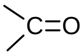
Subject	Topic		Date
C + M + P	Complete Syllabus	CET – 12 – CT	

C+M+P Key Answers:

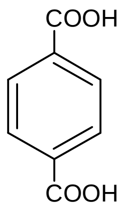
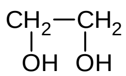
1. c	2. b	3. b	4. d	5. d	6. d	7. d	8. a	9. a	10. a
11. b	12. b	13. c	14. a	15. c	16. c	17. a	18. b	19. a	20. a
21. b	22. c	23. b	24. b	25. b	26. a	27. b	28. d	29. d	30. d
31. a	32. a	33. d	34. d	35. d	36. c	37. b	38. c	39. d	40. b
41. c	42. b	43. d	44. a	45. a	46. b	47. a	48. a	49. c	50. a
51. a	52. d	53. a	54. a	55. c	56. a	57. d	58. a	59. a	60. a
61. c	62. d	63. b	64. b	65. a	66. b	67. d	68. b	69. b	70. b
71. c	72. d	73. d	74. d	75. b	76. b	77. b	78. d	79. b	80. c
81. c	82. a	83. a	84. a	85. a	86. d	87. c	88. a	89. b	90. a
91. d	92. a	93. a	94. a	95. d	96. a	97. d	98. c	99. a	100.d
101.b	102.b	103.b	104.b	105.d	106.d	107.b	108.c	109.c	110.c
111.b	112.a	113.b	114.a	115.c	116.c	117.a	118.c	119.d	120.b
121.a	122.d	123.a	124.d	125.a	126.b	127.d	128.d	129.a	130.c
131.b	132.d	133.c	134.b	135.b	136.d	137.b	138.a	139.b	140.a
141.d	142.c	143.b	144.b	145.c	146.a	147.b	148.a	149.a	150.a
151.a	152.a	153.d	154.d	155.a	156.c	157.c	158.b	159.c	160.b
161.a	162.b	163.b	164.b	165.b	166.b	167.b	168.d	169.c	170.c
171.d	172.c	173.c	174.c	175.b	176.a	177.d	178.b	179.a	180.c

Chemistry Solutions:

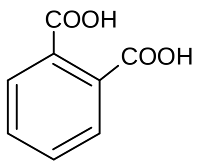
1. $\alpha-$Helix structure of protein is stabilised by hydrogen bonds between $-NH-$ group of one amino acid residue and the  group of another amino acid residue.
2. $KMnO_4$ will oxidise initially formed aldehydes to carboxylic acids.
3. Rate of disappearance of $Q = \frac{1}{2} \times$ Rate of appearance of R
4. Tyndall effect is observed only when these two conditions are satisfied.
5. Buna $-N$:

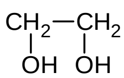
   Polymer of $CH_2 = CH(CN)$ (acrylonitrile) and $H_2C = CH - CH = CH_2$ (butadiene).

Nylon $-6,6$: Polymer of $NH_2 - (CH_2)_6 - NH_2$ (hexa-methylenediamine) and $COOH - (CH_2)_4 - COOH$ (adipic acid).

Dacron (terylene): Polymer of  (terephthalic acid) and  (ethylene glycol)

Glyptal plastic: Polymer of (phthalic acid)



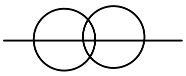
And  (ethylene glycol)

6. Heat evolved from $0.2\,g = 200\times 5 = 1000\,\text{cal} = 1\,\text{kcal}$

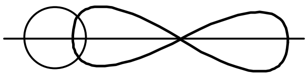
Heat evolved from 1 mole, i.e., $74g$

$= \frac{1}{0.2}\times 74 = 370$ kcal.

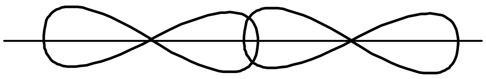
7. Sol:



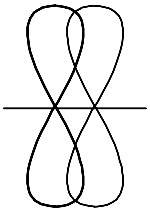
$s-s$ overlapping (σ)



$s-p_z$ overlapping (σ)

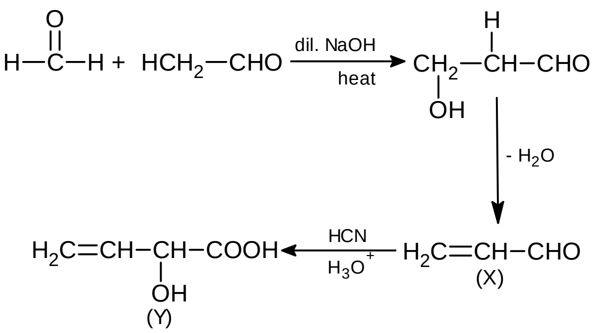


p_z-p_z overlapping (σ)



p_x-p_x overlapping (π)

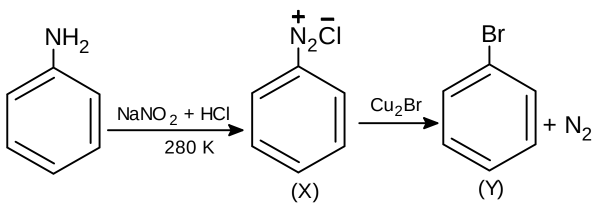
8. Sol:



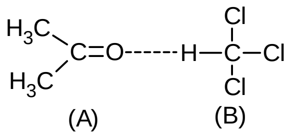
9. $Cu^{2+}_{(aq)} + 4NH_{3(aq)} \rightarrow \left[Cu(NH_3)_4\right]^{2+}_{(aq)}$

deep blue

10. Sol:



11. When acetone and chloroform are mixed together, a hydrogen bond is formed between them which increases intermolecular interactions. Hence $A-B$ interactions are stronger than $A-A$ and $A-B$ interactions.



12. The IE_1, IE_2, IE_3, IE_4 and IE_5 of an element are $7\cdot 1, 14\cdot 3, 34\cdot 5, 46\cdot 8$ and $162\cdot 2\ eV$ respectively. Therefore, the valency of the element is 4 and it is most likely to be silicon.

13. $CH_3NH_2 + CHCl_3 + 3KOH \xrightarrow[\Delta]{} CH_3NC + 3KCl + 3H_2O$

$CH_3CONH_2 + Br_2 + 4KOH \rightarrow$
$CH_3NH_2 + K_2CO_3 + 2KBr + 2H_2O$

$CH_3NH_2 + NaNO_2 + HCl \rightarrow CH_3OH + N_2 + NaCl + H_2O$

$CH_3NC + 4N \xrightarrow{Pt} CH_3NHCH_3$

14. $Z = 64$; outer electronic configuration - $4f^7 5d^1 6s^2$

15. After oxidation, it dissolves in rain water to produce acid rain which reacts with limestone of buildings made up of marble resulting in extensive damage.

$2SO_{2(g)} + O_{2(g)} + 2H_2O_{(l)} \rightarrow 2H_2SO_{4(aq)}$

$CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + CO_2 + H_2O$

16. Coordination number of Ca^{2+} in CaF_2 is 8.

17. $\overset{x\,-2}{(SO_3)^2}$, $x-6=-2,\ x=+4$

$\overset{x\,-2}{(S_2O_4)^{2-}}$, $2x-8=-2,\ x=+3$

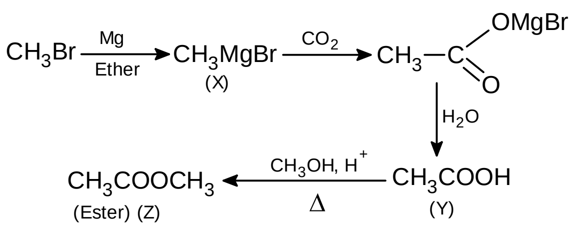
$\overset{x\,-2}{(S_2O_6)^{2-}}$, $2x-12=-2,\ X=+5$

$Pt + Cl_2 \rightarrow Pt^{2+} + 2Cl^-;\ \ E^\circ_{\text{cell}} = 0.15\,\text{V}$

18. $\underline{Pt^{2+} + 2e^- \rightarrow Pt;\ E^\circ = 1.20V}$

$\underline{Cl_2 + 2e^- \rightarrow 2Cl^-;\ E^\circ = 1.35\,\text{V}}$

19. Sol:

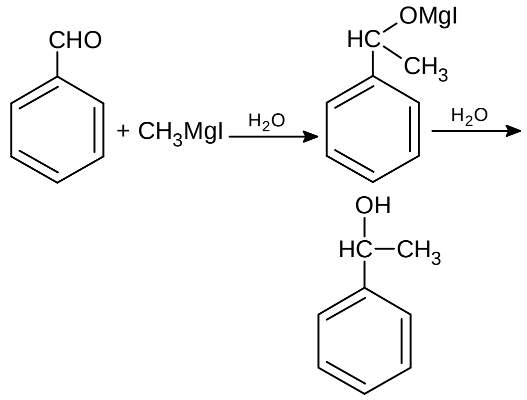


20. $10 = N\times 5.6$ or $N = \frac{10}{5.6}$,

Thus strength $(g/l) = \frac{10}{5.6}\times 17 = 30.35$

21. NH_4^+ ion has no lone pair of electrons thus, cannot act as a ligand.

22. Sol:



1 - Phenylethanol

23. Mass of 1 molecule of haemoglobin

$$=\frac{67200}{6.022\times10^{23}}$$

Mass of iron $=\frac{67200}{6.022\times10^{23}}\times\frac{0.33}{100}$

$=3.682\times10^{-22}\,g$

No. of iron atoms $=\frac{3.682\times10^{-22}\times6.02\times10^{23}}{56}$

$=3.96\approx4$

24. During adsorption, there is always a decrease in residual forces of the surface, i.e., there is decrease in surface energy which appears as heat, therefore, adsorption is an exothermic process.

25. Ionic character of the hydrides increases from LiH to CsH, so thermal stability decreases in the order $LiH > NaH > KH > RbH > CsH$

26. 30#% decomposition means $X=30\%$ of R_0 or

$R=R_0-0.3R_0=0.7R_0$

For first order, $k=\frac{2.303}{t}\log\frac{R_0}{R}$

$=\frac{2.303}{40}\log\frac{10}{7}\text{min}^{-1}=8.918\times10^{-3}\text{min.}^{-1}$

$t_{1/2}=\frac{0.693}{k}=\frac{0.693}{8.918\times10^{-3}\text{min.}^{-1}}=77.7\,\text{min}$

27. $\underset{(X)}{S}+O_2\rightarrow\underset{(Y)}{SO_2}$

$SO_2+Ca(OH)_2\rightarrow\underset{\text{White ppt.}}{CaSO_3}\downarrow+H_2O$

$\underset{\text{Purple}}{2KMnO_4}+\underset{(Y)}{5SO_2}+2H_2O\rightarrow K_2SO_4+\underset{\text{Colourless}}{2MnSO_4}+2H_2SO_4$

$\underset{(Y)}{2SO_2}+O_2\rightarrow\underset{(Z)}{2SO_3}$

$\underset{(Z)}{SO_3}+H_2O\rightarrow H_2SO_4$ (responsible for acid rain)

28. Heat evolved from decomposition of

$1\,g\;NH_4NO_3=1.23\times6.12$ kJ

Heat evolved from 1mol of

NH_4NO_3 (Molar mass $=80\,g$)

$=1.23\times6.12\times80\,\text{kJ}=602$ kJ.

29. For fcc,

$2(r_++r_-)=a$

$2(110+r_-)=508$

$r_-=\frac{580}{1}-110=144\,\text{pm}$

30. $M_B=\frac{K_b\times W_B}{\Delta T_b\times W_A}=\frac{0.52\times12.5}{0.80\times0.185}=43.92\,g\,\text{mol}^{-1}$

$(185\,g=0.185\,kg)$

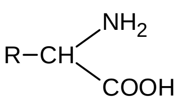
31. Xe atom has large size and lower ionisation potential in comparison to He, Ne, Ar and Kr. Hence it can form compounds with fluorine.

32.

	$2HI$	$\rightleftharpoons$ H_2	+ I_2
Initial	1mole		
After disso.	$1-1/3=2/3$	$1/6$	$1/6$

$K=\frac{(1/6)(1/6)}{(2/3)^2}=\frac{1}{36}\times\frac{9}{4}=\frac{1}{16}$

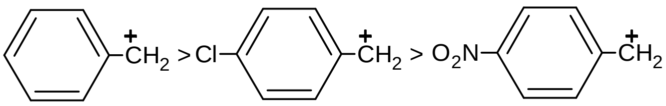
33. In $\alpha-$amino acids, $-NH_2$ and $-COOH$ groups are attached to the same carbon atom



34. It remains constant for a cell.

35. Due to inert pair effect, Pb^{2+} is most stable.

36. The order of stability of benzyl carbocation is:



The electron withdrawing effect of $-NO_2$ group is greater than that of $-Cl$ group.

Thus, the correct increasing order of reactivity with HBr/HCl is: (ii) < (iii) < (i)

37. MgO acts as a basic flux to remove impurities of Si, P and S through slag formation.

$$\underset{\text{Basic flux}}{MgO} + \underset{\text{Acidic impurities}}{SiO_2} \rightarrow \underset{\text{Slag}}{MgSiO_3}$$

$$3MgO + P_2O_5 \rightarrow Mg_3(PO_4)_2$$

$$MgO + SO_2 \rightarrow MgSO_3$$

38. At infinite distance $E_e = 0$. As it comes closer to nucleus, attraction takes place. Hence, energy is released.

39. Only those collisions in which molecules collide with sufficient energy, called threshold energy and proper orientation are effective collisions. Rest of the molecules collide and bounce back.

40. Higher the value of stability constant K, greater is the stability.

41. $2-$pentanone and $3-$pentanone are position isomers.

42. $Ag_2SO_4 \rightarrow Ag^+(4d^{10})-$colourless

$CuF_2 \rightarrow Cu^{2+}(3d^9)-$coloured

$ZnF_2 \rightarrow Zn^{2+}(3d^{10})-$ colourless

$CuCl_2 \rightarrow Cu^+(3d^{10})-$colourless

43. B atom in BF_3 as sp^2 hybridization. Hence planar.

N atom in NF_3 as sp^3 hybridization. Hence pyramidal.

44. Aldehydes are more reactive than ketones.

45. In physisorption, given surface of an adsorbent does not show any preference for a particular gas as the van der Waals forces are universal.

46. Bismuth does not show allotropy.

47. Ca^{2+} and Mg^{2+} ions form insoluble calcium and magnesium soaps respectively when sodium or potassium soaps are dissolved in hard water.

$$\underset{\text{Soap}}{2C_{17}H_{35}COONa} + CaCl_2 \rightarrow 2NaCl + \underset{\substack{\text{Insoluble calcium stearate}\\\text{(Soap)}}}{(C_{17}H_{35}COO)_2Ca}$$

These insoluble soaps separate as scum in water and are useless as cleansing agent.

48. Since compound $'A'$ comes out before $'B'$, $'B'$ is more readily adsorbed to the column and $'B'$ comes out before $'C'$, hence $'C'$ is more readily adsorbed than $'B'$. Hence, the order of adsorption is $C > B > A$

49. $pH = pK_a + \log\frac{[\text{Salt}]}{[\text{Acid}]}$ or

$$5.5 = 4.5 + \log\frac{[\text{Salt}]}{0.1}$$

$$= 4.5 + \log[\text{Salt}] + \log 10$$

$$= 5.5 + \log[\text{salt}] \text{ or}$$

$$\log[\text{Salt}] = 0 \text{ or } [\text{Salt}] = 1\text{M}$$

50. $C_6H_5Br + 2Na + BrCH_2CH_2CH_2CH_3 \xrightarrow{\Delta}$

$\underset{n-\text{Butylbenzene}}{C_6H_5CH_2CH_2CH_2CH_3} + 2NaBr$

51. $\Lambda^{\infty}_{mCH_3COOH} = 390.71 \text{ohm}^{-1}\text{cm}^2\text{equiv}^{-1}$

$\Lambda^{\infty}_{mCH_3COOH} = 14.3 \text{ohm}^{-1}\text{cm}^2\text{equiv}^{-1}$

Degree of dissociation (α)

$$= \frac{\Lambda^c_m}{\Lambda^{\infty}_m} = \frac{14.3}{390.71} = 0.0366 \text{ i.e., } 3.66\%$$

52. $Os-$shows $+8$ oxidation state

$Mn-$ shows $+7$ oxidation state in its oxides.

$Pm-$ Radioactive lanthanoid.

$Ce-$shows $+4$ oxidation state

53. The amount of BOD in water is the measure of the amount of organic matter in water in terms of how much oxygen is required to break it down biologically.

54. I. 1 atom of $O = \frac{16}{6.02\times10^{23}}g$

II. 1 atom of $N = \frac{14}{6.02\times10^{23}}g$

III. 10^{-10} mol of $O_2 = 32\times10^{-10}g$

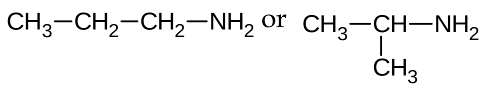
IV. 10^{-10} mol of $Cu = 63\times10^{-10}g$

55. $i = 1-\left(1-\frac{1}{n}\right) = 1-\left(1-\frac{1}{5}\right) = 0.2$

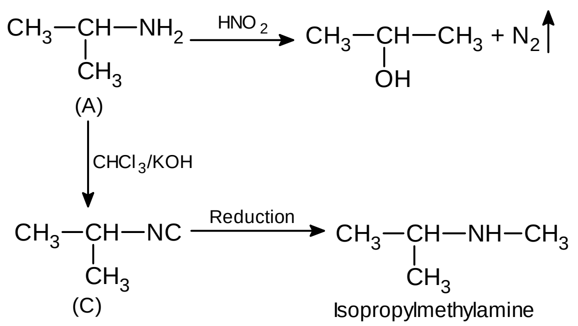
56. As the size of halogen atom increases down the group from F to I, the strength of $H-X$ bond decreases and hence bond dissociation enthalpy decreases. Thus, HF has highest bond dissociation enthalpy.

57. $\underset{\text{Propyl chloride}}{CH_3CH_2CH_2Cl} + KCN \rightarrow \underset{\text{Butane nitrile}}{CH_3CH_2CH_2CN} + KCl$

58. As (A) gives alcohol on treatment with nitrous acid thus it should be primary amine. C_3H_9N has two possible structures with $-NH_2$ group.



As it gives isopropylmethylamine thus it should be isopropyl amine not $n-$propylamine.



59. A rate of diffusion is inversely proportional to the square root of the density at constant pressure. H_2 with lowest molar mass will diffuse out maximum and SO_2 with maximum molar mass will diffuce out minimum. Hence, after 3 hours, p_{H_2} will be maximum and p_{SO_2} will be minimum.

60. $C_4H_{10}O$ can have two structures:

$CH_3CH_2CH_2CH_2OH$ and $C_2H_5OC_2H_5$.

Since it does not react with Na metal, it cannot be an alcohol.

$C_2H_5OC_2H_5 + \underset{(\text{excess})}{HI} \rightarrow 2C_2H_5I + H_2O$

<u>Mathematics Solutions</u>:

61. $\tan^{-1}(1+x)+\tan^{-1}(1-x) = \frac{\pi}{2}$

$\Rightarrow (1+x)(1-x) = 1$

$\Rightarrow 1-x^2 = 1 \Rightarrow x = 0$

62. $\sin(-600°) = -\sin 600°$

$= -\sin(360°+240°)$

$= -\sin 240° \qquad = +\sin 60° = \frac{\sqrt{3}}{2}$

$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}\sqrt{3} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\therefore$ G.E. $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{5\pi}{6} = \frac{\pi}{3} + \frac{5\pi}{6} = \frac{5\pi}{6}$

63. $4\sin^{-1}x + \cos^{-1}x = \pi$

$\Rightarrow 3\sin^{-1}x + \frac{\pi}{2} = \pi \left(\because \sin^{-1}x + \cos^{-1} = \frac{\pi}{2}\right)$

$\Rightarrow 3\sin^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}x = \frac{\pi}{6} \Rightarrow x = \frac{1}{2}$

64. We have ,

$\sin^{-1}\left(\frac{2x}{1-x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$\therefore$ G.E $\Rightarrow 6\tan^{-1}x - 8\tan^{-1}x + 4\tan^{-1}x = \frac{\pi}{3}$

$\Rightarrow 2\tan^{-1}x = \frac{\pi}{3}$

$\Rightarrow \tan^{-1}x = \frac{\pi}{6} \Rightarrow x = \frac{1}{\sqrt{3}}$

65. $\sin^{-1}\left(\frac{x}{5}\right)+\sin^{-1}\frac{4}{5}=\frac{\pi}{2}$

$\Rightarrow \frac{x^2}{25}+\frac{16}{25}=1$

$\Rightarrow x^2+16=25 \Rightarrow x=3$

66. If $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^2-(a+d)A+|A|\cdot I=0$

Now, $A=\begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix}$, $A^2-8A+kI=0$

Observe $1+7=8$. Thus $k=|A|=7$

Or $A^2=\begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ -8 & 49 \end{pmatrix}$

$A^2-8A=\begin{pmatrix} 1 & 0 \\ -8 & 49 \end{pmatrix}-\begin{pmatrix} 8 & 0 \\ -8 & 56 \end{pmatrix}=\begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}$

$A^2-8A+kI=0 \Rightarrow -7I+kI=0 \Rightarrow k=7$

67. Clearly A and B both must be of the order either $3\times n$ or $3\times m$.

We do not have $3\times m$ among the options.

$\therefore$ $(5A-2B)$ is of order $3\times n$

68. By data, $2x+y=7$, $\quad 4x=7y-13$

$5x-7=y \quad 4x=x+6$

Now, $4x=x+6 \Rightarrow x=2$

$x=2$ and $2x+y=7 \Rightarrow y=3$

Clearly these values satisfy remaining equations.

$\therefore$ $x=2,\ y=3$

69. The number of bijection from the set A onto the set B is $m!$, where

$n(A)=n(B)=m$

$\therefore$ the number of bijections $=(108)!$

70. Clearly, $a\neq a$. Thus, relation is not reflexive. If $a\neq b$ then clearly $b\neq a$.

Thus relation is symmetric.

Now $4\neq 5$ and $5\neq 4 \Rightarrow 4=4$

Thus, relation is not transitive.

71. Let x be rational, then

$f(x)=x \Rightarrow f(f(x))=f(x)=x$

Let x be irrational, then $f(x)=1-x$

$\Rightarrow f(f(x))=f(1-x)=1-(1-x)=x$

(if x is irrational then $1-x$ is also irrational)

72. We have, $f:N\to R$, defined by $f(x)=\frac{2x-1}{2}$

Clearly, $f\left(\frac{3}{2}\right)$ is not defined for $\frac{3}{2}\notin N$.

73. We know that, $\Delta=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

By data $\Delta=0$ and $abc\neq 0$

$\therefore \Delta=0 \Rightarrow 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0 \Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=-1$

74. α,β,γ are the roots of $x^3+px+q=0$

$\Rightarrow \alpha+\beta+\gamma=0$

$\Delta=\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix}=0$

(Considering $C_1\to C_1+C_2+C_3$)

75. Let, $A=(-3,0)$, $B=(3,0)$, $(0,k)$

$\Delta ABC=9$

$\Rightarrow \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}=9$

$-3(-k)+1(3k)=18 \Rightarrow 6k=18 \Rightarrow k=3$

76. We have , $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$=\begin{vmatrix} 3x+3y & 3x+3y & 3x+3y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} R_1\to R_1+R_2+R_3$

$=(3x+3y)\begin{vmatrix} 1 & 1 & 1 \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$=(3x+3y)\begin{vmatrix} 1 & 0 & 0 \\ x+2y & -2y & -y \\ x+y & y & -y \end{vmatrix} \begin{matrix} C_2\to C_2-C_1 \\ C_3\to C_3-C_1 \end{matrix}$

$=3(x+y)(3y^2)=9y^2(x+y)$

77. $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \tan^{-1}\left(\frac{2\sin x/2\cos\ x/2}{2\cos^2 x/2}\right)$

$= \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{1}{2}x$

$\therefore\ \frac{dy}{dx} = \frac{1}{2}$

78. Let $u = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right),\quad v = \cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$

Clearly, $u = 2\tan^{-1}x$ and $v = 3\tan^{-1}x$

$\Rightarrow u = \frac{2}{3}v \Rightarrow \frac{du}{dv} = \frac{2}{3}$

79. $y = ax^{n+1} + bx^{-n}$

$\frac{dy}{dx} = a(n+1)x^n - bnx^{n-1}$

$\frac{d^2y}{dx^2} = an(n+1)x^{n-1} + bn(n-1)x^{n-2}$

$x^2\frac{d^2y}{dx^2} = an(n+1)x^{n-1} + bn(n+1)x^n$

$= n(n+1)\left[ax^{n-1} + bx^n\right] = n(n+1)y$

80. $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4} - \ldots.$

$\frac{dy}{dx} = -1 + \frac{2x}{2!} - \frac{3x^2}{3!} + \frac{4x^3}{4!} - \ldots.$

$\frac{d^2y}{dx^2} = 1 - x + \frac{x^2}{2!} - \ldots = y$

81. We have, $f(x) = \frac{x}{1+|x|}$

$\Rightarrow f(x) = \begin{cases} \frac{x}{1-x} & x < 0 \\ \frac{x}{1+x} & x \geq 0 \end{cases}$

Clearly $f(x)$ is differentiable for $x < 0$ and $x > 0$.

Now, $\lim_{x\to 0}\frac{f(x)-f(0)}{x} = \lim_{x\to 0}\frac{1}{1-x} = 1$

$\lim_{x\to 0}\frac{f(x)-f(0)}{x} = \lim_{x\to 0}\frac{1}{1+x} = 1$

$\therefore\ f'(x)$ exists at $x = -$.

Hence $f'(x)$ exists everywhere

82. $\lim_{x\to 3}\frac{2x^3-ax+3}{x-3}$ exists, if $(x-3)$ is factor of $2x^3 - ax + 3$. That is, if $2x^3 - ax + 3 = 0$ for $x = 3$

$\Rightarrow 54 - 3a + 3 = 0 \Rightarrow 3a = 57 \Rightarrow a = 19$

$\therefore\ f(x) = \frac{2x^3 - 19x + 3}{x-3}$

No2, $\lim_{x\to 3} f(x) = f(3)$

$\Rightarrow \lim_{x\to 3}\frac{2x^3-19x+3}{x-3} = f(3)$

$\Rightarrow \lim_{x\to 3}\frac{6x^2-19}{1} = f(3)$

$\Rightarrow f(3) = 54 - 19 = 35$

$\lim_{x\to 3}\frac{2x^3-ax+3}{x-3} = \lim_{x\to 3}\frac{6x^2-a}{1} = 54 - a$

$f(x)$ continuous at $x = 3 \Rightarrow 54 - a = f(3)$

Equality holds only when $q = 19,\ f(3) = 35$

83. $f(x)$ is continuous at $x = \frac{\pi}{2}$

$\Rightarrow f\left(\frac{\pi}{2}\right) = \lim_{x\to\frac{\pi}{2}} f(x)$

$\Rightarrow k = \lim_{x\to\frac{\pi}{2}}\frac{(1-\sin x)}{(\pi-2x)^2}$

$\Rightarrow k = \lim_{x\to\frac{\pi}{2}}\frac{-\cos x}{-4(\pi-2x)}$ (by L'H rule)

$\Rightarrow k = \lim_{x\to\frac{\pi}{2}}\frac{\sin x}{8}$ (by L'H rule)

$\Rightarrow k = \frac{1}{8}$

84. $\int\frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^{\tan^{-1}x}\cdot\left(\frac{1}{1+x^2}\right)dx = e^{\tan^{-1}x} + c$

$\int e^{f(x)}\cdot f'(x)dx = e^{f(x)} + c$

85. Clearly, $\frac{d}{dx}\left[\log(\sin x)\right] = \cot x$

$\therefore\ I = \int\frac{f'(x)}{f(x)}dx = \log f(x) = \log\left[\log(\sin x)\right]$

86. $\int \frac{dx}{(x^2+1)(x^2+4)} = \frac{1}{4-1}\int\left(\frac{1}{x^2+1} - \frac{1}{x^2+4}\right)dx$

$= \frac{1}{3}\left(\tan^{-1}x - \frac{1}{2}\tan^{-1}\frac{x}{2}\right)$

$\Rightarrow \quad k = \frac{1}{3},\ l = -\frac{1}{6}$

87. $I_1 = \int \sin^{-1}x\,dx \quad I_2 = \int \cos^{-1}x\,dx$

$\therefore\ \sin^{-1}\sqrt{1-x^2} = \cos^{-1}x$

$I_1 + I_2 = \int\left(\sin^{-1}x + \cos^{-1}x\right)dx$

$= \frac{\pi}{2}\int I\,dx = \frac{\pi}{2}x$

88. Only point that satisfy both the equations is $(1,2)$.

$\therefore$ Point of contact $=(1,2)$ Or

$y = x+1 \Rightarrow \frac{dy}{dx} = 1$

$y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

By data, $\frac{2}{y} = 1 \Rightarrow y = 2 \Rightarrow x = 1$

$\therefore\ Point = (1,2)$

89. We have, $y^2 = x \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$, slope $=1$ by data

By data $\frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

$\therefore$ Point $\left(\frac{1}{4}, \frac{1}{2}\right)$

90. We have the volume of cylinder

$V = \pi r^2 h \Rightarrow V = 100\pi h \qquad (\because\ r = 100)$

$\Rightarrow \frac{dV}{dt} = 100\pi\frac{dh}{dt}$

$\Rightarrow 314 = 100\pi\frac{dh}{dt} \quad \left(\because\ \frac{dV}{dt} = 314\right)$

$\Rightarrow \frac{dh}{dt} = \frac{7\times 314}{100\times 22} = \frac{2198}{2200}$

$\therefore\ \frac{dh}{dt} = 1\text{m/h}$

91. $f(x) = (x+2)e^{-x}$

$f'(x) = -(x+2)e^{-x} + e^{-x}$

$= -e^{-x}(x+2-1) = -e^{-x}(x+1)$

$f'(x) > 0 \Rightarrow -(x+1) > 0$

$\Rightarrow (x+1) < 0 \Rightarrow x < -1$

$\therefore\ f(x)$ is increasing in $(-\infty, -1)$

$f'(x) < 0 \Rightarrow -(x+1) < 0 \Rightarrow (x+1) > 0 \Rightarrow x > -1$

$\therefore\ f(x)$ is decreasing in $(-1, \infty)$

92. $\int_0^{\pi}\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)dx = \frac{1}{2}\int_0^{\pi}\sin x\,dx$

$= \frac{1}{2}[-\cos x]_0^{\pi}$

$= -\frac{1}{2}[\cos\pi - \cos 0]$

$= -\frac{1}{2}(-1-1) = 1$

93. $\int_0^{\pi/3}\tan x\,dx = \log(\sec x)\Big]_0^{\pi/3}$

$= \log\left(\sec\frac{\pi}{3}\right) - \log(\sec 0)$

$= \log 2 - \log 1 = \log 2$

94. $I = \int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)dx}{\sec(\pi-x)+\cos(\pi-x)}$

$= \pi\int_0^{\pi}\frac{\tan xdx}{\sec x + \cos x} - I$

$I = \frac{1}{2}\int_0^{\pi}\frac{\sin xdx}{1+\cos^2 x}$

$= -\frac{\pi}{2}\tan^{-1}(\cos x)\Big]_0^{\pi} = -\frac{\pi}{2}\left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = \frac{\pi^2}{4}$

95. We have $x^2 - 5x + 6 = (x-2)(x-3)$

$\therefore\ x^2 - 5x + 6 < 0$ for $2 < x < 3$

$\therefore\ |x^2 - 5x + 6| = -\left(x^2 - 5x + 6\right)$ for $2 < x < 3$

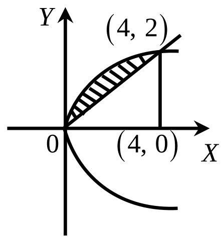
$I = \int_2^3 (x^2 - 5x + 6)dx$

$= -\frac{x^3}{3} + \frac{5x^2}{2} - 6x\Big]_2^3$

$= \left(-\frac{27}{3} + \frac{45}{2} - 18\right) - \left(-\frac{8}{3} + \frac{20}{2} - 12\right)$

$= -\frac{19}{3}+\frac{25}{2}-6=\frac{1}{6}$ Curves $y^2 = x$ and $2y = x$ intersect at $(4,2)$.

96. Required area



$= \int_0^4 \sqrt{x}dx - \int_0^4 \frac{x}{2}dx$

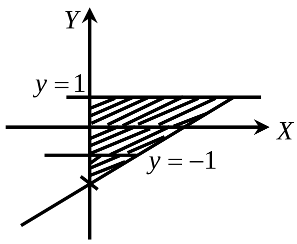
$= \frac{2}{3}x^{3/2}\Big]_0^4 - \frac{x^2}{4}\Big]_0^4$

$= \frac{16}{3}-4=\frac{4}{3}$ sq. units

97. Required area $= \left|\int_0^{\pi/2} \sin x\, dx\right|$

$= \left|\cos x\right]_0^{\pi/2}\Big| = 1$ sq. units

98. Required area



$= \int_{-1}^{1} x\, dy$

$= \int_{-1}^{1} (2y+3)\, dy$

$= y^2+3y\Big]_{-1}^{1}$

$= (1+3)-(1-3)$

$= 4+2=6$ sq. units.

99. Let, $\vec{a} = \lambda i + j + 2k$, $\vec{b} = i + \lambda j - k$, $\vec{c} = 2i - j + \lambda k$

Vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar, if

$\vec{a}.(\vec{b}\times\vec{c}) = 0$

$$\Rightarrow \begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda(\lambda^2-1) - 1(\lambda+2) + 2(-1-2\lambda) = 0$

$\Rightarrow \lambda^3 - \lambda - \lambda - 2 - 2 - 4\lambda = 0$

$\Rightarrow \lambda^3 - 6\lambda - 4 = 0$

$\Rightarrow \lambda = -2$

100. By data, $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow 2 + 2\lambda + 3 = 0 \Rightarrow \lambda = -\frac{5}{1}$

101. $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$

$\Rightarrow |\vec{a}|\cdot|\vec{b}|\cos\theta = |\vec{a}|\cdot|\vec{b}|\sin\theta$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

102. $\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = 1(2) - 2(-2) = 6$

103. $\vec{a}\cdot\vec{p} + \vec{b}\cdot\vec{q} + \vec{c}\cdot\vec{r}$

$$= \frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

$$= \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

$= 1+1+1 = 3$

104. Now, $Z_{(9,0)} = 36$, $Z_{(4,3)} = 16+9 = 25$,

$Z_{(2,5)} = 8+15 = 23$, $Z_{(0,8)} = 24$

Clearly, $Z_{\min}$ at $(2,\ 5)$

105. By data maximum value of Z occurs at $(15,15)$ and $(0,20)$. Thus,

$15p + 15q = 0\cdot p + 20q \Rightarrow 15p = 5q \Rightarrow q = 3p$

106. The equation is not a polynomial equation in terms of its derivatives.

$\therefore$ degree is not defined

107. Equation of the family of circle is

$(x-h)+(y-k)^2 = a^2$

Here, a is a fixed constant, h and k are parameters. Since it has two parameter the order of the differential equation is 2

108. We have, $\frac{dx}{x}+\frac{dy}{y} = 0 \Rightarrow \int\frac{dx}{x} + \int\frac{dy}{y} = \log c$

$\Rightarrow \log x + \log y = \log c$

$\Rightarrow xy = c$

109. We have, $(2y-1)dx - (2x+3)dy = 0$

$\Rightarrow \int\frac{dx}{2x+3} - -\int\frac{dy}{2y-1} = k$

$\Rightarrow \frac{1}{2}[\log(2x+3) - \log(2y-1)] = k \Rightarrow \frac{2x+3}{2y-1} = c$

110.The line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ passes through $Q=(6,7,7)$. Also we have, $P=(1,2,3)$.

Now, $PQ^2=(6-1)^2+(7-2)^2+(7-3)^2$

$=25+25+16=66$

Again, if R is the foot of the perpendicular from P on the line, then QR is the projection of PQ along the line.

Now, d.r's of line are $3,2,-2$

d.r's of line are, $\frac{3}{\sqrt{17}},\frac{2}{\sqrt{17}},\frac{-2}{\sqrt{17}}$

$\therefore\ QR=\frac{3}{\sqrt{17}}(6-1)+\frac{2}{\sqrt{17}}(7-2)-\frac{2}{\sqrt{17}(7-3)}$

$=\frac{15}{\sqrt{17}}+\frac{10}{\sqrt{17}}-\frac{8}{\sqrt{17}}=\frac{17}{\sqrt{17}}=\sqrt{17}$

Now, $PR^2=PQ^2-QR^2$

$66-17=49\Rightarrow PR=7$

111.Equations of a line through $(-2,3,4)$ is of the form

$\frac{x+2}{a}=\frac{y-3}{b}=\frac{z-4}{c}$

This line is parallel to $2x+3y+4z=5$ and $3x+4y+5z=6$

$\Rightarrow\left.\begin{array}{l}2z+3b+4c=0\\3a+4b+5c=0\end{array}\right]\Rightarrow\frac{a}{15-16}=\frac{b}{12-10}=\frac{c}{8-9}$

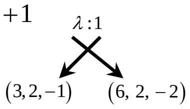
$\Rightarrow\frac{a}{-1}=\frac{b}{2}=\frac{c}{-1}$

Thus the equations are, $\frac{x+2}{1}=\frac{y-3}{-2}=\frac{z-4}{1}$

112.Let P divides the line segment in the ratio $\lambda:1$

$\Rightarrow x-$coordinate of $P=\frac{6\lambda+3}{\lambda+1}$

$\Rightarrow 5=\frac{6\lambda+3}{\lambda+1}\Rightarrow\lambda=2$



Thus, $y-$coordinate $=\frac{2(2)+1(2)}{2+1}=\frac{6}{3}=2$

113.Standard result: d.c's are $\pm\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

114.Equation of the line joining $(4,-5,-2)$ and $(-1,5,3)$ is given by $\frac{x-4}{4+1}=\frac{y+5}{-5-5}=\frac{z+2}{-2-3}$

$\Rightarrow\frac{x-4}{1}=\frac{y+5}{-2}=\frac{z+2}{-1}$

115.Plane through the intersection of the planes is of the form $x+2y-3z+1+\lambda(3x-2y+4z+3)=0$

$\Rightarrow(1+3\lambda)x+2(2-2\lambda)y+(4\lambda-3)z+(1+3\lambda)=0$

This passes through $(1,-1,-1)$

$\Rightarrow 1+3\lambda=1(2-2\lambda)+4\lambda-3+1+3\lambda=0$

$\Rightarrow 12\lambda-3=0\Rightarrow\lambda=\frac{1}{4}$

Thus we have,

$\left(1+\frac{3}{4}\right)x+\left(2-\frac{2}{4}\right)y+(1-3)z+\left(1+\frac{3}{4}\right)=0$

i.e. $7x+6y-8z+7=0$

116.The number of numbers that are divisible by 6 and 8 from among 1 to 90 $=\left[\frac{90}{24}\right]=\lfloor 3.75\rfloor=3$

($\because$ 1c.m. of 6 and 8 is 24)

Thus $n(E)=3$. Also $n(S)=90$.

Required probability $=\frac{3}{90}=\frac{1}{30}$

117.Let, E: event "coin with head",

F: event "coin with tail"

Now, $P(E_1)=PE_2=\frac{1}{2}$. E_1 and E_2 are mutually exclusive and exhaustive.

A: event 'A reports a head appears

$P(A/E)=P$ (head comes and A speaks truth) $=\frac{4}{5}$

$P(A/F)=P$ (tail comes and A speaks lie) $=\frac{1}{5}$

Required probability $=P(E/A)$

$=\frac{P(A/E)\cdot P(E)}{P(A/E)P(E)+P(A/F)\cdot P(F)}$

$=\frac{(4/5)(1/2)}{(4/5)(1/2)+(1/5)(1/2)}=\frac{4}{5}$

118. We have, $P(A \cap B = P(A/B)P(B)$

$= (0.5)(0.2) = 0.1$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{P(A \cup B)'}{P(B')} = \frac{1 - P(A \cup B)}{1 = P(B)}$$

$$= \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - 0.2} = \frac{3}{8}$$

119. We have, $P(A) = \frac{2}{5}, P(B) = \frac{4}{5}, P(B/A) = \frac{3}{5}$

Now, $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow \frac{3}{5} \times \frac{2}{5} = P(A \cap B) \Rightarrow P(A \cap B) = \frac{6}{25}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{5} + \frac{4}{5} - \frac{6}{25} = 0.96$$

120. Here Bernoulli trials with $n = 8$ and

$p = P$ (a head when one coin is tossed) $= \frac{1}{2}$

$\therefore$ required probability $= 8C_3 \cdot p^3 \cdot q^5$

$$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \cdot \frac{1}{8} \cdot \frac{1}{32} \qquad (\because \ q = 1 - p)$$

$$= \frac{7}{32}$$

Physics Solutions:

121. Both speed and velocity are constant in the case of a particle moving the uniform velocity. A particle moving with uniform velocity has zero acceleration.

122. When water is heated from $0°C$ to $10°C$, its volume decreases upto $4°C$. Since density of water is maximum at $4°C$. From $4°C$ to $10°C$, density of water decreases. Therefore volume of water increases.

123. As $K = \frac{1}{2}mv^2$

$$\therefore \frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{2\Delta v}{v} \times 100$$

$$= 2\% + 2 \times 3\% = 8\%$$

124. The most precise device is one whose least count is the least

(a) Least count of vernier callipers

$$= 1\,\text{MSD} - 1\,\text{VSD} = 1\,\text{MSD} - \frac{19}{20}\,\text{MSD}$$

$$= \frac{1}{20}\,\text{MSD} = \frac{1}{20}\,\text{mm} = \frac{1}{200}\,\text{cm} = 0.005\,\text{cm}$$

$(\because 1\,\text{MSD} = 1\,\text{mm})$

(b) Least count of screw gauge

$$= \frac{\text{pitch}}{\text{no. of divisions on circular scale}}$$

$$= \frac{1}{100}\,\text{mm} = \frac{1}{1000}\,\text{cm} = 0.001\,\text{cm}$$

(c) Least count of spherometer

$$= \frac{\text{pitch}}{\text{no. of divisions on circular scale}}$$

$$= \frac{0.1\,\text{mm}}{100} = \frac{1}{1000}\,\text{mm} = 0.0001\,\text{cm}$$

(d) Wavelength of light, $\lambda \approx 10^{-5}\,\text{cm} = 0.00001\,\text{cm}$.

Clearly the optical instrument is the most precise.

125. Since the graph between x and t is a straight line and passing through the origin.

$\therefore x = t$

Since the graph between y and t is a parabola.

$\therefore y = t^2$

$\therefore v_x = \frac{dy}{dt} = 2t$ and $a_y = 2\,\text{m s}^{-2}$

The force acting on the particle is

$F = ma_y = (0.5\,\text{kg})(2\,\text{m s}^{-2}) = 1\,\text{N}$ along $y -$ axis.

126. Here, mass of the stone, $m = 1\,\text{kg}$. As the stone is lying on the floor of the train, its acceleration is same as that of train.

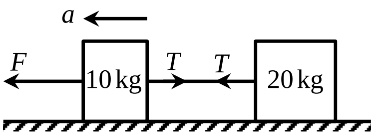
$\therefore$ Force acting on the stone,

$F = ma = (1\,\text{kg})(1\,\text{m s}^{-2}) = 1\,\text{N}$

127. Here, $m_1 = 10\,\text{kg}, m_2 = 20\,\text{kg}, F = 600\,\text{N}$

Let T be tension of the string and a be common acceleration of the system.

$$\therefore a = \frac{F}{m_1 + m_2} = \frac{600\,\text{N}}{10\,\text{kg} + 20\,\text{kg}} = \frac{600}{30}\,\text{m s}^{-2}$$



When a force F is applied on $10\,\text{kg}$ block, then the tension in the string is

$$T = m_2 a = (20\,\text{kg})(20\,\text{m s}^{-2}) = 400\,\text{N}$$

128. According to ideal gas law,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \text{ or } T_2 = T_1\frac{P_2V_2}{P_1V_1}$$

Here, $P_1 = P, V_1 = V, T_1 = T$

$$P_2 = 2P, V_2 = 2V, T_2 = ?\ \therefore T_2 = \frac{T(2P)(2V)}{PV} = 4T$$

129. In thermal equilibrium the macroscopic variables like pressure, volume, temperature, mass and composition do not change with time.

130. For a perfectly rigid body, both Young's modulus and bulk modulus is infinite.

131. According to Stokes' law, viscous drag force,

$F = 6\pi\eta Rv$

132. Out of four given bodies, the centre of mass of a ring lies outside it whereas in all other three bodies it lies within the body.

133. Here, length, $L = 10\,\text{m}$; mass, $M = 5\,\text{g} = 5\times10^{-3}\,\text{kg}$; tension, $T = 80\,\text{N}$

Mass per unit length of the wire is

$$\mu = \frac{M}{L} = \frac{5\times10^{-3}\,\text{kg}}{10\,\text{m}} = 5\times10^{-4}\,\text{kg m}^{-1}$$

Speed of the transverse wave on the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80\,\text{N}}{5\times10^{-4}\,\text{kg m}^{-1}}} = 4\times10^2\,\text{m s}^{-1} = 400\,\text{m s}^{-1}$$

134. According to Kepler's third law,

$T^2 \propto R^3$

$$\therefore \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} \text{ or } R_2 = (R_1)\left(\frac{T_2}{T_1}\right)^{2/3} = (R_1)\left(\frac{16}{2}\right)^{2/3}$$

$= 4R_1 = 4R$ (given $R_1 = R$) … (i)

Orbital velocity, $v_0 = \sqrt{\frac{GM}{R}}$

$$\therefore \frac{v_{02}}{v_{01}} = \sqrt{\frac{R_1}{R_2}} = \sqrt{\frac{R_1}{4R_1}} = \frac{1}{2}$$ (using (i))

or $v_{02} = \frac{1}{2}v_{01} = \frac{1}{2}v_0$

135. It will remain the same as the gravitational force is independent of the medium separating the masses.

136. The acceleration due to gravity at a depth d below the surface of earth is

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right) = g\frac{r}{R}$$ … (i)

where $R - d = r =$ distance of location from the centre of the earth. When $r = 0, g' = 0$,

From (i), $g \propto r$ till $R = r$, for which $g' = g$

For $r > R$, $g' = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{r^2}$ or $g' \propto \frac{1}{r^2}$

Here, $R + h = r$

Therefore, the variation of g with distance r from centre of earth will be as shown in figure (iv).

137. As $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

If $q' = 2q$, then $E' = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

$E' = 2E$, so electric field is doubled

138. As electric flux, $\phi = E\cdot\Delta s$

$\therefore$ Unit of ϕ is $\text{N C}^{-1}\,\text{m}^2$

139. Potential at the centre O of the square due to four equal charges q at four corners.

$$V = \frac{4q}{4\pi\varepsilon_0\left(a\sqrt{2}\right)/2} = \frac{\sqrt{2}q}{\pi\varepsilon_0 a}$$

$$W_{0\to\infty} = -W_{\infty\to0} = -(-q)V = \frac{\sqrt{2}q^2}{\pi\varepsilon_0 a}$$

140. Here, $C_1 = \frac{\varepsilon_0 A}{d} = 10\,\text{pF}$ ... (i)

and $C_2 = K\frac{\varepsilon_0 A}{d/2} = \frac{4\times 2\varepsilon_0 A}{d}$ $(\because K = 4)$

$= 8\times 10 = 80\,\text{pF}$ (using (i))

141. Here, thickness of the slab, $t = \frac{3}{4}d$

Capacitance, $C = \frac{\varepsilon_0 A}{d - t\left(1-\frac{1}{K}\right)} = \frac{\varepsilon_0 A}{d - \frac{3}{4}d\left(1-\frac{1}{K}\right)}$

$= \frac{\varepsilon_0 A}{\frac{d}{4}+\frac{3}{4}\frac{d}{K}} = \frac{\varepsilon_0 A}{\frac{d}{4}\left(1+\frac{3}{K}\right)} = \frac{\varepsilon_0 A}{d}\frac{4K}{(K+3)}$

142. Capacitance of a parallel plate capacitor is

$C = \frac{\varepsilon_0 A}{d}$ ... (i)

Potential difference between the plate is

$V = Ed$ ... (ii)

The energy stored in the capacitor is

$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\left(Ed^2\right)$ (using (i) & (ii))

$= \frac{1}{2}\varepsilon_0 E^2 Ad$

143. As $\vec{F} = \vec{q}\left(\vec{v}\times\vec{B}\right)$

As the electron is stationary, $\therefore$ velocity $\vec{v} = 0$

$\therefore \vec{F} = 0$, so electron will remain stationary.

144. Mobility of charged particle

$\mu = \frac{|v_d|}{E} = \frac{7.5\times10^{-4}}{3\times10^{-10}} = 2.5\times10^{6}\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$

145. The graph (a) is true for conductors whose resistivity increases with temperature. But for semiconductors, resistivity decreases as the number of conducting electrons increase with rise of temperature.

146. When the same potential difference, that is the voltage, is applied as in houses, Power $= VI = \frac{V^2}{R}$.

The smaller resistance consumes greater power. Here $100\,\text{W}$ bulb has less resistance. It should glow more brightly. The $60\,\text{W}$ bulb has more resistance and therefore statement (a) is correct.

147. To get maximum equivalent resistance all resistances must be connected in series.

$\therefore \left(R_{eq}\right)_{max} = R + R + R + \ldots nR = nR$

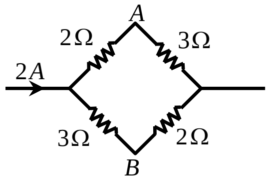
To get minimum equivalent resistance all resistances must be connected in parallel

$\therefore \frac{1}{\left(R_{eq}\right)_{min}} = \frac{1}{R}+\frac{1}{R}+\ldots\frac{1}{n}$

$\frac{1}{\left(R_{eq}\right)_{min}} = \frac{n}{R}$

$\Rightarrow \left(R_{eq}\right)_{min} = \frac{R}{n}$ $\quad\therefore \frac{\left(R_{eq}\right)_{max}}{\left(R_{eq}\right)_{min}} = \frac{nR}{R/n} = n^2$

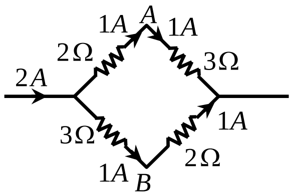
148. Sol:



Resistance of the upper arm $CAD = 2\Omega + 3\Omega = 5\Omega$

Resistance of the lower arm $CBD = 3\Omega + 2\Omega = 5\Omega$

As the resistance of both arms are equal, therefore same amount of current flows in both the arms.



Current through each arm CAD or $CBD = 1A$

Potential difference across C and A is

$V_C - V_A = (2\Omega)(1A) = 2\text{V}$ ... (i)

Potential difference across C and B is

$V_C - V_B = 3\text{V} - 2\text{V} = 1\text{V}$ ... (ii)

Subtracting (i) from (ii), we get

$V_A - V_B = 3\text{V} - 2\text{V} = 1\text{V}$

149. The time period of revolution of a charged particle in a magnetic field is $T = \frac{2\pi m}{Bq}$

For proton, $m_p = m, q_p = q$

$\therefore T_p = \frac{2\pi m}{Bq}$

Now, for $\alpha -$ particle, $m_\alpha = 4m, q_\alpha = 2q$

$\therefore T_\alpha = \frac{2\pi(4m)}{B(2q)} = 2\left(\frac{2\pi m}{Bq}\right) \Rightarrow \frac{T_p}{T_\alpha} - \frac{1}{2}$

150. Here, $N = 90, R = 15\,\text{cm} = 15\times10^{-2}\,\text{m}, B = 4\times10^{-4}\,\text{T}$

$\because B = \frac{\mu_0 NI}{2R}$

$\therefore I = \frac{2RB}{N\mu_0} = \frac{2\times15\times10^{-2}\times4\times10^{-4}}{4\pi\times10^{-7}\times90} = 1.06\,\text{A}$

151. $I = 40\,\text{A}$, $r = 15\,\text{cm} = 15\times10^{-2}\,\text{m}$

$\therefore B = \frac{\mu_o I}{2\pi R} = \frac{2\times10^{-7}\times40}{15\times10^{-2}} = \frac{80}{15}\times10^{-5} = 5.34\times10^{-5}\,\text{T}$

152. The photoelectric emission is possible if the wavelength of the incident light is less than that of yellow light.

153. Here, $\phi_0 = 2.14\,\text{eV}$

Work function, $\phi_0 = h\upsilon_0$ where υ_0 is the threshold frequency

$\upsilon_0 = \frac{\phi_0}{h} = \frac{2.14\times1.6\times10^{-19}}{6.63\times10^{-34}} = 5.16\times10^{14}\,\text{Hz}$

154. The kinetic energy gained by a particle when accelerated through potential difference V, is

$\frac{1}{2}mv^2 = qV$ or $m^2v^2 = 2mqV$ ... (i)

$\therefore$ Momentum, $p = mv = \sqrt{2mqV}$ (using (i))

or $\frac{p_\alpha}{p_p} = \sqrt{\frac{2m_\alpha q_\alpha V}{2m_p q_p V}} = \sqrt{\frac{2\times4m_p\times2e\times V}{2\times m_p\times e\times V}} = \sqrt{8}:1$

155. The torque acting on the coil $|\vec{\tau}| = |\vec{m}\times\vec{B}| = mB\sin\theta$.

Here the circular coil is placed normal to the direction of magnetic field then the angle between the direction of magnetic moment $(\vec{m})$ and magnetic field $(\vec{B})$ is zero, then

$\tau = mB\sin\theta = mB\sin 0 = 0$ $\therefore \tau = 0$

156. Current sensitivity of galvanometer is deflection per unit current, i.e.,

$\frac{\phi}{I} = \frac{NAB}{k}$ ... (i)

Similarly, voltage sensitivity is deflection per unit voltage, i.e.,

$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$ ... (ii)

From (i) and (ii),

Voltage sensitivity $=$ current sensitivity $\times\frac{1}{\text{resistance}}$

Now if current sensitivity is doubled, then the resistance in the circuit will also be doubled since it is proportional to the length of the wire, then voltage sensitivity

$= (2\times\text{current sensitivity})\times\frac{1}{(2\times\text{resistance})}$

$= (\text{current sensitivity})\times\frac{1}{(\text{resistance})}$

Hence, voltage sensitivity will remain unchanged.

157. Domains are partially aligned

158. Balmer series

159. The radius of n^{th} orbit

$r_n = n^2\frac{h^2 4\pi\varepsilon_0}{me^2}$ where $\frac{h^2 4\pi\varepsilon_0}{me^2} = a_0$ (Bohr radius)

Hence, $r_n = n^2 a_0$

160. $A_1 : A_2 = 1:3$

Their radii will be in the ratio

$R_0 A_1^{1/3} : R_0 A_2^{1/3} = 1:3^{1/3}$

Density, $\rho = \frac{A}{\frac{4}{3}\pi R^3}$

$\therefore \rho_{A_1} : \rho_{A_2} = \frac{1}{\frac{4}{3}\pi R_0^3\cdot1^3} : \frac{3}{\frac{4}{3}\pi R_0^3\left(3^{1/3}\right)^3} = 1:1$

Their nuclear densities will be the same.

161.Here $R_0 = 4.0\times10^6\,s^{-1}, R = 1.0\times10^6 s^{-1}, t = 20$ hours,

$t' = 100$ hours

As $\frac{R}{R_0} = \left(\frac{1}{2}\right)^n$ or $\frac{1.0\times10^6}{4.0\times10^6} = \left(\frac{1}{2}\right)^n$

or $\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2 \qquad \therefore n = 2$

$T_{1/2} = \frac{t}{n} = \frac{20}{2} = 10$ hours

Now, $n' = \frac{t'}{T_{1/2}} = \frac{100}{10} = 10$

$\therefore R' = R_0\left(\frac{1}{2}\right)^{n'} = 4.0\times10^6\left(\frac{1}{2}\right)^{10} = 3.91\times10^3\,s^{-1}$

162.Here, $N = 40, A = 4\,cm^2 = 4\times10^{-4}\,m^2$

$q = 2\times10^{-4}C, R = 80\Omega$

As, $q = \frac{\Delta\phi}{R} = \frac{NA(B-0)}{R}$

$\Rightarrow B = \frac{q\times R}{NA} = \frac{2\times10^{-4}\times80}{40\times4\times10^{-4}} = 1\,Wb\,m^{-2}$

163.Here, $l = 20\,cm = 20\times10^{-2}\,m$

$A = 20\,cm^2 = 20\times10^{-4}\,m^2$

$N = 400, I_1 = 2A, I_2 = 0, dt = 10^{-3}s$

As $\varepsilon = \frac{d\phi}{dt} = \frac{d(BAN)}{dt}$

$= \frac{\mu_0 N\,dI\,AN}{l\,dt} \qquad \left(\because B = \frac{\mu_0 NI}{l}\right)$

$= \frac{\mu_0 N(I_1 - I_2)AN}{l\,dt}$

$= \frac{4\pi\times10^{-7}\times(400)^2\times2\times20\times10^{-4}}{20\times10^{-2}\times10^{-3}} = 4\,V$

164.Length of the rod between the axis of rotation and one end of the rod $= \frac{l}{2}$

Area swept out in one rotation $= \pi\left(\frac{l}{2}\right)^2 = \left(\frac{\pi l^2}{4}\right)$

Angular velocity $= \omega\,rad\,s^{-1}$

Frequency of revolution $= \frac{\omega}{2\pi}$

Area swept out per second $= \frac{\pi l^2}{4}\left(\frac{\omega}{2\pi}\right) = \frac{l^2\omega}{8}$

Magnetic induction $= B$

Rate of change of magnetic flux $= \left(\frac{Bl^2\omega}{8}\right)$

Magnitude of induced emf $= \frac{Bl^2\omega}{8}$

Magnitude of induced emf between the axis and the other end is also $\left(\frac{Bl^2\omega}{8}\right)$. These two emf's are in opposite directions. Hence, the potential difference between the two ends of the rod is zero.

165.Here, $P = 2\,kW = 2\times10^3\,W$

$V_{rms} = 223\,V, \tan\phi = -\frac{3}{4}$

As, $P = \frac{V_{rms}^2}{Z} = \frac{(223)^2}{2000} = \frac{49729}{2000} = 24.86\Omega$

or $Z \approx 25\Omega$

$\tan\phi = \frac{X_C - X_L}{R} = -\frac{3}{4}$

$\therefore X_C - X_L = -\frac{3}{4}R$

As, $Z^2 = R^2 + (X_C - X_L)^2$

$\therefore (25)^2 = R^2 + \left(-\frac{3}{4}R\right)^2$

$625 = R^2 + \frac{9}{16}R^2 = \frac{25R^2}{16}$

$R^2 = \frac{625\times16}{25} = 400$

$R = 20\Omega$

166.Here, $V_m = 440\,V$

$\therefore V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{440}{\sqrt{2}} = 311.1V$

167.In a capacitive ac circuits, the voltage lags behind the current in phase by $\pi/2$ radian.

168. Here, transformation ratio, $k = 0.3$

As, $k = \frac{N_s}{N_p} = \frac{V_s}{V_p}$

$\therefore V_s = kV_p = 0.3 \times 220 = 66\,\text{V}$

169. As $n_e n_h = n_1^2$

Here, $n_i = 6 \times 10^8\,\text{per m}^3$ and $n_e = 9 \times 10^{12}\,\text{per m}^3$

$\therefore n_h = \frac{n_i}{n_e} = \frac{\left(6 \times 10^8\right)^2}{9 \times 10^{12}} = 4 \times 10^4\,\text{per m}^3$

170. Since the diode is reversed biased, only drift current exists in circuit which is $20\,\mu\text{A}$. Potential drop across $15\,\Omega$ resistor $= 15\,\Omega \times 20\,\mu\text{A} = 300\,\mu\text{V} = 0.0003\,\text{V}$

Potential difference across the diode

$= 4 - 0.0003 = 3.99 = 4\,\text{V}$

171. Here, $E_g = 2\,\text{eV}$

Wavelength of radiation corresponding to this energy is

$\lambda = \frac{hc}{E_g} = \frac{1240\,\text{eV nm}}{2\,\text{eV}} = 620\,\text{nm}$

Frequency, $\upsilon = \frac{c}{\lambda} = \frac{3 \times 10^8\,\text{m s}^{-1}}{620 \times 10^{-9}\,\text{m}} = 5 \times 10^{14}\,\text{Hz}$

172. The given circuit represents an OR gate when either A or B or both inputs are high, the output C is high.

173. (c)

174. (c)

175. Power of the lens combination is

$P = P_1 + P_2 = \frac{1}{f_1\,(\text{in m})} + \frac{1}{f_2\,(\text{in m})}$

$= \frac{1}{+0.40\,\text{m}} + \frac{1}{-0.25\,\text{m}}$

$= -1.5\,\text{m}^{-1} = -1.5\,\text{D}$

176. Using, $\mu = \frac{\sin\left(A + \delta_m\right)/2}{\sin A/2}$

Here, $A = \frac{\pi}{3} = 60°,\ \delta_m = \frac{\pi}{6} = 30°,\ c = 3 \times 10^8\,\text{m s}^{-1}$

$\therefore\ \mu = \frac{\sin\left(60° + 30°\right)/2}{\sin 60°/2} = \frac{0.7071}{0.50} = 1.414$

Therefore, $v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.414}$

$v = 2.12 \times 10^8\,\text{m s}^{-1}$

177. Here, $\frac{I_{\max}}{I_{\min}} = \frac{25}{9}$

or $\left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{25}{9}$

$\frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1} = \frac{5}{3} \Rightarrow \frac{A_1}{A_2} = 4$

$\therefore$ Width ratio of two slits $\frac{d_1}{d_2} = \frac{A_1^2}{A_2^2} = \frac{16}{1} = 16:1$

178. Fringe width in first case

$\beta_1 = \frac{D\lambda_1}{d}$ … (i)

Fringe width in second case

$\beta_2 = \frac{D\lambda_2}{d}$ … (ii)

Divide equation (ii) by (i),

$\therefore \frac{\beta_2}{\beta_1} = \frac{D\lambda_2/2d}{D\lambda_1/d} = \frac{1}{2} \cdot \frac{\lambda_2}{\lambda_1}$ or $\beta_2 = \frac{1}{2} \cdot \frac{\lambda_2}{\lambda_1} \cdot \beta_1$

$= \frac{1}{2} \times \frac{7500\,\text{Å}}{6000\,\text{Å}} \times 0.8\,\text{mm} = 0.5\,\text{mm}$

179. Aperture of the telescope

$D = \frac{1.22\lambda}{d\theta}$

Here, $\lambda = 5460\,\text{Å} = 5460 \times 10^{-10}\,m,\ d\theta = 4.6 \times 10^{-6}\,\text{rad}$

$\therefore D = \frac{1.22 \times 5460 \times 10^{-10}}{4.6 \times 10^{-6}} = 0.1488\,\text{m}$

180. Using, $\tan i_p = \sqrt{3}$

$\therefore i_p = \tan^{-1}\left(\sqrt{3}\right) = 60°$

As $r = 90° - i_p$

$\therefore r = 90° - 60° = 30°$