Deeksha - K CET

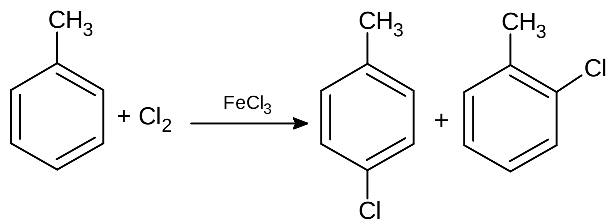
Subject	Topic	**Mock Test – 5**	Date
C + M + P	Complete Syllabus	CET – 12 – CT	

C+M+P Key Answers:

1. b	2. a	3. c	4. c	5. d	6. a	7. c	8. d	9. d	10. d
11. b	12. c	13. a	14. d	15. b	16. d	17. a	18. c	19. b	20. d
21. a	22. a	23. d	24. d	25. b	26. c	27. a	28. d	29. d	30. b
31. d	32. c	33. a	34. a	35. c	36. b	37. b	38. b	39. b	40. a
41. d	42. c	43. c	44. c	45. b	46. d	47. b	48. b	49. c	50. c
51. c	52. a	53. b	54. d	55. a	56. b	57. a	58. c	59. d	60. c
61. d	62. b	63. a	64. b	65. b	66. a	67. b	68. b	69. d	70. c
71. d	72. d	73. b	74. d	75. a	76. a	77. b	78. d	79. a	80. d
81. c	82. a	83. b	84. a	85. d	86. a	87. d	88. c	89. c	90. a
91. c	92. c	93. b	94. b	95. d	96. c	97. a	98. b	99. a	100.c
101.c	102.d	103.b	104.b	105.b	106.d	107.d	108.d	109.a	110.c
111.a	112.c	113.c	114.d	115.b	116.d	117.b	118.b	119.a	120.b
121.a	122.d	123.c	124.b	125.a	126.d	127.a	128.b	129.d	130.a
131.b	132.a	133.a	134.a	135.d	136.d	137.b	138.b	139.d	140.b
141.c	142.d	143.d	144.a	145.d	146.a	147.a	148.d	149.a	150.c
151.b	152.d	153.b	154.a	155.b	156.b	157.b	158.b	159.c	160.d
161.a	162.c	163.a	164.b	165.b	166.a	167.a	168.c	169.d	170.c
171.b	172.d	173.c	174.a	175.d	176.b	177.c	178.c	179.c	180.a

Chemistry Solutions:

1. Deficiency of Vitamin C causes scurvy or bleeding gums.
2. Sol:



3. H_3PO_2 behaves as a strong reducing agent as it contains two $P-H$ bonds.
4. $E_n = 0.529n^2 Å$

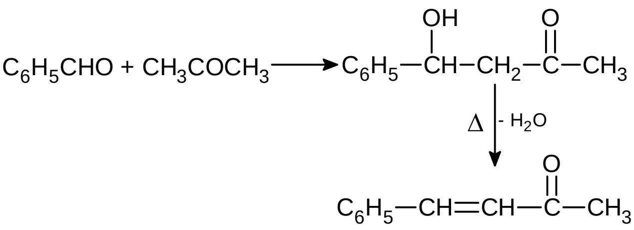
   $\therefore\ r_1 : r_2 : r_3 = 1^2 : 2^2 : 3^2 = 1:4:9$

5. Addition of protective colloid is a method of prevention of coagulation.
6. In (I), Cu^+ is oxidised as well as reduced. In (II), MnO_4^- ions are oxidised as well as reduced.
7. Methanol is also called wood spirit. Ethyl alcohol is obtained by fermentation. $2°$ alcohol can be converted to ketone by heating with copper at $573K$.

   Reaction of alkyl halide with sodium alkoxide is termed as Williamson's synthesis.

8. Inert electrode does not participate in the chemical reaction and acts only as source or sink for electrons and provides surface either for oxidation or for reduction reaction.

9. Sol:



10. During addition of an electron to O_2 to form O_2^-, the additional electron is added to the less stable anti-binding orbital. Therefore, it is easier to remove an electron from O_2^- ion to form neutral O_2 molecule.

Thus IE_1 of O_2^- is the lowest.

11. Rate $= k[N_2O_5]$ (first order as unit of rate constant is s^{-1}

$$[N_2O_5] = \frac{\text{rate}}{k} = \frac{1.4\times10^{-5}\,\text{mol L}^{-1}s^{-1}}{2\times10^{-5}\,\text{s}^{-1}} = 0.7\ \ \text{mol L}^{-1}$$

12. $2.24\times10^{-3}\text{m}^3$ of a gas $= 2.24\text{L}$ at STP

Thus, 2.24L of the gas at STP weigh $= 4.4g$

$\therefore$ 22.4 L of the gas at STP will weigh $= 44g$

But 2.24L of a gas at STP = Molar mass of the gas

$\therefore$ Molar mass of the gas $= 44g$

Molar mass of $C_3H_8 = 36 + 8 = 44$

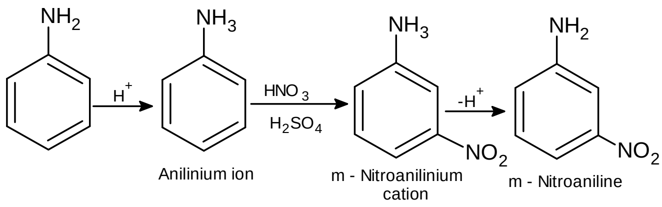
13. The single $N-N$ bond is weaker than the single $P-P$ bond because of high inter-electronic repulsions of the non-bonding electrons, owing to the small bond length.

14. In roasting, definite chemical changes like oxidation, chlorination, etc. take place. Impurities are removed in the form of corresponding volatile oxides.

15. HVZ reaction is given by only those carboxylic acids which have $\alpha-H$ atom. CH_3COOH has three $\alpha-H$ atoms but formic acid does not have $\alpha-$ hydrogen atom hence formic acid cannot be halogenated.

16. Urea is non-electrolyte, hence will not dissociate to give ions.

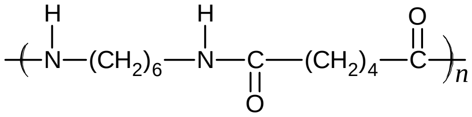
17. Anilinium ion formed by protonation of aniline deactivates $o-$ and $p-$ positions hence substitution takes place at $m-$position.



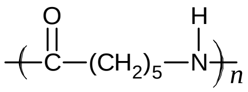
18. $K = k_f / k_b,\ 1.5 = k_f / \left(7.5\times10^{-4}\right)$

or $k_f = 1.125\times10^{-3}$

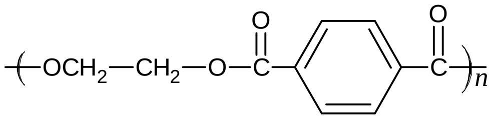
19. Sol:



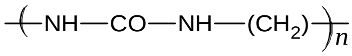
Hexamethylenediamine + adipic acid



- Caprolactam



Ethylene glycol + terephthalic acid



–Urea + formaldehyde

20. Molar conductivity is inversely proportional to molarity.

21. $\underset{(P)}{NH_4NO_3} \xrightarrow{\Delta} \underset{(Q)}{N_2O} + \underset{(R)}{2H_2O}$

$P_4 + \underset{(Q)}{10N_2O} \rightarrow \underset{(S)}{P_4O_{10}} + 10N_2$

22. Thiosuphate $\left(S_2O_3^{2-}\right)$ is a unidentate ligand.

23. By peptization which is the process of converting a precipitate into colloidal sol by shaking it with dispersion medium in the presence of a small amount of electrolyte. The electrolyte used for this purpose is called peptizing agent.

24. $CH_3(CH_2)_{10}CH_2OSO_3^-Na^+$ - Anionic detergent

$\left[CH_3(CH_2)_{15}N(CH_3)_2-CH_3\right]^+Br^-$ - Cationic detergent

$CH_3(CH_2)_{16}COO(CH_2CH_2O)_nCH_2CH_2OH$ - Non-ionic detergent

$C_{17}H_{35}COONa$ - Soap

25. 5 moles of C have a partial pressure of 1.5atm

$\therefore$ 2 moles of A will have partial pressure of

$\frac{1.5}{5}\times 2=0.6$ atm

3 moles of B will have partial pressure of

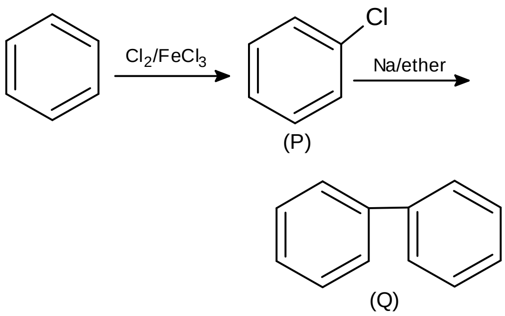
$\frac{1.5}{5}\times 3=0.9$ atm

10 moles of D will have partial pressure of

$\frac{1.5}{5}\times 10=3$ atm

$\therefore$ Total pressure $=1.5+0.6+0.9+3=6\,\text{atm}$

26. Sol:



27. The geometry is tetrahedral.

28. Combination of H_2 and Br_2 to give HBr is a fractional order reaction. The rate law is given by

Rate $=k[H_2][Br_2]^{1/2}$

29. Chlorobenzene does not undergo hydrolysis on treatment with aqueous $NaOH$ at $298K$.

30. $\Delta T_f=\frac{K_f\times W_B}{M_B\times W_A}$

For cane sugar solution, $2.15K=\frac{K_f\times 5}{342\times 0.095}$

$(95\,g \text{ of water}=0.095\,kg)$

For glucose solution, $\Delta T_f=\frac{K_f\times 5}{180\times 0.095}$

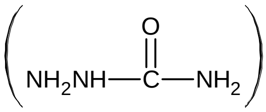
$\frac{\Delta T_f}{2.15}=\frac{K_f\times 5}{180\times 0.095}\times\frac{342\times 0.095}{K_f\times 5}$

$\Delta T_f=\frac{342}{180}\times 2.15=4.085\,K$

Freezing point of glucose solution

$=273.15-4.085=269.07\,K$

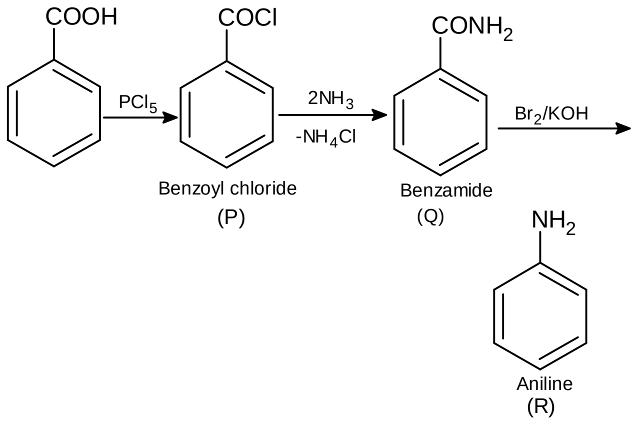
31. Semicarbazone is formed when semicarbazide



reacts with an aldehyde or ketone.

32. $Ca_3(PO_4)_2$ is added to table salt $(NaCl)$ to make it flow freely (moisture free) in rainy season.

33. Sol:



34. Here H_2O_2 reduces Ag_2O to Ag metal and is a reducing agent.

35. Peptide linkage is present in proteins.

Nucleic acid is a polymer of nucleotides.

Hydrolysis of cane sugar is also called inversion.

Starch is a polysaccharide.

36. Mischmetal is an alloy of lanthanoid metal.

$TiCl_4+Al(C_2H_5)_3$ is termed as Ziegler Natta catalyst.

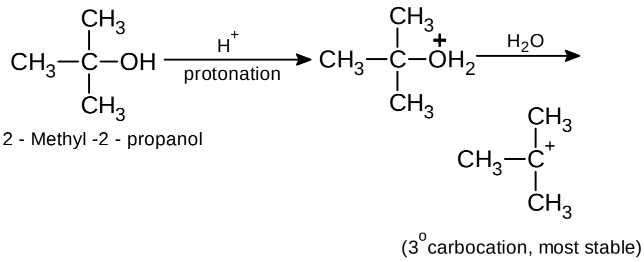
Brass is an alloy of Cu and Zn

Bronze is an alloy of Cu and Sn.

37. (iii) and (iv) will form ideal solutions hence do not form azeotropes. Azeotropes have same composition in liquid and vapour form when distilled.

38. Sulphur shows maximum covalency of six.

39. The tertiary carbocation formed during dehydration of $2-$methyl $-2-$ propanol is most stable.



40. $\Delta_r S^\circ = S^\circ_{XY_3} - \left[\frac{1}{2} S^\circ_{X_2} + \frac{3}{2} S^\circ_{Y_2}\right]$

$= 50 - \left[\frac{1}{2} \times 60 + \frac{3}{2} \times 40\right]$

$= 50 - (30 + 60)$

$= -40 \ \text{JK}^{-1}\text{mol}^{-1}$

$\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$.

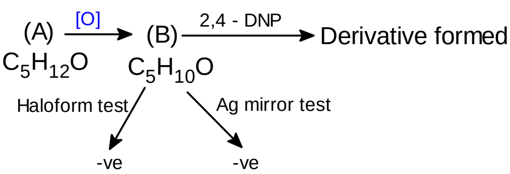
At equilibrium,

$\Delta_r G^\circ = 0$

$\therefore \ T = \frac{\Delta_r H^\circ}{\Delta_r S^\circ} = \frac{-30,000 \,\text{J mol}^{-1}}{-40 \,\text{JK}^{-1}\text{mol}^{-1}} = 750\,\text{K}$

41. $SnCl_2$ is the most commonly used reducing agent in the laboratory.

42. Sol:



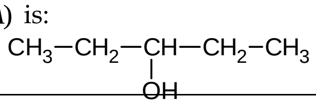
Since (B) on reaction with $2,4-$DNP forms a derivative, it implies that (B) has $>C=O$ Group.

(B) gives -ve Tollen's test, hence it is not an aldehyde, but it is a ketone.

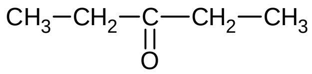
(B) gives -ve haloform test, thus it is not a methyl ketone.

(B) is formed from the oxidation of (A), thus (A) is a 2° alcohol, and among the given options:

(A) is:



and $\therefore (B)$ is



43. At anode : $Fe \rightarrow Fe^{2+}(0.001\text{M}) + 2e^-$

At cathode : $2H^+(1\text{M}) + 2e^- \rightarrow H_2(1\text{atm})$

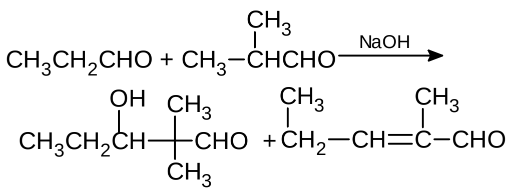
Net reaction: $Fe + 2H^+ \rightarrow Fe^{2+} + H_2$

Nernst equation for the given cell,

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.591}{2}\log\frac{[Fe^{2+}][H_2]}{[Fe][H^+]^2}$$

44. $H_2O_2 + \underset{\text{From } H_2SO_4}{|O|} \longrightarrow H_2O + O_2$

45. Sol:



46. Octahedral sites are created by overlapping two equilateral triangles with apices in opposite directions.

47. 0.2 moles of $AgCl$ are obtained when 0.1mol $COCl_3(NH_3)_5$ is treated with excess of $AgNO_3$ which shows that one molecule of the complex gives two Cl^- ions in solution. Thus, the formula of the complex is $[Co(NH_3)_5Cl]Cl_2$, i.e., $1:2$ electrolyte.

48. $PbI_2 \rightleftharpoons Pb^{2+} + 2I^-$

Thus, Pb^{2+} ion concentration is half of the I^- ion concentration

$[I^-] = 2.6 \times 10^{-3}\text{M} \ \therefore \ [Pb^{2+}] = 1.3 \times 10^{-3}\text{M}$

$K_{sp} = [Pb^{2+}][I^-]^2$

$= (1.3 \times 10^{-3})(2.6 \times 10^{-3})^2 = 8.8 \times 10^{-9}$

49. Potassium dichromate is prepared from chromite, $FeCr_2O_4$.

50. Lower the activation energy, faster is the reaction.

51. For all elements which have atomic number more than 7 (beyond nitrogen) the energy of $\sigma 2p_z$ is lower than $\pi 2p_x$ and $\pi 2p_y$-orbitals.

52. Diethyl ether $(CH_3CH_2OCH_2CH_3)$ and $n-$propyl methyl ether $(CH_3OCH_2CH_2CH_3)$ are metamers.

53. The upper limit concentrations of lead, sulphate, nitrate and fluoride are given by $50\,ppb$, $500\,ppm$, $50\,ppm$ and $10\,ppm$ respectively.

54. Electron affinity of Cl is maximum. The correct trend is $Cl > F > Br > I$.

55. $$\underset{\substack{100 \\ 20g}}{CaCO_3} + \underset{\substack{2\times36.5g \\ 20g}}{2HCl} \rightarrow CaCl_2 + H_2O + \underset{\substack{44g \\ 20\times\frac{44}{100}=8.8g}}{CO_2}$$

$CaCO_3$ is the limiting reagent.

56. Aspartame gets dissociated at cooking temperature.

57. $$CH_3CH_2C\equiv CH \xrightarrow[\Delta]{KOH(alc.)} [CH_3CH=C=CH_2] \rightarrow CH_3C\equiv CCH_3$$

58. The process is known as liquation.

59. Helium is used in in filling tubes of aeroplane tyres.

60. Frenkel defect is exhibited by compound having low coordination number and compound having large difference in size of cation and anion. This defect is not found in alkali metal halides because cations and anions have almost equal size and cations cannot be accommodated in interstitial sites.

Mathematics Solutions:

61. $\cos\left(2\sin^{-1}\frac{1}{2}\right) = \cos\left(2\cdot\frac{\pi}{2}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

62. By observation $x=1$ satisfies the equation

For, $\cos^{-1}1 + \sin^{-1}\frac{1}{2} = 0 + \frac{\pi}{6} = \frac{\pi}{6}$

63. $\cos^{-1}(-1) - \sin^{-1} = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

64. $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$

$\frac{\pi}{2} + \cot^{-1}x = \frac{2\pi}{3}\left(\because\ \tan^{-1}x + \cot^{-1}x = \frac{2\pi}{2}\right)$

$\Rightarrow \cot^{-1}x = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6} \Rightarrow x = \sqrt{3}$

65. Standard result

$p^2 + q^2 + r^2 + 2pqr = 1$

66. A is singular $\Rightarrow |A| = 0$

$\therefore\ 8(7\lambda-16) + 6(-6\lambda+8) + 2(24-14) = 0$

$\Rightarrow 56\lambda - 128 - 36\lambda + 48 + 20 = 0$

$\Rightarrow 20\lambda - 60 = 0 \Rightarrow \lambda = 3$

67. We have, for A to be symmetric,

$x-1=4$ and $2x-3=x+2 \Rightarrow x=5$

68. X is of order $2\times n$ and Z is of order $2\times p$ and $n=p$. Thus $7X-5Z$ is of order $2\times n$ or $2\times p$.

69. Putting $x=0$, we get

$$e = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix}$$

R.H.S. is the det. Of a skew symmetric matrix of odd order $\Rightarrow |A| = 0$

$e = 0$

70. α, β, γ are the roots of $x^3 + px + q = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

Consider, $R_1 \rightarrow R_1 + (R_2 + R_3)$, then we have

$$\begin{vmatrix} \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

71. A matrix A will have inverse iff $|A| \neq 0$.

Consider, $\begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} = 0$

$\Rightarrow 2(6-5) - \lambda(-5) - 3(-2) = 0$

$\Rightarrow 2 + 5\lambda + 6 = 0$

$\Rightarrow 5\lambda = -8 \Rightarrow \lambda = -\frac{8}{5}$

Then, $|A| \neq 0$ for $\lambda \neq -\frac{8}{5}$.

72. If A is an 3×3 matrix then $\det(nA) = n^3(\det A)$

$\therefore\ \det(3A) = 3^3\cdot(\det A)\ \ \therefore k = 3^3 = 27$

73. If $n(A)=n$ and $n(B)=m$, then the number of mapping from A into B is m^n. Here $n=7, m=5$. Thus the required number is 5^7

74. The number of equivalence relation is $2^3=8$.

75. $f(x)=\dfrac{3x+2}{5x-3}$. This is of the form

$f(x)=\dfrac{ax+b}{cx-a}$. Thus $f^{-1}(x)=f(x)$.

76. Now, $f(-1)=3(-1)=-3 \quad (\because -1<1)$

$f(2)=2^2=4 \quad (\because -1<2<3)$

$f(4)=2(4)=8 \quad (\because 4>3)$

$\therefore f(-1)+f(2)+f(4)=9$

77. By data, $\lim\limits_{x\to0} f(x)=k$

$$\lim_{x\to0}\left(\frac{\sin x}{x}+\cos x\right)=k$$

$\Rightarrow 1+1=k \Rightarrow k=2$

78. When x is an integer, $x-[x]=0$. Thus $f(x)$ is discontinuous for all $x\in Z$.

79. $f(x)=\cos^{-1}\left[\dfrac{1-(\log x)^2}{1+(\log x)^2}\right]$

$f(x)=\cos^{-1}\left(\dfrac{1-t^2}{1+t^2}\right); t=\log x$

$f(x)=2\tan^{-1}t=2\tan^{-1}(\log x)$

$f'x=\dfrac{2}{1+(\log x)^2}\cdot\dfrac{1}{x}$

$\Rightarrow f'(e)=\dfrac{2}{2}\cdot\dfrac{1}{e}=\dfrac{1}{e}$

80. $y=\tan^{-1}(\sec x-\tan x)$

$y=\tan^{-1}\left(\dfrac{1-\sin x}{\cos x}\right)$

$y=\tan^{1}\left(\dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$

$y=\tan^{-1}\left(\dfrac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)=\tan^{-1}\left(\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)\right)$

$\Rightarrow y=\dfrac{\pi}{4}-\dfrac{x}{2}\Rightarrow\dfrac{dy}{dx}=-\dfrac{1}{2}$

81. $y=1+\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}+\ldots$ to ∞

By data $|x|>1\Rightarrow\dfrac{1}{x}<1$

$\therefore y=\dfrac{1}{1-\frac{1}{x}}=\dfrac{x}{x-1} \qquad$ (S_∞ of G.P)

$\dfrac{dy}{dx}=\dfrac{(x-1)-x}{(x-1)^2}=\dfrac{1}{(x-1)^{2x}}=-\dfrac{1}{x^2}\cdot\left(\dfrac{x}{x-1}\right)^2$

$\therefore \dfrac{dy}{dx}=-\dfrac{y^2}{x^2}$

82. We have, $\sqrt{\tan y}=\left(e^{\cos 2x}\right)\cdot\sin x \qquad \ldots$(i)

$\Rightarrow\dfrac{\sec^2 y}{2\sqrt{\tan y}}\dfrac{dy}{dx}=e^{\cos 2x}\cdot\cos x-2e^{\cos 2x}\cdot\sin 2x\cdot\sin x$

$\Rightarrow\dfrac{dy}{dx}=\dfrac{2\sqrt{\tan y}}{\sec^2 y}e^{\cos 2x}\sin x[\cot x-2\sin 2x]$

$=\dfrac{2\tan y}{\sec^2 y}(\cot x-2\sin 2x) \qquad$ (from (i))

$=\sin 2y(\cot x-2\sin 2x)$

83. We have, $f(x)=x+2$

$\Rightarrow f'(x)=1\Rightarrow f'(x)=1$ for all x

$\therefore f'[f(x)]=1$

84. Let (x_1,y_1) be the point on the curve $9y^2=x^3$ where the normal to the curve makes equal intercepts.

Slope of the normal $=-1$

$\Rightarrow-\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}=-1\Rightarrow-\dfrac{6y_1}{x_1^2}=-1\Rightarrow x_1^2=6y_1$

Now, $x_1^2=6y_1$

$\Rightarrow 9\left(\dfrac{x_1^4}{36}\right)=x_1^3\Rightarrow x^2(x-4)=0\Rightarrow x=4$

$x = 4 \Rightarrow y^2 = \frac{64}{9} \Rightarrow y = \pm\frac{8}{3}$.

$\therefore$ Point $= \left(4, \pm\frac{8}{3}\right)$

85. Now, $x = e^t \cos t$, $y = e^t \sin t$

$\frac{dy}{dx} = \frac{e^t(\cos t + \sin t)}{e^t(\cos t - \sin t)}$

$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{(2/\sqrt{2})}{0} \Rightarrow$ angle is $90°$.

86. By data $\frac{dr}{dt} = 0.01/\text{sec}$. $r = 12$

Now, Area $A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$= 2\pi \cdot 12 \cdot (0.01)$

$= 24\pi(0.01) = 0.24\pi$ sq.cm/sec.

87. $f(x) = (x+2)e^{-x}$

$f'(x) = -(x+2)e^{-x} + e^{-x}$

$= -e^{-x}(x+2-1) = -e^{-x}(x+1)$

$f'(x) > 0 \Rightarrow -(x+1) > 0$

$\Rightarrow (x+1) < 0 \Rightarrow x < -1$

$\therefore$ $f(x)$ is increasing in $(-\infty, -1)$

$f'(x) < 0 \Rightarrow -(x+1) < 0 \Rightarrow (x+1) > 0 \Rightarrow x > -1$

$\therefore$ $f(x)$ is decreasing in $(-1, \infty)$

88. $\frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right) = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\left(\sqrt{x} + x\sqrt{x}\right)}$

$\therefore$ $I = 2\int e^t \cdot dt$, where $t = \tan^{-1} e^x$

$= 2e^t = 2 \cdot e^{\tan^{-1}\sqrt{x}}$

89. Clearly, $dx\left(xe^x\right) = e^x(1+x)$

$\therefore$ $I = \int \sec^2\left(xe^x\right) \cdot e^x(1+x)dx = \tan\left(xe^x\right)$

90. $\int \frac{dx}{\sqrt{\sin^3 x \cdot \cos x}} dx = \int \frac{dx}{\sqrt{\tan^3 x \cdot \cos^4 x}}$

$= \int \frac{\sec^2 x}{(\tan x)^{3/2}} dx$

$= \int \frac{1}{\frac{1}{2}(\tan x)^{1/2}} = -\frac{2}{\sqrt{\tan x}}$

91. We have, $\frac{d}{dx}\left[2^{2^{2^x}}\right] = 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x \cdot (\log 2)^3$

$\therefore$ $I = 2^{2^{2^x}} \cdot \frac{1}{(\log 2)^3}$

92. $\int_0^1 x(1-x)^n\, dx = \frac{1}{(n+1)(n+2)}$

$\therefore$ $I = \int_0^1 x(1-x)^9\, dx = \frac{1}{10.11} = \frac{1}{110}$

93. $I = \int_0^{\pi/4} \frac{\sqrt{\tan x}}{\tan x \cdot \cos^2 x} dx$

$= \int_0^{\pi/4} \frac{\sec^2 x}{\sqrt{\tan x}} dx = 2\sqrt{\tan x}\Big]_0^{\pi/4} = 2$

94. $\sin x - \cos x < 0$ for $0 < x < \frac{\pi}{4}$

$\sin x - \cos x > 0$ for $\frac{\pi}{4} < x < \frac{\pi}{2}$

$\therefore$ $I = \int_0^{\pi/4} -(\sin x - \cos x)dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx$

$= \cos x + \sin x\Big]_0^{\pi/4} + \Big[-(\cos x + \sin x)\Big]_{\pi/4}^{\pi/2}$

$= \left(\frac{2}{\sqrt{2}} - 1\right) - \left(1 - \frac{2}{\sqrt{2}}\right)$

$= 2\sqrt{2} - 2 = 2\left(\sqrt{2} - 1\right)$

95. $I = \int_{-2}^{2}\left[\tan^{-1}\left(\frac{x+1}{x-1}\right) + \cot^{-1}\left(\frac{x+1}{x-1}\right)\right]dx$

$= \int_{-2}^{2} \frac{\pi}{2} dx = \frac{\pi}{2}[x]_{-2}^{2} = \frac{\pi}{2}(2+2) = 2\pi$

96. $y = 0 \Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x-2)(x-5) = 0$

$\Rightarrow x = 2,\ x = 5$

$A = \int_2^5 y\, dx = \int_2^5 \left(x^2 - 7x + 10\right)dx$

$= \frac{x^3}{3} - \frac{7x^2}{2} + 10x\Big]_2^5$

$$=\left(\frac{125}{3}-\frac{175}{2}+50\right)-\left(\frac{8}{3}-\frac{28}{2}+20\right)$$

$$=\frac{117}{3}-\frac{147}{2}+30=-\frac{9}{2}$$

$$A=\left|-\frac{9}{2}\right|=\frac{9}{2}$$

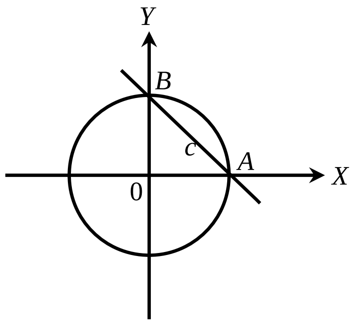
97. Clearly two curves intersect at $x=\frac{\pi}{4}$

$$\therefore \quad A=\int_0^{\pi/4}(\cos x-\sin x)dx$$

$$=\sin x+\cos x]_0^{\pi/4}=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-1=\sqrt{2}-1$$

98. We have, $x^2+y^2=2$ and $x+y=2$

Required area



$=$ Area $OABO-$ Area $\Delta^{le}OAB$

$\frac{1}{4}$ Area of circle $-\frac{1}{2}OA\cdot OB$

$$=\frac{1}{4}\pi(2)^2-\frac{1}{2}(2\times 2)=\pi-2$$

99. Let $\vec{a}=3i-6j+k,\ \vec{b}=2i-4j+\lambda k$.

By data, $\vec{a}$ is parallel to $\vec{b}$

$\Rightarrow \vec{a}=k\vec{b}$ for some scalar k

$\Rightarrow 2k=3,\ -4k=-6,\ k\lambda=1$

$$\Rightarrow k=\frac{3}{2},\ k=\frac{3}{2},\ \therefore\ \frac{2}{3}\left(\because\ k=\frac{3}{2}\right)$$

100.G.E. $=i\cdot i+j\cdot(-j)+k\cdot(k)=1-1+1=1$

101.Let, $\vec{a}=a_1i+a_2j+a_3k$

Now, $\vec{a}\cdot i=a_1,\ \vec{a}\cdot j=a_2,\ \vec{a}\cdot k=a_3$

$\vec{a}\times i=-a_2k+a_3j,\ \vec{a}\times j=a_1k-a_3i$,

$\vec{a}\times k=-a_1j+a_2i$

G.E.

$$=a_1(-a_2k+a_3j)+a_2(a_1k-a_3i)+a_3(-a_1j+a_2i)=\vec{o}$$

102.We have, $\left[(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})\right]^2$

$$=\left[9(\vec{b}\times\vec{a})-(\vec{a}\times\vec{b})\right]^2$$

$$=\left[10(\vec{a}\times\vec{b})\right]^2$$

$$=100\left|\vec{a}\times\vec{b}\right|^2=100\left[|\vec{a}|^2\cdot|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2\right]$$

$$=100[4-2\cos 60^\circ]$$

$$=100\times 3=300$$

103.We have, $(\vec{a}\times\vec{b})^2=|\vec{a}\times\vec{b}|^2$

$$=|\vec{a}|^2\cdot|\vec{b}|^2\cdot\sin^2\theta$$

$$=16\cdot 4\cdot\frac{1}{4}=16$$

104.If the optimal solution occurs at two consecutive vertices of the feasible region, then the optimal solution occurs at every point on the line joining these two points.

105.Clearly, the inequations are

$5x+4y\geq 20,\ x\leq 6,\ y\leq 4,\ x\geq 0,\ y\geq 0$

106.Equation is not a polynomial equation in terms of differential coefficients. Thus degree is not defined.

107.We have, $\frac{dy}{dx}+\frac{2}{x}y=x$.

$$I.F.=e^{\int\frac{2}{x}dx}=e^{\log x^2}=x^2$$

$\therefore$ solution is

$$yx^2=\int x^2dx+c\Rightarrow yx^2=\frac{x^4}{4}+c$$

$$\Rightarrow y=\frac{x^4+k}{4x^2}(k=4c)$$

108.We have, $\int\frac{\sec^2 x}{\tan x}dx+\int\frac{\sec^2 y}{\tan y}dy=\log c$

$$\Rightarrow \log(\tan x)+\log(\tan y)=\log c$$

$$\Rightarrow \tan x\cdot\tan y=c$$

109.We have, $x^y + y^2 - 2ay = 0$ $\quad\ldots(1)$

$$\Rightarrow 2x + 2y\frac{dy}{dx} = 2a\frac{dy}{dx} \quad\ldots(2)$$

(1) $\Rightarrow 2a = \dfrac{x^2+y^2}{y}$. Thus,

(2) $\Rightarrow 2x + 2y\dfrac{dy}{dx} = \left(\dfrac{x^2+y^2}{y}\right)\dfrac{dy}{dx}$

$$\Rightarrow 2xy + 2y^2\frac{dy}{dx} = \left(x^2+y^2\right)\frac{dy}{dx}$$

$$\left(x^2-y^2\right)\frac{dy}{dx} = 2xy$$

110.We have, $3x + 2y + z - 1 + \lambda(2x - 3y - z + 2) = 0$

$$\Rightarrow (3+2\lambda)x + (2-3\lambda)y + (1-\lambda)z - 1 + 2\lambda = 0$$

This is parallel to the plane $5x - y = 0$

$$\Rightarrow \frac{3+2\lambda}{5} = \frac{2-3\lambda}{-1} = \frac{1-\lambda}{0} \Rightarrow \lambda = 1$$

111.Ratio is given by

$$-\left(\frac{4(-2)+5(1)+(-3)\cdot 5-8}{4(3)+5\cdot(3)+(-3)(2)-8}\right)$$

$$= -\left(\frac{-8+5-15-8}{12+15-6-8}\right) = \frac{26}{13} = \frac{2}{1} \text{ i.e. } 2:1$$

112.The planes $2x - y + kz = 0$ and $3x - 5y + 3z = 0$ are perpendicular to each other if

$$2\cdot(3) + (-1)(-5) + k\cdot(3) = 0$$

$$\Rightarrow 11 + 3k = 0 \Rightarrow k = -\frac{11}{3}$$

113.The equation of the plane $3x + 4y - 5z = 4$.

This can be written as $\dfrac{x}{(4/3)} + \dfrac{y}{1} + \dfrac{z}{(-4/5)} = 1$

Thus the intercepts are $\dfrac{4}{3}, 1, -\dfrac{4}{5}$

114.We have, $\dfrac{x}{4} + \dfrac{y}{3} - \dfrac{z}{5} = 1$

$\therefore\ A = (4,0,0),\ B(0,3,0),\ C = (0,0,5)$

$$\vec{AB} = \vec{OB} - \vec{OA} = (-4,3,0)$$

$$\vec{BC} = \vec{OC} - \vec{OB} = (0,-3,5)$$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} i & j & k \\ -4 & 3 & 0 \\ 0 & -3 & 5 \end{vmatrix} = 15i + 20j + 12k$$

$$\Delta ABC = \frac{1}{2}\left|\vec{AB}\times\vec{AC}\right| = \frac{1}{2}\sqrt{225+400+144}$$

$= \dfrac{1}{2}\sqrt{769}$ sq. units

OR

Use TS (9), Area $= \dfrac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}$

$$= \frac{1}{2}\sqrt{144+225+400}$$

$$= \frac{1}{2}\sqrt{769}$$

115.Image of $(3,-4,-3)$ in the plane $x - y + 2z + 2 = 0$ is given by

$$\frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+3}{2} = \frac{-2(3+4-6+2)}{1+1+4}$$

$$\Rightarrow \frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+3}{2} = -1$$

$\therefore\ x = 2,\ y = -3,\ z = 5$

$\therefore$ the image $= (2,-3,-5)$

116.We have, $P(A) = 0.4,\ P(B) = 0.3,\ P(A\cup B) = 0.5$

$$P(A\cap B) = P(A) + P(B) - P(A\cup B)$$

$$= \frac{2}{5} + \frac{3}{10} - \frac{1}{2} = \frac{2}{10}$$

Now, $P(B'\cap A) = P(A) - P(A\cap B)$

$$= \frac{2}{5} - \frac{2}{10} = \frac{1}{5}$$

117.Required probability

$$= P(A\cap B\cap C) + P(A\cap B\cap C) + P(A\cap B\cap C)$$

$$= P(A)\cdot P(B)\cdot P(C) + P(A)\cdot P(B)\cdot P(C)$$

$$+ P(A)\cdot P(B)\cdot P(C)$$

$$= \left(\frac{2}{5}\cdot\frac{3}{10}\cdot\frac{4}{5}\right) + \left(\frac{2}{5}\cdot\frac{7}{10}\cdot\frac{1}{5}\right) + \left(\frac{3}{5}\cdot\frac{3}{10}\cdot\frac{1}{5}\right)$$

$$= \frac{24}{250} + \frac{14}{250} + \frac{9}{250} = \frac{47}{250} = 0.188$$

118.A It is a case of Bernoulli trials with $n=10$

$p=P$ (true/false of the statement) $=\frac{1}{2}$

$q=1-p=\frac{1}{2}$

$\therefore$ Required probability $=P(X=8,9,10)$

$=10C_8\cdot p^8\cdot q^2+10C_9\cdot p^9\cdot q+10C_{10}\cdot p^{10}\cdot q^0$

$=\frac{10\times 9}{1\times 2}\cdot\left(\frac{1}{2}\right)^{10}+10\cdot\left(\frac{1}{2}^{10}\right)+\left(\frac{1}{2}\right)^{10}$

$=(45+10+1)\left(\frac{1}{2}\right)^{10}=\frac{56}{2^{10}}=\frac{7}{2^7}=\frac{7}{128}$

119.It is a case of Bernoulli trials with $n=5$

$p=P$ (a person is a swimmer) $=1-0.3=0.7$

$q=1-p=0.3$

$\therefore$ required probability $=5C_4\cdot(0.7)^4\cdot(0.3)^1$

120.Set, $E:$ event student fails in Physics

$F:$ event student fails in Mathematics

Now, $P(E)=\frac{30}{100}=\frac{3}{10}$ and $P(F)=\frac{25}{100}=\frac{1}{4}$

$P(E\cap F)=\frac{10}{100}=\frac{1}{10}$

Required probability $=P(E/F)$

$=\frac{P(E\cap F)}{p(F}=\frac{(1/10)}{(1/4)}=\frac{2}{5}$

Physics Solutions:

121. $13600\,\text{kg m}^{-3}=13600\times 1000\,\text{g}\times(100\,\text{cm})^{-3}$

$=\frac{136\times 10^5}{10^6}\text{g cm}^{-3}=13.6\,\text{g cm}^{-3}$

122. $H=\frac{u^2\sin^2\theta}{2g}$ and $R=\frac{u^2\sin 2\theta}{g}$

Since $H=R$

$\therefore\frac{u^2\sin^2\theta}{2g}=\frac{u^2\times 2\sin\theta\cos\theta}{g}$

or $\tan\theta=4$ or $\theta=\tan^{-1}(4)$

123.Velocity is the slope of the displacement-time graph

$\frac{v_A}{v_B}=\frac{\tan 30^\circ}{\tan 60^\circ}=\frac{(1/\sqrt{3})}{(\sqrt{3})}=\frac{1}{3}$

124.Let v_w be the velocity of the water and v_b be the velocity of motor boat in still water. If x is the distance covered, then as per question

$x=(v_b+v_w)\times 6=(v_b-v_w)\times 10$

On solving, $v_w=\frac{v_b}{4}$

$\therefore\left[v_b+\frac{v_b}{4}\right]\times 6=7.5v_b$

Time taken by motor boat to cross the same distance in still water is

$t=\frac{x}{v_b}=\frac{7.5v_b}{v_b}=7.5$ hours

125.Here, $K_A=2K_B,\theta_A-\theta_B=36^\circ\text{C}$

Let θ is the temperature of the junction

As $\left(\frac{\Delta\theta}{\Delta t}\right)_A=\left(\frac{\Delta\theta}{\Delta t}\right)_B$

$\therefore\frac{K_AA(\theta_A-\theta)}{x}=\frac{K_BA(\theta-\theta_B)}{x}$

$2K_B(\theta_A-\theta)=K_B(\theta-\theta_B)$

$2(\theta_A-\theta)=\theta-\theta_B$

Add $(\theta_A-\theta)$ on both sides, we get

$3(\theta_A-\theta)=\theta_A-\theta+\theta-\theta_B$

$3(\theta_A-\theta)=\theta_A-\theta_B$

$\theta_A-\theta=\frac{\theta_A-\theta_B}{3}=\frac{36}{3}=12^\circ\text{C}$

$\therefore$ Temperature difference across the layer A

$=\theta_A-\theta=12^\circ\text{C}$

126.According to Newton's law of cooling,

$\frac{\theta_1-\theta_2}{t}=K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$

$\frac{65.5-62.5}{1}=K\left[\frac{65.5+62.5}{2}-22.5\right]$

$3=K(64-22.5)=K(41.5)$ ... (i)

In second case,

$$\frac{46.5-40.5}{t}=K\left[\frac{46.5+40.5}{2}-22.5\right]$$

$$\frac{6}{t}=K(43.5-22.5)=K(21) \quad \ldots \text{(ii)}$$

Divide (i) by (ii), we get

$$\frac{t}{2}=\frac{41.5}{21}\approx 2 \Rightarrow t=4\,\text{min}$$

127.Heat supplied $dQ=nC_p dT$

Heat used for work $=dW=nRdT$

$$\frac{dW}{dQ}=\frac{R}{C_P}=\frac{C_P-C_V}{C_P}=1-\frac{C_V}{C_P}$$

$$=\left(1-\frac{1}{\gamma}\right)=\left(1-\frac{3}{4}\right)=\frac{1}{4}\times 100=25\%$$

128.According to kinetic theory of gases, gas molecules behave as a perfectly elastic rigid spheres.

129. $180°$

130.When heat is supplied the temperature of ice increases from $-10°\text{C to } 0°\text{C}$. It is represented by a straight line inclined to heat axis. At $0°\text{C}$ the heat is used in converting ice into water at $0°\text{C}$. This stage is represented by horizontal straight portion. After that temperature of water rises from $0°\text{C to } 100°\text{C}$. It is represented by a straight line inclined to heat axis. At $100°\text{C}$, the heat is used in converting water into steam. The graph is represented by horizontal straight line.

131.Surface energy $u=S\times 4\pi R^2$

When droplet is splitted into 1000 droplets each of radius r, then

$$\frac{4}{3}\pi R^3=1000\frac{4}{3}\pi r^3 \Rightarrow r=\frac{R}{10}$$

Here, R is bigger drop and S is surface tension.

$\therefore$ Surface energy of all droplets

$$=S\times 1000\times 4\pi r^2=S\times 1000\times 4\pi\left(\frac{R}{10}\right)^2$$

$$=10\left(S4\pi R^2\right)=10u$$

132.Maximum speed $v_m=\omega a$

Maximum acceleration $a_m=\omega^2 a$

$$\Rightarrow \frac{a_m}{v_m}=\omega=\frac{2\pi}{T}\Rightarrow\frac{31.4}{10}=\frac{2\times 3.14}{T}$$

$$\Rightarrow T=\frac{2\times 3.14\times 10}{31.4}\Rightarrow T=2\,\text{s}$$

133.Velocity of transverse wave along string $=v$

$$v=\sqrt{\frac{T}{\mu}} \text{ where } \mu=\text{mass per unit length of string}$$

$$v=\sqrt{\frac{500}{0.2}}=\sqrt{\frac{500\times 10}{2}}=\sqrt{2500}=50\,\text{m s}^{-1}$$

134.For satellite motion, orbital velocity

$$v_0=\sqrt{\frac{GM}{r}}=R\sqrt{\frac{g}{(R+h)}}\left[\text{as } g=\frac{GM}{R^2} \text{ and } r=R+h\right]$$

In this problem, $v_0=\frac{1}{2}v_e=\frac{1}{2}\sqrt{2gR}$ [as $v_e=\sqrt{2gR}$]

$$\therefore \frac{R^2 h}{R+h}=\frac{1}{2}gR\text{, i.e., } 2R=h+R \text{ or } h=R=6400\,\text{km}$$

135.Since surface densities are equal,

$$\frac{q_1}{4\pi r^2}=\frac{q_2}{4\pi R^2}$$

$$\frac{q_1}{r^2}=\frac{q_2}{R^2}=\frac{q_1+q_2}{r^2+R^2}=\frac{Q}{r^2+R^2} \text{ where } q_1+q_2=Q$$

$$q_1=\frac{Q}{R^2+r^2}\times r^2; q_2=\frac{Q}{R^2+r^2}R^2$$

Potential at the common centre is

$$V=\frac{q_1}{4\pi\varepsilon_0 r}+\frac{q_2}{4\pi\varepsilon_0 R}=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R}\right)$$

$$=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R^2+r^2}\times\frac{r^2}{r}+\frac{Q}{R^2+r^2}\frac{R^2}{R}\right)=\frac{Q(R+r)}{4\pi\varepsilon_0\left(R^2+r^2\right)}$$

136. $U=\frac{1}{2}CV^2=\frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$

$$=\frac{1}{2}\times\frac{8.85\times 10^{-12}\times\left(100\times 10^{-4}\right)(200)^2}{2.5\times 10^{-3}}$$

$$U=7.08\times 10^{-7}\,\text{J}$$

137. $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$

$d\vec{s} = 100\hat{k}$

$\phi = \vec{E} \cdot d\vec{s} = \left(8\hat{i} + 4\hat{j} + 3\hat{k}\right) \cdot \left(100\hat{k}\right) = 300$ units

138. Electric dipole moment of a water molecule

$p = 6.4 \times 10^{-30}\,\mathrm{C\,m}$

$p = qd$ where d is the distance between the centre of positive and negative charge of the molecule

$$\Rightarrow d = \frac{p}{d} = \frac{6.4 \times 10^{-30}\,\mathrm{C\,m}}{1.6 \times 10^{-19}\,\mathrm{C}} = 4 \times 10^{-11}\,\mathrm{m}$$

139. According to Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \qquad \ldots \text{(i)}$$

Suppose force between the charges is same when charges are r' distance apart in dielectric

$$\therefore F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{Kr'^2} \qquad \ldots \text{(ii)}$$

From (i) and (ii), $Kr'^2 = r^2$ or $r = \sqrt{K}r'$

Thus distance r' of dielectric is equivalent to $r/\sqrt{K}$ distance of air. In the given situation, force between the charges would be

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\left(\frac{r}{2} + \sqrt{4}\frac{r}{2}\right)^2} = \frac{4}{9}\frac{q^2}{4\pi\varepsilon_0 r^2}$$

$$F' = \frac{4}{9}F$$

140. Potential at the centre = potential on the surface

$= 10\,\mathrm{V}$

141. For the same length and same material,

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} = \frac{3}{1} \text{ or } R_2 = 3R_1$$

The resistance of thicker wire, $R_1 = 10\,\Omega$

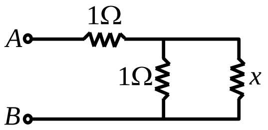
The resistance of thinner wire $= 3R_1 = 3 \times 10 = 30\,\Omega$

Total resistance $= 10\,\Omega + 30\,\Omega = 40\,\Omega$

142. Let $R_{AB} = x$

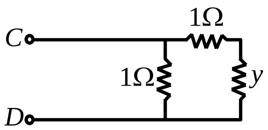
$$R_{AB} = 1 + \frac{x}{1+x} \qquad \text{or} \qquad x = 1 + \frac{x}{1+x}$$

or $x^2 + x = 1 + x + x$ or $x^2 - x - 1 = 0$



$$\therefore x = \frac{1 \pm \sqrt{5}}{2} = \frac{\left(\sqrt{5}+1\right)}{2}\Omega$$

Similarly, let $R_{CD} = \frac{(1+y)}{2+y}$



$$\text{or } y = \frac{(1+y)}{2+y} \qquad \text{or} \qquad y = \frac{\left(\sqrt{5}-1\right)}{2}\Omega$$

$\therefore R_{AB} > R_{CD}$

143. Apply Kirchhoff's first law.

144. Distance between two points $= 60 - 30 = 30\,\mathrm{cm} = 0.3\,\mathrm{m}$

The potential gradient $10\,\mathrm{V\,m^{-1}}$

$\therefore$ Potential difference = potential gradient × length

$= 10 \times 0.3 = 3\,\mathrm{V}$

145. Power, $P = \frac{V^2}{R}$

As R is constant

$$\therefore \frac{P_1}{P_2} = \left[\frac{V_1}{V_2}\right]^2 \qquad \text{or} \qquad P_2 = P_1\left(\frac{V_2}{V_1}\right)^2$$

Substituting the given values, we get

$$P_2 = 500 \times \left(\frac{150}{100}\right)^2 = 1125\,\mathrm{W}$$

146. Here, $\varepsilon = 2.0\,\mathrm{V}$; $n = 6$, $r = 0.015\,\Omega$; $R = 8.5\,\Omega$

Current, $I = \frac{n\varepsilon}{R + nr} = \frac{6 \times 2}{8.5 + 6 \times 0.015} = 1.4\,\mathrm{A}$

Terminal voltage, $V = IR = 1.4 \times 8.5 = 11.9\,\mathrm{V}$

147. $v = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}} = 7.25 \times 10^6\,\mathrm{m\,s^{-1}}$

148. According to Einstein's photoelectric equation,

$\frac{1}{2}mv_{\max}^2 = h\upsilon - h\upsilon_0$ or

$$\frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

where λ is the wavelength of incident radiation and λ_0 is threshold wavelength.

$$\therefore \frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad \ldots \text{(i)}$$

$$\frac{1}{2}m(2v)^2 = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda^0}\right) \quad \ldots \text{(ii)}$$

Divide (i) by (ii), we get

$$\frac{1}{4} = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{\lambda'} - \frac{1}{\lambda_0}} \quad \text{or} \quad \frac{1}{4} = \frac{\frac{1}{480} - \frac{1}{600}}{\frac{1}{\lambda'} - \frac{1}{600}}$$

Solving for λ', we get $\lambda' = 300\,\text{nm}$

149. $I_g = 16 \times 30\,\mu\text{A} = 480 \times 10^{-6} A$

Let G be the resistance of galvanometer and R be the resistance connected in series to convert the galvanometer into voltmeter of range $0 \text{ to } 3\text{V}$. Then

$$G + R = \frac{V}{I_g} = \frac{3}{480 \times 10^{-6}} = 6.25 \times 10^3 = 6.25\,\text{k}\Omega$$

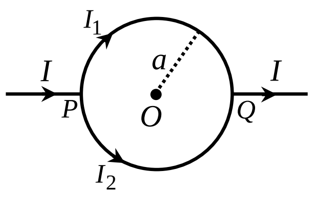
$\therefore$ A resistance R nearly $6\,\text{k}\Omega$ is to be used in series.

150. $\because I_1R_1 = I_2R_2$

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{R_2}{2R_2} = \frac{1}{2}$$

$$\therefore I_2 = 2I_1 \text{ and } I = I_1 + I_2 = I_1 + 2I_1 = 3I_1$$

$$\therefore I_1 = \frac{I}{3} \text{ and } I_2 = \frac{2}{3}I$$



Magnetic field at the centre of coil is

$$B_0 = B_1 \otimes + B_2 \odot$$

$$B_0 = -\frac{\mu_0 I_1 \pi}{4\pi a} + \frac{\mu_0 I_2 \pi}{4\pi a}$$

$$B_0 = \frac{\mu_0 \pi}{4\pi a}(I_2 - I_1) = \frac{\mu_0}{4a}\left(\frac{2}{3}I - \frac{I}{3}\right) = \frac{\mu_0}{4a}\left(\frac{I}{3}\right) = \frac{\mu_0 I}{12a}$$

151. The correct relation is $\mu_r = 1 + \chi$

152. As $H = R\cos\delta$

$$\therefore \cos\delta = \frac{H}{R} = \frac{3 \times 10^{-5}}{6 \times 10^{-5}} = \frac{1}{2}$$

$$\delta = 60°$$

153. The liquid rises up in the part of the tube which is between the poles.

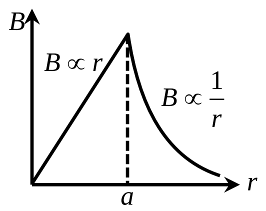
154. Magnetic field due to a long straight wire of radius a carrying current I at a point distant r from the centre of the wire is given as follows:

$$B = \frac{\mu_0 Ir}{2\pi a^2} \text{ for } r < a$$

$$B = \frac{\mu_0 I}{2\pi a} \text{ for } r = a$$

$$B = \frac{\mu_0 I}{2\pi r} \text{ for } r > a$$

The variation of magnetic field B with distance r from the centre of wire is shown in the figure.



155. Here, from small circular coil, number of turns, $N = 10$; area, $A = 1\text{mm}^2 = 1 \times 10^{-6}\text{m}^2$; current,

$$I_1 = \frac{21}{44}A$$

For a long solenoid, number of turns per metre,

$n = 10^3 \text{ per m}$

Current, $I_2 = 2.5A$

Magnetic field due to a long solenoid on its axis is

$$B = \mu_0 n I_2 \quad \ldots \text{(i)}$$

Magnetic moment of a circular coil is

$$M = NAI_1 \quad \ldots \text{(ii)}$$

Torque, $\vec{\tau} = \vec{M} \times \vec{B}$

$\tau = MB\sin\theta = MB \quad (\because \theta = 90° \text{ (given)})$

$\tau = (NAI_1)(\mu_0 n I_2) \quad$ (using (i) and (ii))

$\tau = 10\times1\times10^{-6}\times\frac{21}{44}\times4\times\frac{22}{7}\times10^{-7}\times10^{3}\times2.5$

$=1.5\times10^{-8}\,\text{N m}$

156. $\frac{3h}{2\pi}=n\left(\frac{h}{2\pi}\right)$

$\therefore n=3$

$K_n=\frac{K_1}{(3)^2}=\frac{13.6}{9}=1.51\,\text{eV}$

157. Decrease in charge number due to 8α emissions

$=8\times2=16$

Increase in charge number due to $4\beta^-$ emissions

$=4\times1=4$

Decrease in charge number due to $2\beta^+$ emissions

$=2\times1=2$

Net decrease in charge number $=16-4+2=14$

$\therefore Z$ of resulting nucleus $=92-14=78$

158. Jump to second orbit leads to Balmer series. When an electron jumps from 4th orbit to 2nd orbit shall give rise to second line of Balmer series.

159. Gamma rays are packets of energy. They carry no charge and no mass. Therefore, in gamma ray emission, there is no change in proton number and neutron number.

160. Potential energy $=2\times$ total energy

$=2(-1.5)\text{eV}=-3.0\,\text{eV}$

161. Here, $R=44\Omega, L=8\,\text{mH}=8\times10^{-3}\,\text{H}$,

$C=20\,\mu\text{F}=20\times10^{-6}\,\text{F}$

$\omega_r=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{8\times10^{-3}\times20\times10^{-6}}}$

$\omega_r=\frac{1}{4\times10^{-4}}=\frac{10^4}{4}=2500\,\text{rad s}^{-1}$

$I_0=\frac{V_0}{R}=\frac{\sqrt{2}V_{\text{rms}}}{R}=\frac{\sqrt{2}\times220}{44}=5\sqrt{2}\text{A}$

162. $I=\frac{|\varepsilon|}{R}=\frac{NA\frac{dB}{dt}}{R}=\frac{20\times\left(25\times10^{-4}\right)\times1000}{100}=0.5\,\text{A}$

163. $\tan\phi=\left(\frac{X_L}{R}\right)$

$X_L=\omega L=(2\pi\nu L)=(2\pi)(50)(0.01)=\pi\Omega$

Also, $R=1\Omega$

$\therefore\phi=\tan^{-1}(\pi)$

164. $X_L=2\pi\nu L=2\times3.14\times50\times25.48\times10^{-3}=8\Omega$

$X_C=\frac{1}{2\pi\nu C}=\frac{1}{2\times3.14\times50\times796\times10^{-6}}=4\Omega$

$\therefore$ Impendence of the circuit is

$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{(3)^2+(8-4)^2}=5\Omega$

165. Resonant angular frequency, $\omega_r=\frac{1}{\sqrt{LC}}$

Quality factor, $Q=\frac{\text{Resonant angular frequency}}{\text{Bandwidth}}$

Bandwidth $=\nu_2-\nu_1=\frac{\nu_r}{Q}$ where $\nu_r=$ resonant frequency, $Q=$ quality factor

$\therefore$ Also, $Q=\frac{\omega_r L}{R}$ $\quad\therefore\nu_2-\nu_1=\frac{R}{2\pi\sqrt{LC}\,\omega_r L}=\frac{R}{2\pi L}$

$\nu_2-\nu_1=\frac{R}{2\pi L}=\frac{5}{2\pi\times40\times10^{-3}}=20\,\text{Hz}$

166. Here, $\phi=10t^2-50t+250$

The induced emf is

$\varepsilon=-\frac{d\phi}{dt}=-\frac{d}{dt}\left(10t^2-50t+250\right)=-(20t-50)$

At $t=3\text{s}$, $\varepsilon=-(20\times3-50)=-10\,\text{V}$

167. Voltage gain, $A_v=\beta+\frac{R_0}{R_i}=50\times\frac{5}{1}=250$

168. $Y=\overline{A+B}$ is for 'OR' gate with 'NOT' gate, i.e., of NOR gate

169. Here, $E_g=2.8\,\text{eV}=2.8\times1.6\times10^{-19}\,\text{J}$

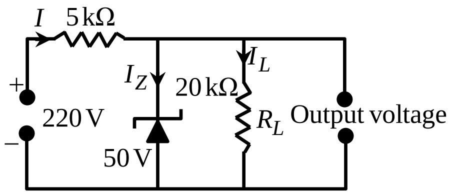
$E_g=\frac{hc}{\lambda}\Rightarrow\lambda=\frac{hc}{E_g}=\frac{6.6\times10^{-34}\times\left(3\times10^8\right)}{2.8\times1.6\times10^{-19}}$

$\lambda=440\times10^{-9}\,\text{m}=440\,\text{nm}$

170. A Zener diode is always connected in reverse bias when it is used. The potential barrier of germanium $p-n$ junction is 0.3V and $Si\ p-n$ junction is $0.7\,\text{V}$. So, statement A is correct but B is wrong.

171.Here, $R_L = 5\times10^3\,\Omega, V_i = 220\,\text{V}$;

Zener voltage, $V_Z = 50\,\text{V}$



Load current, $I_L = \dfrac{V_Z}{R_L} = \dfrac{50}{20\times10^3} = 2.5\times10^{-3}\,\text{A}$

Current through R, $I = \dfrac{220-50}{5\times10^3} = 34\times10^{-3}\,\text{A}$

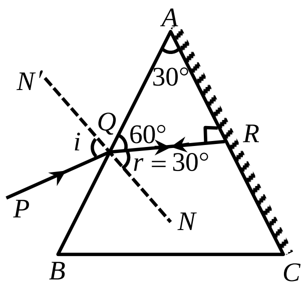
Zener current, $I_Z = I - I_L$

$= 34\times10^{-3} - 2.5\times10^{-3}$

$= 31.5\times10^{-3}\,\text{A} = 31.5\,\text{mA}$

172. $\lambda_{\text{micro}} > \lambda_{\text{IR}} > \lambda_{\text{UV}} > \lambda_{\text{gamma}}$

173.Sol:



It is clear from the figure that the ray will retrace the path when the refracted ray QR is incident normally on the polished surface AC. Thus angle of refraction, $r = 30°$.

$\mu = \dfrac{\sin i}{\sin r}$

$\therefore \sin i = \mu \sin r$

$\sin i = \sqrt{2}\times\sin 30° = \sqrt{2}\times\dfrac{1}{2} = \dfrac{1}{\sqrt{2}}$

$\therefore i = 45°$

174.Here, in this case lens used by person should form the image of distant object at a distance of $40\,\text{cm}$ in front of it.

$\therefore u = -\infty, v = -40\,\text{cm}$

and $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$ or $\dfrac{1}{f} = \dfrac{1}{-40} - \dfrac{1}{-\infty}$

or, $\dfrac{1}{f} = \dfrac{1}{-40}$ or $f = -40\,\text{cm}$

$\text{Power} = \dfrac{100}{f} = \dfrac{100}{-40} = -2.5\,D$

Negative sign shows that lens used is concave lens.

175.In case of diffraction at a single slit, the position of minima is given by

$d\sin\theta = n\lambda$

If θ is small, $\sin\theta = \theta = \dfrac{y}{D}$

So, the position of first minimum relative to centre will be given by

$d(y/D) = \lambda$, i.e., $y = (D/d)\lambda$

Here, $D = 2\text{m}$; $d = 1\times10^{-3}\text{m}$ and $\lambda = 6\times10^{-7}\,\text{m}$

So, $y = \dfrac{2\times6\times10^{-7}}{1\times10^{-3}} = 1.2\,\text{mm}$

$\therefore$ Distance between first minima on either side of central maximum, $\Delta y = 2y = 2.4\,\text{mm}$

176. $\beta = \dfrac{\lambda D}{d} = \dfrac{6000\times10^{-10}\times2}{4\times10^{-3}} = 0.3\times10^{-3}\text{m} = 0.3\text{mm}$

177.The objective of compound microscope forms a real and enlarged image.

178. $\dfrac{W_1}{W_2} = 4 = \dfrac{I_1}{I_2} = \dfrac{a^2}{b^2} \quad \therefore \dfrac{a}{b} = 2$

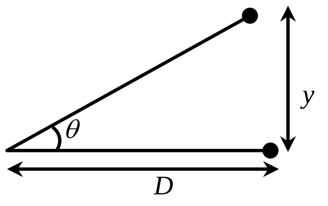
$$\frac{I_{\max}}{I_{\min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{\left(\frac{a}{b}+1\right)^2}{\left(\frac{a}{b}-1\right)^2} = \frac{(2+1)^2}{(2-1)^2} = \frac{9}{1}$$

179.Resolution limit $= \dfrac{1.22\lambda}{d}$

Again, resolution limit $= \sin\theta = \theta = \dfrac{y}{D}$

Where $y =$ distance between two points and $D =$ maximum distance at which dots can be resolved.

$\therefore \dfrac{y}{D} = \dfrac{1.22\lambda}{d}$



or $D = \dfrac{yd}{1.22\lambda}$

or $D = \dfrac{\left(10^{-3}\right)\times\left(3\times10^{-3}\right)}{(1.22)\times\left(5\times10^{-7}\right)} = \dfrac{30}{6.1} \approx 5\,\text{m}$

180. According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For biconvex lens, $R_1 = +R_1; R_2 = -R_2$

$$\frac{1}{0.06} = (1.5-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.06\times0.5} = \frac{100}{3} \qquad \ldots \text{(i)}$$

Now, $\dfrac{R_1}{R_2} = \dfrac{1}{2}$ or $R_2 = 2R_1$

On substituting it in eq. (i), we get

$$\frac{3}{2R_1} = \frac{100}{3}$$

$$R_1 = \frac{9}{200} = 0.045\,\text{m}$$

$$R_2 = 2R_1 = 2\times0.045\,\text{m} = 0.09\,\text{m}$$