

(b) $\frac{1}{f(x)}$

(c) $\left[f\left(\frac{1}{x}\right)\right]^2$

(d) $f(x)$

Sol: Given, $f(x)=4x^3+3x^2+3x+4$

$\therefore\ x^3 f\left(\frac{1}{x}\right)=x^3\left(\frac{4}{x^3}+\frac{3}{x^2}+\frac{3}{x}+4\right)=4+3x+3x^2+4x^3=f(x)$

Ans: (d)

66. If $f(x)$ is polynomial function of the second degree such that, $f(-3)=6$, $f(0)=6$ and $f(2)=11$, then the graph of the function, $f(x)$ cuts the ordinate $x=1$ at the point

Options:

(a) $(1, 8)$

(b) $(1, 4)$

(c) $(1, -2)$

(d) None of these

Sol: Let $f(x)=ax^2+bx+c$

$\because \ f(0)=6 \ \Rightarrow \ c=6$

$\therefore \ f(x)=ax^2+bx+6$

$\therefore \ f(-3)=6$

$\Rightarrow \ 9a-3b+6=6 \ \Rightarrow \ 3a=b$ … (i)

$f(2)=11 \ \Rightarrow \ 4a+2b+6=11$

$\Rightarrow \ 4a+6a=5 \ \Rightarrow \ a=1/2 \ \Rightarrow \ b=3/2$ [from eq. (i)]

$\therefore \ f(x)=\frac{x^2}{2}+\frac{3}{2}x+6$

Now, $f(x)=\frac{1}{2}+\frac{3}{2}+6=8$

Ans: (a)

67. The set $A=\left\{x : x \in R, x^2 \text{ and } 2x=6\right\}$ is equal to

Options:

(a) ϕ

(b) $\{14,3,4\}$

(c) $\{3\}$

(d) $\{4\}$

Sol: Since, $x^2=16 \ \Rightarrow \ x=\pm 4$ and $2x=6 \ \Rightarrow \ x=3$

Hence, no value of x is satisfied.

Ans: (a)

68. Which of the following is correct?

Options:

(a) $A \cap \phi = A$

(b) $A \cap \phi = \phi$

(c) $A \cap \phi = U$

(d) $A \cap \phi = A'$

Sol: $A \cap \phi = \phi$ is correct.

Ans: (b)

69. If $\sin A + \sin B + \sin C = 3$, then $\cos A + \cos B + \cos C$ is equal to

Options:

(a) 3

(b) 2

(c) 1

(d) 0

Sol: Given, $\sin A + \sin B + \sin C = 3$

$\therefore \ \sin A = \sin B = \sin C = 1 \quad [\because \ -1 \le \sin x \le 1]$

$\Rightarrow \ A = B = C = \frac{\pi}{2}$

$\Rightarrow \ \cos A + \cos B + \cos C = 0$

Ans: (d)

70. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}$, $\frac{\pi}{2} < A < 0$ and $-\frac{\pi}{2} < B < 0$, then the value of $2\sin A + 4\sin B$ is

Options:

(a) 4

(b) –2

(c) –4

(d) 0

Sol: Given, $\cos A = \frac{3}{5}$ and $\cos B = \frac{4}{5}$

$\because \ \angle A$ and $\angle B$ lie on IV quadrant

$\therefore \ \sin A = -\sqrt{1 - \frac{9}{25}}$ and $\sin B = -\sqrt{1 - \frac{16}{25}} \Rightarrow \ \sin A = -\frac{4}{5}$ and $\sin B = -\frac{3}{5}$

Now, $2\sin A + 4\sin B = 2\left(-\frac{4}{5}\right) + 4\left(-\frac{3}{5}\right) = -\frac{8}{5} - \frac{12}{5} = -\frac{20}{5} = -4$

Ans: (c)

71. The value of $\frac{\sin 55° - \cos 55°}{\sin 10°}$ is

Options:

(a) $\frac{1}{\sqrt{2}}$

(b) 2

(c) 1

(d) $\sqrt{2}$

Sol: $\frac{\sin 55° - \cos 55°}{\sin 10°} = \frac{\sin 55° - \sin 35°}{\sin 10°} = \frac{2\cos 45° \sin 10°}{\sin 10°} = \sqrt{2}$

Ans: (d)

72. Find the value of $\cos(x/2)$, if $\tan x = 5/12$ and x lies in quadrant III

Options:

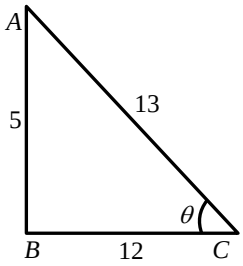
(a) $\frac{5}{\sqrt{13}}$

(b) $\frac{5}{\sqrt{26}}$

(c) $\frac{5}{13}$

(d) $-\sqrt{\frac{1}{26}}$

Sol: Given, $\tan x = \frac{5}{12}$ and x lies in III quadrant



$\therefore \sin x = \frac{-5}{13}$ and $\cos x = \frac{-12}{13}$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow \cos^2\frac{x}{2} = \frac{1}{2}(\cos x + 1) = \frac{1}{2}\left(\frac{-12}{13} + 1\right) = \frac{1}{2}\left(\frac{1}{13}\right) = \frac{1}{26}$

$\therefore \cos\frac{x}{2} = -\sqrt{\frac{1}{26}}$

$\therefore \frac{x}{2}$ lies in II quadrant

Ans: (d)

73. The least value of $3\sin^2\theta + 4\cos^2\theta$ is

Options:

(a) 2

(b) 3

(c) 0

(d) 1

Sol: Consider, $3\sin^2\theta + 4\cos^2\theta$

$= 3\sin^2\theta + 3\cos^2\theta + \cos^2\theta = 3 + \cos^2\theta$

We know that, least value of $\cos^2\theta$ is 0.

Hence, least value of $3\sin^2\theta + 4\cos^2\theta$ is 3

Ans: (b)

74. $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$ is equal to

Options:

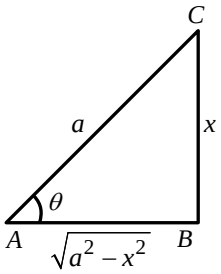
(a) $2\sin^{-1}\left(\frac{x}{a}\right)$

(b) $\sin^{-1}\left(\frac{2x}{a}\right)$

(c) $\sin^{-1}\left(\frac{x}{a}\right)$

(d) $\cos^{-1}\left(\frac{x}{a}\right)$

Sol: Let $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)=\theta \Rightarrow \tan\theta=\frac{x}{\sqrt{a^2-x^2}}$



$\therefore \sin\theta=\frac{x}{a} \Rightarrow \theta=\sin^{-1}\left(\frac{x}{a}\right)$

Ans: (c)

75. If $z_1=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ and $z_z=\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$, then $|z_1z_2|$ is equal to

Options:

(a) 6

(b) $\sqrt{2}$

(c) $\sqrt{6}$

(d) $\sqrt{3}$

Sol: $|z_1|=\sqrt{2}$ and $|z_2|=\sqrt{3}$ $\therefore |z_1z_2|=|z_1||z_2|=\sqrt{6}$

Ans: (c)

76. The set $A=\{x:|2x+3|<7\}$ is equal to the set

Options:

(a) $D=\{x:0<x+5<7\}$

(b) $B=\{x:-3<x<7\}$

(c) $E=\{x:-7<x<7\}$

(d) $C=\{x:-13<2x<4\}$

Sol: Given, set $A=\{x:|2x+3|<7\}$

Now, $|2x-3|<7 \Rightarrow -7<2x+3<7$

$\Rightarrow -7-3<2x<7-3 \Rightarrow -10<2x<4$

$\Rightarrow -5<x<2 \Rightarrow 0<(x+5)<7$

Ans: (a)

77. The number of subsets of $\{1,2,3,...,9\}$ containing at least one odd number, is

Options:

(a) 324

(b) 396

(c) 496

(d) 512

Sol: The total number of subsets of given set is $2^9 = 512$

Case I: When selecting only one even number $\{2,4,6,8\}$. Number of ways $= {}^4C_1 = 4$

Case II: when selecting only two even numbers $= {}^4C_2 = 6$

Case III: When selecting only three even numbers $= {}^4C_3 = 4$

Case IV: When selecting only four even numbers $= {}^4C_4 = 1$

$\therefore$ Required number of ways $= 512 - (4+6+4+1) - 1 = 496$

[here, we subtract 1 due to the null set]

Ans: (c)

78. If ${}^{18}C_{15} + 2\left({}^{18}C_{16}\right) + {}^{17}C_{16} + 1 = {}^{n}C_3$, then n is equal to

Options:

(a) 19

(b) 20

(c) 18

(d) 24

Sol: Given, ${}^{18}C_{15} + 2\left({}^{18}C_{16}\right) + {}^{17}C_{16} + 1 = {}^{n}C_3$

$\Rightarrow {}^{18}C_{15} + {}^{18}C_{16} + {}^{18}C_{16} + {}^{17}C_{16} + {}^{17}C_{17} = {}^{n}C_3$

$\Rightarrow {}^{19}C_{16} + {}^{19}C_{17} = {}^{20}C_{17} = {}^{20}C_3 \Rightarrow n = 20$

Ans: (b)

79. The first four terms of an AP are $a, 9, 3a-b, 3a+b$. The 2011th term of an AP is

Options:

(a) 2015

(b) 4025

(c) 5030

(d) 8045

Sol: Given, first four terms of an AP $a, 9, 3a-b, 3a+b$

$\Rightarrow 2 \cdot 9 = a + 3a - b \Rightarrow 4a - b = 18$ … (i)

and $2(3a-b) = 9 + 3a + b$

$\Rightarrow 6a - 2b = 9 + 3a + b \Rightarrow 3a - 3b = 9$

$\Rightarrow \quad a-b=3$ … (ii)

On solving eqs. (i) and (ii), we get

$3a=15 \Rightarrow a=5$ and $b=2$

The series becomes, 5, 9, 13, 17

First term $=5$, Common difference $=4$

Now, $T_{2011}=5+(2011-1)\cdot 4=8045$

Ans: (d)

80. If a line with $y-$ intercept 2, is perpendicular to the line $3x-2y=6$, then its $x-$intercept is

Options:

(a) 1

(b) 3

(c) -4

(d) 4

Sol: Let the equation of perpendicular line to the line

$3x-2y=6$ be $3y+2x=c$ … (i)

Since, it passes through $(0,2)$

$\therefore \quad c=6$

On putting the value of c in eq. (1), we get

$3y+2x=6 \Rightarrow \frac{x}{3}+\frac{y}{2}=1$

Hence, $x-$intercept is 3

Ans: (b)

81. If the foot of the perpendicular from the origin to a straight line is at the point $(3,-4)$. Then, the equation of the line is

Options:

(a) $3x-4y=25$

(b) $3x-4y+25=0$

(c) $4x+3y-25=0$

(d) $4x-3y-25=0$

Sol: Let $P(3,-4)$ be the foot of the perpendicular from the origin O on the required line. Then, the slope of $OP=\frac{-4-0}{3-0}=\frac{-4}{3}$

Therefore, the slope of the required line is $3/4$

Hence, its equation is : $y-(-4)=\frac{3}{4}(x-3)\Rightarrow 3x-4y=25$

Ans: (a)

82. The distance between the foci of the conic $7x^2 - 9y^2 = 63$ is equal to

Options:

(a) 8

(b) 4

(c) 3

(d) 7

Sol: Given equation of hyperbola is

$\frac{x^2}{9} - \frac{y^2}{7} = 1$

Distance between foci $= 2ae = 2\sqrt{a^2 + b^2}$

$= 2\sqrt{9+7} = 8$

Ans: (a)

83. The angle between a normal to the plane $2x - y + 2z - 1 = 0$ and the $Z-$axis is

Options:

(a) $\cos^{-1}\left(\frac{1}{3}\right)$

(b) $\sin^{-1}\left(\frac{2}{3}\right)$

(c) $\cos^{-1}\left(\frac{2}{3}\right)$

(d) $\sin^{-1}\left(\frac{1}{3}\right)$

Sol: Given equation of plane is $2x - y + 2z - 1 = 0$

So, DR's of normal to the given plane is $(2, -1, 2)$

$\therefore$ DC's of normal to the given plane is

$$\left(\frac{2}{\sqrt{2^2+(-1)^2+2^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+2^2}}, \frac{2}{\sqrt{2^2+(-1)^2+2^2}}\right)$$

i.e. $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$

Also, DC's of $Z-$axis is $(0,0,1)$.

$\therefore$ Angle between normal to plane and $Z-$axis is

$\cos\theta = \frac{2}{3}\times 0 - \frac{1}{3}\times 0 + \frac{2}{3}\times 1 = \frac{2}{3}$

$\therefore \quad \theta = \cos^{-1}\frac{2}{3}$

Ans: (c)

84. The equation of the straight line passing through the points $(4,-5,-2)$ and $(-1,5,3)$

Options:

(a) $\frac{x-4}{1}=\frac{y+5}{-2}=\frac{z+2}{-1}$

(b) $\frac{x+1}{1}=\frac{y-5}{2}=\frac{z-3}{-1}$

(c) $\frac{x}{-1}=\frac{y}{5}=\frac{z}{3}$

(d) $\frac{x}{4}=\frac{y}{-5}=\frac{z}{-2}$

Sol: Equation of straight line passing through $(4,-5,-2)$ and $(-1,5,3)$ is

$$\frac{x-4}{-5}=\frac{y+5}{10}=\frac{z+2}{5} \Rightarrow \frac{x-4}{1}=\frac{y+5}{-2}=\frac{z+2}{-1}$$

Ans: (a)

85. An equation of the plane through the points $(1,0,0),(0,2,0)$ and at a distance $6/7$ units from the origin, is

Options:

(a) $6x+3y+z-6=0$

(b) $6x+3y+2z-6=0$

(c) $6x+3y+z+6=0$

(d) $6x+3y+2z+6=0$

Sol: An equation of plane is $lx+my+nz=d$, where l,m,n are direction cosines and d is perpendicular distance from origin.

$\because\ d=6/7$

$\therefore\ lx+my+nz=\frac{6}{7}$ … (i)

Since, eq. (i) passes through the points $(1,0,0)$ and $(0,2,0)$

$\therefore\ l=\frac{6}{7}$ and $m=\frac{3}{7}$

We know that, $l^2+m^2+n^2=1$

$\Rightarrow \left(\frac{6}{7}\right)^2+\left(\frac{3}{7}\right)^2+n^2=1 \Rightarrow n=\frac{2}{7}$

Hence, equation of plane is $\frac{6}{7}x+\frac{3}{7}y+\frac{2}{7}z=\frac{6}{7}$

$\Rightarrow 6x+3y+2z-6=0$

Ans: (b)

86. The distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$ from the point $(-1,-5,-10)$ is

Options:

(a) 13

(b) 12

(c) 11

(d) 8

Sol: Given line is $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=k$ [say]

Any point on the line is $(3k+2,\ 4k-1,\ 12k+2)$

Since, this point lies on the plane $x-y+z=5$

$\therefore\ 3k+2-(4k-1)+12k+2=5$

$\Rightarrow\ 11k=0\ \Rightarrow\ k=0$

So, the intersection point is $(2,-1,2)$

$\therefore$ Distance between points $(2,-1,2)$ and $(-1,-5,-10)$

$=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}=\sqrt{9+16+144}=13$

Ans: (a)

87. $\lim\limits_{x\to\infty}\left(\frac{1^3+2^3+3^3+...+k^3}{k^4}\right)$ is equal to

Options:

(a) 0

(b) 2

(c) 1/4

(d) 1/3

Sol:

$$\lim_{x\to\infty}\left(\frac{1^3+2^3+3^3+...+k^3}{k^4}\right)=\lim_{x\to\infty}\left(\frac{k^2(k+1)^2}{4}\times\frac{1}{k^4}\right)=\lim_{x\to\infty}\left(\frac{k^4(k+1/k)^2}{4}\times\frac{1}{k^4}\right)=\frac{1}{4}$$

Ans: (c)

88. If $f(5)=7$ and $f'(5)=7$, then $\lim\limits_{x\to\infty}\frac{xf(5)-5f(x)}{x-5}$ is equal to

Options:

(a) 35

(b) –35

(c) 28

(d) –28

Sol: $\lim_{x\to5}\frac{xf(5)-5f(x)}{x-5}=\lim_{x\to5}\frac{f(5)-5f'(x)}{1-0}=f(5)-5f'(5)$

$=7-5\cdot7=7-35=-28$ [using L'Hospital's rule]

Ans: (d)

89. $\lim_{x\to0}\frac{1-\cos^3 x}{x\sin x\cos x}$ is equal to

Options:

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{3}{2}$

(d) $\frac{3}{4}$

Sol: $\lim_{x\to0}\frac{1-\cos^3 x}{x\sin x\cos x}=\lim_{x\to0}\frac{(1-\cos x)(1+\cos^2 x+\cos x)}{x^2\cos x\cdot\frac{\sin x}{x}}=3\lim_{x\to0}\frac{1-\cos x}{x^2}=3\times\frac{1}{2}=\frac{3}{2}$

Ans: (c)

90. If $f(x)=e^x\sin x$, then $f''(x)$ is equal to

Options:

(a) $e^{6x}\sin 6x$

(b) $2e^x\cos x$

(c) $8e^x\sin x$

(d) $8e^x\cos x$

Sol: Given, $f(x)=e^x\sin x \Rightarrow f'(x)=e^x\cos x+\sin xe^x$

$\Rightarrow f''(x)=e^x\cos x-e^x\sin+e^x\sin x+e^x\cos x=2e^x\cos x$

Ans: (b)

91. The value of $\frac{d}{dx}\left[\tan^{-1}\left\{\frac{\sqrt{x}(3-x)}{1-3x}\right\}\right]$ is

Options:

(a) $\frac{3}{2(1+x)\sqrt{x}}$

(b) $\frac{3}{(1+x)\sqrt{x}}$

(c) $\frac{2}{(1+x)\sqrt{x}}$

(d) $\frac{3}{2(1-x)\sqrt{x}}$

Sol: Let $y = \tan^{-1}\left\{\dfrac{3\sqrt{x} - x^{3/2}}{1-3x}\right\}$

Again, let $\sqrt{x} = \tan t$

$\therefore \quad y = \tan^{-1}\left\{\dfrac{3\tan t - \tan^3 t}{1-3\tan^2 t}\right\} = \tan^{-1}(\tan 3t)$

$\Rightarrow \quad y = 3\tan^{-1}\sqrt{x} \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{3}{1+x}\cdot\dfrac{1}{2\sqrt{x}} = \dfrac{3}{2(1+x)\sqrt{x}}$

Ans: (a)

92. Differential coefficient of $\sqrt{\sec\sqrt{x}}$ is

Options:

(a) $\dfrac{1}{4\sqrt{x}}\sec\sqrt{x}\sin\sqrt{x}$

(b) $\dfrac{1}{4\sqrt{x}}\left(\sec\sqrt{x}\right)^{3/2}\cdot\sin\sqrt{x}$

(c) $\dfrac{1}{2}\sqrt{x}\sec\sqrt{x}\sin\sqrt{x}$

(d) $\dfrac{1}{2}\sqrt{x}\left(\sec\sqrt{x}\right)^{3/2}\cdot\sin\sqrt{x}$

Sol: Let $y = \sqrt{\sec\sqrt{x}}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{\sec\sqrt{x}}}\cdot\sec\sqrt{x}\cdot\tan\sqrt{x}\cdot\dfrac{1}{2\sqrt{x}}$

$= \dfrac{1}{4\sqrt{x}}\left(\sec\sqrt{x}\right)^{3/2}\cdot\sin\sqrt{x}$

Ans: (b)

93. If $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$, then $\dfrac{dy}{dx}$ is equal to

Options:

(a) $\dfrac{3y-4x-1}{2y-3x+2}$

(b) $\dfrac{3y+4x-1}{2y+3x+2}$

(c) $\dfrac{3y-4x+1}{2y-3x-2}$

(d) $\dfrac{3y-4x+1}{2y+3x+2}$

Sol: On differentiating given equation w.r.t. x, we get

$4x - 3x\dfrac{dy}{dx} - 3y + 2y\dfrac{dy}{dx} + 1 + 2\dfrac{dy}{dx} - 0 = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{3y-4x-1}{2y-3x+2}$

Ans: (a)

94. If $f(x)=\begin{cases} cx+1, & x\le 3 \\ cx^2-1, & x>3 \end{cases}$ is continuous at $x=3$, then c is equal to

Options:

(a) $1/3$

(b) $2/3$

(c) $3/2$

(d) 3

Sol: At $x=3$

$\text{LHL}=\lim\limits_{x\to 3^-}(cx+1)=\lim\limits_{h\to 0}\left[c(3-h)+1\right]=3c+1$

$\text{RHL}=\lim\limits_{x\to 3^+}\left(cx^2-1\right)=\lim\limits_{h\to 0}\left[c(3+h)^2-1\right]=9c-1$

As $f(x)$ is continuous at $x=3$

$\therefore\ 3c+1=9c-1 \ \Rightarrow\ 6c=1+1 \Rightarrow\ c=\frac{2}{6}=\frac{1}{3} \Rightarrow\ c=\frac{1}{3}$

Ans: (a)

95. If p : 7 is not greater than 4 and q : Paris is in France, are two statements. Then, $\sim(p\vee q)$ is the statement

Options:

(a) 7 is greater than 4 or Paris is not in France

(b) 7 is not greater than 4 and Paris is not in France

(c)) 7 is greater than 4 and Paris is in France

(d) 7 is greater than 4 and Paris is not in France

Sol: $\sim(p\vee q)\equiv\sim p\wedge\sim q$. Hence, 7 is greater than 4 and Paris is not in France

Ans: (d)

96. Mean deviation of 6, 8, 12, 15, 10, 9 from mean is

Options:

(a) 10

(b) 2.33

(c) 2.5

(d) None of these

Sol:

Here, mean $=\frac{6+8+12+15+10+9}{6}=10$

$\therefore$ Mean deviation $=\frac{\sum|x_i-\bar{x}|}{n}=\frac{\left[|6-10|+|8-10|+|12-10|+|15-10|+|10-10|+|9-10|\right]}{6}$

$=\frac{4+2+2+5+0+1}{6}=\frac{14}{6}=2.33$

Ans: (b)

97. In a class, there are 10 boys and 8 girls. When 3 students are selected at random, the probability that 2 girls and 1 boy are selected, is

Options:

(a) $\frac{35}{102}$

(b) $\frac{15}{102}$

(c) $\frac{55}{102}$

(d) $\frac{25}{102}$

Sol: Total number of boys $=10$; Total number of girls $=8$

Number of students have to be selected at random $=3$

If 2 girls and 1 boy are selected, then the required probability

$$=\frac{{}^{8}C_{2}\times{}^{10}C_{1}}{{}^{18}C_{3}}=\frac{\frac{8\times7}{2}\times10}{\frac{18\times17\times16}{3\times2}} \qquad \left[\because {}^{n}C_{r}=\frac{n!}{r!(n-r)!}\right]$$

$$=\frac{4\times7\times10\times6}{18\times17\times16}=\frac{70}{3\times17\times4}=\frac{35}{6\times17}=\frac{35}{102}$$

Ans: (a)

98. If A and B are two events such that $P\left(\overline{A\cup B}\right)=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$ and $P\left(\bar{A}\right)=\frac{1}{4}$, where $\bar{A}$ stands for complement of the event A. Then, events A and B are

Options:

(a) mutually exclusive and independent

(b) independent but not equally likely

(c) equally likely but not independent

(d) equally likely and mutually exclusive

Sol: Given, $P\left(\overline{A\cup B}\right)=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$, $P\left(\bar{A}\right)=\frac{1}{4}$

$P(A\cup B)=1-P\left(\overline{A\cup B}\right)=1-\frac{1}{6}=\frac{5}{6}$ and

$P(A)=1-P\left(\bar{A}\right)=1-\frac{1}{4}=\frac{3}{4}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B) \Rightarrow \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$

$P(B)=\frac{1}{3} \Rightarrow A$ and B are not equally likely.

$P(A\cap B)=P(A)\cdot P(B)=\frac{1}{4}$. So, events are independent.

Ans: (b)

99. If $P(A)=P(B)=x$ and $P(A\cap B)=P(A'\cap B')=\frac{1}{3}$, then x is equal to

Options:

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{1}{6}$

Sol: $P(A'\cap B')=1-P(A\cup B)=\frac{1}{3}$ [given]

$\Rightarrow\ P(A\cup B)=\frac{2}{3}$

$\therefore\ P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\ \frac{2}{3}=x+x-\frac{1}{3}\ \Rightarrow\ x=\frac{1}{2}$

Ans: (a)

100. In a trial, the probability of success is twice the probability of failure. In six trials, the probability of atleast four successes will be

Options:

(a) $\frac{496}{729}$

(b) $\frac{400}{729}$

(c) $\frac{500}{729}$

(d) $\frac{600}{729}$

Sol: Let the probability of success and failure be p and q, respectively.

Then, $p=2q$ and $p+q=1\ \Rightarrow\ 3q=1\ \Rightarrow\ q=\frac{1}{3}$

$\therefore\ p=\frac{2}{3}$

Required probability

$={}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2+{}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)+{}^6C_6\left(\frac{2}{3}\right)^6=\frac{496}{729}$

Ans: (a)

101.If m and σ^2 are the mean and variance of the random variable X, whose distribution is given by

X	0	1	2	3
$P(X)$	1/3	1/2	0	1/6

. Then,

Options:

(a) $m=\sigma^2=2$

(b) $m=1,\ \sigma^2=2$

(c) $m=\sigma^2=1$

(d) $m=2,\ \sigma^2=1$

Sol: Given, distribution is

X	0	1	2	3
$P(X)$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

$\therefore$ Mean, $m=\sum_{i=1}^{4} p_i x_i = 0\times\frac{1}{3}+1\times\frac{1}{2}+2\times 0+3\times\frac{1}{6}=0+\frac{1}{2}+0+\frac{1}{2}=1$

Variance, $\sigma^2=\sum_{i=1}^{4} p_i\left(x_i-m\right)^2$

$=\frac{1}{3}(0-1)^2+\frac{1}{2}(1-1)^2+0(2-1)^2+\frac{1}{6}(3-1)^2$

$=\frac{1}{3}+0+0+\frac{2}{3}=1$

$\therefore\ m=\sigma^2=1$

Ans: (c)

102.If $A=\begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$ is a singular matrix, then the value of $5k-k^2$ is

Options:

(a) 4

(b) 6

(c) -6

(d) -4

Sol: Given, $A=\begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$

Since, the matrix A is singular.

$\therefore\ |A|=0$

$\Rightarrow \begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}=0 \Rightarrow (2-k)(3-k)-2=0$

$\Rightarrow 6-5k+k^2-2=0 \qquad \Rightarrow 4-5k+k^2=0 \qquad \Rightarrow 5k-k^2=4$

Ans: (a)

103. If $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$, then A^2 is equal to

Options:

(a) 0

(b) $-A$

(c) I

(d) $2A$

Sol: Given, $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$

$\therefore\ A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$

$=\begin{bmatrix}1+0+0 & 0+0+0 & 0+0+0\\ 0+0+0 & 0+1+0 & 0+0+0\\ a+0-a & 0+b-b & 0+0+1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$

Ans: (c)

104. If X and Y are 2×2 matrices such that $2X+3Y=O$ and $X+2Y=I$, where O and I denote the 2×2 zero matrix and the 2×2 identity matrix, then X is equal to

Options:

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$

(c) $\begin{bmatrix}-3 & 0\\ 0 & -3\end{bmatrix}$

(d) $\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

Sol: Given, $2X+3Y=O$ … (i)

and $X+2Y=I$ … (ii)

where, $O=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

On solving Eqs. (i) and (ii), we get

$X=-3I=\begin{bmatrix}-3 & 0\\ 0 & -3\end{bmatrix}$

Ans: (c)

105.If a,b and c are cube roots of unity, then $\begin{vmatrix} e^a & e^{2a} & e^{3a}-1 \\ e^b & e^{2b} & e^{3b}-1 \\ e^c & e^{2c} & e^{3c}-1 \end{vmatrix}$ is equal to

Options:

(a) 0

(b) e

(c) e^2

(d) e^3

Sol: $\Delta = \begin{vmatrix} e^a & e^{2a} & e^{3a} \\ e^b & e^{2b} & e^{3b} \\ e^c & e^{2c} & e^{3c} \end{vmatrix} - \begin{vmatrix} e^a & e^{2a} & 1 \\ e^b & e^{2b} & 1 \\ e^c & e^{2c} & 1 \end{vmatrix}$

$= e^a \cdot e^b \cdot e^c \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} + \begin{vmatrix} e^a & 1 & e^{2a} \\ e^b & 1 & e^{2b} \\ e^c & 1 & e^{2c} \end{vmatrix}$

$= e^{(a+b+c)} \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} - \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix}$

$= \left(e^{a+b+c}-1\right) \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} = 0 \qquad [\because\ a+b+c=0]$

Ans: (a)

106.If $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2} \neq 0$, then the area of the triangle whose vertices are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right)$ and $\left(\frac{x_3}{c}, \frac{y_3}{c}\right)$ is

Options:

(a) $\frac{1}{4}abc$

(b) $\frac{1}{8}abc$

(c) $\frac{1}{4}$

(d) $\frac{1}{8}$

Sol: Given, $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2}$

$\Rightarrow 2\begin{vmatrix} a & x_1 & y_1 \\ b & x_2 & y_2 \\ c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2} \Rightarrow \begin{vmatrix} a & x_1 & y_1 \\ b & x_2 & y_2 \\ c & x_3 & y_3 \end{vmatrix} = \frac{abc}{4}$ … (i)

Given vertices of a triangle are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right)$ and $\left(\frac{x_3}{c}, \frac{y_3}{c}\right)$

$\therefore$ Area of triangle

$$= \frac{1}{2}\begin{vmatrix} \frac{x_1}{a} & \frac{y_1}{a} & 1 \\ \frac{x_2}{b} & \frac{y_2}{b} & 1 \\ \frac{x_3}{c} & \frac{y_3}{c} & 1 \end{vmatrix} = \frac{1}{2}\cdot\frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c}\begin{vmatrix} x_1 & y_1 & a \\ x_2 & y_2 & b \\ x_3 & y_3 & c \end{vmatrix}$$

$$= \frac{1}{2abc}\begin{vmatrix} a & x_1 & y_1 \\ b & x_2 & y_2 \\ c & x_3 & y_3 \end{vmatrix} = \frac{1}{2abc}\left(\frac{abc}{4}\right) \quad \text{[from eq. (i)]}$$

$$= \frac{1}{8}$$

Ans: (d)

107. If $\begin{vmatrix} x^2+x & 3x-1 & -x+3 \\ 2x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 3x \end{vmatrix} = a_0 + a_1x + a_2x^2 + \ldots + a_7x^7$, then the value of a_0 is

Options:

(a) 21

(b) 24

(c) 23

(d) 22

Sol: On putting $x = 0$ in the given determinant equation, we get

$$a_0 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix} = 1(0-9) + 3(4+6)$$

$$= 30 - 9 = 21$$

Ans: (a)

108. Three non-zero non-collinear vectors, a, b and c are such that $a+3b$ is collinear with c, $3b+2c$ is collinear with a. Then, $a+3b+2c$ is equal to

Options:

(a) 0

(b) $2a$

(c) $3b$

(d) $4c$

Sol: Given, $a+3b$ is collinear with c.

$\therefore \quad a+3b=\lambda c$

or $a+3b-\lambda c=0 \qquad \ldots$ (i)

and $3b+2c$ is collinear.

$\Rightarrow \quad 3b+2c=\mu a$

$3b+2c-\mu a=0 \qquad \ldots$ (ii)

From eqs. (i) and (ii),

$a+3b-\lambda c=3b+2c-\mu a$

On equating c, we get $\lambda=-2$

On putting $\lambda=-2$ in eq. (i), we get

$a+3b+2c=0$

Ans: (a)

109. If a, b and c are pth, qth and rth terms of a GP, then the vectors $\log a\hat{i}+\log b\hat{j}+\log c\hat{k}$ and $(q-r)\hat{i}+(r-p)\hat{j}+(p-q)\hat{k}$ are

Options:

(a) equal

(b) parallel

(c) perpendicular

(d) None of these

Sol: Let the first term and common ratio of a GP be α and β, then

$a=\alpha\cdot\beta^{p-1}$, $b=\alpha\cdot\beta^{q-1}$ and $c=\alpha\cdot\beta^{r-1}$

$\therefore \quad \log a=\log\alpha+(p-1)\log\beta$

$\log b=\log\alpha+(q-1)\log\beta$

and $\log c=\log\alpha+(r-1)\log\beta$

The dot product of the given two vectors is

$(q-r)\log a+(r-p)\log b+(p-q)\log c$

$\Rightarrow \quad (q-r)\left[\log\alpha+(p-1)\log\beta\right]+(r-p)$

$\left[\log\alpha+(q-1)\log\beta\right]+(p-q)\left[\log\alpha+(r-1)\log\beta\right]$

$\Rightarrow \log\alpha[q-r+r-p+p-q]+\log\beta[(p-q)(q-r)+(r-p)(q-1)+(r-1)(p-q)]$

$=0+0=0$

So, the two vectors are perpendicular.

Ans: (c)

110. If $a=\hat{i}+2\hat{j}+2\hat{k}, |b|=5$ and the angle between a and b is $\pi/6$, then the area of the triangle formed by these two vectors as two sides is

Options:

(a) $\frac{15}{4}$

(b) $\frac{15}{2}$

(c) 15

(d) $\frac{15\sqrt{3}}{2}$

Sol: Area of the triangle $=\frac{1}{2}|a\times b|=\frac{1}{2}\|a\|\|b|\sin\theta$

$=\frac{1}{2}\left[3\times5\times\sin\frac{\pi}{6}\right]$ $\left[\because |a|=\sqrt{1+2^2+2^2}=3\right]$

$=\frac{1}{2}\left[15\times\frac{1}{2}\right]=\frac{15}{4}$ $\left[\because \sin\frac{\pi}{6}=\frac{1}{2}\right]$

Ans: (a)

111. The value of $\left[(\vec{a}-\vec{b})(\vec{b}-\vec{c})(\vec{c}-\vec{a})\right]$ is

Options:

(a) 0

(b) 1

(c) $2\left[\vec{a}\vec{b}\vec{c}\right]$

(d) 2

Sol: $\left[(\vec{a}-\vec{b})(\vec{b}-\vec{c})(\vec{c}-\vec{a})\right]$

$=(\vec{a}-\vec{b})\cdot\left[(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\right]$

$=(\vec{a}-\vec{b})\cdot\left[\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right]$

$=(\vec{a}-\vec{b})\cdot\left[\vec{b}\times\vec{c}-\vec{b}\times a+\vec{c}\times\vec{a}\right]$ $[\because \vec{c}\times\vec{c}=0]$

$=\vec{a}\cdot\left[\vec{b}\times\vec{c}\right]-\vec{a}\cdot\left[\vec{b}\times\vec{a}\right]+\vec{a}\cdot[\vec{c}\times\vec{a}]-\vec{b}\cdot\left[\vec{b}\times\vec{c}\right]+\vec{b}\cdot\left[\vec{b}\times\vec{a}\right]-\vec{b}\cdot[\vec{c}\times\vec{a}]$

$=\left[\vec{a}\vec{b}\vec{c}\right]-0+0-0+0-\left[\vec{b}\vec{c}\vec{a}\right]$

$=\left[\vec{a}\vec{b}\vec{c}\right]-\left[\vec{a}\vec{b}\vec{c}\right]=0$

Ans: (a)

112. The maximum value of $z = 4x + 2y$ subject to constraints $2x + 3y \le 18$, $x + y \ge 10$ and $x, y \ge 0$ is

Options:

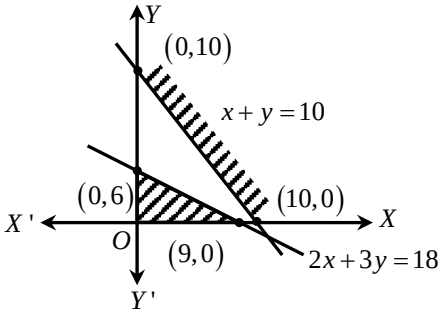
(a) 20

(b) 36

(c) 40

(d) None of these

Sol: From the figure, it is clear that there is no common area.



So, we cannot find maximum value of z

Ans: (d)

113. $\int \frac{1}{\sqrt{7-x^2}}dx$ is equal to

Options:

(a) $\frac{1}{2\sqrt{7}}\log\left|\frac{\sqrt{7+x}}{\sqrt{7-x}}\right| + C$

(b) $\sin^{-1}\left(\frac{x}{\sqrt{7}}\right) + C$

(c) $\log\left|x + \sqrt{x^2 - 7}\right| + C$

(d) $\frac{1}{2\sqrt{7}}\log\left|\frac{x-\sqrt{7}}{x+\sqrt{7}}\right| + C$

Sol: $\int \frac{dx}{\sqrt{7-x^2}} = \int \frac{dx}{\sqrt{\left(\sqrt{7}\right)^2 - x^2}}$

$= \sin^{-1}\left(\frac{x}{\sqrt{7}}\right) + C \qquad \left[\because \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)\right]$

Ans: (b)

114. $\int \frac{x^4+x^2+1}{x^2-x+1}dx$ is equal to

Options:

(a) $\frac{x^3}{3}-\frac{x^2}{2}+x+C$

(b) $\frac{x^3}{3}+\frac{x^2}{2}+x+C$

(c) $\frac{x^3}{3}-\frac{x^2}{2}-x+C$

(d) $\frac{x^3}{3}+\frac{x^2}{2}-x+C$

Sol: $\int \frac{x^4+x^2+1}{x^2-x+1}dx=\int\left(x^2+x+1\right)dx=\frac{x^3}{3}+\frac{x^2}{2}+x+C$

$\because \ \frac{x^4+x^2+1}{x^2-x+1}=x^2+x+1$

Ans: (b)

115. $\int e^{-\log x}dx$ is equal to

Options:

(a) $e^{-\log x}+C$

(b) $-xe^{-\log x}+C$

(c) $e^{\log x}+C$

(d) $\log|x|+C$

Sol: $\int e^{-\log x}dx=\int\frac{1}{x}dx=\log|x|+C$

Ans: (d)

116. $\int \frac{(1+x)^{e^x}}{\sin^2\left(xe^x\right)}dx$ is equal to

Options:

(a) $-\cot\left(xe^x\right)+C$

(b) $\tan\left(xe^x\right)+C$

(c) $\tan\left(e^x\right)+C$

(d) $\cot\left(xe^x\right)+C$

Sol: Let $I=\int\frac{(1+x)e^x}{\sin^2\left(xe^x\right)}dx$

Putting $xe^x = t \Rightarrow \left(1\cdot e^x + x\cdot e^x\right)dx = dt \Rightarrow (1+x)e^x dx = dt$

$\therefore \quad I = \int \frac{dt}{\sin^2 t} = \int \text{cosec}^2 t dt = -\cot t + C \quad = -\cot\left(xe^x\right) + C$

Ans: (a)

117. $\int \frac{dx}{(x+1)\sqrt{4x+3}}$ is equal to

Options:

(a) $\tan^{-1}\sqrt{4x+3} + C$

(b) $3\tan^{-1}\sqrt{4x+3} + C$

(c) $2\tan^{-1}\sqrt{4x+3} + C$

(d) $4\tan^{-1}\sqrt{4x+3} + C$

Sol: Let $I = \frac{dx}{(x+1)\sqrt{4x+3}}$

Putting $4x+3 = t^2 \Rightarrow 4dx = 2tdt \quad \therefore \quad I = \frac{1}{2}\int \frac{tdt}{\left(\frac{t^2-3}{4}+1\right)t} = 2\int \frac{dt}{1+t^2}$

$= 2\tan^{-1} t + C = 2\tan^{-1}\sqrt{4x+3} + C$

Ans: (c)

118. Which of the following is correct?

Options:

(a) $\int_0^1 e^x dx = e$

(b) $\int_0^1 2^x dx = \log 2$

(c) $\int_0^1 \sqrt{x} dx = \frac{2}{3}$ (d) $\int_0^1 x dx = \frac{1}{3}$

Sol:

(a) $\int_0^1 e^x dx = \left[e^x\right]_0^1 = e - 1$

(b) $\int_0^1 2^x dx = \left[\frac{2^x}{\log_e 2}\right]_0^1 = \frac{1}{\log 2}\cdot(2-2^\circ) = \frac{1}{\log 2}$

(c) $\int_0^1 \sqrt{x} dx = \left[\frac{x^{3/2}}{3/2}\right]_0^1 = \frac{2}{3}$

(d) $\int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$

Ans: (c)

119. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then the values of a and b are respectively

Options:

(a) $1,1$

(b) $-1,-1$

(c) $1,-1$

(d) $-1,1$

Sol: $\int_a^b x^3 dx = \frac{x^4}{4}\Big|_a^b = \frac{1}{4}\left(b^4 - a^4\right) = 0 \Rightarrow b^4 - a^4 = 0$ … (1)

$\int_a^b x^2 dx = \frac{x^3}{3}\Big|_b^b = \frac{1}{3}\left(a^3 - b^3\right) = \frac{2}{3} \Rightarrow b^3 - a^3 = 2$ … (2)

Solving (1) & (2)

$a = -1, b = 1$

Ans: (d)

120. $\int_0^{2\pi} \left(\sin x + |\sin x|\right) dx$ is equal to

Options:

(a) 0

(b) 4

(c) 8

(d) 1

Sol: $\int_0^{2\pi} \left(\sin x + |\sin x|\right) dx$

$= \int_0^{\pi} \left(\sin x + \sin x\right) dx + \int_{\pi}^{2\pi} \left(\sin x - \sin x\right) dx$

$= 2\int_0^{\pi} \sin x dx + 0 = -2\left[\cos x\right]_0^{\pi}$

$= -2\left(\cos \pi - \cos 0\right) = -2\left(-1-1\right) = 4$

Ans: (b)

Physics

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60

121. A satellite has kinetic energy K, potential energy V and total energy E. Which of the following statements is true?

Options:

(a) $K = -\frac{V}{2}$

(b) $K = \frac{V}{2}$

(c) $E = \frac{K}{2}$

(d) $E = -\frac{K}{2}$

Sol: $K = \frac{GMm}{2r}$ and $V = -\frac{GMm}{r}$

$\therefore E = K + V = -\frac{GMm}{2r} \Rightarrow K = -\frac{V}{2}$

Ans: (a)

122. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is

Options:

(a) $2Fl$

(b) Fl

(c) $\frac{F}{2l}$

(d) $\frac{Fl}{2}$

Sol: Work done by constant force in displacing the object by a distance l = change in potential energy

$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$= \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times A \times L = \frac{Fl}{2}$

Ans: (d)

123. A wheel has angular acceleration of $3.0\,\text{rad}\,\text{s}^{-2}$ and an initial angular speed of $2.00\,\text{rad}\,\text{s}^{-1}$. In a time of $2\,\text{s}$ it has rotated through an angle (in radian) of

Options:

(a) 10

(b) 12

(c) 4

(d) 6

Sol: Since, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ where α is angular acceleration, ω_0 is the initial angular speed.

$t = 2\,\text{s}$

$\theta = (2\times 2) + \frac{1}{2}\times 3(2)^2 = 4 + 6 = 10\,\text{rad}$

Ans: (a)

124. Spheres of iron and lead having same mass are completely immersed in water. Density of lead is more than that of iron. Apparent loss of weight is W_1 for iron sphere and W_2 for lead sphere. Then $\frac{W_1}{W_2}$ is

Options:

(a) 1

(b) Between 0 and 1

(c) 0

(d) >1

Sol: Density $(\rho) = \frac{\text{Mass}(M)}{\text{Volume}(V)}$

$\rho \propto \frac{1}{V}$ for the same mass

$\therefore \frac{\rho_{\text{lead}}}{\rho_{\text{iron}}} = \frac{V_{\text{iron}}}{V_{\text{lead}}}$ [Given]

As $\rho_{\text{lead}} > \rho_{\text{iron}}$ … (i)

$\therefore \frac{V_{\text{iron}}}{V_{\text{lead}}} > 1$

Using law of floatation,

For iron sphere, $W_1 = V_{\text{iron}}\,\rho_{\text{water}}\,g$ … (ii)

For lead sphere, $W_2 = V_{\text{lead}}\,\rho_{\text{water}}\,g$ … (iii)

Divide (ii) by (iii), we get

$\frac{W_1}{W_2} = \frac{V_{\text{iron}}}{V_{\text{lead}}} > 1$ [Using (i)]

Ans: (d)

125. The ratio of radiant energies radiated per unit surface area by two bodies is $16:1$, the temperature of hotter body is $1000\,\text{K}$, then the temperature of colder body will be

Options:

(a) $250\,\text{K}$

(b) $500\,\text{K}$

(c) $1000\,\text{K}$

(d) $62.5\,\text{K}$

Sol: By Stefan-Boltzmann law, the energy radiated by hot object at $T, E = \sigma T^4$

As, $\frac{16}{1} = \frac{\sigma(1000)^4}{\sigma T^4} \Rightarrow T^4 = \left(\frac{1000}{2}\right)^4 \Rightarrow T = 500\,\text{K}$

Ans: (b)

126. According to kinetic theory of gas, molecules of a gas behave like

Options:

(a) Inelastic rigid sphere

(b) Perfectly elastic non-rigid sphere

(c) Perfectly elastic rigid sphere

(d) Inelastic non-rigid sphere

Sol: Molecules of ideal gas behaves like perfectly elastic rigid sphere.

Ans: (c)

127. In an adiabatic process, the pressure is increased by $\left(\frac{2}{3}\right)\%$. If $\gamma = \frac{3}{2}$, then the volume decreases by nearly

Options:

(a) $\frac{4}{9}\%$

(b) $\frac{2}{3}\%$

(c) 1%

(d) $\frac{9}{4}\%$

Sol: $PV^{3/2} = \text{constant} = K$

$\log P + \frac{3}{2}\log V = \log K$

$\frac{\Delta P}{P} + \frac{3}{2}\frac{\Delta V}{V} = 0$

$\frac{\Delta V}{V} = -\frac{2}{3}\frac{\Delta P}{P}$ or $\frac{\Delta V}{V} = \left(-\frac{2}{3}\right)\left(\frac{2}{3}\right) = -\frac{4}{9}$

Ans: (a)

128. The displacement of particle from the mean position in SHM is given by $x = a\cos\omega t + b\sin\omega t$. If $a = 3, b = 4$ and $\omega = 4$, the amplitude and maximum velocity respectively will be

Options:

(a) 3, 4

(b) 4, 16

(c) 7, 14

(d) 5, 20

Sol: $A = \sqrt{a^2 + b^2} = 5$ and $V_{max} = A\omega = 20$

Ans: (d)

129. The number of beats produced per second by two vibrations, $x_1 = x_0 \sin 646\pi t$ and $x_2 = x_0 \sin 652\pi t$ is of

Options:

(a) 2

(b) 3

(c) 4

(d) 6

Sol: $\omega = 2\pi f \quad \therefore f = \dfrac{\omega}{2\pi}$

$f_1 = \dfrac{646\pi}{2\pi} = 323 s^{-1}$

$f_2 = \dfrac{652\pi}{2\pi} = 326 s^{-1}$

No. of beats/sec = $f_2 - f_1 = 326 - 323 = 3$

Ans: (b)

130. Three point charges Q_1, Q_2, Q_3 are placed equally speed along a straight line. Q_2 and Q_3 are equal in magnitude but opposite in sign. If the net force on Q_3 is zero, the value of Q_1 is

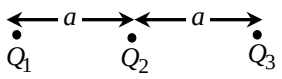
Options:

(a) $Q_1 = 4(Q_3)$

(b) $Q_1 = 2(Q_3)$

(c) $Q_1 = \sqrt{2}(Q_3)$

(d) $Q_1 = (Q_3)$

Sol: 

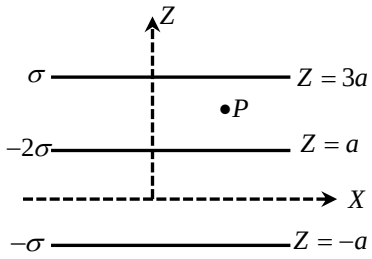
$Q_2 = -Q_3 = Q$

Force on Q_3 due to Q_2 + Force on Q_3 due to $Q_1 = 0$

$\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{-Q^2}{a^2}\right) + \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1 Q}{4a^2} = 0 \Rightarrow Q_1 = 4Q_3$

Ans: (a)

131. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is



Options:

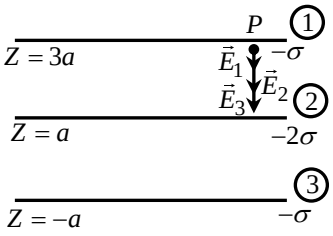
(a) $\frac{2\sigma}{\varepsilon_0}\hat{k}$

(b) $\frac{4\sigma}{\varepsilon_0}\hat{k}$

(c) $-\frac{2\sigma}{\varepsilon_0}\hat{k}$

(d) $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

Sol: Figure shows the electric fields due to the sheets $1,2$ and 3 at point P. The direction of electric fields are according to the charge on the sheets (away from positively charge sheet and towards the negatively charged sheet and perpendicular).

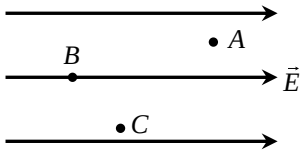


The total electric field, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = E_1\left(-\hat{k}\right) + E_2\left(-\hat{k}\right) + E_3\left(-\hat{k}\right)$

$$= \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right]\left(-\hat{k}\right) = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

Ans: (c)

132. A, B and C are three points in a uniform electric field. The electric potential is



Options:

(a) maximum at B

(b) maximum at C

(c) same at all the three points A, B and C

(d) maximum at A

Sol:

Potential at B, V_B is maximum

$V_B > V_C > V_A$

As in the direction of electric field potential decreases.

Ans: (a)

133. A pendulum bob of mass m carrying a charge q is at rest with its string making an angle θ with the vertical in a uniform horizontal electric field E. The tension in the string is

Options:

(a) $\frac{mg}{\sin\theta}$ and $\frac{qE}{\cos\theta}$

(b) $\frac{mg}{\cos\theta}$ and $\frac{qE}{\sin\theta}$

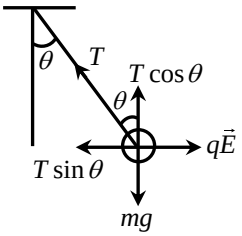
(c) $\frac{qE}{mg}$

(d) $\frac{mg}{qE}$

Sol: In equilibrium,

$T\cos\theta = mg$ … (1)

$T\sin\theta = qE$ … (2)



From (1), $T = \frac{mg}{\cos\theta}$

From (2), $T = \frac{qE}{\sin\theta}$

Ans: (b)

134. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities remain unchanged?

Options:

(a) The charge on the capacitor

(b) The stored energy in the capacitor

(c) The potential difference between the plates

(d) The electric field in the capacitor

Sol: As the capacitor is isolated, so charge on capacitor will not change. Due to insertion of a dielectric slab capacitance increase by K times. The potential difference, the electric field and the stored energy decreases by $\frac{1}{K}$ times.

Ans: (a)

135. A capacitor is charged by a battery. The battery is removed and another identical uncharged capacitor is connected in parallel. The total electrostatic energy of resulting system

Options:

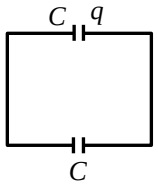
(a) Decreases by a factor of 2

(b) Remains the same

(c) Increases by a factor of 2

(d) Increases by a factor of 4

Sol: When battery is replaced by another uncharged capacitor. As uncharged capacitor is connected parallel,



So, $C' = 2C$ and $V_c = \frac{q_1 + q_2}{C_1 + C_2}$

$V_c = \frac{q + 0}{C + C} \qquad \Rightarrow V_c = \frac{V}{2}$

Initial energy of system, $U_i = \frac{1}{2}CV^2$ … (i)

Final energy of system, $U_f = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2$

$= \frac{1}{2}CV^2\left(\frac{1}{2}\right)$ .. (ii)

From equation (i) and (ii), $U_f = \frac{1}{2}U_i$, i.e., total electrostatic energy of resulting system decreases by a factor of 2.

Ans: (a)

136. Two spheres A and B of radius $4\,\text{cm}$ and $6\,\text{cm}$ are given charges of $80\,\mu\text{C}$ and $40\,\mu\text{C}$ respectively. If they are connected by a fine wire, the amount of charge flowing from one to the other is

Options:

(a) $20\,\mu\text{C}$ from A to B

(b) $16\,\mu\text{C}$ from A to B

(c) $32\,\mu\text{C}$ from B to A

(d) $32\,\mu\text{C}$ from A to B

Sol: Total charge, $Q = 80 + 40 = 120\,\mu\text{C}$

By using the formula, $Q_1^{'} = Q\left[\frac{r_1}{r_1 + r_2}\right]$

New charge on sphere A is $Q_A^{'} = Q\left[\frac{r_A}{r_A + r_B}\right] = 120\left[\frac{4}{4+6}\right] = 48\,\mu\text{C}$

Initially it was $80\,\mu\text{C}$, i.e., $32\,\mu\text{C}$ charge flows from A to B.

Ans: (d)

137. There is an electric field E in $x-$direction. If the work done on moving a charge of $0.2\,\text{C}$ through a distance of $2\,\text{m}$ along a line making an angle $60°$ with $x-$axis is $4\,\text{J}$, then what is the value of E?

Options:

(a) $3\,\text{N C}^{-1}$

(b) $4\,\text{N C}^{-1}$

(c) $5\,\text{N C}^{-1}$

(d) $20\,\text{N C}^{-1}$

Sol: Charge $(q) = 0.2\,\text{C}$; distance $(d) = 2\,\text{m}$; angle $\theta = 60°$ and work done $(W) = 4\,\text{J}$.

Work done in moving the charge $(W) = F \cdot d\cos\theta = qEd\cos\theta$

or, $E = \dfrac{W}{qd\cos\theta} = \dfrac{4}{0.2 \times 2 \times \cos 60°} = \dfrac{4}{0.4 \times 0.5} = 20\,\text{N C}^{-1}$

Ans: (d)

138. A primary cell has an emf of 1.5 volt, when short-circuited it gives a current of 3 ampere. The internal resistance of the cell is

Options:

(a) $4.5\,\text{ohm}$

(b) $2\,\text{ohm}$

(c) $0.5\,\text{ohm}$

(d) $\dfrac{1}{4.5}\,\text{ohm}$

Sol: Short circuit current, $i_{SC} = \dfrac{E}{r} \Rightarrow 3 = \dfrac{1.5}{8} \Rightarrow r = 0.5\,\Omega$

Ans: (c)

139. An electric current passes through a circuit containing two wires of the same material connected in parallel. If the lengths of the wires are in the ratio of $\dfrac{4}{3}$ and radius of the wires are in the ratio of $\dfrac{2}{3}$, then the ratio of the currents passing through the wires will be

Options:

(a) 3

(b) $\dfrac{1}{3}$

(c) $\dfrac{3}{9}$

(d) None of these

Sol: Given: $\dfrac{l_1}{l_2} = \dfrac{4}{3}$ and $\dfrac{r_1}{r_2} = \dfrac{2}{3}$

Since the two wires are connected in parallel, potential remains same, i.e., $V = \text{constant}$, $IR = \text{constant}$

i.e., $I_1R_1 = I_2R_2 \Rightarrow \dfrac{I_1}{I_2} = \dfrac{R_2}{R_1}$ … (i)

But we know that, $R = \dfrac{\rho l}{A}$ $\therefore \dfrac{R_1}{R_2} = \left(\dfrac{l_1}{A_1}\right)\left(\dfrac{A_2}{l_2}\right) = \left(\dfrac{l_1}{l_2}\right)\left(\dfrac{A_2}{A_1}\right) = \left(\dfrac{l_1}{l_2}\right)\left(\dfrac{r_2}{r_1}\right)^2$ (since area, $A = \pi r^2$)

$= \left(\dfrac{4}{3}\right)\left(\dfrac{3}{2}\right)^2 = 3$

Substitute this value in equation (i), we get, $\dfrac{I_1}{I_2} = \dfrac{1}{3}$

Ans: (b)

140. The powers of two electric bulbs are 100 watt and 200 watt. Both of them are joined with 220 volt. The ratio of resistance of their filament will be

Options:

(a) 4:1

(b) 1:4

(c) 1:2

(d) 2:1

Sol: $P_1 = 100\,\text{W},\ P_2 = 200\,\text{W}$

$$R_1 = \frac{V^2}{P_1} = \frac{220\times 220}{100} = 22\times 22\,\Omega$$

$$R_2 = \frac{220\times 220}{200} = 22\times 11\,\Omega$$

$\therefore R_1 : R_2 = 2:1$

Ans: (d)

141. Five equal resistances each of resistance R are connected as shown in the figure. A battery of V volts is connected between A and B. The current flowing in $AFCEB$ will be

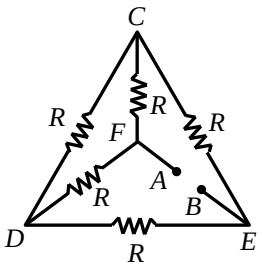
Options:

(a) $\frac{2V}{R}$

(b) $\frac{3V}{R}$

(c) $\frac{V}{R}$

(d) $\frac{V}{2R}$



Sol: A balanced Wheatstone' bridge exists between A and B

$R_{eq} = R$

Current through circuit is $\frac{V}{R}$

Current through $AFCEB = \frac{V}{2}R$

Ans: (d)

142. A cell in secondary circuit gives null deflection of 2.5 m length of potentiometer having 10 m length of wire. If the length of the potentiometer wire is increased by 1m without changing the cell in the primary, the position of the null point now is

Options:

(a) 3.5 m

(b) 3 m

(c) 2.75 m

(d) 2.0 m

Sol: $L \propto l$

$\frac{L_1}{L_2} = \frac{l_1}{l_2}$

$\frac{10}{11} = \frac{2.5}{l_2}$; $10 l_2 = 2.5 \times 11$

$l_2 = \frac{2.5 \times 11}{10} = 2.75\,\text{m}$

Ans: (c)

143. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities remain constant along the length of the conductor is/are

Options:

(a) Current, electric field and drift speed

(b) Drift speed only

(c) Current and drift speed

(d) Current only

Sol: If E be electric field, then current density, $j = \sigma E$. Also we know that current density, $j = \frac{i}{A}$.

Hence j is different for different area of cross-sections. When j is different, then E is also different. Thus, E is not constant. The drift velocity v_d is given by $v_d = \frac{j}{ne} =$ different for different j values. Hence only current i will be constant.

Ans: (d)

144. A conducting wire of length l is turned in the form of a circular coil and a current i is passed through it. For torque due to magnetic field produced at its centre, to be maximum, the minimum number of turns in the coil will be

Options:

(a) 1

(b) 2

(c) 5

(d) of any value

Sol: $\tau_{\max} = MB$ or $\tau_{\max} = ni\pi a^2 B$

Let number of turns in length l is n

$l = n(2\pi a)$ or $a = \frac{l}{2\pi n}$

$\tau_{\max} = \frac{ni\pi Bl^2}{4\pi^2 n^2} = \frac{l^2 iB}{4\pi^2 n_{\min}} = \frac{l^2 iB}{4\pi n_{\min}}$ $\therefore \tau_{\max} \propto \frac{1}{n_{\min}}$, $n_{\min} = 1$

Ans: (a)

145. Circular loop of a wire and a long straight wire carry currents I_c and I_e, respectively as shown in figure. Assuming that these are placed in the same plane. The magnetic fields will be zero at the centre of the loop when the separation H is

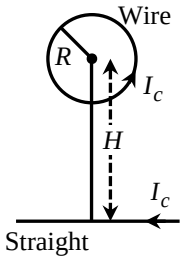
Options:

(a) $\frac{I_e R}{I_c \pi}$

(b) $\frac{I_c R}{I_e \pi}$

(c) $\frac{\pi I_c}{I_e R}$

(d) $\frac{I_e \pi}{I_c R}$



Sol: $\frac{\mu_0 I_c}{2R} = \frac{\mu_0 I_e}{2\pi H} \Rightarrow H = \frac{I_e R}{\pi I_c}$

Ans: (a)

146. A wire of length l m carrying a current I A is bent into a circle. The magnitude of the magnetic moment is

Options:

(a) $\frac{lI^2}{2\pi}$

(b) $\frac{lI^2}{4\pi}$

(c) $\frac{l^2 I}{2\pi}$

(d) $\frac{l^2 I}{4\pi}$

Sol: If r is the radius of the circle, then $l = 2\pi r$ or $r = \frac{l}{2\pi}$

Area $= \pi r^2 = \frac{\pi l^2}{4\pi^2} = \frac{l^2}{4\pi}$ $\therefore$ Magnetic moment $= IA = \frac{Il^2}{4\pi}$

Ans: (d)

147. To convert a $800\,\text{mV}$ range milli voltmeter of resistance $40\,\Omega$ into a galvanometer of $100\,\text{mA}$ range, the resistance to be connected as shunt is

Options:

(a) $10\,\Omega$

(b) $20\,\Omega$

(c) $30\,\Omega$

(d) $40\,\Omega$

Sol: $\frac{i}{i_g}=1+\frac{G}{S}\Rightarrow\frac{iG}{V_g}=1+\frac{G}{S}$

$\Rightarrow\frac{100\times10^{-3}\times40}{800\times10^{-3}}=1+\frac{40}{S}\quad\Rightarrow S=10\,\Omega$

Ans: (a)

148. Let V and H be the vertical and horizontal components of earth's magnetic field at any point on earth. Near the north pole

Options:

(a) $V>>H$

(b) $V<<H$

(c) $V=H$

(d) $V=H=0$

Sol: $H=B\cos\theta, V=B\sin\theta$

Here, $B=$ earth's magnetic field, $\theta=$ angle of dip $=90°$ at north pole

$\Rightarrow H=B\cos90°=0, V=B\sin90°=B$

$\Rightarrow V>>H$

Ans: (a)

149. The distance between the wires of electric mains is $12\,\text{cm}$. These wires experience $4\,\text{mg wt}$ per unit length. The value of current flowing in each wire will be

Options:

(a) $4.85\,\text{A}$

(b) 0

(c) $4.85\times10^{-2}\,\text{A}$

(d) $4.85\times10^{-4}\,\text{A}$

Sol: $\frac{F}{l}=\frac{\mu_0 i^2}{2\pi d}=9.8\times4\times10^{-6}$

$\Rightarrow i=\sqrt{\frac{4\times10^{-6}\times9.8\times0.12}{2\times10^{-7}}}=4.85\,\text{A}$

Ans: (a)

150. When a piece of a ferromagnetic substance is put in a uniform magnetic field, the flux density inside it is four times the flux density away from the piece. The magnetic permeability of the material is

Options:

(a) 1

(b) 2

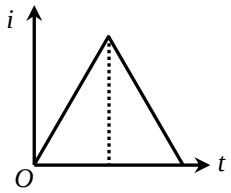
(c) 3

(d) 4

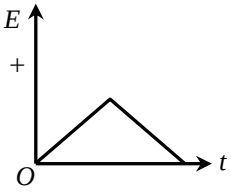
Sol: The magnetic permeability of the material, $\mu = \frac{B}{H} = \frac{4H}{H} = 4$

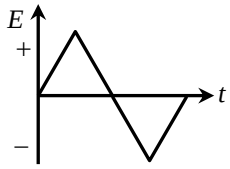
Ans: (d)

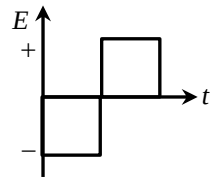
151. The current i in an inductance coil varies with time t according to the graph shown in figure. Which one of the following plots shows the variation of voltage in the coil with time?

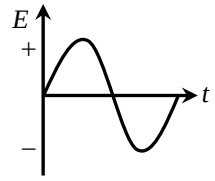


Options:

(a)


(b)


(c)


(d)


Sol: According to $i-t$ graph, in the first half current is increasing uniformly so a constant negative emf induces in the circuit. In the second half current is decreasing uniformly so a constant positive emf induces.

Ans: (c)

152. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\,\text{cm}^2$ and length $= 20\,\text{cm}$. If one of the solenoid has 300 turns and the other 400 turns, their mutual inductance is $\left(\mu_0 = 4\pi \times 10^{-7}\ \text{Tm A}^{-1}\right)$

Options:

(a) $2.4\pi \times 10^{-5}$ H

(b) $4.8\pi \times 10^{-4}$ H

(c) $4.8\pi \times 10^{-5}$ H

(d) $2.4\pi \times 10^{-4}$ H

Sol: $M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi \times 10^{-7} \times 300 \times 400 \times 100 \times 10^{-4}}{0.2} = 2.4\pi \times 10^{-4}\,\text{H}$

$\left[\because \phi = MI \text{ and } \phi = N_1 B_2 A \text{ or } \phi = N_1\left(\frac{\mu_0 N_2 I}{l}\right)\cdot A\right]$

Ans: (d)

153. If instantaneous current is given by $i = 4\cos(\omega t + \phi)$ amperes, then the rms value of current is

Options:

(a) 4 amperes

(b) $2\sqrt{2}$ amperes

(c) $4\sqrt{2}$ amperes (d) zero amperes

Sol: $i_{r\,\text{ms}} = \frac{i_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ ampere

Ans: (b)

154. An inductance L and a resistance R is connected to an alternating source of angular frequency ω. The quality factor Q of the inductance is

Options:

(a) $\frac{R}{\omega L}$

(b) $\left(\frac{\omega L}{R}\right)^2$

(c) $\left(\frac{R}{\omega L}\right)^{1/2}$

(d) $\frac{\omega L}{R}$

Sol: Quality factor $= \frac{V_L}{V_R} = \frac{I\omega L}{IR} = \frac{\omega L}{R}$

Ans: (d)

155. An inductance of $\left(\frac{200}{\pi}\right)$mH, a capacitance of $\left(\frac{10^{-3}}{\pi}\right)$F and a resistance of $10\,\Omega$ are connected in series with an a.c. source $220\,\text{V}\,50\,\text{Hz}$. The phase angle of the circuit is

Options:

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{3}$

Sol: Phase difference between E and $I = \theta$

$\therefore \tan\theta = \dfrac{X_L - X_C}{R}$ Now $X_L = 2\pi fL = 2\pi \times 50 \times \left(\dfrac{200}{\pi} \times 10^{-3}\right) = 20\,\Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{\pi}{2\pi \times 50 \times 10^{-3}} = 10\ \Omega$

$R = 10\,\Omega$

$\therefore \tan\theta = \dfrac{20-10}{10} = \dfrac{10}{10} = 1 = \tan\dfrac{\pi}{4}$

$\therefore \theta = \dfrac{\pi}{4}$. The current will lag by $\dfrac{\pi}{4}$.

Ans: (b)

156. An electromagnetic wave travels along $z-$axis. Which of the following pairs of space and time varying fields would generate such a wave?

Options:

(a) E_x, B_y

(b) E_y, B_x

(c) E_z, B_x

(d) E_y, B_z

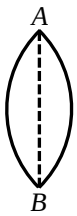
Sol: E_x and B_y would generate a plane EM wave travelling in $z-$direction. $\vec{E}$, $\vec{B}$ and $\vec{k}$ from a right handed system $\vec{k}$ is along $z-$axis. As $\hat{i} \times \hat{j} = \hat{k}$

$\Rightarrow E_x\hat{i} \times B_y\hat{j} = C\hat{k}$, i.e., E is along $x-$axis and B is along $y-$axis.

Ans: (a)

157. The equi-convex lens, shown in figure, has a focal length f. What will be the focal length of each half if the lens is cut along AB?

Options:



(a) $\dfrac{f}{2}$

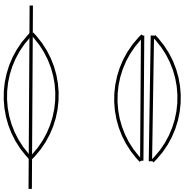
(b) f

(c) $\dfrac{3f}{2}$

(d) $2f$

Sol: $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

This is combination of two lenses of equal focal lengths.



$\therefore \dfrac{1}{f} = \dfrac{1}{f'} + \dfrac{1}{f'} = \dfrac{2}{f'} \Rightarrow f' = 2f$

Ans: (d)

158. An astronomical telescope has a magnifying power 10, the focal length of the eyepiece is $20\,\text{cm}$. The focal length of the objective is

Options:

(a) $\frac{1}{200}\text{cm}$

(b) $\frac{1}{2}\text{cm}$

(c) $200\,\text{cm}$

(d) $2\,\text{cm}$

Sol: The magnifying power of telescope in normal adjustment is given by,

$$M = \frac{f_0}{f_e}$$

$$\Rightarrow 10 = \frac{f_0}{20} \Rightarrow f_0 = 200\,\text{cm}$$

Ans: (c)

159. A convex mirror of focal length f forms an image which is $\frac{1}{n}$ times the object. The distance of the object from the mirror is

Options:

(a) $(n-1)f$

(b) $\left(\frac{n-1}{n}\right)f$

(c) $\left(\frac{n+1}{n}\right)f$

(d) $(n+1)f$

Sol: $m = +\frac{1}{n} = -\frac{v}{u} \Rightarrow v = -\frac{u}{n}$

By using mirror formula, $\frac{1}{f} = \frac{1}{\frac{-u}{n}} + \frac{1}{u} \Rightarrow u = -(n-1)f$

Ans: (a)

160. A plano-concave lens is placed on a paper on which a flower is drawn. How far above its actual position does this flower appear to be?

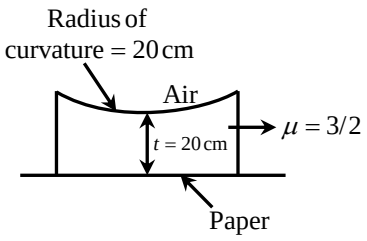
Options:

(a) $10\,\text{cm}$

(b) $15\,\text{cm}$

(c) $50\,\text{cm}$

(d) None of these



Sol: Considering refraction at the curved surface, $u=-20,\ \mu_2=1,\ \mu_1=\frac{3}{2},\ R=+20$

Applying, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$\Rightarrow \frac{1}{v}-\frac{3/2}{-20}=\frac{1-3/2}{20}\Rightarrow v=-10$

i.e., $10\,\text{cm}$ below the curved surface or $10\,\text{cm}$ above the actual position of flower.

Ans: (a)

161. A ray of light traveling inside a rectangular glass block of refractive index $\sqrt{2}$ is incident on the glass-air surface at an angle of incident of $45°$. The refractive index of air is one. Under these conditions the ray will

Options:

(a) Emerge into the air without any deviation

(b) Be reflected back into the glass

(c) Be absorbed

(d) Emerge into the air with an angle of refraction equal to $90°$

Sol: $\sin C=\frac{1}{\mu}=\frac{1}{\sqrt{2}}\quad \therefore C=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45°$

Now, $\frac{\sin C}{\sin r}=\frac{1}{\mu}$ or $\frac{\sin 45°}{\sin r}=\frac{1}{\sqrt{2}}$

$\sin r=1$ or $r=90°$

Ans: (d)

162. If we observe the single slit Fraunhofer diffraction with wavelength λ and slit width b, the width of the central maxima is 2θ. On decreasing the slit width for the same λ

Options:

(a) θ increases

(b) θ remains unchanged

(c) θ decreases

(d) θ increases or decreases depending on the intensity of light

Sol: We know that for maxima, $b\sin\theta=(2n+1)\frac{\lambda}{2}$ or $\sin\theta=\frac{2n+1}{2}\left(\frac{\lambda}{b}\right)$

So on decreasing the slit width, $'b'$, keeping λ same, $\sin\theta$ and hence θ increases.

Ans: (a)

163. If in a photoelectric cell, the wavelength of incident light is changed from $4000\,\text{Å}$ to $3000\,\text{Å}$ then change in stopping potential will be

Options:

(a) $0.66\,\text{V}$

(b) $1.03\,\text{V}$

(c) $0.33\,V$ (d) $0.49\,V$

Sol: $eV_1 = hv_1 - hv_0$

$eV_2 = hv_2 - hv_0$

$$V_2 - V_1 = \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = 12400\left(\frac{1}{3000} - \frac{1}{4000}\right) = 1.03\,eV$$

Ans: (b)

164. If the momentum of electron is changed by P, then the de Broglie wavelength associated with it changes by 0.5%. The initial momentum of electron will be

Options:

(a) $200\,P$

(b) $400\,P$

(c) $\frac{P}{200}$

(d) $100\,P$

Sol: The de Broglie's wavelength associated with the moving electron, $\lambda = \frac{h}{P}$

Now, according to problem, $\frac{d\lambda}{\lambda} = -\frac{dP}{P}$; $\frac{0.5}{100} = \frac{P}{P'}$

$P' = 200\,P$

Ans: (a)

165. Out of the following which one is not a possible energy for a photon to be emitted by hydrogen atom according to Bohr's atomic model?

Options:

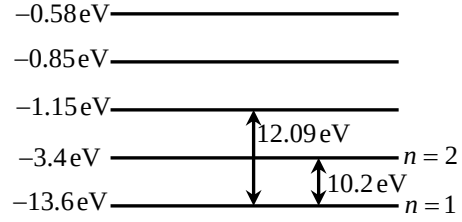
(a) $1.9\,eV$

(b) $11.1\,eV$

(c) $13.6\,eV$

(d) $0.65\,eV$

Sol: Obviously, difference of $11.1\,eV$ is not possible.



Ans: (b)

166. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to

Options:

(a) $\frac{1}{Ze}$

(b) v^2

(c) $\frac{1}{m}$

(d) $\frac{1}{v^4}$

Sol: Kinetic energy of α nucleus is equal to electrostatic potential energy of the system of the α particle and the heavy nucleus. That is,

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha Ze}{r_0}$$

where r_0 is the distance of closest approach

$$r_0 = \frac{2}{4\pi\varepsilon_0}\frac{q_\alpha Ze}{mv^2} \Rightarrow r_0 \propto Ze \propto q_\alpha \propto \frac{1}{m} \propto \frac{1}{v^2}$$

Ans: (c)

167. The ratio of maximum to minimum wavelength in Balmer series is

Options:

(a) 3:4

(b) 1:4

(c) 5:36

(d) 5:9

Sol: $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For Balmer series, $n = 2$

$$\frac{1}{\lambda_{max}} = RZ^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) \text{ and } \frac{1}{\lambda_{min}} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\text{or } \frac{1/\lambda_{min}}{1/\lambda_{max}} = \frac{\left(1/2^2 - 1/3^2\right)}{\left(1/2^2\right)}$$

$$= \frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}} = 1 - \frac{4}{9} = \frac{5}{9} \qquad \Rightarrow \frac{\lambda_{max}}{\lambda_{min}} = \frac{5}{9}$$

Ans: (d)

168. The binding energy per nucleon for $_1^2H$ and $_2^4He$ respectively are 1.1MeV and 7.1MeV. The energy released in MeV when two $_1^2H$ nuclei fuse to form $_2^4He$ is

Options:

(a) 4.4

(b) 8.2

(c) 24

(d) 28.4

Sol: The chemical reaction of process is $2_1^2H \rightarrow {}_2^4He$

Energy released $= 4\times(7.1) - 4(1.1) = 24$ MeV

Ans: (c)

169. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio $2:1$. Ratio of their nuclear sizes will be

Options:

(a) $2^{1/3}:1$

(b) $1:3^{1/2}$

(c) $3^{1/2}:1$

(d) $1:2^{1/3}$

Sol: $p_1 = p_2$

$m_1v_1 = m_2v_2$

$2m_1 = m_2$

$2\rho\cdot\frac{4}{3}\pi R_1^3 = \rho\cdot\frac{4}{3}\pi R_2^2;\ \frac{R_1^3}{R_2^3} = 1:2$

$R_1 : R_2 = 1:2^{1/3}$

Ans: (d)

170. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is

Options:

(a) $0.4\ln 2$

(b) $0.2\ln 2$

(c) $0.1\ln 2$

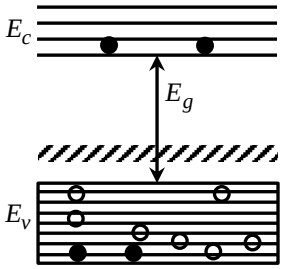
(d) $0.8\ln 2$

Sol: $\lambda = \frac{1}{t}\log_e\frac{A_0}{A} = \frac{1}{5}\log_e\frac{5000}{1250}$

$= \frac{2}{5}\log_e 2 = 0.4\log_e 2$

Ans: (a)

171. In the energy band diagram of a material shown below, the open circles and filled circles denote holes and electrons respectively. The material is



Options:

(a) An insulator

(b) A metal

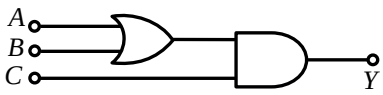
(c) An $n-$type semiconductor

(d) A $p-$type semiconductor

Sol: For a $p-$type semiconductor, the acceptor energy level, as shown in the diagram, is slightly above the top E_v of the volume band. With very small supply of energy an electron from the valence band can jump to the level E_A and ionise acceptor negatively.

Ans: (d)

172. To get output 1 for the following circuit, the correct choice for the input is



Options:

(a) $A=0,\ B=1, C=0$

(b) $A=1, B=0, C=0$

(c) $A=1, B=1, C=0$

(d) $A=1, B=0, C=1$

Sol: The Boolean expression for the given combination is output $Y=(A+B)\cdot C$

Truth table

A	B	C	$Y=(A+B)\cdot C$
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Hence, $A=1, B=0, C=1$

Ans: (d)

173. When a $p-n$ junction diode is reverse biased the flow of current across the junction is mainly due to

Options:

(a) Diffusion of charges

(b) Drift of charges

(c) Depends on the nature of material

(d) Both drift and diffusion of charges

Sol: When $p-n$ junction is reverse biased, the flow of current is due to drifting of minority charge carriers across the junction.

Ans: (b)

174. A $10\,\text{eV}$ electron is circulating in a plane at right angles to a uniform field of magnetic induction $10^{-4}\,\text{Wb}\,\text{m}^{-2}$ $(=1.0$ gauss). The orbital radius of the electron is

Options:

(a) $12\,\text{cm}$

(b) $16\,\text{cm}$

(c) $11\,\text{cm}$

(d) $18\,\text{cm}$

Sol: K.E. of electron $=10\,\text{eV}$

$$\Rightarrow \frac{1}{2}mv^2 = 10\,\text{eV} \Rightarrow \frac{1}{2}\left(9.1\times10^{-31}\right)v^2 = 10\times1.6\times10^{-19} \Rightarrow v^2 = \frac{2\times10\times1.6\times10^{-19}}{9.1\times10^{-31}}$$

$$\Rightarrow v^2 = 3.52\times10^{12} \Rightarrow v = 1.88\times10^{6}\,\text{m}$$

Also we know that for circular motion, $\frac{mv^2}{r} = Bev \Rightarrow r = \frac{mv}{Be} = 11\text{cm}$

Ans: (c)

175. If the units of mass, length and time are doubled, unit of angular momentum will be

Options:

(a) Doubled

(b) Tripled

(c) Quadrupled

(d) 8 times the original value

Sol: $P_1 = \left[ML^2T^{-1}\right]$

$$P_2 = \left[(2M)(2L)^2(2T)^{-1}\right]$$

$$P_2 = 4\left[ML^2T^{-1}\right] = 4P_1$$

Ans: (c)

176. Two cars A and B are moving with same speed of $45\,\text{km h}^{-1}$ along same direction. If a third car C coming from the opposite direction with a speed of $36\,\text{km h}^{-1}$ meets two cars in an interval of 5 minutes, the distance of separation of two cars A and B should be (in km)

Options:

(a) 6.75

(b) 7.25

(c) 5.55

(d) 8.35

Sol: Distance between the cars A and B remains constant. Let the distance by $'x'$

Velocity of C w.r.t. A and B, $V = 45 + 36 = 81\,\text{km h}^{-1}$

$$\text{Distance} = 81 \times \frac{5}{60} = 6.75\,\text{km}$$

Ans: (a)

177. A cricketer hits a ball with a velocity $25\,\text{m s}^{-1}$ at $60°$ above the horizontal. How far above the ground it passes over a fielder $50\,\text{m}$ from the bat (assume the ball is truck very close to the ground)

Options:

(a) 8.2 m

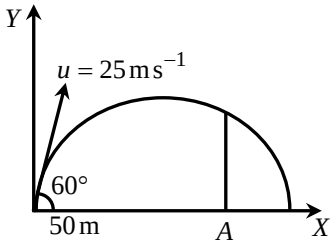
(b) 9.0 m

(c) 11.6 m

(d) 12.7 m

Sol: Horizontal component of velocity, $v_x = 25\cos 60° = 12.5\,\text{m s}^{-1}$

Vertical component of velocity, $v_y = 25\sin 60° = 12.5\sqrt{3}\,\text{m s}^{-1}$



Time to cover $50\,\text{m}$ distance, $t = \frac{50}{12.5} = 4\,\text{s}$

The vertical height y is given by

$$y = u_v t - \frac{1}{2} g t^2 = 12.5\sqrt{3} \times 4 - \frac{1}{2} \times 9.8 \times 16 = 8.2\,\text{m}$$

Ans: (a)

178. A person with his hands in his pockets is skating on ice at the velocity of $10\,\text{m}\,\text{s}^{-1}$ and describes a circle of radius $50\,\text{m}$. What is his inclination with vertical? $\left(g = 10\,\text{ms}^{-2}\right)$

Options:

(a) $\tan^{-1}\left(\frac{1}{10}\right)$

(b) $\tan^{-1}\left(\frac{3}{5}\right)$

(c) $\tan^{-1}(1)$

(d) $\tan^{-1}\left(\frac{1}{5}\right)$

Sol: The inclination of person from vertical is given by

$$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{50\times 10} = \frac{1}{5} \qquad \therefore \theta = \tan^{-1}(1/5)$$

Ans: (d)

179. A block of mass m rests on a rough horizontal surface (coefficient of friction is μ). When a bullet of mass $m/2$ strikes horizontally, and get embedded in it, the block moves a distance d before coming to rest. The initial velocity of the bullet is $k\sqrt{2\mu gd}$, then the value of k is

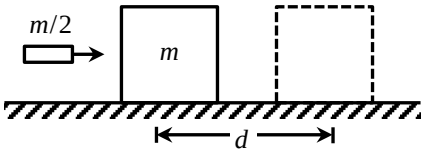
Options:

(a) 2

(b) 3

(c) 4

(d) 5



Sol: Let initial velocity of the bullet be v.

By linear momentum conservation, $\frac{m}{2}v = \left(\frac{m}{2} + m\right)v_1$ (v_1 = combined velocity)

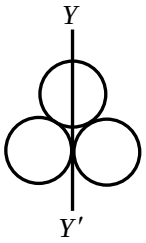
$$v_1 = \frac{v}{3} \qquad \ldots \text{(i)}$$

Retardation $= \mu g$

$$0 = \left(\frac{v}{2}\right)^2 - 2\mu gd \Rightarrow v = 3\sqrt{2\mu gd}$$

Ans: (b)

180. Three rings each of mass M and radius R are arranged as shown in the figure. The moment of inertia of the system about YY' will be



Options:

(a) $3MR^2$

(b) $\frac{3}{2}MR^2$

(c) $5MR^2$

(d) $\frac{7}{2}MR^2$

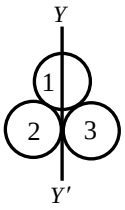
Sol: Moment of inertia of system about YY'

$I = I_1 + I_2 + I_3$

$= \frac{1}{2}MR^2 + \frac{3}{2}MR^2 + \frac{3}{2}MR^2$

$= \frac{7}{2}MR^2$

Ans: (d)



Key Answers:

1. c	2. b	3. a	4. c	5. a	6. d	7. c	8. c	9. c	10. a
11. c	12. b	13. b	14. d	15. a	16. c	17. c	18. b	19. b	20. c
21. d	22. b	23. d	24. a	25. b	26. c	27. c	28. d	29. b	30. d
31. d	32. d	33. c	34. a	35. a	36. b	37. a	38. b	39. b	40. b
41. d	42. a	43. b	44. b	45. a	46. d	47. a	48. d	49. d	50. b
51. d	52. b	53. c	54. c	55. b	56. b	57. b	58. b	59. c	60. a
61. d	62. d	63. d	64. d	65. d	66. a	67. a	68. b	69. d	70. c
71. d	72. d	73. b	74. c	75. c	76. a	77. c	78. b	79. d	80. b
81. a	82. a	83. c	84. a	85. b	86. a	87. c	88. d	89. c	90. b
91. a	92. b	93. a	94. a	95. d	96. b	97. a	98. b	99. a	100.a
101.c	102.a	103.c	104.c	105.a	106.d	107.a	108.a	109.c	110.a
111.a	112.d	113.b	114.b	115.d	116.a	117.c	118.c	119.d	120.b
121.a	122.d	123.a	124.d	125.b	126.c	127.a	128.d	129.b	130.a
131.c	132.a	133.b	134.a	135.a	136.d	137.d	138.c	139.b	140.d
141.d	142.c	143.d	144.a	145.a	146.d	147.a	148.a	149.a	150.d
151.c	152.d	153.b	154.d	155.b	156.a	157.d	158.c	159.a	160.a
161.d	162.a	163.b	164.a	165.b	166.c	167.d	168.c	169.d	170.a
171.d	172.d	173.b	174.c	175.c	176.a	177.a	178.d	179.b	180.d