Subject	Topic	Mock Test – 07	Date
C + M + P	Complete Syllabus	CET – 12 – CT C1220230508	8th May 2023

Max. Marks: 180 **Duration: 3 Hours**

1. **This paper consists of 180 questions with 3 parts of Chemistry, Mathematics and Physics**

   - **Chemistry:** *(Q. No. 1 to 60) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Mathematics:** *(Q. No. 61 to 120) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Physics:** *(Q. No. 121 to 180) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*

2. **The OMR sheet for 200 questions is to be used**

3. **Use of calculators and log tables is prohibited**

4. **Darken the appropriate bubble using a pen in the OMR sheet provided to you. Once entered, the answer cannot be changed. Any corrections or modifications will automatically draw a penalty of 1 mark**

5. **No clarification will be entertained during the examination. Doubts in the paper can be reported to the coordinator after the exam**

6. **If the details in the OMR Sheet are not filled, If the OMR sheet is mutilated, torn, white Ink used, the circles filled and scratched, then the OMR sheet will not be graded**

   All the best!!

Useful Data

At. Wt.:

$N = 14;\ O = 16;\ H = 1;\ S = 32;\ Cl = 35.5;\ Mn = 55;\ Na = 23;\ C = 12;\ Ag = 108;\ K = 39;\ Fe = 56;\ Pb = 207$

Physical Constants:

$h = 6.626 \times 10^{-34}\,\text{Js},\ \text{N}_\text{a} = 6.022 \times 10^{23}\,\text{mol}^{-1},\ \text{c} = 2.998 \times 10^{8}\,\text{m s}^{-1},\ \text{m}_\text{e} = 9.1 \times 10^{-31}\,\text{kg},\ R = 8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$

Chemistry

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. **60 x 1 = 60**

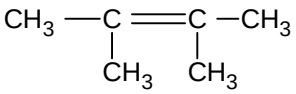
1. Hyper – conjugation is not possible in

   Options:

   (a) $CH_3 - CH = CH_2$

   (b) $CH_2 = CH_2$

   (c) $CH_3 - \overset{+}{C}H - CH_3$

   (d) 

   Sol: In $CH_2 = CH_2$, there are no $-\alpha$ hydrogens

   Ans: (b)

2. When hydrogen chloride gas is treated with propene in presence of benzoyl peroxide, it given

   Options:

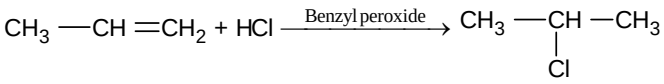
   (a) $2-$Chloropropane

   (b) Allyl chloride

   (c) no reaction

   (d) $1-$Chloropropane

   Sol:

   Peroxide effect is seen only with HBr

   Ans: (a)

3. Which of the following compounds has highest boiling point?

   Options:

   (a) $CH_3CH_2CH_2Cl$

   (b) $CH_3CH_2CH_2CH_2Cl$

   (c) $CH_3CH(CH_3)CH_2Cl$

   (d) $(CH_3)_3CCl$

   Sol: $CH_3CH_2CH_2CH_2Cl$ has higher boiling point

   Ans: (b)

4. Which one of the following forms propane nitrile as the major product?

   Options:

   (a) ethyl bromide + alcoholic KCN

   (b) propyl bromide + alcoholic KCN

   (c) propyl bromide + alcoholic $AgCN$

   (d) ethyl bromide + alcoholic $AgCN$

Sol:

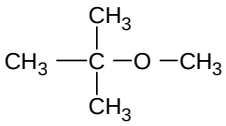
$$CH_3 - CH_2 - Br + KCN\,(alc) \rightarrow CH_3CH_2CN + KBr$$

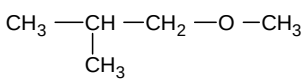
Propane nitrile

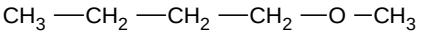
Ans: (a)

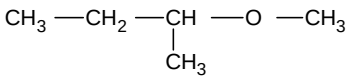
5. Among following ethers, which one will produce methyl alcohol on treatment hot concentrated HI ?

Options:

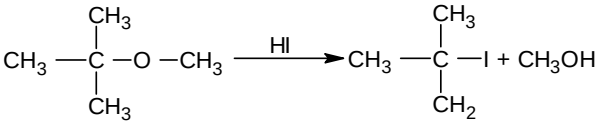
(a) 

(b) 

(c) 

(d) 

Sol:



Ans: (a)

6. Phenol when treated with ex^{ss} of bromine water gives a white precipitate of

Options:

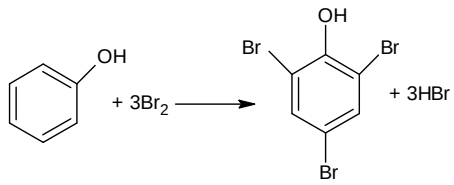
(a) 2, 4, 6 – tribromophenol

(b) o – bromophenol

(c) p – bromophenol

(d) bromobenzene

Sol:



Ans: (a)

7. Vapours of an alcohol X when passed over hot reduced copper, produce an aldehyde, the alcohol is

Options:

(a) primary alcohol

(b) secondary alcohol

(c) tertiary alcohol

(d) dihydric alcohol

Sol: Primary alcohol gives aldehyde

Ans: (a)

8. Benzoquinone is produced by reaction of phenol with

Options:

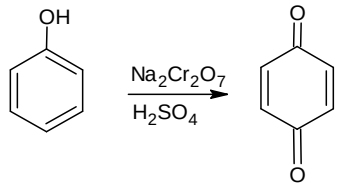
(a) $Na_2Cr_2O_7$, H_2SO_4

(b) $KMnO_4$, H_2SO_4

(c) Na_2CrO_4, HCl

(d) K_2MnO_4, H_2SO_4

Sol:

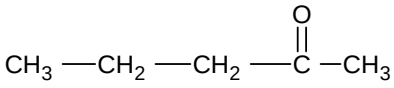


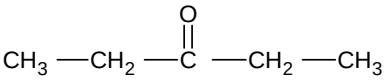
Ans: (a)

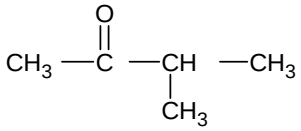
9. Which of the following is the most reactive isomer?

Options:

(a) $CH_3-CH_2-CH_2-CH_2-CHO$

(b) 

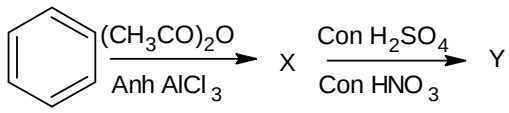
(c) 

(d) 

Sol: Aldehydes are more reactive than ketones

Ans: (a)

10. Identify the products (X) and (Y) in the given reaction:



Options:

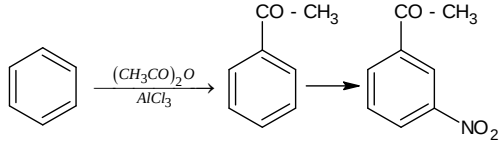
(a) X : Acetophenone Y : $m-$Nitroacetophenone

(b) X : Toluene Y : $p-$Nitrotoluene

(c) X : Acetophenone Y : o and p Nitroacetophenone

(d) X : Benzaldehyde Y : $m-$ Nitrobenzaldehyde

Sol:



Ans: (a)

11. Which of the following compounds undergo Cannizzaro reaction?

Options:

(a) CH_3CHO

(b) CH_3COCH_3

(c) C_6H_5CHO (d) $C_6H_5CH_2CHO$

Sol: Benzaldehyde does not contain $\alpha-$ hydrogen. Hence undergoes Cannizzaro reaction

Ans: (c)

12. Which of the following compounds would have the smallest value for pKa ?

Options:

(a) $CHF_2CH_2CH_2COOH$

(b) $CH_3CH_2CF_2COOH$

(c) $CH_2FCHFCH_2COOH$

(d) $CH_3CF_2CH_2COOH$

Sol: $CH_3CH_2CF_2COOH$ is more acidic and hence it will have smallest pKa value

Ans: (b)

13. Which of the following amines does not react with Hinsberg reagent?

Options:

(a) $CH_3CH_2-NH_2$

(b) $CH_3-NH-CH_3$

(c) $(CH_3)_3N$

(d) $CH_3CH_2CH_2-NH_2$

Sol: Tertiary amines do not react with Hinsberg reagent

Ans: (c)

14. The action of nitrous acid on an aliphatic primary amine gives

Options:

(a) secondary amine

(b) nitroalkanes

(c) alcohol

(d) alkyl nitrite

Sol: The action nitrous acid on aliphatic primary amines gives alcohol

Ans: (c)

15. Which of the following orders is true regarding the basic nature of $-NH_2$ group?

Options:

(a) $O-$Toluidine $>$aniline$>O-$nitroaniline

(b) $O-$Toluidine $<$aniline $>O-$nitroaniline

(c) $O-$Toluidine $>$aniline $<O-$nitroaniline

(d) $O-$Toluidine $>$aniline $<O-$nitroaniline

Sol: $O-$Toluidine is weaker base than aniline because of orthoeffect

Ans: (b)

16. The anomeric carbon in $D(+)$ glucose is

Options:

(a) $C-1$ carbon

(b) $C-2$ carbon

(c) $C-5$ carbon

(d) $C-6$ carbon

Sol: $C-1$ carbon

Ans: (a)

17. In fibrous proteins, polypeptide chains are held together by

Options:

(a) Vander Waal's forces

(b) electrostatic forces of attraction

(c) hydrogen bonds

(d) covalent bonds

Sol: Hydrogen bonds

Ans: (c)

18. A unit in nucleic acid which contains base sugar – phosphate unit is called

Options:

(a) nucleotide

(b) nucleoside

(c) phosphotide

(d) polypeptide

Sol: It is nucleotide

Ans: (a)

19. Which of the following complex has minimum magnitude of Δ_0 ?

Options:

(a) $\left[Cr(CN)_6\right]^{3-}$

(b) $\left[Co(NH_3)_6\right]^{3+}$

(c) $[CoCl_6]^{3-}$

(d) $\left[Cr(H_2O)_6\right]^{3+}$

Sol: magnitude of Δ_0 depends upon spectrochemical series

$CN^- > NH_3 > H_2O > Cl^-$

Ans: (c)

20. The two isomers X and Y with the formula $Cr(H_2O)_5 ClBr_2$ were taken for experiment on depression in freezing point. It was found that one mole of X gave depression corresponding to 2 moles of particles and one mole of Y gave depression due to 3 moles of particles. The structural formula of X and Y respectively are

Options:

(a) $\left[Cr(H_2O)_5 Cl\right]Br_2 \quad \left[Cr(H_2O)_4 Br_2\right]Cl.H_2O$

(b) $\left[Cr(H_2O)_5 Cl\right]Br_2 \quad \left[Cr(H_2O)_3 Cl\ Br_2\right]2H_2O$

(c) $\left[Cr(H_2O)_5 Br\right]BrCl \quad \left[Cr(H_2O)_4 Cl\,Br\right]Br.H_2O$

(d) $\left[Cr(H_2O)_4 Br_2\right]Cl.H_2O \quad \left[Cr(H_2O)_5 Cl\right]Br_2$

Sol: $\underset{X}{\left[Cr(H_2O)_4 Br_2\right]Cl} \rightarrow \left[Cr(H_2O)_4 Br_2\right]+Cl^-$ Two particles

$\underset{Y}{\left[Cr(H_2O)_5 Cl\right]}Br_2 \rightarrow \left[Cr(H_2O)_5 Cl\right]^{2+} + 2Br^-$ Three particles

Ans: (d)

21. Platinum dissolves in aqua regia to form

Options:

(a) $PtCl_4$

(b) H_2PtCl_6

(c) $Pt(NO_3)_4$

(d) $\left[PtCl_2(NO_3)_2\right]$

Sol: H_2PtCl_6

Ans: (b)

22. In $Fe(CO)_5$ the $Fe \leftarrow CO$ σ bond results by the overlap between filled sp hybrid orbital of $C-$atom of CO molecule and vacant

Options:

(a) d^2sp^3 hybrid orbitals of Fe

(b) sp^3 hybrid orbitals of Fe

(c) dsp^3 hybrid orbitals of Fe

(d) dsp^2 hybrid orbitals of Fe

Sol: In $Fe(CO)_5 \rightarrow Fe$ undergoes dsp^3 hybridization

Ans: (c)

23. The acidic, basic and amphoteric nature of Mn_2O_7, V_2O_5 and CrO are respectively

Options:

(a) Acidic, acidic and basic

(b) Basic, amphoteric and basic

(c) Acidic, amphoteric, basic

(d) Acidic, basic, amphoteric

Sol: $Mn_2O_7 \rightarrow$ acidic

$V_2O_5 \rightarrow$ Amphoteric

$CrO \rightarrow$ Basic

Ans: (c)

24. The catalytic activity of transition metals and their compounds is mainly due to

Options:

(a) their ability to adopt variable oxidation state

(b) their chemical reactivity

(c) their magnetic behavior

(d) their unfilled $d-$ orbitals

Sol: Variable oxidation states

Ans: (a)

25. The titanium (atomic number 22) compound that does not exist is

Options:

(a) TiO

(b) TiO_2

(c) K_2TiF_6

(d) K_2TiO_4

Sol: In K_2TiO_4 oxidation state of Ti is

$2(+1)+x+4(-2)=0 \quad x=+6$

Ti electronic configuration is $[Ar]3d^2 4S^2$

It can show a maximum oxidation state of $+4$ only

Ans: (d)

26. For which one of the following metals, the standard potential $\left(E^{\circ}_{M^{2+}/M}\right)$ value has a positive sign?

Options:

(a) $Cu(Z=29)$

(b) $Fe(Z=26)$

(c) $Co(Z=27)$

(d) $Ni(Z=28)$

Sol: $E^{\circ}_{Cu^{2+}/Cu} = +0.34\,V$

Ans: (a)

27. Water is oxidized to oxygen by

Options:

(a) ClO_2

(b) $KMnO_4$

(c) H_2O_2

(d) Fluorine

Sol: Water is oxidized by only fluorine

$$H_2O + F_2 \rightarrow 2HF + \frac{1}{2}O_2$$

Ans: (d)

28. XeF_4 and XeF_6 are expected to be

Options:

(a) oxidising

(b) reducing

(c) unreactive

(d) strongly basic

Sol: XeF_4 and XeF_6 both are oxidizing in nature

Ans: (a)

29. Which of the following is wrong?

Options:

(a) Nitrogen cannot form $d\pi - p\pi$ bond

(b) Single $N-N$ bond is weaker than the single $P-P$ bond

(c) N_2O_4 has two resonance structures

(d) The stability of the hydrides increase from NH_3 to BiH_3

Sol: The stability of the hydrides decreases from NH_3 to BiH_3

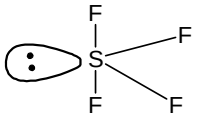
Ans: (d)

30. Which of the following is a mismatch for the compound with its structure or geometry?

Options:

(a) SF_6	Octahedral
(b) SF_4	Square planar
(c) SO_2	Bent
(d) H_2SO_4	Tetrahedral

Sol: For SF_4 geometry is seesaw with sp^3d hybridization



Ans: (b)

31. Which of the following is not correctly matched?

Options:

(a) Butter - gel

(b) Milk - emulsion

(c) Fog - Aerosol

(d) Dust – solid sol

Sol: Dust is an aerosol.

Ans: (d)

32. The correct order of adsorption of the gases studied will be

Options:

(a) $NH_3 > SO_2 > CO_2 > HCl$

(b) $CO_2 > SO_2 > NH_3 > HCl$

(c) $SO_2 > NH_3 > HCl > CO_2$

(d) $HCl > SO_2 > NH_3 > CO_2$

Sol: Higher the intermolecular force higher will be the adsorption

Ans: (c)

33. Which of the following is not a lyophobic sol?

Options:

(a) gold sol

(b) As_2S_3 sol

(c) starch sol

(d) $Fe(OH)_3$ sol

Sol: Starch sol is a lyophilic sol

Ans: (c)

34. The rate of the reaction

$CH_3COOC_2H_5 + NaOH \rightarrow CH_3COONa + C_2H_5OH$ is given by the equation rate

$= k[CH_3COOC_2H_5][NaOH]$

The unit of rate constant is

Options:

(a) $\text{mol}^{-2}L^2S^{-1}$

(b) $\text{mol}\,L^{-1}S^1$

(c) $L\,\text{mol}^{-1}S^{-1}$

(d) S^{-1}

Sol: It's a second order reaction. Unit of rate constant $= L\,\text{mol}^{-1}S^{-1}$

Ans: (c)

35. The half-life of the first order reaction $X \rightarrow Y$ with the initial concentration of X to be $0.01\,\text{mol}\,L^{-1}$ and initial rate to be $0.00352\,\text{mol}\,L^{-1}\,\text{min}^{-1}$ will be

Options:

(a) $19.68\,\text{min}$

(b) $1.968\,\text{min}$

(c) $77.5\,\text{min}$ (d) $7.7\,\text{min}$

Sol: Rate $= k[A]$

$$k = \frac{\text{Rate}}{[\text{A}]} = \frac{0.00352}{0.01} = 0.352\,\text{min}^{-1}$$

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.352} = 1.968\,\text{min}$$

Ans: (b)

36. For a reaction $P + Q \rightarrow 2R + S$, which of the following statements are incorrect?

Options:

(a) rate of disappearance of P = rate of appearance of S

(b) rate of disappearance of $Q = 2\times$ rate of appearance of R

(c) rate of disappearance of P = rate of appearance of R

(d) rate of disappearance of $Q = \frac{1}{2}\times$ rate of appearance of R

Sol: $\text{Rate} = -\frac{dP}{dt} = -\frac{dQ}{dt} = 1/2\frac{dR}{dt} = \frac{dS}{dt}$

Rate of disappearance of $Q = -\frac{dQ}{dt} = 1/2\frac{dR}{dt}$

$= 1/2 \times$ Rate of appearance of R

Ans: (b)

37. What will be the rate equation for the reaction $2X + Y \rightarrow Z$, if the order of the reaction is zero?

Options:

(a) rate $= k[X][Y]$

(b) rate $= k$

(c) rate $= k[X]^{\circ}[Y]$

(d) rate $= k[X][Y]^{\circ}$

Sol: Rate $= k$

Ans: (b)

38. Which of the following is not an application of electrochemical series?

Options:

(a) to compare the relative oxidising and reducing power of substances

(b) to predict evolution of hydrogen gas on reaction of metal with acid

(c) to predict spontaneity of a redox reaction

(d) to calculate the amount of metal deposited on cathode

Sol: The amount of metal deposited on cathode can be calculated using Faraday's law

Ans: (d)

39. The reduction potential for the following half-cell reaction at $298\,K$?

$\left(\text{given}: \left[Ag^{+}\right] = 0.1\text{M and } E^{\circ}_{\text{cell}} = +0.80\,\text{V}\right)$

Options:

(a) $0.741\,\text{V}$

(b) $0.80\,\text{V}$

(c) $-0.80\,\text{V}$

(d) $-0.741\,\text{V}$

Sol: $E = E^{\circ} - \frac{0.591}{n}\log\frac{1}{[M]}$

$= 0.80 - \frac{0.0591}{1}\log\frac{1}{0.1} = 0.80 - 0.0591 = 0.7409 = 0.741\,\text{V}$

Ans: (a)

40. Henry's law constant for molality of methane in benzene at $298\,K$ is 4.27×10^{5} mm Hg. The mole fraction of methane in benzene at $298\,K$ under 760 mm Hg is

Options:

(a) 1.78×10^{-3}

(b) 17.43

(c) 0.114

(d) 2.814

Sol: $P = K_n x$ or $x = \frac{P}{K_n} = \frac{760}{4.27\times10^5} = 1.78\times10^{-3}$

Ans: (a)

41. What is the mole fraction of ethanol in the vapour phase, if the solution contains equimolar mixture of ethanol and methanol? Given($P^{\circ}_{ethanol} = 90$ mm of Hg)

Options:

(a) 0.34

(b) 0.5

(c) 0.66

(d) 0.8

Sol: $P_{\text{total}} = P_E + P_E = x_E P^{\circ}_E + x_M P^{\circ}_M$

$= 0.5\times46 + 0.5\times90 = 23 + 45 = 68$ mm

$\therefore \quad Y_E = \frac{P_E}{P_{\text{total}}} = \frac{23}{68} = 0.34$

Ans: (a)

42. Which of the following solution exhibits highest boiling point?

Options:

(a) 0.1m urea solution

(b) 1m urea solution

(c) 0.01m urea solution

(d) 0.001m urea solution

Sol: Higher the concentration, higher will be the boiling point

Ans: (b)

43. In bcc structure contribution of corner and central atom is

Options:

(a) $\frac{1}{8}, 1$

(b) $\frac{1}{4}, \frac{1}{8}$

(c) $\frac{1}{8}, \frac{1}{2}$

(d) $1, \frac{1}{2}$

Sol: From corner $= \frac{1}{8}$

From central atom $= 1$

Ans: (a)

44. Element B forms CCP structure and 'A' occupies half of the octahedral voids and while oxygen atom occupy all tetrahedral voids. The structure of the compound is

Options:

(a) AB_2O_4

(b) A_4B_2O

(c) A_2B_2O (d) A_2BO_4

Sol: Ratio of atoms

$= A:B:O = \frac{1}{2}\times 4:4:2\times 4$

$= 2:4:8 = 1:2:4$

Or AB_2O_4

Ans: (a)

45. Which of the following defects is also known as dislocation defect?

Options:

(a) Frankel defect

(b) Schottky defect

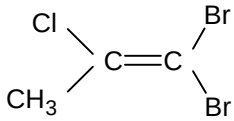
(c) Non-Stoichiometric defect

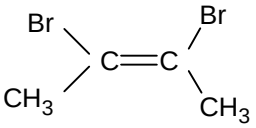
(d) Metal excess defect

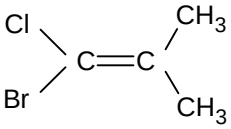
Sol: Frankel defect is known as dislocation

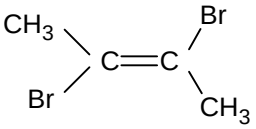
Ans: (a)

46. Which is a pair of geometrical isomers?

I. 

II. 

III. 

IV. 

Options:

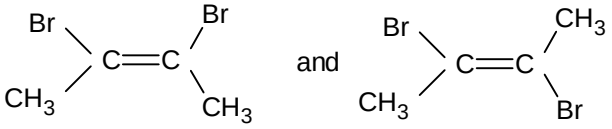
(a) I and II

(b) I and III

(c) II and IV

(d) III and IV

Sol:



Are geometrical isomers

Ans: (c)

47. For the reaction, $CO_{(g)} + Cl_{2(g)} \leftrightarrows COCl_{2(g)}$ the value of Kp / Kc is equal to

Options:

(a) 1.0

(b) RT

(c) $\sqrt{RT}$

(d) $\frac{1}{RT}$

Sol: $Dn_g = 1 - 2 = -1$

$Kp = Kc(RT)^{\Delta ng}$; $Kp / Kc = (RT)^{-1} = \frac{1}{RT}$

Ans: (d)

48. The solubility product of MgF_2 is 7.4×10^{-11} calculate the solubility of MgF_2 in 0.1M NaF solution

Options:

(a) 7.4×10^{-9}

(b) 3.7×10^{-9}

(c) 3.7×10^{-11}

(d) 7.4×10^{-11}

Sol: $K_{sp} = \left[Mg^{2+}\right]\left[F^{-}\right]^2$

$K_{sp} \quad S \quad x^2$

$S = \frac{K_{sp}}{x^2} = \frac{7.4 \times 10^{-11}}{(0.1)^2}$

$= 7.4 \times 10^{-9}$

Ans: (a)

49. A sample of pure compound contains 1.15g of sodium, 3.1×10^{22} atoms of carbon and 0.1mole of oxygen atom. Its empirical formula is

Options:

(a) Na_2CO_3

(b) $NaCO_2$

(c) Na_2CO

(d) Na_2CO_2

Sol: mole ratio

Number of moles of $Na = \frac{1.15}{23} = 0.05$ 1

Number of moles of $C = \frac{3.01 \times 10^{22}}{6.02 \times 10^{23}} = 0.05$ 1

Number of moles of $O = 0.1 \text{mole}$ 2

$EF = NaCO_2$

Ans: (b)

50. Which of the following sets of quantum numbers is not possible?

Options:

(a) $n = 4,\ l = 1,\ m = 0,\ s = +\frac{1}{2}$

(b) $n = 4,\ l = 3,\ m = 3,\ s = -\frac{1}{2}$

(c) $n = 4,\ l = 1,\ m = +2,\ s = -\frac{1}{2}$

(d) $n = 4,\ l = 0,\ m = 0,\ s = -\frac{1}{2}$

Sol: When $l = 1,\ m$ cannot have a value $+2$

$m = 0, +1$ or -1

Ans: (c)

51. Which of the following element is expected to have highest electron affinity?

Options:

(a) $1S^2\ 2S^2\ 2P^6\ 3S^2\ 3P^5$

(b) $1S^2\ 2S^2\ 2P^3$

(c) $1S^2\ 2S^2\ 2P^4$

(d) $1S^2\ 2S^2\ 2P^5$

Sol:

$1S^2\ 2S^2\ 2P^6\ 3S^2\ 3P^5$ -------- Cl

$1S^2\ 2S^2\ 2P^3$ --------- N

$1S^2 2S^2 2P^4$ -------- O

$1S^2\ 2S^2\ 2P^5$ -------- F

Cl has highest electron affinity.

Ans: (a)

52. Which of the following compound has $M = 0$?

Options:

(a) CCl_4

(b) $CHCl_3$

(c) HF (d) NH_3

Sol: CCl_4 has $M = 0$ because of symmetric structure

Ans: (a)

53. Which of the following relationships is true?

Options:

(a) Bond dissociation energy of O_2 and O_2^- are same

(b) Bond dissociation energy of O_2^+ is higher that O_2

(c) Bond dissociation energy of O_2^- and O_2^{2-} are same

(d) Bond dissociation energy of O_2^{2-} is higher then O_2^-

Sol: Higher the bond order, higher is the dissociation energy

O_2^+, $BO = 2.5$; O_2, $BO = 2$, ; O_2^-, $BO = 1.5$, O_2^{2-}, $BO = 1$

O_2^+ is having higher dissociation energy

Ans: (b)

54. Under what conditions gases generally deviate from ideal behaviour?

Options:

(a) At high temperature and low pressure

(b) At low temperature and high pressure

(c) At high temperature and high pressure

(d) At low temperature and low pressure

Sol: At low temperature and high pressure, volume is very low. Hence gases from ideal behaviour

Ans: (b)

55. What will be the heat of reaction for the following reaction? Will the reaction be exothermic or endothermic?

$Fe_2O_3 + 3H_2 \rightarrow 2Fe + 3H_2O$

$\Delta H_f^\circ (H_2O) = -285.83\,kJ\,mol^{-1}$

$\Delta H_f^\circ (Fe_2O_3) = -824.2\,kJ\,mol$

Options:

(a) $-824.2\,kJ$, exothermic

(b) $+33.3$, kJ endothermic

(c) -33.3, kJ exothermic

(d) $+824.2$ kJ, endothermic

Sol: $DH_{\text{reaction}} = \sum \Delta H_f^{\circ}(\text{reactant}) - \sum \Delta H_f^{\circ}(\text{products})$

$= -824.2 - 3(-285.83) = -824.2 + 857.49 = +33.29\,\text{kJ/mol}$

Ans: (b)

56. Which of the following statements is not correct

Options:

(a) For a spontaneous process, ΔG° must be negative

(b) Enthalpy, entropy, free energy etc are state variables

(c) A spontaneous process is reversible in nature

(d) Total of all possible kinds of energy of a system is called internal energy

Sol: A spontaneous process is an irreversible process

Ans: (c)

57. Oxidation number of Sulphur in peroxomonosulphuric acid (H_2SO_5) is

Options:

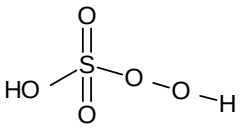
(a) +4

(b) +2

(c) +6

(d) –2

Sol:



$H_2SO_5 \rightarrow 2(+1) + x + 3(-2) + 2(-1) = 0$

$x = +6$

Ans: (c)

58. What is the reaction given below called?

$H_2O(l) + H_2O(l) \leftrightarrows H_3O^+(aq) + OH^-(aq)$

Options:

(a) hydrolysis of water

(b) hydration of water

(c) disproportionation of water

(d) auto – protolysis of water

Sol: Auto – protolysis of water

Ans: (d)

59. A metal M reacts with nitrogen to give nitride which on reaction with water produces ammonia gas. Metal M can be

Options:

(a) Na

(b) K

(c) Li

(d) Rb

Sol: $6Li + N_2 \rightarrow 2Li_3N$

Ans: (c)

60. Which of the following hydroxides is acidic?

Options:

(a) $Al(OH)_3$

(b) $Ga(OH)_3$

(c) $Tl(OH)_3$

(d) $B(OH)_3$

Sol: $B(OH)_3$ is a weak acid

Ans: (d)

Mathematics

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60

61. If R is a relation on the set N, defined by $\{(x, y): 2x - y = 10\}$, then R is

Options:

(a) Reflexive

(b) Symmetric

(c) Transitive

(d) None of these

Sol: Reflexive: As $20 \in N$ but $(20, 20) \notin R$.

So, it is not reflexive.

Symmetric: As $(20, 30) \in R$ but $(30, 20) \notin R$.

So, it is not symmetric.

Transitive: As $(20, 30) \in R, (30, 50) \in R$ but $(20, 50) \notin R$.

So, it is not transitive.

Ans: (d)

62. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then which of the following relations is a function from A to B?

Options:

(a) $\{(1, 2), (2, 3), (3, 4), (2, 2)\}$

(b) $\{(1, 2), (2, 3), (1, 3)\}$

(c) $\{(1, 3), (2, 3), (3, 3)\}$

(d) $\{(1, 1), (2, 3), (3, 4)\}$

Sol: In the given options, only option (c) satisfies the condition of a function.

Ans: (c)

63. The function $f: [0, \infty) \to [0, \infty)$ defined by $f(x) = \frac{2x}{1+2x}$

Options:

(a) one-one and onto

(b) one-one but not onto

(c) not one-one but onto

(d) neither one-one nor onto

Sol: f is one-one: $f(x_1) = f(x_2)$

$$\Rightarrow \frac{2x_1}{1+2x_1} = \frac{2x_2}{1+2x_2} \Rightarrow x_1 + 2x_1x_2 = x_2 + 2x_1x_2 \Rightarrow x_1 = x_2$$

Also, f is not onto as 1 does not have a pre-image.

Ans: (b)

64. The domain of the real function $f(x)=\frac{1}{\sqrt{4-x^2}}$ is

Options:

(a) The set of all real numbers

(b) The set of all positive real numbers

(c) $(-2, 2)$

(d) $[-2, 2]$

Sol: Given, $f(x)=\frac{1}{\sqrt{4-x^2}}$

For domain of $f(x), 4-x^2>0 \Rightarrow x^2<4 \Rightarrow -2<x<2$

$\therefore$ Domain $=(-2, 2)$

Ans: (c)

65. The binary operation $*$ is defined on Q, the set of rational numbers by $a*b=\frac{ab}{5}$, and $(2*x)*3=10$, then x is equal to

Options:

(a) $\frac{5}{3}$

(b) $\frac{25}{3}$

(c) $\frac{125}{3}$

(d) None of these

Sol: $(2*x)*3=\left(\frac{2x}{5}\right)*3=\frac{\frac{2x}{5}\times 3}{5}=\frac{6x}{25}=10 \Rightarrow x=\frac{10\times 25}{6}=\frac{125}{3}$

Ans: (c)

66. Out of 64 students, the number of students taking Mathematics is 45 and number of students taking both Mathematics and Biology is 10. Then, the number of students taking only Biology is

Options:

(a) 18

(b) 19

(c) 20

(d) None of these

Sol: Let M and B denote the number of students taking Mathematics and Biology.

Then, $n(M)=45, n(M\cap B)=10, n(M\cup B)=64$

$\therefore n(B)=n(M\cup B)-n(M)+n(M\cap B)=64-45+10=29 \Rightarrow n(B)=n(B)-n(M\cap B)=29-10=19$

Ans: (b)

67. The value of $\frac{\cot 54^\circ}{\tan 36^\circ}+\frac{\tan 20^\circ}{\cot 70^\circ}$ is

Options:

(a) 0

(b) 2

(c) 3

(d) 1

Sol: $\frac{\cot 54^\circ}{\tan 36^\circ}+\frac{\tan 20^\circ}{\cot 70^\circ}=\frac{\cot(90^\circ-36^\circ)}{\tan 36^\circ}+\frac{\tan 20^\circ}{\cot(90^\circ-20^\circ)}$

$=\frac{\tan 36^\circ}{\tan 36^\circ}+\frac{\tan 20^\circ}{\tan 20^\circ}=1+1=2$

Ans: (b)

68. If $\tan\theta=\frac{1}{\sqrt{7}}$, then $\frac{\left(\operatorname{cosec}^2\theta-\sec^2\theta\right)}{\left(\operatorname{cosec}^2\theta+\sec^2\theta\right)}$ is equal to

Options:

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{5}{4}$

(d) 2

Sol: Given, $\tan\theta=\frac{1}{\sqrt{7}}\Rightarrow\cot\theta=\sqrt{7}$

$\therefore\frac{\left(\operatorname{cosec}^2\theta-\sec^2\theta\right)}{\left(\operatorname{cosec}^2\theta+\sec^2\theta\right)}=\frac{\left(1+\cot^2\theta-1-\tan^2\theta\right)}{\left(1+\cot^2\theta+1+\tan^2\theta\right)}$

$=\frac{\cot^2\theta-\tan^2\theta}{2+\cot^2\theta+\tan^2\theta}=\frac{\left(\sqrt{7}\right)^2-\left(1/\sqrt{7}\right)^2}{2+\left(\sqrt{7}\right)^2+\left(1/\sqrt{7}\right)^2}$

$=\frac{49-1}{7}\times\frac{7}{63+1}=\frac{48}{64}=\frac{3}{4}$

Ans: (b)

69. If $A=35^\circ, B=15^\circ$ and $C=40^\circ$, then $\tan A\tan B+\tan B\tan C+\tan C\tan A$ is equal to

Options:

(a) 0

(b) 1

(c) 2

(d) 3

Sol: $\because \tan(A+B+C)$

$=\dfrac{[\tan A+\tan B+\tan C-\tan A\tan B\tan C]}{[1-\tan A\tan B-\tan B\tan C-\tan C\tan A]}$

$\Rightarrow \tan(90°)=\dfrac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$

$\therefore \tan A\tan B+\tan B\tan C+\tan C\tan A=1$

OR

$A+B=90°-C$

$\tan(A+B)=\cot C$

$\Rightarrow \dfrac{\tan A+\tan B}{1-\tan A\tan B}=\dfrac{1}{\tan C}$

$\Rightarrow \tan A\tan B+\tan B\tan C+\tan C\tan A=1$

Ans: (b)

70. The general value of θ satisfying the equation $2\sin^2\theta-3\sin\theta-2=0$ is

Options:

(a) $n\pi+(-1)^{n+1}\dfrac{\pi}{6}$

(b) $n\pi+(-1)^{n}\dfrac{\pi}{2}$

(c) $n\pi+(-1)^{n}\dfrac{5\pi}{6}$

(d) $n\pi+(-1)^{n}\dfrac{7\pi}{6}$

Sol: $2\sin^2\theta-3\sin\theta-2=0\Rightarrow(2\sin\theta+1)(\sin\theta-2)=0$

$\sin\theta=-\dfrac{1}{2}\ [\because \sin\theta\neq 2]$

$\Rightarrow \sin\theta=\sin\left(-\dfrac{\pi}{6}\right)\Rightarrow\theta=n\pi+(-1)^n\left(-\dfrac{\pi}{6}\right)\Rightarrow\theta=n\pi+(-1)^{n+1}\dfrac{\pi}{6}$

Also, $\sin\theta=-\dfrac{1}{2}=\sin\dfrac{7\pi}{6}$

$\therefore\theta=n\pi+(-1)^n\dfrac{7\pi}{6}$

Ans: (d)

71. The value of $\sin^{-1}\{\cos(4095°)\}$ is

Options:

(a) $-\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $-\frac{\pi}{4}$

(d) $\frac{\pi}{4}$

Sol: $\cos(4095°) = \cos(45\times 90° + 45°) = -\sin 45° = -\sin\frac{\pi}{4} = \sin\left(-\frac{\pi}{4}\right)$

$\therefore \sin^{-1}\{\cos(4095°)\} = \sin^{-1}\left\{\sin\left(-\frac{\pi}{4}\right)\right\} = -\frac{\pi}{4}$

Ans: (c)

72. If $\sqrt{3}+i = (a+ib)(c+id)$, then $\tan^{-1}\left(\frac{b}{a}\right)+\tan^{-1}\left(\frac{d}{c}\right)$ has the value

Options:

(a) $\frac{\pi}{3}+2n\pi, n\in I$

(b) $n\pi+\frac{\pi}{6}.n\in I$

(c) $n\pi-\frac{\pi}{3}, n\in I$

(d) $2n\pi-\frac{\pi}{3}, n\in I$

Sol: Given, $\left(\sqrt{3}+i\right) = (a+ib)(c+id) = (ac-bd)+i(ad+bc)$

On comparing the real and imaginary parts on both sides, we get

$ac-bd=\sqrt{3}$ and $ad+bc=1$

Now, $\tan^{-1}\left(\frac{b}{a}\right)+\tan^{-1}\left(\frac{d}{c}\right) = \tan^{-1}\left(\frac{bc+ad}{ac-bd}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = n\pi+\frac{\pi}{6}, n\in I$

Ans: (b)

73. If $z = \frac{\left(\sqrt{3}+i\right)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

Options:

(a) 8

(b) 2

(c) 5

(d) 4

Sol: Given, $z=\dfrac{\left(\sqrt{3}+i\right)^3\left(3i+4\right)^2}{\left(8+6i\right)^2}$

Now, $|z|=\left|\dfrac{\left(\sqrt{3}+i\right)^3\left(3i+4\right)^2}{\left(8+6i\right)^2}\right|=\dfrac{\left|\left(\sqrt{3}+i\right)^3\right|\left|\left(3i+4\right)^2\right|}{\left|\left(8+6i\right)^2\right|}$ $\left[\because\left|\dfrac{z_1}{z_2}\right|=\left|\dfrac{z_1}{z_2}\right|\right]$

$=\dfrac{\left|\sqrt{3}+i\right|^3\left|3i+4\right|^2}{\left|8+6i\right|^2}$ $\left[\because\left|z^n\right|=|z|^n\right]$

$=\dfrac{\left(\sqrt{3+1}\right)^3\left(\sqrt{9+16}\right)^2}{\left(\sqrt{64+36}\right)^2}=\dfrac{(2)^3(5)^2}{(10)^2}=\dfrac{10^2\cdot 2}{(10)^2}=2$

Ans: (b)

74. The slope of the tangent to the curve $y^2e^{xy}=9e^{-3}x^2$ at $(-1,3)$ is

Options:

(a) $\dfrac{-15}{2}$

(b) $\dfrac{-9}{2}$

(c) 15

(d) $\dfrac{15}{2}$

Sol: Given curve is $y^2e^{xy}=9e^{-3}x^2$

On differentiating both sides w.r.t. we get

$e^{xy}\dfrac{d}{dx}\left(y^2\right)+y^2\dfrac{d}{dx}\left(e^{xy}\right)=9\left[x^2\dfrac{d}{x}\left(e^{-3}\right)+e^{-3}\dfrac{d}{dx}\left(x^2\right)\right]$

$\Rightarrow e^{xy}\left(2y\dfrac{dy}{dx}\right)+y^2\left[e^{xy}\left(y+x\dfrac{dy}{dx}\right)\right]=9\left[x^2\times 0+e^{-3}\left(2x\right)\right]$

Put $x=-1$ and $y=3$, we get

$e^{-1\times 3}\left(2\times 3\dfrac{dy}{dx}\right)+3^2\left[e^{-1\times 3}\left\{3-\dfrac{dy}{dx}\right\}\right]=9\left(-2e^{-3}\right)$

On dividing both sides by e^{-3}, we get

$6\dfrac{dy}{dx}+27-9=-18\Rightarrow -3\dfrac{dy}{dx}=-45\Rightarrow\dfrac{dy}{dx}=15$

Ans: (c)

75. If gas is being pumped into a spherical balloon at the rate of $30\,\text{ft}^3/\text{min}$. Then, the rate at which the radius increases, when it reaches the value $15\,\text{ft}$ is

Options:

(a) $\frac{1}{15\pi}\text{ft/min}$

(b) $\frac{1}{30\pi}\text{ft/min}$

(c) $\frac{1}{20}\text{ft/min}$

(d) $\frac{1}{25}\text{ft/min}$

Sol: Let $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$

$\therefore \frac{dr}{dt}=\frac{30}{4\times\pi\times15\times15}=\frac{1}{30\pi}\text{ft/min} \qquad \left[\because \frac{dV}{dt}=30, r=15\right]$

Ans: (b)

76. The maximum value of $f(x)=\frac{x}{4+x+x^2}$ on $[-1,1]$ is

Options:

(a) $-\frac{1}{3}$

(b) $-\frac{1}{4}$

(c) $\frac{1}{4}$

(d) $\frac{1}{6}$

Sol: Given, $f(x)=\frac{x}{4+x+x^2}$

Let $f(x)=\frac{1}{u}$, then $u=\frac{4+x+x^2}{x}=\frac{4}{x}+1+x$

$\therefore \frac{du}{dx}=-\frac{4}{x^2}+1, \frac{d^2u}{dx^2}=\frac{8}{x^3}$

For maximum or minimum, put $\frac{du}{dx}=0$

$\Rightarrow 1-\frac{4}{x^2}=0\Rightarrow x\pm2$

$\therefore$ At $x=-2, \frac{d^2u}{dx^2}=\frac{-8}{(2)^3}<0$ maximum

At $x=2, \frac{d^2u}{dx^2}=1>0$ minima

$\therefore$ At $x=2$, $f(x)$ is maxima

And at $x=-2$, $f(x)$ is maxima. It is increasing function in the given interval

$\therefore$ The maximum value at $x=1$ is $f(1)=\frac{1}{4+1+1}=\frac{1}{6}$

Ans: (d)

77. If $f(x)=\begin{cases}0, & x=0\\ x-3, & x>0\end{cases}$, then the function $f(x)$ is

Options:

(a) Increasing when $x\geq 0$

(b) Strictly increasing when $x>0$

(c) Strictly increasing at $x=0$

(d) Not continuous at $x=0$ and so it is not increasing when $x>0$

Sol: $f'(x)=1>0$, when $x>0$.

So, $f(x)$ is strictly increasing when $x>0$.

Ans: (b)

78. The number of permutations of the letters of the word CONSEQUENCE in which all the three E's are together is

Options:

(a) 9!3!

(b) $\frac{9!}{2!2!}$

(c) $\frac{9!}{2!2!3!}$

(d) $\frac{9!}{2!3!}$

Sol: The letters in the word 'CONSEQUENCE' are $2C, 3E, 2N, 1O, 1Q, 1S, 1U$.

$\therefore$ Required number of permutations $=\frac{9!}{2!2!}$

Ans: (b)

79. If in the expansion of $\left(3x-\frac{2}{x^2}\right)^{15}$, r th term is independent of x, then the value of r is

Options:

(a) 6

(b) 10

(c) 9

(d) 12

Sol: rth term in the expansion of $\left(3x-\frac{2}{x^2}\right)^{15}$ is $T_r={}^{15}C_{r-1}(3x)^{15-r+1}\left(\frac{-2}{x^2}\right)^{r-1}$

$={}^{15}C_{r-1}(3)^{15-r+1}(-2)^{r-1}(x)^{15-3r+3}$

For the term independent of x, put $15-3r+3=0\Rightarrow r=6$

Ans: (a)

80. The arithmetic mean of 7 consecutive integers starting with a is m. Then, the arithmetic mean of 11 consecutive integers starting with $a+2$ is

Options:

(a) $2a$

(b) $2m$

(c) $a+4$

(d) $m+4$

Sol: $\frac{a+(a+1)+(a+2)+...+(a+6)}{7}=m\ \Rightarrow 7a+21=7m\Rightarrow a+3=m$

$\therefore\frac{(a+2)+(a+3)+(a+4)+...+(a+12)}{11}=\frac{11a+77}{11}$

$=a+7=m-3+7=m+4$

Ans: (d)

81. If 3 and 4 are intercepts of a line $L\equiv 0$, then the distance of $L\equiv 0$ from the origin is

Options:

(a) 5 units

(b) 12 units

(c) $\frac{5}{12}$ units

(d) $\frac{12}{5}$ units

Sol: Equation of line is $\frac{x}{3}+\frac{y}{4}=1$

$\Rightarrow 4x+3y-12=0$

Now, distance from origin $=\left|\frac{4\times 0+3\times 0-12}{\sqrt{3^2+4^2}}\right|=\frac{12}{5}$ units

Ans: (d)

82. Compute the shortest distance between the circle $x^2+y^2-10x-14y-151=0$ and the point $(-7,2)$.

Options:

(a) 0

(b) 1

(c) 2

(d) 4

Sol: We have an equation of circle $x^2+y^2-10x-14y-151=0$

Let us compare this equation with $x^2+y^2+2gx+2fy+c=0$

Clearly, $g=-5$, $f=-7$ and $c=-151$

$\therefore$ Centre of circle $=(-g,-f)=(5,7)$ and radius

$=\sqrt{g^2+f^2-c}=\sqrt{25+49+151}=\sqrt{225}=15$

Now, the distance between centre and the given point $(-7,2)$

$=\sqrt{(5+7)^2+(5)^2}=\sqrt{(12)^2+25}=\sqrt{169}=13$

Clearly, the point $(-7,2)$ lies inside the circle.

$\therefore$ Shortest distance = Radius − Distance between point and centre $=15-13=2$

Ans: (c)

83. If a line in the space makes angles α, β and γ with the coordinate axes, then

$\cos 2\alpha+\cos 2\beta+\cos 2\gamma+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ equals

Options:

(a) -1

(b) 0

(c) 1

(d) 2

Sol: Given, $\cos 2\alpha+\cos 2\beta+\cos 2\gamma+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=\left(\cos^2\alpha-\sin^2\alpha\right)+\left(\cos^2\beta-\sin^2\beta\right)+\left(\cos^2\gamma-\sin^2\gamma+\sin^2\alpha+\sin^2\beta+\sin^2\gamma\right)$

$=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

Ans: (c)

84. The foot of the perpendicular from $(2,4,-1)$ to the line $x+5=\frac{1}{4}(y+3)=-\frac{1}{9}(z-6)$ is

Options:

(a) $(-4,1,-3)$

(b) $(4,-1,-3)$

(c) $(-4,-1,3)$

(d) $(-4,-1,-3)$

Sol: In given options, only option (a) satisfies the given equation of line.

Ans: (a)

85. If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, then the foot of the perpendicular from P to the plane is

Options:

(a) $\left(\frac{8}{3},\frac{4}{3},\frac{-7}{3}\right)$

(b) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$

(c) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$

(d) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

Sol: Distance of point P from plane $=5$

$\therefore 5=\left|\frac{1-4-2-\alpha}{3}\right| \Rightarrow \alpha=10$

Foot of perpendicular is $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=\frac{(1-4-2-10)}{1+4+4}=\frac{5}{3}$

$\Rightarrow x=\frac{8}{3}, y=\frac{4}{3}, z=-\frac{7}{3}$

Thus, the foot of the perpendicular is $A\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$.

Ans: (a)

86. A line with positive direction cosines passes through the point $P(2,-1,2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x+y+z=9$ at point Q. The length of the line segment PQ equals

Options:

(a) 1

(b) $\sqrt{2}$

(c) $\sqrt{3}$

(d) 2

Sol: Since, $l=m=n=\frac{1}{\sqrt{3}}$

$\therefore$ Equation of line is $\frac{x-2}{1/\sqrt{3}}=\frac{y+1}{1/\sqrt{3}}=\frac{z-2}{1/\sqrt{3}}$

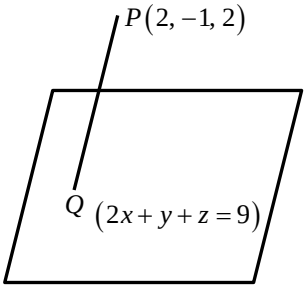
$\Rightarrow x-2=y+1=z-2=r$ [say]

Any point on the line is $Q=(r+2,r-1,r+2)$

Since, Q lies on the plane $2x+y+z=9$.

$\therefore 2(r+2)+(r-1)+(r+2)=0$

$\Rightarrow 4r+5=9 \Rightarrow r=1$



So, coordinates Q are $(3, 0, 3)$.

$\therefore PQ = \sqrt{(3-2)^2 + (0+1)^2 + (3-2)^2} = \sqrt{3}$

Ans: (c)

87. Given, $p = 3\hat{i} + 2\hat{j} + 4\hat{k}, a = \hat{i} + \hat{j}, b = \hat{j} + \hat{k}, c = \hat{i} + \hat{k}$ and $p = xa + yb + zc$, then x, y and z are respectively,

Options:

(a) $\frac{3}{2}, \frac{1}{2}, \frac{5}{2}$

(b) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

(c) $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}$

(d) $\frac{1}{2}, \frac{5}{2}, \frac{3}{2}$

Sol: $p = xa + yb + zc$

$\Rightarrow 3\hat{i} + 2\hat{j} + 4\hat{k} = x(\hat{i} + \hat{j}) + y(\hat{j} + \hat{k}) + z(\hat{i} + \hat{k})$

$\Rightarrow 3\hat{i} + 2\hat{j} + 4\hat{k} = (x + z)\hat{i} + (x + y)\hat{j} + (y + z)\hat{k}$

On comparing both sides the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$x + z = 3$ … (i)

$x + y = 2$ … (ii)

And $y + z = 4$ ... (iii)

On solving equation (i), (ii) and (iii), we get

$x = \frac{1}{2}, y = \frac{3}{2}$, and $z = \frac{5}{2}$

Ans: (b)

88. If $\hat{i} + \hat{j}, \hat{j} + \hat{k}$ and $\hat{i} + \hat{k}$ are the position vectors of the vertices of a ΔABC taken in order, then $\angle A$ is equal to

Options:

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{5}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{3}$

Sol: Let position vector of the vertices be $OA = \hat{i} + \hat{j}, OB = \hat{j} + \hat{k}$ and $OC = \hat{i} + \hat{k}$

Now, $AB = -\hat{i} + \hat{k}$ and $AC = \hat{k} - \hat{j}$

$$\therefore \cos\theta = \frac{(AB)\cdot(AC)}{|AB||AC|} = \frac{\left(-\hat{i}+\hat{k}\right)\cdot\left(\hat{k}-\hat{j}\right)}{\sqrt{1^2+1^2}\sqrt{1^2+1^2}}$$

$$= \frac{\left(\hat{k}\right)^2}{\sqrt{2}\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Ans: (d)

89. $|a| = |b| = 5$ and the angle between a and b is $\frac{\pi}{4}$. The area of the triangle constructed on the vectors $a-2b$ and $3a+2b$ is

Options:

(a) 50

(b) $50\sqrt{2}$

(c) $\frac{50}{\sqrt{2}}$

(d) 100

Sol: We have, $|a| = |b| = 5$ and angle between a and b is $\frac{\pi}{4}$.

Now, area of the triangle constructed on the vectors is given by

$$= \frac{1}{2}\left|(a-2b)\times(3a+2b)\right|$$

$$= \frac{1}{2}\left|3(a\times a)+2(a\times b)-6(b\times a)-4(b\times b)\right|$$

$$= \frac{1}{2}\left|2(a\times b)+6(a\times b)\right| \qquad [\because a\times a = b\times b = 0 \text{ and } a\times b = -b\times a]$$

$$= \frac{1}{2}\left|8(a\times b)\right| = \frac{8}{2}|a\times b|$$

$$= 4|a||b|\left|\sin\frac{\pi}{4}\right| = 4\cdot 5\cdot 5\cdot\frac{1}{\sqrt{2}} = 2\sqrt{2}\times 5\times 5 = 50\sqrt{2} \text{ square units}$$

Ans: (b)

90. If $a = \frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $b = \frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $(2a-b)\cdot\left[(a\times b)\times(a+2b)\right]$ is

Options:

(a) -3

(b) 5

(c) 3

(d) -5

Sol: $a = \frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $b = \frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$

$$\therefore (2a-b)\cdot\{(a\times b)\times(a\times 2b)\}$$

$$= (2a-b)\cdot\{(a\times b)\times a+(a\times b)\times 2b\}$$

$= (2a-b)\cdot\{(a\cdot a)b+(b\cdot a)a+2(a\cdot b)b-2(b\cdot b)a\}$

$= (2a-b)\cdot\{1b-(0)a+2(0)b-2(1)a\}$ [as $a\cdot b=0$ and $a\cdot a=b\cdot b=1$]

$= (2a-b)(b-2a) = -\left(4|a|^2+4a\cdot b+|b|^2\right)-(4+0+1)=-5$

Ans: (d)

91. $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$ is equal to

Options:

(a) $-\dfrac{1}{4}$

(b) $-\dfrac{1}{2}$

(c) 0

(d) $\dfrac{2}{9}$

Sol: $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$

$= \lim\limits_{x\to\infty}\dfrac{x^3(3x+2)-x^2\left(3x^2-4\right)}{\left(3x^2-4\right)(3x+2)}$

$= \lim\limits_{x\to\infty}\dfrac{2x^3+4x^2}{9x^3+6x^2-12x-8}$

$= \lim\limits_{x\to\infty}\dfrac{2+4/x}{9+6/x-12/x^2-8/x^3}=\dfrac{2}{9}$

Ans: (d)

92. $\lim\limits_{x\to0}\dfrac{a^x+a^{-x}-2}{x^2}$ is equal to

Options:

(a) $(\log a)^2$

(b) $\log a$

(c) 0

(d) none of these

Sol: $\lim\limits_{x\to0}\dfrac{a^x+a^{-x}-2}{x^2}=\lim\limits_{x\to0}\dfrac{a^x\log a-a^{-x}\log a}{2x}$ [Using L' Hospital's rule]

$= \lim\limits_{x\to0}\dfrac{a^x(\log a)^2+a^{-x}(\log a)^2}{2}=(\log a)^2$

Ans: (a)

93. The value of $\lim_{x\to 0}\frac{\sin^2 x+\cos x-1}{x^2}$ is

Options:

(a) 1

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) 0

Sol: $\lim_{x\to 0}\frac{\sin^2 x+\cos x-1}{x^2}=\lim_{x\to 0}\frac{\cos x-\cos^2 x}{x^2}$

$=\lim_{x\to 0}\cos x.\frac{1-\cos x}{x^2}=1.\frac{1}{2}=\frac{1}{2}$

Ans: (b)

94. If $f(x)=\begin{cases}\frac{3\sin \pi x}{5x}, & x\neq 0\\ 2k, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is

Options:

(a) $\frac{\pi}{10}$

(b) $\frac{3\pi}{10}$

(c) $\frac{3\pi}{2}$

(d) $\frac{3\pi}{5}$

Sol: Given, $f(x)=\begin{cases}\frac{3\sin \pi x}{5x}, & x\neq 0\\ 2k, & x=0\end{cases}$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(\frac{3\sin \pi x}{5x}\right)=\frac{3}{5}\lim_{x\to 0}\left(\sin\frac{\pi x}{\pi x}\right)\times\pi=\frac{3}{5}\times 1\times\pi=\frac{3}{5}\pi$

Also, $f(0)=2k$

Since, $f(x)$ is continuous at $x=0$.

$\therefore f(0)=\lim_{x\to 0} f(x)\Rightarrow 2k=\frac{3}{5}\pi\Rightarrow k=\frac{3\pi}{10}$

Ans: (b)

95. The number of points of $f(x)=|x-1|+|x-3|+\sin x,\ x\in[0,4)$, where $f(x)$ is not differentiable, is

Options:

(a) 0

(b) 1

(c) 2

(d) 3

Sol: Since, function $|x-1|$ is not differentiable at $x=1$ and function $|x-3|$ is not differentiable at $x=3$.

Hence, $f(x)$ is not differentiable at $x=3$ and $x=1$

Ans: (c)

96. If $y=f\left(x^2+2\right)$ and $f'(3)=5$, then $\frac{dy}{dx}$ at $x=1$ is

Options:

(a) 5

(b) 25

(c) 15

(d) 10

Sol: Given, $y=f\left(x^2+2\right)$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=f'\left(x^2+2\right)\times 2x$$

On putting $x=1$, we get

$$\frac{dy}{dx}=f'\left(1^2+2\right)\times 2=f'(3)\times 2$$

$$=5\times 2 \qquad \left[\because f'(3)=5, \text{given}\right]$$

$$=10$$

Ans: (d)

97. If $y=\sec\left(\tan^{-1}x\right)$, then $\frac{dy}{dx}$ at $x=1$ is equal to

Options:

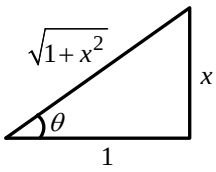
(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{2}$

(c) 1

(d) $\sqrt{2}$

Sol: $y = \sec\left(\tan^{-1} x\right)$



Let $\tan^{-1} x = \theta \Rightarrow x = \tan\theta$

$\therefore y = \sec\theta = \sqrt{1+x^2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \cdot 2x \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{\sqrt{2}}$

Ans: (a)

98. If $y = e^{ax}\sin bx$, then $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y$ is equal to

Options:

(a) 0

(b) 1

(c) $-b^2y$

(d) $-by$

Sol: Given, $y = e^{ax}\sin bx$ … (i)

$\Rightarrow \frac{dy}{dx} = ae^{ax}\sin bx + be^{ax}\cos bx$

$\Rightarrow \frac{dy}{dx} = ay + be^{ax}\cos bx$ … (ii)

$\Rightarrow \frac{d^2y}{dx^2} = a\frac{dy}{dx} + abe^{ax}\cos bx - e^{ax}b^2\sin bx$

$\Rightarrow \frac{a^2y}{dx^2} = a\frac{dy}{dx} + a\left(\frac{dy}{dx} - ay\right) - b^2y$

[From Equation (i) and (ii)]

$\Rightarrow \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y = -b^2y$

Ans: (c)

99. $\frac{d}{dx}\left[\log_e e^{\sin\left(x^2\right)}\right]$ is equal to

Options:

(a) $2\cos\left(x^2\right)$

(b) $2\cos x$

(c) $2x\cdot\cos x$

(d) $2x\cos\left(x^2\right)$

Sol: $\frac{d}{dx}\left[\log_e e^{\sin\left(x^2\right)}\right]=\frac{d}{dx}\left[\sin\left(x^2\right)\right]=\cos\left(x^2\right)2x$

Ans: (d)

100. Five persons A, B, C, D and E are in queue of a shop. The probability that A and E are always together, is

Options:

(a) $\frac{1}{4}$

(b) $\frac{2}{3}$

(c) $\frac{2}{5}$

(d) $\frac{3}{5}$

Sol: Total number of ways $=5!$

And favourable number of ways $=2\cdot 4!$

$\therefore$ Required probability $=\frac{2\cdot 4!}{5!}=\frac{2}{5}$

Ans: (c)

101. The probability of choosing randomly a number c from the set $\{1, 2, 3, ..., 9\}$ such that the quadratic equation $x^2+4x+c=0$ has real roots, is

Options:

(a) $\frac{1}{9}$

(b) $\frac{2}{9}$

(c) $\frac{3}{9}$

(d) $\frac{4}{9}$

Sol: Given, $x^2+4x+c=0$

For real roots, $D=b^2-4ac\geq 0=16-4c\geq 0$

So, $c=1, 2, 3, 4$ will satisfy the above inequality.

$\therefore$ Required probability $=\frac{4}{9}$

Ans: (d)

102. If four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$ and $\frac{1}{8}$. Then, the probability that the problem is solved correctly by atleast one of them, is

Options:

(a) $\frac{235}{256}$

(b) $\frac{21}{256}$

(c) $\frac{3}{256}$

(d) $\frac{253}{256}$

Sol: Required problem $= 1 - P(\text{none solves the problem})$

$= 1 - \left\{\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}\right\} = 1 - \frac{21}{256} = \frac{235}{256}$

Ans: (a)

103. An urn contains 3 red and 5 blue balls. The probability that two balls are drawn in which 2^{nd} ball drawn is blue without replacement is

Options:

(a) $\frac{5}{16}$

(b) $\frac{5}{56}$

(c) $\frac{5}{8}$

(d) $\frac{20}{56}$

Sol: Required probability $= P$ (Ist is red and IInd is blue) $+ P$ (Ist is blue and IInd is also blue)

$= \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{4}{7} = \frac{5}{8}$

Ans: (c)

104. A random variable X has the probability distribution given below

X	1	2	3	4	5
$P(X=x)$	K	$2K$	$3K$	$2K$	K

Its variance is

Options:

(a) $\frac{16}{3}$

(b) $\frac{4}{3}$

(c) $\frac{5}{3}$

(d) $\frac{10}{3}$

Sol: Given distribution is

X	1	2	3	4	5
$P(X=x)$	K	$2K$	$3K$	$2K$	K

$9K=1 \Rightarrow K=\frac{1}{9}$

$\therefore$ Variance $=\sum x_i^2 p-\left(\sum x_i p\right)^2 =(1k+8k+27k+32k+25k)-(k+4k+9k+8k+5k)^2$

$=(93k)-(27k)^2=\left(93\times\frac{1}{9}\right)-\left(27\times\frac{1}{9}\right)^2\left[\because \sum p=1, \text{so } k=\frac{1}{9}\right]$

$=\frac{93}{9}-9=\frac{93-81}{9}=\frac{12}{9}=\frac{4}{3}$

Ans: (b)

105. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards, then the mean of the number of aces is

Options:

(a) $\frac{1}{13}$

(b) $\frac{3}{13}$

(c) $\frac{2}{13}$

(d) None of these

Sol: Let X denote the number of aces. Probability of selecting ace, $p=\frac{4}{52}=\frac{1}{13}$

Probability of not selecting aces, $q=1-\frac{1}{13}=\frac{12}{13}$

$P(X=1)=2\times\left(\frac{1}{13}\right)\times\left(\frac{12}{13}\right)=\frac{24}{169}$

$P(X=2)=\left(\frac{1}{13}\right)^2\cdot\left(\frac{12}{13}\right)^0=\frac{1}{169}$

$\text{Mean}=\sum P_iX_i=\frac{24}{169}+\frac{2}{169}=\frac{2}{13}$

Ans: (c)

106. The set of values of x satisfying $3(2-x)\geq 2(1-x)$

Options:

(a) $\{x: x\in R, x\leq 4\}$

(b) $\{x: x\in R, x<4\}$

(c) $\{x: x\in R, x\geq 4\}$

(d) none of these

Sol: $3(2-x)\geq 2(1-x) \Rightarrow 6-3x\geq 2-2x$

$\Rightarrow -x\geq -4 \Rightarrow x\leq 4$

Ans: (a)

107. If p is the statement 'Ravi races' and q is the statement 'Ravi wins'. Then, the verbal translation of $\sim[p\vee(\sim q)]$ is

Options:

(a) Ravi does not race and Ravi does not win

(b) It is not true that Ravi races and that Ravi does not win

(c) It is not true that Ravi does not race or that Ravi does not win

(d) It is not that Ravi does not race and Ravi does not win

Sol: Given, p : Ravi races, q : Ravi wins

So, the statement of given proposition $\sim[p\vee(-q)]$ which is equivalent to $\sim p\wedge q$, is

Ravi does not race and Ravi wins.

Ans: (c)

108. If given constraints are $5x+4y\geq 2, x\leq 6$ and $y\leq 7$, then the maximum value of the function $z=x+2y$ is

Options:

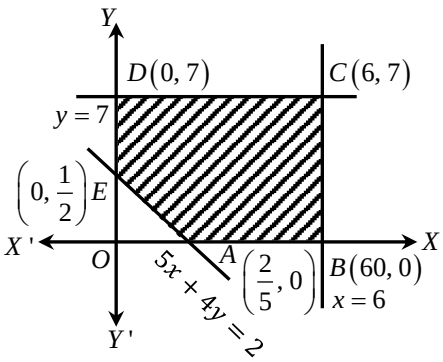
(a) 13

(b) 14

(c) 15

(d) 20

Sol: Feasible region is $ABCDEA$ and $z = x + 2y$.



At point $A\left(\frac{2}{5}, 0\right)$, $z = \frac{2}{3} + 0 = \frac{2}{5}$

At point $B(6, 0)$, $z = 6 + 0 = 6$

At point $C(6,7)$, $z = 6 + 14 = 20$

At point $D(0, 7)$, $z = 0 + 2(7) = 14$

At point $E\left(0, \frac{1}{2}\right)$, $x = 0 + 2\left(\frac{1}{2}\right) = 1$

Hence, maximum value of z is 20.

Ans: (d)

109. If $A = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix, then the value of x is

Options:

(a) 4

(b) 3

(c) –4

(d) –3

Sol: We have $A = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix,

i.e., $A^T = A$

$\Rightarrow \begin{bmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{bmatrix} = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix} \Rightarrow 2x+3 = x-1$

$x = -4$

Ans: (c)

110. If $U = [2 \; -3 \; 4]$, $X = [0 \; 2 \; 3]$, $V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ then $UV + XY$ is equal to

Options:

(a) $[20]$

(b) 20

(c) $[-20]$

(d) –20

Sol: $UV + XY = [2 \; -3 \; 4]\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + [0 \; 2 \; 3]\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

$= [6 - 6 + 4] + [0 + 4 + 12] = [4] + [16] = [20]$

Ans: (a)

111. If $a_1, a_2, a_3, \ldots$ are in an AP, then the value of $\begin{vmatrix} a_1 & a_2 & 1 \\ a_2 & a_3 & 1 \\ a_3 & a_4 & 1 \end{vmatrix}$ is

Options:

(a) $a_4 - a_1$

(b) $\frac{a_1 + a_4}{2}$

(c) 1

(d) 0

Sol: Given, $a_1, a_2, a_3, \ldots$ are in AP.

$\therefore a_2 - a_1 = a_3 - a_2 = \ldots = k \quad \ldots$ (i)

Let $a = \begin{vmatrix} a_1 & a_2 & 1 \\ a_2 & a_3 & 1 \\ a_3 & a_4 & 1 \end{vmatrix}$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 \to R_2$, we get

$$= \begin{vmatrix} a_1 & a_2 & 1 \\ a_2 - a_1 & a_3 - a_2 & 0 \\ a_3 - a_2 & a_4 - a_3 & 0 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & 1 \\ k & k & 0 \\ k & k & 0 \end{vmatrix}$$

[From equation (i)]

$= 0$

[Since, R_2 and R_3 are identical]

Ans: (d)

112. If $x \neq 0, \begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x & 4x+3 & 6x+3 \\ 4x+4 & 6x+4 & 8x+4 \end{vmatrix} = 0$, then $x+1$ is equal to

Options:

(a) x

(b) 0

(c) $2x$

(d) $3x$

Sol: Given, $\begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x & 4x+3 & 6x+3 \\ 4x+4 & 6x+4 & 8x+4 \end{vmatrix} = 0$

$$= \frac{2}{2} \begin{vmatrix} 2x+2 & 4x+2 & 6x+2 \\ 2x & 4x+2 & 6x+2 \\ 2x+2 & 3x+2 & 4x+2 \end{vmatrix} = 0$$

[Multiplying 2 into R_1 and taking common 2 from R_3]

$$\Rightarrow \begin{vmatrix} 0 & x & 2x \\ 2x & 4x+3 & 6x+2 \\ 2x+2 & 3x+2 & 4x+2 \end{vmatrix} = 0$$

[Applying $R_1 \to R_1 - R_3$]

$$\Rightarrow 2\begin{vmatrix} 0 & x & 0 \\ 2x & 4x+3 & -2x-3 \\ 2x+2 & 3x+2 & -2x-2 \end{vmatrix} = 0$$

On applying $C_3 \to C_3 - 2C_2$, we get $-x\left[2x(-2x-2)+(2x+3)(2x+2)\right]=0$

$\Rightarrow 2x\left[x(2x+2)-(2x+3)(x+1)\right]=0$

$\Rightarrow 2x(3x+3)=0$

$\therefore x+1=0 \quad [\because x \neq 0, \text{given}]$

Ans: (b)

113. $\int (x+1)(x+2)^7 (x+3)\,dx$ is equal to

Options:

(a) $\frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + C$

(b) $\frac{(x+1)^2}{2} - \frac{(x+2)^8}{8} - \frac{(x+3)^2}{2} + C$

(c) $\frac{(x+2)^{10}}{10} + C$

(d) $\frac{(x+2)^9}{9} - \frac{(x+2)^7}{7} + C$

Sol: $\int (x+1)(x+2)^7 (x+3)\,dx$

Let $(x+1)(x+3) = (x+2-1)(x+2+1) = (x+2)^2 - 1$

$\therefore \int (x+1)(x+2)^7 (x+3)\,dx = \int \left\{(x+2)^9 - (x+2)^7\right\} dx$

$= \frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + C$

Ans: (a)

114. $\int \frac{2dx}{\left(e^x + e^{-x}\right)^2}$ is equal to

Options:

(a) $\frac{-e^x}{e^{-x} + e^x} + C$

(b) $\frac{e^x}{x+1} + C$

(c) $\frac{xe^x}{x+1} + C$

(d) $e^x\left(\frac{x-1}{x+1}\right) + C$

Sol: Let $I = \int \frac{2dx}{\left(e^x + e^{-x}\right)^2} = \int \frac{2dx}{e^{2x} + e^{-2x} + 2} = \int \frac{2e^{2x}dx}{\left(e^{2x}\right)^2 + 2e^{2x} + 1}$

Put $e^{2x} = t \Rightarrow 2e^{2x}dx = dt$

$\therefore I = \int \frac{dt}{t^2 + 2t + 1} = \int \frac{dt}{(t+1)^2}$

$= \frac{-1}{1+t} + C = \frac{-1}{1+e^{2x}} + C = \frac{-e^{-x}}{e^{-x} + e^x} + C$

Ans: (a)

115. $\int \frac{e^x}{\left(2 + e^x\right)\left(e^x + 1\right)} dx$ is equal to

Options:

(a) $\log\left(\frac{e^x + 1}{e^x + 2}\right) + C$

(b) $\log\left(\frac{e^x + 2}{e^x + 1}\right) + C$

(c) $\left(\frac{e^x + 1}{e^x + 2}\right) + C$

(d) $\left(\frac{e^x + 2}{e^x + 1}\right) + C$

Sol: Let $I = \int \frac{e^x}{\left(2 + e^x\right)\left(e^x + 1\right)} dx$

Putting $e^x = t \Rightarrow e^x dx = dt$

$\Rightarrow I = \int \frac{dt}{(2+t)(t+1)} = \int \left[\frac{1}{(1+t)} - \frac{1}{(2+t)}\right] dt$

$= \log(1+t) - \log(2+t) + C \quad = \log\left(\frac{1+t}{2+t}\right) + C = \log\left(\frac{1+e^x}{2+e^x}\right) + C$

Ans: (a)

116. The value of $\int_0^1 \frac{x^4+1}{x^2+1} dx$ is

Options:

(a) $\frac{1}{6}(3-4\pi)$

(b) $\frac{1}{6}(3\pi+4)$

(c) $\frac{1}{6}(3+4\pi)$

(d) $\frac{1}{6}(3\pi-4)$

Sol: $\int_0^1 \frac{x^4+1}{x^2+1} dx = \int_0^1 \frac{x^4-1+2}{x^2+1} dx = \int_0^1 \left[\frac{x^4-1}{x^2+1} + \frac{2}{x^2+1}\right] dx$

$= \int_0^1 \left(x^2 - 1 + \frac{2}{x^2+1}\right) dx = \left[\frac{x^3}{3} - x + 2\tan^{-1} x\right]_0^1$

$= \left[\frac{1}{3} - 1 + 2\tan^{-1}(1) - 0\right] = -\frac{2}{3} + 2 \cdot \frac{\pi}{4} = \frac{3\pi-4}{6}$

Ans: (d)

117. If $f(x) = \begin{cases} 2x^2+1, & x \le 1 \\ 4x^3-1, & x > 1 \end{cases}$, then $\int_0^2 f(x) dx$ is equal to

Options:

(a) $\frac{47}{3}$

(b) $\frac{50}{3}$

(c) $\frac{1}{3}$

(d) $\frac{47}{2}$

Sol: Given, $f(x) = \begin{cases} 2x^2+1, & x \le 1 \\ 4x^3-1, & x > 1 \end{cases}$

$\therefore \int_0^2 f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx$

$= \int_0^1 \left(2x^2+1\right) dx + \int_1^2 \left(4x^3-1\right) dx$

$$=\left[\frac{2x^3}{3}+x\right]_0^1+\left[\frac{4x^4}{4}-x\right]_1^2$$

$$=\frac{2}{3}(1)^3+1-(0+0)+\left[(2)^4-2-\left\{(1)^4-1\right\}\right]$$

$$=\frac{2}{3}+1+[16-2-0]=\frac{2}{3}+15=\frac{2+45}{3}=\frac{47}{3}$$

Ans: (a)

118. The area (in square units) of the region bounded by $x=-1,\ x=2,\ y=x^2+1$ and $y=2x-2$ is

Options:

(a) 10

(b) 7

(c) 8

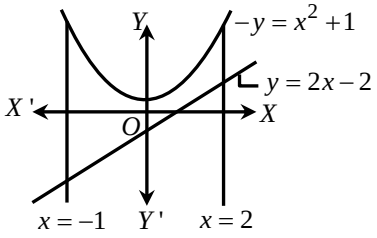
(d) 9

Sol: Given curve is $y=x^2+1\Rightarrow x^2=y-1$ and line $y=2x-2$

The intersection point of curve and line is $x^2=2x-2-1\Rightarrow x^2-2x+3=0$

Now, $b^2-4ac=4-12<0$

Hence, there is no point of intersection.



$\therefore$ Required area $=\int_{-1}^{2}(y_2-y_1)\,dx=\int_{-1}^{2}\left[\left(x^2+1\right)-(2x-2)\right]dx$

$$=\left[\frac{x^3}{3}+x\right]_{-1}^{2}-\left[x^2-2x\right]_{-1}^{2}=\left[\frac{8}{3}+2-\left(-\frac{1}{3}-1\right)\right]-[4-4-(1+2)]$$

$$=\frac{14}{3}+\frac{4}{3}-[-3]=6+3=9$$

Ans: (d)

119. The solution of the differential equation $x\frac{dy}{dx}+y=x\cos x+\sin x,$ given that $y=1$ when $x=\frac{\pi}{2},$ is

Options:

(a) $y=\sin x-\cos x$

(b) $y=\cos x$

(c) $y=\sin x$

(d) $y=\sin x+\cos x$

Sol: Given differential equation can be rewritten as $\frac{dy}{dx}+\frac{y}{x}=\cos x+\frac{\sin x}{x}$

Here, $P=\frac{1}{x}$ and $Q=\cos x+\frac{\sin x}{x}$ $\therefore IF=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

Hence, required solution is $xy=\int(x\cos x+\sin x)\,dx$

$\Rightarrow xy=x\sin x+C$

At $y=1, x=\frac{\pi}{2} \Rightarrow C=0$

$\therefore y=\sin x$

Ans: (c)

120. The slope at any point of a curve $y=f(x)$ is given by $\frac{dy}{dx}=3x^2$ and it passes through $(-1,1)$. The equation of the curve is

Options:

(a) $y=x^3+2$

(b) $y=-x^3-2$

(c) $y=3x^3+4$

(d) $y=-x^3+2$

Sol: Given, $\frac{dy}{dx}=3x^2 \Rightarrow dy=3x^2dx$

On integrating both sides, we get $y=\frac{3x^3}{3}+C \Rightarrow y=x^3+C$

Since, it passes through the point $(-1,1)$.

$\therefore 1=(-1)^3+C \Rightarrow C=2$ Hence, $y=x^3+2$

Ans: (a)

Physics

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60

121. The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite?

Options:

(a) $\left(R^2 g/\omega^2\right)^{1/3}$

(b) $\left(Rg/\omega^2\right)^{1/3}$

(c) $\left(R^2\omega^2/g\right)^{1/3}$ (d) $\left(R^2 g/\omega\right)^{1/3}$

Sol: $T=\frac{2\pi r}{v_0}=\frac{2\pi r}{\left(gR^2/r\right)^{1/2}}=\frac{2\pi r^{3/2}}{\sqrt{gR^2}}=\frac{2\pi}{\omega}$

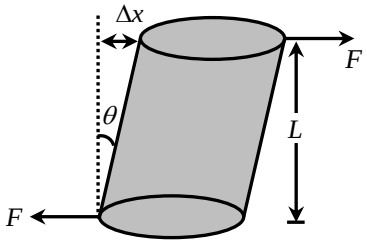
Hence, $r^{3/2}=\frac{\sqrt{gR^2}}{\omega}$ or $r^3=\frac{gR^2}{\omega^2}$

or, $r=\left(gR^2/\omega^2\right)^{1/3}$

Ans: (a)

122. If two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder as shown in the figure, there is a relative displacement between the opposite faces of the cylinder.

The ratio of Δx to L is known as



Options:

(a) Longitudinal strain

(b) Volumetric strain

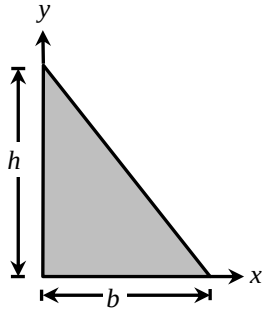
(c) Shearing strain

(d) Poisson's ratio

Sol: Shearing strain $=\frac{\Delta x}{L}$

Ans: (c)

123. The centre of mass of triangle system shown in figure has coordinates, if three equal masses placed at three vertices of the triangle



Options:

(a) $x=\frac{h}{2}, y=\frac{b}{2}$

(b) $x=\frac{b}{2}, y=\frac{h}{2}$

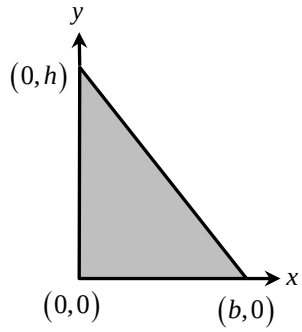
(c) $x=\frac{b}{3}, y=\frac{h}{3}$ (d) $x=\frac{h}{3}, y=\frac{b}{3}$

Sol: As three particles of equal mass m are placed at the corners of triangle.

$\vec{r}_1 = 0\hat{i} + 0\hat{j}, \vec{r}_2 = b\hat{i} + 0\hat{j}$ and $\vec{r}_3 = 0\hat{i} + h\hat{j}$

$\therefore \overrightarrow{r_{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} = \frac{b}{3}\hat{i} + \frac{h}{3}\hat{j}$

i.e., coordinates of centre of mass is $\left(\frac{b}{3}, \frac{h}{3}\right)$



Ans: (c)

124. If the terminal speed of a sphere of gold(density $= 19.5\,\text{kg/m}^3$) is $0.2\,\text{m/s}$ in a viscous liquid (density $= 1.5\,\text{kg/m}^3$), find the terminal speed of a sphere of silver (density $= 10.5\,\text{kg/m}^3$) of the same size in the same viscous liquid

Options:

(a) $0.4\,\text{m/s}$

(b) $0.133\,\text{m/s}$

(c) $0.1\,\text{m/s}$

(d) $0.2\,\text{m/s}$

Sol: Terminal velocity, $v_T = \frac{2r^2(d_1 - d_2)g}{9\eta}$

$\frac{v_{T_2}}{0.2} = \frac{(10.5 - 1.5)}{(19.5 - 1.5)} \Rightarrow v_{T_2} = 0.2 \times \frac{9}{18}$

$\therefore v_{T_2} = 0.1\,\text{m/s}$

Ans: (c)

125. A body initial at $80°C$ cools to $64°C$ in 5 minutes and to $52°C$ in 10 minutes. The tempearture of the body after 15 minutes will be

Options:

(a) $42.7°C$

(b) $35°C$

(c) $47°C$

(d) $40°C$

Sol: From Newton's law of cooling

$\frac{\theta_1-\theta_2}{t}=k\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$ where θ_1 is higher temperature, θ_2 is lower temperature.

$\frac{80-64}{5}=k\left(72-\theta_0\right)$ … (i)

Where θ_0 is temperature of surroundings

$\frac{64-52}{10}=k\left(58-\theta_0\right)$ … (ii)

Dividing (i) and (ii) we get θ_0

$\frac{52-\theta}{15}=k\left(\frac{52+\theta}{2}-\theta_0\right)$ … (iii)

This $\theta=42.7°C$ is found.

Ans: (a)

126. Four mole of hydrogen, two mole of helium and one mole of water vapour form an ideal gas mixture. What is the molar specific heat at constant pressure of mixture? (C_v for water vapour $=3R$)

Options:

(a) $\frac{16}{7}R$

(b) $\frac{7}{16}R$

(c) R

(d) $\frac{23}{7}R$

Sol: C_v for hydrogen $=\frac{5}{2}R$

C_v for helium $=\frac{3R}{2}$

C_v for water vapour $=\frac{6R}{2}=3R$

$\therefore\left(C_v\right)_{mix}=\frac{\left[4\times\frac{5}{2}R+2\times\frac{3}{2}R+1\times3R\right]}{(4+2+1)}=\frac{16}{7}R$

$\therefore C_p=C_v+R$

$C_p=\frac{16}{7}R+R$ or $C_p=\frac{23}{7}R$

Ans: (d)

127. In Carnot engine efficiency is 40% at hot reservoir temperature T. Foe efficiency 50% what will be temperature of hot reservoir?

Options:

(a) $\frac{T}{5}$

(b) $\frac{2T}{5}$

(c) $6T$

(d) $\frac{6T}{5}$

Sol: $\eta = 1 - \frac{T_2}{T_1}$ and $T_1 = T$ (Temperature of hot reservoir)

For $\eta = 40\%$,

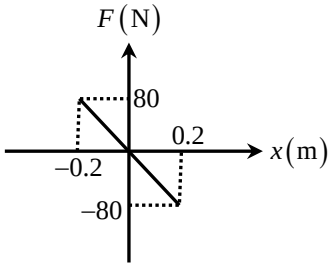
$$\frac{40}{100} = 1 - \frac{T_2}{T} \Rightarrow \frac{T_2}{T} = \frac{3}{5} \Rightarrow T_2 = \frac{3}{5}T$$

For $\eta = 50\%$,

$$\frac{50}{100} = 1 - \frac{\frac{3}{5}T}{T_1} \Rightarrow T_1 = \frac{6T}{5}$$

Ans: (d)

128. A body of mass 0.01kg executes simple harmonic motion about $x = 0$ under the influence of a force as shown in figure. The time period of SHM is



Options:

(a) 1.05s

(b) 0.52s

(c) 0.25s

(d) 0.03s

Sol: Slope of $F - x$ curve $= -k = -\frac{80}{0.2} \Rightarrow k = 400\,\text{N m}^{-1}$,

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 0.0314\text{s}$

Ans: (d)

129. Speed of sound in mercury at a certain temperature is $1450\,\text{m s}^{-1}$. If the density of mercury is $13.6\times10^3\,\text{kg m}^{-3}$, then the bulk modulus for mercury is

Options:

(a) $2.86\times10^{10}\,\text{N m}^{-3}$

(b) $3.86\times10^{10}\,\text{N m}^{-3}$

(c) $4.86\times10^{10}\,\text{N m}^{-3}$ (d) $5.86\times10^{10}\,\text{N m}^{-3}$

Sol: $v=\sqrt{\frac{K}{\rho}}$ $\therefore K=v^2\rho=2.86\times10^{10}\,\mathrm{N\,m^{-3}}$

Ans: (a)

130. Two identical conducting spheres carrying different charges attract each other with a force F when placed in air medium at a distance $'d'$ apart. The sphere are brought into contact and then taken to their original positions. Now the two spheres repel each other with a force whose magnitude is equal to that of the initial attractive force. The ration between initial charges on the spheres is

Options:

(a) $-\left(3+\sqrt{8}\right)$ only

(b) $\left(-3+\sqrt{8}\right)$ only

(c) $-\left(3+\sqrt{8}\right)\text{or}\left(-3+\sqrt{8}\right)$

(d) $+\sqrt{3}$

Sol: $F_1=\frac{kQ_1Q_2}{d^2}$ and $F_2=\frac{k\left(\frac{Q_1-Q_2}{2}\right)^2}{d^2}$

According to question, $F_1=F_2$

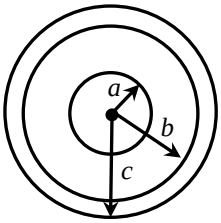
$$Q_1Q_2=\frac{(Q_1-Q_2)^2}{4}\Rightarrow 4Q_1Q_2=Q_1^2+Q_2^2-2Q_1Q_2$$

$$0=Q_1^2+Q_2^2-6Q_1Q_2\Rightarrow\frac{Q_1}{Q_2}=\left(-3\pm\sqrt{8}\right)$$

Ans: (c)

131. A solid conducting sphere of radius a has a net positive charge $2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and has a net charge $-Q$. The surface charge density on the inner and outer surfaces of the spherical shell will be

Options:



(a) $-\frac{2Q}{4\pi b^2},\frac{Q}{4\pi c^2}$

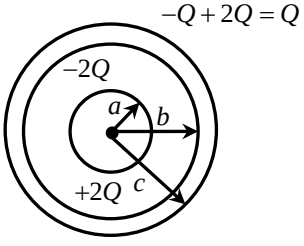
(b) $-\frac{Q}{4\pi b^2},\frac{Q}{4\pi c^2}$

(c) $0,\frac{Q}{4\pi c^2}$

(d) None of these

Sol: Surface charge density $(\sigma)=\frac{\text{Charge}}{\text{Surface area}}$

So, $\sigma_{\text{inner}}=\frac{-2Q}{4\pi b^2}$ and $\sigma_{\text{Outer}}=\frac{Q}{4\pi c^2}$



Ans: (a)

132. A parallel plate air capacitor has a capacitance C. When it is half filled with a dielectric of dielectric constant 5, the percentage increase in the capacitance will be

Options:

(a) 400%

(b) 66.6%

(c) 33.3%

(d) 200%

Sol: $C=\frac{\varepsilon_0 A}{d}$ and $C'=\frac{\varepsilon_0 A}{2d}+\frac{\varepsilon_0(5A)}{2d}$

$=\frac{\varepsilon_0 A}{2d}(1+5)=\frac{6\varepsilon_0 A}{2d}=\frac{3\varepsilon_0 A}{d}$

$\Rightarrow \Delta C=C'-C=\frac{3\varepsilon_0 A}{d}-\frac{\varepsilon_0 A}{d}=\frac{2\varepsilon_0 A}{d}$

Percentage change in capacitance

$\frac{\Delta C}{C}=\frac{\frac{2\varepsilon_0 A}{d}}{\frac{\varepsilon_0 A}{d}}\times 100\%=200\%$

Ans: (d)

133. Three charges are placed at the vertices of an equilateral triangle of side $'a'$ as shown in the following figure. The force experienced by the charge placed at the vertex A in a direction normal to BC is

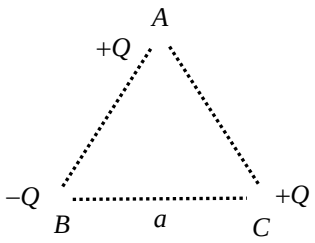
Options:

(a) $Q^2/\left(4\pi\varepsilon_0 a^2\right)$

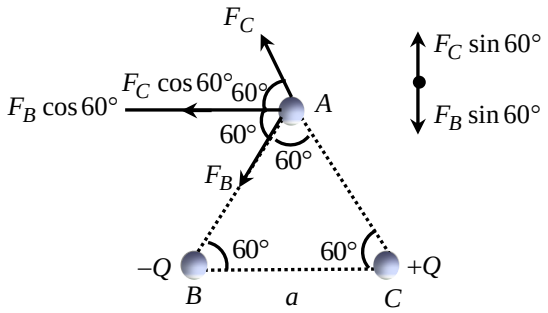
(b) $-Q^2/\left(4\pi\varepsilon_0 a^2\right)$

(c) Zero

(d) $Q^2/\left(2\pi\varepsilon_0 a^2\right)$



Sol: $\left|\overrightarrow{F_B}\right|=\left|\overrightarrow{F_C}\right|=k.\frac{Q^2}{a^2}$



Hence force experienced by the charge as A in the direction normal to BC is zero.

Ans: (c)

134. Each corner of a cube of side l has a negative charge, $-q$. The electrostatic potential energy of a charge q at the centre of the cube is

Options:

(a) $-\frac{4q^2}{\sqrt{2}\pi\varepsilon_0 l}$

(b) $\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 l}$

(c) $\frac{4q^2}{\sqrt{2}\pi\varepsilon_0 l}$

(d) $-\frac{4q^2}{\sqrt{3}\pi\varepsilon_0 l}$

Sol: Length of body diagonal $= \sqrt{3}l$

$\therefore$ Distance of centre of cube from each corner

$r = \frac{\sqrt{3}}{2}l$

P.E. at centre $= 8\times$ Potential Energy due to A

$= 8\times\frac{Kq\times(-q)}{r} = 8\times\frac{1}{4\pi\varepsilon_0\sqrt{3}l}\times 2\times q\times(-q) = \frac{-4q^2}{\sqrt{3}\pi\varepsilon_0 l}$

Ans: (d)

135. The electric potential at a point (x, y) in the $x-y$ plane is given by $V = -kxy$. The magnitude of field intensity at a distance r from the origin varies as (directly proportional)

Options:

(a) r^2

(b) r

(c) $\frac{1}{r}$

(d) $\frac{1}{r^2}$

Sol: Given $V = -kxy$

$\vec{E} = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j}$

$\therefore \vec{E} = ky\hat{i} + kx\hat{j}$

$\therefore |\vec{E}| = k\left(\sqrt{x^2 + y^2}\right) = kr \qquad \Rightarrow E \propto r$

Ans: (b)

136. Three point charges $+q, -2q$ and $+q$ are placed at points $(x=0, y=a, z=0)$, $(x=0, y=0, z=0)$ and $(x=a, y=0, z=0)$ respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are

Options:

(a) $\sqrt{2}qa$ along the line joining points $(x=0, y=0, z=0)$ and $(x=a, y=a, z=0)$

(b) qa along the line joining points $(x=0, y=0, z=0)$ and $(x=a, y=a, z=0)$

(c) $\sqrt{2}qa$ along +ve x direction

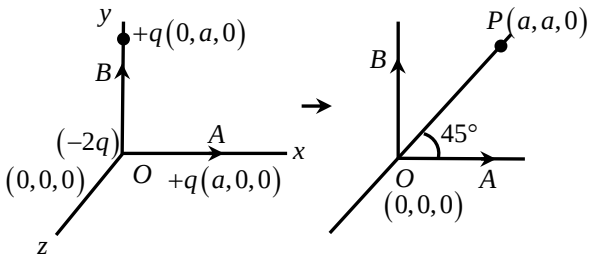
(d) $\sqrt{2}qa$ along +ve y direction

Sol: Three point charges $+q, -2q$ and $+q$ are placed at points

$B(x=0, y=a, z=0), O(x=0, y=0, z=0)$ and $A(x=a, y=0, z=0)$

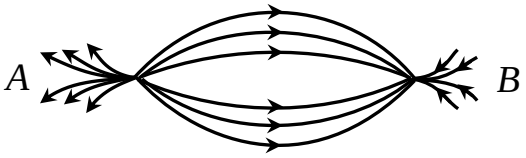
The system consists of two dipole moment vectors due to $(+q$ and $-q)$ and again due to $(+q$ and $-q)$ charges having equal magnitudes qa units. One along $\overrightarrow{OA}$ and other along $\overrightarrow{OB}$. Hence, net dipole moment,

$P_{net} = \sqrt{(qa)^2 + (qa)^2} = \sqrt{2}qa$ along $\overrightarrow{OP}$ at an angle $45°$ with positive $X-$axis.



Ans: (a)

137. The spatial distribution of electric field due to charges (A, B) is shown in figure. Which one of the following statement is correct?



Options:

(a) A is $+ve$ and $B-ve, |A| > |B|$

(b) A is $-ve$ and $B+ve, |A| = |B|$

(c) Both are $+ve$ but $A > B$

(d) Both are $-ve$ but $A > B$

Sol: Since lines of force starts from A and ends at B, so A is $+ve$ and B is $-ve$. Lines of forces are more crowded near A, so $A > B$.

Ans: (a)

138. Five equal resistances each of resistance R are connected as shown in the figure. A battery of V volts is connected between A and B. The current flowing in $AFCEB$ will be

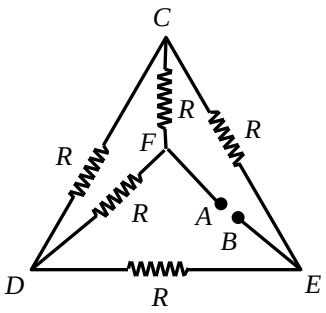
Options:

(a) $\frac{2V}{R}$

(b) $\frac{3V}{R}$

(c) $\frac{V}{R}$

(d) $\frac{V}{2R}$



Sol: A balanced Wheatstone's bridge exists between A & B.

$R_{eq} = R$

Current through circuit is V / R

Current through $AFCEB = V / 2R$

Ans: (d)

139. Two resistors of 6Ω and 9Ω are connected in series to a $120V$ source. The power consumed by 6Ω resistor is

Options:

(a) $384\,W$

(b) $616\,W$

(c) $1500\,W$

(d) $1800\,W$

Sol: In series circuit it is always preferable to use formula I^2R, because I throughout is same.

$I = \frac{120}{6+9} = 8$

$\therefore P = I^2R = 64 \times 6 = 384$

Ans: (a)

140. A 100 watt bulb working on 200 volt has resistance R and a bulb 200 watt bulb working on 100 volt has resistance S then R / S is

Options:

(a) $\frac{1}{8}$

(b) $\frac{1}{4}$

(c) 8

(d) 4

Sol: $P = V^2 / R$

$\Rightarrow R = V^2 / P = 4\times10^4 / 100 = 400\Omega$

$S = 10^4 / 200 = 0.5\times10^2 = 50\Omega$

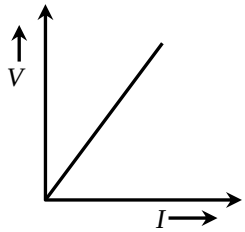
$R / S = 8$

Ans: (c)

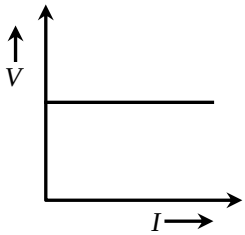
141. Which of the adjoining graphs represents ohmic resistance

Options:

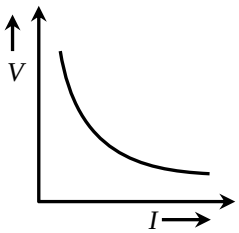
(a)



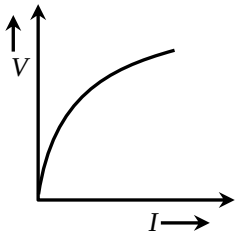
(b)



(c)



(d)



Sol: For ohmic resistance $V \infty i \Rightarrow V = Ri$ (here R is constant)

Ans: (a)

142. Two conductors have the same resistance at $0°C$ but their temperature coefficient of resistance are α_1 and α_2. The respective temperature coefficient of their series and parallel combinations are nearly

Options:

(a) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(b) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

(c) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

(d) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

Sol: $R_1 = R_0[1+\alpha_1\Delta t]$; $R_2 = R_0[1+\alpha_2\Delta t]$

In series, $R = R_1 + R_2$

$= R_0[2+(\alpha_1+\alpha_2)\Delta t] = 2R_0\left[1+\left(\frac{\alpha_1+\alpha_2}{2}\right)\Delta t\right]$

$\therefore \alpha_{eq} = \frac{\alpha_1+\alpha_2}{2}$

In Parallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_0[1+\alpha_1\Delta t]} + \frac{1}{R_0[1+\alpha_2\Delta t]}$

$\Rightarrow \frac{1}{\frac{R_0}{2}(1+\alpha_{eq}\Delta t)} = \frac{1}{R_0(1+\alpha_1\Delta t)} + \frac{1}{R_0(1+\alpha_2\Delta t)}$

$2(1-\alpha_{eq}\Delta t) = (1-\alpha_1\Delta t)(1-\alpha_2\Delta t)$

$\therefore \alpha_{eq} = \frac{\alpha_1+\alpha_2}{2}$

Ans: (d)

143. $125\,\text{cm}$ of potentiometer wire balances the emf. of a cell and $100\,\text{cm}$ of the wire is required for balance, if the terminals of the cell are joined by a 2Ω resistor. Then the internal resistance of the cell is

Options:

(a) $0.25\,\Omega$

(b) $0.5\,\Omega$

(c) $0.75\,\Omega$

(d) $1.25\,\Omega$

Sol: $r = \frac{\ell_1-\ell_2}{\ell_2}\times R\Omega$

Here, $\ell_1 = 125\,\text{cm}, \ell_2 = 100\,\text{cm}, R = 2\Omega$

$\therefore r = 0.5\Omega$

Ans: (b)

144. A charged particle of mass m and charge q travels in a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is

Options:

(a) $\frac{2\pi qB}{m}$

(b) $\frac{2\pi m}{qB}$

(c) $\frac{2\pi mq}{B}$

(d) $\frac{2\pi q^2 B}{m}$

Sol: Equating magnetic force to centripetal force, $\frac{mv^2}{r} = qvB\sin 90°$

Time to complete one revolution, $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

Ans: (b)

145. A charge moving with velocity v in $X-$direction is subjected to a field of magnetic induction in negative $X-$direction. As a result, the charge will

Options:

(a) remain unaffected

(b) start moving in a circular path $Y-Z$ plane

(c) retard along $X-$axis

(d) move along a helical path around $X-$axis

Sol: The force acting on a charged particle in magnetic field is given by

$F = q\left(\vec{v}\times\vec{B}\right)$ or $F = qvB\sin\theta$

when angle between v and B is $180°$,

$F = 0$

Ans: (a)

146. A galvanometer having a coil resistance of 100Ω gives a full scale deflection, when a current of 1mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10A, is

Options:

(a) $0.1\ \Omega$

(b) $3\ \Omega$

(c) $0.01\ \Omega$

(d) $2\ \Omega$

Sol: $I_g G = \left(I - I_g\right)s$

$\therefore 10^{-3}\times 100 = \left(10 - 10^{-3}\right)\times s$

$s = 0.01\Omega$

Ans: (c)

147. At what distance from a long straight wire carrying a current of 12A will the magnetic field be equal to $3\times10^{-5}\ \text{Wb m}^{-2}$?

Options:

(a) 8×10^{-2} m

(b) 12×10^{-2} m

(c) 18×10^{-2} m

(d) 24×10^{-2} m

Sol: Current $(I)=12\text{A}$ and magnetic field $(B)=3\times10^{-5}\,\text{Wb}\,\text{m}^{-2}$

Consider magnetic field $\vec{B}$ at distance r

Magnetic field, $B=\dfrac{\mu_0 I}{2\pi r}$

$$\Rightarrow r=\frac{\mu_0 I}{2\pi B}=\frac{\left(4\pi\times10^{-7}\right)\times12}{2\times\pi\times\left(3\times10^{-5}\right)}=8\times10^{-2}\,\text{m}$$

Ans: (a)

148. If the angles of dip at two places are $30°$ and $45°$ respectively, then the ratio of horizontal components of earth's magnetic field at the two places will be

Options:

(a) $\sqrt{3}:\sqrt{2}$

(b) $1:\sqrt{2}$

(c) $1:\sqrt{3}$

(d) $1:2$

Sol: The horizontal components are

$(B_H)_1=B\cos\phi_1$ and $(B_H)_2=B\cos\phi_2$

$$\therefore\frac{(B_H)_1}{(B_H)_2}=\frac{\cos\phi_1}{\cos\phi_2}=\frac{\cos30°}{\cos45°}=\frac{\sqrt{3}}{2}\times\sqrt{2}=\frac{\sqrt{3}}{\sqrt{2}}$$

Ans: (a)

149. A coil in the shape of equilateral triangle of side $0.2\,\text{m}$ is suspended from the vertex such that it is hanging in a vertical plane between the pole-pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} tesla. The couple acting on the coil when a current of 0.1A is passed through it and the magnetic field is parallel to its plane will be

Options:

(a) $3.28\times10^{-7}\,\text{Nm}$

(b) $5.28\times10^{-7}\,\text{Nm}$

(c) $8.66\times10^{-7}\,\text{Nm}$

(d) $1.23\times10^{-7}\,\text{Nm}$

Sol: The torque on a closed flat current loop of any shape, placed in magnetic field of flux density B is given by $\tau=Bi\ NA\sin\theta$,

According to the question the area of this coil is

$A=(1/2)\,\text{base}\times\text{height}$

$$A=(1/2)(0.2\times0.1732)=1.732\times10^{-4}\,\text{m}^2$$

$\tau = 1\times 01\times 1.732\times 10^{-4}\times\left(5\times 10^{-2}\right)\times 1$

$\tau = 8.66\times 10^{-7}$ N-m

Ans: (c)

150. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is $2s$. The magnet is cut perpendicular to its length into three equal parts and these parts are then placed on each other with their like poles together. The time period of this combination will be

Options:

(a) $2\sqrt{3}$ s

(b) $\frac{2}{3}$ s

(c) 2 s

(d) $\frac{2}{\sqrt{3}}$ s

Sol: $T = 2\pi\sqrt{\frac{I}{M\times B}} = 2\pi\sqrt{\frac{1}{MB}}$ where $I = \frac{1}{12}m\ell^2$

When the magnet is cut into pieces the pole strength will remain the same and

$M.I.(I') = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2\times 3 = \frac{I}{9}$

We have, Magnetic moment (M) = Pole strength $(m)\times\ell$

$\therefore$ New magnetic moment,

$M' = m\times\left(\frac{\ell}{3}\right)\times 3 = m\ell = M$

$\therefore T' = \frac{T}{\sqrt{9}} = \frac{2}{3}$s

Ans: (b)

151. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

Options:

(a) halved

(b) the same

(c) doubled

(d) quadrupled

Sol: Power $P = \frac{e^2}{R}$; Here $e = -\left(\frac{d\phi}{dt}\right)$ where $\phi = NBA$

$\therefore e = -NA\left(\frac{dB}{dt}\right)$ Also $R\propto\frac{l}{r^2}$

where $R=$ resistance, $r=$ radius, $l=$ Length

$$\therefore P \propto \frac{N^2 r^2}{l} \Rightarrow \frac{P_1}{P_2}=1$$

Ans: (b)

152. A copper wire of length $40\,\text{cm}$, diameter $2\,\text{mm}$ and resistivity $1.7\times10^{-8}\Omega\text{m}$ forms a square frame. If a uniform magnetic field B exists in a direction perpendicular to the plane of square frame and it changes at a steady rate $\frac{dB}{dt}=0.02\,\text{T s}^{-1}$, then find the current induced in the frame.

Options:

(a) $9.3\times10^{-2}\,\text{A}$

(b) $9.3\times10^{-1}\,\text{A}$

(c) $3.3\times10^{-2}\,\text{A}$

(d) $19.3\times10^{-2}\,\text{A}$

Sol: Area of the loop $=0.1\times0.1=0.01\text{m}^2$

$$\varepsilon=-\frac{d\phi}{dt}=-\frac{d}{dt}(BA)$$

Magnitude of emf

$$\varepsilon=A\frac{dB}{dt}=\left(0.01\text{m}^2\right)\left(0.02\,\text{T s}^{-1}\right)=2\times10^{-4}\,\text{V}$$

Resistance of the loop is

$$R=\rho\frac{l}{A}=\frac{1.7\times10^{-8}\times40\times10^{-2}}{3.14\times10^{-6}}=2.16\times10^{-3}\Omega$$

Current induced in the loop

$$I=\frac{\varepsilon}{R}=\frac{2\times10^{-4}\,\text{V}}{2.16\times10^{-3}\Omega}=9.3\times10^{-2}\,\text{A}$$

Ans: (a)

153. An inductance of $\left(\frac{200}{\pi}\right)\text{mH}$, a capacitance of $\left(\frac{10^{-3}}{\pi}\right)\text{F}$ and a resistance of 10Ω are connected in series with an a.c. source 220V 50Hz. The phase angle of the circuit is

Options:

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{3}$

Sol: Phase difference between E and $I = \theta$

$\therefore \tan\theta = \dfrac{X_L - X_C}{R}$

Now, $X_L = 2\pi fL = 2\pi \times 50 \times \left(\dfrac{200}{\pi} \times 10^{-3}\right) = 20\Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{\pi}{2\pi \times 50 \times 10^{-3}} = 10\Omega$

$R = 10\Omega$

$\therefore \tan\theta = \dfrac{20-10}{10} = \dfrac{10}{10} = 1 = \tan\dfrac{\pi}{4}$

$\therefore \theta = \dfrac{\pi}{4}$. The current will lag by $\dfrac{\pi}{4}$

Ans: (b)

154. A resistor 30Ω, inductor of reactance 10Ω and capacitor of reactance 10Ω are connected in series to an a.c. voltage source $e = 300\sqrt{2}\sin(\omega t)$. The current in the circuit is

Options:

(a) $10\sqrt{2}$ A

(b) 10 A

(c) $30\sqrt{11}$ A

(d) $\dfrac{30}{\sqrt{11}}$ A

Sol: The current in the circuit

$I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$

$V_{rms} = \dfrac{V_0}{\sqrt{2}} = \dfrac{300\sqrt{2}}{\sqrt{2}} = 300\,\text{V}$

$z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{30^2 + (10-10)^2} = 30\Omega$

$\therefore I_{\text{rms}} = \dfrac{300}{30} = 10\text{A}$

Ans: (b)

155. A 220 volts input is supplied to a transformer. The output circuit draws a current of 2.0 A at 440 volts. If the efficiency of the transformer is 80%, the current drawn by the primary windings of the transformer is

Options:

(a) 3.6 A

(b) 2.8 A

(c) 2.5 A

(d) 5.0 A

Sol: $\frac{V_2}{V_1}=0.8\frac{I_1}{I_2}$ $\Rightarrow\frac{V_2I_2}{V_1I_1}=0.8$

$V_1=220\,\text{V}, I_2=2.0\,\text{A}, V_2=440\,\text{V}$

$I_1=\frac{V_2I_2}{V_1}\times\frac{10}{8}=\frac{440\times2\times10}{220\times8}=5\,\text{A}$

Ans: (a)

156. The electric field part of an electromagnetic wave in a medium is represented by $E_x=0$;

$E_y=2.5\,\text{N C}^{-1}\cos\left[\left(2\pi\times10^6\,\text{rad m}^{-1}\right)t-\left(\pi\times10^{-2}\,\text{rad s}^{-1}\right)x\right]$;

$E_z=0$. The wave is

Options:

(a) moving along x direction with frequency 10^6 Hz and wave length 100m

(b) moving along x direction with frequency 10^6 Hz and wave length 200m

(c) moving along $-x$ direction with frequency 10^6 Hz and wave length 200m

(d) moving along y direction with frequency $2\pi\times10^6$ Hz and wave length 200m

Sol: Comparing with the equation of wave,

$E_y=E_0\cos(\omega t-kx)$

$\omega=2\pi f=2\pi\times10^6$ $\therefore f=10^6$ Hz

$\frac{2\pi}{\lambda}=k=\pi\times10^{-2}\,\text{m}^{-1}$, $\lambda=200\text{m}$

Ans: (b)

157. A thin glass (refractive index 1.5) lens has optical power of $-5D$ in air. Its optical power in a liquid medium with refractive index 1.6 will be

Options:

(a) $-1D$

(b) $1D$

(c) $-25D$

(d) $25D$

Sol: $\frac{1}{f_a}=\left(\frac{1.5}{1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ … (i)

$\frac{1}{f_m}=\left(\frac{\mu_g}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\frac{1}{f_m}=\left(\frac{1.5}{1.6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ … (ii)

Dividing (i) by (ii), $\frac{f_m}{f_a}=\left(\frac{1.5-1}{\frac{1.5}{1.6}-1}\right)=-8$

$P_a=-5=\frac{1}{f_a}\Rightarrow f_a=-\frac{1}{5}$

$\Rightarrow f_m=-8\times f_a=-8\times-\frac{1}{5}=\frac{8}{5}$

$P_m=\frac{\mu}{f_m}=\frac{1.6}{8}\times5=1D$

Ans: (b)

158. For the angle of minimum deviation of a prism to be equal to its refractive angle, the prism must be made of a material whose refractive index

Options:

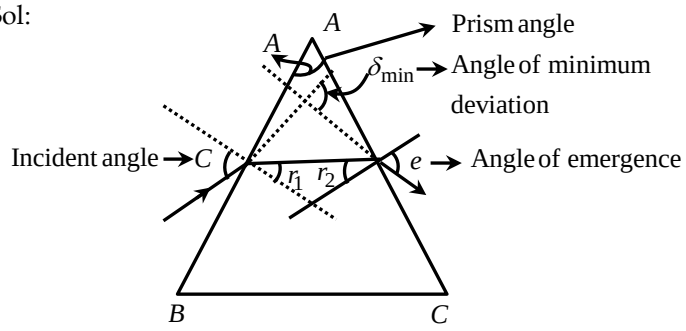
(a) lies between $\sqrt{2}$ and 1

(b) lies between 2 and $\sqrt{2}$

(c) is less than 1

(d) is greater than 2

Sol:



The angle of minimum deviation is given as

$\delta_{\min}=i+e-A$

For minimum deviation

$\delta_{\min}=A$ then

$2A=i+e$

In case of $\delta_{\min} i=e$

$2A=2i\ r_1=r_2=\frac{A}{2}$

$i=A=90°$ from snell's law

$1\sin\ i=n\ \sin\ r_1$

$\sin A=n\sin\frac{A}{2}$

$2\sin\frac{A}{2}\cos\frac{A}{2}=n\ \sin\frac{A}{2}$

$2\cos\frac{A}{2}=n$

When, $A=90°=i_{\min}$

Then, $n_{\min}=\sqrt{2}$

$i=A=0 n_{\max}=2$

Ans: (b)

159. When light travels from one medium to the other of which the refractive index is different, then which of the following will change?

Options:

(a) Frequency, wavelength and velocity

(b) Frequency and wavelength

(c) Frequency and velocity

(d) Wavelength and velocity

Sol: Velocity and wavelength change but frequency remains same.

Ans: (d)

160. A man runs towards a mirror at a speed of $15\,\text{m/s}$. The speed of the image relative to the man is

Options:

(a) $15\,\text{ms}^{-1}$

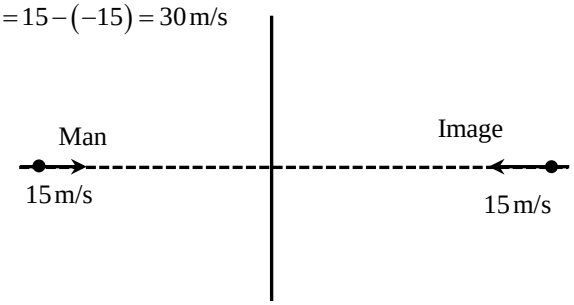
(b) $30\,\text{ms}^{-1}$

(c) $35\,\text{ms}^{-1}$

(d) $20\,\text{ms}^{-1}$

Sol: Relative velocity of image w.r.t. man

$=15-(-15)=30\,\text{m/s}$



Ans: (b)

161. By Huygen's wave theory of light, we cannot explain the phenomenon of

Options:

(a) Interference

(b) Diffraction

(c) Photoelectric effect

(d) Polarisation

Sol: Huygen's wave theory fails to explain the particle nature of light (i.e., photoelectric effect)

Ans: (c)

162. A single slit of width $0.20\,\text{mm}$ is illuminated with light of wavelength $500\,\text{nm}$. The observing screen is placed 80cm from the slit. The width of the central bright fringe will be

Options:

(a) $1\,\text{mm}$

(b) $2\,\text{mm}$

(c) $4\,\text{mm}$

(d) $5\,\text{mm}$

Sol: $\beta = \dfrac{2\lambda D}{d} = \dfrac{2\times500\times10^{-9}\times80\times10^{-2}}{0.20\times10^{-3}} = 4\times10^{-3}\,\text{m} = 4\,\text{mm}$

Ans: (c)

163. Monochromatic light of wavelength $667\,\text{nm}$ is produced by a helium neon laser. The power emitted is 9mW. The number of photons arriving per second on the average at a target irradiated by this beam is

Options:

(a) 3×10^{16}

(b) 9×10^{15}

(c) 3×10^{19}

(d) 9×10^{17}

Sol: $\lambda = 667\times10^{-9}\,\text{m}, P = 9\times10^{-3}\,\text{W}$

$P = \dfrac{Nhc}{\lambda}, N = \text{No. of photons emitted/s}$

$N = \dfrac{9\times10^{-3}\times667\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}}$

$= \dfrac{9\times6.67\times10^{-10}}{3\times6.6\times10^{-26}} \simeq 3\times10^{16}\,/\,\text{s}$

Ans: (a)

164. A proton accelerated through a potential difference of $100\,\text{V}$, has de-Broglie wavelength λ_0. The de-Broglie wavelength of an $\alpha-$particle, accelerated through $800\,\text{V}$ is

Options:

(a) $\dfrac{\lambda_0}{\sqrt{2}}$

(b) $\dfrac{\lambda_0}{2}$

(c) $\dfrac{\lambda_0}{4}$

(d) $\dfrac{\lambda_0}{8}$

Sol: $\frac{\lambda_p}{\lambda_\alpha}=\sqrt{\frac{m_\alpha q_a V_\alpha}{m_p q_p V_p}}=\sqrt{\frac{(4m_p)\times(2e)\times(800)}{(m_p)\times(e)\times(100)}}=8 \qquad \Rightarrow \lambda_\alpha=\frac{\lambda_p}{8}=\frac{\lambda_0}{8}$

Ans: (d)

165. The Rutherford $\alpha-$particle experiment shows that most of the $\alpha-$particles pass through almost unscattered while some are scattered through large angles. What information does it give about the structure of the atom?

Options:

(a) Atom is hollow

(b) The whole mass of the atom is concentrated a small centre called nucleus

(c) Nucleus is positively charged

(d) All the above

Sol: Rutherford concluded from the α -particle scattering experiment that

(i) Most of the space inside the atom is empty because most of the $\alpha-$particles passed through the gold foil without getting deflected.

(ii) Very few particles were deflected from their path, indicating that the positive charge of the atom occupies very little space.

(iii) A very small fraction of α -particles were deflected by 1800, indicating that all the positive charge and mass of the gold atom were concentrated in a very small volume within the atom.

From the data, he also calculated that the radius of the nucleus is about 105 times less than the radius of the atom.

On the basis of his experiment, Rutherford put forward the nuclear model of an atom, which had the following features:

(i) There is a positively charged centre in an atom called the nucleus. Nearly all the mass of an atom resides in the nucleus.

(ii) The electrons revolve around the nucleus in circular paths.

(iii) The size of the nucleus is very small as compared to the size of the atom

Ans: (d)

166. An electron in the hydrogen atom jumps from excited state n to the ground state. The wavelength so emitted illuminates a photosensitive material having work function $2.75\,\text{eV}$. If the stopping potential of the photoelectron is $10\,\text{V}$, the value of n is

Options:

(a) 3

(b) 4

(c) 5

(d) 2

Sol: $KE_{\max}=10\,\text{eV}$

$\phi=2.75$ eV

Total incident energy

$E = \phi + KE_{\max} = 12.75\,\text{eV}$

$\therefore$ Energy is released when electron jumps from the excited state n to the ground state.

$\because E_4 - E_1 = \{-0.85 - (-13.6)\text{eV}\}$

$= 12.75\,\text{eV}$

$\therefore$ value of $n = 4$

Ans: (b)

167. According to Bohr's model of hydrogen atom

Options:

(a) the linear velocity of the electron is quantised

(b) the angular velocity of the electron is quantised

(c) the linear momentum of the electron is quantised

(d) the angular momentum of the electron is quantised

Sol: As per Bohr's model of hydrogen atom, the angular momentum of an electron around the nucleus, is an integral multiple of $\frac{h}{2\pi}$

Thus angular momentum $= mvr = \frac{nh}{2\pi}$ where n is an integer.

Therefore the angular momentum can have only discrete values, i.e. it is quantised.

Ans: (d)

168. Nuclear forces are

Options:

(a) spin dependent and have no non-central part

(b) spin dependent and have a non-central part

(c) spin independent and have no non-central part

(d) spin independent and have a non-central part

Sol: Nuclear forces are spin dependent and have a non-central part

Ans: (b)

169. A nucleus ${}^{m}_{n}X$ emits one $\alpha-$particle and two $\beta-$particles. The resulting nucleus is

Options:

(a) ${}^{m-6}_{n-4}Z$

(b) ${}^{m-6}_{n}Z$

(c) ${}^{m-4}_{n}X$ (d) ${}^{m-4}_{n-2}Y$

Sol: When ${}^{m}_{n}X$ emits one $\alpha-$particle then its atomic mass decreases by 4 units and atomic number by 2. Therefore, the new nucleus becomes ${}^{m-4}_{n-2}Y$. But as it emits two $\beta-$particles, its atomic number increases by 2. Thus the resulting nucleus is ${}^{m-4}_{n}X$.

Ans: (c)

170. The radioactivity of a sample is R_1 at a time T_1 and R_2 at a time T_2. If the half-life of the specimen is T, the number of atoms that have disintegrated in the time $(T_1 - T_2)$ is proportional to

Options:

(a) $(R_1T_1 - R_2T_2)$

(b) $(R_1 - R_2)$

(c) $(R_1 - R_2)/T$

(d) $(R_1 - R_2)T$

Sol: Radioactivity at $T_1, R_1 = \lambda N_1$

Radioactivity at $T_2, R_2 = \lambda N_2$

$\therefore$ Number of atoms decayed in time

$$(T_1 - T_2) = (N_1 - N_2) = \frac{(R_1 - R_2)}{\lambda} = \frac{(R_1 - R_2)T}{0.693} \propto (R_1 - R_2)T$$

Ans: (d)

171. In order to prepare a $p-$ type semiconductor, pure silicon can be doped with

Options:

(a) Phosphorus

(b) Aluminium

(c) Antimony

(d) Germanium

Sol: When we add a trivalent impurity in intrinsic semiconductor (such as B, Al, In), we obtain $P-$ type semiconductors. In $P-$ type semiconductor, the holes are majority carries and electrons are minority carries.

Ans: (b)

172. In figure, the input is across the terminals A and C and the output is across B and D. Then the output is

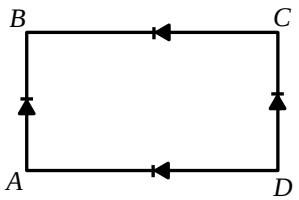
Options:

(a) zero

(b) same as the input

(c) half wave rectified

(d) full wave rectified



Sol: It is the circuit of full wave rectifier.

Ans: (d)

173. A zener diode of voltage $V_Z\,(=6\,\text{V})$ is used to maintain a constant voltage across a load resistance $R_L\,(=1000\Omega)$ by using a series resistance $R_s\,(=100\Omega)$. If the e.m.f. of source is $E\,(=9\,\text{V})$, what is the power being dissipated in Zener diode?

Options:

(a) $0.144\,\text{watt}$

(b) $0.324\,\text{watt}$

(c) $0.244\,\text{watt}$ (d) $0.544\,\text{watt}$

Sol: Here, $E=9\,\text{V}; V_z=6; R_L=1000\Omega$ and $R_s=100\Omega$,

Potential drop across series resistor

$V=E-V_z=9-6=3\,\text{V}$

Current through series resistance R_S is

$$I=\frac{V}{R}=\frac{3}{100}=0.03\text{A}$$

Current through load resistance R_L is

$$I_L=\frac{V_Z}{R_L}=\frac{6}{1000}=0.006\text{A}$$

Current through Zener diode is

$I_Z=I-I_L=0.03-0.006=0.024\,\text{A}$

Power dissipated in Zener diode is

$P_Z=V_Z I_Z=6\times0.024=0.144\,\text{Watt}$

Ans: (a)

174. The length of a solenoid is 0.4m and the number of turns in it is 500. A current of $3\,\text{A}$ is flowing in it. In a small coil of radius 0.01m and number of turns 10, a current of $0.4\,\text{A}$. is flowing. The torque necessary to keep the axis of this coil perpendicular to the axis of solenoid will be

Options:

(a) $5.92\times10^{-6}\,\text{N-m}$

(b) $5.92\times10^{-4}\,\text{N-m}$

(c) $5.92\times10^{-6}\,\text{dyne-cm}$

(d) $5.92\times10^{-4}\,\text{dyne-cm}$

Sol: $B_{\text{solenoid}}=\mu_0 n_s i_s=\dfrac{\mu_0 N_S i_S}{L_S}$,

$$\tau=B_S iNA=\frac{\mu_0 N_S i_S iN\pi r^2}{L_S}$$

$$\tau=\frac{4\pi\times10^{-7}\times500\times3\times0.4\times10\times\pi\times(0.01)^2}{0.4}=5.92\times10^{-6}\,\text{N-m}$$

Ans: (a)

175. If E, m, J and G represent energy, mass, angular momentum and gravitational constant respectively, then the dimensional formula of EJ^2/m^5G^2 is same as that os

Options:

(a) angle

(b) length

(c) mass

(d) time

Sol: $\dfrac{\left[ML^2T^{-2}\right]\left[ML^2T^{-1}\right]^2}{\left[M^5\right]\left[M^{-1}L^3T^{-2}\right]^2}=\left[M^0L^0T^0\right]=\text{angle}$

Ans: (a)

176. A man throws a ball downwards from the roof of a tower of height $400\,\text{m}$. At the same time another ball is thrown upwards with velocity of $50\,\text{m s}^{-1}$ from the surface of the tower, then at which height form the surface of the tower they will meet?

Options:

(a) $100\,\text{m}$

(b) $320\,\text{m}$

(c) $80\,\text{m}$

(d) $240\,\text{m}$

Sol: Let us consider that the two balls meet at height h after time t.

For the ball thrown upwards

$u=50\,\text{m/s}, g=9.8\text{m/s}\,(\text{given})$

$h=ut-\dfrac{1}{2}gt^2=50t-\dfrac{1}{2}gt^2$ … (i)

For the ball thrown downward, $400-h=\dfrac{1}{2}gt^2$

From equation(i), $400-\left(50t-\dfrac{1}{2}gt^2\right)=\dfrac{1}{2}gt^2$

$\Rightarrow 400-50t+\dfrac{1}{2}gt^2=\dfrac{1}{2}gt^2$

$\Rightarrow 50t=400\Rightarrow t=8\,\text{s}$

Again from equation (i),

$h=50\times 8-\dfrac{1}{2}\times 10\times 64=80\text{m}$

Ans: (c)

177. An object is projected with a velocity of $20\,\text{m/s}$ making an angle of $45°$ with horizontal. The equation for the trajectory is $h = Ax - Bx^2$ where h is height, x is horizontal distance, A and B are constants. The ratio $A:B$ is $\left(g = 10\text{ms}^{-2}\right)$

Options:

(a) $1:5$

(b) $5:1$

(c) $1:40$

(d) $40:1$

Sol: Standard equation of projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Comparing with given equation

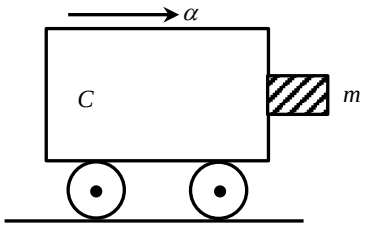
$$A = \tan\theta \text{ and } B = \frac{g}{2u^2\cos^2\theta}$$

So $\frac{A}{B} = \frac{\tan\theta \times 2u^2\cos^2\theta}{g} = 40$

$\left(\text{As } \theta = 45°, u = 20\,\text{m/s}, g = 10\,\text{m/s}^2\right)$

Ans: (d)

178. A block of mass m is in contact with the cart C as shown in the figure.



The coefficient of static friction between the block and the cart is μ. The acceleration α of the cart that will prevent the block from falling satisfies

Options:

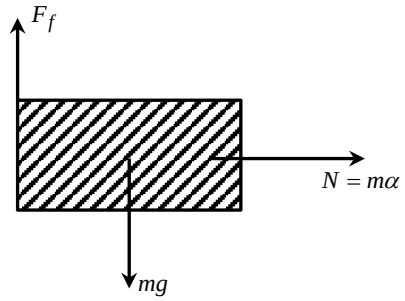
(a) $\alpha > \frac{mg}{\mu}$

(b) $\alpha > \frac{g}{\mu m}$

(c) $\alpha \geq \frac{g}{\mu}$

(d) $\alpha < \frac{g}{\mu}$

Sol: Forces acting on the block are as shown in the figure. Normal reaction N is provided by the force $m\alpha$ due to acceleration α



$\therefore N = m\alpha$

For the block not to fall, frictional force,

$F_f \geq mg$

$\Rightarrow \mu N \geq mg$

$\Rightarrow \mu\, m\alpha \geq mg$

$\Rightarrow \alpha \geq g/\mu$

Ans: (c)

179. A bullet of mass m moving horizontally with a velocity v strikes a block of wood of mass M and gets embedded in the block. The block is suspended from the ceiling by a massless string. The height to which block rises is

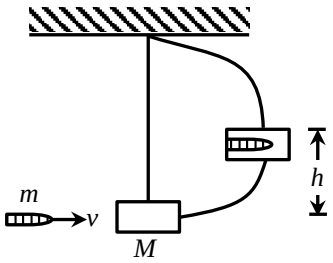
Options:

(a) $\frac{v^2}{2g}\left(\frac{m}{M+m}\right)^2$

(b) $\frac{v^2}{2g}\left(\frac{M+m}{m}\right)^2$

(c) $\frac{v^2}{2g}\left(\frac{m}{M}\right)^2$ (d) $\frac{v^2}{2g}\left(\frac{M}{m}\right)^2$

Sol: The situation is as shown in the figure.



Let V be velocity of the block – bullet system just after collision.

Then by the law of conservation of linear momentum, we get

$mv = (m+M)V$

$V = \frac{mv}{m+M}$

Let the block rises to a height h.

According to law of conservation of mechanical energy, we get

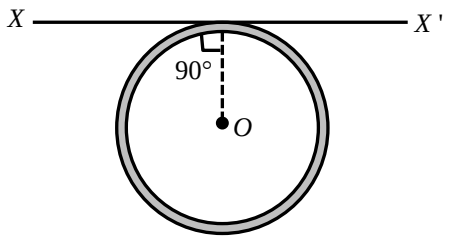
$\frac{1}{2}(m+M)V^2 = (m+M)gh$

$h = \frac{V^2}{2g} = \frac{v^2}{2g}\left(\frac{m}{m+M}\right)^2$

Ans: (a)

180. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is

Options:



(a) $\frac{\rho L^3}{8\pi^2}$

(b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$

(d) $\frac{3\rho L^3}{8\pi^2}$

Sol: Mass per unit length of the wire $= \rho$

Mass of L length, $M = \rho L$ and since the wire of length L is bent in a form of circular loop therefore

$$2\pi R = L \Rightarrow R = \frac{L}{2\pi}$$

Moment of inertia of loop about given axis $= \frac{3}{2}MR^2$

$$= \frac{3}{2}\rho L\left(\frac{L}{2\pi}\right)^2 = \frac{3\rho L^3}{8\pi^2}$$

Ans: (d)

Key Answers:

1. b	2. a	3. b	4. a	5. a	6. a	7. a	8. a	9. a	10. a
11. c	12. b	13. c	14. c	15. b	16. a	17. c	18. a	19. c	20. d
21. b	22. c	23. c	24. a	25. d	26. a	27. d	28. a	29. d	30. b
31. d	32. c	33. c	34. c	35. b	36. b	37. b	38. d	39. a	40. a
41. a	42. b	43. a	44. a	45. a	46. c	47. d	48. a	49. b	50. c
51. a	52. a	53. b	54. b	55. b	56. c	57. c	58. d	59. c	60. d
61. d	62. c	63. b	64. c	65. c	66. b	67. b	68. b	69. b	70. d
71. c	72. b	73. b	74. c	75. b	76. d	77. b	78. b	79. a	80. d
81. d	82. c	83. c	84. a	85. a	86. c	87. b	88. d	89. b	90. d
91. d	92. a	93. b	94. b	95. c	96. d	97. a	98. c	99. d	100.c
101.d	102.a	103.c	104.b	105.c	106.a	107.c	108.d	109.c	110.a
111.d	112.b	113.a	114.a	115.a	116.d	117.a	118.d	119.c	120.a
121.a	122.c	123.c	124.c	125.a	126.d	127.d	128.d	129.a	130.c
131.a	132.d	133.c	134.d	135.b	136.a	137.a	138.d	139.a	140.c
141.a	142.d	143.b	144.b	145.a	146.c	147.a	148.a	149.c	150.b
151.b	152.a	153.b	154.b	155.a	156.b	157.b	158.b	159.d	160.b
161.c	162.c	163.a	164.d	165.d	166.b	167.d	168.b	169.c	170.d
171.b	172.d	173.a	174.a	175.a	176.c	177.d	178.c	179.a	180.d