Subject	Topic	**Mock Test – 03**	Date
C + M + P	Complete Syllabus	CET – 12 – CT	

C+M+P Key Answers:

1. d	2. a	3. c	4. b	5. a	6. a	7. c	8. a	9. c	10. b
11. d	12. c	13. b	14. b	15. c	16. d	17. c	18. c	19. b	20. a
21. b	22. b	23. b	24. c	25. a	26. a	27. c	28. c	29. a	30. b
31. a	32. c	33. b	34. a	35. d	36. b	37. b	38. b	39. d	40. c
41. b	42. a	43. d	44. b	45. d	46. b	47. b	48. c	49. a	50. d
51. c	52. b	53. c	54. b	55. a	56. a	57. c	58. c	59. a	60. c
61. a	62. a	63. d	64. b	65. a	66. c	67. d	68. c	69. d	70. b
71. a	72. b	73. c	74. a	75. c	76. d	77. d	78. b	79. b	80. b
81. c	82. b	83. a	84. c	85. d	86. c	87. a	88. c	89. b	90. c
91. d	92. b	93. a	94. c	95. a	96. a	97. a	98. b	99. b	100.a
101.a	102.c	103.a	104.a	105.a	106.b	107.c	108.a	109.b	110.a
111.d	112.c	113.c	114.a	115.b	116.b	117.c	118.a	119.c	120.b
121.b	122.d	123.d	124.b	125.a	126.a	127.b	128.a	129.b	130.b
131.a	132.b	133.a	134.d	135.c	136.b	137.a	138.b	139.d	140.d
141.c	142.a	143.a	144.b	145.c	146.d	147.c	148.b	149.b	150.c
151.a	152.c	153.c	154.b	155.d	156.c	157.a	158.d	159.d	160.d
161.c	162.d	163.d	164.b	165.d	166.b	167.b	168.d	169.c	170.a
171.b	172.c	173.c	174.b	175.b	176.d	177.b	178.c	179.b	180.b

Chemistry Solutions:

1. All alkali metals are strong reducing agents.
2. In $[NiCl_4]^{2-}$, Ni is in $+2$ oxidation state.

   In $[NiCl_4]^{2-}$: $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\ \underbrace{\overset{4s}{\boxed{xx}}\quad \overset{4p}{\boxed{xx}\boxed{xx}\boxed{xx}}}_{sp^2}$

   [...n = number of unpaired electrons]

   Magnetic moment, $\mu = \sqrt{n(n+2)}$

   $= \sqrt{2(2+2)} = 2.82\,\text{B.M.}$
3. The $cis-$polyisoprene chains re held together by van der Waals forces and has a coiled structure. Thus, it can be stretched like a spring and exhibits elastic properties.
4. The order of boiling points of isometric amines is

   $1°$ > amines > $2°$ amines >$3°$ amines

   Because of absence of $H-$atom vavailable for hydrogen bonding, $3°$ amines do not have intermolecular association. Intermolecular association is more in $1°$ amines than in $2°$ amines as there are two $H-$atoms available for $H-$ bonding. Hydrocarbons are almost nonpolar molecules and possess weak van der waals forces and hence has lowest boiling point i.e., most volatile.
5. For rhombohedral system $a=b=c,\ \alpha=\beta=\gamma\neq 90°$
6. $(NH_4)2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$

   $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

7. Antibiotics which kill or inhibit a wide range of Gram-positive and Gram-Negative bacteria are said to be broad spectrum antibiotics.

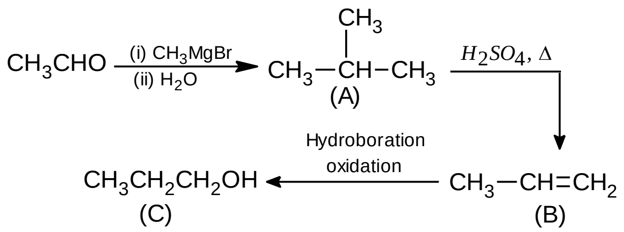
8. Sol:

Element	Percentage	Molar ratio	Relative ration	Simple whole number ratio
Mg	21.9	$21.9/24$ $=0.91$	1	2
P	27.8	$27.8/31$ $=0.90$	1	2
O	50.3	$50.3/16$ $=3.14$	3.48	7

Formula of the compound $= Mg_2P_2O_7$

9. Cryolite reduces the melting point of the mixture and increases its conductivity.

10. Sol:



Compounds (A) and (C) are potential isomers.

11. A sudden large jump between the values of 2nd and 3rd ionisation energies of an element indicates, that the element has two electrons in its valence shell. Then, the possible electronic configuration may be $1s^2, 2s^2, 2p^6, 3s^2$

12. $\alpha-D-$glucose and $\beta-D-$glucose are anomers not the enantiomers.

13. Soda water $\rightarrow$ A solution of gas in liquid
Sugar solution $\rightarrow$ A solution of solid in liquid
German silver $\rightarrow$ A solution of solid in solid
Air $\rightarrow$ A solution of gas in gas
Hydrogen gas $\rightarrow$ A solution of gas in solid palladium

14. The electron withdrawing group $(-NO_2)$ increases the acidity of phenols and the electron donating group $(-OCH_3)$ decreases the acidity of phenols. The effect at $p-$position is greater than at $m-$position. (ii)>(iv)>(iii)>(i)>(v)

15. (A)$\rightarrow$(iii), (B)$\rightarrow$(i), (C)$\rightarrow$(iv), (D)$\rightarrow$(ii)

16. In the solid state it exists as an ionic solid, $[PCl_4]^+[PCl_6]^-$ in which the cation., $[PCl_4]^+$ is tetrahedral and the anion, $[PCl_6]^-$ is octahedral.

17. $\frac{0.693}{t_{1/2}} = \frac{2.303}{t}\log\frac{a}{a-x}$

$\frac{0.693}{10} = \frac{2.303}{100}\log\frac{100}{100-x} \Rightarrow x = 99.9$

18. During the reaction $K_2Cr_2O_7$ is converted to $Cr_2(SO_4)_3$

$K_2Cr_2O_7 + 7H_2SO_4 + 6KI \rightarrow 4K_2SO_4$
$+ Cr_2(SO_3)_3 + 7H_2O + 3I_2$

In $Cr_2(SO_4)_3$, O.S. of Cr is $+3$.

19. $X = \frac{1}{2}(VE + MA - c + a)$

Where, $VE =$ No. of valence electrons on the central atom.

$MA =$ No. of monovalent atoms surrounding the central atom.

$c =$ Charge on the cation

$a =$ Charge on the antion

For NO_2^+, $X = \frac{1}{2}(5+0-1) = 2$ (sp hybrid orbitals)

For NO_3^-, $X = \frac{1}{2}(5+0-1) = 3$ (sp^2 hybrid orbitals)

For NH_4^+, $X = \frac{1}{2}(5+4-1) = 4$ (sp^3 hybrid orbitals)

20. In presence of excess of ammonia, tertiary amine $N,N-$dimethylmethanamine is formed..

21. Packing efficiency,
for sc unit cell $= 54\%$
for bcc unit cell $= 68\%$

for fcc unit cell $= 74\%$

Thus, the order of packing efficiency is $fcc > bcc > sc$

22. $\underset{\substack{\text{Ethyl ethanoate}\\(X)}}{CH_3COOC_2H_5} \xrightarrow{H_2O/H^+} \underset{(Y)}{CH_3COOH} + \underset{(Z)}{C_2H_5OH} \xrightarrow{CrO_3}$

$$\underset{(Y)}{CH_3COOH}$$

23. Canonical structures differ only in the position of electrons not in number of paired and unpaired electrons.

24. Primary alcohols readily form ether when heated with conc. H_2SO_4

25. Cu lies above H_2 in emf series.

26. Number of moles $\propto \dfrac{1}{\text{Molecular mass}}$

Molecular mass of

$CO_2 = 44,\ N_2 = 28,\ CH_4 = 16,\ HCl = 36.5$

CO_2 will have least volume.

27. XeF_2 is an oxidising agent.

28. Mole fraction of $O_2 = \dfrac{\frac{1}{32}}{\frac{1}{32}+\frac{1}{4}} = \dfrac{1}{32}\times\dfrac{32}{9} = \dfrac{1}{9}$

Partial pressure of oxygen

$= P \times x_{O_2} = P_{\text{total}} \times \dfrac{1}{9}$ or $\dfrac{1}{9}P_{\text{total}}$

29. $\Delta H = \Delta E + \Delta n_g RT$

$\Delta n_g = 1 - \dfrac{3}{2} = -\dfrac{1}{2}$ or -0.5

Hence, $\Delta H = \Delta E - 0.5RT$

30. Smaller value of equilibrium constant $\left(< 10^{-3}\right)$ signifies the greater concentration of reactants as compared to that of products.

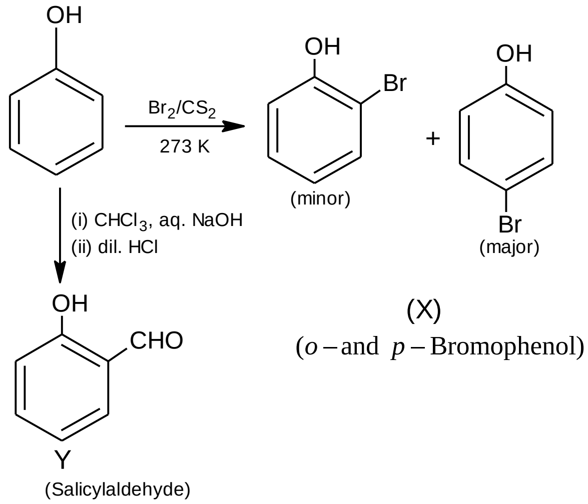
31. $[R] = [R_0] - kt$

For completion of reaction $[R] = 0$ or

$t = \dfrac{[R_0]}{k}$

32. Ionic bond cannot be formed between two similar atoms

33. Sol:



(X)
($o-$and $p-$Bromophenol)

34. $K = 1.67\times10^8\,\text{Pa}$

$P_{CO_2} = 2.5\text{ atm} = 2.5\times101325\,\text{Pa}$

Mass of $CO_2 = ?$

$p = Kx$

From formula,

$x_{CO_2} = \dfrac{p_{CO_2}}{K} = \dfrac{2.5\times101325\,\text{Pa}}{1.67\times10^8\,\text{Pa}} = 1.517\times10^{-3}$

$\therefore\ \dfrac{n_{CO_2}}{n_{H_2O}+n_{CO_2}} = \dfrac{n_{CO_2}}{n_{H_2O}} = 1.517\times10^{-3}$

For $500\,\text{mL}$ of soda water, water present $= 500\,\text{mL}$

$= 500g = \dfrac{500}{18} = 27.78$ moles

$\Rightarrow \dfrac{n_{CO_2}}{27.78} = 1.517\times10^{-3}$

$\therefore\ n_{CO_2} = 42.14\times10^{-3}$ mole

$\therefore$ Mass of $CO_2 = 42.14\times10^{-3}\times44g = 1.854g$

35. $NO \rightleftharpoons \dfrac{1}{2}N_2 + \dfrac{1}{2}O_2;\ K_c = 0.25$

$\dfrac{1}{2}N_2 + \dfrac{1}{2}O_2 \rightleftharpoons NO;\ K_c = \dfrac{1}{0.25} = 4$

36. Basicity of lanthanoid hydroxides decreases across the series due to decrease in ionic radii across the series.

37. Molar mass of

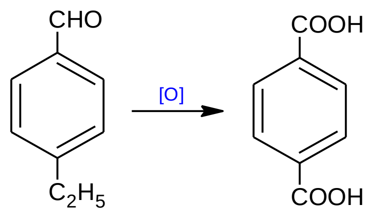
$AgBr = 108 + 80 = 188\, g\, \text{mol}^{-1}$

Percentage of $Br_2 = \frac{80}{188} \times \frac{0.12}{0.15} \times 100 = 34.04\%$

38. Due to highest reduction potential, fluorine is the best oxidising agent.

39. $NaCl, KNO_3, HCl$ are strong electrolytes but the size of H^+ is smallest. Smaller the size of the ions, greater is the conductance and hence greater is the conductivity $(k = G \times \text{cell const.})$

40. Sol:



(X) $1,4-$Benzenedicarboxylic acid

41. At high temperature, carbon and hydrogen react with metals to form carbides and hydrides respectively.

42. The activation energy for the reaction is very high at room temperature hence reaction will not take place.

43. $CrO_2Cl_2 \rightarrow x + (-4) + (-2) = 0 \Rightarrow x = +6$

44. If a lot of organic matter is decomposed, a lot of dissolved oxygen is consumed by microorganisms hence dissolved oxygen is no longer available for aquatic life.

45. Concentration and i factor both are same hence, the solution will be isotonic.

46. Liquid dishwashing detergents are non-ionic type.

47. The transition element having $3d^5$ configuration shows highest magnetic moment as it contains maximum unpaired electrons.

48. $n = 3,\ I = 5\text{A},\ t = 10 \times 60 \sec,\ W = 1.18\, g$

$\because$ Equivalent wt. $(E) = \frac{\text{Molecular mass}(M)}{\text{Valency }(n)}$

$\therefore\ E = \frac{M}{3}$

$1.18 = \frac{M \times 10 \times 60 \times 5}{3 \times 96500}$

$\therefore\ M = \frac{1.180 \times 3 \times 96500}{10 \times 60 \times 5}$

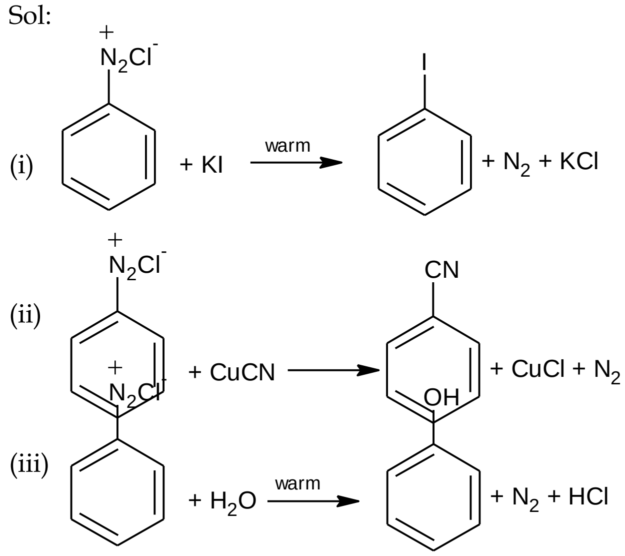
$\therefore\ M 11.87 \approx 114$

$\therefore$ The identity of the meal is Indium.

49. Oxygen atom can form multiple $p\pi - p\pi$ bonds with other oxygen atom on account of small size while $S = S$ is not so table as compared to $-S-S-S-$ chains. Hence S exists as S_8 while O exists as O_2.

50. Chelating ligands form more stable complexes as compared to monodentate ligands.

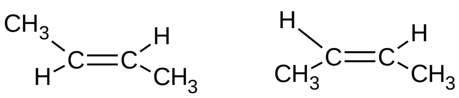
51. Sol:



52. The reactivity of different alkyl halides towards S_N2 reaction decreases in the order:

methyl halides $>$ 1° halides $>$ 2° halides $>$ 3° halides

53. When two groups attached to a double bonded carbon atoms are same, the compound does not exhibit geometrical isomerism. Only $2-$butane exhibits $cis-trans$ isomerism.



trans – 2 – Butene   *cis* – 2 – Butene

54. $CH_3CH_2CH_2OH \xrightarrow{HBr} CH_3CH_2CH_2Br$

$\xrightarrow[\text{Wurtz reacton}]{Na/\text{ether}} \underset{n-\text{hexane}}{CH_3(CH_2)_4CH_3}$

55. Pentan –2 – one and pentan –3 – one can be differentiated by iodoform test..

$$CH_3CH_2-\underset{\underset{O}{\|}}{C}-CH_2CH_3 + I_2 + NaOH \longrightarrow \text{No reaction.}$$

$$CH_3CH_2CH_2-\underset{\underset{O}{\|}}{C}-CH_3 + I_2 + NaOH \longrightarrow \underset{\text{Yellow ppt.}}{CHI_3}$$

56. (A) $\to$ (ii), (B) $\to$ (iv), (C) $\to$ (i), (D) $\to$ (iii)

57. The process of accumulation of colloidal particles together to get the size which ultimately settles as a precipitate is called coagulation. It generally occurs in presence of an electrolyte. Coagulation of soil particles of river in presence of salts present in sea water is responsible for the formation of delta.

58. Structure of white phosphorus is tetrahedral.

59. Primary structure of proteins.

60. For $3d$, $n=3$, $l=2$, $m_l=-2,-1,0,+1,+2$

Mathematics Solutions:

61. $P_1=\dfrac{4}{\sqrt{29}}$, $P_2=\dfrac{1}{2\sqrt{29}}$, $P_3=\dfrac{8}{\sqrt{29}}$

$$\therefore\ P_1+8P_2-P_3=\frac{4}{\sqrt{29}}+8\times\frac{1}{2\sqrt{29}}-\frac{8}{\sqrt{29}}=0$$

62. S.D. $=\dfrac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}$

$$=\left|\frac{(4\hat{j}+4\hat{k})\cdot(4\hat{i}+6\hat{j}+2\hat{k})}{\sqrt{(0)^2+(4)^2+(4)^2}}\right|=\frac{32}{4\sqrt{2}}=4\sqrt{2}$$

63. The plane will be of the form $x+2y+4z=k$

$2+2(3)+4(4)=k\Rightarrow k=24$.

64. Any plane through $(2,\ -1,\ 0)$ is

$a(x-2)+b(y+1)+cz=0$. … (1)

It will pass through $(3,\ -4,\ 5)$ if

$a-3b+5c=0$ … (2)

Also (1) will be parallel to $2x=3y=4z$, i.e.,

$$\frac{x}{\frac{1}{2}}=\frac{y}{\frac{1}{3}}=\frac{z}{\frac{1}{4}}$$

if $a\cdot\dfrac{1}{2}+b\cdot\dfrac{1}{3}+c\cdot\dfrac{1}{4}=0$ or $6a+4b+3c=0$ … (3)

From (2) and (3), $\dfrac{a}{-9-20}=\dfrac{b}{30-3}=\dfrac{c}{4+18}$

i.e., $\dfrac{a}{29}=\dfrac{b}{-27}=\dfrac{c}{-22}$

Hence the plane is $29(x-2)-27(y+1)-22z=0$

65. Any plane through the given line

$2x-y+3z+1+\lambda(x+y+z+3)=0$ (From $S+\lambda S'=0$)

If this plane is parallel to the line $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$, then the normal to the plane is also perpendicular to the above line or

$(2+\lambda)1+(\lambda-1)2+(3+\lambda)3=0$

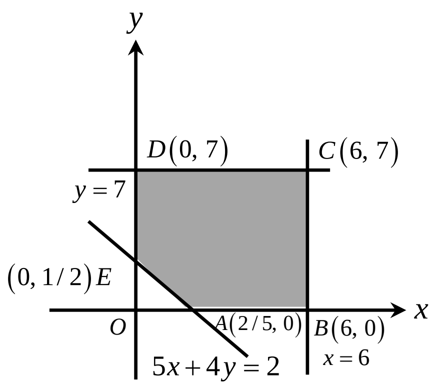
(From $l_1l_2+m_1m_2+n_1n_2=0$)

This gives $\lambda=-\dfrac{3}{2}$ and the required plane is

$x-5y+3z-7=0$

66. $2(4)-4(2)+k=7\Rightarrow k=7$.

67. Sol:



Feasible region is $ABCDEA$ and $z=x+2y$

At point	$A\left(\frac{2}{5},\ 0\right)$	$z=\frac{2}{5}+0=\frac{2}{5}$
At point	$B(6,\ 0)$	$z=6+0=6$
At point	$C(6,\ 7)$	$z=6+14=20$
At point	$D(0,\ 7)$	$z=0+2(7)=14$

At point $E\left(0, \frac{1}{2}\right)$

$$z = 0 + 2\left(\frac{1}{2}\right) = 1$$

$\therefore$ The maximum value of z is 20

68. The optimal value of the objective function is attained at the points given by corner points of the feasible region.

69. $A - B = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) + \cos^{-1}(\pi x) & 0 \\ 0 & \cot^{-1}(\pi x) + \tan^{-1}(\pi x) \end{bmatrix}$

$$= \frac{1}{\pi}\begin{bmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{bmatrix} = \frac{1}{\pi} \times \frac{\pi}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2} I$$

70. $a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2},\ a_{12} = \frac{25}{2}$

$$a_{21} = \frac{16}{2} = 8,\ a_{22} = 18$$

$$\therefore\ A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

71. By matrix multiplication we get

$$\begin{bmatrix} 3x+8-6 & - & - \\ 15+4y+4 & - & - \\ 3-4+2z & - & - \end{bmatrix} = \begin{bmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{bmatrix}$$

Other elements need not be computed.

$3x+2=5 \Rightarrow x=1,\ 4y+19=19 \Rightarrow y=0$ and

$-1+2z=1 \Rightarrow z=1$.

72. $\begin{vmatrix} a-x & a-y & a-z \\ b-x & b-y & b-z \\ c-x & c-y & c-z \end{vmatrix}$

On defining $R_1' = R_1 - R_2$ and $R_2' = R_2 - R_3$, we get

$$\begin{vmatrix} a-b & a-b & a-b \\ b-c & b-c & b-c \\ c-x & c-y & c-z \end{vmatrix}$$

Removing $(a-b)$ and $(b-c)$ as common from the 1st and 2nd rows respectively, we get the first two rows to be identical. Thus, the value of the determinant is 0.

73. Taking $(b-a)$ as common from each of C_1 and C_3

$$\Delta = (b-a)^2\begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

Now applying $C_1 - C_3$ and then C_1 and C_2 become identical.

74. Apply $R_3 \to R_3 - \cos y\, R_1 + \sin y\, R_2$

$$\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ 0 & 0 & \sin y - \cos y \end{vmatrix}$$

Expanding, $\Delta = (\sin y - \cos y)\left(\cos^2 x + \sin^2 x\right)$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin y - \frac{1}{\sqrt{2}}\cos y\right) = \sqrt{2}$$

$$\left[\cos\frac{\pi}{4}\sin y - \sin\frac{\pi}{4}\cos y\right] = \sqrt{2}\sin\left(y - \frac{\pi}{4}\right)$$

$$\therefore -\sqrt{2} \le \Delta \le \sqrt{2} \qquad (\because -1 \le \sin\theta \le 1)$$

75. By using $|kA| = k^3|A|$

$$|3AB| = 3^3|A||B| = 27(-1)(3) = -81$$

76. Number of elements in $A \times A$ is $n \times n = n^2$

Number of relations $=$ number of subsets of

$$(A \times A) = 2^{n^2}$$

77. $f(x) = \sqrt{x^2 - 3x + 2}$

$$x^2 - 3x + 2 \ge 0 \Rightarrow (x-2)(x-1) \ge 0$$

$$\Rightarrow x \le 1 \text{ or } x \ge 2 \ \Rightarrow x \in (-\infty, 1] \cup [2, \infty).$$

78. Given $f(x) = \sin\left[\frac{\pi}{2}[x] - x^5\right],\ 1 < x < 2 \Rightarrow [x] = 1$

$$\therefore\ f(x) = \sin\left(\frac{\pi}{2} - x^5\right) = \cos\left(x^5\right)$$

Differentiating w.r.t. x $\quad f'(x) = -\sin\left(x^5\right)5x^4$

$$\Rightarrow f'\left(\sqrt[5]{\frac{\pi}{2}}\right) = -\sin\left(\frac{\pi}{2}\right)\cdot 5\left(\frac{\pi}{2}\right)^{\frac{4}{5}} = -5\left(\frac{\pi}{2}\right)^{\frac{4}{5}}$$

79. We have $f(x)=\frac{x-1}{x+1}$

Applying componendo and dividendo, we get

$$\frac{f(x)+1}{f(x)-1}=\frac{2x}{-2}\Rightarrow x=\frac{f(x)+1}{1-f(x)}$$

Now $f(2x)=\frac{2x-1}{2x+1}=\frac{2\left(\frac{f(x)+1}{1-f(x)}\right)-1}{2\left(\frac{f(x)+1}{1-f(x)}\right)+1}=\frac{3f(x)+1}{f(x)+3}$

80. LHL $=\lim_{x\to 0} f(x)=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\frac{(0-h)-|0-h|}{(0-h)}$

$=\lim_{h\to 0}\frac{-h-h}{-h}=\lim_{h\to 0} 2=2$

RHL $=\lim_{x\to 0} f(x)=\lim_{h\to 0} f(0+h)=\lim_{h\to 0}\frac{(0+h)-|0+h|}{(0+h)}$

$=\lim_{h\to 0}\frac{h-h}{h}=\lim_{h\to 0}(0)=0$

$\therefore$ LHL $\neq$ RHL $\Rightarrow$ Limit does not exist at $x=0\Rightarrow f(x)$ is not continuous at $x=0$ since LHL $\neq$ RHL.

81. $\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\tan^2\left(\frac{3x}{2}\right)}{x^2}=\lim_{x\to 0}\left[\frac{\tan\left(\frac{3x}{2}\right)}{\frac{2}{3}\times\frac{3x}{2}}\right]^2$

$=\frac{9}{4}\left[\lim_{\frac{3x}{2}\to 0}\frac{\tan\left(\frac{3x}{2}\right)}{\left(\frac{3x}{2}\right)}\right]^2$

$=\frac{9}{4}(1)^2 \lim_{x\to 0} f(x)=\frac{9}{4}$

Now $\lim_{x\to 0} f(x)=f(0)\Rightarrow\frac{9}{4}=k+2\Rightarrow k=\frac{1}{4}$.

82. $\because f$ is differentiable at $x=1$, it is continuous also at $x=1$.

$\therefore$ LHL = RHL ; $\Rightarrow \lim_{x\to 1^-}\left(x^2+3x+a\right)=\lim_{x\to 1^+}(bx+2)$

i.e., $a-b=-2$; Also $\because f$ is derivable at 1,

$Lf'(1)=R\ f'(1)\Rightarrow \lim_{h\to 0^-}\frac{h^2+5h}{h}=\lim_{h\to 0^+}\frac{bh}{h}\Rightarrow b=5$;

$\therefore a=b-2=5-2=3$

83. $\because f(x)=|x|$ is a continuous function,

$f(x)=|x-5|$ is continuous everywhere and hence is continuation at $x=5$.

$L\ f'(5)=\lim_{h\to 0^-}\frac{f(5+h)-f(5)}{h}=\lim_{h\to 0}\frac{-(5+h-5)-0}{h}$

$=\lim_{h\to 0}\frac{-h}{h}=-1$ and

$R\ f'(5)=\lim_{h\to 0^+}\frac{f(5+h)-f(5)}{h}=\lim_{h\to 0^+}\frac{5+h-5-0}{h}$

$=\lim_{h\to 0}\frac{h}{h}=1$ i.e., $L\ f'(5)\neq R\ f'(5)$

$\Rightarrow f$ is not derivable at $x=5$.

84. $f(x)=x(x+3)e^{-\frac{x}{2}}$

$f'(x)=\left(x^2+3x\right)e^{-\frac{x}{2}}\left(-\frac{1}{2}\right)+(2x+3)e^{-\frac{x}{2}}$

$=e^{-\frac{x}{2}}\left\{-\frac{1}{2}\left(x^2+3x\right)+2x+3\right\}=\frac{1}{2}e^{-\frac{x}{2}}\left[x^2-x-6\right]$

Since $f(x)$ satisfies the Rolle's theorem

$f'(c)=0\Rightarrow -\frac{1}{2}e^{-\frac{c}{2}}\left\{c^2-c-6\right\}=0$

$c^2-c-6=0\Rightarrow c=3,\ -2$

$c=3\notin[-3,\ 0]\quad \therefore c=-2$.

85. $f'(x)=e^{\sqrt{3}x}\cos\left(x+\frac{\pi}{3}\right)+\sqrt{3}e^{\sqrt{3}x}\sin\left(x+\frac{\pi}{3}\right)$.

By putting $x=0$ we get, $f'(0)=\frac{1}{2}+\sqrt{3}\cdot\frac{\sqrt{3}}{2}=2$.

86. $y=x^x$, $\log_e y=x\log_e x$, therefore,

$\frac{1}{y}\frac{dy}{dx}=1+\log_e x\Rightarrow\frac{dy}{dx}=x^x\left[1+\log_e x\right]$.

87. $x+y=12$, let $z=xy^2\Rightarrow z=x(12-x)^2$

$\frac{dz}{dx}=(12-x)^2+x\ 2(12-x)(-1)=(12-x)(12-x-2x)$

$=0\Rightarrow(12-x)(12-3x)=0$

$\Rightarrow x=12$ or $x=4$. When $x=12,\ y=0,\ z=0$

When $x=4,\ y=8,\ z$ (maximum) $=256$

$\therefore z$ is maximum at $x=4,\ y=8$.

88. $2yy_1 = 1 \Rightarrow y_1 = \dfrac{1}{2y} \Rightarrow \dfrac{1}{2y} = 1$

$\Rightarrow y = \dfrac{1}{2}$ $\quad\therefore\ x = \dfrac{1}{4}$ $\quad\therefore$ the point is $\left(\dfrac{1}{4}, \dfrac{1}{2}\right)$

89. Put $y^2 = 4x$ in $x^2 + y^2 = 12 \Rightarrow x^2 + 4x - 12 = 0$

$\Rightarrow (x-2)(x+6) = 0 \Rightarrow x = 2$ or $x = -6$

When $x = 2,\ y = 2\sqrt{2}$

$\therefore$ point of intersection is $P\left(2, 2\sqrt{2}\right)$

I curve : $y^2 = 4x \Rightarrow 2y \cdot \dfrac{dy}{dx} = 4 \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{y}$

At P, $\dfrac{dy}{dx} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}} = m_1$

II curve: $x^2 + y^2 = 12 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y}$

At P, $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{2}}$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 + \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)} = \frac{\sqrt{2}}{1/2} = 2\sqrt{2}$$

$\therefore\ \theta = \tan^{-1}\left(2\sqrt{2}\right)$.

90. Let y be volume and $x =$ length

$y = x^3$

Let Δx error in x and Δy error in y

$\dfrac{\Delta x}{x} \times 100 = 1$ (given)

To find $\dfrac{\Delta y}{y} \times 100$

$y = x^3 \Rightarrow \dfrac{dy}{dx} = 3x^2$

$\Delta y = \dfrac{dy}{dx} \cdot \Delta x = 3x^2 \times \dfrac{x}{100} = \dfrac{3x^3}{100} = \dfrac{3y}{100}$

$\therefore$ % error $= \dfrac{\Delta y}{y} \times 100 = \dfrac{100}{y} \times 100 = 3\%$.

91. $\displaystyle\int \frac{x^2}{16 + x^6}\,dx = \int \frac{x^2}{16 + \left(x^3\right)^2}$. Taking $x^3 = t,\ 3x^2 dx = dt$

$$\frac{1}{3}\int \frac{dt}{16 + t^2} = \frac{1}{3} \cdot \frac{1}{4} \tan^{-1}\left(\frac{t}{4}\right) + c = \frac{1}{12}\tan^{-1}\left(\frac{x^3}{4}\right) + c.$$

92. $I = \displaystyle\int \frac{dx}{\cos^2 x (4 + \tan x)^4}$

taking $4 + \tan x = t \Rightarrow \sec^2 x\ dx = dt$

$$I = \int \frac{dt}{t^4} = \int t^{-4}\,dt = \frac{t^{-3}}{-3} + c = -\frac{1}{3t^3} + c = -\frac{1}{3(4 + \tan x)^3} + c$$

93. $\displaystyle\int \frac{dx}{\sqrt{x^2 + 8}} = \int \frac{dx}{\sqrt{x^2 + \left(\sqrt{8}\right)^2}} = \ln\left|x + \sqrt{x^2 + 8}\right| + c$

94. $I = \displaystyle\int \frac{x\left(1 - \frac{1}{x^8}\right)^{\frac{1}{9}}}{x^{10}}\,dx = \int \frac{1}{x^9}\left(1 - \frac{1}{x^8}\right)^{\frac{1}{9}} dx$, upon

substituting $1 - \dfrac{1}{x^8} = t,\ \dfrac{8}{x^9}\,dx = dt$

$$I = \frac{1}{8}\int t^{\frac{1}{9}}\,dt = \frac{1}{8} \cdot \frac{t^{\frac{10}{9}}}{\frac{10}{9}} + c = \frac{9}{80}\left(1 - \frac{1}{x^8}\right)^{\frac{10}{9}} + c$$

95. Let

$$I = \int_0^2 \frac{dx}{\sqrt{8 - 4 + 4x - x^2}} = \int_0^2 \frac{dx}{\sqrt{8 - (x-2)^2}} = \sin^{-1}\left(\frac{x-2}{2\sqrt{2}}\right)\Bigg|_0^2$$

$= \dfrac{\pi}{4}$

96. $I = \displaystyle\int_0^{\pi/2} \frac{\phi(x)}{\phi(x) + \phi\left(\frac{\pi}{2} - x\right)}\,dx$ … (1)

$I = \displaystyle\int_0^{\pi/2} \frac{\phi\left(\frac{\pi}{2} - x\right)}{\phi\left(\frac{\pi}{2} - x\right) + \phi(x)}\,dx$ … (2)

(1) + (2) gives $2I = \displaystyle\int_0^{\pi/2} 1\,dx = \frac{\pi}{2}\ \therefore\ I = \frac{\pi}{4}$.

97. $\displaystyle\int_0^{\pi/2} x\sin x\,dx = -x\cos\Big]_0^{\pi/2} + \int_0^{\pi/2} \cos x\,dx = 0 + \sin x\Big]_0^{\pi/2} = 1$

98. $I = \displaystyle\int_0^{\pi/2} |\sin x - \cos x|\,dx + \int_{\pi/4}^{\pi/2} |\sin x - \cos x|\,dx$

In the interval

$\left(0, \frac{\pi}{4}\right)$, $\sin x < \cos x$ and in the interval

$\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, $\sin x > \cos x$

$$\therefore\ I = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_0^{\pi/2} (\sin x - \cos x)\,dx$$

$$= \sin x + \cos x\Big|_0^{\pi/4} + (-\cos x - \sin x)\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (0+1) - \left(0 + 1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}} - 2 + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}} - 2 = 2\sqrt{2} - 2$$

99. $$\frac{A_1}{A_2} = \frac{2\int_0^a 2\sqrt{a}\sqrt{x}\,dx}{2\int_a^{2a} 2\sqrt{a}\sqrt{x}\,dx} = \frac{\frac{2}{3}x^{3/2}\Big]_0^a}{\frac{2}{3}x^{3/2}\Big]_a^{2a}} = \frac{a^{3/2}}{(2a)^{3/2} - a^{3/2}}$$

$$= \frac{1}{2\sqrt{2}-1} = \frac{2\sqrt{2}+1}{7}.$$

100. $$\int_2^5 \left(x^2 - 7x + 10\right)dx = \frac{x^3}{3} - \frac{7x^2}{2} + 10x\Big]_2^5$$

$$= \frac{1}{3}(125-8) - \frac{7}{2}(25-4) + 10(3)$$

$$= \frac{117}{3} - \frac{7}{2}\cdot 21 + 30 = \frac{234 - 7\cdot 63 + 180}{6}$$

$$= \frac{414-441}{6} = \frac{-27}{6} = -4.5 \quad \Rightarrow \text{ Area } = |-4.5| = 4.5$$

101. $x = 4 - y^2$

$y^2 = -x + 4$, the limits are 0 and 2

$$\text{Area } = \int_a^b x\,dy$$

Required area

$$= 2\int_0^2 \left(4 - y^2\right)dy = 2\left(4y - \frac{y^3}{3}\right)_0^2 = 2\left[4(2-0) - \frac{1}{3}\left(2^3 - 0\right)\right]$$

$$2\left[8 - \frac{1}{3}(8)\right] = 2\left(\frac{24-8}{3}\right) = \frac{16\times 2}{3}$$

$\therefore$ The required area $= \frac{32}{3}$ sq. units.

102. Given equation is $\left(x - e^{\tan^{-1}y}\right)\frac{dy}{dx} = -\left(1+y^2\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{-\left(1+y^2\right)}{x - e^{\tan^{-1}y}} \Rightarrow \frac{dx}{dy} = \frac{-\left(x - e^{\tan^{-1}y}\right)}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}\cdot x = \frac{e^{\tan^{-1}y}}{1+y^2}$$

This is linear in x

$$\text{I.F} = e^{\int P\cdot dy} = e^{\int \frac{1}{1+y^2}\cdot dy} = e^{\tan^{-1}y}$$

Solution is $x\cdot e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2}\cdot e^{\tan^{-1}y}dy + c$

Putting $\tan^{-1}y = t$ in R.H.S

$$xe^{\tan^{-1}y} = \int e^{2t}\cdot dt + c$$

$$\Rightarrow xe^{\tan^{-1}y} = \frac{1}{2}e^{2t} + c \Rightarrow 2xe^{\tan^{-1}y} = e^{2t} + 2c$$

$$\Rightarrow 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$$

103. The equation of the family of circles passing through the origin with their centres on the $y-$axis is

$$x^2 + y^2 + 2fy = 0 \qquad \ldots (1)$$

where f is a parameter.

Differentiating w.r.t x.

$$2x + 2y\frac{dy}{dx} + 2f\frac{dy}{dx} = 0 \Rightarrow 2f\frac{dy}{dx} = -2x - 2y\frac{dy}{dx}$$

$$\Rightarrow 2f = \frac{-2x - 2y\frac{dy}{dx}}{\frac{dy}{dx}} \qquad \ldots (2)$$

Eliminating f from (1) and (2)

$$x^2 + y^2 + y\frac{\left(-2x - 2y\frac{dy}{dx}\right)}{\frac{dy}{dx}} = 0$$

$$\Rightarrow x^2\frac{dy}{dx} + y^2\frac{dy}{dx} - 2xy - 2y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \left(x^2 - y^2\right)\frac{dy}{dx} - 2xy = 0$$

104. The equation of the family of parabolas whose axis is $x-$axis is given by

$$y^2 = 4a(x-k)$$

$$\therefore 2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a \Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

Hence degree is 1 and order is 2.

105. Given equation is $\frac{dy}{dx} = \frac{y\sin x - y^2}{\cos x}$

$$\Rightarrow \frac{dy}{dx} = y\tan x - y^2\sec x \Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$$

Putting

$$-\frac{1}{y} = z \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx} \qquad \therefore \frac{dz}{dx} + \tan x \cdot z = -\sec x$$

This is linear in z.

Integrating factor $= e^{\int \tan x \cdot dx} = e^{\log \sec x} = \sec x$

Solution is $z\cdot\sec x = \int -\sec x\cdot\sec x\,dx$

$z\sec x = -\tan x - c$ But $z = -\frac{1}{y}$

$$\therefore -\frac{1}{y}\sec x = -\tan x - c \Rightarrow \sec x = y(\tan x + c)$$

106. Given that $\tan^{-1}x + \tan^{-1}y = \frac{2\pi}{3}$

$$\Rightarrow \frac{\pi}{2} - \cot^{-1}x + \frac{\pi}{2} - \cot^{-1}y = \frac{2\pi}{3}$$

$$\therefore \cot^{-1}x + \cot^{-1}y = \frac{\pi}{3}.$$

107. $\sec^2\left(\sec^{-1}2\right) - 1 + \operatorname{cosec}^2\left(\operatorname{cosec}^{-1}3\right) - 1$

$= 4 - 1 + 9 - 1 = 11$.

108. $\tan^{-1}\frac{1}{2a^2} = \tan^{-1}\left(\frac{2}{4a^2}\right) = \tan^{-1}\left[\frac{2}{1+\left(4a^2-1\right)}\right]$

$$= \tan^{-1}\left[\frac{(2a+1)-(2a-1)}{1+(2a+1)(2a-1)}\right]$$

$$= \tan^{-1}(2a+1) - \tan^{-1}(2a-1)$$

$$\therefore \sum_{a=1}^{n}\tan^{-1}\left(\frac{1}{2a^2}\right) = \tan^{-1}(2n+1) - \tan^{-1}(1)$$

$$\therefore \lim_{n\to\infty}\tan^{-1}(2n+1) - \tan^{-1}1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

109. Given $\tan^{-1}\left(\frac{1}{a+b}\right) + \tan^{-1}\left(\frac{(a+b)-a}{1+a(a+b)}\right)$

$$= \cot^{-1}(a+b) + \tan^{-1}(a+b) - \tan^{-1}a$$

$$= \frac{\pi}{2} - \tan^{-1}a = \cot^{-1}a.$$

110. $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4} \Rightarrow \frac{x+y}{1-xy} = 1 \Rightarrow x + y + xy = 1$.

111. $\left(a\,\hat{i} + \hat{j} - 2\hat{k}\right).\left(-12\hat{i} - 4\hat{j} + 8\hat{k}\right) = 0$

$$\Rightarrow -12a - 4 - 16 = 0 \Rightarrow a = -\frac{5}{3}.$$

112. Area of the triangle

$$ABC = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\left|\left(\hat{i}+2\hat{j}-3\hat{k}\right)\times\left(2\hat{i}\right)\right|$$

$$= \frac{1}{2}\left|4\hat{j}\times\hat{i} - 6\hat{k}\times\hat{i}\right|$$

$$= \frac{1}{2}\left|-4\hat{k} - 6\hat{j}\right| = \frac{1}{2}\left|\sqrt{16+36}\right| = \frac{\sqrt{52}}{2} = \sqrt{13}.$$

113. Required vector $= 10\frac{\vec{a}\times\left(\vec{b}\times\vec{c}\right)}{\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|}$

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{b}(1+2+1) - \vec{c}(2-1+1) = 6\hat{i} - 8\hat{j} - 2\hat{k}$$

114. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$

Then $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ -1 & 3 & 4\end{vmatrix} = \hat{i}(8-3) - j(4+1) + \hat{k}(3+2)$

$$= 5\hat{i} - 5\hat{i} + 5\hat{k}$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right| = \sqrt{(5)^2 + (-5)^2 + 5^2} = \sqrt{3(5)^2} = 5\sqrt{3}$$

Therefore, unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is given by $\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|} = \frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}}$

Hence, vectors of magnitude of $10\sqrt{3}$ that are perpendicular to plane of $\vec{a}$ and $\vec{b}$ are

$$\pm 10\sqrt{3}\left(\frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}}\right), \text{ i.e. } \pm 10\left(\hat{i} - \hat{j} + \hat{k}\right)$$

115.Projection of $\vec{a}$ on $\vec{b}$ is

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{6+3-3}{\sqrt{9+9+1}}=\frac{6}{\sqrt{19}}$$

116.Let $X=\{\text{even of } A \text{ getting selected}\}$,

$Y=\{\text{even of } B \text{ getting selected}\}$

Then $P(X)=\frac{2}{3}$ and $P(Y)=\frac{3}{5}$

Now, $P(X \text{ or } Y)=P(X)P(\bar{Y})+P(\bar{X})\,P(Y)$

$$=\frac{2}{3}\cdot\frac{2}{5}+\frac{1}{3}\cdot\frac{3}{5}=\frac{4}{15}+\frac{3}{15}=\frac{7}{15}$$

117. $P\left(A\cup B^C\right)=0.8$

$$\Rightarrow P(A)+P\left(B^C\right)-P\left(A\cap B^C\right)=0.8$$

$$\Rightarrow P(A)+1-P(B)-P(A)\cdot P\left(B^C\right)=0.8$$

$$\Rightarrow P(A)+1-\frac{2}{7}-P(A)\cdot\left(1-\frac{2}{7}\right)=0.8$$

$$\Rightarrow P(A)+\frac{5}{7}-P(A)\cdot\frac{5}{7}=0.8$$

$$\Rightarrow \frac{2}{7}P(A)=0.8-\frac{5}{7}=\frac{0.6}{7}\Rightarrow P(A)=\frac{0.6}{2}=0.3$$

118.The prime numbers between 1 to 50 is

$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$

$$\therefore P(A)=\frac{15}{50}=\frac{3}{10}.$$

119.Given, $P(A)=0.3,\ P(B)=0.4,\ P(C)=0.8$,

$P(A\cap B)=0.08,\ P(A\cap C)=0.28$,

$P(A\cap B\cap C)=0.09$

Since, $P(A\cup B\cup C)\ge 0.75$

$$P(A)+P(B)+P(C)-P(A\cap C)-P(A\cap B)$$
$$-P(B\cap C)+P(A\cap B\cap C)\ge 0.75$$

$$0.3+0.4+0.8-0.08-0.28-P(B\cap C)+0.09\ge 0.75$$

$$P(B\cap C)\le 0.48$$

Also, $P(A\cup B\cup C)\le 1$

$$1.23-P(B\cap C)\le 1$$

$$P(B\cap C)\ge 0.23$$

$$0.23\le P(B\cap C)\le 0.48$$

120.It is a case of Bernoullian trails where success is "getting a head" and

$p=P$ (six successes)

$$={}^{10}C_6\ p^6\ q^4={}^{10}C_4\left(\frac{1}{2}\right)^6\left(1-\frac{1}{2}\right)^4$$

$$=\frac{10\times 9\times 8\times 7}{1\times 2\times 3\times 4}\times\frac{1}{2^{10}}=\frac{210}{1024}=\frac{105}{512}.$$

<u>Physics Solutions</u>:

121.Total time $t=\frac{6}{2.5}+\frac{6}{4}=\frac{12}{5}+\frac{3}{2}=\frac{39}{10}$ h

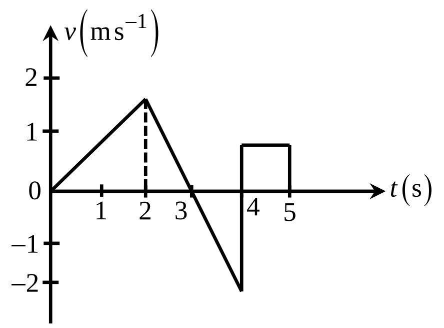
Average speed, $v_{av}=\frac{6+6}{\frac{39}{10}}=\frac{120}{39}=\frac{40}{13}\ \text{km h}^{-1}$

122.Universal gas constant - $\text{J mol}^{-1}\,\text{K}^{-1}$

123.Range of projection, $R=\frac{u^2\sin 2\theta}{g}$

Range will be maximum for which $\sin 2\theta$ is maximum i.e., $\sin 2\theta=1 \ \Rightarrow\ \theta=45°$

124.Sol:



Displacement is algebraic sum of area under velocity time graph

$\therefore$ Displacement in 5 s

$$=\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 1\times 2-\frac{1}{2}\times 2\times 1+1\times 1=3\text{ m}$$

125.Let m grams be the mass of the steam

Heat lost by the steam $=m\times L+m\times 1\times(100-0)$

$$=m\times 540+100m=640m$$

Heat gained by ice

$= m_i \times s \times \Delta T + m_i L$

$= 3200 \times 0.5 \times \left[0-(-10)\right] + 3200 \times 80$

$= 272000$ cal

According to principle of calorimetry

$640m = 272000$ or $m = 425$ g

126. The appearance of a star depends on the wavelength (λ_m) at which it radiates maximum energy. This depends inversely on the surface temperature (T) of the star. Now, $\lambda_{\text{blue}} < \lambda_{\text{yellow}} < \lambda_{\text{red}}$. Thus, star A (bluish) radiates the shortest wavelength and must be at the highest temperature, while star B (reddish) must be at the lowest temperature

127. For $A \to B$, volume is constant

$$\therefore \frac{P_A}{T_A} = \frac{P_B}{T_B}$$

$$P_B = \frac{T_B}{T_A} \times P_A$$

$$P_B = \frac{500}{300} \times 1 = \frac{5}{3} \text{ atm} \quad \dots \text{(i)}$$

For $B \to C$, adiabatic process

$$\therefore \frac{T_C^{\gamma}}{P_C^{\gamma-1}} = \frac{T_B^{\gamma}}{P_B^{\gamma-1}}$$

$$T_C = \left(\frac{P_C}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \times T_B$$

$$= \left[\frac{1}{5/3}\right]^{\frac{(5/3)-1}{(5/3)}} \times 500 \quad \text{(Using (i))}$$

$$= \left(\frac{3}{5}\right)^{2/5} \times 500 \quad \dots \text{(ii)}$$

For $C \to A$, pressure is constant

$$\therefore \frac{V_C}{T_C} = \frac{V_A}{T_A}$$

$$V_C = V_A \times \frac{T_C}{T_A} = 4.9 \times \left(\frac{3}{5}\right)^{2/5} \times 500 \times \frac{1}{300} \quad \text{(Using (ii))}$$

$$= 4.9 \times 0.81 \times \frac{5}{3} = 6.6 \text{ L}$$

128. Velocity of transverse wave along string $= v$

$v = \sqrt{\dfrac{T}{\mu}}$ where $\mu =$ mass per unit length of string

$$v = \sqrt{\frac{500}{0.2}} = \sqrt{\frac{500 \times 10}{2}} = \sqrt{2500} = 50 \text{ m s}^{-1}$$

129. $W = \left(3\hat{i} - 2\hat{j} + 4\hat{k}\right) \cdot \left(-10\hat{i} + 2\hat{j} + 9\hat{k}\right)$

$= -30 - 4 + 36 = 2$ units

130. From $\sqrt{x} + 3 = t,\ x = (t-3)^2$

Now, $v = \dfrac{dx}{dt} = 2(t-3)$

At $t = 0,\ v_1 = 2(-3) = -6$

At $t = 6,\ v_2 = 2(6-3) = 6$

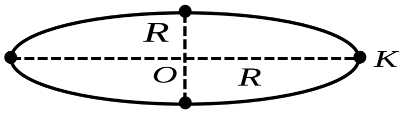
Work done $=$ Change in K.E.

$= \dfrac{1}{2} m\left(v_2^2 - v_1^2\right) = \text{zero}$

131. $L = mr^2\omega$. For given m and ω, $L \propto r^2$. If r is halved, the angular momentum L becomes one-fourth

132. According to the theorem of parallel axes, moment of inertia of disc about an axis passing through K and perpendicular to plane of disc, of following figure is

$= \dfrac{1}{2}MR^2 + MR^2 = \dfrac{3}{2}MR^2$



Total moment of inertia of the system

$= \dfrac{3}{2}MR^2 + m(2R)^2 + m\left(\sqrt{2}R\right)^2 + m\left(\sqrt{2}R\right)^2$

$= (3m + 16M)\dfrac{R^2}{2}$

133. Fundamental frequency of an open pipe $v_o = \dfrac{v}{2l}$ and of a closed pipe of the same length

$$v_c = \frac{v}{4l} = \frac{1}{2}\frac{v}{2l} = \frac{v_o}{2}$$

Here, $3v_c = v_o + 100$

Or $3\dfrac{v_o}{2} = v_o + 100$ or $v_o = 200$ Hz

134. When an source approaches a stationary observer, the frequency heard by the observer is given by

$$\nu = \nu_0\left(\frac{v}{v - v_s}\right)$$

Where,

ν_0 = source frequency

v = speed of sound

v_s = speed of source

As per question

$$\nu_1 = \frac{\nu_0 340}{(340-34)} \text{ and } \nu_2 = \frac{\nu_0 340}{(340-17)}$$

$$\therefore \frac{\nu_1}{\nu_2} = \frac{(340-17)}{(340-34)} = \frac{323}{306} = \frac{19}{18}$$

135. Escape velocity of the planet is

$$v_p = \sqrt{\frac{2GM_p}{R_p}}$$

Where M_p and R_p be the mass and radius of the planet respectively

Escape velocity of the earth is $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$

Where M_e and R_e be the mass and radius of the earth respectively

According to given problem

$v_p = 3v_e$ and $R_p = 4R_e$

$$\therefore \sqrt{\frac{2GM_p}{4R_e}} = 3\sqrt{\frac{2GM_e}{R_e}}$$

$$\frac{M_p}{4R_e} = \frac{9M_e}{R_e}$$

$$M_p = 36M_e = 36 \times 6 \times 10^{24} \text{ kg}$$

$$= 216 \times 10^{24} \text{ kg} = 2.16 \times 10^{26} \text{ kg}$$

136. As is clear from figure given in the question two capacitors each of capacity $\dfrac{\varepsilon_0 A}{d}$ are connected in parallel.

$$\therefore C = \frac{2\varepsilon_0 A}{d}$$

137. A cube has six faces. Therefore, electric flux coming out from any face $= \dfrac{Q \times 10^{-6}}{6\varepsilon_0}$

138. Electrostatic properties of a conductor are as follows:

(i) Electrostatic field is zero inside a charged conductor or neutral conductor

(ii) Electrostatic field at the surface of a charged conductor must be normal to the surface at every point

(iii) There is no net charge at any point inside the conductor and any excess charge must reside at the surface

(iv) Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface

(v) Electric field at the surface of a charged conductor is $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$

Where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction

139. As $\dfrac{4}{3}\pi R^3 = n \times \dfrac{4}{3}\pi r^3 \quad \therefore R = n^{1/3} r$

New potential, $V' = \dfrac{nq}{4\pi\varepsilon_0 R} = \dfrac{nq}{4\pi\varepsilon_0\left(n^{1/3} r\right)}$

$$= n^{2/3}\frac{q}{4\pi\varepsilon_0 r} = n^{2/3} V$$

140. According to Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \qquad \dots (i)$$

Suppose force between the charges is same when charges are r' distance apart in dielectric

$$\therefore F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{Kr'^2} \qquad \dots (ii)$$

From (i) and (ii), $Kr'^2 = r^2$ or $r = \sqrt{K}r'$

Thus distance r' of dielectric is equivalent to $\dfrac{r}{\sqrt{K}}$ distance of air, In the given situation, force between the charges would be

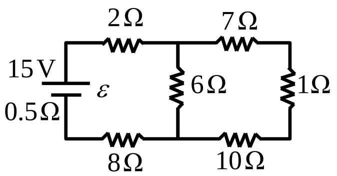
$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\left(\frac{r}{2}+\sqrt{4}\frac{r}{2}\right)^2} = \frac{4}{9}\frac{q^2}{4\pi\varepsilon_0 r^2}$$

$$F' = \frac{4}{9}F$$

141.The direction of electric field intensity $\left(\vec{E}\right)$ at a point on the equatorial line of electric dipole of dipole moment $(\vec{p})$ is perpendicular to equatorial line and opposite to $(\vec{p})$

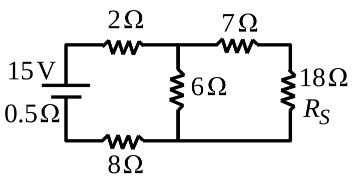
142.The resistances of $7\ \Omega, 1\ \Omega$ and $10\ \Omega$ are in series, their equivalent resistance is

$R_S = 7\ \Omega + 1\ \Omega + 10\ \Omega = 18\ \Omega$



The equivalent circuit is shown in figure (b)

Now as $R_S = 18\ \Omega$ and $6\ \Omega$ are in parallel and is equivalent to $\frac{6\times18}{6+18} = 4.5\ \Omega$



The equivalent circuit diagram is shown in figure (c)

So the equivalent resistance of the circuit will be

$R_{eq} = 0.5 + 2 + 4.5 + 8 = 15\ \Omega$

Current drawn from the battery is

$$I = \frac{V}{R_{\text{eq}}} = \frac{15\ \text{V}}{15\ \Omega} = 1\ \text{A}$$

143.Applying Kirchhoff's first law,

$I = 2 + 2 - 1 - 1.3 = 1.7\ \text{A}$

144. $R = \frac{V^2}{P}$ or $\frac{R_1}{R_2} = \frac{P_2}{P_1} = \frac{300}{500} = \frac{3}{5}$

145. $\frac{P}{Q} = \frac{20}{80} = \frac{1}{4}$ … (i)

$\frac{P+15}{Q} = \frac{40}{60} = \frac{2}{3}$ … (ii)

Divide (i) by (ii), we get

$$\therefore\ \frac{P}{P+15} = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$$

Or $8P = 3P + 45$ or $5P = 45$ or $P = 9\ \Omega$

146.Resistance of arm $AFE = 3 + 3 = 6\ \Omega$

The arms AFE and AE are in parallel, their effective resistance $= \frac{6\times6}{6+6} = 3\ \Omega$

Resistance of arm $AED = 3 + 3 = 6\ \Omega$

Effective resistance between A and D of portions

$AFED = \frac{6\times6}{6+6} = 3\ \Omega$

Effective resistance between A and C of portion

$$AFEDC = \frac{(3+3)\times6}{(3+3)+6} = 3\ \Omega$$

Resistance of arm $AFEDCB = 3 + 3 = 6\ \Omega$

For resistance between A and B, $6\ \Omega$ and $3\ \Omega$ are in parallel

Thus effective resistance, $R_P = \frac{3\times6}{3+6} = 2\ \Omega$

147.Charge for a given time $=$ area under the current-time graph for the given time

$\therefore\ q_1 = 2\times1 = 2\ \text{C},\ q_2 = 1\times2 = 2\ \text{C};$

$q_3 = \frac{1}{2}\times2\times2 = 2\ \text{C}$

$\therefore\ q_1 : q_2 : q_3 = 2:2:2 = 1:1:1$

148.Given initial drift velocity $\left(v_{d_1}\right) = v$; initial current $(I_1) = I$, initial radius $(r_1) = r$, final radius $(r_2) = 2r$ and final current $(I_2) = 2I$. The drift velocity is given by

$$v_d = \frac{1}{neA} = \frac{1}{ne\times\pi r^2}$$

i.e., $v_d \propto \frac{1}{r^2}$

Therefore, $\frac{v_{d_1}}{v_{d_2}} = \frac{I_1}{I_2} \times \frac{r_2^2}{r_1^2} = \frac{I}{2I} \times \frac{(2r)^2}{r^2} = 2$

$\therefore\ v_{d_2} = \frac{v_{d_1}}{2} = \frac{v}{2}$

149.According to Einstein's photoelectric equation

$\frac{hc}{\lambda} = \phi_0 + \text{KE}_{\text{max}}$

$\therefore$ where ϕ_0 is a work function of a metal

$\therefore\ K_1 = \frac{hc}{\lambda_1} - \phi_0$ ... (i)

$K_2 = \frac{hc}{\lambda_2} - \phi_0$ ... (ii)

Or $K_1 - K_2 = hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$

$= hc\left[\frac{1}{3\lambda_2} - \frac{1}{\lambda_2}\right] = -\frac{2hc}{3\lambda_2}$ (Given $\lambda_1 = 3\lambda_2$)

$= -\frac{2}{3}(K_2 + \phi_0)$ (using (ii))

Or $K_1 = K_2 - \frac{2}{3}K_2 - \frac{2}{3}\phi_0 = \frac{K_2}{3} - \frac{2}{3}\phi_0$

Or $K_1 < \frac{K_2}{3}$

150.K.E. gained by a charge q after being accelerated through a potential difference V volt is given by

$qV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2qV}{m}}$ and $mv = \sqrt{2mqV}$

De Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

Now, $\lambda_p = \frac{h}{\sqrt{2m_p q_p V_p}}$

For α particle

$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}}$ $\therefore$ $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$

Putting $V_\alpha = V_p$, we get

$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4\times 2}{1\times 1}} = \sqrt{8} = 2\sqrt{2}$

151.Given, $V_0 = 120$ V

$\therefore\ V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 84.8$ V

152. $I = \frac{|\varepsilon|}{R} = \frac{NA\frac{dB}{dt}}{R}$

$= \frac{20\times\left(25\times 10^{-4}\right)\times 1000}{100} = 0.5$ A

153. $|\varepsilon| = L\frac{dI}{dt} = \frac{0.5(10-0)}{2} = 2.5$ volt

154. $X_L = 2\pi\nu L$

$= 2\times 3.14\times 50\times 25.48\times 10^{-3} = 8\ \Omega$

$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\times 3.14\times 50\times 796\times 10^{-6}} = 4\ \Omega$

$\therefore$ Impedance of the circuit is

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(3)^2 + (8-4)^2} = 5\ \Omega$

155.Here, $X_L = \omega L = 2\pi\nu L = 2\pi\times 50\times 0.7 = 220\ \Omega$

$R = 220\ \Omega$

$Z = \sqrt{R^2 + X_L^2} = \sqrt{220^2 + 220^2} = 220\sqrt{2}\ \Omega$

$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220\text{ V}}{220\sqrt{2}\ \Omega} = \frac{1}{\sqrt{2}}$ A

$\sin\phi = \frac{X_L}{Z} = \frac{220}{220\sqrt{2}} = \frac{1}{\sqrt{2}}$

Wattless current $= I_{\text{rms}}\sin\phi$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$ A $= 0.5$ A

156.Here, $V_R = 100$ V, $R = 1000\ \Omega$

$C = 2\times 10^{-6}$ F, $\omega = 200$ rad s^{-1}

The current in the circuit is

$I = \frac{V_R}{R} = \frac{100\text{ V}}{1000\ \Omega} = 0.1$ A

At resonance

$V_L = V_C = IX_C = \frac{I}{\omega C} = \frac{0.1}{200\times 2\times 10^{-6}} = \frac{1000}{4} = 250$ V

157. $\alpha = 0.96$, $I_E = 7.2$ mA

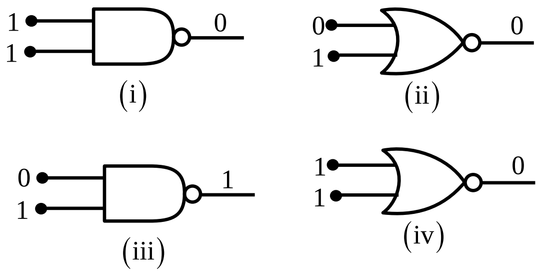
$\alpha = \frac{I_C}{I_E}$ or $\quad I_C = \alpha I_E$

$I_E = 0.96 \times 7.2 \;= 6.91$ mA

$I_E = I_C + I_B$; $I_B = I_E - I_C = 7.2 - 6.91 = 0.29$ mA

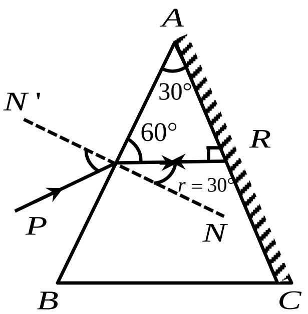
158.To make $n-$type semiconductor, the doping of pentavalent material (i.e., phosphorus) is done in germanium crystal.

159.Sol:



160.FM broadcast is preferred over AM broadcast as the noise is much less and reproduction is of much better quality

161.Sol:



It is clear from the figure that the ray will retrace the path when the refracted ray QR is incident normally on the polished surface AC. Thus angle of refraction, $r = 30°$

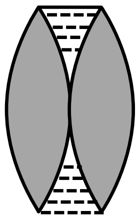
$\mu = \frac{\sin}{\sin r}$

$\therefore \; \sin i = \mu \sin r$

$\sin i = \sqrt{2} \times \sin 30° = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$

$\therefore \; i = 45°$

162.If R is radius of curvature of each surface of convex lens



Then $\frac{1}{f_1} = \frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{R}$

$f_1 = f_2 = R = f$,

For water in between lens

$\frac{1}{f_3} = \left(\frac{4}{3} - 1\right)\left(\frac{-1}{R} - \frac{1}{R}\right) = \frac{1}{3}\left(\frac{-2}{f}\right) = \frac{-2}{3f}$

Using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

We get $\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{2}{3f} \Rightarrow F = \frac{3}{4} f$

163.Here, $d = 0.2 \text{ mm} = 0.2 \times 10^{-3}$ m

$D = 80 \text{ cm} = 80 \times 10^{-2}$ m

$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10}$ m

For a dark fringe

$y_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}$ where $n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$

For fifth dark fringe $n = 4$

$\therefore \; y_4 = \frac{9}{2}\frac{\lambda D}{d}$

$= \frac{9 \times 6000 \times 10^{-10} \times 80 \times 10^{-20}}{2 \times 0.2 \times 10^{-3}} = 10.8$ mm

164.From $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Where, $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

$I_R = I + I + 2I\cos\frac{\pi}{2} = 2I$

At the central bright fringe, $I_R' = 4I$

$\therefore \; \frac{I_R}{I_R'} = \frac{2I}{4I} = 0.5$

165. Let I_0 be the intensity of unpolarised light, then intensity of light from first transparent thin plate of a polaroid is

$$I = \frac{I_0}{2}$$

Now this light will pass through the second similar plate whose axis is inclined at an angle of $30°$ to that of first plate.

According to Malus law, the intensity of emerging light is

$$I' = I\cos^2 30° = \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8}I_0$$

$$\therefore \frac{I'}{I_0} = \frac{3}{8}$$

166. Hysteresis is shown by the ferromagnetic material

167. As $\mu_r = 1 + \chi_m$

$$\therefore \chi_m = \mu_r - 1 = 5500 - 1 = 5499$$

168. $S = \frac{I_g G}{I - I_g} = \frac{0.01 \times 25}{10 - 0.01} = \frac{25}{999}\ \Omega$

169. $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$

Or $I = \frac{2Br}{\mu_0} = \frac{2 \times \pi \times 0.5}{4\pi \times 10^{-7}} = 2.5 \times 10^6\ \text{A}$

170. $\frac{mv^2}{R} = qvB$

Or $B = \frac{mv}{qR} = \frac{\left(9.1 \times 10^{-31}\ \text{kg}\right)\left(10^6\ \text{m s}^{-1}\right)}{\left(1.6 \times 10^{-19}\ \text{C}\right)\left(0.5\ \text{m}\right)}$

$= 1.13 \times 10^{-5}\ \text{T}$

171. $B_{axis} = \frac{\mu_0}{4\pi} \times \frac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}}$

At centre, $B_{centre} = \frac{\mu_0}{4\pi} \times \frac{2\pi I}{R}$

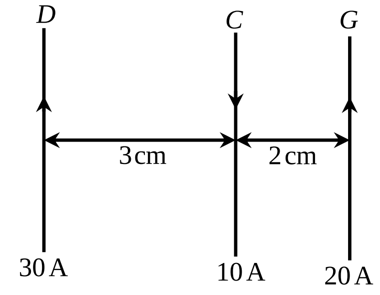
In the given problem,

$$\frac{\mu_0}{4\pi} \times \frac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}} = \frac{1}{8}\left[\frac{\mu_0}{4\pi} \times \frac{2\pi I}{R}\right]$$

Or $\left(R^2 + x^2\right)^{3/2} = 8R^3$

Solving, we get $x = \sqrt{3}R$

172. Sol:



The magnetic field due to wire D at wire C is

$$B_D = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{r} = \frac{10^{-7} \times 2 \times 30}{0.03} = 2 \times 10^{-4}\ \text{T}$$

Which is directed into the page

The magnetic field due to wire G at C is

$$B_G = \frac{10^{-7} \times 2 \times 20}{0.02} = 2 \times 10^{-4}\ \text{T}$$

Which is directed out of the page

Therefore, the field at the position of the wire C is

$$B = B_G - B_D = 2 \times 10^{-4} - 2 \times 10^{-4} = \text{zero}$$

The force on $25\ \text{cm}$ of wire C is

$$F = BIl\sin\theta = \text{zero}$$

173. The magnetic field B at a distance r from a long straight wire carrying current I is given by

$$B = \frac{\mu_0 2I}{4\pi r} = \frac{\mu_0 I}{2\pi r}$$

$$B \propto \frac{1}{r}$$

174. In $_{88}Ru^{226}$ nucleus,

Number of protons $= 88$

Number of neutrons $= (226 - 88) = 138$

175. Jump to second orbit leads to Balmer series. When an electron jumps from 4th orbit to 2nd orbit shall give rise to second line of Balmer series

176. $_{92}U^{238} \longrightarrow {}_{82}Pb^{206}$

Let the number of α and β particles emitted be x and y respectively. Then

$${}_{92}U^{238} \longrightarrow {}_{82}Pb^{206} + x\ {}_{2}He^{4} + y\ {}_{-1}e^{0}$$

Equating the mass and atomic numbers on both sides, we get

$$206 + 4x = 238 \Rightarrow x = \frac{238-206}{4} = 8$$

$$82 + 2x - y = 92 \Rightarrow y = 82 + 2\times 8 - 92 = 6$$

$\therefore$ The number of α and β particles emitted are 8 and 6 respectively

177. $R = R_0\left(\frac{1}{2}\right)^n$

Where n is the number of half-lives

Given: $R = \frac{R_0}{16}$ $\therefore$ $\frac{R_0}{16} = R_0\left(\frac{1}{2}\right)^n$ or $n = 4$

Four half-lives are equivalent to $8\ \text{s}$. Hence, $2\ \text{s}$ is equal to one half-life. So, in one half-life activity will fall half of $1600\ \text{Bq}$, i.e., $800\ \text{Bq}$

178. According to radioactive decay law

$$N = N_0 e^{-\lambda t}$$

Or $t = \dfrac{\ln\left(\frac{N_0}{N}\right)}{\lambda}$

Or $t = \dfrac{T_{1/2}\ln\left(\frac{N_0}{N}\right)}{\ln 2}$ $\therefore$ $t \propto \ln\frac{N_0}{N}$

$$\frac{6}{10} = \frac{\ln\left(\frac{8}{1}\right)}{\ln\left(\frac{N_0}{N}\right)}$$

$$\ln\frac{N_0}{N} = \ln 8^{5/3} = \ln 32$$

Or $\frac{N_0}{N} = 32$ or $\frac{N}{N_0} = \frac{1}{32}$

$\therefore$ Fraction that decays $= 1 - \frac{1}{32} = \frac{31}{32}$

179. Angular momentum

$$L = mvr = \frac{nh}{2\pi} \text{ or } mv = \frac{nh}{2\pi r}$$

Now, $r \propto n^2$

$$\therefore\ mv = p \propto \frac{nh}{2\pi \times n^2}$$

$$p \propto \frac{h}{2\pi n} \text{ or } p \propto \frac{1}{n}$$

Energy, $E \propto \frac{1}{n^2}$

180. $\vec{E}\times\vec{B}$